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AN APPROXIMATION FOR THE TESTING OF PRIVATE PASSENGER LIABILITY TERRITORIAL RATE LEVELS USING STATEWIDE DISTRIBUTION OF CLASSIFICATION DATA

JAMES F. GILL

A previous paper[1] by Philipp K. Stern presented an excellent description of basic ratemaking procedures for Automobile Liability insurance. To utilize this procedure, it is necessary to have territorial exposures by classification. Independent companies writing a large volume of private passenger liability business could use this method to test their private passenger liability territorial rate levels as they would probably have a sufficient volume of data in the necessary detail. Many small independent companies, however, would not have a sufficient volume of their own data and probably would not have a readily available source of data in the detail required by the procedure described in Mr. Stern's paper. A method is needed by statisticians and others responsible for rate levels of small independent companies so they may more easily determine the adequacy of their rate levels.

The purpose of this paper is to explore the possibility of developing a method of testing private passenger liability territorial rate levels by substituting the statewide distribution of classification data for the actual distribution by rating territory. Such a method should be useful to small independent companies and could be used by company actuaries as a basis for estimating the adequacy of their rate levels.

DATA USED IN STUDY

The annual compilation of Automobile experience prepared by the National Association of Independent Insurers provided readily available data for such a study.

[1] Current Rate Making Procedures for Automobile Liability Insurance, *PCAS* XLIII, p. 112.

The NAII calls for Automobile private passenger liability experience require data to be reported by coverage and classification statewide and by coverage and territory for all classifications combined in all states except Louisiana, New Jersey, New York, North Carolina, Texas and Virginia. Private passenger data for these six states are reported by coverage and classification for each territory.

In order to determine the effect of substituting a statewide distribution for the actual territory distribution, statewide and territorial rate level changes were developed for the states of Lousiana, New Jersey, North Carolina and Virginia. The indicated rate level changes for each state were determined both on the basis of a statewide distribution of classification data and classification data by territory. Indicated rate level changes were also determined for the states of Alabama, Arkansas, Florida, Illinois, Ohio and Washington on the basis of each state's distribution of classification data statewide.

It was decided to use only calendar/accident year[2] data and to test the results. The NAII experience data prior to 1958 was compiled on a policy year basis so that calendar/accident year experience was only available for the years 1958, 1959, 1960 and 1961. 1961 rate levels would ordinarily be developed in 1960 from experience data for the years 1957, 1958 and 1959 as this three-year period would be available for review in 1960. As calendar/accident year 1961[3] was to be used for testing the rate levels, the experience data to develop the rate levels was limited to calendar/accident years 1958[4] and 1959.[4] The three-year period 1958, 1959 and 1960 would have been used if calendar/accident year 1962 had been available to test the rate levels. It is suggested that if the statewide distribution method is used, a three-year period should be considered as the longer period should provide a more stable base.

DEVELOPMENT OF EARNED PREMIUM AT PRESENT RATES

Earned premium at present rate level is required in order to determine both the statewide rate level changes and the territorial rate level changes. To compute the earned premium, rates for basic limits are required. The National Bureau of Casualty Underwriters rates were used as they were readily available. However, an independent company's rates or another rating bureau's rates could have been used. The developed rates

[2] Exposures and premium compiled on a calendar year basis. Incurred losses, number of incurred losses and allocated adjustment expense compiled by year of accident.

[3] Incurred losses valued as of March 31, 1962.

[4] Incurred losses valued as of March 31, 1960.

and loss ratios for 1961 would have been substantially the same regardless of the basic limits rates used, provided the rate class differentials were the same. If a different set of basic limits rates had been used, they would have produced different rate level changes by state and territory, however, the application of these rate level changes to the rate schedule utilized should produce approximately the same revised rates as the method actually used. The earned premium was calculated by multiplying N.B.C.U. private passenger car basic limits rates in effect in December of 1960 by the 1958 and 1959 earned car year exposures within each territory for each private passenger classification based on calendar/accident year 1959 distribution by class in each territory. The December 1960 rates were used on the assumption that 1958 and 1959 data would not be available until late in the fourth quarter of 1960.

RATE LEVEL CHANGES

To develop the statewide rate level changes for each of the ten states used in the study, factors were used to include unallocated loss adjustment expenses, to adjust losses to reflect changes in average paid claim costs subsequent to the experience period and to reflect the development of the incurred losses to 63 months for bodily injury and 39 months for property damage. The factors used in this study have been utilized in past rate revisions. However, their accuracy is not important for the purpose of this study as the prime interest at this time is to determine the degree of relationship between rates determined on the basis of the actual class distribution by territory and rates developed by substituting a statewide class distribution for the actual territory distribution by class. The losses used to determine the territorial rate level changes were adjusted to include the unallocated loss adjustment expenses using factors of 1.10 for bodily injury and 1.16 for property damage. The incurred losses including all adjustment expenses used to compute statewide rate level changes were developed to 63 months for bodily injury and 39 months for property damage by application of the following development factors:

Calendar/Accident Year	Bodily Injury	Property Damage
1958	.974	.989
1959	1.006	.962

The calendar/accident year weights, expected loss and loss adjustment ratios, loss trend factors and limits used for each state are as follows:

State	Calendar/ Accident Year Weight		Expected Loss and Loss Adjustment Ratio	Loss Trend Factor		Limits
	1958	1959		B. I.	P. D.	
Alabama	.15	.85	.645	1.051	1.104	5/10/5
Arkansas	.15	.85	.630	1.035	1.053	10/20/5
Florida	—	1.00	.630	1.072	1.020	10/20/5
Illinois	—	1.00	.655	1.029	1.040	10/20/5
Louisiana	.15	.85	.651	1.076	1.045	5/10/5
New Jersey	—	1.00	.636	1.014	1.065	10/20/5
North Carolina	.15	.85	.650	1.119	1.110	5/10/5
Ohio	—	1.00	.625	1.040	1.028	10/20/5
Virginia	.15	.85	.646	1.036	1.087	10/20/5
Washington	—	1.00	.655	1.107	1.054	10/20/5

The various calculations necessary for developing Louisiana statewide and territorial rate levels using classification data by territory are set forth in Exhibits I and II.

Exhibit I shows the development of the following statewide bodily injury and property damage rate level changes:

Bodily Injury	+ 9.1%
Property Damage	— 13.5
Total	— 0.1%

DEVELOPMENT OF 1961 RATES

Exhibit II shows the development of the bodily injury rate level changes by territory. The indicated 1A bodily injury and property damage[5] rates were used to determine rates for the other private passenger classifications by application of the following differentials:

[5] Property damage rates were developed using the procedure shown in Exhibit II for bodily injury.

Differentials to Class IA

Class	Rural and Small City Territories	Large City Territories
1A	1.00	1.00
1B	1.00	1.10
1C	1.45	1.45
2A	1.90	1.90
2C	3.60	3.10
3	1.50	1.50
1AF	.70	.70
2AF	1.33	1.33
2CF	2.52	2.17

The differentials for Classes 1A, 1AF, 1B, 1C and 3 were reduced 20% to reflect the discount for "two or more cars". The differentials were also reduced 10% to reflect the discount for compact cars.[6] All rates were rounded to the nearest dollar except for those states subject to the discount.

Louisiana Statewide Distribution by Class
1959 Calendar/Accident year

Class	Earned Car Years	Percentage
1A	42,811	32.8
1B	53,680	41.1
1C	6,272	4.8
2A	9,702	7.4
2C	1,087	.8
3	4,376	3.3
1AF	10,834	8.3
2AF	1,656	1.3
2CF	270	.2
Total	130,688	100.0

1A rates were also developed for Louisiana using the same procedure except that calendar/accident year 1959 statewide classification distribu-

[6] The compact car discount is not applicable in Louisiana and North Carolina.

tion percentages shown were multiplied by the total 1958 and 1959 earned car year exposure to determine the earned car year exposure by classification within each territory. The assumed earned car year exposures by class and territory were then multiplied by the December 1960 N.B.C.U. basic limits rates to develop earned premium at present rates. The indicated statewide rate level changes for Louisiana on the basis of the statewide distribution by class were as follows:

Bodily Injury	+ 9.4%
Property Damage	— 13.2
Total	+ 0.2%

The effect of using the statewide distribution by class is reflected by an increase of 0.3% for bodily injury and property damage combined.

TESTING OF RATES

The rates developed on both bases were used to determine the earned premium for calendar/accident year 1961. The premium was calculated by multiplying the rates by the earned exposures for each classification within each territory. The 1961 calendar/accident year basic limits incurred losses were adjusted to include unallocated loss adjustment expenses by use of the factor of 1.10 for bodily injury and 1.16 for property damage. The adjusted basic limits losses for calendar/accident year 1961 were then divided by the earned premium to obtain territorial loss ratios on the basis of both sets of rates. The loss ratios were as follows:

Louisiana – Calendar/Accident Year 1961

Territory	Loss and Loss Adjustment Ratio			
	Bodily Injury		Property Damage	
	(a)ø	(b)ø	(a)ø	(b)ø
(01) Jefferson, Orleans, Plaquemines and St. Bernard Parishes	.625	.625	.671	.671
(02) East Baton Rouge and West Baton Rouge Parishes	.632	.616	.631	.601
(03) Bossier, Caddo and De Soto Parishes	.781	.781	.786	.786
(04) Ouachita Parish	.745	.745	.830	.830
(05) Calcasieu Parish	.881	.881	.590	.590
(06) Lafayette Parish	.549	.549	.560	.560
(07) Rapides Parish	.563	.563	.672	.672
(08) Remainder of State	.599	.620	.699	.699
Entire State	.643	.648	.682	.678

(a)ø – Loss ratios computed using earned premiums developed from the actual territorial classification distribution.

(b)ø – Loss ratios computed using earned premiums developed from the statewide classification distribution.

Inspection will indicate a high degree of correlation. Using the standard correlation coefficient formula, the coefficients for Louisiana are as follows:

Bodily Injury	.9964
Property Damage	.9988

The rates developed for the states of New Jersey, North Carolina and Virginia on both a statewide and territorial distribution basis were tested using the same procedure as was used for Louisiana. The correlation coefficients were calculated from the loss ratios developed on both distributions and are as follows:

	Bodily Injury	Property Damage
New Jersey	.9939	.9693
North Carolina	.9999	.9999
Virginia	1.0000	.9726

COMPARISON OF 1961 RATIOS DEVELOPED FROM THE STATEWIDE DISTRIBUTION METHOD AND N.B.C.U. RATES

The procedure used for developing Louisiana rates from the statewide distribution and testing the rates using 1961 calendar/accident year losses was followed to the extent possible in determining the rates and 1961 loss ratios for the states of Alabama, Arkansas, Florida, Illinois, Ohio and Washington. Territory rate levels were developed using calendar/accident years 1958 and 1959 except for the states of Illinois and Ohio. The territory rate levels for these two states were determined on the basis of calendar/accident year 1959 only as Automobile experience for Illinois and Ohio was not collected by the NAII until 1959. The 1959 calendar/accident year statewide distribution by class for each state was used to determine the class distribution by territory. To test the rates developed from calendar/accident years 1958 and 1959, it was necessary to use the 1961 calendar/accident year statewide distribution to determine the assumed car year exposure by class for each territory for 1961 as the distribution by class within each territory was not available for these six states. The 1961 assumed car year exposures were multiplied by the rates developed from calendar/accident years 1958 and 1959 to determine the earned premium. The 1961 calendar/accident year basic limits incurred losses were adjusted to include unallocated loss adjustment expense and were then divided by the earned premium to obtain loss ratios. N.B.C.U. rates in effect in 1961 or revised in 1961 in these six states were used to determine the earned premium by territory and were compared to the earned premium based on rates calculated using NAII data for calendar/accident years 1958 and 1959. The loss ratios on both bases are as follows:

Calendar/Accident Year 1961

Loss and Loss Adjustment Ratio

		Bodily Injury		Property Damage	
State	Expected	(a)ø	(b)ø	(a)ø	(b)ø
Alabama	.645	.572	.497	.620	.479
Arkansas	.630	.643	.604	.582	.523
Florida	.630	.632	.624	.575	.521
Illinois	.655	.619	.837	.661	.597
Ohio	.625	.626	.481	.682	.510
Washington	.655	.653	.525	.668	.527

(a) ø – Loss ratios computed using 1961 earned premiums developed from rates based on the statewide distribution in the NAII compilations.

(b) ø – Loss ratios computed using 1961 N.B.C.U. rates.

The loss ratios computed for each of the six states using 1961 earned premium based on the statewide distribution of NAII experience varied less from the expected loss ratio than those computed from premium on the basis of 1961 N.B.C.U. rates. No criticism of N.B.C.U. rates is implied as they were developed from and for a different book of business. However, independent companies using N.B.C.U. rates may be interested in making similar comparisons.

COMPUTATION OF STATEWIDE DISTRIBUTION BY CLASS

Incidental to determining the percentage of earned cars by territory for those six states, additional computations were necessary as some independent companies do not use the Louisiana classification system in these states. In addition to the nine classifications used in the state of Louisiana, independent companies also wrote the following classes:

1, 1F, 1B and 1C combined, 1BF, 2, 2B and 2BF[7]

The exposures for class 1 were allocated to 1A, 1B and 1C in the same ratio as 1A, 1B and 1C bear to the total of these three classes. 1F and 1BF were added to 1AF. 1B and 1C combined were allocated to 1B and 1C in the same ratio as these two classes bear to their total. Classification 2 was allocated to 2A and 2C in the same manner as 1B and 1C combined. Exposures for 2B were added to 2A and 2BF to 2AF. All classifications were thus combined on a nine classification basis and the percentages were computed for each class. The exposures for code 1900 "other class plan filed" were not used as it was assumed that this classification would follow the same percentage distribution as the nine classifications.

ANALYSIS OF VARIATIONS BY CLASS DUE TO SUBSTITUTION OF STATEWIDE DISTRIBUTION

The results obtained in the states of Louisiana, New Jersey, North Carolina and Virginia show that while the territory loss ratios vary from the expected loss ratio, only slight variations are observed between the loss ratios computed from rates determined on a statewide distribution by class and those computed on the basis of the territory distribution.

At the start of the study, it was believed that use of a statewide distribution would cause sufficient distortion to require the use of a correction factor. An analysis of farm, non-farm, single, multiple and compact car

[7] Automobile Statistical Plan — All Coverages, published by the National Association of Independent Insurers.

classes was made to determine the variation between the statewide distribution and the territory distribution by class. However, there was not a sufficient variation by class to cause a severe distortion. The greatest variation was noted in the farm class 1AF. The Louisiana statewide percentage distribution of class 1AF for calendar/accident year 1959 was 8.3%. Territory 01, 1AF exposure for the same year was 0.6% while territory 08, was 19.1%. The use of the statewide distribution assumes that 8.3% of the earned cars in each territory were class 1AF. The following table indicates the effect of assuming the distribution of class 1AF is 8.3% in both territories 01 and 08:

Louisiana – Calendar/Accident Year 1959

Coverage	Territory	Class 1AF Distribution		Developed 1A Rates based on Distribution		1961 Loss Ratio based on Distribution	
		Actual	Assumed	Actual	Assumed	Actual	Assumed
B. I.	01	0.6%	8.3%	$31	$31	.625	.625
	08	19.1	8.3	31	30	.599	.620
P. D.	01	0.6	8.3	18	18	.671	.671
	08	19.1	8.3	15	15	.699	.699

The amount of error due to the variations in classifications by territory does not seem to justify the use of a correction factor.

APPLICATION OF STATEWIDE DISTRIBUTION METHOD

A statistician who desires to test his company's rate levels and does not have sufficient data or, as often happens, is assigned the task of determining rate levels for a state that his company is planning to enter for the first time could use the statewide distribution method.

Assuming private passenger territorial rate levels were needed for a particular state in January 1963, the 1962 NAII compilation of Automobile experience could be utilized to determine:

1. The total earned car year exposures, basic limits incurred losses and number of incurred claims for calendar/accident years 1959, 1960 and 1961 by territory.
2. The 1961 statewide percentage distribution of earned car year exposures by class.

The total 1959, 1960 and 1961 earned car year exposures for each rating territory could then be distributed by class on the basis of the 1961 statewide percentage distribution. The next step would be to multiply, (1) the company's rates in effect for the state at that time, (2) another company's rates, or (3) a bureau's rates, by the assumed earned cars for each class within each territory to determine earned premium at present rate level for each rating territory and the entire state, separately for bodily injury and property damage. The basic limits incurred losses could then be adjusted to include all unallocated loss expenses by use of a factor determined from the company's own data or the use of 1.10 for bodily injury and 1.16 for property damage currently used in rate revisions by the rating bureaus. The basic data would then be available to determine the statewide rate level changes in the same manner as they were calculated for Louisiana in Exhibit I. The accident year weights may be determined as follows:

1. If the earned premium at present rate level for the latest year for bodily injury and property damage combined, exceeds $20,000,000, apply a weight of 1.00 to the latest year.
2. If the earned premium as computed in (1) is less than $20,000,000 and exceeds $5,000,000, use a weight of .85 applicable to the latest year and a weight of .15 applicable to the prior year.
3. If the earned premium computed in (1) is less than $5,000,000, then use a weight of .70 for the latest year and a weight of .30 for the prior year.

After determining the accident year weights, the incurred losses are developed to 63 months for bodily injury and 39 months for property damage. A loss development factor may be determined from an analysis of the company's loss developments, or the NAII compilation could be used to determine the development of incurred losses from 15 to 27 months. A development to 27 months is not entirely satisfactory but a further development is not readily available. The loss and loss adjustment ratio at present rates is then computed. This ratio may be adjusted to reflect subsequent changes in average paid claim cost and a not unreasonable factor could be developed from the company's quarterly average paid claim cost by the method of least squares. If the company data is not available, NAII average paid claim cost data on an annual basis could be used. The loss and loss adjustment ratio at present rates adjusted to reflect the changes in average paid claim costs should then be divided by the expected loss and loss adjustment ratio to determine the indicated rate level change. The expected loss and loss adjustment ratio could be determined by subtracting the company's provision for expenses in their rates from 1.000. The number of claims for the calendar/accident years used in determining the earned premium at present rates would be used to determine the amount of credibility[8] applicable to the rate level change. After the computation of the statewide rate level changes, the statistician would then compute the territorial rate level changes.

CONCLUSION

The results of this study would seem to indicate territory rate levels could be approximated by the substitution of a statewide distribution of classification data for the territorial distribution, and the statewide distribution method affords independent companies a reasonable basis for checking and comparing rate levels.

[8] Current Rate Making Procedures for Automobile Liability Insurance, *PCAS* XLIII, p. 131.

EXHIBIT I

LOUISIANA
AUTOMOBILE LIABILITY INSURANCE — PRIVATE PASSENGER CARS

Development of Statewide Rate Level Changes†

(1) Coverage	(2) Calendar/ Accident Year	(3) 5/10/5 Limits Earned Premium at Present Rates (a)	(4) 5/10/5 Limits Incurred Losses (b)	(5) Number of Claims	(6) Loss & Loss Adjustment Ratio at Present Rates (4) ÷ (3)	(7) Calendar/ Accident Year Weights
B. I.	1958	$ 3,953,836	$ 2,570,427	2,763	.650	15%
	1959	4,070,745	2,696,312	2,840	.662	85
P. D.	1958	2,696,566	1,528,828	9,181	.567	15
	1959	2,792,999	1,492,154	9,271	.534	85

(8) Coverage	(9) Weighted Loss & Loss Adjustment Ratio at Present Rates Sum of (6) x (7)	(10) Factor to Adjust Losses for 24 months of Subsequent Change in Claim Costs	(11) Experience Loss & Loss Adjustment Ratio (9) x (10)	(12) Expected Loss & Loss Adjustment Ratio	(13) Credibility (based on number of claims)	(14) Indicated Rate Level Change $\left[\frac{(11)}{(12)} - 1.0\right]$ x (13)
B. I.	.660	1.076	.710	.651	1.00	+ 9.1%
P. D.	.539	1.045	.563	.651	1.00	− 13.5
Total						− 0.1

† The calendar/accident year earned cars used to develop the earned premium, incurred losses and number of incurred claims were obtained from data compiled by the National Association of Independent Insurers.

(a) N.B.C.U. rates used to compute earned premium at present rates.

(b) Factors of 1.10 for B. I. and 1.16 for P. D. were applied to the losses and allocated loss adjustment expenses to include unallocated loss adjustment expenses. The calendar/accident year losses have been developed to 63 months for bodily injury and 39 months for property damage by application of the following development factors:

Calendar/Accident Year	Bodily Injury	Property Damage
1958	.974	.989
1959	1.006	.962

EXHIBIT II
Sheet 1

LOUISIANA
AUTOMOBILE LIABILITY INSURANCE — PRIVATE PASSENGER CARS

Development of Bodily Injury Rate Level Changes by Territory†

(1) Territory	(2) Calendar/ Accident Years 1958-1959 Earned Number of Cars	(3) 5/10 Present Average Rate	(4) 5/10 Limits Pure Premium (Incl. All Loss Adj.) for Calendar/ Accident Years 1958 & 1959	(5) Loss & Loss Adjustment Ratio at Present Rates for Calendar/ Accident Years 1958 & 1959	(6) Credibility 1958 and 1959	(7) Formula Loss & Loss Adjustment Ratio at Present Rates [(5) x (6)] + [1.0 – (6)] x .663	(8) Col. (7) as Ratio to Statewide Average	(9) Indicated Territorial Rate Level Changes [(8) x 1.091] – 1.0	(10) Average of Present Differentials to Rate Class 1A	(11) Indicated Class 1A Rate (3) x [1.0 + (9)] ÷ (10)
(01) Jefferson, Orleans, Plaquemines and St. Bernard Parishes	55,774	$ 29.37	$ 21.35	.727	1.00	.727	1.108	+ 20.9	1.163	$ 31
(02) East Baton Rouge and West Baton Rouge Parishes	30,741	38.47	26.22	.682	.80	.678	1.034	+ 12.8	1.169	37
(03) Bossier, Caddo and DeSoto Parishes	27,508	24.19	15.86	.656	.60	.659	1.005	+ 9.6	1.153	23
(04) Ouachita Parish	10,740	23.71	17.40	.734	.40	.691	1.053	+ 14.9	1.128	24
(05) Calcasieu Parish	17,986	29.74	15.88	.534	.50	.599	.913	– 0.4	1.178	25
(06) Lafayette Parish	9,151	33.83	24.10	.712	.40	.683	1.041	+ 13.6	1.089	35
(07) Rapides Parish	11,837	23.37	25.10	1.074	.40	.827	1.261	+ 37.6	1.116	29
(08) Remainder of State	93,158	33.95	20.35	.599	1.00	.599	.913	– 0.4	1.093	31
Entire State	256,895	31.24	20.71	.663		.656				

†The calendar/accident year earned cars, incurred losses and number of incurred claims were obtained from data compiled by the National Association of Independent Insurers.

EXHIBIT II
Sheet 2

NOTES PERTAINING TO DEVELOPMENT OF LOUISIANA BODILY INJURY RATE LEVEL CHANGES BY TERRITORY

Column (3) Average of present rates based on calendar/accident year 1959 distribution by classification in each territory.

Column (4) This column although not essential is included in rate exhibits to show the pure premium.

Column (5) Basic limits incurred losses for calendar/accident years 1958 and 1959 adjusted to include unallocated expenses, divided by the earned premium at present rates for calendar/accident years 1958 and 1959.

Column (6) Credibility[9] – based on number of claims.

Column (7) The formula for calculating this ratio by territory is shown. The ratio for the entire state (.656) is computed as follows:

$$\frac{\text{Sum of Column (2)} \times \text{Column (3)} \times \text{Column (7) for each territory}}{\text{Column (2)} \times \text{Column (3) for entire state}}$$

Please note that if this ratio differs from the loss ratio for the entire state in Column (5) by more than one point, an adjustment should be made in the formula for computing Column (7) by including a factor determined by dividing the actual incurred losses (adjusted to include unallocated expense) by the formula expected losses.

Column (8) Column (7) as a ratio to entire state ratio (.656).

Column (9) Statewide rate level change (Exhibit I – Column 14) applied to each territory.

Column (10) The average of present differentials[10] is computed by multiplying the exposures for each class in each territory by the applicable differential to rate class 1A. The sum of the exposures so extended is divided by the total exposures for each territory.

Column (11) Indicated class 1A rate.

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[9] Current Rate Making Procedures for Automobile Liability Insurance, *PCAS* XLIII, p. 112.

[10] Use of a statewide distribution of class data will require only two computations—one for rural and one for large city territories, assuming the company uses the same differentials to rate class 1A.

DISCUSSION BY PHILIPP K. STERN

A study suggested by the theme of this paper should be of substantial interest beyond the circle of a few experts in the relatively narrow field of ratemaking. The private passenger experience compilations of the National Association of Independent Insurers, which the author undertakes to interpret, comprise about one half of the total private passenger volume in this country, and in many states they account for most of the private passenger business. While Bureau members and subscribers obtain from their rating organization compilations of experience that can readily be used to test the adequacy of Bureau rates, the NAII consolidations are primarily a historical record that may, at best, be interpreted in a qualitative way, and then only with broad assumptions. Mr. Gill has undertaken a substantial amount of work to prove that such experience can be interpreted in a quantitative way, starting with the assumption that a statewide distribution by class can be substituted for the conventional territory distribution. If Mr. Gill has called attention to a method that gives access to a wealth of private passenger experience, he will have performed a valuable service not only for the small companies that report to the NAII, at which the author primarily directs his paper, but also for all those concerned with rate levels and the wide spectrum of automobile liability insurance experience.

Bearing in mind the stated purpose of the paper, "to explore the possibility of developing a method of testing private passenger liability territorial rate levels by substituting the statewide distribution of classification data for the actual distribution by rating territory," one searches the paper for a direct comparison of the results obtained under either method. All other things being equal (expected loss ratios, losses, and the various adjustments introduced to compute loss ratios) the comparison would have been greatly simplified if premiums at manual rates calculated by the two methods had been shown. Instead, Mr. Gill relies more on the practical results of some of the alternatives he uses in his calculations. There is merit to this approach since, after all, private passenger rates are generally rounded to the nearest dollar and, what might appear too great a departure in terms of premiums at manual rates used in ratemaking might be counteracted by the fortuitous effect of successive arithmetic calculations and rounding.

Mr. Gill uses two different methods to prove his point: One for states for which the filing of experience is required in detail by class and territory by all statistical agents, including the NAII; and a different method in several other states for which he quotes some results of his calculations, states in which the NAII experience is filed in class detail on a statewide basis only.

In the calculation of Louisiana rates, using classification distribution by territory and statewide classification distribution, results are obtained that differ by only three tenths of one percentage point in the statewide rate level.

Since the author is also concerned with territory rate levels, it would have been desirable to see the effect by territory. Mr. Gill was good enough to furnish this reviewer the premiums at manual rates he had obtained from which we find the following ratios by territory:

Louisiana – Accident Year 1959

Bodily Injury Earned Premium at Manual Rates

	Based on NAII Classification Distribution		
(1) Territory	(2) By Territory	(3) Statewide	(4) (3) ÷ (2)
01	860,424	846,654	.984
02	598,786	586,489	.979
03	347,054	344,615	.993
04	130,120	127,870	.983
05	281,489	273,539	.972
06	161,200	163,916	1.017
07	142,160	141,734	.997
08	1,549,512	1,570,050	1.013
Entire State	4,070,745	4,054,867	.996

It can be concluded that, all other things being equal, territory rate level changes using the statewide distribution would vary by small amounts from those developed by the conventional method, although it should be noted that the departures from 1.000 are larger than a casual reader may suspect from the statewide effect given by Mr. Gill.

The paper proceeds to test the Louisiana results by presenting territory loss ratios based on 1961 Louisiana losses by territory and premiums at manual rates reflecting the two methods of calculation. If Mr. Gill had used the two sets of premiums at manual rates varying from each other only by virtue of method of calculation (statewide vs. territory classification distribution), the loss ratios would have varied by the factors shown above. Instead, Mr. Gill used two different sets of rates, one set obtained earlier by the use of territory distributions and the other set of rates obtained from the statewide distribution; he applied these rates to the classification distribution by territory. Thus, the results in the table of territory loss ratios are obscured, by the influence of rounding and by the fact that the premiums ultimately were calculated from the territory-class distribution.

By this method the author obtained identical loss ratios in six territories for bodily injury, and in seven territories for property damage out of the eight territories in the state, for the two methods of computation. Obviously, he must have obtained two sets of rates that were similar to the same extent. With somewhat less luck in rounding, the results could have erred considerably, since a one dollar difference in relatively low rates could have affected the territory loss ratio by as much as 5 per cent. The ratemaking practitioner should feel more comfortable knowing the real differences produced by the two methods of premium calculation.

For the other states in this group which Mr. Gill tested, only statewide correlation coefficients are shown in the paper. These factors do not allow a closer scrutiny of the results by territory. In order to complement Mr. Gill's data, this reviewer has calculated bodily injury premiums at manual rates for the state of New Jersey, using the NAII 1961 experience.

A comparison by territory of the premiums at manual rates obtained from the actual territory distribution and the statewide distribution produced ratios ranging from 1.022 to .961. The range for property damage is probably larger, gauged by the statewide correlation factor of .9693 obtained by Mr. Gill in his calculation.

It is apparent from the data shown by Mr. Gill and the few additions we have included that the method of calculating premiums at manual rates from a statewide distribution produces results close enough for the purpose of an estimate, as opposed to ratemaking, for the four states, where this test was made.

Three of these states have the type of rate regulation under which uniform rates apply for all companies, with a uniform classification system. The fourth, New Jersey, although not a uniform rate state, requires the recording and reporting of experience on a uniform classification basis. A close study would be required to determine whether there is in fact greater uniformity in classifications in New Jersey than there is in other non-uniform rate states.

This reviewer wonders whether the greater diversity in classifications and rates in the other states, or the predominance of different types of companies, has a bearing on the relationship of distribution of business by class and territory. If we were sure that this is not the case, we could agree with Mr. Gill that his method of calculating premiums at manual rates is a reasonable basis for the use of NAII data for an approximation for testing of rates.

There is another limiting factor that should be noted. The statewide distribution gives results very close to that obtained by use of territory distribution by class, in the aggregate for a territory, only because of mutually off-setting influences in the relationship of class distribution and classification differentials. For example, in one of the New Jersey territories, we found the actual number of cars for class 1A, Multicar, as 632, while the application of the statewide distribution to the total number of cars in the territory produces 1259 cars for this class. Similarly, we find actual 1531 class 1C cars against 2001 1C cars obtained from the statewide distribution. Thus, the statewide distribution overstates by almost 100% a class written at a differential of .80, and by about 33% a class written at a differential of 1.15. The two over-statements of exposures are partly compensated by virtue of the differentials. Similar differences obtain for other classes, with the final result that in this territory the premium at manual rates obtained under the two methods are almost identical. Any substantial change in the differentials might work in the opposite direction.

When it comes to testing the validity of using a statewide distribution in lieu of the territory distribution for the states for which Mr. Gill did not have available a territory distribution, he was obviously faced with a difficult task. He modified his Louisiana test, but instead of comparing results obtained from rates based on the same body of experience he compared his rates with National Bureau rates. He then applied these two sets of rates to the NAII experience for a year that had not entered into the calculation of either set of rates, and, as a tertium quid, related the respective loss ratios to the expected loss ratio. Since his rates based on one or two prior years were keyed to the same expected loss ratio, all he proves is that the NAII experience had changed relatively little in the following year. The National Bureau rates have no relationship to the NAII experience. While this type of comparison may be useful for other purposes, it has no bearing on the objective of the paper. If the NAII experience had worsened for 1961, Mr. Gill might have come to the conclusion that the National Bureau rates fit the NAII experience better, which would be just as erroneous. As noted above, however, we appreciate Mr. Gill's predicament, since he had no tool available to test his method for states other than those where a territory distribution is available.

In the area of loss development and trend factors, Mr. Gill had a similar problem. Lacking data reflecting the characteristics of NAII experience, he apparently used Bureau factors. Assuming that such factors are available in the ranks outside the Bureau companies, there is a question of whether they are appropriate for NAII data. The influence of these factors on rate level is considerable.

Mr. Gill did not suggest that his method is suitable for ratemaking or rate review, and rightly so. Although it does not produce as close an approximation by territory as the data presented in the paper for the states of Louisiana, New Jersey, North Carolina and Virginia seem to indicate, the suggested method might be useable to establish guideposts that, along with other considerations that motivate independent companies' pricing, could be used for the testing of rates. Since the price differential between the rates of the Bureau and Non-Bureau companies is generally predicated upon loss as well as expense experience, there is more latitude in the degree of required loss level approximation for Mr. Gill's "testing of rates" than is required in ratemaking.

DISCUSSION BY DAVID A. TAPLEY

In the words of the author, the purpose of this paper is to explore the possibility of developing a method of testing private passenger liability territorial rate levels by substituting the statewide distribution of classification data for the actual distribution by rating territory. Also, in his introductory remarks, Mr. Gill gives emphasis to the potential value of such a method to small independent companies and company actuaries as the basis for estimating the adequacy of their rate levels.

In the judgment of this reviewer, Mr. Gill's paper will indeed be of interest to a very large audience. Under current practices, a huge proportion of the total experience of all insured automobiles is accumulated under the statistical plans of the official statistical agents. However, there is a considerable variation from one state to another in the percentage of total business written by members of the Mutual Bureau, members of the National Bureau, members of other local State Bureaus and by independent companies. It seems reasonable to believe that this, or any other, method that will enhance the evaluation of rate levels by territory based on the experience of all companies will find favor in every quarter of our industry.

Mr. Gill's paper will unquestionably be of particular interest to a great many independent companies. In that period in which the all-industry bills were passed, almost 20 years ago, the art and science of automobile insurance ratemaking was practiced by a relatively small number of rating bureaus and large independent companies. Most, if not all, of the medium and small independent companies borrowed heavily from the available rate schedules of the larger organizations. In the years that have elapsed since the mid-nineteen forties, the exposures and premium volumes of the companies have become much larger and the interest of the independent companies in sound ratemaking practices have increased in even larger proportion. Mr. Gill's paper may very well point up a very real opportunity for our Society to capture the interest of even the small independents and to encourage the participation of their personnel in our programs.

The title of this paper properly recognizes that this is a method of approximation. This acknowledgment perhaps relieves in some degree what would otherwise be a necessity for examining the consistency and comparability of both the statistical plans of the N.A.I.I. and the National Bureau, and also the practices of the companies reporting to these organizations. Nevertheless, it should be noted that within the N.A.I.I. some companies follow the definition of a bodily injury claim employed by the National Bureau for all practicable purposes while others count claims only on a per accident basis. This difference alone could have a material effect upon the determination of the credibility factors employed by the author. It is also noted that the scope of testing set forth in this paper is fairly limited, as the author explains.

It may be of interest that in recent years quite a few companies have attempted, in one way or another, to make use of the N.A.I.I. experience accumulations for purposes that are quite similar to those stated by the author. In a panel presentation at one of the Workshop Meetings, some four or five years ago, one of the larger N.A.I.I. companies' personnel presented a rather detailed and somewhat complex method for making a similar kind of evaluation. Mr. Gill has, for many years, been uniquely placed to observe the interest and desire of the independent companies in the sound usage and the proper interpretations of not only the N.A.I.I. statistical accumulations but also those of the entire industry. Throughout his years of service to the many independent companies of every description and employing many varying practices, he has consistently sought to maintain standards of excellence that are in keeping with the object of our Society. It is the sincere hope of this reviewer that this paper will become only the forerunner of additional efforts to make full usage of all the statistical tools of our industry to the mutual benefit of all the companies. This is a thoughtful and stimulating paper devoted, I believe, to a larger purpose than that expressed by the author. Mr. Gill is deserving of our commendation.

DISCUSSION BY PETER B. ZORY

As a representative of a ratemaking organization, I found Mr Gill's paper of particular interest. The essential idea of the paper concerns the calculation of premium at present rates by utilizing a statewide distribution of classification exposures instead of the actual distribution within each rating territory. The standard method of computing premium at present rates, as described in Mr. Stern's paper,[1] requires the use of the actual class exposures within each rating territory. Except for six states, the National Association of Independent Insurers collects automobile experience by territory for all classes combined and only statewide for each classification. Thus, the N.A.I.I.'s compilation of experience does not include the actual classification exposures within each rating territory. Mr. Gill believes an estimate of the exposure by class within each territory, based upon the statewide distribution, would enable small independent companies to use the N.A.I.I.'s compilation of experience to (1) test their rate levels or (2) determine approximate rate levels for a state they are entering for the first time.

In order to demonstrate that his method can closely approximate the actual premium at present rates, Mr. Gill has calculated two rate level changes, for each of four states in which the actual class distributions by territory were available. He calculated one rate level change on the basis of the actual class distribution within each territory and a second based upon the statewide class distribution. The resulting rate level changes were practically identical.

To illustrate further the feasibility of using the statewide class distribution, Mr. Gill performed another test involving six other states. Two sets of premiums at present rates were calculated for each of the six states, using the same exposures but different rates. Both sets of premiums were based upon the N.A.I.I.'s 1961 statewide class exposures, which were distributed by territory using Mr. Gill's approximation method. One set of premiums were calculated at the 1961 N.B.C.U. rates and the second set at the 1960 N.B.C.U. rates, adjusted to reflect the 1958-1959 N.A.I.I.

[1] Current Rate Making Procedures for Automobile Liability Insurance, *PCAS* XLIII.

loss experience. Mr Gill concluded that the second set of premiums, reflecting the N.A.I.I. loss experience, anticipated more closely the premiums indicated by the 1961 N.A.I.I. incurred loss level than did the premiums calculated at the 1961 N.B.C.U. rates. It seems such a result would be inevitable. While Mr. Gill does note that no criticism of N.B.C.U. rates is implied because they were developed from and for a different book of business, it should be pointed out that the 1961 N.B.C.U. rates are those filed and approved and do not necessarily represent a pure statistical formularized approach.

This approximation method however, appears to be a reasonable and useful procedure in helping small independent companies to estimate the adequacy of their rate levels. For example, Company X with insufficient data could check the adequacy of its present rates by utilizing its own present rates in conjunction with the combined N.A.I.I. class exposure and loss experience. Mr. Gill would use the N.A.I.I.'s statewide class distribution in estimating the N.A.I.I.'s class exposures within each territory and these exposures would be multiplied by Company X's rates to produce N.A.I.I. premium at present rates. The standard ratemaking procedures, currently being utilized by the National Bureau, would then be followed to determine indicated rate level changes. If Company X's rates are to be judged on the basis of the combined N.A.I.I. experience, then it would be desirable to have these rates determine originally from a book of business similar to that reported to the N.A.I.I. Also, the Company X's present distribution of business should be fairly representative of the combined N.A.I.I. experience. One possible difficulty in this area is that the N.A.I.I. statistical plan allows the same risks to be reported under different classification codes. For example, companies reporting to the N.A.I.I. may report young driver risks under three class codes, two class codes, or they may combine all of their young drivers under one code.

Considering this approximation method as a precise ratemaking tool, I would have the following comments. Rates must meet statutory requirements that they shall not be excessive, inadequate or unfairly discriminatory. Premium at present rates calculated by utilizing a statewide distribution of class exposure could produce excessive or inadequate premiums both statewide and by territory and also unfair discriminations among territories. The greatest variation between a statewide and a territory distribution cited in this paper produced a difference in bodily injury rates of 3%. In states where there are large differences in classification distribution among territories the resulting rate differences would be larger than 3%. There are numerous examples of states where such differences could result, including states with few rating territories. In one small state, a territory's rates calculated using N.B.C.U. experience and Mr. Gill's approximation method differed by 6.5% from those computed using the actual exposure distribution by class.

Mr. Gill's principal objective however, was to afford small independent companies a reasonable basis for checking and comparing rate levels using the N.A.I.I.'s compilation of experience. It seems to me he has accomplished his purpose and has enabled companies to make more valuable use of the N.A.I.I.'s compilation.

In this connection it might be of interest to consider the National Bureau's compilation of experience. The N.B.C.U. compilation sets forth an experience pure premium and a pure premium underlying the average rate for each rating territory. These average rates have been determined from the actual class exposure within each rating territory and the present rates being used by the members and subscribers of the N.B.C.U. To supplement this information, the National Bureau is now preparing to make available to each of its companies their own experience in the same detail as that shown in the compilation for all companies combined. This will allow each company to compare its own loss experience with that reported for all companies, as well as the pure premium underlying its average rate with the Bureauwide average rate. These pure premium comparisons will facilitate a convenient and precise rate level check for the individual company.

SOME FUNDAMENTALS OF INSURANCE STATISTICS

HARRY M. SARASON

This paper is written for actuarial students, for insurance workers in general, and for non-insurance statisticians. Reference is made to statistical text books in general for much of the elementary mathematics, otherwise the paper is complete in itself.

PART ONE: ELEMENTARY CONSIDERATIONS

CHARACTERISTICS OF INSURANCE STATISTICS

Primarily, insurance statistics have to do with the frequency with which untoward events occur in a class of insured individuals, and with variance in the severity of the effects of those events. Some fundamentals of insurance statistics are as follows: (1) Every risk belongs to a class of risks, but differs from every other risk in the class; (2) All classes of risk are capable of multiple subdivisions; (3) Everything is a time series, often a three dimensional time series: calendar time, age of person or property, time since insurance began; (4) Nearly everything is expressed in dollars and cents; (5) For many kinds of insurance, the class with large amounts insured may be too few in number to produce reliable data; (6) All results are influenced by human actions; (7) Many risks are classified on the basis of representations rather than on facts. Most insurance statistical data are based on second and third hand information; (8) Nearly all insurance statistics for a given class can be compared with statistics for a related class, or with statistics for a somewhat related coverage on what is otherwise the same class; (9) Often insurance statistics are used before claim information is fully developed; (10) Insurance classes are different from classes in the population as a whole because of actions of insureds, actions of the insurers, and because people who buy each kind of insurance are a different class from people who don't buy that kind of insurance; (11) Insurance itself produces changes in the statistics involved; (12) Insurance claim rates "per year" sometimes include grace periods, and are not precise as to the time when coverage actually commenced.

ELEMENTARY ILLUSTRATIONS

If an insurance company could just insure a cross section of the population, the need for underwriting of individual risks would be eliminated or sharply reduced because, to start with, the insured class would be the same as the population class. The need for claim settlement and administration would still be great because insurance changes the action of the people insured. Insureds do not consist of a cross section of the population and this difference between the insured population and the general population results in a difference between general statistics and insurance statistics. In insurance terms, we have self-selection (often equivalent to anti-selection), company selection, effect of insurance upon occurrence of events insured against (or the reporting of the occurrence), and the effect of the insurance upon the amount to be paid. These interrelations and differences of insurance and general statistics are illustrated by considering an actual case.

One of the most important recently developed lines of insurance is Major Medical Insurance. However, before considering the development of Major Medical Insurance in relation to statistics of insurance, we will first consider the development of a much simpler kind of insurance, the insurance against scoring a hole-in-one in a professional golf tournament.

In connection with the hole-in-one in a professional golf tournament recently, a very large prize, $50,000.00, was offered for the scoring of a hole-in-one. The $50,000.00 was the total prize regardless of the number of holes-in-one scored. The statistical application to the insuring of this risk is fairly clear cut to persons who keep up with golf and its statistics. The probable procedure involved illustrates many of the aspects of statistical applications to insurance operations.

In the first place, there is rather readily available a considerable volume of statistics on the number of holes-in-one scored by professionals in their golf tournaments. With some little research, this hole-in-one statistic could be analyzed by difficulty of hole. We would thus have the statistics for holes closely comparable to the one or more to be played for the hole-in-one prize. We would, of course, know how many times the hole or holes were to be played. We would also have a series of statistics for holes of somewhat greater and lesser difficulty as collateral statistics. We would also have a time series which would give us some clue as to whether professionals were tending to score more holes-in-one, or not.

We could also take into consideration some rather remotely related statistical data such as the number of birdies made on par three holes and the number of times when the balls came within a few inches of coming to rest in the hole either off line or too hard or too softly hit. We could also take into consideration similar results in amateur tournaments.

The foregoing would appear to exhaust the statistical data which could be used as a basis of determining the pure premium involved and we now come to the problem of estimating the difference between non-insurance statistics and insurance statistics.

In the case of the golf hole-in-one, there are three parties involved: the insurance company, the promoter, and the professional golfer. In this particular case, and by no means is this always the case, the interest of the insurance company is directly opposed to the interest of the other two parties. We can, therefore, expect that the other parties will do whatever they consider to be ethical in order to provide a hole-in-one with its big prize, and with its collateral publicity which would be considerably to the advantage of the promoter. There is very little risk to the professional golfer himself, since he was going to play in the tournament with corresponding time and expense in any event, so that the golfer has all to gain and little to lose by striving for a hole-in-one.

The first thought in applying the original statistical basis on an uninsured event to the risk when there is insurance is that perhaps there should be some coinsurance. That is, perhaps the promoter should pay some of the prize money even though he has insurance. Coinsurance would reduce the conflict of interest between the promoter and the insurer. We then run the following gamut of thoughts with relation to the effect of the insurance upon the actions of the promoter and the golfers:

1. The golfers would practise for this hole-in-one before the tournament.
2. The golfers will be shooting directly for a hole-in-one on this particular hole rather than shooting more for the center of a green as they might do if they were interested solely in their total scores and how they finish in the tournament.
3. The promoter might easily be tempted to place the hole in the center of the green or in a flat spot, or even in a slight valley on the green, and he might well leave the hole in the same spot during the entire tournament instead of moving it as is customary. He might even leave the grass a little longer or the green a little softer than might well otherwise be the case.
4. The golfers would be somewhat inclined to pair off, or join together in greater numbers to agree to split the prize in the event that one of the group scored a hole-in-one. This is especially true because of the effect of federal income tax upon higher earnings in one year. Correspondingly, the golfers would be inclined to inform each

> other as to the effect of wind and ground conditions on the playing of the hole. They might even be a little less concerned about the avoidance of pressing their feet down into the area immediately surrounding the hole and they might be extremely careful to remove the handicap of ball marks or other irregularities in front of the hole before leaving a green.

The foregoing is merely an elementary illustration of the problems of applying non-insurance statistics to insurance operations and is not intended to indicate that the degrees of conflict between the parties involved are always the same as it is in the case of insurance against the payment of a prize of $50,000.00 for a hole-in-one.

These are factors that we, as amateur golfers and as statisticians, have thought of import. But, as a proverb of Iran states, "Only the hounds of Mazandaran can catch the wolves of Mazandaran." We should have the help of experts in our analysis of the problem; an expert golfer, an expert promoter, a greenskeeper, and, if such there be, an expert golf statistician.

As previously indicated, Major Medical Insurance and its evolution are a better indication of applications of statistics to insurance problems. One of the earliest occurrences related to Major Medical Insurance was that the medical director of a large insurance company suggested to his actuary that individuals be insured against the payment of major medical and hospital expenses on an indemnity basis rather than on a schedule basis. After a moment's reflection, the actuary asked, somewhat incredulously, if this meant that the insurance company would pay whatever the doctors decided to charge. This recognition of the conflict involved between the insurance company and one of the parties involved in Major Medical Insurance ended this attempt to initiate Major Medical Insurance.

There was a large body of incomplete statistics indirectly related to Major Medical Insurance contained in the files of insurance companies on hospital and surgical insurance even though the insurance companies did not pay the major medical expenses. Estimates could be made from the incomplete statistics. There were claim rates by cause of sickness and medical data as to the duration of such sickness. From these, and probably from non-insurance sources, Major Medical Insurance had statistics for its inception. A few years after the initiation of Major Medical Insurance, one of the large insurance companies made a study of the medical expenses of their own employees in great detail. The employees did not have Major Medical Insurance. This analysis was based on questionnaires to the employees, and the results were analyzed and reported by one of the actuaries of that company. Practically all of the problems involved in the statistical application of this experience on this particular class of individuals when they were not covered by Major Medical Insurance, but as applied to insurance operations in different parts of a changing world, were analyzed from a practical viewpoint in that report. (Thaler, *Transactions of the Society of Actuaries,* Volume II.)

The problems involved in that application to Major Medical Insurance can serve as generalizations of many of the problems of applying general statistics to insurance companies.

One of the effects of Major Medical Insurance is to provide not just the funds to pay medical and hospital bills, but to provide an actual increase in the use of medical and hospital facilities. Our hospital insurance and medical insurance, and this includes Major Medical Insurance, helps provide more and better medical and nursing facilities. This means that out of funds provided through insurance, we must be prepared to help build hospitals and improve their facilities, to help employ more nurses (this may mean paying them relatively higher scales and improving their working conditions in accordance with the law of supply and demand), and to reimburse doctors for any added work; all this in the shadow of inflation.

Insurance statisticians, in their consideration of the applications of their statistics, must take into consideration the immediate effect of the insurance and also the sociological and economic forces which, in turn, are largely capable of being understood to a degree only with the help of statistical analysis. Again, experts should help analyze the problem; doctors, hospital administrators, nurses, labor experts and, once the insurance is operating, experts in that line of insurance.

The first statistical application to any somewhat isolated insurance problem involves a great deal of judgment. This judgment, of course, was illustrated by the incident in which the actuary of one large company refused to let doctors and hospitals say how much his company should pay on a claim and, therefore, refused to initiate Major Medical Insurance. In a sense, this judgment was based upon parallels as to what the charges might be for various kinds of services when there was no bargaining between the seller and the buyer. Often, however, we have related coverages or we can extend coverage rates for various other age brackets and may have the statistics for such related insurance.

INTANGIBLES

In any statistical process we are looking at cold figures on paper, but we should be thinking of tangible objects and of live persons. The human element underlies much of the factual material on which statistics are based. The output of one factory manager, one factory inspector, or one factory worker, forms a significant sub-class from the output of other human beings and other human organizations. Insurance is no exception to the impregnation of statistical data with the influence of humans. "Intangibles" we call some of them, but, to those humans near the grass roots they are not intangible. The workers know the influence of their own endeavors and of the endeavors of their fellow workers. To those at the grass roots the reasons for differences are not intangible, they are real. Consider one insurance case. Why did valuable property floater coverage suddenly change from unprofitable to profitable for one large company in a certain large city? The way one appraiser went about his job involved what home office underwriters may call "intangible," but the appraiser could describe precisely how he got more premiums and better experience out of floater insurance. Conversationally – "Mr. Jones, did Mrs. Jones get this dress from Macy's basement?" Mr. Jones (the man that the neighbors are keeping up with) – "No – Saks Fifth Avenue – and the jewelry from Tiffany's, etc. etc." Result – 120% increase in appraised value; 50% drop in loss ratio; 99.44% drop in trouble adjusting claims. Statistical sub-class – property appraised by an appraiser who knew property and who knew people, and who wanted to do a good job. Similar sub-classes – agents who knew; home office underwriters, et al, who knew their business, knew people, and who set high standards of performance for themselves. These generalities of knowledge of people are partly intangible when expressed as generalities, but are tangible to those near the grass roots. The insurance statisticians cannot know all that brokers, agents, appraisers, and underwriters know about a risk. But, translating intangibles into tangibles, picking the significant good or bad subclasses from the commonly recognized class, getting down to individuals, brings statistical figures to life and to extra productivity. *The Ugly American* put it this way – "Have you been out in the boondocks?"

DEFINITIONS AND DESCRIPTIONS OF INSURANCE CLASSES

The precision of our descriptions of insurance statistical classes varies all the way from fairly precise descriptions when detailed research is involved to obviously incomplete descriptions of conglomerates in inter-company investigations. Risks have claim experience because of pertinent characteristics, not just because of the name of a class. The pertinent characteristic of a student applying for accident-medical coverage may be that he is a football player, not that he is a student. The pertinent characteristic of an armed forces lieutenant applying for automobile insurance may be that once a week he races back from the nearest town or entertainment center to his armed forces base, Cinderella-like, to beat the stroke of twelve, or the stroke of two a.m. Statistical thinking is not just thinking about classes; statistical thinking is comparative quantitive thinking about characteristics in an effort, among other things, to come closer to fundamental causes.

UNITS AND VALUES

Statisticians deal in dollars, index numbers, standard deviations, and more purely mathematical standards of comparison. These are technical units or standards of comparison. "The dollar is a unit of value." But is it now? Oh, we all know we get into financial trouble with the growing cost of automobile repairs and property and personal damages in terms of a declining dollar, but insurance is a two-edged sword. The dollar has no value except in what it will buy. Where is the value in a fire deliberately set as an alternative to bankruptcy; in unemployment "benefits" to those who are thereby encouraged to idleness or discouraged from enterprise; and how great the value in rehabilitation, in prevention, in insurance dollars when put to the improvement of human lives, in unemployment benefits that preserve dignity and give hope? Likewise, the cost of paying for the insurance needs measuring in truer values. For example, premiums or taxes to pay for benefits may reduce savings or reduce worthwhile expenditures in other directions. These are indications of true costs and of true values; true statistical measures – truer than the easily expressed and varying unit – the dollar.

THE INEXORABLE APPLICATION OF STATISTICS

Insurance statistics are inexorably translated into accounts, cash accounts and claim reserve accounts, among others, whether they are translated into technically statistical reports or not. In a truly mutual and equitable insurance operation, claims plus expenses are premiums. A bad experience would force premiums to rise very quickly if all companies were limited to just one line of insurance and had very limited amounts of capital. It takes a long time for the facts to catch up with a powerful financial or governmental complex unless it is managed by those who are strongly motivated by competition and by profits and losses, but the facts have a way of catching up with everybody in the long run.

INSURANCE STATISTICS IN THE REALM OF STATISTICS

Insurance statistics comprise a class of statistics which has different characteristics for different kinds of insurance, but, as a broad class, insurance statistics are different from all other statistics in their characteristics and properties. On the other hand, each of the characteristics of insurance statistics is present in certain other statistics.

Insurance statisticians are plagued by: changing economic and social conditions; difficulties in projecting; anti-selection; class selection; the effect of individual agents and other workers; the difficulties of second and third hand reports rather than of direct observations; the use of class labels rather than of characteristics. Other statisticians, who deal with humans, face some or all of these same problems. But scientific statistical thinking has to do with quantitive comparison of characteristics (not classes) in a search for causes. With insurance statistics as an illustration of the shortcoming of much of the statistics about people, no wonder Haroun Al Raschid went about disguised inconspicuously, as a beggar if it suited his purpose, in order to obtain his own statistics about the state of mind of the people of Baghdad at first hand in the days when the *Arabian Nights* were being written.

STATISTICAL THINKING IN THE REALM OF THINKING

Most scientific thinking is comparative quantitive thinking about qualities. The transition of the observations of gravitational effects from Aristotle, to Galileo, to Newton, to Einstein, to the conquest of space, represents the advance from rough statistical thinking to more accurate statistical thinking, to the thinking of pure science, and to statistical thinking again. Aristotle observed that heavy objects fall faster than light objects and, according to his medieval followers, concluded that their speeds were inversely as their weights. Statistically, he had the objects in the wrong statistical classes. He should have divided by the characteristics of surface and shape in proportion to weight, not just by weight. Galileo disproved Aristotle by further experimentation. Newton reached a "cause," a presumably universal law, the gravitational force which attracts objects to each other. Einstein refined our thinking about one of our standards, the "straight line" pursued by a ray of light. But we still have wind tunnels and experimental flights, and weather analyses to observe "statistically" the result of the interplay of the force of gravity with other forces, for example, near the speed of sound in various densities of air. Scientists and scientific statisticians alike are searching for the "causes" of our experience, and none of us has ever reached the ultimate "cause" – What causes gravity?

If Newton and his "cause," the force of gravity, represents thinking more precise than that of statisticians, on the less precise side we have thinking based on commonly observed statistics that we do not even write down, the "judgment" of an experienced actuary or other statistician or business man, for one thing. At the extreme low end of this application of statistical "thinking" we have the emotion that makes us tend to fear anything unfamiliar, or to embrace the familiar so that our home office agency may experience the most generous underwriting and the worst claim results of any of our agencies. There are numerous other instances of a tendency to approve the commonplace. From "general reasoning" it was long thought that the common occurrence of enlargement of the heart in conjunction with a heart murmur was a good sign, as representing good compensation. Actually, the enlargement indicated the seriousness of the heart damage.

Another kind of reasoning related to statistical reasoning is reasoning from a general background and a "statistic" of "one," or from extremely rough statistics. Thus, an insurance company inspector of new electric motors found metal chips inside one motor. He insisted on complete dismantling and found many more chips in six of the seven motors involved, any chip capable of causing a major breakdown. A life insurance statistical worker noticed that a policyholder who died of suicide in the second policy year had been paying seventy-five percent of his income for life insurance. This led to an investigation of the premium-income relationship on other suicide claims, and to new underwriting rules against speculation. Accumulations of statistics often commence with a thorough knowledge of one case, and then another case, and another, before the rules for gathering statistics are set forth. Sometimes, also, incomplete observations, folk lore, point to a truth. Workers in city morgues observed long ago that seven out of eight victims of lung cancer had the thumb and index finger of their right hands heavily stained with nicotine. The other one in eight was left handed.

FACTORS OF SAFETY

Applied mathematics is based in part upon pure mathematics and in part upon many variables which are not expressed mathematically. It is much more complex than pure mathematics, so much so that the "answer" involves expert judgment. The mathematical models used in applied mathematics are approximations: actuarial tables, statistical tables, probabilities assuming random fluctuations only, the average, the range. (What is the worst that can happen?) In engineering, a factor of safety is introduced to allow for approximations to the partly known; the structure may be two or three times as strong as simple mathematical models would indicate. In insurance, except when competition or rate regulation and the urge to expand get out of hand, we have our "margin for safety," "margins for contingencies," "profit margins," or safety factors in calculating credibility. The use of these margins is epitomized by the experience of Andrew R. Davidson, the historian of the Faculty of Actuaries in Scotland. To quote, with unimportant omissions, "But actuaries have always been practical statisticians; they are statisticians but also business men and, however nice the calculations, the men of that time did not forget to apply at the end an ample margin for contingencies when a contract was to be made. In the early part of this century, I can remember a certain chagrin when my official supervisor would add 25% or 33⅓% (his two favorites) to the figures I had ascertained with infinite pains."

This dramatizes the approximations involved in the use of mathematical models in applied mathematics, except that 25% or 33⅓% is not even of the same magnitude as the greatest safety factor used. Applied mathematics is important; it is complicated. No wonder, then, that Count Swedenborg in 1724 declined the chair of mathematics at the University of Uppsala on the ground that it was a mistake for mathematicians to be restricted to pure mathematics.

THE BUSINESS DECISION

Those involved in deciding how to apply statistics include members of state rate regulatory bodies, insurance company presidents, sales vice presidents and statistical experts or near experts, such as actuaries and home office underwriters, brokers or salesmen of individual cases, and, in the case of government "insurance," government executives and administrators, legislators and various classes of voters. All of these bring to the problem their own viewpoints, their own financial interests, their own desires, knowledge and background, their own bargaining abilities and deficiencies, their own emotional involvements and their own concepts of equity and acceptability. The actuary or other statistical expert who has command of the statistical information has the task of dealing with all of these people with their many diverse human qualities and with their specialized knowledge and abilities, and they, in turn, have the task of dealing with him. The task involves understanding the individuals, understanding the sales and acceptability facts, and the realization that there is no easy road to obtaining and analyzing the facts involved in the sales-acceptability and public relations aspects of applying insurance statistics. Sales and public relations aspects are not often investigated as thoroughly as are claim statistics, and we are often dependent on mere opinions of probable effects on sales, colored by very recent experiences with very few cases. Often the claim statistics are the only statistics thoroughly analyzed and recorded in writing. In deciding how to use insurance statistics, not only cold logic and judgment but human emotional reactions are involved – not just in "the decision," but in making the decision as acceptable and as effective as is practicable.

A fundamental difference in individual viewpoint is that for some their background is such, in sales for example, that one "success" in ten attempts leads to an overall success in their major activities. For a particular company decision being made, perhaps one profitable venture in three decisions will lead to an overall profit. On the other hand, some individuals will fear the prospect of even one unprofitable venture in a great number. Nor can people be wholly objective about this. The man who fears one failure is likely to speak of a one in two situation as a one in ten shot. The man who is accustomed to operate successfully with one success in ten attempts is likely to evaluate the probable effect of every likely looking product or rate reduction as a "sure thing," and to think that the power of positive thinking can keep loss ratios down. Perhaps each should continue his own way of thinking in order to offset the bias of each other.

In the application of statistical data, a major portion of the responsibility rests upon the actuary or other statistician who presents the statistical data. Often the statistician is in command of most of the hard facts: the statistics, including related statistical data, probable causes, and the human angles involved. His recommendations will loom large in the minds of others in the future and this may be especially true if the results go awry whether because of unfavorable action or because inactivity seems to be the cause of a lost opportunity to grasp affairs at an optimum time. Often the statistician has a veto or nearly so on the development of new coverages and the changing of other coverages involved in sales expansion. His positive suggestions are listened to with respect. His negative suggestions are sought, or feared. His written and spoken summaries and individual illustrations of the facts are the most important factors in the minds of many as they make their decision, especially his written reports. (When King Ahasueras could not sleep, he commanded the records to be read, and so Mordecai was remembered and honored.)

A business decision involves a weighing of the probabilities and of the stakes involved in various courses of action. The most difficult stakes to weigh are the stakes of goodwill, of respect, and of cooperation; these stakes can be exaggerated or they can be underestimated, or even evaluated in the wrong direction.

Business decisions are not made just at the apparent point of decision. Making a business decision is a developmental process. Pertinent statistics, statistical background and statistical viewpoints are involved at all the stages of developing the ideas and making the decision – the final judgment – not just in the conference room, not just in direct business discussions, but also in informal conversations under various circumstances. The character and characteristics of those making the decisions are among the important factors in decision making – and so is the knowledge that each person has of the characteristics of the others, primarily through personal contact, but including also the statistical knowledge of the characteristics of the various classes of persons to which the individuals involved belong. It is vital to know with whom we are dealing among policyholders, agents and brokers, and the makers of formal decisions.

Kipling wrote: "I keep six honest serving-men (they taught me all I knew). Their names are What and Why and When and How and Where and Who." Insurance statisticians know the importance of "When." Everything is a time series. Our thought is ever "What of tomorrow?" and, also, "Can we beat our competition to the punch?" Rates often vary by geographic area. We know that "Where" is important. We are constantly searching for causes; Why? Why? But, pervading all the other "serving men" of What and Why and How and Where and When in insurance statistics – pervading all our decisions and all the effectiveness of our application, pervading insurance itself, all pervading in our work is the human factor – Who buys? Who sells? Who repairs? Who administers? All insurance involves human beings. Insurance is sold by human beings, sold to human beings, on products built and operated by human beings (or on human beings themselves), for statistical classes into which they are sorted by other human beings, for insurances which are administered by human beings with resulting profits and losses which are studied by human beings, including actuaries and other statisticians, and so on, all involving human beings and things controlled, in part at least, by human beings – except the weather, and we are even talking about controlling that. The result is that, for us at least, it is true that "the proper study of mankind is Man." But, that is just "study." For the end result we turn to Mr. Micawber. "Annual income twenty pounds, annual expenditure nineteen pounds six, result happiness. Annual income twenty pounds, annual expenditure twenty pounds ought and six, result misery."

CONCLUSION TO PART ONE

The science of statistics is based on similarities and on differences. Similarities lead to classes. Differences lead to sub-classes, to frequency distributions and to individuals – unique individuals, persons. Insurance statistics is in the class of human statistics and in the sub-class of business statistics. Insurance statistics of various kinds have their own distinct characteristics; each statistical study has its individual characteristics. One important characteristic of insurance statistics is change; who knows what tomorrow holds, except change? A sudden change like October 1929, or a mathematically smooth change? The application of insurance statistics to insurance operations involves the vital operations of an insurance business; sales, profit making, and the ability to provide the benefits which have been promised. So important are our statistics and our profession that actuaries, quite as a matter of course, appear before and are a part of boards of directors and governmental committees – so important, that the words "actuarially sound" have been used as part of the presidential vocabulary. Actuaries and other insurance statisticians belong to that class of individuals, purveyors of truth, of whom King Solomon wrote in The Book of Proverbs; "Seest thou a man diligent in his business? He shall stand before kings."

PART TWO: MATHEMATICAL MODELS

All applied mathematics is based upon mathematical models and upon approximations implicit in mathematical models.

In sociological and economic sciences, the approximations are very great, especially in the vast majority of cases in which the applications involve the future with all its complexities and uncertainties. For this reason, perhaps, we are often quite content with approximate but easy to apply developments of our mathematical statistical formulas.

Mathematical formulas are more acceptable than free hand or graphic methods partly because free hand and graphic methods more obviously involve personal judgment and thus are more obviously subject to slanting, whether deliberate or not. Thus, even when a supervisory statistician would trust his own judgment in free hand smoothing or in graphing, he sometimes delegates the work as a formulary application, and, when nontechnical men are in a position of responsibility, or when any conflict of interest is involved, the mathematical formula method often becomes a necessity.

All probability and all applications of statistical data are based on partial ignorance. If we knew just how a pair of dice were imperfect, were held, and thrown, and blown, and how the surface on which they bounced reacted, we could predict from tried and true engineering formulas just how the dice would fall. If we knew more about each insurance risk than we do know or even than it is at all practical to determine, we could rate each risk better, and we could build a foundation of statistics which would enable us to rate each risk still better, until, in the ultimate we could predict the actual event insured against so that savings would replace insurance as a means of mitigating the "risk," provided, of course, that our understanding and our knowledge were both built up far beyond the present ability of mankind to know, and to use knowledge.

Our mathematical formulas, per se, assume an ignorance of "major causes." The formulas deal with the assumptions of random fluctuations arising from a multitude of "minor causes" treated in our mathematics as though we did not know the causes. To the extent that we know of probable "major causes," we give less weight to the mathematics of ignorance of causes. On some small insurance classes, such as employees of one employer, we are quite likely to "know" major causes – degree of carefulness in choosing, in training, and in supervising truck drivers, and in providing them with safe vehicles, for example – and to give much weight to such knowledge, especially when it is borne out by correspondingly good or correspondingly poor experience. In a mathematical sense, we put more trust or less trust in the hypothetical claim rate which we use in our purely mathematical calculations, depending upon the extent of our knowledge of the risk. Thus the mathematical "credibility" of poor experience on a small risk often leads simply to an underwriting review of the risk which completely confirms the experience so that 100% "credibility" (in the ordinary sense) replaces 50% or 60% credibility in the mathematical sense.

Since statistical applications are founded upon probability, which is based upon partial knowledge and upon partial ignorance, we have a continual conflict between "good enough" approximations in the mathematical models and somewhat expensive improvements in the models to meet the statistical facts as we see them – "through a glass darkly."

Insurance statistical mathematics is based, in technical terminology, upon frequency and upon variance. Thus we are interested in the frequency with which automobile claims occur and in variance in the size of those claims. In nearly all cases a priori considerations lead us to a numerical probability to be tested as a hypothetical basis for frequency. For variance in severity of claims or occurrence, our bases are a distribution curve which a priori (a priori, and partly on the basis of our general knowledge as well as upon the mathematics of small causes) we would usually expect to be reasonably smooth, but which may have almost any general shape. The third dimension is time. We are always cognizant of the fact that time changes things; and time changes people. In addition, multiple characteristics make multiple sub-classes possible.

Insurance statistical mathematics is based primarily upon the theory *that any insurance statistical class is a sample of a universe.* When the entire earthly class corresponding to the specific insurance class is not large enough to be treated mathematically as a universe, the concept is of enlargement to such a universe by increasing the length of homogeneous time so that our experience hypothetically could enlarge itself to the dimensions of a universe. Secondarily, our sampling theories are for a cross section sample of an insured class, which is a sample of a limited class, the composition of which is changed as each item of the cross section sample is removed from the portion remaining to be sampled.

ELEMENTARY MATHEMATICAL STATISTICS

Elementary mathematical statistics is developed from pure probability theory in a number of text books. The student should choose a text which appeals to his method of thinking; the step by step inductive approach – or the deductive explanation from the formulas of permutations and combinations, whichever suits his fancy. For insurance statistical mathematics, a mechanical model of many sided symmetrical toy logs is superior to the usual model of coins or of six sided dice (cubes). Thus a log with 1000 sides (and two ends) can represent probabilities per thousand. One side can be labelled to show claims with a frequency of one in a thousand for, say, claims of $100.00. For claims of $150.00, or thereabouts, the frequency may be two in a thousand – represented by two sides of a log. The following insurance example presupposes a familiarity with the elementary mathematics.

CROSS SECTION OF EXPOSURES WITH ALL OF CLAIMS – POISSON-HYPERGEOMETRIC

Sampling of exposures deserves special attention. Suppose our attention is focussed on a sub-class – "sub-class A" in an automobile insurance experience. This could happen in several ways. For example:

1. Someone reviewing the distribution of claims against his statistical background may think claims in sub-class A are relatively frequent; or,
2. Certain sub-class A claims may arouse suspicion because of the nature of an accident causing a claim, or because of the method of presenting a claim.

If the claims are apparently frequent, random fluctuation may be a plausible explanation – some sub-class is likely to turn up with an unusual number of claims because of random fluctuation; it may easily just happen to be sub-class A. The whole procedure involves searching for causes and the use of statistical judgment. If sub-class A individual claims looked suspicious, we are already on the trail of a possible cause. In either event, the presumed degree of reliability of the original classification, together with commonplace mathematical models, form a basis for statistical judgment, when and if we know the number of exposed, as well as the number of claims, in sub-class A. Suppose, however, that obtaining the pertinent data on all the exposures is expensive, and that we use a sampling process as illustrated by the following imaginary example.

Suppose that an examination of the claims of sub-class A, which is included in an entire class of young male drivers, gives us cause for suspicion, e.g., the sub-class of non-graduates of high school or grade school who are between 20 and 25 years old and whose automobiles are at least four years old. Suppose we now engage in some research which is expensive enough to limit the number of exposed which we analyze.

As a first step, we distribute the claims and find that there are 15 claims on the sub-class under suspicion. Our information is now as follows, including the date for the "Entire Class."

	Exposure	Claims	Rate
Entire class – young male drivers	3,000	180	0.06
Sub-class A	Unknown	15	Unknown

As the next step, both because we may find something new in the process and also so that we can keep others informed who will enter into any decisions involved, we take a 1% sample of exposures, i.e. 30 of the 3000. The result is as follows:

	Exposure	Claims	Rate
Entire class	3,000	180	0.06
Sub-class A	Unknown	15	Unknown
Sub-class A in 1% sample of 30 from 3000	2	not pertinent	
100 $\times$ 1% sample	200	not pertinent	

A rough idea of the loss rate in the sub-group under suspicion is 15 divided by 200, or 0.075. Additional samples are taken which produce the following in the sub-class, including the first sample — 2 — 0 — 3 — 2 — 1 — 0 — 1 — 0 — 1 — 2 for a total of twelve in a ten percent sample. We now decide to stop, and have the following information:

	Exposure	Claims	Rate
Entire class	3,000	180	0.06
Sub-class A	Unknown	15	Unknown
Sub-class A 10% sample of 300 from 3000	12	not pertinent	
Total in sub-class A	120 (est.)	15 (Tot.)	0.125 (est.)

The hypothesis of random fluctuation in the claims, for this experience segment which produced 15 claims, is the Poisson series. To test the cross section sample result, we may also set up the hypothesis of 250 exposed in sub-class A, corresponding to 15 claims and to the related further hypothesis that the true claim rate for sub-class A is the entire class claim rate of 0.060. For this test of the exposure hypothesis we use the hypergeometric probability function for 3,000 exposed containing 250 of sub-class A, from which a 10% cross section has produced 12 in sub-class A. The two series, the Poisson and the Hypergeometric, may be viewed separately as a basis of judgment.

On the other hand, the two series may be combined by joining the terms in which the probabilities of indicated claim rates are within a range of claim rates arising from the combined hypotheses. The term in the combined series for 8 claims and 6 in the 10% cross section of exposures "indicates" an experience claim rate of $8/60$ or 0.133. This is 221% of the hypothetical claim rate of 0.060. The term in the combined series for 16 "claims" and 12 in the cross section of 3,000 also "indicates" a claim rate of 0.133, or 221%. Since $8/6$ and $16/12$ each "indicate" 221% of the hypothetical 0.060 rate these terms could be combined, and also combined with other nearby terms, "indicative" of, say, 210% to 230% of the hypothetical 0.060 rate. The resulting combined Poisson-Hypergeometric series is illustrated by the term involving 8 claims and 6 sub-class A exposures in the 10% cross section of 3,000 exposures, as follows.

$$\frac{e^{-15}\ 15^8}{8!}\ {}^{300}C_6 \left(\frac{(2{,}750 \cdot \cdot \cdot 2{,}457)\ (250 \cdot \cdot \cdot 245)}{(3000 \cdot \cdot \cdot 2{,}701)}\right) .$$

The problem is of the general class, fluctuation in ratios, which has been solved by the Student t, but the normal curve component of the t is not suited for our small numbers and small claim rates. The foregoing mathematical model, while difficult to calculate, is directly suited to this double uncertainty problem arising from obtaining all claims and a cross section of exposures in an insurance experience. A corresponding double Poisson development would be a more approximate but simpler mathematical model.

The sampling and the research might be extended still further. Suppose the claimant drivers are shown a motion picture in which a driver makes a left turn across the path of another vehicle (which should slow down slightly to avoid a collision); In which the driver of the straight-ahead vehicle also fails to slow down; And a crash occurs. Now suppose that a blood pressure chart is kept of the fifteen claimants, and of twelve other drivers in the 10% sample and that 14 of the 15 show a violent upturn in blood pressure at the time when the left-turn driver starts to turn; while the twelve in the 10% exposure sample happen all to be non-claimants, and they show a similar reaction at the time when it becomes evident that the straight-ahead driver has dangerously delayed slowing down and should therefore be applying his brakes. We suspect a dangerously antagonistic attitude, a causative factor, on the part of the fourteen drivers who reacted violently to the interfering left turn.

We have these data: fourteen of fifteen claimant drivers fail the motion picture-blood pressure test. None of twelve non-claimants tested fail. The analogy is to a sample of an industrial product, with 14 of 15 indicated failures by a test of one production process and no indicated failures in 12 produced by the same test of another production process.

UNWIELDY FORMULAS AND THE PRACTICAL SOLUTIONS

Easily understood but unwieldy formulas, which are understandable mathematical models, are replaced by more tractable but less basic formulas, or by approximate formulas, in our operations. The Bayesian is replaced by a single hypothesis – or by separate consideration of two hypotheses. The complicated formula yields to the Monte Carlo technique, but not without difficulty of its own. The binomial with two parameters implying a volume of tables or an extensive calculation is replaced by the Poisson with only one parameter. The χ^2 test is applied instead of a slightly more complicated formula which could cut down on a bias in its results. The multinomial with amounts and probability is replaced by a binomial or Poisson and a separate consideration of the curve of amounts of claims. A varying homogeneity distribution which would produce several Poisson curves to be dealt with is fitted into a Type III curve to produce the single curve, the negative binomial. All of these represent simplifying the application of our mathematical models and, sometimes, complicating the underlying mathematical philosophy and, sometimes, also, an increase in the degrees of variance from the actual data, in choosing the hypothesis which we test mathematically. The data involves humans and time and changes, so that most of the approximations involved are fully justified because they are overwhelmed by the approximations inherent in our applications.

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For companionship in my travels along these statistical highways and byways, and for keeping me from straying on false trails, I am indebted to Geoffrey Crofts and Norman Rosenberg.

DISCUSSION BY CHARLES C. HEWITT, JR.

This review is directed to an attitude summarized in the paper in the following paragraph:

"All probability and all applications of statistical data are based on partial ignorance. If we knew just how a pair of dice were imperfect, were held, and thrown, and blown, and how the surface on which they bounced reacted, we could predict from tried and true engineering formulas just how the dice would fall. If we knew more about each insurance risk than we do know or even than it is at all practical to determine, we could rate each risk better, and we could build a foundation of statistics which would enable us to rate each risk still better, until, in the ultimate we could predict the actual event insured against so that savings would replace insurance as a means of mitigating the 'risk,' provided, of course, that our understanding and our knowledge were both built up far beyond the present ability of mankind to know, and to use knowledge."

Modern developments in the physical sciences in combination with modern developments in probability and statistics go to the deepest roots of the determinism (expressed by Mr. Sarason) which has dominated Western thought for five hundred years. Results which, two generations ago, were conceived of as the inevitable consequence of known causes are, today, being represented as averages resulting from random juxtaposition of known and unknown factors, both measurable and unmeasurable. Physical laws which were taught as absolute only a generation ago are, today, represented more properly as a macroscopic averaging or balancing of the often erratic individual behavior of a very large number of microscopic particles.

This philosophical retreat from the *certain* to the *averaging of the uncertain* is humbling to all mathematicians. No less a great than Albert Einstein has said, in protest, "I can't believe that God plays dice with the universe." The practical mathematician is reminded that he deals only with mathematical models. No one of his models will ever fit perfectly into an empirical mold. The childlike joy of being absolutely correct is gone, and his subconscious feeling of superiority over other intellectual craftsmen is disturbed.

However, even models and their improvement can and do provide genuine stimulation to the intellect, and their useful adaptation to practical situations can and does provide real challenge. In the fields of probability, statistics and decision theory, a whole new storehouse of mathematical models has become available to the practicing mathematician in the business of insurance – the actuary. Some of these models are beginning to find their way into our *Proceedings* and into the *Transactions of the Society of Actuaries.*

AUTHOR'S REVIEW OF DISCUSSION

Mr. Hewitt's comment on the statistics of causation goes to the root of "natural laws." I stand corrected! We would have to understand the First Cause to be able to understand how ultimate particles of matter act in order to predict with certainty whether or not our "natural laws" would be followed in any specific case.

One of the interesting aspects of the difference between exact reasoning of Mr. Hewitt's kind and ordinary reasoning lies in an analysis of the following question. "Does a human being really have such a thing as free will?"

The pragmatic answer is, "I don't know whether I have a free will or not, but I sure have a lot of fun acting as though I have free will." (And the person who answers is not interested in the specific meaning of the word "acting." "All the world's a stage.")

A GLANCE AT GROUP DENTAL COVERAGE

JAMES H. DURKIN

Is group dental coverage, by insurance or prepayment methods, feasible? Does it have a future? What are the special characteristics and problems that can be adduced at present about this coverage?

This paper is an effort to discuss such questions in a rather general and preliminary way, in the expectation that more precise and specific information and data now accumulating will soon be available for publication.

THE PERSPECTIVE

The subject under discussion, dental coverage as such, of a fairly complete and comprehensive type, is sufficiently different in degree to be different in kind from the limited coverage such as of oral surgery resulting from accident which has long been included in medical expense insurance.

Joseph E. Follman, Jr., director of information and research, Health Insurance Association of America, recently estimated that from 1½ to 2 million people are now receiving dental services under some form of group coverage.

The major forms of coverage are:

Insurance plans offered by at least 25 life and casualty companies.

Plans provided by dental service corporations organized by dental societies in some 30 states.

A number of group health non-profit corporations, several of the Blue Shield type.

Closed-panel or clinic plans maintained by employers, unions or labor-management welfare funds.

In the last two or three years, there has been a growth both in the number of people under group coverage and the number of companies, associations and organizations offering it.

Group dental care coverage, in one form or another, is now available virtually everywhere in the United States.

In estimating the prospects for this coverage, it seems useful to review the growth of group hospital-medical care coverage over the last 25 years. Spectacular expansion of group health insurance and prepayment systems has been attributed to the following elements among others:

1. Existence of a pressing need for more and better medical care, and public recognition of that need.
2. The crisis in financing hospital and medical care which arose from the depression of the 30's, giving rise to the search by hospital boards, doctors, government and the public for more stable and reliable financing methods.
3. The wage-freeze of the World War II period, with its concomitant emphasis on fringe benefits.
4. Adoption by labor and management of the principle of employer contribution to health care for employees.
5. Reaction to proposals for government systems of health care exemplified by the Beveridge Social Security program in Britain and the Wagner-Murray-Dingell bill in the United States.
6. The organization of hospital and medical care prepayment plans, the entry by insurance companies into the field, and the mastery by both types of carriers of the technical and marketing problems involved.
7. The expansion of national income and gross national product, making increasing resources available to provide needed medical services.

The question occurs, to what extent do these economic and social factors exist today; or are there other conditions present promising similar effects?

Authorities agree that the need for more and better dental care is acute, a point which is well demonstrated in the statistics and literature of the subject but is not within the scope of this paper.

Insurance companies, dental service associations and prepayment plans have entered the field and registered some progress. Although the number of people covered thereby is still comparatively small, the evidence is that technical problems of coverage are being satisfactorily solved.

Government's present role appears implicitly to be the encouragement of private forms of group dental care, as evidenced by the application to such systems of the same tax-saving provisions that are applicable to group financing of hospital-medical care, the dissemination of pertinent information by the Federal Public Health Service, and the adoption by numerous states of favorable legislation and regulations.

The dental profession manifests a readiness to encourage and take part in group dental care financing, reflected in decisions of its societies, organization of dental service associations, and cooperation with commercial insurers under insured plans.

On the other hand, there are several important if not decisive unknown elements.

Neither management nor labor has committed itself to adding dental care to the fringe benefits of employees, and voices are heard of opposition to further extension of fringe benefits. This question is of even greater consequence for the dental than for the medical field, for, as will be suggested below, group dental coverage seems impractical without substantial employer contributions to the cost.

Above all, the future of group dental care cannot but be related to the direction taken by the economy. It seems unlikely to the writer that competition by insurers or prepayment organizations for the present payroll or consumer dollar to cover dental care can be highly fruitful, but that a sufficient growth in national income would satisfy the first prerequisite for group dental care to follow a course of growth similar to that of medical coverage. Extension of good dental care to wider sections of the population, whether paid for individually or by some group method, would appear to be a function of the affluence of our society.

OBJECTIVES

The objectives with respect to dental health of the various parties concerned would seem to be:

1. From the viewpoint of the public, to attain improved dental health on a basis reasonable in cost, convenient and practical in method of payment, and mitigatory of the impact of sudden, unexpected large expenses.
2. From the viewpoint of the dental profession, to improve public dental health and see to it that resources are available to pay for adequate dental care provided by the dental profession as it now exists and functions.
3. From the viewpoint of insurers, group health associations and similar enterprises, to enlarge their usefulness and business effectiveness in providing means to meet the aims above-cited of the public and the dental profession.

Any system of group dental coverage must then be measured by whether it conforms to the criteria:

1. Able to provide sufficient resources to cover the costs of better care for more people.
2. Reasonable in cost.
3. Convenient and practical in method of payment.
4. Consistent with present procedures and functions of the dental profession.
5. Responsive to the problem of unexpected large dental expenses.
6. Technically suitable for marketing by insurers and group prepayment organizations.

If insurance or prepayment plans can be devised that meet these criteria, then group dental coverage is feasible. Group coverage plans which have been devised and are in effect so far, at least in a prototype sense, appear to demonstrate such feasibility.

CHARACTERISTICS AND REQUIREMENTS

While the criteria listed above might be generally applicable to any field of insurance, they have their own specific application to dental coverage. I believe it is helpful to analyze the special features of dental care by comparison with surgical-medical expense care with respect to elements significant for insurance purposes.

CHART I

SIMILARITIES AND DIFFERENCES, SURGICAL-MEDICAL CARE AND DENTAL CARE

	Surgical-Medical Care	Dental Care
1. List of defined, distinguishable procedures	Yes	Yes
2. Specific identifiable fees for different procedures	Yes	Yes
3. Generally accepted relative value of different procedures	Yes	Yes(a)
4. Predictable incidence of morbidity	Yes	Yes
5. Variation in utilization by age, sex	Yes	Yes(b)
6. Variation in professional fees by patient income level	Yes	Yes(c)
7. Availability of population utilization data	High	High
8. Incidence of utilization close to incidence of morbidity	Yes	No(d)
9. Significance of pre-existing conditions	Moderate	High(e)
10. Significance of sudden, high-cost treatment need	Moderate	Low
11. Significance of regularly-recurring minimum-treatment need	Low	High
12. Significance of optional element	Low	High
13. Availability of insured utilization data	High	Low(f)

(a) Relative values in dental fees are not as firmly and widely established as in the medical field, but are implicitly reflected in extant fee schedules and in practice.

(b) However, as will be indicated, several other factors seem to influence dental utilization which apply with much less weight to medical utilization.

(c) But a feature of dental care is that not only do charges tend to vary by income of patient for a given treatment, as in the medical profession; but there is a larger area, relatively speaking, where either a more costly or less costly treatment can be chosen for the same condition; e.g., a gold or porcelain filling as against a silicate filling.

(d) As will be discussed, the designation "No" is particularly relevant to an uninsured population, and is subject to modification under insured conditions.

(e) With respect to applicants for coverage who are actively at work when they apply.

(f) There is some published data, and much more is being accumulated.

The most meaningful differences in characteristics between surgical-medical and dental care, which the architect of a group plan must reckon with, are those indicated in the chart above as items 8 through 12.

Follman summarizes some of these obvious problems as follows:

> "One is that a portion of dental work is elective and at times a matter of cosmetics rather than medical necessity. . . .
>
> "The second is that most dental care is not, or need not be, either sudden or sizable in its occurrence. It occurs, or can occur periodically, the cost is regular and not usually sizable, and, hence, more subject to family budgeting in most instances than to an insurance mechanism.
>
> "The third is that often where costly work is needed, it is the result of needs which have accumulated for a period of years prior to the inception of the insurance protection and hence a pre-existing condition which is generally recognized as not being a fit subject for sound insurance practice."

These problems immediately suggest certain general conclusions.

1. The fact that much dental care is repetitive with fairly stable costs from year to year, and that the sudden, unexpected large loss is not a significant factor, tends to minimize the insurance element in group dental care and emphasize the budgeting and service elements. To illustrate, an average patient, with no "back-log" (or "clean-up") problem, through most early and middle years of age will visit the dentist once or twice a year; undergo an examination, including some X-rays; have a cavity or two filled and receive a teeth cleaning. This might mean repetitive annual costs of, say, \$30 – \$40. To this extent, then, dental care coverage by insurance or prepayment contains a large element of dollar-for-dollar exchange. This feature gives rise to the requirements: that if group dental coverage is not to be uneconomical in the insurance sense of that word, its advantages must be demonstrated in the convenience of budgeting costs, in the encouragement of regular dental care that such cost-budgeting begets, in the efficiency and flexibility of service provided by the carrier to the patient and the dentist, and in low administrative expense charges. This characteristic also suggests the necessity of a large employer contribution, for otherwise a consumer of dental services will prefer to pay his own dental bills rather than pay an essentially stable charge for dental services plus a carrier expense charge as well.

Another conclusion that can be drawn from this characteristic of dental care is that the agreement by dentists to provide service benefits (to accept the specified fee provided by the coverage as full payment) could well be a powerful factor in encouraging growth of the coverage. To the prospective purchaser, the quid pro quo of service benefits could well justify the cost of carrier administrative expense included in premiums. A feasible modification of the service benefits approach, and one consistent with the dental practice cited in Note (c) to Chart I, is to provide service benefits for routine and repetitive procedures and for certain standard treatments, while paying indemnity benefits for more complicated dentistry and the more costly options.

2. The optional nature of much dental care dictates other conditions of coverage, for it bears on the important question of the possibility of anti-selection. Optional can be considered in three contexts: (a) freedom of choice by the patient as to whether to go to the dentist at all; (b) freedom of choice by the patient as to when he will go to the dentist; (c) freedom of choice as to the extent and cost of the treatment provided.

Points (a) and (b) are relevant to the question of "back-log" or pre-existing conditions, discussed below.

Point (a) is particularly pertinent to the problem of measuring utilization of dental services as against incidence of dental morbidity. Utilization of dental services, at least by an uninsured population, can be inferred to be a function of a complex of interacting factors: incidence of dental morbidity, income level, cultural-educational level, age, sex, geographical area, and a subjective element that might be called the "Apprehension-of-Pain Deterrent." Tables I and II illustrate the effect of some of these factors.

To take account of the optional element, various devices are at hand. Deductibles, coinsurance, inside limits on a procedure or annual basis, waiting periods, and exclusions of specified procedures, are feasible and in fact one or another such provision is embodied in most current plans.

Special consideration must be given to the use of the deductible provision. A first-dollar deductible can be an obstacle to good dental care and to positive policyholder and public response. This is because one purpose of group dental coverage, that of encouraging visits to the dentist, is defeated if the semi-annual or annual visit to the dentist is not covered. It is precisely the budgeting of normal care that, to many people, will be the most appealing aspect of group dental coverage; an attitude justified by the lesser significance, in this field, of the unexpected, expensive occurrence. Furthermore, in the first year or two of operation of any group plan, it may seem inequitable to policyholders or members who have little or no back-log (who have kept their teeth in shape), that they should enjoy little or no plan benefits while others with heavy cleanup requirements qualify for substantial benefits after the deductible is applied.

One answer is to provide certain annual first-dollar benefits, either in a dollar amount of, say, \$25, or for designated procedures such as examination, X-rays, prophylaxis, and to apply a deductible to expenses thereafter.

Optional element (b) above, freedom of choice in timing dental visits, creates a particular hazard in the possibility that a covered person paying monthly premiums based on annual rates can concentrate needed dental care into a month or two and then cancel participation and premiums, or rapid turnover of employees in an employer-pay group may lead to the same result. This may necessitate the requirement of annual premium payments for all participants, whether or not participation continues for the year, as well as special attention to waiting periods for eligibility. Table III is one illustration of this problem.

As to optional choice of types of treatment: fee schedules, inside limits, coinsurance and annual or lifetime maximum provisions and package programs can be used in many variations and combinations in order clearly to define benefits and costs.

The special circumstances affecting orthodontia (see below), may require not only limits on benefits, but a long waiting period in terms of years before this treatment is covered.

3. The high significance in the area of dental health of pre-existing conditions or back-log must be taken into account. As indicated in Tables IV and V, first-year or initial cost of dental care as against maintenance care is from 1½ to 2½ times more expensive. Several methods present themselves to deal with this characteristic:

A. Exclusion of pre-existing conditions. This seems unsatisfactory, as likely to evoke public dissatisfaction; and for two other reasons. One is that pre-existing conditions may be difficult to define as time passes; the other, correction of currently-incurred conditions may frequently require the treatment of a contiguous or related pre-existing condition. A policyholder unable or unwilling to pay the cost of treatment of a pre-existing condition would in that case be deprived of treatment of the current condition.

B. First-year rates higher than renewal-year rates (with respect to the individual policyholder or member policy year).

C. Higher first-year deductible.

D. Amortization of higher first-year costs over a subsequent period.

All of these methods are presently in use in one form or another.

CLASSES AND SYSTEMS OF COVERAGE

A priori considerations and practice, even at this comparatively early stage, have already given general form to systems of coverage.

For purposes of group coverages, dental care can be broken into three main classifications (admittedly the lines blur in some areas):

I. Basic or simple dentistry. (Simple is not to be construed as implying simple in the skill or technique involved, but rather in contrast to complex as used below.)

This class of treatment or procedures includes:

Dental Examinations
X-rays
Prophylaxis
Extractions
Fillings
Repair of Dentures
Emergency Relief of Pain
Minimal treatment of periodontal disease.

II. Complex, or Restorative Dentistry (essentially Prosthodontia)

Bridges
Crowns
Dentures
Root Canal Work
Other treatments and procedures.

III. Orthodontia

Broadly speaking, Class II services are in the more optional and costly category, and arise less frequently, and with respect to such services there would appear to be a greater insurance element than with respect to Class I services. Class I services are those, generally less costly, which for most people can be expected to be repetitive through youth and early middle age.

Orthodontia is in a class by itself because the optional element is especially significant; the difference between orthodontic treatment required for reasons of oral health and that performed for cosmetic purposes is not easy of determination. Furthermore, orthodontia is almost exclusively applicable to children in early teens.

Use of this rough classification system offers the following possibilities:

1. Class II and III services can be identified as those for which inside limits or annual maxima are especially applicable.
2. Package programs, building up from minimum basic coverage to Comprehensive can be devised by appropriate combinations.
3. Rate calculation and the development of meaningful data may be facilitated.

Two alternative systems of coverage are possible – and extant.

One is similar to surgical-medical expense insurance, in that covered procedures are defined and listed, with corresponding fees. This is typified by the plans offered by the New York Dental Service Association.

The other can be viewed as the Major Medical type, under which covered procedures are defined, but dental charges are paid, without set fees, under provisions for deductible and coinsurance elements. An example is the Continental Casualty Company plan covering employees of the Dentists' Supply Company of New York. (The phrase major medical is somewhat misapplied here, for while the parallel holds with respect to the payment of charges subject to deductible and coinsurance, the catastrophe coverage feature of major medical is relatively absent in dental coverage.)

The writer has not found sufficient information to weigh the relative merits of the two approaches.

RATING

No attempt will be made here to provide either data or precise methods for rate calculation, but some inferences can be drawn from the tables and discussion.

Theoretically, the annual pure premium per individual can be expressed as $\sum_{x=1}^{x=n} p_x f_x$, where p_x is the frequency of utilizing procedure x, when there are n such procedures, and f_x is the fee-schedule fee – or average area fee – for procedure x. Trivial as this formula may be, it does have some utility. It reflects similarities between dental care and surgical expense insurance, and suggests that methods of evaluating fee schedules, combining specific procedures, etc., practiced in the latter field are applicable in the former. It lends itself, too, to precise measurement of variations in cost due to variations in frequency of procedures by age, sex, income and educational level for statistical if not actual ratemaking purposes.

It is of interest to note some of the variations in utilization of different procedures by age and other characteristics. The Group Health Association project identified in the section, "Tables", showed the peak in frequency for fillings to be around age 20, for dentures to be about age 60. In the U. S. population, the National Health Survey (July 1957 – June 1959) exhibited one facet of the difference in utilization due to educational level: those in the highest educational category visited the dentist over three times as frequently per year as those in the lowest educational classification, but among both classes the frequency of visits for extractions was the same.

A more practical rating approach and one especially consistent with the major medical type of program is to express an individual pure premium for a comprehensive coverage in simple form as $(E)(U)$ where E is the expected annual cost per person utilizing dental care, and U is the probability of utilizing dental care at all. E as defined here would seem to be a more meaningful statistic than cost per claim or cost per service because of the difficulty of defining these two terms in the dental field (unless the latter term is construed as cost per procedure). Implicit in this formula is the assumption that some members of any group will not go to the dentist at all in a year, if U is to be less than 1.00. This assumption is borne out by some of the appended tables; and is explained by the factors affecting utilization discussed above. Approximate values for E and U, for different benefits, for male, female, child, and related to age, income and educational levels, can be derived from material presently publicly available.

