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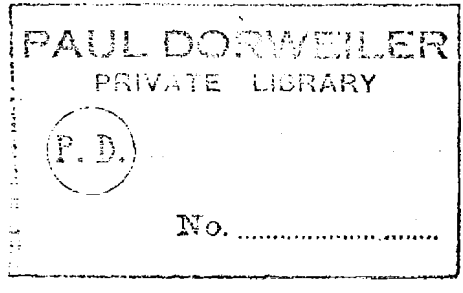

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"There are many angles of vision from which human minds peer at the universe."

"... the Prophets, through their own experience, anticipated Aeschylus' discovery that learning comes through suffering—a discovery which we, in our time and circumstances, have been making too."

"... the learning that comes through suffering caused by the failures of civilization may be the sovereign means of progress."

—*Arnold J. Toynbee*

Vol. XXXVII, Part I No. 67

PROCEEDINGS

May 22, 1950

A MID-CENTURY LOOK AT CASUALTY INSURANCE

PRESIDENTIAL ADDRESS BY HARMON T. BARBER

It seems inevitable that the year 1950 should produce for the *Proceedings* some sort of a comparison of casualty insurance as it existed at the beginning of the present century and as it is today, nearly fifty years later. Whereas I have no special talent as a historian, a preliminary canvass of this idea indicated that it held some promise as a basis for "remarks by the President" as our by-laws graciously term this item in the order of business. Obviously, a comprehensive comparison is out of place on this occasion In fact, even the brief mention of present conditions necessary for comparative purposes could be boresome to an informed audience However, possibly a then-and-now review could develop some comments of interest and a few observations beneficial to our present-day perspective.

As an authentic source record descriptive of casualty insurance shortly after the turn of the century, reference will be made to a Manual of Liability Insurance, published by an individual insurance carrier, and placed in the hands of its agents with the instruction "must not be loaned or shown to any person not in the employ of said Company." The manual became effective July 1, 1902, at a time when liability insurance embraced almost all of the important forms of casualty insurance except accident insurance and its related coverages. Thus, this one manual is the 1902 equivalent of the five-foot shelf of manuals, rating plans, statistical plans, etc. which are the modern tools of the trade. Within its green linen covers are eighty-three printed pages neatly thumb-indexed and containing complete rules and instructions for correspondence, daily reports, policy issuance, cancellation forms, minimum premiums, classifications, and rates for a dozen sub-lines of liability insurance. One cannot help but marvel at the extent of the subject matter described concisely, yet adequately, within the physical limits of a single manual of such diminutive size.

There are many references from this manual which could be cited to convey some of the atmosphere of this post Gay Nineties period. A few will suffice if they are aided by an agile imagination. For example, agents are cautioned to write legibly, an instruction definitely antedating the common use of the typewriter. The classification section for Manufacturers' and Contractors' showing rates for Employers' Liability, Workmen's Collective, and Public Liability coverages occupies about half of the entire manual. Among the classifications which have become obsolete with the passage of time is one for

Coach, Carriage, or Wagon Manufacturing and three separate classifications for Blacking Manufacturing, one each for Harness, Shoe, and Stove. As evidence that this is not an abridged manual, there are seven individual classifications to cover the Button Manufacturing industry—Bone, Celluloid, Ivory, Metal, Pearl, Rubber and N.O.C. What kind and how many Button Manufacturing risks fell within the N.O.C. classification is a question for speculation. A page of classifications is required for the gas industry and there are many covering street railroads, including one classification for "Street Railroad Operation—Surface—Horse."

The classification section for Teams Liability shows for each class the premium per team and an additional premium charge for the "Loading and Unloading" hazard. It is my understanding that this loading and unloading premium was designed to compensate for the extra hazard to the public involved in such a case as a lumber dealer delivering a load of new material at the site of a building project. One of the highest-rated teams classifications is "Police Patrol" with a premium of $100 and,—picture this, there is an additional loading and unloading charge for this classification of $10.

None of the classifications in the manual carries a code number. Evidently, all coding was accomplished either in the home office as an additional operation or possibly it was the intention to compile experience statistics by laborious hand methods on a risk by risk basis.

The first automobile liability policy issued in this country was written about four years prior to the publication of this 1902 manual. Consequently, it is not surprising to find in the manual only one page devoted to Automobile Insurance and rates. Rates were stated to be the same in all parts of the country. There was one classification for private cars, one for commercial cars, and five classifications for public vehicles. One of the latter is for "herdics." The dictionary defines a herdic as a kind of low-hung cab. My imagination fails me in visualizing this type of motor vehicle.

The 1902 manual has an interesting instruction to agents regarding the collection of premiums.

> "Where premiums, either advance or additional, are not reported to the home office within sixty days, the Company may proceed to collect them by special representative. If the premium is collected by the Company's representative, a deduction of 25% will be made from the commission. If it becomes necessary to employ an attorney, 50% of the commission will be deducted. If suit is instituted to collect, no commission will be allowed to the agent."

Evidently, the company was fully aware of the seriousness of the problem of premium collection, which persists to this day, and moved directly and precisely to meet it in the manner indicated.

One who is interested in such matters could spend considerable time selecting other references in the manual which cast sidelights, both amusing and instructive, on the times and on the conditions in the casualty insurance business of the era at the beginning of the century. A more serious view of the situation as revealed by the 1902 manual discloses many features which either are identical with or at least very closely resemble the practice of casualty insurance in 1950. This applies particularly to the kinds of liability insurance

then available. In the old manual, there are sections devoted to Automobile, Contingent Liability insurance for Owners or General Contractors, Elevator, Employers' Liability, General Liability, Physician's and Surgeon's Liability, Manufacturers' and Contractors', Teams, Theatre, Vessel insurance and Workmen's Collective. This list could meet the requirements of a modern multiple line casualty insurance carrier if it were to be supplemented with Products Liability insurance and several property lines, and if Workmen's Compensation insurance were to be substituted for the lines which it has since largely supplanted, namely, Employers' Liability, and to a less extent, Workmen's Collective.

Workmen's Collective insurance was an elementary form of Group Accident "at occupation" insurance with benefits defined in the policy in terms of wages of the injured. A limited choice of optional coverages was offered, including one which is of current interest because of its resemblance in part to New York statutory disability coverage. For an additional rate of 40 cents per $100 of payroll, "occupation only" coverage was extended to "twenty-four hour coverage" with total disability benefits equal to one-half weekly wages not exceeding twenty-six weeks or $500. Note that the coverage is accident only and not accident and sickness as is the case with modern disability coverage. The Workmen's Collective coverage also included lump sum benefits for death and permanent total cases and fractional benefits for specific dismemberment cases. It is rather odd to meet in this old manual a close relative of a subject which is receiving intensive study at the present time.

The basis of exposure for premium determination for each of the liability lines in 1902 is identical with that in use in 1950. Basic limits of liability at a time when eggs were selling for 10 cents a dozen were $5,000 per claim and $10,000 per accident, as is the case today. Higher limits could be obtained for a percentage increase in premium. The General Liability line with a minimum premium of $10 for one building could be written on a three-year basis at a premium equivalent to three annual premiums, less 10% payable in instalments of 50%, 30%, and 20%, a familiar basis which has only recently been revised to a moderate extent. The old manual contained a table of short term insurance rates, very similar in content to our present short rate table. Many of the old classification wordings are not unlike those in use today.

To continue the recitation of similarities would serve no purpose other than to accentuate the wisdom and foresight of the compilers of the 1902 manual. It is a fine tribute to their work that so many fundamentals have withstood the test of half a century of usage and have provided the foundation on which has been built the towering structure of modern casualty insurance. This was a complete, practical, and comprehensive manual of casualty insurance for its time. Today, it serves as a sort of pocket museum disclosing to the reader an interesting and authentic picture of fifty years ago.

HOW WOULD CASUALTY INSURANCE IN 1950 APPEAR FROM THE VIEWPOINT OF 1900?

For an answer to this question, I sought an interview with an imaginary casualty insurance Old Timer who, after the manner of Rip Van Winkle, had emerged from a fifty-year period of suspended animation and who presently was engaged in an intensive objective study of modern casualty insurance.

As might be expected, his first remarks were ones of amazement at the reported aggregate premium volume of the industry, at the development of such lines as automobile and workmen's compensation insurance, and at current office practice with its machine equipment and its specialized organization of personnel. High praise was extended for the benefits to the public from accident prevention and safety activities of the last fifty years and also for the carriers' splendid record of financial stability maintained under the test and strain of war, depression, adversity, and inflation. The spirit of toleration and cooperation between carriers was regarded as a special blessing by one accustomed to the "dog eat dog" philosophy of earlier days. The Old Timer's greatest tribute to the industry was expressed when he spoke at length of the invaluable service which insurance has rendered in the economic development of the nation by softening the blows of misfortune to individuals and corporations.

When queried as to any critical observations of the practice or progress of the business which might have occurred to him, the Old Timer was slow to respond at first but later warmed up to the subject. In any enterprise, true progress is not measured solely by a vast increase in the volume of business but is found in the improvement of product or service with reduced cost to the ultimate consumer. According to this criterion, the progress of the past fifty years, other than with respect to service to policyholders, has not been spectacular. In his opinion, economies from a greater efficiency in the conduct of the business which might have been salvaged and passed along to the purchasers of insurance have been absorbed in part by an unintentioned complication of procedures. This tendency may have been encouraged by allegiance to such ideals as consistency, continuity, and uniformity, which are all valuable attributes of any business in which there is a strong public interest.

To illustrate, he pointed to the comparative physical proportions of the 1902 manual and the corresponding present-day manuals of casualty insurance, and to further emphasize his observation, he handed me a workmen's compensation policy, issued to cover a one-employee risk. Attention was drawn to complications which resulted from the consistent application of procedures evidently designed primarily for the needs of large risks. In this case, it appears that a renewal policy was submitted to the assured by a general agent a month and a half in advance of its effective date, with a bill for the premium. The premium was determined by applying the manual rate to an estimated payroll substantially discounted from the known fact and adding to the product, the necessary loss and expense constants. Part of the advantage from the gesture of using a discounted payroll was lost because the calculated premium was less than the minimum premium of the policy. It was noted that it will be necessary for the assured to pay a few dollars additional premium after expiration of the policy in order to build the total premium up to the amount which could have been established initially as the product of the actual payroll of the one employee and manual rate plus constants. The policy itself consisted of four pages and was literally blanketed by no less than six special endorsements, identified as referring to the illegal employment of minors, board and lodging, inclusion of occupational diseases, overtime and limitation of remuneration, statutory, and an interpretive

endorsement as to "liability over" and the exclusion of contractual liability. Furthermore, once the policy was opened, it was as difficult to refold it properly as a typical automobile road map. "Actually," continued the Old Timer, "what this assured wants is to pay a flat premium for a statement from the carrier that his legal obligations to the employee under the state compensation law and otherwise emanating from the employer-employee relationship are completely assumed by the insurance carrier. He would be quite happy if this could be expressed on a single sheet of paper."

During the last few decades, the industry has made remarkable strides in perfecting its pricing technique and in the adjustment of individual large risk premiums by experience rating and by retrospective rating. This should have provided an opportunity for a simpler and possibly more approximate system of manual rates for small risks, one more in keeping with their actual interest in the subject. Where moderate amounts of premium are involved, the public does not consider it particularly sinister to substitute broad averages for a refined and detailed allocation of insurance costs.

Another instance of the complication arising from the consistent application of large risk procedures to all risks, including small ones, was noted by the Old Timer in connection with his review of unit plan statistical reportings for compensation insurance. Recognizing the advantages of the unit plan as respects the requirements for individually rated risks, he questioned if it were essential that the experience of small risks be reported in the same uniform detail or whether a substitute procedure involving aggregate reporting might produce some net economy in company administration and bureau expense without a material sacrifice in the utility of the reports.

The Old Timer suspects that a critical scrutiny of the details of ratemaking for each line of casualty insurance would reveal points where consistency with the past or within the present has submerged other considerations which might have indicated a different approach. He decries the prevalent tendency to make rates by rote.

According to his observation, the increase in complexity within the industry is partly attributable to a multiplicity of rating bureaus, each with a full complement of committees of company representatives meeting intermittently to polish and improve one facet or another of the business.

The stimulation of state regulation of the casualty insurance brought about by federal interest in the matter is something of a puzzle to the Old Timer with his 1900 background. Noting the large number of special calls for experience, losses and expenses, the regimentation of accounting practice, and many public hearings held on rating questions, he assumes that all must be necessary. He finds it difficult to fully rationalize all these activities with his theory that the true test of essentiality is whether the resulting benefit to the policyholder is commensurate with the time, energy, and expense involved.

At this point, let us bid goodbye to our friend from the past. As students of the business, we are well aware that one of its most urgent needs today is simplification: It is probably true that inertia and resistance to change have developed within the industry in the belief that if consistency in method is maintained, there will be less criticisms and more understanding on the part of the public and supervisory officials But without change, there would be no progress and casualty insurance prides itself on being progressive and adapt-

able to new situations. Some of these qualities should be directed at reform within areas which, up to now, have been overlooked or neglected. As actuaries, we possess a heritage left to us by the founders of this Society who pioneered scientific methods in casualty insurance ratemaking. With it goes an obligation to see to it that the ratemaking systems function smoothly and efficiently. If for no other reason than to keep the faith, we should bestir ourselves now to remedy matters which fall within our own sphere of responsibilities.

CREDIBILITY PROCEDURES LAPLACE'S GENERALIZATION OF BAYES' RULE AND THE COMBINATION OF COLLATERAL KNOWLEDGE WITH OBSERVED DATA

BY ARTHUR L. BAILEY

"If thou canst believe, all things are possible
to him that believeth."
Mark 9:23

The casualty insurance business has used credibility formulas or procedures for many years in making rates or in experience rating plans. These formulas have been used to determine the weight to be given to the indications of actual observations in a combination of such indications with *a priori* expectations which were based either on other actual data, on prior knowledge or on reasonable assumptions made before actual observations were available. Such formulas have invariably provided that the weight to be given to actual observations increase as the volume of such observations increases.

Last December the discussion of a paper by Mr. T. O. Carlson, entitled "Statistical and Actuarial Procedures in Liability Insurance"*[1], pointed out that casualty insurance underwriters and actuaries believe that they are not devoid of knowledge before they have acquired any statistics from observed data, and that this belief results in the use of credibility formulas to produce weighted averages of that prior knowledge and the information provided by the observed data. The remarks made at that time were general and unsupported by any demonstration. In fairness to other statisticians and to students of casualty insurance, it appears desirable to present a complete development from basic principles to show exactly the basis upon which credibility formulas rest and to make evident the point at which the classical statistical theory, particularly that of statistical estimation, departs from that used by casualty actuaries.

The basis for these credibility formulas has been a profound mystery to most people who have come in contact with them. The actuary finds them difficult to explain and, in some cases, even difficult to understand. Paradoxical as it may be, the more contact a person has had with statistical practices in other fields or the more training a person has had in the theory of mathematical statistics, the more difficult it has been to understand these credibility procedures or the validity of their application.

The credibility formulas for casualty insurance have been accepted in the past, and continue to be accepted at the present time, because it appears to most people to be logical and reasonable to give the indications of a large volume of data more consideration or weight than the indications of a small volume of data. How much weight the indications of specific volumes of data

* Numerals in brackets refer to the References at the end of this paper.

are to be given, in the casualty business, has continued to be a matter of individual judgment.

In addition to the relatively simple concept that more consideration or weight should be given to a greater volume of observational data, the casualty actuaries have devised credibility procedures to give more weight to the frequent occurrence of small losses than to the occasional or fortuitous occurrence of large losses of the same total amount. (It should be noted that negative losses can not occur.) For example, the rate making procedure for workmen's compensation insurance separates the actual losses into "Serious," "Non-serious" and "Medical" losses and uses three differing schedules of credibility for the three components of the total loss. Several experience rating plans give a greater schedule of credibility to the first G dollars of each loss than is given to the excess of any loss over G dollars. The "Multi-split Experience Rating Plan" for workmen's compensation insurance carries this even further by providing, in effect, a separate schedule of credibilities for each interval of G dollars of which a loss is composed.

It is at this point in the discussion that the ordinary individual has to admit that, while there seems to be some hazy logic behind the actuaries' contentions, it is too obscure for him to understand. The trained statistician cries "Absurd! Directly contrary to any of the accepted theories of statistical estimation." The actuaries themselves have to admit that they have gone beyond anything that has been proven mathematically, that all of the values involved are still selected on the basis of judgment, and that the only demonstration they can make is that, in actual practice, it works. Let us not forget, however, that they have made this demonstration many times. It does work!

It is the purpose of the technical portion of this paper (1) to show that it is proper to give greater weight to larger volumes of observed data and why; (2) to show that under certain conditions, specifically those prevailing in casualty insurance, it is proper to give greater weight to frequently occurring small values than to infrequently occurring large values and why; and (3) to show that these procedures are universally applicable to all fields of observation and are not peculiar to casualty insurance.

HISTORICAL COMMENTS

It will be realized that all of the problems in which credibilities are used are problems in statistical estimation and that the problem of statistical estimation is a very old problem. One of the first steps in the solution of this problem was made by Bayes [2] resulting in what is known as Bayes' Rule. That rule was initially produced as the solution of a specific case in which, *a priori*, all possible events were equally likely to occur [3]. It appears that statisticians of that day grasped at this as being better than no solution even when the basic condition as to equality of *a priori* probabilities was not met. Laplace in an early paper [4] advocated just that, and the practice appears to have become so well established that the Laplace generalization of Bayes' Theorem [5] (published in 1820) was given very little attention. Laplace's generalization actually provided the solution when, *a priori*, the possible events had varying probabilities of occurring.

It is interesting to note here that the Rev. Richard Price, who presented

Bayes' essay for publication in 1763, was closely connected with the insurance industry and would now be called an actuary. The following quotation from his introductory comments to Bayes' essay is so true today that it could not be improved to introduce the subject at hand:

> "Every judicious person will be sensible that the problem now mentioned is by no means merely a curious speculation in the doctrine of chances, but necessary to be solved in order to assure foundation for all our reasonings concerning past facts, and what is likely to be hereafter. Common sense is indeed sufficient to show us that, from the observation of what has in former instances been the consequence of a certain cause or action, one may make a judgment what is likely to be the consequence of it another time, and that the larger number of experiments we have to support a conclusion, so much the more reason we have to take it for granted. But it is certain that we cannot determine, at least not to any nicety, in what degree repeated experiments confirm a conclusion, without the particular discussion of the beforementioned problem; which therefore, is necessary to be considered by any one who would give a clear account of the strength of analogical or inductive reasoning; concerning which, at present, we seem to know little more than that it does sometimes in fact convince us, and at other times not; and that, as it is the means of acquainting us with many truths, of which otherwise we must have been ignorant; so it is, in all probability, the source of many errors, which perhaps might in some measure be avoided, if the force that this sort of reasoning ought to have with us were more distinctly and clearly understood."

From 1763 to the present time there has been continual argument over the propriety of using Bayes' Theorem in its original form and, possibly because of its apparent complexity, little use made of Laplace's generalization. The advocates of the use of Bayes' original theorem have formalized the process, with its assumption that all possibilities are equally likely, into what they describe as the application of the "Principle of Insufficient Reason." Their opponents have in turn characterized it as the "Assumption of the Equal Distribution of Ignorance," or the "Theory of Equal Ignorance." R. A. Fisher has modified it slightly to produce the "Method of Maximum Likelihood." Others have developed the "Best Unbiased Estimate" by methods which assume that there is only one possibility rather than several or many.

At present, practically all methods of statistical estimation appearing in textbooks on statistical methods or taught in American universities are based on an equivalent to the assumption that any and all collateral information or *a priori* knowledge is worthless. There have been rare instances of rebellion against this philosophy by practical statisticians [6] who have insisted that they actually had a considerable store of knowledge apart from the specific observations being analyzed. Philosophers have recently discussed the credibilities to be given to various elements of knowledge [7], thus undermining the accepted philosophy of the statisticians. However, it appears to be only in the actuarial field that there has been an organized revolt against discarding

all prior knowledge when an estimate is to be made using newly acquired data.

In our own *Proceedings* we have some astounding paradoxes which only serve to show the extent to which the teaching of the Principle of Insufficient Reason has been embedded in the minds of even our own actuaries. In 1918 Mr. Whitney [8] presented the first comprehensive development of credibilities to appear in our *Proceedings*. He assumed that the inherent hazards differed among classifications of risks and assumed a knowledge of the distribution of such hazards. However, in the course of the mathematical development he used Bayes' Rule to obtain a solution, thus reversing his assumption in the middle of the development. Mr. Arne Fisher, in discussing Mr. Whitney's paper [9], took Mr. Whitney to task for using Bayes' Rule, quoted many authorities against the use of it, and then sugges ted another approach which was based on the same philosophy, if not directlyon the same theorem.

From the foregoing it will be appreciated that anyone advocating a return all the way back to the fundamental principles of Laplace's generalization of Bayes' Theorem must look for opposition from many sides. However, Mr. Kendall's recent survey [10] of the current position of probability theory and his plea for progress along practical lines has been accepted by the writer as a definite encouragement to present such a development of the credibilities or weights to be given to observed data in its combination with collateral data or with *a priori* knowledge. Let us be clear in one thing however. Use will be made of Laplace's generalization of Bayes' Rule and not of the original Bayes' Rule.

GENERAL DISCUSSION

Let us define the problem of statistical estimation as that in which it is desired to obtain $E(x \mid H)$, the expected value of a statistic x which corresponds to the origin or cause of an observed event H. Such an expected value is the sum of the products of all possible values of x and the probabilities $P(x \mid H)$; where $P(x \mid H)$ is the probability that the value x was the value corresponding to the origin or cause of the observed event H.

In the insurance business such an expected value is obviously desirable in setting insurance rates, in order that there will be a balance between premiums and losses. The use by an actuary of a "maximum likelihood" estimate would be suicidal because, in many cases, the most likely event is the complete absence of loss. Thus the maximum likelihood estimate would provide nothing for losses and the premium would be, to say the least, inadequate.

The expected value $E(x \mid H)$ is an unbiased estimate of x for a particular value of H. It should be noted, however, that the "Best Unbiased Estimate" of the literature is unbiased for a particular value of x, not of H, under the tacit assumption that there is only one possible value of x, as yet unknown but having a probability or certainty of existing. This only serves to bring out a major difference of approach. The actuary knows that there is more than one possible value of x and is willing to assume that he can approximate the *a priori* probabilities of the existence of such possible values.

The expected value $E(x \mid H)$ will be the "best" estimate, from the least squares point of view, because, if it is used as the estimate of x for all of the

possible cases for which H may occur, the sum of the squares of the errors, $(x - E')$, will be a minimum when $E' = E(x \mid H)$.

It will be noted that $x' = E(x \mid H)$ is the true regression of x on H and that it may be a series of discrete points or a continuous curve, but not necessarily a straight line. In specific cases the discrete points may fall on a straight line or the continuous curve may actually be a straight line. (Several such special cases will be discussed herein.) Whatever the form of the true regression, it will be possible to obtain the best linear regression of x on H, and it is such best linear regressions that have been previously discussed by the writer [11].

The Laplace generalization of Bayes' Rule [3] states that if an event, H, has been produced by one of the mutually exclusive conditions, $F_1, F_2, \cdots, F_j$, and if $K(x)$ is the *a priori* probability that F_x existed, and if $P(H \mid x)$ is the *a priori* probability that when F_x exists the event H will occur, the *a posteriori* probability $P(F_a \mid H)$ that the particular condition F_a was the origin or cause of an observed event H is:

$$P(F_a \mid H) = K(a) \cdot P(H \mid a) \Big/ \sum_x K(x) \cdot P(H \mid x). \tag{1}$$

When the mutually exclusive conditions $F_1, F_2, \cdots, F_j$ are the conditions under which a statistic has the values $1, 2, \cdots, j$, the value of $E(x \mid H)$ can be written as

$$E(x \mid H) = \sum_x x \cdot P(x \mid H) = \sum_x x \cdot K(x) \cdot P(H \mid x) \Big/ \sum_x K(x) \cdot P(H \mid x). \tag{2}$$

It is important to note that, if the event H is the simultaneous occurrence of events $H_1, H_2, \cdots, H_n$, then $E(x \mid H)$ is *not* the average value of $E(x \mid H_i)$, but:

$$E(x \mid H) = \frac{\sum_x x \cdot K(x) \cdot P(H_1 \mid x) \cdot P(H_2 \mid x) \cdot \cdots \cdot P(H_n \mid x)}{\sum_x K(x) \cdot P(H_1 \mid x) \cdot P(H_2 \mid x) \cdot \cdots \cdot P(H_n \mid x)}. \tag{3}$$

In the following developments either formulas (2) or (3) will be utilized as the case requires. For simplicity of expression J_H will be used at times in place of $E(x \mid H)$, $F(H \mid x)$ in place of $K(x) \cdot P(H \mid x)$, and $F(H)$ in place of $\sum_x K(x) \cdot P(H \mid x)$.

The error variance, $\sigma^2_{e,H}$, of using J_H as an estimate of the true value of x in all of the possible cases for which a particular value of H may occur is:

$$\sigma^2_{e,H} = \sum_x (x - J_H)^2 \cdot F(H \mid x) \Big/ F(H),$$

or

$$\sigma^2_{e,H} = \frac{\sum_x x^2 \cdot F(H \mid x)}{F(H)} - J_H^2. \tag{4}$$

The error variance, σ_e^2, of using J_H as an estimate of the true value of x in all of the possible cases for which all possible values of H may occur is:

$$\sigma_e^2 = \sum_H F(H) \cdot \sigma_{e,H}^2 \qquad \text{as} \sum_H F(H) = 1. \tag{5}$$

The mean error for each possible value of H is zero.

At this point it will be desirable to let the mean and variance of the $K(x)$ distribution be indicated by m and σ^2 respectively and to let T^2 indicate the variance of H for all values of x. Thus:

$$m = \sum_x x \cdot K(x)$$

$$\sigma^2 = \sum_x x^2 \cdot K(x) - m^2$$

$$T^2 = \sum_H H^2 \cdot F(H) - \left[\sum_H H \cdot F(H)\right]^2$$

$$\sigma^2 + m^2 = \sum_H \left[\sum_x x^2 \cdot F(H \mid x)\right].$$

Combining (5) with (4) and using this new notation

$$\sigma_e^2 = \sigma^2 + m^2 - \sum_H J_H^2 \cdot F(H). \tag{6}$$

In the special cases for which $\sum_H H \cdot P(H \mid x) = Ax + B$ the regression of H on x is the line $H' = Ax + B$. Irrespective of the form of the true regression of x on H, the best fitting straight line can be obtained readily from knowledge as to the relationship of the coefficients of the best linear regression of x on H and the coefficients of the best linear regression of H on x. Thus, the best fitting straight line can be written as

$$x' = Z\left(\frac{H - B}{A}\right) + (1 - Z)m \tag{7}$$

where

$$Z = A^2 \sigma^2 / T^2. \tag{8}$$

If the true regression of x on H is a straight line, it obviously must be the line expressed in (7), and x' then becomes J_H or $E(x \mid H)$. Such special cases will be considered later, but it should be noted here that in such cases, (6) reduces* to:

$$\sigma_e^2 = \sigma^2(1 - Z). \tag{9}$$

* The algebra of the derivation of (9) from (6) is straightforward but is not shown here because of its length.

It should be noted that equation (7) provides for the combination of the indications of the data, summarized by $(H - B)/A$, and the *a priori* knowledge, summarized by m, through a weighting procedure in which Z is the weight given to the indications of the data and $(1 - Z)$ is the weight given to the *a priori* knowledge.

WHEN $P(H \mid x)$ FOLLOWS THE BINOMIAL DISTRIBUTION

When the event H is the occurrence of H successes out of n trials for each of which the *a priori* probability of a success was the same and equal to x, the value of $P(H \mid x)$ follows the Binomial distribution and is $\binom{n}{H}x^H(1 - x)^{n-H}$. The problem is to obtain the expected value or estimate of the true value of x from the observation that H successes occurred out of n trials and the *a priori* knowledge of the probabilities $K(x)$ of various possible values of x; summarized if possible by the mean, m, and the variance σ^2 of the probability function $K(x)$.

The best straight line regression of x on H may be obtained by the following reasoning.

For a particular value of x, the mean value of H is nx, the variance of H is $nx(1 - x)$ and the mean square of H is $n(n - 1)x^2 + nx$. For all values of x the mean square of H is $n(n - 1)(\sigma^2 + m^2) + nm$; the mean value of H is nm; and T^2, the variance of H, is $n(n - 1)\sigma^2 + nm(1 - m)$. The value of $\sum_H H \cdot P(H \mid x)$ is nx, so that the values of A and B to be used in (7) and (8) are $A = n$ and $B = O$. Thus Z can be obtained from (8) as:

$$Z = \frac{n\sigma^2}{(n - 1)\sigma^2 + m(1 - m)}. \tag{10}$$

This value of Z can be inserted in equation (9) to obtain the best fitting straight line to the regression of x on H.

In general $E(x \mid H)$ will consist of $n + 1$ discrete points which can be calculated from (2) for any known values of $K(x)$. There is one special case for which these $n + 1$ points will all fall on a single straight line. This case occurs when $K(x)$, the *a priori* probabilities of the existence of x, follow the Hardy [12] distribution* as suggested by E. C. Molina in 1946 [6].

Let

$$K(x) = Kx^{a-1}(1 - x)^{b-1} \tag{11}$$

where

$$c = \frac{m(1 - m)}{\sigma^2} - 1$$

and

$$a = mc, \qquad b = (1 - m)c$$

and

$$K = \frac{\Gamma(c)}{\Gamma(a)\Gamma(b)}$$

so that x has a mean of m and a variance of σ^2.

* Note that this is the particular case of the Pearsonian Type I distribution for which the range of x is from 0 to 1 and is also known as the Beta distribution [13].

Inserting these values of $K(x)$ and the Binomial distribution values of $P(H \mid x)$ in (2), except for constants common to both numerator and denominator, gives:

$$E(x \mid H) = \int_0^1 x^{H+a}(1-x)^{n-H+b-1}dx \Bigg/ \int_0^1 x^{H+a-1}(1-x)^{n-H+b-1}dx. \tag{12}$$

$$= \frac{B(H+a+1, n-H+b)}{B(H+a, n-H+b)} = \frac{H+a}{n+a+b}$$

where $B(x, y)$ is the Beta function equal to $\dfrac{\Gamma(x)\,\Gamma(y)}{\Gamma(x+y)}$. When the values of a and b from (11) and the value of Z from (10) are used, (12) becomes:

$$E(x \mid H) = Z\frac{H}{n} + (1-Z)m. \tag{13}*$$

The value Z is thus seen to be the credibility, or percentage of total weight, to be given to the observed ratio of successes to trials in its combination with the *a priori* expectation, m. From (10) it is seen that when n is one, $Z = \sigma^2/m(1-m)$ and that Z increases as n increases, approaching unity as n approaches infinity.

WHEN $P(H \mid x)$ FOLLOWS THE POISSON DISTRIBUTION

When H is the number of events observed in n units of time or space throughout which events are randomly distributed with an average frequency of x events per unit, the value of $P(H \mid x)$ follows the Poisson distribution and is $(nx)^H e^{-nx}/H!$. The problem is to estimate x by obtaining its expected value from the observed value of H and the *a priori* knowledge of the probabilities, $K(x)$, of various possible values of x; summarized if possible by the mean m and the variance σ^2 of the probability function $K(x)$.

The best straight line regression of x on H may be obtained by the following reasoning. For a particular value of x, the mean value of H is nx, the variance of H is nx and the mean square of H is $nx + n^2x^2$. For all values of x the mean square of H is $nm + n^2(\sigma^2 + m^2)$; the mean value of H is nm; and T^2, the variance of H, is $nm + n^2\sigma^2$. The value of $\sum_H H \cdot P(H \mid x)$ is nx, so that the values of A and B to be used in (7) and (8) are $A = n$ and $B = 0$. Thus Z can be obtained from (8) as:

$$Z = \frac{n\sigma^2}{n\sigma^2 + m} \tag{14}$$

which can be inserted in equation (9) to obtain the best fitting straight line to the regression of x on H.

* Note: If the Principle of Insufficient Reason is applied in this case the assumption would be that all values of x from 0 to 1 were equally likely. This would produce $E(x \mid H) = (H+1)/(n+2)$ and not $E(x \mid H) = H/n$ as is frequently used.

In general $E(x \mid H)$ will consist of discrete points corresponding to the discrete values of H from 0 to ∞. These discrete points can be calculated from (2) for any known values of $K(x)$. There is one special case for which these points will all fall on a single straight line. This case occurs when $K(x)$ follows the Pearsonian Type III distribution having a range of x from 0 to ∞;* specifically when:

$$K(x) = \frac{g^{mg} x^{mg-1} e^{-gx}}{\Gamma(mg)} \quad \text{where } g = \frac{m}{\sigma^2} \tag{15}$$

Inserting this value of $K(x)$ and the Poisson distribution values of $P(H \mid x)$ in (2), except for constants common to both numerator and denominator, gives:

$$E(x \mid H) = \frac{\int_0^\infty x^{H+mg} e^{-x(n+g)} dx}{\int_0^\infty x^{H+mg-1} e^{-x(n+g)} dx} \tag{16}$$

$$= \frac{\Gamma\left(H + \frac{m^2}{\sigma^2} + 1\right)}{\left(n + \frac{m}{\sigma^2}\right)^{H+(m^2/\sigma^2)+1}} \cdot \frac{\left(n + \frac{m}{\sigma^2}\right)^{H+(m^2/\sigma^2)}}{\Gamma\left(H + \frac{m^2}{\sigma^2}\right)} = \frac{H + \frac{m^2}{\sigma^2}}{n + \frac{m}{\sigma^2}}.$$

Using the value of Z in (14), this becomes:

$$E(x \mid H) = Z \cdot \frac{H}{n} + (1 - Z)m. \tag{17}\dagger$$

The value Z is seen to be the credibility, or percentage of total weight, to be given to the observed number of events per unit of time or space in its combination with the *a priori* expectation, m. From (14) it is seen that when n is one, $Z = \sigma^2/(\sigma^2 + m)$ and that Z increases as n increases, approaching unity as n approaches infinity.

WHEN H IS THE SUM OF THE INDEPENDENT VARIABLES x AND h

When H consists of the simultaneous occurrence of the values $H_1, H_2, \cdots, H_n$ for the sum of a single value of x and n random values of a variable h, and

* It will be noted that this distribution is closely related to the Chi-square distribution with $2m^2/\sigma^2$ degrees of freedom (see reference [13] and is called the Gamma distribution. It was used by R. Keffer [14] in 1929 with $m = 1$.

† Note: If the Principle of Insufficient Reason is applied in this case it would produce $E(x \mid H) = (H + 1)/n$ and not H/n as is frequently used.

when h is independent of x with a mean of B, a variance of S^2 and a frequency distribution of $\varphi(h)$, the value of $P(H \mid x)$ may be expressed as:

$$P(H \mid x) = \varphi(H_1 - x) \cdot \varphi(H_2 - x) \cdot \cdots \cdot \varphi(H_n - x). \tag{18}$$

The problem is to estimate the value of x included in each of the sums, $H_1, H_2, \cdots, H_n$ by obtaining its expected value from the values $H_1, H_2, \cdots, H_n$ and the *a priori* knowledge of the probabilities, $K(x)$, of various possible values of x; summarized if possible by the mean m and variance σ^2 of the probability function $K(x)$ and the mean, $\overline{H}$, of the values $H_1, H_2, \cdots, H_n$.

Consider the special case when both $K(x)$ and $\varphi(h)$ are normal distributions. Inserting the values of $K(x)$ and of $\varphi(H_i - x)$ in (3), except for constants common to both numerator and denominator, gives:

$$E(x \mid H) = \tag{19}$$

$$\frac{\displaystyle\int_{-\infty}^{\infty} x\, e^{-\left[\frac{(x-m)^2}{2\sigma^2} + \frac{(H_1 - x - B)^2 + (H_2 - x - B)^2 + \cdots + (H_n - x - B)^2}{2S^2}\right]} dx}{\displaystyle\int_{-\infty}^{\infty} e^{-\left[\frac{(x-m)^2}{2\sigma^2} + \frac{(H_1 - x - B)^2 + (H_2 - x - B)^2 + \cdots + (H_n - x - B)^2}{2S^2}\right]} dx}$$

The numerator of (19) is of the form $C \displaystyle\int_{-\infty}^{\infty} U \cdot dV$ where

$$C = e^{-\frac{1}{2}\left[\frac{m^2}{\sigma^2} + \frac{nB^2}{S^2} + \frac{H_1^2 + H_2^2 + \cdots + H_n^2}{S^2} + \frac{2B(H_1 + H_2 + \cdots + H_n)}{S^2}\right]},$$

$$U = e^{x\left[\frac{m}{\sigma^2} + \frac{H_1 + H_2 + \cdots + H_n}{S^2} - \frac{nB}{S^2}\right]}, \text{ and}$$

$$dV = x\, e^{-\frac{x^2}{2}\left[\frac{1}{\sigma^2} + \frac{n}{S^2}\right]} dx.$$

Thus the numerator of (19) may be expressed as $C \cdot U \cdot V]_{-\infty}^{\infty} - \displaystyle\int_{-\infty}^{\infty} C \cdot V \cdot dU.$

The value of $C \cdot U \cdot V]_{-\infty}^{\infty}$ is nil as $C \cdot U \cdot V$ is zero at both ∞ and $-\infty$.

The value of $-\displaystyle\int_{-\infty}^{\infty} C \cdot V \cdot dU$ is the denominator of (19) multiplied by the following quantity, which thus becomes the value of $E(x \mid H)$:

$$E(x \mid H) = \frac{\dfrac{m}{\sigma^2} + \dfrac{H_1 + H_2 + \cdots + H_n}{S^2} - \dfrac{nB}{S^2}}{\dfrac{1}{\sigma^2} + \dfrac{n}{S^2}}. \tag{20}$$

This may be expressed as:

$$E(x \mid H) = Z(\overline{H} - B) + (1 - Z)m \tag{21}*$$

where

$$Z = \frac{n\,\sigma^2}{n\,\sigma^2 + S^2}. \tag{22}*$$

Not only is this special case one for which the true regression of x on H is a straight line when H is a single observed sum of x and h; but it is also one for which all of the knowledge pertinent to the determination of $E(x \mid H)$ is contained in $\overline{H}$ when H is the simultaneous occurrence of n values with such an average. Knowledge of the individual values of $H_1, H_2, \cdots, H_n$ would add nothing to the knowledge provided by $\overline{H}$.

This concentration of knowledge in $\overline{H}$ is the result of assuming that $\varphi(h)$ is a normal distribution function. If that assumption is continued without the assumption that $K(x)$ is a normal distribution function, the regression of x on $\overline{H}$ will not be a straight line although the best fitting straight line will be the line provided by (21) and (22). If $\varphi(h)$ is not a normal distribution function, $E(x \mid H)$ can be calculated from (3) and will involve the individual values of H_i and not only $\overline{H}$.

$E(x \mid H)$ IN TERMS OF $\overline{J}$ *and* $\overline{H}$

Before proceeding to other cases, it will be helpful to investigate the possibility of expressing $E(x \mid H)$ in terms of $\overline{J}$ where H is the concurrent observation of events $H_1, H_2, \cdots, H_n$ and $\overline{J}$ is the average value of $E(x \mid H_1)$, $E(x \mid H_2), \cdots, E(x \mid H_n)$.

In the case where $P(H \mid x)$ follows the Binomial distribution and $K(x)$ follows the Beta distribution, a value $E(x \mid H_i) = J_i$ could be obtained for the result of each of the n trials. It would be:

$$J_i = \frac{\sigma^2}{m(1-m)} \cdot H_i + \left(1 - \frac{\sigma^2}{m(1-m)}\right) \cdot m. \tag{23}$$

Thus

$$\overline{J} = \frac{\sigma^2}{m(1-m)} \cdot \frac{H}{n} + \left(1 - \frac{\sigma^2}{m(1-m)}\right) \cdot m \tag{24}$$

or

* Note that the Principle of Insufficient Reason would, in effect, assume that $\sigma^2 = \infty$ so that Z would become 1 and $E(x \mid H)$ would equal $\overline{H} - B$. Note also that, under that assumption, (8) would produce a larger value, namely $\sigma_e^2 = S^2/n$ instead of $\sigma_e^2 = S^2/\left(n + \frac{S^2}{\sigma^2}\right)$.

$$m = \frac{m(1 - m)\bar{J} - \sigma^2 \cdot \frac{H}{n}}{m(1 - m) - \sigma^2}. \tag{25}$$

Substituting this value of m and the value of Z given in (10) in equation (13) produces:

$$E(x \mid H) = \frac{n - 1}{n} \cdot Z \cdot \frac{H}{n} + (1 - \frac{n - 1}{n} \cdot Z)\bar{J}. \tag{26}$$

In the case where $P(H \mid x)$ follows the Poisson distribution and $K(x)$ follows the Gamma distribution, a value of J_i could be obtained from the number of events observed in each of the n units of time or space. It would be

$$J_i = \frac{\sigma^2}{\sigma^2 + m} \cdot H_i + \left(1 - \frac{\sigma^2}{\sigma^2 + m}\right) \cdot m. \tag{27}$$

Thus

$$\bar{J} = \frac{\sigma^2}{\sigma^2 + m} \cdot \frac{H}{n} + \left(1 - \frac{\sigma^2}{\sigma^2 + m}\right) \cdot m \tag{28}$$

or

$$m = \frac{(\sigma^2 + m)\bar{J} - \sigma^2 \cdot H/n}{m}. \tag{29}$$

Substituting this value of m and the value of Z given in (14) in equation (17) produces:

$$E(x \mid H) = \frac{n - 1}{n} \cdot Z \cdot \frac{H}{n} + (1 - \frac{n - 1}{n} \cdot Z) \cdot \bar{J}. \tag{30}$$

In the case where $P(H \mid x) = P(H_1 \mid x) \cdot P(H_2 \mid x) \cdot \cdots \cdot P(H_n \mid x)$ and where $P(H_i \mid x)$ and $K(x)$ are both normal distributions, a value of J_i could be obtained for each of the n values of H_i. It would be:

$$J_i = \frac{\sigma^2}{\sigma^2 + S^2}(H_i - B) + \left(1 - \frac{\sigma^2}{\sigma^2 + S^2}\right) \cdot m. \tag{31}$$

Thus

$$\bar{J} = \frac{\sigma^2}{\sigma^2 + S^2}(\bar{H} - B) + \left(1 - \frac{\sigma^2}{\sigma^2 + S^2}\right) \cdot m \tag{32}$$

or

$$m = \frac{(\sigma^2 + S^2)\bar{J} - \sigma^2(\bar{H} - B)}{S^2}. \tag{33}$$

Substituting this value of m and the value of Z given in (22) in equation (21) produces:

$$E(x \mid H) = \frac{n - 1}{n} \cdot Z \cdot (\bar{H} - B) + (1 - \frac{n - 1}{n} \cdot Z) \cdot \bar{J}. \tag{34}$$

It is noted that the coefficient $(n - 1)Z/n$ is common to equations (26), (30) and (34) and that, although Z is different in each case, Z is the coefficient in the best straight line regression of x on $\bar{H}$. When J_i is not a linear function of H_i, the coefficient $(n - 1)Z/n$ is obviously proper both when n is one and when n approaches infinity. This suggests that this general relationship is either always true or that it represents a close approximation to the truth even when J_i is not a linear function of H_i. This will be assumed to be the case although it will be clearly understood that it has not been proven.

WHEN H IS THE PRODUCT OF THE INDEPENDENT VARIABLES x AND h

When H consists of the simultaneous occurrence of the values $H_1, H_2, \cdots H_n$ as the product of a single value of x and n random values of a variable h, and when h is independent of x with a mean of 1, a variance of S^2 and a frequency distribution of $\varphi(h)$, the value of $P(H \mid x)$ may be expressed as:

$$P(H \mid x) = \frac{1}{x^n}\varphi\left(\frac{H_1}{x}\right)\cdot\varphi\left(\frac{H_2}{x}\right)\cdot \cdots \cdot\varphi\left(\frac{H_n}{x}\right). \tag{35}$$

This condition is of frequent occurrence in practical applications for which both x and h can have only positive values. The problem is to obtain the expected value of the x which is included in each of the products, H_1, $H_2, \cdots, H_n$, from those values and the *a priori* knowledge of the probabilities, $K(x)$, of various possible values of x; summarized if possible by the parameters of $K(x)$ and $\varphi(h)$ and the means $\bar{H}$ and $\bar{J}$ for the values $H_1, H_2, \cdots, H_n$.

The best fitting straight line to the regression of x on $\bar{H}$ can be shown to be:

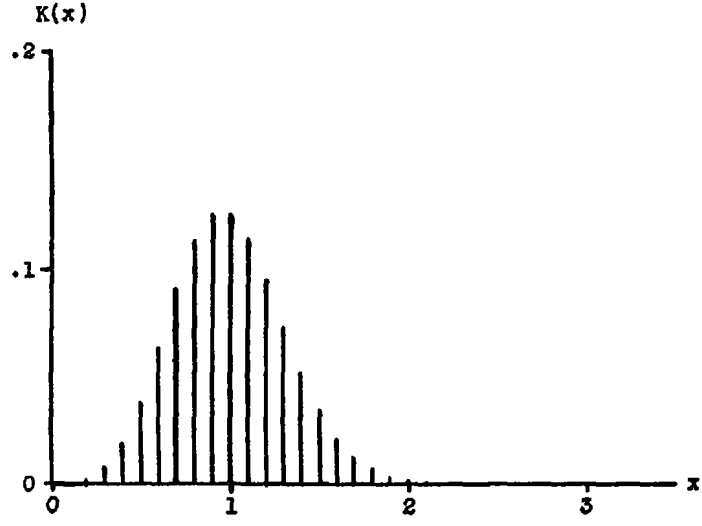
$$x' = Z\cdot\bar{H} + (1 - Z)m \tag{36}$$

where

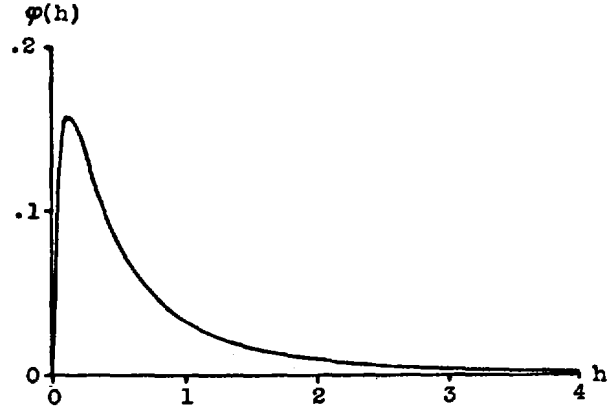
$$Z = \frac{n\cdot\sigma^2}{n\cdot\sigma^2 + S^2(\sigma^2 + m^2)}. \tag{37}$$

This straight line can not, however, be depended upon to give a reliable estimate of x for small values of n for two reasons: first, the true regression must be expected to be far from a straight line in most practical applications; and second, there is usually much more information in the individual values of H_i than is summarized in the average $\bar{H}$.

