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UNDERWRITING PROFIT FROM INVESTMENTS

ROBERT A. BAILEY

Investment income has long been recognized in making rates for life insurance and perpetual fire insurance. Investment income is also recognized in dividend formulas for group accident and health insurance but, with rare exceptions, no formal recognition of underwriting income from investments has been made in fire and casualty insurance.

Investment income in relation to automobile insurance rates has received widespread attention in the past few years. Recently, a few state insurance commissioners have ruled that investment income should be recognized in ratemaking for automobile insurance, but with no uniformity in method or degree. Some ratemakers have written and spoken recently against recognizing investment income in any degree when making rates for fire and casualty insurance.

In the belief that this is not a question of all or nothing but rather a question of degree, my purpose is to suggest some guidelines on how to measure the portion of investment income that is earned on funds held in trust for policyholders. The suggested guidelines are applied to a number of representative insurers and the results are tabulated in total and for each kind of insurance. My hope is that these suggestions and data will contribute to a better understanding of the problems and possible answers regarding how much underwriting profit is realized from investments.

First, we should put this problem in perspective by discussing the difficulty of matching invested assets and uninvested assets, on the one hand, with the funds (liabilities) held in trust for policyholders, and on the other hand, with the funds (capital and surplus) held in trust for stockholders.

A glance at the liabilities in an insurer's balance sheet discloses some liabilities that appear to be held in trust for the policyholders, such as unearned premiums and unpaid losses, while others seem clearly to belong to the stockholders, such as capital and surplus. Some liabilities, such as unpaid taxes, are questionable—do they belong to the policyholders or to the stockholders? To further complicate the problem, many analysts of insurance companies, such as Best's and the CPA's, claim that the statutory annual statement does not give a true valuation of an insurer's liabilities and net worth. They talk about "equity in the unearned premium reserve," "deficiencies" or "redundancies" in the unpaid loss reserves, and "non-admitted assets." Some talk about hidden tax liabilities on unrealized capital gains and on the equity in the unearned premiums. So it is not as easy as it may seem at first glance to divide the liabilities, capital, and surplus neatly into two categories: policyholders' funds and stockholders' funds. It is a little like counting eggs in an omelet.

On the asset side of the balance sheet, some assets are invested and earn income, such as stocks and bonds. Others are not invested, such as cash and balances due from agents. Real estate appears to be an invested asset but, when it consists primarily of the offices occupied by the insurer, the income is sometimes not representative of what would be earned from an arm's-length transaction.

In summary, we have a dual problem: first, to determine the amount of the policyholders' funds and the stockholders' funds; and secondly, to match the invested and uninvested assets with the two funds.

To provide a framework for accomplishing our objective I would suggest that we estimate the amount of investment income that would be earned by the insurer if it ceased its insurance business, becoming solely an investment trust, and allocate that amount of investment income to the stockholders' funds and the remainder to the policyholders' funds.

If an insurer became solely an investment trust, virtually all its assets would be invested. Its balances due from agents would disappear entirely and its need for large cash balances to facilitate the heavy flow of premium, loss, and expense transactions would be greatly diminished. Consequently, to be equitable in matching invested assets against stockholders' and policyholders' funds, we should match invested assets first against stockholders' funds and then only the remainder against policyholders' funds.

Also, if an insurer became solely an investment trust, its surplus would increase due to liquidation of the equity in the unearned premium reserve and liquidation of the other non-admitted assets. The non-admitted assets, such as furniture, equipment, and automobiles, and balances due from unauthorized reinsurers, would no longer be needed for the business of the corporation and would be liquidated. The equity in the unearned premium reserve is actually a form of non-admitted asset. Acquisition expenses and premium taxes are incurred when premiums are put on the books. A corporation subject to generally accepted accounting practices would carry such prepaid expenses as an asset. But insurers, for sound conservative reasons, are not permitted by statute to carry an asset for such prepaid expenses. As a result the insurer's surplus is reduced by the amount of such prepaid expenses. If the insurer let the business run off, the prepaid expenses would flow back into surplus as the unearned premiums become earned. Consequently, in measuring the stockholders' funds, credit should be given for the equity in the unearned premium reserve as well as for other non-admitted assets.

Some would contend that credit should also be given to the stockholders' funds for redundancies (or deficiencies) in loss reserves and for the excess of statutory reserves over the insurer's case basis reserves. In those rare instances where there is conclusive evidence that the insurer's case basis reserves are adequate, credit should be given for the excess of statutory reserves over the case basis reserves. But any evidence of additional redundancies or deficiencies in the loss reserves should be presumed to be inconclusive in the face of the insurer's affirmation that its annual statement is a "full and true" statement.

Another question that must be resolved is what to include in investment income. I advocate that realized capital gains and losses should be included and that unrealized capital gains and losses be excluded.

Realized capital gains and losses on bonds are usually quite deliberate and should be used as an addition or offset to interest income. For example, an insurer may make a practice of buying low yield bonds at a discount. By doing so it deliberately transfers some investment income out of interest income and into capital gains which will be realized at maturity or sale. Capital losses on bonds are often realized when, as happened in 1966, insurers try to take advantage of a decline in bond prices by selling short term bonds at a small loss and buying long term bonds at a large discount. Such a transaction will increase investment income, including realized capital gains, in the future, by far more than the immediate realized capital losses. The inclusion of realized capital gains and losses is therefore necessary to obtain the full story of investment return on bonds.

Similarly with stock, an insurer may make a practice of buying low yield stocks which retain a large part of their corporate earnings. The retained earnings cause such stocks to grow in value. So the return on such investments is partly dividend income and partly capital gains. When such gains are realized they should therefore be included in investment income. Such realized gains and losses are often carefully timed to offset each other or to offset underwriting gains or losses in order to minimize or stabilize income taxes. Accordingly, the realized capital gains and losses are not usually subject to unreasonable fluctuations as unrealized capital gains and losses are.

Investment return on stock in wholly owned subsidiary insurers is often not representative of the earnings of the subsidiary. Accordingly, a truer picture is obtained by using the consolidated data for an insurer and its subsidiaries. Running-mates should also be consolidated if they pool expenses or premiums.

With this background in mind, the amount of the realized investment income attributable to policyholders' funds would equal the total realized investment income times a fraction. The denominator of the fraction would be the total invested assets. The numerator of the fraction would be the total invested assets less the adjusted capital and surplus. The adjusted capital and surplus would be the capital and surplus (including special surplus funds) plus the equity in the unearned premium reserve, the non-admitted assets, unauthorized reinsurance, any voluntary reserves carried "above the line," and any reserves for dividends declared to stockholders. The equity in the unearned premium reserve would be the acquisition expenses and premium taxes incurred on the unearned premiums, which generally can be computed by multiplying the unearned premiums by the ratio of net acquisition expense and taxes to net written premiums, which can be obtained from the insurance expense exhibit. Invested assets equal the sum of items 1-5, 7, and 14 on page 2 of the annual statement for fire and casualty insurers.

For purposes of dividing investment income between stockholders and policyholders, I believe no deduction should be made from capital and surplus for potential federal taxes on unrealized capital gains or on the equity in the unearned premium reserve. Potential tax liabilities are not held in trust for the policyholders, and furthermore, if the insurer became solely an investment trust, it would retain the use of the unrealized capital gains.

There are possibly other minor adjustments or refinements that could be suggested but I believe the method proposed above is realistic in broad terms and gives due consideration to the significant factors. Minor refinements calculated out to the *n*th decimal place are pointless in the face of the large approximations and estimates involved in the reserves for unpaid losses.

A mutual insurer does not have stockholders. All of the funds are held in trust for the policyholders. But we may distinguish between funds held in trust to meet current underwriting obligations to policyholders and funds retained from previous earnings and held in trust for policyholders in their role as owners. Such a distinction of the two roles of policyholders in a mutual insurer permits the same method suggested above for stock insurers to be used for all insurers in determining the portion of investment income attributable to current underwriting operations.

This method will tend to understate the underwriting profit from investments for two reasons. First, realized investment income omits the deferred income accruing in the form of market appreciation of common stocks. Annual increments to such deferred income fluctuate from year to year and are sometimes negative, but in the long run they are on the plus side. Secondly, the reserves for unpaid losses reflect the exposure of some time past. With a rising volume of business the reserves are smaller in relation to current premium volume than they would be with a level volume of business. If the current investment income from loss reserves is related to the current premium volume, the ratio is depressed to the extent of the growth between the date the losses were incurred and the current date. However, in most instances both these influences will be minor in the long run. Any attempt to recognize unrealized income and the effects of premium growth would involve estimates which would be subject to controversy.

The suggested guidelines have been applied to the data reported in the 1966 annual statements and insurance expense exhibits for a number of representative insurers and the results are tabulated below. The total investment income from underwriting, which could also be called the underwriting income from investments, for each company has been allocated to each kind of insurance in proportion first to unpaid losses, and then any remainder in proportion to unearned premium.

Funds for unpaid losses are more available for investment than funds for unearned premiums. Large portions of the funds for unearned premiums are tied up in balances due from agents and in prepaid acquisition expenses, neither of which are available for investment by the insurer.

Accordingly the invested assets in excess of the adjusted capital and surplus were allocated by line of insurance first in proportion to unpaid losses and unpaid loss adjustment expenses and then any remainder in proportion to unearned premiums. The underwriting income from investments was then allocated in the same proportion.

Similar results are obtained whether the computations use only year-end data or use the average of the figures at the beginning and the end of the year. Accordingly, to simplify the computations, only the year-end data was used.

The ratemaking procedures for many property and casualty lines of insurance make various provisions for expected underwriting profit. The actual underwriting profit is usually different from the expected. The actual underwriting profit from investments, shown below, should not be added to the expected underwriting profit margin in the rates as an estimate of the total underwriting profit. Rather, the actual underwriting profit from investments should be combined with the actual other underwriting profit or loss to obtain the total actual underwriting profit or loss. A complete analysis of actual underwriting profit or loss should also recognize dividends to policyholders.

TABLE 1

Underwriting Profit from Investments as a Percent of Earned Premiums *

	State Farm	Allstate	Aetna Casualty	Travelers	INA	Liberty Mutual	Detroit Auto	Michigan Mutual	Total 8 Groups
Fire, EC & Allied Lines	3.3%	2.7%	1.8%	2.0%	2.0%	2.2%		1.6%	2.2%
Homeowners	3.2	2.3	1.7	1.9	2.1	2.3		1.7	2.3
Commercial Multiple Peril	2.9	2.9	1.8	2.1	2.0	3.0		2.8	2.1
Accident & Health	1.0	1.8	1.3	-	2.6	1.3		1.1	1.9
Workmen's Compensation	5.1	3.9	3.5	5.4	4.8	3.9		4.0	4.3
Liability Other than Auto	3.4	3.6	4.4	5.8	6.2	6.8		6.9	5.6
Auto Liability	3.7	4.9	3.1	3.9	4.0	3.8	2.6	4.0	4.0
Auto physical damage	1.4	1.3	0.7	0.7	1.0	0.8	0.9	0.9	1.1
All Other	2.8	1.9	2.1	3.0	2.8	3.3		3.2	2.7
Total	2.9	3.6	2.7	3.6	3.1	3.5	2.1	4.0	3.2

* The data shown in these exhibits are the consolidated data for the groups of insurers listed below:

State Farm Mutual and State Farm Fire and Casualty
Allstate and Allstate Fire
Aetna Casualty & Surety and Standard Fire
Travelers Indemnity, Travelers Insurance (Accident Department) and Charter Oak Fire
Insurance Company of North America
Liberty Mutual and Liberty Mutual Fire
Detroit Automobile Inter-Insurance Exchange and Motor State
Michigan Mutual Liability

TABLE 2

Underwriting Profit from Investments *
(000 omitted)

	State Farm	Allstate	Aetna Casualty	Travelers	INA	Liberty Mutual	Detroit Auto	Michigan Mutual
Invested Assets	1,285,441	1,246,178	1,116,064	1,195,066	1,448,093	1,027,267	107,227	96,278
Unpaid Losses and Loss Expense	473,281	505,364	519,763	598,065	449,947	525,604	52,434	75,133
Unearned Premium	363,624	462,115	338,393	400,841	435,595	164,956	47,267	21,344
Capital and Surplus + Vol. Res. + Res. for stockholders' dividends	447,433	412,057	391,637	345,167	762,165	385,104	29,494	13,941
Acquisition Expense and Tax Ratio	17.4%	20.4%	22.7%	25.7%	24.2%	11.2%	12.2%	10.7%
Non-admitted Assets + Equity in Unearned Premium	72,495	113,403	93,406	142,273	140,254	24,530	5,767	2,763
Adjusted Surplus [(4)+(6)]	519,928	525,460	485,043	487,440	902,419	409,634	35,261	16,704
Fraction [(1)-(7)] ÷ (1)	59.6%	57.8%	56.5%	59.2%	37.7%	60.1%	67.1%	82.7%
Net Investment Gain	51,783	52,016	35,947	48,148	50,765	32,953	2,673	3,450
Und. Profit from Invest. (8)x(9)	30,863	30,065	20,310	28,504	19,138	19,802	1,794	2,853
Proportion of Unpaid Losses & L.E.	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
Proportion of Unearned Premium	80.4%	46.6%	32.9%	27.3%	22.0%	55.8%	41.3%	20.8%
Earned Premium	1,049,319	832,975	760,367	795,185	617,412	567,658	87,471	71,930
Ratio (10)÷(13)	2.9%	3.6%	2.7%	3.6%	3.1%	3.5%	2.1%	4.0%

* See footnote on previous exhibit

DISCUSSION BY A. C. CURRY

The inclusion of investment income in the ratemaking process for the casualty lines is a subject which is difficult to consider objectively. An individual's sentiment on the subject seems to depend upon which half of the facts he chooses to rely. Each of us has heard good arguments both pro and con. It is true, of course, that the carriers do have a source of income in addition to the premiums being charged. It is also true that the insured buys financial protection in an effort to stabilize his financial structure rather than to play the securities market.

From a review of Mr. Bailey's paper, it would appear that the dilemna which he proposes to resolve lies in the answer to two questions:

a) Why should investment income be included? and

b) What is the manner in which it should be included?

Although I can not necessarily endorse some of the figures which Mr. Bailey sets forth, his approach impresses me as being reasonable and a good compromise between both facets of the debate. I would suggest, however, that it is, in fact, a compromise because, even though I can accept his technique in general terms based upon his assumptions, I cannot endorse his assumptions.

Without getting into the question of how the industry got into this debate on investment income, I feel that it is fair to state that most proponents are seeking ultimately to reduce Mr. Public's outlay of total insurance dollars. Mr. Bailey does not propose such a reduction — neither does he propose that the allowance for profits and contingencies be increased by 2% in order to accommodate a 2% investment credit. Suffice to say that I hope we do not protect Mr. Public to the point that he can not obtain financial protection. That this can, in fact, happen has been proven in the recent past in several casualty lines.

Because Mr. Bailey does not comment upon the impact of his proposal on the overall income of the carrier, we should accept or modify his proposal for the specific reasons he gives. This is where I'm afraid he loses me. In essence, he proposes to include the investment income derived from the funds held in trust for the policyholder. This practice I would endorse wholeheartedly, but I can't find any funds held in trust for the policyholder!

Ballentine's law dictionary sets forth a lot of non-actuarial terminology with reference to trusts . . . . . plus the observation that all definitions contain the essential elements that the legal estate must be in one party and the equitable estate in another to constitute a technical trust.

Black's law dictionary also contains a few recognizable passages involving a right of property, real or personal, held by one party for the benefit of another; and any arrangement whereby property is transferred with the intention that it be administered by the trustee for another's benefit.

Crobaugh says that the trustee incurs the obligation to conserve and invest the funds for the beneficiary, and that a trustee is one in whom some estate, interest, or power in property is placed for the benefit of others.

Even in the common vernacular, a trust would have to include an obligation to preserve, not dissipate, or invest. Yet written premiums in an insurance contract represent the consideration which is necessary for the contract to be a binding one. There exists no obligation to return the premiums, conserve the premiums, or invest the premiums. Only if the contract is cancelled is there an obligation or a liability on the part of the carrier to return a portion of the premiums and this potential liability is described in the contract. The manner in which this consideration is entered on the books does not create the liability. The fact that an accounting entry is made for budgetary purposes neither produces a liability nor incurs an obligation to return the premiums paid. A budgetary reserve for the depreciation of office furniture does not create a liability to the cabinetmaker. Neither does a budgetary reserve for deferred expenses create a liability. An unearned premium reserve is merely a budgetary, fictional accounting maneuver designed to theoretically remove some of the funds from the right hand pocket and place them into the left hand pocket to remind us that the opportunity to incur liabilities has not expired. In fact, Uncle Sam, in theory at least, would accept a cash basis of accounting for tax purposes.

One could even develop a line of thinking which begins with . . . . . the annual statement is a measure of the company's condition, the rules can be set at will, etc., etc., ... and ends with the conclusion that the unearned premium reserve is an accounting requirement of the regulatory authorities in order that they may measure the company's ability to meet their future obligations. And even further, this document is neither the only measure nor necessarily the best measure particularly for any other purpose.

If I, as an individual, remove from my pocket $25 per month and put it into a credit union in order to pay my heat bill this winter, I do not in any way incur an obligation to the gas company. If we as a carrier do the same thing with some of our income, we have not incurred an obligation to the purchaser of the contract.

Loss reserves, too, would seem to be a budgetary estimate of future or potential liabilities. Reserves are necessary primarily because either the liability has not been established or the extent of liability has not been established. I would concur that if the extent of liability has been established and payment is not tendered, then some of the funds which offset these budgetary estimates do belong in theory to the *claimant* (who may or may not be the policyholder).

Through this paper, Mr. Bailey has again presented a valuable contribution to the literature of the Casualty Actuarial Society. The blueprint which he advances merits serious consideration by the members of our society. This paper represents a challenge to the rest of us to explore the attendant problems, such as:

a) The assumption of equal liquidity as respects loss reserves and capital investments.
b) Do the policyholders have an equitable estate in the "liabilities" or in the "assets"?
c) Should we act like an investment house if these assumptions are correct?
d) How can the opponents pass off investment gains as easily as the proponents pass off the market setbacks?
e) Should we subject the carriers of other lines to the fluctuations of the casualty business by combining the carriers into groups?
f) Who are we protecting if we encourage the carriers to dissipate their surplus?
g) If we intend to include IBNRs, to whom do we owe that money?

Mr. Bailey's paper in my estimation is a fine attempt to reconcile the diverse opinions available in our industry. Moreover, he has astutely directed his attention toward what will probably be one of tomorrow's facts of life. Before endorsing Mr. Bailey's general premise, however, let's be sure that we aren't about to protect Mr. Public to death by reducing his ability to obtain protection.

DISCUSSION BY RICHARD L. JOHE

Mr. Bailey states his hope that his "suggestions and data will contribute to a better understanding of the problems and possible answers regarding how much underwriting profit is realized from investments." He develops his definitions of investment income, invested assets, and "the stockholders' funds." He states that "we should match invested assets first against stockholders' funds and then only the remainder against policyholders' funds."

"Stockholders' funds" apparently consist of "adjusted capital and surplus" defined as "the capital and surplus (including special surplus funds) plus the equity in the unearned premium reserve, the non-admitted assets, unauthorized reinsurance, any voluntary reserves carried 'above the line,' and any reserves for dividends declared to stockholders." Under this definition, some minor amounts of uninvested funds such as agents' balances over ninety days old would be found in the non-admitted assets, and reserves for dividends declared to stockholders would probably be represented by cash. Therefore, a very large portion of "stockholders' funds" would consist of invested assets. Exhibit A shows investment income on "stockholders' funds" calculated by the author's method compared with with the investment yield percentage applied to "stockholders' funds."

The author does not give his definition of "policyholders' funds" or "funds held in trust for policyholders." However, the arithmetic of his method for developing "the total investment income from underwriting" suggests that the "policyholders' funds" portion of his invested assets approximately consists of the reserve for losses and loss expense plus the reserve for unearned premium less "the equity in the unearned premium reserve," all reduced by the agents' balances admitted asset and by cash (excluding reserves for dividends declared to stockholders). Investment income using this definition is shown in Exhibit B for four insurer groups compared with the investment income produced by the author's suggested method.

In his method for allocation of his "underwriting income from investments" to "kind of insurance," the author appears to substantiate this approximate definition with his statements that:

> "Funds for unpaid losses are more available for investment than funds for unearned premiums. Large portions of the funds for unearned premiums are tied up in balances due from agents and in prepaid acquisition expenses neither of which are available for investment by the insurer."

For the four insurer groups, Exhibit B shows that the reserve for unearned premium excluding "balances due from agents and prepaid acquisition expenses" represented 43.5 percent of the unearned premium reserve (total of the four insurer groups). If we assume that this portion of the unearned premium reserve is completely invested and that the balance of the cash held represents loss and loss expense reserves, we find that 76.5 percent of the author's "funds held in trust for policyholders" consists of the investable portion of loss and loss expense reserves.

Mr. Bailey states that his purpose "is to suggest some guidelines on how to measure the portion of investment income that is earned on funds held in trust for policyholders" and he labels this portion of investment income "underwriting profit from investments." Underwriting income for property and casualty companies has been well defined by court decisions and by the NAIC (National Association of Insurance Commissioners). It does not include any part of the investment income on invested assets.

The author's attempt to re-label a portion of the investment income and call it underwriting profit seems to rest on his assumption that the invested portions of the loss and loss expense reserve and the unearned premium reserve represent "funds held in trust for policyholders." He cites no authority for this assumption even though the past fifty years have seen substantial recorded opinion and decision to the contrary.

His reasoning with respect to loss and loss expense reserves appears to be that if an insurer ceased its insurance business, becoming solely an investment trust, the invested assets represented by the invested portion of these reserves would disappear from the balance sheet. Because of this disappearance, the investment income would also disappear somehow to the benefit of policyholders rather than the stockholders.

If the loss and loss expense reserves were not deficient, but adequate to discharge the liabilities represented by such reserves, it is difficult for this reviewer to understand why subsequent investment income would not accrue to the benefit of the stockholders. This is especially so in view of the author's statement that "any evidence of additional redundancies or deficiencies in the loss reserves should be presumed to be inconclusive in the face of the insurer's affirmation that its annual statement is a 'full and true' statement."

This reviewer also examined the loss reserves held by his company for workmen's compensation and for the automobile lines (B.I., P.D., & physical damage combined). Approximately 30 percent of the current workmen's compensation loss reserves represent long term "pension" reserves. The tabular values prescribed for reporting such cases (e.g., National Council Widow's Benefit Table and Disability Table) for ratemaking purposes already reflect a discounting for interest. In addition, the latest ten year history of our automobile lines showed an underwriting loss of $26,546,697 but an increase of $44,343,535 in loss reserves which could have come about only by a substantial reduction in stockholders' funds.

Mr. Bailey states that "Investment income has long been recognized in making rates for life insurance . . ." and ". . . my purpose is to suggest some guidelines on how to measure the portion of investment income that is earned on funds held in trust for policyholders." He then develops an allocation method which reminded this reviewer of a similar allocation in phase 1 of the Life Insurance Income Tax Act of 1959 (Public Law 86). Phase 1 provides a basis for calculating taxable investment income by splitting investment yield into the policyowners' share and the company's share. The arithmetic in this phase utilizes the lower of the current earnings rate or the five year average earnings rate, both calculated from the ratios of net investment earnings to "all of the assets of the company (including non-admitted assets) other than real and personal property (excluding money) used by it in carrying on an insurance trade or business." The lower of the two earnings rates is used in computing the deduction for interest needed to maintain reserves.

This deduction, the policy interest liability requirement, expressed as a percentage of the investment yield, produces a percentage known as the policyowners' share of the investment yield. The difference between the investment yield and the policyowners' share of the investment yield is known as the company's share of the investment yield.

In spite of the apparent similarity to Mr. Bailey's proposal, there is one very important difference. The phase 1 calculation of the policyowners' share of the investment yield utilizes only the reserves:

1. which are computed or estimated on the basis of recognized mortality or morbidity tables and assumed rates of interest, and
2. which are set aside to mature or liquidate future unaccrued claims.

These are policy reserves of the type found in Exhibit 8 of the Life and A&H companies' annual statement (Association edition). They are comparable to the active life reserves produced by guaranteed renewable and non-cancelable health insurance policies and included in the unearned premium reserve (Part 2B) of the fire and casualty annual statement (Association edition).

As these policy reserves accumulate at interest, they act to reduce the amount at risk and are not in any way comparable to the loss and loss expense reserve liabilities which Mr. Bailey considers to be a very large part of the "funds held in trust for policyholders." Life companies do establish and carry reserve and claim liabilities which are comparable to the loss reserve liabilities held by property and casualty companies. However, the Life Company Income Tax Act of 1959 uses this latter type of reserve as a deduction in phase 2, gain from operations. There is no provision in phase 2 for calculation of the policyowners' share of investment yield on such reserves.

It is certainly true that compound interest plays an extremely important role in the level premium life insurance system. However, the interest element is negligible in rates for non-participating yearly renewable term life insurance compared with the gigantic role interest plays in level premium whole life insurance rates.

Some idea of the magnitude of the deliberate net premium overcharge in the first years of a non-participating whole life insurance contract can be seen from a comparison of age 35 rates of $19.48 per thousand for whole life insurance with a $6.88 rate per thousand for 5 year renewable and convertible term life insurance. This deliberate overcharge in excess of the mortality cost (claim cost) in the first years of a whole life insurance contract gives rise to cash values and other non-forfeiture benefits as well as minimum reserve requirements. The rates charged for most property and casualty lines of business certainly do not contain a deliberate pure premium overcharge of the type found in the level premium life insurance system. The fact that "investment income has long been recognized in making rates for life insurance . . ." is not related to the purpose of the author's paper and has no bearing on the problem for which he presents suggestions and data.

The author states that "Investment income is also recognized in dividend formulas for group accident and health insurance . . .". While the latest versions of such dividend formulas and experience rating formulas are usually closely guarded company secrets, enough information leaks through the screens of secrecy to warrant qualifying the author's statement.

Dividend formulas applying to a participating form of group insurance very well might be found to include an interest element on some types of funds held as reserves against contingencies. An experience rating formula used on non-participating forms of group insurance operates on premiums, incurred claims, and expenses to produce policyholder deficits or indicated excesses which are not usually contractually guaranteed as are retrospective additional or return premiums in other casualty lines of business. As with dividend formulas, experience rating formulas sometimes provide an interest element on some types of contingent reserve funds. However, it is not valid to assume that all dividend and experience rating formulas recognize investment income.

It seems obvious from this discussion that there is much in Mr. Bailey's paper with which this reviewer disagrees. However, there should be no disagreement with the opinion expressed in the author's last paragraph.

During at least the last fifty years, self-appointed critics of the property and casualty insurance business have many times confused themselves and the public with the assertion that investment income is not considered in ratemaking. They really mean that investment income is not, but should be, *directly included* in ratemaking.

This smoke screen is periodically raised, confusing the real issues in the industry's efforts to obtain and maintain adequate rates. For this reason, there are political and public relations reasons for arguing that some part of investment income should be reflected in ratemaking procedures but it has been this reviewer's consistent opinion that such investment income must be restricted to the investment yield on the invested portion of unearned premiums minus the equity in such unearned premiums. This reviewer certainly agrees with Mr. Bailey that such investment income "should not be added to the expected underwriting profit in the rates as an estimate of the total underwriting profit." Rather, it "should be combined with the actual (other) underwriting profit or loss" to obtain an evaluation of the reasonableness of the underwriting profit and contingency allowance to be included in a future rate.

Exhibit A

Comparative Estimates - Investment Income On "Stockholders' Funds"
(000 Omitted)

	U.S.F.&G.	State Farm	Allstate	Aetna Casualty	Travelers	I. N. A.	Liberty Mutual	Detroit Auto	Michigan Mutual
Investment Yield Method									
*(1) Invested Assets	831,600	1,285,441	1,246,178	1,116,064	1,195,066	1,448,093	1,027,267	107,227	96,278
*(2) Net Investment Gain	26,690	51,783	52,016	35,947	48,148	50,765	32,953	2,673	3,450
(3) Investment Yield Percentage (2) ÷ (1)	3.21	4.03	4.17	3.22	4.03	3.51	3.21	2.49	3.58
*(4) Adjusted Surplus	502,999	519,928	525,460	485,043	487,440	902,419	409.634	35,261	16,704
(5) Estimated Investment Income On "Stockholders Funds"(3)X(4)	16,146	20,953	21,912	15,618	19,644	31,675	13,149	878	598
Bailey's Allocation Method									
*(6) "Underwriting Profit From Investments"	10,542	30,863	30,065	20,310	28,504	19,138	19,802	1,794	2,853
(7) Bailey's Estimated Investment Income on "Stockholders Funds" (2) - (6)	16,148	20,920	21,951	15,637	19,644	31,627	13,151	879	597

*These figures, except for USF&G, were taken from Mr. Bailey's paper "Underwriting Profit From Investments". USF&G figures were taken from the 1966 Annual Statement.

Note: This exhibit compares the results of the investment yield method, line (5), with Bailey's allocation method results in line (7). The results of the two methods compare very closely which suggests that the investment yield method applied to "Policyholders' Funds" should produce results which compare closely with Bailey's allocation method if "Policyholders' Funds" can be defined.

Exhibit B

"Policyholders' Funds"
(000 Omitted)

	U.S.F.&G.	Aetna Casualty	Travelers	Liberty Mutual	Totals
Comparative Estimates					
*(1) Bailey's Estimated Investment Income On Policyholders' Funds	10,542	20,310	28,504	19,802	79,158
*(2) Net Investment Gain	26,690	35,947	48,148	32,953	143,738
(3) Investment Yield Method Estimated Investment Income On "Stockholders' Funds", Exhibit A, line (5)	16,146	15,618	19,644	13,149	64,557
(4) Balance As Estimate Of Investment Income On "Policyholders' Funds" (2) - (3)	10,544	20,329	28,504	19,804	79,181
*(5) Reserve For Losses and Loss Expense	255,404	519,763	598,065	525,604	1,898,836
(6) Cash and Bank Deposits	25,957	59,544	56,097	19,707	161,305
(7) Stockholders' Dividends Declared and Unpaid (Reduction of Cash)	2,948	1,883	-	-	4,831
(8) Estimated Investable Unpaid Loss and Loss Expense (5) - (6) + (7)	232,395	462,102	541,968	505,897	1,742,362
*(9) Reserve For Unearned Premium	326,127	338,393	400,841	164,956	1,230,317
*(10) Acquisition Expense and Tax Ratio	27.5	22.7	25.7	11.2	-
(11) Equity In Unearned Premium Reserve (9) X (10)	89,685	76,815	103,016	18,475	287,991
(12) Agents Balances Admitted	123,808	99,448	122,972	61,159	407,387
(13) Estimated Investable Unearned Premium (9)-(11)-(12)	112,634	162,130	174,853	85,322	534,939
(14) Estimated Investable "Policyholders' Funds"(8)+(13)	345,029	624,232	716,821	591,219	2,277,301
(15) Investment Yield Percentage, Exhibit A, Line (3)	3.21	3.22	4.03	3.21	-
(16) Estimated Investment Income on "Policyholders' Funds", Investment Yield Method (14)X(15)	11,075	20,100	28,888	18,978	79,041
(17) Estimated Investable Unearned Premium As Percentage of Reserve (13) ÷ (9)	34.5	47.9	43.6	51.7	43.5
(18) Estimated Investable Unpaid Loss and Loss Expense As Percentage of "Policyholders' Funds"(8)÷(14)	67.4	74.0	75.6	85.6	76.5

*These figures, except for USF&G, were taken from Mr. Bailey's paper "Underwriting Profit From Investments". All other figures were taken from 1966 Annual Statements.

Note: Line(16) compared with lines (1) and (4) show that this definition of "Policyholders' Funds" produces results which are not as good as the results produced by the "Stockholders' Funds" definition. Aetna Casualty and Liberty Mutual are short, indicating Bailey has included some other investable assets, while USF&G and Travelers are over, indicating Bailey's estimate includes some type of reduction in "Policyholders'" investable assets.

DISCUSSION BY W. JAMES MACGINNITIE

Mr. Bailey is to be congratulated for bringing this timely and important subject of underwriting profit from investments before the Society. He has succeeded in pointing out the many facets of this complex subject, and he has avoided many of the pitfalls that less sophisticated commentators have fallen into. He has, for example, distinguished between policyholder and stockholder funds, even in the case of mutuals. He has recognized the understatement of surplus, and also the fact that most cash needs are associated with policyholder funds.

There are, however, two problems that are associated with the author's method of analysis. The first is that allocation of the investment income of a given calendar year to the operations of that year fails to recognize that the underwriting decisions which gave rise to that investment income may have been made several years before. For some some purposes, the calendar year analysis may be acceptable, or even preferable. Furthermore, it is the way insurance companies keep their books and publish annual statements. But for evaluating the adequacy of rates, or the profitability of a book of business, it would seem more appropriate to use a discounted cash flow analysis.

Such an analysis would recognize the time value of money, and would be appropriate whether there had been a stable volume or not. This last problem was recognized by the author, when he stated that a rising volume may lead to an understatement of the underwriting profit from investments. Discounted cash flow also helps with the problem of how to treat capital gains, realized or unrealized. They are important only in determining the interest rate to be used, and a significant variation from one year to the next will not give widely differing results for the underwriting profit from investments.

The other problem associated with the author's analysis is his failure to distinguish between the kinds of investments made with policyholders' funds and those made with stockholders' funds. A quick perusal of several annual statements tends to substantiate the hypothesis that liabilities are kept in bonds, cash, and premium balances, while capital and surplus are kept in common stocks. There are individual company differences, to be sure, but they could well be based on varying interpretations of what constitutes true capital and surplus. The rates of return on these types of investments are significantly different, but the author allocated total investment income without regard for this distinction.

In a discounted cash flow analysis, the appropriate rate would be that obtained on reserves, with recognition of cash requirements and premium balances. The question of appropriate rates of return is a difficult one, though, and it is complicated by tax considerations since many companies deliberately invest in tax-exempt securities. They thereby obtain a lower gross yield but a higher net than would be obtained from a taxable security.

The problems of determining underwriting profits from investments are complex and fascinating, and it will be some time before the methods of analysis appropriate to the various questions in this area have been fully worked out. Mr. Bailey, however, has made an excellent start.

DISCUSSION BY ALLAN L. MAYERSON

Bob Bailey's timely and thought-provoking paper is an important actuarial contribution to the perennial and occasionally emotional debate on whether, and to what extent, investment income should be included in ratemaking. Bob's paper is one of the few discussions of this topic to contribute more light than heat to the controversy.

It is obvious that insurers do earn investment income, not only from the funds contributed by their stockholders, but also from some part of the premiums paid by policyholders. If this were not so, many insurers would surely have withdrawn from certain lines of insurance which have caused persistent underwriting losses for more than a decade. A case can even be made that the solvency of some insurers has, in recent years, depended upon investment income and stock market capital gains.

It seems obvious that investment income is, as implied in Harold Curry's presidential address, taken into account in ratemaking. Whether it is explicitly or implicitly allowed for in the rating formula does not seem too important. The more important question, in my view, is whether the overall profit margin in the rates is adequate or excessive.

Most rating formulas contain an explicit loading for underwriting profit, often 5% or 6% of premiums. That these margins have seldom been realized is due to the actuaries' lack of success in predicting future losses accurately or, having predicted them, in convincing company management or state regulators to approve adequate rates. If actuaries ever become sufficiently expert in time-series analysis to predict loss trends correctly, or if our crystal balls begin to give us better answers, then the adequacy or inadequacy of the profit loading will become very important.

Many industries have a lower profit margin on sales than that built into most property and casualty insurance rates. Grocery store chains often operate on a profit margin of less than 1% of sales. On the other hand, companies with a high ratio of capital investment to sales, such as utility companies, expect to earn at least 6% to 8% on sales.

A better basis for comparison with other industries is probably the profit margin on invested capital. To obtain this ratio, we must consider not only the underwriting profit (assuming that the loss and expense elements of the rate will someday be predicted accurately) but also the investment profit or, as Bob Bailey calls it, the "underwriting profit from investments." For example, if we assume that a property insurer can safely write a premium volume equal to three times its capital and surplus, and that its rates contemplate a 5% underwriting profit while its investment yield on premiums adds 2% to this, its return on invested capital would be 21%. If we assume that a property insurer is only permitted by a state insurance department to write premiums equal to twice its capital and surplus, with a 5% underwriting and 2% investment profit, it would earn only 14% on stockholder equity. To these amounts, of course, must be added the interest dividends and/or capital gains earned on the investment of the stockholder equity itself, in order to determine whether the theoretical return which can accrue to the owners of insurance companies compares favorably or unfavorably to the return available to investors in other businesses. I believe that some careful acturial study of this subject is needed since, under the present system of rate regulation, someone must decide whether the profit allowance in insurance rates should be 2%, 5%, or 10% of premium. Only a careful analysis of the actual return to stockholders, predicated on reasonable assumptions as to investment earnings, volume of business written, and actual (as distinguished from expected) underwriting profit can provide a basis for such judgments. There is considerable work to be done in this field.

Another interesting question raised by Bob Bailey's paper is whether the interest on loss reserves should be considered part of the "underwriting profit from investments." It has often been argued that the interest on loss reserves is used up by inflationary increases in the liabilities, since the longer claim payment is delayed, with consequent higher interest earnings on the reserve, the higher the amount ultimately paid. Whether or not this is really true, and whether or not time-related inflation in claim payments is already included in the loss portion of the premium, is a legitimate and important field for actuarial research. It should have a bearing on whether or not the interest earnings on loss reserves should be considered a part of "underwriting profit from investments."

DISCUSSION BY NICHOLAS F. MILLER, JR.

Investment income has long been recognized by the insurance business when determining insurance premiums. In the property and liability segment of the business, investment income has been taken into account by depressing the underwriting profit and contingency factor built into premium levels.

The 5% (before taxes) provision for underwriting profit and contingencies in automobile liability rate levels is hardly excessive — if it were possible of attainment. And if it were possible of attainment and supplemented by income yields on invested assets, the resulting net earnings would be reasonable when compared with other industries — most certainly far from excessive by any standard of comparison.

An examination of the industry records of the past describes the underwriting and economic climate in which we operate:

> $1.8 billion underwriting loss in the last ten years, a ten year rate of loss over 4%;
>
> a rate of loss greater than 5% in each of four years — years in which the rating formula missed the objective by over 10%.

There is no substance to support a thesis that insurance companies' earnings need be scrutinized for excess profits.

The record does highlight the incongruity of rate regulatory systems which assume that the insurance business is a monopoly which has been required to accede to rate regulation as a consideration for its privileged franchise. Any efforts to establish excessive premium levels will be defeated by the competitive elements of our economy. There is, therefore, no need to develop a formula for taking investment income into account to assure that premiums are not excessive — competition will do the job.

In his paper Mr. Bailey expresses the thought that certain funds are "held in trust for policyholders." While it may be appropriate to interpret this expression in the layman's sense of integrity, confidence, or even hope, the relationship which exists between a policyholder and an insurer is a contractual one and not a fiducial one. Policyholders have no valid claim to investment income earned on unearned premium reserves, and we therefore do not need a formula to take it into account in determining premium levels.

While I cannot agree with the basic purposes for Mr. Bailey's undertaking the development of his paper, I do share his thought that preparation of his paper and our studies and discussions of it should increase our understanding of the characteristics of investment income.

Examining his method from a technical standpoint, some of us observed:

(1) It relates the investment income from an accumulation of years to earned premiums of a particular year. A company that is growing rapidly and building reserves rapidly would have a smaller accumulation of invested "policyholder funds" than would a comparatively stable company and as a result the rapidly growing company would have a smaller element of investment income in its rate making process. The justification for having a smaller amount is not apparent.

(2) The composition of invested assets is ignored. It averages investment income among all assets and between stockholders' and policyholders' interests. The result may be unfair to the shareholders as a company may feel that shareholders' funds may be invested in long-term obligations with the higher yields which normally accompany lack of liquidity.

(3) A company deciding to strengthen its loss reserves would give future policyholders the benefit of a larger investment income element in rate making, all other things being equal. On the other hand, by weakening reserves a company could get a larger allocation of investment income for shareholders.

(4) Another approach would be to measure from actual experience the dates at which funds are received from policyholders and disbursed as expenses and claims. Such a procedure would recognize the lag in premium collections, the payments of commissions and other costs arising when the policy is written, the impact of payroll audit and retrospectively rated business, the spread of certain costs through the term of the policy and the disbursement of losses and expenses over a period of years. These patterns of income and outgo can be expected to vary by line and by company.

There is no preferred way to estimate a segmentation of investment income between that which might be thought of as emanating from insurance-oriented funds and that which may be thought of as emanating from shareholders' funds. While we do not need such a segmentation for rate making purposes, from time to time it is helpful to have one for internal management purposes. The character of the particular purpose will influence the judgment used in the selection of alternatives to be incorporated into the method. I believe Mr. Bailey's paper will serve as a very helpful point of reference.

DISCUSSION BY RUTH SALZMANN

Mr. Bailey's provocative and interesting paper on underwriting profit from investments lends itself to five areas of discussion:

1. THE TERMINOLOGY

The author used the terms *investment income from underwriting, underwriting income from investments,* and *funds held in trust for policyholders.* Such wording confuses and disturbs the reader not only because of the departure from established concepts, but because of the legal implications. This disinterest in wording by the author is unfortunate because such terminology detracts from an otherwise valuable actuarial contribution.

Terminology has been a problem for this reviewer as well. The expressions *premium* and *non-premium funds* are used in this discussion to avoid any inaccurate legal connotations. By definition then, premium funds are moneys received as income from policyholders which immediately become commingled with other corporate funds. The term non-premium funds refers to the "other corporate funds."

2. THE QUANTIFICATION OF INVESTED PREMIUM FUNDS

a. General Comments

Mr. Bailey gives excellent coverage to the problems involved in separating invested assets into premium and non-premium funds. He points out the complications, the judgment areas, and the many considerations that should be made. He makes it clear that this measurement is not an easy one because balance sheet items do not fall neatly into black and white categories.

b. The Method Analyzed

The author's method first establishes an adjusted capital and surplus amount by rearranging old and creating new balance sheet accounts. This adjusted figure becomes the invested non-premium funds, and the invested premium funds are obtained by substracting the non-premium funds from total invested assets. In the beginning of his paper, the author suggests a balance sheet derivation of these two items; the actual calculation is only a simplified version thereof. Although the short-cut used may produce reasonable answers, a full disclosure of what such a short-cut encompasses would have been helpful to the reader. In other words, balance sheet arithmetic says that invested assets equal the sum of liabilities and capital and surplus less non-invested assets. Therefore, what Mr. Bailey obtains by subtraction for invested premium funds is really the sum of the loss and loss expense reserves, the unearned premium reserve and all unspecified liabilities less all non-invested assets. Exhibit 1 attached uses 1966 INA data to show the full significance of Bailey's simplified approach.

c. Criticisms of Bailey's Method

(1) The basis for computing the equity in the unearned premium reserve used by Mr. Bailey is reasonable; but due to the inexact science of allocating expenses by function, the actual measurement of this equity should be studied further. In addition, consideration should be given as to whether the equity in the unearned premium reserve should be calculated by line of insurance or for the company as a whole. The need for such a decision arises because the total of the equities calculated by line of insurance does not equal the equity when it is calculated for all lines combined. This is because the distribution of unearned premium by line of insurance differs materially from the distribution of written premiums by line.

(2) Mr. Bailey's method should incorporate a more adequate basis for distributing the equity in the unearned premium reserve by line of insurance. As noted in Exhibit 1, the equity in the unearned premium reserve is one of the negative components in the compilation of "excess funds." Because these net funds are distributed by line of insurance in proportion to the unearned premium reserves by line, it follows that each component is likewise distributed. Such a distribution would be proper for the equity in the unearned premium reserve if all "equity ratios" were uniform by coverage; but, of course, this is not so. Obviously a more accurate method of assigning this item by line of insurance could be developed from the expense ratios reported by coverage in the Insurance Expense Exhibit.

(3) Uncollected balances is another non-invested asset deduction and likewise is distributed in proportion to the unearned premium reserves by line of insurance. Although uncollected balances are not available by line of insurance, and although the distribution method used by the author is rather ingenious, this reviewer believes that a more sophisticated and accurate basis could be established.

(4) Bailey's method makes no provision for non-premium funds supplied to the insurance operation when premium income is not sufficient to cover underwriting disbursements. If and when operating losses accumulate, additional funds are needed to keep the underwriting operation solvent.

(5) Bailey's method does not use mean invested assets. Because his method develops a fraction rather than a dollar base, the simplification of using year-end invested assets is perhaps justified for illustrative purposes. However, this transgression from a more accurate accounting approach should have been stressed with a recommendation that mean invested assets should be used in any final procedure.

d. An Alternative Approach

Generally speaking, all methods will produce the same answers if the same underlying assumptions and data are incorporated. One method is better than another because of its clarity, its simplicity, and its underlying assumptions. With this in mind, the reviewer has designed a more direct approach which is based upon a cash flow chart analysis. Such a cash flow chart is set forth in Exhibit 2. One can observe from this exhibit that underwriting funds in process come from both premium and non-premium sources. The non-premium funds must be sufficient to keep the underwriting operation solvent and so will equal the equity in the unearned premium reserve and the funds advanced for operating losses. Premium funds will equal the underwriting liabilities (adjusted for unassigned reinsurance funds) less uncollected balances and non-premium funds.

The actual calculation of these funds for INA is shown in Exhibit 3. Using Bailey's measurement of the equity in the unearned premium reserve, the invested premium funds were calculated to be $514,596,000 as of 12-31-66. This compares with $545,674,000 established by Bailey's method. The difference of approximately $31,000,000 is accounted for by the net effect of two items:

(1) Cash and miscellaneous net assets(a credit item) is a smaller figure in my calculation so as to correctly exclude current assets necessary to cover the payment of the current liability for dividends declared to stockholders.

(2) Non-premium funds have been increased in my calculation to provide for deficit operating balances.

With these differences and the incorporation of mean invested assets, the ratio of invested premium funds to total invested assets is 33.4% as compared to 37.7% produced by R. A. Bailey's method.

To include non-premium funds for deficit operating balances, as required by the reviewer's method, calls for the measurement thereof. The reviewer believes that this amount will be properly represented by the largest accumulated operating loss for the latest 5, 6, 7, 8, 9, or 10 calendar years. Stated another way, the deficit operating balance equals the net operating loss in the latest five years, if any, plus the operating losses in the prior five years *not offset by subsequent gains.* Any longer experience period is assumed to be impractical and unnecessary even under the most abnormal underwriting cycle. Operating losses differ from statutory losses, and this difference is defined in Exhibit 3.

3. THE RATE OF INVESTMENT INCOME EARNED ON INVESTED PREMIUM FUNDS

The third area of my discussion relates to the rate of interest earned on invested premium funds once they can be established. I have no quarrel here with Mr. Bailey's thinking except in regard to the inclusion of realized gains and losses. Because of the high degree of risk involved in this phase of the investment operation and because of the significant fluctuations from year to year, the inclusion of these gains and losses is questionable.

In any event, this reviewer is inclined to believe that if investment income is ever included in ratemaking, a fair rate of return (perhaps 3.5%) should be selected. This independently established rate would then be applied to the mean invested premium funds. The deduction for premium funds not invested would also be calculated uniformly by using a stated percentage (perhaps 5%) of mean underwriting liabilities.

4. THE INTEGRATED PROFIT LOADING

Mr. Bailey makes this particularly noteworthy comment in his closing paragraph: "The *actual* underwriting profit from investments should not be added to the *expected* underwriting profit. Rather the *actual* underwriting profit from investment should be combined with the *actual* other profit or loss. . ." (Italics supplied by reviwer.) This point is important and is often conveniently overlooked. The problem of setting rate levels which will produce the profits that such rate levels anticipate remains with us. Only when this goal is accomplished will more refined cost accounting techniques along with more realistic profit standards by line of insurance contribute toward more accurate and adequate pricing in the insurance business.

5. THE PROPRIETY OF INCORPORATING INVESTMENT INCOME IN CASUALTY AND PROPERTY RATEMAKING

Mr. Bailey never poses this question and therefore never directly answers it; however, he does imply that the major stumbling block has been one of quantification. This opinion, if true, is not shared by this reviewer. Certainly the answer to this age-old question involves far more than actuarial considerations; it is even doubtful whether the final decision will be significantly influenced by the availability of scientific measurement criteria.

CONCLUDING COMMENTS

This reviewer believes that company managements today are cognizant of the "business profits" from various lines of insurance, and it is only with this knowledge that such managements are interested in taking the added risk inherent in certain lines of insurance where ultimate loss costs are subject to the uncertainties of future inflation and economic conditions. Therefore, the scientific measurement of investment income on premium funds is very important to all segments of the insurance industry, but most certainly to the stockholders. For this reason Mr. Bailey is to be commended for his many thoughts on the subject and for his suggested guidelines in the measurement of investment income by line of insurance.

Exhibit 1

Distribution of Invested Assets

per R. A. Bailey's Method

Insurance Company of North America

12-31-66

(000 omitted)

	Total	Non-Premium	Premium
Liabilities:			
Loss and Loss Expense	$ 449,947		$449,947
Unearned Premiums	435,595		435,595 *
Voluntary Reserves	0	0	
Stockholder Dividends	6,808	$ 6,808	
Unauthorized Reinsurance	17,827	17,827	
Other Liabilities	49,168		49,168 *
Total	959,345	24,635	934,710
Capital and Surplus - adjusted (1)	877,784	877,784	
Non-Invested Assets:			
Equity in Unearned	105,414		105,414 *
Non-admitted Assets	17,013		17,013 *
Uncollected Balances - admitted	184,961		184,961 *
Cash	33,030		33,030 *
Other	48,617		48,617 *
Total Deductions (2)	389,035		389,035
Invested Assets	1,448,094	902,419	545,675

(1) reported ($755,357) + equity in unearned ($105,414) + non-admitted assets ($17,013)

(2) admitted ($266,608) + equity in unearned ($105,414) + non-admitted assets ($17,013)

* components of the author's "excess funds" which are distributed by line on the basis of unearned premiums

Exhibit 2

Cash Flow Chart

Underwriting Operation

Property-Casualty Insurance

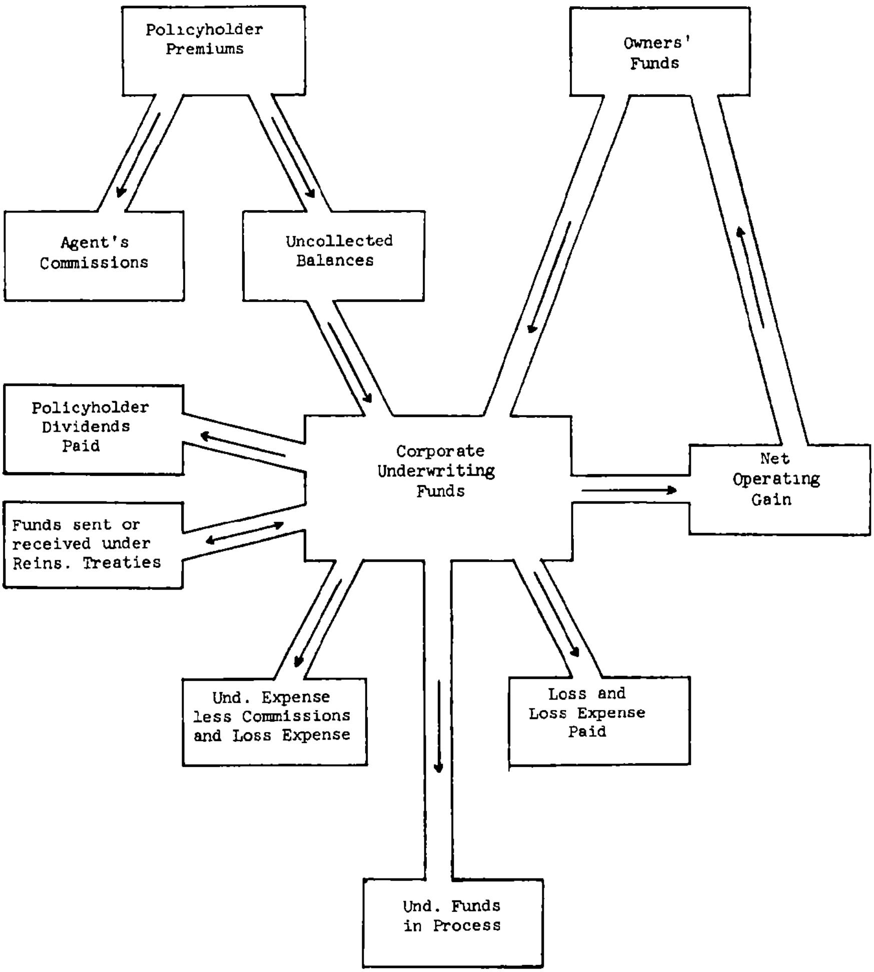

Exhibit 3

Analysis of Mean Underwriting Funds in Process

Insurance Company of North America

1966

(000 omitted)

	12-31-66	12-31-65	Mean
Underwriting Liabilities:			
Loss and Expense Reserves	$ 449,947 *	$ 401,077 *	
Other Und. Expense Reserves	14,603	13,574	
Unearned Premiums	435,595	417,616	
Policyholder Dividends	207	250	
Total	900,352	832,517	
Funds Rec'd. from Reinsurers (+)	16,591	16,731	
Funds Deposited with Reinsureds (-)	18,935	15,301	
Adjusted Total	898,008	833,947	$ 865,977
Uncollected Balances - Ledger	198,315	183,231	190,773
Underwriting Funds in Process	699,693	650,716	675,205
Non-Premium Funds:			
Equity in Unearned Premium	105,414 **	106,075 **	
Net Operating Losses***	37,886 *	27,945 *	
Total	143,300	134,020	138,660
Premium Funds	556,393	516,696	536,545
Cash & Misc. Net Assets	41,797	43,953	42,875
Invested Premium Funds	514,596	472,743	493,670

Check with R. A. Bailey's Method			
Invested Premium Funds - RAB	545,674	-	-
Total Corporate Invested Assets	1,448,094	1,504,751	1,476,422
% - RES	-	-	33.4%
- RAB	37.7%	-	-

* For greater accuracy these figures can be modified as necessary to reflect subsequent developments on loss reserves from Schedules O and P.

** As defined by R. A. Bailey so that methods can be compared. See reviewer's comments on additional studies necessary.

*** Operating Gain or Loss is defined to be the Statutory Gain or Loss less Policyholder Dividends plus the increase in equity in the unearned premium reserve. Non-Premium funds for net operating losses equal the largest accumulated net operating loss for the latest 5, 6, 7, 8, 9 or 10 calendar years, or $0 if none of these periods produces a net operating loss.

AUTHOR'S REVIEW OF DISCUSSIONS

The reviewers have discussed many facets of the subject and have made many helpful suggestions. This is clearly a controversial subject and difficult to discuss without getting involved in the political aspects of it.

One or two of the reviewers let themselves get embroiled in the terminology. I certainly do not deny that all the assets of insurers belong to the insurers. The basic problem is one of cost accounting—of dividing the the investment income into two portions: one derived from the insurance operation and a residual portion. The two portions have to be called something to distinguish them from each other, and it is difficult, as Miss Salzmann discovered, to choose names completely devoid of political and legal implications. I apologize, however, for using such inflammatory terminology as I did.

My objective was to measure, from hindsight, the actual results from underwriting. I do not share Mr. Al Curry's fear that measuring the total results from underwriting will necessarily lead to rate reductions. If what I've heard about results over the past 10 years is true, rate increases are in order. But the first step in any decision making process is to obtain all the pertinent facts. Obtaining all the facts is certainly the function of an actuary.

Several reviewers raised the question whether investment income on loss reserves should be included in underwriting results. Some reserves are discounted for interest. But in casualty insurance when such a claim is finally paid at full value, the increase in the value of the claim at the time of settlement over the original discounted reserve is charged against the underwriting results as a loss. The corresponding investment income should therefore be added back into the underwriting results in order to offset the increase in the claim value and to give a true picture of the underwriting results. The same would apply to inflationary increases in liabilities. If the increase due to inflation is charged against current underwriting operations, the offsetting investment income should be added in also to restore balance to the results.

Mr. Mayerson raised an important point about the need for study as to what the profit ratio in the rates should be. I believe such a study should include consideration of what profit ratio is needed in order to maintain a sound ratio between premium volume and surplus in an expanding and inflationary economy, as well as what is needed to provide an attractive return to stockholders. But I find it hard to see how profits can be artificially pegged at any level in a business where price and product competition is as flourishing as it is in the insurance business.

A THEORETICAL PORTFOLIO SELECTION APPROACH FOR INSURING PROPERTY AND LIABILITY LINES

J. ROBERT FERRARI*

The recent financial literature contains numerous applications of portfolio selection that generally attempt to develop optimal diversification strategies and (perhaps inappropriately)[1] to gauge investment performance. Most of these efforts, however, have been limited to common stock portfolios mainly because equity price movements provide a convenient input to investment models that measure risk by variability of return. The purpose of this paper is to provide a novel application of portfolio selection outside of the investment area. More specifically, it aims at providing an initial report on utilization of portfolio selection techniques to suggest the theoretical, optimal diversification of lines of insurance written by property and liability insurance companies.[2] These results are part of the author's attempts to establish operating criteria for commercial insurance operations.

DIVERSIFICATION AS A DESIRABLE OBJECTIVE

Diversification of investments is generally considered to be a desirable objective and is a widely observed aspect of investment behavior. With particular reference to insurance companies, the primary objective of a number of statutory, quantitative restrictions is to impose some degree of portfolio diversification among and within categories of investments.

Diversification is also closely related to the pooling or averaging aspect of insurance. One of the most obvious examples of "spreading the risk" is the geographical diversification of property coverages, such as fire and

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*The author wishes to gratefully acknowledge the collaboration of Roger Williams, a systems engineer with International Business Machines, who performed the computer work for this paper. The author is also grateful to the Ford Foundation for a summer grant which financed the research for this paper, and is indebted to Drs. Herbert S. Denenberg and Gerald Hartman of the University of Pennsylvania for their valuable suggestions.

[1] Recent criticism of the use of portfolio selection concepts to evaluate investment performance has been expressed by Irwin Friend and Douglas Vickers, "Portfolio Selection and Investment Performance," *Journal of Finance*, XX, No. 3 (September, 1965), pp. 391-396.

[2] This paper will concentrate solely on the insurance portfolio. A logical extension of this effort would be to include investments, as well as insurance, in the model. Underwriting and investment results are obviously interrelated operating criteria and a more advanced model would take into account the insurance-investment interactions.

windstorm, to avoid an undue physical concentration of insureds. Nevertheless, the insurance literature gives little if any notice to the proper allocation of business among the lines of insurance now being offered by multiple-line companies. The theories and techniques of investment diversification are one possibility for explaining and/or prescribing property and liability insurance company operating behavior.

INVESTMENT THEORIES AND DIVERSIFICATION

Traditional economic theories contend that an investor facing alternative opportunities with certain returns or profits (riskless investments) will prefer the investment that offers the maximum return. When uncertainty is introduced, this reasoning is extended to the assumption that an investor will (or should) attempt to maximize the discounted value of expected, future returns.[3] This explanation of rational behavior under uncertainty is considered incomplete because it fails to recognize the aversion of risk which investors possess in varying degrees. Subsequently, the notion that an investor has an aversion to risk in addition to a preference for return was developed. However, the maximization of expected returns is generally preserved as an optimal criterion if anticipated returns included an allowance for risk, or if returns are capitalized at a rate that varies with the individual investment risk.[4] But this theory prescribes the placement of all available cash in the investment having the highest expected return. It offers no explanation for making more than one investment unless the cash available exceeds that single outlet with the maximum expected return. It is surprising that, in spite of the long recognized efforts of investors to diversify, not until 1952 was a theory of investor choice formally introduced that leads directly to diversification and admits it as a desirable goal.

Dr. Harry Markowitz, in a classic article,[5] rejects the maximization of expected returns criterion because it does not recognize investment diversification as a conscious or desirable objective. Markowitz formulates an investment model in which an investor's preference for expected return and aversion toward risk explain the desirability of a diversified investment

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[3] J. B. Williams, *The Theory of Investment Value* (Cambridge, Mass.: Harvard University Press, 1938), pp. 55-75.

[4] Harry Markowitz, "Portfolio Selection," *Journal of Finance*, VII, No. 1 (March, 1952), p. 77. Risk is represented by an appropriate reduction in anticipated returns or an increase in the capitalization (discount) rate. Either, or both, of these adjustments will lower the capitalized value which is the expected return.

[5] *Ibid.*, pp. 77-91.

portfolio. This model also appears to be applicable to the risk-return attributes of the insurance portfolio of a property and liability insurer.

RISK-RETURN CHARACTERISTICS OF LINES OF INSURANCE

Risk and Return Concepts. A property and liability insurance company will have its business diversified among a number of lines of insurance each of which has risk-return characteristics that can be subjected to portfolio selection analysis. Unfortunately, this fact has been largely overlooked primarily because of some confusion in the property-liability area over the concepts of risk and return. The traditional mistake has been to consider the consistently unprofitable lines of insurance as the riskiest business. But a more appropriate view of profitability is as a return concept, ex post, and an expected return concept, ex ante. In addition, a more acceptable and useful measure of risk for a line of insurance is the variability of operating results. Therefore, in the portfolio selection analysis which follows it is assumed that the expected return of a line of insurance is a function of profitability (as measured by loss and expense ratios) and risk is a function of the variability around the expected return.

Input Requirements for an Insurance Portfolio Model. The Markowitz technique has been applied almost exclusively to investment securities, particularly common stock, although, theoretically, any application is possible if expected values and measures of variations are determinable. A by-product of the emphasis on investments is that the available portfolio selection computer programs usually require the inputs for risk to take the form of price movements of securities and expected return is largely a function of capital gains and losses. Therefore, in order to apply portfolio selection techniques to lines of property and liability insurance, the configuration of available computer programs requires that the individual lines be viewed in much the same manner as a category of investment securities. In order to achieve this result, one approach is to use combined loss and expense ratios as determinants of expected return and variability of return.

Comparative Analysis of Lines of Insurance and Investment Securities. The notion of return on an investment security or a category of investments obviously differs from a concept of return on a line of insurance written by an insurer. With a security, the investor relinquishes certain assets with the hope that over time an amount in excess of the original investment will be returned. With insurance, an insurer collects a premium with the hope (at least theoretically) of disbursing less than this amount for expenses and losses.[6] In the insurance transactions the insurer does not relinquish control of any specific assets but it does legally commit a portion of its assets (or its underwriting capacity) in general as resources available for the payment of future possible claims, such claims being estimated by the insurer's reserve liabilities. Similar to investment, a sound insurance operation is based on the expectation that over time there will be a positive return on the initial asset commitment.

Another difference between investment securities and lines of insurance has to do with the potential gain or loss under each type of arrangement. The gain on an investment such as common stock may be theoretically unlimited while the potential loss is limited to the original amount invested. Alternatively, the loss on a line of insurance, particularly a line such as liability, is virtually unlimited up to the amount of the total assets of the company while the gain is limited to the amount of premium income.[7] This difference is not necessarily significant if one is interested in expected returns and the realistically possible, albeit sometimes large, variation from the expectation. Actually, the extreme gain and loss positions of the process of insuring resemble those of common stock investment when equities are sold short. In these transactions one's gain is limited but the potential loss is theoretically unlimited.

Other areas of comparison between lines of insurance and investment securities exist,[8] but these are either not germane to the basic assumptions in this paper or the differences or similarities are reflected in the risk-return measures employed.

Risk and Return Based on Combined Loss and Expense Ratios. The portfolio selection analysis in this paper assumes that the risk-return characteristics of lines of insurance are similar to investment securities and can be based on historical loss and expense ratios. More specifically, it is assumed that each line of insurance generates an annual return based on premium income, operating expenses, and insurance losses. For example, a combined annual loss and expense ratio of .95 or 95 per cent is assumed

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[6] The insurer also can earn interest on funds being held for future disbursement.

[7] In a practical sense the gain will certainly be something less than premiums since expenses will be incurred in acquiring the business.

[8] For example, obvious analogies can be made between the cancellation privileges of both insurer and insured, and the marketability and callability of investment securities. Another is the relative size and variability of the expenses associated with the acquisition of insurance versus the acquisition of investments.

to be a return of 5 per cent on that line of business and, alternatively, a loss-expense ratio of 105 per cent is assumed to be a loss of 5 per cent.

In this paper the determination of risk and return on lines of insurance is made with the assumption that all historical trends and variation will continue in the near future. For this reason, the computer analysis to follow has to be considered illustrative and hypothetical; however, it is somewhat realistic since actual company data are used as a basis for the model input. Utilizing simple linear extrapolation of the most recent, historical, combined loss and expense ratios of one large company, the risk-return assumptions shown in Table I were derived and subsequently used in applying the Markowitz portfolio selection technique to property and liability insurance.

Markowitz also suggests subjective probability beliefs as alternatives to inputs based solely on a past record that may not be representative of the future.[9] He suggests a method for deriving probability beliefs by formulating expectations about the movements (for example, highest, expected, and lowest values) of some relevant index. The analyst would then state the loss-expense ratio (for example, highest, expected, and lowest ratio) for each line of insurance at each of the possible values of the index. When this procedure is carried out the individual lines are said to be "tied" to the index. With these tied estimates, correlations between individual lines of insurance can be determined indirectly from the relationships of the lines to the index. The index can be said to act as a kind of common denominator. Additional correlation factors can be introduced when two lines are more positively correlated or less positively correlated than would be indicated by their relationships with the index.[10]

Another method calls for probability beliefs for each security stated as direct estimates of return and its expected variation. With this method, correlations are determined independently for each pair of lines of insurance. This procedure, and any other input method that does not use estimates tied to an index, has the limitation of necessitating a large number of individual correlations that must be estimated or calculated.[11] Probability beliefs for individual lines of insurance are also subject to the difficulty of formulating statistically consistent estimates.

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[9] Markowitz, *Portfolio Selection, op. cit.*, pp. 26-33.

[10] *Ibid.*, p. 32.

[11] 1,225 correlations for fifty securities and 4,950 correlations for one hundred securities.

APPLICATION OF THE MARKOWITZ PORTFOLIO SELECTION THEORY TO INSURANCE PORTFOLIOS[12]

Expected Return and Risk. In order to apply the Markowitz portfolio selection theory to an insurance portfolio, it must be assumed that an insurer can formulate expectations based on the expected return and risk associated with each line of insurance. The return on each individual line is assumed to be a statistical random variable with a symmetrical probability distribution. The line of insurance can then be viewed as having an expected return which is a statistical average of the probability distribution. The expected return on a portfolio is then the weighted sum of the expected returns of lines of insurance in the portfolio; that is

$$E(R) = \sum_{i=1}^{N} a_i R_i$$

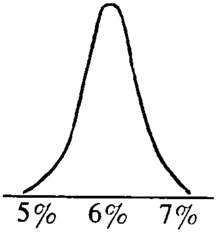
where: $E(R)$ = expected return on a portfolio.

a_i = the proportion of the total portfolio committed to the i^{th} line of insurance.

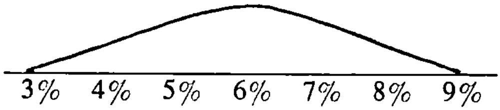
R_i = the expected return of the i^{th} line of insurance.

Therefore, the expected return on the portfolio also is considered a random variable.

The risk on each individual line is assumed to be the variance (V) or standard deviation[13] squared (σ^2) of the return described by the probability distribution for the line. For example, suppose the probability distributions of return on two hypothetical lines of insurance, A and B, can be pictured as follows:



LINE A



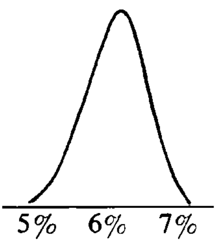
LINE B

[12] This section of this paper is purely theoretical since it ignores many practical constraints which will be taken up in subsequent sections.

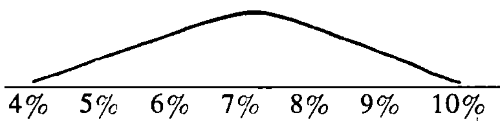
[13] It is helpful to think of the risk in terms of the standard deviation of return when one wishes to draw upon probability theory for a notion of risk. For example, if actual return is assumed to be normally distributed around expected return, then the probability of actual return being less than expected return by more than one, two, or three standard deviations is .16, .02, or .001 respectively.

Both lines have the same expected return of 6 per cent, but line A is considered the safer line because of the lesser variability of return. That is, there is more certainty that the actual return of A will be equal to the expected value. It is assumed that a rational insurer will always prefer A to B since he can expect the same return with less risk.

The situation becomes more complex if an insurer has to make a decision between lines of insurance A and C with probability distribution of return as follows:



LINE A



LINE C

Now the line with the greatest variability of return (risk) also has a higher expected return. The insurer must decide whether A or C is the more desirable line of insurance. With C there is an additional risk for a higher expected return, while with A there is less expected return with more certainty. A preference for one or the other will be a function of the objectives of the insurer and its attitude toward risk and return. That is, the final decision can only be explained by some concept of utility and the introduction of the relevant practical constraints.

The problem of portfolio selection is introduced when an insurer faces a number *(N)* of available lines of insurance that present numerous possible combinations of risk and return. It is assumed that for each individual line an insurer can formalize his beliefs about expected return and risk in the form of a probability distribution.[14] The expected return on an insurance portfolio consisting of any or all of the available lines has already been shown to be the weighted sum of their expected returns; namely

$$E(R) \sum_{i=l}^{N} a_i R_i$$

The risk of such a portfolio, however, is not simply the weighted sum of the individual variances, but it is a function of both the risk of each indi-

[14] The limitations imposed by this assumption are ignored for the time being.

vidual line and the correlation of returns between each pair of lines. This latter phenomenon is termed covariance, and the formula for covariance between two lines of insurance is

$$\sigma_{ij} = E\,[R_i - E(R_i)]\,[R_j - E(R_j)]$$

where: σ_{ij} = the covariance between the i^{th} and the j^{th} lines of insurance.
R_i = the actual return on the i^{th} line of insurance.

In this form covariance is the expected value of the deviation of the return on line i from its mean times the deviation of the return on line j from its mean. An alternative expression is the product of the standard deviation of the i^{th} line times the standard deviation of the j^{th} line times their correlation coefficient, as follows:

$$\sigma_{ij} = \rho_{ij}\,\sigma_i\,\sigma_j$$

where: ρ_{ij} = correlation coefficient for the returns on the i^{th} and j^{th} lines of insurance.
σ_i = standard deviation of return on the i^{th} line of insurance.

The variance of return or risk of a portfolio can be expressed as a weighted sum of the variances of all individual lines plus the weighed sum of the covariances for each pair of lines, as follows:

$$V(R) = \sum_{i=1}^{N} a_i^2\, V_i + 2 \sum_{i=1}^{N} \sum_{j>i}^{N} a_i\, \sigma_{ij}\, a_j$$

$$V(R) = \sum_{i=1}^{N} \sum_{j=1}^{N} \sigma_{ij}\, a_i\, a_j$$

where $V(R)$ = variance of return on the entire portfolio.
a_i = proportion of the total portfolio invested in i^{th} line of insurance.
V_i = variance of return on the i^{th} line of insurance.

The correlation of returns between two lines (covariance) is a primary element in the Markowitz Portfolio Selection Theory. Insurer diversification can be viewed as a procedure for reducing aggregate risk by holding lines of insurance whose returns are not likely to vary in the same direction at the same time. The benefits of diversification are most fully realized by writing business with negative correlation, thus reducing the degree of risk for the insurance company.

The Efficient E-V Criterion. For a given set of available lines of insurance, various E-V (expected return on the portfolio and its variance) combinations are possible depending on the probability beliefs established for the individual returns, variances, and covariances. Purely for illustration, the set of all obtainable E-V combinations might be represented by the circle and the enclosed shaded area in Chart 1.

Markowitz has formulated a rule which states that in surveying all of the possible E-V combinations, one should select only the ones that are "efficient" portfolios. An efficient portfolio is a combination with the minimum variance (or standard deviation) for any given expected return and/or with the highest possible expected return for a given variance (or standard deviation). In Chart 1, the efficient portfolios are those described by the E-V combinations on the arc *ab*. All other combinations are inefficient. For example, it may be seen in Chart 1 that portfolio *c* is inefficient because a higher expected return at the same risk is possible at *d*, while the same expected return at less risk is possible at *e*. Portfolio *f* is inefficient because a higher expected return is available at *e* with the same level of risk. Portfolio *g* is inferior to *d*, because the latter offers the same expected return at less risk. Portfolio *h* is not an obtainable E-V combination given the available investments and their attributes.

It is important to note that no one efficient portfolio (in our hypothetical example those on arc *ab*) is better than any other based on return and risk considerations. Choosing between efficient E-V combinations always involves giving up some expected return for less risk, or alternately, taking more risk for an increase in expected return.

The final choice among efficient portfolios is further limited to those that meet the standards for being "acceptable" portfolios. An "acceptable" efficient portfolio is one that complies with all legal and operating policy constraints imposed on the aggregate commitment to lines of insurance. These constraints can be a minimum commitment to certain lines, a minimum overall rate of return, a maximum allowable proportion in certain lines, etc. Depending on the willingness and ability of an insurer to assume risk, it is necessary to select from among the acceptable, efficient E-V combinations the one that best meets the requirements and objectives of the insurer.

Computational techniques are available that can determine the acceptable, efficient E-V portfolios associated with a given set of expected returns, variances, covariances, and constraints. The procedure is called quadratic programming, which is essentially optimizing (maximizing expected return

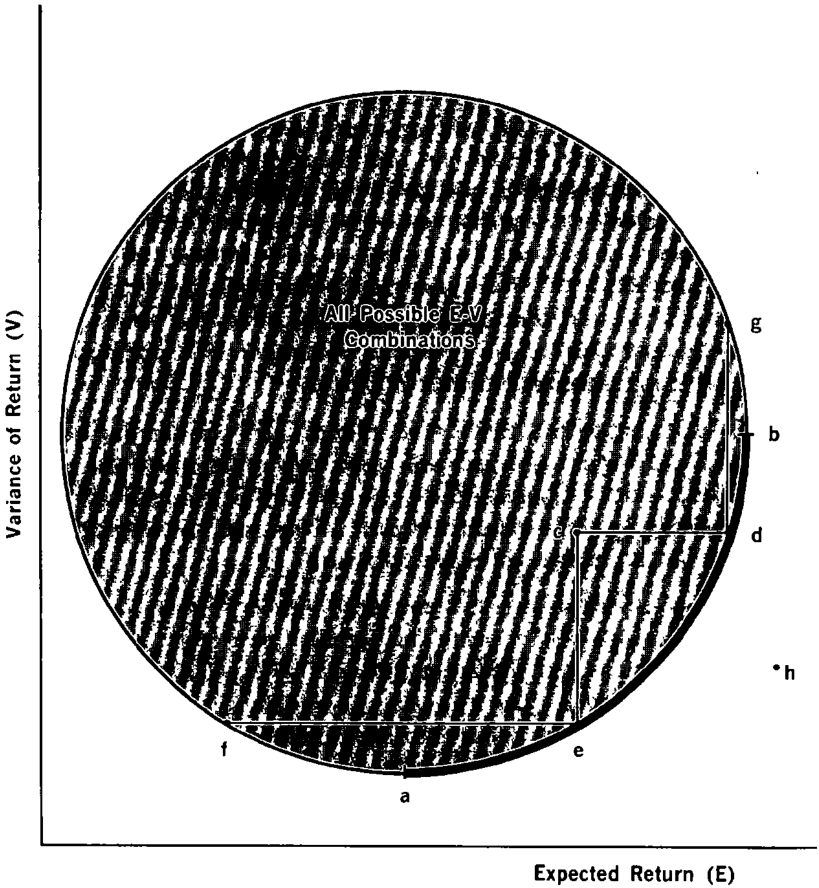


Chart 1. Graphic Representation of the Efficient (E-V) Portfolios in a Hypothetical Situation

given risk or minimizing risk given an expected return) a quadratic function subject to linear constraints. These computations are complex, but these complexities are primarily of mathematical significance. An understanding of mathematical programming procedures is not a prerequisite for using portfolio selection techniques, and the mathematical details of efficient portfolio construction are left to the original sources and will not be duplicated in this study.[15]

Constraints on Property-liability Insurance Company Portfolios. In every portfolio selection application, the combined influence of regulation, managerial policy and practical considerations places constraints on the freedom of action. Such is definitely the case in a property-liability insurance portfolio of an insurer and these constraints must be recognized before realistic results can be obtained from portfolio analysis. The crucial constraints are those aspects of the insurance business that limit the speed with which a company can move from one insurance portfolio to another. A high degree of inflexibility, at least in the short run, stems from an inability or a refusal to radically increase or decrease the percentage composition of a company's insurance business. The obstacles to such action result primarily from the maintenance of agency relationships, the insurance consumption patterns of insureds, and competition among insurers.

