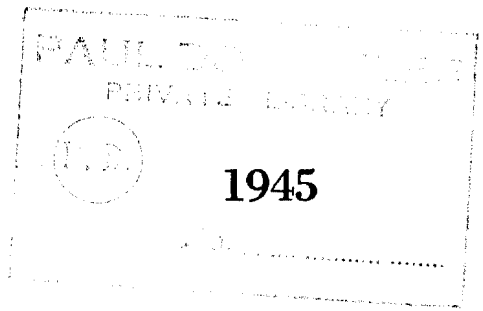
PROCEEDINGS

OF THE

Casualty Actuarial Society

ORGANIZED 1914



Volume XXXII
Number 62—November 16, 1945
1946 Year Book

Printed for the Society by
GLOBE PRINTING COMPANY, Inc.
161 MAIDEN LANE
NEW YORK 7, N. Y.

CONTENTS OF VOLUME XXXII

NOTICE

The Society is not responsible for statements made or opinions expressed in the articles, criticisms and discussions published in these *Proceedings*.

.....“all these things, which thou seest, change immediately and will no longer be; and constantly bear in mind how many of these changes thou hast already witnessed. The universe is transformation: life is opinion.”—*Marcus Aurelius.*

VOL. XXXII No. 62

PROCEEDINGS

NOVEMBER 16, 1945

DEVELOPMENTS UNDER THE NEW LEGAL ENVIRONMENT OF INSURANCE

PRESIDENTIAL ADDRESS BY HAROLD J. GINSBURGH

The march of events during the past year and a half has proceeded in the business of insurance with a rapidity of tempo, with a depth and breadth of significance far exceeding anything experienced in the previous history of the business. The intensity of pressure behind these events still continues. The reconversion from a wartime to a peacetime basis has been and continues to be a tremendous problem in itself, as has been well realized recently by those concerned with automobile insurance underwriting. For the insurance business, however, the reconversion problem has become by force of circumstances not the major problem, but one which must be handled in stride, so to speak, while dealing with the far reaching effects of the fundamental change which has taken place in the legal and political bases of the business. The developments which have occurred thus far as a result of that change are doubtless familiar to many of you. Some of you have had a part in them. However, it seemed to me desirable to recapitulate briefly for the Society the course of events during the past eighteen months, so that we may appraise their significance for the future. We should also, I believe, attempt to arrive at a better understanding of the obligations of the Society to the institution of casualty insurance under the conditions which now prevail and which will exist in the future.

First, let us consider the background to the developments of the more recent past. Paul vs. Virginia, decided in 1869, was a landmark in the insurance business. It established the pattern under which the business grew, was administered and was regulated during the succeeding three-quarters of a century. The decision of the Supreme Court in Paul vs. Virginia was that insurance was not commerce. Therefore the authority of Congress under the Constitution to regulate commerce between the states did not run to the business of insurance, which was left to the sole authority of the several states to regulate. The system of state supervision and regulation which was developed over the ensuing seventy-five years does not require description here. It molded the entire conduct of the insurance business. Its funda-

mental bases suffered no shock until another Supreme Court decision in 1944 completely reversed the situation with respect to the relationship between insurance and Federal authority.

It is true that while regulation of insurance was in its formative period, several proposals for federal regulation of insurance were made in Congress, but the latest of these was made some forty years ago. The Temporary National Economic Committee conducted a study of life insurance in 1938, but did not go into rating questions. This study resulted in twelve recommendations dealing with the conduct of the business and its supervision by the states. In 1942 the Patman Small Business Committee concerned itself with rating questions in the fire insurance field. It discussed monopoly in the industry and discrimination between large and small risks.

Not long after the studies of the Small Business Committee, the Federal Department of Justice was reported to have begun a study of the fire and casualty business, particularly the former, to determine whether there was reason for an anti-trust investigation. On November 20, 1942 a Federal Grand Jury in Atlanta handed up an indictment on two counts charging the South Eastern Underwriters Association, twenty-seven of its officers and one hundred and ninety-eight capital stock fire insurance companies with conspiracy to violate the Sherman Anti-Trust Act. The defendants' demurrer was sustained by the Federal District Court, which dismissed the indictment in August 1943. The Department of Justice took an appeal to the United States Supreme Court. The attorneys general of more than thirty states filed briefs in the Supreme Court favoring upholding the District Court decision. On June 5 the Supreme Court handed down its decision reversing the District Court, denying the demurrer and thus reinstating the indictment. It might be said here that a subsequent petition for rehearing was denied by the Supreme Court. It should be mentioned also that the Polish National Alliance case, decided at the same time as the South Eastern Underwriters Association case, involved a similar extension of Federal authority; it will not be referred to further.

Whatever might happen to the indictment itself, which now became arguable on its merits, the Supreme Court decision of June 5, 1944 on the demurrer in the South Eastern Underwriters Association case reversed the decision in Paul vs. Virginia, of seventy-five years standing, by holding that insurance is commerce and, consequently, is in its interstate aspects subject to the authority of Congress and to the application of all Federal laws relating to commerce. The Federal laws most important for their potential impact on the business of insurance were and are the Sherman Act, the Clayton Act, the Federal Trade Commission Act, and the Robinson-Patman Act. These four acts are generally known as the anti-trust laws. The

immediate question was how to reconcile with the new conditions the practices and organization of the insurance business, developed out of the inherent nature of the institution itself and the politico-legal environment of the past seventy-five years. Here was a great industry, a vital part of the economic structure of the country, charged with a high public interest, called upon to consider a complete and sudden change in the legal reference points of its conduct. It was probably the gravest problem the industry, including those responsible for its administration and those responsible for its supervision in the public interest, had ever been called upon to face.

The first and obvious question in the casualty and fire fields arose with respect to rate-making. In these fields many bureaus had been formed, composed in each case of a number of insurance carriers. Their purpose generally had been the collection and combination of statistics from their members, the making of rates based on such combined statistics, and the administration of these rates for use by their members. Some of these bureaus had been formed under mandate of specific state statutes, some had been formed under direction of state supervisory officials without specific statutory authority, and some were entirely voluntary in character. Now most of these organizations were suspect and might well be held to be in violation of the Sherman Act, since without doubt they constituted private combinations to fix prices, a type of combination which had been held illegal by the Supreme Court. Yet the necessity for pooling experience and for making rates based thereon in concert is inherent in the nature of insurance. It is not surprising, therefore, that the impact of the Sherman Act became the first consideration of the industry, and that proposals were made to obtain from Congress complete exemption for insurance from the application of the Sherman and Clayton Acts.

Even before the South Eastern Underwriters Association Case decision was handed down by the Supreme Court, legislation had been introduced in Congress to provide that the Sherman and Clayton Acts should not apply to the insurance business. On September 20, 1943 identical bills to this effect were introduced in the Senate and the House—Senate Bill 1362 by Senators Bailey and Van Nuys, House Bills 3269 and 3270 by Representatives Hancock and Walter respectively. Another bill, the Anderson Bill, introduced in the House in March 1944, provided similar exemption from the Sherman and Clayton Acts provided that rates made in concert were expressly approved by state officials pursuant to state law, before use. The substantive portion of the Walter Bill was actually passed by the House of Representatives on June 23, 1944, but nothing further was done with the companion bill in the Senate (the Bailey Bill) for meanwhile other factors had come into play.

After the June 5, 1944 Supreme Court decision, further consideration con-

vinced many in the industry and among the insurance commissioners that the legislation then before Congress was neither sufficient for a solution, albeit temporary, to the problem nor desirable even as far as it went. In the first place, it began to be realized that the Robinson-Patman Act and the Federal Trade Commission Act, both of potential importance in their impact on the insurance business, were not touched upon in the bills before Congress. Furthermore, to ask for a complete and unqualified exemption from the Sherman and Clayton Acts for insurance and thus to single it out as an industry beyond the reach of anti-trust regulation might place the industry in an unfortunate position before the public. Moreover, while combination for ratemaking must, in the public interest, somehow be provided for, certain practices such as coercion and boycott must, in the public interest, be denied exemption from application of the anti-trust laws. The industry generally had declared itself in favor of the continuation of state regulation of insurance and upon this basis moved to cooperate with the National Association of Insurance Commissioners in seeking from Congress means to accomplish this end, as well as a temporary suspension of the application of Federal laws to insurance while a satisfactory accommodation of state and Federal authority could be worked out.

During the latter half of 1944 and the early part of 1945, committees of the industry worked with committees of the National Association of Insurance Commissioners in the formulation of suggestions which might serve as the basis for Congressional legislation. The Commissioners' Committee developed a proposal for Federal legislation designed in general to retain for the states the regulation of insurance, to permit combination for ratemaking under supervision of the states, and to suspend for a period of about three years the application of the Federal anti-trust laws to the business of insurance, in order to give the states time to work out a pattern of complete regulation. This was a period of much discussion and considerable disagreement as to the form Federal legislation should take. Some still believed that the best answer lay in complete exemption from all the Federal anti-trust laws, thus leaving to the states the determination of the character and extent of regulation to be imposed on insurance. Others believed the time too early for decision on the various questions involved, and that therefore all that should be done immediately was to suspend application of the anti-trust laws for a limited period to give time for more extended consideration of the problem and adjustment to the new conditions. Still others favored a combination of these two approaches, with eventual qualified exemption from those phases of the anti-trust laws which did not accord with the nature of insurance. Congress finally passed legislation which upon signature March 9, 1945 became Public Law 15 of the Seventy-ninth Congress.

This law provides the Federal legal framework within which the insurance industry is now operating. Until amended or replaced by Congress it must be the reference point for existing and future state legislation. Because of its importance, and since it is not long, for the sake of those not familiar with its terms, I will give here the first three sections of the act (Section 4 leaves the Federal labor laws and the Merchant Marine Act of 1920 fully applicable to the business of insurance, Section 5 merely includes the territories and the District of Columbia within the term "state," and Section 6 is a separability clause):

"AN ACT

To express the intent of the Congress with reference to the regulation of the business of insurance.

"Be it enacted by the Senate and House of Representatives of the United States of America in Congress assembled, That the Congress hereby declares that the continued regulation and taxation by the several States of the business of insurance is in the public interest, and that silence on the part of the Congress shall not be construed to impose any barrier to the regulation or taxation of such business by the several States.

"Sec. 2(a). The business of insurance, and every person engaged therein, shall be subject to the laws of the several States which relate to the regulation or taxation of such business.

"(b). No Act of Congress shall be construed to invalidate, impair, or supersede any law enacted by any State for the purpose of regulating the business of insurance, or which imposes a fee or tax upon such business, unless such Act specifically relates to the business of insurance: Provided, That after January 1, 1948, the Act of July 2, 1890, as amended, known as the Sherman Act, and the Act of October 15, 1914, as amended, known as the Clayton Act, and the Act of September 26, 1914, known as the Federal Trade Commission Act, as amended, shall be applicable to the business of insurance to the extent that such business is not regulated by State law.

"Sec. 3(a). Until January 1, 1948, the Act of July 2, 1890, as amended, known as the Sherman Act, and the Act of October 15, 1914, as amended, known as the Clayton Act, and the Act of September 26, 1914, known as the Federal Trade Commission Act, as amended, and the Act of June 19, 1936, known as the Robinson-Patman Anti-Discrimination Act, shall not apply to the business of insurance or to acts in the conduct thereof.

"(b). Nothing contained in this Act shall render the said Sherman Act inapplicable to any agreement to boycott, coerce, or intimidate, or act of boycott, coercion, or intimidation."

The first section of the act declares the policy of Congress to be the continued regulation and taxation of the business of insurance by the several

states, and Section 2(a) provides that the business shall be subject to the laws of the states. Section 3(a) grants a moratorium until January 1, 1948 on the application of the four anti-trust acts to insurance. Section 3(b) makes an exception both to the moratorium provision and to the provision next to be discussed, by making the Sherman Act applicable at all times, now and in the future, to boycott, coercion and intimidation. Section 2(b) provides that after January 1, 1948 the Sherman Act, the Clayton Act and the Federal Trade Commission Act "shall be applicable to the business of insurance to the extent that such business is not regulated by state law."

Section 2(b), just referred to, is undoubtedly the most important section of Public Law 15. Its provisions and what is done under them may well determine whether the business of insurance will operate in the future primarily under state or Federal regulation, and also what the character and extent of regulation will be. There has been considerable uncertainty concerning the meaning and intent of Section 2(b). For example, the Robinson-Patman Anti-Discrimination Act is not specifically mentioned in this section as being applicable after January 1, 1948, but it is specifically mentioned in the moratorium section. It is now generally agreed that this act, more important in its effect upon insurance than many perhaps originally supposed, will become applicable after January 1, 1948. Probably the most significant of the points considered in connection with the intent of Section 2(b) is the meaning of the phrase "to the extent that such business is not regulated by state law." How much and what kind of state regulation will be necessary in order to make any or all of the Federal anti-trust laws inapplicable? Must state laws and administrative procedures under them cover the entire field embraced within the purview of the Federal laws in order to "oust" the Federal laws? Is this the meaning of "to the extent?" Will the mere existence of a state law covering a given situation be sufficient, regardless of the degree and character of enforcement? Opinions on these matters have differed and still do, but it seems to me that in order for the Federal laws to be inoperative in a state with respect to a given activity or situation in insurance, there must be in existence in the state a law dealing in positive fashion with the activity or situation, and that such law must be actively administered and enforced. It does not seem probable that mere blanket permission by state law to do, without active supervision, things forbidden by Federal law would serve to make the Federal law inapplicable. A laissez faire policy in the states, either legislatively or administratively will not serve, under Public Law 15, to maintain the primacy of state regulation over Federal regulation of insurance. Statements made by the Department of Justice, by members of Congress, and by the President of the United States, while the insurance problem has been under consideration in Washing-

ton, all indicate that the Federal government is presently willing to leave the regulation of the business of insurance to the states provided they regulate in fact. Otherwise it seems almost without question that Congress will step in once more and set up a Federal regulatory system. And the position of the industry in favoring continuation of regulation by the states as opposed to Federal regulation has been due not to fear of Federal regulation as such, but rather to realization that Federal regulation would be superimposed upon the already existing and probably to be extended system of state regulation. The burdens of such a dual system are obvious.

It was in the light of considerations such as those just discussed that the industry and the state insurance commissioners set about immediately after the passage of Public Law 15 to determine a course of action to be followed in preparation for the end of the moratorium period on January 1, 1948. Several courses could be taken with respect to each of the four anti-trust laws, depending upon the nature of the activity or condition affected. State regulatory legislation could be proposed, specifically dealing with matters covered by one of the Federal laws, in order to make that law inoperative, in whole or in part, in the state. If specific legislation in all of the several states appeared impracticable, modification of the Federal law might be sought in its application to insurance. Lastly, any of the anti-trust laws might be left fully applicable to the business of insurance after January 1, 1948.

In many respects the Sherman Act seemed the simplest to deal with of the four anti-trust laws. Passage of rate-regulatory laws in all the several states would serve to lift the price-fixing prohibitions of the Sherman Act from co-operative ratemaking, an activity of vital concern, in all its ramifications, to the institution of insurance and yet most vulnerable to attack under the Act. However, consideration of the extension of state rate regulation brought developments so complex and so highly controversial that I will pass the question by for the moment for later discussion. At this point we can review very briefly the problems related to the other three anti-trust laws. As yet there has been no crystallization of opinion on what to do with respect to any of these laws—whether to seek state regulatory legislation to replace the Federal acts, or to seek modification of the Federal acts, or to do nothing.

The features of the Clayton Act particularly affecting the insurance business are the provisions prohibiting acquisition of the stock of one corporation by another corporation under certain circumstances, and the provisions prohibiting interlocking directorates under certain circumstances. These provisions might affect the operation of company fleets, particularly in the fire field.

As was indicated earlier in this discussion, it appears probable that under

Public Law 15, the Robinson-Patman Anti-Discrimination Act will apply to the business of insurance. This Act is directed at the individual seller and therefore its price discrimination sections will affect the making and charging of rates by an individual insurer. The element of combination to fix rates need not enter into the picture. This feature of the Robinson-Patman Act may well prove to be of major significance in the problem of rate regulation. Another important feature of the Act is the section which deals with brokerage. Under it, unless modified by Federal or state enactment, there might be grave question of the legality of the entire insurance brokerage system.

The primary purpose of the Federal Trade Commission Act was to provide a means for checking unfair trade practices before they could reach the scope of a monopoly or restraint of trade covered by the Sherman Act. There are three general categories into which the provisions of the Federal Trade Commission Act fall—the prevention of unfair methods and practices, the general investigatory powers of the Federal Trade Commission, and the power of the Commission to enforce certain of the other anti-trust laws. It is perhaps more difficult to relate the Federal Trade Commission Act to specific phases of the insurance business than is the case with the other anti-trust laws. The existence of the examination powers of the states indicates the possibility of overlapping investigatory powers. There already are many state laws dealing with rebating, unfair advertising, and other unfair practices in insurance, but such laws certainly do not cover the broad area embraced within the scope of the Federal Trade Commission Act. To pass additional state laws probably would result in additional overlap rather than in making the Federal law inapplicable under Public Law 15. As with the Clayton and Robinson-Patman Acts, there is as yet no generally settled view in the industry on what should be done before January 1, 1948 concerning the Federal Trade Commission Act.

So much, then, for the anti-trust laws other than the Sherman Act, though the brevity of review should not be taken to belittle the problems involved. We can return now to consideration of the impact of the Sherman Act upon the business of insurance, to what has already flowed therefrom and to what is likely to result in the future. The provisions of the Sherman Act relating to combinations fall with peculiar force on the business of insurance, in which co-operative efforts have long been an integral part of the conduct of the business, due to the inherent characteristics of the institution of insurance. The co-operative organizations have various purposes, some of which would probably not be affected by the Sherman Act; but the vitally important activity of co-operative ratemaking would almost certainly be affected. It appeared, however, in the light of certain previous Supreme

Court decisions on price fixing and of statements made by the Attorney General of the United States, that ratemaking in concert might be taken out from under the prohibitions of the Sherman Act if the making and use of the rates were properly regulated under state law—more specifically, were subject to approval under state law.

Even before the Supreme Court decision of June 5, 1944, some states had begun to consider the desirability of enacting insurance rate regulatory laws, in order to be ready for whatever might come out of that decision. New Jersey adopted a fire and casualty rating law in March 1944. After the decision was handed down, the question of state rate regulation assumed increasing importance, particularly in view of the discussions on Congressional legislative proposals to which I have referred previously. It was a practical certainty that regulation of insurance rates, particularly in the fire and casualty fields, would come under consideration in the states holding legislative sessions in 1945, about forty in number. Finally, the passage of Public Law 15, with its Section 2(b), gave added impetus to the movement toward state rate regulation as a means of making the Sherman Act (and probably the Robinson-Patman Act) inapplicable to a business to whose needs in rate making the Act is poorly adapted. While some elements in the industry were still of the opinion that rate regulation generally in the fire and casualty fields was neither necessary nor desirable, the paramount considerations became the nature and the extent of such rate regulation as was to be imposed. In the following discussion I will confine myself to developments in the casualty field.

During the latter part of 1944, recognizing the trend of events, the Association of Casualty and Surety Executives, for its member stock casualty companies, and the National Association of Mutual Casualty Companies, for its member mutual casualty companies, each independently initiated a study of the rate regulatory problem with a view to developing a draft of a bill which might be offered as a suggested basis wherever rate regulatory legislation was to be considered. Both organizations carried through independently of each other to the production of a draft of a rating law. When this point had been reached, there was a growing feeling in both groups that for a problem of such fundamental importance joint rather than separate consideration was desirable. It was decided therefore that the committees of the two organizations would meet jointly to draft a casualty and surety rating bill. Members of the joint committees represented both companies which operated as members of rating organizations and those which did not so operate. I believe that it can and should be said that the members of the committees approached their task with a deep feeling of sincerity and obligation and with the purpose of formulating a rate regulatory law under which the

interests not of one class or type of carriers but of all carriers would be protected and the public interest served, and under which the business of casualty insurance could continue to be carried on efficiently by private enterprise as a business imbued with public interest, under public supervision and in the public welfare.

The work of the joint committees resulted in a draft of a casualty and surety rating bill which was widely distributed. This was done not with the idea that the two organizations which had co-operated in the preparation of the bill were or would be sponsoring rate regulatory legislation, but with the purpose that wherever such regulatory legislation was considered the draft would serve as the pattern for a proper rating law. In most of the ten states which adopted casualty rating laws in 1945, the provisions of the joint bill were followed in whole or in part. This is not surprising in view of the background of the bill as described in the following paragraph of the memorandum which accompanied it:

> "The bill is the result of careful and extended study of the effect of the Supreme Court decision in the South Eastern Underwriters Association case, the application of existing Federal statutes to insurance, the delicate balance of state and Federal power to control interstate commerce, proper principles which should govern state regulation of the cost of insurance, requirements of many types of casualty insurance, and variations in practices of several types of insurance carriers. The conclusions reached after such study have been incorporated in this bill."

The provisions of the joint casualty and surety rating bill are undoubtedly familiar to most of you. The bill is directed at the individual carrier upon whom the primary obligation is placed to file rates which are made in accordance with stated standards. Rating organizations are recognized and regulated. Individual carriers are allowed to fulfill their filing obligations through rating organizations of which they are members and subscribers, but no carrier is compelled to become a member or subscriber of a rating organization. The public interest is protected by requiring approval of rates before use, but the carriers are protected against undue delay in consideration of rate filings by a provision deeming them approved if not acted upon within thirty days. The needs of various types of carriers arising out of their operating methods are recognized by the provision giving the right to a carrier or group of carriers to differ from other carriers or groups in the expense loadings in rates. There is nothing in the bill requiring rate uniformity. The section on expense loadings would certainly indicate that, and it is otherwise made clear. The interests of an individual carrier in a rating bureau or of a minority group therein are protected by a deviation section and by a section giving right of appeal to the supervisory authority by a member or subscriber

of a rating bureau. The effectiveness of rate regulation under the joint bill would depend very largely, as it would under any other regulatory law, on the kind of administration given the law. Opposition to the joint bill has been due in many cases, I believe, to lack of understanding of its provisions, though in some cases it may be due to a reluctance to give opportunity for the kind of administration which could produce effective regulation.

Some carriers, particularly those who do not belong to rating bureaus, have opposed rate regulation of the type embodied in the joint bill. They maintain that, since the Sherman Act applies only to rates made in concert, state rate regulation should apply only to such rates, and that companies which file their rates and operate independently do not need protection from the Sherman Act and consequently should not be subject to rate regulation. Technically, if all that is required is an "umbrella" under the Sherman Act, they may be right, although the Robinson-Patman Anti-Discrimination Act, which is not concerned with combinations, is also in the picture. But to me this seems a very short sighted view. For the ultimate purpose of rate regulation is not to provide an "umbrella" under the Sherman Act and not to control competition between companies, but to protect the public interest. If a rating law is to apply only to bureau companies and not to others, it is obvious that the first group will soon seek the status of the second by ceasing to belong to bureaus; and thus the rating law will become meaningless.

Whatever one may think of rate regulation, it is fairly obvious that there is going to be considerable legislation concerning it within the next few years. State rate regulation presently appears to be the best remedy to correct the "unwisdom" of complete application of the Sherman Act to the business of insurance. The form which rate regulatory laws should take is still under study by the industry and by the National Association of Insurance Commissioners. Before the next series of legislative sessions in the states, new drafts and suggestions will undoubtedly be offered.

I have now completed my summary of events and developments accompanying the ushering in of a new era in the insurance business with the Supreme Court decision of June 5, 1944. We are in a period of transition. By the passage of Public Law 15 the industry and the states have been given time to work out an accommodation of state and Federal authority under which the institution of insurance can continue to perform its function efficiently. One result which will almost certainly come within the next few years is a wide extension of rate regulation by the states.

In what I have said before there is the implication, if not the direct statement, that competent and effective regulation of rates requires competent and enlightened administration of rating laws by supervisory authorities. Such administration must have behind it competent staffs including those with

actuarial training. State insurance departments, called upon to administer rating laws to a tremendously greater extent than ever before, will be looking to build up their staffs. They will be looking for men of sound actuarial training. Beyond all doubt it will be to the interest of the insurance business that they have such men. It may well be, also, that the carriers, operating under a greatly extended system of rate regulation, will have an increased need for actuarially trained staffs. It is the obligation of this Society that, so far as it lies within its power in the casualty field, the demand is met as it should be met. It is our duty to foster the training of competent casualty actuaries. It is our duty as a Society and as individuals to encourage the study of casualty actuarial science and to promote recognition in the industry of the importance of such study. We must at the same time maintain such proper standards of admission to the Society that membership in it will be a hallmark. By doing this the Society will be of service both to the public supervisors of insurance and to the industry.

In a previous address, which is printed in the *Proceedings* of our war proscribed meeting of last November, I referred to the peculiar responsibilities of our members in the formulation of rates and rating systems under the conditions of these times. Now I wish to emphasize the broader obligations of the Society as an institution. There is a task before us.

A GENERALIZED THEORY OF CREDIBILITY

BY

ARTHUR L. BAILEY

INTRODUCTION

Casualty insurance actuaries and statisticians have used credibility factors for many years. They have been satisfied that the use of such a consistently conservative procedure produces much more practical results than any attempt to follow one hundred percent the indications of a limited volume of their statistics. The writer will attempt to show that they have been justified in their pragmatical outlook.

Let it be necessary to obtain estimates of the population means, X_i, of N independent characteristics, for each of which K observations have been made. Let it be assumed that the population variances of all N characteristics are identical but unknown. Let the j-th observation of the i-th characteristic be z_{ij}, and let its departure from the i-th population mean be y_{ij}. Then $z_{ij} = X_i + y_{ij}$ and the unknown population variance of y_{ij} is σ^2_y for all of the N populations.

The non-insurance statistician seeks the "best unbiased estimates" and imposes as the conditions to determine his estimates of the values of X_i:

(1) that the estimate *for each characteristic* be unbiased, and
(2) that the error-variance of the estimates be a minimum *for each characteristic*.

This gives him as his estimate of X_i the average of the K observations of z_{ij} for the particular value of i, or z_i. These estimates have an error-variance of σ^2_y/K for each characteristic.

The insurance statistician seeks the "best unbiased *set* of estimates" and imposes as the conditions to determine his estimates:

(1′) that the average estimate *for all characteristics as a whole* (i.e. the rate level) be unbiased and
(2′) that the error-variance of the estimates be a minimum *for all characteristics combined.*

It will be shown that this gives him as his estimate of X_i the average of the K observations of z_{ij}, for the particular value of i, multiplied by a credibility of C, plus the average of the KN observations of z_{ij}, for all values of i, multiplied by the complement of the credibility, or:

$$\text{Estimated } X_i = Cz_i + (1 - C)\,\bar{z}.$$

These estimates of X_i are not unbiased for each characteristic but the aver-

age for all characteristics combined is unbiased. The value of C, as will be shown later, must be taken as equal to $\frac{K}{K+\frac{\sigma^2_y}{\sigma^2_X}}$ in order to minimize the error-variance of the estimates, and this error-variance becomes $C\sigma^2_y/K$ for all characteristics combined.

There is a further refinement of the above simple case which the insurance statistician finds it necessary to recognize which other statisticians usually neglect, namely, the number of units of exposure represented by an observation. The observations in insurance statistics are of necessity made on risks varying in size of exposure. In other statistical fields the measurements may often be limited to individuals of the same physical magnitude, although not invariably. For example, an agricultural experiment may be laid out to cover plots of a uniform size, say 50 x 50 feet. The results of such an experiment may, however, be indiscriminately compared with a similar experiment involving plots 10 x 10 feet; a comparison which may be quite improper in several respects. In the following more general development proper recognition has been given to variations in the number of units of exposure by letting y_{ij} be the sum of m independent values of u_{ijh} corresponding to the m units of exposure involved. In actual application, each y_{ij} may have a different value of m; but, as m will appear in the formulae to be developed, it will only be necessary to use the proper value of m for the case to which the formulae are applied.

In the following development of the most general case the characteristic X_i will be represented by amx_i, where m represents the number of units of exposure in each observation, a the average value of X_i per unit of exposure for all values of i or $\bar{X}/m$, and x_i the ratio of X_i to $\bar{X}$. In this way the effect of each component of the total variation in X_i will be separately considered and the treatment of special cases will be facilitated.

The Problem and Proposed Solution

Let it be desired to obtain a set of estimates of an element of variation under conditions such that it is impossible to observe that element alone. The procedure presented herein consists, where ax_i represent the elements of variation to be estimated, and where z_i represent the mean values of k observations each covering m units of time or space (i.e. exposure) of the variable z_{ij}, which is the sum of amx_i and y_{ij}, of using as the set of estimates of ax_i the values obtained from the linear regression equation of ax_i on z_i. The required regression coefficient, C, may be evaluated from an analysis of the variance of z_{ij} and, in the insurance business, where several special cases

of the general theory occur, is referred to as the "credibility" of the observation, z_i.

The General Case

Let it be assumed that:

(a) $z_{ij} = amx_i + y_{ij}$;

(b) $y_{ij} = \Sigma\, u_{ijh}$ for values of h from 1 to m;

(c) x_i has a mean of unity and a variance of $\sigma^2{}_x$;

(d) u_{ijh} has a mean of zero and a variance of $\sigma^2{}_{u_i}$ for all values of j;

(e) $\sigma^2{}_{u_i}$ has a mean of $\sigma^2{}_u$ for all values of i;

(f) the m values of u_{ij} are random and independent;

(g) z_i is the average of k random and independent values of z_{ij}.

(h) Except that u_{ijh} and x_i are finite real numbers, no restrictions are placed on their distribution. (In most applications x_i will assume only discrete values).

Under these assumptions it can be shown that:*

(1) $\bar{y} = E(y_i) = 0$, $\bar{z} = am$, and $E(x_i\, y_i) = 0$;

(2) $$\sigma^2{}_z = E(z_i - \bar{z})^2 = E[am(x_i - \bar{x}) + (y_i - \bar{y})]^2$$
$$= a^2 m^2 \sigma^2{}_x + \frac{\sigma^2{}_y}{k} + 2am[E(x_i\, y_i) - \bar{y}E(x_i) - \bar{x}E(y_i) + \bar{x}\bar{y}]$$
$$= a^2 m^2 \sigma^2{}_x + \frac{m\,\sigma^2{}_u}{k};$$

(3) $$U_{xz} = E(x_i - \bar{x})(z_i - \bar{z}) = E(x_i - \bar{x})[am(x_i - \bar{x}) + (y_i - \bar{y})]$$
$$= amE(x_i - \bar{x})^2 + E(x_i\, y_i) - \bar{y}E(x_i) - \bar{x}E(y_i) + \bar{x}\bar{y}$$
$$= am\sigma^2{}_x.$$

It will be seen from (2) that, if z_i/m is used as an estimate of ax_i, the variance of such an estimate will exceed the variance of ax_i by $\sigma^2{}_u/km$. The best unbiased set of estimates of ax_i will be the estimates obtained from the regression of ax_i on z_i/m. Such a regression will be linear if $\sigma^2{}_{u_i}$ is constant for all values of i; otherwise the form of the regression will depend on the relationship between $\sigma^2{}_{u_i}$ and ax_i and on the form of the distribution of x_i.

The best unbiased set of *linear* estimates of ax_i will be the estimates obtained from the equation of linear regression of ax_i on z_i/m, irrespective of the form of the true regression. This linear regression equation, using x'_i to indicate the value of x_i obtained from the regression equation, is:

* E is used to indicate the expected value of that which follows.

$$ax'_i = \frac{amU_{xz}}{\sigma^2_z} \cdot \frac{z_i}{m} + a - \frac{amU_{xz}}{\sigma^2_z} \cdot \frac{\bar{z}}{m} \text{ or, as } a = \frac{\bar{z}}{m} \text{ from (1)}$$

$$ax'_i = C\,\frac{z_i}{m} + (1-C)\,\frac{\bar{z}}{m} \tag{4}$$

where, from (2) and (3):

$$C = \frac{amU_{xz}}{\sigma^2_z} = \frac{a^2 m^2 \sigma^2_x}{a^2 m^2 \sigma^2_x + \frac{m\sigma^2_u}{k}} = \frac{km}{km + \frac{\sigma^2_u}{a^2 \sigma^2_x}}. \tag{5}$$

When z_i/m is used as an estimate of ax_i the error of the estimate, $\xi_i = ax_i - z_i/m$, will have a variance, for all values of i, of:

$$\begin{aligned} \sigma^2_\xi &= a^2 \sigma^2_x + \frac{\sigma^2_z}{m^2} - \frac{2a}{m} U_{xz} \\ &= a^2 \sigma^2_x + \left[a^2 \sigma^2_x + \frac{\sigma^2_u}{mk} \right] - 2a^2 \sigma^2_x \\ &= \frac{\sigma^2_u}{mk}. \end{aligned} \tag{6}$$

When $ax'_i = C\frac{z_i}{m} + (1-C)\frac{\bar{z}}{m}$ is used as an estimate of ax_i the error of the estimate, $\xi'_i = ax_i - C\,\frac{z_i}{m} - (1-C)\frac{\bar{z}}{m}$, will have a variance for all values of i, of:

$$\begin{aligned} \sigma^2_{\xi'} &= a^2 \sigma^2_x + \frac{C^2}{m^2} \sigma^2_z - 2\frac{Ca}{m} U_{xz} \\ &= a^2 \sigma^2_x + C^2 \left(a^2 \sigma^2_x + \frac{\sigma^2_u}{mk} \right) - 2Ca^2 \sigma^2_x \\ &= a^2 \sigma^2_x (1-C)^2 + C^2 \frac{\sigma^2_u}{mk} \\ &= C\,\frac{\sigma^2_u}{mk}, \end{aligned} \tag{7}$$

$$\text{as } a^2 \sigma^2_x = \frac{C}{1-C} \cdot \frac{\sigma^2_u}{mk} \text{ from (5).}$$

It will be noted that $\sigma^2_{\xi'}$ in (7) is less than σ^2_ξ in (6) except in the limiting case when $C = 1$.

An analysis of variance calculated* from K observations of z_{ij}, for which $m = M$, in each of N classifications of i, produces a variance within classification, W, and a variance between classifications, B, for which the expected values are:

* The capitals K, M, and N are used to represent the values of k, m, and n in data used to evaluate C in order to maintain a distinction between the values in such data and the values in data to which C is to be applied.

$$(8) \qquad E(W) = \frac{K}{K-1} E(z_{ij} - z_i)^2 = \frac{K}{K-1} E(y_{ij} - y_i)^2$$
$$= E(\sigma^2_{y_i}) = M E(\sigma^2_{u_i}) = M \sigma^2_u;$$

$$(9) \qquad E(B) = \frac{NK}{N-1} E(z_i - \bar{z})^2 = \frac{NK}{N-1} E[aM(x_i - \bar{x}) + (y_i - \bar{y})]^2$$
$$= \frac{NK}{N-1} [a^2 M^2 E(x_i - \bar{x})^2 + E(y_i - \bar{y})^2$$
$$+ 2aM \{E(x_i y_i) - \bar{y} E(x_i) - \bar{x} E(y_i) + \bar{x}\bar{y}\}]$$
$$= Ka^2 M^2 \sigma^2_x + M \sigma^2_u;$$

and $\frac{MK E(W)}{E(B) - E(W)} = \frac{\sigma^2_u}{a^2 \sigma^2_x}$, so that C may be evaluated by using in (5) the value:

$$(10) \qquad \frac{\sigma^2_u}{a^2 \sigma^2_x} = \frac{MKW}{B - W}.$$

It is interesting to note that, when $k = K$ and $m = M$, as is the case when C is to be applied to the same data from which it is evaluated, the calculated value of C reduces to:

$$(11) \qquad C = 1 - \frac{W}{B},$$

and that, when $k = 1$, C is the intraclass correlation coefficient, and when $m = M$, the calculated value of C becomes the value given by R. A. Fisher* as the unbiased estimate of the intraclass correlation coefficient:

$$(12) \qquad C = \frac{B - W}{B + (K-1) W} = \rho.$$

When K_i varies for different values of i in the actual observations used for the analysis of variance, it is possible to calculate W and B, where $\overline{K}$ represents the average value of K_i, as follows:

$$(13) \qquad W = \frac{\Sigma \frac{K_i}{K_i - 1} (z_{ij} - z_i)^2}{\overline{K} N} \qquad B = \frac{\Sigma K_i (z_i - \bar{z})^2}{\overline{K}(N-1)},$$

and to evaluate C from (10) using $\overline{K}$ in place of K.

It should be noted that no assumption is made in the general case as to the form of the distributions of u_i and x_i except that all values be finite real numbers. When the form of either distribution or the variance of either distribution is known or assumed, special cases arise in many of which more efficient estimates of $\sigma^2_u / a^2 \sigma^2_x$ can be obtained by methods other than the analysis of variance.

* In Section 40 of "Statistical Methods for Research Workers."

SPECIAL CASES

The simplest special case occurs when $\sigma^2_{u_i}$ is assumed to be constant for all values of i. An interesting special case is met in life insurance statistics, where $amx_i + y_{ij}$ is assumed to be distributed according to the binomial distribution with a mean of amx_i and the variance, $\sigma^2_{u_i} = ax_i(1 - ax_i)$. With casualty insurance statistics the special case of the Poisson distribution arises for the number of losses, the special multiplicative case arises for the average amounts of losses, and a combination of these two arises in dealing with the total amount of all losses incurred. The Poisson and multiplicative special cases are outlined below in order to indicate the method of approach to other special cases.

THE POISSON CASE

The number of events, z_i, occurring in m_i units of space are observed for n values of i. It is assumed that the number of events occurring in single units of space are distributed in a Poisson distribution for each value of i but with different means and with variances equal to these means. Denoting the means by ax_i, we have the general case but with $k = 1$ and $\sigma^2_{u_i} = ax_i$, from which $\sigma^2_u = a$. Also m_i is not constant and $\bar{z}/m$ in the regression equation will have to be replaced by $\bar{z}/\bar{m}$.

The regression of ax_i *on* z_i/m_i will not be linear, but the linear regression equation will afford a very close approximation when $am_i x_i$ exceeds 10. The best unbiased set of linear estimates of ax_i, from (4) and (5), is obtained from:

$$ax'_i = C\,\frac{z_i}{m_i} + (1 - C)\,\frac{\bar{z}}{\bar{m}} \tag{14}$$

where:

$$C = \frac{m_i}{m_i + \dfrac{1}{a\,\sigma^2_x}}. \tag{15}$$

An analysis of variance is obviously impossible in this case. Even if it were possible, it would not provide the most efficient estimate of $1/a\,\sigma^2_x$, as the assumption of the Poisson distribution has completely determined $\sigma^2_{u_i}$ in terms of ax_i, thereby making other methods more efficient. If $v_i = z^2_i/m_i$, the expected values of z_i and v_i, for a specified value of m_i, are:

$$E(z_i) = am_i \qquad E(v_i) = \frac{E(z^2_i)}{m_i} = a^2 m_i(\sigma^2_x + 1) + a$$

and, for the observed values of m_i, are:

$$E(z_i) = a\bar{m}\,E(v_i) = a^2\,\bar{m}(\sigma^2_x + 1) + a,$$

so that $1/a\,\sigma^2_x$ can be evaluated from:

$$\frac{1}{a\,\sigma^2_x} = \frac{\bar{z}\bar{m}}{\bar{v}\bar{m} - \bar{z} - \bar{z}^2}. \tag{16}$$

THE MULTIPLICATIVE CASE

The yields of k_i units of variety i are observed as z_{ij} for n varieties. It is assumed that the expected yield per unit of variety i is ax_i and that the variance of the yields of individual units of a variety is proportional to the square of the expected yield and is $ba^2x^2_i$. This is referred to as the multiplicative case because, if $w_{ij} = 1 + \frac{y_{ij}}{ax_i}$, z_{ij} could be represented as: $z_{ij} = ax_i w_{ij}$, where x_i has a mean of unity and a variance of σ^2_x and w_{ij} has a mean of unity and a variance of b.

Here we have the general case with $m = 1$ and $\sigma^2_{u_i} = ba^2x^2_i$, from which $\sigma^2_u = ba^2(\sigma^2_x + 1)$. The best unbiased set of linear estimates of ax_i, from (4) and (5), is obtained from:

$$ax'_i = Cz_i + (1 - C)\bar{z}, \tag{17}$$

where:

$$C = \frac{k_i}{k_i + \frac{b(\sigma^2_x + 1)}{\sigma^2_x}}. \tag{18}$$

If an analysis of variance is made in accordance with (13), C can be evaluated by using in (18) the value:

$$\frac{b(\sigma^2_x + 1)}{\sigma^2_x} = \frac{\bar{k}W}{B - W}. \tag{19}$$

It frequently occurs in this special case that the value of b is known, in which case the evaluation of C can be made as follows. Let $t_i = k_i z_i$ and $v_i = k_i z^2_i$. For a specific value of k_i:

$$E(t_i) = ak_i \qquad E(v_i) = k_i E(z^2_i) = k_i a^2(\sigma^2_x + 1) + ba^2(\sigma^2_x + 1)$$

and, for the observed values of k_i:

$$E(t_i) = a\bar{k} \qquad E(v_i) = a^2(\sigma^2_x + 1)(\bar{k} + b),$$

so that C can be evaluated by using in (18) the value:

$$\frac{(\sigma^2_x + 1)}{\sigma^2_x} = \frac{\bar{v}\bar{k}^2}{\bar{v}\bar{k}^2 - \bar{t}^2(\bar{k} + b)}. \tag{20}$$

In this multiplicative case consideration might be given to the desirability of making the sum of the squares of the percentage errors a minimum rather than the sum of the squares of the arithmetic errors.

CONCLUSION

The conditions which underly and justify the use of credibilities in making a set of estimates from observations based on a limited amount of exposure have been presented. A review of the general case shows such a procedure to

be directly applicable in many fields of practical statistics outside of as well as in the insurance business. Methods of evaluating the proper credibilities have been presented both from an analysis of variance and, in the special applications considered, from more directly available statistics.

One warning should be given the reader. It has *not* been shown that the use of any arbitrarily selected schedule of credibilities is justified. As a matter of fact, if such an arbitrarily chosen credibility, C', is less than:

$$\frac{km - \frac{\sigma^2_u}{a^2\,\sigma^2_x}}{km + \frac{\sigma^2_u}{a^2\,\sigma^2_x}}$$

it can be shown from equation (7) that $\sigma^2{}'_\xi$, is greater than σ^2_u/mk and that the use of the arbitrary credibility has produced a greater error-variance than would have resulted from giving each observation 100% credibility. The converse of this is that, as long as km is less than $\sigma^2_u/a^2\,\sigma^2_x$, any credibility between 0 and 1 will produce a lesser error-variance than the use of 100% credibility.

It should also be mentioned that the presentation herein has been limited to cases where the estimates are to be made for the i-th variable from the observations of the i-th variable and from the observed mean of all variables. The same type of technique is applicable and the theoretical development is similar in the case where a set of estimates based on a large volume of data are to be modified to obtain a set of estimates which reflect the peculiarities of a comparatively limited volume of data for which unknown changes have occurred in the values of the variables x_i. Such techniques become necessary in the revision of insurance rates to reflect the most recent conditions affecting hazards and in the modification of insurance rates to reflect the peculiarities of individual insureds and have been developed specifically for such applications.* Corresponding procedures would appear to be proper in many other fields of statistical application where sets of estimates are desired having a minimum error variance.

---

* See "Sampling Theory in Casualty Insurance" by A. L. Bailey P.C.A.S. Vol. XXIX, page 69—The Fundamentals of Experience Rating, and Vol. XXX, page 64—The Greatest Accuracy Credibility.

NON-RANDOM ACCIDENT DISTRIBUTIONS AND THE POISSON SERIES

BY

JOHN CARLETON

In recent years several papers have appeared in the *Proceedings* in which the Poisson formula has been used to obtain the theoretical distribution of accident frequencies in casualty insurance. In casualty terminology this formula with its required conditions may be expressed somewhat as follows:

> If accidents are distributed individually and collectively at random in a large exposure, the probability of exactly n accidents occurring in a portion of that exposure: i.e., in an individual risk, is given by
>
> $$P_n = \frac{c^n e^{-c}}{n!}$$
>
> where c is the accident expectation for the portion under consideration.

The requisite of individual randomness is that the timing of each accident must be independent of the timing of each other accident.

The requisite of collective randomness is that the accident frequency in any interval within the portion must be independent of the accident frequency in any other such interval not overlapping the first interval.

The formula is applicable even if the accident producing potentialities are changed from interval to interval within the risk period, provided these changes are not made in such a way as to violate either of the two requisites of randomness. To illustrate: the formula applies to risks in which the accident producing potentialities are different during business hours than they are during other hours, but it does not apply to risks in which the occurrence of an accident modifies the accident producing potentialities for an interval immediately following such occurrence. The formula applies to risks for which the hazards are greater during a particular season, but it does not apply to risks in which a high accident frequency early in the policy period has led to effective safety engineering the results of which are operative in the balance of the policy period.

Consideration of the characteristics of casualty insurance risks might lead to the surmise that they do not meet the two requisites of randomness with perfection.

Accidents occurring in connection with defective equipment may remove the equipment from the exposure until such time as it can be repaired. A serious collision involving a particularly antiquated truck of a trucking fleet will remove, at least temporarily, that piece of equipment from the road and correspondingly modify the hazard per unit of exposure. Similarly, if there is validity in the contention that a disproportionate part of the workmen's

compensation hazard inheres in accident prone employees, disability of one of these employees modifies the hazard per unit of payroll for the period of disability.

Again in the workmen's compensation business, in almost any reasonably large risk there will be safety rules and practices, some formal, some informal. It is not unreasonable to believe that the interest in and enforcement of such rules and practices will vary from time to time. It is likely that a peak of interest and enforcement will be immediately following a spectacular accident.

In several lines of casualty insurance significant accidents of an unusual nature will prompt either the insured or the carrier to initiate efforts to prevent the recurrence of that particular type of accident. A catastrophe involving a bus line may result in a rerouting to avoid certain intersections.

In each of these illustrations an accident operates to reduce the risk hazard for a period of time following its occurrence, and, accordingly, the general requirement that accidents must be independent is not perfectly met. The questions are (1) to what extent do these and other similar imperfections actually exist, and (2) to what degree do they modify the theoretical distribution of accidents given by the Poisson formula and thus impair its use, say, in prognosticating loss ratio distributions from accident cost distributions, or in evaluating the accuracy of a rate making system. Preliminary judgment probably would be that the actual influence of accidents upon one another is very small and can be ignored in other than exceptional circumstances. However, effort to explore this problem seems justified, even if only to confirm this preliminary judgment.

The exploratory efforts made in this paper are very limited. To answer the first question would require an analysis of accident timing in actual risk experience, an analysis which would be made very difficult by a lack of precise figures for true inherent hazards of individual risks and a lack of any information on hazard changes during policy periods. Only the second question has been touched, and that only in a way to provide some assistance in forming personal judgments.

The situations cited as examples obviously do not lend themselves to very much precise mathematical calculation. It may be helpful, however, to define an idealized risk in which the requisite of individual randomness is violated in a nice uniform way and to calculate for several sets of conditions the properties of the theoretical accident frequency distributions for such risks. The variances (second moments about the mean) of these distributions can be compared with the variances of Poisson distributions for the same accident expectations. To utilize the results of these calculations, it will be necessary, of course, to subjectively appraise the lack of independence of

accidents occurring in real risks and to compare that appraisal with the conditions assumed for the idealized risks.

The idealized risks selected are assumed to have the following properties: An accident hazard of constant magnitude is operative from the start of the policy period until the occurrence of the first accident. Immediately following the first accident, and immediately following each subsequent accident, the accident hazard becomes negligible for a specific interval. At the end of each such interval the accident hazard immediately resumes its original magnitude and continues at this level until the following accident.

To give these idealized models a touch of reality, they may be considered as corresponding to that portion of a workmen's compensation risk which relates to fatal cases. Whenever a fatal accident happens, factors are brought into play which practically preclude the occurrence of another fatal accident until some time has elapsed. Obviously, in actual risks the hypothetical accident spreading factors would be neither so uniform nor so completely effective. However, they would operate in the general direction of creating a risk with properties similar to those of the mathematical models.

It might be well to mention that this paper is concerned only with the results of a lack of randomness. If the Poisson formula is used with accident expectations derived from casualty insurance rates, discrepancies between actual and theoretical results are usually observed, mainly because casualty insurance rate making does not accurately give the accident expectation of each individual risk. There are a multitude of possible reasons for this theoretical deficiency in casualty rate making, including, conceivably, the possibility that some accidents may modify the risks without correspondingly modifying the rated premiums. In such a situation the discrepancies would arise from at least two sources: inaccurate figures for accident expectations and a lack of randomness in the accidents themselves. This paper is concerned only with the latter.

Let the period of exposure be unity. Let the accident expectation which would prevail if the peak hazard were operative without interruption be a. Let b be the portion of the exposure period for which the hazard is rendered inoperative by an accident.

The average or overall accident expectation which contemplates these intervals of zero hazard will obviously be less than a. It is this average expectation which is involved in what is usually considered to be the inherent hazard of the risk and which is assumed to be reflected in the risk rating. Call this average expectation c.

While the peak hazard is operative, the probability of an accident in any infinitesimal of time dx is adx, and the probability of no accidents occurring for a period of time d is e^{-ad}.

The probability (P_n) of exactly n accidents in a risk as defined above will be obtained by taking the product of a succession of probabilities which account for the n accidents and the exposure time and integrating n times between suitable limits. The product comprises the probability of zero accidents up to point of time t, the probability of an accident at t, the certainty of no accidents between t and $(t+b)$, the probability of zero accidents from $(t+b)$ to u, the probability of an accident at u, etc. The limits of integration are complicated somewhat by the circumstance that the interval of zero hazard occasioned by the last accident is not necessarily confined to the exposure period. This circumstance seems to necessitate $(n+1)$ sets of integrals.

For $nb \leqslant 1$

$$P_0 = e^{-a}$$

$$P_1 = \int_0^{1-b} ae^{-a(1-b)}\,dy + \int_{1-b}^{1} ae^{-ay}\,dy$$

$$P_2 = \int_0^{1-2b}\int_{x+b}^{1-b} a^2 e^{-a(1-2b)}\,dx\,dy + \int_0^{1-2b}\int_{1-b}^{1} a^2 e^{-a(y-b)}\,dx\,dy + \int_{1-2b}^{1-b}\int_{x+b}^{1} a^2 e^{-a(y-b)}\,dx\,dy$$

$$P_3 = \int_0^{1-3b}\int_{w+b}^{1-2b}\int_{x+b}^{1-b} a^3 e^{-a(1-3b)}\,dw\,dx\,dy + \int_0^{1-3b}\int_{w+b}^{1-2b}\int_{1-b}^{1} a^3 e^{-a(y-2b)}\,dw\,dx\,dy + \int_0^{1-3b}\int_{1-2b}^{1-b}\int_{x+b}^{1} a^3 e^{-a(y-2b)}\,dw\,dx\,dy + \int_{1-3b}^{1-2b}\int_{w+b}^{1-b}\int_{x+b}^{1} a^3 e^{-a(y-2b)}\,dw\,dx\,dy$$

$$P_4 = \int_0^{1-4b}\int_{v+b}^{1-3b}\int_{w+b}^{1-2b}\int_{x+b}^{1-b} a^4 e^{-a(1-4b)}\,dv\,dw\,dx\,dy + \int_0^{1-4b}\int_{v+b}^{1-3b}\int_{w+b}^{1-2b}\int_{1-b}^{1} F + \int_0^{1-4b}\int_{v+b}^{1-3b}\int_{1-2b}^{1-b}\int_{x+b}^{1} F$$

$$+\int_0^{1-4b}\int_{1-3b}^{1-2b}\int_{w+b}^{1-b}\int_{x+b}^{1} F+\int_{1-4b}^{1-3b}\int_{v+b}^{1-2b}\int_{w+b}^{1-b}\int_{x+b}^{1} F$$

where $F = a^4\, e^{-a(y-3b)}\, dv\, dw\, dx\, dy$.

Integration reveals that for $0 < n \leqslant \frac{1}{b}$ the probability of exactly n accidents takes the form:

$$P_n = \left[1 + a\,(1-nb) + \frac{a^2\,(1-nb)^2}{2} \cdots + \frac{a^n\,(1-nb)^n}{n!}\right] e^{-a(1-nb)}$$

$$- \left[1 + a\,(1-(n-1)b) + \cdots \frac{a^{n-1}\,[1-(n-1)\,b]^{n-1}}{(n-1)!}\right] e^{-a\,[1-(n-1)b]}$$

It will be noted that when b is made zero, P_n reduces to the Poisson formula. It will also be noted that

$$\sum_{n=0}^{n=s} P_n = 1 \text{ when } sb = 1.$$

The latter result could be anticipated, as under the conditions given it is impossible to have more than s accidents.

Several distributions have been calculated from the above formula and the results are exhibited in the following table:

		$a=2$ $b=0.2$	$a=2$ $b=0.1$	$a=2$ $b=0.05$	$a=10$ $b=0.2$	$a=10$ $b=0.1$
(1)	c or $\Sigma P_n X$	1.4693	1.6805	1.8223	3.5539	5.1509
(2)	$\Sigma P_n x^2$	.7975	1.1877	1.5134	.4889	1.4933
(3)	$\frac{\Sigma P_n x^2}{c}$	.5427	.7067	.8305	.1376	.2899
(4)	$\sqrt{(3)}$	.7375	.8407	.9113	.3709	.5384

Line 1 gives the accident expectations of the risks as defined. These figures approach $\frac{a}{1+ab}$ as b is made smaller, but again because the zero hazard interval following the last accident is not confined to the policy period, they are somewhat larger.

In the Poisson distribution, the variance (second moment about the mean) is equal to the expectation (first moment from the origin). Consequently,

line 1 also gives the variances of Poisson distributions most nearly comparable with the distributions found for the idealized risks.

Line 2 gives the variances calculated for the idealized risk distributions. Line 3 shows the ratios of the two variances, and line 4 shows the square roots of these ratios. Since the standard deviation is the square root of the variance, the figures in line 4 are the ratios of the standard deviations and are intended to show the degree to which these departures from randomness affect dispersion. As might be anticipated, if the values of either a or b are increased, the dispersion is moved further from the value it would have if random conditions prevailed.

It is noted that the conditions assumed: i.e., a reduction of the accident producing potentialities following the occurrence of an accident, tend to reduce the dispersion. Conversely, it may be assumed that the dispersion would be increased by conditions under which an accident renders more likely the occurrence of other accidents. Something analogous to the latter situation is found in the use of claim data as the only available approximation for accident data in estimating loss ratio distributions. When there is the possibility of multiple claim accidents, the instant of time coincident with or immediately following a claim may be considered to have been endowed with a special hazard for other related claims.

With respect to the influences operating in the direction of the idealized risks, it is necessary to inject personal opinion of their magnitude if any effort is to be made to relate the mathematical models to reality. There should be general agreement that this magnitude is sufficiently small so that actual risks, or even the serious portion of actual risks, could not be considered comparable with any of the idealized risks with the possible exception of the one in which $a = 2$ and $b = 0.05$.

If a workmen's compensation risk is of such a size that an average of 1.8 serious accidents occur per year, it is conceivable that the occurrence of one of these would bring forces into play which would preclude a recurrence for a little over two weeks, or at least greatly reduce the hazard for a correspondingly longer period. Such a departure from randomness would reduce the standard deviation of the serious accident distribution by about 9%. Unfortunately, 9% is neither large enough to be startling nor small enough to be dismissed as insignificant.

It is a matter of conjecture just how many risks are subject to such departures from randomness. If it is agreed that these influences are normally much less effective, then it can be concluded that concern over the applicability of the Poisson distribution to casualty insurance accidents can be confined to special situations in which accidents are known definitely to be other than independent.

VALUATION OF NON-CANCELLABLE ACCIDENT AND HEALTH INSURANCE POLICIES

BY

S. F. CONROD

While the existing volume[1] of non-cancellable accident and health insurance in force in this county at the present time is relatively small compared with some of the other accident and health lines the "non-can" business has nevertheless been enjoying a steady growth on a sound basis during the past several years.

During this period there has been a distinct trend away from the aggregate type of policy (whereby an aggregate limit is placed on the total amount of all claims payable during the lifetime of a policy) in favor of policies with a limit on each individual claim, and also a larger number of companies are now issuing policies on business and professional men with sickness benefits payable for a maximum of 50 to 120 months on individual claims. With the longer term benefits now being sold the choice of a proper valuation basis for non-cancellable policies is of paramount importance.

Minimum Standard of Valuation

The Conference Modification of Class (3) Disability Table (or "Conference Table") is now the generally accepted minimum standard for the valuation of non-cancellable accident and health insurance policies. The table was prepared by the Non-Cancellable Reserves Committee of the Health and Accident Underwriters Conference and incorporated in their report, which was adopted by the Conference at their June 1939 annual meeting. The manner in which Class (3) Disability Table was modified and extended to produce the Conference Table is fully set out in the committee's report,[2] and as the report is readily available a description of the modifications made are not included in this paper. The Conference Table was constructed by the committee for valuation purposes only and it is important to note that one of their recommendations is that net premiums derived from the table should not be considered a proper basis for the computation of gross premiums.

The Conference Table was adopted as the minimum standard of valuation by a special committee appointed by the National Association of Insurance

1. Non-can premium income during 1944 was 27 million dollars.
2. Published by the Health and Accident Underwriters Conference, 176 West Adams St., Chicago 3, Illinois.

Commissioners in their report dated July 15, 1941, and since that date it has been adopted by law or regulation in Massachusetts, New York and many other states.

Calculation of Valuation Net Premiums

The basic Conference Table shows the amount of disability in months from date of disablement to the end of each of the first 123 months on the basis of 100,000 lives exposed at each of the quinquennial ages from 20 to 70 inclusive.