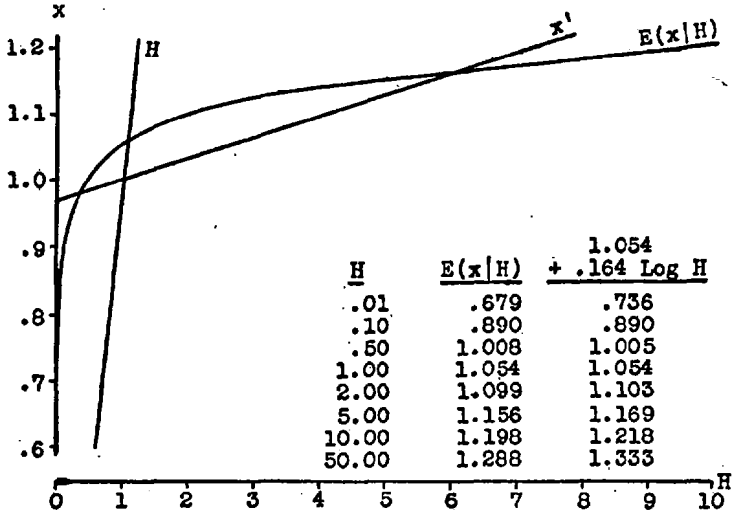
To show the departure from a straight line regression when $n = 1$, an example has been selected in which $\varphi(h)$ is typical of the distribution of losses by size of loss for casualty insurance and in which $K(x)$ is typical of the distribution of classification average claim costs expressed as a percentage of the average for all classes. To simplify calculations $K(x)$ has been taken so that it has values only at the discrete intervals of $x = n/10$ where n is an integer. With $K(x) = e^{-10}\cdot 10^{10x}/(10x)!$ $K(x)$ has a mean of 1, a variance of $\sigma^2 = 1/10$ and the distribution shown in the following diagram. $P(H \mid x)$ has been chosen as equal to $\varphi(\frac{H}{x})/x$ where $\varphi(h)$ has a mean of 1 and a variance of $S^2 = 3$ and



with $\varphi(h)$ following the normal logarithmic distribution shown in the following diagram.



$E(x \mid H)$ has been calculated for a sufficient number of values of H to indicate the relationship to H shown in the following diagram. The lines $x = H$, produced by the application of the Principle of Insufficient Reason, and $x' = ZH + (1 - Z)m$, produced as the best straight line regression, are also shown on the diagram for comparison with the curve $x = E(x \mid H)$. The calculated values of $E(x \mid H)$ are shown together with the values of $1.054 + .164 \log_{10} H$ which appear to reasonably approximate the values of $E(x \mid H)$ in this example.



H	E(x|H)	1.054 + .164 Log H
.01	.679	.736
.10	.890	.890
.50	1.008	1.005
1.00	1.054	1.054
2.00	1.099	1.103
5.00	1.156	1.169
10.00	1.198	1.218
50.00	1.288	1.333

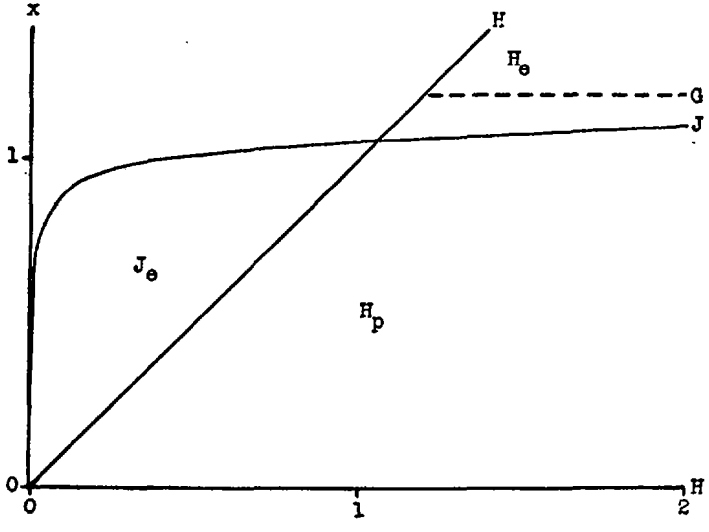
PRIMARY AND EXCESS VALUES

When H_i is the product of the independent variables x and h_i, the value of $E(x \mid H)$ may be expressed in terms of $\overline{H}$ and $\overline{J}$ as:

$$E(x \mid H) = \frac{n-1}{n} Z\overline{H} + (1 - \frac{n-1}{n} Z)\overline{J} \qquad (38)$$

where Z has the value shown in (37). This relationship is exact when the true regression of x on H is a straight line. Let it be assumed to hold when that regression is not a straight line.

Consider now the portions of H_i and J_i illustrated by areas on the following diagram and as defined below:



H_p = the primary portion of H, defined as:
$H_p = H$ if $H \leq J$ and $H_p = J$ if $H > J$;

H_e = the excess portion of H, defined as:
$H_e = 0$ if $H \leq J$ and $H_e = H - J$ if $H > J$;

J_e = the excess portion of J, defined as:
$J_e = J - H$ if $H \leq J$ and $J_e = 0$ if $H > J$.

Noting that $\overline{H} = \overline{H}_p + \overline{H}_e$ and that $\overline{J} = \overline{H}_p + \overline{J}_e$, equation (38) can be written:

$$E(x \mid H) = \overline{H}_p + \frac{n-1}{n} Z\, \overline{H}_e + (1 - \frac{n-1}{n} Z)\overline{J}_e. \qquad (39)$$

It is found that the average of the primary portions of the observations $H_1, H_2, \cdots, H_n$ should be given full credibility (a weight of unity) and that the excess portions of those observations should be given a lesser weight of $(n-1)Z/n$. This coincides with the beliefs of casualty actuaries as expressed in practice in the Multi-split Experience Rating Plan for workmen's compensation insurance. As the *a priori* expected value of $\overline{J}_e$ is equal to that of $\overline{H}_e$, the actuaries have replaced $\overline{J}_e$ in (39) with the *a priori* expected value of $\overline{H}_e$. It is obvious from (39) that such a replacement impairs the accuracy of the estimate of x although such impairment may not be appreciable.

From the diagram it will be seen why the single split of observed values at G may be a sufficiently close approximation. With such a split the definition of primary and excess values would be $H_p = H$ if $H \leq G$ and $H_p = G$ if $H > G$, and $H_e = 0$ if $H \leq G$ and $H_e = H - G$ if $H > G$.

THE UNSOLVED PROBLEM

In casualty insurance, the inherent hazard of an insured, or of a classification of insureds, is the product of an inherent frequency of loss occurrence and an inherent average amount of loss, and it is the value of this product for which an estimate is desired. Such an estimate must be expressed in terms of the amounts of the individual losses which have occurred and the *a priori* knowledge as to average frequencies, average amounts of losses, the distribution of frequencies and loss amounts about such averages *and a priori* knowledge as to the correlation between frequencies of loss and average loss amounts.

The expected value, or estimate, of such a product would, no doubt, be more complicated in form than the results obtained for the simpler cases studied herein. The form such an estimate should take would be very desirable information for the actuary to have, even though, at the present time, there is little or no knowledge as to the correlation between frequencies of loss and average loss amounts in casualty insurance. It is the hope of the writer that someone with a knowledge of the statistical behavior of products will undertake the development of the appropriate procedure. It is for that person's encouragement that Jesus' statement was initially quoted.

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DISCUSSIONS OF PAPERS READ AT PREVIOUS MEETING

UNIFORM ACCOUNTING—A STUDY OF REGULATION

DUDLEY M. PRUITT

Volume XXXVI, Page 22

WRITTEN DISCUSSION BY J. A. MILLS

Mr. Pruitt has done an admirable job of presenting the highlights of events leading up to the adoption of Uniform Accounting Regulations and of the first year of actual experience under these Regulations.

As Mr. Pruitt points out, it is of interest to note that Superintendent Dineen in stating seven objectives of Uniform Accounting Regulations mentioned only one that did not relate to rate making. The Regulations present a major forward step from an accounting standpoint. A good start also has been made towards getting more useful statistics from a rate making standpoint, but a great deal more must be done before they will meet the ultimate objective of producing information ideally suited for rate making purposes.

The Regulations require the use of bases other than premiums for distributing expenses whenever another equally good or better base can be found. The problem of developing a better base than premiums is a difficult one for a company that has not engaged in extensive studies of expenses. When such companies present specific expense allocation problems to Insurance Department men, better bases than premiums are not always readily determinable—with the result that the natural tendency to fall back upon the premium base persists. Over-use of the premium base perpetuates the expense allocation errors already in existence. It is to be hoped that the inability of many companies to immediately devise more appropriate bases will not encourage public officials to require the use of the premium distribution method for the sake of uniformity. If uniformity promotes the *status quo* at the expense of quality, it will have been bought at too high a price.

When the Casualty Expense Exhibit first came into being, there was no requirement to segregate audit expenses from other administration expenses. As a consequence, the judgment methods and pseudo-scientific formulas involving the use of weights by line too often overlooked this element of expense which arose under some lines and not others. This early experience has taught those who are interested in securing better expense statistics for rate making purposes that more accurate assignments to line will result if audit expenses and inspection expenses are classed as specific functions under those lines for which they form an important part of the total cost. These expenses may not be important in relation to the aggregate expenses incurred for all casualty lines combined, but they are important enough to warrant adequate definition and functionalization in the case of compensation, other liability, and boiler & machinery business.

All taxes have been set up as a separate "function" in the Expense Exhibit. Ideally, the Functional Classification System should result in allocating all costs to major insurance operations. The costs assigned to each major insur-

ance operation should approximate those that would obtain if separate legal entities performed each of these major operations. If this reasoning is sound, payroll taxes and other taxes which can be allocated to insurance functions should be so treated, whereas the unclassifiable taxes would remain in a special class of non-functionalized taxes. The aggregate taxes paid by a carrier are readily obtainable by totalling the appropriate nature of expense classifications.

UNIFORM ACCOUNTING—A STUDY OF REGULATION

DUDLEY M. PRUITT

Volume XXXVI, Page 22

WRITTEN DISCUSSION BY T. F. TARBELL

Uniform accounting for Fire and Casualty companies is apparently destined to become of increasing importance in the functioning of Fire and Casualty companies as well as in the regulation of such companies by state authorities, particularly in the rate making field. We are, therefore, much indebted to Mr. Pruitt for taking up this subject in its early infancy and providing both an historical background and pertinent comments and criticisms on the status and trend of uniform accounting thought on the part of supervisory officials.

The writer can add very little to what Mr. Pruitt has written and, accordingly, will confine his remarks to emphasizing certain points which he believes have an important bearing upon the status of uniform accounting at the present time.

The primary object of uniform accounting as interpreted from remarks of Superintendent Dineen of the New York Department is the improvement in rate making and rate regulation. It is reasonable to assume that improvement in rate making means, from the expense standpoint, the proper determination of the costs of the various elements of expense included in the rate. If this conception is correct, the major objective is to provide for uniformity of expense by expense groups or functions.

One criticism by the insurance industry is that to date uniformity of function has taken second place to uniformity of account, expenses by kind have taken precedence over expenses by purpose. This seems to be further substantiated by a seemingly reluctance on the part of supervisory officials to recognize the importance of some of the difficult problems the solution of which is essential to the attainment of uniformity of function. Proper definitions of inspection and payroll audit expenses are cited. The casualty industry is in practically unanimous agreement that proper expenses for these functions are essential to the making of reasonable, adequate and non-discriminatory rates for certain casualty lines and that this should be a major objective of uniform accounting and such definitions incorporated in uniform accounting instructions.

In the last paragraph of Mr. Pruitt's paper he quotes Mr. Morrill, former Deputy Superintendent of the New York Department as follows: "Uniform accounting . . . will produce substantial benefits in the regulation and management of insurance carriers." Initial conception of the purposes of the New York Uniform Accounting Regulation would classify such benefits as minor

or incidental, and perhaps that is the intent; however, such statement coupled with emphasis to date upon uniformity of account rather than uniformity of function, would seem to indicate that possibly the trees are obscuring the forest.

It should be borne in mind that the subject of uniform accounting, from the standpoint of implementation of uniform accounting laws, is still in its infancy and that no claim is being made by supervisory authorities that the existing Uniform Accounting Instructions are the last word on the subject. A committee and sub-committee on uniform accounting of the National Association of Insurance Commissioners and an industry uniform accounting committee have been cooperating in an effort to improve the instructions. While there have been some conflicts of opinion, it is to be hoped that most of the important controversial questions may, in due course, be brought to a successful conclusion.

A DISCUSSION OF GROUP, ACCIDENT AND HEALTH INSURANCE

HAROLD F. LA CROIX, JR.

Volume XXXVI, Page 9

WRITTEN DISCUSSION BY J. H. ROWELL

Mr. La Croix's paper points the necessity for careful planning before a company embarks on a plan of compiling Group Accident and Health Statistics. This necessity arises from the virtually myriad forms of coverage being written, the numerous exposure bases and varying benefits involved in coverages which appear, on the surface, to be very similar.

For example, in the oldest of the coverages, Weekly Indemnity, the various waiting and limiting periods cause such widely differing costs per unit of exposure and such varying patterns of duration distributions that separate studies must be made for each plan. When coupled with the other variables (sex and age) the data may be spread so thin as to produce unreliable results. One expedient often used to overcome this problem is to ignore the age variable on the assumption that there will be only minor variations in age distribution exposures. That this assumption is not entirely correct may be judged by the facts that, first, the average "T" rates used in Group Life Insurance vary from $6.00 or $7.00 in some groups to $40.00 or $50.00 in other groups and second, based on the Inter-Company Group Morbidity Investigation 1931 to 1935* and the Exposure Distribution by Age underlying that study, weekly indemnity rates increase about half as fast as "T" rates starting with the comparable rates in the age bracket 21 to 30. Granting that the average rate in Group Life Insurance results from a weighting by amount of insurance which, often based on salaries or length of service, generally increases with age, Weekly Indemnity Classifications often follow the same pattern with the same resultant weightings.

* "Recent Morbidity upon Lives Insured Under Group Accident and Health Policies and Premiums Based Thereon." By Gilbert W. Fitzhugh. Transactions of the Actuarial Society of America, Volume XXXVIII.

A second expedient often used to overcome the problem of thin exposure spreads is to combine the male and female claims by groups of percentage female exposures, and use an assumption as to the relative cost of female claims as a percentage of male. It would seem that this method would yield better results if it were done separately for maternity claims and other than maternity claims for the reason that the Maternity Benefit does not vary by plan (generally six weeks) and hence, the maternity cost can be separately computed by female percentage brackets on the combination of all plans and the results added to non-maternity results by percentage female brackets. In addition, any significant cost variations of the Maternity Benefit may give useful clues as to the probable female age distributions.

As respects the Maternity Benefits themselves, particularly for hospital and surgical coverages, care must be taken in the original claim handling to charge such claims to the exposures out of which they arise. This means that the accident date (or more properly, the inception date, since the question would otherwise arise "Was the Maternity an Accident?") should be determined by an estimate of the date of pregnancy. It will be found necessary to couple with this rule a rule to provide that the date used must not be prior to the date of coverage for the claimant.

Another matter on which I would like to comment is the Exposure Base for Miscellaneous Hospital Benefits. Mr. La Croix has indicated that a possible base could be the multiple or number of times which the miscellaneous maximum bears to the daily benefit. Another possibility is the miscellaneous maximum amount itself, which will permit the combinations of exposures without reference to the daily benefit amounts.

It may be worthwhile to consider recording the actual charges the hospital makes for daily room and board care and miscellaneous benefits, respectively, and compare these amounts with the amounts actually paid under the coverages. A study has been made of these results on a very limited exposure which shows that if it is desirable that the Insurance Plan pay the same percentage of actual charges for room and board as for miscellaneous benefits, the optimum miscellaneous benefits could be determined by the formula: daily benefits times the quantity daily benefits plus one dollar. Recording actual charges made by surgeons and comparing with amounts paid under the surgical scale in use will likewise permit computation of the factor necessary to apply to a given surgical scale to produce benefits equal to any predetermined percentage of actual charges.

The problem Mr. La Croix posed of finding a method by which to translate taxable payroll limited to the first $3,000.00 of wages per calendar year to Weekly Indemnity Exposure may be illustrated by an extreme example as follows: Considering two groups of ten lives each, written on July 1. The first group, let us say, consists entirely of people whose salary is $3,000.00 per year. The taxable payroll for the period July 1 to December 31 will be $15,000 and the weekly indemnity exposed $250.00 a week for the six months. The second group, we will assume, consists entirely of professional men whose salaries are $6,000.00 per year or more. Since the first $3,000.00 will have been taxed prior to July 1 there is simply no more taxable payroll in that calendar year, but the weekly indemnity exposure is the same amount—$250.00 a week for six months! During the second six months of the policy

year (the next January 1 to June 30) the taxable payrolls will balance out as between the two groups but the first situation will exist during the first six months of the second policy year, with the ultimate result that the second group at the end of any calendar year will have six months more weekly indemnity exposure than taxable payroll exposure. The purpose in drawing this comparison, which of course has been exaggerated to illustrate the point, but on the other hand has been over-simplified by not considering the results of employee turnover, is to point out an important element of the problem. It would indeed be beneficial if someone could come up with an answer to this seeming impasse.

The above discussion is in no way intended to be a criticism of Mr. La Croix's excellent paper. The first section, particularly, describing the principal types of coverage is an unusually concise summary which should prove most valuable to the student of this subject. The second section outlining a possible statistical plan certainly sets out the important elements on which it is difficult to improve.

ON NON-LINEAR RETROSPECTIVE RATING

CHARLES W. CROUSE

Volume XXXVI, Page 35

WRITTEN DISCUSSION BY F. S. PERRYMAN

It was with a great deal of interest that I heard Mr. Crouse present his paper at the November 1949 meeting of the Society. With the aid of a blackboard and a piece of chalk, Mr. Crouse made one of the most interesting presentations I have ever heard of a paper as mathematical as this one. I happen to like mathematical papers and I was very happy to listen to a paper of this kind being explained so clearly. Of course, Mr. Crouse merely went over the main points of his argument and I subsequently spent a good many hours, pleasant hours, reviewing his mathematics in more detail. It is very desirable that from time to time the more mathematically inclined of the Society should take the time to investigate the mathematics of the rating plans and processes that are used in our business. The plans as finally developed for use have, for practical reasons, to require nothing but quite elementary arithmetic but this does not mean that the mathematical concepts behind the plans should not receive a thorough going over and the results recorded in the *Proceedings* of the Society. It is also desirable to have some of the mathematical tools which are now available in the actuarial and statistical field displayed in action in the *Proceedings* of the Society from time to time. One of the benefits which may come from doing this is that other students of the business will be encouraged to investigate and to see what they can do with these mathematical weapons that have been recently brought into use.

With regard to the mathematics of Mr. Crouse's paper I have little comment to offer; for after study I find no reason for disagreeing with the mathematical treatment in the paper. The mathematical techniques are just the kind that should be used in such investigations. However, I would like to make some comments upon the theoretical and practical considerations treated by Mr. Crouse.

In the paper Mr. Crouse commences with a couple of short sections; an introduction dealing with the reasons that caused him to embark on this study of non-linear retrospective rating and a section giving his terminology and notation; this latter section was probably rather terrifying to those not mathematically inclined and requires rather close attention but is perfectly sound and necessary since all mathematical disquisitions should commence by defining precisely what is to be discussed. Next comes Section B with a discussion of conditions of the practicability of retrospective rating plans. One of the arguments put forward here is that unlimited retrospective plans, that is to say, plans without maximum premiums, are not entirely unpracticable; for example, the insured under such a plan can be as much protected as an insured who carries merely excess insurance. This may be so theoretically, but in practice it would seem desirable and not very difficult to arrange for an upper limit to the retrospective premium. Another one of his arguments apparently is in the direction of insisting on the theoretical necessity of the method of rating producing results "exact" to say five or six significant figures. However, in practice since insurance is in any event a question of averaging and all the underlying probabilities are always subject to a certain amount of uncertainty, plans need not and should not be constructed with a verisimilitude of exactitude; all we need to do is to provide a proper degree of rough equity and to see that the progression of values is in the right direction so as to avoid anomalies, e.g. with increasing losses we should have increasing premiums. Thus, even if theoretically the proper curve of a certain function can be determined, in practice it can usually be approximated to by a straight line or a limited number of linear segments.

In the next section, C, Mr. Crouse discusses sectionally linear retrospective rating formulae and deals with the difficulties in choosing the parameters for such plans. Here the question is about a different kind of difficulty. In discussing retrospective plans there are two kinds of difficulties: (a) in deciding just what the maximum and minimum premiums and other parameters shall be and just how the values shall progress from one premium size to another and (b) in determining the proper insurance and expense charges to be put into such plans. The first kind of difficulty is the one which is raised in this Section C, and it must be recognized that this is quite a different kind of difficulty than the second kind which in practice involves the determination of the proper excess pure premium charges. Most of the balance of Mr. Crouse's paper deals with problems analogous to the determination of these excess pure premium charges but in Section C he is concerned with the difficulties of fixing the swing and other characteristics of retrospective plans and the two kinds of difficulties should be sharply distinguished by students.

In the next section, D, Mr. Crouse proposes for consideration a certain type of non-linear retrospective rating formula, one involving an exponential function. It is, of course, not to be wondered at that exponential functions should be brought into the picture. These functions are very closely associated with many of the phenomena investigated by actuaries and other scientists. Thus in the life actuarial field we have the Gompertz and the Makeham formulae for life functions and, of course, in the theory of interest the exponential function is the basis of mathematical treatment. Actually the formula proposed by Mr. Crouse is exactly of the same type as the formula for the value

of an annuity. Thus the tables and mechanics which I gave in my papers on "Tables Adapted for Machine Computation" (PCAS XXV and XXXII) might well prove to be of use in making calculations for Mr. Crouse's formula.

In the last section, E, of his paper Mr. Crouse discusses the calculation of the parameters under his formulae and shows that if the frequency distribution of the (policy) losses of the risks to be retrospectively rated is known the parameters can be determined by use of certain Laplace-Stieltjes transforms and that if the frequency distributions can be satisfactorily described by certain forms of Pearson's Type III (which seems a reasonable hypothesis supported by some of Mr. A. L. Bailey's work) then the parameters can be readily determined, for the Laplace-Stieltjes transforms are comparatively simple. In other words, it seems likely that the kinds of frequency distributions with which we are concerned in our current applications of retrospective plans to casualty insurance are such that the use of a curved and asymptotic retrospective formula of the type suggested in the paper will render simple the calculation of the values of the function that under this scheme takes the place of the excess pure premium ratio. This is a notable advance in the theory of retrospective plans for it is, of course, obvious to all actuaries that the Achilles heel of recent applications and extensions of retrospective plans to different kinds of insurance and combinations thereof is the assumption (which has usually had to be made for practical convenience) that the excess pure premium table calculated for a certain kind of compensation insurance gives satisfactory results when used for other kinds of compensation and other lines of business.

Of course it must not be inferred that the author seriously argues for the necessity of the universal replacement of linear retrospective plans by non-linear ones. He merely shows what could be done in this direction. This is perhaps analogous to certain phases of the problem of credibilities for experience rating plans where it was shown that there should be used certain curves which afterwards were modified so as to be approximated to by more easily handled straight lines. In this credibility problem the curves could possibly have been used as far as the use of the rating plan was concerned for the whole calculation process was reduced to the use of tables anyhow: in the case of retrospective rating plans the situation is different for, because of the need for providing values for so many different premium sizes, the retrospective rating has usually been expressed in the form of a formula and it is obviously very much simpler to do this with a linear plan than with a non-linear one; there are however some non-linear plans in effect, e.g. some of the dividend plans where the dividend rate varies with the premium size and the loss ratio.

I am not sure that I would agree that approximate values of the required functions cannot be obtained for linear retrospective plans or for that matter for non-linear plans of a type different from that suggested by Mr. Crouse. The excess pure premium tables used to date have been obtained by direct observation. If we are going to substitute for that a method of calculation from frequency curves this means that we must first fit frequency curves to the observed distribution of policy losses. I would like to see Mr. Crouse or someone else try his hand at developing practical methods of approximating more directly to the probability values required in ordinary linear retrospective rating plans. I am sorry anyhow that the author didn't give some

numerical examples in his paper; it would have helped students to follow his mathematical arguments and might have highlighted the need for developing a reasonably quick but reasonably accurate means of determining excess pure premiums and similar functions for both linear and non-linear plans.

In conclusion Mr. Crouse's paper is not only interesting but also very stimulating. It is a pleasure to read it and I hope it will encourage others to pursue the lines of investigation opened up and suggested by this worthwhile contribution to casualty actuarial science.

FURTHER REMARRIAGE EXPERIENCE

ROBERT J. MYERS

Volume XXXVI, Page 73

WRITTEN DISCUSSION BY EDWARD S. ALLEN

Mr. Myers' paper is an important step toward the general acceptance of a remarriage table based on American experience. He has utilized in his study a considerable volume of experience which was not available when the American Remarriage Table was developed. It appears, however, that these underlying data must be considered with caution since they are heavily weighted with wartime and postwar experience.

Mr. Myers has noted the fact that the comparisons with expected remarriages are probably affected by the large number of marriages in the early postwar period and by the war in creating relatively more young widows. He also notes that the major part of the upward trend from 1940 to 1946 indicated in Table 8 is undoubtedly due to an actual increase in the rate of remarriage, but that it may result to some extent from the fact that longer durations of widowhood have greater relative weight in the later calendar years. The actual increase in the rate of remarriage may well be due in substantial degree to the inclusion of war widows in the experience. In regard to the increased weight of the longer durations of widowhood, it would appear that this consideration would have a relatively small effect. A review of the ratios of actual to expected remarriages by duration in Table 2 is as follows:

Duration	*Ratio*
0	96%
0 – 1	88
0 – 2	95
0 – 3	95
0 – 4	99
All Durations	106

There is a slight upward trend in the ratios as longer durations are added, but the effect would probably not be great, particularly since durations of five or more years have relatively less weight in Table 8 than in Table 2 from which the above ratios are obtained.

These considerations suggest that some of the data studied reflect a temporary condition and that Mr. Myers' second alternative of a table including

150% of the basic rates in the original American Remarriage Table might be modified downward to some extent.

It is hoped that Mr. Myers' study will encourage reliance upon American remarriage experience and that a further study can be made as soon as it is possible to eliminate experience affected by wartime and immediate post-war conditions.

VALUATION OF DEATH BENEFITS UNDER U. S. LONGSHOREMEN'S AND HARBOR WORKERS' COMPENSATION ACT AS AMENDED

JUNE 24, 1948

SYLVIA POTOFSKY

Volume XXXVI, Page 105

WRITTEN DISCUSSION BY RUSSELL P. GODDARD

The members of Congress displayed a very fine unconcern for actuarial problems when they amended the U. S. Longshoremen's and Harbor Workers' Compensation Act on June 24, 1948. By selecting benefit percentages different from those in either the present or the previous New York law, they made the more important tables in Special Bulletins 207 and 222 unavailable for use in valuing U. S. Longshoremen's death benefits. If the volume of stevedoring business were larger, and if there were to be no further revision of the law, and if the companies could be satisfied with a 3% interest rate, it would be feasible to construct a new set of tables covering all, or almost all, possible situations. These are three big ifs, and Miss Potofsky has met the problem by indicating the New York tables which may be used directly, and by constructing additional tables to cover the remaining cases, except those involving a widow and three or more children. For these cases, which comprise about 14% of the total, she has prepared a set of formulas whereby with a clever manipulation of values already available, new values can be obtained for the third, fourth and fifth youngest children. These formulas, although somewhat cumbersome, probably represent the easiest way out of a difficult situation, and would not present much difficulty to an organization which handled enough such cases to justify the establishment of a routine procedure. Nobody wants to add to Bureau expense, but it would be more economical if the complicated claims could be valued by a rate-making organization rather than by the companies themselves. Miss Potofsky has already saved the companies considerable expense by making available the fruits of her ingenuity and industry.

SEASONAL FLUCTUATION IN LOSS RATIOS FOR AUTOMOBILE BODILY INJURY COVERAGE

JOHN W. CLARKE

Vol. XXXVI, Page 63

WRITTEN DISCUSSION BY CHARLES W. CROUSE

In casualty insurance, as in astronomy, meteorology, demography, economics, and a number of other fields of human endeavor, we are much concerned with statistical series of a kind generally called time series. Yet, unlike astronomers, meteorologists, demographers and economists, casualty actuaries in America have paid almost no attention to the development of mathematical methods in time series analysis, though in that development a number of European actuaries have played important parts. In my opinion, Mr. Clarke's paper is noteworthy principally because, by directing attention to the practical importance of adjustments for seasonal variation in time series of one particular class, it moves us to abandon our complacent neglect of refinements in the analysis of time series in general.

Mr. Clarke's discussion of the application of indexes of seasonal variation in automobile bodily injury liability loss experience, seems to me to be entirely commendable; and I agree with his conclusions relative to the value of the results which may be obtained by their proper use. But there are a few critical comments which I think should be made concerning the method which he employed in constructing the particular sets of index numbers appearing in Table II of his paper.

As he has been very careful to point out, there is an upward trend in his basic data, especially the data for 1946-48, due to increases in the number of policies in force, the number of automobiles per policy, and the average value of claims. No correction for this trend was made. Consequently, the indexes of seasonal variation in Table II (at least those for 1946-48) are distorted: those for January through June are too low and those for July through December are too high. The distortion is—of course—greatest for January and December and possibly negligible for June and July. Proper correction for this trend would have been difficult because of the diversity of its causes. Therefore, it seems to me that, in constructing indexes of the seasonal variation in question, it would have been better to use Persons' *Method of Link Relatives*,* chaining the geometric or the arithmetic means of the link relatives rather than their medians because of the smallness of the numbers of years covered by the basic data. In that method there is what may be called a built-in correction for a trend such as that to which I have just referred.

However, the effect of the saw-tooth wave in the exposure under Massachusetts statutory coverage, would not be eliminated by the Method of Link Relatives any more than it is by the method which Mr. Clarke employed. In order to obtain indexes of seasonal variation free of that effect, it would

*See Persons, W. M. *Correlation of Time Series* in *Handbook of Mathematical Statistics* edited by H. L. Rietz, Houghton Mifflin, Boston, 1924: Davis, H. T. *The Analysis of Economic Time Series*, The Principia Press, Bloomington, Ind., 1941, pp. 237-240; Mills, F. C. *Statistical Methods*, Holt, New York, 1924, pp. 318-321.

be necessary either to make corrections for it in the basic data, or else to eliminate the losses under Massachusetts statutory coverage from the basic data entirely.

In both of his sets of *Basic Ratios*, the minimum is the ratio for April, and there is a dip in November. But in both sets of *Final Corrected Ratios*, the graduation process moved the minimum from April to June and ironed out the November dips completely. There is no reason to regard these alterations as closer approximations to the "truth". There may be something about the holidays in November or the gray November weather which reduces automobile travel in that month below the levels of October and December. I am not criticizing Mr. Clarke's choice of the particular process of graduation which he employed. But rather I am suggesting that indexes of seasonal variation in anything which is subject to so many complex influences as automobile bodily injury liability loss experience, should not be subjected to any process of graduation whatsoever provided they have been constructed on the basis of such a large body of data as that with which Mr. Clarke worked.

Vol. XXXVII, Part II No. 68

PROCEEDINGS

November 17, 1950

THE ENIGMA OF THE PERMISSIBLE LOSS RATIO

PRESIDENTIAL ADDRESS BY HARMON T. BARBER

According to well established precedent, each regular meeting of this Society is destined to start with an address or comments by the President on some subject of his own selection and prepared independently of the review which is imposed by the Committee on Papers on other contributors. The time consumed by this address provides an opportunity for late arrivals to sift into the vacant seats up front, for the Secretary to shuffle his papers and notes so that the balance of the meeting may proceed in an orderly fashion and gives the members a chance to discover by experimentation that there just is no one position of tolerable comfort to be found in the straight backed chairs in which they are expected to spend the day. The time is not wholly wasted, therefore, even though the moral truths or comments on current events expressed by the speaker may be discarded mentally by the audience even before the echoes have subsided.

However, in all sincerity, there is a brief message which I wish to bring to you today—not a new one,—but one which can bear repetition from time to time. It is a plea for a more abundant and spontaneous participation by the members in the writing of papers for our semiannual meetings. The objective of this Society is the promotion of actuarial and statistical science by such means as the presentation and discussion of appropriate papers. Activity of this nature could be unusually strong at the present time, as most observers will agree that there never was a time when the casualty insurance business was more beset with important problems, each requiring the application of some phase of actuarial science for solution and each suggesting the basis for a formal paper. Most of us are inclined to overlook these opportunities in our concern to get on to the next problem. Nevertheless, there is an obligation to leave a permanent record of the thoughts and decisions reached in current studies so that future deliberations may go forward without the necessity of again traversing ground previously covered. The *Proceedings* can furnish such a record but it will be decidedly incomplete unless the facilities of the Society are utilized to a greater extent than has been the case in the recent past.

Some subjects are intricate and involve the theory and practice of specialized branches of mathematics or statistics while others are of an elementary character and can be handled with ease using nothing more than applied common sense. The fundamental question of whether the current experience of a carrier for a line of insurance such as Workmen's Compensation is profitable

or unprofitable is a good illustration of the latter. There is nothing more basic or elementary than this, yet a little reflection will indicate that the problem is far from simple.

For many years it was common practice to compare the actual loss ratio for any group of Workmen's Compensation risks with a permissible loss ratio of sixty percent and if the actual loss ratio was less than this standard, the business was considered to be profitable to a corresponding extent and vice versa, if the actual loss ratio exceeded sixty percent, it was assumed that the carrier was losing money at a rate equivalent to the difference. The application of such an approximate measure may have been acceptable prior to the introduction of graded expense provisions and other factors tending to further disturb the 60/40 relationship of losses and expenses, but today a more accurate analysis is indicated as being essential to a proper interpretation of experience results.

To outline a method for a more exact evaluation of experience is a problem worthy of serious study and effort. Obviously, it will not suffice to merely establish the actual loss ratio of the experience under review for comparison with an average standard permissible loss ratio as such an approach brushes aside important factors which have a definite bearing on the issue. The actual loss ratio itself is surrounded with some uncertainty as to its worth as an index of current experience levels. Incurred but not reported losses, the possible underestimate of the ultimate incurred cost of reported cases, and the extent to which earned premiums are affected by unrealistic advance or deposit premiums are some of the points to be examined to see whether adequate allowances for these have been included in the actual loss ratio so as to make valid and reliable the comparison of the adjusted actual loss ratio with a permissible loss ratio appropriate for the business under review. Much could be written on these and other aspects of the actual loss ratio but the major concern of these comments is the permissible loss ratio and its use for comparison purposes.

The propriety of using a standard permissible loss ratio such as sixty percent for Workmen's Compensation insurance may be challenged from several angles:

1. The permissible loss ratio of minimum premiums is considerably less than that for other than minimum premiums because of the additional provision for expenses in the formula by which the minimum premiums are determined. Therefore, the average permissible loss ratio to be used as the criterion of underwriting profit for any group of risks should be adjusted to reflect the proportion of minimum premium risks in the group.
2. In some states an expense constant of $10 is collected as a part of the premium for risks up to several hundred dollars in premium size. It is evident that the effect of an expense constant on the permissible loss ratio for a very small risk can be substantial and yet be of much less consequence in its effect on the permissible loss ratio of a risk paying several hundreds of dollars of premium. Thus, the appropriate permissible loss ratio for a particular group of risks should be adjusted to reflect the premium from expense constants.

3. Abnormal state premium taxes or special assessments such as for the support of industrial commissions, likewise have a direct bearing on the permissible loss ratio. Abnormal tax loadings are incorporated in manual rates and apply generally to all risks within the state regardless of the risk premium size. From this it follows that the distribution of business by state is an important consideration in adjusting the permissible loss ratio to conform with the experience under review.
4. In some states where graded expense provisions are in effect for risks written on a guaranteed discount basis it is obvious that the permissible loss ratio will be markedly affected by the variations in these provisions for expenses. Therefore, recognition should be given to the proportion of risks of a size sufficient to qualify for these premium discounts and to their departure from normal expense provisions.
5. The proportion of business subject to retrospective rating will likewise have a pronounced effect upon the appropriate average permissible loss ratio for the experience under analysis. The permissible loss ratio for an individual retrospectively rated risk depends upon the size of the risk premium, the character of the risk experience and the status of premium adjustment for the risk. The size of the risk standard premium determines the expense provisions for certain items of expense. The character of the risk loss experience will determine whether the risk pays the minimum retrospective premium, the maximum retrospective premium, or some intermediate amount of premium and yet the provision for certain expenses will be the same in amount under any of these conditions. Therefore, the permissible loss ratio applicable to the risk premium after retrospective adjustment will differ according to the risk's actual losses. Furthermore, the premium initially charged on the carriers records for a retrospective risk is the standard premium and remains at this level until after the first adjustment of premium, which normally does not occur until eighteen months after effective date or later. It is obvious that the permissible loss ratio will change perhaps materially after this first adjustment. If the determination of a suitable permissible loss ratio is difficult for an individual retrospectively rated risk, it is evident that any combination of retrospective risks with or without the addition of non-retrospective risks of varying size could present a baffling problem in the determination of the appropriate permissible loss ratio for the group on any basis other than summing the results of a risk by risk analysis.

There is another angle to this drift away from a standard permissible loss ratio which might have interesting consequences. The loss reserve laws of many states are reflected in the provisions of Schedule P of the annual statement blank. As most of us know, the Schedule P formula loss reserve for compensation insurance for the three most recent policy years is required to be not less than sixty-five percent of the earned premium less losses and loss expenses paid. So long as there was a permissible loss ratio roughly equivalent to sixty percent, the Schedule P formula had some logic to support its use. However, if the true permissible loss ratio for an individual carrier is several points higher than the old standard of sixty percent, the force and effect of

the minimum reserve provisions of the formula reserve are obviously weakened. There appears to be no practical amendment of the Schedule P formula which will recognize such a situation in a manner appropriate for the distribution of business of the individual carrier. It is evident from a review of the preceding comments that actual premiums and losses for Workmen's Compensation insurance taken from the customary company records are not susceptible to immediate and accurate evaluation to determine whether the risks represented in the experience are profitable or unprofitable. The gravity of the situation is probably not widely appreciated, since many of the innovations in rating which have undermined the significance of the old underwriting concept of a standard permissible loss ratio of sixty percent have come into being during a period of years when underwriting results have been favorable more frequently than otherwise. When the reverse situation prevails and greater attention is centered on the loss producing propensities of various subdivisions of the business, it will be disconcerting to discover that the only true test of underwriting results is the combination of loss and expense ratios compared with one hundred percent. Since company expense ratios are usually determined for the countrywide operations of a single line of insurance, it is apparent that such a test is not available for a careful evaluation of the underwriting results for a smaller subdivision of coverage. Therefore, it is evident that there exists a problem of major proportions,—a challenge to restore to the actual loss ratio its former characteristic as a convenient index of underwriting results, or to devise some other means of facilitating estimates of underwriting profits for portions of the business.

There are several possible solutions to the problem which might be explored and evaluated. Some of them involve considerable additional expense and difficulty and hold no certain promise of satisfactory results. For example, one possibility would be to segregate those portions of the business which are most extreme in their variation from normal by coding all such premium items as a sub-line of business. Then if it were found impracticable to analyze this sub-line exactly, at least the balance of the business could be tested with some assurance of its being relatively free from distortion.

A second approach to the problem might be to provide for obtaining premiums at manual or standard rates by calculating the standard premium for each individual risk and either recording this premium in duplicate or by supplementing collectible premiums with a specially constructed record of the differences between standard and collectible premiums. It seems quite evident that such a method would require a material amount of additional work on premium items, including computation, recording, and tabulating of the special premiums.

A third alternative might be to code each premium item to designate the ratio of the provision for losses to premium of the particular item being handled. A two-place direct translation code would suffice for this purpose and, if found practical, extra digits could be added to the code to indicate the provisions for other acquisition costs and for administration and payroll audit costs in percent or tenths of a percent of premium. By mechanical tabulation processes, the aggregate provisions for each of these coded items could be ascertained and suitable average percentages for the entire business thus determined.

Under each of these plans, excepting possibly the first, the amount of additional work presents a discouraging prospect. Not only is the amount of detailed work voluminous but parts of it require careful study or analysis by skilled persons who are experienced in compensation rating methods and practices. None of these suggestions therefore may provide a satisfactory answer to the dilemma of what to do about determining an appropriate permissible loss ratio.

There is another means of dealing with the problem under review which would require a marked change in rating methods for Compensation insurance. It is not a plan which can be adapted to existing procedures and, therefore, may be considered to be somewhat visionary in character. It is of interest as an illustration of how a rearrangement of certain elementary or basic practices occasionally may suggest a way of attaining an objective previously considered to be impracticable.

The principal characteristic of this solution to the problem is a division of all compensation premiums into two parts which for convenience may be designated as Type A and Type B premiums. Every compensation risk would pay a premium composed of these two elements. Type A premium would be determined as the product of risk classification payrolls and a distinctly different system of manual rates so constituted as to provide only for losses, claim adjustment expenses, inspection costs, and taxes on this part of the risk premium. With the exception of taxes, these items all relate to expenditures made directly for the benefit of the insured employer or his employees. There would be no designed expense gradation by risk premium size in this part of the risk premium. It might be necessary to utilize loss constants or some other device as a supplementary source of premium income for small risks in order to equalize loss ratios by size of risk. Prospective experience rating and retrospective rating would be applied to Type A premium but would not be applied to Type B premium.

Type B premium would be developed by the application of a schedule of rates or premium charges to the total risk payroll for all classifications combined. This premium would contain provisions for acquisition, administration and payroll audit expenses with the concurrent taxes for this part of the premium. The items of expense included in Type B premium are necessary costs encountered in providing the protection, benefits and services which are covered by Type A premium. The schedule of Type B rates would provide for a material gradation of premium by risk size and either could consist of a single schedule to be applied uniformly to all risks regardless of risk hazard or could be adjusted to conform with risk hazard on risks of sufficient size (say, risks eligible for experience rating) by applying a separate modification to risk Type B premium. This modification might be equivalent to the ratio of the risk average Type A rate to the statewide average Type A rate, the former being obtained readily by dividing the risk Type A premium by the risk exposure. The choice would depend on the decision reached after careful study of whether a simple method would suffice or whether it would be advisable to adhere to precedent in the matter of making all expense provisions functions of pure premium. It is felt that the complications created in the latter event would not make the method impracticable although it is questionable whether the refinement is otherwise essential. Under either alter-

native the Type B premium could be split by specific percentages to determine the portions allocated for acquisition costs, administration and payroll audit expenses. It might be that the schedule of Type B rates or premium charges could be so established as to avoid the necessity for expense constants and minimum premiums. As stated previously, it is contemplated that Type B premium would not be subject to modification by reason of experience rating or retrospective rating.

For clarification it may be well to indicate the manner in which the special manual rates for Type A premium might be established. Presumably, the Type A classification rates would be based upon pure premiums determined according to present methods. To these pure premiums would be applied a loading for claim adjustment expense and for inspection expense equivalent to the standard provision today, namely (.08 plus .025) divided by .60. The tax provision in Type A rates would be included by using a factor in the pure premium multiplier of 1.000 ÷ .975 = 1.026. Thus, the formula for determining Type A rates in a state employing a forty percent expense loading today would be 1.211* times the pure premium for the classification. To this there might be added a provision for correcting the off-balance created by experience rating. In keeping with present-day practice, the percentage provisions for claim expense and inspection expense need not vary by state nor by risk premium size. Therefore, it should be possible to calculate the permissible losses, the provision for claim adjustment expense, and the provision for inspection expense by applying certain specific percentages to Type A premium for any combinations of states and risk premium sizes. Unusual state taxes might necessitate some exceptional treatment to this general rule.

The elements of Type B premium, on the other hand, are those which are graduated by size of risk premium under present rating methods. Whether Type B premium is determined by use of a single schedule of rates in all states and for all risks or whether Type B premium is adjusted to the pure premium level of the individual risk, Type B premium may be subjected to the principle that producers and carriers would contribute proportionately in the discounting of premium by risk size. Thus commission payments might be determined as a constant percentage of Type B premium for all risk sizes. For instance, it might be decided that Type B premium could be analyzed as follows:

Commissions	40%
Other Acquisition	20%
Administration and P.R. Audit	37½%
Taxes	2½%
Total	100%

Except for minor variations in tax requirements a standard analysis such as this could be applied in all states and even differences due to variations in state taxes might be disregarded in the interests of simplicity.

In order to better visualize the schedule of Type B rates which the author has in mind the following is set forth as illustrative. Obviously Type B rates could be established only after study and investigation including careful con-

*$\frac{.08 + .025}{.60} \times 1.026 = 1.211$

sideration of whether a uniform set of rates or variable rates by risk are to be preferred for this element of premium.

Total Risk Payroll	*Type B Manual Rate*
First $ 5,000 or less	$25.00 Flat Premium
Next 5,000	.50 Per $100
Next 90,000	.30 Per 100
Next 400,000	.15 Per 100
Over 500,000	.05 Per 100

By establishing a flat premium for the first $5,000 of payroll or less, the necessity for separate minimum premiums and expense constants may be avoided. A very low rate for the uppermost payroll bracket avoids an apparent overcharge in premium for extremely large risks such as those in the clerical office group. The rates quoted are merely for illustrative purposes and bear only slight resemblance to those which might be established if the suggested program were to be followed. As mentioned previously, the Type A rates would be modified by experience rating prior to their use in determining the premium for an individual risk. Likewise, if retrospective rating were elected by the risk, it would also be restricted to Type A premium. Whereas no major change would be involved in the application of experience rating to this part of the risk premium, other than possibly a liberalization of credibility, it is apparent that there would have to be a revision in existing retrospective rating plans in order for them to apply properly to Type A premium only. It is contemplated that the separation of the two parts of the risk premium would be carried through the accounting and experience records of the carrier as this would permit a proper interpretation to be made of the usual experience records as respects underwriting profit or loss without encountering too many complications.