The computer program available to the author had the capacity for specifying the maximum percentage of a company's business that could be written in one line. Since the ability to change the insurance mix is a highly variable factor, three sets of maximum percentage constraints were selected so that, compared to the present insurance portfolio, the percentages would represent relatively "low," "average," and "high" degrees of flexibility. These assumed percentages are shown in Table 2 along with the current portfolio composition used as a point of departure.

The maximum percentage constraints are effective in limiting the increase in any one line of insurance to realistic proportions. Unfortunately,

[15] Markowitz describes the procedure called quadratic programming which computes the E-V efficient set of portfolios. See Harry Markowitz, "The Optimization of a Quadratic Function Subject to Linear Constraints," *Naval Research Logistics Quarterly, III* (March-June, 1956), pp. 111-133, and *Portfolio Selection* (New York: John Wiley and Sons, Inc., 1959) Chapter 8. For other related discussions see John Frederick Weston and William Beranek, "Programming Investment Portfolio Construction," *The Analysts Journal,* XI, No. 2 (May, 1955), pp. 51-55; A. D. Martin, Jr., "Mathematical Programming of Portfolio Selections," *Management Science,* I, No. 2 (January, 1954), pp. 152-165; William F. Sharpe, "A Simplified Model for Portfolio Analysis," *Management Science,* IX, No. 2 (January, 1963), pp. 277-293; and Philip Wolfe, "The Simplex Method for Quadratic Programming," *Econometrica,* Vol. 27 (July, 1959), pp. 382-398.

the available computer program did not allow minimum percentages to be specified. Consequently, some of the portfolios derived in the subsequent analysis suggest that certain lines of insurance should be dropped completely or reduced to very small proportions. In many cases this would be either impossible or highly undesirable. Regardless of this deficiency, minimum percentage constraints can still be recognized indirectly by considering as practical only those efficient portfolios that contain a realistic de-emphasis of certain lines of insurance. Using Markowitz's terminology, only the "acceptable" portfolios[16] of the entire "efficient" set can be considered relevant.

Efficient E-V Property-Liability Insurance Portfolios. Using the input items summarized in Tables 1 and 2, the risk and return characteristics of the efficient E-V property-liability insurance portfolios produced by the I.B.M. Portfolio Selection Program (1401-FI-04X) are shown graphically in Chart 2. Tables 3, 4 and 5 contain the same expected returns and variation of returns plus the percentage compositions for representative portfolios under each of the three sets of allocation constraints. In each case, for ease of direct comparison, the attributes and composition of the current portfolio are shown along with the efficient portfolios.

Chart 2 clearly shows that the insurance portfolio of the company whose data were used will be inefficient and non-optimal on the basis of the Markowitz E-V Criterion, if the present percentage composition is maintained. This criterion suggests, for example, that a shift from point P to a point P_I directly horizontal on the efficient portfolio curve under constraint set I would produce a portfolio with the same risk (standard deviation equal to 4.80 per cent) but improve the expected return from .68 per cent to something over 1.1 per cent. Table 3 indicates that this shift would be accomplished largely by relative decreases in extended coverage, ocean marine, and auto property damage liability insurance and relative increases in fire, auto bodily injury liability (surprisingly enough), and treaty reinsurance.

To illustrate the significance of the constraint assumption,[17] consider another horizontal shift of P to P_{III}, one of the portfolios described under the more liberal Constraint Set III. Now, under the assumptions, a portfolio

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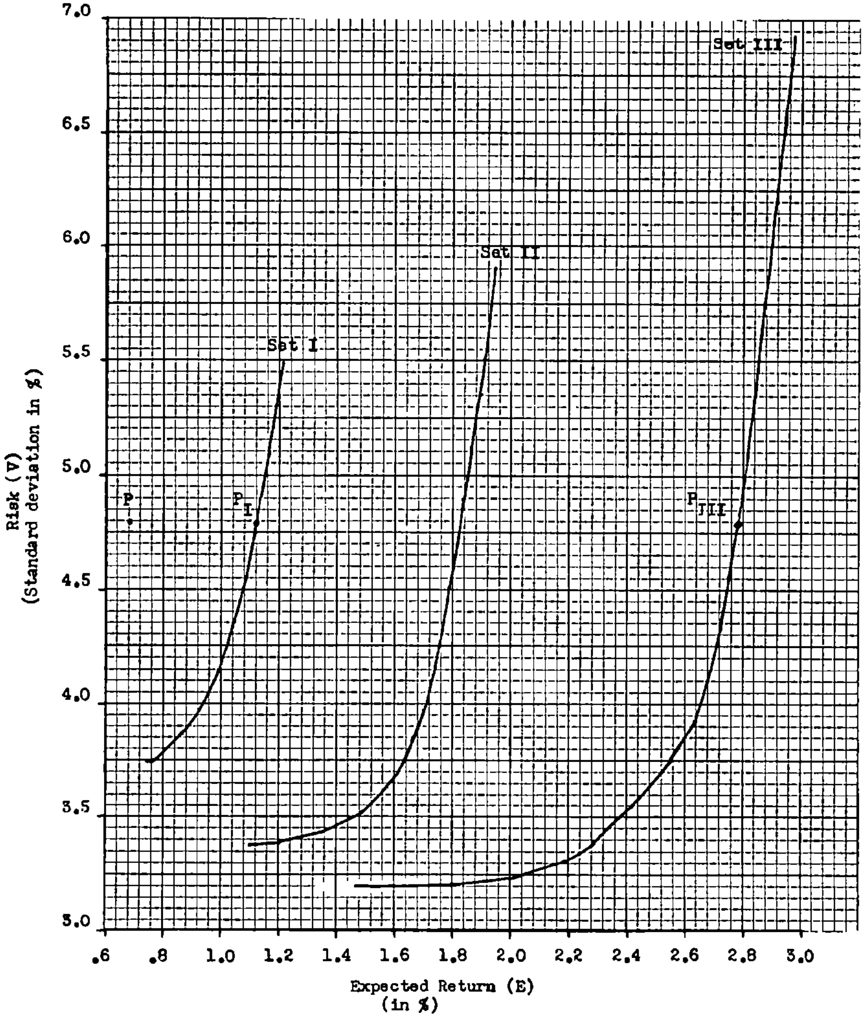
[16] That is, those portfolio that meet various legal, managerial policy, and other constraints that can't be incorporated automatically in the computer computations.

[17] A different kind of limitation on the choice of portfolios that doesn't happen to apply to the portfolios in this paper, but could in other cases, is discussed in William J. Baumol, "An Expected Gain-Confidence Limit Criterion for Portfolio Selection," *Management Science* (October, 1963), pp. 174-186.

CHART 2

Expected Return and Risk of Efficient E-V Insurance Portfolios



is theoretically attainable which has the same risk as current holdings but which increases the expected return from .68 to over 2.7 per cent. While this would represent an admirable improvement under the E-V criterion, the dramatic changes necessary for such a portfolio are evident upon observing Table 5 that fire, extended coverage, ocean marine and auto property damage liability insurance would not be sold. This latter portfolio is in all likelihood not an acceptable one.

Under the Markowitz E-V criterion, given the relevant set of constraints, the movement from point P to any point on an efficient E-V curve is considered to be an improvement recognizing that no point on a curve is ostensibly superior to another on the same curve solely on the risk-return criteria. Only two examples have been illustrated above but a large number of hypothetical portfolio adjustments are summarized in Tables 3, 4 and 5. It must be emphasized that these results were obtained from the data of only one company which in turn were affected by the author's own extra-, polation and constraint assumptions.

LIMITATIONS OF THE MARKOWITZ E-V CRITERION FOR PROPERTY-LIABILITY INSURANCE COMPANIES

There are two general areas of difficulty that definitely limit the theoretical and/or practical application of the Markowitz technique to property-liability company insurance portfolios. The first pertains to the nature of the input assumptions of the model. The second is the uncertain relationship between the Markowitz E-V criterion and the objectives and behavior of non-life insurance companies.

Input Assumptions in the Markowitz Model. The programming solutions applicable to the Markowitz portfolio selection technique are complex. However, they can be performed on digital computers, and they present primarily practical problems of computer storage capacity and calculation time. The technique can be utilized without understanding the complex mathematical programming procedures. But, the preparation of a reliable and acceptable input in the form of an expected return and a variance is a major difficulty in the practical application of portfolio selection.

The formulation of probability beliefs about expected returns and variation of returns on lines of insurance is a complicated task.[18] Un-

[18] Although ostensibly this is not as difficult as the formulation of appropriate risk and return measures for non-common stock investments, particularly private placements and mortgages.

doubtedly, it is difficult for an actuary experienced in statistical methods to make reliable, consistent estimates of future return in the form necessary for the model. Naturally, the procedure is even more difficult for managers less familiar with statistical and probability concepts.

In the case of insurance uncertainties, the historical method described by Markowitz[19] can be used to calculate return, variance, and covariance, and these values then can act as guidelines for quantifying expectations. The historical method, however, is deficient to the extent that it ignores the dynamic aspects of the insurance business. For example, the relative adequacy of future rate levels may differ from that evident in the historical data.[20] This and other similar difficulties can be alleviated by introducing expectations into historical parameters by adjustments based on subjective judgment. The revised historical input still will be deficient to the extent that future developments are unforeseen or that subjective adjustments do not accurately reflect expectations in a quantified form.

Even if a reliable variance of return is available for all lines of insurance, there will still be other fundamental difficulties. Variances of return do not include many factors that are important to actual portfolio selection problems. For example, an insurer will probably be concerned with the skewness or third moment of the probability distribution of losses and expenses on a line of insurance.[21] An insurer, or an investor, might act to maximize the third moment of his probability beliefs (preference for a distribution skewed toward positive returns) since this increases the chance of a large return while decreasing the chance of a large loss.[22]

Recently, insurers seem willing to accept relatively low expected returns in many lines of business and, consequently, in the over-all underwriting operation. Considering the regulatory and actuarial difficulties in obtaining adequate rates, the insurers may (must?) be willing to live temporarily with persistent losses if such losses are relatively stable, offer no significant danger of catastrophic experience, and can be offset by investment results. Portfolios selected with conscious or unconscious recog-

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19 Markowitz, *op. cit.*, pp. 8-26.

20 The possible effect of the relative adequacy of rate levels at various points in time was pointed out to the author by C. A. Hachemeister of the Insurance Company of North America.

21 For a discussion of this concern on the part of investors, see Karl Borch, "A Note on Utility and Attitudes to Risk," *Management Science* (July, 1963), p. 700; and Yale Brogen, "Discussion," *Econometrica* (July, 1951), pp. 325-326.

22 Brozen, *op. cit.*, p. 326.

nition of skewness of results are likely to differ from portfolios consistent with the efficient E-V criterion.

Perhaps the most troublesome problem with the input to a portfolio selection model is that the assumptions of risk and return may not hold up if an attempt is actually made to acquire a prescribed portfolio. Even if a property-liability insurer could significantly alter its insurance portfolio, this action could so seriously affect loss and expense ratios as to destroy the assumptions on which the reallocation was based. For example, since return is measured as a per cent of premiums, variability of return is a function of premium volume. Thus, if the E-V criterion prescribed a reduction in a particular line, such reduction would probably increase the variability of return and this might suggest even further reductions in this line.[23] On the other hand, as business is reduced, more selective underwriting may produce a more profitable book of business, thereby increasing the expected return.

Relationship of the Markowitz E-V Criterion to Company Objectives and Behavior. Intuitively, diversification of insurance, for example, by line and geography, seems desirable for the responsible operation of a property-liability insurance business, and the insurance portfolios of the established multiple-line companies do display a great deal of diversification. The question then remains whether the Markowitz portfolio selection technique can be used to explain or to plan the diversification of non-life insurance portfolios.[24]

The structure of the insurance business is such that non-life insurance companies can attain great diversification by lines of insurance without conscious marginal risk-return decisions. A large company can be expected to establish variety in its lines of insurance simply because of the nature of the marketing channels, the sheer size of the portfolios, and the complementarity of certain lines of insurance,[25] for example, auto bodily injury liability, auto property damage liability, and auto physical damage.

[23] The possibility of this uni-directional movement in individual lines was suggested to the author by William H. Crandall of the Insurance Company of North America.

[24] For an interesting application of a portfolio selection model to the behavior of Mutual Funds, see Donald Eugene Farrar, *The Investment Decision Under Uncertainty* (Englewood Cliffs, N. J.: Prentice Hall, Inc., 1962). For a critique of Farrar's work see Irwin Friend and Douglas Vickers, *op. cit.*

[25] One study follows this line of reasoning to conclude that life insurance company investment behavior is more properly explained by the simple maximization-of-expected-return rule than by a Markowitz portfolio selection theory. See Lawrence Donald Jones, Jr., "Portfolio Objectives, External Constraints and the Post-War Investment Behavior of Life Insurance Companies" (unpublished doctoral dissertation, Department of Economics, Harvard University, 1959).

The application of the Markowitz technique to a financial institution such as a property-liability insurance company is bound to present difficulties, since the model is probably most appropriate for some theoretical individual who has a definite amount of assets to commit for a given time duration. In this sense the model is applicable to static situations and only those in which the prescribed actions will not alter the general market and hence the input assumptions.

The operation of a non-life insurance business is obviously not static, and the continuous marketing activity in a changing environment and the sensitivity of the Markowitz model would probably suggest continual and impractical reallocation of lines of insurance. In addition to the use of constraints, one suggestion for reducing the reallocation problem is to introduce a cost of switching to make the model less sensitive.[26] The inability to reallocate lines of insurance without affecting the market, and the loss and expense assumptions on which the switching is based, has already been discussed.

CONCLUSIONS

The application of portfolio selection techniques to property and liability insurance companies has some interesting theoretical possibilities as well as serious practical limitations. The immediate value of such models appears to stem not so much from the output of optimal insurance portfolios but from the explicit emphasis on the definition and measurability of the crucial variables—risk and return. The analysis stresses the distinction between profitability (return) and variability (risk) and the dependency of both on the portfolio mix. Of more significance than the mechanical production of optimal portfolios is the recognition that decisions related to individual lines of insurance should be considered with regard to their effect on the entire portfolio.

[26] Gordon D. Shellard, "Panel Discussion: Operations Research," *Transactions of the Society of Actuaries* (1966, No. 1), p. D. 333.

Table 1

Assumed Expected Return and Range of Variation of
Return on Lines of Insurance in a Multiple Line Insurance Company*

	Expected	High	Low
Fire	-1.64	5.96	- 9.24
Extended Coverage	-2.61	10.87	-16.09
Home Multiple Peril	-1.10	14.44	-16.64
Commercial Multiple Peril	2.57	61.69	-56.55
Ocean Marine	-4.43	6.07	-14.93
Inland Marine	1.18	8.84	- 6.48
Accident	-1.06	29.98	-32.10
Group A and H	7.93	16.99	- 1.13
Workmans Compensation	- .50	10.52	-11.52
Auto B. I. Liability	.11	10.73	-10.51
Auto P. D. Liability	-2.25	5.19	- 9.69
Auto Physical Damage	- .97	8.97	-10.91
Misc. B. I. Liability	2.49	18.97	-13.99
Misc. P. D. Liability	10.53	22.49	- 2.43
Treaty Reinsurance	-1.37	5.71	- 8.45
Fidelity	7.09	27.47	-13.29
Surety	25.61	44.47	6.75
Burglary and theft	7.58	15.84	- .68

*The range was taken as two standard deviations on either side of the expected value assuming a normal distribution.

Table 2

Maximum Percentage Constraints For A Property-Liability Insurance Portfolio

	Present Composition (P)	Degrees of Relative Flexibility Set I (Low)	Set II (Average)	Set III (High)
Fire	8.95	10.00	12.00	15.00
Extended Coverage	2.58	3.00	4.00	6.00
Home Multiple Peril	11.58	13.00	15.00	17.00
Commercial Multiple Peril	5.51	7.00	8.00	10.00
Ocean Marine	4.73	5.00	6.00	8.00
Inland Marine	4.34	5.00	6.00	8.00
Accident	0.35	0.50	1.00	2.00
Group A. and H.	7.25	8.00	10.00	12.00
Workmen's Compensation	9.07	10.00	12.00	14.00
Auto B.I. Liability	9.60	11.00	12.00	15.00
Auto P.D. Liability	3.54	4.00	6.00	8.00
Auto Physical Damage	4.51	5.00	6.00	8.00
Misc. B. I. Liability	7.35	8.00	10.00	12.00
Misc. P. D. Liability	2.48	3.00	4.00	6.00
Treaty Reinsurance	15.08	17.00	20.00	25.00
Fidelity	1.36	1.50	2.00	4.00
Surety	0.89	1.00	1.50	2.00
Burglary and theft	0.83	1.00	1.50	2.00

Table 3

Efficient E-V Insurance Portfolios Under Constraint Set I

													Present Portfolio
Expected Return	1.21	1.20	1.15	1.10	1.05	1.00	.95	.90	.85	.80	.75		.68
Standard Deviation	5.48	5.43	5.03	4.69	4.39	4.18	4.04	3.94	3.86	3.79	3.74		4.80
	Percentage Composition of Premium Volume											Set I	
Fire	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	8.95
Extended Coverage	--	--	--	--	--	1.69	3.00	3.00	3.00	3.00	3.00	3.00	2.58
Home Multiple Peril	12.00	12.15	13.00	11.56	10.99	9.97	9.38	8.20	7.11	6.01	5.40	13.00	11.58
Commercial Multiple Peril	7.00	6.85	5.61	4.91	4.01	3.34	2.62	2.41	2.22	2.02	1.60	7.00	5.51
Ocean Marine	--	--	--	--	--	--	--	1.39	2.68	3.96	5.00	5.00	4.73
Inland Marine	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	4.34
Accident	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.35
Group A. and H.	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	7.25
Workmans Compensation	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	9.07
Auto B. I. Liability	11.00	11.00	11.00	11.00	11.00	11.00	11.00	11.00	11.00	11.00	11.00	11.00	9.60
Auto P. D. Liability	--	--	--	2.52	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	3.54
Auto Physical Damage	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	4.51
Misc. B. I. Liability	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	7.35
Misc. P. D. Liability	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	2.48
Treaty Reinsurance	17.00	17.00	17.00	17.00	17.00	17.00	17.00	17.00	17.00	17.00	17.00	17.00	15.08
Fidelity	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.36
Surety	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.89
Burglary and theft	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.83

Table 4

Efficient E-V Insurance Portfolios Under Constraint Set II

																	Set II	Present Portfolio
Expected Return	1.95	1.90	1.85	1.80	1.75	1.70	1.65	1.60	1.55	1.50	1.45	1.40	1.35	1.30	1.25	1.20		0.68
Standard Deviation	5.98	5.55	5.05	4.63	4.28	4.00	3.82	3.68	3.59	3.54	3.50	3.47	3.44	3.42	3.40	3.39		4.80
							Percentage Composition of Premium Value											
Fire	--	--	--	1.05	4.30	7.89	10.11	9.28	11.44	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	8.95
Extended Coverage	--	--	--	--	--	--	--	--	--	--	0.65	1.09	1.21	1.33	1.45	1.56	4.00	2.58
Home Multiple Peril	12.26	10.15	9.26	8.15	6.25	3.00	3.00	2.21	1.25	1.30	1.32	1.26	1.13	1.00	0.87	0.74	15.00	11.58
Commercial Muliple Peril	8.00	7.14	5.93	4.80	3.88	4.41	2.30	1.86	1.40	1.33	1.26	1.22	1.19	1.16	1.14	1.11	8.00	5.51
Ocean Marine	--	--	--	--	--	--	--	--	--	--	--	0.38	1.23	2.07	2.92	3.77	6.00	4.73
Inland Marine	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	4.34
Accident	1.00	1.00	1.00	1.00	1.00	0.85	0.43	0.22	--	--	--	--	--	--	--	--	1.00	0.35
Group A. and H.	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	7.25
Workman's Compensation	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	10.90	10.69	10.46	10.19	9.89	9.59	9.30	9.00	12.00	9.07
Auto B. I. Liability	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	9.60
Auto P. D. Liability	--	--	--	--	--	--	1.30	4.99	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	3.54
Auto Physical Damage	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	4.51
Misc. B. I. Liability	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	9.66	8.75	7.78	7.04	6.55	6.07	5.59	5.11	10.00	7.35
Misc. P. D. Liability	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	2.48
Treaty Reinsurance	13.74	16.71	18.82	20.00	19.56	18.84	17.88	16.45	16.11	16.93	17.54	17.84	17.80	17.77	17.73	17.70	20.00	15.08
Fidelity	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	1.36
Surety	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	0.59
Burglary and theft	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	0.83

Table 5

Efficient E-V Insurance Portfolios Under Constraint Set III

																	Set III	Present Portfolio
Expected Return	2.95	2.90	2.80	2.70	2.60	2.50	2.40	2.30	2.20	2.10	2.00	1.90	1.80	1.70	1.60	1.50		0.68
Standard Deviation	6.65	6.05	4.98	4.20	3.84	3.66	3.52	3.40	3.32	3.27	3.24	3.23	3.22	3.21	3.21	3.20		4.80
	Percentage Composition of Premium Volume																	
Fire	--	--	--	--	2.67	3.64	4.66	5.68	8.23	9.08	8.81	8.73	9.04	9.44	9.84	10.16	15.00	8.95
Extended Coverage	--	--	--	--	--	--	--	--	--	0.61	0.89	1.09	1.19	1.25	1.32	1.38	6.00	2.58
Home Multiple Peril	5.82	6.79	5.23	3.67	2.05	1.49	1.23	1.00	0.72	0.52	0.44	0.37	0.35	0.34	0.32	0.31	17.00	11.58
Commercial Multiple Peril	9.41	8.06	5.65	3.24	1.56	1.28	1.17	1.08	1.01	0.95	0.92	0.91	0.91	0.91	0.91	0.91	10.00	5.51
Ocean Marine	--	--	--	--	--	--	--	--	--	0.75	2.07	3.04	3.49	3.78	4.07	4.34	8.00	4.73
Inland Marine	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	4.34
Accident	1.77	1.89	1.36	0.84	0.31	0.14	0.05	--	--	--	--	--	--	--	--	--	2.00	0.35
Group A. and H.	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	11.38	10.56	9.74	8.99	12.00	7.25
Workman's Compensation	14.00	14.00	14.00	14.00	12.09	9.95	9.06	8.39	7.81	7.35	7.15	7.03	7.02	7.04	7.07	7.09	14.00	9.07
Auto B. I. Liability	15.00	15.00	15.00	15.00	15.00	15.00	15.00	14.13	13.19	12.47	12.14	11.96	11.95	11.99	12.03	12.07	15.00	9.60
Auto P. D. Liability	--	--	--	--	--	3.86	5.80	7.81	8.00	8.00	6.00	8.00	8.00	8.00	8.00	8.00	6.00	3.54
Auto Physical Damage	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	7.88	7.74	7.67	7.65	7.69	7.74	7.79	7.85	8.00	4.51
Misc. B. I. Liability	12.00	12.00	12.00	12.00	12.00	11.07	9.35	7.87	6.11	4.83	4.28	3.91	3.76	3.69	3.61	3.55	12.00	7.35
Misc. P. D. Liability	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	5.93	6.00	2.48
Treaty Reinsurance	--	0.26	4.76	9.26	11.88	11.53	11.65	12.01	13.03	13.70	13.96	14.21	14.40	14.58	14.75	14.93	25.00	15.08
Fidelity	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	3.67	3.10	2.86	2.72	2.58	2.46	4.00	1.36
Surety	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	0.89
Burglary and theft	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	0.83

DISCUSSION BY MARTIN BONDY

Professor Ferrari's paper is a thought-provoking one and well worth the reading. However, as is the case with treatises presenting basic concepts, its chief value lies not in its immediate applicability to the solution of problems but in the broad idea it suggests.

It should be clear that the assumptions and constraints set forth in the hypothetical example given in the paper bear almost no relation to reality. In fact, the author has as much as said so in his qualifying comments.

Let us go through some of the more outstanding examples of instances where his model or solutions are not realistic in terms of the insurance world in which we live.

THE ASSUMPTIONS

1. Variability of Results (Risk)

"The return on each individual line is assumed to be a statistical random variable with a symmetrical probability distribution."

While many lines of business may behave in this way, we can easily think of several that do not. Some of these are extended coverage and crop hail, which are seriously affected by weather catastrophes, and bonding, which may suffer extraordinary fluctuations due to economic conditions.

2. Expected Value (Return)

One wonders whether the inherent profitability of a line is a static condition. I believe not. Clearly the course of time changes the expected return. Political considerations impose and release pressures. Competition always attacks profitable places (if there are any) and makes them less profitable. Perhaps more important, the increase or decrease in a company's volume in a selected line will have a substantial effect on results.

3. Table 1

To label the historical results as expectations is to stretch a point quite far. This becomes fairly obvious when we attribute to surety an expected profit margin of 25.61 and a minimum possible of 6.75. (Frankly, I am more than a little envious of the company which regards its expectation in the burglary line as + 7.58.)

Furthermore, such a table denies the efficacy of underwriting. I have become enough of a believer in the past few years to rebel against resignation to an expected loss in certain lines of business based upon past results. We *can* affect our results even within a line of business. To defeatedly accept a loss year after year would make us similar to the famous gambler "Nick the Greek" in the apochryphal story. When his friend approached him and asked him why he got hooked into a crooked card game, he replied that he knew it was fixed but, "It was the only game in town."

THE CONCLUSIONS

1. *Impossibilities*

The application of the author's technique leads him toward a solution which includes increases in fire and auto bodily injury and decreases in extended coverage and auto property damage. The nature of the business forbids such combinations. In order to prevent such a solution he may use one of two approaches. First, he may add constraints which do not permit complementary lines to move in opposite directions. More simply, he may treat complementary lines (such as auto BI and auto PD) in tandem.

2. *Efficacy of Diversification*

The author states:

"Intuitively, diversification of insurance, for example, by line and geography, seems desirable for the responsible operation of a property-liability insurance business . . . ."

I am not certain that one can make such a sweeping statement. Some of us may be more painfully aware than others of instances where diversification has changed a company from a successful specialist into a floundering, uncoordinated mess. There is much to be said for specialization in underwriting and claims handling.

Similarly, geographical diversification, with all its obvious advantages, is not an unmixed blessing. It may go hand in hand with loss of control, the forerunner of poor underwriting results.

3. *The Chosen Path*

Probably inherent in the above criticism is the lack of a road map which will take the hypothetical company from its present position to the desired one. If a change in portfolios is desired, it must be achieved slowly lest the process of arriving at the goal change the nature of the goal. Gradual changes toward the target will disturb the assumptions (and therefore the efficacy of the solution) less than precipitate rushes toward what will turn out to be a mirage.

Sometimes, too, there just ain't no way to get from here to there.

A FINAL WORD

Although I have indicated my disaffection for the example and for the method as it is presently constituted, it still seems to me that the basic approach, as a way of thinking, has a certain appeal. It may indeed be a foundation for an approach which will work. Professor Ferrari is to be commended for presenting his idea despite, I am sure, his knowledge that his example was subject to much criticism. If we focus upon that central concept, we will have extracted the kernel which I feel sure the author has wished to impart.

DISCUSSION BY ROBERT A. RENNIE*

Professor Ferrari's paper sets forth an interesting application of the Markowitz investment model to the problems of portfolio diversification among a number of lines of property-liability insurance. Apart from certain theoretical difficulties noted below, the paper makes several practical contributions. It helps to eliminate the confusion in property-liability insurance over the concepts of risk and return. The expected return of a line is defined in terms of the future profitability of that line. Risk, on the other hand, is a function of the variability around the expected return. Certainly, insurers have tended in the past to concentrate more on precise measures of return than on exact measures of risk.

The paper also shows, at least by inference, how significant the optimal diversification of lines of insurance can be to operating results and to the risk borne by a property-liability insurer. Too often in the past, management has permitted its relative product mix to follow the course of least resistance as dictated by its marketing demands.

At the theoretical level, Professor Ferrari faced a dilemma. His analysis assumed that historical risk-return trends would continue in the near future. The data in his example were based on a linear extrapolation of the recent combined loss and expense ratios of a large company.

The justification for using combined loss and expense ratios and variances over some past period is, of course, that past performance is believed to be

* Mr. Rennie, who is Vice President — Planning, Finance, and Systems of the Nationwide Insurance Company, was a guest reviewer of Professor Ferrari's paper.

indicative of the future. Such a hypothesis may be true of common stocks, but it is certainly questionable as applied to the property-liability insurance lines, particularly in the case of automobile insurance. Professor Ferrari is aware of these difficulties, and suggests that they may be alleviated by introducing expectations based on subjective judgment into the historical parameters.

I suspect, however, that the nature of these difficulties in property-liability insurance is almost fatal to any simple application of the Markowitz model in this area. The original model assumed that common stock returns and variances are independently distributed over time. In its application to insurance, even if modified historical data were used, there is a clear danger that the analysis will not take into account the tendency of the insurance rating mechanism to adjust over time to the past trends and fluctuations in pure premiums and expenses. Thus, if rates and return have been too low in the past, there is likely to be a more concerted effort to secure adequate rates in the future. There is evidence, for example, that automobile insurance has generated alternating cycles of underwriting gain and loss in the past.

Under these circumstances, the immediate past has little linear relevance to our problem. The insurer is primarily interested in the future return for the various lines of insurance. Professor Ferrari sensed this problem when he stated that the revised historical input would still be deficient "to the extent that future developments are unforeseen or that subjective adjustments do not accurately reflect expectations in a quantified form."

A second theoretical issue is raised by the assumption that the expected return and risk on each line of insurance are single valued, regardless of the proportion of the total portfolio committed to that line of insurance. The return on each line is assumed to be a statistical random variable with a symmetrical probability distribution, and the expected return is a statistical average of that distribution.

This assumption of a single-valued expected return may be valid for common stocks, but it must be questioned when applied to lines of insurance. An institutional investor can change the proportions of securities held in his portfolio at relatively uniform prices. Thus, the expected return for a particular stock will remain the same after the reallocation of his assets. However, an insurer cannot change the proportions of the total portfolio committed to specific lines of insurance and expect either the expected returns or the variances to remain the same after he has changed the relative proportions. In the case of auto insurance, for example, if an insurer consciously reduces the percentage of auto premiums in his portfolio, he will undoubtedly seek to eliminate the marginal risks, thereby increasing his expected return and reducing his variance (risk) for that line.

Thus, if a portfolio selection model is to be developed for property-liability insurance, it must be more complex than the Markowitz model. The expected return for a line of insurance is not single-valued, but is a function of the proportion of the total portfolio committed to that line of insurance and the rate of growth of the total portfolio. Likewise, the variance of return of each line of insurance is not single-valued, but a function of the same variables.

A simple example will illustrate this point. Assume that an insurer has one-half of its portfolio in auto insurance, one quarter in homeowners, and one-quarter in commercial fire. All three lines have an expected return of 3 per cent. The insurer becomes concerned about the future risk in auto insurance, and decides to reduce his auto portfolio to 35 per cent. Homeowners is raised to 35 per cent and commercial fire to 30 per cent.

Under the Ferrari-Markowitz model, the expected return on the portfolio would remain unchanged because the expected return on each line of insurance is single-valued, and not related to the proportion of that line in the total portfolio. In fact, however, the expected returns on auto insurance would undoubtedly rise, and those on homeowners and commercial fire would probably fall if premiums were increased in those lines to maintain total premium writings at a constant level. If the expected return on auto rose to 3.5 per cent, and the return on homeowners and commercial fire both fell to 2.8 per cent, the expected return on the total portfolio would rise above 3 per cent. A similar example relating to the variance (risk) could be cited. Both indicate that the assumption of uniform parameter values for risk and return applying to all portfolio proportions oversimplifies the property-liability insurance model.

Finally, I would suggest that any further work on the Ferrari-Markowitz model might also attempt to incorporate the investment portfolio of an insurer within the model as a means of generalizing its application. Certainly, Mr. Ferrari has written an ingenious and interesting paper, and it merits further exploration and analysis by casualty actuaries.

DISCUSSION BY MATTHEW RODERMUND

Professor Ferrari's paper is scholarly, well-written, interesting, and, not least, courageous. The author is welcomed to the Society as an Associate at the November meeting, but his paper was presented to the Society in May by invitation, with the understanding that it be subject to the same treatment accorded papers submitted by members of the Society, that is, acceptance or rejection by the Committee on Review of Papers and exposure to critical review.

By stating these ground rules, the reviewer feels less guilty about being critical of a guest. In the reviewer's opinion the paper is irrelevant to the present or future state of the property-liability insurance industry. A company employing the techniques Professor Ferrari describes might have great fun (that is, if computer time can be spared), like an individual pursuing a solution to the twelve-balls problem;* but when the task is completed, has anything useful been accomplished?

The paper aims at providing "an initial report on utilization of portfolio selection techniques to suggest the theoretical optimal diversification of lines of insurance written by property and liability insurance companies." In his portfolio selection analysis the author employs a risk and return concept and assumes that "the expected return of a line of insurance is a function of profitability (as measured by loss and expense ratios) and risk is a function of the variability around the expected return." The technique is based on a study, by Dr. Harry Markowitz, of investment portfolio diversification; in this paper, however, the Markowitz approach is applied to a portfolio of lines of insurance, and from the application is developed the Markowitz E-V criterion, E-V being a handy abbreviation for "expected return on the portfolio and its variance."

Professor Ferrari is aware of the limited usefulness of his approach to portfolio selection. In his concluding paragraph he states:

"The application of portfolio selection techniques to property and liability insurance companies [Professor Ferrari refers to relatively sophisticated techniques like the one he has described] has some *interesting* theoretical possibilities as well as *serious* practical limitations." (The italics are the revewer's, and they are intended to emphasize Professor Ferrari's choice of adjectives: *interesting* possibilities but *serious* limitations.) Had Professor Ferrari pursued the practical limitations with the thoroughness with which he pursues the theoretical possibilties, possibly he might not have written the paper. If, on the other hand, he had not set forth the limitations as completely and objectively as he does, the reviewer might not have had a solid base for this discussion.

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* The reviewer will be delighted to describe the twelve-balls problem to any reader who has never heard of it.

Professor Ferrari indicates his own doubts frequently. But he does not explore their implications. When he first introduces the subject of the constraints that had to be recognized in his study, he says:

"In every portfolio selection application, the combined influence of regulation, managerial policy and practical considerations places constraints on the freedom of action." In the same paragraph: "The obstacles to such [freedom of] action result primarily from the maintenance of agency relationships, the insurance consumption pattern of insureds, and competition among insurers." Such obstacles are not inconsiderable; their influence is frequently decisive.

According to Professor Ferrari, "There are two . . . . areas of difficulty that . . . . limit the . . . . application of the Markowitz technique to property-liability company insurance portfolios. The first pertains to the nature of the input assumptions of the model. The second is the uncertain relationship between the Markowitz E-V criterion and the objectives and behavior of non-life insurance companies."

With respect to input assumptions, the author admits that the historical method described by Markowitz, which can be used for quantifying expectations of future return, "is deficient to the extent that it ignores the dynamic aspects of the insurance business. For example, the relative adequacy of future rate levels may differ from that evident in the historical data." Then he makes the surprising suggestion that "this and other similar difficulties can be alleviated by introducing expectations into historical parameters by adjustments based on *subjective judgment.*" (Reviewer's italics.) Thus seat-of-pants wisdom, a traditional tool of the underwriter, is introduced to the computer!

Another input problem: "Even if a property-liability insurer could significantly alter its insurance portfolio, this action could so seriously affect loss and expense ratios as to destroy the assumptions on which the reallocation was based." The reviewer agrees.

As to the relationship of the Markowitz E-V criterion to company objectives and behavior, it is here that Professor Ferrari strikes the most telling blow against the feasibility of his theoretical portfolio selection technique:

"The structure of the insurance business is such that non-life insurance companies can attain great diversification by lines of insurance without conscious marginal risk-return decisions. A large company can be expected to establish variety in its lines of insurance simply because of the nature of the marketing channels, the sheer size of the portfolios, and the complementarity of certain lines of insurance, for example, auto bodily injury liability, auto property damage liability, and auto physical damage."

Three comments:

(1) The large company, as Professor Ferrari implies, hardly needs a theoretical portfolio selection approach. The medium size company and the small company have so many practical considerations to resolve—such as the insurance needs of the area in which they operate, the number and quality of employees capable of servicing these needs, the available capital and surplus, the nature of production arrangements, and the strength of the competition—that a theoretical study of profitability becomes too expensive in terms of the limited influence that the results of the study might have on decision-making.

(2) Professor Ferrari refers to the complementarity of certain lines of insurance. Clearly auto bodily injury liability and auto property damage liability will continue to be offered in a package regardless of the profitability of either coverage with respect to the other. Similarly fire and extended coverage in the dwelling field. Complementarity also has its broader aspects. A company writing all lines but specializing throughout its history in personal lines is not likely either to alter the composition of its personal lines portfolio or to move more strongly into commercial lines as the result of a sophisticated profitability study. Nor, if it specializes in commercial lines, will it adjust the distribution of those lines or move into personal lines in a big way. A company writing a good volume of workmen's compensation business, and jealous of its markets, will not alternately increase and decrease its general liability business, auto fleet liability business, or even commercial fire business, *on the basis of profitability studies.* It may move in or out of certain classes, set new capacity limits, or adjust its agency set-up, but generally it will try to hold its markets. And when it does make portfolio adjustments, considerations other than profitability are apt to be the major factors.

Professor Ferrari undoubtedly knows that the package concept is becoming increasingly prevalent in property-liability insurance, but he may not have realized that a company's portfolio can no longer be pictured adequately by a percentage distribution of the lines of business designated in the annual statement. In his Table 2, Professor Ferrari illustrates the percentage composition of the portfolio of an anonymous company by using annual statement lines of business, and applies the Markowitz E-V criterion to that portfolio. Application of the Markowitz technique produces results (as shown in Tables 3, 4, and 5) that he concedes in some respects are not acceptable. He might have fared better by finding out from his anonymous company the dozen or so main types of policies issued by the company, obtaining the premium distribution for those types of policies, and applying his technique to the rearranged portfolio distribution. The limitations of the technique would not have been obviated, but the results might have been less unacceptable.

Incidentally, the author errs in including treaty reinsurance in the portfolio items subject to his computer techniques. Treaty reinsurance is a conglomeration of all the other lines of business (or types of policies) in the portfolio, and therefore the decision as to expansion or contraction of the reinsurance portfolio runs up against countless variables not at all analogous to those involved in decisions affecting primary lines of business.

(3) Finally, with respect to the relationship of the Markowitz E-V criterion to company objectives and behavior, it seems to the reviewer that a company's principal objectives are first and foremost to provide markets and to grow, and its behavior is based on these objectives. Profitability of the business and the degree of variability of operating results are not to be ignored; indeed, without attention to these the company will cease to exist. But there is an old axiom in the insurance business that any risk (subject to certain qualifications inherent in the business) can be insured; the problem is to obtain the proper premium. Thus, a company's first decisions on its portfolio generally relate to what markets it feels itself prepared to provide, and profitability then becomes a function of the premiums it needs to provide the required market. Obtaining a proper premium, under the present regulatory system, is not always easy; but the solution to *that* problem is hardly furnished by a theoretical portfolio selection technique based on profitability. If there is a ready market for a type of insurance coverage, the responsible company is not likely to de-emphasize that coverage because its returns do not meet expectations. More likely the responsible company will devote its energies to servicing that market more efficiently and at the same time working for higher premiums.

The technical aspects of Professor Ferrari's paper are presented in quite understandable terms, intelligible to the lay actuarial reader. Professor Ferrari emphasizes that profitability is not solely a function of combined loss and expense ratios, but also a function of the variability of such ratios around the expected returns. He further indicates that the risk of a given portfolio is not simply the weighted sum of the variances of the individual lines, but a function of both the risk of each individual line and the correlation of returns between each pair of lines. The rationale is well presented and the technique is neat.

The reviewer has prided himself on a progressive attitude toward the introduction of refined actuarial techniques in the management of our business. Now he seems to be rejecting an interesting, forward-looking technique, and saying nonsense, it's not relevant to our business, it's not practical, it will never get off the ground. Undoubtedly the Markowitz E-V criterion has its uses, and the investment portfolio may be a fruitful area for its employment. But the underwriting portfolio, because of the profound practical considerations that Professor Ferrari lists but does not sufficiently evaluate, is not, in the reviewer's opinion, a proper field for effective use of the technique described.

DISCUSSION BY LEROY J. SIMON

The Ferrari paper is one of the most significant papers we have had in the Casualty Actuarial Society *Proceedings.* It will stand as a landmark to be referred to many, many times in the future by researchers and actuaries alike. The paper touches me in a personal way because for at least six years I have carried a note to myself to attempt to develop a "balanced book approach combining profit with stability." This paper is the first significant step in that direction.

The author is a very strict critic of his own work because he never hesitates to point out the areas in which caution must be exercised. He does not offer his paper as a panacea for management or as a computerized substitute for decision making. He does, however, give us an insight into a very powerful tool and shows how it would operate. Particularly impressive is the fact that he has actually applied the technique in a concrete situation and presents the results for the reader to review. As one would suspect, the results do not say "do this" or "don't do that" but rather point in directions where the company would benefit if they would place additional emphasis or impose some restraints. This may give direction to field force efforts, channel advertising themes, or suggest areas for agency contests which the company may wish to pursue. It is rather doubtful that a manager would examine these results and cut out a given line of business merely because of the indications. As the author points out, there are many more factors to be considered other than the results of a statistical analysis. However, management now has an additional signpost pointing in the proper direction which should be a helpful guide in their decision making process.

We must all keep in mind that many procedures and techniques in the actuarial sphere come about through a process of advancing a rough idea and then polishing it by successive improvements. If each of us was required to take each of his ideas to a point where all practical limitations had been removed before the idea was advanced to his colleagues, I fear that we would have a rather slender *Proceedings* and a rather meager body of actuarial theory. Because he so meticulously sets forth the limitations of the method, the author should not be faulted for not having eliminated them. Nor do I believe he should be expected to withhold his paper from the actuarial fraternity because he realizes he does not have a perfect product with complete solutions to the problems presented.

When the author states, "this and other similar difficulties can be alleviated by introducing expectations into historical parameters by adjustments based on subjective judgment," Mr. Rodermund responds, "Thus seat-of-pants wisdom, a traditional tool of the underwriter, is introduced to the computer!" From the general tone of the review and the punctuation of the reviewer's sentence I can only assume this was meant in criticism. My view is quite different. I would say, "Thus subjective judgment, a tool traditionally felt to be outside of the actuary's domain, has been recognized as being subject, in some measure, to mathematical manipulation." If the use of subjective judgment and degrees of belief were removed from the kit of Bayesian statisticians, some of the most important advances by this group would disappear. The author did well to recognize the ability to use subjective judgment in the technique.

Having worked for both a large company and medium-size or small company, I cannot agree with the idea that small companies can't use the Ferrari approach and big companies don't need it. I do not believe that any well-managed, progressive, forward-looking company, regardless of its size, ever feels that it has enough information upon which to base major management decisions. Companies are always striving for profitable operations and attempting to limit the fluctuation of their experience and the author has presented them with another piece of information that will help in reaching these objectives.

I can see a very interesting use of this technique in the reinsurance field. It could provide a valuable adjunct to the reinsurance consultant if he were able to "cookbook" a company (or even an entire industry) and show a client some of his profitability/variability alternatives. Through some further effort he could then show how the function of reinsurance in controlling some of the variability could allow a better combination of profitability/variability for the client.

In his third closing comment with respect to company objectives, Mr. Rodermund, in my opinion, misses the point. The objective of this method is not to tell a company which natural markets it ought to seek, because each company has an operating philosophy and a base of operations which is fundamental to the operation of that company. Much of this basic philosophy can be reflected in Ferrari's approach as shown by the examples. It is a credit to the technique that it is able to accommodate this type of restriction rather than requiring a company to either write a maximum amount of one line or write none of it.

In summary, I believe that this paper represents one of the landmarks in actuarial work and will be referred to many times over the years as actuaries attempt to quantify the decision making processes in the insurance business. Providing more information and eliminating the guesswork in certain areas can only lead to sounder decisions and a greater degree of confidence in the conclusions reached.

AUTHOR'S REVIEW OF DISCUSSIONS

The author is gratified that his paper on portfolio selection inspired comment by four reviewers of considerable stature in the insurance industry. The large body of literature on portfolio selection is no longer void of an application to the property and liability insurance business and the dialogue contained in the reviews is a welcome supplement to the original effort.

Much of the criticism contained in the reviews was predictable since the same limitations of portfolio selection can be found in the financial literature on securities portfolios. Indeed, many of the problems surrounding practical application were suggested in the paper and the reviewer, in some cases, simply expanded on them.

Rennie seems particularly disturbed over "the assumption that the expected return and risk on each line of insurance are single valued, regardless of the proportion of the total portfolio committed to that line of insurance." This is a valid concern but Rennie did not give the author credit for recognizing this problem. The author states that "Perhaps the most troublesome problem with the input to a portfolio selection model is that the assumptions of risk and return may not hold up if an attempt is actually made to acquire a prescribed portfolio," and then goes on to discuss this admittedly troublesome limitation.

The author agrees with Bondy that one should not resign himself to losses in certain lines, but he would also argue that if the likelihood for improvement of an unprofitable line is remote, then an expectation of loss may be a warranted assumption, at least in the short run or until underlying conditions are changed. Bondy also points out that the nature of the insurance business forbids solutions that prescribe movement in opposite directions of complementary lines. He uses as examples increases in fire and auto bodily injury and decreases in extended coverages and auto property damage. This is certainly a constraint on portfolio flexibility, but it also suggests that long-range planning should include recognition of possible undesirable effects of complementarity of coverages. The inflexibility that arises from packaging, for example, may involve marginal costs that have been largely overlooked.

Some of the reviewers reaffirmed the author's concern about inputs to the portfolio selection model based on historical data. In a recent issue of the *Journal of Finance* a more optimistic view is expresed by Keith Smith of the University of California who, in discussing historical inputs for investment portfolio selection, states:[1]

> Although this admittedly has shortcomings, it would seem to be a lower bound on the abilities of security analysts. That is, if portfolio selection and revision are effective using historically generated inputs, then a real-time system, in which subjective factors are incorporated, should work even better.

Rodermund questions the relevancy of the techniques described in the paper and irreverently likens portfolio selection to the twelve-balls problem. Apparently he fails to recognize that portfolio selection techniques, like management gaming, simulation models, and even recreational mathematics, have usefulness not because they always provide answers but because they improve the decision-maker's or problem-solver's ability to identify crucial variables, to detect the impact of constraints and to understand the relationships between alternative decisions and their possible outcomes. To the extent that this is accomplished, the inescapable real-world decision process is improved. The actuary, because of his quantitative orientation, should take the lead in exploring the relevance to insurance of the new techniques in operations research and computer science.

Rodermund attempts to discredit the technique of quantifying subjective judgment by labeling it "seat-of-pants" wisdom. By venturing such an

[1] Smith, Keith V., "A Transition Model for Portfolio Revision," *Journal of Finance*, Vol. XXII, No. 3, September, 1967, p. 431.

unsophisticated objection he appears to be completely unfamiliar with the wide body of literature dealing with the quantification of subjective elements in decision-making. Rodermund is quick to dissent without considering the advantages of attempting to quantify all or part of the decision process such as (1) the focus of attention on pertinent variables or relations that might otherwise be ignored or treated superficially and (2) the testing of assumptions, expectations, and proposed decisions on an experimental basis.[2] Also, the power and versatility of a computer can be best utilized only after a problem has been described in quantitative language. Thus, quantifying a problem may improve decision-making either directly, by facilitating better understanding of the problem, or indirectly, by allowing the high-speed calculating capacity of a computer to aid in the analysis of complex situations. Naturally, there are limitations but as one author has so aptly stated, ". . . quantitative analysis can lead either wittingly or unwittingly to error, but that does not mean that nonquantitative analyses are any less misleading."[3] It is ironic and unfortunate that Rodermund should present his criticism just a few hours before Sterling T. Tooker was to deliver an address to the Society urging the casualty actuary to "change from the comfort of his traditional role and accept an area of responsibility in which his errors can be both seen and quantified, and often corrected."

The actuary has developed little solid theory that either explains or prescribes decisions regarding the composition of a company's insurance portfolio. The paper on portfolio selection was an attempt to fill partially the gap between theory and practice. It is hoped that the potential of portfolio selection techniques in insurance suggests additional questions to the creative actuary such as:

1. Of what practical significance is the fact that portfolio selection analysis can provide a theoretical justification for insuring unprofitable lines?
2. Is portfolio selection theory, with risk measured by variability of returns, more relevant to company decisions than the actuarial theory of ruin, where risk is viewed as the probability that losses will exceed a certain amount?
3. Does portfolio selection analysis offer the potential for a novel look

[2] For a discussion of these and other advantages see, for example, Holt, Charles C., et al., *Planning Production, Inventories and Work Force* (Englewood Cliffs, New Jersey: Prentice-Hall, 1960) p. 10.

[3] Kahn, Herman, *On Thermonuclear War* (Princeton, New Jersey: Princeton University, 1960), p. ix.

at the provocative proposition of including investment return in the rate-making process?

4. Does a company have objectives with regard to the composition of business in its insurance portfolio, or is the portfolio the result of numerous uncoordinated decisions?

The relevance of the original paper should be clear from the broad theoretical and practical issues it raises and the author commends and thanks the Society for inviting him to present it. This investigation of a technique not traditionally used by the actuary hopefully provides an example of the kind of thinking urged by Mr. Tooker when he told this Society, "we urgently need a broader application of your skills and training to contribute to the success of our business."

LOSS RATIO DISTRIBUTIONS
A MODEL

C. C. HEWITT, JR.

1. INTRODUCTION AND SUMMARY

1.1 Historical

Traditionally in casualty insurance loss ratio distributions have been obtained empirically and often at great expense and with great labor [for the most recent such effort see (13)]. Associated with collecting masses of raw data have been serious problems of fitting such data [see (8), (12) and 14)]. The end-product of all of these efforts has been non-analytical; and of value only for use in linear retrospective rating and as a rough guide to loss ratio distributions by size.

1.2 An Analytical Model

In this author's review (11) of (13) he indicated the successful fitting of a mathematical model—the gamma distribution—to actual loss ratio distributions. Also, there was indicated a relationship among the significant parameters for loss ratio distributions at various premium sizes.

1.3 Purpose and Results of this Paper

In this paper are set forth some important mathematical properties of the gamma distribution (Chapter 2) including the very important characteristic of *reproductivity* and *divisibility.* In most instances the development of formulas and lemmas is left to the reference texts, or the reader. The gamma distribution is applied directly to loss ratio distributions as a model (Chapter 3) and a single parameter form is asserted.

In Chapter 4 the method of fitting actual data is explained and the goodness of the fit is discussed. A relationship among parameters at various premium sizes is also asserted. As a corollary it becomes evident that for actual data, loss ratio distributions for larger premium sizes are not equivalent to loss ratio distributions that might have been obtained by taking random samples from smaller premium sizes. An attempt is made to account for this phenomenon.

Finally (Chapter 5) the utility of the new model is discussed for:

(1) Linear retrospective rating.
(2) Non-linear retrospective rating.
(3) Competitive "retro" dividend plans.

1.4 Prospectus

Although significant results are obtained within this paper, the implications go far beyond the answers. For example, the gamma distribution as a loss ratio model for larger risk premium sizes must be the synthesis of:

(1) Distributions of a single loss,
(2) Distributions of occurrence of one or more losses, and
(3) Inherent risk heterogeneity.

It would be interesting to see this analyzed further; such analysis would undoubtedly explain why the goodness-of-fit tests fail for smaller premium sizes. Hopefully, then, this paper will not be an end but merely a beginning.

2. THE GAMMA DISTRIBUTION

2.1 The Gamma Function

2.11 The (Complete) Gamma Function

(a) Definition:

$$\Gamma(r) = \int_0^\infty x^{r-1} e^{-x} dx \; ; \; [r>0] \qquad \text{............(2.111)}$$

(b) The (complete) gamma function has the recursive property:

$$\Gamma(r+1) = r\Gamma(r) \qquad \text{............(2.112)}$$

(c) If r is integral,

$$\Gamma(r+1) = r! \qquad \text{............(2.113)}$$

(d) The (complete) gamma function has a *minimum* when r is approximately 1.4616; the minimum is approximately 0.8856.

As r approaches zero, or increases without limit, the (complete) gamma function increases without limit.

(e) $\Gamma(r) \sim \sqrt{2\pi r}\, e^{-r}\, r^{r-1}$
for large r ....(2.114)

(f) For intermediate, non-integral values of r, use may be made of the recursive property (2.112) and published tables [(1), p. 316].

(g) $$\Gamma(\tfrac{1}{2}) = \sqrt{\pi} \qquad \text{............(2.115)}$$

(h) $$\Gamma(r) = \lim_{n\to\infty} \frac{n!\, n^{r-1}}{r(r+1)(r+2)\ldots.(r+n-1)} \qquad \ldots\ldots.(2.116)$$

[(3), p. 697]

(i) $$\frac{1}{\Gamma(r)} = re^{\gamma r} \prod_{k=1}^{\infty} \left(1 + \frac{r}{k}\right) e^{-\frac{r}{k}} \qquad \ldots\ldots\ldots\ldots(2.117)$$

where γ is the Euler-Mascheroni constant.

$$\gamma = \lim_{n\to\infty} \left(\sum_{k=1}^{n} \frac{1}{k} - \log_e n\right) = -\int_0^\infty e^{-x} \log_e x dx,$$

$\gamma \sim 0.5772157$ [(3), p. 697]

(j) The k^{th} derivative of $\Gamma(r)$ is:

$$\Gamma^{(k)}(r) = \int_0^\infty x^{r-1} (\log_e x)^k e^{-x}\, dx \qquad \ldots\ldots\ldots\ldots(2.118)$$

2.12 *The Incomplete Gamma Function*

(a) Definition:

$$I(u,p) = \frac{\int_0^{u\sqrt{p+1}} x^p e^{-x} dx}{\Gamma(p+1)}; \; (p + 1 = r, \therefore p > -1) \qquad \ldots\ldots\ldots\ldots(2.121)$$

where $u = \dfrac{x_0}{\sqrt{p+1}}$ ....(2.121a)

(b) In the gamma *distribution* a scale parameter, a, is introduced; in this case:

$$u = \frac{ax_0}{\sqrt{p+1}}; \; (a > o) \qquad \ldots\ldots\ldots\ldots (2.121b)$$

(c) Use may be made of published tables, (4). Also see [(5) p. 223] for adaptations from other published tables.

(d) For p near -1, values of the incomplete gamma function may be approximated to a desired degree of accuracy by iteration of the following series [(4, p. xv]:

$$I(u,p) = \frac{\zeta^{(p+1)}}{\Gamma(p+2)} \left\{ 1 - \frac{\zeta(p+1)}{1!(p+2)} + \frac{\zeta^2(p+1)}{2!(p+3)} - \frac{\zeta^3(p+1)}{3!(p+4)} + \ldots \right\}$$

where $\zeta = u\sqrt{p+1}$ ....(2.122)

2.2 *The Gamma Distribution*

2.21 *Basic Properties*

(a) Definition:

$$\Gamma_{a,r}(x) = \frac{a^r}{\Gamma(r)} x^{r-1} e^{-ax}; \begin{bmatrix} x \geqq o \\ a > o \\ r > o \end{bmatrix} \qquad \text{............(2.211)}$$

(b) The gamma distribution is a special case of the more general Pearson Type III distribution:

$f(x) = A(x - \mu)^{r-1} e^{-a(x-\mu)}; \ [\, x > \mu]$

when $\mu = o$. ....(2.212)

(c) a is the trivial scale parameter [(2) II, p. 46]; r is the important parameter.

(d) The *mode* of $\Gamma_{a,r}(x)$ occurs at:

$$\frac{r-1}{a} \qquad ; (r > \ ) \qquad \text{............(2.213)}$$

(e) The characteristic function is

$$\left(1 - \frac{it}{a}\right)^{-r} \qquad \text{............(2.214)}$$

(f) The exponential distribution:

$$ae^{-ax} \qquad \text{............(2.215)}$$

is a special case of $\Gamma_{a,r}(x)$

when $r = 1$

(g) The gamma distribution is the continuous analogue of the negative binomial [(2) II, p. 10].

(h) Estimators [(6), p. 39.]:

If r is given, the maximum likelihood estimator for a is:

$$\hat{a} = \frac{r}{\bar{x}} \ ; \bar{x} = \text{sample mean} \qquad \text{............(2.216)}$$

For large $\underline{n}$, the p. d. f. of $\underline{\hat{a}}$ approaches normality with mean $\underline{a}$ and variance $\frac{a^2}{nr}$. Also the p. d. f. of $\sqrt{r} \log \hat{a}$ approaches normality with mean $\sqrt{r} \log a$ and variance $\frac{1}{n}$.

2.22 *Reproductivity & Divisibility (Theoretical)*

(a) Convolutions:

If x_1 and x_2 are *independent* with p. d. f. Γ_{a,r_1} and Γ_{a,r_2} respectively, then

$$X = x_1 + x_2$$

is gamma-distributed with p. d. f. Γ_{a,r_1+r_2}

[(2) II. p. 46, (6), p. 121, and (5), p. 225]

Similarly $\sum x_i$ under the same conditions would have p. d. f. $\Gamma_{a,\Sigma r_i}$. This is often expressed:

$$\Gamma_{a,r_1} * \Gamma_{a,r_2} * \ldots * \Gamma_{a,r_n} = \Gamma_{a,\sum_1^n r_i} \qquad \text{............(2.221)}$$

Consequently the sum of the values of a random sample of size n from a gamma-distributed population,

$$z = \sum_1^n x_i \text{ has a p. d. f. } \Gamma_{a,nr}.$$

(b) Divisibility:

The "inverse" of this reproductive property of the gamma distribution is infinite divisibility, i.e. $\Gamma_{a,r}$ is the distribution of the sum of n independent random variables with a common p. d. f. $\Gamma_{a,\frac{r}{n}}$

[(2) II, p. 173]

2.23 *Exponential Polynomials — Moments of* x *and* e^x

(a) Functions of x of the form:

$$Ax^n e^{-bx}\, \Gamma_{a,r}(x)\,; \begin{pmatrix} n > -r \\ b > -a \end{pmatrix}$$

are themselves gamma-distributions, $\Gamma_{a+b,r+n}$.

(b) For $\Gamma_{a,r}$

$$E(x^n e^{-bx}) = \left(\frac{a}{a+b}\right)^r \frac{\Gamma(r+n)}{(a+b)^n\,\Gamma(r)} \qquad \text{............(2.231)}$$

(c) Thus the k^{th} moment of x about the origin is:

$$E(x^k) = \frac{(r+k-1)(r+k-2)\ldots.(r)}{a^k} \qquad \text{............(2.231a)}$$

$$E(x) = \frac{r}{a} \qquad \text{............(2.231b)}$$

$$E(x^2) = \frac{(r+1)r}{a^2} \qquad \text{............(2.231c)}$$

etc.

and $$D^2(x) = E(x^2) - \overline{E(x)}^2 = \frac{r}{a^2} \qquad \text{............(2.231d)}$$

(d) Similarly, the k^{th} moment of e^x is:

$$E(e^x)^k = \left(\frac{a}{a-k}\right)^r \quad ; (a > k) \qquad \text{............(2.231e)}$$

and $$E(e^x) = \left(\frac{a}{a-1}\right)^r \quad ;(a > 1) \qquad \text{............(2.231f)}$$

etc.

(e) This latter situation is helpful if the logarithm of a variable, y, is gamma-distributed, i.e.,

if, $\underline{x = \log_e y}$ has p. d. f. $\Gamma_{a,r}(x)$

Since $\underline{y = e^x}$, the moments of y are given in (2.231e). [See (10)]

3. GAMMA-DISTRIBUTION AS A MODEL FOR LOSS RATIO DISTRIBUTIONS

3.1 *Definition of terms:*

L = actual (risk) losses (\$)

ϵ = expected loss ratio

P = (risk) premium (\$)

r^1 = actual (risk) loss ratio $= \dfrac{L}{P}$

$$R = \frac{\text{actual loss ratio}}{\text{expected loss ratio}} = \frac{r^1}{\epsilon}$$

3.2 *The distribution form of R:*

If R is gamma-distributed, its p. d. f. would take the form [see (2.211)]:

$$f(R) = \frac{a^r}{\Gamma(r)} R^{r-1}\, e^{-aR}\, ; \begin{pmatrix} a > o \\ r > o \end{pmatrix} \qquad \text{............(3.21)}$$

with $E(R) = \dfrac{r}{a}$, but if *total actual losses* balance with *total expected losses,*

$E(R) = 1$ by definition,

and $a = r$; therefore

$$f^*(R) = \frac{r^r}{\Gamma(r)} R^{r-1}\, e^{-rR} \qquad \text{............(3.21*)}$$

with r as its own scale parameter.

3.3 *The distribution form of r′:*

From (3.21), (3.21*) and $r' = \epsilon R$
it follows that:

$$g(r') = \frac{\left(\frac{a}{\epsilon}\right)^r}{\Gamma(r)} r'^{r-1} e^{-\frac{a}{\epsilon} r'} \qquad \text{............(3.31)}$$

and

$$g^*(r') = \frac{\left(\frac{r}{\epsilon}\right)^r}{\Gamma(r)} r'^{r-1} e^{-\frac{r}{\epsilon} r'} \qquad \text{............(3.31*)}$$

with $\frac{r}{\epsilon}$ as the scale parameter.

3.4 *The distribution form of L:*

From (3.31), (3.31*) & $L = r'P$
it follows that:

$$h(L) = \frac{\left(\frac{a}{\epsilon P}\right)^r}{\Gamma(r)} L^{r-1} e^{-\left(\frac{a}{\epsilon P}\right)L} \qquad \text{............(3.41)}$$

and

$$h^*(L) = \frac{\left(\frac{r}{\epsilon P}\right)^r}{\Gamma(r)} L^{r-1} e^{-\left(\frac{r}{\epsilon P}\right)L} \qquad \text{............(3.41*)}$$

with $\frac{r}{\epsilon P}$ as the scale parameter.

3.5 *The distribution forms of random samples:*

If *random* samples of size nP are taken (or go to make up a sample of size P) from the risk-population, it follows from the reproductivity and infinite divisibility property of the gamma distribution (see Section 2.22) that L_n, the losses in such random samples, are distributed

$$h(L_n) = \Gamma_{\frac{a}{\epsilon P},\, nr} \qquad \text{............(3.51)}$$

and

$$h^*(L_n) = \Gamma_{\frac{r}{\epsilon P},\, nr} \qquad \text{............(3.51*)}$$

where $(n > o)$

Since $r'_n = \frac{L_n}{nP}$, it follows that

$$g(r'_n) = \Gamma_{\frac{na}{\epsilon},\, nr} \qquad \text{............(3.52)}$$

and

$$g^*(r'_n) = \Gamma_{\frac{nr}{\epsilon}, nr} \qquad \text{............(3.52*)}$$

Also

$$R_n = \frac{r'_n}{\epsilon}, \text{ therefore}$$

$$f(R_n) = \Gamma_{na,nr} \qquad \text{............(3.53)}$$

and

$$f^*(R_n) = \Gamma_{nr,nr} \qquad \text{............(3.53*)}$$

3.6 *Fitting loss-ratio data to the Gamma-distribution:*

(a) As r increases $\Gamma_{a,r}$ approaches the form of the normal distribution.

(b) Furthermore the function:

$$r'g(r')$$

is itself of the gamma distribution form [see 2.23(a)]

$$\Gamma_{\frac{a}{\epsilon}, r+1}$$

(c) Also for *fixed P* (premium size) $r'g(r')$ is proportional to:

$$Lh(L)$$

(d) This combination of (1) increased "normality", (2) primary interest in the distribution of amount ($'s) of loss (as opposed to number of risks), and (3) the convenience of "generated" gamma-distributions of amounts of loss from gamma-distributions by number of risks suggests the following method for determining the important parameter, r:

(1) Use $\bar{r}'$ (= sample mean loss ratio) as an estimator for r'.

(2) Use $\bar{r}''$ (= sample mean loss ratio-weighted by amount of loss in each loss ratio interval—rather than by number of risks) as an estimator for:

$$r'' = E(r'g(r')) = E(\Gamma_{\frac{a}{\epsilon}, r+1})$$

(3) Then from (2.231b)

$$\bar{r}' = r' = \frac{\hat{r}}{a}\epsilon$$

$$\bar{r}'' = r'' = \frac{\hat{r}+1}{a}\epsilon$$

$$\frac{\bar{r}''}{\bar{r}'} = \frac{\hat{r}+1}{\hat{r}}, \text{ and}$$

$$\hat{r} = \frac{\bar{r}'}{\bar{r}'' - \bar{r}'} \qquad \text{............(3.61)}$$

and $\hat{r}$ can be used as an estimator for r.

(e) This uniquely determines the distribution—$f^*(R)$—for a particular premium size. The other distributions in the (*) family, (3.31*) and (3.41*), are known if ϵ is known. Furthermore the non-starred distributions, (3.21), (3 31) and (3.41) can then be determined by using (2.216) to determine a.

(f) From (2.231a) it follows that for $g(r')$:

$$\frac{E(x^{k+1})}{E(x^k)} = \frac{(r+k \ldots . (r)\epsilon^{k+1}}{a^{k+1}} \cdot \frac{a^k}{(r+k-1) \ldots . (r)\epsilon^k}$$
$$= \frac{r+k}{a}\epsilon$$

which, for $k = 1$, gives another way of obtaining $\bar{r}''$.

(g) In general the approach for estimating r which is described in this section is particularly appropriate for highly-skewed loss ratio distributions, since it emphasizes higher-moment-weighted distributions that are more nearly normal.

However, great care must be exercised in fitting higher-moment means to small samples because of the increased effect of infrequent and often erratic large losses and larger loss ratios upon the estimators.

4. FITTING THE GAMMA-DISTRIBUTION TO ACTUAL LOSS-RATIO DISTRIBUTIONS

4.1 The Data

(a) Fortunately, data of unsual homogeneity for its large amount was obtained by using workmen's compensation insurance experience for the single state of California—now our largest state, not only in population, but also in workmen's compensation premium volume. The data is contained in a special unpublished report of the California Inspection Rating Bureau dated January 31, 1963 entitled "California Experience Rating Statistics—Series II—By Interval of Subject Premium Loss Ratio."

(b) The raw data is tabulated in a series of fourteen exhibits by subject premium size:

Exhibit	Subject Premium	Number of Risks
A	Less than $500	2,946
B	$ 500 - $ 749	3,126
C	750 - 999	2,889
D	1,000 - 1,499	3,718
E	1,500 - 2,499	3,576
F	2,500 - 4,999	2,891
G	5,000 - 7,499	939
H	7,500 - 9,999	465
I	10,000 - 14,999	454
J	15,000 - 24,999	316
K	25,000 - 49,999	256
L	50,000 - 99,999	91
M	More than $99,999	55
N	Total	21,722

and is for policies effective in the first nine months of 1958. An extract of the significant portions of the data contained in Exhibit K is shown in Table 1 in the Appendix as an example.

(c) Determining the estimator for r:
Using (3.61) and the data in Table 1 for illustration of the method:

$$\bar{r}' = .578 \text{ (in Table 1, } \frac{\Sigma\,(5)}{\Sigma\,(3)}\text{)}$$

$$\bar{r}'' = .931 \text{ (in Table 1, } \frac{\Sigma\,(5) \times (6)}{\Sigma\,(5)}\text{)}$$

$$\hat{r} = \frac{0.578}{0.931 - 0.578} = 1.639 \text{ [rounded to 1.6 for use as an entry in (4)]}$$

For other premium sizes:

Subject Premium Interval	$\hat{r}$ (rounded)
$5,000 - $7,499	0.45
7,500 - 9,999	0.65
10,000 - 14,999	0.85
15,000 - 24,999	1.3
25,000 - 49,999	1.6
50,000 - 99,999	2.9
More than $99,999	6.2

(d) Goodness of Fit:

In some instances the raw data was adjusted for "contamination," but such changes were minor. Despite the broadness of some of the premium intervals Chi-square tests were met for all of the premium sizes in (c). An example is given in Table 2 in the Appendix.

Premium sizes below $5,000 can *not* be made to satisfy Chi-square tests even with minor smoothing.

(e) Relationship between r and Premium Size:

The results set forth in (c) suggest that there should be a relationship between premium size and the key gamma-distribution parameter, r. Although the goodness of fit for premiums below $5,000 leaves something to be desired, estimators for r were calculated and a logarithmic curve of the form:

$$\log r = \alpha + \beta \log P \qquad \text{............(4.11)}$$

was fitted by least squares. The results are tabulated below:

Average Subject Premium Size (in interval)	$\hat{r}$ Raw	r Using (4.11)*
$ 296	.038	.044
628	.081	.079
869	.096	.102
1,223	.132	.132
1,924	.187	.188
3,481	.326	.298
6,050	.472	.457
8,652	.627	.601
12,265	.868	.787
18,944	1.336	1.104
33,455	1.639	1.710
68,758	2.898	2.985
220,786	6.145	7.362

*(For logarithms to base 10, $\alpha = -3.264$ and $\beta = 0.773$)

The use of average premiums as representative of an entire premium interval is crude particularly in the $100,000-and-over interval. Nevertheless a relationship indicating some predictability does exist. A hasty application of the above methods to the new Table M raw data [see (13)] supports both the use of the gamma-distribution as a model for loss ratio distributions and the use of a logarithmic curve to determine $\acute{r}$.

(f) Reproductivity and Divisibility (Actual):
In Section 3.5 it was shown that, if r is the key parameter for premium size P, then a larger $(n > 1)$ or smaller $(o < n < 1)$ *random* sample from the same risk-population would have the key parameter nr. If the *actual* loss ratio distributions discussed above followed a random sampling pattern then from (4.11)

(A) $\log r = \alpha + \beta \log P$

(B) $\log nr = \alpha + \beta \log nP$

but subtracting (A) from (B)

(C) $\log \frac{nr}{r} = \beta \log \frac{nP}{P}$

(D) $\beta = 1$

However, β was found to be *0.773* for the California data (and logarithm-base 10). Thus, it can be inferred that larger-risk loss ratio distributions can not be obtained by a randomized pyramiding of smaller-risk loss ratio distributions and vice versa. Putting it bluntly—for loss ratio distribution purposes—*two \$50,000 risks don't make a \$100,000 risk.* Nor is a \$100,000 risk for one year the same as a \$50,000 risk for two years.

This result challenges formerly-used methods of arriving at loss ratio distributions for large risks [see (9)]. Also challenged is the present method of equating insurance charges for three-year retrospective rating plans with insurance charges for a one-year plan on a risk three times as large. Similarly, there would appear to be some inaccuracy in calculating the insurance charges (contained in the basic premium) in Retrospective Rating Plan D for premium sizes 50% of, and 200% of, the estimated standard premium by equating such charges to those of risks one-half, and twice, the estimated size of the risk in question.

Since the insurance charge for larger premium sizes is a small portion of the total premium and, since the margin of error in previous and current methods of computing insurance charges would also seem to be small, it is doubtful if any great harm has been, or is being, done by the methods here impugned.

(g) Size Characteristics of Actual Loss Ratio Distributions:

Let r_n be the value of r for a sample of size nP as determined by (4.11), i.e., from actual loss ratio distributions. Then

(A) $\log r = \infty + \beta \log P$

(B) $\log r_n = \infty + \beta \log nP$

(C) $r_n = n^{\beta} r$

On the other hand, if ρ_n is the value of r for a random sample of size nP:

(D) $\rho_n = nr$

From (C) and (D) it follows that:

(E) $\rho_n = n^{(1-\beta)} r_n$

For $o \leq \beta < 1$,

where $n > 1$, $\rho_n > r_n$ ....(4.12a)

where $o < n < 1$, $\rho_n < r_n$ ....(4.12b)

But the variance of $f^*(R)$ [see (2.231d) and (3.21*)] is:

$$\frac{1}{r}$$

Thus a loss ratio distribution of larger risk size obtained by pyramiding a loss ratio distribution of smaller risk size on a *random* basis has a smaller variance $\frac{1}{\rho_n}$ than the *actual* loss ratio distribution $\frac{1}{r_n}$.

There are a number of possible explanations for this conclusion. One such explanation, which would seem logical, would run as follows:

Risk	(1) Exposure (Units)	(2) Frequency (Units)	(3) Severity (Units)	(4) Premium (Units) (1) × (2) × (3)
A	1	1	1	1
B	1	1	2	2

When the premium for Risk A is made equal to that of Risk B by doubling the exposure units of Risk A, it seems clear that the variance of loss ratios for double-units of Risk A would be less than the variance of loss ratios for single-units of Risk B. This is so because the severity for Risk A is only one-half the severity of Risk B.

It is clear, *a fortiori,* that, all other things being equal, risks with larger severities would be in the larger premium size intervals.

5. UTILIZING THE MODEL

5.1 *Linear Retrospective Rating* [see (12) pp. 52-56]

Let S_o = insurance charge for loss ratios exceeding R_o

Then

$$S\hat{o} = \frac{\int_{R_o}^{\infty} (R - R_o) f^*(R) dR}{\int_{o}^{\infty} R f^*(R) dR} \qquad \text{............(5.11)}$$

but

$$\int_{o}^{\infty} R f^*(R) dR = E(R) = 1,$$

and

$$\int_{o}^{\infty} f^*(R) dR = 1$$

therefore,

$$S_o = 1 - \int_{o}^{R_o} R f^*(R) dR - R_o [1 - \int_{o}^{R_o} f^*(R) dR] \qquad \text{............(5.11a)}$$

but

$$\int_{o}^{R_o} f^*(R) dR = I(u_o, p)$$

$$\int_{o}^{R_o} R f^*(R) dR = I(u_1, p + 1)$$

and

$$\int_{o}^{R_o} R^i f^*(R) dR = I(u_i, p + i)$$

where

$$r = p + 1$$

$$u^i = \frac{rR_o}{\sqrt{r + i}}$$

see section 2.12 a & b

Thus

$$S_o = 1 - I(u_1,p+1) - R_o[1 - I(u_o,p)] \qquad \text{............(5.116)}$$

Similarly if S_o' = insurance saving from loss ratios less than R_o

$$S_o' = \frac{\int_0^{R_o} (R_o - R)f^*(R)dR}{\int_0^{\infty} Rf^*(R)dR} \qquad \text{............(5.12)}$$

$$S_o' = R_o \int_0^{R_o} f^*(R)dR - \int_0^{R_o} Rf^*(R)dR \qquad \text{............(5.12a)}$$

and

$$S_o' = R_o I(u_o,p) - I(u_1,p+1) \qquad \text{............(5.12b)}$$

also

$$S_o' = S_o + R_o - 1 \qquad \text{............(5.13)}$$

The advantages of being able to compute insurance charges (or savings) by a relatively simple formula which requires only one parameter, when the parameter is a simple logarithmic function of premium size, are many and obvious. It should be sufficient to point out that parameterization of loss ratio distributions would eliminate huge tables of ratios and charges, would lend itself to computerization and would permit different and more appropriate insurance charges among various lines of insurance, geographical territories, classifications of risk, and even between one year and the next.

