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NOTICE

The Society is not responsible for statements made or opinions expressed in the articles, criticisms and discussions published in these *Proceedings*.

"That which is not good for the swarm, neither is it good for the bee."

—*Marcus Aurelius.*

VOL. XXIX No. 59

PROCEEDINGS

NOVEMBER 20, 1942

THE CASUALTY ACTUARY AND SOCIAL INSURANCE

PRESIDENTIAL ADDRESS BY RALPH H. BLANCHARD

Article II of our Constitution states that "The object of the Society shall be the promotion of actuarial and statistical science as applied to the problems of casualty and social insurance . . ."

While some attention has been given to social insurance in papers and presidential addresses and in the appointment of a Committee on Unemployment Insurance, consideration of that subject has been meager in comparison with discussion of the problems of casualty insurance as a business. Certain of these problems are directly related to social insurance, particularly those of workmen's compensation insurance (without which the Society might not exist). But here the emphasis has been on the technique of the business of implementing workmen's compensation laws rather than on social policy or governmental administration.

I suggest that the casualty actuary, whether motivated by social consciousness or by self-interest, should devote more attention—considerably more—to the social-insurance field. He can be of great usefulness in giving technical guidance to governmental action, in determining the lines to be drawn between social and private insurance, and in conducting private insurance in the light of the present and probable future development of social insurance. Above all, he should avoid shutting himself up in his present corporate bailiwick, where he is insulated from outside and possibly disturbing currents.

It might be well to stop a moment at this point to inquire what is meant by the term *social insurance*. The "definitions" of the term that I have seen all lack the essential quality of a definition—that it *define*. Rubinow devoted an entire chapter of his book on Social Insurance to the "concept of social insurance" but emerged without having defined it. Mowbray intimates in a section heading of his general text that he is about to present a definition, but proceeds to list the "essential features of a complete social insurance scheme." Often the term is used with no attempt at definition.

I have long cherished a rather awkward definition but one which, I submit, does *define*: Social insurance is any form of insurance in which the government goes beyond the regulation of practices and the dissemination of infor-

mation. It may do so by compelling insurance, by shifting the cost, by subsidy, or by becoming itself an insurer. To the extent that it acts in any one of these directions, insurance becomes social insurance, and I should include within its scope compulsory automobile insurance, governmental schemes of war-risk insurance, governmental crop insurance, as well as the more commonly recognized workmen's compensation, unemployment, old-age, and disability insurances. While I believe that this definition properly distinguishes between private and social insurance, it includes certain governmental activities insurancewise which are not generally thought of as "social insurance" and which would be of minor interest to members of the Society. Our attention should be given primarily to those schemes of social insurance which are established or advocated to meet a broad social need, which aim to provide an adequate minimum income, and which are usually compulsory as to membership.

It is particularly important for the membership of this Society to note and ponder all manifestations of a conviction that private initiative, regulated by supervisory authorities, is not adequately meeting the risk problem of the public. Such a conviction has social-insurance legislation as its outlet. We should be prepared to contribute our best judgment and technical ability to social insurance where that is desirable or inevitable, and to conduct private insurance in such fashion as best to satisfy the needs not met by social insurance. In a rationally organized society the two would be complementary rather than competitive.

Social insurance, as a broad national policy, is something of a novelty in the United States, but the rest of the world has long accepted it. If you would learn how far it has been accepted, I recommend that you read "Approaches to Social Security—An International Survey," published this year by the International Labour Office. This study indicates that "There is a strong, broad tendency to bring all persons employed in manual work and the lower ranks of salaried employees within the scope of compulsory insurance in all its branches."[1]

In Great Britain, some 18 months ago, there was appointed an Interdepartmental Committee on Social Insurance and Allied Services, of which Sir William Beveridge was named chairman, to draw up a scheme to meet postwar security problems. The *New York Times* reported recently that he will recommend "the introduction of a comprehensive system of social insurance and the establishment of a national income minimum below which none would fall." More significant for us, there was introduced in Congress on September 9th of this year a bill,[2] providing for a broad federal social-insurance program and adding disability and hospitalization benefits to the

[1] p. 35.
[2] H. R. 7534.

provision already made by the Social Security Act. This bill unquestionably embodies proposals for which there is considerable support, and it, or other similar bills, will probably be given extensive consideration by Congress.

So far, casualty actuaries have had practically no part in the origination, establishment, administration, or development of social insurance, other than workmen's compensation. Life actuaries have had some part, but they have been more or less on the side lines. Social insurance has been in the hands of persons highly interested in *ends* but somewhat impatient with questions of *means*. And the actuaries, particularly the casualty actuaries, have held themselves aloof—or perhaps their interest has not been aroused.

The greater participation of the life actuaries in the discussion and practice of social insurance is at least partly to be explained by the fact that the most generally accepted development in that field is old-age and survivors insurance. There are the further facts that social insurance, actual and proposed, does not threaten and may even benefit life insurance as a business, and that their experience with group insurance has developed their understanding of mass treatment of insurance problems. Granting that old-age and survivors insurance is primarily the concern of the life actuary, it seems to me that unemployment and disability insurance should be very much the concern of the casualty actuary. The hazards of both these fields are the same as, or akin to, those found in casualty insurance.

It may be thought that unemployment insurance does not lend itself to actuarial treatment, and it may be true that it will never be possible to predict losses due to unemployment with even approximate accuracy. But certainly the planning and administration of this form of social insurance would benefit from the type of analysis in which actuaries are skilled. Records should be set up so as to facilitate whatever actuarial analysis is possible, and full use should be made of statistical indications. Such analysis might eventually show how the unemployment risk can be controlled and measured.

Social disability insurance beyond that provided by workmen's compensation has been adopted in only one state. But we shall hear more of it, and of its concomitants, medical care and hospitalization insurance, in the future, probably in the immediate future, in the form of specific legislative proposals, recommendations from high places, and urgings from a variety of sources. In fact, the first efforts to establish a federal scheme of temporary and permanent disability insurance have been made.

Here is a field in which the casualty actuary has basic material and techniques, and in which his services in analyzing proposals and in furnishing guidance could be of outstanding value. But he must understand social insurance as such, its purposes and its essential procedures, if his services are to be really useful. He is cost-minded, realistic, and no-one so well as

he knows what figures indicate, and how misleading or inadequate they may be. One of the greatest services he can render is to point out persistently that social insurance and private insurance, whatever they may have in common, have marked and important divergencies. As Hohaus has said, social insurance "is a new creation . . . requiring an actuarial technique that may sometimes seem rank heresy to the orthodox private insurance actuary."[3]

The private insurer and its insureds may sell, buy, continue, or renew insurance on a mutually voluntary basis—the more important social-insurance schemes are based on compulsorily assembled groups and are operated in accordance with statutory and administrative rules which largely eliminate personal judgment and volition. In the long run, in private insurance, there must be some relationship between the cost of providing protection to the individual insured and the premium charged for it—the contribution of the individual to social-insurance schemes is generally determined by other considerations. The private insurer must make its offering attractive to the buying public—the government is restricted only by the rather tenuous control of the democratic process. Preservation of precise equities for the individual insured is the ideal of private insurance—satisfaction of social needs the aim of social insurance. And where the private and therefore mortal insurer must hold solvency first of its concerns, the government need consider only its ability to collect the special or general taxes necessary to support its scheme.

And still we hear, and shall continue to hear, arguments that this or that should be part of a social-insurance scheme because it is done in private insurance.

If it is proposed that the government furnish an insurance service which is generally needed, there are four tenable answers: that the service is entirely impracticable, that the government cannot properly furnish the service, that private initiative can furnish it to better general advantage, or that it should be furnished by the government, either direct or through the agency of private carriers. In any event the actuary should lend his special competence to the solution of whatever problems may arise.

One last word—when it is reasonably clear that social provision of insurance is desirable or inevitable, the actuary should be among the first to recognize it and to prepare for it.

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[3] "Social Insurance in a Democracy," an address before the American Life Convention, October 7, 1942.

BOILER AND MACHINERY INSURANCE

BY

SEYMOUR E. SMITH

The boiler and machinery lines are two of the basic coverages comprising the casualty insurance field. While they do not produce a premium volume of such size as automobile liability and workmen's compensation, they are nevertheless of considerable importance, and have been given wide acceptance by the general public for many years. To date the proceedings of the Casualty Actuarial Society have not contained any papers or discussions relating to this type of insurance. The most probable reason for the neglect of these lines of insurance by this society is that boiler and machinery insurance have been considered by most people to be the exclusive domain of the engineer. While it is perfectly true that a competent engineering staff is essential to the successful underwriting and inspection of boiler and machinery objects, it will not be amiss for the actuary and statistician to become familiar with these fields of insurance, and to study the ratemaking and statistical procedures that are involved. It is the intent of this paper to outline very briefly the coverage and rating of boiler and machinery objects, to review the ratemaking method used in the recent manual revision, and to suggest a few points in the rating procedure which seem to offer possibilities for further investigation.

Coverage and Type of Object Insured

Modern commercial activity and industrial technique demand machines of constantly greater power, pressure and speed, with the corresponding increase in the potential destructive forces that are released when structural or mechanical failure occurs. This has created the demand for insurance coverage that will provide trained engineering inspection service that will keep accidents to a minimum, and that will indemnify the assured for his loss when an accident does occur.

As an indication of the wide field of coverage offered to the public, the following list enumerates a few of the different objects insured under boiler and machinery policies.

Fire Tube Boilers
Water Tube Boilers
Locomotive Boilers
Tanks
Digesters
Vulcanizers
Economizers
Refrigerating Systems
Fly-wheels
Gear Wheels
Steam Engines
Internal Combustion Engines
Compressors and Pumps
Fans and Blowers
Steam Turbines
Water Turbines
Electric Generators
Electric Motors
Rotary Converters
Transformers

The Conference Form Policy for Boiler or Machinery Insurance, which is used by almost all companies writing this kind of insurance, has five sections agreeing to pay to or on behalf of the assured for different types of direct loss. Three of these sections provide coverage on all policies, while the other two are optional and may be obtained by the assured upon the payment of additional premium. These five sections in the policy are as follows:

"Section I — To PAY the Assured for loss on the property of the Assured directly damaged by such accident (or, if the Company so elects, to repair or replace such damaged property), excluding (a) loss from fire (or from the use of water or other means to extinguish fire), (b) loss from an accident caused by fire, (c) loss from delay or interruption of business or manufacturing or process, (d) loss from lack of power, light, heat, steam or refrigeration, and (e) loss from any indirect result of an accident;

Section II (Optional) — To PAY the Assured, if loss under Section II is stated above as included but not otherwise, for the extra cost represented by items of expense for temporary repair or for expediting the repair of such damaged property of the Assured including overtime and the extra cost of express or other rapid means of transporting material, but if the Company's payment under Section I is $1,000 or less the Company's liability under Section II shall not exceed an amount equal to said payment under Section I, and if said payment under Section I exceeds $1,000, the Company's liability under Section II shall not exceed $1,000 plus 25% of the amount by which the Company's payment under Section I exceeds $1,000; and the Company's liability under Section II shall be a part of and not in addition to the Limit per Accident;

Section III — To PAY, to the extent of any indemnity remaining after payment of all loss as may be required under Sections I and II, such amounts as the Assured shall become obligated to pay by reason of the liability of the Assured for loss on the property of others directly damaged by such accident, including liability for loss of use of such damaged property of others; to DEFEND the Assured against any claim or suit alleging such damage unless or until the Company shall elect to effect settlement thereof;

Section IV (Optional) — To PAY, to the extent of any indemnity remaining after payment of all loss as may be required under Sections I, II and III, if loss under Section IV is stated above as included but not otherwise, such amounts as the Assured shall become obligated to pay by reason of the liability of the Assured, including liability for loss of services, on account of bodily injuries (including death at any time resulting therefrom)

sustained by any person and caused by such accident, except that the indemnity hereunder shall in no event apply to any liability or obligation under any workmen's compensation law; to PAY, if loss under Section IV is stated above as included but not otherwise, irrespective of the Limit per Accident, for such immediate surgical relief as shall be rendered at the time of the accident; to DEFEND the Assured, if loss under Section IV is stated above as included but not otherwise, against any claim or suit alleging such liability unless or until the Company shall elect to effect settlement thereof; and

Section V To PAY, irrespective of the Limit per Accident, all costs taxed against the Assured in any legal proceeding defended by the Company in accordance with Section III or IV, all interest accruing after entry of judgment rendered in connection therewith up to the date of payment by the Company of its share of such judgment, all premium charges on attachment or appeal bonds required in such legal proceedings, and all expenses incurred by the Company for such defense;

PROVIDED the accident happens while the object is in use, or connected ready for use, at the location specified for it in the Schedule."

It will be noted that the words "accident" and "object" as used in the above sections are not defined therein. This is because the definitions are necessarily varied for the individual kinds of objects. To each policy is attached one or more schedules, one for each type of object insured, and the individual schedule lists the object or objects insured, provides a definition of the object and of the word "accident" as applied to that particular type of object. For example, for steel boilers an accident is:

> "a sudden and accidental tearing asunder of the object or any part thereof caused by pressure of steam or water therein, or the sudden and accidental crushing inward of a cylindrical furnace or flue of the object so caused,"

while for steam turbine breakdown coverage an accident is:

> "a sudden and accidental breaking, deforming, burning out or rupturing of the object or any part thereof, which manifests itself at the time of its occurrence by immediately preventing continued operation or by immediately impairing the functions of the object and which necessitates repair or replacement before its operation can be resumed or its functions restored; and clause (a) of Section I is changed to read '(a) loss from fire outside the object (or from the use of water or other means to extinguish fire)'."

The policy coverage mentioned above provides so-called "direct-damage" coverage for the individual objects. In addition the assured may purchase several different forms of "indirect-damage" coverage. The most important of these additional coverages are:

(a) Use and Occupancy
(b) Consequential Damage
(c) Outage (coverage providing a specified indemnity for each hour during which the functions of an insured object are prevented by accident to the object)
(d) Power Interruption

Separate endorsements are provided for (a), (b) and (c) above; and a separate policy for (d).

The balance of the boiler and machinery policy contains the usual provisions for cancelation, subrogation, suits against the company and assured, changes etc., and there is only one section that is unusual compared to other casualty policies. Under the policy conditions, section 5 relates to the inspection of the insured objects and premises, and in addition permits any representative of the company to immediately suspend the insurance by written notice upon the discovery of a dangerous condition with respect to any object insured. The assured is allowed the pro-rata earned premium for the period of such suspension. It is clearly in the public interest that this provision be inserted, since it permits the insurer to stop immediately the operation of an object that is in dangerous condition and which might cause severe damage at any time. The main emphasis in boiler and machinery coverage is placed on inspection service and accident prevention. All insured objects are regularly inspected by trained engineers. An indication of the extent of this accident prevention service is given by the countrywide figures of stock companies entered in New York as reported in the Casualty Experience Exhibit, where the total amount expended for inspection service for boiler and machinery exceeds that expended for workmen's compensation inspection service, even though the boiler and machinery premium is less than 8% of that for compensation.

Rating Procedure

Boiler and machinery insurance does not provide for any experience, equity, retrospective or schedule rating, and all risks are rated in accordance with the manual rules. The manual rates contemplate coverage for three years, and policies written for a lesser period are charged short rates based on the three year premium. Policies may be written for four years by extension of the three year rate at slightly less than pro-rata.

Until the 1942 manual revision, boiler policy premiums were comprised of three parts; a basic charge, charged once for each policy, and similar to a policy fee except that it varied by policy limits; a location charge, charged for each location under the policy and varying by territory and policy limits; and an object charge for each individual object. The manual contains a printed object charge rate for every kind, size and type of insurable object.

The basic and location charges were quite formidable, amounting over all to slightly more than one-third of the total boiler earned premium. These charges performed the three-fold function of expense constants, increased limits factors and territorial differentials in a rather unsatisfactory manner and were subject to severe criticism from the New York Insurance Department.

The recent manual revision has placed the rating of boiler risks on the same basis as machinery risks have been written in the past, with an insurance charge, varying by policy limits, collected for each location under the policy, and an object charge for each individual object. The basic charge and territorial differentials have been eliminated. Graded company expenses and commissions have also been introduced. Separately for boiler and machinery insurance, each policy is subject to a 10% reduction on all manual premium in excess of $500 up to $3,000, and a 30% reduction on all manual premium in excess of $3,000. The full commission rate is payable on the first $2,750 of net collectible premium, and 60% of the normal commission rate on all net collectible premium in excess of $2,750 per policy.

The rating procedure used in the past, although out of line for certain classifications, has produced over all rate-level results that have been reasonably satisfactory. The boiler rate-level has been a little low, but the machinery experience has produced underwriting results more favorable than the expected. The following exhibit (Exhibit A) shows for each line for the five latest available calendar years, the countrywide earned premiums and underwriting results for all stock carriers entered in New York as reported in the Casualty Experience Exhibit. The loss and expense items provided for in the manual rates have been shown so that a comparison of actual and expected results may be made. The experience of stock companies only was used since some mutual companies charge higher policy rates than stock companies for boiler and machinery coverage to assure high dividends, and the inclusion of this experience would tend to distort the underwriting results.

Rate Making Procedure

Since the results of any rate-making procedure are strongly determined by the form of the underlying statistical data, it is not inappropriate to outline briefly the form of the experience data before going to the rate-making procedure used in the latest manual revision.

Boiler and machinery experience is maintained on a calendar year basis. Each company reports its writings on the basis of object months exposure for each of the various types of objects and kinds of coverage. Basic and location charge months exposure for boiler and insurance charge months exposure for machinery are reported in total. Written premiums for the

Exhibit A

BOILER AND MACHINERY EXPERIENCE
Countrywide Results of All Stock Carriers Entered in New York
Calendar Years 1937-1941

Calendar Year	Earned Premium	Incurred Loss Ratio	Incurred Inspection Ratio	Losses and Inspection	Acquisition*	Claim	Administration	Taxes	Profit & Contingency
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
BOILER INSURANCE									
1937	7,239,974	.112	.424	.536	.303	.015	.105	.049	—.008
1938	7,543,695	.111	.441	.552	.314	.014	.101	.055	—.036
1939	7,662,792	.112	.431	.543	.317	.017	.099	.051	—.027
1940	7,735,240	.122	.391	.513	.296	.015	.120	.048	.008
1941	8,008,310	.155	.383	.538	.289	.017	.113	.045	—.002
Total	38,190,011	.123	.413	.536	.303	.016	.108	.050	—.013
Provision in Manual Rates				.490	.300	.020	.090	.050	.050
MACHINERY INSURANCE									
1937	3,466,013	.258	.194	.452	.276	.035	.092	.051	.094
1938	3,656,527	.239	.190	.429	.318	.032	.092	.057	.072
1939	3,710,951	.216	.179	.395	.318	.031	.091	.064	.101
1940	3,915,487	.269	.179	.448	.281	.030	.118	.054	.069
1941	4,452,356	.360	.166	.526	.267	.036	.118	.050	.003
Total	19,201,334	.272	.181	.453	.286	.033	.103	.055	.070
Provision in Manual Rates				.490	.300	.020	.090	.050	.050

* Related to Written Premium.

calendar year are also reported. Object charges written are reported by type of object, indirect damage premiums by kind of coverage, and basic and location charges for boiler, and insurance charges for machinery, are reported in total. From this information calendar year earned premiums and earned object years are calculated by the National Bureau. As an example of the method used consider a steel boiler written for three years on July 1, 1941 with an object charge of $90. The carrier would report its calendar year writings as follows:

	Exposure — Number of Object Months				Object Premiums Written
	1941	1942	1943	1944	
Steel Boilers	6	12	12	6	$90

From this form it is readily seen that the calendar year 1941 exposure is ½ an earned object year with an earned premium of $15. To this are added the earned object years and earned premiums for 1941 on boilers written in previous calendar years, but with exposure carrying over into 1941. In this manner the complete earned object years and earned object charge premiums are determined for the calendar year. Outstanding losses at the beginning and end of the calendar year and losses paid during the year are reported for each of the exposure groups, so that calendar year incurred losses are readily available. The earned premiums for basic and location charges for boiler (these will be eventually replaced by the new insurance charges in future reportings) and insurance charges for machinery are determined in total by the same method of calculation as the object charges. These total basic and location and insurance charges, separately for boiler and machinery, are averaged over all of the direct damage earned premiums, on the assumption that on the average each classification will incur from these charges the same percentage of its total earned premium. In calendar year 1940 the ratio of basic and location charges to object charges for boiler insurance was .5021, and the ratio of insurance charges to object charges for machinery insurance was .1202. The 1940 final statistical data for each classification showed earned premiums which consisted of the actual earned object charges multiplied by 1.5021 for boiler and 1.1202 for machinery.

Since inspection expense is such a large portion of the boiler and machinery pure premium, the correct statistical reporting of inspection costs is of major importance. The procedure used to determine inspection cost by type of object probably varies in small degree between companies, but the method used by one multiple line carrier with an appreciable volume of business will serve as an illustration. It is safe to assume that other methods will not show any major differences. The total company inspection cost is allocated to line of insurance on the basis of inspectors' time sheets for the field and

time estimates and analyses of duties for the home office. The field inspectors' time sheets show the number of hours (including travel time) spent on each particular type of object, and the time spent on inspections for rate surveys, special mechanical services, accident investigation and indirect damage coverage. In the report to the National Bureau, the total inspection expense for boiler and machinery insurance is shown, and this is distributed to type of object on the basis of the time distribution shown in the field inspectors' reports. The effect of this method is to assume that the cost per hour of inspection is the same for each type of object, and to allocate the home office expense as an item of general overhead. The report to the National Bureau also shows the total number for each of the foregoing types of inspections.

From the above statistical information, a compilation of boiler and machinery insurance is able to show the following information for each calendar year:

1. Type of Object
2. Number of earned object years
3. Earned Premium (including loading for basic and location or insurance charges)
4. No. of losses incurred
5. Amount of losses incurred
6. Loss Frequency
7. Loss Ratio
8. Inspection Expense
9. Inspection Expense Ratio
10. Number of Inspections per object year
11. Inspection Cost per object
12. Cost per inspection

The boiler and machinery lines are less affected by many economic changes than the other casualty lines, being influenced to a much smaller degree by wage levels, unemployment, or general prosperity. For this reason the indicated rate level does not usually show the yearly fluctuations that are apt to accur in other lines of casualty insurance, and so rate changes are not a periodic procedure. The last complete revision of the manual was made in 1930, although some individual changes have taken place since that time. However, by the spring of 1941 it was realized that business and economic changes were taking place that might have a seriously adverse effect on the experience. Industrial plants were operating at a greatly increased tempo, many on a 24 hour a day basis, with a greatly increased strain on their power plants. Priorities were being introduced which threatened long delays and greatly increased material and labor costs for replacements and repairs

in the event of accidents. The threat of unavoidable delays in affecting repairs made Use and Occupancy rates seem most in need of review in the light of future possibilities. The experience under Use and Occupancy in the past had been favorable, but it was decided for the purpose of review to increase the indicated losses by 50% and the estimated inspection cost of 5% by 15%. The selected rate increases of 3.2% for Boiler, 32.4% for Fly-wheel, Engine and Turbine and 17.1% for Electrical Machinery were approved by the New York Insurance Department and adopted immediately.

The degree to which boiler and machinery rates should be adjusted over all to compensate for the expected adverse trend in experience was a difficult question that had to be settled purely on the basis of underwriting judgment, since if the carriers waited for the experience to develop the emergency situation calling for higher rates might have passed before these increased rates could become effective, and higher rates would be very difficult to justify after the need for them had passed. It was finally decided that the boiler and machinery experience should be reviewed by classification, and that the indicated premium adjustment should be based on a 50% increase in the experienced losses and a 15% increase in the inspection costs. It was felt that these loadings would serve as proper indications of increased hazard and cost. (Note the sharp rise in the calendar year 1941 loss ratio for both boiler and machinery, as shown in Exhibit A.) The expense loading was left at 51% of the final rate. The experience used was the five latest available calendar years for exposure and losses, and the two latest available calendar years for inspection expense ratios. The following exhibits show the development of the indicated premium and object charge adjustments by class groups on the above basis, the selected object charge adjustments and the effect on the premium at the then effective rates. Since the proposed rate adjustments were to be made only on the object charges, the indicated object charge adjustment shown in column (7) is (for those classifications whose earned premiums were loaded flat for basic and location or insurance charges) equal to the indicated premium adjustment increased by 50.34% for boiler and 12.75% for machinery. These percentages represent the five year average ratio to object charges of basic and location charges and insurance charges respectively.

ANALYSIS OF BOILER EXPERIENCE—ALL COMPANIES—U. S. A.

Exhibit B

EXPERIENCE OF CALENDAR YEARS 1936–1940

Classification Group	Earned Object Years	Earned Premiums at Current Rates	Loss Frequency*	Loss Ratio at Current Rates	1939-40 Average Inspection Ratio	Indicated Premium Adjustment**	Indicated Object Charge Adjustment	Selected Object Charge Adjustment	Effect on Premium at Current Rates	
									Amount	Percent
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1. Steel Boilers—(15 lbs. or less)	347,444	$3,877,239	1.3	2.2%	60.4%	+ 48.6%	+ 73.1%	+ 25.0%†	+ 644,745	+ 16.6%
2. Fire Tube Boilers—(over 15 lbs.)										
(a) All Other than (b) and (c)	437,454	7,603,670	2.9	6.1	53.4	+ 44.1	+ 66.3	+ 40.0	+ 2,023,060	+ 26.6
(b) Oil or Gas Drilling	6,459	238,785	3.6	17.5	53.4	+ 79.0	+ 118.8	+ 40.0	+ 63,532	+ 26.6
(c) Track Locomotives	23,816	457,530	1.1	15.2	53.4	+ 71.8	+ 107.9	+ 40.0	+ 121,732	+ 26.6
Total	467,729	8,299,985	2.8	7.0	53.4	+ 46.7	+ 70.2	+ 40.0	+ 2,208,324	+ 26.6
3. Water Tube Boilers—(Over 15 lbs.)										
(a) 3,750 Sq. Ft. or less	98,949	2,442,313	8.1	11.5	47.2	+ 46.1	+ 69.3	+ 35.0	+ 568,584	+ 23.3
(b) 3,750–10,000 Sq. Ft.	55,417	2,066,772	11.4	12.1	36.9	+ 23.7	+ 35.6	+ 25.0	+ 343,683	+ 16.6
(c) Over 10,000 Sq. Ft.	10,195	1,031,432	29.6	13.7	19.9	− 11.2	− 16.8	—	—	—
Total	164,561	5,540,517	10.5	12.2	37.8	+ 26.1	+ 39.2	+ 24.8	+ 912,267	+ 16.5
4. Cast Iron Boilers—Explosion	410,223	4,052,897	0.9	1.9	47.4	+ 17.1	+ 25.7	+ 20.0†	+ 539,164	+ 13.3
Cast Iron Boilers—Cracking	167,008	4,941,834	40.1	31.1	0.0	− 4.7	− 4.7	− 8.7	− 429,940	− 8.7
Total	410,223	8,994,731	17.2	18.0	22.6	+ 8.2	+ 9.7	+ 1.4	+ 109,224	+ 1.2
5. Unfired Vessels										
(a) Class 1—Excluding Digesters	795,555	4,421,472	0.8	8.8	49.0	+ 42.0	+ 63.1	+ 40.0	+ 1,176,393	+ 26.6
(b) Digesters—Class 1	4,147	287,494	5.1	6.3	11.1	− 54.5	− 81.9	—	—	—
(c) Class 2 and Miscellaneous	103,076	895,744	1.9	8.8	23.4	− 18.2	− 27.4	—	—	—
Total	902,778	5,604,710	0.9	8.6	43.3	+ 28.0	+ 42.1	+ 31.6	+ 1,176,393	+ 21.0
6. Economizers	3,523	65,010	13.9	19.0	29.7	+ 28.0	+ 42.1	—	—	—
7. Refrigerating Systems	23,685	759,336	13.0	19.9	22.6	+ 14.1	+ 21.2	—	—	—
8. Piping	418,723	1,434,546	1.3	8.5	20.8	− 25.1	− 37.7	− 23.1	− 220,420	− 15.4
Total Above (Items 1 through 8)	2,738,666	34,576,074	4.5	10.8	39.6	+ 25.9	+ 36.3	+ 19.6	+ 4,830,533	+ 14.0

(Continued)

*Per 1,000 Object Years.
**Including Loadings of 50% on Losses and 15% on Inspection Cost. Expense Loading = 51%.
†Rates actually increased 17.0%—exclusion of part time operation estimated to bring net increase up to selected.

ANALYSIS OF BOILER EXPERIENCE—ALL COMPANIES—U. S. A. Exhibit B (Continued)

EXPERIENCE OF CALENDAR YEARS 1936–1940

Classification Group	Earned Object Years	Earned Premiums at Current Rates	Loss Frequency*	Loss Ratio at Current Rates	1939-40 Average Inspection Ratio	Indicated Premium Adjustment**	Indicated Object Charge Adjustment	Selected Object Charge Adjustment	Effect on Premium at Current Rates	
									Amount	Percent
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
9. Furnace Explosion	218,385	$1,129,016	4.9	24.6%	5.0(Est.)%	− 12.9%	− 12.9%	− 11.4%	− 128,708	− 11.4%
10. Residence Boilers	45,912	620,757	18.7	18.2	28.5	+ 22.7	+ 22.7	—	—	—
11. Personal Injury	1,684,339	961,809	—	3.5	—	− 89.2	− 89.2	− 50.0	− 480,905	− 50.0
12. Miscellaneous Direct Damage	27,317	105,831	0.3	7.8	5.0(Est.)	− 64.3	− 64.3	—	—	—
(A) Total Direct Damage		37,393,487		11.1	37.3	+ 21.6	+ 29.4	+ 15.4	+ 4,220,920	+ 11.3
13. Boiler Use and Occupancy	37,162	2,180,512	19.0	29.8	5.0(Est.)	+ 3.1	+ 3.1	+ 3.2	+ 69,776	+ 3.2
14. Consequential Damage	3,392	143,942	13.9	13.2	5.0(Est.)	− 47.8	− 47.8	—	—	—
15. Expediting Charges	51,052	163,883	0.9	6.3	5.0(Est.)	− 68.8	− 68.8	− 50.0	− 81,942	− 50.0
16. Outage	537	12,133	29.8	109.5	5.0(Est.)	+ 247.1	+ 247.1	+ 78.0	+ 9,464	+ 78.0
17. Loss of Use—Power Interruption	37	7,821	27.0	0.7	5.0(Est.)	− 85.9	− 85.9	—	—	—
18. Miscellaneous Indirect Damage	884	3,228	24.9	113.1	5.0(Est.)	+ 258.2	+ 258.2	—	—	—
(B) Total Indirect Damage		2,511,519		27.7	5.0(Est.)	− 3.3	− 3.3	− 0.1	− 2,702	− 0.1
Total—All Boiler (A) + (B)		39,905,006		12.2	35.3	+ 20.2	+ 26.9	+ 14.1	+ 4,218,218	+ 10.6

*Per 1,000 Object Years.
**Including Loadings of 50% on Losses and 15% on Inspection Cost. Expense Loading = 51%.

ANALYSIS OF MACHINERY EXPERIENCE—ALL COMPANIES—U. S. A.

Exhibit C

EXPERIENCE OF CALENDAR YEARS 1936–1940

Classification Group	Earned Object Years	Earned Premiums at Current Rates	Loss Frequency*	Loss Ratio at Current Rates	1939-40 Average Inspection Ratio	Indicated Premium Adjustment**	Indicated Object Charge Adjustment	Selected Object Charge Adjustment	Effect on Premium at Current Rates	
									Amount	Percent
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1. Fly-wheels	75,398	$599,123	3.3	19.3%	35.5%	+ 42.4%	+ 47.8%	+ 32.2%	+ $171,079	+ 28.6%
2. Steam Engines	21,528	1,484,138	83.7	46.9	21.8	+ 94.9	+ 107.0	+ 57.0	+ 570,296	+ 50.6
3. Internal Combustion Machines	3,335	741,602	312.4	35.6	16.8	+ 48.4	+ 54.6	—	—	—
4. Compressors and Pumps										
(a) Steam Type	7,537	652,933	50.9	24.9	9.4	− 1.6	− 1.8	—	—	—
(b) Separately Driven Type	28,635	1,021,080	26.3	15.4	18.9	− 8.6	− 9.7	—	—	—
Total	36,172	1,674,013	31.5	19.1	15.4	− 5.3	− 6.0	—	—	—
5. Miscellaneous Machines	18,661	291,194	20.6	40.1	22.3	+ 75.1	+ 84.7	+ 28.0	+ 72,231	+ 24.8
Total Engines (Items 2, 3, 4 and 5)	79,696	4,190,947	54.8	33.3	18.3	+ 44.9	+ 50.6	+ 22.1	+ 822,527	+ 19.6
6. Turbines										
(a) Steam Turbines—All Coverages										
1. 100 Kilowatts or less	9,665	137,239	11.1	18.4	55.4	+ 86.3	+ 97.3	+ 39.3	+ 47,875	+ 34.9
2. 101 to 1,000 Kilowatts	10,859	1,286,294	35.5	19.2	17.6	—	—	+ 7.0	+ 79,423	+ 6.2
3. 1,001 to 9,000 Kilowatts	6,196	2,541,700	52.6	22.0	8.9	− 11.8	− 13.3	− 8.3	− 187,066	− 7.4
4. Over 9,000 Kilowatts (Incl. Mercury Boilers)	1,559	1,896,719	38.5	20.2	4.5	− 27.6	− 31.1	− 15.0	− 252,058	− 13.3
Total Steam Turbines	28,279	5,861,952	31.1	20.7	10.4	− 12.0	− 13.5	− 6.0	− 311,826	− 5.3
(b) Water Turbines	1,885	182,486	96.6	46.5	12.1	+ 70.8	+ 79.8	+ 30.2	+ 48,909	+ 26.8

(Continued)

*Per 1,000 Object Years.
**Including Loadings of 50% on Losses and 15% on Inspection Cost. Expense Loading = 51%.

ANALYSIS OF MACHINERY EXPERIENCE—ALL COMPANIES—U. S. A. Exhibit C (Continued)

EXPERIENCE OF CALENDAR YEARS 1936–1940

Classification Group	Earned Object Years	Earned Premiums at Current Rates	Loss Frequency*	Loss Ratio at Current Rates	1939-40 Average Inspection Ratio	Indicated Premium Adjustment**	Indicated Object Charge Adjustment	Selected Object Charge Adjustment	Effect on Premium at Current Rates	
									Amount	Percent
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
7. Electric Generators										
(a) 100 Kilowatts or less	16,456	$322,030	25.3	12.7%	36.0%	+ 23.5%	+ 26.5%	—	—	—
(b) 101 to 1,000 Kilowatts	9,199	636,901	32.7	18.2	15.8	− 7.1	− 8.0	—	—	—
(c) 1,001 to 9,000 Kilowatts	498	80,952	60.2	42.6	12.4(Est.)	+ 59.6	+ 67.2	—	—	—
(d) Over 9,000 Kilowatts	31	6,351	64.5	13.6	6.0(Est.)	− 44.3	− 49.9	—	—	—
Total	26,184	1,046,234	28.6	18.4	21.6	+ 6.9	+ 7.8	—	—	—
8. Electric Motors										
(a) 5 H.P. or less	41,054	266,541	32.2	15.1	71.2	+ 113.5	+ 128.0	+ 50.0	+ 118,200	+ 44.3
(b) 6 to 25 H.P.	29,286	521,268	32.0	12.3	28.9	+ 5.5	+ 6.2	+ 10.0	+ 46,232	+ 8.9
(c) 26 to 100 H.P.	26,228	944,792	52.3	19.5	17.9	+ 1.8	+ 2.0	+ 2.0	+ 16,759	+ 1.8
(d) 101 to 1,000 H.P.	12,148	882,038	58.6	21.5	11.6	− 6.9	− 7.8	—	—	—
(e) Over 1,000 H.P.	1,020	195,220	61.8	27.1	5.7	− 3.5	− 3.9	—	—	—
Total	109,736	2,809,859	40.1	18.9	21.7	+ 9.0	+ 10.1	+ 7.3	+ 181,191	+ 6.4
9. Rotary Converters	550	99,616	158.2	35.6	11.1	+ 35.1	+ 39.6	+ 32.8	+ 28,956	+ 29.1
10. Transformers and Induction Feed Regulators										
(a) 25 Kilowatts or less	7,462	60,881	10.2	11.5	37.3	+ 22.9	+ 25.8	+ 25.0	+ 13,499	+ 22.2
(b) 26 to 200 Kilowatts	12,149	227,526	10.5	13.4	20.5	− 10.8	− 12.2	+ 10.0	+ 20,180	+ 8.9
(c) 201 to 1,000 Kilowatts	4,424	268,137	15.8	23.1	11.0	− 3.3	− 3.7	− 10.0	− 23,782	− 8.9
(d) Over 1,000 Kilowatts	1,339	163,678	21.7	15.8	8.5	− 31.6	− 35.6	− 25.0	− 36,292	− 22.2
Total	25,374	720,222	11.9	17.4	15.7	− 9.8	− 11.0	− 4.1	− 26,395	− 3.7

(Continued)

*Per 1,000 Object Years.
**Including Loadings of 50% on Losses and 15% on Inspection Cost. Expense Loading = 51%.

ANALYSIS OF MACHINERY EXPERIENCE—ALL COMPANIES—U. S. A. Exhibit C (Continued)

EXPERIENCE OF CALENDAR YEARS 1936–1940

Classification Group	Earned Object Years	Earned Premiums at Current Rates	Loss Frequency*	Loss Ratio at Current Rates	1939-40 Average Inspection Ratio	Indicated Premium Adjustment**	Indicated Object Charge Adjustment	Selected Object Charge Adjustment	Effect on Premium at Current Rates	
									Amount	Percent
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
11. Other Electrical Machinery	59,123	$1,105,966	36.2	18.5%	11.8%	− 15.5%	− 17.5%	—	—	—
Total Electrical Machinery (Items 7-11)	220,967	5,781,897	34.8	18.8	18.8	+ 1.6	+ 1.8	+ 3.6	+ 183,752	+ 3.2
12. Personal Injury	106,381	132,119	—	—	—	− 100.0	− 100.0	− 50.0	− 66,060	− 50.0
13. Miscellaneous Direct Damage	6,794	17,811	0.3	0.2	5.0(Est.)	− 87.6	− 87.6	—	—	—
Total Machinery Direct Damage (A)		16,766,335		23.3	16.1	+ 9.2	+ 10.4	+ 5.7	+ 848,381	+ 5.1
14. Use and Occupancy										
(a) Fly-wheel, Engine & Turbine	8,155	2,174,287	120.3	36.4	5.0(Est.)	+ 23.3	+ 23.3	+ 32.4	+ 704,469	+ 32.4
(b) Electrical Machinery	9,551	1,757,491	100.9	37.8	5.0(Est.)	+ 27.6	+ 27.6	+ 17.1	+ 300,531	+ 17.1
Total	17,706	3,931,778	109.8	37.1	5.0(Est.)	+ 25.5	+ 25.5	+ 25.6	+ 1,005,000	+ 25.6
15. Consequential Damage	1,860	126,800	91.4	42.5	5.0(Est.)	+ 42.0	+ 42.0	+ 25.0	+ 31,700	+ 25.0
16. Expediting Charges	4,515	76,452	2.7	6.8	5.0(Est.)	− 67.3	− 67.3	− 33.3	− 25,459	− 33.3
17. Outage	1,549	108,751	106.5	69.9	5.0(Est.)	+ 125.9	+ 125.9	+ 78.0	+ 84,826	+ 78.0
18. Loss of Use—Power Interruption	427	59,161	796.3	74.5	5.0(Est.)	+ 140.0	+ 140.0	+ 100.0	+ 59,161	+ 100.0
19. Miscellaneous Indirect Damage	45	5,145	177.8	53.9	5.0(Est.)	+ 76.9	+ 76.9	+ 42.0	+ 2,161	+ 42.0
Total Machinery Indirect Damage (B)		4,308,087		38.0	5.0(Est.)	+ 28.2	+ 28.2	+ 26.9	+ 1,157,389	+ 26.9
Total—All Machinery (A)+(B)		21,074,422		26.3	13.8	+ 13.1	+ 14.4	+ 10.4	+ 2,005,770	+ 9.5

*Per 1,000 Object Years.
**Including Loadings of 50% on Losses and 15% on Inspection Cost. Expense Loading=51%.

The foregoing exhibits show that the rates for many classification groups were seriously out of line with the experience indications. However, the relatively small exposure and low accident frequency rate for many groups mean that for these groups very little credence may be given to the loss experience, and the underlying expected losses must be considered as well as the actual losses in determining premium adjustments. This does not hold for inspection expense, however, which should receive full credibility except for those few classes with such small exposure that the fortuitous occurrence or omission of a few inspections during the calendar period would have a measurable effect.

In selecting the actual object charge adjustments to be made, consideration was given to these points, although no formula was employed. This underwriting judgment was influenced to some degree by the desire to avoid disturbing the business. It had been so long since the rates had been changed for many classifications, that most underwriters felt that large indicated increases or decreases should not be effected in one adjustment, but should be spread over several years. The net over all result of the selected rate changes was to produce for boiler insurance 52.5% and for machinery insurance 72.5% of the net increase indicated by the experience with the losses increased 50% and the inspection cost 15%. The boiler rates were increased by 10.6% over all compared with an indicated increase of 20.2%, and the machinery rates were increased 9.5% compared to the indicated of 13.1%.

After the rate level adjustments by classification group had been determined a set of insurance charges was selected to replace the object and location charges for boiler risks. Although an all-company distribution of basic and location charges by accident limits and a policy count were not available, the best judgment of the underwriters, based in part upon an earlier test of these insurance charges, was that the new insurance charges would produce 30% less premium than the basic and location charges. Since these basic and location charges had comprised 33.5% of the total premium for objects with direct damage coverage $\left(1.000 - \frac{1.0000}{1.5034} = .335\right)$, it was necessary to increase the object charges on these classes by 15% to obtain the same amount of premium, as shown in the following calculation:

$$.665X + (.335 \times .70) = 1.00$$
$$X = 1.15$$

The next step was the introduction of graded company expense and acquisition. The statistical plan did not provide a distribution of premium by size of risk, but in May of 1940 five companies had made a study of their risks by premium size covering several years, and then combined their results, so that there was available a boiler and machinery size of risk distri-

bution based upon more than $25,000,000 of premium. The experience for the boiler and machinery lines was combined to produce a uniform basis of expense and acquisition graduation, and the following adjusted distribution was obtained:

First	$ 500	Manual	Premium	48.8%
Next	2,500	"	"	25.7
Over	3,000	"	"	25.5
				100.0

The adjustment made was to offset the increase in the average risk premium produced by the new rates and to allow for the expected size of risk increase due to increased industrial activity. In the actual distribution, 10% of the premium under $500 per risk was transferred on a judgment basis to the higher premium groups and distributed proportionately over these two size groups to produce the final adjusted distribution.

With the premium distribution by size of risk determined, consideration was then given to the degree to which company administration and other production expense should be graded. Although accurate cost records by size of risk are not available and would be extremely difficult to maintain, it is known that there is an appreciable percentage differential in the administration cost between large and small boiler and machinery risks. With the lack of exact information, recourse to judgment was necessary. Experienced opinion was that a 30% increase in the administration and other production provision on the small risks would closely reflect actual costs, and that for the very large risks the provision could be reduced by 50% or slightly more. Commission was to be payable at the regular rate on the net premium determined from the first $3,000 of full manual premium, and at 60% of the regular rate on all net premium over this amount. Based upon the above, and with the provision that the graduation overall should have no effect in the total amount available for administration and other production, losses and inspection and claim, it was calculated that an increase in manual premium level of 8.5%, with a 10% reduction in all manual premium over $500 up to $3,000 and a 31% reduction in all manual premium over $3,000 per risk would produce the desired result. The tax, profit and contingency items were left at 10% of the final premium. The following exhibit (Exhibit D) shows the distribution of the various items in the three premium groups, as well as the net over all effect. It will be noted that the manual premium distribution differs from that shown in Exhibit A, in that acquisition is shown at 25% instead of 30%, while the administration item of 9% in Exhibit A has been changed to administration and other production at 14%. The reason for this is that 5 points out of the 30 available for acquisition are not for commissions, but are for the expense of salaried company boiler and machinery rating experts who assist in the

solicitation and rating of risks. The top boiler and machinery commission paid is 25%, except in a very few instances where a registered producer has his own salaried expert.

Exhibit D

BOILER AND MACHINERY INSURANCE
Distribution of Expense Items — Graded Expenses*

Item	Former Manual Provision	New Manual Provision	Manual Prem. $500-$3,000 per Risk		Manual Prem. Over $3,000 per Risk		Net Overall Result
			Amount	Per cent	Amount	Per cent	
Acquisition	.250	.271	.244	25.0%	.112	15.0%	.224
Admin. & Other Prod.	.140	.182	.137	14.0	.063	8.4	.140
Claim	.020	.020	.020	2.0	.020	2.7	.020
Loss & Inspection...	.490	.503	.478	49.0	.479	63.9	.491
Tax, Profit & Contingency	.100	.109	.098	10.0	.075	10.0	.097
TOTAL.....	1.000	1,085	.977	100.0	.749	100.0	.972

* Ratios shown are in terms of former manual provision.

The slight variation by size of risk in the provision for losses and inspection was necessary to balance out the various expense items with the total premium for each size group, and still maintain the same net overall amount available for administration and other production, losses and inspection, and claim. In the actual application of the plan, the 31% discount on all manual premium over $3,000 has been rounded to 30%.

Since the manual premium level had to be increased by 8.5%, adjustment was made in all manual rates except the boiler and machinery insurance charges. As a result the manual rates for objects with direct damage coverage were increased by 8.5% times 1.5034 for boiler and 1.1275 for machinery. Factors used were rounded to two decimal places and were 1.13 for boiler and 1.10 for machinery direct damage objects and 1.09 for all other manual rates. In the actual determination of manual rates, a single factor for each type of object was used to apply to the old manual rates. This factor was obtained by adding the increase factors for the change in the basic and location charge and the increase factor for the premium graduation to unity and multiplying by the experience adjustment factor. The following example shows the calculation of the multiplier to be applied to the old manual rate for fire tube boilers over 15 lbs. pressure:

Increase factor for change in basic & location charges	.15
Increase factor for premium graduation	.13
Total plus one	1.28
Experience Adjustment Factor ×	1.40
Final factor	1.79

The following exhibit (Exhibit E) shows the various factors making up the final multipliers used in the manual revision by classification for boiler and machinery.

Exhibit E

Boiler and Machinery Rates

Summary showing effect of various rate adjustments

Manual Page No.	Classifications	Experience Adjustment Factor	Manual Rate Multipliers			
			Insurance Charge Factor*	Premium Graduation Factor	Colums (4) + (5)	Total (3) × (6)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
	I. Boiler					
B210	Insurance Charges*.....	New Table	..	..	..	..
B211	Fire Tube Boilers					
	Low Pressure	1.17	1.15	1.13	1.28	1.50
	High Pressure	1.40	1.15	1.13	1.28	1.79
B212	Water Tube Boilers					
	Low Pressure	1.17	1.15	1.13	1.28	1.50
	High Pressure	1.25	1.15	1.13	1.28	1.60
	(Mostly on sizes below 10,000 sq. ft.)					
B213	Cast Iron Boilers					
	Explosion Only	1.17	1.15	1.13	1.28	1.50
	Cracking					
	Horiz. Group 1.....	0.90	..	1.09	1.09	0.98
	Horiz. Group 2.....	0.90	..	1.09	1.09	0.98
	Horiz. Group 3.....	1.20	..	1.09	1.09	1.31
	Horiz. Group 4.....	1.30	..	1.09	1.09	1.42
	Total Cracking	0.91	..	1.09	1.09	0.99
B214	Miscell. Fired Vessels					
	Track Locomotives	1.40	1.15	1.13	1.28	1.79
	All Others	..	1.15	1.13	1.28	1.28
	(Some minor revisions for consistency)					
B215	Economizers	..	1.15	1.13	1.28	1.28
B216	Unfired Vessels — Cl. 1					
	Except Digesters	1.40	1.15	1.13	1.28	1.79
	Digesters	..	1.15	1.13	1.28	1.28
B217	Unfired Vessels — Cl. 2...	..	1.15	1.13	1.28	1.28
B218	Unfired Vessels — Cl. 3...	..	1.15	1.13	1.28	1.28
B219	Refrig. Systems — Comp. .	..	1.15	1.13	1.28	1.28
B219b	Refrig. Systems — Abs....	..	1.15	1.13	1.28	1.28
B220	Auxiliary Piping	..	1.15	1.13	1.28	1.28
B230	Residence Boilers & Vessels					
	Direct Damage	..	..	..	..	..
	Furnace Explosion	0.80	..	..	..	0.80
B240	Furnace Explosion					
	Solid Fuel	0.70	..	1.09	1.09	0.76
	Liquid Fuel	0.90	..	1.09	1.09	0.98
	Gas Fuel	..	..	1.09	1.09	1.09
	Pulverized Fuel	..	..	1.09	1.09	1.09
	Total Furnace Explosion	0.89	..	1.09	1.09	0.97
B250	Personal Injury	0.50	..	..	..	0.50
ID122 to 122b	Use and Occupancy.....	1.03	..	1.09	1.09	1.12
ID222	Consequential Damage	..	..	1.09	1.09	1.09
ID321	Power Interruption					
	Loss of Use.....	..	..	1.09	1.09	1.09
	Spoilage	0.80	..	1.09	1.09	0.87
ID400	Expediting Charges	0.50	..	..	..	0.50
ID522	Outage	1.78	..	1.09	1.09	1.94

* The new Boiler Insurance Charge Table results in a 30% reduction from old Basic and Location Charges.

Exhibit E (Continued)

BOILER AND MACHINERY RATES

Summary showing effect of various rate adjustments

Manual Page No.	Classifications	Experience Adjustment	Manual Rate Multipliers: Premium Graduation Factor	Manual Rate Multipliers: Total (3)×(4)
(1)	(2)	(3)	(4)	(5)
	II. MACHINERY			
M210	Insurance Charges	..	..	1.00
M211	Steam Engines	1.57	1.10	1.73
M212 to M212e	Int. Comb. Engines	..	1.10	1.10
M213 to M213e	I. C. Compressors or Pumps	..	1.10	1.10
M214 to M225	Recipr. Compr. & Pumps	..	1.10	1.10
M227 to M227f	Miscell. Machines			
	Class 1	..	1.10	1.10
	Class 2	1.50	1.10	1.65
	Class 3			
	Type 1	1.50	1.10	1.65
	Type 2	1.25	1.10	1.38
	Type 3	1.50	1.10	1.65
	Type 4	..	1.10	1.10
	Class 4	..	1.10	1.10
M228 & M228a	Wheels			
	Balance Wheels Pulleys & Couplings.	1.30	1.10	1.43
	Gear Wheels	..	1.10	1.10
	All Other	1.50	1.10	1.65
M229	Shafting	1.50	1.10	1.65
M230 to M234	Steam Turbines	0.94	1.10	1.03
	*Breakdown (Gen.)	0.87	1.10	.96
	*Breakdown (Not Gen.)	1.03	1.10	1.13
	*Limited Breakdown	0.75	1.10	.83
	Combined Coverage	1.09	1.10	1.20
	Explosion Coverage	1.31	1.10	1.44
M235 to M239	Water Turbines	1.30	1.10	1.43
	Breakdown (Gen.)	..	1.10	1.10
	Breakdown (Not Gen.)	1.50	1.10	1.65
	Limited Breakdown	..	1.10	1.10
	Combined Cov.	1.30	1.10	1.43
	Explosion Cov.	2.50	1.10	2.75
M240	Electric Generators	..	1.10	1.10
M241 & M242	Electric Motors	1.07	1.10	1.18
	5 H.P. or less	1.50	1.10	1.65
	6 to 25 H.P.	1.10	1.10	1.21
	26 to 100 H.P.	1.02	1.10	1.12
	Over 100 H.P.	..	1.10	1.10
M243	Rotary Converters	1.33	1.10	1.46
	100 K.W. or less	2.00	1.10	2.20
	101 to 1,000 K.W.	1.35	1.10	1.49
	Over 1,000 K.W.	0.80	1.10	.88

* Rate reductions (Breakdown & Ltd. Br.) about 25% over 9,000 kw. Rate increases on all sizes 100 kw. or less. The Multipliers shown are average for all sizes.

Exhibit E (Continued)

Manual Page No.	Classifications	Experience Adjustment	Manual Rate Multipliers: Premium Graduation Factor	Manual Rate Multipliers: Total (3) × (4)
(1)	(2)	(3)	(4)	(5)
	II. MACHINERY (continued)			
M244	Transformers	0.96	1.10	1.06
	25 K.W. or less	1.25	1.10	1.38
	26 to 200 K.W.	1.10	1.10	1.21
	201 to 1,000 K.W.	0.90	1.10	.99
	Over 1,000 K.W.	0.75	1.10	.83
M245	Motor Control Equip.	..	..	1.00
M246	Misc. Elect. Apparatus	..	1.10	1.10
M247	Small Refr. & Comp. Mchs.	..	1.10	1.10
M248	Deep-Well Pump Units	..	1.10	1.10
M249	Air Conditioners	..	1.10	1.10
M250	Personal Injury	0.50	..	0.50
ID122c to ID122f	Use and Occupancy			
	Fly-wheel Engine & Turbine	1.32	1.09	1.44
	Electrical Machinery	1.17	1.09	1.28
ID222	Consequential Damage	1.25	1.09	1.36
ID321 & 322	Power Interruption			
	*Loss of Use	2.00	1.09	2.18
	Spoilage	1.50	1.09	1.64
ID400	Expediting Charges	0.67	..	0.67
ID522	Outage	1.78	1.09	1.94

* Multipliers adjusted so that practically no increase for Group 1, up to 150% increase for Group 5.

At the time of the rate revision, the statistical plan was amended to provide for the reporting of experience on a full manual premium basis, with premium discounts to be reported separately. A size of risk code was provided to be used for all premium, exposure and loss records, and a policy count was set up. With this information available, the manual premium multiplier used to offset the effect of the premium discount may be accurately determined and adjusted when necessary on the basis of all company experience.

At the time of the introduction of graded expense for boiler and machinery insurance, there was some discussion as to whether or not there existed a loss ratio differential by size of risk. Some individuals felt that there probably was, since the large risks frequently employ full time experienced engineers to operate and maintain their equipment, and are also more apt to set up adequate depreciation reserves so that objects will be replaced before they become dangerously old and worn out. However, there was no unanimity on this point, and no statistical experience was available. The amendment

to the statistical plan will provide loss experience by size of risk which will show whether or not a differential exists, and if such be the case, whether such differential is great enough to receive recognition.

Points in the Rating Procedure Offering Possibilities for Further Investigation

1. A review of the indicated object charge adjustments shown in Exhibits B and C indicates the advisability of more frequent truing up of the Boiler and Machinery classification rates. The long intervals of time between complete manual revisions seems to indicate that although attention is paid to the overall rate levels there is not enough credibility assigned to indicated changes in the classification loss and inspection cost relativity. It would seem to be desirable to review the classification experience at regular intervals, every three years for example, and to have a definite formula for the adjustment of classification relativity. One possibility would be to use the five latest calendar years' experience for losses and to apply credibility to the indicated versus the underlying on the basis of a credibility table combining both the number of objects and the loss frequency. Such a table has been shown in a paper by Mr. Barber entitled "Suggested Method for Developing Automobile Rates." This table is shown on pages 219 to 222 of Volume XV of the Proceedings of the Casualty Actuarial Society. For inspection cost the two latest available calendar years combined might be used with 100% credibility applicable in all cases except where the number of earned objects is so small that the inspection cost may be unreliable. In those cases either additional years of experience might be used or else the indicated inspection cost might be modified by analogy to other objects. If such a regular program for the truing up of Boiler and Machinery classification rates were to be introduced, when changes are made, there would not occur the violent fluctuations in the manual premium for individual objects such as was occasioned by the recent manual revision.

2. The second recommendation for possible investigation regards the large number of individual rates contained in the present manual. The small exposure involved in many classifications seems entirely inadequate to justify the great number of separate individual rates for every size and type of insurable object. A great many of these individual rates are based upon very slim loss data and by no inspection expense statistics at all. There may be a theoretical loss and inspection cost differential between all the various types and sizes of objects but considering the very small exposure it would seem to be much more desirable to extend the averaging process by combining many of the present individual objects into a smaller number of classifications for which rates can be justified by statistical data.

3. The third possibility for investigation is in the rates for certain extremely infrequent objects which are tailor-made to fit the power plant requirements of individual risks. An investigation into the cost and hazards involved might indicate that the use of "A" rating under the control of the Bureau, similar to that used in other casualty lines, might produce object premiums more closely akin to the actual risk requirements than that produced at present by the compulsory use of a manual rate. If this were found to be true and the change adopted, the manual would be relieved of a large number of classification rates for which the number of insurable objects is almost negligible and at the same time would produce a loss and inspection pure premium which would provide enough money to cover the object inspections required in the individual case and to cover the estimated loss hazard which would vary from risk to risk.

4. A review of the experience for the personal injury coverage for both Boiler and Machinery shows that the loss ratio is so low and the premium charge for this coverage so small for individual risks that it would probably be desirable to automatically include this coverage in all policies for no premium charge. In many instances the cost of providing this coverage on the policies is almost as much as the premium involved, and it would seem to be more practicable to automatically provide the coverage for all risks and to include the personal injury losses with the property damage losses for each individual classification for experience purposes.

5. Although the recent introduction of insurance charges to replace the old Boiler and Location charges represents a forward step, the present system still seems to offer the opportunity for critical review. It seems quite desirable that an investigation be made into this situation with the object in mind, if the investigation so justifies, of eliminating the present Boiler and Machinery insurance charges entirely and substituting therefor a single manual object premium to be charged for each object insured. This manual object rate would be for a definite standard limit per accident ($5,000 for example), and increase limits premium would be obtained by the application of factors from increased limits tables. The objects could be divided into several groups reflecting the relative seriousness of high limits hazard, with corresponding increased limits tables for each group. The additional inspection dollars on multiple location risks which now consist of a portion of the insurance charge collected for each location might be more appropriately charged through the application of a territorial differential reflecting the accessibility of the risk.

The reason for recommending an investigation as to the feasibility of this change is that in the author's opinion the following drawbacks exist under the present system of Boiler and Machinery insurance charges. The first

drawback is the present method of charges for increased limits. The net effect of current insurance charges is to give a flat dollar premium per location for excess limits regardless of the hazard involved. For certain objects the premium is unwontedly high and in other instances seems to be grossly inadequate, although for all classifications combined the net dollar income for increased limits is amply sufficient to pay for serious accidents when they occur. For example, a risk involving small air tanks spread over a large number of locations is required to pay for each location a charge of $33 for losses in excess of $1,000 per accident up to $50,000 per accident. On the other hand a risk with several large high pressure water tube boilers in one location is required to pay a single premium of only $33 for limits in excess of $1,000 up to $50,000 per accident. It seems patently absurd to charge a much greater premium in the first case where the possibility of an expensive loss is infinitely small compared to the possibility for the risk with the large high pressure boilers. For a great many objects the individual increased limits premium is far out of line with the actual hazard involved and the risk and the company frequently do not have the advantage of a blanket limit per accident since there is a material incentive for assureds to purchase varying limits per object, for separate locations, to avoid the discrimination in increased limits charges for the small objects with very little hazard.

The second drawback is in the collection of increased inspection dollars for multiple location risks. Since a large part of the additional inspection expense for risks of this type is in traveling time, consideration should be given to the accessibility of the risk and its location rather than just to the number of locations. Risks in rural districts or in certain western or southern states are frequently in locations far removed from any inspection office and require appreciably more money to service than the average, while on the other hand a large city school risk involving many locations might require only five or ten minutes traveling time between each location. Further, since the charge for each location also includes excess limits premium it is very difficult to determine overall just how much additional inspection expense provision the company does collect and whether the amount is inadequate or redundant.

The third drawback is in the basis for the determination of manual rates. The all company statistics compiled by the Bureau for the basis of rate making provide that the total dollar income from insurance charges shall be distributed pro rata over the object charge premiums. Although the present Boiler insurance charges are appreciably lower than the old basic and location charges they will still require approximately a 35% flat loading on the earned object premiums for all classifications. This result will produce experience indications for certain classifications that are at an appreciable

variance from the actual conditions. As extremes consider the case of a small air tank where the minimum insurance charge is 100% of the object charge and large high pressure water tube boilers where the insurance charge may be less than 10% of the object charge. Of course, this situation is offset to some degree by the fact that assureds as a rule purchase low limits for low rated objects and high limits for high rated objects, but appreciable distortion of the experience is still present.

AN APPROACH TO A PHILOSOPHY OF SOCIAL INSURANCE

BY

JARVIS FARLEY AND ROGER BILLINGS

Webster defines a philosophy as an integrated and consistent personal attitude toward life or reality, or toward certain phases of it. That is another way of saying that a philosophy is a set of intelligent opinions about a phase of living, based on observation, sound reasoning, and common sense. A philosophy of social insurance consists of a set of intelligent opinions about social insurance—about its purpose, its possibilities, its limitations; about the relative advantages and disadvantages of alternative forms of social insurance; and most important of all, about the relationship of social insurance to our daily life as a whole.

