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CONTENTS OF VOLUME LII

CONTENTS OF VOLUME LII (Cont.)

NOTICE

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THE 1965 TABLE M

LEROY J. SIMON

An Argument Against the Empirical Method
Some haystacks don't even have any needle.

.... William Stafford

Ours did.
We found it.

The purpose of this paper is to set forth in an orderly fashion, a summary of certain aspects of the work done in compiling the 1965 version of Table M. It will be assumed that the reader has a basic knowledge of Table M and its use in obtaining insurance charges for retrospective rating plans.[1] This report will be presented in a fashion that will imply that we went straight as an arrow from the problem to the solution. The reason for doing this is to present a clear and easy to follow line of reasoning. I'm sure it is realized that the work could not and did not proceed in exactly this manner. Many false leads were pursued, and oftentimes decisions would be made near the end of the work which affected things done near the beginning. The earlier work would then be redone. Part of the reason for creating this written record is that the future researcher will have an easier time following the development. Through footnotes and appendices, I hope to indicate some of the areas where we investigated ideas and rejected them, where expediency prompted us to accept the results produced and where we could hope for improvement in the future.

On September 13, 1962, the subject of a revision for Table M was placed on the agenda of the Actuarial Committee of the National Council

[1] Basic reference papers on the compilation of excess pure premium ratios (that is, Table M) are Lewis H. Roberts, "Graduation of Excess Ratio Distributions by the Method of Moments," *PCAS Vol. XLIV*, pg. 45; Nels M. Valerius, "Risk Distributions Underlying Insurance Charges in the Retrospective Rating Plan," *PCAS Vol. XXIX*, pg. 96; and Thomas O. Carlson, "An Actuarial Analysis of Retrospective Rating," *PCAS Vol. XXVIII*, pg. 283. A technical definition of the excess pure premium ratio will be found in this paper as equation (C12) in Appendix C.

on Compensation Insurance. Aetna Casualty and Surety Company, American Mutual Liability Insurance Company, Insurance Company of North America and Liberty Mutual Insurance Company were appointed to a Subcommittee to Review Table M. Also attending all meetings and participating in the discussions was a representative of The Travelers Insurance Company.

Table M was last revised in 1954. Unfortunately, no paper was presented to the Society at the time of this revision, and the only convenient reference is the files of the National Council on Compensation Insurance. At times, it is difficult to differentiate between the proposals made and the ideas that were actually put into effect. However, it was clear that this revision was made to reflect the increased variance in the risk distributions since the 1943 table of the National Council was published. Basically, the table itself was not changed, but the column headings were revised so that the variance of risks having the expected losses shown at the top of a given column was equal to the variance of the underlying risk distribution for that column in Table M. Basic raw data, therefore, was only used to the extent of determining the variance for a given expected loss size, and was not used to actually calculate excess pure premium ratios.

It was decided that the 1965 revision of Table M would be based on actual workmen's compensation risk distributions. Individual risk information was gathered for policy year 1960-61 from all states where the National Council on Compensation Insurance is the filing agency plus the states of California, Hawaii, Massachusetts, Michigan, Montana, New York, North Carolina, Virginia and Wisconsin. The columns of charges[2] in the 1964 Table M were numbered from 1 through 37, and the expected loss ranges were converted to premium ranges by dividing by .596 (the permissible loss ratio most commonly used throughout the country). The risks were then sorted by standard premium and assigned "Old Premium Group" numbers on this basis (risks having a premium less than $1,678 were excluded). Referring to Exhibit A, we see that experience was found in each of the first 36 premium groups of the 1954 Table M. All risks with standard premiums between $1,678 and $1,987 were included in the first group, and the last group included risks with standard premiums between $608,221 and $2,307,046. A total of 112,646 risks were processed with an aggregate premium of $855,278,990.

Four interesting statistics are included on Exhibit A to help visualize

[2] I will use "excess pure premium ratios" and "charges" interchangeably. The European term "stop-loss premium" could also have been used.

the data better: (1) the loss ratio; (2) the charge at a ratio of 1.00, that is, at the mean; (3) the percent of risks falling below the mean and (4) the median value of the ratio. The last three statistics are based on the adjusted ratio of actual losses to expected losses, r, which will be described later.

Exhibit A also shows the raw data average loss per risk which is the quotient of the total actual unlimited losses in the premium group divided by the number of risks. A study of loss trends over the years has indicated that the average cost per case in workmen's compensation is steadily increasing. This is expected because of more costly medical bills, plus the fact that legislation has tended to increase the value of all the benefits. Exhibit B shows a six year trend of workmen's compensation average loss costs. This indicates a 33.7% increase in cost over a five year period. There is to be a five year gap between the period used in collecting the basic statistical information (1960) and the time when the new Table M will first go into effect (1965). Today's workmen's compensation risk, therefore, having a given number of expected losses, will be roughly 35% larger in terms of dollars of expected loss. It is proven in Appendix A that this means that average losses per risk should be modified 35% assuming the coefficient of variation of the claims distribution remains constant. Countering this argument is the slightly improved claim frequency experience resulting from improved safety measures and technological changes. However, it was felt that this was more than offset by other factors.

Another factor behind the modification was that the raw data was based on first reports under the unit statistical plan. It is well-known that the major developments from first to second reports occur in the area of large claims.[3] This is almost certain to increase both the variance and the skewness of the distribution. Hence, we felt that the raw data risk distributions were undoubtedly more compact than the truth would show if it could be known. It was, unfortunately, not financially feasible to make a comparison of risk distributions between first and second reports under the unit statistical plan. Somewhat countering this argument again is the fact that unit reports split large interstate risks into smaller intrastate pieces. Because experience rating is on an interstate basis, the pieces are being adjusted in the direction of the mean of the entire risk. Therefore, we expect the basic data to have a little larger variance than if the smaller pieces had been subject to experience rating on their own. As a result of all these con-

[3] For example, a special study of Serious losses for policy year 1960-61 totaling $150,000,000, showed a development factor of 1.146 from first to second reports.

siderations, we arrived at E, the expected loss (current level) shown on Exhibit A, by modifying the raw data average loss per risk by a factor of 1.35. Three significant digits were retained here and were also used in the final column headings on Table M.

The work now progressed toward calculating and tabulating a column of insurance charges for each of the 36 premium groups. Working on one group at a time, the standard premium (P) for each risk was multiplied by .596, and the ratio of the actual losses (A) to the standard premium times .596 was calculated, rounded to two decimal places and designated R. The risks were then sorted on R, and each premium size group was tabulated as shown in Exhibit C. The standard premium and unlimited losses were shown for reference purposes and for checking. Two additional quantities, Sum 1 and Sum 2, were also calculated on the first pass of the cards. The first is simply a downward accumulation of the number of risks, and the second is the calculated number of "points" of excess over the ratio shown opposite R. This can be expressed as follows:

$$S_{2,i} = S_{2,i+1} + (R_{i+1} - R_i)\, S_{1,i+1}$$

where S_1 and S_2 are Sum 1 and Sum 2 respectively, and $i = 0, 1, 2, \ldots, \omega$; that is, it is a sequential numbering of the R's from the $.00$ end of the table up to the limiting value ω.[4] Sum 2 continues until it reaches R = $.00$. (If a case did not occur with zero losses, such a card was put in the deck with a zero as the number of risks.) $S_{2,0}$ then contained the total number of points of excess over 0, that is, the sum of the frequencies times the ratios. The mean ratio is the quotient of $S_{2,0}$ divided by the total number of risks.

For Table M purposes this mean ratio should be exactly 1.00 and, therefore, a correction factor was applied to R which would adjust the mean to unity. Making this adjustment is equivalent to saying that we would accept the actual loss ratio of the entire group as being the best estimate of the expected loss ratio of the group. It also fitted in properly with theoretical considerations of Table M which will be referred to later. The adjusted ratios were called r and were used hereafter. On the second pass of the cards, two calculations were made at the same time. The first was to calculate r and the second to calculate the Table M charge, $\phi(r)$, by dividing Sum 2 by the entry opposite Sum 2 at $r = .00$.[5]

[4] In actual practice the tabulation was made in a slightly different fashion which included Sum 1 as a part of Sum 2. However, in dealing with the material it was noted that the system outlined could have been used equally well and perhaps with a little advantage in calculating simplicity.

[5] See Appendix B for a proof that the scale translation is permissible and for discussion of the interpolation problem when there are missing values.

A graphic plotting of some of this data across a common entry ratio indicated that there was a fair degree of regularity, but there were certain fluctuations. Exhibit D indicates how these occurred at an entry ratio of $r = 1.60$. It was decided that a preliminary smoothing would be accomplished by graduating the entries across a common entry ratio using the Whittaker-Henderson formula.[6] Although the Whittaker-Henderson formula is based on the assumption of equally spaced data, it was applied to this material. In the lower premium group sizes this is not too disturbing since there are approximately even intervals between the values of E. However, as one goes from Old Premium Group 21 to Old Premium Group 22, the jump in E *is* significantly different than it had been up to that point. There is a similar sharp change at Old Premium Group 32. It will be developed later in this report that the smoothed data under the Whittaker-Henderson smoothing was only used up through Old Premium Group 31. The small circles on Exhibit D indicate the effect of this smoothing on the data. Calculations were made for $r = .20(.20)5.00$. We thus had a compilation of insurance charges smoothed under the Whittaker-Henderson formula which were arrayed in a matrix of 36 premium groups by 25 entry ratios.[7]

The actual production of an entire Table M is a mammoth job even under the best conditions. We expected to compute insurance charges from entry ratios of .00 through 3.00 for publication. We also expected to have in the neighborhood of 50 premium groups requiring the production of some 15,000 insurance charge values. An attempt to do this through any method such as graphing or interpolating between selected points seemed beyond the realms of possibility. We were particularly interested in producing a table as promptly as possible while exercising a minimum number of independent judgments. Constructing this Table M has certainly given all those who were involved in the work a great deal of appreciation and respect for those who constructed the original Table M values. It seemed likely that if we could use the experience data we had

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[6] Valerius, op. cit. pg. 107-110. With the advantage of hindsight, we can now question whether this preliminary graduation across premium groups was altogether necessary. Since we were successful in being able to obtain formulas to represent the columns of charges, we probably would have been just as successful in dealing with the raw data as we were in dealing with the Whittaker-Henderson smoothed data. However, at the time, it seemed that this procedure would give our other formulas a maximum opportunity for success; and at the same time, if they were unsuccessful, would provide the starting point from which the table could be constructed.

[7] Early experience using desk calculators showed that the results using 25 points were as satisfactory as the results using 51 points on the curve fitting technique under test. We did not experiment to see if the 25 could have been reduced further.

accumulated and derive some formulas from it, we could then be in a position to reformulate the premium groups in some fashion that would be more to our liking. Finally, we could foresee the possibility of using high speed computing equipment to evaluate retrospective rating values if the Table M charges could be determined by the computer through the use of mathematical formulas. Because of these many advantages, a considerable amount of effort was spent in the search for "down-the-column" graduation formulas.

Perhaps as many as 25 different general equations were explored as possible mathematical formulas to describe the column of insurance charges, $\phi(r)$. Some were rejected very promptly because they did not appear to offer a sufficient amount of flexibility or because determining the constants necessary appeared to be an insurmountable task. Nine equations seemed to meet most of the subjective requirements, and these were thoroughly tested mathematically in accordance with the development in Appendix C. From a mathematical standpoint and from preliminary desk calculator tests which were made, two formulas were processed for electronic computer programming and thus considered eligible for the final selection. These two formulas were:

$$\phi(r) = \exp\left[-(r + a_2r^2 + a_3r^3 + a_4r^4 + a_5r^5 + a_6r^6 + a_7r^7)\right] \qquad \text{..........(1)}$$

$$\phi(r) = 1/(1 + r + b_2r^2 + b_3r^3 + b_4r^4 + b_5r^5 + b_6r^6 + b_7r^7) \qquad \text{...........(2)}$$

(Work was begun on Equation (C28), Appendix C, but was stopped when we decided to accept another equation as final.)

A computer program was written so that for a given premium group the computer would read in the 25 values of r and the corresponding smoothed data values for the insurance charge. When working with Equation (1), the computer would transform the smoothed charge, $\phi_s(r)$, as follows:

$$Y(r) = -\left[r + \ln \phi_s(r)\right]/r^2$$

We were thus able to rewrite Equation (1) as:

$$Y(r) = a_2 + a_3r + a_4r^2 + a_5r^3 + a_6r^4 + a_7r^5$$

The computer then solved the problem of obtaining the six coefficients a_2 through a_7 such that we would have a least squares best fit using r and $Y(r)$ as the two variables. Using these fitted coefficients, the computer then produced the 25 fitted values. By comparing the fitted values with the original smoothed values, the percentage error was calculated. This is all set out in Exhibit E. It should be noted that in calculating the percentage errors, the computer was working with eight significant digits, although the fitted values which were printed out were only shown to four decimal places. Therefore, the reader will not be able to exactly reproduce the percentage error figures shown in the exhibit.

On the same pass of the data, the information was transformed for Equation (2) as follows:

$$Z(r) = \left[\frac{1}{\phi_s(r)} - 1 - r\right] \Big/ r^2$$

We were thus able to rewrite Equation (2) as:

$$Z(r) = b_2 + b_3 r + b_4 r^2 + b_5 r^3 + b_6 r^4 + b_7 r^5$$

Again, the coefficients were determined by a least squares best fit to the points r and $Z(r)$, the fitted values for the 25 points were produced by the computer and error calculations were made.

As we examine Exhibit E, there is little choice between the two formulas. Out to three decimal places, they produced the same result in nearly every case. However, the pattern shown on that exhibit was generally repeated throughout the first 22 premium groups and indicated that Equation (2) had a slight edge over Equation (1). From Old Premium Groups 23 through 28, Equation (1) was slightly better, and beyond Old Premium Group 28 neither one of the two equations produced satisfactory fits. It was decided that only one general form of equation would be used, and Equation (2) was selected.

In an attempt to extend the area over which the curves would satisfactorily fit, Equation (2) was revised to include an additional term and thus read.[8]

$$\phi(r) = 1/(1 + r + b_2 r^2 + b_3 r^3 + b_4 r^4 + b_5 r^5 + b_6 r^6 + b_7 r^7 + b_8 r^8) \qquad \ldots\ldots(3)$$

An example of this output for Old Premium Groups 10 and 11 is shown on Exhibit F.

The computer run was completed through Old Premium Group 34

[8] Originally, we were unsure whether either Equation (1) or Equation (2) would perform satisfactorily. The original computer programming which was based on orthogonal polynomials dealt with the input in a coded fashion such that the 25 values were sequentially numbered from —12 through +12. The coefficients which the computer actually determined were thus in this coded translation and were not directly usable. It was, therefore, necessary to do some additional programming to get the coefficients into a satisfactory form. We decided we would rewrite the entire job, and at the same time, omit any Equation (1) calculations and extend Equation (2) one more term.

and the results were examined for closeness of fit. In the first two premium groups it was observed that the curves were producing values which implied negative frequencies for the underlying distrubution in the neighborhood of $r = 1.8$. This also occurred in Old Premium Group 3 in the neighborhood of $r = 3.8$. Other than this, the fit of the formula value to the smoothed value was quite good up through Old Premium Group 28. In Old Premium Groups 29, 30 and 31, the formulas were not fitting too closely at $r = 1.4$. The errors were in the direction such that the formulas were coming closer to the raw data on both Old Premium Groups 29 and 31, and we decided to retain the formula approach all the way through Old Premium Group 31. At Old Premium Group 32, it was felt that the results were wholly unsatisfactory and, therefore, nothing beyond this point would utilize the formulas produced by the least squares best fit of the smoothed data.[9]

A basic decision was made at this point about reformulating the entire format of Table M. It was decided that the premium groups would be reformulated on such a basis that the insurance charges at an entry ratio of $r = 1.00$ would be spaced at intervals of .010 between premium groups. For reference purposes the new columns of Table M were referred to as Premium Group .64 through Premium Group .01 where the premium group number identified the first two digits of the charge at an entry ratio of 1.00.[10] The coefficients for the equation of a given premium group were determined by interpolation using the coefficients determined by the raw data. Exhibit G illustrates the method of computation[11] for premium Group .43, and Exhibit H sets forth the coefficients which were so determined for Premium Groups .64 through .21.

We next turned our attention to the problem of extending the tables beyond Premium Group .21. Referring to Exhibit A, we can see that the number of risks became quite small from Old Premium Group 32 onward, and we were not surprised to find random fluctuations playing a larger part. Appendix D gives the development of a technique to pro-

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[9] The formula approach was extended further on a different basis as will be discussed later.

[10] The raw data furnished us with coefficients for Premium Groups .21 through .61. We further extrapolated the coefficients successfully to produce Premium Groups .62, .63 and .64.

[11] A graphic method of interpolation was attempted at one point in our studies, but was rejected when we felt a need for more significant digits than could be so obtained. The coefficients fell along surprisingly smooth curves when plotted against a $1/\phi(1.00)$ abscissa, and we were convinced that the interpolation technique of Exhibit G was quite satisfactory.

duce underlying risk distributions and, hence, charges using Pearson Type III curves.[12] The statistical foundation for this method is also given. In brief, the method consists of graduating the moments of the raw data distributions and then using these moments to produce Type III curves from which the charges are calculated.

In making some of the final checks, we found that the two approaches (reciprocal polynomials and Type III curves) met in such a fashion that the gap between the two could not be bridged in any rational fashion. We found that the cross-differences in the area of $r < 1.00$ were generally too large, and the results produced net insurance charges in retrospective rating which increased as the size of the risk increased. An occasional anomaly of this sort of .001 or .002 might be allowed, but these inversions were both large and frequent. Therefore, to eliminate this problem we used Premium Groups .21 and .11 as fixed end points and performed a linear interpolation between these two groups to produce the values for Premium Groups .20 through .12.[13] The material in Appendix D is included partly for possible future use and partly because our findings closely parallel those reported by Bohman and Esscher.[14]

The final set of premium groups remaining were those from Premium Group .11 through .01. Using the Type III curve we found that the tabled values could only carry us as far as Premium Group .12. The limit of the Pearson tables is at $p = 50.0$ at which value we produced a $\phi(1.00) = .1119$. The curve was beginning to approach normality, and I felt we should swing over to a normal curve at Premium Group .11. This would be out in an area of about $800,000 of expected loss which is beyond the size of the largest risk we had in the raw data. Appendix E sets forth the rationale and the technique used in utilizing the normal curve.

This, then, marked the end of the main effort on producing the columns of charges for the new Table M. Two peripheral areas remained.

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[12] We also tried experimenting in the small expected loss sizes with (a) Pearson curves, (b) the lognormal curve and (c) transformations which would normalize the data. These met with varying degrees of success, but none seemed satisfactory in the final analysis.

[13] We also tried to retain the Type III area and smooth out the irregularity by modifying the polynomials from Premium Groups .28 to .21. Time was very short at this point, and three quickly conceived methods each failed to produce the desired improvement. We did not experiment further.

[14] Bohman, H. and Esscher F., "Studies in Risk Theory with Numerical Illustrations Concerning Distribution Functions and Stop Loss Premiums," *Skandinavisk Aktuarietidskrift*, 1963. A two page summary by Mr. Bohman of this 92 page paper is found in *The Astin Bulletin, Vol. III, Part II*, August, 1964, p. 185.

The first was the problem of extending every column out to a charge of zero. The use of these extended values is so rare that they were dealt with rather arbitrarily by using a straight line set of values of the form:

$$\phi(r) = c - mr \qquad \text{.........(4)}$$

For completeness, these formulas are listed in Exhibit I.

The second peripheral area was the desire to have two more premium groups calculated because of their special nature. Tabular retrospective plans have normally started at an expected loss of $596 (i.e., premium = $1,000). It will later be shown that Premium Group .85 is the one appropriate to $E = \$596$. To help guide our judgment in establishing this premium group, it was observed that if we wanted to place 20 risks at appropriate points along the r scale so they would closely reproduce Premium Group .64, they would be located at *.00, .01, .02, .03, .04, .05, .06, .08, .09, .12, .14, .17, .21, .34, .84, 1.00, 1.27* and at two points which are beyond the maximum usable r of our Equation (3). To formulate Premium Group .85, this information was considered and it was decided to place eight risks at *.00,* five risks at *.05,* four risks at *.10* and one risk at *5.00*. The other two risks were to be at r_{18} and r_{20}. These two points are used to fix the conditions that the mean equals 1.00 and $\phi(1.00) = .850$; that is, $[8(.00) + 5(.05) + 4(.10) + 5.00 + r_{18} + r_{20}]/20 = 1.00$ and $[(5.00 - 1.00) + (r_{20} - 1.00)]/20 = .850$.[15] Solving, $r_{18} = .35$ and $r_{20} = 14.00$.

The column of charges could then be produced from these values and is described in Exhibit J.

We also sought a more or less limiting set of values. A Premium Group .99 was constructed by assuming 99 of 100 risks had a zero entry ratio which meant the one was at $r = 100.00$. This simple column of charges is also described in Exhibit J. An expected loss of $3 was attached to this premium group because one average Workmen's Compensation loss is about $300.

Now that the premium groups have been reformulated from the 1954 premium group numbers to the new system, it becomes necessary to establish the expected loss ranges for the new groups. The problem breaks down into roughly three areas. In the first, we have formula (E1) from

[15] This assumes that r_{20} will lie beyond 1.00 and r_{18} will not lie beyond 1.00 which is borne out as being true by the subsequent solution.

Appendix E which associates the *midpoint* of the premium group with the expected loss size by the formula

$$\bar{E} = 63200/\hat{\sigma}^2$$

where $\bar{E}$ denotes the midpoint value. Let's define g as the premium group number and g' as the number which divides two premium groups, i.e., $g' = g + .005$ divides g from $(g + .01)$, thus establishing the lower limit of the expected loss range for g which will be called E'_g. Hence, we can write:

$$E'_g = 63200/\hat{\sigma}_{g'}{}^2$$

It was observed that, from Appendix E,

$$\hat{\sigma}_g = 2.5g$$

and it was inferred that we could validly write:

$$\hat{\sigma}_{g'} = 2.5g'$$

By substitution,

$$E'_g = 10112/(g + .005)^2$$

This formula applied to Premium Groups .01 through .10. The result for Premium Group .11 (765,000) was modified on a judgment basis (to 725,000)in order to make the transition run smoothly from this segment to the next.

The second area for establishing expected loss ranges was that in which Equation (D5) in Appendix D could be helpful. Although the Type III curves were not used to obtain the insurance charges, they were still the best guide to locating the expected loss ranges. A graph was drawn on semi-logarithmic paper of $1/\phi(1.00)$ vs. $\bar{E}_g$ as calculated from:

$$\bar{E}_g = 53400(p_g + 1)$$

These points appear as small circles on Exhibit K and can be seen to fall almost perfectly along a straight line. The line was drawn on the graph and its equation, using the two point form, was found to be

$$\log E'_g = 3.5510 + .2730/g'$$

where "log" designates the common logarithm. It was used over the range from Premium Group .12 through .24.

The final area of consideration is shown on the graph as lying below $1/\phi(1.00) = 4.0$. The values of E and $\phi(1.00)$, as shown in Exhibit A, were used to plot the small crosses on Exhibit K.[16] It appeared to the

[16] Several other possible plottings were also considered, but this seemed the most satisfactory.

eye that the data fell into two sections where straight lines would fit the points rather well.[17] A formula was preferred over simply drawing a line and reading out the values, because (a) we wanted values to three significant places, and (b) the line could best be drawn statistically. Therefore, Old Premium Groups 27 through 8 were used to determine a best fitting least squares line as

$$\log E_g' = 2.6651 + .4955/g'$$

which was used over the range from Premium Group .25 through .46.

Similarly, Old Premium Groups 7 through 1 were used to determine a best fitting least squares line as

$$\log E_g' = 1.6363 + .9747/g'$$

which was used over the range from Premium Group .47 through .64.[18] Carrying out this entire set of evaluations for expected loss ranges results in Exhibit L.

Very little space can be devoted to commenting on the gigantic task performed by the people who programmed the IBM 7080 in the offices of the Insurance Company of North America. They accomplished a quantity and quality of work which could not have been done by desk calculator methods "in a hundred years." The reader undoubtedly recognizes the scope of the work involved in solving the least squares fit of a sixth degree equation, and we solved 150 of these problems in the course of this project. The calculation of one charge using Equation (3) takes about five minutes with a desk calculator, and we calculated about 50,000 of them during this study. Joan Featherer did the majority of the programming using FORTRAN and programmed the final print-out of the table in such a well designed and executed manner that reviewing the results was made quite easy. The final running and testing of the table was done in a single program which accomplished a number of important steps. Using the polynomial formula, the computer calculated the insurance charge. If the entry ratio was less than 1.00, it also computed the saving from Equation (C23). If the saving was negative, zero was substituted for the calculated value and the charge was set equal to one minus the entry ratio. The values were rounded to three decimal places and were

[17] A single concave parabola might also have fitted closely but was not tried. Looking back, this would have been better since we did run into a little "roughness" where the lines crossed. A single curve would have avoided this.

[18] It was also extrapolated and used to show that $E = \$596$, $g' = .856$ and $g = .851$. Therefore, Premium Group .85 applies to $E = \$596$.

written out on tape. At the same time, the first and second differences down the column of charges were calculated so that we could readily examine the underlying risk distribution. One of the critical tests was to make certain that the charges produced did not imply the existence of negative frequencies. For example, the reason that Premium Group .64 can only use the polynomial equation out to $r = 1.74$ is that beyond that point the polynomial would imply that there was a negative number of cases over certain ranges.

Exhibit M is an extract of the computer output showing the charges and the savings (which are marked with an asterisk on the tabulation). Exhibit N is an extract of the tabulation of the first and second differences running down the column. For ease of examination these were multiplied by 1000 in order to get them to be whole numbers. Finally, Exhibit O shows the results of the third major calculation that was made on this single pass of the data. In order to test the requirement that the charges at a common entry ratio should decrease as the expected loss increases, we calculated the first differences (again multiplied by 1000) between adjacent columns. A quick examination of this run showed that there were no negative figures and, in fact, the pattern seemed to be relatively smooth from one set of differences to the next. Similar calculations were made in the areas in which the polynomial did not apply, but were made using the desk calculators.

Exhibit P is the statement in FORTRAN language of the heart of the computer calculation of the insurance charge in the areas where the polynomial is applicable. It is assumed that computations in a retrospective rating plan evaluation have reached the point where it is necessary to evaluate $\phi(r)$. The formula of Exhibit P is one of the numerous possible ways of writing the statement in IBM 7080 FORTRAN language so as to reproduce the Table M charge. Finally, Exhibit Q is a ready reference for the premium group numbers and the entry ratios over which each of the formulas applies. The possibility of computing Plan D ratings of retrospective risks through the use of computers now seems wholly feasible.

In conclusion, it might be of interest to note the effect of this revision of the Table. One way to do so is to compare $\phi(1.00)$ under the 1954 Table and the 1965 Table. Exhibit R shows this comparison. It can be seen that the change is largest in the small premium sizes and decreases as the size of the risk increases to the point where it is a reduction at the highest sizes. A similar comparison can be made at $\phi(1.60)$ by reference to Exhibit D. A second means of comparison is shown in Exhibit S where we can visualize the underlying distributions. Notice the close agreement between the raw data and the 1965 Table M and the change in shape from the distribution underlying the 1954 Table M.

Work on the 1965 Table proceeded with a sense of urgency because the 1954 Table was known to be deficient. We made a number of quick decisions and resisted revising certain of them because of the time element involved. Despite all this, it took three years of elapsed time to get the revision into effect. The best time to start the next revision of Table M is *now*.

It was evident as we worked on this assignment, that an improved theory of risk variation would have been of great benefit. The approach used was highly empirical, and we were extremely fortunate to find as many haystacks containing needles as we did. To avoid the difficulties and the pitfalls of empiricism, we should try to borrow from the mathematical theory of risk, from Monte Carlo techniques and from operations research, especially in the area of anti-selection. Let's begin pushing out some frontiers today, so we'll be ready to solve tomorrow's problems.

ACKNOWLEDGEMENTS

I have only written about the phase of Table M in which I was principally interested and involved. There are many other facets to retrospective rating that were reviewed and revised at the same time. Others are much better equipped than I to comment on these areas.

There are many who have worked tirelessly on this overall project. I personally know of the substantial contributions by: Roy H. Kallop, John R. Bevan, Harry T. Byrne, Robert Pollack, Robert A. Bailey, George D. Morison, Harry R. Richards, Stephen S. Makgill, James P. Jensen, John P. Welch, Burton Covitz, Fred M. Chorpita, Daniel J. Flaherty, Joseph F. Martorana, Mrs. Joan M. Featherer and John Craig.

WORKMEN'S COMPENSATION INDIVIDUAL RISK EXPERIENCE
UNLIMITED LOSS DATA

Old Premium Group Number	Premium Range	Number of Risks	Standard Premium	Loss Ratio	Raw Data Average Loss per Risk	E Expected Loss (Current Level)	ϕ(1.00) Charge at Actual/ Expected of 1.00	Proportion of Risks Falling Below the Mean	Median Value of Actual/ Expected
1	1678 - 1987	16,950	30,926,469	60.9%	1,112	1,500	.6240	79%	.19
2	1988 - 2429	16,983	37,272,090	60.3	1,324	1,790	.6005	78	.23
3	2430 - 2871	12,089	31,893,890	60.8	1,603	2,160	.5775	77	.26
4	2872 - 3311	8,907	27,443,432	62.7	1,933	2,610	.5592	76	.30
5	3312 - 3974	9,808	35,556,533	58.5	2,121	2,860	.5279	74	.35
6	3975 - 4856	9,153	40,115,384	59.8	2,621	3,540	.5154	73	.38
7	4857 - 5739	6,372	33,614,135	59.2	3,125	4,220	.4865	72	.42
8	5740 - 6623	4,694	28,954,076	62.0	3,823	5,160	.4758	71	.45
9	6624 - 7506	3,674	25,897,532	59.3	4,180	5,640	.4420	70	.53
10	7507 - 8388	2,856	22,636,320	57.9	4,591	6,200	.4320	70	.54
11	8389 - 9271	2,376	20,940,019	58.8	5,183	7,000	.4240	69	.56
12	9272 - 10155	1,874	18,182,410	59.0	5,727	7,730	.4019	65	.59
13	10156 - 11038	1,664	17,619,552	60.6	6,422	8,670	.4022	67	.61
14	11039 - 11920	1,336	15,314,562	60.2	6,898	9,310	.3781	65	.64
15	11921 - 12804	1,245	15,376,031	58.8	7,260	9,800	.3741	66	.66
16	12805 - 13687	1,040	13,771,549	60.3	7,987	10,800	.3743	66	.64
17	13688 - 14569	856	12,077,797	56.4	7,964	10,800	.3461	63	.71
18	14570 - 15454	798	11,980,889	57.7	8,658	11,700	.3531	64	.68
19	15455 - 16336	711	11,290,884	59.8	9,491	12,800	.3314	63	.72
20	16337 - 17219	658	11,042,869	60.8	10,207	13,800	.3594	67	.67
21	17220 - 19934	1,507	27,890,998	56.4	10,442	14,100	.3240	64	.75
22	19935 - 24736	1,842	40,981,259	57.2	12,735	17,200	.3113	65	.75
23	24737 - 29887	1,210	32,839,117	54.7	14,841	20,000	.2868	63	.80
24	29888 - 35429	899	29,240,852	56.7	18,458	24,900	.2788	60	.81
25	35430 - 41346	610	23,385,219	59.4	22,767	30,700	.2884	64	.78
26	41347 - 47340	492	21,821,261	61.3	27,182	36,700	.2755	64	.79
27	47341 - 53831	353	17,821,787	59.5	30,063	40,600	.2523	62	.83
28	53832 - 60821	334	19,096,583	55.3	31,619	42,700	.2482	59	.84
29	60822 - 72427	328	21,601,968	55.9	36,795	49,700	.2294	60	.85
30	72428 - 90603	319	25,640,792	59.4	47,716	64,400	.2340	60	.85
31	90604 - 117449	267	27,426,578	53.3	54,739	73,900	.1943	59	.90
32	117450 - 159395	178	24,179,267	60.7	82,440	111,000	.1842	62	.89
33	159396 - 226509	123	22,861,752	58.2	108,130	146,000	.1611	57	.96
34	226510 - 322986	75	20,381,237	57.4	155,931	211,000	.1655	64	.91
35	322987 - 608220	44	17,547,482	52.6	209,919	283,000	.1348	57	.92
36	608221 - 2307046	21	20,656,415	52.6	517,503	699,000	.1198	54	.99
		112,646	855,278,990	58.5					

TABLE M

EXHIBIT B

COUNTRYWIDE WORKMEN'S COMPENSATION AVERAGE COST PER CASE

ALL CASES, INDEMNITY PLUS MEDICAL

Policy Period	Average Cost	Ratio to Previous Year	Cumulative Change
1956-57	181.03	Base	1.000
1957-58	197.77	1.092	1.092
1958-59	206.46	1.044	1.140
1959-60	223.00	1.080	1.231
1960-61	230.14	1.032	1.270
1961-62	242.40	1.053	1.337

EXHIBIT C

TABULATION OF RAW DATA

Old Premium Group 1

Standard Premium	Unlimited Losses	R Ratio A/(.596P)	No. of Risks	Sum 1	Sum 2	r Adjusted Ratio	ϕ(r) Charge
1950	206614	177.78	1	1	.00	173.58	.0000
1903	100439	88.57	1	2	89.21	86.48	.0051
1684	63829	63.60	1	3	139.15	62.10	.0080
1958	59617	51.09	1	4	176.68	49.88	.0102
1797	44947	41.97	1	5	213.16	40.98	.0123
1820	43504	40.10	1	6	222.51	39.15	.0128
1937	43849	38.00	1	7	235.11	37.10	.0135
1868	41693	37.46	1	8	238.89	36.57	.0138
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
67236	42789	1.07	37	3474	10656.38	1.04	.6138
43350	27296	1.06	24	3498	10691.12	1.03	.6158
50055	31317	1.05	27	3525	10726.10	1.03	.6179
34125	21065	1.04	19	3544	10761.35	1.02	.6199
45683	27925	1.03	25	3569	10796.79	1.01	.6219
43608	26435	1.02	24	3593	10832.48	1.00	.6240
47706	28573	1.01	26	3619	10868.41	.99	.6260
32649	19412	1.00	18	3637	10904.60	.98	.6281
54544	32111	.99	30	3667	10940.97	.97	.6302
54854	31864	.98	30	3697	10977.64	.96	.6323
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
265980	38016	.24	146	7769	14801.98	.23	.8526
284133	38828	.23	155	7924	14879.67	.22	.8571
326246	42676	.22	179	8103	14958.91	.21	.8617
328726	41082	.21	179	8282	15039.94	.21	.8663
302686	36100	.20	166	8448	15122.76	.20	.8711
331543	37399	.19	183	8631	15207.24	.19	.8760
373757	40091	.18	205	8836	15293.55	.18	.8809
386279	39082	.17	212	9048	15381.91	.17	.8860
425014	40408	.16	233	9281	15472.39	.16	.8912
388311	34605	.15	213	9494	15565.20	.15	.8966
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
740516	30809	.07	405	12115	16408.07	.07	.9451
845263	30105	.06	463	12578	16529.22	.06	.9521
836277	24972	.05	460	13038	16655.00	.05	.9594
887957	21086	.04	487	13525	16785.38	.04	.9669
1061074	18927	.03	582	14107	16920.63	.03	.9747
1054247	12388	.02	576	14683	17061.70	.02	.9828
916101	5675	.01	500	15183	17208.53	.01	.9913
3215866	340	.00	1767	16950	17360.36	.00	1.0000
30926469	18844603		16950				

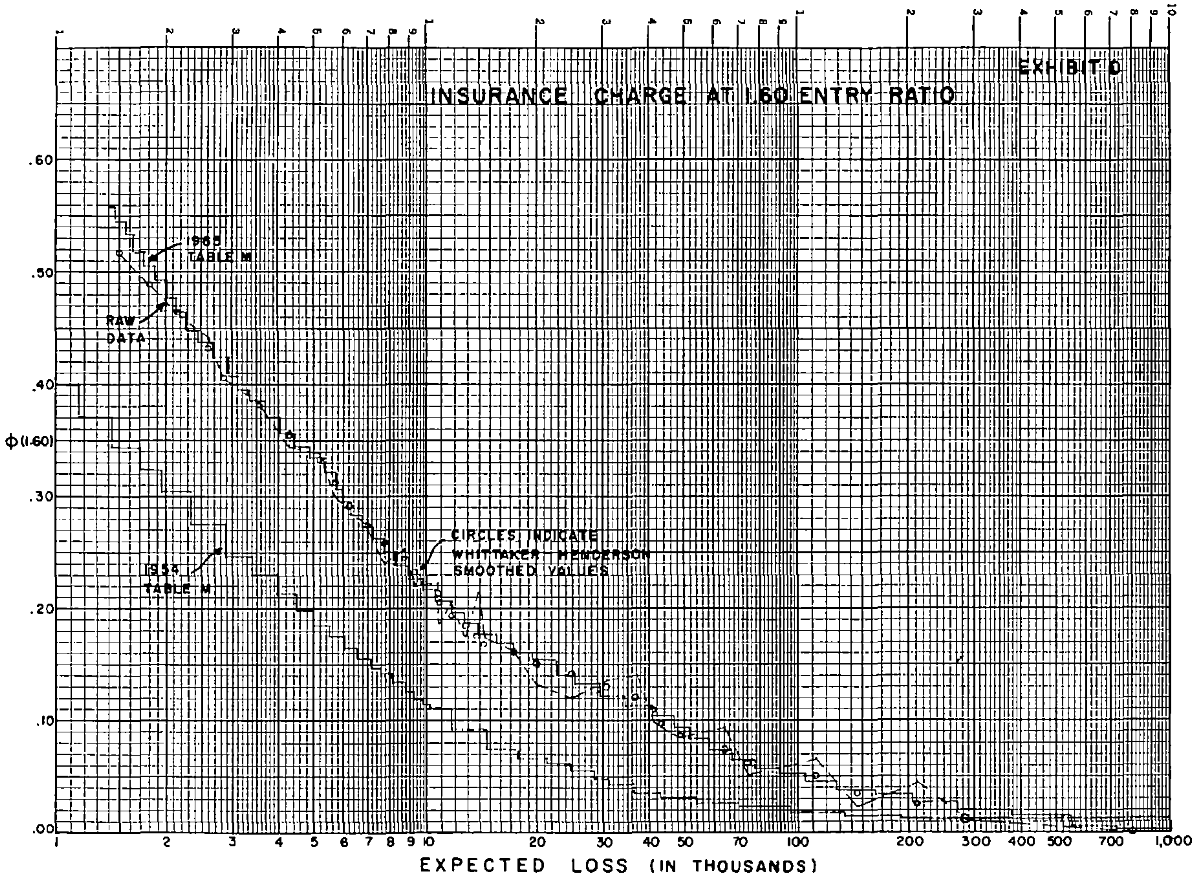


EXHIBIT D
INSURANCE CHARGE AT 1.60 ENTRY RATIO
1965 TABLE M
RAW DATA
1954 TABLE M
CIRCLES INDICATE WHITTAKER HENDERSON SMOOTHED VALUES
φ(1.60)
.60
.50
.40
.30
.20
.10
.00
1 2 3 4 5 6 7 8 9 10 20 30 40 50 70 100 200 300 400 500 700 1,000
EXPECTED LOSS (IN THOUSANDS)

EXHIBIT E

EXAMPLE OF TEST RUN FOR SELECTING EQUATION

Old Premium Group 10

r	Raw Data	Smoothed Data	Fitted Equation (1) Value	Fitted Equation (1) % Error*	Fitted Equation (2) Value	Fitted Equation (2) % Error*
.20	.8270	.8249	.8248	-.2%	.8247	-.6%
.40	.6873	.6901	.6908	.8	.6911	1.1
.60	.5836	.5877	.5867	-.5	.5873	-.2
.80	.4976	.5051	.5042	-.3	.5045	-.2
1.00	.4320	.4377	.4371	-.1	.4368	-.2
1.20	.3752	.3799	.3812	.3	.3805	.1
1.40	.3302	.3325	.3339	.4	.3330	.1
1.60	.2898	.2924	.2934	.4	.2926	.1
1.80	.2579	.2576	.2586	.4	.2582	.2
2.00	.2297	.2293	.2285	-.3	.2288	-.2
2.20	.2067	.2039	.2028	-.5	.2035	-.2
2.40	.1848	.1823	.1808	-.8	.1819	-.2
2.60	.1667	.1631	.1621	-.6	.1633	.1
2.80	.1504	.1470	.1463	-.5	.1471	.1
3.00	.1373	.1329	.1328	-.1	.1331	.2
3.20	.1247	.1197	.1211	1.2	.1208	.9
3.40	.1141	.1094	.1109	1.3	.1100	.6
3.60	.1041	.1034	.1016	-1.7	.1005	-2.9
3.80	.0958	.0915	.0931	1.7	.0919	.5
4.00	.0882	.0837	.0850	1.6	.0843	.7
4.20	.0817	.0771	.0774	.4	.0774	.4
4.40	.0764	.0712	.0704	-1.1	.0713	.1
4.60	.0707	.0660	.0644	-2.4	.0658	-.3
4.80	.0659	.0610	.0600	-1.7	.0609	-.1
5.00	.0612	.0566	.0581	2.7	.0567	.1

* Percentage errors are calculated as the error in the insurance charge for values of $r \geq 1.00$, and as percentage errors in the saving for values of $r < 1.00$, where the saving equals the charge plus the entry ratio minus 1.00.

EXHIBIT F

FINAL CURVE FITTING RUN

Example of Two Premium Groups

	OLD PREMIUM GROUP 10				OLD PREMIUM GROUP 11			
	Raw	Smoothed	Fitted Equation (3)		Raw	Smoothed	Fitted Equation (3)	
r	Data	Data	Value	% Error*	Data	Data	Value	% Error*
.20	.8270	.8249	.8248	-.5%	.8207	.8224	.8223	-.6%
.40	.6873	.6901	.6910	1.0	.6836	.6836	.6845	1.1
.60	.5836	.5877	.5872	-.3	.5810	.5779	.5772	-.4
.80	.4976	.5051	.5044	-.2	.4941	.4921	.4916	-.2
1.00	.4320	.4377	.4368	-.2	.4240	.4229	.4220	-.2
1.20	.3752	.3799	.3806	.2	.3655	.3637	.3643	.2
1.40	.3302	.3325	.3331	.2	.3179	.3154	.3160	.2
1.60	.2898	.2924	.2927	.1	.2763	.2747	.2752	.2
1.80	.2579	.2576	.2583	.3	.2419	.2401	.2406	.2
2.00	.2297	.2293	.2288	-.2	.2134	.2119	.2114	-.3
2.20	.2067	.2039	.2035	-.2	.1906	.1870	.1865	-.3
2.40	.1848	.1823	.1818	-.3	.1697	.1659	.1654	-.3
2.60	.1667	.1631	.1631	-.0	.1511	.1476	.1475	-.1
2.80	.1504	.1470	.1470	-.0	.1351	.1321	.1321	-.0
3.00	.1373	.1329	.1331	.1	.1222	.1188	.1189	.1
3.20	.1247	.1197	.1208	.9	.1110	.1058	.1074	1.6
3.40	.1141	.1094	.1101	.6	.1012	.0968	.0974	.7
3.60	.1041	.1034	.1005	-2.8	.0923	.0918	.0887	-3.4
3.80	.0958	.0915	.0920	.5	.0845	.0804	.0809	.6
4.00	.0882	.0837	.0843	.8	.0771	.0733	.0739	.8
4.20	.0817	.0771	.0774	.4	.0705	.0674	.0677	.5
4.40	.0764	.0712	.0712	.1	.0647	.0621	.0622	.1
4.60	.0707	.0660	.0657	-.4	.0597	.0575	.0573	-.4
4.80	.0659	.0610	.0609	-.2	.0556	.0532	.0530	-.3
5.00	.0612	.0566	.0567	.2	.0517	.0493	.0494	.2

COEFFICIENTS

	Group 10	Group 11
b_2	.3388717	.44044010
b_3	-.16700810	-.22436769
b_4	.15762138	.20788815
b_5	-.042426274	-.058737127
b_6	.0011206270	.0032547560
b_7	.0011646563	.0011371418
b_8	-.00012901262	-.00014028810.

* Percentage errors are calculated as the error in the insurance charge for values of $r \geq 1.00$, and as percentage errors in the saving for values of $r < 1.00$, where the saving equals the charge plus the entry ratio minus 1.00.

EXHIBIT G

INTERPOLATION TO DETERMINE COEFFICIENTS

FOR NEW PREMIUM GROUPS

The coefficients for Premium Group .43 were found by interpolation between the values (shown on Exhibit F) for Old Premium Groups 10 and 11. Harmonic interpolation was used as follows:

	Old Premium Group 10	Old Premium Group 11	New Premium Group .43
ϕ(1.00)	.4368310	.4220344	.430
1/ϕ(1.00)*	2.2892150	2.3694751	2.3255814
Interpolating Proportion	.5468932	.4531068	
b_2	.33887170	.44044010	.3848930
b_3	-.16700810	-.22436769	-.1929981
b_4	.15762138	.20788815	.1803976
b_5	-.042426274	-.058737127	-.0498168
b_6	.0011206270	.0032547560	.0020876
b_7	.0011646563	.0011371418	.0011522
b_8	-.00012901262	-.00014028810	-.0001341

* Determined as 2.0 (i.e., 1 + r) plus the sum of the coefficients b_2 through b_8 for the old premium group columns.

TABLE M

MATRIX OF COEFFICIENTS

Premium Group	b_2	b_3	b_4	b_5	b_6	b_7	b_8	Maximum Usable r
.64	-2.4031906	4.9946221	-5.0352823	2.6534276	-0.7476412	0.1066010	-0.0060366	1.74
.63	-2.2041460	4.5552874	-4.6030607	2.4345125	-0.6881637	0.0984249	-0.0055528	1.74
.62	-2.0171974	4.1473717	-4.1993122	2.2285352	-0.6319215	0.0905791	-0.0051517	1.74
.61	-1.8310552	3.7423805	-3.7973659	2.0229685	-0.5756088	0.0827421	-0.0047170	1.74
.60	-1.6570528	3.3682250	-3.4231837	1.8300687	-0.5223754	0.0752858	-0.0043009	1.74
.59	-1.4832753	2.9961886	-3.0500584	1.6371411	-0.4689893	0.0677905	-0.0038819	3.78
.58	-1.3210878	2.6538260	-2.7035217	1.4562566	-0.4185056	0.0606505	-0.0034801	3.78
.57	-1.1594855	2.3148689	-2.3592562	1.2758234	-0.3679712	0.0534825	-0.0030759	3.78
.56	-1.0269758	2.0494542	-2.0828563	1.1267103	-0.3251918	0.0472951	-0.0027214	3.78
.55	-0.8896476	1.7743882	-1.7964056	0.9721749	-0.2808567	0.0408827	-0.0023540	3.78
.54	-0.7549972	1.5141474	-1.5311006	0.8317601	-0.2411180	0.0351902	-0.0020300	3.78
.53	-0.6159643	1.2463364	-1.2586406	0.6878316	-0.2004408	0.0293691	-0.0016989	5.00
.52	-0.4911550	1.0191766	-1.0283315	0.5666651	-0.1663803	0.0245276	-0.0014256	5.00
.51	-0.3651269	0.7926783	-0.7988756	0.4460618	-0.1325207	0.0197223	-0.0011549	5.00
.50	-0.2502708	0.5970000	-0.5988208	0.3401507	-0.1025966	0.0154505	-0.0009130	5.00
.49	-0.1339706	0.4013138	-0.3983194	0.2338214	-0.0725095	0.0111496	-0.0006690	5.00
.48	-0.0330425	0.2511240	-0.2463034	0.1543364	-0.0502972	0.0080085	-0.0004926	5.00
.47	0.0688215	0.1034968	-0.0973794	0.0767784	-0.0286993	0.0049637	-0.0003220	5.00
.46	0.1540083	0.0062860	-0.0037170	0.0308102	-0.0165737	0.0033337	-0.0002345	5.00
.45	0.2380330	-0.0828200	0.0811232	-0.0097579	-0.0061649	0.0019723	-0.0001634	5.00
.44	0.3140541	-0.1462885	0.1387943	-0.0343862	-0.0006725	0.0013635	-0.0001375	5.00
.43	0.3848930	-0.1929981	0.1803976	-0.0498168	0.0020876	0.0011522	-0.0001341	5.00
.42	0.4520075	-0.2252639	0.2077841	-0.0572532	0.0025717	0.0012534	-0.0001473	5.00
.41	0.5105354	-0.2297982	0.2072574	-0.0497446	-0.0008845	0.0018417	-0.0001828	5.00
.40	0.5578036	-0.1952433	0.1636623	-0.0183729	-0.0108692	0.0032798	-0.0002603	5.00
.39	0.6000823	-0.1393633	0.0963798	0.0270763	-0.0250111	0.0053088	-0.0003703	5.00
.38	0.6373082	-0.0619243	0.0065091	0.0850057	-0.0426207	0.0078055	-0.0005046	5.00
.37	0.6699527	0.0362174	-0.1052290	0.1558492	-0.0643880	0.0109835	-0.0006831	5.00
.36	0.6988077	0.1534788	-0.2401878	0.2446196	-0.0935920	0.0156192	-0.0009676	5.00
.35	0.7225221	0.2925464	-0.3982271	0.3492669	-0.1290635	0.0214321	-0.0013340	5.00

MATRIX OF COEFFICIENTS

TABLE M

Premium Group	b_2	b_3	b_4	b_5	b_6	b_7	b_8	Maximum Usable r
.34	0.7505061	0.4063878	-0.5060154	0.4200164	-0.1536101	0.0254783	-0.0015867	5.00
.33	0.7820579	0.5135850	-0.5980510	0.4804099	-0.1748942	0.0290010	-0.0018055	5.00
.32	0.8143930	0.6003855	-0.6482440	0.5203436	-0.1920786	0.0322248	-0.0020243	5.00
.31	0.8512591	0.6585917	-0.6488186	0.5364166	-0.2044858	0.0350829	-0.0022395	5.00
.30	0.8943599	0.6784657	-0.5853285	0.5210722	-0.2100882	0.0372850	-0.0024331	5.00
.29	0.9448819	0.6556264	-0.4557299	0.4792926	-0.2129519	0.0398448	-0.0026880	5.00
.28	1.0031130	0.5638149	-0.1995380	0.3610718	-0.1933591	0.0390678	-0.0027418	5.00
.27	1.0682040	0.4044798	0.1691869	0.1904578	-0.1632730	0.0374271	-0.0027790	5.00
.26	1.1457775	0.1507550	0.6772723	-0.0367783	-0.1230709	0.0349965	-0.0027982	5.00
.25	1.2431208	-0.2287912	1.3574460	-0.3263899	-0.0751633	0.0326514	-0.0028739	5.00
.24	1.3523489	-0.6733113	2.0671535	-0.5402002	-0.0764166	0.0408023	-0.0037099	5.00
.23	1.4805248	-1.2073324	2.8012469	-0.6334556	-0.1521826	0.0647098	-0.0056850	5.00
.22	1.6498167	-1.9405544	3.6845402	-0.6430773	-0.3019227	0.1055812	-0.0089291	5.00
.21	1.8352317	-2.7436078	4.6519573	-0.6536154	-0.4659240	0.1503452	-0.0124822	5.00

EXHIBIT I

TABLE M

FORMULAS FOR CHARGES BEYOND THE RANGE WHERE EQUATION (3) APPLIES

General Formula: $\phi(r) = c - mr$

PG.	Applicable to Values of r in the range	c	m
.64	1.75 - 3.78	.7190	.10
.64	3.79 - 10.59	.5297	.05
.63	1.75 - 3.78	.7040	.10
.63	3.79 - 10.29	.5147	.05
.62	1.75 - 3.78	.6900	.10
.62	3.79 - 10.01	.5007	.05
.61	1.75 - 3.78	.6750	.10
.61	3.79 - 9.71	.4857	.05
.60	1.75 - 3.78	.6600	.10
.60	3.79 - 9.41	.4707	.05
.59	3.79 - 9.12	.4562	.05
.58	3.79 - 8.77	.4387	.05
.57	3.79 - 8.46	.4232	.05
.56	3.79 - 8.19	.4097	.05
.55	3.79 - 7.95	.3977	.05
.54	3.79 - 7.68	.3842	.05
.53	5.01 - 7.45	.3727	.05
.52	5.01 - 7.25	.3627	.05
.51	5.01 - 7.09	.3547	.05
.50	5.01 - 6.91	.3457	.05
.49	5.01 - 6.76	.3382	.05
.48	5.01 - 6.62	.3312	.05
.47	5.01 - 6.50	.3252	.05
.46	5.01 - 6.38	.3192	.05
.45	5.01 - 6.26	.3132	.05

PG.	Applicable to Values of r in the range	c	m
.44	5.01 - 6.15	.3077	.05
.43	5.01 - 6.05	.3027	.05
.42	5.01 - 5.96	.2982	.05
.41	5.01 - 5.88	.2942	.05
.40	5.01 - 5.79	.2897	.05
.39	5.01 - 5.70	.2862	.05
.38	5.01 - 5.65	.2827	.05
.37	5.01 - 5.59	.2797	.05
.36	5.01 - 5.54	.2772	.05
.35	5.01 - 5.50	.2752	.05
.34	5.01 - 5.47	.2737	.05
.33	5.01 - 5.44	.2722	.05
.32	5.01 - 5.41	.2707	.05
.31	5.01 - 5.38	.2692	.05
.30	5.01 - 5.35	.2677	.05
.29	5.01 - 5.33	.2667	.05
.28	5.01 - 5.30	.2652	.05
.27	5.01 - 5.27	.2637	.05
.26	5.01 - 5.25	.2627	.05
.25	5.01 - 5.21	.2607	.05
.24	5.01 - 5.19	.2597	.05
.23	5.01 - 5.15	.2577	.05
.22	5.01 - 5.13	.2567	.05
.21	5.01 - 5.11	.2557	.05

TABLE OF CHARGES AND SAVINGS - TABLE M

Two Special Premium Groups

r	Premium Group .85 Expected Losses $596 Insurance Charges	Premium Group .99 Expected Losses $3 Insurance Charges
.01	.994	For $r \leq 100.00$, $\phi(r) = 1.000 - .01r$ (Retain all decimal places)
.02	.988	
.03	.982	
.04	.976	
.05	.970	
.06	.967	
.07	.963	
.08	.960	
.09	.956	
For $.10 \leq r \leq .35$	$\phi(r) = .9675 - .15r$	
For $.36 \leq r \leq 5.00$	$\phi(r) = .9500 - .10r$	
For $5.01 \leq r \leq 14.00$	$\phi(r) = .7000 - .05r$	

EXHIBIT K

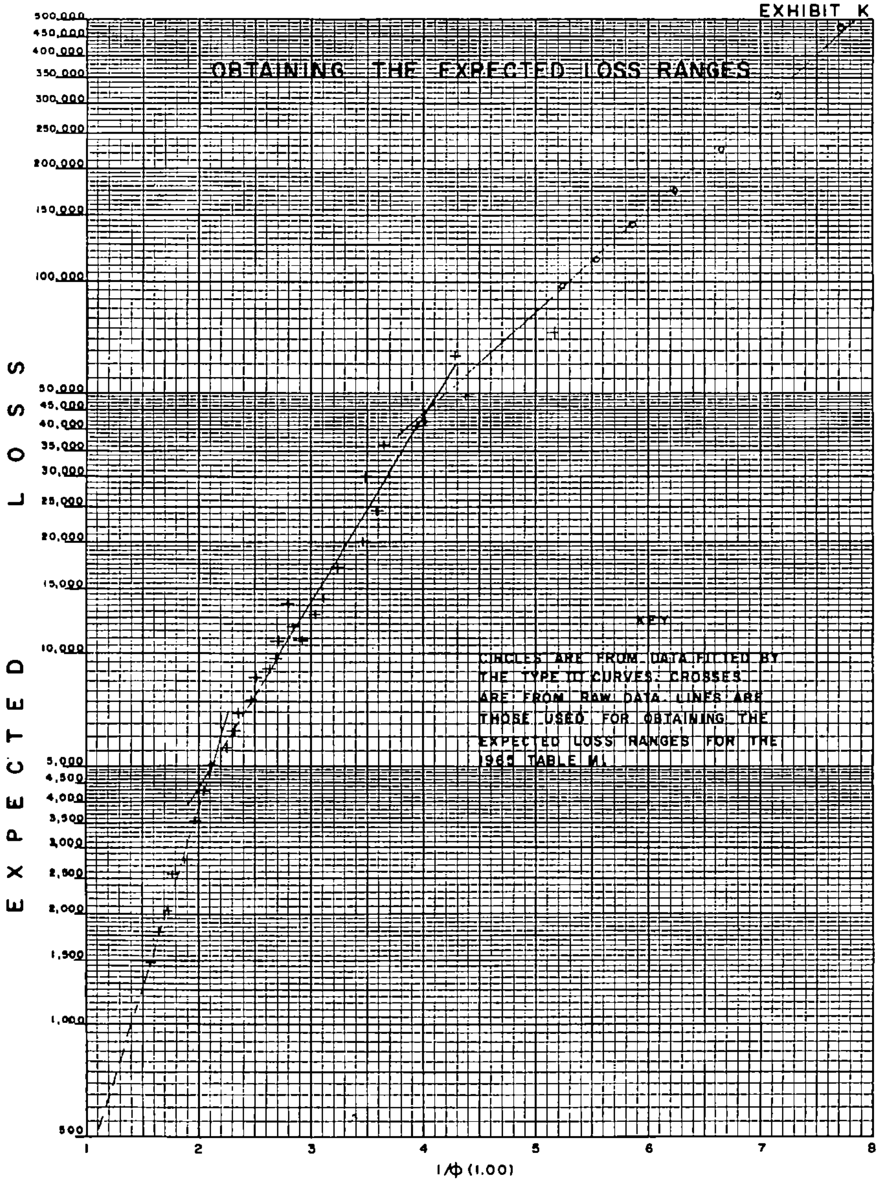

OBTAINING THE EXPECTED LOSS RANGES
EXPECTED LOSS
1/φ (1.00)
KEY
CIRCLES ARE FROM DATA FITTED BY THE TYPE III CURVES. CROSSES ARE FROM RAW DATA. LINES ARE THOSE USED FOR OBTAINING THE EXPECTED LOSS RANGES FOR THE 1965 TABLE M.
500
1,000
1,500
2,000
2,500
3,000
3,500
4,000
4,500
5,000
10,000
15,000
20,000
25,000
30,000
35,000
40,000
45,000
50,000
100,000
150,000
200,000
250,000
300,000
350,000
400,000
450,000
500,000
1
2
3
4
5
6
7
8

EXHIBIT L

EXPECTED LOSS RANGES

Premium Group	Range	Premium Group	Range
.64	$1,400 - $1,479	.32	$15,500 - $17,299
.63	1,480 - 1,569	.31	17,300 - 19,499
.62	1,570 - 1,659	.30	19,500 - 22,099
.61	1,660 - 1,769	.29	22,100 - 25,299
.60	1,770 - 1,879	.28	25,300 - 29,299
.59	1,880 - 2,009	.27	29,300 - 34,299
.58	2,010 - 2,149	.26	34,300 - 40,599
.57	2,150 - 2,299	.25	40,600 - 46,299
.56	2,300 - 2,469	.24	46,300 - 51,599
.55	2,470 - 2,659	.23	51,600 - 58,099
.54	2,660 - 2,869	.22	58,100 - 66,199
.53	2,870 - 3,109	.21	66,200 - 76,299
.52	3,110 - 3,379	.20	76,300 - 89,299
.51	3,380 - 3,689	.19	89,300 - 105,999
.50	3,690 - 4,029	.18	106,000 - 128,999
.49	4,030 - 4,429	.17	129,000 - 160,999
.48	4,430 - 4,879	.16	161,000 - 204,999
.47	4,880 - 5,379	.15	205,000 - 270,999
.46	5,380 - 5,679	.14	271,000 - 373,999
.45	5,680 - 6,009	.13	374,000 - 542,999
.44	6,010 - 6,369	.12	543,000 - 724,999
.43	6,370 - 6,779	.11	725,000 - 916,999
.42	6,780 - 7,229	.10	917,000 - 1,119,999
.41	7,230 - 7,739	.09	1,120,000 - 1,399,999
.40	7,740 - 8,309	.08	1,400,000 - 1,799,999
.39	8,310 - 8,959	.07	1,800,000 - 2,389,999
.38	8,960 - 9,689	.06	2,390,000 - 3,339,999
.37	9,690 - 10,499	.05	3,340,000 - 4,989,999
.36	10,500 - 11,499	.04	4,990,000 - 8,249,999
.35	11,500 - 12,599	.03	8,250,000 - 16,199,999
.34	12,600 - 13,899	.02	16,200,000 - 44,899,999
.33	13,900 - 15,499	.01	44,900,000 & Over

RETROSPECTIVE PLAN D - TABLE OF CHARGES AND SAVINGS - TABLE M

ENTRY RATIO	PREMIUM GROUP 64	63	62	61	60	59	58	57	56	55	54
0.01	0.990	0.990	0.990	0.990	0.990	0.990	0.990	0.990	0.990	0.990	0.990
	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*
0.02	0.981	0.981	0.981	0.981	0.981	0.981	0.981	0.981	0.981	0.981	0.981
	0.001*	0.001*	0.001*	0.001*	0.001*	0.001*	0.001*	0.001*	0.001*	0.001*	0.001*
0.03	0.973	0.973	0.972	0.972	0.972	0.972	0.972	0.972	0.972	0.972	0.971
	0.003*	0.003*	0.002*	0.002*	0.002*	0.002*	0.002*	0.002*	0.002*	0.002*	0.001*
0.04	0.965	0.965	0.964	0.964	0.964	0.964	0.963	0.963	0.963	0.963	0.963
	0.005*	0.005*	0.004*	0.004*	0.004*	0.004*	0.003*	0.003*	0.003*	0.003*	0.003*
0.05	0.957	0.957	0.957	0.956	0.956	0.955	0.955	0.955	0.954	0.954	0.954
	0.007*	0.007*	0.007*	0.006*	0.006*	0.005*	0.005*	0.005*	0.004*	0.004*	0.004*
0.06	0.950	0.950	0.949	0.949	0.948	0.948	0.947	0.947	0.946	0.946	0.946
	0.010*	0.010*	0.009*	0.009*	0.008*	0.008*	0.007*	0.007*	0.006*	0.006*	0.006*
0.07	0.944	0.943	0.942	0.941	0.941	0.940	0.940	0.939	0.938	0.938	0.937
	0.014*	0.013*	0.012*	0.011*	0.011*	0.010*	0.010*	0.009*	0.008*	0.008*	0.007*
0.08	0.937	0.936	0.935	0.935	0.934	0.933	0.932	0.931	0.931	0.930	0.929
	0.017*	0.016*	0.015*	0.015*	0.014*	0.013*	0.012*	0.011*	0.011*	0.010*	0.009*
0.09	0.931	0.930	0.929	0.928	0.927	0.926	0.925	0.924	0.923	0.923	0.922
	0.021*	0.020*	0.019*	0.018*	0.017*	0.016*	0.015*	0.014*	0.013*	0.013*	0.012*
0.10	0.926	0.924	0.923	0.922	0.920	0.919	0.918	0.917	0.916	0.915	0.914
	0.026*	0.024*	0.023*	0.022*	0.020*	0.019*	0.018*	0.017*	0.016*	0.015*	0.014*
0.11	0.920	0.918	0.917	0.915	0.914	0.913	0.911	0.910	0.909	0.908	0.907
	0.030*	0.028*	0.027*	0.025*	0.024*	0.023*	0.021*	0.020*	0.019*	0.018*	0.017*
0.12	0.915	0.913	0.911	0.910	0.908	0.906	0.905	0.903	0.902	0.901	0.900
	0.035*	0.033*	0.031*	0.030*	0.028*	0.026*	0.025*	0.023*	0.022*	0.021*	0.020*
0.13	0.910	0.908	0.906	0.904	0.902	0.900	0.899	0.897	0.896	0.894	0.893
	0.040*	0.038*	0.036*	0.034*	0.032*	0.030*	0.029*	0.027*	0.026*	0.024*	0.023*
0.14	0.905	0.903	0.901	0.898	0.896	0.894	0.893	0.891	0.889	0.887	0.886
	0.045*	0.043*	0.041*	0.038*	0.036*	0.034*	0.033*	0.031*	0.029*	0.027*	0.026*
0.15	0.901	0.898	0.896	0.893	0.891	0.889	0.887	0.884	0.883	0.881	0.879
	0.051*	0.048*	0.046*	0.043*	0.041*	0.039*	0.037*	0.034*	0.033*	0.031*	0.029*
0.16	0.896	0.893	0.891	0.888	0.885	0.883	0.881	0.878	0.877	0.875	0.873

TABLE M

TABLE M - CUMULATIVE DISTRIBUTION AND UNDERLYING FREQUENCY DISTRIBUTION

ENTRY	64		63		62		61		60		59		58		57	
RATIO	D1	D2	D1	D2	D1	D2	D1	D2	D1	D2	D1	D2	D1	D2	D1	D2
0.01	9.		9.		9.		9.		9.		9.		9.		9.	
0.02	8.	1.	8.	1.	9.	0.	9.	0.	9.	0.	9.	0.	9.	0.	9.	0.
0.03	8.	0.	8.	0.	8.	1.	8.	1.	8.	1.	8.	1.	9.	0.	9.	0.
0.04	8.	0.	8.	0.	7.	1.	8.	0.	8.	0.	9.	-1.	8.	1.	8.	1.
0.05	7.	1.	7.	1.	8.	-1.	7.	1.	8.	0.	7.	2.	8.	0.	8.	0.
0.06	6.	1.	7.	0.	7.	1.	8.	-1.	7.	1.	8.	-1.	7.	1.	8.	0.
0.07	7.	-1.	7.	0.	7.	0.	6.	2.	7.	0.	7.	1.	8.	-1.	8.	0.
0.08	6.	1.	6.	1.	6.	1.	7.	-1.	7.	0.	7.	0.	7.	1.	7.	1.
0.09	5.	1.	6.	0.	6.	0.	6.	1.	7.	0.	7.	0.	7.	0.	7.	0.
0.10	6.	-1.	6.	0.	6.	0.	7.	-1.	6.	1.	6.	1.	7.	0.	7.	0.
0.11	5.	1.	5.	1.	6.	0.	5.	2.	6.	0.	7.	-1.	6.	1.	7.	0.
0.12	5.	0.	5.	0.	5.	1.	6.	-1.	6.	0.	6.	1.	6.	0.	6.	1.
0.13	5.	0.	5.	0.	5.	0.	6.	0.	6.	0.	6.	0.	6.	0.	6.	0.
0.14	4.	1.	5.	0.	5.	0.	5.	1.	5.	1.	5.	1.	6.	0.	7.	-1.
0.15	5.	-1.	5.	0.	5.	0.	5.	0.	6.	-1.	6.	-1.	6.	0.	6.	1.
0.16	4.	1.	4.	1.	5.	0.	5.	0.	5.	1.	5.	1.	6.	0.	5.	1.
0.17	4.	0.	5.	-1.	5.	0.	5.	0.	5.	0.	6.	-1.	6.	0.	6.	-1.
0.18	4.	0.	4.	1.	4.	1.	5.	0.	5.	0.	5.	1.	5.	1.	6.	0.
0.19	4.	0.	4.	0.	5.	-1.	4.	1.	5.	0.	5.	0.	5.	0.	5.	1.
0.20	4.	0.	4.	0.	4.	1.	5.	-1.	5.	0.	5.	0.	5.	0.	6.	-1.
0.21	4.	0.	4.	0.	4.	0.	4.	1.	4.	1.	5.	0.	6.	-1.	5.	1.
0.22	3.	1.	4.	0.	4.	0.	5.	-1.	5.	-1.	5.	0.	5.	1.	5.	0.
0.23	4.	-1.	4.	0.	4.	0.	4.	1.	4.	1.	5.	0.	5.	0.	6.	-1.
0.24	3.	1.	4.	0.	4.	0.	4.	0.	5.	-1.	4.	1.	4.	1.	5.	1.
0.25	4.	-1.	3.	1.	4.	0.	4.	0.	4.	1.	5.	-1.	5.	-1.	5.	0.
0.26	3.	1.	4.	-1.	4.	0.	5.	-1.	4.	0.	4.	1.	5.	0.	4.	1.
0.27	4.	-1.	4.	0.	4.	0.	4.	1.	5.	-1.	5.	-1.	5.	0.	5.	-1.
0.28	3.	1.	3.	1.	4.	0.	4.	0.	4.	1.	4.	1.	4.	1.	5.	0.
0.29	3.	0.	4.	-1.	4.	0.	4.	0.	4.	0.	5.	-1.	5.	-1.	5.	0.
0.30	3.	0.	3.	1.	3.	1.	3.	1.	4.	0.	4.	1.	4.	1.	4.	1.
0.31	4.	-1.	4.	-1.	4.	-1.	4.	-1.	4.	0.	4.	0.	5.	-1.	5.	-1.

TABLE M - CROSS COLUMN FIRST DIFFERENCES

ENTRY RATIO	PREMIUM GROUP 64	63	62	61	60	59	58	57	56	55	54	53	52	51	50
0.01	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.02	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.
0.03	0.	1.	0.	0.	0.	0.	0.	0.	0.	1.	0.	0.	0.	0.	0.
0.04	0.	1.	0.	0.	0.	1.	0.	0.	0.	0.	1.	0.	0.	0.	0.
0.05	0.	0.	1.	0.	1.	0.	0.	1.	0.	0.	0.	1.	0.	0.	0.
0.06	0.	1.	0.	1.	0.	1.	0.	1.	0.	0.	1.	0.	1.	0.	0.
0.07	1.	1.	1.	0.	1.	0.	1.	1.	0.	1.	0.	1.	0.	1.	0.
0.08	1.	1.	0.	1.	1.	1.	1.	0.	1.	1.	0.	1.	0.	1.	1.
0.09	1.	1.	1.	1.	1.	1.	1.	1.	0.	1.	1.	1.	1.	0.	1.
0.10	2.	1.	1.	2.	1.	1.	1.	1.	1.	1.	1.	1.	0.	1.	1.
0.11	2.	1.	2.	1.	1.	2.	1.	1.	1.	1.	1.	1.	1.	1.	1.
0.12	2.	2.	1.	2.	2.	1.	2.	1.	1.	1.	2.	1.	1.	1.	1.
0.13	2.	2.	2.	2.	2.	1.	2.	1.	2.	1.	2.	1.	1.	2.	1.
0.14	2.	2.	3.	2.	2.	1.	2.	2.	2.	1.	2.	1.	2.	1.	2.
0.15	3.	2.	3.	2.	2.	2.	3.	1.	2.	2.	2.	1.	2.	1.	2.
0.16	3.	2.	3.	3.	2.	2.	3.	1.	2.	2.	2.	2.	2.	2.	1.
0.17	3.	3.	3.	3.	2.	3.	2.	3.	2.	2.	2.	2.	2.	2.	2.
0.18	4.	3.	3.	3.	3.	3.	2.	2.	3.	2.	2.	3.	2.	2.	2.
0.19	4.	3.	4.	3.	3.	3.	3.	2.	3.	2.	3.	2.	2.	3.	2.
0.20	4.	4.	3.	4.	3.	3.	3.	3.	3.	2.	3.	3.	2.	2.	3.
0.21	4.	4.	4.	4.	3.	3.	4.	3.	2.	3.	3.	3.	3.	2.	3.
0.22	4.	4.	4.	4.	4.	4.	3.	3.	3.	3.	3.	3.	3.	3.	2.
0.23	5.	4.	5.	4.	4.	4.	3.	4.	3.	3.	3.	3.	3.	3.	3.
0.24	5.	4.	5.	4.	5.	4.	4.	3.	3.	4.	3.	3.	3.	3.	3.
0.25	6.	4.	5.	5.	4.	4.	5.	3.	4.	3.	4.	3.	3.	4.	3.
0.26	5.	5.	5.	5.	5.	4.	5.	3.	4.	4.	3.	4.	3.	4.	3.
0.27	6.	5.	6.	4.	5.	5.	4.	4.	4.	4.	4.	4.	3.	4.	3.
0.28	6.	5.	6.	5.	5.	5.	4.	4.	4.	4.	5.	3.	4.	4.	3.
0.29	6.	6.	6.	5.	5.	5.	5.	4.	4.	5.	4.	4.	4.	4.	3.
0.30	7.	6.	6.	5.	6.	5.	5.	4.	4.	5.	4.	4.	5.	4.	3.
0.31	7.	6.	6.	6.	6.	5.	5.	5.	4.	5.	5.	4.	4.	4.	4.

FORTRAN CODING FORM

EXHIBIT P

	Punching Instructions		Page of
Program DETERMINATION OF TABLE M CHARGE	Graphic	Card Form # *	Identification
Programmer JOAN FEATHERER — Date 1965	Punch		73 80

C FOR COMMENT

STATEMENT NUMBER (1–5) | CONT. (6) | FORTRAN STATEMENT (7–72)

```
C    10ASSUME THE 44x7 MATRIX OF COEFFICIENTS IS LOADED AS E(I,M) WHERE I
C     1IS THE PREMIUM GROUP AND M IDENTIFIES COEFFICIENTS B(2) THRU B(8).
C    20ASSUME AN OBTAINED VALUE OF R FALLS WITHIN THE ACCEPTABLE RANGE
C     1FOR APPLICATION OF EQUATION (3) FOR THE OBTAINED VALUE OF I. (NOTE
C     2--IF THE PAIR (R,I) IS NOT IN THE ACCEPTABLE RANGE, EITHER USE A
C     3DIFFERENT EQUATION THRU ADDITIONAL PROGRAMMING OR DETERMINE THE
C     4CHARGE MANUALLY.)
C    30COMPUTE THE CHARGE (CH) AND THE CHARGE ROUNDED TO THREE DECIMAL
C     1PLACES (CHRND) AS FOLLOWS IN 10.
  100 0CH=1./(1.+R+R*R*(E(I,1)+R*(E(I,2)+R*(E(I,3)+R*(E(I,4)+R*(E(I,5)+R*
      1(E(I,6)+R*E(I,7)))))))))
       CHRND=(AINT((CH+.0005)*1000.))*1.E-3
       IF(R+CHRND-1.)20,21,21
   20  CHRND=1.-R
C   21 WRITE OUTPUT STATEMENT TO FIT INDIVIDUAL REQUIREMENTS
```

* A standard card form, IBM electro 888157, is available for punching source statements from this form.

TABLE M

EXHIBIT Q

AREAS OF FORMULA APPLICATION

Equation (3) = $1/(1+ r + b_2r^2 + b_3r^3 + b_4r^4 + b_5r^5 + b_6r^6 + b_7r^7 + b_8r^8)$

Equation (4) = $c - mr$

Premium Group Number:	.64-.60	.59-.54	.53-.21	.20-.12	.11-.01
Expected Loss Range($):	1,400-1,879	1,880-2,869	2,870-76,299	76,300-724,999	725,000 and Over
Critical Entry Ratios					
.00 ... 1.74	(3)	(3)	(3)	A linear interpolation between the charges at Premium Groups .21 and .11 using the premium group number as the argument.	No formula but the charges and savings are symmetrical about r = 1.00.
1.75 ... 3.78	(4)	(3)	(3)		
3.79 ... 5.00	(4)	(4)	(3)		
5.01 ... ω	(4)	(4)	(4)		

ω indicates that the formula works properly out to some limiting value and beyond this the charge is zero.

Equation (3) constants are found in Exhibit H.

Equation (4) constants are found in Exhibit I.

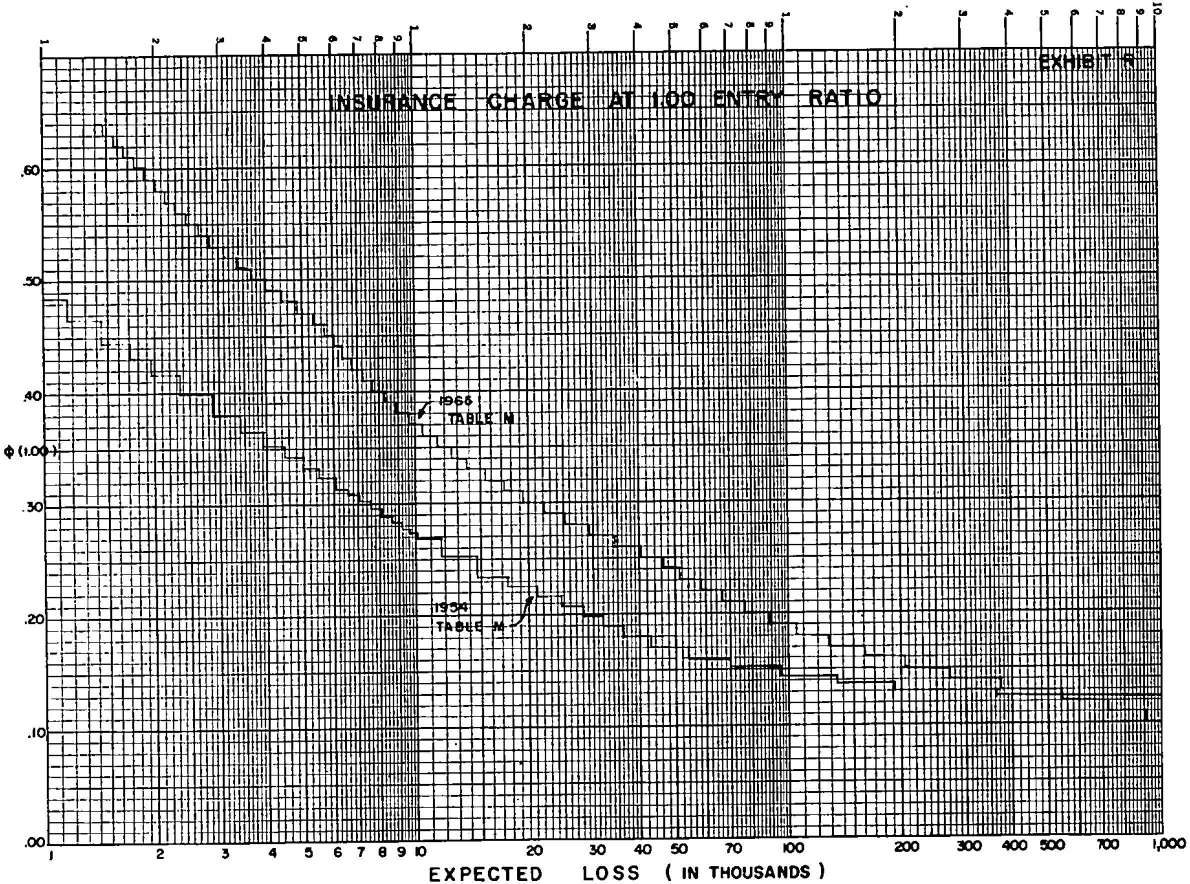
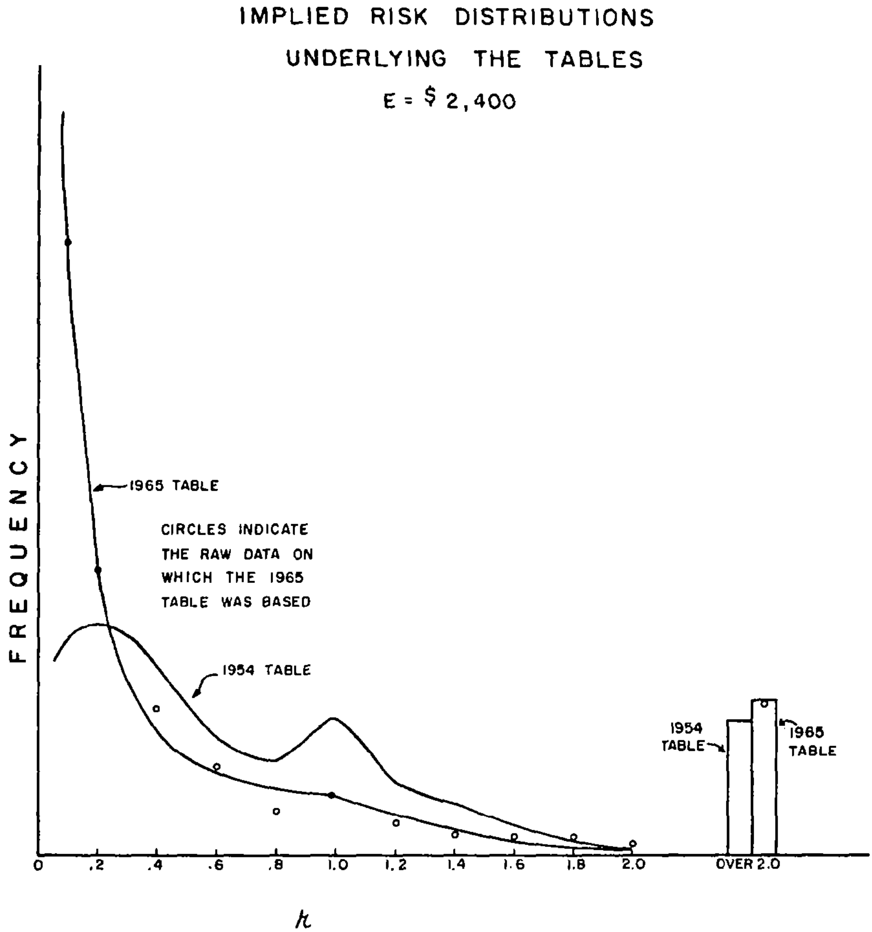


EXHIBIT R
INSURANCE CHARGE AT 1.00 ENTRY RATIO
1966 TABLE M
1954 TABLE M
φ (1.00)
.00
.10
.20
.30
.40
.50
.60
1
2
3
4
5
6
7
8
9
10
20
30
40
50
70
100
200
300
400
500
700
1,000
EXPECTED LOSS (IN THOUSANDS)

EXHIBIT S


IMPLIED RISK DISTRIBUTIONS
UNDERLYING THE TABLES
E = $ 2,400
FREQUENCY
1965 TABLE
CIRCLES INDICATE THE RAW DATA ON WHICH THE 1965 TABLE WAS BASED
1954 TABLE
1954 TABLE
1965 TABLE
0
.2
.4
.6
.8
1.0
1.2
1.4
1.6
1.8
2.0
OVER 2.0
r

APPENDIX A

The formula for the variance of the expected losses can be written[10]:

$$\sigma_E^2 = E^2(1 + V_c^2)/m_f \qquad \text{(A1)}$$

where V_c is the coefficient of variation of the underlying claim distribution and m_f is the mean claim frequency. In terms of the coefficient of variation of the expected losses we'd write:

$$V_E^2 = (1 + V_c^2)/m_f \qquad \text{(A2)}$$

Now, if between year 0 and year 5, we can assume the coefficient of variation of the claims remains constant then

$$\frac{V_{E5}^2}{V_{E0}^2} = \frac{m_{f5}}{m_{f0}} \qquad \text{(A3)}$$

$$V_{E5}^2 = \frac{m_{f5}}{m_{f0}} V_{E0}^2 \qquad \text{(A4)}$$

By general reasoning, we can equate V_{E5}^2 to V_{E0}^2 only if m_{f5} equals m_{f0}; but if the average cost per claim increases by a factor of *1.35*, we can equate m_{f5} to m_{f0} only if $E_5 = 1.35\ E_0$. This straightforward logic completes the argument.

It is interesting to note that if the average cost per claim is brought about by an increase primarily in the higher cost claims (more long term medical, longer life expectancy for injured workers, etc.), a factor larger than *1.35* would be merited due to the increased coefficient of variation of the claims.

APPENDIX B

It is almost obvious that if we change the index from R to r in the manner described, the insurance charge, $\phi(r)$, is the same as the previously calculated $\phi(R)$. By definition we have for some specific R:

$$\phi(R) = (\text{Sum } 2 \text{ at } R)/(\text{Sum } 2 \text{ at } 0)$$

$$= \sum_{i=\omega}^{u} (R_i - R) / \sum_{i=\omega}^{N} R_i$$

where R_i is the value of the ratio starting with the highest value (R_ω) going down to the case which has a value equal to R (R_u), and in the

[10] Bailey, R. A. and Simon, L. J., "Two Studies in Automobile Insurance Ratemaking *PCAS XLVII*, p. 18, equation (14) with slight changes in notation to fit our present definitions.

denominator continuing to the smallest value (R_N). It is apparent from the above equation that if each of the values of R is multiplied by some constant, the value of ϕ remains unchanged except that it will now be shown as $\phi(kR)$, that is, $\phi(r)$.

Care must be exercised in interpolating when the value of r that is being sought is not found in the adjusted ratio column. In Premium Group 28 we were faced with the problem of obtaining the insurance charge for an entry ratio of *.20* when the tab run showed the following information:

R	*r*	ϕ	*Number of Risks*	*Sum 1*	*Sum 2*
.20	.21	.7876	3	324	245.06
.18	.19	.8084	1	325	251.54

To obtain r, we had to multiply R by *1.07347* and Sum 2 at entry of *0* was *311.14*. We now observe that if we were seeking a value for $r = .20$ this would be equivalent to looking for a value of $R = .18631$ (that is, *.20/1.07347*). Between $R = .20$ and $R = .18631$ there would not be any risks and the value of Sum 1 would still be 324. The number of points of excess over *.18631* would increase by $324 \times (.01369)$ since there would be 324 risks which would each contribute this amount in addition to what had already been accumulated as excess points beyond the value of $R = .20$. Hence, Sum 2 at $R = .18631$ would be *249.50*. Thus, $\phi = 249.50/311.14 = .8019$. Another way to obtain this result a little more quickly is to do a straight linear interpolation on ϕ using R as the scale. In this case we would have:

$$(.00631/.02000) \times (.8084 - .7876) = .0066$$

Therefore, the value of ϕ would be $.8084 - .0066 = .8018$. Except for the fact that the values of ϕ are initially rounded off to four decimal places, these two methods produce the same result.

APPENDIX C

Mathematical Testing of Table M Functions

A set of working definitions was established first. Let r be defined as the ratio of actual losses (a random variable) to expected losses for a given risk. We will deal with samples of risks which have the same expected losses (or nearly so). In general $0 \leq r \leq \omega$, where ω is the maximum value that can be assumed and theoretically approaches infinity.

Define $f(r)$ as the density function as pictured in Sketch 1. Notice that $f(0)$ is not necessarily zero since in practice, many risks have no losses. However, we are certain that $f(\omega) = 0$ ....(C1)

and that $f(r) \geq 0$ for $0 \leq r \leq \omega$. ....(C2)

Further, since the area under the curve must equal 1, $\int_0^\omega f(r)dr = 1$. ....(C3)

As discussed in the main text, distributions underlying Table M must have a mean of 1, hence, $\int_0^\omega rf(r)dr = 1$. ....(C4)

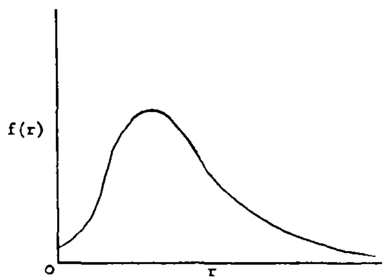
The distribution function, as shown in Sketch 2, will be given by

$$F(r) = \int_0^r rf(r)dr \qquad \text{............(C5)}$$

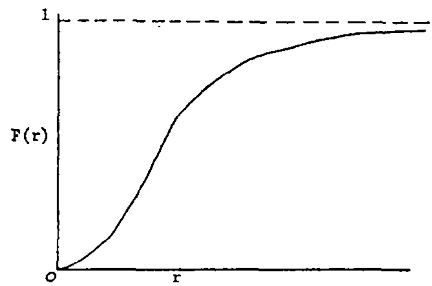
It follows that $F(0) = 0$ ....(C6)

and that $F(r) \geq 0$ for $0 \leq r \leq \omega$. ....(C7)

Statement (C3) may also be restated as $F(\omega) = 1$. ....(C8)



Sketch 1



Sketch 2

Define a special function, Sketch 3, which we will find very useful:

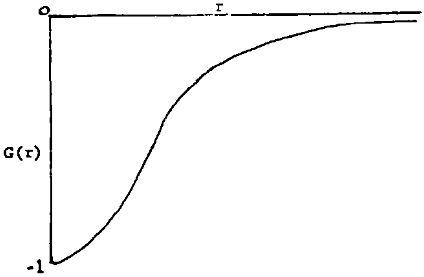
$G(r) = F(r) - 1$. ....(C9)

Hence, $G(0) = -1$ ....(C10)

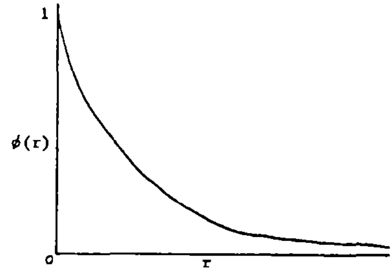
and $G(\omega) = 0$. ....(C11)

Let the Table M function be called $\phi(r)$. See Sketch 4. The basic definition of the Table M function (variously called "the charge," "the column of charges" and "the excess pure premium ratio") is:

$$\phi(r_o) = \int_{r_o}^\omega (r - r_o)f(r)dr \Big/ \int_0^\omega rf(r)dr \qquad \text{............(C12)}$$



Sketch 3



Sketch 4

which will ordinarily be denoted $\phi(r)$ for simplicity of notation. Note that by (C4), the denominator in (C12) equals unity.[20]

It can be shown through straightforward proofs[21] starting with (C12) that

$$\phi(r_0) = 1 - r_0 + \overline{\int\int}_{0}^{r_0} f(r)drdr \qquad \text{(C13)}$$

$$= 1 - r_0 + \int_0^{r_0} F(r)dr \qquad \text{(C14)}$$

$$= 1 + \int_0^{r_0} G(r)dr \qquad \text{(C15)}$$

$$\phi(0) = 1 \qquad \text{(C16)}$$

$$\phi(\omega) = 0 \qquad \text{(C17)}$$

$$\phi'(r_0) = G(r_0) \qquad \text{(C18)}$$

$$\phi''(r_0) = f(r_0) \qquad \text{(C19)}$$

Further, it can be shown that the area under the $f(r)$ curve can be given by

$$[G(r)]_0^\omega \qquad \text{(C20)}$$

and the mean of the $f(r)$ curve can be given by $[rG(r) - \phi(r)]_0^\omega$

$$\text{(C21)}$$

Another basic feature of Table M is the so-called "Saving" defined by:

$$\psi(r_0) = \left[r_0 \int_0^{r_0} f(r)dr - \int_0^{r_0} rf(r)dr \right] \Big/ \int_0^{\omega} rf(r)dr \qquad \text{(C22)}$$

Note that by (C4), the denominator in (C22) equals unity.

[20] The inclusion of the denominator in both (C12) and (C22) was suggested by Mr. Hewitt in his review. To assist the reader, we have agreed to incorporate this distinct improvement in the Appendix.

[21] For a similar development see Carlson, Thomas O., "Observations on Casualty Insurance Rate-Making Theory in the United States," *PCAS; Vol. LI;* p. 294.

