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NOTICE

The Society is not responsible for statements or opinions expressed in the articles, criticisms and discussions published in these *Proceedings*.

PROCEEDINGS

May 23-24, 1957

LESSONS FROM ADVERSITY

PRESIDENTIAL ADDRESS BY NORTON E. MASTERSON

In 1956 the national economy attained new statistical heights in gross national product, personal income, and expenditures. It was a year of great prosperity, full employment, and new high levels in production. Most U. S. industries and commercial enterprises prospered. However, the 1956 records attained by the fire and casualty insurance business were far from satisfactory. Thus, this must be a most unusual presidential address or report, since our Society embraces a segment of our national economy which had one of its worst years in history.

Four unusual losses occurred in 1956, coming with a suddenness to serve as dramatic headlines in a most unusual insurance year. The crash of two airliners over the Grand Canyon, the sinking of the liner Andrea Doria in collision with the steamer Stockholm, the Brooklyn waterfront fire, and the West Coast forest fires were startling events in widely diverse branches of the insurance business. These spectacular catastrophes got the headlines, but it was a steady increase in costs of the general run of claims which plagued most insurance lines.

Historically, cyclical trends have always characterized the fire and casualty insurance business, with periods of unsatisfactory operating results being followed by favorable results due to corrective actions, such as tighter underwriting, better rating practices, adequate rates, and a return to a level of actuarial balance.

One adverse situation was the lack of financial benefit from multiple line operation. Usually, some of the major underwriting classes are profitable and some unprofitable in any given year, but 1956 was completely abnormal as to financial balance by multiple line operation. All classes of business, with few exceptions, were in rising loss cycles during 1956. The fact that adverse underwriting results were not limited to one class of business, one type of company, or one section of the country suggests causal factors of a broad over-all nature.

In multiple line operation there is the temptation to average and standardize underwriting and financial management of the two classes—fire and casualty—to the extent that each loses significant differentiating characteristics.

The business has operated for almost ten years since the expiration of the moratorium period on January 1, 1948 following the SEUA decision of 1944. During that same period we have seen the growth of multiple line underwriting, with a consequent vigorous competition and a growing necessity for fire insurance companies to write casualty lines, and vice versa.

Our Society has the range and scope, both by its constitution and by-laws, and by its diversity of membership to ferret out and appraise those factors which affect all lines of insurance. It becomes a challenge to casualty and fire actuaries to analyze any and all common causes of underwriting losses and separate them from independent causes peculiar to a single line of business.

While the casualty and fire companies felt the full impact of adverse results in 1956, a review of past experience indicates that the trend toward higher losses began in 1955. Taking a broad view of the casualty and fire business, the adverse experience resulted from inadequate rates, insufficient insurance to value, intense competition, broadened coverage without corresponding rate increases, a general wave of carelessness, plus a general inflation in claim cost due both to the decreased purchasing power of the dollar and increased claim consciousness on the part of the public and claimants.

Adverse experience resulting from inadequate rates characterized most of the casualty and fire lines. Being based on past experience, rates do not adequately reflect the effects of today's increased frequency and claim costs. Inflation, increased frequency, increased severity of claims, and higher jury awards have produced an inadequacy of serious proportions in rates for auto bodily injury coverage on private passenger automobiles. Automobile comprehensive coverage also suffered excessive loss ratios because of very high new car and repair costs due to design and construction, particularly new body styles and windshields. Decreased rates on some classes, plus a tendency of amounts of insurance to lag behind higher replacement costs, have plagued fire and allied lines.

A significant contribution to rate inadequacy was a factor not revealed by past statistics. It was an accumulated effect of broadening of coverage, more liberal interpretation of coverage, and changes in laws which, in the aggregate, increased the company liability without corresponding rate adjustment.

We have seen the rapid introduction of so-called package policies—those clever merchandising creations, some of which confound both actuarial laws and geometry, with the result that the "whole becomes less than the sum of its parts!" It is most fortunate that our Society embraces both casualty and fire insurance as only an actuary could "divide the indivisible" premium in this new branch of cut-rate geometry!

Aside from actuarial considerations and financial problems, the most serious national problem confronting our business was the loss of life and property resulting from the operation of a growing number of automobiles on crowded highways. Over 40,000 people were killed in automobile accidents in 1956—more than we lost in three years of the Korean War.

Some serious financial problems are developing in group insurance, other than life, because the very existence of insurance has tended to promote a greater number of claims, and increased medical and hospital costs, with a continuing lag between premiums and sky-rocketing losses.

Generally speaking, competition is a desirable economic tool for commodities, but leaves much to be desired as a regulator of insurance rates. Price competition in the insurance transaction is not the same as that of the market place. This current cycle of rate inadequacy re-emphasizes the characteristics of insurance prices mentioned in my presidential address a year ago. We deal in future contracts, determining our prices prior to performance; while in most other businesses, the product is delivered prior to payment. As between the company and policyholder or claimant, future financial solvency is more important than current net price because the contract has yet to run at time of sale.

Obviously, the 1956 picture was discouraging for our classes of business. Rate increases will give some relief in 1957, but only to the extent that such increased rates are earned and to the extent that they keep pace with adverse developments on pending cases and any increased claim costs that we shall incur in 1957.

There are several lessons to be learned from the adverse results in 1956. Of interest to us are those which relate to the actuarial function and those which can be used by actuaries to urge a return to fundamentals in the future operation of casualty and fire insurance companies.

We have learned that multiple line underwriting is not an automatic financial device for averaging insurance results. We must be alert to the fact that the inter-relationships among separate lines of insurance may be greater and more far-reaching than variations and cyclical trends in individual line loss ratios. In other words, multiple line underwriting may intensify financial problems during certain years or cycles rather than serve as a balancing or hedging mechanism. Inflation causes similar adverse results which cut across definitions separating major types of casualty and fire business. Inflation causes increases in prices, goods, services, and labor required to replace all types of insured property covered by insurance. At the same time, inflationary factors increase the insurance company costs for the various kinds of insurance covering injuries or death to persons, by reason of higher costs for medical care, loss of time, and rehabilitation.

We have seen inflation in actual operation in the casualty and fire insurance business. It is much more pleasant to listen to lectures on

and to study about inflation in the college classroom than it is to experience it. It all seems so logical and matter-of-fact as an academic subject in economics. It usually works out in the same textbook manner in the long run, but it is hard for a nation of millions of individuals to realize what is taking place in the short run. I would suggest that one of our bright young Fellows, well trained as an economist, but who now has to make his living as an actuary, delve into this subject and produce a significant paper for the Society.

Inflation strikes the casualty and fire insurance in ways not typical of price inflation in the usual economic sense relating to goods and services. In most casualty and fire lines, rates are regulated and fixed for annual and longer policy terms, making them relatively rigid and not possible of change on short notice. Even when rates or insurance prices run the gamut of delayed statistics and approvals, such increases on annual policies are not fully earned for two years. Such is the effect on the premium income but the hazard of inflation on the outgo for losses and expenses is also serious. Delays in settlements and prolonged medical care and rehabilitation intensify the effect of inflation after the insured event occurs. In a manufacturing business, there is a constant spiraling interplay between wages and other costs of production, and prices to the buyer, but such inflationary effect does not extend beyond the time of contract or sale. In fact, an inventory of unsold finished products can be protected by up-pricing. However, an "inventory" of unsettled claims remains exposed to the ravages of spiraling costs for several years.

Thus, we need actuarial factors not only to bridge the gap between the indications of our statistics of the past and the present, but also between the present and the future policy period and through the extended settlement period.

If we look back over the period just prior to the problem year of 1956, there is evidence to support the well-known but usually forgotten underwriting theory that we sow the seed for our bad loss ratio years in prosperous years. Favorable underwriting results lead to unsound competitive practices and loose underwriting; and most important of all, a complacency sets in which blinds us from observing and doing something about indications of adverse trends which are bound to grow but which are overshadowed by current rosy financial results.

We have learned the lesson of keeping underwriting independent of investment operations. Unusually favorable accounting results, in the form of unrealized capital gains, should not influence considerations of corrective changes in underwriting operations.

There must be constant expense control within the expense provision of rates. Any tendency to ascend to higher expense levels during a temporary period of low loss ratios serves only to intensify the financial problems when we have high loss ratios.

While some unfavorable factors are still crowding us in 1957, we should ponder well these lessons of adversity and chart a course of actuarially sound rates and rating plans, intelligent responsive reg-

ulation of rates, sound competitive practices, and a constant awareness of future financial hazards where the ultimate course of current results is still subject to future changes.

As individual members of this Society, we can play limited roles only, in solving these problems. We can give advice and counsel in our own companies, bureaus, or insurance departments. But because of the nature of the casualty and fire insurance business in the United States, many of these problems must be solved by joint cooperation and action in bureau jurisdictions and in state insurance departments.

This is a non-partisan professional Society composed of executives and actuaries of insurance companies, rating bureaus, state insurance departments, and consulting actuarial firms. We do not have official powers as an organization but we should carry out the object of this Society, which is the promotion of actuarial and statistical science as applied to the problems of casualty, fire and social insurance. We can establish a favorable *climate* for positive action and solution of some of our problems through official channels.

November 21-22, 1957

**PROFESSIONAL RESPONSIBILITIES OF THE MEMBERS
OF THE CASUALTY ACTUARIAL SOCIETY****PRESIDENTIAL ADDRESS BY NORTON E. MASTERSON**

As I approach the end of my two years as president of this Society, I have my last chance to deliver or present a privileged paper. These past two years have been adverse ones for most of the lines of insurance embraced by our professional society; thus it is more pleasant to look ahead rather than back. I want to comment on two general subjects. The first will relate to professional responsibilities of the members of the Society, and the second will be a short report on the International Congress of Actuaries meeting held last month in the United States and Canada.

Professional Responsibilities

This meeting here in Philadelphia, a city of historical importance and symbolic of freedom and brotherly love, could provide excellent reasons for exploring professional ethics and actuarial freedom. In fact, if there are analogies between actuarial freedom and academic freedom, this would be my final opportunity to make such exploration since a presidential address is not subject to censorship or review by the Committee on Papers.

Any implication that there is any problem of actuarial freedom in our profession—of the importance and scope of academic freedom in the profession of education—would certainly be an exaggeration. It is important that we exercise the obligation we have to speak or write on matters affecting our business. Just as the industrial scientist is today gaining a new birth of freedom and prestige, so also will this period of adversity spotlight the actuaries qualified to do technical research and communicate thereon.

At the recent annual meeting of the Society of Actuaries, the presidential address by Melvin Davis contained the following significant statements, relative to the professional responsibility of actuaries, which are also appropriate for us:

“The rapid succession of new developments . . . has added greatly to the responsibilities of actuaries. The actuary must determine whether a proposed innovation is sound for his particular company. He must be open minded and progressive. He cannot counsel against the adoption of new ideas solely because

they have not been tested. Therefore, in order to give proper counsel, he must carefully analyze and appraise each proposal, gauging its probable effect on his company.

"In making the necessary analyses, an actuary must use his broad knowledge of facts, principles, and practices. Not infrequently, his recommendation must be negative and may be based on complex considerations which are not readily comprehensible to others and, as a result, he may find himself in an unpopular position. Therefore, he must be able to state his position in terms that can be readily understood. He cannot be effective if others cannot understand him. The actuary must be the most practical of men, but his position must always be based on correct theory, extensive knowledge, detailed study, and sound judgment.

"Once the actuary has reached a conclusion as to the soundness of a particular proposal for his company, he must then set forth his own reasoning and conclusion with the utmost clarity and vigor. Unless he meets this responsibility, he is not worthy of his profession, nor is he continuing the tradition of his predecessors who established the insurance business on its sound and reputable basis."

Our Society also has an obligation and responsibility to come forward with the best thinking and ideas. The basic plan of our Proceedings is to create a scientific literature for our business for current and future use and reference. The long formal paper embracing some actuarial study will always form the core of our Proceedings. But we should not overlook the desirability and importance of the short limited-subject scientific paper which would serve as a communication of a new idea. A short paper read by several hundred people has possibilities of being more effective in instilling desire for further research by others than would be a long paper with limited readership. I would urge some use of joint authorship for certain papers. An example would be a paper under joint authorship of two members of the Society—a busy senior actuary with an idea or a proposition, and a young actuary with the time and initiative for research into the subject.

I would like to review certain aspects concerning us as individual actuaries and as a professional organization as outlined in my three previous presidential addresses. I have commented on the fact that this is a most unique organization as to diversity of membership because it includes actuaries and officials in various fields: fire, casualty, surety and some life insurance companies; stock, mutual, state fund, hospital service and reciprocal organizations, state and insurance company rating bureau, state and federal government department, independent consulting actuarial firms; colleges and universities, and a few non-insurance organizations. I have emphasized the fact that a professional society composed of actuaries of companies, actuaries of rating bureaus, and actuaries of state insurance departments, to-

gether with consulting actuaries can do much to explain our insurance rating or pricing to the public with the objective of securing greater understanding. What better place for us to get together for objective self-criticism than in this city of "brotherly love".

In another address I commented on the place of the actuary and his responsibilities in the insurance business. The timing of price changes is one of the most critical areas where actuaries should function to develop new statistical techniques for the benefit of the companies and rating bureaus. The critical underwriting results during the past two years have re-emphasized not only the need for actuaries but more specifically the need to understand actuarial principles and to apply them in company management and state regulation. Adverse experience resulting from inadequate rates characterizes most of the fire and casualty lines. Based as they are on past experience, current rates do not adequately reflect today's increased costs.

One of the pressing problems of the day is that of establishing insurance rates for the diverse and widely differing coverages being written by multiple line companies. Basically, this critical problem is one of predicting future losses. Any individual loss is a fortuitous or chance event which cannot be predicted individually, or even in the aggregate with large numbers of similar chance happenings, with absolute precision. The task of predicting losses for any future period involves the gathering of data of past insurance events, and the analysis of existing trends in line with the principle that experience in the past, properly analyzed and appraised, is or may be a guide to probable future events.

One place where we can better our methods of prediction is in reducing the "time lag". Ours is a futuristic business and when we say our prices are based upon experience in the past, we mean in the casualty coverages, for example, that when we make rates in 1957 we must use data of 1956 and prior, and publish rates for 1958. The greater use of high-speed electronic equipment should be one of our objectives. Perhaps some day the incurring of an automobile bodily injury claim in a western state could, within a short space of time, be reflected and recorded upon an electronic accident statistics board in a New York City rating bureau.

We must eliminate many of the road blocks to successful future prediction. Political considerations disturb the orderly use of statistical trends and sound judgment trends. Political expediency destroys the effectiveness of any statistical tool in doing a good prediction job. Political expediency is inherently inconsistent, and by its very nature causes financial off-balance biased against the financial solvency obligation of the insurance company.

One of the most important fields of prediction in which we need to improve our methods is that of recognizing inflation. We need to classify our various types of casualty and fire insurance as to their vulnerability to inflation and further subdivide those that are vulnerable as between (a) those with premium bases which are flexible to inflation, and (b) those which are relatively rigid.

There is a considerable need for expanded research activity under the sponsorship of the Society. In the Society of Actuaries, significant research papers and tables for the life insurance companies have been developed by groups of actuaries from a limited number of large companies. We have an obligation and opportunity to initiate the research needed to arrive at joint and collective solutions to actuarial problems of our business.

An important "non-mathematical" aspect of the casualty, surety, or fire actuary's job involves the practical use of what is termed "judgment". The dictionary definitions of *judgment* are: "The act or faculty of affirming or denying a conclusion, whether as based upon a direct comparison of objects or ideas, or derived by a process of reasoning," or "the power of arriving at a wise decision or conclusion on the basis of indications and probabilities, when the facts are not clearly ascertained."

The actuarial or rate regulatory aspect of judgment is not a one-way matter. A decision to interpret or project statistical data within broad limits of an established formula is termed the exercise of judgment. However, decisions by an actuary not to use "judgment" or by a regulatory official not to approve the actuary's judgment do not mean that judgment has been eliminated. In the projection of statistical data as a basis for future rates, a decision not to use "judgment" is of itself a matter of *judgment!* The very nature of the trends in loss experience for most of the so-called multiple lines necessitates some *judgment* as a hand-maiden of mathematics and statistics.

Two Fellows of this Society, Mr. T. O. Carlson and Mr. L. H. Longley-Cook, co-authors of the chapters on Ratemaking in "MULTIPLE-LINE INSURANCE*" write as follows about judgment in ratemaking:

"On this whole subject of judgment in rate making there has always been a superfluity of fuzzy thinking and fuzzy talking. The discussion seldom is about judgment unsupported by statistics . . . more often what is at stake is the extent to which judgment may affect the interpretation of statistical data. There is a tendency in these discussions to overlook the fact that every step in the development of a formula is a judgment decision. A formula merely imparts consistency to the interpretation, so that no charges of unfair discrimination may be levied. The demand for formularization can be carried too far. . . . Formularizing is helpful, but it is not a cure-all. Flexibility is as important as the formulas themselves, if equity is to be the goal."

Thus, the intelligent use of *judgment* is an actuarial obligation and responsibility of the highest order.

*By G. F. Michelbacher and Associates—McGraw-Hill Insurance Series

International Congress

The International Congress of Actuaries meeting held in New York, Washington, and Toronto last month was an occasion of importance and significance for our Society. Any international meeting embracing practically all countries of the free world is of itself an historical event. For the first time since 1903 the International Congress was held in the United States and Canada. Actuaries from thirty-two countries were in attendance representing every type of activity requiring the skills of an actuary—private companies, governments, consulting actuaries, and college professors.

The object of the International Congress, which meets every third year, world conditions permitting, is to provide a forum for the world's actuaries in which to discuss new developments and problems, which the actuaries share as a profession.

The most significant aspect of this Congress for the Casualty Actuarial Society, and the reason for including these comments in my presidential address, was the definite and official recognition of the non-life branches of insurance.

In the United States and Canada the Casualty Actuarial Society has been in existence since 1914. A few years ago we expanded the scope of our Society to include fire and allied lines so that today our's is a multiple line organization embracing all but life insurance. In recent years, we have been drawn closer to the Society of Actuaries because of our common interests in accident and health insurance, group insurance, and the impact of government forums of social insurance.

At the International Congress last month a separate section known as "ASTIN" (Actuarial Studies in Non-life) was established on an international basis as a division of the International Congress. This international recognition of our branches of the insurance business will expand the scope of the International Congress in actuarial fields of more direct interest to us than has been true in the past. As has been the case with the International Congress, membership in ASTIN will be basically one for individual action with each member of our Society being eligible to join, participate and pay dues.

The ASTIN management board consists of 7 directors. The representative from the Casualty Actuarial Society for the United States and Canada is Mr. Francis Perryman. Our scientific and editorial representative will be Mr. L. H. Longely-Cook. You will hear more of the ASTIN organization during the next three years prior to the 1960 Congress in Brussels.

This is an opportunity for individual members of our Society to participate in a small way in international cooperation and understanding. The business of insurance or protection against risk is inseparable from the political, social, and economic atmosphere of our own nation, but the risks faced by any nation have international implications.

AUTOMOBILE BODILY INJURY LIABILITY RATE-MAKING ON A PROSPECTIVE BASIS

BY

J. EDWARD FAUST, JR.

I. PURPOSE

Anyone who has worked in the field of Automobile Insurance Rate Making in the last few years is well aware of the necessity of having rates reflect loss costs actually incurred during the period of time that they are in effect. Although this is a perfectly obvious statement, underwriting results of recent years, especially in the Automobile Bodily Injury Liability field, eloquently demonstrate that this goal is rather elusive.

The purpose of this paper is not to supplant current rate making techniques but to supplement them.

Specifically, the purpose of this paper is to present a method for forecasting one year in advance the Incurred Pure Premium for the total writings of a given Company. By knowing this value it is then possible to evaluate the adequacy of rates for a given company so that it may meet its underwriting gain objectives.

Although the method presented here has been developed for the other automobile insurance lines, namely, Property Damage Liability; Collision and Comprehensive coverages; only the results for automobile bodily injury liability are presented since it is the author's purpose to stress method rather than actual results. Furthermore, it is believed that this method must be developed for each carrier separately. The actual results presented here should be construed as being applicable to the data considered, which was of one carrier.

II. METHOD

For the purposes here, Calendar Year (or Accident Year) Incurred Losses are defined as equal to the sum of the losses paid during the calendar year and the increase or decrease in Calendar Year Reserve during the year.

In order to forecast the value of the Incurred Pure Premium, as defined above, our problem resolves itself into determining the following:

- A. An accurate method for forecasting paid calendar year pure premiums; and
- B. An accurate method for forecasting beginning and ending calendar year reserves.

The paid calendar year pure premium, is of course, equal to the product of the average paid claim costs times the paid claim frequency.

Various attempts were made to forecast the pure premium rather than the average claim costs and the claim frequency but it was found that much better results were obtained by attempting to forecast these

two component parts separately. Indeed we are led intuitively to the conclusion that each should be studied separately since each is affected by different influences.

III. FORECASTING AVERAGE PAID CLAIM COST

The object here was to find a criterion for forecasting the above value one year in advance. The most obvious starting point was to try a time series but this produced wholly inadequate and inconsistent results. This led to a search for a better criterion. It was felt that the Consumer Price Index and the Wholesale Price Index, as published by the Bureau of Labor Statistics in Washington, D. C., were worthy of consideration. The values of the Consumer Price Index are based on the base year of 1953 (= 100) and in the case of the Wholesale Price Index, the average value of the years 1947, 1948 and 1949 was taken as equal to 100. In each of the indices the value for a given calendar year was equal to the arithmetic average of the values for each calendar month during the year.

Straight line and multiple correlation methods were used to correlate these economic indices and the average claim cost one year hence.

Values for average claim cost and these indices were found for each calendar year from 1936 through 1954.

On the first attempt it was found that the correlation coefficient between the W.P.I. (Wholesale Price Index) and the average paid claim cost one year hence was 0.986. It was found that the complete W.P.I. produced better results than the use of the index which excludes food and other farm products. The correlation coefficient between the C.P.I. and the average paid claim cost one year hence was 0.964. The multiple correlation coefficient between the W.P.I. and the C.P.I. was found to be 0.991. All results were indeed significant.

Although the degree of correlation was significant, further studies were made due to a rather large difference in some calendar years between the actual and calculated values. It was thought that the method could be improved by eliminating certain years. The years discarded were 1952, 1949, 1947 and 1946. The resultant multiple correlation coefficient was 0.992. The average error between actual and computed values was 3.0%. The maximum positive difference was 6.1% and the maximum negative difference was 6.7%.

Although excellent results were produced, it was observed that the method was tending to underestimate the actual values for recent years. The cause of this was a large increase (at a rate larger than the increase in volume of business) in the number of field claim adjusters in 1951. This demonstrates that any mathematical approach must be tempered by a knowledge of the carrier under study. No mathematical approach will automatically adjust itself for management and operational changes in methods and procedures. On the other hand, the need for this additional knowledge does not disqualify the merits of a sound mathematical and statistical approach in order to quantify the elements affecting losses.

What had been done was discarded and only the results for the years 1952, 1953, 1954 and 1955 were considered.

The following formula was found to produce excellent results :

$$Z_X^E = Z_{X-1}^A \left(\frac{26.649Y_{X-1} - 1895}{26.649Y_{X-2} - 1895} \right)$$

where: Z_X^E = estimated or forecasted value of the average paid claim cost for the year X.

Z_{X-1}^A = actual value of the average paid claim cost for the year X-1.

Y_{X-1} = Average Consumers Price Index, as previously defined, for the year X-1.

Y_{X-2} = Average Consumers Price Index, as previously defined, for the year X-2.

The above formula produced the following results :

<u>Calendar Year (X)</u>	<u>Actual Average Paid Claim Cost</u>	<u>Z_X^E</u>	<u>% Error of Estimate</u>
1952	\$691	\$690	(-) .14%
1953	751	758	(+) .93
1954	765	764	(-) .13
1955	783	770	(-) 1.66

Again it should be stressed that the W.P.I. and the C.P.I. produced good results with the data studied. Perhaps they would be satisfactory for other data or perhaps a different combination of these indices or even other indices would be more satisfactory for other carriers.

IV. FORECASTING PAID CLAIM FREQUENCY

The same method of study was used here as for the Average paid claim cost. That is, certain factors were made to lag the paid claim frequency by one year. The frequencies studied, were for the calendar years 1949 through 1955. The relationship between W.P.I.; C.P.I.; number of passenger car registrations country-wide; the percent of the total autos insured by the carrier etc.; and the paid claim frequency one year hence was studied, but to no avail. The problem lies in the fact that although paid claim frequency has increased each year from 1949 through 1955 that the total increase in these seven (7) years was only 13.6% or less than 2% per year. None of these criteria were sensitive enough to forecast so small a change. As a result, a straight line was established for the data using the years 1952, 1953, 1954 and 1955 by setting the 1952 value on the straight line equal to the actual value with the following results :

<u>Calendar Year</u>	<u>Actual Paid Claim Frequency</u>	<u>Computed Paid Claim Frequency</u>	<u>% Error</u>
1952	0.01132	0.01132	0%
1953	0.01151	0.01149	(-) 0.2
1954	0.01171	0.01166	(-) 0.5
1955	0.01183	0.01183	0

The above computed value equals 0.01132 minus 0.00017 times the calendar year under forecast minus 1952.

V. FORECASTING THE CALENDAR YEAR PURE PREMIUM

The forecasted value of the Calendar Year Paid Pure Premium is equal to the forecasted value of the Average Claim Cost (from III.) times the Paid Claim frequency (from IV.).

The following results were obtained:

<u>Calendar Year</u>	<u>Actual Pure Premium</u>	<u>Forecasted Pure Premium</u>	<u>% Error</u>
1952	\$7.82212	\$7.81080	(-) .14%
1953	8.64401	8.70942	(+) .76
1954	8.95815	8.90824	(-) .56
1955	9.26289	9.10910	(-) 1.66

VI. RESULTS OF APPLYING THIS METHOD TO 1956

This forecasting method was developed in the first half of 1956 and at that time predictions for the entire calendar year of 1956 were made. The reader may be interested in comparing these predictions with the actual values that developed.

<u>Item</u>	<u>Actual Value</u>	<u>Predicted Value</u>	<u>% Error</u>
Average Paid Claim Cost	\$785	\$791	(+) .76%
Paid Claim Frequency	0.01215	0.01200	(-) 1.23%
Paid Calendar Year Pure Premiums	\$9.538	\$9.492	(-) .48%

This method therefore would have predicted at the beginning of 1956 the paid pure premium for the calendar year of 1956 within $\frac{1}{2}$ of 1%.

VII. THE CHANGE IN CALENDAR YEAR RESERVES

Now that we have determined a method for forecasting the paid pure premium the next step is to determine a method for forecasting the increase or decrease in calendar year reserves so that the Incurred pure premium can be forecasted.

It was found that the following method produced substantially better results than by using conventional methods for determining reserves such as case-base estimates, loss development factors, etc. The reserve method determined here was based on the premise that calendar year reserves are equal to the sum of the reserves for each accident year. The same carrier's data was used for the accident years 1942 through 1955.

The following data were available:

- A. The number of claims on reserve at the end of each calendar year exclusive of incurred but not reported reserves.
- B. The matured values of the claims in (A.) above plus incurred but not reported claims.
- C. The number of claims paid during each calendar year.
- D. The amount paid in claims during each calendar year.

Eleven full calendar years of development on 1942 accident year claims were available. It was assumed for the purpose of this study that at least 5 years of development was needed. Therefore, only the data for the accident years 1942 through 1951 were considered.

It was further assumed that we had fully matured experience on 1942 through 1946 accident year claims as of December 31, 1955. The development for the accident years 1942 through 1951 were supplemented by observing the rates of claim development for the accident years 1942 through 1956. On the basis of this "matured" experience for each accident year 1942 through 1951, the number of outstanding claims at the end of the first year, second year, third year, etc., was found along with the matured values of each of these claims.

From this data the average reserve needed was found for each accident year at the end of the current calendar year, the next succeeding calendar year, the second succeeding calendar year, the third succeeding calendar year, etc. These needed reserves at each point of development were divided by the values of the average paid claim cost one year hence as forecasted by methods already presented. The average paid claim cost one year hence was therefore used as the first measure of the average reserve need. This is reasonable for it was found that about 70% of the accident year reserve is paid out in the ensuing calendar year and about 90% within the ensuing two calendar years.

The results were examined and it was found that the following factors produced satisfactory results:

<u>Accident Year</u>	<u>Factor</u>
1st preceding	3.2965
2nd preceding	3.0943
3rd preceding	2.7363
4th & later	2.5616

For example, the reserve need for the accident year 1955 as of December 31, 1956 is equal to 3.2965 times the forecasted average paid claim cost for 1957 times the number of outstanding claims as of December 31, 1956 which were incurred in 1955.

It was found that although the reserve need for the current accident year varies directly with the cost of claims in the next calendar year that there is an inverse relationship with the reported claim frequency for the current calendar year.

For the current accident year it was found that the following factor multiplied by the ratio of the average paid claim cost for the next calendar year to the reported claim frequency for current calendar year gave good results:

$$36.1453(1.052)^{X-1951}$$

where "X" = calendar year under study.

The following table shows the excess or deficiency in the reserve computed by the foregoing method as compared with the true reserve need which developed:

<u>Calendar Year</u>	<u>Percent Difference</u>	<u>Calendar Year</u>	<u>Percent Difference</u>
1945	3.9%	1950	1.8%
1946	(-) 12.1	1951	(-) .2
1947	(-) 5.5	1952	1.8
1948	1.4	1953	(-) .1
1949	(-) .5	1954	.2

The above reserve method, therefore, measures the reserve need against the cost of paying claims in the next calendar year, the year in which about 70% of the reserve will be paid out in claims. It also recognizes the fact that reserve need varies with the age of the claim at the date of evaluation. It furthermore takes cognizance of the observed fact that the short term effect of a higher reported claim frequency is to lower the reserve requirement.

The above explains the determination of year-end reserves. The following discusses the change in calendar year reserve, for if we add this change to the forecasted paid pure premium we shall obtain the Incurred Pure Premium.

The calendar year (or accident year) incurred pure premium for the next calendar year is equal to the forecasted paid pure premium for the next calendar year, plus the calendar year reserve at the end of the next calendar year minus the calendar year reserve at the end of the current calendar year. We have so far determined a method for finding the paid pure premium for the next calendar year and the reserve at the end of the current year. Our next problem is to find the reserve at the end of the next calendar year in terms of the reserves that we know at the beginning of the year. As in the case of the beginning reserve we shall treat the reserve for the current accident year separately from all other accident years.

If: R_X = the reserve per policy for the accident year "X" as of the end of the year "X" then:

$$R_X = 1.052 \times \frac{A_X}{A_{X-1}} \times \frac{N_X}{N_{X-1}} \times R_{X-1}$$

where: R_{X-1} = the reserve per policy for the accident year (X-1) as of the end of the year (X-1).

$\frac{A_X}{A_{X-1}}$ = ratio of Average paid claim cost next year to the Average paid claim cost this year.

$\frac{N_X}{N_{X-1}}$ = ratio of estimated policies in force next year to the number in force this year.

The change in reserve next year due to the accident year represented by year "X" is then:

$$R_X - R_{X-1}$$

The next item to consider is the reserve at the end of next year (X) for all prior accident years.

As of the beginning of the year, the reserve for all prior accident years except the first is given by:

$$\frac{A_X}{N_{X-1}} \left[C_{X-2} \cdot 3.2965 + C_{X-3} \cdot 3.0943 + C_{X-4} \cdot 2.7383 + \sum_{Y=5}^{Y=10} C_{X-Y} \times 2.5616 \right]$$

where: "X" = year under forecast.

C_{X-Y} = number of claims outstanding at the end of the year (X-1) which were incurred in the year (X-Y).

A_X = Average paid claim cost in the year "X".

N_{X-1} = Estimated average policies in force for the year "X-1".

The reserve for all prior accident years, except the current year, as of the end of year "X" is given by:

$$\frac{A_X \cdot A_X}{A_{X-1} \cdot N_X} \left[R_1 \cdot C_{X-1} \cdot 3.2965 + R_2 \cdot C_{X-2} \cdot 3.0943 + R_3 \cdot C_{X-3} \cdot 2.7363 + R_4 \cdot 2.5616 \sum_{Y=4}^{Y=10} C_{X-Y} \right]$$

where: $\frac{A_X \cdot A_X}{A_{X-1}}$ = estimate of the average paid claim cost for the year (X+1).

and R_n = the percent of claims outstanding as of the beginning of the year, which were incurred in the nth preceding calendar year, which still remain unpaid at the end of the year "X".

An examination of the data studied developed the following values of R

$\frac{n}{}$	$\frac{R_n}{}$
1	20%
2	35%
3	40%
4	60%

The change in reserve due to all prior accident years is of course equal to the difference between the ending and beginning values.

The total reserve change would be equal to the reserve change for the next accident year plus the reserve change for all prior accident years.

VIII. MAKING THE FORECAST OF NEXT YEAR'S LOSSES

Three alternatives are available.

First, the losses next year could be estimated by merely considering the actual paid pure premium this year and the forecasted paid premium for next year without regard to the change in calendar year reserves which has been outlined in Section VII. above.

Second, the change in reserve could be approximated by merely considering change in reserve for the accident year coincident with the calendar year under computation. In other words, the Incurred Pure Premium would be taken as equal to the paid pure premium plus the change in the current accident year's reserve.

Third, the losses next year can be estimated by considering paid pure premiums and the change in reserve for all accident years as outlined in Section VII. hereof.

SUMMARY

A method for estimating the change in Calendar Year (or Accident Year) pure premiums has been presented. This statistic can then be used to examine the adequacy of current rates or proposed rates in terms of the losses they will be expected to sustain during the period of time that they are in force.

PRINCIPLES AND PRACTICES IN CONNECTION WITH
CLASSIFICATION RATING SYSTEMS FOR LIABILITY
INSURANCE AS APPLIED TO PRIVATE
PASSENGER AUTOMOBILES

BY

JOSEPH M. MUIR

INTRODUCTION

Automobile registrations in the United States now exceed 67,000,000, an increase of 270% in the last three decades. The premium for bodily injury and property damage liability insurance has increased from \$250 million to more than \$3.0 billion during the same period. Passenger cars have accounted for their proportionate share in the phenomenal growth of the automobile industry and that type of vehicle has produced not less than \$2.3 billion of the liability insurance premium. This multi-million dollar volume of business has been the incentive for the automobile liability insurance industry to exercise all of the techniques and knowledge at its command to effect an equitable and marketable distribution of risk by means of classification rating systems.

More than a quarter of a century has passed since a student of the automobile liability insurance business stated that any advantage occurring from improvement in the loss experience should go to *all* policyholders, and conversely, any adverse development should be apportioned among *all* risks. This theory was predicated on the following precept of insurance:

“A group of persons, each of whom realizes that he is subject to the possibility of some loss, the time and amount of which are matters of uncertainty, create, through justly proportioned contributions, a common fund, from which, in the event of such loss happening to any of them, compensation may be made to the loser and the burden thereof distributed over the entire group.”

The philosophy of distributing loss experience among *all* insureds, irrespective of risk hazard, no longer prevails to any extent. It has been rejected in favor of a policy of fair discrimination with respect to rating criteria which are measurable in terms of loss costs. This development can be attributed to evolutionary changes occurring within the insurance industry. These included the rapid increase in the number of companies organized to write automobile liability insurance, the emphasis placed upon the public welfare by the enactment of legislation affecting the use of automobiles, the competitive measures employed by specialty companies for the purpose of attract-

ing the most desirable portion of the business and, experimentation directed toward a more equitable distribution of hazard within the rapidly growing automobile insurance market. Legislation enacted in the several states has also been influential in encouraging fair discrimination among risks and providing appropriate administrative machinery. The All-Industry Casualty and Surety Rate Regulatory Bill which was approved by the National Association of Insurance Commissioners on June 12, 1946, provided in part, under Section 3(a)3 thereof, that "Risks may be grouped by classifications for the establishment of rates and minimum premiums." A similar permissive grant is found in all of the state laws.

The press and other organs of public information have been instrumental in prompting state supervisory officials, legislators and laymen to debate the merits of private passenger automobile classification rating, primarily from the standpoint of its value in automobile accident prevention. Some effort has been made in this sphere by the insurance companies but the experience gained has not been encouraging. There is little doubt, however, that the automobile insurance industry must meet the ever growing demand for more definitive categories by which risks may be grouped in accordance with variation in hazard. The extent to which conservatism in this area may be overshadowed by a policy to increase premium volume, is reflected by the trend in the indications for rate level adjustments on a state by state basis. A period of rising loss costs will promote greater selectivity and redistribution, and result in a shift of the desirable business to a more favorable rating classification, with a relegation of the less desirable group to a more self-supporting position in the classification system. Favorable underwriting results can lead to a redistribution of the indications within classification divisions, but the means by which this may be accomplished are somewhat more flexible than those utilized when the experience is unfavorable.

The production forces of the industry provide a testing ground for measuring the reasonableness of rating elements which might appear to have all of the desirable qualities for a classification rating system. The producers have materially influenced the further refinements which have been accomplished in the realm of classification distribution of private passenger automobiles.

After three decades of experimentation in this field, it is significant that many of the characteristic features of rating systems which are currently in use by a great majority of automobile insurance companies, were also basic to the systems used thirty years ago, demonstrating their conformity with public interest and soundness from a rating viewpoint.

The development of these systems, together with the principles and practices underlying their evolution, form a composite subject worthy of review and analysis. It is the purpose of the discussion which follows to present and examine some of the important aspects of this phase of automobile liability insurance ratemaking.

PRINCIPLES OF CLASSIFICATION RATING

The fundamental objective underlying any private passenger automobile classification rating system is to establish an equitable distribution of insurance costs so that all risks will be charged their proportionate share of the losses incurred by the company. If the various groups into which the insured population is divided develop the same loss ratio, then no class of business, theoretically, is more desirable than another, from a loss standpoint. Basically, a class of business is or becomes undesirable only because of an inadequate return of premium. The amount of additional expense dollars resulting from a grouping of risks within a high rated class may establish that class as the most desirable business.

The standing of a company in the automobile insurance industry and its determination to maintain that position affects immeasurably its willingness to provide a sound and stable market for a wide range of risks embracing those with low as well as high loss potentials. Full recognition must be given to the fact that a competitor, offering lower rates for substantially the same product, will attract the most desirable business to take advantage of the lower cost. Competitors specializing in writing a class of low hazard business may reject higher loss cost applicants who find a more ready market among the writers of a general class of business. If such competitive lower rate offerings are supported by measurable elements or conditions such as reduced acquisition or other expense costs, extreme care in selection of type of business, restricted territorial solicitation or a rigid renewal policy, those offerings can be sustained and the higher cost companies are obliged to write a disproportionate share of the less desirable business.

In a market involving broad groupings of hazards, with a sufficient volume of business in each group to provide ample writings, a proper loss ratio incurred on a "disproportionate share" would be just as acceptable as on a "proportionate share". Such a market, however, does not remain static. The forces of competition operate to narrow the groupings, with the result that in the interest of public relations, a subsidy—ratewise—is created to compensate for that percentage of the total which develops the highest loss experience.

The more a classification system is refined, the greater must be the reliance placed upon the production forces to assign risks to their proper categories. If the refinement is insufficient, the producer may be placed at a competitive disadvantage. Contraction in the premium volume of a class of exposure which the company has characterized in broad terms may indicate the need to subdivide the elements to create a more attractive classification. The nature and extent of the refinement is limited by the number of measurable elements usable as criteria, the ability of the company to secure proper rating information applicable to those elements, and the effect the system will have in meeting similar methods of a competitive nature. The elements selected may be based upon estimated performances or results in

factual circumstances or determinable conditions, or upon a combination of both. Certification of classification rating information by the insured or by the producer on behalf of the insured is an accepted administrative practice. The system may operate prospectively or retrospectively.

While established ratemaking procedures for automobile liability insurance include the selection of state and territorial rate levels to reflect conditions anticipated during the period the rates will be in force, such rate levels can be unbalanced by superimposing thereon a weighted classification system. When the selected distribution and classification differentials produce such a result, a correction factor may be used to balance the classification rates to the selected level. Classification systems which, as respect premiums, purport to penalize the accident-prone risk or give credit to the accident-free risk, are all subject to rate level balance adjustment. The penalty charges increase the collectible premium and the credits reduce the volume. If the classification system is in balance, the charges and credits will produce the result to which the selected rate level is keyed.

Whether or not classification rating in any form is an incentive to accident prevention has yet to be demonstrated, as such measures involve a complicated problem arising from the variable of human nature. Judgment, foresight, presence of mind, mechanical aptitude, concentration under all kinds of driving conditions, consideration for others—all involve mental alertness and responsiveness to the lessons of experience which do not form a part of man's inherited faculties. Rules for safe driving, safe walking, laws against carelessness in the use of an automobile with fines and penalties commensurate with the offense, educational programs—should all have an effect in reducing automobile accidents, if implemented properly. Fundamentally, however, there are other methods and means better calculated to reduce accidents. While some psychological benefit may be derived from the use of certain elements in classification rating systems for automobile liability insurance, the occasion for accidents could be considerably reduced by city planning, highway construction based upon the most modern and tested safety techniques, traffic control, elimination of grade crossings, dangerous curves and blind intersections, greater segregation of types of traffic—all centered around a national uniform pattern designed to eliminate the present confusion which results from unilateral planning by the individual states. Classification rating for private passenger automobiles could be synchronized with such measures to emphasize the beneficial results which would accrue to policyholders as the result of safer operating conditions.

BACKGROUND

Following the underwriting and rating practice of classifying private passenger automobiles in accordance with their physical characteristics, i.e., horsepower based upon cylinder bore and number of cylinders, to which was added later, wheel base, weight, list price

and several safety factors, a new concept was introduced in 1921—a discount in rate for restricting coverage to exclude use of the automobile for business purposes, coupled with an additional discount if the coverage were limited to an owner-driver exposure. When experience on this classification became available, it did not substantiate the refinement to any reasonable extent. Such result was attributed to the limited spread of the experience among the various classes and rating territories. Furthermore, restricted use of automobiles was an accepted condition of that period, with few hard surface highways, poor quality of tires and numerous mechanical defects contributing to accidents and operating failures. These conditions were recognized as part of the calculated risk assumed in acquiring ownership of an automobile. The restricted classifications were withdrawn in 1924. The next five years brought about changes in automobile design, speed, production quotas, reduction in automobile prices, improvement and expansion in highways, and an increase in the mileage traveled by a population becoming accustomed to locomotion on wheels. Increasing congestion on highways resulted in an increase in accident frequency. During this period, the premium writings for automobile liability insurance increased 100%. Many companies were organized which specialized in writing automobile liability insurance and their policies were issued with rates based upon classification systems of occupational use or accident record.

In 1929, a large segment of the industry introduced a classification rating basis identified as the "Merit Rating Plan" to provide a rate differential between the careful and the accident-prone driver. A credit of 10% was granted on renewal if the insured had not been involved in an accident during a period of 21 months, ending three months prior to the effective date of the policy. This classification rating system was withdrawn after three years because the credit for an accident-free record was being offered to virtually all risks as the result of a breakdown in the administration of the Plan. A prerequisite to the operation of the Plan was the reliance placed upon the insured's declarations and an exchange of information among insurance carriers. Delays in issuing renewals, additional expense attendant upon handling credit adjustments when an insured changed carriers, and the additional work required of the production forces, were all factors which discouraged a willingness to continue the experiment. The 10% surcharge for certification under the then existing Financial Responsibility statutes served as a basis for the "Demerit Plan", the Merit Rating Plan's short-lived replacement.

Under the "Demerit Plan" the manual rates were subject to surcharges. The manual rates, without surcharge, were applicable in the absence of specific motor vehicle convictions and also where the incurred losses resulting from accidents did not exceed \$50.00. The experience period was 21 months. Surcharges of 10%, 25% and 50% were imposed under circumstances involving convictions or accidents, or both. This "Plan" met with strenuous objection from the production forces and was withdrawn on the same day it was released in

1932. Later that same year, the conviction provisions of this rating basis were incorporated into the Financial Responsibility Laws rules, with the 25% and 50% surcharges applied for specified convictions, and the previous 10% charge for financial responsibility certification made applicable to other offenses.

During the succeeding five years, experimentation in classification rating of private passenger cars followed an irregular pattern. Registrations were approaching 25 million and automobile liability insurance premium writings exceeded a third of a billion dollars. Experimentally, a variety of classification elements were introduced: physical characteristics of the automobile; use of the automobile; occupation of the named insured; accident record of the operators; conviction records; age and number of operators; mileage, either actual or estimated—all directed toward the same common objective, i.e., theoretically, to distribute the collectible premium dollars to reflect differences in loss costs. As a practical matter, the purpose was to arrest the steady shift in volume of business away from the principal writers of this line. In the latter part of 1937, frantic efforts were being made by those carriers to cope with the problem, and in December of that year the public was offered a monetary award for safe driving in the form of a 15% premium refund under the "Safe Driver Reward Plan", a form of merit rating.

Under this "Plan", the insured was rewarded with a premium refund of 15% at the end of the policy term if he had operated for a year without an accident. An innovation in the field of automobile liability insurance, this retrospective rating procedure was designed to circumvent the administrative problems which existed in other types of classification systems. However, the cost of making refunds in small amounts to an estimated 88% of the policyholders imposed a financial burden on carriers as well as on producers. Open accounts for non-canceled checks extended over long periods. The principles established by this system were not observed after it was found to be more economical to grant the "reward", in advance, against the likelihood of the insured being involved in an accident during the policy year. Five years later when the Wartime Emergency Rate Program was launched, this retrospective rating system was terminated. It has not been revived generally, although in an isolated quarter it has found some acceptance.

Not all state regulatory authorities or segments of the industry reacted favorably to the "Safe Driver Reward Plan", and although it was used in 34 jurisdictions where approval of rates was not generally required, none of the rate regulated states adopted it. As a competitive tool, it merely served as a forerunner for another refinement in classifications to reflect business or non-business use, estimated mileage and number of operators.

Opposition by various segments of the industry to the retrospective reward system in the State of New York resulted in the development of a penalty classification plan which was approved for all carriers licensed in the state. The "Preferred Risk Rating Plan", as it was

called, included three classifications. The lowest rated, Class 1, applied to risks which had been involved in not more than one property damage accident during a 21 months period ending three months prior to the effective date of the policy. Class 2, rated 10% higher than Class 1, applied if the accident record for the experience period involved one bodily injury or two property damage accidents. A surcharge of 15% of the Class 1 rate was applied to Class 3 risks. Such risks were designated as those having a more adverse accident record than assigned to Class 2. The three classifications were subject to a requirement that the insured complete a rating information form, and the carriers exchanged information covering the past accident experience of the risk. A penalty premium, equal to twice the difference between the premium at which the policy was written and the proper premium, was imposed if the facts were misrepresented by the policyholder. Considering the exigencies of the times, this classification system was unique because it was introduced with the rate level balanced by an off-set for the collectible surcharges.

During the period of approximately three years that the Preferred Risk Rating Plan was in use before it was replaced by the War-time Emergency Rate Program, statistics were compiled which showed that whereas 95.2% of the risks had not more than one property damage accident to mar their record over a year and nine months, 2.7% had one bodily injury or two property damage accidents, and 2.1% were definitely accident repeaters. The administrative detail and expense, the unfavorable public reaction to some claim settlements, the tendency on the part of policyholders to delay reporting accidents, and the opposition registered to carriers' acceptance of liability which was thought by some insureds to be in doubt, were pointed out later to discourage efforts to revive the system after it was withdrawn in 1942.