Accordingly in order to obtain the present value at date of disablement[3] it is necessary to discount each year's disability separately. For example, if $s_x^{0/n}$ is the amount of disability in months incurred at age x and suffered during the n year period immediately following the date of disablement per life exposed at age x, the present value of such disability at date of disablement is

$$s_x^{0/1} + v\, s_x^{1/1} + v^2\, s_x^{2/1} + \cdots\cdots + v^{n-1}\, s_x^{n-1/1} = S_x^{0/n}.$$

As the date of disablement occurs on the average in the middle of a policy year the present value of the disability at the beginning of the policy year in which disability occurs is

$$v^{1/2}\, S_x^{0/n} = \frac{v^{1/2}\, D_x\, S_x^{0/n}}{D_x} = \frac{H_x^{0/n}}{D_x}$$

The single premium at age x at issue for a benefit of 1 per month payable for a maximum of n years or any one claim is

$$\frac{v^{1/2}\, D_x\, S_x^{0/n} + v^{1/2}\, D_{x+1}\, S_{x+1}^{0/n} + v^{1/2}\, D_{x+2}\, S_{x+2}^{0/n} + \cdots\cdots + v^{1/2}\, D_{y-1}\, S_{y-1}^{0/n}}{D_x}$$

$$= \frac{H_x^{0/n} + H_{x+1}^{0/n} + H_{x+2}^{0/n} + \cdots\cdots + H_{y-1}^{0/n}}{D_x}$$

$$= \frac{{}_yK_x^{0/n}}{D_x}, \text{ where } y \text{ is the limiting age of coverage}$$

$$\text{and } {}_yK_x^{0/n} = \sum_x^{y-1} H_x^{0/n}, \text{ or } = {}_zK_x^{0/n} - {}_zK_y^{0/n}$$

where z is the limiting age of coverage in the published commutation column of $K_x^{0/n}$ $(z > y)$.

As values of $S_x^{0/n}$ can be computed directly from the Conference Table for quinquennial ages only it is necessary to interpolate the values of $S_x^{0/n}$ or $H_x^{0/n}$ for intermediate ages.

---

3. The "date of disablement" is the date from which benefits would have accrued if there were no elimination period.

The corresponding annual premium for a benefit of 1 per month is therefore

$$\frac{{}_yK_x^{0/n}}{D_x \cdot \mathrm{a}_{\overline{x:y-n}|}} = \frac{{}_yK_x^{0/n}}{N_x - N_y} = {}_yP_x^{0/n}$$

Where the policy contains an elimination (or waiting) period k

$$\begin{aligned} H_x^{k/n} &= v^{\frac{1}{2}} D_x \left[s_x^{k/1-k} + v\, s_x^{1/1} + v^2 s_x^{2/1} + \cdots + v^{n-1} s_x^{n-1/1} + v^n s_x^{n/k} \right] \\ &= v^{\frac{1}{2}} D_x \left[S_x^{0/n} + v^n s_x^{n/k} - s_x^{0/k} \right] \\ &= v^{\frac{1}{2}} D_x\, S_x^{k/n}. \end{aligned}$$

If the benefit period is not an integral number of years, but is equal to $(n + f)$ years, the general form of H_x (where both k and f are each < 1) becomes

$$\begin{aligned} H_x^{k/n+f} &= v^{\frac{1}{2}} D_x \left[S_x^{0/n} + v^n s_x^{n/k+f} - s_x^{0/k} \right], \text{ where } k + f < 1 \\ &= v^{\frac{1}{2}} D_x \left[S_x^{0/n+1} + v^{n+1} s_x^{n+1/k+f-1} - s_x^{0/k} \right], \text{ where } k + f \gtreqless 1 \\ &= v^{\frac{1}{2}} D_x \cdot S_x^{k/n+f}. \end{aligned}$$

In the formulae given above for $S_x^{0/n}$ each year's disability is discounted from the beginning of the disability year to date of disablement and as a result the value of $S_x^{0/n}$ thus obtained slightly overstates its true value. For practical reasons, however, the slight overstatement can be ignored as its effect on reserves is very small, and is on the side of conservatism.

A more theoretically correct method would be to discount each month's disability separately[4] which gives

$$S_x^{0/n} = \sum_{m=0}^{m=12n-1} v^{\frac{m}{12}}\, s^{\frac{m}{12}} \Big| \frac{1}{12}$$

and

$$\begin{aligned} S_x^{k/n} &= \sum_{m=12k}^{m=12(n+k)-1} v^{\frac{m}{12}}\, s_x^{\frac{m}{12}+k} \Big| \frac{1}{12} \\ &= \sum_{m=0}^{m=12(n+k)-1} v^{\frac{m}{12}}\, s_x^{\frac{m}{12}} \Big| \frac{1}{12} - \sum_{m=0}^{m=12k-1} v^{\frac{m}{12}}\, s_x^{\frac{m}{12}} \Big| \frac{1}{12} \\ &= S_x^{0/n+k} - S_x^{0/k}. \end{aligned}$$

This method has the disadvantage of requiring the construction of a complete table of $S_x^{0/n}$ for each monthly duration up to the largest value of n used, and accordingly it is not a practical method for a company that issues only a few different policy plans. However, once such a table is constructed it has a distinct advantage in that values of $S_x^{k/n}$ or $S_x^{k/n+f}$ can readily be obtained directly from the table by subtracting $S_x^{0/k}$ from $S_x^{0/n+k}$ or $S_x^{0/n+k+f}$; and such a table can also be readily used for computing claim reserves. As our Company issued policies with several different benefit

4. This method was used by Mr. A. W. Larsen in deriving $\mathrm{a}_{[x]}^{\frac{f}{(12)}}$ in R.A.I.A., XXXII, 56.

periods, varying all the way from a minimum of one year to a maximum of ten years we have found it convenient to construct such a table and accordingly the commutation columns appearing in this paper have been computed from this basic table.

Active Life Reserves

The terminal reserve at the end of t policy years for a benefit of 1 per month is

$$\frac{{}_yK^{k/n}_{x+t}}{D_{x+t}} - {}_yP_x^{k/n} \cdot \frac{(N_x - N_y)}{D_{x+t}}$$

As the convention statement requires a Company to set up an unearned premium reserve (i.e. a pro-rate portion of the gross premium) on each policy in addition to the tabular reserve, the reserve used for valuation purposes is a mid-terminal and not a true mean reserve. Accordingly, the reserve for the $(t+1)$th policy year is

$${}_{t+\frac{1}{2}}V = \frac{({}_tV + {}_{t+1}V)}{2}$$

As the terminal reserve is of no direct use to a company, a considerable saving in time can be effected by computing mid-terminal reserves directly, without going to the trouble of first computing terminal reserves. For example,

$${}_{t+\frac{1}{2}}V_x = \tfrac{1}{2}\left[\frac{K_{x+t}}{D_{x+t}} + \frac{K_{x+t+1}}{D_{x+t+1}}\right] - P_x \cdot \tfrac{1}{2}\left[\frac{(N_{x+t} - N_y)}{D_{x+t}} + \frac{(N_{x+t+1} - N_y)}{D_{x+t+1}}\right]$$

Mid-terminal reserves by this formula can be computed just as readily as terminal reserves after columns of mean single premiums and mean annuity values have been calculated by a continuous process.

Effect of an Elimination Period on Reserves

If a company were to compute a separate reserve table for each combination of n, y and k, the computation of the policy reserve would become a very cumbersome calculation for a company that allows the insured the choice of several elimination periods when he applies for a policy, not to mention the calculation of the reserve tables themselves. Fortunately a short elimination period of one or two weeks has a very slight effect on reserves, and for all practical purposes short-term benefits policies (i.e. policies with benefits of two years or less) can be valued on the same basis as policies without any elimination period.[5] Accordingly, the commutation columns included in this

5. See comparisons in tables in Record of the American Institute of Actuaries, XXXII, pages 12 and 72.

paper for valuing short term benefit policies have been computed on the basis of no elimination period. On the other hand, commutations columns for both one and three months elimination periods have been included for the longer term benefits, which are usually sold with a minimum elimination for sickness of at least one month.

Reserve for Waiver of Premium Benefit

Some policies (particularly those with long term benefits) provide for waiver of any premiums falling due during the period for which monthly indemnity is payable provided total disability has existed continuously for a definite period (usually three or four months) immediately prior to the due date of the premium to be waived. A very convenient method of approximating the additional reserve for the t-th policy year is to take the total reserve for the monthly indemnity benefit (using all policies of a particular plan of duration t) and multiply it by the ratio

$$\frac{\frac{1}{12}\text{ total annual premiums (or }\frac{1}{3}\text{ total quarterly premiums)}}{\text{total monthly indemnity}}$$

Unlevel Premiums and Benefits

Some policies provide for a step-rate in the premium at a given attained age (such as age 50) and a reduction in the amount of the benefits at a later age (such as age 60). Other policies particularly those with long-term benefits provide for the termination of monthly income payments on existing claims on the date that coverage under the policy ceases, as for example a policy that pays indemnity for a maximum of ten years under any one claim, but not beyond age 65.

The calculation of net premiums for policies with a step-rate in the premium and a reduction in the amount of benefits is fully discussed by Mr. Farley in his paper in the *Proceedings*, XXVII. The treatment of such policies for valuation purposes is set out in detail in the report of the special committee of the National Association of Insurance Commissioners already referred to. The portion of the committee's report dealing with active life reserves has been reprinted on pages 24 and 25 of Record of the American Institute of Actuaries, XXXII.

In calculating net premiums for policies with reducing benefit periods the proper length benefit should be taken into consideration at each attained age. For example, the net premium for a benefit of 1 per month payable for a maximum of n years, but not beyond age y is $\frac{\sum_{x}^{y-1} H_x^{k/r}}{N_x - N^y}$,

where r is lesser of n or $(y - x - k - \frac{1}{2})$ at each value of x, and the date of disablement is assumed to occur in the middle of the policy year $= \frac{{}_{y}K_{x}^{k/r}}{N_x - N_y}$

In computing reserves the same treatment should be applied to such policies as prescribed by the special committee of the National Association of Insurance Commissioners for policies with step-rate premiums and reduction in amount of benefits after given ages.

Weekly Indemnity Policies

Where indemnity is payable weekly instead of monthly the net premiums and reserves per $1.00 weekly indemnity are the same as for a policy of $4.35 monthly indemnity, provided the other provisions such as length of benefits, etc. are unchanged. For all practical purposes the monthly equivalents of the commoner weekly benefits periods are as follows:

Equivalent Benefit Periods

Weeks	*Months*
52	12
60	14
65	15
100	23
102	24

The corresponding monthly period in each instance slightly exceeds the weekly and accordingly results in a very slight overstatement in the reserve, and hence is on the side of conservatism.

Multiple Reserve Standards

As already pointed out the Conference Table is the minimum valuation standard and is not necessarily "the" standard for valuation to be used by all companies. The report of the special committee of the National Association of Insurance Commissioners specifically states that where the experience of a company indicates that an increase in the reserve should be made, additional reserves must be maintained on a basis having the approval of the commissioners of the different states in which the company operates.

Accordingly a company is perfectly at liberty to use a table based on its own experience as long as it yields reserves in the aggregate at least equal to the reserves by the Conference Table. If a company experience table is used and morbidity rates are derived from mean exposures H_x should theoretically take the form $\bar{D}_x S_x$, where $\bar{D}_x = v^{x+\frac{1}{2}} l_{x+\frac{1}{2}} = \frac{(D_x + D_{x+1})}{2}$ approximately.

A basis used by some companies (particularly those issuing long term benefit policies) is to value according to some multiple of the Conference Table, such as r times Conference disability, or r^1 times first year disability and r^2 times the disability for second and subsequent years. If a straight multiple of the conference disability

$$\overset{/}{K}_x^{k/n} = r \cdot K_x^{k/n},$$

while if a compound multiple basis is used

$$\overset{/}{K}_x^{k/n} = r^1 \cdot K_x^{k/1-k} + r^2 \cdot K_x^{1/n-1+k}$$

where $K_x^{k/1-k} = K_x^{0/1} - K_x^{0/k}$

and $K_x^{1/n-1+k} = K_x^{k/n} - K_x^{k/1-k}$

If the r^1 and r^2 factors are graded by age at disablement it is necessary to first construct a column of $\overset{/}{H}_x^{k/n}$ and then obtain $\overset{/}{K}_x^{k/n}$ by summation.

$$\overset{/}{H}_x^{k/n} = r_x^1 \cdot H_x^{k/1-k} + r_x^2 \cdot H_x^{1/n-1+k}$$

and $${}_y\overset{/}{K}_x^{k/n} = \sum_x^{y-1} \overset{/}{H}_x^{k/n}$$

Claim Reserves

The bases on which claims are to be valued is set out in the disabled life reserve section of the report of the special committee of the National Association of Insurance Commissioners. To quote from the report:

"II *The Disabled Life Reserve*

A. For policies with a waiting period, the duration of disablement shall be considered as dating from the time that benefits would have begun to accrue had there been no waiting period.

B. An interest rate of not more than 3% shall be used.

C. For claims other than life indemnity having a duration of disablement of more than one year and for life indemnity claims, the reserve for each claim shall be established in accordance with the Conference Modification of the Class 3 Experience or shall be an amount equal to the indemnity payable for a period three and one-half times the duration of disablement, whichever is less, and a minimum reserve on each life indemnity claim of seven weeks' indemnity shall be maintained.

D. For claims with a duration of disablement of less than one year under policies not providing life indemnity, and for all unreported and resisted claims, reserves shall be based on the individual company's experience or estimates to be tested by the development of each year's claims over a period

of years as shown in Schedule O, Part 2, of the Convention Form of Annual Statement Blank.

E. For claims on which partial disability is being paid, reserves shall be established for the reduced amount of indemnity using the reserve factors which would have been used if full indemnity were payable for the same period.

F. For claims where the indemnity being paid has been reduced because the assured is not confined to the house, the same reserve factors shall be used as for full indemnity, applying the reduced amount of indemnity to the tabular value.

G. A new disability connected directly or indirectly with a previous disability which had a duration of at least one year and terminated within six months of the new disability shall be considered a continuation of the previous disability."

Tabular claim reserves can readily be computed from a table of $S_x^{0/n}$ in combination with Table Ic of the Non-Cancellable Reserve Committee's Report. (Table Ic shows the number disabled at the end of each month out of 100,000 active lives exposed at each quinquennial age from 20 to 70 inclusive). For example, the value of an annuity of 1 per month to a disabled life age $[x] + t$ for the remainder of the benefit period of $(n - t)$ years is

$$ {}_{12}\mathring{a}^{t(12)}_{[x]+t:\overline{n-t}|} = \frac{(1+i)^t\,[S_x^{0/n} - S_x^{0/t}] \cdot 10^5}{L^t_{[x]+t}} $$

where values of $L^t_{[x]+t}$ are taken from Table Ic referred to above.

From a table of values of ${}_{12}\mathring{a}^{t(12)}_{[x]+t:\overline{n-t}|}$ computed for yearly values of t and n a table of claim reserves can be prepared, the mean claim reserve for the $(t+1)$th year being $\frac{{}_{12}\mathring{a}^{t(12)}_{[x]+t:\overline{n-t}|} + {}_{12}\mathring{a}_{[x]+t+1:\overline{n-t-1}|}}{2} + \frac{1}{2}$ approx. assuming that on the average there will be one-half month accrued indemnity.

When the policy contains an elimination k

$$ {}_{12}\mathring{a}^{t(12)}_{[x]+t:\overline{n+k-t}|} = \frac{(1+i)^t\,[S_x^{0/n+k} - S_x^{0/t}] \cdot 10^5}{L^t_{[x]+t}} $$

and the mean claim reserve for the $(t+1)$th year is

$$ \frac{{}_{12}\mathring{a}^{t(12)}_{[x]+t:\overline{n+k-t}|} + {}_{12}\mathring{a}_{[x]+t+1:\overline{n+k-t-1}|}}{2} + \frac{1}{2} \text{ approx.} $$

In this connection it is important to note that the duration is measured from the date of disablement and not from the end of the elimination period.

If claim reserves are desired for monthly durations of disablement the monthly values can either be obtained by interpolation or can be calculated directly, as the basic formulas apply when t or n (or both) are not integers.

Summary of Tables

In the tables that follow commutation columns are given for benefits of 12, 15, 18, 24, 50, 60, 100 and 120 months in tables 1 to 7 inclusive. In computing values of $H_x^{k/n}$ the quinquennial age values of $S_x^{k/n}$ were first interpolated by a method given by Mr. Camp in *Transactions* of the Actuarial Society of America, XXXVIII and the resulting values of $S_x^{k/n}$ were multiplied by $v^{\frac{1}{2}} D_x$ (C.S.O. Mortality Table) to give the values of $H_x^{k/n}$. In addition commutation columns for benefits of 1 and 3 months are given in Table 8.

Quinquennial values of 100,000 $S_x^{0/n}$ for each of the first 123 months are given in Table 9, while Table Ic of the Non-Cancellable Reserves Committee's Report is given in skeleton form in Table 10.

TABLE 1

CONFERENCE MODIFICATION OF CLASS 3 DISABILITY TABLE
COMMUTATION COLUMNS AT 2½% INTEREST

Age	$H_x^{0/1}$	$H_x^{0\|\frac{15}{12}}$	$H_x^{0\|\frac{18}{12}}$	$H_x^{0/2}$	$H_x^{\frac{1}{12}\|\frac{50}{12}}$	$H_x^{\frac{3}{12}\|\frac{50}{12}}$
20	130669	133462	135860	139955	62659	37876
21	126274	129025	131398	135461	61356	37661
22	122157	124856	127198	131217	60084	37384
23	118292	120941	123240	127210	58842	37049
24	114676	117263	119516	123414	57634	36660
25	111294	113811	116013	119831	56448	36222
26	108097	110545	112692	116430	55292	35729
27	105036	107422	109520	113189	54166	35184
28	102109	104439	106493	110099	53062	34583
29	99308	101588	103603	107145	51985	33939
30	96622	98860	100837	104324	50928	33251
31	94116	96313	98256	101692	49987	32621
32	91848	94004	95913	99299	49240	32138
33	89797	91911	93790	97125	48674	31787
34	87945	90023	91872	95158	48270	31560
35	86283	88324	90139	93373	48013	31441
36	84737	86749	88544	91740	47880	31428
37	83251	85250	87043	90227	47851	31512
38	81817	83820	85622	88819	47914	31683
39	80434	82454	84277	87510	48055	31930
40	79089	81146	82998	86280	48264	32248
41	77814	79906	81796	85139	48561	32640
42	76625	78753	80677	84082	48954	33125
43	75511	77671	79629	83093	49430	33687
44	74466	76654	78642	82162	49973	34314
45	73471	75688	77701	81279	50566	34996
46	72539	74791	76834	80480	51254	35757
47	71674	73973	76059	79788	52069	36636
48	70860	73216	75358	79186	52987	37607
49	70087	72508	74717	78655	53986	38650
50	69343	71836	74113	78175	55039	39746
51	68630	71201	73548	77736	56103	40845
52	67955	70601	73017	77328	57140	41918
53	67301	70018	72502	76932	58134	42947
54	66654	69439	71983	76527	59066	43919
55	65995	68847	71445	76100	59920	44821
56	65386	68308	70965	75737	60860	45798
57	64872	67873	70603	75512	62019	46971
58	64423	67510	70322	75382	63338	48289
59	64003	67183	70082	75301	64751	49696
60	63584	66857	69844	75227	66199	51143
61	63229	66593	69668	75211	67703	52625
62	62984	66434	69590	75286	69269	54131
63	62792	66320	69550	75385	70817	55601
64	62599	66195	69488	75447	72271	56973
65	62355	66006	69350	75410	73562	58194
66	61936	65635	69021	75172	74635	59271
67	61240	64983	68410	74647	75440	60201
68	60258	64035	67498	73806	75917	60916
69	58985	62784	66270	72630	76012	61359
70	57418	61222	64717	71099	75679	61473

TABLE 2

CONFERENCE MODIFICATION OF CLASS 3 DISABILITY TABLE

COMMUTATION COLUMNS AT 2½% INTEREST

Age	$H_x^{\frac{1}{12}\vert 5}$	$H_x^{\frac{3}{12}\vert 5}$	$H_x^{\frac{1}{12}\vert\frac{100}{12}}$	$H_x^{\frac{3}{12}\vert\frac{100}{12}}$	$H_x^{\frac{1}{12}\vert 10}$	$H_x^{\frac{3}{12}\vert 10}$
20	64965	41140	70562	45459	72277	47145
21	63857	41016	69952	45894	71778	47692
22	62746	40806	69264	46173	71187	48068
23	61628	40511	68500	46304	70519	48286
24	60514	40137	67668	46298	69782	48361
25	59395	39680	66775	46155	68972	48301
26	58271	39131	65816	45873	68094	48098
27	57144	38498	64795	45449	67153	47745
28	56014	37788	63714	44884	66145	47260
29	54887	37006	62575	44193	65083	46647
30	53753	36290	61386	43387	63972	45925
31	52738	35757	60331	42656	62998	45280
32	51951	35392	59570	42172	62326	44887
33	51369	35177	59085	41914	61931	44728
34	50974	35103	58847	41865	61792	44774
35	50748	35143	58830	42002	61882	45013
36	50653	35275	58967	42270	62133	45395
37	50661	35496	59195	42618	62483	45869
38	50752	35788	59498	43035	62917	46423
39	50916	36142	59861	43515	63422	47040
40	51143	36596	60278	44043	63989	47715
41	51472	37180	60838	44707	64704	48538
42	51931	37882	61620	45583	65654	49580
43	52503	38677	62593	46643	66806	50817
44	53170	39556	63727	47865	68130	52219
45	53912	40532	64997	49219	69593	53757
46	54755	41615	66372	50667	71155	55380
47	55718	42791	67826	52173	72775	57047
48	56774	44033	69331	53718	74430	58737
49	57902	45321	70860	55278	76092	60425
50	59073	46625	72393	56834	77734	62096
51	60259	47914	73936	58395	79386	63767
52	61427	49170	75492	59970	81063	65458
53	62553	50378	77032	61533	82732	67143
54	63622	51520	78524	63053	84355	68790
55	64610	52762	79940	64508	85904	70372
56	65696	54236	81463	66056	87554	72042
57	67020	55881	83236	67826	89447	73927
58	68513	57638	85180	69747	91502	75952
59	70108	59450	87216	71750	93634	78043
60	71739	61314	89265	73765	95764	80133
61	73424	63219	91363	75811	97948	82257
62	75163	65094	93527	77892	100217	84436
63	76869	66871	95652	79918	102457	86574
64	78465	68489	97640	81810	104560	88577
65	79873	69971	99393	83485	106422	90359

TABLE 3

CONFERENCE MODIFICATION OF CLASS 3 DISABILITY TABLE

COMMUTATION COLUMNS AT 2½% INTEREST

Age	${}_{65}K_x^{0/1}$	${}_{65}K_x^{0\vert\frac{15}{12}}$	${}_{65}K_x^{0\vert\frac{18}{12}}$	${}_{65}K_x^{0/2}$	${}_{60}K_x^{\frac{1}{12}\vert\frac{50}{12}}$	${}_{60}K_x^{\frac{3}{12}\vert\frac{50}{12}}$
20	3736648	3850581	3952957	4134648	2160754	1476248
21	3605979	3717119	3817097	3994693	2098095	1438372
22	3479705	3588094	3685699	3859232	2036739	1400711
23	3357548	3463238	3558501	3728015	1976655	1363327
24	3239256	3342297	3435261	3600805	1917813	1326278
25	3124580	3225034	3315745	3477391	1860179	1289618
26	3013286	3111223	3199732	3357560	1803731	1253396
27	2905189	3000678	3087040	3241130	1748439	1217667
28	2800153	2893256	2977520	3127941	1694273	1182483
29	2698044	2788817	2871027	3017842	1641211	1147900
30	2598736	2687229	2767424	2910697	1589226	1113961
31	2502114	2588369	2666587	2806373	1538298	1080710
32	2407998	2492056	2568331	2704681	1488311	1048089
33	2316150	2398052	2472418	2605382	1439071	1015951
34	2226353	2306141	2378628	2508257	1390397	984164
35	2138408	2216118	2286756	2413099	1342127	952604
36	2052125	2127794	2196617	2319726	1294114	921163
37	1967388	2041045	2108073	2227986	1246234	889735
38	1884137	1955795	2021030	2137759	1198383	858223
39	1802320	1871975	1935408	2048940	1150469	826540
40	1721886	1789521	1851131	1961430	1102414	794610
41	1642797	1708375	1768133	1875150	1054150	762362
42	1564983	1628469	1686337	1790011	1005589	729722
43	1488358	1549716	1605660	1705929	956635	696597
44	1412847	1472045	1526031	1622836	907205	662910
45	1338381	1395391	1447389	1540674	857232	628596
46	1264910	1319703	1369688	1459395	806666	593600
47	1192371	1244912	1292854	1378915	755412	557843
48	1120697	1170939	1216795	1299127	703343	521207
49	1049837	1097723	1141437	1219941	650356	483600
50	979750	1025215	1066720	1141286	596370	444950
51	910407	953379	992607	1063111	541331	405204
52	841777	882178	919059	985375	485228	364359
53	773822	811577	846042	908047	428088	322441
54	706521	741559	773540	831115	369954	279494
55	639867	672120	701557	754588	310888	235575
56	573872	603273	630112	678488	250968	190754
57	508486	534965	559147	602751	190108	144956
58	443614	467092	488544	527239	128089	97985
59	379191	399582	418222	451857	64751	49696
60	315188	332399	348140	376556		
61	251604	265542	278296	301329		
62	188375	198949	208628	226118		
63	125391	132515	139038	150832		
64	62599	66195	69488	75447		

TABLE 4

CONFERENCE MODIFICATION OF CLASS 3 DISABILITY TABLE

COMMUTATION COLUMNS AT 2½% INTEREST

Age	${}_{60}K_x^{\frac{1}{12}\vert 5}$	${}_{60}K_x^{\frac{3}{12}\vert 5}$	${}_{60}K_x^{\frac{1}{12}\vert\frac{100}{12}}$	${}_{60}K_x^{\frac{3}{12}\vert\frac{100}{12}}$	${}_{60}K_x^{\frac{1}{12}\vert 10}$	${}_{60}K_x^{\frac{3}{12}\vert 10}$
20	2296097	1681033	2708045	2011684	2863485	2164742
21	2231132	1639893	2637483	1966225	2791208	2117597
22	2167275	1598877	2567531	1920331	2719430	2069905
23	2104529	1558071	2498267	1874158	2648243	2021837
24	2042901	1517560	2429767	1827854	2577724	1973551
25	1982387	1477423	2362099	1781556	2507942	1925190
26	1922992	1437743	2295324	1735401	2438970	1876889
27	1864721	1398612	2229508	1689528	2370876	1828791
28	1807577	1360114	2164713	1644079	2303723	1781046
29	1751563	1322326	2100999	1599195	2237578	1733786
30	1696676	1285320	2038424	1555002	2172495	1687139
31	1642923	1249030	1977038	1511615	2108523	1641214
32	1590185	1213273	1916707	1468959	2045525	1595934
33	1538234	1177881	1857137	1426787	1983199	1551047
34	1486865	1142704	1798052	1384873	1921268	1506319
35	1435891	1107601	1739205	1343008	1859476	1461545
36	1385143	1072458	1680375	1301006	1797594	1416532
37	1334490	1037183	1621408	1258736	1735461	1371137
38	1283829	1001687	1562213	1216118	1672978	1325268
39	1233077	965899	1502715	1173083	1610061	1278845
40	1182161	929757	1442854	1129568	1546639	1231805
41	1131018	893161	1382576	1085525	1482650	1184090
42	1079546	855981	1321738	1040818	1417946	1135552
43	1027615	818099	1260118	995235	1352292	1085972
44	975112	779422	1197525	948592	1285486	1035155
45	921942	739866	1133798	900727	1217356	982936
46	868030	699334	1068801	851508	1147763	929179
47	813275	657719	1002429	800841	1076608	873799
48	757557	614928	934603	748668	1003833	816752
49	700783	570895	865272	694950	929403	758015
50	642881	525574	794412	639672	853311	697590
51	583808	478949	722019	582838	775577	635494
52	523549	431035	648083	524443	696191	571727
53	462122	381865	572591	464473	615128	506269
54	399569	331487	495559	402940	532396	439126
55	335947	279967	417035	339887	448041	370336
56	271337	227205	337095	275379	362137	299964
57	205641	172969	255632	209323	274583	227922
58	138621	117088	172396	141497	185136	153995
59	70108	59450	87216	71750	93634	78043

TABLE 5

CONFERENCE MODIFICATION OF CLASS 3 DISABILITY TABLE

COMMUTATION COLUMNS AT 2½% INTEREST

Age	$H_x^{\overline{k}\mid 65-x-\frac{1}{12}-k}$	
	$k=\frac{1}{12}$	$k=\frac{3}{12}$
55	83932	67859
56	81703	65582
57	79201	63036
58	76247	60048
59	72588	56372
60	67919	51710
61	62036	45809
62	54582	38278
63	44685	28265
64	27884	11332

TABLE 6

CONFERENCE MODIFICATION OF CLASS 3 DISABILITY TABLE

COMMUTATION COLUMNS AT 2½% INTEREST

Age	${}^{65}_{65}K^{1\,|\,50}_{x\ \ 12\,|\,12}$	${}^{65}_{65}K^{3\,|\,50}_{x\ \ 12\,|\,12}$	${}^{65}_{65}K^{1\,|\,5}_{x\ \ 12}$	${}^{65}_{65}K^{3\,|\,5}_{x\ \ 12}$
20	2416140	1651075	2553203	1856427
21	2353481	1613199	2488238	1815287
22	2292125	1575538	2424381	1774271
23	2232041	1538154	2361635	1733465
24	2173199	1501105	2300007	1692954
25	2115565	1464445	2239493	1652817
26	2059117	1428223	2180098	1613137
27	2003825	1392494	2121827	1574006
28	1949659	1357310	2064683	1535508
29	1896597	1322727	2008669	1497720
30	1844612	1288788	1953782	1460714
31	1793684	1255537	1900029	1424424
32	1743697	1222916	1847291	1388667
33	1694457	1190778	1795340	1353275
34	1645783	1158991	1743971	1318098
35	1597513	1127431	1692997	1282995
36	1549500	1095990	1642249	1247852
37	1501620	1064562	1591596	1212577
38	1453769	1033050	1540935	1177081
39	1405855	1001367	1490183	1141293
40	1357800	969437	1439267	1105151
41	1309536	937189	1388124	1068555
42	1260975	904549	1336652	1031375
43	1212021	871424	1284721	993493
44	1162591	837737	1232218	954816
45	1112618	803423	1179048	915260
46	1062052	768427	1125136	874728
47	1010798	732670	1070381	833113
48	958729	696034	1014663	790322
49	905742	658427	957889	746289
50	851756	619777	899987	700968
51	796717	580031	840914	654343
52	740614	539186	780655	606429
53	683474	497268	719228	557259
54	625340	454321	656675	506881
55	566274	410402	593053	455361
56	506354	365581	528443	402599
57	445494	319783	462747	348363
58	383475	272812	395727	292482
59	320137	224523	327214	234844
60	255386	174827	257106	175394
61	189187	123684	189187	123684
62	127151	77875	127151	77875
63	72569	39597	72569	39597
64	27884	11332	27884	11332

TABLE 7

CONFERENCE MODIFICATION OF CLASS 3 DISABILITY TABLE

COMMUTATION COLUMNS AT 2½% INTEREST

Age	${}^{65}_{65}K_x^{\frac{1}{12}\vert\frac{100}{12}}$	${}^{65}_{65}K_x^{\frac{3}{12}\vert\frac{100}{12}}$	${}^{65}_{65}K_x^{\frac{1}{12}\vert 10}$	${}^{65}_{65}K_x^{\frac{3}{12}\vert 10}$
20	2937555	2156737	3066221	2282697
21	2866993	2111278	2993944	2235552
22	2797041	2065384	2922166	2187860
23	2727777	2019211	2850979	2139792
24	2659277	1972907	2780460	2091506
25	2591609	1926609	2710678	2043145
26	2524834	1880454	2641706	1994844
27	2459018	1834581	2573612	1946746
28	2394223	1789132	2506459	1899001
29	2330509	1744248	2440314	1851741
30	2267934	1700055	2375231	1805094
31	2206548	1656668	2311259	1759169
32	2146217	1614012	2248261	1713889
33	2086647	1571840	2185935	1669002
34	2027562	1529926	2124004	1624274
35	1968715	1488061	2062212	1579500
36	1909885	1446059	2000330	1534487
37	1850918	1403789	1938197	1489092
38	1791723	1361171	1875714	1443223
39	1732225	1318136	1812797	1396800
40	1672364	1274621	1749375	1349760
41	1612086	1230578	1685386	1302045
42	1551248	1185871	1620682	1253507
43	1489628	1140288	1555028	1203927
44	1427035	1093645	1488222	1153110
45	1363308	1045780	1420092	1100891
46	1298311	996561	1350499	1047134
47	1231939	945894	1279344	991754
48	1164113	893721	1206569	934707
49	1094782	840003	1132139	875970
50	1023922	784725	1056047	815545
51	951529	727891	978313	753449
52	877593	669496	898927	689682
53	802101	609526	817864	624224
54	725069	547993	735132	557081
55	646545	484940	650777	488291
56	566605	420432	566845	420432
57	485142	354850	485142	354850
58	405941	291814	405941	291814
59	329694	231766	329694	231766
60	257106	175394	257106	175394
61	189187	123684	189187	123684
62	127151	77875	127151	77875
63	72569	39597	72569	39597
64	27884	11332	27884	11332

TABLE 8

CONFERENCE MODIFICATION OF CLASS 3 DISABILITY TABLE

COMMUTATION COLUMNS AT 2½% INTEREST

Age	$v^{\frac{1}{2}} D_x$ (C.S.O. 2½%)	$H_x{}^{0}\|_{\frac{1}{12}}$	$H_x{}^{0}\|_{\frac{3}{12}}$
20	573538	88376	113669
21	558189	85459	109539
22	543208	82687	105730
23	528587	80049	102218
24	514312	77543	98990
25	500378	75162	96028
26	486768	72874	93226
27	473475	70657	90495
28	460491	68507	87829
29	447799	66427	85230
30	435392	64412	82694
31	423260	62482	80309
32	411396	60660	78157
33	399789	58941	76220
34	388431	57317	74486
35	377309	55781	72934
36	366416	54299	71447
37	355742	52835	69925
38	345278	51388	68368
39	335017	49958	66782
40	324947	48544	65168
41	315062	47177	63614
42	305352	45879	62197
43	295810	44650	60904
44	286428	43480	59717
45	277196	42364	58630
46	268106	41264	57565
47	259153	40153	56456
48	250327	39028	55305
49	241622	37891	54109
50	233030	36742	52872
51	224546	35633	51686
52	216162	34612	50625
53	207874	33663	49674
54	199675	32779	48813
55	191561	31949	48022
56	183529	31130	47251
57	175573	30285	46450
58	167694	29414	45613
59	159888	28516	44732
60	152156	27592	43801
61	144498	26666	42893
62	136916	25759	42063
63	129412	24865	41285
64	121993	23977	40529
65	114664	23086	39770

TABLE 9

CONFERENCE MODIFICATION OF CLASS 3 DISABILITY TABLE

100,000 $S_x^{0/n}$ ON BASIS OF 2½% INTEREST

Months	AGE AT DISABLEMENT										
	20	25	30	35	40	45	50	55	60	65	70
1	15409	15021	14794	14784	14939	15283	15767	16678	18134	20134	22744
2	18663	18008	17740	17926	18418	19208	20325	22081	24847	28991	34415
3	19819	19191	18993	19330	20055	21151	22689	25069	28787	34684	42332
4	20487	19886	19739	20185	21081	22432	24308	27160	31580	38847	48368
5	20973	20386	20268	20779	21835	23362	25533	28779	33772	42160	53293
6	21353	20774	20681	21233	22416	24077	26487	30068	35514	44834	57382
7	21669	21097	21021	21616	22876	24666	27261	31088	36955	47043	60788
8	21941	21376	21309	21935	23255	25146	27905	31902	38171	48906	63679
9	22182	21624	21563	22209	23574	25545	28456	32643	39225	50516	66169
10	22400	21848	21792	22452	23854	25899	28936	33308	40159	51936	68343
11	22599	22053	22000	22669	24106	26215	29364	33903	41004	53213	70277
12	22783	22242	22192	22868	24339	26505	29757	34451	41789	54381	72028
13	22955	22418	22372	23057	24560	26782	30128	34966	42533	55482	73677
14	23117	22586	22543	23236	24769	27048	30483	35461	43248	56541	75263
15	23270	22745	22706	23409	24972	27305	30827	35940	43940	57565	76800
16	23415	22897	22862	23575	25168	27553	31162	36404	44611	58561	78295
17	23554	23044	23013	23735	25358	27795	31487	36856	45265	59533	79756
18	23688	23185	23160	23890	25542	28031	31804	37296	45903	60481	81184
19	23817	23322	23303	24041	25721	28260	32112	37726	46527	61408	82582
20	23942	23455	23442	24188	25895	28484	32413	38145	47136	62315	83950
21	24063	23584	23577	24333	26065	28702	32707	38555	47732	63203	85295
22	24180	23710	23708	24474	26231	28915	32994	38954	48314	64074	86616
23	24293	23831	23836	24612	26394	29121	33274	39344	48883	64928	87914
24	24402	23948	23961	24747	26552	29322	33547	39726	49441	65766	89190
25	24508	24063	24083	24879	26706	29519	33815	40099	49988	66588	90445
26	24610	24175	24202	25008	26857	29712	34076	40465	50525	67395	91680
27	24709	24284	24319	25134	27004	29900	34332	40823	51051	68188	92896
28	24805	24391	24433	25258	27149	30084	34582	41174	51567	68968	94094
29	24897	24495	24543	25380	27290	30263	34827	41519	52074	69736	95273
30	24987	24597	24651	25499	27429	30439	35067	41857	52571	70493	96435
31	25074	24697	24757	25616	27563	30611	35303	42189	53061	71238	97580
32	25158	24794	24861	25730	27695	30780	35535	42516	53543	71972	98709
33	25239	24890	24962	25841	27824	30946	35764	42837	54018	72695	99822
34	25317	24983	25062	25950	27951	31109	35988	43154	54486	73408	100921
35	25393	25074	25160	26057	28075	31268	36209	43465	54948	74111	102005
36	25467	25163	25256	26161	28197	31424	36427	43772	55405	74804	103075
37	25538	25250	25350	26263	28317	31579	36642	44076	55855	75488	104131
38	25607	25335	25443	26364	28434	31731	36854	44375	56300	76165	105174
39	25674	25419	25533	26462	28550	31881	37063	44670	56740	76834	106204
40	25738	25501	25622	26558	28663	32029	37270	44962	57175	77495	107222
41	25801	25581	25709	26653	28774	32175	37474	45250	57604	78148	108228
42	25862	25660	25795	26745	28883	32318	37675	45535	58029	78793	109222
43	25921	25737	25879	26836	28990	32459	37874	45817	58448	79431	110204
44	25978	25813	25961	26925	29095	32598	38070	46095	58863	80062	111175
45	26034	25887	26041	27013	29199	32735	38265	46370	59272	80686	112135
46	26088	25960	26120	27099	29301	32871	38456	46642	59678	81303	113084
47	26140	26031	26197	27184	29402	33005	38646	46910	60079	81913	114023
48	26191	26101	26272	27267	29501	33138	38834	47176	60476	82517	114952

TABLE 9 (Continued)

CONFERENCE MODIFICATION OF CLASS 3 DISABILITY TABLE

100,000 $S_x^{0/n}$ ON BASIS OF 2½% INTEREST

Months	AGE AT DISABLEMENT										
	20	25	30	35	40	45	50	55	60	65	70
49	26240	26169	26346	27349	29599	33269	39020	47440	60868	83114	115871
50	26288	26236	26419	27430	29696	33398	39204	47700	61257	83704	116780
51	26334	26302	26491	27509	29792	33525	39386	47958	61641	84288	117679
52	26379	26367	26561	27586	29886	33651	39566	48214	62022	84865	118568
53	26423	26430	26630	27663	29979	33776	39745	48467	62399	85436	119447
54	26466	26492	26698	27738	30071	33900	39922	48717	62772	86000	120316
55	26508	26552	26764	27812	30161	34023	40097	48965	63142	86558	121175
56	26549	26612	26830	27885	30249	34144	40271	49211	63507	87111	122024
57	26588	26670	26894	27957	30337	34264	40443	49454	63869	87658	122864
58	26627	26727	26957	28028	30424	34382	40614	49695	64227	88200	123694
59	26664	26783	27019	28097	30509	34500	40783	49934	64582	88736	124515
60	26701	26838	27080	28166	30594	34616	40951	50171	64934	89267	125327
61	26736	26891	27140	28234	30678	34732	41117	50406	65282	89792	126130
62	26771	26944	27199	28301	30761	34847	41282	50639	65627	90312	126925
63	26805	26995	27257	28367	30843	34961	41446	50871	65969	90826	127711
64	26838	27046	27314	28432	30925	35073	41609	51100	66307	91335	128488
65	26870	27095	27370	28496	31005	35184	41770	51328	66643	91838	129257
66	26902	27144	27425	28559	31085	35294	41929	51554	66975	92336	130017
67	26933	27191	27479	28622	31164	35404	42088	51778	67304	92828	130769
68	26964	27238	27532	28684	31242	35513	42246	52000	67630	93315	131512
69	26994	27283	27584	28745	31319	35621	42402	52220	67953	93797	132247
70	27024	27328	27636	28806	31395	35729	42557	52439	68272	94273	132973
71	27053	27372	27686	28866	31470	35836	42710	52656	68589	94744	133691
72	27082	27415	27736	28925	31545	35941	42863	52870	68903	95210	134400
73	27110	27457	27785	28983	31619	36046	43015	53083	69213	95671	135102
74	27138	27498	27833	29041	31693	36150	43165	53294	69521	96128	135796
75	27165	27538	27881	29098	31766	36254	43314	53504	69826	96581	136482
76	27191	27577	27927	29155	31839	36357	43462	53712	70128	97029	137160
77	27217	27616	27973	29211	31911	36459	43609	53918	70427	97472	137831
78	27242	27654	28018	29266	31983	36561	43755	54123	70723	97910	138493
79	27267	27692	28063	29321	32054	36662	43900	54326	71016	98343	139148
80	27292	27729	28107	29375	32125	36762	44044	54527	71307	98771	139795
81	27316	27766	28150	29428	32195	36861	44187	54727	71595	99195	140435
82	27339	27802	28192	29481	32264	36960	44328	54925	71880	99615	141067
83	27362	27837	28233	29533	32332	37059	44468	55122	72162	100031	141692
84	27384	27872	28274	29584	32400	37157	44608	55318	72442	100442	142310
85	27406	27906	28314	29635	32468	37254	44747	55512	72719	100849	142921
86	27428	27939	28354	29685	32535	37351	44885	55704	72994	101252	143525
87	27449	27971	28393	29734	32601	37447	45022	55894	73266	101651	144122
88	27470	28003	28432	29783	32667	37542	45157	56083	73534	102046	144711
89	27491	28034	28470	29831	32733	37637	45291	56271	73800	102437	145293
90	27511	28065	28508	29879	32799	37731	45424	56458	74064	102823	145868
91	27531	28095	28545	29926	32864	37824	45557	56643	74325	103205	146436
92	27551	28125	28582	29973	32929	37917	45689	56826	74584	103583	146997
93	27570	28154	28618	30019	32993	38010	45820	57007	74841	103958	147551
94	27589	28182	28654	30065	33057	38103	45950	57187	75095	104329	148099
95	27607	28210	28689	30111	33120	38195	46079	57366	75346	104696	148641
96	27625	28237	28724	30156	33183	38286	46207	57544	75594	105059	149176

TABLE 9 (Continued)

CONFERENCE MODIFICATION OF CLASS 3 DISABILITY TABLE

$100{,}000\,S_x^{0/n}$ ON BASIS OF 2½% INTEREST

Months	AGE AT DISABLEMENT										
	20	25	30	35	40	45	50	55	60	65	70
97	27643	28264	28759	30201	33245	38376	46334	57720	75840	105418	149705
98	27661	28290	28793	30245	33307	38465	46460	57895	76084	105773	150227
99	27678	28316	28827	30289	33368	38554	46585	58068	76325	106124	150742
100	27695	28341	28860	30333	33429	38643	46709	58239	76564	106472	151251
101	27712	28366	28893	30376	33489	38731	46833	58409	76801	106816	151753
102	27729	28391	28926	30419	33549	38819	46956	58577	77035	107156	152249
103	27745	28415	28958	30462	33609	38907	47078	58744	77267	107492	152739
104	27761	28439	28990	30505	33668	38994	47199	58910	77497	107825	153223
105	27777	28462	29022	30547	33727	39080	47319	59075	77725	108154	153700
106	27793	28485	29053	30589	33786	39165	47438	59238	77950	108480	154171
107	27809	28508	29084	30630	33845	39250	47556	59400	78173	108802	154637
108	27824	28531	29115	30671	33903	39334	47673	59561	78394	109121	155097
109	27839	28553	29145	30712	33961	39418	47789	59720	78613	109436	155550
110	27854	28575	29175	30753	34019	39502	47905	59877	78830	109747	155998
111	27869	28597	29205	30794	34076	39585	48020	60033	79044	110055	156440
112	27884	28619	29234	30834	34133	39668	48134	60188	79256	110359	156876
113	27898	28640	29263	30874	34189	39750	48248	60341	79466	110660	157306
114	27913	28661	29292	30914	34245	39832	48361	60493	79674	110957	157730
115	27927	28682	29321	30953	34301	39913	48473	60644	79880	111250	158148
116	27941	28703	29349	30993	34357	39994	48584	60794	80084	111541	158561
117	27955	28724	29377	31032	34412	40074	48694	60942	80286	111828	158968
118	27969	28745	29405	31071	34467	40153	48803	61089	80485	112112	159370
119	27983	28765	29433	31109	34522	40232	48911	61235	80683	112393	159766
120	27997	28785	29460	31147	34576	40311	49018	61379	80878	112671	160157
121	28011	28805	29487	31185	34631	40389	49125	61522	81072	112946	160542
122	28025	28825	29514	31222	34685	40467	49231	61664	81263	113218	160922
123	28039	28844	29541	31260	34739	40544	49336	61805	81452	113487	161296

TABLE 10

CONFERENCE MODIFICATION OF CLASS 3 DISABILITY TABLE

NUMBER DISABLED AT END OF PERIOD SHOWN

BASED ON 100,000 ACTIVE LIVES AT EACH AGE

Months	AGE AT DISABLEMENT										
	20	25	30	35	40	45	50	55	60	65	70
12	182	187	190	199	232	290	389	539	779	1150	1721
18	136	144	150	159	188	241	325	451	654	972	1464
24	112	122	130	140	164	210	284	396	579	871	1328
30	93	107	114	125	145	185	253	356	524	797	1225
36	77	95	102	111	130	167	233	328	488	741	1143
42	65	85	92	100	118	155	218	308	459	698	1076
48	55	76	83	91	109	145	206	292	435	662	1018
54	47	68	75	84	101	137	196	278	414	626	964
60	41	61	68	77	95	131	189	267	395	596	913
66	36	55	63	72	90	126	182	257	378	566	865
72	32	49	57	68	86	122	176	247	362	537	817
78	29	44	53	64	83	119	170	239	346	510	772
84	26	40	49	60	81	116	165	231	330	486	729
90	24	36	45	57	78	113	160	223	315	462	687
96	22	33	42	55	76	110	155	215	301	439	647
102	20	30	40	53	74	108	150	207	287	416	607
108	19	28	38	51	72	105	146	199	274	395	569
114	18	27	36	50	71	103	141	191	261	373	532
120	18	26	35	48	69	100	137	183	248	353	496
123	18	26	35	48	69	99	135	180	242	344	478

For figures for intermediate periods see Table Ic of the Report of the Non-Cancellable Reserves Committee.

PURE PREMIUM TRENDS IN WORKMEN'S COMPENSATION

BY

R. P. GODDARD

Workmen's Compensation Insurance is generally considered to be a line which is subject to unpredictable fluctuations. There are "good periods" of greater or less duration followed by "bad periods" of equally uncertain length. It is recognized that changes in wages and hours of labor as well as changes in the volume of industrial activity may have some effect on compensation costs but precisely what these effects are or when they will be felt are extremely difficult to determine.

The purpose of this paper is to present the results of a method of analyzing a fifteen-year period in two of the more important states. The method followed has been to separate the experience into industry groups and then to determine the indemnity and medical pure premiums as well as the indemnity claim frequency and indemnity average costs for each policy year. The analysis covers the period from 1928 to 1942 for New York and from 1929 to 1943 in Massachusetts. The period chosen in each state coincides with the period during which experience under the Unit Statistical Plan has been published.

The experience has been separated into six industry groups. The manufacturing group comprises all classifications within schedules 5 to 25; contracting, schedules 26 and 27; stevedoring, maritime and shipbuilding, schedules 28 through 30; commercial and clerical, schedules 34 and 35; care and custody, schedule 36 and all other, schedules 1 to 4, 31 to 33 and 37. Classifications which did not use payroll as the exposure base have been excluded throughout the entire period.

The losses used in calculating pure premiums and average costs are the actual losses as incurred without adjustment to reflect current benefit levels. The law amendments which have become effective during the period under review together with the estimated change in benefit level, as calculated by the National Council, are shown below:

Massachusetts		*New York*	
Effective Date	Estimated Effect	Effective Date	Estimated Effect
9/19/35	1.025	4/24/33	1.001
8/27/37	1.037	7/ 1/35	1.028
6/19/39	1.003	4/10/39	1.003
11/ 3/41	1.023	7/ 1/39	1.007
11/15/43*	1.024	7/ 1/40	1.008
		7/ 1/41	1.011
		7/ 1/42**	1.001
		7/ 1/43**	1.008

* Includes several amendments effective in August, 1943.
** Effective date of rate change incorporating amendments.

MASSACHUSETTS

The overall pure premiums which show an almost unbroken descent from policy year 1930 through 1943 are somewhat deceptive since they reflect the relatively rapid decrease in exposure of the high-rated groups such as contracting and stevedoring and maritime. It will be seen, for example, that the drop in total pure premium from 1931 to 1932 ($.72 to $.67) was not shown by any important individual industry group. This illustrates the effect which changes in distribution can have on a too-conglomerate average.

In the manufacturing group a remarkably even decrease in average pure premium is to be noted. Although there was a drop in almost every year, in no case was there a decrease of more than 10% in any one year. Such a condition might conceivably have been caused by a gradual withdrawal of heavy industry from the state but it is doubtful if any such withdrawal could have been gradual enough to produce the results shown. This possibility has been partially investigated by examining the trend of pure premiums for certain of the more important classifications and by calculating the average pure premium for the remainder. The classifications which were individually studied were those covering cotton spinning and weaving, wool spinning and weaving, cloth printing and boot or shoe manufacturing. These classifications accounted for approximately 27% of the total manufacturing payroll in 1929 dropping to 19% in 1941 and 16% in 1942. Since these classes have generally had lower pure premiums than the average, it appears that, if anything, there has been some withdrawal of light industry rather than heavy industry from the state. With these classifications excluded the average manufacturing pure premium dropped from a high of $.86 in 1930 to $.50 in 1942. Here again the decreases from one year to another were always less than 10%. It seems reasonable to conclude, therefore, that the freakishly smooth decrease in total pure premium for this group has not been caused by changes in the relative proportions of high-rated and low-rated classes.

The indemnity pure premium has decreased approximately 40% during the period whereas medical has dropped only about 20%. The average indemnity cost has not shown any decided trend but what trend there is appears to be slightly downward. The indemnity claim frequency calculated in terms of $100,000 of payroll shows roughly the same downward trend as the indemnity pure premiums with a net decrease during the period of approximately 40%. Although the claim frequencies for policy years 1942 and 1943 are somewhat lower than those for the five years immediately preceding, it should be noted that an even larger drop percentage-wise occurred in the three years following 1933.

The contracting industry group, probably because of its smaller volume, does not develop the same smooth pure premium curve as shown by manu-

facturing. Changes from one year to another have generally been less than 20% except in 1940. In this year there was a decided decrease in claim frequency which has continued through 1943. An investigation of the three largest classes, masonry, carpentry, and painting and decorating, which together account for some 25% of the total payroll exposure, indicates that these classes alone were not responsible.

Similarly with respect to stevedoring and maritime, a drop of more than 50% in average pure premium occurred in 1941. This was not due, as might at first be supposed, to the increase in shipbuilding operations but was shared generally by all classifications in this group. For the stevedoring classification alone the total pure premium dropped from $5.23 in policy year 1940 to $2.33 in 1941, probably because of the introduction of the practice of paying double time for the loading of explosives.

The pure premiums for the commercial and all other groups exhibit essentially the same trend as those for manufacturing with a slightly greater fluctuation from year to year. The one industry group in the state which has not indicated a downward trend is that composed of the care and custody classes. The trend for this group appears to be slightly upward through 1938 after which it declined at approximately the same rate as manufacturing.

In general, the Massachusetts pure premiums by industry group as shown on a semi-logarithmic graph present a fairly consistent picture. If we take the year 1932 as the depth of the depression, since in that year the total payroll was at its lowest point, it appears that the effects of the depression were felt primarily by contracting and stevedoring and hardly at all by other industries. It is very difficult to detect from the pure premiums themselves when the law amendments were passed or what effect, if any, the war has had on compensation costs.

New York

Although the total manufacturing payroll in New York has been twice as great as that in Massachusetts and losses have been three times as large, the pure premiums for this group have exhibited somewhat less consistency than in Massachusetts. However, there was only one year in which the change from one year to another was greater than 10%. This was in policy year 1935 when the total pure premium was $.95 as compared with $.86 in 1934. This appears to be due in part to the immaturity of the data which were taken from the second reporting under the Unit Statistical Plan and in part to an increase in the proportion of higher rated classes. It apparently was not caused by the changes in classification phraseology which took place in the manual revision of 1934.

Although the New York manufacturing pure premium was relatively stable throughout these years it would not be proper to assume that every classification within this group enjoyed the same stability. An important exception to the general rule was classification 2501 covering clothing manufacturing. This classification has approximately 20% of the total payroll of manufacturing in New York State and is larger than the manufacturing industry groups in most other states. Because of its importance it has been given a special place in the New York Exhibit.

The average cost of indemnity claims in clothing manufacturing decreased slowly from policy year 1928 through 1932 and then increased quite steadily through policy year 1942. The changes in average indemnity cost for this classification were very similar to those for manufacturing as a whole. As a matter of fact, the average indemnity cost in all industry groups in New York indicated approximately the same rate of decrease and increase throughout the period. The extremely rapid rise in total pure premium for this classification from 1928 through 1932 was caused by the increase in the claim frequency as well as by the increase in medical pure premium. The relationship between claim frequency and average wages is discussed later in this paper.

The contracting pure premiums in New York fluctuate somewhat less widely than in Massachusetts as might be expected in view of the larger volume. The average pure premium decreased approximately 30% from 1941 to 1942 and this decrease did not appear to be attributable solely to any of the more important classes. It is of interest to note that the peak in contracting pure premiums was not reached during the depth of the depression but was reached in 1935 and 1936 with a secondary peak in 1939.

Similarly for stevedoring and maritime the high point in pure premiums was reached in 1939 although this peak was only slightly higher than that reached in 1932 when the indemnity claim frequency also reached its highest point.

The pure premium trends for the other three industry groups, commercial, care and custody, and all other show comparatively little fluctuation from year to year although each appears to be following its own course. The average pure premium for commercial has been almost constant since policy year 1935, that for care and custody has been gradually increasing while that for the all other industry group shows a tendency to decline slightly.

The pure premium curves for each of these two states seem to be straight enough for most industry groups to be dignified as "trends." The pure premium curves for Massachusetts are generally downward whereas those for New York are generally horizontal. The primary reason for this difference between states appears to be that in Massachusetts the average claim cost

has tended to remain constant whereas in New York the average cost has increased with increasing wages.

Effect of Wage Changes

Since compensation benefits are expressed in terms of weekly wages, it appears logical to suppose that changes in wages would directly affect average claim costs, particularly average indemnity costs. This appears to have been the case in New York but not in Massachusetts. Furthermore, if the accident frequency rate per man-hour is a constant there should be an inverse correlation between accident frequency and hourly wages.

Unfortunately, it is difficult to obtain data on hourly wages for most industries since many employees are paid by the piece rather than by the hour. Furthermore, for the purposes of this analysis it is unfortunate that weekly or hourly wages are not available by policy year. This is not too important an obstacle, however, since the average of two calendar years should be roughly equivalent to one policy year. The data which have been obtained are shown on Exhibit III. The weekly wages for New York manufacturing cover representative factories reporting to the New York Department of Labor and are based on the wages of office and shop workers for years through 1934 and on the wages of shop workers alone for 1935 and later years. The weekly wages for clothing manufacturing were also obtained from the New York Department of Labor and are based on data for approximately half of the industry. These figures were compiled on the same basis as those for manufacturing as a whole and include both office and shop workers prior to 1935 and shop workers only for 1935 and later. The weekly wages for Massachusetts manufacturing are based on reports for the entire industry in Massachusetts including shipbuilding and other war industries. In this respect they are not comparable to the indemnity claim frequencies. Furthermore, the weekly wages for both New York and Massachusetts include the effect of overtime. For the later years, therefore, they are approximately 5% higher than they would be if calculated on a straight-time basis.

The hourly wages for Massachusetts contracting are based only on a small proportion of the total contracting industry in the state. This proportion amounted to approximately 16% in the later years. For years prior to 1939 only building construction was included but in later years data were obtained on highway, bridge, marine and other types of construction.

Admittedly, therefore, the weekly and hourly wages obtainable are not ideally suited for the purpose in hand. If reliable data on both weekly and hourly wages by policy year could be obtained, either for industry groups or for individual classifications, there would be no reason why these figures could not be substituted for those which have been used in this paper.