It can be shown, from (C22), that

$$\psi(r_0) = \phi(r_o) + r_o - 1 \qquad \text{.........(C23)}$$

Since $\psi(r_0)$ can never be negative, we have

$$\phi(r_0) \geq 1 - r_0 \qquad \text{..........(C24)}$$

A slightly more difficult proof is the formula for the second moment of the underlying distribution when only the function $\phi(r)$ is known. The equation

$$\mu_2' = \int_0^\omega r^2 f(r)dr$$

can be shown to be

$$\mu_2' = \omega^2\phi'(\omega) - 2\omega\phi(\omega) + 2\int_0^\omega \phi(r)dr$$

by the use of Roberts' ingenious reduction formula.[22] For later use we observe that to keep μ_2' finite we must have $\lim_{\omega \to \infty} \omega\phi(\omega)$ finite ....(C25)

These equations and relationships were used in testing various mathematical equations for acceptability as expressions for $\phi(r)$. For example, a general polynomial was considered of the form

$$\phi(r) = (b_0 + b_1 r + b_2 r^2 + \cdots + b_n r^n)^c \qquad \text{.........(C26)}$$

From (C16) we prove $b_0 = 1$. From (C25) it is apparent that the highest ordered term of (C25), i.e., ω^{cn+1} must remain finite as ω approaches infinity which will only be true if $cn + 1 < 0$. Since n is positive, c must be negative. By (C10) we also show $b_1 = -1/c$. We decided to try an equation with $c = -1$ and this resulted in Equation (3) in the main text. A similar type of analysis led to Equation (1).

As another example, consider

$$\phi(r) = k^{-r^n}$$

where k and n are constants to be determined from the data. As long as k and n are positive real numbers, (C16) and (C17) are satisfied. It is also noted that $\phi(1) = k^{-1}$ and since $\phi(1)$ must lie between 0 and 1, k must be greater than 1. By (C18) we have

$$G(r) = \phi(r)\,[-n(\ln k)r^{n-1}]$$

Now $\ln k$ must be positive (since $k > 1$), and $\lim_{r \to \infty} G(r) = 0$ which is

[22] Roberts, Lewis H., "Graduation of Excess Ratio Distributions between Method of Moments," *PCAS; Vol. XLIV;* p. 52.

in agreement with (C11). However, $G(0) = 0$ and not -1 as it should by (C10). Hence, this equation is inappropriate for expressing $\phi(r)$.

This general approach was used on each equation considered to test to see if it was at all usable and to determine the value of certain constants which were required by the *a priori* conditions of Table M. I am critical of the results found by the use of orthogonal polynomials for Premium Groups .64 through .21 because each value of b_8 is negative. It can be seen that in order to meet condition (C17), we must have $b_8 > 0$ unless we do not allow ω to go to infinity. This forced us to abandon Equation (3) above $r = 5.00$. The press of time did not permit further experimentation.

Many possible curves were discarded because the constants were difficult to determine, and the entire project necessitated a family of about 50 different curves. Two special cases of (C26) were considered carefully because of their simplicity.

$$\phi(r) = \left(1 + \frac{r}{d}\right)^{-d} \qquad \text{............(C27)}$$

One method of establishing d for a given premium group was to get an equation for the variance of (C27) and solve it for d to find $d = (\mu_2 + 1)/(\mu_2 - 1)$. Then by calculating the variance of the raw data, we could determine d for testing purposes. Although a few premium groups were tested this way, we turned to another method. This procedure was to obtain $\phi(1)$ from the raw data and solve (C27) for d. A very good fit was obtained for Old Premium Group 11, but as we tried groups toward the extremes, the system broke down. In fact,

$$\lim_{d \to \infty} \left(1 + \frac{r}{d}\right)^{-d} = \frac{1}{e} = .368$$

By (C25) we can show that $d > 1$ is necessary, so that (C27) could not be used except when,

$$.368 \leq \phi(1) < .500$$

Because this was such a limited range, we modified the formula to get more flexibility:

$$\phi(r) = \left(1 + \frac{r}{d} + b_2 r^2\right)^{-d} \qquad \text{............(C28)}$$

Due to (C2) at $r = 0$, it can be shown that the maximum value for b_2 is $(d+1)/2d^2$ and by (C25) the minimum value for b_2 is $1/2$. Work was progressing on the programming of this equation when we discovered that Equation (3) was producing satisfactory results.

APPENDIX D

After preliminary investigations, the first four moments were calculated for Old Premium Groups 28 through 36 having as our objective the use of Pearson curves to graduate the underlying risk distributions. These moments were calculated using desk calculators so the procedure was shortened as follows:

1. Premium Groups 28, 29, 30 and 31 used 100 values selected from the raw data at the percentile points .995 through .005.
2. Premium Group 32 used one-half of the available 178 cases.[23]
3. Premium Groups 33, 34, 35 and 36 used all of the values available (two *very* exceptional risks were excluded) which gave us 123, 74, 43 and 21 cases respectively.

The results are as follows:

Old Premium Group	Expected Losses (E)	E^{-1}	Mean	V^2	β_1	β_2
28	42,700	23.42×10^{-6}	1.0050	.451167	3.613	7.914
29	49,700	20.12×10^{-6}	.9957	.375437	2.515	6.809
30	64,400	15.53×10^{-6}	.9907	.430462	5.259	11.771
31	73,900	13.53×10^{-6}	1.0000	.271702	2.101	7.020
32	111,000	9.01×10^{-6}	.9751	.277317	6.428	13.857
33	146,000	6.85×10^{-6}	1.0004	.173485	.920	4.719
34	211,000	4.74×10^{-6}	.9607	.145665	1.496	5.140
35	283,000	3.53×10^{-6}	1.0212	.107014	.696	3.002
36	699,000	1.43×10^{-6}	1.0000	.090419	.008	2.123

Roberts'[24] formulas were used for smoothing the moments, except that equal weights were applied to each group. This was done because the total actual losses in each group was considerable and we were willing to assume that the true expected losses for the group was equal to the mean expected losses in the sample. Thus, we wrote $b_3 = \sum E^{-1}\beta_1/\sum(E^{-1})^2$. Based on the data above this resulted in $b_3 = .2069 \times 10^6$; hence $\hat{\beta}_1 = .2069 \times 10^6 E^{-1}$. Similarly, we evaluated $b_4 = \sum E^{-1}(\beta_2 - 3)/\sum(E^{-1})^2 = .3247 \times 10^6$; hence $\hat{\beta}_2 = .3247 \times 10^6 E^{-1} + 3$. The test criterion for Pearson curves is $k \doteq 2\hat{\beta}_2 - 3\hat{\beta}_1 - 6$. In this case we have $k = .287 \times 10^6 E^{-1}$.

[23] Every other case was taken in such a way that the smallest value case was retained in the sample and the largest value case was excluded. Subsequent investigation indicated that the largest case was quite exceptional and, therefore, the decision was a sound one.

[24] Roberts, Lewis H., "Graduation of Excess Ratio Distributions by the Method of Moments," *PCAS; Vol. XLIV;* p. 51-52.

Over the range of E^{-1} shown above, k goes from $.67$ to $.04$. It was concluded that, although a Pearson Type VI curve was indicated, k was close enough to zero to use the Type III curve.

Rather than using Roberts' equation for b_3 as indicated above (and assuming thus that b_4 would be forced to conform to the Type III curve) it was decided that the basic equations for b_3 and b_4 would be solved subject to the additional restriction that the resultant solution would produce a Type III curve.

Let
$$f(E^{-1}) = \sum(\beta_1 - b_3E^{-1})^2$$
$$g(E^{-1}) = \sum(\beta_2 - b_4E^{-1} - 3)^2$$
$$h(E^{-1}) = 2\hat{\beta}_2 - 3\hat{\beta}_1 - 6 = 2b_4E^{-1} - 3b_3E^{-1}$$

The conditions are that $f(E^{-1})$ is to be a minimum, $g(E^{-1})$ is to be a minimum, but this is subject to the restriction that $h(E^{-1}) = 0$. Introducing λ, the undetermined Lagrangian multiplier, these conditions will be met if:

$$\frac{\delta f}{\delta b_3} + \lambda\frac{\delta h}{\delta b_3} = 0$$

$$\frac{\delta g}{\delta b_4} + \lambda\frac{\delta h}{\delta b_4} = 0$$

and
$$2b_4 - 3b_3 = 0$$

Taking the partial derivatives and solving for b_3, we get

$$b_3 = \frac{4\sum\beta_1E^{-1} + 6\sum(\beta_2 - 3)E^{-1}}{13\sum(E^{-1})^2}$$

The raw data gives us

$$b_3 = .2135 \times 10^6; \text{ hence } \hat{\beta}_1 = .2135 \times 10^6E^{-1} \qquad \text{(D-1)}$$

Again, following Roberts' approach for the variance, we solve

$$Na + b_2\sum E^{-1} = \sum V^2$$
$$a\sum E^{-1} + b_2\sum(E^{-1})^2 = \sum E^{-1}V^2$$

Solving, we get $a = .072609$ and $b_2 = .017021 \times 10^6$.

Since the new curves will have their means at 1.00, $\hat{V}^2 = \hat{\sigma}^2$ and we have

$$\hat{\sigma}^2 = .072609 + .017021 \times 10^6E^{-1} \qquad \text{(D-2)}$$

A characteristic of the Incomplete Gamma Function (the Type III curve) is that
$$p = \frac{4}{\beta_1} - 1 \qquad \text{(D-3)}$$

Combining (D1), (D2) and (D3), we can write

$$\hat{\sigma}^2 = .072609 + .31889/(p + 1) \qquad \text{............(D-4)}$$

$$\text{and } E = 53400(p + 1) \qquad \text{............(D-5)}$$

We were now in a position to calculate the Table M charges using the Pearson Type III curve as the underlying distribution of risks. For a trial value of p as used in the Tables of the Incomplete Gamma Function[25] determine $\hat{\sigma}^2$ from Equation (D4) and thence determine u_r from:

$$u_r = \sqrt{p + 1} + (r + .005 - 1)/\hat{\sigma}$$

Enter the Tables of the Incomplete Gamma Function with u_r and p and read out $I(u_r,p)$. Calculate $1 - I(u_r,p)$. Get the accumulation upward times $.01$ and this will be $\phi(r)$. By trial and error, an appropriate value of p was determined so that $\phi(1.00)$ was obtained at the desired value. The values are as follows:

Premium Group:	.19	.18	.17	.16	.15	.14	.13	.12
p:	.8245	1.1687	1.6453	2.3	3.2	4.9	8.0	16.2

The first three groups represent averages of pairs of values found by two trial values of p as follows:

Premium Group .19 is a 268 : 87 weighting of $p = .8$ and $p = .9$
Premium Group .18 is a 77 : 169 weighting of $p = 1.1$ and $p = 1.2$
Premium Group .17 is a 47 : 39 weighting of $p = 1.6$ and $p = 1.7$

A sample of the worksheet used for Premium Group .13 is as follows:

r	u_r	$I(u_r,8.0)$	$1 - I$	$\phi(r)$
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
1.58	4.78	.947	.053	.01079
1.59	4.81	.950	.050	.01026
1.60	4.84	.952	.048	.00976
1.61	4.87	.954	.046	.00928
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.

An interesting corollary to the work in this area of the Table is the relationship between Table M and the χ^2 distribution. From the values

[25] Pearson, Karl, *Tables of the Incomplete Gamma Function*, Cambridge University Press, 1957.

of p above, determine γ, the degrees of freedom used in tabling the χ^2 function by

$$\gamma = 2(p + 1)$$

Also from the above, the value of $\hat{\sigma}$ is available for a given premium group.

Then $$r_{1-\alpha} = \frac{\hat{\sigma}}{\sqrt{2\gamma}} (\chi^2_{\gamma, \alpha} - \gamma) + 1.00$$

where $r_{1-\alpha}$ is the value of r at the α percentile and $\chi^2_{\gamma, \alpha}$ is the tabled value for χ^2 with γ degrees of freedom and at a probability value of α. For example, on Premium Group .13,

$$\gamma = 2(8.0 + 1) = 18$$

$$\hat{\sigma} = [.072609 + .31889/(8.0 + 1)]^{1/2}$$

$$= .32870$$

$$r_{1-\alpha} = .05478\,(\chi^2_{18,\alpha} - 18) + 1.00$$

If we inquire as to the value of r for which 5% of the risks exceed it, we find $\chi^2_{18,\ .05} = 28.8693$ and thus $r_{.95} = 1.595$. Notice how this agrees with the above example. Naturally, the converse question can be asked – for a given value of r, what is the probability that it will be attained or exceeded? In our example, a value of $r = 1.20$ will be exceeded by about 25% of the risks because

$$\chi^2_{18,\alpha} = \frac{1.20 - 1.00}{.05478} + 18 = 21.6501$$

and a reference to the χ^2 table shows this value has a probability of approximately $.250$.

APPENDIX E

As the risk size increases, we expect the shape of the risk distribution to approach the normal curve and we expect the variance to approach zero ultimately. It was decided to use Old Premium Group 36, which had $E = 699{,}000$ and $V^2 = .090419$, as a starting point. Assuming (and this can only be true as an approximation) that the only variance remaining at these large risk sizes is the variance in the claim distribution, and the number of cases (N) is fixed, the ratio of the two standard deviations of the expected losses will be the same as the ratio of the two standard errors of the mean, that is

$$\frac{\hat{\sigma}_E}{\hat{\sigma}_{699000}} = \frac{S/\sqrt{N_E}}{S/\sqrt{N_{699000}}}$$

where S is the standard deviation of the claim distribution. (Because N is so large, I have used N rather than N-1 in the above formula.)

This may be simplified and rearranged as follows:

$$\hat{\sigma}_E = \hat{\sigma}_{699000}\sqrt{N_{699000}\,(Average\ claim\ cost)/N_E\,(Average\ claim\ cost)}$$
$$= \sqrt{(.090419)\,(699000)/E} = \sqrt{63203/E}$$

Thus, as the size of the risk increases, the standard deviation decreases and approaches zero as a limit. Solving for E we have

$$E = 63200/\hat{\sigma}^2 \qquad \text{............(E1)}$$

which was used to set the expected loss ranges for the premium groups using the normal curve.

For a trial value of $\hat{\sigma}$, determine z_r by

$$z_r = (r + .005 - 1)/\hat{\sigma}$$

Enter a table of the normal curve and read out the probability integral value representing the area under the curve to the right of z_r. Get the accumulation (to 3 decimal places) upward times .01 which equals $\phi(r)$. Values of $\hat{\sigma}^2$ which produced the desired values at $\phi(1.00)$ were:

Premium Group:	.11	.10	.09	.08	.07	.06	.05	.04	.03	.02	.01
$\hat{\sigma}$:	.275	.250	.225	.200	.175	.150	.125	.100	.075	.050	.025

A sample of the worksheet used for Premium Group .11 is as follows:

r	z_r	Q_r	$\phi(r)$
.	.	.	.
.	.	.	.
.	.	.	.
1.32	1.1818	.119	.01660
1.33	1.2182	.112	.01541
1.34	1.2545	.105	.01429
1.35	1.2909	.098	.01324
.	.	.	.
.	.	.	.
.	.	.	.

DISCUSSION BY LESTER B. DROPKIN

The current paper by LeRoy Simon, which reports the trials and tribulations, as well as the methods and procedures, by which the National Council's Subcommittee to Review Table M developed the new 1965 Table M is one which is sorely needed and which will avoid the very unfortunate situation which occurred when Table M was modified in 1954 without any concomitant paper appearing in the Proceedings.

This paper is important. It is not to be read casually, commuting to and from work. Anyone who has had, currently has, or will have occasion to become more than just passingly involved with retrospective rating is well advised to set aside a time and a place for a careful reading of the paper. Happily, this need not be a chore – for LeRoy Simon has the faculty of not only being able to be actuarially creative, but also of being able to write well.

Mr. Simon tells us what the scope and nature of the paper is at the very outset. It is to be concerned with only "certain aspects," and the reader is to have "a basic knowledge of Table M and its use." From such suppositions it might be expected that the paper would be rather more difficult to approach than in fact it is. Mr. Simon has wisely, and very nicely, made use of a number of appendices for an expansion upon the more mathematical aspects, thereby allowing the main recitation in the body of the paper to proceed smoothly. This segregation of much of the theory and mathematical details into separate appendices allows the reader, according to his own talents and interests, to pursue more deeply those particular aspects which are of special concern to him. I did feel that at times, however, there was an unfortunate relegation of important material to an appendix. For example, it was rather unexpected to find one of the most fundamental relationships, viz., the definition of the charge in terms of the underlying risk distribution, tucked away as the twelfth equation in Appendix C.

The paper is quite complete in its treatment of the many steps which were involved in the preparation of the new 1965 Table M. In this connection, specific mention should be made of the finely detailed exhibits which accompany the paper. Each important step is illustrated by an appropriate exhibit. The reader, therefore, is able to work along, as it were, with the Committee, and to gain a real feeling for the methods followed. To get the most out of the paper, the reader should not simply read and passively accept. Rather should he actively participate, constantly having pencil in hand, recreating (albeit on a miniature scale) the many details.

One of the first decisions made was to assume that the standard National Council permissible loss ratio applied to all of the data reported. Now while it may be true that this standard permissible loss ratio is most commonly used in terms of numbers of states, it is also true that large and significant portions of the data come from states where a different permissible loss ratio obtains. Since the sorting and grouping of the risks conditions the raw values which form the basis of all subsequent steps, it would have been of interest to know what consideration was given to this point.

Another important early decision was to adjust the data to a more current level. The need for some adjustment cannot be questioned. However, to assume, in effect, that each and every loss increased by a flat amount is open to very serious criticism. In the present context, I doubt whether the Committee had any real choice of an alternative to this simple hypothesis; yet it does point out an area for future investigation. While I fully recognize the difficulties of developing an adequate theory even for static conditions, we will have to concern ourselves increasingly with the complex effects which dynamic changes bring.

Among other actions taken were two which I feel added particularly to the accomplishment. These were: (1) the decision to form the table in such a way that the charges at an entry ratio of unity would be spaced at intervals of .01 between premium groups; and (2) the decision to extend the range of application of the table.

The dominant theme of Mr. Simon's paper is, of course, the search for a formula which would yield columnar charges. The recital of the steps which led to a successful culmination of this endeavor is a monument to the virtue and power of a trial and error, heuristic approach. Monuments, however, are most often erected in memory of what has been and no longer is. It would be most fitting and proper if we could indeed believe that the construction of the next Table M will be achieved by following a quite different route.

A table of charges should be a byproduct, falling out naturally from more fundamental considerations. From a theoretical point of view, the risk distribution of incurred loss amounts is logically prior to the Table M function. It is the analytic expression for the underlying risk distribution that we should be looking for. But even this distribution itself arises out of the interaction of two still more fundamental distributions, viz., the distribution of claim costs and the distribution of claim occurrences.

Investigations into these areas is precisely the subject matter of the mathematical theory of risk. There can be found the general abstract expressions and the symbolic representations of the pertinent mathematical relationships. What we do not yet have are the particular forms and the parametric values of the functions which appear in the equations of the mathematical theory of risk.

Unfortunately, it often appears that the functions which arise in the mathematical theory of risk are characteristically of a complex and intractable nature. No doubt we shall therefore have to call upon a wide variety of computer techniques, approximation techniques, and, in general, upon the whole bagful of methods which have successfully been used elsewhere in arriving at specific numerical results. This may well mean that the final formulas to be used will not be neat and aesthetically satisfying. We may even be surprised to find that an eighth degree reciprocal polynomial is the practical device which corresponds to a theoretically derived Table M function. But consider the difference between using such a formula simply because it happens to work, and using it as a convenient detail in a wide theoretical construct.

The philosophical speculations of the preceding paragraphs are in no sense meant to be a criticism of Mr. Simon and his colleagues. They were meant to express my belief that we are now at a point which will see the rapid development of many new approaches to actuarial problems, and that these new approaches will reflect a much greater degree of mathematical maturity and sophistication than ever before.

In recounting for us the herculean labors of three years, it is understandable that LeRoy Simon would restrict himself to only those aspects in which he was most directly interested and involved. I hope that this means that we can look forward to seeing additional papers which will treat some of the other questions of interest. Among those which immediately suggest themselves are: the question of using the data of all risks vs. the data of experience or retrospectively rated risks; the question of using a table of charges based on workmen's compensation for other lines; the question of one year vs. three year charges; and the question of the effect of a per claim or per accident limitation.

Finally, I can offer only a most heartfelt second to LeRoy Simon's wish that the necessary studies and work on the next Table M be started immediately. It can be truly said: the time *is* now.

DISCUSSION BY CHARLES C. HEWITT, JR.

Mr. Simon's work is a modern "labor of Hercules." Both he and his employer, which made its facilities so readily available for this project, well deserve the epithet "good citizen" from the entire casualty insurance industry.

My remarks are not intended to encompass the job recently completed, but, except for a mathematical note appended, are designed to take up from the point where this paper leaves off. For convenience they may be divided between mathematical and non-mathematical, but are treated in reverse order.

Non-Mathematical

To anyone on the commercial side of the casualty insurance business it has been obvious that net Table M charges have been inadequate in most situations for a long time. What is particularly disturbing, however, is the abundant evidence that the new Table M (even before filing) may already be inadequate in some instances, and almost certainly will become inadequate tomorrow. Intuitively it should be obvious that for "fixed expected loss amounts" the variance of loss ratios will increase as "severities" increase (and "frequencies" decrease). Thus in the normal situation in which selected maximum and minimum ratios produce "charges" in excess of "savings," net insurance charges will be inadequate during any period of increasing severities. Does anyone recall any evidence of decreasing severities in the liability lines in recent years?

Furthermore, there are areas of the commercial liability business in which Table M ratios derived from workmen's compensation experience have been and are now clearly inadequate. A good example in commercial auto liability is long haul trucking. We must produce adequate "Table M"s for all liability lines because retrospective rating (even "retro-type" dividend plans) are being used more and more.

Mr. Simon realizes all of this and suggests a number of constructive steps which ought to be started upon right away. Such a program might include:

(1) Finding an appropriate mathematical model for risk loss-ratio distributions. (Let's rid ourselves of this craven idolatry of raw numbers!)

(2) Determining sets of parameters so that:

(a) values may be substituted in the model for separate lines or even sub-lines of insurance, and

(b) values may be updated frequently without recourse to the arduous labors apparent in the current effort.

(3) When necessary, using convolutions from loss distributions of a single claim developed either analytically, or by approximation or Monte Carlo techniques.

(4) Allowing for the effects of anti-selection, if such anti-selection exists.

Mathematical

Recently I came upon a report of the California Inspection Rating Bureau dated January 31, 1963 entitled "California Experience Rating Statistics – Series II – By Interval of Subject Premium Loss Ratio." With only minor smoothing and ignoring the breadth of the premium intervals, I obtained an excellent fit for loss-ratio distributions by using a Gamma-function (Pearson III); this is the same distribution familiar to us from the recent papers on the negative binomial and referred to in Mr. Simon's current paper. All Chi-square tests were met for subject premium intervals from $5,000 and up. Below $5,000 a problem is created by the substantial frequency of risks with *no losses*. Even so, I developed a Gamma-function parameter for all premium intervals down to and including the less-than-$500 risks.

The interesting point is that I found an empirically-developed relationship among the various Gamma-function parameters of the form:

$$\log (p+1) = a \log P - b$$

where $p =$ the Gamma-function parameter (used in Pearson's tables)

$$P = \text{premium size}$$

and a and b are constants obtained by "least squares".

I hope to expound this point more fully in a future paper, but obviously my ideas have not crystallized sufficiently at this stage. Perhaps someone else may make use of these findings in the meantime.

Mathematical Note on Appendix C

Sketches *1* and *2* are incompatible since

$F(r_0) = Pr\ (r \leq r_0)$, therefore

$F(0) = f(0)$. This error also appears in (C6), which does *not* follow from (C5).

Thus (C7) becomes $F(r) \geq f(0)$ for $(0 \leq r \leq \omega)$ and (C10) becomes $G(0) = f(0) - 1$.

Again our gratitude must be expressed to Mr. Simon for accomplishing this awsesome chore.

A MATHEMATICAL APPROACH TO FIRE PROTECTION CLASSIFICATION RATES

KENNETH L. McINTOSH

I. INTRODUCTION

A. *The Problem.*

The actuarial core of the fire protection classification rate relativity problem is the actuarial core of any fire rating problem: The fire rate structure must be (or, at least, for generations, by custom and usage, has been) refined far beyond the refinement of the fire statistical plan. Entirely apart from the detail of recently-publicized shortcomings of the current most widespread fire statistical plan, the National Board of Fire Underwriters *Standard Classification of Occupancy Hazards,*[1] further refinement of the statistical plan is no answer of itself because, very simply, of credibility considerations. A fact well known to any experienced fire ratemaker has been formalized by Dr. Almer in the statement: "Statistical experience proves that most claims in any branch [of nonlife insurance] will be concentrated in some few statistical risk groups (or tariff partitions), leaving most tariff groups without effective statistics, even if a five-year experience is utilized."[2]

Specifically in present instance, the actuarial problem is to support classification rates and rate relativities for as many as ten or more public fire protection classifications upon a statistical plan which, credibility considerations aside, spans the entire range of protection classifications with only two statistical classifications, "Protected" and "Unprotected." It is submitted that extension of theories already proposed[3,4] not only will permit a mathematical approach to this problem, but also leads to certain working formulas which are completely and immediately practical of application in cook-book fashion to save laborious trial-and-error calculation in rate revision operations.

B. *Fire Protection Classifications.*

In general, the relative efficacy of public fire defenses is evaluated for

[1] Among others, *The National Underwriter,* June 19, 1964, p. 2.

[2] Almer, (11). P. 341. (Bibliography is appended.)

[3] McIntosh, (14).

[4] McIntosh, (15). Specifically the section: *Variable Hazard,* p. 15.

rate making purposes by application of the National Board of Fire Underwriters *Standard Schedule for Grading Cities and Towns of the United States with reference to Their Fire Defenses and Physical Conditions.*[5] This document, seldom designated by its full official title, has been described elsewhere in some detail,[6] but one particular feature is pertinent to what follows here. Application of the *Standard Schedule* to the public fire defense facilities maintained by a given community does *not* produce a protection *classification* directly; it produces a protection *"grading,"* which subsequently is converted to a classification for rate making and underwriting purposes.

In the complete absence of public fire defenses recognizable as such, a maximum grading of 5000 "points of deficiency" is assessed. For recognizable fire defense facilities, the 5000-point maximum is reduced to some lesser figure, depending upon the detail of conditions found by inspection to exist. Theoretical perfection, never yet approached, would result in a point grading of zero. The protection grading actually assigned to any given community will be some number of points of deficiency from zero (theoretically) to 5000; the better the public fire defenses, the lower will be the deficiency-point total, or "grading."

The present significance of this fact is that the protection grading, although necessarily expressed in discrete units, the "points of deficiency," must be considered a *continuous variable.* Any grading from zero to 5000 is theoretically possible, although for practical reasons a grading of less than 1000 points is extremely difficult to achieve, and no city in the United States currently enjoys a grading of 500 points or less. In theory the fire rate must be a *continuous function* of this *continuous variable,* despite the fact that, for obvious reasons, it cannot be treated as such in practice.

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[5] A notable exception is found in the rating system of the State of Texas, whereunder public protection is evaluated by a very different approach. There are other minor exceptions.

[6] Riegel & Miller, (19). p. 564.

The conversion of *grading* to *classification* is illustrated in the table below.

Grading (Point Total)	N.B.F.U. Protection Class*	N.B.F.U. Statistical Class**
0 – 500	1	"Protected"
501 – 1000	2	" " "
1001 – 1500	3	" " "
1501 – 2000	4	" " "
2001 – 2500	5	" " "
2501 – 3000	6	" " "
3001 – 3500	7	" " "
3501 – 4000	8	" " "
4001 – 4500	9	"Unprotected"
4501 – 5000	10	" " "

* According to the N.B.F.U. *Standard Schedule.*
** According to the N.B.F.U. *Standard Classification of Occupancy Hazards.*

Two points should be noted for reference. First, mathematically speaking, the classification is a step function of the continuous grading, hence the rate as a function of classification becomes a step function of the grading. This represents the imposition of an artificially discrete mathematical model upon what in actuality is a continuum. The practical necessity of this departure from actuality is not questioned. Any rating system whereunder the rate must vary with variation of a single grading point anywhere in the 0-5000 range would be impossible of practical application, if for no reason other than that it would drive the ratemaker insane in very short order. But whatever the practical necessity, the artificiality of the model must be recognized, to focus attention upon the problem of just how great a departure from actuality can be tolerated before the inevitable and extremely practical consequences of the fact of the departure itself may become unacceptably severe. In other words, for ratemaking purposes, how refined should the protection classification system be to attain the maximum simplicity of practical operations consistent with avoidance of practical problems of unacceptable severity? The question is not academic.

Secondly, both the number of the classifications and the exact locations of interclass boundaries are arbitrary. Other classification systems can be, and in fact have been, formulated by subdividing the grading range into brackets differing markedly from those shown above. Provided, of course, that stability of the rate structure is not destroyed by too-frequent revision, there is absolutely nothing to prevent the fire ratemaker from establishing protection classifications in whatever number, with inter-class boundaries at whatever locations, may prove most expedient and appropriate to the problem at hand, so long as it is specified just which protection classes are "Protected" and which are "Unprotected" for statistical reporting purposes. The N.B.F.U. protection classifications tabulated above are a generally (but not universally) recognized standard of reference, but it is not unknown for a simplified, variant system to underlie Dwelling rates in the same jurisdiction wherein the N.B.F.U. classes may underlie commercial risk rates. At this writing a six-class system to underlie Dwelling fire rates has been recommended to all fire rating bureaus nationwide.[7]

The importance of these points, first, that any protection classification system is an artificial model and, secondly, that the detail of any such system is arbitrary, will be developed in Section IV. A.2, following.

C. *Designation of Classes.*

Three categories of classifications are involved in what follows. For present purposes, the term *"underwriting class"* will be used to designate either an occupancy class, *e.g.*, "Dwellings," "Metalworkers," etc., or a construction-occupancy class, *e.g.*, "Frame Dwellings," "Brick Metalworkers," etc. The present development is not concerned with relationships between underwriting classes, but only with certain relationships between sub-classes within any given underwriting class.

A *"statistical class,"* or a *"statistical sub-class"* of an underwriting class, will be that sub-class of the underwriting class upon which loss experience is reported separately as "Protected" or, alternatively, as "Unprotected."

A *"protection class"* is a sub-class either of the *"Protected"* or of the *"Unprotected"* statistical class. The precise definition of a given protection class must be in terms of grading-point brackets, as illustrated above, but there will be no occasion in what follows here to specify such brackets, nor even again to refer to the grading except in general terms in discussion of continuity. In particular instance, it must and will be specified which protection classes belong to the "Protected" statistical class,

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[7] Memorandum: *Recommended Schedule of Fire Insurance Rates for Dwellings and Private Outbuildings Appertaining Thereto,* dated December 9, 1959. The Inter-Regional Insurance Conference (now known as The Fire Insurance Research and Actuarial Association). p. 3.

and which to the "Unprotected" statistical class. The term, "protected class" or "unprotected class," *without* capitalization or quotation marks refers to a protection class which is a member of the "Protected" or of the "Unprotected" *statistical* class. Protection classes will be specifically designated by number, and in particular instance the ranges of numbers assigned to protection classes belonging to the "Protected" and "Unprotected" statistical classes, respectively, will be specified. Invariably, the *lower* the numerical designation of a protection class, the *better* the quality of public fire protection associated therewith. *Higher* class numbers denote *inferior* protection.

D. *The Presentation.*

The development proper may be said to begin with the consideration of rate structures in Section IV. Sections II and III are concerned primarily with essential background material, definitions and notation. To support developments presented here, it has been necessary to reformulate in precise mathematical expression certain theoretical material previously presented by McIntosh in somewhat loose statement.[5]

If the working formulas of Section VI, dealing with practical applications, are accepted on faith, then Section VI (page ??) may be read independently of all else save only reference, as necessary, to definitions and notation to be found in Sections II – IV.

II. FIRE PROTECTION CLASSIFICATION RATES AND RATE RELATIVITIES

A. *The Fire Protection Classification Rate*

Fire rating terminology contains no exact equivalents of the casualty terms: "classification rate;" and, "classification rate relativity." Partly this is because terminology must be fitted in particular instance to the detail of a particular rating schedule, and the variations of detail among the several rating schedules in use are too great to permit any sort of standardized expression completely unambiguous out of context. More to the point, however, is the fact that a true "classification rate" is virtually unknown in fire. The fire "classification rate" will be an average rate in nearly all cases. Furthermore, the fire class average rate may reflect fortuitous variation of conditions of hazard completely extraneous to the particular hazards definitive of the class. For example, in a given state, the N.B.F.U. Class x "Mercantile Building" average rate may reflect a significantly disproportionate concentration in Class x of a particular

[5] McIntosh, (14).

type of construction not uniformly distributed among the several fire protection classes. The possible severity of such distortion is exemplified by the fact that, in the State of Louisiana, the average Mercantile Building rate of N.B.F.U. Class 9 is appreciably *higher* than the average rate of N.B.F.U. Class 10, although by provisions of the applicable schedule, the Class 9 Mercantile Building rate must be exactly 5% *lower* than the Class 10 rate wherever and whenever all hazard conditions other than the public protection are equal, whatever those extraneous conditions may be.

That such considerations apply not only to the fire "schedule" rate, but also to the fire "class" rate, may not be obvious. However, many states surcharge the Private Dwelling "class" rate for additional families in occupancy, and whether this be done by "schedule charge" or by separate basis rate tables seems a distinction of convenience without substance. In an actual case known to the author, a disproportionate concentration of multiple-family occupancy in N.B.F.U. Class 3 resulted in a distortion of Dwelling protection classification rate relativities of better than 10%, in any comparisons of Class 3 with other protection classes. If the so-called "loss constant rating method" is used, whereunder the "effective" rate becomes a function of policy size, the distribution of policy size among the several protection classes may not be, and in general will not be, uniform. Again using Louisiana as example, the mean "effective" Brick Dwelling Building rate of N.B.F.U. Class 3 is *lower* than that for Class 2, precisely because the average Dwelling policy size in the City of New Orleans (which dominates Class 3) is appreciably higher than the average policy size elsewhere in the State.

In any given instance, the variation of extraneous hazard conditions from class to protection class may be insignificant; or the variation may be of a nature such that it is not reflected in the rates produced by applicable schedule. Where this is the case, protection classification rate relativities may be determined by direct comparisons among the classification rates themselves. But in many instances direct interprotection-class rate comparison is useless for the purpose of determining the effect upon rate of public fire protection of itself and by itself. Another concept is needed.

For present purposes, the *"protection classification rate"* of Class x is defined to be the appropriately weighted average of individual rates respectively applicable to each of the several risks in Class x. (How this shall be determined in the case of existing rates is of no present concern.) It is these *rates* which *collectively* must be reconciled in the course of rate revision to statistical classification *premiums* developed by application of the rate level adjustment formulas independently of the actual rate revision calculations.

B. *The Fire Protection Classification Normal.*

To isolate for study the protection component of the protection classification rate, the *"protection classification normal"* of Class x in general is defined to be that value which the protection classification rate would have assumed had all extraneous conditions of hazard throughout Class x been identical to those actually existing throughout the highest-numbered class of the protection classification system, *except* when dealing with the "loss constant method" of rating private dwellings. In that particular case, it may be desirable to normalize the effective rate to the statewide mean policy size. The classification *normal* may be and should be conceived as the classification rate "normalized" to a standard set of extraneous conditions.

Choice of the highest-numbered protection class to be in general the standard-of-reference is not entirely arbitrary. Since the rate of this class reflects no recognition whatever of public protection, it already is self-decomposed into an extraneous component equal to the rate itself, and a protection component which (depending upon form of the calculation) will be zero in summations or unity as a factor in products. Entirely apart from any theoretical significance, the self-decomposition of the rate of the highest numbered class may prove extremely convenient in practical calculation involving certain rating schedules.

The difficulty of calculating the classification normals, once the classification rate of the highest-numbered class has been determined, will vary widely according to the detail of the rating schedule. In some cases, precise calculation may be tedious to a point of practical impossibility. In general, where accurate calculation is not practicable, at least acceptably accurate estimates can be made. It is here assumed that either accurate calculation or acceptably accurate estimate of normals can be made in all cases, given adequate data of field conditions which must be obtained in any case.

The *"rate-normal ratio"* is defined to be the quotient of the classification rate divided by the normal, or, alternatively, the reciprocal quotient, of normal divided by rate. Although separate notation for each of these reciprocals will prove convenient to avoid negative exponents, in general discussion there will be no need to distinguish between them, and the term "rate-normal ratio" will be applied indiscriminately to either.

Where distinction may be necessary in particular instance, it will be made by notation if not clear from context.

For what is to follow, it is not sufficient that the rate-normal ratio be obtainable by direct division of the normal into the rate, or *v.v.* Its purpose is to permit calculation of the rate from the normal, or *v.v.*, when *one only* of these quantities is known independently. As with the normal itself, the difficulty of calculating or estimating the rate-normal ratio solely upon the basis of schedule provisions and known field conditions, will vary widely from schedule to schedule. It must be assumed for what follows that calculation or acceptable estimate of the rate-normal ratios can be made by some method *other* than direct division between rate and normal developed independently of each other.

It is further assumed here that the rate-normal ratio will be a constant, characteristic of class and not necessarily the same for all protection classes. No generalizations can be made concerning special methods required when the rate-normal ratio becomes a function of the normal itself, except to say that in the author's experience graphical methods prove expedient and usually will yield satisfactory solutions.

C. *The Rate Revision Problem.*

The *"underwriting target rate"* is defined to be that rate which, if applied indiscriminately to each and every risk of the underwriting class ("Frame Dwelling," "Mercantile Building", etc., etc.), will produce the underwriting classification premium required by the rate level adjustment formulas. (Here assumed to have been pre-determined.)

The *"protected target rate"* is defined by analogy, with specific reference to the "Protected" statistical sub-class of the underwriting class. (Here assumed to have been pre-determined.)

The *"unprotected target rate"* is defined by analogy, with specific reference to the "Unprotected" statistical sub-class of the underwriting class. (Here assumed to have been pre-determined.)

The *"underwriting trial average"* is defined to be the average of the several protection classification rates for the underwriting class over the entire range of protection classifications. This average is to be weighted in accordance with that proportion attributable respectively to each protection class of the total amount of insurance written throughout the underwriting class.[9]

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[9] In practice the exact distribution of liability among the several protection classes may not be ascertainable. In such instance, the distribution is approximated by the best available set of indices, *e.g.* risk count by class, etc.

The *"protected trial average"* is defined by analogy, with specific reference to the "Protected" statistical sub-class of the underwriting class.

The *"unprotected trial average"* is defined by analogy, with specific reference to the "Unprotected" statistical sub-class of the underwriting class.

From the foregoing definitions, it will follow by straightforward algebra (if not obvious) that a given set of protection classification rates will produce upon field application the required classification premiums if and only if the trial averages produced by those rates are respectively equal to the corresponding target rates. The problem of developing adjusted protection classification rates which will produce required underwriting and statistical classification premiums thus resolves itself into the problem of developing adjusted protection classification rates which will produce trial averages equal to pre-determined target rates.

Where only the underwriting target rate is specified, and where pre-existing protection classification rate relativities are to be left undisturbed, the immediate solution is, of course, simply to multiply all existing protection classification rates by the percentage quotient of the required underwriting classification premium divided by the most recently available reported classification premium. However, if separately-specified protected and unprotected targets require respective adjustments in differing percentages, simple multiplication of the protected and unprotected rates by the respectively indicated percentage factors will distort relativities, and may produce inversions such that the rates in a given community will *decrease* if the fire department is disbanded and the fire engines are sold for scrap.[10] In any case, simple multiplication of all existing rates by a constant percentage factor is inappropriate where for any reason the protection classification relativities are to be revised regardless of any premium adjustment. Where uniform percentage adjustment of all protection classification rates is inappropriate, solutions may be obtained by trial and error.

There are less-tedious methods.

D. *General Notation.*

The following general notation will be used throughout what follows, excepting only where superseded by special notation to be defined when introduced.

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[10] McIntosh, (16). Specifically the section: *Rate Adjustment*, p. 131. The principle here involved is not restricted to protection classification rates, but is completely general in application whenever related classes are to be adjusted.

Let:

x: – The class number designating a particular protection class: "Class x."[11]

α: – The highest class number assigned to a protected class. To be specified in particular instance.

β: – The lowest class number assigned to an unprotected class. To be specified in particular instance.

z: – The highest class number assigned under the protection classification system. To be specified in particular instance.[12]

R_x: – The protection classification rate of Class x.

Q_x: – The protection classification normal of Class x. (By definition of Q_x; then: $Q_z \equiv R_z$, *except* when dealing with the "loss constant rating method." Choice of "Q_z" *vs.* "R_z" as appropriate to immediate context.)

$q_x = Q_x/R_x$, $r_x = R_x/Q_x$: – The rate-normal ratio of Class x. (Choice of "r_x" *vs.* "q_x" as convenient. By definition of Q_x; then: $r_z = q_z \equiv 1$, *except* when dealing with the "loss constant rating method.")

$\sum_T = \sum_{x=1}^{z}$, $\sum_P = \sum_{x=1}^{\alpha}$, $\sum_U = \sum_{x=\beta}^{z}$: – For reasons of convenience to become apparent.

T: – The underwriting target rate.

P: – The protected target rate.

U: – The unprotected target rate.

v_x: – The pro-rata portion attributable to Class x of the total amount of insurance written throughout the underwriting class. $\sum_T v_x = 1$

$v_P := \sum_P v_x$; $v_U = \sum_U v_x$

[11] Under some classification systems, the several classes are lettered rather than numbered, but for what follows it is necessary that numbers replace any non-numerical class designations.

[12] For consistency, the Greek omega, "ω", probably should be used here, but this is avoided because of the typographical similarity of "ω" to Roman "w", frequently used in what follows.

$$w_x := \begin{cases} v_x/v_P; \text{ if } 1 \leq x \leq \alpha. \text{ Then: } \sum_P w_x = 1 \\ v_x/v_U; \text{ if } \beta \leq x \leq z. \text{ Then: } \sum_U w_x = 1 \end{cases}$$

$\hat{T} = \sum_T v_x R_x$: – The underwriting trial average rate.

$\hat{P} = \sum_P w_x R_x$: – The protected trial average rate.

$\hat{U} = \sum_U w_x R_x$: – The unprotected trial average rate.

$\hat{T}_Q = \sum_T v_x Q_x$: – The unprotected trial average normal.

$\hat{P}_Q = \sum_P w_x Q_x$: – The protected trial average normal.

$\hat{U}_Q = \sum_U w_x Q_x$: – The unprotected trial average normal.

Rate notation as given above invariably refers to the *"adjusted"* rates, *i.e.* those to be placed into effect upon completion of rate revision calculations. Corresponding notation with reference to the *"existing"* rates in effect immediately prior to rate revision is obtained by superscript, thus:

"R_x^e"; "Q_x^e"; "T^e"; "P^e"; "U^e". (See Section IV.B., following.)

$\overline{Y}(\underline{Y})$: – The maximum (minimum) value of whichever of the foregoing quantities (except "x") may replace "Y", *e.g.* $\overline{R}_x$ ($\underline{R}_x$).

III. RATE VECTORS; PROTECTION CURVES

A. *Sets and Vectors.*

That highly useful concept which permeates the structure of modern mathematics, and which a friend of the author has christened, "The Great God, Set", in impious reference to the fraticidal villain of the Pharaonic pantheon,[13] appears to be the mathematical key to the fire rating problem, just as already it has proved the key to other problems long considered invulnerable to systematic, mathematical attack. In simpler applications, *e.g.* the solution of simultaneous linear equations, the villain need not be formally identified.[14] As the problem becomes more complex, a point is reached where either he must stand forth in his own true shape, or else the development at best becomes interminably tedious and at worst becomes sheer impossibility. It is suggested that the critical point already may have been passed in fire rating theory.[15] In any event, it will be reached here.

A completed jigsaw puzzle presents a picture not inherent to any single one of its pieces, nor even collectively inherent to all of its pieces

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[13] Among others, Müller, (17). p. 114.

[14] But see, for example, Kemeny *et al.*, (6). Ch. 4, Sect. 3, p. 223, for a set-theoretic approach to simultaneous linear equations.

[15] *Cf.* McIntosh, (14).

except when these are arranged in particular relationships to each other. It is *not* the exact value individually assigned to any one protection classification rate, R_x, nor even the *combination* of values assigned respectively to each protection classification rate, which will produce required underwriting classification premiums. It is only by the assignment to each protection classification rate, respectively and *in order,* of the values represented in some *permutation* of some combination of possible values that the rate structure can be reconciled to a premium structure independently pre-determined. Except in special instance, the appropriate permutation will be unique to any given combination, and except in trivial instance[16] the choice of appropriate combinations will *not* be unbounded.

Two puzzles are readily identified and distinguished from each other by reference to the one as, *e.g.,* "the ship picture," and to the other as, *e.g.,* "the horse picture." Equally unambiguous identification and distinction by meticulously cataloging the shape, size, coloration and place in the pattern of each individual piece of each respective puzzle, will prove an endless and fruitless task with any but the simplest of those puzzles designed for amusement of the pre-school-age toddler. A vector exhibits a particular permutation of a particular combination of values. A pair of ordinary Cartesian coordinates, (a,b), which is a very simple vector, does not represent the same point as the pair (b,a) unless it happens that $a = b$ under all possible circumstance. When the vector itself is identified, there is no need to catalogue the individual components, and the latter task may prove quite a chore when these components must be treated as variables to be subsequently evaluated.

Finally, when a jigsaw puzzle must be moved, it is easier and quicker to move it assembled upon a biscuit board than to carry it piece by piece across the room. There will be no need for laborious re-assembly to re-form the picture; and there is no chance of a piece being accidentally dropped in transit, to be unintentionally kicked out of sight under the sofa.

A fire rate structure expressed in terms of rate vectors is easily transformed mathematically from what it is to what it should be. Systematic

[16] With highly specialized underwriting classes, it may happen that in a given territory no risks in class will exist, yet a rate structure for the underwriting class may be desired either for the sake of formal completion of a comprehensive rating schedule, or in anticipation of future establishment of risks in class within the territory. In such cases, normally the ratemaker will incorporate into the schedules the rate levels of other states where the class is represented, but obviously there are no bounds to his judgment in this instance.

mathematical approach to the fire rating problem on any basis other than in terms of rate vectors seems impossible.

B. *Rate Vectors. Notation and Definitions.*

(Superscript Convention: – The convention of tensor notation, omission of the parentheses distinguishing a superscript index, "$R^{(i)}$", will be followed as a matter of convenience. If it is remembered that, throughout what follows, a letter superscript is an index, *not* an exponent, there will be no occasion for confusion.)

$\mathbf{R}^i = (R_1^i, R_2^i, \ldots, R_\alpha^i; R_\beta^i, \ldots, R_z^i)$: – *A rate vector.* The superscript identifies the classification rate, R_x^i, as a component of the vector, $\mathbf{R}^i$. The superscript does *not* designate a predetermined value of R_x. Also, it is quite possible that for some x, then: $R_x^i = R_x^j$, where $\mathbf{R}^i \neq \mathbf{R}^j$. It will be true that $\mathbf{R}^i = \mathbf{R}^j$ only if $R_x^i = R_x^j$ for *all* x. The semicolon indicates the break between the protected and the unprotected classifications.

$\mathbf{R}^{P:j} = (R_1^j, \ldots, R_\alpha^j; O, \ldots, O)$: – *A protected rate vector.* The number of terminal, zero components equals $z - \alpha$ (except as specified later).

$\mathbf{R}^{U:k} = (O, \ldots, O; R_\beta^k, \ldots, R_z^k)$: – *An unprotected rate vector.* The number of initial, zero components equals α (except as specified later).

$\mathbf{R}_P^i = (R_1^i, \ldots, R_\alpha^i; R_\beta^i, \ldots, R_z^i)$: – A *"P-reconciled"* rate vector, such that: $\sum_P w_x R_x^i = \hat{P}^i = P$; *but:* $\sum_U w_x R_x^i = \hat{U}^i \neq U$.

$\mathbf{R}_P^{P:i} = (R_1^i, \ldots, R_\alpha^i; O, \ldots, O)$: – A P-reconciled protected rate vector.

$\mathbf{R}_U^j = (R_1^j, \ldots, R_\alpha^j; R_\beta^j, \ldots, R_z^j)$: – A "$U$-reconciled rate vector, such that: $\sum_U w_x R_x^j = \hat{U}^j = U$; *but:* $\sum_P w_x R_x^j = \hat{P}^j \neq P$.

$\mathbf{R}_U^{U:j} = (O, \ldots, O; R_\beta^j, \ldots, R_z^j)$: – A U-reconciled unprotected rate vector.

$\mathbf{R}_T^i = (R_1^i, \ldots, R_\alpha^i; R_\beta^i, \ldots, R_z^i)$: – A *"T-reconciled"* rate vector, such that: $\sum_T v_x R_x^i = \hat{T}^i = T$; *but:* $\sum_P w_x R_x^i = \hat{P}^i \neq P$; *and:* $\sum_U w_x R_x^i = \hat{U}^i \neq U$.

$\tilde{\mathbf{R}}^i = (R_1^i, \ldots, R_\alpha^i; R_\beta^i, \ldots, R_z^i)$: – A *"feasible"* rate vector, such that: $\sum_P w_x R_x = \hat{P}^i = P$; *and;* $\sum_U w_x R_x = \hat{U}^i = U$.

It follows from definitions that $T = v_P P + v_U U$; whence, if $\tilde{\mathbf{R}}^i$ is feasible, as above, then also: $\sum_T v_x R_x = \hat{T}^i = T$.

(a) $\tilde{\boldsymbol{R}}^i = \boldsymbol{R}_P{}^{P:j} + \boldsymbol{R}_U{}^{U:k} = (R_1{}^j, \ldots, R_\alpha{}^j;\ R_\beta{}^k, \ldots,\ R_z{}^k)$; where possibly but not necessarily: $i = j = k$. If, in the middle member, either the protected vector is *not* P-reconciled; *or,* the unprotected vector is *not* U-reconciled, then $\boldsymbol{R}^i$ in the left member is *not* feasible.

Note that the individual component rates do *not* carry the reconciliation subscript, *"P"*, *"U"* or *"T"*, or the "feasible" tilde, "$\sim$". It is the vector, *as a vector,* which is reconciled, *not* the individual component rates. If $\boldsymbol{R}^i$ is feasible, $\tilde{\boldsymbol{R}}^i$, and $\boldsymbol{R}^j$ is feasible, $\tilde{\boldsymbol{R}}^j$; then $\tilde{\boldsymbol{R}}^k = \boldsymbol{R}_P{}^{P:i} + \boldsymbol{R}_U{}^{U:j}$ will also be feasible, $\tilde{\boldsymbol{R}}^k$. But the vectors:

$$\boldsymbol{R}^l = \boldsymbol{R}_P{}^{P:i} + \boldsymbol{R}^{U:l} = (R_1{}^i, \ldots, R_\alpha{}^i;\ R_\beta{}^j, \ldots, R_\mu{}^i, \ldots, R_z{}^j) \neq \tilde{\boldsymbol{R}}^l$$

and:

$$\boldsymbol{R}^m = \boldsymbol{R}^{P:m} + \boldsymbol{R}_U{}^{U:j} = (R_1{}^i, \ldots, R_\xi{}^j, \ldots R_\alpha{}^i;\ R_\beta{}^j, \ldots, R_z{}^j) \neq \tilde{\boldsymbol{R}}^m$$

will *not* be feasible except possibly in special cases, although every individual component, $R_x{}^i$ or $R_x{}^j$, of $\boldsymbol{R}^l$ and $\boldsymbol{R}^m$ appears also as a component of one or the other of the feasible vectors $\tilde{\boldsymbol{R}}^i$ and $\tilde{\boldsymbol{R}}^j$.

$\boldsymbol{Q}^i = (Q_1{}^i, \ldots, Q_\alpha{}^i; Q_\beta{}^i; \ldots, Q_z{}^i)$: – A *normal vector.*

$\boldsymbol{Q}^{P:j}$; $\boldsymbol{Q}^{U:k}$: — A *protected normal vector;* an *unprotected normal vector.* Definitions by analogy to definitions of $\boldsymbol{R}^{P:j}$ and $\boldsymbol{R}^{U:k}$.

$\boldsymbol{Q}^i \overset{r}{\longleftrightarrow} \boldsymbol{R}^i$: – For all x; then $R_x{}^i = r_x Q_x{}^i$. Then $\boldsymbol{Q}^i$ *"underlies"* its *"resting"* vector, $\boldsymbol{R}^i$. If $\boldsymbol{R}^i$ is T- P- or U-reconciled, $\boldsymbol{R}_T{}^i$, etc.; or is feasible, $\tilde{\boldsymbol{R}}^i$, then $\boldsymbol{Q}^i$ is reconciled, $\boldsymbol{Q}_T{}^i$, etc.; or feasible, $\tilde{\boldsymbol{Q}}^i$, accordingly.

$\boldsymbol{R}^i \overset{q}{\longleftrightarrow} \boldsymbol{Q}^i$: – For all x; then $Q_x{}^i = q_x R_x{}^i$. Then $\boldsymbol{R}^i$ *"rests upon"* its *"underlying"* vector, $\boldsymbol{Q}^i$.

C. *Protection Curves.*

A *"protection curve"* is either a rate curve or a normal curve.

A *"rate curve"* is any smoothly continuous curve passing through the plot of the component rates of a rate vector plotted against class number.

A *"normal curve"* is a smoothly continuous curve passing through the plot of the component normals of a normal vector; provided that the slope of a normal curve must be non-negative throughout the interval $1 \leq x \leq z$.

The reflection in protection classification rates of variation of extraneous hazard conditions may produce negative slope to the rate curve over part of its length. Negative slope to the normal curve indicates *increase* of rate with *improvement* of protection, or *v.v.*, throughout the interval of the protection grading where the negative slope occurs. Remembering that in theory, the rate is a continuous function of grading, which is a continuous variable (see Section I, preceding), this represents a logically indefensible violation of consistency which may result in $Q_\mu < Q_\zeta$, where $\mu > \zeta$, under the classification system currently in use, and is certain to result in $Q_\mu < Q_\zeta$ where $\mu > \zeta$ under some classification system possible of adoption.

A protection curve uniquely "determines" its "defining" vector. A rate vector or normal vector does not define a unique protection curve. In the absence of further specification, the vector defines an entire family of curves, but this is of no practical consequence. The French curves and ships' curves of Mr. Carlson's nostalgic reference[17] are still very much in evidence upon the fire ratemaker's desk. He is sufficiently calloused to the implications to lose no sleep over the fact that a particular squiggle which gives him an appropriate rate pattern will have an infinite number of siblings, any of whom would be equally obliging.

IV. RATE STRUCTURES

A. *Adjusted Rate Structures.*[18]

1. *The Feasible Adjusted Rate Structure.*

The *"feasible adjusted rate structure"*, $\{\tilde{\boldsymbol{R}}\}$, is the set of all feasible adjusted rate vectors. It is completely bounded.

Let:

$$\dot{\boldsymbol{R}}_1{}^P = (P/w_1, \ldots, O;O, \ldots, O) = (\dot{R}_1, \ldots, O;O, \ldots, O)$$

$$\ldots \quad \ldots \quad \ldots \quad \ldots$$

$$\dot{\boldsymbol{R}}_\xi{}^P = (O, \ldots, P/w_\xi, \ldots, O;O, \ldots, O) = (O, \ldots, \dot{R}_\xi, \ldots, O;O, \ldots, O)$$

$$\ldots \quad \ldots \quad \ldots \quad \ldots$$

[17] Carlson, (13). p. 76.

[18] See APPENDIX A for development of equations presented below without proof, and for further discussion of the concepts summarized below. For the practical significance of these concepts, in addition to APPENDIX A, see also Section VI.D, to follow.

$$\left.\begin{array}{l}\dot{\boldsymbol{R}}_{\alpha}{}^{P} = (O, \ldots, P/w_{\alpha}; O, \ldots, O) = (O, \ldots, \dot{R}_{\alpha}; O, \ldots, O) \\ \dot{\boldsymbol{R}}_{\beta}{}^{U} = (O, \ldots, O; U/w_{\beta}, \ldots, O) = (O, \ldots, O; \dot{R}_{\beta}, \ldots, O)\end{array}\right\}; (\beta \equiv \alpha + 1)$$

$$\ldots \quad \ldots \quad \ldots \quad \ldots$$

$$\dot{\boldsymbol{R}}_{\mu}{}^{U} = (O, \ldots, O; O, \ldots, U/w_{\mu}, \ldots, O) = (O, \ldots, O; O, \ldots, \dot{R}_{\mu}, \ldots, O)$$

$$\ldots \quad \ldots \quad \ldots \quad \ldots$$

$$\dot{\boldsymbol{R}}_{z}{}^{U} = (O, \ldots, O; O, \ldots, U/w_{z}) = (O, \ldots, O; O, \ldots, \dot{R}_{z})$$

Then the feasible adjusted rate structure will be formally defined: If $\boldsymbol{R}^{i}$ is feasible, $\tilde{\boldsymbol{R}}^{i}$ (*i.e.*, R^{i} is an element of $\{\tilde{\boldsymbol{R}}\}$), then necessarily:[19]

$$(1) \qquad \tilde{\boldsymbol{R}}^{i} = \boldsymbol{R}_{P}{}^{P:j} + \boldsymbol{R}_{U}{}^{U:k} = \textstyle\sum_{P} a_{x}{}^{P:j} \dot{\boldsymbol{R}}_{x}{}^{P} + \sum_{U} a_{x}{}^{U:k} \dot{\boldsymbol{R}}_{x}{}^{U}$$

$$(a_{x}{}^{P:j} \geq O;\ \textstyle\sum_{P} a_{x}{}^{P:j} = 1. \quad a_{x}{}^{U:k} \geq O;\ \sum_{U} a_{x}{}^{U:k} = 1)$$

(Possibly, not necessarily: $i = j = k$)

and the components of $\tilde{\boldsymbol{R}}^{i}$ will be given by:

$$(1.a) \qquad \begin{array}{l} R_{x}{}^{i} = a_{x}{}^{P:j} \dot{R}_{x} = a_{x}{}^{P:j} P/w_{x};\ (x = 1, \ldots, \alpha.) \\ R_{x}{}^{i} = a_{x}{}^{U:k} \dot{R}_{x} = a_{x}{}^{U:k} U/w_{x};\ (x = \beta, \ldots, z.) \end{array}$$

By implications of definitions given, $P > O$, $U > O$; and for all x, then $w_{x} \geqslant O$ and $R_{x}{}^{i} \geqslant O$. By hypothesis, henceforth for all x, then $w_{x} > O$ in all equations presented. If, for any x, then $w_{x} = O$, *i.e.* if no insurance is written in Class x,[20] then the class must be dropped from all calculation, and the rate must be established by judgment alone, with reference to the rates of other classes. In consequence, $\dot{R}_{x} > O$ for all x. Therefore, for all x, the coefficients, $a_{x}^{:\cdot}$ of Eq.(1) must be non-negative to avoid $R_{x}{}^{i} < O$ for some x.

The restriction that the two sets of coefficients, $\{a_{x}{}^{P:j}\}$ and $\{a_{x}{}^{U:k}\}$, each sum to unity is justified in Appendix A. For the moment it may be noted that since by Eqs. (1.a):

$$\text{and:} \qquad \begin{array}{l} w_{x} R_{x}{}^{i} = a_{x}{}^{P:j} P;\ (x = 1, 2, \ldots, \alpha) \\ w_{x} R_{x}{}^{i} = a_{x}{}^{U:j} U;\ (x = \beta, \ldots, z) \end{array}$$

then the summation-to-unity restriction on $a_{x}^{:\cdot}$ is sufficient to insure that $\tilde{\boldsymbol{R}}^{i}$ will be feasible.

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[19] *Cf. McIntosh*, (14). p. 151, Eq. (8)

[20] If, in a given state, no community is classified as Class x, or if no risks of a given underwriting class are found in any Class x community, then $w_{x} = O$. At this writing, no N.B.F.U. Class 1 city exists in the United States. (See Section VI, to follow.)

Obviously, however, not all vectors possible of calculation by Eq.(1) will be acceptable solutions to the rate revision problem. To begin with, by Eq.(1.a), for one or more x, it is possible that for some i, then $R_x^i = O$, which is an absurdity in practice. Secondly, rate inversions may be produced, *i.e.* it would possible for a given community to suffer *increase* in rates solely by virtue of *improvement* in its fire defenses, or *v.v.*, which again is an absurdity. As noted above in Section II.B, reflection in the rate, R_x, of a disproportionate concentration in Class x of extraneous hazards may properly result in $R_x > R_{x+1}$, but in considering the normals, the condition that $Q_x > Q_{x+1}$ constitutes a serious violation of consistency by the implication therein that *improvement* of protection will *increase* loss expectation, or *v.v.* For many vectors calculable by Eq.(1), it will happen that $R_x/r_x = Q_x > Q_{x+1} = R_{x+1}/r_{x+1}$.

The virtue of defining $\{ \tilde{R} \}$ and of formulating Eq.(1) is to establish a basis for further development.

2. *The Operational Adjusted Rate Structure.*

Consistency, as above, requires only that for all x, then $Q_x \leq Q_{x+1}$, but if $Q_x = Q_{x+1}$ a triviality results. In such a case, Class x and Class $(x + 1)$ should be consolidated into a single class. Therefore, the consistency requirement, that $Q_x \leq Q_{x+1}$, properly may be and should be modified by hypothesis to the strict inequality, $Q_x < Q_{x+1}$, but even this is not sufficient. It has been noted in Section I.B. that the adoption of any protection classification system constitutes imposition of an arbitrarily discrete model upon an actual continuum, which leads to the question of inter-class differentials. In theory the model is inappropriate, whence it follows that the results of application of the model will be inaccurate. Mr. Pruitt's statement that: "*** in this area, as in so many others, simplicity and accuracy are mutually antagonistic. To the degree that we require a mathematical and clearly defined accuracy, we must perforce sacrifice simplicity and ease of operation,"[21] seems entirely appropriate here, although the original context is presently irrelevant. The question, very simply, is: How great a departure from "mathematical and clearly defined accuracy" can be tolerated for the sake of "simplicity and ease of operation"?

If the classification normal relativity Q_x/Q_{x+1}, between adjacent normals, is trivial, the rate structure becomes unnecessarily complex.[22] At

[21] Pruitt, (18). p. 154

[22] It should be noted, however, that in considering elements of the fire rate other than reflection of public protection, it may become necessary to retain trivial rate differ-

the other extreme, if Q_x/Q_{x+1} is excessive, the result well may be Mr. Pruitt's "horse and rabbit stew – 'one rabbit, one horse' ";[23] in any case, the spectrum of hazard within the individual class will be so broad as to constitute open invitation to rate deviation and cream-skimming.[24] There also may be other extremely practical complications of a kind such that some rating jurisdictions on occasion have refined the protection classification system by insertion of additional classes when excessive differences could be reduced in no other fashion. Exact figures at which Q_x/Q_{x+1} passes from "reasonable" to "trivial," or, alternatively, to "excessive" cannot be specified; nevertheless it seems necessary, and on occasion has proved necessary, to establish bounds to Q_x/Q_{x+1}.

Strictly speaking, excess or triviality in this regard cannot be judged on the basis of the ratio Q_x/Q_{x+1} alone. The ratio Q_x/Q_{x+1} and the value of the difference, $Q_{x+1} - Q_x$, must be examined together, for all practical purposes.[25] To incorporate simultaneous consideration of Q_x/Q_{x+1} and $Q_{x+1} - Q_x$ into what follows here, however, would require that the bounds to be hypothecated as applicable to Q_x/Q_{x+1} be made functions of Q_{x+1}, which in turn would materially complicate the development to no good purpose. In practical operations, fore-knowledge of the general level of rates to be obtained (though not, of course, of the exact values) normally allows the ratemaker to estimate ratios which will produce reasonable differences, or *v.v.* if he prefers. What follows in terms of Q_x/Q_{x+1} could have been developed in terms of $Q_{x+1} - Q_x$, though obviously the form of the development would have differed.

To exclude from the rate structure values of Q_x/Q_{x+1} either excessive or trivial, let $c_x = Q_x/Q_{x+1}$, and let the constraint, $0 < \underline{c}_x \leq c_x \leq \overline{c}_x < 1$, be introduced into the calculation. By definition of the rate-normal ratio,

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ences to avoid violation of consistency. The ultimate cause of such circumstances is the fact that the contribution of a given hazard to the total expectation of loss will vary according to the presence or absence of other given hazards. *Cf.* McIntosh, (14), p. 152; also (16), p. 118 *ff.* (The solution given in the latter reference is an alternative to retention of a trivial differential, but is not always practicable.)

[23] Pruitt, (18). p. 153

[24] To untangle the metaphor, cream the rabbit.

[25] For example, the author once was involved in a rather heated controversy with the officials of a certain municipality over the question of whether or not a rate reduction of \$0.20 per \$1,000 of insurance was an insultingly "trivial" return for money spent by the city to improve its protection classification, although the flat sum amounted to 11% of the pre-existing rate of \$1.80, a percentage normally considered quite reasonable.

r_x, it then follows that since f_x is defined by $f_x = R_x/R_{x+1}$, then $O < \underline{f}_x \leq f_x \leq \overline{f}_x < r_x/r_{x+1}$, by the equation:

$$(2) \qquad \underline{f}_x = \underline{c}_x r_x/r_{x+1}; \text{ and: } \overline{f}_x = \overline{c}_x r_x/r_{x+1}$$

The hypothesis of bounds then may be stated completely as the constraint:

$$(I) \qquad O < \underline{c}_x \leq c_x \leq \overline{c}_x < 1; \text{ and: } O < \underline{f}_x \leq f_x \leq \overline{f}_x < r_x/r_{x+1}$$

It should be noted that Constraint (I) also implies that $Q_x < Q_{x+1}$ and that $R_x > O$, as required.

The *"operational adjusted rate structure"*, $op\{\tilde{\boldsymbol{R}}\}$, now may be defined informally as the set of all feasible vectors whose component rates may be appropriate for application, and may be defined formally as a proper subset of the feasible rate structure $\{\tilde{\boldsymbol{R}}\}$ such that if $\boldsymbol{R}^i$ is a member of $op\{\tilde{\boldsymbol{R}}\}$, then:

$$(3) \qquad op\tilde{\boldsymbol{R}}^i = op\boldsymbol{R}_P{}^{P:j} + op\boldsymbol{R}_U{}^{U:k} = \sum_P a_x{}^{P:j}\dot{\boldsymbol{R}}_x{}^P + \sum_U a_x{}^{U:k}\dot{\boldsymbol{R}}_x{}^U$$

$$(a_x{}^{P:j} > O; \sum_P a_x{}^{P:j} = 1. \quad a_x{}^{U:k} > O; \sum_U a_x{}^{U:k} = 1)$$

(For $x \neq a$ or z[26]:

$$\underline{f}_x a_{x+1}^{\cdot\cdot} w_x/w_{x+1} \leq a_x^{\cdot\cdot} \leq \overline{f}_x a_{x+1}^{\cdot\cdot} w_x/w_{x+1})$$

$$(\underline{f}_a a_\beta{}^{U:k} w_a U/w_\beta P \leq a_a{}^{P:j} \leq \overline{f}_a a_\beta{}^{U:k} w_a U/w_\beta P)$$

(Possibly, not necessarily: $i = j = k$)

The component rates, $R_x{}^i$, of $op\boldsymbol{R}^i$, are given by Eqs. (1.a) subject to the restrictions imposed in Eq. (3) upon the coefficients $a_x^{\cdot\cdot}$.

Henceforth, an operational rate vector, $op\tilde{\boldsymbol{R}}^i$ will be denoted simply as "$\tilde{\boldsymbol{R}}^i$", except when it may be necessary to emphasize in particular context that a given vector not only is feasible but also is operational.

Very obviously, the bounds $\underline{f}_x$ and $\overline{f}_x$ are not mathematically rigorous, but the degree of rigidity exhibited will vary with practical circumstance in a particular case.

3. *The Final Adjusted Rate Structure.*

The *"final adjusted rate structure"*, consists of a single vector, $\tilde{\boldsymbol{R}}^*$, which is that particular vector whose components, $R_x{}^*$, are the rates to be placed into effect. Obviously, $\tilde{\boldsymbol{R}}^*$ must be an operational vector.

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[26] By previously-given definition of z, f_z does not exist, hence the restriction cannot apply to a_z.

B. *The Existing Rate Structure.*

The *"existing rate structure"* consists of a single vector, $\boldsymbol{R}^e$, whose components, $R_x{}^e$, are the rates actually in effect at the time the operation of rate revision is initiated. Superscript "e" identifies quantities associated with the existing rate structure, thus: U^e, P^e, $Q_x{}^e$, etc.

The only present concern with the existing rate structure is the utilization of U^e, P^e, $R_x{}^e$, etc., as the parameters and arguments of rating formulas appearing in Section VI, to follow.

V. RATE STRUCTURE ALGEBRA

A. *The Problem.*

It is, of course, obvious that, given pre-determined target rates, U and P, a feasible vector always will result if the components, $R_x{}^i$, of any rate vector, $\boldsymbol{R}^i$ are multiplied by the ratio $P/\hat{P}^i$ for $x = 1,2, \ldots, \alpha$, and by the ratio $U/\hat{U}^i$ for $x = \beta, \ldots, z$. There are, however, circumstances under which this simple solution either is inadequate or produces undesirable side effects, perhaps intolerable side effects.

It may be that for some x, say $x = \zeta$, that the value to be assumed by the final adjusted rate, $R_\zeta{}^*$, is pre-determined within narrow bounds by underwriting or other considerations, and an interminable number of trials with successive rate vectors, $\boldsymbol{R}^i$, $\boldsymbol{R}^j$, . . . may be required before a vector $\boldsymbol{R}^k$ is found such that $R_\zeta{}^* = R_\zeta{}^k P/P^k$ or $R_\zeta{}^* = R^k U/\hat{U}^k$, accordingly as $\zeta \leq \alpha$ or $\zeta \geqslant \beta$, and also such that for all $x \neq \zeta$, the rates $R_x{}^* = R_x{}^k P/\hat{P}^k$ or $R_x{}^* = R_x{}^k U/\hat{U}^k$, as $x \leq \alpha$ or $x \geqslant \beta$, are considered appropriate. The problem becomes particularly difficult if bounding values of two or more of the final adjusted rates are pre-determined by side conditions.

Also, when this method of solution is used, the ratemaker has no control over the boundary ratio, $c_\alpha = Q_\alpha/Q_\beta$. Not only may c_α become either obviously and completely trivial or obviously and intolerably excessive, but uncritical and exclusive reliance upon this method has been known to produce in actual practice the weird situation where a community could secure wholesale fire rate reductions by disbanding the fire department and selling off the apparatus. In theory, remembering that R_x is in actuality a continuous function of protection grading, separate adjustment of the premiums for the "Protected" and "Unprotected" statistical classes (where $P/P^e \neq U/U^e$) should be accomplished by rotating the rate curve, not by breaking it into two pieces and translating each piece up or down the vertical axis independently of the position of the other.[27] Although in

[27] See Note 10, *sup.*

many cases this theory is academic, in other cases it definitely will not be so. Whether or not it is academic will depend entirely on the actual values of P^e, U^e, P/P^e and U/U^e in particular instance.

Two systematic methods of solution which avoid both the theoretical and the practical difficulties involved here are given in Section VI, to follow, but first it may be well to explore the implications of Constraint (I) imposed upon the vectors of the operational rate structure.

B. *The Simplest Non-Trivial Case.*

Assume a system of four protection classes. Classes 1 and 2 belong to the "Protected" statistical class, and Classes 3 and 4 belong to the "Unprotected" statistical class.

It follows from the definition, $f_x = R_x/R_{x+1}$, that since $z = 4$, then:

$$\begin{aligned} R_3 &= f_3R_4 \\ R_2 &= f_2R_3 = f_2f_3R_4 \\ R_1 &= f_1R_2 = f_1f_2R_3 = f_1f_2f_3R_4 \end{aligned} \tag{4}$$

whence:

$$\begin{aligned} \textstyle\sum_P w_xR_x &= (f_1w_1 + w_2)\,R_2 = f_2f_3\,(f_1w_1 + w_2)\,R_4 = \hat{P} \\ \textstyle\sum_U w_xR_x &= (f_3w_3 + w_4)\,R_4 = (f_3w_3 + w_4)\,R_2/f_2f_3 = \hat{U} \end{aligned} \tag{5}$$

whence:

$$\frac{\hat{P}}{\hat{U}} = f_2f_3\,\frac{f_1w_1 + w_2}{f_3w_3 + w_4} = \hat{\mathfrak{p}} \tag{6}$$

where by definition: $\hat{\mathfrak{p}} = \hat{P}^i/\hat{U}^i$; and for reference to follow, let $\mathfrak{p}^* = P/U$; whence it follows that if $\boldsymbol{R}^i$ is feasible, $\tilde{\boldsymbol{R}}^i$, then: $\hat{\mathfrak{p}}^i = \mathfrak{p}^* = P/U$.

It further follows from Eqs. (4) and (5), by rearrangement following direct substitution of corresponding terms, that if $\boldsymbol{R}^i$ is feasible, $\tilde{\boldsymbol{R}}^i$, then:

$$\begin{aligned} R_1{}^i &= \frac{f_1{}^iP}{f_1{}^iw_1 + w_2} = \frac{f_1{}^if_2{}^if_3{}^iU}{f_3{}^iw_3 + w_4} \\ R_2{}^i &= \frac{P}{f_1{}^iw_1 + w_2} = \frac{f_2{}^if_3{}^iU}{f_3{}^iw_3 + w_4} \\ R_3{}^i &= \frac{P}{f_2{}^i\,(f_1{}^iw_1 + w_2)} = \frac{f_3{}^iU}{f_3{}^iw_3 + w_4} \\ R_4{}^i &= \frac{P}{f_2{}^if_3{}^i\,(f_1{}^iw_1 + w_2)} = \frac{U}{f_3{}^iw_3 + w_4} \end{aligned} \tag{7}$$

Imposing the bounds of Constraint (1) upon $f_1{}^*$ and $f_3{}^*$, it follows from Eqs. (7) that extremal values of R_2 and R_3 are given by:

(8) $$\bar{R}_2 = \frac{P}{\underline{f}_1{}^i w_1 + w_2}\text{; and: } \underline{R}_2 = \frac{P}{\bar{f}_1{}^i w_1 + w_2}$$

(9) $$\underline{R}_3 = \frac{\underline{f}_3 U}{\underline{f}_3 w_3 + w_4}\text{; and: } \bar{R}_3 = \frac{\bar{f}_3 U}{\bar{f}_3 w_3 + w_4}$$

whence:

(10) $$f_2{}^i \leq \frac{\bar{R}_2}{\underline{R}_3} = \frac{(\underline{f}_3 w_3 + w_4)\, P}{\underline{f}_3\, (\underline{f}_1 w_1 + w_2)\, U}$$

(11) $$f_2{}^i \geq \frac{\underline{R}_2}{\bar{R}_3} = \frac{(\bar{f}_3 w_3 + w_4)\, P}{\bar{f}_3\, (\bar{f}_1 w_1 + w_2)\, U}$$

The implications of Ineq. (10) are that for *any* choices of $\underline{f}_1 > O$, and of $\underline{f}_3 > O$, as required by Constraint (1), there may be encountered values of U and P, which are beyond the ratemaker's control, such that necessarily $f_2{}^i > r_2/r_3$, whence, by Eq. (IV.A.2),[28] then $Q_2{}^i > Q_3{}^i$ and possibly, even $Q_2{}^i > Q_4{}^i$. Conversely, the implications of Ineq. (11) are that for *any* choice of $\bar{f}_1 < r_1/r_2$ and of $\bar{f}_3 < r_3/r_4$, there will exist values of U and P such that necessarily $\epsilon \geq c_2{}^i > O$, where ϵ is arbitrarily small, which implies that $\eta \geq | Q_3 - Q_2 | > O$, where η is arbitrarily small which is the very essence of triviality. By appropriate rearrangement of Ineqs. (10) and (11), comparable implications concerning the value of $f_1{}^i$ can be demonstrated to result from any choice of bounds to $f_2{}^i$ and $f_3{}^i$, and concerning the value of $f_3{}^i$ for any choice of bounds to $f_1{}^i$ and $f_2{}^i$.

It is to be noted that realization of the possibilities implied, as above, by Ineqs. (10) and (11) depends upon the ratio $\mathcal{P}^* = {}^P/_U$, and not upon the actual value of either P or U. This fact may be turned to practical advantage.

There are eight possible combinations: $\underline{f}_1, \underline{f}_2, \underline{f}_3$; $\underline{f}_1, \underline{f}_2, \bar{f}_3$; etc., of the extremal ratios, $\underline{f}_x$ and $\bar{f}_x$. Entering each of these combinations in turn into Eq. (6), let:

(12)
$$\mathcal{P}^{I} = \underline{f}_2 \underline{f}_3 \frac{\underline{f}_1 w_1 + w_2}{\underline{f}_3 w_3 + w_4} = \mathcal{P}(\underline{f}_1, \underline{f}_2, \underline{f}_3)$$

$$\mathcal{P}^{II} = \mathcal{P}(\underline{f}_1, \bar{f}_2, \underline{f}_3)$$

....

$$\mathcal{P}^{VIII} = \mathcal{P}(\bar{f}_1, \bar{f}_2, \bar{f}_3)$$

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[28] "Eq. (IV.A.2)": – Eq. (2) introduced in Section IV.A.

From the first and last of Eqs. (12), it will be seen that for any given choice of values for $\underline{f}_x$ and $\bar{f}_x$, the value of $\mathcal{P}^I$ is the minimum value and the value of $\mathcal{P}^{VIII}$ is the maximum value which can be assumed by $\hat{\mathcal{P}}^i = \mathcal{P}(f_1{}^i,\ f_2{}^i,\ f_3{}^i)$ subject to Constraint (I). It then follows from Eqs. (7) (whereby it is seen that $R_x{}^i$ is for all x a function of either U or P, together with one or more of the ratios $f_x{}^i$) that if $\mathcal{P}^* < \mathcal{P}^I$ or $\mathcal{P}^* > \mathcal{P}^{VIII}$, no operational rate vector will exist. To be feasible in such instance, there must be associated with the vector $\boldsymbol{R}^i$, values of $f_x{}^i$ such that for at least one x, then $f_x{}^i > \bar{f}_x$ if $\mathcal{P}^* > \mathcal{P}^{VIII}$, or $f_x{}^i < \underline{f}_x$ if $\mathcal{P}^* < \mathcal{P}^I$.

If $\mathcal{P}^* = \mathcal{P}^I$ or $\mathcal{P}^* = \mathcal{P}^{VIII}$, then there will exist exactly one operational rate vector, $\tilde{\boldsymbol{R}}^*$, which may be calculated directly by Eqs. (7), entering as arguments of the equations the values of $\underline{f}_1,\ \underline{f}_2,\ \underline{f}_3$ if $\mathcal{P}^* = \mathcal{P}^I$, and the values of $\bar{f}_1,\ \bar{f}_2,\ \bar{f}_3$ if $\mathcal{P}^* = \mathcal{P}^{VIII}$.

It is to be demonstrated[29] that if $\mathcal{P}^I < \mathcal{P}^* < \mathcal{P}^{VIII}$, then the final rate vector, $\tilde{\boldsymbol{R}}^*$, may be calculated directly as a linear convex combination[30] of certain vectors to be associated with $\mathcal{P}^I, \ldots, \mathcal{P}^{VIII}$, *provided* that side conditions imposed upon $\tilde{\boldsymbol{R}}^*$ (*e.g.*, predetermined bounds to the value to be assumed by $R_x{}^*$ for some one or more x) do not render solution impossible in particular instance. The smaller the value of $\mathcal{P}^* - \mathcal{P}^I$, or, alternatively, of $\mathcal{P}^{VIII} - \mathcal{P}^*$, the more restricted will be the rate-maker's freedom of choice.

C. *The General Case.*

For notational convenience, let:

$$f_x = \begin{cases} R_x/R_{x+1}; & \text{if: } x < z \\ 1; & \text{if: } x = z \end{cases}$$

$$f_{\zeta:\mu} = \prod_{x=\zeta}^{\mu} f_x = \begin{cases} R_\zeta/R_{\mu+1}; & \text{if: } \zeta \le \mu < z \\ R_\zeta/R_z; & \text{if: } \zeta \le \mu = z \end{cases}$$

With the above definition of $f_{\zeta:\mu}$, and extension of the previously-given definition of f_x,[31] under a generalized classification system of Class

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[29] See APPENDIX A and also Section VI.D., to follow.

[30] "Linear convex combination": —*E.g.*, each of the two summations in the right member of Eq.(IV.A.1) is a "linear convex combination" of vectors by virtue of the restrictions upon the coefficients a_x^i. (If these restrictions are removed, the entire right member becomes simply a "linear combination" of vectors.)

[31] It should be noted that this extension of the definition of f_x requires appropriate qualifying extension of the statement of Constraint (I).

1, . . . , Class α; Class β, . . . , Class z, Eqs. (4) become:

$$R_x = f_x R_{x+1} = f_{x:\alpha-1} R_\alpha = f_{x:z} R_z \tag{13}$$

whence Eqs. (5) become:

$$\begin{aligned} \textstyle\sum_P w_x R_x &= \Big(\sum_{x=1}^{\alpha-1} f_{x:\alpha-1} w_x + w_\alpha\Big) R_\alpha \\ &= f_{\alpha:z} \Big(\sum_{x=1}^{\alpha-1} f_{x:\alpha-1} w_x + w_\alpha\Big) R_z = \hat{P} \\ \textstyle\sum_U w_x R_x &= (f_{\alpha:z})^{-1} \big(\textstyle\sum_U f_{x:z} w_x\big) R_\alpha \\ &= \big(\textstyle\sum_U f_{x:z} w_x\big) R_z = \hat{U} \end{aligned} \tag{14}$$

whence:

$$\hat{p} = \frac{\hat{P}}{\hat{U}} = f_{\alpha:z} \frac{\sum_{x=1}^{\alpha-1} f_{x:\alpha-1} w_x + w_\alpha}{\sum_U f_{x:z} w_x} \tag{15}$$

and by analogy to Eqs. (7), for any feasible vector, $\tilde{\boldsymbol{R}}^i$:

$$\begin{aligned} R_\zeta{}^i &= \frac{f_{\zeta:\alpha}{}^i P}{\sum_P f_{x:\alpha}{}^i w_x} = \frac{f_{\zeta:z}{}^i U}{\sum_U f_{x:z}{}^i w_x}; \quad (\zeta \leq \alpha) \\ R_\mu{}^i &= \frac{P}{f_{\beta:\mu}{}^i \left(\sum_P f_{x:\alpha}{}^i w_x\right)} = \frac{f_{\mu:z}{}^i U}{\sum_U f_{x:z}{}^i w_x}; \quad (\mu \geqslant \beta) \end{aligned} \tag{16}$$

Imposing Constraint (I) upon $f_x{}^i$, by analogy to Eqs. (8) and (9):

$$\overline{R}_\alpha = \frac{P}{\sum_{x=1}^{\alpha-1} \underline{f}_{x:\alpha-1} w_x + w_\alpha}; \text{ and: } \underline{R}_\alpha = \frac{P}{\sum_{x=1}^{\alpha-1} \overline{f}_{x:\alpha-1} w_x + w_\alpha} \tag{17}$$

$$\underline{R}_\beta = \frac{\underline{f}_{\beta:z} U}{\sum_U \underline{f}_{x:z} w_x}; \text{ and: } \overline{R}_\beta = \frac{\overline{f}_{\beta:z} U}{\sum_U \overline{f}_{x:z} w_x} \tag{18}$$

From Eqs. (17) and (18), inequalities analogous to Ineqs. (10) and (11) may be formulated, and these inequalities will carry exactly the same implications under a generalized classification system as do Ineqs. (10) and (11) under the 4-class system assumed in Section B, above.

There will be 2^{z-1} possible combinations of the extremal ratios: $\underline{f}_1, \underline{f}_2, \ldots, \underline{f}_{z-1}; \ldots; \overline{f}_1, \overline{f}_2, \ldots, \overline{f}_{z-1}$. Hence the analogue of Eqs. (12) will be a system of 2^{z-1} equations:

$$
(19) \quad
\begin{aligned}
\mathcal{P}^{I} &= \underline{f}_{a}\underline{f}_{\beta:z} \frac{\sum_{x=1}^{a-1} \underline{f}_{x:a-1} w_x + w_a}{\sum_U \underline{f}_{x:z} w_x} \\
&= \mathcal{P}(\underline{f}_1, \ldots, \underline{f}_{a-1}; \underline{f}_a; \underline{f}_\beta, \ldots, \underline{f}_{z-1}) \\
&\ldots \qquad \ldots \qquad \ldots \\
\mathcal{P}^{\phi} &= \mathcal{P}(f'_1, \ldots, f'_{a-1}; f'_a; f'_\beta, \ldots, f'_{z-1}); \quad (f'_x = \underline{f}_x \text{ or } \bar{f}_x) \\
&\ldots \qquad \ldots \qquad \ldots \\
\mathcal{P}^{\Omega} &= \mathcal{P}(\bar{f}_1, \ldots, \bar{f}_{a-1}; \bar{f}_a; \bar{f}_\beta, \ldots, \bar{f}_{z-1}).
\end{aligned}
$$

It will not follow that if $\phi \neq \psi$, then necessarily $\mathcal{P}^{\phi} \neq \mathcal{P}^{\psi}$; and for $I < \phi < \Omega$, the order of relative magnitude among the several $\mathcal{P}^{\phi}$ may vary with the actual values of $\underline{f}_x$, $\bar{f}_x$ and w_x in particular instance. In all cases, however, regardless of the parameter values, the value of $\mathcal{P}^{I}$ will be less than, and the value of $\mathcal{P}^{\Omega}$, greater than, the value of any $\mathcal{P}^{\phi}$ for $\phi \neq I$ or Ω. Also, as under the 4-class system previously displayed, in the completely general case: if $\mathcal{P}^{*} < \mathcal{P}^{I}$ or $\mathcal{P}^{*} > \mathcal{P}^{\Omega}$, there will be no solution to the rate revision problem except in violation of Constraint (1); if $\mathcal{P}^{\phi} = \mathcal{P}^{I}$ or $\mathcal{P}^{\phi} = \mathcal{P}^{\Omega}$ there will be a unique solution to the problem; if $\mathcal{P}^{I} < \mathcal{P}^{\phi} < \mathcal{P}^{\Omega}$, then operational rate vectors may be calculated directly as linear convex combinations of not more than z vectors of certain ones to be associated with the several $\mathcal{P}^{\phi}$;[32] finally, the smaller the value of $\mathcal{P}^{*} - \mathcal{P}^{I}$ or, alternatively, of $\mathcal{P}^{\Omega} - \mathcal{P}^{*}$, the narrower will be the bounds of the operational rate structure, $op\{\tilde{\mathbf{R}}\}$, *i.e.* the more restricted will be the ratemaker's freedom of choice.

See APPENDIX A for further discussion.

VI. RATE CALCULATION

A. *The Classification System.*

Throughout what follows, it is assumed that the protection classification system is the N.B.F.U. system described in Section I.B., preceding. However, as no city in the United States presently is classified as N.B.F.U. Class 1, then $w_1 = 0$, whence Class 2 is the lowest-numbered class to be considered in numerical examples. This is of absolutely no consequence in connection with *Method I,* to follow in Section C, below, except to explain the absence of Class 1 rates throughout the calcula-

[32] Normally, the number of vectors required for this purpose will not exceed three or four, but in no case will more than z vectors be required in the combination. See APPENDIX A and also Section VI.D, to follow.

tion. For the significance of the missing Class 1 with respect to *Method II,* Section D, below, see APPENDIX A, Section 3, following.

B. *Application to "Loss Constant Rates." Data Tables.*

The data used in all numerical examples to follow is given in Tables 1, 2 and 3. The data of Table 1 is to be used in all cases. It will be specified in particular cases whether Table 2 or, alternatively, Table 3 is to be used.

The rates and parameters given in these tables are based upon the Frame Dwelling Building rates in effect as of this writing in the State of Louisiana. The only modification of the actually-existing rate structure has been to eliminate a so-called "Country Dwelling" rate (higher than R_{10}), and to combine the actual weighting factors for "Country" and for N.B.F.U. Class 9 into the value shown for w_9 in Table 1. The true value of w_9 would be less than 2%, since N.B.F.U. Class 9 is virtually non-existent in the state.[33]

The Dwelling rate structure in Louisiana embodies the so-called "loss constant rating method," under which the *"effective rate," E_x*, is given by the formula:

(20) $$E_x = (C_x + D_x V)/V$$

where:

C_x = The *"loss constant."* (Possibly the same for two or more classes.)

D_x = The *"residual rate."*[34]

V = *"Policy size," i.e.* the amount of insurance under a given policy.

Let:

V_x^{av} = The mean policy size in Class x.

V_T^{av} = The mean policy size statewide.

It then follows by Eq. (20) that:

(21) $$E_x^{av} = (C_x + D_x V_x^{av})/V_x^{av} = \text{The mean effective rate of Class x.}$$

(22) $$E_x^{Q} = (C_x + D_x V_T^{av})/V_T^{av} = \text{The mean effective rate of Class x "normalized" to the statewide mean policy size, } V_T^{av}.$$

[33] Whenever an 8th class community in Louisiana slips, it seems to slip all the way through N.B.F.U. Class 9 into Class 10. When a Class 10 town decides to improve its status, momentum usually carries it up through Class 9 into Class 8 or better.

[34] The "residual rate" commonly is denoted by "*R*," rather than by "*D*," in expressing Eq. (20), but in present context this notation obviously would cause confusion.

TABLE 1: — Weighting Factors.

Stat. Class.	"Protected"							"Unprotected"	
Prot. Class	2	3	4	5	6	7	8	9	10
w_x	0.066	0.461	0.052	0.148	0.137	0.079	0.057	0.215	0.785

$w_1 = 0$. min. Class Number in calculation = 2. $\alpha = 8$; $\beta = 9$; $z = 10$.

TABLE 2: — Existing Rates.

Prot. Class	2	3	4	5	6	7	8	9	10
R_x^e	1.81	2.01	2.21	2.89	3.09	3.57	3.77	4.24	5.02
$\underline{f}_x$	0.75	0.75	0.75	0.80	0.80	0.80	0.80	0.80	* *
$\bar{f}_x$	0.90	0.90	0.90	0.95	0.95	0.95	0.95	0.95	* *

$P^e = 2.509$; $U^e = 4.852$. $r_x = q_x = 1$, for all x.

TABLE 3: — Existing Rates, Existing Normals.

Prot. Class	2	3	4	5	6	7	8	9	10
R_x^e	1.72	1.73	2.24	2.75	3.22	3.85	3.98	4.77	5.47
$\underline{f}_x$	0.83	0.64	0.80	0.73	0.77	0.82	0.74	0.83	* *
$\bar{f}_x$	1.00	0.77	0.96	0.87	0.92	0.98	0.88	0.99	* *
Q_x^e	1.81	2.01	2.21	2.89	3.09	3.57	3.77	4.24	5.02
$\underline{c}_x$	0.75	0.75	0.75	0.80	0.80	0.80	0.80	0.80	* *
$\bar{c}_x$	0.90	0.90	0.90	0.95	0.95	0.95	0.95	0.95	* *
r_x	0.951	0.861	1.014	0.952	1.042	1.078	1.056	1.125	1.090
q_x	1.052	1.161	0.986	1.050	0.960	0.928	0.947	0.889	0.917

$P^e = 2.407$; $U^e = 5.320$. $P_Q^e = 2.509$; $U_Q^e = 4.852$.