5.2 *Non-Linear Retrospective Rating*

Inflexibility with respect to arriving at insurance charges is not the only rigidity imposed by linear forms of retrospective rating. Linear retrospective rating implies a minimum premium and a maximum premium with the intermediate values expressed as a linear function of risk losses, i.e.,

$$P = c_1 R + c_0 \text{, but}$$

$$c_1 = \frac{\overline{P} - \underline{P}}{\overline{R} - \underline{R}} \text{ and } c_0 = \frac{\overline{R}\underline{P} - \underline{R}\overline{P}}{\overline{R} - \underline{R}}$$

where $\underline{P}$ = minimum premium

$\overline{P}$ = maximum premium

With $\underline{R}$ and $\overline{R}$ corresponding to the respective P's

These requirements limit the insured and insurer in their choice of values for linear plans.

Why not

$$P = c_1 R^{n_1} e^{-b_1 R} + c_2 R^{n_2} e^{-b_2 R} + \ldots . \qquad ?$$

As long as $n_i > -r$ and $b_i > -r$

[see Section 2.23 (a)], insurance charges are calculable with the knowledge of r.

Of course there are common sense restrictions on

$$P = F(R)$$

such as $\quad F'(R) \geq o \quad ; \quad (o < \underline{R} \leq R \leq \overline{R})$

[see (7)].

5.3 *Competitive "Retro" Dividend Plans*

Finally, it now becomes possible to design a retrospective dividend scale to be most competitive for the most desirable risks. This is not the same as saying most competitive for risks with a zero loss-ratio, since for larger premium sizes there are very few risks with near-zero loss ratios. Rather, if a competitor's dividend formula produces a net premium:

$$P = c_1 R + c_o \quad ; \quad (\underline{P} \leq P \leq \overline{P})$$

then choose a

$$P' = F(R) \qquad \text{such that}$$

$$\psi = \int_{\underline{R}}^{\overline{R}} [c_1 R + c_o - F(R)] f^* R dR + (\underline{P} - \overline{P'}) I(\underline{u},p) + (\overline{P} - \overline{P}')[1 - I(\bar{u},p)]$$

is a maximum. Of course, the correct insurance charges must be made. (The expression for ψ is a deliberate over-simplification since $\underline{\mathrm{R}}$, $\underline{\mathrm{R}}'$, $\overline{\mathrm{R}}$, and $\overline{\mathrm{R}}'$ will almost certainly not be equal. However, the principle is the same).

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CALIFORNIA WORKMEN'S COMPENSATION
EXPERIENCE RATING STATISTICS
Series II - By Interval of Subject Premium Loss Ratio
Exhibit K - Subject Premium 25,000 - 49,999

Policies Effective 1/1/58 - 9/30/58

(1) Subject Premium Loss Ratio Interval	(2) # of Risks	(3) Subject Premium	(4) Avg. Subj. Prem. Size (3) ÷ (2)	(5) Incurred Losses	(6) Loss Ratio (Subj. Prem.) (5) ÷ (3)
.000	2	67,253	33,627	0	.000
.001 - .199	44	1,430,618	32,514	187,072	.131
.200 - .299	33	1,069,378	32,405	262,067	.245
.300 - .399	26	884,548	34,021	313,728	.355
.400 - .499	29	909,445	31,360	406,078	.447
.500 - .599	24	811,911	33,830	443,257	.546
.600 - .699	17	577,790	33,988	373,189	.646
.700 - .799	22	808,761	36,762	601,346	.744
.800 - .899	18	623,852	34,658	530,364	.850
.900 - .999	10	353,662	35,366	338,464	.957
1.000 - 1.249	16	529,073	33,067	586,367	1.108
1.250 - 1.499	8	280,824	35,103	392,025	1.396
1.500 - 1.749	2	65,603	32,802	106,069	1.617
1.750 - 1.999	1	27,352	27,352	48,252	1.764
2.000 - 2.999	2	70,361	35,181	183,178	2.603
3.000 - 3.999	2	54,063	27,032	178,850	3.308
Total	256	8,564,494	33,455	4,950,306	.578

APPENDIX
TABLE 2

CALIFORNIA WORKMEN'S COMPENSATION
EXPERIENCE RATING STATISTICS
Series II - By Interval of Subject Premium Loss Ratio

Exhibit K - Subject Premium 25,000 - 49,999

Policies Effective 1/1/58 - 9/30/58

(1) Subject Premium Loss Ratio Interval	Number of Risks (2) Actual	(3) Theoretical	(4) χ^2 $\frac{[(2) - (3)]^2}{(3)}$
.000 - .199	46	50	0.32
.200 - .299	33	32	.03
.300 - .399	26	29	.31
.400 - .499	29	26	.35
.500 - .599	24	22	.18
.600 - .799	39	34	.74
.800 - .999	28	23	1.09
1.000 & up	31	40	2.03
Total	256	256	5.05

For 7 degrees of freedom:

Level	χ^2
pr= 0.95	2.17
pr= 0.05	14.07

DISCUSSION BY CHARLES A. HACHEMEISTER

The only way in which a working analytical model for retrospective rating will ever be produced is by initially proposing a model which can be criticized and improved. It is extremely difficult to produce a finished working model without having a jumping off place for one's thoughts. We should be grateful indeed to Mr. Hewitt for having given us this paper.

This review will be divided into two main parts. The first is a general discussion of models for loss ratio distributions with particular reference to Table M and the gamma distribution. The second comments on some of the technical aspects of the paper.

MODELS FOR LOSS RATIO DISTRIBUTIONS

We are all aware of the deficiency of assuming that all insureds developing the same expected losses or premium should be subject to the same insurance charges. A large clerical risk and a small oil well drilling risk can produce the same expected losses, but the loss ratio of the clerical risk is much more stable than that of the oil well drilling risk. However, it is not difficult to understand why different tables of insurance charges do not now exist.

If one were to take the time to read through the recent paper "The 1965 Table M"[1] the reason would be eminently clear. It was in his review of this paper that Mr. Hewitt first commented on the difficulties surrounding the use and generation of Table M type statistics. At that time he proposed a program of constructive steps to be taken to do away with these difficulties. The essence of the program was the construction of a mathematical model of the family of loss-ratio distributions. More particularly, the model would contain parameters which would vary by different types of insureds (i.e. sub-line of insurance, class, geographical location, time and size).

The current paper under review expands upon a model first mentioned by the author in his review of "The 1965 Table M." The model was developed by fitting gamma distributions to data in a California Inspection Rating Bureau report containing loss ratio distributions for California experience-rated insureds grouped by premium size. This procedure implies that the composite distribution of loss ratios for all insureds developing the same premium is a gamma distribution. This idea is comparable to the current use of a common Table M for all classes of business being retro-rated.

[1] Simon, LeRoy J., *PCAS*, Vol. LII

A comparison of the parameters of the distributions for different premium sizes showed an apparent theoretical discrepancy. In particular, the ratio of the parameter r to premium size should remain constant. This ratio did not remain constant; it decreased as premium size increased. The reason was correctly assessed to be the different mixes of frequency and severity in different premium size groups.

In the light of this result, the author challenges "the present method of equating insurance charges for three-year retrospective rating plans with insurance charges for a one-year plan on a risk three times as large," in addition to 50% and 200% quotes for Plan D. Let us carry this one step further. If we take any insured and assume that he has just enough exposure to be eligible for retrorating, then his insurance charges will be calculated from the smallest premium group. If then we calculate his insurance charge from the premium group indicated by any increase in exposure, the same problem arises as in the case of the one to three year comparison. If we have assumed that the variability of loss ratios implicit within the smallest premium group is appropriate for this insured, then the variability within the large premium group is too large because of the greater predominance of high severity insureds in this group. In spite of the fact that the larger premium group may exhibit a smaller variability than the smaller group, that variability is larger than would be expected. This problem is independent of whether the gamma distribution is the model or not. Whenever insurance charges are calculated from a model (Table M, gamma, or otherwise) wherein all insureds with the same premium are grouped together in spite of their severity, this problem will arise. If a different criterion, say, perhaps, a combination of class severity and premium were used, more consistent insurance charges could be calculated.

There is some question whether the gamma distribution is an admissible model for loss-ratio distributions, even without considering the fit. Loss ratios are defined continuously over all positive numbers. In addition, zero loss ratios are not only possible, but occur frequently for small premium sizes. Hence the probability of a zero loss ratio must be greater than zero in a realistic model. A model defined strictly in terms of a continuous probability density function, such as the gamma distribution, cannot supply this greater than zero probability mass since $\int_o^o f(x)\,dx = o$, by definition,[2] for any probability density function. At the time Mr. Hewitt

[2] Riemann integration.

first discussed his gamma distribution model, he mentioned that a good fit was obtained for large premium groups. However, for premium groups below $5,000, the fit was unsatisfactory. He ascribed this to the presence of zero loss ratios. The inability of the gamma distribution to properly handle zero loss ratios is certainly a major contributor to the unsatisfactory fit.

Perhaps we need to look one step deeper into the process to overcome this difficulty. Maybe the model for loss ratios should be composed of frequency and severity elements appropriately mixed. One such possibilty would be Poisson frequency, $g(i)$, and gamma severity, $h(x)$, yielding a composite distribution of loss ratios,[3] $f(y)$:

$$f(y) = \begin{cases} e^{-\lambda} & , y = o \\ \sum_{i=1}^{\infty} \dfrac{\lambda^{(r+1)i} r^{ri} y^{ri-1} e^{-\lambda(ry+1)}}{i\,!\,\Gamma(ri)} & , y > o \end{cases}$$

FITTING THE GAMMA DISTRIBUTION

The procedure outined and used in the paper purports to use a more stable procedure than moments by taking advantage of the less skewed moment distribution, $r'g(r')$. Unfortunately, it reduces to one of fitting by moments, since by definition $E_{g(r')}(r'^2) = E_{g(r')}(r') \cdot E_{r'g(r')}(r')$

It is preferable to use maximum likelihood estimates where possible since they are asymptotically minimum variance unbiased estimators. On first blush the maximum likelihood estimates of the gamma parameters look intractable. However, with the aid of a tabulated function of r the solution is straightforward. The maximum likelihood estimate of a is $\frac{\hat{r}}{x}$ even if r is not known. The maximum likelihood estimate of r is the value of $\hat{r}$ such that $\Psi(\hat{r}) - Ln(\hat{r}) = \overline{Lnx} - Ln\bar{x}$

where $\Psi(Z) = \dfrac{d \log \Gamma(Z)}{dZ}$

Note that the maximum likelihood equation for r contains the average of the logarithms of the sample observations, $\overline{Lnx}$. This is only defined for sample values greater than zero. In other words, maximum likelihood estimation cannot be used when the data contains zero values. The gamma

[3] Wadsworth, George P., and Bryan, Joseph G., *Introduction to Probability and Random Variables*, p. 139, Example 5-17, McGraw-Hill, 1960.

distribution can be modified slightly to allow for a zero case by simply introducing a probability, p, of the event zero:

$$f(x) = \begin{cases} p & , x = o \\ \dfrac{(1-p)\, a^r\, x^{r-1}\, e^{-ax}}{\Gamma(r)} & , x > o \end{cases}$$

The maximum likelihood estimates of r and a do not change when this function is fitted. The maximum likelihood estimate, $\hat{p}$, of p is the ratio of the number of zero observations to the total number of observations.

$\Psi(Z)$ is a well tabulated function.[4] For maximum likelihood fitting the gamma distribution this reviewer has tabulated $\Psi(Z) - LnZ$.

I fitted the modified gamma distribution by maximum likelihood to the same C.I.R.B. data. The fit was not improved for the smaller premium size groups. For the larger premium size groups sometimes the fit was better, sometimes worse. However, the estimate for r was consistently lower than that calculated by moments. This leads to the conclusion that perhaps the moment estimator for r is biased on the high size.

A few comments are in order concerning the calculation of the gamma probabilities. Pearson's tables of the incomplete gamma function are used by the author. There is nothing wrong with using these tables; however, the following relationships allow the complete gamma, and the incomplete gamma probabilities can be calculated directly by computer:

$$GAM(r) = \Gamma(r) = \begin{cases} \prod_{i=1}^{[r]+1} (r-i)\, \Gamma(r-[r])/(r-[r]-1), & r > o \\ \Gamma(r-[r])/\prod_{i=1}^{[r]} (r-i+1), & r < o \end{cases}$$

where $[r]$ is the greatest integer less than r.

$$\frac{1}{\Gamma(Z)} = \sum_{k=1}^{\infty} C_k Z^k$$

where C_k are constants.[5]

$$GAMIN(p, a, r) = \int_o^p \frac{a^r x^{r-1} e^{-ax}}{\Gamma(r)} dx = \int_o^{ap} \frac{x^{r-1} e^{-x}}{\Gamma(r)} dx = \sum_{i=1}^{\infty} \frac{\overline{(ap)^{r-1+i}\, e^{-ap}}}{\Gamma(r+i)}$$

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[4] *Handbook of Mathematical Functions,* AMS55 — U.S. Department of Commerce, pp. 267-273.

[5] Ibid p. 256.

Note that when r is an integer this sum reduces to the probability of at least r successes for a Poisson frequency function.

$$\frac{(ap)^{r+i}\, e^{-ap}}{\Gamma(r+i+1)} \bigg/ \frac{ap^{r+i-1}\, e^{-ap}}{\Gamma(r+i)} = \frac{ap}{r+i}$$

Errata in Author's Paper

Section 2.21(h)	p. 39 should read p. 391
Section 3.6(d)	replace r′ by E(r′) replace r″ by E(r″)
Appendix Table 2	Degrees of freedom for Chi-Square is 5 not 7, since one d.f. must be deducted for each parameter estimated.

INVERSE LIABILITY AUTOMOBILE ACCIDENT INSURANCE

JAMES B. M. MURRAY

> "Only a fool will build in defiance of the past. What is new and significant must always be grafted to old roots, the truly vital roots that are chosen with great care from the ones that merely survive"
>
> —Bartok

The spotlight of adverse criticism has in recent years turned full beam on the current system of third party automobile insurance and the methods of compensating those who are injured, and the dependents of those killed on North American roads, methods which have held sway on this continent for over fifty years. That this is a social problem of major magnitude can be easily comprehended from the fact that over 1100 persons were killed every week in automobile accidents during 1966 in the United States and Canada.

The Osgoode Hall Study on Compensation for Victims of Automobile Accidents conducted in Ontario in 1964 found that only 42.9 percent of those hurt or killed received any tort reparation, and only 28.8 percent recovered all of their economic loss.[1] These figures are not surprising when it is realized that a person injured in an automobile accident will only be fully reimbursed for his loss if:

(a) his injury was caused by the negligence of the owner of an automobile; and

(b) he can prove such negligence; and

(c) that owner carries insurance sufficient to cover the whole loss, or alternatively, has sufficient assets to cover the claim;

and even then the unfortunate victim may have to wait months or years until he can successfully pursue a legal action through the courts.

It will be seen therefore that, under the present system, many of those injured and killed have no recourse to recovery because:

(a) the accident was caused by their own negligence; or

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[1] Allen M. Linden, "Peaceful Coexistence and Automobile Accident Compensation." *Canadian Bar Journal,* February 1966.

(b) the accident was inevitable, that is, it was caused by no one's negligence; or

(c) the accident was caused by a negligent motorist but the injured person cannot prove it; or

(d) the negligent motorist carried no insurance and had no assets (In many jurisdictions uninsured motorist coverage or Motor Vehicle Indemnity Funds will be available to take care of such loss up to the statutory minimum limits.); or

(e) the injured person was a passenger in an automobile driven by a negligent motorist in those jurisdictions where passengers have no redress in such circumstances.

In these cases the injured person will have to rely on whatever accident insurance he may have purchased for his own benefit, such as medical payments insurance, accidental death and dismemberment, disability income, and so on, but usually such insurances only cover a woefully small proportion of the loss, and in the vast majority of cases little or no accident insurance is carried. There is in fact no form of accident insurance available at the present time which will provide a full measure of reimbursement of loss to the insured. The dependents of those killed will in many cases be beneficiaries of the life assurance policies which prudent husbands and fathers may have purchased, and this alleviates the hardship to that extent.

The nub of the problem is not in the area where the loss can be measured in hundreds of dollars. Most people can soon recover from such a loss. It is the serious injury cases where years of total disability lie ahead, it is the widows with young families to feed and educate, where the need is greatest. All of the alternative methods proposed for the solution of the automobile victim problem aim their benefits to loss up to $5,000 or $10,000 and leave those who suffer greater loss with all the deficiencies of the present system.

Compensation without Fault.

Compensation without fault is one method which has been considered as an alternative. This system is now in operation in the Canadian province of Saskatchewan. All persons injured or killed in automobile accidents automatically receive compensation, up to the limit of $5,000 any one person, from this government-operated plan, irrespective of who was legally liable for the accident, and their legal rights have been taken away from them up to this amount. If the loss exceeds the $5,000 compensation then they must prove negligence and pursue their claim in the usual way. This plan does not therefore solve the major problem of the serious injury cases involving total disability. Further, the death benefit limit of $5,000 may be much more than an indemnity for a young, single person with no dependents, but would be totally inadequate for a married man with a young family. It should be remembered that the plan was introduced in an endeavor to keep down insurance costs, rather than to provide the best system possible for the victims. It is doubtful whether it has gained its number one objective, since the true costs are partially hidden in the expense of administering the vehicle licensing department, no agency commission is payable, and the premiums are not subject to taxes. It is only fair to say, however, that most residents of the province seem to be reasonably happy with the plan, although this may be partially due to the fact that Saskatchewan is basically a farming community with no large metropolitan centres. The Saskatchewan Plan has been in operation for twenty years but has not been adopted in any other jurisdiction.

Accident Benefits

At the present time there is a proposal in the province of Ontario to include in the standard automobile policy accident benefits providing medical payments, death, dismemberment, and total disability weekly benefit. The death benefit would cover death within 90 days of the accident and the amount would be graded according to the age, sex, and marital status of the deceased—varying from 100% of the Principal Sum for married males up to age 60 (plus 20% of the Principal Sum for each dependent child), down to 5% of the Principal Sum for unmarried children. The percentage of the Principal Sum in the event of dismemberment varies in the usual way. Total disability is provided up to 104 weeks with a waiting period of 7 days. If the injury causes total and permanent disability the weekly benefit continues for a further 104 weeks.

The original proposal by the Select Committee of the Ontario Legislature called for the adoption of these accident benefits, as a mandatory section of the standard third party automobile policy, applicable to any person while an occupant of the insured automobile and any person, not the occupant of an automobile, who is struck in Canada by the insured automobile. The injured person would be deprived of his right to sue the driver or owner of the automobile in which he was riding as a passenger, or by which he was struck, except for any amount in excess of the accident benefits.

The cost of these benefits for a Principal Sum of $5,000 has been estimated at 12.6% of the third party premium for limits of $35,000 inclusive for bodily injury and property damage, assuming benefits are offset against third party liability.[2]

The Ontario plan has been opposed on the ground that the motorist should not be legally obligated to pay the premiums on a policy which provides accident benefits to persons other than occupants of the insured vehicle. If, as now appears, the coverage is to be voluntary, consumer resistance may be expected to the inclusion of third party pedestrians, etc. The Ontario plan is also opposed on the grounds that it requires legislative changes to the common law and that it does not adequately provide for the very serious cases.

Basic Protection Plan

The Basic Protection Plan by Robert E. Keeton and Jeffery O'Connell[3] is the latest proposal for the solution of this urgent social problem. It provides a form of compulsory insurance which compensates victims without regard to fault for economic losses up to $10,000 per person and $100,000 per accident. Legislation is required to exempt the insured from his common law liability to the extent of the compensation. Reimbursement of losses is provided as they accrue so that the victim does not require to await the assessment of his total loss before receiving any payment. In arriving at the amount of net loss, benefits from other sources must be subtracted in order to avoid duplication, and there is a compulsory deductible of $100 or 10% of work loss, whichever is greater. It is perhaps too early to estimate the acceptance of this comprehensive plan. Undoubtedly it would be an improvement on existing methods if the plight of the injured victim is considered. However, as in all compensation plans, there is the necessity to change the common law, and the disadvantage that benefits are limited to small and medium sized losses.

It may be that society is not yet willing to accept the regimentation of fixed and limited benefits in place of unlimited common law rights. The prospective plaintiff would rather take his chance of recovering his full

[2] H. E. Wittick, "Estimating the Cost of Accident Insurance as a part of Automobile Liability Insurance," *PCAS* Vol. LI, 1964.

[3] Keeton & O'Connell, *Basic Protection for the Traffic Victim*, Little, Brown & Co., 1965.

loss at common law rather than be assured of partial compensation. There certainly seems to be a very great reluctance on all sides to endorse any solution which requires, as an essential ingredient, the abrogation of the common law rights of the individual. Perhaps this is not too surprising in a nation where freedom is the individual's birthright defended by the highest courts in the land.

It becomes of interest therefore to seek, if possible, a solution which:

(a) does not subtract from the injured party his right to go to court if he so wishes in order to recover his loss; and

(b) gives the injured party a full indemnity for all economic loss, limited only by the sum insured; and

(c) is payable irrespective of fault; and

(d) provides advance payment of out-of-pocket expenses.

These requirements would be met by a form of accident insurance which would provide an indemnity to the insured and which he can elect to collect from his own insurance in lieu of an action against a wrongdoer, but with provision that the insurer would then be subrogated to the insured's rights, if any, against that wrongdoer.

Inverse Liability

Inverse Liability automobile accident insurance has therefore been designed with these requirements in mind. Simply stated, such a policy would pay to the insured, or in the event of death, to his legal representative, all economic loss suffered as the result of bodily injury in an automobile accident, that is, for the amounts of economic loss which he would have been entitled to collect at law if he had claimed against a responsible third party. Benefits would apply to the insured, and to dependent relatives residing with him, for bodily injury arising out of any automobile accident, whether as drivers, passengers, or pedestrians. Economic loss would include medical, surgical, hospital, and nursing expenses, and loss of income as well as the expense of rehabilitation, excluding any amounts received by the insured under workmen's compensation, Social Security, governmental hospitalization schemes, and so on. In the event of death, the financial loss suffered by the insured's estate because of the accident would be payable for the same amount as would have been recognized as a legitimate claim from the dependents of a deceased third party claimant. Fault does not enter into the question, so that in all cases the insured would be assured of a complete indemnity. Thus where the Inverse Liability insured is himself responsible for the accident, or where no other car is involved, such as in the car-tree type of collision, he would still receive full indemnity under the Inverse Liability policy.

Since the policy is one of indemnity the insurer would be entitled at common law to take over the insured's rights, if any, against any other party responsible in whole or in part for the insured's loss. Thus the Inverse Liability insurer, having agreed to indemnify the insured, would pursue recovery in his name against the wrongdoer's automobile liability insurer.

Where the insured is 100% responsible for the accident the Inverse Liability policy pays the loss but of course has no rights of recovery.

If the insured prefers to pursue an action or make settlement with any other person responsible for the accident, he may of course do so, since that is his legal right, but in that event he would forfeit all benefit under the Inverse Liability policy in exactly the same way as is provided under Uninsured Motorist coverage. In fact Inverse Liability is an extension of the principle of uninsured motorist coverage but of course is not limited to accidents caused by uninsured persons (See the proposed policy wording in the Appendix).

Inverse Liability, being an accident policy, would not be liable to partnerships or corporations — the insured would require to be an individual, but as stated above the benefits would extend to cover relative dependent members of the named insured's household. Like any other accident insurance Inverse Liability could be sold on a group basis for employees on a named schedule.

Payment on Account

Medical, surgical, hospital, and nursing expenses and other out-of-pocket expenses would be paid under Inverse Liability upon production of evidence of payment, subject to a receipt being taken from the insured or his legal representative for the purpose of bringing this into account at the time of final settlement. This is a valuable advantage to the insured and it is an effective answer to the existing problem where victims of accidents who are unable to pay their way are often forced to settle for less than their legal entitlement.

Policy Limits

The insured would select his own limits, the suggested minimum being $100,000 with increased amounts up to $500,000 available at an increased premium. It is important that adequate amounts are available, since small sums insured would suffer from the same defects as limited compensation plans. The availability of alternative amounts enables applicants to purchase coverage in keeping with their dependency obligations and their standing in the community.

Pedestrians

Where the Inverse Liability insured is a pedestrian at the time of an automobile accident he is entitled to the full benefits provided by his Inverse Liability policy, and the insurer would then recover if it can from the automobile owner or driver or his automobile liability insurer. Similarly, where the insured at the time of the accident is a fare-paying passenger in a taxi or omnibus, his loss in the first instance would be paid by his Inverse Liability insurer who would then subrogate against the owner of the taxi or omnibus.

Where the responsible party is insufficiently insured the Inverse Liability insurer would have a net loss of the difference between the full indemnity paid to the insured and the amount recovered from the responsible party's insurer.

Where a greater amount is recovered from the responsible party or his insurer than has been paid by the Inverse Liability insurer to its insured, the excess would belong to the insured on the theory that that is the amount he would have recovered if he had pursued his legal rights against the wrongdoer, instead of claiming under his Inverse Liability policy. In practice this is not likely to arise since in cases involving a responsible third party a final settlement under the Inverse Liability policy is likely to be postponed until the recovery amount has been determined by negotiation with, or by court action against, the responsible third party or his insurer.

Uninsured Motorist Insurance

It will have been appreciated from what has been said that Inverse Liability insurance would include uninsured motorist coverage in the event that the accident is caused by an uninsured motorist or by a hit and run driver, but it is not limited to the statutory minimum limits. The principle is the same—that of the first party insurer acting in the place of the third party insurer for the purpose of determining the amount of the insured's loss. Inverse Liability is also analogous to the Sister Ship Clause of marine insurance whereby if two ships belonging to the same owner are in collision, the liability between them is settled as if the ships belonged to different owners. The principle of Inverse Liability is not therefore new — it is merely an extension to an existing method.

Determination of the Amount Payable

The amount payable under the Inverse Liability policy would be determined by agreement between the insured or his legal personal representative and the company, or failing agreement, by arbitration as defined in the policy. This is also the method adopted by uninsured motorist coverage. No suit by the insured against the company would be valid unless all terms of the policy, including the arbitration condition, are complied with.

Claims under the Inverse Liability policy will fall into one of two main categories—those where some other party was responsible or partly responsible for the accident, and those where the insured was the author of his own misfortune. In the former case the Inverse Liability insurer will be pursuing recovery of its payments to the insured, and final settlement with the insured will not be arrived at until this has been agreed with the third party or his insurer by negotiation or by court judgment. In the latter case the insured would have no means of recovering his loss except by his claim under the Inverse Liability policy, and for this reason he is not likely to be too unreasonable in negotiating a settlement.

However, since the amounts claimable under Inverse Liability relate to economic losses which can be established with reasonable accuracy in most cases, and since the insurer can obtain medical examinations as often as considered necessary, and since the insured must cooperate with the insurer in producing evidence of loss, there should be an amicable settlement reached in the vast majority of cases. There will be controversy in some cases in the same way as these are encountered in all forms of insurance, with the possible exception of total losses under fire and property policies and losses under life policies, where the amount payable is fixed. The amount of loss under Inverse Liability is no less determinate than the amount payable under a Business Interruption policy. Whatever defects can be attributed to Inverse Liability because of the possibility of difficult settlements in some cases should be outweighed by its many advantages, not the least of which is the fact that rehabilitation of injured automobile victims becomes an immediate possibility without all the problems which presently attend the injured third party victim who does not, and can not, share a community of interest with the insurer, because of the fear of prejudicing his legal position. By promoting rehabilitation the Inverse

Liability insurance can make a major contribution to a social problem of national importance.

Voluntary or Mandatory?

There is certainly a very powerful argument for making Inverse Liability a compulsory form of insurance for automobile owners. The state is well within its constitutional rights in requiring the owner of an automobile to produce a guarantee that no person who uses the automobile or who is struck by it, will, by reason of injury following an automobile accident, require the financial aid of the state. This could be accomplished by a combination of an Inverse Liability policy and an automobile liability policy. (Both coverages could be provided in one policy by adding Inverse Liability coverage to the standard automobile policy.)

The usual opposition that such a mandatory requirement calls for one section of the community to pay insurance premiums for benefits which another section of the community receives is scarcely valid, since the insured purchases Inverse Liability for his own protection, and for the protection of the members of his family.

Pedestrians do not usually go without compensation following an automobile accident since in the vast majority of cases the automobile is at fault. (In some jurisdictions the automobile owner is deemed liable unless he can disprove it.) However, pedestrians may in some cases also be automobile owners who have purchased Inverse Liability, and in any event, those who do not own automobiles could still purchase Inverse Liability for their own protection at lower premiums than automobile owners.

It should be noted that if Inverse Liability were made compulsory, there would be no necessity to change the tort liability law, since duplicate reimbursement is avoided by the indemnity-cum-subrogation feature of Inverse Liability.

The disadvantage of a voluntary form of Inverse Liability is of course that it only provides an effective solution to the current problem to the extent that it would be purchased by the motoring public. However, provided the cost can be kept within reasonable bounds, Inverse Liability would likely reach a large section of the community.

The Cost of Inverse Liability

The rating factors to be used for Inverse Liability bear great similarity to those adopted for automobile third party bodily injury insurance, since both premiums are a direct function of the frequency of automobile accidents and the average size of a bodily injury claim. Thus the location, the use of the automobile, the age, sex, and marital status of the drivers, and the accident and conviction record of the insured would all be relevant factors in the rating of Inverse Liability insurance.

Thus it should be possible to relate the cost of Inverse Liaiblity to the cost of the corresponding third party bodily injury liability insurance. In this way maximum use would be made of existing statistics. Superimposed on this base would be a composite factor dependent on the following variables:

(i) the amount of coverage,
(ii) the age of the insured (probably in quinquennial age groups),
(iii) marital status,
(iv) number and ages of dependents,
(v) number of automobiles owned in the household.

It would be necessary in the first instance to set up differentials for these variable factors largely on a judgment basis although reference could be made, for example, to the relativity by age group for disability income insurance, and to the cost of annuities for widows and child dependents. It is recognized that several of these factors may be difficult or impossible to assess accurately in the initial stages since there are many imponderable quantities involved. For example, the married man with a young family would have a larger claim for dependency than an older man whose family were grown up, but on the other hand the young person is not likely to have reached his maximum earning capacity. Again, the older man with long service may not suffer the loss of income to such an extent as the young man, but the young man may make a speedier recovery from his injuries. Notwithstanding the complexity it should be possible to set up a rating structure in each territory based upon the accident statistics which are usually available in considerable detail showing the number of persons injured and killed by age groups. In conjunction with the frequency of accident it will be necessary to arrive at an estimated cost of claim which should bear some reasonable relation to the average cost of a third party bodily injury claim, information on which is available in most, if not all, jurisdictions. In estimating the cost of claim, recognition should be given to the fact that, in a percentage of the cases, some recovery will be made from a responsible third party or his insurer.

As a matter of interest the premium developed along these lines for \$100,000 coverage in the province of Ontario, ignoring recovery possibilities, was of the order of $60, and this compares to an average third party bodily injury and property damage premium of $69 (bodily injury is not recorded separately), and a collision premium of $46. This seems to indicate that the cost of Inverse Liability would be within reasonable limits, although of course, much research would be necessary in order to develop a more detailed rating program. Based on this estimate many people who could not afford both collision and Inverse Liability might choose the latter as being better protection against a financial loss of crippling proportions.

Damages for Pain and Suffering

It will have been observed that the proposed form of Inverse Liability coverage indemnifies the insured for his economic loss, and the question arises as to whether the coverage should be extended to provide an allowance for pain and suffering, loss of future enjoyment of life, mental anguish, and such indefinite items of general damages. These are amounts which a successful plaintiff can include in his claim against a wrongdoer, and the question arises as to whether the Inverse Liability policy which did not pay these amounts is in fact indemnifying the insured, and I think it would have to be conceded that as a purely academic question it provides something less than a full and perfect indemnity. However, from a practical viewpoint, the knowledge that he will be fully reimbursed for all his economic loss including loss of future earning power, plus payments on account, and the absence of worry that these assurances bring, should outweigh to some extent the indefinite amounts recovered in the courts for pain and suffering. In any event, where pain and suffering form a major portion of the insured's claim he can always elect to pursue his claim against the responsible motorist (if there is one) and forego the claim under his Inverse Liability policy. Further, if the Inverse Liability insurer subrogates against a responsible third party and is successful in recovering an amount under the heading of pain and suffering, this would of course be paid to the insured.

There is no reason, in theory at any rate, why pain and suffering could not be provided by Inverse Liability, in the same way as it is offered as an additional coverage under Keeton and O'Connell's Basic Protection Plan, but it does add to the technical problems which might arise.

Unsatisfied Judgment Funds

In several jurisdictions there are government operated funds (financed by the insurance companies in some cases) available for the benefit of persons injured in automobile accidents caused by uninsured motorists or unknown motorists. Usually the regulations prohibit any insurance company benefiting from the funds. The intention here is basically directed to collision insurers but the wording as it stands would also apply to Inverse Liability insurers, who would presumably be unable to subrogate against the fund even if the recovery is pursued in the name of the insured. Actually the Inverse Liability insured has no need of such funds except in the rare event that he had violated the conditions of his Inverse Liability policy, and if Inverse Liability were compulsory, there would be no need for Unsatisfied Judgment Funds.

There are indeed many facets of Inverse Liability which require research for the purpose of relating this new form of coverage to the various jurisdictions.

Deductibles under Inverse Liability

Because of the fact that many persons carry some form of accident insurance which pays some benefits in the event of an automobile accident, such as medical payments, hospitalization, death and dismemberment, disability income, and so on, it becomes of interest to explore the possibility of issuing an Inverse Liability policy subject to a deductible such as $500, $1,000, or more.

From the point of view of reducing the cost to the insured and the avoidance of duplicate insurance, this would seem to be an advisable proposition, but it immediately leads to the question whether the deductible reduces the benefits under Inverse Liability to something less than an indemnity, and thus whether the insurer is entitled to subrogation. As a matter of equity it is entirely reasonable that an insured should not be in the position of recovering a portion of his loss twice over, but it is a fact, unfortunate perhaps, but nevertheless true, that the automobile victim at the present time can claim under any accident policies he possesses and still include these amounts in his claim against a responsible third party.

Undoubtedly the best method is to include a subrogation condition in the policy, and not to rely on common law rights for subrogation. In some jurisdictions it may be necessary to pass enabling legislation to accomplish this.

The Economic Cost of Inverse Liability

It is to be expected that opposition to Inverse Liability will appear in some quarters on the grounds that it increases the cost of insurance to the general public. A deeper consideration of this question, however, will show that the cost is already being borne by the community, either by individuals who have been financially ruined by the effects of serious automobile accidents, or by social or government institutions who are maintaining those who, because of automobile accidents, are unable to meet their own financial obligations. Inverse Liability spreads the existing cost over a large number of insured persons so that no one insured suffers undue economic loss.

Inverse Liability indeed satisfies all the required concepts of an insurable risk as specified by David B. Houston,[4] namely, (1) loss is objective and accidental, (2) exposure units are homogeneous, (3) loss occurring to one exposure unit does not alter the loss expectation of any other exposure unit, and (4) there are a large number of eligible exposure units. In addition Inverse Liability is the low frequency-high possible loss type of insurance which is recognized as one of the most suitable insurable risks. Finally, the occurrence of an accident is easily defined and thus the prospect of fraudulent loss is minimal.

Some Further Objections to Inverse Liability

It might be contended that under Inverse Liability coverage the insured may endeavour to claim for injuries or conditions which were not actually received in the accident. This is undoubtedly true since this is "tried on" by claimants under the third party section of the automobile policy and under general liability policies. I believe, however, that the incidence of such fraudulent claims is lessened under Inverse Liability because of the company's right to examine the insured as often as it is deemed necessary. In addition, the application would contain declarations as to the insured's physical health. Despite the best of safeguards some exaggerated claims will be successful no doubt, in the same way as there are fraudulent or exaggerated fire claims; however, the rate has to be established to include this cost factor.

There may be a tendency for the insured to be uncooperative in proceedings for subrogated recovery from a responsible third party. The insurer, however, should be able to effectively counteract this because of the policy conditions, and also because of the fact that the insured's claim under the Inverse Liability coverage would not be finally settled until the

[4] David B. Houston, "Risk, Insurance and Sampling," *Journal of Risk and Insurance*, Vol. 31, No. 4, 1964.

recovery process is completed. The company, therefore, has a lever to be used where any lack of cooperation is evident.

Another objection may be advanced in the difficulty which may arise in arriving at a settlement figure especially where the insured has himself been responsible for the accident, and therefore no recovery proceedings are possible. Arbitration is admittedly fraught with some difficulties, but it still represents the best method known to us at the present time. The insured of course will undoubtedly take his own lawyer's advice on this subject, and the company also will have the benefit of its own counsel's opinion, so that many cases should be settled by negotiation between the legal advisers of each party without the necessity to refer to an umpire. In actual fact there is probably no existing form of first party insurance where there is not occasionally a conflict between insurer and insured over the amount of settlement. All partial losses under fire and burglary policies involve the necessity to reach an agreement on an indefinite amount of loss, to say nothing of the complexity of loss adjustments under Marine and Business Interruption insurance.

Inverse Liability, if a voluntary form of insurance, will, of course, only solve the problem of the uncompensated victim to the extent that it is bought by the public. There are actually some fairly strong arguments in favour of compulsory Inverse Liability since the state is entitled to be assured that, if it permits a subject to use a potentially dangerous vehicle on the public highways, there will be no uncompensated victims who may become charges on the community dependent upon the financial assistance of the state. The principle here has been established in those states with compulsory uninsured motorist coverage. The combination of third party insurance for injured pedestrians and Inverse Liability for the occupants of the insured automobile effectively ensures that all injured persons would be insured up to the minimum amounts established in each jurisdiction. If Inverse Liability were not mandatory it would have to be admitted that there would continue to be uninsured victims. However, if Inverse Liability can be supplied at reasonable premiums, the uninsured victim is in no different position than the widow of the man who did not buy life assurance.

Conclusion

The automobile insurance industry, rightly or wrongly, is saddled with the task of finding a solution to the uncompensated automobile accident victim, and every effort is being made to find an answer which will at the same time retain the best of the traditional negligence system.

Inverse Liability is one method which, if found to be acceptable, would keep the court and jury system intact as the final arbitrator for the extent of liability and the quantum of damages. It is a modern approach to accident insurance with subrogation which would indemnify the insured automobile victim for economic loss irrespective of fault. It provides cash for current expenses. It provides the insured with complete freedom from financial worry. It enables the insurer to provide the most modern aids to rehabilitation and thus to make a useful contribution to a major social problem in North America today.

APPENDIX

INVERSE LIABILITY AUTOMOBILE ACCIDENT POLICY
PROPOSED POLICY WORDING

WHEREAS an application in writing has been made by the Applicant therein mentioned (and hereinafter called the Insured) to the Company for a contract of Inverse Liability Accident Insurance and the application forms part of this contract of insurance.

NOW THEREFORE, in consideration of the payment of the premium and of the statements contained in the application and subject to the limits, terms and conditions herein stated

THE COMPANY AGREES to pay the Insured or his legal representative the amount of economic loss because of bodily injury, sickness or disease, including death resulting therefrom, hereinafter called bodily injury, sustained by the insured caused by accident during the policy period and arising out of the maintenance, use or operation of an automobile or whilst in, on, or struck by an automobile; provided the amount of such loss shall be determined by agreement between the insured or his legal representative and the Company or, if they fail to agree, by arbitration as defined in this policy.

DEFINITIONS

"Insured" means the named insured and any dependent of the named insured, related to him and resident in the same household. The insurance afforded applies separately to each insured, but the inclusion herein of more than one insured shall not operate to increase the limit of the Company's liability.

"Automobile" includes all self-propelled vehicles, their trailers, accessories and equipment, but not railway rolling stock, watercraft or aircraft of any kind.

EXCLUSIONS

This policy does not apply:

(a) to bodily injury to an insured, or care or loss of services, recoverable by an insured, with respect to which the insured, his legal representative or any person entitled to payment under this policy shall, without written consent of the Company, make any settlement with or prosecute to judgment any action against any person or organization who may be legally liable therefor.

(b) so as to inure directly or indirectly to the benefit of any person or organization other than the insured or his legal representative.

(c) to accidents occurring outside Canada or the continental United States of America.

LIMITS OF LIABILITY

(a) The limit of liability stated herein is the total limit of the Company's liability because of bodily injury as the result of any one accident.

(b) Any loss payable under this policy to or for any person shall be reduced by the amount paid and the present value of amounts payable under any workmen's compensation law, governmental hospitalization or social security.

OTHER INSURANCE

(a) If the Insured has other accident, medical payments or medical or surgical insurance available to him against a loss covered by this policy then this insurance shall be considered as excess insurance over such other insurance.

(b) If the Insured has other similar Inverse Liability Accident insurance available to him against a loss covered by this policy, the Company shall not be liable for a greater proportion of such loss than the applicable limit of liability hereunder bears to the total applicable limits of liability of all valid and collectible Inverse Liability Accident insurance.

ARBITRATION

If any person making claim hereunder and the Company do not agree as to the amount recoverable hereunder then, upon written demand of either, the matter shall be referred to the arbitration of some person to be chosen by both parties, or if they cannot agree on one person, then to two persons, one to be chosen by the Insured and the other by the Company, and a third to be appointed by the persons so chosen, or on their failing to agree, then by a Judge of the County or District Court of the county or district in which the insured resides; and such reference shall be subject to the provisions of The Arbitration Act; and the award shall be conclusive as to the amount payable hereunder; and the question of costs shall be in the discretion of the arbitrators.

TRUST AGREEMENT

In the event of payment to any person under this policy:

(a) the Company shall be entitled to the extent of such payment to the proceeds of any settlement or judgment that may result from the exercise of any rights of recovery of such person against any other person or organization legally responsible for the bodily injury because of which such payment is made;

(b) such person shall hold in trust for the benefit of the Company all rights of recovery which he shall have against such other person or organization because of the damages which are the subject of claims made under this policy;

(c) such person shall do whatever is proper to secure such rights and shall do nothing after loss to prejudice such rights;

(d) if requested in writing by the Company, such person shall take, through any representative designated by the Company, such action as may be necessary or appropriate to recover such payment, as damages from such other person or organization, such action to be taken in the name of such person; in the event of a recovery, the Company shall be reimbursed out of such recovery for expenses, costs and legal fees incurred by it in connection therewith;

(e) such person shall execute and deliver to the Company such instruments and papers as may be appropriate to secure the rights and obligations of such person and the Company established by this provisions.

GENERAL CONDITIONS

1. *Notice.*

Written notice of claims to the Company shall contain particulars sufficient to identify the insured and also reasonably obtainable informtion with respect to the time, place and circumstances of the accident and the names and addresses of the injured and of available witnesses.

If, before the Company makes payment of loss hereunder, the insured or his legal representative shall institute any legal action for bodily injury against any person or organization legally responsible, a copy of the writ, summons, complaint or other process served in connection with such legal action shall be forwarded immediately to the Company by the insured or his legal representative.

2. *Payment of Loss.*

Any amount due is payable (a) to the named insured, or (b) if the insured be a minor to his parent or guardian, or (c) if the insured be deceased to his surviving spouse otherwise (d) to a person authorized by law to receive such payment or to a person legally entitled to recover the damages which the payment represents.

3. *Prohibited Use by Insured.*

The insured shall not drive or operate an automobile:

(a) While under the influence of intoxicating liquor or drugs to such an extent as to be for the time being incapable of the proper control of the automobile; or

(b) unless he is for the time being either authorized by law or qualified to drive or operate the automobile, or while he is under the age of sixteen years or under such other age as is prescribed by the law of the province where he resides at the time the policy is issued; or

(c) for any illicit or prohibited trade or transportation; or

(d) in any race or speed test.

4. *Prohibited Use by Others.*

The insured shall not permit, suffer, allow or connive at the use of an automobile:

(a) by any person under the influence of intoxicating liquor or drugs to such an extent as to be for the time being incapable of the proper control of the automobile; or

(b) by any person, unless such person is for the time being either authorized by law or qualified to drive or operate the automobile, or while such person is under the age of sixteen years or under such other age as is prescribed by law; or

(c) for any illicit or prohibited trade or transportation; or

(d) in any race or speed test.

5. *War Risks.*

The Company shall not be liable for loss or damage that is caused directly or indirectly by bombardment, invasion, civil war, insurrection, rebellion, revolution, military or usurped power, or by operations of armed forces while engaged in hostilities, whether war be declared or not, or by civil commotion arising from any of the foregoing.

STATUTORY CONDITIONS

NOTE: — In those Provinces or Territories lacking Statutory Conditions for Accident Insurance the following shall constitute the Standard Terms and Provisions of the Policy.

1. The Contract — This policy including the endorsements, insertions or riders if any, and the application for the contract if attached to the policy, constitutes the entire contract and no agent has authority to change the contract or waive any of its provisions.

2. Waiver — The insurer shall be deemed not to have waived any condition of this contract, either in whole or in part unless the waiver is clearly expressed in writing signed by the insurer.

3. Material Facts — No statement made by the insured or his application for this contract may be used in defence of a claim under; or to avoid, this contract unless it is contained in the written application for the contract and unless a copy of the application or such part thereof as is material to the contract, is endorsed upon, inserted in or attached to the policy when issued.

4. Termination by Insured — The insured may terminate the contract at any time by giving written notice of termination to the insurer by

registered mail to its head office or chief agency in the province or by delivery thereof to an authorized agent of the insurer in the province and the insurer shall, upon surrender of this policy, refund the amount of premium paid in excess of the short rate premium for the expired time according to the table in use by the insurer at the time of termination.

5. Termination by Insurer.
   (1) The insurer may terminate the contract at any time by giving written notice of termination to the insured and by refunding concurrently with the giving of notice the amount of premium paid in excess of the pro rata premium for the expired time.
   (2) The notice of termination may be delivered to the insured, or it may be sent by registered mail to the latest address of the insured on the records of the insurer.
   (3) Where the notice of termination is delivered to the insured, five days notice of termination shill be given; where it is mailed to the insured, ten days notice of termination shall be given and the ten days shall begin on the day following the arrival of the notice at the post office to which it is addressed.
6. Notice and Proof of Claim — The insured or his agent, or a beneficiary entitled to make a claim or his agent, shall:
   (a) give written notice of claim to the insurer;
      (i) by delivery thereof, or by sending it by registered mail to the head office or chief agency of the insurer in the province or
      (ii) by delivery thereof to an authorized agent of the insurer in the province, not later than thirty days from the date of the accident or the beginning of the disability due to sickness;
   (b) within ninety days from the date of the accident or the beginning of the disability due to sickness for which the claim is made, furnish to the insurer such proof of claim as is reasonably possible in the circumstances of the happening of the accident or sickness and the loss occasioned thereby; and
   (c) if so required by the insurer, furnish a certificate as to the cause and nature of the accident or sickness for which the claim is made and as to the duration of the disability caused thereby, from a medical practioner legally qualified to practice in the province.

7. Failure to Give Notice of Proof — Failure to give notice of claim or furnish proof of claim within the time prescribed in this statutory condition will not invalidate the claim if the notice or proof is given or furnished as soon as reasonably possible and in no event later than one year from the date of the accident or the beginning of the disability due to sickness and if it is shown that it was not reasonably possible to give notice or furnish proof within the time so prescribed.
8. Insurer to Furnish Forms for Proof of Claim — The insurer shall furnish forms for proof of claim within fifteen days after receiving notice of claim but where the claimant has not received the forms within that time he may submit his proof of claim in the form of a written statement of the happening and character of the accident or sickness giving rise to the claim and of the extent of the loss.
9. Right of Examination — The insurer has the right, and the claimant shall afford to the insurer an opportunity, to examine the person of the person insured when and as often as it may reasonably require while the claim hereunder is pending, and also, in the case of the death of the person insured to make an autopsy subject to any law of the province relating to autopsies.
10. Limitation of Actions — An action or proceeding against the insurer for the recovery of a claim under this contract shall not be begun after one year from the date on which the cause of action arose.

DISCUSSION BY J. A. HILLHOUSE

The paper on Inverse Liability Automobile Accident Insurance presented at the May, 1967 meeting of our Society by Mr. J. B. M. Murray is an extremely welcome and a very timely contribution to our *Proceedings*. Seldom can one pick up a newspaper or trade journal today without observing some article leveling adverse criticism towards the current tort liability system. In his presentation as part of a panel discussion on Automobile Compensation Plans at the May, 1966 CAS meeting, Professor Keeton summarized the shortcomings of the present automobile claims system by saying, "It provides too little, too late, unfairly allocated, at wasteful cost, and through means that promote dishonesty and disrespect for law." The degree of consent or opposition toward this statement from various segments of the industry varies quite drastically, although it is generally agreed that some refinement is necessary in the present system of settling third party liability claims.

The material in Mr. Murray's paper is, in my opinion, very well organized and presented in an understandable fashion. Brief introductory statements, setting forth the need for modification in the present system of compensating those who are injured in automobile accidents, precede descriptions of other plans which have been advanced suggesting reformation of the present tort liability system. A new form of automobile accident insurance which the author has titled Inverse Liability is then explained, followed by suggested policy wording including all applicable conditions.

I must say that reviewing this paper was very educational, not only from the knowledge gained as respects the Inverse Liability plan but also from general research on the subject of compensation without fault. It seemed to be a natural tendency to contrast the components of the Inverse Liability plan with the highly publicized Keeton-O'Connell plan. In each case, the primary objective is that of indemnifying the automobile accident victim for economic loss irrespective of fault. The Inverse Liability plan is unique, however, in that it contains a subrogation feature providing that a company having indemnified an insured has recourse against a responsible third party or insurer of the third party. It is an extension of the principle of uninsured motorist coverage except that it is not limited to accidents involving the uninsured person and there is no limitation to the statutory minimum limits inasmuch as the insured may elect whatever limits he desires. The suggested minimum limits are $100,000 with increased amounts available up to $500,000 at an additional premium.

The Inverse Liability approach contemplates preservation of the traditional court and jury system to the extent that an insured may pursue his claim against a responsible third party, in which case any claim under his own policy is forfeited. The plan is devised to be either mandatory or voluntary but the voluntary approach has a distinct disadvantage, for Inverse Liability would be effective only to the extent that it would be purchased by the motoring public. Pain and suffering is not included under the Inverse Liability policy but the author does indicate that it could be offered as an additional coverage. Instead, where pain and suffering constitute a major portion of a claim, an insured may elect to pursue his claim against a responsible third party and forego any compensation under his Inverse Liability policy. The use of deductibles is discussed, and if I interpret the comments correctly, the author does not necessarily recommend a deductible feature under Inverse Liability.

Mr. Murray has submitted a paper incorporating a plan which has required considerable thought and effort in preparing. I have no specific criticism of the paper itself, but for my enlightenment further details or research regarding the cost of Inverse Liability would have been interesting. I suppose I am ultra-conscious of this aspect because of the wide divergence of views which have been expressed in estimating the cost of the Basic Protection plan. It is difficult for me to be as optimistic as the author that the arrival of a figure for resolving a claim under the Inverse Liability policy will result in an amicable settlement in a vast majoriy of cases.

From a purely personal vewpoint, I have some reservation as to the total or partial abandonment of our present liability system. First, I wonder if the adoption of the compensation without fault concept would have an adverse effect on fatalities and accident frequencies because of the tendencies toward more negligent driving habits by automobile operators? I am disturbed also about the inequity of distributing the costs under the compensation without fault plans. It appears that the more prudent and responsible insureds will be assessed higher premiums to subsidize the more negligent drivers who should pay the higher premiums.

On behalf of the Society, I would like to thank Mr. Murray for his fine paper and commend him for sharing his idea with us. On an issue of such great public importance, I hope other members of the Society will be stimulated and encouraged to also share their thoughts or comments with us. It occurs to me that only by pooling and sharing the ideas of several individuals will we be able to arrive at a feasible modification of the traditional tort liability system and one that is acceptable to society.

DISCUSSION BY JACK MOSELEY

Any paper, article, or discussion on the problems attending automobile liability insurance today deserves and generally gets a fair share of attention. Mr. Murray's paper on Inverse Liability Automobile Accident Insurance is one that deserves a lot of attention.

Mr. Murray begins by discussing some of the difficulties involved in recovering damages under the existing tort law. He then discusses several of the short-comings inherent in the compensation without fault system in use in Saskatchewan, Canada; supplementary accident benefits proposed in Ontario, Canada; and the Basic Protection Plan proposed by Professors Keeton and O'Connell. Notable among these short-comings are: (1) the forfeiture of certain legal rights, (2) inadequacy of automobile benefits in the event of serious injury, (3) the probable failure to actually reduce the cost of automobile insurance, and (4) the necessity of substantial and rudimentary changes in statutes as regards the latter two proposals.

Mr. Murray has proposed a most interesting alternative to the three mentioned plans. Particularily appealing is the promise of a limit of coverage sufficient to cover the most serious of injuries, and the fact that statutory changes would not be necessary. It can readily be seen that both of these conditions are distinct advantages over the other plans. In addition, Inverse Liability incorporates the more desirable features such as first party claim settlement, more timely reimbursement for economic loss, the removal of fault as a consideration, and an even more effective elimination of legal actions.

However, I cannot give Mr. Murray an unqualified vote of confidence. By his own admission Inverse Liability would be most effective in answering automobile insurance problems only if it were made mandatory. Mandatory insurance has always been a hard nut to crack and probably will continue that way. Mr. Murray further admits that as a voluntary coverage Inverse Liability "only provides an effective solution to the current problem to the extent that it would be purchased by the motoring public." I believe these two conditions would seriously hamper ready acceptance by the industry. In addition, should the cost estimate as set forth in the paper be reasonably accurate, the public would likely not be enticed to purchase Inverse Liability as a supplementary coverage.

There is one element contained in Inverse Liability, in fact in all compensation without fault type plans, which I believe requires discussion here. All such plans propose that every person injured in an automobile accident, even the grossly negligent operator, be reimbursed for economic loss. Such proposals constitute an implied criticism of the present system, a criticisim which is not justifiable. Every operator of an automobile has a personal responsibility to cover his own economic loss in those instances where his negligence causes an accident, just as he would cover his economic loss in the event of illness. Lack of recovery in these instances should not be levied as a fault of the present system. In fact, the shifting of these losses into the automobile insurance area simply compounds the already impossible problem of price.

Perhaps my most serious reservation stems not from any basic disagreement with Mr. Murray's proposals, but rather from a doubt that the problems of automobile insurance have been sufficiently well defined at this point in time to allow ready access to the most appropriate solution. For example, it has been my impression that the most frequently heard complaint from the public and regulatory authorities is that the cost of automobile insurance is simply too high and is continuing to rise too fast. The high cost coupled with large numbers of cancellations, which are generally a function of price, have been the catalysts precipitating the many investigations that have taken place or are taking place currently. Claim settlement problems have not been a predominant factor in the call for such investigations.

Accordingly, I believe that any solution which the industry might settle on must attack the problem of cost at the outset. It may well be that the insurance industry is unable to materially affect the cost of automobile insurance without substantial changes in driver licensing practices and in law enforcement practices. However, this, if fact, must be made abundantly clear to the authorities in order that all concerned may work together effectively.

Thus, using cost reduction as the measure of success of Inverse Liability, I must conclude that it falls short of the objective. In all fairness to Mr. Murray, he did not suggest that the total cost of automobile insurance would be reduced. Rather he offers Inverse Liability as a form of complete accident protection for the insured, to cover the myriad of instances where recovery for personal injury damages are unavailable. While this is an admirable goal, the cost considerations seem to me to be more imperative.

In conclusion, I congratulate Mr. Murray on the ingenuity of his idea and suggest that Inverse Liability, in my opinion, is a better choice than any of the other plans yet proffered to deal with the social problems of automobile liability insurance. I further suggest that the insurance industry would be well advised to study Mr. Murray's proposal quite carefully even though the question of cost cannot be ignored.

AUTHOR'S REVIEW OF DISCUSSIONS

I welcome the opportunity to thank reviewers Jack Moseley and Jerry Hillhouse for their comments on the subject of Inverse Liability.

They have pointed out two important areas where further research is indicated. The most important of these is the question of cost and I certainly hope some of the members will respond to Jerry Hillhouse's challenge in this respect. I have suggested the affinity of Inverse Liability to third party bodily injury, and since bodily injury claims are separated from property damage claims in the United States it should be possible for you to produce more accurate estimates of average cost than we can in Canada, where bodily injury and property damage are indivisible.

In my estimate of $60 for $100,000 coverage in Ontario I did not take into account the value of subrogation recoveries, and we should not lose sight of the fact that Inverse Liability includes Medical Payments (limited only by the sum insured) and Uninsured Motorist coverage (not limited to statutory minima). It would also be my recommendation that Inverse Liability be excess insurance over any specific accident, disability, major medical, or other insurance, so that Inverse Liability would be an umbrella coverage designed to take care of major losses. If consideration is given to these factors the net cost should be reduced to something under $40.

I also recommend that a deductible coverage be offered provided the insurer can still retain subrogation rights for his proportion of the loss paid, in the same manner as presently obtains under deductible collision coverage.

I agree with Jack Moseley's comments on the question of compulsory insurance. However, his comments are equally applicable to Uninsured Motorist coverage which as you know is now compulsory in several of the states.

The other matter I wish to touch upon briefly is the proposed method of settlement of claims, that is, by agreement with the insured, or, failing agreement, then by arbitration. One of my critics in England has pointed out, and rightly so, that under the present system we sometimes see very widely divergent awards for virtually the same injuries, and that Inverse Liability would engender the same problems. I agree with this comment but believe it should be viewed in the perspective that 98% of third party bodily injury claims in Canada are settled out of court. There will always be those who are difficult to deal with. There will always be a small percentage of people who will fraudulently exaggerate their claim against an insurance company. Loss adjusters tell me their greatest settlement difficulties usually arise with small claims under residence fire and burglary policies, so Inverse Liability is breaking no new ground in this respect! I remain hopeful that the right to call for medical examinations at any time will help to keep problem cases to a minimum.

In conclusion let me say that I am gratified that the broad outline of Inverse Liability is gaining acceptance both here and in Canada. The essence of the problem is to provide first party insurance for an amount which will give the insured an indemnity for economic losses arising out of automobile accidents, without the necessity to change our age-old law of torts, and by a vehicle which will be operated by the private insurance industry. To the best of my knowledge Inverse Liability is the only current proposal which accomplishes all these aims.

SCHEDULE P ON A CALENDAR/ACCIDENT YEAR BASIS

RUTH SALZMANN

INTRODUCTION

Schedule P has been the subject of considerable discussion and criticism over the years. Just recently two NAIC committees—the Actuarial (F5) Subcommittee and the Legislation to Modify Schedule "P" (D1) Subcommittee have been active in the Schedule P area. In the *Report on the Annual Statement* released in 1965 by the Committee on Annual Statement of this Society, Schedule P was listed as one of the subjects of persistent criticism directed toward the annual statement blank by the insurance industry.

The time is appropriate therefore to re-evaluate Schedule P. Serious consideration of ideal solutions and ultimate concepts should continue to be explored, but this paper directs its attention only to those improvements which are practical and feasible at the present time.

CALENDAR/ACCIDENT YEAR EXPLAINED

The proposal in the paper substitutes calendar/accident year data for split policy year data. This concept is not new. It has been inherent in several prior proposals including the one made by the Michigan Insurance Department to the Actuarial (F5) Subcommittee for its meeting on December 5, 1966.

The calendar/accident year basis recommended in the paper is one which assigns loss and loss expense to the year in which the accident occurred and assigns premiums earned to the calendar year in which such premiums were recorded as earned. Thus the earned premiums for each year are the same as the earned premiums reported on page 6 of the annual statement for that year. This means that the earned premiums, against which accident year losses are charged, remain a constant value or are "frozen" as of each year end.

On this basis, changes in accident year ratios from year to year in Parts 1 and 2 of Schedule P are the result of reserve developments entirely. Likewise calendar year loss ratios reported on page 8 can be compared with the loss ratios that subsequently develop for that year in Schedule P—Parts 1 and 2.

There is little question that calendar/accident year loss ratios are theoretically less accurate than policy year loss ratios. This is because the calendar/accident year basis does not match losses against the exact premiums for exposures generating such losses. However, it is to be remembered that the primary purpose of Schedule P is to assist in the determination of adequate reserve levels—not the precise measurement of loss ratios.

An illustration for the XYZ Company is appended to show with hypothetical data how the transition from policy year to calendar/accident year can be accomplished. The illustration is for workmen's compensation only, but a separate Exhibit D shows how the differences for bodily injury liability will be accommodated.

The proposed distributions of unallocated loss expense payments by accident year are the same as the present distributions in Parts 3 and 4 of Schedule P except that they are converted to an accident year basis. The distribution percentages reflect this conversion only. Although the present percentages may be arbitrary and need further study, the transition from policy year to accident year is not dependent upon the completion of such a study.

EXPANSION OF THE CALENDAR/ACCIDENT YEAR BASIS FOR SCHEDULE P

In addition to the redesign of Schedule P on a calendar/accident year basis, the proposal includes rearrangements of old items and the introduction of new items so that the advantages of the calendar/accident year basis can be fully exploited. The major advantages are three-fold:

1. *Simplification*

   With the elimination of policy year data from Schedule P, a simpler format results as can be noted in the appended illustration. Also with calendar/accident year data in Schedule P, the recording of policy year will no longer be necessary for annual statement purposes. This saving can be made without any loss in the real value of Schedule P, because loss and loss expense reserves can be tested equally well on an accident year basis.

2. *Total Loss Developments by Line*

   By rearranging and by adding certain information in Part 5, a retrospective reserve test for each Schedule P line in total can be made available in addition to the present tests by accident year.

   At the present time, loss reserve developments for each Schedule P line in total can be derived with considerable effort from various exhibits in the statement. For instance, developments over the last twelve months can be calculated by subtracting the loss volume for the current accident year in Part 5 from the respective calendar year incurred loss volume on page 8. This arithmetic produces developments on a Schedule O basis. For aggregate developments over a longer period of time than twelve months, Part 5 and Parts 1 and 2 for the current year are needed to produce the developed data from which accumulated losses in prior Parts 1 and 2 are then subtracted. This arithmetic produces loss developments for each successive calendar year similar to the continuation of developments shown in Schedule G.

To eliminate this separate arithmetic, a "prior year" line has been added to Part 5. Also sub-totals have been introduced so that aggregate reserve developments through sixty months will eventually be available. (These changes are shown in Exhibits B-3 and C-3 appended.)

In summary, then, the new Part 5 eliminates policy year detail; it continues loss reserve developments by accident year; but most important, it adds data so that aggregate reserve tests, now available for other lines in Schedules G and O, will be directly available for Schedule P lines.

3. *Prospective Evaluation of Liabilities*

Schedule P in its present form provides for retrospective reserve tests. Some simple uniform prospective test is needed to preempt the introduction of other complicated reserve testing formulas and exhibits. The author has created such a prospective evaluation and recommends that it become a new Part 6 for Schedule P. (See Exhibits B-4 and C-4 appended.)

In this new Part 6, current loss and loss expense reserves can be compared with reserve levels for prior accident years at the same stage of development. Such previous reserve levels are not those actually carried in the statement as of that date, but rather the reserve levels that should have been carried at that time in the light of subsequent developments. For example, to obtain the reserve dollars that should have been carried as of 12-31-61 for 1961 accident year, the payments for 1961 accident year in 1961 calendar year are subtracted from accumulated loss and loss expense incurred as currently reported in Parts 1 or 2. If the current date is 12-31-67, then the recalculated reserve for 1961 accident year as of 12-31-61 has the benefit of six years of hindsight.

Because each additional calendar year provides more information about more of the claims for any accident year, the current estimate of loss and loss expense incurred is more likely to approximate the final value than did any of the previous estimates. Likewise the current estimate should generate reserve levels for earlier stages of development by subtraction which are more accurate than any figure previously established.

Comparisons of current reserve levels with re-established reserves for prior accident years at the same stage of development will be most informative in appraising the reasonableness of current reserve levels. Emphasis is placed on the word "informative" because it must be recognized that a prospective evaluation of reserves, when only dollars are used, does not furnish any conclusive evidence regarding the adequacy of current reserve levels. An increase in paid ratios may reflect higher closing costs, a lower relative earned premium level, or a speeding-up of loss and loss expense payments. In the first two instances, a similar increase should be reflected in the liability; in the third instance, the increase should be offset in the liability. Variations are therefore not fully significant in themselves and cannot be arbitrarily used in measuring the adequacy of current reserve levels. However, such comparisons provide considerably more insight into current reserve levels than exists at present.

Obviously the most sophisticated approach in prospective evaluations of loss reserves is one that includes averages on closed claims, open claims, and total reported claims, and the percentage of claims closed through each stage of development. It is indeed unlikely that evaluations in such detail will ever be possible on a uniform basis in the annual statement. A prospective evaluation in dollars therefore is the more feasible approach.

Part 6 includes loss and loss expense. Losses only could have been used, but the author prefers the more comprehensive evaluation. If losses only were to be included, then of course the entire exhibit could be completed immediately because all of the historical data necessary are available except for earned premiums for homeowners and commercial multiple peril liability coverages in Part 6C.

THE COMPLICATION OF MINIMUM STATUTORY RESERVES

It is apparent from the above discussion that the change to calendar/accident year does not compromise the purpose of Schedule P; the only complication is in the fulfillment of minimum statutory reserve requirements. And this complication may be less formidable than one might at first expect. The following rationale was included on page 3 of Attachment 1 of the Actuarial (F5) Subcommittee proposal:

> "The statutory minimum ratios when applied to the latest 3 accident years produce a more conservative requirement than when applied to latest 3 policy years because the latest 3 accident years include all of the latest 3 policy years plus the premiums earned during the latest 3 years on policies written in previous years."

There is sound logic in the above statement, enough perhaps to accommoderate the recommended change under the existing statutes or at least sufficient encouragement to change the statutes to a calendar/accident year basis. However, if this reasoning is not enough, the author believes that the advantages which will accrue when the calendar/accident year basis is completely exploited will also produce the necessary additional incentives for regulatory authorities to endorse the calendar/accident year basis for calculating minimum statutory reserves. It is only on a calendar/accident year basis that the extra benefits described can be fully attained.

CONCLUDING REMARKS

The redesign of Schedule P proposed in this paper encompasses changes that can be put into effect now—if accident year data can serve as an acceptable basis for calculating minimum statutory reserves. The purpose of this paper was limited to that accomplishment. Further studies, however, are necessary in many areas not touched in this paper, such as:

1. *Lines of Business Included*

   a. Package policies now complicate the isolation of specific Schedule P coverages. The question therefore arises as to whether more meaningful and accurate data would be produced if losses were expanded to match total policy premiums (or easily quantified portions thereof) rather than the present method of apportioning indivisible premiums to get an income figure for Schedule P exposures only.

   b. Reinsurance assumed now complicates the isolation of specific

Schedule P coverages. The question therefore arises as to whether more meaningful and accurate data would be produced if Schedule P were on an "adjusted direct" basis (with or without facultative cessions) rather than the present net basis.

c. International business now complicates the compilation of Schedule P data. The question therefore arises as to whether more meaningful and accurate data would be produced if only domestic business were included.

2. *Distribution of Unallocated Claim Expenses*

The present percentages used to distribute unallocated claims expense by policy year or accident year in Schedule P are arbitrary. Industry studies might be undertaken to determine unallocated claims expense distributions by size of claim and by age of claim.

3. *Elimination of Premiums Earned*

Without earned premiums in Schedule P, many problems associated with the matching of claims against premiums as discussed in above (Item 1) will be eliminated. Schedule P losses could then be strictly limited to bodily injury liability and compensation coverages. The advantages of a Schedule P without premiums will only become possible, of course, if minimum statutory reserve requirements are related to some criterion other than premiums.

Schedule P needs further study in the above areas, and perhaps others as well, before it becomes a truly meaningful Schedule. This does not prevent us, however, from taking some constructive steps now. This paper addresses itself to those constructive steps which should be taken now—changes that will improve Schedule P and will also eliminate much of the present criticism of the Schedule.

APPENDIX

Illustration
XYZ Company
Workmen's Compensation

A. Present Policy Year Basis—December 31, 1966
(Rearranged to facilitate comparisons with Sections B and C)

A-1 Schedule P—Part 2

A-2 Schedule P—Part 5D

B. Initial Year of Transition—December 31, 1967

B-1 Schedule P—Part 2

B-2 Schedule P—Part 4

B-3 Schedule P—Part 5D

B-4 Schedule P—Part 6D

C. Ultimate Calendar/Accident Year Basis—December 31, 1967

C-1 Schedule P—Part 2

C-2 Schedule P—Part 4

C-3 Schedule P—Part 5D

C-4 Schedule P—Part 6D

D. Special Notes for Bodily Injury Liability Coverages

XYZ Company

December 31, 1966

Schedule P - Part 2

(Present Policy Year Basis)

Reserve for Unpaid W.C. Loss & Loss Expense December 31 of Current Year

Year Policies Issued	(1) Premiums Earned	(2) Losses Paid	(3) Losses O/S	(4) Losses Incurred	(5) Losses Ratio	(6) Loss Expense Alloc. Paid	(7) Loss Expense Unalloc. Paid	(8) Loss Expense O/S	(9) Loss Expense Incurred	(10) Loss Expense Ratio	(11) Loss & Loss Expense Incurred	(12) Loss & Loss Expense Ratio
< 1959	$25,000	$14,000	$ 850	$14,850	59.4	$ 650	$1,870	$ 20	$2,540	10.2	$17,390	69.6
1959	2,500	1,500	150	1,650	66.0	70	170	10	250	10.0	1,900	76.0
1960	2,800	1,600	200	1,800	64.3	75	180	15	270	9.6	2,070	73.9
1961	3,000	1,800	300	2,100	70.0	85	195	25	305	10.2	2,405	80.2
1962	3,500	1,900	350	2,250	64.3	95	200	35	330	9.4	2,580	73.7
1963	4,000	2,200	500	2,700	67.5	120	220	45	385	9.6	3,085	77.1
Total first period	40,800	23,000	2,350	25,350	62.1	1,095	2,835	150	4,080	10.0	29,430	72.1
1964	4,700	2,200	1,000	3,200	68.1	105	230	110	445	9.5	3,645	77.6
1965	5,100	1,800	1,800	3,600	70.6	85	215	205	505	9.9	4,105	80.5
1966	2,650	500	1,800	2,300	86.8	10	105	205	320	12.1	2,620	98.9
Total second period	12,450	4,500	4,600	9,100	73.1	200	550	520	1,270	10.2	10,370	83.3
Total	53,250	27,500	6,950	34,450	64.7	1,295	3,385	670	5,350	10.0	39,800	74.7

Computation of Reserve for Unpaid W.C. Loss & Loss Expense

	(13) 65% P.E. in Col. 1	(14) L & LE Payments (2)+(6)+(7)	(15) Remainder (13)-(14)	(16) L & LE O/S (3)+(8)	(17) (15) or (16) whichever is greater	(18)	(19)	(20)	(21)
						← old format →			
1964	$3,055	$2,535	$ 520	$1,110	$1,110				
1965	3,315	2,100	1,215	2,005	2,005				
1966	1,723	615	1,108	2,005	2,005				
Total	8,093	5,250	2,843	5,120	5,120				
(22) Reserve for O/S L & LE, first period					2,500				
(23) Reserve for O/S L & LE, second period					5,120				
(24) Total					7,620				

A-2

XYZ Company

December 31, 1966

Schedule P - Part 5D

(Present Policy Year Basis)

Development of Incurred W.C. Losses

Policy Year	Accident Year	Reserve Date 12-31-61	12-31-62	12-31-63	12-31-64	12-31-65	12-31-66
1961	1961	$1,290	$1,280	$1,270	$1,260	$1,255	$1,250
1961	1962	X	870	865	860	855	850
*1961	1963	X	X	0	0	0	0
*1961	1964	X	X	X	0	0	0
1962	1962	X	1,475	1,450	1,425	1,410	1,400
1962	1963	X	X	880	870	860	850
*1962	1964	X	X	X	0	0	0
*1962	1965	X	X	X	X	0	0
1963	1963	X	X	1,700	1,700	1,700	1,700
1963	1964	X	X	X	1,060	1,030	1,000
*1963	1965	X	X	X	X	0	0
*1963	1966	X	X	X	X	X	0
1964	1964	X	X	X	2,000	2,000	2,000
1964	1965	X	X	X	X	1,250	1,200
*1964	1966	X	X	X	X	X	0
1965	1965	X	X	X	X	2,200	2,200
1965	1966	X	X	X	X	X	1,400
1966	1966	X	X	X	X	X	2,300

* These lines to be filled in only by companies which charge all losses under policies running for a period of more than one year to the original policy year of issue.

XYZ Company

December 31, 1967

Schedule P - Part 2

(Initial Year of Transition)

Reserve for Unpaid W.C. Loss & Loss Expense December 31 of Current Year

Policy Year Policies Issued	Calendar Year Policies Earned	Accident Year Losses Incurred I	Accident Year Losses Incurred II	(1) Premiums Earned	(2) Losses Paid	(3) Losses O/S	(4) Losses Incurred	(5) Losses Ratio	(6) Loss Expense Alloc. Paid	(7) Loss Expense Unalloc. Paid	(8) Loss Expense O/S	(9) Loss Expense Incurred	(10) Loss Expense Ratio	(11) Loss & Loss Expense Incurred	(12) Loss & Loss Expense Ratio
< 1960	< 1967	< 1961	< 1964	$27,500	$15,555	$ 905	$16,460	59.9	$ 725	$2,040	$ 25	$2,790	10.1	$19,250	70.0
1960	"	60 & 61	60 - 64	2,800	1,610	170	1,780	63.6	80	180	10	270	9.6	2,050	73.2
1961	"	61 & 62	61 - 65	3,000	1,865	225	2,090	69.7	85	195	20	300	10.0	2,390	79.7
1962	"	62 & 63	62 - 66	3,500	1,945	300	2,245	64.1	100	200	25	325	9.3	2,570	73.4
1963	"	63 & 64	63 - 66	4,000	2,265	415	2,680	67.0	130	220	35	385	9.6	3,065	76.6
1964	"	64 & 65	64 - 66	4,700	2,490	650	3,140	66.8	155	245	45	445	9.5	3,585	76.3
Total first period				45,500	25,730	2,665	28,395	62.4	1,275	3,080	160	4,515	9.9	32,910	72.3
1965	"	1965 & 1966		5,100	2,390	1,060	3,450	67.7	150	245	100	495	9.7	3,945	77.4
1966	"	1966		2,650	1,330	870	2,200	83.0	50	120	145	315	11.9	2,515	94.9
-	1967	1967		5,800	1,150	2,785	3,935	67.8	20	240	320	580	10.0	4,515	77.8
Total second period				13,550	4,870	4,715	9,585	70.7	220	605	565	1,390	10.3	10,975	81.0
Total				59,050	30,600	7,380	37,980	64.3	1,495	3,685	725	5,905	10.0	43,885	74.3

Computation of Reserve for Unpaid W.C. Loss & Loss Expense

			(13) 65% P.E. in Col. 1	(14) L & LE Payments (2)+(6)+(7)	(15) Remainder (13)-(14)	(16) L & LE O/S (3)+(8)	(17) (15) or (16) whichever is greater	(18)	(19)	(20)	(21)
								← old format			→
1965	< 1967	1965 & 1966	$3,315	$2,785	$ 530	$1,160	$1,160				
1966	"	1966	1,723	1,500	223	1,015	1,015				
-	1967	1967	3,770	1,410	2,360	3,105	3,105				
Total			8,808	5,695	3,113	5,280	5,280				
(22) Reserve for O/S L & LE, first period							2,825				
(23) Reserve for O/S L & LE, second period							5,280				
(24) Total							8,105				

I Companies assigning term policies to each annual policy year involved

II Companies assigning term policies to original policy year

XYZ Company

December 31, 1967

Schedule P - Part 4

(Initial Year of Transition)

Distribution of Unallocated Compensation Claim Expenses

Policy Year	Accident Year I	or	Accident Year II	Current Year Distribution %'s B	or A*	(1) Current Year Paid	(2) Prior Years Paid	(3) Total Paid (Col. 1 + Col. 2)
< 1964	< 65		< 67	0		$ 0	$2,835	$2,835
1964	64 & 65		64 - 66	5		15	230	245
1965	1965 & 1966			10		30	215	245
1966	1966			5		15	105	120
-	1967			80		240	0	240
Total				100	100	300	3,385	3,685

I Companies assigning term policies to each annual policy year involved.
II Companies assigning term policies to original policy year.
A For Companies which have been issuing policies less than 5 years.
B For Companies which have been issuing policies 5 years or more.
* Determine %'s by expanding the %'s in B for the number of years applicable to 100%.