It might properly be expected that, over a period of time, with the expansion of group dental coverage, U should approach 1.00 for a group with average characteristics, ultimately differing from it only by the value of the "Apprenhension of Pain" factor as the deterrent factors related to cost and cultural-educational level wear off.

How much does group dental care cost? Obviously, an accurate estimate would require a full definition of coverage and benefit levels, and take into account group characteristics.

But plans presently operative do indicate at least the order or range of costs. For group coverage of an adult group, at average working age, per person rates run from about $2.00 per month for limited coverage and benefit levels to about $6.00 per month for plans of a relatively comprehensive type.

Variations in rating systems are evident. In some cases age of participants, and female content, are taken into account; in others, only an adult and a child (or children) rate are offered.

In the long run, in view of general trends in group medical insurance, and the particular characteristics of dental care coverage, it seems likely that experience rating and cost-plus rating systems will be the practice.

THE TABLES

The tables are by no means recommended for actual calculation of group dental costs or rates. Their purpose is to reflect some relationships and elements of variation in a broad and approximate fashion. Furthermore, no effort has been made to adjust for differences between cited groups in benefit levels and group characteristics.

Table I, U. S. population data, is from the U. S. National Health Survey covering the two years July 1957 – June 1959.[1] While the figures cover an uninsured population, it seems reasonable to assume their significance would carry over to an insured population, granted greater total utilization in the latter category.

Table II exhibits the proportion of eligible persons who visit a dentist per year. The fact that 60 out of 100 members of the public do not go to the dentist at all in a given year is one of the strong arguments for the view that large numbers of people are receiving insufficient dental care.

The St. Louis Labor Health Institute at the time the study covers (circa 1956) was a dental clinic operated by the Teamsters' Union in which union members received almost all services (except orthodontia and laboratory cost of dentures) without charge.[2]

The I.L.W.U. – P.M.A. plan is a labor-management dental care pro-

[1] Health Statistics; Public Health Service Publication No. 584-B15.

[2] Dental Care in a Group Purchase Plan; Public Health Service Publication No. 684.

gram covering children up to 15 under which payment for all services is provided by the plan. One significant feature is that a systematic and thorough effort is made to see that the eligible children make regular dental visits.[3]

The Naismith data is from experience of a prepaid dental care plan organized by a group of dentists, The Naismith Dental Group. It covered 1925 persons who participated during all or part of the period 1957-1960. Regular monthly "dues" were charged to members, covering a limited program of services. One element that may be reflected in the utilization is that the dues charged were substantially below the value of the services provided.[4]

The data from Group Health Association, Inc., Washington, D. C. is based on 1925 persons over a five-year period in the early 1950's representing 4002 man-years of observation. Services were provided by a professional staff at the Association's building, on a fee-for-service basis. All participants in this group volunteered but were required as a condition of participation to accept all treatment recommended by the examining dentists; so that those receiving services were, to all intents and purposes, all those eligible.[5]

Table III requires no comment.

Table IV illustrates differences between covered group and population utilization, by services; frequencies of different services; and initial as against maintenance costs.

Table V demonstrates again the relation between initial and subsequent year costs. The figures on the Continental Casualty Company plan covering employees of The Dentists' Supply Company of New York are from a press release of Mr. Henry Thornton, president of the latter company. They reflect three years of experience beginning August 1959, involving 2300 persons, employees of the company and their dependents.

Table VI is simply indicative of the types of services identified in fee schedules and of some fee levels. While the fees, as labeled in the table, in no way are intended to represent actual or recommended dental charges, they do convey an idea of approximate relative values of procedures.

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[3] Report on the Dental Program of the ILWU-PMA; Public Health Service Pub. No. 894.

[4] An Experiment in Dental Prepayment; Public Health Publication No. 970.

[5] Comprehensive Dental Care in a Group Practice; Public Health Service Pub. No. 395.

CONCLUSIONS

Following are some conclusions suggested by current information:

1. More and better dental care is needed by the population.
2. Attitude of the dental profession is favorable to the development of group insurance and prepayment plans.
3. There are no insuperable technical obstacles in the way of group dental care; and information, experience and statistics are rapidly being accumulated.
4. The special characteristics of dental morbidity and dental practice, combined with popular attitudes, impose corresponding requirements on the structure of group dental coverage and its financing, some of which this paper has attempted to set forth.
5. Large groups, and substantial employer contributions to cost, are two primary requirements.
6. Public response cannot yet be accurately estimated.
7. The future of group dental coverage depends on: (a) the future course of the economy; (b) the policy decisions of management and labor; (c) the policies and practices adopted by insurers and prepayment plans.

TABLE I

NUMBER OF DENTAL VISITS PER YEAR PER 100 PERSONS, BY VARIOUS CHARACTERISTICS

U. S. Population – From U. S. National Health Survey, July 1957-June 1959

	No. of Visits
Total Population	150
Age	
0 – 4	30
5 – 14	180
15 – 24	220
25 – 44	180
45 – 64	150
65 and over	80
By Sex	
Male	130
Female	170
Family Income	
Under $2,000	70
$2,000 – 3,999	100
4,000 – 6,999	160
7,000 and over	250
Unknown	140
Education of Family Head	
Education under 5 years	60
5 – 8 years	110
9 – 12 years	160
College	240
Unknown	90

TABLE II

ANNUAL UTILIZATION OF DENTAL SERVICES: PERSONS VISITING THE DENTIST AT LEAST ONCE EACH YEAR PER 100 ELIGIBLE

Source of Experience	Utilization
St. Louis Labor Health Institute	27 (1)
U. S. Population	40
Children's Coverage Plan, ILWU-PMA	70
Naismith Plan	85
Group Health Association "Pilot Project"	100

(1) Visits to Institute clinic only.

TABLE III

UTILIZATION BY "SHORT TERM" MEMBERS, NAISMITH DENTAL PLAN

Class of Members	No. Annual Visits Per Member
Average Plan Member	4.24
Members Terminating Membership In Less Than a Year	6.46

TABLE IV

DENTAL SERVICES PER YEAR PER 100 ELIGIBLE RECIPIENTS

A. Group Health Association, Washington, D. C., "Pilot Project"
B. Naismith Dental Plan, San Francisco, California
C. U. S. National Health Survey

		Services		Visits
		Initial Care(1)	Maintenance Care	Initial and Maintenance Care
Examinations	– A	107	129	
	– B	53(2)	51(2)	
	– C			
X-rays	– A	108	132	
	– B	44(2)	39(2)	
	– C			
Prophylaxis	– A	102	123	
	– B	77(2)	63(2)	
	– C			30(3)
Fillings	– A	442	250	
	– B	428	274	
	– C			70
Extractions	– A	44	15	
	– B	36	20	
	– C			30
Dentures	– A	10	2	
	– B	4(4)	2(4)	
	– C			10(5)
Crowns, Bridges	– A	31	11	
	– B	7(4)	4(4)	
	– C			
Other	– A	67	28	
	– B	80	42	
	– C			30
Total	– A	911	690	
	– B	729	491	
	– C			160(6)

(1) Initial care services for A are not on annual basis but for whatever period in which they were performed.
(2) Reported only when no other service performed.
(3) Includes examinations.
(4) Not covered by plan, paid by patient.
(5) Includes bridgework.
(6) Less than the sum of column because one visit may involve more than one type of service.

TABLE V

COST OF SERVICES PER PERSON RECEIVING SERVICES, INITIAL YEAR AND SUBSEQUENT YEAR

Group	First Year Cost	Subsequent Year Cost
Continental Casualty Company Plan, Covering The Dentists' Supply Company	$55.99	$36.26
Childrens Coverage Plan, ILWU-PMA	71.73	46.53
Naismith Plan	63.11	33.43

TABLE VI

SAMPLE LIST OF PROCEDURES AND FEES

THESE ARE FEES PAID BY PLANS AND IN NO WAY INTENDED TO REPRESENT DENTISTS' CHARGES

1. Group Health Dental Insurance Inc. New York
2. California Dental Association (Service Schedule)
3. Illinois Hospital and Health Service, Inc. (Plan C)

	1 (a)	2 (a)	3 (b)
Examination	$10.00 (c)	$ 6.00	$ 5.00
Bite-Wing X-Ray	–	2.00	3.00
Prophylaxis	–	7.00	7.00
Palliative Emergency Treatment	3.00	5.00	5.00
Single extraction, local anesthesia	4.00	6.00	5.00
Apicoectomy	–	35.00	25.00
Filling, one surface, amalgam	4.00	7.00	6.00
Filling, one surface, gold	5.00	25.00	6.00
Porcelain jacket crown	50.00	75.00	75.00
Bridge Pontic, cast gold	25.00	40.00	45.00
Full upper or lower denture, acrylic	90.00	145.00	150.00
Recementing Inlay	–	5.00	5.00
Periodontia Treatment	4.00	10.00	6.00

(a) As of February 1, 1961
(b) As of July 1, 1963
(c) Includes X-rays and Prophylaxis

(Fees for Group Health Dental Insurance Inc. and for California Dental Association from U. S. Public Health Service Publication No. 839; fees for Illinois Hospital and Health Service, Inc. by permission of that company.)

DISCUSSION BY ROGER A. JOHNSON

Mr. Durkin has made a brave effort in tackling what has been, up to now, an untouched field in the annals of the Casualty Actuarial Society. In my opinion, he has taken more than a "glance." It is more like a "long, hard look" at the subject, including a substantial amount of research.

Since few, if any, of us have had any actual experience with this type of coverage, any comments must necessarily be limited to general observations or anticipated problems. There is, for better or worse, an obvious analogy between prepaid dental coverage and prepaid hospital and medical coverage, and Mr. Durkin has adequately discussed the similarities and differences in his paper.

In view of the fact that more and more discussion and thought is being given to prepaid dental programs, it appears that we are on the threshold of a big spurt in the development of such programs. The basic problems, however, still exist. The very character of dental disease is a problem in itself. Some dental conditions occur so often that they are a certainty rather than a risk. Initial care is often the result of needs that have arisen over a long period of time and is, therefore, a pre-existing condition which many feel is of a non-insurable nature. Others feel that the most serious and costly forms of dental care could be included in major medical programs. The public's attitude itself has been such that most dental treatment can be postponed indefinitely. These attitudes are changing. People are becoming more interested in dental health, if not for themselves, at least for their children. Experience has usually proved that, where there is a real unfulfilled need, ways will be found to fulfill that need.

Following are some of the more obvious difficulties which may be encountered along the way:

1. Experience has shown that prepayment of health care increases the use of health services and it is reasonable to assume this will be true of dental care. If so, are there a sufficient number of dentists and technicians to provide the required services? It was estimated that in 1960 the mouths of 180 million Americans contained at least 700 million unfilled cavities brought about by tooth decay. At the rate of $5 per filling, the cost to fill these teeth would be about 3½ billion dollars. Furthermore:

   a. It would take about 5.2 hours for the initial care of the teeth of each U. S. citizen. This would amount to 493,000 Dentist Years of work.

b. It will take about 2.8 hours for annual maintenance care for each citizen. This amounts to 265,000 Dentist Years.
c. There were less than 100,000 dentists active in the practise of dentistry at that time.

2. It appears that the more competent dentists are now working to capacity. Less competent men would therefore have to serve the increasing number of patients due to prepayment, and this could serve to increase the utilization.
3. Mr. Durkin implies that, because of its cost, group dental coverage cannot succeed without substantial employer participation. Yet, while employers generally have accepted participation in group hospital and medical care as a part of the cost of doing business, in order to keep the employee well and on the job and to relieve him from worry about the health of his dependents, employers might react rather strongly to further increases in fringe benefits, for a service which is not absolutely essential.
4. All of Mr. Durkin's figures indicate that the cost of dental coverage is, for obvious reasons, greater in the first year than in subsequent years. This is contrary to normal expectation in other health insurance, where waiting periods and elimination of pre-existing conditions lessen the initial cost. Waiting periods in dental coverage would only delay the treatment of "simple" procedures to the point where they might become "complex" procedures and thus be more costly in the long run.
5. Rating Problems:
   a. Adverse selection – some people are blessed with excellent teeth and never require any services except cleaning. They would not be interested at any price.
   b. Children will require proportionately more treatment than they do under health coverages. This would make a family rate relatively much greater than a family hospital-surgical rate.
   c. Provision for care of retirees should be included from the beginning to avoid problems later on.
6. Service vs. Indemnity Benefits:

   If the purpose of dental coverage is to insure against the cost of dental care, it should be provided on an indemnity basis. If it is to be a real attempt to provide "dental health care," especially for the low and middle income groups, then it should be on a service basis.

DISCUSSION BY GEORGE E. McLEAN

Mr. Durkin's paper on group dental coverage is very timely. The rapid development of hospital-surgical-medical coverage in this country in the last twenty years may well have set a pattern which is about to be followed in the field of dental coverage.

In the perspective, there is very little with which to take issue. Mr. Durkin has done a rather commendable job of outlining the conditions which exist and which impinge on the question of providing group dental coverage. The objectives listed are sound, although I would make one additional point: Wherever there appears any indication of a void in the area of providing the public with health services of any sort, experience in other countries and the current thrust in our own country for adoption of the King-Anderson approach to providing health care for the aged clearly points to outright or attempted government intervention.

The industry and the prepayment plans may have been too late with too little in the way of offerings to avoid some form of participation by the federal government in providing care for the aged and it therefore is most imperative that we develop reasonable alternatives to government programs in the dental field in as much depth and as soon as possible.

Under Characteristics and Requirements, Chart I, citing similarities and differences between surgical-medical and dental care, I considered very well done and it included most of the important considerations.

In stating general conclusions, suggested by certain problems posed by Mr. Follman, this statement is made:

". . . This characteristic [budgeting versus insurance] also suggests the necessity of a large employer contribution, for otherwise a consumer of dental services will prefer to pay his own dental bills rather than paying an essentially stable charge for dental services plus a carrier expense charge as well. . . ."

On the face of it this seems like a logical assumption, but experience in several areas seems to indicate a certain consumer demand for enforced budgeting. One has only to look at the readiness to pay installment charges for what would once have been considered trivial purchases to confirm the public preference. Certainly some employer contribution would be helpful in marketing this coverage but I question whether that contribution needs to be substantial in order for the sale of this program to be successful.

With regard to service benefits, cited by the author as another desirable adjunct to the sale of this coverage, I heartily concur, having had first hand observation of its effect in the surgical-medical field.

Concerning utilization of dental services and Apprehension-of-Pain-Deterrent I should like to make several observations. I think that there possibly is some deterrent because of fear but this will gradually be overcome as experience with new techniques and anesthetics becomes recognized. In my view this argues for a relatively rapid rise in utilization for a number of years once coverage is instituted. In this case familiarity breeds confidence and confidence produces incidence.

In Table I it is interesting to note the close parallel beween the number of dental visits by range of family income and by range of education. I rather suspect that this is not entirely coincidental. There is a known correlation between the education of the family head and family income. In this case I believe that the paramount issue is family income and that the education factor merely underlies the income status.

The author has indicated that the budgeting of normal care may be the most appealing aspect of the coverage and I certainly concur. I do also agree that there is a problem of first year coverage due to back-log. The problem of concentrating the dental care into a month or two and then cancelling could be mitigated to some extent by the use of waiting periods as Mr. Durkin has cited and it seems to me that this is the most acceptable approach because the public has come to recognize this concept, particularly in the hospital-surgical-medical field on such items as maternity care.

Another possible approach is to offer limited benefits during the first year by means of coinsurance which would be successively reduced in each of the following years until it is eliminated. This would allow for a buildup in premium and would mean a greater participation in paying for backlog, while at the same time offering better coverage than a waiting period. The end result may be to discourage care during the first year, thereby leveling experience over the first several years.

The problem cited on orthodontia represents a question in my mind as to whether or not this coverage, together with certain very expensive optional procedures classified under regular dentistry, might best be handled in some sort of Major Dental rider, wherein special attention can be given to waiting periods and/or pre-existing conditions and coinsurance.

The methods presented to deal with the problem of pre-existing conditions probably represent a good outline of various approaches currently in effect. Two of the approaches I would consider somewhat difficult to administer and they are the use of first year rates higher than renewal rates, and the amortization of high first year costs over a subsequent period. A high first year deductible, either in the form of a cash deductible or waiting periods, seems to me to be the most reasonable solution and one which could be most readily administered.

In his definition of classes and systems of coverage Mr. Durkin does admit some blurring of lines so I will not expand on that point, although in general I find his classification quite proper. As I stated earlier, it appears to me that restorative dentistry and orthodontia which the author has categorized as Class II and III services can best be handled by a Major Medical type of approach.

Under the heading of Rating the material presented is so general that it leaves little room for specific comment or criticism. I would merely observe that any attempt to establish rates based upon incidence of individual procedures is probably going to be relatively unrealistic in the beginning. Projection of total utilization and total cost for a particular program will probably provide more reliable rates, although the results could be compared with those obtained by projecting the utilization of individual procedures extending them at their individual costs and summing the resulting requirements.

In the last paragraph under Rating Mr. Durkin indicates that experience rating and cost plus rating systems will very likely be the practice. Certainly the basic coverage (i.e., that encompassing Class I services) lends itself to experience rating because it is a relatively high volume, low average claim cost business which should produce highly credible experience.

Moving now to the conclusions, I would agree with all except conclusion five which states that large groups and substantial contributions to cost are two primary requirements. I fully believe that if sound coverage is developed and properly rated that participation of smaller groups and even those with very modest employer contributions may become possible. In his final conclusion Mr. Durkin states that the future of dental coverage depends upon: a) the future course of the economy; b) the policy decisions of management and labor; c) the policies and practices adopted by insurers and prepayment plans. I would add one last condition and one which may be the most significant of all: the penetration, if any, by the federal government into this area of medical care, particularly with regard to government clinics for lower income persons.

While not much in the way of actuarial treatment has been included in this paper, and while I disagree with a few of the author's positions, in the main I found it an extremely thoughtful and carefully organized presentation of a problem which is going to become very pressing for the industry and the prepayment plans in the immediate future.

MINUTES OF THE MEETING

May 18, 19 and 20, 1964

WENTWORTH-BY-THE-SEA, PORTSMOUTH, NEW HAMPSHIRE

Prior to the formal convening of the meeting on Monday, May 18, a meeting of the Council was held on May 17 and there was an informal buffet supper that evening for early arrivals.

The official attendance record shows the following attendance of 74 Fellows, 30 Associates and 17 Guests including 9 subscribers to the Invitational Program.

FELLOWS

Allen, E. S.
Bailey, R. A.
Barber, H. T.
Barker, G. M.
Bennett, N. J.
Berkeley, E. T.
Berquist, J. R.
Bevan, J. R.
Bornhuetter, R. L.
Boyajian, J. H.
Boyle, J. I.
Brannigan, J. F.
Crowley, J. H.
Curry, H. E.
Dickerson, O. D.
Dropkin, L. B.
Elliott, G. B.
Fairbanks, A. V.
Fitzgibbon, W. J., Jr.
Fowler, T. W.
Gillam, W. S.
Goddard, R. P.
Graham, C. M.
Graves, C. H.
Harwayne, F.
Hazam, W. J.
Hewitt, C. C., Jr.
Hobbs, E. J.
Hope, F. J.
Hunt, F. J., Jr.
Hurley, R. L.
Johe, R. L.
Klaassen, E. J.
Linder, J.
Lino, R.
Longley-Cook, L. H.
MacGinnitie, W. J.
Mackeen, H. B.
Makgill, S. S.
Masterson, N. E.
Matthews, A. N.
Maycrink, E. C.
McClure, R. D.
McNamara, D. J.
Miller, N. F.
Mills, R. J.
Morison, G. D.
Muetterties, J. H.
Murrin, T. E.
Nelson, S. T.
Niles, C. L., Jr.
Otteson, P. M.
Phillips, H. J.
Pollack, R.
Richards, H. R.
Roberts, L. H.
Rodermund, M.
Salzmann, R. E.
Schloss, H. W.
Simon, L. J.
Simoneau, P. W.
Skelding, A. Z.
Smick, J. J.
Smith, E. M.
Tapley, D. A.
Tarbell, L. L., Jr.
Trist, J. A. W.
Trudeau, D. E.
Uhthoff, D. R.
Valerius, N. M.
Walsh, A. J.
Wieder, J. W., Jr.
Wilcken, C. L.
Wolfrum, R. J.

ASSOCIATES

Aldrich, W. C.
Amlie, W. P.
Craig, R. A.
Curry, A. C.
DeMelio, J. J.
Durkin, J. H.
Gill, J. F.
Gillespie, J. E.
Hammer, S. M.
Jensen, J. P.
Jones, N. F.
Lange, J. T.
McDonald, M. G.
McIntosh, K. L.
McLean, G. E.
Muniz, R. M.
Peel, J. P.
Riccardo, J. F.

ASSOCIATES—Continued

Richardson, H. F.	Singer, P. E.	Switzer, V. J.
Roth, R. J.	Stern, P. K.	Webb, B. L.
Scammon, L. W.	Stevens, W. A.	Young, R. G.
Shaver, C. O.	Strug, E. J.	Zory, P. B.

GUESTS

*Benson, C. R.	Hartman, G. R.	McSherry, H.
*Bertram, A. P.	Haught, D. D.	*Nagel, J. R.
*Connolly, C. T.	Hayden, R. C.	*Peterzon, R.
*Donovan, H. S.	*Kedrow, W. M.	*Reiner, J. G.
Fertig, I. J.	Markell, A. S.	Sabbagh, M. J.
*Foody, W. M.	Marshall, R. E.	

*Participants in the Invitational Program.

Although not participating in the full session, the following guests were also present:

Guest Speaker: Shelby Cullom Davis, Managing Partner
Shelby Cullom Davis & Company

Panelists: Robert C. Capasso, Deputy Registrar
Massachusetts Motor Vehicle Bureau
(Mr. Capasso had kindly consented, at extremely short notice, to substitute for Mr. William Colvin, Director of Research, Illinois Secretary of State, who was unable to attend.)

Dr. Charles F. Haner
Grinnell Mutual Reinsurance Company

Dr. Edward M. McAlister, Industrial Psychologist
Rohrer, Hibler and Replogle

Dr. Basil Y. Scott
Director of Motor Vehicle Safety Research
Department of Motor Vehicles
State of New York

The session was convened at 10:10 A.M. on Monday, May 18, 1964 by President Thomas E. Murrin who then turned the meeting over to Vice President Harold E. Curry.

The first item was a panel discussion on the "Availability and Scope of Motor Vehicle Department Registry and Driver Performance Data":

Moderator: Harold E. Curry

Panelists: Robert C. Capasso
William S. Gillam
Dr. Basil Y. Scott

Following the presentations by the panelists, there was an exchange of questions and answers between the members of the panel and from the floor.

This session adjourned at 12:00 noon and reconvened at 2:00 P.M. with Vice President William J. Hazam presiding. The entire session was devoted to a panel presentation relating to "Accreditation of Actuaries" and the current status of the contemplated organization of an American Academy of Actuaries:

Moderator: Laurence H. Longley-Cook

Panelists: Frank Harwayne
Joseph Linder
Daniel J. McNamara

The Chairman and each of the panelists then spoke on various aspects of the problem of accreditation and the anticipated organization of the Academy. Chairman Longley-Cook then summarized the unanimous action taken by the Council at the May 17, 1964 meeting:

1. Adoption of a resolution:

   (a) Endorsing the recommendations of the Joint Committee and the CAS Committee on Professional Status that an American Academy of Actuaries be organized.

   (b) Agreeing that at the 1964 Spring Meeting the Fellows of the CAS present at that meeting would be asked to vote for or against the organization of the Academy and if the vote at the meeting was "yes," the CAS would go on record as being in favor of such organization and would agree to share in the expenses of such organization.

   Note:

   This resolution, summarized above, is reproduced in full as "Exhibit A" in the minutes of the Council Meeting held on May 17, 1964.

2. Nomination of officers and members of Board of Directors and Committee on Admissions:

   Chairman Longley-Cook then informed the gathering that, at the meeting of May 17, the Council had unanimously voted the following nominations for the American Academy of Actuaries:

President	Henry F. Rood
Vice Presidents (4)	John H. Miller
	Laurence H. Longley-Cook
	H. Raymond Strong
	Frank J. Gadient
Secretary	John C. Archibald
Treasurer	George M. Bryce
Admissions Committee	Harold W. Schloss
Board of Directors	Thomas E. Murrin
	Daniel J. McNamara
	Frank Harwayne
	Norton E. Masterson
	William Leslie, Jr.

There followed numerous questions from the membership which were answered by the panel members. It was then announced that, at the Tuesday morning session, the Fellows of the Society would be asked to go on record as approving the action taken by the Council at the May 17 meeting and at that time there would be full opportunity for further discussion by all members present.

There followed a panel discussion on "Fire Insurance Statistics and Ratemaking":

Moderator: David E. A. Carson

Panelists: Norman J. Bennett
Joseph J. DeMelio
Luther L. Tarbell, Jr.

After questions from the floor, answered by the panel members, the May 18 session adjourned at 4:30 P.M.

On the evening of May 18 an enjoyable New England Clambake was held.

The meeting reconvened at 9:50 A.M. on May 19 with President Murrin presiding. In accordance with the announcement made at the Monday panel on Accreditation of Actuaries, the first order of business was the following resolution (identified as Exhibit B) which was moved by the Secretary-Treasurer:

EXHIBIT B

"WHEREAS, the governing boards of the four actuarial bodies in the United States,

CASUALTY ACTUARIAL SOCIETY
CONFERENCE OF ACTUARIES IN PUBLIC PRACTICE
FRATERNAL ACTUARIAL ASSOCIATION
SOCIETY OF ACTUARIES

in order to establish some means whereby actuaries may be accredited, in 1963, appointed a committee consisting of one member from each of the four bodies, known as the Joint Committee on Organization of the Actuarial Profession, and

"WHEREAS, the said Joint Committee, assisted by a larger appointed Subcommittee, after due deliberation, has concluded and recommended that the said accreditation may best be accomplished by the organization under federal charter of an incorporated actuarial body to be known as the American Academy of Actuaries with the expectation that membeship in the Academy will be recognized as a satisfactory standard of accreditation for an actuary, and

"WHEREAS, the said Joint Committee and Subcommittee have submitted to the Council drafts of a Federal Charter and Bylaws for the Academy, which Charter and Bylaws have been reviewed and approved by representatives of the governing boards of each of the four actuarial organizations, and

"WHEREAS, the Society's Committee on Professional Status concurs with the recommendation of the said Joint Committee, and

"WHEREAS, at its meeting of May 17, 1964, the Council of the Casualty Actuarial Society unanimously adopted the following resolution:

That the Council of the Casualty Actuarial Society, having reviewed a draft of the Federal Charter and Bylaws of the proposed American Academy of Actuaries, including the requirements for admission to membership as set forth in Article 1, Section 2, of the said proposed Bylaws, hereby endorses the recommendation of the Joint Committee and of the Committee on Professional Status that an American Academy of Actuaries so constituted be organized and agrees as follows:

1. At the 1964 spring meeting of the Casualty Actuarial Society, the Fellows of the Society present at the meeting will be asked to vote for or against the organization of the American Academy of Actu-

aries. If the vote at the meeting is in favor of the proposition, the Casualty Actuarial Society will go record as being in favor of the organization of the American Academy of Actuaries.

2. If the Casualty Actuarial Society goes on record as being in favor of the organization of the said Academy, the Society will agree to share with the other three actuarial bodies in the expenses incurred in such organization.

"NOW, THEREFORE, BE IT RESOLVED, that the Fellows of the Society go on record as approving the action of the Council."

This resolution was regularly seconded. There then followed a discussion of the motion including questions from the floor. The presiding officer then called for the vote by a show of hands, for or against. It was announced that the resolution carried unanimously.

The gathering then extended a vote of thanks to Laurence H. Longley-Cook for the great amount of time he had expended on behalf of the CAS in connection with the organization of the Academy; and to Joseph Linder, Past Vice President of the CAS who had announced his contemplated retirement at an early date, for his many years of devoted service to the activities, welfare, and advancement of the Society.

The session then heard an extremely interesting address by Shelby Cullom Davis, Managing Partner of Shelby Cullom Davis & Company, relating to an analysis of the fire and casualty insurance business. Mr. Davis indicated that a good title for his more or less extemporaneous remarks, might be "There Is No Such Thing As A Free Lunch." This relates to the point, touched upon in Mr. Davis' remarks, as to what might well be the consequences of "below cost" insurance which fire-casualty companies have been giving the public in recent years.

The gathering then welcomed, as a new Associate, Mr. Bernard L. Webb, who had completed the requirements for enrollment as an Associate by passing the November 1963 General Mathematics examination.

The Spring Meeting was then recessed to reconvene on Wednesday morning, May 20, the remainder of the afternoon being available for Committee meetings and recreation for those not otherwise occupied. In the evening of May 19 there was an informal social hour followed by a banquet.

The May 20 session convened at 9:15 A.M. with Vice President Harold E. Curry presiding.

The President first called to the attention of the membership the need for increased activity with respect to the preparation of new papers for presentation at our meetings.

The previously presented paper "An Approximation for the Testing of Private Passenger Liability Territorial Rate Levels Using Statewide Distribution of Classification Data" by James F. Gill was then reviewed separately by Philipp K. Stern, David A. Tapley, and Peter B. Zory. (In Mr. Tapley's absence his review was read by Alan C. Curry.)

James H. Durkin then presented a summary of his paper "A Glance at Group Dental Coverage" which had previously been distributed to those who had registered for the Spring Meeting.

Matthew Rodermund, Chairman of the Committee on Distribution of Losses, reported:

> "In May 1963 the Committee was given a large volume of automobile excess loss data by the National Association of Independent Insurers, through the cooperation of Vestal Lemmon, General Manager, and Jim Gill, Actuary. Although the data was useful to the Committee, it was a little too thin for the Committee's purposes.
>
> "Accordingly, the Committee has asked for and been granted the use of private passenger excess loss data of the National Bureau of Casualty Underwriters and the Mutual Insurance Rating Bureau. The Committee is grateful to Bill Leslie and Dick Lino of the National Bureau and Joe Muir and Phil Stern of the Mutual Bureau for their cooperation."

Charles C. Hewitt, Jr. reported that the Committee on Annual Statement had rendered a report to the Council which had not as yet had time to review the report.

There followed a panel discussion on "Psychological Testing of Automobile Drivers":

Moderator: Norton E. Masterson

Panelists: Dr. Charles F. Haner
Dr. Edward M. McAlister

After conclusion of the panel presentation and a question and answer period the Spring Meeting was adjourned at 12:15 P. M.

In addition to the formal program for the Spring Meeting there were several activities that had been arranged for the entertainment of the ladies attending the meeting.

This concludes the Minutes of the Spring Meeting of the Casualty Actuarial Society.

VOL. LI, Part II No. 96

PROCEEDINGS

NOVEMBER 18, 19, and 20, 1964

PRESIDENTIAL ADDRESS BY THOMAS E. MURRIN

In this city on November 7, 1914, the Casualty Actuarial Society was founded by a group of 97 men who were active in actuarial and statistical work in the casualty insurance business. We now gather in a vastly different New York City to celebrate the Society's Golden Anniversary. It is indeed a pleasure to have with us practically all of our Past Presidents as well as four of the ten living charter members who attended that first meeting in 1914.

In addition to its significance to the Casualty Actuarial Society, this year is historical for other reasons. It would have been the 75th anniversary of the Actuarial Society of America if it had not merged with the American Institute of Actuaries to form the Society of Actuaries. Moreover, the formation of the American Academy of Actuaries was approved last May by the four national actuarial bodies – life, public practice, fraternal, and property-casualty. When granted by Congress, hopefully at the next session, a Federal Charter will formally give recognition to the actuarial profession and to carefully designed standards which must be met for a person to identify himself or herself as an actuary qualified in one or more fields of insurance. By the able, dedicated, and cooperative efforts of members of the various actuarial bodies, the American Academy of Actuaries is now on the threshold of reality.

It is customary at this point of the meeting for the president to address the membership and our distinguished guests. While an accident of time makes my term as President coincide with the Golden Anniversary of the Society, I deem it a particular privilege to have the honor of addressing you on this historic occasion.

There is no need for me to review in detail the history and accomplishments of the Casualty Actuarial Society. The accomplishments of the Society are contained in the *Proceedings* and in other works of its members; their careers, founded in actuarial science, have reflected advancement into such other positions as company executives and presidents, bureau managers, insurance commissioners, authors, and professors. The history of the Society has been covered more ably than any of my efforts might have done in two excellent papers prepared for this meeting. I recommend you read "Early Actuarial Studies in the Field of Property and Liability Insurance," by immediate past president L. H. Longley-Cook, dealing with the period prior to the formation of our Society, and "The First Fifty Years," by past president D. M. Pruitt.

My remarks will therefore emphasize more the present and future relation of our Society and its members to the broader picture of the insurance industry, with only a few brief references to the past to indicate how far the insurance industry and the actuarial profession have come in the last 50 years.

The founding of the Casualty Actuarial Society occurred at a time when there was a need for a concerted industry approach to a new problem. That problem, of course, was the rapid enactment of Workmen's Compensation laws around the country and the necessity of providing insurance to cover the benefits contemplated by those laws. In the five decades since 1914, the size and complexity of our industry and the country's economy have changed beyond the imagination of even the most foresighted individuals in 1914. From an underwriting point of view, there have been good years and bad years in this span and it is to the credit of actuaries that through the development of sound statistical plans and precise yet responsive ratemaking methods, compensation insurance has been able to keep pace with the economy and introduce indicated rate changes in a timely manner before rates became excessive or seriously inadequate.

At the time of the Society's founding, automobiles were beginning to appear on the scene to replace horse-drawn carriages. For many years, the premium developed from teams liability exceeded that produced by insuring automobiles, but the automobile industry and automobile insurance in turn have grown so rapidly that in 1963 the industry premium volume approximated $7 billion. In this area too, actuaries have contributed much to the development and modernization of ratemaking methods and classification systems.

Property insurance premiums likewise grew during this half century as building, commercial, manufacturing, and industrial activities have expanded to meet the needs of the increased population. It was only during the last 15 years that actuaries became active in this field. Significantly, in recognition of multiple line developments, the scope of the Casualty Actuarial Society was broadened in 1950 to include property lines and the Society even considered a change in its name.

What is the condition of our industry today? We find it suffering serious underwriting losses in several major lines of business. These losses are usually attributed to inadequate rates. Underlying rate inadequacy are many factors. One often overlooked is that of excessive competition. Many colloquial adjectives have been used to describe the competition but their common denominator can perhaps best be described by the term "excessive." Competition is the essence of our private enterprise system, of which the insurance industry can proudly proclaim itself a vital part. We have always had competition, and if we do not have it, our business will no longer be a segment of the private enterprise system. Basically, competition is one of the many sciences of management and it is this "science of competition" I would like to explore with you today.

In glancing through a business publication recently, I read an article on current competitive problems which included the following statement, "The impact of an entirely new producer was reflected quickly in the price structure. Old established producers had to reduce prices to meet the competition provided by the new producer, who offered discounts to attract customers." Reading on, I noted another statement on competition in this article, "New producers continue to enter the market in spite of the decreased profit margins during recent years." And, still again on the same subject, "Often competition is intensified when a large company diversifies through the acquisition of another company. The large company is often able to offer technical, financial and management assistance which feeds the growth of a new subsidiary."

From these statements, I naturally concluded that the author was talking about the insurance business and that he was merely repeating things that we had all heard many times before. It wasn't until I reached the end of the article that I realized that the author had not been talking about the insurance business but rather in the first quotation was referring to the steel business, in the second quotation to the cement business and in the final quotation to the electronics industry. To describe how things are "rough all over" in terms of competition, a final paragraph described still another industry where competition is at an all time high – the toy industry. The competitive problems of the insurance industry are certainly not unique.

A review of the current competitive situation in our business is meaningful only if we review its historical background. For many years price competition among major insurance writers in this country was minimal. The abilty to sell casualty and fire insurance was based to a large extent on individual companies' proximity, reputation, financial standing, and ability to service risks. This situation existed both prior and subsequent to the Southeastern Underwriters decision. After World War II, however, a new type of insurer, the direct writer, emerged as a leading factor in the automobile insurance business. Placing great emphasis on a lower price and utilizing selective underwriting and mass marketing techniques, these companies were successful immediately. Their entrance into the field of insurance is particularly significant for it marked the beginning of a revolution within the industry and the first step in a transition from a seller's to a buyer's market. We all know the result.

Witnessing the rapid success and growth of the direct writers, stock and mutual companies alike responded somewhat belatedly at a critical time when inflation was absorbing the provision for contingencies and profit and was eating into surplus. In attempting to recapture much of the lost market and to prevent further inroads, these companies introduced new classification, marketing, and pricing techniques. This naturally led the direct writers to intensify their efforts. This action and reaction stage is where we find ourselves today. On an industrywide basis, it has resulted in a general underpricing at the worst possible time – when the overall economy is being influenced by an inflationary trend.

Compounding the interplay of competitive zeal and inflation that contributed to underwriting losses of such severe magnitude was the fact that this revolution in merchandising was being accompanied by the practical operation – despite the deemer provision – of the newly adopted prior-approval rating laws. The inevitable adjustment from the "good old days" in pricing, marketing, and classifications was affected and in many cases hindered by the application of these new ground rules. Pressures for their unduly literal interpretation also beset their administrators from various sources, some of which were difficult to understand logically.

Coincidental with the adoption and operational administration of the rating laws was the introduction of multiple line legislation over a period of several years. It was not long before the competition previously felt in automobile lines exploded in the dwelling business with the phenomenal growth of Homeowners contracts. Underwriting results became progressively worse as price competition intensified.

Lest you think I am going to lapse into a lament on our present predicament, relax. You have heard it before but more importantly, I think we all have reason to be optimistic about the future of our industry and the influence of competition on it. My optimism is based upon a belief that the long term trend in the property and casualty business in this country is away from the type of competition that currently exists. I do not mean to imply that we will have a return to the "good old ways." These are gone forever and competition is here to stay. However, I believe it will be drastically different, characterized by maturity and a more scientific approach. There will be price differences but the ability to sell will not be based solely on price. Many of the factors, such as the costs of merchandising (including one stop shopping for all insurance needs), procedures that make it easy for the insured to budget his purchases, and a reputation for intelligent, fair and prompt claims handling will be necessary. Yes, you might even say "image" will be a factor.

Among the reasons why I believe price per se will not be the controlling factor is the fact that the current general market situation is vastly different from that which existed ten years ago – a fact that some may overlook in grappling with day-to-day problems. In the last ten years most companies have adopted modern merchandising techniques so successfully used in other industries. Where not too long ago many companies felt that prosperity was inevitable, all have since learned that aggressive salesmanship is essential for even a minimum of success. In addition, all companies now realize that to compete effectively it is necessary to operate at the lowest possible expense ratio and utilize the most simple and efficient accounting and processing procedures. In general, those differences which characterized the stock, mutual, and independent companies fifteen years ago have faded somewhat. Each segment has learned considerably from the other, even with the result that modifications in coverages, classification plans, rating plans, and marketing systems have been reciprocally adopted. In the process each has obtained a basic respect for the other. From this basic respect evolves the recognition that all segments of the industry are here to stay and compete and that there is sufficient room in our expanding economy for all to prosper. This reason for optimism can become a reality if based on the premise that competition must reflect sound pricing and merchandising methods in our business as in every other business.

An equally important cause for optimism is the fact that reliance on scientific analysis is expanding in the property and casualty insurance business. I think it is axiomatic that as the mathematical knowledge of the true cost of selling a particular type of insurance to an individual in a given class increases, competition based solely on price is correspondingly reduced. You will note that I say "reduced" and not "eliminated" because first, we will never know with certitude the true cost, and second, within any group there will always be risks a little better or worse than average. Accordingly, each company will need to underwrite its business. My point is, that through scientific statistical refinements and sound rating techniques, the experience of any one class will reflect a more homogeneous grouping than ever before. While underwriting will still be necessary, the amount of "skimmable cream" and possible price variation will have been reduced.

The life insurance business affords a good example. Scientifically compiled information in the form of standard mortality tables is the foundation of the proper pricing of life insurance contracts. Competition has certainly been keen but because the expectation of loss for different ages is fairly definitely predictable, competition based *solely on price* has been conducted within very narrow limits and other factors such as coverage innovations, service, reputation, and salesmanship play a major role in the company's ability to attract customers. Contrast this with our business where in the past several years varying discounts utilized by individual companies to attract customers have been based not on analysis of actual loss or expense savings – often this information has been unavailable – but rather on optimism and judgment colored by competitive pressure that "hopefully, with a little luck we will come out ahead." A quick look at the balance sheet for most companies will show that this has not happened. We may never be able to determine our expectation of loss for any particular group of risks as precisely as do life actuaries from mortality studies because there are many other factors – social, economic, and even psychological – which do not lend themselves to exact measurement in our business.

Nevertheless there are many indications that expectation of loss in the property and casualty fields will be measured more accurately than ever before. For example, the new classification plan for private passenger automobiles that was recently announced will enable actuaries and underwriters alike, with the aid of electronic processing equipment, to closely determine the true cost of selling insurance to various classes of insureds.

In the field of fire and allied lines, the traditional compilation of statistics apart from ratemaking requirements has made actuarial study difficult. Actuarial medicines – meaningful statistical plans and sound rating techniques – are necessary for the restoration of health to the fire insurance business. But the indispensable determination for a successful recovery will also require some hard thinking by many with respect to the traditions and practices of the past. Some of the problems involved here are term business written at guaranteed rates, complicated by discounts for pre-payment and even installment payments, proper insurance to value, coinsurance, the use of deductibles, loss adjustment costs, and other expenses.

However, it is encouraging to note, that, more and more, actuarial science is coming to the fire insurance business. Another optimistic sign is the general recognition in the fire business that rate competition is only as sound as the rates themselves and that rates can only be sound if they are developed from a scientifically refined statistical plan designed for ratemaking requirements.

In this changing competitive scene, what can the actuary contribute? First of all, he must be responsive to the changing complexion of the industry. I believe that the actuary should not be concerned solely with statistics and ratemaking formulae but that he must continually broaden his horizons and utilize those economic, legal, and underwriting principles applicable to the industry as a whole which are an essential part of the Society's examinations. Conventional approaches to problems must be re-evaluated. The opportunity of utilizing modern statistical methods in our studies must not be overlooked because we now have the equipment to handle what otherwise would have been tedious mathematical computations. There is an obligation to develop suitable and proper ratemaking refinements heretofore considered impractical and to be optimistic that we can convince our associates in industry and regulatory officials in particular, of the soundness of such refinements.

For example, five full calendar years of classified industry experience are hardly necessary to develop fire or homeowners rates that meet statutory requirements. Yet the inclusion of such a requirement in many state laws suggests that modernization of these laws is needed in this age of computers in which valid results can be achieved by the use of tested and accepted statistical sampling procedures. Improvement in rating laws and their administration could result if these procedures were recognized together with the basic economic principle that competition determines the success of a product and its price in the market place.

Consider how sampling techniques in the recent election were able to accurately predict final results from fragmentary early returns in many of the election contests and how they could be used in fire and homeowner's ratemaking. The biggest challenge in the area of sampling, I believe, is in the commercial package field. In order to insure a profitable outlook for these policies and determine what I described previously as the expectation of loss for particular classes of insureds or coverage forms, it is vitally necessary that we consider ways and means of sampling the early returns under a statistical plan less complex than that currently in effect. We should be optimistic enough to agree that this can be accomplished.

In addition to being responsive, the actuary must continue to develop his ability and desire to counsel other members of management. The simple fact that insurance is a business of numbers is often overlooked. We sell protection or indemnification in dollar amounts for which the consideration is also expressed in dollars. Marketing, underwriting, and claims are the indispensable components of property and casualty insurance but each of these involves numerically quantitive elements, in addition to other important factors. If our industry is to succeed and prosper, our associates in these three areas will need a much greater understanding of the significance of statistical data than was necessary in the past, when coverages and rating systems were more simple and the questions involved in agency evaluation, underwriting and rating policies, and claims management did not have to be made under the pressure of competitive conditions.

The property and casualty insurance industry has certainly changed in the last fifty years. Our charter members as well as our newest members undoubtedly would agree that we will continue to face changes, all basically generated by changes in the science of competition. Insurance has demonstrated itself to be vital to the American economy and actuaries have ably proven themselves to be vital to the insurance business. Future responsibilities will be heavier than those of the past and the challenge will be how to meet them successfully.

I am optimistic enough to believe that, despite the weight of the burdens, success will be achieved. Achievement, however, will require recognition that competition is a factor in pricing. Struthious reaction to the often struthious actions of others is folly. Competitive reaction must be the product of mature, scientific evaluation.

A BAYESIAN VIEW OF CREDIBILITY

ALLEN L. MAYERSON

Until recently, the credibility procedures used by casualty actuaries, and their theoretical justification, were developed apart from, and in isolation from, the methods used by statisticians. Arthur Bailey could write, in 1950:

> "At present, practically all methods of statistical estimation appearing in textbooks on statistical methods or taught in American universities are based on an equivalent to the assumption that any and all collateral information or a priori knowledge is worthless. There have been rare instances of rebellion against this philosophy by practical statisticians who have insisted that they actually had a considerable store of knowledge apart from the specific observations being analyzed. Philosophers have recently discussed the credibilities to be given to various elements of knowledge, thus undermining the accepted philosophy of the statisticians. However, it appears to be only in the actuarial field that there has been an organized revolt against discarding all prior knowledge when an estimate is to be made using newly acquired data." [14]

In 1950 the actuary stood nearly alone in his use of statistical techniques to modify his prior knowledge, instead of treating each new set of data as a separate statistical problem, to be used by itself if the volume of data was large enough to be statistically significant, or discarded if the contrary was the case. Because statistical techniques were not adequate to solve the actuary's problems, he developed his own methods. He ingeniously developed a credibility Z which was used to weight his prior knowledge B, with the current available statistical data A, by the formula $ZA+(1-Z)B$. But to determine Z, since there were no statistical techniques available, he has had to depend on empirical methods which, though they worked in practice, were hard to explain to non-actuaries and even harder to justify mathematically.

Statistical theory has now caught up with the actuary's problems. Starting with the 1954 book by Savage [8], and buttressed by the 1959 volume by Schlaifer [9] and the 1961 book by Raiffa and Schlaifer [7], there has been, among probabilists and statisticians, an organized revolt against the classical approach and a trend toward the use of prior knowledge for statistical inference. Instead of using credibility procedures, however, the Bayesian school of statisticians relies on Bayes theorem to merge the distribution representing prior knowledge with the statistical indications to produce a posterior distribution which reflects both.

At the same time as this revolution in the foundations of statistics, which formally reinstates prior opinion in statistical theory, advances have been made in probability and stochastic processes which result in mathematical techniques which lend themselves to the solution of actuarial problems and which can more easily be used by actuaries.

The relationship between Bayes theorem and credibility was first noticed by Arthur Bailey [14] who showed that the formula $ZA+(1-Z)B$ can be derived from Bayes theorem, either by assuming that the number of claims follow a Bernoulli process, with a Beta prior distribution on the unknown parameter p, or by assuming that the number of claims follow a Poisson process, with a Gamma prior distribution on the unknown parameter m. (The formula for Z differs, however, depending on whether a Bernoulli or a Poisson process is assumed.)

It seems appropriate, in view of the growing interest among statisticians in the Bayesian point of view, to attempt to continue the work started 15 years ago by Bailey, and, using modern probability concepts, try to develop a theory of credibility which will bridge the gap that now separates the actuarial from the statistical world. The purpose of this paper is to summarize the Bayesian point of view, to show its relevance to credibility theory, and to express credibility concepts in terms which are meaningful to a mathematical statistician.

THE "CLASSICAL" VIEW OF CREDIBILITY

As expounded by Whitney [38] in 1918, Perryman [33] and, more recently, Longley-Cook [30], the credibility theory now in use in the United States for fire and casualty insurance ratemaking rests on the following premises:

1. The formula $ZA+(1-Z)B$ can be used to modify the actuary's prior knowledge B (usually the rate currently being charged for a particular classification or, in experience rating, the manual rate) by the latest year's statistical data for the classification or risk in question, A.
2. The probability of an accident is the same for all insureds, namely q, and the total number of claims for n insureds follows a Poisson distribution
$$f(x) = \frac{e^{-m} \; m^x}{x\,!}$$
which has mean and variance both equal to $m = nq$.
3. The Poisson distribution may be approximated by a normal distribution. The normal distribution is a two parameter distribution, but for credibility work it is customary to assume that the mean and variance are both equal to m. Then, if P is the probability that

the actual number of accidents will be within $100k\%$ of the expected number,

$$P = \frac{1}{\sqrt{2\pi}} \int_{-x}^{x} e^{\frac{-t^2}{2}} dt$$

where

$$x = \frac{(m + km) - m}{\sqrt{m}} = k\sqrt{m}.$$

For selected values of P and k, we may determine the value of x from tables of normal curve areas. From the relationship $m = \frac{x^2}{k^2}$ we can then obtain m, the level of expected claims for which the probability is P that the observed number of claims is within $100k\%$ of the expected number.

4. There is a certain number of expected claims which deserves a credibility of 1, and this number is the m determined from the normal curve calculations.
5. If the actual number of claims observed is equal to m, as calculated in 4 above, this set of data may be assigned a credibility of 1.
6. We can ignore the distribution of claim size, or loss severity, and use the number of claims, or loss frequency, to determine our credibilities. Or, if we wish to recognize the fact that the variation in claim severity is at least as great and usually greater than the variation in number of claims, we can do so by using a higher value of P or a lower value of k, thus stiffening our requirements for full credibility.
7. Once the full credibility point m has been settled, partial credibilities, for a volume of data yielding r claims, not large enough to merit full credibility, can be assigned by the formula $Z = \sqrt{\frac{r}{m}}$ or $Z = \frac{r}{r+k}$, where k is a normalizing constant.

It has recently been recognized (Dropkin [23], Simon [37], Bailey and Simon [17]) that assumption 2 is open to question. For example, in automobile insurance the claim frequency varies for different drivers. If we assume that the number of accidents for each driver is Poisson distributed, and that the means of these accident distributions are themselves random variables distributed according to a gamma distribution, the total number of accidents follows the negative binomial distribution, and the probability of exactly x accidents is:

$$f(x) = \binom{x+r-1}{x} p^r (1-p)^x$$

which has mean $\frac{r(1-p)}{p}$ and variance $\frac{r(1-p)}{p^2}$.

Furthermore, the data studied by Harwayne [26] and Dropkin [23] show a mean accident frequency of .163 and variance .193, which casts some doubt upon assumption 3 above. The mean and variance of the distribution of claim frequency are not equal. The data studied by Hewitt [27] also indicates a variance, in each of the classes studied, which differs somewhat from the mean.

THE BAYESIAN VIEWPOINT

The Bayesian view of statistical inference can best be summarized by a quotation from a recent paper by Edwards, Lindman and Savage [25]:

> "Probability is orderly opinion, and inference from data is nothing other than the revision of such opinion in the light of relevant new information."

This view of probability differs radically from that used by most classical statisticians. Most authors define probability in terms of symmetry or as the limit of a series of relative frequencies. For example, one classical definition of probability is:

> "The probability of the occurence of a given event is equal to the ratio between the number of cases which are favorable to this event, and the total number of possible cases, provided that all these cases are mutually symmetric." (Cramer [2])

Another way of expressing this definition is:

> "If an event can occur in N mutually exclusive and equally likely ways, and if n of these outcomes have an attribute A, then the probability of A is the fraction $\frac{n}{N}$." (Mood [6] p. 7)

Some authors embody the limit concept in their definition thus:

> "The proportion of the time that an event takes place is called its relative frequency, and the relative frequency with which it takes place in the long run is called its probability." (Freund [3] p. 124)

Even when probability is treated in the more modern terms of sets and sample spaces, it is usually defined in terms of symmetry i.e. equally likely elementary outcomes:

> "The probability that an event A will occur is the ratio of the number of sample points that correspond to the occurrence of A to the total number of sample points." (Hoel [4] p. 6)

To a believer in an objectivistic definition of probability, the probability of an event may only be estimated by observing a series of trials of the event in question. Such questions as whether it will rain tomorrow, or whether there will be more automobile accidents next year than this year are considered, by holders of the relative frequency view of probability, to be completely outside the scope of probability. Such questions, they would say, have no meaning in probability terms.

By contrast, Bayesians believe that probability concepts may be used to express either the uncertainty of a future event or the uncertainty of unknown existing conditions. For a Bayesian, the probability of an event A is the largest price he would be willing to pay in exchange for the promise of a dollar if A turns out to be true. The probability that it will rain tomorrow is ⅓, for you, if you are willing to pay \$.33 for the right to receive a dollar if, in fact, it does rain tomorrow.

The consistency among the probabilities an individual would assign to various events can be obtained by his being unwilling to accept a combination of bets that assures a loss no matter what happens. Bayesians avoid the apparent contradiction between scientific objectivity and irrational human behavior by postulating an ideal individual who is consistent in this sense. Such a man will confront each of his probabilities with his other beliefs and will maintain consistency between them. The actuary will want to work with a consistent set of probabilities; this is equivalent to requiring that the probabilities assigned to the various events obey the usual mathematical rules of probability.

Such a reasonable and prudent man will not only maintain consistency among his opinions, but will be willing to change them when confronted with new evidence. Furthermore, if there are two reasonable men who initially assign different probabilities (prior probabilities) to a given event, their revised probabilities (posterior probabilities) will draw closer together when they are confronted with external evidence as to the truth or falsity of a given event or proposition. If the evidence is overwhelming (has credibility one), their posterior probabilities will tend to merge, given some degree of initial open-mindedness, no matter how far apart they were before they saw the evidence.

The mechanism by which prior probabilities can be confronted by evidence is Bayes theorem, which states that the conditional probability that the hypothesis H is true, given that data D have been observed, $P(H|D)$, can be expressed as:

$$P(H|D) = \frac{P(D|H)P(H)}{P(D)}$$

where $P(H)$ is the prior probability for hypothesis H. The denominator, $P(D)$ can be expressed as

$$\sum_i P(D|H_i)P(H_i)$$

where $\{H_i\}$ represents a set of exhaustive and mutually exclusive hypotheses of which H is the particular one under examination. If we are only interested in whether H is true or false, then the set H_i compromises only two members, H and $\overline{H}$, and

$$P(D) = P(D|H)P(H) + P(D|\overline{H})P(\overline{H}).$$

The partition $\{H_i\}$ is often arbitrary. For example if H is the hypothesis "the average paid claim cost C for automobile bodily injury liability is \$796 in 1963," the set may consist of only two other members, besides H, namely $C<796$ and $C>796$, or it may consist of a continuum of numbers x, with initial probability densities $f(x)$, such that

$$P(D) = \int P(d|x)f(x)dx$$

where H is the particular interval

$$795.5 < x < 796.5.$$

Bayesians emphasize decision making as the purpose of most statistical work; the purpose of obtaining a statistical estimate of μ is to decide on a certain course of action (e.g. what premium to charge) rather than merely to assert something about μ. By contrast, many statisticians believe that their function is limited to an analysis of the data and that decision making is a separate function; the decision maker, in their view, must combine the statistical results with his own judgment and other relevant factors in deciding what action to take.

The above short explanation of personal probabilities and the use of Bayes theorem is not intended to change the view of anyone who now holds the frequentist view of probability. A more extended and convincing treatment can be found in [25], [21] and [36].

CONJUGATE PRIOR DISTRIBUTIONS

The actuary is rarely interested in testing whether a hypothesis H is true or false. In most problems involving credibility he wants to determine, after seeing claim data for the latest calendar or policy year, whether the current manual rate needs to be modified. Or, his problem may be whether a particular insured should be charged a premium different from the manual rate. His initial point estimate H will usually be the current premium rate in the class under review. He would like to determine whether H must be modified, and to what extent, by the observed data D. Rarely, however, can he decide on the distribution of $P(H)$, the prior probability he is willing to assign to H, purely by introspection.

Fortunately, there is a way out of this dilemma, at least partially, through the theory of conjugate prior distributions, studied in detail by Raiffa and Schlaifer [7]. A prior distribution is said to be conjugate to an experiment when the prior distribution is so related to the conditional distribution that the posterior distribution is of the same type as the prior. For example, if D is viewed as the outcome of a Bernoulli process, and $P(D|H)$ is the binomial distribution, then the choice of a Beta distribution for $P(H)$ will result in a Beta distribution for $P(H|D)$ also, but with different parameters. If D is viewed as the outcome of a Poisson process, and $P(H)$ is chosen as a Gamma distribution, $P(H|D)$ will also be a Gamma distribution. If D is interpreted as the mean of independent normal observations with known variance, and $P(H)$ is assumed to be normal, then $P(H|D)$ will also be normal, but with smaller variance.

Arthur Bailey [14] studied both the Beta-Binomial and the Gamma-Poisson conjugate distributions and showed that, under either assumption, a credibility Z can be obtained, of the form $Z = \frac{n}{n+k}$, so that

$$E(H|D) = Z\,M_D + (1-Z)\,M_H.$$

Contrary to usual actuarial usage, where k is taken as an arbitrary normalizing constant, Bailey's formulas require that k be a specific function of the mean m and variance σ^2 of the prior distribution $P(H)$. If $P(H)$ is taken as a Beta distribution, and $P(D|H)$ is a binomial distribution, then

$$k = \frac{m - m^2 - \sigma^2}{\sigma^2}.$$

If $P(H)$ is assumed to be a Gamma distribution, and $P(D|H)$ is a Poisson distribution, then $k = \frac{m}{\sigma^2}$. It should be noted that Whitney [38] realized that k is not constant, but accepted an invariant k on grounds of expediency and simplicity.

The existence of conjugate prior distributions makes the actuary's job easier. If he thinks that the claim data he observes result from a Bernoulli process, he may, with a sufficient degree of approximation, be able to take $P(H)$ to be a Beta distribution. If he believes that his claim data come from a Poisson process, he may be able to assume that *P(H)* is a Gamma distribution. In either case, he must choose m and σ^2, the mean and variance of *P(H)*, hence the parameters of the prior distribution, so that *P(H)* adequately reflects his belief about H before seeing the observed data D. If there is a sufficient amount of data, the posterior distribution will not depend heavily on the exact form of the prior distribution.

The choice of m will, as a rule, be simple. m will be the pure premium, claim frequency, average claim cost, or whatever other actuarial function H is intended to test, e.g. if H is the hypothesis "the average paid claim cost C for automobile bodily injury liability is $796 in 1963" m would be taken as 796.

The choice of σ^2 is much more difficult, and in its use lies a major departure from present actuarial practice. At present, the current claim frequency, pure premium, etc. is taken as fixed and assigned a credibility $1\text{-}Z$, where Z depends only on the number of claims or the amount of losses observed in D. Actually, the current premium rate, or its component claim frequency or claim cost, is itself only a parameter chosen to represent a distribution which has not only a mean, but also a variance and other moments. The classical view takes H to be an unknown constant which may be estimated, but holds that it is meaningless to speak of probabilities concerning H. The Bayesian, on the other hand, is willing to treat H as a random variable, with a distribution which reflects his current uncertainty regarding H.

THE CONCEPT OF FULL CREDIBILITY

In order to use credibility theory in ratemaking, an actuary must first determine the number of claims required for full credibility (Longley-Cook [30] p. 199). He then uses a formula, often based on the ratio of the number of actual claims in the observed data to some function of the number of claims which would be entitled to full credibility, to assign credibilities to data comprising fewer claims than this magic "full credibility" number. If the observed data for a particular classification results in a greater number of claims than the number required for full credibility, the data are taken at face value and are used for ratemaking, without reference to the previous manual rate or any other auxiliary information.

The concept of full credibility has always been rather difficult, philosophically. Some actuaries believe that no data are entitled to 100% credibility and that the credibility curve should approach 1 asymptotically, without ever reaching it. In Bayesian terms, however, the concept merges with that of partial credibilities in a natural and logical way. The Bayesian poses his credibility problem as that of modifying his prior opinion H by some observed data D. If the data are few, there is no reason for him to change H. $P(H|D)$ remains very close to $P(H)$. As the volume of data increases, $P(H|D)$ becomes more and more dependent on D and, finally, $P(H|D)$ comes to depend almost entirely on D. For a large enough volume of data, the posterior distribution is generally almost independent of the prior distribution.

Thus the Bayesian would pose the question of full credibility as: "For what prior distributions are these data fully credible, i.e. for what prior distributions can we say that, for practical purposes, the posterior distribution is independent of the prior distribution because of overwhelming data?" As we increase the volume of data, we increase the family of prior distributions for which this independence of posterior from prior is substantially true. There will, however, always be some prior distributions for which this is not true. For example, if the actuary (or insurance commissioner) chooses a prior distribution which is rather narrow, with all its mass concentrated in an interval close to last year's claim frequency, no amount of data will be sufficient to make him give up his prior opinion entirely, though he may be willing to modify it somewhat.

Most reasonable people will, however, alter their original beliefs if the data do not appear to support them. In actuarial work in particular, one must be exceedingly stubborn to hold to a narrow prior distribution, in the face of contrary evidence, because of the possibility, or even probability, of trends or secular changes in the underlying situation. Accident rates, average claim costs and other such quantities change with time, and the actuary is not, as a rule, surprised to find that this year's data differs somewhat from last year's.

CHOOSING PRIOR PROBABILITIES

The actuary's choice of his prior probability distribution has traditionally been that underlying the previous rate for the classification in question. In experience rating, he takes the manual rate as the mean of the prior distribution, to be modified by the experience of the individual risk. He has never, however, faced the question: "How much confidence do I have in the current rate?"

One way to achieve meaningful results would be to estimate not only the mean but the variance of the present rate level. Then, after choosing a distribution, we can solve for the number of claims to which our prior knowledge is equivalent, which is a function of the mean and variance.

For example, if we believe our data are the result of a Bernoulli process we may assume that $P(H)$ is a Beta function

$$p(h) = K\, h^{r'}(1-h)^{n'-r'}$$

with r' favorable and n'-r' unfavorable outcomes. Then h has mean

$$m = \frac{r'+1}{n'+2}$$

and variance

$$\sigma^2 = \frac{r'+1}{(n'+2)^2} \cdot \frac{(n'-r'+1)}{(n'+3)} = \frac{m\,(1-m)}{n'+3}$$

(See Raiffa and Schlaifer [7] p. 216). Thus

$$n' = \frac{m\,(1-m)}{\sigma^2} - 3$$

represents the validity of the prior knowledge, and a comparison of n' with the number of exposure units in D will indicate the credibility that D deserves relative to H.

If we assume, for example, that the mean accident frequency underlying our present rates is .10 with σ = .005 then

$$n' = \frac{(.10)\,(.90)}{.000025} - 3 = 3597.$$

If we assume that m = .10 but σ = .02 we would, of course, have much less confidence in H than in the previous example. Here

$$n' = \frac{(.10)(.90)}{.0004} - 3 = 222.$$

If $P(H)$ is a Beta function with parameters n' and r', and if our data has a binomial conditional distribution with parameters n and r, we can approach the credibility problem by treating our prior knowledge H as a sample of size n' and our data as a sample of size n. We will then have

$$m_H = \frac{r'+1}{n'+2} \text{ and } m_D = \frac{r}{n}.$$

If we then combine the two sets of "data" into a single "sample" of size $n + n'$, we have:

$$\begin{aligned} m_{H|D} &= \frac{r+r'+1}{n+n'+2} \\ &= \frac{n}{n+n'+2} \cdot \frac{r}{n} + \frac{n'+2}{n+n'+2} \cdot \frac{r'+1}{n'+2} \end{aligned}$$

which has the form $ZM_D + (1 - Z) M_H$. It should be noted that these expressions for $m_{H/D}$ and for Z are the same as those derived later by means of Bayes Theorem.

It should be noted that n', which measures the validity of the prior knowledge, varies directly with m and inversely with σ. This agrees with our intuitive notion that it takes fewer units exposed to risk to produce a given level of credibility if the claim frequency is high than if it is low. (Here this principle results in greater validity for the underlying experience in a classification with a high claim frequency than in one with a lower frequency.) It also seems logical that the validity of any estimate should vary inversely with its standard deviation, since a larger standard deviation indicates a smaller degree of confidence that the values are clustered around the mean.

USING BAYES THEOREM

Once a prior distribution has been chosen, it is necessary to combine the prior distribution with the distribution of the data, in order to obtain a posterior distribution. This was done in 1950 by Arthur Bailey [14] but, since his notation is rather complicated, it will be helpful to restate his results in terms of modern statistical concepts, in an endeavor to show what assumptions actually underly credibility theory.

Let H be the random variable whose value we would like to estimate and let $p(h)$ be the prior distribution of H, before the data have been obtained. Let D be a random variable whose value can be observed, the data, and $f(H|d)$ the posterior distribution of H, given that $D = d$. Let m_D and m_H be the means of D and H and σ_D^2 and σ_H^2 their respective variances. Let ρ be the correlation coefficient between D and H and σ_{HD} the covariance. $g(D|h)$ is the conditional density of D given that $H = h$. To apply Bayes theorem $p(h)$ and $g(D|h)$ must be known or assumed. $g(D|h)$ will reflect the type of chance variation of the data around the "population parameter" h and $p(h)$, since it is a prior distribution, can be chosen to reflect the actuary's prior knowledge and beliefs about the random variable to be estimated. As we shall see, it is convenient to choose $p(h)$ as a conjugate distribution to $g(D|h)$.

It should be noted that the notion of a correlation coefficient between H and D would not be acceptable in classical statistics, since the former is a "parameter" and the latter a "statistic." In the Bayesian view, however, such a correlation is permissible so long as it makes sense to talk about the joint distribution $f(h,d)$ of H and D.

Bailey suggests that we take as our estimator of H:

$$h' = E(H|d) = \int_{-\infty}^{\infty} h\, f(h|d)\, dh$$

$$= \frac{\int_{-\infty}^{\infty} h\, f(h,d) dh}{f_1\,(d)}$$

$$= \frac{\int_{-\infty}^{\infty} h\, f(h)\, g(d|h)\, dh}{\int_{-\infty}^{\infty} f(h)\, g(d|h)\, dh}$$

It should be noted that the conditional expectation $E(H|d)$ is a function of d alone. $E(H|d)$ may be called the regression function of H on D (Hogg & Craig [5] p. 212). $E(H|d)$ may not be linear. Let $x\, m_H + y\, d$ be the "best fitting" approximation to $E(H|d)$, i.e. choose x and y to minimize

$$\int_{-\infty}^{\infty} [E(H|d) - x\, m_H - y\, d]^2\; f_1(d)\mathrm{d}d$$

where

$$f_1(d) = \int_{-\infty}^{\infty} g(d|h)\, p(h)\, dh$$

is the marginal distribution of d. The minimum is obtained by taking

$$x = 1 - \frac{m_D}{m_H}\,\frac{\sigma_{HD}}{\sigma_D^2} = 1 - \frac{m_D}{m_H}\cdot\rho\cdot\frac{\sigma_H}{\sigma_D}$$

and

$$y = \frac{\sigma_{HD}}{\sigma_D^2} = \rho\,\frac{\sigma_H}{\sigma_D}.$$

Thus

$$E(H|d) \approx \left[1 - \frac{m_D}{m_H}\cdot\rho\,.\,\frac{\sigma_H}{\sigma_D}\right] m_H + \rho\,.\,\frac{\sigma_H}{\sigma_D}.\,d$$

Let $A = \dfrac{\rho^2}{y} = \rho\,\dfrac{\sigma_D}{\sigma_H}$ and $B = m_D - A m_H$, then

$$E(H|d) \approx (1-\rho^2)\, m_H + \rho^2 m_H - \rho^2\,\frac{m_D}{A} + \rho^2\,\frac{d}{A}$$

$$= \rho^2\left(\frac{d-B}{A}\right) + (1-\rho^2)\, m_H$$

and this result is exact if $E(H|d)$ is linear. A and B are the coefficients of the regression line of D on H. In particular, if $A = n$ and $B = 0$, which will be seen to be the case if $E(D|h) = nh$ (which is true if $g(d|h)$ is either a binomial or a Poisson distribution):

$$E(H|d) = \rho^2 \frac{d}{n} + (1 - \rho^2)\, m_H$$
$$= Z\, m_D + (1 - Z)\, m_H.$$

It can be seen, then, that the Z used by actuaries as the credibility to be given to observed data, when the data are combined with prior knowledge, is the square of the correlation coefficient between H and D (called by some the "coefficient of determination"). It should be noted that Z has the desired property $Z \leqq 1$ and, because the "best fitting" approximation $h' = x\, m_H + yd$ is defined in an analogous fashion to a least squares regression line, the error variance $E(h - h')^2$ is minimized by this choice of Z. The exact form of Z depends, in any particular case, on the conditional distribution $g(d|h)$ and on the prior distribution $p(h)$.

THE BETA-BINOMIAL

If $g(d \mid h)$ is a binomial distribution, as appears to be true in many branches of insurance,

$$g\,(d \mid h) = \frac{n\,!}{d\,!\,(n - d)\,!}\, h^d\, (1 - h)^{n-d} \qquad (d \text{ "successes" in } n \text{ trials})$$

then

$$E(D \mid h) = \sum_d d \cdot g(d \mid h)$$
$$= nh = Ah + B,$$

hence $A = n$ and $B = 0$ and

$$E(D^2 \mid h) = \sum_d d^2 \cdot g(d|h)$$
$$= \sum_d [d^{(2)} + d]\, g(d \mid h)$$
$$= n\,(n - 1)\, h^2 + nh$$

and

$$\sigma^2_{D|H} = E(D^2|h) - [E(D|h)]^2$$
$$= n\, h\, (1 - h)$$

If we sum over all values of h we get:

$$E(D) = \sum_h E(D \mid h)\, p(h) = \sum_h n\, h\, p(h) = n\, m_H$$

$$\begin{aligned} E(D^2) &= \sum_h E(D^2 \mid h)\, p(h) \\ &= \sum_h n\,(n-1)\, h^2\, p(h) + \sum_h nh\, p(h) \\ &= n\,(n-1)\, E(H^2) + n\, m_H \\ &= n\,(n-1)\,(\sigma_H^2 + m_H^2) + n\, m_H \end{aligned}$$

and

$$\sigma_D^2 = n\,(n-1)\,\sigma_H^2 + n\, m_H\,(1 - m_H)$$

Although nothing has been said so far about the form of $p(h)$, the prior distribution, it will be helpful to take $p(h)$ as a Beta distribution. If $g(d \mid h)$ is binomial, and $p(h)$ has a Beta distribution, $f(h \mid d)$ will also have a Beta distribution. (See Raiffa & Schlaiffer [7] p. 53.)

If

$$g(d \mid h) = \frac{n\,!}{d\,!\,(n-d)\,!}\, h^d\,(1-h)^{n-d}$$

and

$$p(h) = Kh^{r'}\,(1 - h)^{n'-r'},$$

then

$$\begin{aligned} E(H \mid d) &= \frac{\int_0^1 hg(d \mid h)\;p(h)\;dh}{\int_0^1 g(d \mid h)\;p(h)\;dh} \\ &= \frac{K \int_0^1 h^{r'+d+1}\,(1-h)^{n+n'-d-r'}\,dh}{K \int_0^1 h^{r'+d}\,(1-h)^{n+n'-d-r'}\,dh} \\ &= \frac{B(r'+d+2,\quad n+n'-d-r'+1)}{B(r'+d+1,\quad n+n'-d-r'+1)} \\ &= \frac{r'+d+1}{n+n'+2}. \end{aligned}$$

Since $p(h)$ has mean $m_H = \frac{r'+1}{n'+2}$ and variance $\sigma_H^2 = \frac{m(1-m)}{n'+3}$, and since $E(H \mid d)$ is linear in d, we may write:

$$E(H \mid d) = \frac{n}{n+n'+2} \cdot \frac{d}{n} + \frac{r'+1}{n+n'+2}$$
$$= \frac{n}{n+n'+2} \cdot \frac{d}{n} + \frac{n+2}{n+n'+2} \cdot m_H$$
$$= Z\frac{d}{n} + (1-Z)\, m_H,$$

where $Z = \frac{n}{n+n'+2}$. For a fixed n', Z approaches 1 as n gets very large. We may rewrite Z in terms of the mean and variance of the prior distribution:

$$Z = \frac{n\sigma_H^2}{(n+n'+2)\sigma_H^2}$$
$$= \frac{n\sigma_H^2}{(n-1)\sigma_H^2 + (n'+3)\sigma_H^2}$$
$$= \frac{n\sigma_H^2}{(n-1)\,\sigma_H^2 + m_H(1-m_H)}$$
$$= \frac{n}{n+k},$$

where

$$k = \frac{m_H(1-m_H) - \sigma_H^2}{\sigma_H^2},$$

as previously stated.

THE GAMMA-POISSON

If $g(d|h)$ has the Poisson distribution

$$g(d|h) = \frac{(nh)^d\, e^{-nh}}{d!}$$

then:

$$E\,(D|h) = \sum d g(d|h) = nh$$
$$E\,(D^2|h) = \sum d^2\, g(d|h) = n^2\, h^2 + nh$$
$$\sigma_{D|h}^2 = \mathrm{nh}$$

Summing over all values of h,

$$E\,(D) = nE(h) = nm_H$$
$$E\,(D^2) = n^2\, E(h^2) + n\, E(h)$$
$$= n^2\, (\sigma_H^2 + m_H^2) + nm_H$$
$$\sigma_D^2 = n^2\sigma_H^2 + nm_H$$

If we take $p(h)$ to be the Gamma distribution

$$p(h) = \frac{a^r}{\Gamma(r)} \cdot h^{r-1} \cdot e^{-ah}$$

which has mean $\frac{r}{a}$ and variance $\frac{r}{a^2}$, for $h \geq 0$, letting

$$K = \frac{n^d\, a^r}{d!\,\Gamma(r)}$$

we have:

$$E(H|d) = \frac{\int_0^\infty h \cdot p(h) \cdot g(d|h) \cdot dh}{\int_0^\infty p(h) \cdot g(d|h) \cdot dh}$$

$$= \frac{K \int_0^\infty h^{d+r} \cdot e^{-(n+a)h}\, dh}{K \int_0^\infty h^{d+r-1} \cdot e^{-(n+a)h}\, dh}$$

$$= \frac{\int_0^\infty (n+a)^{d+r} \cdot h^{d+r} \cdot e^{-(n+a)h} \cdot (n+a) \cdot dh}{(n+a) \int_0^\infty (n+a)^{d+r-1} \cdot h^{d+r-1} \cdot e^{-(n+a)h} \cdot (n+a)\, dh}$$

$$= \frac{\int_0^\infty x^{d+r} \cdot e^{-x} \cdot dx}{(n+a) \int_0^\infty x^{d+r-1} \cdot e^{-x} \cdot dx}$$

$$= \frac{(d+r)!}{(n+a)\,(d+r-1)!}$$

$$= \frac{d+r}{n+a}\text{, which is linear in } d.$$

Since $p(h)$ has mean $m_H = \frac{r}{a}$ and variance $\sigma_H^2 = \frac{r}{a^2}$, we may rewrite $E(H|d)$ as:

$$E(H|d) = \frac{d+r}{n+a}$$

$$= \frac{n}{n+a} \cdot \frac{d}{n} + \frac{a}{n+a} \cdot \frac{r}{a}$$

$$= Z\frac{d}{n} + (1 - Z)\, m_H,$$

where $Z = \frac{n}{n+a}$ and $a = \frac{m_H}{\sigma_H^2}$.

THREE UNSOLVED PROBLEMS

The credibility tables commonly used in the United States are based on the normal approximation to the Poisson distribution. As has been shown by Harwayne [26], Dropkin [23], Hewitt [27] and others, the two parameter negative binomial distribution provides a better fit to the data than the Poisson. This would seem to indicate the need for new credibility tables in many branches of insurance.

Such new tables could be based on the negative binomial or on the Beta-binomial distribution. However, both of these ignore a very important factor, the distribution of claim size. Most credibility formulas in use today measure the credibility of a given number of claims. What is really needed, however, is the credibility of the pure premium, which depends on claim severity as well as claim frequency.

Let $X_1, X_2, \ldots, X_n$ represent the amounts of the n claims that occur during a given time period. Let us assume that the amount of each claim is independent of the size of any other claim (which might not be true, for example, in a class of policies containing an aggregate limit on benefits paid) and that the X_i are identically distributed. Let $F(x)$ represent the distribution function of the amount of a single claim. $F(x)$ is the probability that the amount of a claim is $\leq x$, given that a claim has occurred. Let N be the number of claims occurring during the time period in question and $p(n)$ represent the probability that $N = n$. The distribution of the total amount of claims paid by the company, i.e. the probability that this amount is $\leq x$, is

$$\sum_{n=0}^{\infty} p(n) \overset{n*}{F}(x)$$

where $\overset{n*}{F}(x)$ is the n-fold convolution of $F(x)$ with itself.

If we assume that N follows a Poisson or a negative binomial distribution, and $F(x)$ an appropriate claim distribution (which might vary by line of business), the distribution of the total amount of claims or, better still, the distribution of the pure premium, should produce more accurate credibilities than those now in use.

In many branches of property insurance, the distribution of claim size seems to follow a log-normal distribution (Benckert [19] and Bailey [16]). The convolutions of the log-normal, unfortunately, cannot be obtained in closed form. Mathematically, the easiest distribution to use is the Gamma distribution since, if $F(x)$ has a Gamma distribution with parameters (r,a), then $F^{n*}(x)$ has a Gamma distribution with parameters (nr,a). However, the log-normal distribution has greater skewness than the Gamma. Other distributions that may be useful for claim severity are the Pearson Types V and VI and the Pareto distribution, which is a special case of the Pearson Type VI. An analysis of these and many other distributions will be found in Kupper [29].

Unsolved problem number 1 is a statistical problem—to calculate, from insurance company records, the claim distribution $F(x)$ for various branches of property and casualty insurance.

Unsolved problem number 2 is to work out the convolutions, thus determining the joint distribution of claim frequency and claim severity. The Esscher approximation (See Cramér [1] p. 33) is one method of calculating the convolutions. Several methods of numerical integration, using electronic calculators, are described by Bohman and Esscher [20] who used one of these methods, based on the characteristic function of $F(x)$, to compute some claim distributions for life insurance and fire insurance in Sweden.

Unsolved problem number 3 is to obtain the pure premium distribution from the distribution of total claims and use it to compute credibility tables. Presumably it would be possible to choose a prior distribution for the pure premium, obtain some data, and apply Bayes Theorem to compute the posterior distribution from the prior distribution and the conditional distribution of the data. Since the analysis would be more complicated, mathematically, than an analysis involving Poisson, binomial, Beta and Gamma distributions, it may be necessary to use approximate methods. If mathematically more tractable distributions are substituted for the unruly empirical distributions that may result, it will then be necessary to obtain a measure of the error thus introduced.

CONCLUSION

Bayesian statistics, a new approach to the foundations of statistics, has at last enabled the casualty actuary to derive a sound theoretical foundation for his own work in credibility theory and related fields. This paper will have achieved its purpose if it has pointed the way towards the construction of such a foundation and if it has encouraged others to take up the work.

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ESTIMATING THE COST OF ACCIDENT INSURANCE AS A PART OF AUTOMOBILE LIABILITY INSURANCE

HERBERT E. WITTICK

In April 1960, a select committee was appointed by the Legislature of the Province of Ontario "to examine, investigate, enquire, study, and report on all matters relating to persons who suffer financial loss as the result of motor vehicle." In the first interim report made in March 1961, they dealt at considerable length with the various situations which exist in different provinces and states and in particular covered in detail the system used in Saskatchewan where certain basic payments for death, dismemberment and disability are made to persons injured in automobile accidents regardless of fault. The Saskatchewan system is compulsory and is administered by the government insurance office. The legal rights of injured persons to sue under the ordinary rules of negligence are preserved but suit may only be brought for any excess amount which may be recoverable over the amount already received under the basic compensation plan. Thus the system represents a compromise between the ordinary negligence system and a full Workmens' Compensation type plan. The Select Committee were attracted to some basis whereby injured persons would be compensated at least to a certain extent regardless of fault and the following is an excerpt from their report:

> "The Committee is interested in the principle of compensation regardless of fault. Its departure from the traditional concept of fault-liability reflects a view, held in some quarters, that responsibility for automobile accidents rests on society as a whole, rather than on individuals, and that the task of establishing responsibility in this age of many complexities imposes too great a burden on those who settle or adjudicate claims. A compensation plan provides a measure of indemnification for a group who, under the traditional fault-liability system, are not entitled to indemnification. The normal third party liability insurance system provides nothing for the surviving dependents of the negligent party or for the negligent party himself when, as a result of his own injuries, he is disabled for life. There are other situations where the operations of the concept of negligence denies redress to individuals. It is rather surprising to the Committee that, over the years, the insurance industry has not reacted more positively in this particular area of concern."

Actually the companies and the provincial insurance superintendents were already in the process of redrafting the uniform automobile insurance act to permit the inclusion of accident benefits in the automobile policy. This was pointed out to the Select Committee and work on the new act was accelerated. A draft was submitted at the September 1961 Winnipeg meeting of the superintendents with provisions making it possible to include accident insurance for drivers, passengers and pedestrians under automobile liability policies. The insurance contemplated is not only for medical expenses but also for death, dismemberment and weekly indemnity covering all persons injured in or by automobiles. Most importantly the proposed legislation is designed to prevent duplication of payments and thus keep down the additional cost of the universal benefit. Thus:

a. The occupant of an automobile or a pedestrian struck by such automobile will receive the accident benefits from the insurer thereof.

b. The injured person may not sue either the motorist with whom he is riding or by whom he is struck except for any amount which may be recoverable at law in excess of what he has already been paid.

c. The injured occupant may not sue another motorist who carries the coverage except for such excess. Since the Canadian provinces all operate on a basis of comparative negligence and the liability of the driver is imputed to the passenger, it is expected that persons who are only entitled to collect a small percentage of their damage at law will make no claim except for accident benefits. If suit is brought and the amount of damages assessed is less than the accident benefits received, the action is deemed to have failed and the plaintiff is liable for all costs.

d. There will be no subrogation on the part of the insurers against any person also carrying the accident coverage but subrogation is provided against out-of-province motorists and against persons other than motorists who may have caused the accident.

The act will provide that any claimant must disclose the name of the insurer of the automobile of which he was an occupant and the amount of accident benefits received. At the Winnipeg meeting there was considerable discussion of the proposals and agreement was reached on the general principles. In the fall of 1961 and the first part of 1962 various meetings were held by an extended all industry committee set up by the All Canada Insurance Federation working in conjunction with the insurance superintendents.

A revised draft bill was prepared for discussion at the meeting of the superintendents at Regina in September 1962 and this was distributed to all insurers in July. Prior to the meeting there was rather violent objection on the part of some United States companies who felt that this legislation was the opening wedge for a full compensation plan and should be strongly opposed. However because of the definite views of the Ontario Select Committee, the insurers generally agreed opposition was not a practical matter and that a neutral position was the only tenable one. Thus a report was prepared explaining the companies' stand which was that they could not in principle recommend a program which would increase the cost of automobile insurance and that government would have to bear the responsibility for doing so. At the same time the companies made it clear that they were at all times willing and able to provide any form of protection which was required. With this explanation, the draft bill met with the general approval of the meeting and the superintendents likewise approved it in their own closed sessions. Certain changes in the act were still found necessary so final action was deferred for another year.

At this same time the Select Committee approached the All Canada Insurance Federation and asked that the companies estimate the additional cost of providing the basic automobile accident coverage, both on a voluntary basis and on a basis whereby it would become a mandatory part of automobile liability insurance.

The problem was unique because ordinary accident insurance is based on a cost per person and what we had to determine was the cost per automobile. In considering the problem it was evident that the cost of accident benefits would vary by class of risk and territory in the same manner as Third Party Liability costs, since it is largely the frequency of accidents which accounts for the experience differentials. Thus we decided that what we wanted to establish was the additional cost of the accident coverage in terms of a percentage of the standard $35,000 inclusive limit liability premiums. A definite advantage of this approach would be that if accident frequency and severity increased between the time of calculation and the time of application, the insurers would not be bound, to dollar figures but only to percentages. Actually there has been an almost unbelievable 40% increase in persons injured in automobile accidents in Ontario since 1961 and the wisdom of this approach has been clearly demonstrated.

In Ontario, the Department of Transport publishes excellent and detailed statistics of automobile accidents and it is thus possible to quite accurately determine what the cost would have been if all persons had received the benefits proposed under the new legislation. The primary interest of the Select Committee was with respect to private passenger automobiles so we limited our study accordingly. Therefore, what we set out to do was to estimate the total cost of the benefits, separately by drivers, passengers and others, which would have been payable on the actual 1961 accidents. Once the gross cost was determined we could estimate the amounts already being paid under Third Party Liability and so arrive at an increased net percentage cost. We had no precedent for this approach but it did seem to provide a reasonable basis for estimating the extra cost of the benefits. We did consider approaching the problem from the cost of individual accident insurance but this seemed to involve the wide use of unsupported estimates. The total cost basis therefore seemed the best available. The committee agreed that the amount of benefits to be used in calculating the costs would be as follows:

1. Medical Expenses with a limit of $2,000 (except for Funerals) but for hospital amounts only in excess of the provincial plans.
2. Funeral Benefit with a flat amount of $350.
3. Death Benefit:

	Sex and Marital Status		
Age	Married Male	Married Female	Unmarried
4 yrs and under	$ —	$ —	$ 250
over 4 but under 18	5,000	2,500	500
18 but under 60	5,000	2,500	1,000
60 but under 70	3,000	1,500	1,000
70 and over	2,000	1,000	1,000

In addition $1,000 would be payable for each child under age 18 or a child over 18 wholly dependent because of physical or mental infirmity. For the death of a widow or widower the same benefits would be paid as for a married male of equal age. The amount for the death of other unmarried persons over 18 is payable only if there is a wholly dependent parent.

4. Dismemberment Benefit (major losses only):

   $5,000 for loss of two legs, arms or eyes or one of each.
   3,750 for loss of one arm or one leg.
   2,500 for loss of one hand or one foot.

5. Weekly Disability Benefit for Employed Persons:

   Subject to a 7 day waiting period, $5 per day would be payable for 26 weeks total disability based on inability to carry on *his* occupation, then for a further 78 weeks based on inability to carry on *any* occupation and then a further 2 years if total and permanent disability exists. Thus the maximum would be $7,245. Also housewives not otherwise employed are to receive $3.50 per day for 12 weeks if confined to a hospital. No payments whatsoever are to be made to other persons who are not employed.

The persons to whom benefits are payable are (1) the driver and passengers in the insured car, and (2) pedestrians, cyclists, and others while not occupants of an automobile who are struck by the insured car. Also the insured and his dependent relatives residing with him are insured as occupants of automobiles other than those owned in his household and as pedestrians but only for any amounts by which the insured's coverage exceeds that of any other similar insurance.

The first information we decided was necessary for the calculation of the gross cost was as follows:

1. The number of persons killed as (a) Drivers, (b) Passengers, and (c) Pedestrians, cyclists, and others, with each group subdivided by age group and sex.
2. The same break down for persons non-fatally injured.

This information was readily available in the *Accident Facts* publication of the Ontario Department of Transport and the extracts therefrom are shown as Exhibits A and B. A total split only was available by sex so we used the percentages by age group for passengers and pedestrians, and modified slightly the split for drivers. Exhibits C and D show our calculations for the number of persons killed and injured in each group.

The next problem was to determine the average cost of the death or injury of a person of each age group and sex. We first considered the cost of fatal accidents for which the amount payable depended on four factors as follows: (1) sex, (2) age, (3) marital status, and (4) number of dependents. Our calculations are shown on Exhibit E and admittedly there was a considerable margin of judgement used. We had the Canadian Government Statistics to guide us but unfortunately there was not enough detail along some of the lines in which we were particularly interested. In the case of non-fatal injuries, the factors affecting the cost of a claim are: (a) whether the injured person was employed or in the case of an unemployed female, whether she was a housewife, and (b) the average length of disability. Our calculations are shown on Exhibit F.

Our basis for estimating the cost of medical payments was somewhat different because we had insurance statistics which gave the pure premium for coverage as respects drivers and passengers. We applied this pure premium to the total number of private passenger automobiles and arrived at the average cost of claims. By applying this cost to the number of accidents we calculated the total cost for drivers, passengers and others. Exhibit G shows the details. Funeral expense costs were calculated separately under Exhibit H. Having established average values for the cost of fatal and non-fatal injuries to persons of each sex and age group, and medical costs, we proceeded to complete Exhibits H and I.

The final summary involved the combination of the information shown under Exhibits H and I and gave us the estimate of the total gross cost of the accident benefits divided as to drivers, passengers and pedestrians. However it is obvious that a substantial part of these amounts are already being paid under the present negligence system. Since amounts received under the accident plan may not be included in any claim based on negligence, the amount of third party liability claims will be reduced. This was an area in which there was little to guide the Committee except pure judgement. The province of Ontario operates on a comparative negligence system so in theory the average recovery of drivers should be 50% in cases where two automobiles are involved. In practise the percentage of out-of-pocket expenses paid is probably closer to 75% since in settling personal injury claims the discussion of splits of negligence is minimized in order to reach agreement on the amount payable. Also some 23% of the accidents involve collision with fixed objects, railway trains or street cars, or just the upset of the automobile, and in these cases there are no payments being made today under Third Party Liability. As a compromise we decided to use a figure of 40% as the amount by which the gross benefit costs to drivers would be offset by savings under Third Party Liability. Since gratuitous passengers have no rights of recovery in Ontario against the motorist with whom they are riding and may recover against another motorist only to the percentage by which their driver was not negligent, we used the same 40% reduction for passengers. For pedestrians we used a reduction figure of 95% because we agreed that with the onus of disproving negligence lying with the motorist, it was the practise to pay out-of-pocket expenses to pedestrians in practically every case. In total then we estimated that the net cost of providing the benefits would be just slightly over 50% (52% to be exact) of the gross cost.

The only remaining problem was to relate the net cost to the premiums for Third Party Liability insurance. This calculation is shown on Exhibit J along with the summary of the gross and net costs. The final percentage reached was 12.6% of the standard $35,000 inclusive limit premium. This may seem more in line when we realize that of the inclusive limit premium 55% applies to property damage and that in relation to bodily injury charges we are talking about a 28% increase. In 1963 our latest rates vary from $22 for a claim-free farmer to $177 for an unmarried, underage male driver with a claim in the last year. Thus the cost of the accident benefits would be from approximately $3 to $22. The ordinary claim-free class B risk (that is a risk with no underage male driver where the automobile is driven to work less than 10 miles) is now paying around $40 so the extra cost to him would be about $5. Naturally any increase in claim frequency will increase these dollars costs. The 40% increase in the number of persons injured in Ontario during the first quarter of 1963 over the same period of 1961 makes the dollar figures quite obsolete.

All of our costing related to private passenger automobiles and it is recognized that the cost with respect to commercial automobiles will not be the same either in dollars or in percentage. In the first place most drivers and passengers in commercial vehicles receive the benefits of the workmens compensation law and are thus excluded from coverage under the accident plan. Secondly the average number of passengers is much less than in private passenger cars and this will reduce costs per vehicle even when compensation does not apply. Therefore a separate calculation will be necessary. Our calculations were submitted to the Select Committee and it was pointed out that the estimated 12.6% additional cost was for mandatory coverage with benefits offset against Third Party Liability. It was explained that costs on a voluntary basis would be at least twice that of a mandatory plan because there could be no offset and that probably it would be even higher because of a selection of risks against the insurers.

The Select Committee was enthusiastic about the proposed plan and at a meeting with them one member told us that he felt this was the finest plan in existence for providing payments to the victims of automobile accidents. A substantial secondary benefit to the insurers in their approval of this plan was the Select Committee's definite rejection of a proposal to change the guest passenger law. They felt that the extra premium dollars involved in the cost of either a full or gross negligence passenger hazard law would be better spent for accident insurance benefits.

The Ontario Select Committee made their final report in March 1963 and unanimously recommended the adoption of the accident plan as a mandatory part of Third Party Liability insurance. No definite date has been set but it is expected to be effective by January 1, 1965. The insurance companies are now drafting a new standard automobile policy with a special section covering the accident benefits.

In general, the companies in Canada welcome this new approach to automobile bodily injury insurance because the feeling is that it will serve to improve their relationship with the public. With basic benefits paid promptly to everyone injured in an automobile accident by his own insurance company, it is expected that automobile insurance will be viewed in a new and better light. Also it is thought that there will be less litigation with a consequent saving in legal expenses. It will be very interesting to see how this accident plan does work in practise and whether it will spread to other jurisdictions. It will also be interesting to see how close our estimates come to the actual cost of the accident benefits.

