MARRIAGE AND BIRTH INSURANCES IN FRANCE *

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I. History

Birth and marriage insurances prior to the supervisory and regulatory law of May 26, 1921

It is now more than forty years since, as a sequel to writings of certain economists, there developed in France the idea that it would be of social value to create, side by side with the existing combinations of contingencies covered by insurance upon lives, new arrangements which would afford the benefit of insurance at those critical moments in the life of the family when the need for them arises.

These insurances would, for example, guarantee a child in the event of his or her eventual marriage a sum of money to assist in setting up the new household. Or, such insurance might guarantee to a young household a sum which would become due at the birth of each child in the home, so as to compensate in some degree the increase in expenses occasioned by the arrival of the child.

It is to meet these requirements and others that may present themselves in the life of the family, that marriage and birth insurance is designed.

The first French society of the kind here contemplated was founded by Frederic Nogues in 1900 under the name "La Famille Française," but this society had only an ephemeral existence.

Other attempts were made subsequently to launch enterprises of similar nature, but failed before the companies were formed.

* Presented by invitation. Translated into English by Dr. Alfred James Lotka. Incidental notes by E. W. Kopf.

After 1910, there were founded successively several societies for marriage insurance, as follows:

Society	*Year founded*
La Société Anonyme: "L'Épargne Dotale" at Paris	(1913)
La Société Mutuelle: "L'Avenir Familial" at Paris	(1910)
La Société Mutuelle: "La Dotation Française" at Lyons	(1919)
La Société Anonyme: "La Dot" at Lyons	(1920)

The premium systems of these various societies were established on more or less empirical, or at any rate on unpublished technical bases. In other countries, the movement for family insurance has been under way for many decades. Cornelius Walford in his article "Family Insurance" (*Insurance Cyclopedia*) described the early efforts in this direction in England since 1825. In the United States, the idea of family insurances has appeared sporadically, but in each instance approval to conduct such insurances has been denied. Over the next few decades much will be heard of family insurance projects based upon sound statistical and actuarial research. This applies particularly to sickness, accident and medical cost covers. Other historical information is given in the bibliography.

II. The French Supervisory and Regulatory Law of May 26, 1921 *

Since the technical operation of a society practicing this kind of insurance is, in many points, analogous to that of life insurance companies, and since, furthermore, it was desirable that the insured should benefit from the same guarantees as to solvency and the performance of contracts as are given to persons carrying life insurance, the French parliament thought it desirable to extend to societies which undertake to provide marriage and birth insurances the prescriptions of the law of March 17, 1905, which subject the life insurance companies to government control. Hence the regulatory law of May 26, 1921, affecting these marriage and birth insurance societies.

* See text in Paul Sumien's work, noted in bibliography, on the place of birth and marriage insurance in the French program of terrestrial and personal insurance covers.

Decree of June 22, 1921. By the decree of June 22, 1921, made in accordance with the law of May 26, 1921, the Ministry of Labor fixed the actuarial prescriptions for the operation of these societies. These prescriptions by decree comprise more particularly the following:

Article I. The basis for premiums as well as for the corresponding mathematical reserves shall be calculated at minimum as follows:

1. *Interest rate assumed*: The interest rate to be $4\frac{1}{4}$ per cent.
2. *Mortality tables assumed*: Table to be the R. F. Table*; however, in case the payment of the periodic premiums should be subject to the survival of a third person who is not a beneficiary to the amounts payable in the case of marriage or birth of a child, use should be made of the A. F. Table.*
3. *Marriage and birth tables*: The marriage and birth rates shall be deduced from the results of actual experience and of the annual movement of the population dating from the year 1900.
4. *Expense provision*: Management and acquisition costs: A charge of four per cent. of the gross premium for cost of operation, and of six per cent. of the gross premium is allowed for cost of acquisition and of collection.

Article II. The mathematical reserves must not be less than those for net premiums, equal to the gross premiums calculated as stated above, minus the charge estimated to cover the cost of acquisition and collection.

In fact Article II provides that the obligatory mathematical reserves—so-called inventory reserves—must amount to $\frac{4}{90\text{ths}}$ (4.444%), in excess of the mathematical reserves computed on the pure premiums.

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* The abbreviations R. F. and A. F. which are in constant use denote, respectively, the tables *Rentiers Français* and *Assurés Français*, which are drawn from the combined experience of four French companies. Copies of these tables will be found in M. M. Dawson, Practical Lessons in Actuarial Science, 1905, Vol. II, pp. 208-223.

Marriage and Birth Tables

It is evident that the operation of societies of this kind requires the use of marriage and birth tables, hence the Ministry of Labor in Article I, Section 3 of the decree indicates under what conditions these tables should be prepared.

Owing to the very recent origin of the existing societies it would not be possible to construct so-called experience tables, that is, tables based on the statistics of the experience of the companies. It was, therefore, necessary to rest so far as possible upon statistical information available in the official statistics published annually regarding the movement of population in France, and upon the results of the quinquennial censuses of the population of France.

A number of years ago a French actuary, Mr. Eugène Gaillard,* published a thesis presented in candidacy for associate membership in the French Institute of Actuaries, dealing with nuptiality and natality. This was a very complete technical monograph entitled "L'Assurance au Mariage, et l'Assurance a la Natalité" ("Marriage and Birth Insurance"). The author had constructed, upon the basis of statistical data analogous to those indicated by the Ministry of Labor in its decree of June 22, 1921, a marriage table for each of the two sexes; to this he appended commutation tables by means of which various interesting problems in the field of marriage probabilities can be solved. He also gave actuarial formulae regarding the principal kinds of marriage insurance.

Unfortunately, the statistical work of Eugène Gaillard could not be used according to the provisions of the law, for this actuary had based his work on the quinquennial censuses and the annual movement of the population since 1891, whereas the decree prescribes that there shall be used as statistical data only those for the years from 1900 on.

The Ministry of Labor was, therefore, led to undertake for itself the preparation of marriage and birth tables, such as were called for in the provisions of the law, utilizing in the construction of these tables official statistics within the time limits fixed by the decree. The results have been published by the French

* Gaillard's works are in the collection of the Insurance Society of New York, the author's gift transmitted by Mr. Balu.

Ministry of Labor* in a report designated as Recueil No. 10, in which will be found the technical rules for the construction of these tables and also all the actuarial data useful in establishing premiums and for the evaluation of reserves for the principal combinations of contingencies in marriage and birth insurance.

These tables were prepared under the direction of Mr. Louis Weber, the Assistant Chief of the Supervisory Insurance Department of the Ministry of Labor, and Vice-President of the French Institute of Actuaries, who, incidentally, made a summary report on the work to the French Institute of Actuaries. This report has since appeared in the Bulletin of the Society (Bulletin No. 116, March, 1924).

