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PROCEEDINGS

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TWO STUDIES IN AUTOMOBILE INSURANCE RATEMAKING

BY

ROBERT A. BAILEY and LEROY J. SIMON

Section A, Effectiveness of Merit Rating and Class Rating, uses the Canadian experience for private passenger automobiles to show (1) that merit rating is almost as effective as the class plan in separating the better risks from the poorer risks, (2) that both merit rating and class rating leave unanalyzed a considerable amount of variation among risks and (3) that certain available evidence supports the conclusion that annual mileage, which has long been felt to be an important measure of hazard, is a very significant cause of this unanalyzed variation among risks.

Section B, Improved Methods for Determining Classification Rate Relativities, presents a method for obtaining relativities among groups on which a multiple classification system has been imposed. The customary method of calculating class relativities uses the total experience for each class with all subdivisions within the classes added together. With the customary method it is difficult to make a completely accurate adjustment for different distributions by territory or merit rating, because any change in the class relativities disturbs the other sets of relativities and conversely. It is shown that even if such an adjustment were made, the customary method of calculating relativities one set at a time does not reflect the relative credibility of each subgroup and does not produce the best fit to the actual data. Moreover it produces differences between the actual data and the fitted values which are far too large to be caused by chance. In addition, for private passenger automobile insurance in Canada, it is shown that two sets of relativities which are multiplied together cannot produce the best fit to the actual data, and some of the consequences of trying to do so are explained. Some methods are advanced whereby all sets of relativities for classes, merit ratings, territories, and so forth, can be calculated simultaneously, which will overcome all the deficiencies in the customary method. These improved methods use the technique of minimizing a measure (technically known as the Chi-square test) of the differences between the actual data and the fitted values. Some applications to other lines of insurance are mentioned.

SECTION A: EFFECTIVENESS OF MERIT RATING AND CLASS RATING

Introduction

Private passenger automobile insurance uses a multiple classification system. We classify by age (under or over 25 years) and within each age we classify by occupation (farm or non-farm). We also classify by use and sex. On top of all this we classify by territory. And now we have begun to classify by previous accident and conviction record which is popularly called the "merit rating plan." There is no basic difference between merit rating and class rating if the rates for each merit rating group are based on the subsequent experience of cars previously classified according to their accident and conviction record, just as the rates for each class are based on the subsequent experience of cars previously classified according to the characteristics of the class plan. In actual fact, merit rating is a class rating plan and is part of the multiple classification system. However, in this paper, as a matter of convenience, and not implying a basic distinction, we will follow the common usage in the United States by referring to classification according to previous accident and conviction record as "the merit rating plan," and to classification according to age, sex, use and occupation as "the class plan."

A class plan which uses age, sex, use and occupation does not precisely classify each risk according to its true value. Underwriters have long recognized this, and it is further substantiated by the Canadian merit rating experience which shows that risks which have been accident-free for three or more years have better experience in the following year than the average for their class. Likewise a class plan which uses only the previous accident record would not precisely classify each risk[1]. This is shown by the fact that in the Canadian merit rating experience, the cars which qualified for the best merit rating have different accident frequencies depending on which class they are in.

This means that private passenger automobile risks vary considerably from each other and that the class plan and the merit rating plan are both attempts to separate the better risks from the poorer risks. Neither plan is perfect, but we would like to discuss the question, "How do merit rating and class rating compare with each other in their ability to separate the better risks from the poorer risks?" After discussing their comparative effectiveness, we shall then discuss their absolute effectiveness.

Comparative Effectiveness of Merit Rating and Class Rating

Table 1 at the end of this section shows the Canadian automobile experience[2] arranged to show what it would have looked like if there had been (1) merit rating without class rating and (2) class rating without merit rating. The premiums have been adjusted to what they would have been if all the cars had been written at 1B rates, by use of the approximate relativities:

[1] See also "Some Considerations on Automobile Rating Systems Utilizing Individual Driving Records" by Lester Dropkin, CAS XLVI, p. 165.

[2] The Canadian experience includes that of virtually every insurance company operating in Canada and is collated by the Statistical Agency (Canadian Underwriters' Association —Statistical Department) acting under instructions from the Superintendent of Insurance.

Merit Rating Definition	Relativity
A – licensed and accident free three or more years	65
X – licensed and accident free two years	80
Y – licensed and accident free one year	90
B – all others	100
Class Definitions	
1 – pleasure, no male operator under 25	100
2 – pleasure, non-principal male operator under 25	165
3 – business use	165
4 – unmarried owner or principal operator under 25	240
5 – married owner or principal operator under 25	165

The purpose of any classification plan is to reduce the rates for the better risks and to offset this reduction with an appropriate increase in the rates for the more hazardous risks. We will define "effectiveness" of a classification plan in this paper to be the extent to which the plan separates the better risks from the overall average. This definition of effectiveness was applied in making an evaluation of a one-year merit rating plan where the better risks would get only a 1.6% reduction from the average rate if a 15% discount were given for a one-year accident-free record. Because the reduction of 1.6% was so small, the plan was considered to be ineffective.[3]

Since both merit rating and class rating in Canada include about the same proportion, 80%, of the cars in the lowest rated class, a measure of the comparative effectiveness of the two is the percentage reduction of the lowest rated class from the over-all average.

Rating Plan	Reduction of lowest rated class from average	Relative Reduction	Proportion of cars in lowest rated class
Merit rating alone	10.5%	77	80.9%
Class rating alone	13.7%	100	80.1%
Merit and class rating combined	21.4%	156	66.4%

This means that the merit rating plan is 77% as effective as the class plan. The Canadian merit rating plan could be improved by extending it from three years to five (which was done during the latter part of 1959) and by including convictions. Something also could be gained if the merit rating plan gave extra weight to a loss exceeding, say $1000, since it was noted that there is a positive correlation between the loss ratio and the average size of loss. Likewise the Canadian class plan, which is similar to the plans used in the United States, could also be improved. But the point remains that merit rating is almost as effective as the class plan in separating the better risks from the poorer risks and a substantial improvement is realized when they are used in combination.

[3] See Muir, J. M., "Principles and Practices in Connection With Classification Rating Systems for Liability Insurance As Applied to Private Passenger Automobiles", CAS XLIV, pp. 32 and 33.

Our previous paper showed the experience for each class subdivided by merit rating.[4] This was a natural format because the class plan was here first and merit rating was being imposed on top of the already existing class plan. Table 2 at the end of this section shows how the experience would have been presented if merit rating had been here first and the class plan was being imposed on top of the already existing merit rating plan. Losses are used this time instead of number of claims because there is a much greater difference in average claim costs among the classes than among the merit ratings.

The relative loss ratio for Class 1 within each merit rating is slightly lower than the corresponding ratio shown in our previous paper for merit rating A within each class, indicating a greater effectiveness for class rating. The class plan is most effective in the worst merit rating, B, just as merit rating was shown to be most effective in the worst class, 4.

Absolute Effectiveness of Merit Rating and Class Rating

Thus far, this paper has shown, based on the Canadian experience, that merit rating is almost as effective as class rating in separating the better risks from the poorer risks. But it has not shown in absolute terms just how effective either rating plan is.

In order to determine the absolute effectiveness of a rating plan, an analytical expression of the distribution of risks according to their "inherent hazard" is needed. Mr. Dropkin's paper on the negative binomial distribution[5] provides a valuable tool for this purpose. His paper shows that inherent hazards of individual risks are much more widely distributed than was commonly supposed. The class plan reduces this wide distribution very little. This is illustrated by the fact that merit rating will give the best risks a reduction of 10.5% from the average when there is no class plan and will still give the best risks within Class 1 a reduction of 8.9%[6] from the average Class 1 rate. This means that Class 1 has almost as much variation within it as there is among all classes combined.

This demonstrates what has often been recognized, that while merit rating and class rating are effective tools in a relative sense, in an absolute sense both merit rating and class rating are quite ineffective in separating the better risks from the poorer risks. There remains a considerable amount of unanalyzed variation among risks.

Cause of the Unanalyzed Variation Among Automobile Risks

It is one thing to show there is variation among risks and another thing to find the cause of variation.

In our previous paper we listed three possible reasons why the empirical credibilities discussed there for 1, 2 and 3 years of merit rating were not in the expected ratio of 1 : 2 : 3. They were:

[4] Bailey, Robert A. and Simon, LeRoy J., "An Actuarial Note on the Credibility of Experience of a single Private Passenger Car", CAS XLVI; Table 1, p. 162.

[5] *Op. Cit.*

[6] Bailey, Robert A. and Simon, LeRoy J., *Op. Cit.*, Table 4, p. 163.

(1) new risks entering a class,
(2) an individual risk's chance of having an accident varying from year to year, and
(3) a markedly skew distribution of risks.

With the help of the negative binomial distribution, we can check the third alternative. Using the formula derived by Mr. Bailey[7] for the credibility

$$Z = \frac{n}{n+a}$$

where n = number of accident-free years and
a is a parameter in the distribution of risks,
we find that the relative credibilities for 1, 2 and 3 years should be in the ratio of

$$1 : 2\left(\frac{1+a}{2+a}\right) : 3\left(\frac{1+a}{3+a}\right)$$

By setting the one year credibility for Class 1 cars of .055, shown in Table 4 of our previous paper,[8] equal to $\frac{1}{1+a}$, we obtain $a = 17.2$. Therefore the relative credibilities for 1, 2 and 3 years should be in the ratio of 1:1.90:2.70 which are close to 1:2:3 as we had expected. But the actual relative credibilities also shown in Table 4 of our previous paper are in the ratio of 1:1.38:1.62. Therefore while the distribution of risks is definitely skew, it is not skew enough to account for such large discrepancies, and we may cross out the third alternative listed above.

We know that new risks entering the class account for some of the discrepancy, but we do not feel that new risks can account for such large discrepancies. Therefore we feel that the evidence strongly supports the conclusion that the individual risk's chance of having an accident does vary significantly from year to year.

Thus far we have shown that merit rating and class rating are of about equal effectiveness and that a substantial improvement is realized when they are used in combination. However, both of them leave unanalyzed a considerable amount of variation among risks. In our investigation of the characteristics of this unanalyzed variation we have eliminated certain factors from consideration and now feel we have reached the point where we may state that the still unanalyzed cause (or causes) of variation among individual risks:

(1) has a wide dispersion,
(2) varies significantly from year to year for an individual risk, and
(3) is measured only to a limited extent by the class plan and the merit rating plan.

Annual mileage, which has long been felt to be an important measure of hazard,

[7] Bailey, Robert A. Discussion, "Some Considerations on Automobile Rating Systems Utilizing Individual Driving Record", CAS XLVII, p. 152.

[8] *Op. cit.*

fits all these requirements better than any other single cause. The distribution of risks according to mileage is widely dispersed.[9] Mileage varies significantly from year to year. Farmers, for example, have less mileage than average[10] and business use risks have more mileage than average. The discount for two or more cars in one family is a reflection of mileage. Accident frequencies (and even conviction frequencies) are a crude indication of mileage. Mileage is certainly not the whole story because there is conclusive evidence that newly licensed drivers and youthful drivers have a higher accident rate per mile than other drivers and that other things such as drinking and irresponsibility play a part, but the evidence supports the conclusion that mileage is a very significant cause of variation among individual risks.

TABLE 1

Canada excluding Saskatchewan
Policy Years 1957 & 1958 as of June 30, 1959
Private Passenger Automobile Liability - Non Farmers

Merit Rating	Earned Car Years	Earned Prem. at Present 1B Rates	Losses Incurred	Loss Ratio	Relative Loss Ratio
		Classes 1, 2, 3, 4 & 5 Combined			
A	3,356,480	192,881,000	87,094,000	.452	.895
X	175,553	10,518,000	6,233,000	.593	1.174
Y	219,597	13,118,000	8,461,000	.645	1.277
B	398,445	24,152,000	19,633,000	.813	1.610
Total	4,150,075	240,669,000	121,421,000	.505	1.000
A + X	3,532,033	203,399,000	93,327,000	.459	.909
A + X + Y	3,751,630	216,517,000	101,788,000	.470	.931
		Merit Ratings A, X, Y & B Combined			
Class					
1	3,325,714	194,106,000	84,607,000	.436	.863
2	168,998	9,385,000	6,505,000	.693	1.372
3	321,327	20,627,000	13,684,000	.663	1.313
4	252,397	12,390,000	14,199,000	1.146	2.269
5	81,639	4,161,000	2,426,000	.583	1.154
Total	4,150,075	240,669,000	121,421,000	.505	1.000
1A	2,757,520	159,108,000	63,191,000	.397	.786

[9]See DeSilva, Harry R. *Why We Have Automobile Accidents.* John Wiley & Sons, New York, 1942, p. 12.

[10]*Ibid.*, p. 13.

TABLE 2

Canada excluding Saskatchewan
Policy Years 1957 & 1958 as of June 30, 1959
Private Passenger Automobile Liability - Non Farmers

Class	Earned Car Years	Earned Prem. at Present 1B Rates	Losses Incurred	Loss Ratio	Relative Loss Ratio
		Merit Rating A - licensed and accident-free 3 or more years			
1	2,757,520	159,108,000	63,191,000	.397	.878
2	130,535	7,175,000	4,598,000	.641	1.418
3	247,424	15,663,000	9,589,000	.612	1.354
4	156,871	7,694,000	7,964,000	1.035	2.290
5	64,130	3,241,000	1,752,000	.541	1.197
Total	3,356,480	192,881,000	87,094,000	.452	1.000
		Merit Rating X - licensed and accident-free 2 years			
1	130,706	7,910,000	4,055,000	.513	.865
2	7,233	431,000	380,000	.882	1.487
3	15,868	1,080,000	701,000	.649	1.094
4	17,707	888,000	983,000	1.107	1.867
5	4,039	209,000	114,000	.545	.919
Total	175,553	10,518,000	6,233,000	.593	1.000
		Merit Rating Y - licensed and accident-free 1 year			
1	163,544	9,862,000	5,552,000	.563	.873
2	9,726	572,000	439,000	.767	1.189
3	20,369	1,382,000	1,011,000	.732	1.135
4	21,089	1,052,000	1,281,000	1.218	1.888
5	4,869	250,000	178,000	.712	1.104
Total	219,597	13,118,000	8,461,000	.645	1.000
		Merit Rating B - all other			
1	273,944	17,226,000	11,809,000	.686	.844
2	21,504	1,207,000	1,088,000	.901	1.108
3	37,666	2,502,000	2,383,000	.952	1.171
4	56,730	2,756,000	3,971,000	1.441	1.772
5	8,601	461,000	382,000	.829	1.020
Total	398,445	24,152,000	19,633,000	.813	1.000

SECTION B: IMPROVED METHODS FOR DETERMINING CLASSIFICATION RATE RELATIVITIES

Multiple classification systems are quite prevalent in the insurance industry. For example, in fire insurance we classify the simple dwelling risks by town

grading as well as by construction, resulting in a 10 x 2 system (typically). Other lines similarly involve multiple classification systems, but automobile is probably the best example. We have used a class plan and a territorial plan in automobile and now we have introduced the merit rating plan. It has been customary to determine a countrywide set of class relativities. Under the merit rating plan it will be necessary to determine relativities here, too. Assuming these relativities are to continue to be applied in series as multipliers on a "base" pure premium, the problem then arises as to how to determine the best set of relativities. The customary procedure* is to sum over all variables except the one we are interested in and then compute our relativities. For example, to get the class relativities, get the total mass of experience broken down only by class. Then the ratio of the experience for each class (usually adjusted in some manner for differences in the distribution by territory and merit rating) to the overall average experience will give the individual class relativity. The same steps would be followed for the merit rating classes and for territories. The subdivisions within each class are added together because individually they are usually not fully credible. Combining them is a means of obtaining a credible volume of experience. This process of combining subgroups results in a loss of some information because any combination yields less information than the aggregate information yielded by the individual subgroups. A method for obtaining relativities which is able to avoid combining the subgroups and is able to use each subgroup individually would produce a better set of relativities.

For purposes of illustration we'll solve the following problem: What is the best set of class relativities and merit rating relativities to use in Canada? The data is presented in Table B in a loss ratio form (all at Class 1B rates) and in Table D as relative loss ratios. We will assume that the territorial factor is properly reflected in this data because we are dealing with loss ratios. A better way would be to use pure premiums and to work out territorial relativities at the same time as class and merit rating relativities. However, such data is not available to the authors, but the procedure would be similar in either case. To determine what is an acceptable set of relativities we must establish the criteria which a set should meet:

Criterion 1. It should reproduce the experience for each class and merit rating class and also the overall experience; i.e., be *balanced* for each class and in total.

Criterion 2. It should reflect the relative *credibility* of the various groups involved.

Criterion 3. It should provide a minimal amount of *departure* from the raw data for the maximum number of people.

Criterion 4. It should produce a rate for each sub-group of risks which is close enough to the experience so that the differences could reasonably be caused by *chance*.

*For example, see "Current Rate Making Procedures for Automobile Liability Insurance", Stern, Phillip K., CAS XLIII, p. 127ff.

A set which meets these four criteria will be judged to be a "best" set of relativities. If more than one set satisfactorily meets the four criteria, the choice among sets may be made on a non-mathematical basis such as (a) simplicity of application, (b) similarity to existing sets, (c) ease of explanation to non-technical personnel or (d) the actuary's personal preference.

Let us define x_i as the class relativity for the i^{th} class ($i = 1,2,3,4,5$) and y_j as the class relativity for the j^{th} merit rating class ($j = 1,2,3,4$ representing A, X, Y and B respectively). Let r_{ij} be the actual relative loss ratio for persons classified as class i and merit rating class j; $r_{.j}$ is the relative loss ratio of the j^{th} merit rating class where all i classes are combined; $r_{i.}$ is the relative loss ratio of the i^{th} class where all j merit rating classes are combined; and finally $r_{..}$ is the relative loss ratio for all classes and merit rating classes combined and thus equals 1.00. Let us also define n_{ij} as the number of earned car years of exposure. The n_{ij} are shown in Table A.

Relativities calculated by the customary method, which we will call "Method 1", are as follows:

$$\text{and} \qquad \left.\begin{aligned} x_i &= r_{i.} \\ y_j &= r_{.j} \end{aligned}\right\} \qquad (1)$$

and are shown in Table C.

The estimated relative loss ratio is then $x_i y_j$, and, if multiplied by the overall loss ratio, will produce the estimated loss ratio for the i, j class. Or, if $x_i y_j$ is multiplied by the overall pure premium, it would produce the estimated pure premium for the i, j class. The estimated relative loss ratios, $x_i y_j$ obtained by Method 1 are shown in Table D. When compared with the actual relative loss ratios, r_{ij}, also shown in Table D, it is evident that there are some undesirably large differences. Moreover, all $x_i y_1$ are too low and all $x_i y_4$ are too high.

To test the balance (Criterion 1 above) we calculate

$$\Sigma n_{ij} x_i y_j / \Sigma n_{ij} r_{ij} \qquad (2)$$

summing over each i, each j and total.

A set of relativities is balanced if equation (2) equals 1.000. The balance as determined by equation (2) is shown in Table E. Method 1 is out of balance in total and far out of balance for the individual classes. If the off-balance in the total is corrected, the classes will still be far out of balance. The reason why the classes are so far out of balance is that in our calculation of x_i and y_j, no adjustment was made for differences in the distribution by class or merit rating class. This illustrates what happens if a merit rating plan is imposed on an already existing class plan without any adjustment in the class relativities. If we had made some tentative adjustment, the off-balance by class and merit rating class would have been reduced. To make a completely accurate adjustment in the class relativities is difficult, however, because any adjustment in the class relativities disturbs the relativities for the merit rating classes and conversely, thus requiring an adjustment process which zig-zags back and forth. However, even if such an adjustment were made so that Criterion 1 would be satisfied, Method 1 would still not satisfy Criteria 2, 3 and 4, as will be shown later.

Again speaking in general, in order to reflect the relative credibility of the

various groups involved (Criterion 2), the indicated proportional departure of each group

$$\frac{\text{actual loss ratio} - \text{expected loss ratio}}{\text{expected loss ratio}}$$

should be given a weight proportional to the square root of the expected number of losses for the group. This is based on the fact that the indication of each group should be given a weight inversely proportional to the standard deviation of the indication. The standard deviation of the indication is inversely proportional to the square root of the expected number of losses for the group. An equivalent credibility procedure would be to give the square of the indication a weight proportional to the expected number of losses.

Criterion 2 (Credibility) is not met by the customary relativities (Method 1) because when all the data is added together for, say, class i, to obtain $r_{i.}$ each subgroup r_{ij} is given a weight approximately proportional to the expected number of claims instead of the square root of the expected number of claims. This is one of the reasons why Method 1 does not satisfy Criteria 3 and 4. Moreover, if each entry in a row of r_{ij} is of low credibility, the resulting $r_{i.}$ will not be too trustworthy. Nevertheless, the resulting $r_{i.}$ will be treated as 100% credible by Method 1 in the determination of x_i. Methods 2, 3 and 4 developed below will remove these defects. Each r_{ij} will contribute to the final set in proportion to its relative credibility in relationship to all other r_{ij} in the table and not just in relationship to the other members of its row or column, and conversely each x_i and y_j will be influenced by all the r_{ij} and not just by one row or column of r_{ij}.

There is no assurance that Criteria 3 and 4 are met by the customary relativities (Method 1). In the paragraphs that follow we will show clearly that this set of relativities results in an average departure that is far from minimal and further, that the individual departures are too large to be caused by chance.

As a test of a set of relativities for compliance with Criterion 3, let us calculate how much error the average policyholder will have in his estimated relativity by calculating,

$$\sum_{i,j} n_{ij} \mid r_{ij} - x_i y_j \mid / \sum_{i,j} n_{ij} r_{ij} \tag{3}$$

The result of this calculation is shown in Table E.

Equation (3) endeavors to measure how much "inequity" the set has. The farther a policyholder's rate is from the indications of the raw data, the more "inequity" is involved. Anyone who has dealt directly with insureds at the time of a rate increase, knows that you can be much more positive when the rate for his class is very close to the indications of experience. The more persons involved in a given sized inequity, the more important it is.

To test a set of relativities for compliance with Criterion 4 (differences between the raw data and the estimated relativities should be small enough to be caused by chance), the Chi-square test is appropriate. It is shown in the Appendix that in terms of relative loss ratios, exposures and relativities,

$$\chi^2 = K \sum_{i,j} \frac{n_{ij}(r_{ij} - x_i y_j)^2}{x_i y_j} \tag{4}$$

where K is a constant dependent on the data and for the Canadian data, K equals approximately 1/200. The values of χ^2 are shown in Table E.

It should be noticed that the χ^2 formula (4) is equivalent to giving the square of the indication a weight proportional to the expected number of claims.

$$\chi^2 = K \sum_{i,j} \frac{n_{ij}(r_{ij} - x_i y_j)^2}{x_i y_j} = K \sum_{i,j} n_{ij} x_i y_j \left(\frac{r_{ij} - x_i y_j}{x_i y_j} \right)^2$$

This means that a set of x_i, y_j, which is specifically designed to produce a minimum χ^2 will automatically reflect the relative credibility of each group involved (Criterion 2). This is accomplished without a credibility weighting process involving tabular credibilities. Moreover, since a set of x_i, y_j, which produces a minimum χ^2 will very likely also satisfy Criterion 3 (minimal average amount of departure) and will come very close to satisfying Criterion 1 (balance), it seems evident that the best set of relativities will be those which are designed specifically to produce a minimum χ^2. These relativities can be obtained by setting the partial derivatives of χ^2 equal to zero.

$$\frac{\partial \chi^2}{\partial x_i} = K \sum_j n_{ij} y_j - K \sum_j \frac{n_{ij} r^2_{ij}}{x_i^2 y_j} = 0 \tag{5}$$

Solving for x_i, we obtain

$$x_i = \left[\sum_j \frac{n_{ij} r_{ij}^2}{y_j} \Big/ \sum_j n_{ij} y_j \right]^{\frac{1}{2}} \tag{6}$$

and similarly,

$$y_j = \left[\sum_i \frac{n_{ij} r_{ij}^2}{x_i} \Big/ \sum_i n_{ij} x_i \right]^{\frac{1}{2}} \tag{7}$$

This gives us nine equations in nine unknowns. Since the equations are not of a simple, rational form, the easiest way to arrive at a numerical solution is by a method of iteration, as follows:

1. Take $r_{i.}$ (the customary method of obtaining x_i) as the first estimates of x_i.
2. Use these values in the right hand side of (7) to obtain the first estimates of y_j.
3. Use the first estimates of y_j in the right hand side of (6) to obtain the second estimates, of x_i.
4. Repeat this process until two consecutive sets of solutions are identical (or substantially so).

Notice that there are an infinite number of solutions for x_i and y_j, all of which, however, produce the same set of $x_i y_j$. This is true because each x_i may be multiplied by a constant if each y_j is divided by the same constant. The results of this method, which we shall call "Method 2" are shown in Table C. The estimated relative loss ratios, $x_i y_j$, are shown in Table D and the tests of Criteria 1, 3 and 4 are shown in Table E.

It is evident that Method 2, which derives all sets of relativities simultaneously, solves the difficult problem of obtaining relativities which are balanced in total and by class. It automatically satisfies Criterion 2 (credibility) and it also reduces substantially the average error and χ^2 (Criteria 3 and 4). But in spite of this improvement, the average error of .0317 still does not compare very favorably with a profit margin of .050 or thereabouts, especially for a company that writes a disproportion of business in one class. Moreover, χ^2, although much less than for Method 1, is still too high to be the result of chance. This means that a set of factors which are multiplied together, $x_i y_j$, cannot satisfactorily represent the actual data for Canadian private passenger automobiles, although it may be satisfactory for other lines or types of data.

Turning to the actual data, shown in Table D, it can be seen that the *percentage* difference between the lowest and the highest merit rating decreases as the rate for the class increases, ranging from 73% for class 1 down to 39% for class 4. With these conditions present in the basic Canadian data, it is little wonder that the multiplicative relativities do not fit satisfactorily.

A possible method, which we will call "Method 3", is to let the estimated relative loss ratio be $x_i + y_j$, where the relativities are added instead of multiplied. The χ^2 formula becomes

$$\chi^2 = K \sum_{i,j} \frac{n_{ij}(r_{ij} - x_i - y_j)^2}{x_i + y_j} \tag{8}$$

And setting the partial derivatives of χ^2 equal to zero we have:

$$\frac{\partial \chi^2}{\partial x_i} = K \sum_j n_{ij} - K \sum_j \frac{n_{ij} r_{ij}^2}{(x_i + y_j)^2} = 0 \tag{9}$$

For convenience let us write (9) as $f(x_i) = 0$. If we first obtain an estimate of x_i, we can obtain a correction, Δx_i, to be added to x_i by the use of Newton's method; that is,

$$\Delta x_i = -\frac{f(x_i)}{f'(x_i)}$$

where $f'(x_i)$ is the derivative of $f(x_i)$. Using this procedure we obtain

$$\Delta x_i = \frac{\sum_j n_{ij} \left(\frac{r_{ij}}{x_i + y_j} \right)^2 - \sum_j n_{ij}}{2 \sum_j n_{ij} \left(\frac{r_{ij}}{x_i + y_j} \right)^2 \left(\frac{1}{x_i + y_j} \right)} \tag{10}$$

The expression for Δy_j is the same as for Δx_i except that the summations are taken over i instead of j. The x_i and y_j are derived as follows:

1. Select a set of first estimates of x_i and y_j.
2. Use these values in (10) to obtain Δx_i and Δy_j.
3. Add Δx_i to x_i and Δy_j to y_j to obtain the second estimates of x_i and y_j.
4. Repeat this process until all Δx_i and Δy_j are equal to zero.

It should be noted here again that there are an infinite number of solutions for x_i and y_j, all of which, however, produce the same set of x_i+y_j. This is true because a constant may be added to all the x_i if the same constant is subtracted from all the y_j, and this will not change any of the estimated relative loss ratios, x_i+y_j. In fact, if we let the estimated relative loss ratios be (x_i+y_j-1) and alter formula (10) accordingly, we can use the values of x_i and y_j obtained by Method 1 as our first estimates to be used in (10).

It may be well at this point to emphasize how little absolute meaning can be attached to a given set of relativities. Whether they were based on a minimum χ^2 or not, or whether they were multiplicative or additive, a simple transformation can change their individual values and, of course, the happenstance of our choice of initial values in solving either (6) or (10) will produce one solution instead of another. It is quite natural for us to attempt to attach a special meaning to a developed set of relativities; that is, to impart to them some special quality in and of themselves. However, they can only be regarded in relationship to the coordinate system in which they find themselves.

The values of x_i and y_j obtained by Method 3 are shown in Table C, the estimated relative loss ratios, x_i+y_j, are shown in Table D and the tests of Criteria 1, 3 and 4 are shown in Table E.

It is evident that Method 3 not only satisfies Criteria 1 and 2 (balance and credibility) but it also reduces the average error to .0098 which is much better than Methods 1 and 2, and it produces a χ^2 which could very easily be the result of chance. This means that while the actual data cannot be represented satisfactorily by a set of relativities which are multiplied, x_iy_j, the actual data can be satisfactorily represented by a set of relativities which are added, x_i+y_j.

Another method of obtaining relativities which we will call "Method 4" is a compromise between Methods 2 and 3. Let the estimated relative loss ratios be $ax_iy_j-(a-1)$ and then minimize

$$\chi^2=K\sum_{i,j}\frac{n_{ij}[r_{ij}-(ax_iy_j-a+1)]^2}{ax_iy_j-(a-1)} \qquad (11)$$

If $a=1$, (11) reduces to (4) which is Method 2. With the proper selection of a, greater than 1, results can be produced which are very similar to Method 3. It seems that the only practical way to obtain the optimum value of a is by judgment. Basing our judgment on the four corner values of r_{11}, r_{14}, r_{41} and r_{44}, we selected $a=3$. For computational purposes, equation (11) was translated to the form of equation (4) by adding 2 to each r_{ij} and dividing the results by 3. The relativities, x_i and y_j, were then obtained by the iterative process described for Method 2 and are shown in Table C. The estimated relative loss ratios, $3x_iy_j-2$, are shown in Table D and the tests of Criteria 1, 3 and 4 are shown in Table E. It can be seen that Method 4 produces results very similar to Method 3, and for the Canadian data that both Methods 3 and 4 satisfy all four criteria listed at the beginning of this section. Moreover, they both are methods of calculating all sets of relativities simultaneously.

We have developed only a two dimensional problem (x by y) here, but the methods can easily be extended to include more dimensions such as farm versus non-farm and territorial relativities. A small computer would be very useful in performing the tedious calculations which would be involved.

Consequences of Using Multiplicative Relativities

When the attempt is made to fit a set of relativities which are multiplied, $x_i y_j$, to a set of data that should be fitted by a set of relativities which are added, $x_i + y_j$, rates are produced for the lowest rated class that are too high in the lowest merit rating and too low in the highest merit rating and rates are produced for the highest rated class that are too low in the lowest merit rating and too high in the highest merit rating. This can be seen in Table D by comparing Method 2 with the actual data or with Method 3.

It is evident that the same difficulty occurs in private passenger automobile insurance in the United States when a countrywide set of class relativities is multiplied by a set of territory relativities. Several attempts have been made to correct this difficulty. Two sets of countrywide class relativities are used, one for large cities and one for all other territories.* The relativities for large cities have a smaller spread between the lowest and the highest rated classes. This is quite likely not caused by a difference in classification experience between high-rated territories and low-rated territories but it is the result of trying to use two sets of relativities which are multiplied when it is quite likely that the two sets of relativities should be added instead of multiplied. Another example of an attempt to correct this situation is the fact that in New York City, which is about the highest rated territory in the United States and where, in addition, the experience has enough volume to be credible, a special set of class relativities is used which has much less spread between the lowest and highest rated classes than the sets of relativities used elsewhere.

The reason this difficulty has not become more noticeable in other territories is that very few territories have sufficient volume to be credible for each class. But it is very likely that multiplying countrywide class relativities by territory relativities has produced and is producing rates which are too high for Class 1 in very low-rated territories and too low for Class 1 in very high-rated territories. This situation will become worse if three sets of relativities, for territories, classes and merit rating classes, are all multiplied together, $x_i y_j z_k$. The introduction of merit rating makes it all the more important to use a method of obtaining relativities which will satisfy the four criteria listed at the beginning of this section.

The methods developed in this paper, designated Methods 2, 3 and 4, have possibilities of wide application in many lines of insurance. For example, the non-reviewed workmen's compensation classes could be treated on a nationwide basis with relativities established by class and state. General Liability classes, which often involve a limited amount of exposure, could similarly be treated on a nationwide basis with relativities by class and territory. A & H involves many relativities. In automobile insurance itself the excess limits tables could be tested to determine whether the limits changes are, in fact, multiplicative with the basic rates or are more properly included as some other function. One can also visualize Homeowners rate making on a pure premium basis per homeowner with relativities for protection grading, construction and policy size

*See Stern, *Op. Cit.*, p. 154 and Livingston, G. R., & Carlson, T. O., discussion of "Principles and Practices in Connection with Classification Rating Systems for Liability Insurance as Applied to Private Passenger Automobiles". CAS XLV, p. 230.

(this latter item is a quantitative characteristic of the experience and would introduce an interesting facet into the problem). A multitude of similar practical problems could also be solved through this technique.

TABLE A

Array of Number of Earned Car Years of Exposure**
n_{ij} with 000 omitted

		Merit Rating Class				
		A	X	Y	B	
	i\j	1	2	3	4	Total
	1	2758	131	164	274	3327
	5	64	4	5	9	82
Class*	3	247	16	20	38	321
	2	131	7	10	22	170
	4	157	18	21	57	253
	Total	3357	176	220	400	4153

TABLE B

Array of Loss Ratios at 1B Rates**

		Merit Rating Class				
		A	X	Y	B	
	i\j	1	2	3	4	Total
	1	.397	.513	.563	.686	.436
	5	.541	.545	.712	.829	.583
Class*	3	.612	.649	.732	.952	.663
	2	.641	.882	.767	.901	.693
	4	1.035	1.107	1.218	1.441	1.146
	Total	.452	.593	.645	.813	.505

*These classifications have been rearranged so that the "Total" column in Table B is in ascending order.

**Source: Table 2 at end of Section A.

TABLE C

Relativities

		Method 1 (Customary*)	Method 2 (Min χ^2 on $x_i y_j$)	Method 3 (Min χ^2 on $x_i + y_j$)	Method 4 (Min χ^2 on $3x_i y_j - 2$)
	x_1	.863	.881	.869	.958
	x_5	1.154	1.161	1.145	1.049
Class	x_3	1.313	1.309	1.291	1.099
	x_2	1.372	1.367	1.352	1.118
	x_4	2.269	2.125	2.172	1.384
	y_1	.895	.906	−.083	.971
Merit	y_2	1.174	1.113	.135	1.040
Rating	y_3	1.277	1.215	.237	1.076
Class	y_4	1.610	1.462	.512	1.167

*Source: Total column and Total row in Table B divided by .505.

TABLE D

Arrays of Relative Loss Ratios

Actual, r_{ij}
(Table B divided by .505)

Merit Rating Class

		A	X	Y	B
	i\j	1	2	3	4
	1	.786	1.016	1.115	1.358
	5	1.071	1.079	1.410	1.642
Class	3	1.212	1.285	1.450	1.885
	2	1.269	1.747	1.519	1.784
	4	2.050	2.192	2.412	2.853

Method 1, $x_i y_j$
(Customary)

i\j	1	2	3	4
1	.772	1.013	1.102	1.389
5	1.033	1.355	1.474	1.858
3	1.175	1.541	1.677	2.114
2	1.228	1.611	1.752	2.209
4	2.031	2.664	2.898	3.653

Method 2, $x_i y_j$
(Minimum χ^2 on $x_i y_j$)

i\j	1	2	3	4
1	.798	.981	1.070	1.288
5	1.052	1.292	1.411	1.697
3	1.186	1.457	1.590	1.914
2	1.239	1.521	1.661	1.999
4	1.925	2.365	2.582	3.107

Method 3, $x_i + y_j$
(Minimum χ^2 on $x_i + y_j$)

i\j	1	2	3	4
1	.786	1.004	1.106	1.381
5	1.062	1.280	1.382	1.657
3	1.208	1.426	1.528	1.803
2	1.269	1.487	1.589	1.864
4	2.089	2.307	2.409	2.684

Method 4, $3x_i y_j - 2$
(Minimum χ^2 on $3x_i y_j - 2$)

i\j	1	2	3	4
1	.787	.988	1.090	1.354
5	1.057	1.276	1.387	1.675
3	1.198	1.429	1.543	1.846
2	1.255	1.489	1.606	1.915
4	2.029	2.320	2.464	2.845

TABLE E

Tests of Criteria 1, 3 and 4

Criterion 1, Balance

		Method 1	Method 2	Method 3	Method 4
Class	x_1	.9886	1.0007	1.0011	.9979
	x_5	1.0099	1.0014	1.0024	1.0008
	x_3	1.0195	1.0006	.9993	.9982
	x_2	1.0230	1.0027	1.0027	1.0005
	x_4	1.1067	1.0027	.9974	.9994
Merit Rating Class	y_1	.9806	1.0006	1.0015	.9978
	y_2	1.0589	1.0026	1.0083	.9996
	y_3	1.0536	1.0015	1.0020	.9986
	y_4	1.1122	1.0025	.9931	1.0002
Total		1.0103	1.0011	1.0006	.9983
		Criterion 3, Average Error			
Total		.0401	.0317	.0098	.0111
		Criterion 4, Chi-Square			
χ^2		98	34	10	8
Degrees of freedom for χ^2		12	12	12	11
Probability $[\chi^2 > \text{observed}]$		less than .001	about .001	.60	.70

APPENDIX

Harald Cramér in his book, "Mathematical Methods of Statistics", pages 233 and 234, shows that if $\xi_1 \ldots\ldots \xi_n$ are n independent random variables each of which is normal with a mean of 0 and a variance of 1, then

$$\chi^2 = \sum_{i=1}^{n} \xi_i^2$$

is distributed according to the well-known Chi-square distribution.

For the Canadian data,

$$\xi = \frac{\text{actual relative loss ratio-expected relative loss ratio}}{\text{standard deviation of the actual relative loss ratio}}$$

has a mean of 0 and a variance of 1 and is very close to normal when the actual relative loss ratio is based on the average of a large number of car years. The actual relative loss ratio is r_{ij}, the expected relative loss ratio is $x_i y_j$. The variance of the actual relative loss ratio is approximately $\frac{200 x_i y_j}{n_{ij}}$ and is developed as follows:

Letting C = value of claim

n = number of car years

S_p^2 = variance of the pure premium for one car year

S_c^2 = variance of the claim cost

m_f = mean claim frequency per car year

m_c = mean claim cost

m_p = mean pure premium per car year

Then $S_p^2 = \Sigma C^2/n - (\Sigma C/n)^2$ by definition

$$= m_f[(\Sigma C/nm_f)^2 + \Sigma C^2/nm_f - (\Sigma C/nm_f)^2] - (\Sigma C/n)^2$$

$$= m_f(m_c^2 + S_c^2) - m_f^2 m_c^2 \qquad (12)$$

Equation (12) agrees with Mr. R. E. Beard, "Analytical Expressions of the Risks Involved in General Insurance", in Transactions XVth International Congress of Actuaries, 1957, Vol. II, p. 233.

If we let the time interval be less than one year and approach zero as a limit, which is appropriate if S_c^2 is based on a distribution of claims where each claim is listed separately regardless of how close in time they may have occurred, the second term in equation (12) becomes insignificant and we obtain

$$S_p^2 = m_f(m_c^2 + S_c^2) \qquad (13)$$

which agrees with Mr. A. L. Bailey, "Sampling Theory in Casualty Insurance", CAS XXIX, p. 60.

Formula (13) can be written

$$S_p^2 = \frac{m_p^2}{m_f}\left(1 + \frac{S_c^2}{m_c^2}\right) \qquad (14)$$

Equation (14) is the variance of the pure premium for one car year. The variance of the mean pure premium per car year based on a group of n car years, where each mean is divided by the overall mean pure premium, P, is therefore

$$S_r^2 = \frac{m_p^2}{nP^2 m_f}\left(1 + \frac{S_c^2}{m_c^2}\right) \qquad (15)$$

Since $P = M_c M_f$, where M is the overall mean, and m_p is approximately equal to $M_c m_f$, and $x_i y_j = m_p$, equation (15) can be written

$$S_{r_{ij}}^2 = \left(\frac{x_i y_j}{n_{ij}}\right)\left(\frac{1}{M_f}\right)\left(1 + \frac{S_c^2}{m_c^2}\right) \qquad (16)$$

It is estimated that for the Canadian data, which is total limits for BI and PD combined, $1 + \frac{S_c^2}{m_c^2}$ equals approximately 20. This is only a rough estimate

based on the limited data available to the authors. $M_f = .097$. Therefore for the Canadian data

$$\chi^2 = \frac{1}{200} \sum_{i,j} n_{ij} \frac{(r_{ij} - x_i y_j)^2}{x_i y_j}, \text{ approximately.}$$

Notice that the same constant, K, is produced regardless of whether x_i, y_j, are chosen as multiplicative or as some other form.

THE NEGATIVE BINOMIAL AND POISSON DISTRIBUTIONS COMPARED

BY

LEROY J. SIMON

Section I — Preliminaries

For much statistical work the binomial distribution is the most suitable mathematical model. It involves n independent trials, each having a probability of success equal to p. In automobile and other branches of casualty insurance, we are not concerned with a limited number of independent trials, but with an exposure to accident such that n becomes very great while $n \times p$ remains finite and is the number of "successes". In this case, the Poisson distribution is the correct statistical model to produce the probability that 0, 1, 2 . . . . . "successes" will be experienced by a given observational unit (one car, a fleet of cars, all the cars in one territory, etc.).

In computing the probability distribution of the number of experimental units which will have 0, 1, 2 . . . . . "successes", the Poisson distribution is also a good representation if the loss frequency is the same for each element in the group; or in other words the group is isohazardous.[1] In many cases, particularly in automobile insurance, we know that even classified experience is not isohazardous and in such circumstances the negative binomial distribution is the most appropriate model. Considerable interest has been recently stimulated[2] in the negative binomial distribution as it applies to the emergence of claims in automobile insurance. This note shows how the negative binomial distribution compares with the better known Poisson distribution. The method employed is to develop the first four moments of each distribution and then to compare their kurtosis and skewness.

Moment-generating functions (mgf) will be used to develop the moments needed. Let $f(x)$ denote the frequency function being studied. Recall that for discrete data the mgf, designated $M(\theta)$, is defined as follows:

$$M(\theta) = \sum_{x=0}^{\infty} e^{\theta x} f(x) \tag{1}$$

By substituting the power series for $e^{\theta x}$, multiplying through by $f(x)$ and applying the summation to each individual term of the expanded series, it develops that

$$M(\theta) = 1 + \theta\mu_1' + \frac{\theta^2}{2!}\mu_2' + \frac{\theta^3}{3!}\mu_3' + \ldots\ldots\ldots + \frac{\theta^k}{k!}\mu_k' + \ldots\ldots$$

[1] "Isohazardous" is a coined word. *Adj.* [Gr. *isos* equal + O.F. *hasard,* fr. An. *alzahr* the die.] *1.* Having the same or equal inherent hazard. *2.* Homogeneous in propensity for accident involvement. *3. Ins.* Having the same loss frequency potential. The nominative form is "isohazard".

[2] Dropkin, Lester, "Some Considerations on Automobile Rating Systems Utilizing Individual Driving Records", CAS XLVI, p. 165 and discussion thereof in this volume.

where μ'_k is the k^{th} moment about the origin. From this latter form it can be seen that if one takes the k^{th} derivative of $M(\theta)$ with respect to θ, all terms with powers of θ less than k will differentiate to zero. If we then set $\theta=0$, all terms originally having powers of θ greater than k will also go to zero. This will leave the k^{th} moment about the origin with the factorial in the denominator being exactly cancelled by the factorial produced by k differentiations of θ^k. Hence, we may state that

$$\mu'_k = \frac{d^k M}{d\theta^k}\bigg|_{\theta=0} \tag{2}$$

It is further necessary to recall that we reduce moments about the origin to moments about the mean through use of the following equations:

$$\left.\begin{aligned} \mu_2 &= \mu'_2 - (\mu'_1)^2 \\ \mu_3 &= \mu'_3 - 3\mu'_2\mu'_1 + 2(\mu'_1)^2 \\ \mu_4 &= \mu'_4 - 4\mu'_3\mu'_1 + 6\mu'_2(\mu'_1)^2 - 3(\mu'_1)^4 \end{aligned}\right\} \tag{3}$$

Finally, the customary measurement of skewness is $\alpha_3 = \mu_3/\mu_2^{3/2}$ (4)

Kurtosis is measured by $\alpha_4 = \mu_4/\mu_2^2$ (5)

Section II – The Poisson Distribution

The Poisson distribution is described by the following function:

$$f(x) = \frac{m^x e^{-m}}{x!}$$

The moment-generating function for the Poisson would be given by

$$\begin{aligned} M(\theta) &= \sum_{x=0}^{\infty} \frac{m^x e^{-m+\theta x}}{x!} \\ &= e^{-m}\left[1 + me^{\theta} + \frac{(me^{\theta})^2}{2!} + \ldots.\right] \\ &= e^{-m}e^{me^{\theta}} \end{aligned}$$

Hence $M(\theta) = e^{m(e^{\theta}-1)}$

The first four derivatives of $M(\theta)$ are:

$M^{i}(\theta) = M(\theta)me^{\theta}$

$M^{ii}(\theta) = M(\theta)me^{\theta}(1+me^{\theta})$

$M^{iii}(\theta) = M(\theta)me^{\theta}[me^{\theta} + (1+me^{\theta})^2]$

$M^{iv}(\theta) = M(\theta)me^{\theta}[me^{\theta}(3+2me^{\theta}) + (1+me^{\theta})(1+3me^{\theta}+m^2e^{2\theta})]$

Evaluating the above four equations at $\theta=0$, we will produce the first four moments about the origin, that is

$\mu_1' = m$

$\mu_2' = m^2 + m$

$\mu_3' = m^3 + 3m^2 + m$

$\mu_4' = m^4 + 6m^3 + 7m^2 + m$

Substituting these values back in equations (3), we produce the following moments about the mean:

$\mu_2 = m$

$\mu_3 = m$

$\mu_4 = 3m^2 + m$

The measurements of kurtosis and skewness thus become:

$$\alpha_3 = \frac{1}{\sqrt{m}}$$

$$\alpha_4 = 3 + \frac{1}{m}$$

Section III – Negative Binomial Distribution

The negative binomial distribution is described by the following function:

$$f(x) = \left(\frac{a}{1+a}\right)^r \binom{-r}{x} \left(\frac{-1}{1+a}\right)^x$$

The moment-generating function for the negative binomial would be given by

$$M(\theta) = \sum_{x=0}^{\infty} e^{\theta x} \left(\frac{a}{1+a}\right)^r \binom{-r}{x} \left(\frac{-1}{1+a}\right)^x$$

which can be re-written in the form

$$M(\theta) = \left(\frac{a}{1+a}\right)^r \sum_{x=0}^{\infty} \binom{-r}{x} \left(\frac{-e^{\theta}}{1+a}\right)^x (1)^{-r-x}$$

The last term in this equation is inserted merely to permit an analogy to the formula for the expansion of a binomial function. Upon simplification, the mgf can be written

$$M(\theta) = a^r (1 + a - e^{\theta})^{-r}$$

For simplicity of notation, $k = \frac{r+1}{r}$ and $M(\theta)$ is written as M in the formulas below. From $M(\theta)$ we get:

$$M^{i}(\theta) = Mre^{\theta}(1 + a - e^{\theta})^{-1}$$

$$M^{ii}(\theta) = \frac{k(M^{i})^2}{M} + M^{i}$$

$$M^{iii}(\theta) = \frac{2kM^iM^{ii}}{M} - \frac{k(M^i)^3}{M^2} + M^{ii}$$

$$M^{iv}(\theta) = \frac{2k}{M^2}[MM^iM^{iii} + M(M^{ii})^2 - (M^i)^2M^{ii}]$$

$$- \frac{k}{M^4}[3M^2(M^i)^2M^{ii} - 2M(M^i)^4] + M^{iii}$$

Evaluating the above four equations at $\theta = 0$ we will produce the first four moments about the origin; that is

$$\mu_1' = \frac{r}{a}$$

$$\mu_2' = \frac{r}{a^2}(1 + a + r)$$

$$\mu_3' = \left(\frac{2kr}{a} + 1\right)\left(\frac{kr^2}{a^2} + \frac{r}{a}\right) - \frac{kr^3}{a^3}$$

$$\mu_4' = \frac{r^4}{a^4}(6k^3 - 7k^2 + 2k) + \frac{r^3}{a^3}(12k^2 - 6k) + \frac{r^2}{a^2}(7k) + \frac{r}{a}$$

Substituting these values back in equations (3), we produce the following moments about the mean:

$$\mu_2 = \frac{r}{a^2}(a + 1)$$

$$\mu_3 = \frac{r}{a^3}(a + 1)(a + 2)$$

$$\mu_4 = \frac{r(a + 1)}{a^4}[(a + 1)(a + 3r + 5) + 1]$$

The measurements of kurtosis and skewness thus become:

$$\alpha_3 = \frac{a + 2}{\sqrt{r(a + 1)}}$$

$$\alpha_4 = 3 + \frac{a}{r} + \frac{5}{r} + \frac{1}{r(a + 1)}$$

Section IV – A Comparison

In Section II the letter "m" can now be replaced by "r/a" so that a direct comparison may be made of the two distributions. From this we find that the negative binomial α_3 minus Poisson α_3 equals

$$\frac{a+2}{\sqrt{r(a+1)}} - \frac{\sqrt{a}}{\sqrt{r}} = \frac{a+2-\sqrt{a^2+a}}{\sqrt{r(a+1)}}$$

Since a and r are both positive, this latter quantity is always a real, positive number, and it follows that the negative binomial is always more skew to the right than the Poisson distribution.

If we take the negative binomial α_4 minus the Poisson α_4, we obtain after simplification, $\frac{5}{r} + \frac{1}{r(a+1)}$. Since both a and r are positive, this quantity is also positive, which means that the negative binomial is always more peaked than the Poisson.[3]

The table which follows assumes that we are given a population which is distributed as a negative binomial with $r = .8$ and $a = 8$ (thus, mean $= .10$ and variance $= .1125$). The first column of probabilities are those of this negative binomial population. The next column is the result of fitting a Poisson distribution to the first column (i.e., $m = .10$). The last two columns indicate the differences and point out how the Poisson underestimates the probability in all cases except for $x = 1$. It is also apparent that for a first or rough approximation, or for some special purpose, the Poisson distribution is still a fairly good representation even in those cases where the negative binomial is suspected or known to apply. The usefulness of this approximation diminishes as the ratio of the variance to the mean increases and also is much less valid as we move toward the higher number of accidents.

PROBABILITY DISTRIBUTION OF NUMBER OF RISKS

Number of Accidents x	Negative Binomial	Poisson	Difference	Percent Difference
0	.910076	.904837	+.005239	1%
1	.080896	.090484	−.009588	12
2	.008090	.004524	+.003566	44
3	.000839	.000151	+.000688	82
4	.000089	.000004	+.000085	96
5	.000009	.000000	+.000009	100
6	.000001	.000000	+.000001	100
Total	1.000000	1.000000		

[3]The curves are both J-shaped for the small values of r/a which we usually encounter in insurance (ordinarily much less than one accident per year) and the same conclusion would be indicated if we proved the theorem: The number of risks having zero accidents for a negative binomial distribution exceeds that for a Poisson distribution with the same mean. In our notation, this means f(o) for negative binomial $>$f(o) for Poisson, which is true if $[a/(1+a)]^r > e^{-r/a}$. Take the r^{th} root of both sides (r is positive), take the reciprocal and write the series expansion giving

$$1+\frac{1}{a} < e^{1/a} = 1+\frac{1}{a}+\frac{1}{2\,!a^2}+\frac{1}{3\,!a^3}+\cdots.$$

Since a is positive, the theorem is proven, thus proving that for small r/a the negative binomial is more peaked.

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NOVEMBER 16-18, 1960

PRESIDENTIAL ADDRESS BY WILLIAM LESLIE, JR.

It is my privilege to address you today as President of the Casualty Actuarial Society. To those of you who have followed the remarks made on similar occasions by my predecessors it will be clear that I have no easy chore if I am to maintain the high standard set by them. Indeed even without that example I would find it difficult to present to this learned body any new set of ideas or new approaches to old ideas which would be entirely appropriate for the occasion.

A review of some of the Presidential addresses of the past shows that, in a rough way at least, they seem to fall into two categories. One group has viewed the immediate past and related the actuary's role to the questions outlined—often advancing ingenious solutions which later became the order of the day. Another group cast an eye to the future, foretelling an ultimate development of some new idea or technique and once again portrayed the actuary's part in the project. It is intriguing to see how frequently our past Presidents have had almost uncanny foresight in pinpointing future developments in the insurance field. However, both of these approaches have resulted in many of our Presidents' addresses being in part at least about actuaries, their interests and, it should be noted, made to an assemblage of their brother actuaries.

This is pointed out to the end that there be no misunderstanding of why I did not speak out on some of the so-called burning issues of the day within the insurance industry. That there are such issues at this time is not denied and that this President, and I am sure others of your Presidents, have held strong opinions on some of them should be no secret. However, quite apart from the constitutional proscription that the ". . . Society shall take no partisan attitude, by resolution or otherwise, upon any question relating to insurance", there is a much more positive reason for this approach. The honor and responsibility of this office brings one's mind to the contemplation of some of the current concerns of actuaries and especially some of the present problems of this Society. With your permission I should like to restrict my remarks on this occasion to these matters.

At the outset let me say that I think our Society is a strong one and is growing stronger; it is influential and becoming more influential; it is useful and on its way to being more useful. If my remarks seem in part to belie this I assure you they are not so intended but are meant to be helpful toward the goal of an even better organization. Additionally, I find I cannot make these comments in order of presumed importance and in any event they all seem partly inter-related.

I think the Casualty Actuarial Society is misnamed. That is to say it has outgrown its name. "The object of the Society shall be . . . the promotion of

actuarial and statistical science as applied to the problems of insurance, other than life insurance . . ." says our Constitution. It seems to me that the time has now come when so also should say our name. I know there is history here and I am familiar with much of it. However, since 1950 when the Constitution was changed to read as cited above much has been accomplished right here in our meetings and on the pages of our *Proceedings* to give hope that the subject can now be reopened and that it may have a happy resolution.

Since 1950 we have had many valuable papers, many fascinating panel discussions and useful seminars on problems dealing with fire and allied lines and other aspects of property insurance never thought then or now to be "Casualty". Only last month we published a compendium of those papers entitled "Fire Insurance Ratemaking and Kindred Problems" the demand for which has been quite remarkable. Nonetheless, and in spite of some small print in the foreword to our current yearbooks, I have been asked, and I will wager many of you have too, "what is the *Casualty* Actuarial Society doing getting into fire insurance?"

Multiple line operations have been with us now for some time and there may be some who would give this as the principal reason that each and every one of the sixteen largest non-life insurance company groups now employ one or more members of our Society in responsible positions. While the fact that these groups all now write casualty coverages may be part of the answer it is far, far from the whole answer. Our members are assuming increasingly important roles in all lines of non-life insurance and the title "Casualty" is not only non-descriptive, it is almost misleading. These managements are looking and should look to this Society to cultivate and educate the actuarial personnel they so badly need. I feel certain that our somewhat restrictive name is not helping this cause. It may be hurting. I am going to ask the Council to authorize a prompt report concerning this matter. I hope we can make some real progress in resolving the question in a satisfactory manner.

I should like next to say something about the caliber and qualifications of our members. Seymour Smith in his Presidential address of five years ago said ". . . I should like to hold before you . . . an aspect of the . . . Society that goes beyond the realm of technical growth and accomplishment. It is not a concrete thing that can be measured or seen, nor can there even be any proof of its very existence. Nevertheless, I am most firmly of the conviction that it constitutes the major donation which the Society can give to our business. It consists of the contribution that is made to a kind of thinking—to the development of minds that are inquiring and unprejudiced, that can separate fundamental problems from a welter of confusion and detail, that can couple a lively imagination with a grasp of hard reality—minds that are incapable of self-delusion or rigidity, and which are firmly anchored in rigorous mental honesty."

Mr. Smith promptly and properly added "that actuaries are [not] necessarily any more happily endowed" than others in this regard but that our Society is making a most outstanding contribution toward the clarity of thought so necessary for sound progress in all phases of our business.

Do our current practices in the examination and educational field back up our agreement with past President Smith that the actuary should be broad

gauged, informed, intellectually honest, imaginative and creative—a clear thinker? I think they do and I believe that the past decade of emphasis shift in our examination syllabus has been a healthy development. We are and should be attracting the type of man who would serve his profession well and honorably. This man can expect rather rapidly to rise to executive levels to the extent that he has and displays leadership qualities. We owe a great debt of gratitude to those of our members who have served on our Educational and Examination Committees. These are time-consuming tasks and they have been most ably performed.

When our Society was first born in 1914 it was of course necessary to begin with the Charter Members, many of whom were members by examination of other actuarial bodies, and also shortly thereafter to add to our membership by election and by thesis preparation those men qualified to hold themselves out as Casualty actuaries. Shortly after our formation and ever since, however, the principal means for admission has been by examination. Only when the Society itself has broadened its outlook to new lines and fields as we did ten years ago may we expect to find qualified men outside our fold who would be members but who never had previous opportunity or motivation to undertake the examinations. I hope we will continue to hold to the line that the examination route is *the* route to membership—that any other approach to Associateship or Fellowship status in this Society will be a rare exception. Especially I trust we will continue to require that theses submitted in lieu of the Associateship parts must represent original thinking and research into new fields. It should never be enough that the thesis for admission purposes would have been welcomed as a paper had it been submitted by an Associate or Fellow or from a non-member upon invitation. Very properly our *Proceedings* contain descriptive or historical or analytical material related to things going on in the insurance business today. These pages should continue to present some material of this type. This sort of paper is not enough however to stand in the place of our fair but rigorous examinations. There should not be two *standards* for admission even though there will be, for awhile at least, a continuing need for two *avenues* of admission.

There seems to be general agreement that the papers in the *Proceedings* have been both timely and of high caliber in the past few years. The most heartening aspect of this from a long range view it seems to me is the vigorous caliber of many of the papers being submitted by some of our younger members. We seem to have been charged with something of a new life and I am sure we should continue to do all we can to nurture this welcome development.

Our meeting programs have seen some innovations apparently for the better, too. I hope we continue to have seminars on a wide spectrum of timely topics, led where desirable by experts in their field from outside our ranks. The subjects of economics and the fight against inflation; of social insurance and unwelcome government activity in the area of private enterprise; of the ethical practice of law and the rise of the unjustifiable verdict and claim settlement; of corporate good citizenship and the fight for highway safety are among many which could appropriately be subjects for seminars at our meetings and where the discussion leaders might well be nationally known leaders in

their field. Both broad and special problems of taxation, investment, marketing, public relations, advertising, legislation, administration, research, regulation and personnel recruitment are among the areas in which some or all of our members have a quite direct interest. Most of these are fields though where the actuarial aspects are dovetailed with other aspects and, at our meetings, we could well undertake to broaden our understanding and our vision by frank roundtable discussions joined by outside experts.

I mentioned earlier that each of the sixteen largest general insurance groups now employ one or more members of this Society in responsible management positions. Among the remainder of the first fifty such companies only a scattering of our members are employed even though there has been a steady rise in actuarial employment among these companies too. Your officers and the Council have agreed that many managements not yet in a position to make full and productive use of an employed actuary would nonetheless be greatly served if they could send a representative to our meetings and could receive our *Proceedings* and notices of our other publication efforts. As a result the Council of the Society is now preparing to make available, through an invitational program a procedure whereby for an appropriate and modest fee an individual company will receive the *Proceedings* and will have opportunity of sending a representative to our meetings. This representative would be privileged to attend and take part in panels and seminars and would be in a position to keep his management posted on the latest developments in the non-life actuarial field. I am hopeful that a definitive announcement of this new program will be made to the insurance industry soon.

For some little time now our Society has been concerning itself with problems relating to professional conduct and to the certification or licensing of actuaries. Several different events all the way from proposed federal legislation in the labor field to the applicability of unincorporated business taxes in one particular state have heightened our already existing interest in clarifying this matter of the professional status of actuaries. All of these problems come really under that one heading. That is, the general problem of Professional status in the legal or statutory sense embraces all these matters. It also seems clear that the interests of other actuarial bodies are as much or in some cases even more involved in this broad problem. I am very happy to report that in informal talks with representatives of the Society of Actuaries the framework of this problem seems to be agreed. I have not the slightest doubt but that after appropriate consideration of the matter within our respective organizations that the several actuarial bodies concerned can and will undertake a joint effort to generate a sound solution. As this work progresses it must be ever kept in mind that in seeking the benefits and privileges of statutory professional recognition certain additional responsibilities and limitations on actuarial activities will have to be contemplated. In order that this effort be forwarded expeditiously and with the maximum of cooperation among the various actuarial groups interested in it, I am going to suggest to the Council that our Committee on Certification or Licensing of Actuaries be renamed the Committee on Professional Status and given the broad responsibility to proceed to consider all phases of this intriguing and important question.

The last item of Society business that I would like to talk to you about is

the adequate recognition which we should strive to give to our coming Fiftieth Anniversary. On November 7, 1914 this Society came into existence and, as this meeting we are holding in our nation's capital will attest, we are now a strong and flourishing organization. I have already heard several splendid suggestions for activities which will appropriately mark the day in 1964 when we will have had a half century of existence. One of the most engaging of these refers to a special set of anniversary papers designed to give added stature to the Society. It would seem also that we should make special efforts toward bringing as many of our membership as possible together on that occasion. There are, no doubt, other things that should be done to note this anniversary. I hope all of our members will be both free and generous in making suggestions for the successful accomplishment of a truly worthwhile program. In order to get this project properly underway I believe the Council should consider the appointment of a committee of prominent members to be a steering committee with the breadth of vision to encourage us to do this job well. I am sure that we can make this a signal event not only for our Society and profession but for the advancement of actuarial and statistical science generally. In closing these remarks about some of the concerns of actuaries as such and especially of this Society, I wish to thank you for the trust and honor that you have given me in asking me to serve as your President this past year. As this Society nears its fiftieth anniversary it is perhaps not too surprising that we recognize the names of sons and daughters of some of our members and former officers showing up on our rolls of students and associates and fellows. I hope that we will continue to generate a spirit and feeling toward this profession which will encourage our young people to follow in our footsteps. Not surprisingly we see a great deal of this in other professions requiring proof of skill by special education and examination. Think of the doctors' sons who are physicians, or lawyers' sons who are now honored members of the bar. Our actuarial profession should also, we can hope, inspire our sons and daughters to seek the necessary education and to undertake the necessary examinations to follow a family tradition toward the actuarial profession. Not, we can be sure, to attempt some closed and clannish society but rather as stirring proof that we think so highly of the actuarial life and in our time have advanced its best interests to such an extent that we would wish nothing better for our children and those of our friends and neighbors than that they would want to carry on in this work. My intuition runs toward believing that this kind of proof of faith in the opportunities in the actuarial field may do more to augment our ranks with first rate candidates—related and unrelated to present or former members—than some of the more formal activities of promotion in which we engage.

ANY ROOM LEFT FOR SKIMMING THE CREAM?

BY

ROBERT A. BAILEY

In writing private passenger automobile liability insurance there has always been a need for underwriters to select the good business and turn down the poor because the rate classification systems have never been perfect. Within any one class and territory there have always been some risks better or worse than the others. Where the rating plans left off, the underwriters took over in recognizing other factors in risk selection. The operation of rating plans combined with underwriting selection exerts a powerful competitive influence. Advances in underwriting selection are often incorporated into the rating plans so that actually both are part of the same program. Those companies which develop a more effective rating and underwriting selection program are able to "skim off the cream".

Back in 1953 the bureau companies attempted to meet the competitive pressure by expanding the three class plan to six or seven classes and sharply increasing the spread of relativities among the classes. This undoubtedly helped their competitive position but it didn't eliminate the problem. Far from it. So in 1959 the class plan was expanded again to include merit rating in the hope that this would improve their competitive position and would reduce the room for competitors to select the better risks within each rate class.

It is probably safe to say that we will never be able to devise a classification system which will produce a precisely correct rate for each risk, but we attempt to come as close to this ideal as is possible and practical. Considering the new class plan which includes merit rating, how close to the ideal has it come? How much room, if any, is left for skimming the cream?

A generally accepted measure of the relative amount of variation within a group, and an appropriate one for this problem, is the so-called coefficient of variation, which is the standard deviation divided by the mean. If the rate for each class were based on the experience for that class and if the class plan were perfect in assigning a rate to each risk which exactly reflected the inherent hazard of the risk, the coefficient of variation for the rates would be the same as for the risks. If we can measure the coefficient of variation for the risks, we can then compare it with the coefficient of variation for the rates to see how effective the class plan is and how close to the ideal it comes. The less effective the class plan, the more room there is for skimming off the cream.

Using the negative binomial distribution (see "Some Considerations on Automobile Rating Systems Utilizing Individual Driving Records" by Mr. Lester Dropkin, *Proceedings of the Casualty Actuarial Society,* 1959, p. 165) we can estimate the coefficient of variation of the risks from two sources: The California Driver Record Study and the Canadian automobile statistics on merit rating. Using Mr. Dropkin's symbols, the coefficient of variation of T (m), the distribution of the inherent hazard of the risks, is $1 \div \sqrt{r}$. Mr. Dropkin shows that the value of r for the total California population of

licensed drivers is .8927. Using the technique discussed in the review of Mr. Dropkin's paper and using the data shown in Table 1 of "Two Studies in Automobile Insurance Ratemaking", (both in PCAS, 1960) the value of r for the Canadian data can be derived from the credibility of .0682 for one accident-free year for all classes combined and equals 1.3301.

Therefore, the two estimates of the coefficient of variation for the total population of private passenger automobile risks are 1.06 based on the California Driver Record Study and .87 based on the Canadian merit rating data. The result obtained from the California data is a little too high because it is based on licensed drivers which have more variation than licensed automobiles. The result obtained from the Canadian data is a little too low because the technique used to derive the value of r assumes that a risk does not change from one year to the next. Because risks do change, the value of r is overstated and the coefficient of variation is understated. It appears therefore, that the coefficient of variation for risks in private passenger automobile liability insurance is approximately 1.00. This is in close agreement with the value of .977 which M. Pierre Delaporte calculated for the coefficient of variation of pleasure use automobiles in France. (See *Sixteenth International Congress of Actuaries,* 1960, Vol. II, p. 127.)

The next step is to calculate the coefficient of variation for the rates and compare the results with 1.00. For this purpose a distribution of exposures by class and territory is needed. In this paper the exposure distribution written in Pennsylvania during the first quarter of 1960 by a stock agency company and the rates of the same company are used because they were readily available and because Pennsylvania is a fairly representative state. This company's only deviation from National Bureau rates in Pennsylvania during the first quarter of 1960 was in the merit rating plan where it used the same experience period and number of points for accidents and convictions as the National Bureau used in California. Some available data is also shown for the Canadian and Texas merit rating plans.

The exposure distributions and the coefficients of variation are shown in the exhibits at the end of this paper. They are summarized below.

Pennsylvania—Rates as of March 31, 1960

Rating Criteria	*Coefficient of Variation*
Six Class Plan only	.362
Territory only	.273
Discount for Two or More Cars only	.085
Merit Rating only	.050
Farm Versus Non-Farm	.034
Assigned Risk Surcharge only	.030
Driver Training Credit only	.007
Everything above except Territory and Merit Rating	.397
Everything above except Merit Rating	.495
Everything above	.510
Three Class Plan using 1952 Relativities	.190

Canada—Indicated Relativities

Rating Criteria	*Coefficient of Variation*
Five Class Plan only	.352
Merit Rating only	.225
Both Combined	.402

Texas—Rates as of June 30, 1960

Rating Criteria	*Coefficient of Variation*
Merit Rating only	.232

It was interesting to note that the 12,732 cars included in the Pennsylvania sample were distributed among 570 out of the total of 3,675 possible rate classes in Pennsylvania (21 territories times 175 classes in each territory). This means that a sample of this size still left 84% of the rate classes without any exposure. We may be making our rating plans too complex. Let us hope that further advances will be made toward the goal of more effective and, if possible, less complex risk classification systems.

The data shown above permits comparisons to be made among the various rating and merit rating plans, leading to a number of conclusions:

1. The six class plan represents a substantial improvement over the former three class plan although all this improvement did not take place when the six class plan was first introduced in 1953 but developed as the six class plan was improved with experience.

2. The National Bureau merit rating plan in Pennsylvania, which assigns two points for each accident, and uses a three-year experience period, is estimated to have a coefficient of variation of about .10 which is about twice as large as the California-type plan. The small coefficients of variation for these plans may be partially the result of using an exposure distribution for a single company which may not be strictly average; but even allowing for this, the merit rating plans introduced into Pennsylvania, California and other states beginning in 1959 can be made much more effective as can be seen by comparing their coefficients of variation with those of merit rating in Canada and Texas. The Canadian plan, however, started out with an effectiveness of about .10 and attained its present effectiveness gradually as the plan was improved with experience. The most recent Canadian improvement, using a five-year experience period, is not reflected in the data shown in this paper. It is to be expected that substantial improvement will likewise take place in the U. S. plans as experience develops.

3. That the Texas merit rating plan developed a coefficient of variation at inception which is about the same or a little larger than that for the 1959 Canadian plan, is a substantial accomplishment and is attributable to the use of convictions as well as accidents and the use of all the accidents and convictions during the experience period instead of only the most recent one. The measurement of the Texas plan is only tenta-

tive because the relativities in the Texas plan are not yet based directly on experience, although they are probably conservative.

4. The U.S. merit rating plans other than the Texas plan have comparatively little effectiveness as mentioned above in 2. Looking at it another way, under the California-type plan, risks with the lowest merit rating are getting a rate only about 1½% lower than the average merit rating which contrasts with 13% in Texas and 9% in Canada. This points out that the present California-type merit rating plans will have to be improved if they are to continue to justify the work involved in administering them. They presently are less effective in some areas than the discount for two or more cars. They could be made about as effective as the territory rating criteria.

The following conclusions can be drawn from the data for all rating criteria combined and provide an answer to the question raised in the title of this paper:

5. The present multiple classification system in all its complexity takes care of only half of the total variation among risks.

6. The introduction of merit rating has not eliminated the need for careful underwriting and has not eliminated the opportunity to skim off the cream through more effective rating plans and underwriting selection.