It would appear as though the suggested plan has certain advantages. It might be expected that the Type A premium would represent about three-quarters of the total compensation premium and this would be based upon rates which could be considered to be keyed to the requirements of all carriers regardless of type. In other words, in this part of the premium there would be less disparity as to expense provisions than exists today for different types of carriers in total expense provisions. Differences of opinion between types of carriers as to reasonable and adequate provisions for expenses by size of risk might be largely confined to the premium produced by Type B rates. In the event that differing views as to the requirements for certain elements in Type B rates could not be reconciled, it would be entirely practical to establish different schedules of Type B rates for groups of carriers or for individual carriers.

In some respects the suggested program might result in more appropriate premiums for individual risks. The suggested plan limits the application of experience rating to Type A premium exclusively. With a properly designed schedule of Type B rates it should be possible to provide a more appropriate return to agents and a more appropriate provision for the expenses of the carriers in the premiums for small risks. The elimination of expense constants and minimum premiums produces a desirable simplification from present

procedure. It would seem as though certain accounting and commission payment difficulties present under today's procedure will also be diminished. Finally, the expected losses for any block of experience may be readily determined as a fixed percentage of Type A premium, thus restoring the loss ratio to its former position of utility and importance as the keystone of underwriting.

The suggested method has certain disadvantages, the most important of which perhaps stems from the fact that the plan represents a marked departure from present procedure. By limiting the application of retrospective rating to Type A premium it is not practical to make the return of expense savings due to risk size appear to be contingent upon the risk loss experience as was the case under early retrospective rating plans. Likewise, it would not be practical to use undiscounted standard premium as the base for retrospective rating since under the suggested system this premium would not be available without special calculation. It is probable that it might be necessary to make some changes, therefore, in the principles of retrospective rating and this might be considered to be a definite disadvantage. The dual premium system probably would require at least one extra premium entry on every policy and probably would involve extra expense in record keeping. However, there appear to be sufficient advantages to warrant giving further consideration to some of the principles of the suggested system.

It is not the author's intention that these comments be construed as advocating the substitution of the new system for the present one. However, many a new design for a machine has never progressed beyond the drafting room stage yet has proven useful in the creation of some later model. Whether or not the suggestions set forth will have an influence in the development of some future rating system is of little consequence at present. The principal objective and motive in preparing these comments was to show by example how some simple angle of the casualty insurance business may be singled out and subjected to what might be termed actuarial engineering of a crude sort to form the basis for a contribution to the *Proceedings*. There are many opportunities of this nature in the many lines of insurance in which we are interested, as well as other subjects which may be treated in the form of narrative reports of actions taken or decisions reached in various rating conferences, which would considerably enhance the value of our *Proceedings*. A more liberal expression of ideas in the form of papers and resulting discussions would greatly assist the Society in fulfilling its avowed mission of "the promotion of actuarial and statistical science as applied to the problems of casualty and social insurance."

AUTOMOBILE ACCIDENT STATISTICS BY "AGE OF DRIVER"

BY

LAWRENCE W. SCAMMON

In the belief that the recording of the significant original statistical compilations of automobile accident experience by "age of driver" may be of value to the Society and to the Insurance Industry the writer presents the available statistical material on this subject.

With the advent of the speedy light car and the disappearance of the Model T Ford in the early '30's the death knell of the W, X, Y differential rating of private passenger cars by weight, power and purchase price was sounded. Combined W and X differentials sufficed for a while but soon a car was a car for insurance rating purposes and competitive weapons of differing types were forged to entice the accident free car owner.

In February 1938 the safe driver award plan made its debut but when New York failed to go along and the plan ran into other heavy going the first "age of driver", in combination with "mileage" and "business use", plan made its appearance at the National Bureau in April 1939 and later in the year in New York. In this plan the three classes are summarized as follows:

Class A-1—Non-business use, estimated annual mileage not in excess of 7500 miles, no more than two operators in the household none of whom is under 25.
Class A—Non-business use, not eligible for Class A-1.
Class B—Business use.

Backing up "age of driver" as one of the rating elements were certain figures collected by the Connecticut Motor Vehicle Department in 1932-1936 which indicated that drivers under twenty-five were involved in 37% more accidents and 62% more fatal accidents than the average. As far as mileage was concerned this was considered a very tangible measure of hazard which could be easily substantiated with logic and certainly business use would usually mean greater mileage. The differentials chosen were not severe and picked with an idea to possibly widening them as experience indicated the need. The pre-war experience by policy year for states where Classes A-1, A and B were in effect, excluding Indiana and New York, is as follows:

			Bodily Injury			
Policy	*Distribution %*			*Pure Premium*		
Year	*A-1*	*A*	*B*	*A-1*	*A*	*B*
1939	21.2	57.6	21.2	$7.30	$11.97	$17.31
1940	20.8	56.2	23.1	7.64	12.22	14.68
1941	21.4	58.8	19.8	7.46	12.50	13.77
			Property Damage			
1939	22.2	55.3	22.5	2.44	3.83	5.48
1940	21.6	53.9	24.5	2.87	4.31	5.35
1941	22.2	56.8	20.9	3.19	4.81	5.95

Under this plan "age of driver" and "mileage" factors were not differentiatable but no immediate thought was given to separating them. Soon the country was plunged into war and faced with gas rationing making it imperative that insurance premiums vary with amount of gas ration. Statistics later substantiated this rating basis. The war-time statistics in Massachusetts showing a distribution of claim costs under A, B and C gasoline rations so definitely aligned mileage and claim costs that many considered that the whole rating answer was involved in mileage alone. It was not immediately apparent that "age" as a rating factor was practically non-existent during the war with a large proportion of the young drivers in the service. Massachusetts war figures for private passenger cars are as follows:

1943 PRIVATE PASSENGER CAR EXPERIENCE
Statewide

Ration Symbol	*Earned Car Years*	*% Dist.*	*Losses Incurred*	*No. of Claims*	*Claim Freq.*	*Ave. Claim Cost*	*Pure Prem.*
A	298 497.70	47%	2 792 352	9 077	3.0	308	9.35
B	194 010.50	31	2 217 633	7 419	3.8	299	11.43
C	139 321.10	22	2 425 032	8 490	6.1	286	17.41
Total	631 829.30		7 435 017	24 986	4.0	298	11.77

1944 PRIVATE PASSENGER CAR EXPERIENCE
Statewide

Ration Symbol	*Earned Car Years*	*% Dist.*	*Losses Incurred*	*No. of Claims*	*Claim Freq.*	*Ave. Claim Cost*	*Pure Prem.*
A	279 700.60	44%	3 266 920	10 235	3.7	319	11.68
B	198 914.90	32	2 519 433	7 901	4.0	319	12.67
C	154 745.70	24	2 804 622	9 026	5.8	311	18.12
Total	633 361.20		8 590 975	27 162	4.3	316	13.56

Immediately after the war a further attempt was made in Massachusetts to obtain statistics for rate making wherein both age and mileage factors would be recognized. In 1945 and in 1946 the Commissioner of Insurance under dates of September 21, 1945 and August 23, 1946 directed the insurance companies to obtain certain data with respect to the use of private passenger cars segregated by the age of the drivers using each car, by the expected mileage of the car during the subsequent year, and by business or non-business use. In accordance with the Commissioner's directions, in writing all compulsory liability insurance for the years 1946 and 1947 there was obtained from each applicant for insurance a signed questionnaire giving the pertinent

information. The questionnaires were obtained by the brokers and agents from their customers at the same time that the registration application was completed. The completed questionnaires were then transmitted to the insuring company and were utilized by the insuring company in preparing statistics in accordance with Massachusetts Automobile Statistical Plan requirements.

Obviously the base was hereby laid for the accumulation of by far the largest volume of statistics yet available on this subject. These statistics for 1946 policy year, not available until the summer of 1947, shed considerably more light on this whole area of age and mileage as factors in automobile rate making.

MASSACHUSETTS COMPULSORY LIABILITY EXPERIENCE

PRIVATE PASSENGER CARS—POLICY YEAR 1946

Private Passenger Car Experience Segregated Into Significant Age and Mileage Groups

Classification	*Earned Car Years*	*Losses Incurred*	*No. of Claims*	*Claim Freq.*	*Ave. Claim Cost*	*Pure Prem.*
Non-Business Use						
No op. under 25 yrs., under 7500 mi.	285 790.70	4 434 987	13 685	4.8	324	15.52
No op. under 25 yrs., over 7500 mi.	114 849.90	2 072 447	6 097	5.3	340	18.04
An op. under 25 yrs., under 7500 mi.	46 782.70	1 493 970	4 713	10.1	317	31.93
An op. under 25 yrs., over 7500 mi.	21 998.00	793.696	2 157	9.8	368	36.08
Total	469 421.30	8 795 100	26 652	5.7	330	18.74
Business Use						
No op. under 25 yrs., under 7500 mi.	50 261.10	877 725	2 783	5.5	315	17.46
No op. under 25 yrs., over 7500 mi.	75 417.30	1 512 280	4 978	6.6	304	20.05
An op. under 25 yrs., under 7500 mi.	8 080.10	234 641	792	9.8	296	29.04
An op. under 25 yrs., over 7500 mi.	13 363.80	441 454	1 300	9.7	340	33.03
Total	147 122.30	3 066 100	9 853	6.7	311	20.84
Classified to Age Only						
No operator under 25 years	11 622.20	233 613	741	6.4	315	20.10
An operator under 25 years	2 090.00	110 966	271	13.0	409	53.09
Total	13 712.20	344 579	1 012	7.4	340	25.13
Total Classified	630 255.80	12 205 779	37 517	6.0	325	19.37

A further refinement of the Age and Mileage plan was effected in National Bureau states in March 1948 when the Class A-1, A-2, A-3 and B plan became effective summarized as follows:

Class A-1—Individually owned cars—Non-business with no operator under age 25—annual mileage 7500 miles or under.

Class A-2—Individually owned cars—Non-business with no operator under age 25—unlimited mileage.

Class A-3—Individually owned cars—Non-business—no age or mileage limitations.

Class B—All other private passenger automobiles including those owned by corporations, partnerships or unincorporated associations.

The statistical background for this plan lies in the Massachusetts policy year 1946 Private Passenger Car experience segregated into significant age and mileage groups.

It began to be obvious from these statistics that while mileage had some minor bearing that the age factor was the deciding one from a rate making standpoint.

Policy year 1947 statistics, gathered in essentially the same manner under Massachusetts Statistical Plan requirements produced further substantiating results.

MASSACHUSETTS COMPULSORY LIABILITY EXPERIENCE
PRIVATE PASSENGER CARS—POLICY YEAR 1947
Private Passenger Car Experience Segregated Into Significant Age and Mileage Groups

Classification	*Earned Car Years*	*Losses Incurred*	*No. of Claims*	*Claim Freq.*	*Ave. Claim Cost*	*Pure Prem.*
Non-Business Use						
No op. under 25 yrs., under 7500 mi.	267 864.30	3 817 983	11 951	4.5	319	14.25
No op. under 25 yrs., over 7500 mi.	135 736.10	2 261 974	6 923	5.1	327	16.66
An op. under 25 yrs., under 7500 mi.	48 855.50	1 386 402	4 278	8.8	324	28.38
An op. under 25 yrs., over 7500 mi.	33 654.90	986 871	3 124	9.3	316	29.32
Total	486 110.80	8 453 230	26 276	5.4	322	17.39
Business Use						
No op. under 25 yrs., under 7500 mi.	38 975.40	819 337	2 499	6.4	328	21.02
No op. under 25 yrs., over 7500 mi.	68 348.60	1 417 520	4 569	6.7	310	20.74
An op. under 25 yrs., under 7500 mi.	7 060.40	231 585	679	9.6	341	32.80
An op. under 25 yrs., over 7500 mi.	14 732.00	479 866	1 549	10.5	310	32.57
Total	129 116.40	2 948 308	9 296	7.2	317	22.83
Classified to Age Only						
No operator under 25 years	29 770.20	568 821	1 694	5.7	336	19.11
An operator under 25 years	6 523.90	252 643	673	10.3	375	38.73
Total	36 294.10	821 464	2 367	6.5	347	22.63
Total Classified	651 521.30	12 223 002	37 939	5.8	322	18.76

To confirm that a section of the state would produce essentially the same results, the important county of Middlesex representing approximately 22% of the state was segregated from the rest of the statistics and separately analyzed with results so closely in line with the statewide figures that they are not repeated in this record.

It had become quite plain to insurance company executives that these Massachusetts facts pointed to "age of driver" as the determining factor rather than "mileage", hence recommendations to Massachusetts authorities followed this pattern both as respects policy year 1949 and 1950 rates. Classes and selected relativities which went approximately half way of indications were set up as follows:

Class 1—Individually owned cars—Non-business with no operator under age 25.

Selected relativity—75

Class 2—Individually owned cars—Business and Non-business with operator under age 25.

Selected relativity—100

Class 3—All other private passenger automobiles including all private passenger automobiles owned by corporations, partnerships or unincorporated associations.

Selected relativity—85

Failing to convince the Massachusetts authorities that this private passenger classification plan should apply to Massachusetts Compulsory Automobile Insurance, one large company proceeded to apply "age of driver" as a rating basis to Massachusetts Automobile Property Damage rates in 1949 and this basis was followed by practically all companies in 1950 as respects property damage with the same differential relativities as outlined immediately above.

That the effect of the Massachusetts 1947 policy year Private Passenger Car Experience Segregated into Significant Age and Mileage Groups was influencing underwriting opinion in other states is shown by the fact that in 1949 New York established a clearly defined low age group class with a substantial differential. Several other statistical compilations included in the following pages, particularly the "Veness Report" also had a marked influence on this decision in New York. And other jurisdictions were to follow this pattern.

Effective March 20, 1950 the same rating basis as that applicable to automobile property damage in Massachusetts was established for bodily injury and property damage in National Bureau states. The outline of classifications is the same as that set forth immediately above but sharper differentials more in line with statistical indications were selected: Class 1—60, Class 2—100, Class 3—87.5.

Other statistical studies involving smaller volumes of data have been made and are recorded here to complete the rather voluminous statistical record on this subject.

The following figures have been taken in summary form from data prepared by the Registry of Motor Vehicles of the Commonwealth of Massachusetts. The data are based on a sampling of 83,000 1948 and 1949 licensed operators from a total approximating 1,500,000, and from practically the complete record of 1948 accident involvements, totaling 42,709.

Age Group	*Percentage of Licensed Drivers*	*Percentage of Accident Involvements*	*Index Showing Ratio to Average*
16-24	14.8%	17.8%	1.203
25 & over	85.2%	82.2%	.965

The conclusions to be drawn from these figures are that, taken as a whole, operators in the age group 16-24 are involved in 20.3% more than their share of accidents, and conversely, operators in the age group of 25 and over are involved in 3.5% less than their share of accidents.

The study of the New York State Motor Vehicle Department covering 1947 accidents otherwise known as the "Veness Report", appearing in the January 1949 issue of "Best's Magazine" contains the following summary showing the New York Motor Vehicle Department's results from their study of accidents by age of driver.

	Licensed Drivers		*Fatal Accidents*		*Non-Fatal*	
Age Group	*Total*	*%*	*Total*	*%*	*Total*	*%*
Under 18	1 169	1.3	19	1.3	835	1.5
18-20	3 834	4.4	123	8.8	4 287	7.9
21-24	7 588	8.7	258	18.4	7 797	14.4
25-29	11 699	13.4	177	12.6	8 647	16.0
30-39	22 501	25.9	324	23.1	13 139	24.2
40-49	17 655	20.3	234	16.7	9 583	17.7
50-59	13 388	15.3	156	11.1	6 476	11.9
60-64	4 281	4.9	49	3.5	1 763	3.3
65 & over	5 054	5.8	64	4.5	1 677	3.1

		% of		*Above (+) or Below (−) Average*	
Age Group	*% of Operators*	*Fatal Accidents*	*All Accidents*	*Fatal Accidents*	*All Accidents*
Below 25	14.4%	28.5%	23.8%	+97.9%	+65.3%
25 & over	85.6	71.5	76.2	−16.5	−11.0

Additional data appearing on the driving record of youthful automobile operators comes from the New Jersey State Safety Council, Inc. and appears in the August 1949 issue of "Safety Briefs" which is published monthly by this Council. I quote the following from the article in "Safety Briefs" entitled "Fatal Accident Rates of Youthful Drivers":

"The steadily rising trend of accidents involving youthful drivers, especially those between 18 and 24 years of age, is a matter of gravest concern to everyone engaged in highway safety activities.

In 1946 this group of drivers was involved in 24.8 per cent of all fatal accidents. In 1947 the percentage was 25.3. Last year it rose again to 26.9.

In view of the fact that drivers in this age group make up well under 20 per cent of the total of all drivers, these figures are all the more startling. They leave no room for doubt that young drivers are involved in far more than their statistical share of accidents."

"In 1947 youthful drivers between the ages of 18 and 24 years were involved in 166 fatal accidents or 21.62 per cent of the total traffic fatalities in the state. Last year that same age group of drivers was involved in 202 fatal accidents or close to 27 per cent of the total traffic fatalities."

Accident statistics for the Province of Ontario where complete studies covering calendar year 1948 have been made as to the number and percentage of accidents involving drivers of various age groups with the corresponding number and percentage of the total number of licensed drivers is included because of the inescapable facts presented showing how much worse the driver accident record of the 18-24 year old driver age group is in comparison with the average.

Age Group	*No. of Licensed Drivers*		*Drivers Involved in Accidents*		*Index Showing Ratio to Average*
Under 18	49 468	4.1%	1 369	3.2%	.780
18-24	213 557	17.7	11 084	25.7	1.452
25-40	509 158	42.2	18 403	42.7	1.012
41-54	278 710	23.1	8 183	19.0	.823
55-64	108 588	9.0	2 849	6.6	.733
65 & over	47 055	3.9	1 187	2.8	.718
	1 206 536	100.0%	43 075	100.0%	

A comprehensive study of traffic-accident involvement by driver-age was made by the Center for Safety Education at New York University for the State of Connecticut, and later, for Massachusetts and Wisconsin. To these have been added from National Bureau figures the states of Mississippi and Virginia.

(1)	(2)	(3)	(4)	(5)
State	*Age Group*	*Per Cent of Operators*	*Per Cent of Drivers in Accidents*	*Involvement Index* (4) ÷ (3)
Connecticut	Under 25	16.7%	25.4%	1.52
	25 & over	83.3	74.6	.90
Massachusetts	Under 25	14.7	18.0	1.22
	25 & over	85.3	82.0	.96
Wisconsin	Under 25	20.1	28.8	1.43
	25 & over	79.9	71.1	.89
Mississippi	Under 25	17.7	26.5	1.50
	25 & over	82.3	73.5	.89
Virginia	Under 25	18.7	29.3	1.57
	25 & over	81.3	70.7	.87

The most recent development in "age of driver" statistics was a study of Massachusetts Compulsory Bodily Injury claims in the spring of 1950 to positively establish that the youthful driver was actually causing the accidents. It had been alleged that no one knew positively that the young driver caused the accident, that car exposure was segregated in accordance with the potential young driver being in the household. The statistical results speak for themselves in this regard.

It will be noted from this study that segregation of statistical results by actual year by year age of driver reasonably substantiates the age groupings of the various plans which separately rate cars with operators under 25.

The statistical departments of several of the larger company members of the Massachusetts Bureau made samplings of 11,247 bodily injury claims taken from 1949 calendar year settlements in accordance with rigid sampling requirements set down by the Actuarial Committee of the Massachusetts Automobile Rating and Accident Prevention Bureau. The percentage distribution of licenses issued by age of driver was furnished by the Massachusetts Registry of Motor Vehicles from a study completed by the Registry in the spring of 1949.

AGE OF DRIVER AT TIME OF ACCIDENT STUDY

Bodily Injury

(1) Age of Driver	(2) No. of Claims	(3) Amount of Loss	(4) % of Total Claims	(5) % of Total Amount	(6) % of Licenses	Differentials (7) Claims (4)÷(6)	Differentials (8) Amount (5)÷(6)	(9) Average Claim Cost (3)÷(2)
16	119	43 606	1.06	1.16	.45	2.356	2.578	366
17	186	57 102	1.65	1.52	.96	1.719	1.583	307
18	251	86 999	2.23	2.32	1.06	2.104	2.179	346
19	330	120 266	2.94	3.20	1.41	2.078	2.269	364
20	356	127 092	3.17	3.38	1.75	1.811	1.931	357
21	487	162 339	4.33	4.32	2.07	2.092	2.087	333
22	429	151 111	3.81	4.02	2.19	1.740	1.836	352
23	393	147 407	3.50	3.92	2.15	1.623	1.823	375
24	483	152 321	4.29	4.05	2.74	1.566	1.478	315
16—24	3 034	1 048 246	26.98	27.89	14.78	1.825	1.887	345
25	447	135 802	3.97	3.61	2.97	1.337	1.215	303
26	364	127 320	3.24	3.39	2.98	1.087	1.138	349
27	349	132 358	3.10	3.52	3.05	1.016	1.154	379
28	380	111 893	3.38	2.98	3.14	1.076	.949	294
29	283	82 912	2.52	2.20	2.99	.843	.739	307
30	284	121 181	2.53	3.22	2.72	.930	1.184	426
31	270	82 908	2.40	2.21	2.98	.805	.742	307
32	268	95 498	2.38	2.54	2.87	.829	.885	356
33	274	92 156	2.44	2.45	2.94	.830	.833	336
34	246	87 802	2.19	2.34	2.70	.811	.867	356
35—39	1 066	327 210	9.48	8.70	12.92	.733	.673	306
40—44	1 000	283 063	8.89	7.53	11.43	.778	.659	283
45—49	821	255 222	7.30	6.79	9.12	.800	.745	310
50—54	791	289 451	7.03	7.70	7.74	.908	.995	365

AGE OF DRIVER AT TIME OF ACCIDENT STUDY (Cont.)

Bodily Injury (cont.)

(1) Age of Driver	(2) No. of Claims	(3) Amount of Loss	(4) % of Total Claims	(5) % of Total Amount	(6) % of Licenses	Differentials (7) Claims (4)÷(6)	Differentials (8) Amount (5)÷(6)	(9) Average Claim Cost (3)÷(2)
55—59	621	203 629	5.52	5.42	6.06	.911	.894	327
60—64	344	126 117	3.06	3.36	4.08	.750	.824	366
65—69	224	81 981	1.99	2.18	2.47	.806	.883	365
70 and over	181	74 137	1.61	1.97	2.06	.782	.956	409
25 and over	8 213	2 710 650	73.02	72.11	85.22	.857	.846	330
GRAND TOTAL	11 247	3 758 896	100.00	100.00	100.00	1.000	1.000	334

Comparably a similar study was completed of Massachusetts Automobile Property Damage claims with very similar results.

Property Damage

(1) Age of Driver	(2) No. of Claims	(3) Amount of Loss	(4) % of Total Claims	(5) % of Total Amount	(6) % of Licenses	Differentials (7) Claims (4)÷(6)	Differentials (8) Amount (5)÷(6)	(9) Average Claim Cost (3)÷(2)
16	168	11 778	1.11	1.22	.45	2.467	2.711	70
17	316	22 771	2.09	2.35	.96	2.177	2.448	72
18	331	26 787	2.19	2.77	1.06	2.066	2.613	81
19	359	25 465	2.37	2.63	1.41	1.681	1.865	71
20	436	33 266	2.88	3.43	1.75	1.646	1.960	76
21	527	37 115	3.48	3.83	2.07	1.681	1.850	70
22	509	37 343	3.36	3.85	2.19	1.534	1.758	73
23	520	36 934	3.43	3.81	2.15	1.595	1.772	71
24	539	34 926	3.56	3.61	2.74	1.299	1.318	65

AGE OF DRIVER AT TIME OF ACCIDENT STUDY (Cont.)

Property Damage (Cont.)

(1) Age of Driver	(2) No. of Claims	(3) Amount of Loss	(4) % of Total Claims	(5) % of Total Amount	(6) % of Licenses	Differentials (7) Claims (4)÷(6)	Differentials (8) Amount (5)÷(6)	(9) Average Claim Cost (3)÷(2)
16—24	3 705	266 385	24.47	27.50	14.78	1.656	1.861	72
25	527	35 781	3.48	3.69	2.97	1.172	1.242	68
26	458	28 221	3.02	2.91	2.98	1.013	.977	62
27	443	30 371	2.92	3.14	3.05	.957	1.030	69
28	464	29 009	3.06	2.99	3.14	.975	.952	63
29	398	23 875	2.63	2.46	2.99	.880	.823	60
30	394	22 915	2.60	2.37	2.72	.956	.871	58
31	409	26 707	2.70	2.76	2.98	.906	.926	65
32	371	21 672	2.45	2.24	2.87	.854	.780	58
33	335	19 913	2.21	2.06	2.94	.752	.701	59
34	329	20 479	2.17	2.11	2.70	.804	.781	62
35—39	1 557	95 354	10.28	9.84	12.92	.796	.762	61
40—44	1 363	79 475	9.00	8.20	11.43	.787	.717	58
45—49	1 153	73 272	7.61	7.56	9.12	.834	.829	64
50—54	1 143	74 869	7.55	7.73	7.74	.975	.999	66
55—59	799	51 301	5.27	5.30	6.06	.870	.875	64
60—64	618	33 163	4.08	3.42	4.08	1.000	.838	54
65—69	362	18 527	2.39	1.91	2.47	.968	.773	51
70 and over	320	17 449	2.11	1.80	2.06	1.024	.874	55
25 and over	11 443	702 353	75.53	72.50	85.22	.886	.851	61
GRAND TOTAL	15 148	968 738	100.00	100.00	100.00	1.000	1.000	64

The record of the youthful driver, age group 16 through 24, is not a good one and each sizable statistical analysis made, whether it be pre-war or post-war, tells the same story. Analysis of each year by year age segment indicates that the split at age 25 for rating purposes is a proper one. A certain degree of parental control and possibly some effect of driving-training in schools, coupled with smaller volumes of experience, are probably responsible for somewhat varying results among the 16 and 17 year olds but unless several age rate groupings were to be made the present split at age 25 should be continued. Recognition of wider differentials in the March 1950 National revision is very much in line with experience with even higher rates for the under 25 group in relation to the other groups justified.

NEW YORK STATUTORY DISABILITY BENEFITS LAW, COVERAGE, RATES AND RATING PLANS

BY

MAX J. SCHWARTZ

On April 13, 1949, Governor Thomas E. Dewey signed the Mailler-Condon Bill which, upon his signature, became Article IX of the Workmen's Compensation Law, also known as the Disability Benefits Law. The purpose of the law is to provide disability insurance for eligible workers, both employed and unemployed, who are unable to work as a result of non-occupational injury or sickness.

DEFINITIONS

Whenever used in this paper, the following terms shall have the respective meanings hereinafter set forth:

"D.B.L." means Disability Benefits Law.

"CHAIRMAN" means the Chairman of the Workmen's Compensation Board.

"TAXABLE PAYROLL" means the first three thousand dollars of earnings to each employee during the calendar year, to be reported under regulations of the Chairman.

"COVERED EMPLOYER" is an employer who, after July 1, 1949, has four or more persons, not counting excluded employees, in employment on each of at least thirty days in a calendar year.

"COVERED EMPLOYMENT" means employment with a covered employer.

"8-8-13" means waiting period of seven days for both sickness and accident and maximum benefit period of thirteen weeks.

"STATUTORY ASSESSMENTS" are assessments levied by the Chairman for administering the law and to restore the Special Fund for Disability Benefits to the required minimum amount.

GENERAL PROVISIONS

The Law requires that beginning with July 1, 1950 every employer of four or more persons, not counting excluded employees, provide non-occupational disability insurance for his employees by insuring with the State Fund or an insurer authorized in New York State, or by self insurance for (1) Statutory Benefits, or (2) a Plan at Least as Favorable. The coverage provided by the employer insures the employee while employed and during the first four weeks of unemployment. After the first four weeks of unemployment and if disabled within the next 22 weeks, an employee coming from covered employment is automatically insured by the Special Fund for Disability Benefits.

Excluded Employees

Employees not entitled to benefits include the spouse or minor child of the employer; government employees; employees of charitable, religious, scientific, literary or educational non-profit organizations; employees covered by the Federal Railroad Unemployment Insurance Act; crews of vessels on navigable waters; farm laborers; temporary extra employees; casual workers; golf caddies; day students employed part time.

Administration

Responsibility for administration and enforcement of the Law, determination of contested claims, and management of the Special Fund for Disability Benefits have been placed upon the Chairman of the Workmen's Compensation Board.

Contributions to the Cost of Benefits

The employer may deduct ½ of 1% of the wages paid to an employee, but not more than 30 cents per week. The employer pays the excess of the cost, including the cost of statutory assessments.

STATUTORY BENEFITS

Eligibility

An employee of a covered employer is eligible to receive disability benefits:

1. During employment with a covered employer, after the first four consecutive weeks of employment;

2. During a period of four weeks after termination of such employment, but not beyond a day on which the employee again performs work for remuneration or profit;

3. Immediately upon employment with a covered employer, if such employment occurs within the four-week period under "2" above;

4. Immediately upon employment with a covered employer, if such employment occurs when the employee is currently
   (a) receiving unemployment insurance benefits, or
   (b) receiving benefits for disability commencing after the first four weeks of unemployment if he would be, but for disability, eligible for unemployment insurance benefits, or
   (c) eligible for disability benefits commencing during such unemployment, if he is then ineligible for unemployment insurance benefits because he has not accumulated unemployment insurance benefit credits.

An employee regularly in the employment of a single employer on a work schedule less than the employer's normal work week, becomes eligible for benefits on the 25th day of such regular employment.

Benefit Rate

The weekly benefit rate is 50% of the employee's average weekly wage, with a minimum of $10.00, and a maximum of $26.00 per week, but if the average weekly wage is less than $10.00, the benefit is the average weekly wage.

Average weekly wage is determined by dividing the wages paid by the employee's last covered employer for the last eight weeks preceding disability by the number of weeks worked. Discretionary "average weekly wage" redeterminations are authorized if the employee did not work for his last covered employer during all of such eight weeks and if the average weekly wage otherwise determined "does not fairly represent the normal earnings of such employee."

Waiting Period

There is a non-benefit waiting period of the first seven days of disability. Successive disabilities caused by the same or related injury or sickness are deemed a single period of disability, if separated by less than three months.

Duration

The duration of benefits may not exceed thirteen weeks during any period of fifty-two consecutive calendar weeks, or during any single period of disability.

Exclusions

The principal exclusions are occupational injury or illness, pregnancy, disabilities where the employee is not under the care of a licensed physician, self-inflicted injuries, injuries due to an act of war.

PLAN AT LEAST AS FAVORABLE

A Plan may provide cash benefits which differ from the statutory benefits; either as to benefit rate, waiting period, maximum or minimum benefits, duration of benefit period, or in any other respect, and may include hospital, medical and surgical care. The aggregate value of all benefits must be at least as favorable as, i.e., actuarially equivalent to, statutory benefits. By regulation of the Chairman, a Plan, to be acceptable, must meet the following requirements:

1. The aggregate value of benefits for each employee shall be actuarially equivalent to or greater than statutory benefits.

2. The cash disability benefits meet the following minimum requirements:

(a) Equal in value to 60% of statutory benefits.

(b) Shall be at the rate of at least

Fifteen dollars per week for an employee whose average weekly wage is thirty dollars ($30) or more,

Twelve dollars and fifty cents ($12.50) per week for an employee whose average weekly wage is more than twenty dollars ($20) and less than thirty dollars ($30),

Ten dollars ($10) per week for an employee whose average weekly wage is not less than ten dollars ($10) and not more than twenty dollars ($20),

Average weekly wage for an employee whose average weekly wage is less than ten dollars ($10).

(c) A waiting period of not more than seven days.

(d) A duration of benefit period of at least eight weeks of disability during a period of fifty-two consecutive calendar weeks.

3. The plan shall generally cover the employee beginning with the first day of employment and including the first four weeks of unemployment.

The Chairman of the Workmen's Compensation Board prescribed Tables of Evaluation of Plan Benefits to evaluate a Plan to determine whether the Plan is actuarially equivalent to statutory benefits.

BENEFITS TO THE SICK UNEMPLOYED WHOSE DISABILITY BEGINS AFTER THE FIRST FOUR WEEKS OF UNEMPLOYMENT

The benefits payable to the unemployed who become disabled after the first four weeks of unemployment are paid from the Special Fund for Disability Benefits, and benefits payable from the fund are governed by the same provisions as to rate, waiting period, duration, exclusions and reductions as are applicable to disabilities during employment and the first four weeks of unemployment, but:

1. Disability benefits are not payable beyond the twenty-sixth week of unemployment;
2. If the disabled employee is currently receiving unemployment insurance benefits at the time disability commences, he is not subject to the non-benefit waiting period of seven days, and
3. The aggregate of disability benefits payable is thirteen weeks during any period of disability or during a period of fifty-two consecutive calendar weeks, including disability benefits both during employment and unemployment.

SPECIAL FUND FOR DISABILITY BENEFITS

The Special Fund for Disability Benefits is the fund which pays the benefits to the sick unemployed whose disability begins after the first four weeks of unemployment. It is administered by the Chairman of the Workmen's Compensation Board, and is to be financed through assessments levied on carriers after the initial contributions of employees and employers.

The payrolls of all employers who are subject to the Disability Benefits Law were assessed .2 of 1% of the wages paid during the period January 1 to June 30, 1950 but not in excess of 12c per employee per week, to raise an estimated $12,000,000.00 for the Special Fund. The employee paid half of this assessment through deductions from his wages and the employer paid the other half.

STATUTORY ASSESSMENTS

Whenever on April 1 of any year the moneys in the Special Fund for Disability Benefits are $1,000,000.00 below (1) $12,000,000.00, or (2) twice the benefits paid from the Special Fund in the preceding fiscal year, whichever is the greater, the Chairman of the Workmen's Compensation Board levies an assessment to restore the Fund to the above required minimum.

Annually after April 1, the Chairman of the Workmen's Compensation Board levies two assessments on the taxable payroll of all covered employees —an assessment to reimburse the Chairman for expenses for the previous fiscal year April 1 to March 30, and an assessment to bring the Special Fund for Disability Benefits up to the required minimum. The assessments are levied on all carriers and are based on the taxable payrolls for the previous calendar year. A carrier is defined as a self-insured employer, insurance company, State Fund, trustees under a plan or agreement, association or other agency permitted to provide benefits. The liability for assessments is generally assumed by the insurance company which covers the risk, by including an assessment provision in the policy.

POLICY FORMS

The insurance may be written by issuing a separate policy, by attaching a rider to the Workmen's Compensation Insurance Policy, or by attaching a rider to an existing group accident and health policy. The policy form must be approved by the Superintendent of Insurance and accepted by the Chairman. The Superintendent of Insurance and the Chairman have jointly issued Advisory Forms A, B, E, F, G, H, J and K to guide companies in drawing up D.B.L. policy forms.

REQUIREMENTS OF THE N. Y. INSURANCE LAW PERTAINING TO GROUP ACCIDENT & HEALTH RATES AND RATING PLANS

Unlike other casualty lines, which have a regulated rate structure, the Group Accident and Health rate structure, including D.B.L., is a competitive one.

The New York Insurance Law requires that:

1. All insurers file with the Superintendent schedules of premium rates, rules and classification of risks for use in connection with the issuance of Contracts of Group Accident and Health Insurance, and a schedule of maximum commissions, compensation or other allowances to soliciting agents. No insurer shall issue any policy of Group Accident and Health Insurance, the premium rate under which for the first policy year is less than that determined by the schedules of such insurer as then on file with the Superintendent. (Section 221-7)

2. An insurer may not issue a Group Accident and Health Policy which shall not appear to be self-supporting on reasonable assumptions as to morbidity or other appropriate claim rate, interest and expense. (Section 221-6)

3. Any contract of Group Accident and Health Insurance may provide for a readjustment of the rate of premium based on the experience thereunder at the end of the first year or any subsequent year, and such readjustment may be made retroactive for such policy year. (Section 222-9)

4. The Superintendent may disapprove any Group Accident and Health policy if the benefits provided therein are unreasonable in relation to the premiums charged. (Section 154-1)

5. Group Accident and Health rates may not be unfairly discriminatory. (Section 209-2)

GROUP ACCIDENT AND HEALTH RATES

Prior to D.B.L., most Group Accident and Health rates were based on monthly premiums per $10.00 of weekly benefit. Experience of companies writing a large volume of Accident and Health Insurance indicates that the morbidity on female lives is approximately 100% higher than on male lives, if maternity benefits are included, and 50% higher, if maternity benefits are excluded. For certain hazardous industries the morbidity was found to be higher than for non-hazardous industries, even though the coverage was non-occupational. As a result, the basic rate was loaded for female content where the percentage of female exposure was in excess of 10%. The rate was also loaded for certain hazardous industries by specified percentages.

The rates were based on the basic morbidity table developed by Fitzhugh*.

*Gilbert W. Fitzhugh "Recent morbidity upon Lives Insured under Group Accident and Health Policies and Premiums Based Thereon" T.A.S.A. Vol. XXXVIII, Part 2, Oct. 1937.

In part two of his paper, Mr. Fitzhugh developed a table of basic premiums. This table was used by most of the companies to arrive at the gross premiums before loading for sex and industry. These rates were known as "manual" rates. In general, the premium on a risk was adjusted at the end of the policy year depending on the risk experience. In 1949, as a result of publication of a more recent study, some manual rates were modified to bring them in line with the more recent experience.

A typical rate sheet would be as follows:

Non-Occupational Sickness and Accident Coverage
Monthly rate per $10.00 of Weekly Benefits

Day Indemnity Begins		*Benefits extend (weeks)*		
Accident	*Sickness*	*13*	*26*	*52*
8th	8th	$.60	$.72	$.93
1st	8th	.66	.77	.98
4th	4th	.77	.87	1.11
1st	4th	.80	.91	1.14
15th	15th	.52	.64	.83

The above rates would be loaded for female exposure as follows:

Percentage of Benefit Coverage on Women	*Percentage Loading (Maternity benefits up to 6 weeks included)*
Less than 11%	0
11% but less than 21%	15%
21% but less than 31%	25%
31% but less than 41%	35%
41% but less than 51%	45%
51% but less than 61%	55%
61% but less than 71%	65%
71% but less than 81%	75%
81% but less than 91%	85%
91% through 100%	95%

Where maternity benefits are excluded, divide percentage of benefit coverage on women by 2 before using the above table.

For certain hazardous industries the rates might, for example, be loaded in accordance with the following table of industry loadings.

Industry	*Add to Rate*
Breweries and Wine Manufacturers	15%
Distilleries of ethyl or methyl alcohol or of alcoholic beverages	15%
Furriers	15%
Lime, Cement, Gypsum (no quarrying)	15%
Liquor and Wine Wholesalers	15%
Marble and Stone Yards	15%
Mines (surface and underground) and Quarries	40%
Railroads	25%
Tanneries	15%
Woodsmen and Loggers	25%

D.B.L. RATE BASES

The use of the above mentioned rate structure depends on the availability of reasonably accurate data on the average amount of weekly benefit in force during each policy month. For the usual Group Accident and Health case, a record card is kept on each employee showing the amount of weekly benefit to which he is entitled. Periodic summations of these cards, with due allowance for additions and cancellations, produce the totals of weekly benefits in force necessary for premium billing purposes. The record card system is not simple to operate unless changes on individual record cards are kept to a minimum. Under most Group Accident and Health policies, individual changes in amounts of weekly benefits are kept to a minimum by use of a schedule of benefits which is based on very broad classifications as to either earnings or type of position.

Under the New York Disability Benefits Law an employee's weekly benefit is generally 50% of his average earnings and therefore may fluctuate from week to week. The use of record cards, and hence the use of a rate structure based on amounts of weekly benefits, were consequently deemed inappropriate for policies which were to provide statutory benefits. For D.B.L. rates, therefore, three new rate bases were developed:

1. For groups of over 25 lives, and in some cases for all groups, the rate is expressed as a percentage of the employer's unemployment taxable payroll. The unemployment taxable payroll is based on the first $3,000.00 of earnings of each employee during the calendar year. Since most employers subject to D.B.L. are also subject to the unemployment tax, and since these employers must calculate and report their taxable payroll quarterly to the Division of Placement and Unemployment Insurance, the percentage of unemployment taxable payroll billing method is simple and economical to administer. The rate is loaded for sex and hazardous industry.

2. For those employers who elect to deduct ½ of 1% of the wages paid to the employee, but not more than 30c per week, a factor was developed. (Some companies developed a rate which is a percentage of the payroll on which the employee's D.B.L. contributions are based.) The amount of the deductions multiplied by the factor and loaded for sex and hazardous industry is the final rate.

3. Group Accident and Health Insurance is not regularly sold to employers with fewer than 25 employees and normal group underwriting practices are very expensive when applied to groups of this size. Consequently, for the smallest groups, generally groups up to 25 lives, and in some instances not exceeding 50 lives, a statewide per capita rate, regardless of payroll, sex or industry, was developed. The final premium is the per capita rate multiplied by the number of employees on the payroll on the 15th day of each month.

A further simplification of the rate structure was effected by basing the female loadings on the percentage of women employees instead of the percentage of benefit coverage on women. This approximation is probably justified by the labor saved.

PURE PREMIUMS

Coverage under a statutory policy is equivalent to non-occupational (maternity excluded) 13-week coverage with a seven-day waiting period for both accident and sickness with the following main differences: (1) statutory coverage includes the first four weeks of unemployment while ordinary coverage ceases when the employee leaves his employment. (2) statutory coverage is limited to 13 weeks in any 52 consecutive calendar weeks in addition to the limitation of 13 weeks for any one illness. (3) in a D.B.L. policy the insurance company generally assumes the liability for statutory assessments.

The cost of extending coverage to include the first four weeks of unemployment is small during periods of full employment because on the average only a small percentage of employees leave employment each year and the employee is covered only until he finds other employment. The limitation of 13 weeks in any 52 consecutive weeks is insignificant for the younger ages. For the older ages the regular group policies issued prior to D.B.L. contain this limitation. For these reasons it was considered reasonable to use available experience of group accident and health 13-week 7-day waiting period coverage to develop the base rate. This rate was then loaded for statutory assessments. Many companies which had been writing group insurance prior to D.B.L. used their 8-8-13 rate as a basis for the statutory rates. Companies entering the group accident and health field for the first time used the following two morbidity rates to develop their rates:

	Morbidity for 8-8-13
1—1947 Inter-Company Study*	.51 weeks per year—All Male
2—1948 Inter-Company Study†	.48 weeks per year—All Male

Translated into pure premiums the above studies show a net cost for 13 weeks' benefits 7-day waiting period for both accident and illness, of 51c per year per $1.00 of weekly benefits for 1947, and 48c per year per $1.00 of weekly benefits for 1948 for all male groups. To arrive at a gross all-male rate, the pure premium must be loaded for expenses and for statutory assessments.

Loading for Assessments

As indicated above, two assessments will be levied annually by the Chairman; an assessment to cover administration of the law, and an assessment to pay the sick unemployed. The assessment for administration of the law is expected to be fairly constant. The assessment for the sick unemployed, however, is contingent on the unemployment rate. The estimated assessments are usually loaded for commissions and premium taxes but generally not for administration expense. It is to be noted that the assessments are based on taxable payrolls regardless of industry or the sex of the employees. The loading for assessments is therefore not loaded for sex or industry. The loading for assessments is estimated at from .1 of 1% to .15 of 1% of taxable payroll‡, or at from 20c to 30c per person per month.

*"Report of Committee to Prepare Mortality and Morbidity Studies on Group Insurance" Section III T.A.S.A. Vol. XLIX, Part 2, October, 1948.

†"1949 Report of the Committee on Group Mortality and Morbidity" published by the Society of Actuaries, Chicago, Ill.

‡For an explanation as to how these estimates were arrived at see Exhibit A.

DEVELOPMENT OF RATES—STATUTORY BENEFITS

Since the benefit rate and waiting period are substantially equal to the rate and waiting period under the New York Unemployment Insurance Law, it is reasonable to assume that the average weekly benefits under D.B.L. will approximate those under the Unemployment Insurance Law. Taxable payrolls under the Unemployment Insurance Law are defined as the first $3,000.00 of earnings of each covered employee during the calendar year. Thus, available unemployment insurance statistics may be utilized in developing a D.B.L. rate.

The average taxable payroll for the year 1948 was $2,496.00. The average weekly unemployment benefit rate in 1949 was $22.83. (Unemployment benefits are based on earnings the previous year).* It is estimated that the number of women employed in New York State is approximately 35% of all the employed workers.†

Percentage of Payroll Rate

Following is an example of the development of a percentage of payroll rate based on manual rates.

Manual rate for 8-8-13 is 60c per month for each $10.00 of weekly benefits for a group including up to 11% women. Because of savings in expense in handling statutory insurance, a 5% discount is allowed. The rate is then translated into a percentage of payroll rate by multiplying it by 2.283 to arrive at the monthly rate per person, by 12 to arrive at the annual rate per person, and divided by $2,496.00, which is the average taxable payroll. This rate is then loaded for female content and then increased by .1 of 1% of taxable payroll to cover statutory assessments.

$$\frac{\$.60 \times .95 \times 2.283 \times 12}{\$2,496.00} = .626\%$$

Percentage of Payroll on Female Lives		*Final Rate % of Taxable Payroll*
0 to 11%	.63 × 1.000 + .1% =	.73%
11% to 21%	.63 × 1.075 + .1% =	.78%
21% to 31%	.63 × 1.125 + .1% =	.81%
31% to 41%	.63 × 1.175 + .1% =	.84%
41% to 51%	.63 × 1.225 + .1% =	.87%
51% to 61%	.63 × 1.275 + .1% =	.90%
61% to 71%	.63 × 1.325 + .1% =	.93%
71% to 81%	.63 × 1.375 + .1% =	.97%
81% to 91%	.63 × 1.425 + .1% =	1.00%
91% to 100%	.63 × 1.475 + .1% =	1.03%

The above rates, excluding the assessment charge, are loaded for certain hazardous industries.

*These averages were obtained from the Division of Placement and Unemployment Insurance, N. Y. State Department of Labor and are based on the same assumptions which were used to develop Table VIII, Page 151 "Studies in Disability Insurance."

†"Studies on Disability Insurance" Page 134.