In 1926, Mr. Eugène Gaillard, amplifying the first study made and published by him in 1913, and using this time the tables of the Ministry of Labor, published a complete brochure under the title "Control and Technique of Marriage and Birth Insurance", in which he indicates the formulae of the principal combinations of contingencies that arise in connection with such insurances. Mr. Gaillard also gives a number of basic tables.†

III. Developments Subsequent to the Law of May 26, 1921

Subsequent to the promulgation of the law of May 26, 1921, the existing societies requested and obtained registration according to its provisions.

Two further societies were founded at this time, "La Maternelle Française," in Dreux and "La Famille," in Paris. The last-mentioned society had only a very short life. Certain other societies relinquished their portfolios and discontinued their operations.

Since then the Société Anonyme "L'Avenir Familial" has been founded in Paris.

At the present time there remain in existence only three societies, and it is to be noted that each of them functions in con-

* Recueil de documents relatifs aux Assurances sur la vie par le Ministère du Travail. No. 10. Tables de Nuptialité et de Natalité. Paris. Imprimerie Nationale. 1922. In collection of Insurance Society of New York.

† Controle et Technique des Assurances Nuptialité-Natalité. Paris. Librairie des Assurances. 1926.

nection with another society for insurance upon lives or for capitalization (likewise subject to government control).

These three societies are as follows: (1) La Société Anonyme "L'Épargne Dotale", the name of which has just been changed to "La Dotale," which functions in connection with the capitalization society "L'Épargne Mutuelle"; (2) the Société Anonyme "La Maternelle Française" which has taken up the portfolio of the two Lyons societies: "La Dotation Française" and "La Dot". In connection with this society "Maternelle Française" there functions a life insurance company, "La Maternelle-Vie"; (3) the Société Anonyme "L'Avenir Familial", which has taken over the portfolio of the mutual society "L'Avenir Familial". This society is at the present time still under the direction of the founder of the mutual society, Mr. Victor Pougez, who is also the founder of a society called "Avenir Familial Vie".

Status as of December 31, 1930. On December 31, 1930, the sums insured by three companies were about 95,000,000 francs and the 1930 new business amounted to about 20,000,000 francs.

The exposition which will now be made is only an abridged reproduction of the actuarial studies which we have just cited and to which reference can always be made. We shall here limit ourselves to indicating along broad lines the actuarial-technical and practical aspects of the problems of marriage and birth insurance. We have conserved the original notation employed by the authors.

IV. Actuarial Basis of Marriage Insurance

(a) *Annual Nuptiality Rate.* For the technical operation of marriage insurance it is first necessary to know the annual nuptiality rate and to establish a marriage table.

The annual nuptiality* rate λ_x is defined as follows:

$$\lambda_x = \frac{m_x}{W_x} \tag{1}$$

where m_x is the number of marriages taking place between the ages x and $x+1$ among a group of W_x single persons of age x and of the sex under consideration.

* This *nuptiality* rate defined by equation (1) is the probability for a single person of specified sex and at age x of marrying within one year.

If we consider, on the other hand, by way of comparison, a mortality table and denote by V_x the number living at age x, and if we write

$$\alpha_x = \frac{W_x}{V_x} \tag{2}$$

where α_x is the ratio of single persons W_x at age x to the total number V_x of persons of the same sex and same age, and if, further, it is assumed that the mortality is the same for single persons as for the total population, then we have the following relations between the values of α_x and the annual nuptiality rate λ_x:

$$1 - \lambda_x = \frac{\alpha_{x+1}}{\alpha_x} \tag{3}$$

$$\lambda_x = \frac{\alpha_x - \alpha_{x+1}}{\alpha_x} \tag{3a}$$

This relation follows from the equation

$$W_x \, p_x \, (1 - \lambda_x) = W_{x+1} \tag{4}$$

or, since

$$W_x = \alpha_x \, V_x$$

$$\alpha_x \, V_x \, p_x \, (1 - \lambda_x) = \alpha_{x+1} \, V_x \, p_x \tag{4a}$$

where p_x denotes the probability of surviving one year at age x.

(b) *Marriage Tables.* To compute the marriage tables, the Ministry of Labor set out from the expression

$$\lambda_x = \frac{m_x}{W_x} \tag{1}$$

in which the numerator m_x is obtained from the statistics of annual marriages in France given in five year age groups. The denominator W_x, which represents the number of single persons, is derived from quinquennial census reports in five-year age groups.

For smoothing the crude nuptiality rates an interpolation formula has been developed which will be found described in all important details in the Recueil No. 10 (page 7) cited above. By means of this interpolation formula it has been possible to derive adjusted or smoothed annual nuptiality rates which will be found in the Appendix of this paper together with the numerical values of the ratios α_x (see Appendix, Table II and IIa).

Inasmuch as the minimum age required for legal marriage in France is fifteen years for women and eighteen years for men, the nuptiality rates in the table begin at ages fifteen and eighteen, respectively, for females and for males. Furthermore, these rates have been cut off for both sexes at the age of fifty, since above this age the nuptiality rate for both sexes may be regarded as practically zero.

From the annual nuptiality rates λ_x it has been possible to derive step by step the numerical values of the ratios α_x, α_{x+1}, $\alpha_{x+2}, \cdots$ by virtue of the relation (3), it being understood that for the ages fifteen and eighteen, according to sex, and for all ages below this, the corresponding ratio is unity, because below these ages the number of single persons W_x is identical with the number of persons living V_x according to the particular life table chosen.

The numerical value of the ratios α_x, $\alpha_{x+1}, \cdots$ enables us to construct a table of single persons living at age x since we have

$$W_x = \alpha_x V_x \tag{2}$$

As a complement of this table of single persons W_x surviving to age x it is easy to determine for the same age x the number of persons $W_x - W_{x+1}$ which are eliminated during the current year of life, these eliminations being caused on the one hand by marriages m_x and on the other hand by deaths ${}^c d_x$ among single persons, the symbol ${}^c d_x$ denoting the number of single persons of the sex under consideration who die between the ages x and $x + 1$. We have, then,

$$W_x - W_{x+1} = m_x + {}^c d_x \tag{5}$$

or, in view of (1), (2) and (3)

$$m_x = \lambda_x W_x = \frac{\alpha_x - \alpha_{x+1}}{\alpha_x} W_x = (\alpha_x - \alpha_{x+1}) V_x$$

hence
$$m_x = (\alpha_x - \alpha_{x+1}) V_x \tag{6}$$

whence we deduce from (5) and (6)

$$W_x - W_{x+1} = \alpha_x V_x - \alpha_{x+1} V_{x+1} = (\alpha_x - \alpha_{x+1}) V_x + {}^c d_x$$

and from this
$${}^c d_x = \alpha_{x+1} (V_x - V_{x+1}) \tag{7}$$

the expression (5) which defines the law of decrement for the group of single persons is, therefore, given by the formula

$$W_x - W_{x+1} = (\alpha_x - \alpha_{x+1})\, V_x + \alpha_{x+1}\,(V_x - V_{x+1}) \qquad (5a)$$

A complete marriage table should show, it seems to us, the various elements indicated above, that is to say, the number W_x of single persons surviving to age x; the number m_x of marriages taking place between the ages x and $x+1$ and the number of deaths ${}^c d_x$ which occur between these same ages among the members of the group who have remained single during the year.