A new series of classifications, designed to reflect the use of the automobile, was marketed as a companion to the "reward" and "penalty" classifications and was superimposed upon those rating structures in 1939. Some of the rating elements then applied are in current use. Automobiles owned by the insured and used by him in business were rated at manual rates and assigned to Class B. For certification under a Financial Responsibility Law, the Class B rates were surcharged 10%, 25% or 50%, depending upon the offense, and the risk was assigned to Class C. All other private passenger automobiles were divided into two categories, Class A-1 and Class A. Class A-1, with rates 25% less than the Class B manual rates, applied under extremely refined conditions where (1) the number of operators in the same household as the insured did not exceed two, (2) neither of such operators was under 25 years of age, (3) the mileage of the automobile for the previous year was not more than 7500 miles, and (4) the estimated mileage for the policy year did not exceed the same figure. Class A, rated five percentage points higher than Class A-1, embraced those risks which failed to meet those exacting requirements. This means of introducing the "youth-

ful operator" rating element applicable to risks involving operators under 25 years of age, was the forerunner of many classification studies directed at the rising loss costs of a segment of exposure constituting approximately 15% of the private passenger insured risk volume.

Following the withdrawal of the Wartime Emergency Rate Program at the end of World War II, when the nation's motorists took to the highways in unprecedented numbers, the immediate pre-war classifications were restored with some simplification. The lowest rated class, A-1, was made applicable to individually owned, non-business automobiles with no "youthful operator" exposure and with an estimated mileage not over 7500 miles. Class A applied to the balance of the individually owned, non-business cars, and Class B was assigned to those not eligible for Class A or Class A-1. Two years later, in 1948, the Class A group was divided into A-2 and A-3, which was the initial step to determine the extent to which the "youthful operator" risk was being subsidized. The rate for that category, A-3, was set at 5% less than the business use rate. This compared with reductions of 25% and 20% respectively, for the A-1 and A-2 classes. This refinement left unchanged the special classification treatment for farmers and clergymen which had become an integral part of the non-business use classifications.

The next important change was made in 1950 when farmers, as defined, were granted a rate reduction of 15%. Shortly thereafter, the mileage requirement was eliminated by a large segment of the industry, the rate for the preferred non-business use class was reduced in relation to the business class, and the "youthful operator" exposure was rated at 15% above the business class. The revised designations were Classes 1, 2 and 3. This classification rating program was the genesis of the insurance industry's move toward making Class 2 risks self-supporting. By the end of 1952, following the introduction of emergency rate level increases for private passenger automobiles in 1951 and 1952 as a result of the inflationary spiral generated by events in Korea in June 1950, many companies faced a crisis with respect to their private passenger classification rating systems.

On a countrywide basis, approximately 75% of the private passenger automobile business had been written under Class 1, the preferred class; 15% under Class 2, the "youthful operator" class; and 10% under Class 3, the business class. As a result of the cumulative effect of the aforementioned emergency rate increases, the insurance buying public became acutely price conscious; risks of relatively low hazard were seeking a market with carriers using classification systems more refined than the Three Class Plan. The circumstances prompted a course to pursue which encompassed the following considerations:

1. that a substantial volume of business in Class 1 could be distributed to give specific recognition, ratewise, to the most desirable exposure in that class;

2. that the most desirable exposure in Class 1 represented limited use of the automobile through low annual mileage and limited number of operators;
3. that the balance of the exposure in Class 1 could be so divided as to reflect annual mileage, number of operators and use of the automobile in going to or from work;
4. that the Class 2 exposure was still being subsidized on the basis of the indicated Class 2 differential;
5. that the family car risk in Class 2 operated by a "youthful driver" under parental supervision was distinguishable from the risk with an unmarried principal operator or unmarried owner, under 25 years of age;
6. that parenthood among "youthful operators" provided a reasonable basis for classification distinction.

During 1953 the automobile insurance industry focused its attention on private passenger classification refinement. In some quarters, Class 1 was divided into two parts with restrictions on mileage and number of operators weighed against classifications with no such limitations. Class 2 was divided into three groups on the basis of the extent of use by a "youthful operator", the marital status of the operator under 25 years of age, and ownership of the automobile. The most preferred class was rated 45% below the business classification rate, with the balance of the Class 1 exposure continuing at 30% below that rate. The rates for the three subdivisions of Class 2 were 5%, 25% and 50%, respectively, above the business classification rate, which compared with the previous rate of 15% above the business classification rate for all Class 2 exposures.

Another method divided Class 1 into three parts. The first part, 1A, excluded customary use in driving to or from work and was rated 40% less than the business classification rate. The second part, 1B, limited "to and from work" driving to less than ten road miles one way, and made a distinction between urban and rural areas, with the latter rated 30% less than the business classification rate but not more than \$3.00 above the Class 1A rate for bodily injury and property damage combined. "To and from work" driving beyond the ten mile limit was assigned to Class 1C at a rate 15% less than the business classification rate.

Class 2 was also divided into three parts, designated 2A, 2B and 2C. Class 2A the youthful-driver class, rated at 110% of the business classification rate, applied (1) if the operator under 25 years of age was neither the owner nor principal operator, or (2) if the owner or principal operator in the same age group was married and had legal custody of a child. Married owners or principal operators under 25 years of age without legal custody of a child were assigned to Class 2B and rated 125% of the business classification rate. Owners or principal operators under 25 years of age who were not married were rated 150% of the business classification rate. Exceptional rate treat-

ment for farmers and clergymen was continued as a firmly entrenched classification principle.

In the light of practical experience, additional improvements and refinements were introduced quite generally in 1955. A special classification, 2D, was created for family automobiles where the operators under 25 years of age were female and were not the owners or principal operators of the automobile; and for cars owned by married couples where only the wife was under 25 years of age. The rate for this group was reduced substantially. Furthermore, parenthood as a rating element was discontinued and rate recognition was given to driver training courses meeting prescribed requirements. About a year later, the classifications for female owners or operators under 25 years of age were eliminated and the "youthful female" exposure was discontinued as a rating element.

Concurrently, during the latter period, one of the staunchest advocates of average rates—a major carrier specializing in private passenger automobile business—adopted classification rating. Furthermore, the Preferred Risk Rating Plan was restored in modified form in the State of New York. In addition, one of the leading carriers launched a merit and demerit classification experiment on the West Coast.

DIFFERENTIALS AND DISTRIBUTION

The relationship among the classifications is determined by differentials, using one of the classes as unity. For many years it was customary to use the business class, Class 3, as unity and to establish the other classes above or below 1.00. Indications based upon more than 7.1 million car-years of exposure for policy year 1955 demonstrated that substantial differences existed among the classification loss ratios. The basic limits loss and loss adjustment ratios calculated on a Class 3 rate base indicated the relationship which existed among the classifications to produce the proper premium for each classification. Data for policy years 1954 and 1955 substantiated underwriting conclusions that (a) the youthful male owner or principal operator should be rated at not less than twice the Class 3 rate, (b) the family car with incidental use by a male operator under 25 years of age should be rated at approximately 25% above the Class 3 rate, (c) the elimination of the business use and youthful male operator hazards would establish a rate slightly below the Class 3 rate, and (d) further refinement to distinguish between limited use of the automobile in driving to or from work and no such use, would make the lower hazard ratable at 25% to 35% less than the Class 3 rate; the rate for the "to or from work" exposure would then be not more than five points above the lower group rate.

Differences between rate territories have been subject to classification rate recognition on the basis of a comparison of (1) the indicated differentials for large city territories with (2) the indicated differentials for those territories which are predominantly rural or have no city with a population exceeding 40,000. However, data for policy

years 1954 and 1955 eliminated any territorial distinction, except with respect to (a) the "youthful" male owner and principal operator class and (b) the non-business use class with male operators under 25 years of age excluded and "to and from work" operation restricted to not more than ten miles one way.

A representative volume of experience involving more than 200,000 earned car-years of exposure and a bodily injury and property damage premium at basic limits exceeding \$6.8 million for policy year 1954, indicated that mileage limitation and a limitation on the number of operators, as rating elements in a classification, developed a differential in relation to business use which supported rates 45% below the business classification rate. Furthermore, a marked difference in the indicated differential was shown when either limited mileage or a limited number of operators, or both, were not reflected in the classification. An increase of as much as 25 points in the differential resulted when these restrictive hazard features were removed from the classification.

The low hazard differentials reflecting combinations of adult personal and pleasure use, limited mileage, limited number of operators and no trips to or from work, indicated that these elements may be expected to produce substantially the same results so long as business use and "youthful" operator rating provisions follow a common pattern.

The differentials for the subdivisions of Class 2 apply to the "youthful" operator hazard. The marital status of male drivers under 25 years of age who operate the insured automobile extensively establishes whether the risk is a normal family car exposure or whether it belongs in the highest hazard rate group applicable to "youthful" male owners or principal operators. The indicated differential for the highest hazard rate group in the small city areas was virtually double the indicated differential for the exposure with married male operators under 25 years of age, the latter being substantially the same as that for the normal family car risk. In large city areas, the ratio of the indications was approximately 5 to 3.

The practice of using the business use class, Class 3, as unity, and relating the indications of the other classes to that base, has been discontinued. The greater volume of experience in the lowest hazard class, Class 1A, as compared with the volume in the business use class, offers a more stable and reliable base to determine the classification differentials.

If the selected differentials for a classification plan do not vary from the indications on the basis of credible data, theoretically, each class will be self-supporting. Contrarywise, a limitation placed upon the selection, which results in reducing a differential from the indications, will spread the difference over the other classes. This procedure may be elected in order to avoid extreme changes in classification rates or to temper the changes where other adjustments such as those involving rate level or territorial relativity are being made concurrently.

Distributing the exposure among the selected classifications requires sound judgment as well as reliable statistics. In the absence of such statistics, motor vehicle registration figures, licensed operator records, population data, automobile sales, gasoline consumption and the like, aid in estimating the distribution for the purpose of introducing a classification rating system until more indicative data is available. Progressive refinements may be accomplished by using reliable basic data to support a broad classification distribution and sound judgment may be applied to effect the separation into narrower groupings. There would be little advancement in reapportioning the rate level by classification if all of the improvements were to await the development of complete statistical data as supporting information.

Approximately 80% of the total volume of private passenger automobile exposure is now in the non-business category which excludes the hazard of the male operator under 25 years of age. The balance is divided, with approximately 6% to 9% assigned to business use and the remainder to the "youthful" operator exposure classes.

In the large city areas, 34 out of every 100 private passenger risks do not use their cars regularly for business, do not have a young driver exposure and do not drive to or from work. This number is increased to 39 in the rural districts and small city territories. Driving to and from work is a customary operation for 46 out of every 100 risks in the low hazard classes in the large cities. Their counterpart in the rural areas are fewer in number, with 40 out of every 100 driving to work. From such data it may be deduced that while distribution of the low hazard classes is substantially the same in the large city and rural areas, (approximately 80%), the transportation facilities normally found in large cities have not absorbed the highway commuter traffic. The general migration of the populous to the suburban areas has taxed the highway arteries which are used to connect with public transportation. It would appear that a distinction between large city areas, and rural and small city areas is not particularly significant and that a more realistic analysis would be on the basis of zones constructed to give recognition to the comparable operating conditions in various sections of the country.

In order to avoid a rate level off-balance, the selected differentials should be balanced to the classification distribution. This may be done by applying a correction factor to the differentials to adjust them upward or downward while maintaining the same relationship among the classes. If the distribution and differentials are selected with a view to promoting business in a particular classification at a particular level of rates, a rate level off-balance may be accepted in the furtherance of this purpose. Under such conditions, a change in actual distribution is the objective. Opening a market for low hazard exposures by shifting the weight of the differentials among the classifications may be expected to increase the percentage distribution of such exposures, even though the overall volume remains unchanged. This medium as a competitive rating tool has only temporary advantages however, because it fosters similar implementation by competi-

tors, and gives impetus to the search for further refinement. With more than three quarters of the private passenger exposure in the relatively low hazard classes—classes which rely heavily upon rating elements such as low mileage, non-business use and the absence of “youthful” male operators—this portion of the market is the attraction for lower differentials through added limitation elements on the scope of hazard to be insured.

PROSPECTIVE VS. RETROSPECTIVE APPLICATION

The classification rating system that is applied prospectively relies upon the experience of the past, tempered with sound judgment, to establish the proper rate for the period during which the insurance will be in force. Such a system permits a fixed price quotation by the production forces, is relatively simple to deal with from a statistical and accounting standpoint, and does not necessitate rehandling of the business subsequent to the expiration of the policy. Prospective rating may be readily used to introduce rating elements with values developed from sources outside the insurance carrier's own records. This quality can have considerable appeal from a public relations standpoint, particularly if the classifications are designed as an aid to promoting safety. Such a system may have support in an exchange of information among insurance carriers or depend entirely upon factual data secured elsewhere by the carrier of record. Proper balancing of the prospective system, initially coupled with the necessary administrative machinery to assure reasonable safeguards, will produce results with a high degree of accuracy.

Retrospective application of a classification system is not popular, although the rating elements may be not unlike those of a prospective system. The former uses the experience of the policy period to determine the premium for that period, thus in essence establishing the final cost after the product has been consumed. The business must be rehandled after expiration even though the coverage is not renewed and the accounting and statistical operations are increased in connection with a substantial portion of the business written. Although it may be reasoned that such a classification system gives immediate and direct effect to the individual risk's experience, doubt can be cast upon the propriety of experience rating a single car on the basis of its experience for a single year. If a longer period is specified, a question of proper administration arises on the premise that a change of carrier may occur during the experience period.

MERIT AND DEMERIT RATING

An estimate that 20% of all drivers are involved in 80% of all automobile accidents is responsible, in large measure, for the demand which recurs frequently for a form of classification rating that offers a rate reduction for safe driving or penalizes the accident-prone driver. Unquestionably, a system which grants a rate reduction for an accident-free record has considerable appeal psychologically, al-

though there is no basis for concluding that it will solve the problem of the motor vehicle high accident toll. Administrative obstacles inherent in such a rating provision, coupled with the fact that the rate discount, in order to be reasonable and also attractive, is in reality a mere token gesture, discourage any enthusiasm among the insurance carriers for perpetuating this plan.

The imposition of rate penalties for an adverse motor vehicle accident or conviction record has very little public appeal but can be compared with statutory penalties imposed for any law violation. The public is presumed to know the law and for the benefit of the whole, an individual is required to observe it. Similarly, the operation of a motor vehicle is a privilege granted by the state and is not a birth-right. Such a privilege carries with it an obligation to drive safely; and the avoidance of accidents is no more than a fulfillment of that obligation. Penalties, in terms of an increase in the automobile liability insurance premiums, set accident-prone risks apart from their more favored contemporaries.

In accordance with the basic principle of insurance, i.e., spreading the losses of the few on the shoulders of the many, insurance companies should obtain from their motoring policyholders an amount sufficient in the aggregate to cover the collective incurred losses and expenses. If a rate reduction for accident-free experience is granted to some policyholders, the amount of the reduction, in the aggregate, should be charged against the accident-prone risks or should be loaded into the overall rate level. Similarly, surcharged premiums applied in the form of penalties for an adverse experience record should be credited to the overall rate level or used to adjust the differential between the merit and demerit rated business.

A. *Merit Rating*

This form of a classification rating system may be applied prospectively or retrospectively. Prospectively, the premium is reduced if certain conditions with respect to the risk's accident record prior to the issuance of the policy are met. A return of premium at the end of the policy period upon the completion of that period with an accident-free record, is the basis of the retrospective method. Either method presents a problem of public relations stemming from the human inclination to disclaim responsibility for an accident. The determination of "fault" is inherent in liability insurance and when it directly affects the policyholder's insurance costs based on his own involvement, he may protest and pit his judgment against that of his insurance carrier.

Merit rating is actually a form of experience rating. For many years it has been customary to experience rate automobile fleet business by the use of credibility based upon a rating period of several years. A fleet credit of 10% for an accident-free period of three years is not uncommon and has been used by segments of the insurance industry. The credibility to be attributed to a single car would be con-

siderably less than 10% for the same period and if the experience period were to be set at one year to coincide with the term for which automobile liability insurance policies are generally written, the amount would be further reduced. This leads to the conclusion that in order to make a merit rating system attractive to policyholders, the principles of credibility for fleet risks must be discarded and an arbitrary selection must be made. A credit of 5%, or even 10%, carries no particular monetary appeal; so a 15% credit for an accident-free year might be selected to test the propriety of the system.

In order to provide the funds necessary to pay the 15% credit or premium discount to eligible policyholders, an estimate must be made of the number of such policyholders. Looking at this matter from the standpoint of countrywide averages, which will differ from individual state indications, it may be estimated that the accident expectancy of an individual private passenger car risk is one accident in 11 years, thus producing an annual accident frequency of 9%. Therefore, 91% of all private passenger car risks would be subject to the discount of 15%.

Using a nationwide average rate of \$50.00 as approximating the combined bodily injury and property damage liability basic limits charge, it can be demonstrated that the type of merit rating system under discussion virtually requires policyholders to pay their own rewards if the insurance carriers are to receive from all of their risks sufficient funds to pay the total losses and expenses. To allow for the 15% credit to be paid to 91% of the risks, the rate of \$50.00 must be increased by 15.8% to \$57.90. When the 15% credit is applied to this new rate, the result is \$49.22. Since the rate without the merit rating system would be \$50.00, the actual reduction is 1.6% and the accident-free policyholders forego the balance, or 13.4%. A clearer conception of this end result is gathered from noting that if the 15.8% increase in premium is paid by only the 9% who are not accident-free, the total overall premium would be inadequate by 12.22%. It is not to be expected that laymen, making up the policyholders directly affected by this type of classification rating system, have an appreciation of its limited financial incentive, nor do they understand that it is an instrument which is primarily a psychological device.

B. *Demerit Rating*

Unlike its counterpart, demerit rating, to be capable of practical administration, must be applied prospectively. The plan consists of one or more rating conditions which provide for a surcharge in rate for the occurrence of specified incidents during a stipulated rating period prior to the inception of the policy. Recognition may be given to accident frequency, motor vehicle convictions and offenses involving moral turpitude. It may be reasoned that penalty rating as such, from an accident occurrence standpoint, is in reality the imposition of a fine for the very contingency against which the carrier has insured the risk. This may be given some credence if the system fails to take

into account the seriousness of the accident. A flat penalty for all accidents, irrespective of the contributing circumstances and regardless of the character of the damages, might bring about injustices, particularly if the amount of the penalty exceeds the legal liability of the risk.

In addition to the "Preferred Risk Rating Plan" approach previously mentioned, accident-prone risks may be rated to carry the full weight of the loading necessary to compensate for the credits granted to accident-free risks. This can be accomplished by increasing the penalty rate sufficiently to measure the extent to which the large percentage of risks will benefit from a merit rating "award." Using the same data as cited heretofore in relation to merit rating, it will be found that whereas the accident-free risk would be charged a rate of \$42.50, a savings of \$7.50 based upon a credit of 15%, the accident-prone risk would be subject to a rate of \$125.78. This is an increase of 152% for 9% of the business. This example reflects an estimated countrywide average and such an allocation of costs by state and territory would differ from this result.

It may be concluded that both the merit and demerit rating methods virtually resolve into penalty systems. While the demerit approach is undisguised and direct, the merit rating system requires those receiving the credits to pay all but a small fraction of their own awards, thus practically eliminating any difference between the average rate and the "reward" rate.

DRIVER EDUCATION

Private passenger classification rating systems would be incomplete if they failed to provide an incentive to improve the driver education standards of the nation's secondary schools, colleges and universities. With more than 10,000 public high schools offering courses in driver education, and more than 8400 schools offering complete instruction consisting of both classroom and behind-the-wheel training, great strides have been made by the National Education Association—representing all of the state departments of education—in promoting means by which students may be equipped to conduct themselves properly in the use of an automobile.

Rate discounts keyed to the type of course for which the student is certified, are offered generally by the automobile insurance carriers. With minor exceptions, the standards of the National Education Association of 6 hours of classroom study and 30 hours practice driving instruction are the bases for a discount of 10% in rate if all of the male operators of the automobile under 25 years of age, resident in the same household as the insured, are qualified. Simulated practice driving in a device used as a substitute for actual road experience is acceptable in partial satisfaction of the N.E.A. standards.

Primarily, the rate discount for approved driver education is one of public relations on the part of the insurance industry. What limited data is available neither proves nor disproves the theory that driver

education among "youthful" operators results in reduced loss costs. It could be deduced that the steady upward trend in the classification differential for male owners and principal operators under 25 years of age is not indicative of beneficial results from driver education among their groups. However, it may be too early to draw definite conclusions because the National Education Program is growing in scope and quality.

CLASSIFICATION OF SAFETY DEVICES

Periodically there are outbursts of enthusiasm for a classification rating provision to promote highway safety through the medium of a rate discount granted for the use of mechanical safety devices. In some instances, the sponsors may be motivated by civic interest; in others, the monetary return to the manufacturer resulting from widespread use of his device may be the paramount consideration. While it is readily recognized that automobile insurance is imbued with a public interest, the insurance industry should approach all such propositions with great caution. Acceptance of the principle of safety device discounts by a substantial segment of the automobile liability insurance industry might well generate an overwhelming demand that would have far reaching repercussions. Aside from the weighty problem of administering a classification rate discount for safety devices, the proposition is defective in that undue reliance is placed upon mechanical devices to supplant such accident potential influences as emotional disturbances, defective judgment, delayed reflexes, and lax enforcement of traffic regulations.

Irrespective of the safety features that have been built into the modern automobile—such as blow-out proof tires, power brakes, power steering, recessed door handles, crash-proof dash, safety steering wheel, directional signals, less visual obstruction, and seat belts—the fact remains that other factors have contributed to increase the insurance loss costs. Automobile accident frequency has doubled in the last quarter of a century and the average cost per claim is at an all time high. The current economic loss from traffic accidents is estimated to be approaching five to six billion dollars, and highway congestion is aggravated by an increase in the number of multiple car households. More than 15% of the thirty-six million families that now own automobiles have more than one car. Highway fatalities per one-hundred million vehicle miles driven have shown a marked decline, but the death toll in 1956 of 40,000 persons is a near record. Two-thirds of all persons injured in automobile accidents sustained their injuries while occupants of automobiles. One-sixth of that number were pedestrian cases. Speed has accounted for 50 out of every 100 traffic fatalities, and reckless driving has added 13 more to that tally. Private passenger cars are involved in 85 out of every 100 motor vehicle accidents and 82 of those passenger cars are apparently in good condition just prior to the accident.

There is no evidence that the loss level for automobile liability in-

insurance will be reduced by the adoption of rate discounts for safety devices. The price of the coverage is a by-product of physical and psychological conditions which reflect human characteristics. These attributes should be controlled by education, sound licensing laws, and exercise of proper and efficient police power. If these qualities can be imparted to the operator while engineering improvements are built into the machine, the overall favorable experience of the insurance carriers which should result will be reflected automatically in the rate structure.

CLASSIFICATION OF OPERATORS

Approximately 75 million operators of motor vehicles accumulate a total of 583 billion miles annually, an average of 7800 miles per operator. Each year, on the average, 2.5 million new drivers are added to the license rolls. What might appear at the outset as a vast reservoir of potential exposure units does not present a ready-made yardstick for classification rating purposes. Under a system of providing insurance on the basis of the operator instead of the automobile, the unit of exposure is transferred from the automobile to the operator. If automobile insurance were to be written on the basis of providing coverage for named operators instead of having the insurance follow the automobile, a distinction in classification rate between operator-owners and operator-non-owners would be a prerequisite. This stems from the fact that in more than 30 states, vicarious liability statutes are in effect. These laws, which vary somewhat in form, impute liability to the owner of the automobile even though the car is operated by another person at the time of an accident. Further, the law of agency which sets forth the concept of "principal and agent" precludes the adaptation of an exposure base which would necessitate differentiating between liability arising out of the use of an automobile and liability otherwise imposed. If a rate structure were to be established for operator classifications, provision should be made for distinguishing between (1) single and multiple car households, (2) owners who operate and those who do not, (3) individuals who own and operate only private passenger cars and those who own and operate other types, and (4) risks involving multiple types of automobiles.

The matter of coverage is of considerable importance in a change-over from an automobile classification system to one adapted to suit individuals as operators. In general, the automobile liability insurance industry has designed its policy contracts to cover the legal liability of the insured for bodily injury to any person and damage to property of others arising out of the ownership or use of an automobile. The word "insured" is defined so as to apply to the person named in the policy and includes other parties who may use the automobile with the permission of the owner. Protection is extended to the insured and his spouse for their use of non-owned cars and each member of the family has the benefit of the policy coverage on a severable basis. If each operator were required to be classified separately and be written under a separate policy covering him for his use of any automobile,

the case law that has been accumulated and is now available to insurance carriers, insurance authorities and the courts, would be of little value. New legislation would be required in connection with state Financial Responsibility Laws. Amendments would be required in the Compulsory Automobile Insurance laws in effect in Massachusetts, New York and North Carolina.

It could be expected that operator classifications, substituted for automobile classifications, would result in an increase in the carriers' expenses. The additional number of policies issued, along with the rating, typing, mailing and other handling of that volume could increase the expense. This, added to separate policies for physical damage insurance; an increase in the number of certificates filed with the State Bureaus of Motor Vehicles; an upward trend in the number of suspension and cancelation notices; rising printing costs for forms, endorsements, certificates, etc.; rising billing and collection costs; more extensive statistical and accounting records; and a substantial increase in rate administration costs, make an operators classification rating system less attractive than systems now in use.

From a rate standpoint, a transfer of the unit of exposure from a per car basis to an operator basis would require many families to pay substantially more for the family automobile insurance package although the actual protection afforded would be virtually unchanged. Many individuals would be obliged to pay for insurance they did not need or want.

The disadvantages of an operator classification rating system far outweigh the advantages that might accrue to the insuring public through classifying the hazard on the basis of the operating record of individuals. If a new type of classification system is to replace the one which has been in use for more than 35 years, i.e., relating the exposure to the automobile, such replacement should promise substantial economies and a more equitable fulfillment of the public needs. It has not been established that an operator classification rating system will meet those prerequisites.

EFFECT ON COVERAGE

The coverage required by State Financial Responsibility Laws has had a direct influence upon the design of policy provisions under private passenger classification rating systems. In order to avoid the imposition of an "absolute" insurance coverage program by the State Motor Vehicle authorities charged with the responsibility of administering those laws, the insurance carriers devised automobile policy provisions which, for all practical purposes, cover the liability of the policyholder under most circumstances. In doing so, it was recognized that classification requirements could not be applied as coverage warranties if the carriers were to be successful in maintaining their position that the policy defenses are in the public interest and a distinction between certified and non-certified protection is equitable. Most policy contracts contain a Financial Responsibility Laws Condition which conforms the policy to the requirements of state statutes upon

certification of the policy by the company. Although that Condition recites a reimbursement provision accruing to the benefit of the company if the certification requires the waiver of valid defenses otherwise applicable to the company, from a practical standpoint, the reimbursement feature has its principal value in the salutary effect it has upon the policyholder.

Classification rating elements which distinguish between hazards of risk contemplate various degrees of liability to be assumed by the insurance carrier. A warranty with respect to coverage would limit the coverage to the operations contemplated by the classification applied to the risk. Any immediate advantages flowing from such a procedure must be weighed against the magnitude of the problem created by deliberate falsification of classification information, by the use of erroneous information furnished unintentionally, or by other misapplication of the classification system. Experience has demonstrated that the small percentage of error with respect to the application of a reasonable classification rating system does not justify putting the voluntary coverage grant in jeopardy by introducing warranties. The relatively insignificant effect their absence may have upon the carriers' assumed liability can be written off as a "calculated" risk.

In some instances, steps can be taken deliberately to confine a risk's hazard to the area contemplated by the classification applied. The instrument for this purpose is an endorsement excluding the hazard which must be eliminated to make the risk a normal insurable exposure. This practice is accepted quite generally as a reasonable method to deal with those risks which contain some elements that are not in the public interest to insure. By removing those elements, the risk is converted to a normal exposure to fit into the classification rating system.

ASSIGNED RISKS AS A CLASS

All 48 states, the District of Columbia and Hawaii have adopted Automobile Assigned Risk Plans to assist applicants in obtaining automobile liability insurance. The annual return from assigned risk business now approximates 60 million dollars of premium, which represents about two and three tenths percent of the total automobile liability premium volume. These figures demonstrate that assigned risks cannot be looked upon merely as by-products of undesirable business. Primarily, because of complaints to insurance supervisory authorities that the levels of rates charged assigned risks are unfairly discriminatory—levels which vary from company to company depending upon the basis their voluntary business is rated—a movement is gaining ground aimed at bringing about uniformity in assigned risk rates among all carriers. The State of Wisconsin has the distinction of being the first to introduce a uniform system of rating assigned risks while continuing non-uniformity on voluntary business. However, the Wisconsin Rate Regulatory Law is unique in that ample authority is provided for imposing this condition. Paralleling this development is the interest in assigned risk rate uniformity evidenced by the Na-

tional Association of Insurance Commissioners which has appointed a special committee to study the matter.

Assigned risk rates generally have been fitted into the pattern of classifications applied to risks written voluntarily. Aside from the separate surcharges specified in the Plans, the business written through the medium of those Plans has been rated in accordance with the same elements which distinguish between the exposure characteristics of solicited business. In some cases, a carrier uses a competitive classification rating system for solicitation and, for assigned risks, applies a higher rated classification system which it appropriates from another rate filer. This tends to unify assigned risk classifications and rate structures, although the purpose of such a practice is to take full advantage of a major competitor's higher rate levels to be applied to undesirable business while still maintaining a reduced rate level to offer the competitive market. By appropriating the higher rated classification system of another and applying that to assigned risk writings only, a carrier may increase its assigned risk premium income by a substantial percentage.

Classifications for assigned risks should be applied uniformly by the insurance carriers and the rates for assigned risks should not depend upon which carrier receives the assignment. The rates should be computed to reflect the combined experience of all subscribers to the Plan in each state and rate levels should be adjusted to avoid a crossing of rates for voluntary business. Assigned risk classifications should be erected and administered in cooperation with the Motor Vehicle authorities. Multiple minor traffic violations, accidents and major convictions are reliable indices upon which to base rate variables. While it may be reasoned that the price to be paid in overcoming the obstacles to the attainment of assigned risk classification and rate uniformity is not warranted as respect such a relatively insignificant portion of the automobile liability insurance volume, the fact remains that the agitation being created by the present method of rating that portion of the business is rapidly magnifying the existing inequalities. It would behoove the automobile insurance industry to initiate a program of corrective measures of its own. These should include introduction of a standard statistical recording and reporting procedure, arrangements for ratemaking facilities, adoption of proper classifications, a standard coverage program and a means for bridging the gap until the appropriate all-industry machinery is put into operation.

POINT SYSTEMS FOR CLASSIFICATION PURPOSES

The Point System has been adopted in a few states to aid the Motor Vehicle Departments to ferret out those drivers who continually violate traffic laws, and demonstrate the need for remedial treatment. If the system is instrumental in returning better and safer drivers to the road, it has served its purpose, as the ultimate goal is not punishment—it is driver improvement. Encouraged by the beneficial results flowing from the Point System made effective in New Jersey on July

1, 1952, the Motor Vehicle Director of that state reported that the system had a tremendous impact "on the safety consciousness of the motoring public." He added that "the Point System is developing into the most effective driver correction measure ever undertaken by this state." It is admitted, however, that too many members of the public are not acquainted with the traffic law violation penalties in the Point System and, to confuse the situation, the few states with such Systems have not seen fit to coordinate their programs.

The Point System is adaptable to a classification system and while only one state, Massachusetts, temporarily aligned its points with the insurance premium to be paid by policyholders, there is a considerable area for experimentation in this field. The eligibility provisions of the Assigned Risk Plans offer a medium for the creation of classifications which can be dove-tailed with Point Systems as a basis for determining whether or not a risk is entitled to be granted insurance under those Plans. Through this means, the onus of denying the use of the highways is removed from the insurance industry and the responsibility is placed where it belongs—with the State Motor Vehicle Commissioner. This would tend to eliminate the adverse public relations in which insurance carriers can become involved when the issuance of a driver's license or owner's registration is contingent upon a carrier issuing a policy.

CLASSIFYING YOUNG DRIVERS

The evolution of separate classifications for private passenger car risks involving an operator or owner under the age of 25 years continues to show a marked upward trend in the loss and loss adjustment ratios. With an estimated 18% of all drivers falling into this category and being involved in 25% to 30% of all motor vehicle accidents, the industry is obliged to seek further rate increases if this class of business is to pay its own way. During the year 1956, drastic steps were taken which set the classification rate for the young male owner and principal operator at double the business classification rate. Current indications support the conclusion that further increases in this rate can be substantiated.

Since the female exposure is no longer included in the young driver classifications, it would be expected that the male portion remaining would show some upward trend from the combination of the two. Not only is this true, but the male portion also develops adversely when compared with earlier data adjusted to eliminate the tempering effect of the female exposure which was included when that earlier data was compiled.

Although only 4% of the total private passenger exposure is assigned to the young male owner and principal operator classification, the importance of this class cannot be discounted. An available voluntary market is a public relations asset as well as an obligation to the production forces. A voluntary market cannot long survive in an atmosphere of underwriting resistance founded on rate inadequacies. It is incumbent upon the industry to meet this problem with drastic

rating measures where necessary and, coincidentally, to quicken the pace of its media of public education to stimulate acceptance of this objective by the consumer.

A particular troublesome rating area lies in the treatment accorded students away at school who use their own cars or the family car extensively during vacation periods, on weekends or on holidays. This type of use concentrates the youthful driver hazard into a portion of the policy term and requires application of the appropriate classification to take that into account.

Automobile liability classification rates are determined on a per car-year basis. Regardless of the extent of use of an automobile during the full policy year, the experience on the car when insured for the full period is introduced into the ratemaking process as one car-year. To the extent the experience on a volume of classification exposure reflects normal or abnormal lay-up or use, the automobile liability classification rates contemplate similar conditions. It follows, therefore, that if classification rates for students were made to apply for the time students were home from school and using their own car or the family car, there should be two charges; one for the so-called dormant period when the student is at school, and the other for the period of extensive activity when the student is at home. Taken together, the rates should produce the same premium dollars as is produced by the application of a single classification. It is by tempering the high hazard with the low hazard during a policy year that the carriers can offer an average classification rate overall.

Other youthful driver underwriting and classification rating problems include multiple driver non-stop trips; "drag" racing on public thoroughfares; military personnel exposures; and the availability of larger and more powerful vehicles to youthful drivers who do not fully appreciate the potential for injury or damage which is at their command. It is not likely that classification refinement can measure these conditions separately. However, from a rating point of view, this becomes somewhat academic if the present 4% of the total private passenger exposure is adjusted to the proper rate level.

ADMINISTRATION

The backbone of a classification rating system is the administrative machinery established to assure reasonable application of the rating elements. An equitable refinement of hazards with attendant rate differentials, properly related by carefully prepared rule specifications, cannot sustain a system that is devoid of proper administrative characteristics. Poorly constructed classifications which are implemented by a well designed administrative technique, present a greater possibility of survival than improved classifications defectively administered.

It is fundamental that the source of rating information be as unimpeachable as conditions will permit and those conditions are controlled to a large extent by the enthusiasm displayed by the production forces and their willingness and ability to rationalize the classifi-

cation distinctions. Wholehearted support from the field, or at least an absence of resistance to change, is a required condition. Simplicity with respect to refinements lends itself to greater accuracy, minimizes adverse policyholder relations and opens avenues for economies in clerical functions. Signed applications on new business, attesting to the authenticity of rating information, may be obtained from the policyholder or from the producer. Since the renewal ratio on private passenger automobile business is approximately 85%, the information for renewal purposes can be limited to bring forth only basic essential facts involving changes in hazard since the previous survey. This may be in the form of a blanket statement applying to a producer's monthly renewal business, or the policyholder may be required to furnish information on a stamped addressed postal card form.

The reception given to classification discrimination by the policyholder and his acceptance of the rating elements—particularly those applicable to him—may spell success or failure to the system. Distinctions which are generally acknowledged as recognizing differences in hazard, such as business use vs. non-business use, face a minimum of resistance. Likewise, age groups have developed a line of demarcation for classification purposes and policyholder relations have been enhanced thereby. Such means are conducive to promoting public confidence in the insurance industry as they fit into the general economic pattern and are accepted as reasonable. However, the application of measures designed to give weight to estimates of conditions in the future, or of events to happen or not to happen, can undermine public confidence.

To the extent reasonable classification segments can be properly administered, to that extent the individual classes will eventually become self-supporting. However, the experience on those segments which are not amenable to proper administration will become interwoven with the experience of the others, thus creating a distortion and detraction from the credibility of the data. This treatment tends to broaden the exposure base of the low hazard classes and inflate their loss costs.

An exchange of rating information among insurance carriers in connection with the operation of merit or demerit classification systems is suggested from the results reported under the New York State Preferred Risk Rating Plan restored in 1952 after a lapse of ten years. As mentioned previously, the original Plan had a distribution of 95.2% for not more than one property damage accident, 2.7% for one bodily injury or two property damage accidents, and 2.1% for a more adverse accident record. The comparable results under the 1952 restored Plan are 95.9%, 3.4% and 0.7%, respectively. While the second group was broadened in 1952 to include accidents involving both bodily injury and property damage, it appears that the lack of administrative machinery for exchanging information among insurance carriers, such as was in effect under the original Plan, is respon-

sible in large measure for the substantial reduction in the percentage of accident repeaters.

TEST OF SOUNDNESS

Progress in advancing classification elements involving fair discrimination, with an objective of stability in the rate structure, should be inherent in a private passenger classification rating system. Adequacy of the potential market characterized by the classifications, combined with reasonable facilities for servicing that market, will establish an attractive sales inducement. An equitable relationship among the rates for the classifications, together with an underwriting policy of writing a general class of business, should result in an adequate rate level overall as well as a proper return from each of the divisions of separately rated hazards. The rating elements should be realistic and practicable with sufficient scope to avoid overlapping; their descriptions should be clear and concise and be based upon ascertainable facts that may be readily determined by the production forces or by inspection. The classification structure should take cognizance of comparable competitive systems. The administrative details should be arranged to reduce economic waste and promote good public relations to the utmost extent. Statistical facilities should be provided to produce means to check the accuracy of the studied judgment which enters into the classification and rating bases.

These characteristics in a classification rating system indicate a sound approach to refining the exposure to distribute the total insurance costs equitably among insureds and establish rates applicable thereto which are reasonable, adequate and not unfairly discriminatory.

CONCLUSION

It is noteworthy that although three decades of experience in private passenger classification rating have gone into the development of the industry's existing systems, the same problems encountered in the early stages are still very much in evidence. Basically, there are two factors which influence the pattern. One is competition to produce a volume of desirable business and the other is adherence to the philosophy that the industry has an obligation to the public to provide a classification rating system which will distribute the hazard equitably among the insureds. Those who are advocates of the former and ignore the latter tend to keep the rating structure in a state of flux. A few individual carriers may introduce competitive classification systems that will remain competitive so long as the same devices are not applied by the industry generally. However, the competitive value decreases as the field of application increases and eventually the originators reach the point of diminishing returns. The cycle is then repeated in a new vein with the same result occurring in due course.

The Utopian state toward which the advocates of stability may set

their course continues to recede into the future as these conditions recur. Although considerable progress has been made to produce a private passenger classification system that is equitable to all insureds, competition continues to create refinements designed for selective underwriting. Such refinements must stand the test of universal use and public acceptance. Judging from past activity, many classification rating features previously abandoned will be restored as experimentation continues. This is fertile ground for new developments in the underwriting and rating fields.

GRADUATION OF EXCESS RATIO DISTRIBUTIONS BY THE METHOD OF MOMENTS

BY
LEWIS H. ROBERTS

The Place of Excess Ratio Distributions in Casualty Insurance Rate Making

A risk who wishes to be self-insured to a degree, and whose size as measured by expected losses is sufficient to make it practicable, may elect to have his premium based in part on actual losses up to a specified limit. The balance of his premium would consist of charges by the insurance carrier for claim service and other carrier expenses plus a charge for the expected or average value, based on the experience of many risks, of losses in excess of the specified limit. Where the specified limit is a stated percentage of total expected losses, the ratio of expected losses in excess of that percentage to the total expected losses is called an "excess pure premium ratio" or more briefly, an "excess ratio" or "charge." Likewise the risk may elect to forego the full reduction in premium that would otherwise result in event a very low actual loss ratio should be incurred, in which case his premium would be adjusted up to a specified minimum percentage of the standard premium to reflect a saving to the carrier equal to the expected value of losses in excess of actual losses. The difference between the charge and the saving for selected maximum and minimum loss ratios is the net insurance charge. The standard premium is the premium that would be paid in the absence of any plan for basing premium on the actual losses of the risk. A rating plan which bases premium on actual losses is called a "retrospective" rating plan.

In order for such a plan to be equitable it is necessary for the carriers to calculate from a large body of experience the expected ratios to total losses of losses in excess of any specified loss ratio for risks of every size. From these calculations a table of charges and savings can be prepared for rating any risk under a retrospective rating plan. The table used currently for this purpose by the principal carriers is named Table M.

Previous Treatment of the Subject

In his paper entitled "On Graduating Excess Pure Premium Ratios", (P.C.A.S. Vol. XXVIII) Mr. Paul Dorweiler showed how indicated excess ratios calculated directly from actual data could be graduated for varying specified loss ratios for a given amount of expected losses and how they could be graduated for varying expected loss sizes for a given specified loss ratio. It was on the basis of his work that Table M was prepared from the 1934-37 experience of New York State Workmen's Compensation Risks.

In "Sampling Theory in Casualty Insurance", (P.C.A.S. Vol. XXX P. 56)

Mr. Arthur L. Bailey stated the linear relationship that exists between the sum of the charges in Table M and the variance of the loss ratios of risks with corresponding expected losses. (See page 10, *infra*.)

This convenient mathematical relationship permitted adjustment of Table M in 1954 to reflect increases in the variance of loss ratios for risks of a given expected loss size, due in large measure to increased claim costs over the average claim cost of the 1934-37 period and the consequent decrease in the number of claims required to produce a given amount of losses.

For this purpose it was necessary to find a formula for estimating the variance of the probability distribution* of loss ratios for a risk of average size from the experience of a group of risks with varying expected losses. The problem of a formula to use for the purpose arose because grouping of risks by size necessarily involves some spread in the size of risks included in any group. A straightforward calculation of the variance of their loss ratios according to elementary formulas would produce an upward bias in the estimate of the variance for a risk of average size owing to the hyperbolic relationship between expected losses and the expected values of the squares of differences between loss ratios and their expected values. The mathematical details of the relationship are covered in the Appendix, Notes 1 and 1a.

On the basis of Mr. Bailey's studies variances corresponding to various expected losses were calculated from the countrywide experience of Policy Year 1950. Table M was accordingly revised to match the calculated variances based on 1950 experience with the variances underlying the columns of insurance charges in Table M as previously developed.

Advantages of the Method of Moments

It is apparent that the so-called "Method of Moments" has already been of great use in studies of Table M through providing, by means of variance calculations, a simple check on the correctness of the totals of the insurance charges. This check, which tests the graduation of charges by size of expected losses, is sufficient where the charges in each column are believed to stand in the proper proportions to one another.

For a more complete check on the table it is necessary to study the manner in which insurance charges are graded from low loss ratios to high as well as from small risks to large risks. Since the direct computation of a table of excess ratios and their subsequent graduation is quite a laborious undertaking without, in the writer's opinion, a very satisfactory solution from either the practical or the theoretical standpoint, it should be worth while to try to extend the method of moments to cover the grading of charges. This method, which has found wide application in many fields of statistics as a tool for describing probability distributions, should make it possible by calculation of a few parameters to produce a graded table of insurance charges from a listing of individual risk experience. It has the further advantage that the economy of parameters required reduces the sampling error in the finished table. With an electronic calculator the labor would be reduced to very little.

*The probability distribution of loss ratios for a risk of given size is mathematically the same as the theoretical distribution by loss ratio of an infinite population of risks with equal expected losses.

EXPLANATION OF SYMBOLS

*Quantitative
Symbols*

- a* accident cost, except in Eq. (9), where it is a constant in the graduating equation for V_R^2 .
- b*₂ coefficient of u in Eq. (9).
- b*₃ coefficient of u in Eq. (10).
- b*₄ coefficient of u in Eq. (12).
- c* coefficient of u^2 in Eq. (9).
- e* base of natural logarithms.
- f* expected ratio of losses to permissible losses; estimated value of ER .
- g* dummy constant in Eqs. (2.3) et seq.
- h* dummy constant in Eqs. (3.2) et seq.
- i* ordinal subscript retaining same value as a quantity in Eqs. (29.1), (29.2), (29.3).
- m* number of risk size-groups, except in Eqs. (1.5) to (1.7) in which it is the expected value of n .
- n* number of risks in a size group, except:
(1) in Eqs. (29.1), (29.2), (29.3) where values of the argument (x) are numbered from 0 to n and (2) in Note 1a, where n denotes the number of cases in a sample, and (3) in Eq. (14) where it is an exponent.
- n*_a number of accidents.
- u* reciprocal of EL , except in Notes 1 and 1a, where it is a dummy variable used for illustration.
- v* dummy variable used for illustration.
- w* weighting coefficient used in normal equations; equals $f\Sigma X$.
- β_1 measure of skewness; equals μ_3^2/μ_2^3 .
- β_2 measure of kurtosis; equals μ_4/μ_2^2 .
- μ_n n^{th} moment of a variable; equals $E(v - Ev)^n$ if v is the variable.
- φ coefficient of correlation; defined in Note 1, Appendix.
- σ standard deviation; equals square root of μ_2 .
- B** spacing interval for given values of R_0 .
- K** upper limit of the range of a probability distribution; specifically, the lowest value of R for which charges would be shown in Table M as zero.
- L** losses of a risk.
- M** number of possible values of n in Note 1a.
- N** number of risks in all size-groups of risks.

P	premium of a risk.
R	ratio of losses to permissible losses (loss ratio) for a risk; equals L/X .
S	charge in Table M.
S_0	charge in Table M corresponding to R value of R_0 .
S_R	charge in Table M corresponding to R.
V	coefficient of variation, equals the standard deviation divided by the expected value.
X	permissible losses.

Subscripts a subscript adjoined to any symbol denotes that the value of the symbol associated with the subscript is to be used.

Accents

'	shown over a symbol; denotes the value indicated by experience, without graduation.
^	shown over a symbol; denotes graduated values derived from experience.

Operators

—	bar over a symbol; denotes its average value indicated by a sample. In connection with study of experience by size-groups of risks it denotes the average value based on one group.
=	double bar over a symbol; denotes the average value based on all size-groups of risks combined.
<i>av.</i>	average value; equivalent to bar over the symbol.
E	expected value; theoretically true average.
Σ	summation. In connection with size-groups it refers to summation over a single group. Subscripts and superscripts denote limits between which summation is to be taken.
$\Sigma\Sigma$	Used here only to denote summation over all size-groups.

$\int_a^b \dots \int_a^b$ where the number of \int strokes is n , denotes the n -fold integral evaluated at b minus the same evaluated at a .

Formulas for Estimating Moments of the Probability Distribution of Loss Ratios for a Risk of Average Size from Experience of a Group of Risks of Varying Size

To permit combination of the experience of risks with different permissible* loss ratios, actual loss ratios will be expressed as ratios of actual losses to permissible losses, which is the basis on which Table M is constructed. The mean or first moment of loss ratios associated with any risk is therefore assumed under Table M to be unity.