The principles of the Atlantic Charter are here to stay. Any questions about social insurance involve only the method of achieving the highest degree of security for the individual. Individual security is not itself an issue; no advocate of any special method of attaining social security has any monopoly on the desire to improve the welfare of the less fortunate members of society. Our problem is not merely to answer the question, "Is social insurance a good thing to have?", but also to determine the place of social insurance in our daily life. It is not enough to have experts put social insurance under a miscroscope, to analyze it and discuss it in order to be able to say, "This is good" or "That feature is bad." It is not enough to consider social insurance as though it were something entirely independent and separate from the other aspects of our daily life. The expert's study must be made, and the specific opinions must be formed; but in addition we must also have informed opinions about the relationship of social insurance to the other social and economic and cultural values which together make up our life. A complete philosophy of social insurance includes opinions not only on the absolute merit of specific proposals but on the worth of such proposals compared to other values which we want to have as a part of our daily living.

The people who make up a democracy can exercise their free power of choice. They can have social insurance if they want it. Subject to practical limits, they can have as much or as little of it as they want, but in making the choice they must recognize that no magic of government can give something for nothing. If the American people want a broad program of social insurance they will most assuredly have it, but in deciding how much social insurance they want they must recognize that it may not be a question of having social insurance *in addition* to other things which they want. It may be necessary to make a choice between social insurance and something else

on which they set great store. The question is not, "Is social security a thing worth having," but rather "Do we want social insurance enough to pay the price, or would we rather use our labors to buy something else?"

What Is Social Insurance?

A Committee Report from the American Association for Social Security includes the following statement: "The underlying basis for social security is not a new-fangled theory of governmental pampering of the individual, but part and parcel of the deeply rooted primitive desire for social preservation. The undersigned . . . regard social security not as a system negating free enterprise but, on the contrary, as a prerequisite to the healthy functioning of our existing economy."[1]

We are living in a complex economic society in which the direct connection between production and consumption is not always apparent. Man must still labor to produce his food, his shelter, his clothing, and his means of enjoying leisure time. In our modern society few of us work directly to grow our own food or weave the cloth from which our clothes are made—we work at jobs for which we are trained, and we receive payment for that work in the form of money with which we can buy the product of other men's labor. When anything happens to prevent us from working at our jobs, our only sources of money to pay for those things which we need are our own savings or someone else's donations. The savings may be in the form of accumulated capital or they may take the form of payments under a plan of insurance; and the contributions are either direct charity or governmental relief.

The most common causes of loss of income are disability, old age, unemployment, and in the case of those unable or unaccustomed to work outside the home, the loss of the income producer upon whom they are dependent. Our society has developed the device of insurance whereby numbers of people facing a given hazard share among themselves the agreed cost of those individuals for whom the hazard materializes. To an individual, cost is measured in terms of dollars and cents, but for a nation real cost is measured in terms of its standard of living—its hours of work, its hours of leisure, and the material and intangible values which determine the level of life. The cost of the social hazard arises out of the loss of the productive capacity of those who are unable to work, and from the necessity of providing them and their dependents with the elements of decent living, plus an intangible element of cost resulting from the adverse effect on the morale of the unproductive individuals who are forced to rely on relief or charity. Voluntary insurance methods have been established for distributing the social

[1] "Can We Stop a Post-War Depression Now? Social Security in Wartime and After." American Association for Social Security, New York, 1942.

costs resulting from the disability or death of the income producer—and life insurance and disability insurance have firmly established themselves in our society. Insurance, however, requires setting aside current income against current and future contingencies rather than for present enjoyment. Our salesmen of automobiles and radios have been so efficient and our credit facilities so easily available that many people spend their entire income for current enjoyment and fail to make the provision which they can afford to make against the day when the income producer will be too sick or too old to work. The result is that when disability or old age comes the family must look for support to charity or relief, with the corresponding impairment of self-respect and morale which goes hand in hand with dependence on charity.

Disability, unemployment, old age and death will always be with us, together with their accompanying social costs. In order to avoid the discontent and demoralization bred by the omissions and discriminations resulting from haphazard methods of distributing these social costs, compulsory insurance is proposed as a means of budgeting for the costs on a national scale. Social insurance does not necessarily mean free insurance or subsidized insurance. Social insurance is essentially a budgeting device whereby insurance methods are used to enable an entire nation to share the cost of social losses. A social insurance program would compel a minimum level of insurance so that we as individuals would be forced, to the extent of that minimum, to refrain from spending for current enjoyment and to apply the requisite portion of our current income toward meeting our share of the insured hazard, whether it be disability, unemployment, old age or death. If we are compelled to make insurance provision against these hazards, the occurrence of the hazard will not find us destitute and will not force us to apply for relief, save in cases of exceptional need. Income provided by insurance has the advantage that being given as a right it does not produce the relief complex and the impaired morale that goes with providing support in the form of relief. Many advocates of social insurance believe that the federal government itself should undertake to provide and administer this means of national budgeting of the cost of these hazards.

There can be little question that the goal of fair and equitable sharing of the cost of these social hazards is a desirable thing. The questions which arise in connection with social insurance do not concern the goal, but the methods proposed for reaching that goal; whether those methods will in the long run actually produce the desired result, how much they will cost us, and to what extent the probable attainment may be worth the price which must be paid.

Social insurance is distinguished from private insurance by the part which government plays. In private insurance the role of government is limited

to general regulation and supervision, to protect the public against possible abuses and to guard the financial stability of the insurance company. Insurance against these hazards becomes social insurance when the government's role is extended beyond the regulatory function, either by making the purchase of insurance compulsory on certain classes of citizens or by actually underwriting and assuming the risk, or both. Actual assumption of the risk by government is not a necessary characteristic of social insurance. Workmen's compensation insurance, for example, is written in this country primarily by private companies, but it is social insurance because most employers are effectively compelled to arrange the insurance for the benefit of their employes. Any form of insurance against these social hazards becomes social insurance when it is accompanied either by compulsion or by government administration.

Use of the term "social insurance" does not deny the broad social value of the other insurances with which the word "social" is not used, nor does it question the usefulness of voluntary private insurance in distributing the social cost of hazards threatening the economic security of individuals. The word "social" is used in recognition of the part which society, through organized government, plays either in compelling its citizens to make use of the insurance facilities, or in providing those facilities itself.

The most widely known social insurance in this country is that established by the Social Security Act and covering, in the main, only the employes of non-municipal, non-charity, non-agricultural establishments. Agricultural workers, employes of municipal and charitable corporations, self-employed people and unemployed people are not covered. The Old Age and Survivors Insurance section of the present Social Security program provides in effect that those to whom it applies must contribute toward old age annuities underwritten by the federal government. Provision is also made for benefits to widowed mothers and dependent children, and to certain other dependent types of individuals. The Unemployment Compensation section of the program promotes a form of compulsory unemployment insurance underwritten by state governments.

Many advocates of social insurance propose that unemployment insurance be transferred completely to the federal government and that the Social Security program be enlarged to provide federally administered compulsory insurance against long-term disability, against the costs of hospitalization and medical care, and even against occupational injury, thereby replacing workmen's compensation insurance as we now know it. They also propose that the number of people compelled to buy such insurance be increased to include a far greater proportion of the population than is affected by the present law.

The premium for this insurance would be paid in the form of taxes which

might be equal to 15% or 20% or more of the individual's income. Any program so comprehensive and costly, and so full of forces which can make the most tremendous and far-reaching changes in our every day living conditions, puts on its sponsors a responsibility too great to be borne unless the program has the intelligent support and critical understanding of our entire people. Proposed methods of administering the program may be more costly than an alternative method of providing the same benefits. The cost of the entire program may be greater than our people are willing to bear, or even able to bear, if the administrative methods are not efficient. Every citizen has the responsibility of constructive criticism to help define the goal and of honest thought and advice as to how extensive a program we want to pay for. Our leaders have a responsibility to advise us honestly and fully so that as a nation we may better understand the decisions which we must make and what the results of our decisions may be.

The fundamental issue in considering the adoption of social insurance methods for any social hazard is whether insurance against that hazard should be compulsory up to some level or whether we should rely entirely upon voluntary insurance, supplemented by charity and relief, to distribute the cost of providing incomes to the victims of the hazard. A second issue is whether the conduct of the insurance is a proper matter for government administration or whether it should be handled by private enterprise. A third question involves jurisdiction; that it, if the answers to the first questions indicate a desire for social insurance, how should the responsibility for the program be divided between the federal government and the state governments. The answers to these questions may be different for various social hazards.

The Issue of Compulsion

The issue of compulsion requires consideration of the extent of the social need, of the availability of effective methods of meeting the need, and of the relative cost of alternative methods. There can be little doubt that if we can find effective methods of obtaining the advantages of social budgeting of the cost of an expensive hazard, those advantages are worth paying a fair price. Only if the price is too great in terms of reduced living standards or impaired freedom would we hold back from a reasonable attempt to obtain those advantages.

Life insurance, annuities, disability and hospitalization insurance, all on a voluntary basis, have a recognized place in our economy. Group insurance reaches a very large number of our employed citizens, and it has been suggested that it is a short and logical step from group insurance to social insurance. In a very real sense social insurance is simply compulsory group insurance. There is no area of need, however, in which compulsion would

initiate the purpose to provide support for those in want. The purpose is already firmly established. Those income producers who have not made voluntary provision for need must rely on charity or relief when a social hazard strikes, but these sources of support already exist. Compulsion would not create support; the effect of compulsion would be simply to require the use of insurance methods so that the support is provided as a contractual right rather than by charity. There can be no doubt that there is more mental and emotional satisfaction in receiving a check for insurance benefits than in receiving a relief payment of the same amount. For that very reason, however, benefits provided by insurance as a guaranteed right will result in greater social cost than relief benefits, even though the average payment might be identical. Every increase in the level at which society supports its currently unproductive members requires a corresponding contribution from its productive members, and every such increase alters the balance between the incentive to continue working and the temptation to fall back on the benefits which society will provide. If the receipt of benefits is not attractive there will be greater incentive to avoid the condition which qualifies for benefits; but when a not-unattractive benefit is given as a right there is less incentive to continue working. This is an important reason why budgeting of social costs by insurance methods is more expensive than relief in terms of the proportion of the national wealth required to support the social loss.

Advocates of social insurance have urged that if there is only a small proportion of cost to which insurance methods are not already applied, it will make little difference if the insurance is made compulsory, but that if there is a great area of cost to which insurance methods have not been applied, then we need compulsory insurance. If for any hazard voluntary insurance were actually doing most of the cost distribution, then we would have already paid most of the price of introducing insurance methods, and making those methods compulsory would add little in terms of cost—provided that the methods which were made compulsory were efficient and would not themselves bring a further increase in cost. On the other hand, if there is a substantial part of the social cost to which insurance methods have not been applied—and such is quite generally the case—then we must consider not only whether the proposed compulsory methods would be efficient, but also to what extent the mere extension of efficient insurance methods would increase the cost of the hazard. It is a job for experts to estimate the cost of applying compulsory methods to these risks; and it is the obligation of our national leaders neither to overstate the need for the insurance nor to underestimate the cost, but to tell the nation candidly and to the best of their ability the actual facts upon which the decisions must be made. History will deal most kindly with the leader who takes us into

his confidence and guides us down the paths of truth; and our people will not deal kindly with leaders who attempt to persuade us with unsound and over-optimistic appraisals of cost and who refuse to face publicly the possibility of higher cost.

If we decide that we want to pay the price of compulsion we must still make sure that we can feasibly follow through with enforcement. The payment of benefits presumes, first, the establishment of a right to the benefits, and second, the payment, somehow, by someone, of the cost of the benefits. Under some assumptions the establishment of the right to receive the benefits may not be difficult whatever the status of the individual. The payment of the cost, however, presumes either a premium or a tax; and before any right to benefits is established we must be sure that the premium or tax can be collected. Our experience with compulsory insurance has been limited almost entirely to the people from whom the premium or tax can be collected most easily and efficiently by withholding at the source through the use of the formal payroll accounting facilities of employers. The machinery for collecting premiums is relatively simple under these circumstances. Any broad extension of coverage to other groups in the population could not use this collection device. The collection of a premium from every household worker and from every farm laborer, from every doctor, every lawyer and every independent salesman—from all persons whose income is not subject to payroll accounting—would require more costly and elaborate methods of collection and enforcement; and before the entire population could be covered the point might be reached where every additional dollar of tax collected would cost more than a dollar to collect. If this problem can be solved, compulsion can be effective, but unless the problem is solved we must face the possibility that such social insurance as the nation may choose cannot be made available to everyone.

The Issue of Administration

Any estimate of the added cost of social insurance must be based upon stated assumptions as to methods, and the cost of similar benefits may be quite different depending on whether the insurance is administered by government or by private enterprise. Some advocates of government administration have argued that government does not require a profit, and so can provide the insurance more cheaply; and at the Havana Conference[2] the opinion was voiced that social insurance should not be administered by profit-making insurance organizations. The existing Social Security legis-

[2] Second Labor Conference of the American States which are Members of the International Labor Organizations. Havana, Cuba. December, 1939.

lation relies on government administration of the benefits which it established, and the recent proposals for extension of Social Security have assumed government administration.[3]

When should any government extend its activities into the actual administration and conduct of any function? To most Americans it seems that two conditions must exist before any such extension can be justified. First, the function must be considered to be essential to the general welfare; and second, it should be established beyond reasonable doubt that it is not in the public interest to restrict the conduct of the function to private enterprise. If both tests are not recognized in every case then there is no barrier to the government's entering any necessary phase of human economy in competition with private individuals—or to their complete exclusion. Unless both conditions are satisfied in connection with any hazard for which a governmental program of insurance is proposed, then the government can with equal reason enter, or even monopolize, the fields of dairy farming or automobile repair service, or of grocery stores or of labor union management. Unless both conditions are satisfied, the assumption by government of the administration of any insurance becomes socialism and should be entered upon only after full national understanding of that fact, and only if it is the will of the American people that our way of life hereafter include the philosophy of socialism.

Proposals for government's entry into any field may presume originally a very low level of government participation, above which the field would be restricted to private enterprise. This process might prove to be akin to the coming of inflation or the creation of a drunkard. A small inflationary shot, or a first drink, may make the subject feel better without doing noticeable harm—and so with the second and the third. Inflation and alcoholism come when "just one more" so impairs the subject's control that the barriers are lowered and the harmful aspects become acute. The insidious gradual onslaught may be called the "disease" of inflation or of alcoholism; and similarly "just a little socialism" can develop unexpectedly the disease of socialism.

One of the major issues of post-war reconstruction will be whether, for a people capable of self-government, the highest degree of human welfare can best be obtained under socialism or by the operation of enlightened self-interest in a society which allows reasonable scope to private enterprise. A recent publication of the International Labor Office[4] set forth a broad statement of social insurance policy as a part of a program of economic

[3] For example, the so-called Eliot Bill, HR 7534, introduced September 9, 1942; see also "Formulating a Disability Insurance Program," A. J. Altmeyer, published by the Inter-American Committee to Promote Social Security, Montreal, January, 1942.

[4] "Approaches to Social Security," Studies and Reports, Series M (Social Insurance), No. 18, International Labor Office, Montreal, 1942, p. 93.

adjustments aimed to stabilize employment and raise living standards, with the clear implication that the entire program is a responsibility of government. There can be no serious doubt that the promotion of employment, industry, living standards, and so on, is a general goal greatly to be desired. There is serious question, however, of the extent to which direct action by government is either desirable or effective in attaining this goal. Attainment of the ideals suggested by the statement in the pamphlet depends on a program of planning by government, which in turn requires a general surrender of economic freedom and private initiative—for planning is not effective unless the plans are carried out, and state planning can be carried out only if individual freedom of economic action is surrendered in favor of compulsory application of the government's plans.

Our recent high standard of living, and our hope for future higher standards, depend on efficient development and organization of economic endeavor —for only by efficient organization of commerce and industry can we use our natural and human resources to create a high volume of economic values within working periods short enough to leave time for the enjoyment of these values. Less efficient organization would mean either the production of a smaller volume of economic values in the same working time, or else longer hours to produce the same volume—if, indeed, the same high volume can be produced at all with less efficient organization. Either way, living standards are lower; for in the one case there are fewer things to enjoy, and in the other case there is less time left for enjoyment. During the war, and for some time thereafter, we will have a problem to find working-time enough to do the job which must be done—for we have a war to be won and thereafter a peace to be implemented. We have a big stake, therefore, both in the long run and in the short run, in keeping our economy at a high peak of efficiency. The wealthiest man on earth, if he labors sixteen hours a day, does not find in life as much pleasure and reward as a much poorer man with time to enjoy what he has.

Any action which decreases the efficiency of our economic organization automatically lowers the level of the living standard which our economy is capable of maintaining. It does not necessarily follow that every reduction of efficiency is undesirable; for if the economic status of a significant group of our population can be made respectable, or even bearable, only by some sacrifice of the general level, we may prefer to make that sacrifice. To that extent social insurance may itself be a part of the living standard. There is no value, however, in making such a sacrifice if an alternative procedure would improve the status of the less fortunate members at a lower cost in terms of the living standard of the people as a whole; and it would be unfortunate indeed if the desire to eliminate social insecurity led us to adopt measures which so impaired the efficiency of our economy that we

could not longer afford even the systems of charity and relief which today are criticised as inadequate.

Agencies of democratic governments have not been and cannot be truly efficient, if only because there is no way to compel them to be efficient. Unless we are to have a dictatorship we must have "a government of laws and not of men." A government of laws is necessarily relatively inflexible. Routine and red tape take the place of individual judgment, but the organization of an economy will always produce situations which require judgment and not treatment by rule. Even after making every allowance for the difficulties of improvization, our current experiences with wartime rationing and regulation show the extent to which government is forced to sacrifice efficiency to rule and routine—and necessarily so, for absence of rule and routine would open the door to political pressure and abuse of position. Furthermore, however well conceived a program may be, capable administrators are vital to its actual operation. Government has not shown itself capable of attracting and holding such men in sufficient numbers; and often political expediency or jealousy has shackled the abilities of able men who have entered government service. Too often the bureaucrat's primary interest comes to be the nurturing of his own job. To say that government agencies cannot create an economic value as cheaply and efficiently as private enterprise is not a criticism of democratic government, but rather is a recognition of a necessary characteristic of such governments which must be considered in weighing any proposal to extend their functions.

Private industry, on the other hand, must be relatively efficient, for an inefficient concern cannot long exist in the same market with much more highly efficient competitors. Recognition of the profit motive is not protection of special privilege for the selfish benefit of a favored few. With proper safeguards, the profit motive[5] can be an irresistible and socially valuable force to compel a high practical degree of efficiency, which in turn will produce a higher standard of living than could otherwise be possible. The highest values of democracy are attained not by forcing people to act but by recognizing natural incentives so far as possible in such manner that the desired result is obtained by voluntary action. If the welfare of the people is the real aim of government we should allow proper scope to the constructive forces at our disposal and not let any misunderstanding cause us to deny to ourselves the advantages which sound and proper recognition of the profit motive can bring to us. No misunderstanding should be allowed

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5 "Profit," properly defined and understood, is not a residual income made possible by exploiting other agents of production, but is itself an essential cost of production representing primarily the wages of efficient management. The phrase "profit motive" reflects the beneficial effects of such a reward in stimulating the development and maintenance of an efficient economy and full production, which alone can make possible the highest standard of living and the fullest degree of employment.

to be the cause of instituting a degree of control under which the benefits would be outweighed by impaired efficiency and lowered capacity to support our standard of living.

It is true that so long as human nature is what it is the profit motive can result in abuse of economic power and in injury to the rights of others. The system of private enterprise necessarily places a high degree of economic power in the hands of those responsible for organizing and managing industry, and there can be no denial that there have been flagrant cases of abuse of that power. It is a proper responsibility of government to exercise whatever degree of regulation is necessary to curb the abuse which might accompany untrammeled economic freedom. It is also, however, a responsibility of government to serve as an impartial arbiter between the major forces of production. The resolution of the Havana Conference reflecting against the profit motive, and the economic philosophy of the International Labor Organization, seem to be strong evidence of a compelling need for better understanding between labor and management in general, and as such they represent a challenge and an opportunity not only to management but also to our political leaders. If we can avoid misunderstanding and mistrust between labor and management we can rely on the natural desires of each to act as positive incentive toward the creation of the highest possible standard of living, but if we stifle natural incentive the result may be a standard of living so much reduced that the reasonable support of currently unproductive members of society becomes an impossible luxury rather than an accepted fact. It would be naive to assume that the profit motive, alone and unregulated, could be capable both of creating values and distributing them equitably, but it is even more naive to assume that, because a minimum of government regulation is necessary to assure full recognition of social and moral responsibilities, a maximum of government action would be more effective than private enterprise in providing everyone with the highest possible standard of living.

It has been suggested that mutual insurance companies are not subject to the profit motive and that the efficient job which has been done by such companies weakens the argument that the profit motive is necessary as a disciplining force to attain a high degree of efficiency. In actual practice the ownership and control of mutual companies by policyholders rather than by stockholders does not mean that the management of such companies is free from the discipline established by the profit motive. Mutual companies compete directly on the insurance market with stock companies which are unquestionably subject to the profit motive. The buyer of insurance wants value for his money, and inability to compete with the more efficient companies would force an unreasonably inefficient mutual company out of the market just as quickly as any other company operating at too high cost.

The necessity of meeting competition forces efficiency on the management of mutuals just as it does on the management of stock companies, and this result of competition is but one expression of the socially sound operation of the profit system.

The conditions for justification of government operation can be tested against an actual example of social insurance administered by private companies—workmen's compensation insurance, which is this country's oldest major form of social insurance. Legislation effectively compelling employers to provide insurance against occupational injury has been in force for over thirty years, and throughout that period most of the insurance has been underwritten by private companies, although there are also a small number of state insurance funds.

In applying the tests for justification of government operation there can be little question that for insurance against occupational injury the first condition exists—that it be considered essential to the general welfare. Before the days of workmen's compensation insurance the economic responsibility for occupational injuries was governed by the legal rules of negligence. These rules proved inadequate in distributing the social cost of industrial accidents, and it came gradually to be accepted that in most cases of occupational injury there is a significant degree of unquestioned social obligation to the injured employe. That obligation seems to justify legislation fixing arbitrary amounts of compensation for specific injuries and holding the employer responsible for arranging to provide such compensation. With some exceptions, workmen's compensation legislation has been considered a function of state government, and the risk has with few exceptions been underwritten by private insurance companies.

The second test for government operation—that restriction of the function to private companies be either inadequate or against public interest—should not be difficult to apply to workmen's compensation insurance with its long history of administration by private companies. During those years private companies have conducted workmen's compensation insurance efficiently and well, and it would be hard to justify a charge that such operation has been either inadequate or against public interest. There have been charges, made by some advocates of federal insurance, that the companies have centered too much attention on the mechanical aspects of administration and have failed to appreciate the social viewpoint and the social insurance background without which workmen's compensation cannot realize its highest possibilities. This is no proper indictment of private insurance. The immediate concern of the companies is to underwrite and administer the insurance as efficiently as possible, and they are peculiarly well equipped to do this job because of the discipline which the profit motive holds over them. The responsibility for implementing the social viewpoint lies directly

with the policy makers rather than with the administrators—with the government which enacted the legislation defining the risk rather than with the companies which have carried out the economic and accounting distribution of the cost of the risk.

There is a place and a responsibility for both parties. Private enterprise is especially fitted to create economic values and to administer economic services, but it is not as well equipped to define social aims and to police the maintenance of social responsibility. Government, on the other hand, is organized for the particular purpose of policing social responsibilities, but cannot be expected to administer economic functions with as great efficiency as private enterprise. In carrying out any program, government and private industry should each be charged with responsibility for those functions which each is best fitted to perform. If government has not been fully successful in implementing the social viewpoint in the private administration of workmen's compensation insurance, the solution is to be found not in removing from private enterprise the function which it can perform best, but rather in reawakening government to the responsibility which it may have failed to recognize.

Disability insurance provides an extreme example of the difference which may exist between government operation and private operation. Disability insurance—especially insurance against long-term disability—is subject to an extremely high moral hazard. There is not, in practice, any definite line between disability and good health such as there is between life and death. Disability benefits are paid not for an absolute, unquestioned condition, but for whatever condition the claimant is successful in calling disability. For this reason disability claim experience, in practice, is in very large measure a function of the methods of administration. An efficient organization can keep the moral hazard under reasonable control, but every relaxation of efficiency results in the payment of socially unjustified claims. A sound social viewpoint will not allow avoidance of proper claims, but only a highly efficient organization can avoid payment for a very large amount of alleged disability for which the social justification is, at best, doubtful.

For private insurance companies the profit motive, properly safeguarded, can provide the discipline necessary to sustain the socially desirable and practically essential degree of efficiency, but no democratic government can be expected to sustain that degree of efficiency. Government operation could easily lead to a cost of disability tremendously greater than might be expected. A level of benefits so low as to be quite inadequate in meeting the social need might be administered by a government agency without developing too highly exhorbitant disability claim experience, but a program with benefits high enough to provide even a minimum floor of protection would—in the opinion of men familiar with disability insurance—be far

more costly over the long run than may seem possible to those without intimate acquaintance with successful disability insurance. Inefficient operation of a private disability insurance company leads to bankruptcy, which is an effective but socially costly way to force the issue. Government, with its taxing power, is not subject to the same conditions of solvency, and the high cost of inefficiency could accumulate until the burden on our national income forced a politically disastrous modification of the program.

Other countries have established government programs providing cash benefits during disability, and this fact has been put forward to support the possibility of government administration of disability benefits in this country. Before accepting this argument we should be sure that the programs of other countries have actually received an adequate testing. We must recognize also that benefits in other countries have quite generally been set at such a very low level that the moral hazard has been relatively less important. And finally, we should consider whether there is a significant difference in the temperament of the people. In a country most of whose citizens are accustomed to view the government as a superior power whose actions are not subject to question the experience might be quite different from that of a country in which the custom of "writing to the Congressman" is firmly established.

The question, therefore, is really not whether disability insurance should be conducted by government or by private enterprise, but whether government can conduct the insurance at all. If men with a background of practical and theoretical experience in disability insurance question seriously the possibility of successful government operation, then the burden is on any supporter of government operation to prove the experts wrong. Any supporter of federal disability insurance who disregards this opinion may literally be trifling with the stability of our government itself. It would be a major tragedy to set up—even with the highest ideals and intentions—a program designed to improve the welfare of our people and have that program actually produce precisely the opposite result.

The risk of unemployment, on the other hand, is tied rather directly to the business cycle and there is no known method—actuarial or otherwise—of predicting with sufficient accuracy what the expected losses would be under any employment insurance policy. It would be impossible to administer this insurance without the power to compel the payment of premiums, amounting in effect to the taxing power of government. This fact makes unemployment an uninsurable risk except on a compulsory basis, and there has been no unemployment insurance in private companies. The Social Security program of government insurance was the first attempt in this country to apply insurance principles to this risk.[6] It is now, therefore,

[6] Wisconsin and New York had state programs which were incorporated into the Social Security program.

a matter of only academic interest whether private insurance or public insurance against this risk would be more effective; and except for the jurisdictional question discussed below, most of the problems in this field involve technical aspects which are the province of experts.

Although direct medical care was not a part of the Eliot bill and has not been mentioned in connection with specific current proposals of members of the Social Security Board, this country has seen serious proposals for the establishment of nationalized systems of medical care, and the cost of medical care has been one of the subjects to which proponents of social insurance would apply compulsory national budgeting. Compulsory insurance of the cost of medical care would require some method of guaranteeing that the services of a competent physician be available in every case of need, and some method of guaranteeing to the physician an adequate compensation for his services. The suggestions have involved the creation of what has come to be known as "socialized medicine." Medical authorities have severely criticised proposals for socialized medicine on the ground that it would interfere with medical progress and that it would cause actual deterioration of the quality of medical care. That criticism must receive serious consideration, although it is not the part of this paper to discuss the pros and cons of socialized medicine. It is part of the purpose of this paper, however, to point out that the substitution of compulsory insurance and other methods of providing medical care will result in an increase in the total cost. Medical care for sickness has been subject to such a high degree of moral hazard that attempts of private companies to provide insurance protection against the risk have almost universally failed because of excessive claim cost. Conceivably this moral hazard could be curbed by a high degree of government control and regulation of medical practice. There remains, however, the serious question whether such control is desirable—the question of the effect of such control on the quality of medical care, and whether even such control could prevent a very substantial increase in the social cost. A decision on any proposal for government control of medicine requires recognition of the certainty of increased cost, and should be based on consideration of whether the disadvantages and cost are too high a price to pay for the benefits to be gained.

The Issue of Jurisdiction

In the social insurances which we already have there are several different ways of distributing the jurisdiction between the state and federal governments. Workmen's compensation insurance has so far been considered to be entirely within state jurisdiction, with the exception of certain inter-state

and maritime risks governed by federal law. Jurisdiction over Old Age and Survivors Insurance is assumed entirely by the federal government. Unemployment Compensation is provided by state laws enacted under the incentive of a federal statute.

The major argument for federal jurisdiction is that unified responsibility gives us the best chance of developing sound and effective progress. The history of workmen's compensation legislation indicates that wholly independent state plans result in low and irregular development of the possibilities which might make social insurance a valuable instrument. If we are to make the most of the possibilities inherent in social insurance, it seems clear that there must be uniform direction. Social insurance is most easily conceived as a long-range national plan, and the budgeting possibilities can most effectively be realized if it is national in scope.

One of the most impelling reasons for conceiving a plan on a national basis results from the high degree of mobility of labor. If there are forty-nine different independent social security systems, each stopping at state boundaries, John Doe may be fully qualified for benefits in New York where he has been living and working for several years, but upon moving from New York to work in the California shipyards he may have to sacrifice his eligibility for New York benefits without acquiring immediately a similar insurance status in California. The problem of mobility would presumably be very much less in peacetime than today, but at any time a system which penalized reasonable mobility of labor would be unfortunate.

In spite of these arguments for federal jurisdiction there is question whether in actual practice complete centralization would produce the highest attainment of social insurance values. The organization of social insurance under federal jurisdiction *could* be more simple than under state jurisdiction, but it does not necessarily follow that it *would* be so in the long run. Centralization of responsibility and authority means concentration of power, with its long run danger to political stability. The question of federalization hangs directly on the broad issue of state's rights. That issue is too big to allow a serious effort to discuss it here, but probably the strongest argument against complete centralization was stated by President Roosevelt when he said, "We have created powers which in other hands would be dangerous." All history shows that eventually power does change hands, and concentration of power is itself an attraction to men who may abuse that power.

In the short run federal jurisdiction could get results and get them quickly. Social insurance, however, must be conceived in terms of the long run, not of the short run. It would be a tragedy if impatience for quick results produced a situation which in the long run would develop a serious threat to the permanence of the system itself. The major argument for undivided federal jurisdiction is the efficiency which theoretically could be obtained

in that way. Efficiency is also—in theory[7]—one of the attributes of Hitler's government. Sincere advocates of states' rights are not merely jealous of the position and the authority of state governments, but honestly fear the political dangers of too great concentration of power. The dangers are matters of opinion, not subject to definite proof, but they cannot be dismissed lightly. It is not a question merely of safeguarding the rights of the states, but of choosing the best possible means of safeguarding the permanence of our democratic hopes and plans. We who prefer democracy have long recognized that the highest values of democracy can be obtained only by the sacrifice of some measure of political efficiency.

Social insurance, viewed in its broadest aspects, includes not only the budgeting of certain social costs but also measures to keep those costs at a minimum. Prevention and rehabilitation are important aspects of a broad program of social insurance. These aspects have their greatest opportunity in education to help forestall industrial accidents, in public health measures to reduce the chances of becoming disabled, in the direction of job placement for the unemployed, and in the vocational training and rehabilitation of the disabled. There are unquestioned social benefits to be gained if these functions can be performed effectively without offsetting disadvantages. It is possible that this type of activity, if effectively organized, might actually reduce social costs by a greater amount than the cost of the activities themselves. These possible benefits, however, are not in themselves arguments for either complete federal jurisdiction or federal administration of the insurance aspects of a social program. It should be possible to co-ordinate as broad a program as the American people want to afford, utilizing for each phase of the program the functions which federal government, state government and private enterprise are each best fitted to perform, and obtaining a reasonable and practically effective balance which sacrifices some of the possible short-run effectiveness of federal jurisdiction in exchange for a greater prospect of enduring success.

Summary and Conclusion

This paper constitutes a plea to study thoroughly the cost of social insurance proposals as well as their benefits, to consider such proposals against the background of the whole pattern of the way of life we want for America and for the world, and to use the best means at our disposal in our efforts

[7] *Theory* should properly be a clear statement of what is possible in practice. If it is so understood, the word "theory" is misused here. All too often, however, "theorists" overlook certain very real aspects or deficiencies of human nature or morale, so that their "theories" fall short of being statements of what actually is possible in practice. This second meaning—less desirable but more commonly recognized—is behind the usage here. The distinction between the two usages is the distinction between "sound theory" and "unsound theory."

to attain the highest welfare and the greatest degree of security for all people. The paper is written not for the expert but rather for the man who is forming his opinions about the place which social insurance is to have in our way of living. It discusses the background of the three broad questions: "Should insurance methods be made compulsory as a means of distributing the social cost of disability, hospitalization and medical care?", "If so, should government or private enterprise be the insurer?", and "What is the relative place of federal and state governments in a social insurance program?"

In discussing the first of these questions the paper asks for an honest estimate of the cost of substituting insurance methods for relief methods of sharing the cost of social hazards, so that we may make the decision with full recognition of what is involved. The discussion of the second question sets forth the authors' opinions that we cannot attain the highest welfare and the greatest degree of security for all except by using the most efficient tools available to us, and that private enterprise has possibilities of efficiency which government can never be expected to approach. The discussion includes an appeal for better understanding between representatives of management and labor, and of government, so that no misunderstanding or mistrust may prevent our taking full advantage of the powerful incentive to efficient operation which the profit motive—properly regulated—supplies for private enterprise. The paper questions whether disability insurance, with its high moral hazard, can be successfully underwritten by any government agency because of the lack of any powerful disciplinary force such as the profit motive. It would be a major tragedy to set up—even with the highest ideals and intentions—a program designed to improve the welfare of our people and have that program actually produce precisely the opposite result.

In the issue of state versus federal jurisdiction the major argument for centralized federal administration is that unified responsibility gives us the best chance of developing sound and effective progress. In spite of the arguments for federal jurisdiction, however, there is question whether in actual practice complete centralization would produce the highest attainment of social insurance values. Centralization of responsibility and authority means concentration of power, with its long-run danger to political stability. It would be a tragedy if impatience for quick results produced a situation which in the long run would develop a serious threat to the permanence of the system itself. We who prefer democracy have long recognized that the highest values of a democracy can be obtained only by the sacrifice of some measure of theoretical efficiency. It should be within the scope of our ingenuity to devise an effective way of having the advantages of federal leadership in conceiving a national plan without having to embrace at the same time the long-run dangers of too great concentration of power.

The principles of the Atlantic Charter are here to stay. Any questions

about social insurance involve only the method of achieving the highest degree of security for the individual. Individual security is not itself an issue; no advocate of any special method of attaining social security has any monopoly on the desire to improve the welfare of the less fortunate members of society. Our problem is not merely to answer the question, "Is social insurance a good thing to have?", but also to determine the place of social insurance in our daily life. A complete philosophy of social insurance includes opinions not only on the absolute merit of specific proposals but on the worth of such proposals compared to other values which we want to have as a part of our daily living.

Social insurance, despite its unquestioned values, is not an economic cure-all but is only one possible device available to us in our striving for the highest standard of living for all. We must decide on the place which it will have in relationship to other valuable factors; and we must not expect any program of social insurance to do a job beyond its powers. For example, the fullest employment of our people and the most efficient development of resources cannot be attained without a free and full world trade; and a broad social insurance program without a substantial world trade may leave us actually farther from the goal of individual security than a less ambitious social insurance program combined with emphasis on the full development of world trade. We must evaluate both the benefits and the cost of each separate factor in its relationship to the whole before we can make an intelligent decision on any one factor alone.

In striving for the highest welfare and the greatest security for all people, the most important consideration is full and efficient employment—for if the factors of production are efficiently and fully employed, the costs of social hazards will be low and our ability to bear the costs will be high, but every decrease in full, efficient employment decreases our ability to bear the costs at the same time that it increases the costs themselves. Our first concern, therefore, is to create conditions in which employment and productivity can have the best chance of attaining and sustaining the high level which will permit us to afford not only a reasonable social insurance program but also the multitude of other desirable values which only an efficient economy can give us. If the limits of our current productivity and resources make it necessary to choose between an immediate extension of social insurance and an immediate investment in economic reconstruction or in the development of world trade, the welfare of all people will be best served in the long run if we choose first to improve our possibilities of productivity and employment.

Social insurance, like every other valuable commodity, is costly. The complete program of benefits envisioned by the Social Security Board might cost as much as six or eight extra hours of work every week—and more if

their cost estimates are not conservative. The amount which we must pay depends on the amount and kinds of social insurance which we adopt, and on the methods we set up to administer the benefits, and we must recognize that whatever social insurance we decide upon, we must be willing to work these extra hours to pay for it. The major question, therefore, is not whether social insurance is desirable. There can be no denial that social insurance—properly defined and effectively organized—can be intrinsically a valuable addition to social organization. The real question is whether—after honest consideration of the cost and of the relationship to our social and economic structure as a whole—we want social insurance enough to pay for it, and if so how much of it we are willing and able to pay for.

When this war is over the country will have a national debt many times greater than ever before. Whether this debt is to be paid off, or whether—as some economists say—it should be carried without repayment, we will still have at least the interest payments to recognize in our national budgeting. The interest burden alone will require, in effect, that every one of us work several hours more each week. When this war is over the American people will have to decide on their relationship to the other peoples of the world. If we accept the responsibilities of political and economic leadership which will be thrust upon us, the investment in reconstruction and in the establishing of world trade will require extra hours of work each week. It will require extra hours of work to pay for any agricultural program, and for any soil conservation program, and for any flood control program which the people may want. After the war there will be new opportunities for improvement of housing and of transportation—new and better airplanes and automobiles, radios and household comforts—new possibilities for increasing the enjoyment of our leisure time. All of these things must be paid for in terms of working hours if we are to realize the possibilities. We can have them if we want them, but if we are to have any major part of them we must give up, at least for the time being, any thought of a thirty or forty hour work week. The more of these things we want the more of our time must be devoted to working for them and the less of our time will be left for the leisurely enjoyment of them.

The American people must decide in effect how many hours a day they are willing to work, and must buy only those things which that amount of time can pay for. We as a people must recognize that we can have social insurance if we want it, and as much social insurance as we want, but we must first ask ourselves how many of all the valuable choices offered to us we can afford to have, and how much of each. After considering both value and cost, we must decide which of those things we consider most desirable, and if we cannot afford them all we must decide which to buy now and which to forego until we can afford them; just as a family may have to

decide between buying a radio and driving the old car a while longer, or buying a new car and postponing the purchase of the radio. Our leaders must help us to make realistic appraisals of the cost as well as the benefits, so that we may never find ourselves as a nation in the position of a family which bought a new house and a new car and a new radio—all on the installment plan—only to end up poorer than before because the family income was not enough to keep up the payments.

And having decided what we want, we must use the best means at our disposal for production and administration. We want the best that we can get, and as much as we can afford. Our highest hopes can be realized only if we recognize the limits to what we can afford *currently,* and then administer our current program in the most efficient way; for only by today's efficiency can we prepare tomorrow's plenty. If our goal is really the highest degree of economic welfare and social security for all people, social insurance must be recognized as being subordinate to the maintenance of an economy of full employment and efficient production. A man wants a job first. Social insurance itself is not a short cut to social security and must not be allowed to interfere with the conditions which will best bring about true economic and social security. And finally, whatever social insurance we may adopt should make use of the most efficient administrative methods which can be found.

If the American people, under able leadership, consider and decide these problems as a whole and in a way which over the long run will produce the best good for all people, we shall have fulfilled the promise and the responsibility of a democratic nation.

SAMPLING THEORY IN CASUALTY INSURANCE

INTRODUCTION AND PARTS I AND II

BY

ARTHUR L. BAILEY

Introduction

The fundamental concept of insurance is that the insured is relieved of any concern, not only as to what is going to happen, but also as to what could happen but probably will not. Of course, at the time the insurance is written, neither the insured nor the insurer knows what is actually going to happen. But, even when the period of the coverage has expired, and the actual events are determined, both parties still should understand that the coverage provided was against what might have happened rather than against the specific events that actually did happen. Thus the losses paid by an insurer never actually reflect the hazard covered, but are always an isolated sample of all of the possible amounts of losses which might have been incurred.

It is this condition, of never being able to determine, even from hindsight, what the exact value of the inherent hazard of the coverage was, that has brought the casualty actuary into being. It becomes his province to make rates and rating plans such that, in the absence of an unprecedented catastrophe, his company will be able to pay the losses incurred in covering a wide group of such unknown inherent hazards and still stay solvent; or preferably, make a profit for its stockholders or pay a dividend to its policyholders. The myriads of compilations of loss experience, classification refinements, and expedients in general, resulting from the actuary's attempts to achieve this goal, need not be lingered on here, being well enough known. A few of the expedients do need to be referred to specifically.

When the loss experience of one group of insureds was first compiled, separately from that of another group, it was found that it was different. The question immediately arose, "Does this mean that the elusive inherent hazard is different for this group of risks, or does it mean that the hazard is the same but the actual losses of the two groups just happen to be different?" Credibility formulae were designed to provide an answer to this question. These have taken a wide variety of forms during the history of casualty insurance. Some have been based on the soundest of theoretical premises, while others have been purely expedients. Whatever the form, these formulae have been applied in recognition of a condition in which the actual observations are only samples of what might have been. It is the hope of the writer that this paper may serve, among other things, to produce more properly applicable credibility formulae.

Shortly after the problem of classification and territory experience had been met, the question of the insurance-minded large risks arose. Such risks knew that their operations were more efficient than those of their competitors; "otherwise they would not have become so large." They also knew that their loss experience was not exactly that contemplated by the manual rates. "Something ought to be done about it." Experience rating plans were developed as an answer and soon became the accepted thing; and a credibility formula was developed that would produce the same increase, or decrease, depending on the selected value of K, in the accuracy of the modified rate as compared with the unmodified rate, irrespective of the size of the risk. In later years this formula has been departed from as the plans have become more complicated. The implied aim has, of course, continued: that of producing a more accurate rate through application of the rating plan, although no means has been available to determine, before the plan is actually applied, whether or not this has been accomplished. The author hopes to provide a basis for the proper evaluation of the constant K.

The most recent development calling for a sound knowledge of sampling distributions is the retrospective rating plan. It is difficult to understand why this form of gambling remained dormant during the speculative twenties, only to break out in the depression thirties. The present epidemic can no doubt be explained as the direct result of the war hysteria. Whatever the cause, certain elements of the insurance industry desire to depart from rates based on expected averages and explore the possibility of rating on the basis of departures from expected averages. The initial essays in this direction have been made only after the application of some good unactuarial horse-sense by the underwriters, in the selection of the risks. Before, or it may be, while, we embark on an all-out retrospective program, it would seem well to seriously investigate the theoretical principles underlying such a course. It is felt by the author that this paper may serve as a foundation for such an investigation.

Another field where a knowledge of the sampling distributions of losses could be used to advantage is that of the rating procedures for deductible and excess coverages. Such procedures are now based on a very broad grouping of classifications, even including the entire line of insurance in many cases. This results in the necessity for a large safety margin in such rates in order to offset the selection against the company that inevitably results from broad classifications. The effect of such a rating procedure is to exclude, through redundant rates, the normal or subnormal risks from electing such coverages. Rating procedures can be developed, however, with a knowledge of the sampling distribution functions, which would give full rate recognition to differences in the hazard of such coverages by classification. These coverages, which include the real elements of insurance as con-

trasted to the chance variations of retrospective rating, could then be made available to the many large risks who need only this type of coverage.

We must recognize that the only data available to us in casualty insurance is in the form of samples of what may occur. From these samples we are required to measure, as well as we may, the inherent hazard of the coverage provided to a particular insured, or group of insureds. In experience rating, and more especially in retrospective rating, we must also measure the probable distribution of these expected losses among the risks. In order to do this we need a rather complete understanding of the theoretical distribution of losses among samples when various causes of variation are present.

The sampling variation due to pure chance fluctuations is always present in our data. Usually, however, our problems are made more complicated by the presence of other types of variation as well as chance. One of the most important of these types of variation is that between the inherent hazards of risks of the same rate classification and territory. Others are those resulting from errors, due to chance or otherwise, in the rate making procedure, or in the rating plan to be applied. Most problems involve the simultaneous consideration of at least two of these types of variation.

In many problems, however, we are only asked to decide whether or not a particular piece of data could reasonably be attributable to chance variation only. Other types of variation may be present in such data but are not involved in the answer to the problem. If the probabilities are greatly against the event occurring as a result of chance only, we may or may not then want to search for the reason. An extreme example of this kind of problem is presented by the $100,000 premium risk with a loss ratio of more than 100% in each of the past three years. The probability that this series of losses arose only from chance is so small that the underwriter himself would cancel the risk, or double the rate. Cases nearer to the borderline definitely come into the province of the actuary and can be answered only on the basis of a knowledge of sampling distributions resulting from chance variation only.

Thus our first step in the development of the theory of sampling distributions will concern itself only with variations due to chance. When these have been fully explored, we will then have to compound the results when one or more of the other types of variation are also present. The first two parts of this paper deal with the theory of purely chance sampling distributions and with methods of numerically approximating such distributions. Later parts will deal with the inclusion of other types of variation, and with the application of the theory to particular kinds of problems such as individual risk underwriting, rate making, experience rating, retrospective rating, and the rating of deductible and excess coverages.

It is hoped that this analysis of the sampling theory as applicable to casualty insurance will help to bring to light any flaws which may now exist in the rate making or rating structures and thereby serve to make them more accurate. It would be expected that the need for such corrections will be greater in the less highly developed rating of deductible and excess coverages and in the highly sensitive retrospective rating plans than in the older and more time-tested rate making and rating procedures.

It is believed that one conclusion will be drawn immediately from a review of the discussion of the Poisson Distribution. This is that the recording and collection of experience on a per accident basis, rather than on a per claim basis, would greatly assist in the interpretation of the data. This should certainly be done for all classifications of hazard involving any appreciable number of multiple-claim accidents.

The writer would appreciate being advised of any algebraic or arithmetic errors which may be found. As it has not been possible to have any independent check on most of this material, the author will have to assume full responsibility for these. That the symbolism used is different from that more recently presented can only be defended on the ground that the paper has been in progress for several years.

I.

Development of Basic Formulae for the Distribution of Casualty Insurance Statistics Due to Chance Fluctuations Only

A. The Poisson Distribution

The number of accidents in casualty insurance is distributed in accordance with the Poisson Distribution. This is not an assumption, but a demonstrable fact. The assumption, which it will later be necessary to make, is that the number of claims is also distributed in the same way.

We have from the Bernoullian Theorem that: if p is the probability of an event occurring and q is the probability of the event failing to occur, then out of s trials the probabilities of the event occurring 0, 1, 2, ..., $s-1$, s times are given by the terms in the expansion of $(q+p)^s$. It can be seen that the Bernoullian Distribution is not applicable to casualty insurance from the fact that the Bernoullian Theorem is dependent on the condition that there are only two possibilities; namely that the event happens, or it fails to happen. In casualty insurance the event (an accident) may not only happen or fail to happen, but it may also happen more than once.

We can, however, approach the conditions of the casualty business with the Bernoullian Distribution. If only one accident could happen each month, the probabilities of 0, 1, 2, ..., 12 accidents occurring during a year would

be given by the terms in the expansion of $\left(q + \frac{p}{12}\right)^{12}$, where p is the average number of accidents per year and q is equal to $1 - \frac{p}{12}$. Similarly, if more than one accident could happen during a month but only one could happen per day, the probabilities of 0, 1, 2, ..., 365 accidents occurring during the year would be the terms in the expansion of $\left(q + \frac{p}{365}\right)^{365}$, where q is equal to $1 - \frac{p}{365}$.

Finally, we could take the limiting case where the year is divided into an infinite number of parts. In this case the probabilities of 0, 1, 2, ..., to infinity accidents occurring during the year would be the terms in the expansion of:

$$\underset{n \to \infty}{\text{Limit}} \left[\left(1 - \frac{p}{n}\right) + \frac{p}{n}\right]^n$$

Only this limiting case would fit casualty insurance, where accidents can happen in very rapid succession although not at exactly the same time without, by definition, being the same accident. This limiting case is the Poisson Distribution; and the probabilities of 0, 1, 2, ..., n, etc. accidents are:

$$\frac{1}{e^p}, \frac{p}{e^p}, \frac{p^2}{\lfloor 2\, e^p}, \ldots\ldots, \frac{p^n}{\lfloor n e^p}, \ldots\ldots, \text{etc.}$$

Having found that the Poisson Distribution provides the probabilities of the occurrence of 0, 1, 2, ..., n, etc. accidents for an individual risk whose hazard remained constant throughout the year, the probabilities can be determined of 0, 1, 2, ..., n, etc. accidents occurring during a year between two risks having different hazards although both remain constant during the year.

Let the probabilities of 0, 1, 2, ..., n, etc. accidents for the first risk be given by:

$$\frac{1}{e^p}, \frac{p}{e^p}, \frac{p^2}{\lfloor 2\, e^p}, \ldots\ldots, \frac{p^n}{\lfloor n e^p}, \ldots\ldots, \text{etc.}$$

and for the second risk by:

$$\frac{1}{e^q}, \frac{q}{e^q}, \frac{q^2}{\lfloor 2\, e^q}, \ldots\ldots, \frac{q^n}{\lfloor n e^q}, \ldots\ldots, \text{etc.}$$

The probability that neither will have an accident is:

$$\frac{1}{e^p} \cdot \frac{1}{e^q} = \frac{1}{e^{(p+q)}}$$

The probability that there will be only one accident is:

$$\frac{1}{e^p} \cdot \frac{q}{e^q} + \frac{p}{e^p} \cdot \frac{1}{e^q} = \frac{p+q}{e^{(p+q)}}$$

The probability that there will be exactly two accidents is:

$$\frac{1}{e^p} \cdot \frac{q^2}{\lfloor 2 \cdot e^q} + \frac{p}{e^p} \cdot \frac{q}{e^q} + \frac{p^2}{\lfloor 2 \cdot e^p} \cdot \frac{1}{e^q} = \frac{q^2 + 2pq + p^2}{\lfloor 2 \cdot e^{(p+q)}} = \frac{(p+q)^2}{\lfloor 2 \cdot e^{(p+q)}}$$

The probability that there will be exactly n accidents is:

$$\frac{1}{e^p} \cdot \frac{q^n}{\lfloor n \cdot e^q} + \frac{p}{e^p} \frac{q^{(n-1)}}{\lfloor n-1 \cdot e^q} + \cdots\cdots + \frac{p^{(n-1)}}{\lfloor n-1 \cdot e^p} \cdot \frac{q}{e^q} + \frac{p^n}{\lfloor n \cdot e^p} \cdot \frac{1}{e^q}$$

$$= \frac{q^n + npq^{(n-1)} + \cdots\cdots + np^{(n-1)}q + p^n}{\lfloor n \cdot e^{(p+q)}} = \frac{(p+q)^n}{\lfloor n \cdot e^{(p+q)}}$$

Thus it is found that the probabilities of the occurrence of 0, 1, 2, ..., n, etc. accidents among two risks having different hazards, although both remain constant during the year, are likewise given by the Poisson Distribution using the combined hazard of both risks, i.e.:

$$\frac{1}{e^{(p+q)}}, \frac{(p+q)}{e^{(p+q)}}, \frac{(p+q)^2}{\lfloor 2 \cdot e^{(p+q)}}, \cdots\cdots, \frac{(p+q)^n}{\lfloor n \cdot e^{(p+q)}}, \text{ etc.}$$

This combination of hazards can obviously be extended to cover any number of risks having any range of individual hazards and also to cover any variation of hazard during the year. For the general case, where c represents the expected total number of accidents for all risks, the probabilities that the total number of accidents will be 0, 1, 2, ..., n, etc. are given by the terms of the Poisson Distribution:

$$\frac{1}{e^c}, \frac{c}{e^c}, \frac{c^2}{\lfloor 2 \cdot e^c}, \cdots\cdots, \frac{c^n}{\lfloor n \cdot e^c}, \cdots\cdots, \text{ etc.}$$

NOTE: Throughout this paper the expected number of accidents or of claims will be indicated either by a small "c" or a capital "C". No difference is intended between these two.

B. Sampling Distribution of the Number of Claims or Claim Frequency

The assumption will be made that the probabilities of the occurrence of 0, 1, 2, ..., n, etc. claims when c are expected are also given by the terms of the Poisson Distribution. This would seem to provide a very close ap-

proximation unless it is known that the claims usually occur in sizable groups.

In dealing with the various sampling distributions it will be found expedient to deal with the ratio of actual to expected values. In the case at hand the exposure element of the claim frequency cancels out in the ratio of actual to expected claim frequencies, and the ratio becomes identical to the ratio of actual number of claims to the expected number of claims. These ratios can take only the values corresponding to 0, 1, 2, ..., n, etc. claims of:

$$\frac{0}{c}, \frac{1}{c}, \frac{2}{c}, \ldots\ldots, \frac{n}{c}, \ldots\ldots, \text{etc.}$$

In order to prepare tables for the practical use of the sampling distributions, it will be necessary to evaluate the mean, variance, and skewness of these distributions. These are obtained as follows from the totals shown in Table 1 (see page 81):

$$\text{Mean} = \frac{\Sigma r \cdot f(r)}{\Sigma f(r)} = \frac{\text{Total of Column (3)}}{\text{Total of Column (2)}} = \frac{1}{1} = 1$$

$$V_{2:r} = \frac{\Sigma r^2 f(r)}{\Sigma f(r)} = \frac{\text{Total of Column (4)}}{\text{Total of Column (2)}} = \frac{1}{c} + 1$$

$$\text{Variance} = U_{2:r} = V_{2:r} - (\text{Mean})^2 = \frac{1}{c}$$

$$V_{3:r} = \frac{\Sigma r^3 f(r)}{\Sigma f(r)} = \frac{\text{Total of Column (5)}}{\text{Total of Column (2)}} = \frac{1}{c^2} + \frac{3}{c} + 1$$

$$U_{3:r} = V_{3:r} - 3\,(V_{2:r})(\text{Mean}) + 2\,(\text{Mean})^3 = \frac{1}{c^2}$$

$$\text{Skewness} = \frac{U_{3:r}}{(U_{2:r})^{3/2}} = \frac{\frac{1}{c^2}}{\left(\frac{1}{c}\right)^{3/2}} = \frac{1}{\sqrt{c}}$$

C. Sampling Distribution of the Total Cost of a Fixed Number of Claims

Before considering the sampling distributions of other statistics, it will be necessary to record certain data concerning the sampling distribution of the total cost of a fixed number of claims. It will be assumed that these claims occur at random out of an infinite number of equally likely possibilities, and that the moments of this infinite population of possible claims can be estimated from the distribution by size of the claims paid in the past.

Before letting the parent population approach the infinite in size, it will

be assumed to consist of N values whose amounts are $x_1, x_2, x_3, \ldots\ldots, x_N$, and from which the following sums are formed:

Σx = the sum of the N values of x.
Σx^2 = the sum of the N values of x^2.
Σx^3 = the sum of the N values of x^3.
Σxx = the sum of the ${}_NC_2$ possible products two at a time.
Σxxx = the sum of the ${}_NC_3$ possible products three at a time.
Σx^2x = the sum of the $2 \cdot {}_NC_2$ possible products of squares and values.

NOTE: The x's in the last three sums must have different subscripts.

From this population of N values of x, all possible combinations of n values will be formed; there being ${}_NC_n$ such combinations, each of which would be equally likely to occur were a single sample drawn. The total cost of these n claims will be designated as t. The required data are the first three moments of t about the origin, $V_{1:t}$, $V_{2:t}$, and $V_{3:t}$, and the second and third moments about the mean, $U_{2:t}$ and $U_{3:t}$.

In each value of t there are n values of x. In the sum of all ${}_NC_n$ possible values of t there are $n \cdot {}_NC_n$ terms of x's; and, as each of the N different values of x are equally frequent, each of the N values must occur $\frac{n}{N} \cdot {}_NC_n$ times. The average of all possible values of t will therefore be:

$$V_{1:t} = \frac{\frac{n}{N} \cdot {}_NC_n \cdot \Sigma x}{{}_NC_n} = n \cdot \frac{\Sigma x}{N} = nV_{1:x}$$

In each value of t^2 there are n terms of x^2 and ${}_nC_2$ terms of xx, each of which has a coefficient of 2. In the total of the ${}_NC_n$ values of t^2 there are $n \cdot {}_NC_n$ values of x^2 and $2 \cdot {}_nC_2 \cdot {}_NC_n$ values of xx. As there are only N different values of x^2 and ${}_NC_2$ different values of xx, then each value of x^2 occurs $\frac{n}{N} \cdot {}_NC_n$ times and each value of xx occurs $2 \cdot \frac{{}_nC_2}{{}_NC_2} \cdot {}_NC_n$ times. The average value of t^2 therefore is:

$$V_{2:t} = \frac{\frac{n}{N} \cdot {}_NC_n \cdot \Sigma x^2 + 2 \cdot \frac{{}_nC_2}{{}_NC_2} \cdot {}_NC_n \cdot \Sigma xx}{{}_NC_n}$$

$$= \frac{n}{N} \cdot \Sigma x^2 + 2 \cdot \frac{{}_nC_2}{{}_NC_2} \cdot \Sigma xx$$

It is noted here that $(\Sigma x)^2 = \Sigma x^2 + 2\Sigma xx$, so that $(\Sigma x)^2 - \Sigma x^2$ may be substituted for $2\Sigma xx$, giving:

$$V_{2:t} = \frac{n}{N} \cdot \Sigma x^2 + \frac{{}_nC_2}{{}_NC_2} \cdot (\Sigma x)^2 - \frac{{}_nC_2}{{}_NC_2} \cdot \Sigma x^2$$

$$= \left[n - \frac{n(n-1)}{N-1} \right] V_{2:x} + \frac{n(n-1)}{1 - \frac{1}{N}} \cdot {V_1}^2{}_{:x}$$

Letting N approach infinity, the limiting value becomes:

$$V_{2:t} = n \cdot V_{2:x} + n(n-1)\, {V_1}^2{}_{:x}$$

and

$$U_{2:t} = V_{2:t} - {V_1}^2{}_{:t} = n \cdot V_{2:x} - n \cdot {V_1}^2{}_{:x} = n \cdot U_{2:x}$$

In each value of t^3 there are n terms of x^3, $2 \cdot {}_nC_2$ terms of x^2x each with a coefficient of 3, and ${}_nC_3$ terms of xxx each with a coefficient of 6. In the total of the ${}_NC_n$ values of t^3 there are then $n \cdot {}_NC_n$ values of x^3, $6 \cdot {}_nC_2 \cdot {}_NC_n$ values of x^2x, and $6 \cdot {}_nC_3 \cdot {}_NC_n$ values of xxx. As there are only N possible values of x^3, $2 \cdot {}_NC_2$ possible values of x^2x, and ${}_NC_3$ possible values of xxx, each value of x^3 occurs $\frac{n}{N} \cdot {}_NC_n$ times, each value of x^2x occurs $3 \cdot \frac{{}_nC_2}{{}_NC_2} \cdot {}_NC_n$ times, and each value of xxx occurs $6 \cdot \frac{{}_nC_3}{{}_NC_3} \cdot {}_NC_n$ times. The average value of t^3 is therefore:

$$V_{3:t} = \frac{\frac{n}{N} \cdot {}_NC_n \cdot \Sigma x^3 + 3 \cdot \frac{{}_nC_2}{{}_NC_2} \cdot {}_NC_n \cdot \Sigma x^2x + 6 \cdot \frac{{}_nC_3}{{}_NC_3} \cdot {}_NC_n \cdot \Sigma xxx}{{}_NC_n}$$

$$= \frac{n}{N} \cdot \Sigma x^3 + 3 \cdot \frac{{}_nC_2}{{}_NC_2} \cdot \Sigma x^2x + 6 \cdot \frac{{}_nC_3}{{}_NC_3} \cdot \Sigma xxx$$

It is noted here that $\Sigma x^2 \cdot \Sigma x = \Sigma x^3 + \Sigma x^2x$ so that $\Sigma x^2x = \Sigma x^2 \cdot \Sigma x - \Sigma x^3$, and that:

$$(\Sigma x)^3 = \Sigma x^3 + 3 \Sigma x^2x + 6 \Sigma xxx$$

$$= 3 \cdot \Sigma x^2 \cdot \Sigma x - 2 \Sigma x^3 + 6 \Sigma xxx$$

so that $6 \Sigma xxx = (\Sigma x)^3 + 2 \Sigma x^3 - 3 \Sigma x^2 \Sigma x$. These values may be substituted to obtain:

$$V_{3:t} = \frac{n}{N} \cdot \Sigma x^3 + 3 \cdot \frac{{}_nC_2}{{}_NC_2} \left[\Sigma x^2 \Sigma x - \Sigma x^3 \right]$$

$$+ \frac{{}_nC_3}{{}_NC_3} \left[(\Sigma x)^3 + 2 \Sigma x^3 - 3 \Sigma x^2 \Sigma x \right]$$

$$= \left[n - \frac{3n(n-1)}{N-1} + \frac{n(n-1)(n-2)}{(N-1)(N-2)} \right] V_{3:x}$$

$$+ 3 \left[\frac{n(n-1)}{1-\frac{1}{N}} - \frac{n(n-1)(n-2)}{\left(1-\frac{1}{N}\right)\left(N-2\right)} \right] V_{2:x} \cdot V_{1:x}$$

$$+ \left[\frac{n(n-1)(n-2)}{\left(1-\frac{1}{N}\right)\left(1-\frac{2}{N}\right)} \right] V_1{}^3{}_{:x}$$

Letting N approach infinity, the limiting value becomes:

$$V_{3:t} = n \cdot V_{3:x} + 3n(n-1)\ V_{2:x} \cdot V_{1:x} + n(n-1)\ (n-2)\ V_1{}^3{}_{:x}$$

and

$$U_{3:t} = V_{3:t} - 3\ V_{2:t}\ \cdot V_{1:t} + 2\ V_1{}^3{}_{:t}$$

$$= n\ [V_{3:x} - 3\ V_{2:x} \cdot V_{1:x} + 2\ V_1{}^3{}_{:x}]$$

$$= n \cdot U_{3:x}$$

This gives as the skewness (Charlier) of the t distribution:

$$\alpha_{3:t} = \frac{n \cdot U_{3:x}}{(n \cdot U_{2:x})^{3/2}} = \frac{a_{3:x}}{\sqrt{n}}$$

D. Sampling Distribution of Total Losses, Pure Premiums, and Loss Ratios

(1) In the form of the ratio of actual to expected value, the exposure divisor of the pure premiums and the premium divisor of the loss ratios cancel out, leaving only the ratio of actual to expected total losses. Thus only a single sampling distribution is required. Furthermore, as the expected total losses is a constant, the moments of this ratio, R, can be obtained directly from the moments of the total losses, T.