So far as we know these data have not been published. They are essentially of formal interest and without any great practical utility.

(c) *Commutation Table R. F. at 4¼%, Covering Marriage.*

The situation is very different as regards the commutation tables. These have been prepared on the basis of an interest rate of 4¼% and the elements of the R. F. table combined with the nuptiality elements.

These tables naturally end at age fifty in view of the facts set forth above.

If we denote in the usual notation by the letters D_x, N_x, C_x, R_x, M_x, the commutation columns corresponding to a mortality table V_x, the commutation terms, corresponding to the marriage table with $t = .0425$, are as follows:

(d) *Commutation in Case of Survival.*

$${}^c D_x = W_x\,(1+t)^{-x} = \alpha_x\, V_x\,(1+t)^{-x} = \alpha_x\, D_x \qquad (8)^*$$

$${}^c N_x = \sum_{p=0}^{p=50-x} \alpha_{x+p}\, D_{x+p}, \text{ or, more simply, } \Sigma\, \alpha_{x+p}\, D_{x+p} \qquad (8a)$$

Note: In this formula, and the other summation formulae which follow, we shall omit the indications of the limits of summation on the understanding that they remain the same throughout.

* The author's original notation has been retained throughout. For a glossary showing the corresponding English notation see Appendix. The letter t (*taux d' interêt*) corresponds to i of our customary notation.

(e) *Commutation in Case of Death while Unmarried.* In view of (7)

$$ {}^cC_x = {}^cd_x\,(1+t)^{-x-\frac{1}{2}} = \alpha_{x+1}\,C_x \tag{9}$$
$$ {}^cM_x = \Sigma\ {}^cC_{x+p} \tag{9a}$$
$$ {}^cR_x = \Sigma\ {}^cM_{x+p} \tag{9b}$$

(f) *Commutation in Case of Marriage.*

$$ {}^mC_x = m_x\,(1+t)^{-x-\frac{1}{2}} \tag{10}$$
$$ {}^mM_x = \Sigma\ {}^mC_{x+p} \tag{10a}$$
$$ {}^mR_x = \Sigma\ {}^mM_{x+p} \tag{10b}$$

It may be noted that, in view of (6), ${}^mC_x$ may equally well be expressed as a function of the commutation terms in case of survival and in case of death in the unmarried state

$$ {}^mC_x = (1+t)^{-\frac{1}{2}}\,{}^cD_x - {}^cC_x - (1+t)^{\frac{1}{2}}\,{}^cD_{x+1} \tag{10c}$$

(g) *Commutation in Case of Elimination by Marriage or Death.* We deduce from (10c)

$$ {}^sC_x = {}^mC_x + {}^cC_x = (1+t)^{-\frac{1}{2}}\,{}^cD_x - (1+t)^{\frac{1}{2}}\,{}^cD_{x+1} \tag{11}$$

and since $$W_x = \alpha_x\,V_x$$

we have

$$ {}^sC_x = (1+t)^{-\frac{1}{2}}\,\alpha_x\,D_x - (1+t)^{\frac{1}{2}}\,\alpha_{x+1}\cdot D_{x+1} \tag{11a}$$

Furthermore, if we have

$$\alpha_x = \alpha_{x+1} = 1$$

as is the case for ages below the age of nubility, we find the formula

$$C_x = (1+t)^{-\frac{1}{2}}\,D_x - (1+t)^{\frac{1}{2}}\,D_{x+1}$$

Note: It should be recalled that according to common custom the amounts in case of death are supposed to be payable at the time of death, which on the average, is mid-year, whence the factor $(1+t)^{-\frac{1}{2}}$ is introduced in the formulae.

V. Elementary Combinations of Contingencies of Marriage

The aforesaid fundamental data having been established, we shall now proceed to enumerate the most interesting elementary combinations of marriage contingencies and we shall at the same time indicate their actuarial expression by means of the commutation terms defined above.

We shall give in an Appendix certain tables setting forth the numerical values, for each of the two sexes, of several of these combinations.

We shall suppose in the formulae that the attained age at which the insurance is to terminate is the age ω, and, in the numerical applications, this age will be thirty-five years, the age generally adopted as the extreme limit of contracts issued by the companies. In this case $\omega = 35$. The following formulae give the single premium for these combinations:

1. The amount payable in case of survival in the unmarried state to age ω

$$\frac{{}^cD_\omega}{{}^cD_x} \text{(See Appendix, Table IV)} \tag{12}$$

2. The amount payable in case of marriage before age ω

$$\frac{{}^mM_x - {}^mM_\omega}{{}^cD_x} \text{ (See Appendix, Table IV)} \tag{13}$$

3. Amount payable in case of marriage before age ω or, at the latest, in case of survival in the unmarried state at age ω

$$\frac{U_x}{{}^cD_x} = \frac{{}^mM_x - {}^mM_\omega + {}^cD_\omega}{{}^cD_x} \tag{14}$$

In view of the importance of this combination we shall designate it by the expression $\dfrac{U_x}{{}^cD_x}$

4. Amount payable in case of death if death takes place before age ω and if at the moment of death the insured is still unmarried.

In view of the expression (9a) we have

$$\frac{{}^cM_x - {}^cM_\omega}{{}^cD_x} \text{(See Appendix, Table IVa)} \tag{15}$$

5. If, similarly, we consider the case of n annual premiums reimbursable in case of death before age ω in the unmarried state, we have, similarly, in view of expression (9b)

$$\frac{{}^cK_x}{{}^cD_x} = \frac{{}^cR_x - {}^cR_{x+n} - n\,{}^cM_\omega}{{}^cD_x} \tag{16}$$

In view of its importance we designate this expression by

$$\frac{{}^cK_x}{{}^cD_x}$$

6. Similarly, the present worth $\frac{{}^cA_x}{{}^cD_x}$ of a temporary annuity for n years payable in advance, but only so long as the insured remains unmarried, will be

$$\frac{{}^cA_x}{{}^cD_x} = \frac{{}^cN_x - {}^cN_{x+n}}{{}^cD_x} \tag{17}$$

We shall not here enter into the case of the multiple combinations which might be taken into consideration and for which the formulae can readily be established.