NOTE: The B distribution %'s were taken from IASA, Insurance Accounting - Fire and Casualty, p. 168 (2nd Edition)

XYZ Company B-3

December 31, 1967

Schedule P - Part 5D

(Initial Year of Transition)

Development of Incurred W.C. Losses

Policy Year	Accident Year	Reserve Date 12-31-62	12-31-63	12-31-64	12-31-65	12-31-66	12-31-67
< 1962	< 1964	$20,800	$20,700	$20,600	$20,500	$20,400	$20,330
1962	1962	1,475	1,450	1,425	1,410	1,400	1,400
1962	1963	X	880	870	860	850	845
*1962	1964	X	X	0	0	0	0
*1962	1965	X	X	X	0	0	0
1963	1963	X	1,700	1,700	1,700	1,700	1,695
1963	1964	X	X	1,060	1,030	1,000	985
*1963	1965	X	X	X	0	0	0
*1963	1966	X	X	X	X	0	0
1964	1964	X	X	2,000	2,000	2,000	1,990
1964	1965	X	X	X	1,250	1,200	1,150
*1964	1966	X	X	X	X	0	0
1965	1965	X	X	X	2,200	2,200	2,150
1965	1966	X	X	X	X	1,400	1,300
1966	1966	X	X	X	X	2,300	2,200
Sub-total		X	X	X	X	34,450	34,045
-	1967	X	X	X	X	X	3,935

* These lines to be filled in only by companies which charge all losses under policies running for a period of more than one year to the original policy year of issue.

XYZ Company

December 31, 1967

Schedule P - Part 6D

(Initial Year of Transition)

Comparison of Reserves by Accident Year for W.C. Loss & Loss Expense December 31 of Current Year

	Calendar Year (Premiums Earned)/Accident Year (Loss & Loss Expense)													
	1961*	1962*	1963*	1964*	1965*	1966*	1967	1961*	1962*	1963*	1964*	1965*	1966*	1967
				Dollars								Percentages		
						Summary Data from Schedule P - Part 2								
1. Premiums Earned							5,800	100.0	100.0	100.0	100.0	100.0	100.0	100.0
2. Loss & Loss Exp. Incurred							4,515							77.8
						Loss & Loss Expense thru 12 Months								
3. Paid							1,410							24.3
4. Reserve (2)-(3)							3,105							53.5
						Loss & Loss Expense thru 24 Months								
5. Paid							X							X
6. Reserve (2)-(5)							X							X
						Loss & Loss Expense thru 36 Months								
7. Paid						X	X						X	X
8. Reserve (2)-(7)						X	X						X	X
						Loss & Loss Expense thru 48 Months								
9. Paid					X	X	X					X	X	X
10. Reserve (2)-(9)					X	X	X					X	X	X
						Loss & Loss Expense thru 60 Months								
11. Paid				X	X	X	X				X	X	X	X
12. Reserve (2)-(11)				X	X	X	X				X	X	X	X

* The completion of data for these years is optional

SCHEDULE P

Schedule P - Part 2

(Ultimate Calendar/Accident Year Basis)

Reserve for Unpaid W.C. Loss & Loss Expense December 31 of Current Year

Calendar Year Policies Earned/ Accident Year Losses Incurred	(1) Premiums Earned	(2) Losses Paid	(3) Losses O/S	(4) Losses Incurred	(5) Losses Ratio	(6) Loss Expense Alloc. Paid	(7) Loss Expense Unalloc. Paid	(8) Loss Expense O/S	(9) Loss Expense Incurred	(10) Loss Expense Ratio	(11) Loss & Loss Expense Incurred	(12) Loss & Loss Expense Ratio
< 1960	$25,950	$15,000	$ 850	$15,850	61.1	$ 700	$1,955	$ 20	$2,675	10.3	$18,525	71.4
1960	2,700	1,570	170	1,740	64.5	75	170	10	255	9.4	1,995	73.9
1961	3,000	1,775	175	1,950	65.0	80	190	15	285	9.5	2,235	74.5
1962	3,300	1,900	290	2,190	66.4	90	200	25	315	9.5	2,505	75.9
1963	3,700	2,190	350	2,540	68.7	130	200	30	360	9.7	2,900	78.4
1964	4,400	2,435	540	2,975	67.6	135	240	40	415	9.4	3,390	77.0
Total first period	43,050	24,870	2,375	27,245	63.3	1,210	2,955	140	4,305	10.0	31,550	73.3
1965	4,900	2,465	835	3,300	67.3	180	250	60	490	10.0	3,790	77.3
1966	5,300	2,115	1,385	3,500	66.0	85	240	205	530	10.0	4,030	76.0
1967	5,800	1,150	2,785	3,935	67.8	20	240	320	580	10.0	4,515	77.8
Total second period	16,000	5,730	5,005	10,735	67.1	265	730	585	1,600	10.0	12,335	77.1
Total	59,050	30,600	7,380	37,980	64.3	1,495	3,685	725	5,905	10.0	43,885	74.3

Computation of Reserve for Unpaid W.C. Loss & Loss Expense

	(13) 65% P.E. in Col. 1	(14) L & LE Payments (2)+(6)+(7)	(15) Remainder (13)-(14)	(16) L & LE O/S (3)+(8)	(17) (15) or (16) whichever is greater	(18) ← old format	(19) old format	(20) old format	(21) old format →
1965	$ 3,185	$2,895	$ 290	$ 895	$ 895				
1966	3,445	2,440	1,005	1,590	1,590				
1967	3,770	1,410	2,360	3,105	3,105				
Total	10,400	6,745	3,655	5,590	5,590				
(22) Reserve for O/S L & LE, first period					2,515				
(23) Reserve for O/S L & LE, second period					5,590				
(24) Total					8,105				

December 31, 1967

Schedule P - Part 4

(Ultimate Calendar/Accident Year Basis)

Distribution of Unallocated Compensation Claim Expenses

	(1)			(2)	(3)
	Current Year			Prior Years	Total Paid
Accident Year	Distribution %'s		Paid	Paid	(Col. 1 + Col. 2)
	B	or A*			
< 1965	0		$ 0	$2,955	$2,955
1965	10		30	220	250
1966	10		30	210	240
1967	80		240	0	240
Total	100	100	300	3,385	3,685

A For Companies which have been issuing policies less than 5 years.
B For Companies which have been issuing policies 5 years or more.
* Determine %'s by expanding the %'s in B for the number of years applicable to 100%.

NOTE. The B distribution %'s were taken from IASA, Insurance Accounting - Fire and Casualty, p 168 (2nd Edition).

XYZ Company

December 31, 1967

Schedule P - Part 5D

(Ultimate Calendar/Accident Year Basis)

Development of Incurred W.C. Losses

Accident Year	Reserve Date 12-31-62	12-31-63	12-31-64	12-31-65	12-31-66	12-31-67
< 1962	$20,000	$19,880	$19,770	$19,665	$19,565	$19,540
1962	2,275	2,270	2,255	2,245	2,235	2,190
Sub-total	22,275	22,150	22,025	21,910	21,800	21,730
1963	x	2,580	2,570	2,560	2,550	2,540
Sub-total	x	24,730	24,595	24,470	24,350	24,270
1964	x	x	3,060	3,030	3,000	2,975
Sub-total	x	x	27,655	27,500	27,350	27,245
1965	x	x	x	3,450	3,400	3,300
Sub-total	x	x	x	30,950	30,750	30,545
1966	x	x	x	x	3,700	3,500
Sub-total	x	x	x	x	34,450	34,045
1967	x	x	x	x	x	3,935

XYZ Company

December 31, 1967

Schedule P - Part 6D

(Ultimate Calendar/Accident Year Basis)

Comparison of Reserves by Accident Year for W.C. Loss & Loss Expense December 31 of Current Year

	Calendar Year (Premiums Earned)/Accident Year (Loss & Loss Expense)													
	1961	1962	1963	1964	1965	1966	1967	1961	1962	1963	1964	1965	1966	1967
	Dollars							Percentages						
	Summary Data from Schedule P - Part 2													
1. Premiums Earned	3,000	3,300	3,700	4,400	4,900	5,300	5,800	100.0	100.0	100.0	100.0	100.0	100.0	100.0
2. Loss & Loss Exp. Incurred	2,235	2,505	2,900	3,390	3,790	4,030	4,515	74.5	75.9	78.4	77.0	77.3	76.0	77.8
	Loss & Loss Expense thru 12 Months													
3. Paid	740	850	925	1,090	1,235	1,260	1,410	24.7	25.8	25.0	24.8	25.2	23.8	24.3
4. Reserve (2)-(3)	1,495	1,655	1,975	2,300	2,555	2,770	3,105	49.8	50.1	53.4	52.2	52.1	52.2	53.5
	Loss & Loss Expense thru 24 Months													
5. Paid	1,400	1,590	1,790	2,090	2,300	2,440	X	46.7	48.2	48.4	47.5	46.9	46.0	X
6. Reserve (2)-(5)	835	915	1,110	1,300	1,490	1,590	X	27.8	27.7	30.0	29.5	30.4	30.0	X
	Loss & Loss Expense thru 36 Months													
7. Paid	1,690	1,900	2,210	2,595	2,895	X	X	56.3	57.6	59.7	59.0	59.1	X	X
8. Reserve (2)-(7)	545	605	690	795	895	X	X	18.2	18.3	18.7	18.0	18.2	X	X
	Loss & Loss Expense thru 48 Months													
9. Paid	1,840	2,065	2,400	2,810	X	X	X	61.3	62.6	64.9	63.8	X	X	X
10. Reserve (2)-(9)	395	440	500	580	X	X	X	13.2	13.3	13.5	13.2	X	X	X
	Loss & Loss Expense thru 60 Months													
11. Paid	1,940	2,180	2,520	X	X	X	X	64.7	66.1	68.1	X	X	X	X
12. Reserve (2)-(11)	295	325	380	X	X	X	X	9.8	9.8	10.3	X	X	X	X

SCHEDULE P

Special Notes for Bodily Injury Liability Coverages D

1. Schedule P - Part 1

Suit data can be included in Part 1 in its present detail. The amount charged for each suit, however, can be converted to an accident year basis as follows:

Initial Year of Transition					Ultimate Calendar/Accident Year Basis	
Policy Year Policies Issued	Calendar Year Policies Earned	Accident Year Losses Incurred I	Accident Year Losses Incurred II	Amount Charged for each Suit	Calendar Year Policies Earned/ Accident Year Losses Incurred	Amount Charged for each Suit
< 1960	< 1967	< 1961	< 1964	$1,500	< 1960	$1,500
1960	"	60 & 61	60 - 64	1,000	1960	1,250
1961	"	61 & 62	61 - 65	1,000	1961	1,000
1962	"	62 & 63	62 - 66	1,000	1962	1,000
1963	"	63 & 64	63 - 66	850	1963	925
1964	"	64 & 65	64 - 66	850	1964	850
Total first period					Total first period	
1965	"	1965 & 1966		750	1965	800
1966	"	1966		750	1966	750
-	1967	1967		750	1967	750
Total second period					Total second period	
Total					Total	

2. Schedule P - Part 3

The B distribution %'s for Part 3 were taken from IASA, Insurance Accounting - Fire and Casualty, p. 168 (2nd Edition), as were the B distribution %'s for Part 4 shown in exhibits B-2 and C-2. These distribution %'s are as follows.

Initial Year of Transition				Ultimate Calendar/Accident Year Basis	
Policy Year	Accident Year I	Accident Year II	Distribution %'s B	Accident Year	Distribution %'s B
< 1963	< 64	< 66	0	< 1964	0
1963	63 & 64	63 - 66	5	1964	10
1964	64 & 65	64 - 66	10	1965	10
1965	1965 & 1966		10	1966	10
1966	1966		5	1967	70
-	1967		70	Total	100
Total			100		

DISCUSSION BY FRANCIS J. HOPE

It was a pleasure to review this paper, because it is concise, the ideas are clearly stated, and it suggests some things that could be done now and some thought-provoking subjects for the future.

The first stated purpose of the proposed revision of Schedule P is simplification, and with this we can hardly have any quarrel. If the primary purpose of Schedule P is to give some indication of current reserve position, then it may be done equally well on an accident year basis as on a policy year basis, and with less detail in processing. Admittedly the minimum statutory requirements present a hurdle to be overcome.

Certain rationale is cited from the report of the Actuarial (F5) Subcommittee of the N.A.I.C., to the effect that application of the statutory minimum ratios to three years of calendar/accident year premium would produce a more conservative requirement than when applied to three policy year premiums, since the calendar year premium would be greater.

As a technical point it might be noted first that, with respect to policies still in effect at the beginning of the period, such policies would be contributing losses as well as premium into the calendar/accident year period, and thus would not necessarily make the requirement more conservative. Policies which had already expired would quite probably contribute positive amounts to earned premium in the form of audit premiums, but these might be more than offset by negative amounts from retrospective adjustments from time to time.

This technical point should hardly be a deterrent to the use of earned calendar year premiums, since the effect would be quite negligible, and the statutory minimum ratios themselves do not suggest any precise form of measurement.

There is a proposal to revise Part 5 so as to show various sub-totals and aggregate developments by line, and this might be done in a number of ways. If space permits, we would suggest that an additional column be inserted between the columns as proposed, in which the amount of calendar year development could be shown by accident year. This again would merely be a saving in arithmetic, and would show at a glance the amount of contribution to calendar year incurred loss made by each accident year.

The most interesting and challenging part of the paper is the section on prospective evaluation of liabilities. Miss Salzmann would add a new Part 6 to Schedule P, in which she would trace paid and incurred losses by accident year through a series of year-end evaluation dates. By subtracting successive paid amounts from the latest known incurred amount, she obtains what may be considered the most appropriate reserve that could have been established at each prior year end. The two elements of loss would also be expressed as percentages of earned premium as of each evaluation date.

If all elements of loss development maintained a consistent pattern in relation to each other and to earned premium, the later accident years could be projected to ultimate incurred on the basis of older accident year developments, and current reserves evaluated accordingly. Miss Salzmann wisely and properly emphasizes that this is not necessarily so, and that the date would be "informative" but not "conclusive evidence" as to adequacy of current reserves.

To this writer the data would be useful for observing trends, and even more useful in that any significant departure from what appears to be a general pattern should provoke a study in depth, beyond the material in Schedule P. This would include average costs on closed claims, rate of settlement, etc., i.e., the elements named for the "most sophisticated approach" in the paper.

Another approach might be to relate paid losses to the latest known incurred loss, eliminating the factor of premium adequacy, but here again the data could only be informative, because the ratios indicated by a sufficiently mature accident year would not reflect changes taking place since that time.

In her concluding remarks, Miss Salzmann names several other areas which should be studied in a redesign of Schedule P, and among them is the matter of distributing unallocated claim expenses. In such a study, question might be raised as to whether this element of expense should even be included in Schedule P. Recognizing that in all other respects the two types of claim expense must be kept in close association with each other, and with losses, it seems nevertheless that unallocated claim expense is relatively more static, akin to administration expense, and does not belong in an exhibit tracing developments on the more uncertain and volatile elements of loss and allocated claim expense.

To conclude — and as always, it is a pleasure to compliment Miss Salzmann on her paper.

DISCUSSION BY PAUL M. OTTESON

Ruth Salzmann's paper suggests improvements to Schedule P "which are practical and feasible at the present time." With this limitation of subject matter scope in mind the author proposes a calendar/accident year basis to replace the policy year basis now used in establishing the Parts 1 and 2 statutory reserve requirements; and she also presents new exhibits to replace the present Part 5 now used to test and reflect adequacy and accuracy of balance sheet unpaid loss estimates.

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The Schedule P reserve now appearing in the liability section of the balance sheet can consist of either or both of two elements which are completely different in nature: (1) a voluntary reserve established according to no prescribed rules or standards, and (2) a statutory reserve requirement based directly on an incurred loss ratio formula. These two very different types of reserves are included on this "line" singly or in composite and without distinction or identification.

The author chose not to consider the voluntary reserve aspect of the problem nor to consider whether the statutory reserve requirement under either the present or proposed basis really serves a useful purpose. The objective of the paper on this point therefore relates to simplification and economy rather than significant improvement in the finished product. Nevertheless, the simplification contribution is very real and most worthwhile.

The statutory reserve requirements now computed according to a policy year basis are no more meaningful or useful than they would be computed according to the proposed calendar/accident year method; and the policy year basis does involve additional complications and expense.

The author's material comparing calendar/accident year results with policy year results is appropriate and well presented.

DEVELOPMENT OF INCURRED LOSSES

The most important contribution of Miss Salzmann's paper lies in her suggested exhibits pertaining to development of incurred losses with an eye toward the future as well as on the past.

Part 5 of Schedule P now represents a very valuable and important exhibit; the retrospective picture of unpaid loss adequacy and accuracy, however, is not presented as clearly or forcefully as it might be. The complete message comes through "loud and clear" under the "aggregate" development proposed in Miss Salzmann's paper.

Part 6 as proposed should be an extremely valuable addition to the annual statement. However, the reviewer believes very strongly that this exhibit should be on a "losses only" basis the same as the present and proposed Part 5. Since she considers the present Schedule P "timing" percentages for unallocated claims expense as arbitrary it is not clear why the author chooses "the most comprehensive evaluation" basis combining losses and loss expense.

A practical application of these exhibits using actual company data reveals that they will prove to be most effective and useful. Companies will find this type of exhibit very worthwhile whether or not it becomes part of the official annual statement blank.

CONCLUDING REMARKS

The proposals relating to Parts 5 and 6 should be considered for use without delay. The reviewer hopes that a broader study encompassing the entire area of "voluntary reserves" and "statutory reserve requirements" could still be made without interfering with the change in Parts 1 and 2 from a policy year to a calendar/accident year basis.

Miss Salzmann's paper represents a valuable "improved Schedule P" contribution.

DISCUSSIONS OF PAPERS PUBLISHED IN VOLUME LIII

CURRENT RATEMAKING PROCEDURES IN BOILER AND MACHINERY INSURANCE

JAMES F. BRANNIGAN

VOLUME LIII, PAGE 248

DISCUSSION BY ERNEST T. BERKELEY

Papers on ratemaking are read by students probably more than papers on other subjects since they serve as a very convenient and authoritative reference in preparing for the examinations of the Casualty Actuarial Society, but they are also certainly useful to many others, both members and nonmembers of the Society, as a means of keeping informed concerning the contemporary methods of developing rates, which are a vital determinant in the fortunes of our business.

I am reviewing the paper from the viewpoint of an old student who studied for the Society examinations some thirty-five years ago. My reaction to the paper may vary somewhat from that of a present-day student, and yet I am sure there is a common feeling that papers on ratemaking in the Proceedings are most welcome and satisfy a long standing need.

As I read the paper I tried to look at it through the eyes of a young student but I really couldn't do it. Too many years have gone by. I suppose the student looks for a certain kind of format, perhaps, explanations of things I take for granted and illustrations and examples of procedures I would consider to be unnecessary. Another important consideration from anybody's point of view is the fact that due to the size and nature of the boiler and machinery line, one should not expect the degree of refinement found in workmen's compensation ratemaking procedures, for example.

My own feeling is that Mr. Brannigan has made an excellent presentation of the ratemaking procedures currently in use for boiler and machinery insurance. His approach is orderly, logical, and thorough, thus fulfilling in a very satisfactory way the educational intent of the paper. I think Mr. Brannigan is to be complimented on the fine job he has done and the contribution he has made to the growing ratemaking literature of the Society, which already includes similar papers covering some of the better known lines of business such as workmen's compensation, automobile, liability, and fire.

The paper begins with a description of the coverage and the determination of the manual premium, which provides the necessary background for the subsequent description of the ratemaking statistics and finally the details of the rate making procedure itself, by using the actual rate revision of 1961.

As the author indicates, his paper is intended to be wholly descriptive and he makes no attempt to evaluate the procedures described. I am not in a position to make an evaluation either, but I should like to comment briefly on two points that seem to me to have special significance.

The first has to do with credibility, the basis of which the author describes as follows: "The requirement of $7,000,000 of five calendar years of earned premium at present rate level for full credibility was established much the same as the $5,000,000 was for Fire, on a judgment basis. The premium requirements for less than full credibility are calculated using the common partial credibility formula $Z^2 = \frac{P}{N}$ where P is the premium for the object type and N is $7,000,000, or the premium required for 100% credibility." While the word "judgment" may have a number of meanings, I believe it is used here to indicate a basis which is largely non-scientific or non-actuarial in nature. Despite the fact that boiler and machinery differs from most other lines of insurance in that countrywide data are used in ratemaking with no territorial breakdown, thus keeping premium volume at relatively high levels, I believe that a partial credibility factor must come into play very frequently in the determination of the rate level change by object, thus making it important that the full credibility standard be determined as accurately as possible. Credibility problems of various kinds are found in other lines as well, but this does not mean that the credibility procedures generally in use are of questionable value. Rather it is a situation where further research would result in refinements leading to answers of somewhat greater accuracy.

The second point is that I did not notice any reference in the paper to loss development factors, having in mind that losses are on an accident year, calendar year basis. While these factors are probably less important than in some other lines because of the quicker settlement of property claims, I became curious about them and learned on inquiry that another review of rates is currently in progress. A loss development factor is being introduced and probably reflects incurred but not reported losses more than the development of outstanding cases.

On page 261 of the 1966 *Proceedings* a small correction should be noted. In the explanation of the various columns of Exhibit VI the statement is made that: "Column (12) shows the relationship of each of the object formula loss and inspection ratios to that for all objects combined (.593) for this body of experience." The figure (.593) should be (.601).

IMPLICATIONS OF SAMPLING THEORY FOR PACKAGE POLICY RATEMAKING

JEFFREY T. LANGE

VOLUME LIII, PAGE 285

DISCUSSION BY CLYDE H. GRAVES

The making of rates for package policies has required the attention of actuaries in rating bureaus and of company actuaries for a number of years. A knowledge of current procedures for the rating of the automobile package policy, the special multi-peril policy and homeowners policies can be obtained by studying the rate filings made in the various states by the National Bureau of Casualty Underwriters, the Mutual Insurance Rating Bureau, and the Multi-Line Insurance Rating Bureau. Individual companies have also developed and rated multi-line package policies.

The *Proceedings* of the Casualty Actuarial Society is sadly lacking, however, in papers explaining the making of package policy rates. A few papers have dealt with theoretical considerations in the rating of such policies, namely, "Multiple Peril Rating Problems—Some Statistical Considerations" by Robert L. Hurley* and "Commercial Package Policies—Rating and Statistics" by Robert A. Bailey, Edward J. Hobbs, Frederick J. Hunt, Jr., and Ruth E. Salzmann.** Lange's paper "Implications of Sampling Theory for Package Policy Ratemaking," which was presented at the November 1966 meeting of the Casualty Actuarial Society, is a welcome addition to the thinking on this subject.

Half of Mr. Lange's paper is a brief review of certain aspects of sampling theory dealing with the techniques of stratification and ratio estimation. Mr. Lange applies these techniques to the problem of rating multi-line policy forms. As he stated in the introduction to his paper;

> "The essence of the method is that package policy experience will be subdivided by coverage for ratemaking, and will be used in combination with non-package experience in determining rate levels and rate relationships. Differentials will be computed for each coverage between package and non-package data to reflect the differences between these two classes of risks."

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* *Proceedings Casualty Actuarial Society,* Volume XLVI, p. 196
** *Proceedings Casualty Actuarial Society,* Volume L, p. 87

As Mr. Lange pointed out, when a package policy is first introduced, its rates are generally constructed from the non-package rates for component coverages with appropriate discounts. Later experience may develop to the point that the rates for the package policy may be determined on its own experience. This is the history of homeowners ratemaking. As Mr. Lange and others have observed, however, this has led to a problem in the rating of the residual fire dwelling business.

Mr. Lange is suggesting in his paper that the use of ratio estimation could be applied in the rating of both the package policy and the non-package policies by utilizing the experience developed under both forms. It would be interesting to test Mr. Lange's suggestion for rating package policy by using regular dwelling fire and extended coverage experience together with homeowners experience. Would the use of the combined experience result in better rates in the sense that the pure premiums so determined would be better estimates of the expected loss costs, or would the procedure result in inadequate rates for dwelling fire and extended coverage and excessive rates for homeowners?

In recent filings made in Virginia, both the National Bureau and Mutual Bureau proposed changes in the automobile package policy rates which were calculated by the following formula:

(1) The sum of the revised family automobile policy rates for bodily injury at \$10,000/\$20,000 limits, and property damage at \$5,000 limits, and \$1,000 medical payments was determined.

(2) This sum was reduced by applying a 15% discount, and then further reduced by one half to determine a semi-annual rate.

(3) An increased limit factor of 1.05 was then applied to determine a \$35,000 single limit premium.

(4) Uninsured motorists coverage was included at a \$2 flat charge.

It should be noted that this formula does not make use of the special automobile package policy at all but determines the rates entirely on family automobile policy experience. It should be pointed out, however, that the percentage of private passenger automobile liability business written under the special automobile package policy form in Virginia is only about 11% of the total. The loss and loss adjustment ratio for the special automobile package policy in Virginia for accident year 1965, based on the experience of all companies reporting to the National Bureau and Mutual Bureau was .658 compared to a loss and loss adjustment ratio of .670 for the family automobile policy.

The Actuarial Committee of the Mutual Insurance Rating Bureau has recommended to the Automobile Rating Committee that the experience utilized in determining Mutual Bureau private passenger automobile liability rates be based on the combined experience of all companies reporting to the Mutual Bureau and National Bureau on both the family automobile policy form and the package automobile policy form for bodily injury and property damage liability coverages. The medical payment component of the two policy forms will be separately determined. It is quite possible that the suggestion made by Mr. Lange for the use of ratio estimation will be helpful. It will require further study and tests, but certainly the experience developed under the package policy forms should no longer be ignored in the making of package policy rates. This situation also exists in rating the special multi-peril policy forms. There is some hope that with the development of the Commercial Risk Statistical Plan by the National Insurance Actuarial and Statistical Association data will be available to test Mr. Lange's suggestions in the rating of commercial package policies.

DISCUSSION BY DALE NELSON

This paper is another in a series of studies on the application of contemporary mathematical developments to the problems of the actuarial sciences both in terms of providing the theoretical justification for, and introducing new techniques into, actuarial practices. Specifically, this paper is concerned with the application of two techniques of sampling theory—stratification and ratio estimation—to (package) ratemaking. My remarks will be confined to a critique of the statistical theory involved, and I will leave the practical aspects of the implied ratemaking process for others to discuss. It might be observed in passing, though, that Mr. Lange has presented some persuasive arguments in favor of sampling theory in package ratemaking: the ability to incorporate more accurate trend, credibility, and loss development factors as well as to analyze the design of package policies, among others.

The two basic ideas discussed in this paper are in fact, if not in name, well-known to all of us. For example, ratio estimation is used, among other places, in the derivation of loss development factors. Similarly, the classification of risks by territory and class grouping is nothing other than stratification. However, it should be pointed out that this form of stratification has a different purpose from that in statistical sampling. In ratemaking (the non-packaged variety), we are directly interested in the characteristics (e.g. pure premiums) for the various strata and only mildly interested—

if at all—in the aggregate characteristics. This is contrary to the situation in sampling design where the strata are set up primarily to yield a more efficient estimate of the aggregate characteristic. Thus, it would be somewhat misleading to justify this form of actuarial stratification on the grounds that it minimizes the variance of the aggregate estimates, since that part of the statistical theory never comes into play.

Now in package ratemaking, we are interested in the most efficient estimate of the aggregate characteristics; and at first glance it appears that the decomposition of the aggregate experience by coverage or by layer provides the desired stratification for minimizing the sampling variance. However, contrary to Mr. Lange's contention, this decomposition does not necessarily constitute a stratification in the technical sense. The latter term, by definition, is reserved for the decomposition of a population into *mutually exclusive* subpopulations. For example, if we were picking a sample from the population of claims, a breakdown by coverage would be a stratification; on the other hand, the population of policies cannot be uniquely classified by coverage or by kind of loss. Consequently, the decomposition by coverage of the experience compiled by policy—while meaningful—does not satisfy the requirements of statistical stratification.

Well then, what is the justification for making this kind of decomposition; it certainly seems to be a reasonable thing to do. It turns out that the theoretical advantage is tied directly to the other technique discussed by Mr. Lange. Basically, it stems from the fact that this kind of decomposition provides the means for obtaining the maximum efficiency from ratio estimation. In a recent paper in the *Journal of the American Statistical Association,*[1] it was shown that:

> "The precision of ratio estimates is substantially improved if the correlated variables are decomposed into the sum of several components which are pairwise more highly correlated than the original variables."

Thus, it makes sense to split up the package experience by coverage since the latter is presumably more highly correlated to the corresponding non-packaged experience than is the combined experience for the two forms. However, it seems advisable that these presumptions be tested, since the claimed advantages to ratio estimation are dependent on their validity. Unfortunately, I suspect that this would be a rather difficult task to perform.

Finally, I would like to stress what is perhaps the most important

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[1] Robson, D. S. and Vithayasai, C., "Unbiased Componentwise Ratio Estimation," *JASA* Vol. 56, P. 350.

point made by Mr. Lange: that "insurance statistics may be viewed as samples of what might have occurred." As a corollary to this observation, the whole of sampling theory is then available in the actuary's tool kit. How much use can or will be made of this fact is only touched on lightly in the paper under review; it is hoped that future papers, by Mr. Lange and others, will delve into these applications in more detail.

DISCUSSION BY CHARLES F. COOK

It is rare for a paper presenting essentially new material to have such broad scope as this one. The author begins with a lucid discussion of his statistical philosophy of ratemaking and insurance. Next, he presents two technical sections: a discussion of the value of stratification in sampling design, and an introduction to ratio estimation, with special consideration of its power for inference and the control of its bias. Then he applies both techniques to package policy ratemaking in general and to an example, the Special Automobile Package Policy. Leaving the personal lines, he reviews the Bailey, Hobbs, Hunt, and Salzmann paper, "Commercial Package Policies — Rating and Statistics,"[1] mediates the indivisible premium-component rating debate, and shows that the key to the whole problem is a good multiple-line statistical plan.

His presentation of stratification and ratio estimation is clear and accurate; the applications and example are well presented and reasonable. Unfortunately, in the common statistical sense of the terms, the ratio of package to non-package pure premiums is not a ratio estimate, and the subdivision of experience by coverage and layer of loss is not stratification. With this exception (to be developed later) the author is very convincing. Not only am I convinced by what he says, but by several things he does not say. He flirts with some interesting ideas and potential applications of his sampling tools, raising questions in the reader's mind without having space to develop them. I hope he will not mind if I add a bit here.

SAMPLING THEORY AND RATEMAKING

"Those who refuse to go beyond the facts
rarely get as far as facts." — Thomas Huxley

This section presents a good case for treating the entire population of losses as a sample of a greater population of potential losses. It should be

[1] Bailey, R., Hobbs, E., Hunt, F., and Salzmann, R., "Commercial Package Policies — Rating and Statistics," *PCAS*, L, p. 87.

required reading for all skeptics. It is also pleasant reading for actuaries, but our best efforts at precise ratemaking have already made us true believers in the randomness of losses. The interesting questions for the actuary go a little deeper — what are we really trying to estimate, and what kind of a sample do we have?

Are we trying to estimate the parameters of the population, or are we trying to make projections of what will happen in the future? If we are trying to estimate the population mean μ, the estimated variance of our estimate is $S_{\bar{x}_1 - \mu} = S_x/(n_1 - 1)$. If we are trying to estimate the future sample mean $\bar{x}_2$, the estimated variance of our estimate is $S_{\bar{x}_1 - \bar{x}_2} = \dfrac{n_1 + n_2}{(n_1 - 1)(n_2 - 1)} S_x$, or approximately double $S_{\bar{x} - \mu}$ for $n_1 \cong n_2$.

Is each year of experience a random sample from the same population, a non-random sample, or a sample from a stochastic process? If we consider it a random sample, trend in claim costs should be considered a change in the value of money, independent of the loss population, and trend in claim frequency should be disregarded as merely a chance happening. Within a single stationary population, trends in experience can be considered if our model is a not-quite-random sample, in which time is a biasing factor influencing the probabilities of population elements appearing in the sample. The most versatile population model is a general stochastic process $F(X, t)$ in which the population distribution changes over time. This model, mentioned by the author with credit to Cramér, is almost certainly the proper one for a good fit to the real world, but to the best of my knowledge no one in the CAS, including Mr. Lange, has studied the effects on ratemaking of such a non-stationary population model.

APPLICATION TO PACKAGE POLICY RATEMAKING

"Unapplied knowledge is knowledge shorn of its meaning."
— A. N. Whitehead

Perhaps the most significant feature of the paper is the destruction of the indivisible premium ratemaking myth. The author shows (I believe quite conclusively) that with sophisticated analysis of proper statistics, all of the alleged disadvantages of component ratemaking can be overcome. We need no longer debate whether we should analyze package policy experience by policy or by coverage — we can analyze it both ways and gain both sets of advantages. Mr. Lange covered these advantages quite thoroughly; my only addition is that his method, as well as giving due consideration to the policy as a whole, would facilitate the use of different experience periods for different coverages. The failure to do this is the fatal flaw of indivisible premium ratemaking methods such as that used for homeowners.[2] A short experience period in New England may include no hurricane losses, although the premium must obviously provide for them. Conversely, a ten-year experience period is only reasonable if a policy has been in existence, unchanged, for ten years. And even if it has, who wants to use ten-year-old theft and liability experience?

The method also has disadvantages which merit further consideration. To start with a minor point, statistics must be kept in more detail than for indivisible premium ratemaking. I agree with Mr. Lange that the "Indivisible Premium Statistical Plan" presented by Bailey *et al.*[1] would appear to provide an adequate base for his method, but a pure indivisible premium ratemaking scheme could get by with less (e.g. no exposures by coverage).

The ratemaking procedure itself is a more serious problem. The level of technical sophistication demanded of the ratemaker (and of regulatory officials) may be unrealistic. At present most ratemaking is done by non-professional personnel according to fairly straightforward standardized formulas, with only limited high-level supervision. I have strong doubts whether any procedure involving ratios between pure premiums can be properly executed at this level, because such items as trends, loss development, and small fringe coverages, which are likely to be significantly different for package and non-package business, are also likely to have unstable ratios.

Mr. Lange's worry about the need for simultaneous rate revisions is important even if one rejects it. If the ratemaker decides it is unnecessary to make package and non-package rate revisions simultaneously, I can think of only four other choices, all of which add further complications:

1. Use old component data (whatever is freely available) for the packages.
2. Use large trend factors for the components, to bring indications up to the package review date.

[2] It has been correctly pointed out to me that varying experience periods could be used in homeowners ratemaking, by calculating partial loss ratios for each cause of loss. These partial loss ratios could have different experience periods, and their sum would be a homeowners loss ratio which overcomes my objection. However, by the definition set forth by Bailey *et al.*[1] (p. 92), this would be, in effect, a component ratemaking method (even though the premium *per se* is indivisible).

3. Make a partial review of component experience at the package review date, and of package experience at each component review date.
4. Do detail ratemaking with the most recent full data for both package and non-package business, then adjust the over-all level for the item under consideration on the basis of its own later data.

It is good form for an author presenting substantial innovations to cautiously cite any difficulties he may see, lest he oversell his case in a one-sided manner. Mr. Lange having quite properly done this, a reader must be careful not to pay too much attention to the problems and lose sight of the over-all merit of the ratemaking techniques discussed. Indeed, several of the problems and limitations the author mentions, while real, have less importance than it may seem at first reading. The problem of simultaneous rate revisions, discussed in the preceding paragraph, is a good example. Even with the simplest alternative listed, the use of old component data, we would probably get better package rates than we do now. It is a problem only by comparison to mono-line methods — a standard which even a hypothetical 100% efficient package ratemaking method can only approach asymptotically.

Mr. Lange says that his SAP method is not presented as a solution, but only an example of what might be developed. I think he sells his presentation short. Although improvements may be possible, and there are many details to be worked out, the example is essentially a sound, workable framework for package policy ratemaking, which we could profitably use next month.

Another problem he cites is the declining usefulness of the ratio P/N of package to non-package experience as the proportion of package business increases. In part, this problem is based on the assumption that the correlation between them is due to the transfer of risks from N to P, but (as will be shown in a later section) this is not true. What correlation exists is due to common external influences, which will remain roughly constant in their effect. The other basis of the problem, shrinking stability in the denominator, is solved by the author in another section of the paper (for Commercial Packages) by using a pair of ratios, $P/(N + P)$ and $N/(N + P)$, rather than the one ratio P/N. With this simple change the denominator, an average pure premium for all types of policy combined, has the greatest possible stability.

The final point I would like to consider in this section is bias. Although the author shows that ratio estimates may be corrected for bias, this discussion questions whether the ratio of package to non-package experience is a ratio estimate (in the sense the term is used in sampling). This implies a further question whether a correction for bias, such as the Hartley and Ross formula cited by Mr. Lange,[3] will precisely correct for bias in such a ratio. I doubt that it will, but I further doubt whether it matters. Any bias due to the use of ratios will be less than the bias due to time or to loss development, and will be much less than the bias due to completely excluding package policy experience from ratemaking (the present SAP situation). We adjust for time and loss development by simple pragmatic methods and make no adjustment when we exclude presumably superior package experience. Furthermore, a bias in this ratio will only affect the differential between package and non-package experience, having no effect on over-all rate level. In return for a small bias, we get a reduced random error. The net effect should be a reduction of the total mean square error. If so, I think we have come out ahead.

STRATIFICATION

"$1 + 1 = 3$ (for sufficiently large values of one)." Anon.

The author's discussion of stratification is a fine survey. It is doubly valuable, as an introduction and reading guide[4] for those who wish to study further, and more importantly as an outline of the technique for the many actuaries who cannot find the time to study mathematics in detail, but who would like to know what techniques are available, how they work, what they accomplish, and where they have limitations. Mr. Lange has prepared essentially the same material more thoroughly (especially in terms of mathematical development) for the use of an industry subcommittee.[5] Perhaps hé might present the non-overlapping parts to the Society as an Actuarial Note; it would be a useful addition.

The section has one small error:

"Thus, in the case of *small samples* [reviewer's italics] stratification

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[3] Hartley, H. O., and Ross, A., "Unbiased Ratio Estimates," *Nature,* Vol. CLXXIV, pp. 270–271, 1954. This is also covered in Lange[5] and more thoroughly in Cochran,[4] pp. 176–181.

[4] Cochran, W. G., *Sampling Techniques* (2nd ed.), Wiley, 1963. This is the best all around mathematical reference for these subjects I know of, and is recommended by Mr. Lange for further study.

[5] Lange, J. T., *Memorandum to the Sub-Committee on Sampling of the Actuarial Committee,* April 30, 1967, National Bureau of Casualty Underwriters (mimeographed).

will improve precision only if the resulting strata are more homogeneous than the total population."

This statement is in fact true for *any size sample,* and for *any size population.* I suspect from context that the author's intended point is a stronger one, that stratification can result in a decrease of precision if the strata are not more homogeneous than the total population. This point also is independent of sample size; it applies to small populations. Indeed, if a small population is stratified in a manner which improves precision, the percentage improvement in precision is greater for *smaller* samples, due to the action of the finite population correction $(N - n)/N$.

A few simple formulas are an aid in understanding stratification, but those commonly appearing in the literature do not serve well. For one thing, they are intended more for proof than for insight. For another, they are usually expressed in finite terms even when only appropriate for infinite populations. Mr. Lange's formula for the reduction of variance obtained by proportional stratification is a good example — it is precise only for infinite populations, but in that case the terms Nh and N are not defined. In studying the material, especially Cochran,[4] I found it necessary to manipulate some of the formulas, in both form and order of development, to get a clear feeling for what was going on. It is possible that other readers may also find these modifications improve clarity.

Notation[6]

The suffix h denotes a value for stratum h. Notation without the suffix applies to the total population (or total sample). Where formulas are different for finite and infinite populations, the finite case is shown first.

y	a unit in the population
N	total number of units (finite case)
$f(y)$	density function of units (infinite case)
L	number of strata
$\overline{Y} = \frac{\Sigma y}{N} = \int_v yf(y)\,dy$	mean of the population

[6] Slonim, M. J., *Sampling,* Simon and Schuster, 1966 (paperback). For readers who are interested in sampling but choke on mathematics, I recommend skipping the next several pages and instead reading this excellent (and cheap — $1.45) non-mathematical discussion of sampling. In a light, elementary, and very readable style, Mr. Slonim thoroughly describes and evaluates types of sampling (such as simple, cluster, or stratified) as well as methods of drawing samples (such as random, systematic, or accidental).

$S^2 = \frac{\sum(y - \bar{Y})^2}{N - 1}$ $= \int_v y^2 f(y)\, dy - \bar{Y}$	variance of the population (note that the divisor $N - 1$ is used here rather than N, as would be used for σ^2)
$Wh = \frac{Nh}{N} = \int_{y \epsilon h} f(y)\, dy$	proportion of the total population in stratum h
$\bar{S} = \sum Wh\, Sh$	mean stratum standard deviation
$\overline{S^2} = \sum Wh\, Sh^2$	mean stratum variance
n	number of units in the sample
$\bar{y} = \frac{\sum y}{n}$	mean of the sample
$V(\bar{y}) = \frac{S^2}{n}\left(\frac{N - h}{N}\right) = \frac{S^2}{n}$	variance of the sample mean (note that the f.p.c. divisor is N; if σ^2 were used it would be $N - 1$)

Allocation of the Sample to Strata

For proportioned allocation $nh = n\, Wh$; for optimum (Tschuprow-Neyman) allocation $nh = n\, Wh\, Sh/\bar{S}$. The most general method of optimum allocation also gives consideration to the cost ch per unit sampled in stratum h; in this case $nh/n = (Wh\, Sh/\sqrt{ch}) \div \sum (Wh\, Sh/\sqrt{ch})$. One can then determine the total sample size n that will minimize variance at a fixed total cost C, given set-up cost co, by the formula

$$n = (C - co) \sum (Nh\, Sh/\sqrt{ch}) \div \sum (Nh\, Sh \sqrt{ch})$$

or minimize cost for a fixed variance V by the formula

$$n = \sum (Wh\, Sh \sqrt{ch}) \cdot \sum Wh\, Sh/\sqrt{ch} \div (V + \frac{1}{N} \sum Wh\, Sh^2).$$

Due to its complexity, we will not consider optimum cost allocation further, except to note that if ch is constant we get Tschuprow-Neyman allocation, and if Sh is also constant we get proportional allocation.

Estimation of the Population Mean

The whole point of stratification is to improve the estimate of the population mean by using data more nearly representative of the population than a simple random sample. One way to do this is proportional allocation, whereby the sample is made representative in terms of stratum weights. If allocation is not proportional, the estimate must be weighted to make it so, by the formula $\bar{y}_{st} = \sum Wh\,yh$. In the proportional case this formula is not needed because $nh/n = Wh$, so $\bar{y}_{st} = (\sum nh\,\bar{y}h)/n = (\sum y)/n = \bar{y}$. This result points out a common fallacy of sampling: if a simple random sample is drawn and divided into strata, estimating Wh from the sample ratio nh/n, we again get $\bar{y}_{st} = \bar{y}$, the mean of the random sample. *Unless there is advance outside knowledge of the weights Wh, stratification accomplishes exactly nothing.*

The variance of the estimate of the sample mean is also a weighted average of the stratum values:[7]

$$V(\bar{y}_{st}) = \sum Wh^2\, V(\bar{y}h) = \sum Wh^2 \left(\frac{Sh^2}{nh}\right)\left(\frac{Nh - nh}{Nh}\right).$$

Substituting the allocation formulas for nh into the right-hand side of the equation, we get the following formulas for random, proportional, and optimum (Tschuprow-Neyman) allocation respectively, finite cases first:

$$V \text{ ran} = \frac{S^2}{n}\left(\frac{N-n}{N}\right) = \frac{S^2}{n}$$

$$V \text{ prop} = \frac{\sum Nh\,Sh^2}{n\,N}\left(\frac{N-n}{N}\right) = \frac{\sum Wh\,Sh^2}{n}$$

$$V \text{ opt} = \frac{(\sum Nh\,Sh)^2}{n\,N^2} - \frac{\sum Nh\,Sh^2}{N^2} = \frac{(\sum Wh\,Sh)^2}{n}$$

Improvement in Precision due to Stratification

Cochran's formula[8] for the reduction of variance with proportional allocation is based on the identity

$$(N-1)\,S^2 = \sum(Nh-1)Sh^2 + \sum Nh\,(\bar{Y}h - \bar{Y})^2.$$

Substituting this value for S^2 in the formula for V ran, subtracting V prop, and simplifying, he derives

$$V \text{ ran} = V \text{ prop} + \frac{N-n}{nN(N-1)}\left[\sum Nh\,(\bar{Y}h - \bar{Y})^2 - \frac{1}{N}\sum (N - Nh)\,Sh^2\right].$$

[7] The proofs of this formula and the three specific ones to follow can be found in Cochran, *op. cit.*, pp. 90–91.

[8] *Ibid.*, pp. 98–99.

In the infinite case, this simplifies to

$$V \text{ ran} = V \text{ prop} + \sum Wh\,(\bar{Y}h - \bar{Y})^2\,/n.$$

The infinite case is a useful result, but the finite case is nightmarish, especially when the formula for the further reduction for optimum allocation is introduced. This formula (the same for finite or infinite populations) is

$$V \text{ prop} = V \text{ opt} + \frac{1}{n} \sum Wh\,(Sh - \bar{S})^2.$$

Since $(Sh - \bar{S})^2$ and $(\bar{Y}h - \bar{Y})^2$ are non-negative, it follows for the infinite case that V ran $\geqslant V$ prop and for infinite or finite populations that V prop $\geqslant V$ opt. Cochran goes on to show[9] that for a finite population with all Sh equal, say, to Sw, optimum and proportional allocation are identical, and result in higher variance than a simple random sample if

$$\frac{\sum Nh\,(\bar{Y}h - \bar{Y})^2}{L - 1} < Sw^2$$

which is equivalent to an F-ratio less than one in analysis of variance. That is, the mean square between strata is less than the mean square within strata.

I prefer a different approach which produces simpler results by more direct methods. It also has the virtue of parallel forms among the formulas. Using our definitions of $\bar{S}$ and $\overline{S^2}$ we get immediately (finite case first):

$$V \text{ ran} - V \text{ prop} = \frac{N - n}{nN}\,(S^2 - \overline{S^2}) = \frac{1}{n}\,(S^2 - \overline{S^2})$$

$$V \text{ prop} - V \text{ opt} = \frac{1}{n}\,(\overline{S^2} - \bar{S}^2) = \frac{1}{n}\,(\overline{S^2} - \bar{S}^2)$$

$$V \text{ ran} - V \text{ opt} = \frac{S^2 - \bar{S}^2}{n} - \frac{S^2 - \overline{S^2}}{N} = \frac{1}{n}\,(S^2 - \bar{S}^2)$$

In the infinite case, we know that $S^2 \geqslant \overline{S^2} \geqslant \bar{S}^2$, so all of the differences listed are positive. The second part of the inequality $(\overline{S^2} \geqslant \bar{S}^2)$ is also true in the finite case, but it is not necessarily true that $S^2 \geqslant \overline{S^2}$, because S^2 has $N - 1$ degrees of freedom, while $\overline{S^2}$ has $N - L$. Taking the example of a population with L identical strata, each having equal numbers of 0's and 2's, we have $S^2 = N/(N - 1)$, $Sh^2 = Nh/(Nh - 1)$, and $Nh = N/L$. Then $\overline{S^2} = Wh\,Sh^2 = Sh^2 = (N/L)/(N/L - 1) = N/(N - L)$, which is greater than $N/(N - 1)$ for $L > 1$.

With this in mind, we can state the results for the finite case: (1) opti-

[9] *Ibid.*, pp. 99–100.

mum allocation is always at least as good as proportional allocation; (2) proportional stratification results in lower variance than a simple random sample if the population variance is greater than the mean stratum variance; and (3) optimum allocation stratification results in lower variance than a simple random sample if the population standard deviation is greater than the mean stratum standard deviation. Conclusions (1) and (2) follow directly from the formulas, but (3) requires an outline of reasoning:

$$\overline{S^2} \geqslant \bar{S}^2, \text{ therefore } S^2 - \bar{S}^2 \geqslant S^2 - \overline{S^2}.$$

Since we have $N > n$ and $S^2 - \bar{S}^2$, by assumption, all greater than zero, we can multiply inequalities, getting $N(S^2 - \bar{S}^2) > n(S^2 - \overline{S^2})$ which yields

$$\frac{S^2 - \bar{S}^2}{n} > \frac{S^2 - \overline{S^2}}{N}; \text{ therefore}$$

$$\frac{S^2 - \bar{S}^2}{n} - \frac{S^2 - \overline{S^2}}{N} > 0 \text{ if } S^2 - \bar{S}^2 > 0 \qquad \text{Q.E.D.}$$

Other Reasons for Stratification

The foregoing discussion of stratification has centered on stratification as a mathematical technique for increasing the precision of sample estimates. This is the area in which Mr. Lange concentrated his presentation, and is the sense of the term stratification normally considered by statisticians. In the broader sense of "any subdivision of a population for sampling," there are reasons other than reduction of variance. Cochran cites these:[10]

1. Data may be desired for the individual subdivisions in their own right.
2. Subdivision may be administratively convenient.
3. Different sampling problems for the subdivisions may call for different sampling procedures.

Stratification by Layer of Insurance

Cochran begins his chapter on Stratified Random Sampling:[10]

> "In stratified sampling the population of N units is first divided into subpopulations of $N_1, N_2, \ldots, N_L$ units, respectively. These subpopulations are non-overlapping, and together they comprise the whole of the population so that
>
> $$N_1 + N_2 + \ldots + N_L = N$$

[10] *Ibid.*, p. 87.

The subpopulations are called *strata.* To obtain the full benefit from stratification, the values of Nh must be known. When the strata have been determined, a sample is drawn from each, the drawings being made independently in different strata."

A stratification by layer of insurance, such as first $1,000 coverage, next $2,000, . . . clearly forms a set of non-overlapping subpopulations comprising the whole population of coverage, and the means and variances will certainly differ among the strata. It would then seem to be an ideal place for stratification, except that:

1. We do not know anything at all about the value Wh (Wh rather than Nh because this is an infinite case) for the population of coverage which might have been purchased, except the estimate nh/n from our sample. As we discussed in the section *Estimation of the Population Means,* in this case stratification does not reduce variance.
2. Our "drawings" in the different strata are not independent; indeed, they are very highly correlated, policy by policy.

It is possible that the author could show how to improve our knowledge of Wh,[11] and perhaps he could show that the dependence among strata is not harmful, but he did not. Until these objections are answered, I do not believe we can justify rating by layer of insurance on the grounds of decreased variance due to stratification. Note, however, that this argument does not imply that rating by layer does any *harm* to our estimate; if it can be justified by other arguments (such as permitting different credibility factors for higher layers) we should certainly not reject it because stratification as such may not be effective.

Stratification by Coverage

Like stratification by layer of insurance, stratification by coverage suffers seriously from the lack of outside knowledge of Wh. Otherwise, however, it has much more to recommend itself. Considering the section *Other Reasons for Stratification,* it seems that all three apply in this case. Certainly we are interested in data for the individual coverages, for reasons discussed by Mr. Lange. Administratively, company claims personnel and ratemaking

[11] The use of data from prior years to estimate Wh comes immediately to mind, but it is shown in Cochran, *ibid.*, pp. 116–118, that our estimate of Y would then be biased by $\Sigma\,(wh\text{-}Wh)\,\overline{Y}h$, where wh is the estimated weight. This bias is independent of sample size, so for large samples the mean square error of estimate approaches the bias as a limit, whereas the MSE of a simple random sample approaches 0 as a limit.

organizations may be different for the different coverages. Most importantly, the sampling problems are different; reasonable sampling procedures are different by coverage in terms of years in the sample, territorial spread, credibility procedures, and many others. I have only selected a few items here; all of the author's non-mathematical reasons for stratification are valid and could be added.

These reasons apply to stratification in the sense of a convenient subdivision of the population. Mr. Lange's justification in the mathematical sense of reducing variance must still be considered. If for the moment we consider nh/n to be a good enough estimate of Wh, we necessarily have a proportionally allocated stratified sample even with an indivisible premium method which does not identify coverage. In this case we would not know nh/n, but we do not need it because it was shown earlier that for proportional allocation $\bar{y}_{st} = y$. There is therefore no gain by proportional allocation.

Optimum allocation is more interesting. It implies that the sample size for coverage h should be proportional to $Wh\ Sh$. Wh is taken care of by the natural sample of one year of data, so it would seem that the number of years of experience for a coverage should be proportional to its annual standard deviation, or else its standard deviation should be reduced by a credibility procedure to the point where its Sh equals that of the coverage with minimum variance.

We also have an interesting case if package and non-package data are combined. If the total experience is n, and the package n', with nh and nh' units of experience for coverage h, we get $\bar{y}h = (\sum y)/nh$. Estimating the package population weights Wh by nh'/n', our estimate of $\bar{Y}$ is

$$\bar{y}_{st} = \sum_h \frac{nh'}{n'}\left[\sum_i \frac{y_i}{nh}\right].$$

This is exactly the reverse of the normal stratified estimate — the weights are based on a *smaller* volume of experience then the sample — but it is the proper formula to reduce the bias due to varying proportions of package business by coverage. This is the rationale behind averaging FAP class and territory pure premiums using SAP exposures in Mr. Lange's example.

Other Applications of Stratification

To first cite possible application in mono-line ratemaking, territorial subdivision in ratemaking is a form of stratification in the simple subdivision sense. Perhaps these already existing strata could be applied to improving the accuracy of statewide average rates, as well as the intra-stratum territorial rates. The same thing might be done by state or region to improve over-all rates for problem lines such as E.C.E.

In the package area, I am surprised that Mr. Lange did not suggest treating package and non-package data as two strata in setting average rates, then applying his ratios to these averages. It would seem logical to do so if one of our basic goals is accurate over-all rates for broad classes such as insurance on automobiles or insurance on homes, regardless of the specific policy involved.

If the industry should reject Mr. Lange's approach and go the indivisible premium route, stratification may be a key tool. Bailey *et al.* proposed analysis "by type of insured, according to the combination of coverages selected." Stratification by coverage combination is not the same as by coverage, but may be even more important due to the very narrow base of uncommon combinations.

RATIO ESTIMATION

> "Arithmetic tells you how many you lose or win if you know
> how many you had before you lost or won." — Carl Sandburg

Mr. Lange's discussion of the technique itself is very welcome. I suspect I am not the only reader whose college statistics courses never even mentioned ratio estimation. Being unfamiliar with the technique, I can only make a few comments on the author's presentation. First, the primary and auxiliary variables must be measured on the *same units of the population,* because the variables must be correlated, and correlation is only defined for pairs of measurements on the same units. Secondly, the equation presented by the author for determining whether or not a ratio estimate will reduce variance is only valid for large samples, because it relies on substituting the sample mean for the population mean in the ratio estimate variance formula. Thus, "In smaller samples the ratio method probably does not compare so favorably as the [formula] suggests."[12] Finally, I consider it misleading for Mr. Lange to state that "Published comparisons . . . have shown that the variance may be reduced by as much as 50% to 95%." He cites Cochran, page 179, as his source; looking it up, we find that the source is an example, of which Cochran says: "This illustrates an artificial population [of 12 elements] . . . deliberately constructed so that *Rh* varies

[12] *Ibid.*, p. 166.

markedly from stratum to stratum, thus favoring a separate ratio estimate. . . . No general conclusions can be drawn from the example."

Ratio of Package to Non-Package Experience

This ratio is not a ratio estimate in precisely the sense discussed by the author. The raw data are not correlated, as they are not measured on the same units. If we summarize, so that our unit is a class of risks, and our meausrement is the average of risks in the class, we remedy this problem, but add two more:

1. Our sample of summarized units is small. Therefore the bias in the ratio estimate and the understatement of the variance due to the substitution of $\bar{x}$ for $\bar{X}$ (cited above) are of greater signifiance than would be the case if we could keep our sample large.
2. Grouped data tend to show a higher correlation than ungrouped data. Yule and Kendall give a thorough discussion of this effect,[13] and caution that: "Our correlations will accordingly measure the relationship between the variates *for the specified units chosen for the work* . . . [reviewer's italics]. They measure, as it were, not only the variation of the quantities under consideration, but the properties of the unit-mesh which we have imposed on the system in order to measure it." Indeed in the extreme case of two groups, say Class 2 auto vs. all other, the correlation is necessarily unity, no matter what the package and non-package experience may be.

I do not claim to know what effect these problems have on Mr. Lange's ratio estimates, but it would seem that routine formulas may be misleading in our case.

The author states that "For a given coverage, the correlation between package and non-package experience should be fairly high, especially in the early stages, since much of the package business will represent simply a transfer from the non-package policies." I do not understand this. It would seem that when an insured transfers to a package policy, exerting his individual influence on that experience, it will tend to make it like the non-package business was before our insured transferred; but now his influence on non-package experience is gone, and so, it would seem to me, is any correlation due to his transition. The correlation between package and non-package experience by class is due to the influence of the classifica-

[13] Yule, G. U., and Kendall, M. G., *An Introduction to the Theory of Statistics*, (14th ed., 3rd impression), Charles Griffin & Co. (London), 1958, pp. 310-315. The specific quote is from page 312.

tion criterion, which affects the exposure regardless of the type of policy covering it.

The final problem is what to do when the package and non-package coverages are not comparable. For instance, Bailey *et al.* point out that Funeral Directors and Motels in the SMP program are not subdivided by size, but on a non-package basis they may be rated for fire insurance as either dwelling or mercantile class, depending on size. Some careful work may be required to answer the question "Ratio to what?"

The major justification for the use of ratios in this area is a pragmatic one. Where X and Y have large ranges, the difference $\bar{X} - \bar{Y}$ is clearly not the proper model for application — we need a percentage difference. Thus we are left with a model such as $\bar{X}/\bar{Y}$, $(\bar{X} - \bar{Y})/\bar{Y}$, or $2\,\bar{X}/(\bar{X} + \bar{Y})$ by default, and all of these are ratio estimates of sorts.

Other Uses of Ratio Estimation

A simple and logical application is to consider amount of loss as the primary variable, with the first dollar of loss as the auxiliary. The result is to estimate pure premiums by the ratio (average claim cost) multiplied by the auxiliary value (claim frequency).

The most interesting use of ratio estimation is a model for the insurance rating process as a whole. Let the measure of exposure be the auxiliary variable, and losses (loaded for expenses) the primary variable. The ratio is calculated (we call it a premium rate) for our sample of one experience period. During the following year this ratio is applied to the exactly known (at sale or at audit) population values for the auxiliary variable, yielding written premiums — our ratio estimate of losses (loaded for expenses) for the population being rated.

SUMMARY

> "The formulation of a problem is often more essential than its solution." — A. Einstein and L. Infield

The unusual length of this discussion should not be construed as critical of the paper's quality, but of its brevity. The questions raised and models proposed are worthy of more study than can be given by any one author in one paper. I imagine that the implications of sampling theory on all phases of ratemaking will be a lively subject in the *Proceedings* for some time to come, and I hope that Mr. Lange will be able to follow this paper with further study of the same caliber.

AUTHOR'S REVIEW OF DISCUSSIONS

Messrs. Nelson, Cook, and Graves have each discussed my paper from a different point of view, and I will briefly review their discussions separately.

Mr. Nelson does not disagree with the conclusions concerning rate-making in my paper, nor does he quarrel with the idea that sampling theory, and in particular ratio estimation and stratification, have implications for ratemaking. He does, however, feel that the decomposition of aggregate experience by coverage and layer is *not* stratification, as I had contended. Rather, he contends that this decomposition is desirable because it is an example of "componentwise ratio estimation," a term used by Professor Robson in a 1961 paper in the *Journal of the American Statistical Association.*[1]

I cannot fully agree with Mr. Nelson for two reasons. First, I feel that for package policies both premiums and losses may be decomposed by coverage into "mutually exclusive subpopulations," which according to his definition is stratification. Second, I do not feel that Professor Robson's "componentwise ratio estimation" is really any different from a combination of ratio estimation and stratification. I note that in his paper, Robson occasionally uses the terms "stratified" and "componentwise" interchangeably, and that Robson's example of componentwise estimation is cluster sampling with post-stratification.

Mr. Cook, like Mr. Nelson, does not in his review question the conclusions or general approach of my paper, but does have some doubts about certain details and does feel that some additional material is necessary. At the beginning of his review, Mr. Cook states that the "subdivision of experience by coverage and layer of loss is not stratification." Judging from subsequent sections of his review, he feels that subdivision by layer of loss is not stratification, while sub-division by coverage is stratification. He relies upon Cochran's[2] definition of stratification which requires that the population be subdivided into non-overlapping subpopulations whose sum is the total population. Subdivision by layer of loss and by coverage meet this criterion.

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[1] Robson, D. S. and Vithoyasai, C., "Unbiased Componentwise Ratio Estimation," *Journal of the American Statistical Association (JASA),* Vol. LVI, p. 350.

[2] Cochran, W., *Sampling Techniques* (Second Edition), John Wiley & Sons, Inc., New York, 1963, p. 87.

Cochran adds two other desirable criteria for sampling from these strata. First, to obtain full benefit, strata sizes should be known. As will be shown, this criterion is desirable, but not necessary. Second, the sample should be drawn independently within each stratum. This condition can be fulfilled in the case of stratification by coverage, but not necessarily in the case of stratification by layer of loss. As Mr. Cook and others[3] have noted, this latter form of subdivision of experience is actuarially desirable; however, to show that its efficiency derives mathematically from stratification would require some argument beyond that given in my paper. Perhaps one might consider subdivision by layer of loss to be a form of post stratification and then attempt to show that the dependence among strata in drawing the sample is not harmful. I have not pursued that line of reasoning further because subdivision of experience by layer of loss is well established actuarially while the subdivision of package policy data by coverage is the more controversial point.

It would appear from the above definition that sub-division by coverage is a form of stratification. Mr. Cook's question is really whether, from a mathematical view, it reduces the variance. His concern is best summarized by his statement: "Unless there is advance outside knowledge of the weights *Wh,* stratification accomplishes exactly nothing." Mr. Cook reaches this conclusion in his discussion of the estimation of the population mean and later applies it as a criticism of subdivision by layer of loss and by coverage. He arrives at this conclusion after showing that if *Wh* were estimated from the sample (i.e. $Wh = nh/n$) then the sample mean under stratification is equal to the sample mean under simple random sampling. Mr. Cook neglects the fact that the reduction in variance due to stratification arises from the greater homogeneity of each stratum (as compared to the total, unstratified population). Since the procedures are unbiased, we expect identical means. However, the variance under stratification will be less, and the precision greater, than under simple random sampling provided the strata are more homogeneous than the total population. As both Mr. Cook and I have noted when the population (*not* necessarily the sample) is large, the reduction in variance due to the use of stratification is a function of the sum of squared differences of each of the strata means and the grand mean. The fact that the weights, *Wh,* are estimated from the sample will not alter the fact that the reduction in variance is a positive quantity greater than zero. Our state of knowledge with regard

[3] Salzmann, R., "Rating by Layer of Insurance," *PCAS* Vol. L, p. 15.

to *Wh* in ratemaking is probably no more deficient than in other experimental work, where knowledge of "true stratum sizes" is lacking, and estimates must be made from data for prior years, pre-samples, or the like. W. D. Evans, in his paper "On Stratification and Optimum Allocations" in the *Journal of the American Statistical Association,*[4] stated: "Since on the average even random stratification will not reduce precision, stratification may be employed without hesitation whenever there is even slight justification for supposing that the variable under study is related to proposed mode of stratification." I feel that Mr. Cook would agree that there is more than slight justification for supposing that the variables we study in ratemaking are related to coverage.

It would appear that subdivision by coverage not only satisfies the criteria for stratification but that one may expect some reduction in variance. With regard to stratification, Mr. Cook questions whether my statement of caution concerning the improvement in precision is true in general or only for small populations. Mr. Cook claims stratification can result in a decrease in precision, independent of sample size; however, Neyman[5] has proved the converse for large samples, thus contradicting Cook.

Mr. Cook also feels that the ratio of package to non-package pure premiums is not a ratio estimate in the traditional sense (since the primary and auxiliary variables are not measured on the same units of the population) unless the unit is defined to be a class of risks. This reduces the sample size and artificially increases the correlation. He notes that I gave only a large variance formula for the ratio estimate variance; a discussion of the error involved in using this approximate variance has been given by Sukhatme.[6] He further notes that my footnote 18 refers to only one, rather poor published example; perhaps footnote 18 should have read "*Ibid.*, p. 171, 175 and 179," thus including several better examples from Cochran. In addition to preparing the published examples, prior to presenting the paper I undertook several experiments comparing a combination of ratio-estimation and stratification versus simple random sampling in which I used small samples (about 70 units) where the sample unit was in fact a group (a class or territory) and obtained results like those in Cochran. While Mr. Cook's questions were valid, it would appear that the ratio of

[4] Evans, W., "On Stratification and Optimum Allocation," *JASA* Vol. XLVI, p. 95.

[5] Neyman, J., "On the Two Different Aspects of the Representative Method: the Method of Stratified Samplings and the Method of Purposive Selection," *Journal of the Royal Statistical Society* Vol. XCVII, p. 558.

[6] Sukhatme, P. V., "Contribution to the Theory of the Representative Method," *Journal of the Royal Statistical Society* — Supplement — Vol. II, pp. 253-8.

package to non-package experience may be considered a form of ratio estimation.

Addressing himself to the more practical aspects of actuarial science, Dr. Graves gives us an example, from Virginia, of present day automobile package policy ratemaking techniques (simply to ignore the package policy data) and notes that my method might be an improvement. It would seem that the action of the Mutual Bureau Actuarial Committee to use the components of the package policy in ratemaking is almost equivalent to stratification and Dr. Graves feels that some form of ratio estimation might be helpful. It is interesting to note that Mr. Nelson found the ratemaking process implied by my paper persuasive, while Mr. Cook felt we could profitably use it next month. Apparently, Messrs. Cook, Graves, and Nelson agree with my general premise that the estimates (of pure premiums) for package policies would be more precise if the package statistics were decomposed by coverage and if the ratio of package to non-package (or total) experience were used in making the estimates. Mr. Cook and Mr. Nelson find in sampling theory (and in particular stratification and ratio estimation) some justification for my conclusion, although they both find it necessary to redefine some terms and ask for some further elaboration at a few points. Perhaps by limiting the effort (for discussion purposes) to a decomposition by coverage and by properly defining sample units, Messrs. Cook, Nelson, and I might be able to view the suggested ratemaking technique as an example of componentwise (or stratified) ratio estimation. However, my reviewers have made it clear that I have drawn implications from sampling theory and have not proved corollaries from theorems in sampling theory. It is interesting to note that one might have reached the same conclusions concerning package ratemaking by drawing implications from Monte Carlo techniques. In particular, the Monte Carlo method for the numerical evaluation of a multi-dimensional integral may be improved (in the sense of increased precision and efficiency) by arbitrarily breaking up (stratifying) the ranges of integration and by using control variates and regression (ratio) methods.[7] Such an approach might have avoided some of the difficulties raised by Messrs. Cook and Nelson, but, on the other hand, would have necessitated a longer paper since Monte Carlo methods are probably less familiar to most actuaries than sampling. The criticism of my paper for a lack of mathematical rigor may be analogous to the criticism of modern painting which often lacks a clear resemblance to

[7] Hammersley, J. and Handscomb, D., *Monte Carlo Methods*, Methuen & Co. Ltd., London 1964, pp. 50-76.

nature. Paul Klee defended his abstract works by comparing them to the fantastic images one sees through microscope and then asked the question:

> Does then the artist concern himself with microscopy? History? Paleontology?
>
> Only for purposes of comparison, only in the exercise of his mobility of mind. And not to provide a scientific check on the truth of nature.
>
> Only in the sense of freedom. In a sense of freedom, which does not lead to fixed phases of development, representing exactly what nature once was.

UNDERWRITING PROFIT IN FIRE BUREAU RATES

LAURENCE H. LONGLEY-COOK

VOLUME LIII, PAGE 305

DISCUSSION BY STANLEY C. DU ROSE JR.

The author presents an interesting discussion and defense of the hypothesis in fire insurance bureau rate making that, "For rate adequacy, we must limit the data to the experience of stock companies, as otherwise they will not, on the average, experience the underwriting profit assumed in the rating formula."

Mr. Longley-Cook has recited three of the arguments usually given for the exclusion of mutual company underwriting experience from stock underwriting experience in the bureau rate making process. However, there are other equally valid arguments for the inclusion of the experience of all bureau members and subscribers in the rate making process.

I believe the paper would have had better balance and been more convincing if the author had presented his rationale set in a matrix of the legal and actuarial issues involved with rate making in concert.

Consider for example the typical rate making statute under which most rating bureaus operate. The only lawful way in which insurers may act in concert in the making of rates is through the device of a rating bureau. Insurers are relieved of their obligations under the law to file rates by becoming a member or subscriber to a rating bureau. One of the fundamental questions then is whether or not the rate law contemplates that companies making rates in concert may use the underwriting experience of only a portion of the insurers so acting in concert in rate making. This is a question for lawyers to debate, but I suggest that it may be quite difficult to establish as a matter of law that an insurer has a right to use rates predicated upon experience other than its own underwriting experience, without any requirement for a showing that such rates are appropriate for its underwriting and plan of operation.

The rate law contemplates that the rating bureau file rates on behalf of member and subscribers companies. If the bureau were making and filing rates for stock insurers based exclusively on stock insurers underwriting experience, then it would seem that the law would require that the same bureau would make and file rates for non-stock insurers based on the underwriting experience of such insurers. This would be especially true if it could be shown that the volume of such experience was indeed credible. It has yet to be established that member and subscribing companies of a fire insurace rating bureau would agree that they could survive the results of an intensely competitive market wherein a higher rate level were to be promulgated for stock insurers than the rate level promulgated for non-stock insurers by the same bureau.

The author in his paper seems to assume that stock insurers by writing business through the American Agency System are not capable of writing an average cross section of the fire insurance risks placed with all insurers. This assumption is based on empirical data that needs a much greater depth of study and evaluation. A comparison of stock and mutual claim frequency and severity would be helpful. Consideration should be given to the underwriting control that stock insurers can and do exercise, and also to the significant volume of business that is written by non-stock insurers operating through the same American Agency System and not infrequently on the same risks and through the same agents.

In respect to the question of statistically credible differences in loss ratio based on combination by corporate form, I suspect that grouping by other criteria such as Direct Writer vs. American Agency System companies would produce similar statistically credible differences in loss ratio.

It seems to me that an important point which the author has not mentioned is the matter of the manner in which claims are adjusted by stock insurers and non-stock insurers. It is possible that any difference in loss ratio between stock and non-stock insurers could be accounted for by claim adjustment practices and procedures. This in itself would be an interesting study to pursue. The argument could be made that the reason why stock and non-stock loss ratios for workmen's compensation, as presented by the author, are so nearly alike is that there is a rigid framework of law governing claim adjustments.

Some of the same bureau rate making problems just mentioned are also involved with the question of conversion, to a common rate level, of the underwriting experience produced from deviated rates. The basic truths of pure premium rate level calculation should not be arbitrarily abandoned merely because fire rate levels are usually determined by loss ratio rather than by pure premium methods.

If there is a competitive market and if we assume that the bureaus were to make rates only on the underwriting experience of the stock

American Agency System insurer, then it would seem not unreasonable to conclude that at some point in time such insurers would be victims of the process of adverse selection and, therefore, increasing rate level. This would be followed by the stock insurers writing less than a majority of the risks being insured. This raises the question as to when the bureau and the stock insurers would reach the point at which they would have to reverse their position and demand that rates be made on a combined underwriting experience of all members and subscribers to the rating bureau. I do not think it proper to assume that merely because stock insurers at present may have a majority of the business written in some geographic areas and in some risk classifications this will always continue to be true. Is there not a responsibility to determine now the principle that will govern what is to be done when the market shares become more equalized? The automobile insurance business is an interesting case history in the matter of increasing rate levels, adverse selection, and decreasing market share.

A review of the situation that presently exists in the rate making system of the principal physical damage insurance rating bureau is also of interest. A significant percentage of the underwriting experience that is combined for rate making purposes is generated by insurers specializing in the writing of insurance on financed vehicles. For many years, the underwriting experience of such companies has been consistently and substantially poorer than that of all other members and subscribers. In this case, the bureau has rejected any suggestion that the underwriting experience of such companies should be considered separately.

I believe the author has done a service by opening a discussion of a controversial subject that has many facets and about which there is much conversation but all too little thoughtful evaluation and written dissertation. The paper is obviously a valuable contribution to the works of the Society. Indirectly if not directly, it points up items that are urgently in need of further study. It suggests that fire rate making schedules are probably less than adequate in the differentiation and measurement of hazard and risk. This is a subject in need of attention by actuaries. The paper further points up the uncertainty of the requirements of the law relating to rate making by bureau. This needs the attention of both actuaries and the legal profession. In the meantime, insurance regulatory officials should not be condemned if in operating under rate laws that are obsolete they are slow in approving rate making schedules that do not adequately measure risk.

DISCUSSION BY L. L. TARBELL, JR.

Once again the Casualty Actuarial Society is indebted to Mr. Longley-Cook for a paper dealing with the property insurance business. Since this business has traditionally been "non-actuarial," few members of the Society have devoted time to an examination of the procedures used by property insurance rate makers and, therefore, contributions in this area are valuable knowledge for the actuary.

This paper discusses a problem which the property business has recognized for some time. The question phrased in "property" language is, "Should you establish rate levels for the business you presently write or for the business you would like to write?"