AUTOMOBILE ACCIDENTS

ONTARIO 1961

Exhibit A

PERSONS FATALLY INJURED, BY CLASS AND AGE GROUP

	Totals	%	Driver	%	Passenger	%	Pedestrian	%	Bicyclist	%	Motorcycle Driver	%	Motorcycle Pass.	%	Others	%
0-4 Years	59	4.7	-	-	15	3.8	44	14.1	-	-	-	-	-	-	-	-
5-14 "	122	9.6	-	-	29	7.4	68	21.8	23	72.0	-	-	-	-	2	15.4
15-19 "	136	10.7	44	8.8	64	16.3	16	5.1	3	9.4	7	43.7	-	-	2	15.4
20-24 "	159	12.5	97	19.3	50	12.8	5	1.6	-	-	6	37.5	1	-	-	-
25-34 "	188	14.8	105	20.9	62	15.8	14	4.5	2	6.2	3	18.8	-	-	2	15.4
35-44 "	147	11.6	89	17.7	41	10.4	15	4.8	-	-	-	-	-	-	2	15.4
45-54 "	160	12.6	77	15.3	52	13.3	27	8.7	1	3.1	-	-	-	-	3	23.0
55-64 "	122	9.6	50	10.0	36	9.2	32	10.2	2	6.2	-	-	-	-	2	15.4
65 & Over	175	13.8	40	8.0	43	11.0	91	29.2	1	3.1	-	-	-	-	-	-
Not Stated	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Total	1,268	100.0	502	100.0	392	100.0	312	100.0	32	100.0	16	100.0	1	-	13	100.0

PERSONS NON-FATALLY INJURED, BY CLASS AND AGE GROUP

	Totals	%	Driver	%	Passenger	%	Pedestrian	%	Bicyclist	%	Motorcycle Driver	%	Motorcycle Pass.	%	Other	%
0- 4 Years	1,951	5.3	-	-	900	5.8	1,051	18.5	-	-	-	-	-	-	-	-
5-14 "	4,876	13.1	10	.07	1,561	10.2	2,283	40.2	1,001	78.1	1	0.2	6	5.6	14	20.9
15-19 "	4,958	13.3	1,595	11.20	2,644	17.3	294	5.2	160	12.5	191	40.4	68	62.4	6	9.0
20-24 "	5,290	14.2	2,678	18.80	2,248	14.7	187	3.3	16	1.2	133	28.1	24	22.0	4	6.0
25-34 "	6,844	18.4	3,955	27.77	2,432	15.9	317	5.6	24	1.9	99	20.9	7	6.4	10	14.9
35-44 "	5,121	13.8	2,827	19.85	1,925	12.6	321	5.6	14	1.1	23	4.9	1	0.9	10	14.9
45-54 "	3,592	9.7	1,755	12.32	1,451	9.5	335	5.9	26	2.0	15	3.2	1	0.9	9	13.4
55-64 "	2,251	6.1	946	6.64	913	6.0	356	6.3	22	1.7	7	1.5	-	-	7	10.4
65 & Over	1,632	4.4	448	3.15	686	4.5	477	8.4	11	0.9	3	0.6	-	-	7	10.4
Not Stated	631	1.7	28	.20	531	3.5	62	1.0	7	0.6	1	0.2	2	1.8	-	-
Total	37,146	100.0	14,242	100.0	15,291	100.0	5,683	100.0	1,281	100.0	473	100.0	109	100.0	67	100.0

AUTOMOBILE ACCIDENTS - ONTARIO 1961

Exhibit B

VICTIMS, BY AGE GROUP AND SEX	Total	0-4	5-14	15-19	20-24	25-34	35-44	45-54	55-64	65 and over	Not Stated
PERSONS FATALLY INJURED											
1. Male	935	35	88	103	134	150	111	115	76	125	
2. Female	331	24	34	33	25	38	36	45	46	50	
Total	1268	59	122	136	159	188	147	160	122	175	
Persons Non-fatally injured											
1. Male	23514	1169	3134	3187	3738	4728	3101	2025	1274	878	280
2. Female	13632	782	1742	1771	1552	2116	2020	1567	977	754	351
Total	37146	1951	4876	4958	5290	6844	5121	3592	2251	1632	631

Exhibit C

FATALS

1961 NUMBER OF PERSONS KILLED ONTARIO HIGHWAY ACCIDENT STATISTICS

	ALL AUTOMOBILES	PRIVATE (1) PASSENGER	(2) MALE %	(2) MALE NUMBER	(2) FEMALE %	(2) FEMALE NUMBER
DRIVERS		89%				
4 and under	-			-	40%	-
5-14	-			-	28	-
15-19	52	46		34	24	12
20-24	105	94		77	16	17
25-54	284	252		181	25	71
55-64	50	45		26	38	19
65 and over	40	36		24	30	12
TOTAL	531	473		342		131
PASSENGERS		95%				
4 and under	15	14	60%	8	40%	6
5-14	29	28	72	20	28	8
15-19	64	61	76	46	24	15
20-24	51	48	84	40	16	8
25-54	155	148	75	111	25	37
55-64	36	34	62	21	38	13
65 and over	43	41	70	29	30	12
TOTAL	393	374		275		99
PEDESTRIANS & CYCLISTS		89%				
4 and under	43	39	60%	23	40%	16
5-14	93	82	72	59	28	23
15-19	20	18	76	14	24	4
20-24	4	4	84	3	16	1
25-54	56	50	75	38	25	12
55-64	36	32	62	20	38	12
65 and over	92	81	70	57	30	24
TOTAL	344	306		214		92
GRAND TOTAL	1268	1153		831		322

NOTE (1) MOTORCYCLES are treated as NON PASSENGER Automobiles.

(2) See note (2) for NON FATAL accidents concerning adjustment in split for DRIVERS.

Exhibit D

NON FATAL

1961 NUMBER OF PERSONS INJURED ONTARIO HIGHWAY ACCIDENT STATISTICS

	ALL AUTOMOBILES	PRIV. (1) PASS.	PRIV. PASS. +15% (3)	(2) MALE %	NUMBER	(2) FEMALE %	NUMBER
DRIVERS		89%					
4 and under	-				-	40%	-
5-14	11	10	11		7	35	4
15-19	1786	1590	1829		1090	36	739
20-24	2811	2502	2877		1907	30	970
25-54	8703	7745	8907		5204	37	3703
55-64	953	848	975		504	43	471
65 and over	451	401	461		222	46	239
TOTAL	14715	13096	15060		8934		6126
PASSENGERS		95%					
4 and under	900	855	983	60%	590	40%	393
5-14	1567	1487	1710	65	1111	35	599
15-19	2712	2577	2963	64	1896	36	1067
20-24	2272	2158	2482	70	1737	30	745
25-54	6350	6034	6939	63	4372	37	2567
55-64	913	867	997	57	568	43	429
65 and over	686	652	750	54	405	46	345
TOTAL	15400	14630	16824		10679		6145
PEDESTRIANS AND CYCLISTS		86%					
4 and under	1051	903	1038	60%	623	40%	415
5-14	3298	2832	3257	65	2117	35	1140
15-19	460	396	455	64	291	36	164
20-24	207	178	205	70	143	30	62
25-54	1142	981	1128	63	711	37	417
55-64	378	325	374	57	213	43	161
65 and over	495	426	490	54	265	46	225
TOTAL	7031	6041	6947		4363		2584
GRAND TOTAL	37146	33767	38831		23976		14855

NOTE (1) MOTORCYCLES are treated as NON PASSENGER automobiles.

(2) Since commercial drivers are very largely MALE, the split for DRIVERS was adjusted by using for FEMALES the ordinary percentages of the figures for ALL AUTOMOBILES plus a 15% adjustment for non reported injuries, and the figures for MALE was taken as the balance of the adjusted PRIVATE PASSENGER figures.

(3) All figures were increased 15% to cover injuries not shown on the police reports. The Department of Transport statistician concurred in the use of this adjustment.

Exhibit E

ACCIDENT COST SUMMARY SHEET

FATALS (Exc. Funeral Costs)

MALE

AGE	SINGLE WITH DEP. PARENT %		SINGLE OTHER %		MARRIED %		AVERAGE VALUE
4 and under			100	$ 250.			$ 250.
5 - 14			100	500.			500.
15 - 19	2	$1000.	96	300.(a)	2	$5000.	408.
20 - 24	4	1000.	20	-	76	6000.	4600.
25 - 54	6	1000.	10	1200.(b)	84	7000.	6060.
55 - 64	1	1000.	17	-	82	42000.	3454.
65 and over			27	-	73	2300.(c)	1679.

FEMALE

AGE	SINGLE WITH DEP. PARENT %		SINGLE OTHER %		MARRIED %		AVERAGE VALUE
4 and under			100	$ 250.			$ 250.
5 - 14			100	500.			500.
15 - 19	2	$1000.	94	300.(a)	4	$2500.	396.
20 - 24	4	1000.	16	-	80	2500.	2040.
25 - 54	6	1000.	10	1200.(b)	84	2500.	2280.
55-64	6	1000.	19	-	80	2000.	1610.
65 and over			40	-	60	1150.(c)	690.

(a) No payments for ages 18 or 19 except to a DEPENDANT parent.

(b) 20% of SINGLE-OTHER in this age group are calculated as having on the average two dependent children (20% of $6,000 = $1200.)

(c) Principal sum is reduced to 60% at age 60 and to 40% at age 70.

Government population statistics were used to estimate percentages of married and single and the number of children per household.

Exhibit F

NON FATAL		ACCIDENT COST SUMMARY SHEET					WEEKLY INDEMNITY		
	% OF CLAIMS	EMPLOYED PERSONS NO. WEEKS	AMOUNT	VALUE		HOUSEWIVES NO. WEEKS	AMOUNT	VALUE	
MEDICAL ONLY	71.5%								
TEMPORARY TOTAL	27.0	7	$35.	$245.	66.15	7	$25.	$175.	$47.25
PERM. & SERIOUS	1.5	52	35.	1820.	27.30 93.45				
			USE		$93.			USE	$47.

Splits for Medical only, Temporary total and serious were based on figures of the Ontario Workmens Compensation Board but average length of temporary disability was taken as 8 weeks instead of 4 weeks.

AGE	MALE EMPLOYED %		MALE UNEMPLOYED %		AVERAGE VALUE	FEMALE EMPLOYED %		UNEMPLOYED SINGLE %	UNEMPLOYED MARRIED %		AVERAGE VALUE
4 and under	0					0					
5 - 14	0					0					
15 - 19	62	$93.	38	-	$57.66	42	$93.	50	8	$47.	$42.82
20 - 24	92	93.	8	-	85.56	56	93.	2	42	47.	71.82
25 - 54	98	93.	2	-	91.14	30	93.	4	66	47.	58.92
55 - 64	90	93.	10	-	83.70	24	93.	6	70	47.	55.22
65 and over	24	93.	76	-	22.32	4	93.	40	56	47.	30.04

Percentages of persons employed by age groups were obtained from Canadian Government statistics. Persons in the labor force but temporarily unemployed were treated as "Employed".

Exhibit G

MEDICAL PAYMENTS

From Gov't Auto Insurance Statistics		Number of Private Passenger Cars
Number of car years	Farm	93,858
earned (87% of written)	Other	1,170,115
		1,263,973
Car years insured (100%)		1,452,842
Since 91% of autos were insured		
total was		1,596,530
Plus 3% for priv. pass. autos		
insured under fleets, gov't etc.		1,644,426

Medical Payment Pure Prem. from Insurance Statistics	$1.72
Total losses if all cars insured	$ 2,828,413.00
Deduct $500. each for 847 fatals	423,500.00
Total cost for DRIVERS AND PASSENGERS	2,404,913.00
Number of accidents to Drivers & Passengers (per exhibit D)	31,884
Cost per claim	$75.43

DRIVERS	31,884	(15,060	$1,135,975.
PASSENGERS		(16,824	1,269,034.
OTHERS		6,947	524,012.
GRAND TOTAL MEDICAL (EX. FUNERAL)			$2,929,021.

COST SHEET

Exhibit H

FATALS

AGE	MALE NUMBER KILLED	MALE VALUE	TOTAL COST	FEMALE NUMBER KILLED	FEMALE VALUE	TOTAL COST	M & F TOTAL COST
DRIVERS							
4 and under	-	$ 250.	-	-	$ 250.	-	
5 - 14	-	500.	-	-	500.	-	
15- 19	34	408.	13,872.	12	396.	4,752.	
20 - 24	77	4600.	354,200.	17	2040.	34,680.	
25- 54	181	6060.	1,096,860.	71	2280.	161,880.	
55 - 64	26	3454.	89,804.	19	1610.	30,590.	
65 and over	24	1679.	40,296.	12	690.	8,280.	
TOTAL	342		1,595,032.	131		240,182.	1,835,214.
PASSENGERS							
4 and under	8	$ 250.	2,000.	6	$ 250.	1,500.	
5 - 14	20	500.	10,000.	8	500.	4,000.	
15 - 19	46	408.	18,768.	15	396.	5,940.	
20 - 24	40	4600.	184,000.	8	2040.	16,320.	
25 - 54	111	6060.	672,660.	37	2280.	84,360.	
55- 64	21	3454.	72,534.	13	1610.	20,930.	
65 and over	29	1679.	48,691.	12	690.	8,280.	
TOTAL	275		1,008,653.	99		141,330.	1,149,983.
PEDESTRIAN AND CYCLISTS							
4 and under	23	$ 250.	5,750.	16	$ 250.	4,000.	
5 - 14	59	500.	29,500.	23	500.	11,500.	
15 - 19	14	408.	5,712.	4	396.	1,584.	
20 - 24	3	4600.	13,800.	1	2040.	2,040.	
25 - 54	38	6060.	230,280.	12	2280.	27,360.	
55 - 64	20	3454.	69,080.	12	1610.	19,320.	
65 and over	57	1679.	95,703.	24	690.	16,560.	
TOTAL	214		449,825.	92		82,364.	532,189.
GRAND TOTAL	1153		$ 3,053,510.			$463,876.	3,517,386.

	DEATH	*DISMEMBERMENT 5%	D & D	FUNERALS $350. EACH
DRIVERS	1,835,214.	91,761.	1,926,975.	165,550.
PASSENGERS	1,149,983.	57,499.	1,207,482.	130,900.
PEDESTRIANS ETC.	532,189.	26,609.	558,798.	107,100.
	3,517,386.	175,869.	3,693,255.	403,550.

* The cost of Major Dismemberments was taken as 5% of the cost of FATALS based on ordinary accident cost relationships.

Exhibit I

Passenger Cars

COST SHEET

NON FATALS

AGE	MALE NUMBER INJURED	MALE VALUE	MALE TOTAL COST	FEMALE NUMBER INJURED	FEMALE VALUE INC. H'WIVES	FEMALE TOTAL COST	TOTAL
DRIVERS							
4 and under	-	-	-	-	-	-	
5 - 14	7	-	-	4	-	-	
15 - 19	1090	5766.	$ 62,849.	739	4282.	31,644.	
20 - 24	1907	8556.	163,163.	970	7182.	69,665.	
25 - 54	5204	9114.	474,293.	3703	5892.	218,181.	
55 - 64	504	8370.	42,185.	471	5522.	26,009.	
65 and over	222	2232.	4,955.	239	3004.	7,180.	
TOTAL	8934		747,445.	6126		352,679.	1,100,124.
PASSENGERS							
4 and under	590	-		393	-	-	
5 - 14	1111	-		599	-	-	
15 - 19	1896	5766.	109,323.	1067	4282.	45,689.	
20 - 24	1737	8556.	148,618.	745	7182.	53,506.	
25 - 54	4372	9114.	398,464.	2567	5892.	151,248.	
55 - 64	568	8370.	47,542.	429	5522.	23,689.	
65 and over	405	2232.	9,040.	345	3004.	10,364.	
TOTAL	10679		712,987.	6145		284,496.	997,483.
PEDESTRIANS AND CYCLISTS							
4 and under	623	-		415	-	-	
5 - 14	2117	-		1140	-	-	
15 - 19	291	5766.	16,779.	164	4282.	7,022.	
20 - 24	143	8556.	12,235.	62	7182.	4,453.	
25 - 54	711	9114.	64,801.	417	5892.	24,570.	
55 - 64	213	8370.	17,828.	161	5522.	8,890.	
65 and over	265	2232.	5,915.	225	3004.	6,759.	
TOTAL	4363		117,558.	2584		51,694.	169,252.
GRAND TOTAL	23976		1,577,990.	14855		688,869.	2,266,859.

	WEEKLY INDEMNITY	MEDICAL
DRIVERS	1,100,124.	1,135,975.
PASSENGERS	997,483.	1,269,034.
PEDESTRIANS ETC.	169,252.	524,012.
	2,266,859.	2,929,021.

Exhibit J

1961 COSTS

ONTARIO BASIS AUTOMOBILE ACCIDENT BENEFIT SUMMARY

PASSENGER CARS

	DEATH & DIS.	FUNERAL	MEDICAL	WEEKLY INDEMNITY	GROSS TOTAL	EST. NOW PAID *	NET COST
DRIVERS	1,926,975.	154,550.	1,135,975.	1,100,124.	4,328,624.	1,731,450.	2,597,174.
PASSENGERS	1,207,482.	130,900.	1,269,034.	997,483.	3,604,899.	1,441,960.	2,162,939.
PEDESTRIANS AND CYC.	558,798.	107,100.	524,012.	169,252.	1,359,162.	1,291,204.	67,958.
TOTAL	3,693,255.	403,550.	2,929,021.	2,266,859.	9,292,685.	4,464,614.	4,828,071.

(DRIVERS 40%
*(PASSENGERS 40%
(PEDESTRIANS 95%

1961 PRIVATE PASSENGER THIRD PARTY PREMIUMS

87% EARNED	$52,071,000.
WRITTEN (INC. ADJ. OF +3% FOR CARS INSURED UNDER FLEETS AND GARAGES AND GOV'T CARS)	$61,647,000.
ADJ. TO 100% (ACTUAL INSURED 91%)	$67,744,000.
TOTAL LOSSES AT 63% - LIMITS	$42,678,000.
$35,000. Inc. (90% of above)	$38,410,000.
1,644,400 car years.	

ACCIDENT COST AS A % OF

THIRD PARTY LOSSES ($35M.)	
GROSS	24.2%
NET	12.6%

AVERAGE PURE PREMIUM PER AUTO. (1961)	
GROSS	$5.65
NET	$2.94

DISCUSSION BY CARL L. WILCKEN

Mr. Wittick's article is an important addition to our *Proceedings* as it is the first published information on this coverage, which I prefer to call Basic Bodily Injury Accident Benefits.

Mr. Wittick has outlined fully the history behind the coverage in his opening remarks. The quote from the Ontario Government Select Committee contains some of the controversial public-responsibility arguments for this type of coverage. There is some question whether the Insurance Industry should agree with these philosophies or not. Regardless, it is hoped the coverage will provide for our Industry a highly improved public image as regards immediate payment of basic Bodily Injury benefits and in many cases, Property Damage benefits. This intangible benefit to the Industry in conjunction with potential knock-for-knock savings, on which Past-President L. H. Longley-Cook remarked in his Presidential Address of November 1962, makes this coverage a potentially dominating one in North American Automobile Insurance.

Mr. Wittick merits congratulations for first, accepting the request to cost this new coverage and second, doing so in a very short time with little factual information. There are less than a handful of experienced Casualty Actuaries in Canada. As a result Mr. Wittick could neither defer to other Actuaries nor bring many actuarial minds together on the problem. Mr. C. H. Fredrickson, F.C.A.S. retired, reviewed Mr. Wittick's thinking when this initial research was done. The basic approach was agreed upon and it appears to me to be logically sound.

The paper outlines quite clearly the method used to cost the five benefits, if the ten exhibits are followed in sequence. The difficulty I found in understanding some of the exhibits was that Mr. Wittick is often too succinct in his column headings and footnotes. In one respect this adds a note of interest to some exhibits, as the reader must make assumptions and then do some detective work. Usually these assumptions can be verified by interlocking information in other exhibits or the main article. However, to assist the reader in this detective work I have listed at the end of this discussion the Death Benefits for which Average Values are determined in Exhibit E.

There are numerous areas in Exhibits C through J where judgment was used, often of necessity. In this sense, the over-all costing might be criticized as not being rigorous enough. Undoubtedly other individuals would develop different costs, using the same method, for one or more of the five benefits. Individuals who are primarily concerned with the Accident and Health class of business may disagree extensively with some of Mr. Wittick's values. However, to use a British Canadian phrase, the Industry expects the "swings and round abouts" to play their part in this initial cost estimate.

My experience in Accident and Health is very limited and the only two areas I would comment on are the following. In Exhibit E, there are six age groups in the Male and Female Single-Other columns and the two 65 and over age groups in the first column which contain no costs. Some cost is incurred in all eight of these ranges for the small percentage of deceased who had dependents. These costs are undoubtedly small but nevertheless costs. Reference to my list of Death Benefits for Widows, Widowers, or Unmarried Persons outlines these benefits. Even those 65 and over may have living dependent parent(s), Godchildren or children in this age of medicine. In Exhibit F, both % of Claims and No. Weeks might be varied by age and sex, producing varying Values, rather than constant ones, for the Average Value calculations.

The statement that the "cost of accident benefits would vary by class of risk" is not as evident to me as the author suggests. For example, it seems unlikely the automobile of the underage single male will incur three and one-half times as much driver or passenger accident cost as the automobile of those who don't drive to work, and are over 25. The former automobile has few occupants who qualify for dependent death benefits. Also a high percentage of occupants are single and not employed and therefore qualify for no, or minimal, principal sum death benefits and no disability income at all. It is true that frequency is the major factor, but severity is also important. In this same sense there may be less spread in territorial severity for the scheduled benefits of this coverage. In general the class and territory differential spread may be less than the present Bodily Injury spread of differentials. However, given the choice of tying costs to current third party pure premiums or a flat Provincial pure premium per automobile, I prefer the former as being far less discriminatory than a single average premium.

If all Provinces in Canada make this coverage mandatory, all Canadians will be covered for these scheduled benefits by either the Insurance Industry or Provincial Unsatisfied Judgment Funds. Certainly there is apprehension as to the dangers created for the Industry by this blanket coverage of the public. This risk may be heavily outweighed by the following and similar points:

1. A high percentage of small Bodily Injury claims will be settled at cost, or scheduled benefit levels, rather than at today's inflated out-of-court, immediate-release levels.
2. The number of Bodily Injury actions taken for excess of Accident Benefits, should be a fraction of the number of Bodily Injury actions taken now. This should reduce adjustment expenses appreciably.
3. The majority of Bodily Injury actions taken should be settled in much less time and for lower cost per claimant as each claimant receives immediate primary benefits and the courts are far less congested.
4. Many Property Damage claims may be settled more quickly at real-cost and real-liability levels. Claimants will not be able to claim minor Bodily Injury conditions to expedite and inflate Property Damage claims.
5. A British insurance executive remarked that many passengers in Great Britain accept realistic knock-for-knock settlements without further pressure on their driver's insurance company. Also, if these passengers take further liability action, they are more reasonable with their driver's insurer than they would be with the insurer of the driver of the other automobile. Similar changes in attitude of our injured occupants may result, though not to the extent of a full knock-for-knock system.

Many in the Insurance Industry in Canada hope the risk is well taken. In addition some feel Canadians will have the finest Automobile insurance coverage in the world, when this coverage is marketed.

LIST OF DEATH BENEFITS

Deceased		Death Benefits (ex Funeral)	
Description	Age (Years)	Principal Sum	Additional Sum for each dependent Child (i)
Child with parent living	4 and under	$250	None
	5 through 17	$500	None
Married Male	through 59	$5000	$1000
	60 " 69	$3000	$1000
	70 and over	$2000	$1000
Married Female	through 59	$2500	None
	60 " 69	$1500	None
	70 and over	$1000	None
Widow, Widower, or Unmarried Person:			
(a) with Dependent children	through 59	$4000	$1000
	60 " 69	$2000	$1000
	70 and over	$1000	$1000
(b) with Dependent Parent(s) only	All ages	$1000	None
(c) with no Dependents........	18 and over	None	None

(i) Children under 18 and children 18 and over who are fully dependent due to physical or mental infirmity.

HOW TO TELL A PURE ACTUARY FROM A LAY ACTUARY

MATTHEW RODERMUND

Acute powers of observation are hardly required to recognize that the casualty actuarial fraternity is divided, not sharply but nevertheless clearly, between "pure" actuaries and "lay" actuaries. To be sure, there is a degree of overlap between the two classifications: some actuaries covered by the overlap are more pure than lay, some more lay than pure, and some are almost equally pure and lay – they might be termed "pure-lay," or, to simplify the language, "purely" actuaries. At any rate, the intention here is to provide guides so that those of our members who cannot instinctively tell a pure actuary from a lay actuary may become aware of the distinctions and from their newfound knowledge bring more understanding to their relationships with their fellows and associates.

For his approach to this discussion the writer is indebted to E. B. White, for almost forty years a writer for *The New Yorker* magazine and without question one of the great stylists and foremost masters of the casual essay in our time. In the mid-thirties Mr. White wrote a piece for *The New Yorker* entitled "How to Tell a Major Poet from a Minor Poet."[1]

Mr. White said, for example, that any poem starting with "And when" is a serious poem written by a major poet. To illustrate, here are the first two lines of a serious poem:

> And when, in earth's forgotten moment, I
> Unbound the cord to which the soul was bound . . . .

On the other hand, any poem ending with "And how" is light verse, written by a minor poet, as in:

> Placing his lips against her brow
> He kissed her eyelids shut. And how.

Mr. White also told us that all poets who, when reading from their own works, experience a choked feeling, are major, and that all women poets, dead or alive, who smoke cigars are major. And there was more, equally delightful.

But Mr. White's differentiations of major and minor poets are not our concern. The important thing for us is that when a member of the Casualty Actuarial Society brings his wife, or a new Associate, to one of our welcoming Sunday night smorgasbords, and he knows he is going to see Tom Murrin, or Norman Bennett, or Bill Hazam, or Lew Roberts, or Les Dropkin, he will want to introduce them to his guest properly: "This is Mr. So-and-So, the pure actuary," or, "This is my good friend Such-and-Such, the lay actuary," or even, "Here is You-Know-Who, purely actuary." Just to say, "This is Doc Masterson," is hardly satisfactory; it might even be embarrassing.

In the Casualty Actuarial Society it would be difficult to distinguish a pure actuary from a lay actuary by his character, his looks, his title or company affiliation, or the color or quantity of his hair. His drinking capacity, or the amount of sleep he gets, may provide clues, but not reliable ones. The recommendation here is that drinking and sleeping as criteria be discarded as unworthy, for to consider them would involve detached observation under circumstances where detachment is somewhat difficult and not at all conducive to a decent camaraderie.

Fundamentally, by their words shall ye know them, their words as revealed in formal papers, reviews, speeches, and reports. Here are a few rules:

> Any paper whose mathematical demonstration includes a χ^2 test is a paper written by a pure actuary. On the other hand, any paper whose mathematical demonstration is consummated by an expression in x^2 is a paper written by a lay actuary.
>
> A discussion of an actuarial problem that includes the word "stochastic" is a discussion by a pure actuary. A discussion in which the favorite descriptive term is "fantastic" is probably a discussion by a lay actuary.

But this sort of labeling can be tricky. If our lay actuary suddenly throws a "Null Hypothesis" at us we have reason to suspect he may be a pure actuary in disguise. Earlier when he talked about "degrees of freedom" we were content that he was a lay actuary making a sophisticated allusion to a civil rights situation; but the Null Hypothesis alerts us, and if he follows this with "Yates' correction" we have him pegged as pure for sure.

Any actuary who can correctly both pronounce and spell P-o-i-s-s-o-n and B-e-r-n-o-u-l-l-i is a pure actuary.

The simplest way to learn that an actuarial student will live out his career as a lay actuary is to have him ask for a copy of *The Elements of Probability Theory* by Harold Craymer. Of course, if he asks for a copy of *The Elements of Probability Theory* by Harald Crommaire, you can be reasonably sure he is destined to be a pure actuary.

A paragraph in a formal paper that reads, in part,

> "Again speaking in general . . . . the indicated proportional departure of each group . . . . should be given a weight proportional to the square root of the expected number of losses for the group. This is based on the fact that the indication of each group should be given a weight inversely proportional to the standard deviation of the indication. The standard deviation of the indication is inversely proportional to the square root of the expected number of losses for the group. An equivalent credibility procedure would be to give the square of the indication a weight proportional to the expected number of losses,"[2]

– a paragraph like that one has to have been written by a pure actuary. Incidentally, reading that passage aloud can be lots of fun for a lay actuary.

Now, however, if while thumbing through old volumes of the *Proceedings* the unsophisticated reader finds the following lines,

> "The time may never come when the underwriters will consider us as their equals, but let us carry on with the hope that some day they will admit that we are not such bad fellows and associates, after all. Until that happy day, let us be content with our lot. When some one raises the old question, 'What is so peculiar about an actuary?', we shall without malice make a simple but dignified answer, giving a Stoopnagle reverse English twist to the classical one concerning the southern exposure of a horse going north. Yes, our reply shall be, 'There are so many fewer of them than there are underwriters'."[3]

he may be sure that he has been exposed to a lay actuary, one of the most!

The passage quoted above was written by Syd Pinney, who was a president of this Society. But presidents of the C.A.S. have been both pure actuaries and lay actuaries, and purely actuaries too. Their presidential addresses might make one think they are all lay actuaries, but presidential addresses do not represent their total output and the reader can be fooled. Witness the following:

> "The development initially assumes that these respective Poisson distributions are independent; but this complex multiple Poisson distribution of the number of claims reduces to the negative binomial distribution when the parameters of the independent distributions are reduced to two by making them interdependent through the assumed relationship . . . ."[4]

Although the quotation is incomplete, clearly those words are penned by a pure actuary. Now consider this:

> "I think there is merit to an idea that was considered some years ago in connection with boiler and machinery insurance, but which is equally applicable to other lines, that any risk producing an annual premium of $25,000 at manual rates should be subject to (a) rate treatment, that is, individual risk rating on an underwriting judgment basis, possibly with the establishment of certain limitations within which the judgment modification must be contained."[5]

Sounds like a lay actuary, doesn't it, displaying indeed, since it was written in 1951, remarkable prescience of the risk modification programs that are shaking the casualty insurance industry today. Both of the foregoing sentences came from the pen of Tom Carlson, a past president, who can hardly disavow the label, purely actuary.

Mr. Carlson's words of 1951 bring to mind the following paragraphs written in 1959:

> "There remains still the fear that unregulated rates in the face of keen competition will be inadequate rates from the point of view of company solvency, thus endangering the very security of our system. Under today's operating procedures, however, the safety of a carrier is irretrievably given over to the judgment of its underwriting organization through the authority to accept and reject. A company can sink into insolvency with tragic speed through bad risk selection even with every rate charged strictly according to manual. Why should we expect our staffs, which we trust to exercise adequate restraint in risk selection, to cast that restraint to the winds if given some limited discretion in rate assignment?
>
> "Some will accuse me at this point of selling my actuarial profession down the river. I plead 'not guilty.' It has always seemed to me that when the law is too pervasive the atmosphere breeds shysterism. The present regulatory climate makes actuarial shysterism a distinct, though, I hope, as yet an unrealized, possibility."[6]

Those are the words of an actuary's lay actuary, and strong words they are, coming from a presidential address by Dudley Pruitt, who also, apparently, foresaw the rating programs currently being advanced.

From another of our presidents comes the following passage:

> "We may liken our statistics to a large crumbly loaf cake, which we may cut in slices to obtain easily edible helpings. The method of slicing may be chosen in different ways — across the cake, lengthwise down the cake, or even in horizontal slices — but only one method of slicing may be used at a time. If we try to slice the cake more than one way at a time, we shall be left with a useless collection of crumbs."[7]

That is obviously too lay to be pure, yet suggestive of the kind of imagination expected of the pure – a fine example of a clue to an actuary with the "purely" label. Laurie Longley-Cook, who authored the lines, well deserves the label.

No inference should be drawn from the decision not to classify here other presidents of the C.A.S. as pure, lay, or purely; each of our members should be able, if he gives a hoot, to classify past presidents according to his understanding of, or interest in, the criteria in this report.

Here are a few more guidelines in the pure-lay classification system:

Any actuary who presents a paper to the International Congress is a pure actuary. Any actuary who presents song parodies at an actuarial dinner is a lay actuary.

Any actuary named Arthur L. Bailey is a pure actuary.

A rate derived by the method of moments is one fashioned with care by a pure actuary. A rate derived in a matter of moments is one pulled out of the air by a lay actuary.

In E. B. White's essay there is a hint that probably a major poet is the higher form of the species. No similar hint is here intended with respect to pure and lay actuaries. Indeed, a statistical study may very well show that pure and lay actuaries attain executive positions in their organizations roughly in proportion to the relative frequency of pure and lay actuaries on the C.A.S. roster. It is safe to assume, however, that the attainment of the rank of company president is almost exclusively a prospect for a lay actuary, a circumstance which may reveal more about presidents than about lay actuaries.

There must be more ways of telling a pure actuary from a lay actuary, but it is hardly necessary to explore them all. Instead it is fitting to quote Mr. White's closing comment on the various ways to tell a major poet from a minor poet:

"The truth is," he said, "it is fairly easy to tell the two types apart; it is only when one sets about trying to decide whether what they write is any good or not that the thing becomes complicated."

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ON SOME ESSENTIAL PROPERTIES OF A TARIFF CLASS

EDWARD FRANCKX

The celebration of the Casualty Actuarial Society's Golden Jubilee at first seemed an excellent occasion for discussing certain aspects of credibility theory. There is a pertinent reason for that. Credibility theory is a branch of actuarial science created and developed by our American colleagues with the object of justifying the techniques of rating non-life risks. We have, however, abandoned our original project, because the problems of experience rating and credibility theory will be discussed at the next ASTIN Colloquium which will be held at Lucerne in 1965. But the principal reason is more fundamental. In effect the primary problem in the insurance industry is "to be able to meet the expenses resulting from the arrival of claims"; the secondary problem, (because it is in fact complementary) is "the practical realization of the assembly of resources to meet the obligations of the assurer." This second problem, which includes both the problem of rating and the problem of reassurance is from a logical point of view dependent on the first. The first defines the basic fact "what must one expect?" – the second poses the question "how can one prepare for it?"

The object of this note is to reply in a very general form to the first question.

In his excellent introduction to credibility theory Longley-Cook insists on the necessity of basing the theory on an adequate mathematical model. This model must be on the one hand sufficiently precise to represent with the necessary approximation the reality of the risks run and on the other hand sufficiently practical to enable both the particular actuarial cases and the problems of forecasting or security to be treated.

The model of life assurance – Dodson's model based on attained age – is not appropriate for non-life insurance. Nevertheless, the life model belongs to a particular class of stochastic processes and it is within the framework of this general theory that the actuary finds quite naturally the basis of his mathematical calculations. But this theory is complicated and very large; in particular Lundberg's theory of risk and its different developments are also contained in the general theory.

On the other hand, as we have often pointed out, it is remarkable that

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Editor's Note: This paper was presented by invitation. Professor Franckx is president of the Permanent Committee of the International Congress of Actuaries and honorary chairman of ASTIN.

for years practitioners and businessmen have found the reply to the two questions posed in our introduction above without having recourse to any very sophisticated theories. This fact gives food for thought and suggests that a mathematical model can be found which justifies at least to the first approximation current experience and the usual rules of action.

As in every constructive theory – and a model has no other end than to define the structure – we must start from certain basic elements. In non-life insurance we choose our point of departure in the theory of tariff classes.

By definition a tariff class is obtained: (1) By the juxtaposition of a finite number of risks N which belong to k homogeneous classes of risk designated by $C_1, C_2, \ldots, C_k$. (2) However, in spite of the fundamental heterogeneity of the class, we admit that for any specified class the mean claim remains constant. Let S be this amount.

The second hypothesis is equivalent to saying that if one considers the "claims department" as an independent financial organization and if on each arrival of a claim in a class the department is "endowed" with the amount of the mean claim S, then, by the application of the law of large numbers, the statistical equilibrium of the budget of this financial organization will be assured in mean.

The loading to be added to S to meet fluctuations and to provide a security loading is a complementary problem belonging to the theory of risk which we do not touch in this note. In this branch the notion of "feed back control" could probably produce algorithms capable of progressive adaptation which have not been studied up to the present.

On the other hand the notion of a homogeneous class is worthy of further attention. A homogeneous class is composed of elements which are interchangeable from the point of view of risk, that is to say indistinguishable on the basis of statistical observation and a fortiori by the assurer.

Consider such a risk. What distinguishes non-life insurance from life assurance is the fact that in a specific interval of time, located on the time axis by two arbitrary times t_1 and t_2, the number of claims produced by any one contract can vary from nought up to any number n.

This means, for example, if t_1 and t_2 correspond to the beginnings of financial years, that in the course of one year of insurance, the number of claims is a random variable N, such that

$$\text{prob } (N = n) = p_n > 0$$

$$\text{with } \sum_0^\infty p_n = 1 \qquad\qquad n = 0, 1, 2, \ldots$$

It is the choice of this (p_n) which is the decisive act of the actuary. In fact stochastic events are also met with in other domains. The arrival of claims in a company is, to a large extent, comparable to the arrival of telephone calls at an exchange or the arrival of clients at a service hatch. These are the types of events studied by operational research. In this domain, the majority of real situations have been approached with success by means of Poisson variates. This means putting

$$p_n(t) = \frac{(\lambda t)^n e^{-\lambda t}}{n!}$$

t being the time interval of observation, λ the parameter of the Poisson process, which is also the mean value of the number of events which occur in an interval of unit time.

Let us recall that the hypotheses which lead to the Poisson law are:

1. Markov's hypothesis: or the hypothesis that the number of events in two separate time intervals are independent.
2. The hypothesis of rare events, which is that in a very small time interval the probability of the occurrence of two or more events is arbitrarily small in comparison with the occurrence of one event only. Note that this limitation excludes the application of the model to risks with conflagration where the claims occur in groups.

It is remarkable that these hypotheses alone allow a precise answer to the question "what can one expect?"

Let us examine the question mathematically. The global *endowment* which must be made to the claims department is a random figure at the beginning of a financial year depending on the stochastic number of claims which are notified to the Company in the course of a year.

We can, therefore, define this endowment by a random variable D. The question posed, namely, "what can one expect?" is the same as the fixing of this stochastic variable D. It determines the stochastic endowment for a tariff class in the course of the current accounting period and, in a general sense, the stochastic endowment for this same class in the course of an interval of any time t.

Now, because of the hypotheses made for the tariff class, whatever the value of t,

$$\text{prob } (D = nS) = \text{prob (total number of claims} = n)$$

The random variable D is thus completely known as soon as the random variable giving, T, the total number of claims of the tariff class has been ascertained. This is expressable in a very simple manner. If two Poisson variates are added the sum is also a Poisson variate of which the parameter is the sum of the parameters of the variates composing it.

Thus, suppose a contract belongs to a homogeneous class C characterized by a Poisson variate N_1 of parameter λ_i

$$p_{n_1} = \frac{(\lambda_i t)^{n_1}}{n_1!} \exp\{-\lambda_i t\} \tag{1a}$$

and that a second contract belongs to a homogeneous class C_j characterized by the Poisson variate N_2 of parameter λ_j

$$p_{n_2} = \frac{(\lambda_j t)^{n_2}}{n_2!} \exp\{-\lambda_j t\} \tag{1b}$$

then the total number of claims expected under the two contracts will be determined by a Poisson variate N

$$p_n = \frac{[(\lambda_i + \lambda_j)t]^n}{n!} \exp\{-(\lambda_i + \lambda_j)t\} \tag{2}$$

or parameter $\lambda_i + \lambda_j$

This property can be generalized by induction. It is thus sufficient to add the parameters of the ensemble of risks belonging to the tariff class.

Suppose that:

the classes	$C_1, C_2, \ldots, C_i, C_j, \ldots, C_k$
of parameters	$\lambda_1, \lambda_2, \ldots, \lambda_i, \lambda_j, \ldots, \lambda_k$
comprise	$n_1, n_2, \ldots, n_i, n_j, \ldots, n_k$ different

risks with $\sum_1^k n_i = N$

N being the total number in the tariff class. Under these conditions we arrive at the essential property of the endowment variate D for the tariff class.

The variable global endowment B for a tariff class is a Poisson variate.

$$p\ (B = nS) = \frac{(t \sum_1^k n_i \lambda_i)^n}{n!} \exp\{-t \sum_1^k n_i\lambda_i\} \tag{3}$$

of parameter $S = \sum_1^k n_i\lambda_i$ for $t = 1$

This result, valid under the hypothesis defined above, but certainly true as a first approximation represents a solution whose value should be carefully tested. Apart from the fact that it emphasizes once more the fundamental part played in practice by the Poisson law it throws into relief a very important result: *one single parameter is sufficient to characterize the endowment variate for a tariff class.* This reduction to a single parameter, now that we have departed from a multiplicity of homogeneous classes is, perhaps, the most important consequence of the above-mentioned theoretical result.

In practice, the heterogeneity of a tariff class is not always known. Generally, in fact, very little is known about it and what is more it varies from year to year according to the general underwriting policy of the assurer.

But heterogeneity in effect only plays a very relative part in the problem "what can one expect?" Consider two assurers possessing different portfolios of the same tariff class. We can express this fact by the sequence (n_i'), giving the distribution of the insured of the first company into homogeneous classes which is different from the sequence (n_i) of the second company.

However, if $\sum_1^k n_i' \lambda_i = \sum_1^k n_i'' \lambda_i$

the "global endowment" variates of the two classes of risks are strictly identical, which is to say that *from the global point of view, in spite of the differences in heterogeneity, the sums of the endowments are, from the point of view of the two assurers, indistinguishable.*

They are financially equivalent even if the global complements of the two classes are different i.e. $\sum_1^k n_i' \neq \sum_1^k n_i''$.

The possible equivalence of global variates is a remarkable property.

We shall make use of the property of the equivalence of the global variables of the allocations. To this, we give the name *principle of substitution of equivalent partitions.*

Let us consider the population N of the tariff class. Effecting a partition of this population consists of sub-dividing into l sub-classes $E_1, E_2, \ldots, E_j, \ldots, E_l$ comprising the populations $n_1'', n_2'', \ldots, n_j'', \ldots, n_l''$ such that $\sum_{j=1}^{l} n_j'' = 1$.

Let us suppose, *as a working hypothesis,* that each sub-class E_j is conventionally regarded as a homogeneous class. Then, from the risk point of view, we attach to it a Poisson variable with parameter λ_j.

Under these conditions, there corresponds to this partition a global allocation variate of the tariff class, defined by the parameter λ^0

$$\lambda^0 = \sum_{i=1}^{l} n_j'' \tilde{\lambda}_j \tag{4a}$$

From the point of view of the insurer, the global variate defined by (4a) will be identical to that which he must experience in reality, provided that

$$\lambda^0 = \sum_{i=1}^{l} n_i' \lambda_i \tag{4b}$$

n_i' *and* λ_i characterizing the *real partition* of the insurer.

In conclusion, if the partitions satisfy the system (4a) (4b), they must be considered equivalent "from the point of view of allocations to the claims department." This implies that one can substitute for the real partition any equivalent hypothetical partition; and it is this rule which we call the principle of the substitution of equivalent partitions. Let us straightaway give a specific and important application of this in practice. Among the multiplicity of substitutions which one can imagine there is one which possesses a special characteristic. It results from the hypothesis that the total population N of the tariff class constitutes one whole homogeneous class E of parameter $\tilde{\lambda}$.

By virture of the system (4a) (4b), this homogeneous class is equivalent from the allocation point of view if and only if:

$$N \tilde{\lambda} = \sum_{i=1}^{k} n_i' \lambda_i$$

or if $$\tilde{\lambda} = \frac{n_1' \lambda_1 + \ldots + n_i' \lambda_i + \ldots + n_k' \lambda_k}{n_1' + \ldots + n_i' + \ldots + n_k'} \tag{5}$$

Thus, to each class of the tariff there corresponds from the allocation point of view an equivalent homogeneous class having the same population N, if and only if the parameter λ of the homogeneous class is the mean weighted by the population n_i' of the parameters of each homogeneous subclass of the tariff class.

Let us revert to the notion of equivalent partition which we have noted:

$$\begin{cases} \{ n_j'' \lambda_j \} & j = 1, 2, \ldots l \\ \text{with } \sum n_j'' = N \end{cases}$$

From (5) it is clear that the equivalence is only achieved if:

$$\breve{\lambda} = \frac{n_1'' \breve{\lambda}_1 + \ldots + n_i'' \breve{\lambda}_i + \ldots + n_l'' \breve{\lambda}_l}{n_1'' + \ldots + n_j'' + \ldots + n_l''} \tag{6}$$

The totality of all the equivalent partitions is thus characterized by the fact that, whatever the partition, the weighted mean (6) remains invariant.

This invariant, which is the parameter of the equivalent homogeneous class, determines the numerical value of the compatibility relationship (6). It plays, as we shall see further on in this note, a preponderant role in the problem of rating.

In fact, the above considerations result solely from the mathematical model which we have considered. It is time that we established the link between theory and practice. This will be given to us by the classical statistical theory of estimation.

We know from the law of large numbers that during a series of independent tests in relation to one and the same variable the mean value of the values observed converges in probability towards the mean value of this variable. This is particularly so, if this variable is the Poisson variable

$$N_i : p_n^i = \frac{(\lambda_i)^n}{n!} e^{-\lambda_i}$$

with its mean value $En_i = \lambda_i$

On the other hand, if R_i is the total number of claims observed in the course of the year for a homogeneous sub-class the mean of the values observed will be $\frac{R_i}{n_i'}$

Therefore, within the meaning of the law of large numbers, $\frac{R_i}{n_i'}$ is the "statistical estimation" of En_i, that is of the parameter λ_i.

If we substitute these estimations in the relation (5) we shall obtain the estimated value of $\breve{\lambda}$ of the numerical invariant of all the equivalent partitions, which we will denote by λ^*

$$\lambda^* = \frac{\sum_{i=1}^{k} n_i' \frac{R_i}{n_i'}}{\sum_{1}^{k} n_i'} = \frac{\sum_{i=1}^{k} R_i}{\sum_{1}^{k} n_i'} = \frac{R}{N} \tag{7}$$

Thus, from the statistical point of view, *the estimated value of the invariant of the equivalent partitions is precisely the frequency observed in the tariff class.*

This result is remarkable, because the influence of the real heterogeneity of the tariff class has been totally eliminated in (7). On the other hand, we have returned to the "working" figures which practitioners have always used.

Let us demonstrate that they were perfectly right in using such figures from the point of view of rating which, as we have mentioned in our introduction, is the quest for admissible solutions as regards the collection of the necessary means for the financing of the allocation.

When the insurer effects a rating, he in effect achieves a partition of the tariff class, in respect of which he covers the claims. Each sub-class of this class is defined by the property that the *premium asked for is identical for all the insured of this sub-class.*

Rightly or wrongly, the insurer considers that each risk of such a sub-class is for him equivalent. This amounts to saying that this sub-class is *"considered as being homogeneous."*

Let us therefore use the theory of equivalent partitions.

If λ_i is the parameter of the class, the pure premium required for each risk of the sub-class will be:

$$\Pi_i = \sum_{n=1}^{\infty} p_n^i \, n \, S = S \sum_{n=1}^{\infty} n p_n^i = S\lambda_i \tag{8}$$

If we take into account the relationships (6) and (7), we obtain the statistical condition of compatibility:

$$S\frac{R}{N} = \frac{n_1'' \Pi_1 + n_2'' \Pi_2 + \dots + n_l'' \Pi_l}{n_1'' + n_2'' + \dots + n_l''}$$

Let us suppose:

$$\theta_i = \frac{n_i''}{n_1'' + n_2'' + \dots + n_l''} = \frac{n_i''}{N}$$

and let us call this number the *coefficient of credibility belonging to the risk at the level of premium* Π_i.

We thus obtain the *general principal of rating: any method of rating* ($\Pi_1, \Pi_2, \dots, \Pi_l$) *is admissible in practice, if the mean value of the premiums charged within the terms of credibility, is equal to the mean premium of the tariff class.*

$$S\frac{R}{N} = \theta_1 \Pi_1 + \theta_2 \Pi_2 + \dots + \theta_l \Pi_l \tag{9}$$

We have not invented anything; we consider that rating at the average premium is admissible in practice, and in Europe this method has many advocates. But we have justified, without much difficulty, the attitude of our American colleagues who wish to introduce nuances into rating, by introducing different levels of premium. In actual fact, the relationship (9) implies a constraint, the coefficients of credibility cannot be chosen arbitrarily and they must satisfy the condition of statistical compatibility.

In fact, we are pushing open a door which is ajar. For by multiplying (9) by N, we find again the condition of the accounting equilibrium: *the gathering in of premiums en masse must balance the expenses resulting from claims.* And it is this obvious fact with which we conclude our note.

EARLY ACTUARIAL STUDIES IN THE FIELD OF PROPERTY AND LIABILITY INSURANCE

LAURENCE H. LONGLEY-COOK

On this, the Jubilee of the Casualty Actuarial Society, it is tempting to assume that the fifty year period, 1914 to 1964, represents the whole extent of actuarial studies in property and liability insurance. However, many interesting studies in this field were made by actuaries prior to 1914 and it is thought that some short account of these studies should find a place in the *Proceedings* of the Society. No attempt will be made to go beyond the limited field of property and liability insurance, into such areas as accident, health, or disability insurance in which actuaries have always been active; nor will the early work in risk theory be considered within the scope of this study.

PROPERTY INSURANCE

The Reverend David Wilkie published in Edinburgh in 1794 a book *Interest and Annuities*[1] and can, therefore, be reasonably referred to as an actuary, although this was before the formation of any actuarial society. In this work he also attempted to cover the theory of fire insurance. The idea was put forward that higher rates of premium should be charged for larger dwellings because of the increase in the number of possible sources of fire – lamps, fireplaces, servants, etc. While the various schedule rating plans presently employed for rating commercial and industrial risks include charges for such features as area which are in line with the Reverend David Wilkie's thought, the method has never been used for dwellings or other class rated properties in the United States. It is of interest to note, as reported by Paul Johansen, the first President of ASTIN, in a paper presented to the XVth International Congress in New York in 1957,[19] that this rating plan is used for rural farm buildings in Denmark and the experience supports this method of rating. In the United States it has been known for some years that the dollar losses per $1,000 of fire insurance on large and small dwellings are higher than they are for medium size dwellings.

In the *Mémoires of the Société Royale des Sciences, de l'Agriculture et des Arts,* of Lille, France for the year 1834 there appears an article on the application of probability theory to fire insurance by Monsieur Th. Barrois.[2] The article runs to 198 pages and contains a great many complex mathematical formulae. It is tempting to dismiss this early work as of no practical value but, by studying in mathematical form such problems as the spread of fires in buildings of various types, the author points the way to the possible development of certain structural charges for which classified loss data could never be developed. It is of interest to note that in the author's discussion of the problem of losses due to arson he uses Daniel Bernoulli's theory of relative values which has been applied in recent work on reinsurance.[20].

The next study of fire insurance by an actuary which has come to the author's notice was in 1847 by Mr. W. E. Hillman, actuary of Star Assurance Office.[3] He attempted to develop rates from the statistics for fires in London according to various trades over the years 1836-1845 compared with an estimate of the number of buildings exposed for each trade. Although this study did not lead to a practical ratemaking technique, it drew attention to many of the problems involved in scientific ratemaking for fire insurance.

The early issues of the *Journal of the Institute of Actuaries,* first published in London in September 1850 under the title of the *Assurance Magazine* by two eminent actuaries, Charles Jellico and Samuel Brown, contained numerous papers on fire insurance of which a number were presented at meetings of the Institute of Actuaries. However, most of these were not written by actuaries and were largely descriptive in nature so they need not concern us. Note must be made, however, of the work done in this field by Samuel Brown himself. Samuel Brown was the third president of the Institute of Actuaries (1867-1870) and was an indefatigable writer on all aspects of actuarial work. He wrote on mortality studies, including mortality in the U.S.A., probability theory, decimal coinage, sickness insurance, the investment of insurance company funds, as well as fire insurance. His first paper on fire insurance[4] was in 1850 and is a fascinatingly detailed study of the fires in London from 1833 to 1849. We learn, for instance, the frequency of fires not only by month of the year, but by days of the week and by hours of the day. We learn that about 4% of the fires were total losses. Fires are separately analyzed by occupancy and by extent of loss, and an exposure base is also established of the number of buildings in various occupancy groups. Fires are also analyzed by cause.

Samuel Brown again discussed fire statistics at the International Statistical Congress in London in 1851 where he suggested uniformity between countries in the collection of data for various branches of insurance. It would appear that the fire insurance companies resented this mere actuary taking an interest in their ratemaking because he later mentioned to another actuary "his extreme disappointment that the course of his investigations on this subject was stopped by the determination of the fire offices to refuse him all information whatever." (Reported in the discussion of Cornelius Walford's paper of 1879.)[6]

In 1856 another actuary, Thomas Miller, who had considerable experience in fire insurance, made some interesting proposals for the collection of fire insurance statistics.[5] He was well aware of the difficulties of developing an exposure base for fire insurance due to lack of insurance to value, the operations of the average clause, and having more than one company on many risks. He showed how such difficulties could be overcome and ended his paper with these words,

> "there is no reason why the Offices should rest contented with imperfect data, when their own books can supply them accurately, and in the greatest abundance; and if some experienced members of the profession could be induced to cooperate in arranging a comprehensive and simple classification of risks, the individual Offices or such of them as approved of the idea might analyze their own business in conformity with that model; and were it thought advisable, their united statistics might be collected for the benefit of the profession generally."

The next study[6] of interest to actuaries was a review presented to the Institute of Actuaries in London in 1878 of the scientific application of data to the purpose of deducing rates of premium for fire insurance by Cornelius Walford, who was not only an actuary and a statistician but also an attorney. In addition to his numerous and discursive papers, Walford was the author of the great Insurance Cyclopaedia, a mine of information on insurance history, which he never lived to complete. Walford attempts to show how the germ of a scientific approach to fire ratemaking developed over the years quoting not only from the actuaries I have referred to but many other writers on fire insurance on both sides of the Atlantic.

The first volume of the *Transactions of the Actuarial Society of America* contains a paper presented in 1890 by an early member, Walter S. Nichols, on the actuarial elements involved in fire insurance. Despite its title the paper contains little of actuarial interest.

In April 1892 the *Bulletin de l'Institut des Actuaires Francais* contained a note on the mathematical theory of fire insurance by Monsieur P. Soulier.[8] This is of interest because an attempt is made to develop theoretical charges for the various structures due to the spread of fire from one portion to another. The problem of a row of connected houses is considered and also of structures of various number of stories.

In a paper read before the Fire Underwriters' Association of the Pacific in 1904, Albert W. Whitney, then Professor of Mathematics at the

University of California, set out a theory for establishing lines of insurance, that is, the amount of coverage an individual company should carry at its own risk. Professor Whitney, a charter member of the Casualty Actuarial Society and an Associate of the Actuarial Society of America, played a leading role in the development of Workmen's Compensation Ratemaking. This paper is reviewed by Walter S. Nichols in Volume 9 of the *Transactions of the Actuarial Society of America*.[9] Professor Whitney shows how lines must be chosen so that the probable fluctuation of loss shall be proportional to the index of profit-stability of the class. Because some classes have been traditionally more profitable than others, line levels used in practice are more controlled by the attractiveness of the business than by fluctuation of loss considerations, but it seems probable that as fire ratemaking becomes more scientific and we get away from traditionally profitable classes and traditionally unprofitable classes, this actuarial approach to setting line levels may be used.

The next paper in the realm of fire insurance to which reference will be made is also by Professor Whitney and was presented at the VIth International Congress of Actuaries in Vienna in 1909.[10] The general subject matter under discussion was, "Upon what principles and by what working methods should Fire Insurance Statistics be compiled?," a question which is most topical today. Professor Whitney's paper develops, from studies of the distribution of partial losses, the rates which should be charged under a coinsurance clause. The tables contained in the paper show the distribution of partial losses for eight classes of risk based on the loss experience in San Francisco during the years 1899-1903 and provide an interesting comparison with the studies which Miss Salzmann presented at a recent meeting of the Society.[21]

Another paper by Professor Whitney appears in Volume 12 of the *Transactions of the Actuarial Society of America*.[11] This paper discusses the theory of schedule rating and shows how to handle mathematically the three important elements in fire ratemaking – the ignition hazard, the combustion hazard and the damage hazard.

At this time the importance of the actuarial aspects of workmen's compensation was increasing rapidly and those actuaries who were interested in non-life actuarial studies had little time to work on the problems of fire insurance. As a result, the formation of the Casualty Actuarial Society in 1914 coincided rather surprisingly with a decrease in the interest of actuaries in property insurance and very little actuarial work was published in this field from 1914 to 1951.

LIABILITY INSURANCE

The origin of the Casualty Actuarial Society lay in the pressing need in the first and second decades of this century to develop rating plans and loss reserving techniques for liability and workmen's compensation insurance. The ideas of employer's liability and of workmen's compensation stem from England and Germany, with the British Employer's Liability Act of 1880, the German Compensation Law of 1884, and the British Compensation Law of 1897, but soon spread to the rest of Europe and the United States of America. Eleven papers on workmen's compensation insurance were presented at the Second International Congress of Actuaries held in London in 1898 and papers on compensation or employer's liability appear in the *Proceedings* of the IVth Congress (New York 1904), the Vth Congress (Berlin 1906) and the VIth Congress (Vienna 1909). The early literature in this field was devoted almost entirely to a discussion of non-actuarial aspects of the plans, and need not be reviewed here.

In the United States of America in 1896 seven companies engaged in employer's liability insurance set up the "Liability Conference." Some interesting information on the early operations of the Conference can be obtained from the Report on Examination of The Workmen's Compensation Service Bureau by the New York Insurance Department (1913), which tells us:

> "The Liability Conference decided to organize a statistical bureau, to inaugurate standard premium rates and to adopt standard policy forms. The first manual issued by the Conference was not based upon any exact scientific data and represented in a large degree the underwriting judgment of the members of the Conference. The second manual issued in 1898 provided for eight different schedules, classified according to industries, and also contained a differential in the rates charged in the various States. By a comparison of loss ratios, it was discovered that the loss experience in some States was more favorable than in other States; for example, the losses in the eastern States were not as high as compared with the losses sustained in the west and southwest. This was due to the fact that the physical and legal hazards were not equal in all the States and the Conference in constructing the 1898 manual took cognizance of such difference in hazard.
>
> "The manuals issued by the Liability Conference in 1901 and in 1904 were based upon the combined experience of the members of the Conference, after detailed study of the same by the committee. The method of deducing rates from such experience was devised by Mr. Frank E. Law, and has since been published under the title 'A Method of Deducing Liability Rates.' This method was adopted by the Liability Conference in determining premium rates for the use of the members of the Conference for a period of ten years, from 1901 to 1910."

Frank Law was a charter member of the Casualty Actuarial Society and his method of deducing liability rates was published by The Spectator Company in 1908.[13] The kernel of the method was:

"The experience for the country at large was accordingly adopted as the basis, and the differences between States ignored for the time being. This gives the broadest possible basis to work upon and 'smoothes out' many of the inequalities and divergences due to narrow local and State experience. The adoption of this plan necessitates a procedure consisting of two separate and distinct steps: (a) determination of rates or list prices to be printed in the manual, and, (b) determination of the differentials or discounts off of the list prices for each of the several States."

It is noted that Mr. Law recommends that experience be reported by limits, that losses be reported by type, that the policy year method be employed with unallocated claim expenses distributed to year by formula. Development factors for losses, trend factors for experience, and pure premiums, are all part of his plan which was extraordinarily well conceived and has stood the test of time.

In 1909 there was an important conference on workmen's compensation at Atlantic City to which some of the future charter members of the Casualty Actuarial Society, such as M. M. Dawson, contributed and in 1912, two future charter members spoke on workmen's compensation at the Commonwealth Club of California. One of these was Albert H. Mowbray, a Fellow of the Actuarial Society of America and later President of the Casualty Actuarial Society. At this time many of the future charter members of the Casualty Actuarial Society were very actively engaged in getting compensation insurance launched on a technically sound basis.

In 1910, in England, William Penman wrote a detailed paper on the calculation of loss reserves under employer's liability contracts.[14] This was the first contribution of any importance on liability insurance to appear in the *Journal of the Institute of Actuaries*.

In 1913, I. M. Rubinow, the first President of the Casualty Actuarial Society, published a book *Social Insurance*.[15] The book, which runs to over 500 pages, grew out of a series of 15 lectures he gave at the New York School of Philanthropy in the Spring of 1912 and was the leading textbook on Social Insurance for some years to come. This is claimed to be the first university course in Social Insurance and covered Industrial Accidents, Sickness, Old Age, Invalidity, and Death, and Unemployment.

In 1914, A. H. Mowbray presented to the Actuarial Society of America a paper on the criteria for testing the adequacy of rates for workmen's compensation insurance[16] in which we find references to such features as budgeted allowances for acquisition expenses, a feature which was to play an important role in casualty ratemaking for many years. Mr. Mowbray held in answer to the question, "If rates are considered for groups rather than individual companies, for what rates of expense should they provide?"

> "In the writer's opinion there is but one answer to this question. If the rate is to be pronounced adequate for an entire group, it must be adequate for the marginal or least fortunately placed company. Therefore the expense rates used must be not less than those of that company. Otherwise that company must be excluded from the group. Clearly a rate which does not make sufficient provision for its expenses could not be adequate for such a company according to our definition of adequacy."

(Your author expressed a similar view in writing on fire insurance in 1951[18] at a time when he was unaware of Mr. Mowbray's statement.)

In the same volume of the *Transactions*, Harwood E. Ryan, another Fellow of the Actuarial Society of America and a charter member of the Casualty Actuarial Society wrote on "A Method of Determining Pure Premiums for Workmen's Compensation Insurance"[17] and many other persons, who were to be charter members of the Casualty Actuarial Society, were writing and speaking on social insurance and particularly compensation insurance. It has been impossible in this short review to mention the work of many other eminent actuaries, such as J. H. Woodward, actuary, and W. W. Greene, assistant actuary of the New York Workmen's Compensation Commission, William Leslie, actuary of the California State Compensation Insurance Fund, and B. D. Flynn of the Travelers Insurance Company, each of whom later became President of the Casualty Actuarial Society.

The time was clearly ripe for the formation of a new society to provide a forum for these actuaries and others who were concerned to provide a proper scientific basis for casualty insurance. These were men of outstanding ability and inquiring minds. They have set us a tradition of brilliance that we must strive to follow. On November 7, 1914, in the city of New York, the Casualty Actuarial Society was born.

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THE FIRST FIFTY YEARS

DUDLEY M. PRUITT

I. HOW WE BEGAN

In the beginning . . . the earth was without form, and void; and darkness was upon the face of the deep.

—Genesis 1:1-2

Upon carried motion, the president was authorized to appoint a committee. . . .

—Minutes of First Meeting, C.A.S.[1]

Let me say at the beginning that this is not a history of the Casualty Actuarial Society. I have neither the time nor the talent to do the research and analysis necessary for such an undertaking, and more importantly, there are others eminently more qualified if a definitive history is to be written. I hope such a history will be written some day. This is but a footnote to that undertaking and if, in this presentation, you find that I have given a certain skewness to the story, an offbalance of the facts, if the wrong things are emphasized and the right ones omitted, please forgive my wayward pen. There is such a vast amount of material, important and trivial, serious and silly, dull and lively, that I have had to be selective to stay within considered and considerate limitation, and I have selected, quite frankly, what interested or amused me, thinking, I hope rightly, that it would interest or amuse you.

In the span of history fifty years is brief indeed, but in the span of the life of a man or of an actuarial society fifty years encompasses tremendous change, so that the earlier is hardly recognizable in the later. It is fashionable to point out that the past fifty years have witnessed greater changes in the pattern of our lives than had perhaps the preceding fifty decades. Our infant Society was born into an ancient world where horses and beards were still seen on the streets, where the Atlantic Ocean was still a very wide body of water, and where no casualty actuary, to my knowledge, had ever heard of the negative binomial. But the forces of change were on the move and we may well consider the year 1914 as the birth date for a new world, as well as for our new Society.

On May 28, 1914, a group of men, meeting as the Statistical Committee of the Workmen's Compensation Service Bureau, decided that what they needed, in view of the problems presented by the new workmen's compensation laws, was a professional society. One month later the Archduke Francis Ferdinand of Austria was assassinated. On July 27 our organizing committee addressed a call to such persons as might be interested in joining a casualty actuarial and statistical organization, and the next day Austria declared war on Serbia. The organization meeting of the Society was held at the City Club of New York on November 7, the day after Japan took Tsingtao from the Germans. That day our charter members not only founded the Casualty Actuarial and Statistical Society of America*, adopted a constitution and by-laws and elected officers and a council, but also listened to the presentation of three papers (one of which was by our still active member, Win Greene), ate their first Society dinner at 7:00 P.M., and digested it with ten after-dinner speeches. Times have changed!

The First World War, of course, was destined to influence profoundly the course of history for both the world and our Society, but nothing in the record of that first organization meeting indicated even an awareness of its progress. At that time Europe seemed far away indeed.

What was of much more pressing interest, perhaps, to our group of pioneers, was the new spirit of adventure that seemed to be taking hold of American industry. It had been on January 5 of this same year that Henry Ford announced his five dollar minimum wage and his eight hour day, and on July 1, 1914 the broad new New York Workmen's Compensation Law became effective. Mark Sullivan called it a period of "dynamic energy accompanied by a dynamic humanitarianism."[4] A new day was dawning!

Here were the required elements for the founding of a successful actuarial and statistical society: dynamic energy, dynamic humanitarianism, an eight-hour five dollar day, a whole wave of new workmen's compensation laws taking over one state after another, a body of men inspired by and somewhat overwhelmed by the new problems these laws presented, and a few men who were prepared to act boldly.

II. THE PIONEERS

There were giants in the earth in those days.

—Genesis 6:4

You shall sit at the feet of Winfield Greene, that slug-horn tooter tough,
Or become a second Michelbacher—though one is quite enough.

—Clarence W. Hobbs[1]

* This was the original form of our name. The words "and Statistical . . . of America" were amputated in 1921[2] supposedly without prejudice to the statistical element in our membership, though with considerable unhappiness to our founder.[3]

One of the benevolent dispositions of Providence seems to be that when, in the course of human events it becomes necessary to have giants, giants are provided. So it was in the founding of our country, and so it was also in the founding of our Society. Isaac M. Rubinow, James D. Craig, Joseph H. Woodward, Benedict D. Flynn, Albert H. Mowbray, Harwood E. Ryan, William Leslie, Gustav F. Michelbacher, George D. Moore, Winfield W. Greene, Leon S. Senior – these charter members were also elected presidents of the Society and each gave his own unique contribution to its achievements. There were in all ninety-seven charter members, though only forty attended the organization meeting. Many of them were outstanding men and made outstanding contributions, but any selection by me of some would undoubtedly run the risk of omitting others of equal importance. The charter member presidents were giants enough and to spare for the birth of one actuarial and statistical society.

Dr. Isaac M. Rubinow is the acknowledged founder of our Society and the first president. He was what one might call a fortuitous circumstance, a chance occurrence, that had no good excuse for being in the business when our time had come. He belonged in the social sciences, not in business, and he was in business really just long enough to found our Society. Dr. Rubinow was born in Russia and brought up in Manhattan. He took a medical degree, but practiced only a short time, went to Washington in government service for a few years, and in 1911 came to the Ocean Accident and Guarantee Corporation as Chief Statistician. This job lasted less than five years. He then joined the staff of the American Medical Association and after that the Federal Trade Commission, leaving this to become the director of the American Zionist Medical Unit in Palestine, next the director of the Jewish Welfare Society in Philadelphia, and, for the last seven years of his life, the secretary of B'nai B'rith.

One thing that seems clear about Dr. Rubinow was his deep dedication to the cause of social insurance. In 1913, before the organization of our Society, he had already published a book entitled, *"Social Insurance with Special Reference to American Conditions,"* which ran to 525 pages. Win Greene relates that he found the book most useful in the work he did in early 1914 for the purpose of establishing a basis for Workmen's Compensation rates under the New York law effective July 1, 1914. Dr. Rubinow, from all accounts, was a man of strong opinions and of liberal social convictions. Emma Maycrink, who took a course under him at the New York School of Philanthropy, says that, although she has "always opposed socialist tendencies," she found him quiet and one who spoke with authority.

Others have called him "opinionated" and "outspoken," but all agree that he was a man of very real ability, an expert in social statistics when experts were really needed and when our business was first called upon to establish rates and procedures for workmen's compensation insurance. He was chairman of the first statistical committee that laid the foundations for the compensation rate structure. He prepared the "Standard Accident Table" which was the guide for ratemaking until useable experience became available.

He was not, of course, universally admired – what pioneer ever is? And it is told that at least one company threatened to prohibit membership in the Society on the part of its employees if he was to continue as president. This seems to have been because of his "socialist tendencies," and apparently it was no more than a threat. Nevertheless, twenty years later Dr. Rubinow, himself, writes, "I have not altogether forgotten the sharp conflicts and sometimes bitter feelings centering around the term 'social insurance' and its proponents in this country in years gone by. Perhaps if it had not been for that unhappy antagonism I might still be actively in the field, yet happily those days are gone."[2] Or are they?

Of the other ten charter member presidents only seven were college graduates and seven were members of the life actuarial societies, but the correlation between these two was not perfect. Two of those who did not graduate from college, Craig and Flynn, were Fellows by examination of the Actuarial Society of America, Craig serving once as president of that society and Flynn as a member of the Council.

Michelbacher was the youngest, being only twenty-three at the time the Society was organized and thirty-three when he became president. Michelbacher has been the most financially rewarding member of the Society, having allowed us the royalties from his book, *"Casualty Insurance Principles"* for many years. He is the only one of the ten to have become a company president.

Senior, like Rubinow, was born in Russia. He came to this country at the age of fifteen, graduated from New York University at the age of twenty and then had to wait a year till he was old enough to be allowed to take the Bar examinations. He was the master of five languages.

Greene wrote poetry (and perhaps still does) and introduced his papers with literary allusions, whereas Leslie had an engaging way when it came to beguiling insurance commissioners. Woodward had an extremely warm and friendly personality; Ryan had a keen analytical turn of mind. Moore was a practical statistician.

Of the fifteen papers published in Volume 1 of the *Proceedings,* ten were written by these charter member presidents. All of them have contributed to the *Proceedings,* some very frequently, and many did tremendous service in the early development of the science of workmen's compensation ratemaking.

One man must be mentioned here among the pioneers who was neither a charter member nor a president. Richard Fondiller was admitted to membership as a Fellow on February 19, 1915 at the second meeting of the Society and was thereafter the most useful member the Society has ever had. For thirty-five years, from 1918 to 1953, he served as Secretary-Treasurer handling the vast amount of detail of that office with considerable satisfaction to most people, though there was an occasional grumble that the thick lenses of Richard's glasses kept him from seeing what he did not want to see. He also was a member of the New York Bar and a Fellow of the Society of Actuaries. To a young Associate attending his first Society meeting, and to some of us for years after that, his reports on the meetings of the Council made us imagine that the Council had met on Mount Olympus with all the power and prestige of Zeus and the pantheon.

III. SOCIAL INSURANCE

Am I my brother's keeper?

—Genesis 4:9

"In matters of sickness or unemployment insurance, etc., the opportunity is ours and it is before us, always provided we shall succeed in convincing the public that we shall approach it in a spirit of pure scientific inquiry."

—I. M. Rubinow[1]

It is a little difficult for us in this disillusioned and unsettled day to recapture the enthusiasm for progress and social reform that went along with the Ford five dollar day and the bright new workmen's compensation acts. The people and their government were on the move and industry was acting, at least at times, as a cooperative handmaiden. Our Society was born out of the needs of the first great wave of social insurance legislation and many of our charter members had the commitment of their profession to seeing that the new ideas were successful. Emma Maycrink remembers that at that time Joseph Woodward suggested that she sign up for a course in social insurance because it "was the next big move in the insurance world." Dr. Rubinow had great hopes for the Society as an instrument for the advancement of the social welfare, and believed it to be "quite obvious that the United States, having made the first step, is bound to proceed with its ever broadening policy of social provision against the social ills. Throughout the country a powerful propaganda for sickness insurance, maternity insurance, old age pensions, unemployment insurance, and mothers' pensions is rising."[2] This was on February 19, 1915.

Shortly thereafter Dr. Rubinow, by then the secretary of the Social Insurance Committee of the American Medical Association, saw evidence of a growing and friendly interest in social insurance on the part of the medical profession in the publication of a comprehensive AMA committee report on the subject.[3] The AMA's attitude seems to have changed materially in subsequent years.

The propaganda which the doctor saw rising, though it may have been of influence in the medical profession and quite possibly elsewhere in the country, actually had little effect on the production of social insurance papers by the members of our Society, if workmen's compensation is excepted. Our Fellowship examinations, however, carried questions such as the one in 1919 on the "Principles and History of Social Insurance": "(a) What is social insurance?" and "(b) What effect will the unsettled conditions and industrial unrest throughout the world be likely to have in connection with social insurance?"

Perhaps the scarcity of papers on the subject was due partly to the fact that our members were too busy with the problems facing them day by day in the fields of insurance currently being written and partly to the very real lack of enthusiasm for social insurance among some of our outstanding insurance executives of the time. In 1922 Mowbray gave a presidential address on "The Value of the Social Point of View in the Conduct of the Casualty Business," but he was the actuary for the National Council on Compensation Insurance and, therefore, somewhat above the disciplines of the free enterprise system. Under Mowbray's presidency Professor Leo Wolman of the New School for Social Research addressed the Society by invitation on "Unemployment Insurance."[4] The record does not give the reaction to this of the various insurance executives.

Very little more was said in our *Proceedings* about Social Insurance until 1928 when Dr. Rubinow, writing from the professional detachment of his position as Executive Director of the Jewish Welfare Society in Philadelphia, contributed a paper asking the question, "Can Insurance Help the Unemployment Situation?" in which he seemed to be sounding a note of disappointment: "It was always my ambition," he said, "to see this organization of highly trained experts become not only the center of technical information on insurance matters, but also a force for extension of the insurance principle into greater social usefulness."[5]

The great depression came shortly thereafter and presented us with many acute social problems which could not be ignored, but the prevailing point of view among insurance men was, I imagine, fairly expressed in Tom Tarbell's presidential address of May 15, 1931. "Society . . . in the United States," he said, "still places the responsibility of providing food, clothing and shelter upon the individual, provided he is physically and mentally fit to assume that responsibility."[6] Many of us, perhaps, who hold Tom Tarbell in our hearts with great respect and affection, though we may have accepted that point of view at the time, have been forced to move away from it today, however reluctantly.

There were two good papers on Social Insurance in the next decade inspired by the introduction of our national Social Security program. The first, a most learned treatise on "Social Insurance and the Constitution" was by Clarence W. Hobbs,[7] who, of all our members, might properly be allowed to speak from Mount Olympus, since he was what one might call an official's official, being the special representative of the National Association of Insurance Commissioners to the National Council on Compensation Insurance. This paper was presented on November 15, 1935, just three months after the United States Social Security Act had been passed by Congress and signed into law. It was a very thorough discussion of the subject as anything Clarence Hobbs did was thorough. He expressed basic opposition to the national program because to him it seemed "too one-sided," "not conciliatory," and he was convinced it was completely unconstitutional. He was expressing what seemed at the time the prevailing viewpoint of insurance men, or at least of conservative insurance men, and one wondered at that time what other kind there were.

But two years later at the November 1937 meeting we were presented with a refreshingly free wheeling paper, entitled "Social Budgeting," by that most independent of all actuaries, W. R. Williamson, then actuarial consultant to the Social Security Board.[8] It is hard to characterize Bill Williamson either as a liberal or as a conservative. The news magazine *Time* once asserted that he was "too conservative even for the Travelers" from which company he had graduated into the New Deal atmosphere of Washington. This was said of him during one of his many running battles with our National Social Security approach. But a review of his utterances in our Society would hardly indicate conservatism. Most of us, in fact, thought him a bit on the radical side even if he did not agree exactly with the way the New Deal was handling Social Security. We also felt that the subject was too political for actuaries and continued in general to ignore it. The 1939 examinations, for instance, had not a single question on social insurance.

Eleven years passed from the time of Tom Tarbell's statement of rugged individualism at the depression's depth, through the New Deal, to the entry of the United States in the Second World War, and not a single company actuary showed concern through our *Proceedings* for what was happening in the social insurance field until Jarvis Farley stole the show at the November 1942 meeting with his paper, "An Approach to a Philosophy of Social Insurance." It was a considered paper, well planned and well expressed, and most of us in the mood of the time nodded approval as he unfolded his theme. Two brief excerpts will suffice to show how time and technology can play havoc with philosophy.

> "When the war is over the country will have a national debt many times greater than ever before. . . . The interest burden alone will require, in effect, that every-one of us work several hours more each week."[9]

> "The American people must decide in effect how many hours a day they are willing to work, and must buy only those things which that amount of time can pay for. We as a people must recognize that we can have social insurance if we want it, and as much social insurance as we want, but we must first ask ourselves how many of all the valuable choices offered to us we can afford to have, and how much of each."[9]

Comment on Farley's paper did not come from company actuaries, but from two Social Security actuaries, Robert J. Myers, who had just become an Associate of the Society at the time, and our "radical" member Williamson, and also from Professor C. A. Kulp of the University of Pennsylvania, a Fellow of the Society. They all disagreed strongly but gently. A quotation from Williamson makes one wonder how the Travelers could have found him too conservative. "I do not recommend protecting the citizens from securing a fair knowledge of what they may be in for when social budgeting gets under way, nor do I see why they should wait till 'they know all.' Under such caution marriage would be impossible, new enterprises would not arise, the spirit of adventure would die. The times are auspicious for more pioneering, not less, more enterprise, more effective American ingenuity."[10]

At the same meeting that Farley presented his paper, Professor Ralph Blanchard told us:

> "If it is proposed that the government furnish an insurance service which is generally needed, there are four tenable answers: that the service is entirely impracticable, that the government cannot properly furnish the service, that private

initiative can furnish it to better general advantage, or that it should be furnished by the government, either direct or through the agency of private carriers. In any event the actuary should lend his special competence to the solution of whatever problems may arise."[11]

This was the sort of thing one could say from the ivy-covered towers of Columbia University.

The next year, 1943, Williamson wrote once more on Social Security,[12] and then for the next twenty-one years there have been just two papers, an invitational address, the report of one seminar, and a few book reviews dealing with the subject. No president has found it of sufficient moment to make it the subject of his address. An invitational address on medical care insurance was given by Gilbert W. Fitzhugh, President of the Metropolitan Life Insurance Company, at the May 1963 meeting. It expresses the position held by many insurance companies, which is rather parallel with the current position of the American Medical Association and rather far removed from the position that seemed implicit in Dr. Rubinow's hopes and in the employment of Dr. Rubinow by the American Medical Association in 1915.[13] The doctor would, I suspect, have been rather disappointed.

On the other hand our examinations for some years have been doing fair justice to the subject. Dr. Rubinow would be happy at that.

IV. EXAMINATIONS, ADMISSIONS AND MEMBERSHIP

Think it not strange concerning the fiery trial which is to try you, as though some strange thing happened to you.
—II Peter 4:12

With an altruism almost incredible in this practical age, our Society has opened a campaign for the training of our future competitors. There is after all no other meaning to the system of examinations inaugurated a few weeks ago.
—I. M. Rubinow[1]

The First Syllabus: One of the first tasks to which the Society addressed itself was the establishment of a system of examinations for membership. Joseph H. Woodward, Actuary of the New York State Industrial Commission, was the first chairman of the Committee on Examinations, and the first syllabus and rules regarding examinations were adopted by the Council on March 29, 1915.

This first syllabus was ambitious, that for Associateship being in four parts, each part having four sections, or sixteen sections in all. Part I covered elementary algebra, plane trigonometry, analytical geometry and, of all things, double entry bookkeeping. Part II covered advanced algebra, differential and integral calculus, finite differences, and probabilities. Part III included compound interest and annuities certain, statistics, life annuities and assurances, and again, of all things the elements of economics. Lastly Part IV included the practical side of the business: practical problems in statistics, policy forms and underwriting practice in casualty insurance, practical problems in insurance accounting and statistics, and insurance law.

The syllabus for the Fellowship examinations was much simpler, consisting of only two parts, each containing four sections. Part I covered calculation of premiums and reserves, inspection of risks, and the adjustment and settlement of claims, investments of insurance companies, and an all-encompassing section called "current problems." It is a clear indication of the bent of mind of our founders that the whole of Part II, one half of the entire Fellowship examination, was devoted to Social Insurance or its relatives, covering the principles and history of Social Insurance, compilation and use of census or other government statistics, systems of invalidity, old age and unemployment insurance, and the calculation of premiums for and valuation of pension funds.

Although the syllabus was most ambitious in its requirements for Associateship, it was immediately evident that the practical situation at the time made the syllabus unworkable. Announcement was, therefore, made that only Part IV, that part covering the practical side of the business, would be required and given for enrollment as Associate in 1915 and that only Parts III and IV would be required and given in 1916 and 1917. Parts I and II would not be required until 1918. This waiving of Parts I and II was later extended to 1919 and again to 1920.

The first examination, then, of the Casualty Actuarial Society were held October 6, 1915, and consisted of Part IV of the Associateship only. Since there were no Associates presently enrolled there was no one eligible for the Fellowship examinations and none were given. The first question given carried a table of sickness experience of a European Local Sick Benefit Society by principal age periods 1909 and 1912, and asked the candidate, given this table, to discuss the differences in the sickness rate of the two sexes by age. A later question in the same examination is of interest because it gives the first indication anywhere that I can find that the Society knew there was a war on; this question read: "Discuss the probable effect on workmen's compensation experience of the great increase in the manufacture of war materials in the United States. What points should be considered in estimating the catastrophe hazard in the war munitions industries?"

Thirteen candidates passed these examinations and were enrolled as Associates as of October 22, 1915, just sixteen days after they had taken them. This constitutes a record in speed our current examination committee would do well to emulate. Of the thirteen successful enrollees one is still active in our affairs, our "radical conservative," W. Rulon Williamson, then known as William R. Williamson and on the staff of the Travelers Insurance Company.

The next year, 1916, the examinations were shifted to May and have remained there ever since. Parts III and IV of the Associateship only were required and given and Part I only of the Fellowship was given, though Part II was not waived for admission as a Fellow. There is no reason given for this omission and one is led to the conjecture that no one was ready and registered to take it.

One of the questions on the Fellowship examination is of particular interest to us today because, among other implications, it rears the ugly head of Schedule P. It is in three parts:

"a. Explain the uniform rule prescribed by law in several states for computing liability loss reserves. . . .

"b. Is this rule properly applicable to workmen's compensation insurance?

"c. Formulate a rule for computing loss reserves under workmen's compensation insurance policies which would apply with equal justice to a stock company charging low non-participating rates and a mutual company charging high participating rates."

Eight more passed these Associateship examinations and were enrolled as Associates October 27, 1916, and two Associates passed Part I of the Fellowship examinations.

On May 2 and 3, 1917, again the abridged Associateship examinations were given and for the first time a full set of Fellowship examinations. Six more passed the Associateship and two were transferred from Associate to Fellow by examination. The honor of being the first Fellows of the Society to achieve that status by examination went to A. H. Brockway and Robert J. McManus, both characteristically from the Travelers.

The 1921 Revision: But there was developing within the Society a certain uneasiness. To some it seemed a bit anomalous to set up a syllabus for enrollment as Associate, only half of which was actually required year after year. The whole question of examinations was reviewed and a comprehensive report made by the Educational Committee.[2] Apparently some statisticians had been a bit restive and were asking for more than equal treatment. The committee determined, however, that the difference between the actuary and the statistician "was mainly a slight difference in point of view" and that there should be no distinction in examination requirements between the two groups. The committee felt that there was some virtue in pursuing an easier examination policy in the earlier years of the Society with the conscious expectation of tightening up as we grew stronger, citing the example of the Actuarial Society of America as a worthy precedent. Then the committee discussed the distinction between Fellow and Associate, expressing the opinion that the Associateship should be more than merely a step toward the Fellowship, and should "be an evidence of certain qualifications which might justify an executive of a casualty company entrusting certain work definitely to those who had so passed Associate examinations."

The committee then proposed a radical change in the syllabus which abridged the Associateship portion materially, retaining generally the elementary mathematics and the practical insurance problems of the old Part IV though adding the word "simple" in front of "practical problems." The more advanced mathematics, statistics, and life contingencies were transferred to the Fellowship portion and Social Insurance which had been the main thrust of a full half of the old Fellowship examination was reduced to two words in one section which read: "advanced practical problems in compilation and use of statistics relating to casualty (including social) insurance problems." This report was adopted May 28, 1920 to be effective in 1921. Our syllabus had finally become practical indeed.

The 1925 Revision: But this did not last for long. On November 17, 1925, the Council adopted a second complete revision of the syllabus, which concentrated *all* the mathematics sections into the Associateship and *all* the "practical problems" of the insurance business into the Fellowship. Gone was the concept that a casualty company executive might entrust certain work to Associates. Henceforth Associates might gain all the needed know-how from college textbooks.

The 1941 Revision: And here it rested for sixteen years while many of us present-day old-timers sneaked into the Society. In 1941 the Society decided that an Associate should be qualified for an element of trust from the company executive after all and reintroduced insurance practice to the Associateship by adding two non-mathematical sections: Policy Forms and Underwriting Practice, and Casualty Ratemaking Procedures. Social Insurance was most honorably restored to a full section of the Fellowship, but it was no longer considered "practical" enough to share with casualty insurance the "practical problems" questions by the parenthetical "(including social)".

The 1948 and 1955 Revisions: Effective with 1948 Algebra was dropped, and, although Harmon T. Barber[3] in his 1951 presidential address rather warmly lauded the value of mathematical disciplines, mentioning geometry with especial affection, the most radical "de-mathing" of our examinations in our history then followed with the elimination of *all* mathematical sections except Statistics, Probability, and Elementary Life Insurance Mathematics in the 1955 Syllabus. Still more of the "practical" insurance sections, including the one on Social Insurance, were transferred from the Fellowship to the Associateship, and "Machine Methods" was now introduced as a field of questioning for would-be Fellows. This was the high-water mark in "practicality" reached in our fifty year history.

The expressed theory behind this shift in emphasis was that examining a candidate in basic mathematics was unnecessary, since a good working knowledge was implicit in the sections devoted to applied mathematics. Although this theory was probably sound enough, the candidates did not understand it that way, and proceeded to demonstrate, by their wretched showing in the remaining sections, that they, along with some vocal elements in our membership, thought we were letting down the bars.

The 1960 Revision: Then the pendulum swung back. The nature of our mathematical requirements was the subject of an open meeting of the Educational Committee in May, 1956, and also received thorough discussion at several Council meetings. Finally effective with 1960 "General Mathematics" as the first examination was introduced with considerably stiffer mathematical requirements than ever before, and in 1963 we finally achieved mathematical status, or sold out to the competition, depending on your point of view, when this section of the Associateship examination was sponsored jointly with the Society of Actuaries.

The history of our examination syllabus has been a long and confusing story of high theory and practical compromise and the last chapter is not written. There will be many more changes. It can be said honestly, however, that the examination process has done a good job of selection. We who are already in are appalled at the level of learning currently required of candidates, feeling full sure that we could never get in again were we thrown out, but the approach is constantly changing. What was difficult for our parents was easy for us, and *our* high hurdles will be low hurdles to our children. Each generation solves its own problems and wonders why the folks who came before had so much trouble with the problems so conveniently solved today.

The difficulty has not always been in mathematics. Our old friend Charlie Crouse had no trouble with Laplace and Poisson – we called him "Duck Soup" Crouse because of the time he was presenting a summary of a paper before the Society and was progressively covering blackboard after blackboard with the most involved mathematical development when suddenly, apparently sensing the rather dazed and submerged condition of his audience, he turned from the blackboard and said, "Now the rest is duck soup." Duck or chicken, most of us had been in the soup all along. The moment gave comic relief and a battery of august actuaries split their sides. At any rate, Charlie Crouse was denied membership year after year because his very real mathematical aptitude did not help him pass the accounting examination, which he attempted regularly every year. Finally the gods, or the examination committee, took pity on him and he passed.

The generally unsung heroes of the examination system have been the members of the Examination Committee who have put in much time and energy with no reward. It used to be that we had a fairly regular seven year progression. Each new member of the committee started as third man in the Associate section, advancing to become chairman of that section in his third year, then graduated to the Fellowship section for three years, the last as chairman, and in his seventh and last year, if spirit and health held out, he had the ineffable honor of being the general chairman. This practice was highly desirable as providing continuity of content of examinations and also was easy on the President since he had to persuade only one new man to accept service on the committee each year. The system nearly collapsed in 1930 and we almost had a mass resignation when the candidates presenting themselves for the Associateship jumped to sixty-three, more than there had been for the preceding three years combined. Upon investigation it developed that a Professor Warren of the University of Manitoba had given his class in actuarial mathematics the choice of either taking his final examination or one of the examinations of our Society covering the same field. Naturally the students flocked to our examinations as a way both to acquire professional standing and to pass the course for a fee of only five dollars. Norton E. Masterson was chairman of the Associateship section at the time and deserved what came to him, since the young man who suggested the idea to the professor was working for Masterson at the time.

Other Routes to Membership: As an alternative to passing all the examinations set forth in the syllabus the rules have until recently permitted Associates who have passed certain portions of the Fellowship examinations to submit a thesis on an approved subject in lieu of the remaining examinations. In more recent years, Associates of twenty years standing have been permitted to waive *all* the Fellowship examinations by the presentation of an approved thesis. For many years also candidates for Associate membership who have reached a certain level of age and experience in the business were permitted the substitution of a thesis for all the Associateship examinations. Although the so-called "paper route" to membership has not been heavily traveled, it has produced some, though not many, useful papers and valued members. It has also been a source of confusion and embarrassment. There was the question of jurisdiction as between the Examination Committee and the Committee on Review of Papers, since the assumption has usually been made that a paper so submitted should be a useful addition to the *Proceedings*. This was finally resolved in favor of the Committee on Review of Papers. There was also the question as to whether or not the criteria for the acceptance of papers from members for publication in the *Proceedings* should also apply to "paper route" papers. The rule was finally amended to require that a thesis submitted in lieu of Fellowship examinations "shall be of a character which would qualify it for printing in the *Proceedings*."

The Society was also embarrassed from time to time with the assumption by basically unqualified candidates that the "paper route" was a road of admission especially designed for them. The greatest embarrassment of all occurred occasionally when a candidate, usually of some moment in the business, after obtaining approval of the subject, produced with evidence of hard labor, an unacceptable paper. For many years the Society needed both members and papers rather urgently, or so we felt, and the "paper route" served its day. That route was closed in 1962 and no longer may the submission of papers be substituted for the taking of examinations.

The original constitution permitted the Society, upon the recommendation of Council, to admit persons as Fellows without examination by ballot with not more than four negative votes and not less than twenty affirmative votes. This was later changed to three fourths of the Fellows present and voting, and is still in effect. At an early date Council was granted the privilege of waiving the Associateship examinations for candidates who had certain minimum experience qualifications. This privilege was withdrawn in 1962.

The "invitation route" has been at times extensively used by the Society, though now seldom taken. In the early years, however, when we were striving for recognition and the candidates presenting themselves for examination were few, it was a most useful practice to invite into membership prominent insurance executives, many of whom, as Ham Barber expresses it, "had never turned the crank of a Monroe." Most of these gentlemen accepted graciously; in fact some leaders were not averse to letting it be known that they were receptive. They paid their dues, which was important in view of the thin condition of the Society's treasury. Although it is not in the written record, it is had on good authority that the secretary-treasurer in those days would send each of the elected non-actuarial Fellows a full set of the *Proceedings* together with a bill. Apparently the accounts were collected in full. Not only were these members of value financially and in the matter of prestige, but many of them contributed usefully to the *Proceedings* and more particularly to the discussions.

In 1951, when the Society extended its interest to property insurance we added several members from that industry through the "invitation route." The Secretary-Treasurer did not, however, send them the nearly forty volumes of *Proceedings* by then published together with a bill. By then the tables were turned: instead of being a publisher's overstock, early *Proceedings* had become collectors' items.

V. MEETINGS AND PROCEEDINGS

Come now, and let us reason together.

—Isaiah 1:18

A feast is made for laughter, and wine maketh merry.

—Ecclesiastes 10:19

It is quite evident that offices and officers, dues and thoroughly enjoyable dinners, even scientific papers and publications will not alone accomplish all that we hope for, unless all our work is influenced by a few underlying principles.

—I. M. Rubinow[1]

The Pattern: For the first two years of our history we held three meetings a year, settling down thereafter to the basic pattern we have today of a spring and a fall meeting. The only break in this pattern came with the

Second World War when the May meetings were dropped from 1943 to 1946, and no meetings were held whatever during 1944. That year, in view of the emergency situation, we suspended the by-laws quite arbitrarily and continued the officers of the Society for a second year without benefit of election.

The war had a most dramatic and rather permanent effect on our *Proceedings*. For some years the size, if not the quality, of the volumes had been growing to the point that Volume XXVIII, covering the November 1941 and the May 1942 meetings, contained an amazing 651 pages and was three inches thick. The wartime shrinkage was dramatic. Volumes XXIX, XXX, and XXXI contained respectively 208, 127, and 88 pages; the last, being for the year in which we held no meetings, included a presidential address and one paper only. With the resumption of two meetings a year in 1947, the decision was made to have each volume cover both meetings of the same year, so that Volume XXXIV includes the May and November meetings, both of 1947. Since the war the volumes have seldom been more than an inch thick. The question of why our members were more prolific before the war than after has been a matter of considerable concern and in 1954 a Committee on Development of Papers was appointed. In spite of their efforts the quantity of papers seems not to have increased, though quite possibly the quality may be better.

One thing that has remained unchanged for fifty years – the volumes have always been blue.

Business Meetings: Our business meetings, prescribed by the Constitution, have been uniformly dull. Only twice, so far as I have been able to discover, has the breath of life momentarily sparked the sessions. The minutes of the annual meeting of November 15, 1918, show that we elected *three* vice presidents, with the single word "resigned" following one of them.[2] The record gives no more. Yet the circumstance is charged with potential drama and questions keep pressing – why should a candidate resign after he had been nominated and elected, and so soon that his successor could be elected at the same meeting? One can picture the turmoil, the running about, the whispered consultation between the chairman of the nominating committee and the presiding officer. I have queried several members who were listed as present at that meeting, but they just can't seem to remember anything except that there was an unusual hullabaloo. After forty-six years the picture has faded.

The second incident I have not found in the records. It is remembered by Charlie Haugh, though the exact date has faded from memory.

I tell it as he told it to me. It has always been the custom for the Society to accept without question the slate of candidates presented by the Nominating Committee. At times only one candidate for an office has been named and then that candidate is elected. If the Committee decrees that there shall be a contest, it nominates two contenders and the membership duly chooses between the two. Only once and it must have been in the Thirties, some Philistine rose and nominated a candidate for president in addition to the single candidate named by the Committee. Consternation reigned this time also: ballots were now needed; tellers had to be appointed; paper had to be torn into little squares; the Nominating Committee's confidence was shattered. The vote was taken and the count was a tie vote. Again more paper was torn up and passed out. In the run-off the candidate of the Nominating Committee won and orthodoxy has prevailed ever since.

Sociability: Much of the lasting achievement of our Society has not been in the formal meetings nor yet in the printed *Proceedings,* but has developed through the fellowship of the off-hours spent at our semi-annual meetings. Matt Rodermund at the piano; Ham Barber telling a story; Charlie Crouse arguing in a loud voice all night long outside my bedroom door, with whom, I never knew; Arthur Bailey at the hotel bar, late at night, with a soft drink and an attitude toward life that warmed our hearts. We could and did say all manner of nasty things about Arthur Bailey during those years when he was the keeper of our consciences as the actuary of the New York Insurance Department Rating Bureau. But we learned to respect his integrity and stature from knowing him in the after-hours. It is these times we remember best and conjure up when reliving the past fifty years. I regret that time and space restrain me from indulging in a host of reminiscences.

The events of one meeting, however, were so unique that it is still most happily remembered, and was called to my attention by two past presidents. I shall give it here in the words of Charlie Haugh, the central figure in the drama.