VI. Marriage Endowment Policy

We shall conclude by giving the necessary formula for setting up the schedule of rates for the type of policy which is most generally written by the companies and which may be designated by the name *Marriage Endowment Policy*:

Amount payable at the time of marriage or at the latest at age ω.

Premiums payable only during n years and returnable in case of death before marriage or at age ω.

If we designate by π'' the gross annual premium of the policy,

and by $\frac{E_x}{{}^cD_x}$ the part of the loading added to the net premium which is not proportional to the premium (and the rate of which may vary according to the company and the schedule of rates),

if we further designate by $\varepsilon\,\pi''$ the part of this loading which is proportional to the annual premium π'',

and if we take into account the formulae given above, in particular formulae (14), (16), (17) we have

$$(1-\varepsilon)\,\pi'' = \frac{U_x + E_x}{{}^cA_x} + \frac{\pi''\,{}^cK_x}{{}^cA_x} \tag{18}$$

hence

$$\pi'' = \frac{U_x + E_x}{(1-\varepsilon)\,{}^cA_x - {}^cK_x} \tag{19}$$

The pure premium π is

$$\pi = \frac{U_x}{{}^cA_x} + \pi''\frac{{}^cK_x}{{}^cA_x} \tag{20}$$

In the case that the loading is proportional solely to the gross premium, we have $E_x = 0$, and the formula (19) becomes

$$\pi'' = \frac{U_x}{(1-\varepsilon)\, {}^cA_x - {}^cK_x} \tag{20a}$$

and the pure annual premium reduces to

$$\pi = (1-\varepsilon)\, \pi'' \tag{20b}$$

If it is desired to apply the minimum schedule of rates provided by the decree, it is only necessary, in the formulae (20a) and (20b) to put $\varepsilon = .10$.

It should, by the way, be noted that whatever be the charge adopted, the annual inventory premium π' is always

$$\pi' = \left(1 + \frac{.4}{9}\right) \pi = 1.04444\, \pi \tag{21}$$

The Marriage Endowment Policy discussed above is quite frequently modified by the stipulation that the annual premiums shall cease and that the contract shall expire in the case of the death of a third person termed "contractant" (usually the father) who is responsible for the payment of the premiums.

In this case the probability of survival of this person must be introduced. According to the decree, the table known as the A. F. experience table must be applied to this person.

It would then be necessary to construct complementary commutation tables, into which the actuarial elements of this person would enter.

We shall not here discuss the formulae relating to this special form of policy.

VII. Tariffs and Contracts of Marriage Insurance

Although the number of theoretically possible combinations of contingencies is considerable, those combinations which can actually become objects of insurance in practice are rather limited, for in marriage insurance any initial selection, that is to say, selection at the time of signing the contract, would with certainty work to the disadvantage of the companies. It is clear, for example, that if the company accepted marriage insurance on a young girl twenty years of age, there would be strong presumption that this insurance was taken out by the applicant with the prospect of an early marriage in view. This would no longer be insurance, but speculation.

In practice the companies offer only combinations (policies) which approach more or less that which was expounded above under the name of Marriage Endowment Policy, with or without cessation of payment of the annual premiums, and expiration of the contract, in case of decease of a third contracting party responsible for the payment of the premiums.

Lastly, in order to preclude all danger of speculation, the companies accept for insurance only children whose age is sufficiently far removed from the age of marriage. In practice the schedule of rates stops at age ten for females and thirteen for males, that is to say, five years before the insured attains the legal age of marriage.

VIII. Mathematical Reserves

We shall not here discuss the computation of the mathematical reserves, the evaluation of which presents no technical difficulty. We will only draw attention to the fact that the increase with time of the value of the mathematical reserve of a marriage insurance contract is far from presenting the same regular progression as that of a contract of insurance upon lives.

During the first few years of the contract, that is to say, during the deferred period, the period during which, owing to the age of the child, there is no possibility of marriage, the contract is in reality a contract for "Pure Endowment with return."

But when the insured child attains the legal age of marriage and, more particularly, a few years after the age when the effective nuptiality rate is high, the character of the mathematical reserves becomes modified, their value increases only slowly, especially if the contract expires, since a large part of the receipt in premiums and interest is absorbed by the risk of marriage which at certain ages goes so far as to exceed a rate of twenty per cent. annually.

IX. Cash Surrender Value and the Value Applied to Purchase Paid-up Insurance for a Reduced Amount (hereinafter termed Reduction Value)

The same irregularity in the progress-of-time element presents itself in the succession of cash surrender values and reduction values of marriage insurance.

We shall here conclude our summary exposition; marriage insurance is as yet much too recent in France to enable us to foresee its future.

Will the experience of years enable us to establish this insurance on a solid statistical basis? Personally, I rather doubt it; for to a far greater extent than the mortality on which life insurance is based, does the nuptiality rate feel the repercussion of economic conditions and the moral influences of the period covered by the statistics. Now, the frequency of marriages and the variations in the age of one or the other of the two parties contracting marriage, are data which vary from one epoch to another and from one social class to another, and the deductions drawn from the statistics, even though they may be perfectly established on the basis of the recent past, do not enable us to make reliable forecasts for the future.

X. Birth Insurance

(a) *Annual Reproductive Rate* (of married persons).

If we designate by:

n_x the number of legitimate live births to a father or mother of age x (that is to say, between age x and $x+1$)

H_x the number of married persons of age x at risk

then we have by definition of the annual *reproductive** rate of married persons

$$\mu_x = \frac{n_x}{H_x} \tag{22}$$

The number of married persons H_x would be given by the formula

$$H_x = V_x - W_x + \frac{m_x}{4} \tag{23}$$

* This *reproductive* rate defined by equation (22) is the probability for a married person of specified sex and at age x having a child within a year.

The number H_x of married men or married women at age x capable of giving rise to a birth during the course of the year of age x has been determined hypothetically by adding to the difference

$$V_x - W_x$$

between the number of those living at age x and that of single persons of the same age, one-quarter of the number of marriages contracted between the age x and age $x+1$, a hypothesis which amounts to assuming a uniform distribution of marriages during the year of age under consideration.

(b) *Birth Tables*

The brochure published by the Ministry of Labor (Recueil No. 10, *op. cit.*) indicates the ways and means by which it has been possible to determine the annual reproductive rates, primarily the *crude rates* and secondly the *adjusted rates.*

There are appended hereto for each sex the adjusted reproductive rates which, like the annual nuptiality rates, have been carried only to the fiftieth year of age.

(c) *Commutation Tables for Birth Insurance*

For the practical application of the data on reproductive rates it has been necessary to construct commutation tables based on an interest rate of 4¼ per cent.; that is to say, $t = .0425$. In conformity with the usual custom, the amounts are assumed payable at the middle of the year of insurance.