Pennsylvania - First Quarter 1960
Private Passenger Automobile Liability Insurance

Territory Code	Written BI Exposure (Car Months)	BI+PD Rate Class 1A as of 3-31-60
01	3,956	$90
03	11,089	65
05	419	50
06	1,569	34
07	7,293	34
08	3,218	39
09	586	44
10	1,450	38
11	1,863	43
12	6,271	29
13	43,605	44
14	15,727	57
15	7,635	37
16	3,751	55
17,18	4,545	52
19	7,296	34
21	4,989	34
22,23,24,25	7,380	28
25	3,227	41
27,28,29	16,137	40
34	780	52
Total	152,786	
Mean		44.836
Standard Deviation		12.225
Coefficient of Variation		.273

Merit Rating Code		Relativity
9	139,486	85
1	11,628	95
2	1,261	100
3	264	120
4	26	140
5	72	170
6	49	200
Total	152,786	
Mean		86.032
Standard Deviation		4.315
Coefficient of Variation		.050
Assigned Risk Surcharge	2,275	125
No AR Surcharge	150,511	100
Total	152,786	
Mean		100.372
Standard Deviation		3.036
Coefficient of Variation		.030

Class	Written BI Exposure (Car Months)	Relativity
1A	63,781	100
1B Small Cities	36,918	100
1B Large Cities	23,015	110
1C	4,499	145
2A	10,270	190
2C Small Cities	2,001	360
2C Large Cities	1,642	310
3	10,660	150
Total	152,786	
Mean		118.032
Standard Deviation		42.751
Coefficient of Variation		.362
1	128,213	70
2	13,913	115
3	10,660	100
Total	152,786	
Mean		76.191
Standard Deviation		14.452
Coefficient of Variation		.190
Non-Farm	150,852	100
Farm	1,934	70
Total	152,786	
Mean		99.620
Standard Deviation		3.361
Coefficient of Variation		.034
Driver Training Discount	682	90
No D. T. Discount	152,104	100
Total	152,786	
Mean		99.955
Standard Deviation		.719
Coefficient of Variation		.007
Multi-Car Discount	18,764	75
No M-C Discount	134,022	100
Total	152,786	
Mean		96.880
Standard Deviation		8.262
Coefficient of Variation		.085

Pennsylvania - First Quarter 1960
Private Passenger Automobile Liability Insurance

Class	Small City Territories Written BI Exposure (Car Months)	Relativity	Large City Territories Written BI Exposure (Car Months)	Relativity
1A	25,113	100	17,668	100
1A Multi-Car	13,436	75	5,328	75
1A Assigned Risk Surcharge	276	125	243	125
1B	27,931	100	19,262	110
1B M-C	8,710	100	3,596	110
1B A-R Surcharge	277	125	157	138
1C	2,127	145	942	145
1C M-C	1,084	145	178	145
1C A-R Surcharge	120	181	48	181
1AF	930	70	168	70
1AF M-C	415	61	192	61
1AF A-R Surcharge	12	88	0	88
2A	5,865	190	3,595	190
2A A-R Surcharge	127	238	48	238
2A Driver Training	228	171	238	171
2C	1,729	360	1,389	310
2C A-R Surcharge	116	450	145	388
2C Driver Training	108	324	108	279
2AF	157	133	12	133
2AF A-R Surcharge	0	166	0	166
2CF	48	252	0	217
2CF A-R Surcharge	0	315	0	271
3	4,368	150	2,972	150
3 M-C	2,185	150	1,111	150
3 A-R Surcharge	0	188	24	188
Totals	95,362		57,424	
Grand Total	152,786			
Mean		114.789		
Standard Deviation		45.532		
Coefficient of Variation		.397		

All Rating Criteria Except Merit Rating

Mean	51.688
Standard deviation	25.607
Coefficient of variation	.495

All Rating Criteria

Mean	44.581
Standard deviation	22.733
Coefficient of variation	.510

Average Merit Rating .8625

Canada Excluding Saskatchewan - All Companies
Policy Years 1957 and 1958 as of 6-30-59
Private Passenger Automobile Liability Insurance — Non Farmers

Class		Earned Exposure (Car Years)	Indicated Relativity*
(Age	1	3,325,714	.863
Sex	2	168,998	1.372
and	3	321,327	1.313
Use)	4	252,397	2.269
	5	81,639	1.154
Total		4,150,075	
Mean			1.00980
Standard Deviation			.35577
Coefficient of Variation			.352
(Merit	A	3,356,480	.895
Rating)	X	175,553	1.174
	Y	219,597	1.277
	B	398,445	1.610
Total		4,150,075	
Mean			.99566
Standard Deviation			.22354
Coefficient of Variation			.225

Class	Earned Exposure (Car Years)	Indicated Relativity
1A	2,757,520	.786
1X	130,706	1.016
1Y	163,544	1.115
1B	273,944	1.358
2A	130,535	1.269
2X	7,233	1.747
2Y	9,726	1.519
2B	21,504	1.784
3A	247,424	1.212
3X	15,868	1.285
3Y	20,369	1.450
3B	37,666	1.885
4A	156,871	2.050
4X	17,707	2.192
4Y	21,089	2.412
4B	56,730	2.853
5A	64,130	1.071
5X	4,039	1.079
5Y	4,869	1.410
5B	8,601	1.642
Total	4,150,075	
Mean		1.00575
Standard Deviation		.40434
Coefficient of Variation		.402

Texas — All Companies — Second Quarter 1960
Private Passenger Automobile Liability Insurance

Merit Rating	Written Exposure (Car Months)	Relativity
0	4,202,958	.80
1	858,947	1.00
2	551,716	1.20
3	174,319	1.40
4	97,547	1.60
5	31,405	1.80
6	39,740	2.00
Total	5,956,632	
Mean		.90983
Standard Deviation		.21075
Coefficient of Variation		.232

*See "Two Studies in Automobile Insurance Ratemaking", R. A. Bailey and L. J. Simon, PCAS 1960

AUTOMOBILE MERIT RATING and INVERSE PROBABILITIES

BY

LESTER B. DROPKIN

Introduction

The previous paper by this writer[1] started from the fact that the negative binomial distribution provides an analytical expression for risk distribution by number of accidents. The expressions used in that paper did not explicitly introduce the time factor – e.g. the 3-year experience of the California study[2] was treated as a unit entity.

In the discussion of my paper by Mr. R. A. Bailey[3] the negative binomial was utilized to analytically develop an expression for the average frequency of a sub-group having n accident-free years.

It is the purpose of the present paper to set forth (a) the negative binomial distribution with the time element explicitly introduced and (b) a general expression for the probability of x accidents in subsequent years, knowing that a specified number of accidents have occurred in a given time period. This involves the classic problem of inverse probability and its solution is afforded by recourse to Bayes Theorem.

This general expression of risk distribution should find particular application to those automobile merit rating systems which determine credits and debits on the basis of fixed experience periods. However, the required expressions for a system based on the number of accident-free years also fall out in a simple manner.

The Negative Binomial, N(x; t)

We assume, as before, that to each member of the population may be coordinated a measure, m, of inherent accident potential which remains constant for the individual throughout the period involved. Further, we assume that the distribution of m in the population is given by the two parameter function, T(m):

$$T(m) = a^r m^{r-1} e^{-am} / \Gamma(r) \tag{1}$$

This function is independent of the time, has a mean equal to r/a and a variance equal to r/a^2.

For a given time period t, the (forward) probability of the number of accidents equaling x where x=0, 1, . . . is denoted by N (x; t) and is given by:

$$N(x; t) = \int_0^\infty P(x; mt) T(m) \, dm \tag{2}$$

[1]"Some Considerations on Automobile Rating Systems Utilizing Individual Driving Records," CAS XLVI, p. 165.

[2]See Harwayne, F., "Merit Rating in Private Passenger Automobile Liability Insurance and the California Driver Record Study," CAS XLVI, p. 189, for a description of the study and its results.

[3]Page 152.

where $P(x; mt)$ is the Poisson frequency function:

$$P(x; mt) = (mt)^x e^{-mt}/x! \tag{3}$$

Upon substituting (1) and (3) in (2), integrating and simplifying we have that[4]

$$N(x; t) = \left(\frac{a}{a+t}\right)^r \left(\frac{t}{a+t}\right)^x (-1)^x \binom{-r}{x} \tag{4}$$

This distribution, the negative binomial, depends upon t, has a mean equal to rt/a and a variance[5] equal to $\left(\frac{rt}{a}\right)\left(\frac{a+t}{a}\right)$.

Inverse Probabilities and Bayes Theorem

It will be recalled that Bayes Theorem is properly applicable in the following kind of situation. Let us suppose that various mutually exclusive conditions, represented by A, B, C, . . . may exist. Also suppose that the probability of condition A existing is known and given by a, the probability of condition B by b, etc. Further, let us suppose that if condition A exists, then the probability of the happening of an event in which we are interested is known and given by E_a; that if condition B exists, the probability of the happening of this event is E_b; etc. The problem of inverse probabilities arises when we know that the event has occurred but we do not know what condition caused it. We ask, for example, what is the probability that this event arose out of condition A? The probability of condition A existing, knowing that the event has happened is denoted by Prob. (A/event). Bayes Theorem says that

$$\text{Prob. (A/event)} = \frac{a \cdot E_a}{a \cdot E_a + b \cdot E_b + \ldots} \tag{5}$$

$$\text{Similarly, Prob. (B/event)} = \frac{b \cdot E_b}{a \cdot E_a + b \cdot E_b + \ldots}, \text{ etc.}$$

Inverse Probabilities and Automobile Merit Rating

In the model which we have been utilizing, we have supposed the existence of various mutually exclusive conditions. That is, for each individual we have assumed the existence of a particular measure of inherent accident potential, which has a probability given by $T(m)$. These values of $T(m)$ correspond to the a's and b's mentioned above. Now the probability of a given number of accidents occurring, say c accidents, in a time period s, under the condition of an inherent accident potential of m, is given by $P(c; ms)$.

$$P(c; ms) = (ms)^c e^{-ms}/c! \tag{6}$$

These values of $P(c; ms)$ correspond to the E_a's and the E_b's above.

[4]See Appendix A.

[5]Means and variances are neatly determined by the use of moment generating functions. See Simon, L. J., "The Negative Binomial and Poisson Distributions Compared." Page 20.

Now suppose that we have observed the event: c accidents in s years. We ask the question, what is the probability that this event arose out of a particular measure, m? By Bayes Theorem

$$T(m/c, s) = \frac{P(c; ms) \cdot T(m)}{\int_0^\infty P(c; ms) \cdot T(m)\,dm} \tag{7}$$

$$T(m/c, s) = \frac{P(c; ms) \cdot T(m)}{N(c; s)} \tag{8}$$

When the operations indicated by (8) are carried out, we have that

$$T(m/c,s) = (a+s)^{r+c}\, m^{r+c-1}\, e^{-m(a+s)}/\Gamma(c+r) \tag{9}$$

This function has a mean equal to $(r+c)/(a+s)$ and a variance equal to $(r+c)/(a+s)^2$.

This function, T(m/c,s), is the distribution of inherent accident potential for a particular sub-group, viz. those who have been observed to have c accidents in s years. But we are now in a position to determine the (forward) probability of x accidents in the next t years for this sub-group. Denoting this probability by N(x; t/c; s) we have that

$$N(x; t/c; s) = \int_0^\infty P(x; mt) \cdot T(m/c, s)\,dm \tag{10}$$

$$= \int_0^\infty \frac{P(x; mt) \cdot P(c; ms) \cdot T(m)\,dm}{N(c; s)} \tag{11}$$

Upon substituting, integrating and simplifying we have that[6]

$$N(x; t/c; s) = \left(\frac{a+s}{t+a+s}\right)^{r+c} \left(\frac{t}{t+a+s}\right)^{x} (-1)^x \binom{-(r+c)}{x} \tag{12}$$

This distribution has a mean equal to $\frac{t(r+c)}{a+s}$ and a variance equal to $\frac{t(r+c)\,(a+s+t)}{(a+s)^2}$.

[6]See Appendix B.

APPENDIX A

$$N(x;t)=\int_0^\infty P(x;mt)\,T(m)\,dm$$

$$=\frac{a^r t^x}{x!\,\Gamma(r)}\int_0^\infty m^{x+r-1}e^{-m(a+t)}dm$$

$$=\frac{a^r\,t^x}{x!\,\Gamma(r)}\,\frac{\Gamma(x+r)}{(a+t)^{x+r}}=\left(\frac{a}{a+t}\right)^r\left(\frac{t}{a+t}\right)^x\frac{\Gamma(x+r)}{x!\,\Gamma(r)}$$

Since $\Gamma(x+r)=(x+r-1)(x+r-2)\ldots r\,\Gamma(r)$,

$$\frac{\Gamma(x+r)}{x!\,\Gamma(r)}=\frac{r(r+1)\ldots(r+x-1)}{x!}$$

$$=(-1)^x\binom{-r}{x},\text{ and}$$

$$N(x;t)=\left(\frac{a}{a+t}\right)^r\left(\frac{t}{a+t}\right)^x(-1)^x\binom{-r}{x}$$

APPENDIX B

$$N(x;t/c;s)=\int_0^\infty P(x;mt)\cdot T(m/c,s)\,dm$$

$$=\int_0^\infty\frac{P(x;mt)\cdot P(c;ms)\cdot T(m)}{N(c;s)}\,dm$$

$$=\int_0^\infty\frac{(mt)^x e^{-mt}}{x!}\,\frac{(ms)^c e^{-ms}}{c!}\,\frac{a^r m^{r-1}e^{-am}}{\Gamma(r)}\cdot\frac{1}{N(c;s)}\,dm$$

$$=\frac{t^x a^r s^c}{x!\,\Gamma(r)\,c!\,N(c;s)}\int_0^\infty m^{x+r-1+c}e^{-m(s+a+t)}dm$$

$$=\frac{t^x a^r s^c}{x!\,\Gamma(r)\,c!\,N(c;s)}\cdot\frac{\Gamma(x+r+c)}{(s+a+t)^{x+r+c}}$$

Since $N(c;s)=\left(\frac{a}{a+s}\right)^r\left(\frac{-s}{a+s}\right)^c\binom{-r}{c}=\frac{a^r\,s^c\,\Gamma(c+r)}{c!\,\Gamma(r)(a+s)^{c+r}}$

$$N(x;t/c;s)=\frac{t^x\,(a+s)^{c+r}}{(s+a+t)^{x+c+r}}\cdot\frac{\Gamma(x+r+c)}{x!\,\Gamma(c+r)}$$

$$=\left(\frac{a+s}{s+a+t}\right)^{r+c}\left(\frac{t}{s+a+t}\right)^x(-1)^x\binom{-(r+c)}{x}$$

A NEW APPROACH TO INFANT AND JUVENILE MORTALITY

BY

CHARLES C. HEWITT, JR.

Introduction

Considerable space is devoted in the literature of this Society to the problems of evaluating the rights of survivors under the benefit provisions of various Workmen's Compensation statutes. By the very nature of the problem of industrial deaths, survivors are, in a great majority of the cases, widows and orphans. In treating of the life expectancy of orphans, the Casualty Actuary is confronted with the use of life contingency functions developed by the Life Actuary whose primary concern is with individuals who have arrived at maturity or are very close thereto.

There is no satisfactory analytical expression for mortality with respect to infants and juveniles. Values for the various life functions at these ages are, at best, non-analytical and highly empirical.

When the problem of multiple-life contingencies is superimposed upon this situation, the difficulties encountered by the Casualty Actuary in measuring the value of orphans' benefits become considerable.

Summary

This paper offers a different approach to the problem of the analysis of mortality during the early years of life by distinguishing between the force of mortality as a group average and the individual forces of mortality of the various members of the group. It recognizes that the term "force of mortality" as now expressed in the literature on the subject of life contingencies actually means the *average* force of mortality of a group of individual lives.

A simple example is used to illustrate what happens to the group force of mortality when the individual life members of the group have varying individual forces of mortality.

The individual force of mortality is analyzed and is seen to be a combination of three components which for convenience are referred to as "obsolescence", "chance" and "incapacity to sustain life".

An *a priori* expression for the frequency distribution of the "incapacity to sustain life" is introduced. Making use of this *a priori* distribution, a new expression for group mortality rates is obtained, and the component effects of "obsolescence", "chance" and "selection" become evident.

A "complete" expression for the group force of mortality is obtained. From this expression other life functions may be derived.

A relatively crude test of the values obtained by use of the "complete" expression for group mortality shows that the general shape of an actual mortality curve may be quite well represented.

Areas for further development of the principles brought forth in this paper are indicated.

1. Force of Selection

It seems self-evident that expressions of the force of mortality (μ_x) and other functions such as p_x and q_x imply that each is in fact an average for the group of lives whose attained age is x. Actual values for these functions have proven reasonably satisfactory. This undoubtedly results from the circumstance that for those values of x which are most commonly used in life insurance (very roughly speaking from age 17 to age 85), there is relatively little variation of the individual values about the group average.

It might be suspected, however, that at the very beginning of the life span, there exists a considerable dispersion of the individual forces of mortality about the group average, and it is precisely for these ages that there has been the greatest difficulty in expressing the force of mortality either analytically, or with any accuracy or confidence in the values obtained.

New-born children have widely-varying chances of survival. The probability of surviving depends upon physical factors, some hereditary, some environmental, including the circumstances surrounding pregnancy and birth, which will cause wide variations in the probability of living or dying during the early days, weeks, months or even years of life. The question therefore arises as to what effect this wide dispersion in individual probabilities of survival or death will have on the group survival or mortality function. Obviously the child born with the greatest chance of survival will be more likely to live during the first few years of life; the child with a lesser chance of survival will have a greater probability of dying in infancy or early childhood. This "survival of the fittest" has a profound effect upon the group forces of survival and mortality during the early years.

The example given below illustrates, by removing all factors other than selection, the importance of selection when examining group or average mortalities during the early years.

Let us assume for purposes of illustration that in a group of 100,000 births there are 20,000 infants with a constant force of mortality such that each member has a probability of death during one year equal to 4/5ths. Similarly, another 20,000 infants are assumed to have a constant probability of dying during one year equivalent to 3/5ths; 20,000 more are assumed to have a constant one year probability of dying (q_x) of 2/5ths; another 20,000 have a value for q_x of 1/5; and the remaining 20,000 infants have such a strong probability of survival that it will be assumed that, for the purposes of this example, q_x is equal to zero.

It is recognized, of course, that in real life the force of mortality with respect to an individual life normally varies with age. However, in order to make this illustration as simple as possible, it will be assumed that the individual lives referred to have a constant force of mortality throughout the first four years of life as represented by the various values of q_x which have been assigned. It will be recognized that where q_x remains constant, the force of mortality, μ_x, remains constant and can, in fact, be obtained by the expression:

$$\mu_x = \text{colog}_e(1-q_x) \qquad (1.1)$$

Under the assumptions made, the values of l_x and d_x for the first four years of life are set forth in Table 1.

TABLE 1

ILLUSTRATION OF THE EFFECT OF THE FORCE OF SELECTION ON GROUP MORTALITY RATES

Group Number (i)	q^i	1st Year		2nd Year		3rd Year		4th Year	
		l_0	d_0	l_1	d_1	l_2	d_2	l_3	d_3
0	0	20,000	0	20,000	0	20,000	0	20,000	0
1	$\frac{1}{5}$	20,000	4,000	16,000	3,200	12,800	2,560	10,240	2,048
2	$\frac{2}{5}$	20,000	8,000	12,000	4,800	7,200	2,880	4,320	1,728
3	$\frac{3}{5}$	20,000	12,000	8,000	4,800	3,200	1,920	1,280	768
4	$\frac{4}{5}$	20,000	16,000	4,000	3,200	800	640	160	128
Total		100,000	40,000	60,000	16,000	44,000	8,000	36,000	4,672
$\bar{q}_x = \frac{\sum_i d_x}{\sum_i l_x}$			0.400		0.267		0.182		0.130

The resultant mortality table would look as follows:

TABLE 2

GROUP MORTALITY TABLE BASED UPON RESULTS OF TABLE 1

Age	l_x	d_x	$\bar{q}_x$	$\bar{\mu}_x$*
0	100,000	40,000	.400	.652
1	60,000	16,000	.267	.391
2	44,000	8,000	.182	.244
3	36,000	4,672	.130	.160

*See narrative for derivation of $\bar{\mu}_x$.

It is possible to represent the group force of mortality in this illustration by the following expression:

$$\bar{\mu}_x = \frac{\sum_{i=0}^{4} \mu^i l_x^i}{\sum_{i=0}^{4} l_x^i} \tag{1.2}$$

where

$$\mu_x^i = \mu^i = \text{colog}_e \left(1 - \frac{i}{5}\right) \tag{1.3}$$

Since

$$l_x^i = {}_xp_0^i \, l_0^i \tag{1.4}$$

and

$${}_xp_0^i = e^{-\int_0^x \mu^i dt} = e^{-\mu^i x} \tag{1.5}$$

$$\bar{\mu}_x = \frac{l_0^i \sum_{i=0}^{4} \mu^i e^{-\mu^i x}}{l_0^i \sum_{i=0}^{4} e^{-\mu^i x}} = \frac{\sum_{i=0}^{4} \left(1-\frac{i}{5}\right)^x \operatorname{colog}_e \left(1-\frac{i}{5}\right)}{\sum_{i=0}^{4} \left(1-\frac{i}{5}\right)^x} \tag{1.6}$$

Also from this example, it is worth while to examine the change in the frequency distribution of lives with respect to individual rates of mortality:

TABLE 3

FREQUENCY DISTRIBUTION OF LIVES (TABLE 1) BY RATE OF MORTALITY

Group Number (i)	q^i	AGE 0	1	2	3
		FREQUENCY			
0	0	.200	.333	.454	.556
1	$\frac{1}{5}$	.200	.267	.291	.284
2	$\frac{2}{5}$	.200	.200	.164	.120
3	$\frac{3}{5}$	.200	.133	.073	.036
4	$\frac{4}{5}$	.200	.067	.018	.004
Total		1.000	1.000	1.000	1.000

In examining the resulting values for rate of mortality and force of mortality, it must be re-emphasized that, with respect to the individual life members of this group, the force of mortality and the mortality rate remain constant throughout the entire period. Thus, it becomes completely evident that the group force of mortality is favorably affected by the survival of the fittest lives.

The rather drastic effects resulting from the "force of selection" in this example were achieved by the choice, for most individuals, of relatively high rates of mortality and by the selection of a population with a wide dispersion in the rates of mortality of the individual members. It becomes evident that as the process of selection continues the average rates of mortality of the group

diminish and the dispersion of the individual mortality rates about the group average diminishes.

Thus, it may be anticipated that the effect of the "force of selection" while extremely important in the earliest years of life diminishes very rapidly and becomes of relatively minor importance during the middle and later years of the life span.

2. Individual Mortality

In 1825 Benjamin Gompertz[1] stated with respect to the problem of mortality, "The average exhaustion of a man's power to avoid death (is) such that at the end of equal infinitely small intervals of time he (loses) equal portions of his remaining power to oppose destruction which he had at the commencement of these intervals."

Application of this basically philosophical premise led Gompertz to the hypothesis that the individual force of mortality could be expressed as follows:

$$\mu_x = Bc^x \qquad (2.1)$$

It is particularly significant in examining this concept to note that Gompertz referred to the capacity of the *individual* to resist or avoid death.

At the same time that he presented this important concept, Gompertz stated, "It is possible that death may be the consequence of two generally coexisting causes: the one, chance, without previous disposition to death or deterioration; the other, a deterioration, or increased inability to withstand destruction."

In other words, Gompertz foresaw that in addition to the contribution of deterioration to the force of mortality, there was an additional force which he attributed to chance and which was independent of the age of the individual.

The combination of these two forces was stated analytically by Makeham[2]:

$$\mu_x = A + Bc^x \qquad (2.2)$$

Thus, today it is generally recognized that the two major contributions to mortality as expressed in the Makeham formula are the deterioration of the body's ability to resist death (which may be referred to as "obsolescence") and the force resulting from chance causes independent of age (which will be referred to as "chance").

Gompertz' use of the phrase "without previous disposition to death" is interesting and with respect to this paper significant. It has already been asserted here that at the time of birth each individual has his particular ability to sustain life. This ability results from heredity, environment, including the conditions surrounding birth and pregnancy, and from other causes which may be considered metaphysical in nature. Suffice it to state that the individual force of mortality may be viewed as a combination of the forces recognized by Gompertz and Makeham with a third element which recognizes the individual's predisposition to death (or incapacity to sustain life). Using this concept, it is possible to state analytically the force of mortality as follows:

[1]Gompertz, Benjamin, *On the Nature of the Function Expressive of the Law of Human Mortality*. Philosophical Transactions, Royal Society of London, 1825.

[2]Makeham, W. M.:*On the Law of Mortality, and the Construction of Annuity Tables,* Journal of the Institute of Actuaries, Volume 8, 1860.

$$\mu_x = m + A + Bc^x \tag{2.3}$$

where m will be defined as the individual predisposition to death. Thus, for an individual age zero, the force of mortality may be defined as:

$$\mu_0 = m + A + B \tag{2.3a}$$

Consideration should and ultimately must be given to whether the parameters B and c vary from individual to individual as it is assumed that the parameter m does.[3] For the purposes of this paper and in order to simplify further analysis, it will be assumed that all of the parameters except m are the same for all individuals.

Using (2.3) as the expression for the force of mortality, it can be shown that the probability of an individual age x surviving n years is given by the expression:

$${}_np_x = e^{-\left[(m+A)n + \frac{Bc^x(c^n-1)}{\log_e c}\right]} \tag{2.4}$$

and thus in the special case where x equals zero, the probability of an individual surviving to age n from birth is given by the expression:

$${}_np_0 = e^{-\left[(m+A)n + \frac{B(c^n-1)}{\log_e c}\right]} \tag{2.4a}$$

3. Distribution of Individual Mortality

It has been demonstrated by Dropkin[4] that the distribution function of the predisposition of individual drivers to automobile accidents can be very well represented by:

$$T(m) = \frac{a^r}{\Gamma(r)} m^{r-1} e^{-am} \tag{3.1}$$

This distribution function is a Pearson Type III curve with a mean equal to r/a and a variance equal to r/a^2. In this distribution function, $\Gamma(r)$ has the usual meaning:

$$\Gamma(r) = \int_0^\infty e^{-x} x^{r-1} dx$$

(for all positive values of r) (3.2)

and $T(m)$ is defined over the range from m equals 0 to m equals ∞. This distribution function meets the necessary condition that

$$\int_0^\infty T(m)\,dm = 1$$

The success with which Dropkin and others have used this distribution function suggests its use as a distribution function for m where m is assumed

[3]Individual variations in A can be considered as variations in m.

[4]Dropkin, Lester, *Some Considerations on Automobile Rating Systems Utilizing Individual Driving Records,* CAS XLVI, p. 165.

to represent the individual predisposition to death in the expression for individual mortality contained in (2.3).

The range of values for m from 0 to ∞ seems quite meaningful. It should be noted that m equals 0 does not imply that the force of mortality is zero at age zero since from (2.3a)

$$\mu_0 = A + B \qquad (m = 0)$$

On the other hand, m (an annual rate) equals ∞ carries the implication that the new-born child has such serious impairments, either physical or environmental, that it may not be expected to survive for more than an infinitely short period of time. It should be noted that the equivalence of m with ∞ is not intended to imply stillbirth.

Analysis of the distribution function T(m) indicates that the mode occurs when:

$$m = \frac{r-1}{a}$$

Since one of the conditions of the distribution function T(m) is that both r and a must be positive, it can be seen that when r is less than 1, the mode must occur at the point where m equals 0. It will subsequently be seen, when dealing with the group mortality function, that values of r are quite small. Therefore the modal point seems quite reasonable since most new-born children have little or no predisposition to death.

Reference to Table 3 will indicate that the frequency distribution of lives by individual rates of mortality will change with age. In other words, the distribution of lives with respect to the rate of mortality of each life is not the same at age 1 as at age 0, and likewise will not be the same at age 2 as at age 1, and so forth. This change in the distribution of lives with respect to the individual mortality rates results from the "force of selection" already discussed. In the original example it was assumed that the force of mortality with respect to each individual life remained constant and that the only thing that changed with age was the distribution of the number of lives with respect to each individual force of mortality. However, the expression for the individual force of mortality suggested in the previous section:

$$\mu_x = m + A + Bc^x$$

indicates that the individual force of mortality does in fact change with age. From equation (2.4a) it was seen that the probability of an individual life surviving n years after birth is represented by the expression:

$${}_np_0 = e^{-\left[(m+A)n + \frac{B(c^n-1)}{\log_e c}\right]}$$

The distribution function of the surviving individual lives with respect to m after n years may be obtained as follows:

$$T_n(m) = \frac{{}_np_0 \cdot T(m)}{\int_0^\infty {}_np_0 \cdot T(m)\,dm}$$

$$T_n(m)=\left(\frac{a+n}{a}\right)^r e^{-mn}\cdot T(m) \tag{3.3}$$

With respect to $T_n(m)$:

$$\int_0^\infty T_n(m)dm=1$$

the mean is:

$$\int_0^\infty \left(\frac{a+n}{a}\right)^r me^{-mn}\cdot T(m)dm=\frac{r}{a+n} \tag{3.4}$$

and the variance:

$$\left[\int_0^\infty \left(\frac{a+n}{a}\right)^r m^2e^{-mn}\cdot T(m)dm\right]-\left(\frac{r}{a+n}\right)^2=\frac{r}{(a+n)^2} \tag{3.5}$$

It can be seen from (3.4) and (3.5) that the distribution function $T_n(m)$ meets the expectation that the process of selection diminishes the average mortality rate and the dispersion of individual mortality rates about the group average.

4. Group Mortality

Using this distribution function (3.3) with a substitution of age x for the value n, the mean value for the probability of the survival of the lives which have attained age x can be obtained as follows:

$$E({}_np_x)={}_n\bar{p}_x=\int_0^\infty {}_np_x\cdot T_x(m)dm$$

$${}_n\bar{p}_x=\int_0^\infty e^{-\left[(m+A)n+\frac{Bc^x(c^n-1)}{\log_e c}\right]}\left(\frac{a+x}{a}\right)^r e^{-mx}\cdot T(m)dm$$

$${}_n\bar{p}_x=\left(\frac{a+x}{a+x+n}\right)^r e^{-\left[An+\frac{Bc^x(c^n-1)}{\log_e c}\right]} \tag{4.1}$$

and of course from this expression a value for the mean rate of mortality can be obtained:

$${}_n\bar{q}_x=1-{}_n\bar{p}_x=1-\left(\frac{a+x}{a+x+n}\right)^r e^{-\left[An+\frac{Bc^x(c^n-1)}{\log_e c}\right]} \tag{4.2}$$

It is important to distinguish between the probabilities of survival or death for the individual life at any particular age x which are represented by the expressions ${}_np_x$ and ${}_nq_x$, and the group or average probabilities of survival or death which have just been represented by the expressions ${}_n\bar{p}_x$ and ${}_n\bar{q}_x$. It is the latter expressions which actually correspond to the expressions currently in use in life contingencies.

Similarly, it is possible to derive an expression for the average (or group) force of mortality at any age x as follows:

$$E(\mu_x) = \bar{\mu}_x = \int_0^\infty \mu_x \cdot T_x(m)\,dm$$

$$\bar{\mu}_x = \int_0^\infty (m + A + Bc^x) \left(\frac{a+x}{a}\right)^r e^{-mx} \cdot T(m)\,dm$$

$$\bar{\mu}_x = A + Bc^x + \frac{r}{a+x} \tag{4.3}$$

It is this expression for the average (or group) force of mortality which corresponds to the meaning of the term "force of mortality" currently in use in all studies on the subject of life contingencies. Analysis of this new expression shows that the average force of mortality is actually made up of three component forces:

(1) The Makeham component which has been referred to herein as "chance",

(2) The Gompertz component which has been referred to herein as "obsolescence",

(3) A new component which shows the effect on the group force of mortality of the elimination of those lives which are least fit to survive. This new component has been given the name the "force of selection".

In the expression for group force of mortality (4.3) the first two components referred to are independent of the distribution of lives with respect to the individual mortality rate of each life. The Makeham component is independent of age. With respect to the third component, the "force of selection," the value of r will subsequently be seen to be quite small. Thus the expression:

$$\frac{r}{a+x}$$

approaches zero as the age x increases, and the expression for group mortality $\bar{\mu}_x$ approaches the Makeham expression for the force of mortality. As was anticipated, the "force of selection" is of greatest importance and effect at the very earliest ages of life, after which its importance and effect diminishes quite rapidly and approaches zero with increase in age. Thus, it will be seen that the present expressions for the force of mortality and the present mortality tables derived in reliance thereon may be expected to be quite reasonable for ages beyond x equals 15 to 20.

The importance of the new expression for average or group mortality, however, is that it gives a complete mortality function for all ages in a relatively simple form. Furthermore, this new expression for the complete force of mortality meets the condition that at some age (x_1) there is a minimum force of mortality.

$$\left.\frac{d\bar{\mu}_x}{dx}\right|_{x_1} = 0 \qquad \text{results in}$$

$$Bc^{x_1}\log_e c = \frac{r}{(a+x_1)^2} \tag{4.4}$$

(Age x_1 normally occurs during the juvenile years.)

Using the group mortality function (4.3) we can derive the group survival function:

$$\bar{s}(x) = {}_x\bar{p}_0 = e^{-\int_0^x \bar{\mu}_t dt} = e^{-\int_0^x \left[\frac{r}{a+t} + A + Bc^t\right] dt}$$

$$\bar{s}(x) = \left(\frac{a}{a+x}\right)^r e^{-\left[Ax + \frac{B(c^x-1)}{\log_e c}\right]} \tag{4.5}$$

This, of course, agrees with equation (4.1) when the value zero is substituted for x, and x is substituted for n.

Finally, with respect to the specific problem of evaluating benefits to orphans where the joint-life status is involved, the joint-life probability may be expressed as:

$${}_n\bar{p}_{x_1x_2\cdots x_k} = {}_n\bar{p}_{x_1} \cdot {}_n\bar{p}_{x_2} \cdot \cdots\cdots {}_n\bar{p}_{x_k}$$

$${}_n\bar{p}_{x_1x_2\cdots x_k} = \left[\frac{(a+x_1)(a+x_2)\cdots\cdots(a+x_k)}{(a+x_1+n)(a+x_2+n)\cdots(a+x_k+n)}\right]^r e^{-\left[Akn + \frac{B(c^n-1)}{\log_e c}\sum_1^k e^{x_i}\right]} \tag{4.6}$$

and since values for a will be extremely small, an approximation of the joint-life probability is given by the following:

$${}_n\bar{p}_{x_1x_2\cdots x_k} \doteq \left[\frac{x_1 \cdot x_2 \cdots x_k}{(x_1+n)(x_2+n)\cdots(x_k+n)}\right]^r e^{-\left[Akn + \frac{B(c^n-1)}{\log_e c}\sum_1^k e^{x_i}\right]} \tag{4.6a}$$

or even more simply

$${}_n\bar{p}_{x_1x_2\cdots x_k} \doteq \left[\frac{x_1 \cdot x_2 \cdots x_k}{(x_1+n)(x_2+n)\cdots(x_k+n)}\right]^r ({}_np'_w)^k \tag{4.6b}$$

where

$${}_np'_w = e^{-\int_0^n \mu'_{w+t} dt}$$

and

$${}_k\mu'_w = \mu'_{x_1} + \mu'_{x_2} + \cdots + \mu'_{x_k}$$

and

$$\mu'_x = A + Bc^x \qquad \text{[see (2.2)]}$$

5. Test of Group Mortality Functions

It is impossible to find an actual mortality table which would meet any rigorous program for testing the various group mortality functions derived in the previous section.

However, it is interesting to test the general form of the "complete" expression for group mortality to see whether there is an approximate fit against actual data. For this purpose comparison was made with the mortality table for U. S. White Males 1939-41, the figures for which were obtained from the Census of 1940. The comparative values for $\bar{q}_x$ are given in Table 4, and it is interesting to note that the general shape of the actual mortality curve is quite well represented by the group mortality function.

TABLE 4

COMPARISON OF MORTALITY RATES USING "COMPLETE" EXPRESSION FOR GROUP MORTALITY WITH MORTALITY RATES FOR U. S. WHITE MALES 1939-41 (AT QUINQUENNIAL AGES)

Age (x)	Mortality Rates	
	Actual (q_x)	Theoretical ($\bar{q}_x$)
0	.04812	.04812
5	.00138	.00120
10	.00099	.00100
15	.00143	.00110
20	.00211	.00138
25	.00243	.00184
30	.00279	.00257
35	.00363	.00370
40	.00513	.00540
45	.00766	.00792
50	.01155	.01170
55	.01736	.01724
60	.02548	.02566
65	.03684	.03806
70	.05454	.05628
75	.08313	.08336
80	.12472	.12132
85	.18096	.17582
90	.24893	.25112
95	.32071	.35110
100	.38934	.47631
105	.44751	.61998

There are five parameters in the general expression for group mortality, and these parameters were obtained on a relatively crude basis by setting up the following five conditions:

(1) The minimum point of the curve representing the group force of mortality (see 4.4) was assumed to occur at age 10 years and 6 months.

(2) The value of the group force of mortality at this particular age was taken as .001.

(3) The probability of survival of the first year of life was equated to the corresponding probability in the mortality table.

(4) The probability of surviving to age 69 (approximately the median point of the survival function) was equated to the corresponding value in the mortality table.

(5) The probability of surviving an additional 25 years after age 69 was equated to the corresponding value in the mortality table.

It will be recognized that there are better methods of obtaining the parameters, particularly when greater accuracy is desired at the lower ages.

There are a number of reasons why a perfect fit of the data would be impossible:

(1) Values of various parameters used in the group mortality functions are changing from year to year. For example, the lives age 40 at the time of the Census would not have had the same values for the various parameters in the group mortality functions during the first ten years of life as the lives at age 10 in 1940, and so forth.

(2) There are undoubtedly varying individual life values for the parameters B and c. To the extent that it was assumed (in deriving the group mortality functions) that these two parameters are the same for all lives in the group, an accurate fit of the data would be impossible.

(3) The actual method of preparation of the tables[5] involved adaptations and projections for lives age 5 and under, and for lives above age 94, and, therefore, at these ages the presumption that the mortality rates shown in the original tables are more satisfactory than the group mortality rates obtained on the basis of the functions derived in this paper is weakened.

Values used for the parameters are:

$$A = 1.5194 \times 10^{-4}$$
$$B = 1.9722 \times 10^{-4}$$
$$c = 1.08388$$
$$r = 4.0802 \times 10^{-3}$$
$$a = 6.1500 \times 10^{-6}$$

[5] Greville, T. N. E., *United States Life Tables and Actuarial Tables 1939-1941*, 1946.

For T_0 (m) the mean becomes:

$$\frac{r}{a}_{\cdot} = 663$$

the variance:

$$\frac{r}{a^2}_{\cdot} = 108 \times 10^6$$

and the standard deviation:

$$\frac{\sqrt{r}}{a}_{\cdot} = 104 \times 10^2$$

μ_0 has a range from 3.4916×10^{-4} to ∞, with a majority of cases at and near the lower value.

None of these values at age zero should be considered accurate. However, they do illustrate the tremendous dispersion of the initial "incapacity to sustain life" with respect to individual lives. With such tremendous dispersion the group (or average) force of mortality at birth is seen to have little significance. Actually, the most significant value at birth is the minimum value of μ_0 (the mode). This is the force of mortality with respect to healthy infants, and it is these children who can best be expected to survive the early days, weeks and years of life.

6. *Suggested Further Development*

Although the primary purpose of this paper has been to provide a more adequate analytical approach to infant and juvenile mortality, it will be recognized that the concepts of "individual ability to sustain life" and "force of selection" have application at all ages. The further development of these concepts seems to hold more reward for the Life Actuary.

For example, the "individual ability to sustain life" (or its converse, the "predisposition to death") surely does not remain constant for a particular individual throughout life. An "incubator baby" or Siamese twin who survives the early period of infancy certainly does not retain the same "predisposition to death" at age 1 or age 2, that he had at the moment of birth. Even mature individuals will experience physical or environmental changes that increase or diminish the individual ability to sustain life. An asthmatic or tubercular patient can improve his chances of survival by removal to a dry or arid climate. An overweight person can recover normal life expectancy by reducing his weight to normal or slightly below. The reversal of these situations should result in decreased life expectancy.

The use of "select" mortality tables implies recognition of the "force of selection". The rapidity with which "select" mortality rates approach "ultimate" mortality rates is an inverse measure of the effectiveness of the selection. The degree with which this selection is exercised in the underwriting of life insurance is a measure of the recognition that there is dispersion in the individual rates of mortality about the group average.

Further research should develop at least the following:

(1) The manner in which individual "predisposition to death" varies with age,

(2) The extent to which there may be variation of the Gompertz parameters (B and c) among individual lives,

(3) Analytical expressions for, and measures of the effectiveness of selection (underwriting) by life insurers.

THE NEGATIVE BINOMIAL APPLIED TO THE CANADIAN MERIT RATING PLAN FOR INDIVIDUAL AUTOMOBILE RISKS

BY

CHARLES C. HEWITT, JR.

1. Summary of Current Theoretical Developments

Dropkin[1] has shown that an excellent fit of actual automobile accident frequencies is obtained by the use of the negative binomial distribution.

The negative binomial distribution is justified on the assumption that, for a particular mean accident frequency, the Poisson distribution:

$$P(x; mt) = \frac{(mt)^x e^{-mt}}{x!} \tag{1.1}$$

will hold. In the above expression m represents the mean accident frequency for a particular unit period of time (normally it will be assumed that *one year* is the unit of time), and t represents the number of units of time exposed (years). The mean of this Poisson distribution is:

$$E(x; mt) = mt \tag{1.1a}$$

and the variance of this Poisson distribution is:

$$\sigma^2(x; mt) = mt \tag{1.1b}$$

A significant step in Dropkin's approach is the assumption that the mean accident frequency varies among drivers (or cars) and that this variation can be expressed by a Pearson Type III curve of the form:

$$T(m) = \frac{a^r}{\Gamma(r)} m^{r-1} e^{-am} \tag{1.2}$$

(a and r positive)

The mean of this frequency distribution is:

$$E(m) = \frac{r}{a} \tag{1.2a}$$

and the variance of this frequency distribution is:

$$\sigma^2(m) = \frac{r}{a^2} \tag{1.2b}$$

Dropkin shows that if the mean individual accident frequency m is a continuous random variable with a range from 0 to ∞ and with a frequency distribution $T(m)$, then the group probability of exactly x accidents during a unit time interval may be obtained by:

[1] Dropkin, Lester, *Some Considerations on Automobile Rating Systems Utilizing Individual Driving Records*, CAS XLVI, p. 165.

$$N(x;t)=\int_0^\infty P(x;mt)\cdot T(m)\,dm \tag{1.3}$$

This expression takes the form of the negative binomial:

$$(t=1)\qquad N(x;1)=\left(\frac{a}{a+1}\right)^r\binom{-r}{x}\left(\frac{-1}{a+1}\right)^x \tag{1.4}$$

or more generally for a period of time t:

$$N(x;t)=\left(\frac{a}{a+t}\right)^r\binom{-r}{x}\left(\frac{-t}{a+t}\right)^x \tag{1.5}$$

The mean[2] of this latter distribution is:

$$E(x;t)=\frac{r}{a}t \tag{1.5a}$$

and the variance:[2]

$$\sigma^2(x;t)=\frac{r}{a}\frac{a+t}{a}t \tag{1.5b}$$

It is important to remember that, although the mean accident frequencies, m, (for unit time interval) may vary as among individuals, it is assumed that m remains constant over the period of time under consideration for a particular individual.

Bailey and Simon[3] have introduced the concept that the occurrence or non-occurrence of accidents during a particular period of time creates different groupings of individuals by inherent hazard, where the basis of grouping is the individual driving record during the period of time under consideration. These groupings based on driving record have frequency distributions of the group members by inherent hazard, which frequency distributions differ as among groupings and as against the original frequency distribution by inherent hazard of all drivers. This process of creating groupings based upon driving record is in effect a process of selection and is completely random.

It is quite possible that an inherently good risk through bad luck may find himself in a class with risks who have had one or more accidents during the time period under review; on the other hand, an inherently bad risk may find himself in a classification with risks who have had no accidents during a particular period of time. However, in the long run the process of selection on the basis of driving record will result in a greater frequency of good risks in the class with no accidents and a relatively greater frequency of poor risks in a class with one or more accidents.

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[2]For further characteristics of the negative binomial distribution see Simon, L. J., *The Negative Binomial and Poisson Distributions Compared,* CAS XLVII, p. 20.

[3]Bailey, R. A. and Simon, L. J., *An Actuarial Note on the Credibility of Experience of a Single Private Passenger Car,* Appendix 1, CAS XLVI, p. 164.

In an actuarial note currently being presented to the Society, Dropkin[4] shows analytically how this process of selection works and develops an expression for the frequency distribution by inherent hazard in a particular grouping as well as an expression for forecasting claim frequency for this grouping.

Using functions contained in the previous section, it can be shown that for the group of risks who have incurred exactly c accidents in the last s years, the distribution function by inherent hazard, m, for this particular group is given by:

$$T(m/c, s) = \frac{(a+s)^{r+c}}{\Gamma(r+c)} \, m^{r+c-1} e^{-(a+s)m} \tag{1.6}$$

In this system of notation the expression (c, s) will be assumed to stand for the grouping of risks who have had exactly c accidents in the last s years. The mean of this frequency distribution is given by:

$$E(m/c, s) = \frac{r+c}{a+s} \tag{1.6a}$$

and the variance is:

$$\sigma^2(m/c, s) = \frac{r+c}{(a+s)^2} \tag{1.6b}$$

From (1.6), (1.6a) and (1.6b) it is evident that $T(m/c, s)$ is, itself, a Pearson Type III distribution of the form contained in (1.2) where r is replaced by $r+c$ and a is replaced by $a+s$. Furthermore each grouping, (c, s), contains (from (1.5)):

$$N(c; s) = \left(\frac{a}{a+s}\right)^r \binom{-r}{c} \left(\frac{-s}{a+s}\right)^c \tag{1.6c}$$

risks from the initial population.

With respect to future (or forward) accident frequency, the probability of exactly x accidents in a future period of time t is given by a negative binomial expression:

$$N(x; t/c; s) = \left(\frac{a+s}{a+s+t}\right)^{r+c} \binom{-(r+c)}{x} \left(\frac{-t}{a+s+t}\right)^x \tag{1.7}$$

The mean number of accidents during the period of time t will be:

$$E(x; t/c; s) = \frac{r+c}{a+s} t \tag{1.7a}$$

and the variance:

$$\sigma^2(x; t/c; s) = \frac{r+c}{a+s} \cdot \frac{a+s+t}{a+s} t \tag{1.7b}$$

[4]Dropkin, Lester, *Automobile Merit Rating and Inverse Probabilities*. CAS XLVII, p. 37.

The expressions given in (1.6a) and (1.7a) are remarkably simple expressions for the mean forward accident frequency of a grouping of risks who have had exactly c accidents in the last s years. From the mean of all risks (1.2a):

$$E(m)=\frac{r}{a}$$

it is possible to develop an expression for an experience modification on the basis of driving record. This experience modification would have the form:

$$\overline{Mod}_{c,s}=\frac{\frac{r+c}{a+s}}{\frac{r}{a}}=\frac{a(r+c)}{r(a+s)} \tag{1.8}$$

and in the special case where the risk is accident-free during the time interval s, this, of course, indicates the "no claim bonus":[5]

$$1-\frac{ar}{r(a+s)}=\frac{s}{a+s} \tag{1.8a}$$

Of course, such modification is based entirely on accident frequency and ignores the question of accident severity.

Parenthetically it might be noted here that data with respect to accident severity is rather limited. The Canadian information that is available indicates that except for those risks who have had one or more accidents in the past year, there is little or no variation in severity on the basis of driving record. For those risks who have had an accident in the past year, Canadian data indicates that accident severity is approximately 10% greater than for risks in the various accident-free classifications.

2. *Analytical Expressions for Canadian Merit Rating Classes*

The expressions developed in the previous section make is possible to forecast forward accident frequency for any group of risks where the accident history is known. However, the Canadian system of classifying by accident record does not always permit application of these formulae. The Canadian merit rating classes are as follows:

Class A — No claim within past three years (or more)
Class X — No claim within past two years
Class Y — No claim within past year
Class B — One or more claims within past year.

In the case of Class B, the exact value of c is not known since there will undoubtedly be some risks in this class which have had more than one claim in

[5]cf. Written discussion by Bailey, R. A. of *Some Considerations on Automobile Rating Systems Utilizing Individual Driving Records,* Dropkin, L. ibid.

the past year. In the case of Class A, the exact value of s is not known since some risks will have been claim-free for more than the three years necessary to qualify for this class.

In order to permit more complete analysis of the available Canadian data, it is necessary to derive expressions (similar to those developed by Dropkin) with the Canadian merit rating classification system in mind.

The distribution function for risks which have had one or more claims (where the exact number of claims is not stated) during a period of s years may be developed as follows:

$$T(m/>0, s) = \frac{(1-e^{-ms})T(m)}{\int_0^\infty (1-e^{-ms})T(m)\,dm}$$

$$T(m/>0, s) = \frac{1-e^{-ms}}{1-\left(\frac{a}{a+s}\right)^r} T(m) = \frac{T(m)-T(m/0, s)}{1-\left(\frac{a}{a+s}\right)^r} \qquad (2.1)$$

In the above expressions the notation used follows the pattern of the previous analysis. It will be recognized that the number of risks falling into this particular classification is represented by the denominator of (2.1). The mean of this distribution function is given by the expression:

$$E(m/>0, s) = \frac{\frac{r}{a} - \left(\frac{a}{a+s}\right)^r \frac{r}{a+s}}{1-\left(\frac{a}{a+s}\right)^r} \qquad (2.1a)$$

The variance of this distribution function is quite involved and not important to subsequent analysis and is therefore not given here.

The probability of exactly x claims during some period of time t is given by the difference of two negative binomials:

$$\frac{\left(\frac{a}{a+t}\right)^r \binom{-r}{x} \left(\frac{-t}{a+t}\right)^x - \left(\frac{a}{a+s+t}\right)^r \binom{-r}{x} \left(\frac{-t}{a+s+t}\right)^x}{1-\left(\frac{a}{a+s}\right)^r} \qquad (2.2)$$

The mean value of the number of accidents during a future period of time t is:

$$\frac{\frac{r}{a} - \left(\frac{a}{a+s}\right)^r \frac{r}{a+s}}{1-\left(\frac{a}{a+s}\right)^r} t \qquad (2.2a)$$

Again, the variance with respect to this function is quite involved and is not given here. The denominator of expressions contained in (2.2) and (2.2a) represents the number of risks from the original population who fall into the group.

Now, it is necessary to develop a new distribution function consisting of those drivers who have had one or more claims (exact number of claims not stated) in an s year period and who have then been claim-free during a w year period immediately following. This function can be derived and expressed as follows:

$$T(m//0, w/>0, s) = \frac{e^{-mw}T(m/>0, s)}{\int_0^\infty e^{-mw}T(m/>0, s)\,dm}$$

$$T(m//0, w/>0, s) = \frac{[e^{-mw} - e^{-m(s+w)}]T(m)}{\left(\frac{a}{a+w}\right)^r - \left(\frac{a}{a+s+w}\right)^r} \qquad (2.3)$$

The notation $(0, w/>0, s)$ is intended to indicate that the risks in this particular group have had one or more claims in a time interval s and then have experienced no claims in the immediately subsequent time interval w.
The mean value of this frequency distribution is:

$$\frac{\left(\frac{a}{a+w}\right)^r \frac{r}{a+w} - \left(\frac{a}{a+s+w}\right)^r \frac{r}{a+s+w}}{\left(\frac{a}{a+w}\right)^r - \left(\frac{a}{a+s+w}\right)^r} \qquad (2.3a)$$

The probability that the risks in this group will have exactly x accidents in a period of t years in the future is given by the difference of two negative binomials:

$$\frac{\left(\frac{a}{a+w+t}\right)^r \binom{-r}{x} \left(\frac{-t}{a+w+t}\right)^x - \left(\frac{a}{a+s+w+t}\right)^r \binom{-r}{x} \left(\frac{-t}{a+s+w+t}\right.}{\left(\frac{a}{a+w}\right)^r - \left(\frac{a}{a+s+w}\right)^r} \qquad ($$

The mean value of this function is:

$$\frac{\left(\frac{a}{a+w}\right)^r \frac{r}{a+w} - \left(\frac{a}{a+s+w}\right)^r \frac{r}{a+s+w}}{\left(\frac{a}{a+w}\right)^r - \left(\frac{a}{a+s+w}\right)^r} t \qquad (2.4a)$$

In the two expressions immediately above, the number of risks out of the

original population who fall into this particular group is expressed by the denominator:

$$\left(\frac{a}{a+w}\right)^r - \left(\frac{a}{a+s+w}\right)^r \tag{2.4b}$$

It must be remembered that this expression does *not* hold for the group of risks who have had *no* claims in the period of time $s+w$. For this claim-free group, the number of risks is given by the expression:

$$N(0; s+w) = \left(\frac{a}{a+s+w}\right)^r \tag{2.4c}$$

In examining the results during an n-year period ($n=s+w$), the number of risks who have had one or more claims, but have then been claim-free for the most recent w (or more) years is [from (2.4b)]:

$$\left(\frac{a}{a+w}\right)^r - \left(\frac{a}{a+n}\right)^r \tag{2.4d}$$

Similarly the number of risks who have had one or more claims, but have then been claim-free for the most recent $w+1 (<n)$ years (or more) is:

$$\left(\frac{a}{a+w+1}\right)^r - \left(\frac{a}{a+n}\right)^r$$

Therefore the number of risks who have had one or more claims, but have then been claim-free for *exactly* the most recent w $(<n)$ years is:

$$\left(\frac{a}{a+w}\right)^r - \left(\frac{a}{a+w+1}\right)^r \tag{2.5}$$

The average forward probability or claim frequency of such risks is given by the expression:

$$\frac{\left(\frac{a}{a+w}\right)^r \frac{r}{a+w} - \left(\frac{a}{a+w+1}\right) \frac{r}{a+w+1}}{\left(\frac{a}{a+w}\right)^r - \left(\frac{a}{a+w+1}\right)^r} t \tag{2.5a}$$

The weighted average forward probability as represented by the product of the number of risks in each such group times the average forward probability or claim frequency for the group is given by the following expression:

$$\left[\left(\frac{a}{a+w}\right)^r \frac{r}{a+w} - \left(\frac{a}{a+w+1}\right)^r \frac{r}{a+w+1}\right] t \tag{2.5b}$$

TABLE 1

ANALYTICAL EXPRESSIONS FOR CANADIAN PRIVATE PASSENGER AUTOMOBILE DATA

Merit Rating Class	Number of Risks	Average Forward Claim Frequency	Weighted Average Forward Claim Frequency (Number of Risks Times Forward Claim Frequency)
A	$\left(\frac{a}{a+3}\right)^r$	$\frac{r}{a+3}$	$\left(\frac{a}{a+3}\right)^r \frac{r}{a+3}$
X	$\left(\frac{a}{a+2}\right)^r - \left(\frac{a}{a+3}\right)^r$	$\frac{\left(\frac{a}{a+2}\right)^r \frac{r}{a+2} - \left(\frac{a}{a+3}\right)^r \frac{r}{a+3}}{\left(\frac{a}{a+2}\right)^r - \left(\frac{a}{a+3}\right)^r}$	$\left(\frac{a}{a+2}\right)^r \frac{r}{a+2} - \left(\frac{a}{a+3}\right)^r \frac{r}{a+3}$
Y	$\left(\frac{a}{a+1}\right)^r - \left(\frac{a}{a+2}\right)^r$	$\frac{\left(\frac{a}{a+1}\right)^r \frac{r}{a+1} - \left(\frac{a}{a+2}\right)^r \frac{r}{a+2}}{\left(\frac{a}{a+1}\right)^r - \left(\frac{a}{a+2}\right)^r}$	$\left(\frac{a}{a+1}\right)^r \frac{r}{a+1} - \left(\frac{a}{a+2}\right)^r \frac{r}{a+2}$
B	$1-\left(\frac{a}{a+1}\right)^r$	$\frac{\frac{r}{a} - \left(\frac{a}{a+1}\right)^r \frac{r}{a+1}}{1-\left(\frac{a}{a+1}\right)^r}$	$\frac{r}{a} - \left(\frac{a}{a+1}\right)^r \frac{r}{a+1}$
Total	1	$\left[\frac{r}{a}\right]$	$\frac{r}{a}$
A+X	$\left(\frac{a}{a+2}\right)^r$	$\left[\frac{r}{a+2}\right]$	$\left(\frac{a}{a+2}\right)^r \frac{r}{a+2}$
A+X+Y	$\left(\frac{a}{a+1}\right)^r$	$\left[\frac{r}{a+1}\right]$	$\left(\frac{a}{a+1}\right)^r \frac{r}{a+1}$

All that now remains is a summation from a particular value of w to n ($>w$) in order to arrive at expressions for risk groups which have been accident-free for a period of w years or more *including* the group which has been claim-free for the entire n-year period.

These expressions are as follows:

the number of risks who have been accident-free for w years or more is:

$$\left(\frac{a}{a+w}\right)^r \tag{2.6}$$

the average forward claim frequency of risks in this group will be:

$$\frac{r}{a+w} \tag{2.6a}$$

and finally the average weighted forward claim frequency (product of the number of risks times the forward claim frequency) will be

$$\left(\frac{a}{a+w}\right)^r \frac{r}{a+w} \tag{2.6b}$$

This makes it possible to produce analytical expressions for the various Canadian merit rating classes. These are set forth in tabular form in Table 1 opposite.

3. Test of Analytical Expressions Against Canadian Data

It is possible to make a test of the above expressions against actual Canadian data bearing in mind the method in which risks are permitted to enter a particular classification.[6]

The parameters r and a can be determined by solving the simultaneous equations:

$$\frac{r}{a} = \text{forward claim frequency (All classes)}$$

and

$$\frac{a+3}{a} = \frac{\text{forward claim frequency (All classes)}}{\text{forward claim frequency (Class A)}}$$

for the Canadian data for policy years 1957 and 1958.[7]

The parameters used to determine theoretical claim frequencies become:

Class	r	a
1	2.6047	30.076
2	4.3044	35.733
3	4.1665	29.251
4	4.3859	27.065
5	4.5776	41.751

[6]Wittick, H. E., *The Canadian Merit Rating Plan for Individual Automobile Risks,* CAS XLV, p. 214.

[7]Bailey, R. A., and Simon, L. J., ibid, Table 1.

The test of the theoretical expressions against the actual claim frequencies produces the results given in Table 2 immediately below:

TABLE 2

CANADA (EXCLUDING SASKATCHEWAN)
POLICY YEARS 1957 and 1958 (AS OF JUNE 30, 1959)
PRIVATE PASSENGER AUTOMOBILE LIABILITY - NON-FARMERS

Merit Rating Class	Private Passenger Class 1	Class 2	Class 3	Class 4	Class 5
	Theoretical Claim Frequencies				
A	.0787	.1111	.1292	.1459	.1023
X	.1107	.1388	.1629	.1823	.1261
Y	.1142	.1425	.1681	.1887	.1290
B	.1180	.1465	.1738	.1955	.1320
Total	.0866	.1205	.1424	.1621	.1096
A + X	.0812	.1141	.1333	.1509	.1046
A + X + Y	.0838	.1172	.1377	.1563	.1071
	Actual Claim Frequencies				
A	.0787	.1111	.1292	.1459	.1023
X	.1055	.1384	.1698	.1725	.1206
Y	.1183	.1470	.1741	.1716	.1259
B	.1377	.1591	.2008	.2000	.1501
Total	.0866	.1205	.1424	.1621	.1096
A + X	.0800	.1126	.1316	.1486	.1034
A + X + Y	.0820	.1148	.1347	.1511	.1049

With respect to Classes 1 through 5, the standard deviation of risk frequencies in each class may be calculated:

Class	Standard Deviation
1	.0537
2	.0581
3	.0698
4	.0774
5	.0512
Total	.0611

It is interesting to note, in passing, that Classes 1, 2 and 5 produce a standard deviation less than the standard deviation for the entire group of risks while Classes 3 and 4 produce a standard deviation greater than the standard deviation for all risks. However, it should be noted that the claim frequency in Classes 3 and 4 is relatively high. Using a measure of relative dispersion (standard deviation divided by the mean), all of the Classes 1 through 5 show a smaller relative dispersion than the entire group of risks.

It is recognized that the poorest fit of the theoretical expressions for forward claim frequency occurs in merit rating Class B (those risks who have had an accident in the most recent year). Further analysis of the data in this class is certainly warranted. It is possible that part of the difference between theoretical and actual frequencies in this merit rating class is explainable by the inclusion within the class of vehicles where the operator has had a conviction within the past year.

MULTIPLE COVERAGE EXPERIENCE RATING PLAN

BY

ELDON J. KLAASSEN

Background

With the exception of Workmen's Compensation, experience rating plans for casualty lines have not been materially altered for many years despite radical changes in the business to which they have been applied. The pertinent changes which suggest a re-evaluation of our experience rating plans are (1) an ever increasing settlement cost of claims, (2) intensifying competition, (3) a narrowing of profit margins, (4) increasing use of EDP machines, and (5) the trend toward packaging of casualty coverages in a single policy or a limited number of separate policies.

Apropos the increasing cost of claim settlement, we note that virtually all current experience rating plans provide a loss limitation per accident, which, in some cases reaches as low as $200. Yet these plans provide no compensating insurance charge for that portion of the losses which is discarded. While the loss of premium income resulting from this consistent bias may not have been severe at the time these plans were conceived, loss levels since then have increased under the forces of inflation so that a substantial off-balance exists today.

The problems of increasing competition and inadequate manual rate levels have made it difficult for casualty underwriters to achieve an adequate premium income even with a properly balanced experience rating plan. While the underwriter is the dominant factor in producing a profitable loss level, it requires a very astute underwriter to overcome the effect of a biased experience rating plan when rating small and moderate sized business risks for whom limited quantities of data are available.

With the widespread use of high speed computing machinery, it becomes possible to introduce refinements or complexities in rating plans which formerly could not be accommodated because of the burden of clerical work necessary. Because the application of present rating plans – and the one to be proposed here – can be easily reduced to a standardized worksheet, they are quite susceptible to mechanical processing. The initial modification factor, then, can and should be performed mechanically as a part of the normal premium and claims processing.

Assuming that the inherent hazard of any one coverage is correlated with that of other coverages for a given risk, it should be reasonably possible to combine the experience of all lines for a particular risk when determining a rate modification. Greater responsiveness as well as greater stability can be achieved from the use of a greater quantity of data resulting from the combining of coverages, whether or not the several coverages are written in a single policy. While this procedure creates some problems of allocation of premiums by line, the overall effect should be superior. The allocation of premiums is already distorted by the use of such procedures as composite rating plans and retrospective rating plans.

After some initial thought, it was decided, at this time, at least, to restrict the combination of coverages to auto liability, miscellaneous liability, and auto physical damage. Workmen's compensation was excluded because (1) the current rating plans appear to be operating effectively and have been recently revised, (2) regulatory problems might arise from attempting to include Workmen's Compensation and (3) reporting machinery for the NCCI would be disrupted. The minor lines such as burglary, plate glass, and boiler and machinery were excluded because of lack of data.

The principal objective of any experience rating plan should be to obtain the best estimate of a risk's inherent hazard. Corollary objectives should be (1) a reasonable compromise between stability and responsiveness, (2) an adequate overall rate level, and (3) a high degree of equitableness among experience rated risks.

The achievement of these objectives has been attempted with a model which is quite conventional. In order to produce the best estimate of a risk's inherent hazard, a theoretical derivation of credibility values was carried out. This derivation was taken from Arthur L. Bailey's paper, "Sampling Theory in Casualty Insurance," PCAS Volume XXIX, with minor notational changes. The details of this work are shown in Appendix I. The computation of the moments required for the credibility values necessitated the development of frequency functions of claims by size. The source of these data and the final frequency functions for the three coverages under study are shown in Appendix III. The frequency functions were intentionally distorted by including approximately 50% greater frequency of claims in the over $5000 category than the data indicated. This bias was built into the frequency functions for two reasons both based upon my personal opinion. First, it seems reasonable that the larger risks, e.g., those eligible for experience rating, produce relatively more claims in the over $5000 category than small risks that are manually rated. Both groups of risks were represented in the data. Second, I feel that the growth in claim size has been more marked in recent years among the large claims than among small claims. The data used were drawn from claim settlements in 1954 to 1957. This was projected to 1961 by assuming a growth rate of about 5% per year for all claims. The arbitrarily higher base for claims in excess of $5000 has the effect of using a projection factor greater than 5% for this category of claims.

The distorting effect of large losses has to be minimized in order to obtain any degree of stability in a risk's rate level. It appeared to me that the most appropriate device for solution of this problem was the multi-split concept used in Workmen's Compensation. This concept required some modification in order to accommodate the conventional split between basic limits and increased limits in the liability lines. The first $5000 of each claim was adopted as a basis common to all lines. The choices of starting value and discounting formula were largely arbitrary, but the principles by which I was guided were (1) the discounted losses (which I shall refer to as "primary" hereafter) should exceed 75% of basic limits losses, (2) a single maximum claim should produce a modification of not more than 35%, and (3) less than 10% of the claims by number should be subject to discounting. The final choice of formula was:

$$\text{Primary Loss} = 621.33 \log_e \frac{\text{Actual Loss}}{200}$$

where the starting value is $1000. As an additional modifier, I have chosen to use only the first $25,000 of any claim and the corresponding premium for purposes of experience rating.

The choice of $25,000 was made for two reasons: First, virtually all non-personal risks will carry limits to this extent so that a common denominator was established for all risks. Second, the maximum modification produced by this value in the modification formula is quite conventional at about 35%. While this swing is similar to those produced by current plans, it may be noted that the probability of occurrence of a maximum loss is much smaller than in current plans. For example, a risk with $500 of expected basic limits losses will produce a 30% modification from a single accident of $1900 or more under operation of the New York Automobile Liability Experience Rating Plan. The probability of this occurrence is of the order of .04. Under the proposed plan an individual claim of about $15,000 would be required to produce the same modification. The probability of this event (Auto Liability) is about .0036.

Note that the loss limitations discussed above are per claim whereas the traditional method of loss limitation is made per accident. A loss limitation per accident has the greater appeal from a logical standpoint, but I believe a case can be built for the use of a per claim limitation in the current context. First, we are combining coverages with widely varying propensity toward multi-claim accidents, and each coverage for different risks will comprise a widely varying proportion of the total losses. Second, some current ambiguities are eliminated such as coverages written on an occurrence basis and late reported claims for a multi-claim accident which may not get into the current rating. Third, claims may be somewhat more adaptable to machine processing than accidents. Also, as noted in the preceding paragraph, the probability of a maximum loss is considerably smaller under this proposed scheme than under current plans.

The modification formula which I propose includes a provision for rating excess losses for all sizes of risk. While the practical effect is not substantial, there is an aesthetic value resulting from the elimination of discontinuities at the point where excess rating is first introduced and where the primary credibility becomes 1.00. It also makes possible the use of a uniform formula for all risks, which has some value for purposes of machine application.

Other Ideas

During the course of development of this experience rating plan, a number of other ideas were pursued which proved to be unworkable at present. The field of experience rating is I believe in a rudimentary stage of development and much can be accomplished with time and effort. With the idea that someone might wish to investigate some of these possibilities further, I would like to offer some brief comments on several of these concepts.

First, consideration was given to rating by means of two modifications, one

to measure variation in frequency and the second modification to measure variation in size of loss. The measurement of variation by size of loss itself could be subdivided into primary and excess.

The conventional division of losses into "primary" and "excess" is, in itself, an arbitrary choice. Could improve ratings result from the use of three or more "tiers" of losses? It would, in fact, be conceivable that credibility formulas could be developed on a continuous basis differentiating each infinitesimal portion of each loss.

In the formula for rating excess coverage in my proposed plan and most others the excess modification applies to expected losses already modified by the primary modification. This assumes perfect correlation between primary and excess regardless of inherent hazard. This is certainly not true and the proper handling of this concept would involve a weighting of the primary modification and 1.00. I do not know, however, any empirical means of determining the strength of this relationship with the data presently available.

Another consideration was to use different "K" values and "D" ratios for different territories of the country to reflect differences in average size and/or distribution of claims by size in different areas. To the extent to which we were able to develop this concept, differences by territory were adjudged to be too small to warrant the additional complexity, but this might be worth considering if accurate data by state and territory were available.

It was also considered possible to develop credibility formulas which would produce increasing credibility as the difference between the risk's actual experience and the normal increased.

As a means of measuring a risk's propensity toward large losses more accurately some thought was given to modification of excess coverage on the basis of the average primary loss. There should be some correlation between average size of loss and losses in excess of a fixed value. This idea, like most of the others was abandoned, as too little factual matter was available to make a meaningful study.

Another concept that might be introduced in the rating formula is to determine expected basic limits losses by use of a variable expected loss ratio, the expected loss ratio to be determined from the aggregate experience developed by state or territory during the experience period. An additional modifier would then need to be applied to the current manual rates in order to produce an adequate rate level. This, of course, has some obvious non-statistical implications!

The Plan

As it was finally drawn together, this proposed experience rating plan produces greater stability and less responsiveness than the plans to which we are accustomed. The credibility assigned a risk of any given size is approximately equal to credibility assigned a risk one-half as large by the New York Automobile Liability Experience Rating Plan. Also, this plan makes no provision for schedule rating, which is included in the rating plans currently used in most states. While the underwriting factors generally used in schedule rating undoubtedly have some validity, they can be misused and, in any event, I am

no underwriter and hence would not feel adequate to draw up a schedule of debits or credits. No attempt has been made in this plan to refine its language to conform to any state regulation; it is, rather, presented as a model from which it is expected that considerable modification would be required if any company were to use it.

The determination of the minimum size of risk for eligibility was largely arbitrary. It was, however, necessary to set this minimum sufficiently high so that a meaningful modification would be produced. With the minimum eligibility set at $500 of expected basic limits losses, a credit of about 4% would be produced for clear experience. It was considered desirable to use expected basic limits losses instead of premium for this criterion – as well as in the modification formula – to overcome the problem of combining coverages with different expected loss ratios.

Three completed policy years were selected as the normal experience period. The objective in the selection of this period was to obtain as broad a period as possible which would predict the risk's future inherent hazard accurately. While many current plans allow as much as five years for the experience period, it was my opinion, in present times, with business conditions changing rapidly that any experience incurred five or six years ago would not be indicative of the risk's future experience. Even an individual person has a changing inherent hazard from year to year as recently pointed out by LeRoy Simon and Robert Bailey in their paper, "An Actuarial Note on the Credibility of Experience of a Single Private Passenger Car," PCAS XLVI. This variability as well as changing methods of operation apply to a business risk. Experience incurred during the current policy year was excluded for obvious reasons – loss reserves are highly conjectural and premiums, due to late collections or awaiting audit, can only be estimated.

The concept of Expected Basic Limits Losses as applied to Physical Damage may be controversial. I have suggested that 90% of manual premiums written for ACV is equivalent to manual premiums for the first $5000 of coverage per vehicle. In the aggregate, I believe this is reasonably accurate as projected from the experience of my own company. As applied to individual risks, however, there will be many instances where the risk does not have any vehicle with a value in excess of $5000. The error, should, however, be small because the physical damage coverages are unlikely to compose more than 20-25% of the expected losses of any risk.

The rating procedure has been illustrated by means of a sample worksheet, Appendix IV. The procedure is somewhat similar to the rating of Workmen's Compensation risks. It may be noted that although the formula can be applied manually, it represents a substantial increase in clerical work as compared to current experience rating plans other than Workmen's Compensation. With a machine application, however, the processing of this type of procedure should be only slightly more expensive than the procedures presently used. The rather complex experience modification formula used here is derived from the conventional forms of experience modification formulae as shown in Appendix II. The objective in using this type of formula is to avoid the lengthy tables of credibility values and to make it easier to program for machine processing. It may be noted

that no provision is made for departing from the credibility curves to provide for self-rating at some arbitrary point. As a practical matter, this should not be necessary. The uniform processing of all risks regardless of size is advantageous from a machine viewpoint and any reasonable value at which self-rating could be applied would be sufficiently large so that any underwriter would give it considerable individual attention, anyhow.

Both the expected loss ratios and "D" ratios shown are intended to be illustrative rather than recommended values. The "D" ratios were derived directly from the projected frequency functions shown in Appendix III. That is,

$$D = \frac{621.33 \int_{1000}^{25,000} \log_e \frac{x}{200} f(x)\,dx + 3000 \int_{25,000}^{\infty} f(x)\,dx + \int_{1}^{1000} xf(x)\,dx}{\int_{1}^{5000} xf(x)\,dx + 5000 \int_{5000}^{\infty} f(x)\,dx}$$

The denominator of this expression is probably overstated. I have used, in effect, a limit of $5/\infty$ as an approximation to $5/10$ or basic limits. This produces a "D" ratio which is somewhat conservative. The expected loss ratios shown are accurate only for certain states and certain lines. In order to apply this plan efficiently, it would be desirable to make some compromises so that uniform expected loss ratios may be used countrywide, but considerably more attention would have to be given this problem. It might also be desirable to refine the "D" ratios such as using separate values for each line of miscellaneous liability.

Multiple Coverage Experience Rating Plan

Eligibility—Any risk which develops a total of at least $500 of expected basic limits losses from all lines of auto liability, miscellaneous liability, and auto physical damage to be included for rating shall be eligible for application of this experience rating plan.