> "The first meeting of the Society following my election as president was held at the Biltmore, where I had reserved a room for the night before the meeting. The importance of the office in the eyes of the staff of the hotel was evidenced by my waiting until midnight to be assigned my accommodations, which turned out to be a cot in the Turkish bath located in the sub-basement!
>
> "On the day of the meeting, about noon, Richard Fondiller was

called out and returned quite disturbed. He whispered to me that a bartender with a portable bar was outside and asked what we should do. I immediately adjourned the meeting with the announcement that drinks were available in the reception room, and it proved to be a very popular innovation. Later we learned that some organization of women (not the WCTU) had ordered the bar for noon that day, and the Society was billed for a few gallons of martinis and manhattans. Richard seemed to believe that neither the indignity with which the president had been treated the previous night, nor the fact that we had not ordered the bar and therefore might well believe it to have been a friendly gesture on the part of the management warranted our refusal to pay the bill."

Harmon T. Barber says that this event "came near to establishing a precedent which was found very difficult to upset at the next few meetings. It seemed to be much more sociable to imbibe publicly under the lights, than to slink off with a few cronies to a darkened corner of a subterranean lounge." On one thing Ham Barber is misinformed: Actuaries never "slink off."

Our Literary Tradition: Erudite quotation starts with Rubinow, who in his second presidential address broke into Latin with *"Feci quod potui, faciant meliora potentes,"* which he then translated as, "I have done what I could. Let those who can do better."[3] It has been with us most fashionable to open our papers, or even each chapter, with a quotation from classical or other literature. Today I am following a worthy precedent in the pattern of my chapter headings. Two who have been most adept at this sort of thing have been Tom Carlson and Laurie Longley-Cook, who have seldom let an opportunity slip for the apt quotation. Arthur Bailey occasionally quoted from the Bible and Gus Michelbacher had his favorite source, the old mandarin of Christopher Morley.

The most extensive use of quotations will be found in Volume XXXVIII. Tom Carlson in his monumental work, "Rate Regulation and the Casualty Actuary," opened each section with a useful quotation. The paper was a masterpiece for the insurance learning it encompassed, and the quotations added considerable brilliance to the whole. Since Tom was representing the Bureau point of view I felt it encumbent on me in my discussion of his paper to state the case for the Independents, with all the quotations I could muster, aided by Bartlett and any other source I could find. We had fun that day, and I still chuckle a bit at the quotation from *Kon Tiki,* a best seller of the time, which, as used in the discussion, represented the National Bureau as a half-blind shark which had to have the independent pilot fish show him the way to get about.[4]

Some members have excelled in literary parody, the two masters being Win Greene and Clarence Hobbs. These same two gentlemen were equally adept at producing topical skits, bringing us much enjoyment in an evening's light entertainment. Matt Rodermund seems currently to have inherited this mantle. Clarence Hobbs was also our most noted versifier, being given to rhyming at the slightest provocation. A couple of quotations have already been given in this paper, and space does not permit much more. One quatrain from "The Lady Casualty and Her Servitors" presented at the Society's twenty-fifth anniversary should by its content be repeated here:

"So now our goodly Society we hail with three times three,
As it rounds the happy milestone of its quarter century;
And while our Lady's service does not favor longevity,
When the fiftieth anniversary comes, may we all be there to see!" [5]

Many others, besides Clarence Hobbs, are back with us in spirit enjoying our fiftieth anniversary celebrations.

VI. THE SOCIETY'S PROFESSIONAL CONTRIBUTIONS

And ye shall know the truth, and the truth shall make you free.

—John 8:32

In spite of the confident words uttered by Dr. Rubinow in 1914, scientific rate making is still a goal rather than an accomplishment.

—W. W. Greene[1]

Our Society was founded for the purpose, fundamentally, of applying scientific principles to the insurance business. The founders were convinced that Casualty Ratemaking could be made scientific, a conviction shared probably by no one else in the business at the time, and then proceeded upon a very small foundation to build a science. It was an act of considerable faith and courage, and a measure of the men who did it.

The first paper in Volume I of the *Proceedings* was a brave beginning: "Scientific Methods of Computing Compensation Rates" by Dr. Rubinow, our founder. The second paper, "How Extensive a Payroll Exposure is Necessary to Give a Dependable Pure Premium?" by Albert H. Mowbray, has become a classic, the foundation on which much of the subsequent work done on Credibility Theory has been built.

Credibility: In my research for this historical excursion I asked various members what they felt were the more significant contributions made by our Society. There was a considerable consensus that perhaps the most distinctive contribution made has been the development of statistical procedures for recognizing experience too limited to receive full credibility, "the Theory of Non-Credibility" as Russ Goddard put it. Although the work has generally been done by individuals, the Society has provided the incentive and the forum, and the running record in the *Proceedings* has made a steady evolution possible.

From that first work by Mowbray there has been a continuous stream of papers adding new insights, and making it impossible for a reviewer to do justice to them all. I must make a selection and shall unfortunately have to omit mention of many important contributions.

Perhaps one of the most significant meetings of the Society was held the afternoon of May 20, 1918. This was a "credibility" afternoon. First Albert H. Mowbray added further to his earlier work with "A New Criterion of Adequacy of Exposure," followed by "The Theory of Experience Rating" by Albert W. Whitney and "The Practice of Experience Rating" by G. F. Michelbacher. Reading the Whitney paper today one feels the inherent drama in it, though perhaps at the time his audience, like a CAS audience today, was polite and a bit sleepy and uncomfortable in those small hotel chairs. (Actually the meeting was being held at the Yale Club in New York City.)

The first sentence explained, "This paper traces in an informal way the general line of reasoning that was pursued in an investigation into the theory of experience rating which was made recently by the Actuarial Section of the National Reference Committee on Workmen's Compensation Insurance."[2] He did not mention the names of the actuaries, but we find they were Greene, Flynn, Moore, Mowbray and Woodward, every one a charter member of, and destined to be in time, a president of the Casualty Actuarial Society.[3]

The task before this Actuarial Section had been "the problem of experience rating," he said, which "arises out of the necessity . . . of striking a balance between class-experience on the one hand and risk-experience on the other." He then proceeded to give us a step by step description of the committee's work on this problem. To them it seems to have been pretty rough going. Dr. Whitney's paper is studded with such revealing phrases as, "In the first working out of this problem the assumption was made that . . .," and, "Mr. W. W. Greene, chairman . . ., proposed as an alternative treatment the assumption that. . . ." Then, again: "As Mr. A. H. Mowbray has pointed out, however. . . ." Later work on Credibility Theory takes all this for granted, but we must remember that this was the first time through the forest and considerable circling around for direction had to be done and a good deal of underbrush had to be hacked through.

The problem of a workable formula continued to be elusive until "Mr. Greene made the suggestion that in equation (22) the second term of the denominator be taken as a constant." and finally as a result of Mr. Greene's suggestion they gave us

$$Z = \frac{P}{P + K}$$

and behold the formula we have all learned to know and love! "The simplicity of the formula," Dr. Whitney commented, "is remarkable."

Of course $Z = \frac{P}{P + K}$ is not so great a discovery as $E = mc^2$ nor as unalterably true, but it has made life much easier for insurance men for many generations. Mr. Greene must have been a very brash young man to have made so many suggestions considering the fact that he was only 30 at the time, but he must also have shown great promise, since the committee had made him chairman, or was that because he was at the time Special Deputy Commissioner of Banking and Insurance for the State of New Jersey and came all the way from the other side of the river?

The third paper that day, Michelbacher's "The Practice of Experience Rating," picked up where Whitney left off and gave "the development of a practical plan from fundamental theoretical principles."[3]

It was, indeed, quite a day!

And there it rested for over ten years. There were good papers on ratemaking but not much new and original work until Francis Perryman started writing papers in 1932. Ten years after that Arthur L. Bailey appeared on the scene, and from then on there has been a continuing submission of notable papers on Credibility Theory. Tom Carlson has said that Arthur Bailey was "probably the most profound contributor to casualty actuarial theory the United States has produced."[4] It is rather fashionable for the author of a good mathematical paper even today to start with a quotation from the works of Arthur Bailey. Not only were his mathematical developments outstanding but his English text was lucid. His language broke through the fog even for lay actuaries. An example is the following cogent statement of our basic actuarial problem:

> "Thus the losses paid by an insurer never actually reflect the hazard covered, but are always an isolated sample of all of the possible amounts of losses which might have been incurred. It is this condition, of never being able to determine, even from hindsight, what the exact value of the inherent coverage was, that has brought the casualty actuary into being."[6]

Arthur Bailey often expressed amazement at the statistically unorthodox development of credibility theory. He can be quoted to this effect in a dozen different places. Writing of the need for different schedules of credibility for the three compensation loss components, serious, non-serious, and medical, he says, "It is at this point in the discussion that the ordinary individual has to admit that, while there seems to be some hazy logic behind the actuaries' contentions, it is too obscure for him to understand. The trained statistician cries 'Absurd! Directly contrary to any of the accepted theories of statistical estimation.' The actuaries themselves have to admit that they have gone beyond anything that has been proven mathematically, that all of the values involved are still selected on the basis of judgment, and that the only demonstration they can make is that, in actual practice, it works. Let us not forget, however, that they have made this demonstration many times. It does work!"[7]

It has not always been easy to persuade state officials and underwriters that credibility factors were valid. I can recall the occasion when one of the more thorny insurance commissioners remarked rather testily, "You have supported everything else in the filing with actual experience, where is the experience supporting your credibility factor?" Whereupon we hastily changed the subject. Gus Michelbacher tells of Albert Whitney "presenting a mathematical demonstration of the fundamental principles underlying experience rating. One underwriter asked, 'Where did you get that Z factor?' and braced himself expecting a formidable explanation. Mr. Whitney thought for a moment, adding to the suspense of the occasion, and then replied, 'In Michelbacher's dining room!' "[8]

The history of the CAS would be most incomplete without reference to the negative binomial. If the negative binomial had not existed already, I am sure Lester Dropkin or those twins, LeRoy Simon and Bob Bailey, would have invented it. Tom Carlson has called attention to the fact that actually it first appeared in the CAS *Proceedings* in 1942 and that Arthur Bailey derived it again in 1950.[5] But that was all until 1959. Now, for the past five years it has become a basic doctrine in the actuarial neo-orthodoxy of the 1960's and a big help in making automobile merit rating scientific. It was only a few years ago that the experience of a single car was considered by most of us, even some of our more respected members, as being of so little credibility that to allow it to affect the rate was out of the question. Yes, the negative binomial was a great discovery.

The Society is in debt to L. H. Longley-Cook for preparing "An Introduction to Credibility Theory."[9] In this treatise he has brought together in concise and readably simple form the essential elements of credibility theory as they have developed over the past fifty years. This has great value, not only for students for whom it was prepared, but also for fellow actuaries who have neither the facility nor the time to wade through all the papers written on the subject. This is normally the second step in the conquest of the unknown. The pioneers come first hacking their way through the forests, trekking up blind valleys, and doubling back to try a new approach. It is a painful and prolonged process. But after this has been done the cartographer comes along and with a high skill at illumination makes the going clear, or at least clearer, for the rest of us.

Retrospective Rating: Although retrospective rating did not come into use until 1936, it is interesting to note that one of the early professional controversies in the Society was between the advocates of prospective and retrospective rating, with those who did not believe in either taking potshots at both. This was in 1916. Clairvoyance won, and retrospective rating had to wait twenty years. Space prohibits a discussion of the arguments, pro and con, put forth at that time; one gets the impression in reading them now, particularly between the lines, that the chief argument for retrospective rating was that it provided some opportunity for the stock companies to compete with the mutuals on large risks, and the chief argument against it was that the agents would never be able to manage it. The best potshot taken against experience rating in general was provided by Win Greene in 1916.

> "It has been the intention of the writer to indicate in the foregoing pages that in all probability any system of compensation rates dependent upon the experience of the individual risk will be if universally applied so unpopular as to be virtually unworkable; that the chief genesis of the demand for consideration of individual experience in rating compensation risks lies in the hope for competitive advantage on the part of the carrier; and finally that although experience rating plans have sincere advocates among those who feel that such plans may constitute powerful influences toward accident prevention, there is reason to fear that experience rating in any form may harm rather than help the employee through giving the employer a financial interest in minimizing his workmen's claims."[10]

> "The employer should not be encouraged in the false idea that his own experience is a proper criterion for an equitable rate."[10]

Just two years later Win Greene was made chairman of the actuarial committee that developed the experience rating credibility formula. He was a good soldier.

At this early time Dr. Whitney had shown interest in retrospective rating, and it may be that this interest was transferred to his admiring young understudy at the National Workmen's Compensation Service Bureau, Paul Dorweiler. During this twenty year gap, between 1916 and 1936, Paul Dorweiler did considerable work on excess insurance costs, which laid the foundation for the later work underlying the first retrospective plans. In 1927 he presented a paper in which he gave the first treatment in our *Proceedings* of insurance that takes effect in excess of given loss ratios.[11] This paper won the Society's Woodward Prize. His presidential address in November 1933[12] was credited with providing the method used in compiling the experience underlying the insurance charges in the 1936 plan.[13] His 1936 paper, "On the use of Synthetic Risks in Determining Pure Premium Excess Ratios for Large Compensation and Liability Risks," is still read and its techniques admired by students of retrospective theory.[14] And finally in 1941 he presented a paper in which he explained the graduation work that had been done in the name of the Actuarial Committee of the New York Compensation Rating Board.[15] Dorweiler's methods and results were used for the insurance charges of the revised New York retrospective plan and also became the basis of the 1943 National Council retrospective program under which retrospective rating really attained the considerable importance it now holds.

It should be acknowledged that Paul Dorweiler has earned the right to be called the actuarial father of retrospective rating, one of the important achievements of our profession.

The American Remarriage Table: Most of the professional work recorded in the *Proceedings* was done by individual members or industry committees; very little has been done in the name of the Society. One significant contribution made by the Society itself was the development of an American Remarriage Table. This was the work of a committee appointed in 1929 and was completed for presenting to the Society at its May 1933 meeting.[16] Of the seven man committee that did the work Paul is the only one left with us.

Table of Mortality for Disabled Lives: For this work the Society appointed a committee of three in 1937, which was expanded to seven in 1938. Paul Dorweiler was chairman of this committee. The completed work was presented to the Society at the November 1946 meeting.[17]

Schedule P: An area where the Society has very definitely been unsuccessful in making a contribution, in spite of repeated efforts, has been in the improvement of, or hopefully the elimination of, the Schedule P reserve formula for compensation and liability loss reserves. Schedule P is an ancient monstrosity; its basic pattern was with us when the Society was founded, though originally it was designed to apply to liability insurance only. In Volume II of our *Proceedings* we find Robert K. Orr presenting the same basic criticisms of the formula approach to loss reserving as have been given ever since.[18] In 1924 the Society appointed a committee to see what could be done about Schedule P. After six years of hard labor this committee presented its report.[19] This did not go so far as perhaps most members of the Society would have liked, but it did make some valid recommendations, which were ignored completely by supervisory authority.

In 1947 another committee was appointed with Joseph Linder as chairman. This committee's report, released in 1949, was much more sweeping in its recommendations than the former one.[20] To actuaries, in general, it made sense, and it received about as much attention from supervisory authority as the former report had. The problem, of course, is that the Schedule P formula is written into the laws of many states and into the hearts of many state supervisory officials.

VII. OUR LIFE INSURANCE BRETHREN

And Joseph knew his brethren, but they knew him not.
—Genesis 42:8

There is no reason why this Society should not be as valuable to the Casualty business as the Actuarial Society of America has been to that of life insurance.
—Western Underwriter[1]

In spite of the fact that many of our charter members were also active in the life societies, we as an actuarial body were for years held in rather low esteem by those, our professional brothers. We were a bit upstart at the beginning and our scientific stature had yet to be proven. Then, too, an actuary has been generally considered to be "one who makes those calculations as to the possibilities of human life upon which the issuance of life insurance and annuity contracts depends,"[2] and was not thought to include non-life hazards in his field. But we were keen for recognition and a bit of fraternization, and item 2 of the minutes of the Council meeting held September 17, 1919, begins a story: "The Board of Governors of the American Institute of Actuaries was requested to consider the subject of a joint meeting in May 1920. No response having been received, the matter was laid on the table to be taken up at the next Council meeting." Item 1 of the minutes of the next Council meeting ends the story: "The plan to hold the May 20 meeting in Chicago in conjunction with the American Institute of Actuaries was laid indefinitely on the table."

Actually there has always been considerable cordiality shown us by our life friends. At our twenty-five year celebrations both Mr. Ray D. Murphy, President of the Actuarial Society of America, and Mr. R. A. Hohaus, President of the American Institute of Actuaries, were present as official guests, Mr. Murphy being also a Fellow of our Society. At other times, too, a life society president has attended our annual banquet at our official invitation, and we have been proud to have him. But I think it fair to say that the life Societies have in the past made it clear that, much as they liked us, they could not consider us professional equals.

Most of us were not inclined to blame them. We recognized that our general mathematical stature was somewhat lower than theirs, though catching up rapidly. Nevertheless, we held our heads up with the conviction that a Casualty Fellow had to know more about "other things" than a Life Fellow did.

And then, as so often happens, a threatened danger from without has helped to bring about unity within this, our actuarial family. Because so many charlatans were calling themselves "actuaries" without having achieved membership in *any* society and were performing legally required functions as though they were really actuaries, the Society of Actuaries and the Conference of Actuaries in Public Practice took the initiative to approach us as well as the Fraternal Actuarial Association with the thought that something might be done to set up standards of accreditation and that government might then cooperate.[3] (Here *we* suppressed a bit of snobbishness, for these other two organizations did not require *any* examinations for membership.) A CAS committee was appointed in 1958 to meet with representatives from the other organizations. Their work has proceeded with a remarkable degree of harmony. Finally a Joint Committee on Organization of the Actuarial Profession was set up with one member from each of the four societies, L. H. Longley-Cook being our official representative, though the practical work of this committee has required the participation of many members of all four societies, and very considerable work has been done by William Leslie, Jr., Daniel McNamara, and Frank Harwayne. This committee has prepared a charter, by-laws, election procedure, and committee structure for the organization of a new actuarial body, the American Academy of Actuaries, with the expectation that membership in the Academy may be recognized as a satisfactory accreditation for an actuary. To start with, the Academy would take in the entire membership of the four parent bodies, except that Associates would require several years of experience in responsible actuarial work.

At our May 1964 meeting the CAS approved this project, and, the other three bodies having also given their approval, the joint committee is in the process of seeking federal incorporation. To date our bill has passed the Senate, but not the House.

But this is not all. We *have*, in fact, grown more respectable. We no longer invite into membership dues paying executives who have never "turned the crank of a Monroe" and by far the greater part of our membership has had to pass examinations. We now require a general mathematics examination identical with that of the Society of Actuaries, and we have had as our president from 1961 to 1963 a former life actuary who is an unusually able ambassador of good will, and an able actuary to boot. We have had others like him, of course, before Laurie Longley-Cook took up our cause, and they have all helped, but Laurie has really done the job. The relationship we had sought in 1919 when we were young and gauche has now developed in the fullness of time. The 1963 fall meetings of both the Society of Actuaries and the Casualty Actuarial Society were scheduled for consecutive days in the same city with each body inviting the members of the other to its meeting and with a part of both programs on subjects of common interest. This recognition of joint interest and the joint work, mentioned above, which has been done by the four actuarial bodies looking to the formation of the American Academy of Actuaries speaks well for the future of our profession.

VIII. WHAT IS A CASUALTY ACTUARY?

Those that be near, and those that be far from thee, shall mock thee.

—Ezekiel 22:5

There are many who freely condemn the effects of the entrance of the actuarial mind into the development of the compensation business.

—Sanford B. Perkins[1]

One of the more challenging questions the members of our Society have had to grapple with, and one which has generated considerable disputation, has been, in its general form, "What is an Actuary?" and in its more specific form, "What is a Casualty Actuary?" Like Narcissus we have indulged to a greater extent than we sometimes like to admit in gazing at our reflection, and sometimes it has pleased us and sometimes not. More often than not we find that the image has been distorted because the reflecting pool has been roughly agitated by such rude fellows as underwriters. They have not always understood and respected us, and we do like to be understood and respected.

Those Underwriters: Benedict D. Flynn, our fourth president, once wrote:

> "When the Society was organized, the Casualty actuary was generally looked upon with suspicion by underwriters and others connected with the general management of the business. This was due to the fact that the actuaries had very little knowledge of underwriting principles and the underwriters had not been educated to the value of the statistical methods used by the actuary."[2]

It was perhaps out of delicacy that Mr. Flynn spoke of this incompatibility between actuaries and underwriters as in the past. Actually, like Punch and Judy, these two important members of the insurance household have been taking swipes at each other off and on, mostly on, throughout the history of the Society. This has been both bad and good for the business. It has been bad when it has been accompanied by ill will and obstructive behavior; it has been good when it has operated as a natural system of checks and balances between two properly imperfect approaches to truth. An insurance business completely devoid of underwriters or of actuaries and completely dominated by the other, in this complicated world of today, would be carrying within it the seeds of its own destruction. It is interesting to speculate what problems we would have been faced with at the time of the Supreme Court's Southeastern Underwriters Association decision if the casualty business had not had the thirty year benefit of the Casualty Actuarial Society.

Actually there exists a great deal of mutual respect between actuaries and underwriters, and there have been many able insurance executives who have combined the best characteristics of both. William Leslie, Sr., in a presidential address put it well when he said, "The practical actuary and the logically minded underwriter should have no trouble getting along together but the theoretical actuary and the illogically minded underwriter had better keep away from each other."[3]

There have been several historic verbal battles between the two groups in the past, but space permits me to mention only one. One has the feeling in reviewing this particular fracas that both parties had their tongues in their cheeks, for they were both practical and logical men, both Fellows of this Society, the one a chief executive of his company, the other destined to be in a few years.

In his November 1925 presidential address G. F. Michelbacher had told the Society that he did not think much of the use of judgment in rate-making.[4] He contrasted it with the scientific, or, as he called it, the statistical method and said, "It must be obvious that the writer's preference is for the statistical method." Nevertheless he did allow a minor place for judgment, though hardly the kind of judgment exercised by underwriters, rather a refined sort of intellectual process one might call actuarial judgment that interprets facts "as to their adequacy and reliability" and chooses "that particular formula which best meets the requirements."

Frederick Richardson, U. S. Manager of the General Accident, was undoubtedly the most literate and articulate gift Great Britain has ever contributed to American insurance. He presented a written discussion of the Michelbacher paper at the next meeting, in which he gave what has been perhaps the most lavish description of the underwriter's picture of an actuary yet written. Here it is in part:

> "It might not be out of place at this time to express our sense of satisfaction and our fellowship pride in (Michelbacher's) recent appointment to a still more eminent position in the world of insurance. His entry into the arena of practical and competitive business has some significance for us, and will, moreover, have an influence upon his own views concerning the aims, and ambitions of Insurance Companies. Doubtless he will continue to seek the lofty and hyperborean atmosphere of these assemblies, here to renew and refresh his spirit in studying and admiring the lambent fires and coruscations that play about the *aurora borealis* of abstract mathematics. . . . Here we can gather together with our *a's* and our *b's* and our *x, y, z's* and our graphic outlines to postulate the cost of this and the incidence of that, and if our calculations happen to go awry, we, individually, are not a penny the worse. The burden of the experiment falls upon others. . . ."[5]

It took Gus Michelbacher six years to make his reply. I have no explanation of the long delay, save that Frederick Richardson was always a formidable opponent worth training for. At the May 1932 meeting Gus made his reply in a paper he called "Criticisms and Answers."[6] He did not mention Richardson but he made his purpose clear by quoting that stuff about "lambent fires and coruscations." In this good, well-reasoned, document his point was that "the criticisms of the actuary himself might have been in order at one stage of the game, but they are no longer tenable," and that "criticisms of the results produced by actuaries fail to take into consideration the nature of the problem. . . ."

And then Frederick Richardson landed on him with a whole avalanche of quotations, from, among others, a seventh century Chinese poet, Voltaire, and the *Brooklyn Citizen.* It was a sharp and delightful piece presented to the November 1932 meeting as a discussion.[7]

Michelbacher, as the original author, replied, in part, "Ouch," and, "We are not so far apart after all. This may be because I have modified my ideas with the passage of time."[8] An actuary is always a gentleman.

The Fire Actuaries, if any: Early in our Society's career we cast sidelong glances at the fire insurance business. In 1923 Harwood Ryan wrote, "Finally we should begin to look forward to the time when the rates for fire insurance will be statistically determined,"[9] and Edward R. Hardy expressed the hope that fire insurance ratemaking might become some day semi-scientific, though he found considerable resistance within the industry.[10] After that, for more than twenty years, we stopped looking over the fire fence. With the SEUA decision and Public Law 15 it seemed reasonable to expect that fire insurance ratemaking might see the need for at least a veneer of science, if nothing more, to make it acceptable to state regulatory authorities in view of the danger of Federal take-over. The CAS began to hope we might be called in as firemen for a burning house. Though the call was amazingly slow in coming we started our preparations for it. In 1950 we amended our constitution to state that our field of endeavor was "insurance, other than life insurance," instead of the former words "casualty and social insurance," and we tried very hard to find a name for ourselves that would be more inclusive. At one informal discussion session we experimented with such names as, "Property and Casualty Actuarial Society," "The Actuarial Society for Insurance other than Life," and similar monstrosities, with no success whatever. Finally we concluded that our old name was the best name, that, after all, fires were really casualties in the broad sense, and our fire friends would have to take our name if they wanted to take us. In 1951 several fire insurance ratemaking papers were presented to the Society, and we took in by the invitation route six prominent men in the fire insurance ratemaking field. What is a casualty actuary? He may be a fireman.

While it is true that, compared to casualty underwriters, old time fire underwriters are even more intransigent about actuaries, the fire insurance business is gradually getting used to the actuarial invasion it has suffered, and science is creeping into their processes. *Mirabile dictu!*

As We See Ourselves: Casualty actuaries have always fancied themselves as normal people, in spite of popular expressions to the contrary, which we view with a modicum of tolerant amusement. Here we have Syd Pinney's delightful dissertation, given at the celebration of Richard Fondiller's twenty-five years as Secretary-Treasurer, when he asked us in succession:

> "*What* is so peculiar about an actuary?"
> "What *is* so peculiar about an actuary?"
> "What is *so* peculiar about an actuary?"
> "What is so *peculiar* about an actuary?" and
> "What is so peculiar about an *actuary?*"[11]

This last, he maintained, gave the question the proper perspective. He presented, we felt, a superb performance, delivering a measured speech of well over a half hour completely from memory and in the most delightful spirit. When he was finished we were all convinced that there could be nothing possibly peculiar about an actuary, particularly if his name was Syd Pinney.

We are quite proud of our profession, though we have suffered somewhat from the sense of inferiority imposed on us by our older brothers, the life actuaries. But we have insisted that qualities are demanded of us not required of life actuaries. In comparing the two, Francis Perryman said, "Casualty business involves less technical and mathematical work and essentially deals more with what I term 'humanities' and quicker results are looked for. . . ."[12]

Francis Perryman was perhaps one of our very finest casualty actuaries and certainly our most respected actuarial philosopher. He had a high regard for the profession and saw for it a proud future, which he expressed in these words – no one has said it better:

> "His (the actuary's) will be the privilege of using his knowledge and experience, his actuarial tools and methods, so as to solve our modern social problems, our problems of living together in harmony and cooperativeness; for this is sure, that such problems will be solved and they can be dealt with only by scientific methods that are in essence those we use and know as our actuarial ones."[13]

This is the casualty actuary at full stature and we are indebted to Francis Perryman for giving us the dream – a dream not unlike the one Rubinow had when we were founded.

In gathering data for this paper I have had help from a great many people. I fear I cannot acknowledge them all, but I am particularly indebted to HARMON T. BARBER, RALPH BLANCHARD, PAUL DORWEILER, RUSSELL P. GODDARD, WINFIELD W. GREENE, CHARLES J. HAUGH, JOSEPH LINDER, NORTON E. MASTERSON, EMMA C. MAYCRINK, GUSTAV F. MICHELBACHER, MATTHEW RODERMUND, LEROY J. SIMON, *and* NELS M. VALERIUS. *But above all I am indebted to* LAURENCE H. LONGLEY-COOK, *not only for his good counsel, but also for the use of his library, a*

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THE OPTIMAL MANAGEMENT POLICY OF AN INSURANCE COMPANY

KARL BORCH

1. INTRODUCTION

1.1 In this paper we shall discuss some of the decision problems which occur in insurance companies. We shall try to indicate how these problems may be solved by combining the familiar ideas of actuarial mathematics with those of modern theories of scientific management.

In these theories it is generally accepted that the essential function of management is to make decisions. In an insurance company management has to decide what kind of risks the company shall underwrite and if (or how) these risks shall be reinsured. When the results of an underwriting period become known, management will have to decide whether the profits – if any – shall be distributed as dividend or added to the "special reserves" or "catastrophe funds" of the company.

In general, management will have some rules as to how these decisions shall be made. We shall refer to the body of such rules as the *management policy* of the company.

1.2 If a policy shall be general, it must specify which decision should be taken in every possible situation. Mathematically this means that a policy is a function or a mapping from the set of all situations to the set of all possible decisions. A decision may lead to an action which will bring the company into a new situation.

In this paper we shall not consider all aspects of a complete management policy. We shall study only decisions concerning reserve funds and reinsurance. These decisions have particular actuarial interest, and they can be formulated mathematically in a fairly simple way.

In general there will obviously be an infinity of possible policies. This naturally leads us to consider the problem of determining the best among these policies. However the term "best" has no meaning without a scale of values, or a preference ordering. We must therefore assume that management has a preference ordering over the set of situations in which the company can be. The *objective* of management will then be to select the decision which will bring the company to the most preferred among the attainable situations.

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2. DISCUSSION OF A SIMPLE MODEL

2.1 In this Section we shall discuss an extremely simple example, in order to illustrate and clarify the ideas we have presented in general and rather vague terms in the Introduction.

We shall consider an insurance company which in each operating period underwrites a portfolio of insurance contracts. We shall assume the total claim payment in this portfolio can be:

Either 0 with probability p
or 2 with probability $q = 1 - p$

We shall further assume that the company receives a premium of 1 by underwriting the portfolio.

Our assumptions mean that the company in each period engages in a game, where the gain is either $+1$ or -1, with probabilities p and q respectively. We shall assume that the game is favorable to the company, i.e. that $p > q$.

2.2 Let us now assume that the company's initial capital is S_0. There is obviously a possibility that the capital may be lost after some periods of operations, i.e. that the company becomes insolvent or "ruined." However as the game is favorable to the company, the capital can be expected to grow as time goes by.

It is clear that an increase in the capital will reduce the probability of ruin, so that the company will seek to keep a certain amount of capital as a "special reserve." There must however in practice be some limit to the funds which an insurance company is willing to accumulate to meet such contingencies. In the following we shall assume that this limit is given by a constant Z, i.e. if the company's capital should exceed Z, the excess will be paid out as dividends. Z can then be interpreted as the reserve which management considers necessary to conduct insurance in a manner which will meet all possible demands of prudence and security.

2.3 When management decides on a value Z, is really decides on a *dividend policy,* or a rule stating when dividends should be paid. If the reserves of the company amount to S at the end of an underwriting period, the company will according to the rule pay a dividend

$$s = S - Z \text{ if } S > Z$$

and

$$s = 0 \qquad \text{if } S \leq Z$$

This is obviously a very special rule. More generally we could consider dividend policies given by a rule

$$s = s(S)$$

where $s(S)$ is a function giving the amount s which will be paid as dividend if reserves at the end of an underwriting period are S.

2.4 Decisions concerning reserve funds are naturally linked to decisions with regard to reinsurance arrangements.

In the following we shall for the sake of simplicity assume that the only kind of reinsurance arrangements open to our company is quota share cession on "original terms." This means that management will have to decide on a quota k which shall be retained of the portfolio underwritten by the company.

If the company retains a quota k, and cedes a quota $1-k$, it will engage in a game where the stakes are $+k$ and $-k$, and not $+1$ and -1 as in the original game discussed in paragraph 2.1.

In our simple example the company's *risk* policy will consist of a set of rules stating how the numbers Z and k should be chosen when S is given. When the results of an underwriting period become known, these rules will determine the amount, if any, which shall be paid as dividend, and how the portfolio underwritten in the next period shall be reinsured.

2.5 Any pair (Z,k) will determine a complete risk policy in our simple model. It is however desirable, if possible, to single out one pair which is "best" according to some basic principle or objective which the company wants to reach. In the following we shall assume that the company's objective – at least in part – is to maximize the *expected discounted value of the dividend payments* which the company will make during its lifetime.

In itself this assumption does not appear unreasonable. Insurance is a business, and the ultimate purpose of putting money into business is usually to make it grow and return in the form of dividends. However the real test of an assumption lies in deriving its consequences, or implications and checking if these agree with the observations we can make. Introspection may tell us that the objective of an insurance company may well be to maximize expected dividend payments over a finite or infinite period. However we should not accept this unless we can observe that the company actually behaves as it would if it pursued this objective in a rational manner.

2.6 It is easy to see that the problem we have outlined is the classical problem of "the gambler's ruin." This is solved in many textbooks of probability, so we shall just restate the main results in our own terms, following on most points the presentation given by Feller [3].

Let S be the special reserve fund of the company, and asume $0 < S < Z$.

The game described in paragraph 2.1 can then "terminate" in two ways:

(i) S becomes negative, i.e. $S = -1$ in our simple model. In this case the company is ruined and the game is terminated for good.

(ii) S exceeds Z, i.e. in our simple model $S = Z + 1$. In this case the company pays a dividend of 1, and the game continues with reserve funds equal to Z.

2.7 Let now $w(S,n)$ be the probability that the game terminates with a dividend payment after n periods, i.e. that the reserve fund does not become negative, and reaches $Z + 1$ for the first time after n periods.

It is easy to see that this probability must satisfy the condition

$$w(S,n+1) = pw(S+1,n) + qw(S-1,n)$$

This is a difference equation in two variables with the boundary conditions:

$$\begin{array}{lll} \text{(i)} & w(S,0) = 0 & \text{for } 0 \leq S \leq Z \\ \text{(ii)} & w(-1,n) = 0 & \\ \text{(iii)} & w(Z+1,0) = 1 & \\ \text{(iv)} & w(Z+1,n) = 0 & \text{for } 0 < n \end{array}$$

The equation can be solved directly by classical means. We shall however find it more convenient to introduce the generating function

$$W_S(t) = \sum_{n=0}^{\infty} w(S,n)t^n$$

If we multiply our difference equation by t^{n+1} and sum over all $n \geq 0$, we obtain

$$W_S(t) = pt\, W_{S+1}(t) + qt\, W_{S-1}(t)$$

This is a difference equation in only one variable, with the obvious boundary conditions

$$W_{-1}(t) = 0 \text{ and } W_{Z+1}(t) = 1$$

2.8 The last difference equation has the general solution

$$W_S(t) = A_1 r_1^S + A_2 r_2^S$$

where r_1 and r_2 are the roots of the characteristic equation

$$r = ptr^2 + qt$$

i.e.

$$r_1(t) = (1 + \sqrt{1 - 4pqt^2})/2pt$$

$$r_2(t) = (1 - \sqrt{1 - 4pqt^2})/2pt$$

A_1 and A_2 are functions of t which must be determined so that the boundary conditions are satisfied, i.e.

$$A_1 r_1^{-1} + A_2 r_2^{-1} = 0$$

$$A_1 r_1^{Z+1} + A_2 r_2^{Z+1} = 1$$

From these we obtain

$$A_1 = \frac{r_1}{r_1^{Z+2} - r_2^{Z+2}} \text{ and } A_2 = \frac{-r_2}{r_1^{Z+2} - r_2^{Z+2}}$$

which gives the following expression for the generating function

$$W_S(t) = \frac{r_1^{S+1} - r_2^{S+1}}{r_1^{Z+2} - r_2^{Z+2}}$$

2.9 Let us now assume that the company has established the policy of paying dividend only when its capital exceeds a fixed amount Z, and let $V(S,Z)$ be the expected discounted value of the dividends which will be paid under this policy The probability that the first dividend shall be paid after n periods is $w(S,n)$. If this event occurs, the company will then enter the next period with a capital equal to Z. Hence the expected value of the first payment will be

$$w(S,n) \left\{ 1 + V(Z,Z) \right\}$$

which we will discount by the factor v^n. The first payment can take place after $1, 2, \ldots, n, \ldots$ periods, so that we have

$$V(S,Z) = \sum_{n=1}^{\infty} V^n w(S,n) \quad \left\{ 1 + V(Z,Z) \right\}$$

or if we introduce the generating function for $w(S,n)$

$$V(S,Z) = \left\{ 1 + V(Z,Z) \right\} \; W_S(v)$$

Putting $S = Z$, we obtain

$$V(Z,Z) = \frac{W_Z(v)}{1 - W_Z(v)}$$

and for $0 \leq S \leq Z$

$$V(S,Z) = \frac{W_S(v)}{1 - W_Z(v)}$$

or inserting the expressions for the generating function found in paragraph 2.8

$$V(S,Z) = \frac{r_1^{S+1} - r_2^{S+1}}{(r_1^{Z+2} - r_2^{Z+2}) - (r_1^{Z+1} - r_2^{Z+1})}$$

This result has been derived in different contexts by a number of authors, i.e. by Shubik and Thompson [6] who applied it to a problem very similar to the one considered in this paper.

2.10 Let us now consider reinsurance. We noted in paragraph 2.4 that reinsurance of a quota $1-k$ on original terms was the same as reducing the stakes of the game from -1 and $+1$ to $-k$ and $+k$. This is nothing but a change of unit in the original game, i.e. we have to replace S and Z by $\frac{1}{k}S$ and $\frac{1}{k}Z$. For typographical convenience we shall write $\frac{1}{k} = X$. Hence the expected discounted value of the dividend payments when the company selects a policy (Z,k) is given by

$$V(S,Z,X) = \frac{1}{X} \; \frac{r_1^{XS+1} - r_2^{XS+1}}{r_1^{XZ+1}(r_1-1) - r_2^{XZ+1}(r_2-1)} \text{ where } X \geq 1$$

In the following section we shall discuss this result in some detail, and determine the optimal policy.

3. THE OPTIMAL POLICY IN THE SIMPLE MODEL

3.1 Our problem can now be formulated as follows:

For a given $S \geq 0$, determine the values of X and Z which will maximize:

$$V(S,Z,X) = \frac{\frac{1}{X}\left\{ r_1^{XS+1} - r_2^{XS+1} \right\}}{r_1^{XZ+1}(r_1-1) - r_2^{XZ+1}(r_2-1)} = \frac{N(X)}{M(XZ)}$$

subject to

$$X \geq 1 \text{ and } Z \geq 0$$

Differentiating the denominator with respect to XZ we find

$$M'(XZ) = r_1^{XZ+1}(r_1-1)\log r_1 - r_2^{XZ+1}(r_2-1)\log r_2$$

From the expression found in paragraph 2.8 we note that for $v < 1$ we have $r_1 > 1$ and $r_2 < 1$. Hence $M'(XZ)$ is either always positive, or it has a single zero. This means that $M(XZ)$ takes its minimum value, either for $XZ = 0$, or for the single real root of the equation:

$$\left(\frac{r_1}{r_2}\right)^{XZ+1} = \frac{r_2 - 1}{r_1 - 1} \; \frac{\log r_2}{\log r_1}$$

In the following we shall write Y for this root, and we shall assume that it gives the minimum. The case where the minimum is $M(0)$ is actually trivial. It will occur in situations where the best policy is to pay out the initial capital as dividend immediately, without risking it in the insurance business.

3.2 We now consider the numerator. Differentiating with respect to X we find

$$N'(X) = -\frac{1}{X^2}\{ r_1^{XS+1} - r_2^{XS+1} \} + \frac{S}{X}\{ r_1^{XS+1} \log r_1 - r_2^{XS+1} \log r_2 \}$$
$$= \frac{1}{X^2}\{ (XS \log r_1 - 1)\, r_1^{XS+1} - (XS \log r_2 - 1)\, r_2^{XS+1} \}$$

It is easy to see that $N'(X)$ is negative for small values of X, and that it is steadily increasing with X toward $+\infty$. Hence $N(X)$ takes its maximum value either for $X = 1$, or for the largest attainable value of X.

To get an upper limit for X, we note that the equation in paragraph 3.1 gives us $XZ = Y$, where Y depends only on the given parameters. It then follows that Z will decrease with increasing X, but Z cannot become smaller than S, so that we have

$$Z = \frac{Y}{X} \geq S, \text{ or } X \leq \frac{Y}{S}$$

For $S > Z$ our formula is not valid, since we have by definition

$$V(S,Z) = S - Z + V(Z,Z)$$

Hence the largest value of $N(X)$ is either

$$N(1) = r_1^{S+1} - r_2^{S+1} \text{ or } N\left(\frac{Y}{S}\right) = \frac{S}{Y}\left(r_1^{Y+1} - r_2^{Y+1}\right)$$

It is easy to show that

$$N(1) > N\left(\frac{Y}{S}\right) \text{ for all } S < Y$$

Hence $N(X)$ takes its maximum value for $X = 1$, i.e. when the company retains the whole portfolio. This means that in our simple model, reinsurance will not pay, i.e. it is not possible to increase the expected value of the dividend payments by reinsuring on original terms.

3.3 It may be useful to illustrate the preceding results by a simple numerical example.

We shall take $r_1 = 1.1$ and $r_2 = 0.7$ This corresponds to:

$$p = 0.565,\ q = 0.435 \text{ and } v = 0.983$$

Ignoring reinsurance for the time being, we find that the necessary reserves Z_0 are given by

$$\left\{\frac{11}{7}\right\}^{Z_0+1} = \frac{r_2 - 1}{r_1 - 1}\ \frac{\log r_2}{\log r_1} = 11.23$$

which gives $Z_0 = 4.368$

Table 1 gives the value of $V(S,Z)$ for some selected values of S and Z.

TABLE 1

EXPECTED DISCOUNTED VALUE OF DIVIDEND PAYMENTS

S = Initial Funds	Z = Reserves considered necessary							
	0	1	2	3	4	Z_0	5	6
0	1.25	1.49	1.70	1.83	1.89	1.90	1.89	1.82
1	2.25	2.69	3.05	3.30	3.40	3.41	3.40	3.27
2	3.25	3.69	4.19	4.52	4.67	4.68	4.67	4.49
3	4.25	4.69	5.19	5.56	5.79	5.80	5.79	5.56
4	5.25	5.69	6.19	6.56	6.81	6.83	6.82	6.55
Z_0	5.62	6.05	6.55	6.93	7.18	7.21	7.19	6.98
5	6.25	6.69	7.19	7.56	7.81	7.84	7.69	7.50

To illustrate the meaning of this table, let us assume that our insurance company finds itself with funds $S = 3$ at the end of an underwriting period, and that the management considers paying a dividend.

If management decides that $Z = 2$ is sufficient as a special contingency reserve for the future operations of a company, a dividend $s = 1$ will be paid immediately. This decision means that the expected discounted value of the dividends which the company will pay is equal to $V(3,2) = 1 + V(2,2) = 5.19$. If management is prepared to exercise some patience, and postpone dividend payments until the reserves reach $Z = 4$, this expected value will increase to $V(3,4) = 5.79$. However unlimited patience does not pay. If management should decide that reserves in excess of $Z_0 = 4.37$ are necessary, the expected value of dividend payments will decrease from its maximum value of 5.80. For instance if management should set its target as high as $Z = 6$, the expected value of the dividend payments will be reduced to $V(3,6) = 5.56$.

3.4 To illustrate the effect of reinsurance let us assume that the company reinsures 50% of its portfolio on original terms. According to paragraph 3.1, this will reduce reserve requirements by 50%, so that expected dividend payments will be maximized if the company decides to hold an amount 2.18 in reserve.

Using the notation of paragraph 2.11 we find for some values of S

$$V(0,\ 2.18,\ 2) = 0.95$$
$$V(1,\ 2.18,\ 2) = 2.34$$
$$V(2,\ 2.18,\ 2) = 3.42$$

These are considerably smaller than the corresponding values in Table 1, i.e. $V(0,Z_0) = 1.90$, $V(1,Z_0) = 3.41$, $V(2,Z_0) = 4.68$. This illustrates the point made in paragraph 3.2, that reinsurance does not pay.

3.5 Reinsurance plays an important part in real life, so we ought to explain why it does not appear to have any place in our simple model.

Our paradoxical result may be due to the very simplicity of the model. If we consider claim distributions of a more general form, it is possible that reinsurance arrangements may help to increase the expected value of the dividend payments. We shall not take up this problem here, although it certainly merits further study.

To find a solution to our paradox, we shall try to modify our assumptions about the company's objectives. In actuarial literature much – probably too much – attention has been given to the "probability of ruin." This probability has not proved particularly useful in practical work. In the following we shall consider a related concept, the company's "expectation of life," or in less actuarial terms, the "expected duration of the game." We shall assume that this concept enters into the company's objective function.

3.6 Let $D(S,Z)$ be the expected number of periods our company will stay in business, if the initial capital is S, and the company follows the dividend policy determined by Z.

It is easy to see that $D(S,Z)$ must satisfy the difference equation

$$D(S,Z) = pD(S+1,Z) + qD(S-1,Z) + 1 \quad \text{for } 0 \leq S \leq Z$$

with the boundary conditions

$$D(-1,Z) = 0$$
$$D(Z,Z) = D(Z+1,Z)$$

This equation can be solved by methods similar to those used in paragraph 2.8 (see [3] p. 317), and we find:

$$D(S,Z) = \frac{p}{(p-q)^2}\left\{\left(\frac{p}{q}\right)^{Z+1} - \left(\frac{p}{q}\right)^{Z-S}\right\} - \frac{S+1}{p-q}$$

Putting $p = 0.565$ and $q = 0.435$ as in our numerical example, we obtain $D(S,Z) = 33.4 \left\{ (1.3)^{Z+1} - (1.3)^{Z-S} \right\} - 7.7(S+1)$

Table 2 gives the values of the function $D(S,Z)$ for some selected values of S and Z.

TABLE 2

EXPECTED DURATION OF THE GAME

S = Initial Capital	Z = Reserves considered necessary					
	0	1	2	3	4	5
0	2.3	5.3	9.2	14.3	20.9	29.5
1	2.3	7.6	14.6	23.5	35.2	50.2
2	2.3	7.6	16.9	29.0	44.4	64.7
3	2.3	7.6	16.9	31.2	50.0	74.0
4	2.3	7.6	16.9	31.2	52.3	79.3

This table shows that some patience in paying dividend may increase the company's expectation of life in a dramatic manner.

3.7 To compare the Tables 1 and 2, let us consider an insurance company within initial capital 1.

If this company wants to maximize the expected discounted value of the dividends it will pay during its lifetime, it may decide on the policy of not paying any dividend before its capital exceeds 4 (considering this a sufficient approximation to the optimal value 4.368).

The expected value of the dividend payments will then be:

$$V(1, 4) = 3.40$$

This policy will give the company an expected life $D(1, 4) = 35.1$

If the company decides to reinsure 50% of its portfolio, the expected value of the dividend payments will be maximized if the required reserves is set at 2. This maximum value is

$$V(1,2,2) = \frac{1}{2} V(2,4,1) = \frac{1}{2} V(2,4) = 2.34$$

and the expected life of the company is $D(2,4) = 44.7$

3.8 The example just considered illustrates the point we want to make.

If the company reinsures a part of its portfolio, the expected value of the dividend payments will be reduced, but the company will obtain a longer expected life. It is not unreasonable to assume that the policy of an insurance takes both these elements into consideration.

In the terms of paragraph 2.5 this means that the company will select the policy (Z,k) which maximizes some function of two variables

$$kV\left(\frac{S}{k}, \frac{Z}{k}\right) \text{ and } D\left(\frac{S}{k}, \frac{Z}{k}\right)$$

In this paper we shall not embark on a general discussion of the possible shape of this function. We shall however note that one possible rule would be to maximize V subject to the restraint $D \geq M$ where M is some number, which for instance may be imposed by the government as a solvency requirement.

3.9 Returning to our numerical example, let us assume that for some reason we have fixed $M = 50$. From Table 2 we see that this will lead the company to set its reserve requirements at 4, i.e. whenever the company's reserves exceed 4, the excess will be paid out as dividend. From Table 2 we also see that if reserves should fall to 3, the company will not need reinsurance in order to satisfy the restraint $D \geq 50$, since $D(3,4) = 50$.

If however, reserves should fall to 2, something has to be done, because $D(2.4) = 44.4$, so that the restraint is no longer satisfied. If the company reinsures a quota $1-k$, i.e. retains a quota k, its expected life will become $D\left(\frac{2}{k}, \frac{4}{k}\right)$. By rough interpolation in Table 2, we see that the company can satisfy the restraint by reinsuring approximately 10% of its portfolio, i.e. by choosing $k = 0.9$. Should reserves fall to 1, the company's expected life without reinsurance will be $D(1,4) = 35.2$. In this case the company must reinsure a larger quota in order to satisfy the restraint.

It is worth noting that a restraint of the type $D \geq M$ can always be satisfied by reinsurance, since the company can obtain an infinite expectation of life by adopting the policy of always reinsuring its whole portfolio on original terms. However with this policy the company will never be able to pay any dividend.

If the objective of an insurance company is to maximize the expected discounted value of its dividend payments, subject to a restraint of the form $D \geq M$, the company will reinsure heavily when reserves are low, and reinsure less as reserves accumulate after a number of successful underwriting periods. This is very much the way in which insurance companies seem to behave, so our simple model may contain some of the essential elements of the problem which we set out to study.

4. RELATIONS TO THE COLLECTIVE THEORY OF RISK

4.1 The problems we have discussed in the two preceding sections were first studied in a systematic manner by Filip Lundberg at the beginning of this century. Lundberg's ideas are usually referred to as the "collective theory of risk." This name seems rather unfortunate today, but it appeared quite natural 50 years ago, when a term was needed to distinguish Lundberg's radically new approach from the now almost forgotten theory of risk developed by actuaries in the 19th century.

Lundberg attacked the problem in its fullest generality, and this naturally led to a theory of extreme mathematical complexity. Some recent papers by Cramer [2] and Kahn [4] give short surveys of the main results of the theory and fairly complete bibliographies.

It appears from these surveys that most work on collective risk theory has been concerned with mathematical details rather than the basic ideas behind the theory. In this Section we shall apply these ideas to our simple model, and try to show that the ideas also are fairly simple when stripped of their mathematical superstructure.

4.2 Let $u(S,n)$ be the probability that a company with initial capital S shall be ruined after n periods of operations.

Using the same methods as in paragraph 2.8 we find that the generating function

$$U_S(t) = \sum_{n=0}^{\infty} u(S,n)t^n$$

satisfies the difference equation

$$U_S(t) = ptU_{S+1}(t) + qtU_{S-1}(t)$$

with the obvious boundary condition

$$U_{-1}(t) = 1$$

If the company's policy is to pay out as dividend any capital in excess of Z, we get a second boundary condition

$$U_{Z+1}(t) = U_Z(t)$$

The solution of the difference equation is then

$$U_S(t) = \frac{r_1^{Z+1}\, r_2^{S+1}\,(r_1 - 1) - r_1^{S+1}\, r_2^{Z+1}\,(r_2 - 1)}{r_1^{Z+1}\,(r_1 - 1) - r_2^{Z+1}\,(r_2 - 1)}$$

4.3 For $t = 1$ the generating function becomes the probability that the company eventually shall be ruined, $R(S,Z)$.

From paragraph 2.8 we see that for $t = 1$ we have

$$r_1 = 1 \text{ and } r_2 = \frac{q}{p}$$

Inserting these values in the expression for $U_S(t)$, we find

$$U_S(1) = R(S,Z) = 1$$

This means that the company is certain to be ruined – sooner or later. The result holds for all finite values of S and Z, i.e. regardless of how large the initial capital is, and of how high the reserve requirements are set, as long as they are finite.

Our expression of $U_S(t)$ can be written

$$U_S(t) = \frac{(r_1 - 1)\, r_2^{S+1} - (r_2 - 1)\, r_1^{S+1} \left(\frac{r_2}{r_1}\right)^{Z+1}}{r_1 - 1 - (r_2 - 1)\left(\frac{r_2}{r_1}\right)^{Z+1}}$$

From paragraph 2.8 it follows that $r_1 > r_2$, so that as $Z \to \infty$ we have

$$\lim_{Z \to \infty} U_S(t) = r_2^{S+1}$$

For $t = 1$ we obtain the probability of ruin

$$R(S) = \lim_{Z \to \infty} R(S,Z) = \left(\frac{q}{p}\right)^{S+1}$$

which has played such an important part in the collective risk theory. The basic idea is that the company must maintain reserves S, which are so large that the ruin probability $R(S)$ is smaller than a certain acceptable maximum. Should this be impracticable, the ratio q/p must be reduced, either by reinsurance arrangements, or by "loading" the premium.

4.4 The collective risk theory has never found any significant applications in practice. The reasons are fairly obvious. Most insurance companies pay dividends or declare that they would do so if they had sufficient reserves. They will therefore have little use for a theory which presupposes that the company has a firm policy of never paying any dividend – neither to shareholders nor to policyholders.

In practice insurance companies follow policies which ultimately must lead to bankruptcy. Most actuaries realize this, and accept it. Often they add a remark to the effect that it does not really matter if their company is virtually certain to go out of business within the next 10,000 years. This remark, which really dismisses the whole collective risk theory as useless, also points to a more fruitful formulation of the problem. When ruin is certain, like death and taxes, it is natural to ask when it is likely to occur.

This question was first asked by Segerdahl [5], but he has apparently not followed up the idea. In this paper we have tried to show that it may be possible to create a theory of risk which can be used in practice, if we switch our attention from the traditional ruin probability to the time of ruin.

5. CONCLUDING REMARKS

5.1 The main purpose of this paper has been to study the objectives which insurance companies seek to achieve. If the objectives can be spelled out clearly, it will be possible to determine the operating policy which is "best" or "most efficient" in the company's pursuit of these objectives. A set of objectives may however appear quite reasonable on inspection, but imply an operating policy obviously different from the policy followed by any insurance company.

The enthusiastic expert on operations research may then conclude that management has got it all wrong, and insist that the policy should be changed. On this point the expert is right – if the stated objectives completely represent what the managers at the bottom of their hearts want to achieve.

A more mature social scientist may take a different attitude when confronted with management decisions which are obviously irrational under a stated set of objectives. He may admit the possibility that these decisions are quite rational, but under a set of more subtle objectives than the managers have been able to, or bothered to state explicitly. He may be right on this point, although he will probably not rule out the possibility that managers, like other people, may consistently make foolish decisions.

5.2 In the paper we have tried to illustrate these points by discussing a model which represents a drastic simplification of the real insurance world. By this simplification we may have lost, or "assumed away" some aspects which are essential to the real problem.

The methods of difference equations which we used in Section 2 can obviously be applied also when the discrete stochastic variable can take more than two values, but the mathematics will become very cumbersome as the number of possible values increases. In such cases the characteristic equation will be an algebraic equation of high degree, and may have both complex and multiple roots. The function $M(Y)$ introduced in paragraph 3.1 will then contain terms of the form Y^n and sin Y in addition to the terms r^Y, and may have several local minima. This may clearly mean that there is no unique value of Z which maximizes expected dividend payments. In such models there may well be room for reinsurance.

If we consider continuous stochastic variables, the method of difference equations will obviously break down. However the problem can then be formulated in terms of integral equations, an approach which has been explored in another paper [1].

5.3 The assumption that a firm seeks to maximize the expected discounted value of its dividend payments seems a very natural one. The purpose of business is, almost by definition, to make profits, and the earlier the better.

It should be noted that the discount factor v used in our model does not necessarily have anything to do with the market rate of interest. The discount factor $v < 1$ expresses the assumption that an early dividend payment is preferred to a later one. Put another way we can say that $v < 1$ means that the company looks to first things first, i.e. that it attaches greater weight to secure the dividend payment of 1965 than that of 1970.

The assumption implies that the firm assigns some value to "staying in business." This value is however equal to the expected value of the dividends which the firm will be able to pay during its remaining life. It is not unreasonable to assume that some firms, such as insurance companies, may attach a higher value to "staying alive," and this naturally leads us to assume that the expectation of life, i.e. the function $D(S,Z)$

introduced in paragraph 3.6, enters into the objective function of an insurance company.

5.4 In our model we assumed that the probability p was completely known – or in the terminology of American actuaries – that p had 100% credibility. This is probably more unrealistic than any of our simplifying assumptions.

In practice p will not be completely known, and the company's estimate of p may change as experience accumulates. In this case it is not very reasonable to assume, as we did in paragraph 2.3, that the dividend payment at the end of period n depends only on the reserves at that time, i.e. that the dividend policy is given by a function of one variable

$$s_n = s(S_n)$$

The reasonable assumption would be that the whole accumulated experience of the company is taken into account when a dividend payment is considered. This will give us a dividend rule determined by a function of the form:

$$s_n = s(S_n, S_{n-1}, \ldots, S_1, S_0)$$

To some extent credibility theory has been developed apart from the main body of actuarial mathematics. It appears however that if we want a complete and realistic theory for the management of insurance companies, credibility theory must be brought in as an essential element.

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SIZE OF LOSS DISTRIBUTIONS IN WORKMEN'S COMPENSATION INSURANCE

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This paper was generated in the belief that publication of statistical data setting forth actual distributions of incurred loss amounts by size of loss would be of general interest, and that such data should be made freely available for whatever immediate purpose or use might be made of it by others. In the field of workmen's compensation insurance, there have been relatively few papers presented to our Society concerned specifically with size of loss distributions. Furthermore, such information as has been presented has not dealt with the several different type of injury categories separately.

In addition to simply aggregating masses of data to form empirical size of loss distributions which may then be used in the context of a particular problem area, we are often concerned to try to go beyond the observed distribution and to ask questions about the theoretical distribution underlying the specific data.

As an illustration of this, take for example, the determination of the Non-Serious "D" ratio, one of the steps involved in arriving at the rating values of the Workmen's Compensation Experience Rating Plan. Briefly stated, the procedure is to array the Non-Serious claims for a recent experience period by size, discount them according to the multi-split principle or its equivalent, and then compare the aggregate discounted losses with the aggregate undiscounted losses. This process is usually repeated each year in connection with, and as part of, a normal annual workmen's compensation revision. The Non-Serious "D" ratio used in a particular year is thus an empirical figure. The reason for doing this calculation each year is, obviously, to keep the rating values of the Experience Rating Plan on as up-to-date a basis as possible, so that there will be a correspondence between the Actual Primary (i.e., discounted) Losses and the Expected Primary Losses used in the calculation of experience rating modifications.

If, as is reasonable, we consider that the observed distribution represents the "true" distribution coupled with the effects of a random "disturbance" term, then simply using an empirically derived "D" ratio as our estimate has introduced some error into our calculations. If we had suitable information about the underlying distribution, the possibility of improving our estimates would be strengthened.

The foregoing is merely an illustration of one kind of situation which might engender an interest in size of loss distributions and is typical of the kind of problem area in which our objective is knowledge about size of loss distributions in and for themselves. There is, however, another broad area of concern in which our main objective is knowledge about the distribution of the total amount of claims during a time interval. Here the size of loss distribution is a component element to be considered in conjunction with the claim frequency distribution.[1] One of the main reasons that investigations in this area, generally referred to as the mathematical theory of risk, have not been pursued on other than a very formal and abstract basis, has been the lack of readily available information with respect to the distribution of loss size.

The balance of this paper is divided into three sections. First, we describe the data and set forth the observed distributions. Secondly, we consider the question of fitting a curve to the observed distributions, with specific attention, in the case of Permanent Disability and Temporary, to the log-normal curve. Finally, there are a few summary remarks and comments.

THE DATA[2]

The basic data for this paper is the standard coverage California experience of all companies authorized to write workmen's compensation insurance in California for Policy Years 1960 and 1961, as reported under the Unit Statistical Plan.[3]

California's Statistical Plan is basically similar to that of the National Council on Compensation Insurance, and in common with that Plan, provides for identifying each claim as coming under one of the following type of injury categories: Death, Permanent Total, Major Permanent Partial, Minor Permanent Partial, Temporary, or Medical Only. Further, the Plan

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[1] The general form of the cumulative distribution function, $F(y,t)$, of the total amount of claims during a time interval of length t, is given by:

$$F(y,t) = \sum_{0}^{\infty} p_n(t) \cdot G_n(y)$$

where $p_n(t)$ is the probability of the occurrence of n claims;
$G(y)$ is the cumulative size of loss distribution; and
$G_n(y)$ is the n-fold convolution of $G(y)$ with itself.

[2] Although the analysis, procedures and discussions of the paper are based on data reported to the California Inspection Rating Bureau, the manner in which such data has been utilized and any opinions expressed herein are those of the writer and should not be taken to reflect the position of the Bureau, its Members, or its Committees.

[3] It should be noted that U.S. L & H experience and pneumonoconiosis claims under a classification which is subject to a pneumonoconiosis surcharge are not included in the basic data of the paper.

requires the separate listing of each claim, except that a carrier is permitted to group together (by Manual classification) all closed Medical Only claims on which the incurred medical cost is $500 or less. While the Statistical Plan provides for a first, second and third reporting of experience, the manner in which such second and third report data are filed and processed does not, at present, allow for the tabulation of size of loss data on a second or third report basis. Accordingly, the data used here, for both Policy Year 1960 and Policy Year 1961, is on a first report basis, i.e., the losses are valued as of 18 months after the inception date of the policy.

In general the incurred loss for a Death or Permanent Disability case will include Temporary indemnity benefit amounts as well as the amounts arising out of the Death or Permanent Disability rating itself. Also, the size of the incurred losses, as used in this paper, represents the indemnity and medical amounts combined.

Because not all Medical Only claims are individually listed on the Unit Reports, it was not possible to obtain size of loss distributions for this particular type of injury. That is, this paper deals only with claims involving some form of indemnity benefit.

However, it may be of interest to note the corresponding total number and total amount of Medical Only claims. For Policy Year 1960 there were 639,612 Medical Only claims with a total incurred loss amount of $16,160,673; for Policy Year 1961 there were 583,184 claims and a total incurred loss amount of $16,456,429.

The observed size of loss distributions are set forth in Exhibits 1 through 10, as indicated below:

Policy Year	Type of Injury	Exhibit Number
1960	Death	1
1961	Death	2
1960	Permanent Total	3
1961	Permanent Total	4
1960	Major Permanent Partial	5
1961	Major Permanent Partial	6
1960	Minor Permanent Partial	7
1961	Minor Permanent Partial	8
1960	Temporary	9
1961	Temporary	10

Each exhibit shows, for each given incurred loss size interval, the actual average loss size as well as the number of claims within the interval. (Because of the relatively small number of Permanent Total claims in a year, Exhibits 3 and 4 simply list each claim individually.) A column showing relative frequencies has not been included in these exhibits because they are more usefully displayed in the subsequent exhibits.

THEORETICAL SIZE OF LOSS DISTRIBUTIONS

Death Cases: Even a quite casual comparison of the data for the Death cases given in Exhibits 1 and 2 with the data for the other type of injury categories will reveal that the form of the distribution for Death cases is quite different from the form of the other distributions. Accordingly, the procedure followed with respect to the Death type of case was not that which was used for the Permanent Disability and Temporary categories.

Simple histograms were constructed for the Policy Year 1960 and 1961 Death cases, as shown on Exhibits 11 and 12. The three peaks appearing on each of these exhibits reflect the provisions of the California Labor Code with respect to Death benefits. The Labor Code provides that there shall be benefits as follows:

a. Burial expenses, up to $600; and
b. a death benefit to be allowed to the dependents when the employee leaves any person dependent upon him for support; in the case of total dependency, the benefit is $17,500, except that in the case of a surviving widow and one or more dependent minor children it is $20,500.

The three peaks are thus seen to correspond to: the no-dependency death case; total dependency other than widow and children; total dependency, widow and children. The variation about these three specific benefit amounts arises out of several causes, among which are: variation in the amount of temporary indemnity; variation in the amount of medical; partial dependency; compromised cases.

Permanent Disability (Total, Major, Minor) and Temporary Cases: In contrast to the tri-modal distribution of Death cases, the Permanent Disability and Temporary cases exhibit distributions which accord much more nearly with simple probability distributions. That is, histograms for the observed Permanent Disability and Temporary distributions would show that they are uni-modal, have a relatively much wider range, have a "cocked-hat" shape, and are skewed to the right.

The reason for this difference in the nature of the distributions for the Death cases on the one hand, and for the Permanent Disability and Temporary cases on the other, lies in the fact that there are a much larger number of significant variables interacting with each other in the Permanent and Temporary Disability cases as against the situation in the Death cases where the dependency status variable is the prime determinative.

Previous studies on size of loss distributions for lines of insurance other than workmen's compensation have indicated that "for a quite diverse variety of types of insurance, the log-normal curve is a reasonably good fit."[4] Coupled with this as a reason for focussing on the log-normal curve as being the possible theoretical distribution underlying the data, is the fact that the log-normal curve is easy to handle in numerical work. Other possibilities are referred to in the cited article by Dickerson et al.

In deciding whether or not the log-normal curve provides a good theoretical description of the observed data, several (related) approaches can be used. The techniques can conveniently be referred to as being the visual, the tabular and the analytical method, respectively.

Since each of these techniques was used with each of the remaining type of injury categories, a brief description of these approaches is given next, reserving the discussion of specific results to a subsequent portion of the paper.

As a preliminary, it is of course necessary to convert the observed number of claims to relative frequencies and to deal with the logarithm of the loss size.

A good deal of information can often be gained by simply plotting the data on a suitable graph and visually judging the result. Accordingly, the starting point in considering whether the log-normal described the observed data was to plot the data on special probability-log paper. The horizontal axis on this paper is logarithmic, while the vertical scale is adjusted to reflect the probabilities of the normal curve. This graph paper, therefore, has the property that the cumulative distribution function for the log-normal appears as a straight line.[5] When the observed cumulative frequencies are plotted, the result is, of course, a step-function. However, since the number of loss size intervals was fairly large, vertical lines were added to the step-function graphs at the saltus for better visual delineation.

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[4] Dickerson, O. D.; Katti, S. K.; and Hofflander, A. E.; "Loss Distributions in Non-Life Insurance," *The Journal of Insurance*, Vol. XXVIII, No. 3, p. 49.

[5] The particular commercial graph paper I used was 3 cycle, ± 2.05 standard deviation units, which was then extended manually to ± 2.3 standard deviation units.

The next step continued the visual approach and brought in the tabular. This was to fit a log-normal curve to the observed data and to draw the fitted curves on the graphs.

Sheet 1 of each of Exhibits 13 through 20 are the graphs and show both the step-functions and the fitted log-normal distribution functions.[6] Sheets 2 et seq. of these exhibits give the particulars in tabular form. The tabular information shown is as follows: Loss Size Interval; Observed Cumulative Frequency; Theoretical Cumulative Frequency; Absolute Value of Difference between Observed and Theoretical Cumulative Frequencies. In determining the means and standard deviations the actual average loss size within the interval was used. The cumulative frequencies shown correspond to the upper limit of the interval.

Having fitted a log-normal curve to the observed data it is possible to arrive at a judgment as to the goodness of fit, whether based on a visual impression using the graphs, or based on a comparison of the tabular values of the observed and fitted frequencies. For many of the particular areas of interest, it will be sufficient to stop at this point. The question of whether or not there is a significant difference between the observed and fitted curves will be conditioned on the requirements of the individual problem area under consideration. It may be, for example, that the fit overall is not too good, yet the fit may be quite good over a limited portion of the range, or below (or above) a certain point, where, perhaps, our special interest may lie.

On the other hand, there obviously will be times when it is desirable to have an analytical or statistical test of the goodness of fit. Perhaps the most widely used such test is the Chi-Square. There is however, another statistical test which seems to have many advantages over the Chi-Square test. This test, known as the Kolmogorov test, is, like the Chi-Square test, concerned with the problem of testing the hypothesis that a variable (here, the log of the claim size) has a specified distribution (here, the normal) against the alternative that it has some other distribution. However, while the Chi-Square test function is based on the differences between observed and hypothetical frequencies within cells, the Kolmogorov test is based on the observed and hypothetical cumulative distributions.

The test function in the Kolmogorov test is generally designated by D_n and is defined as the maximum of the absolute deviations between the observed and theoretical cumulative frequencies. That is, if $S_n(x)$ is the ob-

[6] Exhibit 14 has Sheets 1a and 1b rather than a Sheet 1. Sheet 1a corresponds to Sheet 1 of the other exhibits. The purpose of Sheet 1b is explained subsequently.

served cumulative relative frequency in a sample of size n corresponding to any given x, and $F(x)$ is the corresponding theoretical frequency, then[7]

$$D_n = \max_{x} | F(x) - S_n(x) |$$

The test itself consists of calculating the sample statistic D_n and then determining whether D_n exceeds a critical value D_n^α. That is, D_n^α is such that the following relation holds:

$$Prob\ (D_n \leqq D_n^\alpha) = 1 - \alpha$$

If we use an $\alpha = .05$, it turns out that for $n > 35$, $D_n^\alpha = \frac{1.36}{n^{1/2}}$. In applying the test at the 95% level, say, all we need do, therefore, is to calculate the statistic D_n and compare it with the value of $\frac{1.36}{n^{1/2}}$ (assuming $n > 35$). If D_n is more than $\frac{1.36}{n^{1/2}}$ we conclude that the fit is not sufficiently good and we reject the hypothesis that $F(x)$ correctly specifies the theoretical distribution.

Although we have not done so in this paper, the critical value D_n^α can also be used to construct a confidence belt with confidence coefficient $1 - \alpha$ about the observed step-function $S_n(x)$. That is, the two step-functions $S_n(x) \pm D_n^\alpha$ give the required belt for $F(x)$.[8]

It was mentioned above that the Kolmogorov test has many advantages. Among these is the fact that it does not involve any extensive calculations and is easy to use. Another is that the Kolmogorov test appears to be a more powerful test than the Chi-Square test; i.e., for a type 1 error of size α, there is a smaller probability of accepting the hypothesis when in fact the hypothesis is not true with the Kolmogorov test than with the Chi-Square test. Also, the Kolmogorov test can be used with relatively small sample sizes.

A few caveats are, nevertheless, in order. The Kolmogorov test is an exact test only when (i) the data is unclassified, and (ii) the parameters of

[7] Technically, D_n is defined as the least upper bound of the absolute deviation of $S_n(x)$ from $F(x)$; from a practical viewpoint this means the maximum.

[8] For $n > 35$, $D_n = \frac{\lambda}{n^{1/2}}$

The values of λ for several values of α are as follows:

α	.20	.10	.05	.01
λ	1.07	1.22	1.36	1.63

For $n < 35$ it is necessary to look up D_n^α in a table.

the hypothetical distribution are not estimated from the data. However, the discrepancy introduced by using grouped data is negligible if the grouping is not too coarse, as we believe is the case here. The second point is more important. If the parameters are estimated from the data, we can correct for the effect of this when a Chi-Square test is used by reducing the degrees of freedom. Unfortunately the effect of estimating the parameters from the data has not been worked out with respect to the Kolmogorov test. The recommended procedure is to correct for this effect by using a critical value smaller than would otherwise be used.[9]

Specific Results – Permanent Disability and Temporary Cases: Before turning to a more detailed consideration of the specific results as set forth in Exhibits 13 through 20, mention should be made of one of the problems that often arises in dealing with a given body of observed data, viz., the possibility that the data has been "contaminated." It will, perhaps, have been noted that among the Permanent Total cases reported for Policy Year 1961 was one case where the incurred loss size was $1,840. Now this is certainly an odd looking figure to find among the Permanent Total cases and it raises some immediate questions. It is, of course, possible that everything is quite legitimate, that it is truly a P. T. case, correctly entered, coded and punched with respect to both type of injury and amount. On the other hand, any one of a number of different types of errors could have occurred. Should the figure be disregarded? It could be argued that one's theory must be broad enough to encompass all possibilities, including mistakes of one sort or another; that mistakes will occur and that in routine handling of data such mistakes will remain unnoticed and uncorrected. This sort of reasoning argues for retaining the figure. One could equally argue for dropping it. The answer really depends on one's particular purposes in a specific context. Since the purpose of this paper is to present information, we have begged the question by including two sets of sheets for Exhibit 14. Those sheets marked with an "a" refer to the unadjusted data of Exhibit 4, Sheet 1; those marked with a "b" refer to the data excluding the $1,840 case.

In visually reviewing the graphs it should be noted that the incurred loss size is expressed in thousands for the Permanent Total and Major Permanent Partial cases; in hundreds for the Minor Permanent Partial cases; and in tens for the Temporary cases.

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[9] A discussion of the Kolmogorov test can be found in Hoel, P. G., *Introduction to Mathematical Statistics*, 3rd ed., Wiley, pp. 345-349; and in Keeping, E. S., *Introduction to Statistical Inference*, Van Nostrand, pp. 256-259.

It will, I think, be generally agreed that the visual impression one gets in reviewing the graphs is that the fit is not unacceptable for each of the categories and for each of the policy years. However, the answer given by the Kolmogorov test of goodness of fit is somewhat different.

Exhibit 21 sets forth the pertinent information for each of the types of injury, for each of Policy Years 1960 and 1961. Shown on this exhibit are the following: Number of Cases *(n)*; the parameters used in fitting a normal curve to the logarithms of the loss sizes, i.e., the mean and standard deviation;[10] the sample statistics D_n; the corresponding critical values $D_n^{.05}$; the result of applying the Kolmogorov test, i.e., accept or reject the hypothesis that the logarithm of the claim size has a normal distribution.

The result of applying the Kolmogorov test at the 95% level, as shown on Exhibit 21, is a rejection of the hypothesis for the Major, Minor and Temporary categories. The fit would appear to be acceptably good for the Permanent Total category. However, in view of the remarks above with regard to estimating parameters from the data one should perhaps say that the fit is just acceptable for the Permanent Total category.

The different conclusions reached by the visual and analytical approaches are only apparent and can be resolved by remembering two facts. The first is that the vertical scale on the graphs is not linear. Therefore, for example, if two given vertical distances are equal, they will not, in general, represent equal portions of the total frequency. That is, one must adjust his visual impressions to the vertical scale. Secondly, the graphs cannot emphasize the dependence of a goodness of fit test on the number in the sample. Thus, for example, while the value of D_n for Temporary for 1960 is much smaller than the value of D_n for Permanent Total for 1961 (something which is ascertainable from the graphs or tables and to be expected given the much larger number of Temporary cases) the graphs or tables by themselves cannot indicate whether the drop in the value of D_n is commensurate

[10] The mean, variance and skewness of the corresponding log-normal curves can be found as follows (assuming logs to base 10 were used in the transformation): If α and $\beta^{1/2}$ stand for the mean and standard deviation as shown on Exhibit 21, then the mean and variance (μ and σ^2) of the log-normal is given by

$$\mu = \exp\left[\frac{\alpha}{c} + \frac{\beta}{2c^2}\right]$$

$$\sigma^2 = \mu^2 \eta^2$$

where $$c = \log e = .43429,$$

and $$\eta^2 = \exp\frac{\beta}{(c^2)} - 1.$$

The skewness is given by $(\eta^3 + 3\eta)$

with the increase in the number of cases. This, of course, is the point and purpose of a "critical value" in an analytical or statistical test.

One additional fact seems to be worthy of specific recognition. Many of the actions and decisions of an Actuary are predicated, explicitly or implicitly, on the assumption that a distribution observed to exist in some past period will continue to be the appropriate distribution in a future period. It is therefore of some interest to note that for each of the type of injury categories, the shape of the observed distribution for Policy Year 1961 is basically the same as that for Policy Year 1960.

SUMMARY

The size of loss data for the various type of injury categories normally recognized in workmen's compensation insurance has been presented in some detail in accordance with the general objective of making available factual material which can then be used in connection with consideration of problems relating to ratemaking, individual risk rating plans, reinsurance and other more specific areas of interest.

The distribution of Death cases has been seen to be directly conditioned by the dependency status variable and the concomitant statutory benefit provisions. Based on the Kolmogorov goodness of fit test at the 95% level, the log-normal distribution does not seem to provide an exact description of the Permanent Disability and Temporary cases, with the possible exception of Permanent Total. Nevertheless, the fact that the log-normal distribution is relatively easy to handle may dictate its use in many areas.

It should again be noted that, while we may not be able to specify exactly what hypothetical distribution underlies an observed distribution, it is still possible to utilize a critical value to construct a confidence belt about the observed distribution, and thereby obtain useful quantitative answers.

The data set forth in this paper, and the specific results described, reflect the experience of two specific years for a specific state. It would clearly be of great value if similar analyses were made of other bodies of data.

I should like to conclude this paper with the following observation: It may be possible to conclude, after a sufficient number of studies, that some given probability function adequately describes the distribution of losses by size. This would be a major achievement. Nevertheless, such a step should be considered as merely a preliminary to the ultimate construction of an appropriate model.

Exhibit No. 1

CALIFORNIA WORKMEN'S COMPENSATION
DISTRIBUTION OF LOSSES FOR DEATH CASES
BY TOTAL LOSS SIZE

Policy Year 1960 - 1st Reports

Loss Size Interval	Number of Cases	Average Loss Size
0 - 499	15	271.53
500 - 999	39	647.28
1,000 - 1,499	14	1,133.71
1,500 - 1,999	11	1,744.73
2,000 - 2,499	6	2,151.00
2,500 - 2,999	4	2,594.25
3,000 - 3,499	6	3,115.00
3,500 - 3,999	1	3,764.00
4,000 - 4,499	11	4,190.09
4,500 - 4,999	2	4,875.00
5,000 - 5,499	9	5,036.89
5,500 - 5,999	4	5,625.00
6,000 - 6,499	7	6,208.71
6,500 - 6,999	2	6,645.00
7,000 - 7,499	3	7,269.33
7,500 - 7,999	9	7,638.00
8,000 - 8,499	4	8,172.75
8,500 - 8,999	5	8,585.40
9,000 - 9,499	4	9,144.25
9,500 - 9,999	2	9,700.00
10,000 - 10,499	14	10,077.14
10,500 - 10,999	8	10,809.38
11,000 - 11,499	5	11,170.80
11,500 - 11,999	2	11,585.00
12,000 - 12,499	1	12,000.00
12,500 - 12,999	2	12,525.00
13,000 - 13,499	4	13,090.75
13,500 - 13,999	1	13,500.00

Loss Size Interval	Number of Cases	Average Loss Size
14,000 - 14,499	4	14,200.00
14,500 - 14,999	4	14,500.00
15,000 - 15,499	6	15,035.83
15,500 - 15,999	1	15,637.00
16,000 - 16,499	2	16,062.50
16,500 - 16,999	1	16,682.00
17,000 - 17,499	8	17,144.50
17,500 - 17,999	10	17,730.00
18,000 - 18,499	83	18,197.81
18,500 - 18,999	19	18,643.79
19,000 - 19,499	13	19,173.69
19,500 - 19,999	13	19,698.46
20,000 - 20,499	9	20,212.33
20,500 - 20,999	15	20,765.13
21,000 - 21,499	188	21,176.41
21,500 - 21,999	24	21,690.75
22,000 - 22,499	11	22,240.09
22,500 - 22,999	8	22,825.50
23,000 - 23,499	6	23,237.67
23,500 - 23,999	5	23,635.80
24,000 - 24,499	2	24,182.50
25,000 - 25,499	1	25,200.00
25,500 - 25,999	3	25,712.00
26,500 - 26,999	1	26,630.00
0 - 26,999	632	15,401.03

Exhibit No. 2

CALIFORNIA WORKMEN'S COMPENSATION
DISTRIBUTION OF LOSSES FOR DEATH CASES
BY TOTAL LOSS SIZE

Policy Year 1961 - 1st Reports

Loss Size Interval	Number of Cases	Average Loss Size	Loss Size Interval	Number of Cases	Average Loss Size
0 - 499	16	312.50	16,000 - 16,499	3	16,200.00
500 - 999	46	644.70	16,500 - 16,999	4	16,945.75
1,000 - 1,499	13	1,188.15	17,000 - 17,499	10	17,204.80
1,500 - 1,999	8	1,646.13	17,500 - 17,999	7	17,662.29
2,000 - 2,499	11	2,145.73	18,000 - 18,499	99	18,176.63
2,500 - 2,999	13	2,656.69	18,500 - 18,999	20	18,708.40
3,000 - 3,499	7	3,169.43	19,000 - 19,499	12	19,188.25
3,500 - 3,999	8	3,590.63	19,500 - 19,999	12	19,658.25
4,000 - 4,499	7	4,269.71	20,000 - 20,499	12	20,279.75
4,500 - 4,999	9	4,660.44	20,500 - 20,999	11	20,693.45
5,000 - 5,499	22	5,085.05	21,000 - 21,499	213	21,175.23
5,500 - 5,999	5	5,592.80	21,500 - 21,999	33	21,640.24
6,000 - 6,499	3	6,205.33	22,000 - 22,499	14	22,157.57
6,500 - 6,999	4	6,762.25	22,500 - 22,999	9	22,636.22
7,000 - 7,499	3	7,093.67	23,000 - 23,499	5	23,253.60
7,500 - 7,999	12	7,558.75	23,500 - 23,999	4	23,689.25
8,000 - 8,499	11	8,138.36	24,000 - 24,499	3	24,352.67
8,500 - 8,999	3	8,670.00	24,500 - 24,999	2	24,895.00
9,000 - 9,499	6	9,079.17	25,000 - 25,499	5	25,310.60
9,500 - 9,999	5	9,726.80	25,500 - 25,999	2	25,765.00
10,000 - 10,499	9	10,117.11	26,000 - 26,499	2	26,367.00
10,500 - 10,999	16	10,624.44	26,500 - 26,999	1	26,981.00
11,000 - 11,499	5	11,124.80	27,000 - 27,499	1	27,254.00
11,500 - 11,999	4	11,661.25	28,000 - 28,499	1	28,400.00
12,000 - 12,499	5	12,144.40	29,500 - 29,999	1	29,790.00
12,500 - 12,999	4	12,647.50	30,500 - 30,999	1	30,750.00
13,000 - 13,499	2	13,125.00	34,000 - 34,499	1	34,000.00
13,500 - 13,999	5	13,729.80	37,000 - 37,499	1	37,222.00
14,500 - 14,999	4	14,691.50	43,000 - 43,499	1	43,312.00
15,000 - 15,499	9	15,021.11	73,000 - 73,499	1	73,090.00
15,500 - 15,999	4	15,759.50	0 - 73,499	770	15,251.35

Exhibit No. 3

CALIFORNIA WORKMEN'S COMPENSATION
DISTRIBUTION OF LOSSES FOR PERMANENT TOTAL CASES
BY TOTAL LOSS SIZE

Policy Year 1960 - 1st Reports

Loss Size	Loss Size	Loss Size
12,380	68,391	147,563
32,499	69,653	147,663
39,348	75,394	159,121
40,299	80,000	161,415
43,624	86,828	164,208
44,977	89,028	165,183
46,000	104,500	174,404
54,825	107,326	179,169
55,338	114,514	199,965
56,000	118,144	206,511
56,001	119,874	280,354
58,506	121,200	292,525
58,600	125,000	4,955,238
59,673	128,985	
62,500	135,844	
63,291	139,845	
67,206	141,564	

No. of Cases = 46
Ave. Loss Size = 107,723

Exhibit No. 4

CALIFORNIA WORKMEN'S COMPENSATION
DISTRIBUTION OF LOSSES FOR PERMANENT TOTAL CASES
BY TOTAL LOSS SIZE

Policy Year 1961 - 1st Reports

Loss Size	Loss Size	Loss Size
1,840	75,000	108,637
33,300	75,500	109,521
46,000	76,823	111,591
48,457	77,711	115,547
50,247	79,304	132,946
53,200	81,969	145,787
53,327	83,000	150,000
53,653	83,481	152,015
55,000	86,690	156,995
59,371	89,000	166,644
62,100	93,410	172,826
62,522	94,816	174,600
63,800	99,187	201,460
64,588	100,187	213,260
64,726	100,340	250,351
65,340	101,090	254,494
68,874	101,312	331,151
70,639	103,515	5,889,192
72,679	107,493	
73,391	108,485	

No. of Cases = 57
Ave. Loss Size = 103,319

Exhibit No. 5
Sheet 1

CALIFORNIA WORKMEN'S COMPENSATION
DISTRIBUTION OF LOSSES FOR MAJOR CASES
BY TOTAL LOSS SIZE

Policy Year 1960 - 1st Reports

Loss Size Interval	Number of Cases	Average Loss Size
0 - 99	6	26.17
400 - 499	1	436.00
700 - 799	3	764.33
900 - 999	2	980.50
1,000 - 1,499	2	1,204.00
1,500 - 1,999	1	1,950.00
2,000 - 2,499	5	2,271.00
2,500 - 2,999	3	2,820.00
3,000 - 3,499	3	3,267.33
3,500 - 3,999	8	3,633.00
4,000 - 4,499	14	4,143.93
4,500 - 4,999	8	4,706.50
5,000 - 5,499	22	5,186.95
5,500 - 5,999	46	5,708.37
6,000 - 6,499	67	6,231.33
6,500 - 6,999	92	6,730.51
7,000 - 7,499	112	7,218.96
7,500 - 7,999	141	7,710.26
8,000 - 8,499	153	8,202.44
8,500 - 8,999	157	8,717.49
9,000 - 9,499	182	9,201.43
9,500 - 9,999	173	9,698.76
10,000 - 10,499	196	10,176.51
10,500 - 10,999	150	10,683.21
11,000 - 11,499	152	11,180.76
11,500 - 11,999	134	11,678.15
12,000 - 12,499	125	12,156.67
12,500 - 12,999	94	12,676.38
13,000 - 13,499	120	13,165.83
13,500 - 13,999	113	13,667.52
14,000 - 14,499	87	14,159.68
14,500 - 14,999	65	14,656.89
15,000 - 15,499	86	15,145.59
15,500 - 15,999	71	15,642.51
16,000 - 16,499	51	16,111.10
16,500 - 16,999	57	16,684.53
17,000 - 17,499	45	17,163.53
17,500 - 17,999	40	17,675.40
18,000 - 18,499	45	18,122.18
18,500 - 18,999	31	18,647.03
19,000 - 19,499	31	19,149.10
19,500 - 19,999	31	19,638.00
20,000 - 20,499	34	20,095.03
20,500 - 20,999	17	20,668.88
21,000 - 21,499	23	21,112.87
21,500 - 21,999	19	21,720.26
22,000 - 22,499	23	22,109.43
22,500 - 22,999	16	22,632.75
23,000 - 23,499	17	23,182.82
23,500 - 23,999	15	23,659.00
24,000 - 24,499	15	24,104.60
24,500 - 24,999	4	24,639.50
25,000 - 25,499	21	25,116.71
25,500 - 25,999	7	25,593.71
26,000 - 26,499	6	26,120.17
26,500 - 26,999	6	26,595.83
27,000 - 27,499	4	27,111.75
27,500 - 27,999	2	27,631.00
28,000 - 28,499	7	28,125.71
28,500 - 28,999	7	28,605.71

Exhibit No. 5
Sheet 2

Loss Size Interval	Number of Cases	Average Loss Size
29,000 - 29,499	2	29,064.00
29,500 - 29,999	5	29,632.80
30,000 - 30,499	5	30,112.00
30,500 - 30,999	3	30,533.33
31,000 - 31,499	2	31,160.00
31,500 - 31,999	1	31,617.00
32,000 - 32,499	3	32,018.33
32,500 - 32,999	3	32,704.00
33,000 - 33,499	3	33,163.00
33,500 - 33,999	4	33,583.75
34,000 - 34,499	2	34,213.50
34,500 - 34,999	1	34,530.00
35,000 - 35,499	4	35,142.00
36,000 - 36,499	4	36,178.75
36,500 - 36,999	1	36,550.00
37,000 - 37,499	2	37,033.00
37,500 - 37,999	1	37,610.00
38,500 - 38,999	2	38,671.00
39,000 - 39,499	1	39,490.00
39,500 - 39,999	1	39,686.00
40,500 - 40,999	4	40,777.00
41,000 - 41,499	1	41,462.00
42,000 - 42,499	2	42,090.00
43,000 - 43,499	2	43,300.50
44,000 - 44,499	2	44,167.00
45,000 - 45,499	1	45,079.00
45,500 - 45,999	3	45,737.67
46,500 - 46,999	1	46,693.00
48,000 - 48,499	1	48,130.00
49,000 - 49,499	1	49,440.00
50,000 - 50,499	3	50,135.33
50,500 - 50,999	1	50,920.00
52,000 - 52,499	1	52,140.00
53,000 - 53,499	3	53,187.00
54,000 - 54,499	1	54,162.00
54,500 - 54,999	1	54,997.00
55,500 - 55,999	1	55,516.00
56,000 - 56,499	1	56,000.00
59,500 - 59,999	3	59,853.67
60,000 - 60,499	2	60,000.00
60,500 - 60,999	1	60,717.00
61,500 - 61,999	1	61,656.00
64,500 - 64,999	1	64,912.00
65,000 - 65,499	2	65,258.00
68,000 - 68,499	1	68,344.00
71,000 - 71,499	1	71,476.00
71,500 - 71,999	1	71,540.00
74,500 - 74,999	1	74,772.00
76,000 - 76,499	1	76,307.00
77,500 - 77,999	1	77,869.00
88,500 - 88,999	1	88,811.00
90,000 - 90,499	1	90,000.00
94,000 - 94,499	1	94,000.00
95,000 - 95,499	1	95,040.00
98,000 - 98,499	1	98,428.00
102,000 - 102,499	1	102,366.00
186,000 - 186,499	1	186,000.00
0 - 186,499	3,271	13,172.79

Exhibit No. 6
Sheet 1

CALIFORNIA WORKMEN'S COMPENSATION
DISTRIBUTION OF LOSSES FOR MAJOR CASES
BY TOTAL LOSS SIZE

Policy Year 1961 - 1st Reports

Loss Size Interval	Number of Cases	Average Loss Size
0 - 99	3	60.67
400 - 499	2	459.00
700 - 799	1	700.00
900 - 999	1	937.00
1,000 - 1,499	2	1,349.00
1,500 - 1,999	2	1,749.00
2,000 - 2,499	2	2,451.50
2,500 - 2,999	2	2,678.50
3,000 - 3,499	11	3,266.18
3,500 - 3,999	18	3,726.61
4,000 - 4,499	23	4,188.87
4,500 - 4,999	26	4,620.62
5,000 - 5,499	45	5,175.00
5,500 - 5,999	63	5,728.59
6,000 - 6,499	112	6,224.45
6,500 - 6,999	124	6,694.15
7,000 - 7,499	155	7,205.90
7,500 - 7,999	173	7,704.00
8,000 - 8,499	185	8,201.86
8,500 - 8,999	217	8,681.21
9,000 - 9,499	220	9,189.52
9,500 - 9,999	213	9,693.54
10,000 - 10,499	230	10,184.08
10,500 - 10,999	202	10,706.73
11,000 - 11,499	192	11,162.41
11,500 - 11,999	177	11,687.40
12,000 - 12,499	207	12,175.69
12,500 - 12,999	167	12,653.72
13,000 - 13,499	181	13,159.60
13,500 - 13,999	145	13,670.69
14,000 - 14,499	141	14,174.25
14,500 - 14,999	135	14,668.33
15,000 - 15,499	133	15,117.36
15,500 - 15,999	100	16,674.01
16,000 - 16,499	103	16,146.57
16,500 - 16,999	74	16,693.46
17,000 - 17,499	76	17,179.34
17,500 - 17,999	74	17,662.42
18,000 - 18,499	80	18,175.70
18,500 - 18,999	44	18,664.34
19,000 - 19,499	50	19,147.78
19,500 - 19,999	59	19,684.10
20,000 - 20,499	54	20,118.19
20,500 - 20,999	33	20,657.36
21,000 - 21,499	38	21,183.79
21,500 - 21,999	34	21,673.65
22,000 - 22,499	25	22,102.08
22,500 - 22,999	23	22,705.91
23,000 - 23,499	24	23,177.88
23,500 - 23,999	16	23,635.94
24,000 - 24,499	20	24,196.55
24,500 - 24,999	17	24,675.00
25,000 - 25,499	21	25,127.95
25,500 - 25,999	13	25,687.77
26,000 - 26,499	13	26,155.54
26,500 - 26,999	7	26,664.71
27,000 - 27,499	7	27,104.29
27,500 - 27,999	13	27,636.54

Exhibit No. 6
Sheet 2

Loss Size Interval	Number of Cases	Average Loss Size
28,000 - 28,499	6	28,054.17
28,500 - 28,999	12	28,682.83
29,000 - 29,499	7	29,278.43
29,500 - 29,999	6	29,777.50
30,000 - 30,499	8	30,179.50
30,500 - 30,999	2	30,750.00
31,000 - 31,499	2	31,277.50
31,500 - 31,999	4	31,779.50
32,000 - 32,499	8	32,305.38
32,500 - 32,999	2	32,666.50
33,000 - 33,499	3	33,156.67
33,500 - 33,999	7	33,790.29
34,000 - 34,499	2	34,244.50
34,500 - 34,999	2	34,690.00
35,000 - 35,499	3	35,252.33
35,500 - 35,999	5	35,695.40
36,000 - 36,499	4	36,115.50
36,500 - 36,999	2	36,822.50
37,000 - 37,499	2	37,233.50
38,000 - 38,499	2	38,073.50
38,500 - 38,999	1	38,590.00
39,000 - 39,499	1	39,462.00
39,500 - 39,999	5	39,754.40
40,000 - 40,499	5	40,193.40
40,500 - 40,999	4	40,767.25
41,000 - 41,499	3	41,137.33
41,500 - 41,999	3	41,800.67
42,000 - 42,499	1	42,256.00
42,500 - 42,999	2	42,865.00
43,000 - 43,499	4	43,233.75
43,500 - 43,999	1	43,830.00
44,000 - 44,499	1	44,000.00
45,000 - 45,499	2	45,168.50
45,500 - 45,999	3	45,758.67
46,000 - 46,499	4	46,186.50
46,500 - 46,999	2	46,679.50
47,000 - 47,499	3	47,108.00
47,500 - 47,999	4	47,697.00
48,000 - 48,499	1	48,087.00
49,500 - 49,999	2	49,739.00
50,000 - 50,499	1	50,257.00
50,500 - 50,999	1	50,527.00
51,500 - 51,999	1	51,564.00
52,000 - 52,499	1	52,025.00
52,500 - 52,999	2	52,707.00
55,000 - 55,499	3	55,237.00
55,500 - 55,999	1	55,900.00
56,500 - 56,999	1	56,624.00
57,000 - 57,499	1	57,433.00
57,500 - 57,999	2	57,596.50
58,000 - 58,499	1	58,490.00
59,000 - 59,499	2	59,270.00
59,500 - 59,999	1	59,581.00
60,000 - 60,499	3	60,000.00
60,500 - 60,999	2	60,695.50
61,500 - 61,999	1	61,659.00
62,000 - 62,499	1	62,000.00
63,000 - 63,499	1	63,146.00
63,500 - 63,999	1	63,858.00
66,000 - 66,499	1	66,051.00
67,000 - 67,499	1	67,340.00
68,500 - 68,999	1	68,887.00
69,500 - 69,999	1	69,500.00
70,000 - 70,499	1	70,238.00
71,500 - 71,999	1	71,829.00
72,000 - 72,499	1	72,100.00
73,000 - 73,499	1	73,158.00
75,000 - 75,499	2	75,010.50
76,000 - 76,499	1	76,100.00
77,000 - 77,499	2	77,187.50
78,500 - 78,999	1	78,757.00
80,500 - 80,999	1	80,683.00
83,000 - 83,499	1	83,472.00
86,500 - 86,999	1	86,500.00
89,000 - 89,499	1	89,167.00
91,500 - 91,999	1	91,925.00
98,000 - 98,499	1	98,204.00
99,000 - 99,499	1	99,197.00
100,000 - 100,499	1	100,404.00
122,000 - 122,499	1	122,272.00
174,500 - 174,999	1	174,998.00
188,000 - 188,499	1	188,418.00
0 - 188,499	4,721	13,687.67

