INSURANCE RATES WITH MINIMUM BIAS

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INTRODUCTION

The paper presents specific methods for obtaining insurance rates that are as accurate as possible for each class and territory and so on. Many of the techniques presented in the paper are already in use by the various bureaus and other ratemakers in one form or another. With the increasing use of electronic computers, there is the opportunity to use them in new ways to improve the accuracy of our ratemaking methods and to reduce the vast mass of statistical detail down to a meaningful set of answers. The methods in this paper are methods that we have used to analyse some of the data in our company.

THE RATEMAKING PROBLEM

In making rates for insurance we are faced with the problem that there are many different classes of risks with a different rate for each class, and that no one class by itself has a sufficient volume of premiums and losses to give a reliable basis for the rate for that class. A simple and practical solution to this problem is to make a rate for each class on the basis of judgment, then to adjust all the class rates up or down by a uniform percentage in order to produce the proper total amount of premium for all classes within one general category. This is a sound procedure under certain conditions and is used in many areas.

It often happens that the classes within one general category can be grouped in such a manner that each group has a sufficient volume of premiums and losses to provide a reliable indication of how much all the rates within each group should be adjusted. An example of this is found in property insurance on dwellings and in Homeowners insurance where the classes are sometimes grouped by type of construction: frame, brick, and fire resistive. Instead of adjusting all dwelling insurance rates by the same percentage, a different adjustment is made for each type of construction. Sometimes the classes in dwelling insurance are grouped by amount of insurance and a different adjustment is made for each amount of insurance. This procedure is better than applying the same adjustment to all classes, but it can only be used when the volume of data is sufficient to provide a reliable indication for each group.

It often happens that the classes within one general category can be grouped in more than one manner. (It should be noted here that we are concerned more with what can be analysed than with what is analysed in every case.) For example, the data for dwelling insurance might be grouped by type of construction and the same data might also be regrouped by amount of insurance. One set of adjustments would be determined for the types of construction and another set for various amounts of insurance. Then each class would receive two adjustments. For example, all the rates for small brick dwellings would receive the adjustment for brick construction and also the adjustment for small amount of insurance. If the same data had also been regrouped into geographical territories and again regrouped by type of fire protection, then each rate would receive four adjustments.

When the same data is successively regrouped in several ways we obtain larger groups with correspondingly greater reliability of the indications, than if we made all the subdivisions simultaneously. For example, the data for all brick dwellings and also for all small dwellings may be sufficient to be reliable whereas the data for small brick dwellings might not be sufficient to be reliable. We naturally would prefer to adjust the rates for small brick dwellings entirely on the basis of the data for small brick dwellings, but if that data is not sufficient to be reliable, we usually find it better to combine the small brick dwelling classes with other groups of classes, as in our example, to produce one adjustment for brick dwellings and another for small dwellings.

Although we may get a more reliable indicated adjustment for brick dwellings by combining all brick classes, and a more reliable indicated adjustment for small dwellings by combining all small dwelling classes, we cannot be so confident that the adjustment for brick dwellings and the adjustment for small dwellings will combine to produce the proper net adjustment for small brick dwellings. The data for small brick dwellings may be insufficient to be fully reliable but it will always provide some information. So we should look at it and take it into consideration. We should try to use a ratemaking system which, instead of producing each set of adjustments successively one after another, produces all sets of adjustments simultaneously. In this way the adjustments for brick dwellings and for small dwellings will both reflect the indication of small brick dwellings as well as the total for brick dwellings and the total for small dwellings. Such a system will produce a better result than a system which ignores the data in each subdivision. Such a system will be set forth in more detail later.

Such a system might possibly be used for fire insurance rates for all commercial risks rated according to the same fire rating schedule, where the data might be subdivided by construction, protection, occupancy, territory, and any other characteristics that are considered important. Such a system could very easily be used in various lines of casualty insurance such as private passenger automobile insurance where the data might be subdivided by territory, class of driver, value of car, age of car, size of deductible, limit of liability, merit rating, whether collision coverage is included or not and so on.