$q_P = 1.069$; $q_U = 0.911$.

It will follow by straightforward algebra that E_x^{av} as calculated by Eq.(21) conforms exactly to the definition of the protection classification rate, R_x, as given in Section II.D., preceding, and hence may be substituted for R_x in any equation so far developed or to be developed below, without affecting the validity of the equation.

In general, as noted in Section II, R_x is "normalized" to the conditions of Class z, *i.e.* of Class 10 in present instance, to obtain Q_x. If V_{10}^{av} is substituted for V_T^{av} in Eq.(22), then E_x^Q will conform exactly to the general definition of Q_x. The fact that it does not so conform to the general definition (unless by coincidence $V_T^{av} = V_{10}^{av}$, which is unlikely) is presently irrelevant. The basic concept of Q_x is simply the normalization of R_x to some common set of extraneous hazard conditions, and the choice of Class 10 in the general case is not mandatory, though convenient. (See Section II.B, preceding) Normalization of E_x^{av} to the statewide average, V_T^{av}, rather than to the Class 10 average, V_{10}^{av}, is not mandatory. However, choice of V_T^{av} normally will give more conveniently-handled values for low-numbered classes, and also a truer picture of rate distribution, than will V_{10}^{av}. In any equation so far developed or to be developed below, E_x^Q may be substituted for Q_x without affecting the validity of the equation in the least.

The values of R_x in Table 2 are the actual values of E_x^Q for Frame Dwelling Buildings, calculated from current Louisiana rates-in-effect on the basis of the actual policy-size sampling underlying the rate structure. These same E_x^Q are entered as "Q_x" in Table 3, wherein E_x^{av} becomes "R_x". Equations (21) and (22) form the bridge which links the present development with the loss constant rating method.

It should be noted that the values of $\underline{f}_x$, $\overline{f}_x$, $\underline{c}_x$ and $\overline{c}_x$ shown in the tables are assumed for illustrative purposes only. There is no intent to suggest that these values are necessarily appropriate in any given instance.

C. *Method I.*[35]

Conditions of Application.

(a) Neither the final value to be assumed by any individual adjusted rate, R_x^*, nor the percentage value of the adjustment to any individual protection class, is pre-determined; *and:*

(b) The bounds to the inter-class ratios, $\underline{f}_x$ and $\overline{f}_x$, or, $\underline{c}_x$ and $\overline{c}_x$, are considered to be extremely elastic.

[35] See APPENDIX B for derivation of all equations employed in this section.

Case 1.

Supplemental Conditions: – The rate-normal ratio, $r_x = R_x/Q_x$, may be taken equal to unity for all x; *and*: the shape of the existing rate curve is to remain unchanged.

Algebraic Solution:

For each protection class in turn, calculate the final adjusted rate, R_x^*, by the equation:

$$R_x^* = R_x^e \frac{U - P}{U^e - P^e} + \frac{PU^e - UP^e}{U^e - P^e} \tag{23.a}$$

Graphical Solution:

(1) Plot the points $(P^e;P)$ and $(U^e;U)$, labeling the horizontal axis, "R_x^e," and the vertical axis, "R_x^*". Draw the straight line through these points.

(2) Read the final adjusted rates, R_x^*, as the ordinates of those points on the line, whose abcissas are the respective existing rates, R_x^e.

EXAMPLE 1.

Premium Adjustments Required: To the "Protected" statistical class: – 10% increase. To the "Unprotected" statistical class: – 25% increase.

Data Reference: Tables 1 and 2.

Algebraic Solution:

The complete rate calculation is shown in Table 4, together with the values of f_x^* and the verification.

The small differences, $\hat{P}^* - P = 0.001$ and $\hat{U}^* - U = 0.003$ are due solely to rounding error, as may be seen by carrying at least five decimals at each stage of the overall calculation. The form of the calculation is exact.

Whether or not the ratio $f_4^* < \underline{f_4}$ is to be accepted is a matter of judgment. In dollars and cents: $\underline{f_4} R_5^* - R_4^* = \$2.422 - \$2.339 = \$0.083$ per $1,000 of insurance.

TABLE 4.
Solution of Example 1.

x	R_x^e		R_x^*	f_x^*	Verification: w_x	Verification: $w_xR_x^*$
*	* *		* *	* *		
2	1.81		1.775	0.863	0.066	0.117
3	2.01		2.057	0.897	0.461	0.948
4	2.21		2.339	#0.709	0.052	0.112
5	2.89		3.299	0.921	0.148	0.488
6	3.09	$\times 1.411 - 0.779 =$	3.581	0.841	0.137	0.491
7	3.57		4.258	0.938	0.079	0.336
8	3.77		4.520	0.872	0.057	0.259
*	* *		* *	* *	$\hat{P}^* = 2.761$	
*	* *		* *	* *	$\hat{P}^* - P = 0.001$	
9	4.24		5.204	0.856	0.215	1.119
10	5.02		6.304	* *	0.785	4.949
$P = 1.10P^e = 2.760$; $U = 1.25U^e = 6.065$					$\hat{U}^* = 6.068$	
$(U\text{-}P)/(U^e\text{-}P^e) = 1.411$; $(PU^e\text{-}UP^e)/(U^e\text{-}P^e) = -0.779$					$\hat{U}^* - U = 0.003$	

$f_4^* = 0.709 < 0.75 = \underline{f}_4$

Graphical Solution:

Figure 1 represents the graphical solution of the problem. The final adjusted rates obtained from the original of the graph are:

$$R_2^* = 1.78\ ;\ R_5^* = 3.30\ ;\ R_8^* = 4.50$$
$$R_3^* = 2.06\ ;\ R_6^* = 3.58\ ;\ R_9^* = 5.20$$
$$R_4^* = 2.34\ ;\ R_7^* = 4.26\ ;\ R_{10}^* = 6.29$$

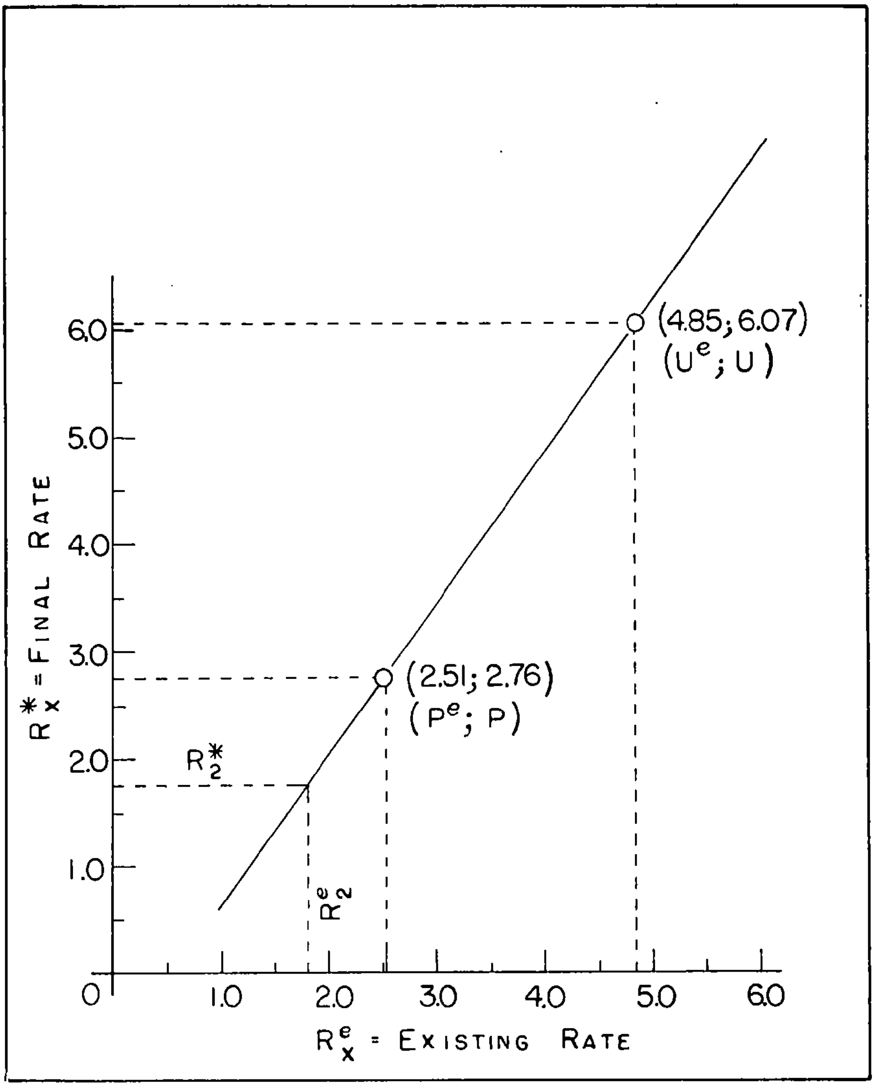
and in verification:

$$\textstyle\sum_P w_xR_x^* = \hat{P}^* = 2.731;\ \text{whence: } \hat{P}^* - P = -0.029$$
$$\textstyle\sum_U w_xR_x^* = \hat{U}^* = 6.016;\ \text{whence: } \hat{U}^* - U = -0.049$$

The graphical solution follows immediately from the fact that Eq. (23.a) is simply the slope-intercept form of a linear equation in R_x^e and R_x^*.

FIGURE 1.

Graphical Solution of Example 1.



Case 2.

Supplemental Conditions: — The rate-normal ratio, r_x, may be taken equal to unity for all *x; but:* the shape of the existing rate curve is to be revised.

Algebraic Solution:

(1) Determine by any convenient method a set of trial rates, $R_x{}^i$, which define a rate curve of the desired shape. Calculate $\hat{U}^i = \sum_U w_x R_x{}^i$ and $\hat{P}^i = \sum_P w_x R^i$.

(2) Substitute $R_x{}^i$, $\hat{U}^i$ and $\hat{P}^i$ respectively for $R_x{}^e$, U^e and P^e in Eq.(23.a), and compute the final rates, $R_x{}^*$, as in Case 1.

Note: Although the exact values of $R_x{}^i$, $\hat{U}^i$ and $\hat{P}^i$ are immaterial, the difference $\hat{U}^i - \hat{P}^i$ should contain at least as many significant figures as does the difference $U - P$. This normally will result if the several $R_x{}^i$ are so chosen that $R_{10}{}^i > U$, $R_8{}^i > P$, and $R_\zeta{}^i < P$ for at least one value of ζ such that $\zeta < 8$.

Graphical Solution:

Proceed as in the graphical solution of Case 1, except that the points to be plotted are $(\hat{P}^i; P)$ and $(\hat{U}^i; U)$, and the horizontal axis is to be labeled "$R_x{}^i$."

EXAMPLE 2.

Premium Adjustment Required: To the "Protected" statistical class: — 25% increase. To the "Unprotected" statistical class: — no adjustment of presently reported premium.[36]

Data Reference: Tables 1 and 2.

Assumption: The following trial rates define a rate curve of the desired shape:

$R_2{}^i = 1.37$; $R_5{}^i = 2.22$; $R_8{}^i = 3.61$

$R_3{}^i = 1.61$; $R_6{}^i = 2.61$; $R_9{}^i = 4.25$

$R_4{}^i = 1.89$; $R_7{}^i = 3.07$; $R_{10}{}^i = 5.00$

Algebraic Solution:

$$U^c = 4.852 \text{ ; whence: } U = 1.000U^c = 4.852$$
$$P^c = 2.509 \text{ ; whence: } P = 1.250P^c = 3.136$$
$$\hat{U}^i = \textstyle\sum_U w_x R_x^i = 4.839$$
$$\hat{P}^i = \textstyle\sum_P w_x R_x^i = 2.066$$

whence by Eq.(23.a) with appropriate substitutions:

(23.a.1) $R_x{}^* = 0.619R_x{}^i + 1.858$.

The final adjusted rates calculated by Eq.(23.a) are:

$$R_2{}^* = 2.706 \text{ ; } R_5{}^* = 3.232 \text{ ; } R_8{}^* = 4.093$$
$$R_3{}^* = 2.855 \text{ ; } R_6{}^* = 3.474 \text{ ; } R_9{}^* = 4.489$$
$$R_4{}^* = 3.028 \text{ ; } R_7{}^* = 3.758 \text{ ; } R_{10}{}^* = 4.953$$

and in verification:

$$\textstyle\sum_P w_x R_x{}^* = \hat{P}^* = 3.136\text{; whence: } \hat{P}^* - P = 0$$
$$\textstyle\sum_U w_x R_x{}^* = \hat{U}^* = 4.853\text{; whence: } \hat{U}^* - U = 0.001$$

The rate curves defined respectively by the trial rates $R_x{}^i$ and the final rates, $R_x{}^*$, are shown in Figs. 2.a and 2.b. The relationship (see the figures) between $R_x{}^i - L_x{}^i$ and $R_x{}^* - \tilde{L}_x$ should be noted.

Case 3.

Supplemental Conditions: — The rate-normal ratio, r_x, cannot be taken equal to unity for all *x; but:* the shape of the normal curve is to remain unchanged.

Algebraic Solution:

(1) Calculate the *"unprotected target normal,"* U_Q, and the *"protected target normal,"* P_Q, by the equations:

(24)
$$U_Q = U_Q{}^c \frac{U - P}{U^c - P^c} + q_U \frac{PU^c - UP^c}{U^c - P^c}\text{ ; } \quad (q_U = \textstyle\sum_U w_x q_x)$$
$$P_Q = P_Q{}^c \frac{U - P}{U^c - P^c} + q_P \frac{PU^c - UP^c}{U^c - P^c}\text{ ; } \quad (q_P = \textstyle\sum_P w_x q_x)$$

---

[36] It may be noted that if the values of R_x shown in Table 2 simply are increased each by 25% for Classes 2-8 while leaving R_x at present value for Classes 9 and 10, the result will be not only: $R_8{}^* = 4.71 > 4.24 = R_9{}^*$; but also: $R_7{}^* = 4.46 > 4.24 = R_9{}^*$. Remembering that by hypothesis, $r_x = 1$, this solution is unacceptable even though the required premium volume would be obtained.

FIGURE 2.a.
Rate Curves. Example 2.

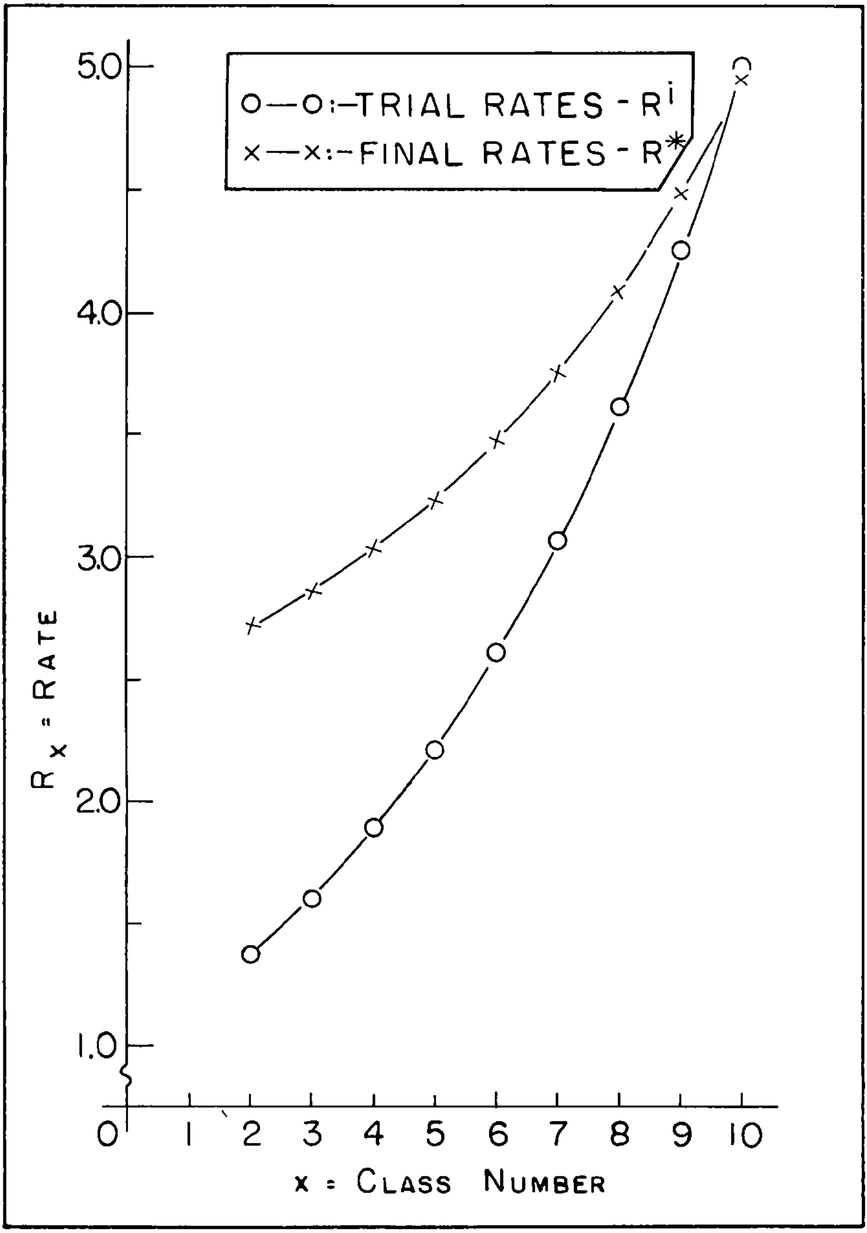
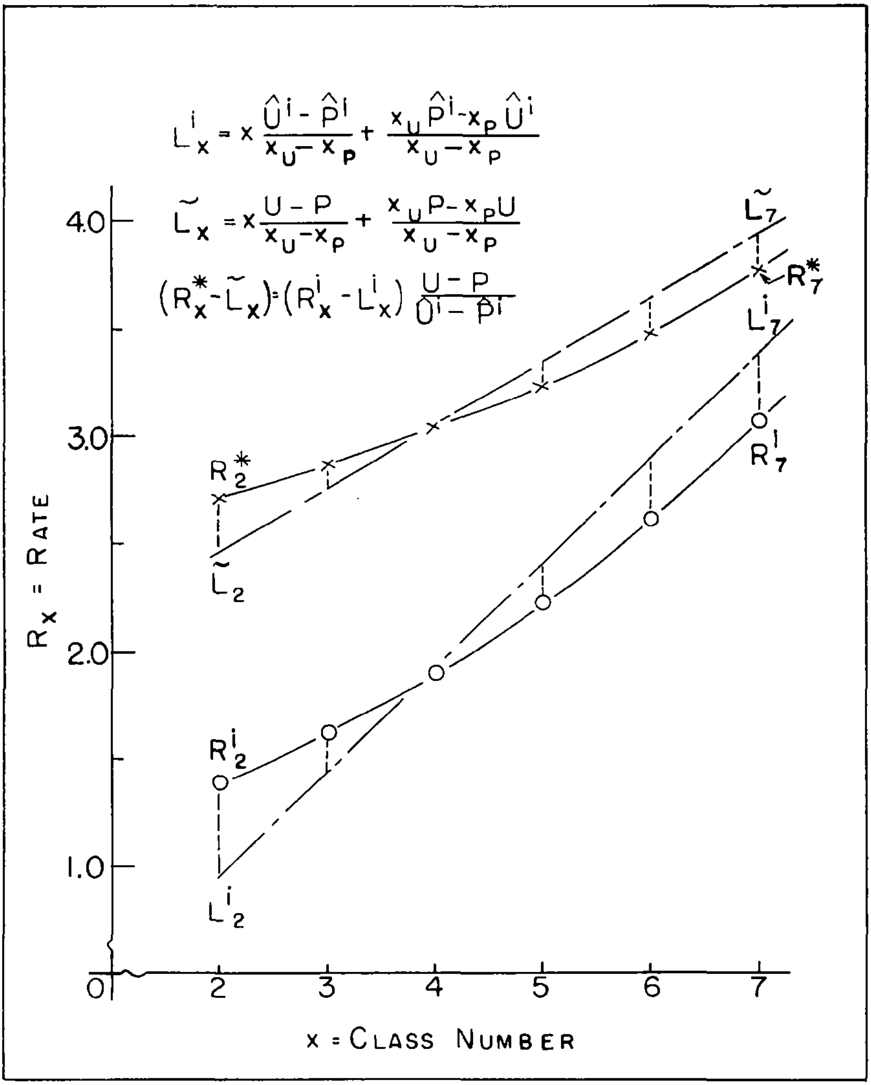


FIGURE 2.b.

Rate Curves. Example 2.
Expanded Scale with Reference Lines.



(2) For each protection class in turn, calculate the final adjusted normals $Q_x{}^*$ by the equation (*cf.* Eq.(23.a)):

$$(23.b) \qquad Q_x{}^* = Q_x{}^e \frac{U_Q - P_Q}{U_Q{}^e - P_Q{}^e} + \frac{P_Q U_Q{}^e - U_Q P_Q{}^e}{U_Q{}^e - P_Q{}^e}$$

(3) Calculate the final adjusted rates by the equation:

$$(25) \qquad R_x{}^* = r_x Q_x{}^*$$

It may be noted that Eqs.(23.b) and (25) can be combined if both sides of Ep.(23.b) are multiplied by r_x, yielding *(Cf.* Eq.(23.a)):

$$(25.a) \qquad R_x{}^* = R_x{}^e \frac{U_Q - P_Q}{U_Q{}^e - P_Q{}^e} + r_x \frac{P_Q U_Q{}^e - U_Q P_Q{}^e}{U_Q{}^e - P_Q{}^e}$$

whence $R_x{}^*$ is obtained directly without intermediate calculation of $Q_x{}^*$. Offsetting the immediate operational economies of Eq.(25.a) is the fact that unless significant changes are to be expected in the distribution of sums insured, as reflected in the several weighting factors, w_x, the final adjusted normals, $Q_x{}^*$, of the current rate revision may be stored to become the existing normals to be used in the next subsequent rate revision. Thus the immediate use of Eqs.(23.b) and (25) in preference to Eq. (25.a) may save calculation at a later date.

It also may be noted that it is possible to obtain a solution by the method of Case 1 which will produce the required premium. However, direct adjustment of $R_x{}^e$ to $R_x{}^*$ by Eq.(23.a) when $r_x \neq 1$ is very likely to result in unacceptable inversion of the normals, *i.e.* for some x, then $Q_x > Q_{x+1}$. If $r_x \neq 1$, then $R_x > R_{x+1}$ is permissible, but never the inconsistency of $Q_x > Q_{x+1}$.

Graphical Solution:

By analogy to the graphical solution of Case 1, the final adjusted normals $Q_x{}^*$ are obtained from the plot of a straight line through the points $(P_Q{}^e; P_Q)$ and $(U_Q{}^e; U_Q)$, where the horizontal axis represents $Q_x{}^e$ and the vertical axis represents $Q_x{}^*$. The final adjusted rates then follow by Eq.(25).

Case 4.

Supplemental Conditions: – The rate normal ratio, r_x, cannot be taken equal to unity for all *x; and:* the shape of the normal curve is to be revised.

Algebraic Solution:

(1) Calculate U_Q and P_Q by Eqs. (24) as in Case 3.

(2) Determine by any convenient method a set of trial normals, Q_x^i, which define a normal curve of the desired shape. Calculate: $\hat{U}_Q^i = \sum_U w_x Q_x^i$; and: $\hat{P}_Q^i = \sum_P w_x Q_x^i$.

(3) Calculate the final adjusted normals, Q_x^*, by Eq.(23.b).

(4) Calculate the final adjusted rates, R_x^*, by Eq.(25), as in Case 3.

Graphical Solution:

Case 4 may be solved graphically for Q_x^*, by analogy to the graphical solution of Case 3, whence R_x^* then follows by Eq.(25).

EXAMPLE 3.

Premium Adjustments Required: To the "Protected" statistical class: – 30% increase. To the "Unprotected" statistical class: – 5% increase.

Data Reference: Tables 1 and 3.

Assumption: The following trial normals define a normal curve of the desired shape:

$$Q_2^i = 1.37\ ;\ Q_5^i = 2.22\ ;\ Q_8^i = 3.61$$
$$Q_3^i = 1.61\ ;\ Q_6^i = 2.61\ ;\ Q_9^i = 4.25$$
$$Q_4^i = 1.89\ ;\ Q_7^i = 3.07\ ;\ Q_{10}^i = 5.00$$

Algebraic Solution:

$$U^e = 5.320;\ \text{whence: } U = 1.05 \times 5.320 = 5.586$$
$$P^e = 2.407;\ \text{whence: } P = 1.30 \times 2.407 = 3.129$$

whence by Eqs.(24):

$$U_Q = 5.092;\ \text{and: } P_Q = 3.289$$

and by hypothesis:

$$\hat{U}_Q^i = \sum_U w_x Q_x^i = 4.839$$
$$\hat{P}_Q^i = \sum_P w_x Q_x^i = 2.066$$

whence by Eq.(23.b) the final adjusted normals are:

$$Q_2^* = 2.836\ ;\ Q_5^* = 3.389\ ;\ Q_8^* = 4.293$$
$$Q_3^* = 2.993\ ;\ Q_6^* = 3.643\ ;\ Q_9^* = 4.709$$
$$Q_4^* = 3.175\ ;\ Q_7^* = 3.942\ ;\ Q_{10}^* = 5.196$$

and in verification of the normals, Q_x*:

$$\sum_P w_x Q_x^* = \hat{P}_Q^* = 3.287;\ \text{whence: } \hat{P}_Q^* - P_Q = -0.002$$
$$\sum_U w_x Q_x^* = \hat{U}_Q^* = 5.091;\ \text{whence: } \hat{U}_Q^* - U_Q = -0.001$$

From the final adjusted normals, as above, the final adjusted rates are, by Eq.(25):

$$R_2^* = 2.694\ ;\ R_5^* = 3.226\ ;\ R_8^* = 4.533$$
$$R_3^* = 2.577\ ;\ R_6^* = 3.795\ ;\ R_9^* = 5.298$$
$$R_4^* = 3.219\ ;\ R_7^* = 4.249\ ;\ R_{10}^* = 5.664$$

and in verification of the final adjusted rates, R_x*:

$$\sum_P w_x R_x^* = \hat{P}^* = 3.124;\ \text{whence: } \hat{P}^* - P = -0.005$$
$$\sum_U w_x R_x^* = \hat{U}^* = 5.585;\ \text{whence: } \hat{U}^* - U = -0.001$$

D. *Method II.*[37]

Conditions of Application.

(a) The bounds to the inter-class ratios, $\underline{f}_x$ and $\bar{f}_r$, are considered relatively inelastic as between one or more pairs of adjacent classes; *or:*

(b) The final values to be assumed by some one or more of the adjusted rates R_x^* are pre-determined by underwriting or other considerations; *and:*

(c) The shape of the final rate curve (or final normal curve) is immaterial.

Pre-calculation of Parameters

Pre-calculate and store for use in successive rate revisions over a period of years the parameter vectors N^ϕ whose component rates are shown in Table 5. For each vector N^ϕ, calculate $P^\phi = \sum_P w_x N_x$. (It will be found that for all ϕ, then $\sum_U w_x N_x^\phi = 1$.) Once calculated, these parameters need not be re-calculated unless and until either significant change occurs in the distribution of sums insured (*i.e.* in the values of w_x) or the extremal ratios $\underline{f}_r$ and $\bar{f}_r$ are revised.

Tables 6A and 6B show a sample calculation of these parameters from data given in Tables 1 and 3. Table 6B serves also as the table of parameters for use in illustrative examples to follow.

[37] See APPENDIX A for full discussion of Method II and derivation of equations to follow.

TABLE 5: – Parameter Formulas.

* *	N_x^{ϕ} $(x \leq 7)$	N_8^{ϕ}	N_9^{ϕ}	N_{10}^{ϕ}	* *	N_x^{ϕ} $(x \leq 7)$	N_8^{ϕ}	N_9^{ϕ}	N_{10}^{ϕ}
N_x^{I}	$\prod_{x=1}^{7} \underline{f}_x N_8^{\phi}$	$\underline{f}_8 \underline{N}_9$	$\underline{N}_9$	$\overline{N}_{10}$	N_x^{V}	$\prod_{x=1}^{7} \overline{f}_x N_8^{\phi}$	$\underline{f}_8 \underline{N}_9$	$\underline{N}_9$	$\overline{N}_{10}$
N_x^{II}		$\underline{f}_8 \overline{N}_9$	$\overline{N}_9$	$\underline{N}_{10}$	N_x^{VI}		$\underline{f}_8 \overline{N}_9$	$\overline{N}_9$	$\underline{N}_{10}$
N_x^{III}		$\overline{f}_8 \underline{N}_9$	$\underline{N}_9$	$\overline{N}_{10}$	N_x^{VII}		$\overline{f}_8 \underline{N}_9$	$\underline{N}_9$	$\overline{N}_{10}$
N_x^{IV}		$\overline{f}_8 \overline{N}_9$	$\overline{N}_9$	$\underline{N}_{10}$	N_x^{VIII}		$\overline{f}_8 \overline{N}_9$	$\overline{N}_9$	$\underline{N}_{10}$

$$\overline{N}_{10} = \frac{1}{\underline{f}_9 w_9 + w_{10}} \qquad \underline{N}_{10} = \frac{1}{\overline{f}_9 w_9 + w_{10}}$$

$$\underline{N}_9 = \underline{f}_9 \overline{N}_{10} \qquad \overline{N}_9 = \overline{f}_9 \underline{N}_{10}$$

TABLE 6A: – Parameter Calculations.

$$\overline{N}_{10} = \frac{1}{0.83 \times 0.215 + 0.785} = 1.0380$$

$$\underline{N}_{10} = \frac{1}{0.99 \times 0.215 + 0.785} = 1.0022$$

$$\underline{N}_9 = 0.83\ \overline{N}_{10} = 0.8615 \qquad \underline{N}_9 \times \begin{cases} 0.88 = 0.7581 = \overline{f}_8 \underline{N}_9 \\ 0.74 = 0.6375 = \underline{f}_8 \underline{N}_9 = \underline{N}_8 \end{cases}$$

$$\overline{N}_9 = 0.99\ \underline{N}_{10} = 0.9921 \qquad \overline{N}_9 \times \begin{cases} 0.88 = 0.8730 = \overline{f}_8 \overline{N}_9 = \overline{N}_8 \\ 0.74 = 0.7341 = \underline{f}_8 \overline{N}_9 \end{cases}$$

TABLE 6B: – Parameter Calculations.

* *	N^{I}	N^{II}	N^{III}	N^{IV}	N^{V}	N^{VI}	N^{VII}	N^{VIII}
$N_{10}\phi$	$\overline{N}_{10}$	$\underline{N}_{10}$	$\overline{N}_{10}$	$\underline{N}_{10}$	$\overline{N}_{10}$	$\underline{N}_{10}$	$\overline{N}_{10}$	$\underline{N}_{10}$
$N_{9}\phi$	$\underline{N}_{9}$	$\overline{N}_{9}$	$\underline{N}_{9}$	$\overline{N}_{9}$	$\underline{N}_{9}$	$\overline{N}_{9}$	$\underline{N}_{9}$	$\overline{N}_{9}$
$N_{8}\phi$	$\underline{f}_8\underline{N}_9$	$\underline{f}_8\overline{N}_9$	$\overline{f}_8\underline{N}_9$	$\overline{f}_8\overline{N}_9$	$\underline{f}_8\underline{N}_9$	$\underline{f}_8\overline{N}_9$	$\overline{f}_8\underline{N}_9$	$\overline{f}_8\overline{N}_9$
$N_{10}\phi$	1.0380	1.0022	1.0380	1.0022	1.0380	1.0022	1.0380	1.0022
$N_{9}\phi$	0.8615	0.9921	0.8615	0.9921	0.8615	0.9921	0.8615	0.9921
$N_{8}\phi$	0.6375	0.7341	0.7581	0.8730	0.6375	0.7341	0.7581	0.8730
	$\times (0.82 = \underline{f}_7)$				$\times (0.98 = \overline{f}_7)$			
$N_{7}\phi$	0.5227	0.6019	0.6216	0.7158	0.6247	0.7194	0.7429	0.8555
	$\times (0.77 = \underline{f}_6)$				$\times (0.92 = \overline{f}_6)$			
$N_{6}\phi$	0.4024	0.4634	0.4786	0.5511	0.5747	0.6618	0.6834	0.7870
	$\times (0.73 = \underline{f}_5)$				$\times (0.87 = \overline{f}_5)$			
$N_{5}\phi$	0.2937	0.3382	0.3493	0.4023	0.4999	0.5757	0.5945	0.6846
	$\times (0.80 = \underline{f}_4)$				$\times (0.96 = \overline{f}_4)$			
$N_{4}\phi$	0.2349	0.2705	0.2794	0.3218	0.4799	0.5526	0.5707	0.6572
	$\times (0.64 = \underline{f}_3)$				$\times (0.77 = \overline{f}_3)$			
$N_{3}\phi$	0.1503	0.1731	0.1788	0.2059	0.3695	0.4255	0.4394	0.5060
	$\times (0.83 = \underline{f}_2)$				$\times (1.00 = \overline{f}_2)$			
$N_{2}\phi$	0.1247	0.1436	0.1484	0.1708	0.3695	0.4255	0.4394	0.5060
* *	$p\phi = \sum_{p} w_x N_x \phi$							
$p\phi$	0.2656	0.3058	0.3160	0.3640	0.4577	0.5272	0.5444	0.6269

Properties of the Parameter Vectors.

Regardless of the values of w_x and of $\underline{f}_x < \overline{f}_x$ for any or all x, it will be found that always: $\mathcal{P}^I < \mathcal{P}^\phi$ for any $\phi \neq I$; $\mathcal{P}^{VIII} > \mathcal{P}^\psi$ for any $\psi \neq VIII$; $\mathcal{P}^I < \mathcal{P}^{III} < \mathcal{P}^{VI} < \mathcal{P}^{VIII}$. Also, for $x < 8$, always: $N_x^I < N_x^{III} < N_x^{VI} < N_x^{VIII}$. The ordering of the remaining $\mathcal{P}^\phi$ and (for $x < 8$) of N_x^ϕ will depend upon the actual values of w_x, $\underline{f}_x$ and $\overline{f}_x$ in a given case.

On the assumption that the extremal ratios $\underline{f}_x$ and $\overline{f}_x$ are rigid bounds to f_x^i; then, letting $\mathcal{P}^* = P/U$; for any value of U and for any values of w_x, $\underline{f}_x$ and $\overline{f}_x$:

(a) If: $\mathcal{P}^{III} \le \mathcal{P}^* \le \mathcal{P}^{VI}$; then:

$$N_x^{III} = \overline{N}_x \ge R_x^*/U = N_x^* \ge N_x^{VI} = \underline{N}_x;\ (x = a = 8;\ \text{or: } x = z = 10)$$

$$N_x^{III} = \underline{N}_x \le R_x^*/U = N_x^* \le N_x^{VI} = \overline{N}_x;\ (x \neq a = 8;\ \text{or: } x \neq z = 10)$$

(b) If: $\mathcal{P}^{VI} < \mathcal{P}^* \leqslant \mathcal{P}^{VIII}$; then:

$$N_x^{VI} < R_x^*/U = N_x^* \le N_x^{VIII},\ (x \le a = 8)$$

(c) If: $\mathcal{P}^I \le \mathcal{P}^* < \mathcal{P}^{III}$; then:

$$N_x^I \le R_x^*/U = N_x^* < N_x^{III};\ (x \le a = 8)$$

(d) If:

$\mathcal{P}^* = \mathcal{P}^I$, then necessarily: $\tilde{\boldsymbol{R}}^* = U\boldsymbol{N}^I$

$\mathcal{P}^* = \mathcal{P}^{VIII}$, then necessarily: $\tilde{\boldsymbol{R}}^* = U\boldsymbol{N}^{VIII}$

(e) If $\mathcal{P}^* < \mathcal{P}^I$ or $\mathcal{P}^* > \mathcal{P}^{VIII}$, there will be *no* solution to the rate revision problem unless and until the bound $\underline{f}_x$ or $\overline{f}_x$ is relaxed for at least one x.

Additional and comparable properties will depend on the values of w_x, $\underline{f}_x$ and $\overline{f}_x$. For example, in the assumed instance of $\mathcal{P}^{VII} > \mathcal{P}^{VI}$ (see Table 6B), if $\mathcal{P}^* > \mathcal{P}^{VII}$, then $N_x^* > N_x^{VII}$ for $x \le 8$, but this would not necessarily be the case were $\mathcal{P}^{VII} < \mathcal{P}^{VI}$, as it might be in particular instance.

Since the exact values of $\underline{f}_x$ and $\overline{f}_x$ depend upon judgment, these bounds may, of course, be relaxed to obtain a solution when and if the ratemaker runs afoul of one of the inequalities above. This will be a matter of judgment in a given case. In extreme cases, revision of the classification system may be necessary. The listed properties can be useful, however, in that before actual rate calculation is started, direct comparison of $\mathcal{P}^*$ with $\mathcal{P}^\phi$ gives immediate indication of what may be expected in the course of the rate revision.

Procedure.

The procedure will be illustrated by examples.

EXAMPLE 4.

Target Rates: $P = 3.129$; $U = 5.586$

Solution:

$$\mathcal{P}^* = 3.129/5.586 = 0.5601$$

Choose any $\mathcal{P}^\phi < \mathcal{P}^*$, say $\mathcal{P}^{VI}$. The only $\mathcal{P}^\psi > \mathcal{P}^*$ in this case is $\mathcal{P}^{VIII}$, so there is no choice, but in general, any $\mathcal{P}^\psi > \mathcal{P}^*$ could be chosen. Let:

(26) $$b\mathcal{P}^\phi + (1 - b)\,\mathcal{P}^\psi = \mathcal{P}^*;\ \mathcal{P}^\phi < \mathcal{P}^* < \mathcal{P}^\psi)$$
$$(0.5272\,b + 0.6269\,(1 - b) = 0.5601)$$

whence:

$$b = 0.6700;\ \text{and: } (1 - b) = 0.3300$$

Then:

(27) $$\tilde{N}^* = bN^\phi + (1 - b)\,N^\psi$$

whence:

(28) $$\tilde{R}^* = U\tilde{N}^*$$

The calculation of this example is completed and verified in Table 7. There are no worksheets other than Table 7 (unless the tape from a standard model desk calculator be counted as such).

It may be noted that this problem is exactly the problem solved in Method I, Case 4, as Example 3, preceding. It will be found that although both solutions are feasible vectors, the solution of this example is such that for all x, then $\underline{f}_x < f_x^* < \overline{f}_x$, which is not the case with the solution of Example 3.

EXAMPLE 5.

Target Rates: $P = 3.129$; $U = 5.586$

Side Condition: R_3^* is to assume the value of $2.000 = R_3^o$.

Solution:

$$\mathcal{P}^* = 3.129/5.586 = 0.5601$$

(29) $$N_3^* = N_3^o = R_3^o/U$$
$$(N_3^o = 2.000/5.586 = 0.3580)$$

There is no solution to this problem. See Property (b) of the parameter vectors, and compare N_3^o with N_3^{VI} from Table 6B.

TABLE 7: – Solution of Example 4.

$$\overset{p^{VI}}{0.5257}\, b + \overset{p^{VIII}}{0.6269}\,(1-b) = \overset{p^{*}}{0.5601} \longrightarrow \begin{cases} b = 0.6700 \\ 1-b = 0.3300 \end{cases}$$

$$\underset{0.6700}{b} \begin{bmatrix} 0.4255 \\ 0.4255 \\ 0.5526 \\ 0.5757 \\ 0.6618 \\ 0.7194 \\ 0.7341 \\ 0.9921 \\ 1.0022 \end{bmatrix}^{N^{VI}} \underset{+\,0.3300}{(1-b)} \begin{bmatrix} 0.5060 \\ 0.5060 \\ 0.6572 \\ 0.6846 \\ 0.7870 \\ 0.8555 \\ 0.8730 \\ 0.9921 \\ 1.0022 \end{bmatrix}^{N^{VIII}} = \begin{bmatrix} 0.4519 \\ 0.4519 \\ 0.5870 \\ 0.6166 \\ 0.7031 \\ 0.7642 \\ 0.7798 \\ 0.9921 \\ 1.0022 \end{bmatrix}^{\tilde{N}^{*}}$$

$$\underset{5.586\tilde{N}^{*}}{U} = \begin{bmatrix} R_2^* = 2.5243 \\ R_3^* = 2.5243 \\ R_4^* = 3.2789 \\ R_5^* = 3.4163 \\ R_6^* = 3.9275 \\ R_7^* = 4.2688 \\ R_8^* = 4.3559 \\ R_9^* = 5.5418 \\ R_{10}^* = 5.5982 \end{bmatrix}^{\tilde{R}^{*}}$$

$$\hat{P}^* = 3.130$$
$$\hat{P}^* - P = 0.001$$
$$\hat{U}^* = 5.586$$
$$\hat{U}^* - U = 0$$

EXAMPLE 6.

Target Rates: $P = 2.680$; $U = 5.586$

Side Condition: $R_3{}^*$ is to assume the value of $2.000 = R_3{}^o$

Solution:

$$p^* = 2.680/5.586 = 0.4800$$
$$N_3{}^o = 2.000/5.586 = 0.3580$$

In general, a solution to this problem will be given by the equation:

$$\tilde{N}^* = \sum b_\phi{}^* N^\phi; \quad (b_\phi{}^* \geq 0; \sum b_\phi{}^* = 1) \tag{30}$$

whence the final rate vector is obtained by Eq. (28)[38]. The coefficients of Eq.(30) are given by:[39]

$$\left.\begin{array}{l} \sum b_\phi{}^* p^\phi + (1 - \sum b_\phi{}^*)\, p^\psi = p^* \\ \sum b_\phi{}^* N_x{}^\phi + (1 - \sum b_\phi{}^*)\, N_x{}^\psi = N_x{}^o \end{array}\right. \quad (b_\phi{}^* \geq 0; \sum b_\phi{}^* \leq 1) \tag{31}$$

Although *a* solution to Eqs. (31) always may be found by choosing not less than three values each of P^ϕ and $N_x{}^\phi$ from among the eight listed in Table 6B, not all combinations of three or more values will give a *non-negative* solution as required. The simplest approach to the problem is as follows:

Choose a value of $N_x{}^{\phi:i} < N_x{}^o$ and a value of $N_x{}^{\psi:i} > N_x{}^o$.[40] Determine b^i by the equation:

$$b^i N_x{}^{\phi:i} + (1 - b^i)\, N_x{}^{\psi:i} = N_x{}^o \tag{32}$$

and calculate:

$$\hat{p}^{*:i} = b^i p^{\phi:i} + (1 - b^i)\, p^{\psi:i} \tag{33}$$

If $\hat{p}^{*:i} = p^*$, the problem is solved by entering b^i and $(1 - b^i)$ as coefficients in Eq.(30). If $\hat{p}^{*:i} \neq p^*$, chose a value of $N_x{}^{\phi:j} < N_x{}^o$ and of $N_x{}^{\psi:j} > N_x{}^o$, where possibly (not necessarily) $\phi{:}j = \phi{:}i$ or $\psi{:}j = \psi{:}i$, but not both. Calculate b^j by Eq.(32) and, thence, $\hat{p}^{*:j}$ by Eq.(33). If $\hat{p}^{*:j} = p^*$, the problem is solved. If $\hat{p}^{*:j} \neq p^*$ and also *either* $\hat{p}^{*:i} < p^*$ and $\hat{p}^{*:j} < p^*$, *or* $\hat{p}^{*:i} > p^*$ and $\hat{p}^{*:j} > p^*$, repeat the

[38] Despite the formal similarity, Eq. (30) does NOT follow by simple change of notation in Eq. (1) of Section IV.A. See APPENDIX A.

[39] *Cf.* McIntosh, (14). p. 152, Eq. (9). Equation (30), above, *DOES* follow from Eq. (9) of the reference by simple change of notation accompanied by re-definition of terms.

[40] If $N_x{}^o < N_x{}^I$, or $N_x{}^o > N_x{}^{VIII}$, the problem is insoluble.

operation until values of $\hat{p}^{*:k}$ and $\hat{p}^{*:l}$ are obtained such that $\hat{p}^{*:k} < p^* < \hat{p}^{*:l}$. (Normally, if: $p^{\phi:i} = p^{I}$ and $p^{\psi:j} = p^{VIII}$, then $\hat{p}^{*:i} < p^* < \hat{p}^{*:j}$ or $\hat{p}^{*:i} > p^* > \hat{p}^{*:j}$; but at most not more than three of four trials should be required to bracket the value of p^* with values of $\hat{p}^*$).

Assume $\hat{p}^{*:i} < p^* < \hat{p}^{*:j}$. Calculate t by the equation:

$$(34) \qquad t\hat{p}^{*:i} + (1 - t)\,\hat{p}^{*:j} = p^*$$

Thence calculate:

$$(35) \qquad \begin{aligned} b_{\phi:i} &= tb^{i};\ \text{and:}\ b_{\psi:i} = t\,(1 - b^{i}) \\ b_{\phi:j} &= (1 - t)\,b^{j};\ \text{and}\ b_{\psi:j} = (1 - t)\,(1 - b^{j}) \end{aligned}$$

Thence a solution to the problem will follow upon entering the coefficients calculated by Eq.(35), together with the associated parameter vectors, $N^{\phi:i}$, etc., into Eq.(30).

The complete solution of Example 6 is given in Table 8.

Extension of Application

The procedures indicated under the Examples 4-6 may be extended to more complicated cases, *e.g.* where values of R_x^o are predetermined for two or more classes, or where for some class the value of f_x^c already is so extreme that any further movement of the value either upward or, alternatively, downward, cannot be tolerated. Such extensions involve techniques of finding directly a non-negative solution of Eqs.(31), and very possibly involve pre-calculation of additional parameter vectors beyond those given in Table 6B. Although a solution to Eqs.(30) and (31) must always exist, utilizing not more than eight parameter vectors under the classification system assumed here for illustrative purposes, the total number of possible parameter vectors, no two of which are equal, will be $2^8 = 256$, and the practical difficulty lies in determining *which* eight out of that total will serve in particular instance.

It seems probable that the full potential of Method II can be exploited in application only if computer facilities are utilized. However, although they cannot be presented simply in empirical fashion, nor be applied properly without at least a basic understanding of theory sum-

TABLE 8: – Solution of Example 6.

$$P = 2.680;\ U = 5.586;\ p^* = 2.680/5.586 = 0.4800$$

$$R_3^* = R_3^o = 2.000;\ N_3^o = 2.000/5.586 = 0.3580$$

$$\overset{N_3^{I}}{0.1503}b^i + \overset{N_3^{VI}}{0.4255}\,(1 - b^i) = 0.3580 \longrightarrow \begin{cases} b^i = 0.2452 \\ (1 - b^i) = 0.7548 \end{cases}$$

$$\overset{N_3^{IV}}{0.2059}b^j + \overset{N_3^{VIII}}{0.5060}\,(1 - b^j) = 0.3580 \longrightarrow \begin{cases} b^j = 0.4931 \\ (1 - b^j) = 0.5069 \end{cases}$$

$$b^i p^I + (1 - b^i)\, p^{VI} = 0.4630 = \hat{p}^{*:i}$$

$$b^j p^{IV} + (1 - b^j)\, p^{VIII} = 0.4971 = \hat{p}^{*:j}$$

$$0.4630t + 0.4971(1 - t) = 0.4800 \longrightarrow \begin{cases} t = 0.5014 \\ (1 - t) = 0.4986 \end{cases}$$

$$0.5014 \times \begin{cases} b^i = 0.1229 = b_I^* \\ (1 - b^i) = 0.3784 = b_{VI}^* \end{cases} \qquad 0.4986 \times \begin{cases} b^j = 0.2458 = b_{IV}^* \\ (1 - b^j) = 0.2527 = b_{VIII}^* \end{cases}$$

$$\overset{U}{5.586}(b_I^* N^I + b_{IV}^* N^{IV} + b_{VI}^* N^{VI} + b_{VIII}^* N^{VIII} = \tilde{N}^*) = \tilde{R}^* = \begin{bmatrix} R_2^* = 1.9327 \\ R_3^o = 1.9986 \\ R_4^* = 2.6794 \\ R_5^* = 2.9354 \\ R_6^* = 3.5415 \\ R_7^* = 4.0688 \\ R_8^* = 4.4324 \\ R_9^* = 5.4502 \\ R_{10}^* = 5.6206 \end{bmatrix}$$

$$\hat{P}^* = 2.660$$

$$\hat{P}^* - P = -0.020$$

$$\hat{U}^* = 5.584$$

$$\hat{U}^* - U = -0.002$$

marized in APPENDIX A, extensions of Method II beyond the elementary applications illustrated above most certainly seem entirely practical.

VII. CONCLUSION

No basic concept new to fire rating theory has been offered in Sections I-VI, preceding, nor is to be offered in appendices to follow. The substance of the entire development is reformulation and extension of theory previously suggested in forms not only incomplete, but also unfortunately imprecise. In Section VI.C., public fire protection facilities simply are treated as a specific example of the variable "hazard r" earlier discussed in general terms by McIntosh, both from a theoretical and from a practical standpoint.[41] Section VI.D., preceding, and APPENDIX A, to follow, are foreshadowed by an earlier application of the theory of polyhedral sets to the fire schedule rating problem on the mathematically acceptable but actuarially unrealistic assumption that the problem will be essentially linear,[42] which it is not. The probable severity of fire loss contingent upon occurrence is not stochastically independent of the probability of occurrence, whence it will follow that the charges and credits of a fire rating schedule cannot be strictly additive, except as an approximation over a very limited range of variation.

The utility of Eqs.(IV. A.1) and (VI.D.31) is that, taken together, these transformations permit reduction of the problem to linear forms,[43] for which ready-made solutions usually will be available by the theorems of linear algebra.[44]

The tool marks can be polished off of the final product. All equations of Section VI.C. are conventional and somewhat elementary algebraic expressions, and the vector equations of Section VI.D. could be replaced by ordinary simultaneous equations at no sacrifice other than of typographical economy. On the other hand, a proof necessary to the support of Method I, Case 4, is complete in five short matrix equations,[45] whereas it is extremely tedious to prove the same result by ordi-

[41] McIntosh, (15), p. 15, and (16), p. 131. Equation (9), p. 17 of the reference is the forerunner of Eqs. (VI. C. 23 –) presented here. The graphical method of curve adjustment cited without description on p. 20 of the reference is a graphical solution of Eqs.(VI.C. 23 –), though not same method presented in Section VI.C., preceding here.

[42] McIntosh, (14). pp. 140-146, & pp. 150-152. See also Note 39, *sup.*

[43] By permitting immediate introduction of curvilinear coordinates. See APPENDIX A.

[44] See BIBLIOGRAPHY, to follow.

[45] Eqs. (B.9) – (B.13) of APPENDIX B.

nary algebra. Further, although it is easily demonstrated by conventional algebra *why* Method II works *when* it works, it is only in terms of the properties of polyhedral convex sets[46] that the conditions under which solution is possible are expeditiously found. If the properties of Eqs. (VI.D.30) and (VI.D.31) (of which Eq.(A.38) of APPENDIX A is the generalized form) can be completely investigated by ordinary algebra, the operation certainly will be interminable, as will be the problem of distinguishing between Eqs.(IV. A.1) and (VI.D.30), which, though similar in form, are by no means equivalent to each other.[47]

Even so, it must be admitted that Method II can be demonstrated in a fashion much simpler than by the full, formal developments to be found in APPENDIX A. The support of Method II is not, however, the sole reason, nor even the primary reason, for APPENDIX A. When the development there presented was begun (in search of a method to define rate bounds under conditions such that all schedule charges are *not* to be assumed as additive) there was no faint suspicion that Method II would begin to take shape on the work-bench almost immediately; the original concept of Section VI was restricted to Section VI.C., before early drafts of APPENDIX A relegated Method I to by-product status.

The main purpose of APPENDIX A is to submit for evaluation, in all detail, an actuarial research tool which seems of not inconsiderable potential utility. Section 1 of APPENDIX A is intended to stand on its own feet. When the transformations defined in that section are further compounded with the particular transformation defined in Section 2 of APPENDIX A, then Method II is the result. However, other transformations can be grafted onto the development of Section 1 as circumstance may dictate; the transformation of the coefficients of Eq.(IV. A.1) into the parameter vectors of Method II is by no means the only direction the extension of Section 1 could have taken.

Once a transformation is defined, it can then be compounded in almost any desired direction to achieve almost any desired result with a minimum of effort. There is no reason why the transformations F^* and F^k of equations (A.28) and (A.32) must be defined in terms of Eqs.(A.26) except for present purposes only. It would be interesting to see what might result from Eq.(A.32) were F^k defined in terms of those equations "not of a simple rational form" which have caused Messrs. Bailey and Simon

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[46] *Cf.* among others, Kemeny *et al.*, (6). Ch. 5.

[47] See APPENDIX A.

to express a plaintive wish for a "small computer."[48] Were F^k to be so re-defined, very obviously the remaining elements of Eq.(A.32) must be appropriately re-defined also, which might prove difficult or perhaps impossible. But the idea seems worth a try.

A second possible line of research would seem to lie in formal recognition of P and of U as the stochastic variables which actually they are, instead of as the constants which here they are unrealistically assumed to be. It does not seem certain that this line is entirely divorced from the problem attacked by Bailey and Simon (cited above), namely that of determining the best set of classification and sub-classification relativities under a multiple-classification system.

APPENDIX A

1. The Adjusted Rate Structures (Sections I.V. A. & V.)

a. *The Feasible Rate Structure,*

Implicitly by definitions given, for all x, then:

$w_x \geqslant 0; R_x \geq 0;$ if $\boldsymbol{R}^{P:j}$ is P-reconciled, $\boldsymbol{R}_P^{P:j}$, then: $\sum_P w_x R_x{}^j = P.$

If $w_\zeta = 0$, drop the $_\zeta{}^{th}$ term from the summation.[49] Then $w_x > 0$ for all x remaining. It is assumed below that $w_x > 0$ for all $x \leq a$.

Let $R_x{}^j = 0$ for all $x \neq \mu$. Then $R_\mu{}^j = P/w_\mu$; whence it follows that, since never: $R_x{}^j < 0$; then never: $R_\mu{}^j > P/w_\mu$; whence always:

$$(A.1) \qquad 0 \leq R_x{}^j \leq P/w_x; \ (x \leq a)$$

Thence it follows that:

(a) *The set* $\{\mathbf{R}_P{}^P\}$ *is bounded.* It is contained in a hypersphere by virtue of Ineq.(A.1).[50]

(b) *The set* $\{\boldsymbol{R}_P{}^P\}$ *is polyhedral and convex.* It is the intersection of the closed half spaces defined by Ineq.(A.1).[51]

Let $\dot{\boldsymbol{R}}_\zeta{}^P = (0, \ldots, P/w_\zeta, \ldots, 0; 0, \ldots, 0)$, where $\zeta \leq a$. Then the vector $\dot{\boldsymbol{R}}_\zeta{}^P$ is an extreme point (or "extremal vector") of $\{\boldsymbol{R}_P{}^P\}$, whence: If and only if $\boldsymbol{R}^{P:j}$ is a member of $\{\boldsymbol{R}_P{}^P\}$, then:

$$(A.2) \qquad \boldsymbol{R}_P{}^{P:j} = \textstyle\sum_P \mathrm{a}_x{}^{P:j} \dot{\boldsymbol{R}}_x{}^P; \ (\mathrm{a}_x{}^{P:j} \geq 0; \sum_P \mathrm{a}_x{}^{P:j} = 1)$$[52]

[48] Bailey & Simon, (12). Specifically: Section B, pp. 11 & 13.

[49] See Section 3, following.

[50] Taylor, (20). p. 70.

[51] Kemeny *et al.* (6). pp. 340-341.

[52] *Ibid.* p. 347.

The vectors $\dot{R}_x^P$ form a basis for the space S_P of all protected rate vectors R^P, and the vectors $\dot{R}_x^P$ are α in number. Hence S_P is α-dimensional and R^P is α-dimensional.[53]

Eq.(A.2) may be rewritten:

$$\text{(A.3)} \quad R_P^{P:j} = \sum_{x=1}^{\alpha-1} a_x^{P:j} (\dot{R}_x^P - \dot{R}_\alpha^P) + \dot{R}_\alpha^P; \quad (a_x^{P:j} \geqslant 0; \sum_{x=1}^{\alpha-1} a_x^{P:j} \leq 1)$$

whence $\{R_P^P\}$ is spanned by the $(\alpha - 1)$ linearly independent vectors $(\dot{R}_x^P - \dot{R}_\alpha^P)$, hence $\{R_P^P\}$ is an $(\alpha - 1)$-dimensional[54] affine[55] subset of S_P.

By exact analogy to the foregoing:

$$\text{(A.4)} \qquad 0 \leq R_x^k \leq U/w_x; \quad (x \geq \beta)$$

The set of all U-reconciled rate vectors, $\{R_U^U\}$, is a $(z - \beta)$-dimensional affine subset of the space S_U of all unprotected rate vectors.

The set $\{R_U^U\}$ is a bounded, polyhedral convex set having as extreme points the $(z - \beta)$ vectors $\dot{R}_\mu^U = (0, \ldots 0; 0, \ldots, U/w_\mu, \ldots, 0)$, where $\mu \geq \beta$, whence: If and only if $R^{U:k}$ is a member of $\{R_U^U\}$, then:

$$\text{(A.5)} \qquad R^{U:k} = \textstyle\sum_U a_x^{U:k} \dot{R}_x^U; \quad (a_x^{U:k} \geq 0; \sum_U a_x^{U:k} = 1)$$

Any feasible vector, $\tilde{R}^i$, may be written uniquely as: $\tilde{R}^i = R_P^{P:j} + R_U^{U:k}$, whence it follows that the feasible rate structure, $\{\tilde{R}\}$ is the direct sum of $\{R_P^P\}$ and $\{R_U^U\}$:

$$\text{(A.6)} \qquad \{\tilde{R}\} = \{R_P^P\} \oplus \{R_U^U\}$$

whence the dimension of $\{\tilde{R}\} = (\alpha - 1) + (z - \beta) = z - 2$.[56]

Equation (IV. A.1) follows from Eqs.(A.2), (A.5) and (A.6).

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[53] Birkhoff & MacLane, (1). pp. 168-169 & 188. It may be noted that this basis is orthogonal.

[54] *Ibid.* pp. 164 & 168-169. Designation of the initial point with subscript "α" is arbitrary here. The usual designation of the initial point is with subscript "zero," but in present instance this would require re-numbering of the vectors, any one of which could have been chosen as initial point.

[55] *Ibid.* p. 291.

[56] *Ibid.* p. 185. (The direct sum is denoted by "$\dot{+}$" in the reference (see p. 472), but "$\oplus$" seems a more common symbol.)

Let:

$$B = \begin{bmatrix} P/w_1 & \dots & 0 & 0 & \dots & 0 \\ \dots & \ddots & \dots & \ddots & \dots \\ 0 & \dots & P/w_a & 0 & \dots & 0 \\ 0 & \dots & 0 & U/w_\beta & \dots & 0 \\ \dots & \ddots & \dots & \ddots & \dots \\ 0 & \dots & 0 & 0 & \dots & U/w_z \end{bmatrix} = \text{The ``\textit{basis matrix}.''}$$

$$i = j = k;\ \text{and:}\ Y = \begin{cases} P;\ (x \le a) \\ U;\ (x \ge \beta) \end{cases}$$

$a_x^{Y:i} = a_x^{i}$; and $\mathbf{a}^i = a_1^i, \dots, a_z^i) =$ A *"primary coefficient vector."*

Then Eq.(IV. A.1) my be written:

(A.7)[57] $\tilde{R}^i = \tilde{\mathbf{a}}^i B;\ (a_x^i \ge 0;\ \sum_P a_x^i = 1;\ \sum_U a_x^i = 1)$

Let:

$$W = \begin{bmatrix} w_1 & 0 \\ \dots & \dots \\ w_a & 0 \\ 0 & w_\beta \\ \dots & \dots \\ 0 & w_z \end{bmatrix} = \text{The ``\textit{weighting matrix}.''}$$

$Y^* = (P\ ;\ U) =$ The *"target vector"*

$\hat{Y}^i = (\hat{P}^i;\ \hat{U}^i) =$ A *"trial average vector."*

Then in general:

(A.8) $R^i W = \mathbf{a}^i B W = \hat{Y}^i;\ (a_x^i \ge 0;\ \sum_P a_x^i = 1;\ \sum_U a_x^i = 1)$ and in particular:

[57] The validity of Eq.(A.7) depends upon the symmetry of the matrix B. The equation is not general for arbitrary choice of basis vectors.

(A.9) $\tilde{R}^i W = \tilde{a}^i B W = \hat{Y}^i = Y^*$; $(a_x^i \geq 0; \sum_P a_x^i = 1; \sum_U a_x^i = 1)$

Where $\tilde{a}^i$ is defined by the condition that $a^i = \tilde{a}^i$ if and only if $R^i = \tilde{R}^i$.

Let B denote the linear transformation whose matrix is $\boldsymbol{B}$, and W denote the linear transformation whose matrix is $\boldsymbol{W}$.

Then:

(A.10) $$a^i B = R^i$$

and the transformation, B, is one-one.[58]

Also:

(A.11) $$R^i W = a^i B W = \hat{Y}^i$$

b. *The Operational Rate Structure, op* $\{\tilde{R}\}$.

By Eqs. (IV. A.1.a) and definition of f_x:

(A.12)
$$f_x^i = (a_x^{P:i} w_{x+1} P)/(a_{x+1}^{P:i} w_x P); \ (x < \alpha)$$
$$f_\alpha^i = (a_\alpha^{P:i} w_\beta P)/(a_\beta^{U:i} w_\alpha U)$$
$$f_x^i = (a_x^{U:i} w_{x+1} U)/(a_{x+1}^{U:i} w_x U); \ (x \geq \beta)$$

whence:

(A.13) $$\underline{f}_x a_{x+1}^i w_x / w_{x+1} \leq a_x^i \leq \bar{f}_x a_{x+1}^i w_x / w_{x+1}; \ (x \neq \alpha)$$

(A.14) $$\underline{f}_\alpha a_\beta^i w_\alpha U / w_\beta P \leq a_\alpha^i \leq \bar{f}_\alpha a_\beta^i w_\alpha U / w_\beta P$$

whence Eq. (IV. A.3) follows from Eq. (IV. A.1) upon imposition of Constraint (1): $0 < \underline{f}_x \leq f_x^i \leq \bar{f}_x < r_x / r_{x+1}$.

By Eqs. (A.2) and (A.5), and by Ineq. (A.12)

(A.15)
$$opR_P^{P:i} = a^{P:i} B$$
$$(\underline{f}_x a_{x+1}^{P:i} w_x / w_{x+1} \leq a_x^{P:i} \leq \bar{f}_x a_{x+1}^{P:i} w_x / w_{x+1}; \sum_P a_x^{P:i} = 1)$$

(A.16)
$$opR_U^{U:i} = a^{U:i} B$$
$$(\underline{f}_x a_{x+1}^{U:i} w_x / w_{x+1} \leq a_x^{U:i} \leq \bar{f}_x a_{x+1}^{U:i} w_x / w_{x+1}; \sum_U a_x^{U:i} = 1)$$

---

[58] Birkhoff & MacLane, (1). p. 121. To avoid notational confusion, see paragraph near the top of the page, beginning: "In the choice of notation for transformations***." The present author will follow Birkhoff & MacLane in writing the point under transformation to the *left* of the transformation symbol.

where, by analogy to definitions of $\boldsymbol{R}^P$ and $\boldsymbol{R}^U$, let:

$$\mathbf{a}^P = (a_1^P, \ldots, a_\alpha^P; 0, \ldots, 0)$$
$$\mathbf{a}^U = (0, \ldots, 0; a_\beta^U, \ldots, a_z^U)$$

By Eqs. (IV. A.3), (A.7), (A.14) and (A.15):

(A.16.a) $op\tilde{\boldsymbol{R}}^i = op\boldsymbol{R}_P^{P:i} + op\boldsymbol{R}_U^{U:i}; \quad (\underline{f}_\alpha a_\beta{}^i w_\alpha U/w_\beta P \leq a_\alpha{}^i \leq \bar{f}_\alpha{}^i a_\beta{}^i w_\alpha U/w_\beta P)$

Note that Ineq. (A.14), appearing as a constraint in Eqs. (IV. A.3) and (A.16.a) does *not* appear either in Eq. (A.15) or in Eq. (A.16). The ratio $f_\alpha{}^i$ is a "link," so to speak, *between* $op\{\boldsymbol{R}_P^P\}$ and $op\{\boldsymbol{R}_U^U\}$; it is not associated exclusively with either subset of the direct sum, $op\{\tilde{\boldsymbol{R}}\}$. It is to be demonstrated that $op\{\tilde{\boldsymbol{R}}\}$ may be the empty set. Let: $\tilde{\mathbf{a}}^i = {}_{op}\tilde{\boldsymbol{a}}^i$ if $a_x{}^i$ conforms to Ineq.(A.13) for all $x \neq \alpha$; *and*: $a_\alpha{}^i$ conforms to Ineq.(A.14). Then by Eq.(A.7):

$$op\tilde{\boldsymbol{R}}^i = {}_{op}\tilde{\mathbf{a}}^i\boldsymbol{B} \qquad \text{(A.17)}$$

Let a *"ratio vector", $\boldsymbol{f}^i$*, be defined by:

$$\boldsymbol{f}^i = (f_1{}^i, \ldots, f_{\alpha-1}{}^i; f_\alpha{}^i; f_\beta{}^i, \ldots, f_{z-1}{}^i)^{59}$$

Then the inner inequalities of Constraint (1): $\underline{f}_x \leq f_x \leq \bar{f}_x$, define a bounded, polyhedral convex set, $op\{\boldsymbol{f}\}$, the extreme points of which are:

$$\boldsymbol{f}^\phi = (f'_1, \ldots f'_{\alpha-1}; f'_\alpha; f'_\beta, \ldots, f'_{z-1}); \quad (f'_x = \underline{f}_x \text{ or } \bar{f}_x)$$

The number of extreme points of $op\{\boldsymbol{f}\}$ is 2^{z-1}, and the set is $(z-2)$-dimensional.[60]

Now, the ratio notation, $f_{\zeta:\mu} = R_\zeta / R_{\mu+1}$, adopted for convenience in Section V.C., may obscure the development henceforward. Returning to conventional product notation and simplifying, Eqs.(V.C.15) and (V.C.16) may be rewritten:

$$f_\alpha{}^i \prod_{x=\beta}^{z-1} f_x \frac{\sum_P (w_x \prod_{(1)})}{\sum_U (w_x \prod_{(2)})} = \frac{\hat{P}^i}{\hat{U}^i} = \hat{p}^i \qquad \text{(A.18)}$$

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[59] The artificial ratio $f_z = 1$, defined in Section V.C. for notational convenience only, may be introduced as the z^{th} component of $\boldsymbol{f}^i$ if desired, but this is not necessary.

[60] The demonstration is analogous to the demonstration by Eqs.(A.2) and (A.3) that $\{\boldsymbol{R}_P{}^P\}$ is $(\alpha - 1)$-dimensional.

(A.19-a) $$R_\zeta{}^i = \frac{\prod_{(1)}\hat{P}^i}{\sum_P(w_x\prod_{(1)})} = R_\zeta(f^i;\hat{P}^i)$$

$(\zeta \leq a)$

-b) $$= \frac{f_a{}^i\prod_{(1)}\prod_{(3)}\hat{U}^i}{\sum_U(w_x\prod_{(2)})} = R_\zeta(f^i;\hat{U}^i)$$

(A.20-a) $$R_\mu{}^i = \frac{\hat{P}^i}{f_a\prod_{(3)}\sum_P(w_x\prod_{(1)})} = R_\mu(f^i;\hat{P}^i)$$

$(\mu \geq \beta)$

-b) $$= \frac{\prod_{(2)}\hat{U}^i}{\sum_U(w_x\prod_{(2)})} = R_\mu(f^i;\hat{U}^i)$$

where:

$$\prod_{(1)} = \begin{cases} \prod_{x=\zeta}^{a-1} f_x \text{ ; if: } \zeta \leq a-1 \\ 1 \quad \text{ ; if: } \zeta = a \end{cases}$$

$$\prod_{(2)} = \begin{cases} \prod_{x=\mu}^{z-1} f_x \text{ ; if: } \beta \leq \mu \leq z-1 \\ 1 \quad \text{ ; if: } \mu = z \end{cases}$$

$$\prod_{(3)} = \prod_{x=\beta}^{\mu} f_x \text{ ; if: } \beta \leq \mu \leq z-1$$

and Eqs.(A.19-) and (A.20-) may be consolidated into:

(A.21) $$\boldsymbol{R}^{i:j} = \boldsymbol{R}(f^i;\hat{\boldsymbol{Y}}^j)$$
$$= (\ldots, R_\zeta(f^i;\hat{\boldsymbol{Y}}^j), \ldots; \ldots, R_\mu(f^i;\hat{\boldsymbol{Y}}^j), \ldots)$$

where possibly, but not necessarily, $j = i$. It does not follow that if $\hat{\boldsymbol{Y}}^j = \hat{\boldsymbol{Y}}^i$, then necessarily $f^j = f^i$.

Define a set $\{\bar{f}\}$ of *"feasible ratio vectors,"* $\bar{f}^j$ by the condition that if and only if $\bar{f}^j$ is a member of $\{\bar{f}\}$, then by Eq.(A.21) necessarily: $\hat{\boldsymbol{Y}}^i = \boldsymbol{Y}^*$. By the previously-given definition of $op\{f\}$, it follows that if f^i is a member of $op\{f\}$, then $\boldsymbol{R}^{i:j}$ will necessarily conform to Constraint (I) but will not necessarily be feasible. By this definition it follows that if f^i is a member of $\{\bar{f}\}$, then $\boldsymbol{R}^{i:j}$ necessarily will be feasible but will not necessarily conform to Constraint (I). Therefore, if a set $op\{\tilde{f}\}$ be defined as the intersection of $op\{f\}$ and $\{\tilde{f}\}$, then necessarily $R^{i:j}$ will conform to Constraint (I) *and* will be feasible. Hence, if f^i is a member of $op\{\tilde{f}\}$, then: $R^{i:j} = opR^{i:*} = op\tilde{R}^i$, where the feasible tilde, "$\sim$". now replaces the second superscript.

If $\tilde{f}$ is a member of $op\{\tilde{f}\}$, let $\tilde{f} = op\tilde{f}$; and if $\tilde{R}^i = op\tilde{R}^i$, let $\tilde{\mathbf{a}}^i = {}_{op}\tilde{\mathbf{a}}^i$. Thence by Eqs. (A.7) – (A.9) and (A.21):

In general:

(A.22) $$R(f^k; \hat{Y}^j) = R^{k:j} = \mathbf{a}^{k:j} B$$

and in particular:

(A.23) $$R(opf^i; Y^*) = op\tilde{R}^i = {}_{op}\tilde{a}^i B$$

It follows from Eqs. (A.8) and (A.9) that a set of *"secondary coefficient vectors,"* $\{\tilde{b}\}$, will exist such that:

(A.24) $$\textstyle\sum_j b_j{}^k (\hat{Y}^j; \mathbf{a}^{k:j}) = (Y^*; \mathbf{a}^{k:*}) = (Y^*; \tilde{\mathbf{a}}^k)$$

where $b_j{}^k$ is the j^{th} component of the vector $\tilde{b}^k$; and it follows further that $\{\tilde{b}\}$ will be the solution set of the system of simultaneous equations:

(A.25) $$\textstyle\sum_j b_j{}^k \hat{P}^j = P \qquad \sum_j b_j{}^k \hat{U}^j = U$$

Hence a feasible rate vector, $\tilde{R}^k$, always may be obtained from two or more estimated trial coefficient vectors, $\mathbf{a}^{k:\gamma}$ $\mathbf{a}^{k:\delta}$, ... This is no guarantee, however, that $\tilde{R}^k$ will conform to Constraint (I), and if Constraint (I) may be violated, then $\tilde{R}^k$ can be obtained directly and with less effort by Method I (Section VI.C. and APPENDIX B), from any one of the trial vectors, $R^{k:\gamma}$, $R^{k:\delta}$, etc., particularly if R^e may be taken as $R^{k:\gamma}$. But if Constraint (I) may not be discarded, or if for some one or more classes, the final adjusted rate, $R_x{}^*$, is to assume a predetermined value, $R_x{}^o$, then Method I will not give $\tilde{R}^*$ directly, except by coincidence or after lengthy trial and error to determine an appropriate trial vector.

By Eqs.(V. A.1.a.), (A.12) and (A.20-b),[61] it will follow in straightforward fashion that the components, a_x^i, of any feasible primary coefficient vector, $\bar{a}^i$, will be given by:

(A.26-a) $$a_z^i = \frac{w_z}{\sum_U (w_x \prod_{(z)})}$$

-b) $$a_\mu^i = \prod_{(z)} a_z^i \left(\frac{w_\mu}{w_z}\right); \quad (\beta \leq \mu \leq z-1)$$

-c) $$a_a^i = f_a^i \prod_{x=\beta}^{z-1} f_x^i a_z^i \left(\frac{w_a}{w_z}\right)\left(\frac{U}{P}\right)$$

-d) $$a_\zeta^i = \prod_{x=\zeta}^{z-1} f_x^i a_z^i \left(\frac{w_a}{w_z}\right)\left(\frac{U}{P}\right); \quad (\zeta \leq a-1)$$

and Eqs.(A.26-) may be consolidated into:

(A.27) $$\bar{\mathbf{a}}^i = \mathbf{a}(\bar{f}^i; \mathcal{P}^*)$$

Equation (A.27) defines a one-one transformation:[62]

$F^*: \bar{f}^i \rightarrow \bar{\mathbf{a}}^i$. The transformation F^* serves to introduce curvilinear coordinates,[63] whereby the linear transformation of Eq.(A.10), $B: \bar{\mathbf{a}} \rightarrow \bar{\boldsymbol{R}}$ may be substituted in calculation for the non-linear transformation of $\{\bar{f}\}$ onto $\{\bar{\boldsymbol{R}}\}$ defined by Eq.(A.21) when $\hat{\boldsymbol{Y}}^i = \boldsymbol{Y}^*$. By Eq.(A.27):

(A.28) $$\bar{f}^i F^* = \bar{\mathbf{a}}^i$$

whence by Eqs.(A.10) and (A.11):

(A.29) $$(\bar{f}^i F^*)B = \bar{\mathbf{a}}^i B = \bar{\boldsymbol{R}}^i$$
$$(\bar{f}^i F^*)BW = \bar{\mathbf{a}}^i BW = \boldsymbol{Y}^*$$

To establish the validity of Eqs.(A.28) and (A.29), it is sufficient to

[61] The practical reasons for selecting Eq.(A.20-b) specifically from among the four equations, Eqs.(A.19-) and (A.20-), will become apparent in Section 3, to follow.

[62] Despite the formidable appearance of the function $\mathbf{a}(f^i; \mathcal{P}^*)$, the demonstration that F^* is one-one, is very easy. If Eqs.(A.26-b) – (A.26-d) are expanded by substitution of the value of a_z^i from Eq.(A.26-a), and a system of simultaneous equations is set up from the recursion formulas obtained by solving Eqs.(A.12) for a_x^i in terms of a_{x+1}^i, then all denominators cancel out immediately, and the rest will follow in simple and straightforward fashion.

[63] Kaplan, (5) pp. 96 & 151; but see also pp. 132 *ff*. For any fixed value of $\hat{\mathcal{P}}^i = \hat{P}^i / \hat{U}^i$, the several ratios f_x^i are functionally dependent.

prove that F^* is one-one; and see *Note* 62, preceding. It is *not* necessary (fortunately) to define the matrix of F^*, thence to proceed by Eq.(A.7) and (A.9).

Upon substitution of $\hat{U}^j/\hat{P}^j = 1/\hat{\mathcal{P}}^j$ for $U/P = 1/\mathcal{P}^*$ in Eqs.(A.26-c) and (A.26-d),[64] the complete generalization of Eqs.(A.27) – (A.29) follows by exact analogy to the development of those equations, whence:

(A.30) $$\mathbf{a}^{k:j} = \mathbf{a}(f^k;\ \hat{\mathcal{P}}^j)$$

(A.31) $$f^k F^j = \mathbf{a}^{k:j}$$

(A.32) $$(f^k F^j)BW = \mathbf{a}^{k:j}BW = \hat{\mathbf{Y}}^j$$

In particular, if $f^k = f^\phi =$ an extreme point of $op\{f\}$, then

(A.33) $$\mathbf{a}^{\phi:\phi} = \mathbf{a}^\phi = \mathbf{a}(f^\phi; \mathcal{P}^\phi)$$

where $\mathcal{P}^\phi$ is calculated by Eq.(A.18), letting $f^i = f^\phi$; and where arbitrarily by convention it is required that $\mathcal{P}$ carry the superscript of f, regardless of the fact that possibly $\mathcal{P}^\phi = \mathcal{P}^\psi$ where $f^\phi \neq f^\psi$.

By Eq.(A.33), the extreme points of $op\{f\}$ are mapped in one-one correspondence onto the vectors $\mathbf{a}^\phi$. Extending to any value of k the convention that $\mathcal{P}$ always must carry the superscript of f, then all remaining points, f^k of the set $op\{f\}$ are mapped by Eq.(A.30) onto the vectors $\mathbf{a}^k$.

Thus there exists a family of transformations, $\{F^k\}$, whereby every f^k belonging to $op\{f\}$ is mapped in one-one correspondence into a coefficient vector, $\mathbf{a}^k$; where possibly but not necessarily, $k = \phi$. By the definition of $op\{f\}$ and the derivation of Eq.(A.30) it follows that any $\mathbf{a}^k$ will conform to Constraint (I) if $\mathbf{a}^k = f^k F^k$ and f^k is a member of $\{f\}$. Let the set of all such coefficient vectors, $\mathbf{a}^k$, be designated $c\{\mathbf{a}\}$.

It can be shown that $c\{\mathbf{a}\}$ is a bounded, polyhedral convex set whose extreme points are the vectors $\mathbf{a}^\phi = f^\phi F^\phi$.[65] Thence it follows that any

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[64] Justified by Eqs.(A.19-) and (A.20-), and by the formal similarity to Eqs. (IV.A.1.a) of the equations: $R_x{}^i = a^{P:i}\hat{P}^i/w_x$, where $x \leq \alpha$; and: $R_x{}^i = a_x{}^{U:i}\hat{U}^i/w_x$, where $x \geq \beta$. The substitution is equivalent to re-definition of the basis vectors in terms of $\hat{P}^i$ and $\hat{U}^i$, though the operation is not identical in concept to a change of basis.

[65] The fact that the set is bounded and convex follows immediately from the facts that $op\{f\}$ is bounded; and that, by its form, $a(f; \mathcal{P})$ is a continuous function. The rest will follow from Eqs.(A.18) and (A.30) by the use of Lagrange multipliers. (Kaplan, (5). p. 128 *ff*.)

$\mathbf{a}^k$ which is a member of $c\{\mathbf{a}\}$ will be given by:

(A.34) $$\mathbf{a}^k = \Sigma_\phi b_\phi{}^k \mathbf{a}^\phi; \quad (b_\phi{}^k \geq 0; \ \Sigma_\phi b_\phi{}^k = 1)$$

Thence it follows that, if by Eq. (A.24):

(A.35) $$\Sigma_\phi b_\phi{}^k (\hat{\mathbf{Y}}^\phi; \mathbf{a}^\phi) = (\mathbf{Y}^*; \tilde{\mathbf{a}}^k); \quad (b_\phi{}^k \geq 0; \ \Sigma_\phi b_\phi{}^k = 1)$$

then necessarily: $\tilde{a}^k = {}_{op}\tilde{a}^k$; whence the rate vector $\widetilde{opR}^k = {}_{op}\tilde{a}^k \boldsymbol{B}$ must be feasible and also must conform to Constraint (I); whence the rate revision problem is solved.

Since by Eq.(A.7), then: $a_x{}^k \geq 0$; $\Sigma_P a_x{}^k = 1$; $\Sigma_U a_x{}^k = 1$; then the system of ordinary simultaneous equations equivalent to Eq.(A.35) need never contain more than z rows, the first two of which give the target rates, U and P. The remaining rows may be formulated to contain as the constant terms not more than $(z - \beta)$ pre-selected values, $a_x{}^{U:o} = w_x R_x{}^o / U$, of the coefficients $a_x{}^{U:k}$, plus $(\alpha - 1)$ values, $a_x{}^{P:o} = w_x R_x{}^o / P$, of the coefficients $a_x{}^{P:k}$,[66] where $R_x{}^o$ is a pre-selected value of $R_x{}^k$.

A solution to Eq.(A.35) always must exist, since, as noted, there will be 2^{z-1} extreme points, f^ϕ, of $op\{f\}$, and hence there will be 2^{z-1} choices among the vectors a^ϕ from which to select at least $(z + 1)$ vectors to give a system of not more than z equations in not less than $z + 1$ unknown secondary coefficients, $b_\phi{}^k$. However, a *non-negative* solution may not exist. By the form of Eq.(A.18) (remembering that for all x, then $f_x > 0$ by Constraint (I), and $w_x > 0$ by hypothesis) it follows that $p^\psi = \max\{p^\phi\}$ when $f_x{}^\psi = \bar{f}_x$ for all x, and $p^\psi = \min\{p^\phi\}$ when $f_x{}^\psi = f_x$ for all x. Thence it will follow that if $p^* = P/U > \max\{p^\phi\}$ or $p^* < \min\{p^\phi\}$, then necessarily $b_\phi{}^k < 0$ for at least one ϕ in the solution of Eq.(A.35); whence $\mathbf{a}^k = \Sigma_\phi b_\phi{}^k \mathbf{a}^\phi$ will not belong to $c\{\mathbf{a}\}$ and hence will not conform to Constraint (I).

If $\min\{p^\phi\} \leq p^* \leq \max\{p^\phi\}$, a non-negative solution to Eq.(A.35) will exist, which will be given by some combination of not more than z of the 2^{z-1} vectors $(\hat{\mathbf{Y}}^\phi; \mathbf{a}^\phi)$. It will be a unique solution if $p^* = \max\{p^\phi\}$ or $p^* = \min\{p^\phi\}$. This follows from the properties of $c\{\mathbf{a}^k\}$ as a bounded, polyhedral, convex set.

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[66] See Subsection a., preceding. The nature of $\{\tilde{\boldsymbol{R}}\}$ as the direct sum of $\{\boldsymbol{R}_P{}^P\}$ and $\{\boldsymbol{R}_U{}^U\}$ precludes the selection of more than $(z - \beta)$ of the $a_x{}^{U:k}$; or of more than $(\alpha - 1)$ of the $a_x{}^{P:k}$.

2. *Rate Calculation Method II.*

For purposes of practical application, Eq.(A.35) is greatly simplified by one final transformation, $G^k: \mathbf{a}^k \rightarrow N^k$; where N^k is an *"abstract rate vector."* Let:

$$\hat{\boldsymbol{\gamma}}^i = \hat{Y}^i/\hat{U}^i = (\hat{\mathfrak{p}}^i; 1); \; \boldsymbol{\gamma}^* = Y^*/U = (\mathfrak{p}^*; 1)$$

and let the matrix of G^k be:

$$G^k = \begin{bmatrix} \hat{\mathfrak{p}}^k/w_1 & \cdots & 0 & 0 & \cdots & 0 \\ \cdots & \ddots & \cdots & & \ddots & \cdots \\ 0 & \cdots & \hat{\mathfrak{p}}^k/w_\alpha & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 1/w_\beta & \cdots & 0 \\ \cdots & \ddots & \cdots & & \ddots & \cdots \\ 0 & \cdots & 0 & 0 & \cdots & 1/w_z \end{bmatrix}$$

where if $\hat{\mathfrak{p}}^k = \mathfrak{p}^*$, then $G^k = G^*$

Then:

(A.36) $\quad (\mathbf{a}^k G)W = N^k W = \hat{\boldsymbol{\gamma}}^k$; (if $\mathbf{a}^k = \tilde{\mathbf{a}}^k$; then $\hat{\boldsymbol{\gamma}}^k = \boldsymbol{\gamma}^*$)

To prove Eq.(36), by definition of G:

(A.37)
$$\sum_{P} w_x N_x^k = \sum_{P} w_x \mathrm{a}_x^k \hat{\mathfrak{p}}^k / w_x = \sum_{P} \mathrm{a}_x^k \hat{\mathfrak{p}}^k = \hat{\mathfrak{p}}^k$$
$$\sum_{U} w_x N_x^k = \sum_{U} w_x \mathrm{a}_x^k / w_x = \sum_{U} \mathrm{a}_x^k = 1$$

whence Eq.(A.36) follows immediately. Equation (A.36) simply is the matrix form of Eqs.(A.37) with the first and second members transposed.

Thence by analogy to Eq.(A.35):

(A.38) $\quad \sum_\phi b_\phi^k (\hat{\boldsymbol{\gamma}}^\phi; N^\phi) = (\boldsymbol{\gamma}^*; \tilde{N}^k)$; $\quad (b_\phi^k \geq 0)$, where $N^\phi = a^\phi G^\phi$.

Since by definitions of $\hat{\boldsymbol{\gamma}}^\phi$ and of $\boldsymbol{\gamma}^*$, the second equation of the System (A.38) always will be $\sum_\phi b_\phi^k = 1$, it is not necessary to include this equation in the constraints.

The discussion following Eq.(A.35), concerning the existence of solutions thereto, applies in its entirety to Eq.(A.38).

It follows from the definition of $\boldsymbol{N}^{\phi}$ that if a non-negative solution to Eq.(A.38) exists, then $\tilde{\boldsymbol{N}}^k$ conforms to Constraint (I), and hence may be written: $op\tilde{\boldsymbol{N}}^k$. And by Eq.(A.36) and definition of $\boldsymbol{\Upsilon}^*$:

$$\text{(A.39)} \qquad U(op\tilde{\boldsymbol{N}}^k)\boldsymbol{W} = U\boldsymbol{\Upsilon}^* = \boldsymbol{Y}^*$$

Thence it follows that if $\boldsymbol{R}^k = U(op\tilde{\boldsymbol{N}}^k)$, then necessarily: $\boldsymbol{R}^k = op\tilde{\boldsymbol{R}}^k$; whence Method II follows immediately.

Letting $N_x{}^o = R_x{}^o/U$, where $R_x{}^o$ is a pre-selected value of $R_x{}^k$, then up to $(\alpha - 1)$ values of $N_x{}^o$ for $x \leq \alpha$, plus up to $(z - \beta)$ values for $x \geq \beta$, may be entered as the constant terms of the system of Eq.(A.38). Also, pre-selected ratios may be substituted for preselected rates, on a one-for-one exchange of choice, *i.e.*, if $f_\zeta{}^o$ is chosen, where $\zeta \leq \alpha$, then only $(\alpha - 2)$ of the protected rates may be pre-selected. If a ratio $f_x{}^o$ is pre-selected, the product $f_x{}^oN_{x+1}{}^k$ is substituted for $N_x{}^k$ in the equation for $N_x{}^k$.

The nature of the entire foregoing development from Eq.(A.7) forward now may be indicated by the compound-transformation equation:

$$\text{(A.40)} \qquad U(f^k\mathsf{F}^k\mathsf{G}\mathsf{B}\mathsf{W}) = Uf^k\mathsf{T}^k = \hat{\boldsymbol{Y}}^k$$

where $\mathsf{T}^k = \mathsf{F}^k\mathsf{G}^k\mathsf{B}\mathsf{W}$; and if $f^k = opf^k$ and $\mathsf{F}^k = \mathsf{F}^*$, then $\hat{\boldsymbol{Y}}^k = \boldsymbol{Y}^*$.

Once the transformations have been appropriately defined, the rest follows in straightforward fashion.

3. *Practical Considerations.*

It follows from the "abstract"[67] nature of $\boldsymbol{P}^{\phi}$ and $\boldsymbol{N}^{\phi}$ that once calculated, the values may be stored and used in the course of successive rate adjustments over a period of years. Re-calculation of these parameters is required only following significant change in the distribution of sums insured among the classes, revision of the original estimates of $\underline{f}_x$ and $\bar{f}_x$ or revision of the classification system itself.

Throughout the formal development, the fixed class numbers, α, β, z, defined in Section II.D as indices, have been used also as parameters, *e.g.* in stating that $\{\tilde{\boldsymbol{R}}\}$ is "$(z - 2)$-dimensional". So long as $w_x > 0$

[67] A more appropriate term here would be "dimensionless," in the sense that a trigonometric sine is a "dimensionless" ratio; but since the abstract vector $\boldsymbol{N}$ is "z-dimensional" in a mathematical sense; the term "abstract" is used to avoid semantic difficulties.

for all $x = 1, 2, \ldots, z$, this presents no problem. In such instance, "a" not only designates a class, but also equals the number of protection classes within the "Protected" statistical class, etc., but if $w_x = 0$ for any x, as in Section VI where all calculations contemplate $w_1 = 0$, then z will not indicate the number of classes as well as the designation of the highest numbered class; and in reference to the incomplete system of Section VI, the feasible rate structure, $\{\tilde{R}\}$, is *not* $(z-2)$-dimensional, but is $(z-3)$-dimensional. Although $z = 10$ still designates, as an index, the highest-numbered class, the number of classes is not $z = 10$ but is $z - 1 = 9$, etc. In practical application, general expressions in which the fixed class numbers, a, β and z, represent the number of classes, rather than indices designating particular classes, must be modified according to circumstances where $w_x = 0$ for one or more of the classes included in the system currently involved.

If $w_\eta = 0, \ldots, w_\theta = 0$, for one or for two or more consecutive class numbers, $\eta, \ldots \theta$, but $w_\zeta > 0$ *and* $w_\mu > 0$, where $\zeta = \eta - 1$ and $\mu = \theta + 1$, a further modification must be made in all expressions involving c_x and f_x to avoid distortion of the results of practical calculations. In such instance, for:

c_ζ; substitute: $d_\zeta = Q_\zeta/Q_\mu$
f_ζ; " : $g_\zeta = R_\zeta/R_\mu$
$\underline{c}, \bar{c}$; " : $\underline{d}, \bar{d}$
$\underline{f}, \bar{f}$; " : $\underline{g}, \bar{g}$

and "$\bar{g} < r_\zeta/r_\mu$" replaces "$\bar{f} < r_x/r_{x+1}$" in Constraint (I).

The choice of Eq.(A.20-b) from which to develop Eqs.(A.26-) rests upon the fact that normally there will be not more than two, or at most three, unprotected classes, *vs.* at least three, and probably six to eight protected classes. Hence in practice, the denominators will be simpler if either Eq.(A.20-b) or Eq.(A.19-b) is used in preference to the other choices. Of these two, choice of Eq.(A.20-b) results in the simplest form of the recursion equations, Eqs.(A.26-), which in turn simplifies the formulas for pre-calculation of the several N^ϕ. In particular cases, it may prove expedient to choose Eq.(A.20-a) or one of Eqs.(A.19-). Theoretically, it makes no difference whatever in the final result, whichever of the four possibilities may be chosen; it is not even necessary that the same one of the four equations, Eqs.(A.19-) and (A.20-) be chosen to calculate each of the several coefficients a_x^k in turn.