Exhibit No. 7

CALIFORNIA WORKMEN'S COMPENSATION
DISTRIBUTION OF LOSSES FOR MINOR CASES
BY TOTAL LOSS SIZE

Policy Year 1960 - 1st Reports

Loss Size Interval	Number of Cases	Average Loss Size
0 - 99	46	50.04
100 - 199	86	150.86
200 - 299	120	252.53
300 - 399	182	349.75
400 - 499	219	445.32
500 - 599	377	544.10
600 - 699	510	647.37
700 - 799	637	745.40
800 - 899	666	846.18
900 - 999	655	941.02
1,000 - 1,499	2,762	1,220.40
1,500 - 1,999	2,280	1,713.34
2,000 - 2,499	1,909	2,205.07
2,500 - 2,999	1,549	2,706.84
3,000 - 3,499	1,418	3,193.23
3,500 - 3,999	1,236	3,695.52
4,000 - 4,499	1,052	4,182.15
4,500 - 4,999	845	4,690.28
5,000 - 5,499	738	5,189.59
5,500 - 5,999	610	5,680.56
6,000 - 6,499	566	6,160.49
6,500 - 6,999	420	6,661.54
7,000 - 7,499	365	7,157.25
7,500 - 7,999	285	7,673.02
8,000 - 8,499	217	8,143.23
8,500 - 8,999	165	8,661.25
9,000 - 9,499	139	9,141.95
9,500 - 9,999	118	9,643.19

Loss Size Interval	Number of Cases	Average Loss Size
10,000 - 10,499	111	10,106.61
10,500 - 10,999	64	10,685.69
11,000 - 11,499	50	11,123.22
11,500 - 11,999	33	11,638.36
12,000 - 12,499	31	12,170.52
12,500 - 12,999	24	12,571.33
13,000 - 13,499	13	13,065.00
13,500 - 13,999	11	13,656.09
14,000 - 14,499	11	14,136.18
14,500 - 14,999	5	14,504.00
15,000 - 15,999	9	15,116.56
16,000 - 16,499	5	16,000.00
16,500 - 16,999	4	16,578.25
17,000 - 17,999	3	17,273.33
18,000 - 18,999	2	18,250.00
19,000 - 19,499	3	19,133.33
20,500 - 23,499	3	21,983.33
0 - 23,499	20,554	3,113.05

Exhibit No. 8

CALIFORNIA WORKMEN'S COMPENSATION
DISTRIBUTION OF LOSSES FOR MINOR CASES
BY TOTAL LOSS SIZE

Policy Year 1961 - 1st Reports

Loss Size Interval	Number of Cases	Average Loss Size
0 - 99	54	58.39
100 - 199	102	149.22
200 - 299	175	253.29
300 - 399	215	350.08
400 - 499	255	447.61
500 - 599	447	545.84
600 - 699	576	646.04
700 - 799	688	745.10
800 - 899	744	843.18
900 - 999	703	942.10
1,000 - 1,499	3,192	1,212.56
1,500 - 1,999	2,685	1,714.55
2,000 - 2,499	2,356	2,207.92
2,500 - 2,999	1,908	2,707.26
3,000 - 3,499	1,687	3,206.31
3,500 - 3,999	1,432	3,689.99
4,000 - 4,499	1,311	4,187.15
4,500 - 4,999	1,077	4,692.27
5,000 - 5,499	875	5,173.41
5,500 - 5,999	745	5,684.97
6,000 - 6,499	593	6,180.70
6,500 - 6,999	523	6,673.59
7,000 - 7,499	432	7,166.75
7,500 - 7,999	344	7,672.03
8,000 - 8,499	320	8,167.00
8,500 - 8,999	232	8,675.17
9,000 - 9,499	219	9,159.14
9,500 - 9,999	140	9,643.50

Loss Size Interval	Number of Cases	Average Loss Size
10,000 - 10,499	132	10,114.25
10,500 - 10,999	94	10,642.41
11,000 - 11,499	81	11,139.68
11,500 - 11,999	51	11,679.49
12,000 - 12,499	46	12,171.07
12,500 - 12,999	41	12,640.80
13,000 - 13,499	26	13,141.62
13,500 - 13,999	24	13,686.29
14,000 - 14,499	16	14,118.75
14,500 - 14,999	10	14,645.30
15,000 - 15,499	14	15,178.21
15,500 - 15,999	6	15,740.67
16,000 - 16,499	5	16,062.20
16,500 - 16,999	5	16,616.40
17,000 - 17,499	9	17,101.67
17,500 - 17,999	5	17,676.00
18,000 - 18,999	4	18,475.00
19,000 - 19,499	2	19,030.00
20,000 - 20,999	2	20,325.00
21,000 - 22,999	3	21,916.67
24,000 - 24,499	2	24,250.00
25,000 - 25,999	3	25,398.33
34,000 - 35,499	2	34,825.00
0 - 35,499	24,613	3,228.46

Exhibit No. 9

CALIFORNIA WORKMEN'S COMPENSATION
DISTRIBUTION OF LOSSES FOR TEMPORARY CASES
BY TOTAL LOSS SIZE

Policy Year 1960 - 1st Reports

Loss Size Interval	Number of Cases	Average Loss Size
0 - 9	96	4.93
10 - 19	192	15.77
20 - 29	441	25.29
30 - 39	777	34.97
40 - 49	1,194	44.57
50 - 59	1,487	54.46
60 - 69	1,622	64.62
70 - 79	1,681	74.50
80 - 89	1,691	84.47
90 - 99	1,597	94.60
100 - 149	7,003	123.33
150 - 199	5,158	173.19
200 - 249	4,083	222.39
250 - 299	3,110	272.63
300 - 349	2,856	322.16
350 - 399	2,236	372.52
400 - 449	2,025	420.60
450 - 499	1,633	472.67
500 - 549	1,476	518.81
550 - 599	1,245	572.77
600 - 649	1,332	621.07
650 - 699	1,119	672.04
700 - 749	1,090	720.84
750 - 799	1,066	771.09
800 - 849	981	820.56
850 - 899	851	871.86
900 - 949	753	918.77
950 - 999	571	970.18

Loss Size Interval	Number of Cases	Average Loss Size
1,000 - 1,499	2,887	1,182.26
1,500 - 1,999	1,092	1,685.73
2,000 - 2,499	634	2,146.55
2,500 - 2,999	405	2,680.94
3,000 - 3,499	264	3,165.98
3,500 - 3,999	176	3,655.49
4,000 - 4,499	133	4,154.05
4,500 - 4,999	88	4,648.36
5,000 - 5,499	78	5,098.14
5,500 - 5,999	62	5,619.89
6,000 - 6,499	38	6,088.66
6,500 - 6,999	21	6,610.71
7,000 - 7,499	33	7,075.06
7,500 - 7,999	20	7,636.05
8,000 - 8,499	21	8,143.29
8,500 - 8,999	13	8,650.00
9,000 - 9,999	7	9,657.43
10,000 - 10,999	14	10,231.29
11,000 - 12,999	7	11,943.43
13,000 - 16,499	6	14,292.33
17,000 - 33,999	7	22,634.29
0 - 33,999	55,372	496.90

CALIFORNIA WORKMEN'S COMPENSATION
DISTRIBUTION OF LOSSES FOR TEMPORARY CASES
BY TOTAL LOSS SIZE

Policy Year 1961 - 1st Reports

Loss Size Interval	Number of Cases	Average Loss Size
0 - 9	71	5.99
10 - 19	183	14.90
20 - 29	466	25.10
30 - 39	830	34.69
40 - 49	1,291	44.76
50 - 59	1,621	54.59
60 - 69	1,830	64.38
70 - 79	1,819	74.46
80 - 89	1,846	84.44
90 - 99	1,757	94.52
100 - 149	7,530	123.03
150 - 199	5,706	172.55
200 - 249	4,421	222.86
250 - 299	3,484	272.94
300 - 349	2,979	321.73
350 - 399	2,446	372.72
400 - 449	2,022	421.44
450 - 499	1,714	472.00
500 - 549	1,634	520.62
550 - 599	1,361	571.50
600 - 649	1,345	621.14
650 - 699	1,188	672.58
700 - 749	1,207	721.48
750 - 799	1,163	770.84
800 - 849	1,053	820.35
850 - 899	955	872.93
900 - 949	818	915.45
950 - 999	635	970.61
1,000 - 1,499	3,333	1,173.53
1,500 - 1,999	1,273	1,663.31
2,000 - 2,499	729	2,177.04
2,500 - 2,999	433	2,582.25
3,000 - 3,499	330	3,137.05
3,500 - 3,999	216	3,663.19
4,000 - 4,499	174	4,133.39
4,500 - 4,999	121	4,675.05
5,000 - 5,499	94	5,123.88
5,500 - 5,999	66	5,639.88
6,000 - 6,499	51	6,158.59
6,500 - 6,999	38	6,660.24
7,000 - 7,499	25	7,101.40
7,500 - 7,999	25	7,650.12
8,000 - 8,499	24	8,076.04
8,500 - 8,999	19	8,618.16
9,000 - 9,499	11	9,173.73
9,500 - 9,999	12	9,627.92
10,000 - 10,499	11	10,095.45
10,500 - 10,999	8	10,695.50
11,000 - 11,999	11	11,218.64
12,000 - 12,999	8	12,410.50
13,000 - 14,999	6	13,500.00
15,000 - 20,499	5	17,280.00
0 - 20,499	60,398	513.80

Exhibit 11

DEATH - 1960

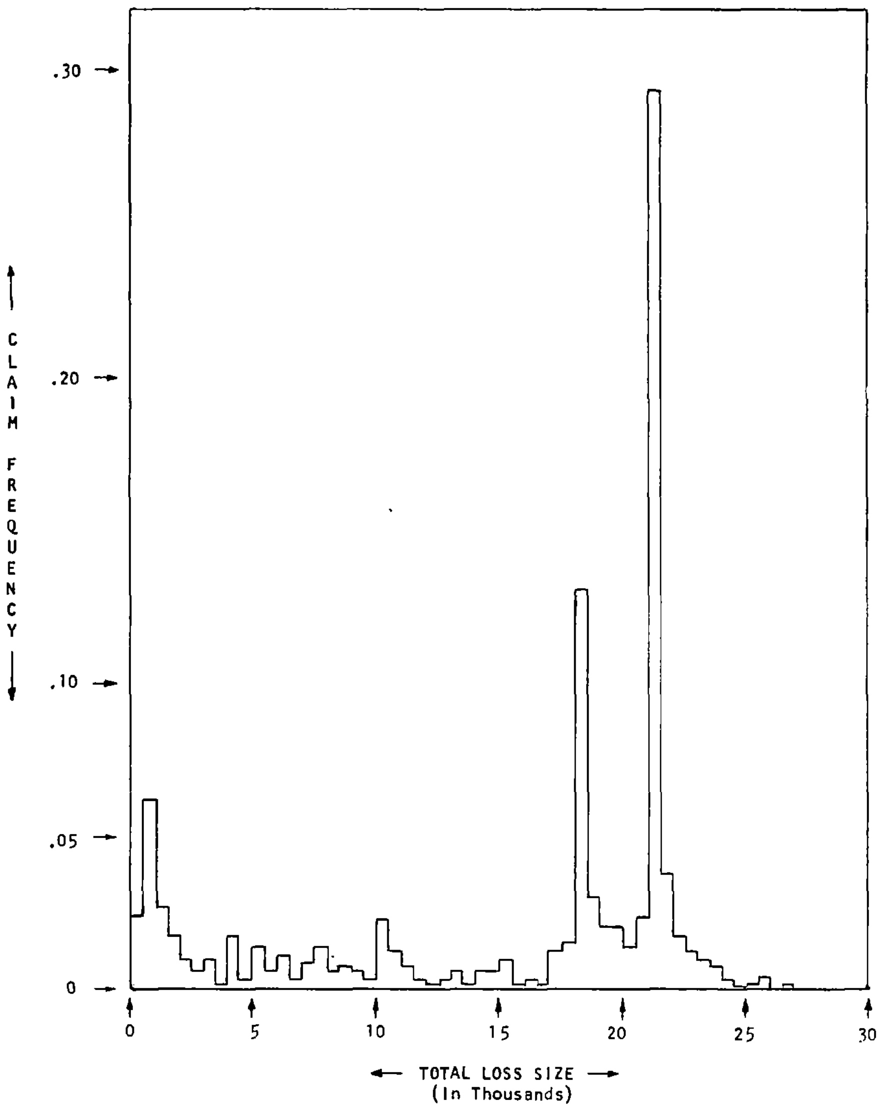

Exhibit 12

DEATH - 1961

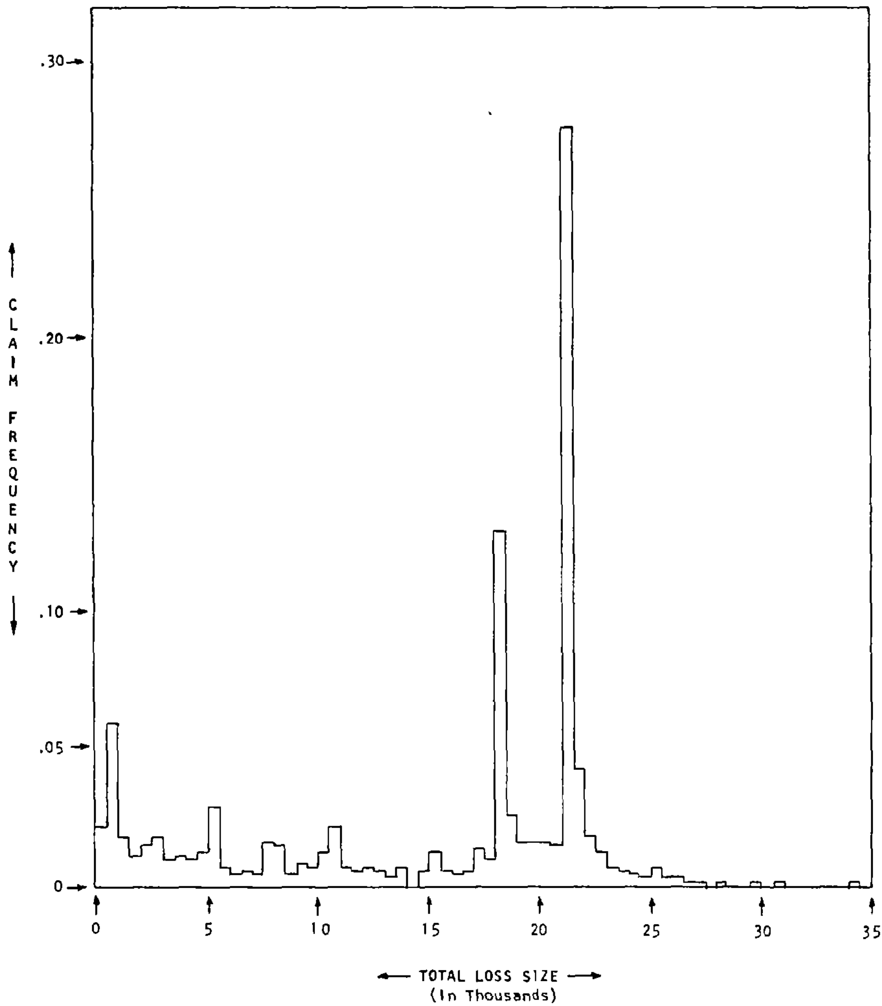

Exhibit 13
Sheet 1

PERMANENT TOTAL - 1960

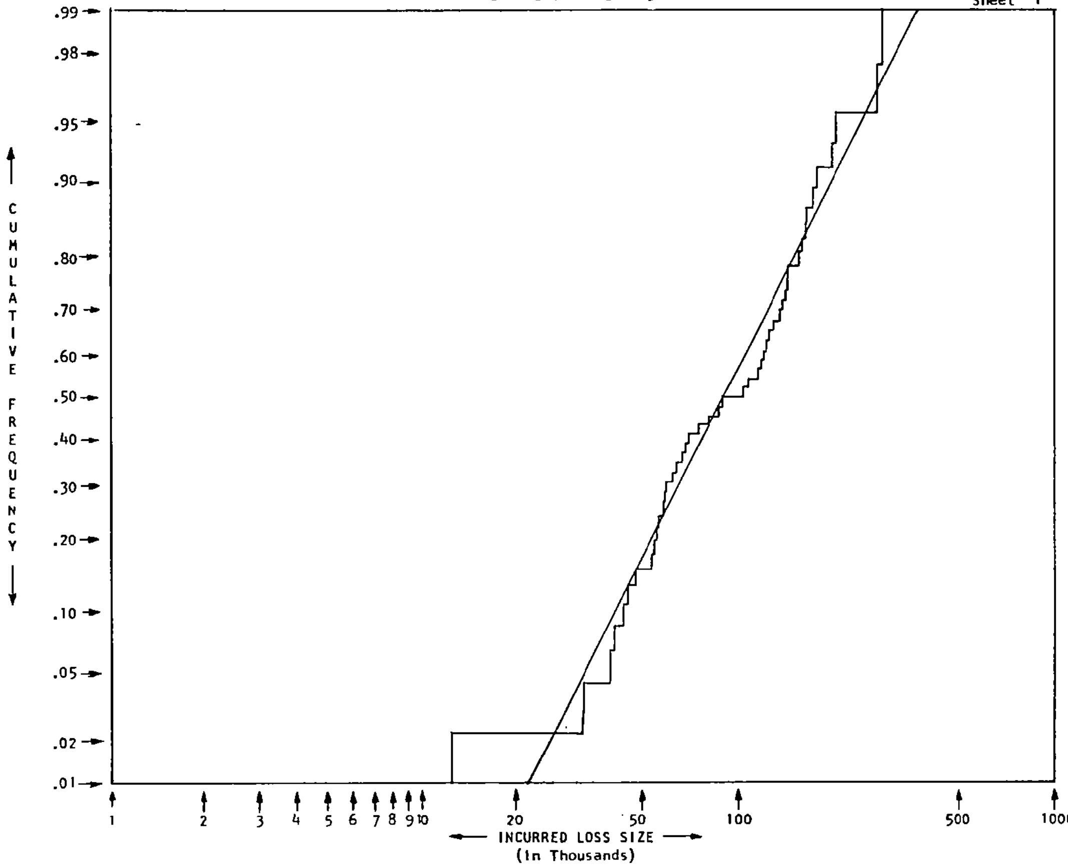

Exhibit 13
Sheet 2

PERMANENT TOTAL - 1960

(1) Loss Size	(2) Cumulative Frequency Observed	(3) Cumulative Frequency Theoretical	(4) Absolute Difference (2)-(3)	(1) Loss Size	(2) Cumulative Frequency Observed	(3) Cumulative Frequency Theoretical	(4) Absolute Difference (2)-(3)
12,380	.0217	.0007	.0210	104,500	.5217	.5910	.0693
32,499	.0435	.0495	.0060	107,326	.5435	.6064	.0629
39,348	.0652	.0901	.0249	114,514	.5652	.6480	.0828
40,299	.0870	.0968	.0098	118,144	.5870	.6664	.0794
43,624	.1087	.1190	.0103	119,874	.6087	.6736	.0649
44,977	.1304	.1292	.0012	121,200	.6304	.6808	.0504
46,000	.1522	.1379	.0143	125,000	.6522	.6985	.0463
54,825	.1739	.2090	.0351	128,985	.6739	.7157	.0418
55,338	.1957	.2148	.0191	135,844	.6957	.7422	.0465
56,000	.2174	.2206	.0032	139,845	.7174	.7580	.0406
56,001	.2391	.2206	.0185	141,564	.7391	.7642	.0251
58,506	.2609	.2420	.0189	147,563	.7609	.7852	.0243
58,600	.2826	.2420	.0406	147,663	.7826	.7852	.0026
59,673	.3043	.2514	.0529	159,121	.8043	.8186	.0143
62,500	.3261	.2743	.0518	161,415	.8261	.8238	.0023
63,291	.3478	.2810	.0668	164,208	.8478	.8315	.0163
67,206	.3696	.3156	.0540	165,183	.8696	.8340	.0356
68,391	.3913	.3264	.0649	174,404	.8913	.8554	.0359
69,653	.4130	.3372	.0758	179,169	.9130	.8643	.0487
75,394	.4348	.3859	.0489	199,965	.9348	.8997	.0351
80,000	.4565	.4207	.0358	206,511	.9565	.9082	.0483
86,828	.4783	.4721	.0062	280,354	.9783	.9656	.0127
89,028	.5000	.4880	.0120	292,525	1.0000	.9706	.0294

Exhibit 14
Sheet 1a

PERMANENT TOTAL - 1961

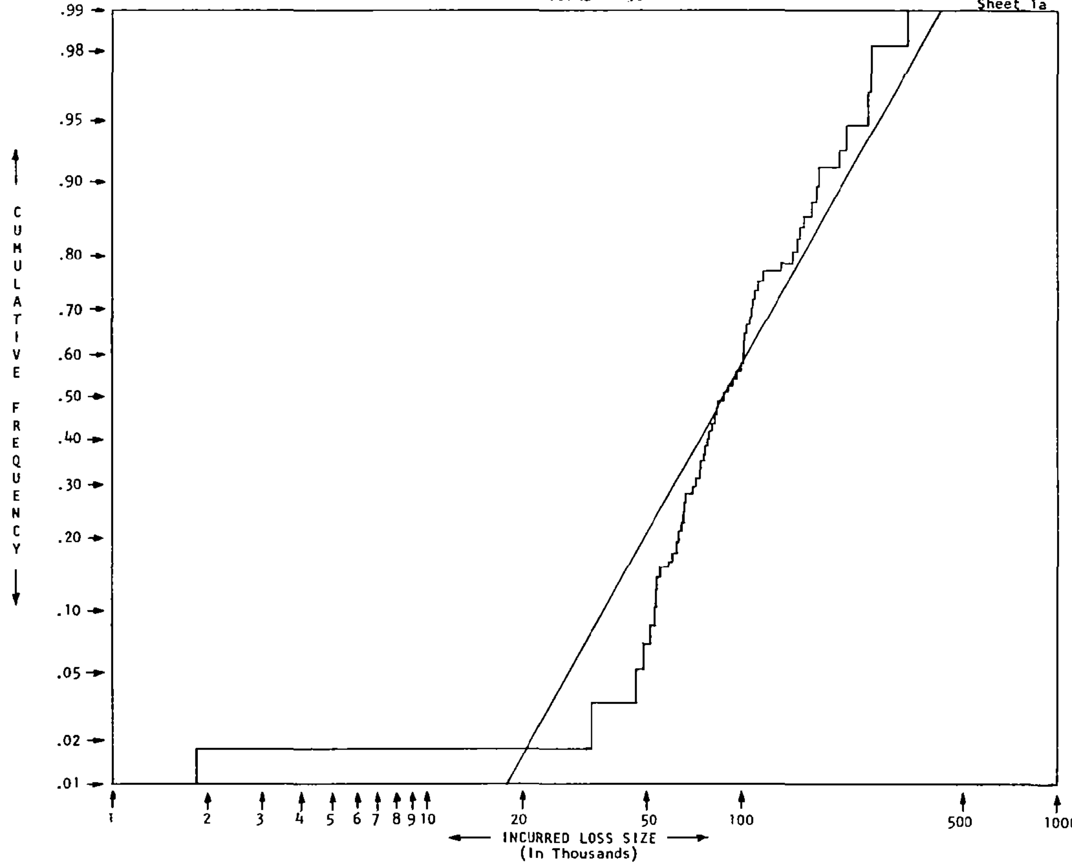

Exhibit 14
Sheet 2a

PERMANENT TOTAL - 1961

(1) Loss Size	(2) Cumulative Frequency Observed	(3) Cumulative Frequency Theoretical	(4) Absolute Difference (2)-(3)
1,840	.0175	.0000	.0175
33,300	.0351	.0838	.0487
46,000	.0526	.1788	.1262
48,457	.0702	.2005	.1303
50,247	.0877	.2148	.1271
53,200	.1053	.2389	.1336
53,327	.1228	.2420	.1192
53,653	.1404	.2420	.1016
55,000	.1579	.2546	.0967
59,371	.1754	.2912	.1158
62,100	.1930	.3121	.1191
62,522	.2105	.3156	.1051
63,800	.2281	.3264	.0983
64,588	.2456	.3336	.0880
64,726	.2632	.3336	.0704
65,340	.2807	.3409	.0602
68,874	.2982	.3669	.0687
70,639	.3158	.3821	.0663
72,679	.3333	.3974	.0641
73,391	.3509	.4013	.0504
75,000	.3684	.4168	.0484
75,500	.3860	.4207	.0347
76,823	.4035	.4286	.0251
77,711	.4211	.4364	.0153
79,304	.4388	.4483	.0097
81,969	.4561	.4641	.0080
83,000	.4737	.4721	.0016
83,481	.4912	.4761	.0151
86,690	.5088	.4960	.0128
89,000	.5263	.5120	.0143
93,410	.5439	.5398	.0041
94,816	.5614	.5478	.0136
99,187	.5789	.5753	.0036
100,187	.5965	.5793	.0172
100,340	.6140	.5793	.0347
101,090	.6316	.5832	.0484
101,312	.6491	.5871	.0620
103,515	.6667	.5987	.0680
107,493	.6842	.6179	.0663
108,485	.7018	.6255	.0763
108,637	.7193	.6255	.0938
109,521	.7368	.6293	.1075
111,591	.7544	.6406	.1138
115,547	.7719	.6591	.1128
132,946	.7895	.7291	.0604
145,787	.8070	.7704	.0366
150,000	.8246	.7823	.0423
152,015	.8421	.7881	.0540
156,995	.8596	.8023	.0573
166,644	.8772	.8238	.0534
172,826	.8947	.8389	.0558
174,600	.9123	.8413	.0710
201,460	.9298	.8869	.0429
213,260	.9474	.9015	.0459
250,351	.9649	.9357	.0292
254,494	.9825	.9382	.0443
331,151	1.0000	.9726	.0274

Exhibit 14
Sheet 1b

PERMANENT TOTAL - 1961

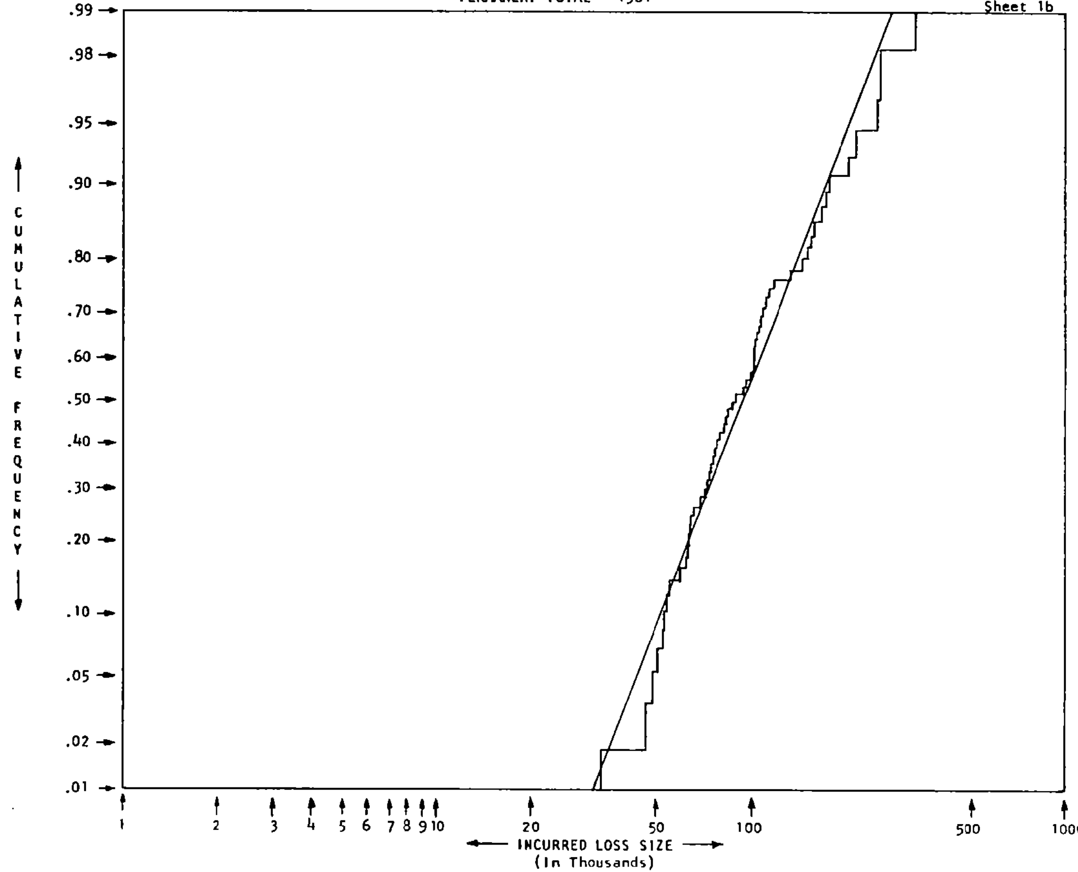

Exhibit 14
Sheet 2b

PERMANENT TOTAL - 1961

(1) Loss Size	(2) Cumulative Frequency Observed	(3) Cumulative Frequency Theoretical	(4) Absolute Difference (2)-(3)	(1) Loss Size	(2) Cumulative Frequency Observed	(3) Cumulative Frequency Theoretical	(4) Absolute Difference (2)-(3)
33,300	.0179	.0143	.0036	89,000	.5179	.4602	.0577
46,000	.0357	.0668	.0311	93,410	.5357	.5000	.0357
48,457	.0536	.0823	.0287	94,816	.5536	.5120	.0416
50,247	.0714	.0951	.0237	99,187	.5714	.5517	.0197
53,200	.0893	.1170	.0277	100,187	.5893	.5596	.0297
53,327	.1071	.1170	.0099	100,340	.6071	.5596	.0475
53,653	.1250	.1210	.0040	101,090	.6250	.5675	.0575
55,000	.1429	.1314	.0115	101,312	.6429	.5714	.0715
59,371	.1607	.1685	.0078	103,515	.6607	.5871	.0736
62,100	.1786	.1949	.0163	107,493	.6786	.6179	.0607
62,522	.1964	.1977	.0013	108,485	.6964	.6255	.0709
63,800	.2143	.2090	.0053	108,637	.7143	.6255	.0888
64,588	.2321	.2177	.0144	109,521	.7321	.6331	.0990
64,726	.2500	.2177	.0323	111,591	.7500	.6480	.1020
65,340	.2679	.2236	.0443	115,547	.7679	.6736	.0943
68,874	.2857	.2611	.0246	132,946	.7857	.7734	.0123
70,639	.3036	.2776	.0260	145,787	.8036	.8289	.0253
72,679	.3214	.2981	.0233	150,000	.8214	.8438	.0224
73,391	.3393	.3050	.0343	152,015	.8393	.8508	.0115
75,000	.3571	.3228	.0343	156,995	.8571	.8665	.0094
75,500	.3750	.3264	.0486	166,644	.8750	.8907	.0157
76,823	.3929	.3409	.0520	172,826	.8929	.9049	.0120
77,711	.4107	.3483	.0624	174,600	.9107	.9082	.0025
79,304	.4286	.3669	.0617	201,460	.9286	.9484	.0198
81,969	.4464	.3936	.0528	213,260	.9464	.9608	.0144
83,000	.4643	.4013	.0630	250,351	.9643	.9821	.0178
83,481	.4821	.4052	.0769	254,494	.9821	.9834	.0013
86,690	.5000	.4364	.0636	331,151	1.0000	.9964	.0036

Exhibit 15
Sheet 1

MAJOR PERMANENT PARTIAL - 1960

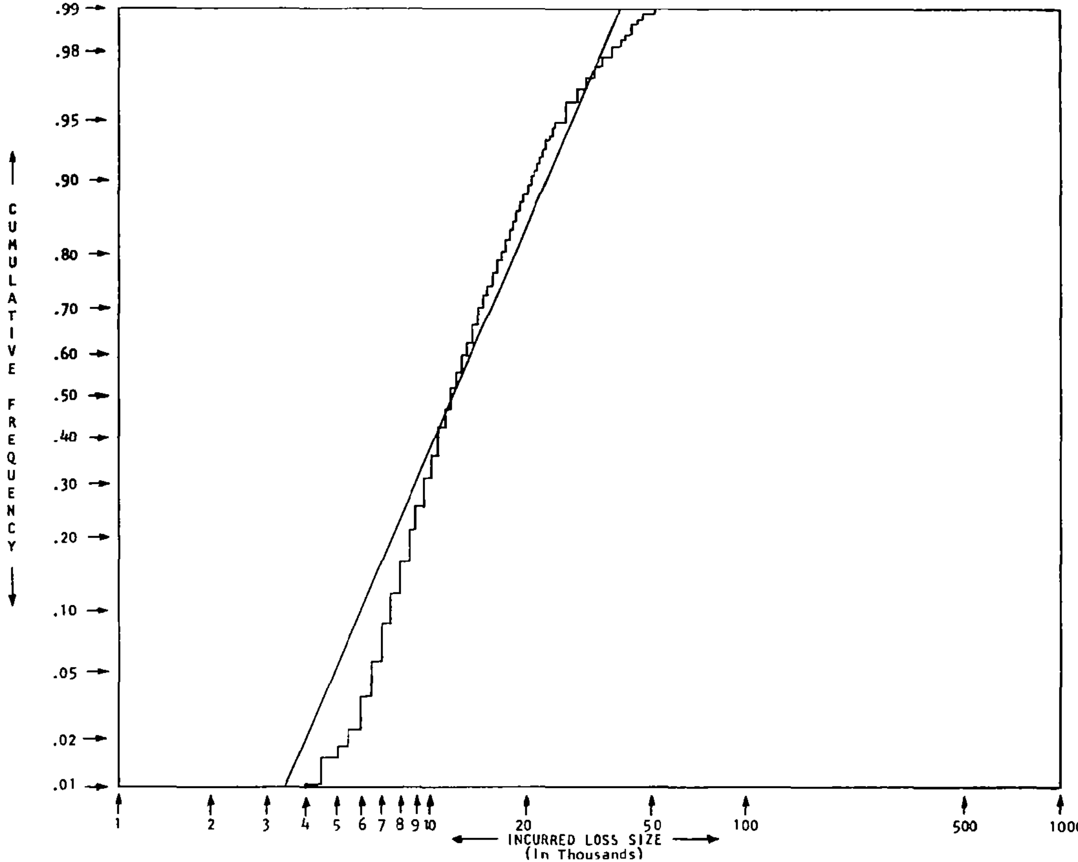

Exhibit 15
Sheet 2

MAJOR PERMANENT PARTIAL - 1960

(1) Loss Size Interval	(2) Cumulative Frequency Observed	(3) Cumulative Frequency Theoretical	(4) Absolute Difference (2)-(3)
0 - 99	.0018	.0000	.0018
400 - 499	.0021	.0000	.0021
700 - 799	.0031	.0000	.0031
900 - 999	.0037	.0000	.0037
1,000 - 1,499	.0043	.0001	.0042
1,500 - 1,999	.0046	.0005	.0041
2,000 - 2,499	.0061	.0019	.0042
2,500 - 2,999	.0070	.0053	.0017
3,000 - 3,499	.0079	.0119	.0040
3,500 - 3,999	.0104	.0223	.0119
4,000 - 4,499	.0147	.0370	.0223
4,500 - 4,999	.0171	.0563	.0392
5,000 - 5,499	.0238	.0799	.0561
5,500 - 5,999	.0379	.1071	.0692
6,000 - 6,499	.0584	.1379	.0795
6,500 - 6,999	.0865	.1711	.0846
7,000 - 7,499	.1208	.2061	.0853
7,500 - 7,999	.1639	.2426	.0787
8,000 - 8,499	.2106	.2800	.0694
8,500 - 8,999	.2586	.3174	.0588
9,000 - 9,499	.3143	.3546	.0403
9,500 - 9,999	.3672	.3913	.0241
10,000 - 10,499	.4271	.4270	.0001
10,500 - 10,999	.4729	.4618	.0111
11,000 - 11,499	.5194	.4952	.0242
11,500 - 11,999	.5604	.5275	.0329
12,000 - 12,499	.5986	.5580	.0406
12,500 - 12,999	.6273	.5871	.0402
13,000 - 13,499	.6640	.6145	.0495
13,500 - 13,999	.6986	.6406	.0580
14,000 - 14,499	.7252	.6653	.0599
14,500 - 14,999	.7450	.6883	.0567
15,000 - 15,499	.7713	.7099	.0614
15,500 - 15,999	.7930	.7301	.0629
16,000 - 16,499	.8086	.7489	.0597
16,500 - 16,999	.8260	.7664	.0596
17,000 - 17,499	.8398	.7829	.0569
17,500 - 17,999	.8520	.7981	.0539
18,000 - 18,499	.8658	.8125	.0533
18,500 - 18,999	.8753	.8259	.0494
19,000 - 19,499	.8847	.8382	.0465
19,500 - 19,999	.8942	.8497	.0445
20,000 - 20,499	.9046	.8601	.0445
20,500 - 20,999	.9098	.8701	.0397
21,000 - 21,499	.9168	.8792	.0376
21,500 - 21,999	.9227	.8879	.0348
22,000 - 22,499	.9297	.8957	.0340
22,500 - 22,999	.9346	.9030	.0316
23,000 - 23,499	.9398	.9097	.0301
23,500 - 23,999	.9444	.9161	.0283
24,000 - 24,499	.9489	.9219	.0270
24,500 - 24,999	.9502	.9273	.0229
25,000 - 25,499	.9566	.9324	.0242
25,500 - 25,999	.9587	.9371	.0216
26,000 - 26,499	.9606	.9414	.0192
26,500 - 26,999	.9624	.9454	.0170
27,000 - 27,499	.9636	.9492	.0144
27,500 - 27,999	.9642	.9526	.0116
28,000 - 28,499	.9664	.9558	.0106
28,500 - 28,999	.9685	.9588	.0097

Exhibit 15
Sheet 3

MAJOR PERMANENT PARTIAL - 1960

(1) Loss Size Interval	(2) Cumulative Frequency Observed	(3) Cumulative Frequency Theoretical	(4) Absolute Difference (2)-(3)
29,000 - 29,499	.9691	.9615	.0076
29,500 - 29,999	.9707	.9642	.0065
30,000 - 30,499	.9722	.9665	.0057
30,500 - 30,999	.9731	.9688	.0043
31,000 - 31,499	.9737	.9708	.0029
31,500 - 31,999	.9740	.9728	.0012
32,000 - 32,499	.9749	.9745	.0004
32,500 - 32,999	.9758	.9762	.0004
33,000 - 33,499	.9768	.9778	.0010
33,500 - 33,999	.9780	.9792	.0012
34,000 - 34,499	.9786	.9806	.0020
34,500 - 34,999	.9789	.9818	.0029
35,000 - 35,499	.9801	.9830	.0029
36,000 - 36,499	.9814	.9851	.0037
36,500 - 36,999	.9817	.9860	.0043
37,000 - 37,499	.9823	.9869	.0046
37,500 - 37,999	.9826	.9877	.0051
38,500 - 38,999	.9832	.9892	.0060
39,000 - 39,499	.9835	.9898	.0063
39,500 - 39,999	.9838	.9905	.0067
40,500 - 40,999	.9850	.9916	.0066
41,000 - 41,499	.9853	.9921	.0068
42,000 - 42,499	.9859	.9931	.0072
43,000 - 43,499	.9865	.9939	.0074
44,000 - 44,499	.9872	.9946	.0074
45,000 - 45,499	.9875	.9952	.0077
45,500 - 45,999	.9884	.9955	.0071
46,500 - 46,999	.9887	.9960	.0073
48,000 - 48,499	.9890	.9966	.0076
49,000 - 49,499	.9893	.9970	.0077

(1) Loss Size Interval	(2) Cumulative Frequency Observed	(3) Cumulative Frequency Theoretical	(4) Absolute Difference (2)-(3)
50,000 - 50,499	.9902	.9974	.0072
50,500 - 50,999	.9905	.9974	.0069
52,000 - 52,499	.9908	.9979	.0071
53,000 - 53,499	.9917	.9981	.0064
54,000 - 54,499	.9921	.9983	.0062
54,500 - 54,999	.9924	.9984	.0060
55,500 - 55,999	.9927	.9986	.0059
56,000 - 56,499	.9930	.9987	.0057
59,500 - 59,999	.9939	.9991	.0052
60,000 - 60,499	.9945	.9991	.0046
60,500 - 60,999	.9948	.9992	.0044
61,500 - 61,999	.9951	.9992	.0041
64,500 - 64,999	.9954	.9994	.0040
65,000 - 65,499	.9960	.9995	.0035
68,000 - 68,499	.9963	.9996	.0033
71,000 - 71,499	.9966	.9997	.0031
71,500 - 71,999	.9969	.9997	.0028
74,500 - 74,999	.9972	.9998	.0026
76,000 - 76,499	.9976	.9998	.0022
77,500 - 77,999	.9979	.9998	.0019
88,500 - 88,999	.9982	.9999	.0017
90,000 - 90,499	.9985	.9999	.0014
94,000 - 94,499	.9988	1.0000	.0012
95,000 - 95,499	.9991	1.0000	.0009
98,000 - 98,499	.9994	1.0000	.0006
102,000 - 102,499	.9997	1.0000	.0003
186,000 - 186,499	1.0000	1.0000	.0000

Exhibit 16
Sheet 1

MAJOR PERMANENT PARTIAL - 1961

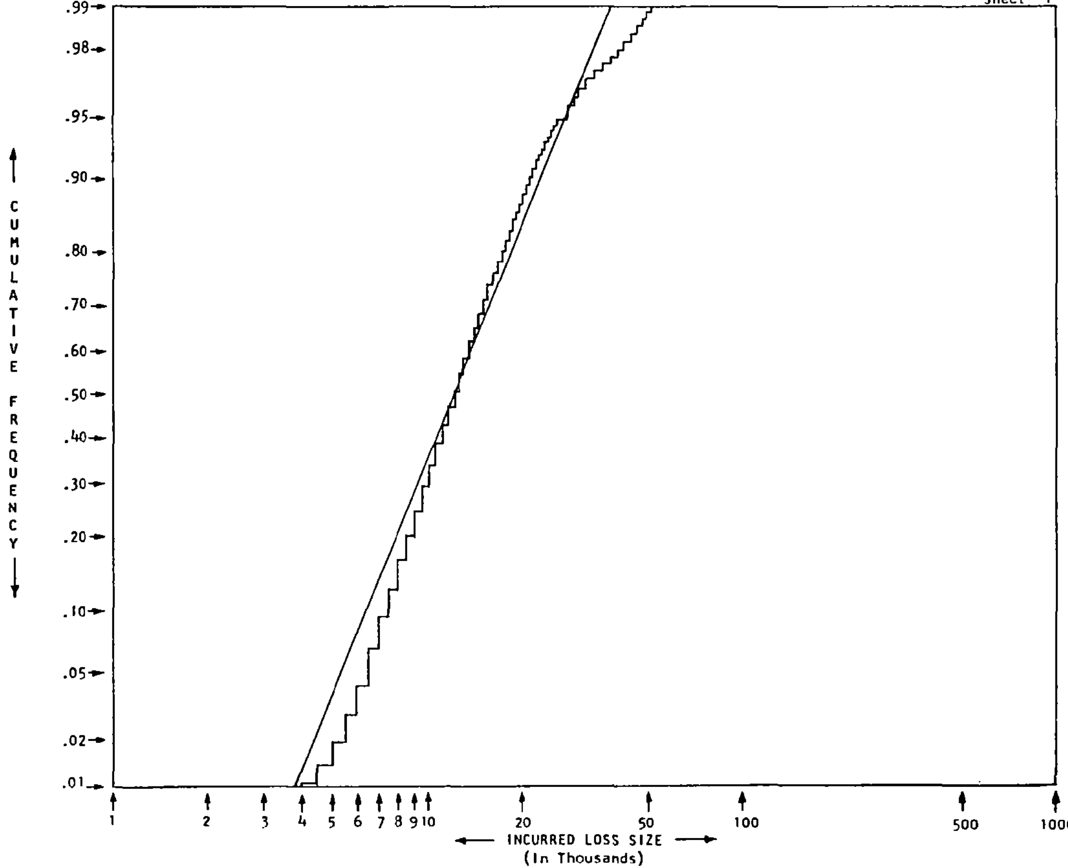

Exhibit 16
Sheet 2

MAJOR PERMANENT PARTIAL - 1961

(1) Loss Size Interval	(2) Cumulative Frequency Observed	(3) Cumulative Frequency Theoretical	(4) Absolute Difference (2)-(3)
0 - 99	.0006	.0000	.0006
400 - 499	.0010	.0000	.0010
700 - 799	.0012	.0000	.0012
900 - 999	.0014	.0000	.0014
1,000 - 1,499	.0018	.0000	.0018
1,500 - 1,999	.0022	.0002	.0020
2,000 - 2,499	.0026	.0007	.0019
2,500 - 2,999	.0030	.0023	.0007
3,000 - 3,499	.0053	.0059	.0006
3,500 - 3,999	.0091	.0123	.0032
4,000 - 4,499	.0140	.0225	.0085
4,500 - 4,999	.0195	.0367	.0172
5,000 - 5,499	.0290	.0554	.0264
5,500 - 5,999	.0423	.0783	.0360
6,000 - 6,499	.0660	.1050	.0390
6,500 - 6,999	.0923	.1353	.0430
7,000 - 7,499	.1252	.1683	.0431
7,500 - 7,999	.1619	.2036	.0417
8,000 - 8,499	.2012	.2404	.0392
8,500 - 8,999	.2473	.2781	.0308
9,000 - 9,499	.2940	.3163	.0223
9,500 - 9,999	.3392	.3545	.0153
10,000 - 10,499	.3880	.3922	.0042
10,500 - 10,999	.4309	.4292	.0017
11,000 - 11,499	.4717	.4651	.0066
11,500 - 11,999	.5093	.4997	.0096
12,000 - 12,499	.5532	.5330	.0202
12,500 - 12,999	.5887	.5647	.0240
13,000 - 13,499	.6271	.5948	.0323
13,500 - 13,999	.6579	.6234	.0345
14,000 - 14,499	.6879	.6503	.0376
14,500 - 14,999	.7165	.6755	.0410
15,000 - 15,499	.7447	.6993	.0454
15,500 - 15,999	.7659	.7214	.0445
16,000 - 16,499	.7877	.7422	.0455
16,500 - 16,999	.8034	.7614	.0420
17,000 - 17,499	.8195	.7795	.0400
17,500 - 17,999	.8352	.7961	.0391
18,000 - 18,499	.8521	.8116	.0405
18,500 - 18,999	.8614	.8260	.0354
19,000 - 19,499	.8720	.8392	.0328
19,500 - 19,999	.8845	.8515	.0330
20,000 - 20,499	.8959	.8629	.0330
20,500 - 20,999	.9029	.8735	.0294
21,000 - 21,499	.9109	.8832	.0277
21,500 - 21,999	.9181	.8921	.0260
22,000 - 22,499	.9234	.9004	.0230
22,500 - 22,999	.9283	.9080	.0203
23,000 - 23,499	.9334	.9151	.0183
23,500 - 23,999	.9368	.9215	.0153
24,000 - 24,499	.9410	.9276	.0134
24,500 - 24,999	.9446	.9331	.0115
25,000 - 25,499	.9490	.9381	.0109
25,500 - 25,999	.9518	.9428	.0090
26,000 - 26,499	.9546	.9472	.0074
26,500 - 26,999	.9561	.9511	.0050
27,000 - 27,499	.9576	.9548	.0028
27,500 - 27,999	.9604	.9583	.0021
28,000 - 28,499	.9617	.9613	.0004
28,500 - 28,999	.9642	.9643	.0001
29,000 - 29,499	.9657	.9669	.0012
29,500 - 29,999	.9670	.9694	.0024
30,000 - 30,499	.9687	.9716	.0029
30,500 - 30,999	.9691	.9737	.0046
31,000 - 31,499	.9695	.9757	.0062
31,500 - 31,999	.9703	.9774	.0071
32,000 - 32,499	.9720	.9791	.0071
32,500 - 32,999	.9724	.9806	.0082
33,000 - 33,499	.9730	.9820	.0090
33,500 - 33,999	.9745	.9833	.0088

Exhibit 16
Sheet 3

MAJOR PERMANENT PARTIAL - 1961

(1) Loss Size Interval	(2) Cumulative Frequency Observed	(3) Cumulative Frequency Theoretical	(4) Absolute Difference (2)-(3)	(1) Loss Size Interval	(2) Cumulative Frequency Observed	(3) Cumulative Frequency Theoretical	(4) Absolute Difference (2)-(3)
34,000 - 34,499	.9749	.9845	.0096	56,500 - 56,999	.9918	.9993	.0075
34,500 - 34,999	.9753	.9856	.0103	57,000 - 57,499	.9920	.9993	.0073
35,000 - 35,499	.9759	.9866	.0107	57,500 - 57,999	.9924	.9994	.0070
35,500 - 35,999	.9770	.9876	.0106	58,000 - 58,499	.9926	.9994	.0068
36,000 - 36,499	.9778	.9885	.0107	59,000 - 59,499	.9930	.9995	.0065
36,500 - 36,999	.9782	.9893	.0111	59,500 - 59,999	.9932	.9995	.0063
37,000 - 37,499	.9786	.9900	.0114	60,000 - 50,499	.9938	.9995	.0057
38,000 - 38,499	.9790	.9913	.0123	60,500 - 60,999	.9942	.9995	.0053
38,500 - 38,999	.9792	.9920	.0128	61,500 - 61,999	.9944	.9996	.0052
39,000 - 39,499	.9794	.9926	.0132	62,000 - 62,499	.9946	.9996	.0050
39,500 - 39,999	.9805	.9931	.0126	63,000 - 63,499	.9948	.9997	.0049
40,000 - 40,499	.9816	.9935	.0119	63,500 - 63,999	.9950	.9997	.0047
40,500 - 40,999	.9824	.9940	.0116	66,000 - 66,499	.9952	.9998	.0046
41,000 - 41,499	.9830	.9944	.0114	67,000 - 67,499	.9954	.9998	.0044
41,500 - 41,999	.9836	.9948	.0112	68,500 - 68,999	.9956	.9998	.0042
42,000 - 42,499	.9838	.9951	.0113	69,500 - 69,999	.9958	.9998	.0040
42,500 - 42,999	.9842	.9954	.0112	70,000 - 70,499	.9960	.9998	.0038
43,000 - 43,499	.9850	.9957	.0107	71,500 - 71,999	.9962	.9998	.0036
43,500 - 43,999	.9852	.9960	.0108	72,000 - 72,499	.9964	.9998	.0034
44,000 - 44,499	.9854	.9963	.0109	73,000 - 73,499	.9966	.9999	.0033
45,000 - 45,499	.9858	.9967	.0109	75,000 - 75,499	.9970	.9999	.0029
45,500 - 45,999	.9864	.9970	.0106	76,000 - 76,499	.9972	.9999	.0027
46,000 - 46,499	.9872	.9972	.0100	77,000 - 77,499	.9976	.9999	.0023
46,500 - 46,999	.9876	.9974	.0098	78,500 - 78,999	.9978	.9999	.0021
47,000 - 47,499	.9882	.9975	.0093	80,500 - 80,999	.9980	1.0000	.0020
47,500 - 47,999	.9890	.9977	.0087	83,000 - 83,499	.9982	1.0000	.0018
48,000 - 48,499	.9892	.9978	.0086	86,500 - 86,999	.9984	1.0000	.0016
49,500 - 49,999	.9896	.9982	.0086	89,000 - 89,499	.9986	1.0000	.0014
50,000 - 50,499	.9898	.9984	.0086	91,500 - 91,999	.9988	1.0000	.0012
50,500 - 50,999	.9900	.9985	.0085	98,000 - 98,499	.9990	1.0000	.0010
51,500 - 51,999	.9902	.9987	.0085	99,000 - 99,499	.9992	1.0000	.0008
52,000 - 52,499	.9904	.9987	.0083	100,000 - 100,499	.9994	1.0000	.0006
52,500 - 52,999	.9908	.9988	.0080	122,000 - 122,499	.9996	1.0000	.0004
55,000 - 55,499	.9914	.9991	.0077	174,500 - 174,999	.9998	1.0000	.0002
55,500 - 55,999	.9916	.9992	.0076	188,000 - 188,499	1.0000	1.0000	.0000

Exhibit 17
Sheet 1

MINOR PERMANENT PARTIAL - 1960

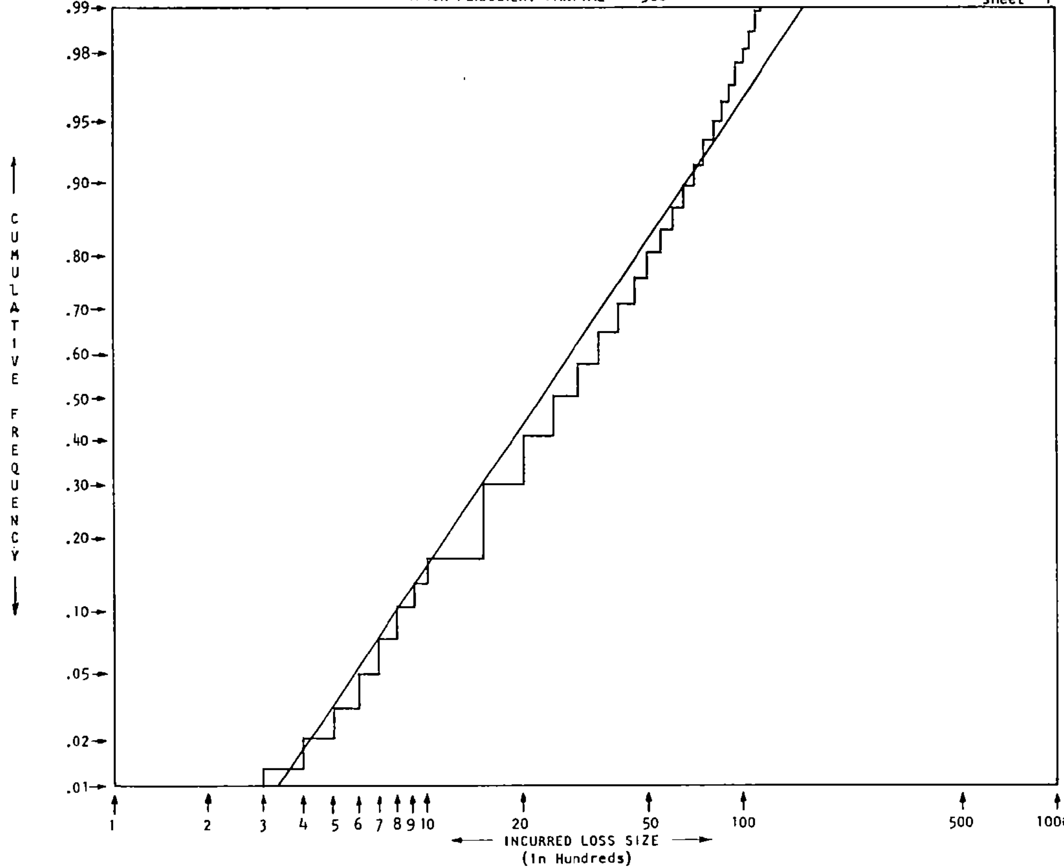

Exhibit 17
Sheet 2

MINOR PERMANENT PARTIAL - 1960

(1) Loss Size Interval	(2) Cumulative Frequency Observed	(3) Cumulative Frequency Theoretical	(4) Absolute Difference (2)-(3)
0 - 99	.0022	.0001	.0021
100 - 199	.0064	.0017	.0047
200 - 299	.0123	.0074	.0049
300 - 399	.0211	.0183	.0028
400 - 499	.0318	.0342	.0024
500 - 599	.0501	.0544	.0043
600 - 699	.0749	.0779	.0030
700 - 799	.1059	.1042	.0017
800 - 899	.1383	.1320	.0063
900 - 999	.1702	.1609	.0093
1,000 - 1,499	.3046	.3067	.0021
1,500 - 1,999	.4155	.4364	.0209
2,000 - 2,499	.5084	.5426	.0342
2,500 - 2,999	.5837	.6278	.0441
3,000 - 3,499	.6527	.6950	.0423
3,500 - 3,999	.7129	.7486	.0357
4,000 - 4,499	.7640	.7913	.0273
4,500 - 4,999	.8051	.8259	.0208
5,000 - 5,499	.8411	.8536	.0125
5,500 - 5,999	.8707	.8762	.0055
6,000 - 6,499	.8983	.8948	.0035
6,500 - 6,999	.9187	.9101	.0086
7,000 - 7,499	.9365	.9226	.0139
7,500 - 7,999	.9503	.9333	.0170
8,000 - 8,499	.9609	.9421	.0188
8,500 - 8,999	.9689	.9497	.0192
9,000 - 9,499	.9757	.9561	.0196
9,500 - 9,999	.9814	.9614	.0200
10,000 - 10,499	.9868	.9661	.0207
10,500 - 10,999	.9899	.9700	.0199
11,000 - 11,499	.9924	.9735	.0186
11,500 - 11,999	.9940	.9765	.0175
12,000 - 12,499	.9955	.9791	.0164
12,500 - 12,999	.9966	.9813	.0153
13,000 - 13,499	.9973	.9833	.0140
13,500 - 13,999	.9978	.9850	.0128
14,000 - 14,499	.9983	.9865	.0118
14,500 - 14,999	.9986	.9879	.0107
15,000 - 15,999	.9990	.9901	.0089
16,000 - 16,499	.9993	.9911	.0082
16,500 - 16,999	.9995	.9919	.0076
17,000 - 17,999	.9996	.9932	.0064
18,000 - 18,999	.9997	.9944	.0053
19,000 - 19,499	.9999	.9949	.0050
20,500 - 23,499	1.0000	.9974	.0026

Exhibit 18
Sheet 1

MINOR PERMANENT PARTIAL - 1961

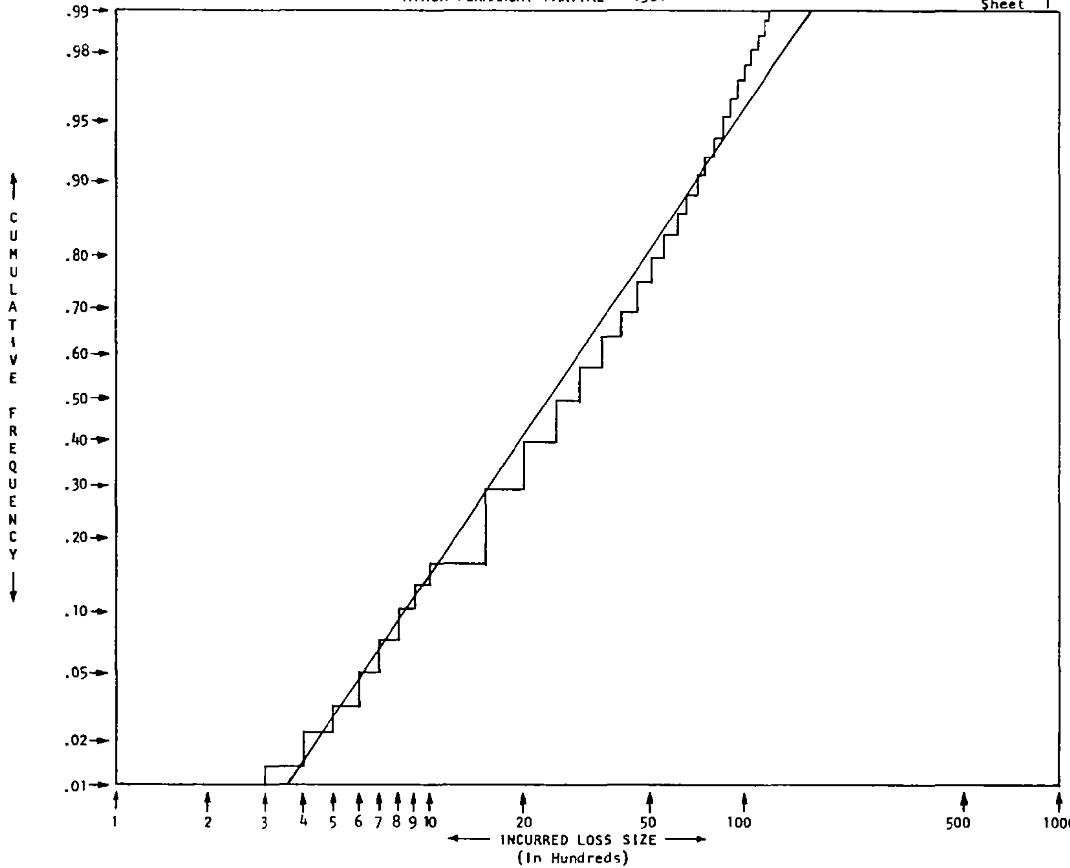

Exhibit 18
Sheet 2

MINOR PERMANENT PARTIAL — 1961

(1)	(2)	(3)	(4)
	Cumulative Frequency		Absolute Difference
Loss Size Interval	Observed	Theoretical	(2)-(3)
0 - 99	.0022	.0000	.0022
100 - 199	.0063	.0018	.0045
200 - 299	.0134	.0073	.0061
300 - 399	.0221	.0179	.0042
400 - 499	.0325	.0332	.0007
500 - 599	.0507	.0526	.0019
600 - 699	.0741	.0754	.0013
700 - 799	.1021	.1005	.0016
800 - 899	.1323	.1272	.0051
900 - 999	.1609	.1551	.0058
1,000 - 1,499	.2905	.2965	.0060
1,500 - 1,999	.3995	.4230	.0235
2,000 - 2,499	.4951	.5278	.0327
2,500 - 2,999	.5725	.6124	.0399
3,000 - 3,499	.6409	.6800	.0391
3,500 - 3,999	.6991	.7343	.0352
4,000 - 4,499	.7524	.7780	.0256
4,500 - 4,999	.7962	.8132	.0170
5,000 - 5,499	.8318	.8420	.0102
5,500 - 5,999	.8621	.8655	.0034
6,000 - 6,499	.8862	.8849	.0013
6,500 - 6,999	.9074	.9011	.0063
7,000 - 7,499	.9250	.9146	.0104
7,500 - 7,999	.9390	.9259	.0131
8,000 - 8,499	.9520	.9354	.0166
8,500 - 8,999	.9614	.9435	.0179
9,000 - 9,499	.9703	.9504	.0199
9,500 - 9,999	.9760	.9564	.0196
10,000 - 10,499	.9814	.9614	.0200
10,500 - 10,999	.9852	.9658	.0194
11,000 - 11,499	.9885	.9696	.0189
11,500 - 11,999	.9906	.9729	.0177
12,000 - 12,499	.9925	.9758	.0167
12,500 - 12,999	.9942	.9783	.0159
13,000 - 13,499	.9953	.9805	.0148
13,500 - 13,999	.9963	.9825	.0138
14,000 - 14,499	.9970	.9842	.0128
14,500 - 14,999	.9974	.9857	.0117
15,000 - 15,499	.9980	.9870	.0110
15,500 - 15,999	.9982	.9883	.0099
16,000 - 16,499	.9984	.9894	.0090
16,500 - 16,999	.9986	.9903	.0083
17,000 - 17,499	.9990	.9911	.0079
17,500 - 17,999	.9992	.9919	.0073
18,000 - 18,999	.9994	.9932	.0062
19,000 - 19,499	.9995	.9938	.0057
20,000 - 20,999	.9996	.9952	.0044
21,000 - 22,999	.9997	.9965	.0032
24,000 - 24,499	.9998	.9972	.0026
25,000 - 25,999	.9999	.9977	.0022
34,000 - 35,499	1.0000	.9993	.0007

Exhibit 19
Sheet 1

TEMPORARY - 1960

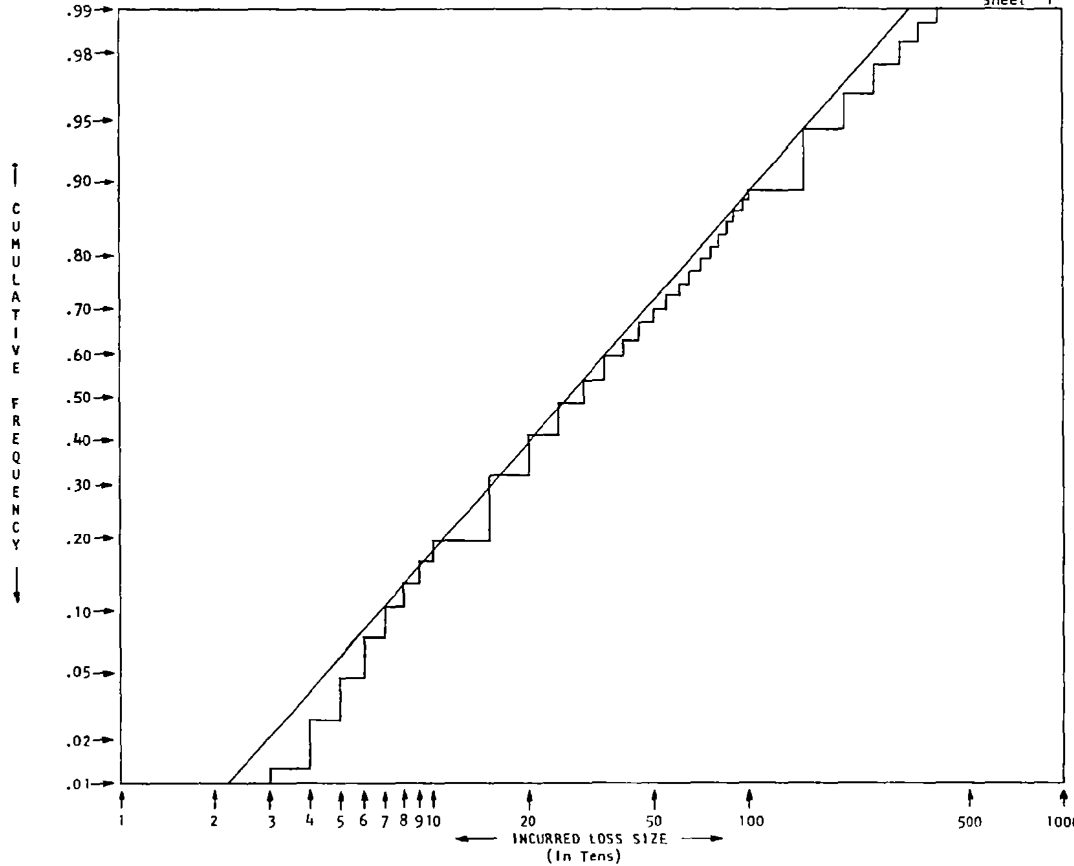

Exhibit 19
Sheet 2

TEMPORARY - 1960

(1) Loss Size Interval	(2) Cumulative Frequency Observed	(3) Cumulative Frequency Theoretical	(4) Absolute Difference (2)-(3)
0 - 9	.0017	.0009	.0008
10 - 19	.0052	.0076	.0024
20 - 29	.0132	.0208	.0076
30 - 39	.0272	.0387	.0115
40 - 49	.0488	.0599	.0111
50 - 59	.0756	.0829	.0073
60 - 69	.1049	.1069	.0020
70 - 79	.1353	.1316	.0037
80 - 89	.1658	.1564	.0094
90 - 99	.1946	.1809	.0137
100 - 149	.3211	.2956	.0255
150 - 199	.4143	.3928	.0215
200 - 249	.4880	.4737	.0143
250 - 299	.5442	.5402	.0040
300 - 349	.5958	.5960	.0002
350 - 399	.6361	.6428	.0067
400 - 449	.6727	.6822	.0095
450 - 499	.7022	.7160	.0138
500 - 549	.7289	.7448	.0159
550 - 599	.7513	.7698	.0185
600 - 649	.7754	.7916	.0162
650 - 699	.7956	.8106	.0150
700 - 749	.8153	.8272	.0119
750 - 799	.8345	.8418	.0073
800 - 849	.8523	.8549	.0026
850 - 899	.8676	.8665	.0011
900 - 949	.8812	.8770	.0042
950 - 999	.8915	.8863	.0052
1,000 - 1,499	.9437	.9428	.0009
1,500 - 1,999	.9634	.9673	.0039
2,000 - 2,499	.9748	.9797	.0049
2,500 - 2,999	.9822	.9866	.0044
3,000 - 3,499	.9869	.9908	.0039
3,500 - 3,999	.9901	.9934	.0033
4,000 - 4,499	.9925	.9952	.0027
4,500 - 4,999	.9941	.9963	.0022
5,000 - 5,499	.9955	.9972	.0017
5,500 - 5,999	.9966	.9978	.0012
6,000 - 6,499	.9973	.9982	.0009
6,500 - 6,999	.9977	.9986	.0009
7,000 - 7,499	.9983	.9989	.0006
7,500 - 7,999	.9986	.9991	.0005
8,000 - 8,499	.9990	.9992	.0002
8,500 - 8,999	.9993	.9994	.0001
9,000 - 9,999	.9994	.9995	.0001
10,000 - 10,999	.9996	.9997	.0001
11,000 - 12,999	.9998	.9998	.0000
13,000 - 16,499	.9999	.9999	.0000
17,000 - 33,999	1.0000	1.0000	.0000

Exhibit 20
Sheet 1

TEMPORARY - 1961

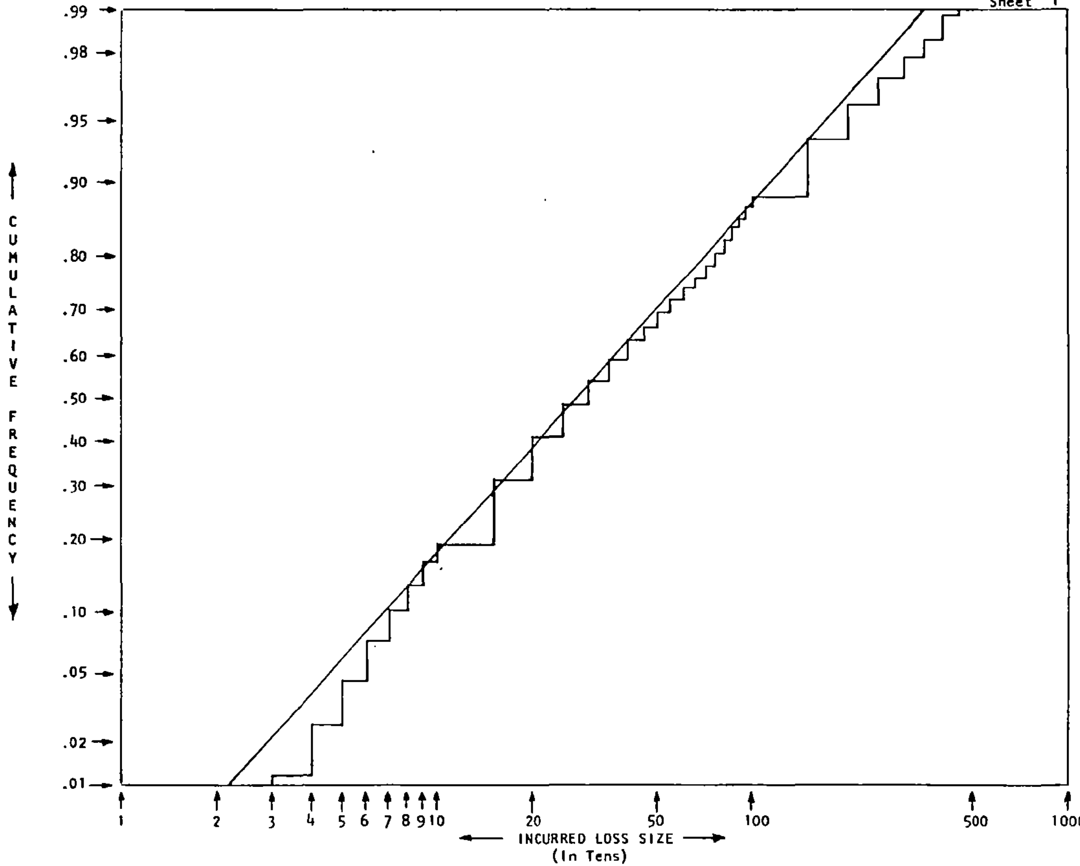

Exhibit 20
Sheet 2

TEMPORARY - 1961

(1) Loss Size Interval	(2) Cumulative Frequency Observed	(3) Cumulative Frequency Theoretical	(4) Absolute Difference (2)-(3)	(1) Loss Size Interval	(2) Cumulative Frequency Observed	(3) Cumulative Frequency Theoretical	(4) Absolute Difference (2)-(3)
0 - 9	.0012	.0010	.0002	900 - 949	.8731	.8720	.0011
10 - 19	.0042	.0078	.0036	950 - 999	.8836	.8815	.0021
20 - 29	.0119	.0209	.0090	1,000 - 1,499	.9388	.9396	.0008
30 - 39	.0256	.0387	.0131	1,500 - 1,999	.9599	.9651	.0052
40 - 49	.0470	.0595	.0125	2,000 - 2,499	.9720	.9781	.0061
50 - 59	.0738	.0823	.0085	2,500 - 2,999	.9792	.9855	.0063
60 - 69	.1041	.1060	.0019	3,000 - 3,499	.9847	.9899	.0052
70 - 79	.1342	.1303	.0039	3,500 - 3,999	.9883	.9928	.0045
80 - 89	.1648	.1547	.0101	4,000 - 4,499	.9912	.9946	.0034
90 - 99	.1939	.1789	.0150	4,500 - 4,999	.9932	.9960	.0028
100 - 149	.3185	.2919	.0266	5,000 - 5,499	.9948	.9968	.0020
150 - 199	.4129	.3879	.0250	5,500 - 5,999	.9959	.9975	.0016
200 - 249	.4861	.4678	.0183	6,000 - 6,499	.9967	.9981	.0014
250 - 299	.5438	.5341	.0097	6,500 - 6,999	.9973	.9984	.0011
300 - 349	.5931	.5895	.0036	7,000 - 7,499	.9977	.9987	.0010
350 - 399	.6336	.6360	.0024	7,500 - 7,999	.9981	.9989	.0008
400 - 449	.6671	.6755	.0084	8,000 - 8,499	.9985	.9991	.0006
450 - 499	.6955	.7093	.0138	8,500 - 8,999	.9988	.9993	.0005
500 - 549	.7226	.7383	.0157	9,000 - 9,499	.9990	.9994	.0004
550 - 599	.7451	.7634	.0183	9,500 - 9,999	.9992	.9995	.0003
600 - 649	.7674	.7853	.0179	10,000 - 10,499	.9994	.9995	.0001
650 - 699	.7871	.8045	.0174	10,500 - 10,999	.9995	.9996	.0001
700 - 749	.8071	.8214	.0143	11,000 - 11,999	.9997	.9997	.0000
750 - 799	.8264	.8364	.0100	12,000 - 12,999	.9998	.9998	.0000
800 - 849	.8438	.8496	.0058	13,000 - 14,999	.9999	.9999	.0000
850 - 899	.8596	.8614	.0018	15,000 - 20,499	1.0000	1.0000	.0000

Exhibit 21

CHARACTERISTICS OF THE
SIZE OF LOSS DISTRIBUTIONS FOR
PERMANENT DISABILITY AND TEMPORARY CASES

Type of Injury	Policy Year	Number of Cases	Mean	Standard Deviation	D_n	$D_n^{.05}$	Result of K-test
Permanent Total	1960	46	4.95667	.26967	.083	.201	accept
	1961 - "a"	57	4.93985	.30200	.134	.180	accept
	1961 - "b"	56	4.96976	.20460	.102	.182	accept
Major	1960	3,271	4.06335	.22971	.085	.024	reject
	1961	4,721	4.07928	.21256	.046	.020	reject
Minor	1960	20,554	3.35888	.36261	.044	.009	reject
	1961	24,613	3.37215	.36719	.040	.009	reject
Temporary	1960	55,372	2.42763	.47380	.026	.006	reject
	1961	60,398	2.43481	.47759	.027	.006	reject

$$D_n = \max_x \left| F(x) - S_n(x) \right|$$

$$D_n^{.05} = 1.36 \div n^{\frac{1}{2}}$$

DISCUSSION BY ROY H. KALLOP

Mr. Dropkin is to be congratulated once again for another fine contribution to the Society. His paper, Size of Loss Distributions in Workmen's Compensation Insurance, represents a forward step toward establishing an appropriate mathematical model which would adequately describe the distribution of losses by size. If such a model is eventually established it would be of real value in calculating "D" ratios, excess loss premium factors, the self-rating point in experience rating, and other calculations which relate to size of loss.

The detailed statistics of loss distributions by type of injury which were available for California enabled the author to analyze at considerable depth the fitting of a curve to observed loss distributions with special emphasis on the log-normal curve applied to disability losses. A significant achievement in Mr. Dropkin's paper is his use of the Kolmogorov test of the goodness of fit. Although the test does not share widespread popularity with the Chi-Square test, it is, nevertheless, a method which is relatively simple and appears to be more suitable than the Chi-Square test for this particular analysis.

It would be most interesting to study the loss distributions of other states and compare the results with Mr. Dropkin's analysis in California. Unfortunately, we do not have readily available loss distributions by type of injury for the states under the National Council jurisdiction but it is entirely possible that this type of data may be made available in the near future. Before such a study of other states is conducted, however, there are some major obstacles that would have to be overcome.

California has the largest volume of compensation experience and, therefore, is an ideal state to analyze since loss distributions can be developed consisting of a large number of claims incurred within a relatively short period of time. Most of the other states have considerably less volume than California and it would be necessary to compile losses over a much longer period in order to secure a sufficient number of claims to review by type of injury. In fact, a number of states have only a handful of permanent total cases each year. This raises serious doubts as to whether a mathematical model can be established for permanent total loss distributions within a state even over a long period of time. Due to noticeable differences in state benefit scales, wage levels, attitudes of adjudicating claim bodies, etc., regional or countrywide distributions have only limited value.

Another question arises as to the effect that the changes in state conditions will have on loss distributions during the period under study. For example, law amendments obviously can affect the characteristics of a loss distribution. Changes in wage level may also affect the shape of the distributional curve because of the maximum and minimum limitations on workmen's compensation benefits. A Supreme Court decision applicable to a particular type of injury is another factor to be taken into account. These changes may not appreciably affect a distribution over a short period of time such as Mr. Dropkin has used in his analysis of California data, but such changes could significantly affect loss distributions over a longer period of time which would be required if other states were being reviewed.

The California Unit Statistical Plan requires that all indemnity cases be listed separately regardless of amount. Under the present National Council rules, all claims which have a total loss (indemnity and medical combined) less than $500 may be lumped together. A good percentage of temporary total cases are under $500 and, are reported on a combined basis. In addition, there are a number of minor permanent partial cases under $500. Hence, loss distributions that might be developed for other states would have as its first interval all claims under $500. This means that a study of the other states would be useful if we are concerned only with the larger loss sizes. This suggests that a mathematical analysis of the upper parts of a loss distribution would involve the theory of extreme values. This could be a good subject for a future paper.

Development of losses beyond a first reporting basis can be significant, particularly for serious injuries. Unfortunately, Mr. Dropkin's analysis had to be confined to first reporting figures, since losses were not available on a per-claim basis on a subsequent reporting basis.

It is hoped that the problems to be faced in analyzing loss distributions for other states can be met with successfully in order that we can augment the very fine work that Mr. Dropkin has initiated in California.

DISCUSSION BY LeROY J. SIMON

We all know what to expect when we read a paper by Mr. Dropkin. We expect to get some new ideas, come interesting information and a careful, precise and correct presentation which mixes both the practical and the theoretical. In his paper, Size of Loss Distributions in Workmen's Compensation Insurance, we are not disappointed. The interesting information this time comes in the form of a series of ten actual distributions of losses in Workmen's Compensation. One of the significant new ideas that we get from the paper is an introduction to the Kolmogorov test. The blending of the theoretical and practical is quite evident in Mr. Dropkin's summary at the end of the paper. Hence, any comments I make will be either supplementary to what the author has said or will look into areas which are outside of the scope that the author set for himself in the original paper.

Basic distributions of losses of this sort are fundamental to the Theory of Risk. If we could have a theory with enough of an empirical basis, we could not only have better based "D" ratios but could also find it quite helpful in constructing Table M, determining excess of loss factors and in establishing *S*-Points. While the Theory of Risk may not be sufficiently advanced in the United States to make its use feasible at this time in the construction of Table M, the determination of excess of loss factors is a rather straightforward calculation which can be easily demonstrated. Exhibit A is a calculation based on policy year 1961 of the excess losses over six different levels of loss. It was necesary to assume that the Medical Only losses were all less than $10,000 which is undoubtedly a safe assumption. It is unfortunate that Mr. Dropkin did not include a distribution of the Medical Only claims, even though he would have been forced to make the first interval extend from 0 to $500. One must be careful when referring to these excess of loss factors to recall that the raw data is from first reports under the Unit Statistical Plan. This means that some of the losses have been evaluated with as little as six months or less (depending upon individual company processing methods) of elapsed time since the accident occurred. The maximum amount of time of evaluation would be 18 months. The general tendency is for certain claims to become more severe as they age and the excess of loss factors are influenced markedly by the presence of large losses. One must view these figures as minimum indications and recognize that they apply only to California in policy year 1961. Extensions beyond this scope can be made, of course, subject to the use of sound actuarial judgment. Both curve fitting techniques and statistical tests of significance could be used in this area to smooth out the irregularities in the raw data and to help decide when a set of excess of loss factors have need of revision.

In our experience rating plans in Workmen's Compensation we attempt to establish a point at which the insured will become entirely self-rated (the *S*-Point). Any system for determining *S* will have a set of controls which will attempt to minimize any sharp variation from one year to the next. One typical technique is to use two or more years of data, dropping out the earliest and adding in the latest year each time. Ignoring these types of controls, one system in use in the past for determining *S* was to make it a function of the average Death and Permanent Total loss in the state. It was found that the variations in this statistic were quite violent (especially in the smaller states), and a considerable degree of arbitrary control had to be imposed upon the result. In an effort to avoid this, the National Council on Compensation Insurance went to a function of the average Serious case (where Serious cases are Death, Permanent Total and Major Permanent Partials). A third method which has been suggested by this reviewer is based upon the percentile values of the distribution of losses excluding the Medical Only cases. The general reasoning is based upon the fact that the *S*-Point should be located at a point where the premium from year to year on risks that are at or above this point will be relatively stable. In other words, if the risk is going to have 100% credibility, he should have a fairly stable premium from one rating period to the next. It is also desirable to have *S* vary from state to state since the laws vary on a state basis and, hence, the distribution of losses will vary this way also. Finally, *S* should change if the law within a state changes in such a way as to make the loss distribution more "dangerous" or less "dangerous." This is another example of the age-old actuarial problem of wanting responsiveness to changing conditions, but wanting protection from unnecessary random fluctuation. Exhibit B consolidates the data from Mr. Dropkin's paper into a single distribution for each policy year. This type of information was used to locate certain percentile values. In each case a careful study of the published distributions by type of injury was made to establish the percentile value as accurately as possible. In actual practice one would use ungrouped data and obviate the need for approximation. In Exhibit C we have a comparison of the variation from one year to the next in the average Death and Permanent Total value, the average Serious case value and each of a number of percentile values. We can see from this exhibit that the two average value figures did not change much, while each of the percentile values moved up sharply. This latter fact indicates that the distribution has become more "dangerous" and, hence, there *should* be an increase in the 100% credibility point. Further experimentation along this avenue of approach to establishing the *S*-Point seems worthwhile.

It is appropriate to point out here that the entire change from 1960 to 1961 cannot be attributable to random fluctuation since there were two law amendments which have an effect upon the two sets of data. The individual losses are included in the loss distributions at the incurred cost to the company and are unadjusted for the effect of any benefit level changes. Mr. Dropkin has written me as follows:

"There was a change in benefit level effective September 15, 1961. The calculated effect was as follows:

Type of Injury	Effect	
Death	1.005	
P.T.	1.001	
Major	1.007	
Serious		1.006
Minor	1.011	
Temp.	1.039	
Non-Serious		1.018
Medical		1.000
Total		1.009

It should be noted that the effects listed above are for the indemnity portion and the total medical portion separately, while the incurred loss size used in the basic distribution represent the indemnity and medical amounts combined. Also, effective October 1, 1962, there was a change in the Official Minimum Medical Fee Schedule which was applicable to all injuries whenever they may have occurred. In calculating the effect in this case we considered only new injuries since there is no good way to measure the effect on old ones. The calculated effect was:

Indemnity	1.000
Medical	1.025
Total	1.008"

The use of the one-sample Kolmogorov test leads rather naturally to the use of the two-sample Kolmogorov test to test the hypothesis that the two samples could have been drawn from the same population. Hoel[1] does not feel that this test possesses any advantages over other non-parametric methods for dealing with this problem. However, Siegel[2] cites it as the most powerful test available for continuous distributions when we wish to test for any kind of difference in the two samples. We proceed in the two-sample Kolmogorov test by evaluating D_{mn} as the maximum absolute deviation between the two observed cumulative frequencies that we are testing. If $S_m(x)$ and $S_n(x)$ are the two observed cumulative relative frequencies in the samples of sizes m and n, then

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[1] Hoel, Paul G., *Introduction to Mathematical Statistics*, Third Edition, John Wiley & Sons, Inc., New York, 1962, p. 349.

[2] Siegel, Sidney, *Nonparametric Statistics for the Behavioral Sciences*, McGraw-Hill Book Company, Inc., 1956, p. 157.

$$D_{mn} = \max_x \left| S_m(x) - S_n(x) \right|$$

For small samples and when $m = n$, published tables are available for the test of significance. However, when the number of cases exceeds 40 in each sample, the critical value can be found by the formula $k\sqrt{\frac{m+n}{mn}}$ where k is 1.3581 at the 5% level and 1.6276 at the 1% level. The test then reduces to comparing the sample value of D_{mn} against the critical value at the level of significance desired. Exhibits D through H set forth the two observed cumulative relative frequency distributions and the differences necessary to determine D_{mn}. Each of the exhibits has been shortened somewhat, especially by coarser grouping in the upper tails, but no significant information has been omitted. Exhibit E of Permanent Total cases is different only because the raw data was set forth for each case separately, while all other distributions were presented on a grouped basis. Column (5) of Exhibit I sets forth the sample value of D_{mn} for each of the types of loss. Columns (3) and (4) show the critical values of D_{mn} at the two significance levels most commonly used by statisticians, and column (6) shows the conclusion reached on the hypothesis that the two samples came from the same population (against the alternative that they came from different populations). The rule used is to accept the hypothesis below the 5% level, to reject it above the 1% level and to remain in doubt when the sample statistic falls between these two levels. In the case of Temporary losses the sample statistic puts us in the doubtful area. With access to the entire raw data, it would be well to go back to the interval 650-699 and investigate more carefully on a case-by-case basis to determine the true maximum value of the statistic D_{mn}. A similar investigation would be made in the interval 800-849. This may very well lead to a value in excess of .0096 and, thus, lead to rejection of the hypothesis at the 1% level. This illustrates one of the problems when dealing with grouped data. If the grouping is too coarse, a significant difference between distributions can be completely masked. If there is any grouping at all and the test statistic gets close to the critical value, the researcher must go back and get more information in order to arrive at his conclusion. This two-tailed Kolmogorov two-sample test is appropriate when one wishes to investigate whether the distributions come from the same population or not.

If we have evidence that one distribution may have arisen from a population with a higher (or lower) distribution, the appropriate test would be a one-tailed test of significance. In the case we have here we know that the policy year 1961 data comes to us with more of the losses subjected to the law with higher benefit levels in it. We may, therefore, wish to test the hypothesis that the distribution of policy year 1961 losses is higher than the distribution of policy year 1960 losses. This can be done by again calculating the sample statistic D'_{mn} where D'_{mn} is the maximum difference between the two observed cumulative relative frequencies *in the desired direction.* From this we calculate

$$\chi^2 = 4\,(D'_{mn})^2\,\frac{mn}{m+n}$$

This time the critical value is the value of Chi-square with two degrees of freedom. The values of χ^2 are set forth in column (8) of Exhibit I and the conclusions with regard to the hypothesis are in column (9). The extremely high probability on Death cases (χ^2 at .99 is .020) makes one suspicious that we are testing the wrong hypothesis, and perhaps some other factors are at work in addition to law level and random fluctuation.

In setting forth a few cautions about the use of the one-sample Kolmogorov test, Mr. Dropkin says that it is exact only when the data is "unclassified" (that is, ungrouped). He also cautions us that if parameters are estimated from the data, the Kolmogorov test is affected; but it is not known exactly what the effect will be. He recommends that to correct for this we use a critical value smaller than would otherwise be used. Another way of saying the same thing would be that if a sample statistic leads to the rejection of the hypothesis, one could be confident that he was safe in rejecting the hypothesis at the given level of significance. However, if the sample value leads to accepting the hypothesis by a rather thin margin of difference between the sample value and the critical value, one would feel a little unsure about accepting the sample at the specified significance level if some of the parameters had to be estimated from the sample data.

Mr. Dropkin opens up the rather interesting area of outliers when he discusses the problems with the case evaluated at $1,840 among his Permanent Total cases for policy year 1961. With a case that stands out as far as this, we can use a rather straightforward approach which is not particularly powerful. If we assume that the sample is, in fact, from a lognormal distribution with a mean and standard deviation as set forth in Exhibit 21 of the paper, we can quickly calculate that a case such as this falls 5.546 standard deviations away from the mean. The probability of such a rare event (or one more rare) occurring in random sampling is .00000003. Since we have a sample of 57 cases, the binomial probability of an event like this (or one more rare) occurring one or more times is:

$$1 - (.99999997)^{57}$$

This equals .000002 and we would conclude that an event such as this in a sample of this size is quite unlikely. Therefore, we would reject the hypothesis that this was a true Permanent Total case which arose solely due to chance fluctuation. It can be treated as an outlier and justifiably excluded from the sample. If the sample were smaller and a rigorous test were still needed for the outlier, tables of the critical values of the studentized extreme deviate are available.[3]

This useful and important paper will undoubtedly be referred to many times. It will be helpful to have such distributions available for ready reference in the solution or approximation of solutions to a number of problems. The use of distribution-free tests in insurance statistics is bound to gain more acceptance as time goes on. Mr. Dropkin's fine description and illustration of the Kolmogorov one-sample test will be a handy reference. The paper was not only interesting but informative; not only theoretical but practical; not only advanced but understandable. It is a fine addition to our actuarial literature.

[3] *Biometrika Tables for Statisticians*, Volume I, Second Edition, Cambridge University Press, 1958.

EXHIBIT A

CALIFORNIA WORKMEN'S COMPENSATION

Policy Year 1961

Type	All Losses	Losses in Excess of: $10,000	$15,000	$25,000	$50,000	$100,000	$250,000
Death	$ 11,743,540	5,403,256	2,794,965	111,616	23,090	0	0
Permanent Total	5,889,192	5,327,352	5,047,352	4,487,352	3,109,595	1,270,247	85,996
Major	64,619,490	21,012,513	10,369,092	4,148,381	1,182,909	186,092	0
Minor	79,462,086	1,249,416	195,332	20,845	0	0	0
Temporary	31,032,492	96,703	11,400	0	0	0	0
Medical Only	16,456,429	0	0	0	0	0	0
Total	209,203,229	33,089,240	18,418,141	8,768,194	4,315,594	1,456,339	85,996
Ratio to All Losses		.1582	.0880	.0419	.0206	.0070	.0004

EXHIBIT B

CALIFORNIA WORKMEN'S COMPENSATION
Distribution of All Types
Policy Year 1960

Loss Size Interval	Number of Cases	Sum Up
0 - 4,999	71,759	79,875
5,000 - 5,499	847	8,116
5,500 - 7,999	2,903	7,269
8,000 - 8,499	395	4,366
8,500 - 12,999	2,183	3,971
13,000 - 16,499	675	1,788
16,500 - 16,999	62	1,113
17,000 - 33,999	935	1,051
34,000 - 45,499	35	116
45,500 - 45,999	3	81
46,000 - 299,999	78	78

Policy Year 1961

Loss Size Interval	Number of Cases	Sum Up
0 - 5,499	80,859	90,559
5,500 - 5,999	879	9,700
6,000 - 9,499	4,084	8,821
9,500 - 9,999	370	4,737
10,000 - 14,999	2,396	4,367
15,000 - 20,999	1,140	1,971
21,000 - 49,999	726	831
50,000 - 53,652	14	105
53,653	1	91
53,327	1	90
53,328 - 339,999	89	89

Note: The loss size intervals have been selected to facilitate location of the percentile values shown in Exhibit C while reducing the length of the exhibit to a minimum.