An example of the development of a percentage of payroll rate based on the 1948 Morbidity Investigation is as follows:

Pure premium—all male—for 8-8-13 is $.48 per year for each $1.00 of weekly benefit. If we assume a loading for expenses and contingencies of 25% of gross premium before loading for assessments, an average benefit rate of $22.83 per week, average taxable annual payroll of $2,496.00, loading for statutory assessments of .1 of 1% of taxable payroll, the gross rate would be:

$$\frac{\$.48 \times 22.83}{.75 \times \$2{,}496.00} + .1\% = .69\% \text{ of taxable payroll}$$

This rate would be loaded for sex and industry as shown above.

Factor Rate

For employers who deduct ½ of 1% of the salaries of their employees but not more than 30c per employee per week a factor was developed. To arrive at the premium, the deductions are multiplied by the factor. To arrive at a factor rate, the same assumptions may be used as were used for developing a percentage of payroll rate except that the average payroll would be slightly higher than $2,496.00, since the maximum taxable payroll is $3,000.00 while the maximum deduction of 30c is ½ of 1% of $60.00 a week or $3,120.00 a year. It is estimated that the average payroll based on a maximum of $3,120.00 a year is $2,518.00 a year*. A factor would be arrived at as follows, using the above assumptions:

$$\frac{\$.60 \times .95 \times 2.283 \times 12}{\$2{,}518.00} + .1\% = .72\%$$

$$\frac{.72\%}{.5} = 1.44$$

The factor to be applied to deductions if the female content is less than 11% is 1.44. The rate is loaded for female content in excess of 11% and for industry as indicated above.

Statewide Per Capita Rate

Following is an example of the development of a statewide per capita rate based on manual rates. Manual rate for 8-8-13 is 60c per month for each $10.00 a week benefit. This rate is loaded 17.5% for an estimated 35% women workers in New York State. The per $10.00 weekly benefit rate is then changed to a per person rate by multiplying it by 2.283 based on an average benefit rate of $22.83 per week. The rate is then increased by 21c which is the loading for statutory assessments.

$$\$.60 \times 1.175 \times 2.283 + \$.21 = \$1.8195$$

Per capita rate per employee per month rounded to the nearest 5c = $1.80.

*This estimate is based on a report dated Feb. 7, 1950 of the Bureau of Research and Statistics of the N. Y. State Department of Labor, which report contains a wage distribution table of employees covered by unemployment insurance. The table lists the distribution up to $3,000.00 per employee per year and states that 21.24% of the employees received over $3,000.00 per year. Exterpolating this table the following figures were arrived at: 6.25% of the employees receive between $3,000.00 and $3,120.00 a year (average $3,060.00) and 15% receive over $3,120.00 a year. Based on these figures an average annual wage of $2,518.00, including wages up to $60.00 per employee, was arrived at.

An example of the development of a per capita rate based on the 1948 morbidity investigation is as follows:

Pure premium—all male—for 8-8-13 is $.48 per year for each $1.00 of weekly benefit. The rate per month is 48c divided by 12. This rate is loaded 17.5% for the estimated 35% women workers in New York State. If we assume a loading for expenses and contingencies of 25% of gross premium before loading for assessments, an average benefit rate of $22.83 per week and a loading of 21c per person per month for statutory assessments, the gross rate would be:

$$\frac{\$.48 \times 1.175 \times 22.83}{12 \times .75} + \$.21 = \$1.64 \text{ per person per month}$$

There are a number of variations of the above rate structures. Some companies have four classes of rates as follows:

0 to 25% Female
25 to 50% Female
50 to 75% Female
Over 75% Female

Others have a male rate and a female rate, the female rate being 50% higher than the male rate before loading for assessments. There are other variations too numerous to list here.

DEVELOPMENT OF RATES FOR PLANS AT LEAST AS FAVORABLE

For the smaller groups, some of the companies developed rates for benefits in excess of statutory, such as extending the maximum benefit period to 26 weeks, no waiting period for accidents, increase of benefit rate to higher than 50% of pay, or increase maximum benefit rate to more than $26.00 per week. Since the liability for assessments is not changed by increasing the rate or period of benefits or reducing the waiting period, all increases in the rate to provide additional benefits are made before the loading for assessments is added.

To develop a statewide percentage of taxable payroll premium rate for a benefit rate and maximum other than statutory, the wage distribution table of the National Council on Compensation Insurance may be used.* This table shows the percentage distribution of wages around the average wage. Column A gives the percentage of employees earning the given percentage of the average wage or less, Column B gives the percentage of the payroll earned by employees who receive the given percentage of the average wage or less. To illustrate, 21.52% of the employees earn 70% of the average wage or less and they earn 12.51% of the total payroll.†

For large groups the existing rate structures which are based on monthly rates per $10.00 of weekly benefits are used. To the final premium a loading of from 20c to 30c per person per month or from .1% to .15% of taxable payroll is added to cover statutory assessments.

*See Table I. This wage distribution table is derived from the Standard Wage Distribution Table Vol. IX P.C.A.S., Page 220.

†For an example, see Exhibit B.

On the larger groups most companies allow a percentage discount graduated by size of premium, and in some cases based on the number of lives. The discount is in accordance with a discount table, which is part of the rate manual, and is based on savings in expense on large groups.

At the end of the policy year, the premium on each large risk generally is adjusted, based on the experience of that risk. The premium rate for the second and subsequent policy years may be modified, based on the experience of the risk.

Some of the companies have a retrospective rating plan, which is similar to the plan used in Workmen's Compensation Insurance.

In conclusion, it must be emphasized that the group accident and health rate structure, including D.B.L., is a competitive one and while all rates must meet the statutory tests that they be self-supporting, reasonable, and not unfairly discriminatory, the rates vary from company to company. Some of the reasons for the variation in rates are lower expense of operation and lower commissions; some companies load the rate for contingencies, others do not; there are a number of different morbidity assumptions on which rates may be based.

At the time this paper was written (October, 1950), no actual experience for D.B.L. coverage was available. It is possible, that experience will prove some of the assumptions made in this paper to be incorrect.

EXHIBIT A
LOADING FOR STATUTORY ASSESSMENTS

Annually, two assessments will be levied by the Chairman, the assessment to administer the Law and the assessment to reimburse the Special Fund for Disability Benefits. The assessment to administer the Law is generally estimated at .02 of 1% of taxable payroll and is expected to remain constant through the years. The assessment to reimburse the Special Fund for Disability Benefits will vary with the rate of unemployment of covered workers since this fund pays the disability benefits to the sick unemployed, whose disability begins after the first four weeks of unemployment. Early in 1950 about 7% of the covered employees were unemployed. At that time unemployment was expected to increase. Most companies based their loading for assessments for this Fund at an unemployment rate of 10%. Some of the companies based their loading on the average unemployment rate over a cycle of 20 years and estimated that average, based on statistics of the past 20 years, at 15%. The cost of payments to the sick unemployed based on an unemployment rate of 10% was estimated at .071 of 1% of taxable payroll and for an unemployment rate of 15% at .106 of 1% of taxable payroll*. Thus, the total cost of the two assessments was estimated at .091 or .126 of 1% of taxable payroll. These estimates were then loaded 10% for commissions and premium taxes giving a gross loading for assessments of .1 or .14 of 1% of taxable payroll.

Assuming an average annual taxable payroll of $2,496.00,† the above loading may be translated into a monthly per capita loading, by multiplying

*See Table XII, Page 153, Studies in Disability Insurance.

†See text of this paper under "Development of Rates—Statutory Benefits."

$2,496.00 by either .1 or .14 of 1% and dividing the product by 12, thus arriving at a loading of either 21c or 29c per person per month.

EXHIBIT B
DERIVATION OF A PERCENTAGE OF TAXABLE PAYROLL PREMIUM RATE FOR BENEFITS IN EXCESS OF 50% OF WEEKLY WAGE AND A MAXIMUM IN EXCESS OF $26.00 A WEEK

Let us assume that the average weekly wage in New York State is $54.00 per week, that the monthly premium rate per $10.00 weekly benefit is 57c and we want to develop a rate where the weekly benefit rate is 60% of the weekly wage with a minimum of $10.00 and a maximum of $35.00 a week.

To simplify the calculations, let us assume that there are 100 employees on the payroll.

Average Taxable Payroll*

$57.70 $\frac{(\$3{,}000}{(\ 52\ )}$ is 105% of the average weekly wage figured to the nearest 5%.

Percentage of payroll paid to employees who receive 105% or less of the average wage is 50.10% (See Table I, Col. B)

Total weekly salary of employees receiving 105% or less of the average weekly salary is ($54.00 × 100) × 50.1% = $2,705.40

Number of employees receiving over $57.70 a week is 100 × (1 — 63.31%) = 36.69 (See Table I, Col. A)

Amount paid to these employees, excluding wages in excess of $57.70 per employee: 36.69 employees × $57.70 = 2,117.01

Total wages paid to 100 employees, excluding wages in excess of $57.70 per employee $4,822.41

Average weekly taxable wage $\frac{\$4{,}822.41}{100} = \$48.22$

Average monthly taxable wage $48.22 × 4.333 = $208.94

Average Benefit Rate

$16.67 ($16.67 × 60% = $10.00) figured to the nearest 5% is $\frac{\$16.67}{\$54.00} = 30\%$ of the average wage.

$58.30 ($58.30 × 60% = $35.00) figured to the nearest 5% is $\frac{\$58.30}{\$54.00} = 110\%$ of the average wage.

Number of employees receiving $16.67 or less per week is .0065 (See Table I, Col. A) × 100 = .65

Benefit exposure of these employees .65 × $10.00 $ 6.50

Payroll of employees earning between $16.67 and $58.30 per week is (.5573-.0017) Table I, Col. B) × $54.00 × 100 = $3,000.24

*References are to Table I which follows.

Benefit exposure of these employees \$3,000.24 × 60% =	1,800.14
Number of employees receiving over \$58.30 a week is (1 — .6841) (Table I, Col. A) × 100 = 31.59	
Benefit exposure of these employers is 31.59 × \$35.00 =	1,105.65
Total benefit exposure of 100 employees	\$2,912.29

Average benefit rate is $\frac{\$2,912.29}{100} = \$29.12$

Percentage of taxable payroll rates excluding loading for statutory assessments is the monthly rate per \$10.00 of weekly benefits (57c) multiplied by the number of \$10.00 units of weekly benefit rate (2.912) divided by the average monthly taxable payroll (\$208.94) or

$$\frac{\$.57 \times 2.912}{\$208.94} = .79 \text{ of } 1\% \text{ of taxable payroll}$$

This rate must be loaded for female exposure, hazardous industry and statutory assessments.

TABLE I

NATIONAL COUNCIL ON COMPENSATION INSURANCE WAGE DISTRIBUTION TABLE

Wage Expressed As Ratio to Average Wage	*COLUMN A* *Percentage of Employees Receiving Wage at Left or Less*	*COLUMN B* *Percentage of Total Wages Paid to Employees Receiving Wage at Left or Less*
10%	.02%	.00%
15%	.06%	.01%
20%	.14%	.02%
25%	.32%	.07%
30%	.65%	.17%
35%	1.24%	.37%
40%	2.20%	.76%
45%	3.66%	1.42%
50%	5.75%	2.46%
55%	8.56%	4.01%
60%	12.14%	6.16%
65%	16.48%	8.98%
70%	21.52%	12.51%
75%	27.13%	16.72%
80%	33.15%	21.54%
85%	39.39%	26.84%
90%	45.68%	32.50%
95%	51.85%	38.36%
100%	57.76%	44.27%
105%	63.31%	50.10%
110%	68.41%	55.73%
115%	73.03%	61.04%
120%	77.15%	65.99%
125%	80.77%	70.51%
130%	83.91%	74.59%
135%	86.61%	78.24%
140%	88.91%	81.46%

TABLE I (*Continued*)

Wage Expressed As Ratio to Average Wage	*COLUMN A* *Percentage of Employees Receiving Wage at Left or Less*	*COLUMN B* *Percentage of Total Wages Paid to Employees Receiving Wage at Left or Less*
145%	90.86%	84.29%
150%	92.49%	86.73%
155%	93.85%	88.84%
160%	94.98%	90.65%
165%	95.91%	92.19%
170%	96.68%	93.50%
175%	97.31%	94.60%
180%	97.82%	95.52%
185%	98.24%	96.30%
190%	98.58%	96.94%
195%	98.86%	97.49%
200%	99.08%	97.93%
205%	99.26%	98.30%
210%	99.41%	98.61%
215%	99.53%	98.88%
220%	99.63%	99.10%
225%	99.71%	99.28%
230%	99.77%	99.42%
235%	99.82%	99.54%
240%	99.86%	99.63%
245%	99.89%	99.71%
250%	99.92%	99.78%
255%	99.94%	99.83%
260%	99.96%	99.88%
265%	99.98%	99.94%
270%	100.00%	100.00%

REFERENCES

Title	*Published By*
Disability Benefits Law	Workmen's Compensation Board 80 Center Street, New York, N. Y.
Regulations under the Disability Benefits Law	Workmen's Compensation Board 80 Center Street, New York, N. Y.
Rules under the Disability Benefits Law	Workmen's Compensation Board 80 Center Street, New York, N. Y.
Table for Evaluation of Plan Benefits under the Disability Benefits Law	Workmen's Compensation Board 80 Center Street, New York, N. Y.
Studies in Disability Insurance (Special Bulletin Number 224, 1949)	State of New York Department of Labor 80 Center Street, New York, N. Y.
Recent Morbidity upon Lives Insured under Group Accident and Health Policies and Premiums Based Thereon (T.A.S.A. Volume XXXVIII, Part 2) October, 1937	Society of Actuaries 208 South LaSalle Street Chicago, Illinois
Report of Committee to Prepare Mortality and Morbidity Studies on Group Insurance (T.A.S.A. Volume XLIX, Part 2) October, 1948	Society of Actuaries 208 South LaSalle Street Chicago, Illinois
1949 Report of the Committee on Group Mortality and Morbidity	Society of Actuaries 208 South LaSalle Street Chicago, Illinois

THE COMBINED FIRE AND CASUALTY ANNUAL STATEMENT BLANK

BY

THOMAS F. TARBELL

INTRODUCTION

At its June 1950 meeting in Quebec, the National Association of Insurance Commissioners adopted the report of the Committee on Blanks of that Association which incorporated a new combined Fire and Casualty annual statement blank, effective for the returns for calendar year 1950.

It is generally conceded that the new form of blank is a definite forward step in the field of insurance reports from the standpoint of insurance departments, insurance carriers and the general public.

From the standpoint of the insurance departments it provides a single form for the reporting of financial condition, operating results and pertinent supporting and collateral information for all types of insurance carriers other than life, fraternal societies, and title insurance companies. It possesses the advantage of greater uniformity and should result in economy by the elimination of the separate forms of blank which it supersedes.

From the standpoint of the companies it provides a form which is more readily understood by company officials not particularly versed in the intricacies of accounting and to whom the peculiarities of the superseded forms are not readily understandable. The pertinent facts disclosing financial condition and operating results are exhibited concisely on three pages (Pages 2, 3 and 4). The previous division of asset items into ledger assets, non-ledger assets, and assets not admitted, has been discontinued and superseded in the new blank by a showing of each type of asset at statement (or allowable) value. The previous division, peculiar to insurance company statements, made it difficult to determine the statement value of certain types of assets. While the liabilities page does not differ materially from those in the previous statement blanks, it has the advantage of conciseness since only essential totals are provided for. The details which are unessential, such as outstanding losses by line of business and status and loss adjustment expenses by line of business, have been properly relegated to supporting exhibits. In case of fleets or groups of companies consisting of both Fire and Casualty carriers a single form of statement blank has its advantages both from a preparation standpoint and uniformity of accounting records.

From the standpoint of the general public the blank has the advantage (Pages 2, 3 and 4), in addition to eliminating unessential statistical detail, of providing exhibits of assets and liabilities in forms more comparable to those of other types of corporations with which an interested public is more familiar, and producing in concise form the year's operating results on a revenue basis and the accounting thereof in the capital and surplus account along conventional lines.

SCOPE

This is the first of two or three installments, or papers, dealing with the following phases of the combined blank.

1. History and development.
2. Financial statement.
3. Exhibits and schedules.

HISTORY AND DEVELOPMENT

While the history and development of the annual reports required from insurance companies in general are available in certain publications, such as Proceedings of National Association of Insurance Commissioners and insurance department reports (particularly New York), and papers prepared for other actuarial societies, it is believed that a retracing of the course will be of some interest. It is also, to a certain degree, pertinent. The historical data will be confined only to that which, in the mind of the writer, is essential. Much of the historical and factual data has been taken from a paper "The 'Convention' Statement of Life Insurance Companies" by C. O. Shepherd, presented at the May 1937 meeting of the American Institute of Actuaries and printed in the Institute's publication The Record, Vol. XXVI, Part 1, No. 53, and it is recommended that those members who are interested in a more detailed account of the history and development of insurance companies' reports refer to this paper.

In the formation of insurance companies of the modern type, fire insurance companies antedated life insurance companies. A substantial number of fire companies were organized in the first and second decades of the 19th century, and a few even earlier, but the first of the modern type life insurance companies was established in 1843. There are minor exceptions to this general statement. Also, prior to 1843 there were certain foreign companies transacting business in this country. The earlier incorporation of fire insurance companies was not a material factor in the development of annual reports but such reports resulted from the development of life insurance written primarily, as indicated in the period prior to 1843, by foreign insurance companies. The reasons for this are obvious. The fire insurance companies were engaged in writing policies of short duration whereas the life insurance companies, as today, wrote contracts for life duration or a comparatively long term of years. Consequently, although solvency of all types of insurance carriers was important, the continued solvency of life insurance companies was of greater concern to those supervisory officials who interested themselves in the subject than the continued solvency of the fire insurance companies.

In 1828 New York State enacted a statute requiring that every moneyed corporation thereafter organized file a financial statement annually with the Comptroller of that state. From 1828 to 1860, when the Insurance Department was organized, the form was prescribed by the Comptroller. In the processes of evolution differentiation was made by type of corporation and forms were developed exclusively for the use of insurance companies. The earlier forms were little more than questionnaires asking for certain information, some of which was merely of a statistical nature. One such form in use during the 1830's carried the caption "A Full and Perfect Statement of the

Affairs of the ———— Company." However, there was no assembly of either resources or obligations and no showing of surplus (net worth). Some further progress was made in the intervening years up to the end of the first half of the century. As of January 1, 1849 a fire company voluntarily supplemented its regular report by a so-called general balance of its books. This balance showed the usual items of assets, including premiums due. Its liabilities, however, were restricted to capital stock, dividends (apparently declared and unpaid), losses by fire unsettled, and profit and loss. No liability was established for unearned premium reserve or unpaid expenses. The company, however, abandoned this supplementary form in its filings for subsequent years.

In 1853 a special form was adopted by the New York Comptroller for Fire companies. This form was in effect until 1859. It was similar to the form previously described. In addition to amount of capital it included statements of assets, liabilities, income, and expenditures. It is significant that assets contemplated statement values as evidenced from the item "Market Value of Stocks." For some reason bonds were not included as a specific item of assets. The liabilities provide for losses due and unpaid (apparently in process of adjustment), resisted, and reported on which no action had been taken. Apparently no reserve was contemplated for losses incurred but not reported. No specific item was included for the unearned premium reserve. The income statement covered cash premiums, notes received for premiums, and income from other sources. Expenditures included losses, dividends, salaries, commissions to agents, taxes, and all other payments and expenditures. There was no balancing of assets and liabilities.

The lack of a reserve for unearned premiums was commented upon by the Comptroller in his report of March 7, 1854 as follows:

> "I have not seen in any statement received at this office from any stock company in this State a deduction from their estimated surplus profits of any sum for unearned premiums, and in all probability the cash received on the 30th day of December last for premiums is counted in the assets, which go to swell the apparent profits of the company . . ."

An Act of 1853 authorized the Comptroller to appoint one or more persons to examine into the affairs of any fire company doing business in the state. This marked a milestone in the development of insurance supervision and annual statement forms. In 1856 a special commission was appointed to examine all fire companies in New York City.

In 1858 the fire blank was amended by inserting a liability item for unearned premium reserve under the following designation:

> "Amount required to safely reinsure all outstanding risks, estimated by the President and Secretary."

The organization of the New York Insurance Department by Act of the 1859 Legislature marks another milestone in the development of insurance supervision and financial statements. The Massachusetts Insurance Department had been organized five years earlier and an exchange of ideas between the supervising officials of the two states culminated in the organization of the National Convention (now Association) of Insurance Commissioners. Between 1860 and 1865 there were numerous changes and improvements in

the annual statement blanks. In 1868 the fire and marine blank incorporated an item of "Surplus" and equalized total of assets and liabilities.

Prior to 1871 there had been no uniformity as to the treatment of accrued and deferred items. In that year, the caption of the assets schedule was changed to "Assets Available." This made specific provision for accrued and deferred income items. The total of the items in the schedule was designated as "Total Admitted Assets." Subscribed was a list of "Items not Admitted as Available Assets," consisting of advances to officers and agents, cash in hands of officers and agents, other items viz: expense accounts, and furniture, fixtures and stationery. The total of such items was designated "Total Unadmitted Assets." It should be pointed out that the foregoing was merely a statement of unadmitted assets that had not been taken credit for as "Assets Available." The division of assets into ledger, non-ledger and not admitted, familiar to the present and several past generations, was not adopted until 1874. It is further interesting to note that many of the then unadmitted asset items are still so recognized and classified by law or ruling.

A form of annual statement blank for casualty companies was adopted by New York in the year 1871. It was adapted from the life blank. This blank with additions and changes necessitated by the extended scope of the casualty business was in use by New York, Connecticut, and other states until 1903 when it attained Convention status by action of the Committee on Blanks and approval of such action by the National Convention (now Association) of Insurance Commissioners. It was officially designated as "Miscellaneous" blank but is referred to in this paper under the more commonly used designation "Casualty" blank.

In 1868 an attempt was made by the then Superintendent of Insurance of New York to require the fire companies to show a balance between assets at the end of the preceding year, income and expenditures of the year and assets at the end of the current year. This did not succeed since the treatment of accrued income items had not been definitely settled and this was used by some companies as an excuse that it was not possible to produce such a balance. As a result, such balance feature was omitted in 1869 and was not reincorporated in the fire blank until 1899. However, such a balance was incorporated and required in the life blank beginning in 1875. The balance feature appeared in the New York casualty blank in 1896, three years prior to its incorporation in the fire blank.

By 1875 the general forms of the financial statements required by New York had become fairly well stabilized. One basic change was the treatment of uncollected premiums. In the fire blank these were carried as a non-ledger asset until 1902 when they were transferred to ledger assets on the familiar "Agents' Balances" basis (net as to commissions) to conform to the established "Account Current" method of reporting by agents. In the casualty blank the non-ledger asset basis was continued until 1909. Prior to the adoption of the Convention edition in 1903 the non-ledger item of premiums in course of collection was on a net basis. The gross premiums, also the commissions payable thereon and the net premiums were shown. In the 1903 blank the non-ledger assets showed the gross premiums and the commissions payable thereon were included on the liabilities page. The only change as respects this item in

1909 was the transfer of the gross premiums in course of collection from a nonledger to a ledger asset, the commissions payable thereon were still included on the liabilities page.

The scope of the blank was gradually extended to meet changes in methods of operation and practices and details were incorporated ostensibly for the purpose of furnishing a better picture of both operations and financial condition.

In the casualty blank schedules were added to implement laws governing the computation of loss reserves for established or new lines of business and tests of adequacy of loss reserves as reported in previous years' statements. The following is a list of the more important schedules with dates of incorporation in the blank.

Schedule		*Year*
G		1903 or prior
O	Part 1	1910
O	Part 2	1941
P	Liability, Parts 1-4	1903
P	Liability, Part 5	1934
P	Compensation, Parts 1-4	1913
P	Compensation, Part 5A	1934

Schedules M, Parts I-IV, designed to elaborate on certain types of company expense, were incorporated in both the fire and casualty blanks in 1946.

The foregoing brief history of the evolution and development of the fire and casualty annual statement blanks is helpful, if not essential, to an understanding of some of the factors which influenced the development and adoption of the 1950 combined blank.

The growing intricacies of the business, particularly casualty business, and the gradual increase in the scope of detailed information in the annual statements and supplementary exhibits and reports reached a stage where companies were hard pressed to make filings with the supervising authorities on the dates required by statute. This inspired thought and study by accountants, particularly fire accountants, looking to an amelioration of the growing burden. As a result, an informal committee of the Insurance Accountants Association, a fire insurance accountants association, developed a stock fire blank in the early 1940's consisting of two parts. Such blank did not involve any basic or material changes but merely divided the existing blank into its two natural divisions. In brief, Part I consisted of the essential details of operations and financial condition, and Part II the various supporting exhibits and schedules. Part I corresponded to pages 2, 3, 4, and 5 of the existing blank, excluding the exhibits of details of premiums written, losses paid, and losses outstanding by line of business which it was proposed to incorporate along with the other supporting exhibits and schedules in Part II.

The idea behind the two-part blank was the hope that if adopted by the National Association of Insurance Commissioners companies would be permitted to file Part I in compliance with statutory requirements and to file Part II at a somewhat later date.

The two-part blank was subsequently submitted to the Association of Casualty and Surety Accountants and Statisticians for its consideration.

The desirability of a united front was obvious. The idea of a two-part blank did not appeal too strongly to that organization, or at least to certain members, as it was felt that it would be desirable, if not essential, that the details of written premiums, paid losses, and outstanding losses, be balanced with the totals, or "controls" reported on pages 2, 3, and 5 of the proposed Part I before filing the same, and consequently the potential saving in time might not be material.

However, informal discussions between representatives of the two associations led to the logical conclusion that it would be desirable to have a joint committee of the associations meet and discuss the specific proposal and any other proposals or ideas of mutual interest or concern involving the respective annual statement blanks. Such committee was appointed by the respective associations and the first meeting was held on November 27, 1945. The result can best be stated by quotations from the minutes thereof:

"After considerable study the following motion was unanimously passed:

'Whereas there is a demand for presenting the Fire and Casualty Annual Statements on a modern and uniform basis, following the generally accepted principals of accounting, it is the consensus of this Committee that we prepare on the accrual basis one form of statement to be used for both Fire and Casualty Companies.'

It was further agreed that the work incident to the necessary studies to be made in connection with suggestions for revised annual statements would progress through the medium of two major sub-committees.

1. Those members located in Hartford and Boston.
2. Those members located in New York, Newark and Philadelphia.

These sub-committees to continue to study and have sub-committee meetings whenever conveniently possible in New York and Hartford. It was agreed that there would be an interchange of results developed from these sub-committee meetings and that an over-all full Committee meeting would be called when conveniently possible sometime in March, 1946."

The motion contemplated the separation of all exhibits and data of a statistical nature from the financial condition and operating results section as well as the development of a statement which would serve the requirements of both stock fire and casualty companies.*

A combined fire and casualty blank was more desirable than essential at the time although there existed some overlapping of coverages, particularly auto collision and personal property floater. Furthermore, the practice of establishing fire or casualty running mates was growing and for such groups a combined blank, permitting a maximum degree of uniformity in accounting systems, was highly desirable.

The decision to recommend a statement exhibiting assets at statement or admitted values, and operating results on the earned-incurred basis in summary form, with supporting details shown in supplementary exhibits and schedules was influenced by a contemplated new form of Life blank which incorporated these features and which was being developed by a joint com-

*Convention Blanks in use at that time and prior to 1950 were: Stock Fire, Mutual Fire (not used in all states), Foreign Fire (U.S. Branches; also not used in all states), Miscellaneous (used for Stock, Mutual and U.S. Branches) and Reciprocal.

mittee of actuaries representing the American Life Convention and the Life Insurance Association of America.

With the foregoing general objectives in mind the two sub-committees developed a rough draft of the proposed blank and this was studied and discussed at a meeting of the full committee held in Stockbridge, Massachusetts in June 1946 and the basic features of the blank were agreed upon. No particular areas of disagreement were encountered as it was found to be a rather simple matter to produce a form which would accommodate the reportings of both fire and casualty companies. The problem incidental to different systems of premium reporting and collection was solved by the logical decision that the "net balance" basis would be adopted, since the casualty companies could readily convert to this basis at the year end for annual statement purposes, whereas it would be impracticable for the fire companies to convert to the "gross premiums in course of collection" basis.

Schedule O presented a minor problem. The fire companies, because of the multiplicity of reinsurance arrangements and treaties, both external and internal (inter-group), would find it impracticable, if not impossible, to compile this schedule. Consequently, it was agreed that the scope of this schedule should be confined to the so-called casualty lines.

Following the Stockbridge meeting a proposed revised form of blank (excluding schedules) was made up, reproduced, and distributed to the supervising insurance officials of all states in the latter part of 1946. The reactions of the officials and their actuaries were mixed as might be anticipated when it is realized that the proposal was somewhat revolutionary, especially as it did not make specific provision for a balance of ledger assets between years. However, there was sufficient sentiment in favor of the proposal to encourage the committee to continue its efforts, and a revised edition of the blank which met some of the objections advanced against the original edition, and also reflected certain constructive criticisms, was issued and distributed in 1947. The second edition was received more favorably but did not gain approval by the Committee on Blanks of the National Association of Insurance Commissioners at its 1948 meeting.

In the meantime, two practically concurrent developments in the industry and in the field of supervision gave impetus to the proponents of the new blank. One was the trend to multiple line underwriting operations and the other the enactment of the New York Uniform Accounting Law. The first made it essential that a blank be adopted which would make provision for both fire and casualty lines and the second made it desirable that uniformity of expense accounting, and consequently of reporting, be attained. The New York Insurance Department had adopted supplementary blanks in 1947 for reporting the details of casualty business written by fire companies and fire business written by casualty companies. The now well known New York Regulation 30 was promulgated in 1948 to become effective with the 1949 annual statements.

As a result of the foregoing developments the time had arrived for a final decision on the proposed blank, or some other form of combined blank. At the April 1949 meeting of the Committee on Blanks the following action was taken by the Committee:

"Multiple Line Blank (Fire and Casualty Lines)

A proposed combined annual statement blank for fire and casualty insurers was submitted in December 1947 and considered at subsequent meetings of this committee. Considerable progress has been made after extended discussions in respect to suggested changes and additions. A sub-committee of six has been appointed to meet with the joint committee of fire and casualty accountants and statisticians to prepare a revised draft."

The meeting contemplated by the above was held in May 1949 and, as a result, a further revised edition was printed and distributed to the state supervisory officials and to the industry in December 1949. It was adopted at the April 1950 meeting of the Committee on Blanks and, as stated in the initial paragraph of this paper, was subsequently approved by the National Association.

A word of explanation of the principal differences of opinion between the industry and the Committee on Blanks is in order. These were two in number. The Committee on Blanks felt that the statement should contain (1)—an exhibit containing an analysis of assets into ledger, non-ledger, non-admitted, and statement (admitted) and (2)—an exhibit providing for a balance of ledger assets between years. The industry felt that such exhibits were not essential and that the preparation of the same would involve extra time and additional expense. The final edition incorporated these two exhibits (Exhibit 1, Page 11 and Exhibit 3, Page 12) at the insistence of the special committee and the blank was approved by the Committee on Blanks in such form.

The blank adopted contains several basic changes from the superseded forms which were not included in the "final edition" of the blank put out by the industry. These will be commented upon in the next installment.

EXCESS LOSS RATIOS VIA LOSS DISTRIBUTIONS

BY

D. R. UHTHOFF

Modifications of the retrospective rating procedures were adopted this year by the National Council on Compensation Insurance. These modifications included the optional provision of limiting ratable losses to stated amounts of $10,000, $15,000, or $25,000, per accident. With such loss limitations, the adopted retrospective premium formula can be expressed as follows: Retrospective premium = basic premium + expected excess losses and claim expense + limited losses and claim expense, subject to the tax multiplier and the appropriate minimum and maximum premium limits.

This paper is intended to describe the method adopted by the Actuarial Committee to determine the expected excess loss portion of this formula. The practical aspects of the problem required the development of a practical and flexible procedure, and the oft-quoted maternal nature of necessity produced a perhaps novel method which may be of general interest, conceivably useful for excess loss problems wherever they may arise.

DESCRIPTION OF END RESULTS

Intuition leads to the conclusion that this material will be more easily understood with a preliminary description of a few Tables, appended, which demonstrate the end product, and its use.

Table I presents the final indicated excess ratios which, when applied to individual state standard premiums, are designed to produce expected excess losses. For Alabama, .6 of 1% of Alabama premiums is equivalent to losses expected to be in excess of $10,000 per accident, and for District of Columbia premiums, 9.3% is required. For the time being, a brief note is necessary: these factors include a calculated *per claim* element, a catastrophe loading of 10% of the per claim element, and a flat catastrophe loading as indicated in the note.

Table II presents certain details in arriving at the factors shown in Table I. In Table II, Columns (2), (5), and (8) give ratios which are entries to Tables III or IV for death cases, Table V for permanent total disability cases, and Table VI for major permanent partials.

For illustration, let us refer to the Indiana values of Table II. The death average value of $6,676 has been calculated from the latest two years of Indiana experience, adjusted to law level. The $10,000 limitation is 50% higher than such average, Column (2), and therefore the entry to Table III is 150%. Table III represents values of a generalized distribution of death cases by size, and leads us to believe that 21.9%, Column (2), of all Indiana death claims will cost an amount equal to, or higher than, 150% of the average cost of all Indiana death claims; i.e., $10,000 or more. These high cost cases would involve 38.60%, Column (3), of total death losses, and the excess ratio, therefore, is 38.6 minus 21.9 $\times$ 1.50, or 5.8% as shown in Column (4).

Similarly, Column (5) of Table II is the entry to Table V; Column (8) the entry to Table VI. We then have ratios of excess losses to total losses, by the

serious injury parts. Weighting these injury ratios by the state portions shown in Columns (10), (11) and (12), the over-all ratios of Column (13) are obtained.

Now it is desired to apply these ratios to individual risks, and as a minimum recognition of risk differences the readily known risk standard premiums were selected as bases to which excess loss *premium* factors would be applied Therefore, multiplication by the state permissible loss ratio underlying standard premiums is necessary, since:

$$\text{Standard Premiums} = \frac{\text{total losses}}{\text{perm. loss ratio}}, \quad \text{and}$$

$$\frac{\text{excess losses} \times \text{perm. loss ratio}}{\text{total losses}} \times \frac{\text{total losses}}{\text{perm. loss ratio}} = \text{excess losses}$$

The distributions of Tables III through VI are for single claims only, and thus no multiple claim values have so far been reflected. This catastrophe element could not be treated formally and the final conclusion was to include part of this element as a 10% loading on the per claim excess indications, to reflect relative benefit levels; thus the 1.10 multiplier in Column (15), and the additional flat loadings shown in Table I, which were selected after a study of catastrophe experiences.

ADVANTAGES OF METHOD

It can be seen that this method allows rather easy revisions of state excess ratios as they may be required, and it is anticipated that the ratios now in effect in many states will be revised periodically. If benefit provisions are amended, the average values used in Table II can be immediately adjusted, with a consequent revision of excess ratios. Also, as new state experiences indicate, as they are now indicating, increasing average costs, the excess ratios can be kept in step. If further tabulations of catastrophe experiences indicate need for adjustment of the catastrophe excess elements, this can easily be done.

The calculations of excess loss ratios, in the past, usually have followed an approach simple in theory but cumbersome in practice, one notable exception being Mr. Elliot's Pennsylvania procedure. In short, this "simple" approach is to tabulate losses by size, adjust each loss as closely as possible to current cost conditions, draw the retention line, compare the excess to total losses, and then, after all that work, use the judgment which is necessary in excess rating problems. To follow that approach for every state, however, appeared impractical for several reasons: First, the relatively lower frequencies of higher cost cases involving excess, particularly catastrophes, require tabulation of quite a few years' experience for each state, for credible results. Second, increasing cost levels have a much greater effect upon losses in excess of a fixed retention value, ratio-wise, than upon total loss volumes, and in these times particularly, excess ratios based upon old loss amounts, without some magical means of adjusting, case by case, to current levels, would be too low. Third, ratios so established can become obsolete merely by enactment of benefit changes, and the only recourse for revision is to repeat the same arduous process. Finally, the anticipated date for introduction of the new retrospective rating procedure did not allow time to do all this.

DEVELOPMENT OF PROCEDURE

Anticipating the need for some short-cut method, the above "simple" approach was followed for a few states selected to provide variance of cost levels and benefit provisions, in the hope that these state excess ratios might lead to a general definition of excess ratios in terms of benefit provisions or average cost levels as parameters. This turned out to be a dead end, however, partly because of occasional catastrophes and high medical costs, and partly because of the ticklish nature of the small excess ratios, and inability to define benefit provisions in standard terms.

It became clear that if any general function were to be developed it would have to be by the three serious injury types separately—deaths, permanent total disabilities, and major permanent partials—and that catastrophes would have to be treated separately. A further conclusion appeared: Excess ratios are, after all, functions of loss distributions by size, and if loss distributions reveal a general pattern, state by state, and that general pattern can be expressed in terms of a parameter such as state average costs, state excess ratios would follow.

Pursuing this idea, the data which we had already tabulated for several states were adjusted, if necessary, not to the *latest* benefit provisions, but to the benefit provisions most common to the experience period. In Illinois, for example, we used claims incurred under 1944 through 1947 policies, and selected the July 17, 1945 benefits as the most common level. Claims incurred prior to that date were adjusted upward, those incurred subsequent and up to July 18, 1947 (the next benefit change) were used at actual cost, and those incurred subsequent to July 18, 1947 were adjusted downward. In this way a minimum of adjustments were required and the distorting effects of other increasing, or decreasing, loss factors, such as increasing medical costs, were minimized. In short, our objective was to get a distribution of experience as homogenous as possible, with respect to loss levels, the values of this distribution to be placed in terms of the average cost of all of those same cases on the common level.

Arrangement of the data in this fashion for the several states indicated the general pattern which had been hoped for, and which would allow proceeding to the details of combining the state experiences to form a general, large volume, multi-state curve, as presented in Tables III, V and VI.

DISTRIBUTION OF DEATH CLAIMS

The eight states' experiences forming the basis of Table III were incurred under benefit provisions having a maximum limit, either monetary or durational, upon total death indemnities. The loss distributions for these states would be expected to have peaks about the points representing the benefit maxima, and this was revealed. The New York benefits are not limited arbitrarily, and the New York experience showed a much smoother distribution—an essentially different type of curve. New York was the only un-limited benefit state for which the loss tabulation was available at the time, and it was assumed that the New York curve would be representative of such other state distributions, such as District of Columbia, which law was amended in 1948 to remove their $7,500 limit. Therefore, it was decided to show the New York values (Table IV) separately, for use in such states.

In the process of obtaining Table III, each state's data were arranged by the same intervals of Column (1), of Table III, the intent being to combine the frequencies according to the sizes expressed as ratios to the individual state averages.

Before proceeding to this combination, however, it was noted that the frequency peaks, due to maximum benefit limits, did not coincide. For example, the Alabama peak occurred at a value 165% of the average death value, and the Georgia peak occurred at 145% of the average. The coincidence, therefore, was not ideal. It appeared that the average value of all cases, both large and small, was not a perfect denominator, and was being influenced by the incidence and value of cases below retention points and in which we could have no direct interest in a study of excess ratios. Inasmuch as our final use of Table III contemplated the use of the full state average value, all cases, we could not hope to entirely eliminate this difficulty, but we could at least improve the coincidence of the upper portions of each curve before combining. This seemed worth-while, since the distributions of the higher-cost cases would be directly responsible for final excess ratios.

By examination of each state distribution, it was found that consideration of only those cases costing 90% or more of the average would include all cases with any possibility of excess over $10,000. By re-arranging the data in terms of the average of cases costing 90% or more of the original all-case average, the origin of the upper part of each state curve was shifted before combination, and the following calculation demonstrates this procedure for the Illinois data:

ILLINOIS

Death Cases Costing 90% or More of Illinois Average Cost of $3,967 And Adjustment to Terms of Average Cost of $5,355

(1) *Size, Ratio to $3,967 Average*	(2) *Claim Frequency*	(3) *Percentage Cost* (1) × (2)	(4) *Size, Ratio to $5,355 Average** (1) ÷ 1.35	(5) *Accum. Freq.* ΣCol. (2)	(6) *Std. Size Ratio*	(7) *Accum. Freq. at Size of* (6)
90%	21	1890	67%	21		
100	24	2400	74	45	70%	31
110	17	1870	81	62	80	60
120	201	24120	89	263		
130	121	15730	96	384	90	280
140	72	10080	104	456	100	420
150	91	13650	111	547	110	534
160	18	2880	119	565		
170	57	9690	126	622	120	573
180	11	1980	133	633	130	628
190	7	1330	141	640	140	639
200	4	800	148	644		
210	2	420	156	646	150	645
220	3	660	163	649	160	648
230	1	230	170	650	170	650
	650	87730				

* 877.30 ÷ 650 × $3,967 = 1.35 × $3,967 = $5,355

In explanation of the above: Columns (1) and (2) represent the upper portion of the original Illinois distribution of all death cases, for which the average value was $3,967.

Column (3) is an expression of cost, the total of which, 87730, divided by the 650 claims, indicates the average cost to be 135% of the 100% average of $3,967, and thus the average of the higher cost cases shown above is 1.35 × $3,967, or $5,355.

Column (4) shows the size groups as percentages of the new denominator of $5,355.

Columns (6) and (7) are for the purpose of standardizing the Illinois values for combination with other state data. Column (6) are merely the 10% size intervals of the final table, Column (7) being interpolated values (straight line) from Columns (4) and (5).

This procedure was followed in each state and the values of Columns (7) for each state were added directly, producing a distribution of higher cost death cases, based upon eight states and 2,327 cases. As this distribution was in terms of the higher average, however, and as it was to be used in terms of all-case averages, it was necessary to transform the size intervals from percentages of higher-case averages to percentages of all-case averages. This was done by a factor of 1.39, the ratio of the all-state higher-case average to the all-case average.

It is probable that this transformation introduces an error which could be avoided if we knew, in each state for which we would use Table III, the probable average cost of death claims in excess of 90% of the average cost of all death cases, or what amounts to the same thing, indicated ratios of these two averages, by state, such as the 1.35 shown above for Illinois.

To complete the curve, the lower-cost portions of the state distributions, in terms of ratios to all-case averages, were combined without adjustment, and only slight smoothing (by inspection) was necessary to obtain the values presented in Table III.

Table V for permanent total disability cases, and Table VI for major permanent partials, are self-explanatory in view of the above discussions of Table III for death cases. The state disability distributions, from which Tables V and VI were compiled, did not exhibit the peaks which were observed in the death distributions about the maximum death limits, and combination of the individual state values was performed without shifting origins of the upper portions of these curves. The Table VI, for major permanent partials, was based upon data for only the three states, Massachusetts, New York and Wisconsin, the only data available to us at the time, but each of these state distributions showed such a basic similarity as to allow the conclusion that any one, or the combination, would be sufficient for our purpose.

Tabulations of less serious claims, minor permanent partials and temporary totals, were not available, but it is probable that losses in excess of $10,000 in those cases, in any state, would have no appreciable influence upon average excess ratios.

APPROXIMATE "TEST" OF DEATH EXCESS RATIOS

It is difficult to imagine how any prediction of a reasonable and proper excess loss premium charge for a particular risk, or even for all risks of any one

state as a whole, could be thoroughly tested from a broad insurance viewpoint. However, since we have used basic indications of several states' data in combined form, such that we cannot say that the death distribution of Table III, for example, is exactly right for any one state, a reasonable question can be anticipated: How do the excess ratios obtained from the combined distributions compare to those we would obtain using the individual state distributions separately?

As at least a partial answer to this question, the following table shows the comparison for death cases—the group having the greatest influence upon these ratios:

	Death Excess Ratios				
State	*(1) From Table III*	*(2) From State Distribution (Approx.)*	*(3) Indicated Error (Approx.) (1) — (2)*	*(4) Ratio, Death To Total Losses*	*(5) Over-All Error, In Premiums (3)×(4)×Perm. (Approx.)*
Ala.	.007	.01	—	.14	—
Ga.	.006	—	+.01	.12	—
Ill.	.011	—	+.01	.06	—
Mass.	.275*	.20	+.08	.07	+.003
Mich.	.113	.08	+.03	.11	+.002
Mo.	.058	.11	−.05	.11	−.003
N. M.	.009	.01	−	.11	—
Wis.	.080	.01	+.07	.07	+.003

*From Table IV, Massachusetts now providing life benefits.

Column (3) shows differences in *death* excess ratios which at first glance seem rather large, particularly as possible errors relative to some correct ratio which might be assumed to be in the neighborhood of Columns (1) or (2). Column (6), however, demonstrates the relative importance of these "errors" from an over-all premium viewpoint.

Although these differences appear small, there is a possible justification in each of the above four states where the differences are notable. Massachusetts amended its law subsequent to the experience period to provide unlimited benefits to a widow, and the future distribution of Massachusetts claims can be expected to be quite different, with greater excess indications. Column (2), therefore, could be expected to be too low for the future. Michigan has also amended its law closer to an unlimited basis, as in Massachusetts.

In Missouri the reverse, a minus error indicating we may be too low, is shown. Missouri recently amended its law such that many claims previously settled under employers' liability will be compensation claims, and the distribution of Missouri death claims can be expected to change considerably. Wisconsin also has seen amendments, and the average value has increased from $6,180 on the 1945 law level to $7,140 on the latest law level.

Also, credibility of the state experiences must be considered, the 1% ratio in Column (2) for Wisconsin, for example, having been based on 301 claims, only 12 of which would have indicated excess on the present level of Wisconsin benefits. Those 12 claims might easily have been a substantially

different number because of different medical aspects only, regardless of other possible influences.

CONCLUSION

No matter how carefully we calculate average excess loss insurance charges, it is obvious that considerable underwriting judgment must be involved in deciding how appropriate such charges might be for particular risks. For example, the 9.3% District of Columbia charge for a $10,000 limitation might seem reasonably high for ordinary risks, but possibly inadequate for a hazardous risk where perhaps 50% of all losses are incurred under death claims. Although the risk standard premium will reflect such hazard, and the 9.3% will produce consequently greater volume of expected excess losses, reference to Table II will show the 9.3% factor was based upon an 18% proportion of death losses, not 50%, and is probably inadequate for such a risk.