The total expectation is the sum, taken over all possibilities, of the product of the probability of an event occurring and the expectation if the event occurs. The average values of the first three powers of the total cost, t, t^2, and t^3, have been obtained in the previous section for a fixed number of claims. The Poisson Distribution will be assumed to give the probabilities of obtaining 0, 1, 2, ..., n, etc. claims. The sum of the products of these will be the first three moments, about the origin, of T, the total cost when c claims are expected.

From the totals on Table 2 (see page 82) are obtained:

$$V_{1:T} = c \cdot V_{1:x}$$

$$V_{2:T} = c \cdot V_{2:x} + c^2 V_1{}^2{}_{:x}$$

$$V_{3:T} = c \cdot V_{3:x} + 3\, c^2 V_{2:x} \cdot V_{1:x} + c^3 V_1{}^3{}_{:x}$$

The moments of R, the ratio of actual to expected losses, are then obtained by dividing these by the first three powers respectively of $c \cdot V_{1:x}$, the expected loss, as:

$$V_{1:R} = 1$$

$$V_{2:R} = \frac{V_{2:x}}{c \cdot V_1{}^2{}_{:x}} + 1$$

$$V_{3:R} = \frac{V_{3:x}}{c^2 V_1{}^3{}_{:x}} + 3\,\frac{V_{2:x}}{c \cdot V_1{}^2{}_{:x}} + 1$$

and:

$$U_{2:R} = \frac{V_{2:x}}{c \cdot V_1{}^2{}_{:x}}$$

$$U_{3:R} = \frac{V_{3:x}}{c^2 V_1{}^3{}_{:x}}$$

with the skewness (Charlier) of:

$$a_{3:R} = \frac{V_{3:x}}{\sqrt{c}\,(V_{2:x})^{3/2}}$$

(2) Under certain conditions we may wish to exclude, or may not have available, the cases for which there were no actual losses. As the proportion of such cases to the total will be e^{-c}, then, designating this select set of ratios by R^1:

$$V_{n:R} = 0\,(e^{-c}) + V_{n:R^1}\,(1 - e^{-c})$$

and

$$V_{n:R^1} = \frac{V_{n:R}}{1 - e^{-c}}$$

Thus the moments of R^1, the ratio of actual to expected losses when cases with no actual losses are excluded are found to be:

$$V_{1:R^1} = \frac{1}{1 - e^{-c}}$$

$$V_{2:R^1} = \frac{1}{1-e^{-c}}\left[\frac{V_{2:x}}{c\,V_1{}^2{}_{:x}} + 1\right]$$

$$V_{3:R^1} = \frac{1}{1-e^{-c}}\left[\frac{V_{3:x}}{c^2\,V_1{}^3{}_{:x}} + 3\,\frac{V_{2:x}}{c\,V_1{}^2{}_{:x}} + 1\right]$$

and

$$U_{2:R^1} = \frac{1}{1-e^{-c}}\left[\frac{V_{2:x}}{c\cdot V_1{}^2{}_{:x}} - \frac{e^{-c}}{1-e^{-c}}\right]$$

$$U_{3:R^1} = \frac{1}{1-e^{-c}}\left[\frac{V_{3:x}}{c^2\cdot V_1{}^3{}_{:x}} - \frac{3\,e^{-c}\cdot V_{2:x}}{(1-e^{-c})\,c\cdot V_1{}^2{}_{:x}} + \frac{e^{-c}\,(1+e^{-c})}{(1-e^{-c})^2}\right]$$

E. The Sampling Distribution of the Average Claim Cost

The first three moments, about the origin, of the total cost, t, of a fixed number of claims were determined in section C as:

$$V_{1:t} = {}_nV_{1:x}$$

$$V_{2:t} = {}_nV_{2:x} + n(n-1)\,V_1{}^2{}_{:x}$$

$$V_{3:t} = {}_nV_{3:x} + 3\,n(n-1)\,V_{2:x}\,V_{1:x} + n(n-1)(n-2)\,V_1{}^3{}_{:x}$$

The first three moments, about the origin, of the average claim cost, a, of a fixed number of claims can be obtained directly from these by dividing respectively by the first three powers of n:

$$V_{1:a} = V_{1:x}$$

$$V_{2:a} = \frac{V_{2:x}}{n} + \frac{n-1}{n}\,V_1{}^2{}_{:x}$$

$$V_{3:a} = \frac{V_{3:x}}{n^2} + 3\,\frac{n-1}{n^2}\,V_{2:x}\,V_{1:x} + \frac{(n-1)(n-2)}{n^2}\,V_1{}^3{}_{:x}$$

The first three moments, about the origin, of the ratio of the actual average claim cost to the expected average claim cost, s, of a fixed number of claims can then be obtained by dividing these respectively by the first three powers of $V_{1:x}$, the expected average claim cost.

$$V_{1:s} = 1$$

$$V_{2:s} = \frac{1}{n}\cdot\frac{V_{2:x}}{V_1{}^2{}_{:x}} + \frac{n-1}{n}$$

$$V_{3:s} = \frac{1}{n^2} \cdot \frac{V_{3:x}}{V_1{}^3{}_{:x}} + 3\,\frac{n-1}{n^2} \cdot \frac{V_{2:x}}{V_1{}^2{}_{:x}} + \frac{(n-1)(n-2)}{n^2}$$

and

$$U_{2:s} = \frac{1}{n}\,\frac{U_{2:x}}{V_1{}^2{}_{:x}}$$

$$U_{3:s} = \frac{1}{n^2}\,\frac{U_{3:x}}{V_1{}^3{}_{:x}}$$

with the skewness (Charlier) of:

$$\alpha_{3:s} = \frac{\alpha_{3:x}}{\sqrt{n}}$$

Combining these with the probabilities of 1, 2, ... , n, etc., claims occurring from the Poisson Distribution, (note that the cases where no losses occur are excluded) the first three moments of the ratio of actual average claim cost to the expected average, for all possible number of claims are obtained as:

$$V_1{:}s = \frac{\sum_{n=1}^{\infty} (1)\left(\frac{c^n}{\lfloor n e^c}\right)}{\sum_{n=1}^{\infty} \frac{c^n}{\lfloor n e^c}} = \frac{1 - \frac{1}{e^c}}{1 - \frac{1}{e^c}} = 1$$

$$V_2{:}s = \frac{\sum_{n=1}^{\infty} \left(\frac{1}{n}\frac{V_{2:x}}{V_1{}^2{}_{:x}} + \frac{n-1}{n}\right)\left(\frac{c^n}{\lfloor n e^c}\right)}{\sum_{n=1}^{\infty} \frac{c^n}{\lfloor n e^c}}$$

$$= \frac{\left(1 - \frac{1}{e^c}\right) + \left(\frac{V_{2:x}}{V_1{}^2{}_{:x}} - 1\right)\left(1 - \frac{1}{e^c}\right)K_{(c)}}{1 - \frac{1}{e^c}} = 1 + K_{(c)}\left(\frac{V_{2:x}}{V_1{}^2{}_{:x}} - 1\right)$$

where $$K_{(c)} = \frac{\sum_{n=1}^{\infty} \frac{c^n}{n \lfloor n\, e^c}}{1 - \frac{1}{e^c}} = \frac{\sum_{n=1}^{\infty} \frac{c^n}{n \lfloor n}}{e^c - 1}$$

NOTE: The only method of determining the values of $K_{(c)}$ and of $G_{(c)}$ below is that of laboriously calculating each term of the series and adding them together. For large values of c an approximation is available as shown in the table at the end of this section.

$$V_{3:s} = \frac{\sum_{n=1}^{\infty} \left(\frac{1}{n^2}\frac{V_{3:x}}{V_1{}^3{}_{:x}} + 3\frac{n-1}{n^2}\frac{V_{2:x}}{V_1{}^2{}_{:x}} + \frac{(n-1)(n-2)}{n^2}\right)\left(\frac{c^n}{\lfloor n e^c}\right)}{\sum_{n=1}^{\infty} \frac{c^n}{ne^c}}$$

$$= \frac{\left(1-\frac{1}{e^c}\right) + 3\left(\frac{V_{2:x}}{V_1{}^2{}_{:x}} - 1\right)\left(1-\frac{1}{e^c}\right)K_{(c)} + \left(\frac{V_{3:x}}{V_1{}^3{}_{:x}} - 3\frac{V_{2:x}}{V_1{}^2{}_{:x}} + 2\right)\left(1-\frac{1}{e^c}\right)G_{(c)}}{1-\frac{1}{e^c}}$$

$$= 1 + 3\left(\frac{V_{2:x}}{V_1{}^2{}_{:x}} - 1\right)K_{(c)} + \left(\frac{V_{3:x}}{V_1{}^3{}_{:x}} - 3\frac{V_{2:x}}{V_1{}^2{}_{:x}} + 2\right)G_{(c)}$$

where

$$G_{(c)} = \frac{\sum_{n=1}^{\infty} \frac{c^n}{n^2 \lfloor n}}{e^c - 1}$$

The moments of this ratio of actual to expected average claim cost about the mean then reduce to:

$$U_2{:}s = K_{(c)} \cdot \left(\frac{V_{2:x}}{V_1{}^2{}_{:x}} - 1\right) = K_{(c)} \cdot \frac{U_{2:x}}{V_1{}^2{}_{:x}}$$

$$U_3{:}s = G_{(c)} \cdot \left(\frac{V_{3:x}}{V_1{}^3{}_{:x}} - 3\frac{V_{2:x}}{V_1{}^2{}_{:x}} + 2\right)$$

and the skewness to:

$$\alpha_{3:s} = \frac{G_{(c)}}{[K_{(c)}]^{3/2}} \cdot \alpha_{3:x}$$

Values of $K_{(c)}$ and $G_{(c)}$

C	$K_{(c)}$	$G_{(c)}$	$\frac{G_{(c)}}{[K_{(c)}]^{3/2}}$
1	.766988	.667235	.993335
4	.329627	.157766	.833642
10	.113021	.015322	.403253
40	.025659	.000677	.165
For larger values of c	$\frac{1}{c-1}$	$\frac{1}{(c-1)(c-2)}$	$\frac{\sqrt{c-1}}{c-2}$

F. A Useful Function of Actual and Expected Losses

In section D of this chapter the moments of the distribution of R (the ratio of actual to expected total losses, pure premiums, or loss ratios) were found to involve the expected number of claims, c, in both second and third moments. In many cases the available data consists of that for individual risks or classifications with c having a different value for each observation. In analyzing such data, the function of actual and expected losses, which is described below, will be found useful. Although its form is such that its first and second moments do not involve the value of c, its value in practical use will not be found to arise from this fact alone. It will be largely due to the effective weighting factor of unity for each observation in the suggested function as contrasted to an effective weighting factor of $\frac{1}{E}$ in the R function, which, therefore, exaggerates the influence of the small risk or small classification experience. The suggested function is:

$$Z = \frac{A-E}{\sqrt{E}} = \left(\frac{A}{E} - 1\right)\sqrt{E} = (R-1)\sqrt{E} = (R-1)\sqrt{c \cdot V_{1:x}}$$

The moments of Z can be determined in terms of the moments of R from the relationship $Z = (R-1)\sqrt{C \cdot V_{1:x}}$ and in terms of the moments of x by substituting the values of the moments of R in terms of those of x as follows:

$$V_{1:Z} = \sqrt{C \cdot V_{1:x}}\,(V_{1:R} - 1) = 0$$

$$V_{2:Z} = C \cdot V_{1:x}\,(V_{2:R} - 2\,V_{1:R} + 1) = \frac{V_{2:x}}{V_{1:x}}$$

$$V_{3:Z} = (C \cdot V_{1:x})^{3/2} \cdot (V_{3:R} - 3\,V_{2:R} + 3\,V_{1:R} - 1) = \frac{V_{3:x}}{\sqrt{C}\,(V_{1:x})^{3/2}}$$

from which:

$$U_{2:Z} = U_{2:R}\,(C \cdot V_{1:x}) = \frac{V_{2:x}}{V_{1:x}}$$

$$\sigma_Z = \sigma_R \sqrt{C \cdot V_{1:x}} = \sqrt{\frac{V_{2:x}}{V_{1:x}}}$$

and

$$U_{3:Z} = U_{3:R}\,(C \cdot V_{1:x})^{3/2} = \frac{V_{3:x}}{\sqrt{C}\,(V_{1:x})^{3/2}}$$

$$\alpha_{3:Z} = \alpha_{3:R} = \frac{V_{3:x}}{\sqrt{C}\,(V_{2:x})^{3/2}}$$

As the moments of R will most often be required in terms of the moments of Z the reverse of these relationships will be given. They are:

$$V_{1:R} = \frac{V_{1:Z}}{\sqrt{C \cdot V_{1:x}}} + 1$$

$$V_{2:R} = \frac{V_{2:Z}}{C\,V_{1:x}} + \frac{2\,V_{1:Z}}{\sqrt{C \cdot V_{1:x}}} + 1$$

$$V_{3:R} = \frac{V_{3:Z}}{(C \cdot V_{1:x})^{3/2}} + \frac{3\,V_{2:Z}}{C\,V_{1:x}} + \frac{3\,V_{1:Z}}{\sqrt{C\,V_{1:x}}} + 1$$

$$U_{2:R} = \frac{U_{2:Z}}{C\,V_{1:x}}$$

$$U_{3:R} = \frac{U_{3:Z}}{(C\,V_{1:x})^{3/2}}$$

$$a_{3:R} = a_{3:Z}$$

It will be noted that, although the first two moments of Z are independent of the amounts of expected losses, that the third moment and $a_{3:Z}$ are still functions of the expected loss ($E = C \cdot V_{1:x}$). As the value of $V_{3:Z}$, as calculated from observations having different values of E, will actually be of the form:

$$V_{3:Z} = \frac{V_{3:X}}{V_{1:x}} \cdot (\text{Average value of } \frac{1}{\sqrt{E}} \text{ in the actual observations})$$

it will be necessary, in order to obtain the value of $V'_{3:Z}$ corresponding to a particular value of E (indicated as E'), to make the adjustment:

$$V'_{3:Z} = \frac{(V_{3:Z} \text{ as calculated from the observations})}{\sqrt{E'} \cdot (\text{Average value of } \frac{1}{\sqrt{E}} \text{ in the actual observations})}$$

It will usually be expedient to make this adjustment directly to the value of $a_{3:Z}$ as:

$$a'_{3:Z} = \frac{(a_{3:Z} \text{ as calculated from the observations})}{\sqrt{E'} \cdot (\text{Average value of } \frac{1}{\sqrt{E}} \text{ in the actual observations})}$$

The "average value of $\frac{1}{\sqrt{E}}$ in the actual observations" would involve a very considerable amount of work to calculate exactly but it can be approximated with an accuracy sufficient for most purposes by the separation of the observations into ten or more groups according to the size of the expected loss, E. For a rough approximation the average $\frac{1}{\sqrt{E}}$ for each group would be assumed to be $\frac{1}{\sqrt{\text{Average } E}}$ and these values weighted by the number of observations in the groups. A closer approximation can be obtained by correcting this estimate, $\frac{1}{\sqrt{\text{Average } E}}$, by the factor $\frac{\sqrt{2r+2}}{1+\sqrt{r}}$, where r is the ratio of the highest value of E in the group to the lowest value of E in the group.

G. The Excess Pure Premium Ratio in Terms of the Loss Ratio Distribution

The excess pure premium ratio (for a loss ratio of B, a premium per risk of P, and a permissible or expected average loss ratio of L) is defined as the ratio of the amount of losses which are expected to be in excess of BP per risk to the total of all expected losses. As the permissible loss ratio is subject to many arbitrary changes, it would seem advisable to eliminate it from the theoretical considerations as well as to construct tables of the excess pure premiums which would be independent of the permissible loss ratio.

This can be done by recognizing the excess pure premium (for a ratio of actual to expected losses of R', and an expected loss per risk of E) as the ratio of the amount of losses which are expected to be in excess of $R'E$ per risk to the total of all expected losses. This can be expressed symbolically as:

$$\chi(R', E) = \frac{\sum_{A=R'E}^{\infty} (A - R'E)}{\sum_{A=0}^{\infty} E}$$

where A represents an actual loss per risk and E the corresponding expected loss per risk.

It should be noted here that there is no specific qualification that the sum of all A's be the same as the sum of all E's. The only conditions necessary to obtain the proper excess pure premium for practical application are that all values of E in this equation are identical and that the values of A in the equation are those which are expected to occur.

As all values of E are the same, we can divide the numerator and denominator by E obtaining:

$$\chi_{(R',\,E)} = \frac{\sum_{A=R'}^{\infty} E\left(\frac{A}{E} - R'\right)}{\sum_{A=0}^{\infty}{}^{(1)}} = \frac{\sum_{R=R'}^{\infty}{}^{(R)} \quad - \quad R' \sum_{R=R'}^{\infty}{}^{(1)}}{\sum_{R=0}^{\infty}{}^{(1)}}$$

Insofar as our expectations are concerned the values that R may take form a continuous function for each value of which the probability that R may take such a value is $F_{(R)}$. Thus, in terms of these probabilities, the excess pure premium ratio is:

$$\chi_{(R',\,E)} = \frac{\int_{R=R'}^{\infty} R \cdot F_{(R)} \cdot dR - R' \cdot \int_{R=R'}^{\infty} F_{(R)} \cdot dR}{\int_{R=0}^{\infty} F_{(R)} \cdot dR}$$

As R may not be negative, we recognize that:

$$\int_{R=0}^{\infty} F_{(R)} \cdot dR = 1,$$

and,

$$\int_{R=R'}^{\infty} R \cdot F_{(R)} \cdot dR = \int_{R=0}^{\infty} R \cdot F_{(R)} \cdot dR - \int_{R=0}^{R'} R \cdot F_{(R)} \cdot dR$$

and as $\int_{R=0}^{\infty} R \cdot F_{(R)} \cdot dR$ is the first moment of R about the origin, then:

$$\int_{R=R'}^{\infty} R \cdot F_{(R)} \cdot dR = V_{1:R} - \int_{R=0}^{R'} R \cdot F_{(R)} \cdot dR$$

Making these substitutions, we have:

$$\chi_{(R',\,E)} = V_{1:R} - \int_{R=0}^{R'} R \cdot F_{(R)} \cdot d\,R - R' \cdot \int_{R=R'}^{\infty} F_{(R)} \cdot d\,R$$

From the formula for integration by parts, we have:

$$\int_{R=0}^{R'} R \cdot F_{(R)} \cdot d\,R = R' \int_{R=0}^{R'} F_{(R)} \cdot d\,R - \int_{R=0}^{R'} \int_{R=0}^{R'} F_{(R)} \cdot d\,R \cdot d\,R$$

which gives us:

$$\chi_{(R',\,E)} = V_{1:R} - R' + \int_{R=0}^{R'} \int_{R=0}^{R'} F_{(R)} \cdot d\,R \cdot d\,R$$

as the actual functional form of the excess pure premium ratio which can then be put in the form of:

$$\chi_{(B,\,P,\,L)} = \frac{V_{1:B}}{L} - \frac{B}{L} + \int_{R=0}^{B/L} \int_{R=0}^{B/L} F_{(R)} \cdot d\,R \cdot d\,R$$

H. The Loss Elimination Ratio in Terms of the Distribution of Individual Losses

The loss elimination ratio, or "K" value, used in determining rates or discounts for deductible insurance, is (for an assured's retention of B dollars) the ratio to total losses of the total of the first $\$B$ of each loss. Thus:

$$K = \frac{(\text{All Losses of less than } \$B) + B \cdot (\text{Number of Losses over } \$B)}{\text{Total of All Losses}}$$

In terms of the distribution of individual losses by size of loss, this becomes:

$$K = \frac{\int_0^B x \cdot f(x) \cdot d\,x + B \int_B^{\infty} f(x) \cdot d\,x}{\int_0^{\infty} x \cdot f(x) \cdot d\,x}$$

which through the substitution of:

$$\int_0^B x \cdot f(x) \cdot dx = B \int_0^B f(x) \cdot dx - \int_0^B \int_0^B f(x) \cdot dx \cdot dx$$

becomes:

$$K = \frac{B\int_0^\infty f(x) \cdot dx - \int_0^B \int_0^B f(x) \cdot dx \cdot dx}{\int_0^\infty x \cdot f(x) \cdot dx}$$

and recognizing $\int_0^\infty x \cdot f(x) \cdot dx$ as $V_{1:x}$ and $\int_0^\infty f(x) \cdot dx$ as unity, we have

$$K = \frac{B - \int_0^B \int_0^B f(x) \cdot dx \cdot dx}{V_{1:x}}$$

as the functional form of the loss elimination ratio.

I. The Fundamentals of Experience Rating

For the purpose of this paper, experience rating will be defined as a procedure to obtain, on the average, better estimates of the inherent hazard of the coverage provided individual risks than that represented by the premium at manual rates. This definition must be recognized as being entirely different from one that would include all methods of partial "self-rating." Many such methods produce premium charges that, on the average, represent poorer estimates of the hazard than the original premium at manual rates.

Obviously, in order for experience rating to be necessary, there must exist either a demonstrable difference in the inherent hazards of risks not ade-

quately measured by the manual rating procedure or appreciable errors in the manual rates. The other basic premise is that the accuracy of an experience rating procedure must be judged on a percentage basis. Otherwise a $100 error on a $1,000 risk could be offset by a $100 error on a $10,000 risk. Because of their simplicity and their firm foundation in practice, the following outline of the experience rating procedure will be based on the linear regression formulae resulting from the methods of least squares.

Figure 1. will help to visualize the experience rating process on this basis. It is representative of all risks for which the premium at manual rates is a specified amount, P, contemplating an expected loss of E'. For such risks the true inherent hazards are represented by $E'(1+m)$, where m varies from risk to risk. The ratio of the true inherent hazard, $E'(1+m)$, to the contemplated hazard, E', is then equal to $(1+m)$, which is measured on the vertical axis. An assumed frequency distribution of risks according to the value of $(1+m)$ is shown along this axis.

The ratio of the actual losses of the risk, A, to E', the expected losses contemplated by the premium at manual rates, is represented by R' and is measured along the horizontal axis. For all risks having a manual premium of P, the frequency distribution of risks according to the value of R' will be a skew distribution such as shown along the horizontal axis (except for very large values of P, when this distribution may be even skew in the other direction). The resulting frequency surface of $(1+m)$ and R' will be approximately as shown by the contour lines.

One very important characteristic of such a frequency surface is that the regression line of R' on $(1+m)$ is always the line: $R' = (1+m)$. This is evident from a consideration of the risks having a particular value of m. For such risks the true inherent hazard is $E'(1+m)$. For risks with such a true inherent hazard the average of the actual losses will be $E'(1+m)$ and the average ratio of the actual losses to the expected losses contemplated by the premium at manual rates will be $(1+m)$. As the regression line of y on x is the straight line, if such a straight line exists, passing through the mean values of y for each particular value of x, then the regression line of R' on $(1+m)$ is the line $R' = (1+m)$.

The regression line of y on x has the formula:

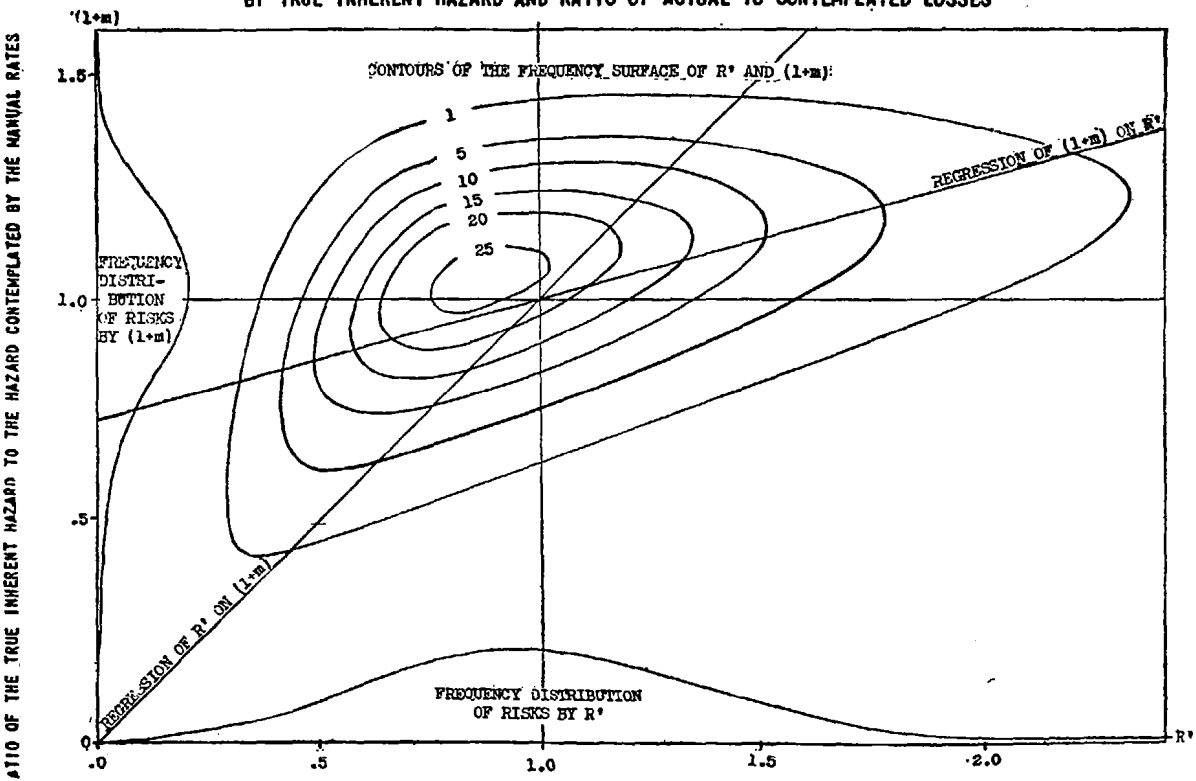
$$y = \left(r_{xy} \cdot \frac{\sigma_y}{\sigma_x}\right) x + \left(V_{1:y} - r_{xy} \cdot \frac{\sigma_y}{\sigma_x} \cdot V_{1:x}\right)$$

in terms of the coefficient of linear correlation between x and y, r_{xy}, and the standard deviations of x and y, σx and σy. This gives us first that

$$r_{R'(1+m)} \cdot \frac{\sigma_{R'}}{\sigma_{(1+m)}} = 1 \text{ or } r_{R'(1+m)} = \frac{\sigma_{(1+m)}}{\sigma_{R'}}$$

FIGURE 1.

DISTRIBUTION OF RISKS, HAVING PREMIUM AT MANUAL RATES OF P CONTEMPLATING LOSSES OF E' BY TRUE INHERENT HAZARD AND RATIO OF ACTUAL TO CONTEMPLATED LOSSES



and secondly that:

$$V_{1:R'} - V_{1:(1+m)} = 0 \text{ or } V_{1:R'} = V_{1:(1+m)}.$$

We can place these values directly in the equation of the other regression line, that of $(1+m)$ on R', of:

$$(1+m) = \left(r_{R'(1+m)}\frac{\sigma_{(1+m)}}{\sigma_{R'}}\right)\cdot R' + \left(V_{1:(1+m)} - r_{R'(1+m)}\frac{\sigma_{(1+m)}}{\sigma_{R'}}\cdot V_{1:R'}\right)$$

to obtain:

$$(1+m) = \left(\frac{U_{2:(1+m)}}{U_{2:R'}}\right)R' + \left(1 - \frac{U_{2:(1+m)}}{U_{2:R'}}\right)V_{1:(1+m)}$$

As $U_{2:(1+m)} = U_{2:m}$ and, if the rate level is assumed to be correct, $V_{1:(1+m)} = 1$, we have as the regression line of $(1+m)$ on R' on a percentage basis:

$$(1+m) = \left(\frac{U_{2:m}}{U_{2:R'}}\right)R' + \left(1 - \frac{U_{2:m}}{U_{2:R'}}\right)$$

and, by multiplying through by E'

$$E'(1+m) = \left(\frac{U_{2:m}}{U_{2:R'}}\right)A + \left(1 - \frac{U_{2:m}}{U_{2:R'}}\right)E'$$

or: Estimated True Inherent Hazard $= Z\cdot A + (1-Z)\cdot E'$, which is recognized as the typical experience rating formula. Furthermore, if we define K as being equal to $E'\left(\frac{U_{2:R'}}{U_{2:m}} - 1\right)$ then $Z = \frac{E'}{E'+K}$

and we have the well established credibility formula originally suggested by Mr. Greene* as a practical approximation to the more complicated formula developed by Mr. Whitney.**

In order to evaluate K let us assume for the moment that, as a result of chance variation only, the actual losses are distributed, for risks for which the true expected losses are E, in such a way that the first two moments about the origin are: $V_{1:T} = E$, and $V_{2:T} = H\cdot E + E^2$. Then, for risks having an inherent hazard of $E'(1+m)$, the corresponding moments would be: $V_{1:T} = E'(1+m)$ and $V_{2:T} = H\cdot E'(1+m) + E'^2(1+m)^2$. The ratios of these actual losses to E', the losses contemplated by the premium at manual rates, would be: $V_{1:R'} = (1+m)$ and $V_{2:R'} = \frac{H\cdot(1+m)}{E'} + (1+m)^2$, for a particular value of m.

* P.C.A.S., Vol. V, page 133.
** P.C.A.S., Vol. IV, page 274.

Averaging these moments for all values of m, but only for risks having losses contemplated by the premium at manual rates of E', we will obtain: $V_{1:R'} = 1$ and $V_{2:R'} = \frac{H}{E'} + 1 + U_{2:m}$ if it is again assumed that the rate level is correct so that $V_{1:m} = 0$. This gives us $U_{2:R'} = \frac{H}{E'} U_{2:m}$ which can be substituted in the formula for K to obtain: $K = \frac{H}{U_{2:m}}$.

Returning to our original supposition involving H we find that:

$$H = \frac{V_{2:T} - E^2}{E} = \frac{V_{2:T} - V_1{}^2{}_{:T}}{E} = \frac{U_{2:T}}{E} = E \cdot U_{2:R} = U_{2:Z}$$

which gives us as the final values of K:

$$K = \frac{U_{2:T}}{E \cdot U_{2:m}} = \frac{E \cdot U_{2:R}}{U_{2:m}} = \frac{U_{2:Z}}{U_{2:m}}$$

It must be definitely understood here that while the second moments of T, R, and Z in the formula for K are the measures of chance variation only. they measure the chance variation of all risks. Thus K is not necessarily a constant but will vary between classifications for at least three reasons: (1) variation in the accuracy of the manual rate, and (2) variation in the diversity of the inherent hazard of risks in the classification, both of which are jointly measured by $U_{2:m}$, and (3) variation in the relative hazards of the classifications as measured by $\frac{V_{2:x}}{V_{1:x}}$. Variation of K by size of E' will also occur as a result of both (2) and (3) as well as a result of variation in loss frequencies within or between classifications. These variations will be studied in subsequent chapters.

For the special case of a group of classifications for which the manual rates are incorrect, but in all of which all risks have the same distribution of losses by size of loss and have the same expected frequency of loss per unit of exposure, we have, by using the values of $U_{2:T}$, $U_{2:R}$, or $U_{2:Z}$ obtained in previous sections of this chapter:

$$K = \frac{V_{2:x}}{V_{1:x}} \cdot \frac{1}{U_{2:m}}$$

as in this case $U_{2:m}$ measures only the errors in the manual rates.

II.

Preparation of Tables of the Normal Sampling Range Due to Chance Fluctuations Only

A. Number of Claims or Claim Frequencies

In other lines of statistical analysis, tables of the normal range of values to be expected as a result of the sampling variation are found to be valuable aids in the interpretation of the significance of observed data. Such a table for the ratio of actual to expected number of claims or claim frequencies would be universally applicable to any line of casualty insurance for which the assumption of the Poisson Distribution is valid.

In Table 3, the probabilities from the Poisson Distribution of obtaining 0, 1, 2, ..., etc., claims are calculated for the values of c, the expected number of claims, of 1, 4, 10, and 40. The ogives of these probabilities are also shown representing the probability of n or less claims occurring. The values of the ratio of actual to expected number of claims corresponding to the .005, .025, and .050 points on these ogives are entered in Table 5.

As c increases, the labor involved in this calculating procedure becomes prohibitive and recourse to an approximation is made. As will be pointed out later, it is believed that this approximation produces the correct result to the number of digits retained in Table 5.

For values of c above 40, the skewness of this sampling distribution $\left(=\frac{1}{\sqrt{c}}\right)$ is comparatively small although significant. For these values of c the Poisson Distribution is closely approximated by the Type III frequency distribution. The ogives of the Type III distribution have been tabulated for 1/10th intervals of skewness.*

In Table 4, the values of the abcissas, measured in standard deviational units from the mean, corresponding to the .005, .025, and .050 points on the ogives, are shown. These are values interpolated from the tables corresponding to the required skewness. The values in Table 5 are calculated directly from these by multiplying by the standard deviation and adding the mean to produce the required results in the scale of the ratio of actual to expected values.

An indication of the accuracy of the approximation of the Poisson Distribution by the Type III distribution is obtained by comparing the values entered

* Note: Although the writer used the tables given in "Introduction to Mathematical Statistics," by J. W. Glover and H. C. Carver, published in mimeograph form in 1926 by Edwards Brothers, Ann Arbor, Michigan, these tables are understood to be available in Volume 2 of the "Annals of Mathematical Statistics" in a paper by L. R. Salvosa.

in Table 5 for c equal to 1, 4, 10, and 40 with the values which would have been obtained from the Type III distribution as calculated in Table 4. For the lower and upper 2½% points, the comparison is:

c	Value of Ratio Corresponding to a Probability of a Lesser Value Occurring of:			
	.025		.975	
	Poisson	Type III	Poisson	Type III
1	.000	—1.000	3.000	3.000
4	.250	.250	2.000	2.000
10	.400	.400	1.700	1.700
40	.700	.700	1.325	1.325

B. Total Losses, Pure Premiums, and Loss Ratios

(1) The various distributions of claims by size of claim are uniform in that they all exhibit a concentration of frequency at the low amounts with a tapering off of the frequencies up to and including very high amounts. This produces a skewness far in excess of that usually encountered in a study of frequency distributions. The only type of theoretical frequency distribution which has been found to fit these distributions of claims by size is the Normal Logarithmic Distribution. Tests of the goodness of fit of this type of distribution have indicated that, except for the concentration of claims at such round-figures values as $50, $100, $500, and $1000, the departures of the actual distributions from the Normal Logarithmic are not greater than would frequently occur in samples of the size tested. (See Table 6 for an example of procedure of fitting such a distribution and the test as to its goodness of fit.)

The only condition necessary to produce a Normal Logarithmic Distribution is that the amount of an observed value be the product of a large number of factors, each of which is independent of the size of any other factor. Reflection as to the conditions entering into the determination of the amount of a claim settlement in casualty insurance, the variations in the seriousness of accidents for which claims are made, and all of the factors eventually recognized in making the final settlement makes it apparent that the necessary condition is at least approximated in the data with which we are concerned. When this condition is met, the logarithms of the observations become the sum of a large number of independent elements, which is the only condition necessary to result in a Normal Distribution. Thus, we shall expect to find the logarithms of the claim amounts normally distributed.

The generalized Normal Logarithmic Distribution, which we shall use, provides an additional degree of freedom in fitting the actual conditions by assuming that only the amount of all observations over and above a fixed amount are distributed in the manner described. Thus, if x represents the

amount of a claim and "a" this fixed amount, it will be assumed that log $(x - a)$ is normally distributed with a mean of l_0 and a standard deviation of σ_e. The quantity:

$$\frac{\log (x - a) - l_0}{\sigma_e}$$

will then be distributed normally with a mean of zero and a standard deviation of unity, permitting the use of available tables of the integral of the normal distribution in fitting this type of distribution to the observed distributions.

Although the necessary transformation from the original scale of observations to the logarithmic distribution is not difficult, the determination of the constants, a, l_0, and σ_e, from the moments of the observed distribution is quite involved. S. D. Wicksell has derived the procedure for the determination of the constants as follows:

If $$s = \frac{-a_3}{2}$$

and $$\eta = \sqrt[3]{-s + \sqrt{s^2 + 1}} + \sqrt[3]{-s - \sqrt{s^2 + 1}}$$

then $$a = M - \frac{\sigma}{\eta},$$

$$l_0 = \log_{10} \frac{(M - a)^2}{\sqrt{U_2 + (M - a)^2}}, \text{ and}$$

$$\sigma_e = \sqrt{2 (\log_{10} e [\log_{10} (M - a) - l_0]}$$

$$= \sqrt{.868589 [\log_{10} (M - a) - l_0]}$$

where: M, σ, U_2, and a_3 represent the Mean, Standard Deviation, the second moment about the mean, and the skewness, respectively, of the distribution to be fitted.

(2) The standard deviation and skewness of the sampling distribution of the ratio of actual to expected losses, pure premiums, and loss ratios corresponding to a particular value of the expected loss, $E = c \cdot V_{1:x}$, are proportional to $\sqrt{\frac{V_{2:x}}{V_{1:x}}}$ and $\frac{V_{3:x}}{V_{2:x}} \div \sqrt{\frac{V_{2:x}}{V_{1:x}}}$ respectively; functions of the distribution of claims by size of claim. The values of these functions vary by line of insurance and may vary by classification or territory. The extent of the variation by line of insurance is shown in the following table, which gives the values for several of the casualty lines calculated from the distribution of claims by size group as reported under the official calls for New York State experience.

Line of Insurance	Coverage	Classifications	$\sqrt{\frac{V_{2:x}}{V_{1:x}}}$	$\frac{V_{3:x}}{V_{2:x}} \div \sqrt{\frac{V_{2:x}}{V_{1:x}}}$
Workmen's Compensation		All	56.89	165.53
Automobile	B.I.	Priv. Pass.	50.34	148.27
"	"	Commercial	55.93	164.32
"	P.D.	"	11.84	60.81
Manufacturers' and Contractors	B.I.	All	70.96	247.34
Manufacturers' and Contractors	P.D.	"	38.86	108.64
Owners', Landlords' and Tenants'	B.I.	Excl. N. Y. C. Apts. and Tenements	36.65	145.44
Product	B.I.	Foodstuffs	15.25	71.93
"	"	All Others	49.17	166.07

To indicate in detail the advocated procedure of calculating the desired table of the normal sampling range, property damage liability coverage on commercial automobiles has been selected as an example. Although the following discussion deals only with this single case, it is believed that the method is equally adaptable to all cases. Comparison of the resulting Table 10 with Table 5 gives a specific comparison of the normal sampling variation in total losses, pure premiums, or loss ratios with that occurring in the number of claims or claim frequencies.

In preparing the desired tables of the normal sampling range of the ratio of actual to expected values of total losses, pure premiums, or loss ratios, the values of M, σ, U_2, and α_3 will be (as found in section D of I):

$$M = 1 \qquad U_2 = \frac{1}{c}\frac{V_{2:x}}{V_1{}^2{}_{:x}}$$

$$\sigma = \frac{1}{\sqrt{c}}\sqrt{\frac{V_{2:x}}{V_1{}^2{}_{:x}}} \text{ and } \alpha_3 = \frac{1}{\sqrt{c}}\frac{V_{3:x}}{(V_{2:x})^{3/2}}$$

For commercial automobile property damage claims we have:

$$\sqrt{\frac{V_{2:x}}{V_1{}^2{}_{:x}}} = 2.050 \text{ and } \frac{V_{3:x}}{(V_{2:x})^{3/2}} = 10.524$$

In fitting available theoretical distributions to this data, we will find three different ranges requiring separate treatment. The first of these will be where the expected number of claims is small (10 or less). In this range, the occurrence of no losses must be recognized as a distinct possibility and the number of such cases set aside before attempting to fit a continuous distribution such as the Normal Logarithmic Distribution to the remaining cases. This pro-

cedure is followed in Table 7, where each successive step has been set out in order to show the algebraic process as well as the arithmetic computation.

The second range is that where the probability of obtaining zero losses is insignificant, although the skewness of the distribution of losses by amount of loss is still a controlling influence. Here the Normal Logarithmic Distribution is fitted directly by omitting steps (2) to (6) inclusive and step (28) as shown in Table 8.

The third range is that for very large values of expected claims. In this range the skewness, although still large enough to preclude the use of the normal distribution, comes to a level recognized by a Type III distribution. The calculation procedure can thus be further reduced to that shown in Table 9.

Table 10 presents the final results of the calculations of Tables 7, 8, and 9 and shows for the ratio of actual to expected total losses, pure premiums, and loss ratios the normal sampling range. This table corresponds for these statistics to Table 5 for the number of claims or claim frequencies.

C. *Average Claim Costs (of a Fixed Number of Claims)*

In actual practice we will usually be concerned only with the sampling variation of the ratio of actual to expected average claim costs for the fixed number of claims that actually occurred.

From section E of part I, we find the necessary statistics to construct the desired table as:

$$M = 1,\ \sigma = \frac{1}{\sqrt{n}} \frac{\sqrt{U_{2:x}}}{V_{1:x}} \text{ and } \alpha_3 = \frac{1}{\sqrt{n}} \alpha_{3:x}$$

which, combined with the values for Commercial Automobiles, P.D., give:

$$M = 1,\ \sigma = \frac{1.7891}{\sqrt{n}} \text{ and } \alpha_3 = \frac{13.972}{\sqrt{n}}$$

In fitting theoretical distributions to this data, it will again be necessary to use the Normal Logarithmic Distribution for the smaller values of n (less than 1440), while the Type III distribution will expedite calculations for larger values of n. The resulting Table 11 is presented, without again showing the details of calculation which are similar to those of Tables 8 and 9.

D. *Average Claim Costs (with c Claims Expected)*

In some few cases we shall be concerned with the sampling variation of the ratio of actual to expected average claim costs when only the expected

number of claims is known. From section E of Part I, we find the necessary statistics from which to calculate the desired table of sampling variation as:

$$M = 1, \qquad \sigma = \sqrt{K_{(c)}} \cdot \frac{\sqrt{U_{2:x}}}{V_{1:x}} \text{ and } a_3 = \frac{G_{(c)}}{K_{(c)}^{3/2}} \cdot a_{3:x}$$

or combined with the values for Commercial Automobiles, P.D.:

$$M = 1, \qquad \sigma = 1.7891 \sqrt{K_{(c)}} \text{ and } a_3 = 13.972 \frac{G_{(c)}}{K_{(c)}^{3/2}}$$

The Normal Logarithmic Distribution will again be found useful in fitting a theoretical distribution for values of c less than 1440. The results are shown in Table 12, where it is found that this table is practically identical with Table 11 for values of c, of 40, or greater.

A very substantial and carefully written paper by Mr. Stefan Peters, designed as a second part to his paper in Volume XXVIII, Part I, Page 105, was approved for the *Proceedings* by the Committee on Papers. Mr. Peters' induction into the armed forces did not permit him to complete and test calculations designed to be annexed, and at his own request, publication is deferred.

—EDITOR.

TABLE 1

RATIO OF ACTUAL TO EXPECTED NUMBER OF CLAIMS OR CLAIM FREQUENCIES

(1) Possible Values of Ratio r	(2) Probability or Frequency $f(r)$	(3) Col.(1) x Col.(2) $r \cdot f(r)$ = Col.(2)*	(4) Col.(1) x Col.(3) $r^2 f(r)$ = $\frac{\text{Col.(2)*}}{c}$ + Col.(2)**	(5) Col.(1) x Col.(4) $r^3 f(r)$ = $\frac{\text{Col.(2)*}}{c^2}$ + $\frac{3\ \text{Col.(2)**}}{c}$ + Col.(2)***
$\frac{0}{c}$	$\frac{1}{e^c}$	0	$0 = 0 + 0$	$0 = 0 + 0 + 0$
$\frac{1}{c}$	$\frac{c}{e^c}$	$\frac{1}{e^c}$	$\frac{1}{ce^c} = \frac{1}{ce^c} + 0$	$\frac{1}{c^2e^c} = \frac{1}{c^2e^c} + 0 + 0$
$\frac{2}{c}$	$\frac{c^2}{\lfloor 2 \cdot e^c}$	$\frac{c}{e^c}$	$\frac{2c}{ce^c} = \frac{c}{ce^c} + \frac{1}{e^c}$	$\frac{2^2c}{c^2e^c} = \frac{c}{c^2e^c} + \frac{3}{ce^c} + 0$
$\frac{3}{c}$	$\frac{c^3}{\lfloor 3 \cdot e^c}$	$\frac{c^2}{\lfloor 2 \cdot e^c}$	$\frac{3c^2}{c\lfloor 2 \cdot e^c} = \frac{c^2}{c\lfloor 2 \cdot e^c} + \frac{c}{e^c}$	$\frac{3^2c^2}{c^2\lfloor 2 \cdot e^c} = \frac{c^2}{c^2\lfloor 2 \cdot e^c} + \frac{3c}{ce^c} + \frac{1}{e^c}$
$\frac{4}{c}$	$\frac{c^4}{\lfloor 4 \cdot e^c}$	$\frac{c^3}{\lfloor 3 \cdot e^c}$	$\frac{4c^3}{c\lfloor 3 \cdot e^c} = \frac{c^3}{c\lfloor 3 \cdot e^c} + \frac{c^2}{\lfloor 2 \cdot e^c}$	$\frac{4^2c^2}{c^2\lfloor 3 \cdot e^c} = \frac{c^3}{c^2\lfloor 3 \cdot e^c} + \frac{3c}{c\lfloor 2 \cdot e^c} + \frac{c}{e^c}$
.....	....	....	.....	.....
$\frac{n}{c}$	$\frac{c^n}{\lfloor n \cdot e^c}$	$\frac{c^{(n-1)}}{\lfloor n-1 \cdot e^c}$	$\frac{nc^{(n-1)}}{c\lfloor n-1 \cdot e^c} = \frac{c^{(n-1)}}{c\lfloor n-1 \cdot e^c} + \frac{c^{(n-2)}}{\lfloor n-2 \cdot e^c}$	$\frac{n^2c^{(n-1)}}{c^2\lfloor n-1 \cdot e^c} = \frac{c^{(n-1)}}{c^2\lfloor n-1 \cdot e^c} + \frac{3c^{(n-2)}}{c\lfloor n-2 \cdot e^c} + \frac{c^{(n-3)}}{\lfloor n-3 \cdot e^c}$
.....	....	....	.....	.....
TOTAL	1	1	$= \frac{1}{c} + 1$	$= \frac{1}{c^2} + \frac{3}{c} + 1$

*Column (2) refers to the item in Column (2) of the table but one line above.
**Column (2) refers to the item in Column (2) of the table but two lines above.

TABLE 2

EXPECTED FIRST, SECOND, AND THIRD POWERS OF TOTAL LOSSES

(1) Number of Claims n	(2) Probability of n Claims	(3) Average Value of T if n Claims occur	(4) (2) x (3) [=CV_1 · Col.(2)*]	(5) Average Value of T^2 if n Claims Occur	(6) (2) x (5) [=CV_2·Col.(2)*+$C^2V_1^2$ Col.(2)**]	(7) Average Value of T^3 if n Claims Occur	(8) (2) x (7) [=$C.V_3$Col.(2)*+$3C^2V_2V_1$.Col.(2)** +$C^3V_1^3$ Col.(2)***]
0	$\frac{1}{e^c}$	0	0	0	0	0	0
1	$\frac{c}{e^c}$	V_1	$C.V_1.\frac{1}{e^c}$	V_2	$C.V_2.\frac{1}{e^c}$	V_3	$C.V_3.\frac{1}{e^c}$
2	$\frac{c^2}{2e^c}$	$2V_1$	$C.V_1.\frac{c}{e^c}$	$2.V_2 + 2.V_1^2$	$C.V_2.\frac{c}{e^c} + C^2V_1^2.\frac{1}{e^c}$	$2.V_3 + 3.2.1.V_2V_1$	$C.V_3.\frac{c}{e^c} + 3\,C^2V_2V_1.\frac{1}{e^c}$
3	$\frac{c^3}{3e^c}$	$3V_1$	$C.V_1.\frac{c^2}{2e^c}$	$3V_2 + 3.2.V_1^2$	$C.V_2\frac{c^2}{2e^c} + C^2.V_1^2\frac{c}{e^c}$	$3.V_3+3.3.2.V_2.V_1+3.2.1.V_1^3$	$C.V_3\frac{c^2}{2e^c} + 3.C^2V_2.V_1\frac{c}{e^c} + C^3.V_1^3\frac{1}{e^c}$
n	$\frac{c^n}{n!e^c}$	nV_1	$C.V_1\frac{c^{n-1}}{(n-1)!e^c}$	$n.V_2+n(n-1)V_1^2$	$C.V_2\frac{c^{n-1}}{(n-1)!e^c} + C^2.V_1^2\frac{c^{n-2}}{(n-2)!e^c}$	$n.V_3+3n(n-1)V_2.V_1+n(n-1)(n-2)V_1^3$	$C.V_3\frac{c^{n-1}}{(n-1)!e^c}+3.C^2.V_2.V_1\frac{c^{n-2}}{(n-2)!e^c} +C^3.V_1^3\frac{c^{n-3}}{(n-3)!e^c}$
Total	1		$C.V_1$		$C.V_2+C^2.V_1^2$		$C.V_3+3C^2.V_2.V_1+C^3.V_1^3$

*Column (2) refers to the item in Column (2) of the table but one line above.
**Column (2) refers to the item in Column (2) of the table but two lines above.
***Column (2) refers to the item in Column (2) of the table but three lines above.

TABLE 3

THE POISSON DISTRIBUTION

FOR c = 1, 4, 10 AND 40

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(1)	(8)	(9)
	c = 1		c = 4		c = 10		c = 40			c = 40 (cont.)	
n	$\frac{c^n}{\lfloor n \cdot e^c}$	Accumulation of Col. (2)	$\frac{c^n}{\lfloor n \cdot e^c}$	Accumulation of Col. (4)	$\frac{c^n}{\lfloor n \cdot e^c}$	Accumulation of Col. (6)	$\frac{c^n}{\lfloor n \cdot e^c}$	Accumulation of Col. (8)	n	$\frac{c^n}{\lfloor n \cdot e^c}$	Accumulation of Col. (8)
0	.3679	.3679	.0183	.0183	.0000	.0000			30	.0185	.0618
1	.3679	.7358	.0733	.0916	.0005	.0005			31	.0238	.0856
2	.1839	.9197	.1465	.2381	.0023	.0028			32	.0298	.1154
3	.0613	.9810	.1954	.4335	.0076	.0104			33	.0361	.1515
4	.0153	.9963	.1954	.6289	.0189	.0293			34	.0425	.1940
5	.0031	.9994	.1563	.7852	.0378	.0671			35	.0485	.2425
6	.0005	.9999	.1042	.8894	.0631	.1302			36	.0539	.2964
7	.0001	1.0000	.0595	.9489	.0901	.2203			37	.0583	.3547
8			.0298	.9787	.1126	.3329			38	.0614	.4161
9			.0132	.9919	.1251	.4580			39	.0629	.4790
10			.0053	.9972	.1251	.5831			40	.0629	.5419
11			.0019	.9991	.1137	.6968			41	.0614	.6033
12			.0006	.9997	.0948	.7916			42	.0585	.6618
13			.0002	.9999	.0729	.8645			43	.0544	.7162
14			.0001	1.0000	.0521	.9166			44	.0495	.7657
15					.0347	.9513			45	.0440	.8097
16					.0217	.9730			46	.0382	.8479
17					.0128	.9858			47	.0325	.8804
18					.0071	.9929		.0001	48	.0271	.9075
19					.0037	.9966	.0001	.0002	49	.0221	.9296
20					.0019	.9985	.0002	.0004	50	.0177	.9473
21					.0009	.9994	.0004	.0008	51	.0139	.9612
22					.0004	.9998	.0007	.0015	52	.0107	.9719
23					.0002	1.0000	.0012	.0027	53	.0081	.9800
24					.0001		.0019	.0046	54	.0060	.9860
25							.0031	.0077	55	.0043	.9903
26							.0047	.0124	56	.0031	.9934
27							.0070	.0194	57	.0022	.9956
28							.0100	.0294	58	.0015	.9971
29							.0139	.0433	59 & over	.0029	1.0000

TABLE 4

Ratio of Actual to Expected

Number of Claims or Claim Frequencies

Calculation of the Normal Sampling Range of the Ratio

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Expected Number of Claims	Mean	Standard Deviation	Skewness	Abscissa in Standard Deviational Units of a Type III Frequency Distribution Having the Skewness Shown in Column (4) Corresponding to					
c	1.	$1/\sqrt{c}$	$1/\sqrt{c}$	$\frac{1}{2}\%$ of Area	$2\frac{1}{2}\%$ of Area	5% of Area	95% of Area	$97\frac{1}{2}\%$ of Area	$99\frac{1}{2}\%$ of Area
1	1.	1.00000	1.00000	-1.67	-1.47	-1.42	+1.88	+2.38	+3.49
4	1.	.50000	.50000	-2.11	-1.72	-1.49	+1.78	+2.19	+3.04
10	1.	.31623	.31623	-2.28	-1.81	-1.55	+1.73	+2.10	+2.88
40	1.	.15810	.15810	-2.43	-1.89	-1.60	+1.72	+2.04	+2.73
90	1.	.10541	.10541	-2.48	-1.91	-1.62	+1.70	+2.01	+2.68
160	1.	.07906	.07906	-2.50	-1.92	-1.62	+1.67	+2.00	+2.65
250	1.	.06325	.06325	-2.52	-1.93	-1.63	+1.66	+1.99	+2.64
360	1.	.05270	.05270	-2.53	-1.94	-1.63	+1.66	+1.99	+2.63
490	1.	.04518	.04518	-2.54	-1.94	-1.63	+1.66	+1.98	+2.62
640	1.	.03953	.03953	-2.54	-1.94	-1.64	+1.66	+1.98	+2.62
810	1.	.03514	.03514	-2.55	-1.95	-1.64	+1.66	+1.98	+2.62
1000	1.	.03162	.03162	-2.55	-1.95	-1.64	+1.66	+1.98	+2.61
1440	1.	.02635	.02635	-2.56	-1.95	-1.64	+1.65	+1.97	+2.60
1960	1.	.02258	.02258	-2.56	-1.95	-1.64	+1.65	+1.97	+2.60
2560	1.	.01976	.01976	-2.56	-1.95	-1.64	+1.65	+1.97	+2.60
3240	1.	.01757	.01757	-2.56	-1.96	-1.64	+1.65	+1.97	+2.59
4000	1.	.01581	.01581	-2.57	-1.96	-1.64	+1.65	+1.97	+2.59
4840	1.	.01437	.01437	-2.57	-1.96	-1.64	+1.65	+1.97	+2.59
5760	1.	.01318	.01318	-2.57	-1.96	-1.64	+1.65	+1.97	+2.59
6760	1.	.01216	.01216	-2.57	-1.96	-1.65	+1.65	+1.97	+2.59
7840	1.	.01129	.01129	-2.57	-1.96	-1.65	+1.65	+1.97	+2.59
9000	1.	.01054	.01054	-2.57	-1.96	-1.65	+1.65	+1.97	+2.59
10240	1.	.00988	.00988	-2.57	-1.96	-1.65	+1.65	+1.97	+2.59

TABLE 5

RATIO OF ACTUAL TO EXPECTED NUMBER OF CLAIMS OR CLAIM FREQUENCIES

Normal Sampling Range of the Ratio [Any Poisson Distribution]

(1)	(2)	(3)	(4)	(5)	(6)	(7)
Expected Number of Claims	Value of the Ratio Corresponding to a Probability of a Lesser Value Occurring of:					
c	.005	.025	.050	.950	.975	.995
1	.000	.000	.000	3.000	3.000	5.000
4	.000	.250	.250	2.000	2.000	2.500
10	.333	.400	.500	1.500	1.700	1.900
40	.625	.700	.750	1.275	1.325	1.425
90	.739	.800	.829	1.179	1.212	1.282
160	.802	.848	.872	1.132	1.158	1.209
250	.841	.878	.897	1.105	1.126	1.167
360	.867	.898	.914	1.088	1.105	1.139
490	.885	.912	.926	1.075	1.089	1.118
640	.900	.923	.935	1.066	1.078	1.104
810	.910	.931	.942	1.058	1.070	1.092
1000	.919	.938	.948	1.052	1.063	1.082
1440	.932	.948	.956	1.044	1.052	1.068
1960	.942	.956	.963	1.037	1.044	1.058
2560	.949	.961	.967	1.033	1.039	1.051
3240	.955	.966	.971	1.029	1.034	1.045
4000	.959	.969	.974	1.026	1.031	1.041
4840	.963	.972	.976	1.024	1.028	1.037
5760	.966	.974	.978	1.022	1.026	1.034
6760	.969	.976	.980	1.020	1.024	1.031
7840	.971	.978	.981	1.019	1.022	1.029
9000	.973	.979	.983	1.017	1.021	1.027
10240	.974	.981	.984	1.016	1.019	1.026

NOTE: Values corresponding to c = 1 to 40 incl. were obtained directly from the Poisson Distribution shown in Table 3.

Values for c = 90 or more were obtained from Table 4, i.e., Col. (2) of Table 5 = Col. (2) plus [Col. (5) × Col. (3)] of Table 4.