Experience Period—The normal experience period shall be the three policy years ending with the last completed policy year. Where, however, the risk has been insured less than three years or experience is not available for three completed policy years, a lesser period may be used subject to a minimum of one completed policy year.

Experience Used—Incurred losses and earned premiums developed by the company from operations of the risk during the experience period in all states and from all lines to be included for rating shall be used. Incurred losses are to include allocated claim expense. Experience of other companies or self-insured experience may be used if available in the form necessary for application of the rating procedure.

Expected Basic Limits Losses—The manual basic limits premium for the experience period multiplied by the expected loss ratio for each line of insurance included for rating is defined as expected basic limits losses. All limits of medical payments, 90% of Ph D written for ACV and the first $5000 of Ph D written for stated amount may be considered basic limits. The expected loss ratio is to be applied to 50% of the manual basic limits premium for elevator liability. Expected loss ratios are .47 for Miscellaneous Liability, .61 for Auto Liability

(Publics & Long Haul), .58 for Auto Liability (all other), .60 for Ph D (Publics & Long Haul), and .55 for Auto Ph D (all other).

Rating Procedure—(1) Determine total ratable losses by recording all incurred claims subject to a limit of $25,000 each. (2) Determine primary losses, Ap, by adding the total of all claims of less than $1000 each to the primary value of all other claims as determined from the Table of Primary Losses. (3) Excess losses, Ae, may be determined by subtracting primary losses from total ratable losses. (4) Determine a composite "D" Ratio by weighting the "D" ratio for each line by the expected basic limits losses for that line. "D" ratios are .759 for Miscellaneous Liability, .802 for Auto Liability, and .809 for Auto Physical Damage. (5) Determine a composite "Y" value by finding the ratio of the sum of total limits expected losses (subject to $25,000 per claim limit) for all lines to the sum of the expected basic limits losses for all lines. (6) Determine expected basic limits losses, E, from the sum of the expected basic limits losses for each line. (7) The experience modification may be found from the following formula, the result of which is to be applied to the manual total limits premium at current rates to determine the renewal premium:

$$M=\frac{(Ap+12{,}000D)(500{,}000Y+DE)+DAe(E+12{,}000)}{DY(E+12{,}000)(E+500{,}000)}$$

TABLE OF PRIMARY LOSSES

Actual Claim		Primary	Actual Claim		Primary
From	To	Value	From	To	Value
$1001	$1016	$1005	$ 3867	$ 3993	$1850
1017	1032	1015	3994	4123	1870
1033	1066	1030	4124	4257	1890
1067	1101	1050	4258	4396	1910
1102	1137	1070	4397	4540	1930
1138	1174	1090	4541	4689	1950
1174	1212	1110	4690	4842	1970
1213	1252	1130	4843	4999	1990
1253	1294	1155	5000	5184	2010
1295	1336	1170	5185	5397	2035
1337	1379	1190	5398	5618	2060
1380	1424	1210	5619	5849	2085
1425	1471	1230	5850	6089	2110
1472	1519	1250	6090	6339	2135
1520	1569	1270	6340	6600	2160
1570	1620	1290	6601	6871	2185
1621	1673	1310	6872	7153	2210
1674	1728	1330	7154	7446	2235
1729	1784	1350	7447	7752	2260
1785	1843	1370	7753	8070	2285
1844	1903	1390	8071	8402	2310
1904	1965	1410	8403	8747	2335
1966	2030	1430	8748	9142	2360
2031	2096	1450	9143	9595	2390
2097	2165	1470	9596	10069	2420
2166	2236	1490	10070	10567	2450
2237	2309	1510	10568	11090	2480
2310	2385	1530	11091	11639	2510
2386	2463	1550	11640	12215	2540
2464	2544	1570	12216	12819	2570
2545	2628	1590	12820	13453	2600
2629	2714	1610	13454	14119	2630
2715	2801	1630	14120	14877	2660
2802	2892	1650	14878	15739	2695
2893	2987	1670	15740	16651	2730
2988	3085	1690	16652	17616	2765
3086	3186	1710	17617	18637	2800
3187	3291	1730	18638	19716	2835
3292	3398	1750	19717	20860	2870
3399	3509	1770	20861	22068	2905
3510	3624	1790	22069	23441	2940
3625	3743	1810	23442	24999	2980
3744	3866	1830	25000 or More		3000

APPENDIX I

Derivation of Credibility Formulas

For a risk with expected losses of E at manual rates during a given experience period, the true inherent hazard of that risk$=E(1+m)$, $m>-1$.

The ratio of inherent hazard to expected hazard, then, $=1+m$.

The ratio of actual losses, A, to expected losses for risks with expected losses of E may be defined as $A/E=R$.

If we plot the frequency of $1+m$ on one axis and the frequency of R on a second axis, we obtain a frequency surface whose marginal distributions $g(1+m)$ and $h(R)$ are skewed normal curves.

The regression line of R on $(1+m)$ is $R=(1+m)$ because for each group of risks with inherent hazard $E(1+m)$, the average actual loss must be $E(1+m)$. Since the usual form for a regression line may be expressed as

$$y_x=\left(r_{xy}\frac{S_y}{S_x}\right)x+\left(V_{1:y}-r_{xy}\cdot\frac{S_y}{S_x}V_{1:x}\right)$$

it is evident that $r_{R(1+m)}\dfrac{S_R}{S_{(1+m)}}=1$

or that $r_{R(1+m)}=\dfrac{S_{(1+m)}}{S_R}$

and $V_{1:R}-r_{R(1+m)}\dfrac{S_R}{S_{(1+m)}}V_{1:(1+m)}=0$

The regression line of $(1+m)$ on R is then

$$(1+m)=r_{R(1+m)}\frac{S_{(1+m)}}{S_R}R+\left(V_{1:(1+m)}-r_{R(1+m)}\frac{S_{(1+m)}}{S_R}V_{1:R}\right)$$

$$=\frac{S^2_{(1+m)}}{S^2_R}R+\left(1-\frac{S^2_{(1+m)}}{S^2_R}\right),\ \text{if we may assume the rate level}$$

to be correct so that $V_{1:(1+m)}=V_{1:R}=1$

$$\text{Then } E(1+m)=\frac{S^2_{(1+m)}}{S^2_R}A+\left(1-\frac{S^2_{(1+m)}}{S^2_R}\right)E$$

$$=ZA+(1-Z)E,\ \text{where } Z=\frac{S^2_{(1+m)}}{S^2_R}$$

$$\text{By defining } K=E\left(\frac{S^2_R}{S^2_{(1+m)}}-1\right),\ Z=\frac{E}{E+K}$$

which is the familiar formula used in determining credibility values. In application to a particular class of insurance, this requires only a determination of S^2_R and $S^2_{(1+m)}$ to determine credibility values.

$$S_R^2 = \frac{S_A^2}{E^2} = \frac{S_A^2}{\bar{n}^2\bar{c}^2}$$

where n is the number of claims variable and c is the size of claim variable. Then the sample variance of A is

$$S_A^2 = \frac{S_{xc}^2}{\bar{x}}\overline{(1+m)}^2 + S_{(1+m)}^2\bar{x}^2\bar{c}^2$$

$$= S_x^2\bar{c}^2 + \frac{S_c^2}{\bar{x}}\bar{x}^2 + S_{(1+m)}^2\bar{x}^2\bar{c}^2, \text{ since } \overline{(1+m)} = 1$$

$= \bar{x}(c^2 + S_c^2) + S_{(1+m)}^2\bar{x}^2\bar{c}^2$, if we assume that x, a purely chance variable defined by $n = x(1+m)$, forms a Poisson Distribution

$$\text{Therefore, } K = E\left(\frac{S_R^2}{S_{(1+m)}^2} - 1\right)$$

$$= \bar{n}\bar{c}\left(\frac{\bar{x}(\bar{c}^2 + S_c^2) + S_{(1+m)}^2\bar{x}^2\bar{c}^2}{\bar{n}^2\bar{c}^2} - 1\right)$$

$$= \frac{\bar{c}^2 + S_c^2}{\bar{c}S_{(1+m)}^2}, \text{ because } \bar{n} = \bar{x}$$

$$= \frac{U_{2:c}}{\bar{c}S_{(1+m)}^2}$$

where $U_{2:c}$ is the second moment about the origin of the distribution of claims by size.

For the purposes of this experience rating plan, $U_{2:c}$ was calculated from the frequency functions shown in Appendix III. Where the functions were not directly integrable, the appropriate integrations were carried out by use of Simpson's rule, using 5 points between c=1,000 and c=5,000, 5 points between c=5,000 and c=10,000, and 7 points between c=10,000 and c=25,000.

No empirical basis is available for computation of $S_{(1+m)}^2$. It is, therefore, necessary to assume an appropriate value. The "K" values shown below have been computed using $S_{(1+m)}^2 = .10$. If we assume, for the moment, that $(1+m)$ is normally distributed, the implication of the choice of $S_{(1+m)}^2 = .10$ is that 50% of all risks have inherent hazard between .79 and 1.21, and that 99% of all risks have inherent hazard between .19 and 1.81. Because the distribution of $(1+m)$ is undoubtedly skewed, the 50% interval is probably being ascribed to a range of about .85 to 1.30 while the 99% range more likely runs from about .40 to 2.50. It may be noted that a computation of $S_{(1+m)}^2$ for New York Workmen's Compensation risks, using their experience modifications as an estimate of $(1+m)$, produced $S_{(1+m)}^2 = .06$ approximately. Because Workmen's Compensation insurance utilizes a more refined classification system than Auto Liability, Miscellaneous Liability, or Physical Damage, we would expect the latter coverages to produce $S_{(1+m)}^2$ greater than that for Workmen's Compensation.

On these bases, K values were computed as follows:

	$Kp^{(1)}$	$Ke^{(2)}$	$Kp^{(3)}$	$Ke^{(3)}$
Auto Liability	9,099	177,786	11,343	581,957
Miscellaneous Liability	10,884	186,304	14,334	463,817
Auto Physical Damage	8,735	152,250	10,801	710,880

(1) in terms of primary losses
(2) in terms of excess losses
(3) in terms of basic limits losses

As working values for application to this Multiple Coverage Experience Rating Plan, I have suggested the use of Kp=12,000 and Ke=500,000.

APPENDIX II

Derivation of Rating Formula

Let the modification of primary losses be:

$$Mp = \frac{Zp\,Ap + DE(1-Zp)}{DE},$$

where Zp = primary credibility
Ap = actual primary losses
E = basic limits expected losses
D = "D" ratio

Let the modification of excess losses be:

$$Me = \frac{Ze\,Ae + (Y-D)E\,Mp(1-Ze)}{(Y-D)E\,Mp}$$

where Ze = excess credibility
Ae = actual excess losses
Y = the composite increased limits factor

Therefore, the total modification would be:

$$Mt = \frac{MpDE + MeMp(Y-D)E}{YE}$$

$$= \frac{Mp(D + MeY - MeD)}{Y}$$

Substituting for Mp and Me in the above equation,

$$Mt = \frac{ZpAp + DE(1-Zp)}{YDE}\left[Y(1-Ze) + ZeD + \frac{ZeAeD}{ZpAp + DE(1-Zp)}\right]$$

Substituting $Zp = \frac{E}{E+Kp}$ and $Ze = \frac{E}{E+Ke}$,

$$Mt = \frac{Ap + DKp}{YD(E + Kp)}\left[\frac{YKe + DE}{E + Ke} + \frac{AeD(E + Kp)}{(E + Ke)(Ap + DKp)}\right]$$

$$= \frac{(Ap + DKp)(YKe + DE) + AeD(E + Kp)}{YD(E + Kp)(E + Ke)}$$

APPENDIX III

Frequency Distributions

Miscellaneous Liability

Interval	Data	Fitted Function	Projected Function
$ 1 – $ 24	.27450	.26517	.20663
25 – 49	.17223	.15740	.13915
50 – 99	.16630	.17223	.16878
100 – 249	.16561	.19070	.21086
250 – 499	.09526	.09415	.11506
500 – 999	.06256	.05626	.07267
1,000 – 1,999	.03375	.03098	.04140
2,000 – 2,999	.01258	.01077	.01467
3,000 – 3,999	.00596	.00551	.00748
4,000 – 4,999	.00316	.00332	.00459
5,000 +	.00809	.01351	.01870

Source of Data: NBCU Special Call – Claims Settled in Calendar Year 1954.

Fitted Function: $f(x) = \frac{68.608}{(x + 95)(x + 46)}$, $x \geqslant 1$

Projected Function: $f(x) = \frac{95.376}{(x + 133)(x + 64)}$, $x \geqslant 1$

Auto Liability

Interval	Data	Fitted Function	Projected Function
$ 1 – $ 24	.18824	.20948	.17678
25 – 49	.20956	.17876	.15689
50 – 99	.22402	.23676	.23085
100 – 249	.21023	.20833	.23840
250 – 499	.08493	.08013	.09405
500 – 999	.04315	.04242	.05032
1,000 – 1,999	.02035	.02184	.02605
2,000 – 2,999	.00711	.00738	.00882
3,000 – 3,999	.00388	.00371	.00444
4,000 – 4,999	.00218	.00223	.00267
5,000 +	.00635	.00896	.01075

Source of Data: NBCU Special Call – Claims Settled in Any 12 Month Period of 1956-57 for BI and Any 3 Month Period of 1956-57 for PD.

Fitted Functions: $f(x) = .0095657 - .000064407x,\ 1 \leqslant x < 100$

$$= \frac{45}{(x+20)^2},\ x \geqslant 100$$

Projected Functions: $f(x) = .0079445 - .000044503x,\ 1 \leqslant x < 120$

$$= \frac{54}{(x+24)^2},\ x \geqslant 120$$

AUTO PHYSICAL DAMAGE

Interval	Data	Fitted Function	Projected Function
$ 1 – $ 99	.62701	.62701	.57970
100 – 499	.26542	.28574	.31226
500 – 999	.05379	.04536	.05657
1,000 – 1,999	.03097	.02189	.02807
2,000 – 2,999	.01153	.00828	.00883
3,000 – 3,999	.00405	.00340	.00420
4,000 – 4,999	.00285	.00194	.00242
5,000+	.00428	.00638	.00795

Source of Data: Continental Casualty Company – Claims Settled in Calendar Year 1957.

Fitted Function: $f(x) = \frac{244.305}{(x+78)^{2.214}},\ x \geqslant 1$

Projected Function: $f(x) = \frac{305.612}{(x+94)^{2.214}},\ x \geqslant 1$

APPENDIX IV

Experience Rating Worksheet - Page 1.

Coverage	Policy Year	Manual Basic Limits Premium	Exp. Loss Ratio	Expected Basic Limits Losses	"D" Ratio	Expected Primary Losses	Incr. Lim. Factor	Total Expected Losses
				E		B		C

D ______ = B/E

Y ______ = C/E

Experience Rating Worksheet - Page 2.

Loss Experience				Rating Procedure		
	Ratable Losses	Policy Year	Primary Losses	(1)	12,000 × D	
				(2)	(1) + Ap	
Total of Claims $1,000 or Less Each				(3)	500,000 × Y	
				(4)	(3) + B	
Individual Claims of More Than $1,000				(5)	E + 12,000	
				(6)	(5) × D	
				(7)	E + 500,000	
				(8)	(7) × Y	
				(9)	(2) × (4)	
				(10)	(6) × Ae	
				(11)	(9) + (10)	
				(12)	(6) × (8)	
				(13)	(11) + (12)	
T		Ap				

Ae [ ] = T - Ap

THE CENSUS METHOD

BY

LAURENCE H. LONGLEY-COOK

A New Approach to the Analysis of Casualty and Property Insurance Statistics for Rate Making

"The old order changeth, yielding place to new."

Introduction

One of the tenets of insurance rate making is that statistics should be developed on the broadest possible base. With simple classification systems and rating plans which were common to all companies, this ensured overall adequacy of the premiums charged and permitted companies to vie one with another to persuade agents to give them a larger share of the better business.

This form of competition is rapidly being replaced by competition at the customer level. Rating plans are devised by individual companies to attract good business by offering lower rates where they can be justified. The development of these new rating plans requires many skills, not the least being the determination of the correct rate levels for new benefits and classification groups. The actuary has had to develop many new techniques and abandon some old tenets to solve these problems. The "broadest possible base" serves no purpose here except as a bench mark. The development of good rate indications from comparatively small bodies of data is a challenging problem. One facet of it is the rapid and convenient analysis of data with numerous classification breakdowns. This brief paper is concerned with this analysis and associated problems.

Present Procedures

The problem of analyzing rate making data must be viewed against the background of recent developments and some short review of the present position seems desirable. In order to clarify our thinking we will consider private passenger automobile insurance written by a company which is a member or subscriber to the National Bureau of Casualty Underwriters. This business provides an excellent example of the problem of numerous classification breakdowns. There are classifications by state, territory within the state, type of automobile, use of automobile, age of automobile, age and sex of driver, type of coverage, liability limits and merit rating class. For very many years such studies for automobile and practically all other classes of casualty insurance were made by what is known as the policy year method. For each classification all policies issued in a particular year, say 1950, were investigated from the date of issue of the policy to the subsequent policy anniversary, the losses arising from accidents occurring in the policy year being compared with the number of cars exposed to risk of loss.

Since for some policies in the 1950 experience the period of investigation will begin on January 1, 1950, and end on January 1, 1951, while for some

other policies it will begin as late as December 31, 1950, and end on December 31, 1951, the 1950 experience cannot be developed until some time in 1952.

In recent years it has been found that the length of time needed to develop loss data and the difficulty in interpreting trends, because a loss occurring in 1951 may belong to either policy year 1950 or 1951, were not acceptable and a different method known as the calendar-accident year method has been generally substituted. Under the calendar-accident year method the losses arising out of accidents occurring in any calendar year are compared with the "earned exposure" in the calendar year. If a policy insuring a single car is effected on August 1, 1960, it will have 5 months exposure to accident, or 5/12 earned exposure in 1960.

Each company reports its writings and exposures quarterly to the National Bureau by classification and term. For every cancelled policy each company must calculate the individual unearned exposure. The method used by the National Bureau proceeds as follows. To calculate the earned exposures for 1960 first take the policies issued for a term of one year and sum the following:

1/8	of	the	policies	issued	in	the	1st	quarter	of	1959
3/8	"	"	"	"	"	"	2nd	"	"	1959
5/8	"	"	"	"	"	"	3rd	"	"	1959
7/8	"	"	"	"	"	"	4th	"	"	1959
7/8	"	"	"	"	"	"	1st	"	"	1960
5/8	"	"	"	"	"	"	2nd	"	"	1960
3/8	"	"	"	"	"	"	3rd	"	"	1960
1/8	"	"	"	"	"	"	4th	"	"	1960

Then do similar calculations for 6 months policies and also for policies of other terms.

Of course the actual calculations can be accomplished more simply on machines, but it should be noted that

1. no allowance is made for uneven distribution of writings over each quarter of a year,
2. cancellations are not handled in a strictly correct manner,
3. companies must calculate individual unearned exposures for canceled policies.

Further, it can be readily appreciated that this is a complicated and expensive procedure for developing earned exposures. The only recent simplification is the exclusion from the reporting instructions of endorsements subsequent to issue on non-audited automobile business. Even with modern electronic data processing equipment the calculation of earned exposures for all the classification breakdowns by this procedure is at least tedious and most likely not practical for an individual company.

The Census Method

The census method uses the calendar-accident year approach but uses a simplified procedure for developing the earned exposure for the calendar

year. Instead of trying to calculate as accurately as possible the earned exposure over the calendar year 1960, we can take a census of the number of automobiles actually insured in the classification on July 1, 1960. Under normal circumstances this will be as accurate a measure of the exposure as the more complicated method just described. However, if greater accuracy is required, we can take the mean of the number of automobiles insured on January 1, 1960, and January 1, 1961, or even use three tabulations of the automobiles insured—or censuses of the in-force as they are better described —one at January 1, 1960, one at July 1, 1960, and one at January 1, 1961. For detailed classifications the greater accuracy obtained by using more than one census is a will-o-the-wisp because the loss data cannot be sufficiently credible. Where total exposure in all classifications is required more than one census may be used or a control maintained with accurately calculated earned premiums. With the census method term does not enter into the development of exposure; canceled policies are simply excluded and endorsements can be ignored.

For a company maintaining an in-force tape or punched card file the method should be considerably cheaper and the classification codes will be really accurate since all, or practically all, the classification coding needed for rate making will be included in the in-force file.[1] Even for a company with more old-fashioned records there should still be appreciable savings.

It may be remarked that the census method has been used for many years for the investigation of the mortality of insured lives and its accuracy has been fully tested in this field.

Census Method Without In-Force File

It is desirable to consider in a little further detail the application of the census method when no detail in-force file is maintained. A company will maintain a file of detail statistical cards (or a corresponding electronic tape). Terminated policies are not excluded from the file which is normally maintained by calendar quarter. When a policy is cancelled the original card is not removed but a cancelled card is added. It is from this file that summary cards are prepared for annual statement and other company records.

Let us suppose we wish to make a detailed study by classifications too numerous to show up on the summary cards. We will further assume that the study is a special one and data for the study have not been accumulated. With the present method of calculating earned exposure and with the census method, we must go to the detail cards and extract all those which were in force (or could have been in force if not cancelled) for the calendar year or years being investigated. Under the present method we must proceed to calculate earned exposures by the method already described. For the census method we need tabulate only the number of cards issued prior to, and terminating after, the date of any census we require, treating cancelled cards as negative items. No term analysis or calendar quarter of issue is required.

[1] The exclusion of impossible codes, common to many electronic programs, only scratches at the surface of inaccurate coding of statistical data; but, with an in-force file used for rating where the coding determines the rate, coding errors are most unlikely.

It is apparent that the census method will be very much less laborious. For an investigation which is made annually both methods are more simple, but the relative advantage of the census method is unchanged.

One difficulty inherent in the present bureau statistical plan for automobile, which can easily be rectified, must be mentioned. At present we record the policy term (by code) and the number of car-months of exposure. Under the census method we do not need the number of car-months but the number of cars. To illustrate we must distinguish readily between two cars insured for 6 months and one car insured for a year, each of which have 12 car-months exposure.

Electronic Processing

Note must be taken of the progress certain companies have made in simplifying the development of statistical data. While some companies have been content to take their standard procedures and put them into their electronic program, others have made radical changes. Since electronic programs are generally keyed to a monthly cycle of recording, the preparation of a statistical tape, corresponding to the file of statistical cards previously developed, can be avoided if the portion of each statistical report corresponding to each month's writings (including endorsements and cancellations) is developed each month. This avoids all sorting of the data from the production order in which it is kept: Sorting is an expensive electronic processing procedure. For each report a tape will be maintained showing earned exposures, losses and other information to be shown on the final report. This tape is fed to the memory units of the machine once a month and all additions and subtractions to the report resulting from the month's operations incorporated. The data are then returned to tape form. In the method the actual earned exposures for the current and subsequent calendar years will be recorded so that no record of unearned premiums or unearned exposures has to be kept. For some reports the number of classifications presents a capacity problem. This can be overcome by recording about 90% of the business in the main classification and taking the remainder to an exception tape for further analysis.[2]

These procedures are directly applicable to the census method. The development of the contributions to a census at a particular date from each month's writings and cancellations is more simple than the development of the contributions to the earned exposure for a particular year.

Endorsements

At a recent meeting of the Society, Mr. Stellwagen stressed the great expense at present involved in coding endorsements. Mr. Harmon Barber has suggested that a great amount of unnecessary coding, key punching and tabulation is wasted on endorsements which can easily be avoided. The National Bureau statistical plan for automobile business does not require the reporting of endorsements other than those at date of issue on non-audited

[2] For instance, since Philadelphia producers will normally come in sequence, territories close to Philadelphia only need be considered in running the portion of the business from Philadelphia agencies.

business and the census method does not require endorsements for its calculation of exposures. However, a large amount of coding, key punching and record keeping is still generally required to include the small monetary sums involved in endorsements in company accounts. It is proposed that for practically all lines of business all monetary endorsements, other than endorsements at issue and audit premiums, should be coded only to state and major lines of business and excluded for all classification studies. All premiums on such endorsements could be considered as earned when written so they will not be involved in unearned premium reserve calculations. Someone will raise the problem of premium tax for townships and municipalities, but since endorsements involving reductions in premium are probably about as common as those involving increases in premium, I believe our accountants could sell the idea of coding endorsements to state only.

Application to Homeowners

Complex detailed classification systems occur only in lines with a large number of units insured and we need consider here only the application to the Homeowners policy, although the census method is applicable generally. Under present statistical procedures the calculation of earned premiums for all classification breakdowns is intolerably involved.

The census method would allow a proper pure premium approach to Homeowners rate making if the amounts of dwelling insurance were recorded, either exactly or in fairly narrow intervals. At present the standard statistical plan provides only for broad group classification of amounts of insurance. Many will desire to continue the loss ratio approach however. In applying the census method to calculate earned premiums for this approach a census of premiums in force is required. Here either 3-year or 1-year premiums can be used but not a combination of both without first dividing the 3-year premiums by three or multiplying the 1-year premiums by three. When there is no in-force record, the "original premium", recorded by many companies on their cards for cancelled business, must be used for the census of the premiums in force. Otherwise the method involves no new problems and could readily be used by an individual company and should not be difficult for Bureau operations.

Conclusions

The adoption of the census method for statistical analysis for rate making and research should have the following advantages:

(1) The analysis of data with complex classification systems would be considerably simplified.

(2) The statistical file would no longer be required for companies maintaining a detail in-force file for policy writing and billing.

(3) No calculation of earned exposures or earned premiums by classification would be needed.

(4) The statistical coding and detail key punching of endorsements

could be completely eliminated for practically all lines of business.

(5) For Bureau reports no classification of business by term or calendar quarter would be required. (Term would be required where the loss ratio method is employed.)

(6) Greater accuracy would result where a detail in-force file is used for statistical work.

(7) The consequent reduction in key punching and processing of statistical data should save the insurance industry many millions of dollars.

COVERAGE AND UNDERWRITING ASPECTS OF BURGLARY INSURANCE

BY

WALKER S. RICHARDSON AND RICHARD J. WOLFRUM

Like Gaul, burglary insurance is susceptible to division into three parts: the first, coverage for banks, the second, coverage for individuals, and the third, coverage for non-banking commercial enterprises. This paper will be confined to a discussion of the coverage, rate structure, and underwriting considerations involved in providing coverage for non-banking commercial enterprises, hereinafter referred to as commercial burglary. The omission of banks and individuals does not mean that these areas of coverage are unimportant, but it does recognize that, today, the major portion of bank coverage is written as part of an indivisible package which fidelity coverage controls, and that coverage for individuals is moving rapidly toward inland marine and multiple peril packages. Both of these latter subdivisions, bank and individual coverage, played a major role in the past, but this role has lessened substantially in the last 10 to 20 years, and there is no reason to assume that the trend will change.

That this is the first paper presented to the Casualty Actuarial Society on this subject is not surprising. Burglary, traditionally, is a casualty line and, as such, is a misfit. Except in three instances, it is two party property coverage, not third party liability coverage as is the preponderance of the casualty market. For this reason, it has been placed in a corner with glass insurance and other miscellaneous property coverages and is handled by people known as burglary and glass specialists. This is true even at the Rating Bureau level. As a result, people involved with workmen's compensation and liability insurance, the majority of casualty people, have seldom been exposed to burglary insurance because the relatively small premium volume supported relatively few specialists; moreover, because this small group did have special knowledge, a mysterious area has been created. In truth, burglary is not complicated, but it is different. Viewed by the uninitiated, it could be forbidding. We hope the following discussion will lessen the supposed mystery.

COVERAGE

The first necessary step is to understand the types of coverage provided by the various standard policy forms. Coverage is fairly standard throughout the industry with variations involving minor areas only. There are two major types of exposure, money and merchandise. Coverage for money in some cases includes coverage for other property (i.e. merchandise), but such coverage is incidental except for jewelers and furriers. The purchase of money coverages for the specific purpose of insuring property other than money is rare. Let us first consider money exposure.

A money loss can occur inside the insured's premises or it can occur outside. It can occur while the premises are open or when they are closed. If the premises are closed, the money is probably kept in a safe. In addition to

the variation in location and time, the loss can be perpetrated in a number of ways: by breaking into the building or into the safe (burglary), by forcibly taking the property from its custodian (robbery), by sneaking the property away from the owner without the owner being aware of the act (theft), or by some other means which does not involve force and is not a voluntary surrender (disappearance). By mixing location, time, and manner of occurrence, we obtain all standard coverage.

The other major exposure, merchandise, is subject to the same location, time, and manner of occurrence variations as are applicable to money exposure. However, merchandise outside the insured's premises is ordinarily considered an inland marine exposure, and, to date, burglary underwriters have generally refused to offer insurance for the disappearance peril. Standard coverage for merchandise exposure is therefore limited to the insured's premises and to burglary, robbery, and theft perils. Mixing the three variables, location, time, and manner of occurrence, again produces standard coverage. The various possible combinations with accepted policy names are summarized as Exhibit I.

A student of coverage might well view the foregoing and ask, "If standard coverage is a combination of three variables, and if there are two types of exposure, why have I studied a dozen policies? Why not two or three?"

It is true that three basic policies would suffice; the Mercantile Open Stock Policy with the Theft Endorsement would cover the merchandise exposure and the Money and Securities Broad Form Policy, or the Mercantile Robbery and Safe Burglary Policy would cover the money exposure. Unfortunately, many special combination burglary policies have been created. Policies including fidelity and forgery are common; fringe coverages are sometimes included; coverages and limits of liability are packaged for particular types of risks. Competition has forced industry acceptance of some of these specialities, but if the student cuts each policy back to its basic concepts, he will find that the burglary coverage falls into the pockets indicated in Exhibit I. Exhibit II demonstrates this approach for all standard commercial burglary policies.

In 1956 the Mutual Insurance Rating Bureau promulgated a policy which offers, on an optional basis, all major commercial burglary coverage. This policy, with such additional fidelity and forgery options as individual companies chose to make, effected a standardization for Mutual Bureau members and provided a means of eliminating six policies. About the same time, the National Bureau of Casualty Underwriters promulgated a Special Coverage Policy and coverage forms enabling its members to eliminate a number of minor policies as well as the recognized commercial buglary policies in favor of a single jacket. This program was also adopted by the Mutual Bureau. This type of action does much to remove the mystery and confusion of buglary coverage, as policies are eliminated and coverages are consolidated in a simple, logical fashion. The industry will benefit most from public acceptance, which will come only with understanding. We suggest making the single, simple approach mandatory rather than optional as at present.

It is evident from Exhibit II that the number of policies results from com-

binations of the time, place, and manner of occurrence variables shown in Exhibit I. Policy nomenclature varies by company. Some companies have additional policy or coverage combinations, arranged to serve a particular need. The only limit on the number of different policies is expense and the administrative difficulty in policy issuing units. This alone has started a movement for simplification and consolidation that may in time substantially reduce the number of policies offered.

Any attempt to codify insurance coverage, as in Exhibit II, necessarily overlooks minor variations and concentrates on major intent. It would seem in order to point out some peculiarities of burglary insurance as follows:

1) Most burglary policies contain broad ownership provisions extending coverage to property held by the insured in any capacity, whether or not he is legally liable for its loss.
2) Although fidelity may be included as a separate insuring agreement in some combinations of coverage, a burglary insuring agreement will generally exclude coverage for acts committed by employees unless such act is one of force (robbery or forced entry).
3) Policy provisions generally reduce insurance if agreed protection does not exist at the time a loss occurs. The Mercantile Open Stock Policy and Safe Burglary Policy may void insurance in some cases.
4) Typical exclusions eliminate coverage (1) unless a loss can be documented, (2) for war risk, (3) for the intrinsic value of printed material and (4) for property of the United States Government.

RATES AND RATING

Most commercial burglary policies produce small premiums averaging less than $100 annually. With the exception of the low limit of liability packages, each coverage must be separately rated. Since normal exposure includes money inside the premises, money outside the premises, and merchandise, there are three separate rating operations. Since classification of business and territorial assignment vary by coverage, at least two facts must be determined for each coverage. Let any of the sixteen possible protective devices exist, and you have a Chinese puzzle. Expand this for a multi-location risk, and your computation takes on the appearance of the application of the Dean Analytical Fire Rating System for a multi-story, multi-occupant building. The possible rate variations are set forth in Exhibit III.

Jurisdiction over burglary rates and forms is exercised by the National Bureau of Casualty Underwriters and by the Mutual Insurance Rating Bureau for their respective memberships for most states. The two bureaus determine rate levels and differentials for the various states. A number of companies act independently for these lines of insurance but their rate levels and rating procedures generally follow the bureau's programs. Bureau rate-making generally proceeds as follows:

Territorial schedules are set up, much in the manner of the National Automobile Underwriters Association. This means a $26 schedule, a $28 schedule, and so forth, with no thought as to which state belongs to a particular sched-

ule. Differentials between schedules are rounded to quarter dollars and are generally less than 120% of the preceding lower schedule. Areas are assigned to schedules according to past experience (with adjustment for credibility), and if a large territory (premiumwise) indicates need for a special schedule, one is set up. This operation is performed for each sub-line of coverage. Rate revisions are made at irregular intervals and more than one year of experience is used.

Although this procedure produces right answers for existing geographical divisions (ones for which statistics are obtained), it does not permit refinement or subdivision of existing territories according to the actual experience of the area. This is questionable if one considers the size of territory involved. Massachusetts is divided into 4 parts: Norfolk, Suffolk, and Middlesex counties, and remainder of state. New York is divided into 11 parts, 7 of which are greater New York City, the others being Buffalo, Rochester, Syracuse, and remainder of state. Twenty of the 50 states have only one territory, and the remaining 30 average only 4.1 territories per state or 3.2 territories if New York, Texas and Virginia are excluded. A city like Worcester, Massachusetts, with a population of 200,000 carries the same rate as isolated Nantucket Island. Albany and Binghamton have the same rates as an Adirondack hamlet. Middlesex County, Massachusetts, includes densely populated Cambridge on the fringe of Boston and Tyngsboro on the New Hampshire Border.

Such territorial problems as outlined above exist in all types of insurance. In a low credibility line, the cure could be worse than the disease. Some will argue that no cure is necessary because the volume in the affected areas is so small. However, if the possible inequity was removed, volume might increase appreciably. It is true that other rate variables enable the underwriter to offset, to some degree, territorial faults, but it does not seem proper to leave a known variable to underwriting judgment.

Class of Business, Type of Safe, Limit of Liability

In 1948, the Statistical plans coded these items for the applicable lines of coverage. Under Mercantile Open Stock, Safe Burglary, and Interior Robbery, the actual business of the insured was coded rather than the broad rate classification. By 1951, Limit of Liability codes had disappeared, and the business codes were consolidated into rate classification codes. In 1954, classification coding was suspended and the present practice of coding to policy form only was adopted. Since classification assignments have changed since 1954, it appears that judgment has the upper hand in some of the present differentials.

The availability of credible statistical data in a small volume line, such as burglary, will always be a problem if one thinks in terms of dollars of loss and dollars of premium. Units of exposure are as difficult to measure in this line as they are in the fire line. Burglary might well be thought of as a fire line with limited volume. Although it might be considered actuarially crude, a rough check on differentials and relativities could be developed from a simple count of number of insured locations and number of losses. Use of

frequency figures of this type overlooks severity and is deficient in this respect, but it would seem that reasonable credibility could be achieved more easily if the wide variations in size of loss were eliminated. Perhaps the result would be no better than a rough guide, but even this should be of value. Of course, the availability of such figures would depend upon coding of the relativity to be measured.

Judgment has a valuable characteristic; it permits instant action. As supermarkets grew bigger in the early 1950's, the concentration of cash presented one of the most desirable burglary and robbery targets ever known. When a scarce supply of small, heat-treated tools existed during the Korean War, a wave of burglaries occurred. Both of these situations were new and different and required action that would have been impossible or greatly delayed under a pure statistical system. However, the drastic action taken hinged on the alertness of a few underwriters, and once the action was taken and time had passed, the question arose as to whether or not any adjustment should be made. With regard to supermarkets, no, for the cash target still exists; but do the taps and dies belong with sporting goods and pens in the highest classification under normal supply and demand conditions? Is judgment classification a one-way street, easy to go up but difficult to come down?

Protection Variables

These variables include Guards, Alarm Systems, Watchmen, Tear Gas, Division of Insurance, Special Cages, Private Conveyance, Relocking Devices, 24-Hour Operation, Messenger Bags, Inside Routes, and Two People on Duty. No recent statistical data have been obtained in this area. Since the value of protection must be expressed in terms of losses avoided or reduced, statistical data are unobtainable except as a comparison of the experience of protected and unprotected risks with regard to both frequency and severity. To the best of our knowledge, this has not been done in recent years. Percentage credits have been established as an estimate of the degree that exposure is reduced by specific protection. This leads to some difficulty in that the specific definition often prevents adequate protection from qualifying for credit. An ingenious underwriter can, in some cases, reduce this problem by persuading the risk to fulfill the deficit or, where permitted, by allowing schedule credits reflecting his own estimate of the existing protection. Frequency figures comparing protected and unprotected risks would be an aid to the underwriter. This is particularly true with regard to guards, watchmen, and alarm systems, since the other types of protection exist infrequently or are of little importance.

Second Exposure

Burglary risk rating provides for a discount to be applied to the premium computed for all locations except for the primary location. This discount is limited to money coverage and is 10% for inside exposures and 50% for outside exposures. It would appear that the discount recognizes expense savings derived from the issuance of a single policy to a multi-location insured. The saving may be overstated since a manual premium for each location must be computed and then discounted so that there is no saving in the rating

operation. There is saving in the policy issuance, accounting, and general administrative functions. The 50% credit for outside money exposure probably anticipates control of severity of loss by virtue of divided exposure. This makes sense if one sends two messengers to the bank, each carrying half the exposure. However, if one messenger is in Boston and the second in New York, there is no reduction in exposure. Actually, if we pursue this avenue, the way to control money exposure would be to increase rates as limit of liability increases. This is not suggested in jest. Probability of loss and size of exposure appear to have a logarithmic relationship. The present system does not reflect this accelerated increase and requires that the underwriter institute protection designed to produce a one-for-one relationship.

The foregoing discussion is not intended as a criticism of the Bureaus. In 1954, some of the relativities used, other than territorial relativities, dated from 1927. Action since 1954 indicates that a thorough review of all relativities has been made. New classification groups have been set up to permit greater refinement. Existing relationships have been modified. However, the basis for such action must rely largely on judgment, for as a state examiner said in 1954:

> "Although we do not wish to belabor the point, it is evident from a perusal of the latest classified experience countrywide for calendar years 1947 to 1951 that rates generally for the listed manual classes of these coverages are based on a premium volume too small for credibility."

Perhaps an indication of the relative volume of commercial burglary insurance is in order. Exhibit IV shows 1956 calendar year earned premium volume for the National and Mutual Bureaus.

Although Exhibit IV omits data on California and Missouri, they are major states for burglary insurance. Inspection of the list of major states shows a common factor, concentrated centers of population. Concentrated population produces concentrated value, which in turn produces crime. Perhaps a sociologist could reduce the relationship to a formula, but it is obvious, however one explains it, that it results in burglary insurance by creating a need and desire to insure. Exposure alone will produce the need, but concentration of value and the resultant crime frequency are necessary to produce the desire. Each sensational loss produces a buying wave. Little losses produce neighborhood buying. Without a loss, only the large concerns or the more conservative people buy burglary insurance. It is often said that most insurance of this type is purchased by people with a demonstrated need, which results in adverse selection. In this line, lightning strikes twice frequently.

It is difficult to make rates for risks of this type, insurance limited to a large degree to accident prone people. Confuse the issue by recognizing the tremendous potential differences between individual similar exposures, similar in location or similar in type of goods or similar in type of activity, but dissimilar in a hundred other ways.

Burglary rating attempts to solve or rise above the differences in exposure by combining territorial rating, classification rating, and protection rating with schedule rating and experience rating. This means that after all the variables previously described have been used to determine a proper manual

rate, additional modifications reflecting the underwriter's evaluation of the risk and the past experience of the risk may be applied.

The experience rating plans merit comment. In New York the Bureaus use a plan with the following basic elements:

1. Premium subject to experience rating is that premium produced by the first $20,000 of coverage per location.
2. The period used for experience rating is the two years and nine months ending three months prior to the effective date of the rating.
3. Eligibility for experience rating requires $4,000 of premium subject. (Plans used in other states vary and have lower requirements with regard to premium subject).
4. The dollar value of a single loss included in a rating is limited to that dollar amount which produces a 50 point change in modification. Assuming a minimum risk with a two-year nine-month premium of $4,000, we find that the maximum single loss permitted is $3,283. If there were no losses during the period the risk would receive a 29% credit. Inclusion of a maximum single loss would change the risk from a 29% credit to a 22% debit. A larger risk, with $10,000 premium subject would receive a 50% credit with no losses, or a 1% credit with a maximum single loss of $4,651. A jumbo risk with $30,000 premium subject would receive a 75% credit with no losses, a 26% credit if there was a maximum loss of $9,300.

If we consider the middle size risk and the fact that a loss remains in the rating for a three year period, a maximum single loss is returned to the carrier through experience rating approximately dollar for dollar in that with a $4,651 loss, the premium which would have been $5,454 for the following three years becomes $10,800. From the buyer's standpoint, the action of this plan may seem extreme. From an underwriting standpoint, salvage of limited losses through experience rating is justified since the true area of insurance is the area in excess of the dollar loss limitation. The problem stems from the wide variations caused by a single loss. While it is true that the use of experience rating plans encourages risk improvement and prevention or protection, premium swings of the magnitude cited resulting from a single occurrence must provoke serious questions with regard to the value of insurance and the methods of insurance companies in the buyer's mind.

UNDERWRITING

Since rates are not made by development of loss and exposure data, underwriting judgment is of more importance in this line than in other casualty lines. Since most compulsion to insure reflects known loss potential and since a very small portion of the total exposure is insured, the underwriter can perform a real service to his employer by carefully selecting the standard of risk he accepts. There is enough freedom in the rating system to permit correction of minor manual excesses or deficiencies, if they are recognized. There is tremendous potential in the physical improvement of risks or loss prevention available to the underwriter who leaves his desk and looks at

his risks. And then there is the gem which the texts refer to as "moral hazard."

The foregoing are generalities. Here are some specifics that illustrate the thought processes that become second nature to an experienced burglary underwriter.

1. A county may or may not be a homogeneous area. If it is not, a better result should be obtained by concentrating writings in the less congested areas.
2. Some types of retail business operate late at night, and generally these classifications reflect an increased charge for money exposed to robbery. A better selection of risks will result if the underwriter concentrates on those risks with shorter than average business hours for these classifications.
3. Merchandise classifications are generally based on the finished product in its normal form. Variations in form may change the relative hazard to loss. Thus, costume jewelry varies with the raw material, gold, silver, or no precious metal. The brass screws for earrings or the brass nose of an artillery shell contains hours of machine work. The potential loss is not pounds of brass, but brass plus labor and overhead, even though the burglar steals brass.
4. Loss potential of merchandise varies with size of package, value, and market. A shortage (real or artificial) will make an otherwise undesirable product a burglar's dream. A market surplus will render a normally hot item relatively harmless. Therefore, the underwriter should be constantly advised as to the supply of and demand for the product he insures.
5. There is more to manufacturing than the finished product, and more to protect. Raw material, scrap, machinery, and fixtures may generate as much or more exposure than the finished product, and these other items are usually less protected. In costume jewelry, the raw material always represents more concentrated value than the finished product. It is difficult to steal an airplane, but the parts and scrap aluminum are most attractive. Our friends on the outside have flexible desires, hate to leave empty-handed, habitually choose the path of least resistance, and are notorious for discovering new uses for materials.
6. Burglary policies include coverage for damage as a result of burglary. Although a safe may not contain money, this fact is not obvious until the door is open. If the office equipment or fragile merchandise gets in the way, the loss may be large despite a lack of money exposure. Damage potential should not be overlooked.
7. Although policy terms require records to substantiate the value of a loss, a great deal of grief can be avoided if the underwriter determines the existence of good bookkeeping and inventory systems at the outset. This may also bear on the moral desirability of the risk.
8. Physical conditions which permit one loss to occur generally indicate

a weakness which may be corrected. Analysis of the material stolen may indicate something new with regard to value or classification. The underwriter should profit, experience-wise, from every loss and should take such steps as are indicated to prevent repetition. Thus, one may learn that rhinestones are Austrian pearls and come from behind the iron curtain, or that telephone poles are as good as ladders if close to a building. Each physical fault can be corrected.

9. In addition to general area variations within a rating territory, hazard may vary by street and within a building. Certainly it is more difficult to obtain access to the 10th floor of a building than to the first, (provided there are no adjoining roofs). A building set back from the street is more prone to loss than one that fronts on the sidewalk. An alley is a more likely starting point than a well-lighted, heavily-travelled street. If the man next door operates 24 hours a day, his neighbor becomes a better insurance risk. The underwriter should use the burglar's outlook in evaluating the risk at hand.
10. Good underwriting sometimes reduces premium income. If you can convert a cash payroll to check, you remove a target, and the petty cash or day's receipts which remain are probably safer. An alarm system will receive a large protection credit, but many burglars will observe the alarm tape on the window or the sign indicating the presence of the alarm and not attempt entry. A burglar-proof chest may be purchased by a few years premium savings if there is a reasonable amount of cash exposure. Loss prevention in burglary insurance is often immediately rewarded by premium reduction, and if the cost of the protection is not offset by the immediate reduction, the prevention can often be sold on the basis of the effect of experience rating, if a serious loss were to occur.

More underwriting axioms could be recorded if we wished to probe deeper. Actually, such axioms are the application of common sense to a known problem. The rules become apparent as the underwriter gains experience. There is no special thought process or secret knowledge. The underwriter, like all underwriters, attempts to select the better-than-average risk, based largely on physical conditions and guided by a criminal outlook.

Of course, things do not always develop as planned. Since no burglary discussion is complete without a case history, here is a favorite: When requested to insure $50,000 of machined brass fittings in dead storage in an isolated country warehouse, the underwriter required that an employee visit the location each day, that certain arrangements be made for a regular state police check, and that the building be secured in such a way that entry would have to be made through the lighted front. The risk complied; the policy was issued. The underwriter successfully prevented a burglary loss, but unfortunately, the employee whose presence was required was lonely and brought a junkman along for company. The resulting fidelity loss probably exceeded the potential burglary loss. No matter how hard one tries, one cannot escape the problem—"Who watches the watcher?"

STATISTICAL PLAN

We have mentioned a number of areas in which we feel the current statistical plan is deficient, and a number of items which have been recorded in the past but are no longer obtained. The following is a description of the present plan:

Premium Cards

Identification number: This is the actual policy number assigned the risk.

Effective date: A three-digit code showing month and year.

Expiration date: A three-digit code showing month and year.

Policy form: A two-digit code indicating the subline of insurance involved, such as safe burglary or robbery inside the premises.

Classification: A four-digit code not currently used except for supermarkets.

Territory: A four-digit code showing state (standard 2 digit) and subdivision. The maximum use of the subdivision code is 11 for any one state.

Premium: Dollars and cents are recorded.

Loss Cards

Loss cards record the same data as premium cards with the exception of the effective and expiration dates and premium data. The following additional information is obtained:

Year of accident: A one-digit code.

Number of claims: A one-digit code to provide a means of keeping an accurate claim count for losses involving more than one payment.

Losses: Dollars and cents are recorded.

Analysis of loss: A one-digit code not currently used.

The usual provisions are made for company identification, entry date, and credit entries. Although a one card layout is used for premiums or losses, only 50 of the standard 80 IBM columns are necessary for the data required.

Separate cards are required for each policy form or subline for each rating territory. Thus, a multi-state, multi-location risk with several coverages would produce a number of cards but not necessarily a card for each location. If a risk had several locations on Manhattan Island, the premium for all such locations could be reported on a single card (for each policy form) since only one rating territory is involved.

Paid losses are reported as the payments are made. A call for incurred losses is issued once a year, 3 months after the close of the calendar year. Territory for losses would be that territory in which the loss was located or which produced the premium insuring the loss. (This is possibly different from the location of the loss since messengers go outside the premises).

RECOMMENDATIONS

Coverage

The trend towards fewer policies is admirable. The standard provisions program is a basic need. It should be encouraged. The number of special purpose policies should be reduced. All of this is welcomed, not just as an expense savings to the companies, but also as an opportunity to increase the buyer's understanding of his coverage.

To this end, we feel consideration should be given to a substantially new approach in presenting or arranging coverage, the approach demonstrated by Exhibit I. If a policy combining all standard coverage by type of property covered, by location of the property, by time, and by peril, could be set up so that the exact grant was indicated (perhaps more important, the grant not taken was indicated), the buyer would understand the product and there would be fewer disputes. Understanding and a straightforward all-in-one presentation should increase premium volume by making the product both easier to sell and easier to buy. This is not a suggestion that present coverage be altered, merely that it be re-arranged.

Another approach can be taken with regard to merchandise exposure. If a multiple peril approach, combining fire and burglary, is offered for contents coverage, it overcomes adverse selection. Other direct damage coverage could also be included. Thus, all normally insurable damage to contents on the insured's premises would be in a single policy, as is the case with the Jewelers Block Policy, the Manufacturers Output Policy, or the Commercial Property Floater. Such combinations are logical and, perhaps, inevitable and have been prevented or retarded only by the division of company authority, which existed prior to multiple line legislation.

Territorial Assignments and Differentials

The single rate with territorial multipliers used in Glass Insurance demonstrates the extreme variations (by street in New York) possible in property insurance. The fire rating of towns is another demonstration of variation. We would recommend an approach similar to that used for fire be applied to burglary insurance on a town basis. The formula should include consideration of density of population, concentration of value, an evaluation of police protection, and loss frequency. Rates for larger geographical areas, determined on the present loss ratio basis, could then be modified within the territory to reflect variations. Local, state, and federal statistics are available for most of the required information.

Class of Business—Protection—Limit of Liability

At the present time, no data of this type is coded (except for supermarkets); although provision is made for a four-digit code. Statistics developed in this area have tended to have low credibility in the past. Effective January 1, 1961, the Bureau Statistical Plan will be revised so that class of business and alarm systems will be coded for merchandise exposure. Class of business and type of safe will be coded for inside money exposure. This represents a major change. It is possible that statistics developed by this plan will lack

credibility and, therefore, be inconclusive. We feel that it is possible that discounted figures, on a basis similar to workmen's compensation multi-split plans, giving heavy weight to frequency and discounting severity, could be used to advantage. If dollar figures prove inadequate, exposure and loss counts, or pure frequency figures would be of value.

The proposed statistical plan revision effective January 1, 1961, gives detailed recognition to class of business for the classified lines. It gives detailed recognition to alarm systems and to types of safe. It gives no recognition to watchmen or to the less important types of protection. It is possible that after a number of years of experience has been obtained on the new basis, the detailed statistical plan will be eliminated and the form codes which existed prior to this time will again become dominant. A statistical plan may be simple or complex. Even under a complex plan, we often fail to obtain all possible data. Attached, as Exhibit 5, is a suggested four-digit classification system to be used for all commercial lines of burglary which we feel would permit an over-all evaluation of the present rating system. This statistical proposal differs from the Bureau 1961 program primarily in that it obtains more data with respect to protection and less data with regard to classes of business. Our proposal classifies risks by rating group only and contains no subdivision of the various rating groups. In this respect it is less complete than the Bureau proposal. However, the subdivisions of classifications or rating groups could be incorporated into our proposal or could be obtained on a sample basis from the records of the larger writers.

We believe that the statistical handling of protection under the Bureau proposal is inferior to that suggested herewith. The Bureau plan calls for absolute detail on alarm systems and no data on the other types of protection. We sincerely question whether the detailed datum on alarm systems is warranted. Only a few high hazard businesses install alarm systems better than Class 3. Therefore, although the 1961 Bureau plan is well laid out, logical and complete, its productivity is doubted. It would seem more to the point to obtain data on the other common forms of protection, watchmen, and, if possible, an over-all evaluation of all protection. We feel that the proposal in Exhibit 5 provides such data.

This type of program, either the Bureau proposal or the proposal included herewith, requires substantial effort on the part of rating and statistical units. The effort may, in fact, be unwarranted. On the other hand, the effort is no more than that required prior to 1951 and, perhaps, is something less than that required prior to World War II.

A number of credits for miscellaneous protection are rarely used. Since schedule or equity rating is available in most states, both the rating system and the value of the statistical system would be improved by eliminating manual credits for this miscellaneous protection, and schedule credits at the discretion of the underwriter be substituted to recognize such protection.

The proposed statistical system, supplemented by voluntary samples or special calls to determine intra-class distributions, would not eliminate judgment in determining differentials, but it would lessen the dependence on judgment. It would produce data heretofore unavailable and open the frequency area for ratemaking and underwriting consideration.

Experience Rating

Because of the catastrophic nature of the line, the small average premium sizes and infrequency of expected losses, burglary insurance does not lend itself to experience rating except, perhaps, from a frequency standpoint. If the line is to be experience rated, the period used should be as long as practicable, and a sizeable portion of the premium should be set aside as a nonrateable element. Although this was not discussed in the statistical section of this paper, it should be noted that part of the statistical recommendation is to obtain limit of liability data. It is possible that such data would make possible actuarial studies leading to refinements of the existing experience rating plans.

CONCLUSION

There is no perfect rating system. In all probability there never will be a perfect rating system. Presumably, we attempt to produce the best possible answer consistent with insurance theory, expense considerations, and equality of treatment between risks insured. Perhaps the last consideration is the most important, since if we do achieve equality of treatment or fair discrimination, the rating system must be considered successful.

Our two basic ideas seem to be to simplify the system and lessen the importance of judgment. This in no way presumes the present system produces wrong answers. Simplification will ease the introduction of a statistical system, and the expanded statistical system will lessen the necessity for reliance on judgment. Desirable by-products would be industry and consumer understanding.

The schedule rating system applied to fire coverage on commercial building has in the past been subject to similar analysis. A detailed statistical system was investigated and not adopted. Possibly the statistical requirements of the plan were considered impractical. It is entirely possible that the same line of reasoning would be applied to Burglary insurance. The line is so small that it receives little actuarial attention. Requirements for credibility preclude the application of actuarial techniques standard to the Casualty insurance field. We have tried to avoid this pitfall by substituting use of frequency data rather than dollar data. Possibly this approach would effectively forestall criticism of the established rating system.

EXHIBIT I

POLICY SOURCE OF

MAJOR COMMERCIAL BURGLARY COVERAGE

	LOSS FROM PREMISES WHILE ---			LOSS AWAY --
	Premises are Open	Premises are Closed	Property in Locked Safe	From Premises
	LOSS OF MONEY			
Burglary	Loss Impossible by Definition	Broad Form Premises	Safe Burglary Broad Form Premises	Loss Impossible by Definition
Robbery	Interior Robbery Broad Form Premises	Loss Impossible by Definition	Loss Impossible by Definition	Messenger Robbery Broad Form Messenger
Theft	Broad Form Premises	Broad Form Premises	Broad Form Premises	Broad Form Messenger
Disappearance	Broad Form Premises	Broad Form Premises	Broad Form Premises	Broad Form Messenger
	LOSS OF MERCHANDISE			
Burglary	Loss Impossible by Definition	Mercantile Open Stock	Mercantile Open Stock Safe Burglary Broad Form Premises	Loss Impossible by Definition
Robbery	Mercantile Open Stock with Theft Endorsemt. Broad Form Premises Interior Robbery	Loss Impossible by Definition	Loss Impossible by Definition	Messenger Robbery Broad Form Messenger
Theft	Mercantile Open Stock with Theft Endorsement	Mercantile Open Stock with Theft Endorsement	Loss Impossible by Definition	No Coverage Available
Disappearance	No Coverage Available	No Coverage Available	No Coverage Available	No Coverage Available

Burglary: Forcible Entry with Physical Evidence Thereof
Robbery : Forceful Taking - Fear or Threat of Violence
Theft : Unauthorized Taking - Custodian Unaware
Disappearance: Method of Removal Undetermined or Non-Human

EXHIBIT II

STANDARD POLICY COVERAGE

Name of Policy	From Open Premises: Money: Robbery	From Open Premises: Money: Theft	From Open Premises: Money: Disap.	From Open Premises: Mdse.: Robbery	From Open Premises: Mdse.: Theft	From Closed Premises: Money: Safe Burg.	From Closed Premises: Money: Theft	From Closed Premises: Money: Disap.	From Closed Premises: Mdse.: Safe Burg.	From Closed Premises: Mdse.: Burglary	From Closed Premises: Mdse.: Theft	Outside Premises: Money: Robbery	Outside Premises: Money: Theft	Outside Premises: Money: Disap.	Outside Premises: Mdse: Robbery	Other Coverage	Fixed Lim. of Liab.	Comments
Mercantile Open Stock									Yes	Yes								
Mercantile Open Stock With Theft End				Yes	Yes				Yes	Yes	Yes							
Broad Form-Premises	Yes	Yes	Yes	Yes		Yes	Yes	Yes	Yes									
Broad Form-Messenger												Yes	Yes	Yes	Yes			
Merc. Robbery-Inside	Yes			Yes														
Merc. Robbery-Mess.												Yes			Yes			
Merc. Rob.-Safe Burg.						Yes			Yes									
Paymaster Robbery	Yes											Yes						Limited to payroll funds
Storekeepers Burglary and Robbery	Yes			Yes		Yes			Yes	Yes		Yes	Yes	Yes	Yes	Yes	Yes	Minor Add'l coverage - package Limit of Liability in Multiples of $250
Storekeepers Broad Form	Yes	Yes	Yes	Yes		Yes	Yes	Yes	Yes	Yes		Yes		Yes	Yes	Yes	Yes	Fidelity included - package Limit of Liability in Multiples of $250
Office Burglary & Robbery	Yes			Yes		Yes			Yes	Yes	Yes	Yes				Yes	Yes	Mdse. limited to Office Equipment package - Limit in Multiples of $250

EXHIBIT II Cont.

STANDARD POLICY COVERAGE

	From Open Premises					From Closed Premises						Outside Premises						
	Money			Mdse.		Money			Mdse.			Money			Mdse.			
Name of Policy	Robbery	Theft	Disap.	Robbery	Theft	Safe Burg.	Theft	Disap.	Safe Burg.	Burglary	Theft	Robbery	Theft	Disap.	Robbery	Other Coverage	Fixed Lim. of Liab.	Comments
Fraud Policy	Yes			Yes		Yes			Yes			Yes			Yes	Yes	Yes	Fixed low limit package
Crime Protective	Yes			Yes		Yes			Yes			Yes			Yes	Yes	Yes	Fixed low limit package
Merchants Protective	Yes			Yes		Yes			Yes			Yes			Yes	Yes	Yes	Fixed low limit package
Church Protective	Yes			Yes		Yes			Yes			Yes			Yes	Yes	Yes	Fixed low limit package
Fraternal Protective	Yes			Yes		Yes			Yes			Yes			Yes	Yes	Yes	Fixed low limit package
Blanket Crime Policy	Yes	Yes	Yes	Yes		Yes	Yes	Yes	Yes			Yes	Yes	Yes	Yes	Yes	Yes	Minimum limit $1,000, Theft of Office Equip. by Endorse.
Comprehensive 3D Policy	Yes	Yes	Yes	Yes		Yes	Yes	Yes	Yes			Yes	Yes	Yes	Yes	Yes		
Innkeepers Legal Liab.	These policies are legal liability policies and Burglary coverage is incidental to other perils.																	
Warehousemans Legal Liab.																		
Valuable Papers Policy	This is an all Direct Damage Policy with Fire consituting largest peril.																	
Accounts Receivable Pol.	Pays for inability to collect amounts owed because of destruction of records - Fire major peril.																	

EXHIBIT III

SUMMARY OF VARIABLES USED IN RATING

COMMERCIAL BURGLARY COVERAGES

Reason for Rate Variation

Subline	Territory	Class of Business	Type of Safe	Limit of Liability	Guards	Second Exposure	Securities Only	Excess Insurance	Alarm Systems	Watchmen	Tear Gas Systems	Merchandise Only	Division of Insurance	Special Cashiers Cage	Private Conveyance	Relocking Device	24-Hour Operation	Messenger Bag	Inside Route	2 People on Duty	Special Schedule	Total Variations
Messenger Robbery	x				x	x	x	x							x			x	x		x	9
Broad Form Messenger	x				x	x	x	x							x			x	x		x	9
Mercantile Open Stock	x	x		x					x	x											x	6
Safe Burglary	x	x	x			x	x	x	x	x	x	x	x			x	x				x	14
Broad Form Premises	x	x	x		x	x	x	x	x	x	x										x	11
Interior Robbery	x	x			x	x	x	x	x		x			x						x	x	11
TOTAL USE	6	4	2	1	4	5	5	5	4	3	3	1	1	1	2	1	1	2	2	1	6	60

EXHIBIT IV

1956 CALENDAR YEAR EARNED PREMIUM

BY MAJOR STATE BY SUBLINE

NATIONAL AND MUTUAL BUREAUS

	Mercantile Open Stock	Safe Burglary	Interior Robbery	Premises Broad Form	Total
New York	3,699,890	385,395	743,893	1,723,130	6,552,308
Illinois	1,474,471	104,939	281,149	765,225	2,625,784
Pennsylvania	675,376	92,323	145,789	651,909	1,565,397
Ohio	481,663	96,187	146,725	652,371	1,376,946
Texas	403,896	229,329	116,711	687,488	1,437,424
Michigan	417,168	81,426	119,037	538,120	1,155,751
Massachusetts	422,786	54,460	82,512	387,132	946,890
All Other *	3,368,186	992,816	822,643	4,768,041	9,951,686
Countrywide *	10,943,436	2,036,875	2,458,459	10,173,416	25,612,186

* Omits California, Missouri, Montana, and Idaho

EXHIBIT V

PROPOSED COMMERCIAL BURGLARY CLASSIFICATION CODES

Standard IBM Card Columns 24, 25, 26, 27

(These codes are applicable to the Policy Forms indicated. All other Policy Forms code 0000 in this field)

Applicable to Policy Form Codes:

Mercantile Open Stock	- 20, 26
Mercantile Safe	- 71
Loss Inside the Premises	- 60, 76, 77
Mercantile Robbery - Inside	- 32
Mercantile Robbery - Outside	- 40
Loss Outside the Premises	- 61, 78, 79
Paymaster Robbery	- 41
Paymaster Broad Form	- 62
Storekeepers Burglary	- 73, 82

TYPE OF BUSINESS - Column 24 - Policy Forms 20, 26, 71, 32, 60, 76, 77, 73, 82

These codes reflect the actual numerical manual classification of the risk.

Manual Classification	Code
Class 1 or Unclassified	1
Class 2	2
Class 3	3
Class 4	4
Class 5	5
Class 6	6
Class 7	7
Class 8	8
Class 9, 10, 11	9
(Non Classified Policy Forms	0)

A. PROTECTION - Column 25, Policy Forms 20, 26, 71, 60, 76, 77

Description of Protection	Code
Certified Central Station Alarm Systems	
Alarm System Only	1
Alarm System & miscellaneous protection other than watchmen	2
Alarm System & watchmen with or without miscellaneous protection	3
Other Alarm Systems - Non-Certified and/or Local	
Alarm System Only	4
Alarm System & miscellaneous protection other than watchmen	5
Alarm System & watchmen - without or with miscellaneous protection	6

EXHIBIT V Cont.

Description of Protection	Code
Watchmen	
Watchmen Only	7
Watchmen & miscellaneous protection	8
Watchmen & alarms - See codes 3 or 6	9
Miscellaneous Protection	
Includes all credited protection except watchmen or alarms	
No Credited Protection	0

******

B. PROTECTION - Column 25 - Policy Forms 32, 40, 61, 78, 79, 41, 62

Guards	
Guards Only	1
Guards & miscellaneous protection	2
Guards & private conveyance only	3
Guards & miscellaneous & private conveyance	4
Private Conveyance	
Private conveyance only	5
Private conveyance & miscellaneous protection	6
Private conveyance & guards - See 3 or 4	
Miscellaneous Protection	
Includes all credited protection except guards and private conveyance	9
No Credited Protection	0

Policy Forms not specified in A or B punch 0 in Column 25

* * * * *

PERCENTAGE PROTECTION CREDIT - Column 26 - All Subject Policy Forms

All credits from manual rates except for experience credit and additional exposure credit should be reflected in this column. The aggregate modification for all other protection should be computed and coded as follows:

Aggregate Modification	Code
1.00	0
.90 to .999	1
.80 to .899	2
.70 to .799	3
.60 to .699	4
.50 to .599	5
.40 to .499	6
.30 to .399	7
.20 to .299	8
less than .20	9

* * * * *

EXHIBIT V Cont.

LIMIT OF LIABILITY - Column 27 - All subject policy forms

Dollar Limit of Liability	Code
0 - 1,000	0
1,001 - 2,000	1
2,001 - 3,000	2
3,001 - 4,000	3
4,001 - 5,000	4
5,001 - 7,500	5
7,501 - 10,000	6
10,001 - 15,000	7
15,001 - 20,000	8
Over 20,000	9

* * * * *

THE RATING OF CROP-HAIL INSURANCE

BY

RICHARD J. ROTH

Contents

Introduction

Crop-hail insurance is the name of that type of coverage which insures a farmer against loss resulting from hail damage to growing crops. Hail, though the basic hazard, is not the only peril insured against, as the crop-hail policy also provides protection, depending upon the crop and state, against fire, lightning, livestock, wind (when accompanied by hail), aircraft, and vehicles.

In addition, experimental coverage called crop-failure insurance is offered in specific counties in a few states. Added as an endorsement to the crop-hail policy, it provides disaster protection against many additional perils such as drought, excess moisture, insects, etc. Very little of this coverage has been written, however, since its introduction in 1956.

Also, to say that crop-hail insurance applies to growing crops only, is not strictly correct. For selected crops and states, crop-hail insurance is extended to cover crops until they have been unloaded at the first place of storage. One special policy covers tobacco while in the curing and pack barns and until delivered to the sales warehouse.

Up until 1948 crop-hail insurance rating was accomplished in a relatively informal manner by committees of company men. Only a few states required filing of crop-hail insurance rates and forms, and stringent regulation for this field had not yet come into being. Public Law 15 gave impetus to the already existing desire to develop a more scientific rate structure. Consequently, on December 5, 1947, the stock fire insurance companies organized the Crop-Hail Insurance Actuarial Association and made its scope national. It was the decision of the companies to have a professional meteorologist in charge of the Association, and the author was hired as its Manager.

A tremendous task faced the Association at its start. Rate and form filings had to be made in all states to meet the January 1, 1948 deadline date set by Congress. Statistical information had to be obtained from the 5 regional organizations then in operation, forms printed, and justifications prepared. This was all accomplished and member companies of the Association met all requirements of the new filing laws when they wrote their 1948 business.

After the initial problems had been solved, there was still much work to be done. The consolidation of the detail statistical data and the conversion of this from manual to punched card records took years. Informal, subjective rate-making methods had to be reworked, changed and put in writing.

This paper covers the present status of crop-hail insurance rating, as accomplished directly for the members and subscribers of the Crop-Hail Insurance Actuarial Association, the bulk of whom are stock fire insurance companies. In 1957, the affiliated companies of the Association wrote 63% of all crop-hail insurance written in the United States, and 73% of the premium volume.

To my knowledge this is the first comprehensive survey ever written regarding the rating of crop-hail insurance. The principles and methods described include those basic developments of the pioneer hail insurance men without which the rating systems of today could not exist, and also the many developments since the formation of the Crop-Hail Insurance Actuarial Association. The future of crop-hail insurance rating is explored, and it becomes apparent that the application of scientific methodology to it is in its infancy, the potentials for future improvement being indeed large.

I. BACKGROUND INFORMATION

A. *History of Crop-Hail Insurance**

Crop-hail insurance is comparatively new in the United States as compared to Europe. As early as 1797, a hail insurance organization known as

* Much of the historical information concerning crop-hail insurance is taken from the writings of James B. Cullison, Jr., first president of the Crop-Hail Insurance Actuarial Association, and pioneer in the development of all phases of the field.

the Mecklenburg Hail Insurance Association was formed. A similar attempt was made in France by a M. Barrau in 1801, although in 1809 a Council of State suppressed the undertaking evidently believing this to be almost an interference with divine Providence. However, the need for protection against hail damage to growing crops was so great that hundreds of associations were formed in Europe and many stock companies started offering coverage during the 19th century.

The International Congress of Hail Insurers reports that almost $55 million in premiums were written during 1957 in 13 European countries and North Africa. The leading countries by premium income were: Germany, $12 million; Italy, $10½ million; France, $10¼ million; Yugoslavia, $9¾ million; North Africa, $3½ million; and Switzerland, $2 million. Other countries writing crop-hail insurance and reporting to the International Congress were Austria, Belgium, Denmark, Spain, Greece, Luxemburg, Netherlands, and Sweden. The $55 million of European writings compares with $69 million written in the United States during 1957.

The first mutual hail insurance companies in the United States were organized in 1879, and many more started in business up to 1900, although the rate of failure was high due to lack of reserves and adequate rate structures.

The first stock fire insurance company entered the crop-hail insurance field in 1883 offering insurance in a few of the prairie states. By 1906 another entered the field and by 1912 there were probably 12 to 15 stock companies, and 35 to 40 mutuals writing this line.

The stock fire insurance companies formed the Western Hail and Adjustment Association in November 1915, and began the collection of statistical experience. At the start only premiums and losses by county were collected, but in 1917 it was decided to add the reporting of liability, and member companies went back in their records to obtain this for 1915 and 1916. Beginning in 1924 statistics were collected by governmental township (6 miles by 6 miles) for the important prairie states.

Other regional hail associations were formed in the early twenties for the Southeast, Pacific Coast states, and Texas, and at a somewhat later time an association for the Eastern states was organized. These associations made rates, devised policy forms, and developed scientific methods of loss adjustment.

The United States premium income for stock companies grew from about $3 million in 1915 to $39 million in 1947. Since an additional $19 million was written by mutual companies in 1947, the grand total of crop-hail insurance premiums for all insurers in 1947 was over $58 million.

B. *Crop-Hail Insurance Actuarial Association*

In December 1947, the Crop-Hail Insurance Actuarial Association was organized by 62 stock fire insurance companies. Originally, its purpose was to operate as a statistical and advisory organization to the state fire insurance rating bureaus giving advice as to crop-hail insurance rates and forms, but in 1953 its Constitution was amended to permit it to act as a rating organization on a national scale. In 1959 the scope of the Association was further en-

larged to permit the rating of rain insurance on public events, business ventures, and private proceedings.

Operating as a non-profit research, statistical and rate-making organization it is now supported by 133 members and subscribers, most of these being stock fire insurance companies. The Association's work consists of not only the preparation and promulgation of rates and policy forms, but also the justification and filing of these with the insurance departments of each of the states. It also acts as the official statistical agent for crop-hail insurance for the states having laws providing for the appointment of same.

The Association receives money for its operating expenses by assessing its supporting companies annually, and each company pays in proportion to the amount of premiums which it wrote during the past growing season. Representatives of member companies meet each December to elect the three non-salaried officers of the Association.

The policy direction and over-all responsibility for Association affairs rests in the hands of the Executive Committee which consists of the three elected officers and eight other appointed members. The principal committee assisting the Executive Committee is the Actuarial and Forms Committee which reviews the technical phases of the Association's work, and is mainly concerned with the preparation of recommended policy forms and endorsements, and the review of rates to be charged. All the work of the Actuarial and Forms Committee is presented to the Executive Committee for final action.

Besides the Actuarial and Forms Committee, the Executive Committee has appointed a Research Committee, which studies all phases of research applying to crop-hail insurance. In addition it is responsible for developing a new experimental coverage which is added to the hail policy by endorsement, and covers growing crops against the hazards of drought, excessive heat, flood, excessive moisture, insect infestation, plant disease, wildlife, wind, tornadoes, sleet, hurricane, frost freeze and snow. A Priority Committee determines the order of states to be rated, and a Rain Insurance Committee deals with the new coverage added in 1960.

In addition to these committees, there are 18 Regional Committees assisting the Association in maintaining local contact all over the United States. These are scheduled to meet periodically to make recommendations concerning their particular areas, and have proved to be indispensable in keeping the Association in close touch with developments of agriculture and insurance in each region.

