## DISCUSSION BY ROBERT L. HURLEY

This paper involves certain aspects of higher algebra which have received an impetus in the last decade from the developments in the field of Linear Programming. As yet these techniques have had limited application to actuarial problems—but this may soon be rectified since the author of this paper, Kenneth L. McIntosh, is an active participant in the CAS recently appointed Special Committee on the Mathematical Theory of Risk.

The nub of the McIntosh paper is the application of linear algebra to the underlying philosophy of fire schedule rating. Undoubtedly the transition to matrices and vectors can be disconcerting for actuaries whose previous interests may have gravitated towards statistical distributions and probability theory. There are certain tangible aspects of probability theory which can be most pleasing.

For example, theoretical developments on the Negative Binomial by members of our Society within the last few years can be, and have been, put to the test of actual loss occurrences. The same situation held in the case of the earlier work by our Society on the Poisson distribution and even in the prior actuarial generation of Arne Fisher, Mowbray, Whitney and H. C. Carver in their investigations into the Pearson curves, the Chi Square test, etc.

It is certainly reassuring to take a highly theoretical mathematical development, apply it to raw insurance statistics, and see that it helps to solve actual rating problems. Unfortunately, Mr. McIntosh could not avail himself of the luxury of insurance data to fit his mathematical schema. This is thought to be the fault neither of Mr. McIntosh, nor of the Fire Insurance Industry.

Now, every author constrained to work with abstruse symbols runs the risk of having his work misinterpreted and what is at first strange often becomes suspect. There is, of course, the classic story, probably apocryphal, of the agnostic Diderot's confusion in being confronted by the mathematician Euler with the challenge "$(a + b^n) / n = x$. Therefore, a God must exist. Refute, Sir, if you can."

This quotation is cited not to imply that the author is thus attempting to convert an unbelieving reviewer afflicted with an invincible ignorance. But seriously, the reviewer considers the McIntosh paper as a sincere effort to explain fire schedule rating in terms of some advanced mathematical techniques.

The schedule rating method, in itself, suggests the linear equation philosophy. The schedule sets up a systematic arrangement of debits and credits for the physical characteristics of the risk. The net result of adding up the various debits and credits is the risk rate.

As Mr. McIntosh points out, a schedule cannot be operated in a laboratory but must be applied to specific risks to develop the policyholders' insurance costs. Therefore, there are practical constraints to the maximum and minimum charges that can be made. Many bureaus have long had minimum or "stop" rates below which any net rate produced by the schedule would be disregarded. While I do not know of any formal maximums there are practical limits beyond which the schedule rate cannot go.

Consequently, in arranging his schedule charges and credits the fire insurance engineer must set his relative values to stay within the plan minimum and the practical maximum charges. At the same time the factors should be

arranged in such a fashion that the preponderance of risks do not cluster at either the top or the bottom limits. Moreover, there must be a relative consistency in the charge for similar hazards from risk to risk.

The Fire Insurance Industry has been operating this type of risk rating plan for many years with obvious success. Probably few of the schedule raters would recognize the association of their work with the McIntosh equations but it is believed that both are reducible to a common philosophy. Mr. McIntosh states in his paper that the fire insurance underwriters do not explicitly rate risks according to his mathematical model. Moreover, he observes that it would be impossible for them to do so at the present time. Basically Mr. McIntosh is saying that the limits which the rater must observe in scheduling his debits and credits are akin to a problem in linear algebra.

The reviewer does not believe that Mr. McIntosh offers his paper as an irrefutable proof of the mathematical basis for fire rates. Actually the value of fire schedule rating has been substantiated in many ways. Of prime importance, each assured is guaranteed that his rate takes into consideration the hazard presented by his own property as measured according to the impartial evaluation of the bureau engineer. The merit of the schedule rating approach is also proved by the fact that the insurance companies have been able to render vital protection to the public and at the same time have been able to segregate sufficient funds as a guarantee that future losses will be paid despite the magnitude of the conflagration hazard.

Lest there may be some misunderstanding of the role played by the fire insurance schedule, the reviewer would probably like to have the record straight. The schedule rate in itself is not susceptible of mathematical demonstration by insurance loss statistics. There are many students of the business who sincerely believe that no such attempt should be made.

This is not to say that fire insurance rates are not responsive to actual loss experience. The overall fire rate level is determined by comparing the actual experience with the balance point loss ratio. Once the rate level has been set for fire insurance within the State, the local rating bureau reviews the National Board tabulations of classified experience to see which types of business should be subject to rate modifications. In a way the schedule might be viewed as the means of distributing the State rate level as modified for the classification experience to the individual risk.

The Society is indebted to Mr. McIntosh for writing a paper on advanced mathematical techniques which have had wide application in many fields other than insurance. It may well be that the Society's "Special Committee on the Mathematical Theory of Risk" will indicate areas of possible application which will encourage other members to undertake writing valuable papers in this potentially important field.

## AUTHOR'S REVIEW OF DISCUSSION

KENNETH L. McINTOSH

Mr. Dropkin's emphasis on the particular method of transforming the initial equations is a little bewildering, granted that in the immediate instance the method he suggests would have been a better choice than that which was