TABLE 6

Fitting the Normal Logarithmic Distribution to the Distribution of Claims by Size Group for New York State—Commercial Cars—Property Damage Liability Coverage

(1) Size Group	(2) Number of Claims $f(x)$	(3) Amount of Claims $x \cdot f(x)$	(4) Average Amount x	(5) (3)(4) $x^2 \cdot f(x)$	(6) Upper Limit of Group x	(7) $x-a$	(8) $\log(x-a)$	(9) $\frac{\log(x-a)-l_0}{\sigma_e}$	(10) Normal Curve Integral Corresponding to (9)	(11) $\Delta(10)$	(12) 34672(11) Calculated Frequency	(13) χ^2
0-10	10,825	68,145	6	408,870	10.5	7	.8451	-.5097	.30514	.30514	10,580	5.67
11-25	11,824	212,296	18	3,821,328	25.5	22	1.3424	.3930	.65284	.34770	12,055	4.43
26-50	7,038	263,691	37	9,765,567	50.5	47	1.6721	.9915	.83927	.18643	6,464	50.97
51-75	2,018	129,485	64	8,287,040	75.5	72	1.8573	1.3276	.90784	.06857	2,377	54.22
76-100	1,192	108,708	91	9,892,428	100.5	97	1.9868	1.5627	.94094	.03310	1,148	1.69
101-200	1,238	178,575	144	25,714,800	200.5	197	2.2945	2.1213	.98305	.04211	1,460	34.37
201-250	189	43,947	233	10,239,651	250.5	247	2.3927	2.2995	.98827	.00822	216	3.38
251-300	123	35,016	285	9,979,560	300.5	297	2.4728	2.4449	.99276	.00349	121	.03
301-400	109	38,715	355	13,743,825	400.5	397	2.5988	2.6736	.99625	.00349	121	1.19
401-500	54	24,949	462	11,526,438	500.5	497	2.6964	2.8508	.99782	.00157	54	-
501-750	40	25,118	628	15,774,104	750.5	747	2.8733	3.1719	.99924	.00142	49	1.65
751-1000	15	12,946	863	11,172,398	1000.5	997	2.9987	3.3996	.99966	.00042	15	-
1001-1500	4	4,645	1,161	5,392,845	1500.5	1,497	3.1752	3.7199	.99990	.00024	8	
1501-2000	3	5,512	1,837	10,125,544	2000.5	1,997	3.3004	3.9472	.99996	.00006	2	.36
2001-2500	1	2,044	2,044	4,177,936						.00004	2	
2501-3000	-	-	-	-								
3001-4000	1	3,500	3,500	12,250,000								
Total	34,672	1,157,292		162,263,334								157.96

$n = 34,672$

$V_{1:x} = 1,157,292 \div 34,672 = 33.378$

$V_{2:x} = 162,263,334 \div 34672 = 4,679.963$

$V_{3:x} = 116,824,363,668 \div 34,672 = 3,369,415.200$

$U_{2:x} = 3,565.662$ and $\sigma_{\bar{x}} = 59.715$

$U_{3:x} = 2,975,165.046$ and $\sigma_{3:x} = 13.972$

$$s = \frac{-\alpha_{3:x}}{2} = -6.986$$

$$\eta = \sqrt[3]{-s+\sqrt{s^2+1}} + \sqrt[3]{-s-\sqrt{s^2+1}} = 1.999$$

$$a = V_{1:x} - \frac{\sigma_x}{\eta} = 3.506$$

$$l_0 = \log_{10} \frac{(M-a)^2}{\sqrt{U_{2:x} - (M-a)^2}} = 1.1259$$

$$\sigma_e = \sqrt{.868589 \left[\log_{10} (M-a) - l_0.\right]} = .5509$$

TABLE 7

(1)	c	1	4	10
(2)	$e^{-c} = \text{antilog}_{10}\left[-.4342945\ (1)\right]$	.367879	.018315	.000045
(3)	$\frac{1}{1-e^{-c}} = \frac{1}{1-(2)}$	1.581976	1.018657	1.000045
(4)	$1+e^{-c} = 1+(2)$	1.367879	1.018315	1.000045
(5)	$U_{2:R'} = (3)\frac{4.200820}{(1)} - (2)(3)$	5.724608	1.050743	.420056
(6)	$U_{3:R'} = (3)\left[\frac{90.607122}{(1)^2} - \frac{3(2)(3)(4.200820)}{(1)} + (2)(3)^2(4)\right]$	133.728407	5.728439	.906100
(7)	$\sqrt{c} = \sqrt{(1)}$	1.000000	2.000000	3.162278
(8)	$\frac{1}{\sqrt{c}} = \frac{1}{(7)}$	1.000000	.500000	.316228
(9)	$\sigma_{R'} = \sqrt{(5)}$	2.392615	1.025058	.648102
(10)	$-S = \frac{(6)}{2(5)(9)}$	4.881744	2.659282	1.664242
(11)	$\sqrt{S^2+1} = \sqrt{(10)^2+1}$	4.983114	2.841069	1.941672
(12)	$-S+\sqrt{S^2+1} = (10)+(11)$	9.864858	5.500331	3.606814
(13)	$-S-\sqrt{S^2+1} = (10)-(11)$	-.101370	-.181807	-.277330
(14)	$\sqrt[3]{-S+\sqrt{S^2+1}} = \sqrt[3]{(12)}$	2.144685	1.765210	1.533443
(15)	$\sqrt[3]{-S-\sqrt{S^2+1}} = \sqrt[3]{(13)}$	-.466268	-.566374	-.652126
(16)	$\eta = (14)+(15)$	1.678417	1.198836	.881317
(17)	$\frac{\sigma}{\eta} = \frac{(9)}{(16)}$	1.425519	.855044	.735379
(18)	$a = M - \frac{\sigma}{\eta} = (3) - (17)$	.156457	.163613	.264666
(19)	$(M-a)^2 = (17)^2$	2.032104	.731100	.540782
(20)	$U_2 = (5)$	5.724608	1.050743	.420056
(21)	$\sqrt{U_2+(M-a)^2} = \sqrt{(20)+(19)}$	2.786087	1.334687	.980213
(22)	$(M-a)^2 \div \sqrt{U_2+(M-a)^2} = \frac{(19)}{(21)}$	.729636	.547699	.551698
(23)	$\ell_o = \log (22)$	9.8631074-10	9.7385419-10	9.7417014-10
(24)	$\log (M-a) = \log (17)$	.1539730	9.9319684-10	9.8666112-10
(25)	$\log (M-a) - \ell_o = (24) - (23)$	.2908656	.1934465	.1248098
(26)	$\sigma_\ell^2 = .868589\,(25)$	.2526428	.1680255	.1084084
(27)	$\sigma_\ell = \sqrt{(26)}$	.502636	.409910	.329254

* $\frac{V_{2:x}}{V_{1:x}^2} = 4.200820$ † $\frac{V_{3:x}}{V_{1:x}^3} = 90.607122$

TABLE 7 (Continued)

(1) C	1	4	10
(28) $P' = 1-(3)\left[1-P\right]$			
P= .005	-*	-*	.00496
.025	-*	.00681	.02496
.050	-*	.03228	.04996
.950	.92090	.94907	.95000
.975	.96045	.97453	.97500
.995	.99209	.99491	.99500
(29) t (from tables of the Normal Distribution)			
P= .005	$-\infty$	$-\infty$	-2.578
.025	$-\infty$	-2.467	-1.961
.050	$-\infty$	-1.848	-1.645
.950	+1.411	1.636	1.645
.975	1.756	1.952	1.960
.995	2.413	2.569	2.576
(30) $t.\sigma_{\bar{x}}+\bar{x}_o$ = (29)(27)+(23)			
P= .005	$-\infty$	$-\infty$	8.8929-10
.025	$-\infty$	8.7273-10	9.0960-10
.050	$-\infty$	8.9810-10	9.2001-10
.950	.5723	.4092	.2833
.975	.7457	.5387	.3870
.995	1.0760	.7916	.5899
(31) x-a = antilog of (30)			
P= .005	*	*	.0871
.025	*	.0534	.1247
.050	*	.0957	.1585
.950	3.7351	2.5657	1.9200
.975	5.5680	3.4570	2.4378
.995	11.9120	6.1887	3.8896
(32) x = (31) + (18)			
P= .005	.0000	.0000	.3518
.025	.0000	.2170	.3894
.050	.0000	.2593	.4232
.950	3.8916	2.7293	2.1847
.975	5.7245	3.6206	2.7025
.995	12.0685	6.3523	4.1543

(7)	$\sqrt{(1)}$	3.162278	9.486833	15.811389	22.135944	25.298222	28.460499	31.622777
(8)	1 ÷ (7)	·316228	·105409	.063246	.045175	.039528	.035136	·031623
(9)	2.04956 (8)	·648128	.216042	·129926	.092589	.081015	.072013	.064813
(10)	5.262140(8)	1.664036	.554677	.332809	.237717	.208002	.184891	.166405
(11)	$\sqrt{(10)^2+1}$	1.941396	1.143533	1.053227	1.027866	1.021403	1.016949	1.013751
(12)	(10)+(11)	5.805431	1.698210	1.388736	1.265583	1.229405	1.201840	1.180156
113)	(10)-(11)	-.277359	-.588856	-.721115	-.790149	-.813401	-.832056	-.847346
(14)	$\sqrt[3]{(12)}$	1.533389	1.193064	1.115145	1.081675	1.071268	1.063201	1.056788
(15)	$\sqrt[3]{(13)}$	-·652150	-·838178	·896744	-·924492	-·933473	-·940656	-·946281
(16)	(14)+(15)	.881239	.354886	.218401	.157185	.137795	.122545	.110487
(17)	(9)÷(16)	.735474	.608765	.593523	.589052	.587959	.587166	.586612
(18)	1 -(17)	.264526	.391235	.406477	.410948	.412061	.412834	.413388
(19)	$(17)^2$	.540922	.370595	.352270	.346962	.345672	.344764	.344114
(20)	$(9)^2$	.420070	.046674	.016803	.008573	.006563	.005186	.004201
(21)	$\sqrt{(20)+(19)}$	.980302	.645964	.607514	.596284	.593494	.591566	.590182
(22)	(19)÷21	.551791	.573708	.579855	.581907	·582436	·582799	.583064
(23)	log (22)	9.7417746	9.7586909	9.7633195	9.7648536	9.7652482	9.7655188	9.7657162
(24)	log (17)	9.8665674	9.7844497	9.7734376	9.7701537	9.7693323	9.7687609	9.7683510
(25)	(24)-(23)	.1247928	.0257588	.0101181	.0053001	.0040841	.0032421	.0026348
(26)	.868589(25)	.108394	.022374	.008788	.004604	.003547	.002816	.002289
(27)	$\sqrt{(26)}$	.329232	.149579	.093754	.067853	.059417	.053066	.047843
(29)	from tables of the Normal Distribution							
	P= ·005	-2.5757	-2.5757	-2.5757	-2.5757	-2.5757	-2.5757	-2.5757
	·025	-1.9600	-1.9600	-1.9600	-1.9600	-1.9600	-1.9600	-1.9600
	·050	-1.6449	-1.6449	-1.6449	-1.6449	-1.6449	-1.6449	-1.6449
	·950	1.6449	1.6449	1.6449	1.6449	1.6449	1.6449	1.6449
	.975	1.9600	1.9600	1.9600	1.9600	1.9600	1.9600	1.9600
	.995	2.5757	2.5757	2.5757	2.5757	2.5757	2.5757	2.5757
(30)	(29) (27)+(23)							
	P= .005	8.893772	9.373420	9.521837	9.590085	9.612208	9.628837	9.642487
	·025	9.0964799	9.4855161	9.5795617	9.6318617	9.6487909	9.6615094	9.6719439
	·050	9.2002209	9.5126484	9.6091035	9.6532422	9.6675132	9.6782305	9.6870192
	·950	.2833283	.0047334	9.9175355	9.8764650	9.8629832	9.8528071	9.8444132
	·975	.3870693	.0518657	9.9470773	9.8978455	9.8817065	9.8695282	9.8594885
	.995	.5897775	.1439615	.0048017	9.9403011	9.9182886	9.9022009	9.8889454
(31)	antilog (30)							
	P= .005	.07832	.236276	.332535	.389131	.409457	.425439	.439023
	·025	.12488	.29209	.37981	.42841	.44544	·45868	·46983
	·050	·15857	·32557	·40654	·45005	·46506	·47668	·48643
	·950	1.9201	1.0110	.82706	.75243	.72943	.71254	.69890
	.975	2.4382	1.1268	.88527	.79040	.76156	.74051	.72358
	.995	3.8885	1.3930	1.0111	.87157	.82849	.79836	.77436
(32)	(31)+(16)							
	P= .005	.343	.628	.739	.800	.822	.838	.852
	.025	.389	.683	.786	.839	.858	.872	.883
	.050	.423	.717	·813	·861	·877	·890	.900
	.950	2.185	1.402	1.234	1.163	1.141	1.125	1.112
	.975	2.703	1.518	1.292	1·201	1.174	1.153	1.137
	.995	4.153	1.784	1.418	1.283	1.241	1.211	1.188

TABLE 9

(1) C	(7) $\sqrt{C}$ = $\sqrt{(1)}$	(8) $1/\sqrt{C}$ = $1 \div (7)$	(9) σ_x = 2.050(8)	(10a) α_3 = 10.524(8)	(29) t, (from table of Type III distribution)*						(31) $x = 1 + t_\alpha \sigma = 1.000 + (9)(29)$					
					P=.005	P=.025	P=.050	P=.950	P=.975	P=.995	P=.005	P=.025	P=.050	P=.950	P=.975	P=.9
810	28.460499	.035136	.0720	.370	-2.23	-1.78	-1.53	1.74	2.13	2.92	.839	.872	.890	1.125	1.153	1.
1,000	31.622777	.031623	.0648	.333	-2.27	-1.80	-1.55	1.74	2.11	2.88	.853	.883	.900	1.113	1.137	1.
1,440	37.947332	.026352	.0540	.277	-2.32	-1.83	-1.56	1.72	2.09	2.84	.875	.901	.916	1.093	1.113	1.
1,960	44.271886	.022588	.0463	.238	-2.36	-1.85	-1.58	1.71	2.07	2.80	.891	.914	.927	1.079	1.096	1.
2,560	50.596443	.019764	.0405	.208	-2.38	-1.86	-1.58	1.70	2.06	2.78	.904	.925	.936	1.069	1.083	1.
3,240	56.920999	.017568	.0360	.185	-2.41	-1.87	-1.59	1.70	2.05	2.75	.913	.933	.943	1.061	1.074	1.
4,000	63.245554	.015811	.0324	.166	-2.43	-1.88	-1.60	1.69	2.04	2.73	.921	.939	.948	1.055	1.066	1.
4,840	69.570109	.014374	.0295	.151	-2.44	-1.89	-1.60	1.69	2.03	2.72	.928	.944	.953	1.050	1.060	1.
5,760	75.894665	.013176	.0270	.139	-2.45	-1.89	-1.60	1.68	2.03	2.71	.934	.949	.957	1.045	1.055	1.
6,760	82.219220	.012163	.0249	.128	-2.46	-1.90	-1.61	1.68	2.02	2.69	.939	.953	.960	1.042	1.050	1.
7,840	88.543776	.011294	.0232	.119	-2.47	-1.90	-1.61	1.68	2.02	2.69	.943	.956	.963	1.039	1.047	1.
9,000	94.868331	.010541	.0216	.111	-2.48	-1.91	-1.61	1.68	2.01	2.68	.946	.959	.965	1.036	1.043	1.
10,240	101.192886	.009882	.0203	.104	-2.48	-1.91	-1.62	1.68	2.01	2.67	.950	.961	.967	1.034	1.041	1.

*The values of t are the interpolated values from the table corresponding to the value of α_3 in column (10a) and the indicated value of P.

TABLE 10

RATIO OF ACTUAL TO EXPECTED

TOTAL LOSSES, PURE PREMIUMS, OR LOSS RATIOS

Normal Sampling Range of the Ratio [New York State Automobile—P.D. Commercial Cars]

(1)	(2)	(3)	(4)	(5)	(6)	(7)
Expected Number of Claims	Value of the Ratio Corresponding to a Probability of a Lesser Value Occurring of:					
(c)	.005	.025	.050	.950	.975	.995
1	.000	.000	.000	3.892	5.725	12.069
4	.000	.217	.259	2.729	3.621	6.352
10	.352	.389	.423	2.185	2.703	4.154
40	.529	.585	.621	1.613	1.817	2.318
90	.628	.683	.717	1.402	1.518	1.784
160	.693	.745	.774	1.296	1.374	1.548
250	.739	.786	.813	1.234	1.292	1.418
360	.773	.817	.840	1.192	1.238	1.336
490	.800	.839	.861	1.163	1.201	1.283
640	.822	.858	.877	1.141	1.174	1.241
810	.838	.872	.890	1.125	1.153	1.211
1000	.852	.883	.900	1.112	1.137	1.188
1440	.875	.901	.916	1.093	1.113	1.153
1960	.891	.914	.927	1.079	1.096	1.130
2560	.904	.925	.936	1.069	1.083	1.113
3240	.913	.933	.943	1.061	1.074	1.099
4000	.921	.939	.948	1.055	1.066	1.088
4840	.928	.944	.953	1.050	1.060	1.080
5760	.934	.949	.957	1.045	1.055	1.073
6760	.939	.953	.960	1.042	1.050	1.067
7840	.943	.956	.963	1.039	1.047	1.062
9000	.946	.959	.965	1.036	1.043	1.058
10240	.950	.961	.967	1.034	1.041	1.054

TABLE 11

Ratio of Actual to Expected Average Claim Costs of a Fixed Number of Claims

Normal Sampling Range of the Ratio

[New York State Automobile—P.D. Commercial Cars]

(1) Actual number of Claims (n)	(2)	(3)	(4)	(5)	(6)	(7)
	Value of the Ratio Corresponding to a Probability of a Lesser Value Occurring of:					
	.005	.025	.050	.950	.975	.995
1	.120	.138	.154	3.331	4.916	10.608
4	.388	.411	.430	2.420	3.221	5.797
10	.508	.536	.556	1.991	2.472	3.895
40	.644	.679	.702	1.534	1.733	2.248
90	.710	.748	.771	1.356	1.469	1.741
160	.756	.792	.814	1.263	1.339	1.513
250	.790	.824	.845	1.207	1.263	1.387
360	.815	.847	.866	1.171	1.215	1.310
490	.835	.866	.883	1.145	1.181	1.257
640	.852	.881	.896	1.125	1.155	1.218
810	.865	.891	.906	1.111	1.137	1.192
1000	.876	.901	.914	1.100	1.123	1.170

TABLE 12

Ratio of Actual to Expected

Average Claim Costs (When c Claims Are Expected)

Normal Sampling Range of the Ratio

[New York S Automobile–Commercial

(1)	(2)	(3)	(4)	(5)	(6)	(7)
Expected Number of Claims	Value of the Ratio Corresponding to a Probability of ε Lesser Value Occurring of:					
(c)	.005	.025	.050	.950	.975	.995
1	.227	.243	.257	3.046	4.434	9.417
4	.451	.464	.476	2.415	3.335	6.550
10	.541	.561	.577	2.004	2.533	4.172
40	.645	.679	.702	1.541	1.746	2.279
90	.711	.748	.771	1.358	1.473	1.749
160	.756	.792	.814	1.264	1.340	1.515
250	.789	.824	.844	1.208	1.265	1.390
360	.815	.848	.866	1.171	1.215	1.310
490	.835	.866	.883	1.145	1.181	1.257
640	.851	.880	.895	1.126	1.156	1.220
810	.863	.890	.904	1.113	1.140	1.196
1000	.876	.901	.914	1.100	1.123	1.172

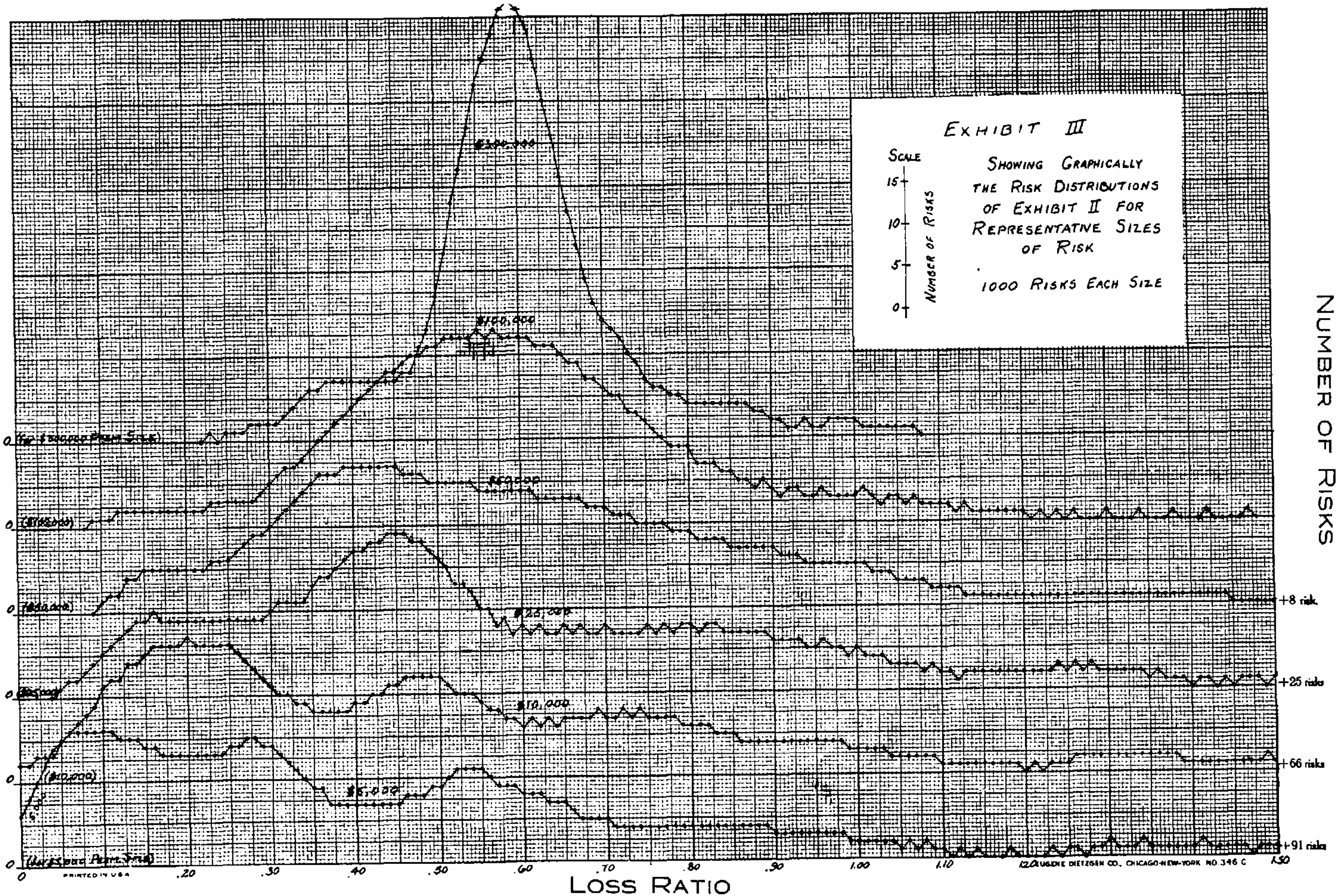
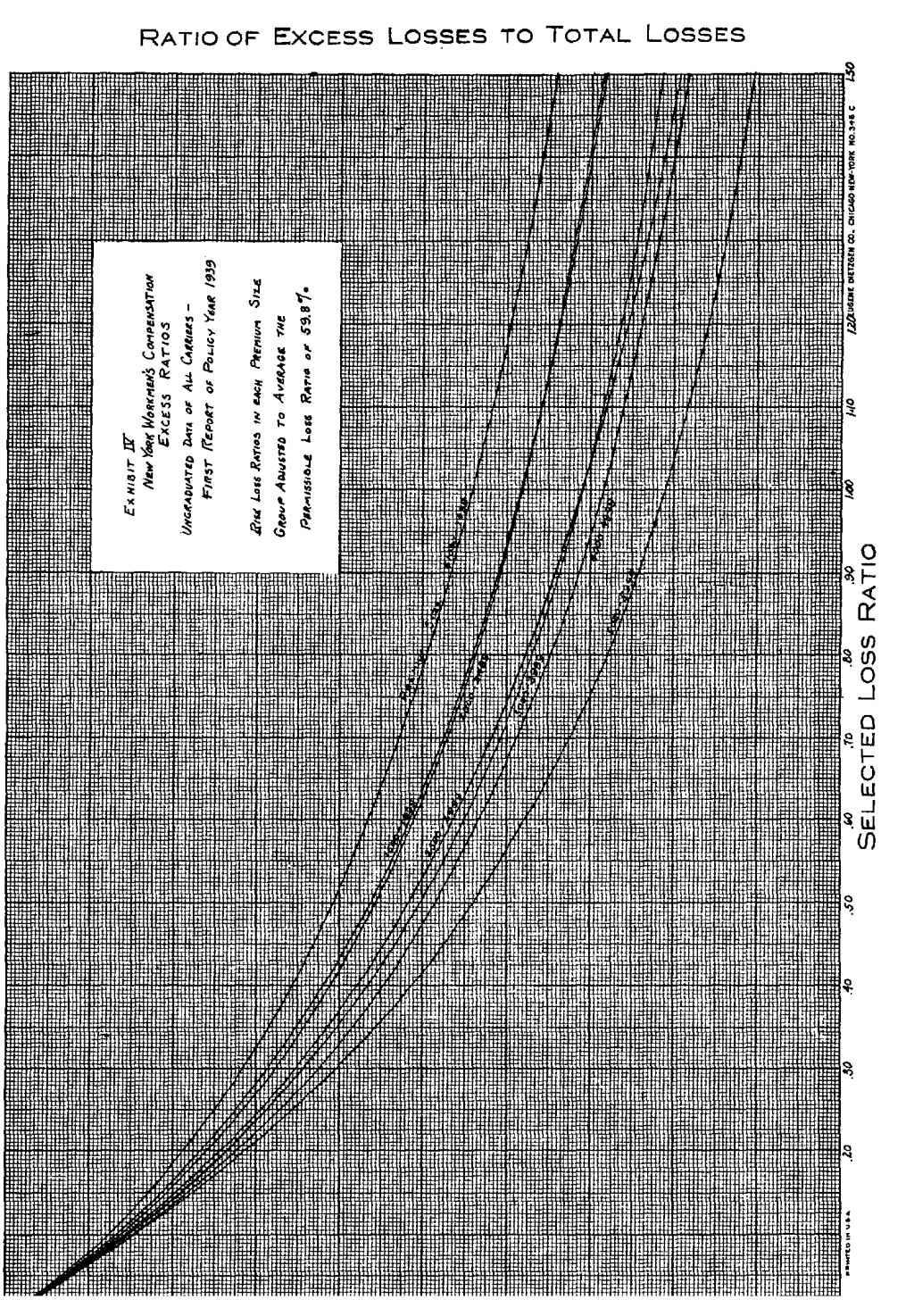


EXHIBIT III
SHOWING GRAPHICALLY THE RISK DISTRIBUTIONS OF EXHIBIT II FOR REPRESENTATIVE SIZES OF RISK
1000 RISKS EACH SIZE
SCALE
15
10
5
0
NUMBER OF RISKS
$500,000
$150,000
$50,000
$25,000
$10,000
$5,000
(For $500,000 Prem. Size)
($150,000)
($50,000)
($25,000)
($10,000)
(For $5,000 Prem. Size)
+8 risks
+25 risks
+66 risks
+91 risks
0
.20
.30
.40
.50
.60
.70
.80
.90
1.00
1.10
1.20
1.50
LOSS RATIO
NUMBER OF RISKS
PRINTED IN U.S.A.
EUGENE DIETZGEN CO., CHICAGO-NEW-YORK NO. 346 C


RATIO OF EXCESS LOSSES TO TOTAL LOSSES
EXHIBIT III
NEW YORK WORKMEN'S COMPENSATION
EXCESS RATIOS
UNGRADUATED DATA OF ALL CARRIERS -
FIRST REPORT OF POLICY YEAR 1939
EXCESS LOSS RATIOS IN EACH PREMIUM SIZE
GROUP ADJUSTED TO AVERAGE THE
PERMISSIBLE LOSS RATIO OF 59.8%
SELECTED LOSS RATIO
.10
.20
.30
.40
.50
.60
.70
.80
.90
100
110
120
150

RISK DISTRIBUTIONS UNDERLYING INSURANCE CHARGES IN THE RETROSPECTIVE RATING PLAN

BY

NELS M. VALERIUS*

Anyone who has reflected in even a non-technical way on retrospective rating in casualty insurance recognizes that there is or should be a connection between the setting of the minimum and maximum premiums and insurance charges of retrospective rating plans and the way the risks are expected to "stack up" as to losses. Perhaps the first thought that occurs is that the minimum and maximum premium limits must be so selected relative to each other that the redundancy of premium from risks turning out very well but paying the minimum premium is expected to be just enough to offset the deficiency from risks turning out badly and not paying their way, as seen retrospectively, because of the maximum limit on premium. There is no theoretical reason why they should not be so selected, and perhaps originally it was intended to select them thus, but this notion has to be modified when net insurance charges are encountered, as they usually are, and it is seen that the redundancy on the one hand does not generally offset exactly the deficiency on the other. It must then be concluded that the minimum premium risks are not sufficient in point of number, low loss ratio or the two combined to offset the relative number, high loss ratio or the two combined of the maximum premium risks. Put in a more statistical style, it is readily recognized that proper insurance charges are dependent on the way the risks are expected to distribute themselves by loss ratio prospectively, in those intervals of the possible range in loss ratio from zero to infinity, where, under the terms of the retrospective rating agreement, the minimum or maximum premium is indicated.

Excess Ratio Charts and Tables

Our non-technical observer takes it for granted that in plans "tailor-made" for individual risks covering several lines of insurance in one agreement, analogy and judgment enter into the setting of premium limits and insurance charges. Risk plans often reflect special requirements of the assured and may even be frankly one-sided, compared to standard premium, to meet unusual hazard conditions. If, however, he has delved[1] into the make-up of

* This study was begun jointly by J. W. Wieder, Jr., a student of the Society, and the writer, but Mr. Wieder was called into the Service before its completion.

[1] The best exposition is S. D. Pinney's paper, "The Retrospective Rating Plan for Workmen's Compensation Risks," P.C.A.S. XXIV, pp. 291-359.

one of the formal plans such as the Retrospective Rating Plan of the National Council on Compensation Insurance, where he has a right to expect the insurance charges would be based on valid statistics, he has found that they are so indeed, but the basic data are derived charts or tables of "excess pure premium ratios" and not the fundamental risk distributions, and, in fact, the risk tables he might have expected to be available for the various sizes of risk are not in existence. He has found the table of excess ratios well adapted to the calculation of the insurance charge for the insurance granted by the provision of a maximum premium. If he has tried to understand entirely the calculation whereby the credit arising from a minimum premium is derived from this same chart or table of excess ratios, and has succeeded, he has demonstrated some degree of actuarial talent. It should be said at this point that there are very good reasons, which will be investigated later herein, for the use, hitherto, of excess ratios rather than risk distributions as basic data for the insurance charges of retrospective rating plans.

In this paper, only insurance charges arising from maximum premiums and modified by the effect of minimum premiums are considered. There are also to be found in formal plans and individual risk plans charges for limitations directly on losses, and hence on derived premium, eliminating from consideration in the plan losses in excess of certain limits per case or per accident, such charges having been usually handled as increments to the other insurance charge, which is first determined on the basis that individual cases and accidents are not limited in cost beyond the limitations that inhere in the standard coverage of the line in question. In the New York compensation plan, since the adoption of a separate New York excess ratio chart in 1941, the per claim limit has been worked into the chart itself by increments on what would have been the excess ratios without loss limitations so that no further account need be taken of the per claim limit in calculating insurance charges.[2] Numbers of rating plans, encountered or proposed to be encountered in casualty insurance, ostensibly with few or no features in common with the prototypal workmen's compensation retrospective rating plan beyond the fact that the final premium as to the individual assured is determined with reference to the experience actually had with his policy or policies involved in the plan, can be transformed to show insurance charges arising from maximum premiums and perhaps modified by minimum premiums, thus coming within the scope of this paper.[3]

It is planned to show herein that any table or chart of excess pure premium ratios as developed in connection with retrospective rating implies corresponding risk distributions which can be worked out fairly accurately from

[2] See Paul Dorweiler, "On Graduating Excess Pure Premium Ratios," P.C.A.S. XXVIII, pp. 138-142.

[3] See T. O. Carlson, "An Actuarial Analysis of Retrospective Rating," P.C.A.S. XXVIII, pp. 283-284.

the excess ratio table; and there is presented a risk distribution table based on the chart of the Compensation Insurance Rating Board of New York issued in May 1941, and subsequent table of readings therefrom, entitled "New York Workmen's Compensation Excess Pure Premium Ratios," for premium sizes from $5,000 to $500,000. The New York chart, as mentioned in the last paragraph, includes increments for a limitation per claim to $10,000. A modified table without the increment for the claim limit has been prepared and used as the basis of insurance charges in a proposed workmen's compensation rating program submitted by the National Bureau of Casualty and Surety Underwriters to various compensation rate-making bodies in June 1942. The table of excess pure premium ratios submitted by the Bureau is the one from which the risk distributions here presented have been worked out and is shown as Exhibit I, herewith.

Generally accepted excess pure premium ratio charts or tables are found only in the compensation line. Tentative charts on other lines can probably be found in some offices. It might be said in passing that a chart for one line may be used in another if the dispersion of losses is not considered too widely different. An adjustment for the difference in permissible loss ratio is accomplished by the device given in Mr. Carlson's previously cited paper, P.C.A.S. XXVIII, on page 319. When both maximum and minimum premiums are involved, even the difference due to dispersion is overcome considerably. Suppose the non-charted line is considered to have its loss ratios dispersed more, that is, not grouped so closely about the permissible (average) loss ratio as the charted line. In that case, the charge for losses over the maximum loss ratio as figured from the chart with the adjustment mentioned might be considered too low for the non-charted line. The credit for the minimum, however, will also be too low and hence the difference or net charge may well be more nearly correct than the two items composing it.

Excess pure premium ratio charts as developed for retrospective rating (they can be used for any kind of aggregate stop loss insurance) show, for various sizes of risk as measured by standard premium and for all loss ratios in an interval of practical usefulness, the expected or average ratio to total losses of losses in excess of any selected loss ratio. Up to this point the reference sources of excess ratios have been called charts and tables somewhat indiscriminately. By "chart" we have meant the original graphical presentation of the ratios. The most frequently used portion of the National chart for compensation presently effective in states other than New York, is reproduced in P.C.A.S. XXIV, page 353, in Mr. Pinney's paper, "The Retrospective Rating Plan for Workmen's Compensation Risks." The newer chart of the Compensation Insurance Rating Board of New York, adjusted to be on a comparable basis by omitting the additional New York retrospective limitation of $10,000 per claim, is shown in P.C.A.S. XXVIII, page 152, in Mr.

Dorweiler's paper, "On Graduating Excess Pure Premium Ratios." In using the term "table" we have had in mind compilations of readings from the charts such as the one presented as Exhibit I, herewith. The entries of Exhibit I may be compared with the chart mentioned above, found in P.C.A.S. XXVIII, page 152.

Corresponding to any selected maximum or minimum premiums there are calculated "selected" loss ratios which indicate the limits of the assured's participation in losses. Thus, under the National Council Plan, a risk of $25,000 standard premium has a minimum premium of 60%, a maximum of 140%, and a basic charge of 30%. Say also that the risk is in a state with a loss conversion factor of 1.12. When the loss ratio is approximately 26.8%, the minimum premium is earned.

$$26.8\% \times 1.12 + 30\% = 60\%$$

When the actual loss ratio falls below 26.8% the assured, by paying the minimum premium, may be said to be paying for a 26.8% loss ratio, despite his actual experience being better. On the other hand, when the loss ratio is 98.2%, the maximum premium is earned.

$$98.2\% \times 1.12 + 30\% = 140\%$$

In case the actual loss ratio exceeds this figure the assured does not pay retrospectively for more than the selected loss ratio of 98.2%, no matter how much higher his loss ratio may be. Mr. Pinney's above cited paper, P.C.A.S. XXIV, gives in full in the appendix, middle of page 341 et seq., the process of calculation of the charge and credit for the $25,000 risk size from the data of the chart, which process we have here followed only to the point of determining the selected loss ratios for entering the chart. The selected loss ratios by which the present charts or tables are entered would also be the reference points for calculations based on risk distributions. In the example above we should get from the corresponding risk table the relative number of risks having loss ratios over 98.2% and their average loss ratio and the relative number of those under 26.8% and their average loss ratio.

It is not claimed here that risk distribution tables will be better than excess ratio tables for purposes of retrospective plans. It is our thought that fairly well defined risk distributions are implicitly involved in the accepted excess ratio tables and that it is of some value and interest to work them out and examine them. We have found them useful in connection with various questions arising on insurance charges and credits, particularly credits, and in the analysis of unusual retrospective rating propositions, such as one that involved two formulas, one applying when the risk's actual loss ratio is below the permissible, the other when it is above. It is our thought

also that the risk distributions implied should be compared with *a priori* notions thereof. Among other things, should they really be unimodal or are several modes in each size of risk appropriate? They also may be compared with standards of risk credibility in experience rating.

Deduction of Risk Distributions from Excess Ratios

It happens that there is a relatively simple relation between the distributions and the excess ratios. When the excess ratios are the given quantities, as in our investigation, we find the second differences of these at the successive loss ratios and multiply them by the permissible loss ratio to arrive at the items of the distribution. Conversely, when we have the observed distribution given, two summing operations will enable us to pass to the excess ratios. An example of the derivation of the implied risk distribution from the excess ratios for the $25,000 premium size follows. We start with excess ratios which we have expanded to five decimal places from the original three place ones (the expansion and the need for it will be discussed later) and work out the few first entries in a frequency distribution by performing the operations indicated in the column headings. The permissible loss ratio is 59.8%.

(1) Selected Loss Ratio x	(2) Graduated Excess Ratio u_x	(3) $-\Delta u_{x-1}$	(4) Ogive $-59{,}800\,\Delta u_{x-1}$	(5) Number of Risks Δ Col. (4)	(6) Final Selected Number	(7) Excess Ratio Based on (6)
0	1.00000	—	(1 000)	—	—	1.000
.01	.98327	.01673	1 000	—	—	.983
.02	.96654	.01673	1 000	—	—	.967
.03	.94981	.01673	1 000	1	—	.950
.04	.93310	.01671	999	—	—	.933
.05	.91640	.01670	999	2	1	.916
.06	.89972	.01668	997	1	2	.900
.07	.88307	.01665	996	3	2	.883
.08	.86646	.01661	993	2	3	.866
.09	.84989	.01657	991	4	4	.850
.10	.83339	.01650	987	..	..	...
..	...	...	...	..	..	...
..	...	...	...	..	..	...
..	...	...	...	..	..	...

In the calculation, it is convenient, before passing to the second difference or frequency distribution, to multiply by the permissible loss ratio times some power of 10 to give an ogive or cumulation of the risks by number mounting up to some convenient power of 10: we chose 1,000 risks. The derivation of the method in the above finite difference form is found in Appendix A. Mr. Stefan Peters has indicated it in terms of infinitesimal calculus in his discus-

sion, P.C.A.S. XXVIII, pp. 588-590, of Mr. Dorweiler's previously cited paper.

The deduction of the frequency distributions from the excess ratios for successive loss ratios depends on knowledge of two statistics, the excess ratios themselves, which are given, and the average loss ratio. In the use of an excess ratio chart or table in retrospective rating, it is assumed that the average loss ratio for any size of risk in the table is equal to the permissible loss ratio. The excess ratios in Exhibit I are assumed to be keyed so that every risk size has an average loss ratio of 59.8%, which was the basic permissible loss ratio for New York at the time of constructing them. While considerable effort was made to have the underlying data correctly adjusted to 59.8%, it is unlikely that they were always within the usual error limits of a three-place figure.[4] Also the underlying ungraduated excess ratios were necessarily worked out for broad groupings of risks by size and only at intervals of loss ratio, to get stability of data and keep the work within bounds, and then extended over the whole range by various processes, including mathematical and a final graphical graduation. It would not be surprising, therefore, if the keying to 59.8% is in reality only approximate in the final result. It would still be correct to use the permissible loss ratio in deriving the implied frequency distributions instead of the unknown approximations, because this accords with the application of the excess ratio table.

Having established that the two necessary statistics are at hand, the average loss ratio and the excess ratios per loss ratio at successive loss ratios, the method still works only in theory because of the shape in which the excess ratios are available. In taking out second differences, the error due to confining the excess ratios to three places of decimals (and the charts do not justify an attempt to read off the values to any more places) is enormously magnified. In fact, when the attempt is made to establish the frequency distributions in this way, taking out second differences and multiplying them by 59.8% and an appropriate power of 10, and the results are graphed, they are unrecognizable as frequency distributions, excepting possibly graphs for the largest sizes of risk. The frequent negative entries suggest some oscillating data are being recorded rather than frequency distributions.

The first differences when plotted, however, constitute ogives, or cumulative frequency distributions of the risks having loss ratios equal to and over the various loss ratios, which yield a good deal of significance. Smoothing or graduation is obviously required, but is also obviously quite feasible, which latter seems hardly the case for the second difference distribution.

Our first approach to the problem, suggested by such graphs, was to attempt graduation of the first differences or ogives by means of the Whittaker-Henderson Graduation Formula A. The results were not unsatis-

[4] See Mr. Peters' cited discussion, P.C.A.S. XXVIII, at top of page 589, or Compensation Insurance Rating Board staff memorandum May 20, 1941.

factory and were retained for some sizes of risk, but the final method adopted was to graduate the excess ratios directly by the same formula, which we have referred to earlier as a process of expanding the ratios from three decimal places to five. In this procedure there appeared to be several advantages. The end conditions gave less trouble, and the relative goodness of fit was under observation throughout the process of graduation, the test of goodness of fit in this case being not how the final distribution measures up to the observed one, because, as we have seen, the graduated one would be expected to be unrecognizably different, but how the excess ratios based upon the final distribution compare with the original. To check the assumption that the distributions produced were approximately those implied by the excess ratios and did not just happen to work more or less fortuitously, graphic comparisons of the distributions, graduated and ungraduated, in ogive form, were made. The graduation process is described in more detail in Appendix B.

* * * * *

In Exhibit II are shown the resulting distributions for all sizes of risks on the bases of 1,000 risks in each size. The general shape of each of these distributions when plotted on graph paper seems not to do violence to our preconception of what they should be. The relationship between the distributions for different sizes is rather satisfactory. The median and the primary mode properly move downward from about permissible loss ratio in the $500,000 premium size toward the zero side as one passes across the sizes downward to the $5,000 size. On the whole, the excess ratios of Exhibit I stand scrutiny from the angle of their implied loss ratio distributions reasonably well. Graphs of the distributions for representative sizes of risk are shown in Exhibit III. (Page 94.)

There is a tendency in all sizes for subsidiary modes or near-modes to appear. Our *a priori* opinion on this point was that the distributions should be unimodal but there are arguments for multiple modes. Perhaps there should be a mode for the risks with normal losses only and others for those with excess losses. For instance, there might be a slight mode for the occasional death case in the $5,000 size somewhere above 150% loss ratio. This is speculation in an area not very much explored at the present time.

As an example of the possible practical use of the results we give in Exhibit II, and also of its approximate equivalence to Exhibit I, we return to the example of the $25,000 risk in Connecticut under the National Council Compensation Retrospective Rating Plan and work it through first as in Mr. Pinney's cited paper, P.C.A.S. XXIV, middle of page 341, et seq., substituting the New York chart by means of our table of readings in Exhibit I for the National chart. We first adjust our selected loss ratios of 26.8% and 98.2% for entry to the table:

$$26.8\% \times \frac{59.8}{62.5} = 25.6\% \qquad 98.2\% \times \frac{59.8}{62.5} = 94.0\%$$

It should perhaps be recalled that the present accepted method of adjusting from the state permissible loss ratio (Connecticut 62.5%) to the chart underlying permissible ratio in referring to the chart or table was not in use at the time of Mr. Pinney's paper and so a corresponding adjustment to the above is not made there. In the former method, however, the chart permissible ratio was used as the factor to convert the excess ratio to an insurance charge in terms of premium, where, in the present method, the state permissible is used. The former calculation may be considered as approximately on the chart level; the present is more exactly on the particular state level throughout.

Reference to Exhibit I gives the excess pure premium ratio for selected loss ratio 94.0% as .079. Expressed in terms of the risk premium this becomes $.079 \times .625 = .049$. Similarly for a 25.6% loss ratio limitation, the excess ratio is .593. Therefore, the ratio to total losses of losses falling below the 25.6% limitation equals $1.000 - .593 = .407$. Related to premium this becomes $.407 \times .625 = .254$. The indicated credit is then $.268 - .254$ or .014 of the risk premium. The net insurance charge becomes $.049 - .014$ or .035 of the risk premium compared to .044 under the former chart and method of calculation.

Next we work out the net insurance charge from Exhibit II, the risk table, and we observe that 149 risks of a thousand will have loss ratios of 95% and above, averaging 125.7%. Also we find that there are 147 risks of 25% loss ratio and below. Their average loss ratio is 17.1%. Keying both these figures back to the 62.5% level, they become

$$125.7\% \times \frac{625}{598} = 131.4\%; \qquad 17.1\% \times \frac{625}{598} = 17.9\%$$

The average loss of $131.4\% - 98.2\%$ on 149 of a thousand cases necessitates an insurance charge of $33.2\% \times .149$ or .049. The offsetting credit to this is $26.8\% - 17.9\%$ or 8.9% realized in 147 cases of a thousand, amounting to $8.9\% \times .147$ or .013. The net insurance charge turns out to be $.049 - .013$ or .036, compared to .035, computed from Exhibit I.

Note on Producing Graduated Excess Ratios

Basically, the reverse of the process outlined above for passing from excess ratios to risk distributions by loss ratio underlies the working out of the excess ratios from risk experiences. The work has usually been done from an accounting point of view with excess loss calculations rather than from a statistical one, and the ease with which loss ratio distributions may be used

with a double summation has seemingly been overlooked. It should be mentioned, however, that in the earliest discussion of excess ratios per loss ratio appearing in the *Proceedings* of the Society, in Mr. Dorweiler's paper "Observations on Making Rules for Excess Compensation Insurance," the work proceeded directly from loss ratio groupings, see P.C.A.S. XIII, page 174-175.

Through the courtesy of the Compensation Insurance Rating Board we were furnished actual risk distributions by loss ratio in certain size of risk groups, for one policy year, 1939, first reports, available in connection with the Board's present study of variations by industry. It was remarkably easy to sum these up twice and produce the excess ratio chart shown as Exhibit IV. The adjustment or keying to permissible loss ratio in each premium size group was done last while graphing the excess ratios produced. Instead of graphing each one at the loss ratio associated with it, it was graphed at the point to which that loss ratio was adjusted by a flat factor: 59.8%/(premium size group loss ratio).

Exhibit IV (Page 95) is introduced in this paper because of one or two interesting features. In the first place, the results are untenable when we compare the different sizes of risk, which was to be expected because of paucity of data (one year). On the other hand, each curve is in itself satisfactorily smooth without graduation of the underlying data. This suggests the point that the process of taking out the excess ratios is a graduating process, in fact, a double unsymmetrical summation graduation of the underlying distributions, and will give smooth results on rather limited data. This is equally true for any legitimate method of deriving the excess ratios as, of course, all methods should give at least approximately the same results.

It becomes quite clear that while the idea of producing finished graduated excess ratios by means of graduated basic frequency distributions in each size or size group of risks makes a very strong appeal because of its theoretically satisfying and attractive quality, yet, in a practical way, the greatest need for graduation methods is across the sizes to line them up consistently with each other, bringing in the weight of the whole experience of all sizes. In graduating across the sizes, the indicated excess ratios appear to be the most convenient basic data. Thus, it would seem Mr. Dorweiler's graduation method[5] made the proper frontal attack. As Mr. Peters says in the discussion "An ideal graduation method would reflect both kinds of relationship" and it is to be hoped the method he outlines will receive further attention. He also says, "The excess pure premium ratios are so closely linked with the distributions of risks of a given premium size by size of loss ratio and, ultimately, with the basic concepts of accident frequency and severity that it is desirable that these relationships be reflected in the graduation method or

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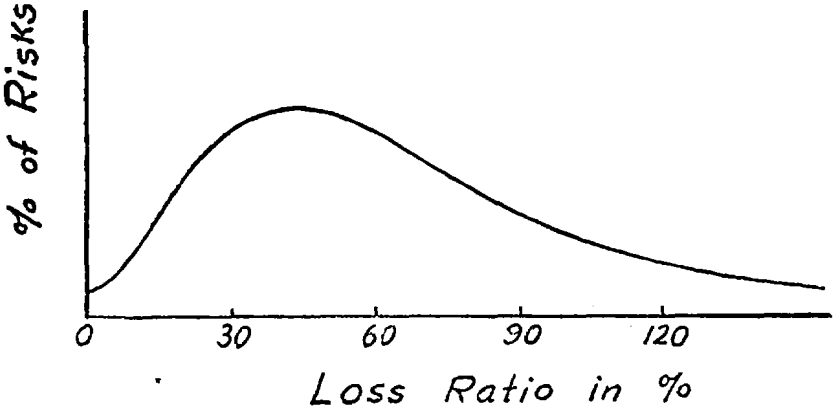
[5] P.C.A.S. XXVIII, pp. 132-157, "On Graduating Excess Pure Premium Ratios," Discussion pp. 586-590.

be used to test its accuracy." One of the objects of this paper is partially to enable the test to be made at least against *a priori* notions of the risk distributions.

Appendix A

Relation of Excess Ratios and Risk Distributions

Assume the risks of one premium size to be arranged in a frequency distribution by size of loss ratio and that the average loss ratio is .60. Thus,



Let x = loss ratio, figured to nearest whole per cent, and regarded as a whole number.

Let y = the % of risks by number at each loss ratio.

Let s = a certain "selected" loss ratio.

Let ρ_s = the losses in excess of the selected loss ratio s, compared to all losses, or excess pure premium ratio per loss ratio s.

Then total losses of all risks = $100\% \times 60 = 6000\%$

$$\text{or} \quad \sum_{0}^{x=\infty} x\,y = 6000\%$$

The losses within the certain selected loss ratio s, compared to all losses, or

$$1-\rho_s = \frac{\sum_{0}^{x=s-1} x\,y + s\sum_{s}^{x=\infty} y}{\sum_{0}^{x=\infty} x\,y}$$

Then

$$\rho_s = \frac{\sum_{0}^{x=\infty} x\,y - \sum_{0}^{x=s-1} x\,y - s\sum_{s}^{x=\infty} y}{6000}$$

$$= \frac{\sum_{s}^{x=\infty} x\,y - s\sum_{s}^{x=\infty} y}{6000} \qquad (1a)$$

When $s+1$ is the selected loss ratio, we have excess ratio per loss ratio $s+1$,

$$\rho_{s+1} = \frac{\sum_{s+1}^{x=\infty} x\ y - (s+1)\sum_{s+1}^{x=\infty} y}{6000}$$

$$= \frac{\sum_{s}^{x=\infty} x\ y - s\ y_s = (s+1)\left[\sum_{s}^{x=\infty} y - y_s\right]}{6000}$$

$$= \rho_s + \frac{-s\ y_s - \sum_{s}^{x=\infty} y + (s+1)\ y_s}{6000}$$

$$= \rho_s + \frac{y_s - \sum_{s}^{x=\infty} y}{6000}$$

$$= \rho_s - \frac{\sum_{s+1}^{x=\infty} y}{6000}$$

so that

$$6000\ [\rho_s - \rho_{s+1}] = \sum_{s+1}^{x=\infty} y$$

or

$$-6000\ \Delta\ \rho_s = \sum_{s+1}^{x=\infty} y \qquad (2a)$$

From the last expression, it is evident that an ogive of the frequency distribution can be formed by differencing of successive excess ratios per loss ratio. If we difference the successive excess ratios twice we should derive the number of risks at each loss ratio, thus

$$6000\ \Delta^2 \rho_s = \sum_{s+1}^{x=\infty} y - \sum_{s+2}^{x=\infty} y = y_{s+1} \qquad (3a)$$

where y_{s+1} = the per cent number of risks at loss ratio $s+1$.

It will be noted that the number of risks at loss ratio zero cannot be established in this way, which might be expected, since the risks without losses are not involved in ratios of excess losses to total losses. The per cent at zero will be 100% minus the sum per cent of all other risks.

We write the expressions (1a), (2a) and (3a), above, in general form by substituting $\bar{x}$, average loss ratio, for the specific 60% above, and we have

$$100\,\bar{x}\,\rho_s = \sum_{s}^{x=\infty} x\,y - s\sum_{s}^{x=\infty} y \tag{1}$$

$$-100\,\bar{x}\,\Delta\,\rho_s = \sum_{s+1}^{x=\infty} y \tag{2}$$

$$100\,\bar{x}\,\Delta^2\rho_s = \sum_{s+1}^{x=\infty} y - \sum_{s+2}^{x=\infty} y = y_{s+1} \tag{3}$$

As the ρ's are always given in decimal form and the x's and s are taken as whole numbers, the y's or number of risks in the right hand sides above will be in terms of per cent of all risks or decimals dependent on whether $\bar{x}$ is taken in per cent or ratio form.

We have treated the loss ratios as though they were always exactly expressible in units, $0, \cdots s-1, s, s+1$, etc. Since they are not actually a discrete series, we take them to the nearest unit, so that if we are using .01 as the unit, as we do in working from the excess ratio tables, we have the number of risks at loss ratio $s = 43$ say are all those with loss ratios .425 to .4349.

Appendix B

Notes on the Graduation

There appears not to have been an occasion for the Whittaker-Henderson Graduation Formula A to be mentioned in the *Proceedings*, so that perhaps a short introductory statement about it should be made before describing the specific application. This formula is more than a formula as the term is usually understood, it is a whole system of graduation, and one would expect it should be fully as useful in casualty insurance as it has been in life insurance, where there is a considerable literature on it.[1] If we let u''_x be the general term of the ungraduated series and u_x the corresponding term of the graduated series, the system proceeds on the basis that

$$\Sigma\,(\Delta^z u_x)^2 + k\,\Sigma\,(u_x - u''_x)^2$$

shall be made a minimum,[2] where z and k are constants to be selected. The first term $\Sigma\,(\Delta^z u_x)^2$ is the measure of smoothness and there is freedom in choosing z, the order of differences to be minimized. The other term is the

[1] The method is described and the literature outlined in Hugh H. Wolfenden, "The Fundamental Principles of Mathematical Statistics," under the heading, "Graduation by the Difference — Equation Method." For a working reference, the paper by Charles A. Spoerl, "Whittaker-Henderson Graduation Formula A," in *Transactions* of the Actuarial Society of America, Vol. XXXVIII, pp. 403-462, should be consulted.

[2] There is also an elaboration with more terms, the "mixed difference" case.

measure of closeness of fit, and a choice between relative smoothness and relative closeness of fit is made through the constant k, as the larger the k, the more the emphasis thrown on the "fit" term. As k approaches zero, a least-squares fitting of a polynomial of degree $z-1$ is approached and as k gets very large, a state of no graduation at all. In between there tends to be for each selection of k a series of successive polynomials of order $z-1$ fitted to the data. As a practical matter z is 2 or 3 and k is a number less than one. Generally speaking the smaller z is and the larger k is, the easier the graduation will be, and the easiest graduation that will reasonably fulfill the requirements of the specific job in hand has considerable claim to be called the best.

In the working out of the series $\{u_x\}$ from the ungraduated series $\{u''_x\}$, an intermediate series is first constructed, $\{u'_x\}$, from $\{u''_x\}$ and then $\{u_x\}$ from $\{u'_x\}$ by means of a formula whose form depends on z and whose coefficients numerically depend on k. The graduation can also be done directly in a linear compound form. The first method seemed the more feasible in this case because of the many points to be graduated. By trial, $z=2$ appeared to give satisfactory values in general. This gives the graduation formulae:

$$u'_x = \frac{2n\,(n+2)}{(n+1)(n+2)}\,u'_{x-1} - \frac{n\,(n+1)}{(n+1)(n+2)}\,u'_{x-2} + \frac{2}{(n+1)(n+2)}\,u''_{x+n}$$

$$u_x = \frac{2n\,(n+2)}{(n+1)(n+2)}\,u_{x+1} - \frac{n\,(n+1)}{(n+1)(n+2)}\,u_{x+2} + \frac{2}{(n+1)(n+2)}\,u'_{x-n}$$

where n is a constant fulfilling the condition $k = \dfrac{4}{n\,(n+1)^2(n+2)}$.

The biggest difficulty in a Whittaker-Henderson Formula A graduation is to get the right start. In this case, referring to the formula, it is seen that the calculation of each term in the intermediate series $\{u'_x\}$ depends on knowing two before it, so there must be two to start with. A seemingly satisfactory device was hit upon in meeting a special condition of this graduation, which is discussed in the next paragraph. The same situation occurs when "turning the corner" and the $\{u_x\}$ series is to be developed from the $\{u'_x\}$ one in reverse order, but the rules of the method provide for this situation.

The series of excess ratios was regarded as a series of observations which missed the true values as any other observations would, although they are results of a graphic graduation, with one exception, the excess ratio at selected loss ratio zero is definitely unity. To insure the final value being unity, the series was extended to negative loss ratios, making it symmetrical about the point: excess ratio $= 1$ and selected loss ratio $= 0$. The graduation method automatically turns up the right answer for this point under these

conditions as the linear compound alternative form of it suggests. By starting out rather far back on the appendage to the series, an error in the start gets worn off before the part that matters is reached. Two successive terms of the u'_x column were assumed to be equal to the corresponding terms in the u''_x column and set down and the start made. For the larger sizes of risk it was not necessary to use this device as the curves became practically identical with the line: selected loss ratio = .598 (1 — excess ratio), considerably to the right of the point, and $\{u''_x\}$, $\{u'_x\}$, and $\{u_x\}$ all merge into it.

Various values of n were used. This constant varying inversely to k, a large value of n goes with relative smoothness and a small one with relative closeness of fit. This shows up in the last term of the working formulae which reach out farther ahead along the series to be smoothed when n is large than when it is taken small. It was decided that on the one hand n should be at least large enough to graduate out all negative risks or actual dips in the observed ogive, that is the first differences must be a descending series. On the other hand, n should not be so large as to iron out what appear to be characteristic modulations in the ogive or to produce a series departing more than an occasional two units in the third place from the given excess ratios, which might be inaccurate to that extent, due to faulty drafting of, or reading off from, the excess ratio chart. It was thought it was in order to attempt to have the greatest smoothness consistent with these rules.

In general an integral value of n was used in calculating the distributions given and varied between 3 and 8. A very convenient value to work with was $n = 3$, for the coefficients become 1.5, .6, and .1. Choice of other n's involves recurring decimal coefficients or divisions by the common denominator $(n + 1)(n + 2)$. With $n = 3$, it was unnecessary to move the carriage of the calculating machine as at the conclusion of the preceding operation with the formula

$$u'_x = 1.5\, u'_{x-1} - .6\, u'_{x-2} + .1\, u''_{x+3}$$

$1.0\, u'_{x-1}$ is already in the machine and it is only necessary to add $.5\, u'_{x-1} - .6\, u'_{x-2} + .1\, u''_{x+3}$. There was, therefore, an inclination to run through with $n = 3$ and then, if sufficient smoothness was not attained, to use the results as observed values and run over again with $n = 3$, which results in a fractional n, about 4.3 in value. One more repetition makes n about 5.3.

While the above method of graduating was generally used, namely applying the second difference formula to the excess ratios, some of the results given in Exhibit II, as was stated in the paper, were derived from our first method of graduating the indicated ogives formed from the first differences of the excess ratios keyed to a thousand-risk total by applying the factor 59,800. Another exception was that for the two largest sizes of risk, where the range in the second differences is greatest, the final selection was based

on a graduation with z taken as 3, that is, a third difference formula graduation.

It has already been said that the graduation of the excess ratios was done in order to expand the ratios from three decimal places to five. The three-place ratios, while they were readings from smoothly turning graduated curves, were treated as though they were rough observations of the true five-place values. It was necessary to have five decimal places in the graduated excess ratios in order to produce second differences with enough significant figures to give distributions of 1000 risks in each size. This is readily seen from the table on Page 100. In the graduations that were done on the ogives, the situation was like the usual graduation problem, as what was required was the production of smooth curves from indications which when plotted presented jagged lines with, however, the inherent trends more or less evident.