Now, though the Executive Committee sets the general policy of the Association, the Manager of the Association and his staff are responsible for putting this policy into action. There are 56 salaried employees working for the Association.

When the Association was organized in December 1947, it assumed statistical, rating and form functions formerly exercised by the various regional hail insurance organizations.* The first major task of the Association was the

* The Hail Insurance Adjustment and Research Association and the Southeastern Hail Conference have continued to operate in the fields of loss adjustment procedures and simulated hail damage research carried on by various agricultural colleges.

consolidation of the statistical information turned over to it by these regional organizations, and the transferring of this data from manual records on to punch cards.

This vast amount of accumlulated data has been kept up-to-date, and added to since 1948. Each year affiliated companies have reported their crop-hail insurance liability, premiums, and losses and this has been tabulated, and separate statistical summaries published annually for each state.

The nationwide crop-hail premium income of the Association's companies has increased from $39 million in 1947 to $77½ million in 1958, and $73 million in 1959.*

C. *The Crop-Hail Insurance Policy*

Crop-hail insurance is fundamentally written as a physical per cent of damage contract.

The basic contract, known as the "percentage policy", provides that the same proportion of insurance will be paid as the proportion of crop destroyed. If 30 per cent of the farmer's crop is destroyed on any insured acre, he will receive in payment 30 per cent of the amount of insurance that he has taken out on that acre. If he has $10.00 insurance applying to that acre, he will collect $3.00. If he has $50.00 insurance, he will be paid $15.00.

If the amount of insurance equals the value of the crop, the farmer will be completely protected. If the amount of insurance equals half of the crop value, the insured will receive payment for one-half of his actual loss. In other words, crop-hail insurance has a 100% coinsurance feature similar to marine insurance.

The usual life of a crop-hail insurance policy is counted in months, being the length of the crop growing season. Generally speaking, the policy attaches when the crops insured are up to a normal stand, and the coverage continues until the crop is harvested. There is also a date in the policy after which the insurance automatically expires, but this is included primarily to protect the company against a farmer abandoning his crop.

Most policies are taken out annually at the start of the growing season. In a few states, however, three-year and five-year policies are issued, but the premium is paid annually and an endorsement is furnished giving the number of acres of each insured crop grown.

Local agents do not issue the policies, but send in applications to the company. Insurance becomes effective 24 hours after the farmer makes application, although the company has the option of rejection.

The application form requires the description of the land on which the crop is grown (county, township, and range), the kind of crop, the per cent interest that farmer has in the crop, the number of acres, and the insurance per acre desired.

Agents are supplied with specimen policy forms so that the farmer may be fully aware of the conditions of the contract for which he is applying.

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* The five leading states ranked by 1959 premium income: North Carolina, $8.2 million; Texas, $7.7 million; Kansas, $7.4 million; Nebraska, $6.8 million; and North Dakota, $5.4 million.

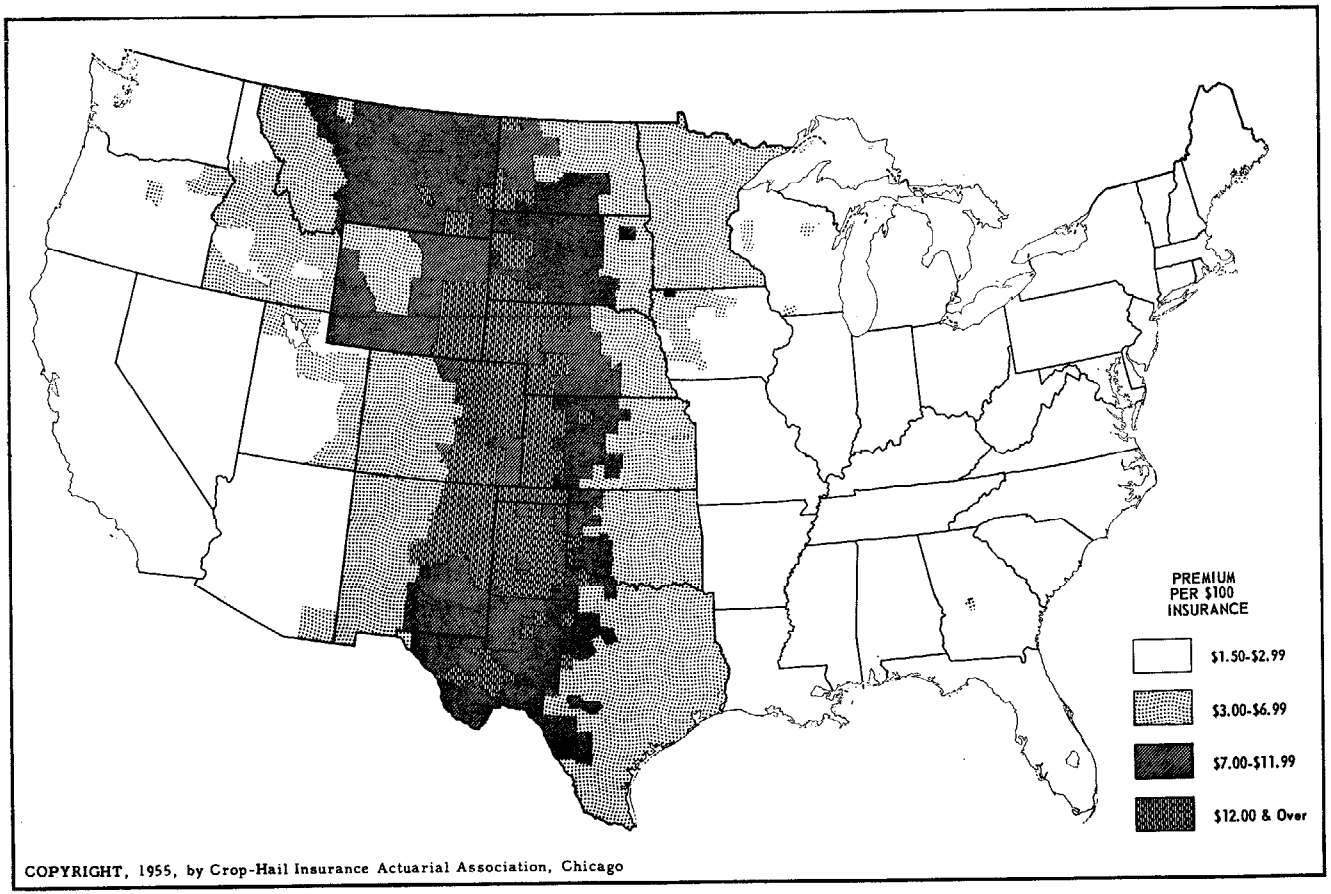

Chart 1. Average 1955 wheat rates by county for the non-deductible policy form.

The average rate charged for crop-hail insurance in the United States in 1958 was $5.14 for every $100 of insurance. The rates, however, vary considerably by geographical location, crop, and policy form. In many states different rates may be charged for each six-mile square government township.

The highest rates are charged in the western parts of Kansas and Nebraska and the eastern portions of Colorado and Wyoming. Chart 1. shows the average county wheat rates in effect in 1955.

All of the rates promulgated by the Crop-Hail Insurance Actuarial Association are based on accumulated insurance experience, as it had soon become evident that U. S. Weather Bureau data of number of days with hail was of little use in establishing usable crop-hail rates.

The method of developing rates is based on loss costs, or "pure premiums" rather than loss ratios. Liability and loss data are available back to 1924, and in many instances back to 1915. The loss cost is obtained by dividing losses by the liability or amount of insurance, and is expressed in dollars and cents per $100 of insurance. Another way of looking at the loss cost is that it is the average loss in dollars per $100 insurance.

II. GATHERING OF EXPERIENCE FIGURES

A. *Method of Reporting*

In earlier years all statistical reporting was accomplished by companies completing a summarized report of their experience by the classifications required. At a central location in each of the regions, the reports of all companies were consolidated.

In 1948, when the Crop-Hail Insurance Actuarial Association undertook the collection of statistics nationwide, this same procedure was followed, although it was provided that companies desiring the Association to summarize its liability, premiums and losses from the original documents could do so at extra cost and on a purely optional basis.

The advantages* of using up-to-the-minute experience in rate calculations became so apparent as time went on that in 1957 the Association inaugurated its current statistical reporting program. This provides for each company sending in copies of applications during the writing season, and copies of proofs of loss as adjustments are completed.

During the summer the Association places data on punched cards for those states which have been designated to be re-rated by the Priority Committee. A closing date is set for each of these states and companies are notified by bulletin. Documents received after the closing date are held until the following year, and are then included as supplemental material separately designated.

Also as part of the program, each of the companies sends in a closing report which gives the total amount of premiums and losses contained in the documents sent to the Association up to the closing date. These are used as control figures to check the data which has been placed on punched cards.

* See Part V, "Other Factors Affecting Crop-Hail Insurance Rating."

Balancing is not required to the penny, but the company totals compared to the Association totals must be within a specified range. The table setting forth the balancing requirements is so designed that the higher the dollar amounts involved, the less the permissible percentage deviation. In no case is a deviation of over 5% allowed to go unexplained, although if it is not possible to clear up a discrepancy immediately it becomes necessary to add supplemental information in the next year's summary.

Closing dates for states not being re-rated are set at a later time and the data is punched during the fall and winter months.

Companies have the option of reporting liability and premium data by punched cards in lieu of sending copies of their application, and in this case they must observe the same rules for closing dates and closing reports. Loss information is not permitted to be reported by punched cards because of the large possibility of error in coding due to the complex nature of proofs of loss.

B. *Machine Processing of Data*

The ability to include the most current experience in the cumulative record for rating purposes is possible only because of modern electronic data processing equipment. The number of crop-hail insurance punched cards to be processed each year varies between 1¼ million and 1½ million, which poses a most difficult problem for standard tabulating equipment.

The Association uses a magnetic tape I.B.M. 650 data processing system which provides extremely rapid and accurate handling of data. A further advantage of magnetic tape is the reduction in storage requirements. The ratio of space required to store magnetic tape as compared to punched cards is about the same as the ratio of space required for microfilm compared to original documents.

Punched cards are used only to enter the magnetic tape system, and are then destroyed. All historical information required to be saved is on magnetic tape.

The 650 system is well adapted to the type of statistical information needed in crop-hail insurance work. By doing many things at once the time expended is greatly reduced. Erroneous rates, faulty computations, and errors in coding are punched out in the initial phases of the work. Later on, standardized individual company reports (upon request) are prepared, and statistical summaries combining all companies experience produced. Rate analysis procedures are also included as part of the operation when re-rating has been specified.

A relatively small clerical staff is used in checking documents for coding prior to punching, and for processing errors which are indicated by the 650 machine. One of the functions of the clerical staff is to see that the totals produced by the machine are in balance with the control totals furnished by the companies.

C. *Publication of Data*

Statistical summaries are produced on a 407 tabulating machine (on line in the 650 system). These summaries are used by member and subscribing

CROP – HAIL INSURANCE

GENERAL, HAIL AND FIRE
HAIL ONLY

TOWNSHIP STATISTICS

KANSAS (STATE) | ALL CROPS (CROP) | 1924-1958 (PERIOD) | HAIL (CAUSE OF LOSS)

COUNTY CODE	COUNTY NAME	TOWNSHIP NUMBER	TOWNSHIP DIR.	RANGE NUMBER	RANGE DIR.	LIABILITY (NEAREST DOLLAR)	LOSSES DOLLARS	LOSSES CENTS	LOSS COST DOLLARS	LOSS COST CENTS
153	RAWLINS	5	S	34	W	693148	1202657	41	17	35
153	RAWLINS	5	S	35	W	596911	55475	86	9	29
153	RAWLINS	5	S	36	W	312095	145611	3	4	67
	TOTAL					1297314 0	10792158	9	8	32
155	RENO	22	S	4	W	1314820	2886347	4	2	19
155	RENO	22	S	5	W	239582	77497	3	3	23
155	RENO	22	S	6	W	1395209	709976	1	5	48
155	RENO	22	S	7	W	1643714	761679	4	4	63
155	RENO	22	S	8	W	2039115	970936	1	4	76
155	RENO	22	S	9	W	1698576	10840004		6	38
155	RENO	22	S	10	W	2940405	1947187		6	61
155	RENO	23	S	4	W	887338	2613411	4	2	97
155	RENO	23	S	5	W	779504	1207006		1	55
155	RENO	23	S	6	W	1158160	5060176		3	20
155	RENO	23	S	7	W	3229095	15789995		4	89
155	RENO	23	S	8	W	2507098	13591713		5	42
155	RENO	23	S	9	W	2157914	1401500	2	6	49
155	RENO	23	S	10	W	994715	3649204		3	67
155	RENO	24	S	4	W	1401050	3021334		2	16
155	RENO	24	S	5	W	628287	996092		1	59
155	RENO	24	S	6	W	1344750	4829678		3	59
155	RENO	24	S	7	W	2572276	14686846	4	5	71
155	RENO	24	S	8	W	2327290	8185509		3	52
155	RENO	24	S	9	W	77548	2667203		3	44
155	RENO	24	S	10	W	67426	1540764		2	29
155	RENO	25	S	4	W	2283362	5305322		2	32
155	RENO	25	S	5	W	1300913	4180697		3	21
155	RENO	25	S	6	W	85651 4	56595	92	6	61
155	RENO	25	S	7	W	1090022	4364349		4	00
155	RENO	25	S	8	W	1749318	9018007		5	16
155	RENO	25	S	9	W	1658072	6957052		4	20
155	RENO	25	S	10	W	549900	1038974		1	89
155	RENO	26	S	4	W	1842658	5687869		3	09
155	RENO	26	S	5	W	1081820	2611851		2	41
155	RENO	26	S	6	W	597960	1614378		2	70
155	RENO	26	S	7	W	449451	737974		1	64
155	RENO	26	S	8	W	1453417	5119488		3	52
155	RENO	26	S	9	W	1976237	6555872		3	32
155	RENO	26	S	10	W	910356	1813472		1	99
	TOTAL					4818620 1	19341 27	25	4	01
157	REPUBLIC	1	S	1	W	1011789	7219325		7	14
157	REPUBLIC	1	S	2	W	48672 3	2666964		5	48
157	REPUBLIC	1	S	3	W	65322 6	1602282		2	45
157	REPUBLIC	1	S	4	W	1133084	3964868		3	50
157	REPUBLIC	1	S	5	W	1146730	5122433	5	4	47
157	REPUBLIC	2	S	1	W	601348	760396		1	26
157	REPUBLIC	2	S	2	W	682087	2555695	4	3	75
157	REPUBLIC	2	S	3	W	509093	1560259		3	11
157	REPUBLIC	2	S	4	W	1101654	3866921		3	51
157	REPUBLIC	2	S	5	W	1195164	4563726		3	82
157	REPUBLIC	3	S	1	W	704798	1994153		2	83
157	REPUBLIC	3	S	2	W	549056	1435300		2	61
157	REPUBLIC	3	S	3	W	420358	856321		2	04
157	REPUBLIC	3	S	4	W	642430	2067514		3	22
157	REPUBLIC	3	S	5	W	1629766	5033023		3	09
157	REPUBLIC	4	S	1	W	908992	2775111		3	05
157	REPUBLIC	4	S	2	W	920114	6053595		6	58
157	REPUBLIC	4	S	3	W	925237	4700998		5	08
157	REPUBLIC	4	S	4	W	628629	2527261		4	02
157	REPUBLIC	4	S	5	W	1036217	2199231		2	12
	TOTAL					16678305	63528437		3	76
159	RICE	18	S	6	W	863281	1678049		1	94
159	RICE	18	S	7	W	1095227	1334675		1	22
159	RICE	18	S	8	W	1144016	1525499		1	33
159	RICE	18	S	9	W	1325097	2165174		1	63
159	RICE	18	S	10	W	925515	3034529		3	28
159	RICE	19	S	6	W	847429	1230333		1	45
159	RICE	19	S	7	W	1590068	2427386		1	53
159	RICE	19	S	8	W	2123703	5641861		2	66
159	RICE	19	S	9	W	1667836	4445260		2	67
159	RICE	19	S	10	W	1400406	8838574		6	31
159	RICE	20	S	6	W	861164	501006			58

Chart 2. A Sample Page of the 1958 Kansas Statistical Summary.

The tabulating machine's print-out sheet is photographed and printed

companies to check their underwriting plans, compare their individual experience with the average of all companies, to determine areas of potential development of future sales of crop-hail insurance, and for various other purposes.

Annually, each Insurance Department receives a published statistical summary for its state, which is not only for information, but also serves as the official report for those states providing for the formal appointment of a statistical agent. (The Crop-Hail Insurance Actuarial Association has been designated as the official statistical agent for crop-hail insurance in all states requiring this.)

The publication of the summaries is simplified by the process of taking reduced photographs of the tabulating machine print-out sheets, and plates for printing are made from these.

A sample page of a statistical summary is shown in Chart 2.

III. RATING METHOD

A. *General Remarks*

Hail damage is the direct result of thunderstorm activity. The lightning, thunder, heavy rain and gusty winds of a severe thunderstorm are frequently accompanied by a deluge of frozen ice balls. These may vary from small pea-size stones of ¼″ in diameter up to the dimensions of a grapefruit, although the average size is about ½″, and it is rare to have stones fall larger than 2″ in diameter.

Hailstorms almost always occur when the temperature at ground level is considerably above freezing, spring and summer being the season of most activity. Since hailstones are frozen water (often with successive layers of clear ice and snowy, cloudy ice), they must be formed at heights where the temperature is below freezing. In summer in the central United States, the freezing level occurs at about 13,000 to 14,000 feet above sea level, and stones are formed in thunderclouds above this level.

There are two theories of formation, one postulating that a nucleus of frozen water is subject to a series of updrafts and downdrafts which transports the stone from the freezing region of cloud to the the warmer regions below. There an additional coating of water is added, and then the stone is carried again up into the freezing region, thus explaining the concentric layers of clear and opaque ice. When the stone grows to a size which cannot be supported by the updrafts, it falls to earth.

Another theory suggests that the frozen nucleus starts to fall and successively encounters supercooled water droplets and snowflakes. There is only one descent, and the amount that the stone grows as it falls depends upon how many droplets and flakes it encounters.

Regardless of how hailstones are formed, it is known that they are products of the violent atmospheric updrafts found in thunderstorms. A storm area however, is actually a region of convective activity made up of a number of thunderstorm cells. Each cell has a life cycle during which its cumulus cloud develops into a cumulonimbus or thundercloud, precipitates rain and possibly

hail, and then dissipates. In a storm area, one cell may be in the cumulus stage, while another is in the mature stage, and a third may be dissipating. There is a tendency for successive cells to reach greater heights as a well-developed thunderstorm area moves across the country.

Opinions differ as to whether every thunderstorm cell contains hail. Certainly, stones do not reach the ground in most cases, but whether they melt in descent or never existed in the first place is still not conclusively proven. Hailstones reaching the ground seem to be associated with cells having higher than average updraft velocities (in excess of 35 m.p.h.).

Hail damage occurs in a path, the width of which averages from one to two miles and may be as much as ten miles wide. The length of the paths, which is dependent on the velocity of the hail-producing cell and the duration of its life cycle, will range from a few miles to 50 miles or more.

Discontinuous paths of hail can be explained by attributing the different portions to different cells, rather than by a theory that the storm cloud precipitates hail, lifts, and then showers down more hail at a later time.

Basically, the extent of damage (except for very severe storms) is relatively local in nature. Recent meteorological research has tended to confirm the long-held opinion of hail insurance men that the frequency and severity of hailstorms may differ significantly within short geographical distances, the influence of local topographic features being held responsible for this variance. However, in addition to the local variability of hail hazard, there is also a broad-scale difference in hail occurrence due to the general weather circulation as affected by large land masses and bodies of water. The local topographic features are superimposed on the large-scale pattern.

In general, meteorological knowledge about hailstorms is relatively limited, significant advances having been made only in recent years. Thus, the physical reasoning which is so useful in arriving at rating classifications in other lines has been of restricted use in crop-hail insurance. Engineering concepts with regard to occupancy, exposure, structure, and protection are vital to fire rating, and the knowledge that the probability of death increases with age is essential to the development of rates for life insurance.

We do not know much about why it hails more in one place than another. We know that in the Great Plains states the elevation of the land above sea level is important. In these same states we have reason to believe that the slope of the land in relation to the direction of hailstorm movement is of significance, although to date not enough conclusive evidence has been produced so that we can use it in our rating methods. We suspect that the presence of large bodies of water will affect surrounding land areas, and have certain other theories, but basically, our approach in crop-hail insurance rating is an empirical statistical one—and in certain areas, entirely so.

The above considerations influence crop-hail insurance rating in the following ways:

1. The number of years' experience used for rating must be as many as possible.
2. Rating zones must be small in area—for many states even a county

division is unsuitable, and rating areas must be divided by township lines.

3. Rates must be revised frequently and must include the experience of the most recent season.

1. *Length of Record*

Hail will not fall at a given location in most years, and the average percent of crop destroyed is determined by a relatively few years of damage. In other words, the annual frequency distribution of hail damage for a limited area (county or township) is very skewed.

This condition, which is true in varying degrees of all "catastrophe" insurance, renders a limited period of record of doubtful value in estimating a "true" mean. Thus, we must use the maximum number of years of record available to us to achieve any degree of predictability.

Township data (a township is 6 miles by 6 miles) is extremely unreliable. Consider the leading township in Kansas according to amount of insurance written from 1924 through 1959: Township 29S, Range 4W, Sedgwick County. The total insurance recorded for this township is $4,985,724, or an average of over $138,000 per year. Over the 36 years of record it has a mean loss cost of $4.61. The estimated standard deviation is $13.29 and the estimated standard error $2.22. If our estimate of the standard deviation is a good one, it would require *2715* years of record to reduce the calculated standard error to a magnitude which would allow us to assert that we were 95% confident that our experienced loss cost was $\pm$ $0.50 from the true mean.

This, of course, renders a township figure useless *by itself*. There are, of course, two ways in which the predictability of the mean may be increased: a) by increasing the length of record and b) by increasing the size of the area.

Fortunately, since the crop-hail coverage is a physical percentage of damage contract, it is not influenced by the declining value of the dollar or by the changing ratios of amount of insurance to value. Therefore, the entire period of record can and must be used for crop-hail insurance rating.

2. *Size of Area*

Although the predictability of the mean increases as the size of the area increases, it is at this point that we run into conflict. Meteorological knowledge and observed experience indicate local variance in hail hazard, and to make rates based on state-wide experience is equivalent to mixing oranges, apples, boxcars, and airplanes together. This is borne out by the early attempts at state-wide rating which resulted in adverse selectivity to an unusual degree: farmers in the higher hazard areas being happy to buy insurance at inadequate rates, and farmers in the low hazard areas refusing to buy at what seemed excessive rates.

The dilemma: small rating areas are necessary to satisfy the basic principle that the rate should reflect the hazard, large rating areas are essential to assure that meaningful conclusions may be drawn from statistical data.

The best approach to the solution lies in the classification of townships according to degree of hazard as determined by meteorological factors. For instance, in Kansas we have a striking correlation of elevation with loss cost.* Each township has been classified according to elevation, and then all townships grouped into like elevation categories. Consequently, instead of 2,561 individual townships, there are 33 elevation rating areas.

The following figures indicate the stability introduced by using elevation areas instead of townships. Listed are the five leading townships according to amount of liability (1924-1959), and the five elevation areas with the most business written.

5 Leading Townships

	County	*Twp.*	*R.*	*Liab. 1924-59 ($1,000)*	*Weighted L.C.*	*Mean L.C.*	*Standard Deviation*	*No. Years for 95% Conf. ±$0.50*
1.	Sedgwick	29S	4W	$4,986	$5.94	$4.61	$13.29	2,715
2.	Sedgwick	26S	3W	3,692	4.25	3.24	9.51	1,391
3.	Doniphan	4S	19E	3,419	1.90	2.40	4.82	357
4.	Sedgwick	28S	2W	3,383	1.59	1.34	3.22	160
5.	Reno	23S	7W	3,367	4.69	5.02	10.28	1,625

Average No. of Years for 95% Confidence ±$0.50 = 1,250 years

5 Leading Elevation Areas

	Elevation Group	*Liab. 1924-59 ($1,000)*	*Weighted L.C.*	*Mean L.C.*	*Standard Deviation*	*No. Years for 95% Conf. ±$0.50*
1.	1300 feet	$155,385	$2.26	$2.23	$1.38	29
2.	1400 feet	145,735	2.59	2.48	1.84	52
3.	1500 feet	113,266	3.30	3.06	2.30	82
4.	1200 feet	95,943	2.29	2.04	1.39	30
5.	1100 feet	73,596	1.32	1.42	1.03	16

Average No. of Years for 95% Confidence ± $0.50 = 42 years

The striking difference between 1,250 years of required record on a township basis and 42 years on an elevation group basis speaks for itself. It should be noticed that the elevation data is arranged by descending order of liability. When placed in order by elevation group, the mean loss costs rank in order from lowest to highest showing the close relationship of average loss cost to elevation.

* Losses divided by liability.

Both the mean and weighted loss costs are shown. The mean loss cost is the average of each year's loss cost irrespective of amount of liability; the weighted loss cost is the average loss cost with each year weighted by the amount of liability written.

Grouping townships by elevation group, then, gives us a large amount of statistical data capable of producing useful predictions, while at the same time each of the townships in the group is assumed to have the same degree of inherent hazard.

As additional meteorological knowledge becomes available, other factors can be used in classifying, and the result should be a net gain in predictability. If, for instance, it becomes established that the slope of the land in relation to the direction of hailstorm movement and loss cost are significantly correlated, each township could be classified by elevation *and by slope,* thus reducing the amount of unexplained variation.

3. *Frequent Rate Revision*

Because of the high degree of reliance which must be placed at present on empirical statistical data and the great length of record needed for predictability, it is essential to revise the rate structure frequently.

Consequently, every state is re-rated at least once every three years, and some states more frequently than this. The Association through its current statistical reporting is able to include the experience of the crop year just ended in the cumulative record. This has the advantage, not only of increasing the length of record an additional year, but also several additional benefits of a practical nature to be mentioned later.

As our physical understanding of hailstorms increases, it will result in more stability of the rate structure, and will reduce the need for frequent rate revisions.

B. *Basic Classifications in Rating*

Crop-hail rates are all applied on a minimum or class basis. However, the process of determining the class rate to charge is similar to that of schedule rating.

A crop-hail rate depends on three variables: 1) geographical location, 2) crop, and 3) policy form. A base rate is assigned to each geographical location and applies without alteration to one specific crop and to one specific form. Rates for other crops and policy forms are determined by percentage surcharges or credits from the base rate.

1. *Geographical*

From both practical and theoretical considerations, rates need to be quoted by subdivisions of a state. For the 1959 growing season, 64% of the nationwide premiums were written in states for which crop-hail insurance rates were quoted by governmental township (6 miles square), and 36% of the premiums were written in states where rates were quoted by county.

The geographical classification is the most important one and a base rate is determined for each location.

2. *Crop*

Within any geographical area different crops may be damaged in different degrees by the same hailstorm. Generally, sugar beets, potatoes, and sorghums are least affected by hail damage. Cotton is somewhat less hazardous than wheat, corn, and oats, and more damageable are barley, rye, soybeans, vegetables, and tobacco. Cantaloupes, cucumbers, tree fruits and nursery crops represent a high degree of hazard and usually take a considerable surcharge above the base rate.

The base rate determined for a geographical area is applied to the major crop grown within a state. Thus the base rate applies to corn in Illinois, wheat in Kansas, tobacco in North Carolina, and cotton in Texas.

The other crops are grouped by classes and the rate for each class is determined by multiplying the base rate by a factor either less than 1.00, or greater than 1.00, depending upon the relative hazard.

Insurance has been written on 194 different crops since 1948.

3. *Policy Form*

Generally speaking, the basic policy form nationwide is known as the Annual Percentage form. As previously explained, this form pays the same percentage of the insurance as the percentage of crop destroyed.

Usually, there is a minimum percentage of 5% (occasionally 10%) below which no payment is made. This is not a deductible, as full payment is made if the loss percentage exceeds the minimum. Thus, if the percent of crop destroyed is 3%, no payment is made; if the percent loss is 6%, the percent of insurance payable is 6%.

The purpose of the minimum loss provision is to keep loss adjustment costs at a reasonable level, and to discourage unjustified loss reporting in the hope of collecting part or all of the premium paid for the policy.

There are several rate-reducing endorsements which may be added to the policy. One of these is the Excess Over 10% Loss Endorsement (other percentages are sometimes used). This form provides that the farmer absorb the first 10% of the loss and the company pay the excess. The 10% is 10% of the insurance applying and is deducted from the total percent of crop destruction. If 35% of the crop is destroyed, the company pays 25% of the amount of insurance.

Another form used widely is the Excess Over 20% Loss—Increasing Payment Endorsement. This operates the same as the straight Excess over Loss form except that it provides that the percentage which the insured absorbs reduces as the percent of crop destruction increases. This is accomplished by deducting the 20% from the crop loss and multiplying the remaining percentage by 1.25. Thus, a 100% actual loss to the crop is computed by multiplying 80% by 1.25, which results in 100% of the insurance being paid.

Comparison of payments under the various rate-reducing forms and the annual percentage form are given below:

	Per Cent of Insurance Payable Under:		
Per cent of Crop Destroyed	*Annual Percentage**	*Excess Over 10% Loss*	*Excess Over 20% Loss—Increasing Payment*
3%	0%	0%	0%
6	6	0	0
10	10	0	0
20	20	10	0
40	40	30	25
60	60	50	50
80	80	70	75
100	100	90	100

* 5% minimum loss provision.

The advantage of the increasing payment provision is that the farmer may collect 100% of the insurance in the event of total loss, while under a straight Excess over 10% Loss form he is able to collect only 90% as a maximum. This raises the question in the mind of some insureds: "Why is the premium calculated by applying the rate to the total amount of insurance, when you can collect only 90% as a maximum?"

The rate for the Excess over 10% Loss form has been promulgated taking this into account, but it is difficult for many people to understand this. The increasing payment provision removes the objection, and there is actually no difference between it and a straight excess over 20% loss coverage, the rate for an Excess over 20% Loss—Increasing Payment form being precisely 25% higher than that for a straight Excess Over 20% Loss Endorsement. At each and every damage level a loss under either form will pay out exactly the same number of dollars per premium dollar received.

There are other types of rate-reducing provisions, but these are variations of the ones explained above.

Generally, the base rate is set for the Annual Percentage form and the rates for the other forms are obtained by multiplying by policy form factors which represent the relative hazard between forms. An exception to using the Annual Percentage form as a base would be in states where a majority of the premiums are written under one of the rate-reducing provisions, in which case the base rate would apply to that form.

C. *Conversion of Losses for Determination of Base Loss Cost*

It is desirable to develop base rates from all available experience regardless of crop insured or policy form written. This may be accomplished by adjusting the losses to a common base.

Since the base rate applies to that policy form and crop for which the

majority of premiums statewide is written,* the losses for all other policy forms and crops are adjusted to this level by using percentage rate differentials.

For instance in Nebraska, policies with the Excess over 10% Loss endorsement attached are considered 20% less hazardous than the Annual Percentage form. The policy form factor is 0.80 and the losses over the period of record for the Excess over 10% Loss form are divided by 0.80.

Generally,

converted losses (policy form) = policy form losses (period of record) ÷ policy form factor

In the same manner losses for crops other than the one to which the base rate applies are converted by dividing by the appropriate crop factor.**

Thus, corn grown in certain counties in Nebraska is considered 20% less hazardous than wheat, the crop to which the base rate applies. The crop factor for corn therefore, is 0.80, and the actual losses over the years for corn would be converted, or adjusted, by dividing by 0.80:

The general formula:

converted losses (crop) = crop losses (period of record) ÷ crop factor

When *both* policy form and crop losses need conversion, the work is simplified by using the formula:

converted losses (policy form and crop) = losses (period of record) ÷ policy form factor x crop factor

At present there are only a few states where crop loss conversions are made, while in the remaining states the losses are considered to be as if occurring on the crop to which the base rate applies. The reason for this is that statistics have been gathered by location and crop for most states only since 1948. Even in states in which crop losses are converted, it must be assumed that losses prior to 1948 are as if occurring to the base crop.

D. *Determination of Base Loss Cost*

Once the geographical area, policy form and crop to which the base rate will apply have been determined, a base loss cost for this rating unit is calculated using the converted losses.

In Kansas an individual base rate applies to wheat written under the Annual Percentage form for a specific governmental township. The base loss cost for each township in Kansas is calculated using three factors:

1. *Individual township loss cost:* 25% of the base loss cost is determined by the all-time loss cost for the township itself. Township statistics have been gathered in Kansas since 1924, and the individual township loss cost is

* Actuarially, the policy form or crop to which the base rates apply does not matter, since the percentage differentials for all the policy forms and crops remain in a constant relationship. However, from a practical viewpoint the use of the base rate for the policy form and crop most widely insured simplifies explanation to insurance departments and the insuring public.

** Conversion of losses is accomplished by using the same crop and policy form factors as used in calculation of the expanded rate schedule. Explanation of how these differentials are developed is explained in "Policy form and crop factors", see pages 135ff.

derived by dividing the accumulated losses (converted) by the accumulated liability.

2. *County loss cost:* 25% of the base loss cost results from the all-time experience of the county within which the township to be rated is located. Accumulated converted losses of all townships within the county are divided by the accumulated liability of the same townships to obtain the county loss cost.

3. *Elevation loss cost:* 50% of the base loss cost is derived from the all-time experience of the elevation group to which the township to be rated belongs.

As mentioned previously, excellent correlation has been attained between the elevation above mean sea-level and township loss cost. Each of the 2,306* townships in Kansas has been assigned to an elevation group, the groups being arranged in 100 foot intervals.

Table 1 shows the accumulated liability, converted losses, and elevation group loss costs for Kansas. Also shown is the smoothed elevation group loss cost obtained by fitting a straight-line (least-squares method) to the actual elevation group loss costs. Chart 3 shows the excellent fit which results, the correlation coefficient being + .98. Charts 4 and 5 show similar information for Nebraska and North Dakota.

The correlations which have been obtained are unusually high, though it must be realized that the calculations involve a correlation of *means* with the elevation, rather than individual township loss costs. This results in higher values for the correlation coefficients; on an individual township basis the correlation coefficient should be somewhat less.

* There are actually 2,561 townships in Kansas, but 255 of these are partial townships having an area of 18 square miles or less. These have been combined with adjacent townships for rate analysis purposes. The resultant rate for the "partial" township is, consequently, the same as for the "master township". In the printed rate schedule all 2,561 townships are shown with base rates applying.

Table 1. Loss Cost by Elevation Group, Kansas, 1924-1959.

X Elevation (in hundred feet)	No. of Townships	Liability (nearest $1000)	Y Loss Cost	Yc Computed Loss Cost*
7	2	250	.40	.24
8	40	10,633	1.05	.59
9	100	27,189	1.04	.94
10	187	51,478	.87	1.29
11	147	73,596	1.32	1.64
12	137	95,943	2.30	1.99
13	168	155,385	2.25	2.34
14	173	145,735	2.59	2.69
15	125	113,266	3.30	3.04
16	78	72,939	3.80	3.39
17	76	63,808	3.93	3.74
18	76	47,440	3.72	4.09
19	73	61,655	4.03	4.43
20	64	57,980	4.16	4.78
21	64	51,197	5.68	5.13
22	66	45,542	5.48	5.48
23	48	25,129	6.03	5.83
24	43	21,388	7.49	6.18
25	57	30,949	6.53	6.53
26	56	32,645	5.70	6.88
27	56	30,556	7.34	7.23
28	54	26,356	8.25	7.58
29	63	28,336	8.69	7.93
30	59	24,889	8.02	8.28
31	48	19,641	8.55	8.63
32	38	14,284	7.93	8.98
33	46	11,977	8.46	9.33
34	38	12,128	10.53	9.68
35	41	14,389	10.57	10.03
36	32	12,029	10.97	10.38
37	21	7,197	10.17	10.73
38	16	5,923	9.95	11.08
39	14	4,219	10.09	11.42
Total and Average for State-	2306**	1,396,071	$ 4.15	

*Yc= 0. 34951X- 2.20603. Each loss cost was weighted by elevation group liability in deriving equation.

** Does not include 255 partial townships. Experience of partial townships, however, is included with that of their "master" townships and is, therefore, accumulated in the above table.

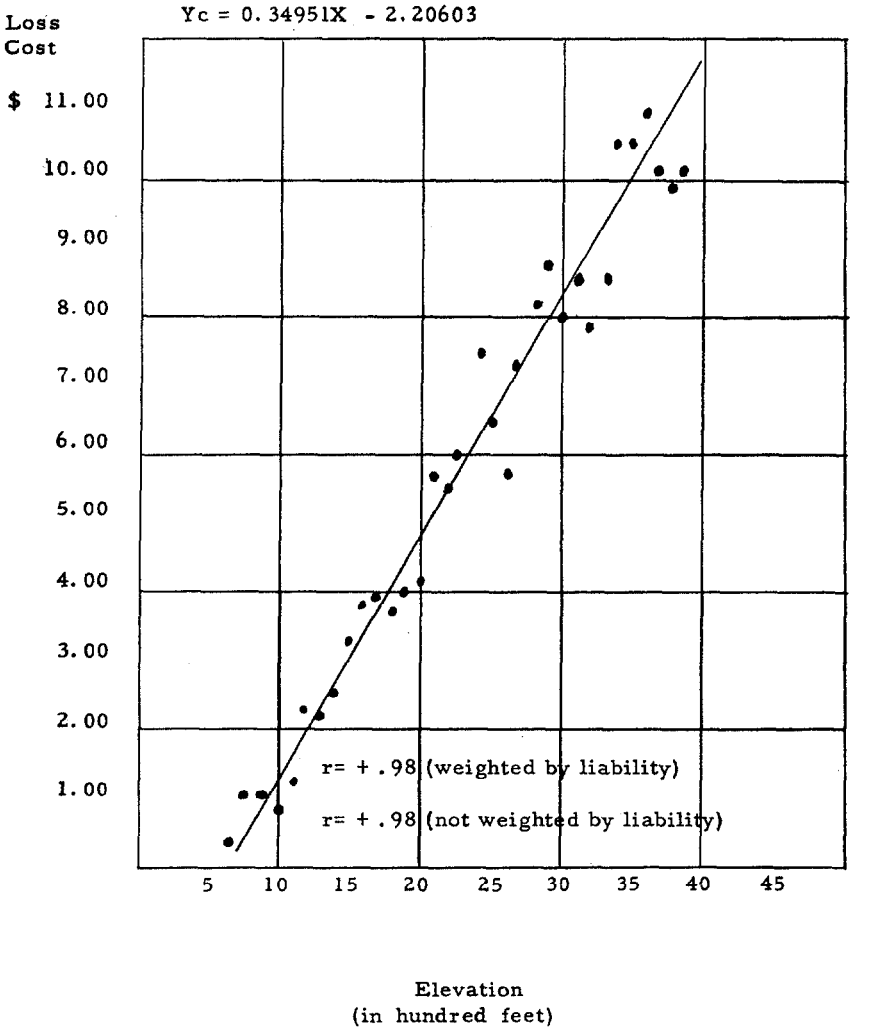

Chart 3. Loss Cost by Elevation Group, Kansas, 1924-1959. Each point represents the loss cost for all townships in that elevation group obtained by dividing the total losses of those township, 1924-1959, by the total liability of the same townships, 1924-1959.

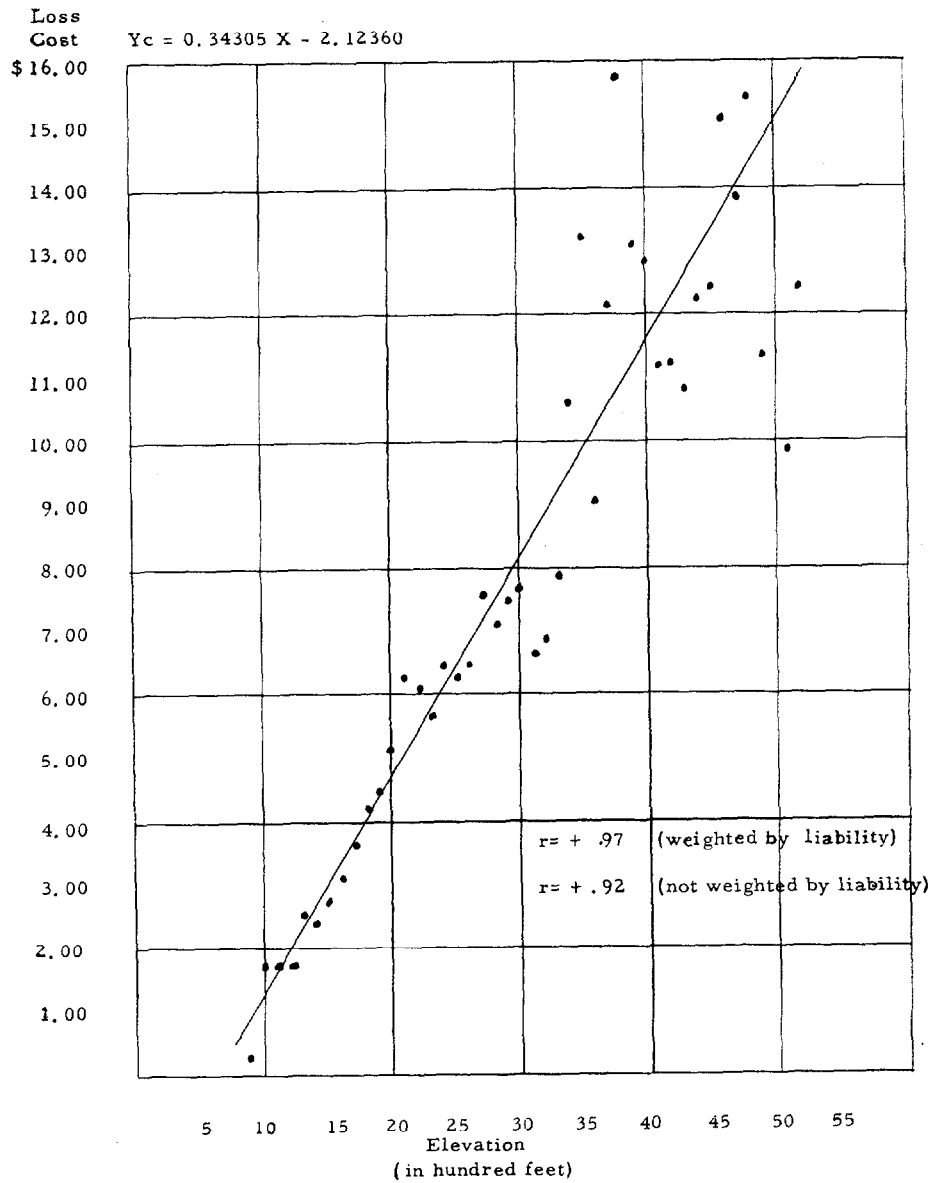

Chart 4. Loss Cost by Elevation Group, Nebraska, 1924-1959. For explanation see Chart 3.

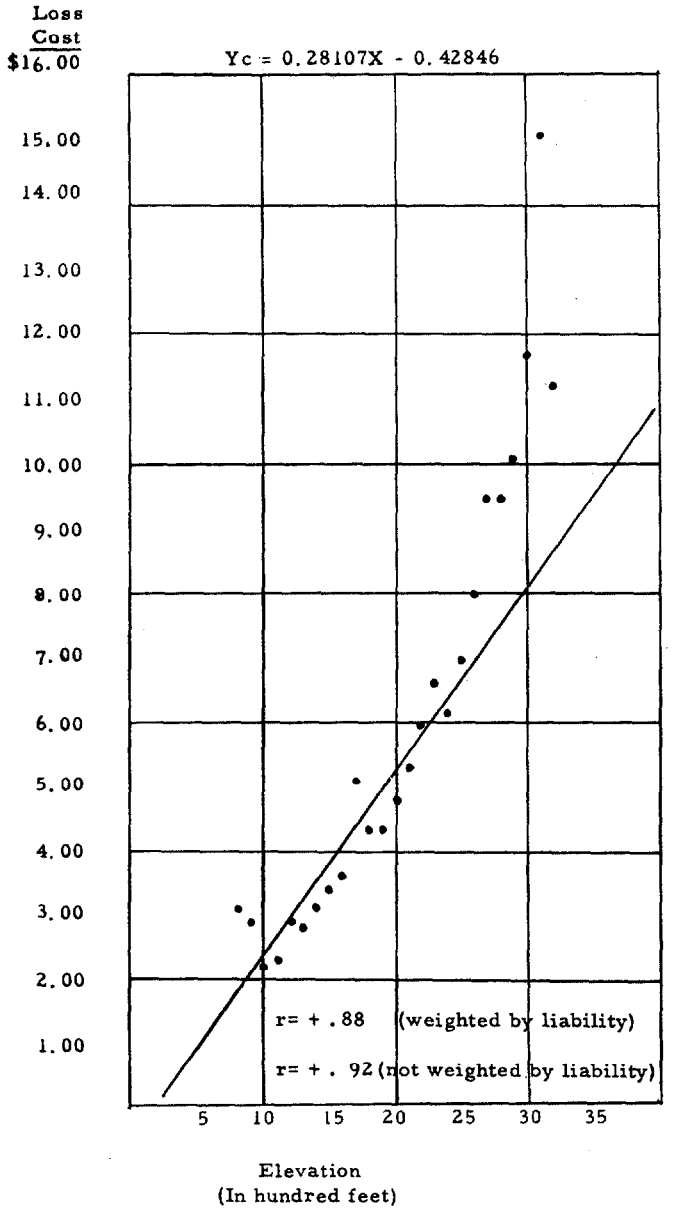

Chart 5. Loss Cost by Elevation Group, North Dakota, 1924-1959. For explanation see Chart 3. A curved line would appear to fit the data better, and would increase the correlation coefficient.

The introduction of county and township loss cost into the rating formula was done in an attempt to partially compensate for possible unknown variance, as well as to satisfy long-established customs in rating by not deviating too radically and too fast from former rating methods.

For Kansas then, the formula for base loss cost is as follows:

$$\begin{aligned}\text{base loss cost} = {} & 25\% \times \text{individual township loss cost} \\ & + 25\% \times \text{county loss cost} \\ & + 50\% \times \text{elevation group loss cost}\end{aligned}$$

Example: *Reno County, 26S, 8W.*

liability, 1924-1959	$1,534,062
converted losses, 1924-1959	53,080.25
individual township loss cost	$3.46

Reno County, all townships

liability, 1924-59	$50,717,707
converted losses, 1924-1959	1,938,542.68
county loss cost	$3.82

Elevation group 1600 ft.
(26S 8W is in this group)
see Table 1
computed elevation loss cost $3.39

$$\text{base loss cost} = (.25)\ (3.46) + (.25)\ (3.82) + (.50)\ (3.39)$$

$$\text{base loss cost} = \$3.52$$

This method of calculating the base loss costs using elevation as a major factor applies only to certain of the prairie states, although in these states 45% of the 1959 crop-hail United States premiums were written.

In the rest of the states base loss costs are derived in various other ways. For instance, in North Carolina the basic geographical area is county, and the basic crop is tobacco. The policy form to which the base rate applies is the annual percentage form. The conversion of losses is done in the usual manner, but the base loss cost for each county is calculated by simply dividing the accumulated losses over the years by the accumulated liability over the same period.

Many states use this method, and in the rest not using the elevation factor there are a few other variations as to the geographical area used. All of these calculate a base loss cost by the same method as used in North Carolina. There is little need to go into further details in these cases as it would add little to what has already been presented.

E. *Expense-Loading and Calculation of Required Base Rate*

The rate to be charged must include, of course, a loading to compensate the insurer for commissions paid to agents, taxes, and company disbursements including field, home office, and other overhead expenses. Loss adjustment

expenses are not included in crop-hail insurance loss figures, so these too must be added. In addition the rate must allow for a fair gain from underwriting and a contribution for a catastrophe reserve.

The average commission paid by all companies varies between states, and by rate classification within certain states. Thus in Kansas for rates $10.00 and under per $100.00 of insurance, the average commission paid by companies is approximately 20%. For rates $10.00 to $15.00 it is 15%; and for rates above $15.00, 10%.

The other company expenses nationwide are estimated at 22% of the premium dollar, and the expected gain from underwriting and contribution to catastrophe reserve at 6%.

Thus, the rates as calculated must anticipate the following loss ratios in Kansas*:

Kansas Rates	*Anticipated Loss Ratio*
$10.00 and under	52%
$10.01 through $15.00	57%
$15.01 and over	62%

The required base rate is obtained by dividing the base loss cost by the anticipated loss ratio (expressed in decimal form). The formula is:

$$\text{required base rate} = \text{base loss cost} \div \text{anticipated loss ratio}$$

The required base rate is usually rounded to the nearest 20¢ below $4.00 to the nearest 50¢ between $4.00 and $8.00, and to the nearest $1.00 above $8.00.

In states with the extra harvesting expense allowance or fire coverage on growing crops, rates are established separately for these additional coverages. They are added to the required hail base rate (calculated to the nearest cent) and the resultant combined required rate is rounded as mentioned in the preceding paragraph.**

Example: Reno County, 26S, 8W

base loss cost $3.52

anticipated loss ratio 52%

$$\text{required base rate} = \frac{3.52}{.52} + .20 \text{ (extra harvesting expense)} + .10 \text{ (fire coverage)}$$

required base rate = 6.77 + .20 + .10 = $7.07

rounded required base rate = $7.00

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* The average loss ratio anticipated for the entire United States is approximately 52%.

** In actual practice a table is used showing ranges of hail loss costs and giving the required rate in rounded form for each range.

F. *Development of Proposed Base Rate*

The calculation of *required* base rates provides the first stepping stone to the promulgation of new proposed rates. The base rates as proposed are not always the same as the required rate for the reasons indicated below:

1. *Judgment rates:* Many of the required rates are for areas where little business has been written, and, consequently, the base loss costs from which they are derived are neither representative nor significant. For instance in Nebraska there are 2179 townships for which required base rates are calculated. 1187 or 54% of these have had 97% of the total insurance written 1924-59. The other 992 townships account for only 3% of the insurance, and each individual township's base loss cost is meaningless due to the sparsity of data.

   Therefore, an arbitrary definition is established to designate "judgment" townships. The method now used consists of taking the cumulative amount of insurance over the period of record for each township. In Nebraska if this figure is under $150,000, the township is rated on "judgment" basis; if $150,000 or over, the township's proposed base rate is developed using all of the pertinent rules and formulas. The proposed base rate for a "judgment" township may be set at any figure, but usually rates of contiguous areas play a large part in its determination.

2. *Minimum and maximum rates:* Another factor which prevents the proposed rate from always equaling the required rate is the minimum and maximum rates set for each state.

   Even eliminating townships with small amounts of cumulative liability written, the required base rates range from very low figures to excessively high values for any state in question. It has been found necessary to establish a minimum base rate and a maximum base rate for each state. For example, in Kansas no proposed base rate may be less than $3.00 per $100.00 of insurance nor more than $20.00.

3. *Percentage limitations on rate changes:* During the development of a methodical method of crop-hail insurance rating, it became apparent that it was not possible from a public relations viewpoint to proceed from the present rate to the required rate in every case. Due to the catastrophic nature of crop-hail insurance this could well involve increases of rates ranging from 100% to 200%.

With regard to rate decreases the same problem did not manifest itself as the all-time loss cost with good experience drops rather slowly from year to year. However, the setting of a maximum percentage increase in rates necessitated that a corresponding maximum percentage decrease be set in order to keep the state-wide average rate at a proper level. To allow every rate decrease without limitation, and at the same time to restrict rate increases produces a constantly deteriorating rate level.

The rules of the Association in most township states provide that the maximum rate increase cannot exceed 60%, and the maximum rate decrease can-

not be more than 30%. The relationship of 60% increase to 30% decrease has been developed from experience as that which is necessary to keep the rate level in balance.

A further development came at a later date. Situations developed where a devastating hailstorm resulted in required rate boosts of more than 100%. A rate boost of 60% was actually given and several years later at a subsequent rate revision, the required rate was still above the rate in effect. However, the experience had been excellent since the last rate revision, even to the point of no losses. At this time the insureds could well ask "Why do you plan another rate increase? Three years ago you raised my rate and we have had no losses since."

To answer this problem the loss ratio since last revision was introduced to influence the magnitude of rate increases and decreases. A bad loss ratio since last rate revision results in a maximum rate increase, a good loss ratio in a lesser increase, or possibly no increase at all.

Similarly, for rate decreases it does not appear sensible for rates to be reduced if a bad loss ratio has ensued since the last change in rates, even if the required rate is less than the present rate.

A further refinement in the percentage limitation table came about through consideration of the relationship of the required rate to the present rate. The further the spread between these two figures, the greater the need for rate adjustment. Consequently, the ratio of the required rate to the present rate was also made part of the table. If the required rate is considerably above the present rate, a larger rate increase is permissible than if they are close together. The same reasoning applies to rate decreases.

A percentage limitation table presently in use for Kansas is shown in Table 2.

A formal table is used only in states where base loss costs are calculated for each township. In states having rates set by county or area it has been found sufficient to use a somewhat less rigorous approach. A typical paragraph in the explanatory manual for a county-rated state reads:

"From a consideration of calculated required rates, amount of liability written over the period and in recent years, rates in effect during the past season, recent loss experience, etc., a rate is recommended for each area."

With a limited number of required rate and present rate combinations it is possible to apply in each individual case the same reasoning outlined above without having rigid rules.

With thousands of townships it is not possible to do this manually and a formal table is used which is adaptable to machine processing. (See Chart 6.)

4. *Exceptions to the rating system:* It is realized that no matter how comprehensive a rating system is, that there are occasions when the rates as determined are not considered as reliable. To take care of this contingency the rate system manuals of the various states have a provision whereby exceptions to the rating system may be made.

   The use of this device, however, must be watched carefully lest the use and acceptance of the rating method be damaged. Exceptions should be rarely made, and when made, supported with sound reasons.

Table 2. *Percentage Limitations on Rate Changes, Kansas, 1960 Filings*

Maximum Increases In Rates

Required Rate— % Higher than Present Rate:	*Loss Ratio Since Last Rate Revision:*	*Maximum Rate Increase:*
0-39%	0-49%	No increase
	50%-up	Increase to required rate
40-69%	0-29%	No increase
	30-49%	20%
	50-79%	40%
	80%-up	Increase to required rate but not more than 60%
70%-up	0-19%	No increase
	20-29%	20%
	30-49%	30%
	50-79%	50%
	80%-up	60%

Maximum Decreases In Rates

Required Rate— % Lower than Present Rate:	*Loss Ratio Since Last Rate Revision:*	*Maximum Rate Decrease:*
0-29%	0-29%	Decrease to required rate but not more than 10%
	30%-up	No decrease
30-59%	0-29%	20%
	30-59%	10%
	60%-up	No decrease
60%-up	0-29%	30%
	30-59%	20%
	60%-up	No decrease

Example: Reno County, 26S, 8W

premiums since last rate revision	$6,066.84
hail losses " " " "	$2,092.32
loss ratio " " " "	34.49%
present base rate	$7.50
required base rate	$7.00
% required rate lower than present rate	6⅔%
maximum decrease in rate permissible	no decrease permissible
Therefore, proposed base rate	$7.50

G. *Policy Form and Crop Factors*

The determining of the proposed rate accomplishes the second major step. The last stage in the production of the final rate schedule involves the expanding of base rates to cover all of the various crops, policy forms, and additional coverages (if any).

1. *Policy form rate factors*

To expand the base rate to apply to each policy form necessitates the determination of the policy form factor.

Where the amount of insurance written on other policy forms is small, the policy form factors are set by judgment. Increasingly, however, statistical analyses which have been developed are used, and these allow a more factual determination.

a. *Percentage loss summary:* One type of analysis involves taking each proof of loss and recalculating it as if another policy form applied. For instance, if a $1000 policy has a 30% loss under the Annual Percentage form, the total loss would be $300. However, if this had been an Excess over 10% Loss form, the loss would be 20% (30% − 10%) or $200. Under an Excess over 20% Loss Increasing Payment form the loss calculation would be (30% − 20%) × 1.25 = 10% × 1.25 = 12½% or $125.00.

Fortunately, we have detail loss records on magnetic tape, and computations are made rapidly. After the individual loss calculations are completed, computed losses are added for each policy form, and the total is expressed as a percentage of the base policy form. This, then establishes the basis for setting policy form factors.

You are able to go only from broader coverage policies to more restricted policies, not reverse. Thus, you may calculate Excess over 10% losses from Annual Percentage form losses, but you cannot compute Annual Percentage form losses from Excess over 10% losses. In the latter case you are missing those instances when the loss percentages are under 10%, and are not reported.

Another caution must be observed. There may be a bias in estimating Excess over 10% losses from Annual Percentage form data due to the human element in loss adjustments. It is not inconceivable that an inexperienced loss adjuster may tend to be more liberal in evaluating a damage to a crop which has an Excess over 10% Loss Endorsement covering, than when full cover attaches. Theoretically, this should not happen and scientific loss adjustment procedures minimize its occurrence, but mistakes and pressures do happen.

Usually the computations are restricted to the base crop, and the policy form relationships are assumed to hold state-wide. However, recently a summary was subdivided by rate area, and this brought out a close relation between rate level and the amount of credit which should be allowed for the excess over loss endorsements: the higher the rate, the less the percentage credit. The Kansas percentage loss summary is shown in Table 3.

b. *Policy form comparison:* Another method of determining policy form factors is to tabulate the actual experience of the various forms over the period

CROP-HAIL INSURANCE ACTUARIAL ASSOCIATION

RATE ANALYSIS

YEAR 1960 STATE Kansas BASED ON PERIOD 1924-1959 PREVIOUS ANALYSIS BASED ON PERIOD 1924-1958

STATE	COUNTY CODE	COUNTY NAME	TOWNSHIP	RANGE	LIABILITY	ELEVATION	SIGNIFICANT YEARS	LOSS RATIO SINCE LAST ANALYSIS	LOSS COST TOWNSHIP	LOSS COST ELEVATION	LOSS COST COUNTY	LOSS COST FINAL AVER.	RATE DESIRED	RATE PRESENT	RATE PROPOSED
15	155	RENO	23S	6W	16632 43	15	34		304	304	382	324	650	700	650
15	155	RENO	23S	7W	33666 09	16	36		469	339	382	388	750	800	750
15	155	RENO	23S	8W	26272 43	17	35		517	374	382	412	800	900	800
15	155	RENO	23S	9W	22592 96	17	35		621	374	382	438	900	900	900
15	155	RENO	23S	10W	10670 17	18	31		342	409	382	386	750	750	750
15	155	RENO	24S	4W	14845 90	14	35	18	208	269	382	282	550	600	550
15	155	RENO	24S	5W	6513 06	15	27		153	304	382	286	600	650	600
15	155	RENO	24S	6W	14031 26	16	34		347	339	382	352	700	800	700
15	155	RENO	24S	7W	26802 18	16	35		548	339	382	402	800	900	800
15	155	RENO	24S	8W	24601 10	16	36		333	339	382	348	700	750	700
15	155	RENO	24S	9W	8092 41	17	31		330	374	382	365	750	800	750
15	155	RENO	24S	10W	6992 17	17	31		220	374	382	338	700	750	700
15	155	RENO	25S	4W	23937 90	15	36		222	304	382	303	600	650	600
15	155	RENO	25S	5W	13672 69	15	34		306	304	382	324	650	700	650
15	155	RENO	25S	6W	9179 18	15	25		618	304	382	402	800	900	800
15	155	RENO	25S	7W	11512 62	16	31	5	381	339	382	360	700	800	700
15	155	RENO	25S	8W	18509 20	16	34	16	489	339	382	387	800	900	800
15	155	RENO	25S	9W	17423 46	16	36		398	339	382	365	750	800	750
15	155	RENO	25S	10W	5659 45	17	27		189	374	382	330	650	700	650
15	155	RENO	26S	4W	19629 88	15	34	10	294	304	382	321	650	700	650
15	155	RENO	26S	5W	11564 02	14	29	2	227	269	382	287	600	600	600
15	155	RENO	26S	6W	6555 84	16	17		246	339	382	327	650	750	700
15	155	RENO	26S	7W	4847 67	16	14		152	339	382	303	600	700	650
15	155	RENO	26S	8W	15340 62	16	36	34	346	339	382	352	700	750	750
15	155	RENO	26S	9W	20668 50	17	36		313	374	382	361	750	800	750
15	155	RENO	26S	10W	9595 01	17	30		189	374	382	330	650	700	650
					507177 07	*									
15	157	REPUBLIC	1S	1W	11122 25	16	35	15	665	339	366	427	900	900	900
15	157	REPUBLIC	1S	2W	5515 21	16	28	131	662	339	366	427	900	900	900
15	157	REPUBLIC	1S	3W	7508 29	16	28	65	277	339	366	330	650	700	700
15	157	REPUBLIC	1S	4W	12265 11	16	32	128	396	339	366	360	700	750	750
15	157	REPUBLIC	1S	5W	13152 70	15	28	33	419	304	366	348	700	750	750
15	157	REPUBLIC	2S	1W	6491 87	15	31	69	145	304	366	280	550	600	600
15	157	REPUBLIC	2S	2W	7526 30	16	29	32	364	339	366	352	700	750	750
15	157	REPUBLIC	2S	3W	5580 39	16	27	53	322	339	366	342	700	750	750
15	157	REPUBLIC	2S	4W	11642 04	15	31	9	336	304	366	328	650	700	650
15	157	REPUBLIC	2S	5W	13451 07	15	31	5	344	304	366	330	650	750	700
15	157	REPUBLIC	3S	1W	7474 03	16	29	9	271	339	366	329	650	750	700
15	157	REPUBLIC	3S	2W	5895 45	16	27	8	251	339	366	324	650	700	650
15	157	REPUBLIC	3S	3W	4668 78	16	27		183	339	366	307	600	700	650
15	157	REPUBLIC	3S	4W	7028 88	15	26		300	304	366	319	650	700	650
15	157	REPUBLIC	3S	5W	17707 31	15	31	2	286	304	366	315	650	700	650
15	157	REPUBLIC	4S	1W	9636 41	14	31		288	269	366	298	600	600	600
15	157	REPUBLIC	4S	2W	9692 46	14	28	5	627	269	366	383	750	800	750
15	157	REPUBLIC	4S	3W	9781 48	14	28	1	487	269	366	348	700	700	700
15	157	REPUBLIC	4S	4W	6810 58	14	24		374	269	366	320	650	650	650
15	157	REPUBLIC	4S	5W	11320 36	15	32		196	304	366	293	600	650	600
					184268 37	*									
15	159	RICE	18S	6W	9023 32	17	27		186	374	278	303	600	650	600
15	159	RICE	18S	7W	11578 52	16	26		115	339	278	268	550	600	550
15	159	RICE	18S	8W	11997 37	17	28		127	374	278	288	600	600	600

Chart 6. A Sample Sheet of the Tabulating Machine Print-Out of Kansas Rate Analysis for the 1960 Season. All computations and application of rules are done by machine for the large township states.

of record. It is necessary to classify the experience by rate area, and then to calculate the percentage relationship for each of these rate levels. The state-wide average is calculated as an average of the computed percentages. Generally, only the experience of the base crop is used.

Because the writings of crop-hail insurance tend to be concentrated in one policy form in a given area, the results of policy form comparison summaries have in most instances been disappointing. The percentage loss summary has produced much more useful results.

2. *Crop factors*

Different crops are assigned to crop classes according to degree of hail hazard. For instance, in Kansas there are about 85 crops divided into 7 crop classes (including a catch-all category for crops not specifically named in the schedule).

A crop factor is determined for each crop class. Again, as with policy form factors, where sufficient experience has not been accumulated, factors are set by judgment.

When ample experience is available, crop comparison summaries are able to be produced similar to the policy form comparison summaries mentioned above. Experience over the period of record for each of the major crops is classified by rate level. Ratios of the loss cost of each crop to the base crop loss cost are calculated for each level, and state-wide average calculated from the ratios.

In contrast to the policy form comparison summary, the results obtained from the crop summary have been most helpful. An example of a crop comparison summary is shown in Table 4.

Table 3. Percentage Loss Summary, Kansas, Wheat, 1951-1957

1958 Rate Area	Ann. % Form Loss Cost	XS 10 Basis Computed Loss Cost *	% of Ann. % Loss Cost	XS 20 IP Basis Computed Loss Cost *	% of Ann. % Loss Cost
$ 3.00	$ .62	$ .32	52%	$ .22	35%
3.25	1.00	.52	52	.37	37
3.50	1.09	.52	48	.36	33
3.75	1.28	.72	56	.53	41
4.00	1.44	.81	56	.62	43
4.50	1.48	.81	55	.57	39
5.00	2.37	1.40	59	1.09	46
5.50	2.17	1.31	60	1.07	49
6.00	2.54	1.55	61	1.26	50
6.50	2.20	1.27	58	.99	45
7.00	2.98	1.80	60	1.39	47
7.50	3.30	1.97	60	1.55	47
8.00	3.61	2.24	62	1.83	51
9.00	3.96	2.44	62	1.98	50
10.00	5.11	3.38	66	2.90	57
11.00	5.09	3.33	65	2.77	54
12.00	8.21	6.09	74	5.71	70
13.00	8.19	5.94	73	5.48	67
14.00	9.03	6.85	76	6.49	72
15.00	6.31	4.44	70	3.96	63
16.00	9.12	6.79	74	6.44	71
17.00	14.48	11.65	80	11.94	82
18.00	15.76	12.60	80	12.49	79
19.00	14.37	11.36	79	11.22	78
20.00	12.86	9.71	76	9.17	71
Entire State	$3.43	$2.27	66%	$1.96	57%

* Annual Percentage Form losses recalculated.

Table 4. Crop Comparison Summary, Iowa, 1948-56, Annual Percentage Form Data.

Rate Area (1956)	Liability: (Base Crop) Corn	Soybeans	Loss Costs: Corn	Soybeans	Soybean Loss Cost as % of Corn Loss Cost:
$1.70	$ 4,340,869	$ 1,555,967	$ .41	$ .66	161%
1.80	5,878,712	636,343	.59	.71	120
1.90	2,894,013	322,528	1.54	2.99	194
2.00	32,456,646	5,189,834	1.21	1.87	155
2.25	9,994,709	2,398,391	.41	1.18	288
2.50	29,276,581	6,303,692	1.31	2.39	182
2.75	11,549,874	2,905,445	.88	1.93	219
3.00	35,609,334	7,768,493	1.59	3.94	248
3.25	17,103,137	3,452,644	2.15	4.48	208
3.50	6,294,437	562,976	2.97	5.93	200
3.75	9,628,456	1,897,193	2.46	5.64	229
4.00	18,954,985	3,135,948	2.54	4.79	189
4.50	6,346,325	958,415	2.22	4.82	217
5.00	7,705,188	1,282,665	3.29	5.17	157
5.50	2,502,851	610,351	3.17	6.03	190
6.00	1,537,098	463,523	1.65	3.29	199
7.00	26,318	7,163	5.96	12.60	211
7.50	1,891,194	577,006	4.59	8.45	184

Average Indicated Crop Factor- 204 % or 2.04
(Weighted by soybean liability)

H. *Additional Coverages*

The basic crop-hail policy has additional coverages which are either included, or may be added on an optional basis, but these vary from state to state.

The extra harvesting expense allowance is included in many states. This provides for an additional loss award when the percent loss to the crop exceeds 70%. The rate for the extra harvesting expense feature is included at the time the required base rate is calculated.

Fire coverage on growing crops is part of the policy in most states, and, again, the rate is included in the calculated required base rate.

Kentucky, Tennessee, and North Carolina have available a policy form which gives protection to the harvested tobacco crop against the perils of windstorm, explosion, riot, riot attending a strike, civil commotion, and vehicles. This is in addition to the perils insured against in the standard crop-hail insurance policy, and coverage on the harvested tobacco cannot be written unless the growing crop is also insured against hail damage.

In this case a flat rate is added to the crop-hail rate, and the final rate is quoted in the rate schedule as a single, indivisible rate.

Similarly, there are 78 counties, situated in Illinois, Indiana, Iowa, Minnesota, and Ohio, for which an experimental coverage is offered against crop failure. Known as Crop Failure Insurance, it gives disaster protection, as a farmer must lose a substantial part of his normal crop before he is eligible to receive loss payment. The perils insured against include drought, excessive heat, flood, excessive moisture, insect infestation, plant disease, wildlife, wind, tornado, sleet, hurricane, frost, freeze, and snow; they are referred to as "B" perils, the "A" perils being those covered in the standard policy. This endorsement, which must be attached to a crop-hail insurance policy, has separate rates quoted and the premium is calculated as an additional amount to be paid along with the crop-hail premium.

I. *Preparation of Expanded Rate Schedule*

The expansion of the rate schedule to cover every crop and all policy forms involves multiplying the base rate for each location by the crop factor, rounding to the nearest 10¢; then multiplying these rates by the policy form factors, and again rounding.

Example: Reno County, 26S, 8W

Crop class:*	Crop factor:	Proposed base rate: $7.50 Annual % form proposed rate:
Class W	1.0	$ 7.50
Class D	1.5	11.30
Class E	2.0	15.00
Class F	2.2	16.50

* Only selected classes used for illustration.

Policy form factors:

Excess over 10% loss	0.71
Excess over 20%—increasing payment	0.62

	W	D	E	F
Ann. %	$7.50	$11.30	$15.00	$16.50
XS 10	5.30	8.00	10.70	11.70
XS 20-IP	4.70	7.00	9.30	10.20

Different ways are used for publishing the rates to be charged. In a state with townships a list of base rates by township is shown, and supplemental tables are used to determine the final rate according to location, policy form, and crop.

In other states where base rates are not so numerous, complete rate tables by location, policy form and crop are set forth which enables the agent to find the appropriate rate immediately.

One limitation is imposed on all schedules. A rate in excess of $24.00 is never quoted; a coverage requiring more than this is listed as "insurance not offered". Also, no rate less than $1.00 is quoted; and in this case, the schedule has a footnote stating that this is a minimum rate.

This, then, with several pages of rules and information, a table of contents and an index, constitutes the crop-hail insurance rate schedule.

IV. RESEARCH TO IMPROVE RATES

There are certain significant dates which stand out in the history of crop-hail insurance representing major steps forward in scientific rating:

1915. The first organized effort of hail-writing companies to gather statistics. The Western Hail and Adjustment Association was formed in this year, and statistics by county gathered.

1924. The realization that experience should be accumulated by geographical areas smaller than counties. Companies reported for certain major-writing states liability, premiums and losses by governmental township (6 miles by 6 miles).

1932. General revision of rating procedures to use township data. Arrangements made to accumulate data by use of tabulating machines. Policy forms and endorsements were clarified by including clauses as to methods of determining losses on specific kinds of crops.

1948. Crop-hail Insurance Actuarial Association started to gather crop-hail insurance statistics nationwide by location, policy form, and crop. Mathematical rating formulas devised and rating system manuals developed. Use of elevation areas: the first instance of using a *physical* classification instead of a strictly *location* classification.

If one would ask the most important difference between fire insurance rating and crop-hail insurance rating, the answer would be that "crop-hail

insurance rates have been based on primarily *statistical* considerations, while fire insurance rates have been developed mainly from a consideration of *physical* factors."

This is not to disparage either method. Indeed, the reasons for the two approaches originated in the unique factors affecting the two types of insurance.

The Analytical System uses a physical classification method based on occupancy, exposure, structure, and protection. Much engineering knowledge was available in earlier days to enable predictions to be made as to which risks were more hazardous than others. On the other hand, the problem of collecting detailed statistics (especially without the aid of modern data processing systems) was enormous. Numerous parameters existed with the further complication that large amounts of insurance were written at specific rates, rather than at class rates. Schedule rating reduced considerably the number of homogeneous statistical units capable of being mathematically analyzed.

Therefore, the approach was primarily to set rates based on *physical* factors and then to use very general statistical data to evaluate total results.

The opposite situation prevailed in crop-hail insurance. Until 1948 there was no knowledge available to indicate why it hails more in one place than another. It was impossible to construct a crop-hail insurance rate schedule on an *a priori* basis. Only after experience was gathered was it possible to make rates in other than a blind, guessing way.