Consideration of risk characteristics such as these reveals room for development of the method described herein, not so much from a retrospective rating point of view, but more for the purpose of contributing to solutions of excess rating problems in general, in other lines as well as compensation. Given any adequate generalized distribution of losses by size, it can be seen that logical variations in excess values, for any retention, can be obtained readily through variations in easily determined factors, varied average values, varied proportions of serious losses, varied catastrophe elements, such that we can proceed to more satisfactory solutions of our perennial excess rating problems.

TABLE I
EXCESS LOSS RATIOS AS FACTORS APPLICABLE TO STANDARD PREMIUMS

State	*Factors, Limitations of* $10,000	$15,000	$25,000	*State*	*Factors, Limitations of* $10,000	$15,000	$25,000
Alabama	.006	.005	.002	Minnesota	.015	.007	.003
Arkansas	.019	.006	.002	Mississippi	.019	.007	.002
California	.031	.016	.008	Missouri	.019	.010	.004
Colorado	.020	.013	.005	Montana	.018	.006	.003
Connecticut	.023	.012	.005	Nebraska	.037	.022	.010
Dist. of Col.	.093	.054	.021	New Hampshire	.008	.005	.002
Florida	.011	.005	.002	New Jersey	.024	.011	.005
Georgia	.006	.005	.002	New Mexico	.029	.014	.005
Hawaii	.025	.013	.006	New York	.061	.033	.009
Illinois	.012	.007	.003	No. Carolina	.013	.007	.003
Indiana	.013	.006	.003	Oklahoma	.011	.005	.002
Iowa	.007	.005	.002	Rhode Island	.020	.010	.005
Kansas	.006	.005	.002	So. Carolina	.007	.005	.002
Kentucky	.014	.006	.002	So. Dakota	.010	.005	.002
Louisiana	.013	.006	.003	Tennessee	.007	.005	.002
Maine	.011	.006	.003	Texas	.012	.005	.002
Maryland	.009	.005	.002	Vermont	.006	.005	.002
Massachusetts	.068	.035	.012	Virginia	.007	.005	.002
Michigan	.025	.009	.003	Wisconsin	.042	.023	.010

Note: Above factors include flat catastrophe elements of .005 for $10,000, .004 for $15,000 and .002 for $25,000.

TABLE II

CALCULATION OF EXCESS LOSS PREMIUM FACTORS FOR $10,000 LIMITATION PER ACCIDENT

State	(1) Death Aver. Value	(2) 10,000 ÷ (1)	(3) Death Excess Ratio	(4) P.T. Aver. Value	(5) 10,000 ÷ (4)	(6) P.T. Excess Ratio	(7) Major Aver. Value	(8) 10,000 ÷ (7)	(9) Major Excess Ratio	(10) Ratios To Total Losses: Death	P.T.	Major	(13) Over-All Excess Ratio	(14) Std. Permiss. Loss Ratio	(15) Premium Ratios With 10% Catast. Loading (13)×(14)×1.10
Alabama	4,801	233%	.007	7,251	138%	.105	2,838	352%	—	.14	.01	.11	.002	.588	.001
Arkansas	6,341	158	.043	8,637	116	.152	5,740	174	.062	.13	.02	.21	.022	.560	.014
California	6,294	159	.043	31,073	32	.685	5,729	175	.061	.07	.03	.27	.040	.597	.026
Colorado	4,900	204	.009	19,797	51	.510	3,875	258	.009	.11	.04	.15	.023	.585	.015
Connecticut	7,026	142	.075	22,249	45	.565	5,119	195	.040	.05	.03	.18	.028	.600	.018
Dist. of Col.	17,827	58*	.490	32,521	31	.695	6,270	160	.083	.18	.04	.18	.133	.610	.089
Florida	5,221	192	.013	9,261	108	.175	4,515	221	.024	.11	.02	.17	.009	.560	.006
Georgia	4,176	239	.006	6,548	153	.080	3,106	322	—	.12	.01	.14	.002	.594	.001
Hawaii	6,457	155	.050	30,358	33	.675	5,822	172	.065	.08	.02	.19	.030	.595	.020
Illinois	5,011	200	.011	14,386	70	.370	4,181	239	.015	.06	.02	.16	.010	.600	.007
Indiana	6,676	150	.058	10,567	95	.225	4,046	247	.012	.10	.02	.18	.012	.600	.008
Iowa	5,112	196	.012	5,061	198	.044	3,728	268	.007	.12	.01	.11	.003	.600	.002
Kansas	3,838	261	.005	6,499	154	.077	3,097	323	—	.10	.02	.18	.002	.570	.001
Kentucky	6,435	155	.050	9,520	105	.185	4,177	239	.015	.13	.02	.22	.014	.593	.009
Louisiana	4,756	210	.009	9,393	106	.180	4,213	237	.017	.09	.04	.18	.011	.620	.008
Maine	3,700	270	.004	11,848	84	.280	4,313	232	.019	.07	.02	.15	.009	.600	.006
Maryland	5,842	171	.025	8,825	113	.160	4,098	244	.013	.08	.01	.20	.006	.600	.004
Massachusetts	10,933	91*	.275	31,084	32	.685	8,780	114	.170	.07	.04	.28	.095	.605	.063
Michigan	7,847	127	.113	12,347	81	.295	5,394	185	.050	.11	.04	.15	.032	.575	.020
Minnesota	6,305	159	.043	20,625	48	.540	4,907	204	.036	.08	.01	.17	.015	.610	.010
Mississippi	(Arkansas values used)													.590	.014
Missouri	6,680	150	.058	17,627	57	.465	3,626	276	.005	.11	.03	.17	.021	.590	.014
Montana	7,608	131	.105	10,814	93	.235	4,705	212	.031	.09	.02	.20	.020	.600	.013
Nebraska	5,540	181	.018	22,450	44	.575	4,995	200	.038	.11	.07	.16	.048	.600	.032
New Hampshire	5,150	194	.013	6,328	158	.075	4,125	242	.014	.05	.01	.18	.004	.575	.003
New Jersey	8,122	123	.135	27,603	36	.650	4,743	211	.031	.06	.02	.23	.028	.608	.019
New Mexico	4,729	211	.009	16,110	62	.425	5,309	188	.045	.11	.06	.22	.036	.600	.024
New York	15,346	65*	.440	21,379	47	.550	8,042	124	.145	.11	.03	.18	.091	.562	.056
No. Carolina	5,278	189	.014	12,316	81	.300	3,565	281	.004	.13	.03	.18	.012	.590	.008
Oklahoma	6,948	144	.080	9,342	107	.180	3,897	257	.010	.07	.01	.25	.010	.590	.006
Rhode Island	7,036	142	.075	19,187	52	.510	4,826	207	.034	.03	.03	.20	.024	.580	.015
So. Carolina	5,181	193	.013	6,061	165	.065	3,448	290	.002	.10	.01	.28	.003	.575	.002
So. Dakota	5,591	179	.018	—	—	—	4,948	202	.037	.23	—	.10	.008	.570	.005
Tennessee	5,548	180	.018	7,617	131	.120	3,647	274	.005	.13	.01	.19	.004	.565	.002
Texas	6,440	155	.050	9,165	109	.170	3,617	276	.005	.15	.01	.20	.010	.608	.007
Vermont	3,577	280	.004	5,301	189	.048	2,969	337	—	.09	.02	.14	.001	.600	.001
Virginia	4,094	244	.006	10,582	95	.225	3,144	318	—	.11	.01	.14	.003	.595	.002
Wisconsin	7,140	140	.080	35,109	28	.725	8,779	114	.170	.07	.03	.18	.058	.579	.037

*Entry to Table IV, unlimited death benefits.

TABLE III—DEATH CASES*

(1)	(2)	(3)	(4)
Ratio to Average (Mid Point)	*% of Total Cases, Cases at or Above Column (1)*	*% of Total Costs, Cases at or Above Column (1)*	*% of Losses in Excess of Col. (1) Per Case (3) — (2) × (1) ÷ 100*
0%	100.0%	100.00%	100.0%
10	98.4	99.96	90.1
20	93.7	99.49	80.8
30	85.8	97.91	72.2
40	81.2	96.53	64.1
50	76.6	94.69	56.4
60	72.4	92.59	49.2
70	68.0	89.95	42.4
80	64.3	87.36	35.9
90	60.3	84.16	29.9
100	55.0	79.39	24.4
110	50.5	74.89	19.3
120	45.0	68.84	14.8
130	38.0	60.44	11.0
140	30.0	50.04	8.0
150	21.9	38.60	5.8
160	14.3	27.20	4.3
170	9.3	18.20	2.4
180	6.1	12.76	1.8
190	4.0	8.98	1.4
200	2.8	6.70	1.1
210	2.0	5.10	0.9
220	1.3	3.63	0.8
230	1.0	2.97	0.7
240	0.8	2.51	0.6
250	0.7	2.27	0.5
260	0.6	2.02	0.5
270	0.5	1.76	0.4
280	0.4	1.49	0.4
290	0.3	1.21	0.3
300 & Over	0.2	.92	0.3

*Based on experiences of Alabama, Georgia, Illinois, Massachusetts, Michigan, Missouri, New Mexico, Wisconsin.

TABLE IV—DEATH CASES* N. Y.

(1) *Ratio to Average (Mid Point)*	(2) *% of Total Cases, Cases at or Above Column (1)*	(3) *% of Total Costs, Cases at or Above Column (1)*	(4) *% of Losses in Excess of Col. (1) Per Case (3) — (2) × (1) ÷ 100*
0%	100.0%	100.00%	100.0%
10	99.9	100.00	90.0
20	99.6	99.97	80.1
30	98.7	99.79	70.2
40	80.0	94.18	62.2
50	76.0	92.58	54.6
60	72.5	90.83	47.3
70	68.5	88.43	40.5
80	64.0	85.28	34.1
90	58.5	80.88	28.2
100	52.0	75.03	23.0
110	44.2	67.23	18.6
120	36.7	58.98	14.9
130	29.7	50.58	12.0
140	23.4	42.39	9.6
150	17.9	34.69	7.8
160	13.8	28.54	6.5
170	9.8	22.14	5.5
180	7.2	17.72	4.8
190	5.7	15.02	4.2
200	4.5	12.74	3.7
210	3.7	11.14	3.4
220	3.0	9.67	3.1
230	2.5	8.57	2.8
240	2.0	7.42	2.6
250	1.6	6.46	2.5
260	1.2	5.46	2.4
270	0.9	4.68	2.3
280	0.6	3.87	2.2
290	0.4	3.31	2.1
300 & Over	0.2	2.73	2.1

*Based on New York experience only.

TABLE V—PERMANENT TOTAL CASES*

(1) *Ratio to Average (Mid Point)*	(2) *% of Total Cases, Cases at or Above Column (1)*	(3) *% of Total Costs, Cases at or Above Column (1)*	(4) *% of Losses in Excess of Col. (1) Per Case (3) — (2) × (1) ÷ 100*
0%	100.0%	100.00%	100.0%
10	98.7	99.87	90.0
20	96.0	99.33	80.1
30	92.3	98.22	70.5
40	87.5	96.30	61.3
50	81.0	93.05	52.6
60	73.6	88.61	44.5
70	65.0	82.59	37.1
80	55.5	74.99	30.6
90	45.5	65.99	25.0
100	36.0	56.49	20.5
110	27.5	47.14	16.9
120	23.3	42.10	14.1
130	19.3	36.90	11.8
140	15.8	32.00	9.9
150	12.5	27.05	8.3
160	9.8	22.73	7.1
170	7.5	18.82	6.1
180	5.5	15.22	5.3
190	4.3	12.94	4.8
200	3.8	11.94	4.3
210	3.0	10.29	4.0
220	2.5	9.19	3.7
230	2.1	8.27	3.4
240	1.8	7.55	3.2
250	1.5	6.80	3.1
260	1.3	6.28	2.9
270	1.1	5.74	2.8
280	0.9	5.18	2.7
290	0.8	4.89	2.6
300 & Over	0.7	4.59	2.5

*Based on Alabama, Georgia, Illinois, Massachusetts, Michigan, Missouri, New Mexico, Wisconsin.

TABLE VI—MAJOR CASES*

(1) *Ratio to Average (Mid Point)*	(2) *% of Total Cases, Cases at or Above Column (1)*	(3) *% of Total Costs, Cases at or Above Column (1)*	(4) *% of Losses in Excess of Col. (1) Per Case (3) — (2) × (1) ÷ 100*
0%	100.0%	100.00%	100.0%
10	99.4	99.94	90.0
20	98.3	99.72	80.1
30	95.6	98.88	70.2
40	87.0	95.44	60.6
50	77.8	90.84	51.9
60	67.6	84.72	44.2
70	55.3	76.11	37.4
80	47.5	69.87	31.9
90	40.5	63.57	27.1
100	35.0	57.07	22.1
110	29.7	51.24	18.6
120	24.7	45.24	15.6
130	20.2	39.39	13.1
140	16.9	34.77	11.1
150	14.7	31.47	9.4
160	12.8	28.44	8.0
170	11.2	25.72	6.7
180	9.6	22.92	5.6
190	8.3	20.45	4.7
200	7.1	18.05	3.8
210	6.2	16.16	3.2
220	5.5	14.62	2.5
230	4.8	13.01	2.0
240	4.1	11.33	1.5
250	3.4	9.62	1.1
260	2.7	7.92	0.9
270	2.3	6.84	0.6
280	1.9	5.72	0.4
290	1.5	4.56	0.2
300 & Over	1.2	3.66	0.1

*Based on Data from Massachusetts, New York and Wisconsin.

DISCUSSION OF PAPERS READ AT THE MAY 22, 1950 MEETING

CREDIBILITY PROCEDURES—LA PLACE'S GENERALIZATION OF BAYES' RULE AND THE COMBINATION OF COLLATERAL KNOWLEDGE WITH OBSERVED DATA

ARTHUR L. BAILEY

Volume XXXVII, Part I

WRITTEN DISCUSSION BY *DR. RICHARD VON MISES, Harvard University

Comments on Statistical Theory of Inference.

1. The basis of any statistical (probability) theory of inference is supplied by the concept, due to Bayes, of a priori and a posteriori probabilities. These may perhaps better be called "over-all" and "inferred" probabilities.

2. To be sure, the generalization, ascribed to Laplace, according to which the over-all probability is not necessarily a constant, but an actual function of the variable parameter has to be incorporated in the Bayes theory.

3. Under certain conditions (which are fulfilled in most practical cases) a decisive supplementation of the Bayes theory is supplied by the following statement: The influence of the over-all probability upon the inferred probability decreases the more the number of trials on which the statistics is based increases; in the limit, for an infinite number of trials, the inferred probability becomes independent of the over-all probability.

4. The a priori or over-all probability should not be confounded with concepts like "credibility" or "degree of confirmation," or "strength of expectation," etc. Whenever it is assigned a numerical value and enters as such a computational formula, it is a frequency limit like any other probability.

5. The fact that in many cases the over-all probability is not exactly known does not preclude the application of Bayes' concepts. One has to introduce reasonable estimates for it and to study the extent (depending on the number of trials), to which the indeterminacy affects the results.

6. The various currently used inference methods must be judged according to their compatibility with Bayes' concepts. Some results are the following:

a) The method of confidence limits (or fiducial limits) is in agreement with Bayes' theory, but it does not answer the question what inference can be drawn from a definite observation.

b) The Nyman-Pearson method of testing hypotheses, if interpreted in the correct way, leads to a weak and in most cases insufficient answer.

c) The likelihood method solves the problem only if the over-all probability is supposed to be constant or some metaphysical principle of "insufficient reason" is applied.

d) The recently developed decision functions of A. Wald are in full agree-

*By invitation.

ment with Bayes' concepts; they solve a more elaborate problem connected with the original inference problem.

7. All so-called small sample theories which derive estimates, decisions, etc., from a small number of observations without taking into account the over-all probability are completely unfounded and unreliable.

8. Many more detailed investigations of the consequences of Bayes' theory would be in order, for instance, that initiated by Mr. Bailey, or the development of approximation formulas according to point 5, or the extension of the range of validity of the statement in point 3, etc.

9. It is to be hoped that those and similar problems will find the attention of competent statisticians, as the unjustified and unreasonable attacks on the Bayes theory, initiated by R. A. Fisher, will fade out.

References

To 3): The statement is first proved in my paper: *Mathematische Zeitschrift* 5 (1919), p. 83, later in my textbook *Wahrscheinlichkeitsrechnung* (now: reprint Rosenberg, New York 1945), p. 185. See also *Lecture notes on probability and statistics*, Harvard University 1946, Chapter VI, 3.

To 5): An example is given in my paper: On the correct use of Bayes' formula, *Annals of Math. Statistics* 13 (1942), pp. 156-165.

To 6a, b, c): See the *Notes* mentioned above, Chapter IX.

To 6a: See also my paper: On the foundations of probability and statistics, *Annals of Math. Statistics* 12 (1941), p. 200.

To 6b): See also my paper: On the problem of testing hypotheses, *Annals of Math. Statistics* 14 (1943), pp. 238-252.

WRITTEN DISCUSSION

BY

*E. C. MOLINA

The privilege of discussing Mr. Bailey's paper is indeed gratifying to one who is not a member of the Casualty Actuarial Society. Moreover, the paper under consideration is of particular interest to one who, for nearly four decades, has been applying inverse probability formulas to problems confronting another great industry.

Inverse, or a posteriori, probability is that branch of probability theory which enables one to draw conclusions regarding the antecedents or causes of observed events. Quoting from the first paragraph of Mr. Bailey's paper, one has recourse to inverse probability theory "to determine the weight to be given to the indications of actual observations in a combination of such indications with a priori expectations which were based on other actual data, on prior knowledge or on reasonable assumptions made before observations were available."

In the literature of probability theory great confusion exists because many authorities have failed to distinguish clearly between the original Bayes inverse theorem and its subsequent generalization by Laplace. The generalized

*By invitation.

theorem embraces, or brings together, both the data obtained from a series of observations and whatever "collateral" information exists in relation to the observed results. We are greatly indebted to Mr. Bailey for the emphasis he has placed on the Laplacian generalization. Its appearance as a sub-title to his "Credibility Procedures" gives one *ab initio* the kernal of Mr. Bailey's analysis.

One whose acceptance of Laplace's generalization for the solution of inverse probability problems is based on an extensive and intensive study of the classics beginning with Bayes' famous Essay and running through the works of Laplace, Poisson, Cournot, Bertrand, Poincaré, Czuber, Borel, Castelnuovo *et al*, finds it difficult to commend in restrained terms the paper on "Credibility Procedures" submitted by Mr. Bailey for your consideration.

WRITTEN DISCUSSION
ON THE UTILIZATION OF DIRECT AS WELL AS COLLATERAL INFORMATION IN THE PROBLEM OF STATISTICAL ESTIMATION

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> "To make a careful estimation means to utilize all relevant knowledge available and to reason well in deriving the estimate from this knowledge."
>
> R. Carnap[1]

Attempts to formulate a general theory of statistical estimation date back as far as the eighteenth century. It was only recently, however, that a method was developed, the method of Maximum Likelihood, which, although not general in the strictest sense, takes care of a relatively large class of problems of estimation. A good number of statisticians seem reluctant, however, to accept this method in general, questioning its appropriateness in specific applications, as well as doubting the soundness of its arbitrary choice of criteria.

It seems questionable to us whether it is at all possible to formulate satisfactory universal principles which define a "best" estimate, not necessarily the same in each case, for every problem of estimation and for every kind of direct or indirect evidence. Indeed, we are doubtful whether it is actually wise to follow the above quotation and consider *all* available information under *all* circumstances.

This does not mean that we are questioning the usefulness and importance of the most recent developments in the generalization of statistical theory, which are due mainly to John von Neumann and Abraham Wald.[2]

In the first part of this paper we shall discuss very briefly our own approach to the subject of credibility. Because of the magnitude of the problem, it is understandably impossible, to present to you today anything but a brief ab-

[1] R. Carnap, *Logical Foundations of Probability*, Univ. of Chicago, 1950.
[2] J. von Neumann, *Theory of Games and Economic Behaviour*, Princeton Univ., 1944 and A. Wald, *Statistical Decision Functions*, New York, 1950.
* By invitation.

stract of these views. The second part of this paper deals with two very short comments on Mr. Bailey's paper, "Credibility Procedures," which was presented to you this spring at the Stockbridge meeting of the Casualty Actuarial Society.

I.

For reasons which will be explained later we shall treat the problem of credibility as a problem of multiple estimation. By multiple estimation we mean the problem of estimating the population parameters of a set of populations which have been chosen as a group because of some property or properties which they may have in common. Such a set of populations might, for example, consist of the various risks which belong to a given classification.

Let us denote the distribution which is associated with each of these populations (risks) by the symbol $f(x_{ij} \mid \theta_i)$. This distribution represents the conditional probability (probability density) of obtaining an observation x_{ij} from the ith population of our class, if the parameter θ_i is a certain fixed constant. The symbol x_{ij} stands for the jth observation taken from the ith population. This symbolism is convenient, if we have more than one observation from each population (risk).

Our problem is to estimate the population parameters θ_i, which, for each population must, of course, be a fixed constant, but which need not be the same for the various elements of our chosen class of populations. Consequently, if we consider the entire class of populations (risks), we can now speak of the distribution $f(\theta)$ of the parameters θ_i *within the chosen class of populations.* Whenever we treat the parameters θ_i as variables, *in this sense,* the subscript i will be omitted.

Given the distribution $f(x_{ij} \mid \theta_i)$ and the distribution $f(\theta)$, we can readily calculate the distribution $f(\theta \mid x_{ij})$ by means of the rule of Bayes-Laplace. The new distribution function $f(\theta \mid x_{ij})$ expresses the probability that a given observation x_{ij} has come from a population whose parameter equals the constant θ. In order to complete the symbolism which we shall use, let O_{n_i} stand for a random sample of n_i observations taken from the ith population and let θ_i' stand for an estimate of θ_i.

In the examples which we shall discuss, it will always be assumed that the direct information consists of random samples O_{n_i} from at least one of the populations. As a matter of convenience (it is by no means necessary), we shall also assume that the n_i are all equal to a constant n. The indirect, collateral, or antecedent information which may be available in our examples will consist of either complete or partial information concerning the distributions $f(\theta)$ and $f(x_{ij} \mid \theta_i)$. Estimates which are based on partial or complete knowledge of the distribution $f(\theta)$ will be called *Credibility Estimates.*

Before we can estimate the parameters θ_i, we must first establish a criterion which defines what we mean by a "good", "best", or "preferred" estimate. This is essential because we can estimate the θ_i in infinitely many ways. As a matter of fact, the method of estimation is completely arbitrary unless we specify some sort of criterion, on the basis of which we can distinguish between the various kinds of estimates with reference to some desirable properties.

This situation is quite similar to the customary problem of fitting a straight line through a given set of points. We can draw, of course, infinitely many

of these lines, and unless we define what we mean by a "good fit", we have no basis for expressing a preference for any one of these lines. Therefore, also in our original problem, we must establish such a criterion before we can estimate the θ_i. It is important to note that a chosen criterion must be such that whatever information is available or can be obtained will be sufficient to perform the method of estimation which has thus been defined. Furthermore, we shall, in general, base our criteria on pragmatic considerations, such as minimizing certain quantities relating to errors or maximizing certain probabilities.

We shall now proceed to discuss a few of the credibility estimates which may be obtained under several conditions regarding the collateral information and under correspondingly different criteria which will define our "preferred" estimates. It must be understood, of course, that by knowledge, collateral or otherwise, we mean empirical and not a priori knowledge. As we shall show later on, we are *not* justified in using the rule of Bayes-Laplace, unless we have an empirical basis for the type of distribution which is to be used for $f(\theta)$.

Case A. *We have complete knowledge of both* $f(\theta)$ *and* $f(x_{ij} \mid \theta_i)$. In this case, where we have the maximum amount of collateral information, short of actually knowing the θ_i, we might suggest two alternative criteria which define our "preferred" estimates. *Criterion 1*:

> "The estimates should be such that if we were to apply this method of estimation to all members of our class of populations (to all risks within the given classification), the direct information being identical in each case, then the error variance $\Sigma(\theta_i' - \theta_i)^2$ should be a minimum."

It can easily be shown that the estimate which is thus defined is simply the mean of the distribution $f(\theta \mid O_{n_i})$ and we shall consequently call this type of estimate a *Mean Estimate.*[3] Therefore in this case

$$\theta_i' = \int \theta . f(\theta \mid O_{n_i}) d\theta$$

and the actual form of the estimate will, of course, depend on the distributions which are being used.

An alternative solution may be obtained by means of *Criterion 2*:

> "The estimate θ_i' should be the value of the parameter θ of the population (within our chosen class of populations) from which the given sample is most likely to have come."

It is important to note that this estimate is *not* a Maximum Likelihood estimate. The estimate which has thus been defined is simply the mode of the distribution $f(\theta \mid O_{n_i})$ and we shall therefore call it a *Modal Estimate.* Consequently in this case

$$\theta_i' = \text{the mode of } f(\theta \mid O_{n_i})$$

and the form of the estimate will again depend on the nature of the distributions which are used in the computation of $f(\theta \mid O_{ni})$. It is an interesting fact that if both of the original distributions are normal distributions, the estimates resulting from the two different criteria will be identically the same.

[3] The term "Mean Estimate" was suggested to us by Prof. H. Reichenbach of the University of California at Los Angeles.

Case B. *We know the distribution* $f(x_{ij} \mid \theta_i)$, *but we have only partial information concerning* $f(\theta)$. *For example, we might know its first two moments, namely* $\overline{\theta}$ and σ_θ^2. The criterion which we shall employ in this case is an adaptation of the "Theory of Adjustment",[4] originally developed for problems in surveying, where we estimate parameters like the sides and angles of a triangle, which, as we know, must satisfy certain trigonometric identities. The criterion, in this case, is the following, *Criterion 3*:

> "The set of estimates θ_i' should be the values of the θ_i which maximize the probability of obtaining the given sample of n observations from each population within our classification, under the condition that the θ_i' must satisfy a given number of functional restrictions."

This means that we must maximize the probability P, where P is given by

$$f(O_{n_1} \mid \theta_1) \cdot f(O_{n_2} \mid \theta_2) \ldots\ldots\ldots\ldots f(O_{n_k} \mid \theta_k)$$

under the condition that, for example, $\Sigma\theta_i' = c$ and $\Sigma\theta_i'^2 = d$, where c and d are known constants. As a suitable name for estimates of this type we would like to suggest *Restricted Maximum Likelihood Estimates*. It is important to note that the objections which Mr. Bailey raised against the method of Maximum Likelihood (on page 6 of his paper) do not apply in this case. The resulting estimates θ_i', which can easily be obtained using the technique of Lagrange Multipliers, will be weighted estimates, very much like those which are obtained with the use of the other criteria.

An alternative approach, in this case, might be to disregard some of the collateral information concerning $f(x_{ij} \mid \theta_i)$ and treat the problem as if it belonged to the case which we shall discuss next.

Case C. *The only knowledge which we have about* $f(x_{ij} \mid \theta_i)$ *is, in this case, its standard deviation* σ *which is assumed to be the same for all populations of our class.*[5] *The only knowledge which we have concerning* $f(\theta)$ *consists of its mean and standard deviation,* $\overline{\theta}$ *and* σ_θ *respectively.* This leads us to Mr. Bailey's "Best Linear Regression". The resulting estimates, which we shall call *Best Linear Estimates*, are defined by the following criterion, *Criterion 4*:

> "The estimates should be such that if we were to apply this method of estimation to all members of our class of populations (risks) for all possible samples from these populations (randomization), the error variance $\Sigma(\theta_i' - \theta_i)^2$ should be a minimum, under the condition that the θ_i' be of the form $A\bar{x}_i + B$, where $\bar{x}_i$ is the sample mean of the ith population."

The properties of this type of estimate are well known, having been developed in detail by Mr. Bailey in one of his earlier papers.[6] It is important to note the distinction between Criterion 4 and Criterion 1. We are now summing on the x_{ij} as well as on θ, whereas we kept the direct information, i.e. the x_{ij}, constant in our formulation of Criterion 1.

Case D. *The only collateral information which we have in this case consists of the mean and standard deviation of* $f(\theta)$, *i.e.* $\overline{\theta}$ *and* σ_θ. A possible estimate

[4] See N. Arley and K. R. Buch, *Introduction to the Theory of Probability and Statistics*, New York, 1950, esp. chapter XII.

[5] This assumption is modified, for example, in the multiplicative case, where the σ's are proportional to the θ's.

[6] See A. Bailey, "A Generalized Theory of Creditibility," *Proceedings* of the Casualty Actuarial Society, Vol. XXXII, 1945.

which suggests itself in this case is, what might reasonably be called a *Restricted Least Square Estimate.* Its properties are defined by *Criterion 5*:

"The estimates θ_i' should be such that they minimize the expression

$$\sum_{i=1}^{k} \sum_{j=1}^{n} (x_{ij} - \theta_i')^2$$

under the restriction that $\Sigma\theta_i'/k = \bar{\theta}$ and $\Sigma\theta_i'^2/k = \sigma_\theta^2 + \bar{\theta}^2$ where k is the number of populations belonging to our chosen classification."[7]

Applying this criterion, we obtain (again with the aid of Lagrange Multipliers) the final result that

$$\theta_i' = C \cdot \bar{x}_i + (1 - C) \cdot \bar{x} + (\bar{\theta} - \bar{x})$$

where $\bar{x}$ is the over-all mean of the sample values of all the populations, and where the constant C is given as

$$C = \sigma_\theta / \sigma_{\bar{x}_i}$$

The criterion used in this case demands that certain conditions which are satisfied by the θ's must also be satisfied by the θ''s. In other words, we have transferred certain properties of the population parameters to their estimates.

It must be evident, that the analysis which we have given in the above discussion is far from being an exhaustive study of the subject of credibility. Indeed, it has been our purpose rather to indicate by means of a few special cases the approach which can be used in obtaining credibility formulae, i.e. the formulae for credibility estimates. The steps to be taken can be summarized as follows:

1. We must specify precisely the nature of the collateral information which might be available or which might be obtained.
2. We must then formulate a principle which defines the preferred properties which we want our estimates to have. It is important that these conditions must be such that they can be satisfied by whatever collateral information is available, and they must also be such that they can be translated into mathematical terms.
3. The final step consists of computing the actual formulas, on the basis of the given criterion, using the collateral information which was specified in step 1. This last step may involve a good deal of mathematical detail, but once the criteria have been established in step 2, the problem is, logically speaking, straightforward.

We have denoted the estimates, which we have developed, as "preferred estimates", rather than as "good" or "best" estimates, because this term seems to be more descriptive of the actual situation. An estimate can be "best" in a variety of different ways, depending on whatever we happen to mean by the word "best". (We could, for instance, call an estimate "best" if its formula looks the "prettiest".) The term "preferred" estimate brings out very clearly

[7] It is necessary, in this example, to have a sample from *each* of the populations. The symbol $\sigma_{\bar{x}_i}$ stands for the standard deviation of the sample means, as computed from the given data.

that the estimate is based on conditions which express a preference which may be based on pragmatic or other considerations.

II.

It seems to us that the basis which Mr. Bailey chose in his development of the theory of credibility in his paper, "Credibility Procedures", is equivalent to what we have described very briefly in Case A, Criterion 1.

In spite of the fact that this equivalence may not be immediately apparent, we feel that the meaning of Mr. Bailey's distribution $K(x)$ must be interpreted in the sense of our $f(\theta)$.

The quantity x, the "true" expected losses of a particular risk (or whatever parameter we are trying to estimate) must be a constant as long as we are speaking about a specific risk. As a matter of fact, it must be *defined* as a constant which, incidentally, belongs to what is called the mathematical model. We, therefore, cannot speak about the probability or certainty of its existence. In his general discussion, Mr. Bailey makes the statement:

> ". . . The actuary knows that there is more than one possible value of x and is willing to assume that he can approximate the a priori probabilities of the existence of such possible values."

We cannot believe that Mr. Bailey means to imply that x can be anything but a constant. If, however, we speak about more than one risk, then the corresponding values of x may, of course, be different, and in this sense we can say that there is more than one possible value of x.[8] Therefore, in order to treat x as a variable, we must embed a given risk within a class of other risks. Indeed, we cannot speak about the probability of obtaining a certain value of x, unless we specify such a class of similar risks, which in probability theory is commonly referred to as the reference class.

The embedding of an event within a class of similar events for the purpose of making predictions or estimations is a common procedure in scientific methodology. Let us suppose, for example, that we wish to predict whether it will rain tomorrow or not. The meteorologist, whom we consult, tells us that the probability that it will rain tomorrow is .65. As it is quite evident that one or the other has to happen, we must interpret his statement as saying that: "In a large class of similar situations, we can expect it to rain about 65 percent of the time." In order to make a meaningful prediction, we had to embed the given situation within a large class of similar situations. This, incidentally, is precisely what is being done when a risk is given a manual rate at the time when it is first insured and when no direct information is available.

In establishing the criterion for his "best" estimate (in the sense of Least Squares), Mr. Bailey says that the error variance is summed

> ". . . for all of the possible cases for which H may occur."

To speak of "all possible cases" is meaningful only if we specify a definite reference class. It seems to us, therefore, that Mr. Bailey's criterion is identical with our criterion 1 in which we also summed the error variance over the entire class within which we have classified the risk.

It is true, of course, that the question of how to formulate a problem

[8] R. von Mises, "On the Correct Use of Bayes' Formula," Annals of Math. Statistics, 1941, p. 191.

and how to state the criteria is a matter of taste and expedience, so long as the formulations are equivalent. The reason why we prefer our own development as presented in the first part of this paper is that it seems to us to be a logically more precise formulation which is a good deal easier to understand.

It is important to note that although the criterion is based on the entire class, this does not mean that we must estimate every element of that class. This can most easily be understood if we refer to the example which we gave before. The prediction that the probability that it would rain is .65, specifies the "best" odds, even though we may be interested in the weather only on one particular day. In the same sense, we have a "best" estimate, even though we may estimate only one of the risks which belong to the chosen class.

Since the type of inference discussed in this problem involves the highly controversial rule of Bayes-Laplace, we would like to add a very brief comment on the justification of this rule. This formula, commonly called simply the "Rule of Bayes", can be derived from the axioms of probability in two or three simple steps. Consequently, the arguments which have been raised against the application of this rule consist basically of the claim that we can never actually know all of the distributions which are involved. If we do not know the distribution $K(x)$, but merely assume its form a priori, we are guilty of distributing our ignorance in some arbitrary fashion, uniformly or otherwise.[9] The important consideration, therefore, is that we must have an *empirical basis* for the type of distribution to be used for $K(x)$; and casualty actuaries are indeed privileged because this type of information is seldom available in other fields of scientific inquiry. Consequently, it seems to us that Mr. Bailey is unnecessarily asking for criticism in his statement (on page 6) that he is considering the weights to be given to

> ". . . observed data in its combination with collateral *or with a priori knowledge.*"

All the indirect information *must* consist of collateral data or of reasonable inferences drawn from such collateral knowledge.

We are certain that it will please Mr. Bailey to hear that a good number of statisticians are disturbed by, what Professor Carnap calls the startling spectacle of unsolved controversies and mutual misunderstandings that appears in most standard treatises on probability and statistics. It is our sincere hope that Mr. Bailey's pioneer work in the field of credibility may lead to the elimination of some of these controversies and to a better understood and more general approach to the problem of statistical estimation.

WRITTEN DISCUSSION BY

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Mr. Bailey's very interesting and important paper presents a novel departure from the conventional philosophies of statistical estimation. Although Mr. Bailey has concerned himself primarily with the derivation of estimation procedures through the application of Bayes' Theorem it seems to me that he

[9] If, for example, we put a priori $K(x)$ equal to the Gamma distribution, we might justifiably be accused of employing the principle of "Gamma—distributed ignorance," analogous to the principle of "Equally distributed ignorance" which is mentioned several times in Mr. Bailey's paper.

* By invitation.

has made an even more basic contribution in his recognition of the fact that the parameters characterizing a group of related probability distributions may properly be considered as stochastic variables under certain conditions. As Mr. Bailey has pointed out this concept has heretofore been almost completely ignored by the recognized authorities in the field of mathematical statistics.

It should be noted that the stochastic variation of a group of distribution parameters will be of a somewhat different sort than that of the variables characterized by the individual distributions. If we consider a group of variables such that broadly similar causal factors apply to all of the members of the group we may expect the distributions of the variables comprising this group to exhibit certain similarities. However, the parameters characterizing these distributions need not be identical for all members of the group since there may be specific influences operating to produce differences among the members. Once the individual variables comprising the group are defined the values of the parameters characterizing the distributions of each of these variables are fixed. However, these values certainly will not be evenly distributed throughout the range of all possible values since their variation is restricted by the underlying casual similarities existing among the variables. Thus we may say that the set of parameters so defined constitutes a sample from the statistical population composed of the values of the parameters of all possible distributions having the same underlying similarities. It is in this sense that we may consider the parameters as stochastic variables.

Since the credibility procedures discussed by Mr. Bailey are basically estimation processes it might have been more logical to derive them from some criterion of accuracy rather than from considerations explicitly involving inverse probabilities. In the following section I will present an outline of such a derivation which does not explicitly involve inverse probabilities and which assumes that *all* of the parameters (rather than just the expected values) of the various distributions are stochastic variables in the sense described above.

II.

This section will be devoted to the development of an estimation procedure predicated on the principles outlined above and to the application of this procedure to insurance statistics. The notation $E[A \mid B_i, (i = 1, 2 \cdots)]$ will be used throughout the following exposition to indicate the conditional expectation of A given the quantities B_1, B_2, B_3, etc. where these quantities B_i represent various parameters of the probability distribution of A.

Let x_{ij} be the ith variable of the jth class of variables where the number of variables in the jth class is n_j and there are N classes in all and where the x's are all mutually independent. The criteria for determining the arrangement of the variables into these classes will usually be such as to insure that each variable in a particular class will have properties more similar to those of other members of the class than to the properties of variables in other classes. The class of all the x's must be determined so that the probability distributions of the x's have certain general characteristics in common. Specifically, this class of variables must be defined so that every possible value of each parameter of the various probability distributions will be associated with a definite probability of occurrence in the sense described in the preceding section.

If all of the moments of a probability distribution are given then the dis-

tribution is completely determined so that we may define the parameters of any distribution in terms of its moments.

Let the parameters t_{ijk}, $(k = 1, 2, \cdots)$ of the distribution of x_{ij} be defined by

$$E[x_{ij}^k \mid t_{ijk},\ (k = 1, 2, \cdots)] = t_{ijk} \quad \text{for } k = 1, 2, \cdots.$$

Now each of these quantities t_{ijk} is associated with a probability determined by the general character of the class of all the x's. Therefore, we may define parameters of the *joint* distribution of all the t's as follows:

$$E[t_{ijk}^s \mid T_{ks},\ S_r,\ (k, s, r = 1, 2, \cdots)] = T_{ks} \text{ for } k, s = 1, 2, \cdots$$
$$\text{and } E[t_{ijr} \cdot t_{vjr} \mid T_{ks},\ S_r,\ (k, s, r = 1, 2, \cdots)] = S_r \text{ for } i \neq v$$
$$\text{and } r = 1, 2, \cdots.$$

These parameters T_{ks} and S_r do not entirely determine the joint distribution of the t's since the higher order product moments are not considered but they will be sufficient for the present investigation.

We will assume that it is desired to estimate the expected value of the arithmetic mean of the n_j variables comprising the jth class. This expected value will be represented by $t_{j1} = \frac{1}{n_j}\sum_i t_{ij1}$, and the estimate of t_{j1} will be represented by t'_{j1}. In order to determine the "best estimate" of t_{j1} we may set up a criterion of accuracy in terms of a minimum error variance. That is, we may minimize

$$\sigma_{j1}^2 = E[(t'_{j1} - t_{j1})^2 \mid T_{ks},\ S_r,\ (k, s, r = 1, 2, \cdots)]$$

with respect to t'_{j1} where t'_{j1} is considered as a function of $\bar{x}_j = \frac{1}{n_j}\sum_i x_{ij}$ and does not involve the t's. If no further restrictions are placed on t'_{j1} we may minimize σ_{j1}^2 as follows:

$$\sigma_{j1}^2 = \int_R \int_{R'} (t_{j1} - t'_{j1})^2\, P dt_{j1} d\bar{x}_j, \qquad \text{where } P \text{ is}$$

the joint probability density function of t_{j1} and x_j, R is the region containing all values of $\bar{x}_j$ and R' is the region containing all values of t_{j1}. For simplicity the variance is shown here as an ordinary double integral as if P were continuous throughout R and R'. Since t'_{j1} is independent of t_{j1} we may write:

$$\sigma_{j1}^2 = \int_R \left[\int_{R'} t_{j1}^2\, P dt_{j1} - 2t'_{j1} \int_{R'} t_{j1}\, P dt_{j1} + t'^2_{j1} \int_{R'} P dt_{j1} \right] d\bar{x}_j$$

Setting the partial derivative with respect to t'_{j1} of the quantity in brackets equal to zero and solving for t'_{j1} we have

$$t'_{j1} = \frac{\int_{R'} t_{j1} P dt_{j1}}{\int_{R'} P dt_{j1}} \qquad \text{as the value of } t'_{j1} \text{ which}$$

minimizes σ_{j1}^2. This is equivalent to $t'_{j1} = E[t_{j1} \mid \bar{x}_j, T_{ks}, S_r, (k, s, r = 1, 2, \cdots)]$ as in Mr. Bailey's derivation from Baye's Theorem. Suppose, however, that we wish to approximate this value of t'_{j1} by a polynomial of mth degree in $\bar{x}_j$. If we represent such a polynomial by $t'_{j1} = a_o + a_1\bar{x}_j + a_2\bar{x}_j^2 + \cdots + a_m\bar{x}_j^m$ we may evaluate the coefficients $a_o, a_1, a_2, \cdots a_m$ in terms of the T's and S's by minimizing σ_{j1}^2 with respect to $a_o, a_1, a_2, \cdots a_m$. The case of particular interest here is that in which the estimate is linear in $\bar{x}_j$. When dealing with insurance statistics a linear estimate is the most practical since the data available is usually insufficient for the evaluation of the constants involved in an estimate of higher degree. Thus, letting $t'_{j1} = a_o + a_1\bar{x}_j$ and minimizing σ_{j1}^2 with respect to a_o and a_1 we have

$$a_o = (1 - a_1)\, T_{11},$$

$$a_1 = \frac{T_{12} - S_1 + n_j\,(S_1 - T_{11}^2)}{T_{21} - S_1 + n_j\,(S_1 - T_{11}^2)}.$$

If a sufficient number of observed values of the variables x_{ij} were available it would be possible to estimate the parameters T_{11}, T_{12}, T_{21} and S_1 and hence obtain values for a_o and a_1. Unfortunately, since the data available to the insurance statistician are not sufficiently detailed for this purpose, further assumptions must be made in order to derive a workable procedure.

The insurance problem is essentially that of the estimation of pure premiums, so that we may define x_{ij} to be the total losses produced by the ith unit of exposure of the jth risk (or territory or manual classification depending on whether the estimate is being made for experience rating or for manual ratemaking purposes). The pure premium for the jth class will then be represented by $\bar{x}_j$. In order to obtain estimated values for a_o and a_1 we must first assume that each of the N classes designated by $j = 1, 2, 3, \cdots N$ is internally homogeneous. This means that $t_{ijk} = t_{vjk}$ for any i and v and for all values of k within the jth class. The parameter S_1 will then be replaced by T_{12} and we will have

$$a_1 = \frac{n_j}{\dfrac{T_{21} - T_{12}}{T_{12} - T_{11}^2} + n_j}$$

It can easily be shown that if the exposure basis is varied, the quantities T_{21}, T_{12}, and T_{11} will vary in such a way that a_1 will remain constant for any particular class. This is in accord with our intuitive conviction that the credibility coefficient associated with an insurance pure premium should be independent of the exposure basis used.

Since the values of x_{ij} are usually not available separately but only in the form of the average, $\bar{x}_j$, it will be necessary to make some assumptions regarding the form of the probability distribution of x_{ij} if we are to obtain some sort of estimates of T_{12} and T_{21}. The assumptions adopted henceforth are as follows:

(a) The probability density function of the losses x_{ij} will be non-negative and will have a discontinuity at zero since the probability that a unit of exposure will produce no losses is a definitite positive quantity.

(b) The claim frequency will follow a Poisson distribution so that the probability that x_{ij} will be zero (i.e., that the ith unit of exposure will produce no losses) will be given by e^{-c_j} where c_j = the expected claim frequency of the jth class.

The probability density function of x_{ij} under these assumptions is

$$f(x_{ij}) = \begin{cases} e_j^{-c}; \text{ for } x_{ij} = 0 \\ (1 - e_j^{-c})\, g(x_{ij}); \text{ for } x_{ij} > 0 \end{cases}$$

where

c_j = the expected value of the claim frequency, and
$g(x_{ij})$ = the probability density function for all losses greater than zero.

The variance of x_{ij} is then given by

$$t_{ij2} - t_{ij1}^2 = t_{ij1}^2 \left[\frac{k_j^2 + e^{-c_j}}{1 - e^{-c_j}} \right]$$

where

k_j^2 = the coefficient of variation associated with $g(x_{ij})$, i.e., the coefficient of variation of the losses greater than zero, and
t_{ij1} = the expected value of x_{ij} as before.

Since the data necessary for the direct evaluation of k_j are usually not available this expression for the variance of x_{ij} must be modified somewhat in order to reduce it to a form more useful for estimation purposes. In order to accomplish this we may make the following assumptions:

1) The coefficient of variation of the distribution of claims by size of claim is constant for a given type of coverage, and
2) The claim frequency is independent of the claim size.

Then it may be shown directly that

$$k_j^2 = \frac{(1 - e^{-c_j})\,(k_o^2 + 1)}{c_j} - e^{-c_j}$$

where k_o is the coefficient of variation of the distribution of claims by size of claim. The expression for the variance of x_{ij} then becomes

$$t_{ij2} - t_{ij1}^2 = t_{ij1}^2 \left[\frac{k_o^2 + 1}{c_j} \right]$$

We now have a foundation which makes possible the estimation of T_{11}, T_{12} and T_{2i} from available data. The value of k_o may be estimated from the appropriat observed distribution of claims by size of claim, and observed values of the pure premiums $\bar{x}_j$ and claim frequencies c_j are usually available.