EXHIBIT C

CALIFORNIA WORKMEN"S COMPENSATION
Some Possible Statistics for Determining the
Self-Rating Point under the Experience Rating Plan

	Policy Year 1960		Policy Year 1961		
	Case Number	Value	Case Number	Value	Change
Average Death and Permanent Total	--	$21,700	--	$21,300	-2%
Average Serious Case	--	$14,600	--	$14,800	+1%
Percentiles					
99.9	80	$45,700	91	$53,500	+17%
99	799	18,100	906	20,300	+12%
98	1,598	13,700	1,811	16,000	+17%
95	3,994	8,470	4,528	9,780	+15%
90	7,988	5,080	9,056	5,870	+16%

CALIFORNIA WORKMEN'S COMPENSATION
Death Cases -- Two Policy Years

Loss Size Interval	Cumulative Observed Frequency 1960	Cumulative Observed Frequency 1961	Difference
0 - 499	.0237	.0208	.0029
500 - 999	.0854	.0805	.0049
1,000 - 1,499	.1076	.0974	.0102
1,500 - 1,999	.1250	.1078	.0172
2,000 - 2,499	.1345	.1221	.0124
2,500 - 2,999	.1408	.1390	.0018
3,000 - 3,499	.1503	.1481	.0022
3,500 - 3,999	.1519	.1584	-.0065
4,000 - 4,499	.1693	.1675	.0018
4,500 - 4,999	.1725	.1792	-.0067
5,000 - 5,499	.1867	.2078	-.0211
5,500 - 5,999	.1930	.2143	-.0213
6,000 - 6,499	.2041	.2182	-.0141
6,500 - 6,999	.2073	.2234	-.0161
7,000 - 7,499	.2120	.2273	-.0153
7,500 - 7,999	.2263	.2429	-.0166
8,000 - 8,499	.2326	.2571	-.0245
8,500 - 8,999	.2405	.2610	-.0205
9,000 - 9,499	.2468	.2688	-.0220
9,500 - 9,999	.2500	.2753	-.0253
10,000 - 10,499	.2722	.2870	-.0148
10,500 - 10,999	.2848	.3078	-.0230
11,000 - 11,499	.2927	.3143	-.0216
11,500 - 11,999	.2959	.3195	-.0236
12,000 - 12,499	.2975	.3260	-.0285
12,500 - 12,999	.3006	.3312	-.0306
13,000 - 13,499	.3070	.3338	-.0268
13,500 - 13,999	.3085	.3403	-.0318
14,000 - 14,499	.3149	.3403	-.0254
14,500 - 14,999	.3212	.3455	-.0243
15,000 - 15,499	.3307	.3571	-.0264
15,500 - 15,999	.3323	.3623	-.0300
16,000 - 16,499	.3354	.3662	-.0308
16,500 - 16,999	.3370	.3714	-.0344
17,000 - 17,499	.3497	.3844	-.0347
17,500 - 17,999	.3655	.3935	-.0280
18,000 - 18,499	.4968	.5221	-.0253
18,500 - 18,999	.5269	.5481	-.0212
19,000 - 19,499	.5475	.5636	-.0161
19,500 - 19,999	.5680	.5792	-.0112
20,000 - 20,499	.5823	.5948	-.0125
20,500 - 20,999	.6060	.6091	-.0031
21,000 - 21,499	.9035	.8857	-.0178
21,500 - 21,999	.9415	.9286	-.0129
22,000 - 22,499	.9589	.9468	.0121
22,500 - 22,999	.9715	.9584	.0131
23,000 - 23,499	.9810	.9806	.0004
23,500 - 23,999	.9889	.9885	.0004
24,000 - 73,499	1.0000	1.0000	.0000

EXHIBIT E

CALIFORNIA WORKMEN'S COMPENSATION
Permanent Total Cases - Two Policy Years

Loss Size 1960	Loss Size 1961	Cumulative Observed Frequency 1960	Cumulative Observed Frequency 1961	Difference
*	*	*	*	*
46,000	46,000	.1522	.0526	.0996
	48,457		.0702	.0820
	50,247		.0877	.0645
	53,200		.1053	.0469
	53,327		.1228	.0294
	53,653		.1404	.0118
54,825		.1739		.0335
	55,000		.1579	.0160
55,338		.1957		.0378
56,000		.2174		.0595
56,001		.2391		.0812
58,506		.2609		.1030
58,600		.2826		.1247
	59,371		.1754	.1072
59,673		.3043		.1289
	62,100		.1930	.1113
62,500		.3261		.1331
	62,522		.2105	.1156
63,291		.3478		.1373
	63,800		.2281	.1197
	64,588		.2456	.1022
	64,726		.2632	.0846
	65,340		.2807	.0671
67,206		.3696		.0889
68,391		.3913		.1106
	68,874		.2982	.0931
69,653		.4130		.1148
	70,639		.3158	.0972
	72,679		.3333	.0797
	73,391		.3509	.0621
	75,000		.3684	.0446
75,394		.4348		.0664
	75,500		.3860	.0488
	76,823		.4035	.0313
	77,711		.4211	.0137
	79,304		.4386	-.0038
80,000		.4565		.0179
	81,969		.4561	.0004
	83,000		.4737	-.0172
	83,481		.4912	-.0347

Loss Size 1960	Loss Size 1961	Cumulative Observed Frequency 1960	Cumulative Observed Frequency 1961	Difference
	86,690		.5088	-.0523
86,828		.4783		-.0305
	89,000		.5263	-.0480
89,028		.5000		-.0263
	93,410		.5439	-.0439
	94,816		.5614	-.0614
	99,187		.5789	-.0789
	100,187		.5965	-.0965
	100,340		.6140	-.1140
	101,090		.6316	-.1316
	101,312		.6491	-.1491
	103,515		.6667	-.1667
104,500		.5217		-.1450
107,326		.5435		-.1232
	107,493		.6842	-.1407
	108,485		.7018	-.1583
	108,637		.7193	-.1758
	109,521		.7368	-.1933
	111,591		.7544	-.2109
114,514		.5652		-.1892
	115,547		.7719	-.2067
118,144		.5870		-.1849
119,874		.6087		-.1632
121,200		.6304		-.1415
125,000		.6522		-.1197
128,985		.6739		-.0980
	132,946		.7895	-.1156
135,844		.6957		-.0938
139,845		.7174		-.0721
141,564		.7391		-.0504
	145,787		.8070	-.0679
147,563		.7609		-.0461
147,663		.7826		-.0244
	150,000		.8246	-.0420
	152,015		.8421	-.0595
	156,995		.8596	-.0770
159,121		.8043		-.0553
161,415		.8261		-.0335
164,208		.8478		-.0118
165,183		.8696		.0100
*	*	*	*	*

EXHIBIT F

CALIFORNIA WORKMEN'S COMPENSATION

Major Permanent Partial Cases - Two Policy Years

Loss Size Interval	Cumulative Observed Frequency 1960	1961	Difference
0 - 4,999	.0171	.0195	-.0024
5,000 - 5,499	.0238	.0290	-.0052
5,500 - 5,999	.0379	.0423	-.0044
6,000 - 6,499	.0584	.0660	-.0076
6,500 - 6,999	.0865	.0923	-.0058
7,000 - 7,499	.1208	.1252	-.0044
7,500 - 7,999	.1639	.1619	.0020
8,000 - 8,499	.2106	.2012	.0094
8,500 - 8,999	.2586	.2473	.0113
9,000 - 9,499	.3143	.2940	.0203
9,500 - 9,999	.3672	.3392	.0280
10,000 - 10,499	.4271	.3880	.0391
10,500 - 10,999	.4729	.4309	.0420
11,000 - 11,499	.5194	.4717	.0477
11,500 - 11,999	.5604	.5093	.0511
12,000 - 12,499	.5986	.5532	.0454
12,500 - 12,999	.6273	.5887	.0386
13,000 - 13,499	.6640	.6271	.0369
13,500 - 13,999	.6986	.6579	.0407
14,000 - 14,499	.7252	.6879	.0373
14,500 - 14,999	.7450	.7165	.0285
15,000 - 15,499	.7713	.7447	.0266
15,500 - 15,999	.7930	.7659	.0271
16,000 - 16,499	.8086	.7877	.0209
16,500 - 16,999	.8260	.8034	.0226
17,000 - 17,499	.8398	.8195	.0203
17,500 - 17,999	.8520	.8352	.0168
18,000 - 18,499	.8658	.8521	.0137
18,500 - 18,999	.8753	.8614	.0139
19,000 - 19,499	.8847	.8720	.0127
19,500 - 19,999	.8942	.8845	.0097
20,000 - 20,499	.9046	.8959	.0087
20,500 - 20,999	.9098	.9029	.0069
21,000 - 21,499	.9168	.9109	.0059
21,500 - 21,999	.9227	.9181	.0046
22,000 - 22,499	.9297	.9234	.0063
22,500 - 22,999	.9346	.9283	.0063
23,000 - 23,499	.9398	.9334	.0064
23,500 - 23,999	.9444	.9368	.0076
24,000 - 24,499	.9489	.9410	.0079

Loss Size Interval	Cumulative Observed Frequency 1960	1961	Difference
24,500 - 24,999	.9502	.9446	.0056
25,000 - 25,499	.9566	.9490	.0076
25,500 - 25,999	.9587	.9518	.0069
26,000 - 26,499	.9606	.9546	.0060
26,500 - 26,999	.9624	.9561	.0063
27,000 - 27,499	.9636	.9576	.0060
27,500 - 27,999	.9642	.9604	.0038
28,000 - 28,499	.9664	.9617	.0047
28,500 - 28,999	.9685	.9642	.0043
29,000 - 29,499	.9691	.9657	.0034
29,500 - 29,999	.9707	.9670	.0037
30,000 - 30,499	.9722	.9687	.0035
30,500 - 30,999	.9731	.9691	.0040
31,000 - 31,499	.9737	.9695	.0042
31,500 - 31,999	.9740	.9703	.0037
32,000 - 188,499	1.0000	1.0000	.0000

EXHIBIT G

CALIFORNIA WORKMEN'S COMPENSATION

Minor Permanent Partial Cases - Two Policy Years

Loss Size Interval	Cumulative Observed Frequency 1960	Cumulative Observed Frequency 1961	Absolute Difference
0 - 99	.0022	.0022	.0000
100 - 199	.0064	.0063	.0001
200 - 299	.0123	.0134	-.0011
300 - 399	.0211	.0221	-.0010
400 - 499	.0318	.0325	-.0007
500 - 599	.0501	.0507	-.0006
600 - 699	.0749	.0741	.0008
700 - 799	.1059	.1021	.0038
800 - 899	.1383	.1323	.0060
900 - 999	.1702	.1609	.0093
1,000 - 1,499	.3046	.2905	.0141
1,500 - 1,999	.4155	.3995	.0160
2,000 - 2,499	.5084	.4951	.0133
2,500 - 2,999	.5837	.5725	.0112
3,000 - 3,499	.6527	.6409	.0118
3,500 - 3,999	.7129	.6991	.0138
4,000 - 4,499	.7640	.7524	.0116
4,500 - 4,999	.8051	.7962	.0089
5,000 - 5,499	.8411	.8318	.0093
5,500 - 5,999	.8707	.8621	.0086
6,000 - 6,499	.8983	.8862	.0121
6,500 - 6,999	.9187	.9074	.0113
7,000 - 7,499	.9365	.9250	.0115
7,500 - 7,999	.9503	.9390	.0113
8,000 - 8,499	.9609	.9520	.0089
8,500 - 8,999	.9689	.9614	.0075
9,000 - 9,499	.9757	.9703	.0054
9,500 - 9,999	.9814	.9760	.0054
10,000 - 10,499	.9868	.9814	.0054
10,500 - 10,999	.9899	.9852	.0047
11,000 - 11,499	.9924	.9885	.0039
11,500 - 11,999	.9940	.9906	.0034
12,000 - 35,499	1.0000	1.0000	.0000

EXHIBIT H

CALIFORNIA WORKMEN'S COMPENSATION

Temporary Cases - Two Policy Years

Loss Size Interval	Cumulative Observed Frequency 1960	1961	Absolute Difference
0 - 9	.0017	.0012	.0005
10 - 19	.0052	.0042	.0010
20 - 29	.0132	.0119	.0013
30 - 39	.0272	.0256	.0016
40 - 49	.0488	.0470	.0018
50 - 59	.0756	.0738	.0018
60 - 69	.1049	.1041	.0008
70 - 79	.1353	.1342	.0011
80 - 89	.1658	.1648	.0010
90 - 99	.1946	.1939	.0007
100 - 149	.3211	.3185	.0026
150 - 199	.4143	.4129	.0014
200 - 249	.4880	.4861	.0019
250 - 299	.5442	.5438	.0004
300 - 349	.5958	.5931	.0027
350 - 399	.6361	.6336	.0025
400 - 449	.6727	.6671	.0056
450 - 499	.7022	.6955	.0067
500 - 549	.7289	.7226	.0063
550 - 599	.7513	.7451	.0062
600 - 649	.7754	.7674	.0080
650 - 699	.7956	.7871	.0085
700 - 749	.8153	.8071	.0082
750 - 799	.8345	.8264	.0081
800 - 849	.8523	.8438	.0085
850 - 899	.8676	.8596	.0080
900 - 949	.8812	.8731	.0081
950 - 999	.8915	.8836	.0079
1,000 - 1,499	.9437	.9388	.0049
1,500 - 1,999	.9634	.9599	.0035
2,000 - 2,499	.9748	.9720	.0028
2,500 - 2,999	.9822	.9792	.0030
3,000 - 3,499	.9869	.9847	.0022
3,500 - 3,999	.9901	.9883	.0018
4,000 - 4,499	.9925	.9912	.0013
4,500 - 4,999	.9941	.9932	.0009
5,000 - 5,499	.9955	.9948	.0007
5,500 - 33,999	1.0000	1.0000	.0000

TESTS OF SIGNIFICANCE

Type	Number of Cases 1960	Number of Cases 1961	Critical Values 5% Level	Critical Values 1% Level	Sample Value of D_{mn}	Hypothesis*	Sample Value of D'_{mn}	χ^2	Hypothesis**
Death	632	770	.0729	.0874	.0347	Accept	.0172	.02	Accept
Permanent Total	46	57	.2695	.3229	.2109	Accept	.1373	.38	Accept
Major	3,271	4,721	.0309	.0370	.0511	Reject	.0511	20.18	Reject
Minor	20,554	24,613	.0129	.0154	.0160	Reject	.0160	11.47	Reject
Temporary	55,372	60,398	.0080	.0096	.0085	Doubt	.0085	8.35	Doubt

* The two samples could have come from populations having the same distribution function; alternative, they come from populations having different distribution functions. D_{mn} is the maximum absolute difference.

** The two samples could have come from populations having the same distribution function; alternative, they come from populations having the 1961 distribution function higher than (that is, to the right of) the 1960 distribution function. Critical values for the one-tailed test are from χ^2 with 2 degrees of freedom; 5.99 at 5% point and 9.21 at 1% point. D'_{mn} is the maximum positive difference.

PANEL DISCUSSION – NOVEMBER 1964 MEETING

MOTOR INSURANCE IN FOREIGN COUNTRIES

Moderator: NORTON E. MASTERSON

One of the features of the Casualty Actuarial Society's Fiftieth Anniversary celebration in November 1964, was a panel discussion on Motor Insurance in Foreign Countries by five noted European actuaries. Taking part, in alphabetical order by country, were Messrs. Edward Franckx of Belgium, B. Christoffersen of Denmark, Robert E. Beard of England, Karl Borch of Norway, and Hans Ammeter of Switzerland. Mr. A. Tröbliger of the German Federal Republic did not attend the meeting but he sent a paper and it is included below. Norton E. Masterson, Past President of the Society and currently Vice Chairman of ASTIN, was Moderator.

Participants in the discussion were requested to organize their remarks around ten specific aspects of motor car insurance in their respective countries, as follows:

1. Extent of compulsory coverage
2. Ratemaking
3. Authority of government over coverage and rates
4. Determination of claim liability
5. No-claim bonus schemes
6. Hit-and-run coverage
7. Reinsurance of large losses
8. Uninsured motorists coverage
9. Typical passenger car rates
10. Significant variations from U. S. automobile insurance.

EDWARD FRANCKX – *Belgium*

Mr. Franckx is head of the Department of Mathematics in the Royal Military College of Belgium and a member of the Royal Association of Belgian Actuaries. Also he is currently President of the Permanent Committee of the International Congress of Actuaries. He was founder of ASTIN and is now Honorary Chairman.

COVERAGES

Third party liability insurance has been obligatory, for all types of motor vehicles, since 1957. Previously it had been required for public and commercial vehicles; in 1957 it became obligatory for motorcycles and private passenger cars.

Policies sometimes include, in addition to third party liability insurance, one or more of the following coverages:

Defense in court *(contre-assurance spéciale)*,
Fire and theft of the vehicle,
Material damage to the vehicle caused by accident.

The obligatory liability insurance provides unlimited coverage except for material damage caused by fire or explosion where coverage is obligatory only up to 5 million francs (about $100,000).

Most policies are written for a term of 10 years, a discount of 10% being generally granted to insureds who subscribe for that term.

SOME FIGURES RELATING TO OBLIGATORY LIABILITY INSURANCE

In 1963 the make-up of total registered vehicles was:

Private passenger cars	874,000
Motorcycles (including motorized bicycles)	632,000
Light or heavy trucks – transport of goods for own account	319,000
Light or heavy trucks – transport of goods for others	23,000
Buses and motor coaches	7,000
Taxicabs	9,000
Total	1,864,000

One hundred forty-one companies, eighty-two Belgian and fifty-nine foreign, share the Belgian market.

Premium income from 1963 operations amounted to 4,929 million Belgian francs ($98.6 million). It was made up of:

	Millions F.B.
Private passenger	2,945
Motorcycles	287
Transport for own account	1,233
Transport for others	314
Buses and motor coaches	102
Taxicabs	48
Total	4,929

Losses incurred in 1963 amounted to approximately 3,784 million F. B. ($75.6 million).

After taking into account general expenes which amount to approximately 15% of income and agents' commissions of approximately 20%, it becomes obvious that total expenses cannot be met out of premium income.

THE FINANCIAL SITUATION

The financial situation is therefore difficult as it is in many countries. The inadequacy of premium income results in a great measure from the inadequacy of tariff rates. Negotiations are in progress with the Minister of Economic Affairs, who has jurisdiction over the regulation of insurance and over price control, in an attempt to obtain authorization for higher tariffs.

But premium inadequacy is also caused to a considerable extent by the play of competition, often unhealthy, which manifests itself in three ways:

(a) using inadequate rates
(b) granting large and unwarranted discounts from current rates
(c) finally, too often, allowing excessively high rates of commission in order to hold agents.

RATE STRUCTURE

Private passenger: Two or three years ago many companies adopted "personalized" rates. Premiums are a function not only of the cubic capacity of the engine but also of the type of use (business or pleasure) and of the driver's occupation. Some companies retain cubic capacity of the engine as the only criterion, and among these are some who adjust the premium retrospectively by a bonus for no accidents.

Motorcycles: Cubic capacity of the engine is the only criterion generally used.

Transport of objects: Cubic capacity of the engine and tonnage both enter into the determination of the premium.

Passenger transport: The number of seats determines the amount of premium.

STATISTICAL NOTE

The continuing deterioration of underwriting results has caused great and widespread concern over this line of insurance for several years. There is an organization in Belgium called STATAU (Statistiques Automobiles) which collects information from its member companies and in their behalf compiles the data needed for determining average loss costs and accident frequencies and calculating pure premiums. But the competitive situation is such that far too little use is made of the indications STATAU supplies.

B. CHRISTOFFERSEN – *Denmark*

Mr. Christoffersen is Managing Director of the Baltica Life Insurance Company of Copenhagen. He is a member of the Danish Association of Actuaries and of ASTIN.

In order to give you an idea of certain local dimensions which may serve to illustrate certain of the points which I am going to mention later on, I would state, by way of introduction, that Denmark covers an area of about 43,000 square kilometres (16,500 square miles) and has a population of about 4.8 million inhabitants distributed over 1.6 million households. Of these households every forth or fifth owns a motor car, and also in Denmark the development goes towards there being more than one motor car in a number of households. The total number of motor cars, including tractors and mo-peds (motor bikes), is today 1.5 million and they represent a premium income of roughly 500 million Danish kroner or $70 million under third party insurance and insurance covering physical damage to the car.

The first Danish legislation on motor cars dates back to the year 1903. When perusing the rules laid down in this our first act on the subject, one is invariably stricken by surprise at the rapid development since then. Article 28 of this Act laid down that in case of collision with a motor car or in case of wayfaring horses shying at the motor car resulting in injury to persons or damage to property, the person responsible for the motor car should make good the damage. The liability could, however, lapse or be reduced if the injured person had himself intentionally or by gross negligence caused the damage, or circumstances showed that the damage could not have been avoided by the prudence and diligence which the driver of the motor car is required to apply. The wording of this Article is in all essentials identical with the present provision of the Road Traffic Act, i.e., that the onus of proof rests with the person in charge of the motor car, or as the saying goes that the principle of the "inverted onus of proof" was already then valid. Article 19 of the same Act lays down: If the motor car has to pass other road users, driving or riding in front of him, the signalling apparatus should be used in due time and the motor car be kept properly to the left side of the street or road. If the other driver signals to the driver of the motor car, the latter is obliged to stop the motor car to await whether any safety measures are necessary, such as alighting or drawing aside. If no such measures are taken, the driver of the motor car is entitled to continue. During the actual passing, the signal horn must not be used and the speed must not exceed what is necessary to pass. If the driver of the motor car encounters persons driving or on horseback, the motor car driver shall keep as much as possible to the right of the road or the street, and it is further incumbent on him to stop the vehicles and the engine (so that no sound is produced) when the other driver or rider makes sign to him or he himself perceives that the horse or horses show sign of taking fright. The driver or horseman shall leave a passable stretch of the road to the driver of the motor car.

From the same period came rules from the Police Regulations to the effect that a driving speed of four Danish miles an hour (about 18 U.S. mph), can be permitted on stretches of roads which are highroads and which belong to the grounds of Copenhagen (Frederiksberg) or a market town or market place. The speed of driving in any other case must not exceed two Danish miles an hour (about 9 U.S. mph).

This first Act on Motor Cars contained no provision dealing with compulsory third party liability insurance. The rules relating to this subject were introduced in 1918, and the maximum amount of compensation according to the insurance was D.kr. 12,000 for motor cars corresponding to about $4,000 based on the rate of exchange prevailing at the time.

In the following the rate of exchange ruling today has been applied. Rate of exchange and purchasing power are not identical. The purchasing power of the Danish krone in Denmark in relation to dollars is roughly 30% to 40% higher than the amount indicated by the rate of exchange. In 1921 the amount of compensation was increased to D.kr. 20,000 (about $3,000), in 1927 to D.kr. 30,000 (about $4,300) and in 1950 to D.kr. 60,000 (about $8,500).

The Act presently in force was adopted in 1959. Amendments were required partly to bring the Act up-to-date in relation to the compulsory insurance cover and partly in an endeavor to attain uniformity in the rules of the Scandinavian countries on the basis of the work of a Scandinavian Committee. Such a uniformity was indeed realized in most respects, but on one important point the Danish legislation still differs from that of the other Scandinavian countries: Denmark has retained the rules of the onus of proof resting with the driver of the motor vehicle whereas objective liability was introduced in the other countries. According to the latest amendments, the amounts of cover were fixed at D.kr. 150,000 (just under

$22,000) per person injured or killed without limitation of the number of injured persons. As far as damage to property is concerned, the insurance cover was fixed at a minimum of D.kr. 60,000 (just under $10,000), but the latter cover has in certain cases proved inadequate, and it is therefore to be expected that the minimum amount of insurance protection in this regard will be increased.

To the Danish insurance companies the revision of the Act implied a most important increase in the companies' commitments on the individual insurance, and it was therefore natural for the companies simultaneously to investigate how they could obtain the simplest, least costly and most effective reinsurance protection.

A considerable number of companies working on the same tariff basis (members of the Danish Association of Motor Vehicle Insurers) solved the problem by establishing a Catastrophe Pool, the conditions of which are briefly that the individual company for its own account covers D.kr. 100,000 (upwards of $14,000) of damage arising out of the same event. Amounts in excess thereof, up to a sum of D.kr. 2,000,000 (just under $300,000), are distributed over the participating companies on the basis of their premium income in the preceding year in motor car business. Any other amount relating to claims resulting from the same event and exceeding D.kr. 2,100,000 ($300,000) is covered by way of an excess of loss arrangement. A few minor companies cover part of the first D.kr. 100,000 ($14,000) by a quota share reinsurance.

In Denmark the duty levied on private motor cars is very high. The size of this duty, which to a certain extent varies with the value of the motor car, may roughly be stated as being equal to the selling price of the motor car before the levy of said duty, i.e., about 100% on the price of a new car. The yearly duty will vary from $10 for the smallest to about $30 for the bigger cars. Gas would cost about 60 cents a gallon, about 60% of which is duty.

This naturally results in the price of a motor car, new or second-hand, being rather high, which in turn entails that a considerable percentage of the total number of cars, about 80%, is insured against damage to the car. However, in the following I will in the main confine myself to compulsory third party insurance. The premiums payable for this type of insurance are fixed on the basis of the following criteria: (a) use of the vehicle, and (b) the place of registry of the vehicle. By way of statistics it has been established that these two criteria provide sufficient differentiation with regard to the mileage driven and the density of traffic in the area in which the vehicle is chiefly used. By way of illustration, the following categories may be mentioned: motor cars for private use, motor cars used commercially, taxicabs, transportation of light goods for own account and for the account of others, transportation of heavy goods – the last-mentioned category being further sub-divided: "for own account," "for the account of others," etc. The geographic areas are: a) Copenhagen with suburbs and surrounding districts; b) major provincial towns; c) minor provincial. towns; d) rural districts.

With regard to insurance against damage to the car a further criterion is added to the tariff. Until recently, the value of the vehicle at the moment the policy was taken out was the decisive factor, but as that criterion led to frequent adjustments owing to the depreciation of motor vehicles, it was decided – when the tariff was last amended two years ago – instead of using the value of the vehicle as a basis, rather to divide the insured vehicles into different classes according to their net weight, subject however to a limitation of value for the individual class; in consequence, the premium payable for insurance against damage to the car is a fixed amount irrespective of the age and value of the insured motor vehicle.

So far, this division into groups only applies to private motor cars and light vans whereas it has not been possible to apply the same system to heavy vans and lorries.

Incidentally, the Association of Danish Motor Vehicle Insurers, to which the majority of Danish companies adhere, works out statistics based on particulars received from the individual company.

From the preceding remarks it appears that in Denmark there is a legal obligation to arrange insurance up to the minimum cover mentioned, but this does not necessarily imply that the liability to pay damages may not exceed the insurance amounts. However, the Danish law regulating the question of damages operates very largely with a concept designated "middle-class average," which means that the Courts when assessing damages to an injured person either do not all, or only to a very slight degree, take into account the claimant's income, so that, as far as I know, it has never yet happened that the insurance amount has been insufficient in case of injury to persons whereas the cover for damage to property has proved inadequate in two or three cases.

With regard to the fixing of premiums, the rule is that the rates of premium are not subject to the approval of the insurance supervision, in this case the Danish Insurance Board, but the companies are obliged to submit their tariffs by way of information to the supervisory authorities.

As regards the question of cartels and the fixing of premiums we do have a special law on these subjects in Denmark (the Supervision of Monopolies). As far as the insurance industry is concerned, this supervision is also carried out by the Insurance Board. It is incumbent on the companies to report to the Supervision of Monopolies all agreements which are estimated to have a restrictive effect on the competition. This rule applies also to the tariff agreements made under the auspices of the Danish Association of Motor Vehicle Insurers. In actual fact, the supervision relating to the fixing of prices, which as mentioned above has also been referred to the Insurance Board, is exercised by this Authority in the way that measures are only taken if the Insurance Board estimates that the existing tariffs do not furnish sufficient elements of competition.

The vast majority of the claims are settled by the companies without intervention by the authorities. The police even go so far as to request the companies not to ask for a police report in case of damage, provided only damage to property is involved and the claim is not estimated to exceed about D.kr. 3,000 ($400).

Further, the large majority of the Danish companies have joined a convention to the effect that they undertake not to exercise any right of recovery against the company having insured the vehicle causing the damage, to the extent of the first D.kr. 5,000 ($700) for any one loss or damage. In case of grave injury to persons or heavy offenses against the Road Traffic Act, the police will always be called upon to take a statement and the question of blame will be settled by the Courts. In such cases, also the amount of damages will generally be fixed either in the Police Court or in the course of an ensuing civil action.

There is in Denmark a certain company, a dairies and farmers mutual, which writes motor car insurance at a fixed premium and without no-claim bonus, but the majority of companies have adopted the following bonus systems:

> Following one year of no claims, the premium will be reduced from 100% to 75%, and if the subsequent year is also free of claims, the premium is further reduced to 60%. In case of claim the premium is increased, irrespective of whether the damage occurred in the first or second year, so as to correspond to the initial premium.
>
> The system is complicated by the fact that in order to counter the competition from the company operating with a fixed premium, a new insured is permitted to start in bonus category No. 2, i. e., he is only

charged 75% of the maximum premium, which in turn results in the insured remaining in category No. 2 for two years.

There is special coverage for "hit and run" claims in case of personal injury; we have been having a few such cases in recent years. Both police and courts follow a very severe course in such cases, of which so far none has remained unsolved. When such cases are cleared up, the Pool has a right of recovery against the company who has insured the vehicle causing the damage, and the company has a similar right against the person responsible for the damage. Claims remaining unexplained will be apportioned to all companies operating in Denmark proportionally with their premium income.

It is very rare for a motor vehicle not to be properly insured, but such claims as might arise in connection with uninsured driving are covered by the companies according to an apportionment similar to that described in connection with "hit and run" coverage.

The schedule below shows the premiums for third party liability insurance, and for third party liability plus full insurance for damage to the car, in the four tariff areas and for cars of four different weights, before application of the no-claim bonus. The premiums are in Danish kroner and may be converted to dollars at a rate of 7 D.kr. for one dollar.

Type of Motor Car and Weight	Cover		Tariff Area I	II	III	IV
V. W., 600-799 kgs	A	Liability	335	290	190	155
	FK	Liability plus Damage	985	940	790	645
Opel Record, 800-999 kgs	A		335	290	190	155
	FK		1035	990	840	675
Mercedes 190, 1200-1399 kgs	A		335	290	190	155
	FK		1135	1090	940	735
Chevrolet Bel Air, 1600 kgs	A		335	290	190	155
and over	FK		1285	1240	1090	825

ROBERT E. BEARD – *England*

Mr. Beard is Assistant General Manager of Pearl Assurance Ltd., of London. He is senior Vice President of the Institute of Actuaries. He has been Secretary and Chairman of ASTIN and is now the Editor.

The need for compulsory cover is laid down by the Road Traffic Act. This Act requires that the insured's liability in respect of death or bodily injury to any person arising out of the use of a vehicle on the road shall be covered by insurance. The liability does not extend to passengers in the vehicle unless they are: (1) being carried for hire or reward or (2) being carried in pursuance of, or by reason of, a contract of employment. In addition, it is necessary to cover hospital charges and emergency treatment charges, which are limited by the Act to £50 and 12/6d respectively for each injured (about $141.75).

The issue of policies which provide only the statutory cover is comparatively rare. The normal type of policy is for comprehensive cover which includes, in addition to the statutory cover, accidental damage to the car, including fire and theft, third party property damage and liability to passengers, as well as some variable fringe benefits, e.g., rugs, clothing and personal effects (limit about £20), personal accident benefits, medical expenses, Continental extensions, etc.

The machinery and method of ratemaking is never divulged but it is safe to assume that there is sufficient classification of results in groups to present suitable adjustment of premiums within those groups. The major tariff divisions are seven geographical locations, three categories of use, eight categories by cubic capacity, and variation of premium according to value of car.

Except as mentioned above, the government has no authority over, nor responsibility for, cover or rates, and there are no cartel laws applicable to insurance. There is a Monopolies Commission concerned with price fixing but the existence of tariff and non-tariff companies takes insurance outside of this area.

The establishment and apportionment of liability is purely a civil matter and is never dealt with by the police. Where it is necessary for liability to be apportioned, this is effected in the main by negotiation with recourse to the civil courts only where it is impossible to reach agreement. The need for apportionment is, however, kept to a minimum by reason of an extensive range of sharing and similar agreements between insurers.

No-claims discount schemes vary from company to company, between 40% and 60% attained over four or more years free of claim. Apart from one insurer the whole earned discount is lost in the event of a claim being made.

There is no "hit and run" cover as such, but the Motor Insurers' Bureau (discussed later) will consider making *ex gratia* payments where it is established that injury has been caused by a motor vehicle which is not traced.

Reinsurance is generally on an excess of loss basis, the retention probably varying between £2,000 and £10,000 (between $5,600 and $28,000).

To provide for cases in which there is, at the time of the accident, no effective insurance to cover the statutory requirements, the Motor Insurers' Bureau was set up in agreement with the Ministry of Transport. The effect of the agreement is, *inter alia,* that the Bureau will deal with the statutory liabilities of the motorist as if such liabilities were insured. By agreement with the separate insurers, each insurer will deal on behalf of the Bureau with claims under policies it has issued but which are not effective in the circumstances at time of accident. Only where there is no policy at all does the Bureau itself handle the claims. The Bureau is financed by the insurers in proportion to premium income.

Typical premiums for comprehensive cover are as follows:

C.C. and value not exceeding		Private Use City	Private Use Rural	Maximum Business Use City	Maximum Business Use Rural
1100 c.c.	£500	£39	£20	£58	£29
2300 c.c.	£750	£59	£31	£88	£46
Over 4500 c.c.	£1,500	£112	£61	£168	£96

A. TRÖBLIGER – *German Federal Republic*

Mr. Tröbliger is Managing Director of the Public Insurance Company of the Savings Banks of Baden in Manheim.

Motorization has made very great progress in the German Federal Republic these last few years.

On January 1, 1964, the number of permits issued for vehicles totalled 10.8 million, apart from so-called "Mopeds" (bicycles with auxiliary motors), which totalled about 1.5 million. This total of 10.8 million is composed as follows:

Motorcycles	1,048,000
Private passenger cars	7,248,000
Trucks	805,000
Vans	503,000
Tractors	1,124,000
Buses	38,000
Cars with special bodies	72,000

There are about 134 private passenger cars per 1,000 inhabitants in the Federal Republic. Thus, the Federal Republic, together with Luxemburg, ranks third on the scale of European countries, after Sweden and France. Sixty-seven per cent of all motor vehicles (Mopeds excluded) are passenger cars. Of these, again 67% are owned by workmen, employees and civil servants, and 33% are company-owned and owned by self-employed persons. This heavy increase in motor vehicles is also reflected in the development of automobile insurance.

The total premium income of insurance companies operating in the Federal Republic amounted to 2,800 million Deutschmarks (about $700 million) for automobile insurance in 1963. Of the total German insurance premium, the percentage of automobile insurance is 22.6%, i.e., as a branch, it is second in volume – after Life.

Of the premium income of automobile insurance, 79.3% is automobile third party liability insurance, 15.9% is hull and 4.8% is accident.

In 1963, loss experience was not very favorable. As compared with 1962, the loss ratio deteriorated considerably in 1963. The claims average in automobile third party amounted to DM 764 ($191) and in hull to DM 988 ($247), full coverage.

Liability for damages caused to a third party by a motor vehicle is regulated according to the Traffic Law which states that the operator of a car must indemnify the damage caused when using a vehicle unless the accident is due to an Act of God (liability on account of hazard).

However, in case other indemnities may be claimed on account of other regulation under Federal Law, the more extended liability (liability on account of negligence) shall apply instead of that under the Traffic Law.

The extent of liability on account of hazard is limited according to the Traffic Law, i.e., in case of death or bodily injury, the person liable to pay damages shall do so only up to a capital sum of DM 50,000 ($12,500), or in annuities of up to DM 3,000 ($750) per annum.

The extent of liability on account of negligence is unlimited. In order to render protection of traffic victims more effective, each operator of a vehicle must buy third party liability insurance for himself and the authorized driver to cover material and personal damages caused by use of the vehicle.

The minimum amounts of this obligatory insurance for personal damages are for:

Vehicles up to 6 seats	DM 100,000 ($25,000)
Vehicles 7 to 10 seats	DM 150,000 ($37,500)
Vehicles 11 to 80 seats plus DM 8,000 ($2,000) each for the 11th and each further seat.	DM 150,000 ($37,500)

The minimum cover for material damage is the 10th part of the minimum sum insured for personal damage.

The legislative body of the Federal Republic is at present studying the increase in minimum coverage according to law. It may be expected that the amount required by law as minimum coverage for private passenger cars (up to 6 seats) will be increased to DM 250,000 ($62,500), the other coverages to be increased accordingly.

Automobile insurance like any other branch of insurance except marine insurance and reinsurance is controlled by the Federal Supervisory Office for Insurance and Building Societies. From the fact that each operator of a car must buy automobile third party liability insurance for his vehicle, the government derives the right to decree tariff principles for automobile insurance and to make tariffs, general conditions of insurance, general and special rating agreements and the tables of rates dependent on its approval. Taking into account that hull insurance of the vehicle and the automobile accident insurance are closely connected with the automobile third party liability insurance, these branches of automobile insurance are included in the procedure of approval.

Commissions have upper limits by decree. Professional agents shall not be paid more than 12% of the premium. The Cartel Bureaus (a kind of Anti-Trust division) see, within the provisions of the law, that automobile insurers do not make agreements among each other which do not correspond to the regulations.

Rates are separated according to types of vehicles (motorcycles, private passenger cars, trucks, tractors, tractor trailers, buses, bus trailers, and vehicles with special bodies). Furthermore, separation is made according to the purposes for which the vehicles in question are used: as to passenger cars, there are those used by the owner, taxis, livery cars and cars used for hire and drive-yourself; as to vehicles used for freight traffic, trucks are separated into those used for interurban hauling, commercial short-distance hauling and commercial long-distance hauling, and those used as furniture removal vans; tractors are separated as to whether they are used in agriculture or in commercial life; buses are separated into buses, spare buses, hotel-owned buses, factory-owned buses and buses used for training purposes. Within this separation according to type of vehicle and purpose for which it is used, vehicles are further classified according to horse-power or capacity of the engine.

For passenger cars and trucks with permits, in places with less than 5,000 inhabitants, the premium will be reduced by 10% as of January 1, 1965. The same reduction also applies to vehicles owned by farmers, irrespective of the number of inhabitants of the place where the permit to drive has been issued. For vehicles owned by national authorities and their employees, the premium will be reduced by 20% as of January 1, 1965.

Statistics show that the premium required to meet claims for vehicles which have been without claim for one year, is considerably less than that required for vehicles with claim. The premium required becomes even smaller, if no claims have been advised during two years. In that case, the indicated reduction is about 40%. For vehicles which have been free from claim during three years or more, the premium required is about 50% of that required for vehicles with claim. This has caused German automobile insurers to introduce as of January 1, 1965, the following scale of no-claim rebate:

> Drivers who did not make a claim during one calendar year have to pay a rate 10% less than they would have to pay in case a claim had been advised.
>
> Drivers who did not make a claim during two calendar years have to pay a rate 20% less than they would have to pay in case a claim had been advised.
>
> Drivers who did not make a claim during three calendar years or more will have to pay a rate 50% less than they would have to pay in case a claim had been advised.

Furthermore, insureds will receive a corresponding part of the so-called technical profit. Each year, automobile insurers must establish a sort of profit and loss account showing on the one side the premium income and on the other the claims paid and the administration costs. Of the profit resulting from this account, insurers may keep for themselves up to 6% of the premium income, the profit exceeding the above-mentioned 6% to be distributed to the insureds.

When calculating the technical profit, administration cost may not be included for their actual value, but in the utmost by 25% of the premium income.

Automobile third party insurance for passenger cars is offered in five forms (in Deutschmarks, where four DM equal one dollar):

	Personal Damage	Material Damage	Property Damage
Form 1	100,000	10,000	5,000
Form 2	250,000	50,000	10,000
Form 3	500,000	100,000	20,000
Form 4	750,000	150,000	30,000
Form 5	General amount of coverage 1 million without limitation as to the particular kind of damage (personal, material or property).		

The extent of automobile third party liability insurance is laid down in the General Conditions of Automobile Insurance (the so-called AKB) according to which this insurance shall satisfy justified claims raised against the insured or co-insureds on the basis of Public Liability regulations, if by using the vehicle which is insured under the policy, persons are injured or killed, things are damaged or destroyed or get lost or in case property damage is caused which has no direct or indirect connection with a personal or a material damage.

Hull insurance comprises damage to, destruction, or loss of the vehicle and of its parts or accessories either fixed to the car or kept under lock.

Hull partial insurance covers damage caused by fire and explosion; by taking away, in particular by theft; unauthorized use by persons who do not belong to the staff; robbery and fraud; by direct effect on the vehicle of windstorm, hail, lightning or floods.

Hull full insurance, furthermore, covers damage caused by accident, i.e. by an event which has a direct sudden effect from outside with mechanical power (damages caused by handling the brakes, natural wear and tear as a consequence of the use of the vehicle, and machinery breakdown are not considered as damages caused by accident), and by willful and malicious acts by persons who do not belong to the staff.

In both these forms of hull insurance, protection also extends to glass breakage (windscreen).

In hull insurance, indemnities are paid according to the depreciation value. For passenger cars and vans, however, compensation is paid during the first year up to the value as new.

Automobile personal accident insurance refers to accidents the proximate cause of which is driving, using, handling, loading and unloading of or parking the vehicle or trailer. The insurance comprises bodily injury which the insured suffers by an accident (an event which has an effect onto his body from outside). Indemnities are paid under the automobile personal accident insurance in case of death and disability, furthermore for daily benefits and medical expenses. The percentage of disability is determined according to a Table stating a certain percentage of disability for each member of the body which is multilated.

Under an automobile insurance, there may also be taken out a baggage insurance referring to objects which the insured or his passengers or the driver are taking along for personal needs, in or attached to the vehicle.

Personal damages caused by persons who cannot be found, are indemnified under an "Indemnity Fund for Traffic Victims." This is constituted by an Association of all automobile third party liability insurers. Indemnities from this Fund are paid by contributions of the members of this Association and are assessed according to the premium income of each member.

On account of the high losses which may occur in particular in automobile third party liability insurance, reinsurance is of particular importance. As loss experience may be subject to considerable variations, the form of reinsurance which is favored as a rule is that of a quota share reinsurance which often is combined with an excess of loss reinsurance. In certain cases, there is also the form of stop loss treaties.

KARL H. BORCH – *Norway*

Dr. Borch, who presented a paper to the Casualty Actuarial Society in 1962, is currently visiting Professor at the Graduate School of Business Administration, U.C.L.A. His permanent position is that of Professor of Insurance at the Norwegian School of Economics and Business Administration in Bergen. He is a member of ASTIN.

In Norway, a new law about automobile liability was passed in 1961. The other Scandinavian countries passed very similar laws at about the same time. We had hoped that the laws would be absolutely identical, but our Scandinavian legislators were not able to agree on this – for a number of reasons.

After three years, the law seems to operate fairly well, although there are a few points which probably will have to be clarified by court decisions. The new law was radically different from the old one, and presented the companies with some very difficult rating problems, which I don't think they have quite solved yet. Before taking a stand, I should at least like to see the operating results for 1964.

The basic idea of the new law is that most traffic accidents really are accidents, where nobody is at fault. The main social problem is then to make certain that those who are injured or suffer property losses in traffic accidents get compensation, without too much fuss and formalities. (Absolute liability.)

To achieve this, the law requires every motor vehicle to be insured in a company which is allowed to operate in Norway. The person who gets hurt in a traffic accident shall then make his claim directly to the insurance company of the automobile which caused the accident.

One of the most interesting elements in the new law is that it places an absolute limit on the amount which can be awarded to one person as compensation for bodily injury, about $40,000 or about 15 times the annual earnings of a Norwegian factory worker. This is very much in line with the welfare state philosophy. The idea is that automobile drivers should pay a premium sufficient to secure the national average standard of living to those who become disabled by traffic accident. Persons who occupy a privileged position far above the national average will not get adequate compensation under this scheme. They will have to protect their earning power by taking personal accident insurance. This may be creeping or leaping socialism, but the idea is popular in insurance circles. It has increased the market for accident insurance considerably. It has simplified the ratemaking problem in automobile insurance. You get rid of the nightmarish question of what would happen if an insured car hits a successful dentist, father of ten, who may sue for millions. This dentist may still be a risk to your company, but he will be where he belongs – in the personal accident portfolio.

I should add that it is a fairly widespread view in Scandinavia, expressed in many court decisions, that owners of valuable property should protect that property by insurance and not by bringing suit against people who happen to damage the property. Wilful or criminal damage obviously comes in a separate class.

Norway is a small country with relatively few automobiles. For this reason there are relatively few classes in our rate system. In the present system non-commercial vehicles fall in three classes according to annual mileage. For the rest, rates depend on a system of no-claim bonus.

According to this system the base premium is reduced by 10% for each year without claims. There are six steps to this scale, so that after six years without claims, the premium will be 40% of the base premium. There are also steps in the other direction. For each claim, the premium goes up by 20% of the base premium. The maximum is 140% of the base premium. Drivers with a record so bad that it should give a higher premium are rated specially by a committee, and insured by a pool set up by the tariff companies. There is no competition for this kind of business.

Some non-tariff companies operate on a different bonus scale, or at least have tried it from time to time.

HANS AMMETER – *Switzerland*

Dr. Ammeter is First Actuary and Vice President of Rentenanstalt, a life insurance company in Zurich. He is a member of the Swiss Actuarial Society and the German Actuarial Society. One of the founders of ASTIN, he has been Editor and Vice Chairman and is now Chairman.

It is a privilege for me to talk before the distinguished members of the Casualty Actuarial Society on such an outstanding occasion and I would like to thank you for giving me the opportunity to address this panel. You will certainly forgive me when I am mainly reading instead of talking to you, and will you also forgive my accent and any shortcomings. As you probably know Switzerland has four different languages – English is obviously not one of them.

Although Switzerland is very small in size and covers an area of approximately one-seventeenth the size of Texas, it also has its problems in motor insurance. As a matter of fact, there is no other branch in Switzerland which is as controversial as motor insurance.

Mr. Masterson has been so kind to give me an outline of all the problems that might interest you. I will follow this outline as closely as possible to give you an opportunity for drawing comparisons with other countries. My remarks will, however, be restricted to private passenger cars. Since the official money exchange would give a false picture all comparisons between Dollars and Swiss Francs have been put on the basis of purchasing power. Therefore one Sfr. is counted as half a dollar.

EXTENT OF COMPULSORY COVERAGE

In Swiss motor insurance there is a split which is certainly quite unusual to the American expert. The driver usually covers only the third party liability risk. Comprehensive policies are rather seldom. The premium volume for liability amounted in 1962 to about $157 million, while for comprehensive it was only $35 million. On the other hand nobody will make a difference between bodily injury and property damage. Both types are automatically included in a liability or comprehensive policy.

Only the automobile liability insurance is compulsory. No car is put into circulation without an insurance certificate. The law provides for the following minimum coverage:

$ 75,000 for an injured person
$500,000 for a claim
$ 20,000 for property damage

RATEMAKING

The rating system is very simple. There are five different classifications for private passenger cars in accordance with their physical characteristics, i.e., horsepower based upon cylinder bore and number of cylinders according to the formula

$$HP = 0.4n\, D^2$$

HP = horsepower
n = number of cylinders
D = cylinder bore

It seems that Switzerland is too small for classifying according to regions. Switzerland, besides the alpine regions, is densely populated and even in the rural territory you can hardly find a place completely deserted of homes and villages. The traffic conditions are therefore not completely different from those in towns. Also the characteristic of the driver is completely disregarded. A small improvement was made when in 1963 a franchise for young drivers below the age of 25 was introduced.

There have been quite a lot of requests these last years to improve the rate structure. As actuary of a life company I regret that the compilation of motor insurance statistics in Switzerland seems to be rather behind times. I think, however, that progress will be achieved within a few years and that other characteristics of the driver will soon form the basis of calculation.

AUTHORITY OF GOVERNMENT OVER COVERAGE AND RATES

The Swiss supervising authorities have no legal power to impose on all companies the same insurance conditions and rates. Nevertheless, this is exactly what is done. There have been only very few cases tried in court and all companies follow the same pattern. The rates and the conditions are set up by an association, the member companies of which write more than 90% of the total business. The other outsider companies, among them a daughter company of Allstate, apply the same rates. The rates in force have to be approved by the government.

DETERMINATION OF CLAIMS LIABILITY

The liability is usually determined by negotiations. For property damage the guilty party is liable; for bodily injury, however, there is interdependency between cause and effect. For heavy accidents there will usually be a police investigation. The police report will then be entered as evidence. Of course, some cases have to be tried in court. A defendant can appeal several times; the case is tried before the Supreme Federal Court where final judgment is pronounced.

NO CLAIM BONUS SCHEME

The scheme is a merit as well as a demerit plan with 21 categories. The careful driver receives a credit of 40% of the initial premium at the most; the prone driver will be penalized in accordance with the following schedule:

Category	Premium in % of Initial Premium
1	60
2	60
3	60
4	80
5	80
6	80
7	100
8	100
9	100
10	100
11	100
12	100
13	140
14	140
15	140
16	200
17	200
18	200
19	280
20	280
21	280

A new applicant for an insurance cover will start in category 9. For each year of accident-free driving he will advance one step, for each claim he will fall back three steps.

HIT-AND-RUN COVERAGE

There is no such coverage. Since the courts are very severe when a hit-and-run driver gets caught, there are amazingly few property damages which are not reported.

REINSURANCE OF LARGE LOSSES

Reinsurance obviously varies with the importance of the company. Although non-proportional reinsurance is very well known, all companies to my knowledge follow the old pattern of proportional reinsurance. The excess of loss treaty usually provides for a retention by the ceding company between $10,000 and $50,000, the smaller amount being kept by the small companies. The percentage of premiums delivered to the reinsurer is about 4%-5% for large companies and 10%-15% for small companies.

UNINSURED MOTORISTS COVERAGE

There is no uninsured motorists coverage to my knowledge.

TYPICAL PASSENGER CAR RATES

As mentioned before Switzerland has no regional rates. The rates are valid for the largest cities and the rural territory as well. Moreover, there is only one characteristic taken into account – horsepower. No characteristic of the driver, neither sex, age, civil status, nor profession or use is taken into consideration.

The rates are as follows for the minimum coverage as provided by law and the unlimited coverage:

Horsepower	Minimum	Unlimited
until 2.09	$137	$146
2.10-4.09	153	162
4.10-7.09	204	216
7.10-15.09	245	260
15.10 and more	374	396

The bonus credits will reduce these rates of about 17% as an average.

These rates are the result of the latest rate revision to which the public highly objected. The large drivers' associations are still threatening the companies to establish their own insurance company. Controversies were brought up because the rates were raised about 33% as an average which is certainly a big step. The insurance companies stated that they had lost $45 million in the years 1960-1962.

SIGNIFICANT VARIATIONS FROM U.S. AUTOMOBILE INSURANCE

Most of the variations have already been mentioned. The type of coverage is quite different; the rates and conditions are exactly the same for all companies. The competition between companies is only in the field of the service rendered to the public for the same premium dollar. The rating structure is, unfortunately, I would say, oversimplified; statistical investigations are scarce. The liability is different according to the type of damage (property damage or bodily injury).

REPORT

OBSERVATIONS ON CASUALTY INSURANCE RATE-MAKING THEORY IN THE UNITED STATES

by

THOMAS O. CARLSON (*United States*)

Writing in 1941, Mr. Arthur L. Bailey, probably the most profound contributor to casualty actuarial theory the United States has produced, observed as follows with regard to his entry into the actuarial profession:

> "The first year or so I spent proving to myself that all of the fancy actuarial procedures of the casualty business were mathematically unsound. They are unsound—if one is bound to accept the restrictions implied or specifically placed on the development of the classical statistical methods. Later on I realised that the hard-shelled underwriters were recognising certain facts of life neglected by the statistical theorists. Now I am convinced that casualty insurance statisticians are a step ahead of those in most fields. This is because there has been a truly epistemological review of the basic conditions of which their statistics are measurements."[1]

In elaboration of these remarks, Mr. Bailey refers to recognition of heterogeneity of populations as opposed to the homogeneity assumed in classical statistics, the imposition of restrictive conditions on groups of estimates considered in the aggregate rather than upon each individual estimate, with consequent reduction in the variances involved, and the development of "credibility" formulas to produce consistent weightings of statistical experience with prior knowledge in the form, for example, of existing rate schedules. While statistical theory has in more recent years been giving attention to the first two of these departures from the classical approach, the third area seems to have escaped investigation outside of actuarial circles.

It appears appropriate in this jubilee year of the Casualty Actuarial Society to review actuarial developments in this country, with particular attention to theory. Certain of these developments will be found to complement the growth of the science elsewhere for reasons which are peculiar to our insurance system. An attempt is made to avoid duplication of material presented in any previous Congress. Presentation will be as nearly as possible topical with illustrative material, where needed, drawn from the automobile lines which contribute more than $6,000,000,000 of the $15,000,000,000 of annual premiums on non-life insurance in our country.

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Editor's Note: This paper was reprinted from the *Transactions* of the XVIIth International Congress of Actuaries held in Great Britain. Mr. Carlson died shortly after returning from Europe where he presented this, his last, technical paper.

Errata are listed following the Bibliography.

Theory in the United States has usually followed and, happily almost invariably, supported practice, echoing the experience voiced by W. Perks of England at the 1951 Scheveningen Congress:

> "I want to stress that the modern developments of statistical theory are less important for actuarial work than for providing a sounder theoretical basis for the traditional actuarial methods. More and more we are finding that our methods are justified by the more precise modern analysis."[2]

IMPACT OF RATE REGULATION

The development of actuarial science in the United States cannot be fully understood without appreciation of the impact of statutory regulation, necessity having nurtured invention.

Rate-making by companies in concert dates from the latter years of the nineteenth century on a basis of voluntary informal association. Co-operation in rate-making became formalised in a rating organisation only in 1910, with the advent of the first workmen's compensation legislation, later repealed, and it was the puzzle of establishing rates for a new compulsory coverage, rather than the attendant regulation, that provided the real incentive. Four years later, in 1914, the Casualty Actuarial Society was established as a direct outgrowth of the committee discussions on the theory of rate-making as applied to workmen's compensation insurance. One of the three papers presented at the historic inaugural meeting remains to this day the classic introduction to the problem of the credibility to be accorded to statistical experience in the determination of classification rates.

The rapid spread of workmen's compensation legislation and the existence of a rating organisation embracing all private and most public carriers of workmen's compensation insurance focused actuarial attention upon this line and made it the testing-ground of theory and practices that were later extended to other lines.

The 1920's and 1930's witnessed the limited *de facto* extension of rate regulation to other casualty lines, and the concentration of rate-making activities in large measure in the hands of rating organisations, with non-affiliated companies establishing rates for the most part by reference. This is in general still the pattern as respects rate determination.

In 1944 the epochal decision of the United States Supreme Court in the South-Eastern Underwriters Association case reversed 75 years of court rulings, and established jurisdiction over the business in the United States Congress. That body returned regulation to the individual states, however, with the threat of federal intervention to fill any gaps, and specifically exempted the insurance industry from the application of the various federal anti-trust laws, except for acts of boycott, intimidation, or coercion, thus permitting companies to continue to pool their statistics and to make rates in concert.

The aftermath was the passage of rate regulatory laws in all 52 jurisdictions within the United States, none of them alike in all particulars [3], creating a flood of problems, but still almost universally considered preferable to federal control.

In 1951 the scope of the Society's considerations and membership was extended to include fire insurance in recognition of the passage of laws permitting a company to write both fire and casualty coverages.

D. B. Martin of Canada remarked at the Brussels Congress that North American actuaries have of necessity stressed practical rather than theoretical aspects. The science has been hammered out upon the anvil of practical experience, with attention restricted largely to day-by-day exigencies. In virtually all the 52 jurisdictions, rates before they can be used must be approved by state officials, some of whom are elected by popular vote and many of whom have had no insurance background; in some states public hearings are held.

The actuary must in consequence be able to explain all formulas on the level of comprehension of the general public, and thus has need for articulateness and common sense in at least as great measure as for mathematical facility and comprehension.

RATE-MAKING—GENERAL

Ratemaking embraces (1) the determination of "manual" or class rates and (2) the development of rating plans for the modification of the class rates for those individual risks large enough so that the deviation of the risk's experience from the class experience, as summarised in the class rate, is significant.

Establishment of class rates is based fundamentally upon an annual review of averages, with any further analysis of distributions restricted to special studies on the fringes of such reviews.

Statistical reports include by class within territory for each coverage the premiums, the amount of losses, the number of claims and, for lines with a third-party interest, the exposures or number of units of the manual rate base, e.g. number of cars for automobile liability. From these are calculated the averages:

loss ratio	=	(amount of losses)/(premiums)
average claim cost	=	(amount of losses)/(number of claims)
claim frequency	=	(number of claims)/(exposures)
pure premium	=	(amount of losses)/(exposures)
	=	(average claim cost) × (claim frequency)

Premiums at present rates are calculated as the summed products of exposures times current basic manual rates (or, in lines like burglary and glass, for which exposures are not reported, as the collected premiums adjusted to reflect the current rate level) and the loss ratio on present level calculated, using the losses within the limits represented by the basic rates.

Supplementary data, such as average claim costs by state for current trend review, are collected, but since this paper is to deal essentially with theory, and such practical devices were fully presented, together with classification structure, statistical bases and other details in the illustrative automobile line, by Mr. Matthews and Mr. McGuinness at the 1960 Brussels Congress [4], [5], I shall refrain from going beyond these descriptive fundamentals.

RATE-MAKING—OBJECTIVES

The primary objective of ratemaking in the United States has always been the establishment of rates that will be proper for the period during which they will be effective. By "proper" we mean adequate to meet the losses and expenses which may normally be expected and to provide what the statutes prescribe as a "reasonable margin for underwriting profit and contingencies", which is almost universally established currently for casualty lines as 5% of the premium (too often not realised). L. Wilhelmsen of Norway said at the 1960 Brussels Congress: "... adjustment to the level of rates following changes in the level of claims ... are part of the rating system in Canada and in U.S.A. In other countries forecasting of the period during which rates should be valid seems not to take place." [6] American actuaries have never thought otherwise than in terms of rates proper for their effective term. An important secondary objective is the establishment in the rate-making procedures of a "best" compromise between the principles of (1) stability in rate-level and (2) responsiveness to current experience indications while maintaining consistency in the interpretation of the statistics. These two objectives of meeting anticipated costs and yet compromising reasonably and consistently between stability and responsiveness are interwoven throughout the entire development of actuarial science, and actuarial procedures can only be understood in the light of these objectives.

At the outset, before statistical information is available, rates have to be based upon underwriting judgement. When statistical information subsequently becomes available, two alternatives are open: either to consider that body of statistical data as the only true information available, i.e. classical theory, or to consider such data together with the information that is embodied in an already established rate structure. The latter, and theoretically unorthodox, approach has been followed in the United States for more than fifty years and has its own niche in actuarial theory known as "credibility theory", which will be examined in detail. As successive revisions develop, it is obvious that the rate structure partakes more and more of the statistical contributions and may become entirely founded upon them, but each revision takes the existing schedule of rates as a springboard with current statistical data providing the impetus for the leap into the unknown territory of loss-and-expense predictions.

CREDIBILITY THEORY

The simple formula for such a weighting with credibility factors is

$$M = ZA + (1 - Z)B \tag{1}$$
$$= B + Z(A - B) \tag{1a}$$

where A is a statistical observation while B is the corresponding value in the reference base with which A is being compared. B is commonly a broader population average, whether this concept be utilised directly as when B, for example, is a countrywide class pure premium with which a local class pure premium A is being compared, or whether the concept is used by implication as when B is a present rate representing a population average that is broader than A either in respect to space (as in individual risk rating where it is in effect a class rate with which the risk's indicated rate is compared) or in respect to time (as in manual rate-making where it embodies the results of previous years of experience with which the current indication is compared). Z is the credibility, the mathematical measure of the credence attached to the statistical observation A. In one branch of the theory Z is determined from specific probability assumptions as to the deviation of the sample average A from the true average and is called a "limited fluctuation" credibility. In another branch Z is determined from parameters of the variables being weighted and is called a "greatest accuracy" credibility.

The credibility principle thus appears in various guises and disguises. The rationale behind its use is several-fold:

(1) One has to use point-estimates rather than interval-estimates, regardless of the standard deviations of the estimates, and a credibility-weighting procedure connecting with some norm or frame of reference introduces in effect reflection of the comparative validity of the point-estimate and at the same time introduces a marked stabilising influence into the procedure—one of our objectives. This stabilising is accomplished not only by restricting the impact of fluctuations in the statistical data so weighted, but also by introducing on a weighted basis the frame of reference, whether this be the existing schedule of rates (e.g. in determining statewide rate level) or averages drawn from a more comprehensive population (e.g. in determining relationships between territories or between classes).

(2) In establishing relationships between classes within a territory, or between territories within a state, the credibility-weighting of the individual indications with the average of all indications produces a series of indices which can be applied to any determined average rate-level change to produce equitable rate-levels for the individual classes or territories, as the case may be, recognising the comparative validity of the point-estimates involved.

(3) Complete consistency in the interpretation of statistical data is obtained by the credibility procedures; they replace the vagaries of underwriting judgement, sound though such may be. This is of the utmost importance in the processing of rates for thousands of classification and territory divisions annually, which must have the stamp of approval from a regulatory official before they can be used; the avoidance of unfair discrimination in rate determination is a universal statutory requirement

(4) There is a five-pronged interrogatory to which every rate-maker must subject himself: what questions may be raised about the revision by (*a*) actuaries on technical aspects, (*b*) company officials who will use the rates, (*c*) regulatory authorities who must approve the rates, (*d*) agents who must sell the rates to the public, and (*e*) the public who, it is hoped, will buy, in a sharply competitive market. The last three of these groups are most of all interested in the relationship of the new rates to the old, and it would be a practical impossibility to obtain approval from the regulatory authorities if one were to discard the present rate structure as information of no value.

Although the question of the dependability of an observation, as deducible from Gauss's law of error, was explored by Woolhouse in England as far back as 1873 [7], the first application in ratemaking resulted from Mowbray's investigations in 1914 [8]. He assumed that probabilities of accident can be represented by the terms of a binomial expansion, which approximates the normal curve as the exponent (here the exposure) becomes very large, and thereby deduced from tables of the indefinite normal integral, values of the variable corresponding to a given probability P that the variation of the observed average from the most probable value will not exceed k per cent. For this problem, the variable limit in the normal integral

$$\mathrm{P}=\frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt \tag{2}$$

is

$$x=\frac{knq}{\sqrt{2npq}} \tag{3}$$

where n = exposure

q = probability a claim $=1-p$.

The exposure required to satisfy the assumed probability level is then

$$n=2\cdot\frac{x^2}{k^2}\cdot\frac{1-q}{q}. \tag{4}$$

Replacement of = by $\geqslant$ expresses the criterion for 100 % credibility. Perryman in 1932 [9] modified this approach to reflect (in effect although not by specific statement) the assumption generally accepted by then that accidents follow a Poisson rather than a binomial distribution, by taking a single trial as a unit of exposure for $\frac{1}{s}$ years rather than or 1 year, so that n becomes ns and q becomes $\frac{q}{s}$ in the binomial formula, then letting s (and consequently the binomial exponent ns) become very large.

Then
$$x = k\sqrt{nq} \tag{5}$$
and
$$nq \geqslant 2 \cdot \frac{x^2}{k^2} \tag{6}$$
in the foregoing notation. Since nq is the annual number of claims this assumption simplifies the credibility criterion. Since $\frac{1}{\sqrt{nq}} = \frac{\sigma}{\mu} =$ coefficient of variation, the above criterion is equivalent to $\frac{\sigma}{\mu} \leqslant \frac{1}{\sqrt{2}} \cdot \frac{k}{x}$.

It should be noted that the selection of assumptions, i.e. of P and k, is arbitrary. The important point is that, once P and k are selected, consistency in the interpretation of statistical data between classes and between territories is ensured.

The criterion so determined is assumed to provide 100% credibility. For partial credibilities, the approach has varied. The procedure commonly adopted is that the relative weights of two experiences, one with exposure entitled to 100% credibility and the other with exposure $\frac{n}{r}$, would be in the ratio of the reciprocals of their standard deviations or as $\frac{\sqrt{n}}{\sigma}$ to $\frac{\sqrt{n/r}}{\sigma}$, that is, as 1 to $\frac{1}{\sqrt{r}}$. Thus credibility $Z(\leqslant 1)$ would be assigned to Z^2n claims; i.e. $Z = \frac{1}{\sqrt{r}}$.

Further considerations on partial credibilities are set forth in Longley-Cook's excellent recent booklet for students [10] where a complete bibliography will be found together with an extended discussion of other aspects of the theory.

Bailey, in 1943, in discussing this "limited fluctuation" credibility, cast light upon the problem of partial credibility without pursuing it to a conclusion [11]. He constructed a table of the normal sampling range of the ratio of actual to expected number of claims (or of frequencies) corresponding to given values of the probability that a smaller value will occur, assuming that the number of claims C follows a Poisson distribution approximated for large numbers of claim by a Pearson Type III distribution with the same skewness. If V_1 and V_2 are values in such a table, entered with E(C), corresponding to $\left(1-\frac{1-P}{2}\right)$ and $\left(\frac{1-P}{2}\right)$, P being interpreted as in the foregoing,

$$2k = Z(V_1 - V_2)$$

and

$$Z = \frac{2k}{V_1 - V_2}.$$

Values of Z (< 1) produced by this evaluation approximate (to the second decimal place for very small values of C and more accurately for large) those produced by the square root rule for the same values of P and k. The explanation lies in the fact that the credibility determined from the sampling range tables varies, for any given P and k, approximately with the reciprocal of the standard deviation of C, i.e. with $\sqrt{C}$; for skewness = 0, the correspondence is of course exact. The same factor $\sqrt{C}$ is the numerator of the expression for the determination of Z by the square root rule, since the actual number of claims is assumed to be equal to the expected in the establishment of distribution tables; the denominator is $\sqrt{\text{no. of claims for } 100\% \text{ credibility}}$, which equals $\frac{x}{k}$ from (5).

Thus the square-root rule is justified as the closest simple approximation, and a very close approximation to the theoretically correct values.

It should be noted that the foregoing formulas, strictly interpreted, refer not to the actual number but to the expected number of claims, although in practice the actual number is customarily used. The reason is not only that the actual number is an unbiassed estimate of the expected number, but also a matter of expediency, since the actual number is immediately available whereas the expected number involves not only an assumption as to the expected frequency but also a subsequent calculation. There is a very fundamental difference in the results of application: the actual number increases credibility for a class with relatively high loss frequency (adverse experience) while giving a low credibility for a class with relatively low loss frequency (favourable experience), while the expected number will give more weight compared with the class with low frequency. Further refinement from theoretical considerations produces formulas impracticably complicated, since these credibilities are used in thousands of calculations annually.

Expected losses are sometimes used, but theoretically this will relax the assumptions for a given credibility level because of the greater variances of distributions of loss amounts as compared with those of numbers of claims. When it is considered, however, that the criteria are determined empirically, and that the important results are consistency in the interpretation and use of statistics and the restriction of the influence of chance fluctuations, the broadening or narrowing of the underlying assumptions are minor considerations. R. A. Bailey in Longley-Cook's booklet [10] sets forth in Appendix C a comparison of the number of claims to maintain, for claim cost, pure premiums and trends, a given level of credibility determined from claim frequencies.

A. L. Bailey has developed the theory of the "greatest accuracy" credibility from regression theory [12] by obtaining the best unbiassed linear estimate of the population mean of a certain characteristic in terms of the observations of this characteristic and of all characteristics. In reviewing the following, consider, for example, the ith characteristic as an individual risk's loss hazard and the combination of all characteristics as the loss hazard of the class comprising all such risks.

Let X_i = population mean (i.e. true value of ith characteristic)

$x_i = X_i/\overline{X}$ where the bar signifies mean value, as usual

m = units of exposure in each observation

$a = \overline{X}/m$, so that $amx_i = X_i$

w_{ij} = jth observation of ith characteristic

y_{ij} = deviation of jth observation of ith characteristic from X_i

so that $w_{ij} = amx_i + y_{ij}$ $\qquad (j = 1, 2, \ldots k)$.

Then, since $E(y_i) = 0$, $E(x_i y_i) = 0$, and $\bar{w} = am$,

$$\sigma_w^2 = a^2 m^2 \sigma_x^2 + \frac{\sigma_y^2}{k}$$

and $$E(x_i - \bar{x})(w_i - \bar{w}) = am\sigma_x^2.$$

The linear regression equation of X_i on w_i gives as the best unbiassed linear estimate of X_i,

$$E(X_i|w_i) = X_i' = \frac{a^2m^2\sigma_x^2}{\sigma_w^2} \cdot w_i + \left(1 - \frac{a^2m^2\sigma_x^2}{\sigma_w^2}\right) . \bar{w}. \tag{7}$$

In other words,

$$E(X_i|w_i) = Zw_i + (1 - Z)\bar{w}$$

which is a weighted average of w_i, the average of all observations of the characteristic, with $\bar{w}$, the average of the observations of all characteristics, where the weight-factor, or credibility, attached to w_i is

$$Z = \frac{a^2m^2\sigma_x^2}{\sigma_w^2} = \frac{k}{k + \dfrac{\sigma_y^2}{a^2m^2\sigma_x^2}} . \tag{8}$$

It is easily shown that the variance of such an estimate is Z times that of w_i. So that we have illustrated here two of the areas of innovation in theory mentioned at the outset of this paper, credibility-weighting and the reduction of variance through the use of unbiassed estimates of averages for characteristics in the aggregate as compared with each of the characteristics individually.

The expression (8) for Z is easily translated to the familiar $\dfrac{P}{P+K}$ formula (where P refers to premium or expected losses) for the credibility used in the rating of individual risks obtained by A.W. Whitney in 1918

[13] and still in use. In such ratings, M in formula (1) is the modification of the class rate resulting from applying the formula to the risk, B represents the class rate, and A the rate indicated by the risk's own experience; if A is the risk's loss ratio and B the expected loss ratio, M becomes a percentage modification,

$$M = 1 - Z + Z \cdot \frac{A}{B} \tag{9}$$

from which it is seen that the credibility is at the same time equal to the credit if there are no losses. This characteristic is utilised in an elegant derivation of $Z = \frac{P}{P+K}$ by R. A. Bailey (A. L. Bailey's son) by developing the expected claim frequency for risks accident-free for n or more years as compared with all risks, assuming that the inherent hazard remains constant for each risk, following a Poisson distribution, with the risk parameter following a Pearson Type III distribution, producing the familiar negative binomial distribution of total claims [14]. The relative frequency so obtained is in the form $\frac{a}{a+n}$ whence the indicated discount from manual rates is $1 - \frac{a}{a+n}$ or $\frac{n}{a+n}$, and multiplication of numerator and denominator by the annual premium produces the $\frac{P}{P+K}$ form. This result was independently obtained by F. Bichsel in 1959 [15].

The relationship to the Gauss theory of error and weighting of observations should be noted. Weights proportional to the reciprocals of standard deviations of w_i and $\bar{w}$, i.e. in the ratio $\frac{1}{\sigma_{w_i}^2} : \frac{1}{\sigma_{\bar{w}}^2}$ would be of the form $Z = \frac{P}{P+K}$ and $1 - Z = \frac{K}{P+K}$.

In another paper [16] Bailey derives the same form (1) from Laplace's generalisation of Bayes' Rule in determining the expected value of a statistic which corresponds to the origin or cause of an observed event H, and shows that, if P(H,x) represents the *a priori* probability connecting H and x, and K(x) the *a priori* probability of the existence of x, then the regression of x on H, or E(x|H), is linear

(a) when P(H,x) follows the Binomial distribution, only when K(x) follows the Beta distribution;

(b) when P(H,x) follows the Poisson distribution, only when K(x) follows the Pearson Type III distribution (producing a negative binomial form).

In summary, the credibility-weighting process, with theoretical justification even in the streamlined way in which it is used, is admirably adapted to provide the necessary balance between stability and responsiveness in the rate structure, and at the same time to provide the necessary link between statistical experience and prior information (whether in the form of the existing rate structure or of broader statistical averages) while ensuring utter consistency in the treatment of the various bodies of statistics involved in the determination of rates.

CREDIBILITY VARIABLES—EXPERIENCE RATING

It will be noted that formulas (4) and (6) show that the "limited fluctuation" credibility depends upon both volume of statistics and frequency of loss, but that the "greatest accuracy" formula in its customary appearance reflects only the volume directly, any reflection of loss frequency being restricted to variation of the K. Ideally we can postulate that any credibility factor should be a function $F(v, q)$ of the volume and loss frequency (volume being understood here in a general sense) such that

I. $0 \leqslant z \leqslant 1$

II. For $z < 1$,

$$\text{(a)}\ \frac{dz}{dv} > 0,\ \frac{dz}{dq} > 0; \qquad \text{(b)}\ \frac{d^2z}{dv^2} < 0,\ \frac{d^2z}{dq^2} < 0.$$

The square root formula for partial credibilities related to the 100% criterion determined by (6), or

$$Z = \sqrt{\frac{nq}{(nq)_{100\%}}} = \sqrt{\frac{1}{r}} \tag{10}$$

satisfies both postulates, as does the formula

$$Z = \frac{P}{P+K} = \frac{f(n).q}{f(n).q+K}. \tag{11}$$

The failure of (11) to reflect loss frequency variations by size of loss (i.e. for different loss severities) has resulted in the development of experience rating plans (wherein the credibility (11) is most often encountered) with a split of the losses into a primary portion which includes the first T_P dollars of each loss and an excess portion T_E which includes the balance of the losses, with credibility on T_P higher than on T_E for $T_P = T_E$. Because of the greater frequencies of loss on property damage liability as compared with bodily injury, credibilities for experience rating of property damage are correspondingly increased by variation of the K value.

Perryman in 1937 refined the theory of credibilities for experience rating and developed the multi-split principle to introduce a diminishing credibility for successive increments of a single loss by including as primary losses the first t dollars of each loss plus $r\%$ of the next t dollars plus $r^2\%$ of the next t dollars and so on in geometric progression, the balance being the excess losses [17]. The maximum primary loss is thus $t(1-r)$ dollars. The proportions of losses thus designated as primary and excess vary by class so that expected losses must be split by class correspondingly. The plan in this form has in actual application been restricted to workmen's compensation risks.

R. A. Bailey has recently re-examined experience rating theory [18], viewing the credibility as a multiple correlation coefficient between the frequencies of losses of different sizes and the total expected losses as modified by the rating plan, which might be called a multi-multi-split approach; with an assumption of comparative ignorance as to the correct tariff rate (which is realistic for a newly established coverage as in the present multiple-peril developments in the United States) he has the paradoxical result of a one-split plan with primary losses self-rated and with zero credibility on the excess losses, conclusions which he notes support the findings of Professor Karl Borch of Norway on excess of loss reinsurance presented in 1960 at Brussels [19].

A significant contribution to credibility theory reflecting the greater variances of distribution of loss amounts by size as the limit of loss increases is a study made by L. H. Roberts of the effect on credibility of using in manual ratemaking the now common automobile liability limits of 10/20 ($10,000 per claim subject to a maximum of $20,000 per accident) as compared with the old 5/10 basic limits [20]. He calculates "that 10/20 experience would require at least 40% more claims for full credibility to retain the same statistical reliability as 5/10 experience". Space does not permit a summary here of the details of the calculation.

RETROSPECTIVE RATING

The rating formula (9) applied to individual risks illustrates "prospective" experience rating, that is, a rating modification developed from past experience on the risk to apply for the coming year. In the 1930's a type of plan termed "retrospective" rating was developed under which the risk's experience is reviewed after expiration and a modification developed for retro-active application. Under such plans the risk's losses are as a rule self-rated within minimum and maximum limits in accordance with the formula

$$\mathrm{M}=\begin{cases}\mathrm{H} & (t\leqslant h)\\ \mathrm{B}+\mathrm{C}t & (h<t<g)\\ \mathrm{G} & (t\geqslant g)\end{cases} \tag{12}$$

where, if M, H, B, and G are ratios to class rates,

M = modification

H = minimum premium

G = maximum premium

t = risk loss ratio

h = minimum loss ratio reflected in rating

g = maximum loss ratio reflected in rating

C = loss conversion factor to include rate variables dependent on the losses

B = basic premium $= e+t'(g)-t''(h)$

where e = provision for other expenses (selling, administration, servicing)

$t'(g)$ = average class ratio of losses in excess of g to total losses

$t''(h)$ = average class ratio of losses less than h to total losses.

Now t' and t'' depend upon the distribution of loss ratios by risk about the average loss ratio (after adjustments to reflect equality between actual and expected losses).

Let E( ) = expected value, as usual

u = risk premium

R_s = excess pure premium ratio over a loss ratio of s

$$=\frac{\Sigma(\text{losses} > su)}{\Sigma E(tu)}$$

t = total loss ratio on each risk.

$$\text{Then } R_s=\frac{\Sigma_{tu=su}^{\infty}(tu-su)}{\Sigma E(tu)} \tag{13}$$

or, for a given size of risk u, writing $t_1=\frac{t}{E(t)}$ and $s_1=\frac{s}{E(t)}$, and further considering t as a continuous variable with distribution $F(t)$,

$$R_s=\frac{\int_{s_1}^{\infty}tF(t)dt-s_1\int_{s_1}^{\infty}F(t)dt}{\int_0^{\infty}F(t)dt=1}$$

Since $$\int_{s_1}^{\infty}tF(t)dt=\int_0^{\infty}tF(t)dt-\int_0^{s_1}tF(t)dt=\mu_t-\int_0^{s_1}tF(t)dt$$

where μ has the usual signification of the mean value,

and further $$\int_0^{s_1}tF(t)dt=s_1\int_0^{s_1}F(t)dt-\int_0^{s_1}\int_0^{s_1}F(t)dt\,dt$$

it follows that $$R_s=\mu_t-s_1+\int_0^{s_1}\int_0^{s_1}F(t)dt\,dt \tag{14}$$

as derived by A. L. Bailey [11].

Formula (12) may be modified so as to sectionalise the range of self-rated losses, or even so as to modify them by a credibility-weighting process, but these variations have not been utilised in common practice.

Excess pure premium ratios vary by size of risk, and the loss ratio variances are so great on small risks that the size of $t'(g)$ makes application impracticable. As u increases, however, $t'(g)$ approaches 0, and the plan is widely used on the larger risks. Carlson [21] has developed the theory in so far as it is concerned with the interrelationships of the variables involved, and has explored various types of plan, and Dorweiler [22] has developed procedures for producing graduated tables of excess pure premium ratios.

DISTRIBUTION THEORY

Little will be said here about distribution theory, because the subject has been so fully developed in the International Congress Transactions and the ASTIN Bulletins. Rate-making in the United States has had a much larger statistical volume at its command than elsewhere, and this has been a factor in what appears to be a more pragmatic approach to rate-making with less dependence upon mathematical models—notwithstanding the fact that the major factor has been, as already indicated, the impact of statutory regulation.

Distribution theory in casualty insurance statistics commonly stems *from the mathematical model assumed for the distribution of the number* of claims. For years this was assumed to follow the Poisson form and it is only recently that the negative binomial has come generally to supersede the Poisson for this purpose. Bailey's paper on sampling theory [11] uses the Poisson distribution to reflect chance fluctuations in claim frequency distributions, but makes allowance for other distributions in the development of expressions for the moments of other statistics, with analytic ramifications that covered almost the entire field of casualty actuarial science twenty years ago.

It is interesting to note that the negative binomial distribution was presented, including its generalised form, in the Proceedings of the Casualty Actuarial Society as early as 1942 as a model reflecting variation of the Poisson parameter to recognise differences from individual to individual in the inherent risk hazard [23]. Bailey derived it again in 1950 in his study of credibility theory as developed from Bayesian considerations, as already noted in this paper. But it did not come into general use in the United States until its application in 1959 by Harwayne [24] and Dropkin [25] in automobile driver record studies, since when there have been a number of papers relating to theory and applications [26]-[29]. Varied interpretations of this model, which is of surprisingly wide versatility, have been reviewed in PCAS [30] and [31], but the recent ASTIN article by Campagne [32] seems to be the most complete in this respect.

SCHEDULE RATING—LINEAR PROGRAMMING APPLICATION

Schedule rating is used as a classification refinement reflecting physical characteristics of individual risks. It was once applied universally in workmen's compensation insurance, but is now retained in only one state since experience rating has almost entirely superseded it. In fire insurance it still constitutes the core of rate differentiation.

The extreme refinement of classification effected by schedule rating and the interaction of the multitude of factors involved has to date placed such factors virtually beyond analysis. The theory of an approach to this very difficult problem has been developed in a recent paper by McIntosh [33] utilising linear programming techniques which with the new electronic speeds of data processing for the first time open the door to the possibility of undertaking the solution of such complex multivariate problems of factor-interdependence.

RISK THEORY

Collective risk theory, which has played such a large role in the literature of the science in Europe and the origination of which dates back close to the inception of the International Congress, has received little attention in America but is currently being examined by a rapidly increasing number of actuaries, and the Casualty Actuarial Society has organised a committee on the mathematical theory of risk. Again, the explanation for previous scanty consideration lies principally in the history of insurance developments in our country, with statutory measures forcing primary attention to the propriety of rates for individual risks. The two approaches need to be blended, for both involve important concepts without which the science is incomplete.

CONCLUSION

In reviewing the theoretical contributions of the Casualty Actuarial Society only those aspects which it is believed may not be familiar ground to actuaries in other countries have been emphasised. It has been possible to mention only a few of the papers that still are significant, and it should be emphasised that the greater number have dealt with practical problems and their solution rather than with theory. We welcome these international exchanges, and assure you of our increasing participation both in the International Congresses and in ASTIN.

I should like to acknowledge the suggestions of a number of colleagues in the Society, but shall name only four: the present immediate-past presidents, L. H. Longley-Cook and William Leslie, Jr., who proposed the general subject; and R. A. Bailey and L. H. Roberts, discussions with both of whom have been more helpful than they can realise.

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ERRATA

Page	*Location*	*Correction*
284	Bottom of page	Delete a*
287	Third paragraph	For "*k* per cent" read "100*k* per cent"
288	Line 3	For "or 1 year" read "for 1 year"
289	Line 9	For $\sqrt{C}$ read $\frac{1}{\sqrt{C}}$
291	Lines 20 and 21	For "standard deviations" read "variances"
292		For $\frac{dz}{dv}$, $\frac{d^2z}{dz^2}$ and $\frac{d^2z}{dq^2}$ read $\frac{\partial z}{\partial v}$, $\frac{\partial^2 z}{\partial z^2}$ and $\frac{\partial^2 z}{\partial q^2}$
294	Lines 2 and 3	For "average class ratio" read "average ratio"
294	Equation (13)	The lower limit of summation should be zero and the condition $tu \geqq su$ should be stated

Greetings from Other Actuarial Organizations

GOLDEN JUBILEE BANQUET

November 19, 1964

The Casualty Actuarial Society was honored on the occasion of its Fiftieth Anniversary celebration by the presence of distinguished actuaries who brought greetings from other actuarial organizations. These greetings were given extemporaneously at our Golden Jubilee Banquet and in recording them in these PROCEEDINGS *it is hoped that their graciousness and flavor have been preserved.*

HANS AMMETER
Chairman, ASTIN Section,
International Congress of Actuaries

I thank you, Sir, for your friendly words of welcome and your kind remarks about ASTIN and its Committee.

I should also like to thank all the gentlemen – I beg your pardon – and ladies, who have taken such pains in the preparations for this golden jubilee and I warmly congratulate them on their brilliant success.

On behalf of ASTIN as well as on behalf of its Committee and the members, I offer our hearty felicitations to the Casualty Actuarial Society on its 50th anniversary. In the last 50 years the Casualty Actuarial Society has made important contributions to the Non-life actuarial science. The Casualty Actuarial Society can proudly look back to 50 years of momentous contributions in this sphere. May it continue its successful activities in the years to come.

When we examine rather closely the relationship between ASTIN and the Casualty Actuarial Society and think of the possibility that in other countries also similar organizations could be founded then we can make a figurative, allegorical comparison, that ASTIN is to a certain extent the mother of such organizations and the national institutions are the daughters. Unfortunately, the mother has only one, sole daughter at present. But what a daughter! Besides, we must not give up the hope that in the course of time further daughters would be born. However, in the present mother-daughter relationship before us, there appears to be an anomaly. Namely, the mother is considerably younger than the daughter. But what does it matter to a loving mother when she thinks of such a well-placed daughter? And strictly speaking, is it not the secret desire of all mothers to be younger than their daughters?

In token of thanks of the mother to the well-placed daughter and as a souvenir for the hour of the birthday today, ASTIN presents this pewter jug to the Casualty Actuarial Society. It bears the inscription

Presented to the
Casualty Actuarial Society
by ASTIN
1964

This pewter jug is associated with many historical memories – memories of the development in the Middle Ages of European towns and of the guilds which were then formed. To some extent, these guilds exist even today – not of course in the original conception of their aims, but as pure social fraternities. In Zurich, there exist even at present thirteen guilds, of shoemakers', tailors' etc. The guild life has been so popular that even in later times, when the political importance of guilds had already disappeared, new guilds came to be founded. Each year in Zurich the guilds celebrate the reawakening of the nature in spring at the "Sechseläuten"; I may roughly translate it as the "Six O'clock Festival". At this festival, the "boggey", representing to a certain extent the winter, is publicly burnt at exactly six o'clock in the evening. This gigantic fire produces an immense thirst in those taking part. At the guild revelries that follow, this immense thirst is quenched in an equally immense measure, perhaps, even in a more immense measure than needed for just the thirst itself.

At these guild revelries, the wine would be poured out of guild jugs just of this type. Naturally, there are diverse forms and dimensions of such guild jugs. The jug before us is the so-called "Wallis" jug because of its octagonal base. In selection, we had a heavy discussion on the subject of the dimensions of thirst of our American friends, especially of the present and future Presidents of the Casualty Actuarial Society. As a result, we came to the conclusion that the grinding work of a President of the Casualty Actuarial Society is of such an extent that it must lead to a strong thirst. We therefore decided on this rather good-sized jug in order to get even with the drinking capacity of the illustrious board.

In presenting you this jug on behalf of ASTIN, I wish your society once again progress and prosperity and raise my glass to the health of you all and in particular of the well-being of the birthday child, to the Casualty Actuarial Society.

ROBERT E. BEARD

Senior Vice President, Institute of Actuaries

Thank you very much indeed for the manner in which you have introduced me this evening. As your remarks have indicated I am here in a dual or perhaps multiple capacity but I would first like to say how happy I am to have the privilege of joining in your 50th birthday party. Ever since the formation of ASTIN during the New York Congress in 1957, a birth which was greatly helped by the members of the C.A.S., the links with ASTIN have been growing stronger and we now have a much greater mutual understanding of our common problems.

Our Chairman, Mr. Ammeter – as we are bilingual he is referred to in French as our President – has eloquently expressed greetings on behalf of ASTIN and I will therefore now speak in my other capacity as Vice President of the Institute of Actuaries. The President, Council and members have charged me with the very pleasant task of expressing our warmest congratulations and greetings on this memorable occasion and wish you continued success in the future.

KARL H. BORCH

Norwegian Society of Actuaries

Finn Alexander, the President of the Norwegian Society of Actuaries has asked me to convey the congratulations from Norwegian actuaries to your society on this occasion.

He has asked me to try to find adequate words for expressing how much he and our members appreciate the professional contacts and personal friendships which we have established with members of the Casualty Actuarial Society during the past half-century. I feel that I am not quite up to this task, and I doubt that adequate words really exist.

In accordance with good traditions in actuarial mathematics I shall refrain from trying to give an existence proof. Instead I shall add my most sincere personal thanks for having been given the opportunity to be here tonight, and also for having been invited to some of the earlier meetings of your society.

Casualty Actuaries seem to have an exceptional ability for combining business and pleasure. The social events and the professional discussions at the meetings of the Casualty Actuarial Society will always be among my most treasured memories from my visits to this very hospitable country.

B. CHRISTOFFERSEN

Danish Association of Actuaries

It is a great honour for me to be present here in New York on the occasion of the 50th Anniversary of the Casualty Actuarial Society, and on behalf of Mrs. Christoffersen and myself I thank the Society for inviting us to attend. I have also been authorised to convey to the assembly the greetings and congratulations of the Association of Danish Actuaries and the Danish Members of ASTIN.

GILBERT W. FITZHUGH

President-elect, Society of Actuaries

It is a real privilege to bring to the Casualty Actuarial Society warm congratulations from its sister organization, the Society of Actuaries. I am particularly pleased to be able to do this as a Fellow of the Casualty Actuarial Society in my own right.

I have been fairly active in the work of the joint committees of the various actuarial societies endeavoring to get the federal charter for the new Academy of Actuaries, and, if this becomes a reality, our organizations will be even more closely related.

This has been an eventful evening for me, and I wouldn't have missed it for anything.

EDWARD FRANCKX

President, Permanent Committee,
International Congress of Actuaries

We are very happy to bring you, in the name of the Office of the Permanent Committee of International Congress, the congratulations that the Casualty Actuarial Society deserves on the occassion of its fiftieth birthday.

It was therefore a half century ago that actuaries in the United States became aware of the merit of applying their professional and scientific talents to other fields besides life insurance, and understood the need for pooling their efforts and exchanging their viewpoints.

A similar concern, hardly ten years ago, brought the actuaries of several countries to set the cornerstone of the ASTIN section in which we are particularly happy to have a large number of American colleagues.

The founding of your association met an obvious need and its fifty years of operations have enabled it to achieve noteworthy progress as much in the field of non-life risks as in the practical applications which derive from it.

For all this the sponsors of the Casualty Actuarial Society are entitled to our gratitude.

Congratulating you once more, Mr. President, we are sending you our sincere wishes for your fruitful work and success and bid you to believe in the assurance of our very distinguished feelings.

MINUTES OF THE GOLDEN ANNIVERSARY MEETING

November 18, 19 and 20, 1964

HOTEL PLAZA, NEW YORK, NEW YORK

The attendance at the Fiftieth Anniversary Meeting of the Casualty Actuarial Society was greater by far than that of any other meeting since the founding of the Society in 1914.

The attendance included four of the ten living Charter Members:

S. Bruce Black	Winfield W. Greene
William Breiby	John S. Thompson

Also present were all of the living Past Presidents with the exception of Gustav F. Michelbacher and Sydney D. Pinney who had telegraphed their felicitations and expressed their disappointment at not being able to attend:

Harmon T. Barber	1949-50	Charles J. Haugh	1945-46
Ralph H. Blanchard	1941-42	William Leslie, Jr.	1959-60
James M. Cahill	1947-48	Laurence H. Longley-Cook	1961-62
Paul Dorweiler	1932-33	Norton E. Masterson	1955-56
Harold J. Ginsburgh	1943-44	Dudley M. Pruitt	1957-58
*Winfield W. Greene	1934-35	Seymour E. Smith	1953-54

*Also a Charter Member.

The following Special Guests were also present:

James B. Donovan – Watters & Donovan
Gilbert W. Fitzhugh – President-Elect, Society of Actuaries
John H. Miller – Past President, Society of Actuaries

It is worthy of note that Messrs. Fitzhugh and Miller are also Fellows of the Casualty Actuarial Society.

In addition there were present the following distinguished foreign actuaries, members of ASTIN:

Hans Ammeter – ASTIN and the Swiss Actuarial Society
Robert E. Beard – Institute of Actuaries, England
Karl H. Borch – Norwegian Society of Actuaries
B. Christoffersen – Danish Association of Actuaries
Edward Franckx – Royal Association of Belgian Actuaries

The detailed attendance by members and guests will be found later in these minutes.

The Golden Anniversary Meeting convened at 2:15 p.m. on Wednesday, November 18, 1964 with President Thomas E. Murrin presiding. The first order of business was an address by Past President Laurence H. Longley-Cook "Early Actuarial Studies In The Field Of Property And Liability Insurance." This was followed by a companion address by Past President Dudley M. Pruitt "The First Fifty Years."

Vice President William J. Hazam then took charge of the program for the remainder of the afternoon. The next order of business was a panel discussion relating to the problems outlined in the report "The Actuarial Problem" which had been submitted, by top executives of 12 leading capital stock agency system companies, to the Executive Committee of the National Association of Insurance Commissioners at its June 1964 session in Minneapolis, Minnesota.

Moderator: Seymour E. Smith

Panelists: John W. Carleton
Harold E. Curry
Frank Harwayne
Richard L. Johe

This concluded the program for the Wednesday, November 18 session. However, in the evening there was an informal, off-the-record get-together dinner attended by Charter Members, Past Presidents and Vice Presidents, Foreign Guests, present Officers, and Accompanying Wives.

The meeting reconvened at 10:00 a.m. on Thursday, November 19, 1964 with Vice President Harold E. Curry presiding. The session opened with a panel discussion "Rate Regulation And The Casualty Actuary – 20 Years After The S.E.U.A. Decision":

Moderator: Leslie P. Hemry

Panelists: James M. Cahill
Laurence H. Longley-Cook
Allen L. Mayerson
Hubert W. Yount

The session then recessed for luncheon. At the luncheon the gathering was addressed by the Honorable Henry R. Stern, Jr., Superintendent of Insurance of the State of New York.

The meeting reconvened with a panel discussion by distinguished foreign actuaries on "Motor Insurance In Foreign Countries."

Moderator: Norton E. Masterson

Panelists: Hans Ammeter – Switzerland
Robert E. Beard – England
Karl H. Borch – Norway
B. Christoffersen – Denmark
Edward Franckx – Belgium

There then followed a panel session "Actuaries and The Insurance Industry — Past, Present and Future."

Moderator: William Leslie, Jr.

Panelists: Harmon T. Barber
S. Bruce Black
Ralph H. Blanchard
William Breiby
Harold J. Ginsburgh
Winfield W. Greene

The session recessed at 5:10 p.m.

In the evening there was a reception and social hour followed by the Golden Anniversary Banquet at which Past President Charles J. Haugh acted as Master of Ceremonies. Mr. Haugh introduced those seated at the head tables: Charter Members, Past Presidents, Distinguished Guests, Present Officers of the Society and Accompanying Wives.

Brief responses were made by the distinguished guests who expressed greetings and congratulations, on behalf of their respective actuarial organizations, to the Casualty Actuarial Society on the occasion of its Golden Anniversary:

Hans Ammeter – ASTIN and Switzerland
Robert E. Beard – England
Karl H. Borch – Norway
B. Christoffersen – Denmark
Gilbert W. Fitzhugh – Society of Actuaries
Edward Franckx – Belgium

At this time Mr. Ammeter presented to the Casualty Actuarial Society a beautiful pewter jug on behalf of ASTIN. President Murrin responded on behalf of the Casualty Actuarial Society and expressed his appreciation that the distinguished guests were able to attend our Fiftieth Anniversary Meeting and to so ably represent their respective organizations.