Fundamentally, then, fire insurance rates have an *a priori* emphasis (deduction of rates from principles assumed), while crop-hail insurance rates have an *a posteriori* emphasis (rates cannot be known except through experience).

Actually, the argument as to which is the best procedure is senseless. Improved scientific rating in either case requires a merging of the two approaches. A physical classification technique without subsequent verification of assumptions by detailed statistical data and analysis is just as faulty as blind reliance on statistical data where *real* differences cannot be distinguished from *random* differences.

The key to improved crop-hail insurance rating lies in the development of much additional meteorological knowledge with regard to why it hails more in one place than another.

The first important breakthrough achieved was the use of the elevation factor in the states to which it was applicable. The use of this physical classification *together with* the excellent statistical data gathered over the period 1924 to date has imparted a degree of stability to rates in those selected states not possible before. Examples of the close relationship of elevation to loss cost have already been given in Part III.

To date the elevation relationship has been found to apply only in the states of Kansas, Oklahoma, Nebraska, South Dakota, North Dakota, Minnesota, and Iowa. In all other rating territories with the exception of Illinois (see below), a statistical approach is the only one that we have had and have.

As mentioned, up until 1948 very little was known in meteorology with regard to hailstorms. Since that time, and especially within the last five years,

the understanding of severe local storms of all kinds, including hail, has increased immensely. A number of scientists have become interested in hailstorms and the outlook for the future is encouraging. One of the main contributing causes of the rising interest in this field has been the constant encouragement of the Crop-Hail Insurance Actuarial Association. Both the Manager and Assistant Manager of the Association are professional meteorologists, and they have consistently kept the importance of hail alive in the minds of other meteorologists with whom they have come into contact.

A significant step was taken by the Association in 1957 when a research contract was negotiated with the Meteorology Division of the Illinois State Water Survey. Headed by a very competent meteorologist versed in the new field of "radar" meteorology, this unit has made many contributions to knowledge about severe local storms.

The Illinois State Water Survey's project includes not only a study of Illinois hailstorms, but the general understanding of hailstorms, and the relationship of the occurrence of these with topographical and other physical parameters. Even with very inadequate statistical experience (township data is only available in Illinois from 1948 on) a marked improvement in the Illinois rating system was made possible for 1960 as a result of their two years of study.

The research program of the Asociation was expanded in 1960 to include the study of two additional states.

Although the study of physical factors affecting the occurrence of hailstorms is the major need for improvement of the crop-hail rate structure, additional progress is also possible by using more advanced methods in the statistical analysis of the vast amount of accumulated crop-hail insurance statistics. To date only the simplest forms of statistical analysis have been used.

Increased use of measures of variance, correlation coefficients, and time series analysis will elucidate relations which are now obscured by the mass of data.

The "normal curve" assumption of conventional statistics, however, does not fit crop-hail insurance data well. Much work will be needed to develop proper techniques to handle the extremely "skewed" nature of hail loss costs. Gumbel's* work on statistics of extremes will be useful in this regard.

Multiple correlations to develop the various interrelations between the variables affecting hail hazard will need to be developed and expanded. Orthogonal polynomials, successfully used in other meteorological applications, is another powerful tool.

V. OTHER FACTORS AFFECTING CROP-HAIL INSURANCE RATING

Even if there were no other considerations involved in determining hail hazard, the task of evaluation of the meteorological and statistical information would be most difficult. In reality, other factors complicate the development of sound rate structures.

* E. J. Gumbel, *Statistics of Extremes,* Columbia University Press, New York, 1958.

A. *Regulation by States*

As with other lines of insurance, all crop-hail insurance rates are subject to the approval of the various Insurance Departments.

Experience has shown that on the whole this has not proven to be hurtful. Indeed, the necessity for providing detailed supporting data many times improves recommendations which might otherwise be based on less conclusive assumptions. But it is a fact that pressure from agents and the public may adversely influence the decisions of regulatory bodies.

The use of current information in rate-making offers an opportunity to minimize unreasonable objections to needed rate increases. Proposing a rate increase immediately following a disastrous experience is to present your case under the most favorable circumstances. The losses are fresh in the minds of the insuring public, and the regulatory body has a minimum of protests to consider.

If your statistical experience is a year behind, however, the climate is no longer favorable. Besides losing the amount of increase for the period of one year, the intervening season may well have been a most profitable one. Even if long-term experience indicates a substantial rate increase is justified, it is much more difficult to successfully attain this. The proper level of the rate structure cannot be maintained if proposed increases are consistently scaled down.

B. *Acceptance of Rates by Insuring Public*

That the insuring public does not protest rate changes to regulatory authorities is important, but even more so is that they realize the equity of the rating and continue to purchase adequate amounts of protection.

A program of current rating accomplishes this aim, and especially so when considerations of loss ratio since last analysis are made part of the system of rate changes (see Part III). Required rates are based on all-time experience, but proposed rates take into account whether the experience in the area under consideration has been favorable or unfavorable since the last time the rates were promulgated. To raise rates after good experience causes resentment, to lower rates after adverse experience suggests irresponsible action in the farmer's mind. Again, if statistics are a year behind, a further complicating factor is introduced when a good season follows a bad season.

C. *Competition*

Vigorous competition exists in crop-hail insurance. Although there tends to be more in one area than another, being somewhat less in very high hazard regions, there exists a constantly balancing safeguard to excessive rates, even if there were a desire to charge such, and even if there were no regulatory agencies.

Rate structures which most adequately fit the actual existing degrees of hazard are potent competitive weapons. If the rate is not in accord with the risk, adverse selection and "skimming the cream" by competitors will lead to steadily worsening loss ratios. On the other hand, if your competitors are

charging too much in some areas, and too little in other areas, judicious underwriting will protect your position.

D. *Weather Cycles*

Speculation on changing weather has probably existed since *Homo sapiens* first became established as a unique species. A favorite question asked today: "Is our weather changing?" must be answered "yes". Our weather is changing over the millenniums, the centuries, the decades, from year to year, day to day, and hour by hour. Some of these changes are rapid, some slow, some hardly perceptible.

But from a practical point of view the crop-hail insurance industry is concerned with the weather here and now, and for a short span of years ahead. In this aspect the weather can be considered as not changing fast enough to matter.* Climatological records give ample proof that our average weather measured over periods of tens of years changes but slowly.

This does not mean that there is no difficulty in estimating the proper level of the rate structure necessary to provide an equitable return. In "catastrophe" insurance the magnitude of the long term mean is determined by the loss experience occurring in a relatively few years out of the many years of record. When we have had 100 years of crop-hail experience, will it then be evident that our general average of rates now is 10% or 20% too low?

The application of the newer statistical techniques such as the "extreme-value" theory may help us obtain a more satisfactory answer than we now possess. An adequate "catastrophe reserve" loading, subject to change as our knowledge increases, will also minimize the consequences of a general inadequacy of rate levels.

E. *Weather Modification and Hail Suppression*

The Advisory Committee on Weather Control, established by act of Congress in 1957, was directed to make "a complete study and evaluation of public and private experiments in weather control for the purpose of determining the extent to which the United States should experiment with, engage in, or regulate activities designed to control weather conditions." The report** was completed and transmitted to President Eisenhower on December 31, 1957.

The Committee surveyed the present status of knowledge in the area of cloud physics and weather modification. It was their conclusion that there was some theoretical basis, but insufficient experimental proof, for the suppression of hailstorms. Unfortunately, due to concentration on other major aspects of weather modification, the Committee was unable to pursue projects directly designed to evaluate the effectiveness of hail suppression techniques. They did, however, produce a special study entitled "Survey and History of Hail Suppression Operations in the United States" (published in Volume II

* Time series analyses, however, may reveal a tendency for persistence of certain patterns of general weather circulation and which may result in a greater probability of a bad hail year following a bad hail year, than vice versa. However, this is in the realm of speculation as no positive proof has been produced to date.

** *Final Report of the Advisory Committee on Weather Control,* Volumes I and II, Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C.

of their report) which states "although proving nothing *per se,* the fact remains that not only does there exist a definite desire to actively combat hail on the part of the subscribers but that once a project has been in operation it apparently has been deemed sufficiently worth-while to be continued in subsequent seasons." The report also expresses the opinion that "the importance of effective hail suppression to the economy of the country cannot be overestimated." It goes on to say that it is hoped that data would be forthcoming from the many hail suppression projects in existence (35 during the period 1949-57).

The Committee also published in Volume II a technical report entitled "A Method for the Evaluation of Hail Suppression" which presents a program for statistical testing.

Neither confirming nor denying evidence as to efficacy of hail suppression was produced by the Committee; only the conclusion that there was a theoretical basis for expecting hail suppression to work. The best attitude for the insurance industry to maintain is an open-minded one, hoping that more positive proof will become available in future years.

VI. CONCLUSION

It should be emphasized that the principles of rate-making which have been set forth are generalized, and exceptions may be found in almost every state. Knowledge of the general system, however, will allow one to take any of the state rating system manuals and to master it quickly.

The necessity for merging statistical techniques and meteorological information has been dwelt on at length. In closing, the following statement taken from an earlier article of the author's is still as *apropos* today as when written in 1948:

"The object of the meteorological-statistical program is to elucidate the underlying principles that determine relative hail damage, and thus be able to develop a rate for each location and type of crop that will be in direct proportion to its risk from hail damage. In order to accomplish this it is necessary to correlate meteorological and physical factors with the accumulated insurance experience. Differences in hail damage from location to location must be explained by physical reasons in order that we may have confidence that the difference is real and not random."

"Research work along these lines is now being carried out. In addition, contact is being maintained with various outside authorities and agencies for assistance and information."

"The problems which are to be solved in the field of crop-hail insurance are complex, and steps toward their solution must be from many directions. These steps are now being taken."

DISCUSSIONS OF PAPERS READ AT THE MAY 1959 MEETING

A REVIEW OF THE EXPERIENCE OF MASSACHUSETTS WORKMEN'S COMPENSATION EXPERIENCE RATED RISKS

BY

WALDO A. STEVENS

Volume XLVI, Page 87

DISCUSSION BY R. P. GODDARD

In these days of Poissons and negative binomials, Mr. Stevens' paper, couched as it is in English words and Arabian numerals, comes as a welcome change of pace. Mr. Stevens brings out the fact, not quite so well known as it ought to be, that the Workmen's Compensation Experience Rating Plan is something like alcohol, it lifts you up (if you're a bad risk), and it lets you down (if you're good). The net result is that the ultimate loss ratios which determine underwriting results (loss ratios based on premium at adjusted rates) are just about as good for debit risks as for credit risks, with not too much variation throughout the whole gamut of experience modifications. The success of the various experience rating plans in exalting valleys and laying low the mountains and hills is well demonstrated in the table below, which begins with a study made by Mr. Dorweiler of New York data for policy year 1931, and continues with similar tabulations made by the Massachusetts Bureau and the New York Board since that time.

		Ratio of Losses to Standard Premium		
State	*Policy Year*	*Credit Risks*	*Debit Risks*	*All Experience Rated Risks*
New York	1931	56.2	57.9	57.0
Massachusetts	1937	43.4	46.8	44.8
"	1938	50.8	49.8	50.1
"	1939	45.9	52.6	48.8
"	1940	45.9	51.4	48.3
"	1941	46.7	50.2	48.2
"	1942	46.7	49.7	48.1
"	1943	52.0	52.2	52.1
"	1944	53.0	54.2	53.6
New York	1941 (2nd half)	54.5	53.6	54.1
Massachusetts	1955 Intrastate	48.1	53.2	51.0
"	1955 Interstate	48.6	49.7	49.1
New York	1956	53.0	54.6	53.7

The New York data for 1956 includes risks subject to interstate rating with the modification based on interstate experience. For the Massachusetts risks, both intrastate and interstate, Mr. Stevens developed modifications based entirely on Massachusetts experience.

Of the thirteen sets of risks exhibited in the above table, the credit risks had a higher average loss ratio in only two instances, 1938 (Mass.) and 1941 (N. Y.), though in 1943 there was not enough difference to talk about. If the proper credibility were given to risk experience, we would expect a more even distribution of high and low loss ratios between the two types of risk. This seems to be happening in New York, as the following breakdown for 1956 shows:

Manual Premium Size Group	*Credit Risks*	*Debit Risks*	*Total*
Under 500	68.7	56.1	64.9
500 — 999	55.7	59.5	56.7
1,000 — 2,499	51.1	57.5	53.6
2,500 — 4,999	56.6	52.2	54.6
5,000 — 9,999	52.8	53.2	53.0
10,000 — 49,999	53.4	52.7	53.1
50,000 & Over	51.4	55.5	52.9
Short Term	49.5	63.8	57.5
Total	53.0	54.6	53.7

With three out of the eight sub-groups showing higher loss ratios for credit risks, no revision of credibility constants seems necessary in New York. A similar breakdown for Massachusetts might prove valuable.

It should be remembered, however, that all of the tabulations of this type have been based on first reports of experience, and the results could be different on the fifth report. In general, the larger risks (which usually have the greater credits) tend to show a greater upward loss development as the experience matures, so that it is quite possible that the ultimate reports would show very little difference in the desirability of credit and debit risks.

Mr. Stevens devotes some time to a discussion of the 1.03 off-balance factor which applies to each experience modification in Massachusetts. In most states, the credit off-balance is at least partially man-made, because the expected losses are usually higher than the actual losses. This was not true in Massachusetts in policy year 1955, but, even with the actual losses equalling the expected losses in the aggregate, the risks with greater credibility had actual losses low enough to produce an overall credit off-balance. If no correction factor had been applied to either rated or non-rated risks, the loss ratios would have looked like this:

Type Of Risk	*Loss Ratio If No Off-Balance Factor Had Been Used*
Rated — Intrastate	.525
Interstate	.506
Total Rated	.516
Non-Rated	.561
Grand Total	.525

With a permissible loss ratio of 60%, it is a bit difficult to determine which group of risks should be subject to an off-balance factor.

AUTOMOBILE PHYSICAL DAMAGE RATEMAKING

BY

LUTHER L. TARBELL, JR.

Volume XLVI, Page 123

DISCUSSION BY C. L. NILES, JR.

Mr. Tarbell's paper on Automobile Physical Damage Ratemaking has been long awaited. It is a logical adjunct to the other ratemaking papers that have recently appeared in the *Proceedings* of the Casualty Actuarial Society. Once we had Mr. Marshall's paper on Workmen's Compensation ratemaking, Mr. Stern's paper on Automobile Liability ratemaking and Mr. Shaver's paper on Property Fire Insurance ratemaking, it was essential that we obtain this paper.

It is generally felt that papers such as Mr. Tarbell's are written primarily for the actuarial student or trainee. I don't completely agree with this general point of view. Certainly papers of this nature are of great interest and of immeasurable assistance to the student. They may be specifically used by the student in preparing himself for the Society's examinations. In this regard they serve as a reference paper. Notwithstanding, I feel it is absolutely necessary to set down on the written page, in this case the P.C.A.S., the current ratemaking procedures of the various lines of insurance.

Not very many of us have such a broad exposure that we are able to know intimately, or even well, all of the various ramifications of all ratemaking procedures for all lines of insurance. Papers on ratemaking are required to fill this void. I would like to think that we of the actuarial profession will always be students. As students we will continually require reference papers of this nature. Regardless of one's experience in the insurance field, Mr. Tarbell's paper is particularly appropriate. Automobile Physical Damage ratemaking is one area where nearly all of us are truly students. Prior to Mr. Tarbell's paper, Automobile Physical Damage ratemaking was an esoteric and confused area in the casualty insurance business; now, thanks to Mr. Tarbell, we have the key to this mystery.

Before this paper, very few beyond the pale of the National Automobile Underwriters Association had anything that approximated a comprehensive knowledge of this phase in ratemaking. Consequently, we can all well appreciate the time and effort expended by Mr. Tarbell. He is to be commended for this. But, most important, he is to be commended for the excellent job he has done. He has set forth the ratemaking procedures of the N.A.U.A. in a most logical and intelligent fashion for us, the students, to follow.

In the past, I had heard it said that the N.A.U.A.'s ratemaking procedures were rough and crude and unworthy of an actuarial imprimatur. I had pondered these words. If indeed their procedures were crude, could we not learn from them? After all, the N.A.U.A. was doing something strange in this general area of automobile insurance. They were promulgating rates that rather consistently made the companies money. This is indeed strange, if

not immoral. Mr. Tarbell's paper indicates that we can learn from the N.A.U.A. He has clearly demonstrated that the N.A.U.A.'s ratemaking procedures are not crude. The N.A.U.A has done an excellent job—one worthy of actuarial approbation.

Once papers such as Mr. Tarbell's are printed in the P.C.A.S., another end is accomplished. We then have something available for all to discuss and to improve upon. This is a most desirable end. Our business is not static and our ratemaking procedures cannot be allowed to become staid or sterile. We must be alert to the requirements of the insuring public—probably the largest public of any American industry. What better way to lay the groundwork for this activity than by a general airing of the facts in the form of papers on ratemaking?

Papers on the fundamental ratemaking procedures of the various casualty, property and fire and accident and health lines have been sorely needed. Is not ratemaking basic to our industry? Is it not the actuary's main stock in trade? Regardless of where we work—for ourselves or for another; a private concern, an insurance department, a rating bureau, or an insurance company; an independent company or a bureau company; a stock company or a mutual company—regardless of our primary concern in our own particular job, do not all of our activities eventully devolve to ratemaking?

A start has been made, but additional papers on ratemaking are still needed. We should have a paper on General Liability ratemaking—an enormous task. The areas of burglary, fidelity and surety also require coverage. An important ratemaking area, almost completely devoid of papers in our *Proceedings,* is the Accident and Health field. We should have ratemaking papers on both Group and Individual Accident and Health. Accident and Health, incidentally, is a most timely and important topic.

These are the thoughts Mr. Tarbell's excellent paper has evoked from me.

DISCUSSIONS OF PAPERS READ AT THE NOVEMBER 1959 MEETING

AN ACTUARIAL NOTE ON THE CREDIBILITY OF EXPERIENCE OF A SINGLE PRIVATE PASSENGER CAR

BY

ROBERT A. BAILEY AND LEROY J. SIMON

Volume XLVI, Page 159

DISCUSSION BY W. J. HAZAM

The authors are to be congratulated for their very valuable contribution to our knowledge of credibility. Presented, as it was, at a time when a large segment of the industry is embarking on merit rating programs for individual private passenger risks, it provides a basis for the actuarial evaluation of plans now available and perhaps many we have yet to see.

While the data underlying the paper are exclusively the results under the

Canadian Merit Rating Plan,[a] the conclusions are not so geographically restricted. The most provocative of these conclusions is that the experience for one car-year has significant and measurable credibility. In the years prior to the current flurry of merit rating plans, this demonstrable fact had been all but lost, if at all recognized, in the generally prevailing opinion that merit rating was unfeasible. Our current plans may yet prove to be unfeasible. However, this paper demonstrates a means or concept by which to measure the actuarial justification for experience credits (credibilities) for one, two, three, etc., claim-free years.

In developing their credibilities, the authors have placed heavy reliance on frequencies in terms of premiums to correct for the maldistribution deriving from the use of an exposure base. I would be remiss as a reviewer to fail to point out that of which the authors are no doubt aware: that a premium base eliminates maldistribution only if (1) high frequency territories are also high premium territories and (2) if territorial differentials are proper. However, premium, although not perfect, is an improvement over exposure as a base for this type of study. The fact that either or both of these inherent assumptions may not always exist does not detract from the qualitative nature of the conclusions but may alter somewhat the basic relative frequencies of Table 1 and the consequent values in Tables 2 and 3.

The authors make the statement, ". . . the credibilities for experience periods of one, two, and three years would be expected to vary approximately in proportion to the number of years." This holds largely true only for low credibilities; large credibilities would render such a statement inaccurate. However, even in a low credibility area such as the authors are working with in the Canadian results, the theoretical relative credibilities would be less than 1.00, 2.00, and 3.00 for one, two, and three years claim free. For example, using the actuarially accepted $\frac{P}{P+K}$ formula for credibility in experience rating, the theoretical relativities to .046 (1 year credibility of class 1—see Table 2) would be as follows (Note: the k value of 2074 used below was derived on the assumption of 100 claims per year producing a one-year credibility of .046):

Credibility	Relative Credibility	Observed Result (Table 3)
$100 \quad \frac{100}{+\ 2074} = .046$	1.00	1.00
$200 \quad \frac{200}{+\ 2074} = .088$	1.91	1.48
$300 \quad \frac{300}{+\ 2074} = .126$	2.74	1.74

[a] See also "The Canadian Merit Rating Plan for Individual Automobile Risks" Herbert E. Wittick, CAS XLV, p. 214.

This observation should be added to the other reasons why the observed relative credibilities in Table 3 are not 1.00, 2.00, and 3.00.

It may be surmised from this approach to the Canadian results that, in a balanced merit rating plan, there is not enough credibility by class to warrant the magnitude of credits now being offered by many U. S. plans. We must remember, however, that these results are based strictly on claim frequencies, not claim frequencies plus convictions frequencies. Adding convictions no doubt helps substantiate larger credits but it is dubious that it will support current merit rating differentials, if the Canadian experience is at all indicative of what we might expect in this country.

This paper with its original concepts sets forth a basis for analysis of current U. S. plans when the data by class becomes available.

SOME CONSIDERATIONS ON AUTOMOBILE RATING SYSTEMS UTILIZING INDIVIDUAL DRIVING RECORDS

BY

LESTER B. DROPKIN

VOLUME XLVI, PAGE 165

Discussion by R. A. Bailey

As Mr. R. E. Beard, secretary and editor of Astin, said,[1]

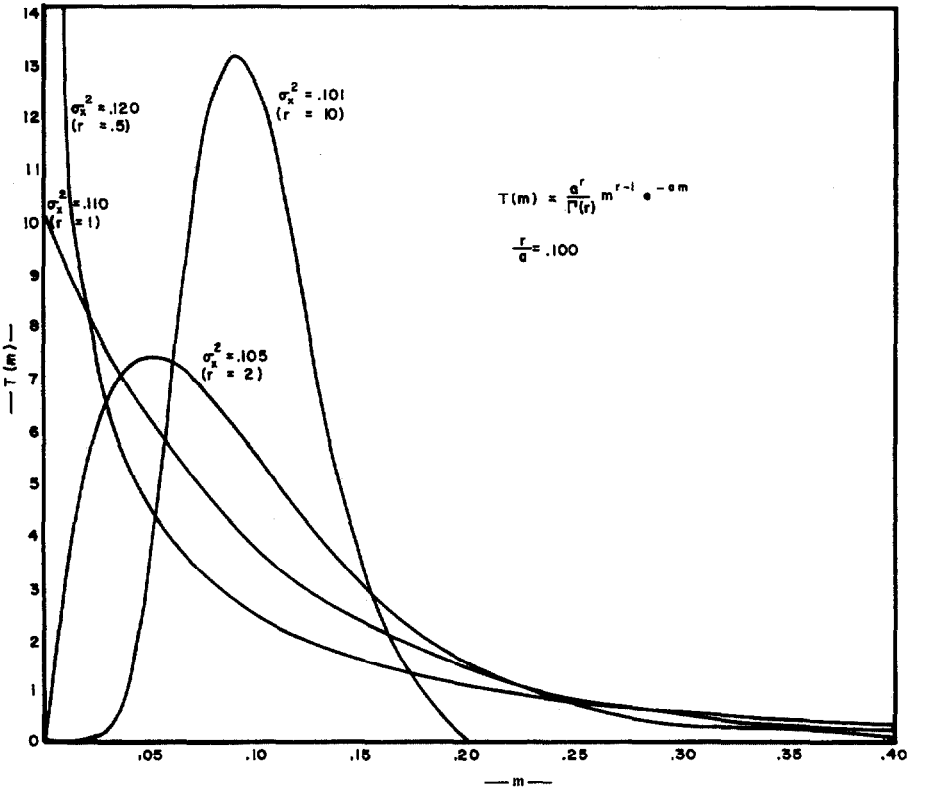
> "The literature in the English language relating to analytical expressions of the risks involved in general insurance is scanty and largely limited to papers presented to International Congresses of Actuaries and the *Proceedings* of the Casualty Actuarial Society. There are, however, a number of contributions to the subject in various other languages, scattered over various journals, mainly, insurance publications of European countries, e.g. *Skandinavisk Aktuarietidskrift* and a few books."

The C.A.S. can rightfully be proud of its contributions in this field which have been ably enhanced by Mr. Dropkin's treatment of the negative binomial distribution.

The analytical expression of risk distributions provides a valuable insight into many practical problems. One of the important results of Mr. Dropkin's paper is a realization of the large amount of variation among individual risks. Automobile risks even within a single class or merit rating group are far from being all alike. In order to help visualize this variation there are shown in Figure 1 the graphs of the distribution of risks which Mr. Dropkin shows to be inherent in the negative binomial distribution. Four graphs are shown, all for an average accident frequency $\frac{r}{a} = .100$, and with variances of the accident frequency (not the variances of m, the inherent hazard) of $.120(r=\frac{1}{2})$, $.110(r=1)$, $.105(r=2)$ and $.101(r=10)$.

[1] Transactions of the XVth International Congress of Actuaries, Volume II, 1957, p. 230.

FIGURE 1



One of the many practical applications to which Mr. Dropkin's development can be applied is the calculation of the discount for n accident-free years. This application was suggested to the writer by Mr. Dropkin's paper because it provided a means of deriving mathematically what had been derived empirically in the paper presented at the same time as Mr. Dropkin's, "An Actuarial Note on the Credibility of Experience of a Single Private Passenger Car", since the discount from the overall average rate for n accident-free years is equal to the "credibility" as defined in the paper just cited.

The chance that any individual risk with inherent hazard (m) will be accident-free for 1 year is e^{-m} where e^{-m} is the value of the Poisson distribution $P(x) = \frac{m^x e^{-m}}{x!}$ when $x = 0$. Mr. Dropkin shows that the total distribution of individual risks can be described by the distribution

$$T(m) = \frac{a^r}{\Gamma(r)} m^{r-1} e^{-am}$$

Therefore the distribution of risks with 1 or more accident-free years is

$$T_1(m) = \frac{T(m)e^{-m}}{\int_0^\infty T(m)e^{-m}dm} = \left(\frac{a+1}{a}\right)^r T(m)e^{-m}$$

Likewise the distribution of risks with 2 or more accident-free years is

$$T_2(m) = \left(\frac{a+2}{a}\right)^r T(m)e^{-2m}$$

This provides us a means of immediately calculating the expected claim frequency of claim-free risks. Mr. Dropkin shows that the claim frequency for all risks $= E(x)$

$$= \sum_{x=0}^{\infty} x \int_0^\infty \frac{m^x e^{-m}}{x!} \frac{a^r m^{r-1} e^{-ma} dm}{\Gamma(r)}$$

$$= \frac{r}{a}$$

Therefore the claim frequency for risks with 1 or more accident-free years

$$= \sum_{x=0}^{\infty} x \int_0^\infty \frac{m^x e^{-m}}{x!} \frac{(a+1)^r m^{r-1} e^{-m(a+1)} dm}{\Gamma(r)}$$

$$= \frac{r}{a+1}$$

Similarly the expected claim frequency for risks with 2 or more accident-free years is $\frac{r}{a+2}$ and for 3 or more accident-free years is $\frac{r}{a+3}$ and so on.

Therefore, the expected claim frequency for risks accident-free for n or more years relative to the expected claim frequency for all risks, assuming that the inherent hazard (m) for each individual risk remains unchanged from one year to the next, is $\frac{a}{a+n}$ and the corresponding discount from the average rate is $\frac{n}{a+n}$. This is the same as saying that these risks are $\frac{n}{a+n}$ better than average. The expression $\frac{n}{a+n}$ is equal to the "credibility" of risks accident-free for n or more years, as defined in the paper cited above, and it is the same result obtained independently by Dr. F. Bichsel, in a paper entitled *Une méthode pour calculer une ristorne adéquate pour années sans sinistres* (A method of calculating an adequate no-claim bonus for years without accidents) presented at the ASTIN Colloquy in La Baule, France, in June, 1959. Furthermore, if this expression for the credibility of the experience of an individual risk for n years

$$Z=\frac{n}{a+n}$$

is multiplied in the numerator and denominator by the premium for one car year, it becomes

$$Z=\frac{P}{P+K}$$

where P is the premium during the experience period and where K is a constant which equals the parameter a multiplied by the premium for one car year. This is the credibility formula derived by Mr. A. W. Whitney in "The Theory of Experience Rating", PCAS, Vol. IV, and used ever since in almost all experience rating plans.

Another application which Mr. Dropkin's development suggested is a comparison of the variation of hazard among licensed drivers and among licensed automobiles. In Appendix B Mr. Dropkin fits the negative binomial to the total distribution of California drivers and obtains $r=.8927$. From the graphs shown in Figure 1 and also from an analysis of the formula for $T(m)$ it can be seen that when $0<r\leq 1$, $T(m)$ is a "J" shaped curve with a maximum height at $m=0$. ($T(m)$, it should be remembered, is the distribution of the inherent hazard of the individual drivers and is to be distinguished from $N(x)$, the distribution of the resulting accidents.) It is reasonable that the California data should be described by a "J" shaped curve since some drivers licensed in California do not drive in California for a number of reasons, such as they do not have a car or they live outside the state. Since such licensed drivers will have an inherent hazard $m=0$, a "J" shaped curve is a reasonable distribution of hazard for licensed drivers. On the other hand, however, the distribution of hazard for licensed automobiles should not be a "J" shaped curve, since practically no automobiles have a hazard $m=0$ and therefore for the distribution of hazard for licensed automobiles, r should be greater than 1.

This proposition can be tested by using the Canadian merit rating experience for insured automobiles. By setting the one-year credibility for Class 1 cars of .055[2] equal to the expression derived above for the one-year credibility, $\frac{1}{a+1}$, we obtain $a=17.2$. Since the average frequency for Class1 $=.087=\frac{r}{a}$, we obtain $r=1.50$ which is greater than 1 as we would expect. From this we can draw the conclusion that there is more variation of hazard among drivers than among cars.

There are undoubtedly many other applications which can be made of Mr. Dropkin's work and we are fortunate to have a development of the negative binomial distribution in the *Proceedings,* especially at this time when merit rating is of such great concern. We are entering a time of great competitive

[2] An Actuarial Note on the Credibility of Experience of a Single Private Passenger Car, CAS XLVI, Table 4, p. 163.

effort in the search for more accurate classification systems, not only in private passenger automobile insurance but in other lines as well, as Mr. Pruitt pointed out so forcefully last November in his presidential address, "St. Vitus's Dance". The negative binomial distribution, which has also been called the "accident proneness" distribution, provides a valuable tool for that search.

THE ACTUARIAL ASPECTS OF BLUE CROSS PLANS

BY

J. EDWARD FAUST, JR.

Volume XLVI, Page 177

DISCUSSION BY M. KORMES

The paper submitted by Mr. Faust describes one rate making technique and it creates the impression that the problem is a rather simple one. This may be the case where the contract benefits are more or less uniform, i.e., where there is only one coverage for group contracts and only one coverage for non-group contracts.

There exists, however, in many Blue Cross plans a multiplicity of contracts which may range from full semi-private coverage to an allowance of $7.00 (or even less depending on the area served) for Room and Board. The ancillary (all other hospital expenses) benefits may be covered in full or there may be some exclusions or monetary limits on certain benefits (such as X-Rays and laboratory or blood plasma). Maternity Coverage may be in full or limited to a fixed amount for regular delivery or all obstetrical admissions. Allowances for private accommodations may vary from group to group. Out-patient benefits may be provided in full or in part or only accident emergency within twenty-four hours. Co-insurance in the form of a flat percentage on all or part of benefits or in the form of various deductibles is used by many plans. In fact, the multiplicity of coverage is so great that the coding of the coverages becomes a serious problem, especially as it is necessary for the member hospitals to know the coverage granted to any subscriber upon admission.

In the introduction the author states that for the plan which serves as a statistical basis of his paper the Underwriting Gain is from 3.5% to 4.0% of Gross Income. As a rule Blue Cross plans have a provision in the rates for additions to the Statutory Surplus (as required by the Insurance Departmen having jurisdiction) of 3.0% to 5.0% of the rates so that only after these amounts are realized after losses and expenses is there a real Underwriting Gain. Since with a few exceptions the Statutory Surplus of the plans is considerably below the required amount there are very few plans having a real Underwriting Gain.

The rate making process described by Mr. Faust is based on the loss ratio method, first by determining the adjustment for the current level of cost, and then projecting to a future level by graphical extrapolation.

As respects his loss development method, it should be pointed out that the percentages depend on the promptness of reporting discharges by the hos-

pitals, the promptness with which the claims are processed and paid by the plan, as well as the most prevalent durations of contract coverage. Thus, for a plan with a coverage of 120 days and an annual loss volume of over $69,000,000, the percentages are as follows:

Month of Period	Per Cent of Ultimate Incurred Losses Represented by Payments to End of Month
1st	.83%
2nd	6.33
3rd	13.68
4th	21.63
5th	29.75
6th	37.96
7th	46.23
8th	54.52
9th	62.81
10th	71.11
11th	79.42
12th	87.73

The estimates of ultimate incurred losses made on the basis of loss payment patterns are extremely accurate and the error of the first estimate seldom exceeds 2% and is usually an overestimate. The value of this method for other casualty losses should be given further study as the papers by Mr. Tapley[1] and Mr. Harwayne[2] indicate that there is a functional relationship between the losses paid at the end of a given period and the ultimate losses incurred.

The rate making as such is much more refined in many plans than that described by Mr. Faust.

For the plans with which the writer is connected the procedure may be briefly described as follows:

(a) Fiscal or calendar year experience for three or four consecutive years is studied to determine the trend of incidence of in-patient days per contract month for each class of contracts and each major coverage. Where there appears to be a definite trend, a projection is made by the method of least squares (straight line or parabola). Where there is no particular trend, an average of the last two years is usually taken but judgment is exercised in this connection.

(b) The per-diem hospital payments are also studied separately by class of contract and by coverage (there is a great deal of variation) and here also a projection is made taking into consideration the fixed and the variable portions and contractual arrangements with hospitals.

(c) The values of (a) and (b) projected to the midpoint of the period during which the rates would be applicable are multiplied to obtain the in-patient pure premiums.

(d) Separate projection is made for out-patient pure premiums calculated

[1]CAS XLIII, p. 166.
[2]CAS XLV, p. 63.

for each of the years of the experience period and each class of contract.

(e) The total pure premiums are then loaded for expenses and additions to Statutory Surplus.

The rates are usually calculated to be sufficient for a period of two or three years and so far the above method has produced extremely satisfactory results.

No discussion of rate making would be complete without a consideration of merit rating plans. While it is true that the majority of Blue Cross plans frown on merit rating as being contrary to the community principles of Blue Cross, several large plans felt that one cannot exist in an abstraction and must prevent the loss of good risks to keen competition. Thus, there have been developed merit rating plans embracing prospective, retrospective and cost-plus features and recognizing gradation of expenses by size of risk. The prospective merit rating plans have a built-in feature of cost projection to the level of the period during which the rates will apply and, in view of the fact that a substantial percentage of the business is merit-rated, have postponed the necessity of general rate revisions in many instances for one or more years.

I do not wish to dwell on the author's analysis of Blue Cross rates or the comparison of costs with that of insurance companies for the following reasons:

1. It is not sound to charge premium rates by age or by the sex distribution of the employees in a group. Any inherent differences are taken care of by the merit rating for larger groups and uniform rates for smaller groups are more desirable from the public relations point of view.
2. A true comparison of costs with those of insurance companies could only be made if the coverages were identical and then the comparison should be made on the basis of the incidence of in-patient days per contract.
3. The results are based on 63,960 contracts and this number subdivided by single males, single females, married males and married females as well as by sex and age groups gives rather thin exposure for some if not all of the subdivisions (2,665 contracts on the average). In fact, as a member of a committee working under the auspices of the Blue Cross Commission, I have participated in a study embracing fourteen Blue Cross plans from areas representative of the entire United States. This study predicated on over 3,400,000 contracts and over 3,850,000 in-patient days showed that the utilization differential for those over 65 as compared with those under 65 is 3.79 for males and 2.17 for females. The results in Mr. Faust's paper seem to indicate identical cost for both sexes in the over 65 bracket and approximately the same average cost for all ages up to 65 combined.

In conclusion, I would like to say that in the development of its rating techniques the Blue Cross plans have leaned heavily on the methods used in casualty insurance business and when some of the refinements have been proved by the test of time they may well be a worth while subject of a paper to be submitted to the Society.

AUTHOR'S REVIEW OF DISCUSSION

It is indeed a pleasure to have a man with Mr. Kormes' experience review my paper.

Mr. Kormes indicates that I created the impression that the problem is a simple one. I don't see how anyone who read the "Foreword" of my paper could obtain this impression. The rating problems are quite complex. In order to keep my paper to a reasonable length, I selected one rating problem.

Mr. Kormes' comments about the statutory additions to surplus in some states is most instructive.

Mr. Kormes' table showing the "Percent of Incurred Losses Represented by Payments to End of Month" is quite different than mine, which is a good demonstration that such a Development Table must be determined for each plan since conditions will vary from one plan to another.

Some of the additional data Mr. Kormes uses is interesting. The rate-making process is subject to so many variables, certainly all pertinent data should be considered.

Mr. Kormes indicates that his process has produced extremely satisfactory results. I am happy to report that my methods have had equally satisfactory results.

I fully agree with Mr. Kormes' comments in regard to merit or single risk rating.

I must disagree with Mr. Kormes that "It is not sound to charge premium rates by the age or the sex distribution of the employees in a group." After a contract is in force it is true that inherent differences are taken care of by merit rating for larger groups. The initial rates, however, are also important from a competitive standpoint. Every body of experience I have seen, clearly shows that losses depend on age and sex. Uniform rates for smaller groups may be more desirable from a public relations viewpoint but such uniform rates are not a more sound rating basis than rates that vary by age and sex. Certainly a group with a high average age can be expected to produce more losses than one with a low average age.

Mr. Kormes states that "a true comparison of costs with those of insurance companies could only be made if the coverages were identical and then the comparison should be made on the basis of the incidence of in-patient days per contract." I presume that he is referring to the Section VI of my paper entitled "Blue Cross Rating System versus Group Hospital Expense Insurance Rating System." I would like to carefully point out that I am not comparing costs. I am comparing rating systems. Insurance companies use an "Employee" rate and an "Employee and Dependent" rate while Blue Cross plans use an "Individual" rate and a "Family" rate. All four (4) of these rates, in my paper, were developed from the same Blue Cross data so that their comparison is quite valid. Those concerned with selling Blue Cross plans in competition with Insurance Company plans should carefully study the results of this comparison.

I appreciate Mr. Kormes reporting of results of a study of a large volume of data. Admittedly my sample was quite small.

MERIT RATING IN PRIVATE PASSENGER AUTOMOBILE LIABILITY INSURANCE AND THE CALIFORNIA DRIVER RECORD STUDY

BY

FRANK HARWAYNE

Volume XLVI, Page 189

DISCUSSION BY J. H. MUETTERTIES

Mr. Harwayne has given us a good brief summary of the Private Passenger Merit Rating history along with an excellent and precise statement summarizing the "con" to merit rating and then the "pro" to merit rating.

After the history and the initial statements about Merit Rating, the author refers to the California type plan as ". . . being in answer to the need for recognition of the driving performance of individuals" and that the cornerstone of this plan is the California Driver Record Study.

In this discussion this writer would like to raise the question as to whether the California type plan "answers the need for recognition" and this writer is also somewhat dubious that the California study is the cornerstone of the plan. The need for recognition of the driving performance of individuals, first and foremost, is in the motor vehicle departments where more enforcement is needed, and second, when a driver with a poor record or near-poor record (four or more accidents and/or abstracts—7.4% of drivers in California; five or more, 4.4%) is given a license, he should be treated as a substandard risk and surcharges should be permitted.

As to the cornerstone comment, a somewhat different concept could be set forth for discussion—the detailed records of the California Motor Vehicle Department and the desire of certain companies to get at the clean risk are probably the fundamental underlying concepts in laying the foundation of the California Merit Rating Plan.

The author states, and most everybody will agree, that the California study is meaningful. He then goes on to give us another good and precise summary of the study. This writer is in full agreement with the summary but there are two questions which although minor are troublesome—there is a consistent tendency toward an increase in the number of accidents with an increase in the number of abstracts. Question: How much does the accident which results in an abstract affect the shown tendency? Should this accident abstract be viewed as equal to one accident, one abstract, or one accident and one separate abstract? Second, should a rating system be tied into a system which probably has a variable enforcement pattern?

Besides the good summaries of the history of merit rating and of the California study, the author has done excellent research in fitting the negative binomial distribution to the California data. This, we understand, is being discussed in another paper and with a more scientific approach than this writer could take. However, comments on the importance of finding a distribution which fits the California data are in order.

It is not denied that individual risk rate adjustments can be made from the California study and that expected distributions can be formed (and by territory) but are all these approaches practical and economical? It is not too difficult to set up theoretical yardsticks, but it is very difficult to collect the actual experience in such a form for a comparison to these theoretical yardsticks.

In fact, this writer will go even so far as to say that it will be very difficult to collect reliable experience by the debit-credit classes for the many merit plans.

Speaking of costs and not being in the position to price the additional costs of handling merit rating, this writer wonders what problems the companies are running into in the way of increased underwriting and handling costs when some policies must be rated twice—first when the application is received, and second after the MVR. (Motor Vehicle Report)

In addition to Mr. Harwayne's proposed uses of the California driver data, a pricing of a merit rating plan can be attempted. Referring to the study, we find that 54.1% of all drivers had no accidents or abstracts; 18.0% had one abstract and 4.2% one accident; and combined 22.2% had one accident or one abstract. Thus, the weights can be obtained for a simple merit rating plan and set-up as follows:

Accidents or Abstracts	Weight	Percent of Manual	
		Plan I	Plan II
0	54.1%	80%	80%
1	22.2	100	90
2	10.8	125	100
3	5.5	150	125
4	3.0	175	150
5	1.6	200	175
6 or more	2.8	250	200
Rate Level	100.0%	103%	94%

The above shows that Plan I will probably produce a 3% increase in the manual level and Plan II a 6% decrease. These two plans do not represent any particular plan and are only presented as examples. Much additional work should be done on the weights as shown because, first of all, the California study is on a per driver basis and private passenger rating is per car. There are about 8,000,000 drivers in California and about 6,500,000 automobiles (excluding trucks and motorcycles). Therefore, the 54.1% for no accident and no abstracts would be a lesser figure. Also, if the merit rating plan charged higher points for more severe abstracts (about 5% of abstracts are severe) and no points for non-fault accidents, additional adjustments must be made. In any case, the California study can be used as a basis for pricing out a merit rating plan.

The next question is: When is a merit rating plan in balance?

1. When the expected distribution times the debit or credit rated risks equals the manual level, or

2. a. When a lower than manual level is produced but the losses are lower to the same degree, and
   b. When a higher than manual level is produced but the losses are higher to the same degree.

Someone more learned than the writer will have to answer this question.

If the general belief of those using merit rating is that it adjusts the premium commensurate with the hazard for the class—maybe it should be added that other measurable characteristics should also be included in rating; such as occupation, environment, and then the age-old symbols of recklessness, of bootees hanging from the sun visor or an extra tail pipe.

Even though this discussion takes the form that the writer is not sold on merit rating—deep down within me there is a yearning for merit rating because of being lucky enough to be without accidents or convictions. (Probably will regret the day this was written). But to get back to my point—the yearning in my case is not so much for merit rating but for a lower cost of insurance—and thereby tells the tale of the real want—lower cost.

AUTHOR'S REVIEW OF DISCUSSION

Mr. Muetterties' discussion of my paper is indeed welcome, for such exchange helps to sharpen the tools of the ratemaker and tailor the price paid by insureds more appropriately to the hazards which they present.

Mr. Muetterties' view that the need for recognition of driving performance is along the lines of more enforcement in the motor vehicle departments is more of a social comment rather than an actuarial or insurance comment. I like to believe that the insurance industry is concerned with objective evaluation of potential risk more than merely finding a plausible but erroneous basis for risk evaluation.

Regarding the inter-relationship of accidents and abstracts, 37.1% of the individuals exhibited a greater number of accidents than abstracts. The following summary of average number of abstracts by number of accidents derived from the California data is noteworthy:

TABLE A

		Increment from Preceding Line	
Number of Accidents	Average Number of Abstracts	Number of Accidents	Average Number of Abstracts
0	0.7	—	—
1	1.6	1	0.9
2	2.5	1	0.9
3	3.2	1	0.7
4	3.8	1	0.6

It should be noted that on the average, those with no accidents during the three-year experience period have 0.7 abstracts; thereafter, an increment

of one accident is associated with an increment of 0.9 abstracts on the average. After two accidents are reached, the increment in the average number of abstracts decreases to 0.7 and then to 0.6. It is almost as if the first abstract is expected to occur unaccompanied by an accident but thereafter, each abstract is accompanied by an accident in most instances.

In the matter of collecting reliable experience for the many merit plans, the answer would appear to lie in the individual company's evaluation of the merit plan which it uses. If, on the other hand, merit rating by the insurance industry is contemplated, there may be great merit to the adoption of a uniform merit rating plan, or at least one with a minimum of variations.

It is clear that for a merit rating plan to be successful, a company must have available to it satisfactory accident and traffic violation records. For the Bureau type plan this means that the Motor Vehicle Department must keep for every driver a record which shows both violations and accidents in such a manner that separate instances may be identified. Such records should be available at nominal cost (say $.50 or less). If reliance is placed on a system of company interchange of information, the resultant expenditure of money and effort would probably be excessive. Of subordinate concern is the question of availability of records from other states. Here major reliance might have to be placed on the signed statement of the insured.

To the degree that the question of "fault" as opposed to "involvement" enters in counting accidents or violations, a plan becomes more costly and more difficult to administer. There are, however, certain situations in which the insured, although involved in an accident, is obviously not at fault—e.g., if his car were parked. Such instances do not present real administrative difficulties and can be recognized in a plan. (The records kept by the Motor Vehicle Department would have to be such as to permit identification of these situations.)

It would be necessary for the carriers concerned to keep statistics in greater detail in order periodically to reevaluate a plan once it becomes effective. This, together with the other elements already mentioned, means that administrative costs will increase. Estimates of this increase could probably be made before embarking on a merit rating program.

Among other administrative problems would be the question of treatment of the new driver, the more-than-one-car risk and in general, the fact that a one-to-one correspondence does not necessarily exist between cars and drivers.

Concerning the balance of a merit rating plan, it appears to me that Mr. Muetterties' two possibilities might be summarized as being either a static balance or a dynamic balance. For my own view, I believe a merit rating plan may be said to be in balance when the sum of the expected losses and expected expenses can reasonably be expected to produce a reasonable profit margin or dividend margin.

Apropos of measurable characteristics, some of these are daily being recognized in premium determination. For example, use is made of such categories as farms, clergymen, territory, driver training, age and sex.

It seems that the majority of individuals yearn for a lower cost of insurance, the lower the better; yet for the most part people tend to be entirely satisfied,

or less dissatisfied if their insurance premiums are lower than that charged their "reckless" neighbors.

I believe the problems posed by merit rating would be more susceptible of evaluation if its consequences could be expressed in quantitative terms. Questions of efficiency, adequacy, discrimination and public interest might come clearly into focus if we had some idea of the numbers and percentages of risks involved.

The following reasonable assumptions may be used to develop a notion of the overall efficiency of grouping by previous accident records for a three-year period:

Let $q = .1$, the annual accident frequency

Let $\frac{\sigma^2}{q} = 1 + b = 1.06$, the ratio of the variance to the mean

Then $b = .06$

The distribution of 100,000 risks and their expected means may then be estimated to form the following table:[1]

TABLE B

No. of Accidents in 3 Years	Risks		Expected Mean (1 Year)		
	Symbol	No.	Value	Indices	
0	A_0	75,893	.085	1.00	.85
1	A_1	19,295	.136	1.60	1.36
2	A_2	3,924	.186	2.19	1.86
3	A_3	732	.237	2.79	2.37
4	A_4	130	.288	3.39	2.88
5	A_5	26	.339	3.99	3.39
		100,000	.1	1.18	1.00

In each category the number of risks associated with the expected mean may be described as having a distribution of accident proneness.[2] It will be seen that risks will have been assigned to a category on the basis of previous accident *record;* some of these risks, however, will have an "inherent accident proneness" which could be better described by the expected mean frequency of a lower or higher category. For purposes of this discussion a figure 20% above or below the expected mean of each category was assumed to separate those risks "misclassified" by the record of past accidents. Tables[3] were used to produce the following results:

[1] See Appendix for formulae used in developing this table.

[2] Corresponding to the Pearson Type III. See L. B. Dropkin, "Some Considerations on Automobile Rating Systems Utilizing Individual Driving Records," CAS XLVI, p. 165.

[3] Ibid. Description is contained in Appendix D therein.

TABLE C

No. of Accidents	No. of Risks	Expected Mean			Percentage of Risks with "Inherent Proneness" Equal to or Less than 80% or 120% of Expected Mean	
		100%	80%	120%	80%	120%
0	75,893	.085	.068	.102	50.5%	70.0%
1	19,295	.136	.109	.163	45.5	70.2
2	3,924	.186	.149	.223	41.6	70.7
3	732	.237	.190	.284	38.4	71.4
4	130	.288	.230	.346	36.	72.
5	26	.339	.271	.407	35.	72.
Total	100,000					

The percentages of the last two columns above may be used to determine the overall efficiency of the merit rating system with the stringent 20% criterion. In this instance we would find 29,921 risks whose "inherent proneness" indicated they belong in a category more than 20% higher than the one to which they have been assigned. At the same time we would find 10,748 risks in other than the 0 accident category whose "inherent proneness" indicated they belong in a category more than 20% lower than the one to which they have been assigned. Lastly, 38,326 risks in the 0 accident category have an "inherent proneness" which indicates they belong in a category more than 20% lower than the lowest category.

If we assume the 38,326 risks in the 0 accident category cannot feasibly be rated any lower, then we conclude that only 40,669 risks out of the 100,000 have been "misclassified," and on a net basis, the number of risks not rated high enough exceed the number not rated low enough by 19,173.

From an individual company viewpoint, lack of the existence of a merit rating program encourages highly selective underwriting. Assuming rates are based on a 10% frequency as above, a company might undertake to write all risks with no accidents in the past three years at an expected mean frequency of .085; in addition it might attempt to identify through other underwriting characteristics, the 45.5% of those involved in one accident in the past three years whose "inherent proneness" is given by the figure .109 or less. Of course, this could leave the 54.5% of those with one accident and all those with more than one accident without a free insurance market at all.

If a plan is to be considered in terms of "balance," rating organization rate levels must be considered; should all or part of industry experience be the basis on which discounts or surcharges are contemplated? Does merit rating involve "unfair" discrimination in some instances?

In this author's view the relative quantitative relationships expressed above may help the individual to decide whether merit rating is a good thing or a bad thing for the insured, the carrier or the rating organization. Whatever inequities may be pointed up by this illustration might be sharply curtailed

if $\frac{\sigma_2}{q}$ could be made more nearly 1.00, that is, if rating territorial, occupational, etc. classifications could be made more homogeneous than they are today.

APPENDIX

Formulae for Computing Distribution of 100,000 Risks and Expected Means on a 3-Year Experience Basis

$(q = .1 \text{ and } b = .06)$

No. of Accidents in 3 Years	Risks		Expected Mean Value (1 Year)	
	Symbol	Formula	Symbol	Formula
0	A_0	$100{,}000\left(\frac{1}{1+3b}\right)^{\frac{q}{b}}$	V_0	$\frac{q}{1+3b}$
1	A_1	$\left(\frac{3q}{1+3b}\right) A_0$	V_1	$V_0 + \frac{b}{1+3b}$
2	A_2	$\frac{3(q+b)}{2(1+3b)} A_1$	V_2	$V_0 + \frac{2b}{1+3b}$
3	A_3	$\frac{3(q+2b)}{3(1+3b)} A_2$	V_3	$V_0 + \frac{3b}{1+3b}$
4	A_4	$\frac{3(q+3b)}{4(1+3b)} A_3$	V_4	$V_0 + \frac{4b}{1+3b}$
5	A_5	$\frac{3(q+4b)}{5(1+3b)} A_4$	V_5	$V_0 + \frac{5b}{1+3b}$

MULTIPLE PERIL RATING PROBLEMS—SOME STATISTICAL CONSIDERATIONS

BY

ROBERT L. HURLEY

Volume XLVI, Page 196

DISCUSSION BY P. M. OTTESON

Mr. Hurley's paper represents a valuable contribution to the literature on multiple peril rating. The paper is most interesting to read and study; it reveals much of the author's thinking and philosophy concerning the general problems of insurance statistics and ratemaking and should provoke thought, study and discussion on a most timely subject.

The Homeowners policy is used to illustrate the multiple peril statistical

and ratemaking problems. No consideration is given to other types of "composite" coverages.

The multiple peril idea as represented by the Homeowners policy is considered as very desirable to the policyholder; this is in contrast to the difficulties it poses from the standpoint of analyzing prospective loss costs for ratemaking purposes. The fact that "multiple peril" does not represent any fixed combination of hazards but is always a "moving target" is brought out very capably.

The problem of the interplay of experience and rates between multiple peril policies and individual policies is discussed thoroughly and well. The danger of differences in rate levels based on pure chance rather than difference in inherent hazard is very real in a policy which includes unpredictable catastrophe windstorm losses in the same composite as high frequency liability claims and minor property losses. The basic question suggested is this: when different policy types of the same classification insure certain hazards which are common to each policy, should the proportion of the rate based on these common hazards consider the combined experience of all policy types? Or on the other hand, should the rates for each policy type stand on its own experience? This question must be answered in determining rate relativities between Homeowners Forms 1 and 2, as well as between Homeowners and individual coverage dwelling policies. A stronger and more positive position on this basic point by the author would have made the presentation more effective.

The weakness of relating loss costs to premiums instead of a fixed type of exposure unit is revealed; the weaknesses of the loss ratio approach are enumerated.

The discussion concerning losses by cause brings out the point that it would be highly desirable in any coverage involving windstorm to be able to segregate a certain percentage of the annual exposure and earmark it for catastrophe windstorm losses. The likelihood of changes in coverage and the need to establish proper rate relativities among even the various forms of Homeowners coverages further establishes the need for exposure information and losses by cause.

The author's review of the statistical devices used in establishing ratemaking statistics brings up the most vital question on this entire problem—what is Homeowners exposure and how can it be defined and measured? Would the auto "unit basis" coupled with an increased limits table to take care of high severity claims prove effective? Is the summation of amount at risk a proper base? Is a combination of bases necessary? Why is the premium for the first $8,000 of insurance higher than for the next $8,000 in Homeowners policies but not in dwelling policies? A more specific proposal or recommendation as to exposure base would have been most desirable.

A subsidiary class plan which would classify risks according to many possible variants such as occupation, income level, etc. is suggested. Some further elaboration as to why such items as income level or occupation would make one risk different from another would seem appropriate.

A most interesting tabulation concerning number and amount of loss at various size classifications for an entire book of fire business is presented. This

information should be most useful in analyzing variations in loss experience on fire coverage overall but there would seem to be some question as to what interpretations could be resolved which would be applicable to Homeowners coverage. The pattern for the dwelling type of risk exclusively would probably run along different lines. For samples of comparable size the consistency should run higher in the dwelling losses because the "probable loss" range would be narrower. It would be exceedingly interesting if this type of tabulation could be made available for dwelling or Homeowners policies within a certain size classification. A size of loss breakdown for these policies where the amount of insurance carried was $10,000 as compared with $20,000 for example would present interesting information for determining the increased premium necessary to take care of additional losses as the amount of insurance increases.

Concerning the element of hazard attributable to tropical windstorms, the 42-year record indicates that the number of storms reaching the United States per year averages 1.88 and that the observed frequency follows the Poisson distribution very closely. Some additional observations as to how these facts could be applied in determining an overall loading for catastrophe losses from this cause in the Homeowners rate would have been appropriate. The author's study further suggests the question as to whether or not we can assume that the tornadoes and hailstorms of the mid-western states would follow a similar distribution. These latter hazards assume substantial proportions in many areas.

The distribution of the number of fire catastrophes over a 45-year period did not fit the Poisson distribution closely. This suggests that the conditions producing catastrophe fires do change over the years and even vary from year to year. Here again, the question arises as to how this data could be interpreted with a view to determining Homeowners expected losses.

A "square root" credibility table is submitted as part of this paper without designation as to where or how this table was constructed. The author feels that a table of this kind may be dangerous and even represent an obstacle to the use of sound judgment in a line of business such as Homeowners because of the catastrophe elements of some of the coverage components. This is a sound conclusion. The swing in losses as a result of chance would not approach the zero point at a $5,000,000 premium level with a reasonable and usual proportion of the book subject to the same catastrophe.

The author definitely saved the best until the last. The increased credibility of individual risk experience resulting from the combination of coverages definitely is an important factor in distinguishing Homeowners from the regular dwelling policies. The average overall frequency of 20 claims per hundred risks makes the Homeowners policy reasonably comparable to an all-coverage automobile policy from a loss frequency standpoint. Although this point is not mentioned specifically as such, the temptation to transfer the merit rating thinking prevalent in the private passenger automobile field over into the Homeowners field is irresistible. A sound basis for a merit rating approach appears to be borne out by the author's research studies. Concerning this phase of the study, further developments appear likely as the competitive struggle for Homeowners business accelerates.

A COMPARISON OF AUTO LIABILITY EXPERIENCE UNDER A COMPULSORY LAW AND UNDER FINANCIAL RESPONSIBILITY LAWS

BY

MILTON G. McDONALD

Volume XLVI, Page 214

DISCUSSION BY L. W. SCAMMON

In his paper "A Comparison of Auto Liability Experience Under a Compulsory Law and Under Financial Responsibility Laws," Mr. M. G. McDonald, Fire and Casualty Actuary of the Massachusetts Insurance Department, has set forth private passenger and commercial car experience for Massachusetts, a compulsory law state, and for Connecticut and New Jersey, financial responsibility law states. In order to make a direct comparison of the experience of these states, it was first necessary to adjust the Massachusetts compulsory experience inasmuch as compulsory coverage in Massachusetts is limited to coverage on the public ways of the Commonwealth and does not include coverage for guest occupants of the insured's automobile.

After making the necessary adjustments to include guest losses and losses off the public ways of Massachusetts, Mr. McDonald very concisely points out that (1) the Massachusetts pure premiums are higher than the New Jersey or Connecticut pure premiums, (2) that the Massachusetts average claim costs are lower than the New Jersey or Connecticut average claim costs, (3) that the Massachusetts claim frequencies are much higher than the New Jersey or Connecticut claim frequencies, and (4) that frequency is, therefore, the reason for the comparatively poor experience for Massachusetts under a compulsory law. Mr. McDonald then quotes from the Report of the Special Commission, Senate No. 466, which investigated automobile insurance in Massachusetts, to the effect that claim consciousness of the inhabitants of the Commonwealth is the reason for the higher claim frequency. Without so stating, therefore, the implication is that the reason for the comparatively poor experience in Massachusetts is the claim consciousness resulting under a compulsory law.

I find myself in substantial agreement with this implication. The Massachusetts figures which Mr. McDonald has cited are no flash results. Comparable figures have been indicated for many years as Massachusetts Compulsory Automobile data have been analyzed. Those of us close to the Massachusetts figures have come to expect a comparable pattern year after year.

Perhaps it should be stated objectively that obvious as it appears to be that claim consciousness under the compulsory law is the principal cause of the comparatively poor experience in Massachusetts, there may be other contributing causes. For example, may not some increase in claim frequency be expected when all vehicles must be insured? It would seem that in a non-compulsory state those who do not carry insurance are somewhat the irre-

sponsible drivers whose irresponsible attitudes must carry over to their driving habits. The frequency of such drivers could well increase the total frequency.

Another possibility of the cause for the higher frequency under a compulsory law is the difference in state enforcement practices. For example, much is made of the New Jersey "no fix" traffic ticket. Safety experts are agreed that where the enforcement index is high the accident index is low and vice versa. Further backing to this possibility is provided by the contraction of claim frequency for nearly a year in Massachusetts with the passage of Merit Rating legislation in July 1953 during which time the threat of accumulation of points against individual driving records served to reduce the number of accidents reported.

While it is generally believed that weather conditions should affect the three states of New Jersey, Connecticut and Massachusetts in reasonably the same way, the flood of claims in Massachusetts in March of 1957, a month when an excess of snow and extremely slippery driving conditions prevailed in Eastern Massachusetts, suggests the thought that the concentration of traffic that exists in and around Boston when subjected to quickly changing winter driving hazards contributes to higher claim frequencies to a greater extent than in the other two states. Two-thirds of the private passenger cars in Massachusetts are concentrated within a thirty mile radius of Boston with its narrow, twisting, cow-path streets. Old New England as typified by Boston and environs was not laid out with an eye to 20th-Century automobile traffic conditions. It is little wonder frequencies of accident are high here.

In other words, although claim consciousness is probably the major reason for the higher claim frequency in Massachusetts under a compulsory automobile insurance law, there may be other reasons contributing to the higher frequency. However, after hearing legislators at a public hearing on compulsory automobile rates state that they cannot blame people for making claims when the opportunity arises because they have to get back the money they paid in premiums due to the high compulsory rates, one cannot help but get the impression that the people are somewhat claim-conscious.

OASDI COST ESTIMATES AND VALUATIONS

BY

ROBERT J. MYERS

Volume XLVI, Page 219

DISCUSSION BY W. RULON WILLIAMSON

Before commenting directly upon Mr. Myers' current paper, I shall set down certain background remarks on "Social Security" and the segment called OASI. I shall largely omit both the Disability segment, with its separate trust fund and tax-base, as well as Medical Care currently being discussed as the next addition to what is called the "Insurance Part" of Social Security.

Freedom Loss

A few months a go a *New Yorker* cartoon showed two free tigers watching a captive tiger being hustled into a cage. One free tiger remarked to the other: "It might not be such a bad life, security-wise."

Recently I finished reading Wilhelm Roepke's "A Humane Society" which bracketed "The Welfare State" and "Inflation" as twin interacting cancerous growths within the modern economy. Both Roepke and Felix Morley have recently discussed Federalism, with its checks and balances, as a protection against the irresponsibilities of over-centralization, reviving both the wisdom of the founding fathers and that of Columbia's Burgess about the close of the last century. Roepke calls attention to Edmond Burke's comment: "Men are qualified for civil liberty in exact proportion to their disposition to put moral chains upon their appetites. . . . It is ordained in the eternal constitution of things, that men of intemperate minds cannot be free. Their passions forge their fetters." Social Security seems to deny all men's capacity to put chains on their appetites and furnishes their amateur bureaucracy for the blacksmithing job. Given this negative attitude toward our citizen's capacity for self-governance, it is not surprising that this formerly functioning Federalism has become a centralized Government of "bigger checks and smaller balances."

Negation

The Marxist ideology has talked about the new synthesis to follow the negation and the negation of the negation. It is that Marxist line that I here follow for a spell, but not into the never-developed new synthesis. Admiral Ben Moreell's pamphlet "To Communism via Majority Vote" presents the frightening series of Marxist negations already piled up within our Republic. One of the most important of these is the National income tax—progressive in its upper tax bands. But as our National Government thus denied more and more fully a man's right to his own property, the initial denial of Constitutional limitations upon the taxing power grew in oppressiveness. The "centrist" Government denied the citizen, with more and more effectiveness, the full returns from his superior performance. Then legal and illegal "tax avoidance" grew, to "negate the negations." The Lasser annual tax guide alone carries a thousand ways of such legal self-protection.

One of the impressive progressive tax reactions has been the whole pension philosophy. Here the employee can postpone the receipt of part of his compensation until the days of reduced income in old age, and thus secure lower tax rates.

To reduce one's reported earnings as a protection from Governmental confiscation is a negation of one's pride of accomplishment, and a step away from straightforwardness. Then OASI financing and the freeing of benefits entirely from the levy of the National income tax—"number 1"—carries the negation of the negation further still. It is also a dubious extension of the National taxing power. It disarms the victim by talking about "contribution toward deferred benefits" and puts both the tax and the benefit payment outside the National budget. Now that the non-budget outlay for OASI has passed ten billion dollars, the situation—like a house set on a hill—cannot be hid.

Postponed OASI Effectiveness—More Negation

The OASI scheme seems to me—and has long seemed to me—about the most unsatisfactory method of "meeting cost" that any country has adopted for its old age benefits. It prated of serious need in old age and then set the first age payments five years off. They did begin three years, instead of five years, after starting, and it did have a lumpsum payment as an apology for the absence of monthly benefits at early death or non-qualification at 65. The first 1937 check was for a few cents. Against the ten billions in benefits of 1959, it took fourteen years to reach the first billion in 1950. Further to indicate how flippantly "need" seemed to register, the monthly benefits to the non-needy have run about three times those to "the needy" under the plan. Further, the payments in the "affluent year 1959" were ten thousand times the payments in the "needy year 1937." "Cupidity" rather than "need" seems the quality recognized.

Benefits Differences

Both England and Canada were more consistent in meeting need, in setting flat benefits. They tried to get their socializing functioning more promptly, but with greater limitations. We followed Bismarck's German lead in establishing a sort of caste system of dole-receivers, where the top men got more than the "lower orders." The explanation sometimes given was "to make it seem less socialistic." The variation has been rationalized here by the phrase "a mixture of equity and adequacy." It seems to be interpreted as "balancing self-support with charity." The range started with a range from $10 to $85, shifted to $10 to $40, but with yearly increments, and is now $33 to $127—all relating to the retired tax-payer. The range from a wife retired at 62 to a whole family would run from $12 to $254. It was initially claimed that the wage-tie would be flexible. It apparently does not easily adjust with a biennial change in formula. It is difficult either to see a clear social purpose or to justify claims of equity.

Contribution Toward Benefits

In their British Welfare State, Beveridge talked of "a shilling for thruppence." Our Social Security Board claimed for a time that "every man got a bargain." Years ago it seemed to me that men were collecting dollars of benefits for nickels of tax. The Curtis Sub-Committee in 1953 cut the nickel to two cents.

Given the happy feeling that nobody pays full cost, Mr. Peterson's recent OASI paper in the Society of Actuaries, uses some of the published studies to show that one of these days, the life-time tax-payer is apt to be overpaying for his prospective buyer-bargain. But if the employer's tax is not allocable to the individual employee's benefit—merely socialized for all employees of the country—we are thrown into a comparison between personal taxes and personal age benefits paid much later—one of those open-end accounts that can't "prove out" till the academic interest in the penny-pinching will have evaporated.

Here Mr. Myers does indicate the complexity involved in "retirement benefits" which include children under 18, their mothers under 62, wives

over 62, wives over 62 but disqualified by work, dependent parents, burial benefits and aged widows. Life insurance benefits at pre-retirement deaths bring in the orphaned children, their widowed mothers, dependent parents, special agreements with the Railroad Retirement Agency, the military organization, state and local governments, compulsory self-employed covered, and some voluntary worker membership. All this scrambling of grades of belonging has been the subject of numerous actuarial studies.

Often when I write or talk on this subject, I am advised "keep it simple." And this is another negative—simplicity has become impossible. Simplicity means significant exclusions. Two men reaching 65 with the same wage record and tax payment may have radically different benefits following 65. One may keep at work till 72, be unmarried, live to 73, while the other with a young wife and children under 18, may retire at 65, live to 80 and leave a "young widow of only 62." The pre-retirement possibilities are very wide too in event of early death. Less than half of these deaths leave minor children under 18. At one death there may be a burial benefit of $250, at another aggregate payments to children and mother that can add up to more than $50,000. When "contributions toward" sound simple, they represent tremendous differences in the personal and family benefits. Protest is coming from spinsters at having to help pay the costs for men with large families.

The initial average tax in 1937, personally paid by the worker, was $9, in 1959 it had passed $60. This early limitation of taxes to so small an amount was due to the denial of benefits to the aged until certain minimum periods in the labor-market with OASI tax-payment should qualify the worker for benefits, and also to the low earnings of the depression period. But in spite of the low tax payments, the initial surplus over benefits exceeded 99% of the first year's taxes. There was some surplus each year for 20 years. In 1957 the "surplus" ran out, and has been missing for three years. In accordance with that Parkinson's Law that says that outgo rises to use up income, surplus is hardly to be expected hereafter. Due to Secretary Morgenthau's insistence the initial tax rate was doubled, making the early taxes *less* ridiculously small, and the period of "coasting" somewhat longer.

Jarvis Farley wrote a paper in this Society—perceptive of coming difficulties—and now Peterson's "Misconceptions and Some Missing Perceptions of our Social Security System" has broken the too-long Actuarial silence. Social Security had been as all-embracing a term as "Sin," against which Calvin Coolidge's minister is said to have inveighed. One has to know an evil to wage intelligent war against it. The ignorance as to OASI has saved it from much intelligent attack.

The Lost Republic

There was the America of de Tocqueville and Lord Bryce, the loss of which has been deplored by Garet Garrett and his friends. It was a Republic of Sovereign States, where *insurance* was not a central Government function, where relief was local charity, where the poorhouse was but little used. One member of the Social Security Board said "We are going to change all that." That intention was a leading negation. The change is still in progress. The radio announcer, out to help the colleges get funds, reports that workers'

real wages have doubled, that college professors have lost 5% in purchasing power. Labor monopoly has been very effective. On two successive days feminine voices over the telephone announce they are "making a survey" and ask the year and make of my car and whether I am out to get a 1960 model. No one has asked me yet which part of the purchasing market I am in—the gainers or the losers, nor whether I intend to pay my own bills. Nor is it openly called silly to keep on assuming that the portion with doubled income is still unable to meet its own bills without major subsidy. The children spending more time in school are being told the "new facts of life" in courses of Social Studies.

The Word Insurance

In two briefs to the Supreme Court, the first in 1937 on the then Old Age Benefits, the second in 1959 dealing with OASI, it has been stated—truly enough—that the benefits are not *insurance,* but rather *gratuities,* presumably needed. In Bismarck's Germany, the phrase *Social Insurance* seems to have been coined after the Franco-Prussian War, as France too quickly finished paying the war indemnity. It came into use again in England between the Boer and First World Wars. It was continued by the International Labor Office, organized following World War I. A temporary emergency led into what was designed to be a permanent burden upon the nations. The word *insurance* seems to have been chosen for its connotations of dependability, self-support, "balanced and actuarially correct finance"—for its very *sound* of soundness.

The terminology was, however, a part of that negation of the individual that Hayek called "The Road to Serfdom." The temporary unbalance was not allowed to right itself, but an artificial remedy claiming to use socialism to fight socialism, sacrificed freedom, in the name of freedom. ("not such a bad life, security-wise")

In the United States 30 years ago we apparently had 1% of the citizens over 65 years of age in the poorhouses. Today they say "To stem the parade to the poorhouse, we had to invent social security. And today in accordance with the structure of the benevolences as outlined in the two briefs, well over half of our aged are in receipt of what are mainly *pauper doles,* either Old Age and Survivors Insurance or Public Assistance. The poorhouse also persists, though hardly "on parade." The "parade" is present in the substitutes.

Three Stages of Growth Chronology

1. We have established—on the statement of dire need—a system of largely deferred benefits to meet that need, and then added benefits for other needs as the negations against prompt qualifications delayed meeting the first need. 23 years of operation have passed. And at the end of that period, three years after the margin in the yearly tax collection disappeared, there were on the benefit rolls some 7½ million "retired primary beneficiaries." They had been called fully insured, and had reported small enough recent earnings to be qualified for monthly benefits or had reached the age of 72. Their "retirements" ranged from the year 1940 to the year 1959 so that the maximum periods of benefit receipts might have been nearly 20 years, and the minimum as short as 1 month. The specific personal records of tax history

were not available, but using over-all average reports, a plausible relationship of aggregate personal tax payment into the Treasury and then to the Fund, to benefits paid the retired person and members of his family, after his retirement, could be deduced. This benefit item was made up of benefits already received by the primary beneficiary or apt to be received later together with all supplemental income to his dependents, now or subsequently, either while he was getting benefits or after his death. But with a considerable potential margin of error, the results appear to show a nickel of tax for "two bucks of benefit." For the cohorts retired in 1940, 1950 and 1959, the nickel would become respectively 4 mills, 2 cents and 7½ cents for "the two bucks." The yearly relationship to the two bucks would show higher ratios for the higher taxpayers, and lower ratios for the lower taxpayers under the "bent formula" that "adapts so clumsily to later modification."