APPENDIX B

RATE CALCULATION METHOD I

(SECTION VI. C.)

By rearrangement of the standard "two-point" formula, the equation of a line through the points $(x_P; \hat{P}^i)$ and $(x_U; \hat{U}^i)$ will be:

$$\text{(B.1)} \qquad L_x{}^i = x\frac{\hat{U}^i - \hat{P}^i}{x_P - x_U} + \frac{x_U\hat{P}^i - x_P\hat{U}^i}{x_P - x_U}$$

where $x_P = \sum_P x\, w_x$ and $x_U = \sum_U x\, w_x$. It follows by straightforward algebra that if x is restricted to integral values, then:

$$\text{(B.2)} \qquad \textstyle\sum_P w_x L_x{}^i = \hat{P}^i; \text{ and: } \sum_U w_x L_x{}^i = \hat{U}^i$$

When $\hat{P}^i = P$ and $\hat{U}^i = U$, let $L_x{}^i = \tilde{L}_x$ in Eqs.(B.1) and (B.2).

It follows immediately from the definition of $\hat{P}^i$ and $\hat{U}^i$, and from Eqs.(B.2), that:

$$\text{(B.3)} \qquad \textstyle\sum_P w_x(R_x{}^i - L_x{}^i) = 0; \text{ and: } \sum_U w_x(R_x{}^i - L_x{}^i) = 0$$

Let:

$$\text{(B.4)} \qquad R_x{}^* = \frac{U - P}{\hat{U}^i - \hat{P}^i}(R_x{}^i - L_x{}^i) + \tilde{L}_x$$

then by Eqs.(B.2) and (B.3):

$$\text{(B.5)} \qquad \begin{aligned} \sum_P w_x R_x{}^* &= \frac{U - P}{\hat{U}^i - \hat{P}^i}\sum_P w_x(R_x{}^i - L_x{}^i) + \sum_P w_x\tilde{L}_x \\ &= \sum_P w_x\tilde{L}_x = P \\ \sum_U w_x R_x{}^* &= \sum_U w_x\tilde{L}_x = U \end{aligned}$$

whence $\tilde{\boldsymbol{R}}^*$ as defined by Eq.(B.4) is feasible.

Substituting into Eq.(B.4) the values of $L_x{}^i$ and $\tilde{L}_x$, and simplifying, it follows immediately that:

$$\text{(B.6)} \qquad R_x{}^* = R_x{}^i\,\frac{U - P}{\hat{U}^i - \hat{P}^i} + \frac{P\hat{U}^i - U\hat{P}^i}{\hat{U}^i - \hat{P}^i}$$

It should be noted that (unless $\hat{U}^i - \hat{P}^i = 0$, in which case the problem is degenerate) the actual values of the trial averages, $\hat{P}^i$ and $\hat{U}^i$

are completely immaterial. Any trial rate vector whatever will be transformed into a feasible vector by Eq.(B.6). Upon substitution of R_x^e for R_x^i, etc., Eq.(VI. C.23.a) is obtained.

Eq.(VI. C.23.b) follows by exact formal analogy to the above derivation of Eq.(VI. C.23.a).

Now let $\boldsymbol{R}^*$ be feasible by hypothesis. Then by definition of q_x and by Eq.(VI. C.23.a):

(B.7) $$q_x R_x^* = q_x R_x^e \frac{U - P}{U^e - P^e} + q_x \frac{PU^e - UP^e}{U^e - P^e}$$

whence by definition of q_x, Eq.(VI. C.24) follows immediately by summation of both sides of Eq.(B.7) following multiplication by w_x:

(B.8) $$\begin{aligned} \textstyle\sum_U w_x Q_x^* = U_Q = U_Q^e \frac{U - P}{U^e - P^e} + q_P \frac{PU^e - UP^e}{U^e - P^e} \\ \textstyle\sum_P w_x Q_x^* = P_Q = P_Q^e \frac{U - P}{U^e - P^e} + q_U \frac{PU^e - UP^e}{U^e - P^e} \end{aligned}$$

where $q_P = \sum_P w_x q_x$ and $q_U = \sum_U w_x q_x$. But for insertion of the summations on the left, Eq.(B.8) is Eq.(VI. C.24).

To prove that for an arbitrary vector, $\boldsymbol{Q}^j$, if $\sum_P w_x Q_x^j = P_Q$ and $\sum_U w_x Q_x^j = U_Q$, then necessarily the rate vector $\boldsymbol{R}^j$ which rests upon $\boldsymbol{Q}^j$ will be feasible, let Eqs.(B.8) be expressed in the form:

(B.9) $$\tilde{\boldsymbol{Q}}^* \boldsymbol{W} = (P_Q; U_Q) = \boldsymbol{Y}_Q^*$$

where $\boldsymbol{W}$ is the weighting matrix defined in Section 1.a., of APPENDIX A. (page .) Let the matrix $\boldsymbol{M}$ be defined as the $z \times z$ matrix whose entries along the main diagonal are r_x, and elsewhere than along the main diagonal, are zero. Then:

(B.10) $$\boldsymbol{R}^* = \boldsymbol{Q}^* \boldsymbol{M}$$

By derivation of Eq.(B.8), $\boldsymbol{R}^*$ is feasible, $\tilde{\boldsymbol{R}}^*$, whence by Eq.(B.10):

(B.11) $$\tilde{\boldsymbol{R}}^* \boldsymbol{W} = (P; U) = \boldsymbol{Y}^* = \tilde{\boldsymbol{Q}}^* \boldsymbol{M} \boldsymbol{W}$$

By hypothesis: $\boldsymbol{Q}^j \boldsymbol{W} = \boldsymbol{Y}_Q^*$; whence by Eq.(B.9):

(B.12) $$\boldsymbol{Q}^j \boldsymbol{W} = \boldsymbol{Y}_Q^* = \tilde{\boldsymbol{Q}}^* \boldsymbol{W}$$

whence immediately:

(B.13) $$\boldsymbol{R}^j \boldsymbol{W} = \boldsymbol{Q}^j \boldsymbol{M} \boldsymbol{W} = \tilde{\boldsymbol{Q}}^* \boldsymbol{M} \boldsymbol{W} = \tilde{\boldsymbol{R}}^* \boldsymbol{W} = \boldsymbol{Y}^*$$

whence $\boldsymbol{R}^j$ must be feasible, $\tilde{\boldsymbol{R}}^j$.

It may be noted that a feasible solution always will result from application of Eq.(VI. C.23.a) to the rates, $R_x{}^i = r_x Q_x{}^i$, without consideration of the normals as prescribed in Cases 3 and 4. However, by definitions given:

(B.14) $\quad c_x = f_x q_x / q_{x+1}$

It follows that if $q_x > q_{x+1}$, it may happen that values of $c_x{}^*$ will be obtained which not only exceed $\bar{c}_x$ but also exceed unity, which implies $Q_x > Q_{x+1}$, which in turn implies *increase* of rate with *improvement* of protection. To illustrate, in Example 2, of Section VI. C., it was assumed that $r_x = 1$, but on the alternative assumption that r_x and q_x are as shown in Table 3, then by Eq.(B.14) it will be found that the c_x ratios associated with the solution of Example 2 are such that $c_5 = 1.02$ and $c_8 = 1.11$. Unless $r_x \doteq r_{x+1} \doteq 1$ for all x, then the normals, rather than the rates themselves, must be used in the calculation to preclude possibility of inversions such as the foregoing; except in the special case where $P\hat{U}^i - U\hat{P}^i = 0$, in which case $f_x{}^*$ will equal $f_x{}^i$ for all x, since the additive term then disappears from Eq.(VI. C.23.a).

A SHORT BIBLIOGRAPHY OF LINEAR ALGEBRA

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* * * * * *

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DISCUSSION BY LESTER B. DROPKIN

For several years now, writers and reviewers of papers presented to this Society have stressed the desirability, and indeed, the inevitability of utilizing theory, methods, techniques and procedures derived from what may be broadly referred to as the field of Finite Mathematics. During these same years the Society has also seen an increasing number of papers dealing with the ratemaking problems of the fire actuary. Recalling Mr. McIntosh's earlier work, it is not unexpected that he would again bring these two lines together in the paper now under review.

In his current paper, "A Mathematical Approach to Fire Protection Classification Rates," Mr. McIntosh deals with the problem of determining a set of rates such that they will, in the language of the paper, simultaneously fulfill the conditions of "feasibility" and "operational constraint." These two terms, although coming from the language of linear programming, represent simple and familiar concepts. The feasibility property will be readily recognized as that old friend: a rate structure in balance by part and in total. The question of operational constraints may similarly be recognized as coming within considerations of rate relativity, albeit the rate relativities here are not specifically given. Rather, each of the rate relativities is fixed only to the extent of having given lower and upper limits, such limits being predetermined by judgment or other outside factors. It is, of course, the simultaneous existence of the feasibility and constraint conditions that make the problem a real and interesting one.

The definition of the problem and the treatment of its solution (including therein those cases where no solution is possible) proceeds via linear algebra and matrix theory. Although this mathematical area may be somewhat unfamiliar to many of us, it is fortunate that much of the terminology is natural, intuitive and extremely suggestive, and that much of the theory has direct geometric analogues.

It is clear, at least to this reviewer, that the presentation of this paper will have several most desirable consequences: First, there are those areas, both within and without the fire field, whose structure is such as to directly parallel the problem treated by Mr. McIntosh. Here the methods of the paper can be lifted bodily and immediately applied with a minimal amount of alteration. In this connection, let us specifically note how well the very detailed and comprehensive illustrative examples have been prepared. The advantage of having many concrete illustrations to follow while working through the theoretical material is obvious.

Second, there is the undoubted stimulus to a wider study of the theory, principles and applications of those many mathematical areas which may be said to come within or be related to the scope of finite mathematics. This, not only for the specific purpose of following the particular mathematics of Mr. McIntosh's paper, but also for the purpose of developing that wider background which will increasingly become more and more necessary if we are to more completely fulfill our actuarial responsibilities. It will, perhaps, be of more than passing interest to note the rising sentiment for modifying the syllabus of the actuarial examinations in this regard.

Third, there will be those generalizations and extensions of the paper's methodology and theory to a wider class of problems. The mathematical discipline of linear algebra, as is well known, has served to unify many formerly separate branches of the mathematics tree, thereby, for example, providing the theoretical base for such applications as linear programming. There would seem to be no reason why the attack on many actuarial problems could not now derive substantial advantage from just such an alternative viewpoint.

The reader, on first coming to the paper, is quite likely to feel himself overwhelmed. The paper is long; the notation and terminology is unfortunately not a familiar one in our *Proceedings;* there are many pages of mathematical symbols unrelieved by normal linguistic intercourse; and the author's style of exposition is at times too much akin to those stream-of-consciousness writers whose elliptical simplicity is sometimes bafflling. These, however, are really unimportant and passing details, for the paper is a fine piece of actuarial work. For the reader willing to give the paper the serious consideration it deserves, there will be ample repayment for any expenditure of time and effort.

It would be neither possible nor desirable to attempt to summarize or paraphrase Mr. McIntosh's paper in the short space of this review, although it may be of some interest to single out some of the most important facets. Before doing so, however, mention should also be made of one approach to the reading of the paper which I found to be quite helpful, viz., a free use of the method of general reasoning. Many of us will recall that we were first introduced to this method in connection with our study of interest and annuities, a subject which suffers no lack of multitudinous symbols.

Assuming then that it will not detract from the paper itself, where the whole theoretical construct that comprises Method II is given, I should like to point out what appears to me to be the one equation which can be identified as going to the heart of the matter, viz., equation A-27 of the Appendix. It will be recalled that this equation defines a transformation, F*, from the ratio vectors to the coefficient vectors. Basically, Method II then results from the fact that this transformation is one-to-one, and in particular that the extreme points of the ratio vectors map into the extreme points of the coefficient vectors under the transformation, together with the basic fact that the points of a convex set can be expressed as a linear convex combination of extreme points, all coupled with the properties of the parameter vectors.

Reference was made to Method II as a "whole theoretical construct." This review would hardly be complete without also mentioning the satisfaction to be derived from a consideration of the manner in which the several different aspects of the paper are brought together into an interlocking harmonious unity.

In reviewing an earlier paper of Mr. McIntosh, I said, ". . . I am sure that this Society will be looking forward to future papers in which he will carry forward the ideas and conceptions of the present notable contribution." This, in full measure, he has done and will no doubt continue to do.

DISCUSSION OF PAPER PUBLISHED IN VOLUME LI

A BAYESIAN VIEW OF CREDIBILITY

ALLEN L. MAYERSON

VOLUME LI, PAGE 85

DISCUSSION BY CHARLES C. HEWITT, JR.

This is a perfectly delightful paper and could not have come at a more propitious time in the development of our Society's research efforts. The paper is not without flaws, but I hope that by pointing out such items I will in no way diminish my appearance of enthusiasm for Professor Mayerson's effort.

Heresy

There is another, and entirely separate reason, why Professor Mayerson's *clarification* of credibility concepts comes at an excellent moment. In listening to a sermon recently I was reminded that in the Anglican Catholic Church one of the four major obligations of the priesthood includes the stamping out of heresy. Gentlemen, I am not an Anglican priest (although my only brother is), but I say that there is heresy amongst us on the very basic issue of the meaning of credibility. One might have hoped that the recent publication and reprint issue of Laurie Longley-Cook's thorough and definitive study on credibility[1] would have dispelled what Arthur Bailey referred to as the "profound mystery" which has surrounded the basis for credibility formulas – at least among our own membership. Alas, such is not the case for we have been confronted just this Spring with an article[2] (in another insurance journal) which purports to explain credibility to non-actuaries, but which hopelessly confuses the term "credibility" with the statistical term "dispersion." (At one point this article refers to the terms "measures of dispersion" and "measures of credibility" as having identical meaning.[3]) Laurie Longley-Cook correctly points out,[4] ". . . credibility is not a simple property of data which can be calculated by some mathematical formula as can the standard deviation or other measures of the effect of chance variation on a body of statistical data. While credibility and statistical variance are related,

[1] An Introduction to Credibility Theory—L. H. Longley-Cook—*PCAS XLIX* (1962).

[2] Actuarial Science and Credibility—John S. McGuinness—*CPCU Annals* (Spring 1965).

[3] Ibid. p. 20.

[4] An Introduction to Credibility Theory—p. 4.

the former is meaningful only against a stated or implied background of the purpose for which the data are to be used and a consideration of the value of the prior knowledge available."

Not to belabor reference to the aforementioned article for non-actuaries, but it is appropriate to this discussion of Professor Mayerson's work to add that he does give us two mathematical formulations for credibility, applicable in specific instances, which make it abundantly clear that credibility may under certain circumstances be a function of:

(1) sample size,

(2) underlying hazard (mean of prior distribution), and

(3) underlying dispersion (variance of prior distribution).

And ironically (but not surprisingly) it turns out that credibility *increases* with variance (of the prior distribution). Thus, imprecise conclusions, such as equating wide dispersion with poor credibility,[5] can be seriously misleading to both professional and lay readers.

Synthesis

The essence of Professor Mayerson's paper is the bringing together of the late Arthur Bailey's pioneering work on credibility with the most up-to-date techniques of statistical decision theory. My advice to all interested persons, who have not already done so, is to read:

1) Mayerson's paper, then
2) *Applied Statistical Decision Theory* by Raiffa and Schlaifer – Harvard University (1961) – with emphasis on Chapter 3 (Conjugate Prior Distributions), Sections 10 and 11 of Chapter 7 (Negative Binomial and Beta-Binomial), and Chapters 9 (Bernouilli Process) and 10 (Poisson Process), and then
3) "Credibility Procedures" by A. L. Bailey – *PCAS XXXVII* (1950) with extremely interesting discussions and author's reply.

You should then have a superior grasp of what credibility formulas are bottomed on, and incidentally a deeper appreciation of Professor Mayerson's perception in conceiving this paper.

Perhaps a further synthesis with the works of Dropkin, et al, on merit rating in private passenger automobile can be achieved with the following illustration:

[5] Actuarial Science and Credibility—p. 19.

Dropkin[6] and Hewitt[7] point out that a risk in a class with mean r/a which has c accidents in s years indicates an expected frequency of $\frac{r+c}{a+s}$. In credibility terms:

$$\frac{r+c}{a+s} = (1 - Z)\frac{r}{a} + Z\frac{c}{s}$$

from which $$Z = \frac{s}{s+a}$$ [8]

but $a = \frac{Class\ Mean}{Class\ Variance}$, and substituting for a in the above expression produces Mayerson's expression for credibility in the Gamma-Poisson case.

The Neglected A. W. Whitney

My attraction to this paper prompted me to correspond with Professor Mayerson concerning the extent to which Albert W. Whitney[9] had been a forerunner of some of the conclusions reached by (A.L.) Bailey and Mayerson. For example Whitney and his colleagues were well aware that the K in

$$Z = \frac{P}{P+K}$$

was not really a constant. Starting with the presumption that the frequency distribution of risks within a particular classification is normal, Whitney arrived at an expression[10] for Z which can be reduced to:

$$Z = \frac{n}{n + \frac{P(1-P)}{\epsilon^2}}$$

n = exposure (to hazard) of a particular risk
P = indicated class hazard
ϵ^2 = variance of risks within the class

How close this comes to Mayerson can be seen by comparing it to Mayerson's Beta-Binomial derivation of Z, which can be expressed:

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[6] Automobile Merit Rating and Inverse Probabilities—*PCAS XLVII* (1960).

[7] The Negative Binomial Applied to the Canadian Merit Rating Plan for Individual Automobile Risks—*PCAS XLVII* (1960).

[8] R. A. Bailey—Discussion of "Some Considerations on Automobile Rating Systems Utilizing Individual Driving Records"—*PCAS XLVII*, p. 155 (1960).

[9] The Theory of Experience Rating—A. W. Whitney—*PCAS IV* (1918).

[10] Ibid. p. 288—Equation (23).

$$Z = \frac{n}{(n-1) + \frac{m_H(1-m_H)}{\sigma_H^2}}$$

$n =$ (as above)
$m_H =$ mean of prior distribution (assumed hazard)
$\sigma_H^2 =$ variance of prior distribution.

How then did our predecessors get trapped into an invariant "K"? Whitney's remarks are revealing.[11]

"We now come to the most difficult question of all, the determination of ϵ^2. It is obviously impossible as a practical matter to determine ϵ^2 statistically in each case."

Further along:

"Mr. [Winfield] Greene made the suggestion that . . . the second term of the denominator be taken as constant."

Whitney, in defense of a constant *K*, says "This brings us to the question of whether it is desirable in actual practice to admit the varying credibility of the class-experience and hence of the manual rate. We know that the manual rates for some classifications are more reliable than for others *and yet it is doubtful whether it is expedient in practice to recognize this fact. . . .*"

In his later work Arthur Bailey acknowledged this earlier effort in a passing reference to Greene's practical approximation of Whitney's "more complicated formula." [12]

Whitney's introductory non-mathematical remarks are so pertinent to a clear understanding of the foundation of credibility in experience rating that portions of them must be re-quoted:

(1) *Risk-exposure*[13]

"It is evident in the first place that the weight of the risk-experience will depend upon the risk-exposure. Other things being equal, the experience of that risk which has the larger exposure will be entitled to the larger degree of consideration. In the case

[11] Ibid. p. 287.
[12] Sampling Theory in Casualty Insurance—A. L. Bailey—*PCAS XXIX* (1942)—p. 72.
[13] The Theory of Experience Rating—p. 275.

of a very large risk the rate may with safety be based almost wholly upon its own experience; in the case of a small risk very little credence can be given to risk-experience and the rate must be based almost wholly upon the experience of the class."

(2) *Hazard*[14]

"Essentially the same relationship holds true in the case of the hazard; the larger the hazard, the larger will be the number of accidents, the exposure remaining the same, and therefore the more trustworthy the average."

(3) *Degree of concentration within class*[15]

"Now it is evident intuitively that if the risks are concentrated within the class, that is, if the standard deviation is small, a risk-experience that departs from the average of the class can be more easily accounted for as due to chance than as due to an inherent difference in the degree of hazard. On the other hand, if the standard deviation is large, that is if the risks are diverse, it is inherently likely that a risk-experience that departs from the average is to be accounted for by a real difference in the hazard."

(4) *Credibility of manual rate*[16]

"Another element that in theory may be taken account of is the varying credibility of the manual rate. The manual rate is established upon experience which in a majority of classifications is insufficient and which in many cases has been supplemented by judgment. It is evident that, other things being equal, the higher the credibility of the manual rate, the greater its weight in establishing the balance between class-experience and risk-experience. If, on the other hand, the manual rate is established upon insufficient experience, we shall be inclined to give greater relative credence to the risk-experience."

Kinds Of Credibility

If I had to choose my major criticism (in the unfavorable sense), it would be that the author leaves the implication that his approach is equally

[14] Ibid.
[15] Ibid. p. 276.
[16] Ibid.

applicable in any credibility situation. But in *practical* casualty actuarial work there are at least two significantly different applications of credibility:

(1) Class pure premium selection,

(2) Experience rating.

In the former situation Mayerson's general approach is directly applicable, and, in fact, provides a quite satisfactory solution to the important question, "What credibility should be assigned to the underlying pure premium?" I think sub-consciously we have known for years that $(1 - Z)$ is *not* a totally honest answer, if Z is solely a function of the volume of the current experience.

But Mayerson's general approach is not so easily applied to experience rating, if we agree with Whitney that the degree of concentration of risks within a class is pertinent. For there is now a three-way credibility problem – what credibility should be assigned to:

(1) Current risk experience,

(2) Previous risk experience, and

(3) Current manual rate.

Frankly no one else has suggested a theoretical answer for this very real problem, but it should be clearly understood that Mayerson's work does not come to grips with this issue, either.

Minor Technical Criticisms

There are several minor mathematical items that can be criticized:

(1) Meanings attached to symbols do not always remain constant (or clear). E.g., the capital letter H is used alternately to represent an *hypothesis, a point,* a *random variable* and a *parameter.* This failure to be more precise is confusing to the reader and may upon occasion have confused the author.

(2) In the discussion of conjugate distributions the statement that m (assumed mean) can be a pure premium, a claim frequency, an average claim cost, or some other actuarial function is too loose. How, for example, do we choose a Beta-distribution for average claim cost or pure premium when the variable must lie within the range *zero* to *one?*

(3) The statement in the last paragraph of "Choosing Prior Probabilities" that n varies directly with m is imprecise for

$$n = \frac{m(1 - m)}{\sigma^2} - 3$$

The author must therefore qualify his statement by adding "for $m < ½$".

* * * * * *

This is one of the most significant papers presented to this Society in many years and, happily, should produce much controversy and further thought in this important area. European actuaries have outstripped us in the classical "theory of risk". Professor Mayerson has distilled the essence of American achievement in the areas of credibility and the Bayesian approach. We may well be proud of what our Society has done and hopeful for what it promises to do in these areas.

MINUTES OF THE MEETING

May 23-26, 1965

SHAWNEE INN, SHAWNEE-ON-DELAWARE, PENNSYLVANIA

Prior to the formal convening of the meeting on May 24 there was a meeting of the Council on the afternoon of May 23 and a buffet supper in the evening for early arrivals.

The Spring 1965 Meeting was called to order at 9:40 a.m., May 24 with President Thomas E. Murrin presiding.

American Academy of Actuaries

John Bloys of the staff of Watters and Donovan presented a résumé on the present status of this project. He stated that, as the proposed legislation had passed the Senate in the previous session, favorable action by the Senate in this session of Congress was anticipated. However, the situation in the House was different. It was believed that amendments to the original bill, recommended by Insurance Superintendent Jordan of the District of Columbia, would necessitate public hearings before committees and subcommittees in the House and this would inevitably slow up action by the full body.

At this point Vice President Harold E. Curry took over the conduct of the meeting.

LeRoy J. Simon presented a résumé of his paper "The 1965 Table M." This paper was reviewed separately by Lester B. Dropkin and Charles C. Hewitt, Jr.

Charles C. Hewitt, Jr. then presented a review of the paper "A Bayesian View Of Credibility" which had been presented by Allen L. Mayerson at the November 1964 Meeting.

The next item consisted of a

Panel on Report of Committee on Annual Statement

Chairman: Joseph Linder

Panelists: John W. Carleton
Robert G. Espie
Ruth E. Salzmann

This report had been distributed to the entire membership of the CAS under date of December 3, 1964.

The panel provoked a lively discussion among the panel members followed by considerable discussion and numerous questions from the audience.

The session then recessed for lunch at 12:30 p.m. and reconvened at 2:00 p.m. with the following two concurrent panels:

(A) *Loss Reserves – Workmen's Compensation and Automobile*

Chairman: Richard J. Wolfrum

Panelists: James R. Berquist
Martin Bondy
Walter J. Fitzgibbon, Jr.
Stephen S. Makgill
Charles L. Niles, Jr.

(B) *Trend and Projection Factors – Automobile and Property Insurance*

Chairman: William S. Gillam

Panelists: Lewis H. Roberts
Paul W. Simoneau
Philipp K. Stern
Luther L. Tarbell, Jr.

The two panels adjourned at about 5:00 p.m.

In the evening, prior to dinner, there was a brief social hour.

The meeting reconvened at 9:00 a.m., May 25 with the following two concurrent seminar discussions (9:00 a.m. – 10:15 a.m.):

(A) *Rate Regulation – 20 Years After the SEUA Decision*

Chairman: Leslie P. Hemry

Participants: James M. Cahill
Laurence H. Longley-Cook
Allen L. Mayerson
Hubert W. Yount

In effect this seminar was a continuation of the seminar discussion on the same topic which was held at the November 1964 Meeting. This subject had been included on the Program for the May 1965 Meeting at the request of several members inasmuch as time limitations at the previous meeting had not permitted the full discussion desired of this interesting and important item.

(B) *Education and Training of Actuaries*

Chairman: William J. Hazam

Participants: Norman J. Bennett
Paul S. Liscord
John W. Wieder, Jr.

There then followed two concurrent seminars (10:30 a.m. – 12:00 noon):

(C) *Workmen's Compensation Rating Developments*

Chairman: LeRoy J. Simon

Participants: Francis J. Hope
Roy H. Kallop
John H. Muetterties

(D) *Accident and Health Developments*

Chairman: John A. Resony

Participants: John R. Bevan
Alfred V. Fairbanks
Paul E. Singer

There was no formal program for the afternoon of May 25 which was left open for Committee meetings and recreation.

It is also noted that on the morning of May 25 there was a sightseeing and antique tour, arranged by the CAS, which was well attended by the ladies.

In the evening there was a brief reception for the entire gathering followed by a banquet.

On Wednesday, May 26, there were held the following two concurrent seminars:

(E) *Package Policy Ratemaking – Automobile Insurance* (9:00 a.m. – 10:15 a.m.)

Chairman: Jack Moseley

Participants: Jeffrey T. Lange
Philipp K. Stern

Parenthetically, it is noted that the program for the meeting had indicated additional participants and that the discussion would embrace both automobile and property insurance. However, Chairman Moseley reported that the group decided this was too broad an area for the limited time available. Therefore, the discussion was confined to automobile insurance and it was suggested that the remaining section of the originally assigned topic be included on the program for the November 1965 Meeting.

(F) *The Actuary and Large Risk Rating – Casualty*

Chairman: Albert J. Walsh

Participants: Edward H. Budd
Robert Pollack

At 10:30 a.m. the gathering reassembled in plenary session with President Murrin presiding.

The President introduced to the gathering a new Associate, John A. Gibson, III.

Vice President Hazam presided over the greater part of the remainder of the May 26 session.

Reports were given by the chairmen on the discussions at various panels and seminars which had been held during the meeting:

Panel A – Loss Reserves – by Richard J. Wolfrum

Panel B – Trend and Projection Factors – by William S. Gillam, read by Ronald L. Bornhuetter in Mr. Gillam's absence

Seminar A – Rate Regulation – by Leslie P. Hemry, read by Laurence E. Longley-Cook in Mr. Hemry's absence

Seminar B – Education and Training of Actuaries – by William J. Hazam

Seminar C – Workmen's Compensation – by LeRoy J. Simon

Seminar D – Accident and Health – by Paul E. Singer substituting for John A. Resony

Seminar E – Package Policy Ratemaking – Jack Moseley

Seminar F – Large Risk Rating – Albert J. Walsh

Kenneth L. McIntosh presented a résumé of his paper "A Mathematical Approach To Fire Classification Rates." It was announced that a review of this paper would be presented by Lester B. Dropkin at the November 1965 Meeting.

President Murrin then informed the gathering of the action of the Council on the Report Of Committee On Annual Statement. That action is duly set forth in the minutes of the Council meeting held on May 23 and 25, 1965.

The foregoing completing the program for the 1965 Spring Meeting, adjournment was taken at 12:05 a.m.

For the purpose of the record it is noted that the following 84 Fellows, 33 Associates and 22 invited guests registered, at the time of the meeting, as being in attendance:

FELLOWS

Aldrich, W. C.
Allen, E. S.
Bailey, R. A.
Balcarek, R. J.
Barker, G. M.
Bennett, N. J.
Berkeley, E. T.
Berquist, J. R.
Blodget, H. R.
Bornhuetter, R. L.
Boyajian, J. H.
Brannigan, J. F.
Budd, E. H.
Byrne, H. T.
Cahill, J. M.
Carleton, J. W.
Coates, C. S.
Crowley, J. H.
Curry, H. E.
Dickerson, O. D.
Dropkin, L. B.
Elliott, G. B.
Espie, R. G.
Fairbanks, A. V.
Finnegan, J. H.
Fitzgibbon, W. J., Jr.
Fowler, T. W.
Gillam, W. S.
Goddard, R. P.
Graves, C. H.
Harwayne, F.
Hazam, W. J.
Hewitt, C. C., Jr.
Hobbs, E. J.
Hope, F. J.
Hughey, M. S.
Hunt, F. J., Jr.
Hurley, R. L.
Johnson, R. A.
Kallop, R. H.
Klaassen, E. J.
Lange, J. T.
Linder, J.
Liscord, P. S.
Longley-Cook, L. H.
MacKeen, H. E.
Makgill, S. S.
Masterson, N. E.
Mayerson, A. L.
McGuinness, J. S.
McNamara, D. J.
Meenaghan, J. J.
Menzel, H. W.
Morison, G. D.
Moseley, J.
Muetterties, J. H.
Murrin, T. E.
Nelson, S. T.
Otteson, P. M.
Phillips, H. J., Jr.
Pollack, R.
Resony, J. A.
Richards, H. R.
Roberts, L. H.
Rodermund, M.
Rosenberg, N.
Salzmann, R. E.
Schloss, H. W.
Simon, L. J.
Simoneau, P. W.
Skelding, A. Z.
Smith, E. M.
Stankus, L. M.
Tarbell, L. L.
Thomas, J. W.
Trudeau, D. E.
Walsh, A. J.
Wieder, J. W., Jr.
Williams, D. G.
Williams, P. A.
Wilson, J. C.
Wittick, H. E.
Wolfrum, R. J.
Yount, H. W.

ASSOCIATES

Brown, W. W., Jr.
Buffinton, P. G.
Crandall, W. H.
Curry, A. C.
Dahme, O. E.
DeMelio, J. J.
Durkin, J. H.
Franklin, N. M.
Gibson, J. A.
Gill, J. F.
Gingery, S.
Gould, D. E.
Harack, J.
Jensen, J. P.
Jones, N. F.
Markell, A. S.
McIntosh, K. L.
Muir, J. M.
Muniz, R. M.
Riccardo, J. F., Jr.
Richardson, H. F.
Roth, R. J.
Scammon, L. W.
Scheel, P. J.
Scheibl, J. A.
Schneiker, H. C.
Singer, P. E.
Stern, P. K.
Stevens, W. A.
Webb, B. L.
Woodworth, J. H.
Young, R. G.
Zory, P. B.

GUESTS

Anderson, R. R.
Battaglin, B. H.
Bechtolt, P. R.
Bloys, J.
Boissier, J. J.
Bühlmann, H.
Donovan, H. G.
Fertig, I. J.
Foody, W. M.
Galban, L. S., Jr.
Hayden, R. C.
Hemry, L. P.
Hoyt, F. A.
Kedrow, W. M.
Marshall, R. D.
McSherry, H.
Nagel, J. R.
Ratnaswamy, R.
Reid, J. N.
Stathos, N.
Wells, C. C.
Zunser, A. J. B.

VOL. LII, Part II No. 98

PROCEEDINGS

NOVEMBER 15 and 16, 1965

PRESIDENTIAL ADDRESS BY THOMAS E. MURRIN

In keeping with the tradition of the Society, I now have the privilege of addressing you upon the completion of my second term as President. Again I wish to thank you for the honor of serving as your President and to acknowledge the accomplishments of our members in the last two years.

During this period, two milestones were passed. The Fiftieth Anniversary of the founding of the Casualty Actuarial Society which was celebrated a year ago gave us an opportunity to pause and reflect upon the many contributions of the membership and to renew our dedication to actuarial science and the vital role it plays in our industry. Last month the American Academy of Actuaries was formed through cooperative research, study, and plain hard work by members of the four actuarial societies. This development will be of great future benefit to the actuarial profession, as well as to the insurance industry and to others who require actuarial services. My position as President-Elect of the Academy is a compliment to the membership of this Society as well as a personal honor for me. I am deeply grateful.

Our past record as a Society is one of which we can all be justifiably proud. Considering that actuarial analysis has been emphasized only recently in many companies, it is significant that many of our members are now in key executive positions. It is appropriate that we pause at the start of our fifty-second year as a Society, to take a hard look at the condition of our industry and at our own position as members of the actuarial profession.

While it has been said many times within the last few years, I would feel remiss if I did not comment on the fact that most companies continue to suffer severe underwriting losses during recent years. The year 1965 will produce unfavorable results for practically all companies and will probably be one of the worst years ever recorded. The causes of this deplorable situation are for the most part inadequate rates and a rate regulatory system that is a proven failure.

However, if the members of this Society, as a group, had had the final authority to make all decisions for the industry during these disastrous years, would different decisions have been made? To what extent must we acknowledge a contribution to the dismal record of the past? Let each of us ask ourselves whether we have always developed objective decisions and held fast even to the point of disagreeing with policy decisions. Do we not have the responsibility as well as the duty to be steadfast in our decisions when the problems under study are so grave and the potential consequences so disastrous? As we grapple with present problems and those of the future, our analyses should as always be based on sound and logical reasoning but our recommendations should also be brutally realistic. We must convince management that the prospects for the future should not be viewed through rose-colored glasses. Hurricanes and other catastrophes merely highlight the necessity of insurance and should not be blamed for adverse results. The industry must be positioned for underwriting profit on all coverages—even on those coverages with wind exposures.

What can we anticipate about the problems of the future? In other words, what can the actuary do to better serve the insurance industry? There is no doubt that in 1965 the world is moving faster than ever before. We have all heard statistical predictions on the estimated distribution of our population by age ten years hence. We know of the increasing amount of leisure, greater mobility and longer life-span of the general population. On the business side, our economy has now reached an unprecedented record of consecutive months of prosperity. Unemployment is at a level thought unattainable only a few years ago. The current economy is characterized by built-in inflation and more active federal government participation. Everywhere we turn we find fantastic new technology resulting in new products, new attitudes and new values.

It is my opinion that all professions, including the actuarial profession, must periodically take stock of themselves if they are to keep up with the rapidly changing world in which we live. Generally accepted ways of doing things must be re-examined and, if found wanting, rejected. The "good old days" are gone forever. As with other professions, we are now requiring more and different types of ability from young men and women entering our field. Should not each of us examine recent developments in our industry and be certain we are expanding our individual talents at a sufficiently fast rate? In his address to the Society of Actuaries in Montreal a few weeks ago, President Henningsen reminded his audience that they have the *continuing* obligation "to preserve and improve the standing of our profession." We must not be diverted from preparing for tomorrow's job by the pressures of today's job. It has been said that the future belongs to those who plan for it and today I would like to review a few areas in which I feel each of us should be planning programs of self-education.

The first major area is electronic data processing. Although computer technology in this country is considerably less than fifteen years old, there are many dramatic examples which indicate that the advent of computers is equivalent to a second industrial revolution. Individual companies in other industries have already mastered computer technology to the point that entire factories are running through directions supplied by a central computer. While certain individual insurance companies have also made dramatic progress in the use of computers, I believe that we are running far behind other industries. Even if computers were to be regarded only as sophisticated bookkeeping machines—a limited and incorrect view—how many companies have been able to free for more productive work the vast number of employees performing essentially clerical tasks? The record seems to indicate that insurance industry computers are producing too much output that merely duplicates reports that were traditional when only mechanical equipment was available. While actuaries, as most other insurance people, have paid lip service to statements that computers are the key to the future, few have taken the time to master the technology involved. While I am not now suggesting that we should all become programmers, we should recognize that the advent of the computer is probably the most important single event that will affect our ultimate role in the overall organizational pattern. It is imperative that each of us examine how much we know regarding the new machines, their capabilities and applications. Automation has also reached the programming function in the sense that machine language programming has been replaced in many ways by simple language systems for addressing the machine. The actuary must not fall behind these and other advances in technology to the extent he finds himself a mere recipient of "what the machine can do." Rather, he should specifically know how they can be used as tools to develop output—whether rate schedules, management exhibits or mathematical models.

The advent of the computer has even affected the art of diplomacy. Traditionally, foreign relations policies have been the function of senior diplomats of individual countries. Within the last ten to fifteen years, however, some of their decision-making has become increasingly complicated because of the difficulty in evaluating defense systems, nuclear capabilities, etc. Each of these subjects is so specialized and scientific that diplomats have often found themselves, because of a lack of technical ability, entirely dependent upon advice and facts furnished by people who have been able to master the new sciences. This latter group, often called technicrats, have to a certain extent taken the decision-making process away from the diplomat and their analyses and recommendations are often the controlling factors upon which diplomatic decisions are made. The similarity to the decision-making process in many fire-casualty insurance companies is very discomforting. The information insurance companies need to function is often furnished by non-insurance people. Actuaries, the traditional analysts of the insurance business, must provide the necessary bridge between information preparation and management decisions based upon sound analysis.

My own conviction on this subject is so strong that in my own company I am instituting a requirement that new trainees receive introductory and basic programming courses which are available on a correspondence and on a classroom basis. This requirement will also be extended to my senior associates in the actuarial unit. I feel it imperative that actuaries, and for that matter, all members of senior management, be fully cognizant of the potentialities of the new electronic marvels.

The growing science of operations research, which relies heavily on the methods that have proved so successful in the physical sciences, is one aspect of efforts to put decision-making on a more objective and routine basis. Seminars on operations research techniques and their application to the insurance industry are being conducted at this meeting. I strongly suggest that we all participate to the fullest extent possible and, upon returning to our individual companies, further pursue the ideas discussed. Operations research will undoubtedly find many applications in the insurance business in future years. Who, more than the actuary, is qualified to extend these mathematical techniques to our business? While many of our members have been interested in mathematical theory individually and through committee studies, this subject should be of vital interest and concern to all of us.

Another area in which we should build upon our current knowledge and capabilities is in the life insurance field. I am not suggesting that we become life actuaries; the founding of the Academy recognizes that there will be major fields of specialties. Very few members of the Actuarial Societies are crossed trained in both fields but we should not overlook the fact that an increasing number of property and casualty companies have launched or acquired life subsidiaries. The development thus far of two common mathematical examinations emphasizes that property/casualty and life actuarial studies have a common origin. While our exams require a fundamental understanding of life insurance, if we are to serve our companies well, continual improvement in our understanding of life insurance problems will be necessary.

Lastly, another area in which actual analysis could be used to greater advantage is in the area of expenses or, more descriptively, costs. I fear that in our business, the present approach to providing for expenses on an average percentage basis does not allow us to pinpoint our merchandising costs, by type of policy and by regional territory. While some companies have done work on the cost of selling and servicing different policies in different regions, generally much work remains to be done by the industry. Considering the fact that many manufacturing firms know their production and merchandising costs almost to the penny, by geographical area, it is almost inconceivable that an industry as large and vital as the insurance industry has done so relatively little by comparison. While expense accounting is a function of the accounting department, each of us in the future should encourage our own companies to undertake or actively pursue studies in this area. The key to proper pricing in the future intrinsically involves knowledge of actual production and servicing costs by kind of policy and by size of policy. Advances in providing for proper loss costs per unit of exposure are continually being developed but proper expense allowances per policy will be necessary if actuaries are to be truly able to properly price the products of tomorrow.

We must continually expand our horizons as a profession if we are to be prepared for tomorrow's economic world. Only if actuaries continue to be extremely progressive and aggressive can we hope to attract the quality young men and women so vital to the future growth in the insurance business. Knowledge and understanding of new situations are attributes we must acquire for ourselves—no one is going to bestow them on us. The learning process must not only be continual, but should constitute a major portion of our business activity. Spare time study is not enough. We owe it to our companies, our profession and ourselves to pursue actively a continual program of self-betterment.

RATEMAKING PROCEDURES FOR AUTOMOBILE LIABILITY INSURANCE

PHILIPP K. STERN

PREFACE

The proceedings of the Society already contain a paper on this subject which was presented by this author at the November meeting in 1956 *(PCAS XLIII)*. In the exposition of the statistical base for ratemaking, that paper stated that "Automobile Liability Insurance is compiled on a policy year basis" with a footnote that reads:

> "Since January 1, 1953, the Statistical Plan provides also for the reporting of statistical detail for the compilation of private passenger and commercial non-fleet experience on a calendar-year accident-year basis. At the time of this writing, this method of compiling experience is in an experimental stage."

A short time after the paper was published, the decision was made by the rating organizations to adopt the accident year basis for private passenger cars and, a few years later, the same basis was extended to commercial cars. The experience of only a small portion of automobile liability insurance, that for garages, public automobiles and miscellaneous classifications, continues on a policy year basis.

The change to the accident year basis for the largest portion of the business was followed by changes in the ratemaking formulae pertaining to the experience periods used for rate level determination; the formula for the development of territory rate levels was modified and a new method was developed to measure loss cost trends. In view of these changes, it became clear to the author that his 1956 paper was in need of up-dating. It was obvious that a complete revision of the prior paper was necessary, rather than a mere substitution of chapters, to give the proper emphasis to the new statistical base of accident year data.

The new paper has the same objective as the paper in 1956, namely, to describe the ratemaking process rather than to evaluate it. The material has been completely reorganized to provide a clearer separation of material pertaining to the gathering and summarization of ratemaking statistics from the actuarial ratemaking procedure. The section on statistics explains the accident year and the policy year bases currently in use. An appendix deals with the *incomplete policy year* even though it is not used at present in everyday work, in order to preserve the concept and as a caution against the misuse of grossly immature experience without the necessary adjustments.

A separate section is included which, for lack of a better name, is called Preliminary Ratemaking Calculations. It explains the various terms found on the ratemaking exhibits and explains the method of calculating the values used in rate level and rate calculations, such as *premiums at manual rates, expected loss ratio, loss development factors,* etc.

After dealing with these details, it was possible to keep the section on the ratemaking procedure relatively short and departures from the mainstream of thought could be avoided. The basic process of ratemaking is explained for private passenger cars dealing with statewide rate level, territory rate level and class rates, followed by additional comments to set forth any differences that apply for commercial cars and garages.

A Miscellaneous section deals with the review of experience on classifications other than the three major classification groups and rates for automobile assigned risks; it also refers to the automobile package policies for which a ratemaking procedure based on experience has yet to be developed.

The new classification plan and rating system for private passenger cars, only recently introduced in many states, is referred to in an appendix to this paper.

INTRODUCTION

This paper presents a description of basic procedures currently used in ratemaking for automobile liability insurance. It is intended to serve as an introduction to ratemaking for this line of insurance. A superficial knowledge, at least, of the automobile manuals and the statistical plan referred to in this paper is expected of the reader. Frequent reference reading from these sources will assist in the comprehension of the material discussed in the following pages.

The making of rates for automobile liability insurance, along with the other lines of casualty and fire insurance, is regulated by laws passed in the various states. These laws establish the standards rates have to meet and set forth the prerequisites for the administration of the rate regulatory function of the states. In most states, the rate regulatory law was patterned after the Casualty and Surety Rate Regulatory Bill (All-Industry Commissioners' Draft) which provides as follows in Section 3:

1. Due consideration shall be given to past and prospective loss experience within and outside this state, to catastrophe hazards, if

any, to a reasonable margin for profit and contingencies, to dividends, savings or unabsorbed premium deposits allowed or returned by insurers to their policyholders, members or subscribers, to past and prospective expenses both countrywide and those specially applicable to this state, and to all other relevant factors within and outside this state;

4. Rates shall not be excessive, inadequate or unfairly discriminatory.

In Section 4, the Bill provides:

> Every insurer shall file with the (commissioner) every manual of classifications, rules and rates, every rating plan and every modification of any of the foregoing which it proposes to use. Every such filing shall state the proposed effective date thereof, shall indicate the character and extent of the coverage contemplated and shall be accompanied by the information upon which the insurer supports the filing.

The Rate Administration section of that Bill provides (Section 13):

> The (commissioner) shall promulgate reasonable rules and statistical plans, reasonably adapted to each of the rating systems on file with him, which may be modified from time to time and which shall be used thereafter by each insurer in the recording and reporting of its loss and countrywide expense experience, in order that the experience of all insurers may be made available at least annually in such form and detail as may be necessary to aid him in determining whether rating systems comply with the standards set forth in Section 3. Such rules and plans may also provide for the recording and reporting of expense experience items which are specially applicable to this state and are not susceptible of determination by a prorating of countrywide expense experience. In promulgating such rules and plans, the (commissioner) shall give due consideration to the rating systems on file with him and, in order that such rules and plans may be as uniform as is practicable among the several states, to the rules and to the form of the plans used for such rating systems in other states. No insurer shall be required to record or report its loss experience on a classification basis that is inconsistent with the rating system filed by it. The (commissioner) may designate one or more rating organizations or other agencies to assist him in gathering such experience and making compilations thereof, and such compilations shall be made

available, subject to reasonable rules promulgated by the (commissioner) to insurers and rating organizations.

Accordingly, statistical plans have been promulgated or approved by the regulatory authorities in almost all states, and statistical agents have been appointed who collect and compile the loss experience which provides the basis for rate review and ratemaking.

The Mutual Insurance Rating Bureau and the National Bureau of Casualty Underwriters function as statistical agents for the states and they are national rating organizations that develop and file rates for automobile liability insurance, and other lines of casualty insurance, on behalf of their respective members and subscribers.[1] Generally, a formula ratemaking procedure is used in the course of this activity; the two Bureaus cooperate in the development of manual rates in a limited number of states; however, they use the same formula generally throughout the country. It is this formula which will be described in this paper.

The reader should be aware of the fact, however, that the rates developed by the two rating organizations are not the only rates used by companies writing automobile liability insurance.

The percentage of total premium written at Bureau rates varies greatly by state, as can be seen from the following examples of distribution of premiums by company groups:

Percent Distributions of Earned Premiums
Automobile Bodily Injury Liability Insurance—Calendar Year 1961

State	Members & Subscribers of N.B.C.U.	Members & Subscribers of M.I.R.B.	Other Companies
Connecticut	49.1%	17.8%	33.1%
Iowa	11.0%	4.8%	84.2%
New York	52.2%	24.0%	23.8%
Washington	14.7%	3.7%	81.6%

Companies that are not members or subscribers of a rating organization file their rates individually.

Moreover, even a member or subscriber of a Bureau may depart from the Bureau rates: by way of a *deviation* it may make application to the

[1] A member company of a rating organization generally utilizes the services of the Bureau for all states and all lines in which the Bureau functions; a subscriber company may select states and lines for which it receives Bureau services, and may function independently in other areas.

rate regulatory authority for a percentage departure (upward or downward) from the rates approved for the rating organization. In recent years, a method has evolved by which the rating organization makes a separate filing that produces rates, rules or classifications different from those of the Bureau on behalf of a member or subscriber requesting it; since the Bureau, in such case, merely acts as a conduit for the company's application, this type of filing is referred to as an *agency filing*. Thus, there may be a variety of rate schedules used in a state at the same time, in addition to those developed by the National Bureau or by the Mutual Bureau.

Yet, even in a state in which only a small proportion of the total business is written at Bureau rates, these rates have a direct effect upon the total rate structure. Many Non-Bureau companies use rates promulgated by a Bureau, frequently with percentage departures from the Bureau rates more in the nature of a deviation, or with selective departures from such rates. Apparently, such filings are supported, though by means different and presumably less exacting than is required of the rating organizations. The use of Bureau manuals by Non-Bureau companies is so extensive that rating organizations in recent years took steps to protect their work product, at the same time making available the manuals to Non-Bureau users at a charge.

In view of this wide use of the Bureau rates, a study of the Bureau rate structure and the methods used in developing Bureau rates is necessary for an understanding of present ratemaking practices for automobile liability insurance in general.

RATEMAKING STATISTICS

Although past experience is only one of the several factors that shall be given "due consideration" in the making of rates, actual practice has given it an eminent role in the ratemaking process. Reliance upon past experience is based upon the expectation that the most recent past experience will repeat itself in the immediately following period during which the rates to be determined will apply.

A rate consists of the expense portion and the loss portion; correspondingly, separate statistics are compiled on expense experience and on loss experience.

The basis for the expense experience is the Insurance Expense Exhibit which provides countrywide premium, loss and expense data by line of insurance, including automobile bodily injury and automobile property damage liability insurance. This paper will make only brief reference to the expense portion of the rate and the expense experience, in connection with the expense loading in manual rates.

Loss Experience

Loss experience is the aggregate of transactions recorded by classification and territory on the company books of (1) the measure of the insured hazard, (2) the premium charged for the insurance coverage, and (3) the payments of indemnity amounts that eventually are made under the insurance contract.

1. The measure of the insured hazard, or the *exposure,* gives a numerical value to the insured object: the exposure of a private passenger automobile used by the owner in the ordinary way is *one car;* but if the private passenger car is owned by a concern that is engaged in renting it to others, the measure of the hazard may be expressed in miles driven, or in rental receipts. For the various types of risks, the exposure base is selected in such manner that it most accurately measures the hazard to which the object is exposed. The commonly used types of exposure bases for automobile liability insurance are:

Exposure	Recorded as
Per car	Car months
Mileage	Number of miles
Receipts	Dollars
Payroll	Dollars

2. The recorded written premium is the premium charged for the policy. The definition of written premium in the Automobile Statistical Plan is self-explanatory.

3. There will be claims for indemnification under the insurance contracts; amounts the company eventually will pay are called *losses.* Some losses are paid almost instantly upon the presentation of the claim, others after a moderate delay while the circumstances of the loss or the extent of the damage or injury are investigated; some claims may require extensive investigation or court litigation, with the result that it will not be known for an extended period of time whether there is liability on part of the insuring company to make any payment, and if so, how large the payment will be. Thus, there are *paid losses* entered on the company records, and *outstanding losses,* the latter reflecting a reserve based on the company's estimate of the ultimate cost of settling a specific claim. In connection

with claims under litigation, substantial amounts are often spent by the company in defense of its insured against whom the claim is made. Certain expenses incurred in connection with claims in suit, as defined in the Automobile Statistical Plan, are susceptible to the same statistical treatment as losses, i.e., they can be assigned to the particular class and territory applicable to the risk. They are called *allocated loss adjustment expenses;* in most of the statistical summaries described here, they are combined with the losses.

In addition to recording loss amounts, the company enters a count of claimants on whose behalf a loss payment is made or a loss reserve is established.

Each company may develop its own set of codes needed for the recording of its experience in a form suitable for the company's internal operations and requirements. Each company is obligated, however, to record its experience at least in such detail as is required by the Commissioner of Insurance in each state in which the company operates. It must follow an accepted set of rules so that the experience, after it is summarized, is meaningful and susceptible to interpretation.

The loss experience used in the ratemaking procedures described in this paper is that gathered by the National Bureau of Casualty Underwriters and the Mutual Insurance Rating Bureau.

Experience reports are received by each Bureau from its member companies for all states, from subscriber companies for the states in which they receive rating service from the Bureau, and from other companies that have elected a Bureau as their statistical agent.

Such reports are prepared in accordance with the statistical plan and periodic instructions issued by the Bureaus to the reporting companies.

The Automobile Statistical Plan

Since January 1, 1963 there is in use the Automobile Statistical Plan that applies to automobile liability and automobile physical damage insurance.[2] The Plan is jointly developed by the Mutual Insurance Rating Bureau, National Automobile Underwriters Association and National Bureau of Casualty Underwriters, and is published and distributed by the latter organization to companies affiliated with either of the three. (Prior

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[2] The Plan applies in all states other than Massachusetts. In that state, a different plan is published by the Massachusetts Automobile Accident Prevention and Rating Bureau applicable to automobile bodily injury liability; the codes in that plan are also used for automobile property damage liability insurance by the Bureaus.

to that date, each of the three organizations published its own plan.) The Plan, after the required approval by the regulating authority in each state, becomes an official Statistical Plan.

Statements in this context will be directed only at the provision in the Plan pertaining to automobile liability insurance.

The Plan is designed to develop private passenger and commercial car experience on an accident year basis and the experience for other automobile classifications on a policy year basis. Experience developed on an accident year basis provides a comparison of the incurred losses on accidents that occur in a given 12 months period with the exposures and premiums earned during the same period. The policy year basis of experience consolidation provides a comparison of the incurred losses that occurred on all policies having an effective date in a given calendar year, with the earned exposures and earned premiums on such policies. The concepts of accident year and policy year statistics will be more fully explained in the presentation of experience consolidation in a subsequent section.

The accident year basis of consolidating experience was adopted first for private passenger experience beginning with accident year 1954 and extended to commercial cars with the consolidation of data for accident year 1959. It has not been adopted for the other classifications, which remain on a policy year basis mainly because the need of dealing with interim policy audits by special procedures would counteract any benefits that might be obtained from the adoption of the accident year basis.

The Plan contains instructions as to the maximum detail by which experience is to be recorded. There are two basic characteristics of detail of experience: classification and territory.

With respect to classification detail, the statistical plan provides, with only minor exceptions, for separate codes for every manual[3] classification for which separate rates are established. For example, if there are 9 private passenger manual classifications for which rates are published, the statistical plan provides for as many statistical codes, viz: classes 111, 112, 113, 115, 121, 123, 125, 127 and 130.

The manual rates are modified for a specific private passenger car by manual rules that provide for multi-car discount, compact-car discount, and driver training credit. These elements are reflected in the codes by

[3] Automobile Casualty Manual and Special-Package Automobile Policy Manual of N.B.C.U. and M.I.R.B. respectively.

the addition of a fourth coding digit to indicate whether any of these modifications or combinations thereof was applied. A fifth coding digit is added to indicate the application of the Safe Driver Insurance Plan or other merit rating plan and the resulting rate modification.

For commercial cars, separate codes apply by rate class and size type, corresponding to the rating criteria in the manual. Separate codes are shown for the various types of public automobiles, the divisions for garage liability, and various miscellaneous classifications and special types of coverages.

Occasionally, the plan may require statistical detail greater than the detail reflected in the rating system, if such detail is required for analytical studies. For example, the statistical plan required for some years the coding and reporting of experience on garages by industry classifications (Dealers, Service Stations, etc.) before a rate distinction was made between these classifications in the Automobile Casualty Manual. Such differentiation was introduced based on the data thus obtained.

Other detail required for analytical studies is sometimes obtained from special calls for experience or from sampling studies.

With respect to territory detail, the plan provides, again with minor exceptions, that all business be recorded by the applicable territory codes. The rate territory code numbers are shown, with the definitions of territories, in the Automobile Casualty Manual and the Special-Package Automobile Policy Manual. (The manuals are arranged so that separate rate schedules are shown for each territory for which separate statistics are to be obtained, even if two or more territories are assigned the same rates.)

As noted above, statistical plans require the approval of the rate supervisory authorities in the various states. After the approval of an original plan, each subsequent change in the plan also requires approval. Changes in the rating system have to be reflected in the statistical plan in order to maintain the correspondence between the detail of the rating system and the detail for its statistical support.

Reports of Experience

Each year, all companies that are due to report their experience receive from the Bureau a set of instructions setting forth the detail in which the data are to be filed; these instructions are referred to as a *Call for Experience*. The content of the Call is developed by the appropriate committees of the Bureau, pursuant to a statistical program that was submitted to and approved by the rate supervisory authority in each state. Therefore, the Call is an official document issued by the Bureau on behalf of the Insurance Commissioners, as well as for the Bureau's ratemaking purposes; compliance with the requirements of the official Call is mandatory. Some of the requirements of the Call go beyond the detail required by the states, compliance with which is a matter of the relationship of the Bureau with its members and subscribers.

Under a typical official Call for automobile liability insurance the following reports are required, separately for bodily injury and property damage:[4] (See Appendix A for recent changes)

Written exposures and written premiums

- Private passenger non-fleet
  - Summary reports by class and territory, for each calender quarter – filed quarterly, or transaction report (options available).
- All other classifications
  - Transaction reports – filed quarterly.

Losses and number of claims

- Paid losses with paid allocated loss adjustment expenses and number of paid claims – monthly transaction reports.
- Outstanding losses and number of outstanding claims with reserves for allocated loss adjustment expenses. Transaction reports twice a year with losses valued as of March 31 and September 30 respectively (the latter is limited to private passenger cars). Loss reports are also required for medical payments coverage.

Individual reports of excess losses

- Such reports are filed in conjunction with the reports of outstanding losses.

Almost all companies file these reports in the form of punch cards; except for the exposure and premium reports for private passenger cars, these punch cards are duplicates of information recorded by the company as each transaction is made. Because of the large volume of private passenger business, options are available to companies for reporting in form of summaries or transactions.[5]

Exhibit 1 shows a facsimile of the punch card used for reporting automobile liability experience to the National Bureau and Mutual Bureau

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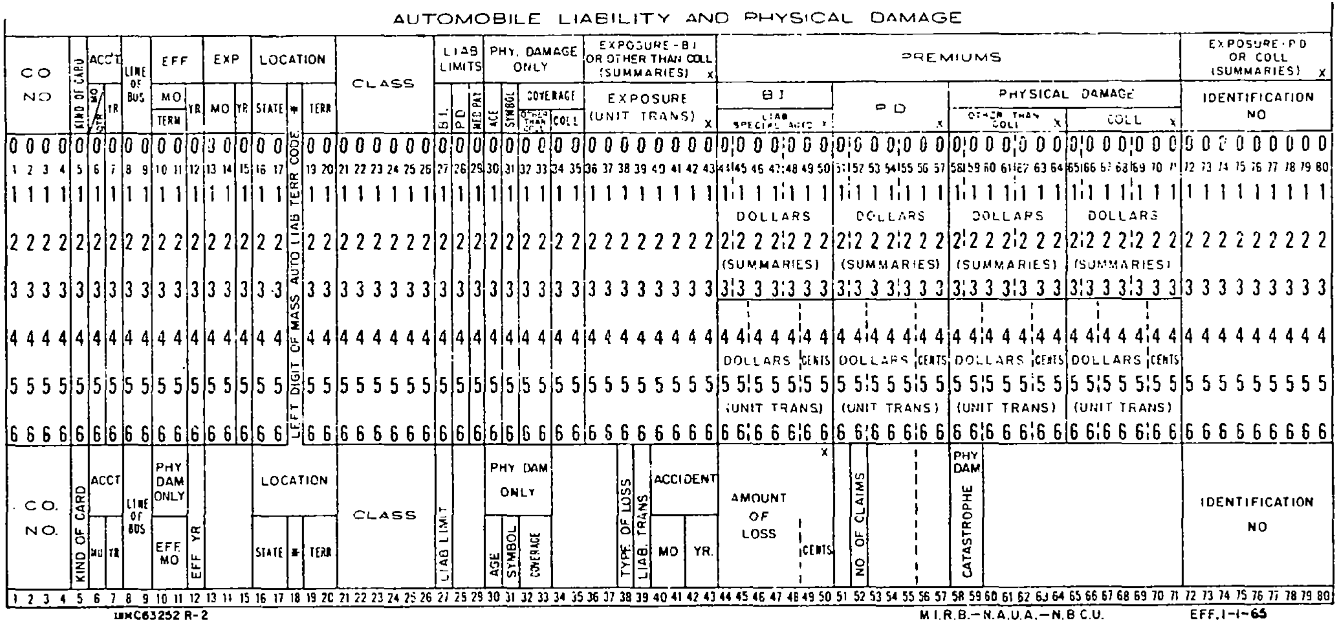
[4] For the sake of clarity, some procedural detail is omitted.

[5] When the transaction method of reporting on punch cards was first adopted in 1953, it was considered the most economic method of reporting; with the increasing use of electronic computers, reporting on tape takes its place as an alternative medium.

Exhibit 1

FACSIMILIE OF PUNCH CARD FOR REPORTING AUTOMOBILE INSURANCE EXPERIENCE



NOTE: Upper headings apply to exposures and premiums. Lower headings apply to losses and number of claims.

and physical damage experience to the National Automobile Underwriters Association.

These reports are filed with the Bureau by the companies at dates fairly evenly spread throughout the year. Written exposures and written premiums are reported for each quarter 60 days after the end of the quarter. Paid losses and paid allocated loss adjustment expenses are reported monthly, 45 days after the end of the month, and outstanding losses with outstanding allocated loss adjustment expenses are filed on May 15 and October 15, with excess loss reports following within one month. (In future references, the term "losses" will be used as losses including allocated loss adjustment expenses, unless otherwise stated.) These are the building blocks from which the Bureau prepares summary tabulations which are discussed next.

As noted previously, the reported data are used to produce accident year experience for some classification groups, and policy year experience for the other classifications. These summaries are generally in detail by class within territory, separately for bodily injury and property damage liability. These two types of summarizations will be dealt with separately.

Accident Year Experience Summaries

This basis of summarization is applied to private passenger car non-fleet and commercial car fleet and non-fleet experience.[6]

The accident year experience, after consolidation, will consist of the exposures earned and premiums earned during a 12 month period, and the incurred losses and number of claims resulting from accidents that occurred during the same period.

The earned exposures and premiums have to be calculated from the reported written exposures and written premiums. It was noted above that written exposures for private passenger cars are reported by the companies, summarized by class and territory, for each quarter year. A quarter year in this context is described as an *accounting quarter,* which means that it includes all written exposure and written premium transactions entered on

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[6] An explanation, at this point, of the terms *fleet* and *non-fleet* is in order. The Automobile Statistical Plan states that a vehicle is part of a fleet if the policy covering it is written under a fleet plan; all other cars are non-fleet. That is not a good definition, but it is generally understood. The Automobile Casualty Manuals of the Bureaus contain a manual rule (Rule 9, General Rules Section) that describes the Fleet Plan. From this rule, characteristics of a fleet can be identified, sufficient for assigning a risk to the fleet category for statistical purposes:

> There are at least 5 cars insured under the policy at its inception date. The policy contains a provision for the automatic coverage of all automobiles owned or leased by the insured during the policy term. The final exposure and premium is determined by audit after expiration of the policy.

the company records during that quarter. Such transactions are the writings on new and renewal business, full or partial cancellations on business previously recorded and corrections of existing entries, regardless of the effective date of the transaction. Since private passenger policies are written for a term of one year as well as for terms less than one year (three, four, six months, etc.) separate summaries are filed by the companies by term of policy.

For commercial cars, the transaction reports received from the companies are summarized by the Bureau into accounting quarter summaries by class and territory.

These written exposure and written premium summarizations, private passenger and commercial – each by class and territory, are now converted into *earned* data.

The concept of *earned* exposure and premium may be explained by the following example: A policy is issued covering one private passenger car, for a premium of $108, with an effective date of July 1, 1963, for a term of one year expiring June 30, 1964. A transaction entry will be made, recording 12 car months of written exposure and $108 of written premium, effective date 7/1/63, term 1 year. On July 31, the company will have provided coverage for 1/12 of the term; it will have earned 1/12 of the annual premium, or $9; the fact that the exposure for one car has been in effect for 1/12 of the policy term is expressed as 1 car month earned. Two months after the effective date, $18 and 2 car months will have been earned, $27 and 3 car months after 3 months, etc. As of December 31, one-half will have been earned: $54 of the written premium and 6 car months of exposure, during the year 1963. The remainder will be earned during the following year, 1964. The remaining $54 of written premium and 6 car months of exposure will be fully earned as of June 30, 1964, the expiration date of the policy.

For the purpose of the Bureau calculations of earned exposures and premiums, it was decided to work from quarterly written data with the assumption that the writings are evenly distributed within each quarter. Barring unusual circumstances, this assumption is reasonable for the degree of accuracy desired.

Thus, all writings during the first quarter of the year are assumed to have an average effective date of February 15. What will be their contributions to earnings as of the end of the year? Contributions will be made at the rate of ¼ during the 4th, 3rd and 2nd quarters, but only ⅛ during the first quarter in view of the assumption that the policies have been in effect only for one-half of that quarter. The remaining ⅛ of the first quarter writings will be earned during the following year. Similarly, the second quarter writings will contribute ⅛ + ¼ + ¼ to the earnings of the current year and ¼ + ⅛ to the earnings of the following year.

The earnings in each year, therefore, contain contributions from the writings during the 4 quarters of the preceding year and from the writings during the 4 quarters of the current year.[7]

Exhibit 2 illustrates the above method of earnings calculations.

Incurred losses for an accident year consist of the losses paid and the losses reserved pertaining to the accidents that occurred during that year. The number of claims are defined in the same way.

In the transaction reports of paid losses filed by the companies each month, loss payments are reported, as they are made, on accidents that occurred in the past. For example, the transactions for the month of January, 1963 may include amounts paid on several accidents that occurred during that month plus those on accidents that occurred during December, 1962, November, 1962 etc., going back any number of months and years. Similarly, the reports of outstanding losses include loss reserves on accidents of relatively new vintage as well as on accidents that may have occurred two, three or more years ago.

All these transaction reports are sorted by the year of accident for the purpose of summarizing accident year incurred losses.

If we take all losses on 1963 accidents that were paid from January to December 1963 and add to these paid losses all reserves established on 1963 accidents not yet settled on December 31 of that year, we would have the incurred losses on accident year 1963 as they are known on December 31, 1963. This type of summary, however, would be quite incomplete. Reports on many accidents that occurred toward the end of the year may not yet have reached the central recording office in the company organization, reports of payments made may still be in the internal reporting channels and information on the severity of recent accidents may be too spotty for a reliable estimate of their expected loss cost.

For these reasons, the cut-off date is moved forward to March 31 of the following year. During these additional three months, much of the lacking information is received on accidents of recent occurrence, and the loss data on accidents of the entire year achieve greater maturity. By

[7] This applies to business written for a term of 1 year. Appropriate modifications have to be made for business written for terms of less than 1 year, such as terms of 3, 4 or 6 months.

CALCULATION OF EARNED PREMIUMS

Exhibit 2

FROM PREMIUMS WRITTEN BY QUARTERS - POLICIES WRITTEN FOR AN ANNUAL TERM

Calendar Year Quarter of Writing	Premium Written	PREMIUMS EARNED FROM QUARTERS OF WRITING									
		EARNED DURING YEAR N					EARNED DURING YEAR N+1				
		1st Qtr.	2nd Qtr.	3rd Qtr.	4th Qtr.	Total	1st Qtr.	2nd Qtr.	3rd Qtr.	4th Qtr.	Total
Year N											
1st Quarter	$10,000	$1,250	$2,500	$2,500	$2,500	$8,750	$1,250				$1,250
2nd Quarter	11,000		1,375	2,750	2,750	6,875	2,750	$1,375			4,125
3rd Quarter	9,000			1,125	2,250	3,375	2,250	2,250	$1,125		5,625
4th Quarter	10,000				1,250	1,250	2,500	2,500	2,500	$1,250	8,750
Total	40,000	1,250	3,875	6,375	8,750	20,250	8,750	6,125	3,625	1,250	19,750
Year N+1											
1st Quarter	10,000						1,250	2,500	2,500	2,500	8,750
2nd Quarter	11,000							1,375	2,750	2,750	6,875
3rd Quarter	9,000								1,125	2,250	3,375
4th Quarter	10,000									1,250	1,250
Total	40,000						1,250	3,875	6,375	8,750	20,250

Factors applied to premiums written during Calendar Year Quarter M to calculate premiums earned:

Earned During Quarter	Factor
M	.125
M+1	.250
M+2	.250
M+3	.250
Current Year	.875
M+4	.125
Total	1.000

Summary

Premium Earned During Year N	
From Year N Writing	$20,250
Premium Earned During Year N+1	
From Year N Writing	19,750
From Year N+1 Writing	20,250
Total	40,000

moving the valuation date to March 31 following, accident year 1963 now includes all loss payments on 1963 accidents made from January 1, 1963 to March 31, 1964 and reserves valued as of the same date on all such accidents not yet settled as of March 31, 1964. For accident year incurred losses so developed the terms *accident year 1963 as of March 31, 1964* or *accident year 1963 as of 15 months* are used.

The losses included as outstanding in this summary will change as time goes on. Some of them will be paid during the following 12 months at the same amount as reserved, some at smaller or larger amounts while some cases may be closed without payment. Some will still be unsettled as of the later date, but the estimate of their ultimate cost may have changed. These changed values are reflected in a new summarization of the 1963 incurred losses, 12 months later, as of March 31, 1965, or, as of 27 months. This new summarization consists of all losses paid from January 1, 1963 to March 31, 1965 plus losses outstanding as of March 31, 1965 on all 1963 accidents. This process is repeated once more to produce the same accident year as of 39 months for bodily injury.[8]

The difference between the incurred losses from one valuation date to the next is called *loss development;* it is usually expressed as a ratio of the amounts at the later date to those at the earlier date, and this ratio is used as a *loss development factor* in ratemaking, which will be explained later.

All that has been said above in reference to losses equally applies to the method used to summarize the reported number of claims for an accident year as of 15, 27 and 39 months.

We have dealt with the incurred losses in total, i.e. the sum of all losses regardless of the size of each individual loss. A separation of these losses into two parts is needed, each to be used in a different phase of the ratemaking process.

Rates published in the Automobile Casualty Manual set the price for coverage at certain basic limits of liability. The basic limit is the lowest limit for which rates are published. For automobile property damage liability insurance, the basic limit is $5,000. For bodily injury liability insurance, the basic limit in some states is $5,000 per claimant subject to a maximum of $10,000 for all claimants in a single accident, and in other

[8] The process is repeated further to 51 months and to 63 months but these summaries are made on a broader basis, statewide or countrywide for groups of classifications, rather than in the full detail by class and territory.

states the limit is $10,000 per claimant subject to a maximum of $20,000 per accident.[9]

Coverage at limits higher than the basic limits is provided at charges in addition to those resulting from the application of the manual rates. The charges for such higher limits are found in the Increased Limits Tables in the Automobile Casualty Manual.

Generally, rates for the standard limits coverage are based on experience that excludes the effect on premiums and losses of coverage provided for limits above basic. Experience on the portion of coverage provided above basic limits is used in the determination of the Increased Limits Tables. The technique of obtaining premium at basic limits is explained later; it is necessary, however, to separate the losses into *basic limits losses* and *excess losses* at the point of experience consolidation.

Of the many thousands of accidents for which paid and outstanding loss transaction reports are filed annually, relatively few involve payments or reserves larger than the basic limit. Companies are required to earmark for a special report every accident with an incurred loss exceeding the basic limit (excluding allocated loss adjustment expenses – such expenses incurred by the company are in addition to the policy limit).

At reporting time, an Individual Report of Excess Losses is filed on each such accident. The Bureau determines from these reports the excess portion for the accident, for bodily injury and for property damage liability separately, For medical payments coverage, a simplified method is used to determine the excess portion.

The excess portions are incorporated in the data which enter into the experience summary.

So far we have defined an accident year as covering a 12 month period with the inference that this period is from January 1 to December 31 of that year. That is the usual understanding when the term *accident year* is used. Any other 12 months period could, of course, be selected, so long as reporting procedures adequately provide for it. The Bureaus use a modification of the accident year concept in consolidating private passenger experience: 12 months periods beginning July 1 and ending June 30 of the following year. This type of consolidation is called *fiscal-accident year experience.*

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[9] These limits correspond to the minimum coverage required by the Financial Responsibility Laws in effect in each state.

For practical reasons, the Bureaus work with sub-summarizations of private passenger experience by semi-annual periods of earned exposures and earned premiums and paid losses. Either accident year or fiscal-accident year data can be produced by combining 2 semi-annual sub-summaries (first plus second half, or second half plus first half of the following year) with the appropriate paid losses; outstanding losses as of March 31 are included for the accident year summaries and, as of September 30, for the fiscal-accident year.

Experience compiled on a fiscal-accident year basis is, by six months, more recent than the last prior compiled experience on the accident year basis. The Bureaus compile private passenger experience for about 15 states on a fiscal-accident year basis, while the experience for all other states is compiled on the accident year basis. This procedure allows for some staggering of the workload in summarization as well as rate review and rate filings, without increasing the lag between the time of review of experience and the experience period. This method is used only for private passenger cars because of the relatively greater importance of this portion of the business.

Policy Year Experience Summaries

For classifications other than private passenger non-fleet and commercial fleet and non-fleet, experience is compiled on a policy year basis. Prior to the adoption of the accident year basis of consolidation, all automobile liability insurance experience was compiled on a policy year basis. For reasons of practicality, the policy year method was retained for private passenger fleets, garages, public automobiles, and numerous miscellaneous classifications.

A policy year experience summary uses the same building blocks as does an accident year summary, only the arrangement of the components differs.

Experience might be compiled for policy years as of 15, 27, 39 months, etc., as is the accident year experience. In the area in which at present the policy year basis is used, however, it is the practice not to compile experience as of 15 months; for these classification groups, policy year data are only compiled as of 27 and as of 39 months in classification detail, and loss development to 51 months and to 63 months is obtained through the summarization of losses by broad groups of classifications.

We may, therefore, direct our attention now to the method of compiling experience for a policy year as of 27 months, and its development to a

39 months basis.[10] By the definition previously given, the policy year experience compares earned exposures and earned premiums on all policies written with effective dates during a calendar year with the losses incurred on the same policies. Policies written to be effective January 1 remain in effect during the entire year and expire December 31; they are fully earned as of December 31. Policies with later effective dates continue to be in effect beyond December 31, with the policies effective on the last day of the year remaining in effect until the end of the following year. On the latter, only one day of the one year exposure and the corresponding fraction of the premiums are earned during the year of the effective date; the remainder is earned during the following year. Thus, 24 months after the first day of the policy year all policies are expired and the written exposures and written premiums are fully earned.[11]

For many of the classifications for which experience at present is compiled on the policy year basis, the exposures and premiums are subject to final determination upon policy expiration based upon an audit of the risk's records, such as classifications for which the experience is measured in payroll, miles or earnings. For example, for an automobile policy covering the premises and operations of a garage, the exposure is the total payroll of the garage employees for the year. At the inception date, only an estimate can be made of the number of mechanics and their salary for the ensuing year, the number of salesmen, etc. These quantities are finally determined from the payroll record of the insured after the policy has expired. As these audits are performed, the final audited exposures and premiums are entered into the statistical records of the company and from there they flow to the statistical agent with the quarterly exposure and premium transaction reports.[12]

In order to allow sufficient time for the inclusion of the audit results, the Bureau includes an additional three months in the exposure and premium policy year summarization; consequently, policy year experience is

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[10] Although of no immediate import on current ratemaking, the concept of experience for a policy year as of 15 months should not be disregarded. Appendix B contains a brief discussion of this subject.

[11] Automobile liability policies are generally not written for a term of more than one year. Some companies write *open-end* policies, providing that the policy remains in effect, unless cancelled by the insured or by the company, upon payment of the renewal premium. For statistical purposes, such policies are treated as policies written for a definite term and have to be reported accordingly to the statistical agent.

[12] The results of audits are usually addditions to or subtractions from previously filed reports of the estimated exposures and premiums.

"as of 27 months" with respect to exposures and premium for all classifications subject to the policy year method of consolidation.

The summarization of the policy year incurred losses differs from that of accident year losses only in the time element. The paid loss and outstanding loss transaction reports include in the identifying information the effective year of the policy under which the loss arose. All loss transactions on policies with the same effective year make up the incurred losses for that policy year. Incurred losses for a completed policy year are valued as of March 31 of the second following year for the first summarization (as of 27 months), e.g. policy year 1963 as of March 31, 1965. A subsequent summarization produces losses valued 12 months later, or as of 39 months.

Excess loss reports are related to the total incurred losses on the same basis as explained earlier for the accident year.

The end product of the operations explained in this section is an ordered tabulation of the experience; an example of a tabulation of accident year experience, in the form usually prepared by the Mutual Insurance Rating Bureau, is attached as Exhibit 3.

Before concluding this discussion on experience summaries, a few remarks are in order to demonstrate what is done by the Bureau to attain the greatest possible degree of accuracy in the consolidated experience.

As noted previously, companies report their data on punch cards and/or tabulations, at various times throughout the year. In the Mutual Bureau alone, more than ten million punch cards are received each year from the companies.

All these fragments are combined by the Bureau to produce the summaries. From the time of recording of each piece of information in the company offices to the last step in the Bureau, the data are processed many times by people and machines; without predetermined safeguards, errors would enter and remain in the system.

One of the most effective safeguards is the requirement of balancing totals which are carried through the system from the beginning to the end. Other means of testing the accuracy of the reported data are comparisons of averages produced by the data with known averages, tests for distributions and comparisons with prior reports. A very large effort in man hours and machine hours, involving substantial expense, is required in this activity.

When the data are finally summarized, tabulations of the experience are filed with the rate regulating authority in each state. The data are now

SPECIMEN OF EXPERIENCE TABULATION

AUTOMOBILE LIABILITY INSURANCE

CALL YEAR	LINE	COMPANY CODE	COVER	ITEM	TERRITORY		CLASS	ACCIDENT YEAR	EARNED EXPOSURE (CAR YEARS)	EARNED PREMIUM	LOSSES INCURRED *		NO. OF CLAIMS INCURRED		EXCESS LOSSES (EXCESS PORTION)	
					STATE	TERR.					LIABILITY	MEDICAL	LIABILITY	MEDICAL	LIABILITY	MEDICAL
4	1	x	1	01	xx	yy	1110	1963	534	26223	5225	2048	13	15		
4	1	x	1	01	xx	yy	1111	196	6026	332997	166488	34835	143	180	42000	4182
4	1	x	1	01	xx	yy	1112	196	3403	144761	63936	9846	71	61	13000	187
4	1	x	1	01	xx	yy	1113	196	500	21030	9859	6166	7	14	543	1205
4	1	x	1	01	xx	yy	1120	196	786	39401	11567	4524	17	38		
4	1	x	1	01	xx	yy	1121	196	7042	405660	152783	33568	225	220	2000	1236
4	1	x	1	01	xx	yy	1122	196	3091	153014	21082	13781	39	68		2850
4	1	x	1	01	xx	yy	1123	196	469	22781	4180	6719	6	44		1580
4	1	x	1	01	xx	yy	1130	196	43	2275	41165	5546	5	8	30000	2000
4	1	x	1	01	xx	yy	1131	196	316	20636	13420	1028	17	10		
4	1	x	1	01	xx	yy	1132	196	153	9494	4350	336	3	3		
4	1	x	1	01	xx	yy	1133	196	37	2128		300		2		
4	1	x	1	01	xx	yy	1150	196	17	412		471		1		
4	1	x	1	01	xx	yy	1151	196	341	9489	25718	5165	16	15	1500	146
4	1	x	1	01	xx	yy	1152	196	365	9179	150	971	1	9		
4	1	x	1	01	xx	yy	1153	196	16	384		110				
4	1	x	1	01	xx	yy	1210	196	241	10369	1471	1482	2	12		
4	1	x	1	01	xx	yy	1211	196	1030	90694	24336	3001	54	34		
4	1	x	1	01	xx	yy	1214	196	26	1833						
4	1	x	1	01	xx	yy	1218	196	54	4523						
4	1	x	1	01	xx	yy	1230	196	46	5464	3321	111	4	4		
4	1	x	1	01	xx	yy	1231	196	393	52279	35086	5827	23	21		226
4	1	x	1	01	xx	yy	1234	196	65	6865	3410		8			
4	1	x	1	01	xx	yy	1238	196	63	8448	6666		8			
4	1	x	1	01	xx	yy	1250	196	2	110						
4	1	x	1	01	xx	yy	1251	196	36	1673						
4	1	x	1	01	xx	yy	1254	196	2	171						
							Total	1963	27,496	1,796,467	803,966	176,129	829	954	107,296	17,619

Explanatory Notes for Identifying Information:

Call Year - 4, i.e. 1964
Line - 1, Automobile Liability
Company Code - not used here
Coverage - 1, stands for bodily injury liability
Item -01, code to identify major classification group private passenger
State & Territory - the appropriate codes would be shown
Class Code
1110 Class 111 (1A) Single Car Risk, cars with compact car discount
1111 Class 111 (1A) Single Car Risk, standard size
1112 Class 111 (1A) Two-or-More Car Risk with multi-car discount, Cars without compact car discount
1113 Class 111 (1A) " " " " " " " Cars with compact car discount
112X Class 112 (1B)
Etc.

* Losses incurred are the total limits losses and they include allocated loss adjustment expenses.

ready to be used for rate review by these authorities, and for ratemaking by the rating organization.

PRELIMINARY RATEMAKING CALCULATIONS

Certain calculations will be explained in this section which are preliminary to the actual analysis of the experience. Dealing mainly with the mechanics of the calculations in this section will allow a more straightforward presentation of the ratemaking formula in the following chapter.

Earned Premiums At Manual Rates

The premiums at present manual rates are calculated by multiplying the earned exposures by the basic limit present (at time of rate review) manual rates. Taking the data from Exhibit 3, as example, which shows accident year 1963 experience for private passenger cars, we find the exposures shown below:

Territory xxyy Bodily Injury Liability

(1) Class Code	(2) Earned Number of Cars	(3) Present Manual Rate	(4) Premiums at Present Manual Rates (2) x (3)
1110	534	$ 90	$ 48,060
1111	6,026	100	602,600
1112	3,403	80	272,240
1113	500	72	36,000
Total	27,496	$119.36	$3,281,923

Class Code 1111 stands for Rate Class 11, a car subject to the manual rate without any modification. This rate is shown on a rate page of the Automobile Casualty Manual. Code 1110 denotes a Class 11 car qualifying for the 10% compact car discount. Code 1112 applies to a Class 11 car that is part of a multi-car risk and obtains the 20% multi-car discount. A car under Class 1113 receives both of these discounts (.80 × .90 = .72), a compact car subject to the multi-car discount. The average rate of $119.36 is obtained by dividing the total premium at present manual rates by the total earned number of cars.

This calculation is repeated for each class within each territory, separately for bodily injury and for property damage.

We could now prepare a new tabulation, containing the same data as the example on Exhibit 3 to which has been added on each line the earned premium at present manual rates. It should be noted that such new tabulation will show two kinds of premiums:

The Earned Premium – more fully described as the Total Limits or Collected Earned Premium

The Earned Premium at Present Basic Limits Manual Rates – often called the Collectible Earned Premium

It would be repetitious to expand the discussion of the differences between these two kinds of premiums. The above explanation of the premium at present basic limits manual rates and the earlier reference to the Automobile Statistical Plan for a complete definition of the premium reported by the companies should suffice.

Loss Adjustment Expenses

The incurred allocated loss adjustment expenses are combined with the incurred losses in the summarization of the experience by class and territory. It has been found desirable by the Bureaus to include, in ratemaking statistics, also the unallocated loss adjustment expenses with these losses. By exhibiting the combined loss and loss adjustment expense amounts, there is shown more clearly how much of the premium dollar is expended by the companies directly on behalf of the insured, by the terms of the insuring agreement.

The unallocated loss adjustment expenses are not reported by the companies under the calls for classified experience, since they cannot be segregated in comparable detail. Total loss adjustment expenses incurred are reported by the companies in the Insurance Expense Exhibit, countrywide by line of insurance. The Bureaus require their companies to report annually, as supplemental information, allocated and unallocated loss adjustment expenses separately, countrywide, and separately for automobile bodily injury and property damage liability. From these data, a loading factor is calculated which is applied to the reported losses and allocated loss adjustment expenses, converting them to losses including all loss adjustment expenses. The factors used at present, as determined from stock company experience by the National Bureau of Casualty Underwriters, are 1.10 for automobile bodily injury liability and 1.16 for automobile property damage liability.

Loss Development Factor

The incurred losses for each accident year include paid losses and outstanding losses. The latter are loss reserves on open cases. The loss reserves represent the best estimates by the companies of the ultimate cost of all open cases, based on all available information as of the reporting date of the outstanding losses. There are, however, differences between these estimates and the actual ultimate cost. The incurred losses, under the Bureau program, are summarized for a number of subsequent valuation dates for each year to a point where they can be considered, for all practical purposes, as having reached the ultimate value. The observed development of the incurred losses on older years is used to adjust the incurred losses of the more recent years to an estimated ultimate value. For bodily injury liability insurance, incurred losses for each year are valued five consecutive times, carrying the development from 15 months to 63 months. For property damage liability, the development is carried to 39 months, from three successive valuations. The difference in the aging required for bodily injury incurred losses as compared with property damage losses is based on the recognition that a greater portion of the former remains in the category of open cases for a longer period.

These development data are obtained from basic limits losses up to 39 months for bodily injury and to 27 months for property damage. For the development beyond these valuations, total limits losses are used.

Exhibit 4 shows the calculation of loss development factors of the type generally used in the present ratemaking procedures. For most states, such factors are based on countrywide data, although in some states with a substantial volume, loss development factors based on the state's own experience are used.

The loss development factor is the ratio of the incurred losses as of the later valuation date to the incurred losses as of the earlier valuation date of the same accident year. For each development period, there are available such ratios for a number of years, and the average of the ratios is used as the loss development factor. On Exhibit 4, the bodily injury loss development from 15 months to 27 months is based on accident years 1958, 1959 and 1960; in the absence of strong counter-indications, the average of the factors, in this case 1.081, is used as the factor to develop losses from 15 months to 27 months. Loss development factors for the other periods are obtained in the same manner. The products of the factors will combine the loss development for longer periods, such as the 1.064 factor to develop losses from 15 months to 63 months.

Exhibit 4

AUTOMOBILE LIABILITY INSURANCE - PRIVATE PASSENGER CARS

CALCULATIONS OF LOSS DEVELOPMENT FACTORS

BODILY INJURY*

Accident	10/20 Limits Incurred Losses As Of:			Loss Development Factors:		
Year	15 Months	27 Months	39 Months	15 to 27	27 to 39	15 to 39
1958	$118,364,408	$126,058,939	$126,063,887	1.065	1.000	
1959	180,893,383	196,976,013	196,043,747	1.089	.995	
1960	218,239,683	237,427,687	xxx	1.088	xxx	
Average:				1.081	.998	1.079

Accident	Total Limits Incurred Losses As Of:			Loss Development Factors:		
Year	39 Months	51 Months	63 Months	39 to 51	51 to 63	39 to 63
1956	$ 55,402,103	$ 54,583,271	$ 54,175,414	.985	.993	
1957	128,338,912	127,565,779	127,105,218	.994	.996	
1958	138,327,181	137,516,031	xxx	.994	xxx	
Average:				.991	.995	.986

Loss Development From 15 to 63 Months = 1.079 x .986 = 1.064
27 to 63 Months = .998 x .986 = .984
39 to 63 Months = .986

PROPERTY DAMAGE**

Accident	Basic Limits Incurred Losses As Of:		Total Limits Incurred Losses As Of:		Loss Development Factors:		
Year	15 Months	27 Months	27 Months	39 Months	15 to 27	27 to 39	15 to 39
1957	$161,559,941	$159,015,151	$159,151,125	$157,762,253	.984	.991	
1958	167,908,153	164,272,502	164,345,870	163,422,273	.978	.994	
1959	171,712,658	168,545,452	168,658,986	167,442,062	.982	.993	
1960	176,582,043	174,791,491	174,886,985	xxx	.990	xxx	
Average:					.984	.993	.977

15 to 39 Months = .984 x .993 = .977
27 to 39 Months = .993

Incurred Losses include allocated loss adjustment expenses.
* All states where bodily injury losses up to 10/20 are used for manual rates.
** Countrywide.

Three such cumulative factors are developed for bodily injury, one each to develop losses from 15 months, 27 months and 39 months respectively, to an ultimate basis. Two factors are needed for property damage. Thus, these factors develop the experience from the point at which the development of the classified experience ends.

Loss development factors for commercial cars accident year experience are obtained in the same manner.

For classifications summarized on a policy year basis, loss development from 27 months to later dates is measured similarly. Appendix B discusses the use of pure premium ratios for the development of the incomplete policy year to a complete policy year basis. Such pure premium ratios are used also for the development of policy year losses valued as of 27 months to a later date for classification groups for which exposures and premiums as well as losses are subject to changes beyond 27 months.

Expected Loss Ratio

The portion of the premium dollar available for the payment of losses and all loss adjustment expenses is the Expected Loss and Loss Adjustment Ratio. Its complement is the portion required for expenses and a provision for underwriting profit and contingencies. The expense ratios can be obtained from the Insurance Expense Exhibit, which shows separate amounts for the various categories of expense.

It is customary to include in the expense items a budgetary provision as Production Cost Allowance, which is generally not based on the past experience from the Insurance Expense Exhibit. At present, the production cost allowance for automobile liability insurance is generally 20% for the major classification groups (private passenger cars, commercial cars and garages) with some departures upward and downward in some areas.

The expense item Taxes is provided for at present by an average allowance of 3.0% to cover state local insurance taxes, licenses and fees, payroll taxes and a variety of miscellaneous taxes, but not including Federal Income taxes. Appropriate departures by state where tax requirements depart from the average are reflected in the expense provision on a state basis.

The provision for the expense item Inspection and Bureau is 1% for private passenger cars and commercial cars, with larger amounts for the other automobile insurance categories, to cover dues, assessments, fees and charges for the various boards and bureaus, statistical and service organizations and other affiliations of companies.

Miscellaneous expenses not specifically assignable to any of the above categories fall into the expense item General Administration. The present provision for this item is 5.5%.

Added to the above items of expense is a provision in the expense loading for underwriting profit and contingencies. This amounts to 5% at present, with exceptions in a few states. It is evident that this 5% of premium is not available as underwriting profit if the losses or expenses are greater than expected; any funds obtained from this theoretical profit provision in the rates becomes a contingency cushion against adverse loss or expense experience. Conversely, better than expected loss and expense experience adds to profit, until rates are adjusted.

The standard expense and loss ratios for private passenger and commercial cars determined by the National Bureau of Casualty Underwriters from the expense experience of its member companies are set forth below. The Mutual Insurance Rating Bureau, in its ratemaking calculations, uses the same expected loss and loss adjustment ratio as that used by the National Bureau.