The dilemma presented in the mixture of stock and non-stock experience is really one of classification and, as Mr. Longley-Cook points out, when the business tends to differentiate between insurance carriers on the basis of monetary considerations, service facilities, friendship with insurance agents or whatever, the type of carrier with which the insured places his business does, in effect, classify the business. Since fire rating bureaus file the same rate level and develop rates for given buildings and occupancies regardless of the type of insurer, the non-stock carrier, because of better selection, underwriting, or other factors, enjoys a margin of underwriting profit potential which contributes toward dividend distributions. Naturally, the most direct and obvious solution to the problem would be the establishment of separate stock and non-stock rate levels. This has been the practice in the casualty lines (except for workmen's compensation) for quite some time and arguments such as those Mr. Longley-Cook has presented in his paper could be presented to substantiate this approach.

The mathematics of this problem are relatively simple; however, the solution is not as uncomplicated. From a practical point of view, the stock insurers are, in my opinion, forced to consider non-stock experience in the establishment of rate levels (either directly by inclusion of the experience or indirectly through judgment) since to do otherwise would tend to force the rate levels higher and allow the non-stock insurer a larger market within which to operate. Using the example which Mr. Longley-Cook presented in his paper, the indicated rate level for stock companies only is some 5.2% higher than the present level or the level indicated by the combined experience. If the higher level were achieved, the mutual companies would have available rates which would allow them to relax their underwriting and selection criteria so as to sweep in a larger market which would still be profitable.

A review of figures from *Spectator* for states where fire bureaus operate on different bases (SEUA where mutual experience is included for rate level, and New England Fire Insurance Rating Bureau where stock only experience is used to set the rate level except for the State of New Hampshire) produces the following:

	1965 Earned Premium (000's Omitted)		
	Stock	*Mutual*	*Total*
SEUA	88,876	27,255	116,131
% of Total	76.5%	23.5%	100.0%
NEFIRB	65,088	32,915	98,003
% of Total	66.4%	33.6%	100.0%

Louisiana, New York, and Texas, where mutual experience is included for rate level, show the following distributions:

	Stock	*Mutual*	*Total*
Louisiana	25,421	2,750	28,171
% of Total	90.2%	9.8%	100.0%
New York	141,311	38,711	180,022
% of Total	78.5%	21.5%	100.0%
Texas	72,676	9,801	82,477
% of Total	88.1%	11.9%	100.0%

These figures would seem to indicate that the stock companies maintain a larger share of the market where the rate levels are determined on a combined basis. However, the figures for the State of California, where rate levels are set on a stock only basis, show that 92.2% of the business is controlled by the stock insurers, and figures for Illinois and Ohio, where rate levels are determined on a similar basis, show 82.1% and 73.7% of the business written by stock insurers. These differences may be accounted for by the geographical distribution of mutual insurance companies which traditionally have been strongest in the eastern regions of the country.

The problem then resolves itself into one of a business decision as to how divergent stock and non-stock rate levels can or should be, and whether or not stock insurers feel that present non-stock business can be attracted to the stock carriers. It is in this area that the actuary for a stock company can best serve his management by analyzing rate level indications and business distributions and advising accordingly.

It would seem that a more thorough investigation of this subject with emphasis both on state rating procedures and market penetration by the various types of carriers would be neecssary in order to fully explore this problem and Mr. Longley-Cook is to be thanked for his paper which opens this area to investigation.

AUTHOR'S REVIEW OF DISCUSSIONS

I greatly appreciate Mr. DuRose's careful review of my paper on profit in fire bureau rates. This is an important subject and Mr. DuRose's comments call for a considered reply. My paper aimed to show that, if mutual and independent loss experience is included in the data used for making bureau rates for stock agency companies, the rates developed may not, in fact, be adequate for such companies and will certainly not provide the 5% profit in the Commissioner's formula.

Mr. DuRose argues that regardless of whether my contention is correct, fire insurance rating bureaus may be required by law to use all the experience of the companies for which they make rates, namely, all members and subscribers. It is unfortunately true that in the regulation of insurance there has been a tendency to seek to determine matters of ratemaking, not on the question of the adequacy of the rate for various companies which would seem to be the intent of the law, but on the literal interpretation of the wording of other sections of the law which were written many years ago without contemplating conditions as they exist today. This is, of course, very understandable because an insurance policy is a legal document and must be interpreted strictly with little regard to intent and it is not unreasonable for the same philosophy to carry over into the field of ratemaking.

A cobbler should stick to his last, and, not being a lawyer, I do not intend to try to argue the law. However, I must point out that if Mr. DuRose's argument is, in fact, the law, it is being otherwise interpreted in many states and in many lines of business; further, the difficulty could be overcome by having separate stock and mutual fire rating bureaus or by a proper interpretation of profit, which should consist of 5% *plus* the differential between the loss experience of stock member companies of the bureau and the loss experience used to make rates.

The reviewer makes the point that, "It is yet to be established that member and subscribing companies of a fire insurance rating bureau would agree that they could survive the results of an intensely competitive market wherein a higher rate level were to be promulgated for stock insurers than the rate level promulgated for non-stock insurers by the same bureau." I did not contemplate two rate levels because the mutual companies anyway distribute their profits to their policyholders and are at liberty, and often do, deviate from the bureau rates. I have many times advocated that, at least for personal lines, competition should be allowed to look after the problem of adequate and non-excessive rates, and that insurance commissioners should concern themselves with discrimination and company solvency. The spread of the direct writer method of merchandising has introduced a variety of rates for nearly every class of personal lines business in the majority of states, and as is apparent from the recent Virginia automobile hearing, the stock agency companies are opposed to the use of all company experience for automobile insurance and do not object to independent filings even by bureau members. It would seem logical, therefore, for the fire rating bureau to promulgate rates at a level which provides the correct profit margin for the stock member companies and for mutual companies to use these rates or such lower rates as they may wish, by deviations or independent filings. Of course, if the stock member companies feel such rates would be uncompetitive, a lower profit margin could be adopted by the bureau for its filing.

Mr. DuRose suggests there is not sufficient data to justify my contention that stock agency companies are not capable of writing an average of fire insurance risks placed with all insurers. There is certainly ample evidence that this is true for private passenger automobile insurance and for homeowners business. For fire business alone it is, I believe, sufficient to note that for every year since 1944, the first year for which data is readily available, the nationwide fire loss ratio of mutual companies entered in New York has been consistently about ten percentage points more favorable than the loss ratio of stock companies. This provides very high credibility to my statement. I fully agree with the reviewer that there is a similar difference between Direct Writer experience and American Agency experience, at least for all personal lines.

It can be argued that this difference in experience could be due to difference in fire class of business written, since it is true even today that some classses are consistently more favorable than others. That this is to some extent true is probable but I have not had the opportunity to research this point although the necessary data are available. However, if any difference in class distribution exists, it would be because the stock agency companies are not capable of writing an average of fire insurance business placed by all insurers. For stock companies this points to the desirability of introducing improved rating techniques. Mr. DuRose makes the point that any difference might be due to claim adjustment practices and procedures. This may be in part true but cannot, I believe, account for anything approaching ten percentage points.

I was, perhaps, remiss in not discusssing the problem of conversion to common rate level in my paper. I did, in fact, include a short discussion in an early draft but to discuss all possible cases becomes complicated and seemed to distract from the point I was making, that if the data used for ratemaking has a different overall loss ratio from the data of those companies who are seeking to earn a profit for their stockholders, then the rate of profit assumed in the formula will, in fact, never be earned. Where a stock company deviates on the ground of expenses, if the company's *actual* expenses enter into the determination of the expense portion of the rate, then the company's *actual* premiums should enter into the determination of the loss portion of the rate; otherwise the standard profit margin will not be preserved. If, on the other hand, a budget expense provision is used, as is the custom for production expenses in most casualty lines of business, then the company's premiums should be restored to common rate level.

It seems to me that with divergent marketing techniques there are differences not only in expense but in loss experience and different kinds of companies must be granted different rate levels, if they so desire. Regulatory authorities should allow companies to operate at profit margins less than 5% for fire insurance if they wish for competitive reasons, but should not force margins of less than 5% on the companies by insisting on the use of all company statistics.

BURGLARY INSURANCE RATEMAKING

STEVEN H. NEWMAN

VOLUME LIII, PAGE 312

DISCUSSION BY MARTIN BONDY

Steve Newman has given us a disturbingly accurate picture of the system of making rates for burglary insurance. The reader need only study the method carefully in order to guess how our results have been. Then a look at the results confirms the guess—disastrous.

A chart on page 325 of Volume LIII shows that National Bureau members have lost an average of 5.6% per year in the period from 1961 to 1965—and the situation is getting worse. To quote the author:

> "The impact of inflation upon buglary loss settlement costs, as well as the increase in the number of burglaries and robberies during this period, have contributed substantially to this situation."

To illustrate this, Steve then presents us with an exhibit entitled "Crime in the United States" which shows that the number of crimes against property has increased by about 40% in the four year period covered by the exhibit.

This chart only confirms numerically what the newspapers scream at us every day.

And yet, strangely, the ratemaking procedure does not recognize this universally known fact. The rates made for providing insurance in 1967 through 1970 are based upon the crime levels of the early 1960's.

To compound this lack, loss severity levels are brought only to the anticipated level of the effective date of the revision. They are somewhat short of what their target should be—the severities which can be expected to prevail at the time the losses will occur under policies affected by the revision.

In my opinion there is one convenient measure of the trend of burglary insurance costs which has the following desirable features:

1. It reflects severity changes.
2. It reflects frequency changes.
3. It reflects changes in insured values.
4. It is based entirely upon insurance data and therefore does not rely upon analogy which is so often open to dispute.

The trend of loss ratios at present rates automatically takes all pertinent factors into account.

In the body of his paper, Steve Newman gives us a numerical example which illustrates the determination of a statewide rate level change. "The actual data were taken from a recent burglary rate filing." We have been told that the loss ratio which is "selected" to underly the proposed change depends upon the relationship among the latest 5 year, the latest 3 year, and the latest 2 year loss ratio. If a consistent upward trend exists among these three, then the latest 2 year loss ratio is selected. If a trend does not exist, then the middle one is selected. In the numerical example the three loss ratios are:

5 year	.531
3 year	.594
2 year	.610

But—the loss ratio selected is not .610. In fact, it is not even .594. It is .580, the loss ratio which will produce a 20% change.

So we magnify the errors discussed above by further compromise.

My only criticism is that Steve has been too matter-of-fact in describing the methodology. This is probably not a fair comment since the paper is an exposition and not a critique of the method. Other than this, the paper is clear and should provide a good reference for students. I hope that it will soon be obsolete.

DISCUSSION BY R. G. OIEN

One of the very nice things about Mr. Newman's paper is that, after his very clear description of burglary insurance ratemaking, he concludes with comments on the current situation for this line. Included in these comments is an exhibit of the underwriting results for a large group of comparable stock companies. The five year composite result indicating an underwriting loss of 5.6% is shown on page 325 of Volume LIII. From a comparison to the 5% provision for profit and contingencies indicated on page 319, we can reasonably conclude that a genuine problem exists for a substantial portion of the industry in this line. It would appear that "contingencies" outweigh "profits" by better than 2 to 1.

Mr. Newman indicates one avenue of possible remedy in suggesting the use of mandatory deductibles; for some sublines, with proper pricing, this may be useful. However, I would suggest that the underwriting result for this line, as well as for many others, is greatly influenced by the fact that rates are calculated for today's circumstance and sold to cover tomorrow's exposure. It is recognized that this is no new opinion, but perhaps it should be said more often. That ratemaking is ideally prospective is something that *should* be accepted for *"shouldness"* sake. In burglary insurance, the need for prospective rating considerations is compound. In addition to the effects of inflation, there is an increasing frequency in the underlying crime events which generate the losses. In other industries contracts may be entered into based on current costs and the ultimate costs may generate a loss, but this is a result due to an inadvertent cost estimate. This is not the "expected" basis for doing business as it is so often in the insurance industry.

What has been said so far was stimulated by Mr. Newman's paper, but does not constitute a review. The subject in this paper was well delineated, placed in perspective, and very well described. To state it simply, in my opinion, the author did his job and did it exceedingly well.

AUTHOR'S REVIEW OF DISCUSSIONS

As mentioned in the presentation of this paper to the Casualty Actuarial Society in November, 1966, its purpose is simply to describe current ratemaking procedures for burglary insurance, and to provide the casualty actuarial student with some insight into the reasons underlying these procedures and why they may differ from those common to other lines of business. In the following discussion, I have tried to clarify certain areas in which interest has been expressed—particularly the development of the Master Rate Table and the use of trend factors.

MASTER RATE TABLE

Background

Prior to August, 1964, the burglary rates applicable to a particular class of risk were determined by reference to a series of rate schedules which were published for each buglary subline. Each territory within a state was rated in accordance with the schedule closest in line with its experience indications. For example, if we assume that for the Money & Securities Broad Form—Inside Premises Coverage, past experience indicated that Territory 3 in State X should use the Money & Securities rate schedule 5, and if we further assume that each rate schedule reflects a 5% increase in rate level over the last numerically lower schedule, then a 12% increase in rate level in Territory 3 for this subline would be translated into the rating structure by assigning Territory 3 to rate schedule 7. Thus only a 10% rate increase would be realized when a 12% increase was indicated. Similarly, an indicated increase in rate level of 13% would result in use of rate schedule 8 for an actual increase of 15%.

It is clear from these examples that this rating procedure was burdensome to handle as well as lacking in precision. The Master Rate Tables, effective in August, 1964, were developed to increase the precision of the rating procedure and to simplify the burglary manual by publishing only one schedule of rates for each subline. The rate relativities underlying the various rate schedules for each subline were retained and are reflected in the applicable Master Rate Table. Territory multipliers were calculated to assure that the rate for each class of risk in each territory would not change as a result of conversion from rating schedules to a Master Rate Table.

Under current ratemaking procedures, rate level changes only affect the territory multipliers of the sublines and have no effect on the Master Rate Tables. Revision of the Master Rate Tables may periodically take place in connection with reviews of classification differentials within each subline. The National Bureau is currently in the process of conducting such a review.

Review of Classification Differentials

Reviews of classification differentials for each subline are based on the countrywide experience of the latest available three years of data, tabulated separately for each class of risk. Earned premiums are adjusted to the level of the base classification by applying the rate differentials underlying the present Master Rate Table. Loss ratios are computed to determine the indicated classification relativities. Set forth below is a simplified hypothetical example to demonstrate this procedure:

Subline Y — Countrywide

(1) Classification	(2) Current Differential	(3) Incurred Losses (000)	(4) Earned Prems. (000)	(5) E.P. on Class 1 Level, (4)/(2) (000)	(6) Loss Ratio at Class 1 Rates (3)/(5)	(7) Indicated Differential
1	1.00	$7,265	$13,576	$13,576	.535	1.00
2	.80	1,726	3,075	3,844	.449	.84
3	.65	443	827	1,272	.348	.65
4	1.25	896	1,745	1,396	.642	1.20

When revised differentials are determined for use in the Master Tables, care is exercised to remove any off-balance that might result.

Graded Rates

For each class of risk under the various burglary sublines, the Master Rate Table sets forth one rate per $1,000 of insurance, with the exception of the Mercantile Open Stock and Residence Theft sublines, for which graded rates are published. The gradations in the rates for these sublines are established by periodic reviews of experience tabulated separately by amount of insurance carried. An illustration of how gradations would be reviewed is set forth below:

(1) Policy Limit	(2) Incurred Losses	(3) Earned Premiums	(4) Loss Ratio	(5) Loss Ratio Index
$ 5,000 or under	$17,600,000	$32,000,000	.550	1.000
$ 5,001 to $10,000	6,267,500	11,500,000	.545	0.991
$10,001 to $15,000	1,769,000	3,200,000	.553	1.005
$15,001 to $20,000	1,493,500	2,900,000	.515	0.936
$20,001 or more	3,948,000	7,000,000	.564	1.025
Total	$31,078,000	$56,600,000	.549	0.998

By comparing the actual loss ratios of the experience summarized by limits carried, the effect of the current rate gradations is automatically included. Therefore, the indices calculated in column 5 indicate the degree of success or failure of the currrent gradations in equalizing loss ratios. Wide variations in loss ratios would indicate the need for revised gradations, whereas uniform loss ratios by amount of insurance would suggest retention of the current gradations.

Graded rates have not been developed for use in the burglary sublines where the primary exposure is money and securities, because under these coverages there is a relatively high probability that total loss to the insured will be the result of each claim. In recognition of this fact the insured usually purchases insurance to value.

In contrast, past experience under Mercantile Open Stock and Residence coverages has shown that when the primary exposure is merchandise, the bulkiness of these items precludes a total loss from theft in most instances. Partial lossses are more common under these coverages, and so experience compiled under these sublines indicates that insureds purchasing policy limits sufficient to cover their maximum possible loss deserve discounts on the rates per $1,000 of insurance in excess of the amount of the most probable loss.

TREND

Rates are based upon the experience of the past, but are to be applied to policies providing coverage in the future. It is therefore necessary to adjust losses incurred in the experience period to the level of costs expected to prevail during the period for which the revised rates will be in effect. For this reason the trend factors computed for application to losses incurred under burglary coverages reflect cost levels anticipated at the average effective date of policies written under the revised rates.

The rationale for the application of trend factors to the lossses incurred under burglary insurance coverages is that a crime committed in the future would be expected to result in greater financial loss to the insured than if the same crime is committed in the present, primarily because of the effects of inflation. As an example, we may select the case of a luggage and leather goods retail store. The actual cash value of merchandise stolen in 1970 would probably be greater than that of similar items stolen in 1965; the cost of labor and materials to repair or replace any damaged goods, furniture, or fixtures would also reflect general inflationary trends.

It has been said that it is improper to reflect trends in loss costs in burglar insurance ratemaking, because inflation will have the same effect on premiums as on losses. This may be true to some extent, but even where premiums *do* increase, they can be expected to lag behind the greater amount of losses and claims paid. In addition, an important factor to consider is that most crimes cause only partial losses to the insured. In our example of the luggage shop, there is a natural limit to the amount of merchandise that may be stolen, because of its bulkiness and the relatively short amount of time available for the theft. If a storeowner buys $3,000 of insurance to cover his estimate of his maximum possible loss, at a time when his most probable loss would be approximately $1,800, he will rarely bother to purchase additional insurance when inflationary trends cause him to revise his estimate of the most probable loss of stock to $1,900. In most cases he would not even be aware of such a change.

In their reviews, both Mr. Bondy and Mr. Oien acknowledge the fact that the trend procedures currently in use fail to reflect the increase in the number of crimes committed in the United States in recent years. Mr. Bondy suggests the use of loss ratios at present rates, rather than the present method of computing trends based on average paid claim cost data, since he believes that the former system "automatically takes all pertinent factors into account."

The National Bureau recognizes the limitations in its current ratemaking procedure, and is constantly studying alternate means of developing rates that will be adequate without becoming excessive or unfairly discriminatory. In particular, the Bureau is presently studying the relative merits of trend factors (for use in ratemaking) based on each of these types of data: average paid claim costs, pure premiums, claim frequencies, and loss ratios at present rates.

With specific regard to burglary insurance, two of the drawbacks of a ratemaking procedure involving trend factors based on either claim frequencies or loss ratios at present rates are discussed below:

1. Claim frequencies are computed by means of a comparison of claims and exposures. However, while burglary insurance is sold using $1,000 of coverage as an exposure basis, no provision is made in the present National Bureau Statistical Plan for recording this figure (only total premium is reported). Even if provision were made in the Statistical Plan to report amounts of insurance purchased, claim frequencies based on these data might not yield a reliable measure of trend for ratemaking purposes, because of credibility considerations.

   The *Uniform Crime Reports* of the Federal Bureau of Investigation, which list total offenses by type, are not based on insurance statistics and therefore do not bear a direct relationship to insurance data, or to the number of claims incurred by insurers under burglary coverages. For this reason trends based on these data are not properly applicable to burglary insurance ratemaking procedures.

2. The trend of average paid claim costs, which measures the effect of inflation on loss costs, is currently computed on a countrywide basis because premium volume is too small and the number of claims too low to permit analysis by state or by subline. The use of countrywide data to compute trends in loss costs does not affect the validity of an individual state rate filing, because the forces of inflation are present throughout all sections of the country. On the other hand, it is widely accepted that the incidence of crime varies in proportion to

a region's degree of urbanization. Small but densely populated areas can be expected to show a much higher incidence of crime than larger but more rural sections of the country.

Although trends should be based on countrywide experience to achieve proper credibility, it must be recognized that burglary rates are computed and filed separately for each state. If countrywide trend factors were to be based on, or reflect in some way, the incidence of crimes, it is clear that the data from cities and urbanized areas would have a disproportionate weight in the indications, since it is these areas that develop more premiums, claims, and losses. It would be unfair to penalize the residents of states with a primarily rural population by having the trends in the incidence of crime in more urban areas reflected in the overall rate level of these less urbanized states.

CONCLUSION

It is most difficult to evaluate fairly the performance of the burglary ratemaking system described in this paper. The underwriting losses of past years do not by themselves tell the whole story, and cannot be completely blamed on the underlying ratemaking system.

In recent years approximately one-half of the statutory underwriting losses developed by the burglary line of insurance resulted from the fact that many companies have overspent the production cost allowance included in the manual rates. Another factor (discussed in the last section of my original paper) which has contributed to poor underwriting results, has been the increasing popularity of multiple-line package policies that include crime insurance coverages. Also of concern to the industry are the rate regulatory practices in some jurisdictions that have an inhibitory effect on justifiable rate increases. At times the industry is not permitted to secure prompt rate relief and is often required to cut back on the level of rates indicated as necessary by the experience in order to secure approval for a needed increase.

Thus the underwriting climate for the burglary insurance business has not been a very favorable one, but we have reason to expect that it will improve in the near future. The industry is becoming increasingly expense-conscious in the face of prolonged underwriting losses, and rate regulatory laws are being reviewed in some states to provide an atmosphere in which necessary rate level changes may be more easily secured.

MINUTES OF THE 1967 SPRING MEETING

May 21-24, 1967

PHEASANT RUN LODGE, ST. CHARLES, ILLINOIS

The meeting formally convened on Monday, May 22, 1967.

Prior to that time there was a meeting of the Council on Sunday, May 21 and, in the evening of that day, there was a social hour for early arrivals.

MONDAY, MAY 22, 1967

The Spring Meeting convened at 9:30 a.m., President Harold E. Curry presiding.

Mr. Curry introduced the Honorable John F. Bolton, Jr., Director of Insurance of the State of Illinois, who welcomed the Casualty Actuarial Society to the State of Illinois for its Spring Meeting.

Vice President Harold W. Schloss then assumed the chair.

The following new papers were presented:

(1) "Underwriting Profit from Investments," by Robert A. Bailey, Chief Actuary, Michigan Department of Insurance.

(2) A guest paper, "A Theoretical Portfolio Selection Approach for Insuring Property and Liability Lines," by Professor J. Robert Ferrari, Wharton School of Finance.

(3) "Loss Ratio Distributions — A Model," by Charles C. Hewitt, Jr., Actuary, Allstate Insurance Company. This paper had also been part of the symposium on the Mathematical Theory of Risk held in conjunction with the November 1966 meeting in Detroit, Michigan.

(4) "Inverse Liability Automobile Accident Insurance," by James B. M. Murray, Casualty Superintendent, Prudential Insurance Company, Ltd., Montreal, Canada.

(5) "Schedule P on a Calendar/Accident Year Basis," by Ruth E. Salzmann, Secretary-Underwriter, Insurance Companies of North America.

This part of the program was followed by reviews of previous papers:

(1) "Current Ratemaking Procedures in Boiler and Machinery Insurance," by James F. Brannigan. Reviewed separately by Ernest T. Berkeley (presented by James F. Richardson) and Augustin J. Cima. Mr. Brannigan commented on these reviews.

(2) "Implications of Sampling Theory for Package Policy Ratemaking," by Jeffrey T. Lange. Reviewed separately by Clyde H. Graves (presented by Robert L. Hurley), Dale A. Nelson, and Charles F. Cook. Mr. Lange's comments were presented in absentia by Mavis Walters.

(3) "Underwriting Profit in Fire Bureau Rates," by Laurence H. Longley-Cook. Reviewed separately by Stanley C. DuRose and Luther L. Tarbell, Jr. (presented by James F. Brannigan). Mr. Longley-Cook's comments on these reviews were read by Frederic J. Hunt, Jr.

(4) "Burglary Insurance Ratemaking," by Steven H. Newman. Reviewed separately by Martin Bondy (presented by Phillip Ben-Zvi) and R. Gustave Oien. Mr. Newman's comments were read by William S. Gillam.

There was then held a panel discussion on "A Current Evaluation of Private Passenger Auto Classification, Merit Rating, and Territory Programs," with the following participants:

Vernon J. Switzer, Moderator
Neill W. Portermain
John S. Trees
Peter B. Zory

Vice President Harold W. Schloss then stated he regretted that time limitations would not permit the discussion period and questions from the floor which had been planned in connection with this panel.

The session then recessed for lunch and the afternoon was devoted to several committee meetings which had been called by the respective chairmen.

TUESDAY, MAY 23, 1967

This session convened at 9:00 a.m. with Vice President Charles C. Hewitt, Jr. presiding.

First, there was held a panel discussion on "Loss and Loss Expense Reserving and Testing Procedures," with the following participants:

John W. Wieder, Jr., Moderator
Robert A. Bailey
Rafal J. Balcarek
F. Lee Herman
Raj Ratnaswamy

After the panel discussion, the gathering broke up into two discussion groups for further consideration and discussion of this topic.

In passing it is noted that a sightseeing bus tour and luncheon had been arranged for the ladies from 10:00 a.m. to 4:00 p.m.

After the luncheon recess, the session reconvened at 1:30 p.m. with Vice President Charles C. Hewitt, Jr. again presiding.

The afternoon session was devoted to a panel discussion on "Government Medical Assistance Programs" with Paul E. Singer as moderator. This discussion was divided into two subdivisions:

(a) "Implications For Tort Liability Systems"
Donald McHugh
DeRoy C. Thomas

(b) "Implications For Private Health Insurance"
Earl F. Petz
Gordon Trapnell

Again the panel discussion was followed by small groups getting together for further discussion of the topics.

In the evening there was a brief social hour followed by a dinner-theatre party.

WEDNESDAY, MAY 24, 1967

After convening of the session at 9:15 a.m., with President Harold E. Curry presiding, the gathering proceeded to consider the conclusions reached by the Council, on the Report of the Constitution Amendment Committee, distributed to all members under date of May 5, 1967.

Daniel J. McNamara, Chairman of the Committee, assisted by Harold E. Curry, Frederic J. Hunt, Jr., Henry W. Menzel, and Thomas E. Murrin, presented the item and answered many questions from the floor. During this discussion many suggestions were made by the membership.

After full discussion, Mr. McNamara stated the Committee would consider all of these suggestions and present a further report for consideration by the Council and the entire subject would be presented to the membership for action at the Annual Meeting in November 1967.

President Harold E. Curry then stated that he was about to appoint the Nominating Committee, with Past President Norton E. Masterson as Chairman, to function in connection with the November 1967 elections. At this point Mr. Masterson asked that he be relieved from serving on the Committee in view of the fact that he had so served for a period of about eight years. Accordingly, President Curry announced the appointment of the following:

William Leslie, Jr., Chairman
Laurence H. Longley-Cook
Thomas E. Murrin

Past President Thomas E. Murrin then reported on matters of interest concerning the American Academy:

(1) There was under consideration a revision of the examination requirements for admission as a member of the Academy.

(2) In view of possible problems that legislative certification of actuaries could cause the property and casualty insurance business, as exemplified by the current Indiana law dealing with the certification of actuaries, the Council of the CAS had voted to inform the American Academy of Actuaries that the Academy should not introduce legislation in any additional states pending a complete discussion and resolution of this matter by the entire membership of the Casualty Actuarial Society.

Following this report, the meeting was adjourned at 12:20 p.m.

It is noted that registrations made at the CAS registration desk during the course of the meeting indicated, in addition to 40 wives, attendance by 90 Fellows, 47 Associates, and 25 invited guests.

FELLOWS

Alexander, L. M.	Berquist, J. R.	Brannigan, J. F.
Allen, E. S.	Bevan, J. R.	Brindise, R. S.
Bailey, R. A.	Blodget, H. R.	Byrne, H. T.
Balcarek, R. J.	Bornhuetter, R. L.	Cima, A. J.
Barker, G. M.	Boyajian, J. H.	Cook, C. F.
Bennett, N. J.	Boyle, J. I.	Crandall, W. H.

FELLOWS

Crane, H. G.
Curry, A. C.
Curry, H. E.
Dahme, O. E.
DeMelio, J. J.
Dickerson, O. D.
Dropkin, L. B.
Eide, K. A.
Elliott, G. B.
Even, C. A., Jr.
Flaherty, D. J.
Foster, R. B.
Gillam, W. S.
Gillespie, J. E.
Graham, C. M.
Hazam, W. J.
Hewitt, C. C., Jr.
Hillhouse, J. A.
Hughey, M. S.
Hunt, F. J., Jr.
Hurley, R. L.
Johe, R. L.
Johnson, R. A.
Kallop, R. H.
Kates, P. B.
Klaassen, E. J.
Linder, J.
Lino, R.
MacGinnitie, W. J.
MacKeen, H. E.
Masterson, N. E.
Mayerson, A. L.
McClure, R. D.
McGuinness, J. S.
Menzel, H. W.
Mills, R. J.
Moseley, J.
Muetterties, J. H.
Murrin, T. E.
Nelson, D. A.
Nelson, S. T.
Niles, C. L., Jr.
Oien, R. G.
Otteson, P. M.
Petz, E. F.
Pollack, R.
Portermain, N. W.
Richards, H. R.
Riddlesworth, W. A.
Roberts, L. H.
Rodermund, M.
Rosenberg, N.
Roth, R. J.
Ruchlis, E.
Salzmann, R. E.
Scheibl, J. A.
Schloss, H. W.
Simon, L. J.
Skelding, A. Z.
Stankus, L. M.
Switzer, V. J.
Trist, J. A. W.
Uhthoff, D. R.
Verhage, P. A.
Walsh, A. J.
Webb, B. L.
Wieder, J. W., Jr.
Wilcken, C. L.
Williams, D. G.
Wilson, J. C.
Wittick, H. E.
Wolfrum, R. J.

ASSOCIATES

Amlie, W. P.
Andrews, E. C.
Ben-Zvi, P. N.
Bickerstaff, D. R.
Bland, W. H.
Blumenfeld, M. E.
Brown, W. W., Jr.
Durkin, J. H.
DuRose, S. C., Jr.
Eliason, E. B.
Farnam, W. E.
Franklin, N. M.
Gibson, J. A. III
Gill, J. F.
Gould, D. E.
Greene, T. A.
Hachemeister, C. A.
Hammer, S. M.
Hanson, H. D.
Harack, J.
Honebein, C. W.
Jensen, J. P.
Mohnblatt, A. S.
Murray, E. R.
Murray, J. B. M.
Naffziger, J. V.
Peel, J. P.
Perreault, S. L.
Raid, G. A.
Ratnaswamy, R.
Richardson, J. F.
Royer, A. F.
Scammon, L. W.
Scheid, J. E.
Schuler, R. J.
Singer, P. E.
Staley, H. B.
Sturgis, R. W.
Toren, C. J.
Trees, J. S.
Walters, M. A.
Welch, J. P.
Wood, D. M.
Wood, D. M., Jr.
Wooddy, J. C.
Young, R. G.
Zory, P. B.

GUESTS

*Battaglin, B. H.
*Benson, L. E.
Bischoff, R.
*Blanc, R.
Bolton, J. F., Jr.
*Brown, P. S.
*Connolly, C. T.
*Davis, R. C.
Ferguson, C. M.
Ferrari, J. R.
Foody, W.
*Griffith, R. W.
Herman, L.
*Kedrow, W. M.
Martin, B.
McHugh, D.
*Nagel, J. R.
*Nightingale, W. R.
*O'Shea, H. J.
Reinbolt, J. B.
*Reiner, J. G.
Song, Y. B.
Thomas, D. C.
Trapnell, G. R.
Zubay, E. A.

Respectfully submitted,

A. Z. SKELDING,
Secretary-Treasurer

*Invitational Program

VOLUME LIV, Part II No. 102

PROCEEDINGS

NOVEMBER 12, 13, 14, 1967

"CHANCE FAVORS THE PREPARED MIND"

PRESIDENTIAL ADDRESS BY HAROLD E. CURRY

One of the most important reasons for my appearance on this program is to acknowledge the fine work done by those members of our Society who have served on the various committees during my terms as President. Committee work requires a great deal of time and effort from the members. Too often, we tend to accept this work as a matter of course and overlook expressing our appreciation to committee members and fellow officers for their contribution to the effective functioning of our Society.

I want to thank every member, personally and on behalf of the Society, for the careful attention given to each task assigned. I would be remiss if I did not mention the names of a few members whose work has been particularly outstanding. Thanks to your careful selection, I have been blessed with two Vice Presidents of great ability and tireless energy. Charlie Hewitt and Harold Schloss have cheerfully accepted the full burden of planning our meeting programs which, I believe you will agree, have been interesting and informative. In addition, on numerous occasions they have volunteered to handle tasks that could have been left for me to do. It is gratifying to note that each of them is being considered for other distinctions of honor.

The Examination Committee, headed by Norm Bennett, has a never ending task. Every member of his committee has a specific assignment for some segment of our examination procedure. They have a heavy responsibility. They must devise examinations that will adequately test the professional proficiency of those seeking membership in our Society and then administer these examinations in a satisfactory manner. To those members who have concluded service on this committee — Don Trudeau, Eldon Klaassen, James Boyle, Darrell Ehlert, and Laurie Longley-Cook — we express our special thanks for their service.

Dan McNamara and his committee who have worked on a revision of our Constitution and By-Laws have faced a difficult task and resolved the many problems in a most capable manner.

Hank Menzel and his Financial Review Committee, after analyzing our financial position, have plotted a course which has made it possible to avoid an increase in annual dues and still preserve our financial stability. Matt Rodermund, our Editor, has contributed to the success of this effort by finding ways to economize in the cost of printing the *Proceedings*.

Al Skelding, with his broad knowledge of our Society's past, has provided sound guidance for my conduct and prevented me from making many mistakes.

The Council is a lively bunch. Far from being an aggregation of "rubber stamp" individuals, they have attacked our many problems with vigor and, through earnest discussion and debate, have resolved them on a just basis.

In his capacity as President of the American Academy of Actuaries, Tom Murrin has demonstrated those qualities of leadership that not only increase his stature as a person but bring renown for our Society from throughout our profession in this country.

The leadership for our profession is not only recognized nationally but also internationally. "Doc" Masterson, through his diligent work as our ASTIN delegate, has been elevated to the highest position in that group.

I could continue on and on citing the good work and accomplishments of our members but these few citations will, I believe, serve to illustrate that our members are providing a substantial body of leadership, not only for our profession but for the insurance industry as a whole. It is gratifying to have this occur and to recognize that we are, as a profession, accepting responsibility for leadership and discharging this responsibility with distinction. I have, many times, told you that ours is a profession that should provide leaders. We are doing just that. In these days when our business is being examined for weaknesses it behooves each one of us to use our knowledge of the business to help mold industry and public opinion along lines that are sound and in keeping with an enduring public policy oriented to the consumer of our services.

A companion of leadership is responsibility — responsibility to analyze a situation accurately and urge a course of action that is fair and equitable to all interests concerned. In order that any situation can be analyzed accurately, we must first assemble all of the facts and arrange them in proper sequence for consideration. Quite candidly, I don't feel that our industry is doing a good job in this area on some of the problems that confront us today and which need to be resolved in the years just ahead. Our profession, I believe, can provide counsel in these situations.

Whether or not we like it our economy has, for some thirty years, been moving toward one that is more completely consumer oriented. I do not allege this is bad but I do assert that, unless we recognize this is occurring and adjust our thinking to this concept, our industry in its many branches — casualty, property, life — will not retain its forceful role in private enterprise economy. Hardly a day goes by when we do not pick up a newspaper and find some comment critical of our business. It may be a letter to the editor complaining about an unexplained cancellation, an editorial about the high costs of insurance, or a headline demanding an investigation of our business. While the easy way out is to ignore these indications of discontent and "trust to luck" that the problem will go away, the enduring answer is to face the problem, analyze the causes for discontent, develop an equitable solution, and take steps to implement the solution.

We seem to have a tendency, as an industry, to either ignore criticism or accept it — at times almost eagerly — but not come to grips with the basic cause for the criticism.

Let's take a quick look at some of the problems confronting us now and see if we are really attacking the basic issues.

1. Auto Insurance Costs Too Much.

We have all read or heard this allegation. Many in the industry seem to accept this allegation as a fact. Many studies have been conducted with the sole objective being to find some means whereby the quantity of premiums needed to maintain the financial stability of our industry and serve the market needs can be diminished. The objective is worthwhile but the force motivating the studies is an assumption open to question. When an item or service is alleged to cost too much the unsaid implication is that the cost is too great in comparison to other items. If we look at the insurance costs of a car in relation to the other cost items involved in car operation and maintenance, you begin to wonder about the validity of an allegation that auto insurance costs too much.

If we use as a span of time the years that have elapsed since the Korean conflict we find that labor costs have moved up 154.9%, typical car repairs have advanced 53.5%, rents have gone up 62.9%, construction costs

94.4%, medical expenses 60.6%, and all living costs 47.8%. Granting that insurance costs have varied in the degree of change, depending on the location and risk classification, the increases tend to hover around the 20% figure — the lowest of any of the indices cited.

As a matter of fact, one calculation I saw recently shows that in relation to the number of miles driven per year the average per mile cost for auto insurance has *decreased* slightly in the past two decades. This is without adjustment for improvements in policy coverage or the inclusion of equipment on vehicles as "standard" nowadays (making them more costly) that twenty years ago was non-existent or available on a limited option basis only. Illustrative of items in this category are air-conditioning, power brakes, power steering and automatic transmissions.

Do figures such as these support the contention that automobile insurance costs too much? Have you noted any clamor in the press for an industry-wide investigation of any of these other cost items? I would direct your attention to the curious fact that the only item of cost for which there is agitation to investigate is the one item subject to strict regulation — insurance rates. Car manufacturers can adjust the prices of new cars, oil companies can increase gasoline costs, and tire prices can go up, without approval from any regulatory authority, and the public will object little, if any — certainly not clamor for an investigation; but if our industry presents a request for a rate increase we are, quite often, confronted with resistance and criticism at the regulatory and legislative levels and by the news media. This is a situation that, I feel, we can and should take steps to correct. This can be done without an appreciable increase in our costs. If, as an industry, we were to take the dollars we spend in trying to establish distinctions between marketing systems and corporate structure and spend them through an integrated, consumer-oriented program of information to the buying public geared to the real causes of needed price changes, I believe we would bring into proper perspective the causative factors that influence the prices (premiums) we find it necessary to charge. The buyer of our insurance services has relatively little interest in the corporate structure of the company from which he buys insurance — whether it be stock, mutual, or reciprocal, what method is used to compensate the agent for his services, or who owns the business. The basic interests of the insurance buyer are what his insurance service costs, whether the company will respond promptly if a loss occurs, and whether the market facility will be available on a continuing basis.

These are facts that have been confirmed many times through private surveys and reiterated through the protestations made to public authorities.

If we are interested in preserving our business as a part of the private enterprise system — and I presume we are — we must shift our thinking to a consumer-oriented economy and make a conscious, concerted effort to provide a market facility for every risk granted a license to drive. The place to advocate what qualifications a person must meet to be accorded a license to drive is in the legislative halls, not the underwriting departments of the companies. It is our responsibility to develop realistic, sound, rating and pricing systems that will provide a market facility for every risk who meets the prescribed licensing standards. To accommodate this concept we must direct our attention to the creation of a regulatory system, regardless of the level, that will foster the existence of market facilities of this breadth on a fair and equitable basis to the insurance buyer *and* to the insurance companies. There is nothing inherent in this concept that dictates uniformity in approach either as to corporate structure, distribution system, coverage, or price. We should awaken to the fact that our statute books, at both the State and Federal levels, contain many laws specifically designed to safeguard the rights of the consumer to select the market facility he desires to support and to have a choice in the price he shall pay. These are not sterile statutes, enacted and forgotten. They are active statutes alertly and vigorously enforced.

I do not know of any industry that offers as many sources of supply or as many variations in coverage or price as does ours. The emphasis on which our regulatory system rests seems to me to be unsupported by sound logic. The emphasis on regulation should be on establishing and maintaining companies on a stable financial basis and to encourage adequate market facilities. The forces of competition will, within the framework of our free enterprise system, prevent prices from attaining levels out of proportion to the services provided. This has been demonstrated innumerable times in our economic history. The key fact is the availability of a market from which the insurance buyer can select the service he desires. Some people overlook this fact and liken our business to those industries that hold a monopolistic franchise to provide a named service to a defined geographical area. I fail to see the need for the same pattern of regulation for our industry that would apply to, for example, a telephone company or a power company that holds an exclusive service franchise to a given area. The fact that each is concerned with the "public interest" — whatever that means — does not persuade that parallel regulatory systems are warranted.

It is my earnest hope that our industry will not postpone an objective analysis of the basic causes of voiced protests about insurance costs, and will adopt a consumer-oriented approach to dispel the misconceptions we have allowed to take root.

2. *The Tort Liability System Is Outmoded.*

This is another of those situations where our industry is alleged to be the root cause of problems the consumer wants solved and, in our role as traditional whipping boy, we accept the allegation as factual and frantically embark in quest of an acceptable substitute rather than analyze the problem in depth, strive to isolate the causes of the problem, and embark on a sound program of reform.

Basically, the American consumer is a staunch advocate of fair play. Being of this conviction, when he encounters situations offensive to this concept, he voices his objection. This is what has happened to our present system of tort liability.

I have not encountered any substantial body of criticism to the basic concept that the wrongdoer should be held accountable for his acts. The voiced protest is primarily a lack of acceptance as to the means used for assessing accountability for wrongdoing.

I have not detected any widespread advocacy of a system whereby persons who are injured or have property destroyed will profit through such an occurrence. The American concept of fair play does, understandably, seek restitution for out-of-pocket expense and loss of earnings — restoration of conditions existing prior to the occurrence of injury or damage to property. The objective is clear-cut. The measures we have taken toward this objective are hesitant and limited. We have not provided market facilities sufficient in scope or adequate in amount to meet this objective. We have been a party to the development of intricate legal terminology involving such terms as "gross negligence," "contributory negligence," and "comparative negligence," none of which are understood by the buying public generally and which, in too many instances, are offensive to his basic concept of fair play. Most of us realize that some accidents will occur so long as human judgment is involved in the operation of an automobile and that, except in rare instances, the occurrence will involve error by only one person. If we were to work away from this concept we can modify our tort liability system to a relatively minor degree — not abandon it — and provide the market facilities to fulfill this objective.

It is alleged our industry is responsible for court congestion. This seems to me to be a confusion of cause and effect. I don't know why we are reluctant to face this problem squarely and advocate realistic remedies because, to the best of my knowledge, no company follows a procedure that dictates that all claims must be adjudicated in court. What causes court congestion? One speaker I heard recently stated, in reference to the New York City situation, that the number of judges hearing civil cases there had not been increased in forty years. If this be true, does this impress you as realistic? Would it not be proper for our industry to try and help correct this situation?

I wonder, too, if our industry should not interest itself in seeking simplification in our laws to the end that it becomes relatively unnecessary for any injured person to retain counsel and file an action in court as a step precedent to a consideration of his claim on its merits. This avenue of study could beneficially be extended to include the ethical procedures by which counsel is retained and the sanctions invoked for departures therefrom.

My whole point is that the basic defect is not with the tort liability concept itself. The defect lies in the practices and procedures that have grown up over the many years. These defects can be defined, analyzed, and proper remedies instituted without destroying a system that is sound in concept and has been a means for measuring justice for many, many years.

Discontent with the tort liability system is currently fostering multiple suggestions for programs to replace it. While I like to think I am an advocate of progress I am reluctant to replace a known quantity, even though defective in some respects, with an unknown quantity, particularly when the cost of the advocated replacement cannot be determined with accuracy and confidence.

As a profession, we have a vital concern in the costing of these suggested programs. Several of our members have spent or are spending a great deal of time and effort in research trying to forecast the cost of these programs, particularly the Keeton-O'Connell plan. I commend each of you for your courage, diligence, and sincerity in this research. You face an almost impossible task for the simple reason there is no body of reliable data presently available to use in forecasting costs. Unfortunately, the Keeton-O'Connell plan advocates are using "lower cost" as a principal selling point. This places our profession in a dangerous position and one of heavy responsibility. If our selection of assumptions are accurate and properly weighted so that the financial result to our industry is favorable we could emerge as heroes. If the results are adverse we can, with some justification, be blamed for destroying an industry. It is my personal conviction that a problem of this magnitude merits the development of meaningful data directly pertinent. This is an industry and professional problem potentially involving billions of dollars annually and the future well-being of our industry and the public.

Recognizing that we may be confronted with a plan of the Keeton-O'Connell type as a legislative reality in the not-too-distant future, I feel our Society would be well advised, without delay, to institute a research program in depth designed to develop the data necessary to accurately forecast the cost of such a plan. This is not an individual company, rating bureau, or trade association problem. It is an industry problem intimately involving our profession. The cost of such a research project would be substantial but of small amount in relation to the potential financial impact.

3. *Involvement of Investment Income in Ratemaking.*

This is another subject in which our profession is deeply involved. In some respects, it is difficult to conceive why this arises as a problem to plague us. Perhaps the fragmentation of investment income, involved in some proposals, has tended to prevent a proper perspective for the entire subject.

Any elementary analysis of our business will establish that there are definite financial criteria that must be met if the needed market facilities are to be provided. The funds necessary to maintain (or establish) companies that are financially strong and able to serve an expanding market must come from one of two sources, or a combination of the two:

(a) contribution of capital funds either through the sale of stock or contribution of surplus, or

(b) funds earned from within the business.

These total financial needs of our business are not augmented or diminished by bookkeeping entries that simply transfer funds from one account to another. Reduced to its simplest terms, this is what happens when so-called investment income (in whole or in part) is transferred to the so-called underwriting account. For example, let us assume a company determines that it needs $50,000,000 in a given year to maintain a suitable relation of surplus to writings and that it is not feasible to seek outside capital. If the company assumes that $40,000,000 of the total needed will be derived from investment income then only $10,000,000 needs to be loaded into the rate level.

If the anticipated results are reversed or blended in any other proportion, the aggregate sum is not altered. However, the rating formula will need to be adjusted to reflect the difference in bookkeeping practice. This is a realistic fact that we need to forcefully convey to the public.

I have just cited three areas involving problems of deep concern to our industry, in which our profession is intimately involved. Many others could be mentioned. I have raised questions hoping to stimulate your thinking. I have expressed ideas pertinent to possible solutions. I do not expect you to endorse them but I do hope that some comment or remark may motivate the study or research from which equitable and sound solutions can be developed.

The topic for these comments — "Chance favors the prepared mind" — is an utterance from an unknown author but it is certainly appropriate to the current situation. I sincerely hope that we will take the steps necessary to attain a "prepared mind."

In closing, I want to thank you from the bottom of my heart for the honor of having been your President. It has been a rewarding experience. I have learned much. The Society has moved forward with vigor. Our membership has increased as has the attendance at our meetings. I acknowledge my mistakes but assure you they occurred because I lack the wisdom of a Solomon and not through deliberate intent. I am proud of our Society and our profession. As time flows on I know we will achieve new goals and accomplishments. Subject to the limitations of my abilities, I want to help.

THE MINIMUM ABSOLUTE DEVIATION TREND LINE

CHARLES F. COOK

> "Since the desired curve is to be used for estimating, or predicting purposes, it is reasonable to require that the curve be such that it makes the errors of estimation small. . . . However, sums of absolute values are not convenient to work with mathematically; consequently [it is required] that the sum of the squares of the errors be a minimum."
>
> —Paul G. Hoel

Two problems arise out of the use of the method of least squares for determining an average claim cost trend line. First, a single odd point in the data has an excessive influence on the fitted line, and second, the oldest and newest points are given excessive weight relative to intermediate points, which may result in an inordinately large change in slope when a new point is added to the data and the oldest is deleted. These problems are not unique to our trend lines, but apply to all lines fitted by the method of least squares. They are the direct result of squaring the deviations between the data and the fittted line, which is simply not as reasonable a criterion of "best fit" as the absolute value of the deviation.

Why then do we use the method of least squares? Simply because absolute values are alleged to be mathematically inconvenient. This is not true; a trend line minimizing the sum of the absolute values of the deviations can be calculated, by the method shown in this paper, more easily than a least squares trend line. I do not mean to claim that all authors of books on mathematical statistics are wrong; but what is mathematically inconvenient to them is not necessarily inconvenient to an actuary. A minimum absolute deviation method of fitting a line is mathematically inconvenient for the following reasons:

1. It will not fit a *curved* line.
2. It requires equal intervals between measurements.
3. The form of calculation is an algorithm of the operations analysis type, rather than a concise mathematical formula.
4. It does not always produce a unique result; rather the minimum may be achieved for any slope a such that $m \leq a \leq n$.
5. It does not necessarily pass through the mean, so that the average deviation may not be zero, as it is for a line fitted by the method of least squares.

Inconveniences (1), (2), and (3) do not apply to our specific problem. Number (4) appears to be only theoretical; in practice, it seems adequate to use $a = (m + n)/2$, the mean of all slopes which produce the minimum.

We have chosen to resolve inconvenience (5) by *defining* our line as the one which minimizes the sum of absolute deviations, *subject to the condition that the average deviation be zero.* Incidentally, this condition not only yields an intuitively more reasonable result, but reduces the computational labor by about one-third.

THE MINIMUM ABSOLUTE DEVIATION ALGORITHM

Given n observations y_i associated with equally-spaced points x_i, the problem is to determine the values a, b such that $\sum_{i=1}^{n} | ax_i + b - y_i |$ is a minimum, subject to the condition $\sum_{i=1}^{n} (ax_i + b - y_i) = 0$.

1. If n is odd, set $x_i = -\left(\frac{n+1}{2}\right) + i$. If n is even, set $x_i = -(n+1) + 2i$.
2. Calculate $\sum_{i=1}^{n} yi/n = \bar{y}$ and $\sum_{i=1}^{n} | x_i |/2 = MX$.
3. Calculate $a_i = \dfrac{y_i - \bar{y}}{x_i}$ for all i.
4. Order the a_i from least to greatest, such that $a_{i1} \leq a_{i2} \leq \ldots \leq a_{in}$.
5. Order the x_i the same way as their associated a_i.
6. Accumulate the $| x_{i_j} |$ to form $Z_k = \sum_{j=1}^{k} | x_{i_j} |$.
7. Find k^*, the least k for which $Z_k \geqslant MX$.
8. If $Z_{k^*} = MX$, then the desired line is $y' = \dfrac{a_{i_{k^*}} + a_{i_{k^*+1}}}{2} x + \bar{y}$.
   If $Z_{k^*} > MX$, then the desired line is $y' = a_{i_{k^*}} x + \bar{y}$.

Example

While at first reading the algorithm may seem complex, it is very simple to perform. All arithmetic except possibly the division $\sum_{i=1}^{n} y_i/n = \bar{y}$ in step (2) may be done mentally. All other divisors are small integers; there is no multiplication or squaring at all. The simplicity of the procedure is illustrated by the following example, in which all work is shown. It may be enlightening for the reader to try fitting a least squares line to the same data without benefit of calculator, slide rule, or scratch paper.

x_i	y_i	$(y_i - \bar{y})$	a_i	Rank	Z_k
−6	110	−3.4	.567	5	18
−5	109	−4.4	.880	10	
−4	112	−1.4	.350	4	12
−3	111	−2.4	.800	8	
−2	115	+1.6	−.800	1	2
−1	112	−1.4	1.400	12	
0	113	−0.4	——	—	
1	114	+0.6	.600	6	19
2	112	−1.4	−.700	2	4
3	116	+2.6	.867	9	
4	114	+0.6	.150	3	8
5	117	+3.6	.720*	7	24*
6	119	+5.6	.933	11	

$\sum |x_i| = 42 \qquad \sum y_i = 1{,}474$

$\frac{\sum |x_i|}{2} = 21 \qquad \bar{y} = 113.4 \qquad y' = .72\,x + 113.4$

Proof of the Algorithm

Lemma 1: If $E(a) = \sum_{j=1}^{n} | ax_i + \bar{y} - y_j |$, $0 < \Delta \leq (a_{i_{k+1}} - a_{i_k})$, and

$$\epsilon_k = E(a_{i_k} + \Delta) - E(a_{i_k}), \text{ then for any } a_{i_k}$$

$$\epsilon_k = \Delta \left(\sum_{j=1}^{k} | x_{i_j} | - \sum_{j=k+1}^{n} | x_{i_j} | \right)$$

Proof: By the definition of ϵ_k we have

$$\epsilon_k = \sum_{j=1}^{n} | (a_{i_k} + \Delta)\, x_j + \bar{y} - y_j | - \sum_{j=1}^{n} | a_{i_k} x_j + \bar{y} - y_j |$$

By substituting with the equation $y_j = a_j x_j + \bar{y}$, we get

$$\epsilon_k = \sum_{j=1}^{n} | (a_{i_k} + \Delta - a_j) x_j | - \sum_{j=1}^{n} | (a_{i_k} - a_j) x_j |$$

$$= \sum_{j=1}^{n} | x_j | \cdot \Big(| a_{i_k} + \Delta - a_j | - | a_{i_k} - a_j | \Big)$$

It is clear that if $a_{i_k} \geqq a_j$, then $(a_{i_k} + \Delta) > a_j$, so that

$$| a_{i_k} + \Delta - a_j | - | a_{i_k} - a_j | = \Delta$$

Likewise if $a_{i_k} < a_j$, then $(a_{i_k} + \Delta) \leqq a_{i_{k+1}} \leqq a_j$, so that

$$| a_{i_k} + \Delta - a_j | - | a_{i_k} - a_j | = -\Delta$$

But by construction $a_{i_k} \geqslant a_j$ for $j = i_1, i_2, \ldots i_k$, and similarly

$a_{i_k} < a_j$ for $j = i_{k+1}, i_{k+2} \ldots i_n$. Therefore

$$\epsilon_k = \sum_{j=1}^{k} | x_{i_j} | \Delta + \sum_{j=k+1}^{n} | x_{i_j} | (-\Delta)$$

$$= \Delta \left(\sum_{j=1}^{k} | x_{i_j} | - \sum_{j=k+1}^{n} | x_{i_j} | \right) \qquad \text{Q.E.D.}$$

Lemma 2: If $a_{i_{k+1}} = a_{i_k}$, then $\epsilon_k = 0$. *If* $a_{i_{k+1}} > a_{i_k}$, then ϵ_k is $> 0, = 0,$ or < 0 according to whether $\left(\sum_{j=1}^{k} | x_{i_j} | - MX \right)$ is $> 0, = 0,$ or < 0, respectively.

Proof: If $a_{i_{k+1}} = a_{i_k}$, then $\epsilon_k = E(a_{i_{k+1}}) - E(a_{i_k}) = 0$.

If $a_{i_{k+1}} > a_{i_k}$, we have by the definition of MX that

$$\sum_{j=1}^{k} | x_{i_j} | + \sum_{j=k+1}^{n} | x_{i_j} | = 2MX$$

$$\sum_{j=1}^{k} | x_{i_j} | - MX = MX - \sum_{j=k+1}^{n} | x_{i_j} | = \tfrac{1}{2} \left(\sum_{j=1}^{k} | x_{i_j} | - \sum_{j=k+1}^{n} | x_{i_j} | \right)$$

$$= \frac{\epsilon_k}{2\Delta} \quad \text{(by Lemma 1)}$$

This completes the proof because $\Delta > 0$ and thus cannot affect the sign.

Theorem 1: If $Z_{k^*} > MX$, then for all $a \neq a_{i_{k^*}}$, $E(a_{i_{k^*}}) < E(a)$.

Proof: For $a_{i_{k^*}} < a \leqq a_{i_{k^*+1}}$, let a $= a_{i_{k^*}} + \Delta$. Then $\epsilon_{k^*} > 0$, because

$$\sum_{j=1}^{k^*} | x_{i_j} | = Z_{k^*} > MX$$

For $a > a_{i_{k^*+1}}$, the argument holds *a fortiori*, because for all $k \geqslant k^*$

$$\sum_{j=1}^{k} | x_{i_j} | \geqslant Z_{k^*}$$

so that each $\epsilon_k \geqslant 0$. At least one such $\epsilon_k > 0$ because $a \neq a_{i_{k^*}}$. Therefore

$$E(a_{i_{k^*}}) \leqslant E(a_{i_{k^*+1}}) \leqslant \ldots < E(a)$$

For $a < a_{i_{k^*}}$ the same arguments hold in reverse; by the algorithm, for all $k < k^*$

$$\sum_{j=1}^{k} | x_{i_j} | < MX$$

so that each $\epsilon_k \leqslant 0$. At least one such $\epsilon_k < 0$ because $a \neq a_{i_{k^*}}$. Therefore

$$E(a) > \ldots \geqslant E(a_{i_{k^*-1}}) \geqslant E(a_{i_{k^*}})$$

Theorem 2: If $Z_{k^*} = MX$, then for all a, b such that $a_{i_{k^*}} \leqq b \leqq a_{i_{k^*+1}}$ and $a < a_{i_{k^*}}$ or $a > a_{i_{k^*+1}}$, $E(b) = E(a_{i_{k^*}}) < E(a)$.

Proof: If $a_{i_{k^*}} = a_{i_{k^*+1}}$, we have $Z_{k^*+1} > MX$ and $b = a_{i_{k^*}} = a_{i_{k^*+1}}$, so that Theorem 1 can be applied. Therefore we need only consider the case in which $a_{i_{k^*+1}} > a_{i_{k^*}}$.

Let $b = a_{i_{k^*}} + \Delta$. Then $\epsilon_{k^*} = 0$ by Lemma 2, since

$$\sum_{j=1}^{k^*} | x_{i_j} | - MX = Z_{k^*} - MX = 0$$

If $a_{i_{k^*+1}} < a < a_{i_{k^*+2}}$, let $a = a_{i_{k^*+1}} + \Delta$. Then $\epsilon > 0$ by Lemma 2, since

$$\sum_{j=1}^{k^*+1} | x_{i_j} | = Z_{k^*} + | x_{i_{k^*+1}} | > MX$$

For $a > a_{i_{k^*+2}}$ and $a < a_{i_{k^*}}$, the proof of theorem 1 is applicable.

A DISCIPLINE FOR THE AVOIDANCE OF UNNECESSARY ASSUMPTIONS

LEWIS H. ROBERTS

INTRODUCTION

Although unnecessary assumptions are something we all try to avoid, advice on how to do so is much harder to come by than admonition. The most widely quoted dictum on the subject, often referred to by writers on philosophy as "Ockham's razor" and attributed generally to William of Ockham, states *Entia non sunt multiplicanda praeter necessitatem* (Entities are not to be multiplied without necessity). As noted in reference (1), however, the authenticity of this attribution is questionable.

The same reference mentions Newton's essentially similar statement in his *Principia Mathematica* of 1726. Hume (3) is credited by Tribus (2c) with pointing out in 1740 that the problem of statistical inference is to find an assignment of probabilities that "uses the available information and leaves the mind unbiased with respect to what is not known." The difficulty is that often our data are incomplete and we do not know how to create an intelligible interpretation without filling in some gaps. Assumptions, like sin, are much more easily condemned than avoided.

In the author's opinion, important results have been achieved in recent years toward solving the problem of how best to utilize data that might heretofore have been regarded as inadequate. The approach taken and the relevance of this work to certain actuarial problems will now be discussed.

BIAS AND PREJUDICE

One type of unnecessary assumption lies in the supposition that a given estimator is unbiased when in fact it has a bias. We need not discuss this aspect of our subject at length here since what we might consider the scalar case of the general problem is well covered in textbooks and papers on sampling theory. Suffice it to say that an estimator is said to be biased if its expected value differs by an *incalculable* degree from the quantity being estimated. Such differences can arise either through faulty procedures of data collection or through use of biased mathematical formulas. It should be realized that biased formulas and procedures are not necessarily improper when their variance, when added to the bias, is sufficiently small as to yield a mean square error lower than the variance of an alternative, unbiased estimator.

As an example of bias due to sampling procedure, suppose we sample a population in a non-random, haphazard manner so that probabilities of selection vary in an unknown way. There is no method by which to calculate the difference between the expected value of the mean of such a sample and the mean of the population. Hence, the sample mean is a biased estimator. On the other hand, if probabilities of selection are known, appropriate weighting will provide an unbiased estimator. An example of bias due to choice of mathematical formula is the use of ratio estimates, as where the ratio of y to x obtained by sampling is multiplied by a known population total of x to estimate the population total of y. The combined bias and standard error of a ratio estimate is often less, however, than the standard error of the best alternative unbiased estimate. An estimator is not considered to be biased if there is any way of removing the bias. Thus, the sum of the means of random samples of x and of y is considered to be an unbiased estimator of the expected value of x if we know the expected value of y. This is because we can subtract the latter quantity leaving $\bar{x} + \bar{y} - \mathcal{E}y$, the expected value of which is clearly $\mathcal{E}x$.

Our concern here is not primarily with point estimations but with complete statistical distributions. We shall consider any distribution function characterized by parameters *or form* not directly derived from the data as "prejudiced." This seems an apt characterization since different analysts may derive different functions from a given set of data if they go beyond the data in their specifications. These differences can inferentially be imputed to differing personal prejudices (perhaps unconscious) in favor of one function over another.

While we presumably exercise no conscious favoritism for one type of distribution function over another and we test all plausible choices impartially, we are necessarily limited to those functions with which we are familiar and which we can handle mathematically. The phenomena we study are not necessarily so constrained. In some problems, however, we are fortunate in that the data include information that a process is involved which can produce only a particular kind of distribution, so there is no possibility of prejudice.

THE LOGICAL INCONSISTENCY OF PREJUDICE

Let us suppose that data X imply conclusions C_X. Let us suppose, further, that we do not quite know how to interpret X and cannot draw any conclusion unless we assume Y also to be true. Then we draw the conclusion C_{XY} and tender it as C_X. That this is clearly a false coin is seen when someone else similarly finds it necessary to make an assumption, say Z, and tenders C_{XZ} as C_X. More embarrasssing, we ourselves may at a later date find assumption W to be more agreeable than Y so we now find ourselves with a different conclusion, C_{XW}, from the same data. Alternatively, we may telescope the process and offer two or more conclusions simultaneously, at the same time admitting their dubious nature by revealing the alternative assumptions we found ourselves obliged to adopt but between which we are at a loss to choose.

The thesis of this paper is that there is a way out of this dilemma in an important class of problems.

ENTROPY

By way of wielding Ockham's razor, we might devise some measure whereby different functions could be compared as to number of *"entia."* Of all functions consistent with the data we might select the one, or ones, requiring the fewest *entia,* i.e., the least information, as being minimally prejudiced. The author joins others, cited in the references hereto, in proposing a measure employed by Shannon (4) in the development of information theory and subsequently adopted by Jaynes (5), Tribus (2), and others in re-derivations of the theorems of statistical mechanics and thermodynamics.

Success in these areas suggested that valid applications might be found in the area of statistical inference (2d, 2e). Shannon's measure, which he called the "entropy" or "uncertainty" of a distribution, is defined by:

$$S = -K \sum p_i \ln p_i \tag{1}$$

where p_i is the probability associated with the i^{th} discrete possibility and the summation is taken over all possibilities having non-zero probability. K is an arbitrary scaling factor. *Ln* refers to natural logarithms although inclusion of a scaling factor would permit use of logarithms to any base.

An amusing sidelight on the naming of this measure is related by Tribus (2f):

> When Shannon discovered this function he was faced with the need to name it, for it occurred quite often in the theory of communication he was developing. He considered naming it "information" but felt that this word had unfortunate popular interpretations that would interfere with his intended uses of it in the new theory. He was inclined towards naming it "uncertainty" and discussed the

> matter with the late John Von Neumann. Von Neumann suggested that the function ought to be called "entropy" since it was already in use in some treatises on statistical thermodynamics . . . Von Neumann, Shannon reports, suggested that there were two good reasons for calling the function "entropy." "It is already in use under that name," he is reported to have said, "and besides, it will give you a great edge in debates because nobody really knows what entropy is anyway." Shannon called the function "entropy" and used it as a measure of "uncertainty," interchanging the two words in his writings without discrimination.

Shannon showed that this measure is unique in satisfying the following criteria:

(a) It should depend only upon the probability distribution, i.e., S is a function of $p_1, p_2 \ldots p_n$.

(b) If all of the p_i are equal, then $p_i = 1/n$ and S is a monotonically increasing function of n.

(c) The measure should be consistent in the sense that if we consider events A and B in the context of a state of knowledge X, then we should have

$$S(AB \mid X) = S(A \mid BX) + S(B \mid X)$$

That is, the entropy ascribed to A and B jointly in the context of X equals the entropy that would be ascribed to A in the context of B and X plus the entropy that would be ascribed to B alone in the context of X. This parallels the law of compound probabilities.

FORMAL RESULTS

Defining the minimally prejudiced distribution function as that for which S is at a maximum, let us look at the derivations of some familiar distributions. These problems will be characterized by the information available and the solution derived by maximizing S. We assume that nothing whatever is known about each distribution beyond what is stated. In practice there might be additional, non-quantitative data that would preclude use of the functions derived here in certain cases. Derivation of the minimally prejudiced distribution subject to common qualitative constraints would be an important extension of presently known results.

In a wide variety of problems, available information may be in the form of averages such as the mean first power, mean square, mean cube, etc. of the variate x. The following results would apply to means of any single-valued continuous functions, for example trigonometric or logarithmic functions, as well as to the usually reported integral power functions. We can denote these various means as

(2) $$\bar{g}_r(x) = \sum p_i g_r(xi)$$

where $r = 1, 2, 3 \ldots m$ for m different functions of x and $\sum p_i = 1$.

The measure just presented enables us to compare statements about a distribution in such a way that we can select that one among all satisfying the given data which, by virtue of maximum entropy, best complies with Ockham's dictum in the sense of asserting the least information. As noted by Tribus, "By using this principle, the observer reduces his subjectivity to the minimum possible value." In problems where this procedure inevitably leads different analysts to the same result, the author considers that subjectivity, or prejudice, has been reduced to zero. The only challenge that might be made to this claim would seem to rest upon the degree of subjectivity entailed in adopting the principle of maximum entropy as a criterion in the first place. Whether the case for adoption of this principle is so overwhelming as to remove all possibility of subjectivity on that point (so that its rejection is outright error) will not be argued here. It does seem clear, however, that as between persons who adopt the principle as a convention, there is no room for personal prejudice. This alone is a strong recommendation for any convention not demonstrably in error.

We now make certain observations concerning Shannon's measure:

1. If the logarithm is taken to base 2 (rather than to the base e) S is equal to the expected number of questions in a taxonomic game, such as Twenty Questions, that would be needed to remove all doubt (2b).
2. In general, S is a measure of the "flatness" of a distribution, hence of the relative equality with which probabilities are assigned. This follows from the intuitive notion that event A should not be assumed, without reason, to be more likely than event B. (It seems obvious that consistent results cannot be expected if probabilities are assigned whimsically.)
3. The measure is differentiable, hence can be maximized by classical methods (i.e., without resort to linear programming or other iterative procedures) to yield minimally prejudiced functions as extremals.
4. The fact that the measure employs a summation of probabilities, rather than an integral, apparently precludes its use in problems

that require continuous distributions. Yet, the class of phenomena involving only a finite number of particles and the emission or absorption of discrete quanta of energy may be sufficiently broad as severely to limit, if not to rule out, the occurrence of physical events for which continuous distributions are strictly appropriate. Physical considerations aside, the digitalization of measurements converts data representing even theoretically continuous distributions into discrete form. This author does not see it as a flaw, therefore, that the measure of entropy has not been defined for continuous distributions.

It is shown by Jaynes and Tribus that the assignment of the p_i for which S is at a maximum (K being an arbitrary constant) is

$$(3) \qquad p_i = exp.\ [- a_o - a_1 g_1(x_i) - a_2 g_2(x_i) - \ldots .]$$

in which the a's are Lagrangian multipliers satisfying the requirements of $\bar{g}_r(x)$ and

$$(4) \qquad a_o = ln \sum_i exp.\ [\ \sum_r a_r g_r(x_i)]$$

while

$$\bar{g}_r(x) = -\ \partial a_o / \partial a_r = \text{mean of } g_r(x)$$

$$var.\ [g_r(x)] = \partial^2 a_o / \partial a_r^2 = \text{variance of } g_r(x)$$

and

$$S = K a_o + K \sum_r a_r\ \bar{g}_r(x)$$

SPECIFIC DERIVED DISTRIBUTIONS

Known Data	Distribution with Maximum Entropy
Range	Uniform
$\sum_{i=1}^{m} p_i = 1$	$p_i = exp.(-\ a_o) = \dfrac{1}{m}$
Mean*	Exponential
$\sum_{i=o}^{\infty} p_i x_i = \bar{x}$	$p_i = exp.(-\ a_o - a_1 x_i)$

Mean and variance*	Truncated Gaussian
$\sum_{o}^{\infty} p_i x_i = \overline{x}$	$p_i = exp.(- a_o - a_1 x_i - a_2 x_i^2)$
$\sum_{o}^{\infty} p_i x_i^2 = x^2$	
Mean and mean logarithm*	Gamma
$\sum_{o}^{\infty} p_i x_i = \overline{x}$	$p_i = exp.(- a_o - a_1 x_i - a_2 x_i^2)$
$\sum_{o}^{\infty} p_i \ln x_i = \overline{\ln x}$	$= x_i^{-a_2} exp.(- a_o - a_1 x_i$
Mean logarithm and mean logarithm of complement where $o \leqq x \leqq 1$	Beta distribution $p_i = exp. [- a_o - a_1 \ln x_i - a_2 \ln (1 - x_i)]$ $= x_i^{-a_1} (1 - x_i)^{-a_2} e^{-a_o}$

$* \sum_{i=o}^{\infty} p_i = 1$

From theory and the foregoing examples it can correctly be inferred that for every distribution there is at least one specification as to the data which must be known for that distribution to be the minimally prejudiced distribution. Also, there is a unique minimally prejudiced distribution for each specification of known data. In general, for $f(x)$ to be the minimally prejudiced distribution, the known data must be the expected value of the natural logarithm of $f(x)$. For example, what data must be known in order that $f(x) = \sin x$ where $o < x < \pi/2$? Evidently we shall have $p_i = exp. (-a_o - a_1 \ln \sin x) = \sin x$ *if* a_o is set equal to zero and $a_1 = 1$.

AN APPARENT PARADOX

An apparent paradox can arise in the fitting of distributions of the generalized exponential type, $p_i = exp. (a_o + a_1 x_i + a_2 x_i^2 + \ldots)$, which more or less typify the system of maximum entropy, when actual distributions are better fitted by some other curve. At such a time we are inclined to ask what is so good about a system that does not give the best fit. The point to remember here is that if we have the distribution function, or if we have a summary of it in the form of grouped data, there is no particular reason to prefer the generalized exponential over any other curve. Equation (3) applies strictly only when our data are limited to the expected values of $g_1(x)$, $g_2(x)$, etc. If we have more information we should use it. Theoretically, of course, by calculating the mean values of a sufficient number of functions of x we can approximate any arbitrary distribution as closely as we please.

The discipline advanced here does not tell us what function best fits a more or less completely specified distribution. It does tell us, however, what data to summarize in order that a given kind of distribution function shall be best characterized by that data. For example, if a class of distributions are found to be of the log-normal type, the data we should be collecting are the mean and variance of $\log x$. Similarly, if the distributions for a certain kind of variable are typified by a Gamma distribution, then we should compile mean values of x and $\log x$, and so on. Such knowledge is economical since necessary data can often be summarized in the course of ordinary processing of cases without the necessary of compiling a great many separate distributions.

It is obviously advantageous, by judicious selection of the function of x to be averaged, to reduce the number of statistics that must be compiled.

Of more importance, in the author's opinion, is that for any given data the criterion of maximum entropy leads to what he believes to be a mathematically optimum compliance with the principles attributed at the outset of this paper to Ockham and Hume for the avoidance of prejudice and unnecessary assumptions.

ENTROPY AS A MEASURE OF HOMOGENEITY

Let a classification plan subdivide a population of risks into n classes such that for any particular layer of loss the probability of occurrence of a loss during a specified time interval is p_i for the i^{th} class. Then for that layer of loss the entropy of this classification scheme is as defined in equation (1). As between two classification plans applied to the same population of risks, the plan for which S is smaller contains the more information (less entropy). As between two populations classified according to the same plan, S is greater for the more homogeneous population. This measure is of interest in comparison with the coefficient of variation, proposed by Bailey (6) as a measure of homogeneity. It is not clear how much advantage, beyond consistency with the general theory advanced here, entropy offers over Bailey's measure.

APPLICATIONS TO COMPOSITE AND CONVOLUTED DISTRIBUTIONS

We define a composite distribution as the result of mixing two or more dissimilar distributions. It is obvious that for the mixture all of the functions x, x^2, x^3, etc., will have as their expected values the weighted averages of the distributions brought together. This enables us to describe the composite distribution without further analysis in terms of equation (3). It does not, however, assure that the distribution so determined will provide a good fit to the data unless the functions being averaged are appropriate to describe each of the separate distributions.

We define an n-fold identically convoluted distribution as the distribution of the sum or mean of n values selected independently from the same (infinite) parent population. The parameters of such a distribution are shown by Kendall (7) to vary as follows:

Parameter	Parent Population	Convolution
Mean	$\bar{x}$	Sum $n\bar{x}$, mean $\bar{x}$
Relative Variance	$V^2 = \sigma^2/\bar{x}^2$	V^2/n
Skewness	$\beta_1 = \dfrac{[E(x-\bar{x})^3]^2}{[E(x-\bar{x})^2]^3}$	β_1/n
Kurtosis	$\beta_2 = \dfrac{E(x-\bar{x})^4}{[E(x-\bar{x})^2]^2}$	$\dfrac{\beta_2 - 3}{n} + 3$

Parameter values shown for the convolution can be used to compute Ex^2, Ex^3, Ex^4, etc., and similarly substituted in equation (3). Of course, if the parent distribution function is known explicitly its convolutions can be calculated by standard methods (8).

COMPARISON WITH OTHER SCHOOLS OF STATISTICAL INFERENCE

The method of minimum prejudice, or maximum entropy, is distinguished from the Neyman-Pearson school of statistical inference in that whereas the latter school sets up hypotheses and judges their plausibility in terms of the probability of occurrence of an observed event given the truth of a hypothesis, the former method goes straight from the data to the answer without any testing whatsoever. No testing is theoretically even possible if the method of maximum entropy has been strictly followed, since all available data will have gone into the calculation and no further information is obtainable, in principle, by testing or otherwise.

As a practical matter, the two approaches apply under different circumstances. If the only available data are several different kinds of means, the distribution with maximum entropy is asserted to be the appropriate distribution *on these data.* As more data, such as a histogram, are acquired, an entirely different curve may be indicated from what was derived from limited data. In principle it should be possible to derive a maximum entropy distribution from any arbitrary data. Very little is known, however, as to just how to go about incorporating data other than averages. This should be a fruitful field for study. Fully developed, it ought to obviate the need for Chi-square and other tests in a great many cases. In the meantime, however, it is entirely possible to conceive of using a Chi-square test, for example, upon receipt of more data, to confirm or revise any earlier choice of curve based upon maximum entropy. It might also be used where a generalized exponential function has been fitted to given data on the basis of selected parameters computed from more detailed data such as a histogram. The necessity for such a mixing of methods is less than satisfying.