EXHIBIT I

NEW YORK WORKMEN'S COMPENSATION—EXCESS PURE PREMIUM RATIOS

Ratio: Losses in Excess of Any Selected Loss Ratio Per Risk
Total Losses
No Limitation on Individual Losses

Loss Ratio	Premium Size						
	$5,000	$7,500	$10,000	$15,000	$20,000	$25,000	$50,000
.01	.983	.983	.983	.983	.983	.983	.983
.02	.967	.967	.967	.967	.967	.967	.967
.03	.950	.950	.950	.950	.950	.950	.950
.04	.934	.933	.933	.933	.933	.933	.933
.05	.918	.917	.917	.917	.917	.917	.916
.06	.903	.901	.901	.901	.901	.901	.900
.07	.888	.886	.885	.885	.884	.884	.883
.08	.872	.870	.868	.868	.867	.867	.866
.09	.857	.854	.852	.851	.850	.850	.849
.10	.843	.838	.836	.835	.834	.834	.833
.11	.829	.823	.820	.818	.817	.817	.816
.12	.815	.808	.805	.802	.801	.800	.799
.13	.801	.793	.789	.786	.785	.784	.783
.14	.788	.779	.774	.770	.768	.767	.766
.15	.775	.765	.759	.754	.752	.751	.749
.16	.762	.751	.745	.739	.736	.735	.732
.17	.748	.737	.730	.724	.721	.720	.716
.18	.736	.724	.716	.710	.707	.705	.700
.19	.724	.711	.703	.696	.693	.690	.684
.20	.712	.698	.689	.682	.679	.676	.669
.21	.700	.686	.676	.668	.665	.661	.653
.22	.688	.673	.663	.654	.651	.646	.637
.23	.677	.662	.650	.640	.637	.631	.621
.24	.666	.650	.638	.627	.623	.616	.605
.25	.654	.638	.625	.613	.609	.601	.589
.26	.643	.626	.613	.600	.595	.587	.574
.27	.632	.614	.601	.587	.581	.573	.558
.28	.621	.603	.590	.575	.568	.560	.543
.29	.611	.593	.579	.562	.554	.546	.528
.30	.602	.583	.569	.550	.541	.532	.513
.31	.592	.572	.558	.538	.528	.519	.498
.32	.583	.562	.547	.526	.515	.505	.483
.33	.574	.552	.536	.514	.502	.492	.468
.34	.565	.542	.526	.503	.489	.479	.454
.35	.557	.533	.516	.492	.477	.467	.440
.36	.548	.523	.506	.481	.465	.454	.426
.37	.540	.514	.496	.470	.453	.441	.413
.38	.531	.504	.486	.459	.441	.429	.400
.39	.522	.494	.476	.448	.429	.417	.387
.40	.514	.485	.467	.438	.418	.405	.374
.41	.506	.477	.458	.428	.408	.394	.362
.42	.498	.468	.448	.418	.398	.383	.350
.43	.490	.460	.439	.408	.387	.372	.339
.44	.482	.451	.430	.399	.377	.361	.326
.45	.475	.443	.421	.389	.367	.351	.316

EXHIBIT I (Cont'd)

NEW YORK WORKMEN'S COMPENSATION—EXCESS PURE PREMIUM RATIOS

Ratio: Losses in Excess of Any Selected Loss Ratio Per Risk
Total Losses
No Limitation on Individual Losses

Loss Ratio	Premium Size						
	$5,000	$7,500	$10,000	$15,000	$20,000	$25,000	$50,000
.46	.468	.436	.413	.380	.358	.341	.305
.47	.460	.428	.404	.371	.349	.332	.294
.48	.452	.420	.396	.362	.340	.323	.284
.49	.445	.412	.388	.354	.332	.314	.274
.50	.438	.404	.380	.346	.324	.306	.264
.51	.431	.397	.373	.338	.316	.298	.255
.52	.424	.390	.366	.330	.308	.290	.245
.53	.418	.383	.359	.323	.301	.282	.236
.54	.411	.376	.352	.316	.294	.275	.227
.55	.405	.369	.345	.309	.287	.268	.218
.56	.399	.363	.338	.302	.280	.261	.210
.57	.394	.357	.332	.295	.273	.254	.202
.58	.388	.350	.325	.288	.266	.247	.194
.59	.383	.344	.319	.282	.260	.240	.186
.60	.378	.338	.313	.276	.253	.233	.179
.61	.373	.332	.307	.270	.247	.227	.172
.62	.368	.327	.301	.264	.241	.220	.164
.63	.363	.321	.295	.258	.235	.213	.158
.64	.359	.316	.289	.252	.229	.207	.151
.65	.354	.311	.283	.246	.223	.201	.145
.66	.349	.306	.278	.240	.217	.195	.138
.67	.344	.301	.272	.234	.211	.189	.133
.68	.340	.296	.267	.229	.205	.184	.127
.69	.336	.291	.262	.224	.200	.178	.122
.70	.332	.287	.257	.219	.194	.173	.116
.71	.328	.283	.252	.214	.189	.168	.112
.72	.324	.278	.247	.209	.184	.162	.107
.73	.320	.273	.242	.204	.178	.157	.102
.74	.316	.269	.238	.200	.173	.152	.097
.75	.312	.265	.233	.195	.168	.148	.093
.76	.308	.261	.229	.190	.163	.143	.088
.77	.304	.257	.225	.186	.159	.138	.084
.78	.300	.253	.221	.181	.154	.133	.081
.79	.297	.249	.217	.177	.150	.129	.077
.80	.293	.245	.213	.173	.146	.125	.074
.81	.290	.241	.209	.168	.141	.121	.070
.82	.287	.238	.206	.164	.137	.117	.067
.83	.284	.234	.202	.160	.133	.113	.064
.84	.280	.230	.198	.156	.130	.110	.061
.85	.277	.227	.195	.152	.126	.106	.058
.86	.274	.224	.192	.149	.123	.102	.055
.87	.271	.221	.189	.145	.119	.099	.052
.88	.268	.218	.186	.142	.116	.096	.050
.89	.265	.214	.182	.138	.113	.093	.047
.90	.262	.211	.179	.135	.110	.090	.045

EXHIBIT I (Cont'd)

NEW YORK WORKMEN'S COMPENSATION—EXCESS PURE PREMIUM RATIOS

Ratio: Losses in Excess of Any Selected Loss Ratio Per Risk
Total Losses
No Limitation on Individual Losses

Loss Ratio	Premium Size						
	$5,000	$7,500	$10,000	$15,000	$20,000	$25,000	$50,000
.91	.259	.208	.176	.132	.107	.087	.043
.92	.257	.206	.174	.130	.105	.085	.041
.93	.254	.203	.171	.127	.102	.082	.039
.94	.251	.200	.168	.124	.099	.079	.037
.95	.249	.198	.165	.121	.096	.077	.036
.96	.247	.195	.162	.118	.094	.074	.034
.97	.244	.192	.159	.116	.092	.072	.032
.98	.242	.190	.157	.114	.089	.069	.030
.99	.239	.187	.155	.112	.087	.067	.029
1.00	.237	.185	.153	.109	.085	.065	.027
1.01	.234	.182	.150	.107	.083	.064	.026
1.02	.232	.180	.148	.104	.081	.062	.025
1.03	.230	.178	.146	.102	.078	.060	.024
1.04	.228	.176	.144	.100	.077	.058	.023
1.05	.225	.173	.141	.098	.075	.056	.022
1.06	.223	.171	.139	.096	.073	.054	.021
1.07	.221	.169	.137	.094	.071	.053	.020
1.08	.219	.167	.135	.092	.069	.051	.019
1.09	.217	.165	.133	.090	.067	.050	.019
1.10	.215	.163	.131	.088	.066	.048	.018
1.11	.213	.161	.129	.086	.064	.047	.017
1.12	.211	.159	.128	.084	.063	.046	.016
1.13	.209	.157	.125	.082	.060	.044	.015
1.14	.206	.154	.123	.081	.059	.043	.015
1.15	.204	.152	.121	.079	.057	.041	.014
1.16	.202	.150	.119	.077	.056	.040	.014
1.17	.201	.149	.118	.075	.054	.038	.013
1.18	.199	.147	.116	.074	.052	.037	.013
1.19	.197	.145	.114	.072	.051	.036	.012
1.20	.195	.143	.112	.071	.049	.035	.011
1.21	.193	.141	.110	.069	.048	.034	.011
1.22	.191	.139	.108	.068	.047	.033	.010
1.23	.189	.137	.107	.066	.045	.031	.010
1.24	.187	.135	.106	.065	.044	.030	.009
1.25	.185	.133	.103	.063	.042	.029	.009
1.26	.183	.132	.102	.062	.041	.028	.008
1.27	.181	.130	.099	.061	.040	.027	.008
1.28	.179	.128	.097	.059	.039	.027	.007
1.29	.177	.126	.096	.058	.038	.026	.007
1.30	.175	.124	.094	.057	.037	.025	.007
1.31	.173	.122	.093	.055	.036	.024	.006
1.32	.171	.120	.092	.054	.035	.023	.006
1.33	.170	.119	.090	.053	.034	.023	.006
1.34	.168	.117	.088	.051	.033	.022	.005
1.35	.166	.115	.087	.050	.032	.021	.005

EXHIBIT I (Cont'd)

NEW YORK WORKMEN'S COMPENSATION—EXCESS PURE PREMIUM RATIOS

Ratio: Losses in Excess of Any Selected Loss Ratio Per Risk
Total Losses
No Limitation on Individual Losses

Loss Ratio	Premium Size						
	$5,000	$7,500	$10,000	$15,000	$20,000	$25,000	$50,000
1.36	.164	.113	.086	.049	.031	.021	.005
1.37	.162	.111	.084	.048	.030	.020	.005
1.38	.161	.110	.083	.046	.029	.020	.004
1.39	.160	.109	.081	.045	.028	.019	.004
1.40	.158	.108	.080	.044	.028	.019	.004
1.41	.156	.106	.079	.043	.027	.018	.003
1.42	.154	.104	.078	.042	.026	.018	.003
1.43	.152	.102	.077	.041	.026	.017	.003
1.44	.150	.101	.075	.040	.025	.017	.003
1.45	.149	.100	.074	.039	.024	.016	.003
1.46	.148	.099	.073	.038	.024	.015	.003
1.47	.146	.098	.072	.037	.023	.015	.003
1.48	.145	.097	.070	.037	.023	.015	.002
1.49	.143	.095	.069	.036	.022	.014	.002
1.50	.141	.094	.068	.035	.022	.014	.002
1.51	.140	.093	.067	.034	.021	.013	.002
1.52	.138	.091	.066	.033	.020	.013	.002
1.53	.137	.090	.065	.033	.019	.012	.002
1.54	.136	.088	.064	.032	.019	.012	.002
1.55	.134	.087	.063	.031	.018	.012	.002
1.56	.133	.086	.062	.031	.018	.012	.001
1.57	.131	.084	.061	.030	.017	.011	.001
1.58	.129	.083	.060	.030	.017	.011	.001
1.59	.128	.082	.059	.029	.017	.010	.001
1.60	.127	.081	.058	.029	.017	.010	.001
1.61	.126	.080	.057	.028	.016	.010	.001
1.62	.124	.078	.056	.028	.016	.010	0
1.63	.123	.077	.056	.027	.016	.009	0
1.64	.121	.076	.055	.027	.016	.009	0
1.65	.120	.074	.054	.026	.015	.008	0
1.66	.119	.073	.054	.026	.015	.008	0
1.67	.118	.072	.053	.025	.014	.008	0
1.68	.117	.071	.052	.025	.014	.008	0
1.69	.116	.070	.051	.024	.014	.008	0
1.70	.115	.069	.051	.024	.013	.007	0
1.71	.113	.068	.050	.023	.013	.007	0
1.72	.112	.067	.049	.023	.013	.007	0
1.73	.111	.066	.048	.022	.012	.006	0
1.74	.110	.065	.047	.022	.012	.006	0
1.75	.109	.064	.047	.022	.012	.006	0
1.76	.108	.063	.047	.022	.012	.006	0
1.77	.106	.062	.046	.022	.012	.006	0
1.78	.105	.061	.045	.021	.011	.006	0
1.79	.104	.061	.045	.021	.011	.006	0
1.80	.103	.060	.044	.021	.011	.006	0

EXHIBIT I (Cont'd)

NEW YORK WORKMEN'S COMPENSATION—EXCESS PURE PREMIUM RATIOS

Ratio: Losses in Excess of Any Selected Loss Ratio Per Risk
Total Losses
No Limitation on Individual Losses

Loss Ratio	Premium Size						
	$75,000	$100,000	$150,000	$200,000	$300,000	$400,000	$500,000
.01	.983	.983	.983	.983	.983	.983	.983
.02	.967	.967	.967	.967	.967	.967	.967
.03	.950	.950	.950	.950	.950	.950	.950
.04	.933	.933	.933	.933	.933	.933	.933
.05	.916	.916	.916	.916	.916	.916	.916
.06	.900	.900	.900	.900	.900	.900	.900
.07	.883	.883	.883	.883	.883	.883	.883
.08	.866	.866	.866	.866	.866	.866	.866
.09	.849	.849	.849	.849	.849	.849	.849
.10	.833	.833	.833	.833	.833	.833	.833
.11	.816	.816	.816	.816	.816	.816	.816
.12	.799	.799	.799	.799	.799	.799	.799
.13	.783	.783	.783	.783	.783	.783	.783
.14	.766	.766	.766	.766	.766	.766	.766
.15	.749	.749	.749	.749	.749	.749	.749
.16	.732	.732	.732	.732	.732	.732	.732
.17	.716	.716	.716	.716	.716	.716	.716
.18	.699	.699	.699	.699	.699	.699	.699
.19	.683	.683	.683	.682	.682	.682	.682
.20	.668	.668	.667	.666	.666	.666	.666
.21	.652	.652	.650	.649	.649	.649	.649
.22	.636	.635	.633	.632	.632	.632	.632
.23	.619	.618	.616	.615	.615	.615	.615
.24	.603	.602	.600	.599	.599	.599	.599
.25	.587	.586	.584	.583	.582	.582	.582
.26	.571	.570	.568	.566	.565	.565	.565
.27	.555	.554	.552	.549	.548	.548	.548
.28	.539	.538	.536	.533	.532	.532	.532
.29	.523	.521	.519	.516	.515	.515	.515
.30	.507	.505	.503	.500	.499	.498	.498
.31	.492	.490	.487	.484	.483	.482	.482
.32	.477	.475	.471	.467	.466	.465	.465
.33	.461	.459	.454	.450	.449	.448	.448
.34	.446	.443	.438	.433	.432	.431	.431
.35	.431	.428	.422	.417	.416	.415	.415
.36	.416	.412	.406	.401	.400	.398	.398
.37	.402	.397	.391	.386	.384	.382	.381
.38	.388	.382	.375	.370	.368	.366	.365
.39	.374	.367	.360	.355	.352	.350	.348
.40	.360	.353	.346	.340	.336	.334	.332
.41	.347	.338	.331	.326	.321	.318	.316
.42	.334	.325	.316	.311	.306	.302	.300
.43	.322	.311	.301	.295	.290	.287	.285
.44	.309	.298	.287	.280	.274	.271	.269
.45	.297	.285	.273	.265	.260	.255	.254

EXHIBIT I (Cont'd)

NEW YORK WORKMEN'S COMPENSATION—EXCESS PURE PREMIUM RATIOS

Ratio: Losses in Excess of Any Selected Loss Ratio Per Risk
Total Losses
No Limitation on Individual Losses

Loss Ratio	Premium Size						
	$75,000	$100,000	$150,000	$200,000	$300,000	$400,000	$500,000
.46	.285	.271	.259	.251	.245	.240	.237
.47	.274	.259	.245	.237	.230	.225	.221
.48	.262	.246	.233	.223	.216	.210	.206
.49	.251	.234	.222	.209	.201	.196	.191
.50	.240	.224	.211	.197	.187	.182	.176
.51	.230	.213	.199	.184	.172	.166	.160
.52	.220	.202	.186	.170	.158	.150	.144
.53	.210	.191	.174	.157	.144	.134	.128
.54	.200	.181	.162	.145	.131	.119	.115
.55	.191	.172	.151	.133	.119	.107	.102
.56	.181	.163	.141	.122	.108	.094	.089
.57	.173	.154	.132	.113	.097	.083	.078
.58	.164	.146	.122	.105	.087	.073	.068
.59	.156	.137	.113	.097	.077	.065	.059
.60	.148	.129	.104	.088	.068	.057	.051
.61	.140	.121	.098	.080	.062	.050	.045
.62	.133	.114	.091	.074	.057	.045	.040
.63	.126	.107	.084	.069	.051	.040	.035
.64	.119	.101	.078	.062	.046	.036	.031
.65	.113	.095	.072	.057	.042	.032	.027
.66	.107	.090	.067	.053	.038	.028	.024
.67	.101	.084	.062	.048	.034	.026	.021
.68	.096	.079	.058	.044	.032	.023	.018
.69	.092	.074	.054	.040	.029	.020	.016
.70	.087	.070	.050	.038	.027	.018	.014
.71	.082	.067	.047	.035	.024	.017	.012
.72	.078	.063	.044	.033	.021	.015	.010
.73	.075	.059	.041	.030	.019	.013	.009
.74	.071	.055	.038	.028	.018	.012	.008
.75	.068	.052	.035	.026	.017	.011	.007
.76	.065	.049	.033	.024	.015	.010	.007
.77	.061	.046	.030	.022	.014	.009	.006
.78	.059	.043	.028	.020	.013	.009	.006
.79	.056	.040	.026	.018	.011	.008	.005
.80	.053	.038	.024	.017	.010	.007	.004
.81	.050	.036	.022	.016	.009	.006	.003
.82	.048	.034	.020	.014	.008	.006	.003
.83	.045	.032	.019	.013	.008	.006	.003
.84	.042	.029	.018	.012	.007	.005	.003
.85	.039	.028	.016	.011	.006	.005	.003
.86	.037	.026	.015	.009	.005	.004	.002
.87	.035	.025	.014	.008	.004	.003	.002
.88	.033	.023	.013	.008	.004	.003	.002
.89	.031	.022	.012	.007	.003	.002	.001
.90	.029	.020	.010	.006	.003	.002	.001

EXHIBIT I (Cont'd)

NEW YORK WORKMEN'S COMPENSATION—EXCESS PURE PREMIUM RATIOS

Ratio: Losses in Excess of Any Selected Loss Ratio Per Risk
Total Losses
No Limitation on Individual Losses

Loss Ratio	Premium Size						
	$75,000	$100,000	$150,000	$200,000	$300,000	$400,000	$500,000
.91	.027	.019	.010	.006	.003	.002	.001
.92	.026	.018	.009	.005	.003	.002	.001
.93	.024	.017	.009	.005	.003	.002	.001
.94	.023	.016	.008	.004	.002	.001	0
.95	.022	.014	.007	.004	.002	.001	0
.96	.021	.013	.006	.003	.001	0	0
.97	.020	.012	.006	.003	.001	0	0
.98	.019	.011	.006	.003	.001	0	0
.99	.018	.010	.005	.002	0	0	0
1.00	.017	.010	.004	.001	0	0	0
1.01	.016	.009	.003	0	0	0	0
1.02	.015	.008	.003	0	0	0	0
1.03	.014	.007	.002	0	0	0	0
1.04	.014	.007	.002	0	0	0	0
1.05	.013	.006	.001	0	0	0	0
1.06	.012	.005	.001	0	0	0	0
1.07	.011	.005	.001	0	0	0	0
1.08	.010	.005	.001	0	0	0	0
1.09	.010	.005	.001	0	0	0	0
1.10	.009	.004	.001	0	0	0	0
1.11	.009	.004	.001	0	0	0	0
1.12	.008	.004	.001	0	0	0	0
1.13	.008	.003	.001	0	0	0	0
1.14	.007	.003	0	0	0	0	0
1.15	.007	.003	0	0	0	0	0
1.16	.007	.003	0	0	0	0	0
1.17	.006	.002	0	0	0	0	0

Compiled
in 1941

EXHIBIT II

DISTRIBUTION OF RISKS BY LOSS RATIO
Corresponding To Excess Ratios In Exhibit I
Based on 1000 Risks in Each Premium Size

Loss Ratio	Standard Premium Size in Thousands of Dollars													
	5	7.5	10	15	20	25	50	75	100	150	200	300	400	500
0	6	3	2	1	1	—	—	—	—	—	—	—	—	—
.01	8	4	2	1	—	—	—	—	—	—	—	—	—	—
.02	10	5	3	1	—	—	—	—	—	—	—	—	—	—
.03	12	6	3	1	—	—	—	—	—	—	—	—	—	—
.04	14	8	3	2	—	—	—	—	—	—	—	—	—	—
.05	15	9	5	2	1	1	—	—	—	—	—	—	—	—
.06	16	10	6	3	2	2	—	—	—	—	—	—	—	—
.07	16	11	7	4	3	2	—	—	—	—	—	—	—	—
.08	16	12	8	5	3	3	—	—	—	—	—	—	—	—
.09	16	12	9	6	6	4	—	—	1	—	—	—	—	—
.10	16	14	11	8	7	5	1	—	1	—	—	—	—	—
.11	16	15	12	9	8	6	2	1	1	—	—	—	—	—
.12	15	15	12	10	10	7	2	1	2	1	—	—	—	—
.13	15	15	14	11	12	8	4	2	2	1	—	—	—	—
.14	15	15	14	12	12	9	4	2	2	1	—	—	—	—
.15	14	15	15	14	14	9	5	3	2	1	—	—	—	—
.16	14	16	16	14	14	10	5	3	2	1	—	—	—	—
.17	13	16	16	14	13	9	5	3	2	1	—	—	—	—
.18	13	16	16	14	13	9	5	3	2	1	—	—	—	—
.19	13	15	16	13	12	9	5	3	2	2	1	—	—	—
.20	13	15	17	13	11	9	5	3	2	2	1	—	—	—
.21	13	15	16	13	9	9	5	3	2	2	1	—	—	—
.22	13	14	16	13	8	9	5	3	2	2	1	—	—	—
.23	13	14	16	12	8	9	6	3	3	2	1	1	—	—
.24	13	13	16	12	6	9	6	4	3	2	1	—	—	—
.25	14	13	16	12	7	9	6	5	3	2	1	1	—	—
.26	14	13	15	12	6	9	7	5	3	2	1	1	—	—
.27	15	12	14	12	7	9	8	5	3	2	1	1	—	—
.28	15	12	13	12	7	9	9	6	3	2	2	2	—	—
.29	14	11	12	12	7	9	9	6	4	2	2	2	—	—
.30	14	11	11	12	9	10	10	8	5	2	3	2	—	—
.31	13	10	10	12	9	11	11	9	6	3	3	2	1	—
.32	12	10	10	11	11	11	12	10	7	4	6	3	2	—
.33	11	10	9	11	11	11	13	11	7	5	7	4	4	1
34	10	10	9	11	12	11	14	12	8	7	8	5	5	2
.35	9	10	8	11	14	13	15	14	9	8	9	6	6	3
.36	9	10	8	11	14	13	16	14	10	9	9	6	7	5
.37	7	10	8	11	16	14	16	15	11	10	9	7	7	6
.38	7	10	8	11	16	15	16	15	12	12	9	7	7	6
.39	7	10	8	12	17	16	17	16	13	12	9	7	7	7
.40	7	9	9	12	17	17	17	16	14	12	9	7	7	6
.41	7	9	9	12	18	17	17	16	15	14	9	7	7	6
.42	7	9	10	12	18	18	17	16	16	15	9	7	7	5
.43	7	9	10	13	18	18	17	17	17	15	9	7	6	4
.44	7	9	11	13	18	19	17	17	18	16	9	7	5	3
.45	7	9	11	14	18	19	17	17	18	17	10	7	3	2

EXHIBIT II (Cont'd)

Loss Ratio	Standard Premium Size in Thousands of Dollars													
	5	7.5	10	15	20	25	50	75	100	150	200	300	400	500
.46	8	9	12	14	18	19	16	17	19	18	11	8	2	2
.47	8	9	12	14	17	18	16	17	20	18	12	8	2	2
.48	8	9	12	14	17	18	16	17	20	18	13	11	2	4
.49	9	9	12	14	16	17	15	17	21	18	15	13	2	7
.50	9	9	12	14	15	16	15	17	21	19	17	17	2	11
.51	10	9	11	13	14	15	15	17	22	19	22	22	10	16
.52	11	9	10	12	13	13	15	17	22	21	26	28	23	23
.53	11	9	10	12	12	13	15	17	22	23	32	32	36	39
.54	11	9	10	12	11	12	15	17	22	25	36	37	47	49
.55	11	9	9	11	9	10	14	18	23	26	41	42	58	58
.56	10	9	8	10	9	10	14	18	22	27	43	45	65	65
.57	9	9	8	9	9	8	14	18	23	29	42	48	69	68
.58	9	9	7	8	7	9	14	18	22	30	41	51	70	70
.59	9	9	7	8	7	7	14	19	22	30	39	52	67	68
.60	8	9	6	7	6	8	14	19	22	30	37	51	62	64
.61	8	9	7	7	6	7	14	20	22	30	34	49	56	56
.62	8	8	6	7	5	8	13	20	21	29	32	45	48	49
.63	7	8	7	7	5	7	13	20	21	29	31	40	41	41
.64	7	8	6	7	5	7	13	20	21	27	29	36	33	33
.65	7	8	7	7	5	8	13	19	20	26	27	31	28	27
.66	6	7	7	7	5	7	13	19	19	24	26	27	22	22
.67	5	7	7	7	6	8	13	19	19	23	24	23	19	19
.68	5	6	7	7	5	7	12	18	17	21	23	19	16	16
.69	5	6	8	7	6	7	12	17	17	20	21	16	15	16
.70	5	6	7	6	6	8	12	16	16	18	18	14	13	14
.71	4	6	7	6	6	7	11	15	15	16	15	13	13	14
.72	4	5	8	5	7	7	11	14	15	14	12	12	11	13
.73	4	5	7	5	7	7	11	13	13	13	9	10	11	12
.74	4	5	8	5	7	7	10	11	13	12	8	9	9	11
.75	4	5	7	5	8	8	10	10	12	11	7	7	9	8
.76	4	5	7	5	8	7	10	9	11	10	6	6	7	6
.77	4	5	7	4	8	8	10	8	10	8	6	6	6	5
.78	4	5	7	5	8	7	9	8	9	8	6	5	4	4
.79	4	5	6	5	8	8	9	7	9	8	6	5	4	3
.80	4	5	6	5	8	8	9	6	9	8	6	4	2	3
.81	4	5	6	6	9	7	8	6	7	7	5	4	2	3
.82	4	5	6	6	8	8	8	6	7	7	5	4	2	3
.83	4	5	5	7	8	8	8	6	7	6	5	4	1	2
.84	4	5	5	7	8	7	8	6	6	5	5	4	1	1
.85	4	5	5	8	8	7	7	6	6	5	5	4	1	1
.86	4	5	4	8	7	7	7	6	5	5	5	4	1	—
.87	4	5	4	8	7	7	7	6	5	5	5	4	1	1
.88	4	4	4	8	7	7	7	6	4	4	5	3	2	—
.89	4	4	4	8	7	7	7	6	5	4	5	3	2	—
.90	3	4	4	7	6	6	7	6	4	4	4	2	2	1

EXHIBIT II (Cont'd)

Loss Ratio	Standard Premium Size in Thousands of Dollars													
	5	7.5	10	15	20	25	50	75	100	150	200	300	400	500
.91	3	4	4	6	6	6	6	6	3	3	3	2	2	1
.92	3	4	4	6	5	6	6	6	4	3	2	1	1	1
.93	3	4	4	5	5	6	6	6	4	3	2	1	1	1
.94	3	3	4	5	5	5	5	6	3	2	1	2	2	2
.95	3	3	4	4	5	5	5	5	3	2	1	1	2	2
.96	3	3	4	3	4	6	5	4	4	2	1	1	2	2
.97	3	3	4	3	5	5	5	4	3	2	1	2	2	2
.98	3	3	4	3	4	5	5	4	3	2	2	2	2	2
.99	2	3	3	3	3	4	5	3	3	2	2	2	2	1
1.00	2	3	3	3	4	5	5	3	3	2	3	2	1	
1.01	2	2	3	3	3	4	5	3	4	2	3	1	2	
1.02	2	2	3	3	3	4	4	2	3	2	3	1	1	
1.03	2	2	3	3	3	4	4	2	2	2	3	1	1	
1.04	2	2	2	2	2	3	4	2	3	3	3	1		
1.05	1	2	2	3	3	3	3	2	3	3	3	1		
1.06	2	2	2	2	2	3	3	2	2	2	2	1		
1.07	1	2	2	2	2	3	3	2	3	2	1	1		
1.08	2	2	2	3	2	2	3	2	2	2	1			
1.09	1	1	2	2	1	3	2	2	2	1	1			
1.10	1	1	1	2	2	2	2	2	2	1				
1.11	—	1	1	2	1	2	2	2	2	1				
1.12	1	1	1	2	2	1	2	2	1	1				
1.13	—	1	1	3	1	2	1	2	2	1				
1.14	1	1	1	2	2	2	1	2	1	1				
1.15	—	1	1	2	1	2	1	2	1	1				
1.16	—	1	1	3	2	2	1	2	1	1				
1.17	—	1	1	2	2	2	1	2	1	1				
1.18	1	1	1	2	2	2	1	1	1	1				
1.19	—	1	1	2	2	2	1	1	1	1				
1.20	—	—	—	2	2	2	1	1	1					
1.21	—	—	1	2	3	2	1	1	—					
1.22	—	1	—	1	2	2	1	1	1					
1.23	1	1	1	2	3	2	1	1	—					
1.24	—	1	1	1	3	3	1	1	1					
1.25	1	1	1	1	2	2	1	1	—					
1.26	1	1	2	1	3	3	1	1	1					
1.27	—	1	2	1	2	2	1	—	—					
1.28	1	1	2	1	3	3	1	—	—					
1.29	1	1	2	1	2	2	1	—	—					
1.30	2	1	2	1	2	2	1	—	1					
1.31	1	1	2	1	2	2	1	—	—					
1.32	1	1	2	1	1	2	1	—	—					
1.33	1	1	2	1	2	2	1	—	—					
1.34	2	2	2	1	2	2	1	—	1					
1.35	1	2	2	1	2	1	1	—	—					

EXHIBIT II (Cont'd)

Loss Ratio	Standard Premium Size in Thousands of Dollars													
	5	7.5	10	15	20	25	50	75	100	150	200	300	400	500
1.36	1	2	2	2	1	2	1	—	—					
1.37	1	2	2	2	2	1	1	—	—					
1.38	1	2	2	2	2	1	1	—	1					
1.39	1	2	1	2	2	1	1	—	—					
1.40	1	2	1	2	2	—	1	—	—					
1.41	2	2	1	2	2	1	1	—	—					
1.42	1	2	1	2	1	1	1	—	1					
1.43	1	1	1	2	1	—	1	—	—					
1.44	1	1	1	2	1	1	1	—	—					
1.45	1	1	1	2	1	—	—	—	—					
1.46	1	1	1	2	1	1	—	—	—					
1.47	—	1	1	2	—	1	—	—	1					
1.48	1	1	1	2	1	1	—	1						
1.49	1	1	2	2	—	—	—	1						
1.50	1	—	1	1	1	1	—	1						
1.51	1	—	1	1	1	1	—	1						
1.52	1	—	2	2	1	1	—	1						
1.53	1	—	1	2	2	—	—	1						
1.54	1	—	2	1	2	1	—	1						
1.55	1	—	1	2	2	1	—	1						
1.56	1	—	2	1	2	—	—	1						
1.57	1	1	1	2	1	—	—	1						
1.58	2	1	2	1	2	1	—							
1.59	1	1	1	1	1	—	—							
1.60	1	1	2	1	1	1	—							
1.61	1	1	2	—	1	—	—							
1.62	1	1	1	1	—	—	1							
1.63	2	1	1	—	—	1	1							
1.64	1	1	1	1	—	1	1							
1.65	1	1	1	—	—	—	1							
1.66	1	1	1	1	—	1	1							
1.67	1	2	—	—	—	—	1							
1.68	1	2	1	1	—	1	1							
1.69	—	2	1	1	—	—	1							
1.70	1	1	1	2	1	1								
1.71	1	2	—	1	—	1								
1.72	—	1	1	2	1	1								
1.73	1	1	—	1	1	1								
1.74	—	1	1	1	1	1								
1.75	1	1	1	1	1	1								
1.76	—	1	—	1	1	1								
1.77	—	1	1	1	1	1								
1.78	1	1	—	—	1	—								
1.79	—	1	—	—	1	—								
1.80	—*	1*	—*	—*	1*	1*								

*Also: $ 5,000 Size: 66 risks beyond 1.80, averaging 2.735 $15,000 Size: 16 risks beyond 1.80, averaging 2.58
$ 7,500 Size: 50 risks beyond 1.80, averaging 2.515 $20,000 Size: 9 risks beyond 1.80, averaging 2.53
$10,000 Size: 37 risks beyond 1.80, averaging 2.51 $25,000 Size: 6 risks beyond 1.80, averaging 2.34

NOTES ON MATHEMATICAL STATISTICS[1]

BY

FRANKLIN E. SATTERTHWAITE

The basis of actuarial science is statistical. For his raw material the actuary has tabulations of the behavior of certain statistics in the past. His task is to predict as accurately as possible the behavior of certain statistics in the future. If such future statistics have had identical counterparts in the past and if a large experience has been collected for these counterparts, the problem is relatively simple. The future will reproduce the past approximately. Unfortunately such an ideal condition seldom exists. Social, economic, and physical conditions change so that adjustments such as law amendment factors are necessary. Also, policy provisions and coverages change so that the statistic under study may not have existed in the past at all.

The problem is similar to that of the chemist who is asked to make nylon. First the chemist must gather together the available raw materials such as air, water, and coal. The raw materials of the actuary are the probabilities of accidents and the distributions of their costs. Next the chemist must combine his raw materials by use of complex processes which may involve intermediate products and catalysts. For the actuary the combination processes are mathematical and involve such operations as summation and integration. It is also sometimes efficient, or even necessary, to use intermediate functions such as moments and characteristic functions to obtain the result.

This paper is a collection of notes on certain mathematical techniques which have been found useful in developing, comprehending, and applying statistical theory. The specific problems taken up and the formulas developed are the same as those covered in Mr. Bailey's paper on sampling theory in this issue of the *Proceedings*. Therefore, it was considered superfluous to include an example of the actual construction of a statistical distribution. Mr. Bailey's examples are excellent and apply as well to this paper.

1. *Expected Value and Moments*: By the expected value of a statistic is meant the weighted arithmetic average value. The weights assigned to each possible value of the statistic are the probabilities that such value of the statistic will occur. Thus if a statistic $y(x)$ is a function of another statistic, x, which has a distribution function, $f(x)$, the probability that $y(x)$ will take on a particular value $y(x')$ is $f(x')$ and the expected value of $y(x)$ will be

$$E[y(x)] = \Sigma\, y(x)\, f(x), \tag{1}$$

[1] A paper on this subject was submitted in April 1941 as a thesis in lieu of parts VII and VIII of the Fellowship examinations for this Society. Because of the identity of subject matter with Mr. Bailey's paper in this volume of the Proceedings, only the technical parts of my paper are being printed at this time.

where the summation extends over all possible values of x. If x has a continuous rather than discrete distribution, the formula becomes

$$E[y(x)] = \int y(x)\, f(x)\, dx, \tag{2}$$

where the integration is taken over all possible values of x.

The operation symbol, E, will be reserved to indicate the expected value. If one will remember the following properties of this operator, he will find it very useful in developing many of the formulas necessary in statistical analysis. These properties are:

i The expected value operator, E, is a linear operator. That is

$$E(ax + b\,y) = a\,E(x) + b\,E(y). \tag{3}$$

ii The expected value of the product of two independent statistics is equal to the product of the expected values. That is

$$E(x\,y) = E(x)\,E(y). \tag{4}$$

iii The expected value of a constant is the constant itself. That is

$$E(a) = a. \tag{5}$$

iv The expected value of the expected value is the expected value. That is

$$E[E(x)] = E(x). \tag{6}$$

The fact that the expected value is a linear operator follows directly from the same property of integrations (or summations). We know that

$$\iint (ax + by)\, f(x, y)\, dx\, dy = a \iint x\, f(x, y)\, dx\, dy + b \iint y\, f(x, y)\, dx\, dy. \tag{7}$$

The actuary is experienced in the manipulation of linear operators through the study of finite differences. He knows that in formal manipulations he may treat the operation symbol as an algebraic quantity except that the operator and a variable may not be permuted. That is

$$\mathrm{E}(x) \neq x\,E. \tag{8}$$

The fact that the expected value of the product of independent statistics is equal to the product of the expected values follows because if two statistics are independent, the probability of their joint occurrence is the product of their probabilities of individual occurrences. Then by the separatability of integrations (or summations) we have

$$\begin{aligned} \iint x\,y\,h\,(x, y)\, dx\, dy &= \iint x\,y\, f(x)\, g(y)\, dx\, dy, \\ &= \int x\, f(x)\, dx \cdot \int y\, g(y)\, dy. \end{aligned} \tag{9}$$

Note that this property of the expected value holds only if the statistics are independent. The other properties hold for dependent as well as independent and single statistics.

The property that the expected value of a constant is equal to the constant

follows from the fact that the integration (or summation) of the distribution function of a statistic over all possible values of a statistic is unity. Since the expected value is a constant, not a statistical variable, it also follows that the expected value of the expected value is the expected value.

Mr. Bailey developed his formulas for the moments of the distributions of actuarial statistics by arithmetic methods. It is very convenient to define moments and to determine relations between moments by use of the expected value notations. Thus the defining equations for the three types of moments are

$$\begin{aligned} \mu''_k &= E(x-a)^k, \\ \mu'_k &= E(x)^k, \\ \mu_k &= E(x-E(x))^k. \end{aligned} \tag{10}$$

In this paper the Greek letter μ, (mu) will always be used to refer to moments. Unprimed it will refer to moments about the mean, with a single prime to moments about zero, and with a double prime to moments about some special origin, a. The reader should verify that the application of the properties of E gives

$$\mu''_0 = \mu'_0 = \mu_0 = 1, \qquad \mu_1 = 0. \tag{11}$$

One should be acquainted with the following parameters and their symbols:

(12)

mean: $m = \mu'_1$,
variance = (standard deviation)^2 = (probable error/.6745)^2:
$\sigma^2 = \mu_2$,
skewness: $\beta_1 = (\alpha_3)^2 = (\mu_3)^2/\sigma^6$,
kurtosis: $\beta_2 = \alpha_4 = \mu_4/\sigma^4$.
For the normal distribution $\beta_1 = 0$ and $\beta_2 = 3$.

Facility in using the notations of expected value to obtain relationships between parameters is very useful. Note that formulas so developed are perfectly general in their application. They hold for statistics with any type of distribution function whatever. However, if the second property of E is used, the different statistics must be independent. We shall give a few examples.

$$\begin{aligned} \sigma_x^2 &= \mu_2, && \text{by (12)}, \\ &= E\,[x - E(x)]^2, && \text{by (10)}, \\ &= E\,[x^2 - 2\,x\,E(x) + \{E(x)\}^2], \\ &= E(x^2) - 2\,E(x)\,E(x) + E\,\{E(x)\}^2, && \text{by (3)}, \\ &= E(x^2) - 2\,\{E(x)\}^2 + \{E(x)\}^2, \\ &= \mu'_2 - m^2, && \text{by (10)}. \end{aligned} \tag{13}$$

If two statistics, x and y, are independent,

(14)

$$\begin{aligned}
\sigma_{x+y}^2 &= E[(x+y)-E(x+y)]^2, && \text{by (12) and (10),}\\
&= E[\{x-E(x)\}+\{y-E(y)\}]^2, && \text{by (3) and (4),}\\
&= E[x-E(x)]^2+2\,E[x-E(x)]\cdot E[y-E(y)]+ E[y-E(y)]^2, && \text{by (3) and (6).}\\
&= \sigma_x^2+\sigma_y^2, \text{ since } E[x-E(x)] = 0,
\end{aligned}$$

The origins from which the statistics are measured are arbitrary. Therefore it will be assumed in the next two developments that they are measured from the mean so that $E(x)$ and $E(y)$ equal zero.

(15)

$$\begin{aligned}
{}_{x+y}\mu_3 &= E(x+y)^3 = E(x^3+3x^2y+3xy^2+y^3), && \text{by (10),}\\
&= E(x^3)+3\,E(x^2)\,E(y)+3\,E(x)\,E(y^2)+E(y^3), && \text{by (3) and (4),}\\
&= E(x^3)+E(y^3) = {}_x\mu_3+{}_y\mu_3, && \text{by (10).}\\
{}_{x+y}\mu_4 &= E(x+y)^4 = E(x^4)+0+6\,E(x^2)\,E(y^2)+0+E(y^4),\\
&= {}_x\mu_4+6\,\sigma_x^2\,\sigma_y^2+{}_y\mu_4.
\end{aligned}$$

The extension of these formulas to the distribution of the sum of n independent statistics each having the same distribution gives the following parameters where the sub-x refers to the parameters of the distribution of x:

(16)

$$\begin{aligned}
m &= n\,m_x,\\
\sigma^2 &= n\,\sigma_x^2,\\
\beta_1 &= {}_x\beta_1/n,\\
\beta_2-3 &= ({}_x\beta_2-3)/n.
\end{aligned}$$

2. *The Poisson Distribution*:

The distribution of the frequency of occurrence of independent events is of fundamental importance in the analysis of insurance statistics. This distribution is the Poisson distribution.

In order to bring out the properties of the Poisson distribution as clearly as possible, a development based on the infinitesimal calculus will be used. In following the development, the reader should remember that the values of any terms which involve infinitesimals of higher order than the first in a variable are immaterial to the argument. They drop out on integration. The

[2] The best tabulation of the Poisson distribution is: Molina, E. C.: Poisson's Exponential Binomial Limit. (1942) Van Nostrand.

notation, $o(dx)$, indicates an infinitesimal of higher order than dx and is read "zero of dx".

Let $p(\alpha)\,d\alpha$ be the probability that an event will occur in the infinitesimal unit of time, α to $\alpha + d\alpha$. Then the probability that no events will occur in this unit of time is

$$1 - p(\alpha)\,d\alpha + o(d\alpha). \tag{17}$$

Expressing this in the form of an exponential gives

$$e^{-p(\alpha)\,d\alpha\,+o(d\alpha)}. \tag{18}$$

If the probability of the occurrence of an event in any unit of time is independent of the occurrence of the event in any other unit of time, the probabilities, (18), for successive infinitesimal units of time may be multiplied together to obtain the probability that no event will occur in the finite interval of time, a to b. This is

$$\begin{aligned} F(0) &= e^{-p(a)\,da+o(da)} \cdot e^{-p(a+da)\,da\,+o(da)} \cdots\cdots e^{-p(b-da)\,da\,+o(da)}, \\ &= e^{-\int p(a)\,da\,+o(da)} = e^{-\int p(a)\,da} \\ &= e^{-P} \end{aligned} \tag{19}$$

where

$$P = \int_a^b p(\alpha)\,d\alpha \tag{20}$$

is the expected number of events.

Because of the independence conditions, $p(\alpha)\,d\alpha$ and e^{-P} may be multiplied together to obtain the probability that one event will occur in the period, α to $\alpha + d\alpha$, and that no other event will occur in the period a to b:

$$p(\alpha)\,d\alpha\,e^{-P}. \tag{21}$$

In this case α is a fixed point in time. If α is now allowed to vary over the period, a to b, the integral of (21),

$$\begin{aligned} F(1) &= \int p(\alpha)\,d\alpha\,e^{-P}, \\ &= P\,e^{-P}, \end{aligned} \tag{22}$$

gives the probability that one and only one event will occur in the period, a to b. The probability that exactly two events will occur in the period, a to b, is

$$\begin{aligned} F(2) &= \frac{1}{2!} \iint [p(\alpha')\,d\alpha'][p(\alpha'')\,d\alpha'']\,e^{-P}, \\ &= \frac{1}{2!}\,P^2\,e^{-P}. \end{aligned} \tag{23}$$

The $2!$ $(= 1 \cdot 2)$ enters because the occurrence of the first event at time α'' and the second at time α' duplicates the occurrence of the first at time α'

and the second at α''. Similarly the probability of the occurrence of exactly n events is

$$F(n) = \frac{1}{n!} P^n e^{-P}, \quad 0! = 1, \; n! = 1.2 \ldots n. \tag{24}$$

This is the Poisson distribution. It is the distribution of the number of events *whenever* the probability of the occurence of an event is *independent* of any other occurrence of the event.

The moments of the Poisson distribution are easily determined. For example

$$\begin{aligned} \mu'_2 = E(x^2) &= \sum_{x=0}^{\infty} x^2 \frac{P^x}{x!} e^{-P}, \\ &= e^{-P} \left\{ 0^2 + 1^2 P + \sum_{x=2}^{\infty} \frac{x + x(x-1)}{x!} P^x \right\}, \\ &= e^{-P} \left\{ P \frac{P^0}{0!} + P \sum_{x-1=1}^{\infty} \frac{P^{x-1}}{(x-1)!} + P^2 \sum_{x-2=0}^{\infty} \frac{P^{x-2}}{(x-2)!} \right\}, \\ &= e^{-P} \{ P e^P + P^2 e^P \}, \\ &= P + P^2. \end{aligned} \tag{25}$$

The straightforward application of formulas (12) and (10) and the properties of the expected value leads to the following parameters of the Poisson distribution:

$$\begin{aligned} m &= P, \\ \sigma^2 &= P, \\ \beta_1 &= 1/P, \\ \beta_2 - 3 &= 1/P. \end{aligned} \tag{26}$$

Notice that no restrictions were placed on $p(\alpha)$ in this development. It can vary in any manner at all with respect to time. For any given expected number of events, P, the Poisson distribution is the same. Thus if we are studying annual accident frequency, it is immaterial to us if $p = .01$ the year around, or if $p = .02$, November to April, and $p = 0$, May to October. The assignment of α as a time variable is also arbitrary. If p (Iowa) $= .01$ and p (Massachusetts) $= .02$, the experiences may still be combined and the combined experiences will follow a Poisson distribution.

In life insurance, a person can not die twice. While he is living the probability of death is, say, $p = .01$. If he dies, it immediately changes to $p = 0$. Thus the probability of the occurrence of death depends on whether the

person has died previously. If exposures are taken to the end of the year of death (as is customary), the independence conditions are not satisfied and the distribution of claim frequencies follows the binomial, not the Poisson distribution (if p is small, it is almost a Poisson distribution). If the exposure is cut off at the time of death, the independence conditions are satisfied and the distribution of the number of deaths is a Poisson distribution. This latter procedure has the added advantage that it is then perfectly proper to combine experiences with different probabilities. The combination of two experiences which follow different binomial distributions does not have a binomial distribution.

3. *Characteristic Functions*:

The characteristic function of the distribution has been found to be a very useful tool in the development of statistical theory. The characteristic function[3] is defined as

$$\phi_x(t) = E(e^{itx}), \qquad i = \sqrt{-1}. \tag{27}$$

For example, the characteristic function of the Poisson distribution is

$$\phi(t) = \sum_{x=0}^{\infty} e^{itx} \frac{P^x}{x!} e^{-P},$$

$$= e^{-P} \sum \frac{(P e^{it})^x}{x!} = e^{-P} \cdot e^{Pe^{it}},$$

$$= e^{-P(1-e^{it})}. \tag{28}$$

The characteristic function converts the distribution function, a function of the statistic, x, into a function of a new variable, t. The characteristic function is frequently used instead of the distribution function to develop statistical theory because of two properties:

i. The characteristic function of the sum of two independent statistics is equal to the product of the characteristic functions. That is

$$\phi_{x+y}(t) = \phi_x(t) \cdot \phi_y(t), \tag{29}$$

since

$$E(e^{it(x+v)}) = E(e^{itx} \cdot e^{ity}) = E(e^{itx})\, E(e^{ity}), \qquad \text{by (4).} \tag{30}$$

[3] If the reader has not studied functions of the complex variable and does not feel secure when manipulating "imaginary" numbers, it is suggested that he take a red pencil and cross out all the "i's" which appear in the formulas which follow. He will then have *moment generating* functions instead of characteristic functions. The reasons the mathematician has for preferring characteristic functions need not concern us here.

ii. The k^{th} moment about zero of the distribution of x is equal to $(\sqrt{-1})^{-k}$ times the k^{th} derivative of the characteristic function at $t = 0$.

That is:

$$(31) \qquad {}_x\mu'_k = (\sqrt{-1})^{-k} \frac{d^k}{dt^k} \phi_x(t)\,|_{t=0}$$

since

$$\frac{d^k}{dt^k} \phi_x(t)\,|_{t=0} = E\left[\frac{d^k}{dt^k} e^{itx}\right]_{t=0} = E\left[(ix)^k e^{itx}\right]_{t=0},$$

$$= i^k E(x^k) = i^k\, {}_x\mu'_k.$$

To illustrate the second property, let us find the first and second moments of the Poisson distribution. Differentiating (28) twice gives

$$(32) \qquad \frac{d}{dt} \phi(t) = e^{-P(1-e^{it})}(-P)(-1)\, e^{it}\,(i),$$

$$\frac{d^2}{dt^2} \phi(t) = e^{-P(1-e^{it})}\{[(-P)(-1)\, e^{it}\,(i)]^2 + [(-P)(-1)\, e^{it}\,(i)^2]\}.$$

Setting $t = 0$ and cancelling out the i's give by (31),

$$(33) \qquad \mu'_1 = e^{-P(1-1)}(-P)(-1)\, e^0 = P$$

$$\mu'_2 = (1)\{[P(1)]^2 + P(1)\} = P + P^2.$$

This value of the second moment agrees with the value found in formula (25) by the direct method.

4. *The Generalized Poisson Distribution:*

In many insurance lines one is only indirectly interested in the number of events. The primary interest is centered around the total cost of the claims which arise from the events. This cost is usually a variable and is not the same for each event. The analysis of this type of situation is quite parallel to the development of the Poisson distribution given above so that the result may well be called a generalized Poisson distribution. To keep the development within bounds, it will be necessary to make use of the characteristic function (27) and its properties which were introduced in (29) and (31).

In the development of the generalized Poisson distribution the same notations and independence assumptions will be used as in the development of the Poisson distribution. Let $f(x, \alpha)$ be the distribution of the cost of a claim at time α. Then the distribution of total cost over the period, α to $\alpha + d\alpha$, is

$$F(x,\alpha) = 1 - p(\alpha)\,d\alpha, \qquad \text{if } x = 0, \tag{34}$$
$$= p(\alpha)\,f(x,\alpha)\,d\alpha + o(d\alpha), \qquad \text{if } x > 0.$$

The characteristic function of the distribution of the total claim cost over the period, α to $\alpha + d\alpha$ is by (27),

$$\Phi(t,\alpha) = (1 - p(\alpha)\,d\alpha) + \int e^{itx}\,p(\alpha)\,f(x,\alpha)\,d\alpha\,dx + o(d\alpha), \tag{35}$$

since $e^{it0} = 1$.
Now let

$$\phi(t,\alpha) = \int e^{itx} f(x,\alpha)\,dx, \tag{36}$$

so that (35) becomes

$$\Phi(t,\alpha) = 1 - p(\alpha)\,d\alpha\,[1 - \phi(t,\alpha)] + o(d\alpha), \tag{37}$$
$$= e^{-p(\alpha)\,d\alpha\,(1-\phi(t,\alpha)) + o(d\alpha)}.$$

Assuming that events in different units of time are independent, the characteristic functions, (37), for the different units may be multiplied, together in the same way as in (19), to obtain the characteristic function of the distribution of the total claim cost over the period, a to b. This gives

$$\Phi(t) = e^{-\int p(\alpha)(1-\phi(t,\alpha))\,d\alpha}, \tag{38}$$
$$= e^{-P(1-\phi(t))},$$

where

$$\phi(t) = \int [p(\alpha)/P]\,\phi(t,\alpha)\,d\alpha, \tag{39}$$
$$= \iint e^{itx}\,[p(\alpha)/P]\,f(x,\alpha)\,d\alpha\,dx,$$
$$= \int e^{itx}\left\{\int [p(\alpha)/P]\,f(x,\alpha)\,d\alpha\right\} dx,$$
$$= \int e^{itx} f(x)\,dx,$$

is (assuming that the reversal of the order of integration was legitimate) the characteristic function of the mean distribution of claims,

$$f(x) = \int [p(\alpha)/P]\,f(x,\alpha)\,d\alpha. \tag{39a}$$

Thus we see that if we know the expected number of events, P, and the mean distribution of the cost of a claim, $f(x)$, the distribution (since it is determined by its characteristic function) of the total claim cost is determinate. We may combine in any way experiences with unlike distributions, $f(x,\alpha)$'s, of the cost of a claim if we can determine $f(x)$.

The application of (31) and (12) to (38) gives the following formulas for the parameters of the generalized Poisson distribution in terms of the parameters of the mean distribution of the cost of a claim. This latter dis-

tribution is referred to by a sub-x and in practice the calculation of certain intermediate parameters indicated by a sub-o will be found convenient:

$$
\begin{aligned}
(40)\qquad & m = P\,m_o, && \text{where} && m_o = {}_x\mu'_1,\\
& \sigma^2 = m\,\sigma_o{}^2, && \text{where} && \sigma_o{}^2 = {}_x\mu'_2/m_o,\\
& \beta_1 = {}_o\beta_1/m, && \text{where} && {}_o\beta_1 = [{}_x\mu'_3/\mathrm{m}_o]^2/\sigma_o{}^6,\\
& \beta_2 - 3 = ({}_o\beta_2 - 3)/m, && \text{where} && ({}_o\beta_2 - 3) = [{}_x\mu'_4/m_o]/\sigma_o{}^4.
\end{aligned}
$$

For the mean, m, a single differentiation of (38) was necessary:

$$(41)\qquad \Phi'(t) = \frac{d}{dt}\, e^{-P\,(1-\phi(t))} = e^{-P\,(1-\phi(t))}\,(-P)\,[-\phi'(t)].$$

Setting $t = 0$ and remembering that by (31)

$$\phi(t)|_{t=o} = {}_x\mu'_o = 1,\ \phi'(t)|_{t=o} = i\,{}_x\mu'_1,$$

gives the above formula. A second differentiation of (41) gives

$$(41a)\qquad \Phi''(t) = e^{-P\,(1-\phi(t))}\,\{(-P)^2\,[-\phi'(t)]^2 + (-P)\,[-\phi''(t)]\},$$

which on the application of (31) gives

$$i^2\,\mu'_2 = i^2\,(P\,{}_x\mu'_1)^2 + i^2\,P\,{}_x\mu'_2.$$

Cancelling the i^2's and applying (12) gives the above formula for σ^2. Similarly the above formulas for β_1 and β_2 can be obtained.

The above development of the generalized Poisson shows its properties but is more or less useless for purposes of calculation. Therefore consider the repeated application of formula (29). This gives the characteristic function of the distribution of the sum of n independent statistics from the same population as ϕ^n where ϕ is the characteristic function of the distribution of a single claim. Now if n is considered a statistical variable which obeys the Poisson distribution, (24), the expected value of ϕ^n with respect to (24) is

$$
\begin{aligned}
(42)\qquad \Phi &= E(\phi^n) = \Sigma\,\frac{1}{n!}\,(P\,\phi)^n\,e^{-P},\\
&= e^{P\phi}\cdot e^{-P},\\
&= e^{-P(1-\phi)},
\end{aligned}
$$

which is identical with the characteristic function of the generalized Poisson given in (38). Thus, to calculate the numerical values of a generalized Poisson distribution:

i. Find the distribution of the sum of zero, one, two, three, . . . , claims.
ii. Multiply each of these by the corresponding terms of (24).
iii. Add together these products.

Also consider the characteristic function of the distribution of the sum of two independent statistics, each of which obeys a generalized Poisson distri-

bution with the characteristic function (38). By the application of (29) this is

$$\Phi = [e^{-P(1-\phi)}]^2 = e^{-2P(1-\phi)}. \tag{43}$$

This is identical with the characteristic function of the generalized Poisson distribution when $2P$ claims are expected instead of P claims. Therefore in calculating the generalized Poisson distribution for $2P$ expected claims, it may be considered as the distribution of the sum of two independent statistics obeying generalized Poisson distributions with P claims expected.

The distributions of the total claim payments in practically all lines of insurance fall in the class of generalized Poisson distributions. They are such directly if each claim payment is independent of the rest. If certain payments are related to each other, they can be combined and the distribution of their sum taken as the element. Thus a group health and accident policy may provide for weekly indemnity, hospital payments, and surgical payments. For a given sickness or accident these three payments are related. However if one first finds the distribution of the total of the three payments for each sickness or accident, this distribution can be used as the basis of a generalized Poisson distribution.

5. *The Hyper-geometric Distribution*:

The Poisson distribution is a one parameter distribution. If the expected number of events, P, is known, one can completely specify the distribution. However in many practical problems one does not know the expected number of events, but only knows an estimate of it. For example, in group life insurance the expected number of deaths under a policy can be calculated in accordance with some general mortality experience. However, this is only an estimate of the true expected number of deaths in the sense of theoretical statistics. It does not make allowance for the individual characteristics of the risk such as its geographical location, the type of personnel hired and working conditions. Allowance can be made for these by assuming that P is a statistical variable distributed about the estimate, N, as a mean. Unless we have a great deal of information available, we can not determine much about the distribution of P. From general reasoning we know it is non-negative and continuous. One of the simplest such distributions is the Type III distribution,

$$g(P)dP = \frac{1}{(b-1)!}\left(\frac{b}{N}\right)^b P^{b-1} e^{-bP/N} dP, \tag{44}$$

which has a variance (in units of the mean) of

$$\sigma^2/m^2 = 1/b. \tag{45}$$

The expected value of the Poisson distribution, (24), with respect to (44) is

$$F(n) = E\left\{\frac{1}{n!}P^n e^{-P}\right\} = \frac{(n+b-1)!}{n!\,(b-1)!}\left(\frac{b}{N}\right)^b\left(\frac{N}{b+N}\right)^{b+n} \tag{46}$$

which is called the hyper-geometric distribution. It has the parameters:

$$\begin{aligned} m &= N, \\ \sigma^2/m^2 &= 1/N + 1/b, \\ \mu_3 &= N + 3\frac{N^2}{b} + 2\frac{N^3}{b^2}. \end{aligned} \tag{47}$$

Similar treatment of the characteristic function of the generalized Poisson distribution, (38), gives the characteristic function of the generalized hyper-geometric distribution,

$$\begin{aligned} \Phi(t) &= E(e^{P(1-\phi)}) \\ &= \left[1 + \frac{N(1-\phi)}{b}\right]^{-b} \end{aligned} \tag{48}$$

The application of (31) and (12) to this gives the parameters:

$$\begin{aligned} m &= N\,m_x \\ \sigma^2 &= m\,\sigma_0^2 + m^2/b, \qquad \sigma_0^2 = {}_x\mu'_2/{}_xm \\ \mu_3 &= N\,{}_x\mu'_3 + 3\frac{N^2}{b}\,{}_x\mu_2'\,m_x + 2\frac{N^3}{b^2}\,m_x^3. \end{aligned} \tag{49}$$

In practice the parameter, b, which appears in the above formulas can be estimated in the following way:

1. Tabulate the actual value, y_j, (viz., total claims under a policy) and the expected value, m_j, (viz., pure premiums) of a number of statistics which follow the desired distribution. It is best that the m_j's should not vary too much.
2. Calculate
$$R^2 = \Sigma\,(y_j - m_j)^2, S^2 = \Sigma\,m_j^2, T = \Sigma\,m_j.$$
3. The expected value of R^2 is

$$\begin{aligned} E(R)^2 &= \mathrm{E}\,[\Sigma\,(y_j^2 - 2\,y_j\,m_j + m_j^2)], \\ &= \Sigma\,[E(y_j^2) - 2\,E(y_j)\,m_j + m_j^2], && \text{by (3)}, \\ &= \Sigma\,[m_j\,\sigma_0^2 + \frac{m_j^2}{b} + m_j^2 - 2\,m_j^2 + m_j^2], && \text{see (49) and (13)}, \\ &= T\,\sigma_0^2 + S^2/b. \end{aligned} \tag{50}$$

4. Set R^2 equal to its expected value and solve for b.

When b is large compared with the expected number of claims under each policy, statistical methods for the determination of b breakdown because the

difference, $R^2 - T\sigma_o^2$, is small compared with the variability of R^2. It is then necessary to fall back on personal judgment in the choice of b. For example, if it is estimated that the true measure of the risk for 95% of the policyholders is within 20% of the rate on which manual premiums are based, the standard deviation, σ, (in units of the mean) of the distributions of the true risks about the manual rate is approximately 10%. Then, applying formula (45), we see that b is approximately 100.

6. *Summary*

The practical problem of the actuary is to forecast the behavior of certain statistics in the future. These statistics can usually be expressed as functions of certain elementary statistics. Fortunately the volume of past experience available for these elementary statistics is usually quite large so that they may be studied in great detail. Using the distributions of these elementary statistics as a basis, it is then possible to do the combining and integrating necessary to find the distribution of the desired statistic by mathematical formula or by mean strength on the calculating machine. The result is obtained more quickly (and often more accurately) than would be possible by waiting for a sizable experience to accumulate for the particular statistic under study.

The distribution of the frequency of events was found to take a very simple and general form, the Poisson distribution. The only information necessary to completely specify this distribution is the expected number of events. If the expected number of events is unknown but can be estimated, the hyper-geometric distribution is used in place of the Poisson. Both the Poisson and hyper-geometric distributions generalize to give the distribution of the total of claim payments. These generalized distributions were found to be functions of the average distribution of claims alone so that questions of seasonal or other variations in the distribution of claims can be ignored.

In some circles there has been a tendency to disparage the actuary as being backward in adapting theoretical methods to his problems. However, those who study the problem more carefully discover that the inadequacy is as much with statistical theory than with the actuary. Mathematical statistics is an infant science which has only reached the stage of rapid development in the last ten or twenty years. It still has many very simple types of problems to solve. It is only beginning to develop methods which are general enough to handle the complex problems to which the actuary must obtain an answer the best he can, inadequate theory being no excuse.

It is hoped that the reader will have been encouraged to reflect on the theoretical basis of his work more often and that maybe he will take some part in expanding that theoretical basis.

ABSTRACT OF THE DISCUSSION OF PAPERS READ AT THE PREVIOUS MEETING

BUDGETING BY CASUALTY INSURANCE COMPANIES

WILLIAM F. DOWLING

VOLUME XXVIII, PAGE 324

WRITTEN DISCUSSION

MR. W. B. BAILEY:

Mr. Dowling's system of budgetary control endeavors to forecast premium income, and plan expenditures in the coming year so that a predetermined profit will result.

The method of premium forecasting is quite involved and calls for statistics which it is believed very few companies are keeping. In a large company the cost of compiling these figures would be quite sizeable. The judgment factor which is supposed to allow for changes in external business during the year would appear to be subject to wide margins of error. We have found it rather difficult to accurately forecast premiums for a period as short as two months.

Expenditures are much easier to estimate. The largest of these, other than for claims, are for commissions and salaries. Commissions depend on the premium forecast. They are not subject to much control by management. Mr. Dowling makes several interesting suggestions in regard to control of salaries that should be helpful to personnel departments, for instance, the one on the grading and classification of clerks. Nevertheless, salary control has very definite limitations in an insurance company. If a reduction in premium volume is forecast, the chances are that the salary ratio to premiums will increase. Salaries are very inflexible over as short a period as one year. However, the budget will have accomplished something if it has brought to the attention of the executives the coming reduction in profits due to reduced volume and fixed salaries.

In studying the forms supplied to department heads for their estimates of salaries and expenses for the year, it was noted that they contained no information on the volume of premiums expected. It would seem that this would be necessary before the department heads could decide what their expenditures for the year would be.

The method used to estimate inspection expense is novel but is so closely

tied in with the premium forecast that any error in that would also reflect itself in the inspection budget.