We have

$$E_x = n_x (1+t)^{-x-\frac{1}{2}} \tag{24}$$

or, taking account of (22)

$$E_x = \mu_x H_x (1+t)^{-x-\frac{1}{2}} \tag{24a}$$

furthermore, the summation term

$$U_x = \Sigma\ E_{x+p} \quad \text{(See Appendix, Table Va)} \tag{25}$$

To these two commutation columns prepared by the Ministry of Labor, it has seemed interesting to add two complementary columns which have been given by Mr. Eugène Gaillard in his brochure "Control and Technique of Marriage and Birth Insurance"

$$_{\mu}D_x = \mu_x D_x\,, \qquad D_x = V_x (1+t)^{-x} \tag{26}$$

and the summation

$$_{\mu}N_x = \Sigma\ _{\mu}D_{x+p} \qquad \text{(See Appendix, Table Vb)} \tag{26a}$$

(d) *Fundamental Character of Birth Insurance*

What distinguishes birth insurance from other insurances such as life insurance and marriage insurance (the latter contemplating in practice only the first marriage of the insured), is that birth insurance is a form of insurance against a risk subject to repetition.

As the statistical documents which have furnished the materials for the construction of the tables indicate only the number of live-born children of a group of men or women married before the age specified, it follows that the reproductive rates deduced from these tables give simply the probability of the birth of a child at that age, regardless of whether that child was preceded or followed by births of other children to the same insured person.

The computations which will follow are, therefore, based essentially on the assumption that the risk of having a child born, as given in the tables, applies to each birth, irrespective of prior births to the same parent. The payment of the amount of insurance, if made upon some one particular birth, still leaves in force the obligation that the amount may have to be paid again on each subsequent birth.

Note: One could conceive of birth insurance in which the amount of the policy were payable only after the birth of a specified order of child, the first, the second, etc., occurring during the state of marriage of a given person (man or woman); but the tables as at present constructed do not enable us to solve this problem.

In the types of birth insurance termed "repetitive" (*à repetition*), which are the only ones that we shall here study, two principal cases may present themselves:

Case I. The insured of age x is married at the time the contract is made.

The risk of a child being born may exist from the date of the original contract.

The single premium insuring the payment of $1 on each and every birth arising from the marriage of the insured persons at age x is evidently

$$M_x = \Sigma \frac{V_{x+p}}{V_x} \mu_{x+p} (1+t)^{-p-\frac{1}{2}} \tag{27}$$

and by virtue of the commutation terms (26) and (26a)

$$\left.\begin{aligned} M_x &= \Sigma\, \frac{\mu_{x+p} D_{x+p}}{D_x} (1+t)^{-\frac{1}{2}} \\ &= \Sigma\, \frac{\mu D_{x+p}}{D_x} (1+t)^{-\frac{1}{2}} \\ &= (1+t)^{-\frac{1}{2}} \frac{\mu N_x - \mu N_\omega}{D_x} \end{aligned}\right\} \tag{28}$$

The assumption is here made that the marital state of the insured is unchanged, for the present computations do not take into account separation, divorce or death of the husband or wife of the insured.

Case II. The insured, of age x, is single on the date of signing the contract.

In order that a birth should take place at age $x+p$ of the insured, it is necessary that at that epoch he or she be married. If H_x is the number of those married at age x, the number of survivors from these persons at age $x+p$ is

$$H_x \frac{V_{x+p}}{V_x}$$

and since the number of those married at age $x+p$ is H_{x+p}, the number of those which entered the married state between the ages x and $x+p$, and which still survive at the age $x+p$, among the group of single persons W_x at age x, is

$$H_{x+p} - H_x \frac{V_{x+p}}{V_x} \tag{29}$$

The single premium insuring the payment of one dollar at each and every birth arising after the marriage of a person as yet single at age x, but before he or she reaches the age ω, will be

$$\frac{1}{W_x} \Sigma \left(H_{x+p} - H_x \frac{V_{x+p}}{V_x} \right) \mu_{x+p} (1+t)^{-p-\frac{1}{2}} \tag{30}$$

Now, by virtue of (24a) and (25) the positive term in (30) becomes

$$\frac{1}{{}^cD_x} \Sigma\, E_{x+p} = \frac{U_x - U_\omega}{{}^cD_x}$$

the negative term in (30) can evidently be written

$$\frac{\mu_x H_x (1+t)^{-x} (1+t)^{-\frac{1}{2}}}{W_x (1+t)^{-x}} \Sigma\, \frac{\mu_{x+p} V_{x+p}}{\mu_x V_x} \frac{(1+t)^{-x-p}}{(1+t)^{-x}}$$

whence by virtue of (8), (24a) and (26) we have

$$\frac{E_x}{{}^cD_x} \Sigma \frac{{}_\mu D_{x+p}}{\mu D_x}$$

and by the summations (25) and (26a) the negative term becomes

$$\frac{E_x}{{}^cD_x} \frac{{}_\mu N_x - {}_\mu N_\omega}{\mu D_x}$$

The complete expression therefore becomes

$$\frac{U_x - U_\omega}{{}^cD_x} - \frac{E_x}{{}^cD_x} \frac{{}_\mu N_x - {}_\mu N_\omega}{\mu D_x} \tag{31}$$

If the insured is not yet at a marriageable age at the time the contract is written, the formula given above remains applicable, but in simplified form, since $E_x = 0$ and the formula (31) accordingly reduces to its positive term alone, that is to

$$\frac{U_x - U_\omega}{{}^cD_x}$$

XI. Combinations of Contingencies Offered by the Insurance Companies

It is understood that in practice companies cannot accept marriage insurance on persons already married, nor, for similar reasons, on single persons who have already passed the age of nubility.

If such insurances were issued, only those would actually take out insurance who already foresaw a practically certain benefit, and the laws of probabilities are inapplicable in such a case.

In point of fact, there is at present only one society which issues birth insurance, and even this, as a rule, merely as a complement to marriage insurance.

The typical contract of combined marriage and birth insurance would be one which assured:

1. At the first marriage of the insured the payment of a stated amount.
2. The payment of another stated amount, usually less than under (1), (or an educational allowance) at the birth of each and every legitimate live-born child of the insured.

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APPENDIX

TABLE OF CONTENTS

Commutation Tables

The tables given below have been constructed on an interest rate of four and a quarter per cent., by the aid of the nuptiality and reproductive rates calculated as has been explained in the foregoing, and on the basis of the life table R. F.

TABLE I.