The changes in weekly wages for New York clothing manufacturing appear to offer no adequate explanation for the rapid increase and decrease in indemnity claim frequency for this classification. There was a decrease of approximately 28% in weekly wages in this industry between 1929 and 1933 but this was no greater than the decrease for New York manufacturing as a whole. In later years average wages in this industry increased somewhat more slowly than wages in other industries but the claim frequency decreased a great deal more rapidly. The changes in claim frequency do not appear to be mere random fluctuations especially in view of the size of this classification. The rapid increase in frequency through policy year 1932 might be ascribed to malingering in view of the fact that the wage scale was lower than that prevailing in New York at the time, were it not for the rapid decrease in frequency following policy year 1932. By 1940 the average wages were approximately as high as in 1929 but the claim frequency was 40% lower. It goes without saying that it was impossible to make rates prospectively for this classification which would produce a 60% loss ratio every year. For policy year 1932 the loss ratio was 148.1% and for policy year 1936 it was 29.4%. Although an adequate explanation for the behavior of this classification is still to be found it may be of some value to know that changes in wages do not provide the answer.

The indemnity claim frequency for New York and Massachusetts manufacturing as well as Massachusetts contracting have been entered on a graph and compared with the reciprocals of weekly or hourly wages. For the two manufacturing groups there appears to be close correlation between wages and claim frequency for the years 1934 through 1940 when wages were increasing fairly slowly. There is considerably less correlation in the period prior to 1934 when wages were decreasing or after 1940 when wages were increasing. In New York, for example, an increase of 42% in wages between 1940 and 1942 was accompanied by a decrease in claim frequency of only 8%. Even after taking into account the effect of overtime in increasing weekly wages it is obvious that there is very little correlation here.

The indemnity claim frequencies for Massachusetts contracting decrease in a fairly straight line from policy year 1932 through 1943 with the exception of policy years 1938 and 1939. The hourly wages also follow a straight line from calendar year 1933 through 1945 but the two lines do not coincide. In policy year 1943 the claim frequency was 32% of what it was in 1932. Such a decrease, to be explained by changes in hourly wages alone, would have required an increase of more than 300%. The actual increase was approximately 60%. There therefore appeared to be some long-term forces working toward the reduction of accidents and it is conceivable that the combined efforts of insurance companies, employers, and manufacturers of prod-

ucts designed to increase industrial safety may account for part of the improvement which is not due to increases in wages. If this is indeed the case it is not logical to assume that this improvement will continue indefinitely.

Indemnity Claim Frequency

In order to facilitate comparison the indemnity claim frequencies already shown in Exhibits I and II have been shown separately in Exhibit IV and in the accompanying graphs. The similarity of the two sets of curves, to the writer's mind at least, is quite striking. This similarity is particularly noteworthy for the years following 1932. From 1932 through 1942 the claim frequency for contracting, according to a straight line of least squares, decreased approximately 60% in Massachusetts and 57% in New York. For manufacturing the decreases in the two states were almost identical, 33% in Massachusetts and 34% in New York. Approximately the same decreases were shown for the commercial group in both states and for the care and custody classes the decrease was approximately 15% in each state. Without considerably more information than we now possess, it is impossible to explain why the decreases were not the same for all industry groups but the fact that the trends were almost identical in the two states appears to indicate that the same explanation, once it is found, will hold good for both states. If similar results were shown for a number of other states these trends would be valuable as guides to the future even though it might be impossible to reduce them to a simple formula.

Summary

A trend by its very nature is a rather amorphous thing, somewhat like an ocean current or a trade wind. It may be none the less real, if it is confirmed by a wide range of observations. In Massachusetts and New York the trends in pure premiums and claim frequency cannot be readily explained by changes in wages or by the rises and falls in industrial activity, as indicated by the total insured payroll. The fact that these trends are not purely fortuitous, however, is demonstrated, if not proved, by the similarity of the trends in both states. One corroborates the other. Since Massachusetts and New York are both large states and the experience studied in this paper covers a reasonably long period, it seems probable that the trends in other states would be similar to those here discussed. It is conceivable, however, that the experience in some states might be similar to that exhibited by clothing manufacturing in New York; this might be expected to be true in states which are dominated by a single industry. Furthermore, we might expect abnormal results in states which have only recently enacted compen-

sation laws, in view of the opinion which has been expressed that compensation costs tend to rise during the first few years after a new law has been passed. This theory could be tested by comparisons among a number of states in which the compensation laws had been in effect for varying periods.

The method used in this paper has been applied principally in automobile insurance, to analyse separately the changes in frequency and average cost. As applied to compensation insurance the method could be made much more extensive, since separate analyses of medical costs and frequencies could be made, as well as of indemnity costs and frequencies by type of injury. Individual classifications or groups of classifications, not necessarily those used in this paper, could be studied separately, if conditions affecting these classifications appeared to be different from those affecting industry generally.

Although the compensation insurance business has been in existence for more than thirty years, during which time a large volume of statistics has been collected, in many respects we are still in the fact-finding stage. It is still possible to bring forward new theories which cannot be proved or disproved by loss ratios alone. If figures similar to those discussed here could be compiled for a number of states we would have available, in usable form, a wealth of material against which such theories could be tested.

EXHIBIT I

MASSACHUSETTS CLASSIFICATION EXPERIENCE

By Industry Group

MANUFACTURING, SCHEDULES 5-25

Pol. Year	No. of Classes	Payroll (in Thousands)	Indemnity		Losses		Pure Premiums		
			Claim Freq.	Avge. Cost	Indemnity	Medical	Ind.	Med.	Total
1929	375	668,054	2.68	191	3,423,426	1,562,199	.51	.24	.75
1930	371	547,533	2.48	221	3,005,285	1,297,746	.55	.24	.79
1931	373	430,549	2.54	197	2,151,486	1,046,946	.50	.24	.74
1932	367	336,561	2.64	183	1,621,671	858,867	.48	.26	.74
1933	368	391,874	2.52	178	1,751,102	986,403	.45	.25	.70
1934	361	424,196	2.23	190	1,794,643	1,014,105	.42	.24	.66
1935	323	469,950	2.24	188	1,979,732	1,125,661	.42	.24	.66
1936	310	540,027	2.10	177	2,002,978	1,249,203	.37	.23	.60
1937	318	516,871	1.82	194	1,832,035	1,137,836	.35	.22	.57
1938	317	478,954	1.79	193	1,652,521	1,055,433	.35	.22	.57
1939	317	548,318	1.78	187	1,823,606	1,272,839	.33	.23	.56
1940	317	662,798	1.77	164	1,920,862	1,461,791	.29	.22	.51
1941	318	927,052	1.77	171	2,802,706	2,043,394	.30	.22	.52
1942	325	1,204,672	1.63	181	3,570,009	2,282,870	.30	.19	.49
1943	326	1,297,173	1.58	204	4,178,548	2,388,826	.32	.19	.51
Total		9,444,582	2.00	188	35,510,610	20,784,119	.38	.22	.60

CONTRACTING, SCHEDULES 26 AND 27

1929	90	97,131	6.64	300	1,933,485	647,092	1.99	.67	2.66
1930	94	82,028	7.14	303	1,774,026	572,836	2.16	.70	2.86
1931	94	58,065	8.36	311	1,510,684	509,261	2.60	.88	3.48
1932	93	32,799	8.66	302	857,263	289,097	2.62	.88	3.50
1933	90	30,369	8.07	264	646,117	243,710	2.13	.80	2.93
1934	90	36,951	7.51	318	881,742	314,236	2.39	.85	3.24
1935	78	40,774	6.29	304	779,340	298,385	1.91	.73	2.64
1936	74	52,500	5.63	331	979,474	354,819	1.87	.67	2.54
1937	72	51,356	5.19	343	914,382	373,276	1.78	.73	2.51
1938	71	54,598	5.78	314	991,944	419,759	1.82	.77	2.59
1939	73	57,075	5.45	335	1,040,485	439,670	1.82	.77	2.59
1940	79	90,968	3.99	304	1,104,604	558,687	1.21	.61	1.83
1941	77	91,384	3.67	320	1,071,799	507,789	1.17	.56	1.73
1942	75	86,814	2.91	342	861,863	385,577	.99	.45	1.44
1943	72	70,044	2.78	333	648,982	273,273	.93	.39	1.32
Total		932,856	5.48	313	15,996,190	6,187,467	1.71	.66	2.38

STEVEDORING AND MARITIME, SCHEDULES 28-30

1929	40	12,041	9.18	270	297,869	87,152	2.47	.73	3.20
1930	40	12,230	8.14	199	198,342	84,100	1.62	.69	2.31
1931	41	7,083	11.63	238	195,841	63,071	2.77	.89	3.66
1932	42	3,040	16.81	149	76,349	30,224	2.51	1.00	3.51
1933	37	3,239	15.71	212	107,894	39,888	3.33	1.23	4.56
1934	32	3,665	11.81	265	114,798	35,326	3.13	.97	4.10
1935	33	4,057	11.93	212	102,675	52,730	2.53	1.30	3.83
1936	32	4,857	10.87	218	114,941	50,339	2.36	1.04	3.40
1937	33	5,200	10.27	233	124,220	54,571	2.39	1.05	3.44
1938	31	4,841	9.44	188	85,903	39,606	1.77	.82	2.59
1939	32	5,595	10.08	202	113,948	50,723	2.03	.91	2.94
1940	31	6,032	8.47	280	143,252	70,925	2.37	1.18	3.55
1941	32	10,089	5.28	212	113,080	63,375	1.12	.63	1.75
1942	32	27,299	3.51	233	222,984	152,648	.82	.56	1.38
1943	28	33,965	3.07	286	298,263	175,429	.88	.51	1.39
Total		143,233	6.98	231	2,310,359	1,050,107	1.61	.74	2.35

COMMERCIAL, SCHEDULES 34 AND 35

Pol. Year	No. of Classes	Payroll (in Thousands)	Indemnity		Losses		Pure Premiums		
			Claim Freq.	Avge. Cost	Indemnity	Medical	Ind.	Med.	Total
1929	89	558,626	.98	181	985,185	506,020	.18	.09	.27
1930	94	540,885	.98	180	951,232	490,256	.18	.09	.27
1931	85	485,289	.92	178	792,808	432,204	.16	.09	.25
1932	83	392,535	.98	166	640,175	393,909	.16	.10	.26
1933	82	399,331	.98	168	655,972	411,487	.17	.10	.27
1934	88	419,126	.92	164	631,016	410,691	.15	.10	.25
1935	80	431,633	.88	177	675,783	419,258	.15	.10	.25
1936	78	467,591	.82	164	627,759	447,994	.13	.10	.23
1937	80	491,162	.80	177	691,617	470,464	.14	.10	.24
1938	83	487,008	.74	187	674,984	463,923	.14	.09	.23
1939	82	511,862	.73	158	590,524	472,241	.12	.09	.21
1940	82	548,270	.71	172	665,212	569,394	.12	.11	.23
1941	82	606,325	.68	167	692,796	543,239	.11	.09	.20
1942	81	647,980	.63	194	793,829	523,388	.12	.08	.20
1943	82	718,436	.61	192	835,433	534,857	.12	.07	.19
Total		7,706,059	.81	175	10,904,325	7,089,325	.14	.09	.23

CARE, ETC., SCHEDULE 36

1929	29	64,207	2.69	169	292,829	135,617	.46	.21	.67
1930	26	64,208	2.66	167	285,856	145,410	.44	.23	.67
1931	26	59,672	2.63	177	277,674	134,380	.46	.23	.69
1932	24	50,802	2.68	165	225,027	114,349	.44	.23	.67
1933	24	50,380	2.87	170	245,980	120,914	.49	.24	.73
1934	27	53,307	2.74	174	254,207	148,300	.48	.28	.76
1935	22	55,815	2.72	149	224,274	150,669	.40	.27	.67
1936	21	60,628	2.86	145	251,546	163,001	.41	.27	.68
1937	20	64,821	2.68	189	329,310	187,717	.51	.29	.80
1938	22	65,314	2.69	188	331,575	192,560	.51	.29	.80
1939	23	67,519	2.58	159	276,464	194,848	.41	.29	.70
1940	22	69,516	2.58	169	303,456	208,192	.44	.30	.74
1941	22	76,553	2.58	135	266,739	207,625	.35	.27	.62
1942	23	86,919	2.28	192	380,391	222,392	.44	.25	.69
1943	23	111,257	1.84	213	435,488	227,520	.39	.21	.60
Total		1,000,918	2.55	171	4,380,816	2,553,494	.44	.25	.69

ALL OTHER

1929	47	104,126	3.92	205	834,662	348,782	.80	.34	1.14
1930	46	100,169	3.65	242	886,970	359,343	.88	.36	1.24
1931	48	92,039	3.56	233	763,797	314,332	.83	.34	1.17
1932	47	77,746	3.35	245	637,978	259,864	.82	.33	1.15
1933	48	75,749	3.60	223	608,635	265,540	.80	.35	1.15
1934	54	73,088	3.03	200	442,927	219,101	.61	.30	.91
1935	48	70,423	2.84	247	493,684	228,370	.70	.33	1.03
1936	46	74,838	2.74	211	431,355	245,164	.57	.33	.90
1937	45	78,445	2.47	221	428,016	244,469	.55	.31	.86
1938	47	80,024	2.66	246	522,905	263,430	.65	.33	.98
1939	49	81,657	2.46	205	410,676	248,882	.50	.31	.81
1940	54	84,487	2.42	252	515,138	275,773	.61	.33	.94
1941	52	94,347	2.33	206	452,897	303,069	.48	.32	.80
1942	48	101,220	2.22	227	510,637	265,473	.51	.26	.77
1943	47	115,581	2.05	240	568,211	287,677	.49	.25	.74
Total		1,303,939	2.88	227	8,508,488	4,129,269	.65	.32	.97

GRAND TOTAL (Excluding Per Capita and Flying Hours)

Pol. Year	No. of Classes	Payroll (in Thousands)	Indemnity		Losses		Pure Premiums		
			Claim Freq.	Avge. Cost	Indemnity	Medical	Ind.	Med.	Total
1929	670	1,504,185	2.44	212	7,767,456	3,286,862	.51	.22	.73
1930	671	1,347,053	2.31	228	7,101,711	2,949,691	.53	.22	.75
1931	667	1,132,697	2.29	220	5,692,290	2,500,194	.50	.22	.72
1932	656	893,483	2.24	202	4,058,463	1,946,310	.45	.22	.67
1933	649	950,942	2.20	192	4,015,700	2,067,942	.42	.22	.64
1934	652	1,010,333	2.00	204	4,119,333	2,141,759	.41	.21	.62
1935	584	1,072,652	1.95	204	4,255,488	2,275,073	.40	.21	.61
1936	561	1,200,441	1.87	197	4,408,053	2,510,520	.37	.21	.58
1937	568	1,207,855	1.67	214	4,319,580	2,468,333	.36	.20	.56
1938	571	1,170,739	1.68	217	4,259,832	2,434,711	.36	.21	.57
1939	576	1,272,026	1.64	204	4,255,703	2,679,203	.34	.21	.55
1940	585	1,462,071	1.61	197	4,652,524	3,144,762	.32	.21	.53
1941	583	1,805,750	1.58	189	5,400,017	3,668,491	.30	.20	.50
1942	584	2,154,904	1.46	201	6,339,713	3,832,348	.29	.18	.47
1943	578	2,346,456	1.37	216	6,964,925	3,887,582	.30	.16	.46
Total		20,531,587	1.83	207	77,610,788	41,793,781	.38	.20	.58

NOTE: All data taken from fourth reportings under the Unit Statistical Plan, except as follows:

Policy Years

1936, 1939 and 1940	Third Report
1937 and 1941	Second Report
1938, 1942 and 1943	First Report

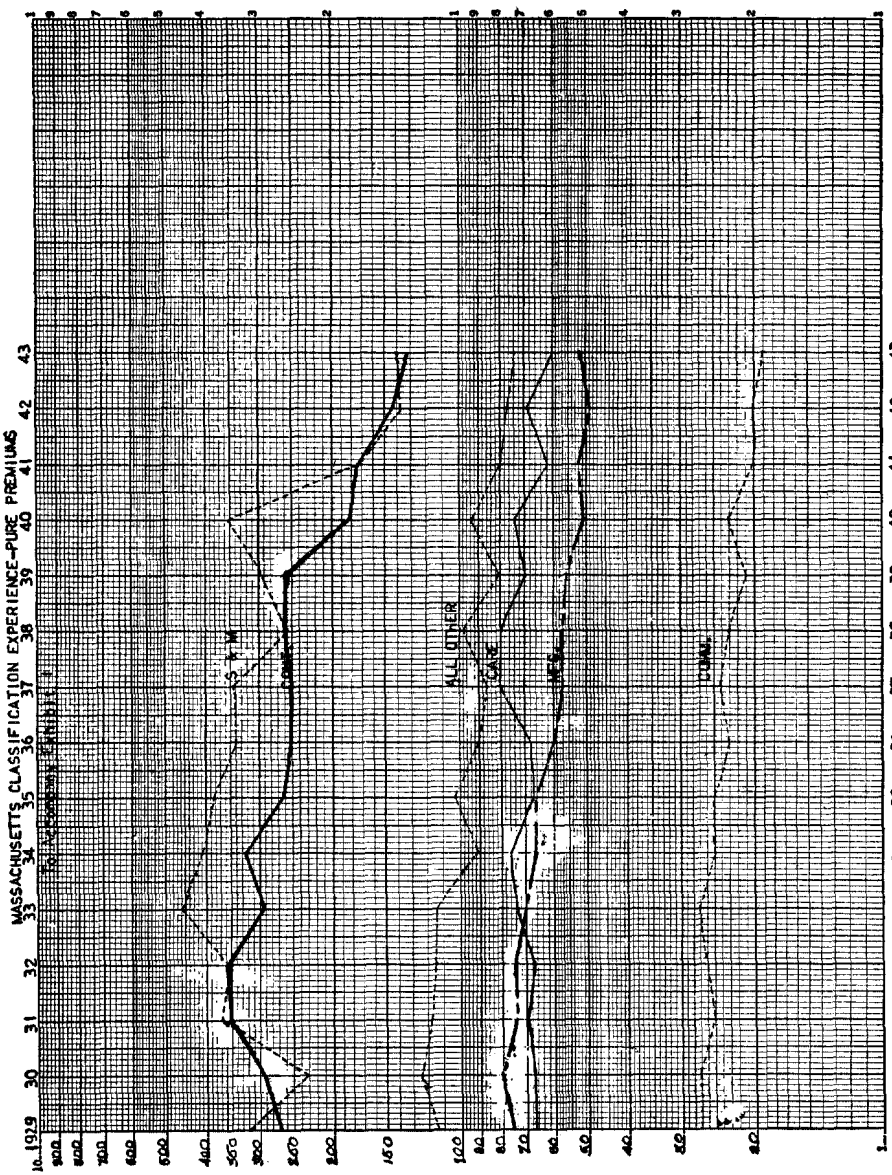

MASSACHUSETTS CLASSIFICATION EXPERIENCE—PURE PREMIUMS
ALL OTHER
CARP.
MFG.
CONST.
S. & W.
DOM.

EXHIBIT II

NEW YORK CLASSIFICATION EXPERIENCE

By Industry Group

MANUFACTURING, SCHEDULES 5-25

Pol. Year	No. of Classes	Payroll (In Thousands)	Indemnity		Losses		Pure Premiums		
			Claim Freq.	Avge. Cost	Indemnity	Medical	Ind.	Med.	Total
1928	445	1,425,462	2.26	269	8,676,608	3,670,468	.61	.26	.87
1929	441	1,382,446	2.26	272	8,515,292	3,645,610	.62	.26	.88
1930	438	1,149,396	2.23	261	6,681,738	3,228,815	.58	.28	.86
1931	435	910,381	2.54	238	5,517,524	2,992,949	.60	.33	.93
1932	435	690,442	2.83	223	4,360,165	2,499,857	.63	.36	.99
1933	438	817,749	2.54	226	4,692,934	2,758,223	.57	.34	.91
1934	432	922,434	2.18	252	5,074,771	2,861,332	.55	.31	.86
1935	393	1,030,431	2.19	283	6,381,056	3,382,550	.62	.33	.95
1936	376	1,222,854	2.12	290	7,534,750	4,053,501	.62	.33	.95
1937	373	1,188,964	1.94	303	6,991,886	3,859,803	.59	.32	.91
1938	374	1,187,363	1.82	311	6,731,326	3,883,066	.57	.32	.89
1939	375	1,300,936	1.80	327	7,663,974	4,379,841	.59	.34	.93
1940	379	1,565,768	1.78	344	9,590,265	5,348,176	.61	.34	.95
1941	384	2,142,258	1.70	354	12,900,946	6,782,987	.60	.32	.92
1942	382	2,802,404	1.64	371	17,052,019	7,422,051	.61	.26	.87
Total		19,739,288	2.02	296	118,365,254	60,769,229	.60	.31	.91

CONTRACTING, SCHEDULES 26 AND 27

1928	103	408,733	5.58	461	10,505,793	2,800,731	2.57	.69	3.26
1929	95	387,290	5.57	454	9,789,356	2,876,392	2.53	.74	3.27
1930	102	323,996	6.09	442	8,726,272	2,822,863	2.69	.87	3.56
1931	99	231,618	7.19	413	6,880,721	2,317,483	2.97	1.00	3.97
1932	98	137,293	7.77	359	3,828,409	1,409,832	2.79	1.03	3.82
1933	96	106,096	8.03	335	2,852,616	1,165,964	2.69	1.10	3.79
1934	92	111,140	7.45	403	3,339,699	1,267,660	3.01	1.14	4.15
1935	83	139,228	7.13	445	4,417,374	1,575,426	3.17	1.13	4.30
1936	80	196,680	6.27	514	6,334,197	2,124,865	3.22	1.08	4.30
1937	78	213,976	5.63	524	6,307,552	2,095,428	2.95	.98	3.93
1938	81	233,222	5.19	591	7,158,561	2,307,924	3.07	.99	4.06
1939	87	262,029	5.03	642	8,455,329	2,522,408	3.23	.96	4.19
1940	84	257,706	4.79	630	7,789,223	2,441,128	3.02	.95	3.97
1941	91	265,150	4.17	665	7,353,230	2,386,506	2.77	.90	3.67
1942	90	249,523	3.21	603	4,837,826	1,553,415	1.94	.62	2.56
Total		3,523,680	5.65	495	98,576,158	31,668,025	2.80	.90	3.70

STEVEDORING AND MARITIME, SCHEDULES 28-30

1928	61	40,021	8.76	327	1,147,294	319,860	2.87	.80	3.67
1929	60	43,789	8.22	351	1,263,735	359,452	2.89	.82	3.71
1930	61	36,182	8.01	362	1,049,977	280,295	2.99	.78	3.68
1931	60	26,433	7.80	333	687,350	201,048	2.60	.76	3.36
1932	56	19,318	9.01	337	586,680	194,185	3.04	1.00	4.04
1933	50	22,919	8.94	330	676,507	197,628	2.95	.86	3.81
1934	36	24,224	7.64	352	650,661	185,640	2.69	.76	3.45
1935	35	28,492	7.45	335	711,401	234,682	2.50	.82	3.32
1936	34	34,680	7.00	412	1,000,128	298,050	2.88	.86	3.74
1937	35	33,899	6.19	415	870,216	273,865	2.57	.80	3.37
1938	35	30,283	6.54	455	902,291	280,496	2.98	.93	3.91
1939	35	33,942	7.27	438	1,082,053	351,513	3.19	1.03	4.22
1940	35	50,122	6.45	411	1,329,422	476,346	2.65	.95	3.60
1941	34	93,181	5.07	459	2,167,744	746,280	2.33	.80	3.13
1942	34	167,295	3.91	473	3,098,193	980,402	1.85	.59	2.44
Total		684,780	6.33	398	17,223,652	5,379,742	2.52	.78	3.30

COMMERCIAL, SCHEDULES 34 AND 35

Pol. Year	No. of Classes	Payroll (in Thousands)	Indemnity		Losses		Pure Premiums		
			Claim Freq.	Avge. Cost	Indemnity	Medical	Ind.	Med.	Total
1928	92	2,054,625	.69	281	3,958,993	1,786,996	.20	.08	.28
1929	95	2,198,325	.69	299	4,568,819	2,087,029	.21	.09	.30
1930	101	2,178,482	.73	269	4,272,698	2,190,786	.20	.10	.30
1931	92	1,958,633	.80	258	4,058,013	2,244,927	.21	.11	.32
1932	96	1,655,836	.83	252	3,444,367	1,959,248	.21	.12	.33
1933	101	1,663,711	.84	279	3,876,204	2,164,170	.23	.13	.36
1934	96	1,724,331	.76	318	4,162,528	2,159,762	.24	.13	.37
1935	90	1,817,930	.77	330	4,608,428	2,434,268	.25	.14	.39
1936	87	1,975,467	.71	340	4,790,403	2,574,990	.24	.13	.37
1937	88	2,080,302	.68	351	4,965,844	2,755,202	.24	.13	.37
1938	89	2,085,144	.65	382	5,150,990	2,857,030	.25	.13	.38
1939	89	2,172,407	.65	366	5,164,317	3,005,476	.24	.14	.38
1940	91	2,343,189	.65	365	5,576,024	3,296,529	.24	.14	.38
1941	94	2,578,955	.60	381	5,898,453	3,380,208	.23	.13	.36
1942	96	2,717,498	.55	409	6,093,115	2,922,125	.22	.11	.33
Total		31,204,835	.70	325	70,589,196	37,818,746	.23	.12	.35

CARE, ETC., SCHEDULE 36

1928	27	342,404	2.26	275	2,124,027	850,676	.62	.25	.87
1929	30	375,074	2.33	282	2,462,892	1,039,091	.65	.28	.93
1930	25	404,922	2.37	256	2,448,964	1,177,831	.61	.29	.90
1931	24	377,382	2.57	249	2,414,968	1,232,745	.64	.33	.97
1932	23	331,061	2.71	236	2,119,630	1,173,673	.64	.35	.99
1933	24	346,814	2.91	241	2,432,008	1,361,863	.70	.39	1.09
1934	27	361,634	2.73	233	2,306,688	1,363,167	.64	.37	1.01
1935	26	381,569	2.71	276	2,852,972	1,587,073	.75	.41	1.16
1936	24	415,565	2.63	271	2,964,237	1,700,178	.71	.41	1.12
1937	24	441,256	2.55	281	3,162,224	1,864,651	.72	.42	1.14
1938	25	450,408	2.49	295	3,300,652	1,985,196	.73	.44	1.17
1939	26	477,540	2.40	314	3,586,299	2,083,800	.75	.44	1.19
1940	26	485,403	2.44	338	4,009,705	2,273,650	.82	.47	1.29
1941	28	497,441	2.53	343	4,322,905	2,464,235	.87	.49	1.36
1942	28	531,620	2.43	371	4,796,489	2,318,212	.90	.44	1.34
Total		6,220,093	2.53	288	45,304,660	24,476,041	.73	.39	1.12

ALL OTHER

1928	58	216,161	3.58	397	3,073,221	948,441	1.42	.44	1.86
1929	59	230,525	3.57	408	3,355,617	1,115,423	1.46	.48	1.94
1930	62	226,687	3.73	402	3,400,057	1,127,922	1.50	.50	2.00
1931	64	204,513	3.77	355	2,737,472	1,088,581	1.34	.53	1.87
1932	67	167,710	3.88	317	2,062,635	889,549	1.23	.53	1.76
1933	69	165,565	3.81	348	2,194,331	864,905	1.33	.52	1.85
1934	69	161,065	3.44	385	2,135,773	874,520	1.33	.54	1.87
1935	59	165,857	3.36	368	2,053,881	893,402	1.24	.54	1.78
1936	59	182,951	3.39	423	2,622,198	1,020,247	1.43	.56	1.99
1937	61	197,952	3.01	412	2,455,157	1,053,637	1.24	.53	1.77
1938	59	204,835	2.87	432	2,538,077	1,042,335	1.24	.51	1.75
1939	59	214,951	2.78	420	2,512,115	1,074,450	1.17	.50	1.67
1940	63	226,278	2.76	468	2,918,289	1,154,427	1.29	.51	1.80
1941	65	237,903	2.65	450	2,839,000	1,118,512	1.19	.47	1.66
1942	61	257,696	2.37	577	3,532,637	1,077,967	1.37	.42	1.79
Total		3,060,649	3.23	409	40,430,460	15,344,318	1.32	.50	1.82

GRAND TOTAL (Excluding Building Wrecking, Per Capita, Cabs, Flying Hours, and Man Days)

Pol. Year	No. of Classes	Payroll (in Thousands)	Indemnity		Losses		Pure Premiums		
			Claim Freq.	Avge. Cost	Indemnity	Medical	Ind.	Med.	Total
1928	786	4,487,406	1.96	335	29,485,936	10,377,172	.66	.23	.89
1929	780	4,617,449	1.92	338	29,955,711	11,122,997	.65	.24	.89
1930	789	4,319,665	1.90	324	26,579,706	10,828,512	.62	.25	.87
1931	774	3,708,960	2.02	297	22,296,048	10,077,733	.60	.27	.87
1932	775	3,001,660	2.04	268	16,401,886	8,126,344	.55	.27	.82
1933	778	3,122,854	1.97	271	16,724,600	8,512,753	.54	.27	.81
1934	752	3,304,828	1.78	301	17,670,120	8,712,081	.54	.26	.80
1935	686	3,563,507	1.81	326	21,025,112	10,107,401	.59	.28	.87
1936	660	4,028,197	1.79	351	25,245,913	11,771,831	.63	.29	.92
1937	659	4,156,349	1.65	361	24,752,879	11,902,586	.59	.29	.88
1938	663	4,191,255	1.58	389	25,781,897	12,356,047	.62	.29	.91
1939	671	4,461,805	1.58	403	28,464,087	13,417,488	.64	.30	.94
1940	678	4,928,466	1.56	406	31,212,928	14,990,256	.63	.31	.94
1941	696	5,814,888	1.49	409	35,482,278	16,878,728	.61	.29	.90
1942	691	6,726,036	1.40	417	39,410,279	16,274,172	.59	.24	.83
Total		64,433,325	1.73	350	390,489,380	175,456,101	.61	.27	.88

NOTE: All data taken from second reportings under the Unit Statistical Plan, except for policy year 1942, for which the first reporting was used.

CLASS 2501—CLOTHING MANUFACTURING

Policy Year	Payroll (In Thousands)) (Incl. Ex-Med.)	Indemnity		Losses		Pure Premiums		
		Claim Freq.	Avge. Cost	Indemnity	Medical	Ind.	Med.	Total
1928	340,469	1.11	156	592,010	368,590	.17	.11	.28
1929	322,794	1.29	141	586,823	382,216	.18	.12	.30
1930	214,939	1.64	140	492,450	325,370	.23	.15	.38
1931	167,347	2.27	135	510,837	371,974	.31	.22	.53
1932	123,596	2.77	129	442,460	332,870	.36	.27	.63
1933	182,149	1.74	131	416,252	346,222	.23	.19	.42
1934	221,976	1.21	160	430,981	363,206	.20	.16	.36
1935	239,600	1.05	206	519,149	391,314	.22	.16	.38
1936	282,592	.87	199	488,036	415,033	.17	.15	.32
1937	254,256	.79	203	405,806	367,466	.16	.14	.30
1938	267,994	.77	242	498,523	419,741	.18	.16	.34
1939	281,686	.73	287	592,827	482,695	.21	.17	.38
1940	334,887	.75	303	759,932	578,931	.23	.17	.40
1941	396,336	.66	331	868,645	596,741	.22	.15	.37
1942	500,053	.57	330	936,926	596,045	.19	.12	.31
Total	4,130,674	1.06	196	8,541,657	6,338,414	.21	.15	.36

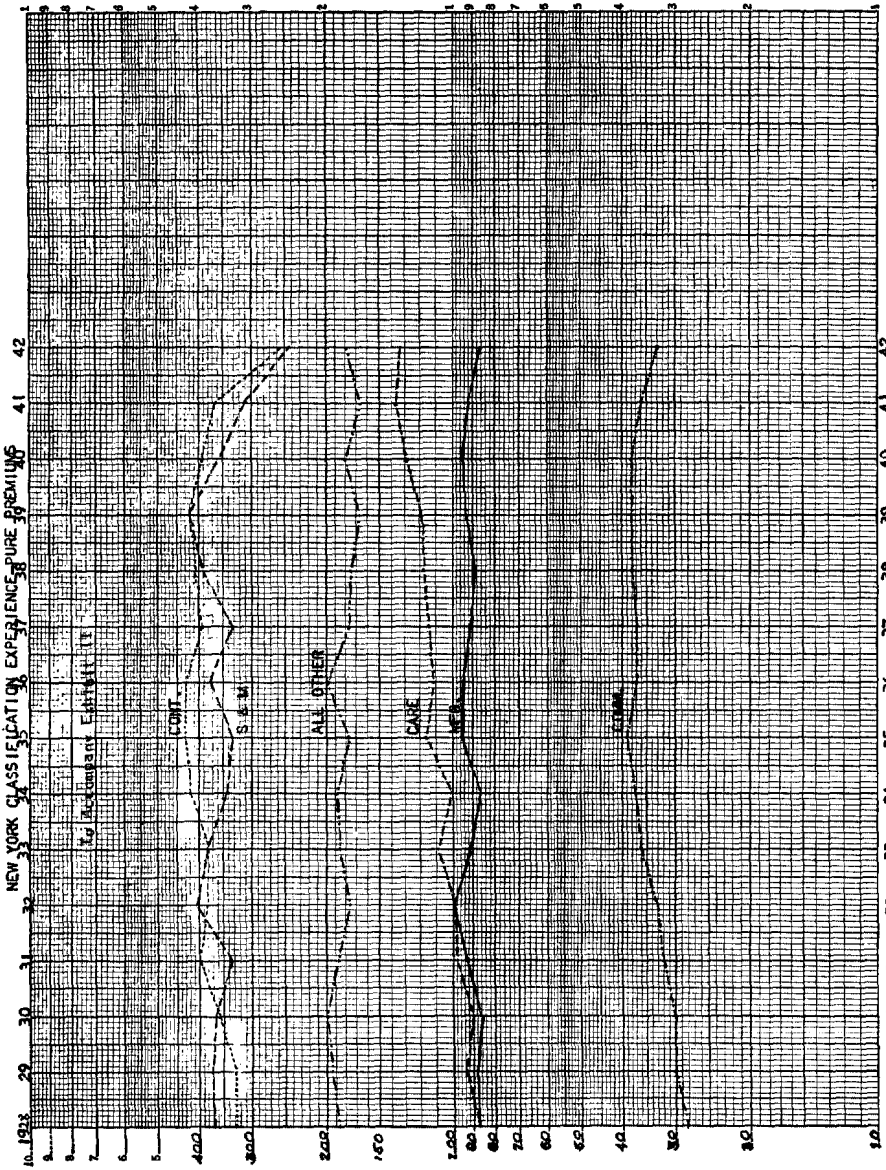

NEW YORK CLASSIFICATION EXPERIENCE PURE PREMIUMS
1 to Accompany Exhibit II
CONT.
S. & M.
ALL OTHER
CONST.
MFG.
CONST.

EXHIBIT III

COMPARISON OF WAGES AND CLAIM FREQUENCY

Year	N. Y. MANUFACTURING		N. Y. CLOTHING MFG.	
	Weekly Wages (Calendar Year)	Indemnity Claim Frequency (Policy Year)	Weekly Wages (Calendar Year)	Indemnity Claim Frequency (Policy Year)
1928	29.44	2.26	25.91	1.11
1929	29.99	2.26	26.00	1.29
1930	28.81	2.23	26.10	1.64
1931	26.42	2.54	23.92	2.27
1932	22.73	2.83	19.72	2.77
1933	21.83	2.54	18.81	1.74
1934	23.19	2.18	20.54	1.21
1935	23.19	2.19	22.71	1.05
1936	24.08	2.12	23.45	.87
1937	25.74	1.94	23.78	.79
1938	24.71	1.82	23.34	.77
1939	25.85	1.80	24.26	.73
1940	27.09	1.78	24.47	.75
1941	31.68	1.70	27.15	.66
1942	38.40	1.64	30.71	.57

Year	MASS. MANUFACTURING		MASS. CONTRACTING	
	Weekly Wages (Calendar Year)	Indemnity Claim Frequency (Policy Year)	Hourly Wages (Calendar Year)	Indemnity Claim Frequency (Policy Year)
1929	23.97	2.68	.986	6.64
1930	22.92	2.48	1.031	7.14
1931	20.99	2.54	.992	8.36
1932	18.34	2.64	.899	8.66
1933	17.10	2.52	.798	8.07
1934	18.54	2.23	.805	7.51
1935	19.35	2.24	.818	6.29
1936	20.56	2.10	.823	5.63
1937	21.57	1.82	.895	5.19
1938	20.53	1.79	.927	5.78
1939	20.80	1.78	.888	5.45
1940	23.59	1.77	.927	3.99
1941	27.38	1.77	1.010	3.67
1942	34.33	1.63	1.218	2.91
1943	39.82	1.58	1.266	2.78

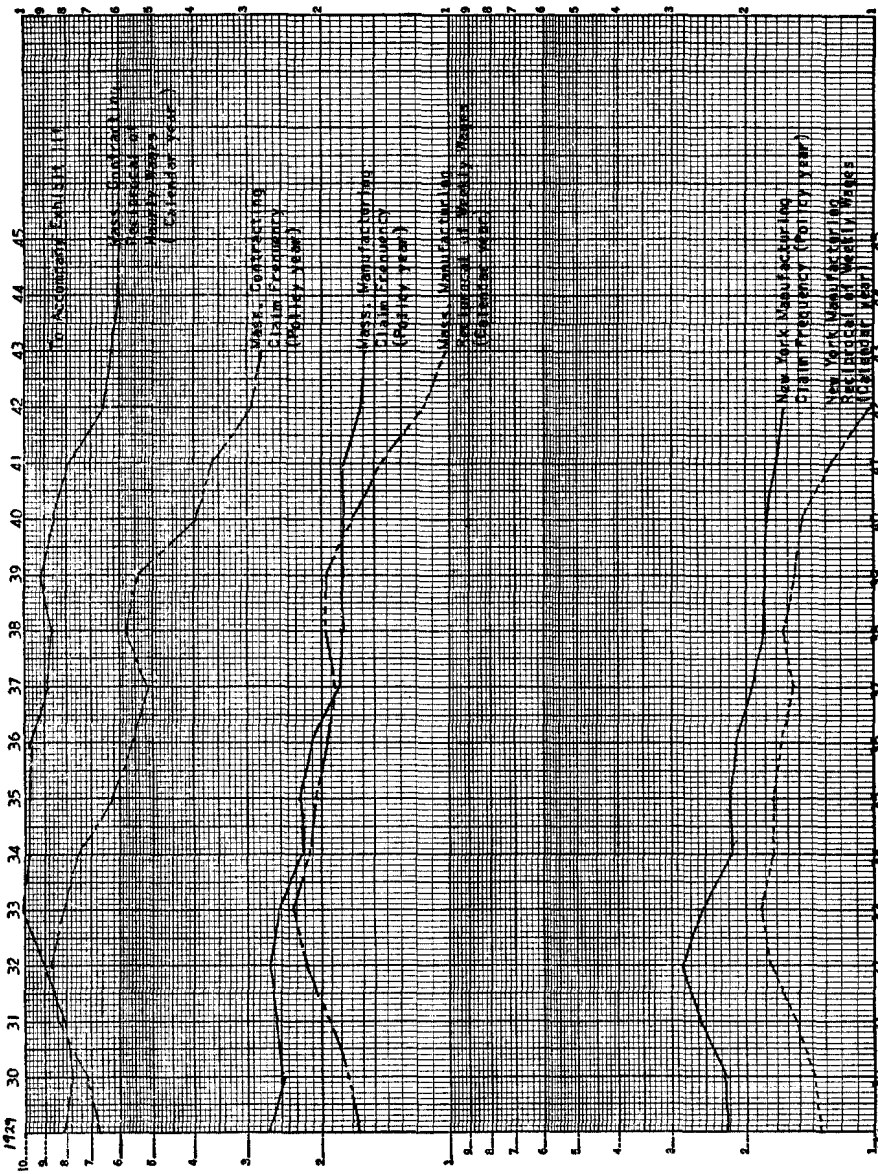

Mass. Contracting
Claim Frequency
(Policy Year)
Mass. Manufacturing
Claim Frequency
(Policy Year)
New York Manufacturing
Claim Frequency (Policy Year)
1929

EXHIBIT IV

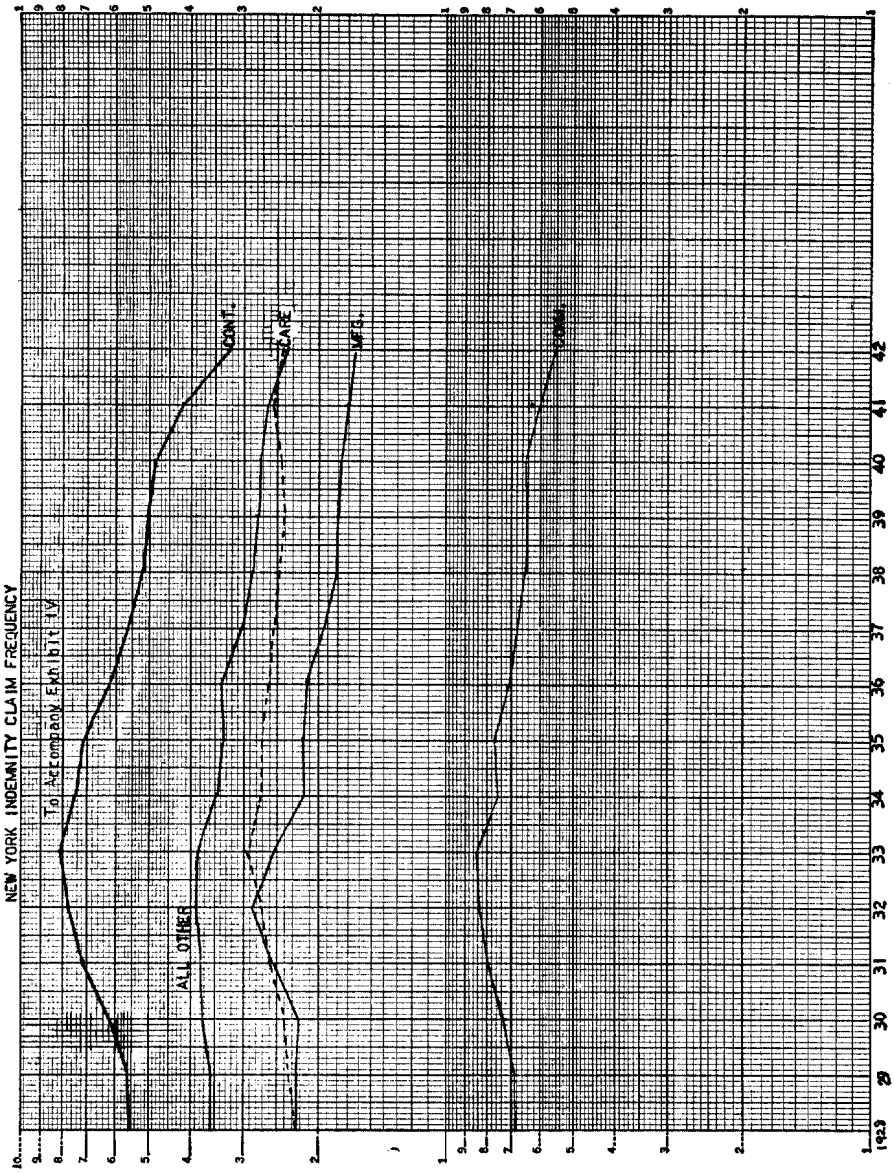
INDEMNITY CLAIM FREQUENCY

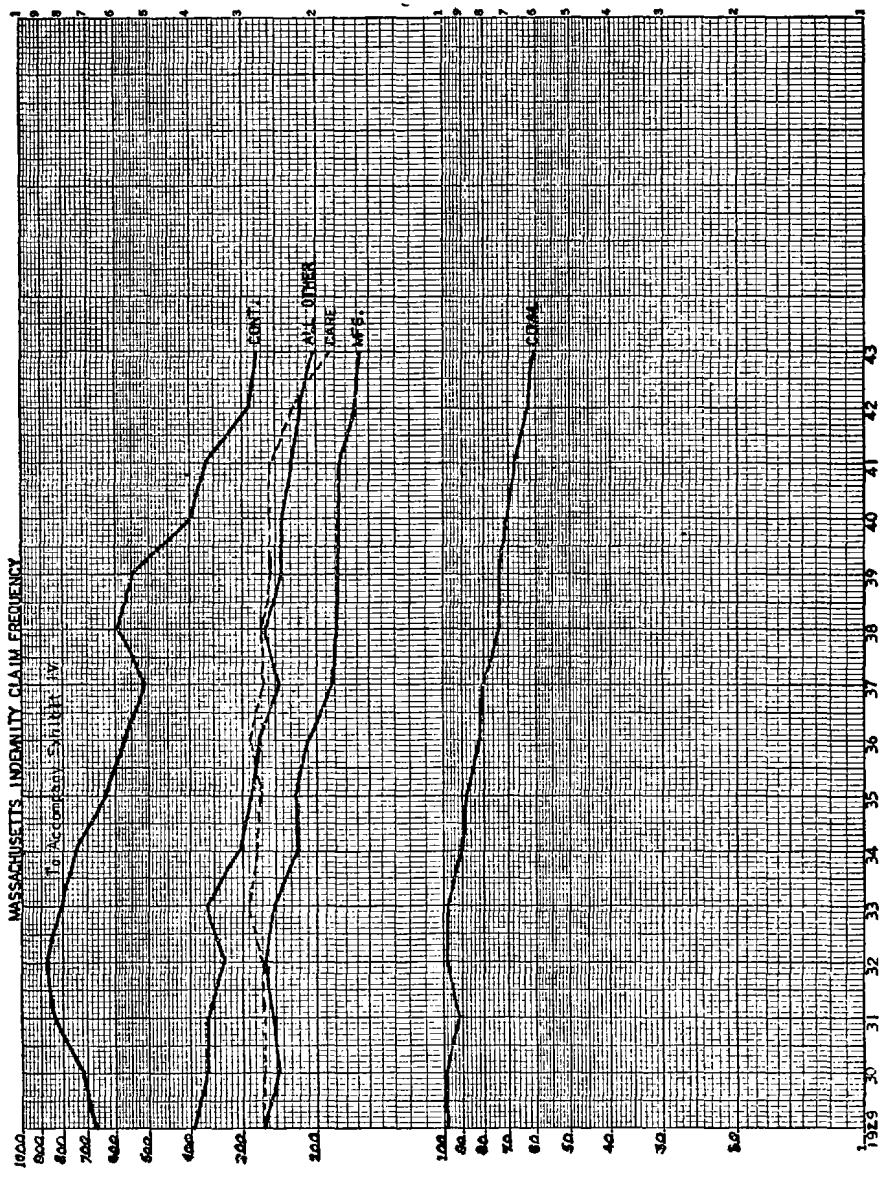
MASSACHUSETTS

Pol. Year	INDUSTRY GROUP					
	Mfg.	Contr.	S. & M.	Comm.	Care, etc.	All Other
1929	2.68	6.64	9.18	.98	2.69	3.92
1930	2.48	7.14	8.14	.98	2.66	3.65
1931	2.54	8.36	11.63	.92	2.63	3.56
1932	2.64	8.66	16.81	.98	2.68	3.35
1933	2.52	8.07	15.71	.98	2.87	3.60
1934	2.23	7.51	11.81	.92	2.74	3.03
1935	2.24	6.29	11.93	.88	2.72	2.84
1936	2.10	5.63	10.87	.82	2.86	2.74
1937	1.82	5.19	10.27	.80	2.68	2.47
1938	1.79	5.78	9.44	.74	2.69	2.66
1939	1.78	5.45	10.08	.73	2.58	2.46
1940	1.77	3.99	8.47	.71	2.58	2.42
1941	1.77	3.67	5.28	.68	2.58	2.33
1942	1.63	2.91	3.51	.63	2.28	2.22
1943	1.58	2.78	3.07	.61	1.84	2.05

NEW YORK

1928	2.26	5.58	8.76	.69	2.26	3.58
1929	2.26	5.57	8.22	.69	2.33	3.57
1930	2.23	6.09	8.01	.73	2.37	3.73
1931	2.54	7.19	7.80	.80	2.57	3.77
1932	2.83	7.77	9.01	.83	2.71	3.88
1933	2.54	8.03	8.94	.84	2.91	3.81
1934	2.18	7.45	7.64	.76	2.73	3.44
1935	2.19	7.13	7.45	.77	2.71	3.36
1936	2.12	6.27	7.00	.71	2.63	3.39
1937	1.94	5.63	6.19	.68	2.55	3.01
1938	1.82	5.19	6.54	.65	2.49	2.87
1939	1.80	5.03	7.27	.65	2.40	2.78
1940	1.78	4.79	6.45	.65	2.44	2.76
1941	1.70	4.17	5.07	.60	2.53	2.65
1942	1.64	3.21	3.91	.55	2.43	2.37


NEW YORK INDEMNITY CLAIM FREQUENCY
To Accompany Exhibit IV
CONST.
MFG.
ALL OTHER
1929
30
31
32
33
34
35
36
37
38
39
40
41
42


MASSACHUSETTS INDEMNITY CLAIM FREQUENCY
Per Accident Subject to Act
CONST.
ALL OTHER
TRADE
MFG.
CONST.
1000
800
700
600
500
400
300
200
100
90
80
70
60
50
40
30
1929
30
31
32
33
34
35
36
37
38
39
40
41
42
43

FURTHER TABLES ADAPTED FOR MACHINE COMPUTATION

BY

F. S. PERRYMAN

In my previous paper, Proceedings, Casualty Actuarial Society, XXV, entitled "Tables Adapted for Machine Computation"—which will be referred to hereafter as "the previous paper"—I gave ten place tables of logarithms adapted to be used on modern calculating (multiplying) machines. I also gave some auxiliary tables of interest functions. The logarithm tables, however, were a little trouble to use, although of course we must expect a little of that with such condensed tables, so I sought to get an arrangement easier to use. This I found and it is described in Part II of this paper. Again, in the previous paper, what I gave by way of interest tables were really only tables of the fundamental values and not tables of values for immediate use: the paper gave the procedure for obtaining the required values from the given fundamental functions. Here again, I was not satisfied and endeavored to get a set of tables giving the final values directly; I was principally interested in tables of weekly annuity values as these occur frequently in Casualty work. The set-up I finally devised is given in Part I of this paper.

Perhaps I should say a word here in anticipation of a type of criticism that may be levelled against the results given here, by some hasty critics, on the grounds that ten place logarithm tables and ready to use weekly annuity tables are unnecessary luxuries or give needless accuracy. I don't regard the matter thus. The tables I give are, I believe, useful additions to the tools of our profession: and it is a fitting example of the principle of division of labor for one person like myself, who is interested in these things and likes working them out, to undertake the work of preparing these tools and presenting them to the profession. If the few pleasant hours I spent in putting this paper together save members of the profession a few minutes work from time to time, then my labor was useful as well as pleasant.

PART I

As stated above, in the previous paper I gave some tables of interest functions which were merely tables of fundamental values that did not give directly the values actually required in practice, such as values of annuities certain.

One example in the previous paper—(8) on p. 142—did give, almost directly, with the aid of a calculating machine, weekly annuity values at

3½% and this example contains the germ of the idea for a table to give such values for any rate of interest. The first part of this paper will develop this idea and give the necessary tables.

I will give the full tables for finding weekly annuities and indicate how they can be used also for annual annuities payable either yearly or at more frequent intervals.

The present value of an annuity certain of 1 per annum payable r times a year for m years at rate of interest i is

$$\frac{1-e^{-\delta m}}{j_r} \qquad \text{(I)}$$

where, as usual, $\delta = \log_e (1+i)$ and $j_r = r\,[(1+i)^{\frac{1}{r}} - 1]$. The present value of an annuity certain of 1 per week, for m weeks at rate of interest i, is

$$\frac{r\,(1-e^{-\delta \frac{m}{r}})}{j_r} \qquad \text{(II)}$$

where δ and j_r have the same values as before and r is the number of weeks assumed to be in a year.

As in the previous paper, we will give tables for $r = 52$ and for $r = 52.1775$ together with the interpolation procedure to be used for other values of r, such as 52⅐.

Both (I) and (II) are of the form $C\,(1 - e^{-Bm})$ where B and C depend only on the rate of interest and the value of r. So we can get annuity values if we have tables of

(A) $\{n\} = 1 - e^{-kn}$ for all values of n, k being a fixed constant.

(B) $B = \frac{\delta}{rk}$ for weekly annuities or $B = \frac{\delta}{k}$ for annual annuities.

(C) $C = \frac{r}{j_r}$ for weekly annuities

or $C = \frac{1}{j_r}$ for annual annuities

for if we calculate $n = Bm$ then the annuity value is

$$C\,(1 - e^{-kn}) \text{ or } C\,\{Bm\}.$$

The tables for B and C are easily constructed and can readily be given for all usual rates of interest and values of r required. On the other hand, the construction of $\{n\}$ requires some preliminary considerations. First of all, in theory, it should proceed from $n = 0$ to $n =$ infinity so as to give annuity values for extended terms, but in practice it is desirable to limit the table to

a reasonable size, say to 200 entries. Fortunately, we can do this conveniently by choosing k, which can be any arbitrary number, so that $e^{-200k} = \frac{1}{2}$. If we do this and tabulate $\{n\}$ for values of n from 0 to 200, the table will

(i) directly give values for periods up to the number of years in which money doubles itself at compound interest at rate i; namely, about $70/i$ years (or about 70 years at 1%, about 35 years at 2%, etc.).

(ii) give an easy formula for n greater than 200: this is because

$$\{n + 200\} = 1 - e^{-(n+200)k} = 1 - \tfrac{1}{2}e^{-nk} = \tfrac{1}{2} + \tfrac{1}{2}\{n\}$$

and $\{200\} = \frac{1}{2}$

Similarly $\{n + 200\,s\} = 1 - \dfrac{e^{-nk}}{2^s} = \dfrac{2^s - 1}{2^s} + \dfrac{\{n\}}{2^s}$

and $\{200s\} = \dfrac{2^s - 1}{2^s}$

So if $n > 200$ $\quad \{n + 200\} = \frac{1}{2} + \frac{1}{2}\{n\}$
if $n > 400$ $\quad \{n + 400\} = \frac{3}{4} + \frac{1}{4}\{n\}$
if $n > 600$ $\quad \{n + 600\} = \frac{7}{8} + \frac{1}{8}\{n\}$, etc.

So we put $e^{-200k} = \frac{1}{2}$ which gives

$$k = \frac{\log_e 2}{200} = .003465735903.$$

then B, which is $\dfrac{\delta}{rk}$, will be approximately $\dfrac{i}{.18}$ and will vary from about .06 for $i = 1\%$ to about .4 for $i = 7\frac{1}{2}\%$.

Also for weekly annuities C which is $\dfrac{r}{j_r}$ will be approximately $\dfrac{52}{i}$ and vary from about 5200 for $i = 1\%$ to about 700 for $i = 7\frac{1}{2}\%$.

Now since $\{n\}$ will go from 0 at $n = 0$ to $\frac{1}{2}$ at $n = 200$, I find it more convenient to multiply $\{n\}$ by 1000 and divide C by 1000 and so tabulate

$$\text{(A)} \qquad \{n\} = 1000\,(1 - e^{-nk})$$

$$\text{(B)} \qquad B = \frac{\delta}{rk}$$

$$\text{(C)} \qquad C = \frac{r}{1000 j_r}$$

Thus $\{n\}$ will go from 0 at $n = 0$ to 500 at $n = 200$, C will go from about 5.2 at 1% to about .70 at $7\frac{1}{2}\%$, while B, as before, will vary from about .06 at 1% to .4 at $7\frac{1}{2}\%$.

One more difficulty remains and that is that of interpolating in the table for $\{n\}$. Even though m is usually a whole number, the number n, with which

we want to enter the table, will usually, except by sheer chance, consist of an integer plus a decimal portion for it is equal to Bm and B is not integral.

Now as $\{n\} = 1000\,(1 - e^{-kn})$

$$\Delta\{n\} = \{n+1\} - \{n\} = 1000\, e^{-kn}\,(1 - e^{-k})$$

$$\Delta^2\{n\} = \Delta\{n+1\} - \Delta\{n\} = -1000\, e^{-kn}\,(1 - e^{-k})^2$$

So $-\Delta^2\{n\}$ is about .012 at $n = 0$ and decreases to about .006 at $n = 200$. Thus if we use ordinary first difference interpolation in the table for $\{n\}$ the maximum error, which is equal to $\left|\frac{\Delta^2\{n\}}{8}\right|$ would be between .0015, the value for $n = 0$ and .00075, the value for $n = 200$.

This is not accurate enough; for example, if n is less than 100 the maximum error may exceed .001 and thus the result is not reliable to 5 significant figures.

We must thus use a more powerful interpolation procedure. We could use second difference interpolation but it is awkward to do this, even with a calculating machine, especially for the inverse interpolation which we have to use to obtain n from $\{n\}$. Fortunately, we can get over the difficulty by a simple procedure based on the fact that the differences of $\{n\}$ are in geometrical progression.