Standard Loss and Expense Items
Private Passenger and Commercial Cars

Production Cost Allowance	20.0%
General Administration	5.5
Inspection and Bureau	1.0
Taxes, Licenses & Fees	3.0
Underwriting Profit & Contingencies	5.0
Sub-Total	34.5
Expected Losses and Loss Adjustment Expenses	65.5
Total	100.0

Credibility

Credibility factors are used in ratemaking to express in numerical values the credence given to the experience. Full credibility is expressed as 1.00, with values below 1.00 assigned to the various intervals of less than full credibility. The criterion upon which credibility is based is volume of experience. For liability insurance, the number of claims has been used for many years for this purpose. For the automobile line of insurance, full credibility is assigned to a volume producing 1084 claims or more during the experience period. The following table is used for the assignment of credibilities:

Number of Claims	Credibility	Number of Claims	Credibility
0-10	0	390-530	.60
11-42	.10	531-693	.70
43-97	.20	694-877	.80
98-172	.30	878-1083	.90
173-270	.40	1084 and over	1.00
271-389	.50		

This table is used in conjunction with the normal ratemaking data. A different table is used for credibilities assigned to paid losses used in trend factor calculations.[13]

Where a body of experience does not meet the standard of full credibility, some other element has to be found to fill the deficiency. This other element is given the complement of the credibility assigned to the particular body of experience. These two elements are averaged by the respective credibility weights to arrive at a value that is accepted as the true value. We may average an experience loss ratio of .60 with an expected loss ratio of .50; if the former is given .70 credibility, the weighted average would be determined as follows: Experience Loss Ratio × Credibility + Expected Loss Ratio × (1.00 − Credibility) or

$$.60 \times .70 + .50 \times (1.00 - .70) = .57$$

Thus, while the experience loss ratio regardless of credibility indicates an increase of 20% (.60 ÷ 50 = 1.20 or + 20%) we give it credence only to the extent of .70, which produces an indicated increase of 14%. In this case, we are giving weight partly to the experience indication and partly to the status quo, that is, present rates or present rate level.

Experience Exhibits

The first summary of experience (Exhibit 3) is a tabulation listing the data, usually in numerical order by class, within territory, separately for each year. For use in ratemaking, further summarizations are required, the form of which varies depending upon the purpose each such summarization serves. For example, if it is desired to compare the experience for the various territories within a state, experience for all classes com-

[13] A full discussion of this subject is contained in L. H. Longley-Cook, "An Introduction to Credibility Theory" — *PCAS XLIX*.

bined within a major classification group may be used by year or for a number of years combined. For another type of review, many territories and the experience of several years may be combined in order to exhibit a larger volume of data in a finer breakdown by classification.

Exhibit 5 presents an example of private passenger experience by territory. It offers an opportunity to introduce frequently used terms not previously presented.

Experience pure premium is the average incurred basic limits loss cost per unit of earned exposures, or for a convenient multiple of units of exposures (commonly called *pure premium*):

$$\text{Pure premium} = \frac{\text{Incurred basic limits losses}}{\text{Earned exposures}}$$

The pure premium is usually expressed in dollars and cents, e.g., \$38.41 per private passenger car on the first line of Exhibit 5. For exposure bases with a low loss cost per unit the pure premium may be expressed in mills, such as \$.545 per \$1,000 of exposure.

Average incurred claim cost is obtained in the following manner:

$$\text{Average incurred claim cost} = \frac{\text{Incurred basic limits losses}}{\text{Number of incurred claims}}$$

Incurred claim frequency is the number of incurred claims per unit of earned exposure or a convenient multiple of units of exposures:

$$\text{Incurred claim frequency} = \frac{\text{Number of incurred claims}}{\text{Earned exposure}}$$

On Exhibit 5, the incurred claim frequency is expressed per 100 earned car years.

It can be seen from the three fractions above that the product of average claim cost and claim frequency is equal to the pure premium.

Incurred loss ratio is the portion of the earned premium set aside for losses (paid and reserves) expressed as:

$$\text{Incurred loss ratio} = \frac{\text{Incurred losses}}{\text{Earned premium}}$$

Pure premiums and average incurred claim costs are usually based on basic limits incurred losses, i.e., incurred losses excluding the excess portion. Loss ratios have to include incurred losses on a basis compatible with the premiums used. In the automobile liability insurance experience reported under the Bureau reporting procedure, the earned premium is the total premium charged for the policy; it includes, in addition to the charge

EXHIBIT 5

AUTOMOBILE LIABILITY INSURANCE EXPERIENCE

ACCIDENT YEARS 1960 - 1962

PRIVATE PASSENGER CARS-ALL CLASSES COMBINED-BY STATISTICAL TERRITORY

BODILY INJURY

Territory Code	Accident Year	Earned Exposure	Earned Premium	Liability: Incurred Losses: Basic Limits *	Liability: Incurred Losses: Excess	Liability: Number of Incurred Claims	Medical Payments Incurred Losses	Total Limits Loss Ratio	Average Claim Cost	Claim Frequency	Pure Premium
11	60	19,371	967,489	744,019	54,899	755	90,076	.92	985	3.9	38.41
11	61	87,030	4,352,881	2,718,941	188,580	2,871	335,978	.75	947	3.3	31.24
11	62	98,506	3,704,034	2,865,292	79,524	2,936	384,403	.90	976	3.0	29.09
	TOTAL	204,907	9,024,404	6,328,252	323,003	6,562	810,457	.83	964	3.2	30.38
12	60	3,612	174,998	52,562		95	12,192	.37	553	2.6	14.55
12	61	19,199	924,312	392,812	21,275	470	69,091	.52	836	2.4	20.46
12	62	23,192	808,762	557,752	26,980	619	87,157	.83	901	2.7	24.05
	TOTAL	46,003	1,908,072	1,003,126	48,255	1,184	168,440	.64	847	2.6	21.81
13	60	1,846	91,347	81,079	45,645	68	8,990	1.49	1,192	3.7	43.92
13	61	9,327	441,007	206,667		231	45,965	.57	895	2.5	22.16
13	62	11,188	397,630	240,415	13,490	290	30,711	.72	829	2.6	21.49
	TOTAL	22,361	929,984	528,161	59,135	589	85,666	.72	897	2.6	23.62
14	60	7,234	342,304	199,174	4,296	230	33,501	.69	866	3.2	27.53
14	61	31,711	1,500,193	826,839	21,522	927	98,434	.63	892	2.9	26.07
14	62	39,347	1,355,679	1,150,229	67,443	1,204	141,583	1.00	955	3.1	29.23
	TOTAL	78,292	3,198,176	2,176,242	93,261	2,361	273,518	.80	922	3.0	27.80
15	60	3,635	170,077	129,051		120	14,491	.84	1,075	3.3	35.50
15	61	18,837	878,655	567,693	64,488	527	67,047	.80	1,077	2.8	30.14
15	62	30,161	1,050,007	858,962	102,308	765	110,780	1.02	1,123	2.5	28.48
	TOTAL	52,633	2,098,739	1,555,706	166,796	1,412	192,318	.91	1,102	2.7	29.56
16	60	130,423	6,831,378	3,070,391	157,717	3,516	387,730	.53	873	2.7	23.54
16	61	26,439	1,306,658	463,973	21,820	482	71,780	.43	963	1.8	17.55
16	62	4,494	155,724	146,218		120	18,809	1.06	1,218	2.7	32.54
	TOTAL	161,356	8,293,760	3,680,582	179,537	4,118	478,319	.52	894	2.6	22.81
17	60	23,743	996,942	404,539	128,761	461	65,476	.60	878	1.9	17.04
17	61	21,667	749,796	359,725	92,626	452	57,177	.68	796	2.1	16.60
17	62	21,717	590,841	368,002	16,775	438	52,223	.74	840	2.0	16.95
	TOTAL	67,127	2,337,579	1,132,266	238,162	1,351	174,876	.66	838	2.0	16.87
18	60	18,594	1,341,290	489,798	2,398	509	94,262	.44	962	2.7	26.34
18	61	17,749	1,077,756	648,726	23,399	524	84,127	.70	1,238	3.0	36.55
18	62	18,259	953,148	554,831	75,406	530	89,840	.76	1,047	2.9	30.39
	TOTAL	54,602	3,372,194	1,693,355	101,203	1,563	268,229	.61	1,083	2.9	31.01

* 10/20

for basic coverage, charges for increased limits, and the bodily injury liability premium includes charges of medical payments insurance when such coverage is provided. This earned premium has to be compared with the losses corresponding to the coverage it represents. The losses have to be the total limits losses (basic limits losses plus excess losses), and the medical payments losses have to be added to the bodily injury losses. On Exhibit 5, the bodily injury loss ratio on the first line is shown as:

$$.92 = \frac{\$744,019 + \$54,899 + \$90,076}{\$967,489}$$

If premiums at basic limits manual rates are used, the resulting loss ratio will reflect only the basic limits liability incurred losses.

The underlying pure premium is the portion of the rate that is available for losses and all loss adjustment expenses. The rate is the premium charged per unit, e.g., the rate for a private passenger car for a given class in a given territory is $100. If the expected loss and loss adjustment ratio is .655, the amount in that rate available for losses is $65.50; that is the underlying pure premium. It can be directly compared with the experience pure premiums. Usually, such comparison is based on broader averages. A comparison may be made of the experience pure premium in a territory for all private passenger classes combined with the pure premium underlying the average rate in the territory. The calculation of the average rate is shown in a prior section in conjunction with the calculation of premiums at manual rates.

Classification Differentials

As will be seen later, the ratemaking formula for the major classification groups first establishes a statewide indicated rate level change, then distributes the change by territory, and finally produces rates for each class in each territory. This last step utilizes percentage relationships between the classes, generally known as classification differentials.

Because of the relatively large number of classifications within the major classification groups, experience for each class in each territory or even on a statewide basis is not sufficiently stable for a rating system that is designed for countrywide application. A stable system of differentials is obtained by the use of classification experience on a very broad basis.

It must be noted at this point that it has become customary, in any such broad experience summarization, to segregate the data between New York and countrywide excluding New York. Although these two components of the total experience very often produce similar results, recognition has to be given to conditions characteristic of New York experience that require its separate review. There are other states that, for a variety of reasons, may not fit into the countrywide pattern and that are treated separately.

The experience is arranged by classification for each year, or a combination of years, of a selected experience period, and by coverage. The raw experience is then used to calculate, for each class, incurred claim frequencies, average claim costs, loss ratios and pure premiums. Realizing that the experience for each class may be different from the other classes by chance alone, the class developing the largest volume of experience is selected as the *base class* against which the other classes are measured. Within the major classification groups, the base classes best meeting the criterion of stability are: Class 11 (1A) for private passenger cars, Class 5CA for commercial cars, and the industry classification Franchised Dealers for garages – hazard 1.

By use of the following simulated data, it may now be demonstrated how classification relativity may be measured:

Private Passenger Class	Pure Premium	Loss Ratio	Indices* Based On Pure Premium	Indices* Based On Loss Ratio
11	$ 33.12	.722	1.000	1.000
12	38.06	.770	1.149	1.066
13	39.29	.603	1.186	.835
21	62.22	.746	1.879	1.033
23	121.78	.800	3.677	1.108
30	46.42	.770	1.402	1.066

* Ratio of each class to Class 11

The pure premium indices above measure the relationship of the loss cost per car for each class to the base class. Consequently, they also indicate how the rate for each class should relate to the rate for the base class, if it is accepted that the expense portion of the rate is obtained by a uniform expense loading. Thus, the rate for Class 12 should be 114.9% of that for Class 11, that for Class 13, 118.6% of the Class 11 rate, etc. However, pure premiums obtained from a consolidation of widely divergent bodies of experience must be used with great caution since they may contain distortions. The above model may contain in Class 11 a proportionally larger share of experience coming from low loss cost territories than is contained in the experience for Class 12. Consequently, a part of the indicated rate differential is purely due to distribution; this distortion due to distribution would have to be corrected for, prior to accepting pure premium indices as true indications of classification relativities.

The loss ratio indices have an entirely different meaning. It will be recalled that the unqualified term *loss ratio* for automobile liability insurrance is the ratio of the total limits incurred losses to the earned premiums resulting from the actual premiums charged during the experience period. The experience used in the model already reflects the fact that the different rate classes were charged different rates, reflecting a system of differentials in effect during the experience period. Let it be assumed that Class 12 rates were 110% of the Class 11 rates. If Class 12 developed losses 10% higher than Class 11, both classes should produce the identical loss ratios. If Class 12 does not produce the same loss ratio as Class 11, the existing 1.10 differential should be changed. The loss ratio index indicates the magnitude of the indicated change; in the above example, the index for Class 12 is 1.066, or the differential for Class 12 should be increased by 6.6%. As in the case of the pure premiums, caution is necessary in using this type of loss ratio for classification relativity. The inclusion of increased limits premiums and excess losses introduces an element usually not included in the determination of manual rates. Bodily injury liability loss ratios also reflect medical payments premiums and losses, which may not necessarily produce the same relativity of a rate class to the base as does the liability experience. There are, however, many advantages in favor of using *collected loss ratios*. These loss ratios can be obtained with relative ease directly from the experience; unlike pure premiums, they are less likely to be distorted by the influence of divergent distributions, since the premiums reflect the different rate and loss levels of the component territories; and finally, loss ratios based on the actual experience have an air of reality, reflecting the over-all underwriting record for each class.

Average incurred claim costs and frequencies may be similarly interpreted. (It should be borne in mind that the product of the two reproduces the pure premium.)

All of these quantities, properly used, have their place in the interpretation of experience.

A different measure of classification relativities, at present most readily accepted, but requiring quite elaborate calculations, is obtained by determining, for each class, the loss ratio the class would produce if the premiums were calculated at the rates in effect for the base class; loss ratios are calculated for each private passenger class at the rates for Class 11. After placing the loss ratios for each class on the level of the base class, indices can be calculated for each class that indicate the relativity to the base required for each class.

When all these calculations are completed, classification differentials are determined by a process of selection rather than by a formula approach. Such selection takes into account what can be gleaned from claim frequency, average claim cost, pure premium and collected loss ratio relationships, as well as the indices obtained from loss ratios at base class rates.

The classification differentials reflected in the present Bureau rating system are set forth in a later section, with additional comments appropriate for each of the major classification groups.

Trend Factors

Considerable time elapses before the latest available experience is reflected in the rates. The last portion of the experience for the latest accident year is reported by the companies approximately six months after the close of that year; several additional months are required for the processing of the data in the Bureau, which includes the summarization of the experience, preparation of rate review exhibits for the Bureau rating committees, review by the committees and the preparation of a rate filing. A means of reducing this time gap is the use in the ratemaking formula of trend factors based upon data that can be obtained for a more recent period.

The reports of paid losses and number of paid claims, filed currently by the companies under the established reporting procedure, provide a ready source of such later information. The Bureaus compile these data for each calendar year quarter from which average paid claim costs are calculated for each state. These data provide a record of the changes that have occurred in average claim costs, for a period subsequent to the experience period that is reflected in the classified experience.

They are used for the calculation of trend factors which are then superimposed upon the classified experience used for rate level determination.

Average paid claim cost data are compiled separately for private passenger cars and for all automobile classifications combined. The former are applied to private passenger experience, while the latter are used for the other classification groups. The influence of large loss payments, presumably involving excess losses, is reduced by excluding from each reported bodily injury loss payment the amount above $5,000; thus, the bodily injury average paid claim cost data are on an approximate basic limits basis. The influence of seasonal fluctuations is eliminated by calculating average paid claim costs for 12 month periods ended with successive quarters, i.e., 12 months ended March 31, June 30, etc.

At present, a three year period is used for the calculation of the average change in average paid claim costs. This three year period provides twelve values, one for each quarter-ended twelve month period. These data are fitted to a straight line by the use of the least squares method. From the values of average paid claim costs on this *line of best fit,* the average annual change in paid claim costs is determined. Multiples of the annual dollar change are used to determine the expected average paid claim cost at a date subsequent to the last period for which actual data are available.

A comparison of the extended value with the value of the last point of the straight line indicates the expected percentage increase or decrease in average paid claim costs. This percentage change is converted into a trend factor which is applied to the accident year or policy year loss experience.

The selection of the point of time to which the straight line is extended depends upon the experience period reflected by the latest available accident year or policy year experience. If, for example, the statewide rate level is to be based upon the latest accident year, the experience reflects the average loss level prevailing during the third quarter of that accident year. In that case, a trend factor reflecting eighteen months of subsequent change in average paid claim costs would adjust the loss level to approximately the date at which it might be expected that revised rates based upon such experience would be introduced. A longer period of subsequent change in average paid claim costs would be required if the rate level were to be determined on the basis of the mean of the latest two accident years, since such mean would reflect the average loss level prevailing at a time further removed from the time the experience is utilized.

Exhibit 6 illustrates the calculation of the average paid claim cost trend factor.

THE MAKING OF RATES

Before rates can be promulgated, a filing has to be submitted to the rate regulatory authority in the state affected, and, in most states, approval of the rates has to be obtained from that authority. The rate filing consists of a memorandum which explains the various steps in the development of the rate revision, supporting statistics and an exhibit of proposed rates.

Exhibit 6
Sheet 1

AUTOMOBILE LIABILITY INSURANCE - TREND FACTORS

BASED ON AVERAGE PAID CLAIM COSTS

Sheet 2 of this exhibit presents an example for trend factor calculation. The line of best fit average paid claim costs are calculated as follows:

BODILY INJURY

Year Ended	x *	Average Paid Claim Cost Actual (y)	x^2	xy	Line of Best Fit
3/31/60	-11	$ 624	121	$-6,864	$600.00
6/30/60	- 9	602	81	-5,418	609.56
9/30/60	- 7	603	49	-4,221	619.12
12/31/60	- 5	620	25	-3,100	628.68
3/31/61	- 3	624	9	-1,872	638.24
6/30/61	- 1	661	1	- 661	647.80
9/30/61	+ 1	669	1	+ 669	657.36
12/31/61	+ 3	672	9	+2,016	666.92
3/31/62	+ 5	678	25	+3,390	676.48
6/30/62	+ 7	670	49	+4,690	686.04
9/30/62	+ 9	690	81	+6,210	695.60
12/31/62	+11	718	121	+7,898	705.16
		7,831	572	+2,737	

Derivation of Line of Best Fit

Mean of y $= \bar{y} = \frac{\Sigma y}{12} = \frac{\$7,831}{12} = \$652.58$

Semi-quarterly increment (z) $= \frac{\Sigma\ xy}{\Sigma\ x^2} = \frac{\$+2737}{572} = \$+4.78$

Value for line of best fit $= \bar{y} + (x)\ (z)$

Example (3/31/60) $652.58 + (-11) ($4.78) = $652.58 - $52.58 = $600.00

* Number of semi-quarterly periods counted from the midpoint of the experience period 8/15/61.

The same calculations are made from countrywide data. The trend factor is the credibility-weighted average of the factor indicated by the state's experience and by the countrywide experience. A credibility table is used by .05 intervals, giving full credibility on the basis of the paid losses for the latest calendar year of $7.5 million for bodily injury and $1.0 million for property damage.

Exhibit 6
Sheet 2

STATE XX

AUTOMOBILE LIABILITY - PRIVATE PASSENGER CARS
Development of Factors to Adjust Accident Year
Data for Subsequent Change in Claim Costs
(Based on Calendar Year Average Paid Claim Cost Data)

(1) Year Ended	(2) Paid Losses*	(3) Number of Paid Claims	Average Paid Claim Cost (4) Actual (2)÷(3)	(5) Line of Best Fit	(6) Paid Losses*	(7) Number of Paid Claims	Average Paid Claim Cost (8) Actual (6)÷(7)	(9) Line of Best Fit
		Bodily Injury			Property Damage			
3/31/60	$6,021,489	9,649	$624	$600.00	$5,946,354	48,385	$123	$125.14
6/30/60	5,975,009	9,933	602	609.56	6,072,887	48,469	125	126.22
9/30/60	6,099,300	10,122	603	619.12	6,249,715	49,047	127	127.30
12/31/60	6,399,391	10,315	620	628.68	6,364,625	49,350	129	128.38
3/31/61	6,782,022	10,872	624	638.24	6,484,178	49,629	131	129.46
6/30/61	7,196,524	10,884	661	647.80	6,368,658	48,382	132	130.54
9/30/61	7,342,247	10,971	669	657.36	6,305,692	47,862	132	131.62
12/31/61	7,528,739	11,204	672	666.92	6,259,689	46,099	136	132.70
3/31/62	7,717,863	11,380	678	676.48	6,411,388	48,109	133	133.78
6/30/62	7,759,403	11,580	670	686.04	6,746,592	49,829	135	134.86
9/30/62	8,066,606	11,690	690	695.60	6,891,570	50,627	136	135.94
12/31/62	8,356,618	11,641	718	705.16	6,996,132	52,032	134	137.02

	Bodily Injury	Property Damage
(10) Average annual dollar change in paid claim costs based upon line of best fit		
a) State	$+38.24	$ + 4.32
b) Countrywide	+11.84	+ 3.92
(11) Average dollar change in paid claim costs in 18 month period (Line 10 times 1.50)		
a) State	$+57.36	$ + 6.48
b) Countrywide	+17.76	+ 5.88
(12) Average change in paid claim costs in 18 month period expressed as percent		
a) State #	+ 8.1%	+ 4.7%
b) Countrywide	+ 2.3	+ 4.1
(13) State credibility	1.00	1.00
(14) Indicated factor 1.0 + [(12a) x (13)] + [(12b) x [1.0 - (13)]]	1.081	1.047

B.I. $57.36 ÷ $705.16
P.D. $ 6.48 ÷ $137.02

* Excluding all loss adjustment expenses.

It is submitted with a letter of transmittal which usually specifies the proposed effective date of the revised rates.

In the following discussion, examples will be used in the form of exhibits usually found in such rate filings. The major steps in the development of the rate revision are:

Determination of statewide rate level
Development of rate level changes by territory
Calculation of classification rates

Generally, rate programs for any given year and group of classifications reflect a pattern which is followed in all states. The pattern established for determination of the statewide rate levels in any given cycle of rate revisions, the method used for the development of territory rate level changes, the evaluation of the experience through the use of credibility tables, and the various other elements of the ratemaking process have the objective of producing consistency in the interpretation of experience.

The aggregate of these procedures is a ratemaking formula that should produce rates that are adequate, not excessive and not unfairly discriminatory. The ratemaker, as well as the rate regulatory official, finds comfort in the formula approach; with each state, territory and class treated alike as the formula works, unfair discrimination has no place in manual ratemaking. There are differences of opinion on the propriety of the present ratemaking formula in meeting the requirement that rates be adequate and not excessive. It is not the purpose of this paper to expand on this discussion. The formula is presented as one that does produce adequate and non-excessive rates, as is stated in the rate filings by the rating bureaus.

The use of a formula does not mean that automobile liability insurance ratemaking should or has become a mechanical process. The ratemaker and the reviewer of rates have to be willing and able to depart from the formula by superimposing upon it such modifications as special circumstances require to give due consideration to all relevant factors, in addition to past experience, as mandated by the rating laws.

Statewide Rate Level

As was noted before, past experience is taken as an indication of the required premium level for the immediate future during which the rates are to apply. It is necessary to select the experience period which is most likely to meet this expectation. While responsiveness of the experience is desirable, it is also necessary to select a base that has stability in order to avoid large fluctuations in the rates from year to year. For some states, the volume of experience for the latest year is large enough to meet both requirements. In others, a balance has to be found between stability and responsiveness by giving weight to the latest two years for statewide rate level determination. Thus, there is in use a schedule (based upon the combined bodily injury and property damage liability premium at manual rates for the latest year) according to which weight is given in any state to the experience for the latest two years. Corresponding to these weights, varying multiples of the average change in average paid claim costs are used to adjust the accident year or policy experience to current level.

These schedules are shown below:

Premium Volume (Premium at Manual Rates)	Weight: Prior Year	Weight: Latest Year	Trend Factor Annual Change Multiples
	Private Passenger Cars		
Under $5,000,000	30%	70%	2.00
$5,000,000 or more, but less than $20,000,000	15%	85%	1.75
$20,000,000 and over	0	100%	1.50
	Commercial Cars		
Under $1,000,000	50%	50%	2.25
$1,000,000 or more, but less than $7,500,000	30%	70%	2.00
$7,500,000 or more, but less than $30,000,000	15%	85%	1.75
$30,000,000 and over	0	100%	1.50
	Garages		
All sizes	30%	70%	2.75

A typical rate level calculation is shown in Exhibit 7, demonstrating the development of proposed statewide rate level changes for private passenger cars. While in this example the statewide premium volume is large enough to warrant basing the rate level on the experience for the latest year, the experience for the preceding year is also shown for comparison purposes. This is a state in which standard coverage requires 10/20/5 limits; this is, therefore, the lowest limit at which rates are published in this state. Consequently, premiums and losses used in ratemaking are at these limits.

Exhibit 7

AUTOMOBILE LIABILITY INSURANCE - PRIVATE PASSENGER NON-FLEET

Development of Statewide Rate Level Changes

(1) Coverage	(2) Accident Year	(3) 10/20/5 Limits Earned Premium at Present Collectible Rates ∅	(4) 10/20/5 Limits Incurred Losses *	(5) Number of Claims	(6) Loss & Loss Adjustment Ratio at Present Rates (4) ÷ (3)	(7) Accident Year Weights	(8) Weighted Loss & Loss Adjustment Ratio at Present Rates
B.I.	1961	$ 15,010,758	$10,506,865	10,679	.700	0%	.694
	1962	15,150,080	10,510,586	11,114	.694	100	
P.D.	1961	10,082,544	7,045,698	43,934	.699	0%	.688
	1962	10,185,639	7,010,762	44,464	.688	100	

(9) Coverage	(10) Factor to Adjust Losses for Subsequent Change of Average Paid Claim Costs	(11) Rate Level Loss Ratio (8) x (10)	(12) Expected Loss & Loss Adjustment Ratio	(13) Credibility	(14) Indicated Rate Level Change $\left[\frac{11}{12} - 1.0\right] \times (13)$
B.I.	1.041	.722	.657	1.00	+ 9.9%
P.D.	1.000	.688	.657	1.00	+ 4.7
Total					+ 7.8

* Including all loss adjustment expenses. Factors of 1.10 for B.I. and 1.16 for P.D. were applied to losses and allocated loss adjustment expenses to include unallocated loss adjustment expenses. The accident year incurred losses have been developed to 63 months for bodily injury and 39 months for property damage by application of the following loss development factors:

Accident Year	B.I.	P.D.
1961	.984	.993
1962	1.064	.977

∅ The earned premium at present collectible rates takes into account the manual rates and rules modifying private passenger rates.

Data are shown separately by year and separately for bodily injury and property damage liability, as identified in Columns 1 and 2. Column 3 shows the premium that would be collected under the rate structure in effect at time of rate review for all units insured during the experience period. Reference is made to the earlier section (p. 160) explaining the calculation of premiums at manual rates. From that earlier example it is noted that the premium at present rates not only reflects the rates that are printed in the Manuals but also the rules that are superimposed upon the rates in the rating of a private passenger car, reflecting, where applicable, rate reductions given to compact cars, multi-car risks, driver training credit for youthful operators, and the application of the Safe Driver Insurance Plan.

Column 4 shows the incurred losses for the two accident years at basic limits and developed by loss development factors as explained in the footnote. The calculation of the loss development factors was previously explained and is set forth in Exhibit 4.

Columns 5, 6 and 7 are self-explanatory.

Column 8 is a simple calculation of weighted averages:

$$\text{B.I.} \quad .700 \times 0 + .694 \times 1.00 = .694$$
$$\text{P.D.} \quad .699 \times 0 + .688 \times 1.00 = .688$$

The factors shown in Column 10 are determined as previously explained. However, the factors developed on Exhibit 6, Sheet 2 were modified in the rate filing, recognizing other relevant underwriting information. Column 11 is described as the rate level loss ratio. It reflects the premium resulting from the rates in effect at time of rate review and the losses incurred during the experience period adjusted for any changes that have occurred in claim costs since the average date reflected by the accident year losses. Thus, if past loss experience will repeat itself, the present rates will produce the loss ratios shown in Column 11 for the immediate future. If the rate level loss ratios are higher than the percentage of the premium dollar available for losses, rates have to be increased; if lower, a decrease is in order. The expected loss and loss adjustment ratio is shown in Column 12. The indicated percent change in rate level is determined by the division shown in Column 14:

$$\text{Indicated rate level change} = \frac{\text{Rate level loss ratio}}{\text{Expected loss ratio}} - 1.00$$

If the statewide experience is given less than full credibility, the credibility factor is applied to the result of the above calculation. (See Credibility Table, Page 166)

Exhibit 7 thus shows that the overall statewide premium level for private passenger cars should be increased by 9.9% for bodily injury and by 4.7% for property damage liability insurance.

The change indicated for the loss portion of the rates, therefore, affects the total premium which provides for the necessary losses and expenses.

It is in order to comment briefly on this inter-relationship of loss and expense requirements. By far the largest part of the expense portion, by its nature, is a percentage function of the total premium. The production cost item reflects an allowance for commission and brokerage and for other acquisition costs. Compensation to producers, for the companies operating through the agency system, is generally determined, as percentage of the total premium, by the contractual relationship between the companies and the producers. Taxes, likewise, are a percent of total premium. Dividends to policy holders, although not a contractual obligation of the company, are paid as a percentage of premium. This leaves only the expense items General Administration and Inspection and Bureau for which a percentage relationship to the loss level is not obvious. To a great extent, these expenses, subject to inflationary influences, have risen with the loss level during the past 20 years. However, these expense requirements are reviewed independently, based on the Insurance Expense Exhibit. If the dollars required for these expense items related to the total premium dollars produce ratios lower than provided for in the expense loading, the percentage expense provision is reduced; or it is increased in the contrary case.

Exhibit 7 demonstrates the rate level calculation for private passenger cars, based upon the experience on private passenger cars that are not insured under a fleet plan (non-fleet). The resulting rates are used for both fleet and non-fleet private passenger cars. The ratemaking procedure does not provide for the inclusion of private passenger fleet experience in the ratemaking data.

The statewide rate level is determined by the same procedure for commercial cars and for garages with some differences, as noted below.

For commercial cars, fleet and non-fleet experience is used in ratemaking. In the calculation of premiums at manual rates, recognition is given to the *graded premium reduction* for fleet risks based upon number of cars in each fleet. (See Automobile Casualty Manual Rule 9.) In order to reflect this rate modification, periodic samples are obtained by the Bureaus of the distribution of exposures by size of fleet. From these distributions, an average reduction factor is obtained which is applied to the fleet premiums at manual rates, usually on a statewide basis. The experience period used for statewide rate level on commercial cars varies in accordance with the table of weights shown earlier in this section.

Garage experience is compiled on a policy year basis, utilizing the two latest complete policy years as of 27 and 39 months respectively for statewide rate level. Since this policy year experience reflects an average loss level further removed in time from the loss level at the time of rate review, a longer period is reflected in the trend factor than is used on the accident year data for private passenger and commercial cars.

The calculation of premiums at manual rates for garages contain some departures from the previously described methods. Garages may be insured on two bases under the Automobile Casualty Manual: Hazard 1 coverage applies to the premises and operations including owned and rented automobiles, as well as automobiles in the custody or control of the insured. Hazard 2 coverage does not include such automobiles.

The manual defines three rate classes for Hazard 1 and one class for Hazard 2 with exposures measured by payroll and, in addition, a minimum premium for each of the two divisions, on a per location basis. The Automobile Statistical Plan does not contain separate classifications corresponding to the payroll classes (a), (b) and (c) for Hazard 1. Through periodic Special Calls, data are obtained by the Bureaus that provide distributions of exposures by payroll class and by size of payroll, which are used in the premium at manual rate calculations. While these samples provide adequate information on the exposure distribution by rate class and size of risk, loss statistics are not available in comparable detail. No satisfactory method has yet been found by which accidents can be reliably related to the payroll class of a garage risk. Therefore, the relationship between the rates for payroll classes (a), (b) and (c) has to be based on judgment.

The above sets forth the calculation of the statewide rate level change by use of the loss ratio (at manual rates) method. Another method used is that based on pure premiums. In that case, the experience pure premium is compared with the underlying pure premium. Algebraically, both methods involve the use of the same quantities and produce the same results. This can be demonstrated as follows:

Formulae for calculation of indicated rate level change I:

Loss Ratio Method

(1) $$I = \frac{\text{Statewide loss ratio at manual rates}}{\text{Expected loss ratio}}$$

Pure Premium Method

(2) $$I = \frac{\text{Statewide experience pure premium}}{\text{Underlying pure premium}}$$

The statewide loss ratio at manual rates is determined as:

$$\frac{\text{Statewide sum of losses}}{\text{Statewide sum of premiums at manual rates}}, \text{ or}$$

(3) $$\frac{\sum l}{\sum er}$$

where e are the exposures for each class within each territory
r are the manual rates corresponding to the exposures
l are the losses

The statewide underlying pure premium is determined as:

Statewide average rate × Expected loss ratio, or

(4) $$\frac{E \cdot \sum er}{\sum e}$$

where E is the expected loss ratio, e and r as defined above.

The statewide experience pure premium is determined as:

(5) $$\frac{\text{Statewide losses}}{\text{Statewide exposures,}} \quad \text{or}$$

$$\frac{\sum l}{\sum e}$$

Substituting in Formula (2) the identities from (5) and (4), we find

(6) $$I = \frac{\sum l}{\sum e} \div \frac{E \cdot \sum er}{\sum e}$$

$$= \frac{\sum l}{\sum e} \times \frac{\sum e}{E \cdot \sum er}$$

$$= \frac{\sum l}{E \cdot \sum er}$$

Substituting in Formula (1), we find

$$(7) \qquad I = \frac{\sum l}{E \cdot \sum er}$$

which is identical with formula (6)

Depending upon circumstances, the ratemaker will use either the loss ratio or the pure premium method. The latter has advantages since the calculation of the underlying pure premium is usually based upon the exposure distribution for the latest year, which saves work in calculating the premium at present rates. Provided no significant change in distribution occurred during the years of the experience period, the underlying pure premium can be compared with the experience pure premium of several years. Fewer clerical operations are involved in averaging pure premiums for two or more years or coverages than in working with loss ratios. The pure premium method was more widely used before the advent of fast calculating machines. At present, the Bureaus generally use the loss ratio method for the major subdivisions of private passenger, commercial cars and garages, while many of the other classifications are reviewed on a pure premium basis.

The two methods described above are used where premiums at manual rates can be calculated. There are situations where data are not available in the required detail for such calculation or where the effort to accomplish this would be disproportionate. Under such circumstances, the rate review is based on total limits loss ratios, i.e., loss ratios based on the total collected earned premiums and the total incurred losses, with such adjustments as are appropriate. Examples for this type of rate review are in a later section of this paper.

Territory Rate Level

The statewide rate level change is next distributed among the territories within the state, in accordance with each territory's contribution to the statewide experience. The territory experience is reviewed on the basis of a longer experience period than is used for statewide rate level. This provides a broader base which reduces the influence of any chance fluctuations in the experience due to the relatively smaller volume on a territory basis. For private passenger cars, three years of territory experience are used at present. For commercial cars and garages, which develop a considerably smaller volume, the latest five years of territory experience are used.

Each state is divided into a number of territories ranging from two territories for small states such as New Mexico to about 70 territories for the state of New York. The territory subdivisions are established by the Bureaus on the basis of surveys and underwriting judgment that take into account characteristics bearing on the loss-producing potential, such as population density, intensity and flow of motor vehicle traffic, frequency and severity of accidents, etc. The existing territory structure is periodically reviewed by the rating organizations and changes are made where required. As was noted before, experience is recorded and reported separately for each of these territories.

The basic approach to the development of rate level changes for each territory within a state is by a formula which is explained by reference to Exhibit 8. Frequently, two or more territories may be combined for the development of a single rate schedule where the experience does not warrant differentiation. It may require one or more test runs of the data, before Exhibit 8 can be produced in its final form.

After decisions have been made on the desired territory combinations, certain basic data are posted on Exhibit 8 from source material for each territory, as identified in Column 1.

The data shown in Columns 2 and 4 are obtained from the territory experience exhibit (Exhibit 5). Column 2 shows the number of earned exposures for the latest year as an indicator of volume in each territory and also for use in weighting of certain data, as will be shown later. Column 4 shows the experience pure premiums for the experience period.

The average manual rate shown in Column 3 is based on the exposures for each class in each territory and the corresponding rates, as explained on page 160. Column 5 shows the loss and loss adjustment ratio at manual rates. It could be obtained from the premiums at manual rates and the incurred losses for each territory. In this exhibit, it is obtained by dividing the experience pure premium by the average rate. From the prior proof regarding the pure premium and the loss ratio methods it can be seen that the same results would be obtained either way.

Column 6 shows the credibility given to the experience in each territory. The credibility is determined from the number of incurred claims shown on Exhibit 5 and the table of credibility factors shown earlier in this paper. The statewide totals are obtained by addition for column 2, as weighted averages for columns 3 and 4, using column 2 as weights, and by the same calculation as for the territory entries for column 5.

AUTOMOBILE LIABILITY INSURANCE - PRIVATE PASSENGER CARS

Development of Rate Level Changes by Territory

Bodily Injury - 10/20 Limits

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
			Accident Years 1960 - 1962							
Territory	Accident Year 1962 Earned Number of Cars	Present Average Rate	Pure Premium (Incl. All Loss Adj.)	Loss & Loss Adj. Ratio at Present Rates (4) ÷ (3)	Credibility	Formula Loss & Loss Adjustment Ratio at Present Rates	Col. (7) as Ratio to Statewide Average	Territorial Rate Level Change [(8) x Statewide Rate Level Factor]-1.0	Average of Present Differentials to Rate Class 1A	Proposed Class 1A Rate
11,12,13 & 16 Combined	137,380	$ 38.65	$ 26.55	.687	1.00	.687	1.027	+12.9%	1.137	$ 38
14,15 & 84 Combined	72,463	36.95	26.06	.705	1.00	.705	1.054	+15.8	1.119	38
17	21,717	25.31	16.87	.667	1.00	.667	.997	+ 9.6	1.150	24
18	18,259	44.09	31.01	.632	1.00	.632	.945	+ 3.9	1.142	45
19	10,894	32.44	22.41	.691	.80	.687	1.027	+12.9	1.119	33
21	21,676	31.67	20.53	.648	1.00	.648	.969	+ 6.5	1.131	30
22	12,111	32.60	26.36	.809	.90	.795	1.188	+30.6	1.124	38
23	39,974	26.98	17.29	.641	1.00	.641	.958	+ 5.3	1.124	25
24	23,790	33.86	22.19	.655	1.00	.655	.979	+ 7.6	1.092	33
25	29,427	25.84	14.63	.566	1.00	.566	.846	- 7.0	1.123	21
26	11,206	31.73	18.62	.587	.70				1.094	
27	3,894	33.00	21.56	.653	.30				1.087	
28	40,767	31.51	19.86	.630	1.00				1.087	
Sub-Total of 26,27,28	55,867	31.66	19.73	.623	1.00	.623	.931	+ 2.3	1.092	30
Total	443,558	34.45	23.06	.669		.669				

NOTE: Territory combinations 11,12,13 and 16, and 14,15 and 84 respectively represent areas in which boundaries were re-defined during the experience period. They have to be used in combination until experience for the separate parts becomes available.

Column 7 is called the Formula Loss and Loss Adjustment Ratio at Present Rates. It is calculated as the weighted average of the territory loss ratio in column 5 and the statewide loss ratio in column 5, giving weight to the territory loss ratio to the extent of the credibility given to the territory, with the complement of that credibility given to the statewide loss ratio. This calculation can be expressed by the following formula:

$$\text{Col. (7)} = \text{Col. (5)} \times \text{Col. (6)} + \text{Col. (5) Statewide Total} \times [1.0 - \text{Col. (6)}]$$

The formula loss ratio in column 7 provides the basic indicator for the share of the statewide rate level change that will eventually be assigned to each territory. A review of the formula and Exhibit 8 shows that, for a territory with full credibility, the loss ratio in column 5 becomes the formula loss ratio in column 7. For territories that have less than full credibility, the territory's own experience is recognized to the extent of the territory's credibility. The inclusion of the statewide average experience in the formula tends to keep fluctuations within narrower limits for territories in which their experience might produce chance fluctuations because of limited volume.

In the example presented in Exhibit 8, three territories (26, 27, 28) were combined for the development of the formula loss ratio in column 7. The sub-totals for columns 2, 3, 4 and 5 were obtained in the same manner as set forth above for the statewide totals; the credibility in column 6 is based on the number of claims for the territory combination. The formula loss ratio in column 7 for this combination was calculated from the sub-total entries in the preceding columns; thus, a territory combination is used as if it were a single territory.

The statewide average for column 7 is calculated from the territory entries in column 7 using as weights the products of (2) and (3). This average is not necessarily the same as the statewide average in column 5, as it happens to be in this example.

In column 8, the quantities shown in column 7 are expressed as ratios to the statewide average. For example, for the first territory entry, the index of 1.027 is obtained by dividing .687 for the territory by the statewide average of .669. Column 8 indicates the percentage departure of the loss ratio of each territory from the statewide average. These indices in column 8, translated into percentage changes, show the indicated change in rate level for each territory, prior to any change in the statewide rate level. (The indices in column 8 average to 1.000 using columns 2 and 3 as weights.) Thus, if it were desired to adjust only the territory rate levels without a change in the statewide premium level, rates would have to be increased 2.7% in territory 19, decreased by 3.1% in territory 21, etc.

Column 9 combines the rate level change indicated for the territory with the previously determined statewide rate level change. In Exhibit 7, it was determined that the statewide bodily injury rate level changes shall be an increase of 9.9%. Consequently, the factor used in column 9 is 1.099.

If there were only one rate in each territory, the precent changes in column 9 applied to that rate would produce the revised rate. Since, however, the territory rate level changes will affect more than one class within each of the major classification groups, additional steps are required before the revised rates can be determined. We shall return to Exhibit 8 for the additional calculations in the succeeding chapter after dealing with classification relativities.

Rates for the Major Classification Groups

Within the major classification groups of private passenger cars and commercial cars, rates for the various classes are related to each other by percentages, referred to as *classification differentials.* The method of determining these differentials has been explained in a prior section. The following tables set forth the differentials reflected in the rates in most states:

PRIVATE PASSENGER CARS

Table of Differentials to Class 1A Rates

TABLE I—STANDARD

Class	Large Cities	Small Cities
111 (1A)	1.00	1.00
112 (1B)	1.10	1.00
113 (1C)	1.45	1.45
115 (1AF)	.70	.70
121 (2A)	1.90	1.90
123 (2C)	3.10	3.60
125 (2AF)	1.33	1.33
127 (2CF)	2.17	2.52
130 (3)	1.50	1.50

COMMERCIAL CARS

Table of Differentials to Class 5CA Rates

Class	Major Cities	All Other
3CA	1.65	1.95
3CB	2.65	3.15
4CA	1.25	1.30
4CB	1.90	2.05
5CA	1.00	1.00
5CB	1.50	1.70
6	.55	.60
7CA	1.45	1.55
7CB	2.15	2.35
8CA	.80	.80
8CB	1.50	1.35
9	.47	.51

For private passenger cars, the table of differentials is expanded to reflect manual rules that modify the rates shown in the manual, viz., the 10% reduction for compact cars, the reduction for multi-cars which is generally 20%, and the reduction of 10% granted to youthful drivers who have completed a driver education course. The rates printed in the manual are the rates prior to these modifications. Thus, a manual rate of $100 for Class 1A would be $90 if the car is a compact car; it would be $80 if there are two standard size cars insured for the same household; the rate for one of these two cars would be $72 if it were a compact car, etc. These modifications superimposed on the rate class differentials produce differentials that reflect all possible combinations.

From the above table it is noted that different sets of differentials apply for large cities compared with other areas. For private passenger cars, territories are defined as large city territories if the territory includes a city with a population of 40,000 or more. This distinction presumably recognizes different driving patterns in the use of the automobile in driving to and from work. While it was based on underwriting judgment when the 6 Class Plan for private passenger cars was first introduced, subsequent experience gave support to this type of territory identification. For commercial cars, large cities are the cities with a population of 500,000 or more.

The base classes to which the differentials are applied are Class 1A for private passenger and Class 5CA for commercial cars. These base rates are developed from the average rate that can be determined for each territory after the territory rate level change has been established. On Exhibit 8, column 9, the percent change for each territory is shown. This percent change, applied to the average rate in column 3, produces the revised average rate. The Class 1A rate is found by dividing the revised average rate by the average differential. The latter is the average of the applicable differentials, each weighted by the corresponding exposure in the territory. This calculation is designed to reproduce in each territory, as closely as possible, the indicated premium level, taking into account the distribution of business by the various classes for the latest year. If there is no change in the existing rate class differentials, the average differential is more directly obtained by dividing the average rate by the Class 1A rate in effect at time of rate review. The extension of differentials by exposures is necessary if, in conjunction with a rate level change, differentials between classes are also revised. In that case, column 10 would show the average of the proposed differentials.[14] This exact calculation is necessary in such case so that the revised rates with the new differentials will in the aggregate reproduce the indicated average rate.

Rates for the other classes within the classification group are obtained by multiplying the revised base rate by the applicable differentials, i.e., the Class 1A rate times the private passenger differentials and the Class 5CA rate by the commercial car differentials. All manual rates for private passenger and commercial cars are rounded to the nearest dollar.

For garages, the rate level change developed for the territory is applied to the existing rates for Classes (a) (b) and (c) for Hazard 1 and the single class for Hazard 2. These rates are shown in dollars and cents in

[14] When differentials are changed, reductions for some classes and increases for others are not necessarily in balance. The average of the proposed differentials compared with the average of the existing differentials indicates the *off-balance* of the new system. Example for calculation of average differential:

Class (1)	Exposure Distribution (2)	Differentials (3)	Product (2) × (3) (4)
A	.35	1.00	.350
B	.20	.90	.180
C	.30	1.50	.450
D	15	2.25	.338
Total	1.00		1.318

1.318 is the average of the differentials in Col. (3)

the manual. The garage minimum premiums are adjusted by the same percentages as the rates, subject to certain limitations.

Rates for Other Classifications

The Automobile Casualty Manual contains rates for many other types of risks in addition to the major classifications discussed in the preceding section. Some of these rates are shown on the rate pages while the premium charges for other classes are set forth in the rules in the various sections of the Automobile Casualty Manual.

For some of these classifications, experience is compiled in detail by class and territory and is reviewed on the basis of loss ratios at manual rates or pure premiums and underlying pure premiums, such as rates for taxicabs. For most of these miscellaneous classifications, however, the experience is relatively sparse and it is compiled only on a statewide basis. Consequently, premiums at manual rates cannot be computed; use is made of total limits loss ratios which are sometimes adjusted to present rate level by average factors that reflect the premium level changes from the time reflected in the experience period to the time of rate review.

For most of these classifications, the hazard of any one class can be related to that of a class within the major classification groups. For example, motorcycles are likely to be used for pleasure or in going to and from work, similar to the use of private passenger cars. This similarity in use suggests a relationship in rates. The percentage relation is obtained by comparing total limits loss ratios for the same experience period for such related classes. If rates for motorcycles are 75% of the private passenger rate and they produce approximately the same loss ratio as do private passenger cars, it can be concluded that the 25% difference in premium properly recognizes a corresponding difference in loss level. If the loss ratios differ significantly, a change in the percentage relationship of rates is indicated.

These relationships are reviewed periodically, but not as frequently as the rates for the major classification groups. Between reviews, the premium charges for the related classes will change with the rates for the classes to which they are related.

Assigned Risk Experience and Rates

Risks that do not meet the ordinary underwriting standards are distributed among the companies on the basis of each company's participation in the total automobile liability business. This distribution of risks is performed by the Automobile Assigned Risk Plan, an instrumentality maintained in each state by all companies writing automobile liability insurance. Over many years, assigned risks have produced, in the aggregate, extremely adverse loss experience, while the number of such risks continues to increase. Approximately 90% of the premiums developed from assigned risks come from private passenger cars. The adequacy of rates charged such risks and the effect of this portion of the atuomobile liability insurance market on the total experience is of particular concern.

The Assigned Risk Plan in every state contains a section dealing with the rates that the company shall apply to risks assigned to it. Initially, this section stated in all states that the company shall apply the rates produced by its own rating system, combined with a provision for additional charges that apply to a risk that has had a record of accidents, or of convictions for violation of the motor vehicle laws. The additional charges varied with the number of accidents and severity of law violations. This provision still applies in several states.

In recent years, a different method of rate treatment has been introduced in a number of states. Under this method, private passenger rates for assigned risks are developed from the assigned risk experience of all companies and these rates are filed by the Bureaus on behalf of their companies and individually by each of the non-Bureau companies. (For classifications other than private passenger, the procedure described in the preceding paragraph is generally in force.) These rates are further subject to additional percentage charges for risks that, during a stated experience period prior to issuance of the policy, have had accidents or have been convicted for motor vehicle law violations. These rates for assigned risks are determined by the ratemaking method described in this paper; because they are based on their own experience, they tend to be more nearly self-supporting than the rates charged assigned risks in states where this method is not used. Unfortunately, assigned risk rates are more nearly self-supporting in states in which the assigned risk premium is relatively small. They have remained inadequate in many states, among them the states accounting for most of the assigned risk premium volume.

In order to maintain an over-all adequate rate level, the private passenger assigned risk experience in most states is combined with the private passenger experience not written through the Assigned Risk Plan (such business is frequently referred to as *voluntary business* to distinguish it from the assigned risk business). In this combination, the premium at present manual rates for voluntary business is determined as explained earlier; that for the assigned risk portion reflects the rates and the additional charges applicable to assigned risks. Also, the expected loss and loss adjustment ratios for voluntary and assigned risks[15] are weighted to produce an average expected loss and loss adjustment ratio to be applied to the total experience. By this procedure, any deficiency[16] in the assigned risk rate level is reflected in the experience used for making manual rates that apply to voluntary risks.

A full explanation of this matter as it affects rates would require a complete exposition of the statistical treatment of assigned risk experience and the variations from state to state. The latter depend upon the degree of adequacy obtained in the assigned risk rates and the readiness of the rate supervisory authorities to approve, for all assigned risks, rates that are higher than those applied to voluntary business. On the latter point, some raise the question of unfair discrimination that may result if assigned risks without accident or conviction records, even though in the aggregate they produce adverse experience, were required to pay higher rates than similar risks accepted in the voluntary market.

Package Automobile Policies

The marketing of a combination of automobile insurance coverages for an *indivisible premium*, in use in the property insurance field since the early fifties, was adopted in 1959 for Bureau companies with the development of the Special Automobile Policy for private passenger cars by the National Bureau of Casualty Underwriters and the National Automobile Underwriters Association and a similar Package Automobile Policy by the Mutual Insurance Rating Bureau.[17] Provision was made in the Automobile Statistical Plan for the separate recording and reporting of experience developed under these policies.

The package policies of the National Bureau and the Mutual Bureau consist of two parts: Part I, paralleling the coverages provided in the Auto-

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[15] The difference lies in the production cost allowance which is lower for assigned risk business than for voluntary business.

[16] We need not concern ourselves with the possibility of redundancy; if assigned risks should develop experience better than average, competition among the companies would soon absorb such risks in the voluntary market.

[17] The above announcement by NBCU was accompanied by the introduction of a refinement in the private passenger classification system based on the accident and traffic law violation record of the individual insured, the Safe Driver Insurance Plan. Both innovations occurred at a time when the relationship of the member and subscriber companies to their respective rating organizations experienced a change in the direction of lesser rigidity and greater recognition of the need for experimentation in the classification of risks, pricing systems, and marketing methods. Several of the Bureau companies developed their own form of package policies and introduced different types of merit rating plans for private passenger cars.

mobile Casualty Manual, and Part II, those in the Physical Damage Manual. Part I provides coverage for bodily injury liability, property damage liability, medical expense and insurance for bodily injury caused by uninsured motorists on a combined basis for a single premium charge. This section will deal with ratemaking for Part I.

For a better understanding of the pricing formula which was used to develop a single premium for this combination of coverages, note should be taken of some of the differences between the coverages included in the Special Automobile Policy (S.A.P.) and the corresponding coverages provided in the Automobile Casualty Manual for the Family Automobile Policy (F.A.P.):

Liability Limits: The basic limits under the Family Automobile Policy are $5,000/$10,000 or $10,000/$20,000 for bodily injury (depending upon the minimum requirements of the financial responsibility laws in each state) and $5,000 for property damage liability. These limits provide larger maximum amounts for bodily injury than for property damage liability, and higher amounts in the case of bodily injury or death to two or more persons than if only one claimant is involved in one accident. The insured has a choice of a variety of other limit combinations.

The Special Automobile Policy provides coverage at a single liability limit, so that the same maximum amount is available for indemnification whether an accident involves one or more injured persons, or whether it involves only bodily injury, only property damage, or both. In a 5/10/5 state, the lowest available single limit is $15,000; in a 10/20/5 state, $25,000. A limited number of higher single limits is available.

Medical Expense Coverage: An insured covered under the Family Automobile Policy may, if he wishes, purchase medical payments insurance at a selected limit in conjunction with the coverage. Available data indicate that about 75% of private passenger cars insured for bodily injury under this policy also carry medical payments insurance. The Special Automobile Policy includes a minimum of $1,000 medical expense coverage. The medical coverages under the two policies, however, are not identical. Under the Family Automobile Policy, medical payments insurance is a separate and distinct coverage. Under the Special Automobile Policy, the medical expense provisions specify that, as a condition of payment, the injured person is required to execute a covenant not to sue any person insured under the liability coverage of the policy, or the insurance company that issued the policy, for the medical expense. The possibility of duplication of payment under the liability and medical expense coverages is thereby eliminated. In addition, medical expense coverage does not apply under the package policy if the expenses are paid or are payable under other forms of insurance affording benefits for medical expenses.

Uninsured Motorist Coverage: At the time of introduction of the Special Automobile Policy, uninsured motorist coverage was available on an optional basis under the Family Automobile Policy in most states.[18] Under the Special Automobile Policy, every insured receives this coverage.

The effects of these differences were given recognition in the design of the formula by which the original rates for the Special Automobile Policy were calculated. In addition, the formula reflected anticipated expense savings in the marketing of automobile insurance and in the processing of the accounting and statistical records.

The component parts of the single premium charge for the lowest available limit were based on the charges for the corresponding coverages in the Automobile Casualty Manual, with certain modifications, as set forth below:

Liability Coverages: The manual 10/20 bodily injury rate is adjusted $25,000 limit. To reflect the increment from 10/20/5 coverage to to 25/25[19] limits and the manual $5,000 property damage rate to the $25,000 single limit coverage, the applicable factor in the Automobile Casualty Manual for bodily injury was used and a selected factor (lower than the manual factor) for property damage, both discounted in accordance with a table shown further below.

Medical Expense Coverage: One half of the medical payments charge applicable under the Automobile Casualty Manual.

Uninsured Motorist Coverage: One half of the uninsured motorist rate applicable under the Automobile Casualty Manual.

The sum of the charges so determined was further reduced by a packaging discount factor; in most states, this discount amounted to 10%. Since

[18] Since that time, laws have been passed or regulations have been issued in many states that require that every automobile liability policy contain this coverage, unless rejected by the insured. Under the Family Policy and the Package Policy, uninsured motorist insurance provides bodily injury coverage at limits corresponding to the limit requirements of the Financial Responsibility Law. (In a few states a limited property damage coverage is also included.)

[19] 5/10 and 15/15 respectively for a $15,000 single limit.

rates in the Automobile Casualty Manual are for annual coverage, while the package policy rates are published for a semi-annual term, the result has to be multiplied by .50; the product is rounded to the nearest whole dollar.

The discounts applied to the manual increased limits factors are obtained from the following table:

B.I. Rate Percent of B.I. & P.D. Combined Rate		Discount for Single Limit	
At Least	But Less Than	$15,000	$25,000
—	15%	.025	.020
15%	20	.035	.025
20	25	.040	.030
25	30	.045	.035
30	35	.050	.040
35	40	.055	.040
40	60	.060	.040
60	65	.055	.040
65	70	.050	.040
70	75	.045	.035
75	80	.040	.030
80	85	.035	.025
85	—	.025	.020

The above discounts were selected by judgment. Assuming that an insured purchases a single limit at least as high as the per accident bodily injury limit he had heretofore available under his split limit coverage, the package policy provides more coverage than was granted under the policy written on a split limit. For example, an insured who carried 20/40/5 coverage is likely to purchase a $50,000 single limit policy. While the company liability also increases with the granting of the higher single limit coverage, it is expected that loss distributions will not change materially. Therefore, if past experience shows that the aggregate premium charged for coverage for the various limit combinations on a split limit basis was adequate to cover excess losses, the premium for the broader single limit coverage can be reduced. The actual selection of the discount factors was made by relating premium charges for various split limit combinations within each bracket of single limit coverage to the single limit charge obtained by a straight application of the increased limits table.

The following example illustrates the calculations of a single limit package policy rate:

Given:

Manual B.I. rate = \$60 (10/20 limit)
Manual P.D. rate = \$40 (\$5,000 limit)
Manual Medical rate = \$9.00
Manual U.M. rate = \$3.00

\$25,000/25,000 increased limits B.I. factor = 1.12
\$25,000 increased limits P.D. factor = 1.08

Applicable single limits discount factor = .96
Packaging discount factor = .90

Single limit \$25,000 rate =
(Liability portion + ½ Medical rate + ½ U.M. rate) × .90 × .50

Liability portion =
(\$60 × 1.12 × .96) + (\$40 × 1.08 × .96) = \$105.98

Single limit \$25,000 rate = (\$105.98 + \$4.50 + \$1.50) × .45 = \$50.39
rounded to \$50

Higher limits are available for the following combinations:

Bodily Injury and Property Damage Liability—Single Limit	Medical Expense Limit
\$ 50,000	\$2,000
100,000	3,000
200,000	4,000
300,000	5,000

The rates for the higher limit combinations are obtained by applying increased limits factors of 1.10, 1.20, 1.25, 1.30, respectively to the \$25,000 rate.

The Special Automobile Policy of NBCU and NAUA and the Package Automobile Policy of MIRB were introduced gradually on a state by state basis. Eventually, the differences in coverages provided in the Mutual Bureau and National Bureau policies in respect to the liability part were reconciled, and the new product is called the Special Package Automobile Form.

Subsequent to the introduction of this policy and the pertinent rates, there were, of course, changes in most states in some or all of the Automobile Casualty Manual rates for the coverages that are combined in the Special Package Policy. As is always the case with innovations in insurance, be it classifications, territories, coverages or combinations thereof, considerable time elapses before data become available from which the judgment used in rate determination may be reviewed and any indicated corrective action may be taken. In the meantime, additional judgment must be used in updating the rates.

The Special Package Automobile Policy poses a particular problem because of the difference in the marketing approach adopted by the various companies within the rating organizations. Some companies have adopted the package as their vehicle for marketing all or most of their private passenger business, while others use it selectively. The ratemaking practices during the five years since the introduction of this policy have tended to widen the gap between the premium charged for the package of coverages and the premium that would be due if these coverages were purchased separately under the rules of the Automobile Casualty Manual. It is not uncommon to find that this difference amounts to 25% to 30% under present manual rate schedules. This widening of the difference was brought about by the practice of changing rates for the package not directly in formula relationship to the change in rates for the separate components; rather, selected changes were frequently applied which reflected less than the average increase in the component rates.

Studies now in progress will help to determine whether the private passenger rate level in any state should be based on the average combined experience developed under the Family and Package Policies, or whether each should determine its own level. Closely connected with this question is that of the expense requirement in the rates for either type of policy. So far, the loss and expense experience developed under package automobile policies has not formally been used in ratemaking for private passenger cars; presumably, whatever information is available in this respect is reflected in the judgment used to adjust the private passenger rate levels. The time may be close, in view of the volume developed from automobile package policies, when this experience will receive formal recognition in the ratemaking procedure.

APPENDIX A

NEW PRIVATE PASSENGER CLASSIFICATION AND RATING SYSTEM—STATISTICS

Effective January 1, 1965, a revised private passenger classification and rating system was introduced, with a refinement in the classifications that will require a new approach to the summarization of experience. The new system was developed by the National Automobile Underwriters Association and the National Bureau of Casualty Underwriters; it was adopted also by the Mutual Insurance Rating Bureau for optional use by its members and subscribers.[20]

The greatest expansion in statistical and rating detail under the new system affects the youthful driver classifications which include male drivers under 25 years of age, as heretofore, and to which are newly added female unmarried drivers under 21 years of age, and the unmarried male owners or principal operators of an automobile, ages 25 to 29. For the youthful operators, rates will vary by year of age, personal status (male or female, single or married), and qualification for driver training credit, and the use of automobile criteria will newly apply. In addition, rate differences recognizing compact car and multi-car credits will apply, as will the sub-classification system under the Safe Driver Insurance Plan or other plans serving the same purpose. There are 4900 distinct rating classes possible in any rate territory for these youthful driver categories (assuming 5 subdivisions under the Safe Driver Insurance Plan). For the remaining adult driver population, the new system has added, as separate categories, the over-65 age group, and females, ages 30 to 64, who are the sole drivers of the automobile. Three hundred separate statistical entities are produced by the system for the adult drivers.

In order to accommodate this classification system, it was necessary to go from a 5 digit to a 6 digit code for private passenger cars. It is apparent that it would be impractical to continue the system of summarizing private passenger experience in complete detail by class and territory. Tabulations of experience in the full detail of the new classification system would be unmanageable by their mere length, the cost of producing them would be prohibitive by present data processing standards, and most of the detail would be too sparse to be of use in analysis and ratemaking.

[20] While the new system applies to automobile liability and automobile medical payments insurance as well as to automobile physical damage insurance, the following comments are directed only at the kinds of insurance under the jurisdiction of NBCU and MIRB.

Consequently, the rating organizations have changed the requirements for the reporting of experience by the companies under the Official Call, and have made tentative plans for a new approach to summarization of that experience for use in rate review and ratemaking, and for filing of such experience with the rate regulatory authorities.

In the area of reporting, the new system encourages the filing of experience by the companies in complete detail without any summarization. Losses (paid and outstanding, with allocated loss adjustment expenses) have been filed for some time by the companies in the form of transaction reports on punch cards; there will be no change in this respect. For the reporting of exposures and premiums, heretofore reported in summaries by class and territory for each accounting quarter, changes in the direction of transaction reports were made. The National Bureau will accept exposure and premium transaction reports on punch cards (or magnetic tape) on all private passenger business. The Mutual Bureau, at this time, will accept such transaction reports on the youthful driver classifications, continuing the summarized form for the other private passenger classes.

In addition to these reporting methods, there is available to the companies a method of reporting exposures and premiums in summarized form on a *limited key* basis, i.e., summarized experience by accounting quarter by selected digits of the 6 digit code. Some of the classification detail is lost in this type of summary. That detail will be obtained from companies using this reporting method by periodic supplementary reports of exposure and premium samples.

The utilization of this experience will be based partly upon data summarized by territory in some of the classification detail, possibly the detail of the limited key referred to above, and partly upon the use of samplings of distributional data for the remaining elements of the classification system. The use of magnetic tape for storage of this vast amount of detail and the use of electronic computers for its processing and analysis is imperative.

A program of this type requires approval and acceptance by rate regulatory authorities, since it also affects the type of information they will receive from the organizations acting as their official statistical agents. To the extent that companies not affiliated with the National Bureau of Casualty Underwriters or the Mutual Insurance Rating Bureau will use the new private passenger classification system, official statistical agents other than these two rating organizations will presumably be affected by the new demands for rate review statistics. It must be recalled that the rating laws provide that companies shall file their loss experience in a form reasonably adapted to and not inconsistent with the rating systems in use.

Eventually, a reasonable summarization program will evolve; it might be expected that the ratemaking procedure will continue to follow the present pattern. Data will be available to calculate premiums at manual rates, partly from summaries of the total experience and partly from the sampling distributions of exposures, for the determination of statewide and territory rate levels. Losses can be correspondingly summarized. Reasonable assumptions will have to be made and techniques will have to be developed for the review of classification experience. In this connection, it should be noted that the new private passenger classification system is compatible with, and its experience can be reduced to, the statistical detail of the private passenger class plan generally in use prior to January 1, 1965 and to be continued in use in some areas.

APPENDIX B

THE INCOMPLETE POLICY YEAR

A policy year, by definition, extends over a period of two calendar years; policies written during the 12 months period of the policy year remain in effect beyond December 31, with the policies written on the last day not expiring until December 31 of the following year. On the average (assuming an even distribution of writing throughout the year), one half of the written premiums are earned during the year of writing; correspondingly, one half of the exposures are earned, i.e., on the average, the insured objects have been exposed to the loss producing hazards for one half of the full annual duration of coverage.

If we want to review the experience on all policies written during 1963, as of December 31, 1963, we could construct the experience with the above assumption as follows: the total incurred losses for policy year 1963 consist of all loss payments from January 1 to December 31, 1963 plus the reserves[21] from all accidents covered by the policies written during 1963. These losses are compared with one half of the premiums earned during 1963 from policies written during 1963. Such experience is called experience of an *incomplete policy year.* Twelve months later, as of

[21] Reserves for outstanding losses are usually valued as of three months later; in this case, as of March 31, 1964. Also, the paid and outstanding losses include amounts on accidents that occurred prior to the cut-off date but were recorded between January 1, 1964 and March 31, 1964.

December 31, 1964, the written premiums will be fully earned, all accidents that are covered by these policies will have occurred; the earned premiums and incurred losses as of that date would present a *complete policy year.*

A more accurate approximation, however, is required if experience for an incomplete policy year is to be used for ratemaking, as was the case prior to the adoption of the accident year method.

The experience for an incomplete year (as of 12 months) was adjusted to a complete basis by applying to it modification factors obtained from the observed development of prior years. This development was measured by use of pure premiums of prior policy years at their successive valuations.

Since the pure premium is a function of exposures and incurred losses, development factors based on pure premiums combine in one step a measure of the development of both. The following illustrates the calculations of such pure premium ratios:

BODILY INJURY LIABILITY

	Number of Written Car Years		Basic Limits Incurred Losses*	
(1) Policy Year	(2) As Of 12 Months	(3) As Of 24 Months	(4) As Of 12 Months*	(5) As Of 24 Months*
1962	2,079,685	2,085,145	$35,369,982	$65,568,694
1963	2,177,435	2,168,448	39,145,075	72,632,151

Pure Premiums

(6) Policy Year	(7) 12 Months (4) ÷ (2)	(8) 24 Months (5) ÷ (3)	(9) Ratio Of Pure Premiums (7) ÷ (8)
1962	17.01	31.45	.541
1963	17.98	33.49	.537
Mean			.539

*Valued as of 15 months and 27 months respectively.

The pure premium ratios used to be called *earned factors;* in order not to mistake them, in this discussion, for the earned premium and exposure fractions used in connection with accident year data, we shall refer to them as *pure premium development factors.*

In the above example, it will be noted that the written exposures, car years in this case, do not change materially during the second half of the policy year. This second half is the run-off of the exposures written during the first half; any changes after December 31 can only be changes on existing policies, such as cancellations, additions of coverage, changes in class or territory, and any new business effective December 31 of the policy year or just prior to that date but recorded after that date.

The average (or mean) of the pure premium development factors of two or three prior years is used to adjust the written exposures and written premiums for the most recent policy year, which is available only as of 12 months, to an earned basis as of 12 months. For example, if the written exposures for the next year, 1964, are 2,201,853, the application of a .539 factor produces 1,186,799 of earned exposures for policy year 1964. If the incurred losses as of December 31, 1964 are $42,560,606, a pure premium of $35.86 for 1964 would result.

The incomplete policy year losses are subject to further adjustment for loss development beyond the 27 months level to which the earned factor adjusts them. Comments on the loss development of policy year experience may be found in the section dealing with this subject.

It is noted that the application of earned factors to the incomplete policy year experience results in a volume approximately one half of what it will be eventually on a complete policy year basis. This has to be taken into account when use is to be made of several policy years, one including an incomplete year, on a weighted basis.

RESERVING FOR RETROSPECTIVE RETURNS

WALTER J. FITZGIBBON, JR.

INTRODUCTION

This paper discusses possible approaches to the problem of establishing reserves for retrospective returns for the annual statement and for statements of company operating results. The first of the formulas explained has produced satisfactory results when applied to the data of one company for policy years from 1956 to 1962. The reserves established by the methods described in this paper do not lend themselves readily to run-off tests. The reasons these reserves are difficult to test and the method which should be used for testing will be discussed in the paper.

AMOUNT OF THE RETROSPECTIVE RETURN RESERVE

An insurance company must display the retrospective return reserve in Column (6) of Part 2B of its annual statement, thus including the amounts in the unearned premium reserve. The retrospective return reserve may be made up of the following two amounts:

1. The net return premium which would be due to policyholders as a result of making retrospective adjustments using premiums and losses as contained in company records as of the statement date for all retrospectively rated risks for which final adjustments have not yet been calculated.
2. The premium due to policyholders as of the statement date as a result of final adjustments which have been calculated but not yet recorded on the company's books.

NEGATIVE RESERVES

Should the retrospective return reserve calculation indicate that additional premiums will be due the company as a result of retrospective adjustments, it is appropriate that the company include negative retrospective reserves in its annual statement. It is only by permitting reserves to become negative that the proper underwriting profit for the calendar year can be reflected.

CHARACTERISTICS OF A GOOD RESERVING METHOD

A system for determining the reserve for retrospective returns should meet the following objectives:

1. The amount of the reserve should be the best estimate of the

probable run-off of retrospective returns, in consideration of premiums recorded as earned and losses known and estimated at the time the reserve is established.

2. The total reserve can be considered to be composed of the sum of a reserve for each line of insurance for each policy year. Each such portion of the reserve should move gradually from the beginning of a policy year (January) to a maximum (absolute value) at about 20 months (August of the second calendar year), and then should gradually go to zero as retrospective returns are disbursed. Throughout the entire life of the reserve for each line for each policy year, the monthly changes should appropriately reflect monthly changes in earned premium, incurred losses and deviation payments, so that underwriting results will not be distorted.
3. At some point in time the reserve for each line for each policy year should become zero. Determination of when this point in time is reached may be somewhat arbitrarily set as the point at which any further reserve, if carried, would be small, equally likely to be plus or minus, and probably unreliable.
4. The reserve produced by the system should be a net reserve, i.e., the net of retrospective return and retrospective additional premiums, for appropriate effect on underwriting results. However, to meet the requirements of any Insurance Department that requires a company to calculate a reserve for returns only, the system must also be able to determine an appropriate reserve for returns only.
5. The method should allow a reserve to be calculated quickly enough to be used for company results; data actually collected to the end of the accounting period should be the basis for the calculation. The relatively short time available between receipt of the data and the necessary completion of the reserve calculation probably rules out a risk by risk calculation for most companies although companies using computers may be able to use this method.

DATA REQUIRED FOR THE CALCULATION

Written premiums, written restrospective adjustment premiums, unearned standard premiums and incurred losses are required by line of business and by policy year. These data must be gathered as frequently as reserves are to be calculated for company results and always at year-end.

FORMULA FOR RESERVING

It seems logical that the reserve for retrospective returns should vary with the loss ratio, increasing when the loss ratio declines and declining, even to becoming negative, when the loss ratio increases.

A monthly calculation of a reserve for Workmen's Compensation will illustrate one method which produces retrospective return reserves which vary with loss ratios.

The following Table I shows policy year experience for retrospectively rated Workmen's Compensation business.

Table I

Ætna Casualty & Surety Company Experience Under Retrospectively Rated Workmen's Compensation Policies

All Policy Years Valued as of 6-30-64
(Thousands of Dollars)

Policy Year (1)	Earned Standard Premium (2)	Incurred Losses (3)	Loss Ratio (4) (3)÷(2)	Retrospective Adjustment Premium (Deviations) (5)	Deviation Ratio (6) (5)÷(2)
1958	24,552	14,447	58.84%	3,726	15.18%
1959	27,359	17,058	62.35	3,350	12.24
1960	29,864	18,904	63.30	4,204	14.08
1961	36,439	21,612	59.31	5,685	15.60
1962	41,956	24,724	58.93	6,575	15.67

Notes:

1. Earned Standard premium includes all premium written for policies which contain retrospectively rated premium i.e. total policy premium would be included even though some premium contained on the policy is not subject to retrospective rating.
2. Incurred losses include all losses paid and unpaid for policies which have contributed their premium to Column (2). The losses contain reserves for incurred but not reported losses omitting losses which are expected to emerge after all retrospective adjustments for the policy year have been considered final. Losses are included at full value and the effect of the loss limitations of the retrospective plans has been ignored. Our studies indicate that little loss of accuracy results from using total losses, and total data are much simpler to gather.

Assuming that such a relationship exists between the loss ratio and the deviation ratio that one increases while the other decreases, a least squares line has been fitted to the data contained in Table I.

With the loss ratio represented by X and the deviation ratio by Y, the equation is:

$$Y = .472 - .539\,X$$

This can be changed to the form:

Indicated deviation =
(.472) Earned standard premium − (.539) Incurred losses

Knowing the premiums and losses each month, an indicated deviation can be calculated. This indicated deviation is compared to the actual deviation premiums recorded to date and the difference held as the reserve.

CALCULATION OF RESERVES FOR POLICY YEAR 1958

Exhibit I shows a sample calculation of a reserve for policy year 1958 at each month end from January 1958 to June 1964.

DIFFICULTY OF A RUN-OFF TEST OF THIS RESERVE

The formula deviation would be expected to reproduce the experience exactly only if losses had been estimated exactly and all premiums earned for policy years prior to the most recent year had been included in company records. Since much earned premium is reported late—audits for example—and since it is impossible to predict losses exactly, formula deviations for each policy year will change as the experience matures.

The reserve for the most recent policy year at year-end is based on incomplete data, a partial policy year, and the actual deviations would equal the formula deviation only if all policies were terminated as of the statement date and the conditions described for prior policy years were fulfilled. As a company continues operations, the premiums earned and losses incurred for the remainder of the policy year add to the data entering the formula. There would be no practical way to test a portion of the policy year.

When the reserve at 12-31-59 was calculated (Exhibit I), ultimate deviations of $3,503,805 were predicted. To June 30, 1964, $3,726,224 were actually returned. At 12-31-59, the loss ratio was 60.7% and premiums earned were $24,213,757. Since that time, late reported premiums have totaled $338,553 and the loss ratio has dropped 1.9 points; both changes have increased the returns and distorted the runoff test.

If excessive loss reserves are held, retrospective return reserves produced by this formula are depressed. Revaluing losses and lowering them should produce more returns. When premiums increase for a policy year, again more returns should be produced. The fact that the formula deviation is not reproduced does not necessarily mean the reserve formula did not work properly. It may only prove that the formula depends for its validity on the accuracy of the data entering into the calculation.

If excessive loss reserves are held, the direct effect will be to understate underwriting gain and the indirect effect through understatement of the retro reserve will be to overstate underwriting gain; thus, an error in the evaluation of losses will give rise to an error of opposite direction in the retro reserve and thereby dampen the effect on underwriting gain.

If earned unreported premiums could be estimated and included in premium income, and if incurred losses could be accurately estimated, a retrospective reserve could be calculated which would be expected to reproduce the ultimate deviations. Since earned unreported premiums cannot be included in premium income, the retro reserve is more truly a measure of what must be set aside out of reported income than a measure of what ultimately will be paid.

The proper way to examine the reserves established by the methods described here begins with a review of the characteristics of the method. The following questions should be answered:

1. Did the same formula, or the formula in use at the time, produce reserves of zero for older policy years several year-ends beyond the end of each policy year?
2. Have the data underlying the present formula been verified and are these data up-to-date?
3. Has the equation developed from the data been tested for goodness of fit?
4. Do changes in premium volume and loss ratio account for the difference between the present reserve level for a policy year and the level of prior policy years at the same age?

If an examiner agrees with the general principles and these questions have been satisfactorily answered, the reserves held may be assumed to have been reasonable. The appropriate tests are mainly of method and formula and not run-off tests of the answers.

OTHER RESERVING FORMULAS

From Table I, it can be observed that for the two most recent policy years 15.60% and 15.67% have been returned to policyholders. A reserve

EXHIBIT 1

RESERVES FOR POLICY YEAR 1958
WORKMEN'S COMPENSATION

Date	Earned Standard Premium	Incurred Losses	Loss Ratio	Indicated Deviation	Deviation Payments	Retrospective Return Reserve
(1)	(2)	(3)	(4)	(5) .472 (2) — .539 (3)	(6)	(7)
1-58	48,873	219,456	4.490	− 95,219	0	− 95,219
2-58	341,501	606,657	1.776	− 165,800	0	− 165,800
3-58	970,886	959,287	.988	− 58,798	0	− 58,798
4-58	1,965,507	1,868,034	.950	− 79,151	0	− 79,151
5-58	3,101,186	2,525,315	.814	102,615	0	102,615
6-58	4,293,575	3,550,857	.827	112,655	0	112,655
7-58	5,742,469	4,430,602	.772	322,351	0	322,351
8-58	6,903,355	5,504,474	.797	291,472	0	291,472
9-58	8,289,701	6,385,123	.770	471,158	0	471,158
10-58	10,469,528	7,917,996	.756	673,817	0	673,817
11-58	13,230,451	8,503,872	.643	1,661,186	0	1,661,186
12-58	14,723,978	9,929,790	.674	1,597,561	0	1,597,561
1-59	16,524,513	10,748,103	.650	2,006,343	2,124	2,004,219
2-59	17,743,166	11,128,578	.627	2,376,471	− 4,767	2,381,238
3-59	18,712,852	12,187,908	.651	2,263,184	− 4,767	2,267,951
4-59	19,720,306	12,582,782	.638	2,525,865	− 5,013	2,530,878
5-59	20,777,043	13,046,836	.628	2,774,520	− 5,160	2,779,680
6-59	21,529,638	13,514,331	.628	2,877,765	− 3,054	2,880,819
7-59	22,298,087	13,942,448	.625	3,009,718	6,634	3,003,084
8-59	22,937,661	14,038,745	.612	3,259,692	31,137	3,228,555
9-59	23,313,567	14,307,320	.614	3,292,358	247,812	3,044,546
10-59	23,532,656	14,438,825	.614	3,324,887	628,988	2,695,899
11-59	24,128,247	14,687,860	.609	3,471,776	1,231,380	2,240,396
12-59	24,213,757	14,703,318	.607	3,503,805	1,450,858	2,052,947
1-60	24,348,037	14,730,053	.605	3,552,775	1,678,470	1,874,305
2-60	24,431,671	14,917,900	.611	3,491,001	2,182,359	1,308,642
3-60	24,519,386	15,020,731	.613	3,476,976	2,336,521	1,140,455
4-60	24,584,549	15,083,072	.614	3,474,131	2,683,030	791,101
5-60	24,602,208	15,171,746	.617	3,434,671	2,822,311	612,360
6-60	24,676,964	15,191,695	.616	3,459,203	3,194,782	264,421
7-60	24,600,050	15,219,361	.619	3,407,988	3,975,650	− 567,662
8-60	24,597,037	15,181,773	.617	3,426,826	4,084,234	− 657,408
9-60	24,593,690	15,168,438	.617	3,432,434	4,145,569	− 713,135
10-60	24,581,555	15,239,416	.620	3,388,449	4,122,280	− 733,831
11-60	24,578,541	14,848,863	.604	3,597,534	4,122,253	− 524,719
12-60	24,582,891	14,781,496	.601	3,635,898	4,140,417	− 504,519

EXHIBIT 1

RESERVES FOR POLICY YEAR 1958
WORKMEN'S COMPENSATION—Continued

Date	Earned Standard Premium	Incurred Losses	Loss Ratio	Indicated Deviation	Deviation Payments	Retrospective Return Reserve
(1)	(2)	(3)	(4)	(5) .472 (2) — .539 (3)	(6)	(7)
1-61	24,585,706	14,784,871	.601	3,635,408	4,078,807	— 443,399
2-61	24,587,919	14,939,962	.608	3,552,858	4,018,580	— 465,722
3-61	24,580,671	14,948,369	.608	3,544,906	3,976,628	— 431,722
4-61	24,581,123	14,500,604	.590	3,786,465	4,049,434	— 262,969
5-61	24,581,615	14,550,033	.592	3,760,054	4,038,984	— 278,930
6-61	24,578,227	15,237,426	.620	3,387,951	3,930,467	— 542,516
7-61	24,579,354	15,344,455	.624	3,330,794	3,910,376	— 579,582
8-61	24,579,177	15,398,985	.627	3,301,319	3,936,715	— 635,396
9-61	24,579,177	15,445,477	.628	3,276,259	3,937,706	— 661,447
10-61	24,579,177	15,374,827	.626	3,314,340	3,895,927	— 581,587
11-61	24,579,177	15,429,452	.628	3,284,897	3,769,020	— 484,123
12-61	24,579,177	15,448,617	.629	3,274,567	3,723,231	— 448,664
1-62	24,579,177	15,396,856	.626	3,302,466	3,715,428	— 412,962
2-62	24,579,177	15,401,391	.627	3,300,022	3,647,753	— 347,731
3-62	24,579,177	15,383,646	.626	3,309,586	3,593,909	— 284,323
4-62	24,579,177	15,377,494	.626	3,312,902	3,544,057	— 231,155
5-62	24,578,149	15,412,716	.627	3,293,432	3,545,993	— 252,561
6-62	24,578,149	15,384,335	.626	3,308,729	3,504,021	— 195,292
7-62	24,573,188	15,420,734	.628	3,286,769	3,487,615	— 200,846
8-62	24,573,188	15,417,290	.627	3,288,625	3,476,186	— 187,561
9-62	24,572,407	15,388,670	.626	3,303,683	3,436,465	— 132,782
10-62	24,553,614	15,291,317	.623	3,347,286	3,474,370	— 127,084
11-62	24,553,498	15,315,849	.624	3,334,008	3,456,161	— 122,153
12-62	24,553,498	15,255,367	.621	3,366,608	3,473,035	— 106,427
1-63	24,553,498	15,099,592	.615	3,450,571	3,449,320	1,251
2-63	24,553,498	15,151,851	.617	3,422,403	3,510,675	— 88,272
3-63	24,553,498	15,082,159	.614	3,459,967	3,532,441	— 72,474
4-63	24,553,498	15,070,661	.614	3,466,164	3,517,778	— 51,614
5-63	24,553,498	15,083,138	.614	3,459,440	3,525,330	— 65,890
6-63	24,553,498	14,912,758	.607	3,551,274	3,566,941	— 15,667
7-63	24,553,498	14,793,725	.603	3,615,433	3,560,924	54,509
8-63	24,553,498	14,788,648	.602	3,618,170	3,573,301	44,869
9-63	24,553,498	14,790,984	.602	3,616,911	3,588,039	28,872
10-63	24,553,498	14,748,939	.601	3,639,573	3,641,677	— 2,104
11-63	24,553,498	14,741,103	.600	3,643,797	3,656,103	— 12,306
12-63	24,553,498	14,698,789	.599	3,666,604	3,619,253	47,351
1-64	24,553,498	14,672,364	.598	3,680,847	3,658,181	22,666
2-64	24,553,498	14,623,380	.596	3,707,249	3,653,977	53,272
3-64	24,552,318	14,592,927	.594	3,723,106	3,658,617	64,489
4-64	24,552,310	14,564,483	.593	3,738,434	3,671,060	67,374
5-64	24,552,310	14,537,192	.592	3,753,144	3,688,916	64,228
6-64	24,552,310	14,446,331	.588	3,802,118	3,726,224	75,894

could be developed by applying a factor of 16%, for example, to earned standard premium, comparing this indicated return to the actual return and carrying the difference as the reserve. This method is simpler to use than the earlier formula but ignores the effect of changes in loss ratio on the reserve. In a period of worsening experience, we would continue to build a large reserve, and in a period of improving experience the reserve would not be large enough.

To develop our next formula, let us suppose that all retrospectively rated business operates the same as one large risk. Only countrywide averages of basic premium ratios, tax multipliers, loss conversion factors, loss limitation charges and a factor to reduce total losses to losses which enter the retro calculation would be required to calculate a reserve. For example, using the following averages which were obtained from an analysis of retrospectively rated one year policies for policy years 1959-1961:

Basic premium ratio including loss limitation charge	.197
Tax multiplier	1.026
Loss conversion factor	1.140
Losses exceeding loss limitation	.019
Losses exceeding maximums minus losses under minimums	.043

The reserving formula will be:

Retro premium = Tax mult. (Basic × Std. prem. + LCF × Losses × Limitation factor)

Deviation = Standard premium − Retro premium

Reserve = Indicated deviation − actual deviations

At 12-59, the reserve for policy year 1958 would have been calculated as follows:

Retro premium = 1.026 (.197 × 24,213,757 + 1.140 × .938 × 14,703,318) = 21,025,471

Deviation = 24,213,757 − 21,025,471 = 3,188,286

Reserve = 3,188,286 − 1,450,858 = 1,737,428

This $1,737,428 compares to $2,052,947 using the earlier formula (Exhibit I).

While this method may give satisfactory results, it is more difficult to use than the least squares approach mainly because it is difficult to keep the required average factors up to date.

LINES OF BUSINESS FOR WHICH THE FORMULA APPROACH SEEMS APPROPRIATE

While retrospective rating is presently being used in the auto physical damage, inland marine, fidelity and burglary lines, other reserving methods are better in those lines than the formula approach because of the small volume. The formula approach seems to be suitable for:

Workmen's compensation
Auto liability B.I. & P.D.
Liability other than Auto B.I. & P.D.

ALTERNATIVE APPROACHES TO RESERVING

A reserve may always be created through use of judgment alone. Using judgment is certainly the simplest method since no calculations are required, and this may be a satisfactory approach for a very small volume of retrospectively rated business. For a line with a few retrospectively rated risks, a risk by risk calculation may be made at each month-end using premium earned and losses incurred to the reserve date. Basic premium ratios and maximum and minimum ratios should be selected for the size of the premium earned to date.

CONVERSION OF NET RESERVE TO RETURNS ONLY RESERVE

Some Insurance Departments may require that retrospective return reserves be established for *returns only*. This means that if one insured has some lines or policies which will produce returns and other lines or policies which will produce additionals, it is proper to hold the net return as the reserve, since only one insured is involved and actual settlements with the insured are on a combined basis, not line by line or policy by policy. If, however, one insured has a return coming while another insured will be billed for additional premium, the reserve held should be for the returns only. Under the returns only reserving approach, it is possible to develop negative reserves for a line of business but not a negative reserve for the total of all lines.

Our company analyzes each retrospective adjustment separating the premium into amounts received for commission, taxes, claim expense, other expenses, profit, insurance charge, excess loss premium and losses. The punch cards which are prepared as part of this analysis contain the standard premium and the retro premium for each risk. Exhibit II shows the data obtained by grouping all policies for one insured together and sorting all risks into "return" and "additional" groups. Risks which had

EXHIBIT II

RETROSPECTIVELY RATED BUSINESS OF THE AETNA CASUALTY & SURETY COMPANY

Policy Years 1958-1961 All Lines of Business Combined

(000 omitted from dollars)

	Risks Producing Additional Premiums		Risks Producing Return Premiums		Deviations		Ratio	
PY	Standard Premium	Retro Premium	Standard Premium	Retro Premium	(4) − (5) Returns	(6) + (2) − (3) Net	$\frac{(7)}{(2)+(4)}$	(6) ÷ (7)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
			One Year Term Policies—1st Adjustments				Y	X
58	5,953	7,113	18,016	13,169	4,847	3,687	.154	1.315
59	9,345	11,024	16,777	12,724	4,053	2,374	.091	1.707
60	7,449	9,099	20,128	14,771	5,357	3,707	.134	1.445
61	7,699	9,260	23,054	17,273	5,781	4,220	.137	1.370
			Three Year Term Policies—Adjustments of 1st Year Only					
58	1,430	1,688	4,901	3,824	1,077	819	.129	1.315
59	1,689	2,093	3,703	2,856	847	443	.082	1.910
60	1,593	1,843	4,134	3,237	897	647	.113	1.386
61	3,179	3,618	6,168	4,257	1,911	1,472	.157	1.298

standard premium equal to the retrospective premium, i.e., risks producing no deviation premium, were considered to be "return" risks.

Fitting a least squares line to the data in Exhibit II we obtain the following equation:

$$Y = .286 - .110\,X$$

where Y represents the ratio of the net deviations to earned standard premium and X is the multiplier which converts net deviations to returns only.

This relationship may be used to convert a net reserve to a reserve for returns only as illustrated in the following example:

Earned standard premium for all lines and all policy years for which reserves are held	=	\$100,000,000
Net retro reserves	=	\$ 7,000,000
Deviations paid to date	=	\$ 3,500,000

Then $Y = .105$ and $X = 1.645$

The net retro reserve would be \$7,000,000 and the amount required to convert to a returns only reserve would be \$4,515,000.

ANNUAL STATEMENT TREATMENT OF THE ADDITIONAL RETRO RESERVE

The additional retro reserve or the reserve correction amount can be added to Page 3 of the Annual Statement of any state requiring a "returns only" reserve as a write-in item. It is preferable to keep the Page 6 unearned premium reserve at its proper net amount so that underwriting results will not be distorted.

SOME CONCLUDING OBSERVATIONS

The methods described here used total losses on retrospectively rated policies. To prevent large losses, which will enter adjustment calculations only at reduced values, from distorting the experience, the system could be modified to remove losses above a certain size. This refinement is of more value in liability lines.

Premiums and losses could be restricted to states and limits which are retrospectively rated rather than being policy totals. Since accounting data were used in the calculations described here, refinements could not be introduced easily.

More important, perhaps, than refining data is the comparison, for reasonableness, of reserve indications to reserves of past years at a similar age. This comparison requires knowledge of the company's current underwriting results.

Any formula reserving method requires a constant effort to keep factors updated. The formula should not be permitted to operate for long periods of time without frequent critical reviews and, of course, a better formula or method should always be sought.

It should be borne in mind by readers that the formulas shown in this paper are not appropriate for use by any company other than the one supplying the data underlying them. Caution should be exercised in using the methods exactly as described here; individual company conditions may dictate modifications to the methods.

SOME OBSERVATIONS CONCERNING FIRE AND CASUALTY INSURANCE COMPANY FINANCIAL STATEMENTS

PAUL OTTESON

INTRODUCTION

Page 1 of the annual statement contains a sworn statement signed by the company president, secretary and treasurer, respectively; part of this statement reads as follows:

> ". . . . and that this annual statement, together with related exhibits, schedules and explanations therein contained, annexed or referred to are a *full and true* statement of all the assets and liabilities and the condition of affairs of the said insurer as of the thirty-first day of December last, and of its income and deductions therefrom for the year ended on that date, according to the best of their information, knowledge and belief, respectively." (Italics added.)

This paper considers and evaluates specified phases of the question as to how well financial statements prepared under present prescribed rules and company practices meet the *full and true* objective; phases considered are:

I. Consolidated statements
II. Valuation of investment securities
III. Incurred losses
IV. Schedule P
V. Unearned premiums and prepaid expenses

Suggestions designed to improve present practices and procedures are proposed.

Two basic unique industry characteristics pertinent to this problem are: (1) the element of risk inherent in the fire and casualty insurance business is high because the determination of expected costs and price is based upon predictions of future events both as to frequency and severity; and (2) present and future solvency of the insurance carrier is of paramount importance because it is essential to the fulfillment of the insurance obligation. Consideration of these characteristics prescribes a conservative tone to the financial statements; this means that on questionable items the bias should oppose any tendency to overstate net income or surplus.

It is recognized that absolute accuracy cannot be obtained from current financial statements in which assets and liabilities are still in an *in use* or *non liquidated* status and when accuracy in valuation cannot be determined in an absolute sense. This situation prevails in all industries to a greater or lesser extent. In most industries the problem of evaluating *in use* or *non liquidated* elements involves assets; in the insurance industry the major problem concerns valuation of liabilities, particularly unpaid losses.