For any group of risks the average loss ratio is $\bar{R} = \Sigma L / \Sigma X$ where L is actual losses, X is permissible losses, and R is L/X for individual risks. EL , the expected losses for any risk, is fX where f is an abbreviation for ER , the expected ratio of L to X. If we have only one group of risks with which to deal and the group is sufficiently large to make \bar{R} statistically significant, \bar{R} can be used as an estimator of f . Use of \bar{R} adjusts for excess or deficiency in rate level on the basis of the experience of the group. Possible alternatives to \bar{R} as an estimator of f are unity and $\bar{\bar{R}}$, where $\bar{\bar{R}} = \Sigma \Sigma L / \Sigma \Sigma X$, the double Σ signs indicating summation over a number of groups. ϕ

Variance

The variance or dispersion of the probability distribution of loss ratios for a given risk is defined:

$$(1) \quad \sigma_R^2 = E(R - f)^2$$

where E denotes expected value.

V_R , the coefficient of variation of R, is defined as σ_R/f . Since the coefficient of variation of any variable is invariant for all multiples of the variable, and L is a multiple of R equal to XR, we have

$$(2) \quad V_R^2 = \frac{E(L - EL)^2}{(EL)^2}$$

Because Table M assumes ER equals unity, the variance of R underlying Table M is the same as V_R^2 in this case. We shall find it most convenient to calculate V_L^2 , knowing that $V_L^2 = V_R^2 (= \sigma_R^2 \text{ for } ER = 1.000)$.

Since EL equals fX we can substitute

$$(3) \quad V_R^2 = \frac{E(L - fX)^2}{(EL)^2}$$

Eq. (3) follows from the definition in Eq. (1) of the variance of R for a given risk. For a risk with expected losses equal to the average loss for the group we can define EL as $\frac{\Sigma L}{n}$. The numerator, however, will require close analysis. It must be estimated on the basis of the experience of all risks in a given group. Only if we can show that the value $\frac{1}{n} \Sigma (L - fX)^2$ based on the group

*The word "permissible," though superseded in current insurance usage by "expected," will be used here to avoid confusion with "expected" in the statistical sense of "average value." In this paper "expected" will be used only in the statistical sense.

ϕ Unity and \bar{R} are incorrect to use for small risks, however, because the ratio of average losses to permissible losses rises sharply for small risks when experience of risks with no losses is excluded, as is done in developing a table of excess ratios.

is a proper estimate of $E(L - fX)^2$ for a risk with expected losses equal to the group average is Eq. (2) a valid estimator of V_R^2 for that risk.

Since the proof of this is rather lengthy it is omitted here and given in Note 1 of the appendix.

On the basis of this proof we can use group experience in Eq. (3) to estimate V_R^2 for a risk of average expected losses.

Our estimate of V_R^2 is therefore

$$(4) \quad \dot{V}_R^2 = \frac{av.(L - fX)^2}{f^2(av. X)^2} = \frac{av. L^2 - f^2(av. X)^2}{f^2(av. X)^2} \quad (*) \phi$$

The operator "av." denotes an estimate of the expected value and is equivalent to the operator Σ/n .

Where f is estimated by the ratio \bar{R} , adjustment must be made for the loss of a degree of freedom by use of the so-called "finite multiplier" $n/(n-1)$.

$$(4a) \quad \dot{V}_R^2 = \frac{av. L^2 - (av. L)^2}{(av. L)^2} \frac{n}{n-1}$$

If f is estimated by the ratio \bar{R} , less than a whole degree of freedom has been lost in any group. The finite multiplier in that case uses the total number of risks in all groups and

$$(4b) \quad \dot{V}_R^2 = \frac{av. L^2 - \bar{R}^2 (av. X)^2}{\bar{R}^2 (av. X)^2} \frac{N}{N-1}$$

If f is estimated to be unity, no degrees of freedom are lost so no finite multiplier is required. Then

$$(4c) \quad \dot{V}_R^2 = \frac{av. L^2 - (av. X)^2}{(av. X)^2}$$

Skewness

The skewness of a probability distribution is measured by the statistic β_1 , which is invariant with respect to the origin or unit in terms of which a variable is expressed. Because of this invariance, β_1 is the same for losses as for corresponding loss ratios. We shall compute β_1 for losses and use it for loss ratios. β_1 is defined as the square of the third moment divided by the cube of the second moment, or μ_3^2/σ^6 . For losses or loss ratios of a risk with average expected losses:

$$(5) \quad \beta_1 = \frac{[E(L - EL)]^2}{\sigma_L^6}$$

$$(6) \quad \dot{\beta}_1 = \frac{[av. (L - fX)]^2}{[av. (L - fX)^2]^3} \quad (**)$$

The derivation of Eq. (6) is given in Note 2 of the appendix.

**See footnote * in Note 2 of the Appendix regarding the applicability of this equation to small risks.

*An accent over a statistic will denote an estimate of its value based on observed values, without graduation.

ϕSee Note 1a of the appendix regarding the accuracy of this equation for small risks.

Kurtosis

The peakedness (or to be more precise, the lack of peakedness) of a probability distribution is measured by the statistic β_2 , which like β_1 is invariant with respect to origin or unit of measurement. We shall compute β_2 for losses and use it for loss ratios. β_2 is defined as the fourth moment divided by the square of the second moment, or μ_4/σ^4 .

$$(7) \quad \beta_2 = \frac{E(L - EL)^4}{\sigma_L^4}$$

For losses or loss ratios of a risk with average expected losses:

$$(8) \quad \hat{\beta}_2 = \frac{av. (L - fX)^4}{[av. (L - fX)^2]^2} - 3V_X^2 \quad (*)$$

where V_X^2 is the squared coefficient of variation of permissible losses within the group. The derivation of Eq. (8) is given in Note 3 of the Appendix.

*Graduation of Indicated Moments**Variance*

The relationship between V_R^2 and the reciprocal of EL is practically linear for large risks. For small risks the curve is concave upward due to the u^2 term in the equation:

$$(9) \quad \hat{V}_R^2 = a + b_2u + cu^2 \text{ where } u = 1/EL$$

See Note 1, Appendix, for derivation of this equation.

The constant term, a , is included because only with as yet unattained perfect rating procedures that precisely estimated in advance the expected losses of each risk, would V_R^2 approach zero.

The weights to be applied to calculated values of u and V_R^2 in fitting Eq. (9) should, according to the Theorem on Observation Weights[#], be inversely proportional to the sampling variances^{**} of the respective observations. Since the principal element of sampling variance is, like V_R^2 , inversely proportional to the expected number of claims underlying the expected losses used, hence to the total expected losses, the weight to be given to each pair of V_R^2 and u values is $f\Sigma X$ for the group from which V_R^2 and u were calculated. Letting $w = f\Sigma X$, the normal ϕ equations for determining a , b_2 and c are:

$$(9a) \quad \Sigma w \hat{V}_R^2 = a\Sigma w + b_2\Sigma wu + c\Sigma wu^2$$

$$(9b) \quad \Sigma wu \hat{V}_R^2 = a\Sigma wu + b_2\Sigma wu^2 + c\Sigma wu^3$$

$$(9c) \quad \Sigma wu^2 \hat{V}_R^2 = a\Sigma wu^2 + b_2\Sigma wu^3 + c\Sigma wu^4$$

Eq. (9) provides \hat{V}_R^2 , the graduated value of V_R^2 , for any given value of EL when the values given to a , b_2 and c are derived from these normal equations.

*See footnotes in Notes 2 and 3 of the Appendix regarding the applicability of this equation to small risks.

ϕ Equations from which values of constants are derived according to the least squares method.

**Mean square error of observed values from their expected values.

#Harold Jeffreys, *Theory of Probability*, Clarendon Press, Oxford, 1948, p. 124, and other authors.

Skewness

The procedure for graduating β_1 values is the same as for V_R^2 values except that no constant term is needed since the distribution of R approaches the normal distribution for very large risks. Graduated values of β_1 are given by the equation

$$(10) \quad \hat{\beta}_1 = b_3 u$$

where b_3 is determined from the normal equation

$$(11) \quad b_3 = \frac{\sum w u \hat{\beta}_1}{\sum w u^2} \quad (*)$$

Kurtosis

β_2 values are graduated in the same way on the basis of the equation:

$$(12) \quad \hat{\beta}_2 = 3 + b_4 u$$

with b_4 determined from the normal equation

$$(13) \quad b_4 = \frac{\sum w (\hat{\beta}_2 - 3) u}{\sum w u^2}$$

The constant, 3, included above represents the kurtosis of a normal distribution, with the term $b_4 u$ measuring the excess of kurtosis in the observed distribution over the normal.

Relationship Between Charges in Table M and the Moments of the Underlying Probability Distribution

As pointed out by Mr. Nels M. Valerius in "Risk Distributions Underlying Insurance Charges" (P.C.A.S. Vol. XXIX), the second differences in a table of charges yield the theoretical frequency distribution of risks by size of entry ratio (ratio of actual losses to expected losses). A double integration of the risk distribution, therefore, provides a table of charges.

The mathematics of this relationship are very interesting and are readily extended to include higher moments. We use the reduction formula:

$$(14) \quad \int x^n f(x) dx = x^n \int f(x) dx - n x^{n-1} \int f(x) dx dx + n(n-1) \int f(x) dx dx dx - \dots + \dots$$

For $n = 2$ we have

$$(15) \quad \int x^2 f(x) dx = x^2 \int f(x) dx - 2x \int f(x) dx dx + 2 \int f(x) dx dx dx$$

The charge in Table M for a selected loss ratio of R_0 is defined mathematically by the equation

$$(16) \quad S_0 = \int_{R_0}^{\infty} R f(R) dR - R_0 \int_{R_0}^{\infty} f(R) dR$$

$$(17) \quad S_0 = 1 - \int_0^{R_0} R f(R) dR - R_0 [1 - \int_0^{R_0} f(R) dR] \quad (\text{since } ER = 1)$$

*Note that the normal equation is *not* $\sum w \hat{\beta}_1 / \sum w u$ as might be supposed by simple averaging. Eq. (11) is derived by minimizing the quantity: $\sum w (\hat{\beta}_1 - b_3 u)^2$ according to the principle of least squares. The same principle applies in connection with the normal equation for any ratio estimate (see β_2 , following).

On application of Eq. (14) this reduces to

$$(18) \quad S_0 = 1 + \frac{R_0}{\int_0^{\infty} f(R)dR} - R_0$$

The charge for a selected loss ratio of R is therefore

$$(19) \quad S_R = 1 + \int_0^{\infty} f(R)dR - R$$

Integration and doubling gives

$$(20) \quad 2 \int_0^{\infty} S_R dR = 2R + 2 \int_0^{\infty} \int_0^{\infty} f(R)dRdR - R^2$$

the constant of integration being zero.

If $f(R)$ is continuous over the finite range $0 \leq R \leq K$ and $\int_0^K f(R)dR = 1$,

the following equations hold. The second specification is met to as close a degree of precision as required by choosing K sufficiently large.

$$(21) \quad 2 \int_0^K S_R dR = 2K - K^2 + 2 \frac{K}{\int_0^K f(R)dR} - 1$$

Since for Table M $\int_0^K f(R)dR = 1$, $\int_0^K Rf(R)dR = 1$,

$$\text{and } \sigma_R^2 = \int_0^K R^2 f(R)dR - \left(\int_0^K f(R)dR \right)^2 = 1 - 1,$$

we have from Eq. (15) on taking the definite integral:

$$(22) \quad \sigma_R^2 = K^2 - 2K \frac{K}{\int_0^K f(R)dR} + 2 \frac{K}{\int_0^K f(R)dR} - 1$$

Again from Eq. (15) and taking the definite integral:

$$(23) \quad \int_0^K Rf(R)dR = K \int_0^K f(R)dR - \frac{K}{\int_0^K f(R)dR} - 1$$

$$(24) \quad \frac{K}{\int_0^K f(R)dR} - 1 = K - \int_0^K Rf(R)dR = K - 1$$

$$(25) \quad \sigma_R^2 = 2K - K^2 + 2 \frac{K}{\int_0^K f(R)dR} - 1 = 2 \int_0^K S_R dR - 1$$

$$= 2 \int_0^{\infty} S_R dR - 1$$

For values of S_R spaced at intervals of 0.1 or more for R the value of

$\int_0^K S_R dR$ should be estimated by Simpson's one-third rule or other non-linear quadrature formulas, but for spacing at intervals of .01 the trapezoidal rule is sufficient. The latter rule gives for a spacing interval of B :

$$(26) \quad \int_0^\infty S_R dR = 2B (\Sigma S_R - \frac{1}{2}) - 1$$

or if the charge (unity) for $R = 0$ is omitted, as in Table M,

$$(27) \quad \int_0^\infty S_R dR = 2B \sum_{R=B}^\infty S_R + B - 1$$

as stated by Mr. Arthur Bailey in the paper previously mentioned.*

The principles used in deriving Eq. (25) when extended to higher moments of R give:

$$(28) \quad \mu_3 = 6[(K - 1) \int_0^K S_R dR - \frac{K}{0} S_R dR dR] + 2$$

$$(29) \quad \mu_4 = 12[(K^2 - 2K + 1) \int_0^K S_R dR + 2(1 - K) \frac{K}{0} S_R dR dR + 2 \frac{K}{0} S_R dR dR dR] - 3$$

Equations (28) and (29) have the disadvantage, for purposes of practical computation, that because the values of μ_3 and μ_4 are derived as differences, accurate calculation of small values of these statistics is subject to considerable relative error unless precise values of the several definite integrals of S_R can be calculated.

In evaluating multiple integrals by single quadrature formulas it is necessary to use the calculated values of the $(n - 1)$ th integral at the selected values of the argument when applying the quadrature formula to estimate the n 'th integral.

*Mr. Bailey (page 56) showed the summation as \sum_0^∞ rather than $\sum_{R=B}^\infty$ but this is apparently an error if the positive sign is given to B . If \sum_0^∞ is used the sign of B must be negative. In its memorandum dated November 12, 1952, in which the method used in the 1953 studies of Table M is described, the National Council on Compensation Insurance indicated the summation \sum_0^∞ and showed sums of charges for various expected loss sizes. The figure shown for \$300,000 expected losses, the only one checked by the writer, reflected summation correctly from $R = .01$.

The formulas shown below are equivalent to repeated application of the trapezoidal rule in accordance with the preceding paragraph. For completeness, the well known rule for single quadrature is shown first:

$$(29.1) \quad \int_{x_0}^{x_n} y dx \doteq h \left[\frac{y_0 + y_n}{2} + \sum_1^{n-1} y_i \right]$$

$$(29.2) \quad \iint_{x_0}^{x_n} y dx dx \doteq h^2 \left[(.5n - .25)y_0 + \sum_1^{n-1} (n - i)y_i + .25y_n \right]$$

$$(29.3) \quad \iiint_{x_0}^{x_n} y dx dx dx \doteq \frac{h^3}{2} \left\{ \left[\frac{2n - 1}{4} + \frac{(n - 1)^2}{2} \right] y_0 + \sum_1^{n-1} [(n - i)^2 + .5] y_i + .25 y_n \right\}$$

Conversion of Graduated Moments into a Table of Charges and Savings

Because of the relationship:

Saving = Entry ratio + Charge - Unity, it is sufficient to calculate a table of charges, from which savings are derived by use of this equation.*

Three principal types of frequency functions are available for calculating the probability distribution from the graduated moments, namely Pearson's system of curves, the Gram-Charlier Series and the Edgeworth Series. Pearson's system is recommended here. Elderton's investigations^φ indicate that Pearson's curves are best adapted to representation of extremely skewed distributions (characteristic of loss ratios for small risks) and approach the normal distribution for such variates as the loss ratios of very large risks. Pearson's curves have the further advantage that they do not develop negative frequencies (as the other series tend to do near the tails of the distribution).

Because the procedure for fitting these curves is published elsewhere^φ, there is no need to repeat it here.

*The saving is defined mathematically by

$$\text{Saving} = R_0 \int_0^{R_0} f(R) dR - \int_0^{R_0} R f(R) dR$$

By application of Eq. (14) this reduces to

$$\text{Saving} = \frac{R_0}{0} \int f(R) dR dR$$

which is the charge (Eq. 18) minus unity plus the entry ratio, R_0 .

^φElderton, Sir W. P., *Frequency Curves and Correlation*, 3rd Edition, Cambridge University Press, explains the procedure in great detail. Many examples of fitting these curves appear in *Biometrika*.

In performing the double integrations of the risk distributions it is essential to add -1 , the constant of integration, to $\int f(R)dR$, and $+1$ to $\int \int f(R)dRdR$ as is seen by differentiating Eq. (19):

$$(29a) \quad dS_R/dR = \int f(R)dR - 1$$

$$(29b) \quad d^2S_R/dR^2 = f(R)$$

Use of the trapezoidal quadrature rule for integration with R spaced at intervals of .01 produces the finished table for selected sizes of expected losses. Charges for intermediate expected losses should be calculated by interpolation.

The Problem of Sampling Error

The question as to whether a given volume of experience is sufficient for derivation of a usable table requires that an estimate be made of the sampling error in the final results. The best way to accomplish this in theory is to divide the available experience into a number of parts or sub-samples selected on a random basis so that a given risk has equal probabilities of being included in any of the several parts, and compute the standard deviations from the several sets of values derived from the sub-samples. Where the values of interest are the end products of a long chain of arithmetical operations, however, this procedure is prohibitive in cost unless electronic calculating equipment is available.

A short cut is to compute the sampling errors of certain key statistics. For this purpose we can best choose \hat{V}_R^2 since V_R^2 for given expected losses is a linear function of the sum of the charges as noted earlier. The coefficient of variation of \hat{V}_R^2 is therefore the relative sampling error in the charges.

The simplest method of calculating sampling errors of the \hat{V}_R^2 values is to compute the values of $(\hat{V}_R^2 - \hat{V}_R^2)^2 = s^2$ for each group, which is to say for each value of u used, and fit a curve to plotted values of s^2 and u . Representing this curve by $f(u)$, the coefficient of sampling error of \hat{V}_R^2 for a given value of u is estimated by:

$$(30) \quad V_{\hat{V}_R^2} = \sqrt{f(u)/[(m-3)g(u)\hat{V}_R^2]}$$

where $g(u)$ is the experience-density function described below, m the number of size-groups and 3 the number of constants in Eq. (9).

Values of s^2 may tend to be larger for large values of u , but this is not necessarily the case. It depends on the numbers of risks in the various size-groups. If all size-groups have equal total expected losses, $f(u)$ should be a straight line with zero slope. The distribution of risks by size, however, will ordinarily prevent use of such size groups without introducing excessive ranges of size within certain groups. If the total expected losses in each group increases in proportion to average expected losses (number of risks in each group constant) the curve should be a straight line with positive slope. This procedure leads to wider individual deviations of \hat{V}_R^2 from \hat{V}_R^2 for large u . The reliability of \hat{V}_R^2 in a given region of u values, however, depends on the total weight given to \hat{V}_R^2 values of that region in the derivation of Eq. (9), that is, on the total expected losses of the region. The grouping of risks by size, therefore, should be done in a regular way so that the total expected losses corresponding

to a given value of u , hence to given average expected losses per risk, will be a smooth function of u , not necessarily expressed algebraically. It can be expressed merely by a graph of the total expected losses for each size-group when plotted against corresponding u values. Denote the function represented by this graph as $g'(u)$. It is also necessary to reflect the spacing of u values. This is done by plotting, against the means ($u_{i+.5}$) of successive u values, the values of $u_{i+1} - u_i$. Denote the function represented by a graph of these points as $\Delta(u)$. The product of the ratio of $g'(u)$ to the average expected losses per group times the ratio of the average separation of u values to $\Delta(u)$ gives the experience density function of u :

$$(31) \quad g(u) = \frac{mg'(u) \cdot [u_{\max.} - u_{\min.}]}{[\Sigma fX] \cdot [m\Delta(u)]} = \frac{g'(u)[u_{\max.} - u_{\min.}]}{\Delta(u)\Sigma fX}$$

Choice of size-group ranges will affect $f(u)$ but this effect will be cancelled by $g(u)$ which works in the opposite direction. Narrow groups in a region of u produce unreliable \hat{V}_R^2 values, hence large values of $f(u)$, but there will be more values of \hat{V}_R^2 in that region so the reliability of \hat{V}_R^2 is not reduced. The presence of $g(u)$ in the denominator of the radical of Eq. (30) expresses this by dividing the $f(u)$ values by a proportionately small number.

Values of \hat{V}_R^2 will not have an approximately normal probability distribution for size-groups with average expected losses as low as \$1,000 unless a good many—say 100 or more—risks are included in the group. The probability distribution of \hat{V}_R^2 , however, can be considered normal since it is a kind of average \hat{V}_R^2 based on all groups, only the smallest of which need be as low as \$1,000 under the present form of Table M. We are therefore justified in using the normal curve to interpret $V_{\hat{V}_R^2}$ values in terms of the probability of stated percentages of sampling error.

APPENDIX

Note 1

Derivation of formula for V_R^2 , the squared coefficient of variation of loss ratios for risks with average expected losses, estimated from experience of a group of risks of varying size.

We define

$$(1.1) \quad \sigma_L^2 = E(L - EL)^2 = E[L - 2LEL + (EL)^2]$$

$$(1.11) \quad = EL^2 - (EL)^2$$

Since $L = n_a \bar{a}$ and $E\bar{a} = E\bar{a}$

$$(1.2) \quad \sigma_L^2 = En_a^2 \bar{a}^2 - (En_a \bar{a})^2$$

We use the coefficient of correlation, $\varphi_{u,v} = \frac{Euv - EuEv}{\sigma_u \sigma_v}$, in Eq. (1.2) to

give

$$(1.3) \quad \sigma_L^2 = En_a^2 E\bar{a}^2 - (En_a E\bar{a})^2 + (\text{terms involving } \varphi) \dagger.$$

Assuming a Poisson probability distribution for n_a , $\sigma^2 n_a = En_a$ but for any

†These terms, dropping the subscript on n , are

$$\varphi_{n^2, \bar{a}^2} \sigma_{n^2} \sigma_{\bar{a}^2} - 2EnE\bar{a}\varphi_{n, \bar{a}} \sigma_n \sigma_{\bar{a}} - \varphi_{n, \bar{a}}^2 \sigma_n^2 \sigma_{\bar{a}}^2$$

Where for a given risk average accident cost (severity) is statistically independent of accident frequency, φ is equal to zero so these terms can be dropped. The assumption of a zero

variable, z , $Ez^2 = (Ez)^2 + \sigma_z^2$, so $En_a^2 = (En_a)^2 + En_a$. Also, for any sample of n with mean \bar{x} drawn from a universe with variance σ_x^2 , $\sigma_{\bar{x}}^2 = \sigma_x^2/n$. Eq. (1.3) may therefore be written, if we neglect the terms involving φ :

$$(1.4) \quad \sigma_L^2 \doteq [(En_a)^2 + En_a] \left[(Ea)^2 + \frac{\sigma_a^2}{En_a} \right] - (En_a Ea)^2$$

Division by $(EL)^2 \doteq (Ea)^2(En_a)^2$ gives, letting $m = En_a$

$$(1.5) \quad V_L^2 \doteq \frac{1 + V_a^2}{m} + \frac{V_a^2}{m^2}$$

$$(1.51) \quad V_L^2 \doteq \frac{(Ea)(1 + V_a^2)}{EL} + \frac{(Ea)^2 V_a^2}{(EL)^2}$$

The second term of Eq. (1.5) is negligible for large risks but not for risks with only a few thousand dollars expected losses since with present average claim costs of about \$700 for Workmen's Compensation, m^2 in such cases is not a large number.

The Poisson assumption regarding the probability distribution of the number of accidents was investigated by Mr. John Carleton (P.C.A.S. Vol. XXXII, p. 26). He stated "concern over the application of the Poisson distribution to casualty insurance accidents can be confined to special situations in which accidents are definitely known to be other than independent." We therefore assume the Poisson distribution ordinarily is valid for use in these equations.

To continue, we define V_R^2 as σ_R^2/f^2 , $f = EL/EX$ so $EL = fEX$, hence $\sigma_L = \sigma_R EX$. Then since $V_L^2 = \sigma_L^2/(EL)^2$, $V_L^2 = [\sigma_R^2/f^2(EX)^2]/(EX)^2 = \sigma_R^2/f^2 = V_R^2$.

V_R^2 is therefore given by dividing Eq. (1.1) by $(EL)^2$:

For a risk with expected losses of X

$$(1.6) \quad V_R^2 = E(L - EL)^2/(EL)^2 = [EL^2 - (EL)^2]/(EL)^2,$$

$$(1.61) \quad = E(L - fX)^2/f^2X^2$$

We substitute for V_R^2 the value in Eq. (1.51) and multiply by $f^2X^2 = (EL)^2$, with $m = EL/Ea$

$$(1.7) \quad f^2X^2 \left[\frac{1 + V_a^2}{m} + \frac{V_a^2}{m^2} \right] = E(L - fX)^2$$

Since $EL = mEa = fX$:

$$(1.8) \quad (Ea)(fX)(1 + V_a^2) + (Ea)^2 V_a^2 = E(L - fX)^2$$

Eq. (1.8) applies to individual risks. The value of $(L - fX)^2$ for each risk is

value for φ for lines of insurance subject to retrospective rating is believed to be justified as a practical and necessary approximation. Although a risk's adoption of a new process may change the nature of the hazard and temporarily produce a correlation between severity and accident frequency (as by increasing the number of small accidents), there appears to be no *a priori* reason to expect a correlation between severity and frequency in the normal fluctuations of experience.

*The exact value of this term is $\sigma_a^2 En_a^{-1}$, rather than σ_a^2/En_a as shown. With a Poisson probability distribution of n , however, En^{-1} for non-zero values of n is closely approximated by $1/En$. The case of small values of En is discussed in Note 1a.

used as an estimate of its own expected value. Summing over the group of n risks:

$$(1.9) \quad f(Ea)(\Sigma X)(1 + V_a^2) + n(Ea)^2 V_a^2 = \Sigma(L - fX)^2$$

$$(1.91) \quad f(Ea)(av.X)(1 + V_a^2) + (Ea)^2 V_a^2 = av.(L - fX)^2$$

Dividing by $[f(av.X)]^2$ we get

$$(2.0) \quad \frac{(Ea)(1 + V_a^2)}{f av. X} + \frac{(Ea)^2 V_a^2}{(f av. X)^2} = \frac{av.(L - fX)^2}{(f av. X)^2}$$

The left member of Eq. (2.0) is recognized as V_R^2 from Eq. (1.51) with EL represented by $f(av.X)$. The required formula is therefore

$$(2.1) \quad \hat{V}_R^2 = \frac{av.(L - fX)^2}{(f av. X)^2} = \frac{av. L^2 - f^2(av. X)^2}{f^2(av. X)^2} \quad \text{Q.E.D.}$$

As noted in connection with Eq. (4a), (4b) and (4c), finite multipliers are necessary if f is estimated from the same experience as used to compute $av.(L - fX)^2$ or from a larger body of experience which includes it.

The formula used in the 1953 studies of Table M to calculate the variance indicated by the experience of a given group of risks for a risk with average expected losses has not, to the writer's knowledge, previously been published. The worksheets for those calculations were based on the formula:

$$(2.2) \quad \hat{V}_R^2 = \frac{av. P av. L^2/P - (av. L)^2}{(av. L)^2} \frac{n}{n - 1}$$

It will be noted that this formula differs from Eq. (4a) in that (1) no recognition is given in this formula for variation between risks in the expected loss ratio, and (2) in the presence of P in the numerator. This writer has been unable to find the theoretical basis for Eq. (2.2) because the expected value of the first term of the numerator is not EL^2 , which is needed in Eq. (1.6) above, but a complex expression involving the coefficients of correlation between $av. P$ and $av. L^2/P$ and between L^2 and $1/P$.

Note 1a

Calculation of V_R^2 for small risks.

It is elementary that for fixed n , $E\bar{u} = Eu$ and $\sigma_{\bar{u}}^2 = \frac{\sigma_u^2}{n}$

With n variable, we achieve sufficient generality by considering n free to take M possible positive integral values, the highest of which is K , not all values of n being necessarily unequal. Then the average of all possible sample means is

$$(2.21) \quad E\bar{u} = \frac{1}{M} \left[u_{11} + \frac{u_{21} + u_{22}}{2} + \dots + \frac{u_{K1} + u_{K2} + \dots + u_{KK}}{K} \right]$$

$$(2.22) \quad = \frac{1}{M} \left[Eu + \frac{2Eu}{2} + \dots + \frac{KEu}{K} \right] \text{ with } M \text{ terms in } [\quad].$$

$$(2.23) \quad = \frac{1}{M} [MEu] = Eu.$$

The expected value of a sample mean is therefore equal to the expected value of the variate, regardless of the probability distribution of the number of cases that comprise the sample.

The average of all possible squared deviations of sample means from the population mean for $n = 1, 2, 3 \dots K$ is

$$(2.24) \quad E(\bar{u} - Eu)^2 = \frac{1}{M} E \left[(\bar{u}_1 - Eu)^2 + (\bar{u}_2 - Eu)^2 + \dots + (\bar{u}_K - Eu)^2 \right]$$

$$(2.25) \quad = \frac{1}{M} [\sigma_u^2 + \sigma_u^2/2 + \dots + \sigma_u^2/K] \text{ with } M \text{ terms in } [\quad]$$

$$(2.26) \quad = \sigma_u^2 E n^{-1}$$

The expected value of the mean square deviation of a sample mean from the population mean is therefore the variance of the variate multiplied by the expected value of the reciprocal of the number of cases in the sample.

For Table M we are concerned with the variance of non-zero losses. Consequently, the expression $E n^{-1}$ refers to the expected number of accidents *provided at least one occurs*, which restriction is essential if the expression is to have a finite value. Likewise the value of $E n$ must reflect the same restriction. For large risks, the probability of zero accidents is negligible, hence for them the restriction against non-zero values is insignificant; but it is important for small risks where zero losses have considerable probability.

We see, therefore, that for non-zero losses the probability distribution of n is not the complete Poisson distribution, but only that portion of it for values of n equal to or greater than one. This considerably complicates the mathematics for small risks. The mean of such a distribution is $m/(1 - e^{-m})$ where m is the mean of the complete Poisson distribution, and the variance is $\frac{m^2 + m}{1 - e^{-m}} - \frac{m^2}{(1 - e^{-m})^2}$, as compared with m for both the mean and variance of the complete Poisson distribution.

Because of these mathematical complications in the way of accurate calculation of \hat{V}_R^2 for small risks when size groups contain a wide variation in size of risk, the most practical solution is to use Eq. (4a), which is considerably more accurate than the simple *av.* $(R - \bar{R})^2$ but still only an approximation, and keep the error down by making size-groups for small risks as narrow as the volume of experience and computing facilities will permit. The resulting scatter of \hat{V}_R^2 values will be ironed out in the graduated values.

Note 2

Derivation of formula for β_1 of loss ratios of risks with average expected losses, estimated from experience of a group of risks of varying size.

Rather than go through detailed calculations similar to those used for V_R^2 in Note 1, which were given at length because the formula advanced in Eq. (2.1) differs from the one used in the past, making it desirable to show its derivation from first principles, we shall simplify the derivation of β_1 by

making use of the known inverse relationship between β_1 of the average of a sample and the number of cases on which the average is based.*

The experience of a risk may be regarded as that of a sum of short term exposures, each with expected losses of one dollar. The number of exposures for a risk is therefore equal to its expected losses. The loss ratio for a risk is therefore the average loss per exposure. Then for each risk

$$(2.3) \quad \beta_1 = \frac{[E(R - ER)]^2}{[E(R - ER)]^3} = \frac{b_3}{fX}$$

Since β_1 is invariant with respect to units of measurement:

$$(2.4) \quad \beta_1 = \frac{[E(L - fX)]^2}{[E(L - fX)]^3} = \frac{b_3}{fX}$$

We have shown in Eq. (1.8) that, except for the relatively small term† $(Ea)^2 \cdot V_a^2$, $E(L - fX)^2$ is proportional to fX so that approximately

$$(2.5) \quad \frac{[E(L - fX)]^2}{g^3(fX)^3} = \frac{b_3}{fX}$$

where g is a constant

$$(2.6) \quad E(L - fX)^3 = fX \sqrt{b_3 g^3}$$

Summing over all risks in the group and dividing by the number of risks, with $(L - fX)^3$ treated as an estimate of its own expected value for each risk and then squaring:

$$(2.7) \quad [av. (L - fX)]^2 = [f av. X]^2 b_3 g^3$$

Division by $(f av. X)^3 g^3$ gives

$$(2.8) \quad \frac{[av. (L - fX)]^2}{g^3(f av. X)^3} = \frac{b_3}{f av. X}$$

The denominator of the left member of Eq. (2.8) is equivalent to $[E(L - fX)]^2$ estimated by $[av. (L - fX)]^2$, hence for the risk with average expected losses

$$(2.9) \quad \beta_1 = \frac{[av. (L - fX)]^2}{[av. (L - fX)]^3} = \frac{b_3}{f av. X} \quad \text{Q.E.D.}$$

Note 3

Derivation of formula for β_2 of loss ratios of risks with average expected losses, estimated from experience of a group of risks of varying size.

*See Kendall, *The Advanced Theory of Statistics*, Vol. 1, page 284 (Chas. Griffin & Sons, Ltd. 1948). The expected values of μ_3 and μ_4 for sample means as given there for sampling from a finite population of N individuals, reduce to β_1/n and $\beta_2/n + 3$ on taking the limit as $N \rightarrow \infty$ and dividing by μ_2^3 and μ_2^2 respectively.

†It should be realized that this term becomes important for risks with small expected losses. With average accident costs of \$700, expected losses of \$700 give the first and second terms the same order of magnitude.

For this reason Eqs. (6) and (3.2) are rather rough approximations for risks with only \$1,000 or so of expected losses when the range of sizes is wide.

The error is minimized by keeping the range of sizes in groups of small risks as narrow as possible. This will reduce the reliability of individual β values but not the reliability of graduated values, since there will be more β values underlying the graduating lines given by Eqs. (10) and (12). The exact formulas for calculating β_1 or β_2 from experience of small risks of varying size for a risk of average size are too complicated to make their use practicable.

$(\beta_2 - 3)$ of sample averages is, like β_1 , inversely proportional to the number of cases. ϕ The derivation here is similar to that in Note 2. For each risk

$$(3.1) \quad \beta_2 - 3 = \frac{E(L - fX)^4}{[E(L - fX)^2]^2} - 3 = \frac{b_4}{fX}$$

Since $E(L - fX)^2$ is proportional* to fX by Eq. (1.8)

$$(3.2) \quad \frac{E(L - fX)^4}{(hfX)^2} - 3 = \frac{b_4}{fX}$$

where h is a constant

$$(3.3) \quad E(L - fX)^4 = 3(hfX)^2 + b_4h^2fX$$

Treating the value of $(L - fX)^4$ for each risk as an estimate of its own expected value, summing for all risks in the group and dividing by the number of risks we get

$$(3.4) \quad av. (L - fX)^4 = 3h^2f^2 av. X^2 + b_4h^2f av. X$$

Dividing by $h^2f^2(av. X)^2$ we have

$$(3.5) \quad \frac{av. (L - fX)^4}{h^2f^2(av. X)^2} - \frac{3 av. X^2}{(av. X)^2} = \frac{b_4}{f av. X} = \dot{\beta}_2 - 3$$

Since $av. X^2 = (av. X)^2 + \sigma_X^2$

$$(3.6) \quad \dot{\beta}_2 = \frac{av. (L - fX)^4}{[av. (L - fX)^2]^2} - 3V_X^2$$

Q.E.D.

ϕ Kendall, loc. cit.

*Remarks in footnote \dagger of Note 2 apply here as well.

REVISION OF RATES APPLICABLE TO A CLASS OF PROPERTY FIRE INSURANCE

BY
C. OTIS SHAVER

THE OBJECTIVE AND DEFINITION OF TERMS USED

Many volumes have been written covering the general subject of property insurance, with emphasis on underwriting, claims settlement, and contractual relationships, but little has been written on the processes involved in rating fire insurance and the subsequent revision of these rates. For the most part only a relatively few informed people connected with insurance departments, rating bureaus, and statistical associations have been in a position to discuss the validity of the methods used in fire insurance rate making.

During the past few years, due to this lack of information, fire insurance rates have been the subject of controversy. The controversy has centered largely around the question of adequacy or inadequacy of the rates as affected by deviations from the prescribed bureau plans.

Objective.—It is to be hoped that an analytical study of the processes involved in a revision of the fire insurance rates for a given class of property will afford enough information from which those who are somewhat informed can draw reasonable conclusions as to the propriety of the principles involved, to the end that those having actuarial inclinations may be challenged to make a deeper investigation into the field of fire rating than is being attempted in this particular study. It is to be noted that the procedure outlined in this study is that which is followed by the bureaus operating in certain midwestern states such as Ohio, Indiana, Michigan, and Kentucky and may differ to some degree from the methods used in the Eastern part of the country. No attempt has been made to cover any differences which may exist as between these different areas.

Since the main problem of ratemaking is the adequacy of the rates, this naturally leads to a discussion of the statistical basis and to questions of discrimination between various classes of risks.

Effective January, 1947, a revised Fire Statistical Plan was adopted. This Plan is generally known as the *Standard Classification of Occupancy Hazards* and has been approved by the National Association of Insurance Commissioners. The Plan contains 115 occupancy classes, which in turn are collected into five major groupings of risks: residential, mercantile, non-manufacturing, manufacturing, and sprinklered.

For purposes of this study the three largest occupancy classes contained in Group I (Residential) of the Plan have been chosen. These classes are as follows:

Class No. 009—Household contents of Dwellings, when contents are written on separate policy.

Class No. 019—Dwelling—Buildings and Contents, when both written on same policy.

Class No. 029—Dwellings—Buildings only when written on separate policy.

These three classes apply entirely to class rated risks, whereas most of the other classes in the Plan are for specifically rated risks.

Fire loss experience is reported in accordance with this class which in turn forms the basis of adjustment by classes. Thus the classified data for the adjustment of the fire loss cost of insurance rates are provided. These fire loss experience data may be used in determining a statewide rate level as well as for classes of risks.

While such expressions as earned premiums, incurred losses, and expense ratio are generally familiar, there are certain expressions peculiar to fire insurance which should be mentioned.

Catastrophe Allowance.—A loading in the rate to compensate for the effects which a conflagration might have upon the normal or expected loss experience.

Protection Classification.—One of a series of categories established by the National Board of Fire Underwriters to identify certain types of risks by kinds of construction in combination with certain town gradings for statistical identification.

Occupancy Class.—One of a series of categories established by the National Board of Fire Underwriters to identify certain types of risks by kinds of occupancy for statistical identification.

The method followed in this study is one that is currently used by some of the midwestern fire rating bureaus in making class rate adjustments and for the most part reflects the effects of the loss experience on the rates.

RATING METHODS

Fire Insurance Rates are separated into two main categories: Class Rates and Specific Rates.

Class Rates.—Class rates apply to risks that are of substantially the same general character, and where minor differences in exposure can be ignored without any material increase in risk occurring. Such risks are classified as to construction, occupancy, and fire protection and include dwellings, small apartments, and sometimes churches, clubs, schools, farms, small mercantiles, and certain special risks.

Dwellings are universally class rated and may make up from 50% to 80% of the number of insurable risks in a town or city. They are, however, mostly small risks and therefore may not produce more than 10% to 25% of the premium volume.

Specific Rates.—A specific rate applies to an individual risk that is not subject to class rates; it is the rate for a particular building, or the rate for the contents of a certain tenant. The main difference in determining specific rates, as opposed to class rates, is the procedure involved. The determination of each specific rate requires an inspection for hazards that may create conditions favorable to fire damage. The deficiencies and hazards are evaluated in accordance with sets of standards, with allowances for protection devices, and a rate for a

specific building and rates for each occupant of this building is formulated.

Statistical Accumulation.—The National Board of Fire Underwriters collects fire insurance premium and loss statistics on a state by state basis from all stock companies and files consolidated reports of these statistics with state insurance departments and rating bureaus for their use in connection with making rate revisions.

In addition to the National Board there are two other statistical organizations which accumulate and report fire insurance statistics to the insurance departments, namely; the Mutual Insurance Advisory Association and the National Association of Independent Insurers. These statistics are not used for rating purposes.

Throughout the country there are several regional advisory organizations which co-ordinate the functions of the rating bureaus in their respective jurisdictions. These organizations afford advice in connection with the development of new coverages, changes in coverages, and changes in rate levels.

Fire rates are made in most states by rating bureaus which have been established by the stock companies. State laws require that these bureaus furnish their services to any other companies desiring them. A few states have bureaus established by statute and all companies operating in such states are required to belong to them.

Rate Make-up.—Fire insurance rates are expressed in terms of the amount of dollars or cents charged for \$100 valuation of property insurance for a period of one year. These rates should result in sufficient aggregate premiums to provide for (1) losses, (2) expenses of conducting business, (3) an allowance for catastrophe, and (4) a reasonable profit. Adjustment expense is included in the operating expenses and not as a part of losses.

While fire insurance rates are promulgated on a statewide basis and follow the same basic pattern as to development, certain differences exist between states as to the proportionment of the rate for losses, expenses, profit, and catastrophe as well as to the application of credits applicable to specifically rated risks. The following formula most nearly reflects the pattern which is in use in the State of New York and the midwestern states mentioned above:

Loss Payment	.475
Underwriting Expense	.465
Conflagration Allowance	.010
Profit (Underwriting)	.050

Underwriting profit as referred to in this formula shall be determined with the use of direct earned premiums and incurred loss and incurred expense figures without regard to reinsurance.

This formula for the most part reflects the expense and loss experience of the stock companies reporting to the National Board, and the profit factor (5% profit plus 1% catastrophe) follows the 1921 Profit Formula of the National Board as modified in the 1949 Subcommittee Report of the N.A.I.C. In connection with adjusting rates, it is assumed that no adjustment shall be made if the indicated profit is

within a tolerance zone of two percentage points above or below such 6% factor.

ACCUMULATION OF EXPERIENCE

In order to assure the accumulation of experience statistics, the Standard Classification requires that the companies furnish annually to the various statistical agencies premium and loss reports of written premiums by occupancy-construction classification and further division by term of policy. These data for premiums written are classified according to term of policy, to be converted to premiums earned for each class of risk and construction group through the application of specially computed fractions or factors.

Annual Call.—This detailed report of loss experience is called the Annual Calendar Year Report and is furnished to the companies' statistical agents. In this report, written premium and paid loss detail is shown by occupancy class and construction-protection class. Such a report would show the following information for one of the residential classes:

<i>Occupancy Class</i>	<i>Construction-Protection</i>	<i>Written Premium</i>	<i>Paid Losses</i>
029	1	\$554,250	\$114,385
029	2	408,100	72,427
029	3	67,039	14,587
029	4	57,165	19,148

This is direct experience. (Gross of reinsurance.) To attempt to use loss experience accumulated on a net basis would present a rather unreal picture in instances where changes have occurred in the reinsurance program during the period covered due to the fact that reinsurance cannot be regulated and further, since the rates are intended to cover the full effects of the losses on the class of business to which they are applicable, it is only proper that direct experience be used.

Pro-rata Earned Premium.—To obtain the pro-rata earned premium to be used for each class of business involved, requires that each transaction be identified as to the policy term and spread by year written and then factored on the basis of five year premium being earned—1/10, 1/5, 1/5, 1/5, 1/5, and 1/10 each year. Three year business—1/6, 1/3, 1/3, and 1/6 each year. One year business—1/2 and 1/2 each year. This array applied in a given year would appear as follows:

Class	Construction- Protection	Year of Writing	Term	Written Premium	Factor	1955 Earned Premium
029	1	1950	5	\$139,643	.1000	\$13,964
029	1	1951	5	147,312	.2000	29,462
029	1	1952	5	121,137	.2000	24,227
029	1	1952	3	182,149	.1667	30,358
029	1	1953	5	139,800	.2000	27,960
029	1	1953	3	210,175	.3333	70,058
029	1	1954	5	138,849	.2000	27,770
029	1	1954	3	210,177	.3333	70,059
029	1	1954	1	63,110	.5000	31,555
029	1	1955	5	188,556	.1000	18,856
029	1	1955	3	275,518	.1667	45,919
029	1	1955	1	90,176	.5000	45,088

1955 Earned Premium \$435,276

Incurred Losses.—As stated previously, incurred losses are to be used to measure the loss severity. Under the National Board Plan incurred losses are available at present for four years, but most bureaus have used the usual formula, net losses paid for period covered, plus outstanding losses at end of period, minus outstanding losses at beginning of period. Earned premium and incurred losses for a five year period must be set up for each occupancy class subject to adjustment and separated by construction and protection group.

Class	Construction- Protection	Earned Premium	Incurred Losses
029	1	\$1,706,717	\$425,989
029	2	1,351,309	329,181
029	3	202,033	53,819
029	4	193,501	51,503

Adjustment of Earned Premiums.—Before loss ratios are calculated, earned premium must be adjusted to current rate levels. This requires that an accurate record be maintained of all rate changes made during the period under study.

The following example will illustrate what is involved in making these adjustments:

A rate study is being undertaken based on the loss experience of the five year period, 1951-1955. For Class 029-frame protected dwellings (construction-protection group 1)—a 3% rate increase was effective January 1, 1952 and a 4% rate increase was effective October 1, 1955. All premium written before October 1, 1955 must be adjusted. The premium written in 1951 would have to be adjusted for both rate increases. This results in a 7% increase in written premium for 1951, and a 4% increase for 1952, 1953, and 1954, and a 3% increase for 1955.

The following table shows the appropriate adjustments to be made:

Year	Class	Construction- Protection	Written Premium	Percent of Adjustment	Adjusted Written Premium
1951	029	1	\$275,720	7	\$295,020
1952	029	1	319,138	4	331,904
1953	029	1	421,087	4	437,930
1954	029	1	412,136	4	428,621
1955	029	1	554,250	3	570,878
Five Year Total			\$1,982,331		\$2,064,353

The relationship between the actual and the adjusted written premium for the five years combined provides a factor to adjust the five year earned premium to current rate levels. The following formula would apply:

$$\frac{(\text{Adjusted Written Premium})}{(\text{Actual Written Premium})} \times \frac{\text{Actual Earned Premium}}{\text{Premium}} = \frac{\text{Earned Premium}}{\text{Adjusted to Current Rate Levels}}$$

Loss Ratio.—Finally then, loss ratios can be computed by dividing incurred losses by the adjusted earned premiums. These loss ratios are ready to be inserted into the rate formula:

$$\frac{\text{Incurred Losses}}{\text{Adjusted Earned Premium}} = \text{Loss Ratio}$$

Expenses.—Although the fire rate adjustment is not based on expenses, expenses nevertheless play a part in determining the allowable loss ratio .475 as reflected in the formula referred to previously in this study. Some states use a 50% allowable loss ratio and some others a different one. This formula reflects for the most part the industry stock company loss and expense experience as reflected in the Loss and Expense Ratios as compiled from insurance expense exhibits as filed with the New York Insurance Department.

Some explanation of the treatment of expenses would seem in order at this time.

Because of the intrinsic part reinsurance plays in the fire insurance operation, net expenses are more indicative of operating costs. Whereas reinsurance recoveries for losses are unpredictable, expenses involved in reinsurance transactions are explicit enough to justify their inclusion.

Countrywide expenses for the most part are used for fire rating purposes. Some few states use taxes and commissions on a state basis and some states use state expenses which probably do not vary greatly from countrywide expenses.

Although fire rates are based primarily on historical data, it is nonetheless important that the expense ratio be indicative of future

expenses. It follows, then, that the most recent year's expenses would probably be more suitable than expenses over the last two, three, or more years period. Serious consideration should also be given to future changes which would affect expenses. It would be misleading and perhaps even dangerous to attempt to say explicitly what technique or period of time should be used to obtain the premium and expense data involved in calculating an expense ratio.

Rating bureaus, faced with the impossible situation of not being able to consider plans and anticipated expenses of their member companies, are forced to rely on expense data over the immediate past five years. This information is available from the New York Insurance Department.