This procedure is arrived at as follows:

If we wish to obtain $\{n+t\}$ where n is integral and t is less than 1, then

$$\{n+t\} - \{n\} = 1000\, e^{-kn}\,(1 - e^{-kt})$$

$$\Delta\{n\} = 1000\, e^{-kn}\,(1 - e^{-k})$$

So $$\frac{\{n+t\} - \{n\}}{\Delta\{n\}} = \frac{1 - e^{-kt}}{1 - e^{-k}}$$

Put $\bar{t}$ for the expression on the right hand side of the last equation. This is independent of n and depends solely on t, as k is fixed. That this is so, arises from the fact that $\Delta\{n\}$ is a geometric series. Thus $\{n+t\} = \{n\} + \bar{t}\,\Delta\{n\}$. In other words, we use first difference interpolation, putting however $\bar{t}$ for t. Now $\bar{t}$ and t are equal if $t = 0$ or 1 and are nearly equal if t is between 0 and 1, as it is.

$$\bar{t} = \frac{kt - \frac{k^2 t^2}{\lfloor 2} + \frac{k^3 t^3}{\lfloor 3} - \cdots}{k - \frac{k^2}{\lfloor 2} + \frac{k^3}{\lfloor 3} - \cdots}$$

$$= t + \frac{t\,(1-t)\,k}{2} + \frac{t\,(1-t)\,(1-2t)\,k^2}{12} + \cdots$$

So if we put $u = \bar{t} - t$

$$u = \frac{t\,(1-t)\,k}{2} + \cdots$$

and the maximum value of u is at $t =$ approximately $\frac{1}{2}$

(more exactly $\frac{1}{2} - \frac{k}{24} - \cdots$)

when $u =$ approximately $\frac{k}{8}$ (more exactly $\frac{k}{8} - \frac{k^3}{526} + \cdots$) or .00043322 $\cdots$

(See the appendix to this paper for mathematical details.)

Thus if we wish to have t and $\bar{t}$ correct to five decimal places, when $\bar{t}\,\Delta\,\{n\}$ will be correct to five significant figures or at least four decimal places, we can get $\bar{t}$ by adding to t a correction u of less than .00044.

So we prepare an auxiliary table giving the ranges of t, to five decimal places, for which an addition of .00001, or .00002, etc. up to .00043 must be made to t to get $\bar{t}$. Naturally, the end points of these ranges are those for which u is equal to .000005, .000015, etc. For example, the range of t for which u equals .00012, that is to say, that in which .00012 must be added to t, runs from the value of t for which $u = .000115$ to that for which $u = .000125$ and the table, Table 2, is so calculated. There will be two ranges for a given value of u, one between 0 and the value of t for which u is a maximum and the other between this value and unity (see Table 2). Table 2 is an example of a so-called "critical table".

Our complete procedure and a simple one can now be set forth. First, we have the following tables given at the end of this Part I:—

Table 1 giving the values of $\{n\}$ and $\Delta\,\{n\}$ for $n = 0, 1, 2$, etc., up to 200.
Table 2 giving the value of u for all values of t and $\bar{t}$.
Table 3 giving the values of B and C for weekly annuities of 1 for $r = 52$ and $r = 52.1775$ for all rates of interest at intervals of $\frac{1}{4}\%$ from $\frac{1}{4}\%$ to $7\frac{1}{2}\%$.

Second, to find the present value of an annuity for 1 per week for m weeks at a given rate of interest, r being specified:

(i) Take out of Table 3, the value of B for the given i and r.
(ii) Multiply m by B and express the result in the form $n + t$ where n is a whole number and t a pure decimal to five places.
(iii) From Table 2, get the addition u to be added to t to get $\bar{t}$.
(iv) From Table 1, calculate $\{n + t\} = \{n\} + \bar{t}\,\Delta\,\{n\}$
(v) Multiply $\{n + t\}$ by C from Table 3 for the given i and r and the result is the required present value.

The answer will be correct to four figures more than the number of figures in the integral portion of $\{n + t\}$ but in the calculation find $\{n + t\}$ to five decimal places.

If n exceeds 200, use the formula given at the foot of Table 1. The process can obviously be reversed to find m, for a given i and r, from the present

value by dividing the given present value by C, entering Table 1 inversely with the result to get $n + \bar{t}$, subtracting the proper value of u to get $n + t$, and dividing $n + t$ by B to get m.

For working purposes, the method can be expressed a little differently and perhaps a little more clearly, as follows:—

To find the present value of P per week for M weeks (r and i being given) take B and C from Table 3 and get

$BM =$ the adjusted number of weeks

and $CP =$ the adjusted weekly payment

Enter Table 1 with the adjusted number of weeks and multiply by the adjusted weekly payment. The result is the required present value, namely $CP\ \{BM\}$.

To find, on the other hand, the number of weeks M, for which the present value has given value X, find the adjusted number of weeks BM from $\{BM\} = \dfrac{X}{CP}$ and then divide by B to get M.

The whole procedure, whether direct or inverse, is very readily done with a calculating machine.

If we wish to use for r a value other than those given in Table 3, this value will be obviously somewhere near 52 and we proceed by interpolating in Table 3 for C. For example, for $r = 52\frac{1}{7}$, if the values of C for $r = 52$ and $r = 52.1775$ are C_1 and C_2 respectively, we use $C = C_1 + \dfrac{\frac{1}{7}}{.1775}(C_2 - C_1)$. As for B we use $\dfrac{52\,B_1}{52\frac{1}{7}}$ where B_1 is the value for $r = 52$.

A similar procedure can be used for annual annuities, i.e. 1 per annum payable annually, half-yearly, quarterly, etc. The only change required is in the B and C values. We must put $B = \dfrac{\delta}{k} = \delta \times 288.5390082$, which does not vary with the frequency of payment, and $C = \dfrac{1}{1000 j_r}$ which does change with the frequency of payment. It is usually, however, easier to use tables of $a_{\overline{n}|}^{m}$ and $\dfrac{i}{j_r}$, of which there are many available, but for completeness I give in Table 4 the necessary information for calculating B and C for annual annuities.

We can also use the tables for obtaining other interest functions. In fact,

given the values of i and r, if for m weeks we calculate the value of $\{n+t\} = Bm$ we have

$\{n+t\} = 1000\,(1-\bar{v}^{\overline{m}})$

so the present value of 1 due m weeks hence is $1 - \dfrac{\{Bm\}}{1000}$

and the amount of 1 accumulated for m weeks is $\dfrac{1000}{1000 - \{Bm\}}$

and the sum of an annuity of 1 per week accumulated for m weeks is $C\,\dfrac{1000\,\{Bm\}}{1000 - \{Bm\}}$

to which we add our first result, the present value of an annuity of 1 per week for m weeks is $C\,\{Bm\}$

We get similar expressions in connection with annual payments.

In Table 5, I have collected these formulas together for ready reference.

Some examples for the use of these tables follow. Most of these deal with the same data as in the interest examples in the previous paper.

EXAMPLES OF THE USE OF THE TABLES

(1) Find the present value at 3¾% per annum compound interest of an annuity certain of 12.83 a week for 400 weeks. (52.1775 weeks to the year).

From Table 3 the "adjusted no. of weeks" is

$$400 \times .20357946 = 81.43178$$

and the "adjusted weekly payment" is

$$12.83 \times 1.4168287 = 18.177912$$

Now $\{81.43178\} = 244.76371 + 2.61292\ (.43178 + .00043)$

$= 245.89304$

so the required present value is $18.177912 \times 245.89304$

$= 4469.822$ to seven significant figures.

(2) Find the accumulated amount of the annuity in (1) at the end of the 400 weeks.

If V is the present value of 1 due 400 weeks hence, with the given data, then $V = 1 - \dfrac{\{81.43178\}}{1000} = .75410696$ and the required amount is the present value found in (1) divided by V or 5927.305.

(3) Find the present value of 1625.14 at 3¾% per annum compound interest due 400 weeks hence (52.1775 weeks to the year).

The value is $1625.14 \times V$ or 1225.529.

(4) Find the amount of 1625.14 accumulated for 400 weeks (52.1775 to the year) at 3¾% per annum compound interest.

The amount is $1625.14 \div V$ or 2155.052.

(5) Find the present value of the annuity in (1) if a year be assumed to have 52 weeks.

The "adjusted no. of weeks" is $400 \times .20427437$ or 81.70975 and the "adjusted weekly payment" is 12.83×1.4120072 or 18.116052 and the present value is

$$18.116052\ \{81.70975\}$$
$$\text{or } 18.116052\ [244.76371 + (.70975 + .00036)\ 2.61292]$$
$$\text{or } 18.116052 \times 246.61917$$
$$\text{or } 4467.766 \text{ to seven significant figures.}$$

(6) For how many weeks will a payment of 1000 suspend an annuity certain for 12 per week, at 3% per annum compound interest, allowing 52.1775 weeks to the year?

If w is the number of weeks we have

"adjusted no. of weeks" is $.16345872 \times w = n$

"adjusted weekly payments" is 12×1.7647103 or 21.176524

and $21.176524\ \{n\} = 1000$.

So $\{n\} = 47.22210$

$n = 13 + (.95770 - .00007)$

for $\{n\} - \{13\} = 3.16742 = .95770 \times \Delta\{13\}$

$= 13.95763$

so $w = 85.3893$

(7) By how many weeks will a payment of 1000 now shorten an annuity certain of 12 per week payable for 300 weeks, at 3% compound interest per annum, 52.1775 weeks to the year?

If x is the number of weeks in the shortened annuity we have

$B = .16345872$

$C = 1.7647103$

and $1000 = 12\ [C\ \{300\ B\} - C\ \{Bx\}]$

$\{Bx\} = -47.22210 + \{300\ B\} = -47.22210 + \{49.03762\}$

$= 109.07210$

So $Bx = 33.32388$

and $x = 203.8673$

So the annuity is shortened by 96.1327 weeks.

(8) Find the present value of an annuity of 1 per week payable for 467 weeks (52.1775 to the year) at 3½% per annum compound interest.

Adjusted no. of weeks $= 88.84124$

Adjusted weekly payments $= 1.5162249$

Present value is $1.5162249\ \{88.84124\}$ or 401.8169.

(9) Find the present value and amount of an annuity of 1, payable half-yearly, for 50 years at 2½% per annum.

B ($= 52\,B$ for weekly annuities 52 weeks to year) $= 7.1247821$

Adjusted number of years $= 356.23911$

$$\{356.23910\} = 500 + \tfrac{1}{2}\,\{156.23910\} = 709.05781$$

Divide this by $1000\,j_{(2)}$ (from Table II of the previous paper) which is 24.845673 and we get the required result of 28.53848.

TABLE 1

n	$\{n\}$	$\Delta\{n\}$	n	$\{n\}$	$\Delta\{n\}$
0	0.00000	3.45974	33	108.07148	3.08584
1	3.45974	3.44776	34	111.15732	3.07516
2	6.90750	3.43584	35	114.23248	3.06452
3	10.34334	3.42396	36	117.29700	3.05392
4	13.76730	3.41210	37	120.35092	3.04336
5	17.17940	3.40030	38	123.39428	3.03282
6	20.57970	3.38854	39	126.42710	3.02234
7	23.96824	3.37681	40	129.44944	3.01187
8	27.34505	3.36513	41	132.46131	3.00146
9	30.71018	3.35349	42	135.46277	2.99107
10	34.06367	3.34189	43	138.45384	2.98072
11	37.40556	3.33032	44	141.43456	2.97041
12	40.73588	3.31880	45	144.40497	2.96014
13	44.05468	3.30732	46	147.36511	2.94989
14	47.36200	3.29588	47	150.31500	2.93969
15	50.65788	3.28447	48	153.25969	2.92951
16	53.94235	3.27311	49	156.18420	2.91939
17	57.21546	3.26179	50	159.10359	2.90928
18	60.47725	3.25050	51	162.01287	2.89921
19	63.72775	3.23926	52	164.91208	2.88919
20	66.96701	3.22805	53	167.80127	2.87918
21	70.19506	3.21688	54	170.68045	2.86923
22	73.41194	3.20575	55	173.54968	2.85930
23	76.61769	3.19466	56	176.40898	2.84941
24	79.81235	3.18361	57	179.25839	2.83955
25	82.99596	3.17259	58	182.09794	2.82973
26	86.16855	3.16162	59	184.92767	2.81993
27	89.33017	3.15067	60	187.74760	2.81018
28	92.48084	3.13978	61	190.55778	2.80046
29	95.62062	3.12892	62	193.35824	2.79077
30	98.74954	3.11809	63	196.14901	2.78111
31	101.86763	3.10730	64	198.93012	2.77149
32	104.97493	3.09655	65	201.70161	2.76191
33	108.07148	3.08584	66	204.46352	2.75234

$\{n + \bar{t}\} = \{n\} + \bar{t}\,\Delta\{n\}$
where $\bar{t} = t + u$ from table 2

$\{n\} = 1000\,(1 - e^{-kn})$ $\quad e^{200k} = 2$ $\quad k = .002465735903$

TABLE 1 (Continued)

n	$\{n\}$	$\Delta\{n\}$	n	$\{n\}$	$\Delta\{n\}$
66	204.46352	2.75234	100	292.89322	2.44640
67	207.21586	2.74283	101	295.33962	2.43794
68	209.95869	2.73333	102	297.77756	2.42951
69	212.69202	2.72388	103	300.20707	2.42110
70	215.41590	2.71446	104	302.02817	2.41272
71	218.13036	2.70506	105	305.04089	2.40438
72	220.83542	2.69570	106	307.44527	2.39605
73	223.53112	2.68638	107	309.84132	2.38777
74	226.21750	2.67709	108	312.22909	2.37951
75	228.89459	2.66782	109	314.60860	2.37127
76	231.56241	2.65859	110	316.97987	2.36307
77	234.22100	2.64940	111	319.34294	2.35490
78	236.87040	2.64022	112	321.69784	2.34674
79	239.51062	2.63110	113	324.04458	2.33863
80	242.14172	2.62199	114	326.38321	2.33054
81	244.76371	2.61292	115	328.71375	2.32247
82	247.37663	2.60388	116	331.03622	2.31444
83	249.98051	2.59487	117	333.35066	2.30693
84	252.57538	2.58589	118	335.65709	2.29845
85	255.16127	2.57694	119	337.95554	2.29050
86	257.73821	2.56804	120	340.24604	2.28258
87	260.30625	2.55914	121	342.52862	2.27468
88	262.86539	2.55029	122	344.80330	2.26681
89	265.41568	2.54147	123	347.07011	2.25896
90	267.95715	2.53268	124	349.32907	2.25115
91	270.48983	2.52391	125	351.58022	2.24336
92	273.01374	2.51518	126	353.82358	2.23561
93	275.52892	2.50648	127	356.05919	2.22786
94	278.03540	2.49781	128	358.28705	2.22016
95	280.53321	2.48917	129	360.50721	2.21248
96	283.02238	2.48055	130	362.71969	2.20482
97	285.50293	2.47197	131	364.92651	2.19719
98	287.97490	2.46342	132	367.12170	2.18960
99	290.43832	2.45490	133	369.31130	2.18201
100	292.89322	2.44640	134	371.49331	2.17447

$\{n + \bar{t}\} = \{n\} + \bar{t}\,\Delta\{n\}$
where $\bar{t} = t + u$ from table 2

TABLE 1 (Continued)

n	$\{n\}$	$\Delta\{n\}$	n	$\{n\}$	$\Delta\{n\}$
134	371.49331	2.17447	167	439.41696	1.93947
135	373.66778	2.16695	168	441.35643	1.93276
136	375.83473	2.15944	169	443.28919	1.92007
137	377.99417	2.15198	170	445.21526	1.91941
138	380.14615	2.14453	171	447.13467	1.91277
139	382.29068	2.13711	172	449.04744	1.90015
140	384.42779	2.12972	173	450.95359	1.89956
141	386.55751	2.12235	174	452.85315	1.89298
142	388.67986	2.11501	175	454.74613	1.88644
143	390.79487	2.10769	176	456.63257	1.87991
144	392.90256	2.10040	177	458.51248	1.87340
145	395.00296	2.09313	178	460.38588	1.86692
146	397.09609	2.08589	179	462.28280	1.86047
147	399.18198	2.07867	180	464.11327	1.85403
148	401.26068	2.07148	181	465.96730	1.84761
149	403.33213	2.06431	182	467.81491	1.84122
150	405.39644	2.05717	183	469.65613	1.83485
151	407.45361	2.05006	184	471.49098	1.82850
152	409.50367	2.04296	185	473.31948	1.82218
153	411.54663	2.03590	186	475.14166	1.81587
154	413.58253	2.02885	187	476.95753	1.80959
155	415.61138	2.02183	188	478.76712	1.80333
156	417.63321	2.01483	189	480.57045	1.79709
157	419.64804	2.00787	190	482.36754	1.79087
158	421.65591	2.00092	191	484.15841	1.78468
159	423.65683	1.99399	192	485.94309	1.77850
160	425.65082	1.98710	193	487.72159	1.77235
161	427.63792	1.98022	194	489.49394	1.76621
162	429.01814	1.97337	195	491.26015	1.76011
163	431.59151	1.96655	196	493.02026	1.75402
164	433.55806	1.95974	197	494.77428	1.74794
165	435.51780	1.95296	198	496.52222	1.74191
166	437.47076	1.94620	199	498.26413	1.73587
167	439.41696	1.93947	200	500.00000	

$\{n+t\} = \{n\} + \overline{t}\,\Delta\{n\}$
where $\overline{t} = t + u$ from table 2

$\{n+200\} = 500 + \frac{1}{2}\{n\}$
$\{n+400\} = 750 + \frac{1}{4}\{n\}$
$\{n+600\} = 875 + \frac{1}{8}\{n\}$
etc.

$$\{n+200\,s\} = 1000 - \frac{1000}{2^s} + \frac{\{n\}}{2^s}$$

TABLE 2

t	u	$\bar{t}$	t	u	$\bar{t}$
.00000	0	.00000	.43100	43	.43143
.00290	1	.00291	.56872	42	.56914
.00873	2	.00875	.60240	41	.60281
.01464	3	.01467	.62748	40	.62788
.02061	4	.02065	.64838	39	.64877
.02667	5	.02672	.66668	38	.66706
.03280	6	.03286	.68317	37	.68354
.03902	7	.03909	.69829	36	.69865
.04532	8	.04540	.71234	35	.71269
.05170	9	.05179	.72552	34	.72586
.05818	10	.05828	.73797	33	.73830
.06476	11	.06487	.74980	32	.75012
.07144	12	.07156	.76109	31	.76140
.07822	13	.07835	.77192	30	.77222
.08512	14	.08526	.78232	29	.78261
.09213	15	.09228	.79237	28	.79265
.09926	16	.09942	.80208	27	.80235
.10526	17	.10543	.81148	26	.81174
.11393	18	.11411	.82061	25	.82086
.12147	19	.12166	.82949	24	.82073
.12917	20	.12937	.83814	23	.83837
.13703	21	.13724	.84657	22	.84679
.14507	22	.14529	.85480	21	.85501
.15329	23	.15352	.86284	20	.86304
.16172	24	.16196	.87071	19	.87090
.17035	25	.17060	.87841	18	.87859
.17923	26	.17949	.88597	17	.88614
.18835	27	.18862	.89464	16	.89480
.19775	28	.19803	.90064	15	.90079
.20746	29	.20775	.90778	14	.90792
.21749	30	.21779	.91480	13	.91493
.22789	31	.22820	.92170	12	.92182
.23871	32	.23903	.92849	11	.92860
.25000	33	.25033	.93518	10	.93528
.26182	34	.26216	.94176	9	.94185
.27426	35	.27461	.94825	8	.94833
.28742	36	.28778	.95464	7	.95471
.30147	37	.30184	.96095	6	.96101
.31659	38	.31697	.96717	5	.96722
.33307	39	.33346	.97331	4	.97335
.35137	40	.35177	.97937	3	.97940
.37226	41	.37267	.98536	2	.98538
.39734	42	.39776	.99127	1	.99128
.43100	43	.43143	.99711	0	.99711

$$\bar{t} = t + u$$

For values of t, or $\bar{t}$, between the values given use for u the value for the next smaller value of t, or $\bar{t}$.

TABLE 3

Values of B and C for Weekly Annuities of 1

i	$r = 52$		$r = 52.1775$	
	B	C	B	C
¼ %	.013854756	20.825489	.013807625	20.896578
½ %	.027675005	10.425478	.027580859	10.461067
¾ %	.041460918	6.9588010	.041319874	6.9825563
1 %	.055212665	5.2254569	.055024840	5.2432955
1¼ %	.068930415	4.1854462	.068695924	4.1997348
1½ %	.082614336	3.4921021	.082333295	3.5040223
1¾ %	.096264594	2.9968534	.095937117	3.0070848
2 %	.10988135	2.6254142	.10950755	2.6343760
2¼ %	.12346478	2.3365147	.12304477	2.3444920
2½ %	.13701504	2.1053930	.13654893	2.1125814
2¾ %	.15053228	1.9629916	.15002020	1.9228345
3 %	.16401668	1.7587053	.16345872	1.7647103
3¼ %	.17746839	1.6253615	.17686467	1.6309113
3½ %	.19088202	1.5110653	.19023820	1.5162249
3¾ %	.20427437	1.4120072	.20357946	1.4168287
4 %	.21762896	1.3253301	.21688861	1.3298558
4¼ %	.23095148	1.2488491	.23016581	1.2531137
4½ %	.24424209	1.1808649	.24341121	1.1848974
4¾ %	.25750094	1.1200358	.25662496	1.1238607
5 %	.27072818	1.0652887	.26980721	1.0689267
5¼ %	.28392397	1.0157546	.28295811	1.0192235
5½ %	.29708846	.97072263	.29607781	.97403786
5¾ %	.31022178	.92960566	.30916645	.93278054
6 %	.32332410	.89191428	.32222420	.89496050
6¼ %	.33639554	.85723741	.33525118	.86016526
6½ %	.34943627	.82522723	.34824754	.82804582
6¾ %	.36244642	.79555093	.36121343	.79830486
7 %	.37542614	.76806409	.37414900	.77068755
7¼ %	.38837557	.74243822	.38705437	.74497420
7½ %	.40129485	.71852009	.39992970	.72097443

Weekly Annuity of 1 for W weeks $= \{W \times B\} \times C$

Values of $\{W \times B\}$ from Tables 1 and 2.

Note: $B = 288.5390082 \times \dfrac{C}{r}$ $\qquad C = \dfrac{r}{1000\, j_r}$

TABLE 4

Annual Annuities

Value of annuity of 1 per annum, payable r times a year for Y years,

$$= \{Y \times B\} \times C$$

where $B = 288.5390082 \times \delta$

$C = \dfrac{1}{1000\, j_r}$

} Value of $\delta\ (= j_\infty)$ and j_r from Table II P.C.A.S. XXV p. 130-1.

Value of $\{Y \times B\}$ from Tables 1 and 2; or, preferably, value $= \dfrac{\{Y \times B\}}{1000\, j_r}$

where $B = 52 \times$ value of B for $r = 52$ in Table 3 and j_r is taken from Table II P.C.A.S. XXV p. 130-1.

TABLE 5

If $a_{\overline{n}|}$, value of an annuity $= C\,\{N\}$

then $s_{\overline{n}|}$, amount of the annuity $= \dfrac{1000}{1000 - \{N\}} \times C\,\{N\}$

and v^n, present value of 1 for the term of the annuity, $= \dfrac{1000 - \{N\}}{1000}$

and $(1 + i)^n$, amount of 1 for the term $= \dfrac{1000}{1000 - \{N\}}$

PART II

In the previous paper I also gave tables for taking out rapidly, with the aid of a calculating machine, logarithms and antilogarithms to ten figures. These tables are based on the well-known factorial method and are reasonably accurate and rapid. Nevertheless, their use requires the splitting of a number into four factors which takes a little time and care. If a method could be devised to reduce the number of factors, to say two, the procedure would be much simplified and speeded up. This method I shall give in this Part II; it rests on the same method of interpolation used in Part I for $\{n\}$. We have seen that this stems from the circumstance that $\Delta\{n\}$ is a geometrical series; if the differences of $\log nk$ for successive values of n formed a geometrical series we could use the same method; however, this is not so but the differences of antilog nk are, since $^*\log^{-1} nk = 10^{nk}$ which is itself a geometric series for successive values of n and therefore its differences are also. In fact $\Delta \log^{-1} nk = \log^{-1}(n+1)k - \log^{-1} nk = 10^{nk}(10^k - 1)$. As a matter of fact $\{n\}$ in Part I is a kind of table of antilogarithms since $\{n\}$ is a thousand times the complement of the antilogarithm of n to the rather unusual base of $e^{-k} = .9965402$.

The circumstances that we have to tabulate antilogarithms instead of logarithms to be able to apply the special method of interpolation to obtain, from a short table, results to a large number of figures, is no disadvantage; for in using tables of logarithms or antilogarithms, we almost always have to enter the tables first directly or inversely to get a logarithm and then later, enter the table the other way to get an antilogarithm. In fact, some modern calculators find advantages in tabulating antilogarithms instead of logarithms. For example, Frederic Deprez, in the introduction to his "Tables for Calculating, By Machine, Logarithms to Thirteen Places of Decimals" published in Switzerland in 1939, states "It is well known that a table of logarithms may be used directly for finding logarithms and inversely for finding antilogarithms. After extensive research, the author has found it better, when viewing the subject with the availability of calculating machines in mind, to produce a table of antilogarithms, which is used directly to find antilogarithms and inversely, to find logarithms. The advantages of this method will be enhanced when the log of the final number desired can be formed from tabulated logarithms, as is usually the case in interest calculations; this means that the tables are used more frequently directly than inversely". Incidentally, it is interesting to note that Mr. Deprez' tables are constructed on the factorial method.

---

* Note: We shall use, as convenient, both of the notations antilog N and $\log^{-1} N$ to denote the antilogarithm of N, i.e. the number whose logarithm is N.

As will be seen, our two tables for the two factors to which we shall reduce the taking out of logarithms and antilogarithms will be, one, a log table and the other an antilog table.

Let us then analyze a table of antilogarithms to base 10 with an interval of 10^{-r}, that is to say, a table of $\log^{-1} 10^{-r} n$ as follows:

$$\log^{-1} .00\cdots(r-1 \text{ zeros})\cdots 01$$
$$\log^{-1} .00\cdots(r-1 \text{ zeros})\cdots 02$$
$$\log^{-1} .00\cdots(r-1 \text{ zeros})\cdots 03$$
$$\text{etc.}$$

(it is, of course, only necessary to tabulate antilogarithms between 0 and 1).

The differences are shown below where R is put for

$$10^{-r} = .00\cdots(r-1 \text{ zeros})\cdots.001$$

	$\log^{-1}$	Δ	Δ^2
Rn	10^{Rn}	$10^{Rn}\ (10^R-1)$	$10^{Rn}\ (10^R-1)^2$
$R\ (n+1)$	$10^{R\ (n+1)}$	$10^{R\ (n+1)}\ (10^R-1)$	$10^{R\ (n+1)}\ (10^R-1)^2$
$R\ (n+2)$	$10^{R\ (n+2)}$	$10^{R\ (n+2)}\ (10^R-1)$	$10^{R\ (n+2)}\ (10^R-1)^2$
		etc.	

A little study of this shows that ordinary first difference interpolation gives about $2r$ places correct. Let us see how to increase this accuracy by a method similar to that in Part I. Let us for shortness put temporarily $[n]$ for 10^{Rn}.

Then
$$[n+1] = 10^{R(n+1)}$$
$$\Delta\,[n] = 10^{Rn}\,(10^R-1)$$
$$\frac{[n+t]-[n]}{\Delta\,[n]} = \frac{10^{Rt}-1}{10^R-1} = \bar{t} \text{ say}$$

and
$$[n+t] = [n] + \bar{t}\,\Delta\,[n]$$

where as before $t > 0$ and < 1 and $\bar{t}$ is nearly equal to t, for putting $k = R \log_e 10 = \mathrm{R} \times 2.30259\cdots$ so that $10^R = e^k$

$$\bar{t} = \frac{e^{kt}-1}{e^k-1} = t - \frac{t\,(1-t)\,k}{2} + \text{etc.}\cdots$$

and
$$u = t - \bar{t} = \frac{t\,(1-t)}{2}\,k - \frac{t\,(1-t)\,(1-2\,t)}{12}\,k^2 + \cdots$$

The maximum value of u is when

$$t = \tfrac{1}{2} + \frac{k}{24} - \cdots$$

and is
$$u = \frac{k}{8} - \frac{k^3}{576} + \cdots$$

Now $k = R \times 2.30259\cdots = .00\cdots(r-1 \text{ zeros})\cdots 0230259\cdots$
So the maximum value of u is approximately $.00\cdots(r \text{ zeros})\cdots 02878\cdots$ which is its value when $t = .500\cdots(r+1 \text{ zeros})\cdots 096\cdots$ approximately.

Thus if in the formula

$$[n+t] = [n] + (t-u)\,\Delta\,[n]$$

we have $[n]$ tabulated to $2r+s$ significant figures then $\Delta\,[n]$ will contain $r+s$ or $r+s+1$ significant figures and $t-u$ should consist of $r+s$ significant figures, i.e. u must be found to s figures.

For example, to take the case of $r=4$ or $R=.0001$, k will equal $.0002302585\cdots$ and the maximum value of u is $.00002878\cdots$, so if we want results to 10 places, we must tabulate log .0001, log .0002, etc., to 10 places which will give first differences with 6 or 7 figures. Then we must have $\overline{t} = t-u$ to 6 places at least, which means we must calculate u to two significant figures and use t with 6 figures. Thus u will not be greater than .000029 and we will need a table of the values of t, for which u is to be taken as .000000, .000001, .000002, etc., up to .000029. What we do is to calculate t to 6 significant figures for $u = .0000005$, etc., in a manner similar to that explained in Part I. These values are given in Table III. As in Part I, there will, of course, be two ranges for which u is equal to a given value, the first between 0 and the value of t for which u is greatest and the other between that value and unity.

We could, when and if we wanted to, proceed similarly for other values of r and s. For example, we could get 14 place antilogarithms by taking $r=5$ and $s=4$ when we would have to calculate the ranges of t for u from .000000001 up to .000002878.

To return to our 10 place tables, all we need now is a table of antilogarithms of .0000, .0001, .0002, etc., up to .9999, together with the first differences (for they are convenient in performing the first difference interpolations to which we have reduced the taking of values out of the table). This would mean a table of 10,000 entries, too many to give in this paper. We can, however, reduce this by using the same factorial method given in the previous paper. The Table III of that paper gave the logarithms to 10 places of 1.00, 1.02, etc., up to 10.00 at varying intervals such that the difference between any two successive logarithms did not exceed the log of $1.022\cdots$ or $.009545\cdots$ so if we use this table—and it is given in this paper as Table 1—and a table of antilogarithms of .0000, .0001, etc., up to say .0096 (about 100 entries)—see Table II—we can rapidly obtain 10 place logarithms and antilogarithms.

The complete procedure is:

First, the tables are:

Table I. Values of $\log N$ for values of N from 1.00 to 10.00

Table II. Values of $\log^{-1} M$ and $\Delta \log^{-1} M$ for values of M by intervals of .0001 from .0000 to .0100.

Table III. Values of t, u and $\bar{t}$.

Second, to find the antilog of a log supposed given to 10 decimal places. Subtract the largest log in Table I that is less than the given log. The remainder will be less than .0096. The first four decimals will be n and the next six t. From Table III take out u and *subtract* it from t to get $\bar{t}$. Then $\log^{-1}(n+t)$ will be $\log^{-1} n + \bar{t}\,\Delta \log^{-1} n$. Multiply this by the number from Table I, whose log was subtracted in the first step and the product to 10 places is the required antilog.

Third, to find the log of a number supposed given to 10 significant figures, we reverse the process. Divide the number by the largest number in Table I that is less than the given number. Take the result to 10 significant figures and enter inversely in Table II getting a result of 10 decimals. Adjust the last six from Table III by *adding* the proper value of u. Finally add the log from Table I of the number used as divisor in the first step. This gives the desired log.

Except for (i) the preparation of dividing by a number or subtracting by a log from Table I and (ii) the completion by adding the corresponding logarithm or multiplying by the corresponding number from Table I, the whole operation consists of a first difference interpolation, either direct or inverse, in Table II using an adjusted fractional part (t adjusted to $\bar{t}$) from Table III and it is decidedly easier and quicker than the fractorial method of the preceding paper.

It might have been theoretically more consistent if instead of the table of logs in Table I, I had given a table of antilogs of .01, .02, etc., up to 1.00, as then we would be using antilog tables throughout and not one log and one antilog table; but in practice this would not be so convenient for when carrying out the multiplication in the last step of taking out an antilog, or the division in the first step of taking out a log, we would be multiplying or dividing by a 10 figure number whereas in the procedure given above the multiplication or division is by a 3 figure number and this is a little easier. Furthermore, as it happened, Table I was already available.

The arrangement of logarithmic tables as given here for 10 places is obviously, and as indicated above, applicable to tables with more or less figures. I trust the 10 figure tables will prove useful. Tables of this power are not very accessible.

A word as to the accuracy of these tables and logarithm tables in particular. No ten figure table is absolutely accurate to the tenth figure; it can't be, from the nature of things. The same is true of five or seven or any other

figure tables. I will illustrate with seven figure logarithms for we can check these with our ten figure table. A complete seven figure table gives, in effect, the logs to seven decimals of all numbers from 1,000,000 to 9,999,999 or 10,000,000 and the logarithms go from .0000000 to 1.0000000: there are 9,000,000 possible numbers and 10,000,000 possible logarithms so there cannot be one unique number for each logarithm. In actual fact the situation is even worse because at the beginning of the table, say just over 1,000,000 the logarithms increase faster than the numbers and at the end of the table, say near 9,999,999 the situation is reversed. Thus we get this sort of thing—

Near beginning of table:

$\log N$	N (10 places)	N (7 places)
.0004004	1.000922380	1.000922
.0004005	1.000922610	
.0004006	1.000922841	1.000923
.0004007	1.000923071	
.0004008	1.000923302	
.0004009	1.000923532	
.0004010	1.000923763	
.0004011	1.000923993	1.000924
.0004012	1.000924224	
.0004013	1.000924454	
.0004014	1.000924685	1.000925

Near end of table:

N	$\log N$ (10 places)	$\log N$ (7 places)
9.600002	.9822713233	.9822713
9.600003	.9822713684	.9822714
9.600004	.9822714136	
9.600005	.9822714588	
9.600006	.9822715039	.9822715
9.600007	.9822715491	
9.600008	.9822715943	.9822716

At the beginning, to one seven-place number there belong 4 or even 5, seven-figure logarithms: at the end, to one seven-figure logarithm there belong 2, or even 3, seven-place numbers.

Absolute accuracy to the last place accordingly is not possible even apart from the errors introduced by rounding in the course of the work. It is remarkable, however, how little inaccurate seven-figure logarithm work is; except for long calculations or special circumstances it is usually reliable to six figures but if absolute seven-figure reliability is needed larger tables must be used. That is one reason why I have prepared the ten-place tables: how-

ever, it must be remembered that, similarly, these do not produce results absolutely accurate to the last, tenth, figure.

In the appendix, I give for the more mathematically minded, some details for the formulae for t and u.

There follow now some examples of the use of these tables: most of them are based on the same data as used in the examples of the previous paper.

Examples of the Use of the Tables

(1) Find log 1.05:

Divide 1.05 by 1.04 the next largest number in Table I: the result is 1.009615385, to ten figures.

Antilog 1.009615385 comes between .0041 and .0042 in Table II. Subtract antilog .0041 from 1.009615385; we get 0.000130083 which we divide by Δ antilog .0041 or 0.000232469 getting .559571. This is $\overline{t}$ which we adjust to t from Table III by *adding* 28 units: $t = .559599$.

So log 1.009615385 = .0041559599 and to get log 1.05 we add log 1.04 from Table I.

So log 1.05 = .0211892992.

(2) Find $\log^{-1} .6$:

Subtracting from .6 the largest possible log from Table I, namely, log 3.92, we get .0067139330.

Then

$\log^{-1} .0067139330 = \log^{-1} .0067 + (.139330 - .000014)\,\Delta \log^{-1} .0067$

where the adjustment (*subtraction*) of .000014 is taken from Table III.

So from Table II $\log^{-1} .0067139330 = 1.015546936 + .000032581$

$= 1.015579517$

and $\log^{-1} .6 = 1.015579517 \times 3.92$

$= 3.981071707$

(3) Evaluate $1.23456789^{9.87654321}$:

$1.23456789 \div 1.22 = 1.011940893$

$\log^{-1} .0051 = 1.011812406$

$.000128487$

Divide by $\Delta \log^{-1} .0051 = .000233005$ and we get .551435 which we adjust by adding .000028.

So log 1.011940893 = .0051551463

Add log 1.22 = .0863598307

log 1.23456789 = .0915149770.

So $\log 1.23456789^{9.87654321} = 1.0915149770 \times 9.87654321$

$= 10.7803948347.$

To find $\log^{-1} .7803948347$ we subtract log 6 and get .0022435843 and from Table III adjust the last six figures downward to 435815.

So $\log^{-1} .002235843 = \log^{-1} .0022 + .435815 \,\Delta \log^{-1} .0022$
$= 1.005179411$

and $\log .7803948347 = 6 \times 1.005179411 = 6.031076466$

Thus $1.23456789^{9.87654321} = 60{,}310{,}764{,}660$ nearly.

(The correct figure is $60{,}310{,}764{,}802.44\cdots$)

(4) Find π^{19}: (a) assuming $\pi = \frac{355}{113}$ (b) using the true value of $3.141592654\cdots$

We find $\log \frac{355}{113} = .4971499094$, $\log \pi = .4971498726$.

Taking the antilogarithms of 19 times these we get

$$\left(\frac{355}{113}\right)^{19} = 2{,}791{,}568{,}434$$

$$\pi^{19} = 2{,}791{,}563{,}937$$

(5) Find a number such that its common logarithm (i.e. to base 10) is one-tenth of the number:

Let N be the required number. Then we have $\frac{N}{10} = \log N$.

If we make N go from 0 to plus infinity we find that $\frac{N}{10} - \log N$ is plus infinity for $N = 0$, and plus infinity for $N =$ plus infinity.

Its differential coefficient with respect to N is $\frac{1}{10} - \frac{\log e}{N}$ which is negative from $N = 0$ to $N = 10 \log e = 4.343\cdots$ and positive from there to $N =$ plus infinity. At $N = 10 \log e$, $\frac{N}{10} - \log N = -.204$ so therefore $\frac{N}{10} - \log N$ decreases continually from a positive value for $N = 0$ to a negative value for $N = 4.343$ and then increases continually to a positive value for $N =$ infinity. So there are two values of N for which $\frac{N}{10}$ equals $\log N$, namely one value below $4.34\cdots$ and one above $4.34\cdots$.

As to the second value we recognize at once it is $N = 10$ since $\log 10 = 1$.

As to the first we find it is greater than 1 since $\frac{1}{10} - \log 1$ is positive.

From Table I we find N is between 1.36 and 1.38

$\frac{N}{10}$	$\log N$	$\frac{N}{10} - \log N$
.136	.1335389··	+ .0024611··
.138	.1398791··	− .0018791··
Difference	.0063402··	.0043402··

Since an increase in $\log N$ in the second column by .0063042 brings a change (decrease) of .0043402 in $\frac{N}{10} - \log N$ in the third column, a change of .0024611 (which will make $\frac{N}{10} - \log N = 0$) will be brought about by an increase in $\log N$ of approximately $.0063402 \times \frac{24611}{43402}$ or .0035952. So $\log N$ must be increased by an amount between .0035 and .0036.

Making use of Table II we get the following

$$\log 1.36 + .0035 = .1370389084 = \log (10 \times .1371004588)$$
$$\log 1.36 + .0036 = .1371389084 = \log (10 \times .1371320311)$$

Now what we want is to find t such that in

$$\log 1.36 + .0001\,(35 + t) = .137089084 + .0001\,t$$
$$= \log 10\,(.1371004588 + \bar{t}\,.0000315723)$$

we have

$$.1370389084 + .0001\,t = .1371004588 + \bar{t}\,.0000315723$$

or $t - .315723\,\bar{t} = .615504$.

As a first approximation we put $\bar{t} = t$; then we get $t = .899495$ and for this value of t the value of u is .000010 and $\bar{t} = t - .000010$. So putting this value of $\bar{t}$ in the equation we get $t = .899492$.

Thus $\log 1.371288576 = \log 1.36 + .0035899492 = .1371288576$.

So $1.371288576\cdots$ is the second value for which $\frac{N}{10} = \log N$.

These two values, 10 and $1.371288575\cdots$, are examples of numbers whose logs to base 10 have the same significant figures as the numbers.

TABLE I

Logarithms of Numbers from 1.00 to 10.00

N	$\log N$	N	$\log N$
1.00	.00000 00000	1.80	.25527 25051
1.02	.00860 01718	1.83	.26245 10897
1.04	.01703 33393	1.86	.26951 29442
1.06	.02530 58653	1.89	.27646 18042
1.08	.03342 37555	1.92	.28330 12287
1.10	.04139 26852	1.95	.29003 46114
1.12	.04921 80227	1.98	.29666 51903
1.14	.05690 48513	2.01	.30319 60574
1.16	.06445 79892	2.04	.30963 01674
1.18	.07188 20073	2.07	.31597 03455
1.20	.07918 12460	2.10	.32221 92947
1.22	.08635 98307	2.13	.32837 96034
1.24	.09342 16852	2.16	.33445 37512
1.26	.10037 05451	2.19	.34044 41148
1.28	.10720 99696	2.22	.34635 29745
1.30	.11394 33523	2.25	.35218 25181
1.32	.12057 39312	2.30	.36172 78360
1.34	.12710 47984	2.35	.37106 78623
1.36	.13353 89084	2.40	.38021 12417
1.38	.13987 90864	2.45	.38916 60844
1.40	.14612 80357	2.50	.39794 00087
1.42	.15228 83444	2.55	.40654 01804
1.44	.15836 24921	2.60	.41497 33480
1.47	.16731 73347	2.65	.42324 58739
1.50	.17609 12591	2.70	.43136 37642
1.53	.18469 14308	2.75	.43933 26938
1.56	.19312 45984	2.80	.44715 80313
1.59	.20139 71243	2.85	.45484 48600
1.62	.20951 50145	2.90	.46239 79979
1.65	.21748 39442	2.95	.46982 20160
1.68	.22530 92817	3.00	.47712 12547
1.71	.23299 61103	3.05	.48429 98393
1.74	.24054 92483	3.10	.49136 16938
1.77	.24797 32664	3.15	.49831 05538

TABLE 1 (Continued)

Logarithms of Numbers from 1.00 to 10.00

N	$\log N$	N	$\log N$
3.20	.50514 99783	5.68	.75434 83357
3.25	.51188 33610	5.76	.76042 24834
3.30	.51851 39399	5.88	.76937 73261
3.35	.52504 48070	6.00	.77815 12504
3.40	.53147 89170	6.12	.78675 14221
3.45	.53781 90951	6.24	.79518 45897
3.50	.54406 80444	6.36	.80345 71156
3.55	.55022 83531	6.48	.81157 50059
3.60	.55630 25008	6.60	.81954 39355
3.68	.56584 78187	6.72	.82736 92731
3.76	.57518 78449	6.84	.83505 61017
3.84	.58433 12244	6.96	.84260 92396
3.92	.59328 60670	7.08	.85003 32577
4.00	.60205 99913	7.20	.85733 24964
4.08	.61066 01631	7.32	.86451 10811
4.16	.61909 33306	7.44	.87157 29355
4.24	.62736 58566	7.56	.87852 17955
4.32	.63548 37468	7.68	.88536 12200
4.40	.64345 26765	7.80	.89209 46027
4.48	.65127 80140	7.92	.89872 51816
4.56	.65896 48427	8.04	.90525 60487
4.64	.66651 79806	8.16	.91169 01588
4.72	.67394 19986	8.28	.91803 03368
4.80	.68124 12374	8.40	.92427 92861
4.88	.68841 98220	8.52	.93043 95948
4.96	.69548 16765	8.64	.93651 37425
5.04	.70243 05364	8.76	.94250 41062
5.12	.70926 99610	8.88	.94841 29658
5.20	.71600 33436	9.00	.95424 25094
5.28	.72263 39225	9.20	.96378 78273
5.36	.72916 47897	9.40	.97312 78536
5.44	.73559 88997	9.60	.98227 12330
5.52	.74193 90777	9.80	.99122 60757
5.60	.74818 80270	10.00	1.00000 00000

TABLE II

M	Antilog M	Δ	M	Antilog M	Δ
.0000	1.000000000	230285	.0025	1.005773063	231614
.0001	1.000230285	230338	.0026	1.006004677	231668
.0002	1.000460623	230391	.0027	1.006236345	231721
.0003	1.000691014	230444	.0028	1.006468066	231775
.0004	1.000921458	230498	.0029	1.006699841	231828
.0005	1.001151956	230550	.0030	1.006931669	231881
.0006	1.001382506	230603	.0031	1.007163550	231935
.0007	1.001613109	230657	.0032	1.007395485	231988
.0008	1.001843766	230709	.0033	1.007627473	232041
.0009	1.002074475	230763	.0034	1.007859514	232095
.0010	1.002305238	230816	.0035	1.008091609	232149
.0011	1.002536054	230869	.0036	1.008323758	232202
.0012	1.002766923	230922	.0037	1.008555960	232255
.0013	1.002997845	230976	.0038	1.008788215	232309
.0014	1.003228821	231028	.0039	1.009020524	232362
.0015	1.003459849	231082	.0040	1.009252886	232416
.0016	1.003690931	231135	.0041	1.009485302	232469
.0017	1.003922066	231188	.0042	1.009717771	232523
.0018	1.004153254	231242	.0043	1.009950294	232577
.0019	1.004384496	231294	.0044	1.010182871	232630
.0020	1.004615790	231348	.0045	1.010415501	232683
.0021	1.004847138	231401	.0046	1.010648184	232737
.0022	1.005078539	231455	.0047	1.010880921	232791
.0023	1.005309994	231508	.0048	1.011113712	232844
.0024	1.005541502	231561	.0049	1.011346556	232898
.0025	1.005773063	231615	.0050	1.011579454	232952

$$\text{Antilog}\,(M + .0001\,t) = \text{Antilog}\,M + \bar{t}\,\Delta$$

where $\bar{t} = t - u$ from Table III

TABLE II (Continued)

M	Antilog M	Δ	M	Antilog M	Δ
.0050	1.011579454	232952	.0075	1.017419366	234297
.0051	1.011812406	233005	.0076	1.017653663	234350
.0052	1.012045411	233059	.0077	1.017888013	234404
.0053	1.012278470	233113	.0078	1.018122417	234459
.0054	1.012511583	233166	.0079	1.018356876	234512
.0055	1.012744749	233220	.0080	1.018591388	234566
.0056	1.012977969	233273	.0081	1.018825954	234621
.0057	1.013211242	233328	.0082	1.019060575	234674
.0058	1.013444570	233381	.0083	1.019295249	234729
.0059	1.013677951	233435	.0084	1.019529978	234782
.0060	1.013911386	233488	.0085	1.019764760	234837
.0061	1.014144874	233543	.0086	1.019999597	234890
.0062	1.014378417	233596	.0087	1.020234487	234945
.0063	1.014612013	233650	.0088	1.020469432	234999
.0064	1.014845663	233704	.0089	1.020704431	235053
.0065	1.015079367	233757	.0090	1.020939484	235107
.0066	1.015313124	233812	.0091	1.021174591	235161
.0067	1.015546936	233865	.0092	1.021409752	235215
.0068	1.015780801	233919	.0093	1.021644967	235270
.0069	1.016014720	233973	.0094	1.021880237	235324
.0070	1.016248693	234027	.0095	1.022115561	235378
.0071	1.016482720	234080	.0096	1.022350939	235432
.0072	1.016716800	234135	.0097	1.022586371	235486
.0073	1.016950935	234189	.0098	1.022821857	235540
.0074	1.017185124	234242	.0099	1.023057397	235595
.0075	1.017419366	234297	.0100	1.023292992	

$$\text{Antilog}\,(M + .0001\,t) = \text{Antilog}\,M + \bar{t}\,\Delta$$

where $\bar{t} = t - u$ from Table III

TABLE III

t	u	$\bar{t}$	t	u	$\bar{t}$
.000000	0	.000000	.450491	29	.450462
.004363	1	.004362	.549529	28	.549501
.013204	2	.013202	.605547	27	.605520
.022209	3	.022206	.640807	26	.640781
.031387	4	.031383	.668857	25	.668832
.040749	5	.040744	.692871	24	.692847
.050305	6	.050299	.714208	23	.714185
.060069	7	.060062	.733605	22	.733584
.070054	8	.070046	.751510	21	.751489
.080277	9	.080268	.768222	20	.768202
.090756	10	.090746	.783953	19	.783934
.101509	11	.101498	.798856	18	.798838
.112561	12	.112549	.813051	17	.813034
.123938	13	.123925	.826629	16	.826613
.135669	14	.135655	.839665	15	.839650
.147791	15	.147776	.852219	14	.852205
.160346	16	.160330	.864341	13	.864328
.173383	17	.173366	.876072	12	.876060
.186962	18	.186944	.887448	11	.887437
.201157	19	.201138	.898499	10	.898489
.216061	20	.216041	.909252	9	.909243
.231792	21	.231771	.919729	8	.919721
.248505	22	.248483	.929952	7	.929945
.266411	23	.266388	.939936	6	.939930
.285808	24	.285784	.949700	5	.949695
.307147	25	.307122	.959255	4	.959251
.331161	26	.331135	.968616	3	.968613
.359212	27	.359185	.977793	2	.977791
.394473	28	.394445	.986798	1	.986797
.450491	29	.450462	.995639	0	.995639

$$\bar{t} = t - u$$

For values of t, or $\bar{t}$, between the values given use for u the value for the next smaller values of t, or $\bar{t}$.

APPENDIX

In Part II we have

$$\left.\begin{aligned} u = t - \bar{t} &= t - \frac{e^{kt}-1}{e^k-1} \\ &= \frac{t(1-t)}{2}k - \frac{t(1-t)(1-2t)}{12}k^2 - \frac{t^2(1-t)^2}{24}k^3 \\ &\quad + \frac{t(1-t)(1-2t)(1+3t-3t^2)}{720}k^4 - \cdots \end{aligned}\right\} \text{(i)}$$

which is the fundamental equation.

In our case k is $.0002302585\cdots$ and comes from $e^k = 10^{.0001}$ so that $k = .0001 \times \log_e 10$.

Let the maximum value of u, for $t > 0$ and < 1, be denoted by U, and let the value of t that produces U be denoted by T.

Differentiating the second equation of (i) by t and setting du/dt equal to zero to get the maximum value we have

$$0 = 1 - \frac{ke^{kT}}{e^k-1}$$

whence $e^{kT} = \dfrac{e^k-1}{k}$

and $T = \dfrac{1}{k}\log_e \dfrac{e^k-1}{k}$

$$= \tfrac{1}{2} + \frac{k}{24} - \frac{k^3}{2880} + \cdots + (-1)^{s-1}\frac{B_{2s}k^{2s-1}}{2s\,\underline{|2s}} + \cdots \qquad \text{(ii)}$$

where the numbers B_{2s} are Bernouilli's numbers.

Putting these values of T and e^{kT} in (i) we get

$$U = T - \frac{1}{k} + \frac{1}{e^k-1}$$

$$= \frac{k}{8} - \frac{k^3}{576} + \cdots + (-1)^{s-1}\frac{(2s+1)B_{2s}k^{2s-1}}{2s\,\underline{|2s}} + \cdots \qquad \text{(iii)}$$

The main problem, however, that we have with equation (i) is to determine the values of t for which u has a given value. We could do this by solving

directly the third equation of (i), by putting $\frac{8u}{k} = 1 - z^2 = w$ and we get

$$t = \tfrac{1}{2} \pm \frac{z}{2} + \frac{w\,k}{24} \pm \frac{w\,(2-w)}{288\,z}\,k^2 + \frac{w\,(2-w)}{4320}\,k^3 \cdots \qquad \text{(iv)}$$

and for our purposes the first three terms of this are accurate enough and give

$$t = \tfrac{1}{2} + \frac{u}{3} \pm \sqrt{\tfrac{1}{4} - \frac{2u}{k}} \qquad \text{(v)}$$

However if z is small, i.e. if u is near to U, the series in (iv) is divergent (this is because $U < \frac{k}{8}$ and so $z = 0$ gives imaginary roots in the third equation of (i)).

The correct method of finding t accordingly is: from the second equation of (i) and the first of (iii) we get

$$k\,(U-u) = e^{k(t-T)} - k\,(t-T) - 1 \qquad \text{(vi)}$$

and we solve this for $t - T$: note it is satisfied by $u = U$, $t = T$.

If $e^\phi - x - 1 = \frac{y^2}{2}$

then $x = y - \frac{y^2}{6} + \frac{y^3}{36} - \frac{y^4}{270} + \frac{y^5}{4320} - \frac{y^6}{17010} + \cdots$

so putting $U - u = \frac{k\,\phi^2}{2}$ and $k\,(U-u) = \frac{k^2\,\phi^2}{2} = \frac{y^2}{2}$

we get $k\,(t-T) = y - \frac{y^2}{6} + \cdots$

and

$$t = T \pm \phi - \frac{U-u}{3} \pm \frac{\phi\,(U-u)}{18}\,k - \frac{2\,(U-u)^2}{135}\,k^2 \pm \cdots$$

$$= \tfrac{1}{2} + \frac{u}{3} - \frac{2\,(U-u)^2}{135}\,k^2 - \cdots \pm \phi\left[1 + \frac{U-u}{18}\,k + \cdots\right] \qquad \text{(vii)}$$

The first three terms of this are accurate for the calculations of this paper and give

$$t = \tfrac{1}{2} + \frac{u}{3} \pm \sqrt{\frac{2\,(U-u)}{k}} \qquad \text{(viii)}$$

which of course is practically the same as (v).

For the Part I calculations we have to change the sign of k, and of u, and we get similarly to the above

$$u = \frac{1 - e^{-kt}}{1 - e^{-k}} - t$$

$$T = \tfrac{1}{2} - \frac{k}{24} + \frac{k^3}{2880} - \cdots$$

$$U = \frac{k}{8} - \frac{k^3}{576} + \cdots$$

$$t = T \pm \phi + \frac{U - u}{3} \pm \cdots$$

$$= \tfrac{1}{2} - \frac{u}{3} \pm \sqrt{\frac{2\,(U-u)}{k}} \quad \text{near}$$

$$\text{or} = \tfrac{1}{2} - \frac{u}{3} \pm \sqrt{\tfrac{1}{4} - \frac{2u}{k}} \quad \text{enough.}$$

THE ACTUARY AS A CONSULTANT

BY

J. J. SMICK

All actuaries have probably at one time or another been asked to explain the nature of their work. As far as the layman is concerned the easiest way of answering a question of this kind is to cite one or more specific examples of not too technical a nature. For those desiring a somewhat fuller explanation the various qualifications and requirements for membership in the societies may be used to explain the matter. It is obvious that a few specific examples are not adequate criteria and the whole syllabus of study for the examinations too broad a gauge to use. A more nearly correct answer would be somewhere in between and this too would omit many intangible factors which are involved in actuarial work and which are nowhere directly covered in the literature of the profession.

A personal example of the difficulty of explaining the work of an actuary may be cited. My wife, upon inquiry, explained to our son that the actuarial profession involves work with numbers. This interesting item of information was broadcast to the neighbors and the many playground companions as "My daddy plays the numbers."

In attempting to prepare a representative list of examples of the work of an actuary, I was impressed by the fact that so much of the work depends on the calls made upon his services by other departments. Rarely is the work of an actuary, and particularly that of an experienced casualty actuary, confined to what one would characterize as a purely actuarial problem; more often is it a problem originating outside of the actuarial department, but which somehow requires the training, experience and background of an actuary for aid in its proper solution.

Following this line of reasoning it occurred to me that a good description of the work could be given by emphasizing the consulting phases of the work, both in company offices and in the work of independent consulting firms. The presentation could be made relatively easy by citing a sufficient number of examples of actual cases. Since nearly all of the examples have been submitted by consultants the paper has been entitled "The Actuary as a Consultant." It could just as easily have been called "Examples of the Nature of Actuarial Work."

The functions of an actuary and the work he performs are in a large measure determined by his own abilities and by the needs of the organization employing him. Certain minimum qualifications are established by the

examination requirements for membership testing both his knowledge and ability. In practice few men actually need or use all of their knowledge and abilities except in a general way. Each man confines his work to the particular needs of the organization employing him. Sometimes the needs are broad, sometimes narrow and highly specialized. If we consider our colleagues engaged in actuarial work, one by one, we can probably arrive at a fair idea of each individual's responsibilities in the particular position he holds. If we think of the actuarial department of any company, and of the men in it, we can also obtain a fair idea of the range of activities of the department.

The functions of an actuary are proven by the fact that many actuaries often become engaged in activities which are not actuarial. An actuary as a result of his work with administrative matters may become an executive. If his company has found the advice he has given to the underwriting department useful, he is quite likely to become an underwriter. If the statistical department requires his services he becomes a statistician. In all of these instances the individual, originally qualified as an actuary, proves more useful to his particular company in some other capacity. He is still of course capable of acting as an actuary. The fact that so many actuaries do become engaged in other phases of insurance work is fairly conclusive proof that many of the basic qualifications of an actuary are of value to other departments.

In examining the rosters of the societies we see that many actuaries have shifted their endeavors to other phases of work. Actuaries have become presidents, secretaries and assistant secretaries, treasurers, statisticians and chief statisticians, comptrollers, general managers, vice presidents, executive vice presidents, first, second, third and fourth vice presidents. Sometimes they also retain the title of actuary and call themselves vice president and actuary or secretary and actuary. I do not know whether this is to emphasize the fact that the company feels that the position or duties of actuary are to be maintained; or whether the individuals retain the title of actuary from a feeling of pride connected with the professional connotation of the word which may serve to place them in a class separate and distinct from the common garden variety of vice presidents and secretaries. Possibly they retain the title of actuary to denote their humble origin as an ordinary working member of the insurance fraternity who has come up the hard way.

When he is employed by an insurance company or organization, the actuary is consulted by the other departments on matters involving actuarial procedure. If he isn't consulted, the company is not making full use of his services. There is of course the additional possibility that he is not capable of giving advice on matters outside of his department. A study of the

requirements for becoming a Fellow would lead one to believe this would be improbable unless only a cursory knowledge of subjects was obtained sufficient for passing the examinations but not for actual use.

In addition to those members who are directly employed as actuaries by companies, and those members who have transferred their activities to other departments we have the independent professional actuary, operating his own office and unaffiliated with any company or recognized association maintained by insurance companies. He is usually called a consulting actuary. Often a number of men are associated and operate as a firm.