That the need for testing can be eliminated may come as a surprise to persons, such as the author, trained under the Neyman-Pearson influence. Yet it is readily apparent that a solution derived strictly according to Bayes' theorem requires no testing. Application of this theorem does, however, require knowledge of prior probabilities. It is only in the attempt to "fudge" an answer in the absence of such knowledge that we find ourselves obliged to resort to confidence tests and the like. The method of maximum entropy, as a logical outgrowth and extension of Bayes' theorem, provides a solution to this dilemma in a wide class of cases.

ACTUARIAL IMPLICATIONS

An obvious actuarial implication arises in the calculation of deductibles under conditions of inadequate data. Given only the mean of a non-negative variable, we know the exponential distribution is the minimally prejudiced estimate of the distribution. Sometimes we may have more information, such as that $f(0) = 0$. This implies that $ln\ x$ has a finite mean.* Hence we might let $f(x) = exp.\ (-a_0 - a_1x - a_2 ln\ x) = (x)\ exp.\ (-a_0 - a_1x)$ if $a_2 = 1$. Whether such a solution is valid is one of the questions to be studied. (If we knew the mean value of $ln\ x$, this equation would be minimally prejudiced — but is it minimally prejudiced when only the existence, not the value, of $E(ln\ x)$ is known? How do we know the exponent of $ln\ x$ should be unity? Does the arbitrary selection of this value for the exponent betray a prejudice?)

It appears that in many important practical cases involving constraints of a form inexpressible as averages, it is not feasible to maximize the entropy through use of the calculus of variations to find extremals. Correct answers in such instances may be calculable only through iterative procedures (9).

In collective risk theory it seems unlikely that we shall ever have satisfactorily specified distributions of the claims arising from heterogeneous portfolios. It may be that equation (3) provides our best estimate of such distributions for practical purposes.

Finally, in such imponderables as the probability distribution of the error in existing rates — which must be estimated if credibility is to be calculated using Gauss's theorem on minimum variance, complete specification of distributions is apparently out of the question. In this and many other cases we must settle for a good deal less information. It seems clear that in such instances, as in others, we are well advised to use such information as we have with a minimum of prejudice and unsupported assumptions.

REFERENCES

(1) The authenticity of such attribution is questionable, as observed by C. Kenneth Brampton, editor of the volume, *The De Imperatum et Pontificum Potestate of William of Ockham* (University Press, Oxford, 1927), who states in a note on page 80: "By a curious fate Ockham is in many quarters known solely for his 'razor', which Mr. W. M. Thorburn ably proves (Mind, no. 107, July 1918) to be an invention of a later age, occurring first in the works of Condillac less than two centuries ago, and introduced into England by Sir William Hamilton in 1852. But Ockham's meaning is clear enough, that if there is no 'humanity' existing apart from the individuals which collectively form it, it is gratuitous to postulate its objective existence (Log. i, cap. lxvi):

* This implication holds without qualification only for discrete distributions, which are the only distributions for which entropy has been defined here.

'frustra fit per plura quod potest fieri per pauciora' (Sent. ii, Dist. 15, 0). These words as Mr. Thorburn points out, are actually quoted by Sir Isaac Newton in his third edition of his *Principia Mathematica* of 1726 (De Mundi Systemate, lib. iii, p. 387). This is Regula i, and continues, 'Natura enim simplex est et rerum causis superfluis non luxuriat': but the garbled version in the form 'entia non sunt multiplicanda praeter necessitatem' was invented by John Ponce of Cork in 1639 and took its present shape for the first time in the Logica Vetus et Nova of John Clauberg of Groningen in 1654. Even in his philosophy there is much that is untrue in the name, weapon, and formula bestowed upon Ockham by posterity."

The *Encyclopaedia Brittanica,* however, says that "The famous dictum, 'pluralites non est ponenda sine necessitae' (multiplicity ought not to be posited without necessity) has become known as 'Ockham's razor' though it had already been stressed by other Scholastics," without commenting upon the variation in wording or challenging the attribution to Ockham. In the following paragraph it says ". . . Ockham did not make much of the philosophical arguments of earlier theologians, and applied to theology his famous 'razor' . . ."

This author relinquishes the task of any further research into the authenticity of Ockham's razor to qualified medievalists.

(2) Tribus, Myron

(a) "The Probability Foundations of Thermodynamics," Myron Tribus and Robert B. Evans, *Applied Mechanics Review,* Vol. 16, No. 10, October 1963.

(b) "Why Thermodynamics Is a Logical Consequence of Information Theory," Myron Tribus, Paul T. Shannon and Robert B. Evans, *A.I.Ch.E. Journal,* March 1966.

(c) "Information Theory as the Basis for Thermostatics and Thermodynamics," Myron Tribus, *Journal of Applied Mechanics,* March 1961.

(d) "The Maximum Entropy Estimate in Reliability" in *Recent Developments in Information and Decision Processes,* Macmillan Co., 1962.

(e) "The Use of Entropy in Hypothesis Testing," Myron Tribus, Robert Evans, and Cary Crellin, paper presented at the Tenth National Symposium on Reliability and Quality Control, January 7-9, 1964.

(f) "Information Theory and Thermodynamics," *Boelter Anniversary Volume,* McGraw Hill Book Co., 1963.

(3) Hume, David, *A Treatise of Human Nature,* 1740. A more pertinent reference, in this author's opinion, is provided in Volume 4 of Hume's *Philosophical Works,* Edition of 1777. This edition was "corrected by the author for the press, a short time before his death, and which he desired might be regarded as containing his philosophical principles," according to the "Advertisement" prefacing Volume 1 of the 1854 reprint, published by Little, Brown and Co. of Boston and by Adam and Charles Black of Edinburgh, of the 1777 edition. Most to the point, perhaps, is Hume's rhetorical question (page 35), "All these suppositions are consistent and conceivable. Why then should we give the preference to one, which is no more consistent or conceivable than the rest?" In what follows he argues that past experience is our only guide where no *a priori* connection can be demonstrated between cause and effect. This author agrees that Hume's discussion of inductive principles is consistent with Tribus's formulation but thinks it may be reading too much into Hume's rather prolix text to find there so clear a statement of the problem as given by Tribus.

(4) Shannon, C. E., "A Mathematical Theory of Communication," *Bell System Technical Journal,* Vol. 27,379,623, 1948.

(5) Jaynes, E. T., "Information Theory and Statistical Mechanics," *Phys. Review,* 106, p. 620 and 108, p. 171 (1957); AMR 11 (1958), Rev. 2293. Other references to Jaynes are given in Tribus (2).

(6) Bailey, R. A., "Any Room Left for Skimming the Cream," *P.C.A.S.* XLVII, 1960.

(7) Kendall, Maurice, *The Advanced Theory of Statistics,* Vol. 1, p. 302, Charles Griffin & Sons, Ltd., 1948.*

(8) Feller, William, *An Introduction to Probability Theory and Its Applications,* Vol. 1, p. 250, John Wiley & Sons, 1950.

(9) Besides linear programming, possible directions such calculations might take are suggested in *Nonlinear Mathematics* by Thomas L. Saaty and Joseph Brom, McGraw Hill Book Co., 1964.

* The expected values M_3 and M_4 for sample means, as given there for sampling from a finite population of N cases, can be reduced to β_1/n and $(\beta_2 - 3)/n + 3$ on taking the limit as $N \to \infty$.

NOTES ON WHITTAKER-HENDERSON FORMULA A

NELS M. VALERIUS

NOTE 1

When Mr. Robert Henderson, 1873-1942, the distinguished life actuary and a Fellow of our Society, put forward "A New Method of Graduation" in 1924, in *Transactions* XXV of the Actuarial Society of America, the problem of starting values was resolved by calculating upward an auxiliary series which he labelled the u_x''' column, alongside the u_x'' column to be graduated. The u_x''' column terminated, in an extension of its upper end, in terms deemed acceptable for starting values. These were then copied to the same positions of the Henderson intermediate u_x' column to start that column off downward.

The formula for calculating the successive terms of the u_x''' column was the same basic formula that was thereupon used to work out the graduation itself, beginning from the starting values thus established; Henderson's intermediate u_x' column being next filled in downward by means of it, then the final u_r column upward.*

These starting values were not highly accurate, and the resulting $\{u_x\}$ was therefore not accurate at the upper end of the column. Mr. Henderson was not greatly concerned, witness his pun regarding "such unprofitable ends as the ends of a graduated mortality table" in his discussion, *T.A.S.A.* XXXIX, page 50, of C. A. Spoerl's comprehensive paper, "Whittaker-Henderson Formula A," *T.A.S.A.* XXXVIII, pp. 403-462.

In casualty and property actuarial work, unlike mortality tables, the ends and extensions of the ends of, for instance, graduated time series may be the more crucial parts. The potential of accuracy at the ends under Formula A, when it is used in this work, is a most attractive feature, not to overlook the preservation of moments inherent in an accurate A graduation.

The determination of accurate starting values has remained troublesome. Thus, the textbook for life actuarial students, sponsored by the Society of Actuaries, the monograph, *Elements of Graduation,* says (of the fourth-order difference equation graduation), "At the zero end of the series, the two needed values of u_r' cannot be determined accurately at the outset except by involved methods. If such methods are not to be resorted to, the

* A version of the formula is given in the Appendix herewith. Algebraic symbols used in the Notes may be identified there also.

first two graduated values must be estimated from the general run of the ungraduated values at the zero end. The graduation is completed using these estimated values and then corrected, if necessary."

Unexpectedly, a return to Henderson's method yields a solution. Henderson's up and down approach to the final column suggests that more of the same might provide any desired accuracy and this proves to be right. Actually, iteration of Henderson's u_x' series in alternately opposite directions will provide as perfected a Henderson's intermediate series $\{u_x'\}$ as one wishes, enabling a graduation with accuracy to as many places as one may have elected of any ungraduated series, short or long, provided it falls within the province of Formula A, that is, it has equal intervals, e.g., data of successive years, and graduation without formal recognition of weights is acceptable. Many series are of this kind.

No additional techniques are involved, obviously, to those used to work out the graduation in the usual way, from starting terms however established.

The graduator runs an auxiliary column like u_x''' upwards or downwards, starting opposite what segment of z consecutive terms of the u_x'' column he deems expedient; then at the extended end of such auxiliary column, he turns about and produces in reverse direction a tentative u_x' column, then another in reverse direction to it, and so on, until the required accuracy results. The process amounts to successive approximations to the final $\{u_x'\}$.

A point is reached where z successive terms in a column duplicate the corresponding terms in the last previous column of like direction and no more improvement is possible. The column between is an accurate u_x' column, as would also be the last partial column if filled out by copying from the previous column of like direction, so that accurate u_x' columns of both directions are available. The final u_x column must, of course, be worked out in the direction opposite to that of the u_x' column selected.

Having appropriated from Henderson's up u_x''' and down u_x' operations, the up and down (or down and up) sequence, his u_x''' column may, if one wishes, be replaced by any other plan for starting terms, including guesses. With approximate starting terms, one goes to work at once on a first tentative u_x' column, following it with others at will. Depending on the accuracy desired, it is not necessary to follow through to the point of maximum improvement. Just to adopt the bottom end of the first tentative u_x' column as the chosen start may be acceptable.

The proposal of this Note will be made clearer by reference to Exhibit I, where the Specimen Graduation by Formula A on page 39 of *Elements of Graduation* is reworked under the method of this Note.

In Exhibit I, to start the auxiliary column u_x''' off upward, z terms, two in this case, u_{16}''' and u_{17}''', were devised by a preliminary smoothing of the corresponding segment of $\{u_x''\}$. The work was carried forward with two places of decimals and so displayed; as one or two extra places beyond those retained are advisable, however (even more if a is quite large), the result is not accurate in the last place.

This judgment may be confirmed by reference to the Society of Actuaries' Study Notes for Part V, wherein the graduation by the "second Henderson method" is given to five places. That result, rounded back to two places, differs, but by no more than .01 or .02, as to most of the terms.

Pursuing the proposal's potential for accuracy, the writer has reproduced the Study Notes' five-place result by the method of Exhibit I, retaining six places of decimals after the initial $u_{16}''' = 105$ and $u_{17}''' = 117$; u_{15}''' becoming 94.166667 in place of 94.17, and so on. It was necessary to go to $\{_3u_x'\}$, for a satisfactory $\{u_x'\}$. The work is not reproduced here, being mentioned only as an example of extreme accuracy achieved by this method.

Exhibit II is included for comparison of the usual method (unless use of matrices and computers should today be called the usual methods). It is the monograph's numerical example, pages 38-39, with slight improvements in the arithmetic and with Spoerl's corrections carried out, for comparison to Exhibit I. Faulty arithmetic on page 39 marred Spoerl's corrections, in the writer's opinion, and the recalculation is provided for an equitable comparison of results. The corrected u_0 and u_1 are given on page 38 of the monograph as 27.34 and 29.76, respectively, instead of the values on Exhibit II of 27.39 and 29.80, which accord with the Study Notes' five-place results of 27.39625 and 29.80043.

In the monograph, and in Exhibit II, an auxiliary column u_x^* is read from a graph drawn among the upper end values of $\{u_x''\}$ and extended upward. The extension is a straight line since $z = 2$, rather than an arc as it would be were $z = 3$ (see Table on page 5). Note that the graph approximates the true u's extended, and, using the fact that true $u_x =$ true u_x' in the outer extension where $x < -a$, thereby approximates the true starting u''s.

It might not be amiss to remind that a large a emphasizes smoothness, a small a, fidelity or closeness of fit. The value of z (1, 2, or 3) fixes the relative simplicity or complexity of the specific Formula A: $z = 1$ being so simple as hardly to be useful at all, "the trivial case," as Spoerl calls it, page 413; $z = 2$ being widely useful; and $z = 3$ being rather elegant. One, two, or three starting terms are required, respectively, and the graduated results tend to be a succession of segments whose first, second, or third differences are zero, respectively. Their respective extensions are exactly such segments, save the inner a terms of the extensions at the starting end.

TABLE

z	$2z$	*Starting Terms*	*Extensions and Tendency*
1	2	1	$\Delta u_x = 0$
2	4	2	$\Delta^2 u_x = 0$
3	6	3	$\Delta^3 u_x = 0$

In this Note, the intermedite variable is u'_{x-a}, like Henderson's u'_{x-n}. In the monograph, the variable is termed u_x', shedding what Mr. Kingsland Camp, F.S.A., has called the "displacement" in his manual. *The Whittaker-Henderson Graduation Processes.* The newer usage is neater and advantageous in several respects, particularly in accommodating fractional a's. Nevertheless, with the alternating sequences proposed, the symmetry of the chosen format seems preferable. It also adheres to Mr. Spoerl's cited paper, an advantage still, as this remains the prime reference for Formula A. Iterating $\{u_x'\}$ is, of course, equally valid either way. Exhibit IIA is included to illustrate the work with u_x' as the intermediate variable.

NOTE 2

Graduations by Formula A share the property that least squares fittings of lines and parabolas exhibit (these may be looked upon as the cases of ultimate smoothness of fourth-order and sixth-order Whittaker-Henderson applications), viz., additive elements of a series, such as an underwriting history of annual losses, expenses, and underwriting gain or loss, and their sum or premium, may be separately graduated with the same choice of constants, with results that are still additive, so that the graduated annual elements add up to the graduated annual sums. Another example might be a history of losses analyzed by kinds. In other words, these multiplicative processes follow the distributive law for multiplication.

NOTE 3

Extensions of the graduated series may be useful. In casualty actuarial work, smoothing or graduation of a time series is quite likely to be used for prediction of the future. Interest concentrates on extension of the series, and the smoothing process is assumed to reveal, more or less, an underlying law.

A straight line or parabola, fitted by least squares, is sometimes calculated and extended. As said in connection with Note 2, these are the extreme cases of Whittaker-Henderson A graduations, resulting when the constant called a herein approaches infinity. If such extensions are valid, extensions derived with lesser a's should have validity also. The difference would be that the crude or observed terms nearest the end influence the extension more than the others, whereas in the usual least squares fitting, all observed terms influence the result alike.

The calculation of valid extensions of any length involves the retention of more places than required for a graduation over the observed interval only, that is, if the extensions are to be considered meaningful and not merely auxiliaries of the graduation process. The reason is that the extended terms are in a difference series of $(z-1)$th order and the rounding error in the difference used cumulates. To illustrate, extensions from Exhibit II and extensions from the "second Henderson" method are given below. In the case of sixth-order graduations, the rounding error would cause a greater divergence.

x	*Exhibit II*	*"Second Henderson"*
−4	17.75	17.77953
−3	20.16	20.18371
−2	22.57	22.58789
−1	24.98	24.99207
0_α	27.39	27.39625
1	29.80	29.80043
17	117.36	117.36378
18_ω	126.74	126.74849
19	136.12	136.13320
20	145.50	145.51791
21	154.88	154.90262
22	164.26	164.28733

APPENDIX

The formula by which the individual terms of the u''', u', and u series are calculated, corresponding to a given u'' series, may be applied in the following forms:

$$
\begin{aligned}
Au'''_x &= Bu'''_{x+1} - Cu'''_{x+2} + Du'''_{x+3} + Eu''_{x-a} \\
Au'_x &= Bu'_{x-1} - Cu'_{x-2} + Du'_{x-3} + Eu''_{x+a} \\
Au_x &= Bu_{x+1} - Cu_{x+2} + Du_{x+3} + Eu'_{x-a}
\end{aligned}
$$

The value z may be 1, 2, or 3 in a Whittaker-Henderson A graduation and the value a may be any positive number.

When $z = 1$, $A = (a + 1)$
$B = a$
$C = 0$
$D = 0$
$E = 1$

When $z = 2$, $A = (a + 1)(a + 2)$
$B = 2a(a + 2)$
$C = a(a + 1)$
$D = 0$
$E = 2$

When $z = 3$, $A = (a + 1)(a + 2)^2(a + 3)$
$B = a(a + 2)(a + 3)(3a + 5)$
$C = a(a + 1)(a + 3)(3a + 4)$
$D = a(a + 1)^2(a + 2)$
$E = 4(2a + 3)$

These are adapted from C. A. Spoerl's paper, page 408. As in Note 1, the intermediate variable in this Appendix is u_{x-a}', like Henderson's u_{x-n}'. If the variable is termed u_x', all a's would drop out of the subscripts.

The subscripts in the formula as shown assume the u''' and u columns are being worked out upward and the u' column is being worked out downward. Since the columns might just as well be worked out in the opposite directions, that is, u_x''' downward, u_x' upward, and u_x downward, and further since Note 1 proposes iteration of the u' series in reverse directions, the subscripts on the right hand may also be:

$$
\begin{aligned}
&x - 1, x - 2, x - 3, x + a \\
&x + 1, x + 2, x + 3, x - a \\
&x - 1, x - 2, x - 3, x + a
\end{aligned}
$$

The coefficients resulting from certain choices of values for z and a are tabulated:

z	a	A	B	C	D	E
1	1	2	1	0	0	1
1	2	3	2	0	0	1
1	3	4	3	0	0	1
2	1	3	3	− 1	0	1
2	2	6	8	− 3	0	1
2	3	10	15	− 6	0	1
2	4	15	24	− 10	0	1
3	1	18	24	− 14	3	5
3	2	60	110	− 75	18	7
3	3	100	210	−156	40	6
3	4	315	714	−560	150	11

The case, $z = 1$, is hardly useful. The case, $z = 2$, being a good deal easier to work with than the case $z = 3$, is widely used. The case, $z = 3$, makes a fine graduation. It is to be used when the differences in some parts of the distribution are large relative to those in other parts, as in a bell-shaped distribution or one with large differences at one or both ends.

Note that some of the choices of constants are more convenient or useful than others. The combination $z = 2$, $a = 1$ makes for a very rapid calculation and would be a good choice for becoming familiar with the process, as well as for practical applications. Where $a = 3$, whether z is 2 or 3, A is a power of 10 so it is not necessary to divide through by A or use fractional multipliers. This is quite convenient, and these are among the most useful combinations.

The problem of starting terms is treated in Note 1. At the end away from the start, the calculation of any of these series but the final one runs into the problem that the u_x''s give out before the series can be completed. The rest of the column is filled in by means of a difference series continuing such a series established by the last z terms it was possible to calculate by formula.

The columns are carried $(x + a)$ terms beyond the end of $\{u_x''\}$ so that the outer z terms may serve as successively corrected starting terms. When one of the columns has been selected as the final u_x' column, back over which the u_x column will be developed, it is usual to "turn the corner" and start $\{u_x\}$ by copying the last z terms of $\{u_x'\}$, excluding any extension, into the u_x column, but no harm is done by turning farther out, as the last z terms of $\{u_x'\}$ proper and the terms of the extension are all of the same difference series.

The terms appended have z^{th} differences of zero and we easily caculated by means of the formulas as next given, prime marks omitted:

If $z = 1$, $u_x = u_{x-1}$,

If $z = 2$, $u_x = 2u_{x-1} - u_{x-2}$,

If $z = 3$, $u_x = 3u_{x-1} - 3u_{x-2} + u_{x-3}$,

provided the columns are downward. Again, when the columns are in reverse direction and developing upward, the subscripts on the right-hand side become $x + 1$, $x + 2$, and $x + 3$.

EXHIBIT I

Specimen Graduation by Formula A
Method of Note 1; Henderson Type Start

x	u'''_x	u''_x	${}_1u'_x$	${}_2u'_x$ (Final u'_x)	${}_3u'_x$	u_x
-4	17.60		17.60	17.74	17.74	
-3	20.05		20.05	20.15	20.15	
-2	22.50		23.60	22.56	23.66	
-1	24.95		25.44	24.97	25.47	
0	27.40	34	27.29	27.38	27.30	27.38
1	29.85	24	30.33	29.79	30.33	29.79
2	32.30	31	31.80	32.20	31.79	32.57
3	34.75	40	35.40	34.61	35.39	35.76
4	39.41	30	39.30	39.22		39.32
5	43.59	49	42.70	43.36		43.44
6	47.75	48	48.45	47.52		47.79
7	53.48	48	52.92	53.33		52.36
8	57.11	67	57.50	57.18		57.15
9	61.67	58	62.71	62.16		61.92
10	66.23	67	67.53	67.41		66.98
11	69.27	75	71.35	71.43		72.42
12	74.60	76	78.37	77.99		78.32
13	79.72	76	85.48	84.44		84.91
14	85.73	102	91.62	91.52		92.29
15	94.17	100	98.59	100.18		100.07
16	105**	101	107.98	109.44		108.38
17	117 *	115	117.37	116.81		117.37
18		134	126.76	126.62		126.76
19			136.15	137.37		
20			145.54	146.75		
21			154.93	154.93		
22			164.32	164.32		
		1275				1274.98

* 1/3 (101 + 115 + 134) = 117

** 1/3 (100 + 101 + 115) = 105

$z = 2;\ a = 2$

$$u'''_x = 1/6\,(8u'''_{x+1} - 3u'''_{x+2} + u''_{x-2})$$

$$u'_x = 1/6\,(8u'_{x\mp 1} - 3u'_{x\mp 2} + u''_{x\pm 2})$$

$$u_x = 1/6\,(8u_{x-1} - 3u_{x-2} + u'_{x+2})$$

EXHIBIT II Specimen Graduation by Formula A
Page 39 of "Elements of Graduation" Reworked
Spoerl's Correction Applied

x	u^*_x	u''_x	u'_x	u_x	u^*_x	u'_x	u_x
-4	14 →		14.00		17.75	17.75	
-3	17 →		17.00		20.16	20.16	
-2	20		21.33		22.57	23.67	
-1	23		23.94		24.98	25.48	
0	26	34	26.42	26.40	27.39	27.30	27.39
1	29	24	29.92	29.27	29.80	30.33	29.80
2		31	31.68	32.36		31.79	32.57
3		40	35.45	35.74		35.39	35.75
4		30	39.43	39.39		39.29	39.30
5		49	42.85	43.54		42.69	43.41
6		48	48.58	47.89		48.44	47.76
7		48	53.02	52.44		52.91	52.34
8		67	57.57	57.21		57.49	57.14
9		58	62.75	61.97		62.70	61.92
10		67	67.55	67.02		67.52	66.99
11		75	71.36	72.45		71.34	72.43
12		76	78.37	78.34		78.36	78.33
13		76	85.48	84.92		85.48	84.92
14		102	91.62	92.30		91.63	92.30
15		100	98.59	100.07		98.60	100.07
16		101	107.98	108.38		107.98	108.38
17		115	117.37 →	117.37		117.36	117.36
18		134	126.76 →	126.76		126.74	126.74
		1275		1273.82			1274.90

Corrections per formula 5.37, page 37, "Elements of Graduation"

Corrected u_0 = 26.40 + 9/2 (26.40 - 26) - 3 (29.27 - 29) = 27.39

Corrected u_1 = 29.27 + 7/3 (26.40 - 26) - 3/2 (29.27 - 29) = 29.80

$z = 2$; $a = 2$

EXHIBIT II A Specimen Graduation by Formula A
Method of Note 1; with Monograph's Start & Placement

x		u^*_x	u''_x	${}_1u'_x$	${}_2u'_x$	${}_3u'_x$	u_x
					17.74		
					20.15		
-4		14			22.56		
-3		17			24.97		
-2		20		14.00	27.38	17.74	
-1		23		17.00	29.79	20.15	
0	α	26	34	21.33	32.20	23.66	27.39
1		29	24	23.94	34.61	25.47	29.80
2			31	26.42	39.22	27.30	32.57
3			40	29.92	43.36	30.33	35.75
4			30	31.68	47.52	31.79	39.30
5			49	35.45	53.33	35.39	43.41
6			48	39.43	57.18	39.29	47.76
7			48	42.85	62.16	42.69	52.34
8			67	48.58	67.41	48.44	57.14
9			58	53.02	71.43	52.91	61.92
10			67	57.57	77.99	57.49	66.99
11			75	62.75	84.44	62.70	72.43
12			76	67.55	91.52	67.52	78.33
13			76	71.36	100.18	71.34	84.92
14			102	78.37	109.44	78.36	92.30
15			100	85.48	116.81	85.48	100.07
16			101	91.62	126.62	91.63	108.36
17			115	98.59	137.37	98.60	117.36
18	ω		134	107.98	146.75	107.98	126.74
19				117.37	154.93		
20				126.76	164.32		
21				136.15			
22				145.54			
				154.93			
				164.32			
Σ			1275				1274.90

SOME REFLECTIONS ON THE EXPANDING CONCEPT OF THE CASUALTY-PROPERTY ACTUARY

STERLING T. TOOKER

I suspect it is something of a mistake to invite a layman to address a professional group, such as the Casualty Actuarial Society. Any attempt to offer a discriminating analysis of inferential statistics, mathematical model-making, or any of the other esoterics of your profession would demonstrate what might be called a modesty of informational exposure. (That's about as nice a definition for ignorance as I've heard in a long time.)

What is left to me is to recount the accomplishments of your profession, which my actuarial friends modestly tell me couldn't possibly be covered in a day, let alone a luncheon talk; or to reflect on the growing concept of the actuary's profession; on some of the needs that are developing in our business and on some of the challenges (as I see them) that lie ahead for the casualty actuary.

Remarks such as these are not designed to induce serenity or self-satisfaction. They set aside all you have accomplished in the interest of generating creative dissatisfaction. That's what comes of inviting a layman, and I apologize in advance if I cause you any intellectual indigestion.

One of the distinctions of the business world in which we all live is that we don't have any report cards. If we did, I think ten years from now the casualty property actuary who has tried to cling to his traditional role might find written across his the most damning of modern euphemisms: *Underachieving.* The word carries with it a compliment, a compliment as to capacity. And it carries with it a criticism, a criticism as to how well the actuary is capitalizing on his abilites. It questions his commitment to his profession, and to himself, in light of his training and his abilities. And it questions his contribution to the company for which he works, in light of his *capacity* to contribute.

Your profession lives in what, to those of us who are outside it, appears to be the best of all possible worlds. A world that is specific; a world that is quantitative, measurable, probable. A world that can be defined, processed, and summarized. A world that has some answers instead of ever, and eternally, more questions. A world in which the answers can be documented and defended.

Editor's Note: Mr. Tooker, President of the Travelers Insurance Companies, presented this address at the Society's luncheon November 13, 1967.

Now, those of us who are outside that world are suggesting that as the world changes, your profession is going to have to change also. Some may say that it's because we can't stand to see someone else happy. Others will admit it is because we urgently need a broader application of your skills and training to contribute to the success of our businesses. I suggest that the actuary must change from the comfort of his *traditional* role or run the risk of becoming irrelevant.

What we ask is a great deal, and we know it. Philosopher Eric Hoffer points out that every change is a crisis in self-esteem. It is difficult, almost to the point of impossibility, to let go, with enthusiasm, of a role in which we are expert, and accept one in which we must run the risk of making major mistakes. It is doubly difficult to let go of an era of expertise in which most of the sins are sins of omission (neither noticeable nor measurable, no matter how lethal to the health and well-being of the corporation) and to accept an area of responsibility in which our errors can be both seen and quantified, and often corrected. To take such a step exposes us to risk, and to criticism. To take such a step calls for character, and personal fortitude.

Today the casualty-property actuary works in the retrospective — setting his rates on historical evidence. He *reacts*, often ignoring the *trends* being indicated by historical evidence, and ignoring, equally, what is going on around him *right now*. Since neither regulators nor ratemakers willingly accept trends in the casualty and property field, preferring to rely on recent past evidence, the casualty-property actuary has had little incentive to develop information on what rates *should* be in light of *all* the available information. Our companies are always in the position of "catching up" in both our rates and our reserves — a "catching up" that has serious economic implications for our business. I suggest this is just one place in which we must make a change. Whether, in fact, we *can* charge the rates we should — at least we ought to know what those rates *should be*.

From this one example of the need to change our approach to rates and reserves I think you can see that I am not suggesting that the casualty-property actuary has to improve his performance, but rather that he must develop a wholly new, and far broader, perspective of his role in the insurance industry. He must change.

It it because I believe that such a step is necessary, and in the hope of conveying this same conviction to you, that I will concentrate my remarks of the next few minutes on three topics:

The first, is what's wrong with the *traditional* role of the actuary?

The second, is what is happening to the environment in which we work and causing us to seek a change in the role of the actuary?

And the third is the implication — to you as a person, and to actuaries as a profession — that can be drawn from the changes now taking place in the insurance industry.

It would be difficult to understate the importance of the actuary in the development of our business. Without him, none of our present-day accomplishments would have been possible. He was, and is, the forerunner of all the men of quantitative skills who are so popular in the business, governmental, and academic worlds today . . . the statisticians, the model makers, the projectionists, and the operations researchers. He took, and often wrote, the laws of probability, of statistical inference, and all the rest, and applied them to ratemaking, reserve policies, and research.

He was the pioneer, but the ultimate test of the pioneer is his ability to keep pace. Looking ten years from now, with its situations and its demands, we have to question whether the *traditional* actuary might not be subject to the following comments:

He takes a precise, but limited, view of life.

He is superbly skilled with statistics, but often fails to include all the relevant data such as social and economic trends, inflation, rising costs, and other available information that is pertinent to his answers.

He is a master of problem solving, but often abdicates the role of defining what the problems are to be solved.

Looking in the opposite direction from the problem, he often wants to end with analysis — offering neither interpretation, projection, nor management recommendation.

The actuary, in love with exactness, has a fear of the intuitive. He knows it is inappropriate to arrive *at an intuitive* answer to a column of figures, and lets this knowledge carry over and contain his creative ability to develop new approaches in improving the operation of our business.

He was, in many cases, the cause and initiator of data processing systems, but often fails to press for the full potential of new developments in data equipment and its applications.

The actuary functions within a business for most of his professional life, but neglects the necessary attitudes of a businessman.

Perhaps it is within this "businessman" concept that we have the key to such broader problems as the casualty-property actuary failing to pay enough attention to the marketing needs of his company, ignoring the implications of political approval necessary to rate structures he recommends, and neglecting to develop all the economies he might for his company, economies that could result from tailoring rating techniques, and the design of statistical requirements to new and sophisticated machine capabilities.

In every case that I have mentioned, there would be neither a question of the capability of the casualty-property actuary, nor a failure to fill the *traditional* obligations of an actuary. There would be only a record reluctance to take on a broader role, a reluctance I suspect sometimes engendered by upper management attitudes.

Ten years from now, the situation will demand that the actuary see his job as more than one beginning with a "given" — a given provided by somebody else — and ending with a "processed probability." The danger of segmentation — of separating the input from the precision of statistical processing, and again from the application of output — the danger, particularly in light of our ever-increasing skill with mathematics and computers, is that we will end up with finely processed misinformation; that we will become victim of the famous "garbage in-garbage out" equation, and fail to realize it until it's too late.

It may well sound, and actually be, that I am overdrawing the picture. If you think so, I suggest you spend a few hours with economist John Kenneth Galbraith's new book, *The New Industrial State.* In it, he cites two major points I have in mind right now. One is the rise of the "technostructure" in the corporation — a layer of technical and technical-management people such as yourselves who, more and more, by their selection and processing of information are controlling the direction and destiny of the organization. The other point I have in mind is the continually increasing size of the commitment an organization must make, in capital, in human resources, and in lead time, to either a continuance or change in marketing policy, and with it the concomitant cost of an error in reading the market.

We are in an age of change, change that can, and already has, heightened our opportunities, and with them our opportunities for error. For example, the factors that have often been significant in determining the market for traditional lines of insurance, population and disposable income, have been growing at an almost phenomenal rate. Population, this month, will pass the 200 million mark in this country, and is currently growing at more than 2¼ million people a year. Median annual family income reached $7400 in 1966, and is expected to top $10,000 by 1975.

Beyond this, our population is getting younger, with a median age of just under 28. It is a better fed, better clothed, healthier, better educated, and a more sophisticated population than ever existed in the history of *this* or any other country. It is dominated by young ideas and changing attitudes — some the product of education, some the product of environment, and many of the product of modern communications.

We have seen rapid shifts in taste — the shift from the full size car, to the compact car, to the quasi-sports car — among a large segment of the buying public. In our own field, we have seen the growth of credit, equities, and of insurance, indicating the increasing importance of the financial field in the economy, and in the eyes of the consuming public.

We have seen government emerge as a major factor of our own environment and that of the total economy. And there is nothing to indicate that there will be any moderation in the pace of change now taking place around us. On the contrary, we can expect more:

more population

more corporate and personal income as we continue beyond the 81st month of economic expansion

more competition

more participation of government

more market demands

more in the way of consumer and public expectations of us, both as a seller of services and a major financial industry, and as a social entity in the community.

This should cause those of us in the insurance industry *no dismay.* The world is forcing us to tailor ourselves and our products to its tastes at an accelerating pace, but in so doing it is enhancing the challenges and the opportunities in our industry, and giving us exactly what the young people on campuses across the country say they must have in their careers. We are no longer spectators of change. We are involved. Our industry should conceive itself as one of the major *causers* of change in contemporary society.

Now this opens a whole range of questions as to what is the proper purview of the insurance industry. Each of our companies is in the process of trying to determine the answers. It is a subject that has been discussed both privately and from the public platform, and at considerable length. I'm not going to add to that discussion right now, except to draw some of the implications that all this change has for you as an individual, and for the actuary as a professional.

The demands of the world in which we function are forcing *us* to change — to redefine our roles and our relationships *within* the organization — as well as that of the organization within the community. The casualty-property actuary is being offered alternatives. The only alternative that is *not* being offered is that of maintaining his *traditional* role. If he tries to do so, he will be relegated — quietly, subtly but efficiently — to the function of a technician. And the actuary will be superseded by a new level of organization — the operations researchers.

This would be a mistake and a waste, because the new role being offered is ideally suited to the training, background, skills, and experience of the actuary, and because we in the insurance industry have an urgent and growing demand for the broadest possible application of all your skills and abilities. In addition, this broadening of challenge is exactly the element which can attract more capable young people to your profession.

The second alternative being offered to you is to extend beyond your traditional functions:

Become involved in the realities of underwriting.

Develop your ability to define the problems facing your company by becoming increasingly aware of economic trends and conditions and all the other factors that affect your market.

Heighten your awareness of the political and social climate so that you can give it a weighting in your judgment in structuring prices, and so that you may participate in product design and modification that relates your products to the needs of society.

Develop a businessman's view of your business so that you will be aware of competitors and what they are doing, the market and where it is going and the opportunities for economies within your business.

Expect, and prepare, to become involved in company planning, both for the benefit of perspective it will offer you in looking at your job, and for the benefit to your corporation that will result from your contribution to planning.

Interject yourself into the marketing end of your business.

And learn to communicate your ideas; and when you know your ideas are right, be willing to fight for them.

As you do all these things, you will make yourselves more indispensable to top management. Let me point out three implications: *One* is that if you are a member of the controlling "technostructure" to which Galbraith refers, you will do a more valuable and effective job of guiding the organization. A *second* is that a greater area of agreement between the actuary and the top management on problems and goals will break down the barriers to communications that exist in all organizations. It will heighten the effectiveness with which management can use its quantitative resources, and again increase the value of your contribution to the company.

The *third* implication is perhaps the most obvious. The job of the actuary will become a more ideal training ground for men who aspire to top management.

The disciplined approach and the quantitative skills that are common to the actuary are becoming imperative in the management of a large corporation. The modern executive should understand the application of computer techniques and quantitative technology both to administer, and to understand the information with which he is being provided to make decisions under uncertainty. But these skills alone will not make an able executive. Too many of the factors that affect the success of his enterprise are outside the walls. He must develop sensitivity to change, social awareness, economic alertness, political perception, and a marketing perspective.

In short, he must become good enough to *be* a top flight actuary.

I am suggesting to many of you that you must change. You will go through a crisis — the crisis of self-esteem of which Eric Hoffer writes. You will make mistakes, but they will be mistakes of action, the mistakes of achievers. You are being presented with the challenge of change, and with an opportunity.

Let me close with one last thought: Opportunity is like a fanatic; if you give it a warm reception when it knocks, it becomes your friend, your servant and your benefactor. If you fail to answer the knock, it *doesn't* go away. It kicks the door down, and walks all over you.

MINUTES OF THE 1967 ANNUAL MEETING

November 12-14, 1967

HOTEL AMERICA, HARTFORD, CONNECTICUT

Prior to the formal convening of the 1967 Annual Meeting on the morning of November 13, the Council met in the Carleton Suite of the Hotel America on Sunday, November 12, from 11:00 a.m. to 5:30 p.m.

In the early evening of that day the Aetna Life & Casualty, the Hartford Insurance Group, and the Travelers Insurance Companies were hosts for a reception and social hour for early arrivals.

Later in the evening Past President Seymour E. Smith, Senior Vice President of the Travelers Insurance Companies, was the host at a social hour and dinner for the President and Past Presidents and Vice Presidents who were able to be present. Those in attendance were:

Harmon T. Barber	Joseph Linder
Ralph H. Blanchard	Laurence H. Longley-Cook
Harold E. Curry	Norton E. Masterson
Paul Dorweiler	Arthur N. Matthews
Harold J. Ginsburgh	Thomas E. Murrin
Charles J. Haugh	Sydney D. Pinney
William J. Hazam	Albert Z. Skelding
Charles C. Hewitt, Jr.	Seymour E. Smith
William Leslie, Jr.	Harry V. Williams

The 1967 Annual Meeting of the Casualty Actuarial Society was called to order at 9:10 a.m. on Monday, November 13 by President Harold E. Curry.

Mr. Curry then called upon Allen L. Mayerson who introduced the Honorable William R. Cotter, Insurance Commissioner of the state of Connecticut. Commissioner Cotter welcomed the gathering to his state and extended his best wishes for a successful and fruitful meeting.

At this point, Vice President Charles C. Hewitt, Jr. took over the Chair as presiding officer.

The following new papers were then presented by the authors:

Charles F. Cook	"The Minimum Absolute Deviation Trend Line"
Lewis H. Roberts	"A Discipline for the Avoidance of Unnecessary Assumptions"
Nels M. Valerius	"Notes on Whittaker-Henderson Formula A"

There followed written reviews of previously presented papers:

(1) "Underwriting Profit from Investments" by Robert A. Bailey was reviewed separately by

Alan C. Curry
Richard L. Johe
W. James MacGinnitie*
Allen L. Mayerson**
Nicholas F. Miller, Jr.
Ruth E. Salzmann

*Read in absentia by Eldon J. Glaassen
**Off-the-cuff remarks

Mr. Bailey commented briefly on the reviews.

(2) "A Theoretical Portfolio Selection Approach for Insuring Property and Liability Lines" by J. Robert Ferrari, reviewed by

Martin Bondy
R. A. Rennie*
Matthew Rodermund**
LeRoy J. Simon

*Read in absentia by R. W. Griffith
**Read by Jack Moseley

Mr. Ferrari commented briefly on these reviews.

(3) "Loss Ratio Distributions — A Model" by Charles C. Hewitt, Jr., reviewed by Charles A. Hachemeister.

(4) "Inverse Liability Automobile Accident Insurance" by James B. M. Murray, reviewed by Jerry A. Hillhouse and Jack Moseley.

Mr. Murray commented on these reviews.

(5) "Schedule P on a Calendar/Accident Year Basis" by Ruth E. Salzmann, reviewed by Francis J. Hope and Paul M. Otteson.

President Curry then resumed the Chair and presented diplomas to the following 7 new Fellows and 20 New Associates:

FELLOWS

Edwin A. Carlson
Edward B. Eliason
John A. Gibson, III
Arnold S. Mohnblatt
Steven H. Newman
Philip O. Presley
Robert J. Schuler

ASSOCIATES

Edward J. Carter, Jr.
Fred M. Chorpita
Rex C. Davis
David P. Flynn
J. Robert Ferrari
Robert W. Gossrow
Robert C. Gowdy*
E. LeRoy Heer
John R. Hunter, Jr.
Terry S. Jacobs
Alan G. Jones
Alan F. Kaur

*In absentia

ASSOCIATES

Charles McDonald*	Michael A. Walters
Joseph A. Plunkett	Michael R. Ward
Edith E. Price	W. Thomas Williams
Darvin A. Torgrimson	Arthur E. Winter

* In absentia

The Secretary-Treasurer then presented his report on the activities of the Council subsequent to the 1966 Annual Meeting. The report, including the financial results for the twelve months ending September 30, 1967, immediately follows these Minutes.

The gathering then arose to stand in memory of the deceased members who had died subsequent to the previous Annual Meeting.

E. Alfred Davies	Dudley M. Pruitt (Past President)
Charles V. R. Marsh	Homer D. Rice
Robert K. Orr (Charter Member)	Richard J. Wolfrum (Past Vice President)

The President announced that the Woodward-Fondiller Prize for 1967 had been awarded to Jeffrey T. Lange, Assistant Secretary of the National Bureau of Casualty Underwriters, for his paper "Implications of Sampling Theory for Package Policy Ratemaking" presented at the November 1966 meeting of the Casualty Actuarial Society.

The meeting recessed at 12:15 p.m. for luncheon.

At the luncheon Sterling T. Tooker, President of The Travelers Insurance Companies, addressed the gathering on the subject "Some Reflections on the Expanding Concept of the Casualty-Property Actuary."

The meeting reconvened at 2:25 p.m. on November 13 with President Curry presiding.

Daniel J. McNamara, Chairman of the Constitutional Amendment Committee, then reported in detail on the proposed revised Constitution, drafts of which had been discussed by the members of the Society at previous CAS meetings. This document, recommended by the Council to become effective January 1, 1968, had been mailed to the membership on October 10, 1967.

Mr. McNamara discussed at length Article IX — "Public Expression of Professional Opinion," which the Council had again considered at great length at its meeting of November 12, 1967.

Mr. McNamara reported that the Council had unanimously reaffirmed its approval of Article IX as it appears in the material mailed to the members on October 10, 1967.

Nevertheless, in accordance with the recommendation of the Council, a vote on the proposed Article IX was first called for in the full realization that rejection of the proposed Article IX would mean that, because of the provisions of the then present Article X, it would not be possible to adopt the proposed revised Constitution at the 1967 Annual Meeting.

A motion to adopt the proposed Article IX was made and seconded. After lengthy discussion in which views pro and con were presented by the members, the Fellows present proceeded to vote. The motion to adopt the proposed Article IX was carried.

It was then moved and seconded that the proposed revised Constitution be adopted to become effective January 1, 1968. This motion was carried unanimously.

A motion was made and seconded that the proposed revised By-Laws, mailed to the members under date of October 10, 1967, be adopted to become effective January 1, 1968. This motion was carried unanimously.

President Curry then reminded the members that there had been extended discussion, both in Council meetings and at Society meetings, on "Council Guide Lines for the Nominating Committee" to assist that Committee in developing an annual slate for the election of officers and members of the Council. Mr. Curry stated that the Council had again wrestled with this problem at the meeting of September 13, 1967. The final action of the Council provides, in part:

(1) The names of those appointed to serve on the Nominating Committee will be announced at each Spring meeting.

(2) The Secretary-Treasurer shall send an informal ballot to each Fellow upon which preference for the various positions may be indicated anonymously. These ballots are to be returned to the Chairman of the Nominating Committee by no later than September 1.

(3) Prior to the Annual Meeting, the Nominating Committee shall prepare for forwarding to the Fellows a slate of candidates, all of whom shall have previously indicated their willingness to serve.

(4) The slate shall normally consist of:

President	One candidate
Vice President	Two candidates
Secretary-Treasurer	One candidate
Council Members	Five to six candidates.

Also, the "Guides" set forth criteria by which the Nominating Committee is to be guided in preparing its slate. The foregoing does not, of course, preclude additional nominations from the floor.

Following this President Harold E. Curry presented his Presidential Address, "Chance Favors the Prepared Mind."

William Leslie, Jr., Chairman of the Nominating Committee, then reported that due to the untimely death of Richard J. Wolfrum who had previously been selected by the Nominating Committee as one of the candidates for Vice President, it was necessary to revise the slate and the Committee had selected William J. Hazam and Daniel J. McNamara as candidates for Vice President.

The slate of the Nominating Committee, therefore, consisted of:

President	Harold W. Schloss
Vice President	William J. Hazam
	Daniel J. McNamara
Secretary-Treasurer	Albert Z. Skelding

There being no additional nominations from the floor, the four foregoing candidates were declared duly elected to the positions indicated.

The Fellows present then proceeded to confirm the election by the Council of the incumbents:

Editor	Matthew Rodermund
Librarian	Richard Lino
General Chairman of the Examination Committee	Norman J. Bennett

The slate of the Nominating Committee for three members to be elected to the Council for a three-year term was then presented:

James R. Berquist	Paul S. Liscord
M. Stanley Hughey	Ruth E. Salzmann
Richard L. Johe	LeRoy J. Simon

There being no other nominations from the floor, ballots were distributed and the President appointed as tellers John R. Bevan, George B. Elliott, John S. McGuinness, and John H. Muetterties. The tellers subsequently reported that 96 valid ballots had been cast and only the following had received a majority of the ballots cast: James R. Berquist, Ruth E. Salzmann, and LeRoy J. Simon. Therefore, no run-off being required, the foregoing were declared duly elected.

There then followed a panel on: Computer Applications other than Data Processing:

P. Adger Williams, Second Vice President and Actuary, The Travelers Insurance Companies — *Moderator*

Robert B. Foster, Associate Actuary, The Travelers Insurance Companies

George L. Hogeman, Vice President, Management Information and Planning Department, Aetna Life Casualty

Eldon J. Klaassen, Associate Actuary, Continental National American Group

It was regretted that, due to time limitations, it was not possible to hold the anticipated question and answer period following conclusion of the panel discusion. P. Adger Williams announced he had arranged a tour of the nearby Data Processing Center of The Travelers Insurance Companies for the afternoon of Tuesday, November 14, for those who were interested.

The meeting was recessed at 5:20 p.m. on Monday, November 13.

In the evening there was a social hour and reception. No formal banquet had been scheduled.

The meeting was reconvened at 9:10 a.m. on Tuesday, November 14 with President Harold E. Curry presiding.

John W. Wieder, Jr., Chairman of the Education Committee, announced that a revised syllabus of examinations would become effective with either the November 1968 or May 1969 examinations. Mr. Wieder stated that details of the new syllabus would be mailed the members and students as soon as the Committee had concluded its deliberations.

The President announced that Richard J. Wolfrum had been designated as the CAS representative on the Committee of Admissions to the American Academy of Actuaries to succeed Harold W. Schloss who had asked to be relieved of this assignment. However, because of the untimely death of Mr. Wolfrum, he had appointed Richard L. Johe to succeed Mr. Schloss.

There was then presented a panel discussion — "Flood Insurance":

Philip G. Buffinton, Vice President, State Farm Fire and Casualty Company — *Moderator*

Donald H. Garlock, Second Vice President, Casualty-Property Department, The Travelers Insurance Companies

Charles C. Hewitt, Jr., Actuary, Allstate Insurance Company

Henry B. Schechter, Director, Office of Economic and Market Analysis, U. S. Department of Housing and Urban Development

At the conclusion of the panel, time permitted of only a few questions from the floor.

Another panel discussion followed — "Regulation of Industries other than Insurance":

Allen L. Mayerson, Professor of Insurance and Actuarial Mathematics, University of Michigan, Graduate School of Business Administration — *Moderator*

Peter T. Beardsley, General Counsel, American Trucking Association

Allan J. Caldwell, Senior Vice President and Cashier, Hartford National Bank and Trust Company

Ernest L. Grove, Jr., Vice President, Northeast Utilities and Affiliated Companies

Here, again, time limitation precluded questions from the floor after conclusion of the panel presentation.

This concluded the scheduled program and, therefore, the 1967 Annual Meeting of the Casualty Actuarial Society was adjourned at 12:15 p.m. on Tuesday, November 14.

As a matter of interest it is noted that the Annual Meeting of the American Academy of Actuaries had been scheduled to tie in with that of the Casualty Actuarial Society. Accordingly, a large number of the members of the CAS found it convenient to attend that meeting which followed immediately after adjournment of the CAS meeting.

A tabulation of those who filed registration cards at the meeting to indicate actual attendance shows, in addition to about 30 wives, attendance by 125 Fellows, 67 Associates, and 23 Invited Guests as follows:

FELLOWS

Aldich, W. C.	Berquist, J. R.	Brannigan, J. F.
Allen, E. S.	Bevan, J. R.	Budd, E. H.
Bailey, R. A.	Blanchard, R. H.	Byrne, H. T.
Balcarek, R. J.	Bondy, M.	Carlson, E. A.
Barber, H. T.	Bornhuetter, R. L.	Cook, C. F.
Barker, L. M.	Boyajian, J. H.	Crane, H. G.
Bennett, N. J.	Boyle, J. I.	Crowley, J. H.

FELLOWS

Curry, A. C.
Curry, H. E.
DeMelio, J. J.
Dorweiler, P.
Drobisch, M. R.
Dropkin, L. B.
Ehlert, D. W.
Eliason, E. B.
Elliott, G. B.
Even, C. A., Jr.
Fairbanks, A. V.
Finnegan, J. H.
Fitzgibbon, W. J., Jr.
Flaherty, D. J.
Forker, D. C.
Foster, R. B.
Fowler, T. W.
Gibson, J. A., III
Gillam, W. S.
Gillespie, J. E.
Ginsburgh, H. J.
Graham, C. M.
Hart, W. Van B., Jr.
Haugh, C. J.
Hazam, W. J.
Hewitt, C. C., Jr.
Hillhouse, J. A.
Hobbs, E. J.
Hope, F. J.
Hunt, F. J., Jr.
Hurley, R. L.
Johe, R. L.
Johnson, R. A.
Klaassen, E. J.
Kormes, M.
Kuenkler, A. S.
Leslie, W., Jr.
Linden, J. R.
Linder, J.
Liscord, P. S.
Livingston, G. R.
Longley-Cook, L. H.
MacKeen, H. E.
Makgill, S. S.
Masterson, N. E.
Matthews, A. N.
Mayerson, A. L.
McClure, R. D.
McGuinness, J. S.
McLean, G. E.
McNamara, D. J.
Meenaghan, J. J.
Miller, N. F., Jr.
Mills, R. J.
Mohnblatt, A. S.
Morison, G. D.
Moseley, J.
Muetterties, J. H.
Munterich, G. C.
Murrin, T. E.
Nelson, D. A.
Newman, S. H.
Niles, C. L., Jr.
Oien, R. G.
Otteson, P. M.
Petz, E. F.
Pinney, A. D.
Pinney, S. D.
Pollack, R.
Portermain, N. W.
Presley, P. O.
Resony, A. V.
Resony, J. A.
Riccardo, J. F., Jr.
Richards, H. R.
Riddlesworth, W. A.
Roberts, L. H.
Rodermund, M.
Roth, R. J.
Salzman, R. E.
Scheibl, J. A.
Schuler, R. J.
Scott, B. E.
Simon, L. J.
Simoneau, P. W.
Skelding, A. Z.
Smick, J. J.
Smith, E. M.
Smith, E. R.
Smith, S. E.
Tarbell, L. L., Jr.
Thomas, J. W.
Uhthoff, D. R.
Valerius, N. M.
Verhage, P. A.
Walsh, A. J.
Webb, B. L.
Wieder, J. W., Jr.
Wilcken, C. L.
Williams, D. G.
Williams, P. A.
Willsey, L. W.
Wilson, J. C.
Wittick, H. E.

ASSOCIATES

Adler, M.
Andrews, E. C.
Ben-Zvi, P. N.
Bland, W. H.
Brian, R. A.
Brown, W. W., Jr.
Buffington, P. G.
Butler, R. H.
Carson, D. E. A.
Carter, E. J.
Chorpita, F. M.
Connor, J. B.
Crawford, W. H.
Crofts, G.
Davis, R. C.
Durkin, J. H.
Faber, J. A.
Farnam, W. E.
Ferrari, J. R.
Flynn, D. P.
Franklin, N. M.
Fulton, C. B.
Gerundo, L. P., Jr.
Getman, R. A.
Gossrow, R. W.
Greene, T. A.
Grossman, E. A.

ASSOCIATES

Hachemeister, C. A.
Hart, W. Van B., Sr.
Heer, E. L.
Holt, W. T.
Hunter, J. R., Jr.
Jacobs, T. S.
Jensen, J. P.
Jones, A. G.
Kaur, A. F.
Kilbourne, F. W.
McIntosh, K. L.
Mokros, B. F.
Munro, R. E.
Murray, E. R.
Murray, J. B. M.
Perreault S. L.
Plunkett, J. A.
Price, E. E.
Quinlan, J. A.
Raid, G. A.
Ratnaswamy, R.
Richardson, H. F.
Richardson, J. F.
Royer, A. F.
Ryan, K. M.
Scheid, J. E.
Schneiker, H. C.
Singer, P. E.
Snader, R. H.
Stein, J. B.
Stern, P. K.
Sturgis, R. W.
Torgrimson, D. A.
Trees, J. S.
Walters, M. A.
Ward, M. R.
Welch, J. P.
Williams, W. T.
Winter, A. E.
Woodworth, J. H.

GUESTS

*Battaglin, B. H.
Beardsley, P. T.
*Blanc, R.
*Brown, P. S.
Caldwell, A. J.
*Connolly, C. T.
Eddins, J. M.
Garlock, D. H.
*Griffith, R. W.
Grove, E. L., Jr.
*Hayden, R. C.
Hogeman, G. L.
Kavanagh, B.
*Kedrow, W. M.
Lawson, H. R.
May, J. W.
*Nagel, J. R.
*O'Shea, H. J.
Reinbolt, J. B.
Scurfield, H. H.
Sitkin, I. J.
*Strong, H. L.
Tooker, S. T.

*Invitational Program

Respectfully submitted,

A. Z. SKELDING
Secretary-Treasurer

REPORT OF THE SECRETARY-TREASURER

Among the items considered by the Council subsequent to the 1966 Annual Meeting are the following:

Meeting of February 6, 1967

Accepted the report of the Committee on Constitutional Amendments regarding the principles recommended up to that time by the Committee with respect to amendments to the Constitution and By-Laws.

Voted to increase the annual remuneration of the Secretary-Treasurer from $1,200 per annum to $1,500 per annum effective January 1, 1967.

Voted that diplomas be given to Associates admitted at or after the May 1967 meeting and that Associates who had achieved such status prior to May 1967 but had not achieved Fellowship status be presented Associateship diplomas upon request.

Voted that President Harold E. Curry be authorized to appoint the CAS representative on the Admissions Committee of the American Academy of Actuaries to succeed our then present representative Harold W. Schloss who had asked to be relieved of this assignment upon the expiration of his present term of office. President Curry later announced that he had appointed Richard P. Wolfrum, Actuary, Allstate Insurance Company, be designated the CAS delegate to Astin to succeed Norton E. Masterson who had asked to be relieved of his assignment upon expiration of his present term of office.

Voted that the President appoint a Committee to consider the feasibility of developing a table on the mortality of disabled lives due to occupational diseases.

Meeting of April 20, 1967

Adopted, with some amendments, an interim report of the Committee on Constitutional Amendments.

Voted that the dues of Associate Hugh P. Ham be waived under the disability provisions of Article IV of the By-Laws.

Meeting of May 21, 1967

Considered an interim report of the Education Committee relating to a revised examination syllabus and requested the Committee to pursue its investigations for a subsequent report to be considered by the Council.

Adopted the report of Editor Matthew Rodermund regarding methods for effecting savings in the future costs of printing the *Proceedings* and authorized Mr. Rodermund to arrange with the Recording and Statistical Corporation of Boston for the printing of future volumes.

Voted that Charles C. Hewitt, Jr. be authorized to discuss with Edward A. Lew, Chairman of the Committee on Research of the Society of Actuaries, a contemplated Research Conference to be conducted at Yale University in November or December 1967. Mr. Hewitt was authorized to commit the CAS to a contribution not to exceed $250 towards the expenses of this conference.

Meeting of September 13, 1967

Voted that the recommendations of the Committee on Constitutional Amendments with respect to

(a) Guides to Nominating Committee

(b) Amended Constitution

(c) Amended By-Laws

as amended at the September 13 meeting be adopted and that the proproposed amendments to the Constitution and By-Laws be mailed to the Fellows for action at the 1967 Annual Meeting of the Society.

(Note: For the purpose of the record it is noted that by mail vote of October 3, 1967 the Council adopted additional changes in the action taken at the September 13 meeting with respect to certain phraseologies in the proposed amendments to the Constitution and By-Laws).

Voted to accept the report of the Examination Committee, Norman J. Bennett, Chairman, and endorsed, as well worthy of study, certain additional recommendations which Mr. Bennett stated he would present to the Examination Committee at the time of the 1967 Annual Meeting.

Voted, that the establishment of a Joint Committee on Review of Education and Examinations consisting of representatives of the American Academy of Actuaries, Canadian Institute of Actuaries, Casualty Actuarial Society, Conference of Actuaries in Public Practice, Fraternal Association of Actuaries, and Society of Actuaries is in the best interests of the actuarial profession and that this committee should be (1) charged with a continuing review of policy matters relating to the education and examination of actuaries and (2) empowered to study these matters and, when appropriate, make recommendations on them to the governing bodies of the organizations represented on this committee.

Meeting of November 12, 1967

Voted to accept the report of the Education Committee which provides for a rather extensive revision of the Syllabus of Examinations. At the May 21, 1967 meeting of the Council the thought was expressed that the new Syllabus would not become effective prior to the May 1969 examinations.

Accepted the report of the Financial Review Committee. This report, in essence,

(1) Expressed the opinion that, barring an unforseen major increase in the cost of printing the *Proceedings*, it can reasonably be expected that receipts will exceed disbursements in the immediate future, and

(2) Recommended that, in the future, the Secretary-Treasurer prepare a budget on an annual basis for submission to the Council at the appropirate time.

Considered, and voted to accept, the reports of ten other Committees.
Reelected the incumbents, Messrs. Rodermund, Lino, and Bennett, to the posts of Editor, Librarian, and General Chairman of the Examination Committee, respectively, subject to the confirmation by the membership as provided by Article V of the Constitution.
Received, no action being required, the advices of the Secretary-Treasurer that Associate Milton J. Wood (November 18, 1927) had been dropped from membership for non-payment of dues in accordance with Article IV of the By-Laws.

OTHER MATTERS

(a) *Future Meetings*

Contemplated future meetings have been scheduled as follows:

1968 Spring Meeting — Monticello, New York

1968 Annual Meeting — Washington, D. C.

1969 Spring Meeting — Tamiment, Pennsylvania

1969 Annual Meeting — Atlanta, Georgia

1970 Spring Meeting — Philadelphia-New Jersey area

1970 Annual Meeting — Chicago, Illinois

(b) *Finances*

The Financial Report of the Secretary-Treasurer, on the following page, shows, for the fiscal period ending September 30, 1967,

Receipts	$31,461.89
Disbursements	27,643.76
Increase in assets	$ 3,818.13

As of September 30, 1967 the assets were

Balance in checking account	$ 5,416.04
Savings accounts	17,751.36
U.S. Treasury Bonds (maturity value)	5,000.00
Total	$28,167,40

FINANCIAL REPORT

Cash Receipts and Disbursements
from October 1, 1966 through September 30, 1967

Receipts

On deposit 10-1-66 (Checking)		$ 2,375.69
On deposit 10-1-66 (Savings-Bowery)		8,570.34
On deposit 10-1-66 (Savings-Chase)		8,403.24
Members, dues	$12,360.00	
Examination fees	3,651.65	
Sale of *Proceedings*	3,997 88	
Sale of Readings	310.00	
Spring and annual meetings	2,948.50	
Registration fees	3,070.00	
Invitational program	1,860.00	
Exchange	24 86	
Bond interest	193.76	
Savings account interest:		
Bowery	436.59	
Chase	341.19	
Michelbacher Fund	722.38	
For Actuaries' Club of N.Y.	857.50	
Contribution by Aetna Life & Casualty	500.00	
Water damage claim-City of N.Y.	200 00	
Miscellaneous	37 30	31,461.89
Total		$50,811.16

Disbursements

Printing and stationery	$14,730.97
Secretary's office	2,325.00
Examination expense	2,650.11
Meeting expense	5,329.83
Library fund	40.01
Insurance	100 00
Refunds	
Lunch and dinners	206.00
Examination fees	30.00
Registration fees	210 00
Out-of-print *Proceedings*	34.50
Membership-Insurance Society of N Y	150.00
Membership-International Congress	25.00
Donation-American Friends' Service Committee	25.00
Legal Expense-Organization American Academy	774.29
Fees-Actuaries' Club of N.Y	907 50
Miscellaneous	105.55
Subtotal	$27,643.76
On deposit 9-30-67	
Checking	5,416.04
Savings (Bowery)	9,006.93
Savings (Chase)	8,744.43
Subtotal	$23,167.40
Grand total	$50,811.16

Assets

Cash in bank 9-30-67	
Checking	$ 5,416.04
Savings (Bowery)	9,006.93
Savings (Chase)	8,744.43
U S. Treasury Bonds	5,000.00
Total	$28,167.40

Liabilities

Surplus (Michelbacher Fund)	$18,405.67
Other surplus	9,761.73
Total	$28,167.40

One U S. Treasury Bond 3⅞ % No. 24277 due for $1,000 on May 15, 1968.

Two U. S. Treasury Bonds 3⅞ % Nos. 3462-3 due for $1,000 each on May 15, 1968.

Two U. S Treasury Bonds 3⅞ % Nos 1673-4 due for $1,000 each on November 15, 1974

Employers' Fire Insurance Company Policy No. F16-10997-81 for $5,000 on books and book cases stored at 200 East 42 Street and $2,000 on material stored in library of Insurance Society of New York. Expires 9-14-70.

Fidelity Bond No. 044571 for $25,000 in Royal Indemnity Company.

Workmen's Compensation Policy No 03-223577 with Coverage B-Employers' Liability endorsement for $25,000. Expires 5-10-69.

Owners', Landlords' and Tenants' Liability Policy No 50-34107 in Maryland Casualty Company for 100,000/300,000/5,000. Expires 4-23-68.

This is to certify that we have audited the accounts, examined all vouchers and the assets shown above, and find same to be correct

Auditing Committee
HOWARD G CRANE, Chairman
J. H BOYAJIAN
THOMAS W. FOWLER

1967 EXAMINATIONS — SUCCESSFUL CANDIDATES

The examinations were held May 10 and 11, 1967. Parts 1 and 2 were jointly sponsored by the Casualty Actuarial Society and the Society of Actuaries. Those who passed were listed in the joint release of the two Societies dated June 26, 1967.

The listing of successful candidates for the remaining CAS examinations follows:

ASSOCIATESHIP EXAMINATIONS

Part 3 (a)

Allen, Park W., II
Bradshaw, John G., Jr.
Carter, Edward J., Jr.
Comey, Dale R.
Ferrari, J. Robert
Flynn, David P.
Fossa, Emilio F.
Grady, David J.
Hardy, Howard R.
Head, Thomas F.
Hearn, Vincent W.
Heer, E. LeRoy
Hilyer, Karen A.
Loeb, Harold A.
Lyon, Linda C.
McClenahan, Charles L.
Olsen, Dennis W.
Spitzer, Charles R.
Wade, Roger C.

Part 3 (b)

Allen, Park W., II
Bergen, Robert D.
Carter, Edward J., Jr.
Comey, Dale R.
Crescio, Joseph
Hardy, Howard R.
Hilyer, Karen A.
Jones, Alan G.
Lyon, Linda C.
Mark, Thomas W.
McClenahan, Charles L.
Moore, James E.
O'Dowd, John J.
Phlamm, James D.
Price, Edith
Rives, Robert A.
Simmons, Joel E.
Spitzer, Charles R.
Towle Richard H.
Walters, Michael A.
Winter, Arthur E.

Part 4

Carter, Edward J., Jr.
Chorpita, Fred M.
Covitz, Burton
Davis, Rex C.
Ferrari, J. Robert
Flynn, David P.
Gossrow, Robert W.
Gowdy, Robert C.
Hannes, Louis N.
Hartman, David G.
Hunter, John R., Jr.
Jacobs, Terry S.
Jones, Alan G.
Jones, Del R.
Jorve, Barry
Kaur, Alan F.
Plunkett, Joseph A.
Price, Edith
Shaw, James E.
Sturgeon, Purser K.
Torgrimson, Darvin A.
Walters, Michael A.
Ward, Michael R.
White, William D.
Williams, W. Thomas
Winter, Arthur E.

FELLOWSHIP EXAMINATIONS

Part 5

Adler, Martin
Brian, Robert A.
Connor, James B.
Faber, James A.
Farnam, Walter E.
Fulton, Clyde B., Jr.
Heer, E. LeRoy
Halt, William T.
Kilbourne, Frederick W.
McDonald, Charles
Presley, Philip O.
Schuler, Robert J.
Sturgis, Robert W.
Walters, Mavis

FELLOWSHIP EXAMINATIONS

Part 6

Ben-Zvi, Phillip N.
Bickerstaff, David R.
Carlson, Edwin
Dickson, Carol D.
Faber, James A.
Farnam, Walter E.
Honebein, Carlton W.
Lowe, Robert F.
Presley, Philip O.
Quinlan, John A.
Ratnaswamy, Raj
Scheid, James E.
Toren, Chester J.
Zory, Peter B.

Part 7

Zen-Zvi, Phillip N.
Eliason, Edward B.
Gibson, John A., III
Hachemeister, Charles A.
Hanson, H. Donald
Hartman, Gerald R.
Naffziger, Joe V.
Schuler, Robert J.
Sturgis, Robert W.

Part 8

Carlson, Edwin
Eliason, Edward B.
Gibson, John A., III
Hartman, Gerald R.
Mohnblatt, Arnold S.
Newman, Steven H.
Presley, Philip O.
Quinlan, John A.
Ryan, Kevin M.
Schuler, Robert J.

NEW ASSOCIATES

The following 20 candidates, having been successful in completing the examinations, were admitted as Associates of the Society at the Annual Meeting November 13, 1967:

Carter, Edward J., Jr.
Chorpita, Fred M.
Davis, Rex C.
Ferrari, J. Robert
Flynn, David P.
Gossrow, Robert W.
Gowdy, Robert C.
Heer, E. LeRoy
Hunter, John R., Jr.
Jacobs, Terry S.
Jones, Alan G.
Kaur, Alan F.
McDonald, Charles
Plunkett, Joseph A.
Price, Edith
Torgrimson, Darvin A.
Walters, Michael A.
Ward, Michael R.
Williams, W. Thomas
Winter, Arthur E.

NEW FELLOWS

The following 7 Associates, having been successful in completing the examinations, were admitted as Fellows of the Society at the Annual Meeting November 13, 1967:

Carlson, Edwin
Eliason, Edward B.
Gibson, John A., III
Mohnblatt, Arnold S.
Newman, Steven H.
Presley, Philip O.
Schuler, Robert J.

BOOK NOTES

D. V. Lindley, *Introduction to Probability and Statistics from a Bayesian Viewpoint, Part 1: Probability, Part 2: Inference,* Cambridge University Press, 2 vols., 1965.

Reviewed by ALLEN L. MAYERSON

Professor Lindley's two volume work is, professedly, the first attempt to attack the first course in probability and statistics from a Bayesian point of view. This should be of considerable interest to the actuary, since the Bayesian approach to statistics helps the actuary both to explain, in statistical terms, some of the techniques he has been using, and to find applications of some statistical methods to actuarial problems.

The first volume of Lindley's text comprises 4 chapters, 259 tightly reasoned pages, and is concerned with probability. Volume 2, 292 pages, contains chapters 5 to 8, dealing with statistical inference. But let no one assume that these slim volumes are easy reading; the level of mathematical sophistication required is high, and the notation and organization very different from that usually used in American introductory textbooks. Professor Lindley's two books may be an excellent introduction to probability and statistics for the pure mathematician, but they make heavy going for the typical actuarial student and even for many practicing actuaries.

The first chapter of volume I deals with the axioms of probability, the concept of probability as a limit of a relative frequency of successes, and the notions of independence and random trials. Bayes theorem, of course, is given a prominent place, and, throughout the book, the notation is such as to emphasize the conditional nature of many probabilities, with $P(A|B)$ being used in many instances where another author would write $P(A)$. The Bayesian bias of the book is also evident in the statement, which appears after the definition of $P(A|B)$ as the degree of belief in proposition A given that proposition B is true (or that even B has occurred), that: "The main subject matter of statistics is the study of how data (events) change degrees of belief; from prior, by observation of B, to posterior." Lindley also makes clearer than most other authors the relationship between the probability $P(A_i|H)$ where A is an event and H a hypothesis and the likelihood $P(A|H_i)$ and states that: "One speaks of the probability of an event or the likelihood of a hypothesis," depending on which is the variable.

Chapter 2 deals with probability distributions of one variable. It introduces the concept of a random variable, certain theorems dealing with expectations, and, of course, the binomial, Poisson, and normal distributions. Chapter 2 also contains material not often found in introductory texts, such as the simple random walk and the introduction of the characteristic function and the cumulant generating function, in addition to the more familiar moment generating function.

Chapter 3 introduces probability distributions in several variables, and derives joint and conditional distribution functions for Poisson and normal random variables, as well as notions of correlation and regression, in a highly condensed and mathematically elegant fashion. The central limit theorem and both the weak and strong laws of large numbers are given and the chapter also contains an erudite discussion of convergence and ergodicity which are well beyond anything normally found in an introductory statistics textbook. The chapter ends with the use of the Cauchy distribution, "the standard skeleton in the statistician's cupboard" to demonstrate the importance, to the central limit theorem, of the existence of the mean and the variance.

Chapter 4, entitled "Stochastic Processes," is of considerable interest to the actuary, and much of it will be unfamiliar to him. It deals with immigration-emigration and queueing processes, as well as renewal theory and random walks; all of these are related to the risk theory which is becoming so important to the actuary. There is also a 20 page introduction to Markov Chains which covers a good deal of material in a rather limited space.

It is interesting to note that, after the first chapter, the Bayesian ideas of prior distributions and subjective probability are not used until volume 2. Chapters 2-4 of volume 1 are independent of the Bayesian viewpoint which Lindley espouses and which he uses throughout volume 2.

Volume 2 contains chapters 5-8, dealing with statistical inference. Chapter 5 is a masterful exposition of the estimation of parameters from sample data when the random variable has a normal distribution, of course from a Bayesian point of view. Lindley discusses confidence intervals for population means and variances, significance tests, and the meaning and use of sufficient statistics. One would have expected that Lindley's Bayesian orientation would lead him to devote considerable space to decision theory, but he allots it only five pages. The description of confidence intervals and the explanation of the non-uniqueness of the confidence interval prescribed by most elementary texts is valuable and the discussion of the Student-Fisher t distribution and how it differs from the normal is explicit and original. Hypothesis testing is largely ignored: those confusing evil spirits which jinx many beginning statistics students — Type I and Type II errors — are conspicuous by their absence. As we would expect in this type of book, considerable use is made of the likelihood function, but, to preserve generality, most of Lindley's Bayesian applications assume a uniform prior distribution corresponding to vague prior information.

Chapter 6 extends the theory of statistical inference to multiple samples taken from different normal distributions. Testing the difference between two means, comparing variances using the F test, and an introduction to analysis of variance are among the topics covered.

Chapter 7, entitled "Approximate Methods," discusses the method of maximum likelihood, chi-square goodness of fit tests, and contingency tables. The Bayesian viewpoint and the importance of prior knowledge are often evident, as in a lucid discussion of why the chi-square test is not as good as the usual type of significance test when the form of the distribution is known to be normal.

Chapter 8, which discusses least squares, regression, correlation, and analysis of variance, requires some knowledge of matrix algebra. Lindley's caveat against unthinking use of the correlation coefficient and his warning that it tends to overemphasize the association between two random variables would be useful to many social scientists.

Professor Lindley's two volumes are remarkable. Their unique and unorthodox approach to probability and statistics will be an eye-opener to those unfamiliar with the Bayesian school of statistics. Lindley's unconventional approach to many statistical topics does not make the classical methods obsolete; rather the two approaches complement each other, and reading Lindley will make many facets of statistics, only dimly perceived in the half-light of classical treatment, come alive.

Although not recommended for the neophyte, Lindley's two volume text is very worthwhile reading for the intellectually curious actuary whose mathematics has not grown too rusty. Volume 2 in particular, especially chapter 5, is almost "must" reading for the actuary who, having passed part 2 of the C.A.S. examinations some years ago, has a smug feeling that he knows quite a bit about statistics. A word of warning may not be amiss, however; the book is so compact and the path it follows so different from the furrow plowed so regularly by the great number of elementary statistics textbooks published each year that many readers will find it very difficult at first reading. Do not despair; read it again and you will find it lucid, perceptive, and very educational.

John H. Magee and Ocsar N. Serbein, *Property and Liability Insurance* (The Irwin series on Risk and Insurance), Fourth Edition, 944 pages, Richard D. Irwin, Inc., Homewood, Illinois, 1967.

Reviewed by ALLEN L. MAYERSON

This is the fourth edition of the late Mr. Magee's classic text, revised by Professor Serbein of the Stanford University Graduate School of Business. Mr. Magee's *Property Insurance,* along with his other books, *General Insurance* and *Life Insurance,* have been standard reference works and college texts for more than 25 years. Their most prominent characteristics have been painstaking attention to detail, especially concerning coverage and policy forms, considerable bulk, and a rather dry, indeed rather dull style of presentation. But for anyone who wants to know the exact wording of the Deviation Clause in the Marine Open-Cargo Policy, or the difference between the Unpaid-Balance Form and the Single-Interest Form of deferred payment fire and allied lines insurance, Magee is the place to look.