Matthew Rodermund then presented a delightful and entertaining *paper,* not listed on the formal program, "How To Tell A Pure Actuary From A Lay Actuary." In introducing his remarks Mr. Rodermund explained that they stemmed from a conversation with the late Thomas O. Carlson, a Past President of the CAS, and therefore, he deemed it appropriate to dedicate his skit to the memory of Mr. Carlson.

The meeting was reconvened at 9:45 a.m. on Friday, November 20, 1964, for a business session, with President Thomas E. Murrin in the chair:

Amendment To Article IV Of By-Laws – Dues

Under date of October 7, 1964 the membership had been advised of a proposed amendment to Article IV of the By-Laws which would be presented for action at the Golden Anniversary Meeting.

It was regularly moved and seconded that the proposed amendment be adopted. Following a voice vote for a "yes" or "no" President Murrin announced that the amendment had been adopted unanimously so that the present second sentence of Article IV will read:

> "Effective November 20, 1964, the payment of dues will be waived in the case of any Fellow or Associate who attains the age of 70 years or who attains the age of 65 years and notifies the Secretary-Treasurer in writing that he has retired from active work."

The substantive effect of the amendment is to change "November 19, 1954" to "November 20, 1964" and to eliminate the phrase "having been a member for at least 20 years."

Report Of Secretary-Treasurer

The Secretary-Treasurer reported on specific actions of the Council subsequent to the 1963 Annual Meeting. The report also noted that during the fiscal period ending September 30, 1964 receipts had exceeded disbursements by $3,105.84. Copies of the detailed Financial Report, which had been examined by the Auditing Committee (Howard G. Crane, Chairman) and certified to be correct, were available to the membership at the rostrum. The Report of the Secretary-Treasurer will be printed in the next volume of the *Proceedings.*

Obituaries

The gathering stood for a moment of silence in memory of the following deceased members:

Fellows	*Date of Death*
Thomas O. Carlson	July 15,1964
Solomon S. Huebner	July 17, 1964
Arthur Hunter	January 27, 1964
Ray D. Murphy	February 24, 1964
Edward C. Stone	June 6, 1964
Associate	*Date of Death*
Robert E. Ankers	March 1, 1964

Academy Of Actuaries

Past President Longley-Cook then acquainted the membership with the current status of this project. The application for a Federal Charter had passed the United States Senate but had failed to be reported out by the Judiciary Committee of the House. Therefore, it would be necessary to start de novo at the coming session of the new Congress.

Presidential Address

President Thomas E. Murrin then delivered his Presidential Address which will appear in the next volume of the *Proceedings.*

Election Of Officers And Three Members Of Council

Seymour E. Smith, Chairman of the Nominating Committee, then presented on behalf of the Committee the following nominations:

President .... Thomas E. Murrin*
Vice President .... Harold E. Curry*
Vice President .... William J. Hazam*
Secretary-Treasurer .... Albert Z. Skelding*

*Incumbent

A motion was made from the floor nominating the foregoing slate and was regularly seconded. A call for other nominations receiving no response, the nominations were closed. The Secretary-Treasurer was requested to cast one ballot for the nominees who were then declared as being duly re-elected to their respective offices for the coming year.

The Nominating Committee then presented its recommendation with respect to three new members of the Council:

Frederic J. Hunt, Jr.	Daniel J. McNamara	Henry W. Menzel

There being no other nominations from the floor, and, following the constitutional procedure, Messrs. Hunt, McNamara and Menzel were declared duly elected.

The gathering then proceeded to re-elect for the coming year, in accordance with the procedure set forth in the second sentence of Article V of the Constitution:

Editor .... Harold W. Schloss
Librarian .... Richard Lino
General Chairman – Examination Committee .... Norman J. Bennett

Admission Of New Fellows And Associates

President Murrin presented diplomas to the following new Fellows:

William C. Aldrich	James E. Gillespie
Lee M. Alexander	Jeffrey T. Lange
Robert A. Craig	George E. McLean

It was also announced that the following, having completed the requirements for admission, would be enrolled as Associates:

William W. Brown, Jr.	Bertram F. Mokros
Edwin A. Carlson	Gary A. Raid
David C. Forker	Robert Schuler
Costandy K. Khoury	Brian E. Scott
Andrew S. Markell	Irwin T. Vanderhoof

Woodward-Fondiller Prize

The President announced that the first award of the Woodward-Fondiller Prize had been given to James H. Durkin, Actuary, Wolfe, Corcoran & Linder, for his paper "A Glance At Group Dental Coverage" which had been presented at the May 1964 meeting.

Papers – Presentations And Reviews

At this point, Vice President William J. Hazam took charge of the remainder of the program which consisted of the presentation of new papers and reviews of new and previous papers.

(a) The previously presented paper "A Glance At Group Dental Coverage" by James H. Durkin was reviewed separately by Roger A. Johnson and George E. McLean.

(b) The previously presented paper "Some Fundamentals of Insurance Statistics" (originally entitled "The Philosophy of Statistical Applications to Insurance Operations") by Harry M. Sarason was reviewed by Charles C. Hewitt, Jr.

(c) The invitational paper "The Optimal Management Policy Of An Insurance Company" was presented by Professor Karl Borch (Norway).

(d) The invitational paper "On Some Essential Properties Of A Tariff Class" was presented by Professor Edward Franckx (Belgium).

(e) The new paper "A Bayesian View Of Credibility" was presented by the Honorable Allen L. Mayerson.

(f) Kenneth L. McIntosh presented in summary form, the paper "Observations On Casualty Insurance Rate Making Theory In The United States" by the late Past President Thomas O. Carlson. Mr. Carlson had presented his paper at the meeting of the XVIIth International Congress of Actuaries held in London and Edinburgh from May 26 thru June 3, 1964.

(g) A new paper "Size Of Loss Distributions In Workmen's Compensation Insurance" was presented by Lester B. Dropkin and reviewed by Roy H. Kallop and LeRoy J. Simon.

(h) A new paper "Estimating The Cost Of Accident Insurance As A Part Of Automobile Liability Insurance" was presented by Herbert E. Wittick and reviewed by Carl L. Wilcken.

This concluded the formal program of the Golden Anniversary Meeting of the Casualty Actuarial Society.

President Thomas E. Murrin then expressed the hope that the attendance at the next meeting of the CAS May 24-26, 1965 at Shawnee-On-Delaware, Pennsylvania, would approach that of our Fiftieth Anniversary Meeting. He extended his thanks to the membership, officers of the Society, committee chairmen, and invited guests whose cooperation had contributed so greatly to probably the most pleasant and successful meeting in the history of the Casualty Actuarial Society. Thereupon, the meeting was adjourned at 12:35 p.m.

In attendance at the Golden Anniversary Meeting were the following 128 Fellows, 49 Associates and 41 Guests.

FELLOWS

Aldrich, W. C.
Alexander, L. M.
Allen, E. S.
Ault, G. E.
Bailey, R. A.
Balcarek, R. J.
Barber, H. T.
Bennett, N. J.
Berkeley, E. T.
Berquist, J. R.
Bevan, J. R.
Black, S. B.
Blanchard, R. H.
Blodget, H. R.
Bondy, M.
Bornhuetter, R. L.
Boyajian, J. H.
Boyle, J. I.
Brannigan, J. F.
Breiby, W.
Budd, E. H.
Burling, W. H.
Cahill, J. M.
Cameron, F. R.
Carleton, J. W.
Coates, C. S.
Corcoran, W. M.
Craig, R. A.
Curry, H. E.
Dickerson, O. D.
Dorweiler, P.
Drobisch, M. R.
Dropkin, L. B.
Eide, K. A.
Elliott, G. B.
Finnegan, J. H.
Fitzgibbon, W. J.
Fitzhugh, G. W.
Foster, R. B.
Fowler, T. W.
Gillam,W. S.
Gillespie, J. E.
Ginsburgh, H. J.
Goddard, R. P.
Graham, C. M.
Graves, C. H.
Greene, W. W.
Hart, W. V. B., Jr.
Harwayne, F.
Haugh, C. J.
Hazam, W. J.
Hewitt, C. C., Jr.
Hobbs, E. J.
Hope, F. J.
Hunt, F. J., Jr.
Hurley, R. L.
Johe, R. L.
Johnson, R. A.
Kallop, R. H.
Klaassen, E. J.
Kormes, M.
LaCroix, H. F.
Lange, J. T.
Leslie, W., Jr.
Linden, J. R.
Linder, J.
Lino, R.
Liscord, P. S.
Livingston, G. R.
Longley-Cook, L. H.
MacGinnitie, W. J.
MacKeen, H. E.
Magrath, J. J.
Makgill, S. S.
Masterson, N. E.
Matthews, A. N.
Maycrink, E. C.
Mayerson, A. L.
McClure, R. D.
McGuinness, J. S.
McLean, G. E.
McNamara, D. J.
Meenaghan, J. J.
Menzel, H. W.
Miller, J. H.
Morison, G. D.
Moseley, J.
Muetterties, J. H.
Murrin, T. E.
Nelson, S. T.
Niles, C. L., Jr.
Oberhaus, T. M.
Otteson, P. M.
Perkins, W. J.
Petz, E. F.
Phillips, H. J., Jr.
Pruitt, D. M.
Resony, J. A.
Richards, H. R.
Riddlesworth, W. A.
Roberts, L. H.
Rodermund, M.
Rosenberg, N.
Rowell, J. H.
Ruchlis, E.
Salzmann, R. E.
Schloss, H. W.
Simon, L. J.
Skelding, A. Z.
Smick, J. J.
Smith, S. E.
Stankus, L. M.
Tarbell, L. L., Jr.
Thomas, J. W.
Thompson, J. S.
Trist, J. A. W.
Trudeau, D. E.
Uhthoff, D. R.
Valerius, N. M.
Wieder, J. W., Jr.
Wilcken, C. L.
Williams, D. G.
Williams, P. A.
Williamson, W. R.
Willsey, L. W.
Wittick, H. E.
Wolfrum, R. J.
Yount, H. W.

ASSOCIATES

Andrews, E. C.
Berkman, J. M.
Brown, W. W., Jr.
Carlson, E. A.
Carson, D. E. A.
Cima, A. J.
Crofts, G.
DeMelio, J. J.
Durkin, J. H.
Even, C. A., Jr.
Forker, D. C.
Franklin, N. M.
Gerundo, L. P., Jr.
Gill, J. F.
Gould, D. E.
Greene, T. A.
Hammer, S. M.
Harack, J.
Hart, W. V. B., Sr.
Hillhouse, J. A.
Jensen, J. P.
Jones, N. F.
Markell, A. S.
McIntosh, K. L.
Mohnblatt, A. S.
Mokros, B. F.
Muir, J. M.
Muniz, R. M.
Raid, G. A.
Richardson, H. F.
Roth, R. J.
Royer, A. F.
Ryan, K. M.
Scammon, L. W.
Scheel, P. J.
Schneiker, H. C.
Schulman, J.
Scott, B. E.
Singer, P. E.
Smith, E. R.
Steinhaus, H. W.
Stern, P. K.
Stevens, W. A.
Uhl, M. E.
Vanderhoof, I. T.
Webb, B. L.
Wooddy, J. C.
Young, R. G.
Zory, P. B.

GUESTS

Ammeter, H.
Anderson, R. R.
Beard, R. E.
Bechtolt, P. R.
Benson, L. E.
Boissier, J. J.
Borch, K. H.
Callahan, W. E.
Caputo, R. N.
Chorpita, F. M.
Christoffersen, B.
Cooper, W. P.
Crane, J.
Donovan, H. G.
Donovan, J. B.
Flaherty, D. J.
Frame, D. P.
Franckx, E.
Galban, L. S., Jr.
Grahn, D. E.
Griffith, R. W.
Hartman, G. R.
Haught, D. D.
Hayden, R. C.
Hemry, L. P.
Hoffman, L. R., Jr.
Hoyt, F. A.
Kahn, P. M.
Kedrow, W. M.
Kemble, J. W.
Marshall, R. E.
Martorana, J. F.
McSherry, H.
Nagel, J. R.
O'Halloran, W. F.
Ratnaswamy, R
Shwide, H.
Smith, C. C.
Smith, C. F.
Strain, R. W.
Van Orman, F.

REPORT OF SECRETARY-TREASURER

The following report summarizes those activities of the Council subsequent to the 1963 Annual Meeting which it is felt will be of particular interest to the membership.

Meeting of February 14, 1964

The Council expressed disagreement with the "Requirements For Admission To Membership" in the draft of the proposed By-Laws of the American Academy of Actuaries. Our representative on the Joint Subcommittee, Past President Laurence H. Longley-Cook, was authorized to bring this disagreement to the attention of the subcommittee looking forward to a revision of the admission requirements in line with the thoughts of the Council. It is a matter of interest to note that the resulting revised version of he By-Laws was accepted by the Council.

Meeting of May 17, 1964

At this meeting the Council voted unanimously to go on record in favor of the organization of an American Academy of Actuaries and to share with the other three actuarial bodies in the expenses incurred in such organization. This action of the Council was ratified by the membership at the Spring Meeting of the CAS held at Wentworth-By-The-Sea in New Hampshire on May 18, 19 and 20, 1964.

The Council unanimously approved the following nominations to the American Academy of Actuaries:

President	Henry F. Rood
Vice Presidents (4)	John H. Miller
	Laurence H. Longley-Cook
	H. Raymond Strong*
	Frank J. Gadient*
Secretary	John C. Archibald*
Treasurer	George M. Bryce
Admissions Committee	Harold W. Schloss
Board of Directors	Frank Harwayne
	William Leslie, Jr.
	Norton E. Masterson
	Daniel J. McNamara
	Thomas E. Murrin

* Not a member of CAS.

The Council voted that, beginning with the 1965 examinations, the fees for the Associateship examinations be increased from $3.00 for a section to $3.75, and from $6.00 for one complete part to $7.50.

The Council reviewed the procedures with respect to invited guests eligible to attend our meetings. The action of the Council was bulletined to all members under date of June 9, 1964.

The Council considered the report of our Committee on Local Actuaries Clubs. This report was later unanimously adopted by Mail Vote and was distributed to the membership under date of June 15, 1964.

Meeting of September 24, 1964

The Council approved the "Petition For Recognition" of the newly formed Actuaries Club of Philadelphia.

The Council voted to amend Article IV of the By-Laws, subject to confirmation by the membership, as required by Article VI of the By-Laws, to provide that a retired member who has reached the age of 65 need no longer meet the present concurrent requirement of at least 20 years membership in order to qualify for waiver of dues.

Meeting of November 18, 1964

At this meeting the Council had before it a revised draft of the Report of the Committee On Annual Statement. The Council took no specific action other than to direct the Secretary-Treasurer to distribute that document to the membership at an early date with the thought that the report might well be one of the items on the program of the May 1965 meeting, perhaps as a panel discussion, before the Council acted formally.

Also, at the November 18, 1964 meeting the Council considered the problem of making available past volumes of the *Proceedings* which are now out of print but which contain papers of considerable interest to the members and to students studying for the examinations. The Secretary-Treasurer and the Editor were instructed to explore this matter with the Johnson Reprint Corporation of New York City.

FUTURE MEETINGS

Securing appropriate facilities for meetings is becoming increasingly difficult unless confirmed reservations are obtained long in advance. The membership will be interested in the following schedule of future meetings which includes some changes from previous announcements:

May 1965 – Shawnee Inn
Shawnee-On-Delaware, Pennsylvania
May 24-26, 1965

Nov. 1965 – Sheraton-Boston Hotel
Boston, Massachusetts
November 15-17, 1965

May 1966 – Cavalier Hotel
Virginia Beach, Virginia
Dates not yet determined

Nov. 1966 – Ann Arbor, Michigan
November 16-18, 1966

May 1967 – Pheasant Run Lodge
St. Charles, Illinois
Dates not yet determined.

Nov. 1967 – Possibly in Baltimore or the Baltimore area.

May 1968 – Site not yet selected.

Nov. 1968 – Hartford, Connecticut will be considered.

The financial report of the Secretary-Treasurer for the fiscal period October 1, 1963 through September 30, 1964 is attached to this report. Copies are now available from the Secretary-Treasurer at this meeting. In summary, this report shows that during the fiscal period ending September 30, 1964 receipts exceeded disbursements by $3,105.84 so that as of that date the assets and surplus of the Society consist of

Checking Account	$ 8,770.97
Savings Accounts	15,624.72
U. S. Bonds (Maturity Value)	5,000.00
Total	$29,395.69

FINANCIAL REPORT

Cash Receipts and Disbursements
from October 1, 1963 thru September 30, 1964

Receipts

On deposit 10-1-63 (Checking)		$11,107.76
On deposit 10-1-63 (Savings)		10,182.09
Members Dues	$11,405.50	
Examination Fees	3,416.00	
Sale of Proceedings	2,281.10	
Sale of Readings	460.56	
Spring & Annual Mtgs.	2,222.75	
Registration Fees	2,610.00	
Invitational Program	1,380.00	
Foreign Exchange	— 6.41	
Bond Interest	193.76	
Savings Acc't. Int.	442.63	
Michelbacher Fund	1,362.31	
For Actuaries' Club N. Y.	660.00	26,428.20
Total		$47,718.05

Disbursements

Printing & Stationery	$12,671.25
Secretary's Office	2,100.00
Examination Expense	2,098.47
Meeting Expense	4,306.75
Library Fund	91.75
Insurance	107.16
Refunds:	
Lunch & Dinners	240.00
Examination Fees	73.00
Registration Fees	170.00
Fees to Actuaries' Club N. Y.	675.00
Miscellaneous	788.98
	$23,322.36
On Deposit 9-30-64	
Checking	8,770.97
Savings	10,556.09
Savings	5,068.63
Total	$47,718.05

Assets

Cash in Bank 9-30-64	
Checking	$ 8,770.97
Savings	10,556.09
Savings	5,068.63
U. S. Savings Bonds	5,000.00
Total	$29,395.69

Liabilities

Surplus (Michelbacher Fund)	$15,819.63
Other Surplus	13,576.06
Total	$29,395.69

One U. S. Treasury Bond 3⅞% No. 24277 due for $1000 on May 15, 1968.
Two U. S. Treasury Bonds 3⅞% Nos. 3462-3 due for $1000 each on May 15, 1968.
Two U. S. Treasury Bonds 3⅞% Nos. 1673-4 due for $1000 each on November 15, 1974.
Employers' Fire Insurance Company Policy No. 31F238562 for $5000 on books and book cases stored at 200 East 42nd Street and $2000 on material stored in library of Insurance Society of New York. Expires 9-14-67.
Fidelity Bond No. 044571 for $25,000 in Royal Indemnity Company.
Workmen's Compensation Policy No. 01-740174 in Maryland Casualty Company. Expires 5-10-65.
Owners', Landlords' and Tenants' Liability Policy No. 52-459648 in Maryland Casualty Company. Expires 4-23-65.

Note: The principal miscellaneous disbursements items are:

Insurance Society of New York — Organization membership.	$150.00
Contribution to American Statistical Association toward printing of "A Career In Statistics."	200.00
Expenses in connection with formation of proposed American Academy of Actuaries.	127.38
Contribution to International Congress.	25.00
Donation to Middlebury College in memory of Past President Thomas O. Carlson	50.00
Total	$552.38

* * *

This is to certify that we have audited the accounts, examined all vouchers and investments shown above and find same to be correct.

HOWARD G. CRANE
Chairman, Auditing Committee

October 22, 1964

OBITUARY

Robert E. Ankers
Thomas O. Carlson
Robert A. Craig
Solomon S. Huebner
Arthur Hunter
Ray D. Murphy
Edward C. Stone

ROBERT E. ANKERS

1882 — 1964

Robert E. Ankers died March 1, 1964, at Clearwater, Florida while on a visit from his native state of Virginia. His death was sudden and unexpected. He had retired from Southland Life Insurance Company in 1957 after 36 years service, having joined Continental Life Insurance Company of Virginia June 1, 1921, which was purchased by Southland Life the latter part of 1950.

He became an Associate of the Casualty Actuarial Society on November 15, 1918, and was also a Contributing Member of the American Institute of Actuaries.

Born in Virginia April 21, 1882, he graduated from the University of Richmond with the class of 1905 and had attended many class reunions. While with Continental Life he was Secretary-Treasurer and Actuary, and he continued under Southland Life as consultant in actuarial, underwriting, special settlement options and settlement of death claims. Prior to entering the life insurance profession he had been examiner for the Department of Insurance of Virginia.

Prior to his death Mr. Ankers and his wife lived at Falls Church, Virgania, and at a summer home named Stonycroft located on the top of a mountain near Bluemont, Virginia. He also had a 250 acre cattle farm near Harpers Ferry, West Virginia. His hobbies included gardening, raising flowers, farming and Hereford cattle. His azaleas around his home in Falls Church were so outstanding that many people driving by would stop to ask about them.

Surviving are his widow, Alice S. Ankers, two sons, Marvin and Robert, Jr., two daughters, Mrs. E. W. Gilkey and Miss Mary Alice Ankers, and a number of grandchildren. His body was returned to Virginia for services at Westover Baptist Church, then to Fort Union, Virginia, for interment near his birthplace.

THOMAS O. CARLSON

1905 — 1964

Thomas O. Carlson, Southeastern Branch Manager of the National Bureau of Casualty Underwriters and Past President of the Casualty Actuarial Society, died of a heart attack in Berkeley County Hospital, Moncks Corner, South Carolina July 15, 1964, at the age of 59.

He had returned to his office in Atlanta, Georgia, two days previously after a visit to Europe. While abroad he attended the XVIIth International Congress of Actuaries and presented a paper entitled "Observations on Casualty Insurance Ratemaking Theory in the United States" to the ASTIN Section.

He was a member of the Society for thirty-five years, serving as Librarian from 1937 to 1947, as Vice President from 1949 to 1951, and as President from 1951 to 1953. His contributions to the Society were numerous, including papers and many reviews of other papers and books.

He took a deep and sincere interest in the aspirants to the Society and through his encouragement succeeded in bringing many into its ranks.

He started in the insurance business as a claim adjuster for the Aetna Casualty and Surety Company in New York. He joined the National Bureau in 1928, became assistant actuary in 1932 and actuary in 1944.

While in the home office of the National Bureau, as well as during his tenure in the Southeastern Branch, he appeared frequently before insurance gatherings as a speaker and wrote many articles for insurance publications, the best known being his contributions to the most recent edition of G. F. Michelbacker's text, *"Multiple Line Insurance,"* and his paper "Rate Regulation and the Casualty Actuary" which was separately printed. Both of these are extensively used in college insurance courses. He contributed much to the development of ratemaking procedures in the United States.

He was born in Shoreham, Vermont and graduated with high honors from Middlebury College in 1925, completing a four year course of study in three years, with election to Phi Beta Kappa. In 1937 he received a Master of Science degree from New York University.

Mr. Carlson also pursued avocations far afield from insurance. He was a member of long standing in the Poetry Society of America; for many years he was an active mountain climer, having scaled Mount Rainier and other mountain peaks in the United States.

He is survived by his widow, Mrs. Reka D. Carlson; a daughter, Norda Dee Carlson; and two sons, Thomas Andrew Carlson and Dana Fordyce Carlson.

ROBERT A. CRAIG

1930 — 1965

Robert A. Craig was one of eighty-four victims of a major air disaster on February 8, 1965 when the airliner on which he was a passenger plunged into the Atlantic Ocean off Jones Beach, New York, minutes after take-off from Kennedy International Airport. He was travelling in his capacity as Actuary of the Multi-Line Insurance Rating Bureau, a position he had held for barely three months, when the tragedy occurred.

Mr. Craig was born in Winsted, Connecticut October 7, 1930 and educated in the Windsor, Connecticut school system, graduating from Windsor High School in 1949. Upon graduation from the University of Connecticut in 1953, he was commissioned a lieutenant in the Air Force and was stationed in Japan for two years as a Radar Officer. More recently, Mr. Craig served as a captain and intelligence officer of the 103rd Squadron of the Connecticut Air National Guard. He was a resident of Bloomfield, Connecticut for the past two and one-half years.

Prior to his association with the Multi-Line Insurance Rating Bureau, Mr. Craig had been an Actuary in the Rating Division of the Connecticut Insurance Department since March 1962. His introduction to the actuarial profession was through The Travelers in June 1956 where he was an Actuarial Assistant before assuming the position with the State Insurance Department.

He was a member of Washington Lodge No. 70, AF and AM of Windsor, Connecticut Consistory, and Sphinx Temple Shrine of Hartford, the Bloomfield Jaycees, and auditor of the Bloomfield Federated Church. Mr. Craig served on the Insurance Advisory Board for the Town of Bloomfield, and was also a member of the Bankers Club of America. He was a brother of Lambda Chi Alpha Fraternity.

Surviving Mr. Craig are his widow, Ruth Burns Craig, and two sons, Scot and Kent, his mother, Mrs. Catherine Craig and a brother, James Irving Craig.

SOLOMON S. HUEBNER

1882 — 1964

On July 17, 1964, Dr. Solomon S. Huebner, internationally prominent University Professor and insurance authority, died of a heart attack at Merion, Pennsylvania.

Known throughout the insurance world as the pioneer of insurance education at the college level and as the teacher who changed an industry, Dr. Huebner, at eighty-two, was enjoying a readjustment to his recently acquired titles of Emeritus Professor of Insurance at the University of Pennsylvania, Emeritus President of the American College of Life Underwriters and Emeritus Chairman of the Board of Trustees of the American Institute for Property and Liability Underwriters.

Dr. Huebner was born March 6, 1882 at Manitowoc, Wisconsin, and received his bachelor and master degrees from the University of Wisconsin. In 1903 he was named a Harrison Fellow in Economics at the University of Pennsylvania where he received the Ph.D. degree in 1905. In 1908 he married Ethel Elizabeth Mudie, who, along with their four children, survives him.

Although Dr. Huebner was on occasion a special lecturer at New York and Columbia Universities, his entire academic life centered at the University of Pennsylvania where he developed the first collegiate level program in insurance and chaired the Department of Insurance from its inception until his retirement in 1953.

A man of limitless vigor and industry and with the intense fervor of an evangelist, he not only aroused thousands of students to superior effort and excellence but also provided them with the tools of study by writing texts on subject matter previously undeveloped. The business world was his laboratory where he cultivated countless sources of information which combined with his vision, imagination and enthusiasm, enabled him to carry to the classroom a brilliance that made him one of the most effective and popular educators of our time. To the undergraduate body of the University of Pennsylvania he was affectionately known as "Sunny Sol" and the senior class repeatedly dedicated its year book to him. His rare talents produced a group of disciples in academic ranks and created a nationwide demand for his frequent appearance on the public platform, from which he inspired and influenced thousands of listeners. His popularity was greatly enhanced by an unusual capacity to diagnose business conditions accurately and give emphatic expression to his political views.

His record as a public servant is distinguished. He has served as a special expert or adviser to congressional committees, the U.S. War Department, the U.S. Department of Commerce and the Civil Aeronautics Board.

As a world traveler he derived great pleasure from a hobby of collecting specimens of rare minerals from all corners of the earth. On his travels and lecture tours he was frequently accompanied by Mrs. Huebner, whose quiet charm is known to thousands.

The name S. S. Huebner will forever be synonymous with two concepts he applied to insurance: first, "The Human Life Value Concept" through which he "changed the life insurance industry" and influenced the financial planning of more families than any other single person, and second, "The Professional Concept in Insurance" through which the standards of performance of insurance practitioners are being raised to a professional level. This latter he achieved, first by founding the American College of Life Underwriters and later by helping to found the American Institute for Property and Liability Underwriters.

Dr. Huebner's leadership qualities are best illustrated by a series of firsts: first in insurance teaching at the college level, chairman of the first college department of insurance instruction at the University of Pennsylvania, first Dean of the American College of Life Underwriters, first President of the American Association of University Teachers of Insurance (now A.R.I.A.), first to receive the John Newton Russell Memorial Award in life insurance and the first living person to be elected to the Insurance Hall of Fame.

Honors bestowed on him during his lifetime are too numerous to mention in detail. In addition to those already identified he was a member of Phi Beta Kappa, Beta Gamma Sigma and Pi Gamma Mu honorary scholastic fraternities, a member of the American Philosophical Society and a Fellow of the Casualty Actuarial Society. In 1931 the University of Pennsylvania conferred on him the honorary degree of Doctor of Science. In 1960, the first building to be erected at the American Center for Insurance Education in Bryn Mawr was named Huebner Hall. The tribute he valued most highly was the establishment of the S. S. Huebner Foundation for Insurance Education in 1940 by the life insurance industry with its principal purpose the development of college teachers of insurance, capable of perpetuating the work he had started and stimulated.

To list all the attributes of this remarkable man would exhaust Webster's descriptive superlatives. His achievements bespeak his skill and determination; his fortitude in resisting the ravages of several serious physical ailments characterizes his courage. To his intimates he was a positive thinker and actionist. His life and deeds will leave an everlasting imprint on the economic behavior of the citizenry of a free society.

Reprinted from *The Annals* of the C.P.C.U., Fall 1964.

HARRY J. LOMAN, PH.D., *President, American Institute for Property and Liability Underwriters*

ARTHUR HUNTER

1869 — 1964

Arthur Hunter, charter member and fellow of the Casualty Actuarial Society, died January 27 at the age of 94 after a long illness.

Dr. Hunter was a member of the Actuarial Society of America for over 63 years and a most active participant in its affairs. His contributions to the actuarial profession and to the life insurance industry during his active years were monumental in extent and had made him one of the most highly respected actuaries on both sides of the Atlantic. He became an Associate of the Society in 1900 and a Fellow in 1903. He was the Society's Secretary during the years 1905 – 1911, a Vice President in 1912 – 13 and President in 1916 – 17.

Dr. Hunter was a member of the Joint Committee of the Medico-Actuarial Mortality Investigation and Chairman of the Central Bureau. This investigation was a most important pioneering inter-company effort. He also served on many of the Society's Committees.

In addition to his fellowship in the Society of Actuaries, Arthur Hunter was a fellow of the Faculty of Actuaries in Scotland, a charter member and fellow of the Casualty Actuarial Society, a corresponding member of the Institute of Actuaries of England, the Institute of Actuaries of France, and the Association of Actuaries of Switzerland, as well as a fellow of the Royal Society of Edinburgh.

It is particularly noteworthy that Dr. Hunter's membership in the Faculty of Actuaries extended over a period of more than 70 years. The minutes of the Council of the Faculty for its meeting of December 9, 1963 contained the following statements with respect to Dr. Hunter's unusually long and distinguished membership in the Faculty.

"The Chairman drew the attention of Council to the fact that Dr. Hunter had been admitted a Fellow of the Faculty in 1893 and that he had thus completed seventy years of membership. This was in itself a rare distinction which few Members achieved but when each of these years represented, as it did in Dr. Hunter's case, a period of sterling and progressively outstanding service to the actuarial profession in two Continents, it had a very special significance.

"The Faculty Council unanimously agreed that this particular anniversary should be marked by reference in the Council Minutes to Dr. Hunter's service to the profession, to his fellow Members' appreciation of the honor he had brought to the Faculty during his seventy years of membership, and to the high regard and esteem in which he was held by all the many Faculty Members who had the privilege of knowing him."

Arthur Hunter was born in Edinburgh, Scotland and graduated from the famous George Watson's College for boys. The University of Edinburgh and Hamilton College conferred honorary degrees upon him. An internationally famous actuary, he began his American career with the Fidelity Mutual Life Insurance Company in 1892. Six years later he joined the New York Life Insurance Company from which he retired in 1941 as Vice President and Chief Actuary.

The New York Life started issuing insurance to substandard risks in 1896. Dr. Hunter created an outstanding milestone in actuarial research when he persuaded the president of his company to allow him to present a paper to the Actuarial Society, giving the company's experience under its substandard risks. Since the company had pioneered in the issuance of insurance to various categories of substandard risks its published experience for many years became a guide for the industry until broader, inter-company studies became possible.

Subsequent to the presentation of Arthur Hunter's first paper, many more followed, in fact about 80 of them. Quite a few were written in collaboration with Dr. Oscar Rogers, Chief Medical Director of New York Life. The Hunter and Rogers papers were also presented to the Medical Directors Association. Incidentally, Arthur Hunter was the only layman who was an honorary member of that association. In conjunction with Dr. Rogers, he also devised the numerical method of rating life insurance risks.

He wrote the Actuarial Society of America's first textbook on Disability Benefits and collaborated in the revision which is currently in use. He was a prolific writer and contributed many actuarial and medico-actuarial articles, not only to the Society but also to other bodies here and abroad and to publications such as the *Encyclopedia Brittanica.*

While conducting his many actuarial research activities, he gave unusual opportunities to the younger actuaries in the company. Many outstanding actuaries both in the United States and Canada owe much of their early training as actuarial students to the broad and excellent experience gained while serving under Arthur Hunter.

He served for many years as a director of the New York United Medical Service, his knowledge of medico-actuarial problems being particularly invaluable.

During World War I, as president of the Actuarial Society of America, he was named chief consulting actuary of the Government and was chairman of the Advisory Committee of the War Risk Insurance Bureau. He had a great deal to do with the planning and development of life insurance for our armed forces. For this work he was commended by Secretary of the Treasury, Carter Glass, who wrote him in 1919 to thank him "for the most splendid service" in the "difficult problems in connection with the planning and administration of the provisions of the War Risk Insurance Act under which more than 4,500,000 American soldiers and sailors were insured for an amount aggregating more than $40 billion."

The Assistant Secretary of the Treasury called his service "unexampled" not only to the Treasury Department but "to the discharged service men and their families."

New York Governor Charles Whitman praised Arthur Hunter in 1917 for "your patriotic spirit with which you gave your time and abilities during the last six months to make the New York State Military Census the extraordinary success that it is."

In 1938 President Franklin D. Roosevelt, through Secretary of State Cordell Hull, selected Arthur Hunter as chairman of a three member American delegation to the International Congress of Actuaries in Paris, France.

In 1939 he received from French President Albert Lebrun the rank of Chevalier of the Legion of Honor.

In 1946 Arthur Hunter received His Majesty's Medal "for service in the cause of freedom" from King George VI of England.

But Dr. Hunter's activities were by no means confined to actuarial problems. He was associated all during his lifetime with cultural, health, religious and welfare activities and gave generously both of his time and money.

In Montclair, New Jersey, where he had lived for over 50 years he was elected a trustee of the Montclair Art Museum in 1927 and served as its president from 1932 to 1938 and again from 1946 to 1951. As mentioned in *The Montclair Times,* his leadership is credited during the depression years with exerting a major influence in not only the maintenance but also the furtherance and expansion of the aims and ideals of the museum. Until recently he had served on the art, music and etching committees of the museum.

In a formal resolution adopted by the trustees of the museum, his accomplishments over a 34 year period were praised in part as follows:

> ". . . tangible evidence of your devoted interest and generosity is shown by your many gifts throughout the years for which you were named a benefactor in 1954. In May 1960 you were named a founder when you established the Ethel Parsons Hunter Fund for the acquisition of paintings by British and American artists . . ."

From 1939 to 1951 he was trustee of Mountainside Hospital and was elected its president in 1947. On his retirement as a trustee, the Board, in a formal resolution, said ". . . interested in the early diagnosis of cancer, he worked diligently toward the establishment of a cancer clinic and in 1948 became chairman of a special trustees' cancer committee . . ."

"As chairman of a special committee he did a great deal of work in establishing social security and a supplemental pension plan for hospital employees . . ."

Besides his interest in the hospital he was, for six years, trustee of the Community Chest, during the major portion of which he served as chairman of the budget committee. He was also formerly a member of the planning committee of the Council of Social Agencies.

Arthur Hunter was a trustee of the Unitarian Church of Montclair for many years. When serving as president of its Board of Trustees from 1918 to 1921 he played a major role in the development of Unity Institute and the concert course and the forum series (now the travel course).

His cultural interests also extended beyond his home town. In his desire to promote a better understanding by the British people of Americans, he provided funds for books on American history and for American historical novels to Edinburgh University, St. Andrew's University and other schools in Scotland.

In 1955 Arthur Hunter established a scholarship in America for a graduate from George Watson's College for boys and made a gift at the time to the English Speaking Union to continue it for a number of years.

He was a member of Pi Gamma Mu Fraternity and of the Century and National Arts Clubs in New York. His membership in the Century Club covered a period of 60 years.

He was an enthusiastic traveler, having been around the world several times. And, being a Scot, he was a keen golfer and brought with him a knowledge of and love for the game. He was one of the first to play the game here and while in the Fidelity Mutual he helped in the development of the first golf course in Philadelphia.

As a Montclair resident, for many years, he guided a "before church" Sunday morning group of golfers on the links of the Montclair Golf Club. He played regularly until several years ago, although he admitted that he played then more for "companionship" than to improve his game.

Also, being a Scot, he had a great love for his native land and served as president of both the St. Andrew's Society and the Burns' Society, both of New York. Up to the end his mind was keen and his eyes bright and he continued to serve as chairman of the Standing Committee of the St. Andrew's Society until the time of his death.

The Pibroch, the magazine of the St. Andrew's Society, published the following poem commemorating his ninetieth birthday:

"The work you've done to help the young
And well deserving Scot
With scholarships in U.S.A.
Will never be forgot.

Your long, long years in Nylic ranks
Stand out in bold relief.
You left behind a record great
As Actuarial chief.

The governments of U.S.A.
Of France and Britain, too,
And from Edina's learned seat
Saw fit to honor you."

A fitting tribute to the "Dean of American Actuaries."

RAY D. MURPHY

1887 — 1964

Ray Dickinson Murphy died February 24, 1964 following a long illness. He was born February 28, 1887 in Springfield, Mass. His long and brilliant career marked him as one of the truly distinguished actuaries and executives.

A gifted student of mathematics, Mr. Murphy entered actuarial work for the Massachusetts Mutual Life shortly after his graduation, magna cum laude, from Harvard in 1908, and became a Fellow of the Actuarial Society within four years. At the age of 23 he became Actuary of the old Hartford Life, transferring briefly to the rolls of the Missouri State Life when it reinsured the Hartford Life, and then joining The Equitable Life Assurance Society of the United States.

Starting in the Equitable as Assistant Actuary in 1913, Mr. Murphy rose through underwriting and actuarial positions, always serving with a superb combination of technical and managerial talents. He became a member of the Board in 1947, President and Chief Executive Officer in 1953, and Chairman of the Board in 1956. He retired from active duty in 1958, but continued to serve on the Board through 1962.

Bacon's dictum that every man is a debtor to his profession, found wholehearted response in Mr. Murphy. Over the years he held almost every major office of The Actuarial Society of America, serving as President in 1938—1939 and later served as a member of the Board of its successor, the Society of Actuaries. He became a Fellow of the Casualty Actuarial Society in 1920 and of the American Institute of Actuaries in 1940. He was also Secretary and Council member of the U.S. Section of the International Congress of Actuaries.

Mr. Murphy wrote a number of papers on underwriting and actuarial subjects, as recorded in *The Record* and the *Transactions of the Society of Actuaries*. He was a co-author of one of the official publications of the Society, that on the Construction of Mortality Tables.

In the larger sphere of business, Mr. Murphy was exceedingly effective in a wide range of activities. He was a Director and President (1955) of the Life Insurance Association of America and was called upon several times to testify before legislative committees in Washington and Albany. Friends remember, also, that he found considerable satisfaction in being one of a small group who formulated the Institute of Life Insurance in 1939. Further, he was a Trustee of many important organizations including the Annuity Fund for Congregational Ministers, the Y.M.C.A. Retirement Fund, the Life Insurance Medical Research Fund, the S.S. Huebner Foundation, and the Chase Manhattan Bank.

Retaining his interest in academic affairs and in Harvard, he served a long period as a member of Harvard's graduate committee on mathematics. Also, he was a member of the Institute of Mathematical Statistics and of the American Mathematical Society.

Mr. Murphy played an active part in social and community service, particularly as a resident of Montclair, N. J. In the capacity of major officer or Director, he was a strong force in a long list of social, philanthropic and cultural institutions. For many years he was active in music, being an accomplished singer.

With character and integrity founded on traditional New England virtues (inherited from ancestry tracing back to the Mayflower), Mr. Murphy was at once firm and gentle, fair-minded and determined, charming and essentially modest. Many actuaries will fondly recall his friendliness, his pleasure in social contacts and the sense of strength and purpose which seemed to permeate his activities. Surely, the imprint which he left on the actuarial profession, on the life insurance business and on the large community in which he moved, constitutes an impressive memorial.

Mr. Murphy is survived by his wife, the former Elizabeth Chapin of Springfield, Mass.; a son, R. Bradford Murphy; a daughter, Mrs. Herbert H. Garrison; and eight grandchildren.

EDWARD C. STONE

1879 — 1964

Edward C. Stone, executive head of The Employers' Group of Insurance Companies from 1926 until 1947, died June 6th in New England Baptist Hospital.

Born in Lexington, Massachusetts, he took his LL. B. degree, (magna cum laude) at Boston University in 1900. Mr. Stone served as a trial lawyer, and for two years was a member of the Massachusetts Legislature prior to entering the insurance business. He became General Counsel for The Employers' Liability Assurance Corporation, Ltd., in 1919 and was appointed Associate United States Manager in 1924. Two years later he was named United States Manager of ELAC as well as President of the American Employers' Insurance Company and The Employers' Fire Insurance Company. Mr. Stone became United States General Manager and Attorney in 1936.

During his long career in the insurance business, Mr. Stone held many positions including Presidency of the Insurance Federation of Massachusetts, the Insurance Institute of America and the International Association of Casualty and Surety Underwriters. He was a member of the executive committees of the Association of Casualty and Surety Underwriters and the National Board of Fire Underwriters.

Edward C. Stone served as a member of the Massachusetts Senate from the Cape Cod and Plymouth District from 1948 to 1962. He was active in the fight to establish the Massachusetts Port Authority and was recognized as an expert on insurance and tax affairs.

Boston University benefitted from Mr. Stone's affiliation as he was instructor and later a lecturer in criminal law at the Law School while he was engaged in the practice of law. Later he served as Chairman of the Trustees of the University from 1953 to 1962.

Mr. Stone is survived by his wife, a son, and three daughters.

EXAMINATION FOR ENROLLMENT AS ASSOCIATE

PART I GENERAL MATHEMATICS

The questions for Part I were prepared and copyrighted by the Educational Testing Service of Princeton, N. J., and cannot be reprinted. Students may obtain a set of similar questions from the Secretary-Treasurer.

PART II SECTION (a)

PROBABILITY

1. A party of three is to be chosen from 7 Americans, 6 Englishmen and 5 Frenchmen. What is the probability that the party will consist of one person of each nationality?

2. In how many ways can the letters of the word CHESS be arranged so that the letters S are not next to one another?

3. What is the probability of obtaining at least 4 heads in 6 throws of a fair coin?

4. There are 8 seats in a row. If 2 seats are chosen at random, what is the probability that they are adjacent to each other?

5. Two different digits are chosen at random from a set of the first eight integers. What is the probability that both digits will exceed 5?

6. In how many ways can 4 men be seated around a circular table with 8 chairs?

7. A die is thrown 3 times. What is the probability that the sum of the numbers thrown is 16 or more?

8. The odds against success in an experiment are 3 to 1. What is the probability of at least one success in four tries?

9. Two dice were tossed simultaneously and showed a total of 8. What is the probability that the two dice showed the same number?

10. A bag contains 5 white and 4 black balls. If they are drawn out one by one without replacement, what is the chance that the two colors will be drawn alternately?

11. A circle of diameter 13″ is inscribed in a square. A coin of 1″ diameter is dropped at random on the square. If only those cases are counted when the coin lies wholly inside the square, what is the probability that the coin is also wholly inside the circle? (Leave answer in terms of π.)

12. An urn contains 6 black, 5 red and 4 white balls. If 4 balls are drawn at random without replacement, what is the probability that exactly 2 of them are black?

13. Seven people are seated at random around a round table. What is the probability that two given people sit next to each other?

14. A, B, and C each fire at a target. The chance that both A and B will hit the target is 1/8; that both B and C will hit is 1/20; that both A and C will hit is 1/10. The chance that all three will hit the target is 1/40. What is the probability that exactly two of A, B, and C will hit the target?

15. A and B alternately toss a die, A tossing first. The first to toss an ace is the winner. What is the probability that B wins?

16. For a certain biased coin, the probability of obtaining at least 1 head in three throws is 63/64. What is the probability of obtaining a head in one throw of this coin?

17. If the occurrence of a 5 or a 6 is called a success when a die is thrown, what is the probability of obtaining 3 successes on a single throw of 5 dice?

18. A man with \$5 tosses three fair coins simultaneously. He receives \$3 if exactly two heads turn up, and pays \$1 in any other event. Find the probability that after 3 such trials he has exactly \$6 left.

19. A bag contains 7 red and 3 blue balls. 5 balls are drawn in succession without replacement. What is the probability that the third blue ball is drawn on the fifth draw?

20. Given the probability density function:

$$\begin{aligned} f(x) &= Ae^{\frac{-x}{5}} && \textit{for } x > o \\ &= 0 && \textit{otherwise,} \end{aligned}$$

what is the probability that the random phenomenon specified by the given function will be between 5 and 10?

21. A die has been loaded in such a way that the chance of getting exactly 2 even numbers in 5 throws is three times the chance of geting exactly 3 even numbers in 5 throws. What is the chance of obtaining an even number in one throw of this die?

22. An urn contains 10 balls of which 4 are white. An integer, n, is chosen at random from the set (1, 2, 3, 4, 5, 6) and n balls are then drawn at random from the urn without replacement. Find the probability that all the balls drawn are white.

23. Two dice and a coin are tossed and the result "two sixes and a tail" did not occur. What is the probability that the coin came up tails?

24. In an infinite series of independent trials of an event with constant probability, p, of success in a single trial, what is the expectation of the number of failures preceding the first success?

25. From a bag containing 5 white and 4 black balls, 3 balls are drawn at random without replacement. From the remaining balls, a second drawing of 3 balls is made (without replacement) and all are found to be black. What is the probability that the first drawing consisted of 3 white balls?

PART II SECTION (b)

STATISTICS

26. The following distribution is binomial. Find its variance.

x	0	1	2	3	4	5
Frequency	1	10	40	80	80	32

27. What is the 3*rd* moment about the mean for the random variable whose first three moments about zero are 2, 4 and 16, respectively?

28. A coin is tossed 3 times and a score equal to the square of the number of heads given. What is the variance of the score?

29. What is the variance of a random variable which has a uniform distribution of probability over the range $-a$ to $+a$?

30. What is the value of k (to the nearest tenth) if you are told that 80% of the area under a normal curve lies between plus and minus k standard deviations about the mean?

31. Two sets of observations yield the following data:

Size	Mean	Standard Deviation
20	15	3
30	10	2

What is the variance of the combined set?

32. A coin is tossed 3 times and a score equal to the product of the number of heads and the number of tails given. If x denotes this score, what is the (cumulative) distribution function for x?

33. A random variable x is normally distributed. Given that the probability that $x < 2$ equals .067 and the probability that $x > 7$ equals .159, find the variance of x.

34. A sample of 64 items is drawn from an infinite population whose first and second moments about the origin are 5 and 125 respectively. What is the probability that the sample mean exceeds 8?

35. The (cumulative) distribution function for a random variable is given by: $1 + \frac{1}{3}(2x\text{-}3)\, e^{-2x}$, $0 \leqq x < \infty$. Find the mode.

36. Calculate the correlation coefficient between x and y from the following data.

x	y
6	6
9	12
3	6

37. Find the equation of the regression line of y on x from the following data.

x	y
2	3
1	1
3	2
4	3

38. If a coin is tossed 400 times, what is the approximate probability that the number of heads will be between 196 and 205 inclusive?

39. A sample of 900 items is drawn from a normal population whose variance is equal to 81. If the mean of the sample is 21.2 find a 95% symmetric confidence interval for the mean of the population.

40. 1,000 experiments are tried, each with a probability of success equal to .002. What is the probability that the number of successes will not be more than 2 standard deviations away from the mean?

41. What is the variance of the distribution specified by the Moment Generating Function, $M(t) = (1 - \frac{t}{2})^{-3}$

42. A value of m is predicted for y when $x = n$, and the value of n is predicted for x when $y = m$, based upon the two least square regression lines determined from the following data:

x	2	6	7	12	16
y	5	10	11	14	17

What is the value of $m + n$?

43. A certain normal population has a mean of 140 and a standard deviation equal to 10. If a sample of 1,000 members is taken, what is the expected number of members in the interval 135-150?

44. Given that in a certain joint distribution of two variables x and y, with standard deviations $Sx = 3$, $Sy = 4$, the variance of $(2x-y)$ is 11. What is the covariance between x and y?

45. What value of Chi-square would you use to test the hypothesis that the following set of observed frequencies is compatible with the given expected frequencies:

Class

	1	2	3	4
obs. freq.	8	30	60	82
exp. freq.	10	20	50	100

46. A sample of 400 shows that 20% of the items are defective. What are the 95% confidence limits for the proportion of defectives in the population? (Ignore the continuity correction.)

47. A random sample of 10 observations has a mean of 52.00; the sum of the square of the deviations of these 10 observations from 52.00 is 40. What are the 95% confidence limits for the mean of the population?

48. Two random samples are drawn independently of each other from a normal population with mean 16 and standard deviation 10. If the sample sizes are 25 and 20, what is the probability that the difference of the sample means is 4 or more?

49. A population has a mean of 68.0 and standard deviation of 3.0. Estimate (using the normal distribution) the smallest number that should be included in a sample in order to be reasonably sure (95% level) that the sample mean is within 0.1 of the population mean.

50. A sample of 10 items is drawn from a normal population whose standard deviation is 3. What is the probability that the sample variance, s^2, exceeds 15.227?

$[s^2 = \frac{1}{10} \text{Sum} (x_i - \bar{x})^2 ; i = 1, 2, \ldots 10]$

PART III SECTION (a)

1. Given: $l_x = 100\sqrt{100-x} \quad 0 \leq x \leq 100$

Find the smallest integral age x greater than m such that the probability that a life age $x+m$ will die before attaining age $x+m+n$ is twice the probability that a life age x will die between age $x+m$ and age $x+m+n$.

2. Express in terms of $\ddot{a}$, with appropriate subscripts each of the following:

(a) the present value at age 35 of an annuity of 1 per annum, the first payment at age 42;

(b) the present value at age 35 of a 15 year temporary annuity due of 10 per month;

(c) the present value at age 40 of an annuity of 1 payable every 6 months, first payment at the end of 3 months.

3. Find the net single premium in commutation symbols for a term insurance issued to (x) which provides a death benefit of $1,000 in the first year increasing by $500 each year thereafter and terminating at age $x+n$.

4. Suppose 100,000 lives age 25 purchase an ordinary insurance of $1. The net annual premium is .025, the interest rate is .03, and the mortality table provides for 1,000 deaths at age 25. Find ${}_1V_{25}$.

5. Write in commutation symbols the present value of a temporary annuity at age 20 which is a series of 25 payments, beginning with a payment of \$10 at age 20 and increasing annually by \$1 for 5 years, the sixth and subsequent payments being \$15.

6. Prove that $v = 1 - \dfrac{\ddot{s}_n - \ddot{a}_n}{\ddot{a}_n \cdot \ddot{s}_n}$

7. Prove that $\dfrac{A_{x+n} - A_x}{1 - A_x} + \dfrac{1 + a_{x+n}}{1 + a_x} = 1$

8. Given $a_{65} = 9.096894$ and $a_{66} = 8.709184$ in a certain mortality table at $2\frac{1}{2}\%$, evaluate q_{65}.

9. Write down in terms of (1) probability, annuity, and interest functions and (2) commutation functions and interest functions, expressions for the following yearly annuities of \$1.
   (a) An annuity to (x) payable for 5 years certain and so long thereafter as he may live.
   (b) An annuity to (x) payable throughout his lifetime and for 5 years thereafter.

10. The net single premium for a pure endowment of \$1,000 issued at a certain age and for a certain period is \$700 with return of net premium in event of death during the period, or \$650 with no return at death. Find the exact net single premium for a pure endowment of \$1,000 issued at the same age and for the same period, if one-half of the net premium is to be returned at death during the period.

PART III Section (b)

1. Probably no two writers will agree upon a definition of insurance. The elements of insurance, however, may be considered to fall under the following headings. Discuss briefly each of them.
   (a) The form
   (b) The purpose
   (c) The means
   (d) The insured's interest
   (e) The insurer's offer
   (f) The insured hazards.

2. Discuss the role of Insurance, other than Life Insurance, in the Theory of Production.

3. Outline briefly what you consider to be the six most important factors involved in formulating investment policy for a casualty and fire insurance company, and tell why.

4. Bonds are issued in a variety of forms, depending upon the circumstances of the issuer. Define each of the following and give one reason or circumstance that would induce a corporation or government to issue that type in preference to others.

   (a) Debentures
   (b) Revenue Bonds
   (c) Collateral Trust Certificates
   (d) General Obligation Bonds
   (e) Guaranteed Bonds
   (f) Convertible Bonds.

5. Briefly describe the effects of each of the following as outlined by Willett. Tell whether you agree or disagree with the conclusions which he reached in these areas.

   (a) Inequalities of risk upon the apportionment of capital
   (b) The law of diminishing utility and its effect on a fair wager
   (c) Society on risk and on prevention
   (d) Increasing the number of risks
   (e) Combining risks.

6. Assume that your company carries fixed income securities as investments approximately equal to the sum of its reserves. In the following circumstances indicate what form(s) of investments should predominate, given a choice among: (a) federal government bonds, (b) municipal bonds, and (c) corporate debentures, and state reasons for your choices.

   (i) Your company incurs persistent heavy underwriting losses, but has a very large surplus relative to its liabilities and premium volume.

   (ii) Your company earns a consistent underwriting profit.

   (iii) Your company's underwriting results generally produce a loss and its surplus is of modest size.

PART IV

SECTION (a)

1. Define each of the following terms as it applies to Workmen's Compensation contracts:

   (a) Coverage A
   (b) Coverage B

   What is the company's maximum liability under each of these coverages?

2. The NBCU Automobile Casualty Manual specifies certain surcharges when a certificate of insurance is required to comply with a financial responsibility law. Indicate some of the situations that necessitate a surcharge and indicate how the surcharge is applied to (a) owned automobiles, and (b) automobile repair shops.

3. Define the following perils and tell whether they are covered in a Homeowners Insurance Policy:

   (a) Explosion
   (b) Smoke
   (c) Breakage of glass

4. Describe the operation of the "Liberalization Clause" in the Standard Fire Policy.

5. In the usual course of planning and construction of a major office building, three types of contract bonds are required. Name and describe the function of each type.

6. As used in Accident and Sickness Insurance, define . . .

   (a) Total Disability
   (b) Partial Disability
   (c) Elective Indemnity
   (d) Proximate Cause of Loss
   (e) Uniform Provisions

7. A signed application is not commonly required in Fire and Casualty Insurance today. Indicate what lines of insurance do require a signed application and discuss the advantages and disadvantages that might accrue if a signed application were always required.

8. Briefly describe the following which are generally found in Automobile Liability and Physical Damage Insurance Policies:
   (a) Two or more automobiles
   (b) Notice of claim
   (c) Appraisal

9. What is a deductible? How are deductibles used in Automobile Physical Damage and Homeowners Policies?

10. A manufacturer carries two general liability policies, one with company A—$25,000 limit—the other with company B—$50,000 limit. If a loss occurs, what amounts will be paid by A and B respectively if the loss is settled for . . .
    (a) $6,000
    (b) $30,000
    (c) $100,000

11. Distinguish between the following terms:
    (a) Condition and exclusion
    (b) Continuous and noncontinuous insurance

12. Explain the differences in coverage among Homeowners Forms 1 to 5.

13. Contrast subrogation and salvage.

14. (a) What two large classes of employees were excluded from coverage under Workmen's Compensation and covered under the Federal Employees Liability Act of 1908?

    (b) What other employees are not usually covered by Workmen's Compensation laws?

15. Distinguish among the following related plans of insurance for providing accident and sickness coverage.
    (a) Group
    (b) Franchise
    (c) Blanket

16. Describe the provisions of the Water Exclusion Clause of an Extended Coverage Endorsement to a Standard Fire Policy.

17. What is the insuring agreement and what are the exclusions in a Comprehensive Glass Policy?

18. Give the rights of Insurer and Insured under a Standard Workmen's Compensation Policy.

19. Define "in flight" and "not in flight" in an Aviation Hull Policy. How might this affect the coverage?

20. What is meant by a "floater" policy? What is meant by a "block" policy?

SECTION (b)

1. Given the following morbidity table and the fact that female rates are to be 150% of tabular, calculate the gross annual premium for a \$30 weekly benefit for a group of 800 males and 200 females requesting 13 weeks maximum benefit, 1 week elimination period. Assume the loading formula to be \$.10 per \$1 of weekly benefit plus 30% of gross premium.

Duration in weeks	Weeks of disability per 10,000 exposure years
1	1790
2	2908
....	....
13	6440
14	6598
15	6749

2. Discuss the extent to which a classification system for private passenger automobile liability insurance should be refined, weighing benefits to be derived from further refinements against shortcomings.

3. Credibility procedures are required to recognize three types of variability—the number of losses, the size of loss, and the inherent loss frequency. Discuss briefly the models used for each of these three causes of variability.

4. Outline the Workmen's Compensation ratemaking procedure as described by R. M. Marshall.

5. Given the following information for State X:

	1960	1961	1962
Premiums Earned, Adjusted to Current Rates	$3,600,000	$3,800,000	$4,000,000
Losses Paid	1,500,000	1,800,000	3,500,000
Loss Adjustment Expense Incurred	180,000	190,000	200,000
Acquisition, Profit, etc.	30%	30%	30%
Taxes	3%	3%	3%
Ratio of Company Expenses to Premiums Earned	.10	.10	.10

and assuming a constant price level, a ratio of paid loss to incurred of 1.0, and a distribution of losses by size which included no abnormal losses in 1960 and 1961 and two hurricanes in 1962 costing $500,000 and $1,200,000;

Calculate the over-all indicated percentage rate level change for Automobile Physical Damage Comprehensive coverage in State X for the latest year (1962).

6. Is it proper to charge the same rate per unit of insurance, regardless of the number of units involved in a given transaction? Explain.

7. Describe how trend and projection factors could be calculated and used in the development of automobile bodily injury and property damage rates.

8. Discuss the control an insured has over his Ocean Marine Cargo or Hull rate.

9. On 9/30/62, Workmen's Compensation benefits were increased 5.0% and on 6/30/63 the benefits were increased 3.0%. Calculate the amendment factors for policy years 1962 and 1963 which are required to adjust losses for these two policy years to the current law level as of 1/1/64. The previous law level change was an increase of 2.5% on 9/1/61.

10. Define the following terms:

    (a) Unit of exposure
    (b) Rate
    (c) Premium
    (d) Underlying pure premium
    (e) Calendar-accident year statistics

EXAMINATION FOR ENROLLMENT AS FELLOW

PART I

SECTION (a)

1. a. In addition to the three general standards which provide that rates shall be adequate, not excessive, and not unfairly discriminatory, most rating laws contain guides to assist the ratemaker and the supervisory official in determining elements to be given consideration in ratemaking. Name four such elements which are to be given due consideration under most rating laws.

   b. Most rating laws contain provisions for administrative machinery which makes possible reasonable policing to assure that the standards for rates contained in the law are actually applied. Explain three important administrative requirements or controls of the supervisory official which apply to rates.

2. In a recent court case in Connecticut, a group of mutual companies were successful in getting a rating plan filed by the National Council on behalf of its stock company members rejected on the basis that no standards were included in the plan for determining price differences between risks. Briefly describe this plan and state three of the major reasons for and three major reasons against such a rating plan.

3. In U. S. vs. S.E.U.A., the United States Supreme Court overturned the system of regulation of the insurance business that had existed for 75 years.

   a. What were the two counts in the indictment of the S.E.U.A. case?

   b. What was the name of the court case which had established the system 75 years earlier?

   c. Although only four justices concurred in the majority decision, two of the three minority judges agreed with the majority on one very important point. What was this point?

4. a. Explain how an assuming carrier which is licensed in all states may pay double premium taxes when reinsurance on a risk located in several states is accepted from a company which is not authorized in one or more of the location states.

   b. Discuss briefly the Model Unfair Trade Practices Bill.

5. a. Explain what is meant by each of the basic criteria for rates:

      (1) not excessive
      (2) not inadequate
      (3) not unfairly discriminatory

   b. Describe the requirements of most rating laws with regard to rating plans which may be used to modify classification rates to produce rates for individual risks.

6. a. Recently, the Internal Revenue Service has indicated its intention to change the basis of taxation of casualty companies as respects cases in litigation. The change is apparently brought over from the method of taxation of life insurance companies. Describe the change. Is there sufficient similarity between life and casualty operations to warrant similar treatment of these cases? Give support for your opinion.

   b. Discuss the advantages and disadvantages of state regulation of the insurance business as compared to federal regulation.

7. Discuss the following provisions of the New York State Insurance Law as respects automobile liability insurance policies.

   a. Insolvency or bankruptcy of the person insured.
   b. Injury to the spouse of an insured.
   c. Uninsured Motorists Protection.

8. Describe the most important provisions of the:

   a. Sherman Act
   b. Clayton Act
   c. The Robinson-Patman Act

Section (b)

9. Describe an unsatisfied judgment fund and appraise the use of such funds in relation to the problem of the uninsured motorist, including in your appraisal four arguments against their use.

10. Most Blue Cross plans use one of two general types of formulae for determining how much the plan should pay to a hospital. Compare these two methods.

11. Given the following data, show how you would arrive at a final rate for a temporary disability benefits insurance plan with benefit provisions similar to the companion unemployment compensation law (assume no employee contribution):

    a. if the rate were expressed as a percentage of taxable payroll and there was no loading for female lives or extra-hazardous industry;

    b. if the rate were per person per month, and was loaded to reflect the fact that female morbidity is 50% higher than male and that 40% of the insured group is female.

    Pure premium (all male)—$.50 per month for each $10 of weekly benefit.

    Average taxable payroll per person—$2,400 per year.

    Average weekly benefit rate in companion unemployment compensation act—$24.00.

    Statutory assessment loading—1/10 of 1% of taxable payroll.

    Loading for expenses and contingencies—25% of gross premium.

12. The funding of the various Unsatisfied Judgment Funds differs considerably from one jurisdiction to another. Describe the method of funding used in:

    a. British Columbia
    b. North Dakota
    c. Manitoba and Alberta
    d. New Jersey

13. With respect to the problem of the uninsured motorist, describe the following:

    a. The Saskatchewan Plan
    b. Impounding law
    c. Safety responsibility law
    d. Financial responsibility law

14. Cite four arguments in favor of integrating a state's disability plan with its unemployment compensation act.

15. Discuss the similarities and differences between state and federal systems of old age assistance and insurance.

16. Define the method used in the Federal Old-Age and Survivors Insurance system in determining if a person is:

    a. Fully insured
    b. Currently insured

    What benefits are payable to a fully insured person which are not payable to one who is currently insured?

17. State five principal arguments in favor of an automobile compensation system.

18. Name and describe five causes of unemployment as classified by Gagliardo.

PART II

SECTION (a)

1. You have been asked to determine whether your company's claim department has been underreserved on individual automobile bodily injury claims (excluding loss adjustment expenses). How might the Annual Statement be used for the necessary analyses?

2. As company actuary you are asked to devise a method for distributing incurred but not reported loss reserves to branch office for use in your company's branch office operating results yearly statement. Discuss the advantages and disadvantages of using each of the following as a base:

   Reserves for unpaid losses on reported claims by line, by branch
   Written Premiums by line, by branch

3. Describe fully "Recapitulation of Premiums" and "Recapitulation of Fire Premiums" of the Fire and Casualty Annual Statement. What are the primary reasons for the existence of these two sections?

4. (a) Describe a method of calculating the reserve for premium credits on business subject to retrospective rating.

   (b) Explain what information is required, and how it is used, to calculate the change in the equity in the unearned premium reserve during a calendar year.

5. Section 326 of the New York State Insurance Law sets forth the formulae for determining statutory reserves in Schedule P. State the formulae for both Workmen's Compensation and liability insurance.

6. Discuss four common methods of determining loss reserves for known cases and give an example for each.

7. Develop a criterion for distinguishing between a true liability and a surplus reserve. Apply this criterion to the following statement items and explain your decision in each instance.

   (a) Unearned premium reserve.

   (b) Excess of Schedule "P" reserve over case reserve.

   (c) Reserve for undeclared dividends.

8. Discuss three methods of reserving for allocated claim expense.

SECTION (b)

The following data were taken from the records of stock insurance company A. Unless otherwise noted, assets and liabilities are as of December 31, 1963. Other items are for the year 1963.

(1)	Premiums earned	$4,180,000
(2)	Unearned premiums	2,014,000
(3)	Losses incurred	2,100,000
(4)	Losses unpaid	2,900,000
(5)	Loss expenses incurred	350,000
(6)	Loss expenses unpaid	450,000
(7)	Other underwriting expenses incurred	1,400,000
(8)	Other expenses unpaid	23,000
(9)	Federal income taxes incurred	30,000
(10)	Taxes, licenses, and fees unpaid	129,000
(11)	Federal income taxes unpaid	35,000
(12)	Dividends to stockholders (cash)	60,000
(13)	Excess of bodily injury liability and compensation statutory and voluntary reserves over case basis and loss expense reserves	95,000
(14)	Net investment income earned	200,000
(15)	Net realized capital gains	400
(16)	Net unrealized capital gains	50,000
(17)	Agents balances or uncollected premiums	500,000
(18)	Net loss from agents balances charged off	2,000
(19)	Increase in non-admitted assets	1,000
(20)	Bonds	5,000,000
(21)	Stocks	2,500,000
(22)	Reinsurance recoverable on loss payments	20,000
(23)	Cash and bank deposits	400,000
(24)	Interest, dividends and real estate income due and accrued	52,000
(25)	Contingency reserve	1,100,000
(26)	Capital paid up	250,000
(27)	Surplus as regards policyholders 12-31-62	2,338,600

In answering Questions 9 through 12, use the item numbers, rather than their descriptions, to conserve time.

9. Prepare the page 4 Statement of Income for Company A's 1963 Annual Statement.

10. Prepare the page 4 Capital and Surplus Account for Company A's 1963 Annual Statement.

11. Prepare the page 2 exhibit of "Assets" for Company A's 1963 Annual Statement.

12. Prepare the page 3 exhibit of "Liabilities, Surplus And Other Funds" for Company A's 1963 Annual Statement.

13. Because of the major emphasis on solvency under the Convention statement there are a number of major differences from generally accepted accounting principles. Name three such differences.

14. Part III of the Expense Exhibit includes the following two items:

    Item 2. Adjustment for Premium Discounts and Retrospective Rating.

    Item 12. Effect of Expense Graduation.

    Explain these two items and state what condition will result in their being equal.

15. Describe the theory upon which premium discount plans are based.

16. Describe the purpose of Schedule O. Demonstrate how this purpose is accomplished by the Schedule.

17. Describe the content of page 14 of the Annual Statement. What is the footnote on this page which is designed to assist carriers in simplifying the calculation of certain figures on that exhibit.

18. A direct writing company's remuneration plan for its employee-salesmen provides that the company will pay these men 10% of the premium written subject to a minimum annual salary of $6,000. The company employed two such salesmen for the entire Calendar Year 1963, one of whom wrote $50,000 in premium while the other wrote $100,000. According to the Instructions for Uniform Classification of Expenses (Regulation 30), how should this expense be allocated by Operating Expense Classification and Expense Group in Part I of the Insurance Expense Exhibit. State your reasons for such allocation.

19. List the five major categories of paid expenses that were analyzed by size of risk in the 1949 study and briefly describe the results of the study.

PART III

Section (a)

1. (a) What are the three functions of an individual risk rating plan?
   (b) Name four types of rating plans and briefly discuss each.

2. (a) What are the three conditions to which credibility must be subject as discussed by Perryman in his paper on "Experience Rating Plan Credibilities"?
   (b) What is the K in the K formula for credibility and how is it usually determined?

3. The Fire Bureaus in a number of states have introduced a "Disappearing Deductible Clause" applicable to most non-dwelling properties. Describe the features of this plan, including a comparison with similar plans in the field.

4. (a) What are D-Ratios as used in workmen's compensation?
   (b) What are excess pure premium ratios?

5. (a) What are the essential features of the premium adjustment portion of the National Defense Projects Rating Plan?
   (b) How is the Standard Premium for Automobile and General Liability insurance determined under this plan?

6. (a) During the last year, casualty type experience and schedule rating plans have been introduced for use with Special Multi-Peril programs. This action has been severely criticized by several different groups. Give three of the arguments generally offered against their continued use.
   (b) What changes would you propose in order to make these plans more acceptable while not hurting their effectiveness as rating plans?

7. You have been asked to prepare a Plan D quotation which includes automobile liability with a $100 deductible applicable to property damage. The following information was given to you:

(1) Property damage $100 deductible discount .40

(2) Loss and expense ratio allowances in the manual rates:

Total production cost allowance	.200
Administration	.055
Inspection and Bureau	.010
Taxes, Licenses and Fees	.030
Underwriting Profit and Contingencies	.050
Loss adjustment factors—B.I.	1.10
P.D.	1.16
Combined B.I. and P.D. expected loss ratio	.580

Determine the following:

(1) The deductible property damage expected loss ratio.

(2) Basic limits property damage deductible expense ratio.

8. Describe the necessary steps involved in developing a composite rate.

9. In P.C.A.S. XXXVII, D. R. Uhthoff describes a practical procedure for computing state excess loss ratios by use of loss distributions developed from claims as reported under the National Council Unit Statistical Plan. After the overall excess ratios are developed as weighted averages of the individual excess ratios for Death, Permanent Total, and Major Permanent Partial, he applies the standard permissible loss ratios to these overall excess ratios. Explain why this is necessary (formula explanation is acceptable).

10. A number of plans introduced recently in the property field have contained a provision essentially as follows:

> "This filing contemplates the standard allowance for expenses. If expenses are less than standard, the rate modification, if a credit, shall be increased or, if a debit, shall be decreased by the amount of reduction in such expense."

Discuss this provision, including its pros and cons and its effect on established procedure.

SECTION (b)

11. In a Presidential Address to the C.A.S., *Multiple Line Underwriting,* J. M. Cahill gave six reasons why the development of "all risk policies" would not come quickly. Give three of these reasons.

12. Discuss the comparative reinsurance needs of the following types of property insurers:

    a. Local county mutuals

    b. Specialized stock company pools or associations such as Factory Insurance Association, Oil Insurance Association, etc.

    c. Stock company operating through independent agency system.

13. You are an Underwriter for a large company specializing in Individual Disability Income Benefits policies. Name six important factors which you would consider in accepting and rating a new risk.

14. Name and briefly define five types of reinsurance covers.

15. Listed below is data for several insurance companies

	(a)	(b)	(c)
Written Premiums	12,000,000	6,000,000	19,500,000
Unearned Premiums	7,000,000	5,000,000	11,000,000
Earned Premiums	12,000,000	6,000,000	18,000,000
Paid Losses	7,000,000	4,500,000	10,000,000
Outstanding Losses	5,000,000	2,000,000	11,000,000
Incurred Losses	7,500,000	4,000,000	11,500,000
Policyholders Surplus	12,000,000	9,000,000	20,500,000

    a. If these were all basically fire companies, how would Roger Kenney probably rank them as to strength?

    b. If basically casualty, how would he rank them?

16. A large single line company is evaluating the possibilities of going into a multiple line operation. What probable effects should the company expect such an action to have on its capital structure?

17. Some companies in issuing interim operating reports (that is, for other than the calendar year) show results for the full twelve months ending with the date of the report. Discuss.

18. Your company has just decided to enter the group insurance field. What recommendations would you make in regard to risks of less than fifty lives to better assure an underwriting profit?

19. List six of the reasons which have been advanced for fire and casualty companies to acquire life companies.

20. Discuss the advantages and disadvantages of multiple line underwriting as encountered in the more recent package policies as compared with the traditional compartmentalized underwriting.

PART IV

Section (a)

1. (a) (8 Points)

What statistics are published by or are available from the National Board of Fire Underwriters? Include in your answer compilations which the National Board regards as "confidential" and supplies "to the insurance departments and rating bureaus of the respective states and also to member and subscriber companies".

(b) (2 Points)

Describe the significant changes which became effective at the beginning of 1963 in the statistical area with respect to the automobile line of insurance.

2. (5 Points)

Describe briefly the contents of each of the following publications and state the source document(s) from which the various publishers obtain the information.

(a) Spectator—Insurance by States

(b) Best's Fire and Casualty Aggregates and Averages

(c) Argus Chart

(d) Spectator Handy Chart

3. (5 Points)

Of what does the Spectator's "Factual Financial Appraisal" as shown in the Fire Index consist and on what is it based?

4. (10 Points)

An objector to a workmen's compensation rate filing refers to compensation underwriting results for stock carriers in Best's Fire and Casualty Aggregates and Averages. As Insurance department actuary, do you think these results should have any bearing on the propriety of the proposed rates? Why?

5. (30 Points)

Sketch a multiple peril policy of your own design. Include specifically the category of risk for which you intend this policy, the major lines which will be mandatory or optional, your method of establishing the initial premium charge, and your proposed subsequent rating treatment of this policy either in the context of existing rating organizations or as an independent venture. Outline a statistical plan which will meet the statutory requirements of a selected state and will provide the basis for your rating treatment or analysis.

6. (20 Points)

By specific reference to a line or kind of insurance for which traditional premium, loss, and exposure statistics in recent years have not been, in your opinion, a satisfactory basis for prospective ratemaking, develop a procedure utilizing external non-insurance data to attempt to correct the deficiencies you have noted. If you are opposed to introducing external data in your ratemaking, set forth your objections and suggest a modification of the existing statistical program which could improve your ratemaking methods.

7. (10 Points)

Some companies have explored the possibility of retaining punched cards as the basic file-keeping medium, while using magnetic tape electronic equipment as the processing medium. What are the advantages and disadvantages of such a system?

8. (a) (8 Points)

Define the following terms as used in Punched Card Data Processing.

a. Collating
b. Control Panel
c. Detail Printing
d. Gang Punching
e. Group Printing
f. Reading
g. Verification
h. Zone Punch

(b) (2 Points)

What are "COBOL" and "FORTRAN"? How do they differ?

SECTION (b)

9. (10 Points)

The Comprehensive Dwelling Policies, the Homeowners Policies and the Commercial Property Coverage Policies are examples of the various types of multiple line coverages that have been developed. Briefly describe the methods used in rating each of these policies.

10. (10 Points)

It has sometimes been suggested that the effect of wage changes should be included in the determination of workmen's compensation rate levels because compensation premiums are based on payrolls and will increase with the increase in payrolls while losses, which are also based on wages, will increase to a lesser degree. Discuss, giving your reasons for agreeing or disagreeing with the suggestion.

11. (10 Points)

In the fire field, rating organizations have introduced in many states a revised method of rating dwellings and some other residential property with the method commonly being referred to as the "loss constant plan". Discuss this plan, including a description of the way it operates, the reasons why is was needed and the appropriateness of its title.

12. (10 Points)

What are the major differences in the rating procedures of the Factory Mutual companies as compared with other rating organizations in the fire field?

13. (10 Points)

How are the rating territories established under Massachusetts compulsory auto rating procedures?

14. (a) (5 Points)

You have been furnished with some data on New York Disability Benefits Law Insurance experience which reveals the following:

(1) Female morbidity is about 1.7 times that of male morbidity.

(2) The covered payroll for women is 22% of the total covered payroll.

(3) The average claim cost per $1.00 of weekly benefits exposed was $.326.

Determine the average claim cost for males for each $1.00 of weekly benefits exposed.

(b) (10 Points)

Experience shows that the claim cost for Statutory D.B.L. Coverage is approximately 60% of the cost of 8th day, 13 weeks plans. What reasons would you give for this difference?

15. (10 Points)

Outline and briefly discuss the procedure discussed by Mr. J. M. Cahill in P.C.A.S. XXVII for developing rates for Workmen's Compensation Excess Coverage (Per Accident Basis) for Self-Insurers.

16. (a) (5 Points)

What are the basic elements entering into premium determination in individual health insurance?

(b) (5 Points)

As used in Bartelson's "Health Insurance," what is the difference between "realistic" and "conservative" assumptions in determining premiums?

(c) (5 Points)

What are the major factors to be recognized in the classification of risks in individual health insurance?

17. (10 Points)

In late 1963 a meeting of representatives of stock agent groups and major stock companies resulted in a set of recommendations with respect to insurance rating which have come to be known as the "Johnson Plan" or "Johnson Principles". With what area of insurance rating are these recommendations concerned? Briefly, what are the general provisions of the recommendations?

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INDEX TO PROCEEDINGS

OF THE

Casualty Actuarial Society

VOLUMES I TO L INCLUSIVE

NEW YORK

1965

Acknowledgment

An index is prepared for every ten volumes of the *Proceedings* but on the occasion of our Fiftieth Anniversary, it was felt desirable to include herein a grand index encompassing the first fifty volumes. Although this is confined principally to papers and authors, it was nevertheless an arduous task. The Society is grateful to Mr. Luther L. Tarbell, Jr., Assistant Editor, for accepting the assignment, and for performing so effectively.

THOMAS E. MURRIN,
President

FIFTY YEAR INDEX

CASUALTY

ACTUARIAL SOCIETY

ORGANIZED 1914

1965 YEAR BOOK

Corrected to December 1, 1964

FOREWORD

The Casualty Actuarial Society was organized in 1914 as the Casualty Actuarial and Statistical Society of America, with 97 charter members of the grade of Fellow; the Society adopted its present title on May 14, 1921.

The roots of actuarial science are found in England, dating back as far as 1792, in the early days of life insurance. Due to the technical nature of the business, the first actuaries were mathematicians and eventually the growth of their numbers resulted in the formation of the Institute of Actuaries in Great Britain in 1848. A similar organization, the Faculty of Actuaries, was founded in Scotland in 1856. This was followed in the United States by the Actuarial Society of America in 1889 and the American Institute of Actuaries in 1909. These two actuarial bodies were merged in 1949 to form the Society of Actuaries.

In the meantime, problems requiring actuarial treatment were emerging in sickness, disability and casualty insurance, particularly workmen's compensation which began in 1911. These problems were quite different from life insurance and led to the organization of the Casualty Actuarial Society in 1914 which was brought about through the suggestion of Dr. I. M. Rubinow who became the first president. Since the problems surrounding workmen's compensation were at that time the most urgent, many of the members played a leading part in the development of the scientific basis upon which workmen's compensation insurance now rests. The object of the Society was, and is, the promotion of actuarial and statistical science as applied to the problems of insurance other than life insurance by means of personal intercourse, the presentation and discussion of appropriate papers, the collection of a library and such other means as may be found desirable.

From its beginning the Society has grown constantly in membership, in the scope of its interests and in its contributions to the formulation of scientific standards for the computation of rates and reserves for the many lines of business in the non-life field. These contributions are found in the original papers prepared by members of the Society and printed in the *Proceedings* which are published annually. Other papers deal with acquisition costs, pension funds, legal decisions, investments, claims, reinsurance, accounting, statutory requirements, loss reserves, statistics, and the examination of insurance companies. The presidential addresses constitute a valuable record of the actuarial problems, some of them still unsolved, which have faced the insurance industry over the years.

At the November 1950 meeting of the Society, the Constitution and By-Laws were amended to enlarge the scope of the Society to include all lines of insurance other than life insurance. The effect of the amendment was to include fire and allied lines insurance, in recognition of the multiple line power granted by many states to both casualty companies and fire companies.

The membership of the Society consists of actuaries who are employed by insurance companies, ratemaking organizations, state insurance departments, and as independent consultants. The Society has two grades of membership comprised of Fellowship and Associateship. Examinations for these two classes of membership are held during the second or third week of May in various cities in the United States and Canada. In addition, the examination for Associateship, Part I, is held in November of each year.

On the inside, front cover of the *Year Book* are listed the Proceedings and other publications of the Society and the prices thereof. The *Year Book* is published annually. *Recommendations for Study* is a pamphlet which outlines the course of study to be followed for admission. These two booklets may be obtained free upon application to the Secretary-Treasurer, Albert Z. Skelding, 200 E. 42nd Street, New York, N. Y. 10017.

CASUALTY ACTUARIAL SOCIETY

NOVEMBER 1, 1964

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Matthew H. McConnell
Herbert E. Wittick

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Lewis H. Roberts
LeRoy J. Simon

NOVEMBER 1, 1964

Those Marked (†) were Charter Members at date of organization, November 7, 1914

Admitted	
Nov. 21, 1930	AINLEY, JOHN W., (Retired), 33 Paxton Road, West Hartford, Conn. 06107
Nov. 20, 1964	ALDRICH, WILLIAM C., Secretary, National Council on Compensation Insurance, 200 East 42nd Street, New York, N. Y. 10017
Nov. 20, 1964	ALEXANDER, LEE M., Assistant Actuary, American Mutual Liability Insurance Company, Wakefield, Mass. 01881
Nov. 14, 1947	ALLEN, EDWARD S., Actuary, The Phoenix of Hartford Insurance Companies, 61 Woodland Street, Hartford 15, Conn.
Nov. 13, 1931	AULT, GILBERT E., Actuary, Church Pension Fund & Church Life Insurance Corporation, 20 Exchange Place, New York 5, N. Y.
Nov. 18, 1955	BAILEY, ROBERT A., Vice President, Bank A Count Corporation, P. O. Box 246, Wisconsin Rapids, Wis. 54494
Nov. 15, 1962	BALCAREK, RAFAL J., Actuary, Reliance Insurance Company, 401 Walnut Street, Philadelphia, Pa. 19106
Nov. 20, 1924	BARBER, HARMON T., (Retired), 18 Ridgewood Road, Windsor, Conn. 06095
Nov. 19, 1954	BARKER, GORDON M., Actuary, Great American Group, 99 John Street, New York, N. Y. 10038
Nov. 14, 1947	BARKER, LORING M., Actuary, Fireman's Fund American Insurance Companies, 3333 California Street, San Francisco, Calif. 94120
Nov. 20, 1942	BART, ROBERT D., Director of Industrial Relations and Assistant Treasurer, The West Bend Company, 400 W. Washington Street, West Bend, Wis. 53095
Nov. 18, 1932	BARTER, JOHN L., 90 Tunxis Road, West Hartford, Conn. 06107
Nov. 13, 1931	BATHO, ELGIN R., Vice President and Actuary, Berkshire Life Insurance Company, 700 South Street, Pittsfield, Mass. 01203
Nov. 14, 1958	BENBROOK, PAUL, Vice President, American General Insurance Company, 2727 Allen Parkway, Houston, Texas 77019
Nov. 16, 1956	BENNETT, NORMAN J., Assistant Secretary and Actuary, Continental Insurance Companies, 80 Maiden Lane, New York, N. Y. 10038
Nov. 22, 1934	BERKELEY, ERNEST T., Actuary, Employers' Group of Insurance Companies, 110 Milk Street, Boston, Mass. 02107
Nov. 22, 1957	BERQUIST, JAMES R., Associate Actuary, Employers Mutuals of Wausau, 407 Grant Street, Wausau, Wis.
Nov. 19, 1953	BEVAN, JOHN R., Assistant Actuary, Liberty Mutual Insurance Company, 175 Berkeley Street, Boston 17, Mass.
†	BLACK, S. BRUCE, Honorary Chairman, Liberty Mutual Insurance Company, 175 Berkeley Street, Boston 17, Mass.
Apr. 20, 1917	BLANCHARD, RALPH H., Professor Emeritus of Insurance, Columbia University, Plympton, Mass. 02367
Nov. 19, 1959	BLODGET, HUGH R., Assistant Actuary, Aetna Casualty and Surety Company & Standard Fire Insurance Company, 151 Farmington Avenue, Hartford 15, Conn.
Nov. 16, 1956	BONDY, MARTIN, Assistant Vice President and Actuary, Consolidated Mutual Insurance Company, 345 Adams Street, Brooklyn 1, N. Y.
Nov. 22, 1957	BORNHUETTER, RONALD L., Associate Actuary, National Bureau of Casualty Underwriters, 125 Maiden Lane, New York N. Y. 10038
Nov. 16, 1956	BOYAJIAN, JOHN H., Actuary, National Board of Fire Underwriters, 85 John Street, New York 38, N. Y.

Admitted	
Nov. 19, 1959	BOYLE, JAMES I., Assistant Actuary, The Travelers Insurance Company, One Tower Square, Hartford, Conn. 06115
Nov. 16, 1961	BRANNIGAN, JAMES F., Assistant Actuary, The Travelers Insurance Company, One Tower Square, Hartford, Conn. 06115
†	BREIBY, WILLIAM, Consulting Actuary, William Breiby, F.S.A., F.C.A.S., Pacific Mutual Life Bldg., 523 West 6th St., Los Angeles, Calif. 90014
Nov. 21, 1952	BRINDISE, RALPH SULLIVAN, Supervisor, Insured Benefit Plans, Standard Oil Company (Indiana), Box 5910A, Chicago, Ill. 60680
Nov. 18, 1927	BROWN, F. STUART, (Retired), Cedar Land Road, Orleans, Mass. 02653
Oct. 22, 1915	BROWN, HERBERT D., (Retired), Glenora-on-Lake Seneca, Dundee, N. Y.
Nov. 16, 1961	BUDD, EDWARD H., Assistant Secretary, The Travelers Insurance Company, One Tower Square, Hartford, Conn. 06115
Nov. 23, 1928	BURLING, WILLIAM H., Manager and Actuary for Canada Group, The Travelers Insurance Company, Suite 1306, 7 King Street, East, Toronto 1, Ontario, Canada
Nov. 19, 1959	BYRNE, HARRY T., Assistant Actuary, Aetna Casualty and Surety Company & Standard Fire Insurance Company, 151 Farmington Avenue, Hartford, Conn. 06115
Nov. 19, 1929	CAHILL, JAMES M., General Manager, National Bureau of Casualty Underwriters, 125 Maiden Lane, New York, N. Y. 10038
Nov. 18, 1932	CAMERON, FREELAND R., Senior Vice President, Swiss National Insurance Company, U. S. A., 901 N. E. Second Avenue, Miami, Fla. 33132
Nov. 17, 1938	CARLETON, JOHN W., Vice President, Liberty Mutual Insurance Company, 175 Berkeley Street, Boston, Mass. 02117
Nov. 18, 1949	CLARKE, JOHN W., President, General Reinsurance Life Corporation, 400 Park Avenue, New York 22, N. Y.
Nov. 15, 1918	COATES, BARRETT N., (Retired), 1007 Cragmont Avenue, Berkeley, Calif. 94708
Nov. 17, 1922	COATES, CLARENCE S., Actuary, Lumbermens Mutual Casualty Company, 4750 Sheridan Road, Chicago, Ill. 60640
Feb. 19, 1915	COLLINS, HENRY, (Retired), Lochbrae, Windermere, Fla.
Nov. 22, 1934	COOK, EDWIN A., President and General Manager, Interboro Mutual Indemnity Insurance Company, 270 Madison Avenue, New York 16, N. Y.
Nov. 18, 1925	CORCORAN, WILLIAM M., Consulting Actuary, Wolfe, Corcoran & Linder, 116 John Street, New York 38, N. Y.
Nov. 20, 1964	CRAIG, ROBERT A., Actuary, Multi-Line Insurance Rating Bureau, 110 William Street, New York, N. Y. 10038
Nov. 19, 1926	CRANE, HOWARD G., Vice President and Treasurer, General Reinsurance Corporation, 400 Park Avenue, New York, N. Y. 10022
Nov. 21, 1952	CRITCHLEY, DOUGLAS, E. B. Savory & Company, London, England.
Nov. 22, 1946	CROUSE, CHARLES W., Consulting Actuary, C. E. Preslan & Company, Inc., 20015 Detroit Road, Cleveland 16, Ohio.
Nov. 18, 1960	CROWLEY, JAMES H., Assistant Secretary, Accounts Dept., Aetna Life Affiliated Companies, 151 Farmington Avenue, Hartford, Conn. 06115
Nov. 19, 1953	CURRY, HAROLD E., Senior Vice President, State Farm Mutual Automobile Insurance Company, 112 E. Washington Street, Bloomington, Ill. 61701
Nov. 18, 1932	DAVIES, E. ALFRED, (Retired), Falls Village, Conn.

Admitted	
Nov. 18, 1927	DAVIS, EVELYN M., Partner, Woodward, Ryan, Sharp & Davis, Consulting Actuaries, 26 Broadway, Room 708, New York 6, N. Y.
May 25, 1956	DAY, ELDEN W., (Retired), 758 Edwards Avenue, Fairhope, Ala. 36532
Nov. 18, 1960	DICKERSON, O. D., Associate Professor, Florida State University, Tallahassee, Fla.
Nov. 16, 1951	DOREMUS, FREDERICK W., (Retired), 120 So. Harrison Street, East Orange, N. J. 07018
Nov. 17, 1920	DORWEILER, PAUL, (Retired), 51 Wethersfield Avenue, Hartford, Conn. 06114
Nov. 22, 1957	DROBISCH, MILES R., Assistant Actuary, California Inspection Rating Bureau, 1453 Mission Street, San Francisco 3, Calif.
Nov. 14, 1958	DROPKIN, LESTER B., Actuary, California Inspection Rating Bureau, 1453 Mission Street, San Francisco, Calif. 94103
Nov. 24, 1933	EDWARDS, JOHN, Consulting Actuary, 91 Arundel Avenue, Toronto 6, Ontario, Canada
Nov. 19, 1959	EIDE, K. ARNE, Statistical Bureau, Actuarial Div., Metropolitan Life Insurance Company, 1 Madison Avenue, New York 10, N. Y.
Nov. 15, 1940	ELLIOTT, GEORGE B., General Manager, Pennsylvania Compensation Rating Bureau, 1819 John F. Kennedy Blvd., Philadelphia, Pa. 19103
Nov. 17, 1922	ELSTON, JAMES S., (Retired), 1640 Palmer Avenue, Winter Park, Fla.
Nov. 15, 1935	EPPINK, WALTER T., 1st Vice President, Treasurer & Actuary, Merchants Mutual Insurance Company, 268 Main Street, Buffalo, N. Y. 14205
Nov. 14, 1958	ESPIE, ROBERT G., Vice President and Assistant Comptroller, Aetna Life Affiliated Companies, 151 Farmington Avenue, Hartford 15, Conn.
Nov. 18, 1955	FAIRBANKS, ALFRED V., Actuary, Monarch Life Insurance Company, 1250 State Street, Springfield, Mass. 01101
†	FALLOW, EVERETT S., (Retired), 28 Sunset Terrace, West Hartford, Conn.
Nov. 15, 1940	FARLEY, JARVIS, First Vice President, First Assistant General Manager and Treasurer, Massachusetts Indemnity & Life Insurance Company, 654 Beacon Street, Boston, Mass. 02215
†	FARRER, HENRY, (Retired), R. D. #3, Box 322, Fleetwood, Pa. 19522
Nov. 18, 1960	FAUST, J. EDWARD, Consulting Actuary, R. R. #1, Zionsville, Ind.
May 25, 1956	FINNEGAN, JOSEPH H., Manager, Actuarial Bureau, National Board of Fire Underwriters, 85 John Street, New York 38, N. Y.
Nov. 16, 1961	FITZGIBBON, WALTER J., JR., Assistant Actuary, Aetna Casualty and Surety Company & Standard Fire Insurance Company, 151 Farmington Avenue, Hartford, Conn. 06115
Nov. 15, 1935	FITZHUGH, GILBERT W., President, Metropolitan Life Insurance Company, One Madison Avenue, New York, N. Y. 10010
Nov. 18, 1955	FOSTER, ROBERT B., Associate Actuary, The Travelers Insurance Company, One Tower Square, Hartford, Conn. 06115
Nov. 18, 1955	FOWLER, THOMAS W., Actuary, North American Reinsurance Corp., 161 E. 42nd Street, New York 17, N. Y.
Nov. 18, 1927	FREDERICKSON, C. H., Consulting Actuary, 3434 Eglinton Ave. E., Scarboro, Ontario, Canada.
Nov. 22, 1934	FULLER, GARDNER V., (Retired), Conover, Wis.

Admitted	
Nov. 22, 1957	GILLAM, WILLIAM S., Secretary and Actuary, National Bureau of Casualty Underwriters, 125 Maiden Lane, New York, N. Y. 10038
Nov. 20, 1964	GILLESPIE, JAMES E., Assistant Actuary, Continental National American Group, 310 South Michigan Avenue, Chicago, Ill. 60604
Nov. 20, 1924	GINSBURGH, HAROLD J., (Retired), 14 Crestview Road, Belmont, Mass. 02178
Nov. 21, 1930	GLENN, JOSEPH B., Actuarial Consultant, Department of Defense, 6110 Valley Road, Washington, D. C. 20034
Nov. 13, 1931	GODDARD, RUSSELL P., Actuary, Bowles, Andrews & Towne, Inc., 200 Park Avenue, New York, N. Y. 10017
†	GOODWIN, EDWARD S., (Retired), Investment Counselor, 96 Garvan Street, East Hartford 8, Conn.
Nov. 19, 1926	GRAHAM, CHARLES M., Fire and Casualty Actuary, Florida Insurance Dept., Carlton Building, Tallahassee, Fla. 32304
Nov. 19, 1953	GRAVES, CLYDE H., Assistant Manager and Actuary, Mutual Insurance Rating Bureau and Mutual Insurance Advisory Association, 733 Third Avenue, New York, N. Y. 10017
†	GREENE, WINFIELD W., President, W. W. Greene, Inc., 32 Cliff Street, New York, N. Y. 10038; Vice President and Treasurer, Old Republic Life Insurance Co. of New York, 99 John Street, New York, N. Y. 10038
Nov. 19, 1953	HALEY, JAMES B., JR., Coates, Herfurth & England, Consulting Actuaries, Crocker Bldg., San Francisco, Calif.
Nov. 16, 1956	HART, W. VAN BUREN, JR., Aetna Insurance Company, 55 Elm Street, Hartford, Conn. 06115
Nov. 17, 1950	HARWAYNE, FRANK, Chief Actuary, New York State Insurance Department, 123 William Street, New York 38, N. Y.
Nov. 19, 1926	HAUGH, CHARLES J., (Retired), 25 LeMay Street, West Hartford, Conn. 06107
Nov. 17, 1950	HAZAM, WILLIAM J., Vice President and Actuary, American Mutual Liability Insurance Company, Wakefield, Mass.
Nov. 16, 1951	HEWITT, CHARLES C., JR., Actuary, Allstate Insurance Company, 7447 Skokie Blvd., Skokie, Ill. 60078
Nov. 16, 1961	HOBBS, EDWARD J., Associate Actuary, Insurance Company of North America, 1600 Arch Street, Philadelphia 1, Pa.
Nov. 22, 1934	HOOKER, RUSSELL O., Russell O. Hooker, & Associates, Consulting Actuaries, 266 Pearl Street, Hartford 3, Conn.
Nov. 17, 1950	HOPE, FRANCIS J., Actuary, Hartford Insurance Group, 690 Asylum Avenue, Hartford, Conn. 06115
Nov. 14, 1947	HUGHEY, M. STANLEY, Executive Vice President, Lumbermens Mutual Casualty Company, 4750 N. Sheridan Road, Chicago, Ill. 60640
Nov. 19, 1959	HUNT, FREDERIC J., JR., Assistant Secretary, Insurance Company of North America, 1600 Arch Street, Philadelphia, Pa. 19101
Nov. 18, 1955	HURLEY, ROBERT L., Actuary, Fire Insurance Research and Actuarial Association, 125 Maiden Lane, New York, N. Y. 10038
Nov. 19, 1954	JOHE, RICHARD L., Vice President and Actuary, United States Fidelit and Guaranty Company, Calvert & Redwood Streets, Baltimore 3, Md.
Nov. 14, 1941	JOHNSON, ROGER A., Actuary, Blue Cross of Greater Philadelphia, 1333 Chestnut Street, Philadelphia, Pa. 19107
Nov. 16, 1939	JONES, HAROLD M., Group Statistician, John Hancock Mutual Life Insurance Company, 200 Berkeley Street, Boston 17, Mass.
Nov. 16, 1956	KALLOP, ROY H., Actuary, National Council on Compensation Insurance, 200 E. 42nd Street, New York 17, N. Y.