Insurance company financial statements are prepared according to a prescribed form and are a statutory requirement. These conditions imply uniformity in the portrayal of financial conditions and operating results. The uniform results can be accomplished only as companies understand and follow well defined accounting rules and a common philosophy concerning financial statement objectives. The rules and philosophy must be sound in principle and support the *full and true* objective.

I. CONSOLIDATED STATEMENTS

A balance sheet consolidating all insurance operations including life insurance should be filed by all companies whenever ownership or financial control *of* or *by* another insurance company is involved.

The consolidated balance sheet is the only method available to reflect properly the financial situation of a group of insurance companies when ownership or financial control by one company over another is involved. It is the only means by which total capital can be compared with and related to the magnitude of the total insurance operation. In cases where the ownership or financial control involves only fire and casualty companies a consolidated income statement should be required; interest on life insurance policy reserves presents complications in consolidating life insurance with fire and casualty insurance income statements.

Generally speaking, consolidated statements are not required by insurance departments. Solvency appears on the surface at least to be a certainty and therefore there is no great concern in the case of large and financially powerful fleets and groups. A consolidated statement from companies in a less favorable position however could point up financial problems long before they show up in the present individual company statements.

Best's Insurance Guide (small book) shows the financial figures on a non-consolidated basis. Best's Insurance Reports (large book) shows the financial figures on both an individual company and consolidated (fire and casualty company only) basis.

Situations in which a number of mutual insurance companies operate under the same management are quite common; here the problem of establishing group or composite financial statements is different than when inter-company ownership is involved. Total surplus for these groups of mutual companies is the aggregate total surplus for all companies combined; in the case of one company owning another company the total surplus of the two companies combined *actually is the surplus only of the company that owns the other company.* This situation is not well understood even by many people in the insurance business.

The problem now assumes a new dimension because of fire and casualty companies organizing life companies. Funds invested by the owning company in the life subsidiary appear to build up the subsidiary company surplus, or safety margin, while at the same time these same funds appear to serve as surplus or safety margin for the owning company. This could lead to an over-extension of the insurance operation (fire, casualty and life combined) in relation to the true consolidated surplus position.

Most stock fire and casualty company Federal tax returns are now on a consolidated basis. Life insurance companies are taxed under a different law so consolidation with fire and casualty companies for Federal tax purposes is not permissible.

II. VALUATION OF INVESTMENT SECURITIES

A. Stocks

Market value is the only proper basis for the valuation of stocks; cost is immaterial in determining current worth and should receive no consideration in financial statement valuation.

Market value will differ from cost and in recent years has generally been substantially higher. The use of market valuation has created a problem in that unrealized gain resulting from increase in market valuation has gone directly to surplus without going through the Federal tax wringer. General accounting practice usually considers income to be earned only when it is realized and thus subject to Federal tax; this unrealized form of increment to surplus is unusual although it is both logical and reasonable.

A practical solution to this problem is to establish a reserve equal to

25 percent of the amount by which the market value exceeds the cost value of the stocks. There is reasonable but not complete justification for reducing the amount of this reserve by the computed tax on a *loss carry forward* from previous years. This loss carry forward must first be applied against future income subject to tax at ordinary rates. On the other hand the loss carry forward will reduce overall future Federal taxes and the offset can be justified on this basis only.

When market value is less than cost a negative reserve is not proper. Under ordinary circumstances these capital losses when realized can be offset only against capital gains.

Only eight out of the 105 companies listed in the 1965 Best's Reproduction of Financial Schedules set up this reserve for Federal tax on unrealized gains.

Companies with Liability Account for Federal Tax on Unrealized Capital Gain—1964 Annual Statement

	Amount of Liability in $000's
American Motorists Insurance Company	683
American Mutual Liability Insurance Company	2,100
Employers Mutual Casualty	500
Employers Mutual Insurance Companies, Wisconsin	7,655
Farmers Insurance Exchange	6,265
Liberty Mutual Insurance Companies	12,542
Shelby Mutual Insurance Co. of Shelby, Ohio	700
Truck Insurance Exchange	1,767

These companies are predominately mutual and reciprocal. Some companies likely did not establish the reserve because of loss carry forwards.

B. Bonds

Bonds of a specified grade or better are valued on an amortized rather than on a market basis. At present the amortized valuation often exceeds the market valuation by a substantial margin.

A statement valuation in excess of market value is difficult to defend under an accounting system which emphasizes convertibility to cash as the basis for asset valuation. An abrupt change in valuation basis would cause a surplus movement hardship for many companies; therefore if any change in valuation basis is made it should be done in steps so that the effect on surplus can be spread over a number of years.

III. INCURRED LOSSES

A. General

The validity of the present accounting system, often referred to as the annual statement method is related in large part to the accuracy of unpaid and incurred loss information contained in the financial statements.

Determination of accuracy of incurred loss figures involves a study of facts rather than of accounting theory or principles. Incurred losses charged against operations in the official company statements represent losses paid plus unpaid losses at the end and minus unpaid losses at the beginning of the period; this formula for incurred loss determination is called the *calendar year basis.* The *developed accident year basis* on the other hand values losses according to the year in which they happen or occur; valuations and revaluations are then made far into the future until certainty in valuation is approached and finally reached.

A statistical study is used to serve as a basis for measuring the degree of accuracy actually present in recent and current calendar year incurred loss figures for a large number of companies.

The lines of business included in this study are workmen's compensation and auto bodily injury; these are considered most difficult from the standpoint of estimating unpaid losses accurately. The companies selected are those included in Best's Reproduction of Principal Schedules from Fire and Casualty Convention blanks and whose calendar year losses incurred for the line involved amount to over $2 million. Additional comparable companies whose annual statements were available are also included.

In setting up this study one problem was encountered immediately. Many companies had to be excluded because their annual statement page 8 was set up on an individual company basis and Schedule P was set up on a group basis. It was thus not possible to make a comparison between the calendar year incurred losses shown on page 8 and the developed accident year losses shown in Schedule P, part 5. The annual statement should require the information contained in Schedule P, part 5 to be completely homogeneous with the page 8 loss information.

Comparison of calendar year losses against developed accident year losses is an effective way to view the degree of accuracy contained in the financial statement incurred loss figures. If the unpaid losses are strictly accurate there will be no difference between the calendar year basis losses incurred and the developed accident year losses. If the degree of inadequacy or redundancy is consistent these two figures should also closely approximate each other and reasonably accurate calendar year basis incurred losses will result. If the degree of redundancy or inadequacy changes, there can be wide differences between the developed accident year and the calendar year losses; underwriting results then can be highly distorted.

The statistical information used in this study includes 71 companies with over $1 billion in annual auto bodily injury losses and 38 companies with $600 million in annual workmen's compensation losses. This sample should be sufficiently large and representative to portray industry aggregate results with reasonable accuracy.

1962 was selected as the year for which to compare losses computed on these two bases. An earlier year would permit a longer loss development period but the information would be less current.

The estimated final valuation is computed by multiplying the 1964 valuation of these 1962 accident year losses by a factor representing for each company the following fraction: $\frac{\text{1960 losses valued December 31, 1964.}}{\text{1960 losses valued December 31, 1962.}}$ This assumes that the 1960 accident year valuation as of December 31, 1964, is a reasonably accurate approximation of the ultimate valuation and also that the development factors from the end of the second year to the end of the fourth year will be the same for accident year 1962 as for accident year 1960. The distortion of results attributable to the inaccuracies of these assumptions is believed to be limited.

The exhibits are listed in detail by individual company; this will permit study of the variations by individual company and consolidation of the data into other selected classification categories.

B. Auto Bodily Injury Recapitulation

Concerning the auto bodily injury (Exhibit A) recapitulation, the amount of losses incurred charged against 1962 operations was too low by (column d − column a = $49,421,000) or 4.5 percent in spite of the fact that the 1962 statement established some redundancy for 1962 accident year losses. The redundancy in total unpaid losses on December 31, 1961 released in 1962 thus was greater than the redundancy established for accident year 1962 in the 1962 statements.

One significant result of this shrinkage in redundancy was to delay the impact of the deterioration of the automobile underwriting situation upon the official financial statements of insurance companies.

According to the estimated ultimate valuation of 1962 losses, it appears that the aggregate 1962 accident year losses were valued very precisely on December 31, 1962 (column b − column d = $11,030,000). The 1962 valuation of these losses was 1 percent less than the valuation of these losses as of December, 1964 and 1 percent more than the estimated final valuation.

The small difference between 1964 calendar year and accident year totals means that the aggregate unpaid loss provisions December 31, 1963 were much more precise than on December 31, 1961. During 1964 the loss development for all companies in aggregate was practically nil. The 1964 development indicates a small redundancy in 1963 unpaid losses for stock companies and a small inadequacy for mutuals and reciprocals.

Considering the high degree of difficulty in the valuation of auto bodily injury unpaid losses, the aggregate industry results appear surprisingly accurate. Many individual companies produced calendar year basis incurred losses which were unusually close to the developed accident year figures. Results for some of the companies were far from precise and the closeness of the aggregate figures does represent some offsetting of redundancies against inadequacies.

C. *Workmen's Compensation Recapitulation*

The recapitulation (Exhibit B) indicates that for the years involved in the study the degree of accuracy in the aggregate calendar year incurred losses was surprisingly high; differences between accident year and calendar year results are much smaller than for auto bodily injury coverage.

Accuracy in calendar year incurred losses assumes special importance in workmen's compensation. Calendar year incurred loss experience is used directly in ratemaking through the rate level adjustment factor. The Insurance Expense Exhibit provides this loss experience data by state. The official source of all calendar year loss experience elements, however, is the line of business breakdowns in the annual statement; these then are tied directly to the official company statement totals for all coverages combined.

D. *Accuracy Improvement*

Effective techniques, adequate time, and meaningful accuracy tests are three important ingredients in the establishment of accurate unpaid and incurred loss information. This study will consider only *time* and *tests*.

Additional time is one form of remedy. One company of medium size, for example, has a rule that no loss experience is ever released without at least one month development. In a very large operation where "averaging" has more opportunity to work this rule may not be necessary; in a small or medium size operation it definitely improves loss experience accuracy. The rule materially reduces the incurred but not reported problem and it permits claims examiners to have some knowledge at least of the severity potential involved in recent claims.

It is possible for the above mentioned company to meet the March 1 annual statement filing date after a loss development of one month. A later filing date may encourage additional companies to improve the accuracy of their unpaid losses through using a development period of one month or longer.

The present annual statement blank affords opportunity of measuring accuracy in unpaid losses to persons very familiar with the blank and who are willing to study and analyze page 8, Schedule O, and parts 1, 2, and 5 of Schedule P. Most company presidents however would have little awareness as to how closely their auto bodily injury losses incurred figures in column 9, page 8 compared with the valuation of losses for the corresponding accident year as developed and reported in part 5 of their Schedule P. This comparison could prove to be of management significance in evaluating operating results.

An exhibit testing the relationship between calendar year basis and developed accident year basis losses incurred would highlight the degree of accuracy of the losses incurred figures used in the official income statement.

Exhibit C illustrates an effective test of unpaid loss provisions which is useful in high claim frequency lines such as auto property damage and accident and health. Information in this exhibit reveals:

1. A repetitive relationship between paid and unpaid losses by individual company; and
2. Wide differences among companies in the timing of loss settlements.

An exhibit of this type will bring a discrepancy to light much earlier than the present Schedule O setup.

E. General Recapitulation

Problems in lines other than auto liability and workmen's compensation are not considered in this study. The problem in general liability would be comparable with auto bodily injury but on a smaller volume.

On the remaining lines the loss valuation problems are considered to be less difficult.

The calendar year basis of incurred loss determination is advantageous when year-to-year accumulation of results is practical and important. Errors in one accounting period are automatically corrected by offsetting errors in future periods. This correction process serves well in establishing income subject to Federal income tax or in evaluating results that are cumulative for a long period of time.

The Exhibit A results emphasize the need for the accident year basis with a development period for auto bodily injury when the timing of loss costs is important, as in ratemaking. These results also suggest the usefulness of individual company reviews of their own financial results at later dates using developed accident year losses for bodily injury and possibly other lines in place of the page 8 calendar year basis incurred losses.

Objective standards of permissible error in unpaid or incurred losses would be difficult to promulgate; variation in amounts and percentages would both have to be considered because of vast differences in size of operations. State regulatory authorities and the Internal Revenue Service have a vital interest in the accuracy of unpaid and incurred losses; these organizations have authority to take appropriate action in particular situations.

IV. SCHEDULE P

Schedule P contains four separate and distinct types of provisions:

1. Statutory reserve requirements;
2. Voluntary reserve provision;
3. Claim expense analysis;
4. Incurred loss development.

Any amount shown under the Schedule P statutory or voluntary provision is called a reserve; this is the only place in the balance sheet where the term "reserve" is used.

The statutory reserve requirement and the voluntary reserve provision are not set out separately in the balance sheet; this lack of separation is unfortunate because the significance and meaning of each type of reserve is entirely different. It is necessary to check the computation of this reserve in Part 1 and Part 2 of Schedule P in order to determine whether the balance sheet liability is a voluntary reserve or a statutory requirement.

The Schedule P statutory requirement does not affect a large proportion of companies. A study has been made of the 71 companies included in Exhibit A and of the 38 companies included in Exhibit B to get some perspective as to the applicability of this requirement. The results of this study are contained in Exhibit D which indicates that only 10 of the 71 Exhibit A companies were required to set up statutory reserves for liability insurance. The statutory requirements for these companies were necessitated by the relatively low loss ratios on general liability as indicated in Section A, Exhibit D. If these companies had written a larger proportion of automobile insurance, their statutory Schedule P requirements would have been decreased or eliminated.

The combination of auto liability and general liability experience for the purpose of computing Schedule P requirements is subject to question. There often is a difference in the loss experience pattern between these two lines of business; the statutory reserve will then depend upon the proportion of these two lines of business written.

Only three of the 38 Exhibit B companies were required to set up statutory reserves on workmen's compensation. The applicability and significance of the Schedule P statutory requirements for workmen's compensation thus is very limited at the present time.

Consideration should be given to the *second period* definition for which the statutory requirement is applicable. At the present time each of the three current years is treated separately and distinctly and each is treated in a similar manner. The degree of certainty in loss valuation for the earliest policy year certainly is far greater than the degree of certainty in the loss valuation for the latest policy year in the period. The statutory reserve requirement could give greater emphasis to this latest policy year. Such a change would serve another useful purpose; it would partly obviate the existing situation whereby the release of Schedule P reserves to surplus at the end of the third year can cause distortions to surplus movement of considerable significance and can even turn a surplus loss into a surplus gain. This distortion in surplus movement resulting from the release of Schedule P reserves is a problem to be recognized; for one of the companies in Section A, Exhibit D this release of reserves changed a 1964 surplus loss of $1,536,000 into a surplus gain of $1,361,000.

A second solution to the two types of problems mentioned above would be to reduce the number of policy years in the second period from three to two. The purpose of this change is to apply the statutory reserve principle to the area where the unpaid loss provisions are least certain.

Much has been said and written about accident year versus policy year and about the proper basis of establishing the loss ratio levels at which the statutory reserves begin to apply; this study will not attempt to probe these phases of the problem.

The voluntary reserves in Schedule P indicated in Section C, Exhibit D amount to more money and involve more companies than do the statutory reserve requirements. A balance sheet mechanism which provides an arrangement for voluntary reserves in the liability section should be questioned; a voluntary reserve is not a liability. On the other hand, if provisions for unpaid losses in excess of the estimated ultimate settlement value are necessary or advisable it is better that they be set up as voluntary loss reserves in Schedule P than as excessive redundancies in the unpaid loss provision. The voluntary amounts in Schedule P will not affect income subject to Federal tax and also will not affect statutory gain or loss; they should not serve as a surplus movement stabilization fund. If the risk of unforeseen circumstances merits a special reserving procedure, such procedure should be mandatory rather than voluntary so that all companies would be involved on a uniform basis.

Both the statutory and voluntary reserves should be reported "below the line" rather than in the liability section of the balance sheet. These reserves should be reported separately so that the regulatory authorities would know whether the incurred loss totals produced by regular unpaid loss provisions were lower than the statutory reserve attachment points; from there on they could analyze each company situation on its own merits.

Parts 3 and 4 of Schedule P concern the allocation of unallocated claims expense to policy year. Unallocated claims expense is not subject to positive, objective definition. Further, the definition of the proportion of total unallocated claims expense already paid on unpaid claims is even less positive and objective. Schedule P definitely would be improved if all claims expense were completely eliminated from consideration; Parts 3 and 4 then could be eliminated.

Part 5 of Schedule P does not affect directly the financial statements themselves but it is one of the most valuable exhibits in the annual statement.

The liability suits section of Schedule P serves no useful purpose and therefore should be eliminated.

V. UNEARNED PREMIUMS AND PREPAID EXPENSES

A unique feature in the fire and casualty insurance company accounting method is the concept of unearned premiums and the establishment of earned premiums (premiums written plus the unearned premiums at the beginning and minus the unearned premiums at the end of the period) as gross underwriting income.

The unearned premium liability is quite generally referred to as the unearned premium reserve but it is of interest to note that both in the langauge of the annual statement blank and in Section 832 (b) (4) of the Internal Revenue Code the use of the term "reserve" is studiously avoided. The implication of the language is that unearned premiums are a liability rather than a reserve. This is in contrast to the language used in the life insurance annual statement blank where the policy reserve is called "aggregate reserve for life policies and contracts."

The Canadian annual statement blank also uses the term "unearned premiums" and omits the term "reserve" but in determining the amount of balance sheet liability the unearned premium is "carried out" at 80% of the total computed value. Although the unearned premiums are carried out at 80% in the balance sheet the full 100% figure is used in determining earned premiums for computing loss ratios by province. Also, the full 100% unearned premium is used to establish income subject to the Canadian Federal income tax.

Arguments frequently are advanced that the unearned premiums in the financial statements should be set up at less than the full 100% value. Certain expenses, particularly agents commissions, are paid when the premium is written rather than when it is earned. The portion of this expense attributable to the unearned portion of the premium is known as prepaid expenses which then is frequently referred to as equity in the unearned premium reserve.

In annual reports to stockholders insurance company management frequently takes the equity in unearned premiums into account and makes interpretative adjustments to earnings figures produced by the basic accounting system. Interpretative adjustments of statement figures are necessary in making an intelligent evaluation of both the *worth* and of the operating results of an organization.

Investment houses and statistical firms analyzing company results are constantly appraising the type of expenditures made today which will increase the earnings tomorrow. An example of this is a sentence taken from the Walston & Co. Market Letter of September 20, 1965, which reads as follows: "The Mead Johnson Research Staff has been more than tripled since 1954; in 1965 research expenditures are up to $5.75 million or around $1 per share." The implication of this statement is that this research expenditure is an investment in future enterprise rather than a cost properly and entirely chargeable against 1965 operations.

Insurance statistical services such as Best & Co. use a regular formula in computing equity in unearned premiums. Also, in computing operating ratios, expenses other than claims are related to premiums written in preference to premiums earned.

The interpretative adjustments of figures by Best & Co. and by company managements in stockholders reports do serve a useful purpose in considering and emphasizing elements which are of importance in the longer range analysis and valuation problems. This subjective form of financial analysis has been considered as supplement to rather than revision of the official financial statements.

The full amount of the unearned premiums represents a liability; the question then is whether the prepaid expenses can properly be capitalized as an asset or as a deduction from the full unearned premium liability.

An argument favoring this capitalization is that these expenses are analyagous to costs contained in goods in process in a manufacturing company; these costs are an integral part of the value determining selling price and reimbursement will be accomplished when the inventory is liquidated. These costs are then included in establishing inventory valuation.

The opposing argument places emphasis on the liquidation value concept which underlies the accounting method for financial institutions, including insurance companies. In the event of a straight liquidation, without sale or reinsurance, expenses paid to company employees including commissions or other remuneration paid by a direct writing company to its salesmen would have no reimbursement value. The asset value of potential reimbursement from agents would be dubious; the agent has earned his remuneration and recovering vast sums would prove very difficult.

A more plausible potential reimbursement of prepaid expenses would be through reinsurance commissions if the unearned premiums were reinsured rather than liquidated. Nearly all insurance unearned premiums have some commission value; cases where loss experience is so bad that the commission value is zero exist but are exceptional. However, commission valuation rules with general applicability are difficult to establish; expected losses and loss expense vary by line and by company and change from year to year. Also, ability to negotiate reinsurance at a given commission rate could never be considered as reasonably certain for all companies.

A valuation study concerning 30 companies in which there is a general interest in the stock was published by The First Boston Corporation in 1965. In this study "estimated liquidating value" is defined as follows: "This consists of the sum of policyholders' surplus (adjusted to reflect market values of all securities owned), unauthorized reinsurance, estimated equity in the unearned premium reserve, and any excess of statutory loss reserves over case basis reserves. In computing the equity in the unearned premium reserve, ratios of 40% for fire insurance business and 35% for casualty insurance business are used in most cases. No allowance is made for contingent Federal income taxes which might be incurred on realization of such equity. The estimated liquidating value bears no necessary relationship to the amount which might be realized in actual liquidation."

The report then compares the estimated liquidating value per share with the market value per share. The market value of these stocks at a later date was added to the information contained in the First Boston report; Exhibit E reproduces this information.

Market value represents liquidation value *through sale;* when this value exceeds the statement value the excess represents going concern value, meaning that the business is worth more than the value of its net assets. When market value is less than statement value the implication must be either that the going concern value is negative or that the statement values produced by the accounting method are excessive.

The value comparisons in Exhibit E should sound a note of caution to the general idea of introducing equity in unearned premiums into the official company balance sheets; the formulas commonly in use would tend to portray an exaggerated rather than a conservative view of financial capacity. In a high risk industry where solvency is a paramount consideration this would be unfortunate.

SELECTED FIRE AND CASUALTY INSURANCE COMPANIES
AUTO BODILY INJURY LOSSES
(In $000's)

Source: Calendar Year Basis — Annual Statement, Page 8, Column 9
Accident Year Basis — Annual Statement, Schedule P, Part 5A

	(a)	(b)	(c)	(d)	(e)	(f)
	1962 Losses				1964 Losses	
		Accident Year — Valued				
Company	Calendar Year	1962	1964	Est. Final	Calendar Year	Accident Year Valued 1964
Aetna Casualty and Surety Company	60,835	65,250	64,247	61,484	77,857	80,702
Aetna Insurance Company	13,627	16,601	14,978	14,349	23,627	24,753
Agricultural Insurance Company	2,871	3,127	3,178	3,083	3,601	3,447
Allstate Insurance Company	136.003	147,988	142,764	138,624	159,105	169,380
American Employers' Insurance Company	6,394	8,397	7,349	7,018	10,088	11,290
American Family Mutual Insurance Co.	7,846	7,989	8,653	8,575	10,269	10,081
American General Insurance Company	1,778	1,826	1,835	1,747	2,796	2,757
American Motorists Insurance Company	9,820	11,037	10,526	9,484	13,125	13,911
American Mutual Liability Insurance Co.	7,927	7,028	8,835	8,119	11,734	11,050
American Re-Insurance Company	4,254	5,902	6,144	6,162	7,498	7,242
American States Insurance Company	4,500	4,779	4,240	4,215	4,598	5,099
Auto Owners Insurance Company	5,069	4,865	5,445	5,385	6,285	5,279
Bituminous Casualty Corporation	2,011	2,244	2,310	2,164	2,083	2,295
Boston Insurance Company	3,303	3,053	3,765	3,682	4,770	4,353
Buckeye Union Casualty Company	6,374	5,867	6,648	6,455	6,448	5,683
California State Automobile Association	12.542	12,091	11,652	11,361	12,098	14,146
Continental Casualty Company	22,148	23,152	20,119	19,636	14,476	16,602
Cosmopolitan Mutual Insurance Company	2,797	1,769	4,392	4,502	6,587	3,433
Detroit Auto. Inter-Ins. Exchange	12,082	12,284	11,359	11,779	15,755	14,981
Employers' Casualty Company	3.079	3,042	3,042	3,002	3,386	3,343
Employers' Mutual Casualty Company	6,124	6,335	6,792	6,595	6,682	6,596
Employers' Mutual Liab. Ins. Co. of Wis.	9,097	9.100	8,755	8,519	9,791	10,437
Employers Reinsurance Corporation	4,267	4,927	5,980	5,053	5,329	5,425
Farmers Insurance Exchange	37,991	35,560	37,947	38,023	42,520	41,958
Federal Insurance Company	4,982	6,204	6,088	5,954	8,624	8,271
Federated Mutual Impl. & Hdwe. Ins. Co.	2,587	2,969	2,836	2,765	4,002	4,138

EXHIBIT A (Cont.)

SELECTED FIRE AND CASUALTY INSURANCE COMPANIES AUTO BODILY INJURY LOSSES—Continued
(In $000's)

Source: Calendar Year Basis — Annual Statement, Page 8, Column 9
Accident Year Basis — Annual Statement, Schedule P, Part 5A

	(a)	(b)	(c)	(d)	(e)	(f)
	1962 Losses				1964 Losses	
		Accident Year — Valued				
Company	Calendar Year	1962	1964	Est. Final	Calendar Year	Accident Year Valued 1964
General Fire and Casualty Company	1,520	1,871	1,820	1,747	3,326	2,921
General Insurance Company of America	8,241	9,547	8,945	8,489	6,846	7,815
General Reinsurance Corporation	6,128	6,556	7,419	7,263	7,678	8,021
Government Employees Insurance Company	24,623	24,409	23,817	23,888	34,176	34,393
Hanover Insurance Company	9,523	10,583	10,022	9,581	11,019	11,752
Harleysville Mutual Casualty Company	4,926	4,721	5,789	5,673	7,267	6,435
Hartford Accident and Indemnity Company	63,336	68,884	66,729	64,460	76,413	79,905
Insurance Company of North America	30,287	31,004	30,701	30,026	37,556	39,404
Interinsurance Exchange of the Automobile Club of Southern California	13,830	15,490	16,706	16,355	19,495	19,293
Keystone Insurance Company	4,838	4,396	5,416	5,167	5,638	4,690
Liberty Mutual Companies	42,774	51,431	45,072	44,531	50,467	55,532
Lumbermen's Mutual Casualty Company	22,279	25,209	25,871	24,396	27,790	28,158
Manhattan Fire and Marine Insurance Co.	1,651	2,413	2,824	2,770	2,295	2,847
Maryland Casualty Company	20,270	20,351	20,278	20,177	20,391	20,463
Merchants Mutual Insurance Company	10,435	10,412	10,541	10,383	11,000	11,485
Michigan Mutual Liability Company	7,462	7,420	7,232	7,124	8,592	8,795
National Grange Mutual Insurance Company	6,992	7,085	8,208	8,027	10,117	9,301
National Union Fire Insurance Company	4,643	3,660	4,402	4,728	6,486	5,858
Nationwide Mutual Insurance Company	58,537	59,324	64,271	66,649	80,682	75,947
New Hampshire Insurance Company	4,947	5,159	5,057	4,819	5,054	4,998
North American Reinsurance Corporation	2,821	3,574	3,246	2,905	3,110	3,530
Northwestern Mutual Insurance Company	5,900	5,764	6,561	6,502	6,810	6,361
Ohio Casualty Insurance Company	13,881	13,421	13,285	13,219	12,885	13,088
Pacific Indemnity Company	5,746	5,512	6,305	6,261	6,670	6,058
Pennsylvania Mfg. Assn. Insurance Co.	1,859	1,965	2,289	2,408	2,818	2,355
Phoenix Insurance Company	4,864	6,241	6,010	5,884	9,796	9,812
Public Service Mutual Insurance Company	1,838	2,670	3,269	3,155	4,432	4,038

SELECTED FIRE AND CASUALTY INSURANCE COMPANIES
AUTO BODILY INJURY LOSSES—Continued
(In $000's)

Source: Calendar Year Basis — Annual Statement, Page 8, Column 9
Accident Year Basis — Annual Statement, Schedule P, Part 5A

	(a)	(b)	(c)	(d)	(e)	(f)
	1962 Losses				1964 Losses	
		Accident Year — Valued				
Company	Calendar Year	1962	1964	Est. Final	Calendar Year	Accident Year Valued 1964
Safeco Insurance Company of America	13,519	14,981	13,130	12,434	15,713	17,838
St. Paul Fire & Marine Insurance Co.	16,941	18,494	17,986	17,374	18,918	19,985
Security Mutual Casualty Company	10,057	7,361	10,527	10,843	12,254	8,374
Sentry Insurance Companies	21,933	21,466	22,286	22,152	20,342	20,678
Shelby Mutual Insurance Company	5,820	5,838	5,520	5,510	6,390	6,333
Southern Farm Bureau Casualty Company	4,340	4,948	4,954	4,924	6,454	6,277
State Auto. Mutual Insurance Company	8,882	8,537	8,926	9,024	11,389	10,812
State Farm Mutual Auto. Insurance Co.	102,301	112,715	112,715	112,715	165,471	163,283*
Swiss Reinsurance Company — U.S. Branch	3,448	4,368	3,973	3,556	3,802	4,315
Trinity Universal Insurance Company	2,851	2,607	3,110	3,088	4,135	3,462
Truck Insurance Exchange	7,264	7,024	8,016	8,096	7,153	7,041
United Pacific Insurance Company	2,486	2,544	2,819	2,746	3,026	2,836
United Services Auto Association	8,968	10,208	11,376	11,183	16,970	14,611
U.S. Fidelity and Guaranty Company	40,195	39,007	41,194	40,988	46,338	42,131
Utica Mutual Insurance Company	8,819	10,707	11,735	10,949	11,254	12,220
Western Casualty and Surety Company	6,475	5,923	7,108	7,030	9,006	8,336
Wolverine Insurance Company	3,049	2,504	2,909	2,836	3,133	2,513
Zurich Insurance Company — U.S. Branch	10,249	10,817	11,813	11,707	14,362	12,692
RECAPITULATION						
Mutual Companies	360,402	380,715	394,231	392,093	489,607	478,766
Reciprocal Companies	92,677	92,657	97,056	96,797	113,991	112,030
Stock Companies	598,977	639,135	630,774	612,587	724,455	742,848
All Companies	1,052,056	1,112,507	1,122,061	1,101,477	1,328,053	1,333,644

* Does not include uninsured motorist losses for policy year 1963, accident year 1964. Full calendar year uninsured motorist losses are included in column (e).

EXHIBIT B

SELECTED FIRE AND CASUALTY INSURANCE COMPANIES
WORKMEN'S COMPENSATION LOSSES

(In $000's)

Source: Calendar Year Basis — Annual Statement, Page 8, Column 9
Accident Year Basis — Annual Statement, Schedule P, Part 5A

	(a)	(b)	(c)	(d)	(e)	(f)
	1962 Losses				1964 Losses	
		Accident Year — Valued				Accident Year
Company	Calendar Year	1962	1964	Est. Final	Calendar Year	Valued 1964
Aetna Casualty and Surety Company	51,429	54,694	53,357	51,756	55,908	61,289
Aetna Insurance Company	5,734	6,277	5,855	5,673	8,111	8,550
American Employers' Insurance Company	5,797	7,370	6,394	6,241	7,987	8,721
American Motorists Insurance Company	8,194	8,334	8,439	8,431	11,031	10,929
American Mutual Liability Insurance Co.	31,528	31,103	34,382	34,832	42,515	40,772
Argonaut Insurance Company	17,137	17,075	16,999	17,288	24,780	24,251
Auto Owners Insurance Company	2,373	2,223	1,935	1,832	1,458	1,312
Continental Casualty Company	21,170	21,764	20,593	21,334	14,003	14,364
Cosmopolitan Mutual Insurance Company	3,646	3,504	4,094	4,340	4,879	3,827
Employers' Mutual Casualty Company	5,005	5,536	5,225	5,199	5,053	5,301
Employers' Mutual Liab. Ins. Co. of Wis.	54,650	54,448	54,543	54,979	66,075	65,951
Federated Mutual Impl. & Hdwe. Ins. Co.	2,800	2,898	2,912	2,973	3,149	3,108
Greater New York Mutual Insurance Co.	3,065	3,581	4,210	4,202	5,275	4,922
Hanover Insurance Company	4,341	4,744	4,042	4,054	4,565	5,023
Hartford Accident and Indemnity Company	38,532	41,264	40,722	41,007	44,137	45,732
Industrial Indemnity Company	28,421	24,175	27,062	26,981	25,808	25,190
Liberty Mutual Insurance Companies	99,972	97,964	99,579	97,986	100,804	105,050
London Guarantee and Accident — U.S.	3,305	3,571	3,435	3,504	3,312	3,532
Lumbermen's Mutual Casualty Company	16,916	16,848	17,876	18,198	19,823	18,197
Maryland Casualty Company	15,433	15,224	15,308	15,660	16,730	16,120
Merchants Mutual Insurance Company	2,300	2,354	2,535	2,558	3,220	3,173
Michigan Mutual Liability Company	17,964	16,170	16,990	17,364	18,907	17,209

SELECTED FIRE AND CASUALTY INSURANCE COMPANIES
WORKMEN'S COMPENSATION LOSSES—Continued

(In $000's)

Source: Calendar Year Basis — Annual Statement, Page 8, Column 9
Accident Year Basis — Annual Statement, Schedule P, Part 5A

	(a)	(b)	(c)	(d)	(e)	(f)
	1962 Losses				1964 Losses	
		Accident Year — Valued				
Company	Calendar Year	1962	1964	Est. Final	Calendar Year	Accident Year Valued 1964
Nationwide Mutual Insurance Company	4,382	4,533	4,701	5,209	5,220	5,484
New Jersey Manufacturers Insurance Co.	18,644	15,523	17,358	16,629	19,700	18,965
Pacific Employers Insurance Company	11,331	10,930	12,392	12,268	16,501	14,975
Pacific Indemnity Company	4,474	4,630	4,829	4,776	5,140	5,178
Pennsylvania Mfg. Assn. Cas. Co.	10,576	10,337	10,177	10,146	11,515	11,064
Phoenix Insurance Company	2,203	2,808	2,627	2,577	4,215	4,131
Public Service Mutual Insurance Company	1,864	2,515	2,599	2,680	3,035	2,718
St. Paul Fire & Marine Insurance Co.	6,733	7,576	6,878	6,830	7,578	8,295
Security Mutual Casualty Company	4,024	3,262	4,095	4,140	4,080	3,351
Sentry Insurance Companies	11,790	11,355	12,307	12,418	14,975	13,986
Texas Employers Insurance Association	17,521	16,884	16,683	16,516	16,599	17,103
Truck Insurance Exchange	6,182	5,994	6,665	6,698	7,992	7,394
U.S. Fidelity and Guaranty Company	30,182	31,664	32,679	32,418	34,714	32,487
Utica Mutual Insurance Company	5,394	6,259	5,256	5,288	6,023	6,874
Western Casualty and Surety Company	3,583	3,623	3,831	3,781	4,225	3,907
Zurich Insurance Company	10,753	11,675	11,059	11,081	12,631	12,580
RECAPITULATION						
Stock Companies	297,972	303,258	304,036	302,435	332,591	335,283
Mutual Companies	285,194	281,437	289,922	290,264	321,090	318,338
Reciprocal Companies	6,182	5,994	6,665	6,698	7,992	7,394
All Companies	589,348	590,689	600,623	599,397	661,673	661,015

EXHIBIT C

SELECTED FIRE AND CASUALTY INSURANCE COMPANIES

Auto Property Damage

Ratio: Ratio Unpaid Losses at End of Year to Losses Paid for that Year
1962 Unpaid Losses from 1963 Annual Statement, Schedule O, Col. 9
1963 Unpaid Losses from 1964 Annual Statement, Schedule O, Col. 9
1964 Unpaid Losses from 1964 Annual Statement, Page 8, Col. 7
Paid Losses for All Years from Annual Statement, Page 8, Col. 6

Company	1962	1963	1964
Aetna Casualty and Surety Company	.55	.56	.51
Aetna Insurance Company	.59	.60	.43
Agricultural Insurance Company	.68	.56	.73
Allstate Insurance Company	.57	.58	.56
American Employers' Insurance Company	.56	.62	.73
American Family Mutual Insurance Co.	.33	.35	.33
American General Insurance Company	.34	.35	.21
American Motorists Insurance Company	.77	.83	.90
American Mutual Liability Insurance Co.	.75	.75	.71
American Re-Insurance Company	.56	.34	.68
American States Insurance Company	.33	.30	.33
Auto Owners Insurance Company	.33	.38	.35
Bituminous Casualty Corporation	.53	.57	.64
Boston Insurance Company	.56	.50	.50
Buckeye Union Casualty Company	.46	.48	.56
California State Automobile Association	.49	.47	.47
Continental Casualty Company	.69	.82	1.11
Cosmopolitan Mutual Insurance Company	2.34	2.63	2.18
Detroit Auto. Inter-Ins. Exchange	.35	.33	.39
Employers' Casualty Company	.47	.42	.49
Employers' Mutual Casualty Company	.37	.40	.42
Employers' Mutual Liab. Ins. Co. of Wis.	.48	.52	.59
Employers Reinsurance Corporation	1.23	.11	.23
Farmers Insurance Exchange	.38	.39	.43
Federal Insurance Company	—	.68	.88
Federated Mutual Impl. & Hdwe. Ins. Co.	.35	.32	.36
General Fire and Casualty Company	1.28	1.36	1.69
General Insurance Company of America	.53	.57	.73
General Reinsurance Corporation	.73	.25	.67
Government Employees Insurance Company	.44	.43	.42
Hanover Insurance Company	.35	.70	.72
Harleysville Mutual Casualty Company	.55	.54	.49
Hartford Accident and Indemnity Company	.56	.57	.58
Insurance Company of North America	.84	.87	.96
Interinsurance Exchange of the Automobile Club of Southern California	.45	.41	.49
Keystone Insurance Company	.41	.45	.31

EXHIBIT C (Cont.)

SELECTED FIRE AND CASUALTY INSURANCE COMPANIES

Auto Property Damage

Continued

Ratio: Ratio Unpaid Losses at End of Year to Losses Paid for that Year
1962 Unpaid Losses from 1963 Annual Statement, Schedule O, Col. 9
1963 Unpaid Losses from 1964 Annual Statement, Schedule O, Col. 9
1964 Unpaid Losses from 1964 Annual Statement, Page 8, Col. 7
Paid Losses for All Years from Annual Statement, Page 8, Col. 6

Company	1962	1963	1964
Liberty Mutual Companies	.81	.79	.80
Lumbermen's Mutual Casualty Company	.73	.77	.80
Manhattan Fire andMarine Insurance Co.	.69	.81	.62
Maryland Casualty Company	.52	.55	.60
Merchants Mutual Insurance Company	.83	.92	1.04
Michigan Mutual Liability Company	.78	.66	.63
National Grange Mutual Insurance Company	.62	.51	.40
National Union Fire Insurance Company	.59	.58	.60
Nationwide Mutual Insurance Company	.45	.46	.40
New Hampshire Insurance Company	.69	.71	.78
North American Reinsurance Corporation	2.68	1.57	2.05
Northwestern Mutual Insurance Company	.44	.40	.31
Ohio Casualty Insurance Company	.35	.37	.41
Pacific Indemnity Company	.57	.51	.62
Pennsylvania Mfg. Assn. Insurance Co.	.64	.87	1.09
Phoenix Insurance Company	1.05	.81	.75
Public Service Mutual Insurance Company	1.58	1.57	1.52
Safeco Insurance Company of America	.51	.54	.64
St. Paul Fire & Marine Insurance Co.	.48	.44	.47
Security Mutual Casualty Company	1.46	1.34	1.12
Sentry Insurance Companies	.51	.54	.56
Shelby Mutual Insurance Company	.51	.53	.51
Southern Farm Bureau Casualty Company	.31	.30	.31
State Auto. Mutual Insurance Company	.37	.48	.34
State Farm Mutual Auto. Insurance Co.	.35	.38	.34
Swiss Reinsurance Company – U.S. Branch	2.72	1.56	2.04
Trinity Universal Insurance Company	.49	.46	.60
Truck Insurance Exchange	.66	.62	.63
United Pacific Insurance Company	.42	.38	.43
United Services Auto Association	.40	.48	.43
U.S. Fidelity and Guaranty Company	.42	.42	.45
Utica Mutual Insurance Company	.73	.71	.72
Western Casualty and Surety Company	.47	.50	.59
Wolverine Insurance Company	.39	.36	.39
Zurich Insurance Company – U.S. Branch	.58	.63	.61

EXHIBIT D

SELECTED FIRE AND CASUALTY INSURANCE COMPANIES

A. Companies Included in Exhibit A that Report Schedule P Liability Statutory Reserve Requirement

December 31, 1964

Company	Schedule P Statutory Reserve (In $000's)	2nd Period Loss Ratio Auto Liab.	2nd Period Loss Ratio Other Liab.
American Re-Insurance Company	$ 703	63.7	54.4
American States Insurance Company	349	68.7	38.8
Cosmopolitan Mutual Insurance Company	1,160	64.8	45.9
Employers Casualty Company	1,078	62.0	42.1
Employers Reinsurance Corporation	183	67.8	40.5
National Union Fire Insurance Company	248	66.9	34.9
Public Service Mutual Insurance Company	594	63.6	54.4
Trinity Universal Insurance Company	1,362	57.8	34.7
United Pacific Insurance Company	1,096	60.4	45.0
United Services Auto Association	1,525	59.6	23.1
	$8,298		

B. Companies Included in Exhibit B that Report Schedule P Workmen's Compensation Statutory Reserve Requirement

December 31, 1964

Merchants Mutual	$ 70
Michigan Mutual Liability Insurance Company	133
Texas Employers Insurance Association	1,176
	$1,379

C. Companies Included in Exhibits A and B that Report Schedule P Voluntary Loss Reserves

December 31, 1964

Company	Liability (In $000's)	Workmen's Compensation (In $000's)
Aetna Casualty & Surety Company	$ 4,000	$ 5,500
American Mutual Liability Insurance Co.	6,606	1,837
Cosmopolitan Mutual Insurance Co.	200	450
Employers Casualty Company	664	136
Employers Mutual Liability Insurance Co. of Wisconsin	4,000	6,000
Harleyville Mutual Casualty Company	250	0
Liberty Mutual Companies	65,420	44,415
Lumbermens Mutual Casualty Company	17,984	7,361
Michigan Mutual Liability Insurance Co.	750	1,367
Nationwide Mutual Insurance Company	20,000	0
Public Service Mutual Insurance Co.	1,406	0
Security Mutual Casualty Co.	1,200	0
State Farm Mutual Auto Insurance Co.	42,500	0
Texas Employers Insurance Exchange	0	3,200
Utica Mutual Insurance Company	200	500

EXHIBIT E

COMPARATIVE CONSOLIDATED STATISTICS

Thirty Fire and Casualty Insurance Companies

Company	(a) Estimated Liquidating Value 12/31/64*	(b) Sept. 23, 1965 Market Price
American Re-Insurance	$ 58.23	46
Boston Insurance	50.30	32⅛
Continental Casualty	53.81	52½
Continental Insurance	105.29	70
Employers' Group	74.07	64
Federal Insurance	56.50	57
Fidelity and Deposit	42.85	41¾
Fireman's Fund	43.33	33¼
General Reinsurance	168.66	196
Glens Falls	79.38	44¼
Great American	111.02	64⅝
Hanover Insurance	63.68	42
Hartford Fire	71.40	63
Home Insurance	113.63	65⅛
Insurance Company of North America	90.23	77¼
National Union Fire	63.26	36¾
New Hampshire	56.05	32
North River	98.11	46
Northwestern National	68.84	49¼
Ohio Casualty	24.42	23¼
Pacific Indemnity	41.46	27
Phoenix Insurance	113.95	52¾
Providence Washington	53.56	26¾
Reliance Insurance	56.34	31
St. Paul Fire and Marine	59.66	69½
Security Insurance	58.75	45¼
Trinity Universal	57.30	30
United States Fidelity and Guaranty	63.02	55½
United States Fire	66.55	30½
Westchester Fire	78.63	35¼

* No deduction made for contingent Federal income tax liability on equity in unearned premiums.

Source: Column (b) from Wall Street Journal; all other information in this exhibit from First Boston Corporation "Data on Selected Life Insurance Company Stocks and Fire and Casualty Insurance Company Stocks (1965)."

INSURANCE COMPANY INVESTMENTS

S. DAVIDSON HERRON, JR.

Investment departments have traditionally been set apart in our industry to protect the assets built up in the business and to earn a return both on these funds and on the additions generated by volume growth and retained earnings. The separate character of the department has developed for a number of reasons.

The nature of the insurance side of the business is such that practically all outlays are viewed as expenses to be controlled, and the nature of the investment side is such that practically all outlays are considered as investments to be encouraged. Thus there is a minimum of competition for the privilege of spending the company's money. The legitimate desire to keep investment results out of insurance rates has also contributed to the isolation of the investment function. For the stock companies, another wedge between underwriting and investment has been the traditional relationship of dividend disbursements to net investment income. For the insurance security analyst, there are often two separate companies under one corporate roof – the insurance company and the investment company. (The stockholder cannot enjoy this sophistication. For him there is only one.) The analyst spends most of his time delving into the distinctions between one company and another in underwriting, but presumes that a particular financial position establishes limits on the investment results of a company from which individual departures are not significant enough to measure. Even if he were to analyze the relative investment result, he might not consider the past to be as reliable an indication of the future as he would in appraising the insurance operations.

From what has been said thus far, it might be concluded that the investment manager has the best of all possible worlds – he is set apart from the problems of the insurance business, his fellow executives make every effort to maximize the money he has to spend, and no one seems to measure the result of his efforts! For any investment man whose view of his responsibilities is this comfortable, I need hardly advise that he should never sell a bond at a loss if it is amortizable, and should always add to common stock carried at a low cost. Neither precept is guaranteed to produce a good result, but will, nonetheless, present a favorable appearance.

Editor's Note: This paper was presented by invitation. Mr. Herron is Vice President of the Insurance Company of North America.

The real job of investment management must be described in more aggressive and less amiable terms. The challenge is to improve the financial position of a company relative to its competitors, so that, year by year, it is gaining on them in its ability to add to premium volume, to stand large insurance exposure, to innovate, to raise capital, to acquire companies, and to increase dividends. Because of the deterioration of underwriting profit margins, investment results seem to be drawn more and more directly into the competitive problems of the industry.

Before the portfolio manager can make useful decisions, there are several major factors he must take into account in pursuing his investment polices. These would be:

(1) State insurance laws
(2) Tax laws
(3) Financial position
(4) Liquidity requirements, and
(5) Market conditions

The first of these – state insurance laws – has a negligible influence on the day-to-day investment activities of responsible companies in the business. Typically, such laws prevent investments which common sense would discourage and require investments that prudence would suggest. For instance, we are required to invest an amount at least equal to the par value of the common stock of our company in U.S. Treasury obligations, state and municipal general obligations or Federal Housing Authority bonds. On the other hand we are prevented from lending money to individuals or investing in insolvent corporations. Investment in real estate must be limited to those properties necessary for the conduct of the insurance business. One state in which we do business influences our investment decisions indirectly by permitting us to avoid certain premium taxes, provided we deposit with it general obligations of that state in an amount related to the premium volume generated within that state. Another state over-reaches in this same effort to encourage investment in its debt by relating the share of premium taxes it will forgive to the share of the company's investments represented by tax-exempts issued from within that state. As you might expect, the encouragement is disregarded in a national company, or, if business in that state is large enough, a separate subsidiary is established there.

One aspect of state regulation of investments which individual companies have examined carefully in recent years has been general discouragement of controlled and managed investments in other businesses. When companies have considered major moves into investments in other financial services, they have frequently found that it would be desirable to establish a holding company that would separate the stock of the insurance company from the other operations. If capital supporting the stronger insurance companies is drained away into these other activities, it should have a profound effect on the business. From time to time there are suggestions that state laws should be more restrictive on investments, and there is always available to frighten small children the mythical company that is recklessly gathering in premiums so it can speculate in the stock market and parlay its inadequate capital funds. Capital is leaving the business, by liquidation, common stock repurchase, and diversification. A tightening of state laws governing the investments of the business would accelerate the withdrawal of responsible capital, weakening the strong and providing a useless remedy for the weak.

Turning now to the impact of tax laws on investment decisions, it is worth noting that neither the stock companies nor the mutual companies benefit from any significant tax concessions, nor do they suffer from any peculiar tax burdens. We pay the full corporate rate (48%) on income from U.S. Government securities, corporate debt securities and foreign securities. As is true for all corporations, domestic state and municipal obligations are exempt from tax and an 85% dividends-received credit reduces the effective tax rate on most domestic preferred stocks and all domestic common stocks to 7.2%. Relative yields tend to reflect these differences, but not fully, so that a profitable insurance company will usually minimize its investment in taxable corporate and foreign bonds, maximize its investment in tax exempts and carry a heavy position in preferreds. One curious effect of institutional buying patterns is that the preferred stocks of many companies yield less than the bonds that are senior to them. Some insurance companies do not make great use of preferred stocks in their portfolio, partly because they must be priced at market, but if this is not a consideration, they usually provide more yield after tax than municipals of equivalent quality.

The impact of the capital gains tax has been manifold. For gains taken in six months, or more, during a year when a company is on a current tax basis, the tax is 25%. If the gain is taken in less than six months, or if the company is on a non-current tax basis, the effective tax is 48%. This tax has encouraged the purchase of yield stocks rather than growth stocks, since most of the dividend can be retained or funneled through, but only three-quarters of the profit can be kept. It has contributed to over-diversification, since few sales seem to make sense at the time if a substantial capital gains tax must be paid. It has justified considerable activity in securities that can be sold at a loss, since in a large portfolio it is almost impossible to avoid realizing gains, which it would be neglectful to fail to try to offset.

As statutory underwriting results have deteriorated, more and more companies' results are not only too poor to be taxable, but so poor that the losses can be applied against the earnings of certain other years. The three years immediately prior to the loss year are the first step in the search and the taxes incurred in those years can be recovered from the Government. If the losses are too great to be absorbed by the profits of these three years, they are eligible to reduce the taxable income of the succeeding five years. If these losses continue, a company will build up a series of carry-overs that expire in different years. At some point in these unfortunate developments, the appeal of tax-exempt securities is lost, and taxable bonds take their place. The shift is a delicate maneuver because it is exhausting carry-over while it is underway. It is also delicate in the sense that it is not evidence of much confidence in the underwriters and producers. The last phase in the transition is the sale and repurchase of common stocks so that capital gains are realized just before a carry-over expires. This activity has the effect of writing up the tax cost of the portfolio. While it may at first seem a useless exercise, a portfolio with a potential tax liability becomes more valuable when that liability is removed. It would seem that an actuarial department would be of considerable assistance to the investment manager groping his way toward a policy that would make sense in the early stages of a carry-over.

Turning now to the influences of the financial position of a company on its investment policies, it could be summarized by citing a recent article by Allan Comrie, of Great American Insurance Company, in which he remarked that "Insurance exposure and investment exposure should vary inversely." You may be familiar with a rule of thumb that says that assets equal to the sum of the loss reserve and the unearned premium reserve should be in cash, agents' balances and fixed income securities. There are a number of variations to this which consider the equity in the unearned premium reserves and redundancy in the loss reserve, or which exclude preferreds or non-rated bonds from the offsetting assets. In other words, the measure is adaptable to management's desires. As the ratio of net worth to liabilities increases, and as the profitability of the business improves, investment judgment should take over and asset allocation should be disregarded. If the company is not financially strong, has poor operating results, and is not generating funds, then a more conservative attitude than that embodied in the rule of thumb may be called for, since capital adequacy is in doubt and liquidity needs become a factor. If fixed income seems called for very heavily, it is interesting that the investment manager will often accept a lower credit portfolio to reach for results ordinarily denied him by the category. Where the manager has a number of options, the temptation to speculate in the bond account is minimal.

Capital adequacy is perhaps the single most important influence on investment policy. Reviewing the record of the past 10 years, it can be shown that most additional capital was raised (either in the market or through acquisition of an overcapitalized or liquidating company) when a company's net worth was between 40 and 45% of its liabilities. This is probably the best measure of minimum capital. If a company in this position were to invest its capital funds in common stocks, and were to suffer simultaneously a sharp market decline and a bad underwriting year, the potential of the company would be severely damaged. Companies with a more substantial capital position can afford the risk associated with full investment of their capital funds in common stock, and as we move toward the over-capitalized companies, it is not uncommon to find the value of equities in the portfolio exceeding the capital funds.

Liquidity is the next factor which was listed at the outset of this discussion. Insurance companies may need money fast for a number of reasons. They may be concerned about the ability to pay off a catastrophe loss quickly. They may want to buy in some of their own stock. They may want flexibility in shifting between taxables and tax-exempts. They may work their cash so hard that there is a frequent call on invested funds. Or, they may simply be losing cash in poor underwriting results without volume growth. Government bonds of any maturity are highly liquid, and an excellent vehicle for any of these requirements. A maturity schedule for all bonds may be established to augment the cash flow. A line of credit may be established for quick access to funds. Now that banks are active borrowers for capital needs, there is every reason to expect that insurance companies will change their attitude about debt. The healthier the company, the less it must consider liquidity, for a strongly positive cash flow should take care of the needs of the business. A maturity schedule in such a case is unnecessary, and only provides the manager with an excuse for hiding behind a formula.

From what has been discussed so far, perhaps some insight has been gained into why there is such a variety of ways in which different companies' investments are distributed. Circumstances change enough that from decade to decade, the portfolio pattern will vary considerably, so that what you have learned today will be obsolete not many years from now. One lesson that may last longer is one we all knew anyway: the rich get richer. More particularly, the weak companies with poor underwriting results must maintain relatively conservative portfolios, and their opposites can be just as aggressive as they please. Whether this advantage is real or imaginary depends on the relative qualities of the managers. You may have heard of the studies supporting the belief that a random selection of equities performs as well as a portfolio professionally administered. Obviously, this is nonsense. I don't know any investment managers who agree with it. Oh, yes, there was one chap – but he was about to retire.

REPORT OF THE COMMITTEE ON ANNUAL STATEMENT

PREFACE

In the belief that a study of the Annual Statement is particularly appropriate at the present time, the President of the Society, with the approval of the Council, appointed a Committee on Annual Statement. The Committee's assignment was general, and during its early deliberations the Committee decided that the best interests of the Society would be served by devoting this initial report primarily to an enunciation and discussion of important principles. Although this report will often refer to such specifics as the Committee feels are necessary to clarify the discussion of principles, no actual recommendations with respect to the content of the Annual Statement are contained herein.

INTRODUCTORY COMMENT

History

In 1950, the National Association of Insurance Commissioners adopted a new *combined* Fire and Casualty Annual Statement Blank. This new Blank, replacing the five Convention Blanks which were then current (Stock Fire, Mutual Fire, Foreign Fire, Miscellaneous, and Reciprocal), resulted from several years of study by the Committee on Blanks of the National Association of Insurance Commissioners and insurance industry committees. Impetus to this move had been given by the trend to multiple line underwriting and the change was hailed as a forward step in meeting this challenge.

Furthermore, some dissatisfaction had been expressed with the lack of uniformity among the Blanks and incompleteness of detail in which expenses were being reported in all Blanks. To meet the latter challenge the combined Blank contemplated that a "uniform classification of accounts" would be observed by all insurers.

A particularly comprehensive paper* on the new combined Annual Statement Blank was presented to the Society by Thomas F. Tarbell. Aside from an explanation of the new Blank and the reasons for its adoption, not the least interesting part of Mr. Tarbell's paper is the concise history of the Blank which goes back to the last century.

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* *P.C.A.S. XXXVII*, 1950 and *XXXVIII*, 1951.

Current Criticism

Since the promulgation of the new combined Blank, various changes of a relatively minor nature have been made, but none of them are considered fundamental. However, in the intervening years there has been a continuing body of criticism directed towards the Blank. The more persistent of these criticisms, not necessarily in order of importance and regardless of this Committee's opinion as to merit, follow:

FINANCIAL RESULTS: The Blank exhibits a balance sheet which, although prepared in accordance with legal requirements, uses criteria for valuing assets and liabilities which appear to differ from valuation criteria considered appropriate for non-insurance enterprises. Therefore, the insurance company operating statement consistent with end-year balance sheets seems not to be comparable with operating statements for non-insurance enterprises.

LINES OF BUSINESS: The present Blank matches neither current marketing nor current management divisions within insurers. It is geared to an era which preceded insurance packages. Distinct lines of business have become increasingly blurred. At present more than twenty-five separate divisions by line add to the cost of recording premiums, losses, and expenses without furnishing information commensurate with such costs.

REINSURANCE: Under many reinsurance treaties it is virtually impossible to ascertain premiums and losses by the line of business categories set forth in the current NAIC statement blank. This situation imposes a reporting difficulty on the ceding and accepting carriers, with no guarantee of consistency in the method of reporting premiums and losses. Also, as presently recorded in the Blank, reinsurance transactions may obscure the results of financial operations.

SCHEDULE P: The entire Schedule P adds measurably to recording and preparation expense without contributing material of equivalent value, and without achieving the purpose for which it was intended.

EXPENSE CLASSIFICATION: In view of the substantial variations among carriers, the benefits obtained from analyzing expenses in the categories required by the blank are minimal.

TERMINOLOGY AND PRESENTATION: Many persons not familiar with the insurance industry have difficulty in understanding the various terms. From a purely public relations standpoint, for an industry requiring public confidence, the Statement in its entirety is too complicated.

ELECTRONIC DATA PROCESSING: The present Blank does not permit utilization of modern data processing equipment to its fullest capabilities.

ECONOMY: Some of the items referred to in the foregoing paragraphs and numerous other items not mentioned result in unnecessary expense in the administering of the business.

The criticisms in preceding paragraphs, both the founded and the unfounded ones, are sufficiently troublesome to warrant an examination of the purpose and principles underlying the Statement.

PURPOSE OF ANNUAL STATEMENT

The prescribed Annual Statement is a report of financial condition, and an operating statement (including an analysis of surplus changes) with supporting schedules and data. It is filed in all states in which a company is licensed.

A fundamental duty of insurance supervision is to issue and to renew the licenses of only those insurers which are clearly deserving of public confidence by demonstrating an ability to fulfill all obligations arising out of an insurance contract. One obvious purpose of the Annual Statement is to test, or to assist in the testing of an insurer's financial ability to meet these obligations. The public interest, the interest of regulatory authorities and the interest of the insurance industry are best served if this assessment of capability is recognized as the primary purpose of the Annual Statement.

It is true that data similar to or closely related to that in the Annual Statement is needed for many other purposes. For example, ratemakers need expense data, as well as premium, loss, and exposure data in considerable detail. Insurance managements need operating results subdivided by division of responsibility. Insurance stockholders and those who advise on insurance investments need bases for determining capital stock equity values and bases for estimating earnings and other possible contributions to such values. *It is neither necessary nor even desirable that the primary purpose of the Annual Statement be compromised to meet any of these needs, pressing and legitimate though they may be.*

CONTROLLING PRINCIPLE
WITHHOLDING FULL RECOGNITION OF EARNINGS AND SURPLUS WHILE MATERIAL UNCERTAINTIES REMAIN

Almost all contemporary accounting practice can be called conservative in the sense that whenever uncertainty exists as to the proper valuation of an asset or a liability it mandates somewhat earlier recognition of the unfavorable than the favorable. The American Institute of Certified Public Accountants expresses this difference in timing among their basic concepts as follows:

> "Sales, revenue and income are not to be anticipated. Recognition ordinarily requires sales and delivery, and all known liability or losses should be recorded regardless of whether definite amounts are determinable."[1]

When the word "conservative" is used in this report, it is intended to imply only an accounting principle under which, *when there are uncertainties that will be eliminated only with the passage of time,* there is required an earlier recognition of not fully determinable liability, outgo, or operating loss items than is permitted for not fully determinable asset, income, and earnings items.

If there were no uncertainties in valuing rights and obligations before their ultimate disposition is determinable, the problems of different recognition rates would not arise. Since there are such uncertainties, more in some businesses than in others, there is need for establishing guidelines in terms of some purpose. It may be important to dissuade the managers of a business from distributing or otherwise diverting earnings that, after time has erased the uncertainties, were found never to have actually existed. In most commercial enterprises this dissuasion will be for the ultimate benefit of the owners and perhaps indirectly for the creditors. For industries imbued with a large public interest, such as banks and insurance companies, the dissuasion may be necessary for the ultimate protection of the customers.

Accounting principles obviously are not entirely within the domain of actuaries. However, actuaries do recognize the concept of conservatism underlying generally accepted accounting principles, do recognize that there are important reasons why the insurance business should not be exempt from this concept and do recognize the kinds of uncertainties with which the insurance business must cope. An objective of insurance accounting, broadly stated, should be to the effect that the portions of revenue from customers released into earnings have a degree of certainty, or finality, comparable to the earnings released by accounting practices in other industries.

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[1] *Inventory of Generally Accepted Accounting Principles*—Grady, Page 36.

In the examination of analogies with other industries one very important contrast reveals itself. When there are material uncertainties in the balance sheet items of other industries, they tend to be on the debit rather than on the credit side. This contrast can be capsuled as follows:

Manufacturer	*Insurer*
Process begins with conversion of money into inventories, whose values are uncertain.	Process begins with acceptance of money in exchange for assumption of uncertain liabilities.
While values of inventories are uncertain, these values are treated conservatively.	While values of liabilities are uncertain, these values are treated conservatively.
Period of uncertainty ends with reconversion of inventories into money at date of sale.	Period of uncertainty ends with release of liabilities at expiration of policy, or with reconversion of liabilities into money at date of settlement of loss.
During period of uncertainty the conservative margin used in valuing inventories is not credited to earnings.	During period of uncertainty the conservative margin used in valuing liabilities is not credited to earnings.
* *	* *
Liabilities are of few types and are comparatively easy to value.	Assets are of few types and are comparatively easy to value.

In non-insurance businesses the manufacturer or vendor most typically buys something and then proceeds to enhance its value by working on it, moving it within the customer's reach, or both. Goods in inventory, as well as other assets with which the value-enhancing activity is carried out, have an uncertain current worth and an ultimate worth which is dependent upon future events and upon the skill with which the management of the business copes with such future events.

Pending the resolution of these uncertainties by actual sales to customers, the inventories are conservatively valued at "cost or market, whichever is lower,"* or on some other standard which approximates this degree of conservatism. Until most of the uncertainties are eliminated by the closing of a sale, earnings are not recognized, even though in many instances the ultimate sale at profitable prices might have been statistically predictable with some accuracy.

The most important liabilities of industrial and mercantile businesses are fixed by contract and present no serious valuation problems. The amount required to discharge them is relatively independent of both future events and the skill of the management of the business that has the liability.

In the business of insurance, on the other hand, many of these circumstances are reversed and the standards for a suitable degree of conservatism require some paraphrasing. Typically the assets, rather than the liabilities, do not pose problems of measurement and can be quantified for a statement date under any suitable method.

In contrast, the amount ultimately required to discharge the major liabilities must be determined partially by events which will occur in the future and depends on the managerial competence of a particular insurer. Further, the period of uncertainty for the insurer is brought to a close by the end of a policy period or by the settlement of the claim, not by the date of policy sale.

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* "Cost or market, whichever is lower" is referred to here as a valuation standard operating to prevent recognition of earnings prior to the elimination of material uncertainties. Commercial accountants, while not denying that it operates to this end, may prefer a more down-to-earth explanation; i.e. the revenues against which costs are to be recognized must first have developed to the state at which they can be considered realized. This method of treating uncertainties suggests that there is an analogy between insurance and commercial practice.

"Cost or market, whichever is lower" is one application of a more general formula for valuing items on both sides of the balance sheet: The general fromula might be worded,

"A or B whichever produces the lower value of surplus (or earnings) when

A is the amount spent historically on the acquisition of the asset (or obtained for the assumption of the liability), and

B is the amount that could be obtained from a hypothetical open market sale of the asset on the statement date (or would be paid to a responsible outside entity for complete relief from the liability in an equally hypothetical open market transaction)."

This generalized form would fit the insurance practice of a liability for unearned premiums that is not discounted either directly or indirectly. It would fit prevailing practices for outstanding loss liabilities to the extent that well managed companies expect to find some margin as the settlements are made and this margin is of the same order of magnitude as the additional amount a reinsurer would charge if it were asked to assume another carrier's open case liability for a fixed and final consideration. It does not fit insurance practices with regard to investments anymore than it does commercial practices with regard to capital assets.

It is recognized that accounting practices must vary from one business to another when the point of taking up revenue must by necessity differ. Quoting from Finney and Miller: "For retail, wholesale and manufacturing businesses, the point of sale is generally regarded as the point when revenue is earned, because (1) it is the point at which a conversion takes place . . . and (2) it is the point at which the amount of revenue is, in the normal case, objectively determinable from a sale price.

"Revenue is earned by service-type businesses as services are performed. In some cases—usually, when the rendering of services extends over a fairly long time period and involves more than one accounting period—estimates may be used in order that revenue may be recorded and reported during the periods when the work is being performed. Practical considerations may lead to the adoption of a policy of postponing the reporting of any revenue services until the services are completed: the amount to be charged for the entire service may not be determinable until completion and, as a consequence, the revenue applicable to services rendered during the periods prior to completion may not be determinable."[2]

The insurance business is more nearly akin to a service type business than to a manufacturing or retailing business, but the completion dates of its contracts can be either policy expiration dates or final settlement dates for intricate third-party claims. In terms of these and other distinctive aspects, a generally appropriate approach to the valuation of assets and liabilities has evolved. The techniques have been and will be in continuous need of updating, but the Committee suggests no change in the basic objective. As stated previously, it is that, when any revenue from customers is released into net earnings, these have the same finality or freedom from uncertainty as earnings released by the accounting practices of other industries.

COLLATERAL PRINCIPLES

There are a number of collateral principles which march toward the fundamental purpose—assessing financial capability.

Informativeness

BALANCE SHEET AND OPERATING STATEMENT. The balance sheet and the operating statement should be in sufficient detail to permit a ready analysis, as well as year-to-year comparisons. Assets, liabilities, and ear-

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[2] Harry A. Finney and Herbert E. Miller, *Principles of Accounting—Introductory*, Sixth Edition, pp. 241-242.

marked surplus items, as well as income, outgo, and changes in ear-marked surplus items, should be in summary form and in mutually exclusive categories. Liquidity of assets and liabilities should be apparent for comparison with one another and with other financial and underwriting commitments. Necessary informative detail should be relegated to supporting schedules, and limited to the degree of refinement necessary to evaluate properly the asset, liability, or operating statement item concerned.

For instance, an analysis of revenue is desirable for assistance in the evaluation of (1) the unearned premium liability, (2) the adequacy of surplus to support the nature and scope of the insurance operation, and (3) the degree of concentration of the company's business both in area and coverage. Such an analysis would require detail which would disclose the business characterized by wide fluctuations in experience, the business requiring specialized expertise, and the business which can be referred to as the bread-and-butter lines. *Losses paid* would of necessity be consistent with *losses outstanding* detail in that one is dependent on the other for any meaningful evaluation. Expense detail should be limited to the refinement necessary to disclose the methods of operation of the company. Details of underwriting gain should also be set forth in a supporting schedule.

For concentration of business by area, an analysis of premiums by state (similar to the present Schedule T) would also assist in analysis of the company's revenue.

PROSPECTIVE AND RETROSPECTIVE EVALUATION OF LIABILITIES. In the Annual Statement, a suitable degree of conservatism superimposes itself upon certain important balance sheet items, chiefly among liabilities and surplus. An operating statement for a fiscal period involves, in part, a differencing of balance sheet items at the beginning and end of such period. Therefore, the desired matching of revenue and expense is inevitably affected by the degree of conservatism, and most significantly by any change in this degree of conservatism during the fiscal period.

In considering the problem of liabilities, particularly the reserve for losses, the question appears to be one not only of intention but of competence. In either event, such retrospective evaluations as are made should be so set forth as to make possible some prospective judgment, even though it is admittedly limited, as to the adequacy of current loss reserves. Where losses incurred are concerned, the constituent elements of paid and outstanding losses should be available. An analysis might require detail which would disclose those losses characterized by lengthy settlements, those not easily appraised, those subject to statutory reserve valuations, and those which are quickly settled.

UNDERWRITING AND FINANCIAL COMMITMENTS. A very important part of assessing an insurer's ability to meet its contractual obligations is knowledge concerning the scope, distribution, and protection of its underwriting and financial commitments.

Is the insurer's business distributed among many lines of insurance, or is it concentrated in one line? Is the business widely distributed geographically or is it all in one state? Can policies be canceled or not renewed or is the insurer locked into the policy contract? Are catastrophe potentials reinsured or absorbed? Can rates be adjusted annually or are they guaranteed for long periods of time? Are pension obligations funded adequately?

The answers to these and many other important questions should be provided in some appropriate manner by the Annual Statement.

FACILITATION OF INSURANCE DEPARTMENT EXAMINATIONS. Periodic examination of insurers by State insurance departments is generally required by State statutes. But, over and above legal requirements, insurance department examinations are a fundamental part of an over-all purpose of enhancing public confidence in the insurance business through public disclosure. The Annual Statement is another very important part of this same purpose. Therefore, there is a vital connection between the Statement and the periodic examinations.

While there is general sentiment in the industry that the Annual Statement contains much useless and redundant information, any effort to reduce the amount or type of information required in the Statement must avoid possible undesirable side effects: (1) public reaction that the insurance business is suddenly trying to conceal something heretofore disclosed; (2) Commissioner feelings that more frequent examinations will become necessary to reveal information no longer found in the Statement; and (3) more time and effort being required by examiners during the conduct of an examination to obtain data previously furnished in the Statement.

In the interim periods between examinations, Annual Statements are reviewed in the Department offices by the staffs of insurance departments for accuracy and internal consistency. If the Statement can assist in these reviews by providing economically assembled data in supporting schedules, the interests of both supervisory authority and insurance industry are served.

Annual Statements are public records. If they were not, it seems probable that Insurance Commissioners would feel their full responsibilities could only be discharged through more extensive examination. It would be false economy to reduce the Statement to the point where office reviews were inhibited or actual examinations were more frequent or more prolonged, and therefore more costly.

SECONDARY USES. The Annual Statement provides a formalized vehicle for public disclosure of all information pertinent to the operation of an insurance company. As such, there should be no objection to its use for secondary purposes provided such adaptation (1) does not interfere with the timeliness, economy, or flexibility achieved in the serving of primary purposes; and (2) cannot be replaced by a better and more economical method.

Examples of such secondary use are: (1) assistance in periodic examinations by insurance departments; (2) assistance in interim Departmental reviews for consistency; (3) basis for levying of some kinds of taxes and assessments; (4) disclosure of names of officers and directors; and (5) such other information as need be displayed and is now included in the general interrogatories.

COMMUNICATION. While the primary purpose of the Annual Statement would seem to be clear, it must be recognized that many individuals use the statement for many purposes. The Annual Statement may not even fulfill its primary purpose unless it has a substantial degree of acceptance outside the insurance business as well as inside. Therefore, its purpose is not served by esoteric terminology or by wording or format totally unfamiliar to accountants more used to phrases and forms of statement found among other industries.

If an insurer is deserving of the confidence of all, a statement which conceals this fact, because it cannot be understood except by a hard core of insiders, is of little value.

Uniformity and Continuity

The responsibility for supervision of insurance is vested in the several states and the District of Columbia. This responsibility continues from year-to-year and from one supervisory official to his successor. A need for uniformity in the Annual Statement is not founded upon mere convenience to insurers doing business in more than one state.

Rather the Annual Statement should maintain reasonable degrees of uniformity and continuity lest the measure of an insurer's ability to meet its obligations appear different to observers in different State Capitals. Contradictory yardsticks create a suspicion as between one another. Annual Statements which conflict in major format or which change radically from year to year would be poor vehicles for inspiring public confidence, where such confidence is otherwise wholly justifiable.

Adaptability

It is to be expected that the insurance business should progress either to meet new needs or to provide a better product at a better price. It would be unfortunate if the Annual Statement operates to prevent such changes from taking place. It would also be unfortunate if the changes take place and the Annual Statement, in failing to recognize them, loses its utility for the purpose described earlier. The need for a degree of adaptability involves a number of things: the statutes, the apparatus whereby the Statement requirements are kept up to date, and the degree of detail deemed necessary. This need for adaptability must be balanced against a conflicting need for some degree of uniformity and continuity.

However, sufficient continuity should be provided by the existing N.A.I.C. apparatus whereby the Statement requirements are kept up to date. The obstacles to change are too great when specific statement requirements are written into the statutes. When statement provisions are written into the statutes, as some liability valuation procedures are, there is the danger that the original language may become obsolete or its reconciliation with the language of other state statutes may become impossible. Revising statutes of a particular state cannot be undertaken without regard to related legislation both within and across state boundaries.

Ideally, state legislation would create the Commissioner's obligation to have Annual Statements filed as public records, but without prescription as to specific detail. To the extent possible, but without the pitfall of specific formulas, it would indicate the desirability of conservatism in asset and liability valuations appropriate for the primary purpose of such a statement. It would facilitate the Commissioner's use of the N.A.I.C. Blanks Committee expertise in the implementing of his responsibility.

Economy

The Annual Statement is prepared as one of many end products of an insurer's data recording, data assembly, and data processing activities.

These activities start with the work stations where premium entries are first recorded and loss reports are first received. They encompass maintenance of agents' accounts, maintenance of policyholders' accounts, all claim and loss record keeping, coding and assembly of ratemaking data, and all other uses of the individual items of income and outgo that must be recorded.

The cost of the physical preparation of the Annual Statement, together with its printing and filing, is thought to be a relatively minor burden on those who buy insurance. However, the requirements of the Annual Statement, both as to detail and timing, influence the entire data recording and processing operation. These requirements, if carelessly drawn, can make it difficult or impossible for insurance managements to control these costs and to keep them within the test of value received.

Electronic data processing equipment is now standard equipment in many insurance operations. Many Annual Statement requirements can and should be adapted to the use of this equipment now and in the future.

Finally, while it is important to have summary figures for the balance sheet and operating statement soon after the close of business, it is true that many of the supporting schedules could just as easily, and certainly more economically, be filed at some later date.

SPECIFIC APPLICATIONS

The reference to legal barriers prompts mention of *Schedule P (Parts 1 and 2)*. Schedule P in turn prompts mention of two related areas where, in the opinion of the Committee, the present Statement is imperfect— the *line of business classification system, and the treatment of reinsurance.*

Schedule P

The theory implicit in Schedule P (Parts 1 and 2) is that of setting an arbitrary minimum value on third-party loss valuations until sufficient time has elapsed to remove most of the uncertainty surrounding these valuations. It was never expected that this Schedule could guarantee that a company was reserved adequately if its rates were too low or its claim handling too inept. It was expected that Schedule P (Parts 1 and 2) could prevent a company from dissipating imaginary earnings before someone discovered such earnings were imaginary. This limited objective is entirely compatible with the principle of conservatism.

Regardless of the worthwhileness of the objective and the classic simplicity of the technique for achieving it, the business has changed so that the present Schedule P (Parts 1 and 2) is too often inoperative, useless, and productive of expensive waste motion in data processing. Permissible loss ratios are no longer uniform among companies. Earned premiums are not always determinable with the same timing as the reporting of claims. Finally, it is becoming increasingly difficult to separate bodily injury (only) premiums on a policy year basis with reasonable economy and plausible accuracy.

If the original objective remains desirable, the Schedule P (Parts 1 and 2) should be redesigned so that these Parts will achieve the objective and can be completed with data more economically assembled.

Line of Business

The line of business classification system appears to have had its origins in the statutes defining the kinds of insurance for which companies may be licensed, even though these statutory definitions are far from uniform among the states. The purpose of the Statement requires some disclosure of the nature of the company's underwriting commitments. Moreover, the techniques of testing the provisions for outstanding losses differ among the different kinds of losses, so a suitable classification is required for this purpose. The present line classification contributes to both of these needs, but inefficiently and expensively. The Committee is persuaded that a more meaningful and less expensive system of classifying premiums and losses probably could be found, if the system or systems were designed specifically for the purposes of the Statement and not borrowed from licensing statutes.

As an illustration consider the possible analysis of premiums. Because the income vehicle in an insurance company is the insurance policy or contract, it would seem to follow that the breakdown of premiums be listed in *type of policy* categories. This would, of necessity, separate direct business from reinsurance contracts. Such breakdown could replace the twenty-five lines of business separation now considered by many to be of no real value, and might run as follows:

(1) Personal liability and property (including present homeowners)
(2) Commercial liability and property (including present commercial multiple peril)
(3) Personal automobile (all perils)
(4) Commercial automobile (all perils)
(5) Personal accident and health

(6) Group accident and health
(7) Workmen's compensation
(8) Bonds
(9) Ocean marine
(10) All other (for specialty companies—specialty to be clearly indicated)

All of the foregoing are easily susceptible to standard definition and would for all practical purposes permit the processing of policy premiums, if incidental coverages were included in the standard definition.

Reinsurance

The present Statement treats reinsurance assumed as though it has essentially the same impact as primary direct business, and reinsurance ceded in a manner suitable for routine facultative cessions. Since the Blank was developed, reinsurance has become a business apart in many companies, has been used to define the relationship among members of fleets, and has acquired a variety of new uses more complicated than mere facultative cessions.

The Committee doubts that the Statement can or should be used to disclose the detail of all reinsurance transactions but does believe that better disclosures of direct business will result if, on many exhibits, reinsurance is dealt with separately.

The classifications of premiums and losses most appropriate for direct business need not, in fact almost certainly will not, be the ones most appropriate for reinsurance. Sufficiently separate treatment of reinsurance would make the classification problem easier.

* * * *

These three specific problems are, of course, interdependent.

CONCLUDING COMMENTS

In this preliminary reporting the Committee has examined the purpose of the Annual Statement and the principle of conservative valuations that is corollary. Some effort has been made to express the degree of conservatism needed in the valuation of uncertain liabilities, which has some analogy, however imperfect, to the familiar one in the valuation of inventories for non-insurance enterprises.

The Committee has also examined the considerations governing the degree of detail and disclosure that are justified in order that the Statement may serve its purpose well. Conceivably the Annual Statement could be either a one-page summarized balance sheet and operating statement or it could involve many hundreds of pages of analytical detail. The Committee has reasons for believing that neither extreme serves the primary purpose.

Also, there has been a review of the need for some uniformity and continuity, and the other need, sometimes in conflict, for adapting the Statement to a changing business. The latter included a look at the apparatus for making changes and some of the legal handicaps under which that apparatus operates.

Three areas (Schedule P, Categories of Business, and Reinsurance) were mentioned briefly in this report because they were thought to be the areas most vulnerable in the present Statement. They were thought vulnerable only to the extent (a) a conservative objective is not being achieved, (b) money is being wasted assembling less meaningful information, and (c) meaningful information is being obscured.

In mentioning these areas this report suggested the directions in which the Committee felt solutions might be found. It is recognized that developing the detail of the solutions would be a long difficult task calling for the additional expertise of people other than actuaries. It is also recognized that the industry and those who supervise the industry have the services of such experts.

John W. Carleton
Howard G. Crane
Robert G. Espie
Clyde H. Graves
Charles C. Hewitt, Jr.
Richard Lino
Ruth E. Salzmann
Harold W. Schloss
Joseph Linder, Chairman

REPORT ON THE AMERICAN ACADEMY OF ACTUARIES

LAURENCE H. LONGLEY-COOK

There have been many statements explaining the need for the proper accreditation of actuaries and I will only touch on this subject briefly in presenting this report. The question of the legal recognition of actuaries has been discussed over the years but it was not until the late 1950's that it received really serious consideration of the various actuarial societies. The need for this accreditation has been well summarized by Walter Klem in his report on The Development of the Actuarial Profession in the United States presented last year to the International Congress of Actuaries.

> "The role of the actuary is becoming increasingly well known today in general circles, and where actuaries are known they are well regarded. This regard could easily be lost, however, by a relatively few instances of public dissatisfaction. As it became clearer that the supply of well qualified actuaries could not keep pace with the expanding public need for their services, it was foreseen that there could well be confusion on the part of the public as to the standing and quality of the rapidly growing body of those describing themselves as actuaries. Anyone in the United States may claim to be a qualified actuary and offer his services as such to the public; and a number of persons do so who cannot be considered fully qualified by any reasonable standard."

The millions of people involved and the immense size of the assets of insurance companies and of pension funds make it essential that the public is protected from unsound actuarial practices.

In our Society the first step toward accreditation was taken at the end of 1957 by Dudley Pruitt, then President, in setting up a Special Committee on Rules and Standards of Professional Conduct under the chairmanship of Doc Masterson. One year later a Committee on Certification or Licensing of Actuaries was formed with the same membership as that of the Professional Conduct Committee, and in 1960 the name of the Committee was changed to the Committee on Professional Status. This committee continues to this day to advise the Council of our Society on this important matter and to work with the other actuarial bodies in seeking accreditation of actuaries.

While a number of informal discussions took place prior to 1963, the first formal joint meeting of representatives of the four actuarial bodies took place in Chicago in February, 1963. On this occasion the Casualty

Actuarial Society was represented by Bill Leslie and myself. This meeting led to the formation of a Joint Committee on the Organization of the Actuarial Profession, with a representative from each of the four societies.

You have already heard of the outcome of this committee's work; the proposal to charter the American Academy of Actuaries, to include not only members of the four societies but other qualified actuaries who were not members of any organization. A bill to provide for incorporation of the Academy was passed by the Senate, but failed to pass the House in the 1964 session of Congress. The bills were reintroduced in the 1965 session and again passed the Senate. At a time when the House was on the point of holding hearings on the bill, the President vetoed a bill which would have conferred a federal charter on another organization and in his veto message raised the question "whether federal charters were being granted by Congress to private organizations without the benefit of clearly established standards and criteria as to eligibility." In view of this message it seemed clear that there would be some delay in obtaining a federal charter and it was decided to proceed with the organization of the American Academy of Actuaries as an unincorporated association with the expectation that, if a federal charter were obtained, this organization could readily be dissolved and we would then continue under a federal charter. If continued delay occurs in obtaining a federal charter, a state charter will be obtained.

The Academy was officially constituted on October 25, 1965 at the time of the Society of Actuaries meeting and many of the leading members of our Society were among the founders. Our Society is strongly represented among the officers of the new Academy. Our president, Tom Murrin, is the president-elect of the Academy, to take office as president in a year's time. I am one of the four vice presidents and Bill Leslie, Doc Masterson, Dan McNamara and Frank Harwayne are directors. All these persons, and also Joe Linder, have given many hours of their time to the problems of the Academy and have attended numerous committee and subcommittee meetings. The officers of the Academy include other members of our Society, but these, who are mentioned below, are more closely affiliated with the Society of Actuaries. The president, Henry Rood, is an Associate of our Society and John Miller, one of the vice presidents, is a Fellow. Gil Fitzhugh, Bob Myers, and Allen Mayerson, all Fellows of our Society, are directors.

Three important committees have been set up: the Committees on Admission, Education, and Professional Conduct. Our representatives on these committees are Harold Schloss, Norman Bennett, and Tom Murrin, and they have each worked hard in their respective spheres.

Details of the new organization have been mailed, or will be mailed shortly, to each one of you. In brief, Fellows of the Casualty Actuarial Society will become members of the Academy unless they submit a written declination within 60 days, and Associates will be eligible for membership if they meet the necessary experience requirements.

It must be realized that the formation of the Academy does not in itself confer on its members any legal professional status. It only opens the door to action by individual states who may pass legislation providing for the certification of actuaries and to action by state and federal regulatory officials who may specify that certain documents and reports must be signed by a member of the Academy.

I cannot end this report without praising the members of the other three societies for their most helpful cooperation at all times and without paying tribute to the inspired leadership of Henry Rood in the formation of the Academy. This marks a significant point in the history of the actuarial profession in this country and the Academy will, I believe, contribute greatly to our future development.

MINUTES OF THE 1965 ANNUAL MEETING

November 15 and 16, 1965

SHERATON-BOSTON HOTEL, BOSTON, MASSACHUSETTS

MONDAY, NOVEMBER 15, 1965

The meeting was called to order at 9:45 a.m. on Monday, November 15, 1965.

A subsequent tally indicated that the following 98 Fellows, 58 Associates and 46 Guests were in attendance:

FELLOWS

Aldrich, W. C.
Alexander, L. M.
Allen, E. S.
Bailey, R. A.
Barker, G. M.
Barker, L. M.
Bennett, N. J.
Berkeley, E. T.
Berquist, J. R.
Bevan, J. R.
Blodget, H. R.
Bondy, M.
Boyajian, J. H.
Boyle, J. I.
Crane, H. G.
Curry, A. C.
Curry, H. E.
DeMelio, J. J.
Dickerson, O. D.
Drobisch, M. R.
Dropkin, L. B.
Ehlert, D. W.
Elliott, G. B.
Finnegan, J. H.
Fitzgibbon, W. J., Jr.
Foster, R. B.
Gillam, W. S.
Gillespie, J. E.
Graham, C. M.
Hart, W. Van Buren, Jr.
Harwayne, F.
Hazam, W. J.
Hewitt, C. C., Jr.
Hobbs, E. J.
Hope, F. J.
Hunt, F. J., Jr.
Hurley, R. L.
Johe, R. L.
Johnson, R. A.
Jones, H. M.
Kallop, R. H.
Kates, P. B.
Klaassen, E. J.
Lange, J. T.
Leslie, W., Jr.
Linden, J. R.
Linder, J.
Lino, R.
Liscord, P. S.
Livingston, G. R.
Longley-Cook, L. H.
MacKeen, H. E.
Makgill, S. S.
Masterson, N. E.
Mayerson, A. L.
McClure, R. D.
McGuinness, J. S.
McLean, G. E.
McNamara, D. J.
Menzel, H. W.
Morison, G. D.
Moseley, J.
Muetterties, J. H.
Murrin, T. E.
Nelson, D. A.
Nelson, S. T.
Niles, C. L., Jr.
Oien, R. G.
Otteson, P. M.
Peters, S.
Petz, E. F.
Phillips, H. J.
Pollack, R.
Portermain, N. W.
Resony, A. V.
Richards, H. R.
Riddlesworth, W. A.
Roberts, L. H.
Rodermund, M.
Rowell, J. H.
Ruchlis, E.
Salzmann, R. E.
Simon, L. J.
Skelding, A. Z.
Smith, E. M.
Tarbell, L. L.
Thomas, J. W.
Trist, J. A. W.
Trudeau, D. E.
Uhthoff, D. R.
Verhage, P. A.
Walsh, A. J.
Webb, B. L.
Wieder, J. W., Jr.
Wilcken, C. L.
Williams, P. A.
Wilson, J. C.
Wolfrum, R. J.

ASSOCIATES

Adler, M.
Amlie, W. P.
Bell, A. A.
Berg, R. A., Jr.
Berkman, J. M.
Bland, W. H.
Brian, R. A.
Brown, W. W., Jr.
Buffinton, P. G.
Carlson, E. A.
Coates, W. D.
Cook, C. F.
Copestakes, A. D.
Crandall, W. H.
Crofts, G.
Dickson, C. D.
Durkin, J. H.
Flaherty, D. J.
Franklin, N. M.
Gerundo, L. P., Jr.
Gibson, J. A. III
Gill, J. F.
Gould, D. E.
Grossman, E. A.
Hachemeister, C. A.
Hammer, S. M.
Hanson, H. D.
Harack, J.
Honebein, C. W.
Jensen, J. P.
Margolis, D. R.
Markell, A. S.
McDonald, M. G.
McIntosh, K. L.
Mohnblatt, A. S.
Mokros, B. F.
Naffziger, J. V.
Newman, S. H.
Peel, J. P.
Perreault, S. L.
Presley, P. O.
Ratnaswamy, R.
Richardson, J. F.
Roth, R. J.
Royer, A. F.
Scammon, L. W.
Scheel, P. J.
Scheid, J. E.
Schuler, R. J.
Scott, B. E.
Shaver, C. O.
Singer, P. E.
Smith, E. R.
Staley, H. B.
Stevens, W. A.
Strug, E. J.
Switzer, V. J.
Young, R. G.

GUESTS

Anderson, R. R.
Anstey, M. P.
Battaglin, B. H.
Bechtolt, P. R.
Bickerstaff, D. R.
Blanc, R.
Bond, T. M.
Callahan, W. E.
Connolly, C. T.
Ferri, L. D.
Forest, J. H.
Fratello, B.
Griffith, R. W.
Hart, J. F.
Hartman, G. R.
Hayden, R. C.
Herron, S. D., Jr.
Hewey, H. V.
Hoskins, R. H.
Hoyt, F. A.
Irish, F. S.
Lewis, S. G.
Luck, T. J.
Marshall, R. E.
McSherry, H.
Murphy, S. W.
Murphy, T. V.
Nagel, J. R.
O'Shea, H. J.
Piersol, D. E.
Plast, L.
Reinbolt, J. B.
Rothbart, H.
Sabbagh, M. J.
Schoomer, B. A.
Shorr, B.
Sohmer, H.
Stocker, R. H., Jr.
Strong, H. L.
Sturgeon, P. K.
Van Orman, F.
Watkins, E., Jr.
Watkins, J. W.
Wight, C. N.
Wise, T. A.
Zubay, E.

After a brief welcoming address to members and invited guests President Murrin announced that the Woodward-Fondiller prize of $200 had been awarded to Kenneth L. McIntosh, Manager, Louisiana Rating & Fire Prevention Bureau, for his paper "A Mathematical Approach to Fire Protection Classification Rates" which had been presented at the meeting of the Society in May of 1965.

Vice President Harold E. Curry then presided for the remainder of this plenary session.

Lester B. Dropkin, Actuary, California Inspection Rating Bureau, read a written review of Mr. McIntosh's paper referred to above.

By invitation S. Davidson Herron, Jr., Vice President, Insurance Company of North America, presented a paper "Insurance Company Investments."

A summary of a new paper by Philipp K. Stern, Actuary, Mutual Insurance Rating Bureau, "Ratemaking Procedures for Automobile Liability Insurance," was presented in Mr. Stern's absence by William W. Brown, Jr.

The session was then recessed to permit the members to attend one or the other of the following concurrent discussions:

A. Package Policy Ratemaking – Property Insurance

Chairman: Jack Moseley, Associate Actuary
United States Fidelity & Guaranty Co.

Participants: Gordon M. Barker, Actuary, Great American Group

Philip G. Buffinton, Vice President,
State Farm Fire & Casualty Co.

Edward J. Hobbs, Associate Actuary,
Insurance Company of North America

John H. Muetterties, Associate Actuary,
Sentry Insurance-Hardware Mutuals Group.

B. Functions of the Actuary

Moderators: Paul E. Singer, Actuary,
Continental National American Group

Dunbar E. Uhthoff, Vice President & Actuary,
Employers' Mutual Liability Insurance Co. of Wisconsin.

A luncheon recess was taken from noon until 2:00 p.m., after which time there was held a general seminar discussion "Operations Research":

Chairman: LeRoy J. Simon, General Manager, National Insurance Actuarial & Statistical Association

Speakers: Dr. Thomas J. Luck, Executive Assistant,
State Farm Mutual Automobile Insurance Co.

Dr. B. Alva Schoomer, Arthur D. Little Co.

Bernard Shorr, Research Department,
Travelers Insurance Co.

Professor Eli Zubay, Georgia State College.

Recess was taken at 5:00 p.m.

In the evening there was a brief social hour followed by a banquet.

TUESDAY, NOVEMBER 16, 1965

The session convened at 9:45 a.m. with President Murrin presiding.

