RATE REVISION ADJUSTMENT FACTORS

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DISCUSSION BY R. L. HURLEY

The paper "Rate Revision Adjustment Factors" by LeRoy J. Simon is essentially an analysis of the mathematics underlying the adjustment of current rates to reflect loss ratios experienced under the premium rate structure formerly in effect. Early in the article, the author points out that the rate revision factor will, most frequently, be of significance with coverages for which the pure premium method is not applicable because the official Stat plans do not provide an exposure base. Probably, fire and allied lines would constitute a most typical environment within which the techniques discussed in this paper might be applied—although there are probably instances when they would be equally pertinent to casualty lines.

At the outset, the "Rate Revision Adjustment Factor" is defined as a number which, when multiplied by a set of collected premiums, will revise or correct these premiums to reflect a new or current set of rates. Under Case A, the paper establishes this number "F" mathematically in its simplest form divested of any of the ramifications encountered in the normal work-a-day situations of rate making. Then a comparison is made between this precise expression and the equivalent equation which would result if common practices were turned into mathematical language.

In subsequent sections, the author relaxes the various restrictions which were initially imposed on his mathematical development in order to present the underlying concepts with a minimum of algebraic distractions. In Case B, the paper analyzes the play of Installment Payment Plans which have, at least in fire insurance, assumed commanding importance. Certainly no mathematical treatment of fire loss ratios could be considered adequate without a careful investigation of this influence. As a consequence of this investigation, the author introduces in "Case C" a mathematical equivalent whereby the effect of rate revisions on 5 year Installment Plans is expressed or "telescoped" into the initial year of the policy.

The previous sections were designed independent of growth, or if you prefer, on the assumption of zero growth. In Cases "D" and "E" a growth factor is superimposed on respectively the Prepaid Policy (i.e., Case "A") and the Installment Policy (i.e., Case "B") including in the latter case the effect of telescoping rate revisions back into the initial year of the Installment Policy. And finally, the author presents a corollary wherein he analyzes methods by which a company with a set of rates differing from those of another carrier or bureau may obtain a composite comparison of the different rate levels between the two organizations.

No mathematical effort ever escapes the logical necessity of making assumptions. By common tests, we customarily demand that the assumptions not outrage our experience of things as we know them. For more theoretical investigations it should suffice that the postulate system be free of any substantive inner contradiction.

It is thought that the author's simplifications of insurance experience are quite straightforwardly presented. He works on written premiums only, although experience for rate making purposes is reviewed on an earned premium basis. He assumes that exposures are distributed evenly over the year whereas there may be reason to suspect a seasonal variation underlying random chance gyrations. Annual installments are treated as constant in respect to the amounts of insurance over the life of the policy and the premiums are considered as paid in equal installments. Neither these nor the other assumptions that serve as the framework on which the mathematics are woven into a multi-phased design rub painfully against the reviewer's appreciation of insurance realities as he understands them.

We should like to consider the paper's conclusions, expressed and implied, under the dual aspects of "factual" and "logical". It may be that other readers will regard such a distinction as tenuous at best; and hold that if any such differentiation is to be made, the reviewer has seemingly reversed the accepted meaning of the terms. Under the connotation of "factual," we do not disagree, but are not distressed, with the author's conclusion that the intuitive approach in adjusting collected premiums for rate changes introduces a constant bias of a maximum order of $1\frac{1}{4}\%$ inadequacy under a 20% rate reduction. We also noted that the commonly used arithmetic mean gives a less accurate answer than the harmonic mean in summarizing the effect of class rate changes on different mixes of business, but a pencil test of a few examples suggest that the variations may not be too wide under typical circumstances.

We begin to become disturbed at the author's demonstration that a significant disparity is introduced by ignoring the effect of five year installment business (cf. equation 34)—but this disquiet may possibly stem chiefly from theoretical considerations. While accepting these factual conclusions, we reflect that one should not be displeased if in its first statistical attempts, fire insurance rate equities attain a rough, frontier-type of justice. Over the years, the schedule approach has proved its value in the fire insurance field, and the future should afford even greater improvements, but it may be a little while before fire rates can be made to a fine degree of statistical precision. Possibly this observation may be extended to certain other coverages for which the rates are influenced by the loss ratio indications.

The reviewer thought that the major contribution of this paper may ultimately prove to be the logical consequences of its mathematical demonstrations. While the substance of the article is within the mathematical requirements of our Society, it demands a careful read-

ing—with a pencil never far from hand. The fundamental ideas are succinctly presented. As the argument unfolds, more difficult concepts are introduced and the algebra becomes somewhat rigorous. The reviewer spent a goodly number of hours on the simplifications in the area of equations (26) through (33) before arriving at the indicated formulas. In general, the notation possesses an inner consistency and a degree of elegance that make the mathematical reasoning a delight for the reader.

In any pursuit founded, as insurance, on statistical science, the more frequently elements significantly deficient in respect to mathematical precision are introduced into the rating procedures the more obscure the logical inter-relationships and the less defensible the procedures— on purely statistical grounds. Few actuaries, we trust, would force this observation to mean that insurance rates are always reducible to set equations. Most practitioners in our profession soon learn that there are seldom mathematical transforms which will automatically turn the specific rating problem into a trim statistical equation. Our theoretical investigations must be counted as successful if they quicken our insight into the noumenal of the insurance transaction. We are fortunate that with Mr. Simon's paper, our *Proceedings* will contain a scholarly research into the inter-relationships underlying the loss ratio method of adjusting rates.

AUTHOR'S REVIEW OF DISCUSSION

LeROY J. SIMON

I appreciate having Mr. Hurley review the paper because I know it represents a thorough and unbiased consideration. While he and I both use the fire insurance business as the principal source of our examples, I know we both agree that the formulas presented in the paper are quite general and may be used in any line of insurance. Wherever rate revision adjustment factors are used, there is no reason to use anything other than the proper formula. To do otherwise is to voluntarily introduce an element of inadequacy into the rate structure.

The factors developed in the paper relate to written premiums only. The preferable way to adjust experience to current rates is to apply these factors to the written premium first and then convert the adjusted premiums to an earned basis. In the fire insurance line, a striking example of the error of reversing the order of this process is given in this volume of the *Proceedings* in Note 7 of the paper "Notes on Some Actuarial Problems of Property Insurance" by L. H. Longley-Cook.

The only difference in Mr. Hurley's conclusions and mine appears to be a matter of degree. He states that he is "not distressed" with the element of inadequacy that is introduced by using the incorrect formula; he is not displeased with the fact that fire insurance rate equities "attain a rough, frontier-type of justice"; and he observes