Here are 7½ million people once anxious to be self-reliant, self-supporting citizens, most of whom "would rather die than enter the poorhouse," now uncomfortable at their role of near-dependency.

2. Somewhat less than 100,000,000 "covered persons," not yet retired, whether because of too much work, or because they have not yet become "fully insured," or are not old enough yet, represent the second stage of working persons with some record of taxed wage payment, still alive. Actuarial studies indicate that the benefits to this group might represent 17.91% of taxable wage, because of their relatively brief period of remaining work-life, as compared with "new entrants," say at the age of 18, with a whole working life ahead of them. There is no expectation that taking their own taxes and those of their employers together they will anywhere near meet prospective benefits by their personally registered contributions. Using their own contributions alone and not counting upon interest earnings (later discussed) probably they are not expected to meet more than 15% to 20% of the prospective family benefits.

3. The prospective "new entrants" who are figured as having future benefits worth at a discounted value 5.23% of their pay, but seem scheduled to contribute 8.88% are left paying for all eternity for the more than 100,000,000 predecessors, who are slated to under-pay.

These later percentages assume that the system can count upon the payment of interest on accumulating funds. But in my own analysis, I have only used the benefits paid after retirement. There has been a tremendous amount of "life insurance protection" before retirement, and were the contributions of the decedents considered as available to be applied toward their death benefits, these sums fall so far short of meeting that cost, that after allotting all the interest on the trust-funds toward the deficiency, the appropriate "claims reserves" could not be maintained. The last three years all the interest accumulations were required toward the current benefit payments, and even then the funds shrank decidedly. So—focusing attention on the "retirement" part of the galaxy of benefits—I see no justification for counting on the availability of interest to help out there. I expect that in accord with the Parkinsonian Law, the accounting will continue to be hand-to-mouth!

Summarizing the three increasingly large groups—the 7½ million retired, the 'somewhat less than 100,000,000 non-retired workers' and the infinity

of future tax-payers stretching out to the end of time—we have one of those confirmations of Holy Writ that so frequently evidence the thoughtfulness of historical observation—this one about visiting the sins of the fathers upon the children unto the third and fourth generations.

That first group of 7½ millions seems to be the band of decoy ducks, presumably content with their bondage—toward which they have contributed so little—2½%—or, to change the figure, the tigers caged in security. Against that subsidy of 97½%, perhaps the next group might be only 80% to 85% subsidised. But then the third group is to be left to foot the unpaid bills. What they let themselves in for is surely "up to them." Peterson, I believe, expects them to object and so does Dr. Frank Dickinson. He knows that youth is capable of rather hard-headed reversals of judgments of previous generations. *I* expect a rather superior smile as they look back upon the current inexplicable foibles of this gullible generation. My generation felt much superior to the assessment and fraternal errors of the generation before. I anticipate that today's children will rediscover some of the sound understanding "we have loved long since and lost awhile."

More Zoology

In the animal field also, the ostrich, with his head in the sand also comes to mind in regarding the dizzy growth rates. But more frequently it is the camel with the "head under the tent-flap." OASI "aged-worker camel," little head inside in 1940, was accompanied by the "dependent-of-aged-worker camel" and the "survivors-of-non-retired-workers camel." All three have got in about up to the hump. The head of the "disablement-of-workers camel" was promptly joined by the head of the dependents-of-disabled workers camel, and now the nose of the "medical-benefits camel" is sniffing at the entrance also. Or, when we watch the flock, the synthesis of hydra-headed or the volatility of the chameleon also come to mind. Peterson's designation of the nonliving "blend" would be simpler, though it misses the growth potential thought of the living organism. His "blend is so bland as to blind us to blunders" says a lot. Whether we reason from the specimens in the *zoo* or consider the dubious broth, the outcome depends on certain decisions over long periods of time by an infinite number of persons.

Public Assistance

Limiting attention to the purely National OASI makes the "Social Security dealing with aged need" seem more heartless than it was. The Staff of the Committee on Economic Security of course knew that, given the existing need in a great depression, mere promise for the future was not enough. The current answer was the Federal subsidy to the State program of Old Age Assistance, Aid to Dependent Children, Aid to the Needy Blind. The start was 1936 for Public Assistance and the National Grants to States reached $80 million—as against the delicate $1 million in 1937 toward starting the ingress of the camels. As a drain on the public funds, the combined demands of the OASI and the Assistances have only grown 150 fold from 1936 to 1959 whereas the OAB AND OASI combined has advanced 10,000 fold from 1937 to 1959. Assistance has taken an intermediary position in "dignified relief"

between the poorhouse and local out-relief and the build-up furnished OASI. The poorhouse is plebian, assistance, bourgeois, OASI aristocratic. It has been the intermediate Assistance that introduced permanent total disability under a new category, well before the aristocratic handling under OASDI. It was Assistance that put in Medical Care, before the Forand Bill conversation about calling the aid "insurance."

The 1937 and 1959 Briefs to the Supreme Court are in accord with this assessment of "cod-fish aristocracy" for the "insurance flock" of camels.

Instability

Each new expansion of OASI is presented with "tongue-in-cheek" claim that *here* is stability. Two years later this claim is dropped. Another "shot-in-the-arm" is needed. Poverty (or at least personal incapacity to budget) has gained on us. After greater thought, it seems "the aged never had it so bad." The OASI tax rates that once were to stop at 3% and 3%, have now been moved up to 4¼% and 4¼%, ignoring both the Disability extras and the intangible extras for Medical Care. The assumption that our overtaxed citizens will continue to submit indefinitely to rising taxes, if only they are labeled *contributions,* and that they enjoy being talked down to, does not quite still the sound of the grumbling, nor silence the rising belief that the chains are cutting in. If we keep on with the biennial "rat-race," the whole system is clearly one of "instability."

Dizzy OASI Growth Rates

Sticking to OAB, turned into OASI, the Fabian-inspired gradualist development has in 23 years "hung up a record." The aggregate of the collected taxes, annually, has risen from $½ billion in 1937 to $8 billion in 1959. That is a 16-fold growth. 16 is 2 x 2 x 2 x 2—or 2 to the 4th power. In the same period the benefits grew from $1 million in 1937 (through $1 billion in 1950—inserted to show the thoughtful application of "gradualness") to $10 billion in 1959. This is a 10,000-fold growth. 10,000 is 10 x 10 x 10 x 10—or 10 to the 4th power.

The growth potential was there all the time. But it was the negation of responsibility or the division of treatment—the vital assistance off in the States, with part of the costs locally met, and only the National subsidy appearing in the National Budget, that permitted the quiet 1000-fold growth in a period of 14 years, to be pyramided further 10-fold in the following ten years. With the inversion of the pyramid, the tip sunk but slightly into the sub-soil, the instability of the edifice in a tremendously mortgaged real-estate below is about to give public concern.

By arranging to pay the major part of the provision from the Assistance account through 1952, instead of from the trust fund amassed from the earmarked OASI taxes, that trust fund rose to $23 billion, and then in the three years 1957 through 1959 has only fallen off to $20 billion. Had those National subsidies to Assistance been paid from the trust fund it would now be completely exhausted.

It is against this situation that the Forand Bill and all the expansionist suggestions must be viewed.

More of the Negative

But as against this painful tracing of past bookkeeping, there must be complete inability to forecast the future progress of costs and taxes. There are scores of interlocking factors—among them the stated reaction of the enemy that we will spend ourselves into ruin, and our own possibility of rediscovering bookkeeping—that will determine what those costs will be.

Roswell Magill in the November Reader's Digest tells us that dropping off all the progressively rising income tax rates above 20% would lose us but 13% of the tax take. Doing just that amputation might release the anesthetized energy of our productive minds to get back to functioning with "their own money!" But in the inflationary reduction of the value of money, which doubly robs us, first in buying power of income, and then in taxing away what are called "gains" in capital, we have had a long-developing dulling of incentives to thrift, to modernising the productive plant through personal nonspending but prudential capital-accumulation. The income-tax is felt by all of the rank-and-file workers. For many of them now, the OASI tax weighs more heavily in dollars than does the first income tax—and they can see that the more taken from their pockets to pass out to others in this "new-assessmentism," the less sound basis for personal choice of money-use remains to them. Even the aged Pauls for whom the Peters are being robbed—insofar as they had been thrifty through savings-bonds, building-and-loan shares, life insurance contracts—find they have lost more through reduced purchasing power, than comes to them through robbing the young Peters. They are longing to recapture the stability of the gold standard!

Verifying the Ready Reckoner

In a more "feet-on-the-ground" period, Carlyle criticized that human curiosity that sets out "to verify the ready-reckoner." Since then we have lost the confidence in some of the "too-ready reckoners." We have to bite the coin to see if it is counterfeit. In this OASI the "half-promises" as to future performance must be met by today's big baby-crop and those to come. That baby-crop seems to me to have been largely swollen by the "take-no-thought-for-the-morrow" of "big-brother's assurance that the State will provide."

The over-dependence of collectivism, heavy corporate taxation, taxing of the very interest that pays for thrift, to slow down the compounding of "the wages of risk-taking," reducing the vigor of competition, over-centralising Government, rewarding idleness, inertia, "feather-bedding" too much—these are items that call for the verification of the complex ready-reckoner that calls liabilities assets.

In a revolution, heads fall, values shift, new machinery is invented, coins are tossed, values change. The re-examination goes to private property, obsolescence, appreciation, conservation, waste, motives, incentives, goals, power and sovereignty. No brief Reader's Digest summary and simplification can replace the wisdom distilled from human experience. More understanding than cupidity must accompany demand.

As I was setting down these comments, a nagging memory sent me back to Browning's "Bishop Blougram's Apology" and the "last chapter of Saint

John." It was almost painfully pertinent. How can we in ignorance sense the "why"—and how, in the fast-moving too-short-days, can we practically remedy the ignorance?

A Few Points on Mr. Myers' Paper

Mr. Myers' paper dealing with the incompleted revolution, with the remaining mortgage against our common property unbelievably great, is a painstaking, though all-too-brief guided tour through a battered area much like our Washington South-west project of "Urban Rehabilitation." He wrote the paper months ago, and has been steadily busy on "the verification project" ever since. I waited for history to unfold so as to have that slight chronological advantage, before I should sum up my comments. They require the previously outlined background.

1. *Perpetuity*

In the 19th and 20th Annual Reports of the Fund Trustees (OASI and Disability) the figure for the OASI "trust-fund end of 1959" differs by $¾ billion. The two reports appeared less than a year apart. The complexity of the program, the various lags in decision, in administration, the bunching of certain decisions at a given time of year make this difference reasonable. I bring it in, because with a 4% difference in one figure in less than a year, it is obvious that certainty out in eternity is completely impossible. I believe that nature abhors perpetuities as she does a vacuum. Hitler only looked ahead 1000 years!

2. *Extrapolation of Long-time Forecasts*

There are scores of factors that enter into the course OASI development will take. Mr. Myers has felt the need to hold some of them constant in the two illustrations he develops. I recently attended a Population Society Meeting. One of the models of population development had to do with "urbanization" in India. A Calcutta of 89 million people showed up in "the projection." Were we to consider these graphs "interesting exercises" rather than *forecasts,* such "adventures of the mind" might be exhilarating. They become dangerous when quoted as though they were "truth" rather than "mental exercises."

3. *Low, High and Game*

The two illustrative "projections" are set down by Mr. Myers with explanations that show their frailty. One cannot know exactly what the course of evolving history may be. The low and high illustrations are to some extent determined by "ideology" of full employment, the need to check wage inflation, and other political gambits. Marx and Keynes have been well-examined lately. It seems to me that the range used is too narrow in considering "the possible"—so that the two prospects might be called *low* low and *low* high. But when the mean of the two "projects" is set down, as not any more dependable than the two boundaries of the low low and the low high, and then is quoted as making this highly suspect system "actuarially sound," "reassurance" has replaced the "need for verification."

4. *The Word Insurance*

The *word* insurance seems to have been used by Bismarck, Lloyd George, the ILO and "the new deal" to bring the *idea* of dependability, sound finance, skilled actuarial and legal direction to the aid of a new element in national life. It never seems to be defined. Challenged, the answer is that it is *social insurance.* I agree, I think, with the final word in a short interchange between the late Senator Taft and "an expert," that final remark being by the Senator: "It isn't *any* kind of insurance and you know it."

5. *Well-Established Actuarial Techniques*

The label of *insurance* could bring along for overtones the old assessment and fraternal methods (probably the most logical), those of individual level-premium life insurance, individual annuity contracts, various group or mass insurances and annuities, non-insurance-company pension trusts. It even seems to suggest bank deposits to some. Mr. Peterson's recent paper on Misconceptions assumes that the overtones of *pensions* hold. My Social Budgeting presented to this Society long ago—illustrated by Canada's flat age pension at 70—dropped the whole *insurance* feeling. Mr. Myers also seems to lean to pension methods. Fortunately these vary so widely that a whole fleet of images comes to mind as different persons consider it. The "accepted actuarial principles" of Secretary Morgenthau's phrase in the Act of 1935 was too clear to him for the comfort of other "planners." It came out. But there is no agreement as to what they are or should be in OASI.

6. *Interest*

In Mr. Myers' space-journeys to eternity, he uses discounts at interest to bring infinity down to finite limitations. As I read the cards, the two decades of margin in tax collections through denial of the value of simple social budgeting, which ended in 1956, have ended the apparent "surplus provision in taxes," and the interest earnings on the fund are cheerfully to be spent for current benefits. Parkinsonian imagination suggests the situation will continue "tight." I have noted that for a long time the underprovision in the taxes of those who die, for their dependents' benefits, is also apt to commandeer any interest allocation if it were there to build a "claims reserve," now only a quarter met. Much as the word "fund" seems safer than "reserve," because "fund" is assets-suggesting and "reserve" suggests huge unmet liability, so "interest" pleases a man when it comes in and pains him when it goes out. The "interest" entering the fund now is a part of the awkward billions budgeted out of our tax-load. I don't believe perpetual payment of interest by the tax-payers will ever be accepted as "the balancing item in blue-sky finance." I think this is a major flaw in "the level-premium figures.

7. *Actuarial Balance*

I deny the validity of any *actuarial balance* brought about by the prospect of tax-rate increases in the years 1963, 1966 and 1969—or even the assumption that today's bureaucrats can force their loosely defined creations indefinitely upon an uninformed or semi-informed electorate. Having had it quoted to me from "high places," I have been impressed by Budget Director

Stans' disapproval of other built-in tax-hungry rising-cost programs. This system merits his greater disapproval. A "very small tail" is expected to "wag a very large dog."

8. *Self-Support*

After three decades of deriding the personal desire to be self-supporting, and under some labor-monopolistic wage adjustment, so that it is *said* labor has double its former *real wages,* I am unimpressed by the claim of the obsolescent term for "the system of OASI." It seems silly to me to tell these double-wage boys that they still need all the subsidies that have been thought up for them. The money comes from the tax-payers—and just as most taxes seem to come out of the "little citizen" earning less than $20,000, I expect that most of the taxes in OASI will be paid by the same group. I guess that here it means to those taxpayers, as a message from the State: "We make the rules. You foot the bills."

9. *Need*

The second word in the paper is "need." Steadily the largest benefits go to the least needy. OAB in the needy year 1937 paid 1/10,000 of what they paid in the year called affluent—1959.

10. *Congressional Cost-Mindedness*

The proof of the pudding is in the eating. My attendance at Congressional Hearings has too little shown that cost-mindedness. I call them fiscally irresponsible. I am not really criticising Mr. Myers. He is in a tight spot. He has worked hard. But "You can't make a silk purse out of the sow's ear of collectivism."

AUTHOR'S REVIEW OF DISCUSSION

Mr. Williamson's extensive discussion of my paper concerns not only the paper, but also the more general topic of the nature of the OASDI system. My reply is confined to those points that I consider most important from an actuarial standpoint.

In general, it seems to me that Mr. Williamson is adversely criticizing development of the OASDI system to the extent that action in any direction had to be "wrong." For instance, he criticizes delay in the effectiveness of the program, pointing out that payments in the early years were very low. In this respect the fact that payments in 1959 were 10,000 times those in 1937 is really not significant because in the latter, the initial year of operation, there was no intention of having sizable payments; the ratio would have been infinity if 1959 is compared to 1936.

The program is now being effective in paying individual benefits in respect to a particular earnings level that are virtually as large as will ever be paid under present law and that go to a majority of the population aged 65 and over. Mr. Williamson ignores this as being a point for praise. Instead, he criticizes the program for paying benefits that are many times the actuarial value of the beneficiaries' contributions.

I am glad to note that Mr. Williamson recognizes the weakness in the argument that Mr. Ray M. Peterson has made about future entrants to the OASDI system paying more than the actuarial value of their benefits in that he includes both the employer and employee taxes in his analysis. As Mr. Williamson recognizes, the employer's tax must be considered as being pooled for the benefit of all covered persons.

In regard to whether the word "insurance" is being correctly used in connection with the OASDI system, I would be the first to agree that there has been misuse in the analogy between OASDI and private insurance. On the other hand, I cannot agree that the cited Supreme Court briefs stated that OASDI is "not *insurance,* but rather *gratuities.*" Rather, the briefs brought out that the benefits are gratuities from an income tax standpoint; but, as I see it, this does not necessarily mean that the program is not social insurance. The 1959 brief stated, in essence, that OASDI is not an insurance program under which premiums are paid to acquire benefit rights, but this does not mean that it is not an insurance program of some other type, such as social insurance involving statutory rights to benefits. The 1959 brief does state that OASDI is "social security 'insurance' ". Furthermore, the Supreme Court, in its decision on the 1959 case, stated that "the social security system may accurately be described as a form of social insurance."

Mr. Williamson criticizes adversely the fact that present beneficiaries have "paid for" less than 10% of their benefits. A pension program of any type cannot provide adequate benefits and be effective within any reasonable period of time unless this situation prevails. Certainly this is true for private pension plans under which those retiring in the early years have contributed very little themselves and yet receive pensions based on prior service credits financed entirely by the employer. The employer may contribute very little for new entrants into such a pension plan—far less than for older employees at the inception of the plan.

Mr. Williamson also criticizes the use of an interest rate in connection with actuarial analysis and consideration of OASDI. Although the system is not fully funded—and I think that nobody would really like to see it on this basis—a significant proportion of the future financing (perhaps as much as 10%) is expected to come from interest earnings. More important, it is sound actuarial practice in comparing payments at different times in the future to use a reasonable interest rate.

In regard to the question of future generations objecting to OASDI tax rates greater for the employer-employee combined than the value of the benefits—I cannot see any problem either from the practical or theoretical standpoint. These individuals will, on the whole, "get their money's worth" in regard to the benefit protection compared with their own contributions. Moreover, when considered on a generation basis, future ones cannot consider the OASDI tax load alone, but also should take into account the many important physical assets bestowed upon them by today's generation.

Mr. Williamson points out that our cost estimates or projections are admittedly illustrations. I agree that these estimates are by no means as precise as valuations made for systems involving a closed group. On the other hand, I do not think that this means that our cost estimates—and particularly the

intermediate estimate, which is merely an average of the low-cost and high-cost estimates, as is quite openly stated—are of little value. I am convinced that they represent a good yardstick for measurement of changes in the cost of the present plan due to variations in the experience and the cost effects of proposed amendments.

The logical result of Mr. Williamson's views would be that no cost estimates at all would be made so that Congress would have no guidance as to cost aspects. Under such circumstances, any changes would be unmeasured even relatively as to cost in both the near-future and more-distant years. In the same way it seems unwise to follow Mr. Williamson's suggestion of disregarding future scheduled tax increases. The result would be that Congress might legislate a tax rate for only a year or two in the future, despite benefit liberalizations involving very high costs after a few years. And, of course, any benefits promised for the future would be difficult, if not impossible, to reduce when the financial pinch came.

Finally, I cannot agree that Congress, and especially the important Congressional committees involved, have not shown adequate cost-mindedness in dealing with OASDI legislation. I believe that on this point Mr. Williamson is coloring his views with his belief that there should not be any type of social insurance program in this area (but rather perhaps only a needs-test system). I have expressed my views on this matter in some detail in the last two paragraphs of my paper, where I point out that in a number of times in the past benefit liberalizations have been turned down for cost reasons.

CREDIBILITY OF 10/20 EXPERIENCE AS COMPARED WITH 5/10 EXPERIENCE

BY

LEWIS H. ROBERTS

Volume XLVI, Page 235

DISCUSSION BY M. BONDY

Lew Roberts has tackled a difficult and important problem with great resourcefulness, making maximum use of a limited quantity of data. The results at which he has arrived provide an excellent general guide to the reliability of excess limits experience and the method evolved should go a long way toward providing the answer to a similar question with respect to General Liability experience. Such guides will be useful to company actuaries in evaluating individual company experience as well as to Bureau personnel in the making of rates.

One question, however, has been raised in my mind with respect to Mr. Roberts' conclusion that "the credibility of 10/20 experience should be somewhat less than 85%, perhaps 80%, as great as the credibility of 5/10 experience." I shall lead up to this question of furnishing a hypothetical example:

Let us suppose that in a given territory, we have just enough experience to warrant 100% credibility on a 5/10 basis. It is now decided to take into account 10/20 experience. Statistics have indicated that for New York private passenger autos, the 10/20 pure premium is split about 87-13 between basic and excess limits. However, we may take as a point of departure the division indicated in the excess limits table of 5/6 — 1/6.

If 10/20 experience is used and the credibility suggested by Mr. Roberts applied, the credibility of this territory's experience would be 80%.

If, alternatively, the experience is split into layers of 5/10 and excess over 5/10, the experience of the territory would be followed to the extent of 83 1/3% on the average even if the excess limits experience received no credibility at all.

I am wondering whether this idea of division into basic and excess layers might not be more easily understood and accepted than would an apparent reduction of overall credibilities caused by the introduction of high limits experience.

A final word on the subject of credibility appears in order at this point. One is occasionally confronted with solutions to the question of how much credibility a given body of experience warrants. However, the complimentary question is often, if not always, left unanswered. "To what shall we apply the (1 — Z) factor?" At times it seems to me that while we are reluctant to give more than, say, 40% credibility to a given body of data, we blithely assign the remaining 60% to a statistic which is not at all reflective of the attribute which we are attempting to measure.

COMMUTATION FUNCTIONS FOR INDIVIDUAL POLICIES PROVIDING FOR HOSPITAL, SURGICAL AND MEDICAL CARE BENEFITS AFTER RETIREMENT

BY

HENRY W. STEINHAUS

Volume XLVI, Page 251

DISCUSSION BY J. J. SMICK

Normally, actuarial tables for the purpose of calculating reserves and costs are the results of the joint work of special committees, groups of insurance companies, organizations such as rating bureaus and similar associations. It is unusual nowadays to have a table prepared by an individual actuary and presented for public use without all of the preliminary steps of appointing committees, issuing a call for data and then proceeding with the work, which is generally considered a major undertaking.

When the work is undertaken by a group of men, the judgment elements used are frequently arrived at on the basis of compromises. In this case Dr. Steinhaus has assumed full responsibility.

We have in this instance a set of tables, reasonably comprehensive in scope, prepared by an individual, suitable for use directly by the industry. Since the tables have not been prepared under the sponsorship of a major organization or committee of actuaries it becomes important to review them rather carefully, keeping the following main elements in mind:

(1) The adequacy of the basic data
(2) The actuarial procedures used in the construction of the tables
(3) The limitations that have to be placed on the uses of these tables.

(1) *The Adequacy of the Basic Data.*

The basic data used is a combination derived in part from the experience of other countries and in part from data compiled in this country. Some specific figures are shown to indicate the total exposure and number of cases considered. We have to assume that as regards actual volume of data the experience used is adequate. Furthermore since the basic data is not shown separately by year we have no means of judging as to whether or not there are any significant trends, or whether there occur any major variations from one year to another. Perhaps these are not too important, but they might have been illuminating.

A brief comment as to the use of the experience of foreign countries for tables to use in this country may be appropriate. Currently we have become accustomed to reliance on tables based entirely on United States experience. It was not always so. We have used the Northampton tables, the Combined Experience Table, the tables prepared by the Friendly Societies, and in determining widows' pen-

sions under workmen's compensation laws we still use the Dutch and Danish Tables. One of the earliest tables in the casualty field was the table prepared by our first president, Dr. I. M. Rubinow, called the Standard Accident Table, which utilized foreign experience.

It is also important to bear in mind that much of the basic data used is American experience, adjusted by European experience where the American experience was missing.

We should therefore not dismiss as inapplicable data derived from foreign sources. Medical and hospital needs are probably similar both in England and Germany to those in this country. The major differences will result from availability of services, utilization, and cost elements, and not from any fundamental differences in either people or their medical requirements.

Appropriate references are included to show the basic sources of the data used whether in foreign publications or in local publications.

(2) *The Actuarial Procedure Used in the Construction of the Tables.*

Actually, standard actuarial procedures have been used in the construction of the table. Unfortunately a separate section usually devoted to a series of definitions and the exposition and development of the formulas, step by step, has not been included. Such a section would have been extremely useful for students and for company actuaries who might wish to substitute some modification of their own. The saving feature in the presentation results from the fact that the net annual claim cost Sx is shown on each of the tables prior to the calculation of Hx & Kx values so that it becomes a simple matter to derive new values by simply substituting alternative sets of annual claim cost figures Sx in order to derive commutation values reflecting changed net annual claim cost figures. I would have preferred to have all of the intermediate calculations and factors shown, including lx figures. I know that this would have increased the printing costs but with the new increased dues, the Society, I am sure, could afford the cost.

There can be very little criticism made of the actuarial techniques involved since they are basically simple and follow standard procedures.

(3) *The Limitations That Have to be Placed on the Tables.*

Since we have no uniform set of charges for medical services, nor for stays in hospitals, the tables can be used only if we remember that costs vary from region to region, that costs and services in rural areas are different from those in city areas, that the actual amount of insurance an individual has often unduly influences the utilization he may make, or the services that may be suggested for him by the physician. Thus I would feel that a $20 per diem for room and board in a hospital would not cost exactly twice what a $10 per diem would

cost. The extra $10 will have an influence on both the utilization and length of stay.

It is my belief that the variations from area to area in this country will prove as great if not greater than the possible difference between the use of European experience and the use of American experience in the construction of the tables.

The second important consideration has to do with trend. Even if the Steinhaus' Tables were exactly comparable to current American experience, will they be valid for use five years or ten years from now? For that matter, will any table constructed on the basis of current experience stay valid for a very long period of time? Not unless there is some form of stabilization reached on hospital charges and surgeons' and physicians' fees. Thus, any actuary using these tables must know what he is doing and must be able to make such adjustments, either in the tables or by overall factors in order to be reasonably sure that the experience of his own company is reflected properly. This in no way detracts from the value or usefulness of the tables. Any basic table will require some adjustment for underwriting procedures and actual company experience.

In view of the recent public discussion of the various alternative proposals for insuring the cost of medical and hospital care for those over 65, Dr. Steinhaus' summary of the probable costs on a net annual basis are of great interest. They are worth repeating at this time.

	Male	Female
For $10 Hospital R & B up to 31 days	$ 26.370	$ 27.670
For Hospital Incidentals up to $150	24.165	25.101
For Surgical Benefits up to $325	12.790	9.910
For Physicians Services, $5 average charge	37.785	43.200
For In-Hospital visits at $3, one a day	3.164	3.320
	$104.274	$109,201

Unless one has a better basis, the above figures represent an informed estimate of the minimum cost of a reasonably adequate program of care for the elderly.

We may summarize the paper presented by Dr. Steinhaus in a few simple words. We now have a set of tables actuarially useful for computing costs and reserves for medical care for those over 65 where none was previously available. We have some basic experience presented supplementing our own meagre data. We have an informed estimate of the net annual cost for those over age 65, extremely useful at this particular time.

It is a most timely and appropriate paper.

DISCUSSION BY M. KORMES

Mr. Steinhaus' paper is a very timely one. The much discussed Forand bill and its Republican counter-measure the Javits bill threaten to

become a serious election issue. The rising cost of hospital care and the limited financial resources of the vast majority of the aged create a serious challenge to the insurance industry and unless it can come up very shortly with a satisfactory solution, government intervention will become inevitable and we are all well aware of the concomitant danger of socialized medicine.

There is no doubt that a level premium is very desirable from the public's point of view but as respects hospitalization coverage which provides a fixed amount of benefits a plan bought at age 65 may become wholly inadequate at age 75 or even age 70, thus necessitating the purchase of additional protection at a higher age at higher rates. Only a few years ago—in 1955—the Prudential Life Insurance Company offered a "Senior Hospital Expense Policy" providing for Room and Board indemnity of $8.00 per day up to 31 days and up to $64.00 indemnity for other hospital charges with an aggregate limit of $2,400. The surgical benefits provided a $200 schedule. The annual level rates which were payable up to age 80 or up to the time when the aggregate limit of $2,400 was reached ranged from $64.08 at age 60 to $84.50 at age 70 (no policies would be issued after that age) for males and from $70.42 at age 60 to $84.50 at age 70 for females. (The higher rates for females most probably due to lower mortality rates.) In the light of present hospital charges this coverage would pay less than 50% of the hospital bill and would be, therefore, totally unsatisfactory.

This leads to another very unsound situation of overinsurance whereby those who can afford it buy policies from several companies and in many cases have also Blue Cross coverage, so that a hospitalization very often results in a financial gain to the insured and having no gainful employment the tendency to extend the hospital stay is very substantial. This fact has been amply demonstrated by a recent study of hospital stays of older persons made by a large Blue Cross organization.

The basic data in Mr. Steinhaus' paper are from European sources and since the ages under 65 show similar claim frequencies to those of the "1957" study, Mr. Steinhaus feels justified to conclude that the extrapolation of the American data should produce satisfactory results. In general, this may be a satisfactory method but the use of claim frequencies will, in my opinion, lead to inadequate net costs.

Several years ago I served on a committee which made quite an extensive study of the cost of hospitalization for persons over 65 based on the experience of fourteen Blue Cross plans comprising a total of 3,411,975 contracts and 3,899,565 days of hospitalization. Each segment of the country was represented (there were three states each in the East and South, and four states each in the Central and Western regions). The data were weighted by the U. S. census distribution for the given state or area of plan operation and the excess of days per 1,000 contracts was calculated for those over 65 as compared with those under 65. In order to determine whether the variations from state to state were merely a matter of chance we applied the statistical methods of analysis of variance. A group of four states, one from each region was considered as a sample and three samples were taken at a time. The within sample and between samples variation was computed and the Snedecor F test applied. No matter what combination of plans were used

the test indicated that the variations were not significant, that is, that the mean of the samples is very close to the mean of the parent population. The results indicated that measured in days of hospitalization per 1,000 contracts, the cost for those 65 and over is from three to four times that for those under 65 (excluding minor dependents).

If we assume that the average age for those over 65 is 75, the net annual cost index S_x of 3.03 shown in Tables 2 and 3 appears on the low side. It is of interest to note that Mr. Steinhaus uses the same cost index S_x for males and for females although the tables in the body of his paper indicate lower cost for aged females. This fact is also borne out by the above mentioned study of Blue Cross plans which indicates an index of 3.79 for males and only 2.17 for females.

I find it somewhat difficult to understand the S_x in Tables 4 and 5 for special services which is rising from 18.6 at age 65 to 45.9 at age 85 remaining constant thereafter. As is indicated in the body of the paper the annual rate of surgery decreases, especially for females, and the cost of other services is lower in medical cases than in surgical cases, so that the values of S_x should be more or less constant unless these values represent an adjustment for the longer duration not expressed in Tables 1 and 2.

The data on the cost of surgical and medical care for the aged available from some Blue Shield plans were not sufficient to form a basis for any definite conclusions. On the whole, they indicated a cost somewhat higher after age 65 but not to any material extent. For this reason I withhold any comments on Tables 6, 7, 8 and 9.

Since the level method of premium requires the setting up of reserves, the problem of rising hospital costs and the resulting inadequacy of coverage could be met by either of the following means:

(a) By using the principle of variable annuities. This method may in specific instances cause hardship but in general should work out fairly well.

(b) By using the principle of increasing insurances whereby the benefits would rise automatically either each year or preferably every five years. This would require the replacement of the K_x column by column $S_x = \sum_{x}^{99} fK_x$ where f would be a factor to adjust for periodic increases in benefits.

In conclusion, I would like to say that the members of the Society should be grateful to Mr. Steinhaus for the presentation of this paper which required not only a great deal of calculation but also research and study of data from this and other countries. I know of no other problem whose timely solution will have a more important impact on the future of a large segment of the insurance industry as well as on the economic welfare of the senior citizen.

TOWARDS STATISTICALLY BASED FIDELITY RATES

BY

ZENAS M. SYKES, JR.

Volume XLVI, Page 271

DISCUSSION BY J. W. WIEDER, JR.

Anyone who has tried to read the fidelity and surety manual and to rate fidelity and surety bonds from them finds a confusing maze of rules and tables that can be quite formidable to the greenhorn.

The author of this paper has done a commendable job in developing the readers interest by citing specific examples of how the present rating system applies.

The study of fidelity losses of one important insurance carrier effectively points up some of the deficiencies of the present rating system and some possible solutions. The loss study indicates, as the author points out, that:

1. Fidelity has a preponderance of low cost losses
2. The rates for low penalty bonds ought to be increased
3. The rates for high penalty bonds may be somewhat redundant.

Thus, this paper demonstrates what fidelity underwriters have generally believed to be true.

As new bond forms have been developed over the years, rates have been set for these forms on a judgment basis by analogy, using the rates for prior or existing forms. Essentially, this is what is causing the fidelity rate structure to become increasingly unsatisfactory and to be in need of drastic revision. As a result of the loss study the author has stated a five-point proposal for rating fidelity bonds, a proposal which can surely serve as a sound basis on which to build a rate structure.

There will obviously be many practical difficulties in actually devising a new rating method, and the author has anticipated these problems. Undoubtedly one of the most difficult areas will be to establish an adequate classification system and much thought will have to be given to this problem by fidelity underwriters.

Perhaps the most difficult area from an actuarial standpoint lies in the treatment of loss salvage in determining loss costs for ratemaking purposes. As the author has noted, the data available to him in making this study were too recent to contain reliable salvage information, and he was forced therefore to make the study on the basis of gross losses. Salvage in the fidelity lines is important for all penalties and types of bonds. Mr. Sykes properly notes that salvage collection may be made soon after loss payments or may be deferred as installments over a period of time. Whether the treatment of salvage in ratemaking partially by an average value approach, as he suggests, will be adequate is a question which requires further study.

Even the definition of what constitutes salvage presents some problems.

For example, there was recently one very large public official loss on which there appeared to be a substantial early salvage collected. However, at least a portion of what appeared to be salvage was actually nothing more than a revision in the estimate of incurred loss. As the investigation of the case proceeded, it developed that the actual monetary obligation of the fidelity carrier was not as great as was first thought, and the reduction in the incurred loss amount was reported as salvage collected.

Another example of the difficulty in defining salvage is found in the case of disappearance of securities. Frequently a fidelity carrier will be issued duplicates of the securities which are held by the carrier for a period of time until it seems evident that the original will not be recovered. When this becomes evident, the carrier disposes of the duplicates for whatever price they will bring, and thus recovers salvage on the loss. The question is how to treat these security holdings prior to the time they produce actual salvage.

Similarly, if money from an embezzlement has been invested in real estate, the fidelity carrier frequently will issue a mortgage on the real estate, and use the monthly mortgage payments as salvage on the original loss. Thus, the salvage is reported over a period of time when actually the value of it is pretty much known soon after the discovery of the loss.

In summary, this paper is a very valuable first step in taking a new approach to fidelity ratemaking. The author has well stated the problems which require further investigation and has made some sound and constructive proposals for basing fidelity rates on statistical information.

THE COMPENSATION EXPERIENCE RATING PLAN—A CURRENT REVIEW

BY

DUNBAR R. UHTHOFF

Volume XLVI, Page 285

DISCUSSION BY R. M. MARSHALL

Mr. Uhthoff's paper serves the admirable purpose of bringing into focus the underlying features of the Experience Rating Plan Manual—1940 for Workmen's Compensation and Employers' Liability Insurance and further enlarging some of these features so that they can be examined for possible defects. Mr. Uhthoff also sets forth some of the considerations of the Subcommittee of the National Council Actuarial Committee regarding remedies for the seeming defects. I feel that the Society is indebted to Mr. Uhthoff for his timely paper.

Having spent many of these "happy hours," referred to by Mr. Uhthoff with the Subcommittee in its consideration of the Plan, it is hoped a report on the conclusions of the Subcommittee and a few comments will be in order.

The main defects of the present Plan appear to be two in number:

(1) It has failed to keep pace with the change in economic conditions, as evidenced by the continuing decrease of the ratio of primary losses to total losses.

(2) Under certain conditions the credibility formulas may give more than 100% weight to "primary" losses, although the average credibility for a risk could not exceed 100%.

With respect to defect number (1), it may be noted that the experience rating plan which was in effect prior to the 1940 Plan had a sort of "built-in inflation corrector" which tended to keep it abreast of economic conditions, at least to the extent they were reflected in the various state compensation acts. Under this previous plan the dividing line between "normal" and "excess" for indemnity losses was set at fifty times the maximum weekly compensation. Thus as economic conditions forced an increase in the maximum weekly compensation, the normal losses as used in experience rating were correspondingly increased. Also defect number (2) was prevented by the actual use of normal credibility and excess credibility, read from a table, in the formula for the experience rating modification.

As indicated by Mr. Uhthoff in his paper, formula (1) $A_p = 500 + 500(2/3) + 500(2/3)^2 + \ldots$ etc., when summed to infinity produces a maximum primary value of only $1,500. Therefore as the average cost per case goes up, the ratio of primary to total goes down. Furthermore, while the maximum primary may be increased by increasing the 500 unit or the 2/3 ratio, or both, it is rather difficult to calculate the effect of such changes. In fact not much of anything can be done without first making up a table of total losses and corresponding primary losses similar to Table I of the 1940 Experience Rating Plan. The Subcommittee experimented with various changes in both values and decided that a formula, $A_p = 750 + 750(4/5) + 750(4/5)^2 +$ etc. would produce about the desired pattern in state average ratios of primary losses to total losses (sometimes referred to as the average D ratio).

However after testing the effect of this revised formula on actual risks, it was felt that this might produce changes which were too drastic to be readily acceptable. It was noted that if a formula such as Mr. Uhthoff's formula (15) were adopted

$$\text{"}A_p = \frac{A}{A + 3{,}000} \times 3{,}750 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)\text{"}$$

the curve showing the relationship of A_p to A was not as steep for the lower range of values of A, nor did it flatten out as fast for the higher range of values of A, as did the geometric series type curve. A look at the appended graph showing the relationship of A_p to A will make this clear. The Subcommittee felt that these were both desirable characteristics. Formula (15) of course has the same maximum value of 3,750 as the geometric series, but due to the different shape of the curve produces somewhat lower D ratios.

Formula (15) as Mr. Uhthoff points out is vastly superior for machine calculation, particularly for electronic computers. For example in summarizing Workmen's Compensation Unit Statistical Plan data at the Council, one of

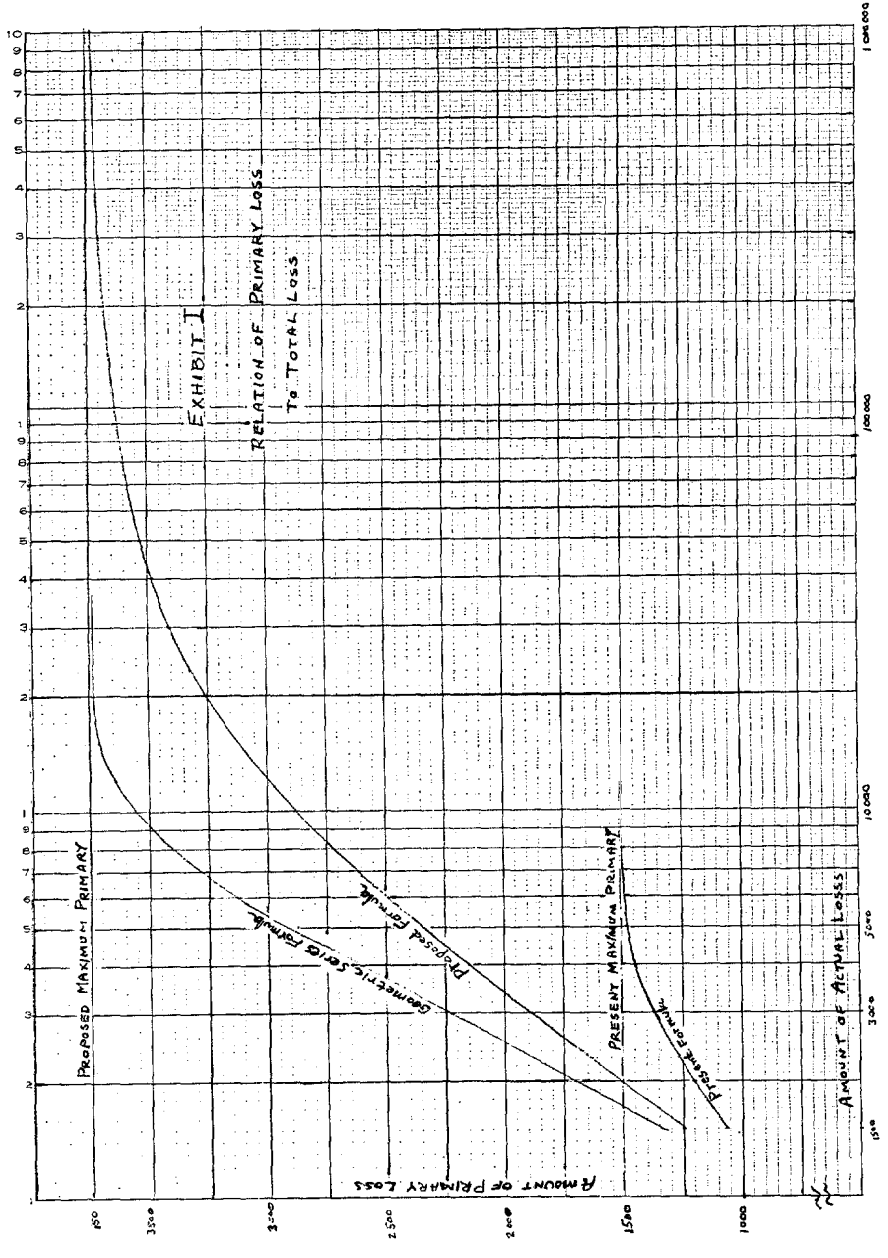


EXHIBIT I.
RELATION OF PRIMARY LOSS
TO TOTAL LOSS
PROPOSED MAXIMUM PRIMARY
PROPOSED FORMULA
GEOMETRIC SERIES FORMULA
PRESENT MAXIMUM PRIMARY
PRESENT FORMULA
AMOUNT OF PRIMARY LOSS
AMOUNT OF ACTUAL LOSS

the steps included in the program could be to add the indemnity and medical on each case and divide the total by the total plus 3,000. The resulting ratio could be either punched out or stored. At the end of the summary process we would have the summation of total losses and summation of the ratio $\frac{A}{A + 3{,}000}$. The summation of this ratio when multiplied by 3,750 gives the summary of primary losses for the state. In this summary process some special treatment is required for cases where the total loss is less than 750, or the primary loss will be greater than the total losses. These cases can be held out from the summary process and added in later as 100% primary, or a further condition put into the computer program that $A + 3{,}000$ must equal at least 3,750.

If this summary is made separately for serious loss, non-serious loss, and non-compensable medical losses (although we would not expect a non-compensable medical case over 750) we have the material for calculating classification D ratios directly from proposed pure premiums, by weighting such partial pure premiums by

$$\text{Serious P. P. Weight} = \frac{\text{Total Primary Serious Losses (Indemnity + Medical)}}{\text{Total Serious Indemnity Losses}}$$

$$\text{Non-Ser. P. P. Weight} = \frac{\text{Total Primary Non-Serious Losses (Indem. + Med.)}}{\text{Total Non-Serious Indemnity Losses}}$$

$$\text{Medical P. P. Weight} = \frac{\text{Non-Compensable and Contract Medical Losses}}{\text{Total Medical Losses}}$$

Before leaving this subject, a comparison of a normal value equal to fifty times the maximum weekly compensation in 1940 and today with the corresponding primary value for various size losses under the present Plan and as proposed may be of interest. In 1940 a maximum weekly compensation of $20 was common, today the figure is nearer $40. The comparison follows:

Size of Total Loss	Normal at 50 Times Maximum Weekly Comp.		Primary Under Multi-Split Plan	
	1940	Now	Present	Proposed
500	500	500	500	500
1,000	1,000	1,000	830	940
2,000	1,000	2,000	1,200	1,500
3,000	1,000	2,000	1,370	1,870
4,000	1,000	2,000	1,440	2,140
5,000	1,000	2,000	1,470	2,340
7,500	1,000	2,000	1,500	2,680
10,000	1,000	2,000	1,500	2,880
20,000	1,000	2,000	1,500	3,260
50,000	1,000	2,000	1,500	3,540
100,000	1,000	2,000	1,500	3,641

Although we still do not have any automatic transition that will keep pace with economic conditions, we at least have a gear shift lever in the formula $A_p = \frac{A}{A + 3{,}000} \times 3{,}750$ which will make it easier to shift up in the future when conditions warrant.

We now come to defect number (2), "the credibility formulas may give more than 100% weight to primary losses." Under the formulas for modification $M = \frac{A_p + K}{E_p + K}$ below the Q-point, or $M = \frac{A_p + WA_e + B}{E_p + WE_e + B}$ above the Q-point, there is no indication as to what the primary credibility or the excess credibility, or even the average credibility, may be. In fact even to calculate the average credibility we resort to the axiom that "the average credibility is equal to the credit for clear experience, i.e. no actual incurred losses". And since we are occassionally exceeding 100% primary credibility without any apparent serious consequences, the question may arise, "Why worry about a primary credibility greater than unity as long as the average credibility is less than unity?"

To the actuarial mind the idea of a credibility greater than unity is unacceptable; it corresponds to the absurdity that the probability of an event happening is greater than certainty. To be actuarially sound the Plan should be corrected so that neither the primary nor the excess credibility can be greater than unity, regardless of whether or not the actual credibility figure may be readily determined.

The question may arise as to how it comes about that primary credibility may be greater than unity. The formulas underlying the Old Experience Rating Plan were comparatively precise. Actual and expected losses were divided into normal and excess, normal and excess losses were assigned separate credibilities from a table depending upon the formulas $Z_n = \frac{P_n}{P_n + K_n}$ and $Z_e = \frac{P_e}{P_e + K_e}$ supplemented by a straight line drawn from an empirically selected self-rating point to be tangent to the curves represented by the above formulas. While being actuarially precise, the rating procedure was rather slow and cumbersome, required table look-ups on every risk, and did not lend itself readily to interstate experience rating.

The 1940 Plan, on the other hand, was greatly simplified and required a minimum of table look-ups. However, when the formula for modification

$$M = \frac{A_p + WA_e + B}{E_p + WE_e + B}$$

were substituted in Mr. Uhthoff's formula (2), the formula for modification

expressed in terms of Z_p and Z_e, it is found that Z_p equals the ungainly expression $\frac{E}{E_p + WE_e + B}$, as shown in Mr. Uhthoff's formula (3).

At and below the Q-point, as Mr. Uhthoff brings out in his paper, $W = O$ and therefore $Z_p = \frac{E}{E_p + B}$. It is evident from this that B must be chosen so that it is always greater than E_e, no matter what the ratio of E_p/E (commonly known as the D ratio) may be. One sure way to accomplish this would be to set B greater than the total expected losses at the Q-point. Other considerations, mainly, that the charge ($\triangle$ M) for a maximum loss for a risk which is of such size as to just qualify for experience rating shall be 25%, prevents the B value (or K value as it is termed for values of E below the Q-point) being set that high.

This suggests the possibility of a variable B value below the Q-point; as the value of E decreases below the Q-point B could also decrease and we would still be assured of Z_p value less than 1.000. This however would seriously detract from the simplicity of the rating formula $M = \frac{A_p + K}{E_p + K}$, where K is constant, which presently applies to the great majority of risks. The most we can do is to try to guess what the minimum ratio of E_p/E will be and juggle our Q-point and K values, and hope for the best.

For simplicity, the above discussion has been restricted to total expected losses below the Q-point. Some figures which were worked out for the Subcommittee showing the minimum value for a classification D ratio at the Q-point for a number of states, may be of interest. Such minimum D ratio is called here the "Critical D Ratio"; if the risk average D ratio falls below this amount, the primary credibility will become greater than 1.000. The table follows:

State	Critical D Ratio	State	Critical D Ratio
Alabama	.36	Kansas	.48
Connecticut	.48	New Mexico	.39
Florida	.45	Virginia	.36
Georgia	.39	Wisconsin	.48

Examination of Table II of the Experience Rating Plan for Compensation shows D ratios for a number of classifications in some of the above states, already below the above critical values.

Above the Q-point the chance of a primary credibility greater than unity is even greater than at the Q-point. A relatively simple calculation will determine the critical value for D at various premium sizes. On the basis of the present rating values for Wisconsin, which assume a "g" value of .40, the minimum allowable D ratio to prevent Z_p exceeding 1.000 varies with the risk expected losses, as follows:

	E	W	B	Critical D
Less than	10,500	.00	5,500	.476
	13,095	.01	6,460	.502
	23,475	.05	10,094	.547
	36,450	.10	14,175	.568
	75,375	.25	23,344	.587
	140,250	.50	28,375	.595
	205,125	.75	20,594	.598
	244,050	.90	9,775	.599
	257,025	.95	5,144	.600
	267,405	.99	1,070	.600
	270,000 and Over	1.00	0	Self-rated

This difficulty could probably be solved by increasing the value of "g" (the maximum excess ratio). This of course would require careful consideration of the effect this would have on the various rating values and resulting modifications.

The Subcommittee however favored the approach set forth by Mr. Uhthoff in Section V of his paper, namely to use a calculated value of $\frac{E_e}{E}$ for each risk in place of "g". The procedure outlined in the paper of replacing $W = \frac{E\text{-}Q}{S\text{-}Q}$ by $W = \frac{E}{S}$, of removing the square of the coefficient of K in Mr. Uhthoff's formula (17) was followed. It was discovered that if these steps were followed Z_p turned out to have the value $\frac{E}{E+K}$ as Mr. Uhthoff points out, and the possibility of a primary credibility greater than unity was thus permanently eliminated. $Z_e = W \cdot Z_p$ as before. Inserting these values of Z_p and Z_e in formula (2) produces revised formulas for modification as follows:

$$M = \frac{A_p + E_e + K}{E_p + E_e + K} \text{ below the Q-point and}$$

$$M = \frac{A_p + WA_e + (1\text{-}W)E_e + B}{E_p + WE_e + (1\text{-}W)E_e + B} \text{ above the Q-point, where } B = K\,(1\text{-}W)$$

These formulas are given in a somewhat different form from the corresponding ones in Mr. Uhthoff's paper, in order to show their similarity to the present formulas for modification. It may be noted that the difference from the present formulas is the presence of a factor $(1\text{-}W)E_e$ in both the numerator and denominator, and, of course, a different formula for B. Many of you will recognize these formulas as those which apply in California. In order to use this formula, it is merely necessary to amend the present intrastate experience rating form by adding a block to include $(1\text{-}W)E_e$ in the numerator and denominator of the fraction representing the modification, where Tables of B and W values would be printed, i.e. adopt the California intrastate form.

Since the revised formula for B is K(1-W) the modification formula may be modified to

$$M = \frac{A_p + WA_e + (1\text{-}W)\ (K+E_e)}{E_p + WE_e + (1\text{-}W)\ (K+E_e)}$$ where $(K+E_e)$ would be calculated for each risk with no table look-up for B.

This in turn suggests a procedure for interstate rating during the transition period until approval of the revised formulas can be secured in all states. Under the present interstate rating procedure B and W values are calculated for each state on the basis of the risk's total expected losses and are then weighted by the expected losses for each state to determine average B and W values to use in the rating. During the transition period, let the term common to numerator and denominator for states using the revised formulas, namely $(1\text{-}W)\ (K+E_e)$, be set equal to B′. A value of B′ for each new-formula state could be calculated using $K = 7{,}500$, E_e equal to the total expected excess losses for the risk using old-formula states and new-formula states, and (1-W) calculated on the basis of a W value for each new-formula state calculated on the basis of the total expected losses for all states included in the rating. Then B′ values for the new-formula states could be averaged with the B values for the old-formula states to determine an average B value. Average W values could be determined as at present, and the calculation of the interstate modification could then proceed in the usual manner.

When all states have adopted the revised formula, it can be demonstrated that the average value of B′, as defined in the previous paragraph, is equal to $(K+E_e)\ (1\text{-}W_{aver.})$. Therefore there would be no need to calculate values of B′ by states; an average W value calculated in the usual manner would be sufficient.

In closing it may be noted that the Subcommittee recommended a universal Q-point of 10,000 and a universal K value of 7,500. If the self-rating points could be consolidated into only a few different values, the number of tables of W and B values required could be greatly reduced from the present number. The Subcommittee is currently investigating the possibility of a revised basis for establishing the self-rating point.

DISCUSSION BY R. A. JOHNSON

Mr. Uhthoff is to be congratulated for a fine technical analysis of the various components of the Multi-Split Experience Rating Plan. Were certain of his suggestions to be adopted, particularly his proposed method of determining primary losses, the Plan could no longer be called by that name, as is proved by the title of his paper. This paper should be, or may already have been, of considerable value to the Subcommittee of the National Council Actuarial Committee on whose shoulders the task of considering possible revisions of the Experience Rating Plan has been placed.

While admiring the excellent handling of technical details on the one hand, this writer failed to be impressed by Mr. Uhthoff's underlying premise, namely, that a major change in the present Plan is required. The school of

thought to which I happen to belong holds that experience rating, particularly for Workmen's Compensation insurance, is a means of producing a better rate for a particular risk using, within certain limitations, the recent past experience of the risk to modify the manual rate which would otherwise be applicable.

The Multi-Split Rating Plan was developed with the expressed purpose of penalizing frequency rather than severity. Besides a substantial cutdown of costly cases in determining primary losses, the Plan also utilized Average Death and P. T. values, which spread the cost of such cases over all risks which incurred them, disregarding the magnitude of a particular case, which was felt to be chiefly fortuitous. Under this Plan, a risk having a single $20,000 case could receive a credit, while a similar risk having twenty $1,000 cases might receive a substantial charge, because frequency is penalized.

It is interesting to note that some years ago, the late Arthur Bailey developed a modification of the experience rating plan in which the first $1,000 of any accident would be primary, the next $9,000 would be regular excess, and anything over $10,000 would be a sort of super excess to be spread over all risks. Here again, the occurrence was the important factor, rather than magnitude. Because of the simplicity of the "split," it was contemplated that the same system could be carried over into manual ratemaking, and eliminate some of the disadvantages of the serious, non-serious and medical categories now used therein.

The other school of thought apparently considers experience rating as a system of rewards and penalties for past experience, and feels that costly cases should be more fully recoverable under the experience rating program. The proponents of this theory were successful several years ago in eliminating Average Death and P. T. values, such cases now being used in rating at their actual values, subject to an extremely high maximum limitation. These people now seem to be disturbed by the gradually decreasing D ratios as indicated by the following quotation from Mr. Uhthoff's paper:

> ". . . the maximum primary loss is $1,500. Probably this limit, and the rapidity with which it is approached, has operated most strongly to accelerate the decrease in D ratios as case costs increased, and also has been the source of most of the discomfiture felt by practical underwriters as they observe the small use of today's high cost cases in a majority of ratings."

Since the advocates of the "rewards and penalties" school were successful in eliminating Average Death and P. T. values, it is likely that they will prevail in revising the Plan to give more emphasis to costly cases. If and when such revision is deemed necessary, Mr. Uhthoff's paper will serve as an excellent guide for accomplishing their desired goal.

DISCUSSION BY E. S. ALLEN

The discussion of the above paper by Mr. Johnson has prompted me to make a few remarks wherein I disagree to some extent.

Mr. Johnson describes a school of thought which "apparently considers experience rating as a system of rewards and penalties for past experience, and feels that costly cases should be more fully recoverable under the experience rating program." As an illustration, he refers to the elimination of the use of Average Death and P. T. values. This change was made in order to better reflect the differences in the averages of high cost cases by classification, since it was maintained that the character of the work in certain classifications required the hiring of highly-paid employees and such employees tended to have more dependents, therefore developing higher death and permanent total claims. In effect, this change limited such losses within a range from zero to twice the average which had previously been included for all such cases. This limitation, combined with the split of losses between Primary and Excess accomplished the objective while still maintaining a reasonable relationship between frequency and severity.

Mr. Johnson also states that this same "school of thought" is likely to "prevail in revising the premium to give more emphasis to costly cases" through a revision which will increase the average D ratios. When the Plan was adopted in the early 1940s, it was an excellent plan and without major change can probably still be considered an excellent plan. However, the eligibility requirements were at that time average annual premiums of $300, $400 and $500, varying by state, and the Multi-Split feature applicable to individual losses was on the basis of full loss up to the amount of eligibility requirement and a two-thirds discount ratio applied as a geometric progression to each successive portion of the loss equal to the eligibility requirement. We can assume, therefore, that a risk which received a specific modification at that time would receive a quite different modification today, using the same rating values, since the expected losses and the actual losses would have changed materially due to increases in payrolls and benefit rates. A revision of the rating values is therefore indicated, not to give more emphasis to costly cases, but to maintain the general principles adopted when the present Experience Rating Plan was introduced.

DISCUSSIONS OF PAPERS READ AT THE MAY 1960 MEETING

TWO STUDIES IN AUTOMOBILE INSURANCE RATEMAKING

BY

ROBERT A. BAILEY AND LEROY J. SIMON

Volume XLVII, Page 1

DISCUSSION BY L. H. ROBERTS

"Two Studies in Automobile Insurance Ratemaking" by Robert A. Bailey and LeRoy J. Simon, Fellows of our Society, is in this reviewer's opinion one

of the most commendable papers ever contributed to our *Proceedings*. The reader of this review is asked not to regard any criticisms that follow here as a qualification of or detraction from this praise. It is a characteristic of new approaches to important problems, particularly when the problems are as difficult as the one tackled in this paper, that much of the value of the work lies in its stimulus to more thought on the subject. Creative thinking inevitably gives rise to further study, which, even if in disagreement with the original work, is indebted to it as a source of ideas.

The authors have clearly demonstrated the weakness of traditional methods of determining classification relativities. The inadequacy of such procedures is well pointed up by the example they have chosen to work with, that of a so-called merit rating classification superimposed on a classification based on use, sex, age and marital status. Seeing beyond their example, however, they have viewed all rating as essentially an N-dimensional classification system in which the "basic" class plan and merit rating classification are only two variables.

More or less incidentally, they have shown that "merit" rating is nearly as effective as the regular class plan in discriminating among risks. We shall, however, devote our attention here to the broader implications of their work, notwithstanding the importance of this conclusion by itself.

If we let $D(i,j.....n)$ be the correct multiplicative differential for a risk in the i'th class of the first dimension, the j'th class of the second dimension, etc., of an N-dimensional classification system, $D(2,3,1)$ would be the differential for, say, Territory 2, Use Class 3, Merit Class 1 in a three-dimensional system. Dimensions may be either qualitative or quantitative. A fourth dimension, suggested by the authors, would be the quantitative measure, mileage. We shall designate the correct average differential for all risks in the the i'th class of one dimension as $D_1(i)$ for the j'th class of a second dimension as $D_2(j)$, etc.

The authors have demonstrated for the two-dimensional system represented by the Canadian basic class plan in conjunction with the merit rating plan used in Canada that $D(i,j)$ is not in general equal to $D_1(i) \cdot D_2(j)$. Stated as a mathematical proposition, this inequality is scarcely surprising. Nonetheless, traditional procedures have implicitly assumed an approximate equality, at least. Although this approximation may be adequate in many applications, we are indebted to the authors for demonstrating that there is an important class of cases where the approximation is not good enough.

What we require, therefore, is a theoretically sound, yet practicable method of determining $D(i,j.....n)$ from a tabulation of classified experience, or better still, from a file of cards or other records in a single pass. The authors have shown that, for the problem they studied, treatment of relativities as additive, i.e., as amounts to be added to or subtracted from the average, gives better results than differential multipliers, while a kind of "compromise" between multiplicative and additive relativities appears to give somewhat better results than purely additive relativities.

This conclusion, too, while important in itself, is a good deal less than the authors are discussing. To them it is only an example of a particular solu-

tion of the problem of discovering the function, f(i,j.....n), which will give the best estimate of D(i,j .....n).

In tackling this broader problem they have stated four criteria, all essentially sound, but which in this reviewer's opinion are partly inconsistent and partly redundant. The first criterion (balance by class as well as in total) does not appear to be strictly compatible with the second criterion (credibility). If a class is too small for its experience to be credible, there is little reason to criticize a set of differentials on the grounds that rates developed for such a class are out of balance with its indications. The authors themselves acknowledge this point in their criticism of the traditional method of determining class relativities. This reviewer is of the opinion that the authors actually regard balance by class as important only to the extent called for by the credibility of the class, with which he fully agrees. Probably the apparent inconsistency is only semantic. The third criterion (minimal departure from raw data for the maximum number of people) is expressed in terms which are not susceptible to precise interpretation except perhaps insofar as they may be regarded as restated in the fourth criterion (departures of rates from experience small enough to be ascribed to chance). The fourth criterion in this reviewer's opinion is sufficient by itself. It seems quite possible that the authors actually regard the first three criteria as stepping stones toward the fourth, since their use of the Chi-Square test is primarily keyed to the fourth criterion, satisfaction of which could hardly be expected where the first three had been disregarded.

In view of the authors' use of the Chi-Square test, it is in order to mention that the method of minimum Chi-Square, since it involves the use of credibility weights inversely proportional to standard deviations rather than to the squares of standard deviations, which is to say variances, does not result in minimum variance for the averages on which class relativities are based. The authors state, without proof, that weights inversely proportional to standard deviations should be used. Actually, use of such weighting, rather than weights inversely proportional to variances in accordance with the Theorem on Observation Weights, results in a loss of information.* The fact that it permits a mathematical test—the Chi-Square test—is of course a partly compensating advantage. Unfortunately, however, the loss of information becomes increasingly serious as the number of classes increases, so that the method of minimum Chi-Square "breaks down for fine grouping" as pointed out by Maurice Kendall in *The Advanced Theory of Statistics* (Hafner Publishing Co., New York, 1951). Although this objection is not serious in the problem treated by the authors, because of the large volume of experience at their disposal and the small number of classifications, it poses an obstacle to unlimited extension of the method to multi-dimensional classification systems.

In this connection it is apropos to mention that exact tests for frequency arrays, not subject to the limitations of the Chi-Square test, are available

* It is not intended here to decry the use of such weighting for credibility purposes where departures are to be measured from an established rate of unknown variance. In that application good grounds exist for weights inversely proportional to standard deviations.

through the application of combinatorial theory. (See *Selected Techniques of Statistical Analysis,* Statistical Research Group, Columbia University, McGraw-Hill, New York, 1947, pp 247-257.) Unfortunately these methods are cumbersome to apply in a multi-dimensional array involving hundreds or even thousands of cells and it would be beyond the scope of this review to go further into that subject.

If the authors' ideas are pursued to their logical conclusion, the problem of classification differentials is perhaps best treated as one of multiple correlation analysis involving quantitative and non-quantitative variables. For those who may wish to pursue this line of investigation, a starting point is provided in Chapter 17 of *Methods of Correlation Analysis* by Mordecai Ezekiel, John Wiley & Sons, New York, 1941.

An approach that would appear to offer promise lies in techniques for the quantification of qualitative data, on which at least one paper appears in the Journal of the American Statistical Association.*

The potential power of such methods is evident when we consider that, once the correct forms for the quantification functions have been derived, calculations in any number of dimensions of classification can be made simultaneously with a loss of credibility measured only by the number of parameters required, as compared with traditional methods where credibility tends to vanish rapidly as the number of classifications is multiplied.

In conclusion, I should like to repeat that this is a very fine paper and a great pleasure to review—but it wasn't easy!

DISCUSSION BY D. B. MARTIN

(Deputy Manager, Royal-Liverpool Insurance Group, Montreal, Canada. Presented by invitation.)

The two "studies" by Messrs. Bailey and Simon are based on Canadian statistics, and while it was not the authors' intention that they should be considered particularly from the point of view of Canadian conditions, we in Canada have been very interested both in the critical review of what we have already done and in the suggestions as to what we should do in the future. I may say that in Canada we sometimes feel that our American friends fall into two classes—those who think of Canada as the 51st State, with no special features of law or custom or race making us any different from the other 50, and those who think of us as the 151st State, socially and economically only slightly in advance of the aboriginal inhabitants of the continent! We are delighted, in consequence, when we see some recognition of the fact that we have a few things up in Canada in which we are on a par with, and may even be slightly in advance of, the rest of the world. We are quite proud of our Automobile Insurance statistics, and we think the Casualty Actuarial Society has every right to be equally proud of the fact that our Statistical Plan was devised and has been operated for very many years by a Fellow of the Society, Mr. C. H. Fredrickson.

In the first of their studies, Bailey and Simon demonstrate that the Cana-

* "The Quantification of Qualitative Data in Discriminant Analysis", Vol. 45, March 1950.

dian classification plan is fairly effective in distinguishing between high-hazard and low-hazard risks. They also show that the merit rating plan is about equally effective and that the combination of a classification plan and a merit rating plan is more powerful than either of the two plans separately, although perhaps not as much so as one would have hoped. The calculations confirm and give quantitative expression to our instinctive feeling about the success of our combination of classification and merit rating plan, but they also show that the combination is not the conclusive answer to the problem of Automobile rate-making, either in your territory or in ours; the authors suggest that a further qualification in the classification plan, a mileage factor, might increase the effectiveness of the present combined classification and merit rating plan to a considerable degree.

I am not convinced about the value of the mileage factor in all cases, although late in 1959 we introduced mileage as part of the definition of a "super-super-select" class, for which we were prepared in Canada to quote rates of premium materially lower even than those appropriate to our then 1-A class. As far as we can at present see, the qualifications of the "super-super-select" class have limited the special rates of premium to insureds of a really satisfactory quality. There were, however, other qualifications which may have been more effective than mileage, notably the requirement that the car insured should not be used for driving to and from work; that the insured should have had five years free of accident; that there should be not more than two adult (i.e., over 25) drivers in the household, and, of course, no under 25 drivers.

But I sometimes wonder whether we are not still missing the real causes of accident-freedom or accident-proneness; whether we are not differentiating by correlated characteristics rather than by causal factors. Professor Poser of McGill University believes that he can identify potentially accident-prone drivers by a series of physical and psychological tests. We have not yet been successful in providing him with an adequate group of test cases and the necessary control population, so that at the present time his theory is unproven. However, I know that it is possible to base a model population on the assumption that it consists of two groups, a relatvely small one with an accident frequency of 20 per hundred per annum, and a very much larger one with an accident frequency of 5 per hundred per annum. If it be assumed that each member of that population acquires a car at the same time, then within a very few years the whole population will divide itself into groups respectively 3 or more, 2 and 1-year claim-free, and with a recent claim record, the groups being proportionately as numerous as, and their claim frequencies being very comparable with, those of the corresponding groups actually found in the Canadian population.

For that model population the combination of a classification and merit rating plan is just as effective as Bailey and Simon have shown it to be for the actual Canadian experience, but it falls a long way short of identification of the 20 per hundred frequency insureds and their appropriate rating. For our model population the combination of classification and merit rating plan is only relatively successful; possibly it is no more so in real life.

The second study considers the method of deriving appropriate premium

differentials for the various class and accident-freedom sub-groups. Four criteria are suggested in the light of which any differential complex should be examined, and some ingenious but quite practical suggestions are offered for calculating one or more complexes which measure up satisfactorily. In Canada we have been using the elementary method of determining column and row differentials separately and then combining them, the actual process being described in some detail in my presentation to the 16th International Congress of Actuaries (Communications of the 16th International Actuarial Congress, Volume 2, Page 37). The examples in that Paper, however, are based on statistics a year older than those used by Bailey and Simon. For a fair comparison I extracted some figures from the calculations made in connection with the development of the 1960 Canadian rate program and set them beside the figures produced in Method 2 of Table C of Bailey and Simon's Paper (Minimum Chi-Square on xy) after these have been adjusted to relate to the Class 1-B rate as 100 . The comparison is—

	Bailey and Simon's Method 2, adjusted to Class 1-B = 100	Canadian Method Original Calculation
Class 1	100	100
" 5	132	133
" 3	149	150
" 2	155	158
" 4	241	245
Merit Rating Class A	62	61
" " " X	76	76
" " " Y	83	83
" " " B	100	100

The two sets of figures show a remarkable resemblance, and it is tempting to claim that the simpler method is just as efficient as the more sophisticated and undoubtedly more laborious one. However, the resemblance may easily be fortuitous, and while we may continue in Canada to use the simpler method for our immediate rate-making purposes, I think we shall, at some stage in the proceedings, be checking to see whether the minimum Chi-square method does give the same result.

Bailey and Simon sound a warning as to the dangers of calculating differentials from the thoroughly heterogeneous data derived from the aggregation of the experience of a number of different rating areas with markedly different basic accident frequencies. We have been conscious of that in Canada, and at times we have been tempted to use different sets of differential complexes, either for different Provinces or for urban as distinct from rural business. However, a single set of differentials has such manifest advantages from the point of view of the "non-mathematical considerations" mentioned by Bailey and Simon, that we would be reluctant to get ourselves involved in the complication of more than one set. We have one advantage over you; the word "discriminatory" does not have such an evil meaning in our Country as it does in yours.

It is those same non-mathematical considerations which have led us to continue the use of multiplicative differentials (Bailey and Simon's Method 1 or 2) as compared with additive differentials (Bailey and Simon's Method 3), although the latter have a great deal to commend them from a common sense point of view and appear to fit the rough data better. In particular, multiplicative differentials facilitate the simple statement "If you've been accident-free for so many years, you save such-and-such a percent of your premium" and the public relations value of that statement is considerable, particularly if the same statement can be made for all relevant coverages, classifications and rating territories. Certainly I would not like to try to explain to a group of producers the reason why of Bailey and Simon's Method 4, even though I admit that this does appear to produce a set of differentials which are mathematically better than those produced by Method 2.

I found the difference between the probabilities quoted in the last line of Bailey and Simon's Table "E" rather surprising, particularly the tremendous difference between the .001 quoted for Method 2 and the .60 and .70 quoted for Methods 3 and 4 respectively. To a very great extent, however, the probabilities depend upon the value of the constant K which on "a rough estimate based on the limited data available" Bailey and Simon calculate as $\frac{1}{200}$. I had available a distribution of actual claim figures (a mixture of B.I. and P.D. as is appropriate to Canadian conditions) sufficiently random for practical purposes and covering just under 1,000 claims. The value of K emerging from that distribution was $\frac{1}{120}$ which reduced the probabilities of Methods 3 and 4 to something of the order of .10, although Methods 1 and 2 remain (or are even more) highly improbable. I think we need to know more about the usual value of K before we conclude that any one of the four methods is, from the standard of the Chi-square test, so very much more satisfactory than any of the others.

Finally, I'd like to stress the importance of Bailey and Simon's four "non-mathematical considerations" and indeed to add a fifth, namely, acceptability to the insuring public. We have not reached finality in distinguishing between the various categories of insureds in relation to accident exposure, and we are not yet so very accurate in our calculation of appropirate rates of premium. I don't think that we ever will reach precision in either respect, and I don't think that it matters. If we present our product to the insuring public, packaged and priced in such a way that the public can see and understand that we have done rough justice both to them and to our Companies, then I think we have done our job. I do not think the public likes it when we introduce complications in our rating methods that they find difficult to understand. Intricacy makes them suspicious, and suspicion is something which is so difficult to allay that we want to avoid it.

I realize that in what I have just finished saying, the "101st State" has been ignoring some of the special difficulties which arise in the first 50; others will, I hope, make up for my short-comings. We in Canada found Bailey and Simon's studies stimulating, and well worth while. We are very grateful for them.