CENTS OR PERCENTS

If the premiums and losses for all classes are combined to produce one adjustment for all classes, it often makes little difference whether we use an adjustment which adds the same number of cents to each rate or an adjustment which increases each rate by the same percent. The relationships among the class rates are not seriously disturbed either way. We can select the type of adjustment which, in our judgment, is more proper for the kind of insurance involved. But when the data is to be divided four different ways with four different adjustments to be applied to each rate, the difference between cents and percents becomes greater. The product of four percents can be materially different than the sum of four amounts of cents. If we produce each set of adjustments successively one after another, we will have to rely entirely on judgment to decide whether each set of adjustments should be cents or percents. But if we produce two or more sets of adjustments simultaneously, we can use the indications of each minor subdivision of the data to tell us which type of adjustment will fit the data better. So an added advantage of computing more than one set of adjustments simultaneously is that we can at the same time determine which type of adjustment is better: cents, percents, a combination of the two, or some other formula relationship among classes.

The Analytic System for the Measurement of Relative Fire Hazard, developed by Mr. A. F. Dean, which is used to establish the rates for commercial buildings in many areas of the United States uses a combination of cents and percents. It is based on fire protection engineering judgment. A system of analysing the premiums and losses developed under such a rating schedule might enable us to test whether cents or percents should be used for several of the more important characteristics recognized by such a schedule.

AN UNBIASED ESTIMATOR WITH MINIMUM VARIANCE

In mathematical statistics the best estimator is defined as the unbiased estimator which has the least variance. For any one mathematical frequency distribution, such as the normal distribution or the Poisson distribution or the negative binomial distribution, there are many unbiased estimators of the mean, sometimes an unlimited number, and the classical problem is to determine which unbiased estimator has the least variance. "Least variance" is equivalent to "most reliable." This problem has been solved for most mathematical distributions.

But in insurance statistics we don't have the luxury of many unbiased estimators to choose from. In fact, we have not yet found even one unbiased estimator. To be sure, when we combine all classes to produce a single adjustment for all classes, the sample mean is unbiased and the resulting adjustment is unbiased in the aggregate, but none of us believe that the resulting rates are unbiased for each class. That is why we subdivide the data when we can. The more we can subdivide the data, the less biased are the resulting rates for each class. But even though we subdivide the data several different ways we are not confident that, for example, the adjustment for young drivers and the adjustment for merit rating combine to produce an unbiased adjustment for young merit rated drivers. So in insurance statistics our big problem is to find the estimator with the least bias.

AN ESTIMATOR WITH MINIMUM BIAS

Suppose that a body of insurance data can be subdivided four different ways into i occupancies, j territories, k constructions and l protections. Suppose further that the total data for each occupancy is considered to be reliable, and similarly for the totals for each territory, each construction and each protection. It is axiomatic, then, that an estimator with minimum bias must produce a total premium for each occupancy exactly equal to the total premium indicated by the total losses for that occupancy, and similarly for each territory, construction and protection. In other words, the estimator with minimum bias must be unbiased in the aggregate for each occupancy, and for each territory, and so on.

If the body of data is only subdivided one way into i occupancies, each of which is considered large enough to be reliable, we simply base the rate for each occupancy on the total for that occupancy. There is only one set of estimators with minimum bias in such a case. But when the data is subdivided in more than one way, such as in the example above with four different ways, there is more than one set of estimators that will be unbiased in the totals. It is possible to devise more than one different set of rates which will produce the same premium totals for each occupancy, each territory, each construction, and each protection. Which set has the minimum bias?

In other words, we seek an estimator that is unbiased for the totals for each occupancy, and so on, and has minimium bias for the multiple subdivisions of the data, where the data is subdivided in all four ways simultaneously. Because the data for each multiple subdivision is not considered fully reliable, we know that the data in each such subdivision will differ from the net adjustment produced for that subdivision. So any set of adjustments will not fit the data in each multiple subdivision at least to the extent of chance variations. Different sets of estimators will differ in different degrees which means that some of them at least will differ more than purely chance variation would account for. So we seek the set of estimators with minimum bias, that is, the set that fits all the data most closely.

Given a certain amount of expected losses for each risk and a certain distribution of actual losses about the mean for each risk, the distribution of actual losses for each class or group of classes will depend on how many risks are included in that class or group of classes. We can see then that the composite distribution of the actual losses about the true population values for the whole body of data and all its subdivisions will be different for every ratemaking study we make and very difficult to calculate. Seeking for an estimator with minimum bias when we are dealing with an unknown distribution which will be different for each set of data we encounter is a problem which will have to be solved in an empirical manner.