Perhaps the term "consulting actuary" should be clarified. In the medical profession a consultant is a doctor whom other doctors consult. A consulting engineer is usually one consulted by other engineers or engineering organizations. A consulting actuary is not in quite the same category. Since the majority of actuaries are employed by insurance firms and organizations, they are usually not in a position to give advice to other organizations or individuals. The consulting actuary may be consulted by insurance companies, by trade associations, by corporations, by brokers and agents, lawyers, governmental agencies, insurance departments, trust officers on estate and tax matters, by retirement and pension funds, by non-profit hospital and medical insurance associations and not infrequently by other actuaries who wish to obtain the benefit of an opinion on problems with which they are not as familiar as is the consulting actuary.

Such consultants are now and have always been an important element in the membership rosters of the Actuarial Society. As independent practitioners they give point to the fact that the actuary is a professional peculiarly qualified by his training and experience to render an important and useful service, whether as an employee or as an independent consultant.

Although many members of the societies know of the qualifications and abilities of the consultants either through personal association when such members worked in company offices prior to becoming independent consultants, or because such members have become fellows or associates by examination, few have definite knowledge of the exact nature of the work such consultants perform. It is assumed that the services of a consultant are somehow related to the work in company offices. There has been little published about the actual work, the methods used and the clients serviced by the consultants.

The fact that so little is known is in part due to the natural desire to keep their business relationships as quiet as possible as a protection from possible competition; also the majority of the recognized firms treat their relationship with their clients as a purely confidential one. They depart from this rule only when they are required to appear in public on behalf of the client.

This has undoubtedly been the principal reason for the relatively little public knowledge of their work.

Part, perhaps the major portion of this paper, will be devoted to actual case problems. Most of the papers in the various journals of the actuarial societies deal with broad general problems or with smaller ones discussed in great detail. In this paper each case will be presented briefly.

Aside from the inherent interest that each case may have, as an illustration of the problems presented to an actuary, the cases are of value as examples of the educational background, technical training and general understanding that an actuary must have of the insurance business as a whole. The actuary not only must be able to solve the purely technical details, but must also understand the administrative, underwriting and statistical relationship of the problem as a whole. He must know the sources of statistical data, the underwriting procedures, the rate making and rating procedure, must understand the manuals, know the effect upon reserves and company solvency of interest and mortality factors, and must also know of the day by day developments in the various fields of insurance and in the social, political and economic life of the country.

I am indebted to a number of the leading consultants for these cases. Each case has been rewritten and edited so as to conceal both the consultant and the client and any insurance companies that might be involved. Each consultant will probably think he recognizes his own case. Sometimes two related problems have been merged as one case if this served as a better illustration. In other instances matter considered as of minor importance or extraneous to the main problem presented by the case has been deleted. In any event, the author assumes responsibility for the treatment. To paraphrase the movies, any similarity of these cases to the problems of any individuals, organizations, insurance companies, living or dead, is purely coincidental.

The cases which treat with similar or related problems have been grouped to facilitate study and comment. The description headings and group titles are correct in only a general way.

GROUP I

CASES INVOLVING RATING PROCEDURES FOR INDIVIDUAL RISKS

CASE 1—*Evaluation of Three Competitive Insurance Proposals.*

A multiple line, interstate risk which had been carried by one carrier for years on a non-competitive basis, desired to place a portion of its business,

amounting to about $150,000 in annual premiums, on an overall retrospective basis. It had received a plan from its carrier, but had instructed its broker to receive bids from two other carriers. After receiving the plans, it was unable to evaluate them, in part because they were not on a comparable basis. The matter was referred to a consulting actuary.

The experience used by the carrier on the risk as the basis for its estimate and that used by the two competing carriers was different. The carrier on the risk when asked for the experience knew that the purpose for which it was to be used was to make a comparative estimate. Undoubtedly it was just an oversight on its part in compiling the data. It was of course a pure coincidence that the experience it had used itself would give a lower overall premium.

It was decided to use the experience of the latest completed policy year as the basis for comparing the three plans. Accordingly a schedule of the experience desired had to be drawn up and obtained from the carrier on the risk which will be designated as A and submitted to the other two carriers who will be designated as B and C. The underwriter in charge of the line and the statistician of carrier A assisted the broker and consultant in developing the experience. The statistician knew his statistics and did a fairly good job in gathering the basic data, which covered a number of different lines, states and locations.

After many conferences and discussions clarifying coverages, three plans with the resulting premiums based on the past experience were available. The estimates submitted by carrier B had to be returned for correction, as the consultant found that the wrong manual of rates had been used for developing the premium on an important line.

Carriers B and C had submitted well constructed plans. Both of these carriers had competent actuarial departments and the results were reflected in the plans submitted. Carrier A did not have an actuarial department and relied on its underwriter and statistician. The statistician, who had only a slight inkling of the purpose of retrospective rating was unsure of himself and was dominated by the underwriter who considered such plans not as modern rating instruments but as purely competitive devices for the purpose of cutting rate structures. As a result Carrier A submitted a "me too" plan. The underwriter was intent on keeping the business and simply adjusted all values to meet competition. The broker or the risk had apparently submitted hints on the contents of the other plans and Carrier A continued to submit revisions of its original plan.

An analysis of the three plans showed that while the "me too" plan submitted by Carrier A might possibly produce the lowest premium, it was basically unsound and did not properly take into account state rate regu-

latory requirements, excess limit losses per case and other factors necessary for the successful operation of such a plan of insurance. Even though it might develop the lowest premium it was bound to cause friction and misunderstanding between the assured and the carrier. The underwriting procedure was such as to make it difficult to determine which losses should and which should not be included in the retrospective premium determination.

The other two plans were sound and workable. However Carrier B was much more conservative and had placed much more of the premium on a guaranteed cost basis. The plan submitted by Carrier C was distinctly more advantageous to the assured, was clear cut and unambiguous and although probably developing, in its basic values, a higher premium than that of Carrier A, was recommended by the consultant.

Contrary to the advice of the consultant the line was placed on a retrospective basis with Carrier A for one year. At the end of the year the consultant was asked to verify some of the computations underlying the premiums. Losses not properly belonging in the retrospective agreement had been included. Of its own volition and without consulting the actuary further, the risk transferred the line to Carrier C.

The underwriter for Carrier A has been since promoted and is now high up in his company. The statistician has never been heard of especially.

Also, purely as a matter of incidental information, the risk had been referred to the consultant by the president of Carrier B.

CASE 2—*Relationship between Underwriting Judgment and Actuarial Procedures.*

A compensation risk had developed very good experience. On the basis of its experience rating it had received a modification of .267 which when applied to the manual rate of $5.00 resulted in a final adjusted rate of $1.335 for the risk.

The underwriter in charge of the risk was not satisfied and desired to lower the rate still further. It was admitted that the $5.00 manual class rate had been established to a large extent on a judgment basis, but it was a newly erected class for a new chemical process and there would not be sufficient experience for some time to come on which to revise the classification rate.

The underwriter insisted on doing something further for the risk and wanted the classification rates reduced. He succeeded in getting a 20% reduction and a new manual rate of $4.00 applicable to the renewal policy. This of course was also largely a matter of underwriting judgment.

By reducing the rate from $5 to $4 the credibility assigned to the risk

experience in the rating was also reduced. The revised modification was .355 which when applied to the $4.00 class rate resulted in a final adjusted rate of $1.420 for the risk. As this was an increase of 6.4% over the previous rate of $1.335 the assured demanded an explanation.

The underwriter is still trying to explain the matter to the assured. He believes it to be the fault of the experience rating plan but cannot locate any mistakes in the rating. As far as he is concerned the original adjusted rate for the risk was too high. He sees no reason why the proper solution should not be a reduction in the classification rate, even if this has to be on the basis of underwriting judgment. It is of course impossible to raise the rate back to $5.00.

The odd part of this story is that the company has a very competent actuarial department in addition to a very competent underwriting department. If the underwriter had consulted the actuarial department on the rate making and experience rating aspects of his problem before he launched on his crusade for lower manual rates on an underwriting judgment basis he would have saved himself and others a lot of trouble.

This is how it happened.

	Original Rate		Revised Rate	
	Risk Experience A	Class Experience B	Risk Experience A	Class Experience B
1. Manual Rate	—	$ 5.00	—	$ 4.00
2. Total Losses	10,000	60,000	10,000	48,000
3. Primary Losses	7,000	42,000	7,000	33,600
4. Excess Losses	3,000	18,000	3,000	14,400
5. "B" Value	7,500	7,500	9,400	9,400
6. "W" Value	.70	.70	.53	.53
7. Rateable Excess Losses (4) × (6)	2,100	12,600	1,590	7,632
8. Total Rateable (3) + (5) + (7)	16,600	62,100	17,990	50,632
9. Indicated Modification $9A \div 9B$		.267		.355
10. Adjusted Rate (1) × (9)		$1.335		$1.420

CASE 3—*An Analysis of the Reasons for Poor Experience and High Rates.*

A concern had its compensation coverage cancelled by a number of carriers, first by stock carriers and then by mutual carriers. Its current policy was in a State Fund. It had been placed in a special group by the Fund so that in effect it was a self rated risk. Its premium rates were still high and it felt that its only remaining recourse was to become a self-insurer and thus save a lot of money. It engaged a consultant to take the necessary steps to enable it to become a self-insurer.

An investigation of the record of its past coverage revealed a deplorable situation as regards accidents. Safety survey after survey and recommen-

dations by the safety engineers of its previous carriers and current carrier were continuously disregarded. There seemed to be a division of authority between the persons responsible for the plant operations and the executive management offices. Instructions to carriers had been to make safety reports directly to the plant officials who in turn filed them away without taking any action. The record of losses and premium payments were however a main office matter. The main office was greatly concerned over the premiums it was paying, but apparently wasn't at all interested in the reasons for this, except that it believed that the insurance companies insisted on charging too much.

It was pointed out to the management that self insurance would not solve the problem but would more likely tend to aggravate it. There was sufficient evidence, based on the survey reports of the insurance carriers to show that the accident record was a management problem. Without these reports there would have been no check on the plant officials.

It was suggested that the main executive offices take direct responsibility for the safety conditions in the plants and see to it that recommendations by company safety engineers be rigidly enforced. It was further pointed out that its own past management or mismanagement was the most convincing argument against self-insurance. The only remedy was proper safety work which it could only learn on the basis of the advice given it by the qualified engineers of the insurance companies.

The management decided not to become a self-insurer.

CASE 4—*Establishing the Value of Self-Insurance in Renegotiation Proceedings.*

A concern operating two large subsidiary corporations was a self-insurer except for some miscellaneous Lloyds and excess policies. It had apparently operated as a self-insurer for a long time and had set aside a fund of $1,000,000 for catastrophe losses. Its operations were such as to make it liable to have losses of catastrophic proportions.

Upon the outbreak of the war it began manufacturing almost exclusively for the Government. The profits on the products sold to the Government were apparently large and finally the Government instituted re-negotiation proceedings to regain some of the excess profits.

The concern desired an estimate of the value of the self insurance which could properly be charged to the Government in the proceedings. Its actual losses and loss adjustment expense had been small, but the concern felt that it had placed its catastrophe funds, accumulated in peace time, at risk and in the event of a catastrophe would have suffered the loss.

Using the standard manuals and classifications and applying various rating plans several estimates were obtained of the premium that would have been payable had the concern been insured with private carriers, The matter was further complicated by the fact that there were two subsidiary corporations and the results varied depending as to whether they were to be treated as a single entity or separate entities for rating purposes. A minimum figure, a maximum figure and a reasonable basis for compromise were developed for the concern to use in the re-negotiation proceedings.

CASE 5—*Establishing the Future Cost of Insurance in Renegotiation Proceedings.*

A metal goods manufacturer was engaged in the production of war supplies for the Navy. He did not avail himself of the special plans available for war contracts but remained under the standard experience rating plan. Due to increase in operations a large number of inexperienced employees were hired and several working shifts were required. As a result the experience became progressively worse. The full impact of the adverse experience would be felt in the years 1946, 1947 and 1948. The manufacturer desired a renegotiation of his government contract to compensate him for excess insurance cost in the post war period.

The problem of determining the extent of a just claim involved a number of assumptions. These assumptions were made as respects the trend of the loss experience had there been no war and what it would be in the reconversion and peacetime period following the cessation of war contract work. Further assumptions were made as respects the payrolls during such a period. Unless these assumptions were reasonable and based on sound judgment and past experience of the risk they could not be justified.

Having made the assumptions it was necessary to calculate the effect of the experience rating plan (again assuming that no essential changes will be made in this plan) and determine in this manner the monetary amount of the excess premium due to wartime activities. In addition, in order to summarize the effect of the wartime experience the retrospective plan was suggested. The results obtained in this fashion were remarkably reasonable and satisfactory.

CRITICAL REVIEW

The above five cases each present a different aspect in connection with problems relating to individual risks. The first three cases involve underwriting judgment and the last two involve accounting. Cases 1 and 2 show the need of actuarial knowledge to supplement the work of the under-

writer. They illustrate the fact that modern casualty rating requires more than a cursory knowledge of the highly technical rating instruments. Rate-making and rating instruments although developed by the actuaries are nevertheless tools used by the underwriter and were in almost every instance established primarily for his use. The underwriter who lost the risk in Case 1 could only blame himself. He was on the risk and should have developed, in an orderly manner and on his own initiative a workable plan before the matter was opened to competition. The carrier and the underwriter lost the risk primarily because of failure to understand the appeal of retrospective rating and the nature of such plans.

Either an underwriter or an actuary would have reached the same conclusion in Case 3. Recognition that high premium rates are due to a poor accident record does not require actuarial training. The value of the advice in the case involved recognition of the basic conditions causing the risk to desire self-insurance.

Cases 4 and 5 present two diametrically opposed situations. In Case 4 the actuary is requested to establish the value, in a claim against the Government, of good experience, while in Case 5 the actuary is requested to establish the cost of poor experience. Although each of these originates as an accounting matter, they are primarily actuarial problems. As such of course the solutions involve many conjectural elements. The reasonableness of the claims depends on the proficiency of the actuary in the use of rating instruments and his grasp of the effect of experience upon rates.

GROUP II

CASES INVOLVING SUPERVISORY AUTHORITIES

CASE 6—*Representation before an Insurance Department on behalf of a Carrier in a Rate Deviation Case.*

A specialty carrier operating in a restricted field was denied its customary deviation from manual by a supervising insurance department, on the grounds that the policy of the department was to grant such deviations only if the requesting carrier could submit evidence that its expense ratio justified such a deviation. In this instance the full deviation requested was not justified on the basis of the expense ratios.

An analysis of the situation disclosed that rates for this particular line had not been revised recently and were in all probability out of line. Fur-

thermore, the method of operation followed by the carrier contemplated relatively large expenditures for safety investigation and legal expenses which in turn were reflected in a very low loss ratio year after year.

It was pointed out, in a brief prepared for the carrier, that the proper basis of determining whether a deviation could be granted was on the basis of the combined loss and expense figure, not merely on the expense ratio. It was shown that the underwriting results amply justified the deviation requested and that it was unsound to base a decision solely on the expense provisions without taking into account the entire rate structure.

The insurance department rescinded its action and granted the request for the deviation.

CASE 7—*Expert Opinion on behalf of an Insurance Commissioner in Life Insurance Company Conservation Proceedings.*

Under the laws of a certain state it was possible to write stipulated premium life insurance. There were a good many concerns organized which were run more in the interest of the management as a quasi-proprietary enterprise than in the interest of the policyholders. Since these concerns incorporated assessment provisions in their policies it was difficult to show on the basis of financial statements alone that they were in a hazardous condition. In attempting to take conservation proceedings the Insurance Commissioner became involved in a court fight.

A consulting actuary was called in as an expert in the proceedings, particularly on the point of the excessive expense at which the concerns were being run. The Actuary took the position that since these concerns were supposedly mutual enterprises and the policyholders were subject to supplementary assessments, if it became necessary, they should do at least as well for their policyholders as non-participating companies. Taking Mr. Cammack's paper on non-participating life insurance premiums (T.A.S.A., Vol. xx) a schedule of reasonable expenses for operating a company was established.

The formula contemplated that with respect to new business up to $6 per policy was allowable for the expense of medical examination and inspection. For the general running of the company including all general overhead $3.00 per policy plus 5% of the premium was allowed. Agency commissions were established at 50% of the first year premiums, nine renewals at 7½%, five renewals after that at 5% and a general collection fee on the subsequent fifteen years of 3%. Most of the companies did not have medical examinations so that no more than the actual expense for this item was allowed. This schedule gave somewhat more than Cammack's but could be considered as

comparable. In order to allow for some leeway 120% of this formula was established as a top limit of reasonable expenses.

Most of the companies involved had expenses that were 160% or more of the established standard.

Another approach was to determine the margin in the gross premiums on the basis of appropriate mortality tables and persistency rates after deducting expense reserve requirements and in this way determine the present value at the end of the first year of the business of the company. This was compared with the cost of putting on the business and it was shown that more was spent for the business than it could bring in.

A comparison of the salaries paid the presidents with that paid other insurance executives of responsible companies was also introduced by the Commissioner. That was probably as persuasive as the actuarial exhibits.

The accumulated evidence influenced the Court to grant the conservation proceedings requested by the Commissioner.

CASE 8—*Determination of the Amount of Deposit for a Self-Insurer.*

A large corporation was a self-insurer as respects Workmen's Compensation in New York State. It desired to recover some of its substantial deposits with the Department of Labor and engaged the services of a consulting actuary to assist it.

The task required a complete review and valuation of several hundred open claims. In order to determine the possibility and probable cost of reopenings it was necessary to make a review of certain claims closed within the last six months and some types of claims closed within the last two years. A valuation of all Death and Permanent Total Disability claims was made on the basis of tabular values, and also of other cases involving life benefits.

In addition, the final report provided for a reserve for incurred but not reported losses, a reserve for underestimates and adverse development of open cases, and a reserve for contingencis.

The report was received favorably by the Department of Labor and a substantial refund granted.

CRITICAL REVIEW

Each of these cases involves a relationship between a carrier and supervisory officials. A judgment or ruling of some supervisory official is to be based on or at least is to be influenced by the opinion and findings of the actuary. In one case the actuary represents an insurance carrier, in another

an Insurance Commissioner and in the third and last case a self-insurer.

Case 7 is probably the most interesting case as well as a most important one. Insurance companies may not operate on an unsound actuarial basis. It is a function of the actuary to determine whether the operational methods are sound.

Group III

Management Problems of Insurance Companies

Case 9—*Advice on the Management of a Union's Accident and Health Department.*

A labor organization operating an insurance department had a management and 100% re-insurance contract with an insurance carrier as respects non-cancellable accident and health certificates. It desired to cancel this arrangement and assume full control of the line.

The insurance department was faced with the problem of installing a statistical system, establishing reserves and preparing the year-end valuation. It desired advice on the procedure to follow and at the same time wished to make a review of its policy forms, endorsements, underwriting procedure and rate structure.

A statistical procedure based on punch cards was installed. Tables for developing reserves on claims based on experience of companies writing similar benefit policies were developed to use for the first valuation, mid-terminal reserves for active lives based on the National Association of Insurance Commissioners Standards, using the morbidity rates of the Conference Modification of the Class (3) table and the mortality rates of the American Men Ultimate Table at 3% were calculated to be used for setting up the active life reserve. An estimate for incurred but not reported losses and a procedure for developing the reserve for the future were developed. Instructions for tabulating the basic data to obtain the year-end valuation figures were prepared. The policy forms, rate structure and endorsements were reviewed and undesirable forms of coverage were eliminatd. A report embodying recommendations for the conduct of the business and future procedure was submitted.

Case 10—*Underwriting Problems of a Casualty Company Losing Business to a Competitor.*

A carrier began losing a number of accounts to a competitor. In each instance the timing of the approach to the risk and the manner of solicitation led the carrier to believe that a former employee with knowledge of the effective dates and details of coverage was somehow involved.

A review of the records indicated that the carrier had been somewhat lax in its method of setting up reserves, which affected the rating of risks. In many instances reserves were too high, or had not been set up on a present value basis. Reserves for medical losses were maintained long after the recovery. Recoveries from subrogation cases were not credited to the risk losses. Classification assignments reflecting changes in the character of the risks operations were not promptly requested. Combinations of policies, where it was possible to make such combinations in the interests of the assured, had not been made.

It was suggested that more care be devoted to the proper underwriting and rating of risks so as to reduce the vulnerability to attack. It was also suggested that as many risks as possible be cancelled prior to the expiration date of the policy and rewritten so as to change the effective dates of the renewal policies. This would to some extent nullify the timing of the competitor's approach and give the carrier time to correct its underwriting methods.

CASE 11—*Services Required by a Small Life Insurance Company.*

A small life insurance company not having a sufficient volume of business to require the permanent services of an actuary decided to retain a firm of consulting actuaries on a yearly basis.

The consulting actuaries were required during the course of the year to perform the following services:

a. Prepare policy forms, premium rates, reserves, and non-forfeiture values necessitated by the passage of new minimum valuation standards.
b. Represent the company in hearings with the Insurance Department of its home State on problems arising out of the Department's examination report.
c. Calculate policy reserves, non-ledger assets and net premiums for use in connection with the Annual Statement.
d. Prepare amortization schedules for all bonds for use in the Annual Statement.
e. Prepare the dividend formula for application to policies expiring the following year, secure the Insurance Department's approval thereof and calculate the dividend rates for policies by year of issue, age and plan.

CASE 12—*Services Required by a Fraternal Benefit Society.*

A medium sized Fraternal Benefit Society decided that it would be more economical to retain the services of a consulting actuary on a yearly basis than to have a full time actuary.

In agreeing to furnish actuarial services throughout the year the consultant was required to perform the following services:

a. Prepare the Society's Annual Valuation Report. This included the calculation of policy reserves, a comparison of actual to expected mortality and the computation of the net rate of interest earned during the year.
b. Prepare the Society's Supplemental Valuation Report as required by the New York State Insurance Department.
c. Set up amortization schedules for bonds.
d. Review Society's constitution and by-laws to bring them up-to-date and simplify their construction and interpretation.
e. Prepare the new Juvenile certificates and calculate the assessments, reserves and non-forfeiture options therefor.

Critical Review

Cases 9, 10, 11 and 12 show various phases of assistance to management that a consulting actuary is in a position to render. They indicate that it may be more economical to have the advice of a competent actuary on a special problem or on a part time basis in place of a full time employee capable of performing the same functions. Cases 11 and 12 illustrating the services required by small life insurance companies may be particularly interesting to casualty men. In the field of casualty insurance so much of the rate-making, rating and statistical work are performed by the central bureaus, that it is often forgotten that life insurance companies do not function in this manner. Case 9—involving accident and health insurance—exemplifies that, if properly trained, both life and casualty actuaries are equipped to handle this lines. Case 10, wherein it was found that lax underwriting and rating procedures were responsible for loss of business, requires no special comment.

Group IV

Technical Advice Required by Insurance Companies

Case 13—*Transfer of Reserves from a 5% to lower interest basis.*

A large compensation carrier had its reserves on a 5% basis. Its investment portfolio yield was in excess of 5% but new investments and the necessity for re-investing maturing issues at a lower rate indicated the advisability of changing the reserve basis from a 5% to a 3% interest assumption.

The management desired to have appropriate reserve tables calculated using a 3% basis and engaged the consultant to prepare such tables.

After developing some test values on the 3% basis, the consulting actuary perceived that the use of the new values would practically wipe out the surplus of the carrier. Not desiring to submit a set of tables whose use would cause embarrassment to the client the consultant discussed the matter with the management. He received the surprising information that the management had always wanted to keep its surplus small. It had sizeable sums in its existing reserves which it could free to bolster its surplus. The management did appreciate being advised of the possible results of using the 3% tables and asked that a program be developed which would allow the surplus to remain at approximately the same level.

An examination of the procedure followed by the company disclosed that its reserve tables did not take into consideration the contingency of remarriage on the part of widows and that in other cases it had used a table of very low mortality rates. Accordingly by constructing reserve tables taking into consideration the remarriage contingency and a different mortality table, and by arranging a program which involved placing reserves for successive blocks of years on a 3% basis, it was possible to place the reserves, over a period of years, on a 3% basis without any marked fluctuation in surplus.

Apparently the carrier was still having trouble with its reserves and surplus because it later asked for tables on a 2½% and 2% basis.

CASE 14—*Rates and Claim Procedure for a Fraternal Sickness Society.*

A fraternal organization writing sickness insurance at a level rate for all ages, desired to have its experience reviewed and recommendations made relative to a possible change in its rate schedule to recognize age groups.

A review of the experience indicated that although on the whole the rate schedule was about right, there was the need for a rather marked increase in the rates for the older age groups. The experience had been getting progressively worse on the older classes. In his recommendations the actuary suggested that the new schedule should not be adopted for all members but only for new members. He pointed out that a sharp increase in the rates for the members now covered would cause many of the more desirable risks to drop their insurance while the less desirable would continue the coverage thus nullifying in a large measure the effect of the rate increase.

In a conference held with the managers the actuary casually asked how the validity of claims was determined. He was told that as respects new members, the character of the individuals proposed was closely investigated and their claims were carefully reviewed, but as respects the members who had been in the fund for many years, why, everybody knew them and their claims were honored as a matter of course.

The actuary suggested that before adopting the revised schedule, a special control group be appointed to review all claims carefully, that claimants be visited and that publicity be given to this procedure. Rates were increased but not to the full extent indicated by the experience.

The experience, reviewed a year or so later, proved much better than that indicated by the earlier study. It was surprising how much the health of the older and trusted members had improved when a fraternal interest was manifested in their welfare.

CASE 15—*Review of Reinsurance Contract.*

A small New York Casualty Insurance Company requested a consulting actuary to review its reinsurance contract covering Workmen's Compensation losses in excess of $10,000. The contract was negotiated many years ago and no cognizance was made of the change in the Law requiring payments into the Aggregate Trust Fund. It was necessary to review the entire experience under the contract and to make a special analysis of the experience during the last ten (10) years revaluing the reinsurance liability in the light of the new law. In view of the relatively small volume of experience, use was made of the study of the cost of excess losses completed by the N. Y. Compensation Insurance Rating Board (see Cahill's paper). The result of the survey was a modernization of the contract and a substantial reduction of the price thereof.

CASE 16—*Audit and Survey of Office Procedure of a Casualty Company.*

A multiple line casualty company anxious to place its operations on a more efficient basis decided to retain the services of consulting actuaries to make periodic detailed audits of the company's books of account and to make a report thereon of each audit.

In making this audit, the actuaries realized that the office system and management could be improved considerably by a complete survey and analysis of the company's operations. Accordingly, it recommended to the company that it authorize the actuaries to make such a survey. This recommendation was accepted and upon completion of the survey, the actuaries suggested new accounting and statistical procedures which improved the company's methods of doing business.

The actuaries were also retained by the company to check the reserves set up on accident and health and workmen's compensation claims and to represent the company before the Insurance Department in the matter of rate deviations and Department examination reports.

Critical Review

This group of cases shows a number of different technical problems presented to the actuary for solution. They include (Case 13) calculation of reserve tables; (Case 14) review of experience and establishment of rates by age groups; (Case 15) details of establishing rates for a reinsurance contract; and (Case 16) audit and survey of office procedures.

It is of interest to note that in some of these a supplementary problem presents itself. Thus in Case 14 in addition to the reserve tables a method of transfer had to be developed and in Case 14 in addition to the rates the method of application and the claim procedure were reviewed.

Group V

Problems Involving Legislative Proposals and Benefits to Employees

Case 17—*Technical Advice on the Cost of Workmen's Compensation Legislation.*

A bill radically amending the administrative and benefit provisions of a compensation act, with strong likelihood of passage, was about to be introduced. Labor was sponsoring the measure and certain Employers' Associations were opposing the measure. This is, of course, not an unusual situation.

One Employers' Association, recognizing that certain changes were inevitable, had prepared a substitute measure and desired an estimate of the effect of both measures on the cost of benefits, desired advice on some of the administrative changes, and desired a schedule of changes that would serve as a basis of compromise and which would also keep the increase in cost within certain limits.

Negotiations between the proponents and opponents of the measure started off in a cordial spirit of distrust and disharmony. No sooner did one side want something than the other would oppose it. If one side said black the other side wanted white. The consulting actuary, who was in the background, originally as a purely technical consultant on the actuarial cost of the measures, finally suggested a procedure to break the deadlock. Each side listed in the order of importance those changes in each bill which it favored. Each side also listed in the same order of importance those changes which it opposed, together with its reasons.

It was found that there was a surprisingly large area of agreement. There was much misunderstanding on the relative importance of certain items. The Employers' Association was much more liberal in its concept of benefit changes than had been anticipated but was opposed to many of the administrative changes.

When it was pointed out to both sides that some of the changes in benefit provisions and definitions would make many of the administrative procedures of the past inoperative anyway, the proponents eased up on some of the demands, and the opponents realizing that some of their opposition was pointless also became conciliatory.

Certain changes in provisions which, while not changing the actual benefits, did make for an easier administrative procedure, were suggested by the actuary and accepted. A bill acceptable to both sides was finally evolved.

CASE 18—*An Analysis of the Cost of Benefits to Seamen.*

In connection with the introduction of bills in the U. S. Senate to extend the Longshoremen and Harborworkers Act to Seamen, the Senate Committee, after public hearings, ordered a study to be conducted to show the comparison of benefits under the Jones Act and the Compensation Act. Shipping concerns were requested to furnish individual reports on each claim made in 1938. An organization representing ship owners engaged a consulting actuary to make a parallel study by furnishing him with the copies of their reports. Some 6,000 such reports had been submitted. These reports had to be audited to make certain that the information was complete and not contradictory. Any questionable item discovered in this manner was adjusted by correspondence. The reports were then evaluated on the basis of the U. S. Longshoremen and Harborworkers Act, coded, punched and tabulated. A number of analyses were prepared; by kind of injury, by method of adjustment (whether with or without attorney) by ratings of the seamen, etc. A separate treatment was accorded to non-compensable illnesses. The results were remarkably close to those found by the governmental statisticians.

CASE 19—*The Determination of the Amounts Due to Beneficiaries in the Settlement of Death Cases.*

A concern operated a fleet of vessels. As respects some of the personnel of the vessels, it was a self insurer. A number of its vessels were sunk and it desired to make settlements with the dependents of deceased crew members on the basis of benefit provisions of a compensation act, to which the dependents were agreeable. No state supervisory officials were involved.

An impartial estimate of the present value of the payments, taking into account a 5% interest rate, the contingencies of death and remarriage, and the durations and weekly amounts specified in the act was desired.

The present value of the payments to each family group and the amounts

due each individual in the family group were determined and were the basis of the settlements made between the concern and the claimants.

In one instance the widow was a young woman aged 22. The present value of the payments was relatively low compared to the gross amount undiscounted for interest, mortality and remarriage. The attorney questioned the value submitted by the actuary. It was explained that the probabilities of remarriage for a widow that young were very good and were reflected in the present value. The actuary also told the attorney that probably the remarriage rates were not correct for the war period when most of the eligible men were away in the services. The attorney remarked rather dryly that he didn't think that would bother the young lady as she was a Wave, and he understood was doing all right.

Critical Review

The actuary is expected to be able to determine the probable cost of compensation benefit provisions, pension funds and similar matters. The three cases included in this group illustrate the nature of such calculations. Case 17 illustrates the manner in which the actuary can and does influence social legislation. The presence of the actuary in the midst of the conflict between the employer and labor groups acted as a conciliating influence and enabled progress to be made and acceptable compromise to be reached. Case 19 involved the determination of the exact amounts due to certain beneficiaries after the basis and principles of the settlement had been agreed upon.

Group VI
Problems of Companies of Other Countries

Case 20—*Workmen's Compensation Problems in a South American Country.*

An insurance company of a country "south of the border" desired some guide rates and procedures based on United States methods of operation. Aside from the difficulty of translating our dollars and cents system to the currency of the country in question there were other problems of somewhat peculiar nature.

The government of the country had a certain definite measure of control over the operations of the company. In the settlement of workmen's compensation claims the company would become involved in a political situation as undoubtedly many claims would be controverted and pressure would be brought to bear on the claim policy of the carrier. At the same time important employers insisted that a strict observance of the benefit provisions be made in order to keep the cost down.

It was suggested that a proper basis whereby the company could separate itself from being a party in the adjudication of claims was to have an independent commission established, similar to the industrial boards and commissions in the states, to make awards and settlements on all claims. It was also suggested that the employers could have a representative at the hearings to see that each case was properly presented and adjudicated so as to assure the employers that the carrier was properly defending claims. The carrier would contribute toward the expenses of the employers' representative.

The larger employers who represented foreign owned corporations, claimed that working conditions, safety measures and other factors contributing toward accidents differed as between large and small employers. A manual rate would therefore not be equitable toward such employers as a group. They did not desire to be rated on an average rate basis. On the other hand the carrier insisted that rates must be adequate and uniform for all classes of risks. It would be political suicide to promulgate a prospective rate for large foreign owned corporations that was lower than the rate for smaller locally owned concerns.

It was suggested that a solution to the problem could be made by establishing a high level of rates and that the records be maintained to show the individual experience of the employers. At the end of the year an adjustment based in part on the size of the risk and in part on the experience would be made, subject to a certain minimum and maximum, and the risk either debited or credited with return premiums. The level would be high enough so that there would be little likelihood of debit premiums. It was in fact a modification of the participating form of operation. The employers were agreeable to this procedure, providing a tightly controlled accounting and statistical procedure was adopted so as to make certain that each employer's record was properly maintained.

CASE 21—*Procedures Followed in a South American Multiple Line Carrier.*

It is not only a source of satisfaction but also a definite thrill to be selected by a client from far-away lands to make a trip to a South American Country. A Columbian company writing life, fire, marine and casualty insurance desired modernization of its methods. A survey was to be conducted and the work done with the aid of an interpreter during the first few weeks at least. It was necessary to learn not only the methods and office routine but also the customs and attitudes of the people. Just a few items will illustrate the antiquity of the system found by the actuary. For every letter written, a copy was made by the use of special ink. Such copies were arranged chronologically and bound in volumes in half leather bindings so

beautiful that they would be the envy of many a decorator. These copies were in addition to the regular file copies.

Another instance was the accounting procedure. For every transaction made by an agent or branch office a separate letter of account was prepared. This letter of account was forwarded to the respective department of the company where it was checked and if found incorrect correspondence initiated. Only after all transactions of a given day were found in order and a complete record made in the given department, were these letters of account sent to the accounting department. As a result the status of agents' balances was not known till as late as two months after a given date. Statistics were kept on a calendar year basis exclusively. A profusion of meaningless analysis was being prepared, but certain essential data were not available. Punch cards and tabulating equipment were in use but were primarily utilized by the life insurance new business department. All policies were signed individually by the principal officers of the company.

In the frame of this digest it is impossible to describe the many details and ramifications of the problems and reforms which had to be made. It will suffice to state that the intelligent and splendid cooperation of the management and staff permitted a rapid and smooth modernization and the resultant savings were quite substantial.

CASE 22—*Review of a Social Security System.*

A Central American Republic desired to review its Social Security System and place it on a sound basis. After consultation with the International Labor Office it decided to engage the services of a consulting actuary.

The work involved a study of old age, invalidity, widows' and orphans' pensions. The law establishing the social security system did not contain provisions establishing definite scales of benefits. The law provided that only those employees earning less than a specified sum were covered and those earning over this amount were therefore excluded. The legislature was persuaded to amend the law to provide coverage for the first few thousand of annual earnings for these employees as well as other needed changes.

It was necessary to construct tables to determine the probable cost of the pensions and health insurance provisions, establish organizational, operating, and administrative procedures.

CRITICAL REVIEW

Cases 20, 21 and 22 show some of the insurance problems of neighboring countries. The three cases show that with modifications such problems are

those which also face companies in the United States. The small and large risk problem, the influence of political considerations, the desire for social security systems, the need for keeping abreast of modern insurance principles and methods are just as prevalent in South American countries as they are here. In all probability the future for actuaries is brighter in these countries where insurance, if it follows the pattern of growth in this country, is still in its relative infancy.

* * * * * *

These twenty odd cases which have been presented can hardly be said to represent the entire range of the work of the actuary. Taken together with the subjects discussed in the technical articles published in the various actuarial journals, they do give a somewhat broader picture of the work.

REPORT OF COMMITTEE ON MORTALITY FOR DISABLED LIVES

The Committee on Mortality for Disabled Lives has completed its study of this subject and herewith submits its final report. The report is composed of three parts, a brief narrative description of the work of the Committee, the resultant mortality table with derived commutation columns, and an appendix which covers some of the technical aspects of the work and which is of more limited interest. The study of the Committee has been hampered by circumstances arising from the war which are largely responsible for the delay in submitting the final report.

Origin of Committee

In November 1937 the Council of the Casualty Actuarial Society, acting on the suggestion of Professor A. H. Mowbray, authorized President Leon Senior to appoint a Committee of Three to report on the feasibility of compiling a mortality table for lives disabled under workmen's compensation acts. On March 15th, 1938 this Committee reported that it desired to avoid giving any impression that very substantial results might be expected from a compilation of the experience at the time; that the subject, however, was of such importance that a start should be made in order to get the carriers to keep appropriate records; and that it was as feasible then as it would be at any later time under the current statistical status to compile a mortality table for lives disabled under workmen's compensation acts.

The Council accepted the report and authorized the appointment by the President of an enlarged committee of seven members. The new Committee organized with the National Council on Compensation Insurance serving as secretary and prepared a Call for disability data for cases prior to policy year 1936 which the National Council sent out in October 1938.

Source of Data

In response to the Call, which was sent to all private insurance carriers and state funds, individual case reports or tabulator listings of the material requested in the Call were submitted to the National Council, where the data were recorded on punch cards and compiled. The Committee, after studying

the reports and compilations, found the early years represented results of only a few carriers, as most of the others were unable to report fully for this period. There appeared evidence too, that for the early years some of the shorter term cases may have been overlooked, leaving the experience biased.

To obtain a more representative experience the Committee decided to restrict the data generally to policy years 1930-1935, or to accidents occurring in calendar years 1930-1935 as to carriers reporting on an accident year basis. The decision was made reluctantly as it limited the data materially, but it seemed necessary to prevent undue weight being given to experience of a few carriers interested in certain industries. Further study of the data convinced the Committee that the experience for the first year of observation was defective. There was a general tendency to consider all cases resulting in deaths during this period as fatal cases and thus fail to report any among permanent total. To overcome this defect the Committee decided to eliminate the experience for the first year from the observation of each case. A tabulation of the experience with these restrictions showed a total exposure of 8,598 man-years and 285 deaths or terminations of permanent total disability.

Development of Tables

The decision of the Committee to restrict the data to cases of the last five years and to eliminate the first year of experience for each case precluded study of the experience on a select and ultimate basis. Extensive studies of the data by type of injury, by industry, and by state also were unwarranted.

After reviewing and compiling the data, the mortality rate designated q_x' was calculated for each age on the basis of the raw data. The calculated q_x' was joined with the q_x of the U. S. Life Table—1930, White Males, at ages above 73 years using decreasing weights from age 74 to age 94 to smooth the transition. For ages 17-21, the q_x of the U. S. Life Table—1930 was first raised to the level of the q_x' using the relativity of the rates shown by the ages 22-26, as a group. The raised rates of the U. S. Table were then combined, on a weighted basis, with the q_x' for ages 17-21 to obtain the modified rates which were used in the graduation. The resultant q_x' was graduated by the Whittaker-Henderson technique and the graduated q_x' was used to construct the Mortality Table for Lives Disabled by Industrial Accidents. Commutation columns were constructed and annuity values calculated with interest rates of 2½%, 3%, 3½%, 4%, 5%, and 6%.

Based on the exposure underlying the table the mortality of the Table

for Lives Disabled by Industrial Accidents is 144% of the U. S. Life Table—1930, 125% of the American Experience Table, and 32% of Hunter's Disabled Lives Table.

Respectfully submitted,

COMMITTEE ON MORTALITY FOR DISABLED LIVES
HARMON T. BARBER
JOHN CARLETON
CHARLES M. GRAHAM
MARK KORMES
RALPH M. MARSHALL
RICHARD M. PENNOCK
PAUL DORWEILER, *Chairman*

DEFINITION OF SYMBOLS

The symbols used are defined as follows:

l_x^t denotes the number of permanent total disabled lives living at the beginning of age x.

d_x^t denotes the number dying between ages x and $x+1$. The few cases of recovery in the experience have been included with deaths. The meanings of "fatal", "mortality rate", "dying", etc., are to be construed as having cases of recovery included with deaths.

$q_x^t = d_x^t \div l_x^t$

$D_x^t = v^x \cdot l_x^t$

$N_x^t = \Sigma D_x^t$

$\overline{N}_x^t = \frac{1}{2}(N_x^t + N_{x+1}^t)$

$\bar{a}_x^t = \overline{N}_x^t \div D_x^t$

MORTALITY TABLE FOR LIVES DISABLED BY INDUSTRIAL ACCIDENTS
Based on Permanent Total Disability Cases
under Workmen's Compensation Acts

x	l'_x	q'_x	d'_x	x	l'_x	q'_x	d'_x
15	100000	.03226	3226	60	33221	.03675	1221
6	96774	.03115	3015	1	32000	.03758	1203
7	93759	.03014	2826	2	30797	.03832	1180
8	90933	.02923	2658	3	29617	.03907	1157
9	88275	.02843	2510	4	28460	.03997	1138
20	85765	.02773	2378	65	27322	.04117	1125
1	83387	.02715	2264	6	26197	.04284	1122
2	81123	.02667	2164	7	25075	.04511	1131
3	78959	.02627	2074	8	23944	.04811	1152
4	76885	.02589	1991	9	22792	.05192	1183
25	74894	.02548	1908	70	21609	.05658	1223
6	72986	.02497	1822	1	20386	.06206	1265
7	71164	.02433	1731	2	19121	.06824	1305
8	69433	.02358	1637	3	17816	.07501	1336
9	67796	.02274	1542	4	16480	.08223	1355
30	66254	.02185	1448	75	15125	.08979	1358
1	64806	.02095	1358	6	13767	.09758	1343
2	63448	.02007	1273	7	12424	.10556	1311
3	62175	.01924	1196	8	11113	.11368	1263
4	60979	.01843	1127	9	9850	.12199	1202
35	59852	.01781	1066	80	8648	.13052	1129
6	58786	.01727	1015	1	7519	.13935	1048
7	57771	.01686	974	2	6471	.14851	961
8	56797	.01662	944	3	5510	.15808	871
9	55853	.01654	924	4	4639	.16812	780
40	54929	.01665	915	85	3859	.17872	690
1	54014	.01693	914	6	3169	.18995	602
2	53100	.01739	923	7	2567	.20190	518
3	52177	.01800	939	8	2049	.21464	440
4	51238	.01873	960	9	1609	.22829	367
45	50278	.01955	983	90	1242	.24294	302
6	49295	.02045	1008	1	940	.25871	243
7	48287	.02139	1033	2	697	.27573	192
8	47254	.02237	1057	3	505	.29413	149
9	46197	.02341	1081	4	356	.31403	112
50	45116	.02449	1105	95	244	.33557	82
1	44011	.02562	1128	6	162	.35887	58
2	42883	.02682	1150	7	104	.38407	40
3	41733	.02809	1172	8	64	.41127	26
4	40561	.02942	1193	9	38	.44056	17
55	39368	.03079	1212	100	21	.47202	10
6	38156	.03216	1227	1	11	.50571	6
7	36929	.03348	1236	2	5	.54166	3
8	35693	.03470	1239	3	2	.57989	1
9	34454	.03580	1233	4	1	.62042	1

COMMUTATION COLUMNS AND LIFE ANNUITIES — 2½%

Based on Mortality Table for Lives Disabled by Industrial Accidents

x	D_x^t	$\overline{N}_x^t$	$\bar{a}_x^t$	x	D_x^t	$\overline{N}_x^t$	$\bar{a}_x^t$
15	69046.56	1319034.86	19.1036	60	7550.588	85600.417	11.3369
6	65189.38	1251916.89	19.2043	1	7095.683	78277.282	11.0317
7	61617.96	1188513.22	19.2884	2	6662.370	71398.255	10.7166
8	58303.14	1128552.67	19.3566	3	6250.828	64941.656	10.3893
9	55218.46	1071791.87	19.4100	4	5860.133	58886.176	10.0486
20	52339.88	1018012.70	19.4500	65	5488.596	53211.811	9.6950
1	49647.48	967019.02	19.4777	6	5134.243	47900.392	9.3296
2	47121.49	918634.54	19.4950	7	4794.485	42936.028	8.9553
3	44745.84	872700.87	19.5035	8	4466.566	38305.502	8.5761
4	42507.83	829074.04	19.5040	9	4147.971	33998.234	8.1964
25	40397.12	787621.56	19.4970	70	3836.756	30005.870	7.8206
6	38407.77	748219.12	19.4809	1	3531.324	26321.830	7.4538
7	36535.58	710747.44	19.4536	2	3231.411	22940.463	7.0992
8	34777.45	675090.93	19.4117	3	2937.433	19856.041	6.7597
9	33129.28	641137.56	19.3526	4	2650.887	17061.881	6.4363
30	31586.11	608779.87	19.2737	75	2373.589	14549.643	6.1298
1	30142.23	577915.70	19.1730	6	2107.781	12308.958	5.8398
2	28790.84	548449.16	19.0494	7	1855.769	10327.183	5.5649
3	27525.06	520291.21	18.9025	8	1619.459	8589.569	5.3040
4	26337.15	493360.11	18.7325	9	1400.396	7079.641	5.0555
35	25219.90	467581.58	18.5402	80	1199.517	5779.685	4.8183
6	24166.55	442888.36	18.3265	1	1017.482	4671.185	4.5909
7	23170.04	419220.06	18.0932	2	854.3079	3735.2899	4.3723
8	22223.81	396523.14	17.8423	3	709.6935	2953.2892	4.1614
9	21321.40	374750.53	17.5763	4	582.9344	2306.9752	3.9575
40	20457.24	353861.21	17.2976	85	473.0926	1778.9617	3.7603
1	19625.82	333819.68	17.0092	6	379.0267	1352.9021	3.5694
2	18823.14	314595.20	16.7132	7	299.5363	1013.6206	3.3840
3	18044.84	296161.21	16.4125	8	233.2608	747.2220	3.2034
4	17287.90	278494.84	16.1092	9	178.7031	541.2401	3.0287
45	16550.23	261575.78	15.8050	90	134.5779	384.5996	2.8578
6	15830.88	245385.22	15.5004	1	99.3702	267.6255	2.6932
7	15128.94	229905.31	15.1964	2	71.8848	181.9980	2.5318
8	14444.19	215118.75	14.8931	3	50.8126	120.6493	2.3744
9	13776.68	201008.31	14.5905	4	34.9467	77.7697	2.2254
50	13126.15	187556.90	14.2888	95	23.3681	48.6123	2.0803
1	12492.35	174747.65	13.9884	6	15.1365	29.3599	1.9397
2	11875.29	162563.83	13.6893	7	9.48022	17.05157	1.7986
3	11274.95	150988.71	13.3915	8	5.69169	9.46562	1.6631
4	10691.04	140005.71	13.0956	9	3.29702	4.97126	1.5078
55	10123.50	129598.44	12.8017	100	1.77760	2.43395	1.3692
6	9572.524	119750.424	12.5098	1	.908411	1.09094	1.2009
7	9038.727	110444.799	12.2191	2	.402843	.435315	1.0806
8	8523.124	101663.873	11.9280	3	.157207	.155290	.9878
9	8026.600	93389.011	11.6349	4	.076686	.033343	.5000

COMMUTATION COLUMNS AND LIFE ANNUITIES — 3%

Based on Mortality Table for Lives Disabled by Industrial Accidents

x	D_x^i	$\overline{N}_x^i$	$\bar{a}_x^i$	x	D_x^i	$\overline{N}_x^i$	$\bar{a}_x^i$
15	64186.19	1128606.15	17.5833	60	5638.703	61373.125	10.8843
6	60306.35	1066359.88	17.6824	1	5273.261	55917.143	10.6039
7	56725.73	1007843.84	17.7670	2	4927.203	50816.911	10.3135
8	53413.55	952774.20	17.8377	3	4600.403	46053.108	10.0107
9	50342.00	900896.42	17.8955	4	4291.930	41606.941	9.6942
20	47486.00	851982.42	17.9418	65	4000.304	37460.824	9.3645
1	44824.62	805827.11	17.9773	6	3723.872	33598.736	9.0225
2	42337.49	762246.06	18.0040	7	3460.563	30006.519	8.6710
3	40007.87	721073.38	18.0233	8	3208.230	26672.122	8.3137
4	37822.32	682158.28	18.0359	9	2964.927	23585.544	7.9548
25	35769.79	645362.23	18.0421	70	2729.161	20738.500	7.5989
6	33843.22	610555.72	18.0407	1	2499.707	18124.066	7.2505
7	32037.26	577615.48	18.0295	2	2276.305	15736.060	6.9130
8	30347.55	546423.08	18.0055	3	2059.173	13568.321	6.5892
9	28768.99	516864.81	17.9660	4	1849.280	11614.094	6.2803
30	27295.77	488832.43	17.9087	75	1647.796	9865.556	5.9871
1	25921.56	462223.76	17.8316	6	1456.165	8313.576	5.7092
2	24639.21	436943.38	17.7337	7	1275.837	6947.575	5.4455
3	23441.60	412902.97	17.6141	8	1107.969	5755.672	5.1948
4	22321.05	390021.65	17.4733	9	953.445	4724.965	4.9557
35	21270.41	368225.92	17.3117	80	812.714	3841.885	4.7272
6	20283.07	347449.18	17.1300	1	686.033	3092.512	4.5078
7	19352.30	327631.49	16.9298	2	573.217	2462.887	4.2966
8	18471.87	308719.41	16.7129	3	473.873	1939.342	4.0925
9	17635.78	290665.58	16.4816	4	387.344	1508.733	3.8951
40	16838.86	273428.26	16.2379	85	312.832	1158.645	3.7037
1	16076.08	256970.79	15.9847	6	249.414	877.522	3.5183
2	15343.73	241260.89	15.7237	7	196.150	654.740	3.3380
3	14637.89	226270.08	15.4578	8	152.008	480.661	3.1621
4	13955.79	211973.24	15.1889	9	115.889	346.713	2.9918
45	13295.44	198347.62	14.9185	90	86.8503	245.3429	2.8249
6	12655.83	185371.99	14.6472	1	63.8176	170.0089	2.6640
7	12035.96	173026.09	14.3758	2	45.9418	115.1292	2.5060
8	11435.41	161290.41	14.1045	3	32.3169	75.9999	2.3517
9	10854.00	150145.70	13.8332	4	22.1182	48.7823	2.2055
50	10291.28	139573.06	13.5623	95	14.7182	30.3641	2.0630
1	9746.817	129554.012	13.2919	6	9.48727	18.26138	1.9248
2	9220.394	120070.406	13.0223	7	5.91320	10.56114	1.7860
3	8711.776	111104.321	12.7533	8	3.53290	5.83809	1.6525
4	8220.506	102638.180	12.4856	9	2.03656	3.05336	1.4993
55	7746.331	94654.762	12.2193	100	1.09269	1.48873	1.3624
6	7289.173	87137.010	11.9543	1	.555690	.664540	1.1959
7	6849.292	80067.777	11.6899	2	.245230	.264080	1.0769
8	6427.231	73429.516	11.4248	3	.0952348	.0938479	.9854
9	6023.424	67204.188	11.1571	4	.0462305	.0231153	.5000

COMMUTATION COLUMNS AND LIFE ANNUITIES — 3½%

Based on Mortality Table for Lives Disabled by Industrial Accidents

x	D_x^i	$\overline{N}_x^i$	$\bar{a}_x^i$	x	D_x^i	$\overline{N}_x^i$	$\bar{a}_x^i$
15	59689.06	971083.94	16.2690	60	4216.88	44117.99	10.4622
6	55810.14	913334.34	16.3650	1	3924.54	40047.28	10.2043
7	52242.87	859307.84	16.4483	2	3649.28	36260.37	9.9363
8	48954.79	808709.01	16.5195	3	3390.78	32740.34	9.6557
9	45916.74	761273.24	16.5794	4	3148.13	29470.89	9.3614
20	43102.56	716763.59	16.6293	65	2920.05	26436.80	9.0535
1	40490.30	674967.16	16.6698	6	2705.13	23624.21	8.7331
2	38058.90	635692.56	16.7029	7	2501.71	21020.79	8.4026
3	35790.98	598767.62	16.7296	8	2308.09	18615.89	8.0655
4	33672.33	564035.97	16.7507	9	2122.75	16400.47	7.7260
25	31691.17	531354.22	16.7666	70	1944.51	14366.84	7.3884
6	29839.43	500588.92	16.7761	1	1772.42	12508.37	7.0572
7	28110.65	471613.88	16.7771	2	1606.22	10819.05	6.7357
8	26499.40	444308.85	16.7668	3	1445.99	9292.95	6.4267
9	24999.65	418559.33	16.7426	4	1292.32	7923.79	6.1314
30	23604.87	394257.07	16.7024	75	1145.96	6704.65	5.8507
1	22308.19	371300.54	16.6441	6	1007.80	5627.77	5.5842
2	21102.15	349595.37	16.5668	7	878.728	4684.506	5.3310
3	19979.48	329054.55	16.4696	8	759.424	3865.430	5.0899
4	18932.52	309598.55	16.3527	9	650.352	3160.542	4.8597
35	17954.22	291155.18	16.2165	80	551.681	2559.526	4.6395
6	17038.11	273659.02	16.0616	1	463.438	2051.966	4.4277
7	16177.71	257051.11	15.8892	2	385.356	1627.569	4.2235
8	15367.10	241278.70	15.7010	3	317.032	1276.375	4.0260
9	14600.67	226294.82	15.4989	4	257.890	988.914	3.8346
40	13873.55	212057.71	15.2850	85	207.274	756.332	3.6489
1	13181.11	198530.38	15.0617	6	164.457	570.467	3.4688
2	12519.87	185679.89	14.8308	7	128.711	423.882	3.2933
3	11886.23	173476.84	14.5948	8	99.2638	309.8948	3.1219
4	11277.60	161894.92	14.3554	9	75.3121	222.6069	2.9558
45	10692.08	150910.08	14.1142	90	56.1682	156.8667	2.7928
6	10128.54	140499.77	13.8717	1	41.0729	108.2462	2.6355
7	9585.92	130642.54	13.6286	2	29.4252	72.9971	2.4808
8	9063.62	121317.77	13.3851	3	20.5986	47.9852	2.3295
9	8561.24	112505.34	13.1412	4	14.0300	30.6709	2.1861
50	8078.17	104185.64	12.8972	95	9.29089	19.01044	2.0461
1	7613.83	96339.64	12.6532	6	5.95993	11.38503	1.9103
2	7167.82	88948.81	12.4095	7	3.69674	6.55669	1.7736
3	6739.71	81995.05	12.1660	8	2.19799	3.60933	1.6421
4	6328.92	75460.73	11.9232	9	1.26092	1.87987	1.4909
55	5935.04	69328.75	11.6813	100	.673262	.912781	1.3558
6	5557.80	63582.33	11.4402	1	.340736	.405782	1.1909
7	5197.18	58204.84	11.1993	2	.149643	.160593	1.0732
8	4853.36	53179.57	10.9573	3	.0578328	.0568549	.9831
9	4526.46	48489.66	10.7125	4	.0279385	.0139693	.5000

COMMUTATION COLUMNS AND LIFE ANNUITIES — 4%

Based on Mortality Table for Lives Disabled by Industrial Accidents

x	D_x^i	$\overline{N}_x^i$	$\bar{a}_x^i$	x	D_x^i	$\overline{N}_x^i$	$\bar{a}_x^i$
15	55526.45	839864.02	15.1255	60	3158.00	31795.24	10.0682
6	51668.43	786266.58	15.2175	1	2924.93	28753.78	9.8306
7	48133.36	736365.68	15.2984	2	2706.71	25937.96	9.5828
8	44887.08	689855.46	15.3687	3	2502.89	23333.16	9.3225
9	41899.06	646462.39	15.4290	4	2312.60	20925.41	9.0484
20	39142.02	605941.85	15.4806	65	2134.74	18701.74	8.7607
1	36593.02	568074.33	15.5241	6	1968.12	16650.31	8.4600
2	34230.29	532662.68	15.5612	7	1811.37	14760.57	8.1488
3	32035.74	499529.66	15.5929	8	1663.14	13023.31	7.8306
4	29994.49	468514.55	15.6200	9	1522.24	11430.62	7.5091
25	28094.00	439470.30	15.6429	70	1387.72	9975.64	7.1885
6	26325.26	412260.67	15.6603	1	1258.82	8652.37	6.8734
7	24680.86	386757.61	15.6703	2	1135.30	7455.31	6.5668
8	23154.34	362840.01	15.6705	3	1017.13	6379.10	6.2717
9	21738.88	340393.40	15.6583	4	904.670	5418.196	5.9891
30	20427.35	319310.29	15.6315	75	798.353	4566.685	5.7201
1	19212.41	299490.41	15.5884	6	698.723	3818.147	5.4645
2	18086.35	280841.03	15.5278	7	606.310	3165.630	5.2211
3	17041.81	263276.95	15.4489	8	521.472	2601.739	4.9892
4	16071.14	246720.47	15.3518	9	444.429	2118.789	4.7674
35	15167.42	231101.19	15.2367	80	375.187	1708.981	4.5550
6	14324.31	216355.33	15.1041	1	313.660	1364.557	4.3504
7	13535.56	202425.39	14.9551	2	259.560	1077.947	4.1530
8	12795.53	189259.85	14.7911	3	212.512	841.911	3.9617
9	12098.91	176812.63	14.6139	4	172.038	649.636	3.7761
40	11441.11	165042.62	14.4254	85	137.607	494.814	3.5958
1	10817.81	153913.16	14.2278	6	108.656	371.682	3.4207
2	10225.73	143391.39	14.0226	7	84.6301	275.0390	3.2499
3	9661.52	133447.76	13.8123	8	64.9543	200.2468	3.0829
4	9122.74	124055.63	13.5985	9	49.0444	143.2474	2.9208
45	8607.51	115190.51	13.3826	90	36.4017	100.5244	2.7615
6	8114.64	106829.43	13.1650	1	26.4907	69.0782	2.6076
7	7642.99	98950.62	12.9466	2	18.8871	46.3893	2.4561
8	7191.81	91533.22	12.7274	3	13.1580	30.3667	2.3079
9	6760.52	84557.05	12.5075	4	8.91901	19.32817	2.1671
50	6348.39	78002.60	12.2870	95	5.87791	11.92971	2.0296
1	5954.71	71851.05	12.0663	6	3.75244	7.11453	1.8960
2	5578.94	66084.22	11.8453	7	2.31633	4.08015	1.7615
3	5220.51	60684.50	11.6242	8	1.37060	2.23668	1.6319
4	4878.75	55634.87	11.4035	9	.782496	1.160133	1.4826
55	4553.13	50918.93	11.1833	100	.415800	.560985	1.3492
6	4243.22	46520.75	10.9635	1	.209424	.248373	1.1860
7	3948.82	42424.73	10.7436	2	.0915315	.0978954	1.0695
8	3669.86	38615.39	10.5223	3	.0352044	.0345274	.9808
9	3406.22	35077.35	10.2980	4	.0169252	.0084626	.5000