DISCUSSIONS OF PAPERS READ AT THE NOVEMBER 1960 MEETING

ANY ROOM LEFT FOR SKIMMING THE CREAM?

BY

ROBERT A. BAILEY

Volume XLVII, Page 30

DISCUSSION BY J. T. LANGE AND R. M. MUNIZ

(National Bureau of Casualty Underwriters, presented by invitation.)

Mr. Bailey is to be commended for the excellent work that he is doing, as revealed by this and other papers of his, in bringing mathematical analysis to bear on the problems of rating systems. These problems are extremely difficult, and the final analytical solutions are still to be made, but every contribution, such as Mr. Bailey's, is another step along the way. In this paper Mr. Bailey considers the problem of measuring the amount of skimmable cream to be found in the classification system for automobile liability insurance. He draws several conclusions from his analysis, his final conclusion being that the present rating system is not perfect and still has skimmable cream in it. No one will disagree about there still being *cream* in the rating system; Mr. Bailey, himself points out that perfection can only be achieved if there is a separate rate for each risk or, more precisely for each group of risks with the same accident-potential. As soon as you combine into one rate-class a group of risks with differing accident-potential, no matter how slight the difference, there will of necessity be some risks that are better than average: thus there will always be cream.

But is it skimmable? More precisely, is there so much cream that there is a sizable danger of some other rating system successfully attracting these better-than-average risks? Mr. Bailey concludes that there is, and he bases his conclusion on a comparison of the variation in the rates of the present rating system with the variation in the inherent hazard in the total population of risks.

There are a number of assumptions underlying Mr. Bailey's analysis that warrant further discussion. To begin with, he develops a figure of 1.00 for the relative variation in the hazard and finds support for this in the figure of .977 computed by M. Delaporte (Sixteenth International Congress of Actuaries, 1960, Vol. II). M. Delaporte's figure, however, is for the inherent hazard in only one particular rate-class in Paris and not for all pleasure-use cars in France as Mr. Bailey states. If .977 represents the variation in one class in one territory, the variation of the hazard for all classes in all territories, that is, for the entire population, must be considerably higher than unity. On the other hand, using the Canadian data Mr. Bailey computes a figure of .87 for the coefficient of variation of the risks in Canada. This figure of .87 is derived from a formula he developed previously, namely, $a/(a + n)$, which represents the expected claim freqency for risks accident-free for n or more years relative to the expected claim frequency for all risks. Specifically, the coefficient of variation of .87 is based on the Canadian relative claim fre-

quency for $n = 1$, that is for one or more accident-free years. For $n = 2$, that is for two or more accident-free years, the same computation gives a coefficient of variation of .72; for $n = 3$, the coefficient of variation is .63. These varying values for the coefficient of variation computed by the same formula from the same body of data raises the question of whether the basic mathematical theory is in fact a proper model for this type of analysis. Mr. Bailey believes that these varying values are accounted for by the fact that there is an assumption in the mathematics being used that the inherent hazard for each individual risk remains unchanged from year to year. Since the .87 refers to the variation of the group already having had one accident-free year he concludes that the coefficient of variation for all risks would be higher, closer to 1.00. This hypothesis sounds reasonable and may be right. However, the computation of the coefficient of variation is based on first evaluating the two parameters, r and a, of the negative binomial distribution which has been assumed as the proper model; there is no indication in this mathematical model that these parameters would vary for the same body of data as they seem to do for the Canadian data.

All the above serves to illustrate that the value of 1.00 which Mr. Bailey uses for the coefficient of variation for risks is only an estimate. Also, it would seem that the coefficient of variation for risks would vary from one population of drivers to another to the degree that some of these populations were more homogeneous; for example, it would seem that there would be less variation among drivers in Iowa (which is largely rural) than among drivers in New York (which includes both rural and urban areas). The overall result of these considerations is to demonstrate that while unity may be used as an estimate of the coefficient of variation of the risks it is still an approximation which might be subject to considerable refinement, and it therefore should not be treated as a universal constant applicable in all situations.

Suppose it could in fact be determined that the relative variation in the inherent hazard is some constant, K. Can this be used as a basis for measuring the effectiveness of a rating system? Mr. Bailey says it can; he says that if the relative variation in the rates is A then the ratio A/K times 100 gives the percent effectiveness of the rating system. There are a number of things against this reasoning. First of all, the absolute variation in the rates will always be less than the absolute variation in the inherent hazard. Theoretically, the inherent hazard has no upper bound, whereas the highest rate that can be charged is limited by practical considerations. More fundamentally, the distribution of the hazard is a continuous function; there are an infinite number of values for inherent hazard. The distribution of the rates, however, is discrete; only a finite number can be established. The variance of a discrete approximation to a continuous function is necessarily less than the variance of the continuous function. In short, for a given number of rates, (and assuming equal means in order to simplify this discussion) the variation, A, is limited by an upper bound which is less than K. Call this upper bound M. Mr. Bailey gets a value for A/K equal to 0.5 and concludes that the rating system is only half as effective as it could be. But A cannot be greater than M; is not A/M a more appropriate ratio? It is possible that A/M would be substantially higher than A/K and thus give a much better picture.

Estimating A, the coefficient of variation of the rates, also presents a problem in that estimates of the coefficient of variation of the risks are based upon broad sets of data (e.g. all drivers in California, all drivers insuring in Canada) and an estimate of A should therefore be based on an equally broad sample if it is to be comparable. If an estimate of the coefficient of variation of the rates is based upon a sample population which is more homogeneous than the total population, then the resulting estimate of the coefficient of variation of the rates will be lower than if it were based upon a broader sample. While it is impossible to compare the exposure distribution Mr. Bailey uses with the actual distribution of cars in Pennsylvania to see if this distribution is typical of that of all cars in Pennsylvania, it is possible to compare it with data compiled by the National Bureau, and with data from other sources. First, it was found that in comparison with the National Bureau's distribution (based upon 1,000,000 cars) in Pennsylvania, Mr. Bailey's distribution (based on 12,000 cars) was biased with respect to territory in that a a large percentage of his total exposure is in a single territory while certain other city territories have a relatively low exposure which leads to a lower coefficient of variation than would result from a National Bureau distribution. With regard to merit rating, Mr. Bailey's distribution would appear to be atypical since over 90% of the risks fall into the lowest rated sub-group. National Bureau data from Pennsylvania indicates that 80% fall into this sub-group while data from California (where the plan has been in effect for a longer period) show closer to 60% in this sub-group. (This data is summarized in Table 1 which appears following the conclusion of this paper.) It is interesting to note that the merit rating plans produce a greater coefficient of variation in states where a large majority of drivers are insured under the plan. In Canada and Texas where all auto insurance is written under a merit rating plan, coefficients of variation of .225 and .232 respectively are produced; using the California merit rating plan with the distribution of drivers having accidents and convictions reported by the California Division of Motor Vehicles a coefficient of variation of .269 is produced. It would seem that Mr. Bailey's conclusions concerning merit rating plans of the type introduced in California are unjustified inasmuch as experience indicates that where the majority of cars are insured under the plan the coefficient of variation of the plans is several times that which Mr. Bailey estimates. It would seem that in some respects Mr. Bailey's distribution in Pennsylvania is not typical, and his total coefficient of variation is probably under-estimated.

Thus far in this discussion two questions have been raised in regard to Mr. Bailey's comparison of the coefficient of variation of the rates with that of the inherent hazard in the risks. Concerning his use of the coefficient of variation of the inherent hazard in the risks, it was pointed out that Mr. Bailey's figure is inconsistent with Mr. Delaporte's, and furthermore that, even for the best classification system that could be designed, the coefficient of variation of the rates must of necessity be less than that of the risks. His use of the coefficient of variation of the rates was questioned on the grounds that in some respects Mr. Bailey's sample appeared to be biased. While Mr. Bailey is justified in saying our present classification is not perfect, his statement that the present classification system takes care of only half of the total

variation among risks is subject to question since this conclusion is based upon figures which do not appear to be wholly representative.

Mr. Bailey's approach, that of looking at the total rating structure, is both interesting and enlightening, but suppose Mr. Bailey's conclusion that there is still cream in the rating structure is accepted. Is this cream really skimmable? The total cream for a rating system must be some sort of sum of the cream in each class. Now this might result from a small amount of cream in every class, which for practical purposes is unavoidable, or on the other hand, this cream may be concentrated in one or two classes from which it is easily skimmed. The coefficient of variation of the rating system as a whole seems to be too all-inclusive a measure to be used to determine whether any of the cream is skimmable. A low coefficient of variation says nothing about the individual classes themselves, whether all of the classes have skimmable cream or just one or two. Neither does this type of measure indicate what can be done to improve the rating system, if improvement is indicated. It would seem, therefore, that the proper way of judging the effectiveness of the rating structure would be to study the individual classes to see if any of these are so ineffective that there is still considerable room to skim off the cream. M. Delaporte, in the article already referred to, suggests that the difference between the mean value of a class and the modal value of that class indicates how different the rate charged the typical risk is from the rate indicated by its inherent hazard. Since the rate for any class is based upon the mean value, if the modal value is significantly lower than the mean, then the typical risk is paying a rate higher than is indicated by his inherent hazard, and he is cream that may be skimmed. This approach might be used to provide a more definite answer to the question, how much room is left to skim off the cream?

TABLE 1

Merit Rating Distributions and the Resulting Coefficients of Variation

PENNSYLVANIA

Merit Rating Code	Mr. Bailey's Exposure Distribution	National Bureau Exposure Distribution	Relativity
9	91.3%	83.6%	85
1	7.6	4.9	95
2	.8	9.3	100
3	.2	.7	120
4	.01	1.1	140
5	.04	.2	170
6	.03	.2	200
Mean Relativity	86.032	88.14	
Standard Deviation	4.315	9.93	
Coefficient of Variation	.050	.113	

Mr. Bailey's exposure distribution results from a merit rating plan which is slightly different from the National Bureau plan in Pennsylvania in that his plan uses the experience period and point system which the National Bureau used in California; therefore, in some respects National Bureau data from California provides a better basis of comparison.

CALIFORNIA

Exposure Distribution

Merit Rating Code	National Bureau Actual	National Bureau Adjusted$^{\phi}$	Dept. of Motor Vehicles*	Relativity
9	68.1%	61%	54.1%	80
1	18.5	22	22.2	90
2	8.7	10	10.8	100
3	2.7	4	5.5	120
4	1.1	1.5	3.0	140
5	.5	1	1.6	170
6	.4	.5	2.8	200
Mean Relativity	86	88.2	93	
Standard Deviation	14	16.3	25	
Coefficient of Variation	.163	.185	.269	

ϕ Adjusted to take into account changes as a result of checking policies with the driver records of the Dept. of Motor Vehicles.

* Based on a driver record study conducted by California Dept. of Motor Vehicles.

DISCUSSION BY L. J. SIMON

Mr. Bailey's paper introduces two advances in actuarial theory that make it another milestone in progress. The first stride forward is in the concept of the coefficient of variation of the rates as a method of measuring the overall effectiveness of a rating plan. This concept is destined to revolutionize our thinking with respect to classification systems because now at last we have the key to comparing two different systems of classification and also a measure which will show us how much increase in precision we will get by superimposing a new rating criterion upon the existing system.

The second advance in actuarial theory made in the paper is almost lost to the reader because it is passed over so quickly. This is the method used for determining the coefficient of variation of the *risks*. Being able to do this from risk distributions, such as the California Driver Record Study, is quite good, but being able to do it from the loss ratio of risks who were claim free the preceding year (which leads to the credibility measure, a value for the parameter "a", a value for "r", and hence to the coefficient of variation of the risks) is of major impact. This same method can be applied under many, many circumstances to determine the coefficient of variation of the risks. This,

of course, will provide the measure on which to judge any rate structure in relationship to the absolute maximum that can be achieved.

It must be recalled repeatedly as the paper is read that the coefficient of variation for the *rates* is a valid measurement only if (1) the rates are an accurate reflection of the experience and (2) the exposure distribution is representative of the population. To illustrate the first point, we could arbitrarily set the following class relativities:

1 A	18
1 B Small Cities	18
1 B Large Cities	74
1 C	269
2 A	521
2 Small Cities	1191
2 Large Cities	1471
3	297

Using Mr. Bailey's exposure distribution we would find the mean is the same as his mean (i. e., 118) but the standard deviation = 235.5 and the coefficient of variation = 2.0. However, this would contradict the known fact that the coefficient of variation of the *risks* is close to 1.00. On the other hand, we could just as arbitrarily set all the class relativities equal to 100 which would indicate the class plan was wholly ineffective. Neither of these conclusions would be remotely near the truth because they neglect the fact that the relativities must be based on the experience in order to be valid measures.

To illustrate the second point we see that the Farm and Non-Farm differential has a value of .034 for this company. Assume the differential to be correct, but suppose some other company has its exposure distributed:

Non-Farm	62,912
Farm	89,874
TOTAL	152,786

In this case we would have mean = 82.353, standard deviation = 14.764, coefficient of variation = .179. Hence for this company the farm criterion would be of much more effectiveness. In fact, with the 100-70 differential, this is the *maximum* coefficient of variation that we could have. On the other hand, if a third company refused to write any risk unless he was a farmer, the coefficient of variation would be zero and, *as a rating criterion,* this factor would have no effectiveness. The first coefficient of variation is undoubtedly too high for the population and the second is unquestionably too low. They illustrate that we must guard against being misled by an exposure distribution which is not typical of the population. An atypical distribution may lead us to either overstate or understate the effectiveness of a rating criterion.

We must be very careful when interpreting or comparing coefficients of variation. If two coefficients are equal, it is safe to say that the rating characteristics are equally effective. If coefficient A is .25 and coefficient B is .50, we can say that B is at least twice as effective as A but might be as

much as three, four, or more times as effective. This is true because the first .25 is much easier to get from a rating criterion than is the second .25 and so on up the scale. I suspect that raising the coefficient from .90 to .99 would be as difficult as raising the speed of a particle from 185,000 to 185,400 miles per second.

The most important feature of the paper for this reviewer is the great use these principles should have among those actuaries who must frequently make critical decisions relative to rates and rating plans. Those rating differentials which, after being based on experience representative of the population, show little or no effectiveness can be and should be dropped from the rate structure. Furthermore, the cost of obtaining the information necessary to properly classify a risk under a given rating plan may be weighed against the effectiveness of that plan.

Mr. Bailey has added a new, original and very valuable tool to the actuaries' working procedures and processes. The paper is indeed a significant one.

DISCUSSION BY L. H. ROBERTS

At the seminar in which the paper was discussed, this writer sided with what appeared to be the consensus (although not unanimous): that the coefficient of variation is a good measure of the efficiency of a classification system. He did, however, mention certain reservations with which he believes the author of that excellent little paper to be in agreement.

It should be emphasized that the absolute value of the C.V. of rates is meaningless as a measure of their propriety. What counts, assuming the overall level is correct, is the spread between rates (the C.V. being a measure of this) as compared with the spread between the hazards of individual risks. This, too, has no significance unless rates are closely related to the experience of the respective classes to which rates apply. Since in a perfect rating system there is a one-to-one correspondence between the rate for a given homogeneous class of risks and the hazard of that class (which might include but a single member), it follows that any rate schedule for which the C.V. of rates is less than the C.V. of hazard in the population of risks will be less than 100% efficient, and the C.V. of the rate schedule will decrease with decreasing efficiency in classification.

If, however, rates are based on judgment rather than on credible experience, the C.V. of rates will not necessarily be related to the efficiency of classification. In such cases it may indeed exceed the C.V. of hazard, as where differentials are established for imaginary or exaggerated differences in hazard. It will often be the case, moreover, that the C.V. of hazard is unknown, since knowledge of this statistic requires analysis of experience by individual risk. For these reasons, the most appropriate use of the C.V. will be often only to compare the efficiency of one class plan with that of another, no attempt being made to estimate their absolute efficiency.

Where the C.V. of hazard is known, a measure of absolute efficiency is provided by dividing the square of the C.V. of indicated rates by the square of the C.V. of hazard. (The same result would be obtained if variances are used.) The quotient, called the coefficient of determination, gives the

proportion of the total variance in the population that is accounted for by the class plan.

It is well to keep in mind that a single statistic cannot possibly provide a complete basis for comparison of class plans with more than two classes. Thus, of three different plans, all with the same C.V., one may isolate a particularly good category of risks, producing a concentration of sweet cream; another may concentrate the sour cream; the third may distribute the cream almost equally among classes. The first two situations are quickly recognized and tend to disappear in subsequent rate revisions. It is the third situation in which the cream is most difficult to skim, and for that reason offers the greatest opportunity for profit to the carrier that finds a way to do so.

AUTHOR'S REVIEW OF DISCUSSIONS

The discussions have contributed some important points, most of which I heartily agree with but on some points I feel it would be helpful to offer some clarification.

Mr. Roberts and Mr. Simon both very properly urged caution in interpreting the coefficient of variation of the rates, and discussed the problems which should be considered in order to make a proper interpretation. Mr. Roberts brought out the point that a measure of absolute efficiency of the indicated rates in a class plan is provided by dividing the square of the C.V. of indicated rates by the square of the C.V. of hazard. (A^2/K^2 in the terminology of Messrs. Lange and Muniz) Mr. Simon made much the same point when he said "If coefficient A is .25 and coefficient B is .50, we can say that B is at least twice as effective as A but might be as much as three, four or more times as effective." Both these observations mean that I was too optimistic in saying that the present class plan takes care of half of the total variation among risks. One quarter might have been a better estimate.

Messrs. Lange and Muniz, however, carried some of the statements in my paper beyond their actual meaning. For example, they said that "Mr. Bailey gets a value for A/K equal to 0.5 and concludes that the rating system is only half as effective as it could be" whereas my conclusion was simply that "the present multiple classification system . . . takes care of only half of the total variation among risks." They ignored the fact that I recognized the practical limitations in classification refinement. Moreover, my paper went into the interpretation of the coefficient of variation very little and did not go so far as to state that the ratio A/K times 100 gives the percent effectiveness of the rating system as Messrs. Lange and Muniz credit me with saying.

Messrs. Lange and Muniz also said that the merit rating distribution in Pennsylvania used in my paper is atypical because it is substantially different from the National Bureau distribution in Pennsylvania. Such a conclusion is unwarranted because in my paper I pointed out that the company whose experience I used was using the California-type merit rating plan in Pennsylvania which differs substantially from the Pennsylvania plan of the National Bureau, in that it has a shorter experience period and assigns only one point per accident instead of two. On the basis of the differences between the two merit rating plans, I estimated that the National Bureau merit rating

plan in Pennsylvania would have a coefficient of variation of about .10 which is about twice as large as the California-type plan. Messrs. Lang and Muniz show that the coefficient of variation of the National Bureau plan in Pennsylvania is .113 which is a very close confirmation of my estimate of .10.

Messrs. Lange and Muniz said "but suppose Mr. Bailey's conclusion that there is still cream in the rating structure is accepted. Is this cream really skimmable?" Such a question reminds me of the farmer who locked the barn door after the horse was stolen. We do not need to resort to theory to find out whether there is cream and whether it is skimmable. All we have to do is look at the underwriting results of some of the independents. The rating refinements introduced recently have raised the coefficient of variation of the total rate structure only a small amount as shown in my paper, thus still leaving cream for those who know how to skim it.

A NEW APPROACH TO INFANT AND JUVENILE MORTALITY

BY

CHARLES C. HEWITT, JR.

Volume XLVII, Page 41

DISCUSSION BY A. L. MAYERSON

Mr. Hewitt's paper attempts to derive an analytic expression suitable for evaluating mortality at infant ages. Noting that the Gompertz and Makeham laws, often used by life actuaries to fit mortality data (the 1941 CSO table was Makehamized at ages 15 to 95 while a Gompertz graduation was fitted to the 1937 Standard Annuity Table), are not applicable at juvenile ages, he derives formulas which may be useful in valuing orphans' benefits, especially where multiple lives are involved.

The rationale used in obtaining the formulas is to split the force of mortality operating at age x into three component parts: (1) the portion attributable to chance causes, independent of age, (2) the portion which depends upon the "obsolescence" or deterioration of the body's ability to resist death, and (3) an element which recognizes the individual's inherent predisposition to death. Mr. Hewitt then expresses the individual force of mortality μ_x as $A + Bc^x + m$ where m measures the 3rd or "inherent predisposition" factor and is a random variable with its own distribution function. A, B and c are the usual Makeham constants and measure the "chance" and "obsolescence" components of mortality. He assumes that μ_x has a Pearson Type III distribution function and, by manipulating this distribution function, determines the average force of mortality for a group of individuals, the function used in life insurance mortality studies, as $\bar{\mu}_x = A + Bc^x + \frac{r}{a + x}$ (r and a are the two parameters of the Pearson Type III curve). The third

term, which he calls the "force of selection", is intended to measure the individual's inherent capacity to survive.

Mr. Hewitt's paper is an interesting approach to the problem of deriving an analytic formula to represent mortality rates. Its underlying rationale, namely, the conception of the force of mortality as an average of widely varying individual rates, resembles that used in the paper entitled "A Theory of Mortality Classes" by Louis Levinson which appeared in the Transactions of the Society of Actuaries Vol. XI (1959). Mr. Levinson divides the factors influencing mortality into three types, which he classifies as those inherent in the nature of man, those due to environmental influences, and those based upon the individual's propensity to survive. Mr. Hewitt uses this approach for a quite different purpose, however.

I detect one error early in Mr. Hewitt's paper. In his first section, he states that "where q_x (the rate of mortality) remains constant, the force of mortality, μ_x remains constant" and then proceeds to calculate an interesting arithmetic example based on the formula $\text{colog}_e\ (1 - q_x) = \mu_x$. In fact, $\text{colog}_e\ (1 - q_x) = \int_0^1 \mu_{x+t}\, dt$ and there is no necessity for μ_x to remain constant over the year of age x to x + 1. Furthermore, the contrary is probably true during the year of age 0 to 1, since μ_0 decreases rapidly during the first year of life. The assumption that μ_x is constant for each age x does not invalidate Mr. Hewitt's mathematics, though it does make his numerical example less realistic.

Near the end of his paper, Mr. Hewitt illustrates his formula by fitting a curve to the 1939-41 U.S. white males mortality table. Though he obtains an excellent fit to the mortality rates shown by this table at ages 5 and 10 (but not very close at ages 15 to 30), he does not demonstrate that the method provides a good fit at ages below 5, which is the range he proposed to investigate. It would also be interesting to know whether as good results would be obtained if his curve were fitted to a more recent mortality table.

Mr. Hewitt's attempt to analyze separately each of the factors influencing human mortality is an interesting and worthwhile excursion into the whys and wherefores of mortality data, and his approach may well be useful in analyzing automobile accident statistics and for other purposes. Whether his formulas will produce a more accurate valuation of orphans' benefits than the methods now used is, however, not yet proven. In particular, his formula (4.6b), which expresses a joint life probability in terms of single life probabilities, is such that the law of uniform seniority may not apply. Since the utility of Makeham's and Gompertz' laws in computing annuity values depends on the fact that not only nPxyz but also a_{xyz} can be expressed in terms of single life values or in terms of values at equal ages, I believe Mr. Hewitt should have gone a bit farther and showed that this is also true for his formula. Unless a law of uniform seniority or some similar labor-saving device can be found, it might be easier to obtain joint life annuity values by programming the job for an electronic computer than to use Mr. Hewitt's methods. Even if his analysis does not lead to easier computations, however, his analysis is original and is a worthwhile contribution to actuarial literature.

MULTIPLE COVERAGE EXPERIENCE RATING PLAN

BY

ELDON J. KLAASSEN

Volume XLVII, Page 66

DISCUSSION BY L. L. TARBELL, JR.

Mr. Klaassen's paper develops an experience rating plan utilizing the multi-split principle for automobile liability, miscellaneous liability and automobile physical damage combined. This combination of coverages for rating is proposed in order to achieve a larger and more stable base upon which to predicate an individual risk's experience modification. Mr. Klaassen's basic assumption underlying the combination of experience of different lines for a particiular risk is that "the inherent hazard of any one coverage is correlated with that of other coverages for a given risk".

Experience rating is an attempt to prospectively measure the deviation of an individual risk's loss exposure (as reflected by the individual risk's loss experience) from the average loss exposure contemplated in the manual classification rates under which the risk is rated. There may exist correlation of this loss exposure between various coverages written for an individual risk. All coverages written for a given risk are subject to certain overall characteristics (e.g., safety programs, good or bad housekeeping and other broad, general categories) and it would seem logical that some degree of correlation would be present. I have no statistics at my disposal which can confirm or deny this assumption; however, I feel that while the correlation between automobile liability and automobile physical damage may be quite good, correlation of these lines with miscellaneous liability may be suspect. If this correlation does exist, it would be of varying degree between coverages and would also vary greatly between individual risks.

While automobile liability and automobile physical damage might possess a high degree of correlation in the area of loss exposure, I would also question the inclusion of this indemnity line in an overall experience rating on the basis that the liability lines, particularly in the bodily injury area, are subject to relatively severe fluctuations due to large losses. This fluctuation would be controlled to a degree through the use of the multi-split approach which Mr. Klaassen advocates; however, it should not be allowed to affect the relative stability of a line of indemnity insurance. Conversely, the stability of the indemnity line should not dampen the effect of fluctations in the liability lines. We have a precedent for the combination of coverages for the purpose of premium determination in Retrospective Plan D; however, this plan develops and allocates indicated premiums (within the area between the minimum and maximum premium) in direct relation to the experience of each line. The fluctuations in experience are reflected by line and the results for each line are readily available.

This area of hazard correlation would require extensive study before any plan of rating which combines the results of different coverages could be inaugurated and offers an excellent field for further actuarial studies.

The present liability experience rating plans have been in use for a number of years with very little change. At the time these plans were introduced, the amount of loss excluded by the application of the maximum loss limitation was not serious. However, with an inflationary economy operating to increase the cost of claims and larger limits of insurance becoming the rule, an experience rating utilizing only a portion of the basic limits experience is producing an experience modification based on a relatively small amount of a risk's actual experience.

A similar situation has existed in Workmen's Compensation insurance where the D ratios have been eroded to the point that from 40% to 50% of the total incurred losses are never considered in the development of the experience modification. This situation has been reviewed by a subcommittee of the Actuarial Committee of the National Council on Compensation Insurance and their proposals of raising the eligibility requirements and increasing the initial primary loss value and the maximum loss value should reverse the trend and help develop D ratios more closely in line with the ratios contemplated by the designers of the plan.

The multi-split approach proposed by Mr. Klaassen is designed to incorporate more of a risk's actual experience in the determination of a risk's experience modification. The automobile and miscellaneous liability experience rating plans for a few states (New York, Louisiana and Texas) have employed the rating of excess limits experience for the larger risks, but the proposed plan goes a step further by rating these excess losses for all risks. The effect upon the rating of the smaller risks is limited by the application of the excess credibility values built into the proposed experience modification formula. The multi-split concept has been used successfully in Workmen's Compensation insurance in the rating of risks without causing undue fluctuations in the risk's rate level and the method would seem well suited to the rating of the liability lines where fluctuations in loss severity are most marked.

The experience modification formula developed by Mr. Klaassen places more emphasis on stability than on responsiveness but achieves a greater recognition of loss severity through a built-in bias in the size of loss distributions used in developing the credibilities. The question of whether an experience rating plan should place emphasis on stability or responsiveness has always been with us, and undoubtedly will remain, but Mr. Klaassen's plan presents a logical and conservative approach to the problem.

THE CENSUS METHOD

BY

LAURENCE H. LONGLEY-COOK

Volume XLVII, Page 81

DISCUSSION BY H. T. BARBER

Aside from the advantages claimed for the census method of compiling exposures as advocated by Mr. Longley-Cook, he has drawn attention to the fact that through innovations in established statistical procedures we might accomplish substantial economies without seriously injuring the value of the resulting data. For many years the old policy year basis for compiling rate-making experience was regarded as the ideal for many important lines of casualty insurance. The policy year basis could be termed the "gold standard" for compiling ratemaking statistics. In recent times we have observed a transition to the calendar year-accident year basis for preparing data and now Mr. Longley-Cook has introduced a further shortcut. It would seem desirable to consider other means by which we might bend our procedures to yield further savings in expense without making too great a sacrifice in the dependability of the resulting experience. One such simplification in the treatment of exposures will be suggested later in these comments.

There are two desirable attributes of the original policy year basis of experience compilation which may have to be compromised in any major departure attempted for reasons of economy. The policy year method permits the rate maker to use the same available data for the dual purposes of class and territory pure premium relativity and of rate level determination. The advantage of continuity is self-evident and is of appreciable value in securing rate approvals. It is observed that if less exact methods of compiling experience are adopted, any inaccuracies so introduced will not be as critical in effect in the determination of relativity as they could be if also used for rate level data. This suggests that the census method of determining exposures might be quite acceptable for relativity purposes but it may appear advantageous to use aggregate data from a different source and more meticulously compiled, as the basis for rate level determination. It is noted that the present calendar-accident year basis for automobile insurance rates has preserved this principle of using essentially the same data for both relativity and rate level.

Secondly, the policy year basis has the desirable characteristic of approaching maximum accuracy with the passage of time. If exposures are developed for a period of time sufficient to allow the corresponding losses to mature, the developed exposures should closely approach ultimate true values. On the other hand, other more approximate methods may involve a freeze based on the calendar period in which the transactions were recorded. For example, under any calendar year exposure method if a material clerical error in assignment should occur late in the period, as in a December 1960 transaction, which is not discovered and corrected until the following month, a three-fold effect is created which might hamper the proper interpretation of the resulting data. In this case, the 1960 experience as originally recorded is in error; the

1961 experience of the same category is in error as it will contain a minus exposure item equivalent to that originally recorded; and the 1961 experience of the proper category includes an item which should be charged to 1960 rather than 1961. This illustration points up one weakness in the suggested census method as the intermittent recording of exposure may be more vulnerable to such a situation than is the usual complete calendar year basis. It is observed that transaction date is not a good statistical peg on which to hang the hat. Effective date of the coverage is better statistically though probably less convenient in many cases.

It probably can be inferred from the paper that the census method may be adaptable only to lines of insurance with large volume, with fixed premiums and with units of exposure and premium per policy which are fairly uniform in number or amount. These limitations are suggested by a desire to limit any undesirable effect of approximation to an acceptable minimum. Automobile liability private passenger par car experience and homeowners coverage as mentioned in the paper seem to offer good opportunities for the suggested procedure.

Casualty insurance ratemaking usually makes extensive use of ratios (pure premiums and loss ratios) which consist of amounts of losses for numerators and exposures for denominators. There may be various reasons for uncertainty as to the significance of the numerators, such as the future development of unsettled losses and the small credibility which attaches to finely subdivided experience. Therefore, certain approximations as respects the significance of the denominators are acceptable providing there is no bias and that the range of error introduced by the approximation is nominal. It may be pointed out that in ratemaking usually several years of experience are combined which diminishes the chance for an inaccuracy in exposure to have a substantial misleading effect.

It was previously suggested that it might be preferable to restrict census method data to the area of class and territory pure premium relativity where errors normally would not be damaging. The problem of rate level determination could be isolated and might be based on a more traditional evaluation of available data obtained from a separate source. For example, in Automobile insurance dependence for rate level might be placed on statewide calendar year incurred loss ratios, adjusted to current or anticipated conditions of loss cost and to current premium levels, for sub-lines of insurance such as private passenger per car, etc. Under such a procedure it is believed that the census method would produce many of the advantages claimed by the author without seriously impairing the reliability of the resulting rates.

If ratemaking were to be conducted by the two-step process of relativity and level it might be advisable to see whether the relativity revision could be accomplished on even a more economical and convenient basis than the suggested census method, such as, for example, using calendar year written exposures combined with accident year incurred losses. It, of course, would be necessary to adjust the current exposures for three-year or longer term policies to an annual basis. The economy and convenience of using calendar year written exposure by class, territory, and other sub-divisions without maintaining an "in-force" record should be self-evident. Also, because writ-

ten records have a greater degree of permanence than do "in-force" records the written basis might have a distinct advantage if a retroactive analysis in greater detail were found to be necessary in some particular area of experience.

It is appreciated that the use of written exposure is an unorthodox suggestion but it may be partially rationalized by pointing to a very close parallel with which the public is quite familiar. This is the principle involved in local property taxes in most municipalities. The net budget of expenditures for the coming fiscal year is determined and this amount is pro-rated among individual property owners on the basis of valuations as they appear on a common date such as October 1. In this way the amount of tax for each property owner is determined. This practice seems to be quite acceptable to all concerned and increases or decreases in the exposure which take place after the valuation date are ignored until the next time around. Thus in local taxation a single picture or one snapshot of exposure on a given date is used as contrasted with a twelve-month motion picture of exposure which is traditional in the insurance business. Why should not the same principle be adaptable in ratemaking within the area of class and territory pure premium relativity? Of course, the use of written exposure as suggested represents a still further departure since use would be made of the effective date of the policy or covage as the valuation date rather than using a single common date for all policies such as January 1 or July 1.

It appears to the writer that this simple analogy between ratemaking and taxation practice suggests that there may be some merit in studying the use of written exposure at the same time as attention is centered on the suggested census method. Both alternatives are worthy of friendly scrutiny in view of the potential rewards of economy and convenience.

There is one final comment which occurs to the writer. Both of the alternatives of census method or written exposure involve a trade where some small measure of accuracy in ratemaking is yielded for evident advantages of economy.

Just how vital is this element of accuracy to the three parties which are critically interested in rates, namely, the carriers, the insuring public, and the State? A small margin of error is of little importance to either the State or the insuring public. As for the carriers, their major interest in accuracy should be that the rate be sufficiently exact to discourage adverse selection either by the public or as might result from the activities of competitors who might be stimulated to raid business otherwise obviously overcharged.

It seems as though we might loosen the reins on approximations in making rates which are prospective and temporary in character. This comment is not intended as an espousal of slipshod methods in actuarial or statistical practice. It is merely a suggestion that with substantial rewards in view in the form of economy, we should be willing to give more ground in the self-imposed demand for perfection in ratemaking. This, I believe, is one of the thoughts which prompted Mr. Longley-Cook to submit his valuable contribution on the census method. It seems to be in order for us to proceed to the proving ground of comparative tests to see how small are the aberrations involved in the census method and other even more approximate procedures.

THE RATING OF CROP-HAIL INSURANCE

BY

RICHARD J. ROTH

Volume XLVII, Page 108

DISCUSSION BY W. J. HAZAM

Mr. Roth's paper is a welcome addition to our *Proceedings,* dealing, as it does, with a phase of property insurance that is somewhat foreign to the scope of the actuarial experience and activity of most of our membership, including myself. His comprehensive survey is, to my knowledge, the first and most complete documentation on the subject of crop-hail insurance ratemaking. We should take a great deal of pride in having this "first" within the publications of our Society.

The historical, meteorological and agricultural background, so necessary to the understanding of the many and diverse problems peculiar to ratemaking for this line of insurance, are clearly and concisely presented. I do not expect any serious difficulty will confront the reader because he may be neither meteorologist nor farmer.

I am impressed by the momentous task, after Public Law 15, facing the newly organized Crop-Hail Actuarial Association in the transition from what appears to have been a subjective process of ratemaking to one showing a generous measure of meteorological and statistical inference.

It may be somewhat surprising to casualty actuaries that here we have a line of insurance that uses as many years of experience as is available in establishing base rates—i. e., the all-time experience. In examples set forth in the paper, as many as the latest 35 years have been used. As a former meteorologist painfully familiar with the promiscuity of weather and hailstorms in particular and as an actuary concerned with rating a catastrophe coverage in a manner to avoid adverse selection, I am completely convinced of the wisdom and necessity for long experience periods as a basic foundation for ratemaking in this line of insurance.

Rates in property lines of insurance are developed mainly from a consideration of physical rather than statistical factors. As Mr. Roth shows, it has been just the reverse for Crop-Hail Insurance. Stemming from *a priori* meteorological considerations, Mr. Roth sets forth a very convincing statistical justification for the recognition of one physical factor,—namely, elevation in the grading of township rates. The correlations of the means of average loss costs of townships of like elevation and the elevation are extremely high. Meteorologically speaking, I would have expected a greater thunderstorm sensitivity to slope of land *per se,* of which perhaps elevation is one form of expression. It will be interesting to see the results of Mr. Roth's intended investigations of loss costs by classifications of townships by elevation *and* by slope.

I can understand the need for refinement of geographical areas into rating zones which reflect, to a reasonable degree, differences in exposure to the

meteorological hazard. This is necessary to avoid adverse underwriting selection. However, it is not clear why the refinement is so small as a township. In developing base loss costs for a particular township, the Township, County and Elevation Loss Cost are weighted 25, 25 and 50% respectively. Since hail will not fall at a given location in most years, chance plays a large part in determining township and county experience during the 25-year period. There is no real recognition of credibility when, for example, the following experience on page 15 for two townships receive identical credibilities of 25%:

Locality	*Township*	*Range*	*Liability (Exposure)*	*Losses*	*Pure Premium*
Reno	22 S	5W	239,582	7,750	3.23
Reno	33 S	7W	3,229,095	157,900	4.89

It would seem that casualty insurance techniques of graded credibility weighting depending on expected loss costs suggest some possible areas of adaptability here. While the concept of rate limitations described later in the paper is reasonable, it probably would not be imposed as often if graded credibilities were used.

With respect to the use of crop differentials in the ratemaking process, I have a few comments to make that will no doubt reveal my city-boy ignorance on matters agricultural. The desirability and reasonability of developing base rates in terms of the major crop grown in a particular state are apparent. The development of crop differentials by which losses in minor crops are converted to the common loss level of the major crop relies on indicated differentials of the state experience exclusively. In private passenger automobile liability, we have our class differentials based largely on countrywide experience. I do not contend this is best either and I do have certain misgivings with the automobile approach. On the other hand, I have similar misgivings with the Crop-Hail approach. Both lines of insurance seem to be at opposite extremes to where *a priori* considerations would say they should be. Perhaps somewhere nearer the middle for both may give a better answer. Again, credibility weighting between state and national experience, if feasible, should be investigated.

Policy form factors of conversion are apparently constant for the state. For example, for the State of Nebraska, the excess over 10% loss endorsement is 80% of the Annual Percentage form for each and every crop. Should there not be differences in policy form factors by type of crop? For example, if the average damage to sugar beets was 20%, the loss relativity of the two forms would be .50 (10% ÷ 20%). But if the average for cantaloupes was 80%, the loss relativity between the two forms would be .875 (70% ÷ 80%). Does not the current rating procedure lead to adverse underwriting selection of policy form?

The method of reporting crop-hail experience for ratemaking purposes arouses a few thoughts. It is stated that "loss information cannot be reported by punched card because of large possibility of error in coding and proper handling". Our automobile bureau companies do report loss information

monthly on punched cards and, I trust, are reasonably satisfied with the accuracy of performance. I am not cognizant of the relative complexity of such a basis of reporting to suggest that it be adopted for crop-hail. I do submit, however, that their system of controls suggests the possibility of serious compensatory errors that might still produce figures in the allowable range of deviation.

Incidentally, the sample page of the 1958 Kansas Statistical Summary of experience should contain some explanation of the columns and abbreviated notations within certain columns.

In general, the underlying research, meteorological and statistical, involved in the development of crop-hail rates speaks well for the Crop-Hail Insurance Actuarial Association's continuing search for more scientific ratemaking techniques. They are to be commended for their constant encouragement of scientific interest into the whys and wherefores of hailstorms in the hope of offsetting devastating results such storms can and do produce.

I hope these comments and thoughts of a casualty actuary, a true stranger in this field, suggest some areas of possible fruitful investigation in this difficult line of ratemaking.

Mr. Roth is to be congratulated for the overall excellence of his paper and for his very valuable contribution to our Society's expressed objectives of having in its *Proceedings* a complete catalogue of papers on ratemaking for all lines of casualty and property insurance.

REPORTS OF THE SEMINARS HELD IN SKYTOP AT THE 1960 SPRING MEETING OF THE SOCIETY

THE THEORY OF PRIVATE PASSENGER AUTOMOBILE MERIT RATING

(SUMMATION BY HARMON T. BARBER, SECOND VICE PRESIDENT AND ACTUARY, THE TRAVELERS INSURANCE COMPANY)

This topic proved to be a popular one and the two sessions scheduled for the afternoon were crowded to capacity. It was found impossible to satisfactorily cover the subject in the allotted time and as a result a continuation session was arranged for the evening. This latter session, best described as a "beer and shirtsleeves session," was strictly voluntary as to attendance and informal in character. Members were free to come and go at will and were free to interject comments at any point in the discussion. The continuation session was also well attended and lasted for hours. Its popularity is recorded here as a precedent to consider at subsequent meetings when a topic of widespread interest is under discussion.

The first question to resolve was the definition of the term "merit rating" for purposes of this discussion. The choice was between the generic or comprehensive concept which includes all elements of risk rating or risk classification, and the more restricted and perhaps more currently popular automobile designations, namely, the measurement of accident-proneness of the operator (or family group) as revealed by accident and conviction records. The election of the broader concept was made to permit discussion of the relationship between traditional class and territory distinctions and risk classification according to driver skill. It is evident that there may be some overlapping in any simple system which involves both of these types of criteria.

The scheduled sessions were opened with brief summaries of the formal papers relating to the subject which were presented at the last meeting of the Society. These papers appear elsewhere in the *Proceedings** as do written reviews which also were outlined to the seminars by the authors or their representatives. These writings relate mainly to one phase of merit rating, namely, the measurement of operator driving ability according to the conviction and accident records of the individual driver. One paper analyzes the relationship between the probability of the occurrence of an accident and driving records as revealed by the California Driver Study and the other draws deductions as to the credibility of experience indications of accident-proneness of a single car's operators based on a review of Canadian merit rating data. One paper proposes the application of a negative binomial type distribution as a useful tool and as a replacement for a Poisson type distribution

* See "Some Considerations on Automobile Rating Systems Utilizing Individual Driving Records" by L. B. Dropkin, CAS, XLVI, p. 165, and Review by R. A. Bailey, CAS, XLVII, p. 152; also "An Actuarial Note on the Credibility of Experience of a Single Private Passenger Car" by R. A. Bailey and L. J. Simon, CAS, XLVI, p. 159, and Review by W. J. Hazam, CAS, XLVII, p. 150.

in the analysis of recorded data. Discussion of the technical development of this point was limited to a few questions as to the theory advanced. There seemed to be agreement that the use of the negative binomial approach was a definite improvement in the analysis of available statistics. In this discussion it was pointed out that there is considerable overlap in contiguous merit rating classes based on driving records. This led to a discussion of the question of whether merit rating of driving ability would ever supplant entirely the customary manual class and territory distinctions. Most proponents of merit rating, based on the driving record of the owner-operator or family group of operators, look upon this phase of rating as a further refinement or extension of traditional manual classifications. However, there was at least one advocate of the thought that if a system of classification based on individual driving records could be perfected, it could largely replace other manual classification criteria such as territory, class, age, etc.

Quite a bit of interest was shown in the problem of how to obtain a prompt and reasonably indicative measure of underwriting results immediately following the introduction of a merit rating plan. The situation is exemplified in Texas where unusual activity in canceling and rewriting policies occurred with the introduction of the plan. Risks with poor records were canceled and rewritten just before the effective date of the plan and risks with good records were canceled and rewritten just after the effective date to secure a premium advantage for the assured. Therefore, a substantial, though temporary, disturbance of experience was created.

There was discussion of the inference in the keynote statement that "merit rating is underwriting by rote." The implication here is that if the potential hazard of each risk can be successfully measured by a complete or comprehensive merit rating system, it might facilitate carriers accepting risks automatically without giving consideration to whether the opportunity for underwriting profit was above or below average as respects an individual risk. Contrary opinion was expressed that a rigorously accurate rating method is unnecessary. All that is required is a plan which approximates the correct premium charge for an individual risk to an extent sufficient to discourage a hot war of competition among carriers and also sufficient to allay any suspicions of the public that someone else is securing an unfair advantage because of the applicable basis of premium adjustment.

There were scattered comments on some suspected points of vulnerability in the theoretical analysis of merit rating, such as, using number of cars rather than premium as a basis for accident frequency measurement; failure to distinguish between the magnitude of claims, accidents, or violations; and the observation that the lack of homogeneity within classes of cars might result in a definite bias in the deductions drawn from a statistical analysis of data.

It was observed that fines, license suspensions, and even confinement for violation of traffic laws are justifiable only on the premise that unsafe driving habits can be corrected. The use of driving records in merit rating is incompatible with this, since merit rating, being prospective in character, tacitly assumes that the bad driver of the past will be a bad driver in the immediate future. Contrary comments were that merit rating does not necessarily deny that a convicted violator may be an improved driver after punishment. It

merely assumes that the violator is a less desirable risk than the driver with a clean record and reflects this in the premium charged.

There was a discussion of the varying degrees of law enforcement which might exist in different territories of a state. However, it was pointed out that the average rate for a given territory should reflect the degree of law enforcement in the area and a merit rating plan based on violations would measure departures from that average.

The thought was raised as to whether some sort of psychiatric test might be devised ultimately which would provide a means of measuring the characteristics of individual operators which indicate a propensity toward accident-proneness. In this connection mention was made of several research studies in progress by various state motor vehicle and other state and university authorities in an effort to shed more light on what causes automobile accidents.

A suggestion was offered that the Society might undertake a detailed study of the causes of automobile accidents and duration of personal injury disabilities as revealed by claim records which would parallel the mortality investigations which have been found so useful in the life insurance field. Presumably this thought will be considered by the Research Committee of the Society.

In summary, it appears that various driver-record merit rating plans have been initiated recently, some of which probably have been designed on a cut-and-try basis, some of which are definitely experimental in nature and with features and values which may have been slanted toward competition for desirable business. Whether the theory of merit rating in the form of the statistical analysis of data will keep pace, or will lead, or will follow, the evolution of these plans as amendments are made largely dictated by practical reasons, remains to be seen. Thus far there is definite statistical support for the existence of accident-proneness in some individuals. There seems to be justification at the present time for the philosophy that moving traffic law violations are sound evidence for merit rating when they are viewed as accidents that almost happened and therefore amenable to combination with accidents that actually did occur. Further, the reasoning that every accident is either the basis for a claim or a very close approach to a claim, may likewise be accepted as pertinent in risk rating. The successful identification and measurement of these components presents an inviting field for serious research in the subject of merit rating. The problem of tying together in appropriate proportions measurements of driving skill with other more tangible and directly related classification data bearing on the occurrence or avoidance of personal injury or property damage claims represents another area for future investigation. This entire subject is definitely one for continued study.

The following outline was provided to suggest to those present some of the areas which might be productive of discussion:

Text: Merit rating is underwriting by rote.

A. Driving skill of the operator (individual or family group) as revealed by records of convictions, accidents and claims.

   1. Is accident-proneness a myth or actuality?

2. How successfully can drivers be classified as to degree of driving skill?
3. How reliably can the future accident record of classified drivers be predicted?
4. Can elements other than frequency be adequately assessed in classifying drivers, e.g., willful vs. inadvertent violations; fault vs. nonfault accidents; extent of resulting injuries or damage?

B. Is driver-skill rating properly a supplement or substitute for manual class and territory distinctions?

C. Importance of other merit rating considerations (may or may not be part of special class or rule which calls for rate adjustment).
1. Size of Car—compact or regular.
2. Equipment (or lack)—power brakes, safety belts, safety glass, padded dash, depressed-center wheel, windshield cleaner, etc.
3. Use of Car—transport to work, other business, pleasure only, touring.
4. Mileage—average in past; anticipated.
5. Operators—occupation; age; sex; marital status; proportionate use; years licensed; financial responsibility certificate; assigned risk; physical impairments; use of alcohol.
6. Multiple Cars—number of cars vs. number of operators.

D. Statistical Data.
1. Presently available—California Driver Study, Canadian merit rating statistics, Swiss sample and others.
2. Future—What provisions should be made for additional essential information not presently recorded?—(Cause of accident study. An accident table for Automobile Injuries.)

E. Cross Examination.

You are invited to submit written statements of the "true or false" variety, which are designed to elicit discussion of any doubts, inaccuracies, or deficiencies in the theory of merit rating as thus far developed—to be discussed by volunteers.

PRACTICAL ASPECTS OF AUTOMOBILE MERIT RATING

(SUMMATION BY WILLIAM S. GILLAM, RESEARCH DIVISION, NATIONAL BUREAU OF CASUALTY UNDERWRITERS)

I think it is very true that in any discussion of either the theoretical or practical aspects of automobile merit rating you tend to get involved in the other aspects. Several people mentioned, after our discussion on the practical aspects, that some of the things that were discussed should have been in the other seminar. Of course, when you're discussing something like merit rating,

you don't want to narrow it down too much. On the other hand, it is a broad subject and, even in the seminar on the practical aspects, we covered a lot of ground and it's somewhat difficult to summarize it.

In any case, we started off our seminar, as the other seminars, with a review of a previously submitted paper, that by Frank Harwayne entitled, "Merit Rating in Private Passenger Automobile Liability Insurance and the California Driver Record Study." The review was presented by John Muetterties.

There is one point in particular upon which I would like to comment that came up in John's review and also in a letter that Frank Harwayne sent to us in reply to John's review. This has to do with the question of when a merit rating plan is in balance. In his review John Muetterties posed the question this way:

When is a merit rating plan in balance?

1. When the expected distribution times the debit or credit rated risks equals the manual level, or
2. a. When a lower than manual level is produced but the losses are lower to the same degree, and
   b. When a higher than manual level is produced but the losses are higher to the same degree.

Frank in his reply indicated that he believes it is fair to say that the first answer conceives of a static balance while the second answer deals with a dynamic balance and that both might be included in the following form: a merit rating plan may be said to be in balance when the sum of the expected losses and the expected expenses may be expected to produce a reasonable profit margin or dividend margin.

This question of when a merit rating plan is in balance, or when it is off-balance, came up for discussion in both sessions of the seminar on the practical aspects of merit rating and there was general agreement with this last statement of Frank Harwayne although the idea was expressed in several different ways.

In this connection I think it would be well to comment that no attempt was made to take a vote on the various questions that came up and it's very difficult for the Chairman to determine the consensus of the group. All we can do is report the consensus of those who spoke up and we have to assume that those who remained silent agreed with the statements that were made.

The Chairman had prepared an outline of subjects for discussion for the guidance of the participants in the seminar. This outline on the practical aspects of automobile merit rating started out by stating that the discussion should proceed on the assumption that the seminar on the theoretical aspects of merit rating had concluded that merit rating for private passenger automobiles was feasible from a theoretical point of view and that the discussion in the seminar on the practical aspects should consider these practical aspects (1) from the point of view of the administration of the plans by companies and producers, and (2) from the point of view of acceptance of the plan by insureds and the public.

Six different subheadings for discussion were set forth: (1) definition of accident, (2) definition of conviction, including the basic question of whether the use of convictions is feasible from a practical point of view in an automobile merit rating plan, (3) length of experience period, (4) administration of the plan, including the use of signed applications and the use of motor vehicle records, (5) political aspects of the plan, including the effect of variations in enforcement of traffic laws, the effect on driving habits and highway safety and the general political aspects of the acceptance of this type of a rating plan, and (6) the effect on loss reporting and loss adjustment.

As you can well understand, the participants in the seminars didn't stick to this prepared outline very closely; they jumped around from here to there but, by and large, in both seminars most of these points were touched on to one extent or another.

In the first seminar the principal point that was discussed was the cost of administering merit rating plans. The seminar got into this discussion when it was pointed out that any calculations of the off-balance of a merit rating plan should take into account the additional administrative expenses of a plan. This led into a rather extensive discussion of the administrative problems and costs involved; these problems were discussed particularly in terms of the complications in connection with mechanical billing and in terms of the cost in a state that has low average rates and high costs for motor vehicle records.

In defense of the additional expense for administering merit rating plans, it was pointed out that the allowance for company administrative expenses in the rates for National Bureau companies is only 6.5% of the total premium. Even allowing for a very substantial increase in that part of this allowance that goes for the particular type of expenses that would be increased in administering a merit rating plan, the net effect on the total premium should be quite small. And this additional expense should be considered desirable by all concerned if it produces a more refined classification of risks.

On the question of off-balance, it was noted that in California initially the debits had acted as an automatic screen which served to induce risks who would have been assigned debits under the California Safe Driver Insurance Plan to seek a market where they wouldn't be called upon to pay the debits. This resulted, for the companies using the Safe Driver Insurance Plan, in a distribution by sub-classification that was weighted very heavily on the credit side—much more heavily than had been anticipated in the estimated distribution that had been made on the basis of the California Driver Record Study. At first this caused some serious concern. But later it was generally realized that the off-balance of this type of plan cannot be measured by examining only its effect on premiums; it is necessary also to examine the effect on losses. No detailed loss ratio data on a consolidated basis is as yet available but individual company loss ratios indicate no need in California for serious concern about the effect of the plan on the premiums.

In the second seminar two principal aspects were discussed. First was the competitive aspect. Statements were made to the effect that the real purpose of stock agency companies in going into merit rating was to reshuffle the business and get back some of the cream that had gone to the low-rate companies.

The second principal aspect discussed was the effect of such plans in depopulating the assigned risk plan. It was stated that such a plan should, in the long run, provide a freer market for poor risks outside of the restrictions of the assigned risk plans. In particular this should apply to those marginal risks currently going into the assigned risk plans. In the discussion it seemed to be generally agreed that this was something to be hoped for; but even if it did come to pass, there still would be need for assigned risk plans for the really "bad" risks.

In conclusion, I believe that those who participated in the discussion on the practical aspects of automobile merit rating would agree that no startling conclusions were reached but we hope that those who have not worked closely with the development and introduction of these plans learned more about them. There is a wide field for future discussions on a number of the different practical aspects of merit rating plans.

RATE MAKING AND STATISTICS FOR MULTIPLE PERIL POLICIES

(SUMMATION BY ERNEST T. BERKELEY, ACTUARY, EMPLOYERS' GROUP)

Our seminar was based on a paper that was presented to the Society last fall by Bob Hurley on "Multiple Peril Rating Problems—Some Statistical Considerations" and the discussion at both of the seminars was opened by a review of that paper by Paul Otteson.

Bob Hurley wasn't able to be there, but Paul did an excellent job in setting the stage for our discussion. Both sessions of the seminar were very well attended and I thought there was excellent audience participation. The seminar concentrated on a Homeowners policy on an indivisible premium basis as a prime example of a multiple peril policy. One of the interesting points that came out was that after a show of hands I discovered practically everybody in the room had a Homeowners policy except myself! I'm not quite sure, but I wonder whether that's why I was chosen to moderate the seminar so that I would see the light of day and get one myself.

The paper and the review were in a sense initial surveys of the proper statistics and rate making for multiple peril policies and were of necessity pretty well confined to general considerations and delineation of the problems involved rather than the proposal of definite solutions.

In the seminars, before undertaking a detailed discussion of the points raised in the paper and review, it seemed advisable to set the stage by recalling briefly the history of the Homeowners policy including its origin, coverage, statistics and rate making. The early pattern, I'm sure, is a familiar one to everybody. The removal of the restrictions of the Appleton Rule in 1949 made it possible to combine fire and extended coverage, theft and liability coverages in a single policy which could be written by either a casualty or a fire company. There is no need to recount the enthusiastic reception on the part of the public, the agents and initially at least the companies.

At this point I want to state that the following allegory can't be blamed in any way on the seminar; I must take full responsibility for it myself:

> The companies fairly quickly realized that in the Homeowners policy there had been created a kind of insurance Frankenstein whose carcass had been made out of various separate members by sewing them together with a stout thread in an indivisible manner. The proto-type of this creature came from the North America laboratories in Philadelphia in 1950 and was joined shortly in 1952 by a twin brother made in the Multiple Peril factory in New York. These two croppers worked diligently in harvesting the lush crop from the Homeowners field, but soon the twin brother, at least, developed a rather disturbing habit of putting more and more of the clover in the bags of the agents.
>
> It wasn't long before a cousin Frankenstein appeared on the scene, again from New York, but this time from Inter-Bureau Incorporated in 1954 in the form of the comprehensive dwelling policy. His life was destined to be brief for the thread that held him together was weak and several years later he literally fell apart into his original pieces.
>
> Our twin brother, heartened by the disintegration of this rival and encouraged by his own amazing growth and stature, demanded a new suit. He got it in 1958 in the form of the "new" Homeowners policy, which was a patchy sort of job, coming partly from his own suit and partly from the suit of his departed cousin, the Inter-Bureau relative.
>
> He had no more shaken the wrinkles out of this clothing than he felt the need of another new suit in 1959—otherwise known as the "new, new" Homeowners policy. But the tailors were running short of cloth and had to ask the agents if they could spare a little, so that the great frame of our Frankenstein might be fully covered.
>
> Busy with his harvesting, Frankenstein suddenly becomes aware of the approach of an intruder and looking up he sees coming down the road from the automobile field another cropper astride of what appears to be a harvesting machine of colossal proportions. Momentarily stunned, Frankenstein quickly remembers his mail-order catalogue and makes a mental note to go through it that night to see if he can find a much larger model of the old-fashioned lawn mower he has been using.

Now back to the seminar.

It was noted that the Homeowners statistical plan in current use (Multi-Peril Insurance Conference (Inter-Regional) is designed to produce calendar year earned premiums and losses incurred by state and policy form with supplementary information available by zone, construction and protection and cause of loss. Rates have been made from three ingredients mixed in certain proportions according to a sort of homemade recipe and containing the ever necessary herb of credibility flavored with an unusual type of seasoning. These ingredients are as follows.

1. The rates currently in effect,
2. The current rates modified to reflect the calendar year loss ratio indications, and

3. The sum of the current rates for the component coverages in the policy suitably discounted for the term feature, loss and expense savings from packaging, etc., and whatever saving there may be from commission assumptions.

This is basically a loss ratio method of rate making, which is not surprising because a very large proportion of the premium on Homeowners policies is accounted for by property coverages, the rates for which are usually made on a loss ratio basis.

Since this method of making Homeowners rates is not strictly the product of actuarial research and study but rather a procedure that has been developed with considerable emphasis on underwriting and production factors, the inquiring actuarial mind has discovered various basic questions that should be answered to make certain that Homeowners rate making is on a sound foundation. The usual reaction of the actuary who makes his first appraisal of this problem is something like the mosquito that has gotten into a nudist colony. He knows what he ought to do but he doesn't quite know where to begin.

After covering the foregoing historical aspects the seminar proceeded with a discussion of the principal points brought out in the paper and review, which may be summarized as follows:

1. The type of exposure that should be used in rate making including the present earned premium base and other possibilities such as the number of policies, the amount at risk or some composite.
2. The use in rate making of information pertaining to the cause of loss. The causes of loss include fire and lightning, windstorm and hail, water damage, theft, liability and miscellaneous property losses.
3. Possible extension of the present classifications of policy form, construction and protection to include other variants like occupation of the insured and perhaps his income level.
4. The ever-present question of credibility with consideration of premiums or number of claims or perhaps losses as a base.
5. Several miscellaneous points including the variation in loss frequency for windstorm versus other coverages and the associated windstorm catastrophe hazard.

The estimated frequency of loss of 20 per 100 Homeowners risks is very similar to the frequency on the all-coverage automobile policy, which raises the very interesting possibility of a merit rating plan for Homeowners as well as automobile.

As can be seen, these are all questions which, quite naturally, cannot be answered either quickly or easily. This poses still another question and that is who is going to undertake the research and study that is essential for sound answers?

Certainly any real progress must rest on a well-planned program and not on the occasional paper contributed by members of our Society nor on the actuarial committee of member companies of a rating organization. The ex-

act shape of such a program is not apparent at this moment, but its development would seem to require the application of much time and thought which might be forthcoming from some generous and well-staffed company or a full-time actuary in a rating organization or some combination of the two. Certainly any line of business that is already producing close to half a billion dollars in annual premium, and is still growing, deserves the benefit of all the actuarial talent it can get.

PREMIUMS AND RESERVES ON NON-CANCELLABLE AND GUARANTEED RENEWABLE A & S POLICIES

(SUMMATION BY JOHN H. MILLER, VICE PRESIDENT AND SENIOR ACTUARY, SPRINGFIELD-MONARCH INSURANCE COMPANIES)

I feel some diffidence in bringing you from the esoteric realms of negative binomials and Poisson distributions to the very pedestrian business of health and accident insurance. I had always thought that Poisson distribution referred to some method of merchandising fish, so I see that I'm going to have to get a little further education on the subject.

Mr. Barber's mention of accident proneness in respect to automobile insurance reminded me of the old chestnut I'm sure you've all heard; but it describes, I think, better than anything else the problems of health insurance. That is, the statement that to collect on a life insurance policy you must die; to collect on an accident insurance policy you must have an accident; to collect on a health insurance policy you must have a policy.

In connection with the auto merit rating plans, something was said about off-balance which is a perpetual state of the health insurance company. There are two general categories of companies in this business; there are those which consistently make a profit, perhaps a nominal one, and are severely castigated for gouging the public and then there is the other class that consistently loses money and they are severely castigated by their stockholders and critics in general for not knowing how to run their business. So you see you just can't win!

In our seminar yesterday there was some discussion of the federal income tax. The new life insurance tax law affects many companies—not only as to the tax on their health and accident insurance but also as to the classification of the company. As I think most of you know, the definition of life insurance reserves in the Federal tax law includes not only life insurance but non-cancellable insurance and adjustable premium guaranteed renewable health and accident insurance. There are companies which write no life insurance at all that are classified as life insurance companies for tax purposes because their reserves on these types of health and accident insurance with renewable guarantees are more than half of their total reserves. If they don't meet that test then they're taxed as stock or mutuals as the case may be, so that health and accident business may be taxed in different ways according to the way the company writing the business is classified.

The title of this seminar gave a little trouble. I was asked if it was correct.

The title is "Premiums and Reserves on Non-Cancellable and Guaranteed Renewable A & S Policies," which brings up the matter of definition of non-cancellable—a question that was debated not without some rancor and contention among opposing groups of companies and finally resolved by the NAIC which said that the term "non-cancellable" could be used either alone or in conjunction with "guaranteed renewable" if the policy was not only non-cancellable and guaranteed renewable but also provided a guaranteed rate. The reason for that primarily was that for 30 or 40 years the term non-cancellable had been associated with a guaranteed premium. The NAIC also recognized the use of the term "guaranteed renewable" qualified by the words "adjustable premium" to recognize policies which reserve to the insurer the right to adjust the rate on a class basis only, but otherwise had the attributes of non-cancellable insurance. So to be perfectly accurate the title should have read "Premiums and Reserves on Non-Cancellable or Non-Cancellable and Guaranteed Renewable Policies and on Adjustable Premium Guaranteed Renewable Policies." But that sounded too much like the title of a 17th century novel and without an explanatory comment the average reader not familiar with this controversy would have felt that the person preparing the program didn't know what he was talking about, so I took the simple measure of replying that I thought the title was just fine.

There was some discussion in the seminar on the annual statement requirements. Traditionally the non-cancellable policies have required, in addition to the pro rata unearned premium reserve, the so-called additional reserve, generally computed as a mid-terminal level premium reserve either on a full preliminary term or net level basis; but in the Task Force 4 recommendations, which were adopted by the NAIC, a little more flexibility was permitted and companies now can combine the pro rata unearned premiums with the additional reserve and set it up on a basis comparable to life insurance or on a basis that's roughly midway between.

Then there was considerable interest in the computation of gross premiums. Here with the increasing development of level premiums for life or level premiums to 65 with a step rate or adjustment at that point, we find quite a departure from traditional or customary fire and casualty procedure. We have a continuous policy, a level premium coupled with a risk cost that usually increases with age, and an expense cost that usually decreases, particularly when non-level commissions are used. With this combination you have a situation more akin to life insurance, particularly long-term life insurance, than to any other form of insurance and it's necessary to bring in mortality, interest and the rate of lapse.

Sources of morbidity rates were discussed, or mainly the lack of sources because there is not a great deal that is presently available. A committee of the Society of Actuaries has been assembling data on disability insurance experience, but thus far the data have not been sufficient in volume, homogeneity or maturity to warrant the development of a new table.

The Task Force 4 report on reserves included tables for hospital and surgical costs. Other papers on individual company experience giving some data on major medical expense insurance have been published and are being used for premium and reserve computations.

With the recent development and emphasis of benefits for hospital and medical expense, an additional contingency was added to the problem of evaluating the cost of future benefits, namely the cost index of medical care. This has led to the development of the adjustable premium guaranteed renewable policy under which the insurer guarantees the continuance of the policy subject to the timely payment of premiums which can be adjusted in the future on a class basis to recognize changes in the price level as well as other changes in the underlying assumptions.

In accepting the Task Force 4's report the NAIC did it with the stipulation that any company choosing to write an adjustable premium guaranteed renewable health policy should maintain fund accounts of each form of policy so that if the time came when the company wished to raise the rates on existing policies because of adverse experience, there would be a historical basis of a fund account to justify that request. Thus, the obligation a company assumes in issuing an adjustable premium guarantee renewable policy is not only that of attempting to determine an adequate rate and maintaining proper reserves but also of keeping a fund account, so that if the initial assumptions prove to be inadequate or if cost of services requires an increase in rate, the company will have something to point to in justification of its revised rates.

Finally, I would like to comment on the new development of an automobile policy containing certain renewal guarantees and suggest the parallels between that and the health and accident policies with renewal guarantees, which may now be defined as non-cancellable or as adjustable premium guaranteed renewable policies, depending on whether the premium is or is not guaranteed. I think there are many parallels with respect to definition, nomenclature, advertising, and also in the principles and practices of ratemaking and maintenance of reserves.

REPORTS OF THE SEMINARS HELD IN WASHINGTON, D. C. AT THE 1960 ANNUAL MEETING OF THE SOCIETY

AUTOMOBILE MERIT RATING

(Summation by Thomas O. Carlson, Manager, Southeastern Branch, National Bureau of Casualty Underwriters.)

I stand before you as an innocent victim of a mouse-trapping Society Vice President who asked me to conduct a seminar on developments in Automobile Merit Rating, saying that there would be a number of papers to carry the session and I would only have to referee the bout. The word "only" was the mouse-trap. When I saw the first papers, deep-fried in a batter of hypergeometric foundation overlaid with negative binomials and coefficients of variation, I hastily reviewed my dues-paying status in the Society in the same frame of mind as the chap down in my new "Yo'-all" neck of the woods who came into the City Hall one morning to inquire whether his marriage license had not already expired. I suggested, when the Vice President cruelly refused to unspring the trap, that all members should be forewarned that this was to be a discussion taking off from a springboard of theory rather than practice, but he felt that the papers distributed in advance of the meeting

would speak for themselves. Nevertheless one soul, who probably had ample company, remarked at the end of the second session: "Did I land in the wrong pew? Or *was* this the session on merit rating?"

Both sessions with unexpected enthusiasm went overtime in considering and discussing papers by Mr. L. B. Dropkin, Mr. C. C. Hewitt, Jr., and Mr. R. A. Bailey, a review of a previous Bailey-Simon paper by Mr. D. B. Martin, and a review of Mr. Bailey's current paper by Mr. L. Roberts, by Mr. L. J. Simon and jointly by two student guests, Messrs. Muniz and Lange. Mr. Roberts had also written a discussion of the previous Bailey-Simon paper but this discussion was not received in time to be included as part of the seminar.

Mr. Dropkin's paper, in the unassuming guise of an actuarial note, on "Automobile Merit Rating and Inverse Probabilities", further develops his important work on the negative binomial distribution by bringing in the time element and by utilizing inverse probabilities to develop a function to determine the probability of x accidents in s years for a sub-group observed to have c accidents in t years.

Mr. Hewitt's paper, "The Negative Binomial Applied to the Canadian Merit Rating Plan for Individual Automobile Risks", applies the developments in Mr. Dropkin's new paper to the Canadian experience.

Mr. Martin's review was of a paper, "Two Studies in Automobile Insurance Ratemaking", presented at a previous meeting, jointly by Mr. R. A. Bailey and Mr. L. J. Simon. Mr. Martin, in remarks that were refreshingly down-to-earth (1) emphasized the importance of current developments of proper mathematical foundations underlying the ultimately more simple practical rules-of-thumb utilized by the underwriters, (2) remarked that the underwriter not infrequently (illustrated by references to the development and testing of the Canadian plan in which Mr. Martin himself played an important role) is racing ahead of the theoretician but is comforted when the mathematician, often breathlessly, catches up with him and supports him, and (3) agreed with the Chairman that we are no longer dealing with merit rating as such and that this nomenclature should be dropped for a phrase emphasizing that we are talking about classification refinement rather than merit rating.

I, of course, am omitting technical details of papers and reviews here, since they will be published in full dress in the *Proceedings*.

Mr. Bailey's paper, with the provocative title "Any Room Left for Skimming the Cream?", took as its objective the establishment of some means of measuring the relative effectiveness of various schemes of classification and the various elements in these schemes, an extremely important and timely project. He chose as his measuring-stick the coefficient of variation, which is the standard deviation divided by the mean. Having first estimated the coefficient of the inherent risk hazard of all private passenger car risks as approximately 1.00, on the basis of studies previously reported in these *Proceedings* and at the 16th International Congress of Actuaries, he then took a representative distribution, one company's exposures in one state, and computed the coefficients of variation of the various elements and combinations of elements involved in the class-and-territory system to compare with the predetermined unity norm.

A critique of this paper not previously seen by the author, offered by two students, Mr. Muniz and Mr. Lange, present by invitation, questioned the validity of the calculation of the norm as 1.00, giving reasons for supporting a value perhaps as low as .75. They further pointed out that since the distribution of rates, or of hazard as represented in tabulated experience or through interpretation of such in rates, is discrete while the distribution of inherent hazard is continuous, and since the variance of a discrete approximation to a continuous function is less than that of the function itself, some of the low coefficients obtained by Mr. Bailey were to be expected for mathematical reasons and consequently lack the significance he attached to them.

Mr. Lange additionally referred to a measure suggested by M. Delaporte at the 16th International Congress based on dispersion about the mode rather than the mean.

Mr. Roberts emphasized that such a measure as Mr. Bailey is seeking should be applied to experience rather than to rates, expressed caution generally in using the coefficient of variation, and indicated certain avenues for further exploration of the problem, in particular use of a measure stemming from the variance rather than from the standard deviation. He also suggested that a single measure might not be found which would properly evaluate a classification system, and that some combination of measures might turn out to do a more adequate job.

Mr. Simon commented to the effect that to be meaningful the coefficient of variation must be calculated on rates that reflect the actual experience or on the experience indications themselves, and further stressed the limitations of Mr. Bailey's study, restricted of necessity to a limited population, pointing out that application to a complete population is desirable to arrive at final conclusions. He further brought out that the coefficient is valuable for comparing two rate structures.

The Chairman presented a table which illustrated concretely a point made by Mr. Simon on the restrictions on upper bounds of the coefficient of variation regardless of actual propriety of rate relativities in the light of experience developments; the table at the same time emphasized the symmetry of the variance contrasting sharply with the asymmetry of the coefficient of variation in cases involving a mere reversal of the proportions represented by two classes in a two-class system, and also showed that volume has no effect on the coefficient, the same results coming from indications based on 10 cars as from 10,000,000 cars. A very brief excerpt from this table is shown below because, alone among the reviews, the Chairman's remarks will not be separately published.

If a and b are the proportions of Classes A and B, so that $a + b = 1.00$, and x and y are the Class hazards, with x/y represented by the parametric constant k,

$$\text{C.V.} = \frac{(1\text{-}k)\ \sqrt{ab}}{a + bk}$$

showing that only the ratio (not the actual values) of x to y affects the C.V., and that the C.V. is asymmetrical with respect to a and b while the standard deviation (which is $(x\text{-}y)\ \sqrt{ab}$) is symmetrical with respect to a and b.