Explanation of Symbols

Marriage Insurance

Author's Notation	English Notation	Definition
x	x	Age of the insured.
V_x	l_x	Number* attaining age x.
W_x	(bl_x)	Number* of single persons attaining age x.
d_x	d_x	Number* dying between ages x and $x+1$.
$q_x = \frac{d_x}{V_x}$	$q_x = \frac{d_x}{l_x}$	Probability at age x of dying within one year.
m_x	$(bm)_x$	Number* marrying between ages x and $x+1$.
$\lambda_x = \frac{m_x}{W_x}$	$(bq)_x^m = \frac{(bm)_x}{(bl)_x}$	Probability at age x of marrying within one year.
$\alpha_x = \frac{W_x}{V_x} = \alpha_{x-1}\,(1-\lambda_{x-1})$	$\alpha_x = \frac{(bl)_x}{l_x} = \alpha_{x-1}\{1-(bq)_{x-1}^m\}$	Proportion single at age x.
${}^cd_x = W_x\,(1-\lambda_x)\,q_x$ $= \alpha_{x+1}\,d_x$	$(bd)_x = (bl)_x\{1-(bq)_x^m\}q_x$ $= \alpha_{x+1}\,d_x$	Number* dying single between ages x and $x+1$.
$D_x = V_x\,(1+t)^{-x}$	$D_x = v^x\,l_x = l_x(1+i)^{-x}$	
$C_x = d_x\,(1+t)^{-x-½}$	$\bar{C}_x = v^{x+½}\,d_x = d_x\,(1+i)^{-x-½}$	
${}^cD_x = \alpha_x\,D_x$	$(bD)_x = \alpha_x\,v^x\,l_x = v^x\,(bl)_x$	
${}^cN_x = \sum_{p=0}^{p=50-x} {}^cD_{x+p}$	${}_{51}(bN)_x = \sum_{p=0}^{p=50-x} (bD)_{x+p}$	
${}^cC_x = {}^cd_x\,(1+t)^{-x-½}$ $= \alpha_{x+1}\,C_x$	$(b\bar{C})_x = v^{x+½}(bd)_x = \alpha_{x+1}\,\bar{C}_x$	

* In life table cohort.

TABLE I.—*Continued*

Marriage Insurance

Author's Notation	English Notation	Definition
${}^{c}M_x = \sum_{p=0}^{p=50-x} {}^{c}C_{x+p}$	${}_{51}(b\overline{M})_x = \sum_{p=0}^{p=50-x} (b\overline{C})_{x+p}$	
${}^{c}R_x = \sum_{p=0}^{p=50-x} {}^{c}M_{x+p}$	${}_{51}(b\overline{R})_x = \sum_{p=0}^{p=50-x} (b\overline{M})_{x+p}$	
${}^{m}C_x = m_x\ (1+t)^{-x-\frac{1}{2}}$	$(bm\cdot\overline{C})_x = v^{x+\frac{1}{2}}\ (bm)_x$	
${}^{m}M_x = \sum_{p=0}^{p=50-x} {}^{m}C_{x+p}$	${}_{51}(bm\cdot\overline{M})_x = \sum_{p=0}^{p=50-x} (bm\cdot\overline{C})_{x+p}$	
${}^{m}R_x = \sum_{p=0}^{p=50-x} {}^{m}M_{x+p}$	${}_{51}(bm\cdot\overline{R})_x = \sum_{p=0}^{p=50-x} (bm\cdot\overline{M})_{x+p}$	

TABLE I.—*Continued*

Birth Insurance

Author's Notation	English Notation	Definition
$H_x = V_x - W_x + \frac{m_x}{4}$	$(ml)_x = l_x - (bl)_x + \frac{(bm)_x}{4}$	Number* attaining age x exposed to risk of having child in wedlock before attaining age $x+1$.
n_x	Not provided for. Use n_x	Number* of births to parent of stated sex between ages x and $x+1$.
$\mu_x = \frac{n_x}{H_x}$	Not provided for. Use $\mu_x = \frac{n_x}{(ml)_x}$	Probability that a married person age x will have a child before reaching age $x+1$.
$E_x = n_x\,(1+t)^{-x-\frac{1}{2}}$	Not provided for. Use $(\overline{nC})_x = v^{x+\frac{1}{2}}\,n_x$	
$U_x = \sum_{p=0}^{p=50-x} E_{x+p}$	Not provided for. Use ${}_{51}(\overline{nM})_x = \sum_{p=0}^{p=50-x} (\overline{nC})_{x+p}$	

Complementary Columns

${}_{\mu}D_x = \mu_x\,D_x$	$D_x^{\mu} = \mu_x\,D_x$	
${}_{\mu}N_x = \sum_{p=0}^{p=50-x} \mu_{x+p}\,D_{x+p}$	${}_{51}N_x^{\mu} = \sum_{p=0}^{p=50-x} \mu_{x+p}\,D_{x+p}$	

* In life table cohort.

NOTE: Compilation by Mortimer Spiegelman.

TABLE II

PROBABILITY AT AGE x OF MARRYING WITHIN ONE YEAR

Age	Males	Females
15 years.....		0.0179
16 —........		0.0245
17 —........		0.0340
18 —........	0.0027	0.0479
19 —........	0.0067	0.0681
20 —........	0.0163	0.0964
21 —........	0.0317	0.1323
22 —........	0.0527	0.1693
23 —........	0.0762	0.1945
24 —........	0.1098	0.1989
25 —........	0.1521	0.1855
26 —........	0.1934	0.1644
27 —........	0.2156	0.1428
28 —........	0.2128	0.1236
29 —........	0.1917	0.1076
30 —........	0.1657	0.0942
31 —........	0.1420	0.0829
32 —........	0.1223	0.0732
33 —........	0.1065	0.0647
34 —........	0.0937	0.0571
35 —........	0.0830	0.0504
36 —........	0.0740	0.0443
37 —........	0.0661	0.0388
38 —........	0.0593	0.0340
39 —........	0.0524	0.0298
40 —........	0.0465	0.0261
41 —........	0.0411	0.0230
42 —........	0.0361	0.0202
43 —........	0.0317	0.0179
44 —........	0.0278	0.0159
45 —........	0.0244	0.0143
46 —........	0.0215	0.0129
47 —........	0.0190	0.0118
48 —........	0.0168	0.0108
49 —........	0.0150	0.0101
50 —........	0.0134	0.0093