The possibility of adjusting expenditures to agree with expected premiums is much greater for expenses than for salaries. Here planned spending can be nicely adjusted toward reaching the predetermined profit. Mr. Dowling rightly states that no person should be charged in the budget with an expenditure over which he does not have major control. The example of claim rent being charged to the financial rather than the claim department is well taken, especially so if the claim department is not allowed to decide where and how much space it should have.

The budget for the printshop would seem to be a necessity if a company is to know that it is getting its money's worth in printing.

On the master budget, form VIII, provision is made for estimating claim payments and changes in reserves. Estimates such as these could be very unreliable.

On the whole it is felt that while a budgetary plan has its usefulness, this must be measured against the cost. In most companies this cost would be rather heavy, especially for keeping statistics used to forecast premiums which forecast is subject to pretty large errors in judgment. The plan is useful in that it makes the executives expense-conscious and also provides for expense adjustments before the anticipated changes in premiums. However, because of the inflexibility in salaries, the largest expense item, the amount of these adjustments would be relatively small. Weighing cost against value it would seem that the latter would hardly justify the expenditures required for the system.

AUTHOR'S REVIEW OF DISCUSSION

MR. WILLIAM F. DOWLING:

In my paper I point out that the inauguration of a budgetary system in an insurance company does not mean that a panacea for all of the ills of the company has been found. I also point out that it cannot be expected that the maximum of efficiency of the budgetary system could be reached in the first year nor in the second year.

The companies that now use the budget plan have found that as the years go on they can improve upon the basic figures required and the method of calculating them with the resultant increase in efficiency and a decrease in the error of judgment. An emergency period such as that in which we now find ourselves is the true test of the value of a budget. The companies that

have been operating on controlled budget basis are continuing to do so despite all the obstacles encountered.

Management is the sole judge as to whether or not it desires to act quickly or to go slowly when adverse trends are shown. It is to be presumed that all department heads involved in the making of a budget are made fully acquainted with the estimated income figures for the period involved. They would then be expected to utilize that knowledge in formulating their own budget requirements.

I am not willing to concede too readily that estimates of losses incurred are at all times unreliable. The figures may be relied upon when the period of study covers a sufficient length of time, where the analyses are in sufficient detail and properly related to premiums.

It is conceded that a budgetary plan costs money, but it is felt that the control, the incentive, the possibility for saving and the knowledge gained would be worth the price.

PREMIUM COLLECTIONS ON PUNCH CARDS

DUDLEY M. PRUITT

VOLUME XXVIII, PAGE 503

WRITTEN DISCUSSION

MR. W. H. CRAWFORD:

Mr. Pruitt's paper is one which strikes a responsive chord in those of us who have had close contact with the application of tabulating equipment to the accounting and statistical problems of casualty insurance. It is quite evident that Mr. Pruitt has partaken of the International Business Machines "table d' hote" dinner, as the procedure he outlines, mechanizes all premium collection functions from soup to nuts. Frankly, there is little, if anything, I can add to this very carefully planned and admirable mechanized system which uses all the most modern tabulating devices.

My comments will be along two lines of thought, viz:—

1. A discussion of the "layer cake" card; its origin and basic purpose.
2. Whether these war times are the most advantageous in which to add to our tabulating functions.

* * * *

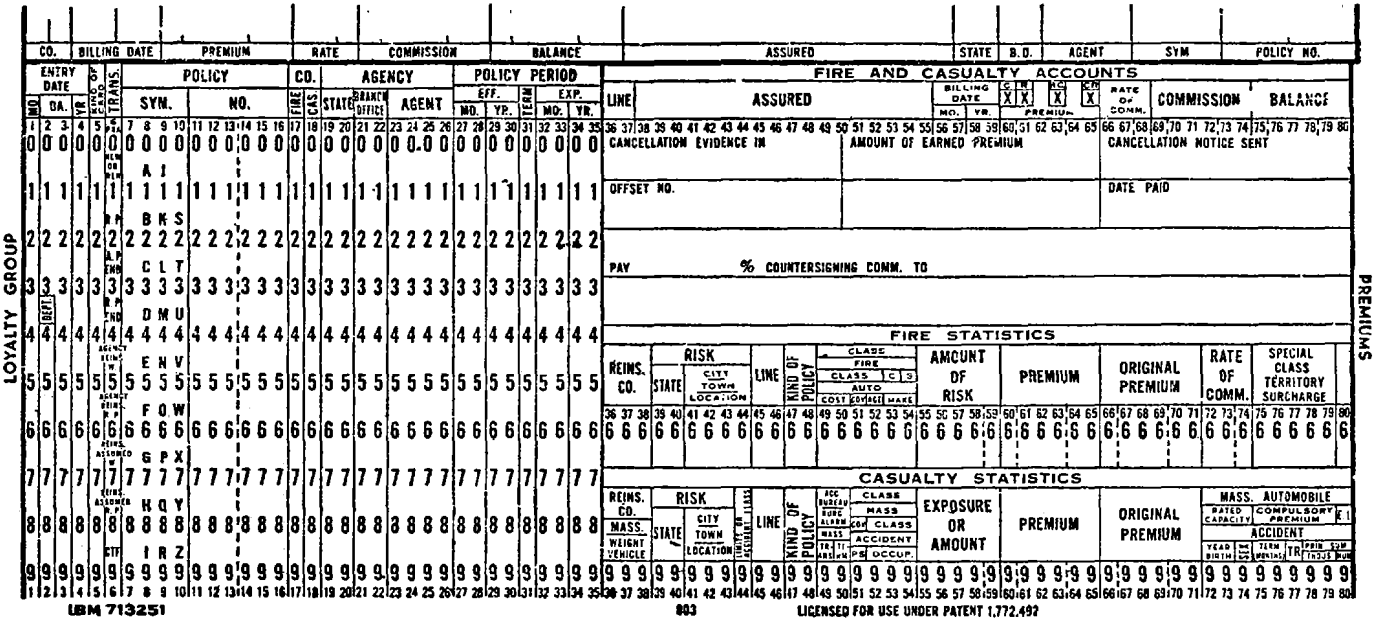
The "layer cake" card is one in which I have a very personal interest, as I first introduced this particular type of card to representatives of the International Business Machines Corporation in August 1939 at conferences held at Endicott, N. Y. The "layer cake" card had its origin in the Western

Department of the Loyalty Group at Chicago, Ill. some months prior to that time. The adaptation of the "layer cake" card discussed in Mr. Pruitt's paper, overlooks the fundamental advantages of this card form. At the time we pioneered the field and originated this form of card, our goal was twofold, viz:—

1. To accelerate the punching, verifying and balancing of the accounting and statistical tabulating cards so as to permit prompt release of the daily report.
2. To have a single coordinated tabulating card form for both Fire and Casualty accounts and statistics, inasmuch as our Fire and Casualty business is handled by coordinated and unified personnel.

After being exhaustively tested in our Western Department for approximately one year, its use was extended to other departmental offices and is now the basic premium tabulating card in our country-wide tabulating system. The card form is shown in Figure 1.

FIGURE 1



IBM 713251 803 LICENSED FOR USE UNDER PATENT 1,772,492

FIGURE 2

TRANS.	CO. FIRE	CO. CAS.	AGENCY State	AGENCY B.O.	AGENCY AGENT	POLICY PERIOD EFFECTIVE MO.	POLICY PERIOD EFFECTIVE YR.	TERM	POLICY PERIOD EXPIRES MO.	POLICY PERIOD EXPIRES YR.	RISK State	RISK CITY TOWN LOCATION	LIMITS	LINE	KIND OF POLICY	CLASS FIRE COST	CLASS FIRE CITY	CLASS FIRE MAKE	CLASS CASUALTY BURG. ALARM	CLASS CASUALTY CLASS	EXPOSURE OR AMOUNT	PREMIUM	RATE OF COMM.	CANCELLED EXPOSURE OR AMOUNT	RETURN PREMIUM	ORIGINAL PREMIUM CANCELLED

It should be noted that this card has but a single punching guide for the first 35 columns—all information common to both the accounting and statistical cards being grouped in these columns. Columns 36 to 80 inclusive, have three punching guides (1) Fire and Casualty Accounts (2) Fire Statistics (3) Casualty Statistics. Punching is done from the daily report directly, thus thru the use of this card we can handle all types of dailies —both fire and casualty—without changing card forms. Our daily reports contain a "code block" which is identical with the tabulating card form so that it permits the key punch operators to punch in the exact sequence as the card form. A sample of the synchronized code block used in automobile dailies is shown in Figure 2.

The first card only is completely punched insofar as columns 6 to 35 inclusive. All additional account and statistical cards are "skeleton" punched, the operator striking the "X" skip key when punching Column 6, which causes the machine to skip to Column 36. As the identical card form is used for both account and statistical cards, the account cards are distinguished from statistical cards by a "Kind of Card" Codes which are punched in Column 5. Thus if a single daily report entry requires two accounting cards and six statistical cards, only the first accounting card would be completely punched. The remaining seven cards would carry no punching in Columns 6 to 35 inclusive, except for the "X" punch in Column 6 which is "cut" when skipping to Column 36. Our punch machines are equipped with an automatic card reversing device to facilitate the verification of the punched cards in the same order as the cards were punched.

After the cards have been punched and verified, we have the fully punched cards *without an "X" in Column 6,* followed by the "skeleton" punched cards for the same entry, *which have an "X" punched in Column 6.* The cards are now put thru a high speed gang summary punch or reproducer which is actuated by the "X" or no "X" in Column 6. This causes the information appearing in Columns 6 to 35 in the fully punched card to be reproduced into the following "skeleton" punched cards. After this step is completed, we have the interspersed account and statistical cards completely punched.

This procedure has the following advantages:

1. Account and Statistical Tabulating cards are punched simultaneously with one handling of the daily report.

2. The manual key punching and certifying is reduced substantially as only one tabulating card is completely punched for each entry. On the remaining cards a maximum of 45 columns is punched.

3. The accuracy of the common information is greatly enhanced as the possibility of error in essential information is reduced to a

minimum, as all common data is mechanically reproduced into both accounts and statistical cards.

It occurred to us that the interspersed account and statistical tabulating cards might become mixed before the common data was gang punched into the "skeleton" punched cards. In such event the "skeleton" punched cards might lose their place from behind the proper completely punched card, and as no identifying information such as policy number appears on the "skeleton" punched cards until after the gang punching operation, a very real problem was present. We solved this satisfactorily by placing our balancing operation—which I will describe later—under control by policy number.

Before the daily report is released, we make certain that the account cards are in absolute balance with the statistical cards. Heretofore this has been a serious problem as it meant comparing two separate listings, one of the account cards and the other of the statistical cards, which is obviously a slow, tedious clerical operation. Having the account and statistical cards interspersed by policy number makes it possible to perform this operation with amazing ease thru the use of a numeric tabulator equipped with one unit of "digit selection." The Tabulator is placed under control as to policy number, and by "digit selecting" the kind of card code, the account cards are caused to *add for each policy* and the statistical cards *subtract from the same counter*. Thus if the account cards do not agree with the statistical cards for any policy number, a difference is shown; if they balance, the counter dial and tabulation are clear. At the same operation we tabulate and print the total amount of the premium being entered for each policy and a grand total of the entries in that particular group, thus obtaining our daily group total, for control purposes.

As our primary concern is to put our daily reports thru our accounting and statistical record with the utmost speed, we route them thru our tabulating room in "pockets" of not more than 5. Thus as soon as a "pocket" is balanced, the daily reports can be released, so that instead of releasing large blocks of dailies sporadically, they are released in a steady flow during the day.

When the cards are balanced and the daily reports released, the cards are sorted so as to segregate the account cards from the statistical cards. The account cards are put thru a multiplying punch which computes the commission and the balance due (Premium, less commission) and punches this data into the proper fields on the account tabulating card. The statistical cards are available for regular statistical functions. The account cards are now interpreted—to permit them to be read by the bookkeepers —and released to the accounts department.

The creation and adoption of the "layer cake" card has enabled us to

achieve our goal. We have greatly accelerated the punching, verifying and balancing of the premium tabulating cards with but one handling of the daily report and have a single coordinated tabulating card form for both Fire and Casualty accounts and statistics. At the same time we have eliminated any variance between Account and Statistical Records in identifying data such as policy number, agent's code, policy period, etc.

* * * *

In his closing remarks, Mr. Pruitt poses a rather debatable point,—"The application of some at least of the principles outlined here should serve to alleviate a few of the difficulties we are all encountering because of war time clerical shortages."

I cannot read Mr. Pruitt's program without being impressed by the complexity of the punching and the multiplicity of the cards. The key punch operator is required to skip the premium field on certain cards if the amount of premium is the same as on the preceding card; the operator must keep in mind that three distinct sets of cards are required for each entry; in addition, she is required to identify by alphabetically punched abbreviations —safe driver rewards, audits, long-haul risks, etc. While I readily agree that to an experienced insurance key punch operator these are minor complications, nevertheless, in these days of untrained operators, simplicity of punching is extremely desirable. Thus I seriously doubt if in these war days, insurance men will be very receptive to any program whereby functions now being performed in their account departments, such as a determination of overdue premiums, maintainence of agent's control records, etc. are placed on tabulating equipment.

Mr. Pruitt speaks of resistance of insurance accountants to the application of machine methods to insurance accounting problems. Altho it has always been my good fortune to work with accountants receptive to machine methods, at various gatherings I have sensed an apathy on the part of some insurance accountants. I suspect this apathy arises from a very understandable caution. Accounting is a rather exact science and its foundations rest on controls and balances. The average insurance accountant has but a cursory knowledge of tabulating equipment, thus he is adamant to any plan transferring basic accounting controls to punched cards, as doing so such familiar records may lose their identity and become involved in that mysterious realm known as "Tabulating Procedure." In consequence, when tabulating procedures "bog down"—as they may do at very inopportune times—the accountant very justifiably would have a sense of hopelessness as his whole accounting structure must await completion because certain

of his basic accounting controls are involved in a mechanized program with which he is quite unfamiliar.

A major personnel problem with insurance companies in particular, is the retention of trained key punch and tabulating machine operators. Already many have been attracted into defense industries, some undoubtedly by patriotism, others by the fantastic salaries being paid. The problem becomes more and more acute. It is not without the realm of possibility that trained key punch and tabulating machine operators as well as tabulating equipment may be drafted into the war effort. In normal times, with competent tabulating personnel, the extension of tabulating equipment to the problems of insurance will find no more ardent advocate than I, but I seriously question that this is the opportune time, even though the job may be more efficiently performed on tabulating machines. I am of the opinion that this is a time when every tabulating function should be meticulously studied and eliminated unless it is absolutely essential to the conduct of a war time insurance business.

* * * *

Mr. Pruitt speaks of the naturally close correlation between companies statistical and accounting functions and points out that it is often difficult to note where the accounting function ends or the statistical function begins. This is very true, and what may be considered an accounting function in one company is considered a statistical function in another. This situation, in my opinion arises from a misconception of statistical functions in an insurance company, as basically, they are comparable to cost accounting in a commercial organization. In essence, the statistical department analyzes and segregates basic accounting control figures, yet, in a great number of companies we have a definite and distinct division as between the accounting and statistical departments, each operating distinctly separate from the other.

I feel very strongly that we will not attain maximum efficiency until every insurance accountant is familiar with statistical functions and every statistician has a working knowledge of insurance accounting, so that they may work together in solving their mutual problems. It naturally follows that the financial officer having supervision of both accounting and statistical functions must have not only a thorough knowledge of insurance accounting and statistical functions but also a good working knowledge of the functions of tabulating equipment inasmuch as it plays such an important role in the accounting and statistical program of the average insurance company.

Again, I compliment Mr. Pruitt on his timely and admirable paper. He has certainly presented a very constructive thought provoking program and I have little doubt that when we resume the insurance business as usual, many companies will embark on some or all of the phases of mechanized accounting he suggests. I am particularly hopeful that Mr. Pruitt's paper

will encourage members to explore the possibilities of office machinery designed to expedite the assembling of basic actuarial data and present their findings to the Society. To date, such contributions have, in my opinion, been all too few. Some members may question the propriety of such discussions in Actuarial proceedings. My answer is that practically all actuarial studies have their basis in the mass of statistical data accumulated by the various insurance carriers. Hence, I suggest it is extremely important to the Society to encourage any studies having to do with the assembling and reporting of accurate and current statistical data. This is particularly true as to the post-war situation when pre-war statistical data will lose most of its credibility and the need for current post-war data will be urgent.

This war will give tabulating equipment and procedures a tremendous impetus. Tabulating equipment is being used extensively by governmental agencies and hundreds of young men are being trained by the government in the operation of tabulating equipment. When these men and the others now in war industries, find their way back to peace time employment, I have no doubt that whatever strides we have taken up to now will seem insignificant in light of post-war developments.

AUTHOR'S REVIEW OF DISCUSSION

DUDLEY M. PRUITT:

The Society is under obligation to Mr. Crawford for giving a more detailed account of the mechanical operation of the "layer cake" card than I felt I had space for in the original paper. Some question may be raised, however, as to his statement that the paper "overlooks the fundamental advantages of this card form". From a careful study of Mr. Crawford's exposition it would seem that his use and application of the "layer cake" card is fundamentally the same as that set forth in the paper. In so far as card form and card punching routine are concerned, except for a few minor details, the two operations are as alike as two peas in a pod. When Mr. Crawford complains of "the multiplicity of cards" and the fact that "the operator must keep in mind that three distinct sets of cards are required for each entry," he is leveling a criticism at his own routine, for the two routines are identical in this respect. It is indeed gratifying to find that another company has so closely paralleled our course and that its findings confirm our own record of beneficial results.

With regard to the feasibility in war time of making systems changes, there is no one more conscious of the restrictions imposed than I. For most of us, of course, radical changes are impossible. We cannot, however, afford to take a "do nothing" attitude, particularly under present conditions. While key punch operators are undoubtedly hard to get, other types of personnel

are scarce too. It is becoming increasingly essential that we examine *all* procedures and routines with a view to efficiency and economy. And in that examination the services tabulating equipment can perform should not be overlooked. Properly planned machine operations can substitute for clerical operations and we should be alert for every opportunity that presents itself for making that substitution within the limits imposed by the present emergency.

RECENT DEVELOPMENTS IN CONNECTION WITH THE SPECIAL FUNDS UNDER THE NEW YORK WORKMEN'S COMPENSATION LAW

ELSIE KARDONSKY

VOLUME XXVIII, PAGE 515

WRITTEN DISCUSSION

MR. GRADY H. HIPP:

The paper presented by Miss Kardonsky has great practical value for Compensation insurance carriers in particular and generally for persons interested in the administration or status of the various special funds. There have been numerous developments in connection with the special funds during the last several years. Miss Kardonsky's paper provides information in convenient form relative to the present status of the various funds.

An outstanding development was the establishment of the Special Funds Conservation Committee and the designation of an attorney of the Committee to defend claims brought against the Reopened Case Fund. The ultimate accomplishments of this cooperative Committee representing insurance carriers and self-insurers will be watched with great interest.

For convenience of reference, I am showing in Exhibit A a summary of the amounts for which carriers have been liable in each no dependency death case award in New York State for payment into the special funds, other than the Workmen's Compensation Security Funds, during various periods since the effective date of the New York Workmen's Compensation Law.

A comparison of the financial status of the Second Injury Fund, the Reopened Case Fund, the Vocational Rehabilitation Fund and the Aggregate Trust Fund at the end of June or December, 1936 and as of the latest date for which information is now available is shown in Exhibit B. This exhibit indicates what, if any, progress has been made in rehabilitating the various special funds.

In discussing the law amendments to Section 25-a of the New York Compensation Law, Miss Kardonsky expresses confidence that the estimate previously made of the annual income to the Reopened Case Fund resulting from such amendments will be realized. The following table shows that the income of the Reopened Case Fund has not yet reached the original estimate

made by the Actuarial Committee of the Compensation Insurance Rating Board of New York:

	Actuarial Committee's Original Estimate	Actually Received During Year Ended 6/30/42
From no dependency death cases	$140,000	$110,275
From low cost dependency cases	50,000	8,617
From $5.00 payments on schedule awards	85,000	93,140
TOTAL	$275,000	$212,032

It is true that there is a considerable time lag in realizing the full effect of certain of the law amendments. Furthermore, the payments into the Reopened Case Fund on schedule awards were increased from $5.00 to $10.00 for each case in which an award is made by reason of injuries sustained during the period July 1, 1942 and June 30, 1947. When the full effect of all law amendments is realized, the income of the Reopened Case Fund may reach the original estimate. Even if that desired objective is attained, it must nevertheless be borne in mind that the loss payments from this Fund may increase to an amount above the present level of payments. Furthermore, as the law now reads, the $10.00 payments on schedule awards will be discontinued altogether on cases based on injuries sustained after June 30, 1947. It is therefore evident that the question of extending the period for these payments must be reexamined at a later date.

In discussing the development of plans to place the Reopened Case Fund on a solvent basis, Miss Kardonsky states that a middle course or modified reserve plan was adopted to provide funds for current loss payments and for known liabilities incurred. She states that this was done as a practical matter in order not to place too heavy a burden on the carriers. It has never appeared to the writer that a plan for placing the Reopened Case Fund on a sound actuarial reserve basis would be unduly burdensome on the carriers for the reasons that provision could be made for the accomplishment of the desired objective by a gradual process extending over a period of years and the necessary additional contributions in any event would be provided for through an increase in the level of rates. It is recognized that there may be some merit in the arguments against building up large reserve funds to cover contingent liabilities where the claims may be expected ultimately to reach a more or less fixed level but I cannot agree that such arguments are necessarily conclusive and final.

If a reserve is not set up to cover contingent liabilities, we should not deceive ourselves by statements to the effect that the Fund is solvent. The estimated deficit of $1,179,450 in the Reopened Case Fund as shown in Exhibit B is based on liabilities which exclude certain contingent liabilities.

The estimated liabilities of the Reopened Case Fund as of June 30, 1940 include a reserve of $862,880 for cases on which awards have been made and charged against the Fund and a reserve of $528,561 which is the estimated cost of known cases reopened by the Industrial Board and under its consideration as of June 30, 1940 for possible awards against the Fund. Unfortunately, Section 25-a, Subdivision 4, of the New York Compensation Law provides that the Superintendent of Insurance shall ascertain the liability of the Fund only upon all cases in which awards of compensation have been made and charged against the Fund. It seems to me to be just as important to estimate the cost of cases reopened by the Industrial Board and still under its consideration as of the valuation date.

In addition, a substantial amount would be required to cover the estimated present value of awards which may be made in the future in cases involving original accidents which have already occured prior to the date of valuation and which may be reopened thereafter. No reserve for this contingent liability is included in the liabilities of the Reopened Case Fund shown in Exhibit B. As a practical certainty, we know that some of the many thousands of cases based on accidents which have already occurred will be reopened and that substantial awards will be made in some cases against the Reopened Case Fund. We do not know exactly how many such cases will be reopened nor do we know the amount of the awards which will be made on such cases. A rate of emergence of such claims, however, could be calculated on which a reserve accurate to a reasonable degree could be based.

EXHIBIT A

COSTS TO CARRIERS OF EACH NO DEPENDENCY DEATH CASE AWARD

(Summary of the amounts for which carriers have been liable in each no dependency death case award in New York State during various periods)

	7/1/14 to 6/1/16	6/1/16 to 5/13/20	5/13/20 to 7/1/22	7/1/22 to 5/21/23	5/21/23 to 4/24/33	4/24/33 to 7/1/40	7/1/40 to Date
Second Injury Fund (§15-8)	$ ...	$ 100	$ 100	$ 500	$ 500	$ 500	$ 500
Reopened Case Fund (§25-a)	...	...	....	...	...	300	1,000**
Vocational Rehabilitation Fund (§15-9)	...	...	900	500	500	500	500
Funeral Expenses (§16-1)	100*	100*	100*	100*	200*	200*	200*
TOTALS	$ 100	$ 200	$1,100	$1,100	$1,200	$1,500	$2,200**

* Funeral expenses not to exceed this amount.

** In addition to the $1,000 contribution in each no dependency death case award for injuries on or after July 1, 1940, the carriers and self-insurers are required to pay into the Reopened Case Fund the following amounts:

(1) The difference between the sum of $2,000 and the amount of the compensation, exclusive of funeral benefits, actually paid to beneficiaries in cases where the amount of the compensation paid to the beneficiaries is less than the $2,000, exclusive of funeral benefits.

(2) $5.00 for each case in which an award is made by reason of injuries sustained between July 1, 1940 and June 30, 1942, inclusive, and $10.00 for each case in which an award is made by reason of injuries sustained between July 1, 1942 and June 30, 1947, inclusive, pursuant to the provisions of paragraphs a to s, inclusive, of Section 15, Subdivision 3 (schedule awards for permanent partial disability).

The above additional payments are required by Chapter 686, Laws of 1940 and Chapter 378, Laws of 1942.

The following lump sum payments have been made to the Reopened Case Fund:

(a) $250,000 was transferred to this fund from the Vocational Rehabilitation Fund under the provisions of Chapter 384, Laws of 1933.
(b) $150,000 levied on insurance carriers and self-insurers under the provisions of Chapter 252, Laws of 1939.
(c) $150,000 levied on insurance carriers and self-insurers under the provisions of Chapter 686, Laws of 1940.
(d) $100,000 levied on insurance carriers and self-insurers under the provisions of Chapter 376, Laws of 1941.

Subdivision 3 of Section 25a of the Compensation Law as amended, contains the following provision with reference to the assessment for expenses of administering the Workmen's Compensation Law and the payments required to be made into the Reopened Case Fund:

"Such assessment and the payment made into said fund shall constitute an element of loss for the purpose of establishing rates for workmen's compensation insurance as provided in the insurance law".

EXHIBIT B

SUMMARY OF FINANCIAL STATUS OF SPECIAL FUNDS (EXCLUDING SECURITY FUNDS)

	Statement As of	Assets	Liabilities (Excluding Certain *Contingent Liabilities)	*Surplus (or Deficit)
Second Injury Fund.....	12/31/36	$ 992,167	$ 1,222,782	—$ 230,615
" " "	12/31/41	1,501,011	1,241,919	259,092
Reopened Case Fund.....	12/31/36	201,703	658,498	—456,795
" " "	6/30/40	211,991	**1,391,441	—1,179,450
Vocational Rehabilitation Fund.	6/30/36	746,366	250,000	496,366
" " "	6/30/41	333,998	***	***
Aggregate Trust Fund.....	12/31/36	3,370,115	3,313,687	56,428
" " "	12/31/41	12,519,721	12,749,161	—229,440

* In calculating the liabilities and surplus of the Second Injury and Reopened Case Funds there has been excluded (1) from the liabilities of the Second Injury Fund the estimated present value of awards which may be made in the future in cases involving original accidents which occurred prior to the respective dates of valuation, and (2) from the liabilities of the Reopened Case Fund the estimated present value of awards which may be made on claims cases which may be reopened in the future, which cases would be based on accidents which had occurred prior to the respective valuation dates.

** The liabilities of the Reopened Case Fund as of 6/30/40 include the amount of $528,561 which is the estimated cost of known cases reopened by the Industrial Board and under its consideration as of June 30, 1940 for possible awards against the Fund.

*** The amount of the liabilities of the Vocational Rehabilitation Fund as of 6/30/41 is not available but it was probably small.

	Assets 3/1/42	Approximate Goal
Workmen's Compensation Security Fund—Stock..	$2,630,516	$3,000,000
" " " " —Mutual	1,399,379	1,500,000

Furthermore, we should not overlook the contingent liability of the Second Injury Fund. A substantial amount, although smaller than in the case of the Reopened Case Fund, would be required to cover the estimated present value of awards which may be made in the future in cases involving original accidents which occurred prior to the date of valuation. A rate of emergence of such claims could be estimated without much difficulty on which a reserve calculated to a reasonable degree of accuracy could be based. The contingent liability of the Second Injury Fund is none the less real even though the annual payments from such Fund may level off approximately around a certain amount.

In any discussion of the Aggregate Trust Fund the fact should be borne in mind that while the stock and mutual companies are required to pay into the Aggregate Trust Fund the present values of all awards made on or after July 1, 1935 for death benefits and for certain types of permanent total disability and permanent partial disability, there still remain many cases which rest in the discretion of the Industrial Board. The assets and liabilities

of the Aggregate Trust Fund have each been increasing at a rate of more than $1,500,000 a year.

While the writer is happy to have had some part in formulating the plans which were designed to restore the solvency of the Aggregate Trust Fund, I am by no means certain that the steps which have been taken will accomplish fully the desired objective. There is a reasonable hope, however, that the steps already taken will, over a period of years, reestablish this Fund on a sound basis. In any event, it does not appear likely that the deficit in the Aggregate Trust Fund will increase to any marked extent, if at all.

The operations of the Aggregate Trust Fund afford a good practical test of the adequacy of the mortality and remarriage tables prescribed by law for the valuation of death cases and certain types of permanent disability cases. The gains or losses from the following sources are calculated each year by the Actuary of the Aggregate Trust Fund:

(a) Mortality and remarriage experience under annuities to dependents in death cases.

(b) Mortality experience under annuities on permanently disabled lives. The greater proportion of these cases consists of dismemberment cases.

Fairly complete and extensive reports on and analyses of the operations of the Aggregate Trust Fund are made annually. The comments made in Miss Kardonsky's paper again focus attention on the need for periodical reports covering more fully and completely the financial operations and status of the Reopened Case Fund, the Second Injury Fund and the Vocational Rehabilitation Fund.

Section 25-a, Subdivision 4, of the New York Compensation Law, provides in part that the Commissioner of Taxation and Finance, as custodian of the Reopened Case Fund, shall furnish a report annually to the Industrial Commissioner, which report shall show the income, disbursements and the balance of the fund remaining to its credit on June thirtieth. The law also provides that the Superintendent of Insurance shall report biennially to the Industrial Commissioner on the liability of the Reopened Case Fund "upon all cases in which awards of compensation have been made and charged against said fund." In his report on examination of the Reopened Case Fund dated June 30, 1940, Examiner John D. Byrne of the State Insurance Department expressed the opinion that the statement covering the receipts and disbursements of the fund for the fiscal year ended June 29, 1940 as submitted by the Commissioner of Taxation and Finance was not in accordance with the provisions of the law.

Section 15, Subdivision 8, of the Compensation Law provides in part that

the Commissioner of Taxation and Finance upon vouchers signed by the Second Injury Fund but it appears that the law contains no requirement for reports covering the operations or status of this fund.

Section 15, Subdivision 9, of the Compensation Law provides in part that disbursements from the Vocational Rehabilitation Fund shall be paid by the Commissioner of Taxation and Finance upon vouchers signed by the Commissioner of Education or the Deputy Commissioner of Education but it appears that the law contains no requirement for reports covering the operations or status of this fund.

The Vocational Rehabilitation Fund which was in a flourishing financial condition six years ago has now reached the point where its financial solvency may become endangered within the next year or two. This Fund apparently has been considered as a source of easy money for purposes entirely foreign to the objectives of the Fund. A study of the accomplishments effected through money spent for rehabilitating and educating injured employees would probably be most enlightening. The expenditures from this Fund are sufficient in amount to warrant periodical reports from official sources furnishing pertinent information on the operations and accomplishments of the Fund.

REVIEWS OF BOOKS AND PUBLICATIONS

CLARENCE A. KULP, BOOK REVIEW EDITOR

Casualty Insurance Principles. Revised Ed. G. F. Michelbacher. McGraw-Hill Book Co., New York, 1942. Pp. xix, 705.

In the revision of this standard work, well-known to most if not all of our members, the basic plan of cooperative writing by a group of experts and of functional treatment of the casualty companies' operations has been retained. The strength and weakness of such a plan has been pointed out by Kulp in his review of the first edition (*Proceedings* XVI, pp. 362-366). Of course, the material has been brought up to date.

A rather remarkable thing is that though 12 years have intervened since the first edition so relatively little change has been necessary. This does not seem to be due to the fact that in general the same experts contributed and treated the same subjects as in the first edition. It is doubtful whether there would have been important changes had experts from the mutual side of the field contributed. Even if this had been the case the changes would probably have been only in those chapters in which the competitive aspects come to the fore. And here the change would have been due to differences in viewpoint rather than to lapse of time. Kulp hailed the first edition as "A sign that the business of casualty insurance is growing up." We of the Casualty Actuarial Society know that stability is not a sign of ossification. Knowing that the same old problems arising out of competition are still present and realizing the power of the individual's private interest to color his judgment even on technical questions, we hope that the absence of change does not indicate that these problems cannot be solved. Some of the chapters seem to indicate a belief that this is the case. But the closing chapter, *The Future,* in which our genial Fellow does his crystal gazing, is little changed and is hopeful in general outlook.

"Mich" has given over the revision of the chapters on *Rate Making* and *Merit Rating* to Perryman, but he did the job so well the first time that the revision is mainly in the way of bringing the record up to date. He has also introduced as a new chapter (*Statistical Misrepresentations*) the gist of his 1938 paper on *Watch Your Statistics.* It is good to have this here, we suspect, even for actuaries occasionally, and doubly good if the book is read by laymen or used in class instruction.

The number of Appendices has been cut from 22 to 12 and the pages devoted to them from 109 to 61, but their value has not greatly diminished. The big omission is former Appendix VIII, *Plan of Operation and Rules of Conference on Acquisition and Field Supervision Cost for Casualty Insurance.* In no other way, it seems to this reviewer, are the complexity

of the competitive situation and the difficulty of its control so clearly brought out.

The book is good and, read in conjunction with the first edition, serves well to bring out what is passing and what is permanent in the casualty insurance business.

A. H. MOWBRAY.

Financial Compound Interest and Annuity Tables. Charles H. Gushee, editor. Financial Publishing Company, Boston. Pages i-v, 884.

This publication includes the most complete set of compound interest and annuity tables known to the reviewer. The tabular values extend to 10 decimal places. Values of s, $s_{\overline{n}|}$, $1/s_{\overline{n}|}$, v^n, $a_{\overline{n}|}$ and $1/a_{\overline{n}|}$ are given for the following periods and interest rates:

To 360 periods for 33 interest rates from $\frac{1}{12}\%$ to $\frac{5}{6}\%$
To 240 periods for 35 interest rates from $\frac{7}{8}\%$ to 5%
To 120 periods for 14 interest rates from $5\frac{1}{4}\%$ to 10%

On each page, in bold-face type in the outside margin, is given the nominal annual rate if the period rate is compounded annually, semiannually, quarterly or monthly, expressed fractionally and decimally.

A unique feature is introduced in a set of auxiliary tables setting forth for various fractional parts of a unit period and multiples of a unit period, and for each of the 82 interest rates in the main tables, the amount of 1; the payment at the end of each time interval equivalent to 1 at the end of each period; and the payment at the end of each period equivalent to 1 at the end of each time interval.

Nine brief chapters present examples and solutions illustrating the application of the tables to a wide variety of problems. These are stated with commendable clarity and no sparing of detail, so that the comparative layman should be able to find his way with a minimum of difficulty. Two additional chapters treat of extension of the tables and interpolation within them, and a twelfth deals with application to non-financial problems.

The chapter on interpolation is unusually complete for a book of this kind, containing in addition to instructions for simple interpolation, factors for use with 4, 5, 6, 7 and 8 entries respectively. Inverse interpolation with two and with 4 entries is also covered.

The final pages contain charts setting forth the mutual relationships between the most important compound interest functions and formulas for calculation of tabular entries for one greater or one less period than any given entry. These charts are more elaborate than any with which the reviewer is familiar.

The editor and the publisher are to be congratulated on the conception

and execution of a project as complete as this one and on the clarity with which the tables and the applications of the tables are presented.

THOMAS O. CARLSON.

Financial Qualifications by States for Casualty, Surety and Miscellaneous Companies. The Spectator, Philadelphia, 1942. Pp. vii, 164.

This is a manual of the statutory requirements of a financial nature imposed by the several states for the transaction of the casualty and surety lines by the several types of insurer. It gives the lines a company may write, capital and surplus requirements, other requirements peculiar to particular kinds of companies, deposit and bond requirements, and notes the presence or absence of a retaliatory or reciprocal law. The requirements listed seem to be drawn from the several insurance laws. There are, of course, in the compensation acts some special bond and deposit requirements which are in some cases made for the authority to transact compensation business, and these do not seem to have been included.

The book is a special production for the benefit of a rather limited field of company functionaries. For those who have in charge the obtaining of admission of a casualty company to do business in a number of states it is very probably useful.

CLARENCE W. HOBBS.

The Fundamental Principles of Mathematical Statistics. Hugh H. Wolfenden. Published for the Actuarial Society of America, by the Macmillan Company of Canada, Toronto, 1942. Pp. xv, 379.

The appearance of a real book on the subject of mathematical statistics was indeed welcome to all statisticians. The fact that it appeared under the authorship of Mr. Hugh H. Wolfenden, one of the outstanding actuaries of today, was sufficient guarantee that it would be a worthwhile book and not "just another book on statistics" attempting to explain mathematically the fundamental principles of the subject.

Mr. Wolfenden has presented his subject matter in a rather unusual form. In order that one may get a grasp of the whole field without becoming lost in the derivation of some more or less abstruse mathematical formula that he wishes to use, he reserves to a late section in the book the mathematical proofs of such formulae.

The subject matter is presented in the first 148 pages of the text and is divided into 11 chapters. It is not the reviewer's purpose to enumerate all the topics discussed. One might say broadly that every worthwhile topic

discussed in modern statistical journals is treated here and references are given to the papers of various authors in which the reader can find a full discussion.

He first gives the classical approach to the subject of statistics. Then follows the theory of random sampling, covering simple sampling, the theory of Lexis, the theory of small samples, the Bayes-Laplace theorem, noting the contributions to the subject by "Student" and Fisher. In chapter VII we have a comprehensive treatment of the various kinds of curves that may be used to represent frequency distributions; in chapters VIII and IX the various methods used in fitting such curves to observed data and the tests that may be applied to determine the "goodness of fit" of the curve fitted.

In chapter XI is given an outline of a course in graduation, a summary which will certainly be welcomed by students reading for actuarial examinations in that subject. This course covers graduation by mathematical formulae, graphic methods, finite difference methods, linear compounding and the difference-equation method.

In the second part of the text are several appendices. Three of these he calls Sections A, B and C. Section A, covering 26 pages, gives a brief history of the development of the theory of statistics. We find in chronological order how each step forward has been taken from the discovery of the normal curve to the modern theory on tests for "goodness of fit" and "degrees of freedom."

In Section B, covering 81 pages, he gives the mathematical proofs and interpretations of many of the formulae used in the first part of the book. These proofs are presented in clear and logical form. References are given to other writers where some of the proofs given or alternative proofs may be found. These references prove the author's most exhaustive knowledge of the literature on the subject. We are indeed indebted to him for bringing togther in one place such clear-cut demonstrations of the fundamental formulae previously available only by hunting far and wide through countless journals.

In Section C, consisting of 83 pages, the author gives some applications of the theory. Many of his illustrations are taken from the field of life contingencies. His illustrations of the Lexis theory are particularly interesting as are also those of the use of the Poisson exponential and many others.

There is added a bibliography of 20 pages. This gives two lists of publications. The first, in chronological order, with dates, is a list of publications on the subject of statistics of historical value and interest. The second is a shorter list of texts and papers that the author considers of value in a present-day study of the subject. This latter is arranged alphabetically by authors. At the end of the book there is an index of 7 pages.

Seldom if ever has the reviewer found such a wealth of material contained

in a book of this size. Moreover, it is so well arranged and so many cross-references are given that one can find readily the treatment of any topic in which he is interested. We have here the fruits of years of labor on the part of the author and the actuarial profession and all statisticians owe him a deep debt of gratitude.

L. A. H. WARREN.

Principles of Punch-card Machine Operation. How to Operate Punch-card Tabulating and Alphabetic Accounting Machines. Harry Pelle Hartkemeier. Thomas Y. Crowell Company, New York, 1942. Pp. xiv, 269.

This is a manual in loose-leaf form designed to provide the beginning student with the text and illustrative material necessary to a proper understanding of the fundamentals of punch-card machine operation. The author is Professor of Business Statistics in the School of Business and Public Administration of the University of Missouri and this manual is the result of teaching and experimentation in his introductory course in statistics.

The author points out that many people confuse the study of the fundamentals of punch-card machine operation with the mere training of machine operators and that this serious error is partially responsible for the failure of the teaching profession to give sufficient attention to this subject. He states that while the business executive need not himself develop speed or proficiency in the use of the machines, he can hardly be considered competent to take charge of modern statistical and accounting records unless he is familiar with these machines and their uses.

The book opens with a clear, concise explanation accompanied by illustrative diagrams of the mechanical principles underlying the operation of the machines; i.e., how the punched hole controls the transmission of "impulses" to the machine and cause it to perform an operation. This is followed by a comprehensive explanation of various features of numeric and alphabetic machines and how to use them. The text is liberally illustrated by wiring charts and types of reports produced by the machines when wired in accordance with these charts. It also contains numerous problems for the student to use in familiarizing himself with the operation of the machines and with the results obtainable therefrom.

Since the author is dealing particularly with the fundamentals of punch-card accounting he does not discuss auxiliary equipment such as the various types of punches, multipliers and collators.

The treatment is based excusively on the use of International Business Machines equipment and hence covers much the same ground as is covered by the pamphlets of the International Business Machines Corporation dealing with machine methods of accounting. The book is written in lay-

men's language and is of distinct value to the student, to machine operators and to executives who wish to familiarize themselves with machine methods of accounting.

HOWARD G. CRANE.

Reinsurance. Kenneth R. Thompson. Commerce Clearing House, Inc., Chicago, 1942. Pp. vii, 275.

This is a comprehensive work dealing with the practice and the legal aspects of reinsurance in the United States by a member of the New York and Federal Bars. The literature on this subject consists in large part of articles on various phases of reinsurance practice which have appeared in insurance periodicals, of papers in proceedings of the actuarial societies and of chapters on reinsurance in general insurance text-books. The author has amplified his own discussion of reinsurance practice by liberal quotation from these sources. As the author says, there is no set formula in matters of reinsurance practice—parties arrange their own contracts and treat particular problems in different ways, and it is well to have each person to whom reference is made and who discusses a point speak in his own words. The author's discussion of the law of reinsurance is amply annotated with citations and quotations from legal decisions. This book is well written. It brings together in a single volume a vast amount of information from many sources and is a valuable addition to the literature on reinsurance.

HOWARD G. CRANE.

Social Insurance and Allied Services. Report by Sir William Beveridge. The Macmillan Company, New York, 1942. Pp. 299, plus conversion table of English to American money.

The importance of the Beveridge report for United States citizens is that it brings to social insurance a unified approach, with a comprehensive economic perspective, sound social judgment and what I call a social budgeting philosophy.

Sir William Beveridge's fundamental interest has been employment and unemployment, and underlying this whole report is his recognition of the priority of "full employment" over benefits for unemployment.

The goal of "freedom from want" requires a basic floor of protection. In a country whose living costs, he assumes, will have advanced some 25 per cent over those of 1938, he visualizes that floor as requiring 40s. a week for two adults, though he has hesitations about so high a benefit, particularly for the aged. He believes subsistence levels to be both "the most" and "the least" for such a national program.

He recommends a single administration for (1) old-age grants, (2) unemployment grants, (3) disability benefits including workmen's compensation, (4) children's benefits, (5) a series of benefits for the married woman, at marriage, in event of maternity, in event of widowhood, (6) funeral grants, (7) health services, (8) training and rehabilitation activities and (9) the assistances. Thus, we have relief, "insurances" to replace relief, and prevention and cure of the malady of joblessness. The case for children's benefits is well presented, and yet is most debatable.

The importance of the *national* approach is that the long-standing overemphasis on benefits to members of the "lower working classes" has given place to a broader, more social scheme. Retained however from the narrower earlier administrative base are the complicated attempts to recognize considerable individual equity, particularly for old age, marriage grants and workmen's compensation. These remain particularly in cumbersome rules for eligibility and for contributions elaborately divided among the employee, the employer and the Exchequer.

The least convincing portions of the *Report* are the sections and the appendices having to do with industrial insurance and with the Friendly Societies which are represented as virtually the satellites of the basic program, rather than as vigorously administered private enterprises. The author seems less familiar with these insurances, and the American reader finds himself at sea because he is not familiar with British insurance either.

The Actuary's report, Appendix A, quotes a single itemized figure for the social security budget for 1945, 1955 and 1965. Well-prepared as this report is, it would have been better had it brought out even more forcefully the undependability of specific yearly budgets. This could have been shown by the use of as least two such budgets, high and low, for each year. Sir George Epps can hardly expect a 1945 budget for unemployment compensation of £110 million, nor of £57 million for disability. He tells exactly how he gets the figures, but only intimates how little credence he places on them. The actual budgets will be what develop from the tremendous number of interlocking factors that will determine England's post-war conditions.

It is not robbing Sir William of any credit to say that much of the preliminary spadework had been done in the P.E.P. reports, particularly in the 1937 survey of the British social services. The little P.E.P. report of July 14, 1942, which was not required to itemize the details of the program so completely, was in certain respects a clearer goal for the British program than is Sir William's meticulously compiled roster of separate benefits. Probably the best characterization to apply to the whole report is "maturity." When a country has handled social services so long as Great Britain has, when these programs have been so much a part of the life, and even the character of the people, no outsider can ever find the *written* narrative

and data from which to build up a report like this. Only a native son, more or less closely connected with the administration of social insurance, could have realized both how much needed to be said and how to say it with such economy of words.

If Great Britain adopts the policy of recognizing only necessary benefits and of year-by-year budgeting for those benefits on a minimum subsistence basis, she will have caught the soul of social security; she will recognize that there remain large incentives to the recipient of benefits in his return from benefit status to earning status. Under vigorous, healthy work conditions, the costs may well be considerably below the cautious budgets of page 199. Without such full employment they could also be much higher.

Part VI contains a summary of aspirations and basic intents and many highly quotable sentences. The concluding paragraph is:

> Freedom from want cannot be forced on a democracy or given to a democracy. It must be won by them [sic]. Winning it needs courage and faith and a sense of national unity: courage to face facts and difficulties and overcome them; faith in our future and in the ideals of fair-play and freedom for which century after century our forefathers were prepared to die; a sense of national unity overriding the interests of any class or section. The Plan for Social Security in this Report is submitted by one who believes that in this supreme crisis the British people will not be found wanting, of courage and faith and national unity, of material and spiritual power to play their part in achieving both social security and the victory of justice among nations upon which security depends.

W. R. WILLIAMSON.

War and Post-War Security. American Council on Public Affairs, Washington, 1942. Pamphlet. Pp. 89.

The authors of this symposium are men engaged in the administration of social security and men in the labor movement obviously sensing social security as a part of that movement. The report includes stimulating presentations of general positions, but in spite of editing and the careful selection of authors no clear American philosophy and no program for the post-war world emerge either from the individual contributors of the group.

This reviewer feels that social security in the American democracy must be a program for all the citizens—a sincere endeavor to continue the wholehearted cooperation that is to win the war. Social security must carry forward the capacity of working together over to the challenging problems of the peace. The limitations of this discussion of social security are (a) an almost exclusively labor approach; (b) a continued emphasis on the limitations of the depression years; (c) the continued use of scrambled contribu-

tions; (d) reliance on subsidies for the laboring class at the expense of everyone else; (e) the continued emphasis upon individual equity, glorification of the seniority principle.

In spite of the difficulty of applying his principles Sir William Beveridge does give the impression that he has a clear view of the way to go. This report naturally is not such a challenging presentation. Rather it sets forth varied objectives; it is a factual outline of our limited accomplishments to date, of our will to learn and of our intention to continue our search for the better way in social insurance.

W. R. WILLIAMSON.

Workmen's Compensation in North Carolina. J. Maynard Keech. Duke University Press, Durham, 1942. Pp. ix, 193.

This book is a brief and very well-written descriptive sketch of the North Carolina Compensation Act. The introductory chapter giving the historical background of workmen's compensation is brief and concise. There follows a very interesting chapter on the legislative history of the North Carolina Act, and one could wish that there were available similar studies for the other states. The background out of which the acts arose has no little bearing on the course of their development. The third chapter, on *Administration,* and the fourth chapter, on *Accident Prevention,* give a very fair idea of the North Carolina Industrial Commission, its duties and practices. Chapter V, on *Workmen's Compensation Insurance,* is a bit too sketchy and descriptive to be closely accurate. On page 125, quotation is made of a very loose and probably inaccurate statement of Mr. J. C. Root that the cost of insurance is 38 per cent less to self-insurers than to employers using outside carriers. It is certainly inaccurate as respects the larger mutual carriers, and seems pretty strong as respects stock carriers. The retrospective rating plan, which seems not to have entered the picture within the time covered by the study, makes the differential materially less. On page 137 appears a statement: "Of the premiums collected in the state approximately 17 per cent to 18 per cent receive some credibility for state experience." This must be based on a misapprehension of what was said by the author's informant who certainly never intended to convey this idea. The percentage figures are about right to express the proportion of manual classifications which get some credibility for state experience. But the premiums developed from those classifications constitute about 90 per cent of the total. The description of the process of modifying formula pure premiums on page 139 is hardly accurate. In calculating pure premiums on state indications, factors are introduced to give effect to loss development, i.e., changes in total losses from reporting to reporting, and to law amendments. This could be done simultaneously with

the formula process but for convenience is done earlier. But factors representing wage and industrial changes have not been used in a long time. Formula pure premiums are occasionally modified though not often. Once in a while the formula produces a premium greater or less than either the national or the state indications. In such cases, an adjustment has to be made for the formula result ought to lie between the two. Once in a while too, the state indications are so very far from the national, or exhibit such a marked trend during the latter years, as to make underwriters reluctant to accept the results of the formula. But occasions for those modifications are very rare. The formula results are in the main very acceptable and in a goodly number of revisions are used throughout.

One other point may be mentioned. On page 88, he quotes Heinrich's statement that 98 per cent of industrial accidents are preventable. In the 1941 edition, Heinrich states (page 19) that 2 per cent of accidents are unpreventable, 50 per cent are practically preventable, 98 per cent are "of a preventable type."

These points are mentioned in no captious mood. Indeed the book as a whole exhibits an admirably judicious spirit and contains some really interesting information. Separate studies of the several compensation acts, their history and their administration are well worth while, for each act is a thing in itself, developing on more or less individualistic lines. Too much so perhaps for the good of the business or for the good of the system. The standardization of wages and hours under federal auspices must, if continued, at some time include a standardization of the average weekly wage formulas and the maximum and minimum limits of the compensation acts, and some standardizations of administrative practice. There is also real need for a standard extra-territorial provision. Standardization will of course rob the acts of some picturesque features but these differentiations are often a prime nuisance.

CLARENCE W. HOBBS.

The Pathology of Trauma. Alan Richards Moritz, M.D. Lea and Febinger, Philadelphia, 1942. Pp. 386.

The author calls attention to the absence of any text book entirely devoted to the pathological processes following injury and in this book an attempt is made to describe typical lesions in tissue following trauma.

The author states that a mechanical injury is one that occurs from force and that the "resulting disruption of normal anatomical relationships is usually characterized as a wound". The wound may be cutaneous or subcutaneous and subcutaneous wounds "may take the form of a bruise, a fracture, or a laceration".

He describes force and states that perfect muscular co-ordination can control the force of a blow; he describes the jumper landing on semi-flexed legs and thighs, the boxer rolling his body with the blow and the baseball player moving his gloved hand backward with the caught ball.

Next to complete muscle coordination in the prevention of serious injury, is complete muscular relaxation and he states that an infant or an intoxicated person may sustain a violent trauma with little tissue damage because of the relaxation of muscles.

The author describes the effects of trauma in terms of (1) length of duration of the force, (2) the area of tissue involved and (3) condition of the part involved.

The immediate local reaction to injury is dilatation of capillaries and stasis of blood in them. There follows increased permeability of the vessels and exudation of plasma and cells. It is claimed that a substance is liberated in damaged cells which sets up the reactive changes of inflammation but the exact nature of the substance is still not definitely decided.

The author then describes bleeding from wounds incurred after death and describes differential points as compared with bleeding from ante mortem wounds.

The second phase of inflammation is the proliferation of fibroblasts and the formation of new blood vessels which repair the defect created by cellular destruction. This is the stage of repair. The healing of wounds is described: (1) without granulation tissue when the edges of the wound are in close contact, (2) with granulation tissue when the wound is more marked and the edges separated. The processes of repair are described. Many factors may delay wound healing and a deficiency in the ascorbic acid content of the blood is particularly important.

Skin, mucous membrance, glandular organs and those of hemopoietic and lymphatic tissues regenerate readily. Smooth muscle, bone and cartilage occupy an intermediate position in regeneration. Striate muscle and nervous tissue have the least regenerative capacity.

The author describes wounds in detail. From a careful study, he shows that it is often possible to picture the type of instrument used, the position of the person, whether it was the result of a blow or fall and whether the blow was inflicted ante or post mortem. The medico-legal value is evident. He also describes methods of ascertaining whether or not the wounds are self inflicted. Under *Bullet Wounds,* the author discusses "the physical principles of firearms and some of the physical and chemical characteristics of projectiles and the substances responsible for their discharge". There are two types of firearms, rifled and smoothbore. Rifled weapons include rifles, revolvers, pistols and machine guns. They are the cause of the great majority by far of firearm wounds. Smoothbore weapons include shotguns and

"toy" revolvers and pistols. Description is given of the effects of black powder and smokeless powder, of the metallic substances, of the liberated gases and their effects on tissue, and of the distance from the target. The recognition of metallic substances and gases is described in detail.

Wounds of entrance are likely to be smaller than the bullet that produced them while wounds of exit are likely to be larger. Exceptions to the rule exist, however, and are described. Internal injuries produced by bullets depend on (1) velocity, weight and shape of the bullet, (2) the character of its motion in flight, (3) the density and cohesion of the tissues through which it passes.

The necessity for the removal of the bullet, the study of the position of the victim and his clothes are emphasized from a medico-legal standpoint.

Shotgun injuries are also described.

A particularly interesting topic is the discussion of the internal and external body injuries which may follow the detonation of high explosives. Among the causative factors are listed:

1. Flying missiles
2. Liberation of burning gas and corrosive chemicals
3. Rapid atmospheric movement
4. Elevation followed by reduction of atmospheric pressure
5. Asphyxiation by irrespirable gases

"Of particular interest", says the author in describing the detonation and resultant injuries, "are the casualties in which the severity of the internal injuries is entirely out of proportion to that of the external wounds". There are two physical disturbances which are likely to cause internal injuries: (1) impact against the body of markedly increased atmospheric pressure, followed by (2) sudden drop in atmospheric pressure. The principal clinical forms of such injuries are: (1) primary and secondary shock, (2) cerebral concussion, (3) pulmonary damage, (4) deafness and (5) neuro-psychiatric disorders.

The most common site of atmospheric damage due to blast impact is the lungs. The pathological lesions vary from scattered foci to intra-alveolar bleeding to gross lacerations of lungs and pleura. Bleeding from the mucous membranes other than the lungs is common.

In the chapter on *Trauma and Infection,* the author opens by stating that "a mechanical injury may augment the severity of an already existing infection, may create a portal of entry for new infection, or may establish a locus of diminished resistance to subsequent infection". He describes "silent lesions" as a locus of infection without subjective evidence of its existence; such as apical abscesses of teeth, salpingitis, liver abscess, etc. He notes the effect of trauma as it "affects the disturbance of the intact inflammatory

barrier by causing an activation of already existing pathology and of dormant infections". Healthy tissue removed aseptically often show a great number of bacteria which the author feels were trapped in the tissue and circulated in the blood. He quotes Reith and Squier in stating that in samples of blood from 293 apparently healthy persons positive cultures were obtained in 64. The organisms recovered included streptococci, diptheroid, bacilli, micrococci, colon bacilli and obligative anaerobic rods.

There appears to be ample evidence that pathogenic bacteria commonly circulate in the blood stream without producing disease and that these bacteria are rapidly destroyed by the liver, spleen, lymph nodes, lungs, bone marrow and kidneys. If an injury is sustained during a period of benign bacteraenemia, then an infectious process is more likely to develop. The author names the habitat of organisms invading wounds:

From skin and mucous membranes

streptococci	influenza
staflococci	B. coli
proteus vulgaris	Cl. welchie
pneumococci	actinomycosis
meningococci	

From soil

Cl. tetani	Cl. oedematis
Cl. welchie	Cl. sporogenes
Cl. septique	

Infections carried from person to person or animal to person are likely to be most severe. A human bite wound is likely to be dangerously infected as are wounds from infected animals and insects. Since many infections are spore-bearing and since spore-forming organisms may live for a long time outside the body, anthrax infections from improperly sterilized shaving brushes have been reported.

All accidental wounds are contaminated. Some contaminated wounds go on to infection. There is a variable lag period between contamination and infection. Other wounds never develop gross infection and heal by primary union, and in these wounds organisms can often be found.

There is no relationship between a single trauma and a malignant tumor. The author states that "there is no evidence that a crushing or a cutting injury of a tumor predisposes either to local acceleration in growth or to metastasis. Intermittent pressure or massage may predispose to metastasis". He further states that "there is overwhelming experimental and clinical evidence to indicate that a tumor may be caused by chronic irritation" and he cites carcinoma of bladder in aniline workers, of breast following chronic mastitis, of the skin in tar workers, etc.

The author divides injuries to the cardiovascular system in 3 classes:

1. Neurogenic: anal dilatation, etc.
2. Hematogenic: interference with quality or quantity of blood
3. Organogenic: direct damage by mechanical force

Under neurogenic the author describes shock and syncope, neurocirculatory asthenia and then post-traumatic vasoconstriction and vasodilatation without damage to blood vessels; under hematogenic, secondary shock, hemorrhage, thrombosis and embolism; under organogenic, cardiac injury resulting from excessive physical exertion, chronic overwork, work and arterial disease, varicose veins and direct injuries to the heart, veins and arteries.

Mechanical injuries of the viscera are described in detail. Damage to bone, diseases of bone, joint injuries and diseases and bursitis are discussed.

The book makes interesting and easy reading. A great number of references are quoted. It should be of particular interest to the surgeon and to all interested in the effects of trauma. It is of particular interest to the medico-legal expert.

*Henry H. Ritter, M.D.

* Guest reviewer.

Publications Received

Calculus Made Easy. Silvanus P. Thompson. Second edition, enlarged. Macmillan Company, New York City, 1942.

Construction of War Damage Corporation Insurance. Abe J. Goldin. U. S. Review Publishing Company, Philadelphia, 1942.

Life-Saving Measures for Merchant Seamen in Time of War. Studies and Reports, Series P (Seamen), No. 4. International Labour Office, Montreal, 1942.

Odd Numbers, or Arithmetic Revisited. Herbert McKay. Cambridge University Press, Macmillan Company, New York City, 1943.

Why We Have Automobile Accidents. Harry DeSilva. Wiley and Sons, New York City, 1942.

OBITUARY

ROBERT J. HILLAS
1858 - 1940

Robert J. Hillas, a charter member of the Casualty Actuarial Society, died at his home in Morristown, New Jersey, on May 17, 1940.

Mr. Hillas was born in Green Bay, Wisconsin, on October 25, 1858. After completing grammar school, he began his business career in a law office. He entered the casualty insurance field in 1876 when he became an office boy with the Fidelity and Casualty Company of New York, in whose service he remained for sixty-four years. In 1880 he was appointed assistant secretary, succeeded to the secretaryship in 1888 and added the office of treasurer to his other duties in 1892. In 1903 Mr. Hillas became vice-president of the company, and in 1910 was named to the presidency, a position which he held until his retirement in 1929. He remained a director until his death.

By constant study, Mr. Hillas achieved a thorough knowledge of all aspects of the casualty insurance business. Foresight and sound executive ability marked his long and successful career.

OBITUARY

WILLIAM ANTHONY GRANVILLE
1863 - 1943

William Anthony Granville, Ph.D., L.L.D., vice-president and director of the Washington National Insurance Company, Evanston, Illinois, passed away at his home in Chicago on February 4, 1943, at the age of 79. Death resulted from a heart attack. He is survived by his widow, two daughters, and a granddaughter.

Dr. Granville performed his duties right up to the day of his passing. Services were held at Holy Trinity Lutheran Church in Chicago Saturday, February 6.

Dr. Granville was nationally known as an outstanding mathematician, and several of his textbooks on calculus and other mathematical subjects are used in schools and colleges throughout the country. He was a professor of mathematics at Yale from 1895 to 1910, following which he served as president of Gettysburg College until 1923, and for several years he also occupied the post of president of the American Federation of Lutheran

Brotherhoods. On March 1, 1923 he became affiliated with the Washington National Insurance Company.

Those of his colleagues who had the privilege of knowing Dr. Granville personally, knew him to be a man of boundless energy, keenly interested in his work to the end and extremely successful in any task he set out to accomplish. They knew him as a person of broad knowledge not only of his own work but of all important subjects of the day. Dr. Granville was a very able and convincing public speaker.

With all his high attainments, his colleagues especially like to remember him as a friend who was never too busy to help the cause of others.

OBITUARY

WILLIAM ANDERSON HUTCHESON

1868 - 1942

William Anderson Hutcheson, a Fellow of the Casualty Actuarial Society for twenty-one years, died at his home in Gladstone, New Jersey, November 19,1942, after a brief illness.