The logic and technique involved after the basic information is made available becomes more confining. For example, it is generally agreed that it is not practical to develop a statutory expense ratio where all expenses are related to earned premium. This exaggerated illustration will show why:

*A company beginning its property fire operation in 1955 writes \$200,000 premium during its first year. However, only \$50,000 premium is earned during the year. If a 25% commission is paid, the commission expense alone equals the earned premium.

It is obvious from this illustration that relating commission expense to earned premium is misleading. While it is true that the distortion has been magnified, nevertheless this same type distortion is a problem to many fast-growing insurance companies. To establish an expense ratio suitable for the rate formula, therefore, it is necessary to consider each classification of expense individually and decide whether it should be related to written or earned premium.

Loss adjustment expenses incurred should be related to earned premium because losses are incurred during the term in which the premium is being earned.

As has been shown in the above illustration, (*), commission and brokerage expenses incurred should be related to written premium.

Other acquisition, field supervision, and collection expenses incurred and general expenses incurred both contain elements that are more appropriately related to written premium. On the other hand, the rest of the expenses in this category are more appropriately related to earned premium. A more conservative approach for the expanding company would be to relate these expenses to earned premium.

Taxes, licenses, and fees incurred are paid on the basis of written premium for the most part; thus these expenses are related to written premium.

Here is an example of how an expense ratio computed on this basis would appear:

Net Written Premium	\$27,481,443	
Net Earned Premium	23,648,225	
*Loss Adjustment Expenses Incurred	1,921,776	8.1%
**Commission and Brokerage Expenses Incurred	(493,644)	(1.8%)
*Other Acquisition, Field Supervision, and Collection Expenses Incurred	4,755,206	20.1%
*General Expenses Incurred	5,260,898	22.2%
**Taxes, Licenses, and Fees Incurred	932,828	3.4%
Expense Ratio		52.0%
*Ratio to Net Earned Premium		
**Ratio to Net Written Premium		

In order to illustrate what has been covered so far in this portion of the study, a series of tables showing actual experience for the residential Class 029 has been developed. To be more in line with present day rating bureau practices, expenses for the five year period, 1951-1955, are shown. No attempt has been made to consider anticipated developments bearing on the expense picture.

TABLE I

Direct Written Premium and Paid Losses
for Occupancy Class 029
Construction and Protection Groups 1-4
Years 1951-1955 and Combined

<u>Class</u>	<u>Const.- Prot.</u>	<u>1951</u>		<u>1952</u>		<u>1953</u>	
		<u>Written Premium</u>	<u>Paid Losses</u>	<u>Written Premium</u>	<u>Paid Losses</u>	<u>Written Premium</u>	<u>Paid Losses</u>
029	1	\$3,295,152	\$664,180	\$3,723,547	\$1,027,224	\$4,832,931	\$1,011,049
	2	3,241,324	601,261	3,573,779	799,536	4,658,253	698,780
	3	346,866	34,613	411,300	56,238	560,765	166,542
	4	423,328	47,395	488,182	39,180	655,107	123,354
	Total	\$7,306,670	\$1,347,449	\$8,196,808	\$1,922,178	\$10,707,056	\$1,999,725
<u>Class</u>	<u>Const.- Prot.</u>	<u>1954</u>		<u>1955</u>		<u>Combined Years</u>	
		<u>Written Premium</u>	<u>Paid Losses</u>	<u>Written Premium</u>	<u>Paid Losses</u>	<u>Written Premium</u>	<u>Paid Losses</u>
029	1	\$4,691,506	\$1,052,453	\$6,246,690	\$1,281,378	\$22,789,826	\$5,036,284
	2	4,376,318	1,486,595	5,247,732	881,730	21,097,046	4,467,902
	3	534,420	160,637	730,134	155,131	2,583,485	573,161
	4	595,931	183,219	721,776	259,603	2,884,324	652,751
	Total	\$10,198,175	\$2,882,904	\$12,946,332	\$2,577,842	\$49,355,041	\$10,730,098

TABLE I reflects written premium and paid losses as contained in annual reports to a statistical agent, covering a five year period, 1951 through 1955, for occupancy Class 029 (dwellings, and includes a portion of Class 019 covering dwellings and contents)

TABLE II

Earned Premium and Incurred Loss Statistics
Years 1951-1955 Used to Arrive at Factor

	<u>Written Premium</u>	<u>Earned Premium</u>	<u>Paid Losses</u>	<u>Incurred Losses</u>
1951	\$17,232,421	\$14,471,410	\$4,902,979	\$4,899,828
1952	19,076,963	16,261,932	7,221,197	6,618,526
1953	23,761,125	19,245,504	6,464,635	6,376,314
1954	23,309,997	21,520,416	7,748,783	7,962,982
1955	27,334,299	23,822,248	7,615,891	8,182,120
Total	\$110,714,805	95,321,510	33,953,485	34,039,770
Factor		.8609644392		1.00254127

This table (II) indicates the relationship between written and earned premium on a statewide basis for the five year period, 1951-1955. The relationship between paid and incurred losses for this period is also shown, since the beginning pending (12-31-50) is not available in the necessary detail.

The earned premium factor is arrived at by dividing the five year earned premium by the five year written premium. The incurred loss factor is arrived at by dividing the five year incurred losses by the five year paid losses.

TABLE III

STATEWIDE DIRECT EARNED PREMIUM AND INCURRED
LOSSES FOR OCCUPANCY CLASS 029
CONSTRUCTION AND PROTECTION GROUPS 1-4
COMBINED YEARS 1951-1955

<i>Class</i>	<i>Construction- Protection</i>	<i>Earned Premiums</i>	<i>Losses Incurred</i>
029	1	\$19,621,230	\$5,049,083
	2	18,164,116	4,479,256
	3	2,224,289	574,618
	4	2,483,300	654,410
	Total	\$42,492,935	\$10,757,367

This table (III) shows the earned premiums and incurred losses for Class 029, for which rates are being adjusted. The experience reflected in this table is the product of the earned premium and incurred loss factors shown in TABLE II times the written premium and paid losses shown in TABLE I.

This completes the conversion of the written premium and paid loss experience to an earned premium and incurred loss basis.

TABLE IV

RATE CHANGES FOR CLASS 029
1951-1955

1. Frame protected rates increased average of 3% effective January 1, 1952.
2. Brick protected rates decreased average of 5% effective July 1, 1953.
3. Frame unprotected rates increased average of 4% effective January 1, 1954.
4. Brick unprotected rates decreased average of 3% effective July 1, 1954.
5. Frame protected rates increased 4% effective October 1, 1955.

This table (IV) reflects the percentage effects of hypothetical rate adjustments over the past five years. It is necessary to adjust the earned premiums to reflect current rate levels before calculating loss ratios.

TABLE V
Adjustment of Actual Written Premium to Current Rate Levels

<u>Class</u>	<u>Const.- Prot.</u>	<u>Year</u>	<u>Act. W.P.</u>	<u>Factor</u>	<u>Adj. W.P.</u>	<u>Ratio of Adj. to Act.</u>	
029	1	1951	\$3,295,152	1.070	\$3,525,813	1.0416	
		1952	3,723,547	1.040	3,872,489		
		1953	4,832,931	1.040	5,026,248		
		1954	4,691,506	1.040	4,879,166		
		1955	6,246,690	1.030	6,434,091		
		Total	\$22,789,826		\$23,737,807		
	2	1951	\$3,241,324	1.040	\$3,370,977		1.0218
		1952	3,573,779	1.040	3,716,730		
		1953	4,658,253	1.040	4,844,583		
		1954	4,376,318	1.000	4,376,318		
		1955	5,247,732	1.000	5,247,732		
		Total	\$21,097,406		\$21,556,340		
	3	1951	\$346,866	.950	\$329,523		.9799
		1952	411,300	.950	390,735		
		1953	560,765	.975	546,746		
		1954	534,420	1.000	534,420		
1955		730,134	1.000	730,134			
	Total	\$2,583,485		\$2,531,558			
4	1951	\$423,328	.970	\$410,628	.9806		
	1952	488,182	.970	473,537			
	1953	655,107	.970	635,454			
	1954	595,931	.985	586,992			
	1955	721,776	1.000	721,776			
	Total	\$2,884,324		\$2,828,387			

TABLE V shows the manner of adjusting written premium to current rate levels and shows the relationship of actual written (TABLE I) to adjusted premium.

Adjusted Written Premium = Ratio of Adjusted to Actual

TABLE VI

Earned Premium Adjusted to Current Rate Levels

Incurred Losses and Loss Ratio
1951-1955

<u>Class</u>	<u>Const.- Prot.</u>	<u>Actual Earned Premium</u>	<u>Factor</u>	<u>Adjusted Earned Premium</u>	<u>Losses Incurred</u>	<u>Loss Ratio</u>
029	1	\$19,621,230	1.0416	\$20,437,473	\$5,049,083	24.7
	2	18,164,116	1.0218	18,560,094	4,479,256	24.1
	3	2,224,289	.9799	2,179,581	574,618	26.4
	4	2,483,300	.9806	2,435,124	654,410	26.9
	Total	\$42,492,935		\$43,612,272	\$10,757,367	24.7

This table (VI) shows the process by which the adjusted earned premium is obtained, that of multiplying the earned premium from TABLE III by the factors shown in TABLE V.

DETERMINATION OF NEED FOR REVISION

In the previous portion of this study, it was shown how the loss and expense ratios are established. It was necessary to determine the loss ratio for each statistical subdivision, i.e., each construction-protection group within the occupancy Class 029. Only one expense ratio was used.

These ratios, then, along with the conflagration allowance and profit percentage provide all that is needed for establishing the percent of change needed to bring the rate in line with the actual experience.

Determination of Amount of Change.—The percent of the premium dollar intended for conflagration allowance and profit are, of course, predetermined by industry practice or by individual companies, and a rate change would not affect these percentages. Similarly, the expense ratio will be only partially affected by a rate change since commissions and taxes are paid as a percentage of premium. Therefore, by adjusting the rate we are aiming at changing only the loss ratio.

According to the formula which is set forth in this paper, the allowable loss ratio is .475. It is obvious that any substantial deviation from this ratio would necessitate a rate change but when the actual loss ratio differs only slightly from the permissible it is necessary to apply some arbitrary rule to "draw the line." A common practice is to make a rate adjustment only if the actual loss ratio differs from the permissible by two or more percentage points. Therefore, with a 47.5% permissible loss ratio, a rate adjustment would not be made unless the actual ratio is (1) 45.5% or less or (2) 49.5% or more.

TABLE VII
DETERMINATION OF NEED FOR RATE CHANGE

<i>Class</i>	<i>Const.- Prot.</i>	<i>Permissible Loss Ratio</i>	<i>Actual Loss Ratio</i>	<i>Rate Change</i>
029	1	45.5%-49.5%	24.7%	Rate Decrease Indicated
029	2	45.5%-49.5%	24.1%	Rate Decrease Indicated
029	3	45.5%-49.5%	26.4%	Rate Decrease Indicated
029	4	45.5%-49.5%	26.9%	Rate Decrease Indicated

TABLE VII shows the actual loss ratios for Class 029 and indicates whether or not a rate change should be made.

CALCULATING AMOUNT OF CHANGE

To calculate the amount of adjustment to be made, it is only a matter of comparing the permissible loss ratio for each class and protection group combination to the actual loss ratio. This can be done by dividing the actual loss ratio by the permissible and apply-

ing the resulting factor to each rate involved in the particular classification. If, for example, the experience indicates a 5% increase for Class 029, construction-protection code 1 (Dwellings—Buildings only—frame protected,) it would be necessary to apply the 5% increase to the rates for the following Class 029 combinations:

<i>Class of Bldg.</i>	<i>Town Class</i>	<i>No. of Fam.</i>	<i>Occ. Class</i>	<i>Const.-Prot.</i>	<i>Rate</i>
Frame approved roof	1 to 4	1 to 2	029	1	.12
Frame approved roof	1 to 4	3 to 4	029	1	.14
Frame approved roof	5 and 6	1 to 2	029	1	.13
Frame approved roof	5 and 6	3 to 4	029	1	.15
Frame approved roof	7 and 8	1 to 2	029	1	.15
Frame approved roof	7 and 8	3 to 4	029	1	.17
Frame unapproved roof	1 to 4	1 to 2	029	1	.16
Frame unapproved roof	1 to 4	3 to 4	029	1	.18
Frame unapproved roof	5 and 6	1 to 2	029	1	.17
Frame unapproved roof	5 and 6	3 to 4	029	1	.19
Frame unapproved roof	7 and 8	1 to 2	029	1	.19
Frame unapproved roof	7 and 8	3 to 4	029	1	.21

Due to the fact that the rates consist of two digits only, the 5% increase may not actually change some of the rates. Moreover, the basic rating structure already in existence would normally not be disturbed. That is, the relationship between the rates for the various construction, protection, and number of family combinations is maintained.

TABLES VIII, IX, X and XI which follow, show the adjustments made in the rates for each construction-protection group combination. From these tables, it can be determined if there is a 2% plus or minus variance from the allowable loss ratio of 47.5% which would necessitate an adjustment.

The proposed rate for each group needing adjusting is developed by dividing the actual loss ratio by the permissible loss ratio of 47.5% and multiplying the result by the current rate.

TABLE XII reflects an array of the proposed rate structures for all the Class 029 groups of business. The purpose of this table is to determine if the construction-protection relationship has been maintained throughout. For instance, had the loss ratio for frame buildings been considerably lower than brick buildings, a lower rate for frame buildings might have resulted which would be inconsistent with the policy of maintaining the basic rate structure.

TABLE VIII

Class 029 Brick Protected Approved and Unapproved Roof
 Business Divided by Town Class and Number of Families
 Showing Current Rates and Proposed Rates

<u>Class</u>	<u>Actual Loss Ratio</u>	<u>Permissible Loss Ratio</u>	<u>Factor (Actual Loss Ratio ÷ 47.5%)</u>	<u>Type of Roof</u>	<u>Town Class</u>	<u>No. of Families</u>	<u>Current Rate</u>	<u>Factor X Current Rate</u>	<u>Proposed Rate</u>
029	26.4%	45.5%-49.5%	.56	Approved	1 to 4	1 and 2	.08	.045	.05
	"	"	"	"	1 to 4	3 and 4	.10	.056	.06
	"	"	"	"	5 and 6	1 and 2	.09	.050	.05
	"	"	"	"	5 and 6	3 and 4	.11	.062	.06
	"	"	"	"	7 and 8	1 and 2	.11	.062	.06
	"	"	"	"	7 and 8	3 and 4	.13	.073	.07
	"	"	"	Unapproved	1 to 4	1 and 2	.12	.067	.07
	"	"	"	"	1 to 4	3 and 4	.14	.078	.08
	"	"	"	"	5 and 6	1 and 2	.13	.073	.07
	"	"	"	"	5 and 6	3 and 4	.15	.084	.08
	"	"	"	"	7 and 8	1 and 2	.15	.084	.08
	"	"	"	"	7 and 8	3 and 4	.17	.095	.10

This table (IX) indicates the process whereby the proposed rate is developed.

47.5% = Permissible Loss Ratio

(Actual Loss Ratio ÷ Permissible Loss Ratio) X Current Rate = Proposed Rate.

TABLE IX

Class 029 Frame Protected Approved and Unapproved Roof
 Business Divided by Town Class and Number of Families
 Showing Current Rates and Proposed Rates

<u>Class</u>	<u>Actual Loss Ratio</u>	<u>Permissible Loss Ratio</u>	<u>Factor (Actual Loss Ratio+47.5%)</u>	<u>Type of Roof</u>	<u>Town Class</u>	<u>No. of Families</u>	<u>Current Rate</u>	<u>Factor X Current Rate</u>	<u>Proposed Rate</u>
029	24.7%	45.5%-49.5%	.52	Approved	1 to 4	1 and 2	.12	.062	.06
	"	"	"	"	1 to 4	3 and 4	.14	.073	.07
	"	"	"	"	5 and 6	1 and 2	.13	.068	.07
	"	"	"	"	5 and 6	3 and 4	.15	.078	.08
	"	"	"	"	7 and 8	1 and 2	.15	.078	.08
	"	"	"	"	7 and 8	3 and 4	.17	.088	.09
	"	"	"	Unapproved	1 to 4	1 and 2	.16	.083	.08
	"	"	"	"	1 to 4	3 and 4	.18	.094	.09
	"	"	"	"	5 and 6	1 and 2	.17	.088	.09
	"	"	"	"	5 and 6	3 and 4	.19	.099	.10
	"	"	"	"	7 and 8	1 and 2	.19	.099	.10
	"	"	"	"	7 and 8	3 and 4	.21	.109	.11

TABLE X

Class 029 Brick Unprotected Approved and Unapproved Roof
Business Divided By Town Class and Number of Families
Showing Current Rates and Proposed Rates

<u>Class</u>	<u>Actual Loss Ratio</u>	<u>Permissible Loss Ratio</u>	<u>Factor (Actual Loss Ratio÷47.5%)</u>	<u>Type of Roof</u>	<u>Town Class</u>	<u>No. of Families</u>	<u>Current Rate</u>	<u>Factor X Current Rate</u>	<u>Proposed Rate</u>
029	26.9%	45.5%-49.5%	.57	Approved	9	1 and 2	.22	.125	.13
	"	"	"	"	9	3 and 4	.24	.137	.14
	"	"	"	"	10	1 and 2	.24	.137	.14
	"	"	"	"	10	3 and 4	.26	.148	.15
	"	"	"	Unapproved	9	1 and 2	.28	.160	.16
	"	"	"	"	9	3 and 4	.30	.171	.17
	"	"	"	"	10	1 and 2	.30	.171	.17
	"	"	"	"	10	3 and 4	.32	.182	.18

TABLE XI

Class 029 Frame Unprotected Approved and Unapproved Roof
Business Divided By Town Class and Number of Families
Showing Current Rates and Proposed Rates

<u>Class</u>	<u>Actual Loss Ratio</u>	<u>Permissible Loss Ratio</u>	<u>Factor (Actual Loss Ratio÷47.5%)</u>	<u>Type of Roof</u>	<u>Town Class</u>	<u>No. of Families</u>	<u>Current Rate</u>	<u>Factor X Current Rate</u>	<u>Proposed Rate</u>
029	24.1%	45.5%-49.5%	.51	Approved	9	1 and 2	.28	.143	.14
	"	"	"	"	9	3 and 4	.30	.153	.15
	"	"	"	"	10	1 and 2	.30	.153	.15
	"	"	"	"	10	3 and 4	.32	.163	.16
	"	"	"	Unapproved	9	1 and 2	.34	.173	.17
	"	"	"	"	9	3 and 4	.36	.184	.18
	"	"	"	"	10	1 and 2	.36	.184	.18
	"	"	"	"	10	3 and 4	.38	.194	.19

TABLE XII

Verification of Proposed Rates

<u>BUILDING</u>					
	<u>Type of Roof</u>	<u>Town Class</u>	<u>No. of Families</u>	<u>Proposed Brick Rates</u>	<u>Proposed Frame Rates</u>
Protected	Approved	1 to 4	1 and 2	.05	.06
	"	1 to 4	3 and 4	.06	.07
	"	5 and 6	1 and 2	.05	.07
	"	5 and 6	3 and 4	.06	.08
	"	7 and 8	1 and 2	.06	.08
	"	7 and 8	3 and 4	.07	.09
	Unapproved	1 to 4	1 and 2	.07	.08
	"	1 to 4	3 and 4	.08	.09
	"	5 and 6	1 and 2	.07	.09
	"	5 and 6	3 and 4	.08	.10
	"	7 and 8	1 and 2	.08	.10
	"	7 and 8	3 and 4	.10	.11
	Unprotected	Approved	9	1 and 2	.13
"		9	3 and 4	.14	.15
"		10	1 and 2	.14	.15
"		10	3 and 4	.15	.16
Unapproved		9	1 and 2	.16	.17
"		9	3 and 4	.17	.18
"		10	1 and 2	.17	.18
"		10	3 and 4	.18	.19

EFFECTS OF RATE REVISIONS

The statistics which have been developed in this study indicate the need for a substantial reduction in rates for nearly all the class combinations involved, and were it not for the consideration which must be given to certain factors relating to other classes, adjustments could be made as indicated.

Consideration must be given to the fact that there may be certain classes of risks which do not have adequate rates and which because of certain credibility limitations cannot be adjusted on the basis of their own experience, to bring them in line. In order to guard against causing an unreasonable imbalance to occur, it is many times necessary to scale down what would otherwise be a sizeable reduction for a given class.

A fire rate maker takes a good deal of liberty in applying judgment in connection with adjusting class rates, keeping in mind that it is his primary aim to maintain the basic rate levels, and that his adjusting is largely that of testing these levels so the approved relationship existing in the basic levels will be maintained.

To determine the degree of adjustment which should be made, it is necessary to ascertain the percent of change required to produce a permissible loss ratio for the combined classes being considered for adjustment. This can be determined by combining the premiums and losses for the various classes involved and arriving at a loss ratio for the total.

As long as the total experience of the business being rated produces the permissible results, it is possible to establish individual rates with some degree of flexibility. Consequently the proposed rates should be checked to determine if the application of them to the business written will produce, in total, the desired results.

SUMMARY AND CONCLUSIONS

From the foregoing discourse, certain facts stand out, which should be mentioned in summarization and from which certain conclusions can be drawn.

Basis for Rates.—It is evident that fire rates do not stem from a purely statistical basis, but rather from arithmetical calculations based upon organized quantitative data compiled by either the companies or the National Board of Fire Underwriters covering the volume of premiums written and losses paid to the end that a permissible loss ratio can be established, which in turn becomes the yardstick for determining the need for adjustment.

By following this principle and including the N.A.I.C. profit factor of 6%, a rather universal formula has been developed with which to determine the adequacy of fire rates.

It is apparent that the groundwork is being laid which will permit a closer approach to statistical rating, but it should be kept in mind that fire insurance rate making is not likely to become an exact science.

Classified experience statistics, no matter how applied, can only tell a story of what has occurred during the period covered by such experience, giving no clues as to what the future holds. However, to paraphrase the National Board statement of principles, by proper application of judgment which gives adequate consideration to economic trends, social conditions, new processes, and inventions, such data can be of great value in predicting what may occur in the immediate future from a loss experience standpoint.

Rate Adequacy.—It is trite, but nevertheless true to state that the fire rate must produce enough premiums for companies to pay their losses and expenses of operations. The pattern followed in this study indicates that adjustments made in keeping with this pattern will afford a rate level adequate to meet the above requirements, keeping in mind of course that this applies on the basis of average if it is the rating bureau which is making the adjustment and not a single company.

Rate Fairness.—Rate regulations in most states prohibit the use of rates which are “unfairly” discriminatory between risks of essentially the same hazard. This restraint, in essence, affords the basis for the fire rate makers’ adherence to the principle of maintaining the uniform relationship between classes of risks as indicated in the processes involved in this study.

Whatever else may be said for or against the fire rating system and the propriety of the base rate make-up, the system for the most part affords a fair degree of consistency as applied to maintaining the basic relationship between classes.

Credibility.—No consideration has been given to the credibility factor in this study, the reason being that no acceptable measure of credibility for fire risks exists. Some rating bureaus take credibility into consideration, but this practice is not universal.

As the “Standard Classification” gains wider usage and sufficient bodies of statistical data are accumulated, credibility factors can possibly be developed which would be satisfactory.

Expenses.—As has been indicated, fire rating does not take into consideration expense costs by class of risk, which can be taken to indicate a degree of unfairness, at least in principle, and it will take a lot of work and study to establish a plan that will properly apportion such costs to the separate risks involved.

It is to be hoped that present efforts being made in this direction will bear fruit, and ultimately eliminate cause for complaint in this one area.

DISCUSSION OF PAPERS READ AT THE
MAY 1956 MEETING

A REVIEW AND COMPARISON OF WORKMEN'S COMPENSATION EXPERIENCE

IN

NEW YORK STATE AND WISCONSIN

FRANK HARWAYNE

Volume XLIII, Page 8

DISCUSSION BY W. W. GREENE

This paper demonstrates conclusively that compensation loss cost in Wisconsin per \$100 of payroll is substantially lower than the corresponding loss cost in New York in spite of the fact that Wisconsin benefits are over all approximately on a par with New York benefits. It is further demonstrated that this difference in pure benefit cost can be accounted for by lower accident frequency in Wisconsin only to a minor, though by no means negligible, extent.

The fact that differences between states as to actual loss cost are not always or even generally consistent with law differentials based on theoretical valuation of the respective benefit schedules was recognized very early in American compensation rate making, and realization that this situation existed led to the use of what were then termed "reduction factors" or "experience differentials." In deriving these experience differentials the technique initially employed was identical with that used by Mr. Harwayne in his paper.

As far as I know, the present syllabus and recommendations for study do not make any reference to experience differentials. This is doubtless entirely justified by the fact that these devices are no longer used in rate making. However, since Mr. Harwayne has worked out experience differentials between New York and Wisconsin, it would appear to be in order to direct students, or at least those who have a leaning toward the historical perspective, to the part which these gadgets once played in practical rate making.

Undoubtedly Benedict Flynn had in mind the disparity which oft exists between theory and reality when in 1914 he recommended that the first New York compensation rates reflect a differential of 1.90 to be applied to Massachusetts pure premiums, as opposed to a differential of 2.58 which the writer had computed on basis of a strictly desk-chair comparison of benefit schedules, there being of course no actual New York experience at that time. As I recall it, Ben's disagreement with my recommendation was due in large part to his opinion that in the initial stages of the New York Law, employees would not fully avail themselves of its benefits. Evidently subsequent events did not justify his expectation (see Leon Senior's reference

to this matter in his address at the 25th Anniversary Meeting of this Society, P.C.A.S. XXVI pp. 154-155), for, according to Leon the New York 1914-16 experience justified the 2.58 multiplier. Apparently the New York Law was several years old when it was born! In the years which immediately followed it was customary when combining experience from a number of states to employ a factor for the "aging of the act" in conjunction with theoretical law differentials. This procedure constituted recognition, however incomplete, that comparative loss costs cannot be measured solely by a study of benefit provisions.

In the summer of 1918 the writer had occasion to observe a striking example of the difference between theoretic differentials and those based on experience. The theoretic law differential for New Jersey was 98% (Ratio of theoretic New Jersey cost to theoretic cost of the original Massachusetts Act), yet the application of Massachusetts pure premiums to New Jersey payrolls indicated a "direct" experience differential of 64%. The "inverse" calculation indicated 69% (P.C.A.S. VI p. 11).

The writer may have gotten the idea of experience differentials from the late Dr. E. H. Downey. In the Pennsylvania rate revision of 1918 explicit recognition was made of the probable permanency of discrepancies between theoretic differentials and differentials based on experience, for reduction factors based on comparison of actual costs were used to some extent in converting the experience of other states to the Pennsylvania level (P.C.A.S. V p. 243 et seq. "The Revision of Pennsylvania Compensation Insurance Rates, 1918").

In the New Jersey rate revision of 1920 the experience of several states was combined by use of experience differentials. To the best of the writer's recollection these experience differentials were computed by the use of a formula set forth in a paper presented to this Society on November 21, 1919, entitled "Upon Combining Compensation Experience From Several States" (P.C.A.S. VI p. 10 et seq.). This formula, which was referred to by Dr. Downey as "mathematical hocus pocus," provoked some rather spirited discussion, which is to be found in the same volume of the Proceedings beginning with page 310. The use of experience differentials was evidently standard National Council procedure as late as 1926, according to "The 'Permanent' Rate Making Method Adopted by the National Council on Compensation Insurance" (P.C.A.S. XII p. 253). The co-authors, Bill Roeber and the writer, wisely put quotation marks around the word "Permanent!"

Experience differentials or reduction factors (as they were sometimes called) served a useful purpose in the early days of compensation rate making, but I thought these once familiar and friendly gimmicks had passed into the "limbo of forgotten things" until I read Mr. Harwayne's paper!

I hope that these remarks, though not necessarily of any immediate practical value, may prove entertaining to those who are interested in the ancient history of our business.

DISCUSSION OF PAPERS READ AT THE
NOVEMBER 1956 MEETING

THE RATE LEVEL ADJUSTMENT FACTOR
IN
WORKMEN'S COMPENSATION RATEMAKING
MARTIN BONDY

Volume XLIII, Page 106

DISCUSSION BY M. G. McDONALD

The evaluation by Mr. Bondy of the effectiveness of the Rate Level Adjustment Factor in New York has stimulated considerable research on the part of the industry as well as in several State Departments.

The National Council completed an analysis of the policy year experience for eight states which indicated the incurred loss ratio came closer to the permissible in 24 policy years out of 42 (57%). This data includes results as of the first reporting, which were not considered conclusive. However, omitting first reporting indications, the incurred loss ratio came closer to the permissible in 19 cases out of 34 (56%). The adjusted loss ratio, as defined in Mr. Bondy's paper, was closer to the permissible in 43% and 44% of the cases respectively. Tests in Massachusetts indicate similar results.

When the rate level adjustment factor was first introduced in Massachusetts in early 1950, the neutral zone idea was incorporated in the formula, and the use of the neutral zone was disapproved by the Deputy Commissioner who conducted the Hearing. The rate level adjustment factor employing the neutral zone was .987, without it .977.

A test indicates Mr. Bondy's "New" Rate Level Adjustment Formula would have produced slightly better rate levels in Massachusetts than the formula employed. However, in the case where the "New" formula produces a higher rate level than the old, most Departments will be reluctant to approve of the change.

In those states where the present methods produce loss ratios deviating substantially from permissibles, it is suggested that further study be given to other elements in the rate making which have greater effect than the rate level adjustment factor, such as the cost of law amendments as compared with the valuation employed in the revision and the development of losses.

DISCUSSION BY G. B. ELLIOTT

Mr. Bondy's paper discusses one of the elements in the ratemaking procedure for workmen's compensation insurance as used in New

York and most other states. The Rate Level Adjustment Factor is a subject which has received the continuing study of ratemaking bodies and regulatory authorities alike, particularly during the past ten years. Its history and development have been summarized a number of times, most recently in the 34th Annual Report of the National Council on Compensation Insurance (pages 9-13).

Mr. Bondy's thesis is that the Rate Level Adjustment Factor, as used in the New York ratemaking procedure, has tended to distort, rather than to improve, the ratemaking process, and his paper contains several exhibits in support of this point. The experience used in his analysis was that contained in the New York Board's rate filing of July 1, 1956, namely that of the five policy years beginning July 1, 1948. The experience is broken down into ten six-month periods, and an analysis of this experience indicates why the Rate Level Adjustment Factor has had what might be termed an undesirable influence on the results for some of the periods. For the first six periods the unadjusted loss ratios were at or slightly above the permissible, ranging from .563 to .609. There followed a sharp and continuing drop in loss ratios for the four latest periods: .504, .464, .414 and .411. Thus, the Rate Level Adjustment Factors based on the unfavorable experience of the earlier periods (presumably the calendar year experience was also unfavorable, although this experience was not exhibited) produced an increase in premiums for some of the later periods, making the loss ratios lower than they would otherwise have been. There is some question as to whether the effect of the Rate Level Adjustment Factor would have been as noticeable if this sharp change in loss ratios had not been experienced. It would be of interest if Mr. Bondy or some other member of the Society would apply the same method he has used to analyze the effect of the Rate Level Adjustment Factor on the experience in a number of other states.

It is interesting to note that the removal of the effect of the Rate Level Adjustment Factor, while having a noticeable effect on the loss ratios for some of the individual periods, makes only a minor change in the loss ratio for the entire five-year experience period. That is, the loss ratio on a reported basis for the entire period was .520; after application of the loss development factors and removal of the effect of the Rate Level Adjustment Factor, the loss ratio becomes .528—a difference of only .008

Mr. Bondy's comments with respect to the unreliability of calendar year experience are well-taken. This fact has been recognized, to a greater or lesser degree, by the manner in which such experience has been used in determining indicated changes in rate level. Some years ago the effect of calendar year experience was minimized by the establishment of a "neutral zone", while in more recent years such experience has been given a weight equal to the permissible loss ratio. This procedure was further modified last year in National Council states by assigning equal weights to the calendar year and policy year loss ratios, and it is understood that this modified proce-

ture will be used in the July 1, 1957 rate revision in New York. Even this latest change may not be sufficient to give full recognition to the unreliability of calendar year experience, and Mr. Bondy's suggested change in procedure would seem to warrant careful study. However, in order to make such a study, a more complete exposition of Mr. Bondy's proposal would be helpful, at least to this writer.

It is not clear just how the formula outlined in the paper would be used in actual practice. Mr. Bondy sets forth the following conditions:

Permissible Loss Ratio	=	.565
Maximum Credibility	=	.40
Maximum RLAF	=	1.10
Minimum RLAF	=	.90

To determine the loss ratio necessary to produce the maximum Rate Level Adjustment Factor of 1.10, the following formula is used:

$$1.10 = \frac{.40 \text{ Loss Ratio (Max.)} + .60}{.565}$$

$$\text{Loss Ratio (Max.)} = .706$$

The Neutral Zone is then derived in the following manner:

$$\text{RLAF} = \text{Loss Ratio} - \text{PLR} \pm \text{NZ} + 1$$

$$1.10 = .706 - (.565 \pm \text{NZ}) + 1$$

$$\text{NZ} = \pm .041 = \pm .040 \text{ (rounded)}$$

An attempt was made to test the formula by assigning a credibility of .20, keeping the three other conditions constant. However, this calculation produced a maximum loss ratio of .848 and a neutral zone of $\pm .183$. It therefore seems obvious that the formula was not intended to be applied in this way, since the stated objective is to increase the credibility as the deviation from the "normal", or permissible, loss ratio increases; whereas in the calculation just mentioned, the assignment of a lower credibility resulted in a higher loss ratio, that is, a greater rather than a lesser deviation from normal.

Another approach was then tried, based on the assumption that as the credibility increases from 0 to .40 the Rate Level Adjustment Factor increases from 1.00 to 1.10. For example, a Rate Level Adjustment Factor of 1.05 would correspond to a credibility of .20. Inserting these values in the above formula produces the same maximum loss ratio of .706, with a neutral zone of $\pm .091$. These results, too, are rather puzzling and it is hoped that Mr. Bondy may be able to shed some light on just how the formula is intended to be applied.

Some comment seems to be appropriate on the neutral zone produced in the example outlined above. If the permissible loss ratio is .565, a neutral zone of $\pm .04$ means that if the calendar year loss ratio lies between .525 and .605, the Rate Level Adjustment Factor would be unity. In an extreme case, therefore, it would be possible for the loss ratio to increase by 8 points in a single year without any

recognition being given to this fact in the ratemaking procedure. Sooner or later the increase would presumably be reflected in the policy year experience, but since one purpose of the Rate Level Adjustment Factor is to recognize trends beyond the policy year experience, it would appear that such a wide neutral zone would tend to defeat this purpose. This criticism could be eliminated, of course, simply by modifying the formula so as to produce a narrower neutral zone within which the Rate Adjustment Factor would be unity.

It is to be hoped that Mr. Bondy and others interested in the workmen's compensation ratemaking procedure will continue to study the problem of how best to use calendar year statistics, and that his proposed procedure will be thoroughly tested in order to determine whether it will result in improved ratemaking methods.

AUTHOR'S REVIEW OF DISCUSSION

MARTIN BONDY

Mr. Elliott's interesting discussion reveals, among other things, that I have not gone into sufficient detail in describing the Neutral Zone formula. I shall attempt to do this in the following paragraphs.

The most important thing to keep in mind is that certain values are to be fixed in advance and do not change from year to year. This is true of any formula we may decide upon. For example, under the existing Rate Level Adjustment Factor formula, we set the condition that the credibility to be allowed is 50%. This is not a value which will change annually. It is based upon certain underwriting considerations and is expected to remain in force until these no longer apply.

Similarly, in the example given in my paper, I have set two conditions. They are:

1. The *maximum* credibility to be allowed is 40%.
2. The *maximum* effect on rate level produced by the Factor is 10 points.

Given these two fixed conditions, the remaining elements of the formula are automatically derived. To determine the loss ratio which will produce the maximum Factor, the formula cited by Mr. Elliott is used:

$$\text{Max. RLAF} = \text{Max. credibility} \times \frac{\text{Loss Ratio underlying max. RLAF}}{\text{Permissible Loss Ratio}} + (1.0 - \text{Max. credibility}) \times \text{Unity}$$

Substituting

$$1.10 = \frac{.40 \times \text{Loss Ratio underlying max. RLAF}}{.565} + .60$$

Solving

$$\text{Loss Ratio underlying max. RLAF} = .706$$

From this, the Neutral Zone is uniquely determined by using the relationship

$$\begin{aligned} \text{RLAF} &= \text{Loss Ratio} - \text{Permissible} \pm \text{NZ} + 1 \\ 1.10 &= .706 - (.565 \pm \text{NZ}) + 1 \\ \text{NZ} &= \pm 0.41 \end{aligned}$$

To summarize, the requirements that maximum Rate Level Adjustment Factor shall equal 1.10 and maximum credibility shall be 40% will produce the following formula:

$$\text{RLAF} = \text{Loss Ratio} - .565 \pm .04 + 1.0$$

The credibilities granted under this formula run according to the following table:

<u>Loss Ratio *</u>	<u>Credibility (%) **</u>
.565-.605	0
.615	11
.625	19
.635	24
.645	28
.655	31
.665	34
.675	36
.685	38
.695	39
.705	40

* This is a symmetrical table about .565

$$** \text{Credibility} = \frac{\text{RLAF} - 1}{\frac{\text{Loss Ratio} - 1}{\text{PLR}}}$$

As Mr. Elliott points out, a Neutral Zone of 4 points on each side of unity may be somewhat insensitive. This is one of the underwriting considerations which must be taken into account in setting up a formula of this type. While I feel that a swing of a few points in calendar year experience is not necessarily significant, nevertheless conditions assigned may be too stringent. It should be kept in mind that this was only used as an example. If the maximum credibility were taken at 50%, the resulting formula would have a Neutral Zone of about 1.5 points on each side of unity.

Concerning Mr. Elliott's comment on the first section of the paper, it appears in order to elaborate on certain points which have not been made sufficiently clear. In the very first place, my thesis is that the Rate Level Adjustment Factor represents *no improvement* in the rate structure. As a matter of fact, in New York for the period exhibited, the consequences of using this Factor were inferior rates. I do not believe that the continuance of the Factor would have been justified if there were neither deterioration nor improvement flowing from it. In that case, it would be sort of a neutral Factor. The Factor was introduced not to be neutral but to be of positive assistance in setting the rate level. Moreover, it should be of the greatest utility in times of stress and change. When conditions are on an even keel, there is no urgent need for the introduction of such a Factor.

It may interest the reader to know that shortly after the presentation of the paper, the effects of the Factor were tested in some states other than New York. In the 34 cases where a Rate Level Adjustment Factor formula with no Neutral Zone was used, the rate level was better in 17 cases and worse in 17 cases than if no Rate Level Adjustment Factor had been used. Of the 16 cases where the

Factor fell within the Neutral Zone (4 points), the rates were improved in 5 cases and made worse in 11. This would tend to reinforce the belief that a calendar year rate level falling close to the permissible should not be used as a forecasting device. It has been pointed out that the average reported loss ratio for the entire period (presented in the paper) was .520. The average adjusted loss ratio was .528. These are indeed close. However, it is not surprising that any fairly reasonable and unbiased method would produce answers which, over the long pull, hover about the permissible loss ratio. A better test of the efficacy of the procedure would be to compare the average variation about the permissible from year to year.*

*For the years reviewed, the average variation of the actual loss ratios exceeded that of the adjusted figures.

In conclusion, I should like to offer my sincere thanks to Mr. Elliott for pointing out the shortcomings in those areas which required clarification of presentation.

CURRENT RATE MAKING PROCEDURES FOR
AUTOMOBILE LIABILITY INSURANCE

PHILIPP K. STERN

VOLUME XLIII, PAGE 112

DISCUSSION BY T. E. MURRIN

A paper on automobile liability insurance ratemaking has been long overdue in appearing in the Proceedings of the Casualty Actuarial Society. Although the need for a paper on this subject has been felt for many years by students particularly, it is welcome as a basic reference for insurance men as well. That the task of writing the paper fell to Mr. Stern is a happy coincidence because of his native ability for clarity of expression and logical discussion. His paper, augmented by many illustrative exhibits, is a valuable contribution to the Society Proceedings.

Mr. Stern's stated purpose of composing a descriptive presentation for the student without any evaluation of ratemaking procedures disarms the reviewer to some extent because controversial matters are thereby avoided. As this paper will be a source of information for students principally, my observations are intended primarily to clarify what Mr. Stern has left unexplained or unsaid rather than criticize what he has said. In reading the paper I tried to keep myself in the place of the student and not read between the lines or recall unsaid things that are familiar to most members of this Society.

As he mentioned in his opening paragraph, Mr. Stern explains many technical terms that appear in the paper, but not always the

first time they occur. In reading the paper I found many terms, which are common to the jargon of our business used without any definition or explanation, such as, transaction reports, summarized reports, statistical program, specified car basis and Fleet Plan.

In discussing ratemaking statistics at the beginning of his paper, Mr. Stern rightly explains the importance of ratemaking statistics, citing applicable language of the rate regulatory statutes regarding statistics and statistical plans. The function of statistics would have been brought into sharper focus I believe, if in his opening statement that the loss portion and the expense portion of the rates are based on experience, Mr. Stern had referred to the provision in the rate regulatory laws providing generally, that in determining rates "Due consideration shall be given to past and prospective loss experience within and outside this state, to catastrophe hazards, if any, to a reasonable margin for underwriting profit and contingencies, to dividends, savings or unabsorbed premium deposits allowed or returned by insurers to their policyholders, members or subscribers, to past and prospective expenses both countrywide and those specially applicable to this state, and to all other relevant factors within and outside this state." Only in the last sentence of the paper did he refer to the basic criteria for rates, namely, that rates shall be adequate, not excessive and not unfairly discriminatory.

My most serious criticism centers on Mr. Stern's presentation of expense provisions in the manual rates and the expected loss ratio. It is unfortunate that Mr. Stern made only a brief reference to this phrase of the ratemaking process as he indicates earlier in his paper. He states " 'the expected loss ratio' . . . represents the portion of the premium dollar available for losses . . . after the requirements for expenses including a stated provision for underwriting profit and contingencies are met." Would that this were literally true! For the benefit of the student Mr. Stern should have accorded fuller discussion to this important element in the manual rates which accounts for a sizeable portion of the premium dollar. In addition to providing for loss payments, the premium dollar also provides for the expenses of selling, underwriting and servicing insurance policies and for taxes. The provision for underwriting profit and contingencies is only theoretical. There will always be losses and expenses but not so with the margin for underwriting profit and contingencies. The nominal margin for underwriting profit and contingencies is solely theoretical because if the losses and expenses combined exceed the premiums, there is no profit and the difference must come out of company surplus.

Mr. Stern correctly points out that expense provisions are determined from countrywide data as reported in the Insurance Expense Exhibit and then unfortunately proceeds to show the New York provisions as being different from the provisions applicable in other states except for the production cost allowance, without making any comment on the differences. In addition, the unallocated loss adjustment item is shown as an expense and expressed as a percentage of

premium in the breakdown of the premium dollar in New York. In the exhibit of "standard" provisions, the unallocated loss adjustment item is expressed in terms of losses and is relegated almost to obscurity in the footnote applicable to the expected loss and loss adjustment ratio. I am afraid that the student will have considerable difficulty in understanding this important element in the ratemaking procedure and be unable to comprehend that the difference between the expected loss ratio in New York and the higher expected ratio applicable to other states, is due almost entirely to the fact that the former ratio excludes, and the latter ratio includes, unallocated loss adjustment expenses. Those familiar with automobile liability ratemaking in New York and other states know that unallocated loss adjustment is handled with losses and expressed as an expense item in terms of premium only in New York and is treated as a function of losses and included with them in ratemaking in all other states. The slight differences in the provisions for administration, inspection, audit and bureau, between New York and other states reflect the unique New York requirement that this provision be reduced slightly to offset the additional dollars that would be collected for these items if the extra assessments for administering the Security Fund and the Safety Responsibility Act were loaded in the rates as a flat percentage. Also it should be pointed out that the basic provision for underwriting profit and contingencies in New York is 3.5% which is less than the standard provision of 5.0% effective in 43 other states and the District of Columbia.

Mr. Stern goes into considerable detail in discussing the current private passenger plan and the preceding plans. In order to understand the change in classification differentials in his example, it was admittedly necessary for Mr. Stern to explain briefly the present plan and the immediate preceding plan. The tremendous amount of additional detail which is not relevant to the topic of the paper will confuse the student, and what is worse, might discourage him from reading the full paper. It would have been far more preferable in my opinion to eliminate the detailed discussion on differences in various classification plans and devote that space to a fuller treatment of the important element of expense provisions.

In his discussion of the statewide rate level Mr. Stern mentions that incurred losses in excess of basic limits are excluded from the experience used in basic manual ratemaking. In his definition of excess losses he also touches the fundamental distinction between basic limits and excess limits losses. Nowhere in his paper however, does he explain clearly that basic limits rates (whether for 5/10 limits or 10/20 limits) are based on the experience in the state for this portion of the coverage on all policies and that the excess limits coverage above basic limits is reviewed separately, at longer intervals than for basic limits coverage, on essentially a countrywide basis in accordance with the applicability of the excess limits tables.

Perhaps it would have been better to add the words "in New York" to the title of the paper and eliminate all reference to what is done

outside of New York because the differences are essentially matters of detail and pointing them up in the paper can confuse rather than clarify the matter in the minds of students. For example, territory relativities, are based on the three latest years in New York and Mr. Stern refers to the use of five years in other states in his discussion of territorial combinations. Furthermore, the section on statewide rate level is unduly complicated by the discussion of earned factors and loss development for the increment of coverage between \$5,000/10,000 and \$10,000/20,000 in New York. In this connection, Mr. Stern also seems to subordinate the importance of the development of claim costs and claim frequencies to the rate at which exposures and premiums are earned in the development of the earned factor. In addition, the fact that New York State is treated as two states (the three boroughs of New York City and the balance of the state) and the added complication of reflecting the offset for the Preferred Risk Rating Plan in the development of present average rates will add to the bewilderment of the uninitiated.

While Mr. Stern specifically noted many exceptions to the general procedure that he was discussing, for the most part they were so minor that they could have been omitted without hindering the students' understanding of the subject.

As I have mentioned earlier in this discussion, my remarks were intended to supplement what Mr. Stern has said and clarify some parts of his paper in the hope that students would benefit. They were not intended to criticize Mr. Stern's careful handling of a technical and complicated subject. The Proceedings of our Society are richer by the addition of his fine paper which will be beneficial to students and others who consult the Proceedings for knowledge.

DISCUSSION BY E. T. BERKELEY

This paper, like Mr. Marshall's recent paper on Workmen's Compensation rate-making, has been written primarily for actuarial students, particularly those who are preparing themselves for Part IV (b) of the Associateship Examinations of the Society, which covers the general principles of rate-making.

Papers of this type are very welcome for they make readily accessible to the students authentic information relating to the fundamental rate-making methods utilized in two of the major classes of business. Mr. Stern deserves a vote of thanks for the time and thought he obviously had to devote to the preparation of this paper and the excellent product he succeeded in turning out.

Mr. Stern develops a logical explanation of the various steps in the rate-making process as respects bodily injury and property damage rates, drawing upon the latest New York rate revision for illustrative exhibits. He explains the source of statistics, defines terminology and then sets forth the basic steps in a rate revision in detail, namely:

A. Determination of state-wide rate level.

- B. Development of rate-level change by territory.
- C. Calculation of classification rates.

Thus, the conscientious reader should succeed in acquiring a satisfactory knowledge of the method used by the National Bureau of Casualty Underwriters and the Mutual Insurance Rating Bureau for the making of automobile liability rates in a state like New York.