The Secretary-Treasurer presented a report on important actions of the Council subsequent to the November 1964 meeting and also presented a financial report covering the period October 1, 1964 thru September 30, 1965.

It was voted that the report be accepted and made part of the Minutes.

The gathering then stood in a moment of silence in memory of the deceased members whose passing had occurred subsequent to the 1964 Annual Meeting:

Gilbert E. Ault
Leo D. Cavanaugh
Robert A. Craig
Winfield W. Greene*
Grady Hayne Hipp
Samuel F. Milligan

Laurence H. Longley-Cook then read a report on the present status of the American Academy of Actuaries. In addition to other observations the report noted that, as an interim step looking forward to the obtaining of a federal charter, the Academy had been organized as of October 25, 1965 as an unincorporated association.

Mr. Longley-Cook then informed the gathering that there were available at the registration desk copies of some remarks on "Investment Income" which he had put in writing.

It was announced that due to time limitations the contemplated seminar "Dual Insurance Company Operations – Investments and Underwriting" had been postponed to a future meeting.

There was then held an open forum discussion "The New Statistical Plan for Homeowners, Comprehensive Dwelling and Dwelling Policies." John H. Boyajian, Actuary, National Insurance Actuarial & Statistical

* Past President and a Charter Member of the CAS.

Association, led off by describing the details of the new Plan, the reasons therefor and the objectives sought. There were a number of questions from the floor.

At this point Vice President Harold E. Curry took over the duties of presiding officer for the remainder of the session.

Jack Moseley then summarized the discussions in the Monday morning colloquium "Package Policy Ratemaking – Property Insurance." This was followed by a summary, presented by Paul E. Singer, on the second discussion held at that time, "Functions of the Actuary."

The following new papers were then presented by the authors:

Walter J. Fitzgibbon, Jr. – "Reserving For Retrospective Returns."

Paul M. Otteson – "Some Observations Concerning Fire & Casualty Insurance Company Financial Statements."

Thomas E. Murrin then gave his Presidential Address.

Following this, President Murrin introduced the following 17 new Associates:

Martin Adler	Daniel J. Flaherty	Stephen L. Perreault
Allan A. Bell	Charles A. Hachemeister	Philip O. Presley
William H. Bland	H. Donald Hanson	Rajaratnam Ratnaswamy
Robert A. Brian	Carlton W. Honebein	James F. Richardson
Charles F. Cook	Joseph V. Naffziger	James E. Scheid
Carol D. Dickson	Steven H. Newman	

In presenting diplomas to the following 9 new Fellows, President Murrin gave a brief biographical sketch on each individual:

Alan C. Curry	Darrell W. Ehlert	Neill W. Portermain
Joseph J. DeMelio	Dale A. Nelson	Paul A. Verhage
Stanley A. Dorf	Robert G. Oien	Bernard L. Webb

Norton E. Masterson then presented the recommendations of the Nominating Committee (William Leslie, Jr., Norton E. Masterson, Seymour E. Smith, Chairman) for nomination for the offices of President, Vice President, Secretary-Treasurer and three members of the Council, based on the returns from the informal ballot distributed to the Fellows under date of October 14, 1965.

There being no further nominations from the floor, the Fellows present proceeded to vote on the following slate as presented by the Nominating Committee:

President – Harold E. Curry

Vice Presidents – Charles C. Hewitt, Jr.
Harold W. Schloss

Secretary-Treasurer – Albert Z. Skelding

Members of Council – Lester B. Dropkin
William S. Gillam
Allen L. Mayerson

A vote was then taken on each position and the foregoing were declared duly elected.

The gathering was then informed that the Council, acting under the provisions of Article V of the Constitution, had elected the following officers:

Editor – Matthew Rodermund

Librarian – Richard Lino

General Chairman of the Examination Committee – Norman J. Bennett.

The action of the Council on these three officers was subject to confirmation by vote at the November 1965 meeting.

Upon a call for a vote the above three individuals were declared duly elected.

Thereupon, the 1965 Annual Meeting was adjourned at 12:00 noon on November 16, 1965.

Beginning at 1:30 p.m. there was held before an overflow audience an extracurricular joint seminar on credibility sponsored by the Casualty Actuarial Society and the Society of Actuaries.

The program and the names of the participants follow:

Moderator – Laurence H. Longley-Cook, FCAS

(1) Credibility — Its Meaning and History

Allen L. Mayerson, FCAS, FSA
"A Survey of the Field"

Donald A. Jones, ASA
"The Posterior Premium"

Charles C. Hewitt, Jr., FCAS
"Reconciliation of Some Ideas of Albert W. Whitney, Arthur L. Bailey, Allen L. Mayerson."

(2) Credibility In Experience Rating

Arthur G. Weaver, FSA
"Credibility in Group Insurance"

Ernest A. Arvanitis, FSA
"Credibility Considerations for Group Underwriting and Group Dividends"

Nathan F. Jones, ACAS, FSA
"Comments on Theory Versus Practice in Experience Rating"

Robert A. Bailey, FCAS
"Credibility as a Correlation Between Past and Future Experience."

(3) Credibility in Classification Rate Making

Lewis H. Roberts, FCAS
"Generalized Theory of Credibility"

LeRoy J. Simon, FCAS
"Credibility Concepts in the Property Insurance Field"

Jeffrey T. Lange, FCAS
"Credibility Concepts in Casualty Insurance."

Following the presentations by the above listed participants there were numerous additional comments from the audience as well as questions directed to individual members of the seminar. Due to time limitations it was necessary to terminate the discussions and adjourn the seminar at approximately 4:30 p.m.

Respectfully submitted,
ALBERT Z. SKELDING,
Secretary-Treasurer

REPORT OF THE SECRETARY-TREASURER

The following report summarizes those activities of the Council, subsequent to the 1964 Annual Meeting, which it is felt will be of particular interest to the membership.

Mail Vote of January 21, 1965

The Council approved the petition for recognition of a contemplated local actuarial club to be known as the Midwestern Actuarial Forum, an organization which is now in being.

Meeting of May 23, 1965

The Council voted to receive the draft report of the Committee on Annual Statement, which had been distributed to the entire membership under date of December 3, 1964. In passing it is noted that this draft was the subject of a lively panel discussion at the May 1965 meeting of the CAS. The Council also voted to delegate to the Committee on Annual Statement, with power, the task of incorporating in a final report any changes from the draft which the Committee agreed upon.

The Council authorized the Librarian to enter into negotiations with the Insurance Society of New York and the Society of Actuaries for reciprocal borrowing privileges from the libraries of the three organizations. As noted in the recently revised edition of the "Recommendations For Study" this arrangement has been ratified by the three societies.

The Council accepted with thanks and sincere regret the resignation of Harold W. Schloss as Editor. It then appointed Matthew Rodermund to succeed him.

Mail Vote of August 3, 1965

The Council voted to approve the recommendation of the Educational Committee that, beginning with the May 1966 examinations, Associate Part 2 be a joint identical examination with the Society of Actuaries.

OTHER ITEMS OF INTEREST

Future Meetings

It has been necessary for various reasons to make several changes with respect to sites and dates of previously scheduled future meetings. The schedule as it now stands follows:

May 22-25, 1966—Cavalier Hotel, Virginia Beach, Virginia.

November 14-16, 1966—Sheraton-Cadillac, Detroit, Michigan in lieu of the previously scheduled Ann Arbor meeting.

May 24-26, 1967—Pheasant Run Lodge, St. Charles, Illinois.

November 15-17, 1967—Hotel America, Hartford, Connecticut.

May 1968—Due to fact that an advance booking of some six or seven years is required, Williamsburg, Virginia has been abandoned as a possible meeting site.

The examination Syllabus has been revised to become effective with the May 1966 examinations. Copies of the revised edition have been mailed to all members.

It is hoped that before the end of the year, or shortly thereafter, the Johnson Reprint Corporation will be able to fill orders for reprint copies of back numbers of the *Proceedings* in accordance with the following tentatively established prices:

Volumes I-XXIX—Cloth bound	—	$640.00 per set
Volumes I-XXIX—Paper bound	—	$570.00 per set
Volumes I-XXVIII—Paper bound	—	$ 20.00 each
Volume XXIX—Paper bound	—	$ 10.00 each

The thanks of the Society are due to the American Mutual Insurance Alliance, the Insurance Information Institute and the National Association of Independent Insurers for a contribution of $300.00 each toward defraying the printing costs of our brochure *A Career as a Casualty Actuary*.

Thanks also are due to the National Council on Compensation Insurance for donating office space and the facilities of the National Council in connection with conducting the business end of the office of the Secretary-Treasurer.

The financial report of the Secretary-Treasurer for the fiscal period October 1, 1964 through September 30, 1965 is attached to this report. Copies of the financial report are available at this meeting.

It will be noted that during this period disbursements ($33,681.74) exceeded receipts ($32,460.42) by $1,221.32. This result is largely influenced by the fact that on the occasion of our gala, once-in-a-lifetime, Fiftieth Anniversary meeting in November of 1964, expenses exceed receipts by $1,870.54.

As of October 1, 1965 the assets and surplus of the Society consist of

Checking account	$ 6,902.98
Two savings accounts	16,271.39
U. S. savings bonds (maturity value)	5,000.00
Total	$28,174.37

FINANCIAL REPORT

Cash Receipts and Disbursements
from October 1, 1964 through September 30, 1965

Receipts

On deposit 10-1-64 (Checking)		$ 8,770.97
On deposit 10-1-64 (Savings)		10,556.09
On deposit 10-1-64 (Savings)		5,068.63
Members dues	$11,650.00	
Examination fees	3,456.28	
Sale of Proceedings	2,709.74	
Sale of Readings	202.25	
Spring & annual meetings	5,703.00	
Registration fees	3,211.00	
Invitational program	1,860.00	
Foreign exchange	—24.36	
Bond interest	193.76	
Savings account interest	646.67	
Michelbacher Fund	1,380.58	
For Actuaries' Club N. Y.	555.00	
Miscellaneous	916.50	32,460.42
Total		$56,856.11

Disbursements

Printing & stationery	$17,728.96
Secretary's office	2,100.00
Examination expense	1,305.73
Meeting expense	10,043.89
Library fund	7.80
Insurance	101.74
Refunds:	
Lunch & dinners	429.50
Examination fees	88.53
Registration fees	263.45
Fees to Actuaries' Club N.Y.	540.00
Miscellaneous	1,072.14
	$33,681.74
On deposit 9-30-65	
Checking	6,902.98
Savings	10,983.91
Savings	5,287.48
Total	$56,856.11

Assets

Cash in bank 9-30-65	
Checking	$ 6,902.98
Savings	10,983.91
Savings	5,287.48
U. S. Savings Bonds	5,000.00
Total	$28,174.37

Liabilities

Surplus (Michelbacher Fund)	$17,200.21
Other surplus	10,974.16
Total	$28,174.37

One U. S. Treasury Bond 3⅞% No. 24277 due for $1,000 on May 15, 1968.

Two U. S. Treasury Bonds 3⅞% Nos. 3462-3 due for $1,000 each on May 15, 1968.

Two U. S. Treasury Bonds 3⅞% Nos. 1673-4 due for $1,000 each on November 15, 1974.

Employers' Fire Insurance Company Policy No. 31 F 238562 for $5,000 on books and book cases stored at 200 East 42 Street and $2,000 on material stored in library of Insurance Society of New York. Expires 9-14-67.

Fidelity Bond No. 044571 for $25,000 in Royal Indemnity Company.

Workmen's Compensation Policy No. 01-814744 in Maryland Casualty Company. Expires 5-10-66.

Owners', Landlords' and Tenants' Liability Policy No. 52-523796 in Maryland Casualty Company. Expires 4-23-66.

NOTES

The miscellaneous receipts item of $916.50 consists principally of a contribution of $300 each from the American Mutual Insurance Alliance, the Insurance Information Institute and the National Association of Independent Insurers towards the reprinting costs of "A Career as a Casualty Actuary."

The principal miscellaneous disbursement items are:

Insurance Society of New York — Organization membership	$ 150.00
American Academy of Actuaries — CAS pro rata share of legal and *organization expenses*	634.46
Contribution to International Congress	25.00
Floral tribute in memory of Past President Winfield W. Greene	26.00
Refunds on orders for out-of-print Proceedings	40.00
Purchase of adding machine	123.76
Non-reimbursed photographic expense — 50th Anniversary meeting	37.00
Total	$1,036.22

* * *

This is to certify that we have audited the accounts, examined all vouchers and investments shown above and find same to be correct.

Auditing Committee
H. G. CRANE, Chairman
THOMAS W. FOWLER
MATTHEW RODERMUND

BOOK NOTES*

James L. Athearn, *General Insurance Agency Management* (The Irwin Series in Risk and Insurance), 390 pages, Richard D. Irwin, Inc., Homewood, Illinois, 1965.

This book describes the principles, procedures, profit techniques, and management information necessary for the successful operation of a general insurance agency. The purposes of the book apparently are to provide one consolidated source of agency management information for those who are experienced in the insurance business as well as for those who are new, to serve as a reference guide to those who work in an insurance agency, to advise the reader of the sources of information about insurance agency management, and to provide a comprehensive and informative view of the nature of the operations of a general insurance agency to readers who have had no previous experience or training in that field.

The author is Dean of the School of Business Administration, Montana State University.

Ralph H. Blanchard, *Risk and Insurance and Other Papers*, 210 pages, University of Nebraska Press, Lincoln, 1965.

The author of this book has been a Fellow of the Casualty Actuarial Society since 1917 and is a Past President. Now Professor Emeritus of Insurance, Graduate School of Business, Columbia University, Doctor Blanchard has collected for this book thirty-five of his major articles and addresses on the fundamental principles of risk-bearing. He considers here such disparate but related matters as ratios and public relations, premium rates and insurance terminology, constructive competition and government and insurance.

Doctor Blanchard explores in all its aspects the risk element in the American adult's personal and business life and the use to be made of insurance in solving such risk problems.

John D. Long and Davis W. Gregg, Editors, *Property and Liability Insurance Handbook* (The Irwin Series in Risk and Insurance), 1265 pages, Richard D. Irwin, Inc., Homewood, Illinois, 1965.

This *Handbook* is an encyclopedic treatment of property and liability

* A traditional feature of the *Proceedings*, Reviews of Publications, has been discontinued. Since the *Proceedings* is issued only once a year, books that are of interest to members of the Casualty Actuarial Society have usually been reviewed in many other organs of the insurance press before any discussion of them can have appeared in the *Proceedings*. Henceforth, therefore, the *Proceedings* will merely list new books of interest with a few words describing the contents but with no critical opinions.

insurance. Planned and edited by Doctor Long, Professor of Insurance of the Graduate School of Business at Indiana University (Doctor Long also authored two of the chapters), and Doctor Gregg, President of the American College of Life Underwriters, this book was written by seventy-four outstanding American insurance authorities in addition to Doctor Long, three of whom are Fellows of the Casualty Actuarial Society: James M. Cahill, Harold J. Ginsburgh, and Charles J. Haugh. Fifty-eight others, including seven members of the Casualty Actuarial Society, served as Consulting Editors.

Detailed attention is given in this book to each line of property and liability insurance, including fidelity and surety bonds. The treatment includes the functions of insurance; structure of the business, including insurance pools, reinsurance, company and agency management, professionalism, marketing, and nonadmitted markets; the concept of risk management; policy forms, underwriting principles, and ratemaking; accounting and tax problems; public relations; and almost every other conceivable aspect of the property and liability insurance business. The reader is aided by indexes and numerous cross-references.

Walter O. Menge and Carl H. Fischer, *The Mathematics of Life Insurance,* 491 pages, The Macmillan Company, New York, 1965.

This is a revision of a textbook, *An Introduction to the Mathematics of Life Insurance,* published in 1935 by Doctor Menge and James W. Glover. Doctor Menge, a Fellow of the Society of Actuaries, is Chairman of the Board of the Lincoln National Life Insurance Company, and Doctor Fischer, also a Fellow of the Society of Actuaries, is Professor of Insurance and Actuarial Mathematics in the Graduate School of Business Administration at the University of Michigan.

Much of the material in the earlier book has been rewritten and new exercises for the student have been added. The rewriting accommodates recent actuarial practice and deals primarily with single life contingencies and applications of these contingencies in combination with the theory of interest. The authors introduce their subject with a discussion of the theory of probability and then go on to explain mortality tables, annuities, net premiums, net level reserves, modern reserve systems, and gross premiums. The text also supplies the latest statutory mortality tables (1958 C.S.O. with 3% interest).

OBITUARIES

GILBERT EUGENE AULT

LEO DAVID CAVANAUGH

WINFIELD W. GREENE

GRADY HAYNE HIPP

SAMUEL MILLIGAN

WILLIAM F. SOMERVILLE

GILBERT EUGENE AULT

1896 — 1965

Gilbert E. Ault, Actuary of The Church Pension Fund, died on April 13, 1965 in Englewood Hospital, New Jersey. He was 68 years of age.

The lifework of Gilbert Ault was pensions. Although his achievements in casualty and life insurance were notable, his major effort was in the construction and maintenance of pension plans, enabling many thousands of people to enjoy the retirement which was denied him.

Mr. Ault stayed on the job to the end, completing his 42nd Annual Statement as Actuary of The Church Pension Fund from his hospital bed. He died a month later. His early work for the Pension Fund (which administers the national pension system for Episcopal clergymen) was on a consulting basis, from 1923 to 1939, when he was appointed Actuary. He was also Actuary of Church Life Insurance Corporation, an affiliate of the Fund.

Previous connections were: Guardian Life Insurance Company (1919 to 1922), Woodward, Fondiller, & Ryan (1922 to 1931), Colonial Life Insurance Company (1931 to 1939). He constructed the Valuation Tables for the New York State Workmen's Compensation Act, also the far-flung pension system of International Telephone and Telegraph Corporation.

Mr. Ault was born on October 7, 1896 in State Line Mills, Pennsylvania. He received his A.B. (with special honors in mathematics) in 1918 from the University of Rochester. He then pursued graduate study in mathematics at Columbia University. He was a Fellow of the Casualty Actuarial Society from 1931, an Associate of the Society of Actuaries from 1924, an Active Member of the Fraternal Actuarial Association from 1924, and a Senior Member of the Actuaries Club of New York.

Among his published papers are "Methods of Financing Pension Plans" in the Handbook of Business Administration (American Management Association) and "Group Annuities" in the *Record* of American Institute of Actuaries, Volume XVIII. He made useful contributions to pension theory in the early years. He also devised programming methods which utilized computers to unusual extent.

By application of a brilliant mathematical intellect to complex actuarial projects, Mr. Ault achieved some meticulous procedures which left negligible room for error. His associates sometimes marveled at his elegant methods and are left with the desire to pursue his theories and purposes insofar as possible.

Church and college benefited from Mr. Ault's efforts as Trustee and Treasurer of the First Congregational Church of Hackensack, Treasurer of the Greater Hackensack Everymember Canvas, Vice President of Rochester Alumni Association. That he had a generous nature was attested by a number of individuals who spoke gratefully after his death of aid given them over the years.

His family includes a son, Wilbur E. Ault; three daughters, Mrs. Charles (Mary) Morris, Mrs. William (Dorothy) Stone, Mrs. David (Ruth) Hadley; and eleven grandchildren. His wife, Constance, predeceased him in 1943.

LEO DAVID CAVANAUGH

1889 — 1965

Leo David Cavanaugh, an Associate of the Casualty Actuarial Society since 1922, died in Chicago July 18, 1965. He was chairman of the board of the Federal Life Insurance Company until his retirement in 1959. He had been associated with that company for 45 years.

He was elected actuary of the company less than a year after he became associated with it and his progress was rapid. In 1923 he became vice president and actuary; in 1931, executive vice president and actuary; in 1939 he succeeded Isaac Miller Hamilton and became the company's second president; and, in 1954 he became chairman of the board.

During his long service in the insurance business he was a former president of the Accident and Health Underwriters Conference; president and chairman of the Insurance Federation of Illinois; a member of the executive committee of the American Life Convention; a member of the board of the Institute of Life Insurance; chairman of the finance committee of the American Life Convention and treasurer of the American Service Bureau.

WINFIELD W. GREENE

1887 — 1965

Winfield W. Greene, a Charter Member and Past President of the Casualty Actuarial Society, and a Fellow of the Society of Actuaries, died on March 26, 1965 at St. Elizabeth's Hospital in New York.

At the time of his death Mr. Green was President of W. W. Greene, Incorporated, Reinsurance Intermediaries and Actuarial Consultants, of New York City, and also a Vice President and Director of the Old Republic Life Insurance Company of New York.

Mr. Greene was born in Surrey, Maine, on May 22, 1887. After graduating from Brown University in 1910 he became associated with the New York Life Insurance Company, Actuary's Department. In 1913 he joined the examination force of the New York State Insurance Department. Later he became Assistant Actuary of the New York Workmen's Compensation Commission. At a still later date he organized the Colorado State Workmen's Compensation Fund and served as Manager of that organization in its infancy. He then was called upon to serve as Manager of the Compensation Rating and Inspection Bureau of New Jersey. He later was an Underwriter and Consulting Actuary of the Employers Mutual Insurance Company of New York. Following this he became Actuary of the young National Council on Compensation Insurance.

In October 1925 he joined the General Reinsurance Corporation as Controller and was Executive Vice President of that organization from 1945 to 1952.

Mr. Greene was a pioneer and a leading contributor in the development of the application of scientific techniques to the problems of workmen's compensation insurance ratemaking and rating procedures when that branch of the insurance industry came into being in the United States. Although there have been many changes in those procedures since those days, the current workmen's compensation ratemaking methods retain many of the basic elements, including the idea of individual classification credibility, developed by Mr. Greene and embodied in the paper on Workmen's Compensation Ratemaking which appeared in Volume XII of the *Proceedings of the Casualty Actuarial Society.*

From the time of the organization of the Casualty Actuarial Society in 1914 Mr. Greene maintained a lively interest in the affairs of the Society. He contributed many outstanding papers to the *Proceedings* beginning with "Valuation of the Death Benefits Provided by the New York Compensation Law," one of the three papers presented at the first meeting of the Casualty Actuarial Society held on November 7, 1914.

The passing of Win, as he was known to his colleagues in the casualty actuarial profession, leaves a gap which cannot be filled. His outstanding abilities and pre-eminence in the fields of workmen's compensation and casualty reinsurance, coupled with his never failing availability for discussion of problems with all who sought him out, were sources of inspiration and comfort to actuaries old and young.

Mr. Greene is survived by his widow, Grace Lau Greene; two sons, Winfield K. and Thomas A.; a daughter, Mrs. Ludwig Saskor; and three grandchildren.

GRADY HAYNE HIPP

1893 — 1965

Grady Hayne Hipp died June 25, 1965 in Greenville, South Carolina. He had retired in 1959 as Executive Vice President of the Liberty Life Insurance Company of Greenville. He was admitted as an Associate of the Casualty Actuarial Society on November 17, 1927. He also was an Associate of the American Institute of Actuaries and the Fraternal Actuarial Association.

Grady Hipp was born in Old Town, South Carolina, on February 3, 1893. He graduated from Newberry College in 1911 and took post graduate work at the Universities of Virginia and Wisconsin in mathematics and political economy. Thereafter he was employed by the Insurance Departments of the State of Wisconsin from 1912 to 1919 with the title of Assistant Actuary, and the State of New York from 1919 to 1930 with the title of Actuary.

In 1930 Mr. Hipp came to the New York State Insurance Fund as Actuary and remained there until 1943 at which time he joined the Liberty Life Insurance Company.

Since his retirement from the Liberty Life Insurance Company Mr. Hipp had been living in Greenville. He is survived by his wife, Thelma Bush Hipp.

SAMUEL MILLIGAN

1887 — 1965

Samuel Milligan, a Charter Member of the Casualty Actuarial Society and a Fellow of the Society of Actuaries, died on August 8, 1965, at the age of 78. He was born in Londonderry in County Derry, Ireland, on May 12, 1887, and was graduated from Foyle College there. In 1906 he came to this country and joined the Metropolitan Life Insurance Company on December 31st of that year as an Audit Clerk.

Later he transferred to the Company's Actuarial Division and was appointed an Assistant Actuary of the Company in 1920.

In 1926 he was given responsibility for the operation of the Company's Ordinary Department with the title of Third Vice-President. He was advanced to Second Vice-President in 1936, Vice-President in 1944, and Administrative Vice-President in 1951. In 1953 he was elected Senior Vice-President and a member of the Company's Board of Directors. He retired from active service with the Company on December 31, 1957.

In 1949 Mr. Milligan was awarded the Ulster-Irish Honor Medal by the Ulster-Irish Society of New York. The medal is awarded annually to a person of Ulster birth or descent "who has enriched the arts, sciences and life of the people of the United States." The citation accompanying the award read in part,

> "Your superb achievements in the world of finance, your invaluable service to our Government in National emergencies, are worthy of honor, but it is what you are, rather than what you have done, that brings us together tonight to acclaim you.
>
> "The high esteem in which you are held by the tens of thousands of the Metropolitan staff, your unfailing courtesy to, and consideration of the humblest worker, your exemplification of all that is fine in Ulster character, your true democracy in that you with the kings of finance yet keep the common touch, make you the ideal recipient of this medal.
>
> "We honor ourselves in honoring you."

At the time of the award, the late Leroy A. Lincoln, then President of the Metropolitan, expressed the following tribute to Mr. Milligan:

> "I know you will speak for all of us on the many facets of Mr. Milligan's character and abilities. I have often said, and have not yet

been challenged, that Samuel Milligan is the most valuable, all-around Officer that we have in the Metropolitan. His genial personality and common-sense attitude toward things are accompanied by a deep sense of responsibility and of a desire to secure the correct solution of any question."

Sam Milligan was an extremely generous man, with both his time and money. The extent of his generosity is little known, since he always insisted on anonymity. But he was a true example of the man who would give the shirt off his back if he thought it were needed.

Mr. Milligan was an honorary member of the Home Office Life Underwriters Association. He was also a member of the New York Athletic Club, the American-Irish Historical Society and the Friendly Sons of St. Patrick.

An ardent fisherman and hunter, he had for many years maintained a home on Lake Champlain, where he resided at the time of his death.

A bachelor, Mr. Milligan is survived by his sister, Miss May R. Milligan of Westport, N. Y.

WILLIAM F. SOMERVILLE

1885 — 1965

William F. Somerville died November 12, 1965 at St. Luke's Hospital in Kansas City, Missouri. He had retired as Secretary of the St. Paul-Mercury Indemnity Company in January of 1951. He was an Associate of the Casualty Actuarial Society, having been admitted November 19, 1926.

Mr. Somerville was born on August 9, 1885 in Birkenhead, England. He started his career in London in 1903; in 1914 he came to America and a little later became affiliated with the Hartford Accident and Indemnity Company in its home office. Later he went to the Lumbermen's Mutual Casualty Company in Chicago, and then joined the St. Paul-Mercury Indemnity Company in 1930. He was appointed Assistant Secretary in 1937, Secretary in 1943, and Director of that company in 1950.

Mr. Somerville lived after his retirement in Excelsior Springs, Missouri. He is survived by his wife, and one daughter, Mrs. Max Mann.

EXAMINATION FOR ENROLLMENT AS ASSOCIATE

PART I GENERAL MATHEMATICS

The questions for Part I were prepared and copyrighted by the Educational Testing Service of Princeton, N. J., and cannot be reprinted. Students may obtain a set of similar questions from the Secretary-Treasurer.

PART II SECTION (a)

PROBABILITY

1. In three throws with a pair of dice, what is the probability of throwing doublets at least once?

2. If the letters of the word BECOMING are arranged in a random fashion, what is the probability that the O and the I will be separated by exactly one letter?

3. Jerry tosses 3 coins while Tom tosses 2 coins. What is the probability that Tom will turn up more heads than Jerry?

4. There are three mutually exclusive events, A, B and C, one of which must happen. The odds are 8 to 3 against A; 5 to 2 against B. Find the odds against C.

5. One urn contains 5 red and 3 yellow balls; a second urn contains 2 red and 4 green balls. One ball is transferred at random from the first to the second urn. What is the probability that a ball now drawn at random from the first urn is red?

6. If at least one of the three events, A, B, C, must occur; and if the events A, B and C are not mutually exclusive; what is the probability that all three events occur, given that Prob (A) $=$ .4, Prob (B) $=$ .5, Prob (C) $=$ .46, Prob (AB) $=$ .14, Prob (AC) $=$ .16, Prob (BC) $=$ .16?

7. A and B stand in a circle with ten other persons. If the arrangement of the twelve persons is at random, find the probability that there are exactly three persons between A and B.

8. An experiment fails 3 times as often as it succeeds. What is the probability that there will be at least 2 successes in the next 4 attempts?

9. Suppose 3 bad light bulbs get mixed with 12 good ones. If you start testing the bulbs one by one until you have found all the bad ones, what is the probability of finding the last defective on the seventh testing?

10. A game is played by drawing a card at random from a special deck of 10 cards consisting of 4 aces, 2 kings, 2 queens, a jack and a ten. The game pays as follows: nothing if an ace is drawn; $1 if either a king or queen is drawn; $2 if either the jack or ten is drawn. What is the fair price for playing this game?

11. An urn contains a white ball, a black ball and a red ball. A sequence of 5 drawings is made with replacement after each drawing. What is the probability that the white ball will appear twice, the black twice and the red once?

12. Three numbers are randomly drawn one after the other from the set (1,2,3,4,5). The three different digits so obtained are written down in the order in which they were drawn. What is the probability that the resulting three-digit number turns out to be even?

13. Consider a square sheet of tin, 20 inches wide, that contains 10 rows and 10 columns of circular holes, each 1 inch in diameter, with centers evenly spaced at a distance 2 inches apart. What is the probability that a particle of sand (considered as a point) blown against the tin sheet will fall upon one of the holes and thus pass through?

14. If a die is tossed 5 times, what is the probability that all numbers will be greater than 2, but not all will be greater than 4?

15. The time, measured in minutes, required by a man to travel from his home to a train station is a random phenomenon obeying a uniform probability law over the interval 20 to 25. If he leaves his home promptly at 7:05 A.M., what is the probability that he will catch a train that leaves the station promptly at 7:28 A.M.?

16. An urn contains balls of 4 different colors, each color being represented by the same number of balls. Four balls are drawn with replacement. What is the probability that at least 3 different colors are represented in the sample?

17. Consider a sequence of 4 games such that r coins are tossed simultaneously for the r^{th} game ($r = 1,2,3,4$). What is the probability that the total number of heads in all 4 games combined is exactly 4?

18. A bridge player announces that his hand (of 13 cards) contains the ace of hearts. What is the probability that it will contain another ace? (Leave your answer in factorial notation.)

19. Twenty-five raffle tickets contain two winning tickets. What is the smallest number of tickets a person must buy in order to have a 95% or better chance of winning at least one prize?

20. Suppose that 75% of the ranchers raising cattle include a special vitamin supplement in the diet of their herds. When the animals are sold, each is classified as either Grade A or Grade B. If the vitamin supplement is not included in the diet of an animal, the probability that the animal will become Grade A is 1/2; if the vitamin supplement is included in the diet, the probability that the animal will become Grade A is 4/5. What is the probability that an animal sold as Grade A had been raised with the vitamin supplement included in its diet?

21. A and B play a game which consists of playing a series of hands. At each play of a hand the probability that A wins the hand is 1/4; that B wins the hand is 1/2; that there is a tie is 1/4. A is declared the winner of the game if he wins at one hand before B wins at two hands. Otherwise B is the winner of the game. What is the probability that A wins the game?

22. A random phenomenon has a probability function specified by the following (cumulative) distribution function:

$$
\begin{aligned}
F(x) &= \tfrac{1}{2}e^{-(x/50)^2} \text{ for } x \geq 0 \\
&= 1 - \tfrac{1}{2}e^{-(x/50)^2} \text{ for } x \leq 0
\end{aligned}
$$

What is the probability that the random phenomenon specified by the given function will be greater than 50?

23. Consider n particles, each of which may be found with the same probability, 1/N, in any one of N cells (N>n). What is the probability that n of the cells each contain one particle? Assume that all the particles are distinguishable and that any number of them from 0 up to n can be found in each of the cells.

24. For a certain line of insurance, individual insureds are classified as being either Class A or Class B. The probability of an insured having an accident during a one-year period is p if he is in Class A; 2p if he is in Class B. The line of insurance is such that an insured can have at most one accident each year. If the probability that a Class A insured will have no accidents in a 3 year period is .216, what is the probability that a Class B insured will have only 1 accident in a 2 year period?

25. An urn contains some black and some white balls. When 2 balls are drawn at random, the probability of their both being white is 5 times the probability of their both being black. Also, it is 6 times as likely that the balls are of different colors as that they are both black. How many black and how many white balls are in the urn?

PART II SECTION (b)

STATISTICS

26. A binomial distribution of 400 items has a mean of 80. What is the standard deviation?

27. Find the coefficient of variation for the following distribution:

Variable	3	4	5	6	7
Frequency	2	4	7	6	1

28. A random variable x has a mean equal to 20 and a standard deviation equal to 8. For what values of R and S does the random variable $y = Rx - S$ have a mean of zero and standard deviation equal to 1?

29. A random variable has the probability density function $f(x) = 1/k^2, o \leq x \leq k^2$. Find its standard deviation.

30. If, on the average, 400 cubic centimeters of water will have 8 bacteria in the solution, what is the probability that, if 20 cubic centimeters are drawn off, there will be not more than 1 bacterium present in the 20 cubic centimeters?

31. For a certain normal distribution the median is 89.0 and the fifth percentile is 56.1. What is the standard deviation?

32. A random variate x has a binomial distribution with parameters n = 50, p = 1/3. Given the probability that x = 23 is .0202, find the probability that x = 24, correct to four decimal places.

33. The recorded heights and weights for 100 males produced the following information:

Variate	*Mean*	*Stand. Dev.*	*Coeff. of Correlation*
Weight	160 lbs.	21 lbs.	.60
Height	69 in.	3 in.	

Estimate, from a regression line, the weight of a 6-foot male.

34. A die is tossed 50 times. Counting a five or a six as a success, what is the approximate probability of getting exactly 20 successes?

35. Calculate the correlation coefficient between s and t from the following data:

s	6	10	3	1	8
t	4	0	7	9	2

36. The weights of 1000 bearings are normally distributed. If all those bearings which weigh 5.18 ounces or more number 115, and all those weighing 5.20 ounces or more are found to number 36, what is the average weight of all the bearings?

37. The random variable x has a distribution specified by the Moment Generating Function $e^t\ (1-t)^{-2}$. What is the Moment Generating Function of the random variable y, where $y = 3x + 2$?

38. A random sample of five items is drawn, two from a normal distribution with mean 3 and standard deviation 3, and three from a

Poisson distribution with mean 2. What is the variance of the sample mean?

39. If you wish to estimate the proportion of Democrats in a certain district, and want your estimate to be correct within .02 units of the true proportion with a probability of .95, how large a sample should you take if, on the basis of preliminary samples, you have estimated the true proportion to be near 4/10?

40. Find the 80% confidence interval for the mean of a normal distribution whose standard deviation is 2, if a sample of size 8 gave the following values: 9, 14, 10, 12, 7, 13, 11, 12.

41. What size sample should be taken if it is desired to reject, at the 5% level of significance, a deviation of 5 units in a sample mean if the mean and standard deviation of the population are 50 and 10 respectively? (Normal distribution is not to be assumed for the sample mean.)

42. A control chart is to be constructed for a manufacturing process which produces 1000 parts a day. From experience it has been found that 4% of the parts coming off the production line are defective. What control limits should be set for the number of defective parts per day, based on a symmetric critical region of size .01?

43. If the heights of soldiers are normally distributed with a mean of 70 inches and a variance of 18 inches, what is the probability that if two soldiers are selected at random one of these is at least 4.8 inches taller than the other?

44. Given the probability density function $f(x) = x/2$, $0 \leqq x \leqq 2$. Find the probability density function for y, where $y = x^2/2$.

45. What is the largest critical region which can be chosen without rejecting the hypothesis that the observed data in the table below fit the theoretical distribution? (Do not interpolate.)

Class	I	II	III	IV	V
Expected Freq.	10	10	10	10	10
Observed Freq.	11	12	10	5	12

46. An urn contains an equal quantity of each of 3 different types of balls, labeled 1, 2, and 3. If three drawings (with replacement) are made, what is the variance of the sum of the numbers on the balls extracted?

47. Random samples were drawn from two normal populations with unknown means and common variance. Find the 90% confidence limits for the difference of the two population means, given:

	Sample Size	*Sample Mean*	*Sample St. Dev.*
Sample 1	5	40	18
Sample 2	6	30	15

(Each sample variance calculated as the average of the squared differences from sample mean.)

48. Given a frequency function of the form $\frac{\theta}{10} e^{-\theta x} \quad 0 \leq x \leq \infty$, 0 elsewhere.

What is the maximum likelihood estimate of θ for the following random sample values: 2.2, 2.4, 3.7, 2.8, 3.9?

49. At what level does the difference in standard deviations between the following 2 sets of observed data become significant? (Experience indicates that $\sigma_1 \geq \sigma_2$)

	I	II
Size of Sample	9	13
Average Value	40	20
Variance	13	3

50. What is the multiple correlation coefficient of z predicted from x and y if it is known that x, y, and z are related to one another by a correlation coefficient in each case of 0.5?

PART III SECTION (a)

1. (3 points)

A mortality table assumes that out of 100 people born at the same time, one dies each year until there are no survivors. If three men, aged 20, 30 and 60, are alive today, what is the probability, on the assumption that this mortality table is applicable, that they will all be alive three years hence?

(A numerical answer is required but no attempt should be made to reduce fractions to lowest possible terms or to multiply out the answer.)

2. (5 points)

The probability that a man aged x will die within a year is 1/10. Find the probability that out of two men, A, and B, each aged x, A will die within the year and will be the first of the two men to die.

3. (4 points)

Express the expectation of life of a life aged x in terms of (a) the l function of the life table and (b) the p function.

4. (3 points)

Prove the identity

$$i = id + d$$

5. (5 points)

Express in terms of the functions $\ddot{a}$ and v, in a form suitable for evaluation, the value of a series of 10 annual payments, the first payment being due in two years' time, consisting of $1,000 a year for 5 years and $2,000 a year thereafter.

6. (5 points)

A company is installing data processing equipment. It can rent the equipment for $100,000 a year (payable at the beginning of each year) or buy it outright for $950,000. It is considered that the equipment will require replacing after 10 years and will then have a sale value of $250,000. The company can earn 5% interest ($v^{10} = .61391$ and $a_{\overline{9}|} = 7.10782$). Show whether it is cheaper for the company to buy or rent.

7. (4 points)

A agrees to pay B $100 one year hence, $110 two years hence, $121 three years hence, etc. Each payment is ten percent greater than the preceding payment. If ten payments are made in all, calculate the present value of these payments to B at 10% interest.

8. (5 points)

Define in terms of l, v and d

$$D_x,\ N_x,\ C_x,\ R_x$$

9. (5 points)

Give an expression in terms of Commutation Functions for the net single premium to a life aged x for an assurance to pay \$1,000 in the event of death in the first five years, \$1,100 in the event of death in the sixth year, \$1,200 in the event of death in the seventh year, increasing to \$2,000 in the event of death in the fifteenth year and remaining constant at this figure for the remainder of life.

10. (5 points)

A man aged x is entitled to an income at 5% (payable annually at the end of each year) from a fund of \$100,000 so long as he lives up to age x + 10. At age x + 10 he will receive his last payment of income and the principal of the fund. At 4% interest, give an expression for the present value of these payments.

11. (5 points)

A policy is issued to a man aged x providing \$10,000 life insurance for 10 years followed by \$5,000 for the next 15 years. After this period, the insurance ceases. Give expressions for:

(a) The net single premium for the policy.
(b) The net annual premium payable for 15 years.

12. (6 points)

Demonstrate the relationship

$$P_{x:\overline{n}|} = \frac{1}{\ddot{a}_{x:\overline{n}|}} - d$$

and give a verbal explanation for the expression.

13. (5 points)

A policy provides \$10,000 on death or survival to age 65. The premium for the first five years is to be one-half the subsequent premiums. Give an expression in commutation functions for the net premiums to a life aged x payable during the first five years for such a policy.

14. (5 points)

A policy provides $10,000 on death before age 65 and an annuity of $1,000 payable yearly in advance, from age 65. Give an expression in commutation functions for the net premium for this policy to a life aged 40, with premiums limited to 20 years.

15. (6 points))

Express in terms of commutation functions the terminal reserve after 5 years for a 10 year endowment insurance of $1,000 to a life aged x.

(a) Using the prospective method.
(b) Using the retrospective method.

16. (6 points)

A deferred annuity is issued to a man aged 55 for $1,000 a year payable yearly in advance from age 65. The annuity is purchased by annual premiums and there is no return of the premiums in the event of death prior or after age-65.
Calculate the terminal reserve for the policy at age 60.

17. (5 points)

Distinguish between Initial, Mean and Terminal Reserves.

18. (6 points)

If the value of an immediate annuity to a life age x = 9.876, what is the value of

(a) An annuity due payable semiannually.
(b) An immediate annuity payable monthly to the same life.

19. (5 points)

Give an expression suitable for evaluating an annuity to a life aged x payable quarterly under which the first payment is made at the end of 9 months.

20. (7 points)

Develop a formula in terms of commutation functions for net premiums for an assurance providing $1,000 in the first year, $900 in the next year and $100 less each subsequent year under which the premium, payable annually in advance, is in proportion to the amount of insurance.

PART III SECTION (b)

1. (10 points)

Discuss the requirements necessary for insurability of a given hazard. How does crop-hail insurance meet these requirements?

2. (9 points)

Explain the differences among the following types of policy :—

(a) Comprehensive
(b) Combination
(c) Schedule

3. (9 points)

"Insurance is unproductive as it is ismply a transfer of funds from the public to insurers and vice versa, without any concrete addition to the National Output." Discuss the validity of this statement.

4. (10 points)

According to Willett, what determines the extent to which society attempts to prevent accidental loss and reduce uncertainty?

5. (12 points)

Economists often differ as to the role of the Risk-Bearer in the production process. Explain in detail "Insurer's Profits" as defined by :—

(a) Hawley
(b) Clark

6. (7 points)

The public utilities have been described as having a "natural" tendency towards monopoly. Explain and discuss.

7. (7 points)

One of the distinguishing characteristics in the history of Bank Stocks has been the feature of Double Liability. Explain.

8. a. (9 points)

Compare common stocks, preferred stocks and corporate bonds using the following criteria:

(1) level of income
(2) regularity of income
(3) security of principal

b. (6 points)

What other important differences exist among these types of securities?

9. (9 points)

Define investment companies. What are their advantages to an individual investor? How do they differ from holding companies?

10. (12 points)

It is necessary that Fire/Casualty Insurance Companies have a different investment policy than Life Insurance Companies.

(a) In what way are the policies different?
(b) Excluding Statutory Requirements, explain why such a difference is necessary in the overall operations of a Fire/Casualty Company.

PART IV SECTION (a)

1. (8 points)

Both property and casualty forms contain provisions relating to the existence of other insurance. Describe the main types of such provisions, indicating the forms in which they can be found.

2. (4 points)

Explain the Replacement Cost Coverage feature of the Dwelling and Contents Broad Form and the Dwelling Building Special Form.

3. (6 points)

Name the two forms used for writing Business Interruption insurance and discuss the main differences in their coverage.

4. (4 points)

Distinguish between a "blanket policy" and a "floating policy" as used in property insurance.

5. (4 points)

What are the exclusions under Coverage B of the Workmen's Compensation insurance policy?

6. (5 points)

Describe the purpose of the endorsement to Workmen's Compensation policy commonly known as Coverage C.

7. (4 points)

What are the "Standard Exceptions" to the normal classification procedures in the Workmen's Compensation manual? Explain why special handling is proper for "Standard Exceptions"?

8. (5 points)

To whom does the term "insured" as used in the Basic Automobile Policy refer?

9. (4 points)

Explain how coverage under (a) Hazard 1 and (b) Hazard 2 of the Garage Liability Policy differ.

10. (6 points)

In automobile physical damage insurance the losses may be settled on an "Actual Cash Value" basis and a "stated amount" basis. Describe the advantages and disadvantages of these two approaches.

11. (4 points)

In regard to O. L. & T. liability coverage discuss the major differences between the definition of an "accident" and that of an "occurrence".

12. (6 points)

In the M & C manual published by the National Bureau of Casualty Underwriters, the symbols "c", "u", "x" appear with the rates for Property Damage Coverage. Explain the meaning of these symbols.

13. (4 points)

What does the "Pair and Set" Clause of the Personal Articles Floater provide?

14. (8 points)

Classify and discuss the various types of renewal provisions used in the individual Accident and Health policies.

15. (4 points)

What is the difference between a primary commercial blanket bond and a blanket position bond?

16. (5 points)

Name the two types of insurance allocation provisions in Coverage A of the broad form theft insurance. Which do you think is more expensive and why?

17. (4 points)

Describe the four basic types of ocean marine contracts.

18. (8 points)

Under the Motel Policy of the Special Multi-Peril Program, what are:

A) The mandatory coverages
B) Three Optional coverages

19. (3 points)

In addition to normal direct damage, the typical Glass insurance policy includes an agreement to assume the cost of three types of indirect damage. What are these three types of indirect damage?

20. (4 points)

Boiler and Machinery insurance contracts fall into four categories, depending on the nature of the loss covered. Name the four categories.

Section (b)

1. (10 points)

The management of your company decides to deviate from the "bureau" rates. What percentage deviation would you recommend if (a) your loss experience is about the same as provided for in the "bureau" rates (b) the provision for profit and contingency is 5% and (c) your actual expenses and the "bureau" rate expense provisions are as follows:

	Actual Expenses Your Company	Bureau Rate Expense Provision
	%	%
Loss Adjustment	6.0	7.0
Commissions and Brokerage	15.0	17.5
Other Acquisition	5.0	7.0
General	5.0	7.0
Taxes, Fees, etc.	3.0	3.5
Total	34.0	42.0

2. (10 Points)

What are the basic requirements for a satisfactory medium of exposure? Do you consider that "Car-Year" in Automobile insurance satisfies these requirements?

3. (15 Points)

Given a group health insurance contract whose benefits were as follows:

$16 Daily Room and Board
$300 Maximum Miscellaneous
$250 Surgical Schedule
$10,000 Major Medical

Your analysis indicates the following composite monthly claim cost per employee:

	1962	*1963*	*1964*
Room & Board	$4.00	$4.25	$4.50
Miscellaneous	3.00	3.50	4.00
Surgical	2.00	2.00	2.00
Major Medical	5.00	6.00	7.00

You are asked to promulgate a rate for 1965 including a change in benefit from $16 Daily Room and Board to $20. What gross rate would you recommend, given a 12% retention?

4. (10 Points)

In state "A" your analysis indicates the following experience on homeowners' coverage:

	1962	*1963*	*1964*
Loss Incurred	$ 5,000,000	$ 7,000,000	$ 9,000,000
Expenses	4,000,000	5,000,000	6,000,000
Premiums Earned	10,000,000	12,000,000	14,000,000

Your management tells you that they have the opportunity to reinsure a block of homeowners' business constituting a probable $5,000,000 of earned premium from state "A" in 1965. What ceding commission would you recommend? Why?

5. (7 Points)

What is the basic purpose of a trend factor in the calculation of rate changes for Automobile Insurance? Describe one such factor currently or formerly in use, and indicate the extent to which you believe this purpose has been accomplished.

6. (10 Points)

Describe the criteria for 100% credibility used by the National Council for the determination of classification pure premiums. What is the formula used to establish partial credibility?

7. (7 Points)

In rate revisions for fire insurance, it has been customary to convert actual earned premiums to present rate levels. Discuss the advantages and disadvantages of converting losses to present cost/price levels.

8. (7 Points)

Describe the basic principles for rate level adjustments as originally recommended by Inter-Regional Insurance Conference.

9. (9 Points)

Explain the meaning of the following terms as used in Workmen's Compensation ratemaking procedure:

(a) Standard Premium
(b) Premiums at Present Collectible Rates
(c) Proposed Pure Premiums.

10. (15 Points)

The statewide rate level adjustment for Automobile Liability has been determined to be +10%. Assume 1,000 claims is required for 100% credibility and permissible loss ratio is 62½%. What rate level adjustment is required for each of the following territories?

Territory	Number of Claims	Pure Premium Acc. Years 1962-63	Present Average Rate
A	360	$60.00	$80.00
B	640	30.00	80.00
C	250	50.00	80.00

EXAMINATION FOR ENROLLMENT AS FELLOW

PART I

SECTION (a)

1. (6 Points)

Compare the treatment of domestic insurers with the treatment of foreign and alien insurers under New York law with respect to licensing.

2. (10 Points)

Discuss the advantages and disadvantages of the premium tax as a source of revenue.

3. (5 points)

What constitutes insurable interest in property?

4. (5 Points)

What is the relationship between valued policies and the principle of indemnity?

5. (15 Points)

Explain why the following cases are important to the history of insurance regulation:

a. Prudential Insurance Company vs. Benjamin
b. Robertson vs. California
c. The Todd Shipyards Case
d. The Travelers Health Case
e. Paul vs. Virginia
f. U.S. vs. Insurance Board of Cleveland

6. (10 Points)

Evaluate the statement "Since the regulation of insurance companies was considered within the domain of the states before the S.E.U.A. decision and again after Public Law 15 was passed, the entire series of events which took place between June 5, 1944, and June 30, 1948, might just as well not have happened."

7. (10 Points)

The following is a quote from Mr. Donovan's article "The New Era of Casualty Rate Regulation":

"Reasonable men may differ in their views as to the probable success or failure of vesting in the several states such comprehensive powers over an interstate industry. What all may agree upon is that the system can succeed only if industry and government approach their respective responsibilities with an intelligent altruism."

How well have the industry and government handled their respective responsibilities? Give the reasons for your answer.

8. (12 Points)

At the December, 1963, meeting of the National Association of Insurance Commissioners, a report was made which outlined some of the reasons for insurance company insolvencies and which also proposed several methods of preventing these. With the report as background, comment on the insolvency problem—the reasons for it and the possible solutions.

9. (12 Points)

Briefly discuss each of the following:

a. The remedies available to a victim of an unfair market practice.
b. The duties or obligations of a common carrier.
c. The preferred status of a "Holder in Due Course" of a negotiable instrument.
d. The liability of an agent on a contract that he makes for a principal.

10. (15 Points)

Outline the unfavorable developments in rate regulation discussed by Mr. Carlson in his paper "Rate Regulation and the Casualty Actuary", and tell in what respects his comments would be appropriate today.

SECTION (b)

11. (9 Points)

Describe the steps to be followed in arriving at a "factor rate" under the New York Statutory Disability Benefits Law.

12. (8 Points)

Outline the end result produced by the Advisory Committee with respect to the Distribution and Assignment of Risks section of the Uniform Automobile Assigned Risk Plan.

13. (10 Points)

Give two methods which are used by a large number of states to eliminate the "Saturday-afternoon-clerk" type of worker from unemployment benefit consideration. Explain which of the methods is becoming more widely used and tell why.

14. (a) (6 Points)

Cite two features of the California Disability Insurance Plan which tend to create a favorable financial balance and two which tend to produce an unfavorable balance.

(b) (3 Points)

Give the definition of "disabled" as it applies in the Rhode Island Cash Sickness Compensation Law and in the New York Disability Insurance Law.

15. (15 Points)

Discuss Individual Equity versus Social Adequacy in Social Insurance.

16. (15 Points)

Compare the concept of actuarial soundness as applied to private long-range benefit programs and as applied to the Old-Age Survivors and Disability Insurance system.

17. (8 Points)

List five of the possible solutions to the problem of the uninsured motorist. Select the one which you feel is best and explain why you think so.

18. (8 Points)

What four general principles of organization are applied by the Blue Cross Commission to plans seeking approval to be identified with the movement?

19. (9 Points)

Describe the three most common experience rating formulas adopted by the states for unemployment insurance.

20. (9 Points)

Over a 14-year period from 1937 to 1951, rates for Automobile Bodily Injury Insurance decreased in Massachusetts while increasing in the bordering states of Connecticut and New York. What factors could reasonably have contributed to this situation? Discuss.

PART II

Section (a)

1. (10 Points)

Discuss the advantages and disadvantages of using each of the following as a base on which the IBNR Reserve calculation is made:

1. Premiums in force.
2. Earned Premiums.
3. Reported Outstanding Losses.
4. Reported Incurred Losses.

2. (10 Points)

Explain the "ratio" method, the "projection" method, and the "dollar" method for setting Workmen's Compensation Unallocated Loss Expense Reserves.

3. (5 Points)

In Workmen's Compensation insurance, why should reopened cases be reserved separately from the late reported cases?

4. (20 Points)

As Actuary, you are asked to set up a simpler system for the reserving of property claims under $200. In the past every case has been reported to the Home Office and reserved individually:

(a) Describe a system you would introduce.
(b) Describe the initial problems.

5. (10 Points)

Your current method for reserving the return premiums on retrospectively rated policies is to take the ratio of return premiums to earned premiums by policy year and apply that ratio to current earned premiums. This method has produced redundant reserves in the past few years. What might cause this redundancy and what changes could be incorporated in the method to produce a more accurate reserve?

6. (15 Points)

You are a consulting actuary and are presented with the following set of circumstances regarding two medium-sized casualty-fire insurance companies with normal rates of premium growth:

(1) Company A for the past few years has experienced underwriting losses and its surplus is moderate.
(2) Company B for the past few years has shown an underwriting profit and its surplus position is excellent.

How would you advise each of the companies with respect to the method to be used on term policies in booking premiums (assuming commission and allied acquisition expenses would be similarly incurred)? Make your reasons explicit.

7. (10 Points)

Line 16 on Page 3 of the Convention form of Annual Statement Blank reads:

"Excess of bodily injury liability and compensation statutory and voluntary reserves over case basis and loss expense reserves (Schedule P)."

Explain why this line is desirable.

8. (a) (15 Points)

Describe the procedure generally followed in a "runoff test" of loss reserves in the liability lines. Since the test is made "after the fact", of what value is it?

(b) (5 Points)

Give the advantages and disadvantages of the projection method of loss reserving when applied to the Automobile Property Damage line.

SECTION (b)

9. (15 Points)

Calculate the surplus as regards policyholders of a casualty insurance company for the current year given the following information pertaining to that company:

(1)	Agents' balances over 90 days due—prior	$ 150.000
(2)	Liability for unauthorized reinsurance—prior	375,000
(3)	Furniture and fixtures—prior	65,000
(4)	Unrealized capital gain	25,000
(5)	Dividends to stockholders	85,000
(6)	Paid in capital—current	500,000
(7)	Net underwriting gain	1,200,000
(8)	Agents' balances over 90 days due—current	400,000
(9)	Liability for unauthorized reinsurance—current	750,000
(10)	Furniture and fixtures—current	195,000
(11)	Net investment income and other income	800,000
(12)	Surplus as regards policyholders—previous year	7,500,000
(13)	Federal and foreign income taxes incurred	0

10. (15 Points)

You have been asked to convert an average fire and casualty insurance company's earnings for the year 1964 and surplus as of year end from a statutory basis to a basis which more nearly corresponds to that generally used for other types of business. What items would you adjust and in what ways would you adjust them?

11. (a) (6 points)

Outline apportionment criteria which apply to joint intercompany expenses as provided by Uniform Accounting Instructions.

(b) (9 Points)

What are the allocation bases or procedures which apply in the Uniform Accounting Instructions with respect to Allocation of Salaries?

12. (15 Points)

Given the following list of items, prepare *Assets and Liabilities, Surplus and Other Funds* pages of the Convention Form of Annual Statement blank of a casualty-fire insurance company for the year 1964: (You may use item numbers to conserve time.)

(1)	Bonds	$80,000,000
(2)	Cash and bank deposits	10,000,000
(3)	Unearned premiums	30,000,000
(4)	Reserve for taxes, licenses and fees	3,000,000
(5)	Capital paid up	5,000,000
(6)	Ceded reinsurance balances payable	1,000,000
(7)	Losses Incurred in 1964	40,000,000
(8)	Stocks, Ledger	50,000,000
(9)	Stocks, Non-ledger	10,000,000
(10)	Special Reserve for General Contingencies	20,000,000
(11)	Losses Paid in 1964	35,000,000
(12)	Agents' balances or uncollected premiums	5,000,000
(13)	Reinsurance recoverable on loss payments	1,500,000
(14)	Reserve for contingent commissions and other similar charges	300,000
(15)	Dividends to Stockholders declared and unpaid	1,000,000

(16)	Bills receivable, taken for premiums	200,000
(17)	Losses Unpaid, previous year	50,000,000
(18)	Capital	10,000,000

13. (10 Points)

Discuss the handling of unauthorized reinsurance in the annual statement and explain the reason it is handled as it is.

14. (10 Points)

Contrast the announcement of underwriting results on a statutory basis with a basis which combines ratios of incurred loss and loss expense to earned premium and other expenses to written premium.

15. (10 Points)

Compare the approach taken in Schedule G to measure the development of bond loss reserves with the approach taken in Schedule P, Part 5, to measure the development of liability and compensation loss reserves.

16. (10 Points)

In the year in which a stock company begins to issue participating policies, what changes would be made in its annual statement with respect to:

(a) The Balance Sheet?
(b) The Income Statement?
(c) The General Interrogatories?
(d) Schedules L and T?

PART III

SECTION (a)

1. (12 Points)

Discuss the Multiple Location Rating Plan for Buildings and Equipment, and include in your discussion a description of how a risk is rated, eligibility requirements, classes of property that are eligible for rating, and coverage that may be rated under the Plan.

2. (10 Points)

An Insured with a \$100,000 annual compensation standard premium subject to Retrospective Rating Plan D wishes to purchase a loss limitation which will limit theamount of ratable losses resulting from a single accident. What accident limitation amounts are available to him and what facts determine the cost of such an accident limitation?

3. (10 Points)

Discuss the Composite Rating Plan, including in your discussion objectives of the Plan and a description of the way in which the Plan operates.

4. (10 Points)

You are given the following information regarding a Retrospective Rating Plan:

1.	Annual Standard Premium	\$100,000
2.	Basic Premium excluding Tax	25,000
3.	Maximum Premium Factor	1.110
4.	Minimum Premium Factor	.613
5.	Tax Multiplier	1.03
6.	Expected Loss Ratio	.600
7.	Loss Conversion Factor	1.15

Ratio of Rated Loss to Expected Loss	*Table M Charge*	*Ratio of Rated Loss to Expected Loss*	*Table M Charge*
.30	.700	.90	.210
.40	.605	1.00	.159
.50	.511	1.10	.119
.60	.421	1.20	.088
.70	.341	1.30	.067
.80	.271	1.40	.049

Determine:

(a) Loss Ratio underlying Maximum Premium.

(b) Loss Ratio underlying Minimum Premium.

(c) The net insurance charge in the Basic Premium.

(d) The amount that will be obtained for expenses (excluding tax) and contingencies if the risk develops a loss ratio equal to the Expected Loss Ratio.

5. (10 Points)

Explain the following terms, which relate to the Workmen's Compensation Experience Rating Plan:

Expected Loss Rate
D Ratio
B Value
W Value
Q Point
S Point

6. (8 Points)

Out of a sample of 1,000 Liability losses totaling $270,000, 550 losses are found to be $200 or less in size and these 550 losses total $45,000. If the full coverage rate of $1 provides .55 for losses and allocated claim expense (of which .05 is for allocated claim expense); .30 for Acquisition, Tax, and Profit; and .15 for Other Expenses, calculate the rate for $200 deductible coverage.

7. (10 Points)

The tabular Retrospective Rating Plans of the National Council (Plans A, B, C and J) provide for a Non-Stock Adjustment Factor:

(a) What is the purpose of this factor?
(b) How is it determined?
(c) How does the factor for a $5,000 risk compare with the factor for a $25,000 risk? Why?

8. (15 Points)

You have dscovered that the New York Automobile Liability Experience Rating Plan of the National Bureau of Casualty Underwriters has been consistently producing a credit off-balance on intrastate risks. Discuss four reasons why this might have happened.

9. (15 Points)

Develop a building rate for a hypothetical fire risk under the Dean Analytic System, considering the following risk characteristics:

Standard first-aid fire appliances
Deficient wall materials (concrete block)
Defective wiring
Non-standard skylights
Standard standpipe and hose

Deficient wall thickness (brick, 8 in.)
Painting on wood floor
Rubbish in basement
Floor openings unprotected
Gasoline in unapproved receptacle

Start with a base rate of $1.00. Fill in values of your own choosing for the above items. Assume a 25% credit for an 80% coinsurance provision.

SECTION (b)

10. (10 Points)

Effective January 1, 1965, the National Bureau has filed a new classification plan for private passenger automobile insurance in many states. Describe the changes proposed in this plan.

11. (12 points)

Recently, a new actuarial and statistical organization has been established to try to develop sounder ratemaking practices in the property insurance lines. Briefly discuss six of the major actuarial shortcomings of the current fire insurance ratemaking system which proponents have been stressing as justification for this new organization.

12. (14 Points)

Reinsurance appears principally in the following forms:
(a) Facultative Reinsurance
(b) Portfolio Reinsurance
(c) Excess of Loss Reinsurance
(d) Retrocession Reinsurance
(e) Quota Reinsurance
(f) Fixed Treaty Reinsurance
(g) Open and Optional Treaty Reinsurance
Briefly define each form.

13. (14 Points)

One of the outstanding recent developments in the casualty and property insurance business is the establishment of the Johnson Principles. Assuming that these principles will gradually be accepted by the industry, discuss the future implications of their adoption.

14. (9 Points)

Describe and justify a method of comparing the cost of Workmen's Compensation in a given state with costs in other states, and name three reasons why there are marked differences in costs between states.

15. (10 Points)

Discuss Kenney's basis for determining the amount that a Fire and Casualty Company should pay as stockholders' dividends, as well as the theory behind it.

16. (12 Points)

Discuss the problems which are generated by the integration of Fire, Life, and Casualty operations of an insurance company group.

17. (10 Points)

Discuss those forces which encourage, as well as those which discourage, the marketing of group property and casualty insurance.

18. (9 Points)

Discuss the factors which generally determine credibility in group health insurance rate adjustment formulas, as well as the two most common methods for applying credibility in such formulas.

PART IV

Section (a)

1. (10 Points)

State the definition of allocated loss adjustment expense in the Automobile Statistical Plan of the NBCU. Also state those items of loss expense specified as not to be included in allocated. How does this definition differ from that used by the NBCU for General Liability and Burglary insurance?

2. (10 Points)

State the definition of an Excess Loss (i.e., above basic limits) on each of the following lines in the Statistical Plans of the National Bureau:

1. Private Passenger Auto
2. O.L. & T. B.I.
3. Elevator Collision
4. Malpractice and Professional Liability

3. (5 Points)

How does the use of an experience and schedule rating plan, a retrospective rating plan, or the Account Rating Plan affect the amount of premium reported under the NBCU Statistical Plans?

4. (10 Points)

Outline the requirements for the reporting of premiums and losses under the NBFU Homeowners Statistical Plan. What changes would you make in the plan? Why?

5. (10 Points)

Describe the NBFU Statistical Plan for Expenses for Fire and Allied Lines. In what ways can the data compiled under this plan be used in establishing rate levels?

6. (15 Points)

The New York Insurance Department's publication "Loss and Expense Ratios" shows expense ratios on a different basis than those shown on the companies' Insurance Expense Exhibits (Part II—Section A). State these differences and comment on the reasonableness of the New York approach. Does this method produce reasonable answers for both direct writing (i.e., employee salesmen) and agency companies?

7. (5 Points)

You have been asked to prepare a report showing, by state, annual workmen's compensation premiums by company group. Describe how you would prepare this report, naming the sources of external statistics which you would use.

8. (5 Points)

Methods of converting to an electronic data processing system have been described as falling into three broad categories. Name and briefly define each of these categories.

9. (15 Points)

How does Canning define the "systems engineering" approach to data processing, and what steps are involved in the establishment of a data processing system under the approach?

10. (a) (5 Points)

Name and describe the three major components of a computer's central processor.

(b) (10 Points)

Explain what the "binary" numbering system is and how the number 78 would be denoted under this system.

SECTION (b)

11. (10 Points)

Under Compulsory Automobile insurance ratemaking in Massachusetts it has been argued that it would be more reasonable to have expense loading of a uniform amount in each territory, rather than a uniform state-wide percentage expense loading. Discuss.

12. (8 Points)

Give four reasons which have been advanced for the more favorable experience under the New York Disability Benefits Law as opposed to comparable group coverage.

13. (12 Points)

Discuss in detail the elements of the fire ratemaking problem which exists as regards low-valued dwellings, and name several suggestions or changes which have been made in an effort to correct the situation.

14. (10 Points)

Name and describe four criteria for determining whether surety rates are excessive or inadequate, and state the shortcomings of the use of each.

15. (12 Points)

The NBCU uses a trend factor in automobile liability ratemaking to update the average claim costs implicit in the accident years used as an experience base in developing rates. It has been argued that only the paid portion of these accident years should be updated by the trend factor in that the outstandings already contemplate current cost levels. Discuss the relative merits of applying the trend factors to (1) incurred and (2) paid losses.

16. (8 Points)

From the following data, develop an expression to show the annual employee pure premium per $1 of hospital daily benefit for a 2-day deductible, 30-day maximum duration hospital plan, assuming 25% females.

	Annual Data on Plan with 32-day Max. Duration	
	Male Employees	*Female Employees*
a. Exposure in lives	10,000	10,000
b. Number of claims	1,000	1,100
c. Average length of stay	7 days	8 days
d. No. of claims lasting exactly one day	200	250
e. No. of claims lasting exactly two days	150	200

17. (12 Points)

L. H. Longley-Cook (PCASXLVII) has suggested that the insurance industry investigate the possible use of a Census Method in ratemaking. Describe how he would use such a system and the possible benefits to insurance ratemaking.

18. (12 Points)

Outline the method normally used currently by the National Bureau in developing rates for the liability coverages under the Special (Package) Automobile Policy. Comment on the adequacy of the rates for this policy in light of this ratemaking system.

19. (16 Points)

In recent years, the insurance industry's Workmen's Compensation loss ratios (to standard premium) have been worse than the expected loss ratio implicit in manual rates. This situation has persisted long enough so that it is extremely unlikely that the poor loss ratios can be ascribed entirely to the cyclical nature of the business. Briefly describe some of the areas of the current ratemaking system which you might consider changing in light of this condition. Also discuss briefly the types of data you would want to review in formulating your decision as to possible changes you might recommend.

INDEX TO VOLUME LII

INDEX TO VOLUME LII (Cont.)

CASUALTY ACTUARIAL SOCIETY

ORGANIZED 1914

1966 YEAR BOOK

Corrected to December 1, 1965

FOREWORD

The Casualty Actuarial Society was organized in 1914 as the Casualty Actuarial and Statistical Society of America, with 97 charter members of the grade of Fellow; the Society adopted its present title on May 14, 1921.

The roots of actuarial science are found in England, dating back as far as 1792, in the early days of life insurance. Due to the technical nature of the business, the first actuaries were mathematicians and eventually the growth of their numbers resulted in the formation of the Institute of Actuaries in Great Britain in 1848. A similar organization, the Faculty of Actuaries, was founded in Scotland in 1856. This was followed in the United States by the Actuarial Society of America in 1889 and the American Institute of Actuaries in 1909. These two actuarial bodies were merged in 1949 to form the Society of Actuaries.

In the meantime, problems requiring actuarial treatment were emerging in sickness, disability and casualty insurance, particularly workmen's compensation which began in 1911. These problems were quite different from life insurance and led to the organization of the Casualty Actuarial Society in 1914 which was brought about through the suggestion of Dr. I. M. Rubinow who became the first president. Since the problems surrounding workmen's compensation were at that time the most urgent, many of the members played a leading part in the development of the scientific basis upon which workmen's compensation insurance now rests. The object of the Society was, and is, the promotion of actuarial and statistical science as applied to the problems of insurance other than life insurance by means of personal intercourse, the presentation and discussion of appropriate papers, the collection of a library and such other means as may be found desirable.

From its beginning the Society has grown constantly in membership, in the scope of its interests and in its contributions to the formulation of scientific standards for the computation of rates and reserves for the many lines of business in the non-life field. These contributions are found in the original papers prepared by members of the Society and printed in the *Proceedings* which are published annually. Other papers deal with acquisition costs, pension funds, legal decisions, investments, claims, reinsurance, accounting, statutory requirements, loss reserves, statistics, and the examination of insurance companies. The presidential addresses constitute a valuable record of the actuarial problems, some of them still unsolved, which have faced the insurance industry over the years.

At the November 1950 meeting of the Society, the Constitution and By-Laws were amended to enlarge the scope of the Society to include all lines of insurance other than life insurance. The effect of the amendment was to include fire and allied lines insurance, in recognition of the multiple line power granted by many states to both casualty companies and fire companies.

The membership of the Society consists of actuaries who are employed by insurance companies, ratemaking organizations, state insurance departments, and as independent consultants. The Society has two grades of membership comprised of Fellowship and Associateship. Examinations for these two classes of membership are held during the second or third week of May in various cities in the United States and Canada. In addition, the examinations for Associateship Parts 1 and 2 are held in November of each year.

On the inside, front cover of the *Year Book* are listed the Proceedings and other publications of the Society and the prices thereof. The *Year Book* is published annually. *Recommendations for Study* is a pamphlet which outlines the course of study to be followed for admission. These two booklets may be obtained free upon application to the Secretary-Treasurer, Albert Z. Skelding, 200 E. 42nd Street, New York, N. Y. 10017.

CASUALTY ACTUARIAL SOCIETY

NOVEMBER 1, 1965

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FELLOWS OF THE SOCIETY

NOVEMBER 1, 1965

Those Marked (†) were Charter Members at date of organization, November 7, 1914

Admitted	
Nov. 21, 1930	AINLEY, JOHN W. (Retired), 33 Paxton Road, West Hartford, Conn. 06107
Nov. 20, 1964	ALDRICH, WILLIAM C., Secretary, National Council on Compensation Insurance, 200 East 42nd Street, New York, N. Y. 10017
Nov. 20, 1964	ALEXANDER, LEE M., Actuary, Massachusetts Workmen's Compensation Bureau, Massachusetts Automobile Rating and Accident Prevention Bureau and Massachusetts Motor Vehicle Assigned Risk Plan, 89 Broad Street, Boston, Mass.
Nov. 14, 1947	ALLEN, EDWARD S., Actuary, Phoenix of Hartford Insurance Companies, 61 Woodland Street, Hartford, Conn. 06115
Nov. 18, 1955	BAILEY, ROBERT A., Chief Actuary, Department of Insurance, State of Michigan, Lansing, Mich. 48913
Nov. 15, 1962	BALCAREK, RAFAL J., Assistant Secretary and Actuary, Reliance Insurance Company, 4 Penn Center Plaza, Philadelphia, Pa. 19103
Nov. 20, 1924	BARBER, HARMON T. (Retired), 18 Ridgewood Road, Windsor, Conn. 06095
Nov. 19, 1954	BARKER, GORDON M., Actuary, Great American Group, 99 John Street, New York, N. Y. 10038
Nov. 14, 1947	BARKER, LORING M., Actuary, Fireman's Fund American Insurance Companies, 3333 California Street, San Francisco, Calif. 94120
Nov. 20, 1942	BART, ROBERT D., Director of Industrial Relations and Assistant Treasurer, The West Bend Company, 400 W. Washington Street, West Bend, Wis. 53095
Nov. 18, 1932	BARTER, JOHN L., 90 Tunxis Road, West Hartford, Conn. 06107
Nov. 13, 1931	BATHO, ELGIN R., Vice President and Actuary, Berkshire Life Insurance Company, 700 South Street, Pittsfield, Mass. 01203
Nov. 14, 1958	BENBROOK, PAUL, Vice President, American General Insurance Company, 2727 Allen Parkway, Houston, Texas 77019
Nov. 16, 1956	BENNETT, NORMAN J., Assistant Secretary and Actuary, Continental Insurance Companies, 80 Maiden Lane, New York, N. Y. 10038
Nov. 22, 1934	BERKELEY, ERNEST T., Actuary, Employers' Group of Insurance Companies, 110 Milk Street, Boston, Mass. 02107
Nov. 22, 1957	BERQUIST, JAMES R., Associate Actuary, Employers Mutuals of Wausau, 407 Grant Street, Wausau, Wis. 54402
Nov. 19, 1953	BEVAN, JOHN R., Assistant Actuary and Group Actuary, Liberty Mutual Insurance Company, 175 Berkeley Street, Boston, Mass. 02117
†	BLACK, S. BRUCE, Honorary Chairman, Liberty Mutual Insurance Company, 175 Berkeley Street, Boston, Mass. 02117
Apr. 20, 1917	BLANCHARD, RALPH H., Professor Emeritus of Insurance, Columbia University, Plympton, Mass. 02367
Nov. 19, 1959	BLODGET, HUGH R., Assistant Secretary, Aetna Casualty and Surety Company & Standard Fire Insurance Company, 151 Farmington Avenue, Hartford, Conn. 06115
Nov. 16, 1956	BONDY, MARTIN, Assistant Vice President and Actuary, Crum & Forster Insurance Group, 110 William Street, New York, N. Y. 10038
Nov. 22, 1957	BORNHUETTER, RONALD L., Secretary and Actuary, General Reinsurance Corporation, 400 Park Avenue, New York, N. Y. 10022
Nov. 16, 1956	BOYAJIAN, JOHN H., Actuary, National Insurance Actuarial and Statistical Association, 85 John Street, New York, N. Y. 10038

Admitted	
Nov. 19, 1959	BOYLE, JAMES I., Assistant Actuary, The Travelers Insurance Company, One Tower Square, Hartford, Conn. 06115
Nov. 16, 1961	BRANNIGAN, JAMES F., Assistant Actuary, The Travelers Insurance Company, One Tower Square, Hartford, Conn. 06115
†	BREIBY, WILLIAM, Chairman of Board, Western Travelers Life Insurance Company, 2701 Beverly Blvd., Los Angeles, Calif. 90057
Nov. 21, 1952	BRINDISE, RALPH S., *Group Insurance Supervisor, Standard Oil (Indiana)*, 910 S. Michigan Avenue, Chicago, Ill. 60680
Nov. 18, 1927	BROWN, F. STUART (Retired), Cedar Land Road, Orleans, Mass. 02653
Oct. 22, 1915	BROWN, HERBERT D. (Retired), Glenora-on-Lake Seneca, Dundee, N. Y.
Nov. 16, 1961	BUDD, EDWARD H., Second Vice President, The Travelers Insurance Company, One Tower Square, Hartford, Conn. 06115
Nov. 23, 1928	BURLING, WILLIAM H., Manager and Actuary for Canada Group, The Travelers Insurance Company, Suite 1306, 7 King Street, East, Toronto 1, Ontario, Canada
Nov. 19, 1959	BYRNE, HARRY T., Assistant Actuary, Aetna Casualty and Surety Company & Standard Fire Insurance Company, 151 Farmington Avenue, Hartford, Conn. 06115
Nov. 19, 1929	CAHILL, JAMES M., General Manager, National Bureau of Casualty Underwriters, 125 Maiden Lane, New York, N. Y. 10038
Nov. 18, 1932	CAMERON, FREELAND R., Senior Vice President and Actuary, Swiss National Insurance Company, U.S.A., P. O. Box 3131, Miami, Fla. 33101
Nov. 17, 1938	CARLETON, JOHN W., Vice President, Liberty Mutual Insurance Company, 175 Berkeley Street, Boston, Mass. 02117
Nov. 18, 1949	CLARKE, JOHN W., Executive Vice President and Director, Hartford Life Insurance Company, Box 137, Essex Station, Boston, Mass. 02112
Nov. 15, 1918	COATES, BARRETT N. (Retired), 1007 Cragmont Avenue, Berkeley, Calif. 94708
Nov. 17, 1922	COATES, CLARENCE S., Actuary, Lumbermens Mutual Casualty Company, 4750 Sheridan Road, Chicago, Ill. 60640
Feb. 19, 1915	COLLINS, HENRY *(Retired), Commercial Union Insurance Company*, 200 Park Avenue, New York, N. Y. 10017
Nov. 22, 1934	COOK, EDWIN A., President and General Manager, Interboro Mutual Indemnity Insurance Company, 270 Madison Avenue, New York, N. Y. 10016
Nov. 18, 1925	CORCORAN, WILLIAM M., Consultant, Peat, Marwick, Mitchell & Co., Wolfe Corcoran Division, 70 Pine Street, New York, N. Y. 10005
Nov. 19, 1926	CRANE, HOWARD G., Vice President and Treasurer, General Reinsurance Corporation, 400 Park Avenue, New York, N. Y. 10022
Nov. 21, 1952	CRITCHLEY, DOUGLAS, E. B. Savory & Co., Basildon House, Moorgate, London, England
Nov. 22, 1946	CROUSE, CHARLES W., Consulting Actuary, C. E. Preslan & Company, Inc., 20015 Detroit Road, Cleveland 16, Ohio.
Nov. 18, 1960	CROWLEY, JAMES H., Assistant Secretary, Accounting Dept., Aetna Life and Casualty, 151 Farmington Avenue, Hartford, Conn. 06115
Nov. 16, 1965	CURRY, ALAN C., *Actuary, State Farm Mutual Automobile Insurance* Company, 112 East Washington Street, Bloomington, Ill. 61701
Nov. 19, 1953	CURRY, HAROLD E., Senior Vice President, State Farm Mutual Automobile *Insurance Company*, 112 E. *Washington Street*, Bloomington, Ill. 61701
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Nov. 18, 1927	DAVIS, EVELYN M., Partner, Woodward, Ryan, Sharp & Davis, 26 Broadway, New York, N. Y. 10004
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Nov. 16, 1965	DEMELIO, JOSEPH J., Secretary, The Home Insurance Company, 59 Maiden Lane, New York, N. Y. 10008
Nov. 18, 1960	DICKERSON, O. D., Associate Professor of Insurance, Florida State University, Tallahassee, Fla. 32306
Nov. 16, 1951	DOREMUS, FREDERICK W. (Retired), 111 South Harrison Street, Apt. 309, East Orange, N. J. 07018
Nov. 16, 1965	DORF, STANLEY A., Principal Actuary, New York Insurance Dept., 123 William Street, New York, N. Y. 10038
Nov. 17, 1920	DORWEILER, PAUL, (Retired), 51 Wethersfield Avenue, Hartford, Conn. 06114
Nov. 22, 1957	DROBISCH, MILES R., Assistant Actuary, California Inspection Rating Bureau, 1453 Mission Street, San Francisco, Calif. 94103
Nov. 14, 1958	DROPKIN, LESTER B., Actuary, California Inspection Rating Bureau, 1453 Mission Street, San Francisco, Calif. 94103
Nov. 24, 1933	EDWARDS, JOHN, Consulting Actuary and Chartered Accountant, 91 Arundel Avenue, Toronto 6, Ontario, Canada
Nov. 16, 1965	EHLERT, DARRELL W., Actuary, Allstate Insurance Company, 7447 Skokie Blvd., Skokie, Ill. 60078
Nov. 19, 1959	EIDE, K. ARNE, Metropolitan Life Insurance Company, 1 Madison Avenue, New York, N. Y. 10010
Nov. 15, 1940	ELLIOTT, GEORGE B., General Manager, Pennsylvania Compensation Rating Bureau, 1819 John F. Kennedy Blvd., Philadelphia, Pa. 19103
Nov. 17, 1922	ELSTON, JAMES S. (Retired), R. D. #1, Moodus, Conn. 06469
Nov. 15, 1935	EPPINK, WALTER T., 1st Vice President, Treasurer & Actuary, Merchants Mutual Insurance Company, 250 Main Street, Buffalo, N. Y. 14240
Nov. 14, 1958	ESPIE, ROBERT G., Vice President and Comptroller, Aetna Life and Casualty, 151 Farmington Avenue, Hartford, Conn. 06115
Nov. 18, 1955	FAIRBANKS, ALFRED V., Actuary, Monarch Life Insurance Company, 1250 State Street, Springfield, Mass. 01101
†	FALLOW, EVERETT S. (Retired), 28 Sunset Terrace, West Hartford, Conn. 06107
Nov. 15, 1940	FARLEY, JARVIS, President, Treasurer and General Manager, Massachusetts Indemnity and Life Insurance Company, 654 Beacon Street, Boston, Mass. 02215
†	FARRER, HENRY (Retired), R. D. #3, Box 322, Fleetwood, Pa. 19522
Nov. 18, 1960	FAUST, J. EDWARD, JR., Consulting Actuary, R. R. #1, West Gray Road, Zionsville, Ind. 46077
May 25, 1956	FINNEGAN, JOSEPH H., National Insurance Actuarial and Statistical Association, 85 John Street, New York, N. Y. 10038
Nov. 16, 1961	FITZGIBBON, WALTER J., JR., Assistant Actuary, Aetna Casualty and Surety Company & Standard Fire Insurance Company, 151 Farmington Avenue, Hartford, Conn. 06115
Nov. 15, 1935	FITZHUGH, GILBERT W., President, Metropolitan Life Insurance Company, One Madison Avenue, New York, N. Y. 10010
Nov. 18, 1955	FOSTER, ROBERT B., Associate Actuary, The Travelers Insurance Company, One Tower Square, Hartford, Conn. 06115
Nov. 18, 1955	FOWLER, THOMAS W., Actuary, North American Reinsurance Corporation, 161 East 42 Street, New York, N. Y. 10017

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Nov. 18, 1927	FREDRICKSON, C. H., Consulting Actuary, Galiano Island, P. O. Box 40, British Columbia, Canada
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Nov. 22, 1957	GILLAM, WILLIAM S., Secretary and Actuary, National Bureau of Casualty Underwriters, 125 Maiden Lane, New York, N. Y. 10038
Nov. 20, 1964	GILLESPIE, JAMES E., Assistant Actuary, Continental National American Group, 310 South Michigan Avenue, Chicago, Ill. 60604
Nov. 20, 1924	GINSBURGH, HAROLD J. (Retired), 14 Crestview Road, Belmont, Mass. 02178
Nov. 21, 1930	GLENN, JOSEPH BRYAN, Actuarial Consultant, Department of Defense, Washington, D. C.
Nov. 13, 1931	GODDARD, RUSSELL P., Actuary, Bowles, Andrews & Towne, Inc., 200 Park Avenue, New York, N. Y. 10017
†	GOODWIN, EDWARD S. (Retired), Investment Counselor, 96 Garvan Street, East Hartford 8, Conn.
Nov. 19, 1926	GRAHAM, CHARLES M., Fire and Casualty Actuary, South Carolina Insurance Department, 1401 Hampton Street, Columbia, S. C. 29201
Nov. 19, 1953	GRAVES, CLYDE H., Assistant Manager and Actuary, Mutual Insurance Rating Bureau and Mutual Insurance Advisory Association, 733 Third Avenue, New York, N. Y. 10017
Nov. 19, 1953	HALEY, JAMES B., JR., Coates, Herfurth & England, Consulting Actuaries, Crocker Bldg., San Francisco, Calif.
Nov. 16, 1956	HART, W. VAN BUREN, JR., Actuary, Aetna Insurance Company, 55 Elm Street, Hartford, Conn. 06115
Nov. 17, 1950	HARWAYNE, FRANK, Chief Actuary, New York State Insurance Department, 123 William Street, New York, N. Y. 10038
Nov. 19, 1926	HAUGH, CHARLES J. (Retired), 25 LeMay Street, West Hartford, Conn. 06107
Nov. 17, 1950	HAZAM, WILLIAM J., Vice President and Actuary, American Mutual Liability Insurance Company, Wakefield, Mass. 01881
Nov. 16, 1951	HEWITT, CHARLES C., JR., Actuary, Allstate Insurance Company, 7447 Skokie Blvd., Skokie, Ill. 60078
Nov. 16, 1961	HOBBS, EDWARD J., Actuary, Insurance Company of North America, 1600 Arch Street, Philadelphia, Pa. 19101
Nov. 22, 1934	HOOKER, RUSSELL O., Russell O. Hooker, & Associates, Consulting Actuaries, 266 Pearl Street, Hartford 3, Conn.
Nov. 17, 1950	HOPE, FRANCIS J., Actuary, Hartford Insurance Group, 690 Asylum Avenue, Hartford, Conn. 06115
Nov. 14, 1947	HUGHEY, M. STANLEY, Executive Vice President, Lumbermens Mutual Casualty Company, 4750 N. Sheridan Road, Chicago, Ill. 60640
Nov. 19, 1959	HUNT, FREDERIC J., JR., Assistant Secretary, Insurance Company of North America, 1600 Arch Street, Philadelphia, Pa. 19101
Nov. 18, 1955	HURLEY, ROBERT L., Actuary, Fire Insurance Research and Actuarial Association, 125 Maiden Lane, New York, N. Y. 10038
Nov. 19, 1954	JOHE, RICHARD L., Vice President and Actuary, United States Fidelity and Guaranty Company, Calvert & Redwood Streets, Baltimore, Md. 21203
Nov. 14, 1941	JOHNSON, ROGER A., Actuary, Blue Cross of Greater Philadelphia, 1333 Chestnut Street, Philadelphia, Pa. 19107
Nov. 16, 1939	JONES, HAROLD M., Group Statistician, John Hancock Mutual Life Insurance Company, 200 Berkeley Street, Boston, Mass. 02117
Nov. 16, 1956	KALLOP, ROY H., Actuary, National Council on Compensation Insurance, 200 East 42 Street, New York, N. Y. 10017

Admitted	
Nov. 22, 1957	KATES, PHILLIP B., Actuary, Pilot Fire and Casualty Company, Box #1320, Greensboro, N. C.
Nov. 19, 1926	KELTON, WILLIAM H. (Retired), 122 Arundel Avenue, West Hartford, Conn. 06107
Nov. 19, 1959	KLAASSEN, ELDON J., Associate Actuary, Continental National American Group, 310 S. Michigan Avenue, Chicago, Ill. 60604
Nov. 14, 1941	KOLB, MORRIS B., Director of Planning and Data Processing, The State Insurance Fund, 199 Church Street, New York, N. Y. 10007
Nov. 24, 1933	KORMES, MARK, President, Actuarial Associates, Inc., 405 Lexington Avenue, New York, N. Y. 10017
Nov. 19, 1953	KUENKLER, ARTHUR S., Executive Vice President, Security Insurance Company of Hartford, 1000 Asylum Avenue, Hartford, Conn. 06101
Nov. 18, 1949	LACROIX, HAROLD F., Second Vice President, The Travelers Insurance Company, One Tower Square, Hartford, Conn. 06115
Nov. 20, 1964	LANGE, JEFFREY T., Manager, Research Division, National Bureau of Casualty Underwriters, 125 Maiden Lane, New York, N. Y. 10038
May 5, 1961	LATIMER, MURRAY W., Industrial Relations Consultants, 1625 K Street, N. W., Washington, D. C. 20006
Nov. 17, 1950	LESLIE, WILLIAM, Jr., Vice President and Actuary, Continental Insurance Companies, 80 Maiden Lane, New York, N. Y. 10038
Nov. 16, 1961	LINDEN, JOHN R., Assistant Actuary, Aetna Casualty and Surety Company and Standard Fire Insurance Company, 151 Farmington Avenue, Hartford, Conn. 06115
Nov. 20, 1924	LINDER, JOSEPH, Consulting Actuary, 200 Park Avenue, New York, N. Y. 10017
Nov. 16, 1956	LINO, RICHARD, Actuary, National Bureau of Casualty Underwriters, 125 Maiden Lane, New York, N. Y. 10038
Nov. 18, 1955	LISCORD, PAUL S., Second Vice President and Actuary, The Travelers Insurance Company, One Tower Square, Hartford, Conn. 06115
Nov. 17, 1950	LIVINGSTON, GILBERT R., Casualty Actuary, Connecticut Insurance Department, State Office Bldg., Hartford 15, Conn.
Nov. 16, 1951	LONGLEY-COOK, LAURENCE H., Vice President and Actuary, Insurance Company of North America, 1600 Arch Street, Philadelphia, Pa. 19101
Nov. 13, 1936	LYONS, DANIEL J., President, Guardian Life Insurance Company of America, 201 Park Avenue South, New York, N. Y. 10003
Nov. 1, 1963	MACGINNITIE, W. JAMES, Actuary, Seguros Bolivar, Apartado Aereo 4421, Bogota, Colombia.
Nov. 19, 1954	MACKEEN, HAROLD E., Assistant Actuary, The Travelers Insurance Company, One Tower Square, Hartford, Conn. 06115
Nov. 18, 1958	MAGRATH, JOSEPH J. (Retired), 47 Woodland Avenue, Summit, N. J. 07901
Nov. 22, 1957	MAKGILL, STEPHEN S., Associate Actuary, Travelers Insurance Company, One Tower Square, Hartford, Conn. 06115
Nov. 23, 1928	MARSHALL, RALPH M. (Retired), Catts Corner, Worton, Kent County, Md.
Nov. 18, 1927	MASTERSON, NORTON E., Vice President and Actuary, Sentry Insurance—Hardware Mutuals Group, 200 Strongs Avenue, Stevens Point, Wis. 54482
Nov. 19, 1926	MATTHEWS, ARTHUR N., 475 Poquonoch Avenue, Windsor, Conn. 06095
May 19, 1915	MAYCRINK, EMMA C. (Retired), 32 Chittenden Avenue, Crestwood, N. Y. 10707

Admitted	
Nov. 14, 1958	MAYERSON, ALLEN L., Professor of Insurance and Actuarial Mathematics, The University of Michigan, Graduate School of Business Administration, Ann Arbor, Mich.
Nov. 1, 1963	MCCLURE, RICHARD D., Underwriting Executive Department, Lumbermens Mutual Casualty Company, 4750 N. Sheridan Road, Chicago, Ill. 60640
Nov. 15, 1935	MCCONNELL, MATTHEW H., Superintendent, Compensation & Liability Dept., General Accident Fire and Life Assurance Corporation Ltd., 414 Walnut Street, Philadelphia, Pa. 19106
Nov. 18, 1960	MCGUINNESS, JOHN S., Consultant in Actuarial Science and Management, 41 Horicon Avenue, Glens Falls, N. Y. 12801
Nov. 20, 1964	MCLEAN, GEORGE E., Actuary, Massachusetts Hospital Service, Inc. and Massachusetts Medical Service, 133 Federal Street, Boston, Mass. 02106
Nov. 15, 1962	MCNAMARA, DANIEL J., Secretary, National Bureau of Casualty Underwriters, 125 Maiden Lane, New York, N. Y. 10038
Nov. 15, 1962	MEENAGHAN, JAMES J., Associate Actuary, Fireman's Fund American Insurance Companies, 3333 California Street, San Francisco, Calif. 94120
Nov. 18, 1955	MENZEL, HENRY W., Actuary, New York Compensation Insurance Rating Board, 200 East 42nd Street, New York, N. Y. 10017
†	MICHELBACHER, GUSTAV F. (Retired), 15201 Quito Road, Saratoga, Calif. 95070
Nov. 17, 1938	MILLER, JOHN H., Executive Vice President, Monarch Life Insurance Company, 1250 State Street, Springfield, Mass. 01101
Nov. 1, 1963	MILLER, NICHOLAS F., JR., Assistant Secretary, Executive Department, The Aetna Casualty & Surety Company and Standard Fire Insurance Company, 151 Farmington Avenue, Hartford, Conn. 06115
Nov. 18, 1937	MILLS, JOHN A. (Retired), Point Placid, Reeds Spring, Mo.
Nov. 22, 1957	MILLS, RICHARD J., Assistant Actuary, Lumbermens Mutual Casualty Company, 4750 Sheridan Road, Chicago, Ill. 60640
Nov. 15, 1962	MORISON, GEORGE D., Assistant Actuary, Aetna Casualty and Surety Company & Standard Fire Insurance Company, 151 Farmington Avenue, Hartford, Conn. 06115
Nov. 16, 1961	MOSELEY, JACK, Associate Actuary, United States Fidelity and Guaranty Company, Calvert and Redwood Streets, Baltimore, Md. 21203
Nov. 17, 1920	MUELLER, LOUIS H., 2845 Lake Street, San Francisco 21, Calif.
Nov. 16, 1956	MUETTERTIES, JOHN H., Associate Actuary, Sentry Insurance—Hardware Mutuals Group, 1421 Strongs Avenue, Stevens Point, Wis. 54482
Nov. 17, 1950	MUNTERICH, GEORGE C., Assistant Secretary, Hartford Insurance Group, 690 Asylum Avenue, Hartford, Conn. 06115
Nov. 19, 1954	MURRIN, THOMAS E., Vice President and Actuary, Fireman's Fund American Insurance Companies, 3333 California Street, San Francisco, Calif. 94120
Nov. 19, 1959	MYERS, ROBERT J., Chief Actuary, Social Security Administration, Department of Health, Education, and Welfare, Washington, D. C. 20201
Nov. 16, 1965	NELSON, DALE A., Assistant Actuary, State Farm Mutual Automobile Insurance Company, 112 East Washington Street, Bloomington, Ill. 61701
Nov. 1, 1963	NELSON, S. TYLER, SR., Casualty Manager—Actuary, American Agricultural Mutual Insurance Company, Room 1000, Merchandise Mart Plaza, Chicago, Ill. 60654
Nov. 14, 1958	NILES, CHARLES L., JR., Assistant General Manager and Actuary, General Accident Group, 414 Walnut Street, Philadelphia, Pa. 19105

Admitted	
Nov. 15, 1935	OBERHAUS, THOMAS M., Vice President, Woodward and Fondiller, 420 Madison Avenue, New York, N. Y. 10017
Nov. 16, 1965	OIEN, R. GUSTAVE, Research Actuary, St. Paul Insurance Companies, 385 Washington Street, St. Paul, Minn. 55102.
†	ORR, ROBERT K. (Retired), 725 E. Palmetto, Lakeland, Fla.
Nov. 22, 1957	OTTESON, PAUL M., Vice President and Actuary, Federated Mutual Implement and Hardware Insurance Company and Federated Life Insurance Company, 129 East Broadway, Owatonna, Minn. 55060
Nov. 21, 1919	OUTWATER, OLIVE E. (Retired), 2404 Loring Street, San Diego, Calif. 92109
Nov. 15, 1962	PARLIN, R. W., Research Associate, University of Minnesota, Laboratory of Physiological Hygiene, Stadium Gate 27, Minneapolis, Minn. 55455
Nov. 18, 1960	PENNYCOOK, ROD B., Health Insurance Secretary, The Great-West Life Assurance Company, 60 Osborne Street, N., Winnipeg 1. Manitoba, Canada.
Nov. 22, 1957	PERKINS, WILLIAM J., F.C.I.A., Assistant Group Actuary, London Life Insurance Company, London, Ontario, Canada
Nov. 14, 1941	PETERS, STEFAN, F.C.A.S., F.S.A., Arthur D. Little, Inc., 35 Acorn Park, Cambridge, Mass. 02140
Nov. 21, 1952	PETZ, EARL F., Associate Actuary, Lumbermens Mutual Casualty Company, 4750 N. Sheridan Road, Chicago, Ill. 60640
Nov. 19, 1959	PHILLIPS, HERBERT J., JR., Associate Actuary, The Employers' Group of Insurance Companies, 110 Milk Street, Boston, Mass. 02107
Nov. 24, 1933	PICKETT, SAMUEL C. (Retired), 40 Niles Road, Poquonock, Conn.
Nov. 22, 1957	PINNEY, ALLEN D., Assistant Secretary, Group Department, The Travelers Insurance Company, One Tower Square, Hartford, Conn. 06115
Nov. 17, 1922	PINNEY, SYDNEY D. (Retired), 290 Wolcott Hill Road, Wethersfield, Conn. 06109
Nov. 19, 1959	POLLACK, ROBERT, Assistant Vice President and Associate Actuary, American Mutual Liability Insurance Company, Wakefield, Mass. 01881
Nov. 16, 1965	PORTERMAIN, NEILL W., Assistant Actuary, Mutual Service Insurance Company, 1919 University Avenue, St. Paul, Minn. 55104
Nov. 13, 1931	PRUITT, DUDLEY M., Executive Secretary, Middle Atlantic Region, American Friends Service Committee, 1500 Race Street, Philadelphia 2, Pa.
Nov. 18, 1955	RESONY, ALLIE V., Assistant Secretary, Hartford Insurance Group, 690 Asylum Avenue, Hartford, Conn. 06115
Nov. 18, 1949	RESONY, JOHN A., Secretary, The Travelers Insurance Company, One Tower Square, Hartford, Conn. 06115
Nov. 16, 1951	RICE, HOMER D. (Retired), 1731 Morningside Drive, Mount Dora, Fla. 32757
Nov. 1, 1963	RICHARDS, HARRY R., Assistant Actuary, The Travelers Insurance Company, One Tower Square, Hartford, Conn. 06115
Nov. 1, 1963	RIDDLESWORTH, WILLIAM A., Assistant Actuary, Aetna Casualty and Surety Company & Standard Fire Insurance Company, 151 Farmington Avenue, Hartford, Conn. 06115
May 24, 1921	RIEGEL, ROBERT, Professor Emeritus of Statistics and Insurance, State University of New York at Buffalo, 3435 Main Street, Buffalo, N. Y. 14214
Nov. 14, 1958	ROBERTS, LEWIS H., Actuary, Woodward & Fondiller, 420 Madison Avenue, New York 17, N. Y.