TABLE IIa

RATIO AT AGE x OF NUMBER SINGLE TO NUMBER LIVING

Age	Males	Females
15 years.....		1
16 —........		0.9821
17 —........		0.9580
18 —........	1	0.9255
19 —........	0.9973	0.8812
20 —........	0.9907	0.8212
21 —........	0.9745	0.7420
22 —........	0.9436	0.6439
23 —........	0.8939	0.5349
24 —........	0.8258	0.4308
25 —........	0.7352	0.3451
26 —........	0.6233	0.2811
27 —........	0.5027	0.2349
28 —........	0.3943	0.2013
29 —........	0.3104	0.1765
30 —........	0.2509	0.1575
31 —........	0.2093	0.1426
32 —........	0.1796	0.1308
33 —........	0.1577	0.1212
34 —........	0.1409	0.1134
35 —........	0.1277	0.1069
36 —........	0.1171	0.1015
37 —........	0.1084	0.0970
38 —........	0.1012	0.0933
39 —........	0.0952	0.0901
40 —........	0.0902	0.0874
41 —........	0.0861	0.0851
42 —........	0.0825	0.0832
43 —........	0.0795	0.0815
44 —........	0.0770	0.0800
45 —........	0.0749	0.0788
46 —........	0.0730	0.0776
47 —........	0.0715	0.0766
48 —........	0.0701	0.0757
49 —........	0.0689	0.0749
50 —........	0.0679	0.0742
51 —........	0.0670	0.0735

TABLE III

Marriage Insurance
Commutation Table
Males

4.25%

X	${}^{c}D_x = \alpha_x D_x$	${}^{c}N_x$	${}^{c}C_x = \alpha_{x+1} C_x$	${}^{c}M_x$	${}^{c}R_x$	${}^{m}C_x = m_x \times (1.0425)^{-x-\frac{1}{2}}$	${}^{m}M_x$	${}^{m}R_x$	X
1	2	3	4	5	6	7	8	9	10
Yrs.									Yrs.
0	1.000.000	14.903.704,3	35.273,2	156.458,4	1.137.734,2		238.973,8	6.480.483,8	0
1	924.685,8	13.903.704,3	24.893,3	121.185,2	981.275,8		238.973,8	6.241.510,0	1
2	862.608,2	12.979.018,5	17.617,1	96.291,2	860.090,6		238.973,8	6.002.536,2	2
3	810.187,6	12.116.410,3	12.493,7	78.674,8	763.798,7		238.973,8	5.763.562,4	3
4	764.922,1	11.306.222,7	8.889,8	66.181,1	685.123,9		238.973,8	5.524.588,6	4
5	725.031,4	10.541.300,6	6.371,9	57.291,3	618.942,8		238.973,8	5.285.614,8	5
6	689.233,2	9.816.269,2	4.637,3	50.919,4	561.651,5		238.973,8	5.046.641,0	6
7	656.593,1	9.127.036,0	3.472,0	46.282,1	510.732,1		238.973,8	4.807.667,2	7
8	626.425,1	8.470.442,9	2.721,1	42.810,1	464.450,0		238.973,8	4.568.693,4	8
9	598.222,3	7.844.017,8	2.270,7	40.089,0	421.639,9		238.973,8	4.329.719,6	9
10	571.610,4	7.245.795,5	2.038,0	37.818,3	381.550,9		238.973,8	4.090.745,8	10
11	546.311,4	6.674.185,1	1.956,8	35.780,3	343.732,6		238.973,8	3.851.772,0	11
12	522.123,2	6.127.873,7	1.978,0	33.823,5	307.952,3		238.973,8	3.612.798,2	12
13	498.900,3	5.605.750,5	2.062,2	31.845,5	274.128,8		238.973,8	3.373.824,4	13
14	476.541,8	5.106.850,2	2.176,6	29.783,3	242.283,3		238.973,8	3.134.850,6	14
15	454.982,6	4.630.308,4	2.296,1	27.606,7	212.500,0		238.973,8	2.895.876,8	15
16	434.185,4	4.175.325,8	2.400,8	25.310,6	184.893,3		238.973,8	2.656.903,0	16
17	414.133,4	3.741.140,4	2.473,8	22.909,8	159.582,7		238.973,8	2.417.929,2	17
18	394.827,4	3.327.007,0	2.498,6	20.436,0	136,672,9	1.044,1	238.973,8	2.178.955,4	18
19	375.261,7	2.932.179,6	2.465,4	17.937,4	116.236,9	2.462,3	237.929,7	1.939.981,6	19
20	355.166,4	2.556.917,9	2.361,5	15.472,0	98.299,5	5.670,0	235.467,4	1.702.051,9	20
21	332.803,4	2.201.751,5	2.183,3	13.110,5	82.827,5	10.332,6	229.797,4	1.466.584,5	21
22	306.975,0	1.868.948,1	1.940,6	10.927,2	69.717,0	15.844,3	219.464,8	1.236.787,1	22
23	277.050,4	1.561.973,1	1.659,7	8.986,6	58.789,8	20.676,3	203.620,5	1.017.322,3	23

25	206.940,6	1.041.038,5	1.071,6	5.963,8	42.476,3	30.827,5	156.717,2	630.757,6	25
26	167.241,6	834.097,9	832,52	4.892,20	36.512,53	31.678,5	125.889,7	474.040,4	26
27	128.568,4	666.856,3	629,64	4.059,68	31.620,33	27.148,5	94.211,2	348.150,7	27
28	96.116,70	538.287,93	478,39	3.430,04	27.560,65	20.032,2	67.062,7	253.939,5	28
29	72.111,57	442.171,23	373,58	2.951,65	24.130,61	13.539,0	47.030,5	186.876,8	29
30	55.546,50	370.059,66	301,40	2.578,07	21.178,96	9.014,5	33.491,5	139.846,3	30
31	44.152,48	314.513,16	250,41	2.276,67	18.600,89	6.140,4	24.477,0	106.354,8	31
32	36.097,36	270.360,68	213,11	2.026,26	16.324,22	4.323,8	18.336,6	81.877,8	32
33	30.194,87	234.263,32	184,85	1.813,15	14.297,96	3.149,6	14.012,8	63.541,2	33
34	25.697,29	204.068,45	162,77	1.628,30	12.484,81	2.358,2	10.863,2	49.528,4	34
35	22.180,99	178.371,16	145,31	1.465.53	10.856,51	1.803,2	8.505,0	38.665,2	35
36	19.368,30	156.190,17	131,09	1.320,02	9.390,98	1.403,7	6.701,8	30.160,2	36
37	17.070,00	136.821,87	119,48	1.189,13	8.070,76	1.105,0	5.298,1	23.458,4	37
38	15.169,50	119.751,87	109,90	1.069,65	6.881,63	880,95	4.193,11	18.160,26	38
39	13.580,73	104.582,37	101,98	959,75	5.811,98	697,05	3.312,16	13.967,15	39
40	12.243,00	91.001,64	95,512	857,766	4.852,226	557,62	2.615,11	10.654,99	40
41	11.116,53	78.758,64	89,943	762,254	3.994,460	447,43	2.057,49	8.039,88	41
42	10.129,39	67.642,11	85,322	672,311	3.232,206	358,09	1.610,06	5.982,39	42
43	9.279,550	57.512,715	81,525	586,989	2.559,895	288,04	1.251,97	4.372,33	43
44	8.541,495	48.233,165	78,360	505,464	1.972,906	232,52	963,93	3.120,36	44
45	7.893,074	39.691,670	75,610	427,104	1.467,442	188,58	731,41	2.156,43	45
46	7.305,183	31.798,596	73,442	351,494	1.040,338	153,89	542,83	1.425,02	46
47	6.791,454	24.495,413	71,544	278,052	688,844	126,43	388,94	882,19	47
48	6.316,955	17.701,959	69,988	206,508	410,792	103,88	262,51	493,25	48
49	5.887,155	11.385,004	68,756	136,520	204,284	86,518	158,631	230,744	49
50	5.497,849	5.497,849	67,764	67,764	67,764	72,113	72,113	72,113	50