When I first went through this paper I thought it might have been improved by the inclusion of comments on the evolution of the various procedures, the reason for the adoption of particular methods and similar phases of the subject, but a second and more careful reading convinced me that the material presented by Mr. Stern is adequate for the immediate needs of the reader concerned with the general principles of rate-making. Later on, after a few years of general experience in the business and dealing with actuarial problems, the student should have developed a more mature viewpoint, permitting a fuller appreciation of the historical aspects of the rate-making procedure, which should be covered properly and more effectively, I believe, in another paper.

In such a paper, besides some of the matters just mentioned, there could well be included for the benefit of both the casual reader and the student alike a discussion of questions similar to the following, which might have occurred already to the alert and inquisitive mind after reading Mr. Stern's paper:

1. Can the reasons for a developed loss ratio higher or lower than the expected loss ratio be determined and appropriate changes made in the rate-making procedure for future revisions?
2. How reliable are the rates in a state where the member companies of the rate-making organization write only a small portion of the total business?
3. If the provision for underwriting profit and contingencies is required entirely for contingencies, do the companies still make a profit, from interest earnings?

I have been asked questions of this sort numerous times and I feel sure my experience is not unique. Since this indicates a general need for answers other than the discussion of general principles to be found in the Society's references for study in connection with a few such questions, the value of an integrated presentation in a sequel to Mr. Stern's paper becomes apparent.

MONTH OF LOSS DEFICIENCY RESERVES FOR AUTOMOBILE
BODILY INJURY LOSSES INCLUDING RESERVES FOR
INCURRED BUT NOT REPORTED CLAIMS

DAVID A. TAPLEY

VOLUME XLIII, PAGE 166

DISCUSSION BY N. M. VALERIUS

As a given interval of time moves off into the past, the accidents happening in that time become reported to the insurance company, are estimated as to cost, re-estimated if necessary, and are eventually settled, or closed without payment. In the process, the aggregate incurred loss to the company from those accidents firms up into the ultimate figure. Mr. Tapley's thesis is that this comes about according to a development pattern, primarily dependent on the company's claim practices, that can be studied and relied on for estimating final incurred cost for other later periods of time whose losses have not yet matured.

The paper offers an unorthodox approach to the problem of reserves for incurred but not reported claims. In the first place, it emphasizes that "the interplay of loss transactions" must be recognized, that is, the offset of late reported claims, reopenings and individual claim reserve increases against reserve reductions, settlements, and claims closed without payments. In other words, it attacks all pluses and minuses with one statistical treatment.

The traditional approach has been to have the statistical, actuarial or accounting departments, that are responsible for the annual statement, estimate the incurred but not reported losses and to hold the claim department responsible for adequacy of reserves of reported cases. Did this custom of divided responsibility give basis for the odd statement in the paper concerning two early treatments of the subject, "neither . . . advanced the premise that the incurred but not reported claim reserve together with reserves for reported losses would offset the total liability of the company for losses incurred but not disposed"? In spite of mandatory schedules of recent years that exhibit and emphasize total incurred loss developments, from all sources, in particular Part 5 of Schedule P of the annual statement, the divided approach persists.

In the second place, the method is unorthodox in operating with month of loss where others operate with year of loss. Furthermore, it has the unexpected result that only very recent months require any reserve for unknowns, that is, in the author's company.

The development pattern is found to be such that only the losses of the last three (shifting to four recently) accident months need any deficiency reserve in addition to individual accident cost estimates. For all more mature accident months, the reserves for known

cases are good estimates of the known and the hidden future liabilities.

The method at the time of writing the detailed description was specifically as follows:

Expected Incurred cost of month just ending = case reserves \div .500

Expected Incurred cost of month prior = case reserves \div .760

Expected Incurred cost of month next prior = case reserves \div .840

Expected Incurred cost of all previous accident months — paid losses plus case reserves.

Under this method, it is necessary to maintain month of loss analyses, of course. These provide valuable current run off tests for any company and are more and more feasible to maintain as electronic equipment comes into use.

The development pattern would preferably be applied, as it is derived, as the pattern of the developing known incurred cost, that is, projection factors would be applied to the sum of paid losses and case reserves instead of using alternate factors applied to case reserves alone. But "it is procedurally difficult to obtain fully detailed data appropriate to the current month of loss in the short time available before monthly closing entries must be made." Remember that breakdown by month of loss is required. Only reported reserves are available in time.

Furthermore, one must estimate the paid losses of the three last accident months, as the actual figures are not available soon enough, in order to derive the deficiency reserve from the equation,

Expected incurred cost — case reserves — estimated paid = deficiency reserve.

The expected paid losses are derived from the development pattern as 6%, 8%, and 12% of ultimate losses for the first, second, and third months respectively. We have, as the combined result:

Deficiency reserve of month just ending = 88% of case reserve

Deficiency reserve of month prior = 21.1% of case reserve

Deficiency reserve of month next prior = 4.8% of case reserve

It interests this reviewer to find Mr. Tapley's new method under the necessity to base hidden liability reserve on the case outstanding. Down through the years in our own company, as we have been criticized from time to time for basing our incurred but not reported reserve on the case outstanding, it has been our clincher that these figures come first to hand and other bases would be too late. It is fair to say now, in passing, that the time of arrival is being affected by the new machines and, in our case, there may be consequent changes.

In an addendum within the paper, the author states that recent higher average claim costs have moved the point of stability out beyond the fourth month. It is noted also that there are large fluctuations in the monthly losses and so in the hidden liability reserve re-

quirement. Possible causative factors are discussed. The author hopes longer acquaintance with the monthly analyses will help to explain what happens.

The author feels quite definitely that an improvement in estimating losses has been achieved. Nevertheless his company continues to keep incurred but not reported estimates at hand. I concur in his feeling that the month of loss analysis separates the loss data into convenient packages from which much can be learned as to loss behaviour. We have been running similar analyses over the same period as a part of information to management and for comparison I show here in the same way as the author's January 1954 losses at the bottom of the first page, the reported incurred losses for the January 1954 month of loss, excluding our New York Office and Massachusetts business.

<i>Date of Evaluation</i>	<i>Reported Loss Reserves</i>	<i>Cumulative Paid Losses</i>	<i>Reported Losses</i>	<i>Ratios</i>
1-31-54	\$ 822,996	\$ 18,375	\$ 841,371	.555
2-28-54	1,140,857	88,122	1,228,979	.810
3-31-54	1,228,486	208,943	1,432,429	.944
4-30-54	1,179,911	337,372	1,517,283	1.000
5-31-54	1,148,967	414,795	1,563,762	1.081
6-30-54	1,104,188	493,144	1,597,332	1.053
9-30-54	945,993	699,203	1,645,196	1.084
12-31-54	725,093	924,237	1,649,330	1.087
3-31-55	559,652	1,055,277	1,614,929	1.064
6-30-55	431,865	1,159,005	1,590,870	1.048
9-30-55	380,732	1,207,913	1,588,645	1.047

This paper is stimulating and informative. It does seem to the reviewer, however, that the writer has been too optimistic in his appraisal, being based on so short an acquaintance with the method and its results. He should certainly continue to produce the usual incurred but not reported reserve alongside the subject method, as he is doing.

A theoretical appraisal may be stated as follows, setting down first a restatement of the thesis:

- 1) The case basis incurred value (paid losses plus estimated unpaid losses) for claims reported up to the point of stability (the end of the 4th or 5th month starting with the month of the accidents) follows a fairly definite pattern, this pattern being characterized by a maximum at the point of stability with some tailing off thereafter.
- 2) The incurred losses which have not emerged before the point of stability but come to light later are matched by the redundancy in the reserves of known cases being currently closed.

It therefore remains only to determine during the first 3 (or 4)

months a "deficiency reserve," intended to bridge the gap between the case basis value of known claims and the case basis value of known claims at the end of the 4th (or 5th) month. The basis for determining such "deficiency reserves" is to apply a factor to the incurred value of known claims at the end of the first, second, third, etc. months, this factor being determined from a study of past relationships between the values of then known claims at the ends of these early months and the value at the end of the 4th (or 5th) month. Actually the factor is applied to outstanding value of known claims, not the incurred value, because of procedural difficulties.

Mr. Tapley suggests that the pattern of the total incurred value of known claims for this company may not be valid for other companies and that the data from which he derives his "deficiency reserves" during the first 3 or 4 months may also not be valid for other companies. He does not, however, mention what appears to be a basic theoretical flaw in his method, namely, that there is no logical relationship between the patterns of development of the incurred value of reported losses and of the value of incurred but unreported losses. It is implicit in his theory that the incurred value of reported losses be overstated at the end of the 4th or 5th month by the then value of incurred but unreported claims.

Is there a logical reason why this relationship should exist? It is obvious that the basis for this supposed relationship would be destroyed if the claims adjusters who establish the value of known claims refine their estimating processes to a point where the value at the end of the 4th or 5th month is a true ultimate value. In other words, the whole fabric would be destroyed by a change in the estimation practices followed by the adjusters.

Similarly we may criticize the projection of the "deficiency reserve" during the first months from the incurred value of known claims. There is no necessary logical relationship. In fact, any tendency to delay unduly the reporting of claims would tend to decrease reported claims and thereby decrease "deficiency reserve" just at a time when the "deficiency reserve" ought to increase.

While this method of developing "deficiency reserves" and of assuming that incurred but not reported reserves are taken care of by over-estimate of reported reserves after the 4th or 5th month may be valid while conditions continue to follow the pattern they have displayed recently, in general, the deduction of laws from observed phenomena is only valid when it is reasonable to assume that logical relationships exist among the observed phenomena.

DISCUSSION BY L. J. SIMON

Mr. Tapley's paper is very interesting and clearly presented. A second reading is strongly recommended because it will reveal a number of subtle points that may have been slighted in the first reading. The philosophy of the deficiency reserve approach is quite stimulating to the imagination and should provoke some interesting dis-

cussions among actuaries as well as within companies. People concerned with claims procedures and those concerned with developing figures for the financial statements of the company are often not actuaries, and this approach will take a great deal of salesmanship on our part.

My remarks will be chiefly directed to the area of statistical experimentation and testing of hypothesis based on the data presented in Mr. Tapley's paper. To restate the author briefly, the month of loss deficiency reserve is the amount of reserve needed to complement case reserves and paid losses in order to give a complete evaluation of incurred losses for an accident month. The deficiency is due to the company not having complete information on losses which have occurred either because the loss has not been investigated thoroughly enough to permit an accurate case basis estimate to be made, or because the loss has not been reported as yet. To establish this month of loss deficiency reserve one must somehow estimate or predict the total loss for the given accident month, called "base" loss (also referred to by some as "ultimate" loss.) Then by deducting payments to date and case basis reserves outstanding from the predicted "base" loss one arrives at the deficiency reserve for the month in question.

The first phase of this discussion will be to develop regression equations which will be usable in predicting the deficiency reserve for a given month at each stage of its development. The second phase will be an analysis of variance to test the homogeneity of the years and the months. Let me hasten to add immediately that these techniques are by no means suggested as a mathematical substitute for the oft discussed "judgment". Rather they are designed to give us more facts and clearer guides to the correct answer. With more complete actuarial analyses of the facts, we have a fuller knowledge and hence can make better judgments.

To illustrate the use of regression equations in predicting the unknown values, let's define

Y = the "base" loss

P_i = the amount paid to date on a month, i is the age or stage of development and equals 1, 2, . . .

R_i = the case reserves as of the end of the i^{th} month of development where i equals 1, 2, . . .

D_i = the deficiency reserve as of the end of the i^{th} month of development where i equals 1, 2, or 3.

These symbols will carry superscripts of one prime to indicate the predicted value as of the end of month 1, two primes for the month 2 prediction, and three primes for month 3. For example, Y'' is the predicted "base" loss for a certain accident month where the prediction is made just after the close of month 2. As another example, P_3' is the amount paid through the close of month 3 on a certain accident month. In this notation the author uses the following formulae in making his deficiency reserve predictions:

$$Y' = \frac{R_1}{.500} \text{ and } P_1' = .06Y' \quad \text{so } D_1' = Y' - P_1' - R_1 = .8800R_1$$

$$Y'' = \frac{R_2}{.760} \text{ and } P_2'' = .08Y'' \quad \text{so } D_2'' = Y'' - P_2'' - R_2 = .2105R_2$$

$$Y''' = \frac{R_3}{.840} \text{ and } P_3''' = .12Y''' \quad \text{so } D_3''' = Y''' - P_3''' - R_3 = .0476R_3$$

To establish equations similar to these using least squares principles, the monthly data for 1954 from Tapley's Exhibits II, V, and VI was recorded in Table I rounded off to thousands. Table II contains the summary statistics, and the only symbol not defined there is N , the number of months in the sample.

We are setting out with an objective of getting the best predictions of Y , P_1 , P_2 , and P_3 which we recognize will be made by those variables which most closely correlate with them. These correlation coefficients were calculated from the general formula:

$$r = \frac{\sum XZ}{\sqrt{\sum X^2 \cdot \sum Z^2}}$$

As a test of significance (see reference 1, page 193) :

$$t = \frac{r(N-2)}{\sqrt{1-r^2}} \quad \text{with } (N-2) \text{ degrees of freedom.}$$

The results are as follows:

<i>To be used for month:</i>	<i>Variables</i>	<i>r</i>	<i>t</i>	<i>Probability that r is zero is less than:</i>	
1	R ₁ & Y	.855	5.21	.001	At month 1 we have no choice but to predict Y and P ₁ by using R ₁ . Fortunately the correlations are fairly good and are statistically significant. At month 2, however, we have R ₁ , P ₁ , and R ₂ available so predictions can be made using these singly or in combination. To predict Y, (P ₁ +R ₂) is quite good, but R ₂ will be used by itself because it is easier to use and has nearly the same correlation with Y. There is no really good way to predict P ₂ so one could just use the mean of the observed values. As a matter of personal judgment, it is felt preferable to use P ₁ .
1	R ₁ & P ₁	.751	3.59	.01	
2	R ₂ & Y	.969	12.42	.001	
2	(P ₁ +R ₂) & Y	.702	3.12	.02	
2	P ₁ & Y	.974	13.53	.001	
2	P ₁ & P ₂	.478	1.72	.2	
2	R ₂ & P ₂	.333	1.12	.3	
2	(P ₁ +R ₁) & P ₂	.462	1.65	.2	
2	(P ₁ +R ₂) & P ₂	.344	1.16	.3	
3	R ₃ & Y	.964	11.48	.001	
3	P ₂ & Y	.470	1.68	.2	
3	(P ₂ +R ₂) & Y	.980	15.56	.001	
3	(P ₂ +R ₃) & Y	.990	22.00	.001	
3	P ₂ & P ₃	.946	9.18	.001	
3	R ₃ & P ₃	.310	1.03	.4	
3	(P ₂ +R ₂) & P ₃	.676	2.90	.02	

Here is a point where actuaries will disagree among themselves because there is no clear evidence of what choice should be made.

At the close of month 3 we know R₁, R₂, R₃, P₁, and P₂ and wish to predict Y and P₃. For P₃, there is no question that P₂ is the best predictor. To predict Y, the strongest correlations are with R₃, R₂, (P₂+R₂) and (P₂+R₃) in that order. Because (P₂+R₂) is available from last month's punched card runs and correlates very highly with Y, it will be selected. Notice how judgment plays a part in this process, but how clearly the actuarial tools guide its use. The general form of a regression equation is:

$$Z' = \bar{Z} + \frac{\sum xz}{\sum x^2} (X - \bar{X})$$

This form of regression will be called Method A and produces a probable error of

$$.700 \sqrt{\frac{\sum Z^2 - \frac{(\sum XZ)^2}{\sum X^2}}{N-2}}$$

where .700 is used instead of the customary .674 because we only have 10 degrees of freedom.

Another regression equation may be established which is similar to the type used by the author in that no constant is involved and the line thus passes thru the origin. This will be called Method B and the general form of the regression equation is:

$$Z' = \left(\frac{\sum XZ}{\sum X^2} \right) X$$

and the probable error of prediction is:

$$.697 \sqrt{\frac{\sum Z^2 - \frac{(\sum XZ)^2}{\sum X^2}}{N-1}}$$

where .697 is used due to having 11 degrees of freedom.

The results of the two methods are:

The Best Least Squares Equation

Probable Error of Prediction

To Predict	Method A	Method B	As a Percent of Mean Prediction			
			In Units		Prediction	
			Method A	Method B	Method A	Method B
Y'	1.5895R ₁ +727.96	2.0770R ₁	166.50	186.09	5.5%	6.1%
P ₁ '	0.0374R ₁ -9.49	0.0310R ₁	5.68	5.76	12.6	12.8
Y''	1.4146R ₂ -248.53	1.3095R ₂	79.31	82.79	2.6	2.7
P ₂ ''	3.0011P ₁ +92.78	4.9340P ₁	47.45	50.25	20.8	22.1
Y'''	1.2935(P ₂ +R ₂)-261.35	1.1928(P ₂ +R ₂)	63.88	68.38	2.1	2.2
P ₃ '''	1.0939P ₂ +130.96	1.6145P ₂	20.25	35.65	5.3	9.4

To conclude the discussion of single variable regression equations, it appears that method B produces very nearly as good results as method A and has the advantage of simplicity and logical clearness. It would, undoubtedly, be the method to use in the practical situation.

Having progressed this far one cannot help but wonder what would occur if every possible shred of loss and reserve evidence were used at each stage to make the best possible linear multiple regression prediction of the value D itself. Rather than going through all the calculations necessary to get the needed sums of squares and cross products, the formulae for D make it just a matter of algebraic manipulation to get the values.

By definition,

$$D_1 = Y - P_1 - R_1$$

$$D_2 = Y - P_2 - R_2$$

$$D_3 = Y - P_3 - R_3$$

At the end of month 1 only R_1 is available so we proceed as before to produce $D_1' = .5521R_1 + 737.45$ with a probable error of 164.65 which is 10.7% of the mean prediction. To protect against the undesirable effects of being under-reserved it might be advisable to cover one probable error by applying a 10% additional "safety factor" to D_1' .

At the end of month 2 we have R_1 , P_1 , and R_2 available for predicting D_2'' . The most complex linear combination envisioned is:

$$D_2'' = aR_1 + bP_1 + cR_2 + d(R_1 + P_1) + e(R_2 + P_1) + f(R_2 - R_1) + g$$

There is considerable overlap here, but it is planned to eliminate all variables that do not contribute significantly to the regression. A multiple regression equation such as this can be solved by a number of methods. Personal preference led to the use of Doolittle's method (see reference 2, page 327) because it provides a systematic way to test the statistical significance of the regression coefficients and eliminate those that are not significant. The solution is rather tedious and will not be presented here, but the resultant equation is:

$$D_2'' = .0057R_1 + 5.6855R_2 + .0570R_2 + .0184(R_1 + P_1) + .0628(R_2 + P_1) + .1785(R_2 - R_1) + 240.11$$

However, most of these coefficients have no statistical significance whatsoever. Eliminating non-significant variables one by one resulted in $D_2'' = 4.9312P_1 + .2023R_2 - 204.08$ with a probable error of 59.66 or 12.2% of the mean prediction. The multiple correlation coefficient is .793 and the test of significance on the regression coefficients results in probabilities of less than .10 that the coefficients equal zero. This is not very encouraging, but they are the best available. Here again it is suggested that a safety factor be employed of, say, 12%.

At the end of month 3 we know P_1 , P_2 , R_1 , R_2 , and R_3 . The following equation was tried, $D_3''' = hP_2 + iR_3 + j(P_2 + R_2) + k(P_2 + R_3) + m(R_2 + P_2 - R_1 - P_1) + n$.

Proceeding as before, the equation was solved, but the results showed no significant regression coefficients. The most promise was held by P_2 which has a probability of .15. The equation is: $D_3''' = .4293P_2 + 46.88$ with a probable error of 41.22 or 28.5% of the mean prediction. Here again a loading of about 30% would be applied to the prediction as a "safety factor".

It is interesting to note how the size of the probable error decreases as more information becomes available, but the size of the prediction decreases even faster so that our percentage error becomes quite large.

In closing the phase on least squares regression equations as a means of predicting, it would be well to pinpoint the two primary advantages over the judgment method (where one looks at a series of factors and selects one that looks reasonable) or the simple arithmetic method (adding up a series of factors and dividing by N).

The first advantage is that it provides a statistical method of selecting among the various criteria available for predicting, thus allowing the actuary to reject those which are of no significance and permitting him to select the best among the remaining indicators. Secondly, the range of error in the prediction may be specified using this method and the actuary has a clear concept of the likely fluctuation in his prediction.

The second phase of the analysis of Mr. Tapley's data dwelt on testing the two hypotheses (1) there is no difference in loss amounts between the years 1954 and 1955 and (2) there is no difference among the various means of the months January through July. These hypotheses may both be tested by an Analysis of Variance and for this purpose the "base" losses from 1954 and 1955 for the months January through July were arrayed as shown in Table III.

There are several excellent sources for information on Analysis of Variance along with working models (see reference 3, page 24) and this is one of the simpler types. Summarizing the results from Table III in a convenient table:

ANALYSIS OF VARIANCE TABLE

<i>Variance</i>	<i>Degrees of Freedom</i>	<i>Sum of Squares</i>	<i>Mean Square</i>	<i>F</i>
Between years	1	922,631	922,631	31.40
Between months	6	659,570	109,928	3.74
Error	6	176,304	29,384	
TOTAL	13	1,758,505		

The hypotheses were tested in the following manner:

(1) Hypothesis: There is no significant difference between years.

$$F = \frac{922,631}{29,384} = 31.40. \text{ Enter } F \text{ table with } n_1 = 1 \text{ and } n_2 = 6.$$

The .01 value of F is 13.74 and therefore we reject the hypothesis.

(2) Hypothesis: There is no significant difference between the $\frac{109,928}{29,384}$ months. $F = 3.74$. Enter F table with $n_1 = 6$ and $n_2 = 6$. The .05 value of F is 4.28 and therefore we accept the hypothesis.

The conclusion reached then is that the seven months are homogeneous but there is a significant difference between years. Our own knowledge and experience in the field tells us that this difference between years might be attributed to an increased volume of business or due to an increase in loss costs on the line of insurance. The author was kind enough to furnish me with the fact that between the two years the average increase in earned exposure was 12.1%. The effect of this increase was eliminated from the data by dividing each X_{2i} by 1.121 and again running the analysis of variance. The results are:

ANALYSIS OF VARIANCE TABLE

<i>Variance</i>	<i>Degrees of Freedom</i>	<i>Sum of Squares</i>	<i>Mean Square</i>	<i>F</i>
Between Years	1	88,166	88,166	3.38
Between Months	6	588,229	98,038	3.76
Error	6	156,602	26,100	
TOTAL	13	832,997		

The hypotheses were tested in the following manner :

- (1) Hypothesis: There is no significant difference between years.

$$F = \frac{88,166}{26,100} = 3.38. \text{ Enter } F \text{ table with } n_1 = 1 \text{ and } n_2 = 6.$$

The .05 value of F is 5.99 and therefore we accept the hypothesis.

- (2) Hypothesis: There is no significant difference between the

$$\text{months. } F = \frac{98,038}{26,100} = 3.76. \text{ Enter } F \text{ table with } n_1 = 6 \text{ and}$$

$n_2 = 6$. The .05 value of F is 4.28 and therefore we accept the hypothesis.

This analysis shows that when we take account of the increase in exposure, the entire group of data may be considered homogeneous both as to month and as to year.

A myriad of other statistical questions arise as a result of this paper. Can a smaller company utilize these techniques? If we had accurate earned exposure on a monthly basis and could calculate accurate pure premiums, would we still find the data to be homogeneous? Could a method such as this be used in lieu of establishing case reserves, especially in lines with a smaller variance than bodily injury? What could be done if a company were not so fortunate as to hit a stability point at month 4, but instead had quite variable results over a long period? In smaller companies, would earned exposure and earlier information on paid losses be available soon enough and wouldn't it improve the predictions considerably? Could claim count and reserve count be introduced to additionally sharpen the prediction? Many of the answers are self-evident, but may serve to stimulate other approaches and variations in technique. Three questions of a more imponderable nature are (1) Will actuaries be sufficiently persuasive within their own companies to establish this method as an increase in accuracy and an expense saver if it is used in lieu of punched cards; (2) Will rate makers be able to establish it as an integral part of rate making methods and thus relieve company tabulating departments of bothersome detail and simultaneously increase accuracy; (3) Will supervisory officials accept such a formula approach especially if it is used in lieu of case basis reserves? Let's

work for the best and retain the highest possible standards in this field of endeavor.

TABLE I RAW DATA

	Y	P ₁	P ₂	P ₃	R ₁	R ₂	R ₃
1954 January	2645	44	172	315	1220	2096	2232
February	2601	28	130	264	1076	2045	2219
March	2592	51	248	389	1282	1950	2078
April	2529	40	147	290	1454	2051	2160
May	3188	31	195	338	1453	2592	2777
June	2808	35	364	507	1430	2116	2207
July	3052	38	195	379	1454	2349	2485
August	2948	48	198	336	1505	2183	2421
September	3205	49	328	445	1332	2336	2472
October	3523	55	207	385	1582	2598	3001
November	3313	46	204	341	1592	2578	2860
December	4081	74	343	570	2078	3006	3279
TOTAL	<u>36485</u>	<u>539</u>	<u>2731</u>	<u>4559</u>	<u>17458</u>	<u>27900</u>	<u>30191</u>

TABLE II SUMMARY STATISTICS

MEANS

(General Notation: \bar{X})

Y	P ₁	P ₂	P ₃	R ₁	R ₂	R ₃
3040.4167	44.9167	227.5833	379.9167	1454.8333	2325.0000	2515.9167

SUMS OF SQUARES AND CROSS PRODUCTS OF VALUES

(General Notation: $\sum XZ$)

	Y	P ₁	P ₂	P ₃	R ₁	R ₂	R ₃
Y	113,243,231						
P ₁	1,682,350	25,873					
P ₂	8,486,401	127,658	687,045				
P ₃	14,148,456	212,290	1,109,222	1,819,603			
R ₁	54,144,698	809,217	4,068,661	6,790,734	26,068,562		
R ₂	86,362,034	1,277,818	6,438,376	10,758,395	41,304,743	65,952,172	
R ₃	93,647,797	1,387,077	6,954,194	11,636,199	44,770,274	71,489,635	77,556,739

SUMS OF SQUARES AND CROSS PRODUCTS REDUCED TO DEVIATIONS ABOUT THE MEANS

(General Notation: $\sum xz = \sum XZ - \bar{N} \cdot \bar{X} \cdot \bar{Z}$)

	Y	P ₁	P ₂	P ₃	R ₁	R ₂	R ₃
Y	2,313,628.92						
P ₁	43,565.42	1,662.92					
P ₂	183,023.08	4,990.58	65,514.92				
P ₃	287,196.42	7,514.92	71,669.58	87,562.92			
R ₁	1,065,103.83	25,061.83	95,511.17	158,148.88	670,081.67		
R ₂	1,534,409.00	24,643.00	88,801.00	158,720.00	714,893.00	1,084,672.00	
R ₃	1,854,577.42	30,997.92	83,225.58	116,139.92	847,400.83	1,295,560.00	1,598,698.92

TABLE III RAW DATA AND SOME CALCULATIONS

Month <i>t</i>	Years			
	X _{1t}	X _{2t}	$\sum_s X_{st}$	$\bar{X}_{.t}$
X _{s1}	2645	3023	5668	2834.0
X _{s2}	2601	2834	5435	2717.5
X _{s3}	2592	3343	5935	2967.5
X _{s4}	2529	3453	5982	2991.0
X _{s5}	3188	3540	6728	3364.0
X _{s6}	2808	3254	6062	3031.0
X _{s7}	<u>3052</u>	<u>3562</u>	<u>6614</u>	3307.0
$\sum_t X_{st}$	19415	23009	42424	
$\bar{X}_{.s}$	2773.57	3287.00		3030.29

The notation employed designates the value in the s^{th} year and the t^{th} month as X_{st} where $s = 1, 2$ and $t = 1, 2, \dots, 7$. Means are denoted as $\bar{X}_{.s}$ to indicate that it is the mean of all t values for a particular s and by $\bar{X}_{.t}$ to denote that it is the mean of all s values for a particular t . The grand mean of the entire table will be denoted $\bar{X}_{..}$. Finally, N_s is the number of cases of s (2 in our example), $N_t = 7$ and $N_{st} = 14$. Let's call the sum of squares between years χ_1^2 , between months χ_2^2 , and the residual or error χ_a^2 . Then:

$$\chi_1^2 = \sum_s \sum_t (\bar{X}_{.s} - \bar{X}_{..})^2 = (2773.57 - 3030.29)^2 + (3287.00 - 3030.29)^2 = 922,631 \text{ with degrees of freedom} = N_s - 1 = 1$$

$$\chi_2^2 = \sum_t \sum_s (\bar{X}_{.t} - \bar{X}_{..})^2 = (2834.00 - 3030.29)^2 + \dots + (3307.00 - 3030.29)^2 = 659,570 \text{ with degrees of freedom} = N_t - 1 = 6$$

$$\chi_a^2 = \sum_s \sum_t (X_{st} - \bar{X}_s - \bar{X}_t + \bar{X}..)^2 = (2645 - 2773.57 - 2834.00 + 3030.29)^2 + \dots + (3562 - 3287.00 - 3307.00 + 3030.29)^2 = 176,304 \text{ with degrees of freedom} = (N_s - 1)(N_t - 1) = 6$$

Finally, as a check on the calculations

$$\chi_1^2 + \chi_2^2 + \chi_a^2 = \sum_s \sum_t (X_{st} - \bar{X}..)^2 = (2645 - 3030.29)^2 + \dots + (3562 - 3030.29)^2 = 1,758,505 \text{ with degrees of freedom} = N_{st} - 1 = 13$$

Other more convenient computational formulae can be developed and actually were employed by the writer.

- Reference 1: Fisher, R. A., *Statistical Methods for Research Workers*, 10th ed., Edinburgh: Oliver and Boyd Ltd., 1948
- Reference 2: Johnson, P. O., *Statistical Methods in Research*, New York: Prentice-Hall, Inc., 1949
- Reference 3: Jackson, R. W. B., *Application of the Analysis of Variance and Covariance Method to Educational Problems*. Department of Educational Research, University of Toronto, Bulletin No. 11, 1940

AUTHOR'S REVIEW OF DISCUSSION

DAVID A. TAPLEY

Mr. Simon's discussion of regression equations and of the need for continuously testing the homogeneous character of the data we are tabulating is more than just pertinent. It brings into effective focus an understanding of the substantial amount of detailed and continuous testing that is being carried forward in connection with these data. The purpose of these tests is twofold. Initially each monthly tabulation must be reviewed to determine known procedural change effects. Secondly, and only after such validation, it must be combined with prior data and current values for the various factors used procedurally must be re-computed and tested.

It may be observed that these continuing processes of analyses are far too cumbersome and expensive to maintain for the sole purpose of producing deficiency reserves. The additional companywide products of our program include the testing of current loss levels, the testing of total reserve levels, the early determination of loss and procedural trends and other items we need not mention here. In all these connections, the early statistical reflections of change, the knowledge that something is different to a measured degree is often of great assistance.

While actuaries and statisticians may have individual preferences as to the detailed form of these testing processes, Mr. Simon has given a clear indication of their general characteristics. In addition

we generally concur with his stated views on the use of regression equations. At the time the paper was written, we were dissatisfied with the sheer lack of samples available for such purposes. However, the simple factors we employed to obtain "guide" projections have worked out better than we had any statistical reason to expect.

The discussion by Mr. Valerius is quite broad and raises certain fundamental questions. Without indulging in repetitious quotation, several comments are made with the intent of showing that the month of loss reserve concept is particularly susceptible to specific dangers which are inherent in almost any reserve process.

Within any given body of collected loss experience exist the unchangeable loss components underlying every analytic method whether it be based on calendar, policy or accident period. We can vary our methods but we cannot alter the data once it is established in the record, either in total or with respect to any individual component thereof. The several components of incurred losses appear, as Mr. Valerius suggests, quite erratic and independent; that is, there does not seem to be any logical and necessary relationships among them except that they are all components of total incurred losses. Furthermore their separate values are constantly interchanging under development. Unreported losses become reported losses. Reported losses become variable reserves, or payments, or reserves closed without payment. After some extended period they all convert to a single total of claims paid under both the policy and accident period forms of analysis.

The lack of logical relationship among the several components of total incurred losses means that we are utterly dependent upon sound statistical concepts in the evaluation of all loss data, and this particularly is true in dealing with that portion of our total losses which are said to be incurred but not reported. It also means that no single component can be demonstrated as a logical function of, or necessarily to change consistently in relation to, any other component. Thus any formula or procedure which evaluates one component on the basis of data for a second component is not based upon a logical relationship. Instead it is based upon temporary consistencies in observed data that are subject to change.

The lack of logical relationships among the several components of total incurred losses obviously creates particular requirements that must be met by any method designed to evaluate losses and loss components. Let us assume momentarily that:

- (1) $X_1, X_2, X_3, \dots, X_n$ represent total incurred losses on either a policy or an accident period basis at succeeding dates of evaluation for a given period of loss, and
- (2) $A_1, A_2, A_3, \dots, A_n$
and $B_1, B_2, B_3, \dots, B_n$ represent a division of components of total incurred losses as above,

for every given date of evaluation, $A_1 + B_1 = X_1$, where there is no

demonstrable relationship between the A and B components and where the matured or X_n value of total incurred losses can only be proven by development.

We have no choice but to assume the stated $A_1 + B_1 = X_1$ relationships exist. They are inherent in every evaluation of total incurred losses whether for ratemaking or for annual statements.

Similarly we are forced to assume that successive values for X must be maintained as accurately and also as consistently as possible. Any assumption to the contrary makes it difficult to support ratemaking techniques which exclude retrospective adjustment factors.

Now if we attempt to determine successive values for A_1 , (incurred but not reported losses) separately from B_1 , (known losses), we have only one possible way in which to test the accuracy of the total values so obtained. We must examine the resulting values of X_1 , for stability. Under the month of loss concept this testing process is carried forward continuously.

The lack of logical relationships among the several components of loss is the cause of what has been termed an implicit theoretical flaw in our method. Unquestionably, known losses are understated in early periods of development because of the then value of incurred but not reported claims. By the same standard they are understated by the then value of claims that will reopen and by the additional reserves needed on claims reported but not investigated. They are also overstated by the then value of reserves that will eventually close without payment. This type of "flaw" is implicit in the data, not in the method of analysis. Customary methods make little effort to define and evaluate such "flaws". One major objective of the deficiency reserve plan is to prevent such unavoidable "overlappings" from being reflected as large fluctuations in the developing value of total incurred losses and, in the event such variations do occur, to expose them in fullest detail to the eyes of management.

Procedural distortions, such as result from a change in the estimation practices of adjusters, are reflected in the components of loss making up any body of recorded experience. We cannot eliminate them as potential sources of error from the basic data. We can average them over an annual period but this does not prevent their accumulation in one direction in times of trend or change. By adopting less than annual periods of evaluation we, potentially, if not actually, increase the ranges of probable variation due to reduced reliability of the smaller segments of data. In contrast, however, we also benefit from a greater definition of components and more flexibility of method. This facilitates the early discovery of all variations, helps to measure them effectively and assists in indicating any necessary adjustments needed in our procedures. There is unquestionably some minimum size of exposures, losses and reserve need that will not satisfy the test of statistical significance. But this criterion is fundamental to all reserves regardless of the method of analysis. There are obviously many territories, both rating and statistical, that develop less than minimum exposures required to produce acceptable

reliability. Special techniques and procedures have historically been adopted to insure a meaningful and dependable interpretation of the loss experience that is recorded in such areas. Such special techniques and procedures are not unavailable to the interpretation of month of loss analyses.

One final point of clarification is pertinent to these discussions. Under the processes employed for developing policy year losses, it is customary to project first reportings to acceptable maturity. The projection factors so used are primarily designed to accomplish an adequate estimation of total losses incurred even though only about one half of the total exposures have then expired. The basic concepts which govern this phase of the policy year loss development processes are the same concepts which underlie the deficiency reserve program with but a single exception; namely, the deficiency reserve program has no application to losses that will be incurred in the future. Also, in a broad sense, the methods of the policy year development process have been employed under the deficiency reserve program, and these methods have been altered only as required to employ accident periods instead of policy period, and monthly instead of annual analyses. Finally, the determination of static values for all unknown losses at given dates of evaluation are obtained as the sum of such losses appropriate to all immature months of loss. Such total evaluations of unknown losses are subsequently tested by development, and may be readily reconciled to accident year rating data. They make it possible to obtain an accurate check upon adequacy of the sum of the incurred but not reported loss reserves and the known losses as reported in the Annual Statement. Here the deficiency reserve plan provides a test of the adequacy of the estimated total incurred losses determined by the sum of the incurred but not reported reserve and the reported incurred losses. Furthermore, the deficiency reserves so used may be completely reconciled to accident year statistics employed for rate-making.

It may now be seen that the deficiency reserve concept is neither in conflict, nor necessarily in competition, with customary practices. It is equally subject to the vagaries of chance variations in underlying loss components and equally responsive to sound statistical principles. Its usefulness depends entirely upon the extent to which it is adaptable to management needs and the observance of sound practice in using the data so provided. I fully share the concern of Mr. Simon and Mr. Valerius that we safeguard the soundness and high standards of our actuarial processes. I also believe that the accident period form of analysis offers considerable promise in supplementing our existing kinds of experience. It is certainly deserving of fair appraisal and adequate testing. In the light of recent industrywide developments such treatment now appears assured.

REVIEWS OF PUBLICATIONS

JOHN W. WIEDER, JR.,
Book Review Editor

Multiple-Line Insurance, G. F. Michelbacher, McGraw-Hill Book Company, Inc., New York City, New York, 1957 Edition, pp. xiii, 660.

As the revolutionary changes in casualty, surety, fire and marine insurance are at the point where the first and largest steps have been taken — this book is not only timely, but practically a necessity for those who want an up-to-date text. The author points out that you could wait until eternity for conditions to “jell”, but I think he has written this first “step” at precisely the right time. It is apparent that the author and the contributors are Casualty men and that this is a revision of a Casualty book. However, as revisions take place, as they must with books of this type, the emphasis upon certain lines may not disappear, but emphasis of importance by subject will occur.

The organization of material is generally good, and there is no loss of words or use of extra ones, which this reader always appreciates. Parts of the book give the impression that all casualty, surety and fire business has gone multiple-line, whereas only a small percentage of the business is written in some kind of “package”. Companies may now be writing all lines, but such functions as underwriting, rating, and adjusting are normally separated by line. There are also specialty companies which are here to stay.

Chapter 5 — Rate Making — Manual Rates, first explains the problem of the rate maker and the basic elements of rate making. The theoretical approach is fine, but a practical application could have been discussed. Rate making in some cases is in reality the adjusting of last year's good loss ratio up to a permissible, or adjusting last year's poor loss ratio down to a permissible. This is probably what is wrong with some rate making today — changes are so fast and varying that the rates are bound to be inadequate or may be excessive. As there are a number of ways of making rates, and as each line of insurance is somewhat different, it would be helpful if explanations of rate making by line were presented. More emphasis might have been given to the fact that the rates must be sufficient to pay all losses incurred under the policy, cover expenses, and make provision for profit; also, that past experience can be used only as a guide to make the rates for the future.

Methods of collecting experience are explained in the book; however, some confusion always seems to exist. In many attempts at explaining the differences between calendar year, policy year and accident year experience, this reviewer has found the drawing of diagrams the best way. Not only do diagrams show basically the different kinds of experience, but they also show time relativities. The policy year diagram is shown as a parallelogram. The initial or incomplete policy year used in auto private passenger and commercial

non-fleet rate making is half of the parallelogram — a 45° right triangle. For the accident year experience, a square with a diagonal line is used, which represents the two portions of the accident year experience, i.e., premiums and losses on last year's policies and on this year's policies. The policy year and accident year diagrams must include loss valuation dates. The calendar year is represented by a square drawn with broken lines.

In the chapter on Statistics it was helpful to find the different kinds of calendar year statistics defined. However, it is hazardous to assign any kind of calendar year experience to a line. Also, it should be pointed out that probably the only reliable calendar year loss ratio is the loss ratio computed on an earned and incurred basis. A written to paid loss ratio has its place but probably only on a comparative basis. Rate level adjustment factors are entirely based upon calendar year statistics on an earned and incurred basis.

On page 424 — Internal Statistics, the comment is made that the "net" basis should be used in reviewing risks as only the net amount is at risk. An underwriter should view the acceptability of a risk on a gross basis, as he has a responsibility to the reinsurer.

On the subject of divisible vs indivisible premiums, both sides of the argument could have been outlined and perhaps a compromise suggested. The compromise could be a formula breakdown. On page 426 it is implied that the MPIRO statistical plan met the challenge of providing reliable statistics. However, the question of the rerating by classification has not been answered. The problem is, once the original grouping is made, how can statistics ever be produced which will show whether the original grouping remains correct.

In the very good chapter on Government supervision one discussion was omitted. Some independents have stated that the rating law is applicable mainly to rating bureaus. The basis of their premise is that rating laws would not exist if it weren't for the concerted actions of bureau members. It might also be brought out that it is quite difficult to regulate rates of independent insurers as their small sample of business can vary substantially from year to year. Should a regulatory official compel an independent insurer to decrease rates when experience is good, or increase rates when experience is bad? Increasing rates may result in rates higher than bureau rates, thus practically putting the independent insurer out of the business. Except for large independent insurers, regulation of independents is a perplexing problem.

Particularly good explanations appear on page 67 — Choice of insurers by the insured; page 86 — Credibility; page 111 — Experience rating for individually owned private passenger automobiles; and the statistical analysis of an insurance company in the latter part of Chapter 18 and also in Chapter 21.

In Chapter 4, in defining the different insurers, it is important to point out that the participating companies are both stock and mutual, and that mutuals may be non-assessable. There are also direct writers

and agency writers in both the stock and mutuals. On page 53 it is stated that governmental managed insurers have made little headway. However, in a number of states, State Funds write a substantial amount of Workmen's Compensation—this can be considered as an inroad into the private ownership of insurers.

It was refreshing to read the whole history of a subject in one place. Aside from using this book for reference purposes, the reviewer believes that advanced college students and those in the beginning and in the midst of their insurance careers, will find its many down to earth discussions and its many practical answers of great value to them, but even more to those studying for the Society or CPCU examinations.

JOHN H. MEUTERTIES

"The First Thirty Years" Casualty Insurance Companies Serving Massachusetts, Boston, 1957. Pp. 53

While this factual commentary on the operation of the Massachusetts Compulsory Automobile Liability Insurance Act is intended primarily for the benefit of the Massachusetts motoring public, it deserves attention outside the state as well.

The purpose of the volume is to "clarify some of the common misconceptions and misunderstandings sown among the Massachusetts public for so many years". There is no attempt made to marshal arguments either for or against the principle of compulsory automobile insurance.

Separate chapters are devoted to the requirements of the law regarding public rate hearings and rate making authority, rising insurance costs, the making of compulsory rates, the under-25 driver problem, and the operation of territorial rating. There is also a chapter devoted to some of the many proposals for changing the law that have been offered at various times. Among these proposals are merit rating, insuring the driver, compulsory with a deductible provision, and allowing deviations downward from the maximum rates set by the Commissioner. Two recommendations currently being considered are, a change to allow the surcharging of assigned risks, and a revision of the law to include property damage liability as a compulsory coverage. The volume recommends the surcharging of assigned risks and points out that this is allowed by every other state in the country. Regarding the inclusion of property damage liability under the law no mention is made of any advantages or disadvantages.

The last chapter is entitled "Reducing Human Suffering and Insurance Costs". The chapter recommends a system of "no-fix" traffic enforcement. Under this system when a motorist receives a ticket, the ticket becomes the property of the court, and any attempt to "fix" the ticket from that point on is regarded as a prima facie case of contempt of court. Statistics indicating a downward trend in claim

frequency for New Jersey, which has such a law, are cited as evidence that comparable results could be attained in Massachusetts with such an enforcement program.

The reviewer believes that more strict enforcement, including a "no-fix" ticket law, is only one of the areas where improvements could reduce human suffering and insurance costs. Perhaps the last chapter could well have included recommendations for improved driver education and examination, uniform traffic regulations, frequent vehicle inspections, and safer highways.

The appendix includes an informative comparison of the New York Compulsory Motor Vehicle Financial Security Act with the Massachusetts Compulsory Automobile Liability Insurance Act.

The insurance industry in the past has done too little in the way of public relations. If the volume helps to fill this gap at all it will have been worthwhile.

H. T. BYRNE

OBITUARY

JOHN M. BLACKHALL

1914-1957

John M. Blackhall, Assistant Actuary of California-Western States Life Insurance Company, passed away on November 14, 1957, in his home at Sacramento, California. His death was due to a sudden heart attack. He was 43 years of age.

Mr. Blackhall was a native of Toronto, Ontario, Canada. He was a graduate with honors from the Mathematics Course of the University of Toronto, Class of '36. After serving two years with Professors Mackenzie and Shepherd, consulting actuaries in Toronto, Mr. Blackhall spent the next four years in actuarial work with the Monarch Life Insurance Company of Springfield, Massachusetts. From 1942 to 1946 he served as a weather forecaster with the Royal Canadian Air Force.

Mr. Blackhall became associated with the California-Western States Life Insurance Company in 1946 and was Assistant Actuary at the time of his death.

Mr. Blackhall was an Associate Member of the Society of Actuaries as well as an Associate of the Casualty Actuarial Society. He was currently Secretary of the Actuarial Club of the Pacific States.

Mr. Blackhall took a very active interest in religious, civic and charitable organizations. Among his affiliations were the Serra Club, Golden Empire Council of the Boy Scouts of America, Toastmaster Club and United Crusade Drive.

Mr. Blackhall's keen, analytical mind and friendly personality will be missed by all his many friends and associates. He will be remembered by the many younger associates whom his enthusiasm inspired with the desire to learn more about the insurance business.

He is survived by his wife, Oretta, two sons, John, Jr. and Malcolm, and his father, Wilmot R. Blackhall, of Toronto.

OBITUARY
EDMUND S. COGSWELL

1883-1957

Edmund S. Cogswell, a fellow of the Casualty Actuarial Society since 1916, died at his home in Wenham, Massachusetts, on April 25, 1957. A graduate of Harvard University, 1906, Mr. Cogswell started in the actuarial field with the New England Mutual Life Insurance Company in 1906. In 1913, he became the first secretary of the Massachusetts Teachers' Retirement Board. From 1916 to 1929, Mr. Cogswell served the Commonwealth of Massachusetts in various capacities; he was General Manager of the National Association of Mutual Casualty Companies for four years; he also engaged in the consulting business during this period.

In 1929, Mr. Cogswell was appointed Second Deputy Commissioner of Insurance in Massachusetts and in 1932, he was promoted to First Deputy Commissioner. In April, 1953, he was appointed Commissioner of Insurance, which post he held until his retirement on July 31, 1953. From that time to his decease, he engaged in the practice of Consulting Actuary. He is survived by his widow, two sons and two daughters.

Those who knew him feel strongly the loss of a personal friend and competent adviser.

OBITUARY
CLARENCE ARTHUR KULP
1895-1957

Dr. C. Arthur Kulp, a fellow of the Casualty Actuarial Society, died August 20, 1957 in Philadelphia. He was born on August 23, 1895.

Dr. Kulp was Dean of the Wharton School of Finance and Commerce, at the University of Pennsylvania. He was a native of Chalfont, Bucks County, and had been a member of the Wharton faculty since 1919. He was an authority on social and casualty insurance, and he initiated at the Wharton School what is believed to be the first course on social insurance taught in the U. S.

Later, he served as an adviser to formulate the Social Security system. He served the Federal Government in various other ways, including membership on the Federal Advisory Council to the Department of Labor, and the U. S. Railroad Retirement Board.

