through the application of combinatorial theory. (See *Selected Techniques of Statistical Analysis,* Statistical Research Group, Columbia University, McGraw-Hill, New York, 1947, pp 247-257.) Unfortunately these methods are cumbersome to apply in a multi-dimensional array involving hundreds or even thousands of cells and it would be beyond the scope of this review to go further into that subject.

If the authors' ideas are pursued to their logical conclusion, the problem of classification differentials is perhaps best treated as one of multiple correlation analysis involving quantitative and non-quantitative variables. For those who may wish to pursue this line of investigation, a starting point is provided in Chapter 17 of *Methods of Correlation Analysis* by Mordecai Ezekiel, John Wiley & Sons, New York, 1941.

An approach that would appear to offer promise lies in techniques for the quantification of qualitative data, on which at least one paper appears in the Journal of the American Statistical Association.*

The potential power of such methods is evident when we consider that, once the correct forms for the quantification functions have been derived, calculations in any number of dimensions of classification can be made simultaneously with a loss of credibility measured only by the number of parameters required, as compared with traditional methods where credibility tends to vanish rapidly as the number of classifications is multiplied.

In conclusion, I should like to repeat that this is a very fine paper and a great pleasure to review—but it wasn't easy!

DISCUSSION BY D. B. MARTIN

(Deputy Manager, Royal-Liverpool Insurance Group, Montreal, Canada. Presented by invitation.)

The two "studies" by Messrs. Bailey and Simon are based on Canadian statistics, and while it was not the authors' intention that they should be considered particularly from the point of view of Canadian conditions, we in Canada have been very interested both in the critical review of what we have already done and in the suggestions as to what we should do in the future. I may say that in Canada we sometimes feel that our American friends fall into two classes—those who think of Canada as the 51st State, with no special features of law or custom or race making us any different from the other 50, and those who think of us as the 151st State, socially and economically only slightly in advance of the aboriginal inhabitants of the continent! We are delighted, in consequence, when we see some recognition of the fact that we have a few things up in Canada in which we are on a par with, and may even be slightly in advance of, the rest of the world. We are quite proud of our Automobile Insurance statistics, and we think the Casualty Actuarial Society has every right to be equally proud of the fact that our Statistical Plan was devised and has been operated for very many years by a Fellow of the Society, Mr. C. H. Fredrickson.

In the first of their studies, Bailey and Simon demonstrate that the Cana-

* "The Quantification of Qualitative Data in Discriminant Analysis", Vol. 45, March 1950.

dian classification plan is fairly effective in distinguishing between high-hazard and low-hazard risks. They also show that the merit rating plan is about equally effective and that the combination of a classification plan and a merit rating plan is more powerful than either of the two plans separately, although perhaps not as much so as one would have hoped. The calculations confirm and give quantitative expression to our instinctive feeling about the success of our combination of classification and merit rating plan, but they also show that the combination is not the conclusive answer to the problem of Automobile rate-making, either in your territory or in ours; the authors suggest that a further qualification in the classification plan, a mileage factor, might increase the effectiveness of the present combined classification and merit rating plan to a considerable degree.

I am not convinced about the value of the mileage factor in all cases, although late in 1959 we introduced mileage as part of the definition of a "super-super-select" class, for which we were prepared in Canada to quote rates of premium materially lower even than those appropriate to our then 1-A class. As far as we can at present see, the qualifications of the "super-super-select" class have limited the special rates of premium to insureds of a really satisfactory quality. There were, however, other qualifications which may have been more effective than mileage, notably the requirement that the car insured should not be used for driving to and from work; that the insured should have had five years free of accident; that there should be not more than two adult (i.e., over 25) drivers in the household, and, of course, no under 25 drivers.

But I sometimes wonder whether we are not still missing the real causes of accident-freedom or accident-proneness; whether we are not differentiating by correlated characteristics rather than by causal factors. Professor Poser of McGill University believes that he can identify potentially accident-prone drivers by a series of physical and psychological tests. We have not yet been successful in providing him with an adequate group of test cases and the necessary control population, so that at the present time his theory is unproven. However, I know that it is possible to base a model population on the assumption that it consists of two groups, a relatvely small one with an accident frequency of 20 per hundred per annum, and a very much larger one with an accident frequency of 5 per hundred per annum. If it be assumed that each member of that population acquires a car at the same time, then within a very few years the whole population will divide itself into groups respectively 3 or more, 2 and 1-year claim-free, and with a recent claim record, the groups being proportionately as numerous as, and their claim frequencies being very comparable with, those of the corresponding groups actually found in the Canadian population.

For that model population the combination of a classification and merit rating plan is just as effective as Bailey and Simon have shown it to be for the actual Canadian experience, but it falls a long way short of identification of the 20 per hundred frequency insureds and their appropriate rating. For our model population the combination of classification and merit rating plan is only relatively successful; possibly it is no more so in real life.