COMMUTATION COLUMNS AND LIFE ANNUITIES — 5%

Based on Mortality Table for Lives Disabled by Industrial Accidents

x	D_x^t	$\overline{N}_x^t$	$\bar{a}_x^t$	x	D_x^t	$\overline{N}_x^t$	$\bar{a}_x^t$
15	48101.71	636990.69	13.2426	60	1778.50	16637.27	9.3547
6	44333.28	590773.19	13.3257	1	1631.56	14932.24	9.1521
7	40906.74	548153.18	13.4001	2	1495.45	13368.74	8.9396
8	37784.54	508807.54	13.4660	3	1369.67	11936.18	8.7146
9	34933.42	472448.56	13.5243	4	1253.49	10624.60	8.4760
20	32323.93	438819.89	13.5757	65	1146.06	9424.82	8.2237
1	29931.13	407692.36	13.6210	6	1046.54	8328.52	7.9581
2	27731.89	378860.85	13.6616	7	954.021	7328.244	7.6814
3	25706.78	352141.51	13.6984	8	867.608	6417.429	7.3967
4	23839.57	327368.34	13.7321	9	786.541	5590.355	7.1075
25	22116.41	304390.35	13.7631	70	710.206	4841.981	6.8177
6	20526.63	283068.83	13.7903	1	638.104	4167.826	6.5316
7	19061.16	263274.93	13.8121	2	570.008	3563.770	6.2521
8	17711.91	244888.40	13.8262	3	505.814	3025.859	5.9822
9	16470.79	227797.05	13.8304	4	445.604	2550.150	5.7229
30	15329.68	211896.81	13.8227	75	389.491	2132.603	5.4754
1	14280.62	197091.66	13.8013	6	337.638	1769.038	5.2395
2	13315.59	183293.56	13.7653	7	290.192	1455.123	5.0143
3	12427.07	170422.23	13.7138	8	247.210	1186.422	4.7992
4	11607.65	158404.87	13.6466	9	208.680	958.477	4.5930
35	10850.59	147175.75	13.5638	80	174.491	766.892	4.3950
6	10149.84	136675.53	13.4658	1	144.487	607.403	4.2039
7	9499.61	126850.81	13.3533	2	118.426	475.946	4.0189
8	8894.72	117653.64	13.2274	3	96.0371	368.7148	3.8393
9	8330.36	109041.10	13.0896	4	77.0055	282.1935	3.6646
40	7802.43	100974.71	12.9414	85	61.0077	213.1869	3.4944
1	7307.10	93419.94	12.7848	6	47.7137	158.8262	3.3287
2	6841.38	86345.70	12.6211	7	36.8092	116.5647	3.1667
3	6402.35	79723.84	12.4523	8	27.9824	84.1689	3.0079
4	5987.74	73528.79	12.2799	9	20.9271	59.7142	2.8534
45	5595.77	67737.04	12.1050	90	15.3845	41.5584	2.7013
6	5225.11	62326.60	11.9283	1	11.0893	28.3215	2.5539
7	4874.53	57276.78	11.7502	2	7.83100	18.86135	2.4085
8	4543.10	52567.96	11.5709	3	5.40365	12.24403	2.2659
9	4229.98	48181.42	11.3905	4	3.62789	7.72826	2.1302
50	3934.28	44099.29	11.2090	95	2.36814	4.73024	1.9974
1	3655.17	40304.57	11.0267	6	1.49741	2.79747	1.8682
2	3391.89	36781.04	10.8438	7	.91552	1.59100	1.7378
3	3143.74	33513.22	10.6603	8	.53658	.86495	1.6120
4	2909.96	30486.37	10.4766	9	.30342	.44495	1.4664
55	2689.87	27686.46	10.2929	100	.15969	.21339	1.3363
6	2482.92	25100.06	10.1091	1	.079666	.093715	1.1763
7	2288.64	22714.28	9.9248	2	.034488	.036638	1.0623
8	2106.70	20516.61	9.7387	3	.013138	.012825	.9762
9	1936.74	18494.89	9.5495	4	.0062562	.0031281	.5000

COMMUTATION COLUMNS AND LIFE ANNUITIES — 6%

Based on Mortality Table for Lives Disabled by Industrial Accidents

x	D_x^i	$\overline{N}_x^i$	$\bar{a}_x^i$	x	D_x^i	$\overline{N}_x^i$	$\bar{a}_x^i$
15	41726.51	490978.66	11.7666	60	1007.07	8789.14	8.7274
6	38094.73	451068.04	11.8407	1	915.149	7828.034	8.5538
7	34818.75	414611.30	11.9077	2	830.894	6955.012	8.3705
8	31857.81	381273.02	11.9680	3	753.827	6162.652	8.1752
9	29176.04	350756.09	12.0221	4	683.376	5444.050	7.9664
20	26741.93	322797.11	12.0708	65	618.914	4792.905	7.7441
1	24528.74	297161.77	12.1148	6	559.840	4203.528	7.5084
2	22512.05	273641.38	12.1553	7	505.532	3670.842	7.2613
3	20671.25	252049.73	12.1933	8	455.405	3190.374	7.0056
4	18988.94	232219.63	12.2292	9	408.957	2758.193	6.7445
25	17450.20	214000.06	12.2635	70	365.784	2370.822	6.4815
6	16043.05	197253.44	12.2953	1	325.548	2025.156	6.2208
7	14757.14	181853.34	12.3231	2	288.064	1718.350	5.9652
8	13583.18	167683.18	12.3449	3	253.210	1447.713	5.7174
9	12512.21	154635.49	12.3588	4	220.965	1210.626	5.4788
30	11535.49	142611.64	12.3629	75	191.318	1004.484	5.2503
1	10644.70	131521.54	12.3556	6	164.283	826.684	5.0321
2	9831.74	121283.32	12.3359	7	139.866	674.609	4.8233
3	9089.13	111822.89	12.3029	8	118.025	545.664	4.6233
4	8409.71	103073.47	12.2565	9	98.690	437.306	4.4311
35	7787.06	94975.08	12.1965	80	81.743	347.090	4.2461
6	7215.44	87473.83	12.1231	1	67.048	272.694	4.0671
7	6689.49	80521.37	12.0370	2	54.437	211.952	3.8935
8	6204.44	74074.40	11.9389	3	43.728	162.869	3.7246
9	5755.96	68094.20	11.8302	4	34.732	123.639	3.5598
40	5340.32	62546.06	11.7120	85	27.257	92.645	3.3989
1	4954.11	57398.85	11.5861	6	21.116	68.458	3.2420
2	4594.60	52624.49	11.4536	7	16.137	49.832	3.0881
3	4259.19	48197.60	11.3161	8	12.151	35.687	2.9370
4	3945.79	44095.11	11.1752	9	9.0019	25.1108	2.7895
45	3652.70	40295.86	11.0318	90	6.5553	17.3322	2.6440
6	3378.57	36780.23	10.8863	1	4.6805	11.7143	2.5028
7	3122.16	33529.86	10.7393	2	3.2741	7.7370	2.3631
8	2882.42	30527.57	10.5910	3	2.2379	4.9810	2.2257
9	2658.44	27757.14	10.4411	4	1.4883	3.1179	2.0949
50	2449.28	25203.28	10.2901	95	.96236	1.89254	1.9666
1	2254.04	22851.62	10.1381	6	.60277	1.10998	1.8415
2	2071.95	20688.63	9.9851	7	.36506	.62606	1.7150
3	1902.26	18701.52	9.8312	8	.21194	.33756	1.5927
4	1744.18	16878.30	9.6769	9	.11872	.17223	1.4507
55	1597.06	15207.68	9.5223	100	.061891	.081929	1.3238
6	1460.28	13679.01	9.3674	1	.030584	.035691	1.1670
7	1333.32	12282.21	9.2117	2	.013115	.013841	1.0554
8	1215.75	11007.68	9.0542	3	.0049490	.0048090	.9717
9	1107.12	9846.24	8.8936	4	.0023345	.0011673	.5000

APPENDIX

Some of the technical aspects of the investigation will be discussed in the appendix under these headings:

Call for Experience
Compilation and Tabulation of Reports
Graduation of Mortality Rates
Comparison of Mortality Rates

supplemented with these Exhibits:

Exhibit I	Form for reporting experience NC 159
Exhibit II	Illustrative Work Sheet for compiling exposure and fatalities by age at entry and year of disability
Exhibit III	Illustrative work sheet showing exposure and deaths by age at entry and year of disability
Exhibit IV	Table showing exposure, number dying during the year and the mortality rate for each age
Exhibit V	Table showing crude mortality rates adjusted for the high and low ages by averaging them with the U. S. Life Table 1930
Exhibit VI	Table showing the graduation process
Exhibit VII	Graph showing the adjusted crude mortality rates and the graduated rates
Exhibit VIII	Table showing comparison of actual number of deaths and expected number from graduated mortality rates
Exhibit IX	Table showing comparison of the results of the Casualty Actuarial Society table with other mortality tables

Call for Experience

The Committee's Call for experience as issued by the National Council on Compensation Insurance in October 1938 requested these data:

Compensation Cases Requested. Reports of following workmen's compensation cases were requested.

1. Permanent total disability
2. Permanent partial disability on non-dismemberment amounting to 50% or more of permanent total disability
3. Temporary total disability involving a duration of more than 18 months

The Committee in its final compilations restricted the data to permanent total disability and non-dismemberment permanent partial cases.

States Covered by Call. All states having workmen's compensation laws were covered by the Call.

Period for the Call. The Call covered all years up to and including policy year 1935 with the years 1930-1935 as a minimum. In the compilations later the cases were restricted generally to those of the minimum period by the Committee. The Committee also eliminated the experience of the first year of each case as it became evident that the data were biased because of inadequate reporting of cases terminated early.

Insurance Carriers. All insurance carriers, private and state funds, were asked to file reports. The final results included returns from the following:

	Number
Private Carriers	66
State Funds	9
Second Injury Fund	1

Form of the Call. The form used in the Call—NC 159—is shown as Exhibit I.

Instructions for Preparation of Report.

1. *The Carrier*—Name either in full or abbreviated should be filled in.
2. *Case Identification*—Each carrier should use its own claim number or any other method of identification so that it may later identify the case if necessary.
3. *Policy Year*—The Carrier should record the year of issue of the policy covering the case reported.
4. *State*—Record the state under whose workmen's compensation law the case was adjusted.
5. *Classification Code*—Give the code number of the classification to which the case was assigned.
6. *Sex*—Indicate by check mark whether male or female.
7. *Date of Birth*—Fill in the month, day and year of birth of the injured. If the date of birth is not available, give the age of the injured at date of accident. The dates in items 7-12 inclusive should, if possible, give the month, the day of the month, and the year.
8. *Date of Accident*—Fill in the date on which the accident occurred.
9. *Date Permanent Total Disability Began*—Fill in the date on which the total disability began.
10. *Date of Last Observation*—If the case is still open fill in the date on which the case was last observed as shown in your file.
11. *Date of Death*—Fill in the date on which the injured died.
12. *Date of Termination*—Fill in the date on which the case terminated according to your file. Check the method of termination in the proper blank.

13. *Nature of Injury*—Fill in the nature of injury as you would on the individual reports accompanying unit risks reports or Schedule "Z".
14. *Remarks*—Any special features that you think require clarification should be explained under remarks, using reverse side if necessary. In permanent partial cases, give degree of disability under "Remarks".

Compilation and Tabulation of Reports

The compensation insurance carriers filed the experience on form NC159 —see Exhibit I—with the National Council, except that a few carriers filed tabulator listings of the essential data. At the National Council these forms or listings were reviewed, coded, and recorded on punch cards from which all tabulations of the Committee were made.

From the punch cards the terminations of exposure and the deaths were tabulated for each age of entry, that is, the age of the injured at the time the case was first considered permanent total. This was the basic tabulation from which the data were entered on working sheets, one for each age of entry like that in Exhibit II for age at entry 25. The number of terminations not resulting from death was entered in column 2, the number of fatalities in column 3, and the number of months of exposure in the last year of observation for non-fatal cases was entered in column 5. In column 6 the exposures given in column 5 were expressed in terms of years. Column 4 shows the number of survivors for each year and is obtained by cumulating columns 2 and 3 upward. The number exposed at the beginning of each year of disability is obtained by adding columns 3, 4 and 6 as indicated in column 7.

The data in columns 3 and 7 from the work sheets—Exhibit II—were then transferred to Exhibit III, showing the number exposed at the beginning of the year and the fatalities during the year for each age of entry by select year of disability. By summing Exhibit III along the diagonal lines, omitting the first year of experience for each age, data were obtained for the form shown here as Exhibit IV, which gives the number exposed at the beginning of each year of age, the number that died during the year, and the crude mortality rate, with the first year of disability omitted from consideration because of inadequate reporting. Fractional year exposures resulting from terminations other than death are recorded as fractional numbers of persons exposed.

In feeling its way in the work the Committee did not proceed in all instances in as direct a manner as outlined. The first tabulations were made substantially as indicated. Later as more data were obtained more tabulations were made and the results were combined with those already made. In the Committee's investigation there were various tabulations, as by kind of injury, by industry, by geographical area, and by type of carrier.

Graduation of Mortality Rates

The crude mortality rates of Exhibit IV were adjusted at the ends where the exposure is sparse by averaging them with the U. S. Life Table—White Males—1930 mortality rates, using weights as shown in Exhibit V, preliminary to graduating. For each age over 73 the weight used was the ratio of the exposure of the given age to the exposure for age 73, the last year for which no adjustment was made. The complement weight was given to the U. S. 1930 mortality rate. For the ages under 22 the mortality rates of the U. S. Life Table were first raised to the general level of the mortality of the disabled lives by multiplying the U. S. Life rates by 9.1678, which is the factor by which the mortality rates of the U. S. 1930 table for ages 22-26 must be raised to reproduce the death experience among the disabled lives in the five year age group 22-26. The actual rates for ages under 22 were then adjusted by weighting them against the raised U. S. mortality rates, giving each actual rate a weight equal to the ratio of the exposure of the given age to the exposure for age 22, the earliest year used without adjustment, and giving the complement weights to the U. S. mortality rates. The crude mortality rates with the ends thus adjusted were graduated by using the Whittaker-Henderson technique as shown in Exhibit VI.

Comparison of Mortality Rates

The graduated mortality rates and the adjusted crude mortality rates are shown in graphical form in Exhibit VII.

Exhibit VIII is a comparison of the actual number of deaths and the expected number from graduated mortality rates. In column 3 are shown the expected deaths obtained by applying the graduated mortality rates column 4, Exhibit VI to the exposures column 2, Exhibit IV, by age. The actual fatalities column 3, Exhibit IV are shown in column 2. In columns 4 and 5 respectively the actual deaths and expected deaths are cumulated down for convenience in comparison.

In Exhibit IX is shown a comparison by five year age groups of the actual deaths with the expected deaths obtained by applying the mortality rates of six different tables to the exposure on which this investigation was based. The indexes in the last line show that when using the C. A. S. mortality rate as the standard of measure, the U. S. Life—1930 White Males shows a mortality rate of 69.3%; Hunter's Disabled Lives, 312.9%; Survivorship Annuitants, 56.3%; American Experience, 79.7%; and Standard Annuity, 49.0%.

EXHIBIT I

NATIONAL COUNCIL ON COMPENSATION INSURANCE

(In Cooperation with the Casualty Actuarial Society)

PERMANENT TOTAL DISABILITY REPORT

1.	Carrier _____				Leave this column blank
2.	Case Identification _____				
3.	Policy Year _____				_____
4.	State _____				_____
5.	Classification Code _____				_____
6.	Sex (check), Male ___ Female ___	Mo.	Day	Yr.	_____
7.	Date of Birth, if not available give age at date of accident _____	___	___	___	
8.	Date of Accident	___	___	___	
9.	Date Perm. Total Dis. Began	___	___	___	_____
	If case is still open				
10.	Date of last observation	___	___	___	_____
	If case is closed				
11.	Date of death if deceased	___	___	___	_____
12.	Date of termination	___	___	___	
	Method of termination (Check)				
	Died from Accident		_____		
	Recovered		_____		
	Followed by Perm. Partial		_____		
	Benefit Period ended		_____		
	Lump Sum Settlement		_____		
	Any Other (Explain in "Remarks")		_____		
13.	Nature of Injury				

14. Remarks

NC 189

EXHIBIT II

WORK SHEET FOR CALCULATION OF NUMBER EXPOSED AT BEGINNING OF EACH YEAR OF DISABILITY FOR AGE AT ENTRY 25

x Year of Disability	Number of Terminations		Number of Survivors $\Sigma\,[(2)_{x+1}+(3)_{x+1}]$	Exposure of Non-fatal Cases in last year of Observation		No. Exposed at Beginning of Year
	Non-fatal	Fatal		In Months	In Years	(3)+(4)+(6)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	—	—	20	0	.0	20.0
2	2	2	16	18	1.5	19.5
3	4	—	12	13	1.1	13.1
4	1	—	11	6	.5	11.5
5	2	—	9	18	1.5	10.5
6	—	1	8	0	.0	9.0
7	2	—	6	11	.9	6.9
8	3	—	3	10	.8	3.8
9	2	—	1	6	.5	1.5
10	—	—	1	0	.0	1.0
11	—	—	1	0	.0	1.0
12	—	—	1	0	.0	1.0
13	1	—	—	10	.8	.8
14	—	—	—	—	—	—
15						
16						
17						
18						
19						
20						
21						
22						
23						
24						

Columns 2, 3, and 5 are filled in from the basic tabulation sheets; other columns are derived as indicated.

Exhibit III — Mortality for Disabled Lives Investigation. C. A. S.

Exposure Unit = 0.1 Man-Year — Territory ALL — Industry ALL — Injury ALL KINDS

Exposure at Start of Year, below – Fatals during year, above.

X Age at Entry	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	CHECK TOTALS
16	30	30	~0	~0	70	~0	~0	~0	~0	14	10	10	7													~41
17	50	41	30	30	~0	~0	~0	~0	19	10	5															1 ~73
18	50	50	50	30	30	~0	10	10	5																	1 ~55
19	120	120	10~	85	61	60	50	35	4																	3 637
20	80	80	78	64	56	45	~0	~0	12	9																464
21	140	1 ~7	108	83	7~	70	47	38	~5	~0	19	5														1 749
22	~10	~10	188	1 170	1 13~	1 91	70	64	40	38	3~	30	30	~6	~0	~0	18	10	4							3 1 403
23	~80	1 ~7	~07	194	171	158	132	1 59	~1	~1	~1	~1	~1	~0	1 10											3 1 559
24	~50	1 ~50	1 ~36	2 ~07	150	1 ~7	105	87	51	31	30	~~	~0	~0	~0	18	10	1 10								5 1 644
25	~00	2 195	131	115	105	1 90	69	38	15	10	10	10	8													3 996
26	310	~93	1 ~41	~07	190	178	1 155	129	91	1 61	50	1 50	36	30	1 30	~0	~0	~0	1 15							6 ~121
27	~04	177	1 170	117	100	1 97	65	55	40	30	17	6														2 1 073
28	1 3~8	305	1 ~83	~45	198	153	117	100	67	~8	10	8														2 1 85~
29	~68	~40	2 ~3~	1 173	116	9~	65	60	44	30	~8	1~	10	10	~											3 1 38~
30	353	1 33~	1 315	~14	3 ~3~	1 167	111	88	60	48	33	30	1 30	~	10	10	10	10	10	10	6					6 ~ 156

EXHIBIT IV

Mortality of Disabled Lives—Data of Exhibit III
Select Data—First Year of Disablement Omitted

(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
Age	Exposed	Died	Mortality Rate	Age	Exposed	Died	Mortality Rate
15				65	113.0	5	.04425
16				66	121.7	5	.04108
17	3.0	—	.00000	67	123.4	7	.05673
18	6.1	1	.16393	68	121.2	4	.03300
19	10.0	—	.00000	69	116.6	5	.04288
20	22.0	1	.04545	70	115.9	2	.01726
21	26.0	—	.00000	71	113.2	7	.06184
22	35.5	1	.02817	72	103.0	11	.10680
23	50.3	—	.00000	73	95.3	6	.06296
24	66.2	2	.03021	74	80.8	6	.07426
25	83.7	3	.03584	75	67.2	8	.11905
26	90.7	5	.05513	76	60.7	5	.08237
27	97.9	3	.03064	77	53.0	9	.16981
28	97.1	1	.01030	78	45.5	6	.13187
29	113.9	1	.00878	79	39.5	3	.07595
30	113.9	3	.02634	80	34.0	2	.05882
31	134.1	3	.02237	81	30.8	6	.19481
32	136.8	3	.02193	82	22.8	4	.17544
33	135.3	—	.00000	83	18.1	2	.11050
34	146.0	6	.04110	84	16.3	1	.06135
35	148.8	3	.02016	85	13.1	3	.22901
36	158.3	1	.00632	86	8.7	1	.11494
37	177.3	2	.01128	87	5.9	—	.00000
38	186.5	5	.02681	88	3.0	1	.33333
39	195.5	4	.02046	89	2.0	—	.00000
40	205.3	3	.01461	90	2.0	—	.00000
41	210.9	2	.00948	91	2.0	1	.50000
42	209.3	2	.00956	92	1.0	—	.00000
43	224.9	4	.01779	93	1.0	—	.00000
44	233.5	6	.02570	94	1.0	1	1.00000
45	225.4	6	.02662		8597.7	285	
46	236.3	4	.01693				
47	230.2	6	.02606				
48	223.1	4	.01793				
49	213.5	6	.02810				
50	205.5	4	.01946				
51	229.0	9	.03930				
52	223.3	4	.01791				
53	229.9	5	.02175				
54	222.0	5	.02252				
55	208.8	6	.02874				
56	184.7	6	.03249				
57	173.3	6	.03462				
58	161.2	7	.04342				
59	150.9	2	.01325				
60	141.3	8	.05662				
61	131.5	6	.04563				
62	130.8	7	.05352				
63	115.4	5	.04333				
64	111.1	3	.02700				

EXHIBIT V

MORTALITY RATES OF DISABLED LIVES EXPERIENCE MERGED (FROM AGES 17 TO 21 INCLUSIVE AND FROM AGE 74 ON) WITH MORTALITY RATES OF U. S. LIFE TABLE 1-A 1930 WHITE MALES

(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
Age	Disabled Lives Mortality Rates	Weight	U. S. Life, Table 1-A. 1930 Mortality Rates	Weighted Average	Age	Disabled Lives Mortality Rates	Weight	U. S. Life, Table 1-A. 1930 Mortality Rates	Weighted Average
10			.00147		60	.05662	100– 0	.02644	.05662
11			.00149		61	.04563	" "	.02838	.04563
12			.00157		62	.05352	" "	.03052	.05352
13			.00171		63	.04333	" "	.03297	.04333
14			.00190		64	.02700	" "	.03568	.02700
15			.00213		65	.04425	" "	.03865	.04425
16			.00241		66	.04108	" "	.04196	.04108
17	0	08–92	.02439*	.02244	67	.05673	" "	.04558	.05673
18	.16393	17–83	.02622*	.04963	68	.03300	" "	.04949	.03300
19	0	28–72	.02760*	.01987	69	.04288	" "	.05362	.04288
20	.04545	62–38	.02915*	.03926	70	.01726	" "	.05796	.01726
21	0	73–27	.03099*	.00837	71	.06184	" "	.06252	.06184
22	.02817	100– 0	.00353	.02817	72	.10680	" "	.06740	.10680
23	0	" "	.00361	0	73	.06296	100– 0	.07271	.06296
24	.03021	" "	.00366	.03021	74	.07426	85–15	.07861	.07491
25	.03584	" "	.00371	.03584	75	.11905	71–29	.08526	.10925
26	.05513	" "	.00375	.05513	76	.08237	64–36	.09274	.08610
27	.03064	" "	.00381	.03064	77	.16981	56–44	.10105	.13956
28	.01030	" "	.00390	.01030	78	.13187	48–52	.11013	.12057
29	.00878	" "	.00402	.00878	79	.07595	41–59	.11983	.10184
30	.02634	" "	.00413	.02634	80	.05882	36–64	.12997	.10436
31	.02237	" "	.00426	.02237	81	.19481	32–68	.14043	.15783
32	.02193	" "	.00442	.02193	82	.17544	24–76	.15117	.15699
33	0	" "	.00463	0	83	.11050	19–81	.16214	.15233
34	.04110	" "	.00486	.04110	84	.06135	17–83	.17333	.15429
35	.02016	" "	.00510	.02016	85	.22901	14–86	.18468	.19089
36	.00632	" "	.00535	.00632	86	.11494	09–91	.19618	.18887
37	.01128	" "	.00563	.01128	87	0	06–94	.20780	.19533
38	.02681	" "	.00597	.02681	88	.33333	03–97	.21967	.22308
39	.02046	" "	.00636	.02046	89	0	02–98	.23211	.22747
40	.01461	" "	.00679	.01461	90	0	02–98	.24550	.24059
41	.00948	" "	.00727	.00948	91	.50000	02–98	.26017	.26497
42	.00956	" "	.00776	.00956	92	0	01–99	.27629	.27353
43	.01779	" "	.00825	.01779	93	0	01–99	.29397	.29103
44	.02570	" "	.00874	.02570	94	1.00000	01–99	.31332	.32019
45	.02662	" "	.00929	.02662	95		0–100	.33445	.33445
46	.01693	" "	.00988	.01693	96		" "	.35745	.35745
47	.02606	" "	.01052	.02606	97		" "	.38243	.38243
48	.01793	" "	.01122	.01793	98		" "	.40951	.40951
49	.02810	" "	.01198	.02810	99		" "	.43879	.43879
50	.01946	" "	.01278	.01946	100		" "	.47037	.47037
51	.03930	" "	.01365	.03930	101		" "	.50436	.50436
52	.01791	" "	.01459	.01791	102		" "	.54087	.54087
53	.02175	" "	.01566	.02175	103		" "	.58001	.58001
54	.02252	" "	.01687	.02252	104		" "	.62187	.62187
55	.02874	" "	.01819	.02874	105		" "	.66656	.66656
56	.03249	" "	.01966	.03249					
57	.03462	" "	.02125	.03462					
58	.04342	" "	.02290	.04342					
59	.01325	100– 0	.02461	.01325					

*Mortality rate ×9.1678.

EXHIBIT VI

GRADUATION, BY WHITTAKER-HENDERSON FORMULA A, OF DISABLED LIVES MORTALITY RATES
(As Adjusted in Exhibit V)

Constants: $z=3$, $n=5$ in notation of T.A.S.A. XXXVIII p. 408

$$2352\ u'_x = 5600\ u'_{x-1} - 4560\ u'_{x-2} + 1260\ u'_{x-3} + 52\ u''_{x+5}$$

$$2352\ u_x = 5600\ u_{x+1} - 4560\ u_{x+2} + 1260\ u_{x+3} + 52\ u'_{x-5}$$

(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
x	u''_x	u'_x	u_x	x	u''_x	u'_x	u_x
9		.0410765					
10		.0393524		60	.05662	.0389782	.0367524
11		.0377305		61	.04563	.0403010	.0375765
12		.0360404		62	.05352	.0416266	.0383180
13		.0348384		63	.04333	.0425871	.0390713
14		.0337264	.0334760	64	.02700	.0432311	.0399711
15		.0329325	.0322622	65	.04425	.0430459	.0411742
16		.0318713	.0311505	66	.04108	.0428566	.0428373
17	.02244	.0307258	.0301409	67	.05673	.0441039	.0451067
18	.04963	.0290078	.0292334	68	.03300	.0463722	.0481051
19	.01987	.0272376	.0284280	69	.04288	.0495175	.0519201
20	.03926	.0258644	.0277317	70	.01726	.0540360	.0565821
21	.00837	.0255331	.0271469	71	.06184	.0593996	.0620559
22	.02817	.0259168	.0266677	72	.10680	.0662767	.0682407
23	0	.0262874	.0262656	73	.06296	.0742527	.0750058
24	.03021	.0262148	.0258922	74	.07491	.0823693	.0822266
25	.03584	.0259172	.0254808	75	.10925	.0899707	.0897853
26	.05513	.0254601	.0249718	76	.08610	.0977880	.0975827
27	.03064	.0249001	.0243341	77	.13956	.1059929	.1055568
28	.01030	.0238087	.0235772	78	.12057	.1143415	.1136834
29	.00878	.0229597	.0227358	79	.10184	.1225428	.1219873
30	.02634	.0222913	.0218485	80	.10436	.1310883	.1305225
31	.02237	.0214552	.0209486	81	.15783	.1399621	.1393459
32	.02193	.0204152	.0200679	82	.15699	.1490588	.1485097
33	0	.0195453	.0192360	83	.15233	.1587047	.1580774
34	.04110	.0189021	.0184786	84	.15429	.1688856	.1681232
35	.02016	.0183708	.0178140	85	.19089	.1795879	.1787235
36	.00632	.0177734	.0172673	86	.18887	.1910375	.1899535
37	.01128	.0170382	.0168636	87	.19533	.2031924	.2018967
38	.02681	.0163433	.0166194	88	.22308	.2160547	.2146432
39	.02046	.0159690	.0165439	89	.22747	.2298921	.2282889
40	.01461	.0160516	.0166453	90	.24059	.2447280	.2429409
41	.00948	.0163874	.0169280	91	.26497	.2606229	.2587149
42	.00956	.0170281	.0173859	92	.27353	.2776696	.2757336
43	.01779	.0177671	.0179979	93	.29103	.2959869	.2941272
44	.02570	.0186892	.0187300	94	.32019	.3157122	.3140280
45	.02662	.0196041	.0195543	95	.33445	.3369945	.3355672
46	.01693	.0208292	.0204451	96	.35745	.3599880	.3588746
47	.02606	.0219934	.0213875	97	.38243	.3848471	.3840725
48	.01793	.0229652	.0223742	98	.40951	.4117228	.4112707
49	.02810	.0236952	.0234057	99	.43879	.4407598	.4405627
50	.01946	.0243104	.0244861	100	.47037	.4720950	.4720235
51	.03930	.0249633	.0256229	101	.50436	.5057284	.5057090
52	.01791	.0257633	.0268228	102	.54087	.5416600	.5416570
53	.02175	.0269264	.0280917	103	.58001	.5798898	.5798898
54	.02252	.0278274	.0294226	104	.62187	.6204178	.6204178
55	.02874	.0291050	.0307913	105	.66656	.6632440	.6632440
56	.03249	.0307802	.0321581				
57	.03462	.0329489	.0334758	$\sum_{17}^{x=105}$			
58	.04342	.0353238	.0346995		10.77520		10.7749718
59	.01325	.0373101	.0357960				

EXHIBIT VII

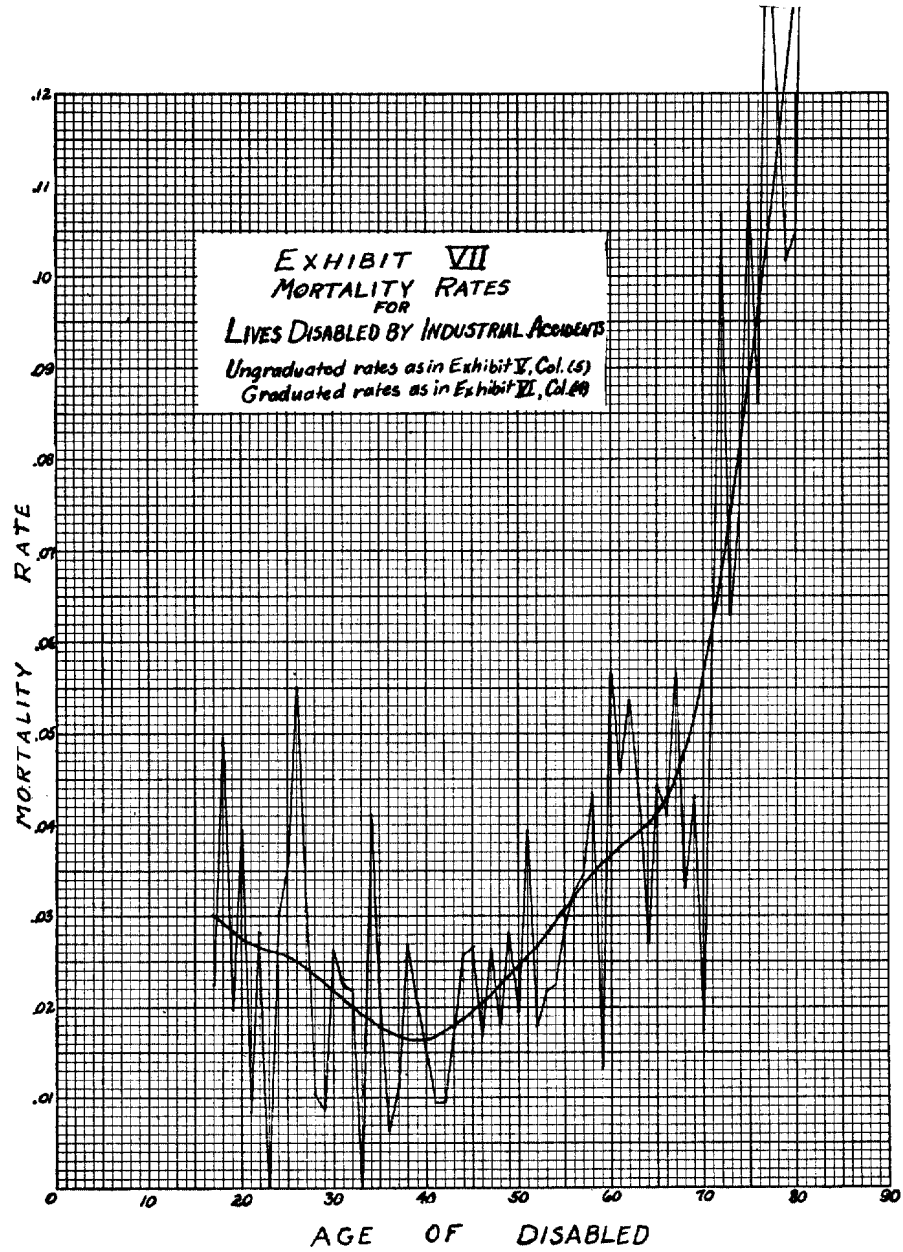


EXHIBIT VII
MORTALITY RATES
FOR
LIVES DISABLED BY INDUSTRIAL ACCIDENTS
Ungraduated rates as in Exhibit V, Col. (5)
Graduated rates as in Exhibit VI, Col. (4)
MORTALITY RATE
.12
.11
.10
.09
.08
.07
.06
.05
.04
.03
.02
.01
0
10
20
30
40
50
60
70
80
90
AGE OF DISABLED

EXHIBIT VIII

COMPARISON OF ACTUAL NUMBER OF DEATHS AND EXPECTED NUMBER FROM GRADUATED MORTALITY RATES — COLUMN 4, EXHIBIT VI.

Column 2—Actual number of deaths
Column 3—Expected number from graduated rates
Column 4—Actual number cumulated down
Column 5—Expected number cumulated down

Age	Number Deaths		Col.2 Cum.	Col.3 Cum.	Age	Number Deaths		Col.2 Cum.	Col.3 Cum.
	Actual	Expected				Actual	Expected		
(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
15	-	-	-	-	55	6	6.4	124	124.9
6	-	-	-	-	6	6	5.9	130	130.8
7	-	.1	-	.1	7	6	5.8	136	136.6
8	1	.2	1	.3	8	7	5.6	143	142.2
9	-	.3	1	.6	9	2	5.4	145	147.6
20	1	.6	2	1.2	60	8	5.2	153	152.8
1	-	.7	2	1.9	1	6	4.9	159	157.7
2	1	.9	3	2.8	2	7	5.0	166	162.7
3	-	1.3	3	4.1	3	5	4.5	171	167.2
4	2	1.7	5	5.8	4	3	4.4	174	171.6
25	3	2.1	8	7.9	65	5	4.7	179	176.3
6	5	2.3	13	10.2	6	5	5.2	184	181.5
7	3	2.4	16	12.6	7	7	5.6	191	187.1
8	1	2.3	17	14.9	8	4	5.8	195	192.9
9	1	2.6	18	17.5	9	5	6.1	200	199.0
30	3	2.5	21	20.0	70	2	6.6	202	205.6
1	3	2.8	24	22.8	1	7	7.0	209	212.6
2	3	2.7	27	25.5	2	11	7.0	220	219.6
3	-	2.6	27	28.1	3	6	7.1	226	226.7
4	6	2.7	33	30.8	4	6	6.6	232	233.3
35	3	2.7	36	33.5	75	8	6.0	240	239.3
6	1	2.7	37	36.2	6	5	5.9	245	245.2
7	2	3.0	39	39.2	7	9	5.6	254	250.8
8	5	3.1	44	42.3	8	6	5.2	260	256.0
9	4	3.2	48	45.5	9	3	4.8	263	260.8
40	3	3.4	51	48.9	80	2	4.4	265	265.2
1	2	3.6	53	52.5	1	6	4.3	271	269.5
2	2	3.6	55	56.1	2	4	3.4	275	272.9
3	4	4.0	59	60.1	3	2	2.9	277	275.8
4	6	4.4	65	64.5	4	1	2.7	278	278.5
45	6	4.4	71	68.9	85	3	2.3	281	280.8
6	4	4.8	75	73.7	6	1	1.7	282	282.5
7	6	4.9	81	78.6	7	-	1.2	282	283.7
8	4	5.0	85	83.6	8	1	.6	283	284.3
9	6	5.0	91	88.6	9	-	.5	283	284.8
50	4	5.0	95	93.6	90	-	.5	283	285.3
1	9	5.9	104	99.5	1	1	.5	284	285.8
2	4	6.0	108	105.5	2	-	.3	284	286.1
3	5	6.5	113	112.0	3	-	.3	284	286.4
4	5	6.5	118	118.5	4	1	.3	285	286.7

EXHIBIT IX

COMPARISON OF ACTUAL DEATHS WITH EXPECTED DEATHS UNDER VARIOUS MORTALITY TABLES. BASED ON EXPOSURE—COL. (2)—UNDERLYING CASUALTY ACTUARIAL SOCIETY MORTALITY TABLE

			EXPECTED DEATHS					
Age Group	Exposure	Actual Deaths	C.A.S. Disabled Lives	U.S. Life 1930 White Males	Hunter's Disabled Lives	Survivorship Annuitants	American Experience	1937 Standard Annuity
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
15-19	19.1	1	.6	-	4.3	.1	.1	-
20-24	200.0	4	5.2	.7	35.3	1.1	1.6	.2
25-29	483.3	13	11.7	1.9	63.2	2.6	3.9	.8
30-34	666.1	15	13.3	3.0	64.0	4.0	5.8	1.5
35-39	866.4	15	14.7	4.9	75.7	5.5	8.0	3.0
40-44	1 083.9	17	19.0	8.4	92.6	7.9	11.1	5.6
45-49	1 128.5	26	24.1	11.9	99.8	9.7	13.6	8.3
50-54	1 109.7	27	29.9	16.3	104.9	12.2	17.2	12.1
55-59	878.9	27	29.1	18.5	91.9	13.1	18.7	13.7
60-64	630.1	29	24.0	19.2	71.5	13.5	19.7	14.5
65-69	595.9	26	27.4	27.4	68.5	19.7	28.6	20.0
70-74	508.2	32	34.3	34.0	58.9	26.0	37.1	24.4
75-79	265.9	31	27.5	26.4	32.5	21.6	29.1	18.4
80-84	122.0	15	17.7	17.8	20.7	15.6	20.7	12.1
85-89	32.7	5	6.3	6.5	9.0	6.4	9.0	4.4
90-94	7.0	2	1.9	1.9	4.2	2.3	4.2	1.4
15-94	8 597.7	285	286.7	198.8	897.0	161.3	228.4	140.4
Index		.994	1.000	.693	3.129	.563	.797	.490

REVIEWS OF BOOKS AND PUBLICATIONS

CLARENCE A. KULP, BOOK REVIEW EDITOR

Employee Retirement Plans. Birchard E. Wyatt, Walter Bjorn, William R. Williamson and Dorrance C. Bronson. Graphic Arts Press, Washington, 1945. Pp. vii, 110.

Anyone who professes or aspires to acquaintance in the field indicated by the title of this little book will do well to read it with care. It might be classed as a handbook or guide or syllabus, and, while an index would have helped, careful attention to the items in the table of contents will permit its use for reference purposes.

Many of the topics covered in these 110 pages—and most of the subjects that have bothered pension students receive some attention—have been highly controversial during the past 20 years. Practices and views regarding some of them have been affected, perhaps temporarily, by the war, and others reflect the excess profits taxes of recent years.

In the early pages the book undertakes to be theoretical or philosophical, and perhaps rightly so; but it soon shifts to more attention to practical considerations. To illustrate, on the first page we find a statement that seems difficult to defend:

> As actual wages or salaries paid during the working life of an employee rarely follow the curve of his efficiency cycle, the need for pensions grows out of what are in effect wage maladjustments which have gradually developed during his period of service.

This seems to indicate that if it were not for "wage maladjustments" the need for pensions would not exist, but this can hardly be true if the value of services generally decreases from age 50 on, as indicated in a chart on Page 2. The practical reason for retirement claimed by the authors is stated on page 3 as follows:

> Eventually, the spread between value and compensation reaches the point where retiring the employee on a fair and properly determined pension is cheaper than keeping him on the payroll.

Chapters II, *Essential Provisions of a Retirement Plan,* and III, *Methods of Administering and Financing Retirement Plans,* contain very reliable discussions of many subjects that will be recognized as of outstanding importance by anyone who has been giving thought to retirement plans. Whether or not the reader agrees with the authors on particular points is immaterial. The important fact is that these chapters give food for thought and present carefully thought-through arguments pro and con on many

topics. Who shall be covered by a retirement plan? What about retirement age? How long a period of service should precede participation? Should workers in their twenties be asked or permitted to participate? How large should benefits be and what is the best time incidence of their purchase? To what extent should employees contribute and to what extent should retirement benefit expectations vest in the employee?

The relative merits of the money purchase plan and the unit of benefit plan are discussed. The importance of termination benefits seems somewhat slighted but is by no means omitted.

Different methods of administration and financing of benefits are discussed, including the use of group annuity contracts, individual annuity or insurance policies and trusteed arrangements with or without contracts of an insurance company.

The importance of provisions of the Internal Revenue Act and rulings and interpretations regarding it are repeatedly mentioned and a final chapter is entitled *Tax Considerations*. Certainly this element is essential in present-day pension developments.

To do more than mention the topics of discussion in this book would lead to a review comparable in length with the book itself. The reviewer can say with emphasis that he found it well worth reading and expects to come back to particular paragraphs and sections when the topics involved are of immediate interest.

RAINARD B. ROBBINS

Fidelity and Surety Reinsurance. Vincent J. McCarthy. Insurance Institute of America, New York, 1945. Pp. 88.

This pamphlet of 88 pages is a thesis submitted in connection with the requirements for Fellowship in the Insurance Institute of America. It serves a very useful purpose in bringing together in one place a lot of material, only portions of which were previously available, and then in scattered places. It contains a brief discussion of the general practices of reinsurance, the difference between reinsurance and coinsurance as practiced in the bonding and casualty fields (the use of the term coinsurance in this connection must not be confused with its use in coinsurance clauses in insurance policies) and an account of reinsurance practices in the binding business. The reasons for the prevalence of facultative reinsurance and share reinsurance and a detailed examination of the standard reinsurance agreements are given. This is followed by an account of treaty reinsurance. The appendices reproduce the text of various forms and agreements.

Throughout the text the history of the different phases of the subject is presented quite fully. Mr. McCarthy has done a good job and produced a work that will not only be useful to the student but a valuable reference source to those in the fidelity and surety business.

This reviewer did not expect to and did not find in Mr. McCarthy's pamphlet any advocacy of a program of reform in the practices of bonding reinsurance. Such a program of reform is desirable, in this reviewer's opinion, and is overdue, but possibly the impetus for it must come from outside the surety business.

F. S. PERRYMAN

Insurance as Interstate Commerce. Elmer Warren Sawyer. McGraw-Hill Book Co., Inc., New York, 1945. Pp. xi, 169.

It has been said that a problem well stated is half solved. If this be true this book should be most welcome to all in the insurance business as well as to insurance commissioners and legislators working for a solution of the problem posed by the Supreme Court decision in the Southeastern Underwriters case and the subsequent passage by the 79th Congress of Public Law 15. For rarely has this reviewer seen an intricate problem and its setting so clearly and succinctly presented as in the 13 chapters of this book.

In the first chapter, *The Task Confronting Insurance,* after noting the decision and the moratorium to January 1, 1948, on the application of certain federal laws applicable to interstate commerce in general, the author says:

> The next eighteen months will be the most important period in the history of insurance in the United States. During this period the future of insurance regulation will be determined. If state regulation or any substantial part of it is to be preserved, a staggering amount of work must be done. The task confronting insurance is to create a new system of cooperative regulation, preserving state regulation within the field in which Congress has invited the states to take over regulation. *This tasks presents obstacles that may prove insuperable.* (Italics the reviewer's.)

He then indicates what in his judgment must be done, briefly outlining his reasons. He thinks there must be (1) agreement on state power, (2) agreement on principles of regulation by all the states, (3) similar agreement among the branches of the insurance business, (4) appropriate state legislation and (5) uniformity in practices.

In the second chapter, *Dual Control of Commerce,* the author discusses the American philosophy of government and the origin and extent of state and federal powers in general and of their respective powers over commerce.

This may be well-known to members of the legal profession. It seems to this reviewer, not a member of that profession, an unusually illuminating exposition, drawing out 14 specific principles from the mass of decisions discussed. These are 62 citation notes, several of which cite more than one case.

The third chapter, *Insurance under Exclusive State Control, 1868 to 1944,* outlines the case of *Paul* v. *Virginia,* quoting the salient part of Justice Field's decision and notes also the subsequent cases of *Hooper* v. *California* (1895) and *New York Life Ins. Co.* v. *Deer Lodge Co. Mont.* (1913) in which the non-commerce doctrine was upheld. The failure of the efforts of Theodore Roosevelt and others to procure federal regulation are indicated to have been due to the acceptance of that doctrine by Congress. The growth of state supervision in reliance on that doctrine is also noted though it is observed that "The degree of regulation established in the several states varied widely". Closing the chapter with a quotation from Justice Black's opinion in the S.E.U.A. case the author observes that the figures quoted "are testimony to the success of state regulation uneven as its 'quality' may have been".

The next chapter reviews the *Southeastern Underwriters Assn.* case and the results of the decision.

Chapter V is titled *Impact Upon State Regulation.* The intent of this chapter is indicated by the following observation:

> Before attempting to determine in detail what will be irretrievably lost and what may be salvaged, it is necessary to note the immediate general results of the decision in its application to regulation of the insurance business.

These are stated to be:

1. Insurance as commerce immediately became subject to all the rules established by the Supreme Court relative to the concurrent powers of the federal government and the state governments over commerce.

2. Insurance as commerce immediately became subject to all existing Congressional acts regulating interstate commerce insofar as such acts were applicable to insurance.

3. Insurance as commerce became subject to such acts as Congress might thereafter adopt relative to it directly or in general terms regulating commerce.

4. The validity of every state statute, court decision and departmental ruling relating to regulation or taxation of insurance became immediately questionable.

The remainder of the chapter considers possible courses of action by the federal and state governments and by the insurance business; and the arguments for federal and for state control.

Chapter VI discusses the application to insurance of existing federal statutes: The Sherman Act, The Clayton Act, The Robinson-Patman Antidiscrimination Act, The Federal Trade Commission Act, The National Labor Relations Act, The Fair Labor Standards Act and the Merchant Marine Act.

In the next very brief chapter (VII) are reproduced Public Law 15 of the 79th Congress and the President's statement when he signed the act with notation of the one point in the act for disagreement in interpretation: "to the extent that such business is not regulated by State law".

Chapter VIII analyses in the light of Supreme Court decisions Public Law 15 as a pattern for cooperative regulation by the states and Congress and discusses the meaning of the term "regulated".

In Chapter IX, titled *State Power to Regulate Insurance,* attention is called to the fact that there are now 3 areas of control, one in which the federal government retains exclusive control, one in which control is exclusively state and one which might be exclusively federal but has been thrown open to the states until January 1, 1948, and left with them thereafter if they elect to regulate. Several leading opinions of the Supreme Court are cited to show the nature of state police power, particularly when it may affect interstate commerce. Limitations on the police power, constitutional and other, are examined and 8 general principles for state regulation are set forth.

As to the application of these principles the author says in the next chapter: "Creation in each state of a system of regulation that meets local requirements consistent with systems in other states and fulfills the requirements of the Congressional plan for cooperative regulation is an extremely complex and arduous task." He emphasizes the need for essential uniformity in state regulation. "The degree of regulation may vary but there should be no variation in the nature of regulation". A basic outline of a plan of state regulation is set forth with comments on the problems presented. The author insists that he is not overstressing the problems. "This discussion is but a cursory review of the field. Many of the problems that will be encountered are not even mentioned". He feels that "One who studies the many facets of this complicated situation is forced to the conclusion that state regulation has little prospect of survival unless the problems of state regulation are clearly recognized and frankly discussed".

Chapter XI, *Federal Regulation* v. *State Regulation,* emphasizes that these terms do not now mean what they did before the Supreme Court held insurance to be commerce.

> Discussion of the comparative merits of federal and state regulation is no longer a contrasting of one system of regulations with another. Rather it is contrasting the merits of direct regulation by Congress of all aspects of insurance with direct regulation by Congress of some aspects and indirect regulation of others, indirect regulation being accomplished by utilization of state help upon matters of local concern.

The author feels that if the business is not to "find itself regulated by a program in the making of which it has had no voice" it must not only decide which of the two it prefers and if it prefers state regulation must meet the problems involved in preserving it; it must also "give some consideration to a system of federal regulation of all insurance activities and have a program at least under consideration".

The author makes it plain in Chapter XII that in his opinion the key words in Public Law 15 are "in the public interest" and that "It is the public interest with which the business of insurance is affected that differentiates it from business having only commercial aspects and warrants its regulation in a manner in which the manufacturer of shoes, for example, could not be legally regulated". He indicates that the extent to which the business may escape burdensome regulation depends on the extent to which it recognizes that interest and conforms its conduct to it.

The author does not present a specific solution to the present dilemma and this may be a disappointment to some who hope for such help. In the concluding chapter he does offer his idea of the best way for the business to help itself find the solution.

The value of this book as a contribution to the current discussion is, in the reviewer's opinion, unquestionable. The citation and comment on many cases on the extent of federal and state power over commerce should, he thinks, make it useful as a reference whatever turn events take.

A. H. MOWBRAY

Risk Appraisal. Harry Dingman, M.D. National Underwriter Company, Cincinnati, 1945. Pp. VII, 824.

The 824 pages of Dr. Dingman's *Risk Appraisal* sound forbidding in these busy days, yet once the actuary, the student or the layman picks up this book, he will read it right through with avid interest. Starting with the historical background and running rapidly yet thoroughly through the background of life insurance and health and accident coverage, Dr. Dingman gives a very clear picture of the need for and the technical factors in risk appraisal in general.

Particularly to the casualty actuary who has any contact with accident and health insurance the book is a "must". The salesman, the underwriter, the claim adjuster and everyone else who delves into the accident and health field, should not only read this book but keep it as a permanent reference.

If you are selling the general background plus the many statistics will be helpful. If you are in the technical end of the business you should understand, as Dr. Dingman points out, how morbidity increases as mortality decreases. The explanation of the waiver idea is helpful to both the salesman and the technician.

By far the greater part of the book is an analysis from an underwriting standpoint of the ailments of the human body. Pointing out early in the book that the human body is chemically worth less than a dollar, the author tells in detail the many trials and tribulations that are encountered before the chemical auction block is reached. Each ailment or injury of the human body is covered in as brief a manner as possible but never at the expense of the entire picture. For example, bronchitis, in the chapter on respiratory ailments, gives an outline of the condition, its cause and symptoms and the follow-through on the credence that must be given to a history of this ailment in underwriting accident and health or life insurance.

The book has the unusual qualities of being a technical treatise yet vitally interesting; of being a book filled with general information but with sufficient definite, specific, encyclopedic data to be kept as a permanent reference. It should be added as a permanent fixture to your library.

ARMAND SOMMER

Relief and Social Security. Lewis Meriam. Brookings Institution, Washington, D. C., 1946. Pp. xx, 912.

This is a report on a thoroughly live subject by a lifelong student of the social sciences. It is a serious contribution, both to the treatment of costs and to the philosophical or psychological aspects of relief and social security. The book is carefully divided into 3 parts. The first part traces the development of relief and social security programs from the Hoover Administration up till about 1945. In considerable detail, it deals with the special agricultural policies, with WPA, CCC, NYA, and with all the social insurances, as well as the special provisions for railroad employees. From this first part there stands out the fact that basic policies are not easily deduced from the actual operation of a program but that the over-all impression of this entire group of activities remains one of piecemeal approach, lacking any clear, unifying concepts.

The second part of the book analyzes two universal programs of social

security and relief. The prospective program of Great Britain is basically presented by Sir William Beveridge in his famous report. The second is the existing program of New Zealand, that utilizes very largely the economies which follow from the use of the means test.

From these two illustrations he draws the objectives of a national social security program of universality, comprehensiveness and coordination; his purpose, already apparent in his treatment of New Zealand, and more thoroughly developed in the third part, is to define social security as a gigantic relief program rather than a gigantic insurance program.

It is in the third part of the report that the backgrounds so carefully built up in the first two parts are drawn on to map out the objectives of an American social security program. It is in the third part that some constructive cost analysis is made—an analysis which steadily indicates how wide the range of potential costs may be under virtually every component of the total social security system. Mr. Meriam contrasts the very high costs of the universal benefits to be given without needs tests with the economy of almost universal application of needs tests, or means tests, as developed under the New Zealand system.

This reviewer considers the background material excellent, not only for the purpose of backing up a specific conclusion, but also because it brings out the following points:

(1) Generally governmental projects occur or evolve with the appearance of having been unavoidable.

(2) There is commonly a postponement in governmental projects of a clean-cut definition of policy or purpose; possibly because of the inertia of the plan's sponsors, possibly because of a wish to avoid estranging any considerable proportion of the electorate.

(3) The factual information essential to a sound appraisal of a given program does not become commonly available to those outside the administering agency, so that whenever the critics of the agency wish for ammunition they cannot get it. Equally true is the fact that friends of the agency cannot get it either.

(4) In his emphasis on universality and singleness of purpose, Mr. Meriam, in reviewing the combination of relief on the basis of needs tests and so-called insurance benefits avoiding needs tests, feels called on to choose between the two and to build up a consistent system based on that choice.

(5) His insistence on cost analysis and the pertinence of cost analysis is a welcome contrast to the writers who seem rather free with other people's

money. He backs up his position by quoting constitutional authority. He lines up a good case for the indivisibility of human and property rights.

Having gone thus far in agreement with the method of analysis developed by Mr. Meriam, I found the following points of specific criticism as an actuary and an insurance man, and as a self-appointed reporter on the American scene:

(1) I find his use of the terms, private insurance, and insurance, inadequate in many cases where definiteness and clarity are required for sound analysis. This is not entirely Mr. Meriam's fault. Even insurance specialists tend to forget that most insurance is provision currently made for current risks: a clear sharing among potential beneficiaries, with most of the contributors receiving no benefit payment. Life insurance and annuities are in fact a special case; in the former for example the contingent saving feature too often obscures the primary feature of current protection.

(2) Keyed in with the author's discussion of insurance is that of the allied subject of costs and the handling of the deferred commitments introduced by the savings feature of life insurance. As the author of various discussions on the wisdom of the elimination of the deferred type of benefit from social security, the reviewer found the cost discussion incomplete.

(3) The conclusion that social workers, meeting federally imposed standards under State direction, can competently handle great masses of relief recipients, seems to me an over-simplification of the problem. As the Calhoun report brings out rather forcefully, the 3 categorical relief programs granting benefits to the needy aged, the needy children, and the needy blind, are not satisfactory either in theory or in practice.

(4) I believe there is a flaw in the conclusion that dealing with fewer beneficiaries is sure to save money. In the operation of our old age assistance program, the limited number of recipients has seemed to provide a clear excuse for the payment of higher benefits. As has been outlined in my little story of the 16 rectangles (available on request), the illusion of saving that results when we deal with only part of the problem seems to lead directly to more of the bargain counter philosophy, that having saved a good deal we can now spend a lot. This is a definite danger in any program that underscores current low cost but implies a more comprehensive effectiveness later on.

(5) The major difference of opinion between Mr. Meriam and the reviewer lies in his selecting relief as against insurance or budgeting arrangements. One of the major motivations for social security has been the effort to escape from the admission of failure in personal living, and the wish to escape

from the prying or snooping that goes with relief administration. Apparently the essential simplicity of social budgeting and its contribution to re-establishment of most of the normal incentives in American life have not been adequately presented to Mr. Meriam.