He also served the Commonwealth of Pennsylvania in advisory capacities, and helped set up unemployment organizations for the States of Pennsylvania, New York, New Hampshire and Massachusetts.

He was the author of a book on casualty insurance, which is considered the authoritative text in the field. At the time of his death, he was vice president of the Casualty Actuarial Society.

Dr. Kulp was graduated from Lansdale High School, and earned a bachelor of science in economics degree at the University of Pennsylvania in 1917, his master of arts in 1921 and Ph.D. in 1924.

He is survived by two sons, Robert A., of Lexington, Va., and Donald A., of Beardstown, Ill., and a brother, Mervin S. Kulp, of Drexel Hill.

OBITUARY
JOHN ROBERT LANGE

1892-1957

John R. Lange, fellow of the Casualty Actuarial Society and former Commissioner of Insurance of the state of Wisconsin, died on April 12, 1957 in Madison. Mr. Lange was born on April 18, 1892 in Lomira, Wisconsin. He was educated at Fond du Lac High School, Superior State College and the University of Wisconsin. He lived in Wausau before attending the University of Wisconsin where he received a B.A. Degree in 1918. He taught mathematics in South Milwaukee and Hayward, Wisconsin high schools and was employed with the Great Northern Life Insurance Company at Wausau before joining the Wisconsin Insurance Department on January 28, 1920, as Assistant Actuary. He became Chief Actuary in April, 1930. On December 1, 1948 he was appointed Commissioner of Insurance by Governor Oscar Rennebohm. He was reappointed on July 1, 1951 by Governor Walter Kohler and served in that capacity until his retirement in July, 1955. Mr. Lange served his state for over 35 years.

He was a veteran of World War I and a member of the American Legion. He was also a member in Madison of the First Congregational Church, the Black Hawk Country Club and the Elks Lodge. He was a member of the Federation of Insurance Counsels.

He became a fellow of the Casualty Actuarial Society on November 24, 1933 and was one of the few men in insurance history who, as a career employe in an insurance department and fellow of the Casualty Actuarial Society, was appointed to the office of Commissioner of Insurance.

Surviving are his wife, a daughter, Mrs. T. G. Budd of South Bend, Indiana, and a son, John Robert Jr., a research electrical engineer in Chicago.

OBITUARY
JOHN L. SIBLEY
1887-1957

John L. Sibley, age 69, Assistant Secretary and Assistant Treasurer of the United States Casualty Company, died after a long illness at his home in Haverhill, Massachusetts.

Born in St. Louis, Missouri on August 9, 1887, he attended and graduated from Erasmus High School in Brooklyn, New York. Early in his business career, he entered the employ of the Empire State Surety Company and on August 14, 1911, he was employed by the United States Casualty Company to head its Statistical Department. He continued in that capacity until his retirement because of illness in 1947.

Mr. Sibley was a charter member of the Casualty Actuarial Society and the Association of Casualty and Surety Accountants and Statisticians. He was a life member of Sandolphin Lodge, F. & A. M., located in Brooklyn, New York.

Mr. Sibley will be remembered by his many friends in the Actuarial field and in the insurance industry for his contribution on the subject of casualty company accounting and for his competency and integrity in the keeping of statistical records of his company. He endeared himself to all of his associates for his friendly qualities.

Mr. Sibley is survived by his wife, Jean, and two daughters, Jean and Elsie, and one sister, Mrs. Bert Colgrove.

MINUTES OF THE MEETING

May 23 and 24, 1957

FRENCH LICK-SHERATON HOTEL, FRENCH LICK, INDIANA

The Spring Meeting of the Casualty Actuarial Society was held at the French Lick-Sheraton Hotel in French Lick, Indiana on May 23 and 24, 1957. An informal buffet supper was held on the evening of May 22nd for early arrivals.

The meeting convened at 10:00 A. M. on Thursday, May 23rd with an interesting panel discussion on "Fire and Extended Coverage Ratemaking" moderated by Francis S. Perryman, Assistant U. S. Manager and Actuary of the Royal-Liverpool Insurance Group. The panel consisted of the following five members: Frederick W. Doremus, Manager, Eastern Underwriters Association; Clyde H. Graves, Actuary, Mutual Insurance Rating Bureau; M. Stanley Hughey, Second Vice-President, Lumbermens Mutual Casualty Company; Robert L. Hurley, Actuary, Liberty Mutual Fire Insurance Company; Laurence H. Longley-Cook, Actuary, Insurance Company of North America.

The meeting recessed at 12:30 P. M. for luncheon. The members gathered again at 6:30 P. M. for a brief social hour followed by Dinner. The guest speaker, Eugene F. Gallagher, Manager of the Planet Insurance Company, Chicago, was introduced to the gathering by M. Stanley Hughey. Mr. Gallagher's humorous talk, couched in a pseudo-serious framework, was thoroughly enjoyed by the audience.

The May 24th session was called to order at 10:00 A. M. by President Masterson. The registration showed the following 46 Fellows and 16 Associates present:

FELLOWS

ALLEN, E. S.	HURLEY, R. L.
BARBER, H. T.	JOHNSON, R. A.
BERKELEY, E. T.	LACROIX, H. F.
BEVAN, J. R.	LESLIE, W., JR.
CARLSON, T. O.	LINDER, J.
COATES, C. S.	LINO, R.
CURRY, H. E.	LISCORD, P. S.
DOREMUS, F. W.	LIVINGSTON, G. R.
ELLIOTT, G. B.	LONGLEY-COOK, L. H.
FOSTER, R. B.	MACKEEN, H. E.
FOWLER, T. W.	MASTERSON, N. E.
FULLER, G. V.	MATTHEWS, A. N.
GRAVES, C. H.	MAYCRINK, E. C.
HART, W. VAN BUREN, JR.	MILLS, J. A.
HARWAYNE, F.	PERRYMAN, F. S.
HAZAM, W. J.	PETZ, E. F.
HEWITT, C. C.	PRUITT, D. M.
HUGHEY, M. S.	RODERMUND, M.

SALZMANN, R.
 SCHLOSS, H. W.
 SIMON, L. J.
 SKILLINGS, E. S.
 TAPLEY, D. A.

THOMAS, J. W.
 VALERIUS, N. M.
 WIEDER, J. W., JR.
 WITTICK, H. E.
 WOLFRUM, R. J.

ASSOCIATES

BERQUIST, J. R.
 BLACK, N. C.
 COATES, W. D.
 FAUST, J. E., JR.
 FURNIVALL, M. L.
 GILDEA, J. F.
 HARACK, J.
 KIRK, C. L.

KLAASSEN, E. J.
 MAYERSON, A. L.
 McDONALD, M. G.
 NICHOLSON, E.
 OTTESON, P. M.
 SCAMMON, L. W.
 SCHWARTZ, M. J.
 WILSON, J. C.

In addition, there were also present a number of invited guests. President Masterson read his Presidential Address "Lessons from Adversity" which will be reproduced in Volume XLIV of the "Proceedings", as will the following written discussions of previous papers which were then presented:

The Rate Level Adjustment Factor in Workmen's Compensation Ratemaking

BY MARTIN BONDY

Reviewed by: M. G. McDonald
 G. B. Elliott

Current Rate Making Procedures for Automobile Liability Insurance

BY PHILIPP K. STERN

Reviewed by: T. E. Murrin
 E. T. Berkeley

Month of Loss Deficiency Reserves for Automobile Bodily Injury Losses Including Reserves for Incurred but Not Reported Claims

BY DAVID A. TAPLEY

Reviewed by: N. M. Valerius
 L. J. Simon

This completed the program, and, upon motion, the meeting was adjourned at 12:00 P. M.

MINUTES OF THE MEETING

November 21 and 22, 1957

SHERATON HOTEL, PHILADELPHIA, PENNSYLVANIA

The annual meeting of the Society was held at the Sheraton Hotel, Philadelphia, Pennsylvania on November 21 and 22, 1957.

The meeting convened at 2:00 P. M. on November 21 with President Norton E. Masterson presiding. The following 66 Fellows and 30 Associates were in attendance:

FELLOWS

ALLEN, E. S.	LIVINGSTON, G. R.
BAILEY, R. A.	LONGLEY-COOK, L. H.
BARBER, H. T.	MACKEEN, H. E.
BARKER, G. M.	MAKILL, S. S.
BENNETT, N. J.	MASTERTON, N. E.
BERQUIST, J. R.	MATTHEWS, A. N.
BONDY, M.	MAYCRINK, E. C.
BORNHUETTER, R. L.	MCCONNELL, M. H.
DAY, E. W.	MENZEL, H. W.
DOREMUS, F. W.	MILLS, J. A.
DROBISCH, M. R.	MILLS, R. J.
ELLIOTT, G. B.	MURRIN, T. E.
FAIRBANKS, A. V.	OTTESON, P. M.
FINNEGAN, J. H.	PERKINS, W. J.
FOSTER, R. B.	PINNEY, A. D.
FOWLER, T. W.	PRUITT, D. M.
GILLAM, W. S.	RESONY, A. V.
GODDARD, R. P.	RESONY, J. A.
GRAVES, C. H.	RODERMUND, M.
GREENE, W. W.	ROWELL, J. H.
HALEY, J. B., JR.	SALZMANN, R. E.
HAZAM, W. J.	SCHLOSS, H. W.
HEWITT, C. C.	SIMON, L. J.
HOPE, F. J.	SKELDING, A. Z.
HURLEY, R. L.	SMICK, J. J.
JOHE, R. L.	ST. JOHN, J. B.
JOHNSON, R. A.	TAPLEY, D. A.
KORMES, M.	THOMAS, J. W.
LACROIX, H. F.	TRIST, J. A. W.
LESLIE, W., JR.	UHTHOFF, D. R.
LINDER, J.	WIEDER, J. W., JR.
LINO, R.	WILLIAMS, P. A.
LISCORD, P. S.	WOLFRUM, R. J.

ASSOCIATES

ALEXANDER, L. M.	BITTEL, W. H.
ANDREWS, E. C.	BOYLE, J. A.

BYRNE, H. T.
 COATES, W. D.
 DANIEL, C. M.
 EGER, F. A.
 FAUST, J. E., JR.
 FELDMAN, M.
 FLACK, P. R.
 HARACK, J.
 HUNT, F. J., JR.
 JONES, N. F.
 KLAASSEN, E. J.
 McDONALD, M. G.
 MUIR, J. M.

NICHOLSON, E.
 NILES, C. L., JR.
 PHILLIPS, H. J., JR.
 ROBERTS, L. H.
 SCHNEIKER, H. C.
 SCHULMAN, J.
 SHAVER, C. O.
 SMITH, E. M.
 STELLWAGEN, H. P.
 TARBELL, L. L., JR.
 WILCKEN, C. L.
 WILSON, J. C.
 WRIGHT, B.

In addition, there were also present a number of invited guests.

The first topic was a panel discussion on the subject "Recent Developments in Automobile Ratemaking."

The panel members were Norman J. Bennett, Clyde H. Graves, Milton G. McDonald, Thomas E. Murrin, all members of the Casualty Actuarial Society, and W. D. Hall, Actuary, National Automobile Underwriters Association. Joseph Linder, Past Vice-President of the Society, acted as moderator.

The panel discussion took up the entire afternoon and was followed by a cocktail hour. This was succeeded by an informal dinner in the evening. Elden W. Day, acting as Master of Ceremonies, introduced the Guest Speaker, Ambrose B. Kelly, General Counsel of the Associated Factory Mutual Fire Insurance Companies. Ambrose held the attention of the gathering with an interesting talk on "Is Insurance Ready for Science?"

The meeting reconvened at 9:45 A. M. on November 22, with President Masterson presiding.

The Secretary-Treasurer presented a report on the cash receipts for the period October 1, 1956 through September 30, 1957. This report, which has been certified as correct by the Auditing Committee, is attached and will be printed in the next volume of the Proceedings.

In reviewing the financial report, attention was called to the generosity of Past President G. F. Michelbacher in donating to the Society for many years the royalties on his book "Casualty Insurance Principles." The gathering was also informed that the Society would continue to receive royalties on the new book "Multiple Line Insurance." It was voted that the Secretary write a letter to Mr. Michelbacher expressing the appreciation of the Society.

The President then announced the passing during the year of the following members of the Society:

John M. Blackhall
 Edmund S. Cogswell

Clarence A. Kulp
 John R. Lange

John L. Sibley

The following new Associates were presented by name to the Society:

Abel, F. E. (Miss)	Houston, D. B.
Alexander, L. M.	Hunt, F. J., Jr.
Boyle, J. I.	Niles, C. L., Jr.
Bragg, J. M.	Muir, J. M.
Byrne, H. T.	Schneiker, H. C.
Church, H. M.	Shaver, C. O.
Feldman, M. F.	Wilcken, C. L.

The President then presented diplomas to the following new Fellows:

Berquist, J. R.	Mills, R. J.
Bornhuetter, R. L.	Otteson, P. M.
Drobisch, M. R.	Perkins, W. J.
Gillam, W. S.	Pinney, A. D.
Kates, P. B.	Williams, P. A.
Makgill, S. S.	

The gathering also confirmed the action of the Council in electing the following for the coming year

EditorEdward S. Allen (re-elected)
 LibrarianGilbert R. Livingston (re-elected)
 General Chairman —
 Examination Committee. William J. Hazam

The Nominating Committee then presented the following nominations to be voted on at this meeting:

PresidentDudley M. Pruitt
 Vice PresidentJohn W. Carleton
 Vice PresidentWilliam Leslie, Jr.
 Secretary-TreasurerAlbert Z. Skelding
 Member of Council — 3 year term — Stanley M. Hughey
 Member of Council — 3 year term — Matthew Rodermund
 Member of Council — 3 year term — John W. Wieder, Jr.
 Member of Council — 2 year term — Ruth E. Salzmann

No further nominations being offered from the floor, it was voted that nominations be closed and that the above slate be declared elected by the meeting.

Mr. Masterson then read his Presidential Address "Professional

Responsibilities of the Members of the Casualty Actuarial Society.”

The following new papers were presented:

- J. Edward Faust, Jr., — “Automobile Bodily Injury Liability Ratemaking on a Prospective Basis.”
- Joseph M. Muir — “Principles and Practices in Connection with Classification Rating Systems for Liability Insurance As Applied to Private Passenger Automobiles.”
- Lewis H. Roberts — “Graduation of Excess Ratio Distributions by the Method of Moments.”
- C. Otis Shaver — “Revision of Rates Applicable to a Class of Property Fire Insurance.”

D. A. Tapley commented on the reviews of his paper “Month of Loss Deficiency Reserves for Automobile Bodily Injury Losses Including Reserves for Incurred but not Reported Claims” previously presented by L. J. Simon and N. M. Valerius.

W. W. Greene presented a written discussion of F. Harwayne’s paper “A Review and Comparison of Workmen’s Compensation Experience in New York State and Wisconsin”.

M. Bondy commented on the reviews of his paper “The Rate Level Adjustment Factor in Workmen’s Compensation Ratemaking” previously presented by M. G. McDonald and G. B. Elliott.

Upon motion, the meeting adjourned at 12:00 Noon.

For the purpose of record, there is appended a list of those who passed the examinations held by the Society on May 9 and 10, 1957.

1957 EXAMINATIONS — SUCCESSFUL CANDIDATES

Following is a list of those who passed the examinations held by the Society on May 9 and 10, 1957:

ASSOCIATESHIP EXAMINATIONS

PART I (a)	Balcarek, R. J.	McDonald, C.	Pollack, R.
	Blodget, H. R.	Meilahn, J. E.	Ratnaswamy, R.
	Copestakes, A. D.	Morrison, D. I.	Scheibl, J. A.
	Craig, R. A.	Moseley, J.	Schlenz, J. W.
	Dickerson, O. D.	Nelson, L.	Smith, Charles P.
	Gelb, M. R.	Niles, C. L., Jr.	Smith, Edward Rolph
	Gillespie, J. E.	Parry, A. E.	Thompson, P.
Herman, F. L.	Peel, J. P.	Tucker, T. F.	
PART I (b)	Balcarek, R. J.	Flanagan, R. M.	Pollack, R.
	Beard, A. R.	Morrison, D. I.	Randall, D. J.
	Berkman, J.	Moseley, J.	Ratnaswamy, R.
	Blodget, H. R.	Niles, C. L., Jr.	Smith, Charles P.
	Craig, R. A.	Oien, R. G.	Smith, Edward Rolph
Dickerson, O. D.	Peel, J. P.		
PART II (a)	Abel, F.	Linden, J. R.	Pollack, R.
	Balcarek, R. J.	Mathews, E. G.	Randall, D. J.
	Bannister, D. W.	Mohnblatt, A. S.	Sarnoff, P. E.
	Beard, A. R.	Morrison, D. I.	Simoneau, P. W.
	Blumenfeld, M. E.	Moseley, J.	Strug, E. J.
	Cherlin, G.	Nadler, H.	Wahlstrom, R. W.
	Gold, M. L.	Niles, C. L., Jr.	Willsey, L. W.
	Houston, D. B.		
PART II (b)	Balcarek, R. J.	Hanssler, H. W.	Piersol, D. E.
	Bannister, D. W.	Houston, D. B.	Pollack, R.
	Beard, A. R.	Linden, J. R.	Richards, H. R.
	Blumenfeld, M. E.	McGuinness, J. S.	Richardson, W. S.
	Craig, R. A.	Mohnblatt, A. S.	Sarnoff, P. E.
	Dickerson, O. D.	Nadler, H.	Simoneau, P. W.
	Fitzgibbon, W. J., Jr.	Niles, C. L., Jr.	Weber, D. C.
	Flaten, L. G.	Parry, A. E.	Willsey, L. W.
		Peterson, H. M.	
PART III	Bannister, D. W.	Crowley, J. H., Jr.	Sarnoff, P. E.
	Berkman, J.	Dickerson, O. D.	Simoneau, P. W.
	Blodget, H. R.	Feldman, M. F.	Snowden, H. N.
	Byrne, H. T.	Houston, D. B.	Stankus, L. M.
	Carrick, W. R., Jr.	Hunt, F. J., Jr.	Willsey, L. W.
		Royer, A. F.	
PART IV	Alexander, L. M.	Byrne, H. T.	Hunt, F. J., Jr.
	Bannister, D. W.	Church, H. M.	McMaster, D. J.
	Boyle, J. I.	Coen, F. J.	Schneiker, H. C.
	Bragg, J. M.	Dickerson, O. D.	Wilcken, C. L.
		Houston, D. B.	

FELLOWSHIP EXAMINATIONS

PART I	Boyle, J. I.	Dorf, S. A.	Schulman, J.
		Phillips, H. J., Jr.	

PART II	Bornhuetter, R. L.	Pinney, A. D.	Tarbell, L. L., Jr.
	Kates, P. B.	Roberts, L. H.	Tucker, T. F.
	Makgill, S. S.	Smith, E. M.	Williams, P. A.
PART III	Bornhuetter, R. L.	McGuinness, J. S.	Tarbell, L. L., Jr.
	Drobisch, M. R.	Niles, C. L., Jr.	Williams, D. G.
	Gillam, W. S.	Perkins, W. J.	Williams, P. A.
	Makgill, S. S.	Pinney, A. D.	Wright, B.
		Smith, E. M.	
PART IV	Berquist, J. R.	Makgill, S. S.	Niles, C. L., Jr.
	Bornhuetter, R. L.	McGuinness, J. S.	Otteson, P. M.
	Coates, W. D.	McNamara, D. J.	Pinney, A. D.
	Dropkin, L.	Mills, R. J.	Williams, P. A.

NEW ASSOCIATES

The following candidates, having been successful in completing the examinations, will be admitted as Associates of the Society as of the date of the Annual Meeting in November 1957:

Abel, F. E.	Byrne, H. T.	Hunt, F. J., Jr.
Alexander, L. M.	Church, H. M.	Niles, C. L., Jr.
Boyle, J. I.	Feldman, M. F.	Schneiker, H. C.
Bragg, J. M.	Houston, D. B.	Wilcken, C. L.

NEW FELLOWS

The following Associates, having been successful in completing the examinations, will be admitted as Fellows of the Society as of the date of the Annual Meeting in November 1957:

Berquist, J. R.	Mills, R. J.
Bornhuetter, R. L.	Otteson, P. M.
Drobisch, M. R.	Perkins, W. J.
Gillam, W. S.	Pinney, A. D.
Kates, P. B.	Williams, P. A.
Makgill, S. S.	

CASUALTY ACTUARIAL SOCIETY

Cash Receipts and Disbursements
from October 1, 1956 to September 30, 1957

<i>Income</i>		<i>Disbursements</i>	
On deposit in Chase Manhattan October 1, 1956	\$3,314.96		
Members Dues	\$7,230.00	Printing & Stationery	\$ 8,501.05
Sale of Proceedings	1,752.90	Secretarial Work	600.00
Examination Fees	1,025.50	Examination Expense	669.28
Luncheons & Dinners	2,645.00	Luncheons & Dinners	2,829.40
Interest on Bonds	187.50	Library Fund	49.40
Sale of Reprints	1,348.09	Insurance	59.83
Michelbacher Fund	217.77	Refunds	128.00
Foreign Exchange	3.15	Miscellaneous	126.50
Total	<u>14,409.91</u>	Total	<u>\$12,963.46</u>
	<u>\$17,724.87</u>	On deposit 9-30-57 in Chase Manhattan	4,761.41
		Total	<u>\$17,724.87</u>

<i>Assets</i>		<i>Liabilities</i>	
Cash in Bank		Michelbacher Fund	\$7,732.71
9-30-57	\$4,761.41	Other Surplus	2,028.70
U. S. Savings Bonds	5,000.00	Total Liabilities & Surplus	<u>\$9,761.41</u>
	<u>\$9,761.41</u>		

* * * * *

One 12 Yr. U. S. Savings Bond 2-½% Series G No. M6,756,060G due for \$1,000 on Nov. 1, 1960.

Four 12 Yr. U. S. Savings Bonds 2-½% Series G Nos. M7,228,102G-103G-104G-105G-due for \$4,000 on October 1, 1961.

U. S. Fire Insurance Company Policy No. 109221 for \$5,000 on Proceedings stored at 229 Fourth Avenue, New York, N. Y.; \$2,000 on books kept in N. Y. Insurance Society Library. Expires September 14, 1962.

Surety Bond for \$5,000 in the Royal Indemnity Company.

* * * * *

This is to certify that we have audited the accounts and examined all the vouchers and investments and find same to be correct.

Emma Maycrink
Matthew Rodermund

November 8, 1957

EXAMINATION FOR ENROLLMENT AS ASSOCIATE

PART I SECTION (a)

1. (a) Distinguish between:
- (i) Standard deviation.
 - (ii) Standard error of the mean.
 - (iii) Standard error of estimate.
- (b) Explain the meaning of each of the following terms used in connection with the testing of the validity of an assumption:
- (i) Type I and Type II errors.
 - (ii) Null Hypothesis.
 - (iii) Level of significance.
2. At the 2% level of significance should the following data be treated as if it had come from a normal distribution?

Length	Frequency	Length	Frequency
10	10	15	180
11	40	16	110
12	90	17	40
13	220	18	10
14	300		

Given:

Degrees of Freedom	$\chi_{.10}^2$	$\chi_{.05}^2$	$\chi_{.02}^2$
5	.75	11.07	13.38
6	1.13	12.59	15.03
7	1.56	14.07	16.62
8	2.03	15.51	18.17
9	2.53	16.92	19.68
10	3.06	18.31	21.16

Normal Curve Areas

<i>z</i>	<i>Area</i>	<i>z</i>	<i>Area</i>	<i>z</i>	<i>Area</i>
0.0	.000	1.1	.364	2.1	.482
0.1	.040	1.2	.385	2.2	.486
0.2	.079	1.3	.403	2.3	.489
0.3	.118	1.4	.419	2.4	.492
0.4	.155	1.5	.433	2.5	.494
0.5	.191	1.6	.445	2.6	.495
0.6	.226	1.7	.455	2.7	.497
0.7	.258	1.8	.464	2.8	.497
0.8	.288	1.9	.471	2.9	.498
0.9	.316	2.0	.477	3.0	.499
1.0	.341				

3. (a) The 1951 premium volume of an insurance company with 131 agents was \$1,070,000. In 1952 and each year thereafter, the company licensed 25 new agents and wrote the following premium volume:

1952	\$ 800,000	1954	\$2,060,000
1953	\$1,460,000	1955	\$1,760,000

Find and discuss the significance of the coefficient of correlation between the premium volume and the number of agents.

- (b) A farmer separated 400 cows into 2 equal herds by random selection. He tried to keep all conditions identical except that each herd was given different foods. One herd yielded an average of 36 quarts per cow with a standard deviation of 5.4, while for the same period the other herd yielded an average of 39 quarts per cow with a standard deviation of 4.5. Is there any significant difference between the average yield of the two herds?
4. For the following data, find the equation of the regression line of y on x and use it to compute the predicted y_i . (Compute coefficients to one decimal place).

x	11	20	17	12	10	0	6	5	8	11
y	24	4	7	17	28	43	30	34	25	22

SECTION (b)

1. (a) One card of an ordinary pack has been lost. From the remainder of the pack 13 cards are drawn at random and are found to consist of 2 spades, 3 clubs, 4 diamonds, and 4 hearts. What are the respective chances that the missing card is a spade, a club, a diamond, or a heart?
- (b) A box contains four dice two of which are true, and the others are so loaded that with either of them the chance of throwing six is $1/4$ and the chance of throwing an ace is $1/12$. Two dice are taken at random out of the box and thrown. If they turn up sixes, find the odds against both dice being loaded.

PART I SECTION (b)

1. (a) One card of an ordinary pack has been lost. From the remainder of the pack 13 cards are drawn at random and are found to consist of 2 spades, 3 clubs, 4 diamonds, and 4 hearts. What are the respective chances that the missing card is a spade, a club, a diamond, or a heart?
- (b) A box contains four dice two of which are true, and the others

are so loaded that with either of them the chance of throwing six is $1/4$ and the chance of throwing an ace is $1/12$. Two dice are taken at random out of the box and thrown. If they turn up sixes, find the odds against both dice being loaded.

2. (a) The chance of one event happening is the square of the chance of a second event, but the odds against the first are the cube of the odds against the second. Find the chance of each.
 - (b) At a chess tournament the players are divided into two classes, the second class having twice as many members as the first. Each player has an even chance of defeating another member of his team but the odds are 2 to 1 in favor of a member of the first class in a game with a member of the second. A player is observed to win and a bet of 7 to 10 is made that he belongs to the first class. If this bet is fair, how many players are there?
3. A club composed of 20 members is electing a new president, but the current president cannot succeed himself and therefore can receive no votes. Each of the other 19 members is as likely as not to vote for himself; otherwise he votes at random. If the current president casts his vote for the current vice-president, what are the odds that a member who receives exactly five votes voted for himself, if:
- (i) the member is the current vice-president?
 - (ii) the member is the current treasurer?
4. (a) Ten clubs compete annually for a cup which is to become the absolute property of the club which wins it for 3 years in succession. Assuming that all the clubs are of equal skill, find the chance that last year's winner, not having won the previous year, will ultimately win the cup.
- (b) If three numbers are selected at random out of the first 100 integers, what is the probability that their sum is divisible by three? (Leave your answer in symbolic form.)

PART II SECTION (a)

1. (a) Prove the following identities:

$$(i) P_x = v - \frac{a_x}{\ddot{a}_x}$$

$$(ii) {}_tV_x = \frac{A_{x+t} - A_x}{1 - A_x}$$

$$(iii) \sum_{t=1}^{t=\infty} {}_tq_x \cdot a_t = \frac{N_{x+1}}{D_x}$$

(b) A 10-pay 25 year endowment policy with a face amount of \$1,000 provides that in the event of death during the 25 years the net premiums paid will be refunded along with the payment of the face amount. Express the net annual premium in terms of commutation symbols.

2. (a) A man aged 40 pays \$1,000 for a policy which provides a death benefit of \$1,000 in the event of death within 25 years. In the event of survival to age 65, the policy is to be exchanged for a contract which provides a life annuity of \$x per year, the first payment to be made at age 65 and the first 10 annuity payments guaranteed whether the annuitant lives or dies. Find x in terms of commutation symbols.

(b) Express in commutation symbols the prospective and retrospective reserves for the fifteenth year for a \$1,000 policy issued at age 50 under each of the following plans. Simplify the final expression as much as possible and show for each plan that the two reserves are equal.

- (i) Ordinary Life
- (ii) 20 Year Term
- (iii) 10 Pay Life

3. (a) A man aged x offers a single premium of $\frac{\bar{a}_{x-n} - \bar{a}_{x-n:2n}}{{}_nE_{x-n}}$ for a deferred life annuity, first payment at age $x+n$. Find the annual rent of the annuity.
- (b) Under a certain mortality table $A_x = 0.01x$ for all values of x when the rate of interest is 4%. Find numerical expressions in terms of x for:
- (i) \bar{a}_x
 - (ii) P_x
4. A 20-payment life policy issued 5 years ago at age 35 is to be changed to an endowment at age 60 policy. The new policy is to have the same reserve on the date of change as the old policy. If the amounts of the net premiums are to be continued unchanged, and if they will continue to be paid to age 60, determine the amount for which the new endowment policy may be issued. Express your answer in the most compact form in commutation symbols.

PART II SECTION (b)

1. (a) Discuss briefly the advantages and disadvantages of "trading on equity" as it applies to corporation financing.
- (b) What are the various types of preventive and protective effort? Give examples of each type.
2. (a) The following descriptions of certain securities appear in the annual statement of a non-life insurance company. Define these securities.
 - (i) \$4.75 Cum. Conv. Pfd.
 - (ii) 1st Mtg. Revenue
 - (iii) Equip. Part. ctfs.
 - (iv) Income Deb.
 - (v) 1st Mtg. & Coll. Trust
- (b) What are the major features of federal regulation of securities markets?

3. You are asked to appraise the financial condition of various fire and casualty companies to be used as a guide to your company in the placing of reinsurance for fire and casualty lines. Determine criteria upon which you can base your appraisals and explain their significance.
4. Does the accumulation of a large number of similar risks by an insurance company actually reduce the total risk? Discuss.

PART III

SECTION (a)

1. (a) An insurable interest is one of the elements essential to the making of a valid insurance contract.
 - (i) Explain the meaning of the term insurable interest.
 - (ii) Explain the status of an insurance contract issued without an insurable interest.
 - (iii) What are the other elements essential to the making of a valid insurance contract?
- (b) State the base or bases upon which the following types of insurance companies are taxed by the Federal Government under the Internal Revenue Code:
 - (i) Stock Fire and Casualty
 - (ii) Mutual Fire and Casualty
2. (a) In 1868, in the case of Paul vs Virginia, the United States Supreme Court held that insurance is not commerce within the meaning of the Federal Constitution and this doctrine was subsequently upheld in a number of other cases.
 - (i) How did this doctrine affect the regulation of the insurance business?

- (ii) How was this doctrine affected by the 1944 opinion of the United States Supreme Court in the South-Eastern Underwriters Association case?
 - (b) Discuss the effects of Public Law 15 on the regulation of the insurance business.
3. (a) Describe the provisions of the Model Rate Regulatory Bill which assist the Casualty and Fire Insurance Industry and the Rate Regulatory Authorities in meeting the rating problems presented by interstate risks.
- (b) Prior to the South-Eastern Underwriters Association decision it was customary for rating organizations to insist upon country-wide adherence to their manuals by member carriers. Discuss the effect of post South-Eastern Underwriters Association legislation on this custom.
4. (a) The Convention Form Annual Statement is one of the primary tools used by supervisory officials in keeping informed on the financial condition and operations of insurers. Schedule P (Parts 1 and 2) and Schedules G and O of this Statement provide certain loss experience information. Name the lines of business included in each of these schedules and briefly describe each such schedule.
- (b) In addition to the Annual Statement, there are several other tools available to supervisory officials to assist them in the protection of policyholders against insolvency of insurers. Name three such tools.

SECTION (b)

5. (a) Section 214 of the New York Disability Benefits Law sets up a "Special Fund for Disability Benefits."
- (i) What specific benefits are paid out of this fund?

- (ii) How was the fund initially accumulated and how is it financed?
- (b) Describe the general characteristics of the Federal Old-Age and Survivors Insurance System.
6. Governmental activity in the insurance business is normally restricted to supervision and regulation. However, in some instances, governmental activity has been extended into the actual conduct of an insurance operation.
- (a) What two conditions should exist to justify such an extension of governmental activity?
- (b) Discuss the extent to which Workmen's Compensation insurance meets these two conditions.
7. List the arguments for and against compulsory health insurance.
8. Assume that your state has proposed an Unsatisfied Judgment Fund Law covering "injury or damage arising out of the ownership, maintenance, or use of a motor vehicle in the state". All default actions and hit-and-run cases are to be assigned to insurers for investigation and defense. The Fund is to be financed by a \$3.00 assessment on persons registering an uninsured motor vehicle, \$1.00 on persons registering an insured motor vehicle, and an assessment on insurers of 0.5% of Automobile Bodily Injury and Property Damage net direct written premiums. A further yearly assessment on insurers is to be made to keep the Fund in operation but no further assessments are to be made on persons registering motor vehicles. State whether you would support or oppose this proposed law and discuss the reasons for your choice.

PART IV**SECTION (a)**

NOTE: Answer any seven of the questions numbered 1 through 10 and then answer questions 11 or question 12.

1. For each of the following events, state whether the loss is or is not covered by the 1943 standard form fire insurance contract and give the reasons on which you base your answer.
 - (a) An overheated furnace causes loss by melting candy exposed to the heat.
 - (b) Smoke and soot escaping from a damaged stovepipe ruin several bolts of silk goods.
 - (c) A chimney catches fire resulting in spoilage of a shipment of sugar.
 - (d) A church steeple is severely damaged by lightning but no fire results.
 - (e) Insured carelessly allows oily rags to accumulate in a wooden box and resulting fire causes substantial loss.

2. Distinguish between the terms "unoccupancy" and "vacancy" as respects the standard fire policy. For what type of risks may unlimited vacancy and/or unoccupancy be granted?

3. (a) An owner has a stock of goods consisting of music valued at \$500 and books valued at \$3,000. Company "G" had insured both of these items under a general policy for \$2,000, and Company "S" had insured the books only for \$1,000. Loss by fire is entirely of books and amounts to \$900. No co-insurance or limiting clauses are involved. State the "Page Rule" for the division of this loss between the two companies. How much would each company pay?
 - (b) An insured under the 1943 standard form fire-insurance contract sustains
 - (i) a loss of \$5,000 by damage to his building,
 - (ii) \$5,000 in addition because he is required by law to use better material for repairs than those which have been previously used in constructing the building, and
 - (iii) \$10,000 due to suspension of production, all caused by fire.

He is insured to value and the amount of his insurance is \$40,000. How much may he collect? Explain.

4. (a) State the eligible classes of property which may be covered under a "Personal Articles Floater".
- (b) Describe the coverage provided by the "Personal Articles Floater" and explain how the premium is calculated.
5. Describe the "Open Cargo Forms" type of policy and "Blanket" policy in connection with Ocean Marine Insurance.
6. (a) Explain the operation of the coinsurance clause under the Mercantile Open Stock Burglary Policy.
- (b) State the insurance company's liability under the Mercantile Stock Burglary Policy for each of the following three conditions:

	(1)	(2)	(3)
Value of Merchandise	\$10,000	\$10,000	\$10,000
Coinsurance Percentage	40%	40%	40%
Coinsurance Limit	\$ 5,000	\$ 3,000	\$ 4,000
Amount of Insurance			
Purchased	\$ 3,000	\$ 4,000	\$ 5,000
Loss of Merchandise	\$ 3,000	\$ 3,500	\$ 6,000

7. (a) Briefly state what is meant by each of the following terms used in Workmen's Compensation Insurance:
 - (i) Governing Classification
 - (ii) N.O.C.
 - (iii) N.P.D.
- (b) Give three examples of each of the following terms used in Workmen's Compensation insurance:
 - (i) General Inclusions
 - (ii) General Exclusions
 - (iii) Standard Exceptions
8. (a) Name six factors upon which the premium rate for glass insurance is based.
- (b) Name three factors which should be considered in underwriting glass insurance.
9. The following terms refer to coverages given in either Workmen's Compensation Insurance or Accident and Health Insurance. Distinguish between each of the following terms and indicate the line of business involved.

- (a) Principal Sum and Capital Sum.
 - (b) Total Disability and Partial Disability.
 - (c) Coverage A and Coverage B.
 - (d) Ex-Medical Endorsement and Additional Medical Endorsement.
10. Name four general types of Surety Bonds and briefly describe the circumstances under which each type of bond would be required.
11. Describe the major changes in coverage effected by the new Family Automobile Liability policy
- (a) as promulgated by the National Bureau of Casualty Underwriters
 - (b) as promulgated by the National Automobile Underwriters Association.
12. Mr. Smith's five-year old son and a five-year old playmate named Bobby constructed an imitation fireplace in Mr. Smith's living room, using small wooden blocks. They then started a fire in the imitation fireplace which ignited the clothing of both boys, causing very serious burns, and the subsequent fire and smoke damaged Mr. Smith's furniture and dwelling. Mr. Smith carried Fire and Extended Coverage on his dwelling and contents, and also a Comprehensive Personal Liability policy.
- (a) Discuss the possibility of the fire insurer denying liability on the grounds of:
 - (i) arson,
 - (ii) fire not hostile,
 - (iii) fire not accidental,
 - (iv) fire caused by negligence.
 - (b) If Bobby's father sued Mr. Smith for injuries to Bobby as a consequence of the fire, would Mr. Smith's comprehensive personal liability insurer have to defend him? Why?
 - (c) Assuming an award of damages is made against Mr. Smith for the injuries to Bobby, is Mr. Smith's comprehensive personal liability insurer liable? Explain the policy provisions on which your decision is based.
 - (d) Assume Bobby's father also carried a Comprehensive Personal Liability contract and Mr. Smith sued him for injuries to his son. To what extent, if any, would the comprehensive liability insurer be liable? Explain the policy provisions on which your decision is based.

SECTION (b)

NOTE: Answer all of the questions numbered 13 through 16.

13. (a) Your company has been writing non-occupational Weekly Indemnity under a Statutory Disability Law. Effective at "x" date an amendment to the law would increase the plan from 8-8-13 to 8-8-20 and increase the maximum weekly benefit from \$33 to \$40. You have only the following information:

(i) Experience for a period of two calendar years, ending 1 year prior to the "x" date. The experience is on small risks all bearing the manual rate effective during the experience period. The experience is under an 8-8-13 plan at a \$30/wk. maximum weekly benefit and is as follows:

Earned Premiums	\$3,000,000
Incurred Losses	\$1,800,000

(ii) The present manual rate is 10% below that of the experience period.

(iii) The cost of an 8-8-20 plan is 20% higher than that of an 8-8-13 plan.

(iv) The average weekly benefit is as follows:

\$30/Wk. Maximum	\$33/Wk. Maximum	\$40/Wk. Maximum
\$25.00	\$27.50	\$30.00

(v) The permissible loss ration is 70%.

What change, if any, considering the change in the law and the experience, would you recommend in the present manual rate level? What credibility assumptions would you make? What comment would you make as regards female exposure? Are there any other considerations involved?

(b) Under Group Major Medical Expense Insurance, four inherent characteristics of the group to be insured have a direct bearing on the employee rate. Name each of these characteristics and explain why it should influence the rate.

14. (a) Contrast the different factors involved in determining a method for making changes in rate level for:

(i) dwelling extended coverage insurance.

(ii) fire insurance for mercantile risks.

- (b) Until recently, in some jurisdictions when a fire loss was paid, the premium on the amount of loss was considered fully earned and the insured had to pay an additional premium to restore the policy to its original amount. Show that, theoretically, the elimination of this practice for no additional premium would cost the insurance companies $r/2\%$ of premium, where r is the fire rate per \$100 of exposure.
15. Automobile Insurance Rates for private passenger cars vary by geographical territory and by classification in order to reflect the differences in conditions that might exist.
- (a) Name the conditions which would cause automobile liability rates to vary by geographical territories.
 - (b) Name the conditions which would cause automobile liability rates to vary by classification.
 - (c) Which of your answers to (a) and (b) for Automobile Liability would also apply to Automobile Physical Damage?
 - (d) Name any other conditions which would have an effect on rates for Automobile Physical Damage.
16. (a) What items are used by the National Council on Compensation Insurance in order to obtain the final manual rates for non-reviewed classifications and to what are these items applied?
- (b) Discuss the purpose of each of the factors or loadings which are applied to the final pure premiums in obtaining manual rates for reviewed classifications under the ratemaking procedure used by the National Council on Compensation Insurance.

EXAMINATION FOR ENROLLMENT AS FELLOW

PART I**SECTION (a)**

1. (a) Develop an algebraic formula for the method used to compute the total reserve for unpaid compensation losses and loss adjustment expense as required in Part 2 of Schedule P. Define clearly all symbols used.
 - (b) What two changes in this method are necessary to meet the requirements of Schedule P — Part 1?
 - (c) In the report of the Casualty Actuarial Society's Committee on Compensation and Liability Loss and Loss Expense Reserves, what method of computing compensation loss reserves was proposed? Discuss the advantages and disadvantages of this method as opposed to the present Schedule P method.
2. (a) You have been asked to develop an Automobile Property Damage Liability reserve for incurred but unreported claims at the end of the 1956 year. Assuming the availability of the following data, calculate the reserve which you would recommend:
 - (i) Number of Claim Notices received in October, November, and December of 1955 — 1000.
 - (ii) Number of Claim Notices received in corresponding three-month period of 1956 — 1200.
 - (iii) Average incurred cost per Claim Notice corresponding with claims in (i) above — \$50.
 - (iv) Average incurred cost per Claim Notice corresponding with claims in (ii) above — \$55.
 - (v) Actual amount of Incurred but Unreported Claims at the end of 1955 year as shown by subsequent tabulations — \$500,000.

On what other lines of insurance would you consider using a formula reserve approach? Why?

- (b) Develop an approximate formula for computing state calendar year earned premiums based on the country-wide unearned premium reserve.
3. (a) A newly formed company, which desires to minimize the drain on its surplus from its new business writings, asks your opinion as to whether it should write its fire insurance policies on a one-year or three-year basis. Outline the answer you would give and illustrate with a numerical example, in which you show a comparison of the expected effect on surplus at the end of the first year of (a) a policy written on January 1 for one year and (b) one written on January 1 for three years. In your example assume:
- (i) Premium of \$120 is paid in advance for the one-year policy and \$300 in advance for the three-year policy.
 - (ii) Losses for the first year will be \$60 under either policy.
 - (iii) Acquisition expenses and taxes of 25% of written premium are incurred at the inception of the contract.
 - (iv) All other expenses for the one-year policy are 20% of earned premium.
 - (v) All other expenses for the three-year policy are 20% of earned premium for the first year, and 10% of earned premium for the second and third years.
- (b) With reference to loss and loss expense reserves, discuss the maxim that a dollar in surplus is as good as a dollar in reserve.
4. (a) You are sitting on a committee which is considering, for a given state, uniform tabular methods of establishing reserves for Unit Statistical Plan purposes on Workmen's Compensation death cases. The state compensation law provides that, in the event of the death of an employee, benefits will be paid to his widow until her death or re-marriage. At the present time, reserves for this type of case are set up as individual estimates after careful consideration of the circumstances surrounding each claim.
- (i) Discuss any advantages of a change to tabular reserves and list available sources of statistical data which could be utilized in preparing these tables.

- (ii) Sketch a practical reserve table which could be used by a statistical clerk to evaluate the reserve for a weekly benefit (W) to a widow age (X). Label each column of your table precisely and express the total reserve in terms of the symbols you have chosen.
- (b) In the past a company has developed an unearned premium reserve for retrospectively rated risks by application of the "retrospective" formulae to each unadjusted individual risk. This company now decides that this method is too time-consuming and wishes to develop an approximate formula or method in its place based on past "retrospective" experience. Outline a possible method which could be used.

SECTION (b)

- 5. In your position as Actuary of the New York Insurance Department, you have been asked to justify the adoption of Regulation No. 30. Draft the answer you would give, citing at least six specific objectives of such legislation.
- 6. (a) In the expense study by size of risk undertaken in 1949 at the request of the NAIC, five major categories of paid expenses were analyzed by size of annual premium. List these categories and outline the results which were demonstrated by this study. How did the results of stock companies differ from those of mutual companies?
 - (b) In this study of expenses by size of risk, the Industry Committee found that the \$10 Expense Constant seemed to be seriously inadequate for Workmen's Compensation premiums under \$100. Rather than increase the Expense Constant, however, it was decided to attempt to reduce the costs of handling small policies. What were some of the suggestions made for reducing such costs?
 - (c) If you were asked to study the expenses involved in the writing of such small risk policies in your company, how would you propose to allocate salaries of your Unit Reporting Section, Tabulating Section, Underwriting Department, and Storage File Section to this category of policies? Give reasons for your recommendations.

7. (a) Describe and explain the reasons for the difference between sections A and B of Part II of the Insurance Expense Exhibit.
- (b) What is the purpose of Part III of the Insurance Expense Exhibit? What records or tabulations are necessary to complete this part of the Insurance Expense Exhibit?
8. The following data (in thousands) have been taken from the records of Company X, a mutual casualty company, and comprise the assets and liabilities as of December 31, 1956.

1. Bonds.....	\$175,000
2. Reserve for Dividends not Declared.....	4,000
3. Agents' Balances or Uncollected Premiums....	10,000
4. Unpaid Losses.....	120,000
5. Stocks.....	18,000
6. Unpaid Loss Adjustment Expense.....	10,000
7. Other Unpaid Expenses.....	800
8. Interest or Dividends Due and Accrued.....	1,000
9. Unearned Premiums.....	33,000
10. Dividends to Policyholders Declared but Unpaid	7,000
11. Contingent Commissions to Agents.....	100
12. Real Estate.....	3,100
13. Unredeemed Loss Drafts.....	225
14. Guaranty Funds.....	625
15. Cash and Bank Deposits.....	9,000
16. Reserve for Investment Fluctuations.....	8,000
17. Unassigned Funds (Surplus).....	?
18. Unpaid Taxes, Licenses and Fees.....	5,000

Prepare Page 2, "Assets" and Page 3, "Liabilities, Surplus and Other Funds" of the Annual Statement. In order to save time, use the number of each item above rather than its description.

PART II

SECTION (a)

NOTE: Answer any four of the questions numbered 1 through 6.

1. Discuss the problems underlying the development of an individual risk rating plan for fire insurance. Refer particularly to:

- (a) Credibility
 (b) Fluctuating insurance values by location
 (c) Expense Credits
 (d) Deductible coverage
2. The classification of a town has been changed from National Board Class 8 to National Board Class 7. Describe the basis on which this change was made, and state the effects on dwelling Fire Insurance rates for this town assuming all other conditions remain the same.
3. (a) In times of rising average loss cost, what changes should be made in experience rating procedures with regard to eligibility and credibility requirements based upon premium. Explain the reasons therefor.
- (b) If
- $$Z = \frac{E}{E + K} \quad \text{where} \quad \begin{array}{l} Z = \text{credibility} \\ E = \text{expected losses} \\ K = \text{a constant,} \end{array}$$
- determine the value of K so that, on a risk for which the expected losses amount to \$1,000, the maximum credit will be 10%. Assume a no-split rating plan.
4. Outline the provisions of the Collision Fleet Rating Plan of the National Automobile Underwriters Association as respects the following:
1. Eligibility.
 2. Definition and effect of "catastrophe" loss experience.
 3. Premium and losses used in the loss ratio determination.
 4. Basic experience period and procedure in applying modification when it is based on experience for lesser periods.
5. An analysis of your Automobile Liability experience for commercial lines shows that, all other factors being equal, the total premium for experience-rated and non-rated risks is less than what the manual rates would have produced for these same risks had there been no experience rating plan.