A body of data that is subdivided four different ways may have a thousand different sets of estimators that are unbiased for the totals for each occupancy, territory, and so on. For practical reasons we will not compute all possible sets. We will probably be satisfied if we compute three or four different sets and test each one for its degree of bias.

RATES THAT ARE UNBIASED IN THE AGGREGATE

As mentioned above, there are usually more than one set of estimated rates that are unbiased in the aggregate. If we can calculate several such sets we can then test them to see which one has the least bias for the multiple subdivisions of the data. An efficient way to calculate a set of estimated rates that are unbiased in the aggregate for each occupancy, each territory, and so on is to set up a formula for the average deviation of the estimated rates from the data for each occupancy, set the average deviation equal to zero, and derive a formula for the estimator for each occupancy. Using a predetermined set of estimators for each territory, construction, and protection, we can solve the formula for the estimator for each occupancy. We can then use these calculated estimators for each occupancy to calculate a revised set of estimators for each territory using a similar formula, and continue this process until the estimators stabilize. Examples of the formulas that might be used are shown in the appendix. Needless to say, if there are many subdivisions of the data, this problem is better done on electronic computers than by hand.

MEASURES OF BIAS

In order to compare several sets of estimators to find which one fits the data better, we cannot use the average bias because we used the average bias to compute the estimators. All sets of estimators should have an average bias of zero.

A very practical and easily understood measure is the average absolute difference between the estimated rates and the data for each multiple subdivision of the data. The differences, without regard to sign, are weighted by the number of risks or amount of premium in each subdivision. The usual disadvantage of the average absolute difference is that the derivation of its mathematical distribution is more difficult than for other measures. This is not a disadvantage in our problem here because we are only comparing one estimator with another. We are not trying to derive any mathematical distributions.

A measure of bias which uses the squares of the differences is a good supplement to the average absolute difference, especially if each subdivision has a large volume of data in it so that the distribution of sample values about the true population value is not too different from a normal distribution. The chi-square test is probably the most appropriate such measure. Since the distribution of losses is not normal, the value computed for chi-square will be much larger than for a normal distribution. But this will not be a problem as long as we are simply comparing one set of estimates with another.

If the data is subdivided too finely for the amount of data available, chance variations will overshadow true variations to such an extent that it will be difficult to tell, from any measure of bias, which relationship is better – cents, percents, or anything else. In such cases the sets of adjustments will have to be analysed two or three sets at a time to determine how the adjustments should be interrelated so as to produce minimum bias. Once the measures of bias have been used in this way to determine how the various sets of adjustments should be interrelated, the actual adjustments can then all be calculated simultaneously.

INCREASING THE RELIABILITY OF THE DATA

We have seen that the more we can subdivide the data, the less biased the resulting rates will be. However, we are limited in our subdivisions by the requirement that the total data in any one subdivision must be sufficient to be reliable. For some kinds of insurance it is possible to increase the reliability of the data by making rates in layers. For example, if the total data for one class of Workmen's Compensation insurance is not fully reliable, perhaps the first $1,000 of each loss would be fully reliable. In Workmen's Compensation insurance in the U.S.A., about half of the rate is for the first $1,000 of each loss. It would be better to base half of a rate on a fully reliable indication of the experience for the first layer for the class, and base the remainder of the rate on some overall indication, than to base the entire rate on an average of the overall indication and an unreliable indication of the total experience for the class.For a thorough discussion of the advantages of using layers rather than percentages of the total experience, see "An Attempt to Determine the Optimum Amount of Stop Loss Reinsurance" by K. Borch, *XVI International Congress of Actuaries,* 1960, Vol. I, p. 597. The principles developed by Mr. Borch are applicable here as well as in reinsurance.

Suppose we divide the losses into three or four layers, for example, the first $1,000 of each loss, the next $2,000, and all losses in excess of $3,000. Then we can subdivide the data in the first layer into much finer detail than we can subdivide the total data and still get fully reliable estimators. This technique of making rates in layers is especially effective when a large proportion of the total losses are small losses.

The combination of the layer technique and the technique outlined above for obtaining rates with minimum bias is a very powerful tool for squeezing every last drop of information out of the data available.