Admitted	
Nov. 14, 1947	RODERMUND, MATTHEW, Vice President-Actuary, Munich Reinsurance Company, 410 Park Avenue, New York, N.Y. 10022
Nov. 14, 1947	ROSENBERG, NORMAN, Executive Assistant, Farmers Insurance Group, 4680 Wilshire Boulevard, Los Angeles 54, Calif.
Nov. 14, 1947	ROWELL, JOHN H., Vice President, Marsh & McLennan, Inc., 231 South LaSalle Street, Chicago, Ill. 60604
Nov. 17, 1938	RUCHLIS, ELSIE, Actuarial Supervisor, National Bureau of Casualty Underwriters, 125 Maiden Lane, New York, N.Y. 10038
Nov. 14, 1947	SALZMANN, RUTH E., Group Actuary, Insurance Companies of North America, 1600 Arch Street, Philadelphia, Pa. 19101
Nov. 1, 1963	SARASON, HARRY M., Managing Actuary, Woodward and Fondiller, Inc., 3625 W. 6th Street, Los Angeles, Calif. 90005
Nov. 19, 1948	SCHLOSS, HAROLD W., Vice President and Actuary, Royal-Globe Insurance Companies, 150 William Street, New York, N.Y. 10038
Nov. 18, 1937	SHAPIRO, GEORGE I., 934 East 9 Street, Brooklyn, N.Y. 11230
Nov. 13, 1931	SILVERMAN, DAVID, Consulting Actuary, Peat, Marwick, Mitchell & Company, Wolfe Corcoran Division, 70 Pine Street, New York, N.Y. 10005
Nov. 19, 1954	SIMON, LEROY J., General Manager, National Insurance Actuarial and Statistical Association, 85 John Street, New York, N.Y. 10038
Nov. 18, 1960	SIMONEAU, PAUL W., Assistant Actuary, Aetna Casualty & Surety Company & Standard Fire Insurance Company, 151 Farmington Avenue, Hartford, Conn. 06115
Nov. 19, 1929	SKELDING, ALBERT Z., Secretary-Treasurer, Casualty Actuarial Society, 200 E. 42nd Street, New York, N.Y. 10017
Nov. 19, 1929	SKILLINGS, E. SHAW (Retired), 3036 Central Street, Evanston, Ill. 60201
Nov. 18, 1932	SMICK, J. J., Consulting Actuary, Smick & Co., Inc., 135 East 42nd Street, New York, N.Y. 10017
Nov. 14, 1958	SMITH, EDWARD M., Associate Actuary, The Travelers Insurance Company, One Tower Square, Hartford, Conn. 06115
Nov. 15, 1940	SMITH, SEYMOUR E., Senior Vice President, The Travelers Insurance Companies, One Tower Square, Hartford, Conn. 06115
Nov. 15, 1962	STANKUS, LEO M., Director of Executive Information, Allstate Insurance Company, 7447 Skokie Boulevard, Skokie, Ill. 60078
Nov. 24, 1933	ST. JOHN, JOHN B., Consulting Actuary, Box 57, Penllyn, Pa.
Nov. 19, 1959	SYKES, ZENAS M., Actuary, Social Security Administration, United States Department of Health, Education and Welfare, Washington 25, D. C.
May 25, 1956	TAPLEY, DAVID A., Senior Vice President, Wolverine Insurance Company, Wolverine-Federal Bldg., Battle Creek, Mich. 49016
Nov. 14, 1958	TARBELL, LUTHER L., JR., Associate Actuary, The Travelers Insurance Company, One Tower Square, Hartford, Conn. 06115
Nov. 16, 1956	THOMAS, JAMES W., Assistant Actuary, The Travelers Insurance Company, One Tower Square, Hartford, Conn. 06115
†	THOMPSON, JOHN S., Newark Athletic Club, Newark 2, N. J.
Nov. 19, 1953	TRIST, JOHN A. W., Assistant Controller, Insurance Company of North America, 1600 Arch Street, Philadelphia, Pa. 19101
Nov. 15, 1962	TRUDEAU, DONALD E., Assistant Actuary, The Travelers Insurance Company, One Tower Square, Hartford, Conn. 06115
Nov. 14, 1947	UHTHOFF, DUNBAR R., Vice President and Actuary, Employers' Insurance of Wausau, 407 Grant Street, Wausau, Wis. 54402

Admitted	
Nov. 23, 1928	VALERIUS, NELS M., Associate Actuary, Aetna Casualty and Surety Company and Standard Fire Insurance Company, 151 Farmington Avenue, Hartford, Conn. 06115
Nov. 21, 1919	VAN TUYL, HIRAM O. (Retired), 1411 Dexter Drive, Clearwater, Fla. 33516
Nov. 16, 1965	VERHAGE, PAUL A., Assistant Actuary, Sentry Insurance—Hardware Mutuals Group, 1421 Strongs Avenue, Stevens Point, Wis. 54482
Nov. 16, 1951	VINCENT, LEWIS A., Vice President, American Insurance Association, 85 John Street, New York, N. Y. 10038
Nov. 17, 1920	WAITE, ALAN W., 16 Penwood Road, Bloomfield, Conn.
Nov. 19, 1962	WALSH, ALBERT J., Vice President, Reliance Insurance Company, 4 Penn Center Plaza, Philadelphia, Pa. 19103
Nov. 16, 1965	WEBB, BERNARD L., Actuarial Specialist, Sentry Insurance Group, 1421 Strongs Avenue, Stevens Point, Wis. 54482
Nov. 14, 1947	WIEDER, JOHN W., JR., Actuary, Aetna Casualty and Surety Company & Standard Fire Insurance Company, 151 Farmington Avenue, Hartford, Conn. 06115
Nov. 18, 1960	WILCKEN, CARL L., Actuary, Canadian Underwriters' Association, 31 Prince Andrew Place, Don Mills, Ontario, Canada
Nov. 1, 1963	WILLIAMS, DEWEY G., Assistant Secretary, Employers' Insurance of Texas, P. O. Box 2759, Dallas, Texas 75232
Nov. 15, 1935	WILLIAMS, HARRY V., President, The Hartford Insurance Group, 690 Asylum Avenue, Hartford, Conn. 06115
Nov. 22, 1957	WILLIAMS, P. ADGER, Second Vice President and Actuary, The Travelers Corporation, One Tower Square, Hartford, Conn. 06115
Nov. 14, 1941	WILLIAMSON, W. RULON, Research Actuary, 3400 Fairhill Drive, Washington 23, D. C.
Nov. 18, 1960	WILLSEY, LYNN W., Assistant Secretary, Group Department, The Travelers Insurance Company, One Tower Square, Hartford, Conn. 06115
Nov. 16, 1961	WILSON, JAMES C., *Vice President and Actuary, Casualty Division,* Security Life and Trust Companies, Winston-Salem, N.C. 27102
Nov. 13, 1931	WITTICK, HERBERT E., Vice President and General Manager, Pilot Insurance Company, 1315 Yonge Street, Toronto 7, Ontario, Canada.
Nov. 18, 1949	WOLFRUM, RICHARD J., Actuary, Liberty Mutual Insurance Company, 175 Berkeley Street, Boston 17, Mass.
Nov. 16, 1951	WOODALL, JOHN P., Manager, South-Eastern Underwriters Association, P. O. Box 5048, Atlanta, Ga. 30302
Nov. 14, 1958	WRIGHT, BYRON, Actuary, Department of Banking and Insurance, State of New Jersey, State House Annex, Trenton 25, N. J.
Nov. 19, 1953	YOUNT, HUBERT W. (Retired), 54 Waban Avenue, Waban, Mass. 02168

NOVEMBER 1, 1965

Admitted	
Nov. 15, 1918	**ACKERMAN, SAUL B., 405 Lexington Avenue, New York 17, N. Y.**
Nov. 16, 1965	ADLER, MARTIN, Actuarial Assistant, The Home Insurance Company, 59 Maiden Lane, New York, N. Y. 10008
Nov. 16, 1939	**AIN, SAMUEL N., Consulting Actuary, 120 Broadway, New York 5, N. Y.**
Apr. 5, 1928	ALLEN, AUSTIN F. (Retired), 4815 Royal Lane, Dallas, Texas 75229
Nov. 15, 1962	AMLIE, WILLIAM P., Actuarial Assistant, Employers' Liability Assurance Company, 110 Milk Street, Boston, Mass. 02107
Nov. 18, 1955	ANDREWS, EDWARD C., Associate Actuary, The Travelers Insurance Company, One Tower Square, Hartford, Conn. 06115
Nov. 21, 1930	ARCHIBALD, A. EDWARD, Vice President, Investors Diversified Services, Inc., Investors Building, Minneapolis, Minn. 55402
Nov. 19, 1959	BANNISTER, DAN W., Executive Vice President and General Manager, Horace Mann Insurance Group, 216 East Monroe Street, Springfield, Ill. 62701
Nov. 23, 1928	BATEMAN, ARTHUR E., Pine Grove Rest Home, Marlboro, Mass.
Nov. 15, 1940	BATHO, BRUCE W., Executive Vice President-Administration, Life Insurance Company of Georgia, 573 West Peachtree Street, N.E., Atlanta, Ga. 30308
Nov. 16, 1965	BELL, ALLAN A., Actuarial Division, Aetna Casualty and Surety Company and Standard Fire Insurance Company, 151 Farmington Avenue, Hartford, Conn. 06115
Nov. 16, 1956	BERG, ROY A., JR., Assistant Actuary, Old Republic Life Insurance Company, 307 North Michigan Avenue, Chicago, Ill. 60601
Nov. 14, 1958	BERNAT, LEO A., Consultant, Minnesota Research Associates, 503 15th Avenue, S.E., No. 2, Minneapolis, Minn. 55414
Nov. 18, 1925	BITTEL, W. HAROLD, Chief Actuary, Department of Banking and Insurance, State of New Jersey, Trenton, N. J. 08625
Nov. 16, 1965	BLAND, WILLIAM H., Actuarial Assistant, The Travelers Insurance Company, One Tower Square, Hartford, Conn. 06115
Nov. 14, 1958	BLUMENFELD, M. EUGENE, Actuary-Group and A & H, Standard Life & Accident Insurance Company, P. O. Box 1097, Oklahoma City, Okla. 73101
Nov. 22, 1934	BOMSE, EDWARD L., Manager—Liability, Royal-Globe Insurance Companies, 150 William Street, New York, N. Y. 10038
Nov. 22, 1957	BRAGG, JOHN M., Vice President and Chief Actuary, Life Insurance Company of Georgia, 573 W. Peachtree Street, N.E., Atlanta, Ga. 30308
Nov. 16, 1965	BRIAN, ROBERT A., Fire and Casualty Associate Actuary, Connecticut Insurance Department, State Office Building, Hartford, Conn. 06115
Nov. 20, 1964	BROWN, WILLIAM W., Actuarial Assistant, Liberty Mutual Insurance Company, 175 Berkeley Street, Boston, Mass. 02117
Nov. 15, 1962	BUFFINTON, PHILIP G., Vice President, State Farm Fire and Casualty Company, 112 East Washington Street, Bloomington, Ill. 61701
Nov. 20, 1924	BUGBEE, JAMES M. (Retired), 115 Hawthorn Road, Baltimore, Md. 21210
Mar. 31, 1920	BURT, MARGARET A., Office of George B. Buck, Consulting Actuary, 60 Worth Street, New York 13, N. Y.
Nov. 19, 1959	BUTLER, RICHARD H., Second Vice President, The Travelers Insurance Company, One Tower Square, Hartford, Conn. 06115

Admitted	
Nov. 20, 1964	CARLSON, EDWIN A., Actuarial Assistant, The Travelers Insurance Company, One Tower Square, Hartford, Conn. 06115
Nov. 15, 1962	CARSON, DAVID E. A., Actuary, Hartford Insurance Group, 690 Asylum Avenue, Hartford, Conn. 06115
Nov. 18, 1927	CHEN, S. T., Consulting Actuary, The Wing On Life Assurance Company, Ltd., Wing On Life Bldg., 22 Des Voeux Road, Central, Hong Kong.
Nov. 16, 1961	CHERLIN, GEORGE, Vice President and Actuary, National Health and Welfare Retirement Association, Inc., 800 Second Avenue, New York, N. Y. 10017
Nov. 22, 1957	CHURCH, HARRY M., Coates, Herfurth & England, 301 E. Colorado Avenue, Pasadena, Calif. 91101
Nov. 1, 1963	CIMA, AUGUSTIN J., Pricing Research Manager, Allstate Insurance Company, 7447 Skokie Blvd., Skokie, Ill. 60078
Nov. 18, 1955	COATES, WILLIAM D., Vice President-Actuary, National-Ben Franklin Insurance Company, 360 W. Jackson Blvd., Chicago, Ill. 60606
Nov. 19, 1953	CONTE, JOSEPH P., Assistant to the President, Berman's Motor Express, P. O. Box 1209, Binghamton, N. Y. 13902
Nov. 16, 1965	COOK, CHARLES F., Continental Insurance Companies, 80 Maiden Lane, New York, New York 10038
Nov. 19, 1959	COPESTAKES, A. D., Assistant Vice President—Reports, American Mutual Liability Insurance Company, Wakefield, Mass. 01881
May 21, 1963	CRANDALL, WILLIAM H., Actuarial Assistant, Insurance Company of North America, 1600 Arch Street, Philadelphia, Pa. 19101
Nov. 24, 1933	CRAWFORD, WILLIAM H., Vice President and Treasurer, Industrial Indemnity Company, 155 Sansome Street, San Francisco, Calif. 94104
Nov. 19, 1953	CROFTS, GEOFFREY, Dean and Director, Graduate School of Actuarial Science, Northeastern University, 360 Huntington Avenue, Boston, Mass. 02115
Nov. 1, 1963	DAHME, ORVAL E., Senior Assistant Actuary, State Farm Mutual Automobile Insurance Company, 112 E. Washington Street, Bloomington, Ill. 61701
Nov. 21, 1952	DANIEL, C. M., Data Processing Manager, Fisher Governor Company, Marshalltown, Iowa 50158
Nov. 18, 1925	DAVIS, MALVIN E. (Retired), 150 East 69 Street, New York, N. Y. 10021
Nov. 16, 1965	DICKSON, CAROL D., Actuarial Division, Aetna Casualty and Surety Company and Standard Fire Insurance Company, 151 Farmington Avenue, Hartford, Conn. 06115
Nov. 14, 1941	DOWLING, WILLIAM F., 77 Brook Street, Garden City, N. Y. 11535
Nov. 1, 1963	DURKIN, JAMES H., Actuary, Peat, Marwick, Mitchell & Company, Wolfe Corcoran Division, 70 Pine Street, New York, N. Y. 10005
Nov. 14, 1958	DUROSE, STANLEY C., JR., Deputy Commissioner of Insurance, State of Wisconsin, 4802 Sheboygan Avenue, Madison, Wis. 53702
Nov. 19, 1954	EATON, KARL F., Controller, Guarantee Mutual Life Company, 8271 Indian Hills Drive, Omaha, Neb. 68114
June 5, 1925	EGER, FRANK A., (Retired), 1119 Prospect Ridge Blvd., Haddon Heights, N. J. 08035
Nov. 15, 1962	EVEN, CHARLES A., JR., The Travelers Insurance Company, One Tower Square, Hartford, Conn. 06115
Nov. 22, 1957	FELDMAN, MARTIN F., Associate Actuary, New York State Insurance Department, 123 William Street, New York, N. Y. 10038
Nov. 16, 1961	FERDEN, STEIN, Undelstadlia 8, Asker, Norway

Admitted	
Nov. 15, 1962	FINKEL, DANIEL, Associate Actuary, The State Insurance Fund, 199 Church Street, New York, N. Y. 10007
Nov. 16, 1956	FLACK, PAUL R., Actuarial Assistant, General Accident Fire and Life Assurance Corporation, Ltd., 414 Walnut Street, Philadelphia, Pa.
Nov. 16, 1965	FLAHERTY, DANIEL J., Actuarial Assistant, Insurance Company of North America, 1600 Arch Street, Philadelphia, Pa. 19101
Nov. 16, 1923	FLEMING, FRANK A. (Retired), c/o Mutual Insurance Rating Bureau, 733 Third Avenue, New York 17, N. Y.
Nov. 20, 1964	FORKER, DAVID C., The Travelers Insurance Company, Group Underwriting Department, One Tower Square, Hartford, Conn. 06115
Nov. 21, 1952	FRANKLIN, NATHAN M., Actuary, The Surety Association of America, 110 William Street, New York, N. Y. 10038
Nov. 19, 1954	GAINES, NATHANIEL, Associate Actuary, Office of George B. Buck, Consulting Actuary, 60 Worth Street, New York, N. Y. 10013
Nov. 15, 1962	GERUNDO, LOUIS P., JR., The Travelers Insurance Company, One Tower Square, Hartford, Conn. 06115
Nov. 18, 1932	GETMAN, RICHARD A., Assistant Actuary, Life Dept., The Travelers Insurance Company, One Tower Square, Hartford, Conn. 06115
May 26, 1965	GIBSON, JOHN A., III, Phoenix of Hartford Insurance Companies, 61 Woodland Street, Hartford, Conn. 06115
Nov. 17, 1922	GIBSON, JOSEPH P., JR., (Retired), 2970 Lorain Road, San Marino, Calif. 91108
Nov. 16, 1923	GILDEA, JAMES F. (Retired), 236 Nott Street, Wethersfield, Conn.
Nov. 1, 1963	GILL, JAMES F., Actuary, National Association of Independent Insurers, 30 West Monroe St., Inland Steel Bldg., Chicago, Ill. 60603
Nov. 14, 1947	GINGERY, STANLEY W., Vice President and Associate Actuary, The Prudential Insurance Company of America, Prudential Plaza, Newark, N. J. 07101
Nov. 19, 1959	GOLD, MELVIN L., Gold Associates, 39 South Fullerton Avenue, Montclair, N. J. 07042
Nov. 16, 1961	GOULD, DONALD E., Senior Statistician, New York State Insurance Fund, 199 Church Street, New York, N. Y. 10007
Nov. 18, 1927	GREEN, WALTER C., A.S.A. (Retired), 923 South 23 Street, East Salt Lake City, Utah 84108
Nov. 16, 1961	GREENE, THOMAS A., Assistant Vice President, American Re-Insurance Company, 99 John Street, New York, N. Y. 10038
Nov. 15, 1940	GROSSMAN, ELI A., Senior Vice President, The Great Eastern Life Insurance Company, 10 Dorrance Street, Providence, R. I. 02903
Nov. 15, 1935	GUERTIN, ALFRED N., Actuarial Consultant, Suite 2017, Field Building, 135 South LaSalle Street, Chicago, Ill. 60603
Nov. 16, 1965	HACHEMEISTER, CHARLES A., Actuarial Assistant, Royal-Globe Insurance Companies, 150 William Street, New York, N. Y. 10038
Nov. 16, 1939	HAGEN, OLAF E., Senior Assistant Actuarial Supervisor, Metropolitan Life Insurance Company, One Madison Avenue, New York, N. Y. 10010
Nov. 17, 1922	HALL, HARTWELL L. (Retired), 34 Lincoln Avenue, West Hartford, Conn. 06117
Nov. 13, 1936	HAM, HUGH P., President, The Western Assurance Company, 40 Scott Street, Toronto 1, Ontario, Canada
Nov. 1, 1963	HAMMER, SIDNEY M., Assistant Actuary, The Home Insurance Company, 59 Maiden Lane, New York, N. Y. 10008

Admitted	
Nov. 16, 1965	HANSON, H. DONALD, Actuarial Assistant, Continental National American Group, 310 South Michigan Avenue, Chicago, Ill. 60604
Nov. 19, 1953	HARACK, JOHN, Actuary, Health Service, Inc., and Medical Indemnity of America, Inc., 200 N. Michigan Avenue, Chicago 1, Ill.
Mar. 24, 1932	HARRIS, SCOTT, Executive Vice President, Joseph Froggatt & Company, Inc., 74 Trinity Place, New York, N. Y. 10006
Mar. 25, 1924	HART, WARD VAN B., 49 Robbins Drive, Wethersfield 9, Conn.
Nov. 21, 1919	HAYDON, GEORGE F., Manager Emeritus, Wisconsin Compensation Rating Bureau, 623 N. 2nd Street, Milwaukee 3, Wis.
Nov. 19, 1953	HEAD, GLENN O., Executive Vice President, First Investors Life Insurance Company, 120 Wall Street, New York 5, N. Y.
Nov. 19, 1959	HICKMAN, JAMES C., Associate Professor, Department of Statistics, University of Iowa, Iowa City, Iowa 52240
Nov. 15, 1962	HILLHOUSE, JERRY A., Associate Actuary, State Farm Mutual Automobile Insurance Company, 112 E. Washington Street, Bloomington, Ill. 61701
Nov. 16, 1965	HONEBEIN, CARLTON W., Assistant Actuary, Illinois Blue Cross-Blue Shield, 425 North Michigan Avenue, Chicago, Ill. 60690
Nov. 16, 1961	HOROWITZ, MILTON, Principal Actuary, The State Insurance Fund, 199 Church Street, New York, N. Y. 10007
Nov. 19, 1929	JACOBS, CARL N., Chairman of the Board, Hardware Mutual Casualty Company, Hardware Dealers Mutual Fire Insurance Company and Sentry Life Insurance Company, 1421 Strongs Avenue, Stevens Point, Wis. 54482
Nov. 18, 1921	JENSEN, EDWARD S. (Retired), 158 Orvil Way, Fallbrook, Calif.
Nov. 15, 1962	JENSEN, JAMES P., Actuarial Assistant, Liberty Mutual Insurance Companies, 175 Berkeley Street, Boston, Mass. 02117
Nov. 21, 1919	JONES, LORING D. (Retired), 64 Raymond Avenue, Rockville Centre L. I., N. Y.
Nov. 21, 1952	JONES, NATHAN F., Associate Actuary, The Prudential Insurance Company of America, Prudential Plaza, Newark, N. J. 07101
Nov. 20, 1964	KHOURY, COSTANDY K., Instructor, Ohio State University, Columbus, Ohio
Nov. 15, 1935	KITZROW, ERWIN W. (Retired), P. O. Box 313, Pasadena, Calif.
Nov. 19, 1959	KROEKER, JOHN, 1166 Dartmouth Avenue, Ottawa 8, Ontario, Canada
Nov. 19, 1959	LEIGHT, ARTHUR S., Actuarial Associate, Metropolitan Life Insurance Company, 1 Madison Avenue, New York, N. Y. 10010
Nov. 14, 1947	LUFKIN, ROBERT W., Manager of Home Office, Craftsman Life Insurance Company, 851 Boylston Street, Boston, Mass. 02116
Nov. 18, 1925	MALMUTH, JACOB, Chief-Rating Bureau, N. Y. Insurance Department, 123 William Street, New York, N. Y. 10038
Nov. 16, 1961	MARGOLIS, DONALD R., Assistant Actuary, Life Insurance Company of North America, 1600 Arch Street, Philadelphia, Pa. 19101
Nov. 20, 1964	MARKELL, ANDREW S., Actuary, Transamerica Insurance Company, Occidental Center, Los Angeles, Calif. 90015
Mar. 24, 1927	MARSH, CHARLES V-R (Retired), Fidelity & Deposit Company of Maryland, Baltimore, Md. 33705
Nov. 16, 1956	MATHWICK, LLOYD F., Manager, Group Division, Employers Mutual of Wausau, 407 Grant Street, Wausau, Wis. 54402
Nov. 18, 1936	MAYER, WILLIAM H., JR., Manager, Group Contract Bureau, Metropolitan Life Insurance Company, 1 Madison Avenue, New York 10, N. Y.
May 26, 1955	McDONALD, MILTON G., Chief Actuary, Massachusetts Insurance Department, 100 Cambridge Street, Boston, Mass. 02202

Admitted	
Nov. 16, 1961	MCINTOSH, KENNETH L., Manager, Louisiana Rating & Fire Prevention Bureau, P. O. Box 60730, New Orleans, La. 70160
Nov. 13, 1931	MILLER, HENRY C., Comptroller, California State Compensation Insurance Fund, 525 Golden Gate Avenue, San Francisco 1, Calif.
Nov. 18, 1937	MINOR, EDUARD H., Associate Actuary, Metropolitan Life Insurance Company, 1 Madison Avenue, New York 10, N. Y.
Nov. 18, 1960	MOHNBLATT, ARNOLD S., Actuarial Assistant, Consolidated Mutual Insurance Company, 345 Adams Street, Brooklyn, N. Y. 11201
Nov. 20, 1964	MOKROS, BERTRAM F., Policyholder Research Manager, Allstate Insurance Company, 7447 Skokie Blvd., Skokie, Ill. 60078
Nov. 17, 1922	MONTGOMERY, JOHN C. (Retired), 165 Westervelt Avenue, Tenafly, N. J.
May 25, 1923	MOORE, JOSEPH P., 115 St. Catherine Road, Outremont, Quebec, Canada
Nov. 16, 1961	MOSS, ROBERT G., Vice President and Actuary, Marsh & McLennan, Inc., 515 Olive Street, St. Louis, Mo. 63101
Nov. 22, 1957	MUIR, JOSEPH M., General Manager, Mutual Insurance Advisory Association & Mutual Insurance Rating Bureau, 733 Third Avenue, New York 17, N. Y.
Nov. 1, 1963	MUNIZ, ROBERT M., National Bureau of Casualty Underwriters, 125 Maiden Lane, New York, N. Y. 10038
Nov. 16, 1965	NAFFZIGER, JOSEPH V., Senior Assistant Actuary, State Farm Mutual Automobile Insurance Company, 112 East Washington Street, Bloomington, Ill. 61701
Nov. 16, 1961	NELSON, ROLAND E., Actuary, State Farm Life and Accident Assurance Company, 112 East Washington Street, Bloomington, Ill. 61701
Oct. 27, 1916	NEWELL, WILLIAM (Retired), 1225 Park Avenue, New York, N. Y. 10028
Nov. 16, 1965	NEWMAN, STEVEN H., Actuarial Division, National Bureau of Casualty Underwriters, 125 Maiden Lane, New York, N. Y. 10008
Nov. 18, 1925	NICHOLSON, EARL H., Actuary and Deputy Insurance Commissioner, Nevada Insurance Division, Nye Building, Carson City, Nevada 89701
May 23, 1919	OTTO, WALTER E., Chairman of the Board, Michigan Mutual Liability Company, 28 West Adams Avenue, Detroit, Mich. 48226
Nov. 19, 1926	OVERHOLSER, DONALD M., 30 Fairlawn Street, Ho-ho-kus, N. J.
Nov. 16, 1961	PEEL, JERALD P., Actuary, Security Mutual Casualty Company, 309 W. Jackson Blvd., Chicago, Ill. 60606
Nov. 20, 1924	PENNOCK, RICHARD M. (Retired), 12 E. Lodges Lane, Bala-Cynwyd, Pa.
Nov. 16, 1965	PERREAULT, STEPHEN L., Actuarial Department, Hartford Insurance Group, 690 Asylum Avenue, Hartford, Conn. 06115
Nov. 14, 1947	PERRY, ROBERT C., Executive Vice President, State Farm Life Insurance Co., 112 East Washington St., Bloomington, Ill. 61701
Nov. 19, 1929	PHILLIPS, JOHN H. (Retired), 915 Steuben Street, Wausau, Wis. 54401
Nov. 17, 1920	PIKE, MORRIS (Retired), 531 East 20th Street, New York, N. Y.
Nov. 17, 1922	POORMAN, WILLIAM F., Chairman of the Board, Central Life Assurance Company, 611 Fifth Avenue, Des Moines, Iowa 50309
Nov. 13, 1936	POTOFSKY, SYLVIA, Senior Actuary, The State Insurance Fund, 199 Church Street, New York 7, N. Y.
Nov. 16, 1965	PRESLEY, PHILIP O., Actuarial Assistant, American Mutual Liability Insurance Company, Wakefield, Mass. 01881

Admitted	
Nov. 20, 1964	RAID, GARY A., Senior Assistant Actuary, State Farm Mutual Automobile Insurance Company, 112 East Washington Street, Bloomington, Ill. 61701
Nov. 16, 1965	RATNASWAMY, RAJ., Actuary, Detroit Automobile Inter-Insurance Exchange, 150 Bagley Avenue, Detroit, Mich. 48226
Nov. 15, 1918	RAYWID ,JOSEPH, Woodward & Fondiller, 420 Madison Avenue, New York, N. Y. 10017
Nov. 18, 1960	RICCARDO, JOSEPH F., Aetna Casualty & Surety Company and Standard Fire Insurance Company, 151 Farmington Avenue, Hartford, Conn. 06115
Nov. 19, 1932	RICHARDSON, HARRY F. (Retired), 413 Ackerman Avenue, Ho-ho-kus, N. J. 07423
Nov. 16, 1965	RICHARDSON, JAMES F., Actuarial Assistant, Employers' Group of Insurance Companies, 110 Milk Street, Boston, Mass. 02107
Nov. 19, 1953	RICHMOND, OWEN D., Controller, Business Men's Assurance Company, B. M. A. Tower, Kansas City, Mo. 64141
Nov. 18, 1960	RIPANDELLI, JOHN S., Chief Actuary, Florida Insurance Department, State Capitol, Tallahassee, Fla. 32304
Nov. 18, 1932	ROBERTS, JAMES A., Actuarial Statistician, The Travelers Insurance Company, One Tower Square, Hartford, Conn. 06115
Nov. 15, 1962	ROOD, HENRY F., President, Lincoln National Life Insurance Company, 1300 South Harrison Street, Fort Wayne, Ind. 46801
Nov. 18, 1960	ROTH, RICHARD J., Actuary, American International Underwriters Corp., 102 Maiden Lane, New York, N. Y. 10005
Nov. 19, 1959	ROYER, ALAN F., Actuary, Multi-Line Insurance Rating Bureau, 110 William Street, New York, N. Y. 10038
Nov. 1, 1963	RYAN, KEVIN M., Assistant Actuary, Industrial Indemnity Company, 155 Sansome Street, San Francisco, Calif. 94104
Nov. 14, 1958	SARNOFF, PAUL E., Assistant Actuary, The Prudential Insurance Company of America, Prudential Plaza, Newark, N. J. 07101
Nov. 16, 1923	SAWYER, ARTHUR, (Retired), 13751 St. Andrews Drive, Leisure World, Apt. 1-I, Seal Beach, Calif.
Nov. 14, 1947	SCAMMON, LAWRENCE W., Manager, Massachusetts Automobile Rating & Accident Prevention Bureau, Massachusetts Workmen's Compensation Rating & Inspection Bureau, & Massachusetts Motor Vehicle Assigned Risk Plan, 89 Broad Street, Boston, Mass.
Nov. 1, 1963	SCHEEL, PAUL J., Senior Actuarial Assistant, U. S. Fidelity & Guaranty Company, Calvert and Redwood Streets, Baltimore, Md. 21203
Nov. 16, 1961	SCHEIBL, JEROME A., Assistant Actuary, Employers Mutuals of Wausau, 407 Grant Street, Wausau, Wis. 54402
Nov. 16, 1965	SCHEID, JAMES E., Actuarial Department, Hartford Insurance Group, 690 Asylum Avenue, Hartford, Conn. 06115
Nov. 14, 1958	SCHLENZ, JOHN W., Senior Vice President and Actuary, Federal Life and Casualty Company, Wolverine-Federal Tower, Battle Creek, Mich. 49016
Nov. 22, 1957	SCHNEIKER, HENRY C., Associate Actuary, The Home Insurance Company, 59 Maiden Lane, New York, N. Y. 10008
Nov. 20, 1964	SCHULER, ROBERT J., Assistant Actuary, Blue Cross of Western Pennsylvania, One Smithfield Street, Pittsburgh, Pa. 15222
Nov. 19, 1954	SCHULMAN, JUSTIN, Group Leader, Mathematical Analysis, Programming, Kollsman Instrument Corporation, 80-08 45 Avenue, Elmhurst, N. Y. 11373
Nov. 14, 1947	SCHWARTZ, MAX J., Chief Accident & Health Rating Section, N. Y. State Insurance Department, 324 State Street, Albany, N. Y. 12210

Admitted	
Nov. 20, 1964	SCOTT, BRIAN E., Aetna Casualty and Surety Company and Standard Fire Insurance Company, 151 Farmington Avenue, Hartford, Conn. 06115
Nov. 20, 1930	SEVILLA, EXEQUIEL S., President, Manager and Actuary, National Life Insurance Company of the Philippines, Regina Bldg., P.O. Box 2056, Manila, Philippines.
Nov. 22, 1957	SHAVER, C. OTIS, Second Vice President and Actuary, Nationwide Mutual Fire Insurance Company, 246 North High Street, Columbus, Ohio 43216
Nov. 20, 1924	SHEPPARD, NORRIS E., Professor of Mathematics, University of Toronto, Toronto 5, Canada.
Nov. 1, 1963	SINGER, PAUL E., Actuary, Continental National American Group, 310 South Michigan Avenue, Chicago, Ill. 60604
Nov. 15, 1962	SMITH, EDWARD R., Assistant Actuary, Hartford Insurance Group, 690 Asylum Avenue, Hartford, Conn. 06115
Nov. 18, 1925	SOMMER, ARMAND, Vice President, Continental Casualty Company, 310 South Michigan Avenue, Chicago, Ill. 60604
Nov. 15, 1918	SPENCER, HAROLD S. (Retired), 8 Chelsea Lane, West Hartford, Conn.
Nov. 1, 1963	STALEY, HARLOW B., Vice President and Director of Administration, Farm Bureau Mutual Insurance Company, 10th and Grand Streets, Des Moines, Iowa. 50307
Nov. 19, 1959	STEIN, JOAN M., Assistant Actuary, National Bureau of Casualty Underwriters, 125 Maiden Lane, New York, N. Y. 10038
Nov. 19, 1959	STEINHAUS, HENRY W., President, Henry Steinhaus Association, Inc., 225 East 46 Street, New York, N. Y. 10017
Nov. 20, 1924	STELLWAGEN, HERBERT P., Director, Insurance Company of North America, 721 Mount Pleasant Road, Bryn Mawr, Pa. 19010
Nov. 15, 1956	STERN, PHILIPP K., Actuary, Mutual Insurance Rating Bureau, 733 Third Avenue, New York, N. Y. 10017
Nov. 19, 1959	STEVENS, WALDO A., Director, Actuarial Division, National Association of Blue Shield Plans, 425 North Michigan Avenue, Chicago, Ill. 60611
Nov. 16, 1923	STOKE, KENDRICK (Retired), 11052 McKinney, Detroit, Mich. 48224
Nov. 19, 1959	STRUG, EMIL J., Assistant Actuary, Massachusetts Hospital Service, Inc. and Massachusetts Medical Service, 133 Federal Street, Boston, Mass. 02106
Nov. 21, 1930	SULLIVAN, WALTER F., Actuary, State Compensation Insurance Fund, 525 Golden Gate Avenue, San Francisco, Calif. 94101
Nov. 15, 1962	SWITZER, VERNON J., Associate Actuary, State Farm Mutual Automobile Insurance Company, 112 E. Washington Street, Bloomington, Ill. 61701
Nov. 1, 1963	THOMPSON, PHILIP R., Statistician, Federated Mutual Implement and Hardware Insurance Company, 129 E. Broadway, Owatonna, Minn. 55060
Nov. 21, 1919	TRENCH, FREDERICK H. (Retired), 5 Bonnie Brae, Utica, N. Y.
Nov. 20, 1924	UHL, M. ELIZABETH (Retired), 320 E. 53rd Street, New York, N. Y. 10022
Nov. 14, 1958	VAN CLEAVE, MARVIN E., Chief, Rate Division, Wisconsin Insurance Department, 4802 Sheboygan Avenue, Madison, Wis. 53702
Nov. 20, 1964	VANDERHOOF, IRWIN T., Senior Vice President and Actuary, Standard Security Life Ins. Co. of N. Y., 111 Fifth Avenue, New York, N. Y. 10003
Nov. 19, 1959	WEBER, DONALD C., Fellow, Institute of Statistics, North Carolina State University, P. O. Box 5457, Raleigh, N. C. 27606
Nov. 18, 1932	WEINSTEIN, MAX S., Actuary, New York State Employees' Retirement System, 90 S. Swan Street, Albany 1, N. Y.

Admitted	
Nov. 18, 1925	WELLMAN, ALEX C., Senior Vice President, Protective Life Insurance Company, P. O. Box 2571, Birmingham, Ala. 35202
Nov. 21, 1930	WELLS, WALTER I. (Retired), West Sterling, Mass. 01565
Nov. 18, 1927	WHITBREAD, FRANK G., Second Vice President, The Lincoln National Life Insurance Co., 1301 South Harrison Street, Fort Wayne, Ind. 46801
Nov. 19, 1948	WHITE, AUBREY, Consulting Actuary, Ostheimer & Co., 1510 Chestnut Street, Philadelphia, Pa. 19102
Nov. 16, 1939	WITTLAKE, J. CLARKE, Executive Vice President, Business Men's Assurance Company of America, One Penn Valley Park, B. M. A. Tower, Kansas City, Mo. 64141
Oct. 22, 1915	WOOD, DONALD M., Partner, Childs & Wood, 175 West Jackson Boulevard, Chicago, Ill. 60604
Nov. 18, 1937	WOOD, DONALD M., JR., Partner, Childs & Wood, 175 West Jackson Boulevard, Chicago, Ill. 60604
Nov. 18, 1927	WOOD, MILTON J., Consulting Actuary, 16 Wardwell Road, West Hartford, Conn. 06107
Nov. 17, 1950	WOODDY, JOHN C., Actuary, North American Reassurance Company, 161 East 42 Street, New York, N. Y. 10017
Nov. 22, 1934	WOODWARD, BARBARA H., Assistant Secretary and Regional General Counsel, The Reuben H. Donnelly Corporation, 235 East 42 Street, New York, N. Y. 10017
Nov. 16, 1956	WOODWORTH, JAMES H., Assistant Secretary, The Hartford Insurance Group, 690 Asylum Avenue, Hartford, Conn. 06115
Nov. 18, 1925	WOOLERY, JAMES M., Senior Vice President-Actuary, Occidental Life Ins. of N. C., Cameron Village, Raleigh, N. C. 27605
May 5, 1961	YOUNG, ROBERT G., Actuary, Michigan Mutual Liability Company, 28 West Adams Avenue, Detroit, Mich. 48226
Nov. 1, 1963	ZORY, P. B., Actuarial Department, National Bureau of Casualty Underwriters, 125 Maiden Lane, New York, N. Y. 10038

Elected	*President*	*Vice-Presidents*	
1914-1915	*Isaac M. Rubinow	*Albert H. Mowbray	*Benedict D. Flynn
1916-1917	*James D. Craig	*Joseph H. Woodward	*Harwood E. Ryan
1918	*Joseph H. Woodward	*Benedict D. Flynn	*George D. Moore
1919	*Benedict D. Flynn	*George D. Moore	*William Leslie
1920	*Albert H. Mowbray	*William Leslie	*Leon S. Senior
1921	*Albert H. Mowbray	*Leon S. Senior	*Harwood E. Ryan
1922	*Harwood E. Ryan	Gustav F. Michelbacher	*Edmund E. Cammack
1923	*William Leslie	Gustav F. Michelbacher	*Edmund E. Cammack
1924-1925	Gustav F. Michelbacher	*Sanford B. Perkins	Ralph H. Blanchard
1926-1927	*Sanford B. Perkins	*George D. Moore	*Thomas F. Tarbell
1928-1929	*George D. Moore	Sydney D. Pinney	Paul Dorweiler
1930-1931	*Thomas F. Tarbell	*Roy A. Wheeler	*Winfield W. Greene
1932-1933	Paul Dorweiler	*William F. Roeber	*Leon S. Senior
1934-1935	*Winfield W. Greene	Ralph H. Blanchard	Charles J. Haugh
1936-1937	*Leon S. Senior	Sydney D. Pinney	*Francis S. Perryman
1938-1939	*Francis S. Perryman	Harmon T. Barber	*William J. Constable
1940	Sydney D. Pinney	Harold J. Ginsburgh	James M. Cahill
1941	Ralph H. Blanchard	Harold J. Ginsburgh	James M. Cahill
1942	Ralph H. Blanchard	Albert Z. Skelding	Charles J. Haugh
1943-1944	Harold J. Ginsburgh	Albert Z. Skelding	Charles J. Haugh
1945-1946	Charles J. Haugh	James M. Cahill	Harry V. Williams
1947-1948	James M. Cahill	Harmon T. Barber	Russell P. Goddard
1949-1950	Harmon T. Barber	*Thomas O. Carlson	Norton E. Masterson
1951-1952	*Thomas O. Carlson	Joseph Linder	Seymour E. Smith
1953-1954	Seymour E. Smith	Dudley M. Pruitt	John A. Mills
1955-1956	Norton E. Masterson	*Clarence A. Kulp	Arthur N. Matthews
1957-1958	Dudley M. Pruitt	John W. Carleton	William Leslie, Jr.
1959-1960	William Leslie, Jr.	Ernest T. Berkeley	Laurence H. Longley-Cook
1961-1962	L. H. Longley-Cook	Thomas E. Murrin	Richard J. Wolfrum
1963-1964	Thomas E. Murrin	Harold E. Curry	William J. Hazam
1965	Harold E. Curry	Charles C. Hewitt, Jr.	Harold W. Schloss

Secretary-Treasurer

1914-1917 . . . *C. E. Scattergood
1918-1953 . . . . *R. Fondiller
1954-1965 . . . . A. Z. Skelding

Editor

1914 . . . . *W. W. Greene
1915-1917 . . . . *R. Fondiller
1918 . . . . *W. W. Greene
1919-1921 . . . . G. F. Michelbacher
1922-1923 . . . . O. E. Outwater
1924-1932 . . . . . *R. J. McManus
1933-1943 . . . . *C. W. Hobbs
1944-1954 . . . . . . E. C. Mayerink
1955-1958 . . . . E. S. Allen
1959-1960 . . . . R. P. Goddard
1961-1964 . . . . H. W. Schloss
1965 . . . . . . Matthew Rodermund

General Chairman Examination Committee

1949-1951 . . . . R. A. Johnson
1952-1956 . . . . J. W. Wieder, Jr.
1957-1961 . . . . W. J. Hazam
1962-1965 . . . . N. J. Bennett

Librarian

1914 . . . . *W. W. Greene
1915 . . . . *R. Fondiller
1916-1921 . . . . L. I. Dublin
1922-1924 . . . . *E. R. Hardy
1925-1936 . . . . W. Breiby
1937-1947 . . . . . . *T. O. Carlson
1948-1950 . . . . *S. M. Ross
1951-1957 . . . . . G. R. Livingston
1958-1965 . . . . R. Lino

*Deceased.

The (†) denotes charter members at date of organization, November 7, 1914.

Admitted		*Died*
Nov. 13, 1931	Gilbert E. Ault	Apr. 13, 1965
Nov. 19, 1948	Arthur L. Bailey	Aug. 12, 1954
May 23, 1924	William B. Bailey	Jan. 10, 1952
†	Roland Benjamin	July 2, 1949
May 24, 1921	Edward J. Bond	Nov. 12, 1941
May 19, 1915	Thomas Bradshaw	Nov. 10, 1939
June 5, 1925	William Brosmith	Aug. 22, 1937
†	George B. Buck, Sr.	Apr. 12, 1961
†	William A. Budlong	June 4, 1934
Nov. 18, 1932	Charles H. Burhans	June 15, 1942
Apr. 20, 1917	William H. Burhop	Oct. 11, 1963
Feb. 19, 1915	F. Highlands Burns	Mar. 30, 1935
†	Edmund E. Cammack	Dec. 17, 1958
Nov. 21, 1930	Thomas O. Carlson	July 15, 1964
†	Raymond V. Carpenter	Mar. 11, 1947
Feb. 19, 1915	Gorden Case	Feb. 4, 1920
Oct. 27, 1916	Edmund S. Cogswell	Apr. 25, 1957
Nov. 23, 1928	Walter P. Comstock	May 11, 1951
Nov. 22, 1934	William J. Constable	Apr. 19, 1959
†	Charles T. Conway	July 23, 1921
†	John A. Copeland	June 12, 1953
†	Walter G. Cowles	May 30, 1942
†	James D. Craig	May 27, 1940
†	James McIntosh Craig	Jan. 20, 1922
Nov. 20, 1964	Robert A. Craig	Feb. 8, 1965
May 26, 1916	Frederick S. Crum	Sept. 2, 1921
†	Alfred Burnett Dawson	June 21, 1931
†	Miles Menander Dawson	Mar. 27, 1942
†	Elmer H. Dearth	Mar. 26, 1947
†	Eckford C. DeKay	July 31, 1951
May 19, 1915	Samuel Deutschberger	Jan. 18, 1929
†	Ezekiel Hinton Downey	July 9, 1922
May 19, 1915	Earl O. Dunlap	July 5, 1944
†	David Parks Fackler	Oct. 30, 1924
†	Edward B. Fackler	Jan. 8, 1952
Feb. 19, 1915	Claude W. Fellows	July 15, 1938
†	Benedict D. Flynn	Aug. 22, 1944
Feb. 19, 1915	Richard Fondiller	Apr. 29, 1962
†	Charles S. Forbes	Oct. 2, 1943
May 26, 1916	Lee K. Frankel	July 25, 1931
†	Charles H. Franklin	May 1951
Feb. 25, 1916	Joseph Froggatt	Sept. 28, 1940
†	Harry Furze	Dec. 26, 1945
Feb. 19, 1915	Fred S. Garrison	Nov. 14, 1949
†	Theodore E. Gaty	Aug. 22, 1925
May 19, 1915	James W. Glover	July 15, 1941
Oct. 22, 1915	George Graham	Apr. 15, 1937
Oct. 22, 1915	Thompson B. Graham	July 24, 1946
†	William J. Graham	Feb. 11, 1963

Admitted		Died
May 25, 1923	William A. Granville	Feb. 4, 1943
†	Winfield W. Greene	Mar. 26, 1965
†	William H. Gould	Oct. 28, 1936
†	Robert Cowen Lees Hamilton	Nov. 15, 1941
†	H. Pierson Hammond	Apr. 10, 1963
Oct. 27, 1916	Edward R. Hardy	June 29, 1951
Oct. 22, 1915	Leonard W. Hatch	Nov. 23, 1958
Nov. 21, 1919	Robert Henderson	Feb. 16, 1942
†	Robert J. Hillas	May 17, 1940
Nov. 15, 1918	Frank Webster Hinsdale	Mar. 18, 1932
May 23, 1924	Clarence W. Hobbs	July 21, 1944
Nov. 19, 1926	Charles E. Hodges	Jan. 22, 1937
Oct. 22, 1915	Lemuel G. Hodgkins	Dec. 26, 1951
†	Frederick L. Hoffman	Feb. 23, 1946
Oct. 22, 1915	Charles H. Holland	Dec. 28, 1951
Nov. 21, 1919	Carl Hookstadt	Mar. 10, 1924
Nov. 18, 1932	Solomon S. Huebner	July 17, 1964
†	Charles Hughes	Aug. 27, 1948
Nov. 19, 1929	Robert S. Hull	Nov. 30, 1947
†	Burritt A. Hunt	Sept. 3, 1943
†	Arthur Hunter	Jan. 27, 1964
Nov. 28, 1921	William Anderson Hutcheson	Nov. 19, 1942
Feb. 25, 1916	Charles William Jackson	Sept. 21, 1959
Nov. 19, 1929	Henry Hollister Jackson	May 27, 1955
May 19, 1915	William C. Johnson	Oct. 7, 1943
Nov. 23, 1928	F. Robertson Jones	Dec. 26, 1941
Nov. 18, 1921	Thomas P. Kearney	Feb. 11, 1928
Nov. 19, 1926	Gregory Cook Kelly	Sept. 11, 1948
Oct. 22, 1915	Virgil Morrison Kime	Oct. 15, 1918
†	Edwin W. Kopf	Aug. 3, 1933
Nov. 23, 1928	Clarence Arthur Kulp	Aug. 20, 1957
Feb. 17, 1915	John M. Laird	June 20, 1942
Nov. 13, 1931	Stewart M. LaMont	Aug. 22, 1960
Feb. 19, 1915	Abb Landis	Dec. 9, 1937
Nov. 24, 1933	John Robert Lange	Apr. 12, 1957
Nov. 17, 1922	Arnette Roy Lawrence	Dec. 1, 1942
†	James R. Leal, Sr.	Dec. 26, 1957
†	William Leslie	Dec. 12, 1962
Nov. 18, 1921	James Fulton Little	Aug. 11, 1938
Nov. 23, 1928	Edward C. Lunt	Jan. 13, 1941
Feb. 19, 1915	Harry Lubin	Dec. 20, 1920
†	William N. Magoun	Dec. 11, 1954
Nov. 16, 1923	D. Ralph McClurg	Apr. 27, 1947
May 23, 1919	Alfred McDougald	July 28, 1944
Oct. 31, 1917	Robert J. McManus	Aug. 15, 1960
Feb. 15, 1915	Franklin B. Mead	Nov. 29, 1933
Apr. 20, 1917	Marcus Meltzer	Mar. 27, 1931
†	David W. Miller	Jan. 18, 1936
†	Samuel Milligan	Aug. 8, 1965
†	James F. Mitchell	Feb. 9, 1941

Admitted		*Died*
†	Henry Moir	June 8, 1937
Nov. 18, 1921	Victor Montgomery	May 2, 1960
Feb. 19, 1915	William J. Montgomery	Aug. 20, 1915
Nov. 19, 1926	William L. Mooney	Oct. 21, 1948
†	George D. Moore	Mar. 11, 1959
May 19, 1915	Edward Bontecou Morris	Dec. 19, 1929
†	Albert H. Mowbray	Jan. 7, 1949
†	Frank Mullaney	Jan. 22, 1953
May 28, 1920	Ray D. Murphy	Feb. 24, 1964
†	Lewis A. Nicholas	Apr. 21, 1940
†	Edward Olifiers	May 13, 1962
†	Stanley L. Otis	Oct. 12, 1937
Nov. 13, 1926	Bertrand A. Page	July 30, 1941
Nov. 18, 1921	Sanford B. Perkins	Sept. 16, 1945
Nov. 15, 1918	William Thomas Perry	Oct. 25, 1940
Nov. 21, 1930	Francis S. Perryman	Nov. 30, 1959
†	Edward B. Phelps	July 24, 1915
Nov. 19, 1926	Jesse S. Phillips	Nov. 6, 1954
†	Charles Grant Reiter	July 30, 1937
†	Charles H. Remington	Mar. 21, 1938
May 23, 1919	Frederick Richardson	July 22, 1955
Nov. 19, 1926	Otto C. Richter	Feb. 17, 1962
Nov. 16, 1923	William F. Roeber	Mar. 21, 1960
Nov. 17, 1943	Samuel M. Ross	July 24, 1951
†	Isaac M. Rubinow	Sept. 1, 1936
†	Harwood Eldridge Ryan	Nov. 2, 1930
†	Arthur F. Saxton	Feb. 26, 1927
†	Emil Scheitlin	May 2, 1946
†	Leon S. Senior	Feb. 3, 1940
Nov. 24, 1933	Robert V. Sinnott	Dec. 15, 1952
Apr. 20, 1917	Charles Gordon Smith	June 22, 1938
Nov. 18, 1927	Edward C. Stone	June 6, 1964
Feb. 19, 1915	John T. Stone	May 9, 1920
Feb. 25, 1916	Wendell Melville Strong	Mar. 30, 1942
Oct. 22, 1915	William R. Strong	Jan. 10, 1946
†	Robert J. Sullivan	July 19, 1934
Nov. 17, 1920	Thomas F. Tarbell	July 2, 1958
Nov. 22, 1934	Walter H. Thompson	May 25, 1935
Nov. 18, 1921	Guido Toja	Feb. 28, 1933
†	John L. Train	June 12, 1958
Nov. 17, 1922	Antonio Thomas Traversi	Apr. 20, 1961
Nov. 19, 1948	Paul A. Turner	Jan. 30, 1961
Nov. 15, 1935	Harry V. Waite	Aug. 14, 1951
Nov. 18, 1925	Lloyd A. H. Warren	Sept. 30, 1949
May 23, 1919	Archibald A. Welch	May 8, 1945
Nov. 19, 1926	Roy A. Wheeler	Aug. 26, 1932
†	Albert W. Whitney	July 27, 1943
†	Lee J. Wolfe	Apr. 28, 1949
†	S. Herbert Wolfe	Dec. 31, 1927
May 24, 1921	Arthur B. Wood	June 14, 1952
†	Joseph H. Woodward	May 15, 1928
†	William Young	Oct. 23, 1927

Admitted		Died
May 23, 1924	Milton Acker	Aug. 16, 1956
Nov. 15, 1918	Robert E. Ankers	Mar. 1, 1964
Oct. 22, 1915	Don A. Baxter	Feb. 10, 1920
Nov. 17, 1920	Nellas C. Black	Dec. 24, 1962
Nov. 15, 1940	John M. Blackhall	Nov. 14, 1957
Nov. 15, 1918	Helmuth G. Brunnquell	June 3, 1958
Oct. 22, 1915	Louis Buffler	July 19, 1963
Nov. 17, 1922	Leo D. Cavanaugh	July 18, 1965
May 25, 1923	Harilaus E. Economidy	Apr. 13, 1948
Nov. 20, 1924	John Froberg	Oct. 11, 1949
Nov. 19, 1929	Maurice L. Furnivall	June 16, 1962
Nov. 22, 1934	John J. Gately	Nov. 3, 1943
Nov. 14, 1947	Harold J. George	Apr. 1, 1952
Nov. 19, 1929	Harold R. Gordon	July 8, 1948
Nov. 18, 1921	Robert E. Haggard	July 26, 1958
Nov. 20, 1924	Leslie LeVant Hall	Mar. 8, 1931
Nov. 17, 1927	Grady Hayne Hipp	June 25, 1965
Oct. 31, 1917	Edward T. Jackson	May 8, 1939
Nov. 17, 1922	Rosswel A. McIver	Apr. 1, 1959
Nov. 21, 1919	Rolland V. Mothersill	July 25, 1949
Nov. 19, 1929	Fritz Muller	Apr. 27, 1945
Nov. 23, 1928	Karl Newhall	Oct. 24, 1944
Nov. 15, 1918	John L. Sibley	Mar. 10, 1957
Nov. 18, 1921	Arthur G. Smith	May 2, 1956
Nov. 19, 1926	William F. Somerville	Nov. 12, 1965
Nov. 18, 1927	Alexander A. Speers	June 25, 1941
Mar. 23, 1921	Arthur E. Thompson	Jan. 17, 1944
Nov. 21, 1919	Walter G. Voogt	May 8, 1937
May 23, 1919	Charles S. Warren	May 1, 1952
Nov. 18, 1925	James H. Washburn	Aug. 19, 1946
Nov. 17, 1920	James J. Watson	Feb. 23, 1937
Nov. 18, 1921	Eugene R. Welch	Jan. 17, 1945
Nov. 16, 1951	Michael T. Wermel	Feb. 6, 1962
Mar. 21, 1929	Charles A. Wheeler	July 2, 1956
Nov. 15, 1918	Albert Edward Wilkinson	June 11, 1930
Oct. 22, 1915	Charles E. Woodman	Dec. 16, 1955

SCHEDULE OF MEMBERSHIP, NOVEMBER 16, 1965

	Fellows	Associates	Total
Membership, November 20, 1964	213	184	397
Additions:			
By Election	...	...	...
By Reinstatement	...	...	...
By Examination	9	18	27
	222	202	424
Deductions:			
By Death	4	3	7
By Withdrawal	...	...	...
By Transfer from Associate to Fellow	...	9	9
	218	190	408

(As Amended November 16, 1962)

Article I:—*Name.*

This organization shall be called the Casualty Actuarial Society.

Article II.—*Object.*

The object of the Society shall be the promotion of actuarial and statistical science as applied to the problems of insurance, other than life insurance, by means of personal intercourse, the presentation and discussion of appropriate papers, the collection of a library and such other means as may be found desirable.

The Society shall take no partisan attitude, by resolution or otherwise, upon any question relating to insurance.

Article III.—*Membership.*

The membership of the Society shall be composed of two classes, Fellows and Associates. Fellows only shall be eligible to office or have the right to vote.

The Fellows of the Society shall be the present Fellows and those who may be duly admitted to Fellowship as hereinafter provided. The Associates shall be the present Associates and those who may be duly admitted to Associateship as hereinafter provided.

Any person may, upon nomination to the Council by two Fellows of the Society and approval by the Council of such nomination with not more than two negative votes, become enrolled as an Associate of the Society, provided that he shall pass such examination as the Council may prescribe.

Any person who shall have qualified for Associateship may become a Fellow on passing such final examination as the Council may prescribe. Otherwise, no one shall be admitted as a member unless recommended by a duly called meeting of the Council with not more than two negative votes in a secret ballot, followed by at least a three-fourths secret ballot of the Fellows present and voting at a meeting of the Society.

Article IV.—*Officers and Council.*

The officers of the Society shall be a President, two Vice-Presidents, a Secretary-Treasurer, an Editor, a Librarian, and a General Chairman of the Examination Committee. The Council shall be composed of the active officers, nine other Fellows and, during the four years following the expiration of their terms of office, the ex-Presidents and ex-Vice-Presidents. The Council shall fill vacancies occasioned by death or resignation of any officer or other member of the Council, such appointees to serve until the next annual meeting of the Society.

Article V.—*Election of Officers and Council.*

The President, Vice-Presidents, and the Secretary-Treasurer shall be elected by a majority ballot at the annual meeting for the term of one year and three members of the Council shall, in a similar manner, be annually elected to serve for three years. The President and Vice-Presidents shall not be eligible for the same office for more than two consecutive years nor shall any retiring member of the Council be eligible for re-election at the same meeting.

The Editor, the Librarian and the General Chairman of the Examination Committee shall be elected annually by the Council at the Council meeting preceding the annual meeting of the Society. They shall be subject to confirmation by majority ballot of the Society at the annual meeting.

The terms of the officers shall begin at the close of the meeting at which they are elected except that the retiring Editor shall retain the powers and duties of office so long as may be necessary to complete the then current issue of *Proceedings*.

ARTICLE VI.—*Duties of Officers and Council.*

The duties of the officers shall be such as usually appertain to their respective offices or may be specified in the by-laws. The duties of the Council shall be to pass upon candidates for membership, to decide upon papers offered for reading at the meetings, to supervise the examination of candidates and prescribe fees therefor, to call meetings, and in general, through the appointment of committees and otherwise, to manage the affairs of the Society.

ARTICLE VII.—*Meetings.*

There shall be an annual meeting of the Society on such date in the month of November as may be fixed by the Council in each year, but other meetings may be called by the Council from time to time and shall be called by the President at any time upon the written request of ten Fellows. At least two weeks notice of all meetings shall be given by the Secretary.

ARTICLE VIII.—*Quorum.*

Seven members of the Council shall constitute a quorum. Twenty Fellows of the Society shall constitute a quorum.

ARTICLE IX.—*Expulsion or Suspension of Members.*

Except for non-payment of dues, no member of the Society shall be expelled or suspended save upon action by the Council with not more than three negative votes followed by a three-fourths ballot of the Fellows present and voting at a meeting of the Society.

ARTICLE X.—*Amendments.*

This constitution may be amended by an affirmative vote of two-thirds of the Fellows present at any meeting held at least one month after notice of such proposed amendment shall have sent to each Fellow by the Secretary.

BY-LAWS

(AS AMENDED NOVEMBER 20, 1964)

ARTICLE I.— *Order of Business.*

At a meeting of the Society the following order of business shall be observed unless the Society votes otherwise for the time being:

1. Calling of the roll.
2. Address or remarks by the President.
3. Minutes of the last meeting.
4. Report by the Council on business transacted by it since the last meeting of the Society.
5. New Membership.
6. Reports of officers and committees.
7. Election of officers and Council (at annual meetings only).
8. Unfinished business.
9. New business.
10. Reading of papers.
11. Discussion of papers.

ARTICLE II.—*Council Meetings.*

Meetings of the Council shall be called whenever the President or three members of the Council so request, but not without sending notice to each member of the Council seven or more days before the time appointed. Such notice shall state the objects intended to be brought before the meeting, and should other matter be passed upon, any member of the Council shall have the right to re-open the question at the next meeting.

ARTICLE III.—*Duties of Officers.*

The President, or, in his absence, one of the Vice-Presidents, shall preside at meetings of the Society and of the Council. At the Society meetings the presiding officer shall vote only in case of a tie, but at the Council meetings he may vote in all cases.

The Secretary-Treasurer shall keep a full and accurate record of the proceedings at the meetings of the Society and of the Council, send out calls for the said meetings, and, with the approval of the President and Council, carry on the correspondence of the Society. Subject to the direction of the Council, he shall have immediate charge of the office and archives of the Society.

The Secretary-Treasurer shall also send out calls for annual dues and acknowledge receipt of same; pay all bills approved by the President for expenditures authorized by the Council of the Society; keep a detailed account of all receipts and expenditures, and present an abstract of the same at the annual meetings, after it has been audited by a committee appointed by the President.

The Editor shall, under the general supervision of the Council, have charge of all matters connected with editing and printing the Society's publications. The *Proceedings* shall contain only the proceedings of the meetings, original papers or reviews written by members, discussions on said papers and other matter expressly authorized by the Council.

The Librarian shall, under the general supervision of the Council, have charge of the books, pamphlets, manuscripts and other literary or scientific material collected by the Society.

The General Chairman of the Examination Committee, shall, under the general supervision of the Council, have charge of the examination system and of the examinations held by the Society for the admission to the grades of Associate and of Fellow.

ARTICLE IV.—*Dues.*

The Council shall fix the annual dues for Fellows and Associates. Effective November 20, 1964, the payment of dues will be waived in the case of any Fellow or Associate who attains the age of 70 years or who attains the age of 65 years and notifies the Secretary-Treasurer in writing that he has retired from active work. Fellows and Associates who have become totally disabled while members may upon approval of the Council be exempted from the payment of dues during the period of disability.

It shall be the duty of the Secretary-Treasurer to notify by mail any Fellow or Associate whose dues may be six months in arrears, and to accompany such notice by a copy of this article. If such Fellow or Associate shall fail to pay his dues within three months from the date of mailing such notice, his name shall be stricken from the rolls, and he shall thereupon cease to be a Fellow or Associate of the Society. He may, however, be reinstated by vote of the Council upon payment of arrears in dues, which shall in no event exceed two years.

ARTICLE V.—*Designation by Initials.*

Fellows of the Society are authorized to append to their names the initials F.C.A.S.; and Associates are authorized to append to their names the initials A.C.A.S.

ARTICLE VI.—*Amendments.*

These by-laws may be amended by an affirmative vote of two-thirds of the Fellows present at any meeting held at least one month after notice of the proposed amendment shall have been sent to each Fellow by the Secretary.

GUIDES TO PROFESSIONAL CONDUCT

(As amended November 20, 1959)

In order to assist the Council of the Society in resolving questions that might be raised as to the professional conduct of members, and more importantly to guide members of the Society when they encounter questions of professional conduct, the following "Guides to Professional Conduct" have been prepared by order of the Council. The actuary has professional responsibilities to society at large, to his client or employer, and to his professional associates. As is true of codes of ethics generally, these guides deal with precepts and principles only. They are not precise rules and are subject to interpretations in relation to the variety of circumstances that occur in practice.

Any member wishing advice on the application of these guides to a particular set of facts is urged to present his case to the Council of the Society. The Council has the power to consider and take action with respect to questions that may be raised as to the professional conduct of members. Any disciplinary action by the Council must be in accord with Article IX of the Constitution.

The Council assumes that every member of the Society earnestly desires to serve his client or employer properly, to protect the public, and to maintain the prestige of the Society and its members. Accordingly, the Council sets forth the following principles by which, in its opinion, every member should be guided in his practice of the actuarial profession.

1. The member will promote a wider understanding of the significance of membership in the Society and will maintain the high standards of the Society by avoiding even the appearance of any questionable practice.

2. The member will conduct his professional competition on a high plane. He will avoid unjustifiable or improper criticism of others and will recognize that there is substantial room for honest differences of opinion on many matters.

3. The member will act in professional matters for each client or employer with scrupulous attention to the trust and confidence that the relationship implies and will have due regard for the confidential nature of his work.

4. The member will bear in mind that the actuary acts as an expert when he gives professional advice, and he will give such advice only when he is qualified to do so.

5. The member will not provide actuarial service for, or associate professionally with, any person or organization if he has reason to believe that the results of such service or association are likely to be used in a manner inimical to the public interest or the interests of the actuarial profession or to evade the law.

6. The member will submit unqualifiedly an actuarial calculation, certificate, or report only if he knows it to be based on sufficiently reliable data and on actuarial assumptions and methods that, in his judgment, are consistent with the sound principles expounded in the course of study of the Society, or in recognized texts, sources or precedents relevant to the subject at hand.

7. The member will recommend for the use of his client or employer, premium rates, rating plans, dividends or other related actuarial functions only if, in his opinion, they are based on adequate and appropriate assumptions and methods.

8. The member will not make or sponsor any actuarial calculation, certificate, statement, report, or comparison, or give any testimony or interview on such matters, which he has reason to believe is false, materially incomplete, or misleading.

9. Where appropriate for the objective use of a certificate or report, or in any event on the request of his employer or client, the member will include a statement of the principal actuarial assumptions and the general methods adopted for his computations.

10. The member will recognize his ethical responsibilities to the person or organization whose actions may be influenced by his professional opinions or findings. When it is not feasible for the member to render his opinions or findings direct to such person or organization, he will act in such manner as to leave no doubt that the member is the source of the opinions or findings and to indicate clearly the personal availability of the member to provide supplemental advice and explanation.

11. The member will not serve more than one client or employer where a conflict of his professional interest may be involved unless there be a full disclosure to all parties concerned, and such parties request and acquiesce in the engagement of his services.

12. The member will sign actuarial recommendations, certificates, and reports if he be acting as an employe, only over a title conferred by his employer if any title is used. Nevertheless, in any capacity, the member may append to his signature the designation: "Fellow of the Casualty Actuarial Society" or "FCAS," or "Associate of the Casualty Actuarial Society" or "ACAS," as the case may be. The member will not use as a signature title the designation "Member of the Casualty Actuarial Society". The member will use a designation dependent upon elective or appointive qualification within the Society such as "President," or "Member of the Council," only when he is acting in such capacity on behalf of the Society.

13. The member will recognize his personal responsibilities under these guides whether he acts as an individual or through a partnership or his employer.

(AS AMENDED DECEMBER 1, 1965)

Method of Review. All papers and reviews of papers are reviewed by the Committee on Review of Papers. The Committee consists of members appointed by the President, plus, ex officio, the Editor of the *Proceedings.* Unanimous vote of the regular Committee is necessary for acceptance of a paper or a review, except that if there is only one vote for rejection, the paper or review will be reviewed by the Editor and accepted if he approves.

Scope and Standards.—1. Broad latitude will be allowed in the choice of a subject, provided it is a subject of interest to property and casualty actuaries. However, it must be clearly suitable for inclusion in the *Proceedings.*

2. The paper must contain original ideas or new material of reasonable value, unless it has a definite educational value for other reasons.

3. When a paper includes material that the Committee finds it is not qualified to review, the Committee will seek advice or opinion from other members of the Society or from recognized experts outside of the Society.

4. Disagreement by the Committee with opinions of the author or reviewer of a paper will not be a bar to acceptance of an otherwise suitable paper or review. Where, however, the Committee believes a paper or review to be fallacious in logic or misleading in matters of fact the Committee may reject it. Reviews of papers are expected to be free of criticism of a personal nature. Opportunity will be given to the authors of papers to respond to reviews. Authors' replies will also be reviewed by the Committee and will be treated in the same manner as reviews.

5. The paper or review should show care in preparation. A reasonable minimum standard will be required as to form, clarity, and literary quality. When a paper or review, otherwise acceptable, does not meet these standards, the Committee may return it to the author or reviewer and invite resubmission after editing or rewriting. The Committee may also make suggestions to the author as to possible improvements in an accepted paper.

6. Papers and reviews should be kept within the general limits of length indicated by past acceptances, ordinarily about twenty printed pages for papers and two or three pages for reviews.

Procedures and Regulations.—1. Papers may be submitted only by Fellows or Associates of the Casualty Actuarial Society, except that papers may be submitted by non-members of the Society upon invitation of the President. A member may collaborate in joint authorship with a non-member who possesses particular qualifications in respect to the subject of a paper.

2. Papers and reviews of papers should be submitted in quintuplicate to the Secretary-Treasurer of the Society. The Secretary-Treasurer is authorized to return to the author or reviewer copies of a paper or a review that in his opinion are not legible.

3. The name of the author should not appear on the copies of the paper submitted to the Secretary-Treasurer but should be included in the covering letter. However, names of the reviewers should be identified on the copy of the review.

4. In submitting a paper, the author must answer the following questions on a separate sheet attached to each of the five copies of the paper:

(a) Name of paper.

(b) Has the paper been published elsewhere, in whole or in part, in identical or similar form?

(c) Is the paper being simultaneously submitted elsewhere, or will it be so submitted before decision by the Committtee on Review of Papers?

(d) In the case of co-authorship with a non-member, to what extent has the Society member contributed?

(e) If the paper contains factual data from some organization, has the organization given the author permission to publish it?

5. Papers and reviews should be typed double-spaced on letter-size stationery, on one side of each sheet. Tables and footnotes may be single-spaced. Pages should be numbered. Footnotes should be numbered consecutively throughout the paper.

6. Major captions should be centered and typed in capitals; subcaptions should appear in the left-hand margin in italics (single underscore). In technical papers paragraphs may be numbered to simplify reference; in non-technical papers paragraphs should not be numbered.

7. So far as possible, tables should be arranged so that they can be printed on a single page of the *Proceedings* without undue reduction in size of type. Column headings must be clear and concise.

8. *All mathematical formulas and symbols should be handwritten in ink* rather than typewritten. They must be legible especially as to subscripts and superscripts. There must be no possibility of confusion between, for instance, dx and d_x; $\times$ (the sign for multiplication) and x; a and α (alpha). The exclamation point (!) should be used to indicate factorials in binomial expansions. Where necessary, instructions to the printer may be inserted in pencil on the manuscript. The Committee strongly recommends that authors of mathematical papers refer to the Style Manual of the American Institute of Physics for precise information on preparation of a manuscript. A copy of the Style Manual may be borrowed from the Editor of the *Proceedings* or it may be purchased from the Editor for one dollar. When life contingency symbols are applicable the International Actuarial Notation should be used. This code is described in the *Proceedings*, Vol. XXVI, page 123.

9. References to books and periodicals and to proceedings of professional societies, should be sufficiently complete to permit obtaining a copy of the source without additional research.

10. If the manuscript has been prepared carefully in accordance with the foregoing suggestions, there should be only a few minor corrections necessary. The paper as originally submitted should not be considered simply as a draft to which extensive alterations can be made.

11. Authors will be notified of the acceptance or rejection of their papers by the Secretary-Treasurer. If a paper is rejected, original and copies will be returned. The Committee does not promise a decision on a paper submitted fewer than forty-five days prior to the meeting for which the paper has been prepared. Reviews of a paper are to be submited to the author and the Secretary-Treasurer thirty days in advance of the meeting at which the paper is to be reviewed. A review of a paper will be considered to have been accepted by the Committee unless the reviewer is otherwise notified.

12. Authors of accepted papers are requested to notify the Secretary-Treasurer whether or not they can supply additional copies for use at meetings or for further distribution prior to publication. (Photographic reproduction is less expensive than printing and insures accuracy.)

13. After acceptance of a paper and before its reproduction, the author should have the following statement typed at the bottom of the first page: "Presented at the (date) meeting of the Casualty Actuarial Society at (city and state). Reproduction in whole or in part without acknowledgment to the Casualty Actuarial Society is specifically prohibited."

14. Except on recommendation of the Committee, no accepted paper will be read in its entirety at a meeting of the Society. The author will be expected to prepare for oral presentation a two or three minute abstract, stating the purposes of his paper and its conclusions.

15. The Editor of the *Proceedings*, in consultation with the author or reviewer, may edit the paper or review prior to publication.

WOODWARD - FONDILLER PRIZE

This award made in commemoration of Joseph H. Woodward and Richard Fondiller is intended to stimulate original thinking and research and will be made to the best eligible paper each year submitted by an Associate or Fellow who has attained his designation within the last five years. To be eligible the paper must show evidence of ability for original research and the solution of advanced insurance problems. If no paper is considered eligible in a given year, the award shall not be made. Papers previously submitted to the Society or elsewhere, shall not be eligible.

The amount of the prize will be $200 and the papers will be judged by the Society's Committee on Review of Papers whose decision will be final.

The announcement of the award will be made at the November meeting each year, based on papers submitted to the Society at the previous November and May meetings.

RULES REGARDING EXAMINATIONS FOR ADMISSION TO THE CASUALTY ACTUARIAL SOCIETY

1. Dates of Examinations.

Examinations for all parts will be held in May each year in such cities as will be convenient. In addition, examinations for Associateship Parts 1 and 2 will also be held in November each year. The exact dates will be set by the Secretary-Treasurer.

2. Filing of Application.

A candidate who wishes to take Associateship examination Part 1 or Part 2, or both, must make application on the Society's application form, which may be obtained from the Secretary-Treasurer.

A candidate who has previously submitted his application on the Society's application form, and who wishes to take one or more examinations other than Associateship Parts 1 or 2, need not again make use of the Society's application form, but may simply write to the Secretary-Treasurer, stating the part or parts for which he is applying.

Each application must be accompanied by the appropriate examination fee, in check or money order payable to the Casualty Actuarial Society.

Applications must be received by the Secretary-Treasurer by April 1 for the Spring examinations and by October 1 for the Fall examinations.

3. Associateship and Fellowship Examinations.

There are four parts of the examinations which the candidate must pass in order to become an Associate of the Casualty Actuarial Society. These consist of five actual examinations:

Part 1	3 hours
Part 2	3 hours
Part 3 Section (a)	1½ hours
Part 3 Section (b)	1½ hours
Part 4 Sections (a) and (b)	3 hours

The examinations for Part 1—General Mathematics, and Part 2—Probability and Statistics, are jointly sponsored with the Society of Actuaries. Credit for passing these examinations will be given by both Societies regardless of the Society through which the candidate registers. One pass list showing the successful candidates (without identification as to the Society through which they register) will be published.

A candidate may write any one or more of the five examinations and will receive credit for those passed, except that Parts 1 and 2 must be taken in numerical order.

There are four examinations which a candidate must also pass to become a Fellow of the Casualty Actuarial Society. Each Fellowship Part consists of two sections, but is a single 3 hour examination. A candidate may present himself for one or more of the Fellowship examinations either if he has previously passed the Associateship examinations or if he concurrently presents himself for and submits papers for all unpassed Associateship examinations. Subject to the foregoing requirements, a candidate will be given credit for any examination which he may pass.

4. Fees.

The examination fee for the Associateship examination is $3.75 for a section, $7.50 for one complete part; subject to a minimum of $7.50 for each year in which the candidate presents himself. The examination fee for the Fellowship examination is $10.00 for each part. Examination fees are payable to the order of the Society and must be received by the Secretary-Treasurer before April 1 for the Spring examinations, or before October 1 for the Fall Associateship examinations.

5. Prize Awards.

The Casualty Actuarial Society and the Society of Actuaries jointly will award one $200 and four $100 prizes to the five successful undergraduates ranking highest in the General Mathematics examination. These prize awards will be granted twice each year, i.e., for both the Spring and Fall examinations.

6. Credit for Examination Parts under Former Syllabus.

A candidate who has passed, or been credited with, one or more of the Associateship or Fellowship examinations under the 1964 Syllabus will receive credit for the corresponding examinations of the 1966 Syllabus. Partial examinations will be given to those candidates requiring them in accordance with such credits, except that beginning with the 1966 examinations, no candidate will be permitted to write only a portion of Associateship Part 2, and any prior credit for one Section of this examination expires.

7. Waiver of Examinations for Associateship.

Waiver of the following Associateship examinations will be allowed for a candidate who has passed or been credited with the corresponding examinations of the Society of Actuaries:

Casualty Actuarial Society	Society of Actuaries
Part 1	Part 1
Part 2	Part 2
Part 3 (a)	Part 4

Candidates who take the Advanced Mathematics test of the Graduate Record examinations may apply for credit for the General Mathematics examination (Associateship Part 1). Credit will be granted if the candidate's score on the Graduate Record Advanced Mathematics test is equivalent, as determined by the Casualty Actuarial Society, to the passing score on the Society's General Mathematics examination. To be eligible for such credit the candidate must apply for credit within two years of the date on which he takes the Graduate Record Advanced Mathematics test. An application to the Casualty Actuarial Society for such credit may be completed either in advance of taking the Graduate Record Advanced Mathematics test or within the two years after taking it. The necessary application form may be secured from the Secretary-Treasurer of the Casualty Actuarial Society.

The Council may waive, subject to such other requirements as it may prescribe, any examinations of the Casualty Actuarial Society which it deems equivalent to examinations required by another recognized actuarial organization which have been passed by an applicant while not a resident of the United States or Canada, or during his first year of temporary or permanent residence in the United States or Canada.

8. **Recommendations for Study.**

To assist candidates in preparation for the examinations, Recommendations for Study have been prepared. The pamphlet lists the texts, readings, and technical material which must be mastered by the candidates. The references listed in the Recommendations for Study are contained in the Society's library. Copies of the Recommendations for Study may be obtained from the Secretary-Treasurer.

LIBRARY

All candidates registered for the examinations of the Casualty Actuarial Society and all members of the Casualty Actuarial Society have access to all the library facilities of the Insurance Society of New York, the Casualty Actuarial Society, and the Society of Actuaries. These libraries, with combined operations, are located at 150 William Street, New York, New York 10038.

Registered candidates may have access to the library by receiving from the Society's Secretary-Treasurer the necessary credentials. Books and manuals may be withdrawn from the library for a period of one month without charge. The Insurance Society is responsible for postage and insurance charges for sending books to out-of-town borrowers, and borrowers are responsible for the safe return of the books.

Address requests for books to:

Librarian
Insurance Society of New York
150 William Street
New York, New York 10038

SYLLABUS OF EXAMINATIONS

(Effective with 1966 Examinations)

ASSOCIATESHIP

Part	*Section*	*Subject*
I		General Mathematics.
II		Probability and Statistics.
III	(a)	Elementary Life Insurance Mathematics.
	(b)	General Principles of Insurance; Insurance Economics and Investments.
IV	(a)	Insurance Coverages and Policy Forms.
	(b)	General Principles of Ratemaking.

FELLOWSHIP

I	(a)	Insurance Law; Supervision, Regulation, and Taxation.
	(b)	Statutory Insurances.
II	(a)	Premium, Loss, and Expense Reserves.
	(b)	Insurance Accounting and Expense Analysis.
III	(a)	Individual Risk Rating.
	(b)	Problems in Underwriting and Administration.
IV	(a)	Insurance Statistics and Machine Methods.
	(b)	Advanced Problems in Ratemaking.

AMERICAN ACADEMY OF ACTUARIES

The American Academy of Actuaries was organized on October 25, 1965 as the culmination of efforts on the part of the four actuarial bodies of the United States—The Casualty Actuarial Society, The Conference of Actuaries in Public Practice, The Fraternal Actuarial Association, and The Society of Actuaries. The Academy is the vehicle which will lead eventually to the legal recognition of actuaries. Fellows of the Casualty Actuarial Society as of October 25, 1965 who are residents of the United States automatically became members of the Academy unless they submitted a written declination prior to December 24, 1965. Members of the Casualty Actuarial Society who did not automatically become members of the Academy on October 25, 1965, but who have had seven years of experience in responsible actuarial work, may become members of the Academy by submitting an application for membership. Applications may be obtained from the Secretary.

OFFICERS

President	HENRY F. ROOD*
President-Elect	THOMAS E. MURRIN*
Vice Presidents	FRANK J. GADIENT LAURENCE H. LONGLEY-COOK* JOHN H. MILLER* H. RAYMOND STRONG
Treasurer	ROBERT E. BRUCE
Secretary	GEORGE M. BRYCE *(Lincoln National Life Insurance Co. Fort Wayne, Indiana 46801)*

BOARD OF DIRECTORS

TERM OF 3 YEARS	TERM OF 2 YEARS	TERM OF 1 YEAR
WILLIAM E. GROVES	EDWARD D. BROWN, JR.	DONALD F. CAMPBELL
VICTOR E. HENNINGSEN	GEORGE B. BUCK, JR.	MARY M. CUSIC
WILLIAM LESLIE, JR.*	FRANK HARWAYNE*	GILBERT W. FITZHUGH*
DANIEL J. MCNAMARA*	ALLEN L. MAYERSON*	WALTER KLEM
WALTER L. RUGLAND	WENDELL A. MILLIMAN	NORTON E. MASTERSON*
ANDREW C. WEBSTER	JOSEPH MUSHER	ROBERT J. MYERS*

* Fellows or Associates of the Casualty Actuarial Society.
Messrs. Rood, Miller, and Fitzhugh are more closely affiliated with the Society of Actuaries.

Committees of the American Academy of Actuaries and their Casualty Actuarial Society representatives are:

Committee on Admissions, HAROLD W. SCHLOSS
Committee on Education, NORMAN J. BENNETT
Committee on Professional Conduct, THOMAS E. MURRIN
Committee on State Accreditation, DANIEL J. MCNAMARA

INTERNATIONAL CONGRESSES OF ACTUARIES

The first International Congress of Actuaries was held in 1895 in Brussels. Since that time numerous congresses have been held, and many actuaries from the United States and Canada have been benefited by attendance at the congresses and by the printed *Proceedings*, in which numerous valuable articles have appeared.

Continuity in the arrangements for successive congresses is achieved by the maintenance of the Comité Permanent des Congrès Internationaux d'Actuaires with headquarters in Brussels, membership of which is on an international basis.

Membership in the Permanent Committee on this continent is divided into two sections, a United States section and a Canadian section. Individual actuaries can support the work of the Permanent Committee by becoming members in their section. Inquiries regarding the Permanent Committee should be directed to Pearce Shepherd, Secretary for the United States Section, Prudential Insurance Company, Newark, New Jersey 07101, or to Donald M. Ellis, Secretary for the Canadian Section, Canada Life Assurance Co., Toronto, Ont.

According to the revised regulations adopted by the New York Congress in 1957, the objects of the Permanent Committee are:

1. To promote or to conduct work or research of interest to the science or practice of the Actuary. For this purpose sections formed by a number of members for study of special problems may be recognized. Each section will have its own regulations, previously approved by the Council; it will elect its Committee, except for the member appointed by the Council on the Committee.

2. To publish periodically a *Bulletin: (a)* bringing together technical, legislative, statistical, and juridical information relating to actuarial science; *(b)* reviewing publications and works which appear in various countries, bearing upon actuarial matters.

3. To cooperate with the Organizing Committees in preparing the work of International Congresses, and in the publication of their *Proceedings*.

An invitation to hold the next Congress, planned for 1968, in Germany, was extended at the 1964 Congress.

Membership on the Permanent Committee is one of the requirements for membership in a Congress. In 1964 the number of full members was limited to 800. A record of continuous membership will be a favorable factor in considering eligibility if it is necessary to impose limitations in the future. The annual dues for membership are 150 Belgian francs. The Permanent Committee wishes to enlist members as broadly as possible.

ASTIN Section

ASTIN (Actuarial Studies in Non-Life Insurance) is the first section of the Permanent Committee to be formed under the modification of the rules approved at the XVth International Congress in New York and is for the study of the application of modern statistical and mathematical methods in the field of non-life insurance. It has grown from the desire expressed by many members of the XIVth Congress held in Madrid to provide an effective interchange of ideas on an international basis.

It has as its object the promotion of actuarial research in general insurance and establishes contact between actuaries, groups of actuaries, and other suitably qualified persons interested in this field.

This section, from time to time, publishes papers on topics related to its objects and also publishes a *Bulletin* containing notes of general interest to members.

Meetings are held every four years, during the course of the International Congress of Actuaries. Between meetings colloquia are held on topics of interest to the Section, and these are hosted by national actuarial bodies. The 1966 Colloquium will be held in Arnhem, Netherlands, September 28 to October 1, sponsored by the Dutch Actuarial Institute.

The members of the Committee of ASTIN are:

Chairman	Ammeter, Hans—Switzerland
Vice Chairman	Masterson, Norton E.—U.S.A.
Treasurer	Thyrion, Paul—Belgium
Members	Johansen, Paul—Denmark
	Ottaviani, Guiseppe—Italy
	Sousselier, Jean—France
	Sternberg, Ingvar—Sweden
	Welten, C. P.—Netherlands
Secretary	Beard, Robert Eric—Great Britain

Membership fees, which are payable in the same manner as the annual dues for membership on the Permanent Committee, are 250 Belgian francs. Inquiries regarding membership in the ASTIN Section should be directed to Albert Z. Skelding, Secretary-Treasurer, Casualty Actuarial Society, 200 East 42nd Street, New York, N. Y. 10017.

FUTURE MEETINGS OF THE CASUALTY ACTUARIAL SOCIETY

1966 Spring Meeting —	May 22, 23, 24, 25 Cavalier Hotel Virginia Beach, Va.
1966 Annual Meeting —	November 14, 15, 16 Sheraton-Cadillac Hotel Detroit, Mich.
1967 Spring Meeting —	May 24, 25, 26 Pheasant Run Lodge St. Charles, Ill.
1967 Annual Meeting —	November 15, 16, 17 Hotel America Hartford, Conn.

1966 EXAMINATIONS

May 11, 12, 1966

November 9, 1966