In order to obtain the necessary relationships we may first note that

$$E\,[x_j \mid T_{ks}, (k, s = 1, 2, \cdots)] = T_{11},$$

$$E\,[x_j \mid T_{ks},\ (k, s = 1, 2, \cdots)] = \frac{T_{21}}{n_j} + \frac{n_j - 1}{n_j} \cdot T_{12}, \text{ and}$$

$$E\,[t_{i2} - t'_{i1} \mid T_{ks},\ (k, s = 1, 2, \cdots)] = T_{21} - T_{12}$$

Now, replacing these expected values by the corresponding observed averages and representing the observed values of x_j, c_j and k_o by X_j, C_j and K_o respectively and the estimates of T_{11}, T_{12} and T_{21} by T'_{11}, T'_{12}, and T'_{21}, respectively, we may derive the following equations for evaluating T'_{11}, T'_{12} and T'_{21}:

$$(1) \qquad \frac{\sum_j n_j \overline{X}_j}{\sum_j n_j} = T'_{11}$$

$$(2) \qquad \frac{1}{N}\sum_j n_j \overline{X}^2_j = T'_{21} + T'_{12}\left[\frac{\sum_j n_j}{N} - 1\right]$$

$$(3) \qquad (K_o^2 + 1)\frac{\sum_j \frac{n_j \overline{X}^2_j}{C_j}}{\sum_j n_j} = T'_{21} - T'_{12}$$

These estimates may be neither unbiased nor efficient but they probably represent the best that can be done with the available information.

The estimation formulae may now be put into the form:

$$(4) \qquad t'_{j1} = Z_j \overline{X}_j + (1 - Z_j)\, T'_{11}$$

$$(5) \qquad Z_j = \frac{n_j}{\frac{T'_{21} - T'_{12}}{T'_{12} - T'^2_{11}} + n_j}$$

where Z_j is the "credibility" of $\overline{X}_j$.

Equations (4) and (5) are of the form usually associated with experience rating credibility procedures but they could easily be adapted for use in manual ratemaking and should give more accurate results than the present ratemaking credibilities based solely on the observed number of claims.

Many experience rating plans have incorporated credibility tables based on relationships very similar in form to (5) except that the premium volume is substituted for the risk exposure n_j which, of course, is what would be obtained by multiplying the numerator and denominator of (5) by [T'_{11} ÷ permissible loss ratio]. However, because of the maximum single loss provision usually incorporated in experience rating plans, equations (4) and (5) are not strictly identical with the operations performed in experience rating. The quantity corresponding to $\overline{X}_j$ in experience rating is derived from a truncated distribution in which the losses are not allowed to exceed a certain prescribed value so that the expected value of these modified observations is no longer equal to

the quantity being estimated. If we let $\overline{Y}_j$ be the observed pure premium derived under the maximum single loss provision, equations (4) and (5) become:

$$(6) \qquad t'_{j1} = Z_j(\overline{Y}_j + W_j) + (1 - Z_j)\, T'_{11}$$

$$(7) \qquad Z_j = \frac{n_j}{A_j + B_j n_j}$$

where W_j represents a correction for the bias introduced by the maximum single loss provisions and where A_j and B_j are functions of the T's and also depend on the manner in which the maximum single loss is determined for the jth risk.

In practice the difference between (4) and (6) may be offset to some extent by the fact that the quantity corresponding to T'_{11} will actually be based on the average experience of all risks of a particular type, whereas there may be a selection in favor of the group of risks which are experience rated. Thus, if the average experience for the group of experience rated risks is better than that for all risks of the same type a certain upward bias will be produced by using the experience for all risks (as reflected in the manual rates) as the estimate of T_{11}.

The quantities of A_j and B_j in (7) cannot easily be evaluated from available information and about all that can be said at present is that A_j approaches $\frac{T'_{21} - T'_{12}}{T'_{12} - T'^{2}_{11}}$ and B_j approaches 1 as the allowable maximum loss is increased indefinitely.

I would like to emphasize the fact that this whole approach to the problem of credibility procedures is predicated primarily on considerations of accuracy and does not take note of the stability requirements which are surely necessary from the point of practicability. In fact, previous derivations of credibility procedures have been concerned mainly with obtaining sufficiently stable estimates with considerations of accuracy being strictly secondary. Since credibilities designed to produce maximum accuracy do not bear any close relationship to the expected relative amount of chance variation of the individual pure premiums, the use of such credibilities might not produce a set of estimates which could be readily used to establish a set of stable rates.

In this connection I would like to suggest that since the expected number of claims may be shown to be directly related to the expected chance fluctuation of the corresponding pure premium, perhaps credibility tables based jointly on the observed number of claims and the exposures (or premium volumes) would give results consistent with both accuracy and stability.

WRITTEN DISCUSSION BY *WILFRED PERKS

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As a convinced supporter of the principles of inverse probability my sympathies are naturally with Mr. Bailey's approach. Whether a particular problem of statistical estimation involves prior ignorance or prior knowledge the one system of Bayes' theorem meets the requirements of the problem.

*By invitation.

With appropriate invariant rules to express prior ignorance, the results of Bayes' theorem in certain important cases are identical with those of confidence intervals and associated techniques. In cases where the prior knowledge is a precise statement of a prior probability distribution all schools would, I suggest, use Bayes' theorem, although these cases have been labelled "trivial" by certain statisticians. It is in the cases where the prior knowledge is imprecise that serious difficulties arise, both in principle and in practical application and it is with cases of this kind that Mr. Bailey's paper is concerned. There is much to be said for Professor Jeffreys' judgment that vague prior knowledge might well be ignored and an appropriate indifference rule applied. I judge, however, that Mr. Bailey's problems involve rather more than "vague" prior knowledge, although it is still "imprecise".

I am in complete agreement with Mr. Bailey that we should, if it is appropriate, try to express, even if only approximately, our prior knowledge in the form of the hypothetical results of a set of hypothetical past trials. This leads at once to the use of the beta distribution for prior probabilities in the problem of estimating the binomial parameter and to the use of the gamma distribution (a limiting case of the beta distribution) for the problem of estimating the Poisson parameter (a limiting case of the binomial) i.e. to formulae (12) and (17) respectively of the paper.

I am, however, troubled about three things:—

1. Is the prior knowledge that we are assumed to have prior knowledge about a super-population from which a particular population is supposed to have been selected at random? That is to say are we estimating the parameter of the particular binomial distribution selected from a known distribution of binomial populations?
2. Or is the prior knowledge that we are assumed to have, prior knowledge about the particular population? That is to say, have we made a prior estimate of the parameter?
3. Have the underlying conditions of operation and observation remained unchanged throughout and as between the circumstances applicable to the prior knowledge and those applicable to the past and future observations? That is to say are there any reasons to suppose that there is a secular or other systematic variation in the parameter concerned?

Even if we have no "knowledge" of kind (1) above, there must be a starting point for the prior probabilities to be used in the application of Bayes' theorem, although any significant amount of "knowledge" of kind (2) would tend to swamp the importance of the particular form of "knowledge" of kind (1). I can understand that in practice we may have good reason for assuming a particular value for the mean of the prior probability distribution but Mr. Bailey's processes call for an assumption about the standard deviation of the distribution or, what is the same thing, an assumption about the total number of hypothetical observation as well as the proportion of hypothetical successes i.e. we need to know the value of the indices in the beta distribution as well as their ratio. It is the basis upon which this standard deviation can suitably be judged that I am not clear about. This was the difficulty that long confused

the problem of a self-consistent indifference rule. A mean value of 1/2 was satisfactory but the standard deviation arising out of a uniform distribution led to trouble. The invariant rule independently devised by Prof. Jeffreys and myself has now got over this difficulty for the indifference case, but the problem still remains in Mr. Bailey's case. This standard deviation is, of course, the vital factor in determining the weights for combining the prior estimate of the parameter with the observed frequency ratio.

If there is a systematic variation of the kind mentioned under (3) above, the use of Bayes' theorem is inappropriate. In practice, unfortunately, the situation is all too often complicated in this way.

At the end of the paper Mr. Bailey refers to the "unsolved problem" of the frequency distribution of claims losses. This is essentially a multinomial problem which can perhaps be formulated in several ways. It is, however, the problem of estimating the $p_i(\Sigma p_i = 1)$ in a multinomial distribution.

If the p_i are linked by a mathematical formula the problem becomes one of estimating the parameters in the formula. Otherwise, the whole set of values of p_i have to be estimated jointly. I have examined the indifference problem in this case (J.I.A. LXXIII, 285) and R. E. Beard and I (J.I.A. LXXV, 75) have indicated the relative insignificance in practice of the correlation effect referred to by Mr. Bailey. It is usually sufficient in practice to assume that each p_i gives rise to an independant Poisson variable.

I should make it clear that I am not familiar with the rather extensive specialized literature in America on Credibility Procedures. My comments arise out of a reading of Mr. Bailey's paper alone and I realize that the points I have made may not be new and may have been answered already in the literature. Indeed, I cannot be sure that I am not misconceiving the problem altogether.

WRITTEN DISCUSSION BY L. H. LONGLEY-COOK

The author is to be congratulated not only on a most interesting and stimulating paper on credibility procedures in casualty actuarial work but also on an important contribution to the subject of inverse probability. Inverse probability has been considered in relation to actuarial work on a number of occasions and when Mr. Perks presented a paper to the Institute of Actuaries on the subject a few years ago a most interesting discussion resulted. My remarks, however, will be limited to the discussion of credibility procedures.

I fear this paper will be found difficult by most students and I have been wondering if the principal results can be brought out in a more simple manner without the use of inverse probability with all its pitfalls or too much loss of rigour. I hope the following demonstration will be of some assistance in this respect.

Following the author's development and using his notation, we first consider the case of the proportion of losses where an investigation shows H "successes" out of n "trials". It is desired to make the best estimate of the true loss frequency taking into account the prior knowledge but ignoring all question of trends, that is giving equal weight to all data. In the simplest form the prior knowledge will be H successes out of $\overline{n}$ trials and the best estimate of the loss frequency is clearly

$$\frac{\overline{H} + H}{\overline{n} + n}$$

which can be written

$$Z\frac{H}{n} + (1 - Z)m$$

where

$$Z = \frac{n}{\overline{n} + n} \quad \text{and} \quad m = \frac{\overline{H}}{\overline{n}}.$$

Since Z increases as n increases relative to $\overline{n}$, this shows that in the usual credibility formula greater weight should be given to larger volumes of observed data. It will be noted however that in this case if $\overline{n}$ is large compared to n practically no weight should be given to the current knowledge.

In practice the data making up a class are not homogeneous and we can imagine the prior knowledge being split up into a number of sub-groups with loss frequencies x_1, x_2, x_3, etc. Let the mean of these values be m and the variance σ^2. Although it may be unreasonable to assume that the distribution of the x's will follow any law, the best estimate which can be made, on the basis of prior knowledge alone, of the true loss frequency for some new sub-group is m subject to a variance σ^2. Also if the observed loss frequency of a new sub-group is H/n, the best estimate which can be made, on the basis of current knowledge alone, of the true loss frequency, q, is H/n with a variance, on the assumption of a Poisson distribution of $\{\sqrt{nq}/n\}^2$.

For rate making purposes we can use a combination of these two estimates

$$Z\frac{H}{n} + (1 - Z)m$$

The variance of this combination is

$$Z^2\{\sqrt{nq}/n\}^2 + (1 - Z)^2\sigma^2$$

Differentiating with respect to Z we find the condition for minimum variance is

$$2Z\{\sqrt{nq}/n\}^2 + (-2 + 2Z)\sigma^2 = 0$$

Using the approximation $q = m$, this becomes

$$Z = \frac{n\sigma^2}{n\sigma^2 + m}.$$

Hence we see that, even when the prior knowledge is large compared to the current data, if the current data consist of the experience of a sub-group and the sub-groups are not homogeneous one with another then more weight should be given to the experience of larger sub-groups.

Turning to the case where we are concerned with the dollar amount of losses instead of their number only, we can subdivide the total number of losses into groups according to size. Taking first the simple case of homogeneous data for losses of amount t, we have the proportion of claims of this size in the current data is H_t/n, and in the prior knowledge $\overline{H}_t/\overline{n}$. Hence the weight to be given to current knowledge is $n/(\overline{n} + n)$ whatever the size of the loss and no more weight should be given to the frequently occurring small losses. The

position is different when the prior knowledge can be divided into a number of sub-groups each with a slightly different experience. The formula is then

$$Z = \frac{n\sigma_t^2}{n\sigma_t^2 + m_t}$$

where σ_t^2/m_t is the ratio of the variance to the mean of the number of claims of size t. σ_t^2/m_t will normally decrease as t increases and hence, since for any sub-group n will be constant, Z will decrease as t increases. From this we see that in these circumstances more weight should be given to frequently occurring small losses.

It seems desirable to warn students that while standard credibility procedures are both necessary and desirable when a routine practice can be introduced, as for instance in Workmen's Compensation rate making, it is not generally practicable to replace actuarial judgment by credibility rules of thumb. If the actuary will see that he has a real knowledge of the data he is handling, how compiled, possibility of errors, changes in conditions which have occurred, etc., and will keep before him as yardsticks the square root of the number of claims and an approximate frequency distribution of claims by size, he will usually obtain a more satisfactory estimate of the rate he may expect in the future than by the blind application of any credibility formula.

REPLY TO DISCUSSIONS BY ARTHUR L. BAILEY

Dr. von Mises has provided us with a commentary on the theory of inference that only one with his broad knowledge of the many proposed solutions to the problems of statistical inference could state so concisely yet completely. It should be read and read carefully, preferably before reading my original paper.

Mr. Molina has been very kind in his comments. His contributions to the literature of mathematical statistics are almost unique because they evidence a determination to mold the mathematics to the practicalities of the case; instead of the reverse. His refusal to discard prior knowledge or collateral information in the analysis of observations has made him an outstanding advocate for inverse probabilities. I, as you should know, am personally very indebted to him for his kindness in going over an early draft of my paper and for the contributions he made to it, especially to the historical background of inverse probability theory.

The comments of Messrs. Perks, Longley-Cook and Freund and my recent reading of "Theory of Probability" by Harold Jeffreys, has convinced me that an estimate of x based on observations O_n (Freund's use of O_n instead of H' for the n observations is a distinct improvement in symbolism) should be symbolized as $E(x \mid O_n, K, L, C)$ where K represents the degree of prior knowledge as to the prior probability function $K(x)$, or the hypothesis substituted for such knowledge; where L represents the degree of prior knowledge as to the likelihood probability $P(O_n \mid x)$ or the hypothesis substituted for such knowledge; and where C represents the criteria selected as the basis of the estimate of the conditions imposed on the estimate.

Dr. Jeffreys has used a symbolism that expresses every probability in terms of the hypotheses made and has stressed the need for completely specifying all such hypotheses. He shows clearly that any evaluation of a posterior probability must be proportional to the product of the prior probability and the

likelihood probability. Similarly it could be shown that any use of an estimate based on an observation must involve either knowledge of or hypothesis as to both the prior probabilities and the likelihood probabilities as well as acceptance of the criteria utilized. It appears that much of the past confusion as to the relative merits of estimation procedures would have been avoided if the hypotheses regarding these probabilities as well as the criteria on which the estimates were based were always specified.

Mr. Freund calls attention to the fact that a chosen criterion must be such that whatever information is available or can be obtained will be sufficient to perform the method of estimation which has been defined. I would like to add another note of caution to this. Criteria should be avoided if they impose any conditions over and above what is necessary to provide the estimate. If the conditions are too broad, they may prevent the statistician from employing certain reliable and justifiable prior knowledge or collateral information.

Freund's use of the term "preferred" instead of "good" or "best" brings out only that tastes differ—and rightly so. For example, the "restricted" estimates produced by the criteria Freund suggests in Cases B and D would appeal to me in much the same way that "restricting" square pegs to round holes would. The condition that the variance of the estimates equal the variance of the thing being estimated, is in my opinion, an unsound one, especially when the correlation between the estimate and the thing being estimated is low. Although I have expressed this repeatedly to Freund, he still likes it proving that tastes differ and that "preferred" has no more useful meaning than "best". Let us then simply state what the criteria for an estimate is without characterizing it.

The real heart of the problem is that, to whatever extent knowledge is lacking as to the prior probabilities or the likelihood probabilities, the lack must be made up by hypotheses. One of the difficulties has been that criteria have been selected at times so as to completely hide the hypotheses implicitly made but not expressed. Take Freund's Case C as an example. His statement of the scope of the assumed knowledge, of the criteria applied, and of the results obtained are correct; but the simple condition that the estimate be a linear function of $\bar{x}i$ implied the hypothesis that the prior probabilities followed one specific distribution when the likelihood probabilities followed another specific distribution as I have shown in the paper now under discussion. The Beta and Binomial, the Gamma and Poisson, and the Normal and Normal were shown to be such paired hypotheses produced by the condition that the estimate be a linear function. An important reason for my writing the paper was to show what hypothesis as to the prior probabilities was involved in that apparently innocuous condition.

One of the most easily lost hypotheses is that implied in the use of a maximum likelihood estimate. The procedure is one that completely disregards the prior probabilities but produces an estimate in a form that requires the user of the estimate to assume that $K(x) = k$ for all possible values of x. The statistician refuses to make the hypothesis, but forces his client to make it.

Running throughout my paper and the discussions is the confusing difference in the concept of probability when we are dealing with a heterogeneous instead of a homogeneous population. Most probability theory and most statistical methods assume a homogeneous population, for each individual of

which the probability is the same—some constant value, known or unknown. In casualty insurance our basic assumption is that the insurance hazards differ from risk to risk as well as from classification to classification. We have only heterogeneous populations, for each individual of which the probability is different—a variable whose value is never known although we frequently wish to estimate it. When I deal, as I do, with the probability of a probability, I am dealing with a concept that never occurs in homogeneous populations and, therefore, with a concept that is disturbing to any newcomer to the field of heterogeneous populations.

Mr. Johns actually was the original cause of my paper. In the fall of 1949 we jointly undertook to determine the most effective split of losses between "normal" and "excess". We were stymied in that project by a philosophical snag. His training, which not only exceeded mine but was twenty years more up to date, would not permit treating a parameter as a variable and required that he impose as a condition, $E(x/B) = B$, to obtain an "unbiased" estimate of B. To proceed along the lines of my previous work on credibility violated his training. To follow his training meant that no split of losses was justifiable. We deserted the original project to study the philosophies of probability theory.

In textbook after textbook the only acknowledgment of the prior knowledge or collateral information that actuaries recognize in the credibility formulas was to be found in the one or two paragraphs covering the theory of inverse probability. Starting with this, guided by Mr. Molina's paper showing a practical application of inverse probability theory, and fortified with a recent paper by Mr. Freund showing that the generalized Bayes' Rule was still alive, my paper evolved.

If I had then had the 1948 edition of Jr. Jeffreys' book I could have shown Mr. Johns that the generalized Bayes' Rule (Mr. Jeffreys' theorem 10) was the basis of all evaluations of probabilities from observations, all tests of significance, and of all estimates. It would have been quite apparent that all of the accepted procedures taking up 99.8 per cent of the space in statistical texts are based on, or can be derived from, the theory of inverse probability in combination with one of the following three assumptions:

(1) The number of observations is so great that the effect of any prior knowledge or collateral information is trivial and can be disregarded.

(2) There is no prior knowledge or collateral information of any value and the theory of equal ignorance, for which $K(x) = k$, or the theory of equal indifference recently devised by Mr. Perks, for which $K(x) = k/x$, is applicable.

(3) We are dealing with a homogeneous population so that x has only one value, say A, and $K(x) = 0$ except that $K(A) = 1$.

Each of these three assumptions produce a credibility of 100 per cent for the indications of the observations; but, they are the only ones that will.

Mr. Johns has made two very substantial contributions. First, he has completely generalized the development of estimation procedures when parameters of sub-populations are treated as variables. Secondly, although he has dealt again with the case when the observation, H, is the product of the parameter, x, which is to be estimated, and an independent variable, h, with the restriction that the variance of h is constant for all values of x, his

procedure is such that the restriction can readily be removed. To do so would be of especial importance to us because it would produce a credibility formula for classification pure premiums or risk experience rating modifications not heretofore available.

In my paper I developed the procedures without regard to the source of the knowledge as to the prior probabilities or of the values of m and σ^2 into which that knowledge was to be concentrated. Mr. Perks has pointed out, and rightly so, that we should be much concerned with the source of such knowledge in any particular application. I have indicated in previous papers the general sources of such knowledge and will try to summarize them briefly here.

When we have no prior knowledge as to the values of x or of the values m and σ^2 it is contemplated that, if we have made observations for each of N individuals, we select the values of m and σ^2 which would lead us to expect to obtain the observed mean and variance of H. Such a selection will even evaluate the prior probabilities if we accept the suggested functional forms. When we do have previous estimates of the values of x, say y, it is intended that a new unknown be estimated namely $x^1 = x/y$ from adjusted observations of $H^1 = H/y$. The mechanics of performing such evaluations from collateral information and prior estimates is by no means settled and considerable work needs to be done along those lines.

Mr. Longley-Cook has indeed simplified the presentation of a demonstration that credibility procedures should be used in dealing with heterogeneous populations. Both as to his closing remarks on the desirability of being bound to the use of a mechanically applied credibility formula, and as to Mr. Johns' remarks on the desirability of maintaining stability in rates, I can only comment that the present matter under study is how to evaluate the indications of the statistical experience and not how to use such evaluations in making rates for the future. The combination of the indications of the past with actuarial judgment, or even with biased opinion, is another and very different study involving personal rather than mathematical equations.

REVIEWS OF PUBLICATIONS

CLARENCE A. KULP, Book Review Editor

Fundamentals of Fire and Casualty Insurance Strength. Roger Kenney. Kenney Insurance Studies, Dedham, 1949. Pp. 14, 246.

The author of this book is Roger Kenney, Insurance Editor of the *U. S. Investor*. His purpose in writing this book was to help the policy-buying public and investors to analyze the financial strength of fire and casualty insurance companies. With a background of war and inflation, and with rigid control of rates and policy forms now resting with the state as the indirect result of the recent trend in the insurance business away from "action in concert", the author feels that managements are faced with a heavy responsibility of marking out a course of individual action in an almost uncharted area of operation. It is the author's hope that this book will not only promote a better understanding of fire and casualty financial statements on the part of the public, but will promote a better appreciation of the skill required to operate a fire and casualty company successfully.

Even though the primary purpose of the book was to serve the policy-buying public and the investors and to present a very technical subject in language which the layman can understand, this book is a "must" for fire and casualty actuaries. An actuary is often charged with the task of explaining his company's financial statements in non-technical terms and of justifying loss reserve practices. It is the prime responsibility of actuaries to keep their companies financially sound. On the assumption that every actuary has or will read this important addition to the library of our business, this review will be limited to a discussion of his rules for measuring financial strength.

The Kenney theory of fire insurance strength is: "In its essence, the formula provides that other things being equal, the ratio of policyholders' surplus to the unearned premium reserve determines the relative strength of fire insurance companies." Taking the unearned premium reserve as the measure of the potential outstanding liability of the company in the form of risks accepted, Kenney concludes that the larger the cushion of safety (policyholders' surplus) against the unearned premium reserve the greater the strength of the organization. The author says that he likes to see a ratio of one to one between policyholders' surplus and the unearned premium reserve of a fire insurance company. However, unlike many other analysts and sponsors of financial formulae, this author "is not unmindful of the fact that there are conditions where a lower ratio may be approved. And one of these conditions is a persistently low loss and expense ratio over a period of years." In appraising this one-to-one ratio, the author also cautions that you must consider the liquidity of the company's assets. He also states that "it all comes down to the fact that to get the whole story of fire insurance strength, you really ought to have one eye on the operating account, as well as the balance sheet."

One chapter gives 6 rules for discerning how a fire insurance company laboring under a huge premium volume stands financially, and underwriting-wise: (1) look well to the investment exhibit; (2) if the company is owned by another, scrutinize the surplus of the parent company; (3) compare the loss ratio in the latest report with that of the preceding 4 or 5 years; (4) make inquiry as to the caliber and character of the management; (5) make judicious inquiry into the reinsurance arrangements of the company; (6) examine carefully the area of operation for concentration of writings in congested and hazardous areas.

In appraising the financial strength of a casualty company, the author makes some very significant statements. He warns that "you cannot legislate a casualty company into solvency or into good underwriting practice. Management still plays an all-important role in determining whether a casualty company will be a success, both from the policyholders' and the stockholders' point of view". Some years ago the author coined the slogan: "A casualty company is no stronger than its loss reserve policy". An important chapter in this book is entitled, *Loss Reserves—The Heart of a Casualty Statement.*

The author gives 7 rules for gauging the strength of a casualty company. Briefly, these are: (1) determine the adequacy of loss reserves, for Schedule P lines, with particular reference to Schedule P—Parts 5 and 5A; (2) examine carefully Schedule O to study the run-off of loss reserves on lines other than liability and workmen's compensation; (3) scrutinize the suit record of the company; (4) reconcile the reported underwriting results with the indicated savings or deficiencies in loss reserves; (5) measure the ratio of policyholders' surplus to premium volume in the light of underwriting results adjusted to the indicated equity or deficiency in the loss reserve; (6) examine the balance sheet for liquidity to ascertain if there is a "jingle" in the loss reserve; (7) study carefully the entire investment portfolio.

As a companion to the Kenney one-to-one theory for a fire insurance company, the author advocates a "2 for 1 casualty rule". Briefly, this rule is that a casualty company should have one dollar of policyholders' surplus for every two dollars of premium volume. The author is, however, very careful to qualify that rule with the statement that there are conditions where a somewhat higher premium volume can safely be written—such as a persistently low (and proven) loss ratio over a protracted period".

He also cautions that "practically every alternative measuring stick of casualty strength loses its value unless something is known about the underwriting habits of the management, particularly as regards premium volume".

These two Kenney rules—one-to-one fire and two-for-one casualty—have been the subject of much argument. The basis of the adverse criticism is not the rules as the author uses them, but with their use as automatic tests of financial strength without the important qualifications which permit of variations for such factors as good management, low loss ratio record, sound investment portfolio and adequate loss reserve practices.

N. E. MASTERSON

REVIEWS OF PUBLICATIONS

Inland Marine & Transportation Insurance. William H. Rodda. Prentice-Hall, Inc., New York City 11, New York. Pp. xvi, 539.

This book, the first to appear on the subject in 15 years, very ably fills the need for an up-to-date and easy-to-read treatise on inland marine insurance.

The rapid growth of the inland marine lines during the past 30 years has been nothing short of stupendous. This rapid expansion kept pace with the growth of the wealth of our country, as well as of our systems of transportation, particularly motor truck and the airplane. Growing pains have necessitated changes in policy conditions and rates. Governmental and self-regulation have become important factors. Both the fire and casualty fields have felt the development of these lines and it might be said that inland marine insurance has become a bridge between the two.

The author in an interesting manner touches on every phase of this subject, beginning with its colorful history and heritage from fire, casualty and ocean marine insurance, and ending with a chapter on governmental regulation. The intervening chapters describe the following: transportation policies, business covers other than transit, bailee and bailee customers' covers and personal covers. Mr. Rodda states, "The principal parts of the book are devoted to a comprehensive discussion of the problems facing the underwriter in providing inland marine coverage to policy holders." The text is based on the new inland marine forms which became mandatory on March 1, 1950.

The author has had 25 years experience in insurance engineering and rate-making and is Secretary-Treasurer of the Transportation Insurance Rating Bureau. He is also Chairman of the Truck Transportation Committee of The National Fire Protective Association. Out of many years experience he has written a book which can well serve as a modern text and reference volume on this important subject.

WILLIAM F. DOWNS*

PUBLICATIONS RECEIVED

Life Insurance Mathematics. Robert E. Larson and Edwin A. Gaumnitz. John Wiley and Sons, Inc., New York, 1951. VII, 184.

* Guest reviewer.

ABSTRACT FROM THE MINUTES OF THE MEETING

May 22 and 23, 1950

The semi-annual meeting of the Casualty Actuarial Society was held at the Red Lion Inn, Stockbridge, Massachusetts, on Monday and Tuesday, May 22 and 23, 1950.

President Barber called the meeting to order at 2:30 P.M. on May 22nd, the roll was called, showing the following 43 Fellows and 18 Associates present:

FELLOWS

ALLEN, E. S.
BAILEY, A. L.
BARBER, H. T.
BARTER, J. L.
BATHO, E. R.
BERKELEY, E. T.
BURLING, W. H.
CAHILL, J. M.
COATES, C. S.
COGSWELL, E. S.
CONSTABLE, W. J.
CROUSE, C. W.
EDWARDS, J.
ELLIOTT, G. B.
FONDILLER, R.
FULLER, G. V.
GINSBURGH, H. J.
GODDARD, R. P.
GRAHAM, C. M.
HOOKER, R. O.
JOHNSON, R. A.
KOLE, M. B.
KORMES, M.
LINDER, J.
MCCONNELL, M. H.
MASTERSON, N. E.
MATTHEWS, A. N.
MAYCRINK, E. C.
MILLER, J. H.
OBERHAUS, T. M.
PERRYMAN, F. S.
PRUITT, D. M.
RESONY, J. A.
RODERMUND, M.
SALZMANN, R.
SCHLOSS, H. W.
SMICK, J. J.
TARBELL, T. F.
VALERIUS, N. M.
WIEDER, J. W., JR.
WILLIAMS, H. V.
WITTICK, H. E.
WOLFRUM, R. J.

ASSOCIATES

DOWLING, W. F.
FURNIVALL, M. L.
GILDEA, J. F.
GROSSMAN, E.
HART, W. VAN B.
HARWAYNE, F.
HAZAM, W. J.
HOPE, F. J.
LESLIE, WM., JR.
LUFKIN, R. W.
MACKEON, H. E.
MALMUTH, J.
MUNTERICH, G.
PENNOCK, R. M.
SCAMMON, L. W.
SCHWARTZ, M. J.
SMITH, A. G.
STOKE, KENDRICK

By invitation, a number of officials of Casualty Insurance Companies and other organizations were present.

The reading of the minutes of the meeting held November 18, 1949 was dispensed with by motion.

An informal dinner had been held on the evening of May 21, 1950, also on the evening of May 22, 1950.

President Barber delivered his Presidential address. This was followed by the presentation of a formal paper by Mr. A. L. Bailey.

President Barber turned the meeting over to Vice President Masterson for informal discussion of the following topics, which was participated in by members of the Society and by representatives of insurance organizations:

1. The new combined annual statement blank for Fire and Casualty Companies.

2. Are present excess limit rates for bodily injury liability coverage adequate for current excess loss costs?
3. New York Statutory Disability Benefits Law rates and rating plans.

Recess was then declared until the following day.

On May 23rd there were presented several written discussions of papers delivered at the last annual meeting, President Barber presiding.

He then turned the meeting over to Vice President Masterson. The informal discussion of Topic 3 above was concluded and it was continued for:

4. The current investigation of company expenses by size of risk.
5. What are the causes of the recent upward trend in the development of compensation losses beyond the first reporting and what steps may be taken to correct the situation?

Upon motion, the meeting adjourned at noon.

ABSTRACT FROM THE MINUTES OF THE MEETING

November 17, 1950

The annual meeting of the Casualty Actuarial Society was held at the Hotel Biltmore, New York, on Friday, November 17, 1950. An informal dinner had been held on Thursday evening, November 16, 1950 at the Hotel Biltmore: the dinner group was addressed by Mr. Roy A. Duffus of Rochester, New York, National Director for the New York State Association of Insurance Agents.

President Barber called the annual meeting to order at 10:20 A.M., the roll was called, showing the following 56 Fellows and 23 Associates present:

FELLOWS

Ainley	Gardiner	Mills
Allen	Ginsburgh	Munterich
Ault	Goddard	Oberhaus
Barber	Graham, C. M.	Perryman
Berkeley	Graham, W. J.	Pruitt
Brown, F. S.	Harwayne	Resony
Burling	Hazam	Rodermund
Carlson	Hope	Salzmann
Cogswell	Johnson	Schloss
Comstock	Kardonsky	Silverman
Constable	Kormes	Sinnott
Crouse	Linder	Skillings
Davies	LaCroix	Smick
Elliott	Leslie, Jr.	Tarbell
Elston	Livingston	Uhthoff
Eppink	Masterson	Valerius
Fallow	Maycrink	Van Tuyl
Fondiller	McConnell	Waite, A.
Fuller		Wieder

ASSOCIATES

Black, N. C.	Kitzrow	Nicholson
Boyajian	Longley-Cook	Sawyer
Critchley	MacKeen	Scammon
Dowling	Mayerson	Schwartz
Eger	Menzel	Stoke
Grossman	Montgomery, J. C.	Uhl
Hart	Murrin	Vergano
Hewitt		Wooddy

By invitation, a number of officials of casualty insurance companies and insurance organizations were present.

Mr. Barber read his presidential address.

The reading of the minutes of the meeting held May 22nd and 23rd, 1950 was dispensed with by motion.

The Secretary-Treasurer (Richard Fondiller) read the report of the Council and upon motion it was adopted by the Society. Frank Harwayne, William J. Hazam, Francis J. Hope, William Leslie, Jr., Gilbert R. Livingston and George Munterich had passed the examinations and had been admitted as Fellows: a diploma was presented to each by the President. John H. Boyajian, Douglas Critchley, James B. Haley, Jr., Laurence H. Longley-Cook, Allen L. Mayerson, Henry W. Menzel, Thomas E. Murrin, John A. W. Trist and John C. Wooddy had passed the examinations and had been admitted as Associates. A prospectus entitled "The Casualty Actuarial Profession," had been published and distributed to the members of the Society, colleges and universities, and to those interested in entering the profession by examination. Certain amendments to the Constitution and By-laws necessary to effect the extension of the scope of the Society had been approved by the Council for presentation to the Society's annual meeting.

The Auditing Committee (Howard G. Crane, Chairman) reported that the books of the Secretary-Treasurer had been audited and his accounts verified.

The report of the Secretary-Treasurer was read and accepted. The report on Finances follows:

CASUALTY ACTUARIAL SOCIETY
ANNUAL REPORT ON FINANCES

Cash Receipts and Disbursements from October 1, 1949 to September 30, 1950

Income			Disbursements	
On deposit in Marine Midland on October 1, 1949		$6,453.20	Printing & Stationery	$4,829.99
			Postage, Tel., Exp., etc.	153.61
			Secretarial Work	521.60
Members Dues	$4,510.00		Examination Expense	697.99
Sale of Proceedings	1,700.50		Luncheons & Dinners	1,821.60
Examination Fees	1,185.00		Library	16.06
Luncheons & Dinners	1,450.00		Storage of Proceedings	86.05
Michelbacher Fund	1,019.90		Fire Ins.—Proc. & Libr.	24.37
Reprints—Loss			Purchase of Bonds	4,000.00
Reserve Report	77.00		Prospectus	294.25
Interest on Bonds	75.00		Fondiller Prize	100.00
Miscellaneous	5.00		Miscellaneous	70.20
Foreign Exchange	—6.09			
Total income		$10,016.31	Total disbursements	$12,615.72
			On deposit September 30, 1950 in Marine Midland Trust Co.	$3,853.79
Total		$16,469.51	Total	$16,469.51

ANNUAL REPORT OF FINANCES (*continued*)

Assets		Liabilities		
Cash in Bank	$3,853.79	Unpaid Bills:		
Bonds Owned	5,000.00	Prospectus	$709.93	
		Printing	81.60	
				$791.53
		Michelbacher Fund		4,018.68
		Total Liabilities		$4,810.21
		Surplus		4,043.58
Total Assets	$8,853.79	Total Liabilities and Surplus		$8,853.79

The Examination Committee (Roger A. Johnson, General Chairman) submitted a report of which the following is a summary:

1950 EXAMINATIONS—SUCCESSFUL CANDIDATES

The following is a list of those who passed the examinations held by the Society on May 9 and 10, 1950:

ASSOCIATE EXAMINATIONS

PART I:	R. S. Brindise	J. S. Hermistone	W. R. Mullens
	C. M. Daniel	S. J. Huse	G. J. Offrowich
	R. F. Flanders	J. D. Hutcheson	T. H. Pate
	R. B. Foster	N. B. King	O. D. Richmond
	T. W. Fowler	J. W. Kroeker	L. J. Simon
	R. R. Gallagher	L. Landes	B. W. Straight
	W. S. Gillam	L. Leckie	J. R. Taylor
	J. B. Haley, Jr.	L. H. Longley-Cook	R. D. Tofte
	B. R. Hazlehurst	P. J. Mize, Jr.	H. P. Valker, Jr.
PART II:	J. S. Acheson	G. W. Holland	W. R. Mullens
	G. M. Barker	R. L. Hurley	R. D. Murray
	H. E. Brower	S. J. Huse	H. O. Noyd
	A. C. Cragoe	J. D. Hutcheson	E. O'Boyle
	C. M. Daniel	R. L. Johe	T. H. Pate
	L. H. Deitchler	M. V. Johns, Jr.	E. F. Petz, Jr.
	K. F. Eaton	L. H. Johnson	J. R. Reimer
	D. Eckersley	M. Kazakoff	L. J. Simon
	G. V. Etherington	J. W. Kroeker	P. J. Spellman
	R. B. Foster	S. Kuryliw	P. C. Spoolstra
	K. W. Fuerste	L. Landes	B. W. Straight
	R. R. Gallagher	R. Lino	J. R. Taylor
	W. F. Hancock	L. H. Longley-Cook	H. P. Valker, Jr.
	J. S. Hermistone	M. C. McMillan	E. R. Vogt
	A. J. Hillman	J. M. Moscovitch	A. Wind, Jr.

PART III:	J. S. Acheson	S. J. Huse	J. R. Reimer
	R. H. Bent	L. H. Johnson	O. D. Richmond
	J. J. Bond	J. W. Kroeker	D. J. Smith
	C. M. Daniel	S. Kuryliw	P. C. Spoolstra
	D. Eckersley	R. Lino	B. W. Straight
	G. V. Etherington	L. H. Longley-Cook	J. R. Taylor
	A. V. Fairbanks	H. W. Menzel	R. D. Tofte
	R. R. Gallagher	W. R. Mullens	H. P. Valker, Jr.
	J. B. Haley, Jr.	R. D. Murray	E. R. Vogt
	W. F. Hancock	T. E. Murrin	A. E. Whiton
	B. R. Hazlehurst	H. O. Noyd	J. C. Wooddy
		R. B. Pennycook	
PART IV:	W. O. Bailey	D. Critchley	A. L. Mayerson
	M. T. Bergan	C. H. Graves	R. D. Morse, Jr.
	J. H. Boyajian	J. B. Haley, Jr.	E. F. Petz, Jr.
	R. N. Caputo	J. S. Hermistone	J. A. W. Trist
		L. H. Longley-Cook	

FELLOWSHIP EXAMINATIONS

PART I:	J. H. Boyajian	W. J. Hazam	A. L. Mayerson
PART II:	J. H. Boyajian	W. J. Hazam	W. Leslie, Jr.
	F. Harwayne		A. L. Mayerson
PART III:	F. Harwayne	W. Leslie, Jr.	H. E. MacKeen
	W. J. Hazam	G. R. Livingston	G. Munterich
	F. J. Hope		E. Vergano
PART IV:	F. Harwayne	F. J. Hope	W. Leslie, Jr.
	W. J. Hazam		G. R. Livingston

The Secretary-Treasurer announced that the Council had elected the following officers:

Editor.....re-elected.....Emma C. Maycrink
Librarian.....re-elected.....Samuel M. Ross
Chairman—Examination Committee.....Roger A. Johnson

In accordance with Constitutional requirements, notice of the following proposed amendments was given. These amendments were, on motion, adopted to read as follows:

Constitution—Article II

The object of the Society shall be the promotion of actuarial and statistical science as applied to the problems of insurance (other than life insurance) by means of personal intercourse, the presentation and discussion of appropriate papers, the collection of a library and such other means as may be found desirable.

The Society shall take no partisan attitude, by resolution or otherwise, upon any question relating to insurance.

Constitution—Article III—Third paragraph (second sentence)

Such examination may be waived in the case of a candidate who for a period of not less than two years has been in responsible charge of the Statistical or Actuarial Department of an insurance organization (other than life insurance organization) or has had such other practical experience in insurance (other than life insurance) as, in the opinion of the Council renders him qualified for Associateship.

By-Laws—Article III—Third paragraph

The Secretary-Treasurer shall also send out calls for annual dues and acknowledge receipt of same; pay all bills approved by the President for expenditures authorized by the Council of the Society; keep a detailed account of all receipts and expenditures, and present an abstract of the same at the annual meetings, after it has been audited by a committee appointed by the President.

The annual elections were then held and the following officers and members of the Council were elected:

President	Harmon T. Barber
Vice-President	Thomas O. Carlson
Vice-President	Norton E. Masterson
Secretary-Treasurer	Richard Fondiller
Editor	Emma C. Maycrink
Librarian	Samuel M. Ross
Chairman—Examination Committee	Roger A. Johnson

Members of the Council:

Edward S. Allen	1953
Clarence A. Kulp	1953
John A. Mills	1953

The papers appearing in this Volume were presented.

Recess was taken for lunch at the Hotel until 2:15 P.M.

President Barber turned the meeting over to Vice-President Carlson. Informal discussion of the following topics were participated in by the members of the Society and by representatives of insurance companies and organizations:

Actuarial, statistical, accounting and underwriting problems in connection with:

1. Multiple line underwriting.
2. Legal liability coverage for property in care custody or control of insured.

Upon motion, the meeting adjourned at 4:40 P.M.

EXAMINATION FOR ENROLLMENT AS ASSOCIATE

PART I

1. (a) Find the arithmetic, geometric, and harmonic means of $1,2,4,8 \cdots 2^{n-1}$.
   (b) The line of regression of Y on X for a set of variates (X_i, Y_i) is $Y = mX + b$. If $\rho_i = Y_i - (mX_i + b)$ prove that $\Sigma\rho^2 = \Sigma Y^2 - b\Sigma Y - m\Sigma XY$.

2. The following table shows the number of accidents which occurred during one year in a number of machine shops of equal size:

Territory A		*Territory B*	
Shop No.	*No. of Accidents*	*Shop No.*	*No. of Accidents*
1	5	11	2
2	0	12	0
3	2	13	4
4	5	14	2
5	1	15	2
6	1	16	2
7	4		
8	6		
9	0		
10	1		

   (a) Determine the mean accident frequency per risk and the standard deviation of the accident frequency per risk in each territory.
   (b) Compare the mean accident frequencies in territory A and territory B and determine whether the difference of the two mean frequencies is significant.

3. (a) Prove that the correlation coefficient r_{xy} of n pairs of variates (X_1, Y_1), $(X_2, Y_2) \cdots (X_n, Y_n)$ is independent of the unit in which either the X's or the Y's are expressed and remains unchanged if all the X's are increased by a constant a and all the Y's are increased by a constant b.
   (b) Given the following data, set up column headings for the necessary steps to compute theoretical frequencies by fitting a normal curve to the data, assuming you have available a table of areas of the normal curve $A \int_0^t \phi$.

X	$f(X)$
.0180 - .01839	6
.0184 - .01879	30
.0188 - .01919	42
.0192 - .01959	66
.0196 - .01999	94
.0200 - .02039	120
.0204 - .02079	102
.0208 - .02119	60
.0212 - .02159	54
.0216 - .02199	14
.0220 - .02239	12
	600

$M_x = .0202$ $\sigma_x = .00085$

4. The following observations, T, were obtained from laboratory tests according to the independent variable θ:

θ	T	θ	T
0	0	30	76
10	2	40	147
20	27	50	241

By the method of least squares fit an appropriate curve to the data. State the reason for your choice of the type of curve.

Note: In the following questions, the new international actuarial notation has been used. The only change affecting these questions is the use of $i^{(m)}$ in place of the former $j_{(m)}$. The student may use either notation in presenting his solutions.

5. (a) A certain workmen's compensation law provides that benefit payments due at the end of each week, for a specified number of weeks, may be commuted to a lump sum settlement, but these payments " ··· shall not be discounted at a rate greater than eight per centum per annum." Using 52 weeks to a year, assume the law intends use of a simple discount rate of 8% per annum, and derive a formula for the present value of \$1 per week for n weeks. For what number of weeks does the present value become negative?

   (b) The present value of \$672 due in a certain time is \$126, interest being compounded at the rate of $4\frac{1}{6}\%$ per year. In how many years (to the nearest year) is the \$672 due? Given $\log 2 = .30103$ and $\log 3 = .47712$.

6. At a time when the investment rate of interest is x, A and B enter into a financial contract according to which A pays to B the sums of $a_1, a_2, \ldots a_m$ at the beginning of the 1st, 2nd, ... mth year of the contract respectively, in return for which B promises to pay to A the sums of $b_{m+1}, b_{m+2}, \ldots b_{m+n}$ at the beginning of the $(m+1)$st, $(m+2)$nd, ... $(m+n)$th year of the contract respectively.

   (a) What relation must exist between the a's and b's so that, if the investment rate of interest remains x, neither A nor B will make a profit?

(b) Prove that, at the end of the t th contract year ($t \leqq m$) the amount accumulated by B from the payments of A at the investment rate of interest x equals exactly the fund which B should have in his possession at that time in order to be able to make his payments to A when due, if he accumulates this fund, and additional payments by A when received, at the rate of interest x.

7. (a) A deposit is made at the end of every three months into an account paying $3\frac{1}{2}\%$ effective. If the first 41 of these deposits are each \$200, what must be the amount of the 42nd deposit in order to bring the account to an even \$10,000? Indicate, but do not perform, the arithmetic operations.

Given: $i^{(4)} = .03455$ $(3\frac{1}{2}\%)$
$s\overline{|10|} = 11.7314$ $(3\frac{1}{2}\%)$

(b) Determine the relative error in the use of a continuous annuity formula in computing the approximate present value of an annuity certain for n years, payable weekly, with interest at 5% effective. Assume 52 weeks to the year.

Given: $(1.05)^{1/52} = 1.0009387$
$\log_e (1.05) = .04879$

8. (a) A plant consists of 3 parts with costs, scrap values and probable lives as follows:

Part	*Cost*	*Scrap Value*	*Probable Life*
A	\$75,000	\$15,000	30 years
B	30,000	4,000	18 years
C	10,000	500	10 years

Replacement is to be provided for by payments at the end of each year of a given amount into a sinking fund earning 3% annually. How much will be in the fund at the end of 8 years?

Given: $s\overline{|8|} = 8.89234$

$\frac{1}{a\overline{|10|}} = .11723$

$\frac{1}{a\overline{|18|}} = .07271$

$\frac{1}{s\overline{|30|}} = .02102$

(b) Determine the approximate investment rate (yield) of a \$100 bond having an interest rate of 4% per annum payable semi-annually, purchased at 90 and redeemable at par in 15 years.