The report on the whole seems to me one of the most constructive in the field of social security that has yet appeared, because it discusses cost relationships seriously. It shows a spirited individualistic orientation. It appreciates the priorities of full protection and over-all efficiency over relief benefits and the mopping up process. It is, in spite of its 900 pages, one of the important treatises on social security that no serious student can ignore. It suggests that the contributors to social insurance may desire to be "included out" when they sense the magnitude of the costs. It is desirable, however, that some one shall prepare a similar discussion of social budgeting.

W. R. WILLIAMSON.

STATISTICAL NOTES

A Statistical Approach to Compensation Benefits

GREGORY C. KELLY

The following tabulations, although made for the purpose of valuing a legislative change in Pennsylvania Death benefits, nevertheless indicate how compensation law changes in general may be based on statistical information.

In the following tabulations there are certain interesting facts to be noted. Fatalities are usually compensated in family groups and the ages of widows and dependent children should be reviewed in these family groups for discussion of benefit changes and for valuation or comparison of compensation acts. For example, it is very advisable for society, which in the long run pays the bill, to compensate widows until the youngest child is sixteen or eighteen years old, in order to keep families together during the school years. On the other hand, twenty per cent of the unencumbered widows are under forty years of age and it does not seem reasonable to pay compensation for life in such cases. The tabulation "Death and Remarriage of Widows" is reliable only within the 300 week period provided in the Pennsylvania Act for compensation to widows. It is apparent that there are remarriages of inclination and remarriages of necessity. The fifteen widows without children and under thirty-nine years of age may have remarried from choice, but the widow in her forties, with four children, given the opportunity to remarry perhaps had little choice. This tabulation further indicates that only seventy-five in one thousand dependent widows remarry, with the remarriages occurring between one and two years after the death of the husband.

FATALITIES

PENNSYLVANIA EXPERIENCE

Policy Years 1939, 1940, 1941 and 1942

Widow Under Observation for 300 Weeks — Children Under 16 Years

Dependency (1)	No. of Cases (2)	Average Age of Widow (3)	AVERAGE AGE OF CHILDREN								
			1st (4)	2nd (5)	3rd (6)	4th (7)	5th (8)	6th (9)	7th (10)	8th (11)	9th (12)
ALL	1635										
No Dependents	320										
Widows Only	521	50.1									
All Widows and Children:	534	36.5									
Widow and 1 child	234	37.0	8.5								
" " 2 children	155	35.7	9.9	6.1							
" " 3 "	74	36.6	11.4	8.9	5.5						
" " 4 "	37	37.9	12.8	10.8	8.0	5.0					
" " 5 "	16	34.4	12.9	10.7	7.7	4.9	1.7				
" " 6 "	13	34.8	12.0	9.7	7.7	6.0	4.0	1.7			
" " 7 "	3	34.0	15.0	13.7	12.3	10.3	8.7	7.3	4.7		
" " 8 "	1	33.0	14.0	12.0	10.0	8.0	6.0	4.0	1.0	0.0	
" " 9 "	1	33.0	16.0	14.0	13.0	11.0	9.0	8.0	6.0	2.0	0.0
All Other Children:	52										
1 Orphan Child	21		10.2								
2 Orphan Children	14		11.7	8.1							
3 " "	10		12.5	9.6	5.9						
4 " "	4		12.3	9.5	7.5	5.8					
5 " "	2		12.5	10.5	6.5	5.0	2.0				
6 " "	..										
7 " "	1		7.0	5.0	4.0	2.0	1.0	1.0	0.0		
Both Parents:	96										
Father	18										
Mother	78										
Brothers, Sisters and Others:	1										

NOTE: 15 cases of Unknown Dependents are not included in the tabulation.

Not included also are 111 cases which appeared in the four policy years here shown; 5 of these cases were due to Asbestosis, 93 to Silicosis, 5 to Lead Poisoning, with a different scale of benefits. 4 of these 111 cases were compensated under the United States Longshoremen's and Harbor Workers' Act, and in 11 cases the age was not given.

It is chance that the number for dependency of both parents (96) is precisely equal to the sum of the cases of dependent fathers or mothers, since in each of the four years there was a difference.

DEATH OR REMARRIAGE OF WIDOW — FATAL CASES

PENNSYLVANIA EXPERIENCE — Policy Years 1939, 1940, 1941 and 1942

Widows Under Observation for Five Years*

Age of Widow at Death of Husband	No. of Widows	Death or Remarriage		Death (No.)	Remarriage (No.)
		No.	Rate Per 100		
(1)	(2)	(3)	(4)	(5)	(6)
ALL	1055	105	10	30	75
Under 20	16	3	19	..	3
20 to 29	181	27	15	..	27
30 to 39	251	28	11	5	23
40 to 49	239	19	8	5	14
50 to 59	228	13	6	5	8
60 to 69	115	10	9	10	..
70 to 79	23	5	22	5	..
80 to 89	2	..	..	..	..

* Compensation for widows runs for 300 weeks. Probably 6 more remarriages and 4 more deaths will be added when the reports for 1942 Policy Year have eventually been revised. No reports were made of remarriage or death later than five years — possibly an omission in reporting, due to the slight change in the compensation cost of cases.

DEATH OF WIDOW

PENNSYLVANIA EXPERIENCE — Policy Years 1939, 1940, 1941 and 1942

Widow Under Observation for 300 Weeks — Children Under 16 Years

Age of Widow at Death of Husband	No. of Widows	No. Dying	Interval to Death (Years)					Family Group				
			Under One Year	One and Under Two	Two and Under Three	Three and Under Four	Four and Under Five	Widow Only	Widow and One	Widow and Two	Widow and Three	Widow and Four
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
No. of Widows	1055	..	..	..	..	..	..	521	234	155	74	37
No. Dying	..	30	9	7	6	6	2	23	2	2	2	1
15 to 19	16	..	..	..	..	..	..	..	..	..	..	..
20 to 24	90	..	..	..	..	..	..	..	..	..	..	..
25 to 29	91	..	..	..	..	..	..	..	..	..	..	..
30 to 34	127	3	3	..	..	..	..	..	1	..	2	..
35 to 39	124	2	..	..	1	1	..	2	..	..	..	..
40 to 44	111	..	..	..	..	..	..	..	..	..	..	..
45 to 49	128	5	1	1	2	1	..	1	1	2	..	1
50 to 54	126	3	..	2	1	..	..	3	..	..	..	..
55 to 59	102	2	..	..	..	2	..	2	..	..	..	..
60 to 64	65	4	1	2	..	1	..	4	..	..	..	..
65 to 69	50	6	3	2	1	..	..	6	..	..	..	..
70 to 74	20	4	..	..	1	1	2	4	..	..	..	..
75 to 79	3	1	1	..	..	..	..	1	..	..	..	..
80 to 84	2	..	..	..	..	..	..	..	..	..	..	..
85 to 89	..	..	..	..	..	..	..	..	..	..	..	..

REMARRIAGE OF WIDOW

PENNSYLVANIA EXPERIENCE — Policy Years 1939, 1940, 1941 and 1942

Widow Under Observation for 300 Weeks — Children Under 16 Years

Family Group	No. of Widows	No. Remarrying	Age of Widow at Death of Husband					Interval to Remarriage (Years)					
			Under 20 Years	20 to 29	30 to 39	40 to 49	50 to 59	Under One Year	One and Under Two	Two and Under Three	Three and Under Four	Four and Under Five	Five and Under Six
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
All Widows	1055	..	16	181	251	239	228	..	..	..	..	..	..
No. Remarrying	..	75	3	27	23	14	8	12	28	15	15	5	..
Widow Only	521	31	1	11	3	9	7	4	10	9	7	1	..
Widow and One Child	234	22	1	9	8	3	1	3	9	3	6	1	..
Widow and Two Children	155	11	1	4	5	1	..	2	5	1	1	2	..
Widow and Three Children	74	5	..	1	4	..	..	2	..	1	1	1	..
Widow and Four Children	37	5	..	2	2	1	..	1	3	1	..	..	..
Widow and Five Children	16	1	..	..	1	..	..	..	1	..	..	..	..
Widow and Over Five Children	18	..	..	..	..	..	..	..	..	..	..	..	..

STATISTICAL NOTES

CASUALTY ACTUARIAL SOCIETY — REMARRIAGE TABLE

Official sanction to the use of this table has been given by one of the higher courts of the country. The United States Tax Court has held that it is proper to use this table in determining the value, for estate tax purposes, of alimony to a widow payable for life or until her remarriage.

Since this decision establishes a precedent, it is expected that it will be followed for use in similar problems both by the Tax Court and other judicial bodies.

Maresi 6 T. C. No. 74 Prentice-Hall Fed. Tax 74,340

OBITUARY

SANFORD BENHAM PERKINS

1889 - 1945

Sanford Benham Perkins, a former president of the Society, died at the Hartford Hospital, on the evening of September 16, 1945, shortly after he was taken there, following a sudden illness while playing golf that afternoon.

Mr. Perkins was born in Torrington, Connecticut, on July 26, 1889, and attended the public schools in that city. He was graduated from Sheffield Scientific School, in 1910, with the Bachelor of Science degree. From 1911 to 1917, Mr. Perkins engaged in teaching mathematics and kindred subjects at Sheffield and the Berkeley Preparatory School of New Haven, of which latter institution he was the active head. He entered the employ of The Travelers Insurance Company, in the Casualty Actuarial Department, on August 1, 1917. Subsequently, he was appointed Casualty Actuary on January 1, 1920 and was elected Assistant Secretary of the Compensation and Liability Department in September 1923. In 1940, he became Secretary of that Department and, on August 20, 1945, less than a month prior to his untimely death, he was elected Vice President of The Travelers Insurance and Indemnity Companies.

Endowed with an exceptionally agile, analytical mind, plus the faculty for expressing himself with logical lucidity, Mr. Perkins early established himself as a leader in his profession. Adhering strictly to theoretical fundamentals, he labored constantly to translate these to serve practical underwriting requirements and contributed greatly in strengthening the actuarial and underwriting structure of the casualty insurance business. He was held in high esteem, in this country and in Canada, for the leading part which he played in the deliberations of numerous actuarial, rating, manual and executive committees of various state rating boards and bureaus, the National Council on Compensation Insurance, the National Bureau of Casualty and Surety Underwriters and the Association of Casualty and Surety Executives, and in state and national insurance hearings and conventions.

Mr. Perkins was especially noted for his ability to resolve knotty problems involving differences of opinion between conflicting interests. Outstanding contributions were made by him in the development of premium

differentials by size of risk; individual risk rating plans, including retrospective rating and the Comprehensive Rating Plan for National Defense projects. In recent years, in addition to his other activities, Mr. Perkins devoted much time to the cause of safety education and conservation work, serving with unflagging zeal on the Advisory Committees of both the Center for Safety Education, New York University, and the National Conservation Bureau. As an appropriate memorial to him, these organizations have established, at New York University, the Sanford B. Perkins Fellowship, carrying with it an annual stipend of $1500, for graduate work and research in the field of safety and conservation.

Admitted as a Fellow of the Casualty Actuarial Society, by examination, in November 1921, Mr. Perkins was elected its Vice President for the years 1924 to 1926, and President, from 1926 to 1928. He contributed a number of papers and written discussions to the Proceedings and took an active interest in the affairs of the Society, particularly during the first two decades of its existence. He inspired the younger men to become members and was ever willing to lend a hand in shaping the policies of the Society.

A big man in every sense, Mr. Perkins exemplified a philosophy which admitted of no meanness. He took success with modesty and failure with a smile. He believed in working hard and playing hard. He took an active interest in athletic contests, especially football, and was a low-handicap golfer. In his dealings with others, he was readily approachable and the warmth of his personality, coupled with a generous and sympathetic nature, made for friendships in greater number than is the lot of most men. His quick wit and mastery of the art of repartee made him a favorite after-dinner speaker and a welcome guest at all gatherings. Beneath his hearty manner, there was a depth of feeling and a sensitivity to the misfortunes of others, although he made it a point, almost to a fault, of concealing personal pain and loss. He was most courageous in facing the loss of his son, Lt. Sanford Benham Perkins, Jr., a Navy fighter pilot, who was reported missing while engaged in a mission against the enemy in the South Pacific, in December 1944.

Standing on the threshold of further success, after a career which marked him as a brilliant and able insurance executive, Mr. Perkins will long be remembered by his many friends and business associates, whose sorrow at his passing is enhanced by its having occurred so prematurely and with such unbelievable suddenness.

OBITUARY

EUGENE R. WELCH

1890 - 1945

Eugene R. Welch, an Associate of the Society since 1921, died on January 17, 1945. He was born in San Jose, California, in 1890, and received his early education there. He was graduated from the University of California in the class of 1912, taking a B.A. degree. In 1913 he was employed with the Industrial Accident Board of California at about the time the Workmen's Compensation Act was enacted. In this employment he was associated with G. F. Michelbacher on the Special Committee to develop a Permanent Disability Rating Schedule, prior to the effective date of the Workmen's compensation Act. He was later transferred to the State Compensation Insurance Fund, becoming associated with the late Claude W. Fellows, who was the first manager of the Fund. Mr. Welch successively became Superintendent of Claims, Assistant Secretary, and Secretary of the Fund.

In 1922, when Mr. Fellows organized the Associated Industries Insurance Corporation, later to become the Associated Indemnity Corporation, Mr. Welch went with him, and served as Secretary of that company until 1928. It was then that Mr. Welch decided to branch out for himself and he became an independent broker, in which endeavor he remained until his death. Even as a broker, Mr. Welch kept close ties with his old associates in the Associated Indemnity Corporation, and up to the time of his death maintained his offices in the same building.

While never an active member of the Society, Mr. Welch was always interested in its welfare and advancement, and he always encouraged those around him to take an interest in it. He had a high sense of patriotism, and served in the California State Guard during World War II.

Surviving him are his wife, one son, and two daughters.

Mr. Welch was a person of very congenial temperament and had a host of friends, not only in California, but throughout the nation. He was always a source of encouragement and enlightenment to the younger persons in the industry.

CASUALTY ACTUARIAL SOCIETY

NOVEMBER 16, 1945

THE COUNCIL

**Officers:* CHARLES J. HAUGH ..... *President*
JAMES M. CAHILL ..... *Vice-President*
HARRY V. WILLIAMS ..... *Vice-President*
RICHARD FONDILLER ..... *Secretary-Treasurer*
EMMA C. MAYCRINK ..... *Editor*
THOMAS O. CARLSON ..... *Librarian*

†*Ex-Presidents:* SYDNEY D. PINNEY 1946
RALPH H. BLANCHARD 1948
HAROLD J. GINSBURGH 1949

†*Ex-Vice-Presidents:* ALBERT Z. SKELDING 1949

†*Elected:* ROBERT V. SINNOTT 1946
ARTHUR N. MATTHEWS 1946
WILLIAM F. ROEBER 1946
NELS M. VALERIUS 1947
WILLIAM R. WILLIAMSON 1947
THOMAS F. TARBELL 1947
ARTHUR E. CLEARY 1948
RUSSELL P. GODDARD 1948
RAINARD B. ROBBINS 1948

*Terms expire at the annual meeting in November 1946.
†Terms expire at the annual meeting in November of the year given.

COMMITTEES

Committee on Admissions

Thomas F. Tarbell (Chairman)
Gustav F. Michelbacher
William J. Constable
Hiram O. Van Tuyl
Francis S. Perryman

Auditing Committee

Howard G. Crane (Chairman)
Charles M. Graham
Joseph Linder

Editorial Committee

Emma C. Maycrink (Chairman)

Assistant Editors

Clarence A. Kulp
Jack J. Smick

Educational Committee

Thomas O. Carlson (Chairman)
Clarence A. Kulp
Daniel J. Lyons
Nels M. Valerius
Norton E. Masterson
Freeland R. Cameron

Examination Committee

John A. Mills (General Chairman)

Fellowship

George B. Elliott (Chairman)
Seymour E. Smith
Ernest T. Berkeley

Associateship

Roger A. Johnson, Jr. (Chairman)
Samuel M. Ross
Franklin E. Satterthwaite

Committee on Papers

Arthur N. Matthews (Chairman)
Paul Dorweiler
Ralph H. Blanchard
Emma C. Maycrink (*ex-officio*)

Committee on Program

Charles J. Haugh, Chairman (*ex-officio*)
James M. Cahill
Gregory C. Kelly
Harry V. Williams
Richard Fondiller (*ex-officio*)

COMMITTEE ON PUBLICATIONS

CHARLES J. HAUGH, CHAIRMAN (*ex-officio*)
RICHARD FONDILLER
EMMA C. MAYCRINK
THOMAS O. CARLSON

SPECIAL COMMITTEE

COMMITTEE ON SOCIAL INSURANCE

FRANCIS S. PERRYMAN (CHAIRMAN)
HARMON T. BARBER
JAMES M. CAHILL
HAROLD J. GINSBURGH
CLARENCE A. KULP
WILLIAM R. WILLIAMSON
JARVIS FARLEY

ABSTRACT FROM THE MINUTES OF THE MEETING NOVEMBER 16, 1945

The annual meeting of the Casualty Actuarial Society was held at the Hotel Biltmore, New York, on Friday, November 16, 1945.

President Ginsburgh called the meeting to order at 10:20 A.M., the roll was called, showing the following forty-six Fellows and thirteen Associates present:

FELLOWS

AINLEY	GINSBURGH	OBERHAUS
AULT	GODDARD	PERRYMAN
BARTER	GRAHAM, C. M.	ROEBER
BERKELEY	GRAHAM, W. J.	ROSS
BLANCHARD	HAUGH	SATTERTHWAITE
CAHILL	JOHNSON	SHAPIRO
CAMERON	KELLY, G. C.	SILVERMAN
CARLSON	KIRKPATRICK	SKILLINGS
COGSWELL	KORMES	SMICK
CONSTABLE	LINDER	SMITH, S. E.
CORCORAN	MARSHALL	TARBELL
ELLIOTT	MASTERSON	VALERIUS
EPPINK	MATTHEWS	VAN TUYL
FARLEY	MAYCRINK	WILLIAMS
FONDILLER	MCCONNELL	WILLIAMSON
	MILLS	

ASSOCIATES

BAILEY	CROUSE	HOLZINGER
BLACK, N. C.	DOWLING	SPENCER
BUFFLER	GIBSON	STOKE
BUGBEE	GROSSMAN	UHTHOFF
	HIPP	

By invitation, a number of officials of casualty insurance companies and other organizations were present.

By unanimous vote, the Society ratified the action of the Council in omitting the 1944 annual meeting of the Society due to the War Emergency, and also ratified all acts of the Council during that period as printed in *Proceedings*, Number 61.

Mr. Ginsburgh read his Presidential Address.

The Secretary-Treasurer (Richard Fondiller) read the report of the Council and upon motion it was adopted by the Society. Ernest Holzinger had passed the necessary examinations and had been admitted as an Associate. Examination Rules 4 and 6 had been amended in order to assist returned veterans pursuing actuarial studies. The report of the Committee on Mortality for Disabled Lives (Paul Dorweiler, Chairman), had been accepted with the thanks of the Society and appears in this number of the *Proceedings*.

The President announced the deaths, during the last year, of one Fellow, Sanford B. Perkins, and one Associate, Eugene R. Welch. Obituary notices appear in this number of the *Proceedings*.

The report of the Secretary-Treasurer was read and accepted. The report on finances follows:

CASUALTY ACTUARIAL SOCIETY

ANNUAL REPORT ON FINANCES

Cash Receipts and Disbursements from October 1, 1944 to September 30, 1945

INCOME

On Deposit October 1, 1944 in Marine Midland Trust Company		$ 225.50
Members' Dues	$2,185.00	
Sales of *Proceedings*	1,529.72	
Examination Fees	294.00	
Michelbacher Fund	135.50	
Sale of Hobbs' Reprint	22.00	
Miscellaneous	—13.24	4,152.98
Total		$4,378.48

DISBURSEMENTS

Printing & Stationery	$1,554.74
Postage, Telephone, Express, etc	66.93
Secretarial Work	423.69
Examination Expense	248.93
Insurance	12.10
Miscellaneous	25.25
Total	$2,331.64
On deposit on September 30, 1945 in Marine Midland Trust Co.	2,046.84
Total	$4,378.48

RECONCILIATION

Cash in Bank—September 30, 1944		$ 225.50
Income	$4,152.98	
Disbursements	2,331.64	
Excess of Income over Disbursements		1,821.34
Cash in Bank—September 30, 1945		$2,046.84
*Bonds Owned September 30, 1945		3,750.00
Total Assets September 30, 1945		$5,796.84

* Michelbacher Fund of $1,698.86 invested in above Bonds.

The Auditing Committee (F. R. Cameron, Chairman) reported that the books of the Secretary-Treasurer had been audited and his accounts verified.

The Examination Committee (Arthur E. Cleary, Chairman) submitted a report of which the following is a summary:

1945 Examinations — Successful Candidates

The following is a list of those who passed the examinations held by the Society on April 4 and 5, 1945:

Associateship Examinations

PART I:	Geoffrey Crofts	Robert C. Morrow
	Alfred V. Fairbanks	Joseph Musher
	J. B. Gardiner	C. S. Stunkel
	Robert Larson	V. S. Zarowski
	John B. Masiowski	
PART II:	Douglas Bond	Robert C. Morrow
	Geoffrey Crofts	Joseph Musher
	J. B. Gardiner	F. E. Nemmers
	Robert Larson	John H. Rowell
	John B. Masiowski	V. S. Zarowki
PART III:	Francis E. Abel	Fred W. Tallman
	Ruth Salzmann	John W. Wieder, Jr.
PART IV:	Robert Larson	A. M. Sutherland
	Joseph Musher	Fred W. Tallman
	R. C. Perry	
PART V:	Frances E. Abel	Ernest Holzinger
	George A. Hedden, Jr.	

Fellowship Examinations

PART I: Charles W. Crouse

The Council's re-election of Emma C. Maycrink as Editor, and of Thomas O. Carlson as Librarian, was announced.

The annual elections were then held and the following officers and members of the Council were elected:

President	CHARLES J. HAUGH
Vice President	JAMES M. CAHILL
Vice President	HARRY V. WILLIAMS
Secretary-Treasurer	RICHARD FONDILLER
Editor	EMMA C. MAYCRINK
Librarian	THOMAS O. CARLSON

Members of Council:

ARTHUR E. CLEARY, RUSSELL P. GODDARD, RAINARD B. ROBBINS
(terms expire in 1948)

NELS M. VALERIUS (term expires in 1947)

In accordance with Constitutional requirements, notice of the following proposed amendment to the By-Laws was given. This amendment was, on motion, adopted to read as follows:

ARTICLE IV—DUES — *First Paragraph*

Fellows and Associates who have become totally disabled while members may upon approval of the Council be exempted from the payment of dues during the period of disability.

The papers appearing in this Number were presented.

Recess was taken for lunch at the Hotel until 2:15 P.M.

Informal discussion of the following topics was participated in by the members of the Society and by representatives of insurance organizations:

1. Essential Characteristics of an Adequate System of Individual Risk Rating Plans for Casualty Insurance under State Regulation.
2. Bases of Rating Commercial Automobiles for Liability Insurance.

Upon motion, the meeting adjourned at 4:30 P.M.

1946 EXAMINATIONS OF THE SOCIETY

APRIL 3 AND 4, 1946

EXAMINATION FOR ENROLLMENT AS ASSOCIATE

PART I

1. (a) Solve the equation:

$$\frac{x+3}{x+4} - \frac{x+4}{x+5} = \frac{x+5}{x+6} - \frac{x+6}{x+7}$$

(b) Solve:

$x^4 - 4x^2 + 8x + 35 = 0$, given that $2 + \sqrt{-3}$ is a root.

(c) Solve the equation:

$$xy + x + y = 23$$
$$xz + x + z = 41$$
$$yz + y + z = 27$$

2. (a) A train traveling at the rate of 30 miles an hour starts from a town A to a town B; 12 minutes later a second train traveling at 40 miles per hour starts from B to A, and goes half a mile beyond the middle point between A and B before it meets the first train; find the distance between A and B.

(b) The first two terms of an infinite geometric progression are together equal to 5, and every term is three times the sum of all the terms that follow it. Find the series.

3. (a) In the expansion of $(a + b + c)^8$ find
   (1) the number of terms
   (2) the sum of the coefficients of the terms.

(b) Show that $\left(\frac{21}{20}\right)^{100}$ is greater than 100.

given $\log. 2 = .30103$
$\log. 3 = .47712$
$\log. 7 = .84510$

4. (a) How many combinations of four letters each can be made from the letters of the word "provocative"?
   (b) (i) Five men, A, B, C, D, E are going to speak at a meeting. In how many ways can they take their turns if A is to speak before B?
   (ii) In how many ways, so that A speaks immediately before B?

5. (a) (i) Define nominal rate of interest.
   (ii) Define effective rate of interest.
   (iii) Define force of interest.
   (b) What nominal rate converted quarterly will give the same yield as 5% compounded semi-annually? Compute answer to four decimal places.

6. (a) Derive a formula for the time required to pay a non-interest bearing debt P by means of annual payments, R, allowed to accumulate at effective rate i.
   (b) A house is listed for sale at \$10,000. The seller agrees to take \$4,000 cash and \$1,000 per annum for six years without interest! If money is worth 6% effective in such transactions, what reduction was made in the price of the house?

7. (a) An automobile has an original value of \$1,750. Its depreciation is to be covered by a sinking fund at 6% under the conditions that the auto will be worth \$250 at the end of eight years. Find the book value at the end of five years.
   (b) A man borrows \$3,000 with interest at 6% compounded semi-annually. At the end of 5½ years he makes the first of eight equal semi-annual payments which completely discharge his obligation. Find the semi-annual payment.

8. (a) A man purchases a house worth \$15,000 cash, with interest at 5%, payable semi-annually. He signs a contract to pay \$5,000 cash, and to discharge the balance, principal and interest included, by equal payments at the end of 6 months for 10 years. At the end of 4 years, this contract was sold to yield the investor 7%, payable semi-annually. Find the price he paid.

(b) The interior of a room can be painted at a cost of \$10 and the painting must be repeated at the end of each 2 years. If money is worth 6%, how much could a householder afford to pay for papering the room, if the paper would need renewal at the end of each 4 years?

NOTE: The following values will be useful in solving problems 5 to 8 inclusive:

$(1.025)^{\frac{1}{2}}$	$= 1.01242$	$a_{\overline{10}\rceil}$ (at 3%)	$= 8.5302$
$(1.025)^2$	$= 1.05063$	$\frac{1}{a_{\overline{20}\rceil}}$ (at 2½%)	$= .06415$
$v6$ (at 6%)	$= .70496$	$a_{\overline{12}\rceil}$ (at 3½%)	$= 9.6633$
$\frac{1}{a_{\overline{8}\rceil}}$ (at 6%)	$= .16104$	$a_{\overline{4}\rceil}$ (at 6%)	$= 3.4651$
$s_{\overline{6}\rceil}$ (at 6%)	$= 5.6371$	$a_{\overline{2}\rceil}$ (at 6%)	$= 1.8334$
$a_{\overline{18}\rceil}$ (at 3%)	$= 13.7535$		

PART II

1. (a) If the sum of the surfaces of a sphere and a cube is given, show that the sum of their volumes is least when the diameter of the sphere is equal to the edge of the cube.

$$\text{Area of sphere} = 4\pi r^2$$
$$\text{Volume of sphere} = 4/3\,\pi r^3$$

(b) Find the critical values and points of inflection of the following curve and draw a rough sketch of the curve

$$y = x^3 - 27x + 6$$

2. (a) Find the area bounded by the curve $y^2 = 16x^2 - x^4$.

(b) Expand a^x by Maclaurin's series as far as the term involving x^3.

3. (a) Differentiate

$$y = X^x + (\sin x)^x$$

(b) Find $\frac{d^2y}{dx^2}$ from:

$$y = e^{ax} \sin bx$$

4. (a) Integrate:

$$\int \frac{x^2\,dv}{x^4+x^2-2}$$

(b) Integrate:

$$\int e^x \operatorname{Sin} 2x\,dx$$

5. (a) Given $U_3=3$, $U_2=12$, $U_1=15$, $U_{-1}=-21$, find the general expression for U_x.

(b) Obtain the following relation between the seven terms of the series represented by $U_1, U_2, \cdots\cdots U_7$, and state what assumption is involved in this relation.
$U_4=1/20\ (U_1+U_7)-3/10\ (U_2+U_6)+3/4\ (U_3+U_5)$.

6. Derive Bessel's interpolation formula and use it to find $f(35)$, given values of $f(x)$ at 20, 30, 40, 50 to be 1313, 1727, 2392, 3493 respectively.

7. Given

$$\begin{aligned} U_0 &= 1 \\ U_5 &= 16 \\ U_{10} &= 56 \\ U_{15} &= 121 \end{aligned}$$

Find $S_{16}=U_0+U_1+U_2+\cdots\cdots+U_{15}$.

8. (a) Given

$$\begin{aligned} U_0 &= 1.7942 \\ U_{10} &= 1.7939 \\ U_{20} &= 1.8109 \\ U_{30} &= 1.8465 \\ U_{40} &= 1.9044 \\ U_{50} &= 1.9907 \end{aligned}$$

Find U_{25}.

(b) By a method of finite differences, sum to n terms the series whose X^{th} term is $X(X+2)\ (X+4)$.

PART III

1. Determine from the following data the value of the median, mode, mean, standard deviation, quartile deviation, coefficient of variation and skewness:

Class Interval	*Frequency*
4.5 - 24.4	5
24.5 - 44.4	8
44.5 - 64.4	16
64.5 - 84.4	30
84.5 - 104.4	20
104.5 - 124.4	10
124.5 - 144.4	7
144.5 - 164.4	4
	Total frequency 100

2. (a) What is meant by "least squares" in curve-fitting?

   (b) By the method of least squares, fit a second degree parabola to the following data:

Year	1880	1890	1900	1910	1920
Population (in thousands)	175	318	414	673	783

3. (a) Give the formula for Fisher's "Ideal" index number and show whether it meets the three tests of validity.

   (b) The heights of two groups of soldiers were measured and the following results were secured:

GROUP I	GROUP II
$N = 100$	$N = 100$
$M = 67.51$ inches	$M = 62.24$ inches
$\sigma = 2.20$ inches	$\sigma = 2.25$ inches

   Determine whether the difference in the means is sufficient to warrant belief that the two groups were chosen from different races.

4. Compute the coefficient of correlation and its probable error for the following data:

x	3	5	6	7	9
y	−2	1	1	2	3

5. The balance sheet of the business of A. B. Rawson on January 1, 1946 was as follows:

ASSETS		LIABILITIES	
Cash	$ 3,000	Mortgage	$15,000
Merchandise	15,000	Notes Payable	5,000
Accounts Receivable	2,500	Accounts Payable	3,000
Notes Receivable	1,500	A. B. Rawson, Capital	19,000
Buildings	20,000		
	$42,000		$42,000

The following transactions occur during January: Goods are sold on account, $1,800 and for cash, $2,600. Customers gave promissory notes to apply on accounts, $800. Cash was collected from customers on accounts, $2,200 and on promissory notes, $1,500. Merchandise was purchased on account $2,000: Cash was paid to creditors on account, $3,000 and on notes $2,500. Cash was borrowed from the banks on promissory notes $2,000. An addition to the building was paid for by increasing the mortgage $5,000. Various expenses of operating the business were paid for in cash, $1,000. The value of goods on hand at the end of January was $14,600.

Record the summarized transactions, debit and credit, on ledger "T" accounts and prepare a new balance sheet as of January 31, 1946.

6. (a) Indicate in which ledger account the respective debit and credit entries should be made for each of the following transactions:

   (1) Note received from a customer to apply on his account.
   (2) Sale of merchandise on account.
   (3) Proprietor's personal expenses paid.
   (4) Supplies used in Sales office.
   (5) Income taxes paid.

   (b) In taking a trial balance, the books are out of balance by $184.14. State two possible sources of the error.

7 and 8.

The following is the unadjusted trial balance of John Jones on December 31, 1946.

	Dr.	Cr.
John Jones, Capital	$	$16,000
Advertising	100	
Cash	6,250	3,100
Accounts Payable	4,500	5,500
Accounts Receivable	5,200	3,700
Equipment — Cost	2,000	
Equipment — Allowance for Depreciation		250
General Expense	400	
Insurance	200	
Inventory—Merchandise (Dec. 1)	10,000	
Salaries	900	
Purchases	4,300	
Rent	200	
Sales		5,500
TOTAL	$34,050	$34,050

Adjustments are to be made for the following:

Merchandise inventory on December 31, $11,000; depreciation on equipment, 10% of original cost; accrued salaries, $100; unexpired insurance, $150.

Make the adjusting and closing journal entries and prepare a Balance Sheet and a Profit and Loss Statement.

PART IV

1. (a) Three men, *P*, *Q*, and *R*, are each exactly 96. Find the chance that they will die at different ages last birthday in the order *P*, *Q*, *R*.

Given:

Exact Age	*Chance of Dying Before Next Birthday*
96	1/2
97	2/3
98	3/4
99	1

(b) Compare the chances of throwing:

(a) 4 with one die
(b) 8 with two dice
(c) 12 with three dice

2. (a) The probability that it will rain on a given day is 1/10 if the preceding day was clear, and 1/5 if the preceding day was rainy. What is the expected claim cost on an insurance policy that will pay \$1,000 on each rainy day on two successive days?

   (b) If a coin turns up heads on ten tosses, what is the probability that it is a coin with two heads when the a priori probability that the coin is a true one is 9/10?

3. (a) If a die is thrown 1000 times, how many times can you expect to get exactly three sixes in succession?

   (b) Two identical urns contain respectively m and n balls; the first urn contains A white balls and the second B. If a ball is extracted from one of the two urns, what is the probability that it is white?

4. (a) In a certain competition, the chances are 7 to 3 against one competitor A, 11 to 5 against B, and 13 to 7 against C. What is the chance that one of these three wins?

   (b) A bag contains 5 balls of unknown color; a ball is drawn and replaced twice, and in each case is found to be red. If two balls are now drawn simultaneously, find the chances that both are red.

5. (a) The annual premium on a whole life policy at age 30 is \$25.00. What would be the equivalent premium payable for 20 years?

   (b) An annuitant now age 30 is receiving \$1,000 per year. He wishes to waive the payment now due and also the next 19 payments. What adjustment should be made in the payments thereafter?

   Given: $N_{30} = 5968$
   $N_{50} = 1816$

6. (a) Show that the net single premium for an increasing insurance on the life of (x) is

$$\frac{hM_x + kR_x + 1}{D_x}$$

   if the death benefit in the first year is h, and the death benefit increases by k per year.

(b) A whole life annuity immediate on the life of (x) provides for a sequence of payments of 1.035 at the end of one year, $(1.035)^2$ at the end of the second year, $(1.035)^3$ at the end of the third year, and so on. Show that the present value of this annuity at $3\frac{1}{2}\%$ is e_x.

7. (a) Give the formula for the Gompertz law of mortality and prove that a single life may be substituted for joint lives in calculating the probabilities of survival and death.

(b) A class of 10 college men age 22 desire a policy under which they pay premiums until the first death, when the proceeds are to be payable into a scholarship fund named after the deceased member. Determine the age to be used with single life tables to calculate the premium assuming Gompertz law.

Given: $\log_{10} c = .04$

8. Show algebraically that the retrospective and prospective methods give identical values for the t^{th} terminal reserve for an ordinary life policy issued at age x.

PART V

1. (a) For those states in which they are applicable, how are Loss and Expense Constants applied under the Standard Workmen's Compensation Policy on interstate risks?

(b) How are Premium Discounts determined under an interstate Workmen's Compensation Policy where the discount percentages vary by state?

2. (a) What inclusions and exclusions of coverage apply under manufacturers and contractors classifications in connection with the transportation of contractors' equipment and appliances from one location to another?

(b) An automobile policy is issued in a state having a financial responsibility law at limits in excess of those required under the law. A loss occurs involving an amount exceeding the limits under the law but less than the policy limits. Is the insurer liable for the payment of the entire amount under all circumstances? Discuss.

3. (a) An employer decides to carry fidelity protection and buys a blanket position bond from Company A. As of its first renewal date, he cancels Company A's bond and gives the risk to Company B. Should a superseded suretyship rider be attached to one or both of these bonds? Why?

   (b) A double benefit clause is often included in personal accident policies. Give the origin of this clause and discuss its value under present day conditions.

4. (a) Explain the difference between use and occupancy coverage and outage coverage as used in steam boiler and machinery insurance.

   (b) Outline briefly the coverage provided under the Comprehensive Personal Liability Policy and explain wherein it differs from that normally provided under liability policies.

5. Discuss the possibility of computing two sets of manual rates for workmen's compensation insurance, one for non-experience rated risks and one for experience rated risks, as a means of meeting the problem of the off balance of the experience rating plan as it exists in the present rate making procedure.

6. If a particular state should pass legislation providing for compensation and medical benefits on a definite schedule basis for persons injured in automobile accidents, regardless of fault, outline a method which you would propose for a rate-making procedure to be followed after adequate experience had been developed for this state following the passage of such legislation.

7. (a) Policy-year data used in the making of rates for workmen's compensation insurance represent experience of the past and are not necessarily indicative of current conditions. What steps are taken in the rate-making procedure to correct for this condition?

   (b) Automobile rates generally are set up by territory within each state. What factors are taken into account in determining the number and extent of such territories?

8. (a) The value of product (or sales receipts) is commonly used as an exposure medium in product insurance. Discuss the possible use of this medium in workmen's compensation and manufacturer's and contractor's insurance, keeping in mind the qualifications which an exposure medium must have to serve as a desirable basis of premium.

   (b) In a state in which the tax provision is 2.5%, the application of a $10 Expense Constant reduces the normal Workmen's Compensation expense loading from 40% to 37.5%. It is assumed that $5 of the Expense Constant is applied to administration expense and $3 to payroll audit. Determine the composition of the 37.5% expense loading.

EXAMINATION FOR ADMISSION AS FELLOW

PART I

1. (a) To what extent may a casualty insurance company in New York invest in the stock of other insurance companies?

   (b) List five types of investments prohibited to a casualty insurance company under the New York Insurance Law.

2. Discuss the effect which current low investment yields might have upon the underwriting policy of a casualty insurance company.

3. (a) Do you believe railroad securities have become more attractive for the investment of funds of insurance carriers? Discuss the favorable and unfavorable trends that are developing currently.

   (b) What is the intent and what are the general provisions of Second Injury Fund Laws?

4. (a) Describe the legal characteristics of a representation in an insurance contract.

   (b) What is the so-called "Appleton Rule"? To what extent and on what grounds was this rule recently relaxed?

5. At the last meeting of the National Association of Insurance Commissioners, model rating bills were adopted with the

recommendation that they be used by the several states as legislative guides. These bills involved the "prior rate approval" theory. Similar bills were drafted by the all-industry committee, composed of representatives of the insurance business, but these bills involved the "subsequent disapproval" theory. Explain what is meant by these two terms. In your opinion, which type of bill will most effectively accomplish the purposes of Public Law 15? State your reasons.

6. What, in your opinion, will be the effect of Public Law 15 upon the rating of interstate Workmen's Compensation risks?

7. Name and discuss four ways in which the state usually exercises control over the financial condition of insurance companies.

8. (a) Name and discuss four requirements of insurable hazard.

   (b) From the viewpoint of the economic theory of risk, contrast the value of insurance to large concerns as compared with small ones. What, in your opinion, are the steps that must be taken by casualty insurance carriers to make insurance attractive to the larger business enterprises?

PART II

1. Describe four methods of calculating claim reserves for known cases. Indicate the advantages of each method and state the conditions under which each should be used.

2. (a) Describe the method by which the convention annual statement blank provides a check on the adequacy of loss reserves in prior years for lines other than compensation and liability. For compensation and liability.

   (b) Assuming that premiums are charged up annually as they become due, outline a common method of computing the unearned premium reserve on three-year liability policies written on a 50-30-20 premium installment basis which results in the creation of a reserve equal to the amount that would be paid back to the policyholder in the event of cancellation of the policy.

3. The annual statement blank of the National Association of Insurance Commissioners requires an unearned premium reserve for retrospective returns based upon experience. Describe two methods that might be employed to calculate this reserve.

4. (a) What information is made available to a rate-making body by the Unit Statistical Plan for workmen's compensation insurance?

   (b) Design a punch card which a multiple line casualty company could use to keep a continuous record of both calendar year and policy year incurred losses by line of insurance on an individual estimate basis. Explain such items as you consider necessary.

5. If you were asked to suggest a program of expanded production for a small, growing multiple line casualty company as respects state, city, or territory and line of insurance, what external statistics would you study to formulate a recommendation?

6. There has been an increase in the latest calendar year incurred loss ratio for workmen's compensation insurance for your company. The company has an appreciable volume of business written under the Retrospective Rating Plan, as well as qualifying for Premium Discounts. Outline the procedure you would follow in analyzing the calendar year experience to determine whether your current underwriting results are favorable or not.

7. Discuss the effect of the unearned premium reserve requirements upon the incurred expense ratio of a casualty company with a rapidly expanding volume of business. How would this vary between a company writing only workmen's compensation insurance and a company whose main volume of business consists of writing private passenger automobile risks?

8. (a) Name and briefly describe the information required in five schedules, other than Schedule P, contained in the convention annual statement blank.

   (b) What information is called for in the new Casualty Insurance Expense Exhibit?

PART III

1. Discuss the differences between the Pennsylvania and the National Council prospective experience rating plans for Workmen's Compensation risks.

2. Discuss the insurance charge as used in the retrospective rating plan, commenting on its purpose, computation and relationship to minimum and maximum loss limitations and size of risk. In general, would you expect the insurance charge to be redundant or deficient? Give reasons.

3. (a) Explain the purpose and calculation of the "D" ratio as used in the National Council experience rating plan for Workmen's Compensation Risks.

   (b) Is the Retrospective Rating Plan suitable for application to casualty insurance lines other than Workmen's Compensation? Give your reasons.

4. In view of the situation that has developed since the cessation of hostilities, discuss the relative merits of State vs. Federal administration of unemployment insurance.

5. Two of the basic problems outlined in the national health program presented by President Truman to Congress last fall were as follows: (1) The high cost of medical care to the individual, and (2) the loss of individual income due to sickness and disability. Describe briefly the solutions to these two problems as proposed by the President, and give your reasons for either favoring or opposing them.

6. It has been suggested that rate regulation be applicable only to the loss portion of casualty rates, and that the individual carriers be permitted to use their own expense provisions in accordance with their method of operation. Discuss the merits of this proposal in relation to current regulatory problems.

7. (a) Do you believe the present method of using a uniform percentage expense loading, irrespective of the amount of the classification rate, produces equitable results for risks whose rates are substantially higher or lower than the average? Discuss.

(b) Explain how you would calculate the liquidating or intrinsic value of the stock of a casualty carrier. What other information would you seek to appraise the investment value of this stock?

8. The Multiple Line Underwriting Committee submitted a report to the Sub-Committee on Laws and Legislation of the National Association of Insurance Commissioners, making five specific recommendations. List the recommendations of the Committee, and state whether you think they should be adopted. Give your reasons.

INDEX TO VOLUME XXXII

CASUALTY ACTUARIAL SOCIETY

ORGANIZED 1914

1946 YEAR BOOK

(Addendum to Volume XXXII of the *Proceedings*)

Corrected to February 1, 1946 **No. 25**

FOREWORD

The Casualty Actuarial Society was organized November 7, 1914 as the Casualty Actuarial and Statistical Society of America, with 97 charter members of the grade of Fellow. The present title was adopted on May 14, 1921. The object of the Society is the promotion of actuarial and statistical science as applied to the problems of casualty and social insurance by means of personal intercourse, the presentation and discussion of appropriate papers, the collection of a library and such other means as may be found desirable.

Prior to 1914 little technical study was given to the actuarial and underwriting problems of most of the branches of casualty insurance. The organization of the Society was brought about through the suggestion of Dr. I. M. Rubinow, who became the first president. The problems surrounding workmen's compensation were at that time the most urgent, and consequently many of the members played a leading part in the development of the scientific basis upon which workmen's compensation insurance now rests.

The members of the Society have also presented original papers to the *Proceedings* upon the scientific formulation of standards for the computation of both rates and reserves in accident and health insurance, liability, burglary, and the various automobile coverages. The presidential addresses constitute a valuable record of the current problems facing the casualty insurance business. Other papers in the *Proceedings* deal with acquisition costs, pension funds, legal decisions, investments, claims, reinsurance, accounting, statutory requirements, loss reserves, statistics, and the examination of casualty companies. The Committee on Compensation and Liability Loss Reserves submitted a report which has been printed in *Proceedings* No. 35 and No. 36, The Committee on Remarriage Table submitted a report including tables. printed in *Proceedings* No. 40. The Special Committee on Bases of Exposure submitted a report which is printed in *Proceedings* No. 43. The "Recommendations for Study" appear in *Proceedings* No. 54 and are in effect for the 1945 and 1946 examinations.

The lower grade of membership in the Society is that of Associate. Examinations have been held every year since organization; they are held on the first Wednesday and following Thursday in April, in various cities in the United States and Canada. The membership of the Society consists of actuaries, statisticians, and executives who are connected with the principal casualty companies and organizations in the United States and Canada. The Society has a total membership of 277, consisting of 154 Fellows and 123 Associates. The annual meeting of the Society is held in New York in November.

The Society issues a publication entitled the *Proceedings* which contains original papers presented at the meetings. The *Proceedings* also contain discussions of papers, reviews of books and current notes. This Year Book is published annually and "Recommendations for Study" is a pamphlet which outlines the course of study to be followed in connection with the examinations for admission. These two booklets may be obtained free upon application to the Secretary-Treasurer, 90 John Street, New York 7, N. Y.

CASUALTY ACTUARIAL SOCIETY

NOVEMBER 16, 1945

THE COUNCIL

**Officers:*	CHARLES J. HAUGH	*President*
	JAMES M. CAHILL	*Vice-President*
	HARRY V. WILLIAMS	*Vice-President*
	RICHARD FONDILLER	*Secretary-Treasurer*
	EMMA C. MAYCRINK	*Editor*
	THOMAS O. CARLSON	*Librarian*
†*Ex-Presidents:*	SYDNEY D. PINNEY	1946
	RALPH H. BLANCHARD	1948
	HAROLD J. GINSBURGH	1949
†*Ex-Vice-Presidents:*	ALBERT Z. SKELDING	1949
†*Elected:*	ROBERT V. SINNOTT	1946
	ARTHUR N. MATTHEWS	1946
	WILLIAM F. ROEBER	1946
	NELS M. VALERIUS	1947
	WILLIAM R. WILLIAMSON	1947
	THOMAS F. TARBELL	1947
	ARTHUR E. CLEARY	1948
	RUSSELL P. GODDARD	1948
	RAINARD B. ROBBINS	1948

*Terms expire at the annual meeting in November 1946.

†Terms expire at the annual meeting in November of the year given.

COMMITTEES

Committee on Admissions

Thomas F. Tarbell (Chairman)
Gustav F. Michelbacher
William J. Constable
Hiram O. Van Tuyl
Francis S. Perryman

Auditing Committee

Howard G. Crane (Chairman)
Charles M. Graham
Joseph Linder

Editorial Committee

Emma C. Maycrink (Chairman)

Assistant Editors

Clarence A. Kulp
Jack J. Smick

Educational Committee

Thomas O. Carlson (Chairman)
Clarence A. Kulp
Daniel J. Lyons
Nels M. Valerius
Norton E. Masterson
Freeland R. Cameron

Examination Committee

John A. Mills (General Chairman)

Fellowship

George B. Elliott (Chairman)
Seymour E. Smith
Ernest T. Berkeley

Associateship

Roger A. Johnson, Jr. (Chairman)
Samuel M. Ross
Franklin E. Satterthwaite

Committee on Papers

Arthur N. Matthews (Chairman)
Paul Dorweiler
Ralph H. Blanchard
Emma C. Maycrink (*ex-officio*)

Committee on Program

Charles J. Haugh, Chairman (*ex-officio*)
James M. Cahill
Gregory C. Kelly
Harry V. Williams
Richard Fondiller (*ex-officio*)

COMMITTEE ON PUBLICATIONS

CHARLES J. HAUGH, CHAIRMAN (*ex-officio*)
RICHARD FONDILLER
EMMA C. MAYCRINK
THOMAS O. CARLSON

SPECIAL COMMITTEE

COMMITTEE ON SOCIAL INSURANCE

FRANCIS S. PERRYMAN (CHAIRMAN)
HARMON T. BARBER
JAMES M. CAHILL
HAROLD J. GINSBURGH
CLARENCE A. KULP
WILLIAM R. WILLIAMSON
JARVIS FARLEY

MEMBERSHIP OF THE SOCIETY, NOVEMBER 16, 1945

FELLOWS

Those marked (†) were Charter Members at date of organization, November 7, 1914.

Those marked (*) have been admitted as Fellows upon examination by the Society.

Admitted	
*Nov. 21, 1930	AINLEY, JOHN W., The Travelers Insurance Company, 700 Main Street, Hartford 3, Conn.
*Nov. 13, 1931	AULT, GILBERT E., Actuary, Church Pension Fund and Church Life Insurance Corporation, 20 Exchange Place, New York 5, N. Y.
May 23, 1924	BAILEY, WILLIAM B., Economist, The Travelers Insurance Company, 700 Main Street, Hartford 3, Conn.
*Nov. 20, 1924	BARBER, HARMON T., Associate Actuary, Casualty Department, The Travelers Insurance Co., 700 Main Street, Hartford 3, Conn.
*Nov. 20, 1942	BART, ROBERT D., West Bend Aluminum Co., West Bend, Wis.
*Nov. 18, 1932	BARTER, JOHN L., Vice-President, Hartford Accident & Indemnity Co., Hartford 15, Conn.
*Nov. 13, 1931	BATHO, ELGIN R., Assistant Actuary, Berkshire Life Insurance Co., Pittsfield, Mass.
†	BENJAMIN, ROLAND, Treasurer, Fidelity & Deposit Company of Maryland and American Bonding Company, Baltimore 3, Md.
*Nov. 22, 1934	BERKELEY, ERNEST T., Superintendent, Actuarial Department, Employers Liability Assurance Corporation, Boston 9, Mass.
†	BLACK, S. BRUCE, President, Liberty Mutual Insurance Company, 175 Berkeley Street, Boston 16, Mass.
Apr. 20, 1917	BLANCHARD, RALPH H., Professor of Insurance, School of Business, Columbia University, New York 27, N. Y.
†	BREIBY, WILLIAM, Vice-President, Pacific Mutual Life Insurance Company, Los Angeles 14, Cal.
*Nov. 18, 1927	BROWN, F. STUART, Secretary, Carolina Casualty Insurance Co., Burlington, N. C.
Oct. 22, 1915	BROWN, HERBERT D., (Retired), Glenora, Yates County, New York.
†	BUCK, GEORGE B., Consulting Actuary for Pension Funds, 150 Nassau Street, New York 7, N. Y.

FELLOWS

Admitted	
Apr. 20, 1917	BURHOP, WILLIAM H., Executive Vice-President, Employers Mutual Liability Insurance Company, Wausau, Wis.
*Nov. 23, 1928	BURLING, WILLIAM H., Assistant Actuary, The Travelers Insurance Company, 700 Main Street, Hartford 3, Conn.
*Nov. 19, 1929	CAHILL, JAMES M., Secretary, National Bureau of Casualty & Surety Underwriters, 60 John Street, New York 7, N. Y.
*Nov. 18, 1932	CAMERON, FREELAND R., Actuary, American Surety Company, 100 Broadway, New York 5, N. Y.
†	CAMMACK, EDMUND E., Vice-President and Actuary, Aetna Life Insurance Company, Hartford 15, Conn.
*Nov. 17, 1938	CARLETON, JOHN W., Liberty Mutual Insurance Company, 175 Berkeley Street, Boston 16, Mass.
*Nov. 21, 1930	CARLSON, THOMAS O., Actuary, National Bureau of Casualty & Surety Underwriters, 60 John Street, New York 7, N. Y.
†	CARPENTER, RAYMOND V., (Retired), 66 Park Avenue, New York 16, N. Y.
Mar. 20, 1941	CARVER, HARRY C., Professor of Mathematics, University of Michigan, Ann Arbor, Michigan.
*Nov. 13, 1936	CLEARY, ARTHUR E., Actuary, Hospital Service Inc., 230 Congress Street, Boston 6, Mass.
*Nov. 15, 1918	COATES, BARRETT N., Coates and Herfurth, Consulting Actuaries, 660 Market Street, San Francisco 4, Calif.
*Nov. 17, 1922	COATES, CLARENCE S., Assistant Secretary, Lumbermens Mutual Casualty Company, Mutual Insurance Bldg., Chicago 40, Ill.
Oct. 27, 1916	COGSWELL, EDMUND S., First Deputy Commissioner of Insurance, 100 Nashua Street, Boston 14, Mass.
Feb. 19, 1915	COLLINS, HENRY, Manager and Attorney, Ocean Accident & Guarantee Corporation and President, Columbia Casualty Company, 1 Park Avenue, New York 16, N. Y.
*Nov. 23, 1928	COMSTOCK, W. PHILLIPS, Statistician, London Guarantee & Accident Company, 55 Fifth Avenue, New York 3, N. Y.
*Nov. 22, 1934	CONSTABLE, WILLIAM J., Secretary, Lumbermens Mutual Casualty Company, 342 Madison Avenue New York 17, N. Y.
*Nov. 22, 1934	COOK, EDWIN A., Assistant Secretary, Interboro Mutual Indemnity Insurance Company, 270 Madison Avenue, New York 16, N. Y.
†	COPELAND, JOHN A., Consulting Actuary, Candler Building, Atlanta, Ga.
*Nov. 18, 1925	CORCORAN, WILLIAM M., Consulting Actuary, Wolfe, Corcoran & Linder, 116 John Street, New York 7, N. Y.
*Nov. 19, 1926	CRANE, HOWARD G., Treasurer, General Reinsurance Corporation, 90 John Street, New York 7, N. Y.
*Nov. 18, 1932	DAVIES, E. ALFRED, Associate Comptroller, Liberty Mutual Insurance Company, 175 Berkeley Street, Boston 17, Mass.
*Nov. 18, 1927	DAVIS, EVELYN M., Woodward, Ryan, Sharp & Davis, Consulting Actuaries, 41 Park Row, New York 7, N. Y.

FELLOWS

Admitted	
†	DEARTH, ELMER H., (Retired), 1409 Clark St., Des Moines, Iowa.
†	DEKAY, ECKFORD C., President, DeKay & Company, 84 William Street, New York 7, N. Y.
'Nov. 17, 1920	DORWEILER, PAUL, Actuary, Aetna Casualty & Surety Company, Hartford 15, Conn.
'Nov. 24, 1933	EDWARDS, JOHN, Casualty Actuary, Ontario Insurance Department, 91 Arundel Avenue, Toronto 6, Ontario, Canada.
'Nov. 15, 1940	ELLIOTT, GEORGE B., Compensation Actuary, Pennsylvania Insurance Department, 938 Public Ledger Bldg., Philadelphia 6, Pa.
'Nov. 17, 1922	ELSTON, JAMES S., Assistant Actuary, Life Department, The Travelers Insurance Co., 700 Main Street, Hartford 3, Conn.
'Nov. 15, 1935	EPPINK, WALTER T., Vice-President, Merchants Mutual Casualty Co., Casualty Insurance Building, Buffalo 5, New York.
†	FACKLER, EDWARD B., Consulting Actuary, Fackler & Company, 8 West 40th Street, New York 18, N. Y.
†	FALLOW, EVERETT S., Actuary, Accident Department, The Travelers Insurance Co., 700 Main Street, Hartford 3, Conn.
'Nov. 15, 1940	FARLEY, JARVIS, Actuary and Asst. Treasurer, Massachusetts Indemnity Co., 632 Beacon Street, Boston 15, Mass.
†	FARRER, HENRY, Insurance Company of North America, 99 John Street, New York 7, N. Y.
'Nov. 15, 1935	FITZHUGH, GILBERT W., Asst. General Manager, Canadian Office, Metropolitan Life Insurance Co., 180 Wellington Street, Ottawa, Ontario, Canada.
Feb. 19, 1915	FONDILLER, RICHARD, Woodward and Fondiller, Consulting Actuaries, 90 John Street, New York 7, N. Y.
'Nov. 22, 1934	FULLER, GARDNER V., Manager, Special Risk Department, Lumbermens Mutual Casualty Co., Mutual Insurance Bldg., Chicago 40, Ill.
†	FRANKLIN, CHARLES H., (Retired), 5157 Latimer Place, Seattle 5, Washington.
'Nov. 18, 1927	FREDRICKSON, CARL H., Actuary, Canadian Underwriters Association, 55 York Street, Toronto, Canada.
†	FURZE, HARRY, (Retired), 42 Douglas Road, Glen Ridge, N. J. (Deceased December 26, 1945)
Feb. 19, 1915	GARRISON, FRED S., Secretary, The Travelers Indemnity Co., 700 Main Street, Hartford 3, Conn.
'Nov. 20, 1924	GINSBURGH, HAROLD J., Vice-President, American Mutual Liability Insurance Co., 142 Berkeley Street, Boston 16, Mass.
'Nov. 21, 1930	GLENN, J. BRYAN, 5214 First Street, N. W., Washington 11, D. C.
'Nov. 13, 1931	GODDARD, RUSSELL P., Assistant Actuary, American Mutual Liability Insurance Company, 142 Berkeley Street, Boston 16, Mass.
†	GOODWIN, EDWARD S., 750 Main Street, Hartford 3, Conn.