Born in Greenock, Scotland, July 13, 1868, Mr. Hutcheson was educated at the Greenock Academy and at Merchiston Castle School. In 1887 he went directly from school to the Scottish Widows' Fund and Life Assurance Society. There followed a remarkable business career of more than fifty years in the life insurance field. He attained Fellowship in the Institute of Actuaries in 1894 and in the Faculty of Actuaries in 1895.

In 1899 he came to the United States as Associate Actuary of The Mutual Life Insurance Company of New York, under Emory McClintock, whom he succeeded as Actuary in 1911. He became Second Vice President in 1917 and Vice President in 1931. He retired in 1940.

Mr. Hutcheson's actuarial career was unique in that, attaining Fellowship by examination in The Actuarial Society of America in 1902, he was the first to attain thus the three degrees, F.I.A., F.F.A., and F.A.S., to which were subsequently added Fellowship in the American Institute of Actuaries and in this Society.

In addition to the many responsibilities imposed by his executive duties in a large company, he took an active interest in the numerous pension funds of New York City, serving for several years as chairman of the Committee of Actuaries of the Committee on Pensions. During the first World War he headed a committee of the Y.M.C.A. which arranged a broad life, accident, and health coverage for Y.M.C.A. Secretaries overseas. He was President of The Actuarial Society of America from 1920 to 1922.

A man of absolute integrity, gracious and friendly bearing, vast and varied insurance experience, Mr. Hutcheson will be deeply missed in actuarial circles.

OBITUARY

ARNETTE ROYCE LAWRENCE

1888 - 1942

Arnette Royce Lawrence, a Fellow of this Society for twenty years, died at his home in Montclair, New Jersey, on December 1, 1942.

Mr. Lawrence was born October 21, 1888 in Yonkers, New York and graduated from Stevens Institute of Technology in 1911 with the degree of Mechanical Engineer. Almost his entire business career was devoted to workmen's compensation rate administration, a field in which he distinguished himself. After brief periods of employment with the New York Edison Company and the Fidelity and Casualty Company, he was employed in 1914 by the Compensation Insurance Rating Board shortly after its organization, becoming its Assistant Chief Inspector. In 1917 he became Assistant General Manager of the Pennsylvania Compensation Rating and Inspection Bureau, and the following year organized and became Manager of the Workmen's Compensation Inspection Rating Bureau of Virginia.

In 1921 he was appointed Manager of the Compensation Rating and Inspection Bureau of New Jersey, as well as Special Deputy Commissioner of Banking and Insurance, and continued until his death in this dual capacity. As such he shaped the course of compensation rate regulation in New Jersey and built up the Bureau into one of the outstanding rating organizations. He was a pioneer in the compensation business and a leader in its development from the beginning. His broad experience in this field and his thorough knowledge of the subject made him especially valuable in helping solve many of the problems which have arisen in this branch of insurance.

CASUALTY ACTUARIAL SOCIETY

NOVEMBER 20, 1942

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*Terms expire at the annual meeting in November 1943.
†Terms expire at the annual meeting in November of the year given.

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ABSTRACT FROM THE MINUTES OF THE MEETING
NOVEMBER 20, 1942

The twenty-ninth annual (fifty-ninth regular) meeting of the Casualty Actuarial Society was held at the Hotel Biltmore, New York, on Friday, November 20, 1942.

President Blanchard called the meeting to order at 10:15 A.M. The roll was called, showing the following forty-two Fellows and twelve Associates present:

FELLOWS

BARBER	FARLEY	MAYCRINK
BARTER	FONDILLER	MILLS
BERKELEY	GODDARD	MULLANEY
BLANCHARD	GRAHAM, C. M.	OBERHAUS
BROWN, F. S.	GREENE, W. W.	PETERS
CAHILL	HOBBS	PRUITT
CARLETON	JACKSON, H. H.	SATTERTHWAITE
CARLSON	JOHNSON, R. A., JR.	SINNOTT
CLEARY	KARDONSKY	SKELDING
CONSTABLE	KORMES	SKILLINGS
CORCORAN	LAWRENCE	SMICK
CRANE	LYONS	VALERIUS
ELLIOTT	MARSHALL	VAN TUYL
EPPINK	MASTERSON	WILLIAMSON

ASSOCIATES

BAILEY, A. L.	GUERTIN	ROSS
BLACK	HIPP	SPENCER
DOWLING	MALMUTH	STOKE
FITZ	MARSH	SULLIVAN

By invitation a number of officials of casualty insurance companies and other organizations were present.

The minutes of the meeting held May 15, 1942, were approved as printed in the *Proceedings*.

The Secretary-Treasurer (Richard Fondiller) read the report of the Council and upon motion it was adopted by the Society.

The following Associates had passed the necessary examinations and had been admitted as Fellows:

ROBERT D. BART F. E. SATTERTHWAITE

The following candidate had passed the necessary examinations and had been enrolled as an Associate:

SAMUEL M. ROSS

Diplomas were then presented by the President to Robert D. Bart and F. E. Satterthwaite, who had been admitted as Fellows under the 1942 examinations.

The President announced the deaths, since the last meeting of the Society, of three fellows: Walter G. Cowles, William A. Hutcheson, and John M. Laird.

The report of the Secretary-Treasurer was read and accepted. The Council had decided that no May meeting of the Society would be held for the duration of the war. This decision was arrived at in the interest of economy and in line with the policy of other scientific societies. The report on finances follows:

CASUALTY ACTUARIAL SOCIETY

ANNUAL REPORT OF FINANCES

Cash Receipts and Disbursements from October 1, 1941 to September 30, 1942.

INCOME

On Deposit on October 1, 1941 in the Marine Midland Trust Company		$ 401.88
Members' Dues	$2,570.00	
Sales of *Proceedings*	1,420.36	
Examination Fees	524.00	
Luncheons and Dinners	296.00	
Interest and Miscellaneous	38.82	
Michelbacher Fund	16.50	4,865.68
Total		$5,267.56

DISBURSEMENTS

Printing and Stationery	$2,559.16
Postage, Express, etc.	120.00
Stenographic Services	420.00
Library Fund	7.02
Luncheons and Dinners	335.42
Examination Expense	455.80
Insurance	45.47
Miscellaneous	91.62
Total	$4,034.49
On deposit on September 30, 1942 in Marine Midland Trust Company	1,233.07
Total	$5,267.56

Income	$4,865.68	
Disbursements	4,034.49	
Excess of Income over Disbursements		$ 831.19
1941 Bank Balance		401.88
1942 Bank Balance		$1,233.07

ASSETS

Cash in Bank	$1,233.07
*Bonds	4,750.00
Total Assets	$5,983.07
*Includes Michelbacher Fund	$1,380.34

The Auditing Committee (W. P. Comstock, Chairman) reported that the books of the Secretary-Treasurer had been audited and his accounts verified.

The Examination Committee (R. V. Sinnott, Chairman) submitted a report of which the following is a summary:

1942 EXAMINATIONS — SUCCESSFUL CANDIDATES

The following is a list of those who passed the examinations held by the Society on April 8 and 9, 1942:

ASSOCIATESHIP EXAMINATIONS

PART I:
J. F. Amsden
Loring M. Barker
Abraham S. Baronowitz
V. E. Bonander
John M. Bragg
Herman Chernoff
David J. Cohen
Florence Conrad
M. S. Hughey
Robert F. Rich
Matthew Rodermund
Ruth Salzmann
Jonas J. Schreiber
Harold J. Silver
Purser K. Sturgeon
Dean A. Thomas
Elia Vergano
John W. Wieder, Jr.
C. J. Woodley

PART II:
John M. Bragg
Herman Chernoff
Charles W. Crouse
Daniel Finkel
Marvin J. Forray
Irene Harnish
Ernest Holzinger
S. M. Ross
Harold Schloss
Paul A. Turner
John W. Wieder, Jr.

PART III:
Loring M. Barker
Abraham S. Baronowitz
John M. Bragg
Milton F. Chauner
M. S. Hughey
John J. Marcus
Harold Schloss
Dean A. Thomas
Paul A. Turner
C. J. Woodley

PART IV:
Abraham S. Baronowitz
John E. Bilsborrow
John M. Bragg
Milton F. Chauner
Herman Chernoff
Charles W. Crouse
K. Arne Eide
Ernest Holzinger
G. R. Livingston
Robert W. Lufkin
John J. Marcus
George C. Munterich
S. M. Ross
Ruth Salzmann
Max J. Schwartz
D. Uhthoff
C. J. Woodley

PART V:
Joseph Ian
G. R. Livingston
F. E. Satterthwaite

FELLOWSHIP EXAMINATIONS

PART I:	EDWARD S. ALLEN	K. ARNE EIDE
	ROBERT D. BART	F. E. SATTERTHWAITE
PART II:	ROBERT D. BART	
PART III:	ROBERT D. BART	

The Council's re-election of Clarence W. Hobbs as Editor and of Thomas O. Carlson as Librarian, was announced.

The annual elections were then held and the following officers and members of the Council were elected:

President	RALPH H. BLANCHARD
Vice-President	ALBERT Z. SKELDING
Vice-President	CHARLES J. HAUGH
Secretary-Treasurer	RICHARD FONDILLER
Editor	CLARENCE W. HOBBS
Librarian	THOMAS O. CARLSON

Members of Council (terms expire in 1945):

ROBERT V. SINNOTT ARTHUR N. MATTHEWS WILLIAM F. ROEBER

The proposed amendment to Article 4, Dues, First Paragraph, after discussion, was tabled.

The new papers appearing in this Number were presented.

Recess was taken for lunch at the Hotel until 2:15 P.M.

Informal discussion of the following topic was participated in by members and representatives of insurance organizations:

"What modifications in the casualty statistical program should be made to offset the shortage of machines and man-power and to recognize the extreme changes in conditions in:

Compensation Insurance
Automobile Insurance
Public Liability Insurance".

Upon motion, the meeting adjourned at 4:30 P.M.

INVITED GUESTS PRESENT AT THE MEETING

Joseph F. Collins, Chief of Rating Bureau, New York Insurance Department, New York.

Alvin F. Comstock, Statistician, Century Indemnity Company, Hartford, Conn.

Charles W. Crouse, Actuary, Manufacturers' Casualty Insurance Company, Philadelphia, Pa.

Ernest A. Erickson, Underwriter and Statistician, Utilities Mutual Insurance Company, New York.

R. L. Inglis, Resident Vice-President, Associated Indemnity Corporation, New York.

Peter H. May, Vice-President-Comptroller, Maryland Casualty Company, Baltimore, Md.

J. P. Kerrigan, Manager, Hartford Steam Boiler Inspection and Insurance Company, New York.

Frederick C. Kessler, Secretary-Treasurer, Consolidated Taxpayers Mutual Insurance Company, Brooklyn, N. Y.

Walter Konther, Treasurer, Utilities Mutual Insurance Company, New York.

F. B. Schroeter, Underwriter, Zurich General Accident and Liability Insurance Company, New York.

B. H. Zimels, Statistician, Consolidated Taxpayers Mutual Insurance Company, Brooklyn, N. Y.

1942 EXAMINATIONS OF THE SOCIETY

APRIL 8 AND 9 1942

EXAMINATION FOR ENROLLMENT AS ASSOCIATE

PART I

1. (a) Solve the equation:

$$\frac{3x^4+x^2-2x-3}{3x^4-x^2+2x+3}=\frac{5x^4+2x^2-7x+3}{5x^4-2x^2+7x-3}$$

(b) There are two sets of numbers each consisting of three terms in A.P., and the sum of each set is 30. The common difference of the first set is greater by 1 than the common difference of the second set, and the product of the terms of the first set is 50 less than the product of the terms of the second set. Find the numbers.

2. (a) How many terms of the following series must be taken such that their sum be greater than 20.

$$7, \frac{14}{3}, \frac{28}{9}, \frac{56}{27}$$

Given $\log 2 = .3010$
$\log 3 = .4771$
$\log 7 = .8451$

(b) Find the coefficient of x^{100} in the expansion of

$$\frac{3-5x}{1-2x+x^2}$$

(c) If x is a real quantity, what numerical values may the following expression have?

$$\frac{x^2+2x-11}{2(x-3)}$$

3. (a) Sum the following series to infinity:

$$\frac{5}{7}-\frac{6}{7^2}+\frac{5}{7^3}-\frac{6}{7^4}+\frac{5}{7^5}-\frac{6}{7^6}\cdots\cdots$$

(b) In terms of i, the effective interest rate, express the present value of a series of payments due as follows:

one dollar at the end of the first year, two dollars at the end of the second year, three dollars at the end of the third year, etc. in perpetuity.

(c) n men can be grouped into C combinations of 3 each. $(n-1)$ men can be grouped into $(C-15)$ combinations of 3 each.

What is the value of n?

4. (a) A company of soldiers consists of 3 officers, 4 sergeants, and 60 privates.

(i) In how many ways can a detachment be made consisting of an officer, 2 sergeants, and 20 privates?

(ii) In how many of these ways will the Captain and the Top Sergeant appear?

(Do not take time to evaluate the factorials.)

(b) (i) How many signals can be made by hoisting 4 flags of different colors one above the other, when any number of them may be hoisted at one time?

(ii) How many with 5 flags?

5. (a) A perpetuity of \$1,000 per year is changed to an annuity due for 20 years. If money is worth 4%, find the annual rent to the nearest dollar.

Given $\frac{1}{s_{\overline{19}|}} = .0361$

(b) A father leaves \$50,000 to his two sons, aged 8 and 10 years. From the accumulation of this amount at interest 4% effective, each son is to receive an annuity for 10 years beginning when he is 21 (first payment at age 22). If both receive the same annuity, what is the annual payment (to the nearest dollar)?

$a_{\overline{11}|} = 9.11090$

$v^{11} = .64958$

$v^{13} = .60057$

6. A man owes a debt of \$6,000 which he may pay off either in 10 annual installments of \$777.03 each or in 9 annual installments of \$844.14 each. If he chooses to pay it in 9 installments, what part of the fifth payment will be for interest and what part will apply against principal?

Given $v^5 = .783526$.

7. (a) A certain type of pavement costs $\$X_1$ per square yard and must be replaced every t_1 years. Superior pavement costs $\$X_2$ per square yard and must be replaced every t_2 years. The assumed rate of interest is i.

$$X_2 > X_1;\ t_2 > t_1$$

If the capitalized costs (K_1 and K_2) of both pavements are found to be the same, express X_1 in terms of X_2 and annuities of t_1 and t_2 years.

(b) If the going rate of interest decreased, what would happen to the ratio of capitalized costs $\frac{K_1}{K_2}$? Explain.

8. (a) Construct a table showing the amortization of a debt of $2,500 to be paid off in 5 annual payments with interest at 5%.

$$\frac{1}{a_{\overline{5}|}} = .230975.$$

(b) Find the purchase price for a $1,000 bond paying 3% interest annually, to be redeemed at par in twelve years, to yield 2% effective.

$$v^{12} \text{ at } 2\% = .78849.$$

PART II

1. (a) Find the critical values and points of inflection of the following curve, and draw a rough sketch of the curve:

$$y = \tfrac{1}{2}x^4 - 3x^2 + 2.$$

(b) Evaluate $\lim_{x \to n} \dfrac{\log \frac{x}{n}}{n - x}$

2. (a) A piece of wire 10 feet long is to be cut into two pieces, one of which is to be made into a square and the other into a circle. In order that the sum of the areas of the two figures should be a minimum, what will be the length of the piece of wire for the square?

(b) Calculate the radius of curvature at the point (x, y) on the following curve:

$$y^{\frac{1}{4}} + x^{\frac{1}{4}} = a^{\frac{1}{4}}$$

3. (a) Evaluate $\int \dfrac{dx}{1 + \sqrt[3]{x} + 1}$

(b) Find the first two terms in the expansion of $\tan x$ by Maclaurins' Theorem.

4. (a) Prove that the area of any segment of a parabola cut off by a chord perpendicular to the axis of the parabola is $\frac{2}{3}$ of the area of the circumscribing rectangle.

(b) Determine by Simpson's rule (parabolic rule) the approximate area between the curve $y = \frac{12}{1+x}$, the x axis, and the ordinates 0 and 6.

5. (a) The function indicated below is a rational integral function of less than the fifth degree. Of what degree is the function? What is the value of u_{16}?

u_1	u_2	u_3	u_4	u_5	u_6	u_7
30	30	28	25	22	20	20

(b) By means of Lagrange's formula, show from given values u_5, u_3, u_{-3} and u_{-5}, that $u_1 = -.3u_5 + u_3 + .5u_{-3} - .2u_{-5}$.

6. (a) From the following data, obtain values for $f(51)$ and $f(52)$.

$x =$	50	51	52	53	54	55	56
$f(x) =$	3.684			3.756	3.780	3.803	3.826

(b) Represent the function $x^4 - 12x^3 + 42x^2 - 30x + 9$ and its successive differences in factorial notation.

7. Given the following table of values of $u_{x:y}$, estimate the value of $u_{23:17}$

x	$y = 15$	$y = 20$	$y = 25$
20	5.947	4.418	3.547
25	6.046	4.530	
30	6.144		

8. Sum the following series to n terms by the method of finite differences.

$$1 + 3 \cdot \underline{|2} + 7 \cdot \underline{|3} + 13 \cdot \underline{|4} + 21 \cdot \underline{|5} + \cdots$$

PART III

1. Name and define five commonly recognized types of averages of a group of measurements, and outline briefly the characteristics of each.

2. (a) Define the "standard deviation." What is its significance?

   (b) The calculated linear correlation coefficient between values of X and corresponding values of Y is r. The standard deviation of the X series is a, that of the Y series is b.

   Derive an expression, in terms of a, b, and r, for the standard deviation of Z, where Z equals the sum of corresponding values of X and Y.

3. (a) By the method of least squares, fit a straight line and a second degree parabola to the following data:

Year	1935	1936	1937	1938	1939
Gold Stock (Billion $)	10	11	13	16	20

   (b) By a comparison of the standard error of estimate for the straight line with that of the parabola, determine which formula gives the closest fit to the raw data in Problem 3.(a).

4. Index numbers for "installment payments" and "bond interest paid" are as follows over a six month period:

	Jan.	*Feb.*	*Mar.*	*Apr.*	*May*	*June*
Installment payments	90	94	98	103	106	110
Bond Interest Paid	102	103	104	105	106	107

   A correlation coefficient has been calculated from these figures. What points would you consider in appraising the significance of such calculation, and such calculations in general?

5. Prepare journal entries for the following transactions:

Jan. 2 Purchase of goods worth $500 from Doe, on account—terms 2%, 10 days.
" 3 Payment of wages of $100, in cash.
" 4 Cash sales of $50.
" 5 Sale to Roe on account, $200—terms 1%, 10 days.
" 6 Doe's bill is paid.
" 8 Advance $10 to petty cash.
" 10 Roe returns goods worth $40.
" 11 Jones bought goods worth $300, paying $150, the balance being on account—terms 1%, 10 days.
" 14 Donation of $25 cash to charity.

Jan. 15 Advance of $60 to Smith, a salesman.
" 16 Telephone bill of $75 is paid.
" 20 Roe pays his bill.
" 21 Jones pays his bill.

6. Assuming that there are no other special journals, set up an analytic cash journal adequate to handle such of the transactions in 5 as involve the cash account, and make the appropriate entries.

7. A company organized January 1, 1939 suffers a loss by fire of its merchandise just before taking inventory December 31, 1941. The inventory made immediately after the fire is $25,000. The books disclose the following:

	1939	1940	1941
Inventory at the beginning of year	80,000	50,000	40,000
Purchases	220,000	255,000	305,000
Purchase Returns	20,000	5,000	5,000
Sales	466,000	525,000	482,000
Sales Returns	6,000	5,000	2,000

Determine the cost to the Company of the merchandise destroyed by fire.

8. It has been said that the normal use of machinery can be regarded as a process of converting the asset "Machinery" into the asset "Cash." Answer the following problem in such a way as to demonstrate the truth of this saying.

A company buys a machine for $10,000, paying for it out of current cash resources. The company assumes that each year the machine will depreciate 10% of its original cost, and at the end of ten years the machine is actually discarded as worthless. Give examples of journal entries involving each account in which the machine figures during its progress from purchase to discarding. (Disregard expenses of installation and removal). Explain why the $10,000 is not a net loss to the company, assuming normal accounting procedure and normally profitable business operations.

PART IV

1. (a) A table is divided into six squares numbered 1 to 6. A player places a coin on a certain square. Three dice are thrown. If the number thus backed appears, once, twice, or thrice, the player receives back his own coin together with one, two or three other respectively of the same value. In any other event, he loses his stake. Show that the advantage in the long run lies with the "banker".

(b) Six cards are chosen at random from a pack of 52. Find the chance that:

(1) Three will be black and three red.

(2) At least one will be black.

Do not take time to evaluate factorials.

2. (a) Two players of equal skill are playing a set of games. They stop playing when A needs 3 points to win and B needs 2 points. If the stake is \$80, what share ought each to take?

(b) A man has 10 coins, all identical except that one coin has 2 heads. He takes one coin at random and tosses it 5 times. It falls heads each time. What is the chance that it is the coin with 2 heads?

3. (a) Five coins are tossed, and n of them fall heads. These n coins are tossed again, and three fall heads. What is the probability that $n = 4$?

(b) P makes a bet of \$8 to \$120 that three races will be won by three horses A, B, C, against which the betting is 3 to 2, 4 to 1, and 2 to 1, respectively. The first race having been won by A, and it being known that the second race was won either by B, or by a horse D against which the betting was 2 to 1, find the value of P's expectation.

4. (a) There are two purses, the first of which contains 2 pennies and 9 dimes, the second 8 pennies and 3 dimes. The purses are placed in a bag and one is drawn at random. Three coins are removed, and are found to be one penny and two dimes. The other purse is then given to a child who is told to draw one coin and keep it. What is his expectation?

(b) A certain stake is to be won by the first person who throws an ace with an octahedral die. If there are four persons, what is the chance of the fourth?

5. (a) Given m_x, the central death rate, is .01. What is the value of p_x?

(b) What is the difference between the complete expectation of life and the probable lifetime of a person of age x?

(c) What is the Law of Uniform Seniority?

6. (a) Interpret in words the following expressions:

1.) ${}_nE_y - {}_nE_{xy}$ 3.) ${}_{10}P_{25:20|}$

2.) ${}_{m|n}a_x$

(b) Represent the expressions in Problem 6. (a) in terms of commutation symbols.

7. (a) The New York Workmen's Compensation Law in fatal cases provides benefits to widows equal to 30% of the deceased's wages, and to each child 10% of the wages until he reaches the age of 18 providing the widow is alive and does not remarry. Upon the death or remarriage of the widow, each child receives benefits equal to 15% of the deceased's wages until the age of 18. Total benefits payable to all dependents at any one time are limited to 66⅔% of the deceased's wages. The table for the present value of benefits to the fourth youngest child in Bulletin 190 of the New York Department of Labor is calculated from the following formula:

$$15\ {}_1\bar{a}_{\overline{18-y_4|}} - 5\ {}_1\bar{a}_{x':\overline{18-y_4|}} - 3\tfrac{1}{3}\ {}_4\bar{a}_{x':\overline{18-y_4|}}$$

If the age of the widow is x, and that of the fourth youngest child y_4, express in words the meaning of this formula.

(b) A person age 30 wants to provide for himself at age 60 an annuity certain for \$1,000 for 10 years, followed by a contingent annuity of \$1,000 for 10 years. If the policy is to be paid for in 30 annual payments, determine the net annual premium.

$N_{30} = 580{,}000$	$D_{60} = 7{,}000$
$N_{60} = 80{,}000$	$D_{70} = 4{,}000$
$N_{70} = 26{,}000$	$D_{80} = 1{,}000$
$N_{80} = 4{,}000$	$a_{\overline{10}|} = 8.3$

8 Prove the equivalence of the prospective and retrospective methods of valuation of life insurance policies.

PART V

1. (a) Differentiate between the application of minimum premiums and minimum charges as set forth in the Compensation Manual. Justify application of minimum charges.

(b) What are the important provisions of the Standard Exception Rule as set forth in the Manufacturers and Contractors Liability Manual?

2. (a) Give the standard exclusions which are contained in the garage liability policy.

(b) What operations or transactions and what parties in interest call for the issuance of the following types of bonds:

(1) Fidelity
(2) Contract
(3) Fiduciary

3. (a) Give details of application of co-insurance principle as it is applied in the writing of Mercantile Open Stock Policies.

(b) If an accident insurance policyholder changes to a more hazardous occupation without notifying the Company, what would be the basis of settlement in the event of a claim?

4. (a) An insured has a boiler policy including personal injury coverage which contains a limit of $25,000. Through the insured's negligence an accident occurs to the insured's boiler and the claim adjuster obtains the following facts:

(1) fire damage to the insured's property $10,000

(2) damage to neighbor's property $5,000

(3) damage to insured's property not by fire but including boiler $10,000

(4) personal injury losses involving members of the public as follows: injured A, $10,000; injured B, $6,000; and injured C, $4,000

(5) insured has public liability policy containing standard limits.

Develop total cost to carrier of the boiler policy and give the order in which amounts will be payable.

(b) Briefly explain need for Grantors' Protective Liability Insurance.

5. (a) Why are losses segregated into serious, non-serious and medical divisions in developing Workmen's Compensation Manual Rates?

(b) What are the criteria for 100% state credibility which are used in the classification pure premium calculation of the Workmen's Compensation Ratemaking procedure?

6. Outline the fundamental steps followed in determining loss constants and their off-setting reductions. In this calculation do you feel exceptional treatment should be given in the handling of experience developed on short term policies?

7. (a) For what lines of insurance are costs other than Indemnity and Medical payments included as a part of the pure premium? For what lines do these costs represent a substantial portion of the pure premium?

(b) Outline the general functions of National Pure Premiums as they are used in casualty rate making.

8. Give four different bases of exposure used in casualty insurance rate making. Select a line or classification of business where each exposure base is used and explain why that particular exposure base was used in preference to others.

EXAMINATION FOR ADMISSION AS FELLOW

PART I

1. Two important considerations in the analysis of fixed interest bearing securities are:

   (a) Earning capacity
   (b) Property Value

   Discuss the relative merits of these considerations in the determination of the investment desirability of such securities.

2. (a) Do you believe the investment of the assets of an insurance company in the capital stock of another insurance company is desirable?

   (b) Discuss the insurance company as an intermediary investment institution.

3. The following exhibit shows the distribution of assets of the average casualty company as of December 31, 1933.

   Estimate the distribution as of December 31, 1941 and give the reasons underlying the changes.

	% of Total
Real Estate	5.3%
Mortgages	5.2
United States Government Bonds	12.2
State, Municipal and Other Government Bonds	10.8
Railroad Bonds	13.6
Public Utility Bonds	11.3
Miscellaneous Bonds	4.7
Total Bonds	52.6

	% of Total
Insurance Stocks	2.5%
Bank Stocks	2.2
Railroad Stocks	2.8
Public Utility Stocks	3.9
Miscellaneous Stocks	6.2
Total Stocks	17.6
Cash	6.3
Premiums in transmission	9.6
Other Assets	3.4
Total Admitted Assets	100.0

4. (a) Define the boundaries of the Maritime Jurisdiction of the United States.

   (b) Many states impose time limits within which a claim for compensation may be presented. Would such limitations ordinarily be valid when applied to claims in existence at the time the limitations are made effective?

5. What are the essential elements of a valid written insurance contract?

6. (a) Define:
   (1) Express warranties
   (2) Descriptive warranties
   (3) Representations

   (b) A judgment of $35,000 is rendered against a policyholder having a liability policy providing $50,000/50,000 limits. The primary carrier, a New York Company, is found to be insolvent and able to pay fifty cents on the dollar. What is the liability of a New York reinsurance carrier which provided the primary carrier with excess insurance coverage over $25,000/25,000 limits and why?

7. Distinguish between chances and uncertainties and state the relationship of the chances to the uncertainties when the chances of an occurrence are zero and even, respectively.

8. Discuss the following, considering only the economic theory of risk and insurance:
   (1). Contingency loading in Workmen's Compensation Rate Making procedure
   (2) Safe Driver Reward Plan
   (3) Merit Rating Plans.

PART II

1. (a) What changes in compiling Workmen's Compensation and Employer's Liability Unearned Premium Reserves are required under a recent ruling of the Massachusetts Insurance Commissioner?

   (b) Outline the shortcomings of the Dutch Remarriage Table in figuring Lump Sum Settlements and Loss Reserves for Workmen's Compensation Cases where remarriage is a factor.

2. Discuss the relative merits of using a daily, monthly or yearly pro rata method in setting up Unearned Premium Reserves for Financial Statement purposes, and give an example under which each method should be used in order to produce a reasonably accurate Financial Statement.

3. (a) Comment on the desirability of carrying special "Depression" Loss Reserves on Bonding Lines. Briefly outline a method for accumulating such Reserves.

   (b) Discuss the desirability of setting up a Special Reserve for Reopened Claims. Outline a method that might be used for determining the amount that should be set aside in such a Reserve.

4. (a) The result of the application of Retrospective Rating to Compensation Risks show that the Retrospective Premium is substantially less than the Standard Premium. If a company has a large amount of its Compensation Premium Volume written on a Retrospective Basis, would special consideration have to be given to this factor in the calculation of its Unearned Premium Reserve? Explain.

   (b) What changes were made in Schedule "P", Part I, during 1940, and what was their effect on the Aggregate Loss Reserve for the average company?

5. (a) Describe at least six internal reports which should be compiled by the statistician of a Multiple Line Casualty Company and briefly describe the purpose of each report.

(b) Discuss the relative value of Loss Ratios and Accident Frequency Ratios in judging the inherent loss producing elements of small and large Compensation Risks, respectively.

6. Assume that you have been requested to make suggestions that would reduce the work in both company offices and ratemaking bureaus in connection with the compilation of Automobile Rate Statistics as an economy for the duration of the present emergency. What changes would you suggest to achieve this end with the minimum of loss in the value of the results?

7. (a) What Bureaus or Organizations compile statistics on the following subjects:
   (1) Factory Employment and Payrolls By Industries
   (2) Commercial Failures By Districts
   (3) Index Of Industrial Production

   (b) Define "Convention Values" and justify their use.

8. What is the theory underlying the Minimum Reserve Requirement of Schedule "P"? What is the principal advantage of Schedule "P" and what is its principal disadvantage from the standpoint of an insurance commissioner interested in the solvency of insurance carriers?

PART III

1. (a) Explain and justify the action taken by the National Council on Compensation Insurance as respects the use of experience developed on National Defense Contracts which are written under the Comprehensive or War Department Rating Plan.

   (b) Describe the method currently in use for determining discounts in connection with deductible coverages for the General Liability Lines.

2. (a) Outline the principal features of the so-called "Comprehensive or War Department Rating Plan."

   (b) Discuss the fundamental principles underlying the Chemical and Dyestuff Rating Plan.

3. Outline the principal features of the Pennsylvania Retrospective Rating Plan, describing how it differs from the Workmen's Compensation Retrospective Rating Plan now in use in many of the states under the jurisdiction of the National Council.

4. (a) Much has been said to the effect that the Casualty Companies should be relieved of the possibility of loss under their Compensation Policies which would result from enemy attack. Would you advocate that the Compensation Laws be amended to exclude such injuries because the loss costs for such injuries are not contained in the Compensation Rates now in effect? If you do not favor this as a solution to the problem, what would you advocate?

   (b) It has been recently advocated in the Congress of The United States that the Social Security Administration pay indemnity to dependents of captured island base contractor's employees. Do you believe that this is a function of the Social Security Administration? Explain.

5. (a) Discuss the advantages and disadvantages of experience rating as applied to Unemployment Compensation.

   (b) What is the Ohio Formula for the computation of Unemployment Benefits?

6. Outline the principal features of the new "New York Motor Vehicle Safety-Responsibility Act" and discuss its advantages and disadvantages from the standpoint of the general public.

7. (a) Discuss the possible future effects on non-defense business rating practices of the adoption of the Comprehensive or War Department Rating Plan.

   (b) Taking into consideration the following:

      (1) Gasoline and tire rationing;
      (2) Blackouts;
      (3) Discontinuance of new car and part manufacturing;
      (4) Decrease in the driving population between the ages of 21 and 35.

   What changes would you recommend in the Private Passenger Liability and Property Damage Rates?

8. Give the principal features of what you would consider to be an ideal Excess Reinsurance Agreement for a Multiple Line Casualty Company having $10,000,000 in Assets, $2,500,000 in Surplus to Policyholders and $8,000,000 of Premium Writings divided in the following manner:

(1) Automobile Casualty Insurance	40%
(2) Workmen's Compensation and Employer's Liability (incl. O. D.)	40%
(3) Fidelity and Surety	5%
(4) Accident and Health	5%
(5) Boiler and Machinery	5%
(6) Public Liability and Property Damage	5%

1943 EXAMINATIONS OF THE SOCIETY

APRIL 7 AND 8, 1943

EXAMINATION FOR ENROLLMENT AS ASSOCIATE

PART I

1. (a) Using the binomial theorem, evaluate $\sqrt[3]{528}$ to four places of decimals.

   (b) Prove the equality of the following binomial expansion:

$$1-\frac{1}{2}\cdot\frac{1}{3}-\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{1}{3^2}-\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{3}{6}\cdot\frac{1}{3^3}-\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{3}{6}\cdot\frac{5}{8}\cdot\frac{1}{3^4}-\cdots\cdots$$

$$=1-\frac{1}{2}\cdot\frac{1}{2}+\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{1}{2^2}-\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{5}{6}\cdot\frac{1}{2^3}+\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{5}{6}\cdot\frac{7}{8}\cdot\frac{1}{2^4}-\cdots\cdots$$

2. (a) If the arithmetic mean between a and b is twice as great as the geometric mean, show that

$$a:b=(2+\sqrt{3}):(2-\sqrt{3})$$

   (b) Prove by induction:

$$1+3+5+\cdots\cdots+(2n-1)=n^2$$

3. (a) Solve:

$$\frac{x-a}{b}+\frac{x-b}{a}=\frac{b}{x-a}+\frac{a}{x-b}$$

   (b) For what values of m will the following equation have roots which are equal in magnitude and opposite in sign?

$$\frac{x^2-bx}{ax-c}=\frac{m-1}{m+1}$$

4. (a) How many numbers of six digits each can be formed if neither 0 nor 1 is to occupy either end position and no repetitions of digits are allowed?

   (b) Find the number of combinations that can be made by taking four letters from the word "combinations".

(c) Find the number of permutations that can be made by taking four letters from the word "permutations".

5. (a) A paving assessment of \$845.19 is to be paid, principal and interest at 5%, in 10 equal annual installments. What is the indebtedness to the city at the time the sixth payment is due but not paid?

(b) What is the purchase price of a \$10,000 bond bearing half-yearly coupons of \$300 each and repayable in 25 years at a price of 110, when the investor is to realize 5% convertible semi-annually?

6. (a) A man buys a team of horses for \$500 and a wagon and harness for \$150. The team will have to be replaced in 10 years and the wagon and harness in 20 years. Feed and blacksmith bills cost \$200 a year, taxes and insurance \$20, and wages of driver \$600. What total sum will be required to purchase the equipment, to provide for its periodic replacements, and to assure payment of the annual expenses, assuming the costs remain constant and that money is worth 5 per cent?

(b) Derive a relationship between successive values of $s_{\overline{n}|}$ by which a table of $s_{\overline{n}|}$ could be constructed.

7. (a)* A house priced \$5,000 is sold on a basis of equal monthly payments to extend through 7 years, the first payment to be made at the end of the first month. What is the monthly payment if money is worth 5% effective?

(b)* What is the monthly payment for problem 7(a) if money is worth 5% compounded semi-annually?

8. (a)* A man buying a piece of property is offered the three following methods of payment:

(i) An annuity of \$1,900 for four years.

(ii) A four payment annuity of \$2,200 deferred for four years.

(iii) Four payments of \$2,100 at the end of two, four, six and eight years.

What is the present value of each method, money being worth 5 per cent?

(b)* In each case of problem 8(a), what additional payment at the end of the second year would extinguish the unpaid portion of the debt?

*NOTE: Questions marked with asterisk (*) — the answer may be stated in the form of an expression in which all numerical values are entered but not multiplied.

Given Values:

$j_{(12)}$ (at 5%) $= 0.0489$
v^2 (at 5%) $= 0.9070$
v^4 (at 5%) $= 0.8227$
v^6 (at 5%) $= 0.7462$
v^8 (at 5%) $= 0.6768$
v^{50} (at 2½%) $= 0.2909$
$(1.050625)^{\frac{1}{2}} = 1.0041$
$(1.050625)^{-7} = 0.7077$
$\frac{1}{a_{\overline{7}|}}$ (at 5%) $= 0.1728$
$\frac{1}{a_{\overline{10}|}}$ (at 5%) $= 0.1295$
$\frac{1}{a_{\overline{20}|}}$ (at 5%) $= 0.0802$

PART II

1. (a) Differentiate:

$$y = x^{e^x} \text{ and express in terms of } x.$$

(b) Given $\log_e 2 = 0.693$, find $\int_0^1 \frac{x^3\,dx}{1+x^4}$

(c) By the trapezoidal rule, evaluate approximately $\int_0^3 \frac{x^2\,dx}{1+x}$

2. (a) Evaluate $\int x^n \log x\,dx$.

(b) The altitude of a circular cone is 100 inches, and decreases at the rate of 10 inches per second; and the radius of the base is 50 inches, and increases at the rate of 5 inches per second. At what rate is the volume of the cone changing?

$$(V = \tfrac{1}{3}\pi r^2 h)$$

3. (a) Expand e^x by Maclaurin's series and determine for what values of x the series is convergent.

(b) A third degree curve has a maximum at $-1, 4$ and a minimum at $1, 0$. Find the equation of the curve, and the point of inflection.

4. (a) Find the volume of the oblate spheroid generated by revolving the area bounded by the ellipse $b^2x^2 + a^2y^2 = a^2b^2$ about the y axis.

(b) Using Taylor's series, expand $\tan x$ in powers of $\left(x - \frac{\pi}{4}\right)$ to three terms.

5. (a) Find the sixth term of the series 1, 37, 61, 77, ····· and prove by extending the table of differences.

(b) From the following data estimate the missing values by a method of finite differences:

Time—	1 P.M.	2 P.M.	3 P.M.	4 P.M.	5 P.M.	6 P.M.	7 P.M.	8 P.M.
Temperature—	53	54	—	49	47	—	40	35

6. Sum, to twenty terms, $5 + 10 + 17 + 28 + 47 + 82 + \cdots\cdots$ (The answer may be left as a numerical expression without evaluating powers.)

7. Evaluate to four decimal places the root of the equation

$$x^3 + x - 3 = 0$$

which lies between 1.20 and 1.24, using Stirling's formula and two approximations; given that

$$1.20^3 = 1.728 \qquad 1.22^3 = 1.816$$
$$1.21^3 = 1.772 \qquad 1.23^3 = 1.861$$
$$1.24^3 = 1.907$$

8. (a) Use Newton's divided difference formula to find the form of the function $y = U_x$; given that

$$U_0 = 8 \qquad U_4 = 68$$
$$U_1 = 11 \qquad U_5 = 123$$

(b) Using Lagrange's formula, find U_4; given that

$$U_1 = 7$$
$$U_3 = 5$$
$$U_6 = 1$$

PART III

1. (a) Criticize the presentation of the data in the following table:

Amount of Rent Paid for Each Apartment in a Certain City

Rent (in dollars)	Number of Apartments (00 omitted)
20. to 30.	2
30. to 40.	14
40. to 50.	21
50. to 60.	8
60. to 70.	5
Total	50

(b) From the data of the table shown in problem 1(a) above (disregarding your criticism) compute the mode and the median, and give a formula for the geometric mean in terms of logarithms.

(c) From the data of the table shown in problem 1(a) above (disregarding your criticism) compute the standard deviation and the mean deviation.

2. Find the standard error of estimate in terms of the standard deviation and the Bravais-Pearson Coefficient of Correlation; given

$$\Sigma \rho^2 = m^2 \Sigma x^2 - 2m \Sigma xy + \Sigma y^2$$

$$\text{where } m = \frac{\Sigma xy}{\Sigma x^2}$$

3. Assuming that a second degree parabola is to be fitted to a mass of observed data, use the method of moments to derive equations which may be used to evaluate the constants in the equation of the curve.

4. (a) Fit a second degree parabola to the following data:

x	1	2	3	4	5
y	0	2	5	8	11

(b) Two types of electric bulbs were observed as to the length of life, and the following data were secured:

Type 1	Type 2
$N_1 = 100$	$N_2 = 100$
$M_1 = 1224$ hours	$M_2 = 1036$ hours
$\sigma_1 = 36$ hours	$\sigma_2 = 40$ hours

Determine the probable error of the difference between the means. Is the difference in the two means sufficient to warrant the conclusion that type 1 is superior to type 2? State your reasons.

5. (a) The trial balance of the ledger accounts of the X Company is given below as it appeared on December 31, 1941, prior to adjustment and closing. Adjustment items are given below. Show all adjusting and closing journal entries in connection with Operating Wages and Salaries, Depreciation, Bad Debts, and Rent.

Trial Balance—12-31-41

Cash	250,000	
Accounts Receivable	130,000	
Inventory—Finished Goods (1-1-41)	50,000	
Inventory—Goods in Process (1-1-41)	25,000	
Inventory—Raw Materials (1-1-41)	40,000	
Machinery and Equipment	1,040,000	
Reserve for Depreciation		200,000
Accounts Payable		58,000
Bonded Indebtedness		400,000
Surplus		500,000
Capital Stock		250,000
Sales		1,605,000
Sales Returns	100,000	
Discount on Sales	5,000	
Purchases	506,000	
Freight in on Purchases	4,000	
Discount on Purchases		3,000
Purchase Returns		7,000

Operating Wages and Salaries.....	237,000	
Rent	11,000	
Heat, Light and Power.....	15,000	
General Manufacturing Expense.....	30,000	
Advertising	50,000	
Selling Expense.....	250,000	
Administrative Expense.....	200,000	
Interest on Bonded Debt.....	20,000	
Dividends	60,000	
	3,023,000	3,023,000
Adjustment data:		
Inventories 12-31-41:		
Finished Goods.....		60,000
Goods in Process.....		40,000
Raw Materials.....		50,000
Accrued Operating Wages and Salaries (Unpaid on 12-31-41).....		3,000
Depreciation during 1941.....		40,000
Estimated Loss on Bad Debts.....		30,000
Estimated Federal Income Tax Payable		50,000
Rent Prepaid.....		1,000

(b) Set up sample ledger accounts (T accounts), and post the adjusting and closing entries, for the following accounts of the X Company:

Operating Wages and Salaries
Reserve for Depreciation
Estimated Loss on Bad Debts
Rent

6. & 7. Prepare a Profit and Loss Statement for the X Company for the year ending December 31, 1941.

8. (a) Prepare a Final Balance Sheet for the X Company as of December 31, 1941.

(b) A "book audit" is concerned only with the accuracy of the books of account, and requires no investigation of the basis for determining the amounts which are entered in those accounts which require the exercise of judgment. If you, as a stockholder, were told that the Balance Sheet and the Profit and Loss Statement of the X Company had been subjected to a book audit, what additional information would you want to have in order to judge the Company's actual financial condition?

PART IV

1. (a) Six persons throw for a stake of $100, which is to be won by the player who first throws heads with a penny. If they are to throw in succession, find the expectation of each player.

(b) In five throws with a single die, what is the chance of throwing the following:

(i) three aces exactly
(ii) three aces at least

2. (a) Three cards are drawn from an ordinary pack of 52 cards. What are the respective probabilities of the following results if the cards are drawn one at a time and replaced:
   (i) A spade each time
   (ii) At least one spade
   (iii) Three cards of the same suit
   (iv) Three cards of different suits
   (v) Two cards of same color and the other different.

   (b) If a coin is tossed 1000 times, how many times can you expect to get exactly five heads in succession?

3. From a bag containing n balls, all either white or black, all numbers of each being equally likely, a ball is drawn which turns out to be white; this is replaced, and another ball is drawn, which also turns out to be white. If this ball is replaced, prove that the chance of the next draw giving a black ball is $\frac{1}{2}(n-1)(2n+1)^{-1}$; given that

$$\sum_1^n r = \tfrac{1}{2} \cdot n(n+1)$$

$$\sum_1^n r^2 = \tfrac{1}{6} \cdot n(n+1)(2n+1)$$

$$\sum_1^n r^3 = \tfrac{1}{4} \cdot n^2(n+1)^2$$

4. (a) A speaks the truth 2 out of 3 times, B 4 times out of 5; they agree in the assertion that from a bag containing 6 balls of different colors a red ball has been drawn. Find the probability that the statement is true.

   (b) In a lottery all the tickets are blanks but one; each person draws a ticket and retains it; show that each person has an equal chance of drawing the prize.

5. (a) The present value per \$100 of annual wages for future payments to each of the three youngest children of a surviving widow, given in Table IV of Special Bulletin No. 207 of the N. Y. Department of Labor, is computed from the following formula:

$$15\ {}_1\bar{a}_{\overline{18-y}|} - 5\ {}_1\bar{a}_{x'} : \overline{{}_{18-y}|}$$

   If x is the widow's age and y the child's age, explain briefly the meaning of each term of the formula, and tell why this table can be used for only the three youngest children.

(b) Show that

$$1 + e_x = q_x + p_x (1 + q_{x+1}) + {}_2p_x (1 + q_{x+2}) + \cdots\cdots$$

6. (a) (i) Show algebraically that $a_{x:\overline{n}|} < a_n$

(ii) Express the annual premium for a 20 year term insurance entirely in terms of temporary life annuity symbols and the rate of interest.

(iii) Interpret in words the symbol ${}_nE_{xy}|_z$ and express its value in terms of pure endowments.

(b) Compute the present value of a temporary life annuity due on the life of an individual now aged 30 providing for a sequence of payments of \$500 a year for fifteen years followed by a sequence of ten payments of \$1000 each year; given that

$$N_{30} = 596800 \qquad N_{55} = 125000$$
$$N_{45} = 253700 \qquad D_{30} = 30400$$

7. Find the equal age w by means of the Gompertz-Makeham law of mortality, and prove the Law of Uniform Seniority.

8. (a) A special 20 year endowment policy issued at age x provides, in event of the death of the insured during the twenty year period, for a benefit of \$1.00 and the return, without interest, of all net premiums paid. If the insured survives the twenty year period the policy matures, the insured receiving only the face amount of \$1.00. Show that the net annual premium may be written in the form

$$\frac{M_x - M_{x+20} + D_{x+20}}{N_x - N_{x+20} - R_x + R_{x+20} + 20\,M_{x+20}}$$

(b) Show that Fackler's Accumulation Formula can be written in the form

$${}_{t+1}V_x = ({}_tV_x + P_x - c_{x+t})\,u_{x+t}$$

given $c_x = \dfrac{C_x}{D_x}$

PART V

1. (a) Outline the provisions for determining the excess limits premium for Occupational Disease (Paragraph 1 b) Coverage which are contained in the Compensation Manual of the National Council on Compensation Insurance.

(b) Explain the purpose and principles underlying the Workmen's Compensation and Employers' Liability minimum premiums and the rules for determining them when several classifications and states are involved.

2. (a) Under what conditions would a steam boiler insurance policy cover a fire loss resulting from a boiler explosion or from a machinery breakdown?

   (b) Under what conditions may additional insureds be included under a fidelity schedule, fidelity blanket or forgery bond, and what are the special premium charges, if any?

3. (a) A certain company is issuing both can and non-can accident and health policies. In accordance with current practice and existing laws, what steps can it take to protect itself against moral hazard resulting from an economic depression?

   (b) What additional safeguards do you think it advisable to adopt?

4. (a) Describe the New York Workmen's Compensation Premium Discount Plan.

   (b) What coverage is contemplated under a Manufacturers' and Contractors' policy for "after completion of operations"?

   (c) What is the difference in the application of the N.P.D. rules between (1) mercantile, mining, construction or erection operations, and (2) other operations?

5. (a) What was the result anticipated when the contingency loading program was introduced in Workmen's Compensation rate-making?
   What have been the actual results?

   (b) Discuss briefly some of the problems of "account current" methods of establishing rate levels.

6. Discuss the feasibility of making Workmen's Compensation rates on a governing classification basis.

7. (a) What is meant by saying that a rating classification is homogeneous? Compare the following lines of insurance by degree of homogeneity of classification:

      (a) Automobile, (b) Workmen's Compensation, (c) Manufacturers' and Contractors' Liability, (d) Burglary, (e) Fidelity.

   (b) Under present circumstances, what would you consider the causes of an observed indicated increase in average settlement costs for automobile liability insurance?

8. What is the purpose of a rate level factor? To what extent is it a trend factor? Does its function vary in any way in the different lines of insurance? Under what circumstances could it cause an unwarranted increase in the rate for a given classification; (a) in Automobile Insurance, (b) in Workmen's Compensation Insurance?

EXAMINATION FOR ENROLLMENT AS FELLOW

PART I

1. (a) The Treasury Department in recent financings has indicated a desire to place as many of its bonds as possible in the hands of individuals, institutions, banks other than those accepting demand deposits, and insurance companies.
   (1) What effect will this have upon inflationary trends?
   (2) Upon insurance company portfolios?

   (b) Discuss the effect upon low coupon long term bonds if interest rates should reach higher levels. How would this affect the balance sheets of casualty insurance companies?

2. Under what circumstances do you think bonds held by an insurance company should be amortized. Suggest rules for determining the propriety of amortizing individual bonds considering market value, financial ratings, and interest rates.

3. (a) Outline the analysis on income, disbursements, assets, and liabilities which should be made to determine the optimal amount of the assets of a carrier writing only Workmen's Compensation Insurance which should be held in cash or short maturities if it is the policy of the carrier to seek the best available yield on its assets.

   (b) Do you consider it advantageous for a company writing only Workmen's Compensation Insurance to hold all of its assets in cash or short maturities?

4. (a) Give the legal meanings of "waiver", "estoppel", and "election".
   (b) An insured has a Paymaster Robbery Policy. A holdup occurred at which time the money was in possession of a guard accompanying the custodian. What rights for recovery has the insured under the Policy? Give reasons.

5. Outline the provisions of the New York Code for capital and surplus of a casualty insurance company. Do you believe these requirements to be proper for the different lines in all cases?

6. (a) What types of governmental bodies levy taxes on casualty insurance premiums? What are the purposes and justification of these taxes?

   (b) Do you think that the two year carry-over provision of the excess profit section of the 1942 Federal Revenue Act would operate, under normal conditions, with equal fairness to a multiple line Casualty Insurance Company and a large manufacturing concern?

7. Discuss the desirability of establishing uniform supervised rates for (a) automobile liability insurance, (b) fidelity bonds, and (c) accident and health insurance.

8. Discuss the feasibility of private insurance carriers assuming the hazards of war damage in the United States.

PART II

1. (a) Discuss the desirability of carrying a special loss reserve for undisclosed occupational disease claims. What method or methods would you suggest for accumulating such a reserve?

   (b) Would you favor accumulating a special reserve from the catastrophe loading collected in Workmen's Compensation rates? Give reasons.

2. (a) Under Schedule "P", unallocated loss expenses are required to be allocated to policy year on the basis of specified percentages. What effect has this requirement on the "case basis" policy year incurred loss and loss expense ratios of (1) a rapidly growing company and (2) a rapidly retrogressing company?

   (b) Outline a method that might be used to test the adequacy of the aggregate loss and loss expense reserves established by a carrier in its financial statement of two years ago on its liability and compensation business assuming that only the information reported in the carrier's last three annual statements is available.

3. (a) What changes would you consider necessary and practical to be incorporated into the unit statistical plan for alleviating the manpower problem of the rate-making bureaus and of the insurance carriers?

   (b) What changes have been made in the statistical plans covering automobile casualty insurance for the purpose of alleviating the manpower problem in the rate-making bureaus and the offices of the carriers?

4. Discuss the advantages and disadvantages of converting the Convention Form of Annual Statement Blank for miscellaneous companies to an earned-incurred basis.

5. What changes would you make in the assets and liabilities of a casualty company as shown on the Convention Form of Annual Statement Blank for miscellaneous companies in order that the capital and surplus accounts would represent the true worth of the carrier as a going concern?

6. (a) What statistical and accounting records of subrogation should be maintained in automobile liability insurance?

   (b) Give two methods by which reinsurance recoverable on paid losses may be shown in the annual statement of miscellaneous companies.

7. (a) Discuss the validity and utility of experience statistics currently being collected on casualty insurance lines.

   (b) What reserves of a casualty insurance company should be given special attention because of current conditions? Why?

8. Design a punch card suitable for continuous accumulation of the "case basis" reserves on reported claims for a large multiple line casualty company, and describe the treatment that would be accorded the various types of transactions entering into the net reserves.

PART III

1. Outline in detail how you would estimate the underwriting loss to a carrier covering a large volume of Workmen's Compensation Insurance on the Retrospective Rating Plan if we assume that the ratio of losses to standard premiums on this business will be 80%. Assume that the items of expense of the carrier including claims expense and miscellaneous taxes approximate the allowances for company expenses in the Retrospective Rating Plan.

2. (a) What modifications are made in the formula for Retrospective Rating for ex-medical risks?

   (b) Outline the principal features of the automobile experience rating plan used in New York.

3. What new principles were introduced into the writing and rating of casualty insurance by the War Department Comprehensive Rating Plan? Discuss the desirability of applying these principles to all large casualty risks.

4. (a) Under what circumstances and to what extent do you consider that compulsory methods are justified in distributing any social cost? Discuss the application of your opinions to

   (a) automobile injury
   (b) unemployment
   (c) disability

   (b) In some industries the principle underlying the price structure is to charge what the traffic will bear. Discuss the application of this principle to casualty insurance rates.

5. Present in broad outline a social program to meet the minimum needs of the average man and discuss the probable effect of such a program on the economic structure of this country.

6. (a) What are the three guiding principles underlying the recommendations in the Beveridge Report?

   (b) What would be the effect on private casualty insurance if such a plan as the Beveridge Plan should be adopted?

7. A stock casualty insurance carrier writing Workmen's Compensation and Automobile Liability Insurance is offered for sale. Outline the calculations which the seller should make in determining a fair value for the net worth. What other factors should be considered in the actual purchase price?

8. Certain causes have contributed to the creation of the present problem of the rating of interstate risks. What are the causes and how have they created the problem? What solutions have you to offer keeping in mind the demands of the buyers of insurance for such interstate risks?

INDEX TO VOLUME XXIX

CASUALTY ACTUARIAL SOCIETY

ORGANIZED 1914

1943 YEAR BOOK

(Addendum to Volume XXIX of the *Proceedings*)

Corrected to February 1, 1943 **No. 22**

FOREWORD

The Casualty Actuarial Society was organized November 7, 1914 as the Casualty Actuarial and Statistical Society of America, with 97 charter members of the grade of Fellow. The present title was adopted on May 14, 1921. The object of the Society is the promotion of actuarial and statistical science as applied to the problems of casualty and social insurance by means of personal intercourse, the presentation and discussion of appropriate papers, the collection of a library and such other means as may be found desirable.

Prior to 1914 little technical study was given to the actuarial and underwriting problems of most of the branches of casualty insurance. The organization of the Society was brought about through the suggestion of Dr. I. M. Rubinow, who became the first president. The problems surrounding workmen's compensation were at that time the most urgent, and consequently many of the members played a leading part in the development of the scientific basis upon which workmen's compensation insurance now rests.

The members of the Society have also presented original papers to the *Proceedings* upon the scientific formulation of standards for the computation of both rates and reserves in accident and health insurance, liability, burglary, and the various automobile coverages. The presidential addresses constitute a valuable record of the current problems facing the casualty insurance business. Other papers in the *Proceedings* deal with acquisition costs, pension funds, legal decisions, investments, claims, reinsurance, accounting, statutory requirements, loss reserves, statistics, and the examination of casualty companies. The Committee on Compensation and Liability Loss Reserves submitted a report which has been printed in *Proceedings* No. 35 and No. 36. The Committee on Remarriage Table submitted a report including tables, printed in *Proceedings* No. 40. The Special Committee on Bases of Exposure submitted a report which is printed in *Proceedings* No. 43. The "Recommendations for Study" appear in *Proceedings* No. 54.

The lower grade of membership in the Society is that of Associate. Examinations have been held every year since organization; they are held on the first Wednesday and following Thursday in April, in various cities in the United States and Canada. The membership of the Society consists of actuaries, statisticians, and executives who are connected with the principal casualty companies and organizations in the United States and Canada. The Society has a total membership of 291, consisting of 165 Fellows and 126 Associates.

The annual meeting of the Society is held in New York in November and the semi-annual meeting is held in May. The Society has decided to discontinue its May meeting for the duration. The twenty-fifth anniversary of the Society was appropriately celebrated in New York on November 16 and 17, 1939.

The Society issues a publication entitled the *Proceedings* which contains original papers presented at the meetings. The *Proceedings* also contain discussions of papers, reviews of books and current notes. This Year Book is published annually and "Recommendations for Study" is a pamphlet which outlines the course of study to be followed in connection with the examinations for admission. These two booklets may be obtained free upon application to the Secretary-Treasurer, 90 John Street, New York.

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NOVEMBER 20, 1942

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THOMAS O. CARLSON

SPECIAL COMMITTEES

COMMITTEE ON MORTALITY FOR DISABLED LIVES

PAUL DORWEILER (CHAIRMAN)
RALPH M. MARSHALL
MARK KORMES
HARMON T. BARBER
S. BRUCE BLACK
CHARLES M. GRAHAM
RICHARD M. PENNOCK

COMMITTEE ON UNEMPLOYMENT INSURANCE

FRANCIS S. PERRYMAN (CHAIRMAN)
HARMON T. BARBER
JAMES M. CAHILL
HAROLD J. GINSBURGH
GRADY H. HIPP
CLARENCE A. KULP
WILLIAM R. WILLIAMSON

MEMBERSHIP OF THE SOCIETY, NOVEMBER 20, 1942

FELLOWS

Those marked (†) were Charter Members at date of organization, November 7, 1914.

Those marked (*) have been admitted as Fellows upon examination by the Society.

Date Admitted	
*Nov. 21, 1930	AINLEY, JOHN W., The Travelers Insurance Company, 700 Main Street, Hartford, Conn.
*Nov. 13, 1931	AULT, GILBERT E., Actuary, Church Pension Fund and Church Life Insurance Corporation, 20 Exchange Place, New York.
May 23, 1924	BAILEY, WILLIAM B., Economist, The Travelers Insurance Company, 700 Main Street, Hartford, Conn.
*Nov. 20, 1924	BARBER, HARMON T., Assistant Actuary, Casualty Actuarial Department, The Travelers Insurance Co., 700 Main Street, Hartford, Conn.
*Nov. 20, 1942	BART, ROBERT D., (American) Lumbermen's Mutual Casualty Company, 4750 Sheridan Road, Chicago, Ill.
*Nov. 18, 1932	BARTER, JOHN L., Secretary, Hartford Accident & Indemnity Co., Hartford, Conn.
*Nov. 13, 1931	BATHO, ELGIN R., Assistant Actuary, Equitable Life Insurance Company of Canada, Waterloo, Ontario, Canada.
†	BENJAMIN, ROLAND, Treasurer, Fidelity & Deposit Company of Maryland and American Bonding Company, Baltimore, Md.
*Nov. 22, 1934	BERKELEY, ERNEST T., Superintendent, Actuarial Department, Employers Liability Assurance Corporation, Boston, Mass.
†	BLACK, S. BRUCE, President, Liberty Mutual Insurance Company, 175 Berkeley Street, Boston, Mass.
Apr. 20, 1917	BLANCHARD, RALPH H., Professor of Insurance, School of Business, Columbia University, New York.
†	BREIBY, WILLIAM, Vice-President, Pacific Mutual Life Insurance Company, Los Angeles, Cal.
*Nov. 18, 1927	BROWN, F. STUART, Statistician, Indemnity Insurance Co. of North America, 1600 Arch St., Philadelphia, Pa.
Oct. 22, 1915	BROWN, HERBERT D., (Retired), Glenora, Yates County, New York.
†	BUCK, GEORGE B., Consulting Actuary for Pension Funds, 150 Nassau Street, New York.