The second study considers the method of deriving appropriate premium

differentials for the various class and accident-freedom sub-groups. Four criteria are suggested in the light of which any differential complex should be examined, and some ingenious but quite practical suggestions are offered for calculating one or more complexes which measure up satisfactorily. In Canada we have been using the elementary method of determining column and row differentials separately and then combining them, the actual process being described in some detail in my presentation to the 16th International Congress of Actuaries (Communications of the 16th International Actuarial Congress, Volume 2, Page 37). The examples in that Paper, however, are based on statistics a year older than those used by Bailey and Simon. For a fair comparison I extracted some figures from the calculations made in connection with the development of the 1960 Canadian rate program and set them beside the figures produced in Method 2 of Table C of Bailey and Simon's Paper (Minimum Chi-Square on xy) after these have been adjusted to relate to the Class 1-B rate as 100 . The comparison is—

	Bailey and Simon's Method 2, adjusted to Class 1-B = 100	Canadian Method Original Calculation
Class 1	100	100
" 5	132	133
" 3	149	150
" 2	155	158
" 4	241	245
Merit Rating Class A	62	61
" " " X	76	76
" " " Y	83	83
" " " B	100	100

The two sets of figures show a remarkable resemblance, and it is tempting to claim that the simpler method is just as efficient as the more sophisticated and undoubtedly more laborious one. However, the resemblance may easily be fortuitous, and while we may continue in Canada to use the simpler method for our immediate rate-making purposes, I think we shall, at some stage in the proceedings, be checking to see whether the minimum Chi-square method does give the same result.

Bailey and Simon sound a warning as to the dangers of calculating differentials from the thoroughly heterogeneous data derived from the aggregation of the experience of a number of different rating areas with markedly different basic accident frequencies. We have been conscious of that in Canada, and at times we have been tempted to use different sets of differential complexes, either for different Provinces or for urban as distinct from rural business. However, a single set of differentials has such manifest advantages from the point of view of the "non-mathematical considerations" mentioned by Bailey and Simon, that we would be reluctant to get ourselves involved in the complication of more than one set. We have one advantage over you; the word "discriminatory" does not have such an evil meaning in our Country as it does in yours.

It is those same non-mathematical considerations which have led us to continue the use of multiplicative differentials (Bailey and Simon's Method 1 or 2) as compared with additive differentials (Bailey and Simon's Method 3), although the latter have a great deal to commend them from a common sense point of view and appear to fit the rough data better. In particular, multiplicative differentials facilitate the simple statement "If you've been accident-free for so many years, you save such-and-such a percent of your premium" and the public relations value of that statement is considerable, particularly if the same statement can be made for all relevant coverages, classifications and rating territories. Certainly I would not like to try to explain to a group of producers the reason why of Bailey and Simon's Method 4, even though I admit that this does appear to produce a set of differentials which are mathematically better than those produced by Method 2.

I found the difference between the probabilities quoted in the last line of Bailey and Simon's Table "E" rather surprising, particularly the tremendous difference between the .001 quoted for Method 2 and the .60 and .70 quoted for Methods 3 and 4 respectively. To a very great extent, however, the probabilities depend upon the value of the constant K which on "a rough estimate based on the limited data available" Bailey and Simon calculate as $\frac{1}{200}$. I had available a distribution of actual claim figures (a mixture of B.I. and P.D. as is appropriate to Canadian conditions) sufficiently random for practical purposes and covering just under 1,000 claims. The value of K emerging from that distribution was $\frac{1}{120}$ which reduced the probabilities of Methods 3 and 4 to something of the order of .10, although Methods 1 and 2 remain (or are even more) highly improbable. I think we need to know more about the usual value of K before we conclude that any one of the four methods is, from the standard of the Chi-square test, so very much more satisfactory than any of the others.

Finally, I'd like to stress the importance of Bailey and Simon's four "non-mathematical considerations" and indeed to add a fifth, namely, acceptability to the insuring public. We have not reached finality in distinguishing between the various categories of insureds in relation to accident exposure, and we are not yet so very accurate in our calculation of appropirate rates of premium. I don't think that we ever will reach precision in either respect, and I don't think that it matters. If we present our product to the insuring public, packaged and priced in such a way that the public can see and understand that we have done rough justice both to them and to our Companies, then I think we have done our job. I do not think the public likes it when we introduce complications in our rating methods that they find difficult to understand. Intricacy makes them suspicious, and suspicion is something which is so difficult to allay that we want to avoid it.

I realize that in what I have just finished saying, the "101st State" has been ignoring some of the special difficulties which arise in the first 50; others will, I hope, make up for my short-comings. We in Canada found Bailey and Simon's studies stimulating, and well worth while. We are very grateful for them.