- (a) What is this phenomenon called? Discuss possible reasons for its existence.
 - (b) Outline two methods for correcting this situation, one method to be applied to only experience-rated risks and the other to apply to all risks. Which would you recommend?
6. The current retrospective rating formula for Workmen's Compensation Insurance may be expressed in standard symbols as:

$$H < R = [B + C \cdot L] T < G$$

- (a) Define carefully each term in the above expression.
- (b) How is the credibility criterion defined for this plan?
- (c) If all other conditions remain the same, describe and explain the effect on B of:
 - (i) an increase in C
 - (ii) a decrease in H

SECTION (b)

NOTE: Answer any four of the questions numbered 7 through 12.

7. Many states have now approved the writing of 3-Year Fixed Rate Workmen's Compensation Policies. Briefly describe this program and comment on its possible effect on the operation of a carrier's statistical, underwriting and sales departments.
8. During the past year, the insurance industry has made some progress in making available third party bodily injury and property damage liability insurance covering privately operated nuclear energy facilities. Summarize developments to date and in your answer touch upon the organizations which have been formed, the intended dollar capacity and objectives of such organizations, and the probable method of rating the hazard involved.
9. You have been asked to prepare an exhibit, which is to be used by a legislative committee, comparing the differences in costs for Workmen's Compensation Insurance among several states. Describe

how you would prepare such an exhibit and explain carefully the uses to which it may be put, and those for which it would be improper.

10. A system of Automobile Compensation Insurance, similar in many respects to our Workmen's Compensation Acts, has been advanced as a means of solving many of the current problems in this field. Outline what you consider to be the advantages and disadvantages of such a system.
11. Outline the major provisions of the Federal Flood Insurance Act of 1956.
12. Fire Insurance premiums for dwelling properties today are directly proportional to the amount of the policy. Recently there has been strong agitation for a system of dwelling fire rates which would provide lower unit costs as the total policy amount increased. This effect could be accomplished under a single schedule of rates for a given construction and protection class by dividing total policy liability into segments; viz, first \$5,000, next \$5,000, next \$5,000, etc., with successively lower rates applying to each segment. Describe the theoretical conditions, particularly with reference to a distribution of partial losses, under which such system might justifiably evolve. Under the conditions you have outlined, would you expect a mandatory co-insurance clause to be of any help?

PART III

SECTION (a)

1. Name twelve "fields" of the standard premium and exposure punch card of a multiple line company and briefly describe the purpose of any six of them.
2. (a) What is "Automatic Programming" for an Electronic Computer?
(b) What are the advantages and disadvantages of such "Automatic Programming"?
3. Describe, in general terms, the important points of a program that a company should follow in preparation for ultimate electronic mechanization.

4. List and describe each of the five general classifications of operations which can be performed by a punch card collator (for example: IBM type 077 or 089).

SECTION (b)

NOTE: Answer any four of the questions numbered 5 through 10.

5. The unearned premium reserves as of December 31 for two insurance companies are compared to their written premiums for the year just ended with the following results:

Carrier A: Ratio — .40

Carrier B: Ratio.— .60

What conclusions about the comparative adequacy of the unearned premium reserve of each can be drawn from this information? What factors might account for this difference in ratios between Carrier A and Carrier B?

6. (a) Outline the data that can be reported on a policy year basis for private passenger cars in accordance with the Automobile Liability Statistical Plan published by the National Bureau of Casualty Underwriters.
 - (b) Summarize briefly the instructions of the Statistical Plan for Liability Insurance Other Than Automobile published by the National Bureau of Casualty Underwriters with respect to the following:
 - (i) Three Year Policies
 - (ii) Type of Loss Codes and Cause of Loss Codes
 - (iii) Definition of Allocated Claim Adjustment Expense
7. The "Uniform Statistical Plan for Fire and Allied Lines" does not correspond exactly with the rating system used. What suggestions have been offered for bringing the two into closer harmony? Give at least ten.

8. In the Burglary and Glass Insurance Statistical Plans published by the National Bureau of Casualty Underwriters, what are the instructions with respect to:
 - (a) Reinsurance
 - (b) Purpose of the Plans
 - (c) Losses

9. What five factors are used in determining a company's "General Policyholders' Rating" as published in Best's Insurance Reports? Briefly indicate the tests made in evaluating each of these factors.

10. (a) In reviewing a filing for increased automobile bodily injury liability rates, a state supervisory official might say to the filing agent "How do you reconcile your request for higher rates with the fact that 'Insurance by States' as published by The Spectator shows an automobile liability loss ratio of 40% in this state last year for the companies you represent?" How would you answer this objection?
 - (b) Name and describe briefly the contents of each of three well known annual insurance statistical publications other than the one referred to in Part (a) of this question.

PART IV

SECTION (a)

NOTE: Answer any four of the questions numbered 1 through 6.

1. Discuss the pros and cons of using a wage-trend factor in the determination of Workmen's Compensation rate levels.

2. In the "McCullough Report", what was the criticism of the 1921 standard profit formula for its treatment of investment income? Under the applications of the McCullough theory regarding investment income, what would be the general effect on the operations of stock fire insurance companies?

3. Discuss the problems involved in surety rate making, particularly in attempting to use the statistics as presently gathered by the Surety Association of America.

4. With respect to fire insurance rate-making, briefly discuss:
 - (a) The principal criticism of the present system, and
 - (b) Why it is difficult to determine suitable rate differentials among types of construction and among classes of risks.

5. In an attempt to produce a method for stabilizing rate levels for classifications having a comparatively small volume of experience, the Pennsylvania Compensation Rating Bureau undertook a study which resulted in a credibility technique based upon the number of Temporary cases. Describe briefly the study, the results thereof, and the actual technique used as a result of this study.

6. Given the following definition relating to Hospital Service Insurance:

h_n = number hospitalized for exactly n days on the basis of an exposure of 1,000,000 life-years.

- (a) Derive an expression in terms of h_n for a Net Premium to provide \$A per day up to a limit of n days for any one hospital stay.

- (b) Derive an expression in terms of h_n for a Net Premium to provide for any one hospital stay:
 - \$X for a one-day stay
 - \$Y for a two-day stay
 - \$Z for a three-day stay
 - \$A per day from the first through the n^{th} day for stays in excess of three days
 - \$D per day from the $(n+1)^{\text{th}}$ to the $(n+m)^{\text{th}}$ day, inclusive

SECTION (b)

Outline and write an essay on any one of the following topics:

- (1) Past, present and future of insurance stock prices compared with general stock market trends.
- (2) Liability Insurance ratemaking in an inflationary economy.
- (3) Insurance and privately-owned atomic energy projects.
- (4) Ratemaking statistics for the various multiple peril, or so-called "package" policies and the interpretation thereof.
- (5) Rate competition based primarily on differences in acquisition costs.

Show your outline of the topic clearly.

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ILLUSTRATION OF HOW EARNED PREMIUMS FOR ANY GIVEN MAJOR PERIL AND SUBCODE IN ANY GIVEN STATE ARE COMPUTED

Fractions and Amounts Earned in Years Shown

<i>Premiums Written</i>	<i>1949</i>	<i>1950</i>	<i>1951</i>	<i>1952</i>	<i>1953</i>	<i>1954</i>	<i>1955</i>	<i>1956</i>
1949—1 Yr. \$50	$\frac{1}{2}$ or 25	$\frac{1}{2}$ or 25						
3 Yr. \$60	$\frac{1}{6}$ or 10	$\frac{1}{3}$ or 20	$\frac{1}{3}$ or 20	$\frac{1}{6}$ or 10				
5 Yr. \$70	$\frac{1}{10}$ or 7	$\frac{1}{5}$ or 14	$\frac{1}{5}$ or 14	$\frac{1}{5}$ or 14	$\frac{1}{6}$ or 14	$\frac{1}{10}$ or 7		
1949 Earnings on 1949 Writings	\$42							
1950—1 Yr. \$60		$\frac{1}{2}$ or 30	$\frac{1}{2}$ or 30					
3 Yr. \$72		$\frac{1}{6}$ or 12	$\frac{1}{3}$ or 24	$\frac{1}{6}$ or 24	$\frac{1}{6}$ or 12			
5 Yr. \$80		$\frac{1}{10}$ or 8	$\frac{1}{5}$ or 16	$\frac{1}{5}$ or 16	$\frac{1}{6}$ or 16	$\frac{1}{6}$ or 16	$\frac{1}{10}$ or 8	
1950 Earnings on 1949 and 1950 Writings		\$109						
1951—1 Yr. \$50			$\frac{1}{2}$ or 25	$\frac{1}{2}$ or 25				
3 Yr. \$60			$\frac{1}{6}$ or 10	$\frac{1}{3}$ or 20	$\frac{1}{3}$ or 20	$\frac{1}{6}$ or 10		
5 Yr. \$70			$\frac{1}{10}$ or 7	$\frac{1}{5}$ or 14	$\frac{1}{5}$ or 14	$\frac{1}{6}$ or 14	$\frac{1}{6}$ or 14	$\frac{1}{10}$ or 7
1951 Earnings on 1949-1950 and 1951 Writings			\$146					

The above illustration is not a complete one but it does show how premiums earned for any given year are obtained. It will be noted that all of the 1949 writings will be earned by the end of 1954 and will not be included in premiums earned for any year after 1954.

For converting the written premiums by term to earned premiums, it might have been possible to use the statutory fractions such as used in the previous example but for the purpose of recognizing the effect of cancellations a set of factors slightly different from the statutory fractions are computed each year and used instead. The factors are obtained from an annual report supplied by companies entitled "Summary Direct Premiums Written and Contributions to In Force," which for the year 1954 appears as follows:

Study Explanatory Notes on Reverse Side Before Preparing This Report

SUMMARY
DIRECT PREMIUMS WRITTEN
AND CONTRIBUTIONS TO
IN FORCE

For the Year Ended
December 31, 1954

Company or Group or Reporting Association

EFFECTIVE YEARS	TERMS	FIRE — Major Perils 10 & 11		Ext. Cov. Major Perils 20 to 28	
		Direct Premiums Written 1954	Direct Contributions To In Force 1954*	Direct Premiums Written 1954	Direct Contributions To In Force 1954*
1954	1 yr. or less	\$	\$	\$	\$
1954	2 years				
1954	3 years				
1954	4 years				
1954	5 yrs. or over				
1953	1 yr. or less		XXXXXX		XXXXXX
1953	2 years				
1953	3 years				
1953	4 years				
1953	5 yrs. or over				
1952	1 yr. or less		XXXXXX		XXXXXX
1952	2 years		XXXXXX		XXXXXX
1952	3 years				
1952	4 years				
1952	5 yrs. or over				
1951	1 yr. or less		XXXXXX		XXXXXX
1951	2 years		XXXXXX		XXXXXX
1951	3 years		XXXXXX		XXXXXX
1951	4 years				
1951	5 yrs. or over				
1950	1 yr. or less		XXXXXX		XXXXXX
1950	2 years		XXXXXX		XXXXXX
1950	3 years		XXXXXX		XXXXXX
1950	4 years		XXXXXX		XXXXXX
1950	5 yrs. or over				
All prior eff. years	all terms		XXXXXX		XXXXXX
Advance Premiums**					
All years	all terms		XXXXXX		XXXXXX
Reporting Assn. Prems.**					
All years	all terms		XXXXXX		XXXXXX
Canadian & Other Foreign Prems.**					
All years	all terms		XXXXXX		XXXXXX
		\$	\$	\$	\$
TOTALS		To agree with Page 6, Line 1, Column 1	XXXXXX	To agree with Page 6, Line 2, Column 1	XXXXXX

See Reverse Side for Explanatory Notes.

SIGNED _____ TITLE _____

ERRATA

The attached corrected pages should be pasted over pages 88 and 89 of volume XLIII of the proceedings for 1956.

CASUALTY
ACTUARIAL SOCIETY

ORGANIZED 1914

1958 YEAR BOOK

Foreword

Officers, Council and Committees

List of Fellows and Associates

Officers of the Society since Organization

List of Deceased Members

Constitution and By-Laws

Examination Requirements

(Addendum to Volume XLIV of the *Proceedings*)

Corrected to February 1, 1958

FOREWORD

The Casualty Actuarial Society was organized November 7, 1914 as the Casualty Actuarial and Statistical Society of America, with 97 charter members of the grade of Fellow. The present title was adopted on May 14, 1921. The object of the Society is the promotion of actuarial and statistical science as applied to the problems of casualty and social insurance by means of personal intercourse, the presentation and discussion of appropriate papers, the collection of a library and such other means as may be found desirable. The organization of the Society was brought about through the suggestion of Dr. I. M. Rubinow, who became the first president. The problems surrounding workmen's compensation were at that time the most urgent, and consequently many of the members played a leading part in the development of the scientific basis upon which workmen's compensation insurance now rests.

The members of the Society have also presented original papers to the *Proceedings* upon the scientific formulation of standards for the computation of both rates and reserves in accident and health insurance, liability, burglary, and the various automobile coverages. The presidential addresses constitute a valuable record of the current problems facing the casualty insurance business. Other papers in the *Proceedings* deal with acquisition costs, pension funds, legal decisions, investments, claims, reinsurance, accounting, statutory requirements, loss reserves, statistics, and the examination of casualty companies. "The Recommendations for Study" appear in *Proceedings* Vol. XLI and are in effect for the 1955 examinations and thereafter. The Report of the Committee on Mortality for Disabled Lives together with commutation tables and life annuities has been printed in *Proceedings* No. 62. The Committee on Compensation and Liability Loss and Loss Expense Reserves submitted a report which appears in Volume XXXV.

At the November 1950 meeting of the Society the Constitution and By-Laws were amended to enlarge the scope of the Society to include all lines of insurance other than life insurance. The effect of the amendment was to include fire insurance and allied lines in recognition of multiple line writing powers granted by many states to both casualty companies and fire companies.

The lower grade of membership in the Society is that of Associate. Examinations have been held every year since organization; they are held during the second or third week of the month of May, in various cities in the United States and Canada. The membership of the Society consists of actuaries, statisticians, and executives who are connected with the principal casualty companies and organizations in the United States and Canada. The Society has a total membership of 332 consisting of 186 Fellows and 146 Associates.

The Society issues a publication entitled the *Proceedings* which contains original papers presented at the meetings. The *Proceedings* also contain discussions of papers, and reviews of books. This Year Book is published annually. "Recommendations for Study" is a pamphlet which outlines the course of study to be followed in connection with the examinations for admission. These two booklets may be obtained free upon application to the Secretary-Treasurer, Albert Z. Skelding, 200 Fourth Avenue, New York 3, N. Y.

CASUALTY ACTUARIAL SOCIETY

NOVEMBER 22, 1957

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 RUTH E. SALZMANN 1959
 JOHN A. RESONY 1959
 M. STANLEY HUGHEY 1960
 MATTHEW RODERMUND 1960
 JOHN W. WIEDER, JR. 1960

*Terms expire at the annual meeting in November 1958.

†Terms expire at the annual meeting in November of the year given.

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MEMBERSHIP OF THE SOCIETY, NOVEMBER 22, 1957
FELLOWS

Those marked (†) were Charter Members at date of organization, November 7, 1914

Admitted	
Nov. 21, 1930	AINLEY, JOHN W., Statistician, Casualty, Fire & Marine Actuarial Department, The Travelers Insurance Company, Hartford 15, Conn.
Nov. 14, 1947	ALLEN, EDWARD S., Assistant General Manager and Actuary, New York Compensation Insurance Rating Board, 100 E. 42nd Street, New York 17, N. Y.
Nov. 13, 1931	AULT, GILBERT E., Actuary, Church Pension Fund and Church Life Insurance Corporation, 20 Exchange Place, New York 5, N. Y.
Nov. 18, 1955	BAILEY, ROBERT A., Assistant Actuary, Hardware Mutual Casualty Company, 200 Strongs Ave., Stevens Point, Wis.
Nov. 20, 1924	BARBER, HARMON T., Second Vice President and Actuary, The Travelers Insurance Co., Hartford 15, Conn.
Nov. 19, 1954	BARKER, GORDON M., c/o Bowles, Andrews & Towne, 1004 North Thompson Street, Richmond, Va.
Nov. 14, 1947	BARKER, LORING M., Actuary, Fireman's Fund Group, 3333 California Street, San Francisco, Calif.
Nov. 20, 1942	BART, ROBERT D., Comptroller and Assistant Treasurer, West Bend Aluminum Co., 92 Island Avenue, West Bend, Wis.
Nov. 18, 1932	BARTER, JOHN L., Vice-President, Hartford Accident & Indemnity Co., 690 Asylum Avenue, Hartford 15, Conn.
Nov. 13, 1931	BATHO, ELGIN R., Vice President and Actuary, Berkshire Life Insurance Co., 7 North Street, Pittsfield, Mass.
Nov. 16, 1956	BENNETT, NORMAN J., Actuary and Deputy Commissioner, Florida Insurance Department, Tallahassee, Florida.
Nov. 22, 1934	BERKELEY, ERNEST T., Actuary, Employers' Liability Assurance Corporation, Ltd., American Employers' Insurance Company and Employers' Fire Insurance Company, 110 Milk Street, Boston 7, Mass.
Nov. 22, 1957	BERQUIST, JAMES R., Assistant Actuary, Employers' Mutual Liability Insurance Co. of Wisconsin, Wausau, Wis.
Nov. 19, 1953	BEVAN, JOHN R., Assistant Actuary, Liberty Mutual Insurance Company, 175 Berkeley Street, Boston 17, Mass.
†	BLACK, S. BRUCE, Chairman, Liberty Mutual Insurance Co., 175 Berkeley Street, Boston 17, Mass.
Apr. 20, 1917	BLANCHARD, RALPH H., Professor Emeritus of Insurance, Graduate School of Business, Columbia University, Plympton, Mass.
Nov. 16, 1956	BONDY, MARTIN, Principal Actuary, New York State Insurance Department, 123 William Street, New York 38, N. Y.
Nov. 22, 1957	BORNHUETTER, RONALD L., Assistant Actuary, National Bureau of Casualty Underwriters, 60 John Street, New York 38, N. Y.
Nov. 16, 1956	BOYAJIAN, JOHN H., Actuary, California Inspection Rating Bureau, 500 Sansome Street, San Francisco 11, Calif.

FELLOWS

Admitted	
†	BREIBY, WILLIAM, Vice President, Pacific Mutual Life Insurance Company, Box 6050 Metropolitan Station, Los Angeles 55, Calif.
Nov. 21, 1952	BRINDISE, RALPH S., Casualty Actuary, Standard Oil Company (Indiana) 910 So. Michigan Ave., Chicago 80, Ill.
Nov. 18, 1927	BROWN, F. STUART, Electronics Committee, American Insurance Group, 15 Washington Street, Newark 2, N. J.
Oct. 22, 1915	BROWN, HERBERT D., (Retired), Glenora-on-Lake Seneca, Dundee, New York.
†	BUCK, GEORGE B., Consulting Actuary, 150 Nassau Street, New York 38, N. Y.
Apr. 20, 1917	BURHOP, WILLIAM H., President, Employers Mutual Liability Insurance Company, 407 Grant Street, Wausau, Wis.
Nov. 23, 1928	BURLING, WILLIAM H., Secretary, Group Department, The Travelers Insurance Company, 700 Main Street, Hartford 15, Conn.
Nov. 19, 1929	CAHILL, JAMES M., Secretary, National Bureau of Casualty Underwriters, 60 John Street, New York 38, N. Y.
Nov. 18, 1932	CAMERON, FREELAND R., Vice President and Actuary, American-Equity Insurance Group, P.O. Box 3131, Miami, Florida.
†	CAMMACK, EDMUND E., Vice-President and Actuary, Aetna Life Insurance Company, Hartford 15, Conn.
Nov. 17, 1938	CARLETON, JOHN W., Vice President and Actuary, Liberty Mutual Insurance Company, 175 Berkeley Street, Boston 17, Mass.
Nov. 21, 1930	CARLSON, THOMAS O., Actuary, National Bureau of Casualty Underwriters, 60 John Street, New York 38, N. Y.
Nov. 18, 1949	CLARKE, JOHN W., Vice President, Gulf Life Insurance Co., Jacksonville 1, Florida.
Nov. 15, 1918	COATES, BARRETT N., 1007 Cragmont Avenue, Berkeley 8, Calif.
Nov. 17, 1922	COATES, CLARENCE S., Second Vice-President, Lumbermens Mutual Casualty Company, 4750 Sheridan Road, Chicago 40, Ill.
Feb. 19, 1915	COLLINS, HENRY, (Retired), Lochbrae, Windermere, Florida.
Nov. 22, 1934	CONSTABLE, WILLIAM J., 45 Pondfield Road, West, Bronxville 8, N. Y.
Nov. 22, 1934	COOK, EDWIN A., President and General Manager, Interboro Mutual Indemnity Insurance Company, 270 Madison Avenue, New York 16, N. Y.
Nov. 18, 1925	CORCORAN, WILLIAM M., Partner, Wolfe, Corcoran & Linder, 116 John Street, New York 38, N. Y.
Nov. 19, 1926	CRANE, HOWARD G., Vice President and Treasurer, General Reinsurance Corporation, 400 Park Avenue, New York 22, N. Y.
Nov. 21, 1952	CRITCHLEY, DOUGLAS, E. B. Savory & Co., London, England.
Nov. 22, 1946	CROUSE, CHARLES W., Consulting Actuary, C. E. Preslan & Co., Inc., 20015 Detroit Road, Cleveland 16, Ohio.
Nov. 19, 1953	CURRY, HAROLD E., Vice President, State Farm Mutual Automobile Insurance Co., 112 East Washington Street, Bloomington, Ill.
Nov. 18, 1932	DAVIES, E. ALFRED, (Retired), Falls Village, Conn.

FELLOWS

Admitted	
Nov. 18, 1927	DAVIS, EVELYN M., Woodward, Ryan, Sharp & Davis, Consulting Actuaries, 55 Broadway, New York 6, N. Y.
May 25, 1956	DAY, ELDEN W., Resident Secretary, Lumbermens Mutual Casualty Co., 342 Madison Avenue, New York 17, N. Y.
Nov. 16, 1951	DOERMUS, FREDERICK W., Manager, Eastern Underwriters Association, 85 John St., New York 38, N. Y.
Nov. 17, 1920	DORWEILER, PAUL, (Retired), 51 Wethersfield Avenue, Hartford 14, Conn.
Nov. 22, 1957	DROBISCH, MILES R., Statistician, California Inspection Rating Bureau, 500 Sansome Street, San Francisco 11, Calif.
Nov. 24, 1933	EDWARDS, JOHN, Actuary, Ontario Department of Insurance, 1st floor, 145 Queen Street West, Toronto 1, Ontario, Canada.
Nov. 15, 1940	ELLIOTT, GEORGE B., General Manager, Pennsylvania Compensation Rating Bureau, 315 Chestnut Street, Philadelphia 6, Pa.
Nov. 17, 1922	ELSTON, JAMES S., (Retired) 1640 Palmer Avenue, Winter Park, Fla.
Nov. 15, 1935	EPPINK, WALTER T., Treasurer and Actuary, Merchants Mutual Insurance Company, 268 Main Street, Buffalo 5, N. Y.
Nov. 18, 1955	FAIRBANKS, ALFRED V., Assistant Actuary, Monarch Life Insurance Co., 365 State Street, Springfield 1, Mass.
†	FALLOW, EVERETT S., (Retired), 28 Sunset Terrace, West Hartford, Conn.
Nov. 15, 1940	FARLEY, JARVIS, Secretary, Treasurer and Actuary, Massachusetts Indemnity and Life Insurance Co., 654 Beacon Street, Boston 15, Mass.
†	FARRER, HENRY, (Retired), 1352 Overlea Street, Clearwater, Fla.
May 25, 1956	FINNEGAN, J. H., Manager, Actuarial Bureau, National Board of Fire Underwriters, 85 John Street, New York 38, N. Y.
Nov. 15, 1935	FITZHUGH, GILBERT W., Second Vice-President, Metropolitan Life Insurance Co., 1 Madison Avenue, New York 10, N. Y.
Feb. 19, 1915	FONDILLER, RICHARD, Consulting Actuary, Woodward and Fondiller, 200 W. 57th Street, New York 19, N. Y.
Nov. 18, 1955	FOSTER, ROBERT B., Assistant Actuary; Casualty, Fire & Marine Actuarial Department, The Travelers Insurance Co., Hartford 15, Conn.
Nov. 18, 1955	FOWLER, THOMAS W., Associate Actuary, New York State Insurance Department, 324 State Street, Albany 10, N. Y.
Nov. 18, 1927	FREDERICKSON, CARL H., Actuary, Canadian Underwriters Association, 12 Upjohn Road, Don Mills, Ontario, Canada.
Nov. 22, 1934	FULLER, GARDNER V., Resident Secretary, Lumbermens Mutual Casualty Co., and American Motorists Insurance Co., 4750 Sheridan Road, Chicago 40, Ill.
Nov. 19, 1948	GARDINER, JAMES B., Assistant Actuary, Metropolitan Life Insurance Co., 1 Madison Avenue, New York 10, N. Y.
Nov. 22, 1957	GILLAM, WILLIAM S., Research Unit, National Bureau of Casualty Underwriters, 60 John Street, New York 38, N. Y.
Nov. 20, 1924	GINSBURGH, HAROLD J., Senior Vice-President, American Mutual Liability Insurance Company, Vice-President, American Policyholders' Insurance Company and Allied American Mutual Fire Insurance Company, 142 Berkeley Street, Boston 17, Mass.

FELLOWS

Admitted Nov. 21, 1930	GLENN, JOSEPH B., Consulting Actuary, 6110 Valley Road, Washington 14, D. C.
Nov. 13, 1931	GODDARD, RUSSELL P., Assistant to the President, Pennsylvania Manufacturers Association Casualty Insurance Co., Finance Building, Philadelphia, Pa.
†	GOODWIN, EDWARD S., (Investment Counselor, Retired) 96 Garvan Street, East Hartford 8, Conn.
Nov. 19, 1926	GRAHAM, CHARLES M., Consulting Actuary, 552 Oakhurst Road, Largo, Florida.
†	GRAHAM, WILLIAM J., Consultant, 1070 Park Ave., New York 18, N. Y.
Nov. 19, 1953	GRAVES, CLYDE H., Actuary, Mutual Insurance Rating Bureau and Assistant Manager, Mutual Insurance Advisory Association, 111 Fourth Avenue, New York 3, N. Y.
†	GREENE, WINFIELD W., President, W. W. Greene, Inc., Reinsurance Intermediaries and Actuarial Consultants, 68 William Street, New York 5, N. Y.
Nov. 19, 1953	HALEY, JAMES B., JR., Actuary, Argonaut Insurance, 250 Middlefield Road, Menlo Park, Calif.
†	HAMMOND, H. PIERSON, (Retired), 22 Vanderbilt Road, West Hartford 7, Conn.
Nov. 16, 1956	HART, W. VAN BUREN, JR., Analyst-Programmer, Aetna Insurance Group, SPAN Electronic Processing Center, Hartford 5, Conn.
Nov. 17, 1950	HARWAYNE, FRANK, Chief Actuary, New York State Insurance Department, 123 William Street, New York 38, N. Y.
Oct. 22, 1915	HATCH, LEONARD W., (Retired), 425 Pelham Manor Road, Pelham Manor, New York.
Nov. 19, 1926	HAUGH, CHARLES J., Vice President, The Travelers Insurance Co., and The Travelers Indemnity Company, Hartford 15, Conn.
Nov. 17, 1950	HAZAM, WILLIAM J., Assistant Vice President and Associate Actuary, American Mutual Liability Insurance Co., 142 Berkeley Street, Boston, Mass.
Nov. 16, 1951	HEWITT, CHARLES C., JR., c/o Bowles, Andrews & Towne, 156 William Street, New York 38, N. Y.
Nov. 22, 1934	HOOKE, RUSSELL O., Consulting Actuary, and President and Actuary, Insurance City Life Co., 750 Main Street, Hartford 3, Conn.
Nov. 17, 1950	HOPE, FRANCIS J., Assistant Secretary, Hartford Accident and Indemnity Company, 690 Asylum Avenue, Hartford 15, Conn.
Nov. 18, 1932	HUEBNER, SOLOMON STEPHEN, Chairman of Board, The American Institute for Property and Liability Underwriters, 3924 Walnut St., Philadelphia 4, Pa., also President Emeritus of The American College of Life Underwriters, Emeritus Professor of Insurance, University of Pennsylvania.
Nov. 14, 1947	HUGHEY, M. STANLEY, Second Vice-President, Lumbermens Mutual Casualty Company, 4750 Sheridan Road, Chicago 40, Ill.
†	HUNTER, ARTHUR, (Retired), 124 Lloyd Road, Montclair, N. J.
Nov. 18, 1955	HURLEY, ROBERT L., Actuary, Liberty Mutual Fire Insurance Co., 175 Berkeley Street, Boston 17, Mass.

FELLOWS

Admitted Feb. 25, 1916	JACKSON, CHARLES W., (Retired), 801 Meadowlark Lane, Glenview, Ill.
Nov. 19, 1954	JOHE, RICHARD L., Assistant Actuary, United States Fidelity and Guaranty Company, Baltimore, Md.
Nov. 14, 1941	JOHNSON, ROGER A., Actuary, Utica Mutual Insurance Co., P. O. Box 530, Utica, N. Y.
Nov. 16, 1939	JONES, HAROLD M., Group Research Division, John Hancock Mutual Life Insurance Company, 200 Berkeley Street, Boston 17, Mass.
Nov. 16, 1956	KALLOP, ROY H., Assistant Actuary, National Council on Compensation Insurance, 200 Fourth Avenue, New York 3, N. Y.
Nov. 22, 1957	KATES, PHILLIP B., Vice President and Actuary, Southern Fire and Casualty Company, P. O. Box 1966, Knoxville 1, Tenn.
Nov. 19, 1926	KELTON, WILLIAM H., Actuary, The Travelers Insurance Company, Hartford 15, Conn.
Nov. 14, 1941	KOLE, MORRIS B., Principal Actuary, State Insurance Fund, 199 Church Street, New York 7, N. Y.
Nov. 24, 1933	KORMES, MARK, Consulting Actuary, 285 Madison Avenue, New York 17, N. Y.
Nov. 19, 1953	KUENKLER, ARTHUR S., Executive Vice President, Security Insurance Co. of New Haven, 175 Whitney Avenue, New Haven, Conn.
Nov. 18, 1949	LA CROIX, HAROLD F., Associate Actuary, The Travelers Insurance Co., Hartford 15, Conn.
Nov. 13, 1931	LA MONT, STEWART M., (Retired), Hotel Claremont, Berkeley, Calif.
†	LEAL, JAMES R., (Retired).
†	LESLIE, WILLIAM, General Manager, National Bureau of Casualty Underwriters, 60 John Street, New York 38, N. Y.
Nov. 17, 1950	LESLIE, WILLIAM, JR., Secretary and Actuary, America Fore Insurance Group, 80 Maiden Lane, New York 38, N. Y.
Nov. 20, 1924	LINDER, JOSEPH, Consulting Actuary, Wolfe, Corcoran & Linder, 116 John Street, New York 38, N. Y.
Nov. 16, 1956	LINO, RICHARD, Assistant Actuary, National Bureau of Casualty Underwriters, 60 John Street, New York 38, N. Y.
Nov. 18, 1955	LISCORD, PAUL S., JR., Assistant Actuary; Casualty, Fire & Marine Actuarial Department, The Travelers Insurance Co., Hartford 15, Conn.
Nov. 17, 1950	LIVINGSTON, GILBERT R., Associate Actuary, National Bureau of Casualty Underwriters, 60 John Street, New York 38, N. Y.
Nov. 16, 1951	LONGLEY-COOK, LAURENCE H., Actuary, Insurance Company of North America, 1600 Arch Street, Philadelphia 1, Pa.
Nov. 13, 1936	LYONS, DANIEL J., Vice President, Guardian Life Insurance Company, 50 Union Square, New York 3, N. Y.
Nov. 22, 1957	MAKGILL, STEPHEN S., Assistant Actuary, The Travelers Insurance Company, Hartford 15, Conn.
Nov. 19, 1954	MACKEEN, HAROLD E., Assistant Actuary; Casualty, Fire & Marine Actuarial Department, The Travelers Insurance Co., Hartford 15, Conn.
Nov. 23, 1928	MARSHALL, RALPH M., Assistant Actuary, National Council on Compensation Insurance, 200 Fourth Ave., New York 3, N. Y.
Nov. 18, 1927	MASTERTSON, NORTON E., Vice-President and Actuary, Hardware Mutual Casualty Co. and Hardware Dealers Mutual Fire Insurance Co., 200 Strongs Avenue, Stevens Point, Wis.

FELLOWS

Admitted	
Nov. 19, 1926	MATTHEWS, ARTHUR N., Actuary, The Travelers Insurance Company, Hartford 15, Conn.
May 19, 1915	MAYCRINK, EMMA C., 32 Chittenden Avenue, Crestwood, N. Y.
Nov. 15, 1935	MCCONNELL, MATTHEW H., Superintendent, Compensation and Liability Department, General Accident Fire and Life Assurance Company, Fourth and Walnut Sts., Philadelphia 5, Pa.
Oct. 31, 1917	McMANUS, ROBERT J., (Retired), 8 Ridgebrook Drive, West Hartford, Conn.
Nov. 18, 1955	MENZEL, HENRY W., Actuary, Springfield Insurance Companies, 1250 State Street, Springfield, Mass.
†	MICHELbacher, GUSTAV F., President, Great American Indemnity Company, 99 John Street, New York 38, N. Y.
Nov. 17, 1938	MILLER, JOHN H., Vice President and Senior Actuary, Monarch Life Insurance Company, Springfield 1, Mass.
†	MILLIGAN, SAMUEL, Senior Vice-President, Metropolitan Life Insurance Co., 1 Madison Avenue, New York 10, N. Y.
Nov. 18, 1937	MILLS, JOHN A., Vice-President and Actuary, Lumbermens Mutual Casualty Co., American Manufacturers Mutual Insurance Company and American Motorists Insurance Co., Mutual Insurance Bldg., 4750 Sheridan Road, Chicago 40, Ill.
Nov. 22, 1957	MILLS, RICHARD J., Statistical Department, Lumbermens Mutual Casualty Company, 4750 Sheridan Road, Chicago 40, Ill.
Nov. 18, 1921	MONTGOMERY, VICTOR, President, Pacific Employers Insurance Co., California Union Insurance Co., 1033 South Hope Street, Los Angeles 15, Calif.
†	MOORE, GEORGE D., Actuary, 13 Emerson Street, E. Orange, N. J.
Nov. 17, 1920	MUELLER, LOUIS H., 2845 Lake Street, San Francisco 21, Calif.
Nov. 16, 1956	MUETTERTIES, JOHN H., Casualty Actuary, Industrial Indemnity Company, 155 Sansome Street, San Francisco 4, Calif.
Nov. 17, 1950	MUNTERICH, GEORGE C., Assistant Secretary, Hartford Accident and Indemnity Company, 650 Asylum Avenue, Hartford 15, Conn.
May 28, 1920	MURPHY, RAY D., Chairman of the Board, Equitable Life Assurance Society of the U.S., 393 Seventh Avenue, New York 1, N. Y.
Nov. 19, 1954	MURRIN, THOMAS E., Associate Actuary, National Bureau of Casualty Underwriters, 60 John Street, New York 38, N. Y.
Nov. 15, 1935	OBERHAUS, THOMAS M., Consulting Actuary, Woodward and Fondiller, 200 West 57th Street, New York 19, N. Y.
†	OLIFIERS, EDWARD, Consulting Actuary, Caixa Postal 8, Petropolis, Rio, Brazil.
†	ORR, ROBERT K., (Retired), 316 E. Lenawee Street, Lansing, Mich.
Nov. 22, 1957	OTTESON, PAUL M., Vice President and Actuary, Federated Mutual Implement and Hardware Insurance Company, 129 East Broadway, Owatonna, Minn.
Nov. 21, 1919	OUTWATER, OLIVE E., (Retired), Harbert, Michigan.
Nov. 22, 1957	PERKINS, WILLIAM J., Senior Actuarial Assistant, The London Life Insurance Company, London, Ontario, Canada.

FELLOWS

Admitted	
Nov. 21, 1930	PERRYMAN, FRANCIS S., Assistant United States Manager and Actuary, Royal-Globe Insurance Group, 150 William Street, New York 38, N. Y.
Nov. 14, 1941	PETERS, STEFAN, Actuary, Connell, Price and Co., 161 Devonshire Street, Boston 9, Mass.
Nov. 21, 1952	PETZ, EARL F., Statistical Department, Lumbermens Mutual Casualty Company, 4750 N. Sheridan Road, Chicago 40, Ill.
Nov. 24, 1933	PICKETT, SAMUEL C., (Retired), Macktown Road, Windsor, Conn.
Nov. 22, 1957	PINNEY, ALLEN D., Assistant Actuary, Casualty, Fire & Marine Actuarial Department, The Travelers Insurance Company, Hartford 15, Conn.
Nov. 17, 1922	PINNEY, SYDNEY D., 290 Wolcott Hill Road, Wethersfield 9, Conn.
Nov. 13, 1931	PRUITT, DUDLEY M., Assistant General Manager and Actuary, General Accident Fire & Life Assurance Corp., Fourth & Walnut Sts., Philadelphia 5, Pa.
Nov. 18, 1955	RESONY, ALLIE V., Assistant Actuary, Hartford Accident and Indemnity Co., 690 Asylum Avenue, Hartford 15, Conn.
Nov. 18, 1949	RESONY, JOHN A., Actuarial Assistant, Accident and Group Actuarial Department, The Travelers Insurance Company, Hartford 15, Conn.
Nov. 16, 1951	RICE, HOMER D., (Retired), 1731 Morningside Drive, Mount Dora, Florida.
Nov. 19, 1926	RICHTER, OTTO C., Chief Actuary, American Telephone & Telegraph Co., 195 Broadway, New York 7, N. Y.
May 24, 1921	RIEGEL, ROBERT, Professor of Statistics and Insurance, University of Buffalo, Buffalo 14, N. Y.
Nov. 14, 1947	RODERMUND, MATTHEW, Assistant Secretary, Interboro Mutual Indemnity Insurance Company, 270 Madison Avenue, New York 16, N. Y.
Nov. 14, 1947	ROSENBERG, NORMAN, Executive Assistant, Farmers Insurance Group, 4680 Wilshire Blvd., Los Angeles 54, Calif.
Nov. 14, 1947	ROWELL, JOHN H., Actuary, Health Service Inc., Medical Indemnity of America, Inc., 200 North Michigan Avenue, Chicago 1, Ill.
Nov. 17, 1938	RUCHLIS, ELSIE, National Bureau of Casualty Underwriters, 60 John Street, New York 38, N. Y.
Nov. 14, 1947	SALZMANN, RUTH E., Associate Actuary, Hardware Mutual Casualty Company, Hardware Dealers Mutual Fire Insurance Co., 200 Strongs Ave., Stevens Point, Wis.
Nov. 19, 1948	SCHLOSS, HAROLD W., Secretary, Actuarial Department, Royal-Globe Insurance Group, 150 William Street, New York 38, N. Y.
Nov. 18, 1937	SHAPIRO, GEORGE I., 934 E. 9th Street, Brooklyn 30, N. Y.
Nov. 13, 1931	SILVERMAN, DAVID, Partner, Wolfe, Corcoran & Linder, 116 John Street, New York 38, N. Y.
Nov. 19, 1954	SIMON, LeROY J., Associate Actuary, Insurance Company of North America, 1600 Arch Street, Philadelphia 1, Pa.
Nov. 19, 1929	SKELDING, ALBERT Z., Associate General Manager, National Council on Compensation Insurance, 200 Fourth Avenue, New York 3, N. Y.
Nov. 19, 1929	SKILLINGS, E. SHAW, Assistant Vice-President and Actuary, Allstate Insurance Co., 7447 Skokie Blvd., Skokie, Ill.
Nov. 18, 1932	SMICK, JACK J., Consulting Actuary, 38 Park Row, New York 7, N. Y.

FELLOWS

<i>Admitted</i>	
Nov. 15, 1940	SMITH, SEYMOUR E., Vice-President and Actuary, The Travelers Insurance Co., Hartford 15, Conn.
Nov. 16, 1951	SNOW, A. J., Manager, Oregon Insurance Rating Bureau, 329 S.W. 5th Avenue, Portland, Ore.
Nov. 24, 1933	St. JOHN, JOHN B., Consulting Actuary, Box 57, Penlynn, Pa.
Nov. 18, 1927	STONE, EDWARD C., Chairman of the Board, American Employers' Insurance Company, 40 Central Street, Boston 9, Mass.
May 25, 1956	TAPLEY, DAVID A., Actuary, State Farm Mutual Automobile Insurance Co., 112 E. Washington St., Bloomington, Ill.
Nov. 17, 1920	TARBELL, THOMAS F., (Retired), 42 Linwold Drive, West Hartford 7, Conn.
Nov. 16, 1956	THOMAS, JAMES W., Assistant Actuary; Casualty, Fire & Marine Actuarial Department, The Travelers Insurance Co., Hartford 15, Conn.
†	THOMPSON, JOHN S., (Retired), Vice-Chairman, Mutual Benefit Life Insurance Co., Newark, N. J., Military Park Hotel, 16 Park Place, Newark 2, N. J.
†	TRAIN, JOHN L., President, Utica Mutual Insurance Co., Box 530, Utica, N. Y.
Nov. 17, 1922	TRAVERSI, ANTONIO T., 59 Barry St., Neutral Bay, Sydney, Australia.
Nov. 19, 1953	TRIST, JOHN A. W., Statistical Department, Lumbermens Mutual Casualty Co., DeForest Avenue, Summit, N. J.
Nov. 19, 1948	TURNER, PAUL A., 435 South La Cienega Boulevard, Los Angeles 48, Calif.
Nov. 14, 1947	URTHOFF, D. R., Associate Actuary, Employers Mutual Liability Insurance Co. of Wisconsin, Wausau, Wis.
Nov. 23, 1928	VALERIUS, NELS M., Assistant Actuary, Aetna Casualty and Surety Co., Hartford 15, Conn.
Nov. 21, 1919	VAN TUYL, HIRAM O., (Retired), 17 Coolidge Ave., White Plains, N. Y.
Nov. 16, 1951	VERGANO, ELIA (Retired), 390 Central Park, W., New York 25, N. Y.
Nov. 16, 1951	VINCENT, LEWIS A., General Manager, National Board of Fire Underwriters, 85 John Street, New York 38, N. Y.
Nov. 17, 1920	WAITE, ALAN W., Secretary, The Aetna Casualty and Surety Co. 151 Farmington Ave., Hartford 15, Conn.
Nov. 14, 1947	WIEDER, JOHN W., JR., Assistant Actuary, Aetna Casualty and Surety Company, 151 Farmington Avenue, Hartford 15, Conn.
Nov. 15, 1935	WILLIAMS, HARRY V., Vice-President, Hartford Accident and Indemnity Co., 690 Asylum Ave., Hartford 15, Conn.
Nov. 22, 1957	WILLIAMS, PHILIP A., Assistant Actuary, Casualty, Fire & Marine Actuarial Department, The Travelers Insurance Company, Hartford 15, Conn.
Nov. 14, 1941	WILLIAMSON, W., RULON, Research Actuary, 3400 Fairhill Drive, Washington 23, D.C.
Nov. 13, 1931	WITTICK, HERBERT E., Vice President and General Manager, Pilot Insurance Company, 1315 Yonge Street, Toronto 7, Ontario, Canada.
Nov. 18, 1949	WOLFRUM, RICHARD J., Assistant Actuary, Liberty Mutual Insurance Company, 175 Berkeley Street, Boston 17, Mass.
Nov. 16, 1951	WOODALL, JOHN P., Manager, South-Eastern Underwriters Association, 327 Trust Co. of Georgia Building, Atlanta 2, Ga.
Nov. 19, 1953	YOUNT, HUBERT W., Executive Vice President, Liberty Mutual Insurance Company, 175 Berkeley Street, Boston 17, Mass.

ASSOCIATES

Admitted	
Nov. 22, 1957	ABEL, FRANCES E., Actuarial Division, National Bureau of Casualty Underwriters, 60 John Street, New York 38, N. Y.
Nov. 15, 1918	ACKERMAN, SAUL B., 250 West 57th Street, New York 19, N. Y.
Nov. 16, 1939	AIN, SAMUEL N., Consulting Actuary, 120 Broadway, New York 5 N. Y.
Nov. 22, 1957	ALEXANDER, LEE M., Casualty, Fire & Marine Actuarial Department, The Travelers Insurance Company, Hartford, 15 Conn.
Apr. 5, 1928	ALLEN, AUSTIN F., Chairman of the Board, Texas Employers' Insurance Association, P. O. Box 2759, Dallas 21, Texas.
Nov. 18, 1955	ANDREWS, EDWARD C., Associate Actuary, Casualty, Fire & Marine Actuarial Department, The Travelers Insurance Company, Hartford 15, Conn.
Nov. 15, 1918	ANKERS, ROBERT E., 414 East Broad Street, Falls Church, Va.
Nov. 21, 1930	ARCHIBALD, A. EDWARD, Director, Management Controls, Investors Diversified Services, Inc., Minneapolis 2, Minn.
Nov. 24, 1933	BARRON, JAMES C., 220 Mountain Road, Pleasantville, N. Y.
Nov. 23, 1928	BATEMAN, ARTHUR E., Pine Grove Rest Home, Marlboro, Mass.
Nov. 15, 1940	BATHO, BRUCE, Vice President and Comptroller, Life Insurance Company of Georgia, 573 West Peachtree Street, N.E., Atlanta 8, Ga.
Nov. 16, 1956	BERG, ROY A., JR., Assistant Actuary, Old Republic Life Insurance Co., 307 No. Michigan Avenue, Chicago 1, Ill.
Nov. 18, 1925	BITTEL, W. HAROLD, Chief Actuary, Department of Banking and Insurance, Trenton 7, N. J.
Nov. 17, 1920	BLACK, NELLAS C., (Retired), 4310 Norwood Road, Baltimore 18, Md.
Nov. 22, 1934	BOMSE, EDWARD L., Assistant Manager, Foreign Department, Royal-Liverpool Insurance Group, 150 William Street, New York 38, N. Y.
Nov. 23, 1928	BOWER, PERRY S., Assistant General Manager and Treasurer, The Great-West Life Assurance Company, 177 Lombard Street, Winnipeg, Manitoba, Canada.
Nov. 22, 1957	BOYLE, JAMES I., Casualty, Fire and Marine Actuarial Department, The Travelers Insurance Company, Hartford 15, Conn.
Nov. 22, 1957	BRAGG, JOHN M., Actuary, Life Insurance Company of Georgia, 573 West Peachtree Street, N. E., Atlanta 8, Ga.
Nov. 15, 1918	BRUNNQUELL, HELMUTH G., (Retired), 1013 East Circle Drive, Milwaukee 17, Wis.
Oct. 22, 1915	BUFFLER, LOUIS, Underwriting Director, The State Insurance Fund, 199 Church Street, New York 7, N. Y.
Nov. 20, 1924	BUGBEE, JAMES M., Vice President, Maryland Casualty Company, Box 1228, Baltimore 3, Md.
Mar. 31, 1920	BURT, MARGARET A., Office of George B. Buck, Consulting Actuary, 150 Nassau Street, New York 38, N. Y.