Admitted	
Nov. 22, 1957	KATES, PHILLIP B., Executive Vice President, Southern Fire & Casualty Company, P. O. Box 240, Knoxville 1, Tenn.
Nov. 19, 1926	KELTON, WILLIAM H., (Retired), 122 Arundel Avenue, West Hartford, Conn. 06107
Nov. 19, 1959	KLAASSEN, ELDON J., Associate Actuary, Continental National American Group, 310 S. Michigan Avenue, Chicago, Ill. 60604
Nov. 14, 1941	KOLE, MORRIS B., Director of Planning and Data Processing, The State Insurance Fund, 199 Church Street, New York, N. Y. 10007
Nov. 24, 1933	KORMES, MARK, President, Actuarial Associates, Inc., 405 Lexington Avenue, New York, N. Y. 10017
Nov. 19, 1953	KUENKLER, ARTHUR S., Executive Vice President, Security Insurance Group, 175 Whitney Avenue, New Haven, Conn. 06505
Nov. 18, 1949	LACROIX, HAROLD F., Second Vice President, The Travelers Insurance Company, 700 Main Street, Hartford 15, Conn.
Nov. 20, 1964	LANGE, JEFFREY T., Assistant Actuary, Research Division, National Bureau of Casualty Underwriters, 125 Maiden Lane, New York, N. Y. 10038
May 5, 1961	LATIMER, MURRAY W., Murray W. Latimer Industrial Relations Consultants, 1625 K Street, N. W., Washington, D. C. 20006
Nov. 17, 1950	LESLIE, WILLIAM, Jr., Vice President and Actuary, Continental Insurance Companies, 80 Maiden Lane, New York, N. Y. 10038
Nov. 16, 1961	LINDEN, JOHN R., Assistant Actuary, Aetna Casualty and Surety Company, 151 Farmington Avenue, Hartford, Conn. 06115
Nov. 20, 1924	LINDER, JOSEPH, 25 Roosevelt Terrace, Bayonne, N. J. 07002
Nov. 16, 1956	LINO, RICHARD, Actuary, National Bureau of Casualty Underwriters, 125 Maiden Lane, New York, N. Y. 10038
Nov. 18, 1955	LISCORD, PAUL S., Actuary, The Travelers Insurance Company, One Tower Square, Hartford, Conn. 06115
Nov. 17, 1950	LIVINGSTON, GILBERT R., Casualty Actuary, Connecticut Insurance Department, State Office Bldg., Hartford 15, Conn.
Nov. 16, 1951	LONGLEY-COOK, LAURENCE H., Vice President and Actuary, Insurance Company of North America, 1600 Arch Street, Philadelphia, Pa. 19101
Nov. 13, 1936	LYONS, DANIEL J., President, Guardian Life Insurance Company of America, 201 Park Avenue South, New York, N. Y. 10003
Nov. 1, 1963	MACGINNITIE, W. JAMES, Actuary, Seguros Bolivar, Apartado Aereo 4421, Bogata, Colombia.
Nov. 19, 1954	MACKEEN, HAROLD E., Assistant Actuary, The Travelers Insurance Company, One Tower Square, Hartford, Conn. 06115
Nov. 18, 1958	MAGRATH, JOSEPH J., (Retired), 200 Springfield Avenue, Summit, N. J. 07901
Nov. 22, 1957	MAKGILL, STEPHEN S., Assistant Actuary, The Travelers Insurance Company, One Tower Square, Hartford, Conn. 06115
Nov. 23, 1928	MARSHALL, RALPH M., (Retired), Catts Corner, Worton, Kent County, Md.
Nov. 18, 1927	MASTERSON, NORTON E., Vice President and Actuary, Sentry Insurance—Hardware Mutuals Group, 200 Strongs Avenue, Stevens Point, Wis. 54482
Nov. 19, 1926	MATTHEWS, ARTHUR N., 475 Poquonoch Avenue, Windsor, Conn. 06095
May 19, 1915	MAYCRINK, EMMA C., 32 Chittenden Avenue, Crestwood, N. Y.
Nov. 14, 1958	MAYERSON, ALLEN L., Commissioner of Insurance, State of Michigan, Lewis Cass Bldg., Lansing, Mich.

Admitted	
Nov. 1, 1963	McClure, Richard D., Vice President, Citizens Casualty Company of New York, 33 Maiden Lane, New York, N. Y. 10038
Nov. 15, 1935	McConnell, Matthew H., Superintendent, Compensation & Liability Dept., General Accident Fire and Life Assurance Corporation Ltd., 414 Walnut Street, Philadelphia, Pa. 19106
Nov. 18, 1960	McGuinness, John S., Consultant in Actuarial Science and Management, 41 Horicon Avenue, Glens Falls, N. Y. 12801
Nov. 20, 1964	McLean, George E., Actuary, Massachusetts Hospital Service, Inc. and Massachusetts Medical Service, 133 Federal Street, Boston, Mass. 02106
Nov. 15, 1962	McNamara, Daniel J., Secretary, National Bureau of Casualty Underwriters, 125 Maiden Lane, New York, N. Y. 10038
Nov. 15, 1962	Meenaghan, James J., Associate Actuary, Fireman's Fund American Insurance Companies, 3333 California Street, San Francisco, Calif. 94120
Nov. 18, 1955	Menzel, Henry W., Actuary, New York Compensation Insurance Rating Board, 200 East 42nd Street, New York, N. Y. 10017
†	Michelbacher, Gustav F., (Retired), 15201 Quito Road, Saratoga, Calif. 95070
Nov. 17, 1938	Miller, John H., Executive Vice President, Monarch Life Insurance Company, 1250 State Street, Springfield, Mass. 01101
Nov. 1, 1963	Miller, Nicholas F., Jr., Aetna Casualty and Surety Company, 151 Farmington Avenue, Hartford, Conn. 06115
†	Milligan, Samuel, (Retired), 15 W. 55th Street, New York, N. Y. 10019
Nov. 18, 1937	Mills, John A., (Retired), Point Placid, Reeds Spring, Mo.
Nov. 22, 1957	Mills, Richard J., Assistant Actuary, Lumbermens Mutual Casualty Company, 4750 Sheridan Road, Chicago, Ill. 60640
Nov. 15, 1962	Morison, George D., Assistant Actuary, Aetna Casualty and Surety Company & Standard Fire Insurance Company, 151 Farmington Avenue, Hartford, Conn. 06115
Nov. 16, 1961	Moseley, Jack, Associate Actuary, United States Fidelity and Guaranty Company, Calvert and Redwood Streets, Baltimore, Md. 21203
Nov. 17, 1920	Mueller, Louis H., 2845 Lake Street, San Francisco 21, Calif.
Nov. 16, 1956	Muetterties, John H., Associate Actuary, Sentry Insurance—Hardware Mutuals Group, 200 Strongs Avenue, Stevens Point, Wis. 54482
Nov. 17, 1950	Munterich, George C., Assistant Secretary, Hartford Insurance Group, 690 Asylum Avenue, Hartford, Conn. 06115
Nov. 19, 1954	Murrin, Thomas E., Vice President and Actuary, Fireman's Fund American Insurance Companies, 3333 California Street, San Francisco, Calif. 94120
Nov. 19, 1959	Myers, Robert J., Chief Actuary, Social Security Administration, Department of Health, Education, and Welfare, Washington, D. C. 20201
Nov. 1, 1963	Nelson, S. Tyler, Casualty Division Manager and Actuary, American Agricultural Insurance Company, Room 1000 Merchandise Mart Plaza, Chicago, Ill. 60654
Nov. 14, 1958	Niles, Charles L., Jr., Assistant General Manager and Actuary, General Accident Group, 414 Walnut Street, Philadelphia, Pa. 19105
Nov. 15, 1935	Oberhaus, Thomas M., Vice President, Woodword and Fondiller, Inc., 420 Madison Avenue, New York, N. Y. 10017
†	Orr, Robert K., (Retired), 725 E. Palmetto, Lakeland, Fla.

Admitted	
Nov. 22, 1957	OTTESON, PAUL M., Vice President and Actuary, Federated Mutual Implement and Hardware Insurance Company, 129 E. Broadway, Owatonna, Minn. 55060
Nov. 21, 1919	OUTWATER, OLIVE E., (Retired), 2404 Loring Street, San Diego, Calif. 92109
Nov. 15, 1962	PARLIN, R. W., Research Associate, University of Minnesota, Laboratory of Physiological Hygiene, Stadium Gate 27, Minneapolis, Minn. 55455
Nov. 18, 1960	PENNYCOOK, ROD B., Health Insurance Secretary, The Great-West Life Assurance Company, 60 Osborne Street, N., Winnipeg 1, Manitoba, Canada.
Nov. 22, 1957	PERKINS, WILLIAM J., Assistant Group Actuary, The London Life Insurance Company, London, Ontario, Canada.
Nov. 14, 1941	PETERS, STEFAN, Consultant, Arthur D. Little, Inc., 35 Acorn Park, Cambridge, Mass.
Nov. 21, 1952	PETZ, EARL F., Associate Actuary, Lumbermens Mutual Casualty Company, 4750 N. Sheridan Road, Chicago, Ill. 60640
Nov. 19, 1959	PHILLIPS, HERBERT J., JR., Assistant Actuary, Employers' Liability Assurance Corporation, Ltd., 110 Milk Street, Boston, Mass. 02107
Nov. 24, 1933	PICKETT, SAMUEL C., (Retired), 126 Macktown Road, Windsor, Conn.
Nov. 22, 1957	PINNEY, ALLEN D., Assistant Secretary, The Travelers Insurance Company, 700 Main Street, Hartford 15, Conn.
Nov. 17, 1922	PINNEY, SYDNEY D., (Retired), 290 Wolcott Hill Road, Wethersfield, Conn. 06109
Nov. 19, 1959	POLLACK, ROBERT, Associate Actuary, American Mutual Liability Insurance Company, Wakefield, Mass. 01881
Nov. 13, 1931	PRUITT, DUDLEY M., Executive Secretary, Middle Atlantic Region, American Friends Service Committee, 1500 Race Street, Philadelphia 2, Pa.
Nov. 18, 1955	RESONY, ALLIE V., Assistant Secretary, Hartford Accident & Indemnity Company, 690 Asylum Avenue, Hartford 15, Conn.
Nov. 18, 1949	RESONY, JOHN A., Secretary, Group Dept., The Travelers Insurance Company, 700 Main Street, Hartford 15, Conn.
Nov. 16, 1951	RICE, HOMER D., (Retired), 1731 Morningside Drive, Mount Dora, P. O. Box 1017, Fla. 32757
Nov. 1, 1963	RICHARDS, HARRY R., Assistant Actuary, The Travelers Insurance Company, One Tower Square, Hartford, Conn. 06115
Nov. 1, 1963	RIDDLESWORTH, WILLIAM A., Assistant Actuary, Aetna Casualty and Surety Company & Standard Fire Insurance Company, 151 Farmington Avenue, Hartford, Conn. 06115
May 24, 1921	RIEGEL, ROBERT, Professor Emeritus of Statistics and Insurance, State University of New York at Buffalo, 3435 Main Street, Buffalo, N. Y. 14214
Nov. 14, 1958	ROBERTS, LEWIS H., Actuary, Woodward & Fondiller, 420 Madison Avenue, New York 17, N. Y.
Nov. 14, 1947	RODERMUND, MATTHEW, Vice President-Actuary, Munich Reinsurance Company, 410 Park Avenue, New York, N. Y. 10022
Nov. 14, 1947	ROSENBERG, NORMAN, Executive Assistant, Farmers Insurance Group, 4680 Wilshire Boulevard, Los Angeles 54, Calif.
Nov. 14, 1947	ROWELL, JOHN H., Assistant Vice President, Marsh & McLennan, Inc., 231 S. LaSalle Street, Chicago, Ill. 60604
Nov. 17, 1938	RUCHLIS, ELSIE, Actuarial Supervisor, National Bureau of Casualty Underwriters, 125 Maiden Lane, New York 38, N. Y.
Nov. 14, 1947	SALZMANN, RUTH E., Associate Actuary, Insurance Company of North America, 1600 Arch Street, Philadelphia, Pa. 19101

Admitted	
Nov. 1, 1963	SARASON, HARRY M., Managing Actuary, Woodward and Fondiller, Inc., 3625 W. 6th Street, Los Angeles, Calif. 90005
Nov. 19, 1948	SCHLOSS, HAROLD W., Secretary and Actuary, Royal-Globe Insurance Companies, 150 William Street, New York, N.Y. 10038
Nov. 18, 1937	SHAPIRO, GEORGE I., 934 E. 9th Street, Brooklyn 30, N. Y.
Nov. 13, 1931	SILVERMAN, DAVID, Consulting Actuary, Wolfe, Corcoran & Linder, 116 John Street, New York 38, N. Y.
Nov. 19, 1954	SIMON, LEROY J., Actuary, Insurance Company of North America, 1600 Arch Street, Philadelphia, Pa. 19101
Nov. 18, 1960	SIMONEAU, PAUL W., Assistant Actuary, Aetna Casualty & Surety Company & Standard Fire Insurance Company, 151 Farmington Avenue, Hartford, Conn. 06115
Nov. 19, 1929	SKELDING, ALBERT Z., Secretary-Treasurer, Casualty Actuarial Society, 200 E. 42nd Street, New York, N. Y. 10017
Nov. 19, 1929	SKILLINGS, E. SHAW, (Retired), 831 Ingleside Place, Evanston, Ill.
Nov. 18, 1932	SMICK, J. J., Consulting Actuary, Smick & Co., Inc., 135 East 42nd Street, New York, N. Y. 10017
Nov. 14, 1958	SMITH, EDWARD M., Associate Actuary, The Travelers Insurance Company, 700 Main Street, Hartford 15, Conn.
Nov. 15, 1940	SMITH, SEYMOUR E., Vice President and Actuary, The Travelers Insurance Company, One Tower Square, Hartford, Conn. 06115
Nov. 15, 1962	STANKUS, LEO M., Product Development Director, Allstate Insurance Company, 7447 Skokie Boulevard, Skokie, Ill. 60078
Nov. 24, 1933	ST. JOHN, JOHN B., Consulting Actuary, Box 57, Penllyn, Pa.
Nov. 19, 1959	SYKES, ZENAS M., Actuary, Social Security Administration, United States Department of Health, Education and Welfare, Washington 25, D. C.
May 25, 1956	TAPLEY, DAVID A., Senior Vice President, Wolverine Insurance Company, Wolverine-Federal Bldg., Battle Creek, Mich. 49016
Nov. 14, 1958	TARBELL, LUTHER L., JR., Associate Actuary, The Travelers Insurance Company, One Tower Square, Hartford, Conn. 06115
Nov. 16, 1956	THOMAS, JAMES W., Assistant Actuary, The Travelers Insurance Company, One Tower Square, Hartford, Conn. 06115
†	THOMPSON, JOHN S., Newark Athletic Club, Newark 2, N. J.
Nov. 19, 1953	TRIST, JOHN A. W., Assistant Secretary, Insurance Company of North America, 1600 Arch Street, Philadelphia, Pa. 19101
Nov. 15, 1962	TRUDEAU, DONALD E., Assistant Actuary, The Travelers Insurance Company, One Tower Square, Hartford, Conn. 06115
Nov. 14, 1947	UHTHOFF, DUNBAR R., Vice President and Actuary, Employers' Mutual Liability Insurance Company of Wisconsin and Employers' Mutual Fire Insurance Company, 407 Grant Street, Wausau, Wisconsin
Nov. 23, 1928	VALERIUS, NELS M., Associate Actuary, Aetna Casualty and Surety Company & Standard Fire Insurance Company, 151 Farmington Avenue, Hartford 15, Conn.
Nov. 21, 1919	VAN TUYL, HIRAM O., (Retired), 1411 Dexter Drive, Clearwater, Fla. 33516
Nov. 16, 1951	VINCENT, LEWIS A., Vice President, American Insurance Association, 85 John Street, New York, N. Y. 10038
Nov. 17, 1920	WAITE, ALAN W., 16 Penwood Road, Bloomfield, Conn.
Nov. 19, 1962	WALSH, ALBERT J., Assistant Vice President, Liberty Mutual Insurance Company, 175 Berkeley Street, Boston, Mass. 02117
Nov. 14, 1947	WIEDER, JOHN W., JR., Actuary, Aetna Casualty and Surety Company & Standard Fire Insurance Company, 151 Farmington Avenue, Hartford, Conn. 06115

Admitted	
Nov. 18, 1960	WILCKEN, CARL L., Actuary, Canadian Underwriters' Association, 31 Prince Andrew Place, Don Mills, Ontario, Canada
Nov. 1, 1963	WILLIAMS, DEWEY GENE, Assistant Secretary, Texas Employers Insurance Association and Employers Casualty Company, P. O. Box 2759, Dallas, Texas 75221
Nov. 15, 1935	WILLIAMS, HARRY V., Vice President, Hartford Insurance Group, 690 Asylum Avenue, Hartford, Conn. 06115
Nov. 22, 1957	WILLIAMS, P. ADGER, Actuary, The Travelers Insurance Company, 700 Main Street, Hartford, Conn. 06115
Nov. 14, 1941	WILLIAMSON, W. RULON, Research Actuary, 3400 Fairhill Drive, Washington 23, D. C.
Nov. 18, 1960	WILLSEY, LYNN W., Assistant Secretary, Group Dept., The Travelers Insurance Company, 700 Main Street, Hartford 15, Conn.
Nov. 16, 1961	WILSON, JAMES C., Vice President & Actuary, Security General Insurance Company and Security Fire and Indemnity Company, 639 W. Fifth Street, Box 3099, Winston-Salem, N. C.
Nov. 13, 1931	WITTICK, HERBERT E., Vice President and General Manager, Pilot Insurance Company, 1315 Yonge Street, Toronto 7, Ontario, Canada.
Nov. 18, 1949	WOLFRUM, RICHARD J., Actuary, Liberty Mutual Insurance Company, 175 Berkeley Street, Boston 17, Mass.
Nov. 16, 1951	WOODALL, JOHN P., Manager, South-Eastern Underwriters Association, P. O. Box 5048, Atlanta, Ga. 30302
Nov. 14, 1958	WRIGHT, BYRON, Actuary, Department of Banking and Insurance, State of New Jersey, State House Annex, Trenton 25, N. J.
Nov. 19, 1953	YOUNT, HUBERT W., (Retired), 54 Waban Avenue, Waban, Mass. 02168

NOVEMBER 1, 1964

Admitted	
Nov. 15, 1918	ACKERMAN, SAUL B., 405 Lexington Avenue, New York 17, N. Y.
Nov. 16, 1939	AIN, SAMUEL N., Consulting Actuary, 120 Broadway, New York 5, N. Y.
Nov. 16, 1961	ALDRICH, WILLIAM C., Secretary, National Council on Compensation Insurance, 200 E. 42 Street, New York 17, N. Y.
Nov. 22, 1957	ALEXANDER, LEE M., Actuarial Assistant, American Mutual Liability Insurance Company, Wakefield, Mass. 01881
Apr. 5, 1928	ALLEN, AUSTIN F., (Retired), 4815 Royal Lane, Dallas, Texas 75229
Nov. 15, 1962	AMLIE, WILLIAM P., Actuarial Assistant, Employers' Liability Assurance Company, 110 Milk Street, Boston, Mass. 02107
Nov. 18, 1955	ANDREWS, EDWARD C., Associate Actuary, The Travelers Insurance Company, One Tower Square, Hartford, Conn. 06115
Nov. 21, 1930	ARCHIBALD, A. EDWARD, Vice President, Investors Diversified Services, Inc., Minneapolis 2, Minn.
Nov. 19, 1959	BANNISTER, DAN W., Executive Vice President and General Manager, Horace Mann Mutual Insurance Company and Horace Mann Life Insurance Company, 216 E. Monroe Street, Springfield, Ill. 62701
Nov. 23, 1928	BATEMAN, ARTHUR E., Pine Grove Rest Home, Marlboro, Mass.
Nov. 15, 1940	BATHO, BRUCE W., Executive Vice President-Administration, Life Insurance Company of Georgia, 573 West Peachtree Street, N.E., Atlanta, Ga. 30308
Nov. 16, 1956	BERG, ROY A., JR., Assistant Actuary, Old Republic Life Insurance Company, 307 N. Michigan Avenue, Chicago 1, Ill.
Nov. 19, 1959	BERKMAN, JOAN M., Assistant Actuary, National Bureau of Casualty Underwriters, 125 Maiden Lane, New York, N. Y. 10038
Nov. 14, 1958	BERNAT, LEO A., Consultant, Minnesota Research Associates, 503 15th Avenue, S.E., No. 2, Minneapolis 14, Minn.
Nov. 18, 1925	BITTEL, W. HAROLD, Chief Actuary, Department of Banking and Insurance, State of New Jersey, Trenton, N. J. 08625
Nov. 14, 1958	BLUMENFELD, M. EUGENE, Actuary-Group and A & H, Standard Life & Accident Insurance Company, P. O. Box 1097, Oklahoma City, Okla. 73101
Nov. 22, 1934	BOMSE, EDWARD L., Manager-C&L, Casualty Underwriting Planning Department, Royal-Globe Insurance Companies, 150 William Street, New York, N. Y. 10038
Nov. 22, 1957	BRAGG, JOHN M., Vice President and Chief Actuary, Life Insurance Company of Georgia, 573 W. Peachtree Street, N.E., Atlanta, Ga. 30308
Nov. 20, 1964	BROWN, WILLIAM W., JR., Actuarial Assistant, Liberty Mutual Insurance Company, 175 Berkeley Street, Boston, Mass. 02117
Nov. 15, 1962	BUFFINTON, PHILIP G., Vice President, State Farm Fire and Casualty Company, 112 East Washington Street, Bloomington, Ill. 61701
Nov. 20, 1924	BUGBEE, JAMES M., Vice-President, Maryland Casualty Company, P. O. Box 1228, Baltimore, Md. 21203
Mar. 31, 1920	BURT, MARGARET A., Office of George B. Buck, Consulting Actuary, 60 Worth Street, New York 13, N. Y.
Nov. 19, 1959	BUTLER, RICHARD H., Second Vice President, The Travelers Insurance Company, One Tower Square, Hartford, Conn. 06115
Nov. 20, 1964	CARLSON, EDWIN A., Actuarial Assistant, The Travelers Insurance Company, One Tower Square, Hartford, Conn. 06115

Admitted	
Nov. 15, 1962	CARSON, DAVID E. A., Actuary, Hartford Fire Insurance Company and Hartford Accident and Indemnity Company, 690 Asylum Avenue, Hartford, Conn. 06115
Nov. 17, 1922	CAVANAUGH, LEO D., Consultant-Life Insurance Management, 55 E. Washington Street, Chicago 2, Ill.
Nov. 18, 1927	CHEN, S. T., Consulting Actuary, The Wing On Life Assurance Company, Ltd., Wing On Life Bldg., 22 Des Voeux Road, Central, Hong Kong.
Nov. 16, 1961	CHERLIN, GEORGE, Vice President and Actuary, National Health and Welfare Retirement Association, Inc., 800 Second Avenue, New York, N. Y. 10017
Nov. 22, 1957	CHURCH, HARRY M., Coates, Herfurth & England, Consulting Actuaries, 325 North Lake, Pasadena, Calif.
Nov. 1, 1963	CIMA, AUGUSTIN J., Pricing Research Manager, Allstate Insurance Company, 7447 Skokie Blvd., Skokie, Ill. 60078
Nov. 18, 1955	COATES, WILLIAM D., Vice President-Actuary, National-Ben Franklin Insurance Company, 360 W. Jackson Blvd., Chicago, Ill. 60606
Nov. 19, 1953	CONTE, JOSEPH P., Assistant to the President, Berman's Motor Express, P. O. Box 1209, Binghampton, N. Y. 13902
Nov. 19, 1959	COPESTAKES, ARTHUR D., Assistant Vice President, American Mutual Liability Insurance Company, Wakefield, Mass.
Nov. 19, 1959	CRAIG, ROBERT A., Associate Actuary, Connecticut Insurance Department, State Office Building, Hartford, Conn. 06115
May 21, 1963	CRANDALL, WILLIAM H., Administrative Assistant, Insurance Company of North America, 1600 Arch Street, Philadelphia, Pa. 19101
Nov. 24, 1933	CRAWFORD, WILLIAM H., Vice President and Treasurer, Industrial Indemnity Company, 155 Sansome Street, San Francisco 4, Calif.
Nov. 19, 1953	CROFTS, GEOFFREY, Dean and Director, Graduate School of Actuarial Science, Northeastern University, 360 Huntington Avenue, Boston, Mass. 02115
Nov. 15, 1962	CURRY, ALAN C., Actuary, State Farm Mutual Automobile Insurance Company, 112 E. Washington Street, Bloomington, Ill. 61701
Nov. 1, 1963	DAHME, ORVAL E., Senior Assistant Actuary, State Farm Mutual Automobile Insurance Company, 112 E. Washington Street, Bloomington, Ill. 61701
Nov. 21, 1952	DANIEL, C. M., Applied Science Representative, International Business Machines Corporation, 2116 Grand, Des Moines 12, Iowa
Nov. 18, 1925	DAVIS, MALVIN E., Executive Vice President, Metropolitan Life Insurance Company, One Madison Avenue, New York, N. Y. 10010
Nov. 18, 1960	DEMELIO, JOSEPH J., Assistant Secretary and Actuary, Home Insurance Company, 59 Maiden Lane, New York, N. Y. 10008
Nov. 16, 1956	DORF, STANLEY A., Associate Actuary, New York State Insurance Department, 123 William Street, New York 38, N. Y.
Nov. 14, 1941	DOWLING, WILLIAM F., 77 Brook Street, Garden City, N. Y. 11535
Nov. 1, 1963	DURKIN, JAMES H., Actuary, Wolfe, Corcoran & Linder, 116 John Street, New York, N. Y. 10038
Nov. 14, 1958	DUROSE, STANLEY C., JR., Assistant Deputy Commissioner, Wisconsin Department of Insurance, 4802 Sheboygan Avenue, Madison, Wis. 53702
Nov. 19, 1954	EATON, KARL F., Controller, Guarantee Mutual Life Company, 8721 Indian Hills Drive, Omaha 14, Neb.
June 5, 1925	EGER, FRANK A., (Retired), 1119 Prospect Ridge Blvd., Haddon Heights, N. J. 08035
Nov. 16, 1961	EHLERT, DARRELL W., Actuary, Allstate Insurance Company, 7447 Skokie Blvd., Skokie, Ill. 60078
Nov. 15, 1962	EVEN, CHARLES A., JR., The Travelers Insurance Company, One Tower Square, Hartford, Conn. 06115

Admitted	
Nov. 22, 1957	FELDMAN, MARTIN F., Associate Actuary, New York State Insurance Department, 123 William Street, New York 38, N. Y.
Nov. 16, 1961	FERDEN, STEIN, Undelstadlia 8, Asker, Norway
Nov. 15, 1962	FINKEL, DANIEL, Associate Actuary, The State Insurance Fund, 199 Church Street, New York, N. Y. 10007
Nov. 16, 1956	FLACK, PAUL R., Actuarial Assistant, General Accident Fire and Life Assurance Corporation, Ltd., 414 Walnut Street, Philadelphia, Pa.
Nov. 16, 1923	FLEMING, FRANK A., (Retired), c/o Mutual Insurance Rating Bureau, 733 Third Avenue, New York 17, N. Y.
Nov. 20, 1964	FORKER, DAVID C., The Travelers Insurance Company, Group Underwriting Department, One Tower Square, Hartford, Conn. 06115
Nov. 21, 1952	FRANKLIN, NATHAN M., Actuary, The Surety Association of America, 110 William Street, New York, N. Y. 10038
Nov. 19, 1954	GAINES, NATHANIEL, Associate Actuary, Office of George B. Buck, Consulting Actuary, 60 Worth Street, New York, N. Y. 10013
Nov. 15, 1962	GERUNDO, LOUIS P., JR., The Travelers Insurance Company, One Tower Square, Hartford, Conn. 06115
Nov. 18, 1932	GETMAN, RICHARD A., Assistant Actuary, Life Dept., The Travelers Insurance Company, One Tower Square, Hartford, Conn. 06115
Nov. 17, 1922	GIBSON, JOSEPH P., JR., (Retired), 2970 Lorain Road, San Marino, Calif. 91108
Nov. 16, 1923	GILDEA, JAMES F., (Retired), 236 Nott Street, Wethersfield, Conn.
Nov. 1, 1963	GILL, JAMES F., Actuary, National Association of Independent Insurers, 30 West Monroe St., Inland Steel Bldg., Chicago, Ill. 60603
Nov. 18, 1960	GILLESPIE, JAMES E., Actuarial Assistant, Continental National Insurance Group, 310 S. Michigan Avenue, Chicago, Ill. 60604
Nov. 14, 1947	GINGERY, STANLEY, Vice President & Associate Actuary, The Prudential Insurance Company of America, Prudential Plaza, Newark 1, N. J.
Nov. 19, 1959	GOLD, MELVIN L., Consulting Actuary, 29 Lakeview Drive, West Orange, N. J.
Nov. 16, 1961	GOULD, DONALD E., Senior Statistician, The State Insurance Fund, 199 Church Street, New York 7, N. Y.
Nov. 18, 1927	GREEN, WALTER C., Consulting Actuary, Walter C. Green and Associates, 1405 S. Main Street, Salt Lake City, Utah.
Nov. 16, 1961	GREENE, THOMAS A., Assistant Vice President, American Re-Insurance Company, 99 John Street, New York, N. Y. 10038
Nov. 15, 1940	GROSSMAN, ELI A., Senior Vice President, The Great Eastern Life Insurance Company, 10 Dorrance Street, Providence, R. I. 02903
Nov. 15, 1935	GUERTIN, ALFRED N., Actuary, American Life Convention, 230 N. Michigan Avenue, Chicago 1, Ill.
Nov. 16, 1939	HAGEN, OLAF E., Senior Assistant Actuarial Supervisor, Metropolitan Life Insurance Company, One Madison Avenue, New York, N. Y. 10010
Nov. 17, 1922	HALL, HARTWELL L., (Retired), 34 Lincoln Avenue, West Hartford 7, Conn.
Nov. 13, 1936	HAM, HUGH P., President and General Manager, The Western Assurance Company, 40 Scott Street, Toronto 1, Ontario, Canada
Nov. 1, 1963	HAMMER, SIDNEY M., Assistant Actuary, The Home Insurance Company, 59 Maiden Lane, New York, N. Y. 10008
Nov. 19, 1953	HARACK, JOHN, Actuary, Health Service, Inc., and Medical Indemnity of America, Inc., 200 N. Michigan Avenue, Chicago 1, Ill.

Admitted	
Mar. 24, 1932	HARRIS, SCOTT, Executive Vice President, Joseph Froggatt & Company, Inc., 74 Trinity Place, New York 6, N. Y.
Mar. 25, 1924	HART, WARD VAN B., 49 Robbins Drive, Wethersfield 9, Conn.
Nov. 21, 1919	HAYDON, GEORGE F., Manager Emeritus, Wisconsin Compensation Rating Bureau, 623 N. 2nd Street, Milwaukee 3, Wis.
Nov. 19, 1953	HEAD, GLENN O., Executive Vice President, First Investors Life Insurance Company, 120 Wall Street, New York 5, N. Y.
Nov. 19, 1959	HICKMAN, JAMES C., Associate Professor, Department of Mathematics, State University of Iowa, Iowa City, Iowa 52240
Nov. 15, 1962	HILLHOUSE, JERRY A., Associate Actuary, State Farm Mutual Automobile Insurance Company, 112 E. Washington Street, Bloomington, Ill. 61701
Nov. 17, 1927	HIPP, GRADY HAYNE, 216 Pine Forest Drive, Greenville, S. C. 29601
Nov. 16, 1961	HOROWITZ, MILTON, Principal Actuary, The State Insurance Fund, 199 Church Street, New York 7, N. Y.
Nov. 19, 1929	JACOBS, CARL N., Chairman of the Board, Hardware Mutual Casualty Company, Hardware Dealers Mutual Fire Insurance Company & Sentry Life Insurance Company, 200 Strongs Avenue, Stevens Point, Wis.
Nov. 18, 1921	JENSEN, EDWARD S., Assistant Vice President, Occidental Life Insurance Company of Calif., Occidental Center, 12th Street at Hill, Los Angeles, Calif. 90054
Nov. 15, 1962	JENSEN, JAMES P., Actuarial Assistant, Liberty Mutual Insurance Company, 175 Berkeley Street, Boston 17, Mass.
Nov. 21, 1919	JONES, LORING D., (Retired), 64 Raymond Avenue, Rockville Centre, L. I., N. Y.
Nov. 21, 1952	JONES, NATHAN F., Associate Actuary, The Prudential Insurance Company of America, Prudential Plaza, Newark 1, N. J.
Nov. 20, 1964	KHOURY, COSTANDY K., 5705 D. Tacoma Road, Columbus, Ohio 43224
Nov. 15, 1935	KITZROW, E. W., Reinsurance Actuary, Farmers Insurance Group, 4680 Wilshire Boulevard, Los Angeles 54, Calif.
Nov. 19, 1959	KROEKER, JOHN, Senior Actuary, Department of Insurance, 770 Heron Road, Ottawa 8, Ontario, Canada
Nov. 16, 1961	LANGE, JEFFREY T., National Bureau of Casualty Underwriters, 125 Maiden Lane, New York 38, N. Y.
Nov. 19, 1959	LEIGHT, ARTHUR S., Actuarial Associate, Metropolitan Life Insurance Company, 1 Madison Avenue, New York 10, N. Y.
Nov. 14, 1947	LUFKIN, ROBERT W., Manager of Home Office, Craftsman Life Insurance Company, 851 Boylston Street, Boston, Mass. 02116
Nov. 18, 1925	MALMUTH, JACOB, Chief-Rating Bureau, N. Y. Insurance Department, 123 William Street, New York, N. Y. 10038
Nov. 16, 1961	MARGOLIS, DONALD R., Assistant Actuary, Life Insurance Company of North America, 1600 Arch Street, Philadelphia, Pa. 19101
Nov. 20, 1964	MARKELL, ANDREW S., Actuary, Transamerica Insurance Company, Occidental Center, Los Angeles, Calif. 90015
Mar. 24, 1927	MARSH, CHARLES V-R., (Retired), Fidelity & Deposit Company of Maryland, Baltimore, Md. 33705
Nov. 16, 1956	MATHWICK, LLOYD F., Assistant Manager, Group Department, Employers Mutuals of Wausau, 407 Grant Street, Wausau, Wis. 54402
Nov. 13, 1936	MAYER, WILLIAM H., JR., Manager, Group Contract Bureau, Metropolitan Life Insurance Company, 1 Madison Avenue, New York 10, N. Y.
May 26, 1955	MCDONALD, MILTON G., Chief Actuary, Massachusetts Insurance Department, 100 Nashua Street, Boston, Mass. 02114

Admitted	
Nov. 16, 1961	McIntosh, Kenneth L., Manager, Louisiana Rating & Fire Prevention Bureau, P. O. Box 60730, New Orleans 60, La.
Nov. 16, 1961	McLean, George E., Actuary, Massachusetts Hospital Service, Inc., 133 Federal Street, Boston 6, Mass.
Nov. 13, 1931	Miller, Henry C., Comptroller, California State Compensation Insurance Fund, 525 Golden Gate Avenue, San Francisco 1, Calif.
Nov. 18, 1937	Minor, Eduard H., Associate Actuary, Metropolitan Life Insurance Company, 1 Madison Avenue, New York 10, N. Y.
Nov. 18, 1960	Mohnblatt, Arnold S., Actuarial Assistant, Consolidated Mutual Insurance Company, 345 Adams Street, Brooklyn 1, N. Y.
Nov. 20, 1964	Mokros, Bertram F., Policyholder Research Manager, Allstate Insurance Company, 7447 Skokie Blvd., Skokie, Ill. 60078
Nov. 17, 1922	Montgomery, John C., (Retired), 165 Westervelt Avenue, Tenafly, N. J.
May 25, 1923	Moore, Joseph P., 115 St. Catherine Road, Outremont, Quebec, Canada
Nov. 16, 1961	Moss, Robert Graham, Actuary and Assistant Vice President, Marsh & McLennan, Inc., 515 Olive Street, St. Louis, Mo. 63101
Nov. 22, 1957	Muir, Joseph M.. General Manager, Mutual Insurance Advisory Association & Mutual Insurance Rating Bureau, 733 Third Avenue, New York 17, N. Y.
Nov. 1, 1963	Muniz, Robert M., National Bureau of Casualty Underwriters, 125 Maiden Lane, New York, N. Y. 10038
Nov. 1, 1963	Nelson, Dale A., Assistant Actuary, State Farm Mutual Automobile Insurance Co., 112 E. Washington Street, Bloomington, Ill. 61701
Nov. 16, 1961	Nelson, Roland E., Associate Actuary, State Farm Life and Accident Assurance Company, 112 E. Washington Street, Bloomington, Ill.
Oct. 27, 1916	Newell, William, (Retired), 1225 Park Avenue, New York 28, N. Y.
Nov. 18, 1925	Nicholson, Earl H., Actuary and Deputy Insurance Commissioner, Nevada Insurance Department, State Office Bldg., Carson City, Nevada 89701
Nov. 16, 1961	Oien, R. Gustave, Actuary, Mutual Service Life Insurance Company, 1919 University Avenue, St. Paul, Minn. 55104
May 23, 1919	Otto, Walter E., Chairman of the Board, Michigan Mutual Liability Company, 28 W. Adams Avenue, Detroit 26, Mich.
Nov 19, 1926	Overholser, Donald M., Actuary for Pension Funds, George B. Buck, 60 Worth Street, New York, N. Y. 10013
Nov. 16, 1961	Peel, Jerald P., Actuary, Security Mutual Casualty Company, 309 W. Jackson Blvd., Chicago, Ill. 60606
Nov. 20, 1924	Pennock, Richard M., (Retired), 12 E. Lodges Lane, Bala-Cynwyd, Pa.
Nov. 14, 1947	Perry, Robert C., Executive Vice President, State Farm Life Insurance Co., 112 East Washington St., Bloomington, Ill. 61701
Nov. 19, 1929	Phillips, John H., (Retired), 915 Steuben Street, Wausau, Wis.
Nov. 17, 1920	Pike, Morris, (Retired), 531 East 20th Street, New York, N. Y.
Nov. 17, 1922	Poorman, William F., Chairman of the Board, Central Life Assurance Company, 611 Fifth Avenue, Des Moines, Iowa 50309
Nov. 15, 1962	Portermain, Neill W., Assistant Actuary, Mutual Service Life Insurance Co., 1919 University Avenue, St. Paul, Minn. 55104
Nov. 13, 1936	Potofsky, Sylvia, Senior Actuary, The State Insurance Fund, 199 Church Street, New York 7, N. Y.

Admitted	
Nov. 20, 1964	RAID, GARY A., Assistant Actuary, State Farm Mutual Automobile Insurance Co., 112 E. Washington St., Bloomington, Ill. 61701
Nov. 15, 1918	RAYWID, JOSEPH, Vice President, Woodward & Fondiller, Inc., 322 W. 72nd Street, New York, N. Y.
Nov. 18, 1960	RICCARDO, JOSEPH F., Aetna Casualty and Surety Company, General Accounts Department, 151 Farmington Avenue, Hartford, Conn. 06115
Nov. 19, 1932	RICHARDSON, HARRY F., (Retired), 413 Ackerman Avenue, Hohokus, N. J. 07423
Nov. 19, 1953	RICHMOND, OWEN D., Controller, Business Men's Assurance Company, B. M. A. Tower, Kansas City, Mo. 64108
Nov. 18, 1960	RIPANDELLI, JOHN S., Consulting Actuary, Lewis State Bank Building, Tallahasee, Fla. 32301
Nov. 18, 1932	ROBERTS, JAMES A., (Retired), 118 Mill St., Apt. B, Wethersfield, Conn. 06109
Nov. 15, 1962	ROOD, HENRY F., President, Lincoln National Life Insurance Company, 1300 South Harrison Street, Fort Wayne, Ind. 46801
Nov. 18, 1960	ROTH, RICHARD J., Actuary, American International Underwriters Corp., 102 Maiden Lane, New York, N. Y. 10005
Nov. 19, 1959	ROYER, ALAN F., Chief Actuary, Department of Insurance, State of Illinois, Springfield, Ill. 62706
Nov. 1, 1963	RYAN, KEVIN M., Assistant Actuary, Industrial Indemnity Company, 155 Sansome Street, San Francisco, Calif. 94104
Nov. 14, 1958	SARNOFF, PAUL E., Assistant Actuary, The Prudential Insurance Company of America, Prudential Plaza, Newark, N. J. 07101
Nov. 16, 1923	SAWYER, ARTHUR, (Retired), 13751 St. Andrews Drive, Leisure World, Apt. 1-I, Seal Beach, Calif.
Nov. 14, 1947	SCAMMON, LAWRENCE W., Manager, Massachusetts Automobile Rating & Accident Prevention Bureau, Massachusetts Workmen's Compensation Rating & Inspection Bureau, & Massachusetts Motor Vehicle Assigned Risk Plan, 89 Broad Street, Boston, Mass.
Nov. 1, 1963	SCHEEL, PAUL J., Actuarial Assistant, U. S. Fidelity & Guaranty Company, Calvert and Redwood Streets, Baltimore, Md. 21203
Nov. 16, 1961	SCHEIBL, JEROME A., Assistant Actuary, Employers Mutuals of Wausau, 407 Grant Street, Wausau, Wis. 54402
Nov. 14, 1958	SCHLENZ, JOHN W., Senior Vice President and Actuary, Federal Life and Casualty Company, Wolverine-Federal Tower, Battle Creek, Mich. 49016
Nov. 22, 1957	SCHNEIKER, HENRY C., Associate Actuary, The Home Insurance Company, 59 Maiden Lane, New York, N. Y. 10008
Nov. 20, 1964	SCHULER, ROBERT J., Assistant Actuary, Blue Cross of Western Pennsylvania, One Smithfield Street, Pittsburgh, Pa. 15222
Nov. 19, 1954	SCHULMAN, JUSTIN, Mathematician, Fairchild Space and Defense Systems, Robbins Lane, Syosset, Long Island, N. Y.
Nov. 14, 1947	SCHWARTZ, MAX J., Chief Accident & Health Rating Section, N. Y. State Insurance Department, 324 State Street, Albany, N. Y. 12210
Nov. 20, 1964	SCOTT, BRIAN E., Actuarial Division, Aetna Casualty and Surety Company & Standard Fire Insurance Company, 151 Farmington Avenue, Hartford, Conn. 06115
Nov. 20, 1930	SEVILLA, EXEQUIEL S., President, Manager and Actuary, National Life Insurance Company of the Philippines, Regina Bldg., P.O. Box 2056, Manila, Philippines.

Admitted	
Nov. 22, 1957	SHAVER, C. OTIS, Second Vice President and Actuary, Nationwide Mutual Fire Insurance Company, 246 North High Street, Columbus, Ohio 43216
Nov. 20, 1924	SHEPPARD, NORRIS E., Professor of Mathematics, University of Toronto, Toronto 5, Canada.
Nov. 1, 1963	SINGER, PAUL E., Assistant Vice President, Continental National American Group, 310 South Michigan Avenue, Chicago, Ill. 60604
Nov. 15, 1962	SMITH, EDWARD R., Assistant Actuary, Hartford Insurance Group, 690 Asylum Avenue, Hartford, Conn. 06115
Nov. 19, 1926	SOMERVILLE, WILLIAM F., 300 Lena Street, Excelsior Springs, Mo. 64024
Nov. 18, 1925	SOMMER, ARMAND, Vice President, Continental Casualty Company, 310 S. Michigan Ave., Floor 19-W, Chicago, Ill. 60604
Nov. 15, 1918	SPENCER, HAROLD S., (Retired), 8 Chelsea Lane, West Hartford, Conn.
Nov. 1, 1963	STALEY, HARLOW B., Vice President and Director of Administration, Farm Bureau Mutual Insurance Company, 10th and Grand Streets, Des Moines, Iowa. 50307
Nov. 19, 1959	STEINHAUS, HENRY W., Partner, Smick and Steinhaus, Consulting Actuaries, 135 E. 42nd Street, New York 17, N. Y.
Nov. 20, 1924	STELLWAGEN, HERBERT P., Director, Insurance Company of North America, 1600 Arch Street, Philadelphia, Pa. 19101
Nov. 16, 1923	STERN, PHILIPP K., Actuary, Mutual Insurance Rating Bureau, 733 Third Avenue, New York 17, N. Y.
Nov. 19, 1959	STEVENS, WALDO A., Director, Actuarial Division, National Association of Blue Shield Plans, 425 North Michigan, Chicago, Ill. 60611
Nov. 19, 1959	STOKE, KENDRICK, (Retired), 11052 McKinney, Detroit, Mich. 48224
Nov. 16, 1956	STRUG, EMIL J., Assistant Actuary, Massachusetts Hospital Service, Inc., 133 Federal Street, Boston 6, Mass.
Nov. 21, 1930	SULLIVAN, WALTER F., Actuary, State Compensation Insurance Fund, 525 Golden Gate Avenue, San Francisco, Calif. 94102
Nov. 15, 1962	SWITZER, VERNON J., Associate Actuary, State Farm Mutual Automobile Insurance Company, 112 E. Washington Street, Bloomington, Ill. 61701
Nov. 1, 1963	THOMPSON, PHILIP R., Statistician, Federated Mutual Implement and Hardware Insurance Company, 129 E. Broadway, Owatonna, Minn. 55060
Nov. 21, 1919	TRENCH, FREDERICK H., (Retired), 5 Bonnie Brae, Utica, N. Y.
Nov. 20, 1924	UHL, M. ELIZABETH, (Retired), 320 E. 53rd Street, New York, N. Y. 10022
Nov. 14, 1958	VAN CLEAVE, MARVIN E., Chief, Rate Division, Wisconsin Insurance Department, Hill Farms State Office Bldg., Madison, Wis. 53702
Nov. 20, 1964	VANDERHOOF, IRWIN T., Senior Vice President and Actuary, Standard Security Life Ins. Co. of N. Y., 111 Fifth Avenue, New York, N. Y. 10003
Nov. 15, 1962	VERHAGE, PAUL A., Actuarial Analyst, Sentry Insurance-Hardware Mutuals Group, 200 Strongs Avenue, Stevens Point, Wis. 54482
May 19, 1964	WEBB, BERNARD L., Marketing Specialist, Hardware Mutuals-Sentry Life, 200 Strongs Avenue, Stevens Point, Wis. 54482
Nov. 19, 1959	WEBER, DONALD C., Fellow, Institute of Statistics, North Carolina State College, P. O. Box 5457, Raleigh, N. C. 27606
Nov. 18, 1932	WEINSTEIN, MAX S., Actuary, New York State Employees' Retirement System, 90 S. Swan Street, Albany 1, N. Y.

Admitted	
Nov. 18, 1925	WELLMAN, ALEX C., Senior Vice President, Protective Life Insurance Company, P. O. Box 2571, Birmingham, Ala. 35202
Nov. 21, 1930	WELLS, WALTER I., Second Vice President, State Mutual Life Assurance Company of America, 440 Lincoln Street, Worcester, Mass.
Nov. 18, 1927	WHITBREAD, FRANK G., Second Vice President, The Lincoln National Life Insurance Co., 1301 South Harrison Street, Fort Wayne, Ind. 46801
Nov. 19, 1948	WHITE, AUBREY, Consulting Actuary, Ostheimer & Co., 1510 Chestnut Street, Philadelphia, Pa. 19102
Nov. 16, 1939	WITTLAKE, J. CLARKE, Executive Vice President, Business Men's Assurance Company of America, BMA Tower, Penn Valley Park, Kansas City, Missouri 64141
Oct. 22, 1915	WOOD, DONALD M., Partner, Childs & Wood, 175 W. Jackson Boulevard, Chicago 4, Ill.
Nov. 18, 1937	WOOD, DONALD M., JR., Partner, Childs & Wood, 175 W. Jackson Boulevard, Chicago 4, Ill.
Nov. 18, 1927	WOOD, MILTON J., Vice President and Actuary, Life, Accident and Group Actuarial Dept., The Travelers Insurance Company, One Tower Square, Hartford, Conn. 06115
Nov. 17, 1950	WOODDY, JOHN C., Actuary, North American Reassurance Company, 161 E. 42nd Street, New York 17, N. Y.
Nov. 22, 1934	WOODWARD, BARBARA H., Assistant Secretary and Regional General Counsel, The Reuben H. Donnelley Corporation, 466 Lexington Avenue, New York 17, N. Y.
Nov. 16, 1956	WOODWORTH, JAMES H., Assistant Secretary, The Hartford Insurance Group, 690 Asylum Avenue, Hartford, Conn. 06115
Nov. 18, 1925	WOOLERY, JAMES M., Senior Vice President-Actuary, Occidental Life Ins. of N. C., Cameron Village, Raleigh, N. C. 27605
May 5, 1961	YOUNG, ROBERT G., Actuary, Michigan Mutual Liability Company, 28 West Adams Avenue, Detroit, Mich. 48226
Nov. 1, 1963	ZORY, P. B., Actuarial Department, National Bureau of Casualty Underwriters, 125 Maiden Lane, New York, N. Y. 10038

Elected	*President*	*Vice-Presidents*	
1914-1915	*Isaac M. Rubinow	*Albert H. Mowbray	*Benedict D. Flynn
1916-1917	*James D. Craig	*Joseph H. Woodward	*Harwood E. Ryan
1918	*Joseph H. Woodward	*Benedict D. Flynn	*George D. Moore
1919	*Benedict D. Flynn	*George D. Moore	*William Leslie
1920	*Albert H. Mowbray	*William Leslie	*Leon S. Senior
1921	*Albert H. Mowbray	*Leon S. Senior	*Harwood E. Ryan
1922	*Harwood E. Ryan	Gustav F. Michelbacher	*Edmund E. Cammack
1923	*William Leslie	Gustav F. Michelbacher	*Edmund E. Cammack
1924-1925	Gustav F. Michelbacher	*Sanford B. Perkins	Ralph H. Blanchard
1926-1927	*Sanford B. Perkins	*George D. Moore	*Thomas F. Tarbell
1928-1929	*George D. Moore	Sydney D. Pinney	Paul Dorweiler
1930-1931	*Thomas F. Tarbell	*Roy A. Wheeler	Winfield W. Greene
1932-1933	Paul Dorweiler	*William F. Roeber	*Leon S. Senior
1934-1935	Winfield W. Greene	Ralph H. Blanchard	Charles J. Haugh
1936-1937	*Leon S. Senior	Sydney D. Pinney	*Francis S. Perryman
1938-1939	*Francis S. Perryman	Harmon T. Barber	*William J. Constable
1940	Sydney D. Pinney	Harold J. Ginsburgh	James M. Cahill
1941	Ralph H. Blanchard	Harold J. Ginsburgh	James M. Cahill
1942	Ralph H. Blanchard	Albert Z. Skelding	Charles J. Haugh
1943-1944	Harold J. Ginsburgh	Albert Z. Skelding	Charles J. Haugh
1945-1946	Charles J. Haugh	James M. Cahill	Harry V. Williams
1947-1948	James M. Cahill	Harmon T. Barber	Russell P. Goddard
1949-1950	Harmon T. Barber	*Thomas O. Carlson	Norton E. Masterson
1951-1952	*Thomas O. Carlson	Joseph Linder	Seymour E. Smith
1953-1954	Seymour E. Smith	Dudley M. Pruitt	John A. Mills
1955-1956	Norton E. Masterson	*Clarence A. Kulp	Arthur N. Matthews
1957-1958	Dudley M. Pruitt	John W. Carleton	William Leslie, Jr.
1959-1960	William Leslie, Jr.	Ernest T. Berkeley	Laurence H. Longley-Cook
1961-1962	L. H. Longley-Cook	Thomas E. Murrin	Richard J. Wolfrum
1963-1964	Thomas E. Murrin	Harold E. Curry	William J. Hazam

Secretary-Treasurer

1914-1917...*C. E. Scattergood
1918-1953........*R. Fondiller
1954-1964.......A. Z. Skelding

Editor

1914 ....W. W. Greene
1915-1917........*R. Fondiller
1918............W. W. Greene
1919-1921....G. F. Michelbacher
1922-1923.......O. E. Outwater
1924-1932.....*R. J. McManus
1933-1943.......*C. W. Hobbs
1944-1954......E. C. Mayerink
1955-1958..........E. S. Allen
1959-1960.......R. P. Goddard
1961-1963.......H. W. Schloss

General Chairman Examination Committee

1949-1951.......R. A. Johnson
1952-1956....J. W. Wieder, Jr.
1957-1961 ....W. J. Hazam
1962-1964.......N. J. Bennett

Librarian

1914............W. W. Greene
1915.............*R. Fondiller
1916-1921.........L. I. Dublin
1922-1924........*E. R. Hardy
1925-1936...........W. Breiby
1937-1947......*T. O. Carlson
1948-1950..........*S. M. Ross
1951-1957.....G. R. Livingston
1958-1963.............R. Lino

*Deceased.

The (†) denotes charter members at date of organization, November 7, 1914.

Admitted		Died
Nov. 19, 1948	Arthur L. Bailey	Aug. 12, 1954
May 23, 1924	William B. Bailey	Jan. 10, 1952
†	Roland Benjamin	July 2, 1949
May 24, 1921	Edward J. Bond	Nov. 12, 1941
May 19, 1915	Thomas Bradshaw	Nov. 10, 1939
June 5, 1925	William Brosmith	Aug. 22, 1937
†	George B. Buck, Sr.	Apr. 12, 1961
†	William A. Budlong	June 4, 1934
Nov. 18, 1932	Charles H. Burhans	June 15, 1942
Apr. 20, 1917	William H. Burhop	Oct. 11, 1963
Feb. 19, 1915	F. Highlands Burns	Mar. 30, 1935
†	Edmund E. Cammack	Dec. 17, 1958
Nov. 21, 1930	Thomas O. Carlson	July 15, 1964
†	Raymond V. Carpenter	Mar. 11, 1947
Feb. 19, 1915	Gorden Case	Feb. 4, 1920
Oct. 27, 1916	Edmund S. Cogswell	Apr. 25, 1957
Nov. 23, 1928	Walter P. Comstock	May 11, 1951
Nov. 22, 1934	William J. Constable	Apr. 19, 1959
†	Charles T. Conway	July 23, 1921
†	John A. Copeland	June 12, 1953
†	Walter G. Cowles	May 30, 1942
†	James D. Craig	May 27, 1940
†	James McIntosh Craig	Jan. 20, 1922
May 26, 1916	Frederick S. Crum	Sept. 2, 1921
†	Alfred Burnett Dawson	June 21, 1931
†	Miles Menander Dawson	Mar. 27, 1942
†	Elmer H. Dearth	Mar. 26, 1947
†	Eckford C. DeKay	July 31, 1951
May 19, 1915	Samuel Deutschberger	Jan. 18, 1929
†	Ezekiel Hinton Downey	July 9, 1922
May 19, 1915	Earl O. Dunlap	July 5, 1944
†	David Parks Fackler	Oct. 30, 1924
†	Edward B. Fackler	Jan. 8, 1952
Feb. 19, 1915	Claude W. Fellows	July 15, 1938
†	Benedict D. Flynn	Aug. 22, 1944
Feb. 19, 1915	Richard Fondiller	Apr. 29, 1962
†	Charles S. Forbes	Oct. 2, 1943
May 26, 1916	Lee K. Frankel	July 25, 1931
†	Charles H. Franklin	May 1951
Feb. 25, 1916	Joseph Froggatt	Sept. 28, 1940
†	Harry Furze	Dec. 26, 1945
Feb. 19, 1915	Fred S. Garrison	Nov. 14, 1949
†	Theodore E. Gaty	Aug. 22, 1925
May 19, 1915	James W. Glover	July 15, 1941
Oct. 22, 1915	George Graham	Apr. 15, 1937
Oct. 22, 1915	Thompson B. Graham	July 24, 1946
†	William J. Graham	Feb. 11, 1963

Admitted		Died
May 25, 1923	William A. Granville	Feb. 4, 1943
†	William H. Gould	Oct. 28, 1936
†	Robert Cowen Lees Hamilton	Nov. 15, 1941
†	H. Pierson Hammond	Apr. 10, 1963
Oct. 27, 1916	Edward R. Hardy	June 29, 1951
Oct. 22, 1915	Leonard W. Hatch	Nov. 23, 1958
Nov. 21, 1919	Robert Henderson	Feb. 16, 1942
†	Robert J. Hillas	May 17, 1940
Nov. 15, 1918	Frank Webster Hinsdale	Mar. 18, 1932
May 23, 1924	Clarence W. Hobbs	July 21, 1944
Nov. 19, 1926	Charles E. Hodges	Jan. 22, 1937
Oct. 22, 1915	Lemuel G. Hodgkins	Dec. 26, 1951
†	Frederick L. Hoffman	Feb. 23, 1946
Oct. 22, 1915	Charles H. Holland	Dec. 28, 1951
Nov. 21, 1919	Carl Hookstadt	Mar. 10, 1924
Nov. 18, 1932	Solomon S. Huebner	July 17, 1964
†	Charles Hughes	Aug. 27, 1948
Nov. 19, 1929	Robert S. Hull	Nov. 30, 1947
†	Burritt A. Hunt	Sept. 3, 1943
†	Arthur Hunter	Jan. 27, 1964
Nov. 28, 1921	William Anderson Hutcheson	Nov. 19, 1942
Feb. 25, 1916	Charles William Jackson	Sept. 21, 1959
Nov. 19, 1929	Henry Hollister Jackson	May 27, 1955
May 19, 1915	William C. Johnson	Oct. 7, 1943
Nov. 23, 1928	F. Robertson Jones	Dec. 26, 1941
Nov. 18, 1921	Thomas P. Kearney	Feb. 11, 1928
Nov. 19, 1926	Gregory Cook Kelly	Sept. 11, 1948
Oct. 22, 1915	Virgil Morrison Kime	Oct. 15, 1918
†	Edwin W. Kopf	Aug. 3, 1933
Nov. 23, 1928	Clarence Arthur Kulp	Aug. 20, 1957
Feb. 17, 1915	John M. Laird	June 20, 1942
Nov. 13, 1931	Stewart M. LaMont	Aug. 22, 1960
Feb. 19, 1915	Abb Landis	Dec. 9, 1937
Nov. 24, 1933	John Robert Lange	Apr. 12, 1957
Nov. 17, 1922	Arnette Roy Lawrence	Dec. 1, 1942
†	James R. Leal, Sr.	Dec. 26, 1957
†	William Leslie	Dec. 12, 1962
Nov. 18, 1921	James Fulton Little	Aug. 11, 1938
Nov. 23, 1928	Edward C. Lunt	Jan. 13, 1941
Feb. 19, 1915	Harry Lubin	Dec. 20, 1920
†	William N. Magoun	Dec. 11, 1954
Nov. 16, 1923	D. Ralph McClurg	Apr. 27, 1947
May 23, 1919	Alfred McDougald	July 28, 1944
Oct. 31, 1917	Robert J. McManus	Aug. 15, 1960
Feb. 15, 1915	Franklin B. Mead	Nov. 29, 1933
Apr. 20, 1917	Marcus Meltzer	Mar. 27, 1931
†	David W. Miller	Jan. 18, 1936
†	James F. Mitchell	Feb. 9, 1941

Admitted		Died
†	Henry Moir	June 8, 1937
Nov. 18, 1921	Victor Montgomery	May 2, 1960
Feb. 19, 1915	William J. Montgomery	Aug. 20, 1915
Nov. 19, 1926	William L. Mooney	Oct. 21, 1948
†	George D. Moore	Mar. 11, 1959
May 19, 1915	Edward Bontecou Morris	Dec. 19, 1929
†	Albert H. Mowbray	Jan. 7, 1949
†	Frank Mullaney	Jan. 22, 1953
May 28, 1920	Ray D. Murphy	Feb. 24, 1964
†	Lewis A. Nicholas	Apr. 21, 1940
†	Edward Olifiers	May 13, 1962
†	Stanley L. Otis	Oct. 12, 1937
Nov. 13, 1926	Bertrand A. Page	July 30, 1941
Nov. 18, 1921	Sanford B. Perkins	Sept. 16, 1945
Nov. 15, 1918	William Thomas Perry	Oct. 25, 1940
Nov. 21, 1930	Francis S. Perryman	Nov. 30, 1959
†	Edward B. Phelps	July 24, 1915
Nov. 19, 1926	Jesse S. Phillips	Nov. 6, 1954
†	Charles Grant Reiter	July 30, 1937
†	Charles H. Remington	Mar. 21, 1938
May 23, 1919	Frederick Richardson	July 22, 1955
Nov. 19, 1926	Otto C. Richter	Feb. 17, 1962
Nov. 16, 1923	William F. Roeber	Mar. 21, 1960
Nov. 17, 1943	Samuel M. Ross	July 24, 1951
†	Isaac M. Rubinow	Sept. 1, 1936
†	Harwood Eldridge Ryan	Nov. 2, 1930
†	Arthur F. Saxton	Feb. 26, 1927
†	Emil Scheitlin	May 2, 1946
†	Leon S. Senior	Feb. 3, 1940
Nov. 24, 1933	Robert V. Sinnott	Dec. 15, 1952
Apr. 20, 1917	Charles Gordon Smith	June 22, 1938
Nov. 18, 1927	Edward C. Stone	June 6, 1964
Feb. 19, 1915	John T. Stone	May 9, 1920
Feb. 25, 1916	Wendell Melville Strong	Mar. 30, 1942
Oct. 22, 1915	William R. Strong	Jan. 10, 1946
†	Robert J. Sullivan	July 19, 1934
Nov. 17, 1920	Thomas F. Tarbell	July 2, 1958
Nov. 22, 1934	Walter H. Thompson	May 25, 1935
Nov. 18, 1921	Guido Toja	Feb. 28, 1933
†	John L. Train	June 12, 1958
Nov. 17, 1922	Antonio Thomas Traversi	Apr. 20, 1961
Nov. 19, 1948	Paul A. Turner	Jan. 30, 1961
Nov. 15, 1935	Harry V. Waite	Aug. 14, 1951
Nov. 18, 1925	Lloyd A. H. Warren	Sept. 30, 1949
May 23, 1919	Archibald A. Welch	May 8, 1945
Nov. 19, 1926	Roy A. Wheeler	Aug. 26, 1932
†	Albert W. Whitney	July 27, 1943
†	Lee J. Wolfe	Apr. 28, 1949
†	S. Herbert Wolfe	Dec. 31, 1927
May 24, 1921	Arthur B. Wood	June 14, 1952
†	Joseph H. Woodward	May 15, 1928
†	William Young	Oct. 23, 1927

Admitted		Died
May 23, 1924	Milton Acker	Aug. 16, 1956
Nov. 15, 1918	Robert E. Ankers	Mar. 1, 1964
Oct. 22, 1915	Don A. Baxter	Feb. 10, 1920
Nov. 17, 1920	Nellas C. Black	Dec. 24, 1962
Nov. 15, 1940	John M. Blackhall	Nov. 14, 1957
Nov. 15, 1918	Helmuth G. Brunnquell	June 3, 1958
Oct. 22, 1915	Louis Buffler	July 19, 1963
May 25, 1923	Harilaus E. Economidy	Apr. 13, 1948
Nov. 20, 1924	John Froberg	Oct. 11, 1949
Nov. 19, 1929	Maurice L. Furnivall	June 16, 1962
Nov. 22, 1934	John J. Gately	Nov. 3, 1943
Nov. 14, 1947	Harold J. George	Apr. 1, 1952
Nov. 19, 1929	Harold R. Gordon	July 8, 1948
Nov. 18, 1921	Robert E. Haggard	July 26, 1958
Nov. 20, 1924	Leslie LeVant Hall	Mar. 8, 1931
Oct. 31, 1917	Edward T. Jackson	May 8, 1939
Nov. 17, 1922	Rosswel A. McIver	Apr. 1, 1959
Nov. 21, 1919	Rolland V. Mothersill	July 25, 1949
Nov. 19, 1929	Fritz Muller	Apr. 27, 1945
Nov. 23, 1928	Karl Newhall	Oct. 24, 1944
Nov. 15, 1918	John L. Sibley	Mar. 10, 1957
Nov. 18, 1921	Arthur G. Smith	May 2, 1956
Nov. 18, 1927	Alexander A. Speers	June 25, 1941
Mar. 23, 1921	Arthur E. Thompson	Jan. 17, 1944
Nov. 21, 1919	Walter G. Voogt	May 8, 1937
May 23, 1919	Charles S. Warren	May 1, 1952
Nov. 18, 1925	James H. Washburn	Aug. 19, 1946
Nov. 17, 1920	James J. Watson	Feb. 23, 1937
Nov. 18, 1921	Eugene R. Welch	Jan. 17, 1945
Nov. 16, 1951	Michael T. Wermel	Feb. 6, 1962
Mar. 21, 1929	Charles A. Wheeler	July 2, 1956
Nov. 15, 1918	Albert Edward Wilkinson	June 11, 1930
Oct. 22, 1915	Charles E. Woodman	Dec. 16, 1955

SCHEDULE OF MEMBERSHIP, NOVEMBER 20, 1964

	Fellows	Associates	Total
Membership, November 1, 1963......	212	181	393
Additions:			
By Election	...	...	...
By Reinstatement	...	1	1
By Examination	6	11	17
	218	193	411
Deductions:			
By Death	5	1	6
By Withdrawal	...	2	2
By Transfer from Associate to Fellow	...	6	6
	213	184	397

CONSTITUTION

(As Amended November 16, 1962)

Article I.—*Name.*

This organization shall be called the Casualty Actuarial Society.

Article II.—*Object.*

The object of the Society shall be the promotion of actuarial and statistical science as applied to the problems of insurance, other than life insurance, by means of personal intercourse, the presentation and discussion of appropriate papers, the collection of a library and such other means as may be found desirable.

The Society shall take no partisan attitude, by resolution or otherwise, upon any question relating to insurance.

Article III.—*Membership.*

The membership of the Society shall be composed of two classes, Fellows and Associates. Fellows only shall be eligible to office or have the right to vote.

The Fellows of the Society shall be the present Fellows and those who may be duly admitted to Fellowship as hereinafter provided. The Associates shall be the present Associates and those who may be duly admitted to Associateship as hereinafter provided.

Any person may, upon nomination to the Council by two Fellows of the Society and approval by the Council of such nomination with not more than two negative votes, become enrolled as an Associate of the Society, provided that he shall pass such examination as the Council may prescribe.

Any person who shall have qualified for Associateship may become a Fellow on passing such final examination as the Council may prescribe. Otherwise, no one shall be admitted as a member unless recommended by a duly called meeting of the Council with not more than two negative votes in a secret ballot, followed by at least a three-fourths secret ballot of the Fellows present and voting at a meeting of the Society.

Article IV.—*Officers and Council.*

The officers of the Society shall be a President, two Vice-Presidents, a Secretary-Treasurer, an Editor, a Librarian, and a General Chairman of the Examination Committee. The Council shall be composed of the active officers, nine other Fellows and, during the four years following the expiration of their terms of office, the ex-Presidents and ex-Vice-Presidents. The Council shall fill vacancies occasioned by death or resignation of any officer or other member of the Council, such appointees to serve until the next annual meeting of the Society.

Article V.—*Election of Officers and Council.*

The President, Vice-Presidents, and the Secretary-Treasurer shall be elected by a majority ballot at the annual meeting for the term of one year and three members of the Council shall, in a similar manner, be annually elected to serve for three years. The President and Vice-Presidents shall not be eligible for the same office for more than two consecutive years nor shall any retiring member of the Council be eligible for re-election at the same meeting.

The Editor, the Librarian and the General Chairman of the Examination Committee shall be elected annually by the Council at the Council meeting preceding the annual meeting of the Society. They shall be subject to confirmation by majority ballot of the Society at the annual meeting.

The terms of the officers shall begin at the close of the meeting at which they are elected except that the retiring Editor shall retain the powers and duties of office so long as may be necessary to complete the then current issue of *Proceedings.*

Article VI.—*Duties of Officers and Council.*

The duties of the officers shall be such as usually appertain to their respective offices or may be specified in the by-laws. The duties of the Council shall be to pass upon candidates for membership, to decide upon papers offered for reading at the meetings, to supervise the examination of candidates and prescribe fees therefor, to call meetings, and in general, through the appointment of committees and otherwise, to manage the affairs of the Society.

Article VII.—*Meetings.*

There shall be an annual meeting of the Society on such date in the month of November as may be fixed by the Council in each year, but other meetings may be called by the Council from time to time and shall be called by the President at any time upon the written request of ten Fellows. At least two weeks notice of all meetings shall be given by the Secretary.

Article VIII.—*Quorum.*

Seven members of the Council shall constitute a quorum. Twenty Fellows of the Society shall constitute a quorum.

Article IX.—*Expulsion or Suspension of Members.*

Except for non-payment of dues, no member of the Society shall be expelled or suspended save upon action by the Council with not more than three negative votes followed by a three-fourths ballot of the Fellows present and voting at a meeting of the Society.

Article X.—*Amendments.*

This constitution may be amended by an affirmative vote of two-thirds of the Fellows present at any meeting held at least one month after notice of such proposed amendment shall have sent to each Fellow by the Secretary.

BY-LAWS

(AS AMENDED NOVEMBER 20, 1964)

ARTICLE I.— *Order of Business.*

At a meeting of the Society the following order of business shall be observed unless the Society votes otherwise for the time being:

1. Calling of the roll.
2. Address or remarks by the President.
3. Minutes of the last meeting.
4. Report by the Council on business transacted by it since the last meeting of the Society.
5. New Membership.
6. Reports of officers and committees.
7. Election of officers and Council (at annual meetings only).
8. Unfinished business.
9. New business.
10. Reading of papers.
11. Discussion of papers.

ARTICLE II.—*Council Meetings.*

Meetings of the Council shall be called whenever the President or three members of the Council so request, but not without sending notice to each member of the Council seven or more days before the time appointed. Such notice shall state the objects intended to be brought before the meeting, and should other matter be passed upon, any member of the Council shall have the right to re-open the question at the next meeting.

ARTICLE III.—*Duties of Officers.*

The President, or, in his absence, one of the Vice-Presidents, shall preside at meetings of the Society and of the Council. At the Society meetings the presiding officer shall vote only in case of a tie, but at the Council meetings he may vote in all cases.

The Secretary-Treasurer shall keep a full and accurate record of the proceedings at the meetings of the Society and of the Council, send out calls for the said meetings, and, with the approval of the President and Council, carry on the correspondence of the Society. Subject to the direction of the Council, he shall have immediate charge of the office and archives of the Society.

The Secretary-Treasurer shall also send out calls for annual dues and acknowledge receipt of same; pay all bills approved by the President for expenditures authorized by the Council of the Society; keep a detailed account of all receipts and expenditures, and present an abstract of the same at the annual meetings, after it has been audited by a committee appointed by the President.

The Editor shall, under the general supervision of the Council, have charge of all matters connected with editing and printing the Society's publications. The *Proceedings* shall contain only the proceedings of the meetings, original papers or reviews written by members, discussions on said papers and other matter expressly authorized by the Council.

The Librarian shall, under the general supervision of the Council, have charge of the books, pamphlets, manuscripts and other literary or scientific material collected by the Society.

The General Chairman of the Examination Committee, shall, under the general supervision of the Council, have charge of the examination system and of the examinations held by the Society for the admission to the grades of Associate and of Fellow.

ARTICLE IV.—*Dues.*

The Council shall fix the annual dues for Fellows and Associates. Effective November 20, 1964, the payment of dues will be waived in the case of any Fellow or Associate who attains the age of 70 years or who attains the age of 65 years and notifies the Secretary-Treasurer in writing that he has retired from active work. Fellows and Associates who have become totally disabled while members may upon approval of the Council be exempted from the payment of dues during the period of disability.

It shall be the duty of the Secretary-Treasurer to notify by mail any Fellow or Associate whose dues may be six months in arrears, and to accompany such notice by a copy of this article. If such Fellow or Associate shall fail to pay his dues within three months from the date of mailing such notice, his name shall be stricken from the rolls, and he shall thereupon cease to be a Fellow or Associate of the Society. He may, however, be reinstated by vote of the Council upon payment of arrears in dues, which shall in no event exceed two years.

ARTICLE V.—*Designation by Initials.*

Fellows of the Society are authorized to append to their names the initials F.C.A.S.; and Associates are authorized to append to their names the initials A.C.A.S.

ARTICLE VI.—*Amendments.*

These by-laws may be amended by an affirmative vote of two-thirds of the Fellows present at any meeting held at least one month after notice of the proposed amendment shall have been sent to each Fellow by the Secretary.

GUIDES TO PROFESSIONAL CONDUCT

(As amended November 20, 1959)

In order to assist the Council of the Society in resolving questions that might be raised as to the professional conduct of members, and more importantly to guide members of the Society when they encounter questions of professional conduct, the following "Guides to Professional Conduct" have been prepared by order of the Council. The actuary has professional responsibilities to society at large, to his client or employer, and to his professional associates. As is true of codes of ethics generally, these guides deal with precepts and principles only. They are not precise rules and are subject to interpretations in relation to the variety of circumstances that occur in practice.

Any member wishing advice on the application of these guides to a particular set of facts is urged to present his case to the Council of the Society. The Council has the power to consider and take action with respect to questions that may be raised as to the professional conduct of members. Any disciplinary action by the Council must be in accord with Article IX of the Constitution.

The Council assumes that every member of the Society earnestly desires to serve his client or employer properly, to protect the public, and to maintain the prestige of the Society and its members. Accordingly, the Council sets forth the following principles by which, in its opinion, every member should be guided in his practice of the actuarial profession.

1. The member will promote a wider understanding of the significance of membership in the Society and will maintain the high standards of the Society by avoiding even the appearance of any questionable practice.

2. The member will conduct his professional competition on a high plane. He will avoid unjustifiable or improper criticism of others and will recognize that there is substantial room for honest differences of opinion on many matters.

3. The member will act in professional matters for each client or employer with scrupulous attention to the trust and confidence that the relationship implies and will have due regard for the confidential nature of his work.

4. The member will bear in mind that the actuary acts as an expert when he gives professional advice, and he will give such advice only when he is qualified to do so.

5. The member will not provide actuarial service for, or associate professionally with, any person or organization if he has reason to believe that the results of such service or association are likely to be used in a manner inimical to the public interest or the interests of the actuarial profession or to evade the law.

6. The member will submit unqualifiedly an actuarial calculation, certificate, or report only if he knows it to be based on sufficiently reliable data and on actuarial assumptions and methods that, in his judgment, are consistent with the sound principles expounded in the course of study of the Society, or in recognized texts, sources or precedents relevant to the subject at hand.

7. The member will recommend for the use of his client or employer, premium rates, rating plans, dividends or other related actuarial functions only if, in his opinion, they are based on adequate and appropriate assumptions and methods.

8. The member will not make or sponsor any actuarial calculation, certificate, statement, report, or comparison, or give any testimony or interview on such matters, which he has reason to believe is false, materially incomplete, or misleading.

9. Where appropriate for the objective use of a certificate or report, or in any event on the request of his employer or client, the member will include a statement of the principal actuarial assumptions and the general methods adopted for his computations.

10. The member will recognize his ethical responsibilities to the person or organization whose actions may be influenced by his professional opinions or findings. When it is not feasible for the member to render his opinions or findings direct to such person or organization, he will act in such manner as to leave no doubt that the member is the source of the opinions or findings and to indicate clearly the personal availability of the member to provide supplemental advice and explanation.

11. The member will not serve more than one client or employer where a conflict of his professional interest may be involved unless there be a full disclosure to all parties concerned, and such parties request and acquiesce in the engagement of his services.

12. The member will sign actuarial recommendations, certificates, and reports if he be acting as an employe, only over a title conferred by his employer if any title is used. Nevertheless, in any capacity, the member may append to his signature the designation: "Fellow of the Casualty Actuarial Society" or "FCAS," or "Associate of the Casualty Actuarial Society" or "ACAS," as the case may be. The member will not use as a signature title the designation "Member of the Casualty Actuarial Society". The member will use a designation dependent upon elective or appointive qualification within the Society such as "President," or "Member of the Council," only when he is acting in such capacity on behalf of the Society.

13. The member will recognize his personal responsibilities under these guides whether he acts as an individual or through a partnership or his employer.

(As amended December 1, 1964)

Method of Review. All papers and reviews of papers are reviewed by the Committee on Review of Papers. The Committee consists of members appointed by the President, plus, ex officio, the Editor of the *Proceedings.* Unanimous vote of the regular Committee is necessary for acceptance of a paper or a review, except that if there is only one vote for rejection, the paper or review will be reviewed by the Editor and accepted if he approves.

Scope and Standards.—1. Broad latitude will be allowed in the choice of a subject, provided it is a subject of interest to property and casualty actuaries. However, it must be clearly suitable for inclusion in the *Proceedings.*

2. The paper must contain original ideas or new material of reasonable value, unless it has a definite educational value for other reasons.

3. When a paper includes material that the Committee finds it is not qualified to review, the Committee will seek advice or opinion from other members of the Society or from recognized experts outside of the Society.

4. Disagreement by the Committee with opinions of the author or reviewer of a paper will not be a bar to acceptance of an otherwise suitable paper or review. Where, however, the Committee believes a paper or review to be fallacious in logic or misleading in matters of fact the Committee may reject it. Reviews of papers are expected to be free of criticism of a personal nature. Opportunity will be given to the authors of papers to respond to reviews. Authors' replies will also be reviewed by the Committee and will be treated in the same manner as reviews.

5. The paper or review should show care in preparation. A reasonable minimum standard will be required as to form, clarity, and literary quality. When a paper or review, otherwise acceptable, does not meet these standards, the Committee may return it to the author or reviewer and invite resubmission after editing or rewriting. The Committee may also make suggestions to the author as to possible improvements in an accepted paper.

6. Papers and reviews should be kept within the general limits of length indicated by past acceptances, ordinarily about twenty printed pages for papers and two or three pages for reviews.

Procedures and Regulations.—1. Papers may be submitted only by Fellows or Associates of the Casualty Actuarial Society, except that papers may be submitted by non-members of the Society upon invitation of the President. A member may collaborate in joint authorship with a non-member who possesses particular qualifications in respect to the subject of a paper.

2. Papers and reviews of papers should be submitted in quadruplicate to the Secretary-Treasurer of the Society. The Secretary-Treasurer is authorized to return to the author or reviewer copies of a paper or a review that in his opinion are not legible.

3. The name of the author should not appear on the copies of the paper submitted to the Secretary-Treasurer but should be included in the covering letter. However, names of the reviewers should be identified on the copy of the review.

4. In submitting a paper, the author must answer the following questions on a separate sheet:

(a) Name of paper.

(b) Has the paper been published elsewhere, in whole or in part, in identical or similar form?

(c) Is the paper being simultaneously submitted elsewhere, or will it be so submitted before decision by the Committtee on Review of Papers?

(d) In the case of co-authorship with a non-member, to what extent has the Society member contributed?

(e) If the paper contains factual data from some organization, has the organization given the author permission to publish it?

5. Papers and reviews should be typed double-spaced on letter-size stationery, on one side of each sheet. Tables and footnotes may be single-spaced. Pages should be numbered. Footnotes should be numbered consecutively throughout the paper.

6. Major captions should be centered and typed in capitals; subcaptions should appear in the left-hand margin in italics (single underscore). In technical papers paragraphs may be numbered to simplify reference; in non-technical papers paragraphs should not be numbered.

7. So far as possible, tables should be arranged so that they can be printed on a single page of the *Proceedings* without undue reduction in size of type. Column headings must be clear and concise.

8. All mathematical formulas and symbols should be handwritten in ink rather than typewritten. They must be legible especially as to subscripts and superscripts. There must be no possibility of confusion between, for instance, dx and d_x; $\times$ (the sign for multiplication) and x; a and α (alpha). The exclamation point (!) should be used to indicate factorials in binomial expansions. Where necessary, instructions to the printer may be inserted in pencil on the manuscript. The Committee strongly recommends that authors of mathematical papers refer to the Style Manual of the American Institute of Physics for precise information on preparation of a manuscript. A copy of the Style Manual may be borrowed from the Editor of the *Proceedings* or it may be purchased from the Editor for one dollar. When life contingency symbols are applicable the International Actuarial Notation should be used. This code is described in the *Proceedings*, Vol. XXVI, page 123.

9. References to books and periodicals and to proceedings of professional societies, should be sufficiently complete to permit obtaining a copy of the source without additional research.

10. If the manuscript has been prepared carefully in accordance with the foregoing suggestions, there should be only a few minor corrections necessary. The paper as originally submitted should not be considered simply as a draft to which extensive alterations can be made.

11. Authors will be notified of the acceptance or rejection of their papers by the Secretary-Treasurer. If a paper is rejected, original and copies will be returned. The Committee does not promise a decision on a paper submitted fewer than forty-five days prior to the meeting for which the paper has been prepared. Reviews of a paper are to be submited to the author and the Secretary-Treasurer thirty days in advance of the meeting at which the paper is to be reviewed. A review of a paper will be considered to have been accepted by the Committee unless the reviewer is otherwise notified.

12. Authors of accepted papers are requested to notify the Secretary-Treasurer whether or not they can supply additional copies for use at meetings or for further distribution prior to publication. (Photographic reproduction is less expensive than printing and insures accuracy.)

13. After acceptance of a paper and before its reproduction, the author should have the following statement typed at the bottom of the first page: "Presented at the (date) meeting of the Casualty Actuarial Society at (city and state). Reproduction in whole or in part without acknowledgment to the Casualty Actuarial Society is specifically prohibited."

14. Except on recommendation of the Committee, no accepted paper will be read in its entirety at a meeting of the Society. The author will be expected to prepare for oral presentation a two or three minute abstract, stating the purposes of his paper and its conclusions.

15. The Editor of the *Proceedings*, in consultation with the author or reviewer, may edit the paper or review prior to publication.

WOODWARD - FONDILLER PRIZE

This award made in commemoration of Joseph H. Woodward and Richard Fondiller is intended to stimulate original thinking and research and will be made to the best eligible paper each year submitted by an Associate or Fellow who has attained his designation within the last five years. To be eligible the paper must show evidence of ability for original research and the solution of advanced insurance problems. If no paper is considered eligible in a given year, the award shall not be made. Papers previously submitted to the Society or elsewhere, shall not be eligible.

The amount of the prize will be $200 and the papers will be judged by the Society's Committee on Review of Papers whose decision will be final.

The announcement of the award will be made at the November meeting each year, based on papers submitted to the Society at the previous November and May meetings.

RULES REGARDING EXAMINATIONS FOR ADMISSION TO THE CASUALTY ACTUARIAL SOCIETY

1. Dates of Examinations.

Examinations for all parts will be held in May each year in such cities as will be convenient. In addition, Associateship Part I will also be held in November each year. The exact dates will be set by the Secretary-Treasurer.

2. Filing of Application.

The initial application for admission to examinations must be made on the Society's official form which may be obtained from the Secretary-Treasurer. No application will be accepted unless accompanied by the appropriate examination fee. Checks must be made payable to the Casualty Actuarial Society.

A student who has once registered to take the examinations need not again register on the official form in order to sit for subsequent examinations. However, in that event, he must notify the Secretary-Treasurer in writing of which subsequent examinations he desires to take, and must concurrently remit the appropriate examination fee.

All applications, whether for the initial registration or for a subsequent examination, must be received by the Secretary-Treasurer prior to April 1 for the Spring examinations or prior to October 1 for the Fall examinations, and must be accompanied by the appropriate examination fee. No application which does not comply with these requirements will be accepted.

3. Associateship and Fellowship Examinations.

There are four parts of the examinations which the candidate must pass in order to become an Associate of the Casualty Actuarial Society. These consist of five actual examinations:

Part I		3 hours
Part II		3 hours
Part III	Section (a)	1½ hours
Part III	Section (b)	1½ hours
Part IV	Sections (a) and (b)	3 hours

Part I of the Associateship examinations is a General Mathematics examination jointly sponsored with the Society of Actuaries. Credit for passing this examination will be given by both Societies regardless of the Society through which the candidate registers. One pass list showing the successful candidates (without identification as to the Society through which they register) will be published.

A candidate may write any one or more of the five examinations and will receive credit for those passed.

There are four examinations which a candidate must also pass to become a Fellow of the Casualty Actuarial Society. Each Fellowship Part consists of two sections, but is a single 3 hour examination. A candidate may present himself for one or more of the Fellowship examinations either if he has previously passed the Associateship examinations or if he concurrently presents himself for all unpassed Associateship examinations. Subject to the foregoing requirements, a candidate will be given credit for any examination which he may pass.

4. Fees.

The examination fee for the Associateship examination is $3.75 for a section, $7.50 for one complete part; subject to a minimum of $7.50 for each year in which the candidate presents himself. The examination fee for the Fellowship examination is $10.00 for each part. Examination fees are payable to the order of the Society and must be received by the Secretary-Treasurer before April 1 of the current year for the Spring examinations, or before October 1 for the Fall Associateship Part I examination.

5. Prize Awards

The Casualty Actuarial Society and the Society of Actuaries jointly will award one $200 and four $100 prizes to the five successful undergraduates ranking highest in the General Mathematics Examination. These prize awards will be granted twice each year, i.e., for both the Spring and Fall examinations.

6. Credit for Examination Parts under Former Syllabus.

A candidate who has passed, or been credited with, one or more of the Associateship or Fellowship examinations under the 1963 Syllabus will receive credit for the corresponding examinations of the 1964 Syllabus. Partial examinations will be given to those candidates requiring them in accordance with such credits.

A candidate who has passed or been credited with only one Section of Associateship Part II (either Section (a) or Section (b) under the 1963 Syllabus will be permitted to write the remaining Section in 1965, upon application to the Secretary-Treasurer. The time allowed for writing the remaining Section will be 1½ hours. Beginning with the 1966 examinations, no candidate will be permitted to write only a portion of Associateship Part II, and any prior credit for one Section of this examination will expire.

7. Waiver of Examinations for Associateship.

Waiver of the following Associateship examinations will be allowed for a candidate who has passed or been credited with the corresponding examinations of the Society of Actuaries:

Casualty Actuarial Society	*Society of Actuaries*
Part I	Part 1
Part II	Part 2
Part III (a)	Part 4

Candidates who take the Advanced Mathematics Test of the Graduate Record Examinations may apply for credit for the General Mathematics Examination, (Associateship Part I). Credit will be granted if the candidate's score on the Graduate Record Advanced Mathematics Test is equivalent, as determined by the Casualty Actuarial Society, to the passing score on the Society's General Mathematics Examination. To be eligible for such credit the candidate must take the Graduate Record Advanced Mathematics Test while a full time undergraduate or graduate student at a college or university, or if he ceases his full-time schooling in May or June he may take the Graduate Record Advanced Mathematics Test in the following July. An application to the Casualty Actuarial Society for credit may be completed either in advance of taking the Graduate Record Advanced Mathematics Test or within two years after taking it. The necessary application form may be secured from the Secretary-Treasurer of the Casualty Actuarial Society.

The council may waive, subject to such other requirements as it may prescribe, any examinations of the Casualty Actuarial Society which it deems equivalent to examinations required by another recognized actuarial organization which have been passed by an applicant while not a resident of the United States or Canada, or during his first year of temporary or permanent residence in the United States or Canada.

LIBRARY

All candidates registered for the examinations of the Casualty Actuarial Society and all members of the Casualty Actuarial Society have access to all the library facilities of the Insurance Society of New York and of the Casualty Actuarial Society. These two libraries, with combined operations, are located at 150 William Street, New York, New York 10038.

Registered candidates may have access to the library by receiving from the Society's Secretary-Treasurer the necessary credentials. Books and manuals may be withdrawn from the library for a period of one month without charge. The Insurance Society is responsible for postage and insurance charges for sending books to out of town borrowers, and borrowers are responsible for the safe return of the books.

Address requests for books to:

LIBRARIAN
Insurance Society of New York
150 William Street
New York, New York 10038

SYLLABUS OF EXAMINATIONS

(Effective with 1964 Examinations)

ASSOCIATESHIP

Part	*Section*	*Subject*
I		General Mathematics.
II		Probability and Statistics.
III	(a)	Elementary Life Insurance Mathematics.
	(b)	General Principles of Insurance; Insurance Economics and Investments.
IV	(a)	Insurance Coverages and Policy Forms.
	(b)	General Principles of Ratemaking.

FELLOWSHIP

I	(a)	Insurance Law; Supervision, Regulation and Taxation.
	(b)	Statutory Insurances.
II	(a)	Premium, Loss and Expense Reserves.
	(b)	Insurance Accounting and Expense Analysis.
III	(a)	Individual Risk Rating.
	(b)	Problems in Underwriting and Administration.
IV	(a)	Insurance Statistics and Machine Methods.
	(b)	Advanced Problems in Ratemaking.

INTERNATIONAL CONGRESSES OF ACTUARIES

The first International Congress of Actuaries was held in 1895 in Brussels. Since that time numerous congresses have been held, and many actuaries from the United States and Canada have been benefited by attendance at the congresses and by the printed *Proceedings*, in which numerous valuable articles have appeared.

Continuity in the arrangement for periodic congresses and for the intervening support and management of the central office located in Brussels is achieved by the maintenance of a Permanent Committee of international membership.

Membership in the Permanent Committee on this continent is divided into two sections, a United States section and a Canadian section. Individual actuaries can support the work of the Permanent Committee by becoming members in their section. Inquiries regarding the Permanent Committee should be directed to Pearce Shepherd, Secretary for the United States Section, Prudential Insurance Company, Newark, New Jersey or to Ben T. Holmes, Chairman of the Canadian Section, Confederation Life Association, 321 Bloor Street, East, Toronto 5, Ontario.

According to the revised regulations adopted by the New York Congress in 1957, the objects of the Permanent Committee are:

1. To promote or to conduct work or research of interest to the science or practice of the Actuary. For this purpose sections formed by a number of members for study of special problems may be recognized. Each section will have its own regulations, previously approved by the Council; it will elect its Committee, except for the member appointed by the Council on the Committee.
2. To publish periodically a *Bulletin: (a)* bringing together technical, legislative, statistical, and juridical information relating to actuarial science; *(b)* reviewing publications and works which appear in various countries, bearing upon actuarial matters.
3. To co-operate with the Organizing Committees in preparing the work of International Congresses, and in the publication of their *Proceedings*.

The XVIIth Congress was held in Great Britain in 1964.

With these purposes in mind the Permanent Committee wishes to enlist members as broadly as possible. The annual dues for membership are now 150 Belgian francs. Membership on the Permanent Committee is one of the requirements for membership in a Congress.

ASTIN Section

ASTIN (Actuarial Studies in Non-Life Insurance) is the first section of the Permanent Committee to be formed under the Modification of the rules approved at the XVth International Congress in New York and is for the study of the application of modern statistical and mathematical methods in the field of non-life insurance. It has grown from the desire expressed by many members of the XIVth Congress held in Madrid to provide an effective interchange of ideas on an international basis.

It has as its object the promotion of actuarial research in general insurance and establishes contact between actuaries, groups of actuaries, and other suitably qualified persons interested in this field.

This section, from time to time, publishes papers on topics related to its objects and also publishes a *Bulletin* containing notes of general interest to members.

Meetings are held every four years, during the course of the International Congress of Actuaries. Between meetings colloquia are held on topics of interest to the Section and these are hosted by national actuarial bodies. The 1965 colloquium will be held in Lucerne, Switzerland, June 9-12, sponsored by the Swiss Actuarial Society.

The members of the Committee of ASTIN are:

Chairman	Ammeter, Hans—Switzerland
Vice Chairman	Masterson, Norton E.—U.S.A.
Treasurer	Thyrion, Paul—Belgium
Members	Johansen, Paul—Denmark
	Ottaviani, Guiseppe—Italy
	Sousselier, Jean—France
	Sternberg, Ingvar—Sweden
	Welten, C. P.—Netherlands
Secretary	Beard, Robert Eric—Great Britain

Membership fees, which are payable in the same manner as the annual dues for membership on the Permanent Committee, are 200 Belgian francs. Inquiries regarding membership in the ASTIN Section should be directed to Albert Z. Skelding, Secretary-Treasurer, Casualty Actuarial Society, 200 East 42nd Street, New York, N. Y. 10017.

FUTURE MEETINGS OF THE CASUALTY ACTUARIAL SOCIETY

1965 Spring Meeting — May 24, 25, 26
Shawnee Inn
Shawnee On Delaware, Pa.

1965 Annual Meeting — November 15, 16, 17
Sheraton-Boston Hotel
Boston, Mass.

1966 Spring Meeting — Dates not yet determined.
Cavalier Hotel
Virginia Beach, Va.

1966 Annual Meeting — November 16, 17, 18
Ann Arbor, Mich.

1965 EXAMINATIONS

May 12, 13, 14, 1965