ing—with a pencil never far from hand. The fundamental ideas are succinctly presented. As the argument unfolds, more difficult concepts are introduced and the algebra becomes somewhat rigorous. The reviewer spent a goodly number of hours on the simplifications in the area of equations (26) through (33) before arriving at the indicated formulas. In general, the notation possesses an inner consistency and a degree of elegance that make the mathematical reasoning a delight for the reader.

In any pursuit founded, as insurance, on statistical science, the more frequently elements significantly deficient in respect to mathematical precision are introduced into the rating procedures the more obscure the logical inter-relationships and the less defensible the procedures—on purely statistical grounds. Few actuaries, we trust, would force this observation to mean that insurance rates are always reducible to set equations. Most practitioners in our profession soon learn that there are seldom mathematical transforms which will automatically turn the specific rating problem into a trim statistical equation. Our theoretical investigations must be counted as successful if they quicken our insight into the noumenal of the insurance transaction. We are fortunate that with Mr. Simon's paper, our *Proceedings* will contain a scholarly research into the inter-relationships underlying the loss ratio method of adjusting rates.

AUTHOR'S REVIEW OF DISCUSSION

LEROY J. SIMON

I appreciate having Mr. Hurley review the paper because I know it represents a thorough and unbiased consideration. While he and I both use the fire insurance business as the principal source of our examples, I know we both agree that the formulas presented in the paper are quite general and may be used in any line of insurance. Wherever rate revision adjustment factors are used, there is no reason to use anything other than the proper formula. To do otherwise is to voluntarily introduce an element of inadequacy into the rate structure.

The factors developed in the paper relate to written premiums only. The preferable way to adjust experience to current rates is to apply these factors to the written premium first and then convert the adjusted premiums to an earned basis. In the fire insurance line, a striking example of the error of reversing the order of this process is given in this volume of the *Proceedings* in Note 7 of the paper "Notes on Some Actuarial Problems of Property Insurance" by L. H. Longley-Cook.

The only difference in Mr. Hurley's conclusions and mine appears to be a matter of degree. He states that he is "not distressed" with the element of inadequacy that is introduced by using the incorrect formula; he is not displeased with the fact that fire insurance rate equities "attain a rough, frontier-type of justice"; and he observes

that it may be a little while before fire insurance rates can be made "to a fine degree of statisical precision". As we encounter increasing rate competition in each of the insurance lines and as we find tighter and tighter rate regulation, we are being forced to eliminate any loose techniques wherever we find them. When we speak of a 1¼% inadequacy in an overall rate structure, I feel that we must be gravely concerned because this represents a full one-fourth of a 5% theoretical profit loading in the rate. If we look back at the actual profit realized over the past few years, any unnecessary bias that produces a consistent inadequacy takes on an even greater importance. Even in the fire insurance industry where rate making methods are perhaps in their most elementary stage, such an improvement is one of a number of steps forward that must be made. Remember, we might be dealing with the overall rate level in a given state and slight errors might result in many thousands of dollars in their effect. Perhaps, through making simple refinements like this we can further improve the accuracy of our rate making in many lines and accelerate the introduction of better actuarial methods into such lines as fire insurance.

In the six-month interval from the time the paper was originally presented I have had a few additional thoughts which may be of help to those who have had the interest and perseverance to read through the paper, the review, and now the reply. On page 199 in the original paper, some interpretations are made of the values found in Appendix B. Rather than stating that the error is equal to $(1 - C)$, it would be better to calculate the amount of error to be equal to $(1 - 1/C)$. In this way we could then say that the true amount multiplied by 1 plus this error factor will produce the incorrect answer. This is the more usual interpretation we place upon the concept of a percentage of error.

To overcome the distortions referred to in the footnote on page 200, there are three methods available. The first of these is to use the full term reporting method under which term business paid on an installment plan is recorded on the company books as a single entry at the inception of the policy. This is the method advocated by the writer of the article referred to in the footnote. The second method uses the annual reporting system where installments are recorded each year as they fall due, but the amount of "surcharge" in the first installment is entered for the full term of the policy. This method is explained by Dudley M. Pruitt in a paper entitled "Unearned Premium Reserve on Fire Installment Premium Policies", which appeared in The Interpreter (the monthly publication of the Insurance Accounting and Statistical Association) for August 1951. Another method also based on the annual reporting system is covered by Paul Otteson in the Proceedings of the Insurance Accounting and Statistical Association for 1951, page 352. This method does not require the use of a full term premium tabulating card entry for the "surcharge" but it does require more coded information to determine which installment payment of the series is being considered. When the first of these three methods

is used, equation (6) is appropriate in dealing with rate revision adjustment factors (under the assumption of a level premium volume). However, when either Mr. Pruitt's or Mr. Otteson's formula is used, the more complex equation (14) would be the starting point for installment business.

Equation (37), which sets forth a formula for comparing rate levels between two different organizations, can also be used to good advantage to determine the value of "d" itself, which is used extensively throughout the earlier equations in the paper. In the denominator of equation (37) there is a ratio of the Bureau rate divided by the company rate. If this ratio is replaced by the old rate divided by the new rate, we then have a formula for determining the average rate level change. Notice that the weights used in this equation are based upon premium volume and not upon exposure units. (Remember that if exposure units are available, one would simply extend the exposures at old rates and then extend them at new rates and make the comparison in this fashion, thus avoiding the computational complexity of equation (37)).

THE CANADIAN MERIT RATING PLAN FOR INDIVIDUAL AUTOMOBILE RISKS

HERBERT E. WITTICK

VOLUME XLV, PAGE 214

DISCUSSION BY A. D. PINNEY

Automobile insurance rates have been a matter of great concern to both the Insurance Industry and the insuring public during the past few years. Many solutions have been proposed, but the one put forth most often is Merit Rating. Mr. Wittick's paper on "The Canadian Merit Rating Plan For Individual Automobile Risks" is, therefore, very timely and of keen interest to most of us.

He has presented to the Society a clear and concise description of what the present Canadian plan is and how it evolved over a number of years. In addition, Mr. Wittick has exhibited data which clearly substantiates the theory that risks which have produced claims are more likely to have losses in the following year than those which are claim free.

In his conclusions, Mr. Wittick makes the following statement in reference to the advantages of this merit rating plan:

> "It permits a low rate for the select risk, and that is what the insuring public demands."

What this plan actually provides is a discount, not a low rate. It will be recalled that the base rate is applied in full for a risk having an accident during the past year, and discounts of 10%, 20%, and 35%, if accident free for one, two or three years. The off-balance that re-