ASSOCIATES

Admitted	
Nov. 22, 1957	BYRNE, HARRY T., Casualty Statistical Department, Aetna Casualty and Surety Company, 151 Farmington Avenue, Hartford, Conn.
Nov. 17, 1922	CAVANAUGH, LEO D., Chairman, Federal Life Insurance Company, 6100 No. Cicero Avenue, Chicago 30, Ill.
Nov. 18, 1927	CHEN, S. T., Consulting Actuary, The Wing On Life Assurance Co., Ltd., Wing On Life Building, 22 Des Voeux Road, Central, Hong Kong.
Nov. 22, 1957	CHURCH, HARRY M., Coates, Herfurth & England, Consulting Actuaries, 325 North Lake, Pasadena, Calif.
Nov. 18, 1955	COATES, WILLIAM D., Assistant Actuary, Accident and Health Department, Continental Casualty Company, 310 S. Michigan Avenue, Chicago 4, Ill.
Nov. 19, 1953	CONTE, JOSEPH P., Secretary-Treasurer, Columbian Mutual Life Insurance Co., 305 Main Street, Binghamton, N. Y.
Nov. 24, 1933	CRAWFORD, WILLIAM H., Vice President and Treasurer, Industrial Indemnity Company, 155 Sansome Street, San Francisco 4, Calif.
Nov. 18, 1932	CRIMMINS, JOSEPH B., Associate Actuary, Metropolitan Life Insurance Co., 1 Madison Avenue, New York 10, N. Y.
Nov. 19, 1953	CROFTS, GEOFFREY, Associate Professor of Actuarial Science, Occidental College, Los Angeles 41, Calif.
Nov. 21, 1952	DANIEL, C. M., Applied Service Representative, International Business Machines Corp., 2116 Grand, Des Moines 12, Iowa.
Nov. 18, 1925	DAVIS, MALVIN E., Vice-President and Chief Actuary, Metropolitan Life Insurance Co., 1 Madison Avenue, New York 10, N. Y.
Nov. 16, 1956	DORF, STANLEY, Actuarial Department, Royal-Liverpool Insurance Group, 150 William Street, New York 38, N. Y.
Nov. 14, 1941	DOWLING, WILLIAM F., President, New York Mutual Casualty Insurance Co., 260 Fourth Avenue, New York 10, N. Y.
Nov. 16, 1956	DROPKIN, LESTER B., Associate Actuary, New York State Insurance Department, 123 William Street, New York 38, N. Y.
Nov. 19, 1954	EATON, KARL F., Electronics Analyst, Business Men's Assurance Company, 215 Pershing Road, Kansas City 41, Mo.
June 5, 1925	EGER, FRANK A., Secretary-Comptroller, Indemnity Insurance Co. of North America, 1600 Arch Street, Philadelphia 1, Pa.
Nov. 19, 1954	EIDE, K. ARNE, Statistical Bureau, Actuarial Division, Metropolitan Life Insurance Company, 1 Madison Avenue, New York 10, N. Y.
Nov. 16, 1956	FAUST, J. EDWARD, JR., Group and Casualty Actuary, Nelson & Warren Inc., Consulting Actuaries, 111 S. Bemiston, St. Louis, Mo.
Nov. 22, 1957	FELDMAN, MARTIN F., Senior Actuary, New York State Insurance Department, 123 William Street, New York 38, N. Y.
Nov. 16, 1956	FLACK, PAUL R., Actuarial Assistant, General Accident Fire & Life Assurance Corp. Ltd., 414 Walnut St., Philadelphia 5, Pa.

ASSOCIATES

Admitted	
Nov. 16, 1923	FLEMING, FRANK A., (Retired), c/o Mutual Insurance Rating Bureau, 111 Fourth Ave., New York 3, N. Y.
Nov. 21, 1952	FRANKLIN, NATHAN M., Actuary, The Surety Association of America, 60 John Street, New York 38, N. Y.
Nov. 19, 1929	FURNIVALL, MAURICE L., Associate Actuary, The Travelers Insurance Company, Hartford, Conn.
Nov. 19, 1954	GAINES, NATHANIEL, Actuary, Pension Planning Company, 260 Madison Avenue, New York 16, N. Y.
Nov. 18, 1932	GETMAN, RICHARD A., Assistant Actuary, Life Department, The Travelers Insurance Co., 700 Main St., Hartford 15, Conn.
Nov. 17, 1922	GIBSON, JOSEPH P., JR., President, American Mutual Reinsurance Co., 919 North Michigan Ave., Chicago 11, Ill.
Nov. 16, 1923	GILDEA, JAMES F., (Retired), 17 Allen Place, Hartford, Conn.
Nov. 14, 1947	GINGERY, STANLEY W., Associate Actuary, The Prudential Insurance Co., Newark, N. J.
Nov. 18, 1927	GREEN, WALTER C., Consulting Actuary, 455 East 4th South, Salt Lake City 11, Utah.
Nov. 15, 1940	GROSSMAN, ELI A., Assistant Vice President and Associate Actuary, Beneficial Standard Life Insurance Company, 756 South Spring Street, Los Angeles 14, Calif.
Nov. 15, 1935	GUERTIN, ALFRED N., Actuary, American Life Convention, 230 N. Michigan Avenue, Chicago 1, Ill.
Nov. 16, 1939	HAGEN, OLAF E., Metropolitan Life Insurance Company, 1 Madison Avenue, New York 10, N. Y.
Nov. 18, 1921	HAGGARD, ROBERT E., (Retired), 922 The Alameda, Berkeley 7, Calif.
Nov. 17, 1922	HALL, HARTWELL L., Chief Examiner, Connecticut Insurance Department, 165 Capitol Avenue, Hartford 2, Conn.
Nov. 13, 1936	HAM, HUGH P., General Manager, The British America Assurance Company, 40 Scott Street, Toronto 1, Ontario, Canada.
Nov. 19, 1953	HARACK, JOHN, Manager, Technical Assistance Division, Blue Cross Commission, 425 North Michigan, Chicago 11, Ill.
Mar. 24, 1932	HARRIS, SCOTT, Executive Vice-President, Joseph Froggatt & Co., Inc., 74 Trinity Place, New York 6, N. Y.
Mar. 25, 1924	HART, WARD VAN B., Associate Actuary, Connecticut General Life Insurance Company, 55 Elm Street, Hartford 15, Conn.
Nov. 21, 1919	HAYDON, GEORGE F., Manager Emeritus, Wisconsin Compensation Rating Bureau, 623 North 2nd Street, Milwaukee 3, Wis.
Nov. 19, 1953	HEAD, GLENN O., Vice President and Actuary, The United States Life Insurance Co., 84 William Street, New York 38, N. Y.
Nov. 17, 1927	HIPP, GRADY H., Underwriting Vice President, Liberty Life Insurance Company, Wade Hampton Blvd., Greenville, South Carolina.
Nov. 22, 1957	HOUSTON, DAVID B., Acting Assistant Professor of Insurance, University of California, School of Business Administration, Los Angeles 24, Calif.
Nov. 22, 1957	HUNT, FREDERIC J., JR., Assistant Actuary, Insurance Company of North America, 1600 Arch Street, Philadelphia 1, Pa.

ASSOCIATES

Admitted	
Nov. 19, 1929	JACOBS, CARL N., President, Hardware Mutual Casualty Co. and Hardware Dealers Mutual Fire Insurance Co., 200 Strong's Avenue, Stevens Point, Wis.
Nov. 18, 1921	JENSEN, EDWARD S., Assistant Vice-President, Group Department, Occidental Life Insurance Co. of California, 1151 So. Broadway, Los Angeles 55, Calif.
Nov. 21, 1930	JONES, H. LLOYD, (Retired), 9 Midland Gardens, Bronxville, N. Y.
Nov. 21, 1919	JONES, LORING D., (Retired), 64 Raymond Avenue, Rockville Centre, Long Island, N. Y.
Nov. 21, 1952	JONES, NATHAN F., Associate Actuary, The Prudential Insurance Company of America, Newark, N. J.
Nov. 17, 1922	KIRK, CARL L., Deputy U.S. Manager, Zurich Insurance Co., 135 South LaSalle Street, Chicago 3, Ill.
Nov. 16, 1956	KLAASSEN, ELDON J., Assistant Actuary, Continental Casualty Company, Chicago, Ill.
Nov. 14, 1947	LUFKIN, ROBERT W., Office Manager, Craftsman Insurance Co., 137 Newbury St., Boston, Mass.
Mar. 24, 1932	MAGRATH, JOSEPH J., Secretary, Federal Insurance Company, 90 John Street, New York 38, N. Y.
Nov. 18, 1925	MALMUTH, JACOB, Principal Examiner, New York State Insurance Department, 123 William Street, New York 38, N. Y.
Mar. 24, 1927	MARSH, CHARLES VAN R., (Retired), Fidelity and Deposit Company, Charles & Lexington Streets, Baltimore, Md.
Nov. 16, 1956	MATHWICK, L. F., Group Rate Analyst, Employers' Mutual Liability Insurance Co. of Wisconsin, 407 Grant St., Wausau, Wis.
Nov. 13, 1936	MAYER, WILLIAM H., JR., Associate Manager, Group Contract Bureau, Metropolitan Life Insurance Co., 1 Madison Avenue, New York 10, N. Y.
Nov. 17, 1950	MAYERSON, ALLEN L., Assistant Professor of Mathematics and Insurance, University of Michigan, Ann Arbor, Mich.
May 26, 1955	MCDONALD, MILTON G., Casualty Actuary, Department of Banking and Insurance, 100 Nashua Street, Boston 14, Mass.
Nov. 17, 1922	McIVER, R. A., Actuary, Washington National Insurance Co., 1630 Chicago Avenue, Evanston, Ill.
Nov. 13, 1931	MILLER, HENRY C., Comptroller, California State Compensation Insurance Fund, 450 McAllister Street, San Francisco 1, Calif.
Nov. 18, 1937	MINOR, EDUARD H., Assistant Actuary, Metropolitan Life Insurance Co., 1 Madison Avenue, New York 10, N. Y.
Nov. 17, 1922	MONTGOMERY, JOHN C., (Retired), 165 Westervelt Ave., Tenafly, N. J.
May 25, 1923	MOORE, JOSEPH P., 115 St. Catherine Road, Outremont, Quebec, Canada.
Nov. 22, 1957	MUIR, JOSEPH M., General Manager, Mutual Insurance Rating Bureau, and Acting General Manager, Insurance Advisory Association, 111 Fourth Avenue, New York 3, N. Y.
Nov. 18, 1937	MYERS, ROBERT J., Chief Actuary, Social Security Administration, Washington 25, D.C.

ASSOCIATES

Admitted	
Nov. 15, 1935	NELSON, S. TYLER, Manager, Casualty Division, American Agricultural Mutual Insurance Company, Room 2300 Merchandise Mart, Chicago 54, Ill.
Oct. 27, 1916	NEWELL, WILLIAM, (Retired), 1225 Park Avenue, New York 28, N. Y.
Nov. 18, 1925	NICHOLSON, EARL, Actuary, Joseph Froggatt & Co., Inc., 74 Trinity Place, New York 6, N. Y.
Nov. 22, 1957	NILES, CHARLES L., JR., Actuarial Assistant, American Mutual Liability Insurance Co., 142 Berkeley Street, Boston 17, Mass.
May 23, 1919	OTTO, WALTER E., President, Michigan Mutual Liability Co., 28 West Adams Avenue, Detroit 26, Mich.
Nov. 19, 1926	OVERHOLSER, DONALD M., Office of George B. Buck, Consulting Actuary, 150 Nassau Street, New York 7, N. Y.
Nov. 20, 1924	PENNOCK, RICHARD M., (Retired), 12 E. Lodges Lane, Cynwyd, Pa.
Nov. 21, 1952	PENNYCOOK, RODERICK B., Assistant to the Executive Director, Manitoba Hospital Service Association, 116 Edmonton Street, Winnipeg, Man., Canada.
Nov. 14, 1947	PERRY, ROBERT C., First Vice-President, State Farm Life Insurance Company, Bloomington, Ill.
Nov. 16, 1956	PHILLIPS, HERBERT J., JR., Actuarial Assistant, Employers' Liability Assurance Corp. Ltd., 110 Milk Street, Boston 7, Mass.
Nov. 19, 1929	PHILLIPS, JOHN H., Vice-President and Actuary, Employers' Mutual Liability Insurance Co., and Employers' Mutual Fire Insurance Company, 407 Grant Street, Wausau, Wis.
Nov. 17, 1920	PIKE, MORRIS, Vice President, John Hancock Mutual Life Insurance Company, Boston 17, Mass.
Nov. 23, 1928	PIPER, K. B., Vice-President, Provident Life and Accident Insurance Co., 721 Broad Street, Chattanooga 2, Tenn.
Nov. 17, 1922	POORMAN, WILLIAM F., President, Central Life Assurance Company, 611 Fifth Avenue, Des Moines 6, Iowa.
Nov 13, 1936	POTOFSKY, SYLVIA, Senior Actuary, The State Insurance Fund, 199 Church Street, New York, N. Y.
Nov. 15, 1918	RAYWID, JOSEPH, Woodward and Fondiller, Inc., 200 West 57th Street, New York 19, N. Y.
Nov. 19, 1932	RICHARDSON, HARRY F., (Retired), Seven Oaks, Bozman, Maryland.
Nov. 19, 1953	RICHMOND, OWEN D., Department Head, IBM Department, Business Men's Assurance, 215 Pershing Road, Kansas City, Mo.
Nov. 18, 1932	ROBERTS, JAMES A., Group Statistician, The Travelers Insurance Company, Hartford, 15 Conn.
Nov. 16, 1956	ROBERTS, LEWIS H., Actuarial Trainee, National Bureau of Casualty Underwriters, 60 John Street, New York 38, N. Y.
Nov. 18, 1927	SARASON, HARRY M., Consulting Actuary, 1060 South Broadway, Los Angeles 15, Calif.
Nov. 16, 1923	SAWYER, ARTHUR, (Retired), 217 San Antonio West, San Clemente, Calif.

ASSOCIATES

Admitted	
Nov. 14, 1947	SCAMMON, LAWRENCE W., Manager, Massachusetts Automobile Rating & Accident Prevention Bureau, Massachusetts Workmen's Compensation Rating & Inspection Bureau and Massachusetts Motor Vehicle Assigned Risk Plan, 89 Broad Street, Boston, Mass.
Nov. 22, 1957	SCHNEIKER, HENRY C., Associate Statistician, Mutual Insurance Rating Bureau, 111 Fourth Avenue, New York 3, N. Y.
Nov. 19, 1954	SCHULMAN, JUSTIN, Statistical Department, Greater New York Mutual Insurance Co., 111 Fourth Avenue, New York 3, N. Y.
Nov. 14, 1947	SCHWARTZ, MAX J., Principal Actuary, New York State Insurance Department, Albany 10, N. Y.
Nov. 20, 1930	SEVILLA, EZEQUIEL S., President, Manager and Actuary, National Life Insurance Co. of the Philippines, Regina Building, P.O. Box 2056, Manila, Philippines.
Nov. 22, 1957	SHAVER, C. OTIS, Actuary, Nationwide Mutual Fire Insurance Company, 246 North High Street, Columbus 16, Ohio.
Nov. 20, 1924	SHEPPARD, NORRIS E., Professor of Mathematics, University of Toronto, Toronto 5, Canada.
Nov. 16, 1956	SMITH, EDWARD M., Casualty, Fire & Marine Actuarial Department, The Travelers Insurance Co., Hartford 15, Conn.
Nov. 19, 1926	SOMERVILLE, WILLIAM F., (Retired), 648 Sibley Highway, St. Paul 18, Minn.
Nov. 18, 1925	SOMMER, ARMAND, Vice President, Continental Casualty Co., Transportation Insurance Co., and United States Life Insurance Co., 310 So. Michigan Avenue, Chicago 4, Ill.
Nov. 15, 1918	SPENCER, HAROLD S., (Retired), 8 Chelsea Lane, West Hartford, Conn.
Nov. 20, 1924	STELLWAGEN, H. P., Executive Vice-President, Indemnity Insurance Company of North America, 1600 Arch Street, Philadelphia 1, Pa.
Nov. 16, 1956	STERN, PHILIPP K., Actuary, Mutual Insurance Rating Bureau, 111 Fourth Avenue, New York 3, N. Y.
Nov. 16, 1923	STOKE, KENDRICK, Actuary, Michigan Mutual Liability Company, 28 W. Adams, Detroit 26, Mich.
Nov. 21, 1930	SULLIVAN, WALTER F., Actuary, State Compensation Insurance Fund, 450 McAllister Street, San Francisco 1, Calif.
Nov. 16, 1956	TARBELL, LUTHER L., JR., Assistant Actuary, Casualty, Fire & Marine Actuarial Department, The Travelers Insurance Company, Hartford 15, Conn.
Nov. 21, 1919	TRENCH, FREDERICK H., Budget Director, Utica Mutual Insurance Co., Utica 1, N. Y.
Nov. 20, 1924	UHL, M. ELIZABETH, National Bureau of Casualty Underwriters, 60 John Street, New York 38, N. Y.
Nov. 18, 1932	WEINSTEIN, MAX S., Actuary, New York State Employees' Retirement System, 90 South Swan Street, Albany 1, N. Y.
Nov. 18, 1925	WELLMAN, ALEXANDER C., Senior Vice-President, Protective Life Insurance Co., Birmingham, Ala.

ASSOCIATES

Admitted	
Nov. 21, 1930	WELLS, WALTER I., Secretary, Sickness & Accident Division, State Mutual Life Assurance Company, 440 Lincoln Street, Worcester, Mass.
Nov. 16, 1951	WERMEL, MICHAEL T., Vice President, Woodward and Fondiller, Inc., 417 South Hill Street, Los Angeles 13, Calif.
Nov. 18, 1927	WHITBREAD, F. G., Assistant Vice-President, Lincoln National Life Insurance Company, 1301-27 S. Harrison Street, Fort Wayne, Ind.
Nov. 19, 1948	WHITE, AUBREY, Vice President and Actuary, Ostheimer & Co., 1510 Chestnut St., Philadelphia 2, Pa.
Nov. 22, 1957	WILCKEN, CARL L., Casualty, Fire and Marine Actuarial Department, The Travelers Insurance Company, Hartford 15, Conn.
Nov. 19, 1954	WILLIAMS, DEWEY G., Assistant Actuary, Texas Employers' Insurance Association, Dallas 1, Texas.
Nov. 18, 1955	WILSON, JAMES C., Actuary, Wolverine Insurance Co., Battle Creek, Mich.
Nov. 16, 1939	WITTLAKE, J. CLARKE, Vice President, Business Men's Assurance Co., B.M.A. Bldg., Kansas City 10, Mo.
Oct. 22, 1915	WOOD, DONALD M., Partner, Childs & Wood, 175 W. Jackson Blvd., Chicago 4, Ill.
Nov. 18, 1937	WOOD, DONALD M. JR., Partner, Childs & Wood, 175 West Jackson Blvd., Chicago 4, Ill.
Nov. 18, 1927	WOOD, MILTON J., Vice-President and Actuary, Life, Accident and Group Actuarial Department, The Travelers Insurance Co., 700 Main Street, Hartford 15, Conn.
Nov. 17, 1950	WOODY, JOHN C., Associate Actuary, North American Reassurance Co., 161 East 42nd Street, New York 17, N. Y.
Nov. 22, 1934	WOODWARD, BARBARA H., Regional General Counsel, The Reuben H. Donnelley Corporation, 305 East 45th Street, New York 17, N. Y.
Nov. 16, 1956	WOODWORTH, JAMES H., Superintendent, Rating Division of Actuarial Department, Hartford Accident and Indemnity Co., 690 Asylum Avenue, Hartford 15, Conn.
Nov. 18, 1925	WOOLERY, JAMES MYRON, Vice-President and Actuary, Occidental Life Insurance Company, Raleigh, N. C.
Nov. 19, 1954	WRIGHT, BYRON, Actuary, Department Banking and Insurance, State of New Jersey, State House Annex, Trenton 7, N. J.

OFFICERS OF THE SOCIETY

Since Date of Organization

<i>Elected</i>	<i>President</i>	<i>Vice-Presidents</i>	
1914-1915	*Isaac M. Rubinow	*Albert H. Mowbray	*Benedict D. Flynn
1916-1917	*James D. Craig	*Joseph H. Woodward	*Harwood E. Ryan
1918	*Joseph H. Woodward	*Benedict D. Flynn	George D. Moore
1919	*Benedict D. Flynn	George D. Moore	William Leslie
1920	*Albert H. Mowbray	William Leslie	*Leon S. Senior
1921	*Albert H. Mowbray	*Leon S. Senior	*Harwood E. Ryan
1922	*Harwood E. Ryan	Gustav F. Michelbacher	Edmund E. Cammack
1923	William Leslie	Gustav F. Michelbacher	Edmund E. Cammack
1924-1925	Gustav F. Michelbacher	*Sanford B. Perkins	Ralph H. Blanchard
1926-1927	*Sanford B. Perkins	George D. Moore	Thomas F. Tarbell
1928-1929	George D. Moore	Sydney D. Pinney	Paul Dorweiler
1930-1931	Thomas F. Tarbell	*Roy A. Wheeler	Winfield W. Greene
1932-1933	Paul Dorweiler	William F. Roeber	*Leon S. Senior
1934-1935	Winfield W. Greene	Ralph H. Blanchard	Charles J. Haugh
1936-1937	*Leon S. Senior	Sydney D. Pinney	Francis S. Perryman
1938-1939	Francis S. Perryman	Harmon T. Barber	William J. Constable
1940	Sydney D. Pinney	Harold J. Ginsburgh	James M. Cahill
1941	Ralph H. Blanchard	Harold J. Ginsburgh	James M. Cahill
1942	Ralph H. Blanchard	Albert Z. Skelding	Charles J. Haugh
1943-1944	Harold J. Ginsburgh	Albert Z. Skelding	Charles J. Haugh
1945-1946	Charles J. Haugh	James M. Cahill	Harry V. Williams
1947-1948	James M. Cahill	Harmon T. Barber	Russell P. Goddard
1949-1950	Harmon T. Barber	Thomas O. Carlson	Norton E. Masterson
1951-1952	Thomas O. Carlson	Joseph Linder	Seymour E. Smith
1953-1954	Seymour E. Smith	Dudley M. Pruitt	John A. Mills
1955-1956	Norton E. Masterson	*Clarence A. Kulp	Arthur N. Matthews
1957	Dudley M. Pruitt	John W. Carleton	William Leslie, Jr.

Secretary-Treasurer

1914-1917	*C. E. Scattergood
1918-1953	R. Fondiller
1954-1957	A. Z. Skelding

Editor†

1914	W. W. Greene
1915-1917	R. Fondiller
1918	W. W. Greene
1919-1921	G. F. Michelbacher
1922-1923	O. E. Outwater
1924-1932	R. J. McManus
1933-1943	*C. W. Hobbs
1944-1954	E. C. Maycrink
1955-1957	E. S. Allen

*General Chairman**Examination Committee*

1949-1951	R. A. Johnson
1952-1956	J. W. Wieder, Jr.
1957	W. J. Hazam

Librarian†

1914	W. W. Greene
1915	R. Fondiller
1916-1921	L. I. Dublin
1922-1924	*E. R. Hardy
1925-1936	W. Breiby
1937-1947	T. O. Carlson
1948-1950	*S. M. Ross
1951-1957	G. R. Livingston

*Deceased.

†The offices of Editor and Librarian were not separated until 1916.

FELLOWS WHO HAVE DIED

The (†) denotes charter members at date of organization, November 7, 1914.

Admitted		Died
Nov. 19, 1948	Arthur L. Bailey	Aug. 12, 1954
May 23, 1924	William B. Bailey	Jan. 10, 1952
†	Roland Benjamin	July 2, 1949
May 24, 1921	Edward J. Bond	Nov. 12, 1941
May 19, 1915	Thomas Bradshaw	Nov. 10, 1939
June 5, 1925	William Brosmith	Aug. 22, 1937
†	William A. Budlong	June 4, 1934
Nov. 18, 1932	Charles H. Burhans	June 15, 1942
Feb. 19, 1915	F. Highlands Burns	Mar. 30, 1935
†	Raymond V. Carpenter	Mar. 11, 1947
Feb. 19, 1915	Gorden Case	Feb. 4, 1920
Oct. 27, 1916	Edmund S. Cogswell	Apr. 25, 1957
Nov. 23, 1928	Walter P. Comstock	May 11, 1951
†	Charles T. Conway	July 23, 1921
†	John A. Copeland	June 12, 1953
†	Walter G. Cowles	May 30, 1942
†	James D. Craig	May 27, 1940
†	James McIntosh Craig	Jan. 20, 1922
May 26, 1916	Frederick S. Crum	Sept. 2, 1921
†	Alfred Burnett Dawson	June 21, 1931
†	Miles Menander Dawson	Mar. 27, 1942
†	Elmer H. Dearth	Mar. 26, 1947
†	Eckford C. DeKay	July 31, 1951
May 19, 1915	Samuel Deutschberger	Jan. 18, 1929
†	Ezekiel Hinton Downey	July 9, 1922
May 19, 1915	Earl O. Dunlap	July 5, 1944
†	Edward B. Fackler	Jan. 8, 1952
†	David Parks Fackler	Oct. 30, 1924
Feb. 19, 1915	Claude W. Fellows	July 15, 1938
†	Benedict D. Flynn	Aug. 22, 1944
†	Charles S. Forbes	Oct. 2, 1943
May 26, 1916	Lee K. Frankel	July 25, 1931
†	Charles H. Franklin	May 1951
Feb. 25, 1916	Joseph Froggatt	Sept. 28, 1940
†	Harry Furze	Dec. 26, 1945
Feb. 19, 1915	Fred S. Garrison	Nov. 14, 1949
†	Theodore E. Gaty	Aug. 22, 1925
May 19, 1915	James W. Glover	July 15, 1941
Oct. 22, 1915	George Graham	Apr. 15, 1937
Oct. 22, 1915	Thompson B. Graham	July 24, 1946
May 25, 1923	William A. Granville	Feb. 4, 1943
†	William H. Gould	Oct. 28, 1936
†	Robert Cowen Lees Hamilton	Nov. 15, 1941
Oct. 27, 1916	Edward R. Hardy	June 29, 1951
Nov. 21, 1919	Robert Henderson	Feb. 16, 1942
†	Robert J. Hillas	May 17, 1940
Nov. 15, 1918	Frank Webster Hinsdale	Mar. 18, 1932
May 23, 1924	Clarence W. Hobbs	July 21, 1944
Nov. 19, 1926	Charles E. Hodges	Jan. 22, 1937
Oct. 22, 1915	Lemuel G. Hodgkins	Dec. 26, 1951
†	Frederick L. Hoffman	Feb. 23, 1946
Oct. 22, 1915	Charles H. Holland	Dec. 28, 1951

FELLOWS WHO HAVE DIED—*Continued*

Admitted		Died
Nov. 21, 1919	Carl Hookstadt	Mar. 10, 1924
†	Charles Hughes	Aug. 27, 1948
Nov. 19, 1929	Robert S. Hull	Nov. 30, 1947
†	Burritt A. Hunt	Sept. 3, 1943
Nov. 28, 1921	William Anderson Hutcheson	Nov. 19, 1942
Nov. 19, 1929	Henry Hollister Jackson	May 27, 1955
May 19, 1915	William C. Johnson	Oct. 7, 1943
Nov. 23, 1928	F. Robertson Jones	Dec. 26, 1941
Nov. 18, 1921	Thomas P. Kearney	Feb. 11, 1928
Nov. 19, 1926	Gregory Cook Kelly	Sept. 11, 1948
Oct. 22, 1915	Virgil Morrison Kime	Oct. 15, 1918
†	Edwin W. Kopf	Aug. 3, 1933
Nov. 23, 1928	Clarence Arthur Kulp	Aug. 20, 1957
Feb. 17, 1915	John M. Laird	June 20, 1942
Feb. 19, 1915	Abb Landis	Dec. 9, 1937
Nov. 24, 1933	John Robert Lange	Apr. 12, 1957
Nov. 17, 1922	Arnette Roy Lawrence	Dec. 1, 1942
Nov. 18, 1921	James Fulton Little	Aug. 11, 1938
Nov. 23, 1928	Edward C. Lunt	Jan. 13, 1941
Feb. 19, 1915	Harry Lubin	Dec. 20, 1920
Nov. 16, 1923	D. Ralph McClurg	Apr. 27, 1947
May 23, 1919	Alfred McDougald	July 28, 1944
†	William N. Magoun	Dec. 11, 1954
Feb. 15, 1915	Franklin B. Mead	Nov. 29, 1933
Apr. 20, 1917	Marcus Meltzer	Mar. 27, 1931
†	David W. Miller	Jan. 18, 1936
†	James F. Mitchell	Feb. 9, 1941
†	Henry Moir	June 8, 1937
Nov. 19, 1926	William L. Mooney	Oct. 21, 1948
Feb. 19, 1915	William J. Montgomery	Aug. 20, 1915
May 19, 1915	Edward Bontecou Morris	Dec. 19, 1929
†	Albert H. Mowbray	Jan. 7, 1949
†	Frank Mullaney	Jan. 22, 1953
†	Lewis A. Nicholas	Apr. 21, 1940
†	Stanley L. Otis	Oct. 12, 1937
Nov. 13, 1926	Bertrand A. Page	July 30, 1941
Nov. 18, 1921	Sanford B. Perkins	Sept. 16, 1945
Nov. 15, 1918	William Thomas Perry	Oct. 25, 1940
Nov. 19, 1926	Jesse S. Phillips	Nov. 6, 1954
†	Edward B. Phelps	July 24, 1915
†	Charles Grant Reiter	July 30, 1937
†	Charles H. Remington	Mar. 21, 1938
May 23, 1919	Frederick Richardson	July 22, 1955
Nov. 17, 1943	Samuel M. Ross	July 24, 1951
†	Isaac M. Rubinow	Sept. 1, 1936
†	Harwood Eldridge Ryan	Nov. 2, 1930
†	Arthur F. Saxton	Feb. 26, 1927
†	Emil Scheitlin	May 2, 1946
†	Leon S. Senior	Feb. 3, 1940
Nov. 24, 1933	Robert V. Sinnott	Dec. 15, 1952
April 20, 1917	Charles Gordon Smith	June 22, 1938
Feb. 19, 1915	John T. Stone	May 9, 1920
Feb. 25, 1916	Wendell Melville Strong	Mar. 30, 1942
Oct. 22, 1915	William R. Strong	Jan. 10, 1946
†	Robert J. Sullivan	July 19, 1934

FELLOWS WHO HAVE DIED—*Continued*

<i>Admitted</i>		<i>Died</i>
Nov. 22, 1934	Walter H. Thompson	May 25, 1935
Nov. 18, 1921	Guido Toja	Feb. 28, 1933
Nov. 15, 1935	Harry V. Waite	Aug. 14, 1951
Nov. 18, 1925	Lloyd A. H. Warren	Sept. 30, 1949
May 23, 1919	Archibald A. Welch	May 8, 1945
Nov. 19, 1926	Roy A. Wheeler	Aug. 26, 1932
†	Albert W. Whitney	July 27, 1943
†	Lee J. Wolfe	Apr. 28, 1949
†	S. Herbert Wolfe	Dec. 31, 1927
May 24, 1921	Arthur B. Wood	June 14, 1952
†	Joseph H. Woodward	May 15, 1928
†	William Young	Oct. 23, 1927

ASSOCIATES WHO HAVE DIED

<i>Admitted</i>		<i>Died</i>
May 23, 1924	Milton Acker	Aug. 16, 1956
Oct. 22, 1915	Don A. Baxter	Feb. 10, 1920
Nov. 15, 1940	John M. Blackhall	Nov. 14, 1957
May 25, 1923	Harilaus E. Economidy	Apr. 13, 1948
Nov. 20, 1924	John Froberg	Oct. 11, 1949
Nov. 22, 1934	John J. Gately	Nov. 3, 1943
Nov. 14, 1947	Harold J. George	Apr. 1, 1952
Nov. 19, 1929	Harold R. Gordon	July 8, 1948
Nov. 20, 1924	Leslie LeVant Hall	Mar. 8, 1931
Oct. 31, 1917	Edward T. Jackson	May 8, 1939
Nov. 21, 1919	Rolland V. Mothersill	July 25, 1949
Nov. 19, 1929	Fritz Muller	Apr. 27, 1945
Nov. 23, 1928	Karl Newhall	Oct. 24, 1944
Nov. 15, 1918	John L. Sibley	Mar. 10, 1957
Nov. 18, 1921	Arthur G. Smith	May 2, 1956
Nov. 18, 1927	Alexander A. Speers	June 25, 1941
Mar. 23, 1921	Arthur E. Thompson	Jan. 17, 1944
Nov. 21, 1919	Walter G. Voogt	May 8, 1945
May 23, 1919	Charles S. Warren	May 1, 1952
Nov. 18, 1925	James H. Washburn	Aug. 19, 1946
Nov. 17, 1920	James J. Watson	Feb. 23, 1937
Nov. 18, 1921	Eugene R. Welch	Jan. 17, 1945
Mar. 21, 1929	Charles A. Wheeler	July 2, 1956
Nov. 15, 1918	Albert Edward Wilkinson	June 11, 1930
Oct. 22, 1915	Charles E. Woodman	Dec. 16, 1955

SCHEDULE OF MEMBERSHIP, NOVEMBER 22, 1957

	Fellows	Associates	Total
Membership, November 16, 1956	181	148	329
Additions:			
By Election
By Reinstatement
By Examination	11	14	25
	192	162	354
Deductions:			
By Death	3	2	5
By Withdrawal	3	3	6
By Transfer from Associate to Fellow	11	11
	186	146	332

CONSTITUTION

(AS AMENDED NOVEMBER 17, 1950)

ARTICLE I.—*Name.*

This organization shall be called the CASUALTY ACTUARIAL SOCIETY.

ARTICLE II.—*Object.*

The object of the Society shall be the promotion of actuarial and statistical science as applied to the problems of insurance, other than life insurance, by means of personal intercourse, the presentation and discussion of appropriate papers, the collection of a library and such other means as may be found desirable.

The Society shall take no partisan attitude, by resolution or otherwise, upon any question relating to insurance.

ARTICLE III.—*Membership.*

The membership of the Society shall be composed of two classes, Fellows and Associates. Fellows only shall be eligible to office or have the right to vote.

The Fellows of the Society shall be the present Fellows and those who may be duly admitted to Fellowship as hereinafter provided. The Associates shall be the present Associates and those who may be duly admitted to Associateship as hereinafter provided.

Any person may, upon nomination to the Council by two Fellows of the Society and approval by the Council of such nomination with not more than one negative vote, become enrolled as an Associate of the Society, provided that he shall pass such examination as the Council may prescribe. Such examination may be waived in the case of a candidate who for a period of not less than two years has been in responsible charge of the Statistical or Actuarial Department of an insurance organization (other than life insurance) or has had such other practical experience in insurance (other than life insurance) as, in the opinion of the Council, renders him qualified for Associateship.

Any person who shall have qualified for Associateship may become a Fellow on passing such final examination as the Council may prescribe. Otherwise, no one shall be admitted as a Fellow unless recommended by a duly called meeting of the Council with not more than three negative votes, followed by a three-fourths ballot of the Fellows present and voting at a meeting of the Society.

ARTICLE IV.—*Officers and Council.*

The officers of the Society shall be a President, two Vice-Presidents, a Secretary-Treasurer, an Editor, a Librarian, and a General Chairman of the Examination Committee. The Council shall be composed of the active officers, nine other Fellows and, during the four years following the expiration of their terms of office, the ex-Presidents and ex-Vice-Presidents. The Council shall fill vacancies occasioned by death or resignation of any officer or other member of the Council, such appointees to serve until the next annual meeting of the Society.

ARTICLE V.—*Election of Officers and Council.*

The President, Vice-Presidents, and the Secretary-Treasurer shall be elected by a majority ballot at the annual meeting for the term of one year and three

members of the Council shall, in a similar manner, be annually elected to serve for three years. The President and Vice-Presidents shall not be eligible for the same office for more than two consecutive years nor shall any retiring member of the Council be eligible for re-election at the same meeting.

The Editor, the Librarian and the General Chairman of the Examination Committee shall be elected annually by the Council at the Council meeting preceding the annual meeting of the Society. They shall be subject to confirmation by majority ballot of the Society at the annual meeting.

The terms of the officers shall begin at the close of the meeting at which they are elected except that the retiring Editor shall retain the powers and duties of office so long as may be necessary to complete the then current issue of *Proceedings*.

ARTICLE VI.—*Duties of Officers and Council.*

The duties of the officers shall be such as usually appertain to their respective offices or may be specified in the by-laws. The duties of the Council shall be to pass upon candidates for membership, to decide upon papers offered for reading at the meetings, to supervise the examination of candidates and prescribe fees therefor, to call meetings, and in general, through the appointment of committees and otherwise, to manage the affairs of the Society.

ARTICLE VII.—*Meetings.*

There shall be an annual meeting of the Society on such date in the month of November as may be fixed by the Council in each year, but other meetings may be called by the Council from time to time and shall be called by the President at any time upon the written request of ten Fellows. At least two weeks notice of all meetings shall be given by the Secretary.

ARTICLE VIII.—*Quorum.*

Seven members of the Council shall constitute a quorum. Twenty Fellows of the Society shall constitute a quorum.

ARTICLE IX.—*Expulsion or Suspension of Members.*

Except for non-payment of dues, no member of the Society shall be expelled or suspended save upon action by the Council with not more than three negative votes followed by a three-fourths ballot of the Fellows present and voting at a meeting of the Society.

ARTICLE X.—*Amendments.*

This constitution may be amended by an affirmative vote of two-thirds of the Fellows present at any meeting held at least one month after notice of such proposed amendment shall have been sent to each Fellow by the Secretary.

BY-LAWS

(AS AMENDED NOVEMBER 19, 1954)

ARTICLE I.—*Order of Business.*

At a meeting of the Society the following order of business shall be observed unless the Society votes otherwise for the time being:

1. Calling of the roll.
2. Address or remarks by the President.
3. Minutes of the last meeting.
4. Report by the Council on business transacted by it since the last meeting of the Society.
5. New Membership.
6. Reports of officers and committees.
7. Election of officers and Council (at annual meetings only).
8. Unfinished business.
9. New business.
10. Reading of papers.
11. Discussion of papers.

ARTICLE II.—*Council Meetings.*

Meetings of the Council shall be called whenever the President or three members of the Council so request, but not without sending notice to each member of the Council seven or more days before the time appointed. Such notice shall state the objects intended to be brought before the meeting, and should other matter be passed upon, any member of the Council shall have the right to re-open the question at the next meeting.

ARTICLE III.—*Duties of Officers.*

The President, or, in his absence, one of the Vice-Presidents, shall preside at meetings of the Society and of the Council. At the Society meetings the presiding officer shall vote only in case of a tie, but at the Council meetings he may vote in all cases.

The Secretary-Treasurer shall keep a full and accurate record of the proceedings at the meetings of the Society and of the Council, send out calls for the said meetings, and, with the approval of the President and Council, carry on the correspondence of the Society. Subject to the direction of the Council, he shall have immediate charge of the office and archives of the Society.

The Secretary-Treasurer shall also send out calls for annual dues and acknowledge receipt of same; pay all bills approved by the President for expenditures authorized by the Council of the Society; keep a detailed account of all receipts and expenditures, and present an abstract of the same at the annual meetings, after it has been audited by a committee appointed by the President.

The Editor shall, under the general supervision of the Council, have charge of all matters connected with editing and printing the Society's publications. The *Proceedings* shall contain only the proceedings of the meetings, original papers or reviews written by members, discussions on said papers and other matter expressly authorized by the Council.

The Librarian shall, under the general supervision of the Council, have charge of the books, pamphlets, manuscripts and other literary or scientific material collected by the Society.

The General Chairman of the Examination Committee, shall, under the general supervision of the Council, have charge of the examination system and of the examinations held by the Society for the admission to the grades of Associate and of Fellow.

ARTICLE IV.—*Dues.*

The Council shall fix the annual dues for Fellows and Associates. Effective November 19, 1954, the payment of dues will be waived in the case of any Fellow or Associate who attains the age of 70 years or who, having been a member for at least 20 years, attains the age of 65 years and notifies the Secretary-Treasurer in writing that he has retired from active work. Fellows and Associates who have become totally disabled while members may upon approval of the Council be exempted from the payment of dues during the period of disability.

It shall be the duty of the Secretary-Treasurer to notify by mail any Fellow or Associate whose dues may be six months in arrears, and to accompany such notice by a copy of this article. If such Fellow or Associate shall fail to pay his dues within three months from the date of mailing such notice, his name shall be stricken from the rolls, and he shall thereupon cease to be a Fellow or Associate of the Society. He may, however, be reinstated by vote of the Council upon payment of arrears in dues, which shall in no event exceed two years.

ARTICLE V.—*Designation by Initials.*

Fellows of the Society are authorized to append to their names the initials F.C.A.S.; and Associates are authorized to append to their names the initials A.C.A.S.

ARTICLE VI.—*Amendments.*

These by-laws may be amended by an affirmative vote of two-thirds of the Fellows present at any meeting held at least one month after notice of the proposed amendment shall have been sent to each Fellow by the Secretary.

RULES REGARDING EXAMINATIONS FOR ADMISSION TO THE CASUALTY ACTUARIAL SOCIETY

1. Dates of Examination.

Examinations will be held on two successive days during the second or third week of the month of May each year in such cities as will be convenient for three or more candidates. The exact dates will be set by the Secretary-Treasurer.

2. Filing of Application.

Application for admission to examinations should be made on the Society's blank form, which may be obtained from the Secretary-Treasurer. No applications will be considered unless received before the first day of March preceding the dates of examination. Applications should definitely state for what parts the candidate will appear.

3. Associateship and Fellowship Examinations.

The examination for Associateship consists of four parts, each of which has two sections. A candidate may now write any or all sections covering Parts I and II and will receive credit for any section passed. This arrangement is restricted to Associateship Parts I and II.

A candidate may present himself for part or all of the Fellowship examinations either if he has previously passed the Associateship examinations or if he concurrently presents himself for and submits papers for all unpassed parts of the Associateship examination. Subject to the foregoing requirements, the candidate will be given credit for any part or parts of either examination which he may pass.

*4. Fees.

The examination fee is \$1.50 for a section, \$3.00 for a complete part; subject to a minimum of \$5.00. Examination fees are payable to the order of the Society and must be received by the Secretary-Treasurer before the first day of March preceding the dates of examination.

5. Credit for Examination Parts under Former Syllabus.

The new Syllabus of examinations effective in 1955 represents a considerable rearrangement of study materials. In order to simplify the process of transition and assure maximum equity among candidates, the following procedure has been established:

A candidate who has passed, or been credited with, one or more parts of the Associateship or Fellowship examinations under the Syllabus effective in 1948 and/or the Syllabus effective in 1953 will receive credit for the corresponding parts of the new Syllabus in accordance with the following table:

*Beginning with 1959 examinations, the fee is \$2.50 per section, subject to a minimum of \$5.00

*Parts Passed or Credited
Under Old Syllabus
(Effective in 1948 and/or 1953)*

Associateship, Part I
 " " II
 " " III
 " " IV
 Fellowship, Part I
 " " II
 " " III
 " " IV

*Parts Credited Under
New Syllabus
(Effective in 1955)*

Associateship, Part I (a) and II (b)
 " " III
 " " I (b) and II (a)
 " " IV
 Fellowship, Part IV
 " " II (a) and III (a)
 " " I (a) and III (b)
 " " I (b) and II (b)

Partial examinations will be given to those candidates requiring them in accordance with the foregoing credits.

6. Waiver of Examinations for Fellowship:

The examinations for Fellowship will be waived under Article III of the Constitution in part or in whole for those candidates who meet the qualifications and requirements set forth below.

1. WAIVER OF FELLOWSHIP PARTS III AND IV

(a) The candidate shall present himself in the same year for Fellowship Parts I and II, or shall have previously passed Parts I and II.

(b) The candidate shall present an original thesis on an approved subject relating to insurance (other than life insurance). Such thesis must show evidence of ability for original research and the solution of advanced insurance problems comparable with that required to pass Fellowship Parts III and IV. The thesis shall be of a character which would qualify it for printing in the *Proceedings*.

(c) Candidates electing this alternative should communicate with the Secretary-Treasurer and obtain through him approval of the Committee on Papers of the subject of the thesis and also of the thesis. In communicating with the Secretary-Treasurer, the candidate should state, in addition to the subject of the thesis, the main divisions of the subject and the general method of treatment, the approximate number of words and the approximate proportion to be devoted to data of an historical nature. All theses shall be in the hands of the Secretary-Treasurer before the examinations are held in May of the year in which they are to be considered. No examination fee will be required in connection with the presentation of a thesis.

2. FULL WAIVER

(a) The candidate shall have completed twenty years as an Associate member of this Society.

(b) The candidate shall present an original thesis on an approved subject relating to insurance (other than life insurance). The thesis shall be of a character which would qualify it for printing in the *Proceedings*.

(c) Candidates electing this alternative should communicate with the Secretary-Treasurer and obtain through him approval by the Com-

mittee on Papers of the subject of the thesis and also of the thesis. No examination fee will be required in connection with the presentation of a thesis.

7. Waiver of Examinations for Associateship.

The examinations for Associateship will be waived under Article III of the Constitution in part or in whole for those candidates who meet the qualifications and requirements set forth below.

1. PARTIAL WAIVER

Associateship Part I will be waived for a candidate who has passed Parts 1, 2 and 3 of the examinations of the Society of Actuaries.

2. FULL WAIVER

(a) The candidate shall be at least thirty-five years of age.

(b) The candidate shall have at least ten years' experience in actuarial or statistical work in insurance (other than life insurance) or in a phase of such insurance which requires a working knowledge of actuarial or statistical procedure or in the teaching of the principles of insurance (other than life insurance) in colleges or universities.

(c) For the two years preceding date of application, the candidate shall have been in responsible charge of the actuarial or statistical department of an insurance organization (other than a life insurance organization) or shall have occupied an executive position in connection with the phase of insurance (other than life insurance) in which he is engaged, or, if engaged in teaching, shall have attained the status of a professor.

(d) The candidate shall have submitted a thesis approved by the Committee on Papers. Such thesis must show evidence of analytical ability and knowledge of insurance (other than life insurance) sufficient to justify waiver of examinations.

(e) Refer to Paragraph 1 (c) of Rule 6 for details of submission.

LIBRARY

All students registered for the examinations of the Casualty Actuarial Society and all members of the Casualty Actuarial Society have access to all the library facilities of the Insurance Society of New York and of the Casualty Actuarial Society. These two libraries, with combined operations, are located at 107 William St., New York 38, New York and are under the supervision of Mrs. Ruby Breitner.

Registered students may have access to the library by receiving from the Society's Secretary-Treasurer the necessary credentials. Books may be withdrawn from the library for a period of one month without charge. The Insurance Society is responsible for postage and insurance charges for sending books to out of town borrowers, and borrowers are responsible for the safe return of the books.

Address requests for books to:

Librarian
Insurance Society of New York
107 William St.
New York 38, New York

SYLLABUS OF EXAMINATIONS

(Effective with 1955 Examinations)

ASSOCIATESHIP

<i>Part</i>	<i>Section</i>	<i>Subject</i>
I	(a)	Statistics.
	(b)	Probability.
II	(a)	Elementary Life Insurance Mathematics.
	(b)	General Principles of Insurance; Insurance Economics and Investments.
III	(a)	Insurance Law; Supervision, Regulation and Taxation of Insurance.
	(b)	Social Insurance.
IV	(a)	Policy Forms and Underwriting Practice.
	(b)	General Principles of Rate-making; Credibility.

FELLOWSHIP

I	(a)	Determination of Premium, Loss and Expense Reserves.
	(b)	Insurance Expense Analysis and Accounting.
II	(a)	Individual Risk Rating.
	(b)	Advanced Problems in Underwriting and Administration.
III	(a)	Machine Methods.
	(b)	Advanced Problems in Insurance Statistics.
IV	(a)	Advanced Problems in Rate-making.
	(b)	Current Insurance Problems.