hand, the assumption of independence is incompatible with the "contagion" process, the second avenue of approach to the negative binomial. An excellent opportunity to explicitly bring out the difference between the two approaches was afforded when Mr. Simon set out the two models for generating the negative binomial. It was, therefore, somewhat disturbing to see Mr. Simon characterize the second model as being only "a more elaborate model" than the first.

Mr. Simon closes his paper with the remark: "There is a great deal of exploration and application yet to do and I hope our Proceedings will contain much of the good work in the future." There is no doubt in my mind that Mr. Simon's paper will be an essential instrument in making possible this hoped for future.

## DISCUSSION BY LEWIS H. ROBERTS

We are favored to receive this bibliography on the negative binomial distribution, all the more so for its concise resumes and evaluations of references. The value of this work is much enhanced by the authors well conceived selectivity in choice of entries since so many discussions have been published on this distribution and on the related subject of accident proneness. The student who attempts to survey the entire literature is confronted with a large number of sources, many being redundant, some misleading and some irrelevant to insurance problems. The first work mentioned in this bibliography, for example, lists eighty four other references!

A point made by several writers, and properly emphasized in this paper, is the danger of estimating rate differentials from the negative binomial parameters derived from the distribution of risks by the number of claims incurred during a single period of experience. The negative binomial can arise from other causes than heterogeneity of risks, and the apparent degree of heterogeneity can be distorted by other factors.

Even the use of a bivariate negative binomial with two periods of experience does not necessarily lead to proof of heterogeneity since, as pointed out, interdependence of accidents can also yield that distribution.

The author mentions a paper by Edwards and Gurland which, because of its particular attention to the treatment of experience for separate time intervals, should be of special interest to actuaries. They show, first, that such experience can sometimes be well represented by a bivariate negative binomial. They next develop a more general function of which the negative binomial is a special case. As might be expected because of its greater generality, the latter distribution may give a better fit than the former, but at the cost of introducing at least one additional independent parameter. I hope a shorter name will be found for this distribution. These authors call it a "compound correlated bivariate Poisson."

With the mathematics of general insurance in its present stage of development, there is no ready formula for every problem. If he is to be more than just a theoretician, the actuary must draw upon the a priori knowledge provided by practical experience. The existence of classes with consistently dissimilar loss experience conclusively demonstrates that heterogeneity does exist in the general population of risks. To suppose that this characteristic

stops at the boundaries of our class definitions not only imputes perfection to our class plans, but implies that the many underwriting factors which are necessarily disregarded in designing a workable classification are immaterial. The practical question is not whether a class is heterogeneous, but whether it is so to a degree that warrants a refinement of treatment. If the answer is yes, we have next the problems of how to identify, measure and reflect variations.

Studies of accident proneness distinguish between variations in personal susceptibility to accident and variations in environment. For most rating purposes (except, for example, where a risk has moved) these factors operate jointly and the distinction does not concern us. In calculating expected losses we do not need to know whether a risk is worse than average because of poor driving or because the roads in his neighborhood are hazardous. In accident prevention such distinctions are important, but in rating it is usually sufficient to measure variation without analyzing its cause. The exceptions are where a change in hazard has occurred.

On the other hand, we do need to know whether a debit based on past experience should continue beyond the next rating period. We are concerned whether an individual is more or less prone to accident for a while after one has occurred or whether he is characteristically worse or better than average. It is insufficient to show merely that his recent experience identifies him as belonging to a category of risks for which a debit is justified at the next rating. We also need to know whether his immediate expectation of loss reflects a temporary condition or whether it is representative of his expectation in the long run. The studies mentioned in this bibliography point up the difficulties in the way of answering this question but suggest avenues of approach.

The negative binomial distribution, or any other mathematical model, is at best only an idealistic simplification of reality. As such, it may enable us better to describe the essential features of a complex phenomenon, and it may thereby have some predictive value. As more accurate formulas are discovered we are tempted to over-exploit them and to rely more upon mathematics and less upon painstaking analysis of the facts in each case. This, however, does not gainsay the value of studies such as the author has recommended. To be of use, even the most thorough analysis of facts must be capable of appropriate expression. These mathematical functions not only afford means of such expression but provide a guide to analysis by suggesting what to look for in the data.