Given: $a\overline{30}| = 22.3965$ (2%)

$a\overline{30}| = 21.6453$ (2¼%)

$a\overline{30}| = 20.9303$ (2½%)

PART II

1. (a) Differentiate $\dfrac{\sqrt{2x-1}\,\sqrt{(2x-5)^5}}{(2x-3)^3}$

   (b) A window of perimeter p feet is in the form of a rectangle surmounted by an isosceles right triangle whose hypotenuse is equal to the width of the window. Show that the window will admit the most light when the sides of the rectangular portion of the window are equal to the sides of the right triangle.

2. (a) Find the value of $\dfrac{e^x - e^{-x} - 2x}{x - \sin x}$ as x approaches 0.

   (b) Find the angle of intersection between the curve $\rho = 6\cos\theta$ and the curve $\rho = 2(1 + \cos\theta)$.

3. (a) Show that $\sin h^{-1} x = \log_e (x + \sqrt{x^2+1})$.

   (b) Integrate $\displaystyle\int \frac{(x^2+2)dx}{x^3+1}$.

4. (a) Calculate by double integration the finite area bounded by the curve $2y = 4x - x^2$ and the line $2y = x - 4$.

   (b) Find the volume of the figure F obtained by rotating an isosceles right triangle whose hypotenuse has the length 4 around the bisector of the right angle.

5. (a) Taking the interval of differencing as unity, find the value to m terms of

$$\Delta x^m - \tfrac{1}{2}\Delta^2 x^m + \frac{1\cdot 3}{2\cdot 4}\Delta^3 x^m - \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\Delta^4 x^m + \cdots$$

   (b) Obtain a polynomial $F(x)$ whose first difference is equal to $f(x) = x^3 + 7x + 10$.

6. (a) Given $u_0 = -4$, $u_1 = -1$, $u_5 = 31$, $u_9 = 95$, find u_4 by the method of divided differences.

(b) Using Simpson's Rule for approximate integration, find an approximate value of $\int_1^3 e^{-1/2x^2}\,dx$, given $e^{-1/2} = .607$, $e^{-2} = .135$.

7. (a) Find the relation between a, β, γ ($\gamma \neq 0$) in order that $a + \beta x + \gamma x^2$ may be expressible in one term in factorial notation.

(b) Sum to n terms $0, -2, -3, -2, 3, 16, 45, \ldots$

8. (a) Assuming the validity of the Gauss formulae, derive Stirling's central difference formula to five terms.

(b) Derive a general formula for interpolation of functions of two independent variables corresponding to the advancing difference formula for one independent variable.

PART III

1. (a) Two men put up a stake of \$1 each and then throw for it, equal throws to divide the stake. A uses an ordinary die, but B uses a die marked 2, 3, 4, 5, 6, 6. Show that B thereby increases his expectation by 5/18ths.

(b) q_x is the probability that a man aged x will die in a year. Find the probability that out of five men, A, B, C, D, E, each aged x, A will die in a year and will be the first to die.

2. (a) A pack of cards is dealt in the usual way to four players of whom two and two are partners. The dealer turns up his last card, and the suit of this card becomes trump. If honors are defined as the ace, king, queen, and jack of a suit, find the chance that each pair of partners shall have two honors of the trump suit.

(b) A purse contains 10 coins, each of which has been taken from either of two bags at random, one bag containing sovereigns and the other bag containing shillings. A coin is drawn from the purse and found to be a sovereign. What is the chance that this was the only sovereign in the purse?

3. (a) A bag contains three red and three green balls. A person draws out three at random and discards them without looking at them. He then drops three blue balls into the bag and again draws out three at random. What are the odds in favor of the balls of the second draw all being of different colors?

(b) A person throws two dice, one the common cube with faces marked 1 to 6, and the other a regular tetrahedron with faces marked 1 to 4, the number on the lowest face being taken in the case of the tetrahedron. What is the chance that the sum of the numbers thrown is not less than 7?

4. A bag contains 13 balls, of which 4 are white and 9 black. If a ball is drawn n times successively and replaced after each drawing, show that the chance that no two successive drawings shall give white balls is

$$\frac{16 \cdot 12^n - (-3)^n}{15 \cdot 13^n}.$$

Note: The Casualty Actuarial Society has adopted the new International Actuarial Notation. However, in answering questions 5 through 8, the student may use either the old or the new notation.

5. (a) The American Remarriage Table assumes a radix of 100,000 at age 18 and gives the following values:

Number Remarrying		*Number Dying Unmarried*	
$m^r_{(18)}$	= 6880	$d^r_{(18)}$	= 343
$m^r_{(18)+1}$	= 15642	$d^r_{(18)+1}$	= 328
$m^r_{(18)+2}$	= 10215	$d^r_{(18)+2}$	= 301
$m^r_{(18)+3}$	= 8598	$d^r_{(18)+3}$	= 278

Calculate the probability that a widow aged 19, whose husband died one year ago, marries in the second year from now.

(b) Develop an approximate formula, in terms of m and the present value of a life annuity immediate with annual rent of 1, for the present value of a life annuity immediate with annual rent 1, payable m times per year in equal installments of $\frac{1}{m}$ each.

6. (a) Obtain the 15th terminal reserve in terms of commutation symbols, of a 20 year endowment policy for \$1000 issued at age 40 by both the retrospective and prospective methods and prove that the two reserves thus obtained are identical.

(b) Assuming you are working with a Makehamized table of mortality, express the annuity $a_{50/20,25}$ in terms of pure annuities for persons of equal ages, and determine the equal ages involved to the nearest integer.

Given:

c	= 1.1	c^{35}	= 28.10
c^{20}	= 6.73	c^{36}	= 30.91
c^{21}	= 7.40	c^{37}	= 34.00
c^{22}	= 8.14	c^{38}	= 37.40
c^{23}	= 8.95	c^{39}	= 41.14
c^{24}	= 9.85	c^{40}	= 45.26
c^{25}	= 10.84	c^{50}	= 117.40

7. An insurance policy issued to a person aged 35 provides for the following benefits:

(a) \$1000 if death occurs in the first 10 policy years.

(b) \$2000 if death occurs in the next 10 policy years.

(c) The return of the total gross premiums paid to time of death, if death occurs in the first 20 policy years.

(d) A life annuity due for \$120 per year if the insured lives to the end of the 20th policy year.

Premiums are payable annually for 15 years. The gross annual premium, π, and the net annual premium, P, are connected by the relation $\pi = 1.10P + 5$. Assuming that the same mortality table is applicable throughout, express the net annual premium, P, in terms of commutation symbols.

8. An insured has a 20 year endowment policy for \$5000 which he took out when he was aged x. At the end of the fifth policy year he decides to convert it to a 20 payment life policy as of the original date. The insurance company grants him a paid up life policy for \$$S$ at net rates, plus a regular 20 payment life policy for \$5000 $-$ S as of the original date, for which the same net annual premium is payable for the remaining 15 years as would have been payable if these premiums had been paid for the past five years. Express the amount S in commutation symbols.

PART IV

1. (a) What is meant by the term "Automatic Coverage" as used in many of the casualty lines?

   (b) Define and explain the purpose of subrogation.

2. (a) What hazard is covered by Contractors' Protective Liability insurance?

   (b) Name and describe the qualifications for each of the three private passenger automobile classifications of Automobile Bodily Injury and Property Damage liability insurance used for most states by the National Bureau of Casualty Underwriters.

3. (a) What are the main points considered by an underwriter in deciding upon the acceptability of an application for a personal accident policy providing the customary benefits including weekly indemnity?

   (b) Outline the four basic coverages and the standard limits provided by a Comprehensive Glass policy.

4. (a) An applicant for a \$50,000 surety bond can provide \$40,000 collateral. According to the manual collateral rule, how will the annual premium be affected? Do you think the rule fair as it applies in this case?

   (b) An interstate Workmen's Compensation Insurance policy issued by a member carrier of the National Council on Compensation Insurance develops a total premium of \$40,000, distributed equally among four states under the jurisdiction of the National Council. It is not retro-

spectively rated, and premium discounts apply in three states, no discounts in the other. Illustrate how to determine the amount of premium discount, in total and by state.

5. (a) Given the average number of weeks of disability from disease per year of exposure, at age 49, to be .69 for total disability and .20 for partial disability, calculate the net one year term premium for a health policy with a benefit of $1 per week of disability, up to a limit of 52 weeks, if half benefit is payable for partial disability and if 3½% interest is earned by the carrier.

   (b) During the 1930's the decreasing number of automobile liability assureds and increasing loss ratios provoked discussion of elements contributing to these conditions, including discussion of yardsticks which might be applied to measure accident proclivities of drivers. Which of these yardsticks has been demonstrated by experience and is being used in current ratemaking?

6. Explain the use of loss development factors in Workmen's Compensation and Liability Insurance ratemaking, and, with respect to Workmen's Compensation, discuss the propriety of applying these factors in addition to law amendment factors.

7. Explain the purpose of the use of classifications in determining and applying premium rates in casualty insurance, and state conditions which, in your opinion, a good classification system ought to satisfy.

8. Describe the nature of the Rate Level Adjustment Factor, based upon calendar year experiences, as it is used in Workmen's Compensation Insurance ratemaking, and discuss the purpose of this factor and its possible defects.

EXAMINATION FOR ENROLLMENT AS FELLOW

PART I

1. All premiums paid by entrepreneurs for insurance against loss or destruction of capital, are ultimately collected from consumers as a part of the prices for the goods and services which capital is employed to produce. In the long run, the sum of such premiums cannot be less than the sum of all the losses of capital covered by insurance, *plus* the wages of the labor employed in the insurance business. But approximately the same losses would have occurred and presumably would have been borne by consumers as a part of the sum of all prices paid for goods and services, *even if* there had been no insurance. How, then, can it be argued that the institution of insurance operates to reduce the prices which consumers must pay for goods and services?

2. Discuss the effects of inflation on the underwriting results of insurance carriers for:
   a. Accident and Health.
   b. Workmen's Compensation.
   c. Automobile Liability.
   d. Glass.

3. Discuss briefly the function of reinsurance from the standpoint of a primary carrier.

4. Briefly compare the investment characteristics of bonds and stocks. To what extent are casualty insurance companies permitted to invest in common stocks under the New York law?

5. Describe the safeguards contained in the All-Industry Casualty and Surety Rate Regulatory Bill for the protection of:
   a. The policyholder.
   b. The insurance company.

6. *A* is named as the insured in a policy of burglary insurance issued by company *C*. Preliminary investigation of a loss reported by *A* leads *C* to suspect that there has been a breach of one of the conditions of the policy. *C* then declines to make any further investigation of the loss until after *A* has signed an agreement reading substantially as follows: "No action taken by any representative of *C* in investigating or ascertaining the amount of loss or damage to the property of *A*, shall waive or invalidate any of the conditions expressed in the policy." What is the legal nature of this agreement? What defects do you find therein? Discuss the limitations upon the extent to which it will protect *C*.

7. (a) What does the Statute of Frauds provide, and what are the principal classes of contracts to which it applies? Discuss its application to: (i) corporate surety bonds, and (ii) policies of liability insurance written for a term of three years.
   (b) Define the term *subrogation* and discuss the possibilities of its use in:
      (i) Health Insurance.
      (ii) Workmen's Compensation Insurance.

8. What are the advantages and disadvantages of rate regulation for casualty insurance as opposed to free rate competition?

PART II

1. (a) Write a general definition of the term "schedule rating."
   (b) Discuss the history of schedule rating in the field of casualty insurance,

commenting upon the stages in the development of rating practice during which schedule rating serves a useful purpose, the reasons why it has been abandoned in respect to certain classes of risks, and the extent to which it is practiced today.

2. (a) How is the primary actual loss value corresponding to any particular claim, defined in a "multi-split" experience rating plan, *e.g.*, the Experience Rating Plan of 1940 issued by the National Council on Compensation Insurance.

(b) From the definition given in response to (a) above, derive a formula expressing the total actual loss on any one claim, in terms of the corresponding primary actual loss value.

3. (a) From the following formulae: $M = \dfrac{ZA + (1 - Z)E}{E}$ and $Z = \dfrac{E}{E+K}$, where M is the Experience Modification Factor, A denotes Actual Losses, E denotes Expected Losses, and K is a constant, derive a single simple formula for M which may be used in rating risks under a "no split" experience rating plan, without referring to any table of credibility values.

(b) Determine the value of K so that, on a risk for which the Expected Losses within the experience period amount to \$200, the "credit" for clear experience will be 5%.

(c) Derive a formula for the limit which must be placed upon the amount of loss which shall be included in the value of A on account of any one accident, in order that—for any risk—the increase in premium rate resulting from a single accident shall not be greater than 25% of the manual premium rate.

4. Suppose that you are constructing a retrospective rating plan for workmen's compensation risks in a state in which the composition of manual premium rates is as follows: Expected Losses, .598; Acquisition Cost, .175; Administration, .075; Payroll Audit, .020; Inspection and Bureau Expense, .025; Claim Investigation and Adjustment, .080; Taxes, Licenses and Fees, .027; that in the plan, the Loss Conversion Factor is to be 1.10, applicable to all actual losses without limitation in respect to each accident; and that the plan is to reflect the following gradation in expense allowances:

	For Acquisition	*For Administration and Payroll Audit*
Of first \$ 1,000 of Standard Premium	17.5%	9.5%
Of next 4,000 of Standard Premium	12.5%	4.1%
Of next 95,000 of Standard Premium	7.5%	4.1%
Of Standard Premium in excess of \$100,000	6.0%	4.1%

Determine the Tax Multiplier and compute the ratios (to Standard Premium) of the Basic Premium (B), the Minimum Retrospective Premium (H), and the Maximum Retrospective Premium (G) for risks developing Standard Premiums of \$50,000, if the allowance for losses in H is to be 30% of Expected Losses and the allowance for losses in G is to be 170% of Expected Losses; given the following values of the Excess Pure Premium Ratio, ρ_s, for such risks, where s is the ratio of losses (without limitation) to Standard Premium and .598 is the expected value of s.

s	ρ_s
.17	.716
.18	.700
1.01	.026
1.02	.025

5. (a) Describe briefly the theory underlying the classification groupings in the Chemical and Dyestuff Rating Plan.
   (b) Discuss briefly some of the considerations involved in the selection of an appropriate retrospective rating plan for a particular risk.

6. In the President's message to Congress, he has advocated a national health insurance program. Discuss briefly the principal features of such a program and the arguments for and against it.

7. Discuss briefly the advantages and disadvantages of compulsory automobile liability insurance from the standpoint of (a) the insurance company, (b) the insured, and (c) the injured party.

8. The recently enacted Disability Benefits Law in New York is part of the Workmen's Compensation Law and is to be administered by the Workmen's Compensation Board. Discuss the advantages of associating this type of benefit with Workmen's Compensation benefits.

PART III

1. Company C has compiled the following statistics relative to its case-basis reserves for losses and allocated claim expenses on account of automobile bodily injury liability (a.b.i.l.) claims.

Year	N	a	b	F	p	s
1947	100	2,800	101	90	2,430	122
1948	110	3,080	163	102	2,856	200
1949	120	3,360	220	112	3,080	314

N and F are expressed in hundreds of claims; a, b, p and s are expressed in thousands of dollars. N is the number of claims reported within the year; a is the sum of the initial reserves posted on claims reported within the year; b is the net increase due to changes in reserves on open claims and to the re-opening of claims within the year; F is the number of claims closed

within the year, including claims closed without payment; p is the total amount of a.b.i.l. losses and allocated claim expenses paid within the year; s is the net "saving" in reserves, *i.e.*, the net excess of reserves released by payments and by the closing of claims without payment within the year, over losses and allocated claim expenses paid within the year.

What conclusion relative to the adequacy or inadequacy of C's case-basis reserves as of December 31, 1949, for a.b.i.l. losses and allocated claim expenses, can you draw from these statistics? Outline the reasoning by which you support your conclusion.

2. (a) What factors may prompt a carrier to set up voluntary or contingency reserves?

   (b) Discuss the particular need, in Fidelity and Surety lines, for establishing voluntary underwriting reserves.

3. State the position you would take in a discussion of the question: Should Schedule P—Part 2 of the annual statement blank for casualty companies, be revised, left unchanged, or eliminated entirely? Outline the principal points which you would make in defense of your position.

4. Outline a method of setting up reserves for future compensation payments in individual cases of disability before it is known whether the disabled workers will recover, suffer permanent, partial or total disability, or die from the injuries.

5. If the indicated loss developments from first to third reports on Workmen's Compensation, losses reported on a case estimate basis have increased substantially over the past several years, outline the course of an investigation to determine the reason for such increase.

6. Outline a procedure for testing the adequacy of the increased limits tables for liability insurance.

7. (a) Under what condition is an estimate of the value of a parameter in a distribution function, called an *unbiased* estimate of that parameter?

   (b) State the principle of the *method of maximum likelihood.*

8. (a) On the following assumptions: (i) that for risks of a certain class C, the number of claims arising on any 100 units of exposure is a variable having a Poisson distribution and a mean of 2.00, and (ii) that the mean and the variance of the amount of loss on account of a single claim on any risk of class C, are 300 and 630,000, respectively, determine the variance of the ratio of actual to expected losses for risks of class C, each comprising 1,000 units of exposure.

(b) On the same assumptions as those set forth in (a) above, estimate the number of risks of class C, each comprising 1,000 units of exposure, which a company would have to assume in order that the probability shall be not greater than .01 that the ratio of its actual to expected losses on such risks will exceed 1.05 or be less than .95.

PART IV

1. Discuss the important differences between the Annual Statement of a casualty insurance company and the corresponding statement of a mercantile or industrial firm.

2. Item 34 under INCOME on page 2 of the Annual Statement blank for casualty insurance companies, reads as follows: "Increase in liabilities during the year on account of reinsurance treaties", and item 44 under DISBURSEMENTS on page 3 reads: "Decrease in liabilities during the year on account of reinsurance treaties". Describe a set of circumstances under which an entry would appear in one of these items, and give the reason for such entry.

3. On December 30, 1949 Company A paid a bodily injury liability loss of $100,000 incurred in policy year 1946, on account of which it had $60,000 recoverable from its reinsurers, which it did not receive from them until January 5, 1950. If the company treated the amount so recoverable as a non-ledger asset in its Annual Statement for the year ended December 31, 1949, then how should the loss payment and the amount so recoverable be reflected in the exhibit of Disbursements (page 3) and in Schedule P—Part 1 of that statement? Discuss.

4. Outline a study which could be made to determine the proper amount to be charged as an expense constant for each Workmen's Compensation policy.

5. (a) Distinguish between "facultative" and "treaty" reinsurance.

   (b) Discuss the various problems which would arise from the introduction of a single limit policy for Automobile Liability Insurance.

6. The New York Disability Benefits Law provides for a fund from which benefits are payable to disabled unemployed persons who, except for such disability, would be entitled to unemployment insurance benefits. This fund will be maintained primarily by assessments based on total payrolls of the preceding calendar year. Outline a method by which an insurance carrier can properly provide for such assessments in view of the cyclical nature of the unemployment situation.

7. Assume that Company X is a primary insurer, transacting business in all states east of the Mississippi River, having an annual premium volume of $20,000,000 distributed approximately as follows: Workmen's Compensa-

tion $8,000,000; Automobile (including Liability, Fire, Theft and Collision) $8,000,000; all forms of Liability Insurance other than Automobile $3,000,000; Burglary and Theft $800,000; Glass $200,000. Make and set forth whatever assumptions you please concerning the Company's modes of operation and methods of risk acquisition. Then draw a chart outlining what you consider to be a sound plan of organization for Company X, indicating officers, major departments and lines of responsibility. On a subsidiary chart or charts, show in some detail the plan of organization of those departments engaged in underwriting, actuarial, statistical, and accounting functions.

8. Your company has been asked to provide Workmen's Compensation Insurance covering a corporation engaged exclusively in the manufacture of textile machinery, having its only plant located in North Carolina. The applicable Experience Modification Factor is 1.150. The estimated annual Standard Premium is $10,000. The ratios of losses incurred to Standard Premium earned for the risk in policy years 1947, 1948, and 1949, are .70, .75, and .82, respectively. The present carrier of the risk has declined to renew its policy. Set forth the questions to which you would require the answers, and indicate in general the kinds of information you would need, before making a decision for your company as to whether or not it will accept the risk. Under what conditions and on what terms (if any) would you find the risk acceptable?

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CASUALTY ACTUARIAL SOCIETY

ORGANIZED 1914

1951 YEAR BOOK

(Addendum to Volume XXXVII of the *Proceedings*)

Corrected to January 16, 1951

No. 30

FOREWORD

The Casualty Actuarial Society was organized November 7, 1914 as the Casualty Actuarial and Statistical Society of America, with 97 charter members of the grade of Fellow. The present title was adopted on May 14, 1921. The object of the Society is the promotion of actuarial and statistical science as applied to the problems of casualty and social insurance by means of personal intercourse, the presentation and discussion of appropriate papers, the collection of a library and such other means as may be found desirable. The organization of the Society was brought about through the suggestion of Dr. I. M. Rubinow, who became the first president. The problems surrounding workmen's compensation were at that time the most urgent, and consequently many of the members played a leading part in the development of the scientific basis upon which workmen's compensation insurance now rests.

The members of the Society have also presented original papers to the *Proceedings* upon the scientific formulation of standards for the computation of both rates and reserves in accident and health insurance, liability, burglary, and the various automobile coverages. The presidential addresses constitute a valuable record of the current problems facing the casualty insurance business. Other papers in the *Proceedings* deal with acquisition costs, pension funds, legal decisions, investments, claims, reinsurance, accounting, statutory requirements, loss reserves, statistics, and the examination of casualty companies. "The Recommendations for Study" appear in *Proceedings* No. 64 and are in effect for the 1950 examinations and thereafter. The Report of the Committee on Mortality for Disabled Lives together with commutation tables and life annuities has been printed in *Proceedings* No. 62. The Committee on Compensation and Liability Loss and Loss Expense Reserves submitted a report which appears in Volume XXXV.

At the November 1950 meeting of the Society the Constitution and By-Laws were amended to enlarge the scope of the Society to include all lines of insurance other than life insurance. The effect of the amendment was to include fire insurance and allied lines in recognition of multiple line writing powers granted by many states to both casualty companies and fire companies.

The lower grade of membership in the Society is that of Associate. Examinations have been held every year since organization; they are held on the second Tuesday and following Wednesday during the month of May, in various cities in the United States and Canada. The membership of the Society consists of actuaries, statisticians, and executives who are connected with the principal casualty companies and organizations in the United States and Canada. The Society has a total membership of 278, consisting of 157 Fellows and 121 Associates. The annual meeting of the Society is held in New York in November.

The Society issues a publication entitled the *Proceedings* which contains original papers presented at the meetings. The *Proceedings* also contain discussions of papers, and reviews of books. This Year Book is published annually. "Recommendations for Study" is a pamphlet which outlines the course of study to be followed in connection with the examinations for admission. These two booklets may be obtained free upon application to the Secretary-Treasurer, 60 John Street, Room 901, NewYork 38, N.Y.

CASUALTY ACTUARIAL SOCIETY

NOVEMBER 17, 1950

THE COUNCIL

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*Terms expire at the annual meeting in November 1951.
† Terms expire at the annual meeting in November of the year given.

COMMITTEES

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MEMBERSHIP OF THE SOCIETY, NOVEMBER 17, 1950

FELLOWS

Those marked (†) were Charter Members at date of organization, November 7, 1914.
Those marked (*) have been admitted as Fellows upon examination by the Society.

Admitted	
*Nov. 21, 1930	AINLEY, JOHN W., Supervising Underwriter, The Travelers Insurance Company, 700 Main Street, Hartford 15, Conn.
*Nov. 14, 1947	ALLEN, EDWARD S., Actuary, Compensation Insurance Rating Board, 125 Park Avenue, New York 17, N. Y.
*Nov. 13, 1931	AULT, GILBERT E., Actuary, Church Pension Fund and Church Life Insurance Corporation, 20 Exchange Place, New York 5, N. Y.
Nov. 19, 1948	BAILEY, ARTHUR L., Actuary, New York Insurance Department, 61 Broadway, New York 6, N. Y.
May 23, 1924	BAILEY, WILLIAM B., (Retired), 52 West Hill Drive, West Hartford, Conn.
*Nov. 20, 1924	BARBER, HARMON T., Actuary, Casualty Actuarial Department, The Travelers Insurance Co., 700 Main Street, Hartford 15, Conn.
*Nov. 14, 1947	BARKER, LORING M., Actuary, Firemen's Fund Insurance Group, 401 California Street, San Francisco 20, Calif.
*Nov. 20, 1942	BART, ROBERT D., Office Manager, West Bend Aluminum Co., 92 Island Avenue, West Bend, Wis.
*Nov. 18, 1932	BARTER, JOHN L., Vice-President, Hartford Accident & Indemnity Co., 690 Asylum Avenue, Hartford 15, Conn.
*Nov. 13, 1931	BATHO, ELGIN R., Associate Actuary, Berkshire Life Insurance Co., 7 North Street, Pittsfield, Mass.
*Nov. 22, 1934	BERKELEY, ERNEST T., Actuary, Employers Liability Assurance Corporation, Ltd. and American Employers Insurance Company, 110 Milk Street, Boston 7, Mass.
†	BLACK, S. BRUCE, President, Liberty Mutual Insurance Company, 175 Berkeley Street, Boston 17, Mass.
Apr. 20, 1917	BLANCHARD, RALPH H., Professor of Insurance, School of Business, Columbia University, New York 27, N. Y.
†	BREIBY, WILLIAM, Vice-President, Pacific Mutual Life Insurance Company, 523 West 6th St., Los Angeles 14, Cal.
*Nov. 18, 1927	BROWN, F. STUART, Systems Analyst, American Insurance Group, 15 Washington Street, Newark 2, N. J.
Oct. 22, 1915	BROWN, HERBERT D., (Retired), Glenora-on-Lake Seneca, Dundee, New York.
†	BUCK, GEORGE B., Consulting Actuary, 150 Nassau Street, New York 7, N. Y.

FELLOWS

Admitted	
Apr. 20, 1917	BURHOP, WILLIAM H., Executive Vice-President, Employers Mutual Liability Insurance Company, 407 Grant Street, Wausau, Wis.
*Nov. 23, 1928	BURLING, WILLIAM H., Assistant Secretary, Group Department, The Travelers Insurance Company, 700 Main Street, Hartford 15, Conn.
*Nov. 19, 1929	CAHILL, JAMES M., Secretary, National Bureau of Casualty Underwriters, 60 John Street, New York 38, N. Y.
*Nov. 18, 1932	CAMERON, FREELAND R., Vice-President and Comptroller, American Title and Insurance Company, 37 N.E. First Avenue, Miami 32, Florida.
†	CAMMACK, EDMUND E., Vice-President and Actuary, Aetna Life Insurance Company, Hartford 15, Conn.
*Nov. 17, 1938	CARLETON, JOHN W., Actuary, Liberty Mutual Insurance Company, 175 Berkeley Street, Boston 17, Mass.
*Nov. 21, 1930	CARLSON, THOMAS O., Actuary, National Bureau of Casualty Underwriters, 60 John Street, New York 38, N. Y.
Nov. 18, 1949	CLARKE, JOHN W., Associate Actuary, Life Actuarial Department, The Travelers Insurance Company, 700 Main St., Hartford, 15, Conn.
*Nov. 13, 1936	CLEARY, ARTHUR E., 102 Pierce Road, Watertown, Mass.
*Nov. 15, 1918	COATES, BARRETT N., Coates and Herfurth, Consulting Actuaries, 620 Market Street, San Francisco 4, Calif.
*Nov. 17, 1922	COATES, CLARENCE S., Third Vice-President, Lumbermens Mutual Casualty Company, 4750 Sheridan Road, Chicago 40, Ill.
Oct. 27, 1916	COGSWELL, EDMUND S., First Deputy Commissioner of Insurance Department of Banking and Insurance, Division of Insurance, 100 Nashua Street, Boston 14, Mass.
Feb. 19, 1915	COLLINS, HENRY, (Retired), Timberlane, Route 4, Easton, Md.
*Nov. 23, 1928	COMSTOCK, W. PHILLIPS, Statistician, Preferred Accident Insurance Company, 80 Maiden Lane, New York 7, N. Y.
*Nov. 22, 1934	CONSTABLE, WILLIAM J., President and Treasurer, Excess Insurance Company of America, 99 John Street, New York 7, N. Y.
*Nov. 22, 1934	COOK, EDWIN A., Assistant General Manager and Secretary, Interboro Mutual Indemnity Insurance Company, 270 Madison Avenue, New York 16, N. Y.
†	COPELAND, JOHN A., Consulting Actuary, 1520-21 Candler Building, Atlanta, Ga.
*Nov. 18, 1925	CORCORAN, WILLIAM M., Partner, Wolfe, Corcoran & Linder, 116 John Street, New York 7, N. Y.
*Nov. 19, 1926	CRANE, HOWARD G., Vice-President and Treasurer, General Reinsurance Corporation, and North Star Reinsurance Corporation, 90 John Street, New York 7, N. Y.
*Nov. 22, 1946	CROUSE, CHARLES W., Actuary, Manufacturers Casualty Insurance Company, 1617 Pennsylvania Boulevard, Philadelphia 3, Pa.

FELLOWS

Admitted	
*Nov. 18, 1932	DAVIES, E. ALFRED, (Retired), Falls Village, Conn.
*Nov. 18, 1927	DAVIS, EVELYN M., Woodward, Ryan, Sharp & Davis, Consulting Actuaries, 41 Park Row, New York 7, N. Y.
†	DEKAY, ECKFORD C., President, DeKay & Company, 84 William Street, New York 7, N. Y.
*Nov. 17, 1920	DORWEILER, PAUL, Actuary, Aetna Casualty & Surety Company, Hartford 15, Conn.
*Nov. 24, 1933	EDWARDS, JOHN, Actuary, Ontario Insurance Department, Parliament Buildings, Toronto 2, Ontario, Canada.
*Nov. 15, 1940	ELLIOTT, GEORGE B., General Manager, Pennsylvania Compensation Rating and Inspection Bureau, 620 Packard Building, 15th at Chestnut Street, Philadelphia 2, Pa.
*Nov. 17, 1922	ELSTON, JAMES S., Associate Actuary, Life Actuarial Department, The Travelers Insurance Co., 700 Main Street, Hartford, 15, Conn.
*Nov. 15, 1935	EPPINK, WALTER T., Vice-President and Actuary, Merchants Mutual Casualty Co., Merchants Mutual Building, Buffalo 5, N. Y.
†	FACKLER, EDWARD B., Vice-President and Actuary, The Western and Southern Life Insurance Co., Cincinnati, Ohio.
†	FALLOW, EVERETT S., (Retired), 28 Sunset Terrace, West Hartford, Conn.
*Nov. 15, 1940	FARLEY, JARVIS, Actuary and Assistant Treasurer, Massachusetts Indemnity Co., 632 Beacon Street, Boston 15, Mass.
†	FARRER, HENRY, (Retired), 4 North Ave., Fanwood, N. J.
*Nov. 15, 1935	FITZHUGH, GILBERT W., Third Vice-President, Metropolitan Life Insurance Co., 1 Madison Avenue, New York 10, N. Y.
Feb. 19, 1915	FONDILLER, RICHARD, Consulting Actuary, Woodward and Fondiller, 524 W. 57th Street, New York 19, N. Y.
†	FRANKLIN, CHARLES H., (Retired), 6225 Princeton Way, Hawthorne Hills, Seattle, Washington.
*Nov. 18, 1927	FREDERICKSON, CARL H., Actuary, Canadian Underwriters Association, 55 York Street, Toronto, Canada.
*Nov. 22, 1934	FULLER, GARDNER V., Third Vice-President and Manager, Special Risks and Risk Experience Departments, Lumbermens Mutual Casualty Co., and American Motorist Insurance Co., 4750 Sheridan Road, Chicago 40, Ill.
*Nov. 19, 1948	GARDINER, JAMES B., Manager, Group Contract Bureau, Metropolitan Life Insurance Co., 1 Madison Avenue, New York 10, N. Y.
*Nov. 20, 1924	GINSBURGH, HAROLD J., Vice-President, American Mutual Liability Insurance Co., 142 Berkeley Street, Boston 16, Mass.
*Nov. 21, 1930	GLENN, J. BRYAN, 5214 First Street, N.W., Washington 11, D.C.
*Nov. 13, 1931	GODDARD, RUSSELL P., Actuary, Pennsylvania Manufacturers' Association Casualty Insurance Co., Finance Building, Philadelphia, Pa.

FELLOWS

Admitted	
†	GOODWIN, EDWARD S., 962 Main Street, East Hartford 8, Conn.
*Nov. 19, 1926	GRAHAM, CHARLES M., Chief Self-Insurance Examiner, New York State Workmen's Compensation Board, 80 Center Street, New York 13, N. Y.
†	GRAHAM, WILLIAM J., Consulting Actuary and Insurance Advisor, 1070 Park Avenue, New York 18, N. Y.
†	GREENE, WINFIELD W., Executive Vice-President, General Reinsurance Corporation, 90 John Street, New York 7, N. Y.
†	HAMMOND, H. PIERSON, (Retired), 22 Vanderbilt Road, West Hartford, Conn.
Oct. 27, 1916	HARDY, EDWARD R., (Retired), 235 East 22nd Street, New York 10, N. Y.
*Nov. 17, 1950	HARWAYNE, FRANK, Assistant Actuary, National Bureau of Casualty Underwriters, 60 John Street, New York 7, N. Y.
Oct. 22, 1915	HATCH, LEONARD W., (Retired), 425 Pelham Manor Road, Pelham Manor, New York.
*Nov. 17, 1950	HAZAM, WILLIAM J., Assistant Actuary, American Mutual Liability Co., 142 Berkeley Street, Boston 16, Mass.
*Nov. 19, 1926	HAUGH, CHARLES J., Secretary, Compensation and Liability Department, The Travelers Insurance Co., 700 Main Street, Hartford 15, Conn.
Oct. 22, 1915	HODGKINS, LEMUEL G., (Retired), 5 Whitman Road, Worcester 5, Mass.
Oct. 22, 1915	HOLLAND, CHARLES H., Suite 2001, 165 Broadway, New York 6, N. Y.
*Nov. 22, 1934	HOOKER, RUSSELL O., Actuary and Director of Examinations, State of Connecticut Insurance Department, Hartford 2, Conn.
*Nov. 17, 1950	HOPE, FRANCIS J., Actuarial Department, Hartford Accident and Indemnity Co., 690 Asylum Avenue, Hartford 15, Conn.
Nov. 18, 1932	HUEBNER, SOLOMON STEPHEN, Professor of Insurance, University of Pennsylvania, Philadelphia 4, Pa.
*Nov. 14, 1947	HUGHEY, M. STANLEY, Procedures Co-ordinator, Lumbermens Mutual Casualty Company, 4750 Sheridan Road, Chicago 40, Ill.
†	HUNTER, ARTHUR, (Retired), 124 Lloyd Road, Montclair, N. J.
Feb. 25, 1916	JACKSON, CHARLES W., (Retired), 74 Quimby Avenue, White Plains, N. Y.
*Nov. 19, 1929	JACKSON, HENRY HOLLISTER, Vice-President, National Life Insurance Co., 131 State Street, Montpelier, Vt.
*Nov. 14, 1941	JOHNSON, ROGER A., Actuary, Utica Mutual Insurance Co., 185 Genesee Street, Utica, N. Y.
*Nov. 16, 1939	JONES, HAROLD M., Group Research Division, John Hancock Mutual Life Insurance Company, 197 Clarendon Street, Boston 17, Mass.
*Nov. 17, 1938	KARDONSKY, ELSIE, 66 Corbin Place, Brooklyn, N. Y.

FELLOWS

Admitted	
*Nov. 19, 1926	KELTON, WILLIAM H., Associate Actuary, Life Actuarial Department, The Travelers Insurance Co., 700 Main Street, Hartford 15, Conn.
*Nov. 21, 1919	KIRKPATRICK, A. LOOMIS, Manager Insurance Department, Chamber of Commerce of the U. S. A., 1615 H Street, N.W., Washington 6, D.C.
*Nov. 14, 1941	KOLE, MORRIS B., Associate Actuary, State Insurance Fund, 625 Madison Avenue, New York 22, N. Y.
*Nov. 24, 1933	KORMES, MARK, Consulting Actuary, 285 Madison Avenue, New York 17, N. Y.
Nov. 23, 1928	KULP, CLARENCE A., Professor of Insurance, University of Pennsylvania, Logan Hall, 36th Street and Woodland Avenue, Philadelphia 4, Pa.
*Nov. 18, 1949	LA CROIX, HAROLD F., JR., Assistant Actuary, Accident and Group Actuarial Department, The Travelers Insurance Co., 700 Main Street, Hartford 15, Conn.
Nov. 13, 1931	LA MONT, STEWART M., (Retired), Hotel Claremont, Berkeley, Calif.
*Nov. 24, 1933	LANGE, JOHN R., Commissioner of Insurance, State of Wisconsin, State Capitol, Madison 2, Wis.
†	LEAL, JAMES R., Vice-President and Secretary, Interstate Life and Accident Co., Interstate Building, 540 McCallie Avenue, Chattanooga 3, Tenn.
†	LESLIE, WILLIAM, General Manager, National Bureau of Casualty Underwriters, 60 John Street, New York 38, N. Y.
*Nov. 17, 1950	LESLIE, WILLIAM, JR., Assistant Manager, National Council on Compensation Insurance, 45 East 17th Street, New York 3, N.Y.
*Nov. 20, 1924	LINDER, JOSEPH, Consulting Actuary, Wolfe, Corcoran & Linder, 116 John Street, New York 7, N. Y.
*Nov. 17, 1950	LIVINGSTON, GILBERT R., Assistant Actuary, National Bureau of Casualty Underwriters, 60 John Street, New York 38, N. Y.
*Nov. 13, 1936	LYONS, DANIEL J., Second Vice-President, The Guardian Life Insurance Co. of America, 50 Union Square, New York 3, N. Y.
†	MAGOUN, WILLIAM N., (Retired), 33 Fearing Road, Hingham, Mass.
*Nov. 23, 1928	MARSHALL, RALPH M., Assistant Actuary, National Council on Compensation Insurance, 45 East 17th Street, New York 3, N. Y.
*Nov. 18, 1927	MASTERSON, NORTON E., Vice-President and Actuary, Hardware Mutual Casualty Co. and Hardware Dealers Mutual Fire Insurance Co., 200 Strongs Avenue, Stevens Point, Wis.
*Nov. 19, 1926	MATTHEWS, ARTHUR N., Associate Actuary, Casualty Actuarial Department, The Travelers Insurance Co., 700 Main Street, Hartford 15, Conn.
May 19, 1915	MAYCRINK, EMMA C., Secretary-Treasurer, Association of New York State Mutual Casualty Companies, 60 East 42nd Street, New York 17, N. Y.
*Nov. 15, 1935	MCCONNELL, MATTHEW H., General Accident Fire and Life Assurance Company, Fourth and Walnut Sts., Philadelphia 5, Pa.

FELLOWS

Admitted	
*Oct. 31, 1917	McMANUS, ROBERT J., Assistant Actuary, Casualty Actuarial Department, The Travelers Insurance Co., 700 Main Street, Hartford 15, Conn.
†	MICHELBACHER, G. F., President, Great American Indemnity Co., 1 Liberty Street, New York 5, N. Y.
*Nov. 17, 1938	MILLER, JOHN HAYNES, Vice-President and Actuary, Monarch Life Insurance Company, 365 State St., Springfield 1, Mass.
†	MILLIGAN, SAMUEL, Vice-President, Metropolitan Life Insurance Co., 1 Madison Avenue, New York 10, N. Y.
*Nov. 18, 1937	MILLS, JOHN A., Vice-President and Actuary, Lumbermens Mutual Casualty Co. and American Motorists Insurance Co., Mutual Insurance Bldg., 4750 Sheridan Road, Chicago 40, Ill.
*Nov. 18, 1921	MONTGOMERY, VICTOR, President, Pacific Employers Insurance Co., 1033 So. Hope Street, Los Angeles 15, Calif.
†	MOORE, GEORGE D., Actuary, 13 Emerson Street, E. Orange, N. J.
*Nov. 17, 1920	MUELLER, LOUIS H., 2845 Lake Street, San Francisco 21, Calif.
†	MULLANEY, FRANK R., Financial Vice-President and Secretary, American Mutual Liability Insurance Co., 142 Berkeley Street, Boston 16, Mass.
*Nov. 17, 1950	MUNTERICH, GEORGE C., Actuary, Manhattan Casualty Co., 1775 Broadway, New York 19, N. Y.
May 28, 1920	MURPHY, RAY D., Executive Vice-President and Actuary, The Equitable Life Assurance Society of the U. S. A., 393 Seventh Avenue, New York 1, N. Y.
*Nov. 15, 1935	OBERHAUS, THOMAS M., Consulting Actuary, Woodward and Fondiller, 524 West 57th Street, New York 19, N. Y.
†	OLIFIERS, EDWARD, Consulting Actuary, Caixa Postal 8, Pertopolis, Rio, Brazil.
†	ORR, ROBERT K., (Retired), 226 S. Logan Street, Lansing 15, Mich.
*Nov. 21, 1919	OUTWATER, OLIVE E., Actuary, Benefit Association of Railway Employees, 901 Montrose Avenue, Chicago 13, Ill.
*Nov. 21, 1930	PERRYMAN, FRANCIS S., Assistant U. S. Manager and Actuary, Royal-Liverpool Insurance Group, 150 William Street, New York 38, N. Y.
*Nov. 14, 1941	PETERS, STEFAN, Associate Professor of Insurance, School of Business Administration, 114 South Hall, University of California, Berkeley 4, Calif.
Nov. 19, 1926	PHILLIPS, JESSE S., Director, Great American Indemnity Co., 1 Liberty Street, New York 5, N. Y.
*Nov. 24, 1933	PICKETT, SAMUEL C., Rating Supervisor, Insurance Department, State of Connecticut, Hartford 2, Conn.
*Nov. 17, 1922	PINNEY, SYDNEY D., 290 Wolcott Hill Road, Wethersfield 9, Conn.
*Nov. 13, 1931	PRUITT, DUDLEY M., Actuary, General Accident Fire & Life Assurance Corp., Fourth & Walnut Sts., Philadelphia 5, Pa.

FELLOWS

Admitted	
*Nov. 18, 1949	RESONY, JOHN A., Casualty Rate Analyst, Connecticut Insurance Department, State Office Building, Hartford 2, Conn.
May 23, 1919	RICHARDSON, FREDERICK, (Retired), Coombe, Bradford Abbas, Sherborne, Dorset, England.
*Nov. 19, 1926	RICHTER, OTTO C., Chief Statistician, American Telephone & Telegraph Co., 195 Broadway, New York 7, N. Y.
May 24, 1921	RIEGEL, ROBERT, Professor of Statistics and Insurance, University of Buffalo, Buffalo 14, N. Y.
*Nov. 14, 1947	RODERMUND, MATTHEW, Assistant Secretary, Interboro Mutual Indemnity Insurance Company, 270 Madison Avenue, New York 16, N. Y.
*Nov. 14, 1947	ROSENBERG, NORMAN, Supervising Rate Analyst, California Insurance Department, 621 South Hope St., Los Angeles, Calif.
*Nov. 17, 1943	ROSS, SAMUEL M., Assistant Actuary, National Bureau of Casualty Underwriters, 60 John Street, New York 38, N. Y.
*Nov. 14, 1947	ROWELL, JOHN H., Lumbermens Mutual Casualty Company, 4750 Sheridan Road, Chicago 40, Ill.
*Nov. 14, 1947	SALZMANN, RUTH E., Assistant Actuary, Hardware Mutual Casualty Company, Hardware Dealers Mutual Fire Insurance Co., 200 Strongs Ave., Stevens Point, Wis.
*Nov. 20, 1942	SATTERTHWAITE, FRANKLIN E., Quality Control Engineer, Product Service Division, General Electric Company, 1285 Boston Ave., Bridgeport 2, Conn.
*Nov. 19, 1948	SCHLOSS, HAROLD W., Superintendent, Actuarial Department, Royal-Liverpool Insurance Group, 150 William Street, New York 38, N. Y.
*Nov. 18, 1937	SHAPIRO, GEORGE I., 934 E. 9th Street, Brooklyn 30, N. Y.
*Nov. 13, 1931	SILVERMAN, DAVID, Partner, Wolfe, Corcoran & Linder, 116 John Street, New York 7, N. Y.
*Nov. 24, 1933	SINNOTT, ROBERT V., Secretary, Hartford Accident and Indemnity Company, 690 Asylum Ave., Hartford 15, Conn.
*Nov. 19, 1929	SKELDING, ALBERT Z., Assistant Manager, National Council on Compensation Insurance, 45 East 17th St., New York 3, N. Y.
*Nov. 19, 1929	SKILLINGS, E. SHAW, Assistant Vice-President and Actuary, Allstate Insurance Co., 20 North Wacker Drive, Chicago 6, Ill.
*Nov. 18, 1932	SMICK, JACK J., Consulting Actuary, 38 Park Row, New York 7, N. Y.
*Nov. 15, 1940	SMITH, SEYMOUR E., Secretary, Casualty Department, The Travelers Insurance Co., Hartford 15, Conn.
*Nov. 24, 1933	ST. JOHN, JOHN B., Consulting Actuary, Box 57, Penllyn, Pa.
Nov. 18, 1927	STONE, EDWARD C., Chairman of the Board, American Employers' Insurance Company, 33 Broad Street, Boston 9, Mass.
*Nov. 17, 1920	TARBELL, THOMAS F., Chief Actuary, Casualty and Fire Actuarial Departments, The Travelers Insurance Co., 700 Main Street, Hartford 15, Conn.