FELLOWS

Admitted	
*Nov. 19, 1926	GRAHAM, CHARLES M., Chief Self-Insurance Examiner, Workmen's Compensation Board of N. Y., 80 Center Street, New York 13, N. Y.
Oct. 22, 1915	GRAHAM, THOMPSON B., Fourth Vice President, Metropolitan Life Insurance Co., 1 Madison Avenue, New York 10, N. Y.
†	GRAHAM, WILLIAM J., Vice-President, Equitable Life Assurance Society, 393 Seventh Avenue, New York 1, N. Y.
†	GREENE, WINFIELD W., Vice-President, General Reinsurance Corporation, 90 John Street, New York 7, N. Y.
†	HAMMOND, H. PIERSON, Actuary, Life Department, The Travelers Insurance Co., 700 Main St., Hartford 3, Conn.
Oct. 27, 1916	HARDY, EDWARD R., Secretary-Treasurer, Insurance Institute of America, Inc., 80 John Street, New York 7, N. Y.
Oct. 22, 1915	HATCH, LEONARD W., (Retired), 425 Pelham Manor Road, Pelham Manor, New York.
*Nov. 19, 1926	HAUGH, CHARLES J., Secretary, Compensation and Liability Dept., The Travelers Insurance Co., Hartford 3, Conn.
Oct. 22, 1915	HODGKINS, LEMUEL G., (Retired), 5 Whitman Road, Worcester 5, Mass.
†	HOFFMAN, FREDERICK L., Consulting Statistician, 1978 Sunset Boulevard, San Diego 3, California. (Deceased February 23, 1946.)
Oct. 22, 1915	HOLLAND, CHARLES H., Suite 2001, 165 Broadway, New York 6, N. Y.
*Nov. 22, 1934	HOOKER, RUSSELL O., Actuary, Connecticut Insurance Department, Hartford 2, Conn.
Nov. 18, 1932	HUEBNER, SOLOMON S., Professor of Insurance, University of Pennsylvania, Philadelphia 4, Pa.
†	HUGHES, CHARLES, (Retired), 285 Smith St., Freeport, N. Y.
Nov. 19, 1929	HULL, ROBERT S., Unemployment Compensation Division, Social Security Board, Washington, D. C.
†	HUNTER, ARTHUR, (Retired), 124 Lloyd Road, Montclair, N. J.
Feb. 25, 1916	JACKSON, CHARLES W., Consulting Actuary, Woodward and Fondiller, 90 John Street, New York 7, N. Y.
*Nov. 19, 1929	JACKSON, HENRY H., Vice President & Actuary, National Life Insurance Co., Montpelier, Vt.
*Nov. 14, 1941	JOHNSON, ROGER A., JR., Actuary, Compensation Insurance Rating Board, 125 Park Avenue, New York 17, N. Y.
*Nov. 16, 1939	JONES, HAROLD M., Liberty Mutual Insurance Company, 175 Berkeley Street, Boston 16, Mass.
*Nov. 17, 1938	KARDONSKY, ELSIE, 66 Corbin Place, Brooklyn, 29, N. Y.
Nov. 17, 1938	KELLY, GREGORY C., General Manager, Pennsylvania Compensation Rating & Inspection Bureau, 938 Public Ledger Bldg., Philadelphia 6, Pa.
*Nov. 19, 1926	KELTON, WILLIAM H., Assistant Actuary, Life Department, The Travelers Insurance Co., 700 Main Street, Hartford 3, Conn.
*Nov. 21, 1919	KIRKPATRICK, A. LOOMIS, Manager Insurance Department, Chamber of Commerce of the U. S. A., 1615 H Street, N.W., Washington 6, D. C.
*Nov. 14, 1941	KOLODITZKY, MORRIS, State Insurance Fund, 625 Madison Avenue, New York 22, N. Y.
*Nov. 24, 1933	KORMES, MARK, Consulting Actuary, 341 Madison Avenue, New York 17, N. Y.

FELLOWS

Admitted	
Nov. 23, 1928	KULP, CLARENCE A., Professor of Insurance, University of Pennsylvania, Logan Hall, 36th Street and Woodland Avenue, Philadelphia 4, Pa.
Nov. 13, 1931	LA MONT, STEWART M., (Retired), Hotel Claremont, Berkeley, Cal.
*Nov. 24, 1933	LANGE, JOHN R., Chief Actuary, Wisconsin Insurance Department, State House, Madison 3, Wis.
†	LEAL, JAMES R., Vice-President and Secretary, Interstate Life and Accident Co., Interstate Building, 540 McCallie Avenue, Chattanooga 3, Tenn.
†	LESLIE, WILLIAM, General Manager, National Bureau of Casualty & Surety Underwriters, 60 John Street, New York 7, N. Y.
*Nov. 20, 1924	LINDER, JOSEPH, Consulting Actuary, Wolfe, Corcoran & Linder, 116 John Street, New York 7, N. Y.
*Nov. 13, 1936	LYONS, DANIEL J., Associate Actuary, Guardian Life Insurance Co., 50 Union Square, New York 3, N. Y.
†	MAGOUN, WILLIAM N., (Retired), 33 Fearing Road, Hingham, Mass.
*Nov. 23, 1928	MARSHALL, RALPH M., Assistant Actuary, National Council on Compensation Insurance 45 East 17th Street, New York 3, N. Y.
*Nov. 18, 1927	MASTERSON, Norton E., Vice-President and Actuary, Hardware Mutual Casualty Co., Stevens Point, Wis.
*Nov. 19, 1926	MATTHEWS, ARTHUR N., Asst. Actuary, Casualty Department, The Travelers Insurance Co., 700 Main Street, Hartford 3, Conn.
May 19, 1915	MAYCRINK, EMMA C., Secretary-Treasurer, Association of New York State Mutual Casualty Companies, 60 East 42nd Street, New York 17, N. Y.
*Nov. 16, 1923	MCCLURG, D. RALPH, Secretary and Treasurer, National Equity Life Insurance Co., Little Rock, Ark.
*Nov. 15, 1935	MCCONNELL, MATTHEW H., JR., Underwriter, Employers Mutual Liability Ins. Co., 12 S. 12th Street, Philadelphia 7, Pa.
May 23, 1919	MCDOUGALD, ALFRED, Ellerslie, Beddington Gardens, Wallington Surrey, England.
*Oct. 31, 1917	MCMANUS, ROBERT J., Statistician, Casualty Department, The Travelers Insurance Co., 700 Main Street, Hartford 3, Conn.
†	MICHELBACHER, GUSTAV F., Vice-President and Secretary, Great American Indemnity Co., 1 Liberty Street, New York 5, N. Y.
*Nov. 17, 1938	MILLER, JOHN H., Vice President and Actuary, Monarch Life Insurance Company, Springfield, Mass.
†	MILLIGAN, SAMUEL, Vice-President, Metropolitan Life Insurance Co., 1 Madison Avenue, New York 10, N. Y.
*Nov. 18, 1937	MILLS, JOHN A., Vice President and Actuary, Lumbermens Mutual Casualty Co., and American Motorists Insurance Co., Mutual Insurance Bldg., Chicago 40, Ill.
*Nov. 18, 1921	MONTGOMERY, VICTOR, President, Pacific Employers Insurance Co., 1033 So. Hope Street, Los Angeles 15, Calif.
Nov. 19, 1926	MOONEY, WILLIAM L., (Retired), 4 Pleasant Street, West Hartford 8, Conn.

FELLOWS

Admitted	
†	MOORE, GEORGE D., Comptroller, Aero Insurance Underwriters, 111 John St., New York 7, N. Y.
†	MOWBRAY, ALBERT H., Consulting Actuary, 806 San Luis Road, Berkeley 7, Calif.
*Nov. 17, 1920	MUELLER, LOUIS H., President, Associated Insurance Fund, 332 Pine Street, San Francisco 4, Calif.
†	MULLANEY, FRANK R., Vice-President and Secretary, American Mutual Liability Insurance Co., and American Policyholders' Insurance Co., and Vice-President, Allied American Mutual Fire Insurance Co., 142 Berkeley Street, Boston 16, Mass.
May 28, 1920	MURPHY, RAY D., Vice-President and Actuary, Equitable Life Assurance Society, 393 Seventh Avenue, New York 1, N. Y.
*Nov. 15, 1935	OBERHAUS, THOMAS M., Associate Actuary, Woodward and Fondiller, Consulting Actuaries, 90 John Street, New York 7, N. Y.
†	OLIFIERS, EDWARD, Consulting Actuary, Caixa Postal 8 Pertopolis, Rio, Brazil.
†	ORR, ROBERT K., 226 S. Logan Street, Lansing, Mich.
*Nov. 21, 1919	OUTWATER, OLIVE E., Actuary, Benefit Association of Railway Employees, 901 Montrose Avenue, Chicago 13, Ill.
*Nov. 21, 1930	PERRYMAN, FRANCIS S., Vice President and Actuary, Eagle Indemnity Co., Globe Indemnity Co. and Royal Indemnity Co., 150 William Street, New York 8, N. Y.
*Nov. 14, 1941	PETERS, STEFAN, 3031 Hillegass Ave., Berkeley 5, Calif.
Nov. 19, 1926	PHILLIPS, JESSE S., Chairman of Board, Great American Indemnity Co., 1 Liberty Street, New York 5, N. Y.
*Nov. 24, 1933	PICKETT, SAMUEL C., Assistant Actuary, Connecticut Insurance Department, Hartford 2, Conn.
*Nov. 17, 1922	PINNEY, SYDNEY D., 290 Wolcott Hill Road, Wethersfield 9, Conn.
*Nov. 13, 1931	PRUITT, DUDLEY M., Actuary, General Accident Fire & Life Assurance Corp., Fourth & Walnut Sts., Philadelphia 5, Pa.
May 23, 1919	RICHARDSON, FREDERICK, Chairman of the Board, General Accident Fire and Life Assurance Corporation, Perth, Scotland.
*Nov. 19, 1926	RICHTER, OTTO C., Asst. Chief Statistician, American Telephone & Telegraph Co., 195 Broadway, New York 7, N. Y.
May 24, 1921	RIEGEL, ROBERT, Professor of Statistics and Insurance, University of Buffalo, Buffalo, New York.
*Nov. 16, 1939	ROBBINS, RAINARD B., Pension Counsellor, also Vice-President, Teachers Insurance and Annuity Association, 19 West 44th Street, New York 18, N. Y.

FELLOWS

Admitted	
*Nov. 16, 1923	ROEBER, WILLIAM F., General Manager, National Council on Compensation Insurance, 45 East 17th Street, New York 3, N. Y.
*Nov. 17, 1943	ROSS, SAMUEL M., Asst. Actuary, National Bureau of Casualty and Surety Underwriters, 60 John Street, New York 7, N. Y.
*Nov. 20, 1942	SATTERTHWAITE, FRANKLIN E., Group Division, Aetna Life Ins. Co., Hartford 15, Conn.
†	SCHEITLIN, EMIL, Treasurer, Globe Indemnity Co., 150 William Street, New York 7, N. Y. (Deceased May 2, 1946)
*Nov. 18, 1937	SHAPIRO, GEORGE I., First Vice President and General Manager, Public Service Mutual Insurance Co., 342 Madison Avenue, New York 17, N. Y.
*Nov. 13, 1931	SILVERMAN, DAVID, c/o Wolfe, Corcoran & Linder, 116 John Street, New York 7, N. Y.
*Nov. 24, 1933	SINNOTT, ROBERT V., Assistant Secretary, Hartford Accident and Indemnity Company, 690 Asylum Ave, Hartford 15, Conn.
*Nov. 19, 1929	SKELDING, ALBERT Z., Actuary, National Council on Compensation Insurance, 45 East 17th Street, New York 3, N. Y.
*Nov. 19, 1929	SKILLINGS, E. SHAW, Actuary, Allstate Insurance Co., 20 North Wacker Drive, Chicago 6, Ill.
*Nov. 18, 1932	SMICK, JACK J., Associate Actuary, Woodward and Fondiller, Consulting Actuaries, 90 John Street, New York 7, N. Y.
*Nov. 15, 1940	SMITH, SEYMOUR E., Casualty Department, The Travelers Insurance Co., Hartford 3, Conn.
*Nov. 24, 1933	ST. JOHN, JOHN B., Associate Actuary, Towers, Perrin, Foster & Crosby, Inc., 12 South 12th Street, Philadelphia 7, Pa.
Nov. 18, 1927	STONE, EDWARD C., U. S. General Manager and Attorney, Employers' Liability Assurance Corporation, Limited, and President, American Employers' Insurance Company, 110 Milk Street, Boston 9, Mass.
Oct. 22, 1915	STRONG, WILLIAM RICHARD, No. 4 "Sheringham," Cotham Road, Kew, Victoria, Australia.
*Nov. 17, 1920	TARBELL, THOMAS F., Actuary, Casualty Department, The Travelers Insurance Co., 700 Main Street, Hartford 3, Conn.
†	THOMPSON, JOHN S., President, Mutual Benefit Life Insurance Co., 300 Broadway, Newark 4, N. J.
†	TRAIN, JOHN L., President and General Manager, Utica Mutual Insurance Co., 185 Genesee Street, Utica 1, New York.
Nov. 17, 1922	TRAVERSI, ANTONIO T., Consulting Actuary and Accountant, Bank of Adelaide Chambers, Margaret St., Sydney, Australia.
*Nov. 23, 1928	VALERIUS, NELS M., Aetna Casualty and Surety Co., Hartford 15, Conn.
*Nov. 21, 1919	VAN TUYL, HIRAM O., Supt., Accounts Department, London Guarantee & Accident Co., 55 Fifth Avenue, New York, 3 N.Y.
*Nov. 17, 1920	WAITE, ALAN W., Secretary, Aetna Casualty and Surety Co., Hartford 15, Conn.
*Nov. 15, 1935	WAITE, HARRY V., Actuary, The Travelers Fire Insurance Co., 700 Main Street, Hartford 3, Conn.

FELLOWS

Admitted	
*Nov. 18, 1925	WARREN, LLOYD A. H., Professor of Actuarial Science, University of Manitoba, 64 Niagara Street, Winnipeg, Manitoba, Canada.
*Nov. 15, 1935	WILLIAMS, HARRY V., Assistant Secretary, Hartford Accident and Indemnity Co., Hartford 15, Conn.
Nov. 14, 1941	WILLIAMSON, WILLIAM R., Actuarial Consultant, Social Security Board, Washington 25, D. C.
*Nov. 13, 1931	WITTICK, HERBERT E., Secretary, Pilot Insurance Co., 199 Bay Street, Toronto, Canada.
†	WOLFE, LEE J., Consulting Actuary, Wolfe, Corcoran & Linder, 116 John Street, New York 7, N. Y.
May 24, 1921	WOOD, ARTHUR B., President and Managing Director, Sun Life Assurance Company of Canada, Montreal, Canada.

ASSOCIATES

Those marked (*) have been enrolled as Associates upon examination by the Society.

Numerals indicate Associateship Part V and Fellowship examination parts credited.

Admitted	
May 23, 1924	ACKER, MILTON, Manager, Compensation and Liability Department, National Bureau of Casualty and Surety Underwriters, 60 John Street, New York 7, N. Y.
*Nov. 15, 1918	ACKERMAN, SAUL B., Professor of Insurance, New York University, 90 Trinity Place, New York 6, N. Y.
*Nov. 16, 1939	AIN, SAMUEL N., Office of George B. Buck, Consulting Actuary, 150 Nassau Street, New York 7, N. Y.
Apr. 5, 1928	ALLEN, AUSTIN F., President and General Manager, Texas Employers Insurance Association and Employers Casualty Co., Dallas 1, Texas.
Nov. 15, 1918	ANKERS, ROBERT E., Secretary and Treasurer, Continental Life Insurance Co., Investment Building, Washington 5, D. C.
*Nov. 21, 1930	ARCHIBALD, A. EDWARD, Vice President and Actuary, Volunteer State Life Insurance Company, Chattanooga 1, Tenn. (V, I.)
*Nov. 16, 1939	BAILEY, ARTHUR L., Statistician, American Mutual Alliance, 60 E. 42nd Street, New York 17, N. Y.
*Nov. 24, 1933	BARRON, JAMES C., Asst. Treasurer, General Reinsurance Corporation, 90 John Street, New York 7, N. Y. (V, I, III.)
*Nov. 23, 1928	BATEMAN, ARTHUR E., Liberty Mutual Insurance Company, 175 Berkeley Street, Boston 16, Mass. (V, I.)
*Nov. 15, 1940	BATHO, BRUCE, Actuary, Country Life Insurance Company, 608 So. Dearborn St., Chicago 5, Ill.
*Nov. 18, 1925	BITTEL, W. HAROLD, Actuary, Department of Banking and Insurance, Trenton 7, New Jersey.
Nov. 17, 1920	BLACK, NELLAS C., Manager, Statistical Department, Maryland Casualty Co., Baltimore 3, Md.
*Nov. 15, 1940	BLACKHALL, JOHN M., Monarch Life Insurance Co., Springfield, Mass.
*Nov. 22, 1934	BOMSE, EDWARD L., Supt. New York Met. Special Risks, Royal Indemnity Co., 150 William Street, New York 8, N. Y.
*Nov. 23, 1928	BOWER, PERRY S., Great West Life Assurance Company, Winnipeg, Manitoba, Canada.
*Nov. 15, 1918	BRUNNQUELL, HELMUTH G., Assistant Actuary, The Northwestern Mutual Life Insurance Co., Milwaukee 2, Wis.
*Oct. 22, 1915	BUFFLER, LOUIS, Director, Underwriting Department, State Insurance Fund, 625 Madison Avenue, New York 22, N. Y.
*Nov. 20, 1924	BUGBEE, JAMES M., Manager, Automobile Department, Maryland Casualty Co., Baltimore 3, Md.
Mar. 31, 1920	BURT, MARGARET A., Office of George B. Buck, Consulting Actuary, 150 Nassau Street, New York 7, N. Y.
Nov. 17, 1922	CAVANAUGH, LEO D., President, Federal Life Insurance Co., 168 N. Michigan Avenue, Chicago 1, Ill.

ASSOCIATES

Admitted	
*Nov. 18, 1927	CHEN, S. T., Actuary, China United Assurance Society, 10 Bubbling Well Road, Shanghai, China.
*Nov. 18, 1927	CONROD, STUART F., Actuary, Loyal Protective Life Insurance Co., 19 Deerfield Street, Boston 15, Mass.
*Nov. 24, 1933	CRAWFORD, WILLIAM H., Secretary, Fireman's Insurance Co. of Newark, N. J. & Affiliated Fire & Casualty Co's Pacific Dept., 220 Bush Street, San Francisco 4, Cal. (V, I.)
*Nov. 18, 1932	CRIMMINS, JOSEPH B., Metropolitan Life Insurance Co., 1 Madison Avenue, New York 10, N. Y. (V, I.)
*Nov. 1, 1944	CROUSE, CHARLES W., Actuary, Manufacturers Casualty Insurance Co., 16th Street and Pennsylvania Boulevard, Philadelphia 3, Pa.
*Nov. 18, 1925	DAVIS, MALVIN E., Actuary, Metropolitan Life Insurance Co., 1 Madison Avenue, New York 10, N. Y.
*Nov. 24, 1933	DAVIS, REGINALD S., Assistant Comptroller, State Compensation Insurance Fund, San Francisco 2, Calif. (V, I.)
*Nov. 14, 1941	DOWLING, WILLIAM F., Asst. Treasurer, Lumber Mutual Casualty Co., 41 E. 42nd Street, New York 17, N. Y.
May 25, 1923	ECONOMIDY, HARILAUS E., 2402 Boulevard, Galveston, Texas.
June 5, 1925	EGER, FRANK A., Secretary, Indemnity Insurance Co. of North America, 1600 Arch Street, Philadelphia 3, Pa.
*Nov. 16, 1923	FITZ, L. LEROY, Group Department, John Hancock Mutual Life Insurance Company, Boston 17, Mass. (V, I.)
*Nov. 18, 1927	FITZGERALD, AMOS H., Assistant Actuary, The Prudential Insurance Company of America, Newark 1, N. J. (V, I.)
*Nov. 16, 1923	FLEMING, FRANK A., Actuary, American Mutual Alliance, 60 East 42nd Street, New York 17, N. Y.
Nov. 20, 1924	FROBERG, JOHN, Manager, California Inspection Rating Bureau, 500 Sansome Street, San Francisco 11, Calif.
*Nov. 13, 1936	FRUECHTEMEYER, FRED J., Liberty Mutual Insurance Co., 175 Berkeley Street, Boston 16, Mass. (V, I.)
*Nov. 19, 1929	FURNIVALL, MAURICE L., Assistant Actuary, Accident Department, The Travelers Insurance Co., 700 Main Street, Hartford 3, Conn. (V, I.)
*Nov. 18, 1932	GETMAN, RICHARD A., Life Actuarial Department, The Travelers Insurance Co., 700 Main Street, Hartford 3, Conn. (V, I.)
*Nov. 17, 1922	GIBSON, JOSEPH P., JR., Vice President, and General Manager, Excess Insurance Company of America, 99 John Street, New York 7, N. Y.
*Nov. 16, 1923	GILDEA, JAMES F., The Travelers Insurance Co., 700 Main Street, Hartford 3, Conn.
Nov. 19, 1929	GORDON, HAROLD R., Managing Director, Health & Accident Underwriters Conference, 176 West Adams Street, Chicago 3, Ill.
*Nov. 18, 1927	GREEN, WALTER C., Consulting Actuary, 211 West Wacker Drive, Chicago 6, Ill.
*Nov. 15, 1940	GROSSMAN, ELI A., Actuarial Department, United States Life Insurance Co., 84 William St., New York 7, N. Y.
*Nov. 15, 1935	GUERTIN, ALFRED N., Actuary, American Life Convention, 230 N. Michigan Ave., Chicago 1, Ill. (V, I.)
*Nov. 16, 1939	HAGEN, OLAF E., Metropolitan Life Insurance Company, 1 Madison Avenue, New York 10, N. Y.

ASSOCIATES

Admitted	
*Nov. 18, 1921	HAGGARD, ROBERT E., Superintendent, Permanent Disability Rating Department, Industrial Accident Commission, State Building, San Francisco, Calif.
*Nov. 17 1922	HALL, HARTWELL L., Associate Actuary, Connecticut Insurance Department, Hartford 2, Conn.
*Nov. 13, 1936	HAM, HUGH P., Automobile Manager & Asst. Secretary, British America Assurance Co., 22 Wellington St. E., Toronto Ontario, Canada. (V, I.)
Mar. 24, 1932	HARRIS, SCOTT, Vice-President, Joseph Froggatt & Co., 74 Trinity Place, New York 6, N. Y.
*Mar. 25, 1924	HART, WARD VAN BUREN, Assistant Actuary, Connecticut General Life Insurance Co., Hartford 2, Conn. (V, I.)
Nov. 21, 1919	HAYDON, GEORGE F., General Manager, Wisconsin Compensation Rating & Inspection Bureau, 715 N. Van Buren Street, Milwaukee 2, Wis.
Nov. 17, 1927	HIPP, GRADY H., Executive Vice-President, Liberty Life Insurance Co., Greenville, S. C.
*Nov. 16, 1945	HOLZINGER, ERNEST, Pension Planning Company, 527 Fifth Avenue, New York 17, N. Y.
Nov. 19, 1929	JACOBS, CARL N., President, Hardware Mutual Casualty Co., Stevens Point, Wis.
*Nov. 18, 1921	JENSEN, EDWARD S., Supt., Group Department, Occidental Life Insurance Co., Los Angeles 55, Calif. (II, III.)
Nov. 21, 1930	JONES, H. LLOYD, Deputy General Attorney, of Phoenix-London Group, Vice-President, Phoenix Indemnity Company, and Deputy United States Manager, London Accident & Guarantee Co., 55 Fifth Avenue, New York 3, N. Y.
*Nov. 21, 1919	JONES, LORING D., (Retired) 64 Raymond Ave., Rockville Centre, Long Island, N. Y.
*Nov. 15, 1940	KELLY, ROBERT G., 2127 California St., Washington 8, D. C.
*Nov. 17, 1922	KIRK, CARL L., Assistant U. S. Manager, Zurich General Accident & Liability Insurance Co., 135 South LaSalle Street, Chicago 3, Ill.
*Nov. 15, 1935	KITZROW, ERWIN W., Vice-President, Hardware Mutual Casualty Co., Stevens Point, Wis. (V, I.)
*Nov. 16, 1939	KNOWLES, FREDERICK, 5724 Mountain Sights Ave., N. D. G., Montreal, Canada.
*Nov. 18, 1937	LASSOW, WILLIAM, 185 206th St., Bronx 58, New York. (V.)
*Nov. 17, 1938	LIEBLEIN, JULIUS, 2710-29th, Street, S. E., Washington 20, D. C.
*Nov. 13, 1931	MACKEEN, HAROLD E., The Travelers Insurance Co., 700 Main Street, Hartford 3, Conn. (V, I.)
Mar. 24, 1932	MAGRATH, JOSEPH J., Executive Assistant, Chubb & Sons, 90 John Street, New York 7, N. Y.
*Nov. 18, 1925	MALMUTH, JACOB, Examiner, New York Insurance Department, 61 Broadway, New York 6, N. Y.
Mar. 24, 1927	MARSH, CHARLES V. R., Comptroller and Assistant Treasurer, Fidelity & Deposit Co. and American Bonding Co., Baltimore 3, Md.
*Nov. 13, 1936	MAYER, WILLIAM H., JR., Actuarial Department, Metropolitan Life Insurance Co., 1 Madison Avenue, New York 10, N. Y.
*Nov. 17, 1922	MCIVER, ROSSWELL A., Actuary, Washington National Insurance Co., 610 Church Street, Evanston, Ill.

ASSOCIATES

Admitted	
*Nov. 17, 1922	MICHENER, SAMUEL M., Actuary, Columbus Mutual Life Insurance Co., 303 East Broad Street, Columbus 15, Ohio. (V, I.)
*Nov. 13, 1931	MILLER, HENRY C., Comptroller and Actuary, State Compensation Insurance Fund, 450 McAllister Street, San Francisco 2, Calif. (V, I.)
*Nov. 19, 1926	MILNE, JOHN L., Actuary, Presbyterian Ministers' Fund for Life Insurance, 1805 Walnut Street, Philadelphia 3, Pa.
*Nov. 18, 1937	MINOR, EDUARD H., Accident and Health Department, Metropolitan Life Insurance Company, 1 Madison Avenue, New York 10, N. Y.
Nov. 17, 1922	MONTGOMERY, JOHN C., Secretary and Assistant Treasurer, Bankers Indemnity Insurance Co., 15 Washington Street, Newark 2, N. J.
May 25, 1923	MOORE, JOSEPH P., President, North American Accident Insurance Co., 455 Craig Street, W., Montreal, Canada.
*Nov. 21, 1919	MOTHERSILL, ROLLAND V., President, Anchor Casualty Co., Anchor Insurance Building, 2700 University Avenue, St. Paul 4, Minn. (II, III.)
*Nov. 18, 1937	MYERS, ROBERT J., Actuarial Mathematician, Social Security Board, Washington 25, D. C.
*Nov. 19, 1929	MULLER, FRITZ, Director, Agrippina Life Insurance Stock Co., Berlin, W. 30 Mackensenstr. 16, Germany.
*Nov. 15, 1935	NELSON, S. TYLER, Actuary, Utica Mutual Insurance Co., 185 Genesee Street, Utica 1, New York.
*Oct. 27, 1916	NEWELL, WILLIAM, Secretary, Assigned Risk Pool, 60 John Street, New York 7, N. Y. (V, I.)
*Nov. 18, 1925	NICHOLSON, EARL H., Actuary, Joseph Froggatt & Co., 74 Trinity Place, New York 6, N. Y.
May 23, 1919	OTTO, WALTER E., President, Michigan Mutual Liability Co., 163 Madison Avenue, Detroit 26, Mich.
*Nov. 19, 1926	OVERHOLSER, DONALD M., Office of George B. Buck, Consulting Actuary, 150 Nassau Street, New York 7, N. Y.
Nov. 20, 1924	PENNOCK, RICHARD M., Actuary, Pennsylvania Manufacturers' Association Casualty Insurance Co., Finance Building, Philadelphia 2, Pa.
Nov. 19, 1929	PHILLIPS, JOHN H., Vice-President and Actuary, Employers' Mutual Liability Insurance Co., Wausau, Wis.
*Nov. 17, 1920	PIKE, MORRIS, Associate Actuary, John Hancock Mutual Life Insurance Co., Boston 17, Mass.
*Nov. 23, 1928	PIPER, KENNETH B., Vice-President and Actuary, Provident Life and Accident Insurance Co., Chattanooga 2, Tenn (V, I.)
*Nov. 18, 1927	POISSANT, WILLIAM A., The Travelers Insurance Co., 700 Main Street, Hartford 3, Conn.
*Nov. 17, 1922	POORMAN, WILLIAM F., Vice-President and Actuary, Central Life Assurance Society, Fifth and Grand Avenues, Des Moines 6, Iowa. (V, I.)
*Nov. 13, 1936	POTOFSKY, SYLVIA, State Insurance Fund, 625 Madison Avenue New York 22, N. Y. (V.)

ASSOCIATES

Admitted	
Nov. 17, 1922	POWELL, JOHN M., President, Loyal Protective Life Insurance Co., 19 Deerfield Street, Boston 15, Mass. (V, I.)
*Nov. 15, 1918	RAYWID, JOSEPH, President, Joseph Raywid & Co., Inc., 92 William Street, New York 7, N. Y.
Nov. 19, 1932	RICHARDSON, HARRY F., Secretary-Treasurer, National Council on Compensation Insurance, 45 East 17th Street, New York 3, N. Y.
*Nov. 18, 1932	ROBERTS, JAMES A., Life Actuarial Department, The Travelers Insurance Co., 700 Main Street, Hartford 3, Conn. (V, I.)
*Nov. 15, 1940	ROSENBERG, NORMAN, Actuary, Public Service Mutual Insurance Co., 342 Madison Avenue, New York 17, N. Y. (I.)
*Nov. 18, 1927	SARASON, HARRY M., Associate Actuary, General American Life Insurance Co., 1501 Locust Street, St. Louis 3, Mo.
Nov. 16, 1923	SAWYER, ARTHUR, Globe Indemnity Co., 150 William Street, New York 7, N. Y.
*Nov. 20, 1930	SEVILLA, EXEQUIEL S., Actuary, National Life Insurance Co., P. O. Box 2856, Manila, Philippine Islands.
*Nov. 20, 1924	SHEPPARD, NORRIS E., Professor of Mathematics, University of Toronto, Toronto 5, Canada. (V, I.)
Nov. 15, 1918	SIBLEY, JOHN L., Assistant Secretary, United States Casualty Co., 60 John Street, New York 7, N. Y.
*Nov. 18, 1921	SMITH, ARTHUR G., Assistant General Manager, Compensation Insurance Rating Board, Pershing Square Bldg., 125 Park Avenue, New York 17, N. Y.
*Nov. 19, 1926	SOMERVILLE, WILLIAM F., Secretary, St. Paul Mercury Indemnity Co., St. Paul 2, Minn. (V, I.)
*Nov. 18, 1925	SOMMER, ARMAND, Supt. of Agencies, Continental Casualty Co., 910 So. Michigan Avenue, Chicago 5, Ill.
*Nov. 15, 1918	SPENCER, HAROLD S., Statistician, Aetna Casualty and Surety Co., Hartford 15, Conn.
Nov. 20, 1924	STELLWAGEN, HERBERT P., Executive Vice-President, Indemnity Insurance Company of North America, 1600 Arch Street, Philadelphia 3, Pa.
*Nov. 16, 1923	STOKE, KENDRICK, Actuary, Michigan Mutual Liability Company, 163 Madison Avenue, Detroit 26, Mich.
*Nov. 21, 1930	SULLIVAN, WALTER F., Asst. Actuary, State Compensation Insurance Fund, 450 McAllister Street, San Francisco 2, Calif. (V, I.)
*Nov. 21, 1919	TRENCH, FREDERICK H., Manager, Underwriting Department, Utica Mutual Insurance Co., 185 Genesee Street, Utica 1, N. Y. (V, I.)
*Nov. 20, 1924	UHL, M. ELIZABETH, National Bureau of Casualty & Surety Underwriters, 60 John Street, New York 7, N. Y. (V, I.)
*Nov. 1, 1944	UHTHOFF, DUNBAR R., National Council on Compensation Insurance, 45 East 17th Street, New York 3, N. Y.
May 23, 1919	WARREN, CHARLES S., Secretary, Massachusetts Automobile Rating and Accident Prevention Bureau, 89 Broad Street, Boston 10, Mass.
Nov. 18, 1925	WASHBURN, JAMES H., Actuary, 1501 Gale Lane, Nashville 4, Tenn.

ASSOCIATES

Admitted	
*Nov. 18, 1932	WEINSTEIN, MAX S., Actuary, New York State Employees' Retirement System, 256 Washington Ave., Albany 1, N. Y.
*Nov. 18, 1925	WELLMAN, ALEXANDER C., Vice-President, Protective Life Insurance Co., Birmingham, Ala.
*Nov. 21, 1930	WELLS, WALTER I., Asst. Actuary, State Mutual Life Assurance Co., Worcester 8, Mass. (V, I.)
Mar. 21, 1929	WHEELER, CHARLES A., Chief Examiner of Casualty Companies, New York Insurance Department, 61 Broadway, New York 6, N. Y.
*Nov. 18, 1927	WHITBREAD, FRANK G., Associate Actuary, Great West Life Assurance Co., Winnipeg, Manitoba, Canada.
*Nov. 16, 1939	WITTLAKE, J. CLARKE, Assistant Actuary, Business Men's Assurance Company, Kansas City 10, Mo.
*Oct. 22, 1915	WOOD, DONALD M., Childs & Wood, General Agents, Royal Indemnity Company, 175 W. Jackson Blvd., Chicago 4, Ill.
*Nov. 18, 1937	WOOD, DONALD M., JR., Childs & Wood, 175 West Jackson Blvd., Chicago 4, Ill.
*Nov. 18, 1927	WOOD, MILTON J., Assistant Actuary, Life Actuarial Department, The Travelers Insurance Co., 700 Main Street, Hartford 3, Conn.
*Oct. 22, 1915	WOODMAN, CHARLES E., (Retired), 75 Norman Place, Tenafly, N. J.
*Nov. 22, 1934	WOODWARD, BARBARA H., Hughes, Hubbard & Ewing, 1 Wall Street, New York 5, N. Y.
*Nov. 18, 1925	WOOLERY, JAMES M., Actuary, Union Labor Life Insurance Co., 570 Lexington Ave., New York 22, N. Y.

SCHEDULE OF MEMBERSHIP, NOVEMBER 16, 1945

	Fellows	Associates	Total
Membership, November 1, 1944	155	123	278
Additions:			
By election	..		..
By reinstatement	..		..
By examination	..	1	1
	155	124	279
Deductions:			
By death	1	1	2
By withdrawal	..	..	..
By transfer from Associate to Fellow	..	..	..
Membership, November 16, 1945	154	123	277

OFFICERS OF THE SOCIETY

Since Date of Organization

Elected	*President*	*Vice-Presidents*	
1914–1915	*I. M. Rubinow	A. H. Mowbray	*B. D. Flynn
1916–1917	*J. D. Craig	*J. H. Woodward	*H. E. Ryan
1918	*J. H. Woodward	*B. D. Flynn	G. D. Moore
1919	*B. D. Flynn	G. D. Moore	W. Leslie
1920	A. H. Mowbray	W. Leslie	*L. S. Senior
1921	A. H. Mowbray	*L. S. Senior	*H. E. Ryan
1922	*H. E. Ryan	G. F. Michelbacher	E. E. Cammack
1923	W. Leslie	G. F. Michelbacher	E. E. Cammack
1924–1925	G. F. Michelbacher	S. B. Perkins	R. H. Blanchard
1926–1927	S. B. Perkins	G. D. Moore	T. F. Tarbell
1928–1929	G. D. Moore	S. D. Pinney	P. Dorweiler
1930–1931	T. F. Tarbell	*R. A. Wheeler	W. W. Greene
1932–1933	P. Dorweiler	W. F. Roeber	*L. S. Senior
1934–1935	W. W. Greene	R. H. Blanchard	C. J. Haugh
1936–1937	*L. S. Senior	S. D. Pinney	F. S. Perryman
1938–1939	F. S. Perryman	H. T. Barber	W. J. Constable
1940	S. D. Pinney	H. J. Ginsburgh	J. M. Cahill
1941	R. H. Blanchard	H. J. Ginsburgh	J. M. Cahill
1942	R. H. Blanchard	Albert Z. Skelding	Charles J. Haugh
1943-1944	H. J. Ginsburgh	Albert Z. Skelding	Charles J. Haugh
1945	C. J. Haugh	J. M. Cahill	H. V. Williams

Secretary-Treasurer

1914–1917.....*C. E. Scattergood
1918–1945.....R. Fondiller

Editor†

1914.....W. W. Greene
1915–1917.....R. Fondiller
1918..... W. W. Greene
1919–1921....G. F. Michelbacher
1922–1923.....O. E. Outwater
1924–1932.....R. J. McManus
1933–1943.....*C. W. Hobbs
1944–1945.....E. C. Maycrink

Librarian†

1914.....W. W. Greene
1915.....R. Fondiller
1916–1921.....L. I. Dublin
1922–1924.....E. R. Hardy
1925–1937.....W. Breiby
1937–1945.....T. O. Carlson

*Deceased.
†The offices of Editor and Librarian were not separated until 1916.

FELLOWS WHO HAVE DIED

The (†) denotes original membership at date of organization, November 7, 1914

Admitted		*Died*
May 24, 1921	Edward J. Bond	Nov. 12, 1941
May 19, 1915	Thomas Bradshaw	Nov. 10, 1939
June 5, 1925	William Brosmith	Aug. 22, 1937
†	William A. Budlong	June 4, 1934
Nov. 18, 1932	Charles H. Burhans	June 15, 1942
Feb. 19, 1915	F. Highlands Burns	Mar. 30, 1935
Feb. 19, 1915	Gordon Case	Feb. 4, 1920
†	Charles T. Conway	July 23, 1921
†	Walter G. Cowles	May 30, 1942
†	James D. Craig	May 27, 1940
†	James McIntosh Craig	Jan. 20, 1922
May 26, 1916	Frederick S. Crum	Sept. 2, 1921
†	Alfred Burnett Dawson	June 21, 1931
†	Miles Menander Dawson	Mar. 27, 1942
May 19, 1915	Samuel Deutschberger	Jan. 18, 1929
†	Ezekiel Hinton Downey	July 9, 1922
May 19, 1915	Earl O. Dunlap	July 5, 1944
†	David Parks Fackler	Oct. 30, 1924
Feb. 19, 1915	Claude W. Fellows	July 15, 1938
†	Benedict D. Flynn	Aug. 22, 1944
†	Charles S. Forbes	Oct. 2, 1943
May 26, 1916	Lee K. Frankel	July 25, 1931
Feb. 25, 1916	Joseph Froggatt	Sept. 28, 1940
†	Theodore E. Gaty	Aug. 22, 1925
May 19, 1915	James W. Glover	July 15, 1941
Oct. 22, 1915	George Graham	Apr. 15, 1937
May 25, 1923	William A. Granville	Feb. 4, 1943
†	William H. Gould	Oct. 28, 1936
†	Robert Cowen Lees Hamilton	Nov. 15, 1941
Nov. 21, 1919	Robert Henderson	Feb. 16, 1942
†	Robert J. Hillas	May 17, 1940
Nov. 15, 1918	Frank Webster Hinsdale	Mar. 18, 1932
May 23, 1924	Clarence W. Hobbs	July 21, 1944
Nov. 19, 1926	Charles E. Hodges	Jan. 22, 1937
Nov. 21, 1919	Carl Hookstadt	Mar. 10, 1924
†	Burritt A. Hunt	Sept. 3, 1943
Nov. 28, 1921	William Anderson Hutcheson	Nov. 19, 1942
May 19, 1915	William C. Johnson	Oct. 7, 1943
Nov. 23, 1928	F. Robertson Jones	Dec. 26, 1941
Nov. 18, 1921	Thomas P. Kearney	Feb. 11, 1928
Oct. 22, 1915	Virgil Morrison Kime	Oct. 15, 1918
†	Edwin W. Kopf	Aug. 3, 1933
Feb. 17, 1915	John M. Laird	June 20, 1942
Feb. 19, 1915	Abb Landis	Dec. 9, 1937
Nov. 17, 1922	Arnette Roy Lawrence	Dec. 1, 1942
Nov. 18, 1921	James Fulton Little	Aug. 11, 1938
Nov. 23, 1928	Edward C. Lunt	Jan. 13, 1941
Feb. 19, 1915	Harry Lubin	Dec. 20, 1920
Feb. 15, 1915	Franklin B. Mead	Nov. 29, 1933

FELLOWS WHO HAVE DIED—Continued

Admitted		*Died*
Apr. 20, 1917	Marcus Meltzer	Mar. 27, 1931
†	David W. Miller	Jan. 18, 1936
†	James F. Mitchell	Feb. 9, 1941
†	Henry Moir	June 8, 1937
Feb. 19, 1915	William J. Montgomery	Aug. 20, 1915
May 19, 1915	Edward Bontecou Morris	Dec. 19, 1929
†	Lewis A. Nicholas	Apr. 21, 1940
†	Stanley L. Otis	Oct. 12, 1937
Nov. 13, 1926	Bertrand A. Page	July 30, 1941
Nov. 18, 1921	Sanford B. Perkins	Sept. 16, 1945
Nov. 15, 1918	William Thomas Perry	Oct. 25, 1940
†	Edward B. Phelps	July 24, 1915
†	Charles Grant Reiter	July 30, 1937
†	Charles H. Remington	Mar. 21, 1938
†	Isaac M. Rubinow	Sept. 1, 1936
†	Harwood Eldridge Ryan	Nov. 2, 1930
†	Arthur F. Saxton	Feb. 26, 1927
†	Leon S. Senior	Feb. 3, 1940
Apr. 20, 1917	Charles Gordon Smith	June 22, 1938
Feb. 19, 1915	John T. Stone	May 9, 1920
Feb. 25, 1916	Wendell Menville Strong	Mar. 30, 1942
†	Robert J. Sullivan	July 19, 1934
Nov. 22, 1934	Walter H. Thompson	May 25, 1935
Nov. 18, 1921	Guido Toja	Feb. 28, 1933
May 23, 1919	Archibald A. Welch	May 8, 1935
Nov. 19, 1926	Roy A. Wheeler	Aug. 26, 1932
†	Albert W. Whitney	July 27, 1943
†	S. Herbert Wolfe	Dec. 31, 1927
†	Joseph H. Woodward	May 15, 1928
†	William Young	Oct. 23, 1927

ASSOCIATES WHO HAVE DIED

Admitted		*Died*
Oct. 22, 1915	Don A. Baxter	Feb. 10, 1920
Nov. 22, 1934	John J. Gately	Nov. 3, 1943
Nov. 20, 1924	Leslie LeVant Hall	Mar. 8, 1931
Oct. 31, 1917	Edward T. Jackson	May 8, 1939
Nov. 23, 1928	Karl Newhall	Oct. 24, 1944
Nov. 18, 1927	Alexander A. Speers	June 25, 1941
Mar. 23, 1921	Arthur E. Thompson	Jan. 17, 1944
Nov. 21, 1919	Walter G. Voogt	May 8, 1937
Nov. 17, 1920	James J. Watson	Feb. 23, 1937
Nov. 18, 1921	Eugene R. Welch	Jan. 17, 1945
Nov. 15, 1918	Albert Edward Wilkinson	June 11, 1930

CONSTITUTION

(As Amended November 15, 1940)

Article I.—*Name.*

This organization shall be called the Casualty Actuarial Society.

Article II.—*Object.*

The object of the Society shall be the promotion of actuarial and statistical science as applied to the problems of casualty and social insurance by means of personal intercourse, the presentation and discussion of appropriate papers, the collection of a library and such other means as may be found desirable.

The Society shall take no partisan attitude, by resolution or otherwise, upon any question relating to casualty or social insurance.

Article III.—*Membership.*

The membership of the Society shall be composed of two classes, Fellows and Associates. Fellows only shall be eligible to office or have the right to vote.

The Fellows of the Society shall be the present Fellows and those who may be duly admitted to Fellowship as hereinafter provided. The Associates shall be the present Associates and those who may be duly admitted to Associateship as hereinafter provided.

Any person may, upon nomination to the Council by two Fellows of the Society and approval by the Council of such nomination with not more than one negative vote, become enrolled as an Associate of the Society, provided that he shall pass such examination as the Council may prescribe. Such examination may be waived in the case of a candidate who for a period of not less than two years has been in responsible charge of the Statistical or Actuarial Department of a casualty insurance organization or has had such other practical experience in casualty or social insurance as, in the opinion of the Council, renders him qualified for Associateship.

Any person who shall have qualified for Associateship may become a Fellow on passing such final examination as the Council may prescribe. Otherwise, no one shall be admitted as a Fellow unless recommended by a duly called meeting of the Council, with not more than three negative votes, followed by a three-fourths ballot of the Fellows present and voting at a meeting of the Society.

Article IV.—*Officers and Council.*

The officers of the Society shall be a President, two Vice-Presidents, a Secretary-Treasurer, an Editor, and a Librarian. The Council shall be composed of the active officers, nine other Fellows and, during the four years following the expiration of their terms of office, the ex-Presidents and ex-Vice-Presidents. The Council shall fill vacancies occasioned by death or resignation of any officer or other member of the Council, such appointees to serve until the next annual meeting of the Society.

CONSTITUTION

ARTICLE V.—*Election of Officers and Council.*

The President, Vice-Presidents, and the Secretary-Treasurer shall be elected by a majority ballot at the annual meeting for the term of one year and three members of the Council shall, in a similar manner, be annually elected to serve for three years. The President and Vice-Presidents shall not be eligible for the same office for more than two consecutive years nor shall any retiring member of the Council be eligible for re-election at the same meeting.

The Editor and the Librarian shall be elected annually by the Council at the Council meeting preceding the annual meeting of the Society. They shall be subject to confirmation by majority ballot of the Society at the annual meeting.

The terms of the officers shall begin at the close of the meeting at which they are elected except that the retiring Editor shall retain the powers and duties of office so long as may be necessary to complete the then current issue of *Proceedings*.

ARTICLE VI.—*Duties of Officers and Council.*

The duties of the officers shall be such as usually appertain to their respective offices or may be specified in the by-laws. The duties of the Council shall be to pass upon candidates for membership, to decide upon papers offered for reading at the meetings, to supervise the examination of candidates and prescribe fees therefor, to call meetings, and, in general, through the appointment of committees and otherwise, to manage the affairs of the Society.

ARTICLE VII.—*Meetings.*

There shall be an annual meeting of the Society on such date in the month of November as may be fixed by the Council in each year, but other meetings may be called by the Council from time to time and shall be called by the President at any time upon the written request of ten Fellows. At least two weeks' notice of all meetings shall be given by the Secretary.

ARTICLE VIII.—*Quorum.*

Seven members of the Council shall constitute a quorum. Twenty Fellows of the Society shall constitute a quorum.

ARTICLE IX.—*Expulsion or Suspension of Members.*

Except for non-payment of dues no member of the Society shall be expelled or suspended save upon action by the Council with not more than three negative votes followed by a three-fourths ballot of the Fellows present and voting at a meeting of the Society.

ARTICLE X.—*Amendments.*

This constitution may be amended by an affirmative vote of two-thirds of the Fellows present at any meeting held at least one month after notice of such proposed amendment shall have been sent to each Fellow by the Secretary.

BY-LAWS

(As Amended November 16, 1945)

Article I.—*Order of Business.*

At a meeting of the Society the following order of business shall be observed unless the Society votes otherwise for the time being:

1. Calling of the roll.
2. Address or remarks by the President.
3. Minutes of the last meeting.
4. Report by the Council on business transacted by it since the last meeting of the Society.
5. New membership.
6. Reports of officers and committees.
7. Election of officers and Council (at annual meetings only).
8. Unfinished business.
9. New business.
10. Reading of papers.
11. Discussion of papers.

Article II.—*Council Meetings.*

Meetings of the Council shall be called whenever the President or three members of the Council so request, but not without sending notice to each member of the Council seven or more days before the time appointed. Such notice shall state the objects intended to be brought before the meeting, and should other matter be passed upon, any member of the Council shall have the right to re-open the question at the next meeting.

Article III.—*Duties of Officers.*

The President, or, in his absence, one of the Vice-Presidents, shall preside at meetings of the Society and of the Council. At the Society meetings the presiding officer shall vote only in case of a tie, but at the Council meetings he may vote in all cases.

The Secretary-Treasurer shall keep a full and accurate record of the proceedings at the meetings of the Society and of the Council, send out calls for the said meetings, and, with the approval of the President and Council, carry on the correspondence of the Society. Subject to the direction of the Council, he shall have immediate charge of the office and archives of the Society.

The Secretary-Treasurer shall also send out calls for annual dues and acknowledge receipt of same; pay all bills approved by the President for expenditures authorized by the Council of the Society; keep a detailed account of all receipts and expenditures, and present an abstract of the same at the annual meetings, after it has been audited by a committee of the Council.

The Editor shall, under the general supervision of the Council, have charge of all matters connected with editing and printing the Society's publications. The *Proceedings* shall contain only the proceedings of the meetings, original papers or reviews written by members, discussions on said papers and other matter expressly authorized by the Council.

The Librarian shall, under the general supervision of the Council, have charge of the books, pamphlets, manuscripts and other literary or scientific material collected by the Society.

ARTICLE IV.—*Dues.*

The dues shall be ten dollars for Fellows payable upon entrance and at each annual meeting thereafter, except in the case of Fellows not residing in the United States, Canada, or Mexico, who shall pay five dollars at the time stated. The dues shall be five dollars for Associates payable upon entrance and each annual meeting thereafter until five such payments in all shall have been made; beginning with the sixth annual meeting after the admission of an Associate as such the dues of any Associate heretofore or hereafter admitted shall be the same as those of a Fellow. The payment of dues will be waived in the case of Fellows or Associates who have attained the age of seventy years or who, having been members for a period of at least twenty years, shall have attained the age of sixty-five years. Fellows and Associates who have become totally disabled while members may upon approval of the Council be exempted from the payment of dues during the period of disability.

It shall be the duty of the Secretary-Treasurer to notify by mail any Fellow or Associate whose dues may be six months in arrears, and to accompany such notice by a copy of this article. If such Fellow or Associate shall fail to pay his dues within three months from the date of mailing such notice, his name shall be stricken from the rolls, and he shall thereupon cease to be a Fellow or Associate of the Society. He may, however, be reinstated by vote of the Council, and upon payment of arrears of dues. Fellows and Associates who have become totally disabled while members may upon approval of the Council be exempted from the payment of dues during the period of disability.

ARTICLE V.—*Designation by Initials.*

Fellows of the Society are authorized to append to their names the initials F. C. A. S.; and Associates are authorized to append to their names the initials A. C. A. S.

ARTICLE VI.—*Amendments.*

These by-laws may be amended by an affirmative vote of two-thirds of the Fellows present at any meeting held at least one month after notice of the proposed amendment shall have been sent to each Fellow by the Secretary.

EXAMINATION REQUIREMENTS

SYLLABUS OF EXAMINATIONS

Effective 1941 and thereafter

ASSOCIATESHIP

Part	*Sections*	*Subjects*
I	1	Algebra.
	2	Compound Interest and Annuities Certain.
II	3	Differential and Integral Calculus.
	4	Calculus of Finite Differences.
III	5	Descriptive and Analytical Statistics.
	6	Elements of Accounting, Including Corporate Accounting.
IV	7	Probabilities.
	8	Life Contingencies, Life Annuities and Life Assurances.
V	9	Policy Forms and Underwriting Practice in Casualty Insurance.
	10	Casualty Insurance Rate Making Procedure.

FELLOWSHIP

I	11	Investments of Insurance Companies.
	12	Insurance Law and Legislation.
	13	Insurance Economics.
II	14	Determination of Premium, Loss and Expense Reserves.
	15	Advanced Problems in Casualty Insurance Statistics.
	16	Advanced Problems in Casualty Insurance Accounting.
III	17	Individual Risk Rating.
	18	Social Insurance.
	19	Advanced Problems in the Underwriting and Administration of Casualty Insurance.

RULES REGARDING EXAMINATIONS FOR ADMISSION TO THE SOCIETY

1. **Dates of Examination.**

Examinations will be held on the first Wednesday and following Thursday during the month of April in each year, except that if such dates are in the week preceding Easter, the examinations will be held on the second Wednesday and following Thursday of April. The examinations will be held in such cities as will be convenient for three or more candidates.

2. **Filing of Application.**

Application for admission to examination should be made on the Society's blank form, which may be obtained from the Secretary-Treasurer. No applications will be considered unless received before the fifteenth day of January preceding the dates of examination. Applications should definitely state for what parts the candidate will appear.

3. **Fees.**

The examination fee is $2.00 for each part, with a minimum of $5.00 for each year in which the candidate presents himself; thus for one or two parts, $5.00, for three parts, $6.00, etc. Examination fees are payable to the order of the Society and must be received by the Secretary-Treasurer before the fifteenth day of January preceding the dates of examination.

4. **Associateship and Fellowship Examinations.**

(a) The examination for Associateship consists of five parts and that for Fellowship consists of three parts. A candidate may take any one or more of the five parts of the Associateship Examination. No candidate will be permitted to present himself for

any part of the Fellowship Examination unless he has previously passed, or shall concurrently present himself for and submit papers for, all parts of the Associateship Examination and all preceding parts of the Fellowship Examination. Subject to the foregoing requirement, the candidate will be given credit for any part or parts of either examination which he may pass.

(b) A candidate who has passed Associateship Parts I-IV prior to 1941, but who has not been enrolled as an Associate because of lack of the experience qualifications required by the examination rules effective prior to 1941, will be enrolled as an Associate upon passing Part V. Such a candidate may also take Fellowship Examination Parts I-III in the same year as Associateship Part V, subject to the provisions of paragraph (a) above.

(c) An Associate who has passed no part of the Fellowship Examination under the Syllabus effective prior to 1941 is required, in order to qualify for admission as a Fellow, to pass Associateship Examination Part V and Fellowship Examination Parts I-III.

(d) In the case of a candidate for admission as Associate who has served for a period of at least two years in the armed forces of the United States or Canada or services directly connected therewith, and who shall have completed courses in any of the sections included in Parts I, II and III, of the Associate Examinations at a college or university of recognized standards, the examinations in such sections may be waived at the discretion of the Examination Committee subject to the following conditions and limitations.

1. Such courses shall have been completed within 10 years prior to date of application for such waiver.
2. The courses shall have been at least equivalent to those required to qualify the candidate to meet the examination requirements.
3. The grades obtained shall have been satisfactory.

The foregoing waiver of examinations provision shall not apply to candidates applying for admission subsequent to January 15, 1948. Any such waiver made shall lapse if the candidate shall fail to attain Associateship status prior to December 1, 1950.

5. Alternative to Passing of Fellowship Parts II and III.

As an alternative to the passing of Parts II and III of the Fellowship Examination, a candidate may elect to present an

original thesis on an approved subject relating to casualty or social insurance. Such thesis must show evidence of ability for original research and the solution of advanced problems in casualty insurance comparable with that required to pass Parts II and III of the Fellowship Examination, and shall not consist solely of data of an historical nature. Candidates electing this alternative should communicate with the Secretary-Treasurer and obtain through him approval by the Examination Committee of the subject of the thesis. In communicating with the Secretary-Treasurer, the candidate should state, in addition to the subject of the thesis, the main divisions of the subject and general method of treatment, the approximate number of words and the approximate proportion to be devoted to data of an historical nature. All theses must be in the hands of the Secretary-Treasurer before the first Wednesday in April of the year in which they are to be considered. Where Part I of the Fellowship Examination is not taken during the same year, no examination fee will be required in connection with the presentation of a thesis. All theses submitted are, if accepted, to be the property of the Society and may, with the approval of the Council, be printed in the *Proceedings*.

6. **Waiver of Examinations for Associate.**

The examinations for Associate will be waived under Article III of the Constitution only in case of those candidates who meet the following qualifications and requirements:

(a) The candidate shall be at least thirty-five years of age.

(b) The candidate shall have had at least ten years' experience in casualty actuarial or statistical work or in a phase of casualty insurance which requires a working knowledge of actuarial or statistical procedure or in the teaching of casualty insurance principles in colleges or universities. Experience limited exclusively to the field of accident and health insurance shall not be admissible. In the case of a candidate who has spent at least one year in the armed forces of the United States or Canada or services directly connected therewith, the period covered by such service shall be included in the ten years experience qualification period provided for above.

(c) For the two years preceding date of application, the candidate shall have been in responsible charge of the actuarial or statistical department of a casualty insurance organization or of an important division of such department or shall have occupied an executive position in connection with the phase of casualty work in which he is engaged, or, if engaged in teaching, shall have attained the status of a professor. In the case of a candidate who has spent at least one year in the armed forces of the United States or Canada or services directly connected therewith, the two years requirement stated above shall be reduced to one year.

(d) The candidate shall have submitted a thesis approved by the Examination Committee. Such thesis must show evidence of original research and knowledge of casualty insurance and shall not consist solely of data of an historical nature. Candidates electing this alternative should communicate with the Secretary-Treasurer and obtain through him approval by the Examination Committee of the subject of the thesis. In communicating with the Secretary-Treasurer, the candidate should state, in addition to the subject of the thesis, the main divisions of the subject and general method of treatment, the approximate number of words and the approximate proportion to be devoted to data of an historical nature.

RECOMMENDATIONS FOR STUDY

To assist students in preparation for the examinations, Recommendations For Study have been prepared. This lists the texts, readings and technical material which must be mastered by the candidates. Textbooks are loaned to registered students by the Society. By "registered students" is meant candidates who have signified their willingness to take the examinations by the payment of their examination fees.

LIBRARY

The Society's library contains all of the references listed in the Recommendations for Study with the exception of certain periodicals and publications subject to periodical revision. It also

contains numerous other works on casualty actuarial matters. Registered students may have access to the library by receiving from the Society's Secretary the necessary credentials. Books may be withdrawn from the library for a period of two weeks upon payment of a small service fee and necessary postage.

The library is in the immediate charge of Miss Mabel B. Swerig, Librarian of the Insurance Society of New York, 107 William Street, New York 7, N. Y.