FELLOWS

Date Admitted	
Apr. 20, 1917	BURHOP, WILLIAM H., Executive Vice-President, Employers Mutual Liability Insurance Company, Wausau, Wis.
*Nov. 23, 1928	BURLING, WILLIAM H., Assistant Actuary, The Travelers Insurance Company, 700 Main Street, Hartford, Conn.
*Nov. 19, 1929	CAHILL, JAMES M., Actuary, Compensation Insurance Rating Board, 125 Park Avenue, New York.
*Nov. 18, 1932	CAMERON, FREELAND R., Assistant Manager, Automobile Department, American Surety Company, 100 Broadway, New York.
†	CAMMACK, EDMUND E., Vice-President and Actuary, Aetna Life Insurance Company, Hartford, Conn.
*Nov. 17, 1938	CARLETON, JOHN W., Liberty Mutual Insurance Company, 175 Berkeley Street, Boston, Mass.
*Nov. 21, 1930	CARLSON, THOMAS O., Assistant Actuary, National Bureau of Casualty & Surety Underwriters, 60 John Street, New York.
†	CARPENTER, RAYMOND V., (Retired), 66 Park Avenue, New York.
Mar. 20, 1941	CARVER, HARRY C., Professor of Mathematics, University of Michigan, Ann Arbor, Michigan.
*Nov. 13, 1936	CLEARY, ARTHUR E., Actuary, Massachusetts Insurance Department, 100 Nashua Street, Boston, Mass.
*Nov. 15, 1918	COATES, BARRETT N., Coates and Herfurth, Consulting Actuaries, 582 Market Street, San Francisco, Calif.
*Nov. 17, 1922	COATES, CLARENCE S., Assistant Secretary, Lumbermens Mutual Casualty Company, Mutual Insurance Bldg., Chicago, Ill.
Oct. 27, 1916	COGSWELL, EDMUND S., First Deputy Commissioner of Insurance, 100 Nashua Street, Boston, Mass.
Feb. 19, 1915	COLLINS, HENRY, Manager and Attorney, Ocean Accident & Guarantee Corporation and President, Columbia Casualty Company, 1 Park Avenue, New York.
*Nov. 23, 1928	COMSTOCK, W. PHILLIPS, Statistician, London Guarantee & Accident Company, 55 Fifth Avenue, New York.
*Nov. 22, 1934	CONSTABLE, WILLIAM J., Secretary, Lumbermens Mutual Casualty Company, 342 Madison Avenue, New York.
*Nov. 22, 1934	COOK, EDWIN A., Assistant Secretary, Interboro Mutual Indemnity Insurance Company, 270 Madison Avenue, New York.
†	COPELAND, JOHN A., Consulting Actuary, Candler Building, Atlanta, Ga.
*Nov. 18, 1925	CORCORAN, WILLIAM M., Consulting Actuary, Wolfe, Corcoran & Linder, 116 John Street, New York.
*Nov. 19, 1926	CRANE, HOWARD G., Treasurer, General Reinsurance Corporation, 90 John Street, New York.
*Nov. 18, 1932	DAVIES, E. ALFRED, Asst. to Treasurer, Liberty Mutual Insurance Company, 175 Berkeley Street, Boston, Mass.
*Nov. 18, 1927	DAVIS, EVELYN M., Woodward, Ryan, Sharp & Davis, Consulting Actuaries, 41 Park Row, New York.

FELLOWS

Date Admitted	
†	DEARTH, ELMER H., (Retired), 1407 Clark St., Des Moines, Iowa.
†	DEKAY, ECKFORD C., President, DeKay & Company, 84 William Street, New York.
*Nov. 17, 1920	DORWEILER, PAUL, Actuary, Aetna Casualty & Surety Company, Hartford, Conn.
May 19, 1915	DUNLAP, EARL O., Third Vice President, Metropolitan Life Insurance Company, 1 Madison Avenue, New York.
*Nov. 24, 1933	EDWARDS, JOHN, Casualty Actuary, Ontario Insurance Department, 91 Arundel Avenue, Toronto, Ontario, Canada.
*Nov. 15, 1940	ELLIOTT, GEORGE B., Compensation Actuary, Pennsylvania Insurance Department, 938 Public Ledger Bldg., Philadelphia, Pa.
*Nov. 17, 1922	ELSTON, JAMES S., Assistant Actuary, Life Actuarial Department, The Travelers Insurance Co., 700 Main Street, Hartford, Conn.
*Nov. 15, 1935	EPPINK, WALTER T., Vice-President, Merchants' Mutual Casualty Co., Casualty Insurance Building, Buffalo, New York.
†	FACKLER, EDWARD B., Consulting Actuary, Fackler & Company, 8 West 40th Street, New York.
†	FALLOW, EVERETT S., Actuary, Accident Actuarial Department, The Travelers Insurance Co., 700 Main Street, Hartford, Conn.
*Nov. 15, 1940	FARLEY, JARVIS, Actuary and Asst. Treasurer, Massachusetts Indemnity Co., 632 Beacon Street, Boston, Mass.
†	FARRER, HENRY, Insurance Company of North America, 99 John Street, New York.
*Nov. 15, 1935	FITZHUGH, GILBERT W., Assistant Actuary, Metropolitan Life Insurance Co., 1 Madison Avenue, New York.
†	FLYNN, BENEDICT D., Vice-President and Actuary, The Travelers Insurance Co., 700 Main Street, Hartford, Conn.
Feb. 19, 1915	FONDILLER, RICHARD, Woodward and Fondiller, Consulting Actuaries, 90 John Street, New York.
†	FORBES, CHARLES S., Treasurer, Smyth, Sanford and Gerard, Inc., Insurance Brokers, 68 William Street, New York.
*Nov. 22, 1934	FULLER, GARDNER V., Secretary, National Council on Compensation Insurance, 45 East 17th Street, New York.
†	FRANKLIN, CHARLES H., (Retired), 6015 Vassar Ave., Hawthorne Hills, Seattle, Washington.
*Nov. 18, 1927	FREDRICKSON, CARL H., Actuary, Canadian Underwriters Association, 55 York Street, Toronto, Canada.
†	FURZE, HARRY, (Retired), 42 Douglas Road, Glen Ridge, N. J.
Feb. 19, 1915	GARRISON, FRED S., Secretary, The Travelers Indemnity Co., 700 Main Street, Hartford, Conn.
*Nov. 20, 1924	GINSBURGH HAROLD J., Vice-President, American Mutual Liability Insurance Co., 142 Berkeley Street, Boston, Mass.

FELLOWS

Date Admitted	
*Nov. 21, 1930	GLENN, J. BRYAN, 5214 First Street, N. W., Washington, D. C.
*Nov. 13, 1931	GODDARD, RUSSELL P., American Mutual Liability Insurance Company, 142 Berkeley Street, Boston, Mass.
†	GOODWIN, EDWARD S., 750 Main Street, Hartford, Conn.
*Nov. 19, 1926	GRAHAM, CHARLES M., Associate Actuary, State Insurance Fund, 625 Madison Avenue, New York.
Oct. 22, 1915	GRAHAM, THOMPSON B., Fourth Vice President, Metropolitan Life Insurance Co., 1 Madison Avenue, New York.
†	GRAHAM, WILLIAM J., Vice-President, Equitable Life Assurance Society, 393 Seventh Avenue, New York.
May 25, 1923	GRANVILLE, WILLIAM A., Vice-President, Washington National Insurance Co., 610 Church Street, Evanston, Ill. (Deceased February 4, 1943.)
†	GREENE, WINFIELD W., Vice-President, General Reinsurance Corporation, 90 John Street, New York.
†	HAMMOND, H. PIERSON, Actuary, Life Actuarial Department, The Travelers Insurance Co., 700 Main St., Hartford, Conn.
Oct. 27, 1916	HARDY, EDWARD R., Secretary-Treasurer, Insurance Institute of America, Inc., 80 John Street, New York.
Oct. 22, 1915	HATCH, LEONARD W., (Retired), 425 Pelham Manor Road, Pelham Manor, New York.
*Nov. 19, 1926	HAUGH, CHARLES J., Actuary, National Bureau of Casualty & Surety Underwriters, 60 John Street, New York.
Nov. 17, 1920	HEATH, CHARLES E., 145 Sussex Drive, Strathmore Village, Manhasset, L. I., N. Y.
May 23, 1924	HOBBS, CLARENCE W., Special Representative of the National Association of Insurance Commissioners, National Council on Compensation Insurance, 45 East 17th Street, New York.
Oct. 22, 1915	HODGKINS, LEMUEL G., Secretary, Massachusetts Protective Association and Massachusetts Protective Life Assurance Co., Worcester, Mass.
†	HOFFMAN, FREDERICK L., Consulting Statistician, 1978 Sunset Boulevard, San Diego, California.
Oct. 22, 1915	HOLLAND, CHARLES H., Bennett & Palmer, 165 Broadway, New York.
*Nov. 22, 1934	HOOKER, RUSSELL O., Actuary, Connecticut Insurance Department, Hartford, Conn.
Nov. 18, 1932	HUEBNER, SOLOMON S., Professor of Insurance, University of Pennsylvania, Philadelphia, Pa.
†	HUGHES, CHARLES, Principal Insurance Report Auditor, New York Insurance Department, 61 Broadway, New York.
Nov. 19, 1929	HULL, ROBERT S., Unemployment Compensation Division, Social Security Board, Washington, D. C.
†	HUNT, BURRITT A. (Retired), 88 Porter St., Manchester, Conn.
†	HUNTER, ARTHUR, (Retired), 124 Lloyd Road, Montclair, N. J.
Feb. 25, 1916	JACKSON, CHARLES W., Consulting Actuary, Woodward and Fondiller, 90 John Street, New York.
*Nov. 19, 1929	JACKSON, HENRY H., Vice President & Actuary, National Life Insurance Co., Montpelier, Vt.
*Nov. 14, 1941	JOHNSON, ROGER A., JR., Assistant Actuary, Compensation Insurance Rating Board, 125 Park Avenue, New York.

FELLOWS

Date Admitted	
May 19, 1915	JOHNSON, WILLIAM C., Vice-President, Massachusetts Protective Association and Massachusetts Protective Life Assurance Co., Worcester, Mass.
*Nov. 16, 1939	JONES, HAROLD M., Liberty Mutual Insurance Company, 175 Berkeley Street, Boston, Mass.
*Nov. 17, 1938	KARDONSKY, ELSIE, 66 Corbin Place, Brooklyn, N. Y.
Nov. 17, 1938	KELLY, GREGORY C., General Manager, Pennsylvania Compensation Rating & Inspection Bureau, 938 Public Ledger Bldg., Philadelphia, Pa.
*Nov. 19, 1926	KELTON, WILLIAM H., Assistant Actuary, Life Actuarial Department, The Travelers Insurance Co., 700 Main Street, Hartford, Conn.
*Nov. 21, 1919	KIRKPATRICK, A. LOOMIS, Insurance Editor, Chicago Journal of Commerce, 12 East Grand Avenue, Chicago, Ill.
*Nov. 14, 1941	KOLODITZKY, MORRIS, State Insurance Fund, 625 Madison Avenue, New York.
*Nov. 24, 1933	KORMES, MARK, Consulting Actuary, 341 Madison Avenue, New York.
Nov. 23, 1928	KULP, CLARENCE A., Professor of Insurance, University of Pennsylvania, Logan Hall, 36th Street and Woodland Avenue, Philadelphia, Pa.
Nov. 13, 1931	LA MONT, STEWART M., (Retired), 305 Sheldon Avenue, New Rochelle, New York.
*Nov. 24, 1933	LANGE, JOHN R., Chief Actuary, Wisconsin Insurance Department, State House, Madison, Wis.
Nov. 17, 1922	LAWRENCE, ARNETTE R., Special Deputy Commissioner of Banking and Insurance, 60 Park Place, Newark, N. J. (Deceased December 1, 1942).
†	LEAL, JAMES R., Vice-President and Secretary, Interstate Life and Accident Co., Interstate Building, 540 McCallie Avenue, Chattanooga, Tenn.
†	LESLIE, WILLIAM, General Manager, National Bureau of Casualty & Surety Underwriters, 60 John Street, New York.
*Nov. 20, 1924	LINDER, JOSEPH, Consulting Actuary, Wolfe, Corcoran & Linder, 116 John Street, New York.
*Nov. 13, 1936	LYONS, DANIEL J., Assistant Actuary, Guardian Life Insurance Co., 50 Union Square, New York.
†	MAGOUN, WILLIAM N., (Retired), 33 Fearing Road, Hingham, Mass.
*Nov. 23, 1928	MARSHALL, RALPH M., Assistant Actuary, National Council on Compensation Insurance, 45 East 17th Street, New York.
*Nov. 18, 1927	MASTERSON, Norton E., Vice-President and Actuary, Hardware Mutual Casualty Co., Stevens Point, Wis.
*Nov. 19, 1926	MATTHEWS, ARTHUR N., The Travelers Insurance Co., 700 Main Street, Hartford, Conn.
May 19, 1915	MAYCRINK, EMMA C., Secretary-Treasurer, Association of New York State Mutual Casualty Companies, 60 East 42nd Street, New York.

FELLOWS

Date Admitted	
*Nov. 16, 1923	McCLURG, D. RALPH, Secretary and Treasurer, National Equity Life Insurance Co., Little Rock, Ark.
*Nov. 15, 1935	McCONNELL, MATTHEW H., JR., 122 So. Prospect St., Verona, N. J.
May 23, 1919	McDOUGALD, ALFRED, Ellerslie, Beddington Gardens, Wallington Surrey, England.
*Oct. 31, 1917	McMANUS, Robert J., Statistician, Casualty Actuarial Department, The Travelers Insurance Co., 700 Main Street, Hartford, Conn.
†	MICHELBACHER, GUSTAV F., Vice-President and Secretary, Great American Indemnity Co., 1 Liberty Street, New York.
*Nov. 17, 1938	MILLER, JOHN H., Vice President and Actuary, Monarch Life Insurance Company, Springfield, Mass.
†	MILLIGAN, SAMUEL, Second Vice-President, Metropolitan Life Insurance Co., 1 Madison Avenue, New York.
*Nov. 18, 1937	MILLS, JOHN A., Secretary and Actuary, Lumbermens Mutual Casualty Co., and American Motorists Insurance Co., Mutual Insurance Bldg., Chicago, Ill.
*Nov. 18, 1921	MONTGOMERY, VICTOR, President, Pacific Employers Insurance Co., 1033 So. Hope Street, Los Angeles, Calif.
Nov. 19, 1926	MOONEY, WILLIAM L., (Retired), 4 Pleasant Street, West Hartford, Conn.
†	MOORE, GEORGE D., 13 Emerson Street, East Orange, N. J.
†	MOWBRAY, ALBERT H., Consulting Actuary, 806 San Luis Road, Berkeley, Calif.
*Nov. 17, 1920	MUELLER, LOUIS H., President, Associated Insurance Fund, 332 Pine Street, San Francisco, Calif.
†	MULLANEY, FRANK R., Vice-President and Secretary, American Mutual Liability Insurance Co., and American Policyholders' Insurance Co., 142 Berkeley Street, Boston, Mass.
May 28, 1920	MURPHY, RAY D., Vice-President and Actuary, Equitable Life Assurance Society, 393 Seventh Avenue, New York.
*Nov. 15, 1935	OBERHAUS, THOMAS M., Assistant Actuary, Woodward and Fondiller, Consulting Actuaries, 90 John Street, New York.
†	OLIFIERS, EDWARD, Actuary and Managing Director, Previdencia do Sul, Caixa Postal 76, Porto Alegre, Brazil.
Nov. 18, 1927	O'NEILL, FRANK J., President, Royal Indemnity Co., and Eagle Indemnity Co., 150 William Street, New York.
†	ORR, ROBERT K., 226 S. Logan Street, Lansing, Mich.
*Nov. 21, 1919	OUTWATER, OLIVE E., Actuary, Benefit Association of Railway Employees, 901 Montrose Avenue, Chicago, Ill.

FELLOWS

Date Admitted	
*Nov. 18, 1921	PERKINS, SANFORD B., Secretary, The Travelers Insurance Co., 700 Main Street, Hartford, Conn.
*Nov. 21, 1930	PERRYMAN, FRANCIS S., Secretary and Actuary, Royal Indemnity Co., and Eagle Indemnity Co., 150 William Street, New York.
*Nov. 14, 1941	PETERS, STEFAN, Assistant Actuary, Compensation Insurance Rating Board, 125 Park Avenue, New York.
Nov. 19, 1926	PHILLIPS, JESSE S., Chairman of Board, Great American Indemnity Co., 1 Liberty Street, New York.
*Nov. 24, 1933	PICKETT, SAMUEL C., Assistant Actuary, Connecticut Insurance Department, Hartford, Conn.
*Nov. 17, 1922	PINNEY, SYDNEY D., Associate Actuary, Casualty Actuarial Department, The Travelers Insurance Co., 700 Main Street, Hartford, Conn.
*Nov. 13, 1931	PRUITT, DUDLEY M., Actuary, General Accident, Fire & Life Assurance Corp., Fourth & Walnut Sts., Philadelphia, Pa.
May 23, 1919	RICHARDSON, FREDERICK, Deputy Chairman of the Board, General Accident Fire and Life Assurance Corporation, Perth, Scotland.
*Nov. 19, 1926	RICHTER, OTTO C., American Telephone & Telegraph Co., 195 Broadway, New York.
May 24, 1921	RIEGEL, ROBERT, Professor of Statistics and Insurance, University of Buffalo, Buffalo, New York.
*Nov. 16, 1939	ROBBINS, RAINARD B., Vice President and Secretary, Teachers Insurance and Annuity Association, 522 Fifth Avenue, New York.
*Nov. 16, 1923	ROEBER, WILLIAM F., General Manager, National Council on Compensation Insurance, 45 East 17th Street, New York.
*Nov. 20, 1942	SATTERTHWAITE, FRANKLIN E., 34 Carol Drive, Manchester, Conn.
†	SCHEITLIN, EMIL, Treasurer, Globe Indemnity Co., 150 William Street, New York.
*Nov. 18, 1937	SHAPIRO, GEORGE I., First Vice President and General Manager, Public Service Mutual Casualty Co., 342 Madison Avenue, New York.
*Nov. 13, 1931	SILVERMAN, DAVID, c/o Wolfe, Corcoran & Linder, 116 John Street, New York.
*Nov. 24, 1933	SINNOTT, ROBERT V., Hartford Accident and Indemnity Company, 690 Asylum Avenue, Hartford, Conn.
*Nov. 19, 1929	SKELDING, ALBERT Z., Actuary, National Council on Compensation Insurance, 45 East 17th Street, New York.
*Nov. 19, 1929	SKILLINGS, E. SHAW, Actuary, Allstate Insurance Co., 20 North Wacker Drive, Chicago, Ill.
*Nov. 18, 1932	SMICK, JACK J., National Council on Compensation Insurance, 45 East 17th Street, New York.

FELLOWS

Date Admitted	
*Nov. 15, 1940	SMITH, SEYMOUR E., Casualty Actuarial Department, Travelers Insurance Co., Hartford, Conn.
*Nov. 24, 1933	ST. JOHN, JOHN B., Social Security Board, Bureau of Old Age Insurance, 801 Equitable Building, Baltimore, Md.
Nov. 18, 1927	STONE, EDWARD C., U. S. General Manager and Attorney, Employers' Liability Assurance Corporation, Limited, and President, American Employers' Insurance Company, 110 Milk Street, Boston, Mass.
Oct. 22, 1915	STRONG, WILLIAM RICHARD, No. 4 "Sheringham," Cotham Road, Kew, Victoria, Australia.
*Nov. 17, 1920	TARBELL, THOMAS F., Actuary, Casualty Actuarial Department. The Travelers Insurance Co., 700 Main Street, Hartford, Conn.
†	THOMPSON, JOHN S., Vice-President and Mathematician, Mutual Benefit Life Insurance Co., 300 Broadway, Newark, N. J.
†	TRAIN, JOHN L., President and General Manager, Utica Mutual Insurance Co., 185 Genesee Street, Utica, New York.
Nov. 17, 1922	TRAVERSI, ANTONIO T., Consulting Actuary and Accountant, London Bank Chambers, Martin Place, Sydney, Australia.
*Nov. 23, 1928	VALERIUS, NELS M., Aetna Casualty and Surety Co., Hartford, Conn.
*Nov. 21, 1919	VAN TUYL, HIRAM O., Supt., Accounts Department, London Guarantee & Accident Co., 55 Fifth Avenue, New York.
*Nov. 17, 1920	WAITE, ALAN W., Assistant Secretary, Aetna Casualty and Surety Co., Hartford, Conn.
*Nov. 15, 1935	WAITE, HARRY V., Statistician, The Travelers Fire Insurance Co., 700 Main Street, Hartford, Conn.
*Nov. 18, 1925	WARREN, LLOYD A. H., Professor of Actuarial Science, University of Manitoba, 64 Niagara Street, Winnipeg, Manitoba, Canada.
†	WHITNEY, ALBERT W., (Retired) 19 Gilbert Place, White Plains, N. Y.
*Nov. 15, 1935	WILLIAMS, HARRY V., Rating & Research Dept., Hartford Accident and Indemnity Co., Hartford, Conn.
Nov. 14, 1941	WILLIAMSON, WILLIAM R., Actuarial Consultant, Social Security Board, Washington, D. C.
*Nov. 13, 1931	WITTICK, HERBERT E., Secretary, Pilot Insurance Co., 199 Bay Street, Toronto, Canada.
†	WOLFE, LEE J., Consulting Actuary, Wolfe, Corcoran & Linder, 116 John Street, New York.
May 24, 1921	WOOD, ARTHUR B., President and Managing Director, Sun Life Assurance Company of Canada, Montreal, Canada.

ASSOCIATES

Those marked (*) have been enrolled as Associates upon examination by the Society.

Numerals indicate Associateship Part V and Fellowship examination parts credited.

Date Enrolled	
May 23, 1924	ACKER, MILTON, Manager, Compensation and Liability Department, National Bureau of Casualty and Surety Underwriters, 60 John Street, New York.
*Nov. 15, 1918	ACKERMAN, SAUL B., Professor of Insurance, New York University, 90 Trinity Place, New York.
*Nov. 16, 1939	AIN, SAMUEL N., Office of George B. Buck, Consulting Actuary for Pension Funds, 150 Nassau Street, New York.
Apr. 5, 1928	ALLEN, AUSTIN F., President and General Manager, Texas Employers Insurance Association and Employers Casualty Co., Dallas, Texas.
Nov. 15, 1918	ANKERS, ROBERT E., Secretary and Treasurer, Continental Life Insurance Co., Investment Building, Washington, D. C.
*Nov. 21, 1930	ARCHIBALD, A. EDWARD, Actuary, Volunteer State Life Insurance Company, Chattanooga, Tenn. (V, I.)
*Nov. 16, 1939	BAILEY, ARTHUR L., Statistician, American Mutual Alliance, 60 E. 42nd Street, New York.
*Nov. 24, 1933	BARRON, JAMES C., Asst. Treasurer, General Reinsurance Corporation, 90 John Street, New York. (V, I, III.)
*Nov. 23, 1928	BATEMAN, ARTHUR E., Liberty Mutual Insurance Company, 175 Berkeley Street, Boston, Mass. (V, I.)
*Nov. 15, 1940	BATHO, BRUCE, Actuary, Country Life Insurance Company, 608 So. Dearborn St., Chicago, Ill.
*Nov. 18, 1925	BITTEL, W. HAROLD, Associate Actuary, Woodward, Ryan, Sharp, & Davis, 41 Park Row, New York.
Nov. 17, 1920	BLACK, NELLAS C., Statistician, Maryland Casualty Co., Baltimore, Md.
*Nov. 15, 1940	BLACKHALL, JOHN M., Monarch Life Insurance Co., Springfield, Mass.
*Nov. 22, 1934	BOMSE, EDWARD L., National Bureau of Casualty & Surety Underwriters, 60 John Street, New York.
*Nov. 23, 1928	BOWER, PERRY S., Great West Life Assurance Company, Winnipeg, Manitoba, Canada.
*Nov. 15, 1935	BRERETON, CLOUDESLEY R., Dominion Department of Insurance, Ottawa, Ontario, Canada.
*Nov. 15, 1918	BRUNNQUELL, HELMUTH G., Assistant Actuary, The Northwestern Mutual Life Insurance Co., Milwaukee, Wis.
*Oct. 22, 1915	BUFFLER, LOUIS, Director, Underwriting Department, State Insurance Fund, 625 Madison Avenue, New York.
*Nov. 20, 1924	BUGBEE, JAMES M., Asst. Manager, Automobile Department, Maryland Casualty Co., Baltimore, Md.
Mar. 31, 1920	BURT, MARGARET A., Office of George B. Buck, Consulting Actuary, 150 Nassau Street, New York.
Nov. 17, 1922	CAVANAUGH, LEO D., President, Federal Life Insurance Co., 168 N. Michigan Avenue, Chicago, Ill.

ASSOCIATES

Date Enrolled	
*Nov. 18, 1927	CHEN, S. T., Actuary, China United Assurance Society, 104 Bubbling Well Road, Shanghai, China.
*Nov. 18, 1927	CONROD, STUART F., Actuary, Loyal Protective Life Insurance Co., Co., 19 Deerfield Street, Boston, Mass.
*Nov. 24, 1933	CRAWFORD, WILLIAM H., Secretary, Fireman's Insurance Co. of Newark, N. J. & Affiliated Fire & Casualty Co's Pacific Dept., 220 Bush Street, San Francisco, Cal. (V, I.)
*Nov. 18, 1932	CRIMMINS, JOSEPH B., Metropolitan Life Insurance Co., 1 Madison Avenue, New York. (V, I.)
*Nov. 18, 1925	DAVIS, MALVIN E., Associate Actuary, Metropolitan Life Insurance Co., 1 Madison Avenue, New York.
*Nov. 24, 1933	DAVIS, REGINALD S., Comptroller-Actuary, State Compensation Insurance Fund, San Francisco, Calif. (V, I.)
*Nov. 14, 1941	DOWLING, WILLIAM F., Asst. Treasurer, Lumber Mutual Casualty Co., 41 E. 42nd Street, New York.
May 25, 1923	ECONOMIDY, HARILAUS E., Comptroller, Associated Employers Lloyds, Neil P. Anderson Building, Fort Worth, Texas.
June 5, 1925	EGER, FRANK A., Secretary-Comptroller, Insurance Company of North America and Affiliated Companies, 1600 Arch Street, Philadelphia, Pa.
*Nov. 16, 1923	FITZ, L. LEROY, Group Underwriter, American Mutual Liability Insurance Co., 142 Berkeley Street, Boston, Mass. (V, I.)
*Nov. 18, 1927	FITZGERALD, AMOS H., Assistant Actuary, The Prudential Insurance Company of America, Newark, N. J. (V, I.)
*Nov. 16, 1923	FLEMING, FRANK A., Actuary, American Mutual Alliance, 60 East 42nd Street, New York.
Nov. 20, 1924	FROBERG, JOHN, Manager, California Inspection Rating Bureau, 500 Sansome Street, San Franciso, Calif.
*Nov. 13, 1936	FRUECHTEMEYER, FRED J., Liberty Mutual Insurance Co., 175 Berkeley Street, Boston, Mass. (V, I.)
*Nov. 19, 1929	FURNIVALL, MAURICE L., Assistant Actuary, Accident Actuarial Department, The Travelers Insurance Co., 700 Main Street, Hartford, Conn. (V, I.)
*Nov. 22, 1934	GATELY, JOHN J., General Reinsurance Corporation, 90 John Street. New York. (V, I.)
*Nov. 18, 1932	GETMAN, RICHARD A., Life Actuarial Department, The Travelers Insurance Co., 700 Main Street, Hartford, Conn. (V, I.)
*Nov. 17, 1922	GIBSON, JOSEPH P., JR., Vice President, Excess Insurance Company of America, 99 John Street, New York.
*Nov. 16, 1923	GILDEA, JAMES F., The Travelers Insurance Co., 700 Main Street, Hartford, Conn.
Nov. 19, 1929	GORDON, HAROLD R., Executive Secretary, Health & Accident Underwriters Conference, 176 West Adams Street, Chicago, Ill.
*Nov. 18, 1927	GREEN, WALTER C., Consulting Actuary, 211 West Wacker Drive, Chicago, Ill.
*Nov. 15, 1940	GROSSMAN, ELI A., 26 California Street, Mount Vernon, New York.
*Nov. 15, 1935	GUERTIN, ALFRED N., Actuary, New Jersey Department of Banking and Insurance, Trenton, N. J., (V, I.)

ASSOCIATES

Date Enrolled	
*Nov. 16, 1939	HAGEN, OLAF E., Metropolitan Life Insurance Company, 1 Madison Avenue, New York.
*Nov. 18, 1921	HAGGARD, ROBERT E., Superintendent, Permanent Disability Rating Department, Industrial Accident Commission, State Building, San Francisco, Calif.
*Nov. 17 1922	HALL, HARTWELL L., Associate Actuary, Connecticut Insurance Department, Hartford, Conn.
*Nov. 13, 1936	HAM, HUGH P., British America Assurance Co., 807 Electric Railway Chambers, Winnipeg, Manitoba, Canada. (V, I.)
Mar. 24, 1932	HARRIS, SCOTT, Vice-President, Joseph Froggatt & Co., 74 Trinity Place, New York.
*Mar. 25, 1924	HART, WARD VAN BUREN, Assistant Actuary, Connecticut General Life Insurance Co., Hartford, Conn. (V, I.)
Nov. 21, 1919	HAYDON, GEORGE F., General Manager, Wisconsin Compensation Rating & Inspection Bureau, 715 N. Van Buren Street, Milwaukee, Wis.
Nov. 17, 1927	HIPP, GRADY H., Executive Vice-President, Liberty Life Insurance Co., Greenville, S. C.
Nov. 19, 1929	JACOBS, CARL N., President, Hardware Mutual Casualty Co., Stevens Point, Wis.
*Nov. 18, 1921	JENSEN, EDWARD S., Asst. Secretary, Occidental Life Insurance Co., Los Angeles, Calif. (II, III.)
Nov. 21, 1930	JONES, H. LLOYD, Deputy General Attorney, of Phoenix-London Group, Vice-President, Phoenix Indemnity Company, and Deputy United States Manager, London Accident & Guarantee Co., 55 Fifth Avenue, New York.
*Nov. 21, 1919	JONES, LORING D., (Retired) 64 Raymond Ave., Rockville Centre, Long Island, N. Y.
*Nov. 15, 1940	KELLY, ROBERT G., 723 North 64th Street, Philadelphia, Pa.
*Nov. 17, 1922	KIRK, CARL L., Assistant U. S. Manager, Zurich General Accident & Liability Insurance Co., 135 South LaSalle Street, Chicago, Ill.
*Nov. 15, 1935	KITZROW, ERWIN W., Vice-President, Hardware Mutual Casualty Co., Stevens Point, Wis. (V, I.)
*Nov. 16, 1939	KNOWLES, FREDERICK, 5534 Trans-Island Ave., N.D.G., Montreal, Canada.
*Nov. 18, 1937	LASSOW, WILLIAM, 185 206th St., Bronx, New York. (V.)
*Nov. 17, 1938	LIEBLEIN, JULIUS, 2095 Honeywell Ave., Bronx, New York.
*Nov. 13, 1931	MACKEEN, HAROLD E., The Travelers Insurance Co., 700 Main Street, Hartford, Conn. (V, I.)
Mar. 24, 1932	MAGRATH, JOSEPH J., Executive Assistant, Chubb & Sons, 90 John Street, New York.
*Nov. 18, 1925	MALMUTH, JACOB, Examiner, New York Insurance Department, 61 Broadway, New York.
Mar. 24, 1927	MARSH, CHARLES V. R., Comptroller and Assistant Treasurer, Fidelity & Deposit Co. and American Bonding Co., Baltimore, Md.
*Nov. 13, 1936	MAYER, WILLIAM H., JR., Actuarial Department, Metropolitan Life Insurance Co., 1 Madison Avenue, New York.
*Nov. 17, 1922	MCIVER, ROSSWELL A., Actuary, Washington National Insurance Co., 610 Church Street, Evanston, Ill.

ASSOCIATES

Date Enrolled	
*Nov. 17, 1922	MICHENER, SAMUEL M., Actuary, Columbus Mutual Life Insurance Co., 303 East Broad Street, Columbus, Ohio, (V., I)
*Nov. 13, 1931	MILLER, HENRY C., Comptroller, State Compensation Insurance Fund, 450 McAllister Street, San Francisco, Calif. (V, I.)
*Nov. 19, 1926	MILNE, JOHN L., Actuary, Presbyterian Ministers' Fund for Life Insurance, 1805 Walnut Street, Philadelphia, Pa.
*Nov. 18, 1937	MINOR, EDUARD H., Accident and Health Department, Metropolitan Life Insurance Company, 1 Madison Avenue, New York.
Nov. 17, 1922	MONTGOMERY, JOHN C., Secretary and Assistant Treasurer, Bankers Indemnity Insurance Co., 15 Washington Street, Newark, N. J.
May 25, 1923	MOORE, JOSEPH P., President, North American Accident Insurance Co., 455 Craig Street, W., Montreal, Canada.
*Nov. 21, 1919	MOTHERSILL, ROLLAND V., President, Anchor Casualty Co., Anchor Insurance Building, 758 So. Mississippi River Boulevard, St. Paul, Minn. (II, III.)
*Nov. 18, 1937	MYERS, ROBERT J., Senior Actuarial Mathematician, Social Security Board, Washington, D. C.
*Nov. 19, 1929	MULLER, FRITZ, Director, Agrippina Life Insurance Stock Co., Berlin, W. 30 Mackensenstr. 16, Germany.
*Nov. 15, 1935	NELSON, S. TYLER, Utica Mutual Insurance Co., 185 Genesee Street, Utica, New York.
*Oct. 27, 1916	NEWELL, WILLIAM, Secretary, Assigned Risk Pool, 60 John Street, New York. (V., I.)
*Nov. 23, 1928	NEWHALL, KARL, Group Department, The Travelers Insurance Co., 700 Main Street, Hartford, Conn.
*Nov. 18, 1925	NICHOLSON, EARL H., Actuary, Joseph Froggatt & Co., 74Trinity Place, New York.
May 23, 1919	OTTO, WALTER E., President, Michigan Mutual Liability Co., 163 Madison Avenue, Detroit, Mich.
*Nov. 19, 1926	OVERHOLSER, DONALD M., Office of George B. Buck, Consulting Actuary for Pension Funds, 150 Nassau Street, New York.
Nov. 20, 1924	PENNOCK, RICHARD M., Actuary, Pennsylvania Manufacturers' Association Casualty Insurance Co., Finance Building, Philadelphia, Pa.
Nov. 19, 1929	PHILLIPS, JOHN H., Vice-President and Actuary, Employers' Mutual Liability Insurance Co., Wausau, Wis.
*Nov. 17, 1920	PIKE, MORRIS, Vice-President and Actuary, Union Labor Life Insurance Co., 570 Lexington Avenue, New York.
*Nov. 23, 1928	PIPER, KENNETH B., Actuary, Provident Life and Accident Insurance Co., Chattanooga, Tenn. (V, I.)
*Nov. 18, 1927	POISSANT, WILLIAM A., The Travelers Insurance Co., 700 Main Street, Hartford, Conn.
*Nov. 17, 1922	POORMAN, WILLIAM F., Vice-President and Actuary, Central Life Assurance Society, Fifth and Grand Avenues, Des Moines, Iowa. (V, I.)
*Nov. 13, 1936	POTOFSKY, SYLVIA, State Insurance Fund, 625 Madison Avenue, New York. (V.)

ASSOCIATES

Date Enrolled	
Nov. 17, 1922	POWELL, JOHN M., President, Loyal Protective Insurance Co. and Loyal Life Insurance Co., 19 Deerfield Street, Boston, Mass. (V, I.)
*Nov. 15, 1918	RAYWID, JOSEPH, President, Joseph Raywid & Co., Inc., 92 William Street, New York.
Nov. 19, 1932	RICHARDSON, HARRY F., Secretary-Treasurer, National Council on Compensation Insurance, 45 East 17th Street, New York.
*Nov. 18, 1932	ROBERTS, JAMES A., Life Actuarial Department, The Travelers Insurance Co., 700 Main Street, Hartford, Conn. (V, I.)
*Nov. 15, 1940	ROSENBERG, NORMAN, Actuary, Public Service Mutual Casualty Co., 342 Madison Avenue, New York. (I.)
*Nov. 20, 1942	ROSS, SAMUEL M., Actuarial Department, National Bureau of Casualty & Surety Underwriters, 60 John Street, New York.
*Nov. 18, 1927	SARASON, HARRY M., Assistant Actuary, General American Life Insurance Co., 1501 Locust Street, St. Louis, Mo.
Nov. 16, 1923	SAWYER, ARTHUR, Globe Indemnity Co., 150 William Street, New York.
*Nov. 20, 1930	SEVILLA, EXEQUIEL S., Actuary, National Life Insurance Co., P. O. Box 2856, Manila, Philippine Islands.
*Nov. 20, 1924	SHEPPARD, NORRIS E., Professor of Mathematics, University of Toronto, Toronto, Canada. (V, I.)
Nov. 15, 1918	SIBLEY, JOHN L., Assistant Secretary, United States Casualty Co., 60 John Street, New York.
*Nov. 18, 1921	SMITH, ARTHUR G., Assistant General Manager, Compensation Insurance Rating Board, Pershing Square Bldg., 125 Park Avenue, New York.
*Nov. 19, 1926	SOMERVILLE, WILLIAM F., Assistant Secretary, St. Paul Mercury Indemnity Co., St. Paul, Minn. (V, I.)
*Nov. 18, 1925	SOMMER, ARMAND, Assistant to Vice-President, Continental Casualty Co., 910 So. Michigan Avenue, Chicago, Ill.
*Nov. 15, 1918	SPENCER, HAROLD S., Statistician, Aetna Casualty and Surety Co., Hartford, Conn.
Nov. 20, 1924	STELLWAGEN, HERBERT P., Executive Vice-President, Indemnity Insurance Company of North America, 1600 Arch Street, Philadelphia, Pa.
*Nov. 16, 1923	STOKE, KENDRICK, Actuary, Michigan Mutual Liability Company, 163 Madison Avenue, Detroit, Mich.
*Nov. 21, 1930	SULLIVAN, WALTER F., Insurance Accountant, State Compensation Insurance Fund, 450 McAllister Street, San Francisco, Calif. (V, I.)
Mar. 23, 1921	THOMPSON, ARTHUR E., Chief Statistician, Globe Indemnity Co., 150 William Street, New York.
*Nov. 21, 1919	TRENCH, FREDERICK H., Manager, Underwriting Department, Utica Mutual Insurance Co., 185 Genesee Street, Utica, N. Y. (V, I.)
*Nov. 20, 1924	UHL, M. ELIZABETH, National Bureau of Casualty & Surety Underwriters, 60 John Street, New York. (V, I.)
May 23, 1919	WARREN, CHARLES S., Secretary, Massachusetts Automobile Rating and Accident Prevention Bureau, 89 Broad Street, Boston, Mass.
Nov. 18, 1925	WASHBURN, JAMES H., Actuary, 1501 Gale Lane, Nashville, Tenn.

ASSOCIATES

Date Enrolled	
*Nov. 18, 1932	WEINSTEIN, MAX S., Examiner, New York Insurance Department, 61 Broadway, New York.
*Nov. 18, 1921	WELCH, EUGENE R., Associated Indemnity Corporation, 332 Pine Street, San Francisco, Calif.
*Nov. 18, 1925	WELLMAN, ALEXANDER C., Vice-President and Actuary, Protective Life Insurance Co., Birmingham, Ala.
*Nov. 21, 1930	WELLS, WALTER I., Supervisor of Applications, Massachusetts Protective Association, Worcester, Mass. (V, I.)
Mar. 21, 1929	WHEELER, CHARLES A., Chief Examiner of Casualty Companies, New York Insurance Department, 61 Broadway, New York.
*Nov. 18, 1927	WHITBREAD, FRANK G., Assistant Actuary, Great West Life Assurance Co., Winnipeg, Manitoba, Canada.
*Nov. 16, 1939	WITTLAKE, J. CLARKE, Actuarial Department, Business Men's Assurance Company, Kansas City, Mo.
*Oct. 22, 1915	WOOD, DONALD M., Childs & Wood, General Agents, Royal Indemnity Company, 175 W. Jackson Blvd., Chicago, Ill.
*Nov. 18, 1937	WOOD, DONALD M., JR., Childs & Wood, 175 West Jackson Blvd., Chicago, Ill.
*Nov. 18, 1927	WOOD, MILTON J., Assistant Actuary, Life Actuarial Department, The Travelers Insurance Co., 700 Main Street, Hartford, Conn.
*Oct. 22, 1915	WOODMAN, CHARLES E., Assistant Manager, Ocean Accident & Guarantee Corporation and Comptroller, Columbia Casualty Co., 1 Park Avenue, New York.
*Nov. 22, 1934	WOODWARD, BARBARA H., Hughes, Hubbard & Ewing, 1 Wall Street, New York.
*Nov. 18, 1925	WOOLERY, JAMES M., Actuary, North Carolina Insurance Dept., Raleigh, N. C.

SCHEDULE OF MEMBERSHIP, NOVEMBER 20, 1942

	Fellows	Associates	Total
Membership, November 14, 1941	173	127	300
Additions:			
By election	..		..
By reinstatement	..		..
By examination	2	1	3
	175	128	303
Deductions:			
By death	9	..	9
By withdrawal	1	1	2
By transfer from Associate to Fellow	..	1	1
Membership, November 20, 1942	165	126	291

OFFICERS OF THE SOCIETY

Since Date of Organization

Elected	*President*	*Vice-Presidents*	
1914–1915	*I. M. Rubinow	A. H. Mowbray	B. D. Flynn
1916–1917	*J. D. Craig	*J. H. Woodward	*H. E. Ryan
1918	*J. H. Woodward	B. D. Flynn	G. D. Moore
1919	B. D. Flynn	G. D. Moore	W. Leslie
1920	A. H. Mowbray	W. Leslie	*L. S. Senior
1921	A. H. Mowbray	*L. S. Senior	*H. E. Ryan
1922	*H. E. Ryan	G. F. Michelbacher	E. E. Cammack
1923	W. Leslie	G. F. Michelbacher	E. E. Cammack
1924–1925	G. F. Michelbacher	S. B. Perkins	R. H. Blanchard
1926–1927	S. B. Perkins	G. D. Moore	T. F. Tarbell
1928–1929	G. D. Moore	S. D. Pinney	P. Dorweiler
1930–1931	T. F. Tarbell	*R. A. Wheeler	W. W. Greene
1932–1933	P. Dorweiler	W. F. Roeber	*L. S. Senior
1934–1935	W. W. Greene	R. H. Blanchard	C. J. Haugh
1936–1937	*L. S. Senior	S. D. Pinney	F. S. Perryman
1938–1939	F. S. Perryman	H. T. Barber	W. J. Constable
1940	S. D. Pinney	H. J. Ginsburgh	J. M. Cahill
1941	R. H. Blanchard	H. J. Ginsburgh	J. M. Cahill
1942	R. H. Blanchard	Albert Z. Skelding	Charles J. Haugh

Secretary-Treasurer

1914–1917.....*C. E. Scattergood
1918–1942.....R. Fondiller

Editor†	*Librarian*†
1914.....W. W. Greene	1914.....W. W. Greene
1915–1917.....R. Fondiller	1915.....R. Fondiller
1918..... W. W. Greene	1916–1921.....L. I. Dublin
1919–1921....G. F. Michelbacher	1922–1924.....E. R. Hardy
1922–1923.....O. E. Outwater	1925–1937.....W. Breiby
1924–1932.....R. J. McManus	1937–1942.....T. O. Carlson
1933–1942.....C. W. Hobbs	

*Deceased.
†The offices of Editor and Librarian were not separated until 1916.

FELLOWS WHO HAVE DIED

The (†) denotes original membership at date of organization, November 7, 1914

Enrolled		*Died*
May 24, 1921	Edward J. Bond	Nov. 12, 1941
May 19, 1915	Thomas Bradshaw	Nov. 10, 1939
June 5, 1925	William Brosmith	Aug. 22, 1937
†	William A. Budlong	June 4, 1934
Nov. 18, 1932	Charles H. Burhans	June 15, 1942
Feb. 19, 1915	F. Highlands Burns	Mar. 30, 1935
Feb. 19, 1915	Gordon Case	Feb. 4, 1920
†	Charles T. Conway	July 23, 1921
†	Walter G. Cowles	May 30, 1942
†	James D. Craig	May 27, 1940
†	James McIntosh Craig	Jan. 20, 1922
May 26, 1916	Frederick S. Crum	Sept. 2, 1921
†	Alfred Burnett Dawson	June 21, 1931
†	Miles Menander Dawson	Mar. 27, 1942
May 19, 1915	Samuel Deutschberger	Jan. 18, 1929
†	Ezekiel Hinton Downey	July 9, 1922
†	David Parks Fackler	Oct. 30, 1924
Feb. 19, 1915	Claude W. Fellows	July 15, 1938
May 26, 1916	Lee K. Frankel	July 25, 1931
Feb. 25, 1916	Joseph Froggatt	Sept. 28, 1940
†	Theodore E. Gaty	Aug. 22, 1925
May 19, 1915	James W. Glover	July 15, 1941
Oct. 22, 1915	George Graham	Apr. 15, 1937
†	William H. Gould	Oct. 28, 1936
†	Robert Cowen Lees Hamilton	Nov. 15, 1941
Nov. 21, 1919	Robert Henderson	Feb. 16, 1942
†	Robert J. Hillas	May 17, 1940
Nov. 15, 1918	Frank Webster Hinsdale	Mar. 18, 1932
Nov. 19, 1926	Charles E. Hodges	Jan. 22, 1937
Nov. 21, 1919	Carl Hookstadt	Mar. 10, 1924
Nov. 28, 1921	William Anderson Hutcheson	Nov. 19, 1942
Nov. 23, 1928	F. Robertson Jones	Dec. 26, 1941
Nov. 18, 1921	Thomas P. Kearney	Feb. 11, 1928
Oct. 22, 1915	Virgil Morrison Kime	Oct. 15, 1918
†	Edwin W. Kopf	Aug. 3, 1933
Feb. 17, 1915	John M. Laird	June 20, 1942
Feb. 19, 1915	Abb Landis	Dec. 9, 1937
Nov. 17, 1922	Arnette Roy Lawrence	Dec. 1, 1942
Nov. 18, 1921	James Fulton Little	Aug. 11, 1938
Nov. 23, 1928	Edward C. Lunt	Jan. 13, 1941
Feb. 19, 1915	Harry Lubin	Dec. 20, 1920
Feb. 15, 1915	Franklin B. Mead	Nov. 29, 1933
Apr. 20, 1917	Marcus Meltzer	Mar. 27, 1931
†	David W. Miller	Jan. 18, 1936

FELLOWS WHO HAVE DIED—Continued

Enrolled		*Died*
†	James F. Mitchell	Feb. 9, 1941
†	Henry Moir	June 8, 1937
Feb. 19, 1915	William J. Montgomery	Aug. 20, 1915
May 19, 1915	Edward Bontecou Morris	Dec. 19, 1929
†	Lewis A. Nicholas	Apr. 21, 1940
†	Stanley Otis	Oct. 12, 1937
Nov. 13, 1926	Bertrand A. Page	July 30, 1941
Nov. 15, 1918	William Thomas Perry	Oct. 25, 1940
†	Edward B. Phelps	July 24, 1915
†	Charles Grant Reiter	July 30, 1937
†	Charles H. Remington	Mar. 21, 1938
†	Isaac M. Rubinow	Sept. 1, 1936
†	Harwood Eldridge Ryan	Nov. 2, 1930
†	Arthur F. Saxton	Feb. 26, 1927
†	Leon S. Senior	Feb. 3, 1940
Apr. 20, 1917	Charles Gordon Smith	June 22, 1938
Feb. 19, 1915	John T. Stone	May 9, 1920
Feb. 25, 1916	Wendell Menville Strong	Mar. 30, 1942
†	Robert J. Sullivan	July 19, 1934
Nov. 22, 1934	Walter H. Thompson	May 25, 1935
Nov. 18, 1921	Guido Toja	Feb. 28, 1933
May 23, 1919	Archibald A. Welch	May 8, 1935
Nov. 19, 1926	Roy A. Wheeler	Aug. 26, 1937
†	S. Herbert Wolfe	Dec. 31, 1927
†	Joseph H. Woodward	May 15, 1928
†	William Young	Oct. 23, 1927

ASSOCIATES WHO HAVE DIED

Enrolled		*Died*
Oct. 22, 1915	Don A. Baxter	Feb. 10, 1920
Nov. 20, 1924	Leslie LeVant Hall	Mar. 8, 1931
Oct. 31, 1917	Edward T. Jackson	May 8, 1939
Nov. 18, 1927	Alexander A. Speers	June 25, 1941
Nov. 21, 1919	Walter G. Voogt	May 8, 1937
Nov. 17, 1920	James J. Watson	Feb. 23, 1937
Nov. 15, 1918	Albert Edward Wilkinson	June 11, 1930

CONSTITUTION

(As Amended November 15, 1940)

Article I.—*Name.*

This organization shall be called the Casualty Actuarial Society.

Article II.—*Object.*

The object of the Society shall be the promotion of actuarial and statistical science as applied to the problems of casualty and social insurance by means of personal intercourse, the presentation and discussion of appropriate papers, the collection of a library and such other means as may be found desirable.

The Society shall take no partisan attitude, by resolution or otherwise, upon any question relating to casualty or social insurance.

Article III.—*Membership.*

The membership of the Society shall be composed of two classes, Fellows and Associates. Fellows only shall be eligible to office or have the right to vote.

The Fellows of the Society shall be the present Fellows and those who may be duly admitted to Fellowship as hereinafter provided. The Associates shall be the present Associates and those who may be duly admitted to Associateship as hereinafter provided.

Any person may, upon nomination to the Council by two Fellows of the Society and approval by the Council of such nomination with not more than one negative vote, become enrolled as an Associate of the Society, provided that he shall pass such examination as the Council may prescribe. Such examination may be waived in the case of a candidate who for a period of not less than two years has been in responsible charge of the Statistical or Actuarial Department of a casualty insurance organization or has had such other practical experience in casualty or social insurance as, in the opinion of the Council, renders him qualified for Associateship.

Any person who shall have qualified for Associateship may become a Fellow on passing such final examination as the Council may prescribe. Otherwise, no one shall be admitted as a Fellow unless recommended by a duly called meeting of the Council, with not more than three negative votes, followed by a three-fourths ballot of the Fellows present and voting at a meeting of the Society.

Article IV.—*Officers and Council.*

The officers of the Society shall be a President, two Vice-Presidents, a Secretary-Treasurer, an Editor, and a Librarian. The Council shall be composed of the active officers, nine other Fellows and, during the four years following the expiration of their terms of office, the ex-Presidents and ex-Vice-Presidents. The Council shall fill vacancies occasioned by death or resignation of any officer or other member of the Council, such appointees to serve until the next annual meeting of the Society.

ARTICLE V.—*Election of Officers and Council.*

The President, Vice-Presidents, and the Secretary-Treasurer shall be elected by a majority ballot at the annual meeting for the term of one year and three members of the Council shall, in a similar manner, be annually elected to serve for three years. The President and Vice-Presidents shall not be eligible for the same office for more than two consecutive years nor shall any retiring member of the Council be eligible for re-election at the same meeting.

The Editor and the Librarian shall be elected annually by the Council at the Council meeting preceding the annual meeting of the Society. They shall be subject to confirmation by majority ballot of the Society at the annual meeting.

The terms of the officers shall begin at the close of the meeting at which they are elected except that the retiring Editor shall retain the powers and duties of office so long as may be necessary to complete the then current issue of *Proceedings*.

ARTICLE VI.—*Duties of Officers and Council.*

The duties of the officers shall be such as usually appertain to their respective offices or may be specified in the by-laws. The duties of the Council shall be to pass upon candidates for membership, to decide upon papers offered for reading at the meetings, to supervise the examination of candidates and prescribe fees therefor, to call meetings, and, in general, through the appointment of committees and otherwise, to manage the affairs of the Society.

ARTICLE VII.—*Meetings.*

There shall be an annual meeting of the Society on such date in the month of November as may be fixed by the Council in each year, but other meetings may be called by the Council from time to time and shall be called by the President at any time upon the written request of ten Fellows. At least two weeks' notice of all meetings shall be given by the Secretary.

ARTICLE VIII.—*Quorum.*

Seven members of the Council shall constitute a quorum. Twenty Fellows of the Society shall constitute a quorum.

ARTICLE IX.—*Expulsion or Suspension of Members.*

Except for non-payment of dues no member of the Society shall be expelled or suspended save upon action by the Council with not more than three negative votes followed by a three-fourths ballot of the Fellows present and voting at a meeting of the Society.

ARTICLE X.—*Amendments.*

This constitution may be amended by an affirmative vote of two-thirds of the Fellows present at any meeting held at least one month after notice of such proposed amendment shall have been sent to each Fellow by the Secretary.

BY-LAWS

(As Amended November 13, 1936)

Article I.—*Order of Business.*

At a meeting of the Society the following order of business shall be observed unless the Society votes otherwise for the time being:

1. Calling of the roll.
2. Address or remarks by the President.
3. Minutes of the last meeting.
4. Report by the Council on business transacted by it since the last meeting of the Society.
5. New membership.
6. Reports of officers and committees.
7. Election of officers and Council (at annual meetings only).
8. Unfinished business.
9. New business.
10. Reading of papers.
11. Discussion of papers.

Article II.—*Council Meetings.*

Meetings of the Council shall be called whenever the President or three members of the Council so request, but not without sending notice to each member of the Council seven or more days before the time appointed. Such notice shall state the objects intended to be brought before the meeting, and should other matter be passed upon, any member of the Council shall have the right to re-open the question at the next meeting.

Article III.—*Duties of Officers.*

The President, or, in his absence, one of the Vice-Presidents, shall preside at meetings of the Society and of the Council. At the Society meetings the presiding officer shall vote only in case of a tie, but at the Council meetings he may vote in all cases.

The Secretary-Treasurer shall keep a full and accurate record of the proceedings at the meetings of the Society and of the Council, send out calls for the said meetings, and, with the approval of the President and Council, carry on the correspondence of the Society. Subject to the direction of the Council, he shall have immediate charge of the office and archives of the Society.

The Secretary-Treasurer shall also send out calls for annual dues and acknowledge receipt of same; pay all bills approved by the President for expenditures authorized by the Council of the Society; keep a detailed account of all receipts and expenditures, and present an abstract of the same at the annual meetings, after it has been audited by a committee of the Council.

The Editor shall, under the general supervision of the Council, have charge of all matters connected with editing and printing the Society's publications. The *Proceedings* shall contain only the proceedings of the meetings, original papers or reviews written by members, discussions on said papers and other matter expressly authorized by the Council.

The Librarian shall, under the general supervision of the Council, have charge of the books, pamphlets, manuscripts and other literary or scientific material collected by the Society.

ARTICLE IV.—*Dues.*

The dues shall be ten dollars for Fellows payable upon entrance and at each annual meeting thereafter, except in the case of Fellows not residing in the United States, Canada, or Mexico, who shall pay five dollars at the time stated. The dues shall be five dollars for Associates payable upon entrance and each annual meeting thereafter until five such payments in all shall have been made; beginning with the sixth annual meeting after the admission of an Associate as such the dues of any Associate heretofore or hereafter admitted shall be the same as those of a Fellow. The payment of dues will be waived in the case of Fellows or Associates who have attained the age of seventy years or who, having been members for a period of at least twenty years, shall have attained the age of sixty-five years.

It shall be the duty of the Secretary-Treasurer to notify by mail any Fellow or Associate whose dues may be six months in arrears, and to accompany such notice by a copy of this article. If such Fellow or Associate shall fail to pay his dues within three months from the date of mailing such notice, his name shall be stricken from the rolls, and he shall thereupon cease to be a Fellow or Associate of the Society. He may, however, be reinstated by vote of the Council, and upon payment of arrears of dues.

ARTICLE V.—*Designation by Initials.*

Fellows of the Society are authorized to append to their names the initials F. C. A. S.; and Associates are authorized to append to their names the initials A. C. A. S.

ARTICLE VI.—*Amendments.*

These by-laws may be amended by an affirmative vote of two-thirds of the Fellows present at any meeting held at least one month after notice of the proposed amendment shall have been sent to each Fellow by the Secretary.

EXAMINATION REQUIREMENTS

SYLLABUS OF EXAMINATIONS

Effective 1941 and thereafter

ASSOCIATESHIP

Part	*Sections*	*Subjects*
I	1	Algebra.
	2	Compound Interest and Annuities Certain.
II	3	Differential and Integral Calculus.
	4	Calculus of Finite Differences.
III	5	Descriptive and Analytical Statistics.
	6	Elements of Accounting, Including Corporate Accounting.
IV	7	Probabilities.
	8	Life Contingencies, Life Annuities and Life Assurances.
V	9	Policy Forms and Underwriting Practice in Casualty Insurance.
	10	Casualty Insurance Rate Making Procedure.

FELLOWSHIP

I	11	Investments of Insurance Companies.
	12	Insurance Law and Legislation.
	13	Insurance Economics.
II	14	Determination of Premium, Loss and Expense Reserves.
	15	Advanced Problems in Casualty Insurance Statistics.
	16	Advanced Problems in Casualty Insurance Accounting.
III	17	Individual Risk Rating.
	18	Social Insurance.
	19	Advanced Problems in the Underwriting and Administration of Casualty Insurance.

RULES REGARDING EXAMINATIONS FOR ADMISSION TO THE SOCIETY

1. **Dates of Examination.**

Examinations will be held on the first Wednesday and following Thursday during the month of April in each year, except that if such dates are in the week preceding Easter, the examinations will be held on the second Wednesday and following Thursday of April. The examinations will be held in such cities as will be convenient for three or more candidates.

2. **Filing of Application.**

Application for admission to examination should be made on the Society's blank form, which may be obtained from the Secretary-Treasurer. No applications will be considered unless received before the fifteenth day of January preceding the dates of examination. Applications should definitely state for what parts the candidate will appear.

3. **Fees.**

The examination fee is $2.00 for each part, with a minimum of $5.00 for each year in which the candidate presents himself; thus for one or two parts, $5.00, for three parts, $6.00, etc. Examination fees are payable to the order of the Society and must be received by the Secretary-Treasurer before the fifteenth day of January preceding the dates of examination.

4. **Associateship and Fellowship Examinations.**

(a) The examination for Associateship consists of five parts and that for Fellowship consists of three parts. A candidate may take any one or more of the five parts of the Associateship Examination. No candidate will be permitted to present himself for

any part of the Fellowship Examination unless he has previously passed, or shall concurrently present himself for and submit papers for, all parts of the Associateship Examination and all preceding parts of the Fellowship Examination. Subject to the foregoing requirement, the candidate will be given credit for any part or parts of either examination which he may pass.

(b) A candidate who has passed Associateship Parts I-IV prior to 1941, but who has not been enrolled as an Associate because of lack of the experience qualifications required by the examination rules effective prior to 1941, will be enrolled as an Associate upon passing Part V. Such a candidate may also take Fellowship Examination Parts I-III in the same year as Associateship Part V, subject to the provisions of paragraph (a) above.

(c) An Associate who has passed no part of the Fellowship Examination under the Syllabus effective prior to 1941 is required, in order to qualify for admission as a Fellow, to pass Associateship Examination Part V and Fellowship Examination Parts I-III.

5. Alternative to Passing of Fellowship Parts II and III.

As an alternative to the passing of Parts II and III of the Fellowship Examination, a candidate may elect to present an original thesis on an approved subject relating to casualty or social insurance. Such thesis must show evidence of ability for original research and the solution of advanced problems in casualty insurance comparable with that required to pass Parts II and III of the Fellowship Examination, and shall not consist solely of data of an historical nature. Candidates electing this alternative should communicate with the Secretary-Treasurer and obtain through him approval by the Examination Committee of the subject of the thesis. In communicating with the Secretary-Treasurer, the candidate should state, in addition to the subject of the thesis, the main divisions of the subject and general method of treatment, the approximate number of words and the approximate proportion to be devoted to data of an historical nature. All theses must be in the hands of the Secretary-Treasurer before the first Wednesday in April of the year in which they are to be considered. Where Part I of the Fellowship Examination is not taken

during the same year, no examination fee will be required in connection with the presentation of a thesis. All theses submitted are, if accepted, to be the property of the Society and may, with the approval of the Council, be printed in the *Proceedings*.

6. **Waiver of Examinations for Associate.**

The examinations for Associate will be waived under Article III of the Constitution only in case of those candidates who meet the following qualifications and requirements:

(a) The candidate shall be at least thirty-five years of age.

(b) The candidate shall have had at least ten years' experience in casualty actuarial or statistical work or in a phase of casualty insurance which requires a working knowledge of actuarial or statistical procedure or in the teaching of casualty insurance principles in colleges or universities. Experience limited exclusively to the field of accident and health insurance shall not be admissible.

(c) For the two years preceding date of application, the candidate shall have been in responsible charge of the actuarial or statistical department of a casualty insurance organization or of an important division of such department or shall have occupied an executive position in connection with the phase of casualty work in which he is engaged, or, if engaged in teaching, shall have attained the status of a professor.

(d) The candidate shall have submitted a thesis approved by the Examination Committee. Such thesis must show evidence of original research and knowledge of casualty insurance and shall not consist solely of data of an historical nature. Candidates electing this alternative should communicate with the Secretary-Treasurer and obtain through him approval by the Examination Committee of the subject of the thesis. In communicating with the Secretary-Treasurer, the candidate should state, in addition to the subject of the thesis, the main divisions of the subject and general method of treatment, the approximate number of words and the approximate proportion to be devoted to data of an historical nature.

RECOMMENDATIONS FOR STUDY

To assist students in preparation for the examinations, Recommendations For Study have been prepared. This lists the texts, readings and technical material which must be mastered by the candidates. Textbooks are loaned to registered students by the Society. By "registered students" is meant candidates who have signified their willingness to take the examinations by the payment of their examination fees.

LIBRARY

The Society's library contains all of the references listed in the Recommendations for Study with the exception of certain periodicals and publications subject to periodical revision. It also contains numerous other works on casualty actuarial matters. Registered students may have access to the library by receiving from the Society's Secretary the necessary credentials. Books may be withdrawn from the library for a period of two weeks upon payment of a small service fee and necessary postage.

The library is in the immediate charge of Miss Mabel B. Swerig, Librarian of the Insurance Society of New York, 107 William Street, New York City.