Mr. Bailey stated he felt the characteristic of asymmetry in the C.V. is reasonable because of the difference between x/y and y/x. The Chairman reserved opinion on this point. The table extract follows:

Differentials		$a = .1$ $b = .9$	$a = .3$ $b = .7$	$a = .5$ $b = .5$	$a = .7$ $b = .3$	$a = .9$ $b = .1$
90%-100%	S.D.	3.0	4.583	5.0	4.583	3.0
	C.V.	.030	.047	.053	.049	.033
70%-100%	S.D.	9.0	13.747	15.0	13.747	9.0
	C.V.	.093	.151	.176	.174	.123
50%-100%	S.D.	15.0	22.913	25.0	22.913	15.0
	C.V.	.158	.270	.333	.353	.273
30%-100%	S.D.	21.0	32.078	35.0	32.078	21.0
	C.V.	.226	.406	.537	.629	.568

Mr. Martin commented that too close an approach to the unity (or other) norm for the inherent risk hazard of all cars by the coefficient of variation of a particular classification distribution would mean that we had reached the point of too great a refinement, i.e., too close an approach to recognition of the inherent hazard of every individual risk, too close to that ultimate refinement in squeezing out the cream expressed by your Chairman as the a-rating of every individual car.

Dr. Dickerson also indicated certain areas of investigation, particularly a more careful and complete analysis of the coefficient of variation itself, by application of the analysis of variance.

I gathered the impression that there was general agreement that the coefficient of variation, while serving as a guide in making comparisons of two systems, is far from a complete indicator and should be used with extreme caution, and that the search for a more adequate measure or measures constituted a promising field of mathematical investigation by the young crop of technicians in the Society.

Mr. Hewitt arose to comment on the large areas of investigation still awaiting the mathematician in our Society's bailiwick, by way of illustration pointing out that while the Poisson distribution applied to accidents assumes a constant inherent hazard from risk to risk, and the negative binomial distribution goes a step further by assuming a Pearson Type III distribution as representing variation in the inherent hazard from risk to risk, there is a further refinement to be explored in the variation of this hazard within the individual risk according to time and circumstances.

And at that point, with your Chairman standing on a platform of quivering variances, trying to support only standard deviations to the exclusion of mere modes, dodging the lethal impacts of gammas, betas and other coefficients, dizzy from multitudinous variations and their pseudo-measures, slipped and skidded precariously down a high-contact Pearson Type III curve, to land in the sharply-cusped coils of a highly irregular fifth degree moment from which he was able to extricate himself only by hanging onto a negative binomial developed by expansion of a characteristic function externalized in purely

imaginary quantities; and that extraordinary mathematical abstraction commonly labeled "t" but known to mere *hoi polloi* as time came to the rescue of all participants and non-participants and the session was adjourned, with the Chairman expressing silent thanks that he has already received credit for passing the mathematical sections of the examinations for admission to the Society.

GUARANTEED RENEWABLE AUTOMOBILE INSURANCE

(Summation by Leo M. Stankus, Actuary, Allstate Insurance Company)

The subject was introduced by presenting the details of two plans—the Allstate Plan and the National Bureau Plan proposed for New York. Both plans guarantee that, except for certain specified reasons, the Liability coverages will not be canceled during the guarantee period. For Allstate, the guarantee period is five years for new business. Under the Bureau Plan, the guarantee period is for one year; however, the policyholder is also guaranteed that he may renew his policy unless a notice of intent not to renew has been mailed to him at least 45 days prior to the renewal date.

For new business, both plans incorporate a "qualification period" during which the underwriter can check the inspection report and Motor Vehicle Department records in order to determine that accurate information has been given on the application. Also, under both plans the Company retains the right to cancel the Liability coverages for certain specified reasons—mainly "public policy reasons" which involve conduct on the part of the policyholder which is detrimental to the public interest.

A good deal of the discussion was devoted to the underwriting problems that are involved in providing this type of a guarantee. When this program had been introduced by Allstate, it was made applicable to all policies which had been in force for at least 90 days on the effective date of the plan, and to all recently issued policies as they complete their 90-day "qualification period." However, for administrative reasons the guarantee periods for in-force policies varied from one to five years, depending upon how long they had been insured with the Company.

The question as to the actuarial aspects of establishing the cost of Liability insurance issued on a guaranteed-renewable basis revealed that the consensus was that such insurance should be issued with some form of "merit rating" plan. All of the Allstate plans have been issued in conjunction with a merit rating plan, and it is understood that the Bureau program in New York also contemplates merit rating.

The possibility of extending a guarantee to coverages other than the Liability coverages was also discussed. It was explained that the guarantee was first offered with respect to the Liability coverages because of the far greater need for this form of protection—both on the part of the policyholder and for the protection of the general public.

HOSPITAL AND SURGICAL BENEFITS FOR PERSONS AGE 65 — PRIVATE INSURANCE OR SOCIAL SECURITY?

(Summation by Harold F. LaCroix, Secretary, The Travelers Insurance Company)

The chairman took the liberty of broadening the subject of this seminar to "Hospital and Surgical Benefits for Persons Age 65 and Over — Private Insurance or Governmental Action?" We believed it necessary to thus broaden our horizons because of the many approaches other than Social Security which were considered by the last Congress.

The seminar was conducted in two parts. The first part consisted of three short statements to "set the stage."

The first statement was made by the chairman and outlined the progress that private insurance has made in insuring persons over age 65 against hospital and surgical expenses. Such progress has come about through many avenues—first, the insuring of persons over age 65 (both as active employees and as retirees) under Group Health and Welfare plans; second, the conversion privilege under a group policy which entitles the retired employee to an individual policy without evidence of insurability; third, the many group association plans which enable retirees to obtain medical care coverage at reasonable group rates (the American Association of Retired Persons was then cited as an example of this type of association); fourth, the various standard individual policy "senior citizen" programs such as that provided on a state-wide basis with specified open enrollment periods by the Continental Casualty Company and the Mutual Benefit of Omaha, Nebraska; and fifth, the variety of individual policies now available for the senior citizen, many now on a guaranteed renewable basis.

The second statement by Mr. Robert J. Myers, Chief Actuary of the Social Security Administration, reviewed recent congressional attitudes and bills which involved medical care for the aged and listed some of the advantages and disadvantages of providing medical care for the aged under the Social Security System. Among those congressional bills reviewed by Mr. Myers were the Forand and the Anderson-Kennedy bills by which coverage would be provided under the Social Security System, the administration's bill which utilized the so-called "general insurance" approach and, of course, the Kerr-Mills bill which was finally passed by both houses of Congress and which allows states to expand medical care for the aged under the Old Age Assistance programs.

The third statement was by Mr. W. Rulon Williamson, now an independent Research Actuary but formerly Actuarial Consultant of the Social Security Board and leading and long-time advocate of financial responsibility in the Social Security Administration. Mr. Williamson's statement dealt with some of the history of social legislation in the medical care field in other countries, and with the efforts which are being made to keep the "camel's nose" of medical care coverage out of the taxpayer's tent.

The second part of the seminar was spent in general discussion of this prob-

lem including methods of improving the financing of private insurance medical care coverages for the aged. Particular mention was made of the need for definite Federal Income Tax rulings in the field of pre-funding hospital and surgical benefits for retirees similar to those now in effect in the pension area.

STATISTICS FOR RATING AND RESEARCH

(Summation by Norman J. Bennett, Actuary, America-Fore Loyalty Group)

Seminar D was conceived at 1:45 P.M. yesterday and suffered a gestation period of almost an hour before appearing to those of us who awaited it and vicariously shared its pangs of birth. Our first hour was an interval during which we probably did what was expected of us, but an unnatural truce zone lying between the author and the moderator prevented the somewhat fiercer exchange of views which seems to make these affairs so worth-while. Between performances, however, as do many producers during the opening nights in Bridgeport, we revised some lines, cut out the second act, and in general tightened up our production. While no one could possibly claim that the scene finally reached a raucous and uncontrollable level, I think we made a good start in introducing Mr. Longley-Cook's fine new concept which is certainly going to receive a lot of attention in the future.

Everyone agreed from the start that the cost of submitting and processing information for statistical agencies is becoming a burden sufficiently large for members, subscribers, and designators alike to warrant a serious look at any method which promises relief. There was also general assent to the idea that with growing individual responsibilities for company action, some method must be evolved to permit a company a broader, more inclusive, and cheaper look at its own experience in whatever form *it may elect itself*. The present methods of computing earned statistics in the detail required for close analysis within a company were universally condemned as physically impossible.

The problem was thus defined; it was only the solution which vexed us, and the solution offered yesterday was the so-called census method. This method depends on an active in-force file. Although the author offered an alternative use of the statistical file where an in-force file does not exist (and it is interesting to note that only one company present admitted to having such a complete file), no discussion at all centered about the use of statistical files.

Instead we tended to project ourselves forward in time to an era when we will all have, as a natural outgrowth of electronic data processing, a single in-force file combining statistical and corporate functions. Several questions immediately arose in our minds which still remain unanswered. What of the relative movement of companies toward this mechanized era? Should the faster-converting companies be penalized in the meantime by methods applicable to the slower-moving companies? Even with the ultimate availability of such a file, do modern computers really care in terms of speed and expense which method we use? Is one method more accurate than another? (Here we discussed the differing degrees of accuracy of input to the two files. An in-force file apparently ages well in terms of accuracy.) What expense savings

exist for one method over the other? This was a question for which there was no agreement over the expense savings of the census versus conventional methods, only agreement that there should be major savings because of mechanization itself.

There seemed to be some concern that in many instances, such as a rapidly changing volume of business, changes in territory and classification definitions, or changes in the distribution of business caused by the introduction of either a financial responsibility or compulsory law, the census method might be seriously inadequate. Mr. Longley-Cook was emphatic, however, that his proposal was aimed primarily at the personal lines field and that real or imagined difficulties in adjusting it to unusual conditions should not disguise its extraordinary usefulness in analyzing multiple breakdowns of large-scale experience.

There was an extremely interesting review of this paper by Mr. Barber which was ended by the stunning proposal that we drop the habits of a lifetime and consider forgetting earned statistics and develop rating methods using written statistics. A much closer scrutiny of this suggestion will be made, I am certain, in the months to come.

In brief, we discussed a short and deceptively simple paper. We were intrigued, repelled, attracted, and now we're not so sure. We are sure, however, that our problems are still with us and we hope that playing devil's advocates to this fresh new thought will start us on our way to a solution.

REVIEW OF PUBLICATIONS

ALLEN L. MAYERSON, Book Review Editor

H. Jerome Zoffer, *The History of Automobile Liability Insurance Rating*, University of Pittsburgh Press, Edwards Brothers Inc., Ann Arbor, Michigan — 1959.

As the author states in the foreword, this book is intended to set forth the various rating methods for automobile liability insurance which have been utilized by the insurance industry since the inception of this type of coverage. With this fairly limited objective, Mr. Zoffer has accomplished his purpose.

For a person who is not intimately acquainted with the detailed study that enters into the presentation of a "Bureau" type of automobile liability rate filing, this book is highly recommended. The subject matter covers the public liability rating of automobiles from the advent of this means of conveyance up to the year 1957. The methods of ratemaking, classification and other rating problems are traced through this period and the solutions developed by the National Bureau are presented in a complete form. Perhaps there is too much space devoted to the finer points of rating and not enough to a discussion of the general principles involved. The author has presented the factual information well but it would have been interesting to read his comments on the procedures.

Following the excellent presentation of rating and classification methods utilized by the National Bureau, the author includes several chapters on topics directly related to automobile liability rating. Such subjects as profit allowance, earned factors, statistical plans, the compensation approach to automobile liability rating, and others are briefly discussed. The chapter on the profit allowance is especially good since it gives a comprehensive presentation of the entire subject.

Since almost all of the test is historical, as was intended by the author, there are few if any points that can be criticized as to content. The one area which should be clarified is that dealing with a comparison of the loss ratio and pure premium approaches to ratemaking. The author implies that the loss ratio method is not so exact as the pure premium approach and is only "broadly reliable". This is not true because the loss ratio method, as it is commonly known today, produces the same rate level indications as the pure premium approach. Current National Bureau filings utilize the actual loss ratio, at present rate level, and expected loss ratio in the determination of rate level changes for a given state.

Since the text covers only rating problems to 1957, many current items which the author could not completely foresee have not been included except for passing comments. With the advent of refined classification plans, merit rating plans, and with special types of policy forms being formulated by all segments of the industry, enough material is available for a complete second volume on this subject. However, by the time such a text is prepared and printed it could very well be outdated.

In conclusion, Mr. Zoffer's book is certainly recommended to the casualty

insurance student since it provides a wealth of information on the subject of automobile liability insurance ratemaking. Furthermore, for those within the industry who are actively engaged in developing new methods, a review of this text will serve as a sharp reminder of past problems, and point out that the solutions have not always been simple or easily found.

RONALD L. BORNHUETTER

OBITUARY

STEWART M. LaMONT
1931-1960

Stewart M. LaMont, who became a Fellow of the Casualty Actuarial Society in 1931, died on August 22, 1960 at the age of 91 in Berkeley, California.

Prior to his retirement in 1938, Mr. LaMont headed the Accident and Health Division of the Metropolitan Life Insurance Company in New York City. Previously he had been Assistant Secretary of the Indemnity Insurance Company of North America and was instrumental in organizing the Accident Department of that company.

Mr. LaMont was a past Vice President of the International Claim Association, a member of the Board of Governors of the Bureau of Accident and Health Underwriters, and chairman of the Hooper-Holmes Bureau for 15 years.

Mr. LaMont is survived by two daughters, Marguerite and Mrs. Russell A. Fairbain.

OBITUARY

VICTOR MONTGOMERY
1891 - 1960

Victor Montgomery, a founder, longtime president, chairman of the board of the Pacific Employers Group of insurance companies and a Fellow of the Casualty Actuarial Society of America, died on May 2, 1960 at his home in Beverly Hills, California following an extended illness.

Mr. Montgomery was a dedicated public servant, serving as a past president of the Hollywood Bowl Association; director of Greater Los Angeles Plans Incorporated; a treasurer and vice president of the Friends of Colleges of Claremont; vice president of Tennis Patrons Association of Southern California; a charter member of the Youth Tennis Foundation of Southern California; and a member of the executive committee of the Southern California Symphony Association.

He was born on June 2, 1891 in Albion, Nebraska, and educated at the University of California at Berkeley where he received the degree of Bachelor of Science in 1916.

Shortly after his graduation from college, Mr. Montgomery became an assistant actuary and later actuary for the California Insurance Department, a position which he held until 1921. He served as a deputy insurance commissioner of California from 1921 until 1923.

Following this, he organized the Pacific Employers Insurance Company in 1923 which he developed from a small California company to one of the nation's leading groups of casualty insurance companies operating in 43 states.

In addition to his role in the Pacific Employers Group of insurance companies, comprising Pacific Employers Insurance Company, Meritplan Insurance Company, Allied Compensation Insurance Company, California Union Insurance Company and California Food Industry Insurance Company, Mr. Montgomery headed the Victor Montgomery General Agency Incorporated, Montgomery and Collins Incorporated, and Pacific General Agency Incorporated in Seattle. He was also a director of the Security Title Insurance Company.

Nationally recognized as a man who pioneered many of today's progressive insurance ideas, Mr. Montgomery was founder-president of the Western Insurance Information Service, Incorporated. He was also a member of the California Club, the Los Angeles Country Club, the Bohemian Club in San Francisco and the Lake Arrowhead Yacht Club of which he was at one time the commodore.

OBITUARY

FRANCIS SPENCER PERRYMAN
1896-1959

Francis Spencer Perryman died November 30, 1959 in Lancaster, Ohio while visiting a daughter. He was sixty-three years old.

Born December 3, 1896 in London, England, he was educated at Christ's College, London and London University from which he received a B.Sc. degree. He entered the employ of the Royal Insurance Company in London in 1914. After serving as a Lieutenant in the British Army from 1915 to 1919, he resumed his career with the Royal Insurance Company and in 1922 became a Fellow of the Institute of Actuaries.

He came to the United States in 1924 as Assistant Actuary of the casualty companies of the Royal-Globe Insurance Companies and later was elected Vice President and Actuary of those companies. In 1949 he became Assistant United States Manager, Vice President and Actuary of all of the companies of the Royal-Globe Insurance Companies operating in the United States.

Mr. Perryman's professional attainments were of the highest order. In addition to his Fellowship in the Institute of Actuaries, he was a Fellow and past President of the Casualty Actuarial Society and an Associate of the Society of Actuaries. He was also a member of the American Statistical Association, the Mathematical Association of America, and the American Mathematical Society. At the time of his death he was Vice-Chairman of the ASTIN Section of the International Congress of Actuaries.

Francis Perryman was an actuary's actuary and was respected by his colleagues for his broad intellectual attainments and interests, his analytical ability and his calm and considered business judgment. His patience with detail without making it detrimental to the overall solution of a problem was one of his outstanding characteristics. He was strongly admired for his outstanding mathematical capabilities which he combined with a very practical approach to problems. Since he completely understood theory, he knew just how far one could go with short-cuts and what would be their consequences. For this reason he was popular with non-actuaries and was able to serve as liaison between them and actuaries.

At actuarial conferences Mr. Perryman was a mathematical doodler and his notes were often interspersed with the working of mathematical processes such as the expansion of determinants.

To his profession he left a legacy of papers in these *Proceedings* which reflected his disciplined mind and his wealth of experience. His mathematical papers on Credibility and Retrospective Rating are classics, and his non-mathematical papers were also of a fundamental nature which have provided

a structure upon which later writers could build. One of these writers said, "Of all the voices crying in the wilderness Mr. Perryman's seems to have been charged with the most prophetic quality. His paper . . . is a masterly analysis. . . . In rereading his words today one is struck with the same sense of vague familiarity expressed by a habitué of Tin Pan Alley in hearing a musical composition of an old master. Some of his phrases have been unduly syncopated, there has been perhaps too much vapid repetition of his more catchy tunes, but the inspiration is manifest."

The respect Mr. Perryman was accorded as an actuary was matched by the warm feelings which people had for him personally. Not an outgoing person, he was regarded by his associates as a delightful companion and among those who worked for him he was universally regarded with affection.

In one of his papers Mr. Perryman characterized himself: "Perhaps I should say a word here in anticipation of a type of criticism that may be levelled against the results given here, by some hasty critics, on the grounds that ten place logarithm tables and ready to use weekly annuity tables are unnecessary luxuries or give needless accuracy. I don't regard the matter thus. The tables I give are, I believe, useful additions to the tools of our profession: and it is a fitting example of the principle of division of labor for one person like myself, who is interested in these things and likes working them out, to undertake the work of preparing these tools and presenting them to the profession. If the few pleasant hours I spent in putting this paper together save members of the profession a few minutes work from time to time, then my labor was useful as well as pleasant."

One of the things which is least known is that Mr. Perryman was an avid student of religion as well as mathematics. He was a member of the Church Club of New York, the St. George's Society and the Pilgrims of the United States. He was formerly the efficient Treasurer and dedicated Church Warden of the Church of Saint James the Less in Scarsdale, New York.

His ability and personality made him one of the most influential persons in the actuarial development of fire and casualty insurance in this country in the past three decades. He will be missed by his contemporaries but his influence will be felt for years to come by the industry through those who never had the opportunity of meeting him but who will know him through his papers.

He is survived by two daughters, Mrs. Douglas R. Burke of Lancaster, Ohio and Mrs. John C. Hazelwood of Roanoke, Virginia, and by six grandchildren.

OBITUARY

WILLIAM F. ROEBER
1901-1960

William F. Roeber, a past Vice President of the Casualty Actuarial Society, died in Saratoga, California, at the age of 59 on March 21, 1960.

Mr. Roeber was born in Newark, New Jersey and after serving in the U. S. Infantry during World War I, graduated from the University of California in 1921. He began his insurance career as a payroll auditor with the California State Compensation Insurance Fund, and was subsequently special agent, resident branch manager and assistant comptroller before resigning in 1923 to accept a position in the actuarial department of the newly formed National Council on Compensation Insurance. He was made assistant actuary of the National Council in 1924, actuary in 1926, assistant manager in 1929 and general manager in 1930. He retired in 1950 and for a number of years resided in Manchester, Vermont, before moving to California.

Mr. Roeber was widely recognized as one of the leading actuaries on workmen's compensation insurance in the United States. During his period of employment with the National Council he contributed greatly, in the infancy of workmen's compensation insurance in this country, to the development of individual risk rating plans and the establishment of scientific principles for the making of workmen's compensation rates.

At the time of his death Mr. Roeber was survived by his widow Reta; two sons, William and James, and by several grandchildren.

MINUTES OF THE MEETING

May 23, 24 and 25, 1960

SKYTOP LODGE, SKYTOP, PENNSYLVANIA

The registration list showed the following 75 Fellows and 22 Associates in attendance:

FELLOWS

Allen, E. S.
Bailey, R. A.
Barber, H. T.
Barter, J. L.
Batho, E. R.
Bennett, N. J
Berkeley, E. T.
Berquist, J. R.
Bevan, J. R.
Blodget, H. R.
Bondy, M.
Bornhuetter, R. L.
Boyle, J. I.
Carlson, T. O.
Curry, H. E.
Day, E. W.
Dropkin, L. B.
Elliott, G. B.
Fairbanks, A. V.
Farley, J.
Foster, R. B.
Fowler, T. W.
Fredrickson, C. H.
Gillam, W. S.
Goddard, R. P.
Graham, C. M.
Graves, C. H.
Hart, W. V. B., Jr.
Hazam, W. J.
Hewitt, C. C., Jr.
Hope, F. J.
Hughey, M. S.
Johe, R. L.
Johnson, R. A.
Kallop, R. H.
Klaassen, E. J.
Kormes, M.
LaCroix, H. F.
Leslie, W., Jr.
Lino, R.
Liscord, P. S.
Livingston, G. R.
Longley-Cook, L. H.
MacKeen, H. E.
Makgill, S. S.
Menzel, H. W.
Miller, J. H.
Muetterties, J. H.
Munterich, G. C.
Niles, C. L., Jr.
Otteson, P. M.
Phillips, H. J., Jr.
Pinney, A. D.
Pruitt, D. M.
Resony, A. V.
Resony, J. A.
Roberts, L. H.
Rodermund, M.
Rosenberg, N.
Rowell, J. H.
Ruchlis, E.
Salzmann, R. E.
Schloss, H. W.
Skelding, A. Z.
Simon, L. J.
Smick, J. J.
Sykes, Z. M., Jr.
Tapley, D. A.
Tarbell, L. L., Jr.
Thomas, J. W.
Trist, J. A. W.
Valerius, N. M.
Williams, P. A.
Williamson, W. R.
Wittick, H. E.

ASSOCIATES

Abel, F. E.
Alexander, L. M.
Andrews, E. C.
Balcarek, R. J.
Berg, R. A., Jr.
Craig, R. A.
Fitzgibbon, W. J., Jr.
McNamara, D. J.
Moseley, J.
Muir, J. M.
Royer, A. F.
Scammon, L. W.
Schneiker, H. C.
Schwartz, M. J.
Steinhaus, H. W.
Stellwagen, H. P.
Stern, P. K.
White, A.
Wilcken, C. L.
Willsey, L. W.
Wilson, J. C.
Woodworth. J. H.

Following a scheduled luncheon, the opening session convened at 1:30 P. M. on Monday, May 23rd. This session consisted of four simultaneous seminars as follows:

Seminar A 1

"The Theory of Private Passenger Automobile Merit Rating," combined with a review of the previously presented papers "An Actuarial Note on the Credibility of Experience of a Single Private Passenger Car" (authors Robert A. Bailey and LeRoy J. Simon) and "Some Considerations on Automobile Rating Systems Utilizing Individual Driving Records" (author Lester B. Dropkin). Moderator of A 1: Harmon T. Barber, Second Vice President and Actuary, Travelers Insurance Company.

Seminar B 1

"Practical Aspects of Automobile Merit Rating," combined with a review of the previously presented paper "Merit Rating in Private Passenger Automobile Liability Insurance and the California Driver Record Study" (author Frank Harwayne). Moderator of B 1: William S. Gillam, Research Unit, National Bureau of Casualty Underwriters.

Seminar C 1

"Rate Making and Statistics for Multiple Peril Policies," combined with a review of the previously presented paper "Multiple Peril Rating Problems—Some Statistical Considerations" (author Robert L. Hurley). Moderator of C 1: Ernest T. Berkeley, Actuary, Employers' Group.

Seminar D 1

"Premiums and Reserves on Non-Cancellable and Guaranteed Renewable A & S Policies." Moderator of D 1: John H. Miller, Vice President and Senior Actuary, Monarch Life Insurance Company.

After a 15-minute intermission these seminars were repeated, beginning at 3:00 P.M. For this second session, the seminars had been designated A 2, B 2, C 2 and D 2 respectively. Recess for the day was taken at 4:30 P.M.

The sessions were resumed at 9:40 A.M. on May 24th, President Leslie presiding, with the moderators presenting a report on the activities and discussion at their seminars. At the time of these reports there was additional discussion from the floor together with a question and answer period.

Following the conclusion of this part of the program, President William Leslie, Jr., announced that Past President Dudley Pruitt had informed him that later this year he was retiring from his present position with the General Accident Fire and Life Assurance Corporation, Ltd., to take up new duties in Japan with the American Friends Service Committee. The gathering extended a standing ovation to Dudley in recognition of his devoted services to the C A S for many years.

The gathering then enjoyed a most interesting talk "Casualty Insurance In The Sixties" by Herbert P. Stellwagen, Executive Vice President, Indemnity Insurance Company of North America.

The meeting was then recessed for luncheon with no planned activities for the rest of the afternoon.

A Social Hour, followed by a Banquet was held in the evening of May 24th.

The meeting reconvened at 9:45 A.M. on May 25th with Vice President Longley-Cook presiding. At this time, although there was some overlapping from other sessions including the seminars, the first item was the review of papers presented at previous meetings.

(1) R. A. Bailey and L. J. Simon—"An Actuarial Note on the Credibility of Experience of a Single Private Passenger Car"—Reviewed by W. J. Hazam and also at Seminar A.

(2) L. B. Dropkin—"Some Considerations on Automobile Rating Systems Utilizing Individual Driving Records"—Reviewed by R. A. Bailey and also at Seminar A.

(3) F. Harwayne—"Merit Rating in Private Passenger Automobile Liability Insurance and the California Driver Record Study"—Reviewed by J. H. Muetterties and also at Seminar B.

(4) F. Harwayne—Mr. Harwayne's comments on the foregoing review.

(5) R. L. Hurley—"Multiple Peril Rating Problems—Some Statistical Considerations"—Reviewed by P. M. Otteson and also at Seminar C.

(6) L. L. Tarbell—"Automobile Physical Damage Ratemaking"— Reviewed by C. L. Niles, Jr.

(7) H. W. Steinhaus—"Commutation Functions for Individual Policies Providing for Hospital, Surgical and Medical Care Benefits After Retirement"—Reviews by J. J. Smick and M. Kormes.

(8) H. W. Steinhaus—Mr. Steinhaus' comments on the foregoing reviews.

(9) L. H. Roberts—"Credibility of 10/20 Experience as Compared with 5/10 Experience"—Reviewed by M. Bondy.

(10) J. E. Faust, Jr.—"The Actuarial Aspects of Blue Cross Plans"—Reviewed by M. Kormes.

(11) R. J. Myers—"OASDI Cost Estimates and Valuations"—Reviewed by W. R. Williamson.

(12) D. R. Uhthoff—"The Compensation Experience Rating Plan—A Current Review"—Reviews by R. A. Johnson and R. M. Marshall.

(13) E. S. Allen—Mr. Allen's comments on the foregoing review by Mr. Johnson.

(14) M. G. McDonald—"A Comparison of Auto Liability Experience Under a Compulsory Law and Under Financial Responsibility Laws" —Reviewed by L. W. Scammon.

(15) O. D. Dickerson—"A Probability Analysis of the Safe Driver Insurance Plan," published in the Annals of the CPCU. This item, not on the formal agenda for the meeting, consisted of comments by LeRoy Simon on Professor Dickerson's paper. Those comments will not be printed in the *Proceedings* until Professor Dickerson has had an opportunity to review and discuss Mr. Simon's comments.

Norman J. Bennett, Chairman of Committee on Finances, assisted by Vice President Longley-Cook then reported on the activities of his Committee which is engaged in a study of ways and means for improving the finances of the Society.

Following this report, the following new papers were presented:

(1) L. J. Simon—"Actuarial Notes—The Negative Binomial and Poisson Distributions Compared."

(2) L. J. Simon and R. A. Bailey—"Two Studies in Automobile Insurance Ratemaking:

(A) Effectiveness of Merit Rating and Class Rating.

(B) Improved Methods for Determining Classification Rate Relativities."

Upon conclusion of the presentation of new papers the Spring 1960 meeting of the CAS adjourned at 12 Noon.

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MINUTES OF THE MEETING

November 16, 17 and 18, 1960

STATLER HILTON HOTEL, WASHINGTON, D. C.

The session started at 1:30 P.M. on November 16 with participation by those in attendance in one of the following seminars in accordance with assignments made prior to the meeting:

A—Automobile Merit Rating
Thomas O. Carlson, Chairman
B—Guaranteed Renewable Automobile Insurance
Leo M. Stankus, Chairman
C—Hospital And Surgical Benefits For Persons Age 65—
Private Insurance Or Social Security?
Harold F. LaCroix, Chairman
D—Statistics For Rating And Research
Norman J. Bennett, Chairman

The foregoing seminars were repeated, beginning at about 3:00 P.M. and ending at about 4:30 P.M.

The meeting reconvened at 9:40 A.M. on November 17 with President Leslie presiding and with the following 83 Fellows and 32 Associates in attendance in addition to a number of invited guests:

FELLOWS

ALLEN, E. S.
BAILEY, R. A.
BARBER, H. T.
BENNETT, N. J.
BERKELEY, E. T.
BERQUIST, J. R.
BEVAN, J. R.
BONDY, M.
BORNHUETTER, R. L.
BOYAJIAN, J. H.
BOYLE, J. I.
BYRNE, H. T.
CARLSON, T. O.
COATES, C. S.
CROWLEY, J. H.
CURRY, H. E.
DICKERSON, O. D.
DOREMUS, F. W.
DROPKIN, L. B.
ELLIOTT, G. B.
ESPIE, R. G.
FAIRBANKS, A. V.
FINNEGAN, J. H.
FONDILLER, R.
FOWLER, T. W.
GILLAM, W. S.
GODDARD, R. P.
GRAHAM, C. M.
GRAVES, C. H.
HARWAYNE, F.
HAZAM, W. J.
HEWITT, C. C., JR.
HOPE, F. J.
HUNT, F. J., JR.
HURLEY, R. L.
JOHE, R. L.
JOHNSON, R. A.
KALLOP, R. H.
KLAASSEN, E. J.
KORMES, M.
LACROIX, H. F.
LESLIE, W., JR.
LINDER, J.
LINO, R.
LISCORD, P. S.
LONGLEY-COOK, L. H.
MACKEEN, H. E.
MAGRATH, J. J.
MAKGILL, S. S.
MASTERSON, N. E.
MATTHEWS, A. N.
MCGUINNESS, J. S.
MENZEL, H. W.
MUETTERTIES, J. H.
MYERS, R. J.
NILES, C. L., JR.
OTTESON, P. M.
PENNYCOOK, R. B.
PETZ, E. F.
POLLACK, R.
ROBERTS, L. H.
RODERMUND, M.
ROWELL, J. H.
RUCHLIS, E.
SALZMANN, R. E.
SCHLOSS, H. W.

FELLOWS *(Continued)*

SIMON, L. J.
SIMONEAU, P. W.
SKELDING, A. Z.
SMITH, S. E.
SYKES, Z. M., JR.
TARBELL, L. L., JR.
THOMAS, J. W.
TRIST, J. A. W.
UHTHOFF, D. R.
WIEDER, J. W., JR.
WILCKEN, C. L.
WILLIAMS, P. A.
WILLIAMSON, W. R.
WILLSEY, L. W.
WITTICK, H. E.
WOLFRUM, R. J.
WRIGHT, B.

ASSOCIATES

ABEL, F. E.
ALEXANDER, L. M.
ANDREWS, E. C.
BALCAREK, R. J.
BARRON, J. C.
BLUMENFELD, M. E.
BRANNIGAN, J. F.
BUGBEE, J. M.
BUTLER, R. H.
DEMELIO, J. J.
CRAIG, R. A.
GILLESPIE, J. E.
GROSSMAN, E. A.
HARACK, J.
JONES, N. F.
LATIMER, M. W.
LINDEN, J. R.
MCDONALD, M. G.
MCNAMARA, D. J.
MILLER, N. F., JR.
MOHNBLATT, A. S.
MOSELEY, J.
MUIR, J. M.
RICCARDO, J. F. JR.
RICHARDS, H. R.
ROTH, R. J.
SCAMMON, L. W.
SCHNEIKER, H. C.
STANKUS, L. M.
STERN, P. K.
STEVENS, W. A.
VAN CLEAVE, M. E.

It was announced that the Secretary-Treasurer had been informed of the death of the following subsequent to the previous annual meeting:

Victor Montgomery—President of the Pacific Employers Insurance Co.
William F. Roeber—Retired—Past Vice President of the C.A.S.

The meeting was then placed in charge of Vice President Laurence H. Longley-Cook who gave a preliminary report on the sales of the publication "Fire Insurance Rate Making And Kindred Problems." It was also announced that the next project of the Educational Committee was a publication on automobile rate making.

Messrs. Carlson, Stankus and Bennett made a report on the activities and discussions in Seminars A, B and D respectively. In the absence of Mr. LaCroix, Bob Myers gave a similar report with respect to Seminar C. It is expected that these four reports will be printed in the next volume of the *Proceedings*.

Following the seminar reports William Leslie, Jr., presented his Presidential Address which will appear in Volume XLVII of the *Proceedings*.

In the evening there was a short social hour followed by a banquet. The guest speaker at the banquet was Thomas C. Morrill, Vice President, State Farm Mutual Automobile Insurance Company, whose interesting and stimulating talk was entitled "The Common Denominator." In his talk Mr. Morrill suggested the formation of a "summit council" of fire and casualty insurance industry leaders which could exert the power of its prestige toward the maintenance of the highest standards of conduct which would entitle the insurance industry to the unqualified respect and confidence of the public.

The session reconvened at 9:40 A.M. on November 18 with Mr. Leslie presiding.

The first order of business was consideration of the report of the Secretary-Treasurer relating to income and disbursements during the 12 months ending September 30, 1960. That report is attached hereto.

The President then introduced the following new Associates to the gathering:

J. F. BRANNIGAN
C. F. & M. Actuarial Department
Travelers Insurance Company
700 Main Street
Hartford 15, Connecticut

EDWARD H. BUDD
Travelers Insurance Company
700 Main Street
8 MS Building
Hartford 15, Connecticut

J. J. DEMELIO
Actuary
Home Insurance Company
59 Maiden Lane
New York 38, New York

J. E. GILLESPIE
Continental Casualty Co.
310 South Michigan Avenue
Chicago 4, Illinois

EDWARD J. HOBBS
Actuarial Department
Insurance Co. of North America
1600 Arch Street
Philadelphia 1, Pennsylvania

JOHN R. LINDEN
Actuarial Department
Aetna Casualty & Surety Co.
Hartford 15, Connecticut

JAMES J. MEENAGHAN
Actuarial Division
National Bureau of Casualty Und.
125 Maiden Lane
New York 38, New York

N. F. MILLER, JR.
Actuarial Supervisor
National Bureau of Casualty Und.
125 Maiden Lane
New York 38, New York

ARNOLD S. MOHNBLATT
Accident & Health Department
Royal-Globe Insurance Group
150 William Street
New York 38, New York

R. WILLIS PARLIN
Actuary
Mutual Service Insurance Companies
1919 University Avenue
St. Paul 4, Minnesota

JOSEPH F. RICCARDO, JR.
Actuarial Department
Aetna Casualty & Surety Company
Hartford 15, Connecticut

HARRY R. RICHARDS
C. F. & M. Actuarial Department
Travelers Insurance Company
700 Main Street
Hartford 15, Connecticut

JOHN S. RIPANDELLI
Consulting Actuary
2020 Chuli Nene
Tallahassee, Florida

RICHARD J. ROTH
Asst. Secy. & Manager
Crop-Hail Insurance Actuarial Asso.
209 West Jackson Boulevard
Chicago 6, Illinois

Diplomas were then presented to the following new Fellows:

JAMES H. CROWLEY
Actuarial Department
Aetna Casualty & Surety Company
Hartford 15, Connecticut

O. D. DICKERSON
Associate Prof.
Florida State University
Tallahassee, Florida

J. E. FAUST, JR.
V. P. & Actuary
Universal Automobile Insurance Co.
1000 N. Delaware
Indianapolis, Indiana

JOHN S. MCGUINESS
Casualty Actuary
Bankers Life & Casualty Co.
4444 W. Lawrence Avenue
Chicago 30, Illinois

R. B. PENNYCOOK
Asst. Commissioner
Manitoba Hospital Services Plan
116 Edmonton Street
Winnipeg, Manitoba, Canada

PAUL W. SIMONEAU
Actuarial Department
Aetna Casualty & Surety Co.
Hartford 15, Connecticut

CARL L. WILCKEN
C. F. & M. Actuarial Department
Travelers Insurance Company
700 Main Street
Hartford 15, Connecticut

LYNN W. WILLSEY
Group Department
Travelers Insurance Company
700 Main Street
Hartford 15, Connecticut

The report of the Nominating Committee was read by Seymour Smith. There being no additional nominations from the floor, the meeting proceeded to elect the following:

President	William Leslie, Jr.*
Vice President	Ernest T. Berkeley*
Vice President	Laurence H. Longley-Cook*
Secretary-Treasurer	Albert Z. Skelding*
Editor	Russell P. Goddard*
Librarian	Richard Lino*
General Chairman-Examination Committee	William J. Hazam*
Member of Council	Harold E. Curry
Member of Council	Frank Harwayne
Member of Council	LeRoy J. Simon

*Re-elected.

At various times during the session the following new papers, reviews of new papers and reviews of previous papers were presented:

NEW PAPERS AND REVIEWS THEREOF

(1) The Rating of Crop-Hail Insurance—Richard J. Roth.
Reviewed by William J. Hazam.

(2) The Census Method—A New Approach to the Analysis of Casualty and Property Insurance Statistics for Rate Making—Laurence H. Longley-Cook.
Presented at Seminar D and reviewed by Harmon T. Barber.

(3) The Negative Binomial Applied to the Canadian Merit Rating Plan for Individual Automobile Risks—Charles C. Hewitt, Jr.
Presented at Seminar A.

(4) Automobile Merit Rating and Inverse Probabilities—Lester B. Dropkin.
Presented at Seminar A.

(5) Any Room Left for Skimming the Cream?—Robert A. Bailey.
Presented at Seminar A and reviewed jointly by Jeffrey T. Lange and Robert M. Muniz.

(6) A New Approach to Infant and Juvenile Mortality—Charles C. Hewitt, Jr.

(7) Burglary Insurance—Richard J. Wolfrum and Walker S. Richardson.

(8) Mutiple Coverage Experience Rating Plan—Eldon J. Klaassen.
Reviewed by Luther L. Tarbell, Jr.

Reviews of Previously Presented Papers

(1) Two Studies in Automobile Insurance Ratemaking—Robert A. Bailey and LeRoy J. Simon.
Reviewed at Seminar A by D. B. Martin and Lewis H. Roberts.

(2) Towards Statistically Based Fidelity Rates—Zenas M. Sykes, Jr.
Reviewed by John W. Wieder, Jr.

This completed the program and the meeting was declared adjourned.

Attached: Financial Report of Secretary-Treasurer.
Results of 1960 Examinations.

CASUALTY ACTUARIAL SOCIETY

Cash Receipts and Disbursements
from October 1, 1959 to September 30, 1960

Income			*Disbursements*	
On deposit in Chase Manhattan—October 1, 1959		$5,672.70	Printing & Stationery	$13,259.06
Members Dues	$10,305.00		Postage & Telegraph	8.70
Sale of Proceedings	1,709.00		Secretarial Work	900.00
Examination Fees	1,805.00		Examination Expense	2,304.49
Luncheons & Dinners	2,640.00		Luncheons & Dinners	2,996.55
Interest on Bonds	125.00		Library Fund	21.16
Sale of Reprints	79.25		Insurance	29.45
Michelbacher Fund	943.26		Refunds	190.00
Foreign Exchange	4.90		Miscellaneous	298.43
Registration Fees	1,210.00			$20,007.84
Miscellaneous	.....	18,821.41	On deposit 9-30-60 in Chase Manhattan	4,486.27
Total		$24,494.11	Total	$24,494.11

Assets		*Liabilities*	
Cash in Bank 9-30-60	$4,486.27	Michelbacher Fund	$11,477.47
U. S. Savings Bonds	5,000.00	Deficit	1,991.20
Total	$9,486.27	Total	$9,486.27

* * *

One 12 Yr. U. S. Savings Bond 2½% Series G No. M6,756,060G due for $1,000 on November 1, 1960.

Four 12 Yr. U. S. Savings Bonds 2½% Series G Nos. M7,228,102G-103G-104G-105G due for $4,000 on October 1, 1961.

Employers' Fire Insurance Company Policy No. 31F169622 for $5,000 on Proceedings stored at 200 East 42nd Street, New York, N. Y. and $2,000 on Books kept in New York Insurance Society Library. Expires September 14, 1962.

Surety Bond for $10,000 in the Royal Indemnity Company.

* * *

This is to certify that we have audited the accounts, examined all vouchers and investments shown above and find same to be correct.

(Sgd.) *H. G. Crane*
Chairman, Auditing Committee

October 19, 1960

1960 EXAMINATIONS—SUCCESSFUL CANDIDATES

Following is a list of those who passed the examinations held by the Society on May 19 and 20, 1960:

ASSOCIATESHIP EXAMINATIONS

PART I	Abbey, W. P.	Honebein, C. W.	Nields, N. B.
	Aldrich, W. C.	Horowitz, M.	Oien, R. G.
	Bacher, W. C.	Jensen, J. P.	Perreault, S. L.
	Baumwart, N.	Kilbourne, F. W.	Portermain, N. W.
	Blaha, J. M., Jr.	Lewis, A. L.	Reilly, F. V.
	Brown, L. E., Jr.	Masterson, W. E., Jr.	Rubin, R. H.
	Carson, D. E. A	McClure, R. D.	Scheel, P. J.
	Cassel, D. L.	Mokros, B. F.	Scheibl, J. A.
	Chao, B.	Morison, G. D.	Smith, C. P.
	Denisoff, B. A.	Morrison, D. I.	Switzer, V. J.
	Ferden, S.	Muniz, R. M.	Taft, R. L.
	Garrett, S. B.	Nelson, H.	Trees, J.
	Gerundo, L. P., Jr.		Trudeau, D. E.
	Hillhouse, J. A.		Walsh, A. J.
PART II (a)	Aldrich, W. C.	Gerundo, L. P., Jr.	Muniz, R. M.
	Bogue, J. L.	Gould, D. E.	Rubin, R. H.
	Cassel, D. L.	Horowitz, M.	Scheel, P. J.
	Crandall, W. H.	Jensen, J. P.	Taft, R. L.
	Curry, A. C.	Kaminoff, H.	Trudeau, D. E.
	Ferden, S.	Morison, G. D.	Walsh, A. J.
PART II (b)	Aldrich, W. C.	Garrett, S. B.	Muniz, R. M.
	Batista, S.	Gerundo, L. P., Jr.	Oien, R. G.
	Cassel, D. L.	Morison, G. D.	Rogers, D. J.
	Ferden, S.		Trudeau, D. E.
PART III (a)	Cima, A.	Leinwand, H.	Scheel, P. J.
	Crandall, W. H.	Maidanick, C. I.	Singer, P. E.
	Gibson, J. A.	Miller, P. V.	Smith, C. P.
	Gould, D. E.	Morison, G. D.	Walsh, A. J.
	Horowitz, M.	Muniz, R. M.	Webb, J. C.
PART III (b)	Bartik, R. F.	Ehlert, D. W.	Nelson, R. E.
	Burney, C. T.	Gould, D. E.	Rubin, R. H.
	Carson, D. E. A.	Meenaghan, J.	Selig, J. G.
	Davidson, W. G.	Miller, P. V.	Singer, P. E.
	DeMellio, J. J.	Muniz, R. M.	Smith, E. R.
	Durkin, J.		Thompson, P.

PART IV	Brannigan, J. F.	Lofgren, P. G.	Parlin, R. W.
	Budd, E. H.	Lorman, W. E.	Piersol, D. E.
	Crandall, W. H.	Meenaghan, J.	Riccardo, J. F., Jr.
	Ehlert, D. W.	Miller, N. F., Jr.	Richards, H. R.
	Even, C. A., Jr.	Mohnblatt, A. S.	Riddlesworth, W. A.
	Greene, T. A.	Naffziger, J. V.	Ripandelli, J. S.
	Linden, J. R.	Nagel, J. R.	Young, R. G.

FELLOWSHIP EXAMINATIONS

PART I	Gillespie, J. E.	Hobbs, E. J.	Meenaghan, J.
PART II	Balcarek, R.	Fitzgibbon, W. J., Jr.	Leight, A. S.
	Budd, E. H.	Hobbs, E. J.	McGuinness, J. S.
	Craig, R. A.	Latimer, M. W.	Pennycook, R. B.
	Faust, J. E., Jr.		Stankus, L. M.
PART III	Brannigan, J. F.	Dickerson, O. D.	McClure, R. D.
	Crowley, J. H., Jr.	Fitzgibbon, W. J., Jr.	Wilson, J. C.
PART IV (a) and (b)	Crowley, J. H., Jr.		
*PART IV (a)	Pennycook, R. B.	Simoneau, P. W.	Willsey, L. W.
		Wilcken, C. L.	
*PART IV (b)	Alexander, L. M.		

*Credit for other section previously granted.

NEW ASSOCIATES

The following 13 candidates, having been successful in completing the examinations, will be admitted as Associates of the Society as of the date of the Annual Meeting in November 1960:

Brannigan, J. F.	Hobbs, E. J.	Parlin, R. W.
Budd, E. H.	Linden, J. R.	Riccardo, J. F., Jr.
DeMelio, J. J.	Meenaghan, J.	Richards, H. R.
Gillespie, J. E.	Miller, N. F., Jr.	Ripandelli, J. S.
	Mohnblatt, A. S.	

NEW FELLOWS

The following 8 Associates, having been successful in completing the examinations, will be admitted as Fellows of the Society as of the date of the Annual Meeting in November 1960:

Crowley, J. H., Jr.	McGuinness, J. S.	Simoneau, P. W.
Dickerson, O. D.	Pennycook, R. B.	Wilcken, C. L.
Faust, J. E., Jr.		Willsey, L. W.

EXAMINATION FOR ENROLLMENT AS ASSOCIATE

PART I GENERAL MATHEMATICS

The questions for Part I were prepared and copyrighted by the Educational Testing Service of Princeton, N. J., and cannot be reprinted. Students may obtain a set of similar questions from the Secretary-Treasurer.

PART II SECTION (a)

1. (a) If the letters of the word CASUALTY are arranged at random, what is the probability that there will be exactly two letters between the C and the U?

   (b) A closet is 2 feet by 3 feet and is floored with boards 2 inches wide, the boards running the long way. A penny (diameter $\frac{3}{4}$ inch) is dropped on the floor of the closet. What is the probability that it does not rest on a joint in the floor boards?

2. (a) If $P\{A\} = \frac{1}{4}$, $P\{B\} = \frac{2}{3}$, and $P\{A + B\} = \frac{5}{6}$, find the value of $P\{B/A\}$.

   (b) A variate has the probability density: $f(x) = 0$ for $x < 0$; $f(x) = (x-1)^2$ for $0 \leqslant x < 1$; $f(x) = 0$ for $x \geqslant 1$. Find the expected value of x and the standard deviation of x.

3. (a) If, on the average, one automobile in ten is involved in some type of accident in one year, what is the probability that, if ten cars are selected at random, two or less will be involved in an accident in one year? The answer may be left in the form of powers of numbers.

   (b) Six coins are tossed and n of them fall heads. These n coins are tossed again and two fall heads. What is the probability that n equals four?

4. (a) A, B, and C each contribute \$0.91 to a \$2.73 stake, all to be won by the first to throw an ace with a single die. If they throw in the order ABC, what net profit or loss should be expected by each player?

(b) Nine cards are drawn at random from a set of cards. Each card is marked with one of the numbers 1, 0 or –1, and it is equally likely that any of the three numbers will be drawn. Find the chance that the sum of the numbers thus drawn is –6.

EXAMINATION FOR ENROLLMENT AS ASSOCIATE

PART II SECTION (b)

1. (a) Find the median, the quartiles and the quartile deviation of the distribution:

Class Mark	35	45	55	65	75
Frequency	3	10	17	8	2

(b) Show that the second moment about the mean is equal to the second moment about the origin minus the square of the first moment about the origin.

2. (a) If for a normal distribution $N = 200$, $\mu = 60$, $\sigma = 15$, how many values lie between 70 and 75?

(b) A physician treats 100 patients suffering from a certain disease and 42 of them die. The mortality rate in this disease, based on thousands of reported cases, is 36%. Can the sample be regarded as exceptional? Why or why not?

3. (a) If θ is the probability of success on a single trial and S is the number of trials, prove that the Poisson distribution is a good approximation to the binomial distribution if θ is small, S large, and $S\theta$ is a number of the order of unity.

(b) The mean fire loss sustained in a simple random sample of fifty losses was \$9,500 with a standard deviation of \$200. What are the 95% confidence limits for the mean fire loss?

4. (a) A trend line on semi-log paper is a straight line passing through the points (1, 1) and (3, 100). What is the equation of this functional relation?

(b) The coefficient of correlation between two sets of data X and Y has been computed and found to be .98. The X values have a mean of 100 and a standard deviation of 10. The Y values have a mean of 25 and a standard deviation of 6. Find the equation of the regression line.

TABULAR VALUES AS MAY BE NEEDED FOR QUESTIONS ABOVE

Z	Area of the Normal Curve
.33	.12930
.67	.24857
1.00	.34134
1.25	.39435
1.96	.47500

EXAMINATION FOR ENROLLMENT AS ASSOCIATE

PART III SECTION (a)

The solutions to all problems should be expressed in symbolic form.

1. An individual aged 25 pays \$5,000 to a life insurance company. The company agrees that (a) if the individual should die before age 65, the company shall pay to his heirs the \$5,000 plus interest accumulated to the date of his death; (b) if he lives to age 65, the company will pay to him a certain amount at the beginning of each year as long as he lives. Find the annual payment beginning at age 65.

2. A man aged 25 agrees to pay to a life insurance company \$1,000 at the beginning of every year for 20 years or until prior death. In return, the life insurance company agrees to pay a sum of money every year as long as the man is alive with the first payment due at age 60. Find the amount of the annual payment to be made by the company. Express your answer in terms of commutation symbols.

3. Prove the identity

$$ {}_{n|t}\ddot{a}_x = \frac{{}_n u_{x+t} \cdot {}_{2n+t}| \ddot{a}_x}{{}_n| \ddot{a}_{x+n+t}} $$

4. Show that the t^{th} terminal reserve for an ordinary life policy issued at age x can be written in the form:

$$ \frac{A_{x+t} - A_x}{1 - A_x} $$

5. A 30-pay life insurance policy provides for a death benefit of \$20,000 if death occurs within 20 years and a death benefit of \$10,000 if death occurs after 20 years. What is the net annual premium for this policy at age 25? Express your answer in terms of commutation symbols.

6. Express in commutation symbols the prospective reserve for the tenth year for a 15-pay life \$1,000 policy issued at age 30.

7. Determine the approximate bi-monthly (every second month) net level premium for an ordinary life insurance policy of \$1000 issued at age x. Express your answer as a formula involving $\ddot{a}_x$ and A_x.

8. Prove that $\ddot{a}_x < \frac{1}{iv}$

9. Given: $5(u_x) = 51(k_x)$ and $i = .02$
   Find p_x.

EXAMINATION FOR ENROLLMENT AS ASSOCIATE

PART III SECTION (b)

1. (a) What are the three basic differences between Willet's concept of "static" and "dynamic" types of loss?

(b) What are the essential features of economic insurance as defined by Willet?

2. (a) Briefly distinguish between an open end and closed end investment company.

(b) Briefly describe the process of "Factoring" as a method of short term financing.

3. (a) Describe the Law of Negligence.

(b) What lines of insurance did it give rise to?

(c) Where was it found to be inadequate and what was done to remedy the situation?

4. (a) Describe the law of diminishing utility and explain its influence on the reluctance to incur risk.

(b) Define speculative risks and pure risks and give an example of each.

5. (a) With bond tables available, what information must be known in order to determine the yield of a bond?

(b) What accounting procedure has been adopted by life insurance companies to eliminate the effect of market price fluctuations of bonds?

EXAMINATION FOR ENROLLMENT AS ASSOCIATE

PART IV

SECTION (a)

1. What are the components of the two basic classes for male operators under 25 years of age in the National Bureau of Casualty Underwriters Private Passenger Classification Plan effective in most states?

2. When a hull policy is written in Aviation Insurance, what coverages may be included on a named perils form?

3. What unusual provisions as respects policy term and minimum premium apply under an all risk Manufacturers' Output Policy?

4. What is the difference between a "three fourths value clause" and a "three fourths loss clause" in a fire insurance contract?

5. What is a "liberalization clause" in a fire insurance contract?

6. What four basic costs of repair are included under the comprehensive glass policy?

7. In ocean marine insurance what is the difference between an "actual total loss" and a "constructive total loss"?

8. What are the four basic divisions or classes of Inland Marine Insurance?

9. For the following forms of public liability insurance name the measure of exposure and unit of exposure used in premium determination.

   (a) Hospital Professional — name 2
   (b) Druggists Liability — name 1
   (c) Owners', Landlord and Tenants — name 3
   (d) Owners' or Contractors' Protective Liability — name 1

10. Describe the difference between an "occurrence" and "accident" as used in general liability insurance. Give an example of an occurrence which may result in an incurred loss.

11. Briefly define the following terms as utilized in Workmen's Compensation Insurance:

   (a) Governing Classification
   (b) N.O.C.
   (c) N.P.D.

12. An insured has a son in college, and some of his clothing is destroyed by a fire in his dormitory. Is this covered under the insured's personal property floater? If so, how much would be paid for such a loss if the policy was issued with a $25 deductible clause and the value of the damaged clothing is $350?

13. A $10,500 judgment is rendered, for injury to one person, against an insured who has automobile liability insurance with $10,000/$20,000 limits. The attorney's bill and other legal expenses total $1,200. For what amount is the company liable? Explain.

14. In proving a claim under a residence and outside theft policy, is it necessary in every case to show evidence of the residence having been broken into? Explain.

15. Name the uninsurable items listed in the standard fire policy.

16. Name 5 perils covered under the additional extended coverage endorsement.

17. What is the fixed order of settlement of loss in a boiler and machinery policy?

18. What remedy, if any, under a workmen's compensation policy, is afforded an injured workman if the employer becomes insolvent?

19. Under what conditions may an insured under auto bodily injury insurance "except at his own cost, voluntarily make any payment"?

20. Define subrogation.

SECTION (b)

1. (a) What tests should a good classification system meet?

   (b) What are the two major qualifications an exposure medium should possess as discussed in Dorweiler's paper "Notes on Exposure and Premium Bases"? Discuss each briefly.

2. (a) Given the following information from a workmen's compensation rate revision:

Policy year collectible loss and loss expense ratio	.780
Permissible loss and loss expense ratio	.650
Change in correction for off balance	1.010
Calendar year loss ratio	.702

   Incorporating the methods described in the addendum to Marshall's article, find:

   (i) Indicated change in collectible level

   (ii) Policy year indicated change in manual rate level

   (iii) Rate level adjustment factor

   (iv) Change in manual rate level

   (b) What purpose is served by loss and expense constants in workmen's compensation insurance? How is the additional premium charge for this provision reflected in the development of statewide rate level changes under the procedure utilized by the National Council on Compensation Insurance?

3. (a) What is the purpose of an earned factor as used in automobile liability insurance ratemaking? From the following data calculate the average earned factor:

Policy Year	*Pure Premium*	
	12 Months	*Final*
1955	$5.00	$10.00
1956	6.70	13.40
1957	8.40	14.00

(b) Compare the advantages of calendar-accident year statistics with policy year statistics as a basis for ratemaking.

4. (a) In arriving at a fire insurance rate, the loss ratios were adjusted to prevent the experience of a single year from increasing or decreasing the final rate more than 10.0%. Derive a formula for the maximum and minimum annual loss ratios which effectuates this limitation. Using a "permissible loss ratio" of 47.5% and a class credibility of 90.0%, calculate the maximum and minimum annual loss ratio.

(b) What effect would the elimination of the "co-insurance" provision in fire insurance have on the present rate structure with respect to the adequacy of rates for various limits of coverage?

5. (a) Explain how statistics other than insurance experience statistics are used in rate revisions for auto physical damage and glass insurance.

(b) What two sources of loss funds, as described by Crist, are available for the payment of losses in suretyship?

(c) What unique problem is present as respects the experience to be utilized in surety ratemaking?

EXAMINATION FOR ENROLLMENT AS FELLOW

PART I

SECTION (a)

1. What general recommendations were contained in the Casualty and Surety Rate Regulatory Bill (All-Industry Bill) sponsored by the NAIC for:
   (a) Expense provisions
   (b) Classification rates and modifications thereof
   (c) Recording and reporting of loss and expense experience

2. (a) Distinguish between "Domestic," "Alien," and "Foreign" insurers as defined in Article I Section 4 of the Insurance Law of the State of New York.
   (b) Specify the differences, if any, in the license issued by the State of New York to "Domestic," "Alien," and "Foreign" Insurers.
   (c) What limitations of risk, in general, are imposed by the State of New York upon "Domestic," "Alien," and "Foreign" Insurers?
   (d) What are the differences, if any, in the licensing of Reinsurance Companies compared to Direct Insurance Companies in the State of New York?

3. Compare the regulation, if any, of policy forms for Workmen's Compensation, Automobile Liability, Liability other than Automobile and Inland Marine.

4. It has been stated that reasonable competition has not been fully permitted under state regulation since the McCarran Act, in that there are limitations imposed by state regulation on the ability of an independent carrier to reduce rates for the benefit of customers. Discuss the accuracy of this viewpoint, with particular regard to provisions in the various rate regulatory laws.

5. Discuss the divisibility of the insurance contract with respect to misrepresentation in a multiple line contract with a single gross premium such as Homeowners.

6. Given the following amounts that have been adjusted to an "income tax" basis:

(a)	Net written premiums less dividends declared to policyholders	$100,000,000
(b)	Net Capital Gains	0
(c)	Wholly exempt interest earned	100,000
(d)	Underwriting profit less dividends declared to policyholders	15,000
(e)	Gross investment income earned	3,000,000
(f)	Net investment income earned	2,450,000
(g)	Effective U. S. income tax rates:	
	Normal tax	30%
	Surtax (on excess of $25,000)	22%

What is the total U. S. income tax incurred on the year's operation (before foreign tax credit) if taxed as a Stock Fire & Casualty Company; if taxed as a Mutual Fire & Casualty Company?

SECTION (b)

1. Evaluate the need of Assigned Risk Plans for each of the following lines of insurance:
   (a) Workmens Compensation
   (b) Automobile Liability
   (c) Liability other than Auto
   (d) Fire Insurance
   (e) Personal Jewelry Floater Policies

2. Briefly discuss the equity of the present distribution of assignments of automobile assigned risks wherein rates and coverages vary among carriers.

3. There are several basically different systems through which a state may assure coverage of wage earners by cash sickness insurance. How may these systems be classified and briefly describe how each system works.

4. One of the arguments against a system of contracting-out such as is allowed under the Disability Insurance laws in California and New Jersey is that there is adverse selection against the state fund. Describe three methods by which this can be and is prevented.

5. Under the provisions of the Social Security Act, grants-in-aid are made to states having old-age assistance laws approved by the Secretary of Health, Education and Welfare. What are the more important provisions that a state program must have in order to be approved?

6. In PCAS Volume XLII, N. Gaines defines "h" (i.e., the probability of a worker unemployed at the beginning of an interval being hired at least once sometime before the end of the interval) as equal to $\frac{U_o - U_t}{U_o}$ where U_o represents the number of workers unemployed at the beginning of the interval and U_t is the number of workers continuously unemployed up to the point in time "t."

   Show that "h" equals $(1 - e^{-rt})$ where:
   "e" equals 2.72 (approx.)
   "r" is a constant

7. Discuss the effects that "Long-duration" unemployment might have on the present programs for unemployment insurance.

EXAMINATION FOR ENROLLMENT AS FELLOW

PART II

SECTION (a)

1. A workmen's compensation policy is written for an estimated annual premium of $12,000 with interim audits to be made quarterly. Develop the unearned premium on this policy at the end of the fourth month where the deposit premium charge is 50% of the estimated annual premium and the first interim audit develops $4000 of premium. Explain each step of your method.

2. Outline three methods by which a reserve for retrospective rating refunds may be established.

3. Develop a formula for a tabulating machine calculation of the unearned premium reserve using semi-monthly pro rata factors, using the following symbols:

F = semi-monthly pro-rata factor

P = premium in force

U = unearned premium

V = valuation month

M = expiration month

Y = expiration year

C = valuation year

T = term in years

4. What are the purposes of Schedule "O" Part 1 and Schedule "P" Parts 5, 5A, and 5B of the Fire and Casualty Annual Statement Blanks? Specify the lines of coverage treated in each schedule.

5. A company processes auto bodily injury claims by delaying these cases in the field for 30 days in order to allow its adjusters time to set up more accurate claim estimates. How may the delay in reporting to the home office be reserved for and where in the Annual Statement should these amounts be shown? Explain.

6. Compare the reasonableness of establishing a formula loss reserve for initial evaluations (i.e., cases reported within the latest six months period) on Fire and Workmen's Compensation losses.

7. (a) Explain the "Case Basis" approach for establishing a reserve for allocated loss adjustment expense and give a method by which reserves set in this manner may be checked.

(b) Comment on the following formula for computing the reserve for unallocated claims expense reserve on automobile liability outstanding losses:

$$\text{Reserve} = L_i \cdot \frac{\sum_{n=1}^{5} U_n}{\sum_{n=1}^{5} P_n} - F_i$$

where

L_i = total loss and unallocated loss expense incurred in each year (i) of the four latest policy years

$\sum_{n=1}^{5} U_n$ = sum of the unallocated loss expense paid on the five latest policy years

$\sum_{n=1}^{5} P_n$ = sum of the losses plus allocated loss expense paid on the five latest policy years

F_i = Unallocated loss expense paid on each year (i) of the four latest policy years as distributed by the schedule "P" factors in the Annual Statement Blank.

SECTION (b)

1. The following data for Company X show calendar year 1959 summary totals with all reserve items valued as of December 31, 1959, unless otherwise noted. In the computations involving these data, it will suffice to identify the particular items by the given numbers (e.g., "2" for Loss adj. exp. res.).

1.	Loss Reserve	$20,000,000
2.	Loss adjustment expense reserve	3,000,000
3.	Excess of liability statutory reserves over case basis	500,000
4.	Paid losses	16,000,000
5.	Cash	1,000,000
6.	Written premiums	32,000,000
7.	Uncollected premiums	3,700,000
8.	Interest due & accrued	300,000
9.	Cash dividends to stockholders	1,000,000
10.	Unearned premium reserve	13,000,000
11.	Reserve for investment fluctuations	2,000,000
12.	Ceded reinsurance balances payable	200,000
13.	Real estate	5,000,000
14.	Capital paid up	1,000,000
15.	Reinsurance recoverable on loss payments	200,000
16.	Bonds	30,000,000
17.	Tax reserve	400,000
18.	Unassigned funds	?
19.	Incurred taxes	500,000
20.	Net investment income	1,000,000
21.	Loss adjustment expense incurred	3,000,000
22.	Underwriting expense reserve	600,000
23.	Net realized capital losses	500,000
24.	Incurred underwriting expense	5,000,000
25.	Increase in agents' balances over three months due (current year end over previous year end)	100,000
26.	Stocks	10,000,000
27.	Loss reserve December 31, 1958	19,000,000
28.	Unearned premium reserve December 31, 1958	11,000,000

Develop the schedule of Assets, Liabilities, Surplus and Other Funds as required on pages 2 and 3 of the Annual Statement Blank.

2. Using the data in question 1, compute the net addition to surplus during the calendar year 1959.

3. Distinguish between the following insurance accounting terms:

   (a) Gross Premium and In-Force Premium
   (b) Account Current and Installment Premiums
   (c) Non Ledger and Non Admitted Assets
   (d) Direct & Net Paid Losses per Part 3 of Annual Statement

4. What general ledger entries by a ceding company would be made for the following monthly transactions: $100,000 of net premiums are ceded by the company to its authorized reinsurer at a commission rate of 40% with 30% of the net premiums withheld, and $50,000 is paid on reinsured claims with $25,000 paid by the reinsurer out of funds held on deposit by the company. Indicate whether each item is an asset, liability, income or disbursement.

5. Many companies analyze their underwriting results by relating certain items of expense to written premiums and other items to earned premiums. Under this system, which are related to written premiums and which to earned and for what reasons?

6. Comment briefly on each of the following observations:

   (a) Uniform Accounting is a misnomer — its objectives are neither to control the accounting procedures of the carriers nor to establish uniform methods for Insurance Accounting.
   (b) The "Uniform Accounting Regulations" will stimulate competition among insurance companies.

7. Under Uniform Accounting Regulations how should the following items of expense be assigned by operating expense classifications:

(a) Company's share of cost of operations of Underwriting pools.
(b) Independent adjusters' fees.
(c) Policy and membership fees paid to agents.
(d) Electricity, water and heat.
(e) Company cafeteria.
(f) Travel expenses allowed to managers, agents and brokers.
(g) Rentals of automobiles and equipment.
(h) Costs of moving company offices.
(i) Advertising as required by law.
(j) Workmen's compensation insurance.

The operating expense classifications are:

1. Claim adjustment services
2. Commission and brokerage
3. Allowances to managers and agents
4. Advertising
5. Boards, bureaus and associations
6. Surveys and underwriting reports
7. Audit of assureds' records
8. Salaries
9. Employee relations and welfare
10. Insurance
11. Directors' fees
12. Travel and travel items
13. Rent and rent items
14. Equipment
15. Printing and stationery
16. Postage, telephone and telegraph, exchange and express
17. Legal and auditing
18. Taxes, licenses and fees
19. Real estate expenses
20. Real estate taxes
21. Miscellaneous

EXAMINATION FOR ENROLLMENT AS FELLOW

PART III

SECTION (a)

1. What are the three functions or purposes of individual risk rating?

2. What are the four basic requirements of an Individual Risk Rating Plan?

3. With respect to the Multiple Location Rating Plan:

   (a) Who may be insured?
   (b) What are the minimum location requirements?
   (c) How are miscellaneous locations included in the calculation of the basic annual rates?

4. (a) Give the formula for determining the modification factor under the Workmen's Compensation Experience Rating Plan.

   (b) What is the function of the B factor?

   (c) What is the function of the W factor?

5. Briefly describe a practical and flexible procedure for computing excess loss ratios by use of loss distributions comprised of the total incurred loss for each claim as reported under the National Council Unit Statistical Plan.

6. Outline the procedure applicable for experience rating of risks written on a deductible basis under the Automobile Liability Experience Rating Plan of the National Bureau of Casualty Underwriters in determining:

   (a) Actual losses

   (b) Experience Loss Ratio, Credibility and Maximum Loss

   (c) Increased Limits

7. The National Bureau of Casualty Underwriters' Composite Rating Plan for Automobile Liability, Burglary, General Liability and Glass Insurance sets forth two different methods for the computation of the composite rate. Explain in detail these two methods for computing the composite rate.

8. What are the essential differences between the Boiler and Machinery Premium Adjustment Rating Plan and Retrospective Rating Plan D?

SECTION (b)

Answer any 5 of the questions numbered 1 through 6

1. A risk complains that the manual rate for the classification under which he pays compensation premium has been increasing even though the actual loss ratio for this class has been low recently. You investigate and find that this classification is non-reviewed. How do you explain the increase to the risk? What reasons do you advance why no credence can be placed on the experience of this classification?

2. A year ago the National Bureau of Casualty Underwriters and the National Automobile Underwriters Association introduced an experimental plan to reward the good driver.
   (a) How does this plan differ from previous merit rating plans?
   (b) What is the theory behind this plan?

3. A bill before Congress last year would add to the existing Social Security Program a plan for hospital, nursing home, and surgical care covering all individuals receiving or eligible to receive retirement or survivor benefits.
   List four reasons why this measure was opposed by the insurance industry.

4. Describe how the Nuclear Energy Liability Insurance Association and the Mutual Atomic Energy Reinsurance Pool permit maximum use of the underwriting capacities of their members. How have they avoided situations which might arise in which the liability of the companies might greatly exceed their capacity?

5. Summarize the impact that multiple line underwriting has had and will have on the capital structure of insurance companies.

6. Explain in detail how reinsurance would reduce the strain on surplus caused by a rapid growth of premiums in a fire insurance company.

EXAMINATION FOR ENROLLMENT AS FELLOW

PART IV

SECTION (a)

Answer any 7 of the questions numbered 1 through 8.

1. The Health Insurance Association of America Statistical Plan has been designed for use in experience studies under Accident and Health and Hospital and Medical policies by companies writing either uniform (commercial type) or level premium policies or both. The particular items a company decides to incorporate in its plan will be influenced to some extent by the procedure it establishes for drawing off exposures. Illustrate.

2. An objector to a workmen's compensation rate filing refers to compensation underwriting results for stock carriers in Best's Fire and Casualty Aggregates and Averages. As insurance department actuary, do you think these results should have any bearing on the propriety of the proposed rates? Why?

3. The original plan for the reporting of classified experience for fire insurance called for a classification of losses and amounts at risk.

   (a) How does the current statistical plan of the National Board of Fire Underwriters differ from the original?

   (b) Why were these changes instituted?

4. With regard to the statistical plan for Homeowners experience:

   (a) How shall the premiums be reported?

   (b) How are deviations treated?

   (c) In what manner will the Calls require the reporting of losses?

5. Where would you obtain information on the following:

   (a) Commodity price indices

   (b) Comparative international statistics — vital statistics, economic indices, mineral products, etc.

   (c) Wage adjustments made during the year by a specified company

   (d) Gross hours and earnings of production workers by industry

   (e) Labor turnover rates

   (f) Estimated sales of durable goods stores

6. (a) What five components do all computers have?

   (b) There are only two types of computers. Name them and indicate what each type does. Which type is used by the insurance industry?

7. Once electronic data processing equipment has been fully harnessed and organizations realigned to make use of their capabilities most effectively, what results will be achieved?

8. Comparisons of the cost of sorting punched cards with the corresponding cost of sorting on magnetic tape frequently show that tape sorting is more costly. However, it can be very misleading to consider the sorting operation by itself. Give six other factors which bear on the method of sorting to be used.

SECTION (b)

Answer any 7 of the questions numbered 1 through 8.

1. Comment on the following observation:

   It is as necessary to base rates on the record of prior years expense experience under a uniform classification of functional expenses by line of coverage, as it is to base rates on prior loss experience of risk classifications within the various lines of coverage.

2. In Multiple Line ratemaking, the drafters of what is now known as the Commercial Property Coverage faced a situation where a package of coverage for mercantile occupancies included not only existing coverages, for which rates were available, but also included a broad form of coverage for which there was no established rate. Describe how rates were established for the Commercial Property Coverage.

3. (a) What are the major differences in the rating procedure of the Factory Mutuals as compared with that of other organizations?

   (b) The rating plan of the Factory Mutuals has been developed in order to meet the specialized rating problems of its member companies.

   What are these rating problems?

4. In a recent paper in the Proceedings, a procedure, which is used by bureaus operating in midwestern states for revising rates applicable to a class of property fire insurance, was described. Describe how loss ratios which are used in the rating formula are obtained.

5. (a) For many types of surety bonds the duration of the risk affects the results the surety company may expect. Explain.

   (b) Does the duration of risk affect the earned premium on a surety bond? Explain.

   (c) How is this question of duration of the risk handled in the rate structure of most construction contract bonds?

6. In determining the extent to which individual company loss experience can be used as a basis for hospital therapeutic benefits rates, consideration must be given to the effectiveness of such loss experience. Do you believe hospital therapeutic benefits as well as most other group accident and health coverages represent business on which statistics are unusually effective? Give reasons.

7. Your company is planning on entering the noncancellable health and accident field. As actuary, what factors will you consider before adopting an experience table to be used for ratemaking?

8. Discuss the merits of a wage factor in workmen's compensation ratemaking.

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