As is usual in successive editions of a well known text, new material has been added by Professor Serbein while very little has been deleted. Magee's second edition, published in 1947, had 725 pages and the third, dated 1955, 767 pages; Professor Serbein has expanded the fourth edition to 944 pages, including a 44 page appendix of policy forms.

The first four chapters of the book, comprising 100 pages, discuss the meaning of risk, how insurance copes with risk, the history and development of property and liability insurance, a brief introduction to contract law, and definitions of many of the "words of art" that comprise our insurance jargon. Chapters on types of insurers, insurance practices, and insurance finance (including such topics as loss reserving, insurance accounting, etc.) have wisely been moved from the prominent place given them by Mr. Magee and relegated to the end of the book, while such esoteric details as the form of a Broker of Record letter have been deleted. (Magee's third edition had a *picture* of such a letter on page 51 of the book, with arrows pointing out that it is written on the insured's letterhead and bears the signature of a company officer!)

Chapters 5 to 10, comprising 156 pages, deal with fire, allied lines and business interruption insurance. They are followed by two chapters on ocean marine insurance, two on inland marine insurance and one each on theft insurance and fidelity and surety bonds. General liability insurance and workmen's compensation received a chapter (35 pages) each, as does automobile insurance; aviation insurance is treated in 38 pages and boiler and machinery in 25. For each line of insurance, the principal features and many of the details of every conceivable policy form are given, in the true Magee tradition.

Six chapters on multiple peril contracts, one each on homeowners, farm, commercial, industrial, institutional, and professional policies, comprise Prof. Serbein's principal contribution to the enterprise, and add 110 pages to the text. The last section, entitled "Operational Problems of the Property and Liability Insurer," includes not only Magee's chapters on types of insurers, rates, and finance, but also chapters of marketing, loss adjustment, and government supervision, and a detailed chapter which lists more than a hundred insurance organizations, some forty of them described in detail, ranging from the rather ineffectual ARIA Commission on Insurance Terminology and the Griffith Foundation for Insurance Education (hardly a major factor in property and liability insurance) to the Casualty Actuarial Society and the American Academy of Actuaries.

The fourth edition of *Property and Liability Insurance* has the virtues of the previous three: an encyclopedic listing of coverages and policy forms updated and expanded, in sufficient detail to tell most people more than they really want to know about almost any type of property insurance. The book has, however, the defects of the previous editions, principally a rather turgid style (though the new chapters written by Dr. Serbein are an improvement in style and readability) and a rather unbalanced presentation in some areas. For example, the chapter on Rates devotes 30 pages to the most detailed explanation of fire insurance rates and classifications I have ever seen, while workmen's compensation experience and retrospective rating get three pages, the manufacturers output policy four pages, and rates for automobile insurance, homeowners, marine insurance and other lines are not discussed at all.

The chapter on automobile insurance consists almost entirely of a clause by clause description of the coverage of the family automobile policy and such related matters as fleet, hired car, and nonownership coverage, as well as garage liability insurance. The reader will look in vain for the rating and classification system; he will never find out, even if he reads all 944 pages, that automobile insurance rates vary with the age of the driver and the use of the car, or that the cost of collision insurance depends in part on the make and model of the car. Neither will he find any discussion (other than the statement that: "The substitution of the compensation for the negligence principle creates many problems.") of tort liability vs. compensation, financial responsibility vs. compulsory insurance, or most of the other controversial questions that make the study of insurance alive and interesting for college students. Actuaries too, might cavil at the chapter on Insurance Finance, retained almost verbatim from the third edition, especially the uncritical acceptance of the Roger Kenney standards on company financial strength, and the erroneous implication that, after reading the chapter, the layman can, by following nine handy rules, understand an insurer's financial statement and judge its financial strength by himself.

Nevertheless, Professor Serbein has done us a great service by bringing up-to-date this standard reference work on property insurance coverage and policy forms.

Dennis F. Reinmuth, *The Regulation of Reciprocal Insurance Exchanges* (The Huebner Foundation Series), 234 pages, Richard D. Irwin, Inc., Homewood, Illinois, 1967.

Reviewed by RAJ RATNASWAMY

Like the American Indian, with his intangible political status, the reciprocals constitute a quasi-separate segment of the insurance industry whose corporate legal status is neither fully defined nor definable, representing a peculiarity not uncommon in the American way of doing things. In fact, in chapter 1, Dr. Reinmuth does state that reciprocals are indeed typically and exclusively American in origin and existence. There are few books on the subject of reciprocals, and so this author deserves special congratulations and thanks for his effort.

On page 1 of the book, the author describes vividly the image of six New York dry goods merchants discussing over lunch their common problems in obtaining fire insurance. Later, each signed a memorandum agreeing to pay each of the other five a maximum of $2,000 in case of loss by fire, all without forming any corporation.

Other firms later joined this arrangement, soon necessitating a committee and then an office manager and finally an attorney-in-fact who would handle and sign all the transactions on behalf of the subscribers. Funds were established through collection of "deposits" subject to refundability of unused moneys called "savings."

Similar reciprocals were formed in several parts of the country, and by 1925 Best's Insurance News showed six classes of reciprocals, such as sprinklered risks, mercantile, lumber, automobile, etc., a total of 117 reciprocals with $45 million in premiums. In 1964 the top eleven reciprocals listed by Best's had totel assets of $782 million and premium writings of $668 million, of which $537 million was in automobile insurance lines.

In chapter 2, The Legal Status of Reciprocal Exchanges, Professor Reinmuth describes the legal differences between stock companies, mutuals, and reciprocals; but unfortunately his discussion is not as comprehensive as this reviewer would have liked. In this chapter, also, he shows concern over the considerations that the attorneys-in-fact of some of the reciprocals are proprietary in nature, like a management corporation, and that the subscribers' contracts are drawn up by the attorneys-in-fact. These contracts authorize his control, limit his liabilities, and agree to fixed allowances for expenses; they permit few flexibilities and afford limited recourse to the subscribers for actions comparable to elections by stockholders or policyholders in stock or mutual companies.

The author does acknowledge, however, that the large auto club inter-insurance exchanges, which write the great bulk of the total premiums of all reciprocals combined, have non-proprietary attorneys-in-fact, and that these reciprocals do enjoy exemplary low expense ratios.

There have been few scandals, if any, involving reciprocals. However, the author's concern is again reflected in his final chapter, Conclusions, wherein he states, "The apparent legal ability of the attorney-in-fact to manipulate, sell or merge with other management interests, including a corporation owned by him, are a few of several possible types of abuses of subscribers' interests."

Note the words "apparent" and "possible." This reviewer concedes that there are dangers and complicated legal problems; also that reciprocals are indeed involved in insurance activity affecting the public interest and welfare, and that abuses are conceivable. These are justifications for suitable general supervision and regulation as with other types of insurers, but are not reasons enough for dissolving all existing reciprocals or converting them to other forms of insurers.

In his chapter on liquidation, conversion, and merger of reciprocals, the author concedes that many aspects are subject to regulation. He enumerates most of the few that pose special problems. For example, in California, where prior approval of the Insurance Commissioner is not required for assessments, it is possible for the attorney-in-fact to declare assessments prior to liquidation; and in the case of proprietary reciprocals, he can determine how much should come from the subscribers and how much from the attorney-in-fact. There are also questions regarding time allowed for assessments to determine which set of policyholders will be assessed. States other than New York do not specify that claimants and creditors shall be given preferential treatment over the subscriber as such.

The Insurance Commissioner has general authority over liquidations, conversions, and mergers of reciprocals; but in several states there are no statutory provisions governing some of the pertinent aspects. Also, certain reciprocals can be liquidated through reinsurance without subscribers' consent.

Many of the problems the author cites, however, do relate to potentialities involving proprietary attorneys-in-fact, and involve only a few states. Passage of new statutes can overcome many of these problems.

The book is highly readable, with well-ordered appendix, bibliography, and index of cases. It makes worthwhile reading for all students of insurance and certainly for actuaries. It succeeds in broadening understanding of many of the concepts involved in mutuals versus stocks, insurer versus insured, fixed liability versus assessability, policyholder versus stockholder, the ownership of surplus in case of liquidations, tax considerations, and other items.

The actuary will find this book no less interesting nor any less diverse than the lawyer, for much of it offers true challenges for quantifications and financial analyses. The reviewer recommends it.

Gerald R. Hartman, *Ratemaking for Homeowners Insurance* (The Huebner Foundation Series), 304 pages, Richard D. Irwin, Inc., Homewood, Illinois, 1967.

Reviewed by RICHARD H. SNADER

Dr. Hartman's book is well organized, well written, and should be useful to students of insurance. The scope of the book goes far beyond the subject matter suggested by its title. Ironically, the author devotes most of his talent to the events leading up to the current state of the art of Homeowners ratemaking, but his description of the procedures now in use is the weakest part of his book.

The earlier chapters are concerned with the fundamentals of insurance ratemaking. In these chapters the author introduces the term "rating" which he uses throughout the book apparently to mean *any* technique used to determine the proper rates to be charged. He also introduces the term "insurance coverage units." This term is not defined but is apparently intended to be a synonym for exposure units defined by the amount of insurance purchased.

The author devotes two chapters completely to rate regulation. The first authoritatively discusses, among other things, the Merritt Report, the Lockwood Report, the SEUA Case, Public Law 15, and All-Industry Bills. The other chapter is concerned with multiple-line rate regulations and discusses the Diemand Report, implementing legislation, the M-1 Report, and important administrative and judicial decisions. These chapters are an excellent summary of the events leading to rate regulation in the form known to us today and should provide the perspective needed by the actuarial student struggling to consolidate the knowledge he has gained from several diverse sources.

Only three of the book's twelve chapters are directly concerned with the ratemaking procedures used for Homeowners. The first of these chapters contains a section on the methods of determining initial multiple-line rates from the rates charged for the components of the Homeowners package; but its real value is contained in the discussion of such topics as the methods for providing multiple-line coverages, the economic motivation for multiple-line insurance, and the justification for package discounts.

The second of the chapters or ratemaking contains an interesting section on the use of credibility and seasoning factors used in the early filings. In this chapter the author uses the term *trend factors* to describe the weighting factors now in use by FIRAA and MLIRB. The term was in vogue during the period covered by the chapter, but it certainly seems a misnomer today. The author correctly explains trend and projection factors, as the terms are currently used, in an earlier chapter.

The third of the ratemaking chapters describes current MLIRB procedures, and contains an excellent description of the pricing of the Section 1 deductible. In fact, seven of the chapter's 15 pages are devoted to this aspect. Although the chapter contains an adequate description of form and policy amount relativities, it is weak in other respects. The author's explanation of projection factors based on the Composite Current Cost Index is not complete. The explanation of the adjustment of premiums to the current rate level is not clear. If an average actuarial student, after reading this chapter, were given all of the ingredients of a rate level calculation, he would probably be unable to apply the principles learned here unless he could deduce the proper applications from his prior knowledge of FIRAA's recommended procedure. Many of the book's inadequacies were respect to current ratemaking procedures could have been eliminated if a recent rate filing had been included as an appendix.

OBITUARIES

F. STUART BROWN	DUDLEY M. PRUITT
E. ALFRED DAVIES	HOMER D. RICE
CHARLES V. R. MARSH	HAROLD S. SPENCER
ROBERT K. ORR	RICHARD J. WOLFRUM

F. STUART BROWN

1896 — 1967

F. Stuart Brown, a Fellow of the Casualty Actuarial Society since 1927, died in Natick, Massachusetts, on October 21, 1967.

Stuart Brown was born April 6, 1896. He started to work at the Maryland Casualty Company in Baltimore at age 13 and continued with that company until 1928. Thereafter he was employed in a number of insurance organzations and finally joined the American Insurance Company of Newark in 1948, where he remained until his retirement in 1959.

Mr. Brown's principal activities in the insurance business involved statistics, methods and procedures, and data processing. He was greatly interested in the electronic data processing developments of the 1950's.

Mr. Brown was heavily engaged in church work and started a chapter of the Christian Businessmen's Committee on Cape Cod after he retired. He was also a member of the Masonic Order and the American Legion.

He is survived by his wife, Eva; a son, Charles; and a daughter, Carol Fricke.

E. ALFRED DAVIES

1885 — 1967

E. Alfred Davies, retired associate comptroller of Liberty Mutual Insurance Companies and a Fellow of the Casualty Actuarial Society, died January 14, 1967. He was 81 years old.

Mr. Davies' career, before joining Liberty Mutual in 1922, spanned half the world: Siberia shortly after the Bolshevik Revolution, Peking, and Tientsin during the famine in China.

He was born in Brighton, Sussex, England He came to the United States in 1909 and received his bachelor's degree in commercial science from New York University.

In 1917, during World War I, he joined the Canadian Army Medical Corps to work in the paymaster's office and serve as managing editor of the regimental newspaper. His unit shipped to Vladivostok, Russia, shortly after the outbreak of the Bolshevik Revolution.

After his tour of duty with the Canadian Army was over, Mr. Davies joined the American Red Cross and took an active part in relief operations in Vladivostok, Omsk in Siberia, and later Peking, where he set up complete accounting procedures for the relief operation and traveled frequently to Tientsin to audit accounts. When the Red Cross ended its relief work in China in 1921, a friend at the Red Cross put Mr. Davies in touch with Liberty Mutual in Boston, and in 1922 he was named auditor of disbursements for the company.

In 1929 Mr. Davies was named budget director, in 1941 he was promoted to assistant to the treasurer, and in 1943 was named associate comptroller, a position he held until his retirement in 1948.

After retiring from Liberty Mutual, Mr. Davies and his wife, Harriet Bosworth Davies, moved to Falls Village, Connecticut, and became extremely active in community and church affairs. The townspeople, in 1957, elected him their representative in the State Legislature where he served on the Insurance and Banking Committee.

CHARLES V. R. MARSH

1879 — 1967

Charles V. R. Marsh, retired comptroller of the Fidelity and Deposit Company of Baltimore, died in St. Petersburg, Florida, September 12, 1967. He had been an Associate of the Casualty Actuarial Society since 1927.

Born in Brooklyn, New York, Mr. Marsh joined a subsidiary of the Fidelity and Deposit in 1895. He went with the parent company in 1905 when it opened a branch office in New York. In January, 1921, he was named the company's assistant treasurer and moved to the home office in Baltimore. Several years later he was elected comptroller of the company. He retired on December 15, 1947, after 52 years with the firm.

Mr. Marsh then moved to Florida and had lived in St. Petersburg for about nine years.

While in Baltimore, Mr. Marsh was a member of the Masonic Order and of the Boumi Temple. He was a member of the Lakewood Methodist Church in St. Petersburg.

He is survived by his wife, the former Myne Woodring; one son, Alva V. R. Marsh, of Coral Gables, Florida; a sister, Mrs. Elizabeth Cooley, of White Plains, New York; and one grandchild.

ROBERT K. ORR

1879 — 1967

Robert K. Orr, a charter member of the Casualty Actuarial Society, died at the age of 88 in Lansing, Michigan, on October 5, 1967. Mr. Orr was the first president of the Michigan Employers Casualty Company, which specialized in workmen's compensation insurance. He founded this company in 1917.

He was also the president of the Wolverine Insurance Company, which he founded on June 1 of 1920. In 1927 these companies were merged, the surviving company being the Wolverine Insurance Company, which is still in existence. Mr. Orr continued as president of the Wolverine Insurance Company until 1944 and since that time has been retired, living in Lakeland, Florida.

Mr. Orr married Emma Gibbs in 1897, who predeceased him by several years. He was survived by two sons, Ivan and Wilfred, and a daughter, Lucile Doris.

DUDLEY M. PRUITT

1902 — 1967

Dudley M. Pruitt, past president of the Casualty Actuarial Society, died on June 27, 1967 at the age of 64. He will be remembered for the clarity of his writing, his quiet charm, and his great humanitarian interests.

The son of a missionary, he was born in the Shantung Province of China in 1902 and spent his early years there. He graduated from Haverford College in 1923 as a Phi Beta Kappa member and was in teaching until 1926 when he entered the insurance industry, first with the consulting firm of Woodward, Fondiller and Ryan, then with the Pennsylvania Indemnity

Corporation, and from 1938 to 1942 as statistician of the Fireman's Fund Indemnity Company in New York. In 1942 he joined the staff of the General Accident Fire and Life Assurance Corporation, where be became Assistant General Manager and Actuary.

Mr. Pruitt became an Associate of the Casualty Actuarial Society in 1928 and a Fellow in 1931. He served on the Council and on a number of committees and was Vice President in 1953 and 1954 and President in 1957 and 1958. He was also President of the Insurance Accounting and Statistical Association (1953).

His major actuarial interests were data processing, accounting, and reserves, on which he wrote papers, and took part in discussions, in the Proceedings of the Casualty Actuarial Society, the Insurance Accounting and Statistical Association, and other insurance accounting associations. As chairman of the Research Committee of the Society, he wrote a *Progress Report on Electronics* in 1954 which greatly influenced the property and casualty insurance industry in its use of electronic data processing equipment. He also wrote the chapter on Loss Accounting in the New York Insurance Department's *Examination of Insurance Companies.*

He will be remembered best by actuaries for his two presidential addresses, "The Seat of Wisdom" (1958) and "St. Vitus's Dance" (1959) where his light touch and wit provided a pleasant contrast to the more serious addresses of most presidents. However, his remarks were no less important because of the humorous vein in which they were written, and much that he said then is still appropriate to today's problems. In 1964 he was asked to prepare a history of the Casualty Actuarial Society ("The First Fifty Years") for the jubilee celebration of the Society. In this paper he skillfully combined the lighter as well as the more serious sides of the development of casualty actuarial work.

In 1951 in review of a paper by Tom Carlson, he wrote:

> "A certain charm about the paper comes from the happy selection of literary quotations at the chapter headings. Or, as Samuel Butler put it, the author
>
> 'Cheer'd up himself with ends of verse
> And sayings of philosophers.'
>
> The reader will pardon me if I proceed to use, or abuse, this technique, but not for the same reason. I find myself to be one of those who, for want of more original material,
>
> '. . . lard their lean books with the fat of others' works.'
>
> Robert Burton"

And so, with Dudley's charm and modesty he enlivened his actuarial writings from then on for the pleasure of us all.

Mr. Pruitt and his wife were active Quakers. He was a former clerk of the Radnor Friends Meeting, chairman of the Friends Peace Committee of Philadelphia, and district commissioner of the Main Line District, Boy Scouts of America. In 1960 he retired from actuarial work to devote all his energies to promote peace and international and interracial understanding. He and his wife went to Tokyo for two years where he was field director of the American Friends Service Committee in Japan. He gave a most interesting account of his experiences there at the Philadelphia meeting of the Society in November, 1962. On his return to Philadelphia he became executive director of the Middle Atlantic Region of the American Friends Service Committee.

In his death the Casualty Actuarial Society loses an outstanding past president and a kindly friend of many. He is survived by his wife, the former Grace Garner; two sons, Drs. Dean G. and John D. Pruitt; a sister, Miss Ida Pruitt; and five grandchildren.

HOMER D. RICE

1892 — 1967

Homer D. Rice, a Fellow of the Casualty Actuarial Society since 1951, died May 12, 1967 at the age of 75. At the time of his retirement in 1953 he was General Manager of the New York Fire Insurance Rating Organization.

He was a native of the western part of New York State and was educated in schools at Buffalo. His early introduction to the business of insurance was with the Buffalo Association of Fire Underwriters, a predecessor to the New York Fire Insurance Rating Organization in that part of New York State. Mr. Rice obtained a broad experience in the insurance field in the nineteen twenties and early nineteen thirties as a representative of insurance companies and later as a licensed insurance agent of a prominent agency located in the City of Buffalo. In 1935 he was asked by a special committee of the insurance companies to return to the bureau side of the business and assume the position of Manager of the Buffalo Division of the New York Fire Insurance Rating Organization. Six years later he was advanced to Assistant General Manager of the Rating Organization, with offices at New York City. He was named General Manager on January 1, 1949.

Mr. Rice served on a number of industry committees during his career. The rating studies directed by him led to a number of new statistical approaches in rate making.

Serious illness in 1953 required his retirement. He moved to Mount Dora, Florida where he resided until his death this year.

He is survived by his son, Victor M. Rice, who resides in Phoenix, Arizona.

HAROLD S. SPENCER

1883 — 1968

Harold S. Spencer, an Associate of the Casualty Actuarial Society since 1918, died March 18, 1968.

He was born in Hartford and spent his business career with the companies now known as Ætna Life & Casualty, starting at age nineteen and serving almost fifty years until his retirement in 1949. He was in charge of the Casualty Statistical Department.

Mr. Spencer had a wide range of interests outside business. His interest in church and in theatrical matters led to his serving as religion editor of The Hartford Courant at one time and often contributing reviews to its columns; he was also a founder and past president of the Ætna Dramatic Club.

Another avocation was an interest in Early Americana, developing into a silent partnership with Mrs. Spencer in an antique business for many years.

Harold S. Spencer is survived by his wife, Grace T. Moore Spencer, with whom he observed the sixtieth anniversary of their marriage in 1967.

RICHARD J. WOLFRUM

1920 — 1967

Richard J. Wolfrum, Past Vice President of the Casualty Actuarial Society and Actuary of the Liberty Mutual Insurance Company, died suddenly in Boston, Massachusetts on October 31, 1967.

Dick Wolfrum was born in Boston and graduated from Harvard University *magna cum laude* in 1941. Upon graduation he entered the United States Army and during World War II saw military service in the European theater as a captain in the Infantry.

He was employed by Liberty Mutual in 1945, was named Assistant Actuary of the Company in 1948, and became a Fellow in the Casualty Actuarial Society in 1949. In 1959 he was appointed Actuary of Liberty Mutual.

During his abbreviated actuarial career his involvement in significant actuarial affairs was impressive. As Past Vice President and an active member in the Society, he contributed generously to its activities and was instrumental in broadening its influence and prestige. In addition he assumed a leading role in the many industry actuarial committees of which he was a member. Dick was Chairman of the Automobile Committee of the American Mutual Insurance Alliance, an active participant in National Council and Mutual Bureau rating and actuarial committees, and will be remembered as an effective industry spokesman for the common-sense application of practical actuarial techniques to complex industry problems. He was also active in several other professional organizations including the American Academy of Actuaries, the International Congress of Actuaries, and the Actuaries Club of Boston.

Dick's sudden passing was shocking in its impact on all of his friends and associates. The high regard in which he was held was due in no small measure to his warmth and wit, combined with a steady, unfailing competence. His judgment, dedication, and integrity qualified him as an outstanding member of our profession.

He leaves his wife, Mrs. Audrey D. Wolfrum; three daughters, Mrs. Mary Audrey Mullen of North Easton, and Carol Ann and Jan Marie Wolfrum of Dedham; his mother, Mrs. Bertha Wolfrum of West Roxbury; and a brother, Alfred H. Wolfrum of West Roxbury.

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CASUALTY ACTUARIAL SOCIETY

ORGANIZED 1914

1968 YEAR BOOK

Corrected to January 1, 1968

FOREWORD

The Casualty Actuarial Society was organized in 1914 as the Casualty Actuarial and Statistical Society of America, with 97 charter members of the grade of Fellow; the Society adopted its present name on May 14, 1921.

Actuarial science originated in England in 1792, in the early days of life insurance. Due to the technical nature of the business, the first actuaries were mathematicians; eventually their numerical growth resulted in the formation of the Institute of Actuaries in England in 1848. The Faculty of Actuaries was founded in Scotland in 1856, followed in the United States by the Actuarial Society of America in 1889 and the American Institute of Actuaries in 1909. In 1949 the two American organizations were merged into the Society of Actuaries.

In the United States problems requiring actuarial treatment were emerging in sickness, disability, and casualty insurance—particularly workmen's compensation, introduced in 1911. The differences between the new problems and those of life insurance led to the organization of the Casualty Actuarial Society in 1914. Dr. I. M. Rubinow, who was responsible for its formation, became the Society's first president. Since the problems of workmen's compensation were the most urgent, many members played a leading part in developing the present scientific basis for that line of insurance. The object of the Society was, and is, the promotion of actuarial and statistical science as applied to the problems of insurance other than life insurance by means of personal communication, presentation and discussion of appropriate papers, collection of a library, and by other desirable means.

From its beginning the Society has grown constantly in membership, scope of interests, and scientific and related contributions to the non-life field. These contributions are found in original papers prepared by members of the Society and published in the annual *Proceedings*. The presidential addresses constitute a valuable record of actuarial problems, some of them still unsolved, that have faced the insurance industry over the years.

In November 1950 the Constitution and By-Laws were amended to enlarge the scope of the Society to include all lines of insurance other than life insurance (specifically, fire and allied lines) in recognition of the multiple line powers granted by many states to both casualty and fire companies.

The membership of the Society includes actuaries employed by insurance companies, ratemaking organizations, and state insurance departments, and as independent consultants. The Society has two grades of membership, Fellowship and Associateship. Examinations for the two grades are held in May and November in various cities in the United States and Canada.

On the inside front cover of the *Year Book* are listed the *Proceedings* and other publications of the Society and their respective prices. The *Year Book* is published annually. *Recommendations for Study* is a pamphlet outlining the course of study recommended for examination. The two booklets may be obtained free upon request to the Secretary-Treasurer, Albert Z. Skelding, 200 E. 42nd Street, New York, N. Y. 10017.

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THE COUNCIL

JANUARY 1, 1968

* *Officers:*

Harold W. Schloss .... *President*
William J. Hazam .... *Vice President*
Daniel J. McNamara .... *Vice President*
Albert Z. Skelding........................ *Secretary-Treasurer*
Matthew Rodermund .... *Editor*
Richard Lino .... *Librarian*
Norman J. Bennett .... *General Chairman-Examination Committee*

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† *Ex-Vice President:*

Charles C. Hewitt, Jr 1969

† *Elected:*

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William S. Gillam 1968
Allen L. Mayerson 1968
Harry T. Byrne 1969
Charles L. Niles, Jr. 1969
Robert Pollack 1969
James R. Berquist 1970
Ruth E. Salzmann 1970
LeRoy J. Simon 1970

*Terms expire at the Annual Meeting in November 1968.
†Terms expire at the Annual Meeting in November of the year given.

COMMITTEES

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Lewis H. Roberts

SCHEDULE OF MEMBERSHIP, NOVEMBER 13, 1967

	Fellows	Associates	Total
Membership, November 18, 1966	229	192	421
Increase By:			
Election	....	...	....
Reinstatement	....	....	....
Examination	7	20	27
	236	212	448
Decrease By:			
Death.......................	6	1	7
Withdrawal	....	2	2
Transfer from Associate to Fellow....	. ..	7	7
	230	202	432

NOVEMBER 13, 1967

Those Marked (†) were Charter Members at date of organization, November 7, 1914

Admitted	
Nov. 21, 1930	AINLEY, JOHN W. (Retired), Travelers, 33 Paxton Road, West Hartford, Conn. 06107
Nov 20, 1964	ALDRICH, WILLIAM C., Secretary and Counsel, National Council on Compensation Insurance, 200 East 42nd Street, New York, N. Y. 10017
Nov 20, 1964	ALEXANDER, LEE M , Actuary, Massachusetts Workmen's Compensation & Inspection Bureau, 89 Broad Street, Boston, Mass. 02110
Nov. 14, 1947	ALLEN, EDWARD S., Actuary, Phoenix of Hartford Insurance Companies, 61 Woodland Street, Hartford, Conn. 06115
Nov. 18, 1955	BAILEY, ROBERT A., Director, Insurance and Actuarial Section, Insurance Bureau, State of Michigan, 111 N. Hosmer Street, Lansing, Mich. 48913
Nov. 15, 1962	BALCAREK, RAFAL J., Assistant Vice President and Actuary, Reliance Insurance Company, 4 Penn Center Plaza, Philadelphia, Pa 19103
Nov. 20, 1924	BARBER, HARMON T. (Retired), 18 Ridgewood Road, Windsor, Conn. 06095
Nov 19, 1954	BARKER, GORDON M., Actuary, Great American Group, 99 John Street, New York, N. Y. 10038
Nov. 14, 1947	BARKER, LORING M., Actuary, Fireman's Fund American Insurance Companies, 3333 California Street, San Francisco, Calif. 94120
Nov 20, 1942	BART, ROBERT D., Vice President-Services & Employee Relations. The West Bend Company, 400 Washington Street, West Bend, Wis. 53095
Nov. 18, 1932	BARTER, JOHN L. (Retired), 90 Tunxis Road, West Hartford, Conn 06107
Nov. 13, 1931	BATHO, ELGIN R., Vice President and Actuary, Berkshire Life Insurance Company, 700 South Street, Pittsfield, Mass. 01203
Nov. 14, 1958	BENBROOK, PAUL, Executive Vice President, Maryland Casualty Company, Box 1228, Baltimore, Maryland 21203
Nov. 16, 1956	BENNETT, NORMAN J., Assistant Secretary and Actuary, Continental Insurance Companies, 80 Maiden Lane, New York, N Y. 10038
Nov. 22, 1934	BERKELEY, ERNEST T , Actuary, The Employers' Group of Insurance Companies, 110 Milk Street, Boston, Mass 02107
Nov. 22, 1957	BERQUIST, JAMES R , Associate Actuary, Employers Insurance of Wausau, 2000 Westwood Drive, Wausau, Wis. 54401
Nov 19, 1953	BEVAN, JOHN R., Assistant Actuary and Group Actuary, Liberty Mutual Insurance Company, 175 Berkeley Street, Boston, Mass. 02117
†	BLACK, S. BRUCE, Chairman Emeritus, Liberty Mutual Insurance Company, 175 Berkeley Street, Boston, Mass 02117
Apr 20, 1917	BLANCHARD, RALPH H , Professor Emeritus of Insurance, Columbia University, Plympton, Mass 02367
Nov. 19, 1959	BLODGET, HUGH R , Assistant Secretary, Data Processing Development Dept., Aetna Life & Casualty, Hartford, Conn. 06115
Nov. 16, 1956	BONDY, MARTIN, Assistant Vice President and Actuary, Crum & Forster Insurance Group, 110 William Street, New York, N Y. 10038
Nov. 22, 1957	BORNHUETTER, RONALD L , Vice President and Actuary, General Reinsurance Corporation, 400 Park Avenue, New York, N Y 10022
Nov. 16, 1956	BOYAJIAN, JOHN H., Actuary, New Jersey Manufacturers Insurance Company, Sullivan Way, Trenton, New Jersey 08607
Nov. 19, 1959	BOYLE, JAMES I., Associate Actuary, The Travelers Insurance Companies, One Tower Square, Hartford, Conn. 06115

Admitted	
Nov 16, 1961	BRANNIGAN, JAMES F , Associate Actuary, The Travelers Insurance Companies, One Tower Square, Hartford, Conn 06115
†	BREIBY, WILLIAM, Chairman of Board, Western Travelers Life Insurance Company, 2701 Beverly Blvd., Los Angeles, Calif 90057
Nov. 21, 1952	BRINDISE, RALPH S , Group Insurance Supervisor, Standard Oil (Indiana), 910 S. Michigan Avenue, Chicago, Ill 60680
Oct. 22, 1915	BROWN, HERBERT D (Retired), Glenora-on-Lake Seneca, Dundee, N Y 14837
Nov. 16, 1961	BUDD, EDWARD H , Vice President, The Travelers Insurance Companies, One Tower Square, Hartford, Conn 06115
Nov 23, 1928	BURLING, WILLIAM H., Manager and Actuary for Canada Group, The Travelers Insurance Company, Canadian Dept., 101 Richmond Street W., Toronto 1, Ontario, Canada
Nov. 19, 1959	BYRNE, HARRY T , Associate Actuary, Aetna Life & Casualty, Hartford, Conn 06115
Nov 19, 1929	CAHILL, JAMES M , General Manager, Insurance Rating Board, 125 Maiden Lane, New York, N Y 10038
Nov 18, 1932	CAMERON, FREELAND R , Senior Vice President, Horace Mann Insurance Company, 150 S E. Third Avenue, Miami, Florida 33131
Nov. 17, 1938	CARLETON, JOHN W . Vice President, Liberty Mutual Insurance Company, 175 Berkeley Street, Boston, Mass 02117
Nov 13, 1967	CARLSON, EDWIN A , Assistant Actuary, The Travelers Insurance Companies, One Tower Square, Hartford, Conn 06115
Nov 18, 1966	CIMA, AUGUSTIN J , Associate Actuary, Allstate Insurance Company, Allstate Plaza, Northbrook, Illinois 60062
Nov 18, 1949	CLARKE, JOHN W., President, Hartford Life Insurance Company, Hartford Plaza, Hartford, Conn. 06115
Nov 15, 1918	COATES, BARRETT N. (Retired), 1007 Cragmont Avenue, Berkeley, Calif. 94708
Nov. 17, 1922	COATES, CLARENCE S (Retired), 1730 Washington Avenue, Wilmette, Illinois 60091
Feb. 19, 1915	COLLINS, HENRY (Retired), Commercial Union Insurance Company, 200 Park Avenue, New York, N. Y. 10017
Nov 18, 1966	COOK, CHARLES F , Associate Actuary, General Accident Group, 414 Walnut Street, Philadelphia, Pa. 19105
Nov. 22, 1934	COOK, EDWIN A., President and General Manager, Interboro Mutual Indemnity Insurance Company, 270 Madison Avenue, New York, N. Y 10016
Nov 18 1925	CORCORAN, WILLIAM M. (Retired), 9 Park View Drive, Bronxville, New York 10708
Nov 18, 1966	CRANDALL, WILLIAM H., Associate Actuary, Insurance Company of North America, 1600 Arch Street, Philadelphia, Pa 19101
Nov 19, 1926	CRANE, HOWARD G., Vice President and Consultant, General Reinsurance Corporation, 400 Park Avenue, New York, N. Y 10022
Nov. 21, 1952	CRITCHLEY, DOUGLAS, E B. Savory & Co , Basildon House, Moorgate, London, England
Nov. 22, 1946	CROUSE, CHARLES W , Consulting Actuary, C E Preslan & Company, Inc , 20015 Detroit Road, Cleveland, Ohio 44116
Nov 18, 1960	CROWLEY, JAMES H., Assistant Secretary, Accounting Dept., Aetna Life & Casualty, Hartford, Conn. 06115
Nov 16, 1965	CURRY, ALAN C., Senior Actuary, State Farm Mutual Automobile Insurance Company, 112 East Washington Street, Bloomington, Illinois 61701
Nov. 19, 1953	CURRY, HAROLD E., Senior Vice President, State Farm Mutual Automobile Insurance Company, 112 E Washington Street, Bloomington, Ill. 61701
Nov. 18, 1966	DAHME, ORVAL E , Associate Actuary, State Farm Mutual Automobile Insurance Company, 112 E. Washington Street, Bloomington, Illinois 61701

Admitted	
Nov. 18, 1927	DAVIS, EVELYN M., Partner, Woodward, Ryan, Sharp & Davis, 26 Broadway, New York, N. Y. 10004
May 25, 1956	DAY, ELDEN W. (Retired), 199 North Ingleside Avenue, Fairhope, Ala 36532
Nov. 16, 1965	DEMELIO, JOSEPH J., Secretary, The Home Insurance Company, 59 Maiden Lane, New York, N Y. 10008
Nov. 18, 1960	DICKERSON, O. D , Professor, Risk and Insurance, Florida State University, Tallahassee, Fla. 32306
Nov. 16, 1965	DORF, STANLEY A., Supervising Actuary, New York Insurance Dept , 123 William Street, New York, N Y. 10038
Nov. 17, 1920	DORWEILER, PAUL (Retired), 51 Wethersfield Avenue, Hartford, Conn. 06114
Nov. 22, 1957	DROBISCH, MILES R., Assistant Actuary, California Inspection Rating Bureau, 1453 Mission Street, San Francisco, California 94103
Nov. 14, 1958	DROPKIN, LESTER B., Assistant General Manager and Actuary, California Inspection Rating Bureau, 1453 Mission Street, San Francisco, California 94103
Nov. 24, 1933	EDWARDS, JOHN, 16 Brentwood Road South, Toronto 18, Ontario, Canada
Nov 16, 1965	EHLERT, DARRELL W., Director of Actuarial Research, Allstate Insurance Company, 321 Middlefield Road, Menlo Park, California 94025
Nov 19, 1959	EIDE, K ARNE, Assistant Statistician, Metropolitan Life Insurance Company, One Madison Avenue, New York, N. Y 10010
Nov. 13, 1967	ELIASON, EDWARD B , Assistant Actuary, Aetna Life & Casualty, Hartford, Conn. 06115
Nov. 15, 1940	ELLIOTT, GEORGE B., General Manager, Pennsylvania Compensation Rating Bureau, 1819 John F. Kennedy Blvd , Philadelphia, Pa 19103
Nov. 17, 1922	ELSTON, JAMES S. (Retired), 1640 Palmer Avenue, Winter Park, Florida 32789
Nov. 15, 1935	EPPINK, WALTER T , 1st Vice President, Treasurer & Actuary, Merchants Mutual Insurance Company, 250 Main Street, Buffalo, N. Y 14240
Nov. 14, 1958	ESPIE, ROBERT G., Vice President and Comptroller, Aetna Life & Casualty, Hartford, Conn 06115
Nov 18, 1966	EVEN, CHARLES A., JR., Assistant Actuary, The Travelers Insurance Companies, One Tower Square, Hartford, Conn. 06115
Nov 18, 1955	FAIRBANKS, ALFRED V., Assistant Vice President and Actuary, Monarch Life Insurance Company, 1250 State Street, Springfield, Mass 01101
†	FALLOW, EVERETT S. (Retired), 28 Sunset Terrace, West Hartford, Conn. 06107
Nov. 15, 1940	FARLEY, JARVIS, President, Treasurer and General Manager, Massachusetts Indemnity and Life Insurance Company, 654 Beacon Street, Boston, Mass 02215
†	FARRER, HENRY (Retired), R D. #3, Box 322, Fleetwood, Pa. 19522
Nov. 18, 1960	FAUST, J EDWARD, JR., Consulting Actuary, R. R. #1, West Gray Road, Zionsville, Ind. 46077
May 25, 1956	FINNEGAN, JOSEPH H., Assistant to the General Manager, National Insurance Actuarial and Statistical Association, 110 William Street, New York, N. Y. 10038
Nov. 16, 1961	FITZGIBBON, WALTER J., JR., Assistant Actuary, Aetna Life & Casualty, Hartford, Conn. 06115
Nov 15, 1935	FITZHUGH, GILBERT W., Chairman of the Board, Metropolitan Life Insurance Company, One Madison Avenue, New York, N. Y. 10010
Nov. 18, 1966	FLAHERTY, DANIEL J., Associate Actuary, Insurance Company of North America, 1600 Arch Street, Philadelphia, Pa. 19101

Admitted	
Nov. 18, 1966	FORKER, DAVID C., Assistant Actuary, Group Dept , The Travelers Insurance Companies, One Tower Square, Hartford, Conn. 06115
Nov. 18, 1955	FOSTER, ROBERT B., Actuary, The Travelers Insurance Companies, One Tower Square, Hartford, Conn 06115
Nov. 18, 1955	FOWLER, THOMAS W., Actuary, North American Reinsurance Corporation, 245 Park Avenue, New York, N Y. 10017
Nov 18, 1927	FREDRICKSON, C. H , Consulting Actuary, Galiano Island, P O. Box 40, British Columbia, Canada
Nov. 22, 1934	FULLER, GARDNER V. (Retired), Conover, Wis 54519
Nov 13, 1967	GIBSON, JOHN A., III, Assistant Actuary, Reliance Insurance Company, 4 Penn Center Plaza, Philadelphia, Pa 19103
Nov. 22, 1957	GILLAM, WILLIAM S., Secretary - Research, Insurance Rating Board, 125 Maiden Lane, New York, N. Y 10038
Nov. 20, 1964	GILLESPIE, JAMES E., Assistant Actuary, Continental Casualty Company, 310 South Michigan Blvd., Chicago, Ill. 60604
Nov. 20, 1924	GINSBURGH, HAROLD J (Retired), 14 Crestview Road, Belmont, Mass. 02178
Nov 21, 1930	GLENN, JOSEPH BRYAN, Actuarial Consultant, Department of Defense, Washington, D C.
Nov. 13, 1931	GODDARD, RUSSELL P., Actuary, Bowles, Andrews & Towne, Inc , 1389 Peachtree Street, N. E., Atlanta, Georgia 30309
Nov. 19, 1926	GRAHAM, CHARLES M , Fire and Casualty Actuary, South Carolina Insurance Department, 1401 Hampton Street, Columbia, S. C 29201
Nov. 19, 1953	GRAVES, CLYDE H., Assistant Manager and Actuary, Mutual Insurance Rating Bureau and Mutual Insurance Advisory Association, 733 Third Avenue, New York, N. Y 10017
Nov 19, 1953	HALEY, JAMES B., JR , Actuary, Coates, Herfurth & England, 320 California Street, San Francisco, California 94104
Nov. 16, 1956	HART, W. VAN BUREN, JR., Actuary, Aetna Insurance Company, 55 Elm Street, Hartford, Conn. 06115
Nov. 17, 1950	HARWAYNE, FRANK, Consulting Actuary, 3 Stuyvesant Oval, New York, N Y. 10009
Nov 19, 1926	HAUGH, CHARLES J. (Retired), 25 Le May Street, West Hartford, Conn. 06107
Nov. 17, 1950	HAZAM, WILLIAM J , Vice President and Actuary, American Mutual Insurance Companies, Quannapowitt Parkway, Wakefield, Mass 01880
Nov. 16, 1951	HEWITT, CHARLES C., JR , Actuary, Allstate Insurance Company, Allstate Plaza, Northbrook, Ill 60062
Nov 18, 1966	HILLHOUSE, JERRY A., Actuary, State Farm Mutual, 112 E. Washington Street, Bloomington, Ill. 61701
Nov. 16, 1961	HOBBS, EDWARD J , Vice President, Insurance Company of North America, 1600 Arch Street, Philadelphia, Pa. 19101
Nov. 22, 1934	HOOKER, RUSSELL O., Consulting Actuary, Russell O Hooker & Associates, 100 Constitution Plaza, Hartford, Conn 06103
Nov. 17, 1950	HOPE, FRANCIS J., Actuary, Hartford Insurance Group, Hartford Plaza, Hartford, Conn. 06115
Nov. 14, 1947	HUGHEY, M STANLEY, Executive Vice President, Lumbermens Mutual Casualty Company, 4750 N Sheridan Road, Chicago, Ill 60640
Nov 19, 1959	HUNT, FREDERIC J., JR., Assistant Secretary, Insurance Company of North America, 1600 Arch Street, Philadelphia, Pa 19101
Nov 18, 1955	HURLEY, ROBERT L , Actuary, Fire Insurance Research and Actuarial Association, 125 Maiden Lane, New York, N. Y 10038
Nov 19, 1954	JOHE, RICHARD L , Vice President and Actuary, United States Fidelity and Guaranty Company, Calvert and Redwood Streets, Baltimore, Md. 21203
Nov. 14, 1941	JOHNSON, ROGER A , Actuary, Blue Cross of Greater Philadelphia, 1333 Chestnut Street, Philadelphia, Pa 19107

Admitted	
Nov. 16, 1939	JONES, HAROLD M , Group Statistician, John Hancock Mutual Life Insurance Company, 200 Berkeley Street, Boston, Mass. 02117
Nov. 16, 1956	KALLOP, ROY H , Actuary, National Council on Compensation Insurance, 200 East 42 Street, New York, N. Y. 10017
Nov. 22, 1957	KATES, PHILLIP B , Vice President and Actuary, First of Georgia Insurance Company, P O. Box 905, Augusta, Georgia 30903
Nov 19, 1926	KELTON, WILLIAM H. (Retired), 122 Arundel Avenue, West Hartford, Conn. 06107
Nov. 19, 1959	KLAASSEN, ELDON J., Associate Actuary, Continental National American Group, 310 S. Michigan Avenue, Chicago, Ill. 60604
Nov 14, 1941	KOLE, MORRIS B . Director of Accounts and Finance, The State Insurance Fund, 199 Church Street, New York, N. Y. 10007
Nov 24, 1933	KORMES, MARK, President, Actuarial Associates, Inc., 415 Lexington Avenue, New York, N Y 10017
Nov. 19, 1953	KUENKLER, ARTHUR S , Consultant, 942 Orange Center Road, Orange Conn. 06477
Nov. 18, 1949	LACROIX, HAROLD F., Vice President, The Travelers Insurance Companies, One Tower Square, Hartford, Conn 06115
Nov 20, 1964	LANGE, JEFFREY T , Secretary, Insurance Rating Board, 125 Maiden Lane, New York, N. Y 10038
May 5, 1961	LATIMER, MURRAY W , Industrial Relations Consultants, 1625 K Street, N. W., Washington, D. C. 20006
Nov 17, 1950	LESLIE, WILLIAM, JR., Vice President and Actuary, Continental Insurance Companies, 80 Maiden Lane, New York, N. Y. 10038
Nov. 16, 1961	LINDEN, JOHN R., Assistant Actuary, Aetna Life & Casualty, Hartford, Conn. 06115
Nov. 20, 1924	LINDER, JOSEPH, Consulting Actuary, 200 Park Avenue, New York, N. Y 10017
Nov. 16, 1956	LINO, RICHARD, Actuary, Insurance Rating Board, 125 Maiden Lane, New York, N. Y. 10038
Nov. 18, 1955	LISCORD, PAUL S , Vice President and Actuary, The Travelers Insurance Companies, One Tower Square, Hartford, Conn. 06115
Nov. 17, 1950	LIVINGSTON, GILBERT R., Casualty Actuary, Connecticut Insurance Department, State Office Bldg., Hartford, Conn. 06115
Nov. 16, 1951	LONGLEY-COOK, LAURENCE H , Consultant, Special Lecturer and Research Consultant, Department of Insurance, Georgia State College, 33 Gilmer Street S. E., Atlanta, Georgia 30303
Nov 13, 1936	LYONS, DANIEL J., President, Guardian Life Insurance Company of America, 201 Park Avenue South, New York, N. Y 10003
Nov. 1, 1963	MACGINNITIE, W JAMES, Director, Corporate Planning and Development, Continental National American Group, 310 South Michigan Avenue, Chicago, Ill. 60604
Nov. 19, 1954	MACKEEN, HAROLD E , Assistant Actuary, The Travelers Insurance Companies, One Tower Square, Hartford, Conn. 06115
Nov 18, 1958	MAGRATH, JOSEPH J. (Retired), 47 Woodland Avenue, Summit, N. J. 07901
Nov. 22, 1957	MAKGILL, STEPHEN S , Associate Actuary, The Travelers Insurance Companies, One Tower Square, Hartford, Conn 06115
Nov 23, 1928	MARSHALL, RALPH M. (Retired), Catts Corner, Worton, Kent County, Md 21678
Nov. 18, 1927	MASTERSON, NORTON E. (Retired), Consulting Actuary, 1516 Clark Street, Stevens Point, Wis. 54481
Nov. 19, 1926	MATTHEWS, ARTHUR N. (Retired), 475 Poquonock Avenue, Windsor, Conn 06095
May 19, 1915	MAYCRINK, EMMA C (Retired), 32 Chittenden Avenue, Crestwood, N Y 10707
Nov. 14, 1958	MAYERSON, ALLEN L , Professor of Insurance and Actuarial Mathematics, The University of Michigan, Graduate School of Business Administration, Ann Arbor, Mich

Admitted	
Nov. 1, 1963	McClure, Richard D, Assistant Actuary, Lumbermens Mutual Casualty Company, 4750 N. Sheridan Road, Chicago, Ill. 60640
Nov. 15, 1935	McConnell, Matthew H. Superintendent, Compensation & Liability Dept., General Accident Fire and Life Assurance Corporation Ltd., 414 Walnut Street, Philadelphia, Pa 19106
Nov. 18, 1960	McGuinness, John S., President, John S McGuinness Associates, Consultants in Actuarial Science and Management, 15 Kevin Rd., Scotch Plains, New Jersey 07076
Nov. 20, 1964	McLean, George E, Actuary, Massachusetts Blue Cross, Inc., Massachusetts Medical Service, 133 Federal Street, Boston, Mass 02106
Nov. 15, 1962	McNamara, Daniel J, Assistant General Manager, Insurance Rating Board, 125 Maiden Lane, New York, N Y. 10038
Nov 15, 1962	Meenaghan, James J., Actuary, Fireman's Fund American Insurance Companies, 3333 California Street, San Francisco, Calif. 94120
Nov. 18, 1955	Menzel, Henry W., General Manager & Actuary, New York Compensation Insurance Rating Board, 200 East 42nd Street, New York, N. Y. 10017
†	Michelbacher, Gustav F (Retired), 15201 Quito Road, Saratoga, Calif. 95070
Nov 17, 1938	Miller, John H., Actuarial Consultant, Monarch Life Insurance Company, 1250 State Street, Springfield, Mass 01101
Nov 1, 1963	Miller, Nicholas F., Jr., Assistant Secretary, Executive Dept, Aetna Life & Casualty, Hartford, Conn 06115
Nov. 18, 1937	Mills, John A. (Retired), Point Placid, Reeds Spring, Mo. 65737
Nov 22, 1957	Mills, Richard J, Assistant Actuary, Lumbermens Mutual Casualty Company, 4750 N Sheridan Road, Chicago, Ill 60640
Nov 13, 1967	Mohnblatt, Arnold S, Assistant Actuary, National Insurance Actuarial and Statistical Association, 110 William Street, New York, N Y 10038
Nov 15, 1962	Morison, George D., Associate Actuary, Aetna Life & Casualty, Hartford, Conn. 06115
Nov. 16, 1961	Moseley, Jack, Assistant Vice President and Associate Actuary, United States Fidelity and Guaranty Company, Calvert and Redwood Streets, Baltimore, Md. 21203
Nov 17, 1920	Mueller, Louis H., 2845 Lake Street, San Francisco, Calif. 94121
Nov. 16, 1956	Muetterties, John H., Associate Actuary, Sentry Insurance-Hardware Mutuals Group, 1421 Strongs Avenue, Stevens Point, Wis. 54481
Nov. 17, 1950	Munterich, George C, Assistant Secretary, The Hartford Insurance Group, Hartford Plaza, Hartford, Conn. 06115
Nov 19, 1954	Murrin, Thomas E, Vice President and Actuary, Fireman's Fund American Insurance Companies, 3333 California Street, San Francisco, Calif 94120
Nov 19, 1959	Myers, Robert J, Chief Actuary, Social Security Administration, Department of Health, Education, and Welfare, Washington, D. C. 20201
Nov. 16, 1965	Nelson, Dale A, Actuary, State Farm Mutual Automobile Insurance Company, 112 East Washington Street, Bloomington, Ill. 61701
Nov. 1, 1963	Nelson, S Tyler, Consulting Actuary, 542 Hillside Avenue, Glen Ellyn, Ill 60137
Nov 13, 1967	Newman, Steven H, Assistant Secretary, Insurance Rating Board, 125 Maiden Lane, New York, N Y 10038
Nov. 14, 1958	Niles, Charles L., Jr. Deputy General Manager and Actuary, General Accident Group, 414 Walnut Street, Philadelphia, Pa. 19105
Nov 15, 1935	Oberhaus, Thomas M, Vice President, Woodward and Fondiller, Inc., 730 Fifth Avenue, New York, N Y 10019
Nov. 16, 1965	Oien, R. Gustave, Research Actuary, St. Paul Insurance Companies, 385 Washington Street, St Paul, Minn. 55102

Admitted	
Nov 22, 1957	OTTESON, PAUL M., Vice President and Actuary, Federated Mutual Implement and Hardware Insurance Company and Federated Life Insurance Company, 129 East Broadway, Owatonna, Minn. 55060
Nov. 21, 1919	OUTWATER, OLIVE E (Retired), 2404 Loring Street, San Diego, Calif. 92109
Nov. 15, 1962	PARLIN, R W , Research Associate, University of Minnesota, Laboratory of Physiological Hygiene, Stadium Gate 27, Minneapolis, Minn. 55455
Nov. 18, 1960	PENNYCOOK, ROD B., Health Insurance Secretary, The Great-West Life Assurance Company, 60 Osborne Street, N., Winnipeg 9, Manitoba, Canada
Nov. 22, 1957	PERKINS, WILLIAM J , Assistant Group Actuary, London Life Insurance Company, 255 Dufferin Avenue, London, Ontario, Canada
Nov. 14, 1941	PETERS, STEFAN, Consultant, Arthur D. Little, Inc., 35 Acorn Park, Cambridge, Mass 02140
Nov. 21, 1952	PETZ, EARL F., Associate Actuary, Lumbermens Mutual Casualty Company, 4750 N Sheridan Road, Chicago, Ill. 60640
Nov. 19, 1959	PHILLIPS, HERBERT J , JR., Associate Actuary, The Employers' Group of Insurance Companies, 110 Milk Street, Boston, Mass. 02107
Nov. 24, 1933	PICKETT, SAMUEL C. (Retired), Connecticut Rating Supervisor, State of Connecticut, Hartford, Conn 06115
Nov. 22, 1957	PINNEY, ALLEN D , Associate Actuary, The Travelers Insurance Companies, One Tower Square, Hartford, Conn 06115
Nov 17, 1922	PINNEY, SYDNEY D. (Retired), 290 Wolcott Hill Road, Wethersfield, Conn. 06109
Nov. 19, 1959	POLLACK, ROBERT, Vice President and Actuary, American Maturity Insurance Company, Colonial Penn Building, Philadelphia, Pa. 19102
Nov. 16, 1965	PORTERMAIN, NEILL W., Assistant Actuary, American Mutual Insurance Companies, Quannapowitt Parkway, Wakefield, Mass. 01880
Nov. 13, 1967	PRESLEY, PHILIP O., Assistant Actuary, American Mutual Insurance Companies, Quannapowitt Parkway, Wakefield, Mass. 01880
Nov. 18, 1955	RESONY, ALLIE V., Assistant Secretary, The Hartford Insurance Group, Hartford Plaza, Hartford, Conn 06115
Nov. 18, 1949	RESONY, JOHN A., Second Vice President and Actuary, The Travelers Insurance Companies, One Tower Square, Hartford, Conn 06115
Nov. 18, 1966	RICCARDO, JOSEPH F , JR , Assistant Secretary, Aetna Life & Casualty, Hartford, Conn. 06115
Nov. 1, 1963	RICHARDS, HARRY R., Associate Actuary, The Travelers Insurance Companies, One Tower Square, Hartford, Conn 06115
Nov. 1, 1963	RIDDLESWORTH, WILLIAM A., Assistant Actuary, Aetna Life & Casualty, Hartford, Conn 06115
May 24, 1921	RIEGEL, ROBERT, Professor Emeritus of Statistics and Insurance, State University of New York at Buffalo, 4244 Ridge Lea Road, Amherst, N. Y. 14226
Nov. 14, 1958	ROBERTS, LEWIS H , Vice President and Actuary, Woodward and Fondiller, Inc., 730 Fifth Avenue, New York, N. Y. 10019
Nov. 14, 1947	RODERMUND, MATTHEW, Vice President-Actuary, Munich Reinsurance Company, 410 Park Avenue, New York, N. Y. 10022
Nov 14, 1947	ROSENBERG, NORMAN, Assistant Vice President-Actuary, Farmers Insurance Group, 4680 Wilshire Boulevard, Los Angeles, Calif 90054
Nov. 18, 1966	ROTH, RICHARD J., Actuary, American International Underwriters Corporation, 102 Maiden Lane, New York, N. Y. 10005
Nov. 14, 1947	ROWELL, JOHN H., Vice President, Marsh & McLennan, Inc., 231 South LaSalle Street, Chicago, Ill. 60604
Nov. 17, 1938	RUCHLIS, ELSIE, Assistant Actuary, Insurance Rating Board, 125 Maiden Lane, New York, N. Y. 10038

Admitted	
Nov 14, 1947	SALZMANN, RUTH E., Vice President and Actuary, Sentry Insurance Group, 1421 Strongs Avenue, Stevens Point, Wisconsin 54481
Nov 1, 1963	SARASON, HARRY M, Editor, BICAT, 1246 (A) Chelsea, Santa Monica, California 90404
Nov 18, 1966	SCHEIBL, JEROME A, Assistant Actuary, Employers Insurance of Wausau, 2000 Westwood Drive, Wausau, Wisconsin 54401
Nov 19, 1948	SCHLOSS, HAROLD W., Vice President and Actuary, Royal-Globe Insurance Companies, 150 William Street, New York, N Y 10038
Nov 13, 1967	SCHULER, ROBERT J, Assistant Actuary, Blue Cross of Western Pennsylvania, One Smithfield Street, Pittsburgh, Pa. 15220
Nov. 18, 1966	SCOTT, BRIAN E., Assistant Actuary, Aetna Life & Casualty, Hartford, Conn 06115
Nov 18, 1937	SHAPIRO, GEORGE I., 934 East 9 Street, Brooklyn, N Y. 11230
Nov. 13, 1931	SILVERMAN, DAVID, Consulting Actuary, Peat, Marwick, Mitchell & Company, 70 Pine Street, New York, N. Y 10005
Nov. 19, 1954	SIMON, LEROY J., General Manager, National Insurance Actuarial and Statistical Association, 110 William Street, New York, N. Y 10038
Nov 18, 1960	SIMONEAU, PAUL W., Associate Actuary, Aetna Life & Casualty, Hartford, Conn 06115
Nov 19, 1929	SKELDING, ALBERT Z, Secretary-Treasurer, Casualty Actuarial Society, 200 E. 42nd Street, New York, N. Y 10017
Nov 19, 1929	SKILLINGS, E. SHAW (Retired), 3036 Central Street, Evanston, Ill. 60201
Nov. 18, 1932	SMICK, J. J, Consulting Actuary, Smick & Co., Inc., 300 E. 46th Street, New York, N. Y. 10017
Nov. 14, 1958	SMITH, EDWARD M., Associate Actuary, The Travelers Insurance Companies, One Tower Square, Hartford, Conn. 06115
Nov 18, 1966	SMITH, EDWARD R., Associate Actuary, The Hartford Insurance Group, Hartford Plaza, Hartford, Conn. 06115
Nov. 15, 1940	SMITH, SEYMOUR E., Senior Vice President, The Travelers Insurance Companies, One Tower Square, Hartford, Conn 06115
Nov. 15, 1962	STANKUS, LEO M., Director of Executive Information, Allstate Insurance Company, Allstate Plaza, Northbrook, Illinois 60062
Nov. 24, 1933	ST. JOHN, JOHN B, Consulting Actuary, Box 57, Penllyn, Pa. 19458
Nov 18, 1966	SWITZER, VERNON J, Health Actuary, State Farm Mutual Automobile Insurance Co., 112 E. Washington Street, Bloomington, Ill. 61701
May 25, 1956	TAPLEY, DAVID A., President, Transamerica Insurance Company, Occidental Center, Suite 2100, 1150 S. Olive Street, Los Angeles, Calif. 90015
Nov 14, 1958	TARBELL, LUTHER L., JR., Actuary, The Travelers Insurance Companies, One Tower Square, Hartford, Conn. 06115
Nov. 16, 1956	THOMAS, JAMES W., Assistant Actuary, The Travelers Insurance Companies, One Tower Square, Hartford, Conn. 06115
†	THOMPSON, JOHN S., Newark Athletic Club, Newark, N J 07102
Nov. 19, 1953	TRIST, JOHN A. W., Associate Actuary, Insurance Company of North America, 1600 Arch Street, Philadelphia, Pa. 19101
Nov. 15, 1962	TRUDEAU, DONALD E., Consultant and Actuary, Consolidated Underwriters, 1907 Grand Avenue, Kansas City, Mo. 64108
Nov. 14, 1947	UHTHOFF, DUNBAR R., Vice President and Actuary, Employers Insurance of Wausau, 2000 Westwood Drive, Wausau, Wis. 54401
Nov. 23, 1928	VALERIUS, NELS M., Associate Actuary, Aetna Life & Casualty, Hartford, Conn. 06115
Nov. 21, 1919	VAN TUYL, HIRAM O. (Retired), 125 56th Avenue, South, St. Petersburg, Florida 33705
Nov 16, 1965	VERHAGE, PAUL A., Assistant Actuary, Sentry Insurance-Hardware Mutuals Group, 1421 Strongs Avenue, Stevens Point, Wis 54481

Admitted	
Nov 16, 1951	VINCENT, LEWIS A , Vice President, The Continental Insurance Companies, 80 Maiden Lane, New York, N. Y. 10038
Nov. 17, 1920	WAITE, ALAN W., 16 Penwood Road, Bloomfield, Conn 06002
Nov. 19, 1962	WALSH, ALBERT J., Vice President, Reliance Insurance Company, 4 Penn Center Plaza, Philadelphia, Pa. 19103
Nov. 16, 1965	WEBB, BERNARD L., Assistant Professor of Actuarial Science and Insurance, Georgia State College, 33 Gilmer Street, S E , Atlanta, Ga. 30303
Nov 14, 1947	WIEDER, JOHN W., JR., Actuary, Aetna Life & Casualty, Hartford, Conn 06115
Nov. 18, 1960	WILCKEN, CARL L., Actuary, Canadian Underwriters' Association, 31 Prince Andrew Place, Don Mills, Ontario, Canada
Nov. 1, 1963	WILLIAMS, DEWEY G., Vice President, Actuary, Texas Employers' Insurance Association, Employers Casualty Company, 423 So. Akard Street, P. O. Box 2759, Dallas, Tex 75221
Nov 15, 1935	WILLIAMS, HARRY V , Chairman of the Boards and President, The Hartford Insurance Group, Hartford Plaza, Hartford, Conn. 06115
Nov 22, 1957	WILLIAMS, P ADGER, Vice President, The Travelers Insurance Companies, One Tower Square, Hartford, Conn. 06115
Nov. 14, 1941	WILLIAMSON, W. RULON, Research Actuary, 3400 Fairhill Drive, S. E., Washington, D. C. 20023
Nov. 18, 1960	WILLSEY, LYNN W., Actuary, The Travelers Insurance Companies, One Tower Square, Hartford, Conn. 06115
Nov. 16, 1961	WILSON, JAMES C., Vice President and Actuary, Security General Insurance Company, Winston-Salem, N. C. 27102
Nov 13, 1931	WITTICK, HERBERT E., Vice President and General Manager, Pilot Insurance Company, 1315 Yonge Street, Toronto 7, Ontario, Canada
Nov. 16, 1951	WOODALL, JOHN P., Manager, South-Eastern Underwriters Association, P O. Box 5048, Atlanta, Ga. 30302
Nov 14, 1958	WRIGHT, BYRON, Actuary, Department of Banking and Insurance, State of New Jersey, State House Annex, Trenton, New Jersey 08625
Nov. 19, 1953	YOUNT, HUBERT W. (Retired), Box 489, Amherst, Mass. 01002

ASSOCIATES OF THE SOCIETY

NOVEMBER 13, 1967

Admitted	
Nov. 15, 1918	ACKERMAN, SAUL B , S. B. Ackerman Associates, 405 Lexington Avenue, New York, N. Y. 10017
Nov 16, 1965	ADLER, MARTIN, Associate Actuary, Woodward and Fondiller, Inc. 730 Fifth Avenue, New York, N Y. 10019
Nov. 16, 1939	AIN, SAMUEL N , Consulting Actuary, 120 Broadway, New York, N. Y 10005
Apr. 5, 1928	ALLEN, AUSTIN F. (Retired), 4815 Royal Lane, Dallas, Texas 75229
Nov. 15, 1962	AMLIE, WILLIAM P., Assistant Actuary, The Employers' Group of Insurance Companies, 110 Milk Street, Boston, Mass. 02107
Nov. 18, 1955	ANDREWS, EDWARD C , Associate Actuary, The Travelers Insurance Companies, One Tower Square, Hartford, Conn. 06115
Nov. 21, 1930	ARCHIBALD, A EDWARD, c/o George V. Stennes & Associates, 2112 First National Bank Bldg., Minneapolis, Minn 55402
Nov 19, 1959	BANNISTER, DAN W., President, Horace Mann Insurance Group, 216 East Monroe Street, Springfield, Ill. 62704
Nov 23, 1928	BATEMAN, ARTHUR E., Pine Grove Rest Home, Marlboro, Mass 01752
Nov. 15, 1940	BATHO, BRUCE W , Executive Vice President-Administration, Life Insurance Company of Georgia, 573 West Peachtree Street, N. E , Atlanta, Georgia 30308
Nov 16, 1965	BELL, ALLAN A , Data Processing Development Division, Aetna Life & Casualty, Hartford, Conn 06115
Nov. 18, 1966	BEN-ZVI, PHILLIP N , Actuarial Assistant, Royal-Globe Insurance Companies, 150 William Street, New York, N. Y 10038
Nov 16, 1956	BERG, ROY A., JR., Assistant Actuary, Old Republic Life Insurance Company, 307 North Michigan Avenue, Chicago, Ill. 60646
Nov 14, 1958	BERNAT, LEO A., Executive Director, Minnesota Research Associates, 503 15th Avenue, S E., No. 2, Minneapolis, Minn. 55414
Nov. 18, 1966	BICKERSTAFF, DAVID R., Actuary, Southern Farm Bureau Casualty Insurance Company, 515 E Amite St , P O. Box 78, Jackson, Mississippi 39205
Nov 18, 1925	BITTEL, W HAROLD, Chief Actuary, Department of Banking and Insurance, State of New Jersey, Trenton, N. J 08625
Nov. 16, 1965	BLAND, WILLIAM H., Actuarial Assistant, The Travelers Insurance Companies, One Tower Square, Hartford, Conn. 06115
Nov. 14, 1958	BLUMENFELD, M. EUGENE, Assistant Actuary, Bankers Life and Casualty Company, 4444 W Lawrence Avenue, Chicago, Ill 60630
Nov. 22, 1934	BOMSE, EDWARD L., Manager—Liability, Comm'l Lines-Research & Development, Royal-Globe Insurance Companies, 150 William Street, New York, N Y 10038
Nov. 22, 1957	BRAGG, JOHN M., Vice President and Chief Actuary, Life Insurance Company of Georgia, 573 W Peachtree Street, N. E., Atlanta, Ga. 30308
Nov 16, 1965	BRIAN, ROBERT A., Actuary, Connecticut Insurance Department, State Office Building, Hartford, Conn. 06115
Nov 20, 1964	BROWN, WILLIAM W , JR., Actuarial Assistant, Liberty Mutual Insurance Company, 175 Berkeley Street, Boston, Mass. 02117
Nov. 15, 1962	BUFFINTON, PHILIP G., Vice President, State Farm Fire and Casualty Company, 112 East Washington Street, Bloomington, Ill. 61701
Nov. 20, 1924	BUGBEE, JAMES M. (Retired), 115 Hawthorn Road, Baltimore, Md. 21210
Mar. 31, 1920	BURT, MARGARET A , Office of George B. Buck, Consulting Actuary, 60 Worth Street, New York, N. Y. 10013
Nov 19, 1959	BUTLER, RICHARD H., Second Vice President, The Travelers Insurance Companies, One Tower Square, Hartford, Conn. 06115
Nov. 15, 1962	CARSON, DAVID E. A , Vice President and Actuary, The Hartford Insurance Group, Hartford Plaza, Hartford, Conn. 06115
Nov. 13, 1967	CARTER, EDWARD J., JR , Actuarial Analyst, United Services Automobile Association, USAA Building, 4119 Broadway, San Antonio, Tex. 78215

Admitted	
Nov 18, 1927	CHEN, S. T., Consulting Actuary, The Wing On Life Assurance Company Ltd., Wing On Life Bldg., 22 Des Voeux Road, Central, Hong Kong
Nov. 16, 1961	CHERLIN, GEORGE, Vice President and Actuary, National Health and Welfare Retirement Association, Inc., 800 Second Avenue, New York, N. Y. 10017
Nov. 13, 1967	CHORPITA, FRED M., Actuarial Assistant, National Council on Compensation Insurance, 200 East 42nd Street, New York, N. Y 10017
Nov 22, 1957	CHURCH, HARRY M., Consulting Actuary, Coates, Herfurth & England, 301 E. Colorado Avenue, Pasadena, Calif. 91101
Nov. 18, 1955	COATES, WILLIAM D., Vice President, National-Ben Franklin Life Insurance Corp., 360 W. Jackson Blvd., Chicago, Ill. 60606
Nov 18, 1966	CONNER, JAMES B., Actuarial Assistant, The Travelers Insurance Companies, One Tower Square, Hartford, Conn. 06115
Nov. 19, 1953	CONTE, JOSEPH P , Assistant to the President, Berman's Motor Express, P. O. Box 1209, Binghamton, N. Y. 13902
Nov 19, 1959	COPESTAKES, A. D., Assistant Vice President-Reports, American Mutual Insurance Companies, Quannapowitt Parkway, Wakefield, Mass. 01880
Nov. 24, 1933	CRAWFORD, WILLIAM H., Financial Consultant, Industrial Indemnity Company, 155 Sansome Street, San Francisco, Calif. 94104
Nov 19, 1953	CROFTS, GEOFFREY, Dean and Director, Graduate School of Actuarial Science, Northeastern University, 360 Huntington Avenue, Boston, Mass. 02115
Nov. 21, 1952	DANIEL, C. M., Data Processing Manager, Fisher Governor Company, Marshalltown, Iowa 50158
Nov. 13, 1967	DAVIS, REX C., Associate Actuary, Allstate Insurance Company, Allstate Plaza, Northbrook, Ill. 60062
Nov. 16, 1965	DICKSON, CAROL D. (Mrs.), 34 Brookline Drive, West Hartford, Conn. 06107
Nov. 14, 1941	DOWLING, WILLIAM F. (Retired), 77 Brook Street, Garden City, New York 11530
Nov. 1, 1963	DURKIN, JAMES H., Consulting Actuary, Peat, Marwick, Mitchell & Company, 70 Pine Street, New York, N. Y. 10005
Nov. 14, 1958	DUROSE, STANLEY C., JR., Deputy Commissioner, Office of the Commissioner of Insurance, 4802 Sheboygan Avenue, Madison, Wis. 53702
Nov. 19, 1954	EATON, KARL F., Vice President and Controller, National Fidelity Life Insurance Company, 1012 Walnut, Kansas City, Mo. 64106
June 5, 1925	EGER, FRANK A. (Retired), 1119 Prospect Ridge Blvd., Haddon Heights, N. J. 08035
Nov. 18, 1966	FABER, JAMES A., Senior Actuarial Assistant, Insurance Company of North America, 1600 Arch Street, Philadelphia, Pa 19101
Nov. 18, 1966	FARNAM, WALTER E., Actuarial Department, Aetna Life & Casualty, Hartford, Conn. 06115
Nov. 22, 1957	FELDMAN, MARTIN F., Associate Actuary, New York State Insurance Department, 123 William Street, New York, N Y. 10038
Nov. 16, 1961	FERDEN, STEIN, Undelstadlia 8, Asker, Norway
Nov. 13, 1967	FERRARI, J. ROBERT, Assistant Professor of Insurance, University of Pennsylvania, Wharton School of Finance & Commerce, Philadelphia, Pa. 19104
Nov 15, 1962	FINKEL, DANIEL, Associate Actuary, The State Insurance Fund, 199 Church Street, New York, N. Y 10007
Nov. 16, 1956	FLACK, PAUL R , Actuarial Assistant, General Accident Group, 414 Walnut Street, Philadelphia, Pa. 19105
Nov. 16, 1923	FLEMING, FRANK A (Retired), c/o Mutual Insurance Rating Bureau, 733 Third Avenue, New York, N Y. 10017
Nov 13, 1967	FLYNN, DAVID P., Fireman's Fund American Insurance Companies, 3333 California Street, San Francisco, Calif. 94120

Admitted	
Nov. 21, 1952	FRANKLIN, N MATTHEW, Actuary, National Insurance Actuarial and Statistical Association, 110 William Street, New York, N. Y 10038
Nov. 18, 1966	FULTON, CLYDE B., JR , Actuarial Assistant, The Travelers Insurance Companies, One Tower Square, Hartford, Conn 06115
Nov. 19, 1954	GAINES, NATHANIEL, Associate Actuary, Office of George B Buck, Consulting Actuary, 2 Pennsylvania Plaza, New York, N. Y 10001
Nov 15, 1962	GERUNDO, LOUIS P., JR., Actuarial Assistant, The Travelers Insurance Companies, One Tower Square, Hartford, Conn. 06115
Nov. 18, 1932	GETMAN, RICHARD A , Assistant Actuary, Life Dept., The Travelers Insurance Companies, One Tower Square, Hartford, Conn. 06115
Nov. 17, 1922	GIBSON, JOSEPH P., JR., (Retired), 2970 Lorain Road, San Marino, Calif. 91108
Nov. 16, 1923	GILDEA. JAMES F (Retired), 236 Nott Street. Wethersfield, Conn. 06109
Nov. 1, 1963	GILL, JAMES F., Vice President and Actuary, National Association of Independent Insurers, 30 West Monroe St., Chicago, Ill. 60603
Nov 14, 1947	GINGERY, STANLEY W., Vice President and Associate Actuary, The Prudential Insurance Company of America, Prudential Plaza, Newark, N. J 07101
Nov. 19, 1959	GOLD, MELVIN L., Consulting Actuary, Gold Associates, 39 South Fullerton Avenue, Montclair, N. J. 07042
Nov. 13, 1967	GOSSROW, ROBERT W., Rate Analyst, Allstate Insurance Company, Allstate Plaza, Northbrook, Ill 60062
Nov 16, 1961	GOULD, DONALD E , Assistant Manager-Research, Insurance Rating Board, 125 Maiden Lane, New York, N. Y. 10038
Nov. 13, 1967	GOWDY, ROBERT C., Administrative Assistant, Planning Department, Industrial Indemnity Company, 155 Sansome Street, San Francisco, Calif. 94101
Nov. 18, 1927	GREEN, WALTER C. (Retired), 923 South 23 Street East, Salt Lake City, Utah 84108
Nov. 16, 1961	GREENE, THOMAS A., Vice President, General Reinsurance Corporation, 175 West Jackson Boulevard, Chicago, Ill. 60604
Nov. 15, 1940	GROSSMAN, ELI A., Senior Vice President, The Great Eastern Life Insurance Company, 10 Dorrance Street, Providence, R. I. 02903
Nov. 15, 1935	GUERTIN, ALFRED N., Actuarial Consultant, 2 Pennsylvania Plaza, New York, N Y. 10001
Nov. 16, 1965	HACHEMEISTER, CHARLES A , Director, Actuarial Research, Insurance Company of North America, 1600 Arch Street, Philadelphia, Pa 19101
Nov 16, 1939	HAGEN, OLAF E., Senior Assistant Actuarial Supervisor, Metropolitan Life Insurance Company, One Madison Avenue, New York, N. Y 10010
Nov. 17, 1922	HALL, HARTWELL L. (Retired), 34 Lincoln Avenue, West Hartford, Conn 06117
Nov. 13, 1936	HAM, HUGH P (Retired), Apt. 901 "A", 1141 Royal York Road, Islington, Toronto, Ontario, Canada
Nov. 1, 1963	HAMMER, SIDNEY M , Assistant Actuary, The Home Insurance Company, 59 Maiden Lane, New York, N. Y. 10008
Nov. 16, 1965	HANSON, H DONALD, Assistant Actuary, Continental National American Group, 310 S. Michigan Avenue, Chicago, Ill. 60604
Nov 19, 1953	HARACK, JOHN, Vice President-Actuary, Health Service, Inc. and Medical Indemnity of America, Inc., 200 N. Michigan Avenue, Chicago, Ill. 60601
Mar. 24, 1932	HARRIS, SCOTT, Executive Vice President, Joseph Froggatt & Company, Inc., 74 Trinity Place, New York, N. Y. 10006
Mar. 25, 1924	HART, WARD VAN B., 49 Robbins Drive, Wethersfield, Conn 06109
Nov. 21, 1919	HAYDON, GEORGE F., General Manager Emeritus, Wisconsin Compensation Rating Bureau, 623 N. 2nd Street, Milwaukee, Wis. 53203