APPENDIX

Let us define x_i as the estimated rate factor for the ith occupancy and y_j, z_k and w_l as the estimated factors for the jth territory, the kth construction and the lth protection, respectively. Let r_{ijkl} be the combined factor indicated by the actual losses and exposures for the n_{ijkl} risk in the ith occupancy, jth territory, kth construction and lth protection.

If all the factors are percents and the estimated rate corresponding to r_{ijkl} is $x_i y_j z_k w_l$:

The average difference for the ith occupancy equals

$$\frac{\sum_{jkl} n_{ijkl}\,(r_{ijkl} - x_i y_j z_k w_l)}{\sum_{jkl} n_{ijkl}\, r_{ijkl}}$$

and similarly for each territory, construction and protection.

The average difference for all classes equals

$$\frac{\sum_{ijkl} n_{ijkl}\,(r_{ijkl} - x_i y_j z_k w_l)}{\sum_{ijkl} n_{ijkl}\, r_{ijkl}}$$

The average absolute difference equals

$$\frac{\sum_{ijkl} n_{ijkl}\,\left| r_{ijkl} - x_i y_j z_k w_l \right|}{\sum_{ijkl} n_{ijkl}\, r_{ijkl}}$$

The chi-square is proportional to

$$\sum_{ijkl} \frac{n_{ijkl}\,(r_{ijkl} - x_i y_j z_k w_l)^2}{w_i y_j z_k w_l}$$

(See the 1960 *PCAS*, page 17 for the derivation of this chi-square formula.)

Setting the average difference for the ith occupancy equal to zero and solving for x_i we obtain

$$x_i = \frac{\sum_{jkl} n_{ijkl}\, r_{ijkl}}{\sum_{jkl} n_{ijkl}\, y_j z_k w_l}$$

and similarly for y_j, z_k, *and* w_l.

If all the factors are cents and the estimated rate corresponding to r_{ijkl} is $x_i + y_j + z_k + w_l$:

The average difference for the ith occupancy equals

$$\frac{\sum_{jkl} n_{ijkl}\,(r_{ijkl} - x_i - y_j - z_k - w_l)}{\sum_{jkl} n_{ijkl}\, r_{ijkl}}$$

and similarly for each territory, construction and protection.

The average difference for all classes equals

$$\frac{\sum_{ijkl} (r_{ijkl} - x_i - y_j - z_k - w_l)}{\sum_{ijkl} n_{ijkl}\, r_{ijkl}}$$

The average absolute difference equals

$$\frac{\sum_{ijkl} n_{ijkl} \left| r_{ijkl} - x_i - y_j - z_k - w_l \right|}{\sum_{ijkl} n_{ijkl} r_{ijkl}}$$

The chi-square is proportional to

$$\sum_{ijkl} \frac{n_{ijkl} (r_{ijkl} - x_i - y_j - z_k - w_l)^2}{x_i + y_j + z_k + w_l}$$

Setting the average difference for the *i*th occupancy equal to zero and solving for x_i we obtain

$$x_i = \frac{\sum_{jkl} n_{ijkl} (r_{ijkl} - y_j - z_k - w_l)}{\sum_{jkl} n_{ijkl}}$$

and similarly for y_j, z_k and w_l.

If the factors are some combination of cents and percents, or are based on some other relationship, appropriate formulas can be set up.

DISCUSSION BY JAMES R. BERQUIST

Mr. Bailey's latest paper is, indeed, a timely contribution to the proceedings of our Society. Timely, not only because it provides a method of calculating rates with minimum bias, but also because it provides ideal computer application. Without the aid of a computer the method is, in fact, impractical.

The technique presented in the paper bears careful study by every ratemaker who has the task of calculating territorial or class differentials, and what ratemaker doesn't? Mr. Bailey's technique is designed to calculate the differentials which provide the best "fit" of the data. He solves for each of the various differentials by setting what he defines as the average difference equal to zero, then, by successive approximation he arrives at the set which provides the best fit.

Mr. Bailey goes on to provide an outline of a method of testing the resultant differentials, or "estimators" for minimum bias. The advantage of this system over the systems presently in use is that the differentials so calculated will yield rates which are most nearly correct for, say, "small brick buildings" as well as small buildings in total and brick buildings in total.

It is interesting to note the similarity between this method and "Method 2" advanced by Bailey and Simon in "Two Studies in Automobile Insurance Ratemaking," *PCAS,* Vol. XLVII, which, I believe, should be read in conjunction with this paper.