FELLOWS

Admitted	
†	THOMPSON, JOHN S., President, The Mutual Benefit Life Insurance Co., 300 Broadway, Newark 4, N. J.
†	TRAIN, JOHN L., President, Utica Mutual Insurance Co., 185 Genesee Street, Utica 2, N. Y.
Nov. 17, 1922	TRAVERSI, ANTONIO T., 9 Balfour Street, Wollstonecraft, Sydney, Australia.
*Nov. 19, 1948	TURNER, PAUL A., 553 So. St. Andrew Place, Los Angeles 5, Calif.
*Nov. 14, 1947	UHTHOFF, D. R., Assistant Actuary, Employers' Mutual Liability Insurance Co. of Wisconsin, Wausau, Wis.
*Nov. 23, 1928	VALERIUS, NELS M., Assistant Actuary, Aetna Casualty and Surety Co., Hartford 15, Conn.
*Nov. 21, 1919	VAN TUYL, HIRAM O., Superintendent, Internal Audit Department, London Guarantee & Accident Co., 55 Fifth Avenue, New York 3, N. Y.
*Nov. 17, 1920	WAITE, ALAN W., Secretary, The Aetna Casualty and Surety Co., 151 Farmington Ave., Hartford 15, Conn.
*Nov. 15, 1935	WAITE, HARRY V., (Retired), 938 Ridge Road, Wethersfield 9, Conn.
*Nov. 14, 1947	WIEDER, JOHN W., JR., Aetna Casualty and Surety Company, Hartford 15, Conn.
*Nov. 15, 1935	WILLIAMS, HARRY V., Secretary, Hartford Accident and Indemnity Co., 690 Asylum Ave., Hartford 15, Conn.
Nov. 14, 1941	WILLIAMSON, W., RULON, Senior Actuarial Consultant, The Wyatt Company, 3400 Fairhill Drive, Washington 20, D.C.
*Nov. 13, 1931	WITTICK, HERBERT E., Secretary, Pilot Insurance Co., 199 Bay Street, Toronto 1, Canada.
*Nov. 18, 1949	WOLFRUM, RICHARD J., Assistant Actuary, Liberty Mutual Insurance Company, 175 Berkeley Street, Boston 17, Mass.
May 24, 1921	WOOD, ARTHUR B., Chairman of the Board, Sun Life Assurance Company of Canada, Montreal, Canada.

ASSOCIATES

Those marked (*) have been admitted as Associates upon examination by the Society.

Admitted	
May 23, 1924	ACKER, MILTON, Manager, General Liability Division, National Bureau of Casualty Underwriters, 60 John Street, New York 38, N. Y.
*Nov. 15, 1918	ACKERMAN, SAUL B., Professor of Insurance, School of Commerce, New York University, Washington Square, New York 6, N. Y.
*Nov. 16, 1939	AIN, SAMUEL N., Consulting Actuary, 120 Broadway, New York 5, N. Y.
Apr. 5, 1928	ALLEN, AUSTIN F., President, Texas Employers' Insurance Association, 530 Interurban Building, P.O. Box 2759, Dallas 1, Texas.
Nov. 15, 1918	ANKERS, R. E., Vice-President and Treasurer, Continental Life Insurance Co., Inc., Investment Building, 15 and K Streets., N.W., Washington 5, D.C.
*Nov. 21, 1930	ARCHIBALD, A. EDWARD, Vice-President and Actuary, Volunteer State Life Insurance Company, Chattanooga 1, Tenn.
*Nov. 24, 1933	BARRON, JAMES C., Asst. Treasurer, General Reinsurance Corporation, 90 John Street, New York 7, N. Y.
*Nov. 23, 1928	BATEMAN, ARTHUR E., c/o Arthur Q. Melendy, Southboro, Mass.
*Nov. 15, 1940	BATHO, BRUCE, Associate Actuary, Life Insurance Company of Georgia, 573 W. Peachtree St., N.E., Atlanta 1, Georgia.
*Nov. 18, 1925	BITTEL, W. HAROLD, Chief Actuary, Department of Banking and Insurance, Trenton 7, N. J.
Nov. 17, 1920	BLACK, NELLAS C., Manager, Statistical Department, Maryland Casualty Co., Baltimore 3, Md.
*Nov. 15, 1940	BLACKHALL, JOHN M., California-Western States Life Insurance Company, 10th & J Sts., Sacramento, Calif.
*Nov. 22, 1934	BOMSE, EDWARD L., Supt. New York Met. Special Risks, Royal Indemnity Co., 150 William Street, New York 38, N. Y.
*Nov. 23, 1928	BOWER, P. S., Assistant General Manager and Treasurer, The Great-West Life Assurance Company, Winnipeg, Manitoba, Canada.
*Nov. 17, 1950	BOYAJIAN, JOHN H., Actuarial Department, National Council on Compensation Insurance, 45 East 17th St., New York 3, N. Y.
*Nov. 15, 1918	BRUNNQUELL, HELMUTH G., (Retired), 1013 East Circle Drive, Milwaukee 2, Wis.
*Oct. 22, 1915	BUFFLER, LOUIS, Director, Underwriting Department, State Insurance Fund, 625 Madison Avenue, New York 22, N. Y.
*Nov. 20, 1924	BUGBEE, J. M., Manager, Automobile Department, Maryland Casualty Co., Box 1228, Baltimore 3, Md.
Mar. 31, 1920	BURT, MARGARET A., Office of George B. Buck, Consulting Actuary, 150 Nassau Street, New York 7, N. Y.

ASSOCIATES

Admitted	
Nov. 17, 1922	CAVANAUGH, L. D., President, Federal Life Insurance Co., 168 N. Michigan Avenue, Chicago 1, Ill.
*Nov. 18, 1927	CHEN, S. T., Actuary, China United Assurance Society, 104 Bubbling Well Road, Shanghai, China.
*Nov. 24, 1933	CRAWFORD, W. H., Secretary, Fireman's Insurance Co. of Newark, N. J. & Affiliated Fire & Casualty Co.'s Pacific Dept., 220 Bush Street, San Francisco 6, Calif.
*Nov. 18, 1932	CRIMMINS, JOSEPH B., Assistant Actuary, Metropolitan Life Insurance Co., 1 Madison Avenue, New York 10, N. Y.
*Nov. 17, 1950	CRITCHLEY, DOUGLAS, Actuarial Department, Royal-Liverpool Group, 150 William Street, New York 8, N. Y.
*Nov. 18, 1925	DAVIS, MALVIN E., Actuary, Metropolitan Life Insurance Co., 1 Madison Avenue, New York 10, N. Y.
*Nov. 24, 1933	DAVIS, REGINALD S., 878 El Dorado Way, Sacramento, Calif.
*Nov. 14, 1941	DOWLING, WILLIAM F., Asst. Treasurer, Lumber Mutual Casualty Co., 260 Fourth Avenue, New York 10, N. Y.
June 5, 1925	EGER, FRANK A., Secretary-Comptroller, Indemnity Insurance Co. of North America, 1600 Arch Street, Philadelphia 1, Pa.
*Nov. 16, 1923	FITZ, L. LEROY, Group Department, John Hancock Mutual Life Insurance Company, Boston 17, Mass.
*Nov. 16, 1923	FLEMING, FRANK A., General Manager, Mutual Insurance Rating Bureau, 60 East 42nd Street, New York 17, N. Y.
*Nov. 13, 1936	FRUECHTEMEYER, FRED J., Assistant to Comptroller, The Andrew Jergens Company, 2535 Spring Grove Ave., Cincinnati 14, Ohio.
*Nov. 19, 1929	FURNIVALL, MAURICE L., Associate Actuary, Accident and Group Actuarial Department, The Travelers Insurance Co., 700 Main Street, Hartford 15, Conn.
*Nov. 14, 1947	GEORGE, HAROLD J., Assistant Actuary, National Life Insurance Co., 131 State Street, Montpelier, Vt.
*Nov. 18, 1932	GETMAN, RICHARD A., The Travelers Insurance Co., 700 Main Street, Hartford 15, Conn.
*Nov. 17, 1922	GIBSON, JOSEPH P., JR., Manager, Casualty Department, American Foreign Insurance Association, 80 Maiden Lane, New York 7, N. Y.
*Nov. 16, 1923	GILDEA, JAMES F., The Travelers Insurance Co., 700 Main Street, Hartford 15, Conn.
*Nov. 14, 1947	GINGERY, STANLEY W., Assistant Actuary, The Prudential Insurance Co., Newark, N. J.
*Nov. 18, 1927	GREEN, WALTER C., Consulting Actuary, Continental Bank Building, Salt Lake City, Utah.
*Nov. 15, 1940	GROSSMAN, ELI A., Actuary, Union Labor Life Insurance Co., 200 East 70th Street, New York 21, N. Y.
*Nov. 15, 1935	GUERTIN, ALFRED N., Actuary, American Life Convention, 230 N. Michigan Avenue, Chicago 1, Ill.

ASSOCIATES

Admitted	
*Nov. 16, 1939	HAGEN, OLAF E., Metropolitan Life Insurance Company, 1 Madison Avenue, New York 10, N. Y.
*Nov. 18, 1921	HAGGARD, ROBERT E., Supervisor, Permanent Disability Rating Bureau, Industrial Accident Commission, 965 Mission Street, San Francisco 3, Calif.
*Nov. 17, 1950	HALEY, JAMES B., JR., Actuarial Department, Fireman's Fund, 401 California Street, San Francisco, Calif.
*Nov. 17, 1922	HALL, HARTWELL L., Associate Actuary, Connecticut Insurance Department, 165 Capitol Avenue, Hartford 2, Conn.
*Nov. 13, 1936	HAM, HUGH P., Assistant General Manager, The British American Assurance Company, 22 Wellington St. East, Toronto 1, Canada.
Mar. 24, 1932	HARRIS, SCOTT, Executive Vice-President, Joseph Froggatt & Co., Inc., 74 Trinity Place, New York 6, N. Y.
*Mar. 25, 1924	HART, WARD VAN B., Associate Actuary, Connecticut General Life Insurance Co., 55 Elm Street, Hartford 15, Conn.
Nov. 21, 1919	HAYDON, GEORGE F., General Manager, Wisconsin Compensation Rating & Inspection Bureau, 715 N. Van Buren Street, Milwaukee 2, Wis.
*Nov. 19, 1948	HEWITT, CHARLES C., JR., New Jersey Manufacturers Casualty Insurance Co., 363 W. State Street, Trenton, N. J.
Nov. 17, 1927	HIPP, GRADY H., Executive Vice-President, Liberty Life Insurance Co., Greenville, S. C.
*Nov. 16, 1945	HOLZINGER, ERNEST, Actuary, Pension Planning Company, 30 Broad Street, New York 4, N. Y.
Nov. 19, 1929	JACOBS, CARL N., President, Hardware Mutual Casualty Co., 200 Strongs Avenue, Stevens Point, Wis.
*Nov. 18, 1921	JENSEN, EDWARD S., Assistant Secretary, Group Department, Occidental Life Insurance Co. of California, 1151 So. Broadway, Los Angeles 55, Calif.
Nov. 21, 1930	JONES, H. LLOYD, United States Manager and Attorney, Phoenix-London Group, 55 Fifth Avenue, New York 3, N. Y.
*Nov. 21, 1919	JONES, LORING D., (Retired), 64 Raymond Avenue, Rockville Centre, Long Island, N. Y.
*Nov. 17, 1922	KIRK, CARL L., Deputy Manager, Zurich General Accident & Liability Insurance Co., 135 South LaSalle Street, Chicago 3, Ill.
*Nov. 15, 1935	KITZROW, E. W., Hardware Mutual Casualty Co., Raymond-Commerce Building, Newark, N. J.
*Nov. 17, 1950	LONGLEY-COOK, LAURENCE H., Actuary, Insurance Company of North America, 1600 Arch Street, Philadelphia 1, Pa.
*Nov. 14, 1947	LUFKIN, ROBERT W., Statistician, Liberty Mutual Insurance Co., 175 Berkeley Street, Boston 17, Mass.
*Nov. 13, 1931	MACKEEN, HAROLD E., Casualty Actuarial Department, The Travelers Insurance Co., 700 Main Street, Hartford 15, Conn.
Mar. 24, 1932	MAGRATH, JOSEPH J., Administrative Assistant, Chubb & Son, 90 John Street, New York 7, N. Y.

ASSOCIATES

Admitted	
*Nov. 18, 1925	MALMUTH, JACOB, Associate Examiner, New York State Insurance Department, 61 Broadway, New York 6, N. Y.
Mar. 24, 1927	MARSH, CHARLES V. R., (Retired), 617 E. Surf Road, Ocean City, N. J.
*Nov. 13, 1936	MAYER, WILLIAM H., JR., Group Contact Referee, Metropolitan Life Insurance Co., 1 Madison Avenue, New York 10, N. Y.
*Nov. 17, 1950	MAYERSON, ALLEN L., Actuarial Assistant, National Surety Corporation, 4 Albany Street, New York, N. Y.
*Nov. 17, 1922	McIVER, R. A., Actuary, Washington National Insurance Co., 610 Church Street, Evanston, Ill.
*Nov. 17, 1950	MENZEL, HENRY W., Actuarial Department, National Bureau of Casualty Underwriters, 60 John Street, New York 38, N. Y.
*Nov. 13, 1931	MILLER, HENRY C., Comptroller-Actuary, California State Compensation Insurance Fund, 450 McAllister Street, San Francisco 2, Calif.
*Nov. 19, 1926	MILNE, JOHN L., Vice-President and Actuary, Philadelphia Life Insurance Company, 111 North Broad Street, Philadelphia 7, Pa.
Nov. 17, 1922	MONTGOMERY, JOHN C., Secretary and Treasurer, Bankers Indemnity Insurance Co., 15 Washington Street, Newark 2, N. J.
May 25, 1923	MOORE, JOSEPH P., Mutual Life and Citizens Assurance Co., Ltd., P.O. Box 1770, Place D'arms, Montreal, Canada.
*Nov. 17, 1950	MURRIN, THOMAS E., Actuarial Department, National Bureau of Casualty Underwriters, 60 John Street, New York 38, N. Y.
*Nov. 18, 1937	MYERS, ROBERT J., Chief Actuary, Social Security Administration, Washington 25, D.C.
*Nov. 15, 1935	NELSON, S. TYLER, Casualty Actuary, Department of Insurance, State Capitol Building, Springfield, Ill.
*Oct. 27, 1916	NEWELL, WILLIAM, (Retired), 1225 Park Avenue, New York 28, N. Y.
*Nov. 18, 1925	NICHOLSON, EARL, Actuary, Joseph Froggatt & Co., Inc., 74 Trinity Place, New York 6, N. Y.
May 23, 1919	OTTO, WALTER E., President, Michigan Mutual Liability Co., Associated General Fire Co., 163 Madison Avenue, Detroit 26, Mich.
*Nov. 19, 1926	OVERHOLSER, DONALD M., Office of George B. Buck, Consulting Actuary, 150 Nassau Street, New York 7, N. Y.
Nov. 20, 1924	PENNOCK, RICHARD M., (Retired), 12 Lodges Lane, Cynwood, Pa.
*Nov. 14, 1947	PERRY, ROBERT C., Vice-President and Actuary, State Farm Life Insurance Company, Bloomington, Ill.
Nov. 19, 1929	PHILLIPS, JOHN H., Vice-President and Actuary, Employers' Mutual Liability Insurance Co., 407 Grant Street, Wausau, Wis.
*Nov. 17, 1920	PIKE, MORRIS, Associate Actuary, John Hancock Mutual Life Insurance Co., Boston 17, Mass.

ASSOCIATES

Admitted	
*Nov. 23, 1928	PIPER, K. B., Vice-President, Provident Life and Accident Insurance Co., 721 Broad Street, Chattanooga 2, Tenn.
*Nov. 17, 1922	POORMAN, WILLIAM F., President, Central Life Assurance Society (Mutual), Fifth and Grand Avenues, Des Moines 6, Iowa.
*Nov. 13, 1936	POTOFSKY, SYLVIA, Senior Actuary, The State Insurance Fund, 625 Madison Avenue, New York 22, N. Y.
*Nov. 15, 1918	RAYWID, JOSEPH, President, Joseph Raywid & Co., Inc., 92 William Street, New York 7, N. Y.
Nov. 19, 1932	RICHARDSON, HARRY F., General Manager, National Council on Compensation Insurance, 45 East 17th Street, New York 3, N.Y.
*Nov. 18, 1932	ROBERTS, JAMES A., Life Actuarial Department, The Travelers Insurance Co., 700 Main Street, Hartford 15, Conn.
*Nov. 18, 1927	SARASON, HARRY M., Statistician, Occidental Life Insurance Company of California, Box 2101, Terminal Annex, Los Angeles 54, Calif.
Nov. 16, 1923	SAWYER, ARTHUR, Actuarial Department, Royal-Liverpool Insurance Group, 150 William Street, New York 38, N. Y.
*Nov. 14, 1947	SCAMMON, LAWRENCE W., Actuary, Massachusetts Automobile Rating and Accident Prevention Bureau, Massachusetts Workmen's Compensation Rating and Inspection Bureau, 89 Broad Street, Boston 10, Mass.
*Nov. 14, 1947	SCHWARTZ, MAX J., Associate Actuary (Casualty), New York State Insurance Department, Albany 1, N. Y.
*Nov. 20, 1930	SEVILLA, EXEQUIEL S., Manager and Actuary, National Life Insurance Co. of the Philippines, Regina Building, P.O. Box 2056, Manila, Philippines.
*Nov. 20, 1924	SHEPPARD, NORRIS E., Professor of Mathematics, University of Toronto, Toronto 5, Canada.
Nov. 15, 1918	SIBLEY, JOHN L., (Retired), 225 Amesburg Road, Haverhill, Mass., c/o Eielson.
*Nov. 18, 1921	SMITH, ARTHUR G., Assistant General Manager, Compensation Insurance Rating Board, Pershing Square Bldg., 125 Park Avenue, New York 17, N. Y.
*Nov. 19, 1926	SOMERVILLE, WILLIAM F., Secretary, St. Paul Mercury Indemnity Co., St. Paul 2, Minn.
*Nov. 18, 1925	SOMMER, ARMAND, Supt. of Agencies, Continental Casualty Co., 910 So. Michigan Avenue, Chicago 5, Ill.
*Nov. 15, 1918	SPENCER, HAROLD S., Statistician, Aetna Casualty and Surety Co., 151 Farmington Avenue, Hartford 15, Conn.
Nov. 20, 1924	STELLWAGEN, H. P., Executive Vice-President, Indemnity Insurance Company of North America, 1600 Arch Street, Philadelphia 1, Pa.
*Nov. 16, 1923	STOKE, KENDRICK, Actuary, Michigan Mutual Liability Company, 163 Madison Avenue, Detroit 26, Mich.
*Nov. 21, 1930	SULLIVAN, WALTER F., Assistant Actuary, State Compensation Insurance Fund, 450 McAllister Street, San Francisco 1, Calif.

ASSOCIATES

Admitted	
*Nov. 21, 1919	TRENCH, FREDERICK H., Manager, Underwriting Department, Utica Mutual Insurance Co., 185 Genesee Street, Utica 1, N. Y.
*Nov. 17, 1950	TRIST, JOHN A. W., Statistical Department, Lumbermens Mutual Casualty Company, Mutual Insurance Bldg., 4750 Sheridan Road, Chicago 40, Ill.
*Nov. 20, 1924	UHL, M. ELIZABETH, National Bureau of Casualty Underwriters, 60 John Street, New York 38, N. Y.
*Nov. 14, 1947	VERGANO, ELIA, Assistant Actuary, Compensation Insurance Rating Board, 125 Park Avenue, New York 17, N. Y.
May 23, 1919	WARREN, CHARLES S., Secretary, Massachusetts Automobile Rating and Accident Prevention Bureau, 89 Broad Street, Boston 10, Mass.
*Nov. 18, 1932	WEINSTEIN, MAX S., Actuary, New York State Employees' Retirement System, 256 Washington Avenue, Albany 1, N. Y.
*Nov. 18, 1925	WELLMAN, ALEXANDER C., Vice-President, Protective Life Insurance Co., Birmingham, Ala.
*Nov. 21, 1930	WELLS, WALTER I., Assistant Actuary, State Mutual Life Assurance Co., 340 Main Street, Worcester 8, Mass.
Mar. 21, 1929	WHEELER, CHARLES A., Chief Examiner of Casualty Companies, New York State Insurance Department, 61 Broadway, New York 6, N. Y.
*Nov. 18, 1927	WHITBREAD, F. G., Vice-President, Reliance Life Insurance Company, Room 412, Farmers Bank Building, Pittsburgh 22, Pa.
*Nov. 19, 1948	WHITE, AUBREY, Ostheimer & Co., 1500 Chestnut St., Philadelphia, Pa.
*Nov. 16, 1939	WITTLAKE, J. CLARKE, Assistant to President, Business Men's Assurance Company, B.M.A. Building, Kansas City 10, Mo.
*Oct. 22, 1915	WOOD, DONALD M., Partner, Childs & Wood, 175 W. Jackson Blvd., Chicago 4, Ill.
*Nov. 18, 1937	WOOD, DONALD M., JR., Childs & Wood, 175 West Jackson Blvd., Chicago 4, Ill.
*Nov. 18, 1927	WOOD, MILTON J., Chief Actuary, Accident and Group Actuarial Department, The Travelers Insurance Co., 700 Main Street, Hartford 15, Conn.
*Oct. 22, 1915	WOODMAN, CHARLES E., (Retired), The Brunswick, Waterville, N. Y.
*Nov. 22, 1934	WOODWARD, BARBARA H., Assistant to Vice-President and General Counsel, The Rueben H. Donnelley Corporation, 305 East 45th Street, New York, N. Y.
*Nov. 17, 1950	WOODDY, JOHN C., Actuarial Statistician, American Telephone and Telegraph Company, 195 Broadway, New York 7, N. Y.
*Nov. 18, 1925	WOOLERY, JAMES MYRON, Vice-President and Actuary, Occidental Life Insurance Company, Raleigh, N. C.

SCHEDULE OF MEMBERSHIP, NOVEMBER 17, 1950

	Fellows	Associates	Total
Membership, November 18, 1949	151	119	270
Additions:			
By Election	...	...	...
By Reinstatement	1	...	1
By Examination	6	9	15
	158	128	286
Deductions:			
By Death	...	...	...
By Withdrawal	1	1	2
By Transfer from Associate to Fellow		6	6
Membership, November 17, 1950	157	121	278

OFFICERS OF THE SOCIETY

Since Date of Organization

Elected	*President*	*Vice-Presidents*	
1914-1915	*Isaac M. Rubinow	*Albert H. Mowbray	*Benedict D. Flynn
1916-1917	*James D. Craig	*Joseph H. Woodward	*Harwood E. Ryan
1918	*Joseph H. Woodward	*Benedict D. Flynn	George D. Moore
1919	*Benedict D. Flynn	George D. Moore	William Leslie
1920	*Albert H. Mowbray	William Leslie	*Leon S. Senior
1921	*Albert H. Mowbray	*Leon S. Senior	*Howard E. Ryan
1922	*Harwood E. Ryan	Gustav F. Michelbacher	Edmund E. Cammack
1923	William Leslie	Gustav F. Michelbacher	Edmund E. Cammack
1924-1925	Gustav F. Michelbacher	*Sanford B. Perkins	Ralph H. Blanchard
1926-1927	*Sanford B. Perkins	George D. Moore	Thomas F. Tarbell
1928-1929	George D. Moore	Sydney D. Pinney	Paul Dorweiler
1930-1931	Thomas F. Tarbell	*Roy A. Wheeler	Winfield W. Greene
1932-1933	Paul Dorweiler	William F. Roeber	*Leon S. Senior
1934-1935	Winfield W. Greene	Ralph H. Blanchard	Charles J. Haugh
1936-1937	*Leon S. Senior	Sydney D. Pinney	Francis S. Perryman
1938-1939	Francis S. Perryman	Harmon T. Barber	William J. Constable
1940	Sydney D. Pinney	Harold J. Ginsburgh	James M. Cahill
1941	Ralph H. Blanchard	Harold J. Ginsburgh	James M. Cahill
1942	Ralph H. Blanchard	Albert Z. Skelding	Charles J. Haugh
1943-1944	Harold J. Ginsburgh	Albert Z. Skelding	Charles J. Haugh
1945-1946	Charles J. Haugh	James M. Cahill	Harry V. Williams
1947-1948	James M. Cahill	Harmon T. Barber	Russell P. Goddard
1949-1950	Harmon T. Barber	Thomas O. Carlson	Norton E. Masterson

Secretary-Treasurer

1914-1917....*C. E. Scattergood
1918-1950.....R. Fondiller

Editor†

1914.....W. W. Greene
1915-1917.....R. Fondiller
1918.....W. W. Greene
1919-1921....G. F. Michelbacher
1922-1923.....O. E. Outwater
1924-1932.....R. J. McManus
1933-1943.....*C. W. Hobbs
1944-1950.....E. C. Maycrink

Librarian†

1914.....W. W. Greene
1915.....R. Fondiller
1916-1921.....L. I. Dublin
1922-1924.....E. R. Hardy
1925-1937.....W. Breiby
1937-1947.....T. O. Carlson
1948-1950.....S. M. Ross

Chairman—Examination Comm.

1949-1950.....Roger A. Johnson

*Deceased.
†The offices of Editor and Librarian were not separated until 1916.

FELLOWS WHO HAVE DIED

The (†) denotes charter members at date of organization, November 7, 1914.

Admitted		Died
†	Roland Benjamin	July 2, 1949
May 24, 1921	Edward J. Bond	Nov. 12, 1941
May 19, 1915	Thomas Bradshaw	Nov. 10, 1939
June 5, 1925	William Brosmith	Aug. 22, 1937
†	William A. Budlong	June 4, 1934
Nov. 18, 1932	Charles H. Burhans	June 15, 1942
Feb. 19, 1915	F. Highlands Burns	Mar. 30, 1935
†	Raymond V. Carpenter	Mar. 11, 1947
Feb. 19, 1915	Gorden Case	Feb. 4, 1920
†	Charles T. Conway	July 23, 1921
†	Walter G. Cowles	May 30, 1942
†	James D. Craig	May 27, 1940
†	James McIntosh Craig	Jan. 20, 1922
May 26, 1916	Frederick S. Crum	Sept. 2, 1921
†	Alfred Burnett Dawson	June 21, 1931
†	Miles Menander Dawson	Mar. 27, 1942
†	Elmer H. Dearth	Mar. 26, 1947
May 19, 1915	Samuel Deutschberger	Jan. 18, 1929
†	Ezekiel Hinton Downey	July 9, 1922
May 19, 1915	Earl O. Dunlap	July 5, 1944
†	David Parks Fackler	Oct. 30, 1924
Feb. 19, 1915	Claude W. Fellows	July 15, 1938
†	Benedict D. Flynn	Aug. 22, 1944
†	Charles S. Forbes	Oct. 2, 1943
May 26, 1916	Lee K. Frankel	July 25, 1931
Feb. 25, 1916	Joseph Froggatt	Sept. 28, 1940
†	Harry Furze	Dec. 26, 1945
Feb. 19, 1915	Fred S. Garrison	Nov. 14, 1949
†	Theodore E. Gaty	Aug. 22, 1925
May 19, 1915	James W. Glover	July 15, 1941
Oct. 22, 1915	George Graham	Apr. 15, 1937
Oct. 22, 1915	Thompson B. Graham	July 24, 1946
May 25, 1923	William A. Granville	Feb. 4, 1943
†	William H. Gould	Oct. 28, 1936
†	Robert Cowen Lees Hamilton	Nov. 15, 1941
Nov. 21, 1919	Robert Henderson	Feb. 16, 1942
†	Robert J. Hillas	May 17, 1940
Nov. 15, 1918	Frank Webster Hinsdale	Mar. 18, 1932
May 23, 1924	Clarence W. Hobbs	July 21, 1944
Nov. 19, 1926	Charles E. Hodges	Jan. 22, 1937
†	Frederick L. Hoffman	Feb. 23, 1946
Nov. 21, 1919	Carl Hookstadt	Mar. 10, 1924
†	Charles Hughes	Aug. 27, 1948
Nov. 19, 1929	Robert S. Hull	Nov. 30, 1947
†	Burritt A. Hunt	Sept. 3, 1943
Nov. 28, 1921	William Anderson Hutcheson	Nov. 19, 1942
May 19, 1915	William C. Johnson	Oct. 7, 1943
Nov. 23, 1928	F. Robertson Jones	Dec. 26, 1941
Nov. 18, 1921	Thomas P. Kearney	Feb. 11, 1928

FELLOWS WHO HAVE DIED—*Continued*

Admitted		*Died*
Nov. 19, 1926	Gregory Cook Kelly	Sept. 11, 1948
Oct. 22, 1915	Virgil Morrison Kime	Oct. 15, 1918
†	Edwin W. Kopf	Aug. 3, 1933
Feb. 17, 1915	John M. Laird	June 20, 1942
Feb. 19, 1915	Abb Landis	Dec. 9, 1937
Nov. 17, 1922	Arnette Roy Lawrence	Dec. 1, 1942
Nov. 18, 1921	James Fulton Little	Aug. 11, 1938
Nov. 23, 1928	Edward C. Lunt	Jan. 13, 1941
Feb. 19, 1915	Harry Lubin	Dec. 20, 1920
Nov. 16, 1923	D. Ralph McClurg	Apr. 27, 1947
May 23, 1919	Alfred McDougald	July 28, 1944
Feb. 15, 1915	Franklin B. Mead	Nov. 29, 1933
Apr. 20, 1917	Marcus Melzer	Mar. 27, 1931
†	David W. Miller	Jan. 18, 1936
†	James F. Mitchell	Feb. 9, 1941
†	Henry Moir	June 8, 1937
Nov. 19, 1926	William L. Mooney	Oct. 21, 1948
Feb. 19, 1915	William J. Montgomery	Aug. 20, 1915
May 19, 1915	Edward Bontecou Morris	Dec. 19, 1929
†	Albert H. Mowbray	Jan. 7, 1949
†	Lewis A. Nicholas	Apr. 21, 1940
†	Stanley L. Otis	Oct. 12, 1937
Nov. 13, 1926	Bertrand A. Page	July 30, 1941
Nov. 18, 1921	Sanford B. Perkins	Sept. 16, 1945
Nov. 15, 1918	William Thomas Perry	Oct. 25, 1940
†	Edward B. Phelps	July 24, 1915
†	Charles Grant Reiter	July 30, 1937
†	Charles H. Remington	Mar. 21, 1938
†	Isaac M. Rubinow	Sept. 1, 1936
†	Harwood Eldridge Ryan	Nov. 2, 1930
†	Arthur F. Saxton	Feb. 26, 1927
†	Emil Scheitlin	May 2, 1946
†	Leon S. Senior	Feb. 3, 1940
April 20, 1917	Charles Gordon Smith	June 22, 1938
Feb. 19, 1915	John T. Stone	May 9, 1920
Feb. 25, 1916	Wendell Menville Strong	Mar. 30, 1942
Oct. 22, 1915	William R. Strong	Jan. 10, 1946
†	Robert J. Sullivan	July 19, 1934
Nov. 22, 1934	Walter H. Thompson	May 25, 1935
Nov. 18, 1921	Guido Toja	Feb. 28, 1933
Nov. 18, 1925	Lloyd A. H. Warren	Sept. 30, 1949
May 23, 1919	Archibald A. Welch	May 8, 1945
Nov. 19, 1926	Roy A. Wheeler	Aug. 26, 1932
†	Albert W. Whitney	July 27, 1943
†	Lee J. Wolfe	Apr. 28, 1949
†	S. Herbert Wolfe	Dec. 31, 1927
†	Joseph H. Woodward	May 15, 1928
†	William Young	Oct. 23, 1927

ASSOCIATES WHO HAVE DIED

Admitted		Died
Oct. 22, 1915	Don A. Baxter	Feb. 10, 1920
May 25, 1923	Harilaus E. Economidy	Apr. 13, 1948
Nov. 20, 1924	John Froberg	Oct. 11, 1949
Nov. 22, 1934	John J. Gately	Nov. 3, 1943
Nov. 19, 1929	Harold R. Gordon	July 8, 1948
Nov. 20, 1924	Leslie LeVant Hall	Mar. 8, 1931
Oct. 31, 1917	Edward T. Jackson	May 8, 1939
Nov. 21, 1919	Rolland V. Mothersill	July 25, 1949
Nov. 19, 1929	Fritz Muller	Apr. 27, 1945
Nov. 23, 1928	Karl Newhall	Oct. 24, 1944
Nov. 18, 1927	Alexander A. Speers	June 25, 1941
Mar. 23, 1921	Arthur E. Thompson	Jan. 17, 1944
Nov. 21, 1919	Walter G. Voogt	May 8, 1945
Nov. 18, 1925	James H. Washburn	Aug. 19, 1946
Nov. 17, 1920	James J. Watson	Feb. 23, 1937
Nov. 18, 1921	Eugene R. Welch	Jan. 17, 1945
Nov. 15, 1918	Albert Edward Wilkinson	June 11, 1930

CONSTITUTION

(As Amended November 17, 1950)

Article I.—*Name.*

This organization shall be called the Casualty Actuarial Society.

Article II.—*Object.*

The object of the Society shall be the promotion of actuarial and statistical science as applied to the problems of insurance, other than life insurance, by means of personal intercourse, the presentation and discussion of appropriate papers, the collection of a library and such other means as may be found desirable.

The Society shall take no partisan attitude, by resolution or otherwise, upon any question relating to insurance.

Article III.—*Membership.*

The membership of the Society shall be composed of two classes, Fellows and Associates. Fellows only shall be eligible to office or have the right to vote.

The Fellows of the Society shall be the present Fellows and those who may be duly admitted to Fellowship as hereinafter provided. The Associates shall be the present Associates and those who may be duly admitted to Associateship as hereinafter provided.

Any person may, upon nomination to the Council by two Fellows of the Society and approval by the Council of such nomination with not more than one negative vote, become enrolled as an Associate of the Society, provided that he shall pass such examination as the Council may prescribe. Such examination may be waived in the case of a candidate who for a period of not less than two years has been in responsible charge of the Statistical or Actuarial Department of an insurance organization (other than life insurance) or has had such other practical experience in insurance (other than life insurance) as, in the opinion of the Council, renders him qualified for Associateship.

Any person who shall have qualified for Associateship may become a Fellow on passing such final examination as the Council may prescribe. Otherwise, no one shall be admitted as a Fellow unless recommended by a duly called meeting of the Council with not more than three negative votes, followed by a three-fourths ballot of the Fellows present and voting at a meeting of the Society.

The General Chairman of the Examination Committee, shall, under the general supervision of the Council, have charge of the examination system and of the examinations held by the Society for the admission to the grades of Associate and of Fellow.

Article IV.—*Officers and Council.*

The officers of the Society shall be a President, two Vice-Presidents, a Secretary-Treasurer, an Editor, a Librarian, and a General Chairman of the Examination Committee. The Council shall be composed of the active officers, nine other Fellows and, during the four years following the expiration of their terms of office, the ex-Presidents and ex-Vice-Presidents. The Council shall fill vacancies occasioned by death or resignation of any officer or other member of the Council, such appointees to serve until the next annual meeting of the Society.

ARTICLE V.—*Election of Officers and Council.*

The President, Vice-Presidents, and the Secretary-Treasurer shall be elected by a majority ballot at the annual meeting for the term of one year and three members of the Council shall, in a similar manner, be annually elected to serve for three years. The President and Vice-Presidents shall not be eligible for the same office for more than two consecutive years nor shall any retiring member of the Council be eligible for re-election at the same meeting.

The Editor, the Librarian and the General Chairman of the Examination Committee shall be elected annually by the Council at the Council meeting preceding the annual meeting of the Society. They shall be subject to confirmation by majority ballot of the Society at the annual meeting.

The terms of the officers shall begin at the close of the meeting at which they are elected except that the retiring Editor shall retain the powers and duties of office so long as may be necessary to complete the then current issue of *Proceedings*.

ARTICLE VI.—*Duties of Officers and Council.*

The duties of the officers shall be such as usually appertain to their respective offices or may be specified in the by-laws. The duties of the Council shall be to pass upon candidates for membership, to decide upon papers offered for reading at the meetings, to supervise the examination of candidates and prescribe fees therefor, to call meetings, and in general, through the appointment of committees and otherwise, to manage the affairs of the Society.

ARTICLE VII.—*Meetings.*

There shall be an annual meeting of the Society on such date in the month of November as may be fixed by the Council in each year, but other meetings may be called by the Council from time to time and shall be called by the President at any time upon the written request of ten Fellows. At least two weeks notice of all meetings shall be given by the Secretary.

ARTICLE VIII.—*Quorum.*

Seven members of the Council shall constitute a quorum. Twenty Fellows of the Society shall constitute a quorum.

ARTICLE IX.—*Expulsion or Suspension of Members.*

Except for non-payment of dues, no member of the Society shall be expelled or suspended save upon action by the Council with not more than three negative votes followed by a three-fourths ballot of the Fellows present and voting at a meeting of the Society.

ARTICLE X.—*Amendments.*

This constitution may be amended by an affirmative vote of two-thirds of the Fellows present at any meeting held at least one month after notice of such proposed amendment shall have been sent to each Fellow by the Secretary.

BY-LAWS

(As Amended November 17, 1950)

Article I.—*Order of Business.*

At a meeting of the Society the following order of business shall be observed unless the Society votes otherwise for the time being:

1. Calling of the roll.
2. Address or remarks by the President.
3. Minutes of the last meeting.
4. Report by the Council on business transacted by it since the last meeting of the Society.
5. New Membership.
6. Reports of officers and committees.
7. Election of officers and Council (at annual meetings only).
8. Unfinished business.
9. New business.
10. Reading of papers.
11. Discussion of papers.

Article II.—*Council Meetings.*

Meetings of the Council shall be called whenever the President or three members of the Council so request, but not without sending notice to each member of the Council seven or more days before the time appointed. Such notice shall state the objects intended to be brought before the meeting, and should other matter be passed upon, any member of the Council shall have the right to re-open the question at the next meeting.

Article III.—*Duties of Officers.*

The President, or, in his absence, one of the Vice-Presidents, shall preside at meetings of the Society and of the Council. At the Society meetings the presiding officer shall vote only in case of a tie, but at the Council meetings he may vote in all cases.

The Secretary-Treasurer shall keep a full and accurate record of the proceedings at the meetings of the Society and of the Council, send out calls for the said meetings, and, with the approval of the President and Council, carry on the correspondence of the Society. Subject to the direction of the Council, he shall have immediate charge of the office and archives of the Society.

The Secretary-Treasurer shall also send out calls for annual dues and acknowledge receipt of same; pay all bills approved by the President for expenditures authorized by the Council of the Society; keep a detailed account of all receipts and expenditures, and present an abstract of the same at the annual meetings, after it has been audited by a committee appointed by the President.

The Editor shall, under the general supervision of the Council, have charge of all matters connected with editing and printing the Society's publications. The *Proceedings* shall contain only the proceedings of the meetings, original papers or reviews written by members, discussions on said papers and other matter expressly authorized by the Council.

The Librarian shall, under the general supervision of the Council, have charge of the books, pamphlets, manuscripts and other literary or scientific material collected by the Society.

The General Chairman of the Examination Committee, shall, under the general supervision of the Council, have charge of the examination system and of the examinations held by the Society for the admission to the grades of Associate and of Fellow.

ARTICLE IV.—*Dues.*

The Council shall fix the annual dues for Fellows and for Associates. The payment of dues will be waived in the case of Fellows or Associates who have attained the age of seventy years or who, having been members for a period of at least twenty years, shall have attained the age of sixty-five years. Fellows and Associates who have become totally disabled while members may upon approval of the Council be exempted from the payment of dues during the period of disability.

It shall be the duty of the Secretary-Treasurer to notify by mail any Fellow or Associate whose dues may be six months in arrears, and to accompany such notice by a copy of this article. If such Fellow or Associate shall fail to pay his dues within three months from the date of mailing such notice, his name shall be stricken from the rolls, and he shall thereupon cease to be a Fellow or Associate of the Society. He may, however, be reinstated by vote of the Council, and upon payment of arrears of dues.

ARTICLE V.—*Designation by Initials.*

Fellows of the Society are authorized to append to their names the initials F.C.A.S.; and Associates are authorized to append to their names the initials A.C.A.S.

ARTICLE VI.—*Amendments.*

These by-laws may be amended by an affirmative vote of two-thirds of the Fellows present at any meeting held at least one month after notice of the proposed amendment shall have been sent to each Fellow by the Secretary.

SYLLABUS OF EXAMINATIONS

(Effective 1948 and Thereafter)

ASSOCIATESHIP

Part	*Section*	*Subject*
I	1	Descriptive and Analytical Statistics.
	2	Compound Interest and Annuities Certain.
II	3	Differential and Integral Calculus.
	4	Calculus of Finite Differences.
III	5	Probabilities.
	6	Life Contingencies, Life Annuities and Life Assurances.
IV	7	Policy Forms and Underwriting Practice in Casualty Insurance.
	8	Casualty Insurance Rate Making Methods.

FELLOWSHIP

I	9	Insurance Economics.
	10	Insurance Law and Regulation.
II	11	Individual Risk Rating.
	12	Social Insurance.
III	13	Determination of Premium, Loss and Expense Reserves.
	14	Advanced Problems in Casualty Insurance Statistics.
IV	15	Advanced Problems in Casualty Insurance Accounting.
	16	Advanced Problems in the Underwriting and Administration of Casualty Insurance.

RULES REGARDING EXAMINATIONS FOR ADMISSION TO THE CASUALTY ACTUARIAL SOCIETY

1. Dates of Examination.

Examinations will be held on the second Tuesday and following Wednesday during the month of May in each year in such cities as will be convenient for three or more candidates.

2. Filing of Application.

Application for admission to examination should be made on the Society's blank form, which may be obtained from the Secretary-Treasurer. No applications will be considered unless received before the fifteenth day of February preceding the dates of examination. Applications should definitely state for what parts the candidate will appear.

3. Fees.

The examination fee is $3.00 for each part or portion thereof taken, subject to a minimum of $5.00 for each year in which the candidate presents himself; thus for one part, $5.00, for two parts, $6.00, etc. Examination fees are payable to the order of the Society and must be received by the Secretary-Treasurer before the fifteenth day of February preceding the dates of examination.

4. Associateship and Fellowship Examinations.

(a) The examination for Associateship consists of four parts and that for Fellowship consists of four parts. A candidate may take any one or more of the four parts of the Associateship Examination. A candidate may present himself for part of the Fellowship Examination either (a) if he has previously passed the Associateship Examination and all preceding parts of the Fellowship Examination, or (b) if he concurrently presents himself for and submits papers for all unpassed parts of the Associateship Examination and all preceding unpassed parts of the Fellowship Examination. Subject to the foregoing requirements, the candidate will be given credit for any part or parts of either examination which he may pass.

(b) A candidate who has passed the Associateship Examination Parts I-IV prior to 1941, but who has not been enrolled as an Associate because of lack of the experience qualifications required by the examination rules effective prior to 1941, will be enrolled as an Associate upon passing the current Associateship Examination Part IV.

(c) An Associate who has passed no part of the Fellowship Examination under the Syllabus effective prior to 1941 is required, in order to qualify for admission as a Fellow, to pass the current Associateship Examination Part IV and Fellowship Examination Parts I-IV.

(d) A candidate who has passed one or more parts of the Associateship or Fellowship Examinations under the Syllabus effective prior to

1948 will receive credit for the corresponding parts of the new Syllabus in accordance with the following table:

Parts Passed Under Old Syllabus (Effective Prior to 1948)	*Parts Credited Under New Syllabus (Effective in 1948)*
Associateship, Part I	Associateship, Part I—Section 2
" " II	" " II
" " III	" " I—Section 1
" " IV	" " III
" " V	" " IV
Fellowship, Part I	Fellowship, Part I
" " II	" Parts III & IV—Section 15
" " III	" Parts II & IV—Section 16

Partial examinations will be given to those students requiring same in accordance with the foregoing credits.

5. Alternative to Passing of Fellowship Parts III and IV.

As an alternative to the passing of Parts III and IV of the Fellowship Examination, a candidate may elect to present an original thesis on an approved subject relating to casualty or social insurance. Such thesis must show evidence of ability for original research and the solution of advanced problems in casualty insurance comparable with that required to pass Parts III and IV of the Fellowship Examination, and shall not consist solely of data of an historical nature. Candidates electing this alternative should communicate with the Secretary-Treasurer and obtain through him approval by the Committee on Papers of the subject of the thesis and also of the thesis. In communicating with the Secretary-Treasurer, the candidate should state, in addition to the subject of the thesis, the main divisions of the subject and general method of treatment, the approximate number of words and the approximate proportion to be devoted to data of an historical nature. All theses must be in the hands of the Secretary-Treasurer before the second Tuesday in May of the year in which they are to be considered. No examination fee will be required in connection with the presentation of a thesis. All theses submitted are, if accepted, to be the property of the Society and may, with the approval of the Council, be printed in the *Proceedings*.

6. Waiver of Examination for Associate.

The examinations for Associate will be waived under Article III of the Constitution in part or in whole only in case of those candidates who meet the following qualifications and requirements:

1. PARTIAL WAIVER

In case of a candidate who, for a period of at least two years preceding date of application, has been in responsible charge of the actuarial or statistical department of a casualty insurance organization and who has passed examinations of other recognized Actuarial Societies at least

equivalent to Parts I, II and III of the Associateship examinations of this Society, the passing of such parts of the Associateship examinations of this Society will be waived upon approval of the Examination Committee.

An organization whose operations or functions are limited to Accident and Health insurance, or Life Accident and Health insurance, shall not qualify as a casualty insurance organization.

2. FULL WAIVER

(a) The candidate shall be at least thirty-five years of age.

(b) The candidate shall have at least ten years' experience in the casualty actuarial or statistical work or in a phase of casualty insurance which requires a working knowledge of actuarial or statistical procedure or in the teaching of casualty insurance principles in colleges or universities. Experience limited exclusively to the field of accident and health insurance shall not be admissible.

(c) For the two years preceding date of application, the candidate shall have been in responsible charge of the actuarial or statistical department of a casualty insurance organization or shall have occupied an executive position in connection with the phase of casualty work in which he is engaged, or, if engaged in teaching, shall have attained the status of a professor.

(d) The candidate shall have submitted a thesis approved by the Committee on Papers. Such thesis must show evidence of original research and knowledge of casualty insurance and shall not consist of data of an historical nature.

Candidates electing this alternative should communicate with the Secretary-Treasurer and obtain through him approval by the Committee on Papers of the subject of the thesis. In communicating with the Secretary-Treasurer, the candidate should state, in addition to the subject of the thesis, the main divisions of the subject and general method of treatment, the approximate number of words and the approximate proportion to be devoted to data of an historical nature.

LIBRARY

The Society's library contains all of the references listed in the Recommendations for Study, including the books noted as being out of print with the exception of certain periodicals and publications subject to periodical revision. It also contains numerous other works on casualty actuarial matters. Registered students may have access to the library by receiving from the Society's Secretary-Treasurer the necessary credentials. Books may be withdrawn from the library for a period of two weeks upon payment of a small service fee and necessary postage.

The library is in the immediate charge of Miss Mabel B. Swerig, Librarian of the Insurance Society of New York, 107 William Street, New York 38, N. Y.