Admitted	
Nov. 19, 1953	HEAD, GLENN O., President, First Investors Life Insurance Company, 120 Wall Street, New York, N. Y. 10005
Nov. 13, 1967	HEER, E. LEROY, Actuarial Department, Aetna Life & Casualty, Hartford, Conn. 06115
Nov. 19, 1959	HICKMAN, JAMES C, Professor, Department of Statistics, University of Iowa, Iowa City, Iowa 52240
Nov. 18, 1966	HOLT, WILLIAM T, Actuarial Assistant, The Travelers Insurance Companies, One Tower Square, Hartford, Conn 06115
Nov. 16, 1965	HONEBEIN, CARLTON W., Assistant Actuary, Fireman's Fund American Insurance Companies, 3333 California Street, San Francisco, Calif. 94120
Nov. 16, 1961	HOROWITZ, MILTON, Principal Actuary, The State Insurance Fund, 199 Church Street, New York, N Y 10007
Nov. 13, 1967	HUNTER, JOHN R., JR, Assistant Actuary, Mutual Insurance Advisory Association and Mutual Insurance Rating Bureau, 733 Third Avenue, New York, N. Y 10017
Nov 19, 1929	JACOBS, CARL N., Honorary Chairman of the Boards, Hardware Mutual Casualty Company, Hardware Dealers Mutual Fire Insurance Company and Sentry Life Insurance Company, 1421 Strongs Avenue, Stevens Point, Wis 54481
Nov. 13, 1967	JACOBS, TERRY S, Actuarial Assistant, U. S. Fidelity and Guaranty Co., Calvert and Redwood Streets, Baltimore, Md. 21203
Nov. 15, 1962	JENSEN, JAMES P, Actuarial Assistant, Liberty Mutual Insurance Company, 175 Berkeley Street, Boston, Mass. 02117
Nov. 13, 1967	JONES, ALAN G, Actuarial Assistant, Liberty Mutual Insurance Company, 175 Berkeley Street, Boston, Mass. 02117
Nov. 21, 1919	JONES, LORING D (Retired), 64 Raymond Avenue, Rockville Centre, N Y. 11570
Nov. 21, 1952	JONES, NATHAN F., Associate Actuary, Prudential Insurance Company, Prudential Plaza, 745 Broad Street, Newark, N. J 07101
Nov. 13, 1967	KAUR, ALAN F., Actuarial Assistant, Continental Casualty Company, 310 S. Michigan Boulevard, Chicago, Ill. 60604
Nov. 18, 1966	KILBOURNE, FREDERICK W., Consulting Actuary, Milliman & Robertson, Inc, 16 North Marengo, Pasadena, Calif. 91101
Nov. 15, 1935	KITZROW, ERWIN W (Retired), P. O Box 313, Pasadena, Calif. 91102
Nov 19, 1959	KROEKER, JOHN, Actuary, James E. Coughlin and Associates, Ltd., 904 Lady Ellen Place, Ottawa 3, Ontario, Canada
Nov 19, 1959	LEIGHT, ARTHUR S., Assistant Actuary, Guardian Life Insurance Co., 201 Park Avenue South, New York, N Y 10003
Nov. 18, 1966	LOWE, ROBERT F, Assistant Actuary, Fireman's Fund American Insurance Companies, 3333 California Street, San Francisco, Calif 94120
Nov 14, 1947	LUFKIN, ROBERT W., Administration, The Hanover Life Insurance Company, 851 Boylston Street, Boston, Mass 02116
Nov 18, 1925	MALMUTH, JACOB, Chief-Rating Bureau, N. Y Insurance Department, 123 William Street, New York, N. Y. 10038
Nov. 16, 1961	MARGOLIS, DONALD R., Assistant Actuary, Life Insurance Company of North America, 1600 Arch Street, Philadelphia, Pa. 19101
Nov. 20, 1964	MARKELL, ANDREW S., Consulting Actuary, Bowles, Andrews & Towne, Inc, 1389 Peachtree Street, N. E., Atlanta, Ga. 30309
Nov 16, 1956	MATHWICK, LLOYD F, Manager, Group Division, Employers Insurance of Wausau, 2000 Westwood Drive, Wausau, Wis 54401
Nov. 13, 1936	MAYER, WILLIAM H., JR., Manager, Group Contract Bureau, Metropolitan Life Insurance Company, One Madison Avenue, New York, N. Y. 10010
Nov. 13, 1967	McDONALD, CHARLES, Associate Actuary, Texas Employers' Insurance Association, P O Box 2759, Dallas, Tex. 75221
May 26, 1955	McDONALD, MILTON G., Chief Actuary, Massachusetts Insurance Department, 100 Cambridge Street, Boston, Mass. 02202

Admitted	
Nov. 16, 1961	McINTOSH, KENNETH L., Property and Casualty Actuary, Arkansas Insurance Department, University Towers, Little Rock, Ark. 72204
Nov 13, 1931	MILLER, HENRY C (Retired), 35 Lower Crescent, Sausalito, Calif. 94965
Nov 18, 1937	MINOR, EDUARD H., Associate Actuary, Metropolitan Life Insurance Company, One Madison Avenue, New York, N Y. 10010
Nov. 20, 1964	MOKROS, BERTRAM F., Underwriting Research Manager, Allstate Insurance Company, 321 Middlefield Road, Menlo Park, Calif. 94025
Nov. 17, 1922	MONTGOMERY, JOHN C. (Retired), 165 Westervelt Avenue, Tenafly, N. J. 07670
May 25, 1923	MOORE, JOSEPH P., 115 St Catherine Road, Outremont, Quebec, Canada
Nov 16, 1961	MOSS, ROBERT G., Vice President and Actuary, Marsh & McLennan, Inc., 515 Olive Street, St. Louis, Mo. 63101
Nov 22, 1957	MUIR, JOSEPH M., General Manager, Mutual Insurance Advisory Association and Mutual Insurance Rating Bureau, 733 Third Avenue, New York, N. Y. 10017
Nov. 1, 1963	MUNIZ, ROBERT M., 19 Muller Street, Norwalk, Conn. 06851
Nov 18, 1966	MUNRO, RICHARD E., Senior Actuarial Assistant, Insurance Company of North America, 1600 Arch Street, Philadelphia, Pa. 19101
Nov. 18, 1966	MURRAY, EDWARD R., Actuarial Assistant, Royal-Globe Insurance Companies, 150 William Street, New York, N. Y. 10038
Nov. 18, 1966	MURRAY, JAMES B. M., Casualty Superintendent, Prudential Assurance Co., Ltd of England, 635 Dorchester Boulevard West, Montreal, Quebec, Canada
Nov 16, 1965	NAFFZIGER, JOSEPH V., Actuary, State Farm Mutual Automobile Insurance Company, 112 E. Washington Street, Bloomington, Ill 61701
Nov. 16, 1961	NELSON, ROLAND E., Actuary, State Farm Life and Accident Assurance Company, 112 East Washington Street, Bloomington, Ill. 61701
Oct 27, 1916	NEWELL, WILLIAM (Retired), 1225 Park Avenue, New York, N. Y 10028
Nov. 18, 1925	NICHOLSON, EARL H., Actuary and Deputy Insurance Commissioner, Nevada Insurance Division, Nye Building, Carson City, Nevada 89701
May 23, 1919	OTTO, WALTER E., Consultant and Member of the Board of Directors, Michigan Mutual Liability Company, 28 West Adams Avenue, Detroit, Michigan 48226
Nov. 19, 1926	OVERHOLSER, DONALD M., 30 Fairlawn Street, Ho-ho-kus, N J 07423
Nov. 16, 1961	PEEL, JERALD P., Vice President Reinsurance, Security Mutual Casualty Company, 309 W. Jackson Blvd, Chicago, Ill. 60606
Nov 20, 1924	PENNOCK, RICHARD M. (Retired), 12 E. Lodges Lane, Bala-Cynwyd, Pa 19004
Nov. 16, 1965	PERREAULT, STEPHEN L., Assistant Actuary, The Hartford Insurance Group, Hartford Plaza, Hartford, Conn 06115
Nov. 14, 1947	PERRY, ROBERT C., Executive Vice President, State Farm Life Insurance Co, 112 East Washington St., Bloomington, Ill. 61701
Nov. 19, 1929	PHILLIPS, JOHN H (Retired), 915 Steuben Street, Wausau, Wis. 54401
Nov. 17, 1920	PIKE, MORRIS (Retired), 19 Old Mamaroneck Road, Apt 2G, White Plains, New York 10605
Nov. 13, 1967	PLUNKETT, JOSEPH A., American Re-Insurance Company, 99 John Street, New York, N Y. 10038
Nov. 17, 1922	POORMAN, WILLIAM F., Chairman of the Board, Central Life Assurance Company, 611 Fifth Avenue, Des Moines, Iowa 50306
Nov. 13, 1936	POTOFSKY, SYLVIA, Senior Actuary, The State Insurance Fund, 199 Church Street, New York, N. Y 10007
Nov. 13, 1967	PRICE, EDITH E (Mrs), Statistical Actuarial Department, Lumbermens Mutual Casualty Company, 4750 N. Sheridan Road, Chicago, Ill. 60640
Nov 18, 1966	QUINLAN, JOHN A., Assistant Actuary, The Hartford Insurance Group, Hartford Plaza, Hartford, Conn 06115

Admitted	
Nov. 20, 1964	Raid, Gary A., Actuary, Northwestern Mutual Insurance Company, 217 Pine, Seattle, Wash. 98101
Nov. 16, 1965	Ratnaswamy, Raj., Actuary, Detroit Automobile Inter-Insurance Exchange, and Motor State Insurance Company, 150 Bagley Avenue, Detroit, Mich. 48226
Nov. 15, 1918	Raywid, Joseph, Consultant, 322 W. 72nd Street, New York, N. Y. 10023
Nov. 19, 1932	Richardson, Harry F. (Retired), 413 Ackerman Avenue, Ho-ho-kus, N. J. 07423
Nov. 16, 1965	Richardson, James F., Assistant Actuary, The Employers' Group of Insurance Companies, 110 Milk Street, Boston, Mass. 02107
Nov. 19, 1953	Richmond, Owen D., Controller, Business Men's Assurance Company, P. O. Box 458, Kansas City, Mo. 64141
Nov. 18, 1960	Ripandelli, John S., Actuary & Consultant, P. O. Box 3552, Tallahassee, Fla. 32303
Nov. 18, 1932	Roberts, James A., Actuarial Statistician, The Travelers Insurance Companies, One Tower Square, Hartford, Conn 06115
Nov 15, 1962	Rood, Henry F., President, The Lincoln National Life Insurance Company, 1300 South Harrison Street, Fort Wayne, Ind 46801
Nov. 19, 1959	Royer, Alan F., Actuary, Multi-Line Insurance Rating Bureau, 110 William Street, New York, N. Y. 10038
Nov. 1, 1963	Ryan, Kevin M., Actuary, Industrial Indemnity Company, 155 Sansome Street, San Francisco, Calif. 94104
Nov. 14, 1958	Sarnoff, Paul E., Assistant Actuary, The Prudential Insurance Company of America, Prudential Plaza, Newark, N J. 07101
Nov. 16, 1923	Sawyer, Arthur (Retired), 13751 St Andrews Drive, Leisure World, Apt. 1-I, Seal Beach, Calif 90740
Nov. 14, 1947	Scammon, Lawrence W., Manager, Massachusetts Automobile Rating & Accident Prevention Bureau, Massachusetts Workmen's Compensation Rating & Inspection Bureau, and Massachusetts Motor Vehicle Assigned Risk Plan, 89 Broad Street, Boston, Mass. 02110
Nov. 1, 1963	Scheel, Paul J., Senior Actuarial Assistant, U S. Fidelity and Guaranty Company, Calvert and Redwood Streets, Baltimore, Md. 21203
Nov. 16, 1965	Scheid, James E., Assistant Actuary, The Hartford Insurance Group, Hartford Plaza, Hartford, Conn. 06115
Nov. 14, 1958	Schlenz, John W., Senior Vice President & Actuary, Federal Life and Casualty Company, 78 W Michigan Avenue, Battle Creek, Michigan 49016
Nov. 22, 1957	Schneiker, Henry C, Manager, Actuarial Dept, The Home Insurance Company, 59 Maiden Lane, New York, N. Y. 10008
Nov 19, 1954	Schulman, Justin, Group Leader, Mathematical Analysis, Programming, Kollsman Instrument Corporation, 80-08 45 Avenue, Elmhurst, N. Y 11373
Nov 14, 1947	Schwartz, Max J., Chief Accident & Health Rating Section, N Y. State Insurance Department, 324 State Street, Albany, N. Y. 12210
Nov. 20, 1930	Sevilla, Exequiel S., President, Manager and Actuary, National Life Insurance Company of the Philippines, Regina Bldg., P O. Box 2056, Manila, Philippines
Nov 20, 1924	Sheppard, Norris E., Professor of Mathematics, University of Toronto, 100 St. George St., Toronto 5, Canada
Nov 1, 1963	Singer, Paul E, Vice President and Actuary, Continental National American Group, 310 South Michigan Avenue, Chicago, Ill 60604
Nov 18, 1966	Snader, Richard H, Actuarial Assistant, United States Fidelity and Guaranty Co., Calvert and Redwood Sts. Baltimore, Md. 21203
Nov. 18, 1925	Sommer, Armand, Vice President, Continental Casualty Company, 310 South Michigan Avenue, Chicago, Ill. 60604
Nov. 15, 1918	Spencer, Harold S (Retired), 8 Chelsea Lane, West Hartford, Conn. 06119

Admitted	
Nov. 1, 1963	STALEY, HARLOW B., Vice President and Director of Administration, Farm Bureau Mutual Insurance Company, 10th and Grand Streets, Des Moines, Iowa 50307
Nov. 19, 1959	STEIN, JOAN BERKMAN, Assistant Actuary, Insurance Rating Board, 125 Maiden Lane, New York, N Y. 10038
Nov. 20, 1924	STELLWAGEN, HERBERT P., Director, Insurance Company of North America, 721 Mount Pleasant Road, Bryn Mawr, Pa. 19010
Nov. 15, 1956	STERN, PHILIPP K , Actuary, Insurance Rating Board, 125 Maiden Lane, New York, N. Y. 10038
Nov. 19, 1959	STEVENS, WALDO A., Vice President, National Association of Blue Shield Plans, 211 E Chicago Avenue, Chicago, Ill 60611
Nov. 16, 1923	STOKE, KENDRICK (Retired), 11052 McKinney, Detroit, Mich 48224
Nov. 19, 1959	STRUG, EMIL J., Assistant Actuary & Manager, Actuarial-Statistical Division, Massachusetts Blue Cross, Inc , 133 Federal Street, Boston, Mass. 02106
Nov. 18, 1966	STURGIS, ROBERT W., Actuarial Department, Aetna Life & Casualty, Hartford, Conn 06115
Nov. 21, 1930	SULLIVAN, WALTER F., Actuary, State Compensation Insurance Fund, 525 Golden Gate Avenue, San Francisco, Calif 94101
Nov. 1, 1963	THOMPSON, PHILIP R., Statistician, Federated Mutual Implement and Hardware Insurance Company, 129 E. Broadway, Owatonna, Minn. 55060
Nov. 18, 1966	TOREN, CHESTER J., Secretary, Zurich-American Insurance Companies, 111 West Jackson Boulevard, Chicago, Ill. 60604
Nov. 13, 1967	TORGRIMSON, DARVIN A , Assistant Actuary, Employers Insurance of Wausau, 2000 Westwood Drive, Wausau, Wis. 54401
Nov. 18, 1966	TREES, JOHN S., Pricing Director & Actuary, Allstate Insurance Company, Allstate Plaza, Northbrook, Ill. 60062
Nov. 21, 1919	TRENCH, FREDERICK H. (Retired), 1629 Genesee Street, Apt. B-4, Utica, New York 13501
Nov 20, 1924	UHL, M. ELIZABETH (Retired), 320 E. 53rd Street, New York, N.Y. 10022
Nov. 14, 1958	VAN CLEAVE, MARVIN E , Chief, Rates Division, Office of the Commissioner of Insurance, State of Wisconsin, Madison, Wis 53702
Nov. 20, 1964	VANDERHOOF, IRWIN T., Senior Vice President and Chief Actuary, Standard Security Life Insurance Co of New York, 111 Fifth Avenue, New York, N. Y. 10003
Nov. 18, 1966	WALTERS, MAVIS A , Assistant Actuary, Insurance Rating Board, 125 Maiden Lane, New York, N Y. 10038
Nov. 13, 1967	WALTERS, MICHAEL A., Assistant Manager-Research Division, Insurance Rating Board, 125 Maiden Lane, New York, N Y. 10038
Nov 13, 1967	WARD, MICHAEL R., Actuarial Assistant, The Travelers Insurance Companies, One Tower Square, Hartford, Conn. 06115
Nov. 19, 1959	WEBER, DONALD C , Lecturer, Miami University, Dept. of Mathematics, Oxford, Ohio 45056
Nov. 18, 1932	WEINSTEIN, MAX S., Consulting Actuary, 29 Elk Street, Albany, New York 12207
Nov. 18, 1966	WELCH, JOHN P., Senior Actuarial Assistant, Insurance Company of North America, 1600 Arch Street, Philadelphia, Pa. 19101
Nov. 18, 1925	WELLMAN, ALEX C., Vice Chairman of the Board, Protective Life Insurance Company, P O. Box 2571, Birmingham, Ala. 35202
Nov. 21, 1930	WELLS, WALTER I. (Retired), West Sterling, Mass. 01565
Nov. 18, 1927	WHITBREAD, FRANK G , Second Vice President, The Lincoln National Life Insurance Co., 1301 South Harrison Street, Fort Wayne, Ind 46801
Nov. 19, 1948	WHITE, AUBREY, Ostheimer, Peat, Marwick & Co , 1500 Walnut Street, Philadelphia, Pa 19103
Nov. 13, 1967	WILLIAMS, W. THOMAS, Consultant, The Wyatt Company, 1900 Republic National Bank Tower, Dallas, Tex. 75201

Admitted	
Nov. 13, 1967	WINTER, ARTHUR E., Actuarial Assistant, The Travelers Insurance Companies, One Tower Square, Hartford, Conn. 06115
Nov. 16, 1939	WITTLAKE, J. CLARKE, Executive Vice President, Business Men's Assurance Company, P. O. Box 458, Kansas City, Mo. 64141
Oct. 22, 1915	WOOD, DONALD M., Partner, Childs & Wood, 175 West Jackson Boulevard, Chicago, Ill. 60604
Nov. 18, 1937	WOOD, DONALD M., JR., Partner, Childs & Wood, 175 West Jackson Boulevard, Chicago, Ill. 60604
Nov. 17, 1950	WOODDY, JOHN C., Actuary, North American Reassurance Company, 245 Park Avenue, New York, N. Y. 10017
Nov. 22, 1934	WOODWARD, BARBARA H., Assistant Secretary & Assistant General Counsel, The Reuben H. Donnelley Corporation, 235 East 42nd Street, New York, N. Y 10017
Nov 16, 1956	WOODWORTH, JAMES H., Assistant Secretary, The Hartford Insurance Group, Hartford Plaza, Hartford, Conn. 06115
Nov. 18, 1925	WOOLERY, JAMES M., Consultant, 3207 Sussex Road, Raleigh, N. C 27607
May 5, 1961	YOUNG, ROBERT G., Special Projects Director, League Life Insurance Company, 13300 Woodrow Wilson, Detroit, Mich. 48238
Nov. 1, 1963	ZORY, PETER B., Actuary, State Farm Mutual Automobile Insurance Company, 112 East Washington Street, Bloomington, Ill. 61701

DECEASED FELLOWS

The (†) denotes charter members at date of organization, November 7, 1914.

Admitted		Died
Nov. 13, 1931	Gilbert E. Ault	Apr. 13, 1965
Nov. 19, 1948	Arthur L. Bailey	Aug. 12, 1954
May 23, 1924	William B. Bailey	Jan. 10, 1952
†	Roland Benjamin	July 2, 1949
May 24, 1921	Edward J. Bond	Nov. 12, 1941
May 19, 1915	Thomas Bradshaw	Nov. 10, 1939
June 5, 1925	William Brosmith	Aug. 22, 1937
Nov. 18, 1927	F. Stuart Brown	Oct. 21, 1967
†	George B. Buck, Sr.	Apr. 12, 1961
†	William A. Budlong	June 4, 1934
Nov. 18, 1932	Charles H. Burhans	June 15, 1942
Apr. 20, 1917	William H. Burhop	Oct. 11, 1963
Feb. 19, 1915	F. Highlands Burns	Mar. 30, 1935
†	Edmund E. Cammack	Dec. 17, 1958
Nov. 21, 1930	Thomas O. Carlson	July 15, 1964
†	Raymond V. Carpenter	Mar. 11, 1947
Feb. 19, 1915	Gorden Case	Feb. 4, 1920
Oct. 27, 1916	Edmund S. Cogswell	Apr. 25, 1957
Nov. 23, 1928	Walter P. Comstock	May 11, 1951
Nov. 22, 1934	William J. Constable	Apr. 19, 1959
†	Charles T. Conway	July 23, 1921
†	John A. Copeland	June 12, 1953
†	Walter G. Cowles	May 30, 1942
†	James D. Craig	May 27, 1940
†	James McIntosh Craig	Jan. 20, 1922
Nov. 20, 1964	Robert A. Craig	Feb. 8, 1965
May 26, 1916	Frederick S. Crum	Sept. 2, 1921
Nov. 18. 1932	E. Alfred Davies	Jan. 14, 1967
†	Alfred Burnett Dawson	June 21, 1931
†	Miles Menander Dawson	Mar. 27, 1942
†	Elmer H. Dearth	Mar. 26, 1947
†	Eckford C. DeKay	July 31, 1951
May 19, 1915	Samuel Deutschberger	Jan. 18, 1929
†	Ezekiel Hinton Downey	July 9, 1922
May 19, 1915	Earl O. Dunlap	July 5, 1944
†	David Parks Fackler	Oct. 30, 1924
†	Edward B. Fackler	Jan. 8, 1952
Feb. 19, 1915	Claude W. Fellows	July 15, 1938
†	Benedict D. Flynn	Aug. 22, 1944
Feb. 19, 1915	Richard Fondiller	Apr. 29, 1962
†	Charles S. Forbes	Oct. 2, 1943
May 26, 1916	Lee K. Frankel	July 25, 1931
†	Charles H. Franklin	May 1951
Feb. 25. 1916	Joseph Froggatt	Sept. 28, 1940
†	Harry Furze	Dec. 26, 1945
Feb. 19, 1915	Fred S. Garrison	Nov. 14, 1949
†	Theodore E. Gaty	Aug. 22, 1925
May 19, 1915	James W. Glover	July 15, 1941
†	Edward S. Goodwin	Jan. 27, 1966
Oct. 22, 1915	George Graham	Apr. 15, 1937
Oct. 22, 1915	Thompson B. Graham	July 24, 1946
†	William J. Graham	Feb. 11, 1963
May 25, 1923	William A. Granville	Feb. 4, 1943

Admitted		Died
†	Winfield W. Greene	Mar. 26, 1965
†	William H. Gould	Oct. 28, 1936
†	Robert Cowen Lees Hamilton	Nov. 15, 1941
†	H. Pierson Hammond	Apr. 10, 1963
Oct. 27, 1916	Edward R. Hardy	June 29, 1951
Oct. 22, 1915	Leonard W. Hatch	Nov. 23, 1958
Nov. 21, 1919	Robert Henderson	Feb. 16, 1942
†	Robert J. Hillas	May 17, 1940
Nov. 15, 1918	Frank Webster Hinsdale	Mar. 18, 1932
May 23, 1924	Clarence W. Hobbs	July 21, 1944
Nov. 19, 1926	Charles E. Hodges	Jan. 22, 1937
Oct. 22, 1915	Lemuel G. Hodgkins	Dec. 26, 1951
†	Frederick L. Hoffman	Feb. 23, 1946
Oct. 22, 1915	Charles H. Holland	Dec. 28, 1951
Nov. 21, 1919	Carl Hookstadt	Mar. 10, 1924
Nov. 18, 1932	Solomon S. Huebner	July 17, 1964
†	Charles Hughes	Aug. 27, 1948
Nov. 19, 1929	Robert S. Hull	Nov. 30, 1947
†	Burritt A. Hunt	Sept. 3, 1943
†	Arthur Hunter	Jan. 27, 1964
Nov. 28, 1921	William Anderson Hutcheson	Nov. 19, 1942
Feb. 25, 1916	Charles William Jackson	Sept. 21, 1959
Nov. 19, 1929	Henry Hollister Jackson	May 27, 1955
May 19, 1915	William C. Johnson	Oct. 7, 1943
Nov. 23, 1928	F. Robertson Jones	Dec. 26, 1941
Nov. 18, 1921	Thomas P. Kearney	Feb. 11, 1928
Nov. 19, 1926	Gregory Cook Kelly	Sept. 11, 1948
Oct. 22, 1915	Virgil Morrison Kime	Oct. 15, 1918
†	Edwin W. Kopf	Aug. 3, 1933
Nov. 23, 1928	Clarence Arthur Kulp	Aug. 20, 1957
Feb. 17, 1915	John M. Laird	June 20, 1942
Nov. 13, 1931	Stewart M. LaMont	Aug. 22, 1960
Feb. 19, 1915	Abb Landis	Dec. 9, 1937
Nov. 24, 1933	John Robert Langė	Apr. 12, 1957
Nov. 17, 1922	Arnette Roy Lawrence	Dec. 1, 1942
†	James R. Leal, Sr.	Dec. 26, 1957
†	William Leslie	Dec. 12, 1962
Nov. 18, 1921	James Fulton Little	Aug. 11, 1938
Nov. 23, 1928	Edward C. Lunt	Jan. 13, 1941
Feb. 19, 1915	Harry Lubin	Dec. 20, 1920
†	William N. Magoun	Dec. 11, 1954
Nov. 16, 1923	D. Ralph McClurg	Apr. 27, 1947
May 23, 1919	Alfred McDougald	July 28, 1944
Oct. 31, 1917	Robert J. McManus	Aug. 15, 1960
Feb. 15, 1915	Franklin B. Mead	Nov. 29, 1933
Apr. 20, 1917	Marcus Meltzer	Mar. 27, 1931
†	David W. Miller	Jan. 18, 1936
†	Samuel Milligan	Aug. 8, 1965
†	James F. Mitchell	Feb. 9, 1941
†	Henry Moir	June 8, 1937
Nov. 18, 1921	Victor Montgomery	May 2, 1960

Admitted		Died
Feb. 19, 1915	William J. Montgomery	Aug. 20, 1915
Nov. 19, 1926	William L. Mooney	Oct. 21, 1948
†	George D. Moore	Mar. 11, 1959
May 19, 1915	Edward Bontecou Morris	Dec. 19, 1929
†	Albert H. Mowbray	Jan. 7, 1949
†	Frank Mullaney	Jan. 22, 1953
May 28, 1920	Ray D. Murphy	Feb. 24, 1964
†	Lewis A. Nicholas	Apr. 21, 1940
†	Edward Olifiers	May 13, 1962
†	Robert K. Orr	Oct. 5, 1967
†	Stanley L. Otis	Oct. 12, 1937
Nov. 13, 1926	Bertrand A. Page	July 30, 1941
Nov. 18, 1921	Sanford B. Perkins	Sept. 16, 1945
Nov. 15, 1918	William Thomas Perry	Oct. 25, 1940
Nov. 21, 1930	Francis S. Perryman	Nov. 30, 1959
†	Edward B. Phelps	July 24, 1915
Nov. 19, 1926	Jesse S. Phillips	Nov. 6, 1954
Nov. 13, 1931	Dudley M. Pruitt	June 27, 1967
†	Charles Grant Reiter	July 30, 1937
†	Charles H. Remington	Mar. 21, 1938
Nov. 16, 1951	Homer D. Rice	May 12, 1967
May 23, 1919	Frederick Richardson	July 22, 1955
Nov. 19, 1926	Otto C. Richter	Feb. 17, 1962
Nov. 16, 1923	William F. Roeber	Mar. 21, 1960
Nov. 17, 1943	Samuel M. Ross	July 24, 1951
†	Isaac M. Rubinow	Sept. 1, 1936
†	Harwood Eldridge Ryan	Nov. 2, 1930
†	Arthur F. Saxton	Feb. 26, 1927
†	Emil Scheitlin	May 2, 1946
†	Leon S. Senior	Feb. 3, 1940
Nov. 24, 1933	Robert V. Sinnott	Dec. 15, 1952
Apr. 20, 1917	Charles Gordon Smith	June 22, 1938
Nov. 18, 1927	Edward C. Stone	June 6, 1964
Feb. 19, 1915	John T. Stone	May 9, 1920
Feb. 25, 1916	Wendell Melville Strong	Mar. 30, 1942
Oct. 22, 1915	William R. Strong	Jan. 10, 1946
†	Robert J. Sullivan	July 19, 1934
Nov. 17, 1920	Thomas F. Tarbell	July 2, 1958
Nov. 22, 1934	Walter H. Thompson	May 25, 1935
Nov. 18, 1921	Guido Toja	Feb. 28, 1933
†	John L. Train	June 12, 1958
Nov. 17, 1922	Antonio Thomas Traversi	Apr. 20, 1961
Nov. 19, 1948	Paul A. Turner	Jan. 30, 1961
Nov. 15, 1935	Harry V. Waite	Aug. 14, 1951
Nov. 18, 1925	Lloyd A. H. Warren	Sept. 30, 1949
May 23, 1919	Archibald A. Welch	May 8, 1945
Nov. 19, 1926	Roy A. Wheeler	Aug. 26, 1932
†	Albert W. Whitney	July 27, 1943
†	Lee J. Wolfe	Apr. 28, 1949
†	S. Herbert Wolfe	Dec. 31, 1927
Nov. 18, 1949	Richard J. Wolfrum	Oct. 31, 1967
May 24, 1921	Arthur B. Wood	June 14, 1952
†	Joseph H. Woodward	May 15, 1928
†	William Young	Oct. 23, 1927

Admitted		Died
May 23, 1924	Milton Acker	Aug. 16, 1956
Nov. 15, 1918	Robert E. Ankers	Mar. 1, 1964
Oct. 22, 1915	Don A. Baxter	Feb. 10, 1920
Nov. 17, 1920	Nellas C. Black	Dec. 24, 1962
Nov. 15, 1940	John M. Blackhall	Nov. 14, 1957
Nov. 15, 1918	Helmuth G. Brunnquell	June 3, 1958
Oct. 22, 1915	Louis Buffler	July 19, 1963
Nov. 17, 1922	Leo D. Cavanaugh	July 18, 1965
Nov. 18, 1925	Malvin E. Davis	Aug. 26, 1966
May 25, 1923	Harilaus E. Economidy	Apr. 13, 1948
Nov. 20, 1924	John Froberg	Oct. 11, 1949
Nov. 19, 1929	Maurice L. Furnivall	June 16, 1962
Nov. 22, 1934	John J. Gately	Nov. 3, 1943
Nov. 14, 1947	Harold J. George	Apr. 1, 1952
Nov. 19, 1929	Harold R. Gordon	July 8, 1948
Nov. 18, 1921	Robert E. Haggard	July 26, 1958
Nov. 20, 1924	Leslie LeVant Hall	Mar. 8, 1931
Nov. 17, 1927	Grady Hayne Hipp	June 25, 1965
Oct. 31, 1917	Edward T. Jackson	May 8, 1939
Nov. 18, 1921	Edward S. Jensen	Sept. 2, 1966
Mar. 24, 1927	Charles V. R. Marsh	Sept. 12, 1967
Nov. 17, 1922	Rosswel A. McIver	Apr. 1, 1959
Nov. 21, 1919	Rolland V. Mothersill	July 25, 1949
Nov. 19, 1929	Fritz Muller	Apr. 27, 1945
Nov. 23, 1928	Karl Newhall	Oct. 24, 1944
Nov. 22, 1957	C. Otis Shaver	June 15, 1966
Nov. 15, 1918	John L. Sibley	Mar. 10, 1957
Nov. 18, 1921	Arthur G. Smith	May 2, 1956
Nov. 19, 1926	William F. Somerville	Nov. 12, 1965
Nov. 18, 1927	Alexander A. Speers	June 25, 1941
Nov. 19, 1959	Henry W. Steinhaus	Aug. 8, 1966
Mar. 23, 1921	Arthur E. Thompson	Jan. 17, 1944
Nov. 21, 1919	Walter G. Voogt	May 8, 1937
May 23, 1919	Charles S. Warren	May 1, 1952
Nov. 18, 1925	James H. Washburn	Aug. 19, 1946
Nov. 17, 1920	James J. Watson	Feb. 23, 1937
Nov. 18, 1921	Eugene R. Welch	Jan. 17, 1945
Nov. 16, 1951	Michael T. Wermel	Feb. 6, 1962
Mar. 21, 1929	Charles A. Wheeler	July 2, 1956
Nov. 15, 1918	Albert Edward Wilkinson	June 11, 1930
Oct. 22, 1915	Charles E. Woodman	Dec. 16, 1955

 OFFICERS OF THE SOCIETY SINCE ORGANIZATION

Elected	*President*	*Vice Presidents*	
1914-1915	°Isaac M. Rubinow	°Albert H. Mowbray	°Benedict D. Flynn
1916-1917	°James D. Craig	°Joseph H. Woodward	°Harwood E. Ryan
1918	°Joseph H. Woodward	°Benedict D. Flynn	°George D. Moore
1919	°Benedict D. Flynn	°George D. Moore	°William Leslie
1920	°Albert H. Mowbray	°William Leslie	°Leon S. Senior
1921	°Albert H. Mowbray	°Leon S. Senior	°Harwood E. Ryan
1922	°Harwood E. Ryan	Gustav F. Michelbacher	°Edmund E. Cammack
1923	°William Leslie	Gustav F. Michelbacher	°Edmund E. Cammack
1924-1925	Gustav F. Michelbacher	°Sanford B. Perkins	Ralph H. Blanchard
1926-1927	°Sanford B. Perkins	°George D. Moore	°Thomas F. Tarbell
1928-1929	°George D. Moore	Sydney D. Pinney	Paul Dorweiler
1930-1931	°Thomas F. Tarbell	°Roy A. Wheeler	°Winfield W. Greene
1932-1933	Paul Dorweiler	°William F. Roeber	°Leon S. Senior
1934-1935	°Winfield W. Greene	Ralph H. Blanchard	Charles J. Haugh
1936-1937	°Leon S. Senior	Sydney D. Pinney	°Francis S. Perryman
1938-1939	°Francis S. Perryman	Harmon T. Barber	°William J. Constable
1940	Sydney D. Pinney	Harold J. Ginsburgh	James M. Cahill
1941	Ralph H. Blanchard	Harold J. Ginsburgh	James M. Cahill
1942	Ralph H. Blanchard	Albert Z. Skelding	Charles J. Haugh
1943-1944	Harold J. Ginsburgh	Albert Z. Skelding	Charles J. Haugh
1945-1946	Charles J. Haugh	James M. Cahill	Harry V. Williams
1947-1948	James M. Cahill	Harmon T. Barber	Russell P. Goddard
1949-1950	Harmon T. Barber	°Thomas O. Carlson	Norton E. Masterson
1951-1952	°Thomas O. Carlson	Joseph Linder	Seymour E. Smith
1953-1954	Seymour E. Smith	°Dudley M. Pruitt	John A. Mills
1955-1956	Norton E. Masterson	°Clarence A. Kulp	Arthur N. Matthews
1957-1958	°Dudley M. Pruitt	John W. Carleton	William Leslie, Jr.
1959-1960	William Leslie, Jr.	Ernest T. Berkeley	Laurence H. Longley-Cook
1961-1962	L. H. Longley-Cook	Thomas E. Murrin	°Richard J. Wolfrum
1963-1964	Thomas E. Murrin	Harold E. Curry	William J. Hazam
1965-1967	Harold E. Curry	Charles C. Hewitt, Jr.	Harold W. Schloss

Secretary-Treasurer

1914-1917	°C. E. Scattergood
1918-1953	°R. Fondiller
1954-1967	A. Z. Skelding

Editor

1914	°W. W. Greene
1915-1917	°R. Fondiller
1918	°W. W. Greene
1919-1921	G. F. Michelbacher
1922-1923	O. E. Outwater
1924-1932	°R. J. McManus
1933-1943	°C. W. Hobbs
1944-1954	E. C. Maycrink
1955-1958	E. S. Allen
1959-1960	R. P. Goddard
1961-1964	H. W. Schloss
1965-1967	M. Rodermund

General Chairman Examination Committee

1949-1951	R. A. Johnson
1952-1956	J. W. Wieder, Jr.
1957-1961	W. J. Hazam
1962-1967	N. J. Bennett

Librarian

1914	°W. W. Greene
1915	°R. Fondiller
1916-1921	L. I. Dublin
1922-1924	°E. R. Hardy
1925-1936	W. Breiby
1937-1947	°T. O. Carlson
1948-1950	°S. M. Ross
1951-1957	G. R. Livingston
1958-1967	R. Lino

°Deceased.

CONSTITUTION

(As amended January 1, 1968)

Article I. — *Name*

This organization shall be called the Casualty Actuarial Society.

Article II. — *Objects*

The objects of the Society shall be to advance the knowledge of actuarial science as applied to the problems of insurance, other than life insurance, and to promote and maintain high standards of conduct and competence within the actuarial profession. The Society shall further these ends by holding meetings, by personal communication, by the presentation, discussion and publication of appropriate papers, by promoting educational activities in the actuarial sciences for its students and members, and by such other means as may be found desirable.

Article III. — *Membership*

The membership of the Casualty Actuarial Society shall be composed of two classes, Fellows and Associates. Fellows only shall be eligible to hold office, make nominations, or have the right to vote.

The Fellows of the Society shall be the present Fellows and those who may be duly admitted to Fellowship as hereinafter provided. The Associates shall be the present Associates and those who may be duly admitted to Associateship as hereinafter provided.

Any applicant shall be enrolled as an Associate at a meeting of the Society provided that:

(i) the applicant passes the examinations prescibed by the Council for Associateship and complies with any further requirements the Council may prescribe;

(ii) the applicant, upon fulfilling all the requirements outlined in (i), is approved by a majority vote of the Council.

An Associate shall be enrolled as a Fellow of the Society at an annual meeting on passing the examinations prescribed by the Council for Fellowship, subject to any further requirements the Council may prescribe.

Otherwise no one shall be admitted as an Associate or a Fellow unless recommended at a duly called meeting of the Council with not more than two negative votes followed by an affirmative vote in a secret ballot of at least three-fourths of the Fellows present and voting at a meeting of the Society.

The Council may waive, subject to such other requirements as it may prescribe, any examination of the Casualty Actuarial Society if the applicant has passed an examination required by another recognized actuarial organization that the Council deems equivalent to such examination of the Casualty Actuarial Society.

Article IV. — *Officers and Council*

The Officers of the Society, all of whom shall be Fellows, shall consist of a President, two Vice Presidents, a Secretary-Treasurer, an Editor, a Librarian, and a General Chairman of the Examination Committee. The Council shall consist of the Officers, nine other Fellows and, for the two years following the expiration of their terms of office, the ex-Presidents and ex-Vice Presidents. The Council may fill vacancies occasioned by death or resignation of any Officer or other member of the Council, such appointees to serve until the expiration of the term of office of the Officer or Council member being replaced.

Article V. — *Election of Officers and Council*

The President, Vice Presidents, and the Secretary-Treasurer shall be elected by a majority vote in a secret ballot of the Fellows present and voting at the annual meeting for the term of one year, or until their qualified successors shall be duly elected. The President and Vice Presidents shall not be eligible for the same office for more than two consecutive years. Three members of the Council shall, in a similar manner, be annually elected to serve from the close of the annual meeting for the, term of three years Any retiring elected member of the Council shall not be eligible for re-election at the same meeting.

A majority of the votes cast shall be required for election as an elected member except that, in the event of a second or subsequent ballot, Fellows receiving the greatest number of votes shall be elected, provided the number of votes received is not less than one-third of those cast.

The Editor, the Librarian, and the General Chairman of the Examination Committee shall be elected annually by the Council at the Council meeting preceding the annual meeting of the Society. They shall be subject to confirmation by a majority ballot of the Fellows present and voting at the annual meeting.

The terms of the Officers shall begin at the close of the annual meeting at which they are elected except that the retiring Editor shall retain the powers and duties of office so long as may be necessary to complete the then current issue of the *Proceedings*.

Article VI. — *Duties of Officers and Council*

The duties of the Officers shall be such as are customarily incident to their respective offices and such other duties as specified in the By-Laws. The duties of the Council shall be to pass upon candidates for membership, to elect annually the Editor, Librarian, and General Chairman of the Examination Committee, to decide upon the publication of papers presented at meetings of the Society, to supervise the examination of candidates and prescribe fees for such examinations, to call meetings, to ratify such committees as may be appointed by the President, and, in general, to manage the affairs of the Society.

Article VII. — *Meetings*

There shall be an annual meeting of the Society on such date in the month of November as may be fixed by the Council in each year, but other Society meetings may be called by the Council from time to time and shall be called by the President at any time upon the written request of twenty Fellows. At least two weeks notice of all Society meetings shall be given by the Secretary-Treasurer.

Article VIII. — *Quorum*

Eleven members of the Council shall constitute a quorum. Forty Fellows of the Society shall constitute a quorum at every meeting of the Society.

ARTICLE IX.—*Public Expression of Professional Opinion*

No opinion with respect to questions of public interest shall be publicly expressed by, or on behalf of, the Casualty Actuarial Society, the Council, or any committee except on matters within the special professional competence of actuaries and then only in accordance with authority given and procedures determined in each instance by the Council and in accordance with the following conditions:

(i) An opinion of the Casualty Actuarial Society shall require advance approval by an affirmative vote of at least ninety percent of the Fellows who vote in a mail ballot.

(ii) An opinion of the Council or a committee authorized by the Council to express an opinion shall indicate that it does not purport to represent the views of the Casualty Actuarial Society, but only of the Council or the committee, as the case may be.

ARTICLE X.—*Resignation and Discipline of Members*

Any member who is not in default in payment of dues, and against whom no complaints or charges are pending, may at any time file his resignation in writing with the Secretary-Treasurer. Notwithstanding the foregoing, the Council may, in its discretion, permit the resignation of a member against whom a complaint or charge is pending. The Council, on written application of any member who has resigned while in good standing, may reinstate such member subject to such conditions as it may prescribe.

No member of the Society shall be disciplined, suspended, or expelled except upon action of the Council and the membership as provided for in the By-Laws of the Society.

ARTICLE XI.—*Amendments*

This Constitution may be amended by an affirmative vote of two-thirds of the Fellows present at any meeting held at least one month after notice of such proposed amendment shall have been sent to each Fellow by the Secretary-Treasurer.

BY-LAWS

(As amended January 1, 1968)

Article I. — *Order of Business*

At a meeting of the Society the following order of business shall be observed unless an agenda is sent to the members prior to a meeting:

1. Calling of the roll
2. Address or remarks by the President
3. Minutes of the last meeting
4. Report by the Council on business transacted by it since the last meeting of the Society
5. New membership
6. Reports of Officers and committees
7. Election of Officers, Council members, and confirmation of Council election of Editor, Librarian, and General Chairman of Examination Committee
8. Unfinished business
9. New business
10. Reading of papers
11. Discussion of papers

Article II. — *Council Meetings*

Meetings of the Council shall be called whenever the President or three members of the Council so request, but not without sending notice to each member of the Council seven or more days before the time appointed. Such notice shall state the objects intended to be brought before the meeting, and should other matter be passed upon, any member of the Council shall have the right to reopen the question at the next meeting.

Article III. — *Duties of Officers*

The President, or, in his absence, one of the Vice Presidents, shall preside at meetings of the Society and of the Council. At the Society meetings, the presiding officer shall vote only in case of a tie, but at the Council meetings he may vote in all cases. The President shall appoint all committees and shall perform all duties customarily incident to the office of President and such other duties as may be prescribed by the Council from time to time Each of the Vice Presidents shall have such duties as may be assigned to him by the President or the Council.

The Secretary-Treasurer shall keep a full and accurate record of the proceedings at the meetings of the Society and of the Council, and send out notices for such meetings. Subject to the direction of the Council, he shall have immediate charge of the office and archives of the Society.

The Secretary-Treasurer shall collect the annual dues of members, pay all bills for ordinary expenditures incurred by the Society and any other bills as authorized by the Council, keep a detailed record of all receipts and expenditures, and present an accounting of the same at the annual meetings, after it has been audited by a committee appointed by the President. The Secretary-Treasurer shall perform all duties customarily incident to the office of Secretary-Treasurer and such other duties as may be assigned to him from time to time by the President or by the Council.

The Editor shall, under the general supervision of the Council, have charge of all matters connected with editing and printing the Society's publications. The *Proceedings* shall contain only the proceedings of the meetings and the original papers, reviews or discussions on said papers by members that may be expressly authorized by the Council to appear in such *Proceedings*. The *Proceedings* may also contain any other matter expressly authorized by the Council.

The Librarian shall, under the general supervision of the Council, have charge of the books, pamphlets, manuscripts, and other literary or scientific material collected by the Society.

The General Chairman of the Examination Committee shall, under the general supervision of the Council, have charge of the examination system and of the examinations held by the Society for admission to the grades of Associate and Fellow.

ARTICLE IV.—*Discipline of Members*

The Council shall have the power to consider and take action, as herein provided, with respect to all questions which may arise as to the conduct of a member of the Casualty Actuarial Society in his relations to the Society or its members, or in his profession, or in the practice thereof, or affecting the interests of the actuarial profession. The Council may, on its own initiative, investigate and take action with respect to any such question, and may also receive and hear any complaint relating to the conduct of a member preferred in writing and subscribed to by a member. In the course of dealing with questions and complaints relating to the conduct of members, the Council may appoint, from among the Fellows of the Casualty Actuarial Society, committees and boards vested with the powers specified herein:

(a) Investigating committees empowered to investigate questions and complaints and to prefer charges against a member;

(b) Prosecuting committees empowered to prosecute charges against a member at hearings before the Council or a disciplinary board;

(c) Disciplinary boards empowered to hear evidence relating to questions and complaints and to make findings with respect to such evidence.

The procedures for such committees and boards shall be prescribed by the Council. The Council may retain counsel for the assistance of the Council and of committees and boards appointed by it.

In any hearing before the Council or a disciplinary board, a member proceeded against shall have the right to appear personally and by counsel, to be informed of the nature and content of the question or complaint, to examine the evidence presented, to examine adverse witnesses, and to present witnesses and evidence in his behalf. Any member preferring a complaint may appear personally and by counsel. Witnesses called in the course of hearings involving conduct shall vouch for the truth of their statements on their word of honor.

In all proceedings under this Article, the Council shall decide, directly or upon review of the findings of a body appointed by it, whether or not misconduct has occurred. If the Council finds that misconduct has occurred, it may warn, admonish, reprimand, suspend, or expel the member, provided that no order reprimanding, suspending, or expelling a member shall be issued except after a hearing before the Council or a disciplinary board.

A member against whom an order of suspension or expulsion has been rendered shall, upon application to the Council, within thirty days thereafter, be entitled to appeal to the Fellows attending a meeting of the Casualty Actuarial Society upon the following conditions:

(a) All rights and privileges of membership shall be suspended during the pendency of the appeal, and

(b) The notice of appeal shall be in writing and shall stipulate that the appealing member consents to the mailing to the Fellows of a transcript of the evidence and copies of exhibits in the form approved by a majority of the Council, and

(c) The appealing member shall, within ten days after an invoice of the amount due is sent to him, deposit with the Secretary-Treasurer the cost of transcribing and printing the transcript of the evidence and copies of any and all exhibits. In the event the decision of the Council shall be set aside, the Secretary-Treasurer shall return to the appealing member the amount of the deposit. Otherwise, the deposit shall be retained by the Casualty Actuarial Society.

In the event of an appeal to the Fellows, the decision of the Council may be affirmed, modified, or set aside by the vote of a majority of the Fellows present and voting at a meeting of the Casualty Actuarial Society.

The Council may, in its discretion, reinstate to membership at any time a member suspended or expelled under this Article, provided in the event the suspension or expulsion has been affirmed by the Fellows, the reinstatement shall not take effect unless and until confirmed at a meeting of the Casualty Actuarial Society by a vote of a majority of the Fellows present and voting.

Except as otherwise provided, all proceedings under this section shall be deemed confidential and kept secret. The Council, however, shall notify the members of its action in all instances in which the Council orders the suspension or expulsion of a member. Such notification shall not be given until the time to appeal has expired or, in the event of an appeal, until a majority of the Fellows present at a meeting of the Society have voted in favor of suspension or expulsion. At the same time notification is given to the members, the Council may also give notice of such suspension or expulsion to such newspapers or journals as it may select.

In the event of subsequent reinstatement of the member, the Council shall give notice of such action to the members of the Society and to any newspapers or journals previously advised by the Council of the member's suspension or expulsion.

Article V. — *Indemnification of Officers, Council Members, and Committee Members*

Each person who at any time shall serve, or shall have served, as an Officer, member of the Council, committee member, or member of any disciplinary board of the Society (and his heirs, executors, administrators, and personal representatives) shall be indemnified by the Society against all costs and expenses (including but not limited to legal fees, amounts of judgments paid, and amounts paid in settlements) reasonably incurred in connection with the defense of any claim, action, suit, or proceeding, whether civil, criminal, administrative, or other, in which he or they may be involved by virtue of such person being or having been an Officer, member of the Council, committee member, or member of any disciplinary board of the Society, or in connection with any appeal therein; provided, however, that in the event of a settlement the indemnification herein provided shall apply only when the Council approves such settlement; and provided further that such indemnity shall not be operative with respect to any matter as to which such person shall have been finally adjudged liable in such claim, action, suit, or proceeding on account of his own wilful misconduct.

The rights accruing to any person under this Article shall be without prejudice to any rights or benefits given by the Council inconsistent therewith in special cases and shall not exclude any other rights or benefits to which he may be lawfully entitled.

ARTICLE VI. — *Dues*

The Council shall fix the annual dues for Fellows and Associates. The payment of dues will be waived in the case of any Fellow or Associate who attains the age of 70 or who attains the age of 65 and notifies the Secretary-Treasurer in writing that he has retired from active work. Fellows and Associates who have become totally disabled while members may, upon approval of the Council, be exempted from the payment of dues during the period of disability.

It shall be the duty of the Secretary-Treasurer to notify by mail any Fellow or Associate whose dues may be six months in arrears, and to accompany such notice by a copy of this Article. If such Fellow or Associate shall fail to pay his dues within three months from the date of mailing such notice, his name shall be stricken from the rolls, and he shall thereupon cease to be a Fellow or Associate of the Society. He may, however, be reinstated by a majority vote of the Council upon payment of dues in arrears which shall in no event exceed five years.

ARTICLE VII. — *Designation by Initials*

Fellows of the Society are authorized to append to their names the initials F.C.A.S.; and Associates are authorized to append to their names the initials A.C.A.S.

ARTICLE VIII. — *Amendments*

These By-Laws may be amended by an affirmative vote of two-thirds of the Fellows present at any meeting held at least one month after notice of the proposed amendment shall have been sent to each Fellow by the Secretary-Treasurer.

GUIDES TO PROFESSIONAL CONDUCT

(As amended November 20, 1959)

In order to assist the Council of the Society in resolving questions that might be raised as the the professional conduct of members, and more importantly to guide members of the Society when they encounter questions of professional conduct, the following "Guides to Professional Conduct" have been prepared by order of the Council. The actuary has professional responsibilities to society at large, to his client or employer, and to his professional associates. As is true of codes of ethics generally, these guides deal with precepts and principles only. They are not precise rules and are subject to interpretations in relation to the variety of circumstances that occur in practice.

Any member wishing advice on the application of these guides to a particular set of facts is urged to present his case to the Council of the Society. The Council has the power to consider and take action with respect to questions that may be raised as to the professional conduct of members. Any disciplinary action by the Council must be in accord with the Constitution and By-Laws of the Society.

The Council assumes that every member of the Society earnestly desires to serve his client or employer properly, to protect the public, and to maintain the prestige of the Society and its members. Accordingly, the Council sets forth the following principles by which, in its opinion, every member should be guided in his practice of the actuarial profession:

1. The member will promote a wider understanding of the significance of membership in the Society and will maintain the high standards of the Society by avoiding even the appearance of any questionable practice.

2. The member will conduct his professional competition on a high plane He will avoid unjustifiable or improper criticism of others and will recognize that there is substantial room for honest differences of opinion on many matters.

3. The member will act in professional matters for each client or employer with scrupulous attention to the trust and confidence that the relationship implies and will have due regard for the confidential nature of his work.

4. The member will bear in mind that the actuary acts as an expert when he gives professional advice, and he will give such advice only when he is qualified to do so.

5. The member will not provide actuarial service for, or associate professionally with, any person or organization if he has reason to believe that the results of such service or association are likely to be used in a manner inimical to the public interest or the interests of the actuarial profession or to evade the law.

6. The member will submit unqualifiedly an actuarial calculation, certificate, or report only if he knows it to based on sufficiently reliable data and on actuarial assumptions and methods that, in his judgment, are consistent with the sound principles expounded in the course of study of the Society, or in recognized texts, sources, or precedents relevant to the subject at hand.

7. The member will recommend for the use of his client or employer, premium rates, rating plans, dividends, or other related actuarial functions only if, in his opinion, they are based on adequate and appropriate assumptions and methods.

8. The member will not make or sponsor any actuarial calculation, certificate, statement, report, or comparison, or give any testimony or interview on such matters, which he has reason to believe is false, materially incomplete, or misleading.

9. Where appropriate for the objective use of a certificate or report, or in any event on the request of his employer or client, the member will include a statement of the principal actuarial assumptions and the general methods adopted for his computations.

10. The member will recognize his ethical responsibilities to the person or organization whose actions may be influenced by his professional opinions or findings. When it is not feasible for the member to render his opinions or findings direct to such person or organization, he will act in such manner as to leave no doubt that the member is the source of the opinions or findings and to indicate clearly the personal availability of the member to provide supplemental advice and explanation.

11. The member will not serve more than one client or employer where a conflict of his professional interest may be involved unless there be a full disclosure to all parties concerned, and such parties request and acquiesce in the engagement of his services.

12. The member will sign actuarial recommendations, certificates, and reports if he be acting as an employee, only over a title conferred by his employer if any title is used. Nevertheless, in any capacity, the member may append to his signature the designation "Fellow of the Casualty Actuarial Society" or "FCAS," or "Associate of the Casualty Actuarial Society" or "ACAS," as the case may be. The member will not use as a signature title the designation "Member of the Casualty Actuarial Society." The member will use a designation dependent upon elective or appointive qualification within the Society such as "President," or "Member of the Council," only when he is acting in such capacity on behalf of the Society.

13. The member will recognize his personal responsibilities under these guides whether he acts as an individual or through a partnership or his employer.

GUIDES FOR THE SUBMISSION OF PAPERS

(AS AMENDED JANUARY 1, 1968)

Method of Review. All papers and reviews of papers are reviewed by the Committee on Review of Papers. The Committee consists of members appointed by the President, plus, ex officio, the Editor of the *Proceedings*. Unanimous vote of the regular Committee is necessary for acceptance of a paper or a review, except that if there is only one vote for rejection, the paper or review will be reviewed by the Editor and accepted if he approves.

Scope and Standards. — 1. Broad latitude will be allowed in the choice of a subject, provided it is a subject of interest to property and casualty actuaries. However, it must be clearly suitable for inclusion in the *Proceedings*.

2. The paper must contain original ideas or new material of reasonable value, unless it has a definite educational value for other reasons.

3. When a paper includes material that the Committee finds it is not qualified to review, the Committee will seek advice or opinion from other members of the Society or from recognized experts outside of the Society.

4. Disagreement by the Committee with opinions of the author or reviewer of a paper will not be a bar to acceptance of an otherwise suitable paper or review. Where, however, the Committee believes a paper or review to be fallacious in logic or misleading in matters of fact, the Committee may reject it. Reviews of papers are expected to be free of criticism of a personal nature. Opportunity will be given to the authors of papers to respond to reviews. Authors' replies will also be reviewed by the Committee and will be treated in the same manner as reviews.

5. The paper or review should show care in preparation. A reasonable minimum standard will be required as to form, clarity, and literary quality. When a paper or review, otherwise acceptable, does not meet these standards, the Committee may return it to the author or reviewer and invite resubmission after editing or rewriting. The Committee may also make suggestions to the author as to possible improvements in an accepted paper.

6. Papers and reviews should be kept within the general limits of length indicated by past acceptances, ordinarily about twenty printed pages for papers and two or three pages for reviews.

Procedures and Regulations — 1. Papers may be submitted only by Fellows or Associates of the Casualty Actuarial Society, except that papers may be submitted by non-members of the Society upon invitation of the President. A member may collaborate in joint authorship with a non-member who possesses particular qualifications in respect to the subject of a paper.

2. Papers should be submitted in quintuplicate to the Secretary-Treasurer of the Society. The name of the author should not appear on the copies of the paper submitted to the Secretary-Treasurer, but should be included in the covering letter. The Secretary-Treasurer is authorized to return to the author copies of a paper that in his opinion are not legible.

3. Reviews of papers and authors' replies to reviews should be submitted in quintuplicate to the Chairman of the Committee on Review of Papers. Names

of reviewers should be identified on the copies of their reviews. The Chairman will return to the reviewer or to the author copies of a review or of an author's reply that in his opinion are not legible.

4. In submitting a paper, the author must answer the following questions on a separate sheet attached to each of the five copies of the paper:

(a) Name of paper.

(b) Has the paper been published elsewhere, in whole or in part, in identical or similar form?

(c) Is the paper being simultaneously submitted elsewhere, or will it be so submitted before decision by the Committee on Review of Papers?

(d) In the case of co-authorship with a non-member, to what extent has the Society member contributed?

(e) If the paper contains factual data from some organization, has the organization given the author permission to publish it?

5. Papers and reviews should be typed double-spaced on letter-size stationery, on one side of each sheet. Tables and footnotes may be single-spaced. Pages should be numbered. Footnotes should be numbered consecutively throughout the paper.

6. Major captions should be centered and typed in capitals; subcaptions should appear in the left-hand margin in italics (single underscore). In technical papers paragraphs may be numbered to simplify reference; in non-technical papers paragraphs should not be numbered.

7. So far as possible, tables should be arranged so that they can be printed on a single page of the *Proceedings* without undue reduction in size of type. Column headings must be clear and concise.

8. All mathematical formulas and symbols should be handwritten in ink rather than typewritten. They must be legible especially as to subscripts and superscripts. There must be no possibility of confusion between, for instance, dx and $d_{\times}$; $\times$ (the sign for multiplication) and x; a and α (alpha). The exclamation point (!) should be used to indicate factorials in binomial expansions. Where necessary, instructions to the the printer may be inserted in pencil on the manuscript. The Committee strongly recommends that authors of mathematical papers refer to the Style Manual of the American Institute of Physics for precise information on preparation of a manuscript. A copy of the Style Manual may be borrowed from the Editor of the *Proceedings* or it may be purchased from the Editor for one dollar. When life contingency symbols are applicable the International Actuarial Notation should be used. This code is described in the *Proceedings*, Vol. XXVI, page 123.

9. References to books and periodicals and to proceedings of professional societies should be sufficiently complete to permit obtaining a copy of the source without additional research.

10. If the manuscript has been prepared carefully in accordance with the foregoing suggestions, there should be only a few minor corrections necessary. The paper as originally submitted should not be considered simply as a draft to which extensive alterations can be made.

11. Authors will be notified of the acceptance or rejection of their papers by the Secretary-Treasurer. If a paper is rejected, original and copies will be returned. The Committee does not promise a decision on a paper submitted fewer than sixty days prior to the meeting for which the paper has been prepared. Reviews of a paper are to be submitted to the author and the Chairman of the Committee on Review of Papers in advance of the meeting at which the paper is to be reviewed. A review of a paper will be considered to have been accepted by the Committee unless the reviewer is otherwise notified.

12. Authors of accepted papers are requested to notify the Secretary-Treasurer whether or not they can supply additional copies for use at meetings or for further distribution prior to publication. (Photographic reproduction is less expensive than printing and insures accuracy.)

13. After acceptance of a paper and before its reproduction, the author should have the following statement typed at the bottom of the first page: "Presented at the (date) meeting of the Casualty Actuarial Society at (city and state). Reproduction in whole or in part without acknowledgment to the Casualty Actuarial Society is specifically prohibited."

14. Except on recommendation of the Committee, no accepted paper will be read in its entirety at a meeting of the Society. The author will be expected to prepare for oral presentation a two or three minute abstract, stating the purposes of his paper and its conclusions.

15. The Editor of the *Proceedings*, in consultation with the author or reviewer, may edit the paper or review prior to publication.

WOODWARD - FONDILLER PRIZE

This award, made in commemoration of Joseph H. Woodward and Richard Fondiller, is intended to stimulate original thinking and research and will be made to the best eligible paper each year submitted by an Associate or Fellow who has attained his designation within the last five years. To be eligible the paper must show evidence of ability for original research and the solution of advanced insurance problems. If no paper is considered eligible in a given year, the award shall not be made. Papers previously submitted to the Society or elsewhere shall not be eligible.

The amount of the prize will be $200 and the papers will be judged by the Society's Committee on Review of Papers, whose decision will be final.

The announcement of the award will be made at the November meeting each year, based on papers submitted to the Society at the previous November and May meetings.

RULES REGARDING EXAMINATIONS FOR ADMISSION

(Effective with 1969 Examinations)

1. Dates of Examinations

Examinations for Parts 1 and 2 will be held twice yearly, in May and November. Effective with the 1969 Syllabus, examinations for Parts 3, 4, 5, 6, 7, 8, and 9 will be held in May 1969. Examinations for Parts 4, 6, and 8 will be held in November 1969 and once a year in November thereafter. Parts 3, 5, 7, and 9 will be held in May 1970 and once a year in May thereafter.

The Secretary-Treasurer will announce the exact dates on which the examinations will be given. It is customary to hold the examinations in such cities as will be convenient.

2. Filing of Application

A candidate who wishes to take Part 1 or Part 2, or both, must make application on the Society's application form, which may be obtained from the Secretary-Treasurer.

A candidate who has previously submitted his application on the Society's application form, and who wishes to take one or more examinations other than Parts 1 and 2, need not again make use of the Society's application form, but may simply write to the Secretary-Treasurer, stating the part or parts for which he is applying.

Each application must be accompanied by the appropriate examination fee, in check, draft, or money order payable to the Casualty Actuarial Society.

Applications must be received by the Secretary-Treasurer by April 1 for the Spring examinations and by October 1 for the Fall examinations.

3. Associateship and Fellowship Examinations

There are five examinations which the candidate must pass in order to become an Associate of the Casualty Actuarial Society. Part 1, the General Mathematics examination, and Part 2, the Probability and Statistics examination, are jointly sponsored by the Casualty Actuarial Society and the Society of Actuaries. Successful candidates will be given credit for these examinations by both Societies regardless of the Society through which the candidate registers.

A candidate may write any one or more of the five examinations and will receive credit for those passed, except that Parts 1 and 2 must be taken in numerical order.

There are four examinations which a candidate must also pass to become a Fellow of the Casualty Actuarial Society. A candidate may present himself for one or more of the Fellowship examinations either if he has previously passed the Associateship examinations or if he concurrently presents himself for and submits papers for all unpassed Associateship examinations given during that examination period. Subject to the foregoing requirements, a candidate will be given credit for any examination which he may pass.

4. Fees

The examination fee schedule is as follows:

Parts 1-3	$ 7.50 for each Part
Parts 4-9	$15.00 for each Part

The examination fees for the partial Part 4 examinations which will be given in 1969 to those candidates requiring them are as follows:

Part 4(a)	$10.00
Part 4(b)	$10.00

Examination fees are payable each time the candidate presents himself. Check, draft, or money order payable to the order of the Casualty Actuarial Society must be received by the Secretary-Treasurer before April 1 for the Spring examinations, or before October 1 for the Fall examinations.

5. Prize Awards

The Casualty Actuarial Society and the Society of Actuaries will jointly award one $200 and four $100 prizes to the five successful undergraduates ranking highest in the General Mathematics examination. These prize awards will be granted for both the Spring and Fall examinations.

6. Credit for Examination Parts under Former Syllabus

A candidate who has passed, or been credited with, one or more of the Associateship or Fellowship examinations under the 1967 Syllabus will receive credit for the corresponding examinations of the 1969 Syllabus in accordance with the following table:

Parts Passed or Credited under 1967 Syllabus	Parts Credited under 1969 Syllabus
Associateship, Part 1	Associateship, Part 1
Associateship, Part 2	Associateship, Part 2
Associateship, Part 3(a)	Associateship, Part 3
Associateship, Part 3(b)	Associateship, Part 4(a)
Associateship, Part 4	Associateship, Parts 4(b) and 5
Fellowship, Part 5	Fellowship, Part 6
Fellowship, Part 6	Fellowship, Part 7
Fellowship, Part 7	Fellowship, Part 8
Fellowship, Part 8	Fellowship, Part 9

Upon application to the Secretary-Treasurer, partial Part 4 examinations will be given in May 1969 and November 1969 to those candidates requiring them in accordance with such credits. Beginning with the 1970 examinations, no candidate will be permitted to write only a portion of the Part 4 examination and any partial credits will expire.

7. Waiver of Examinations for Associateship

Waiver of certain Associateship examinations will be allowed for a candidate who has passed or been credited with corresponding examinations of the Society of Actuaries, in accordance with the following:

Casualty Actuarial Society	Society of Actuaries
Part 1	Part 1, General Mathematics, passed prior to 1963 (before joint sponsorship)
Part 2	Part 2, Probability and Statistics, passed prior to 1966 (before joint sponsorship)
Part 3	Part 4, Life Contingencies, passed prior to 1969
Part 3	Parts 3 and 4 both, if Part 4 is passed after 1968

Candidates who take the Advanced Mathematics test of the Graduate Record examinations may apply for credit for Part 1. Credit will be granted if the candidate's score on the Graduate Record Advanced Mathematics test is equivalent, as determined by the Casualty Actuarial Society, to the passing score on Part 1. An application to the Casualty Actuarial Society for such credit may be completed either in advance of taking the Graduate Record Advanced Mathematics test or within two years after taking it. The necessary application form may be secured from the Secretary-Treasurer of the Casualty Actuarial Society.

The Council may waive, subject to such other requirements as it may prescribe, any examinations of the Casualty Actuarial Society which it deems equivalent to examinations required by another recognized actuarial organization which have been passed by an applicant while not a resident of the United States or Canada, or during his first year of temporary or permanent residence in the United States or Canada.

LIBRARY

All candidates registered for the examinations of the Casualty Actuarial Society and all members of the Casualty Actuarial Society have access to all the library facilities of the Insurance Society of New York, the Casualty Actuarial Society, and the Society of Actuaries. These libraries, with combined operations, are located at 150 William Street, New York, New York 10038.

Registered candidates may have access to the library by receiving from the Society's Secretary-Treasurer the necessary credentials. Books and manuals may be withdrawn from the library for a period of one month without charge. In general, not more than two references may be in the hands of a borrower at one time. The Insurance Society is responsible for postage and insurance charges for sending books to out-of-town borrowers, and borrowers are responsible for the safe return of the books.

Address requests for books to:

Librarian
Insurance Society of New York
150 William Street
New York, New York 10038

SYLLABUS OF EXAMINATIONS

(Effective with 1969 Examinations)

ASSOCIATESHIP

Part	*Time Allowed*	*Subject*
1	3 hours	General Mathematics (jointly sponsored with the Society of Actuaries)
2	3 hours	Probability and Statistics (jointly sponsored with the Society of Actuaries)
3	2 hours	Compound Interest and Life Contingencies
4	3 hours	(a) Principles of Economics: Theory of Risk and Insurance (b) Insurance Coverages and Policy Forms
5	3 hours	(a) Principles of Ratemaking (b) Insurance Statistics and Data Processing

FELLOWSHIP

6	3 hours	(a) Insurance Law; Supervision, Regulation, and Taxation (b) Statutory Insurances
7	3 hours	(a) Insurance Accounting and Expense Analysis (b) Premium, Loss, and Expense Reserves
8	2 hours	Individual Risk Rating
9	3 hours	Advanced Insurance Problems

1967 SYLLABUS

(Applicable in 1967 and 1968)

ASSOCIATESHIP

Part	*Section*	*Subject*
1		General Mathematics
2		Probability and Statistics
3	(a)	Elementary Life Insurance Mathematics
	(b)	General Principles of Insurance; Insurance Economics and Investments
4	(a)	Insurance Coverages and Policy Forms
	(b)	General Principles of Ratemaking

FELLOWSHIP

5	(a)	Insurance Law; Supervision, Regulation, and Taxation
	(b)	Statutory Insurances
6	(a)	Premium, Loss, and Expense Reserves
	(b)	Insurance Accounting and Expense Analysis
7	(a)	Individual Risk Rating
	(b)	Problems in Underwriting and Administration
8	(a)	Insurance Statistics and Machine Methods
	(b)	Advanced Problems in Ratemaking

INTERNATIONAL CONGRESS OF ACTUARIES

The first International Congress of Actuaries was held in 1895 in Brussels. Since that time numerous Congresses have been held, and many actuaries from the United States and Canada have been benefited by attendance at the Congresses and by the printed *Proceedings*, in which numerous valuable articles have appeared.

Continuity in the arrangements for successive Congresses is achieved by the maintenance of the Comité Permanent des Congrès Internationaux d'Actuaires with headquarters in Brussels, membership of which is on an international basis.

Membership in the Permanent Committee on this continent is divided into two sections, a United States section and a Canadian section. Individual actuaries can support the work of the Permanent Committee by becoming members in their section. Inquiries regarding the Permanent Committee should be directed to Edward A. Lew, Secretary for the United States Section, Metropolitan Life Insurance Company, New York, New York 10010, or to Samuel Eckler, Secretary for the Canadian Section, Eckler Brown & Company, Ltd., 789 Don Mills Road, Don Mills, Ontario.

According to the revised regulations adopted by the New York Congress in 1957, the objects of the Permanent Committee are:

1. To promote or to conduct work or research of interest to the science or practice of the actuary. For this purpose sections formed by a number of members for study of special problems may be recognized. Each section will have its own regulations, previously approved by the Council; it will elect its Committee, except for the member appointed by the Council on the Committee.

2. To publish periodically a *Bulletin:* (a) bringing together technical, legislative, statistical, and juridical information related to actuarial science; (b) reviewing publications and works which appear in various countries, bearing upon actuarial matters.

3. To cooperate with the Organizing Committees in preparing the work of International Congresses, and in the publication of their *Proceedings*.

The XVIIIth International Congress of Actuaries will be held in Munich, Germany, from June 4 to June 11, 1968.

Membership on the Permanent Committee is one of the requirements for membership in a Congress. For the 1968 Congress the number of full members is limited to about 800. A record of continuous membership on the Permanent Committee is a favorable factor in considering eligibility for membership in a Congress. The annual dues for membership are 150 Belgian francs. The Permanent Committee wishes to enlist members as broadly as possible.

ASTIN SECTION

ASTIN (Actuarial Studies in Non-Life Insurance) is the first section of the Permanent Committee to be formed under the modification of the rules approved at the XVth International Congress in New York and is for the study of the application of modern statistical and mathematical methods in the field of non-life insurance. It has grown from the desire expressed by many members of the XIVth Congress held in Madrid to provide an effective interchange of ideas on an international basis.

It has as its object the promotion of actuarial research in general insurance and establishes contact between actuaries, groups of actuaries, and other suitably qualified persons interested in this field.

This section, from time to time, publishes papers on topics related to its objects and also publishes a *Bulletin* containing notes of general interest to members.

Meetings are held every four years, during the course of the International Congress of Actuaries. Between meetings colloquia are held on topics of interest to the section, and these are hosted by national actuarial bodies. The 1966 Colloquium was held in Arnhem, Netherlands, September 28 to October 1, sponsored by the Dutch Actuarial Institute.

The next ASTIN meeting will be held during the meeting of the International Congress of Actuaries, June 4 to 11, 1968, in Munich, Germany. Following this meeting of the Congress there will be a continuing ASTIN Colloquium in West Berlin, Germany, June 12 to 14, 1968.

The members of the Committee of ASTIN are:

Chairman	Norton E. Masterson — U.S.A.
Vice Chairman	Jean Sousselier — France
Secretary	Peter J. H. Green — Great Britain
Treasurer	Paul Thyrion — Belgium
Members	Hans Ammeter — Switzerland
	Paul Johansen — Denmark
	Giuseppe Ottaviani — Italy
	Ingvar Sternberg — Sweden
	C. P. Welten — Netherlands

Membership fees, which are payable in the same manner as the annual dues for membership on the Permanent Committee, are 250 Belgian francs. Inquiries regarding membership in the ASTIN Section should be directed to Albert Z. Skelding, Secretary-Treasurer, Casualty Actuarial Society, 200 East 42nd Street, New York, N. Y. 10017.

AMERICAN ACADEMY OF ACTUARIES

The American Academy of Actuaries was organized on October 25, 1965 as the culmination of efforts on the part of the four actuarial bodies of the United States — the Casualty Actuarial Society, the Conference of Actuaries in Public Practice, the Fraternal Actuarial Association, and the Society of Actuaries. The Academy is the vehicle which will lead eventually to the legal recognition of actuaries. Fellows of the Casualty Actuarial Society as of October 25, 1965 who are residents of the United States automatically became members of the Academy unless they submitted a written declination prior to December 24, 1965. Members of the Casualty Actuarial Society who did not automatically become members of the Academy on October 25, 1965, but who have had seven years of experience in responsible actuarial work, may submit applications for membership in the Academy. Applications may be obtained from the Secretary, 208 South LaSalle Street, Chicago, Illinois 60604.

OFFICERS

Past President	Henry F. Rood
Past President	Thomas E. Murrin
President	John H. Miller
President-Elect	Wendell A. Milliman
Vice Presidents— Two-Year Term	Edward D. Brown, Jr.
	William Leslie, Jr.
Vice Presidents— One-Year Term	Frank J. Gadient
	Allen L. Mayerson
Secretary	Norton E. Masterson
Treasurer	Robert E. Bruce

DIRECTORS: In addition to the Officers, for terms expiring at the following Annual Meetings:

1970	1969	1968
Harold E. Curry	Donald F. Campbell	W. E. Groves
J. Stanley Hill	Gilbert W. Fitzhugh	Victor E. Henningsen
Morton D. Miller	Walter Klem	William Leslie, Jr.
John S. Rudd, Jr.	Robert J. Myers	Daniel J. McNamara
Pearce Shepherd	Paul T. Rotter	Walter L. Rugland
H. Raymond Strong	Mary Cusic Wilson	Andrew C. Webster

FUTURE MEETINGS
OF THE CASUALTY ACTUARIAL SOCIETY

1968 Spring Meeting—May 19, 20, 21, 22
Kutsher's Country Club
Monticello, New York

1968 Annual Meeting—November 17, 18, 19
Marriott Motor Hotel
Twin Bridges
Washington, D. C.

1969 Spring Meeting—May 25, 26, 27, 28
Tamiment Resort Hotel
Tamiment, Pa.

1969 Annual Meeting—November 16, 17, 18
Regency Hyatt House
Atlanta, Georgia

1968 EXAMINATIONS

May 8, 9, 10, 15, 1968
November 13, 1968 (Parts 1 and 2)