NOTE: In Table III the French system of decimal notation has been retained. In that system periods are used to separate large whole numbers in sets of three figures, and a comma is used where the English system uses a decimal point.

TABLE IIIa

Marriage Insurance
Commutation Table
Females

4.25%

X	${}^cD_x=\alpha_x D_x$	${}^cN_x$	${}^cC_x=\alpha_{x+1}C_x$	${}^cM_x$	${}^cR_x$	${}^mC_x=m_x \times (1.0425)^{-x-\frac{1}{2}}$	${}^mM_x$	${}^mR_x$	X
1	2	3	4	5	6	7	8	9	10
Yrs.									Yrs.
0	1.000.000	13.928.148,2	35.273,2	149.897,1	981.120,4		285.644,6	6.628.014,4	0
1	924.685,8	12.928.148,2	24.893,3	114.623,9	831.223,3		285.644,6	6.342.369,8	1
2	862.608,2	12.003.462,4	17.617,1	89.730,6	716.599,4		285.644,6	6.056.725,2	2
3	810.187,6	11.140.854,2	12.493,7	72.113,5	626.868,8		285.644,6	5.771.080,6	3
4	764.922,1	10.330.666,6	8.889,8	59.619,8	554.755,3		285.644,6	5.485.436,0	4
5	725.031,4	9.565.744,5	6.371,9	50.730,0	495.135,5		285.644,6	5.199.791,4	5
6	689.233,2	8.840.713,1	4.637,3	44.358,1	444.405,5		285.644,6	4.914.146,8	6
7	656.593,1	8.151.479,9	3.472,0	39.720,8	400.047,4		285.644,6	4.628.502,2	7
8	626.425,1	7.494.886,8	2.721,1	36.248,8	360.326,6		285.644,6	4.342.857,6	8
9	598.222,3	6.868.461,7	2.270,7	33.527,7	324.077,8		285.644,6	4.057.213,0	9
10	571.610,4	6.270.239,4	2.038,0	31.257,0	290.550,1		285.644,6	3.771.568,4	10
11	546.311,4	5.698.629,0	1.956,8	29.219,0	259.293,1		285.644,6	3.485.923,8	11
12	522.123,2	5.152.317,6	1.978,0	27.262,2	230.074,1		285.644,6	3.200.279,2	12
13	498.900,3	4.630.194,4	2,062,2	25.284,2	202.811,9		285.644,6	2.914.634,6	13
14	476.541,8	4.131.294,1	2.176,6	23.222,0	177.527,7		285.644,6	2.628.990,0	14
15	454.982,6	3.654.752,3	2.255,0	21.045,4	154.305,7	7.976,4	285.644,6	2.343.345,4	15
16	426.413,5	3.199.769,7	2.300,0	18.790,4	133.260,3	10.232,2	277.668,2	2.057.700,8	16
17	396.739,8	2.773.356,2	2.289,5	16.490,4	114.469,9	13.211,2	267.436,0	1.780.032,6	17
18	365.412,8	2.376.616,4	2.207,8	14.200,9	97.979,5	17.142,6	254.224,8	1.512.596,6	18
19	331.575,8	2.011.203,6	2.043,6	11.993,1	83.778,6	22.115,3	237.082,2	1.258.371,8	19
20	294.400,6	1.679.627,8	1.798,1	9.949,5	71.785,5	27.795,5	214.966,9	1.021.289,6	20
21	253.401,9	1.385.227,2	1.489,9	8.151,4	61.836,0	32.834,4	187.171,4	806.322,7	21
22	209.475,6	1.131.825,3	1.161,2	6.661,5	53.684,6	34.733,8	154.337,0	619.151,3	22
23	165.784,0	922.349,7	865,82	5.500,28	47.023,12	31.580,9	119.603,2	466.814,3	23
24	127.228,5	756.565,71	639,82	4.634,46	41.522,84	24.784,6	88.022,3	345.211,1	24

25	97.137,09	629.337,21	483,27	3.994,64	36.888,38	17.647,8	63.237,7	257.188,8	25
26	75.423,74	532.200,12	389,01	3.511,37	32.893,74	12.144,2	45.589,9	193.951,1	26
27	60.077,04	456.776,38	321,45	3.122,36	29.382,37	8.402,3	33.445,7	148.361,2	27
28	49.069,98	396.699,34	272,03	2.800,91	26.260,01	5.940,1	25.043,4	114.915,5	28
29	41.004,16	347.629,36	234,51	2.528,88	23.459,10	4.321,2	19.103,3	89.872,1	29
30	34.868,77	306.625,20	205,35	2.294,37	20.930,22	3.217,0	14.782,1	70.768,8	30
31	30.081,91	271.756,43	182,37	2.089,02	18.635,85	2.442,5	11.565,1	55.986,7	31
32	26.289,17	241.674,52	163,79	1.906,65	16.546,83	1.884,8	9.122,6	14.421,6	32
33	23.206,20	215.385,35	148,77	1.742,86	14.640,18	1.468,2	7.237,8	35.299,0	33
34	20.681,85	192.179,15	136,26	1.594,09	12.897,32	1.156,6	5.769,6	28.061,2	34
35	18.568,11	171.497,30	125,95	1.457,83	11.303,23	916,65	4.613,03	22.291,63	35
36	16.788,07	152.929,19	117,31	1.331,88	9.845,40	728,46	3.696,38	17.678,60	36
37	15.274,81	136.141,12	110,15	1.214,57	8.513,52	580,55	2.967,92	13.982,22	37
38	13.985,32	120.866,31	104,02	1.104,42	7.298,95	465,65	2.387,37	11.014,30	38
39	12.853,19	106.880,99	98,813	1.000,401	6.194,526	375,18	1.921,72	8.626,93	39
40	11.862,95	94.027,80	94,403	901,588	5.194,125	303,18	1.546,54	6.705,21	40
41	10.987,41	82.164,85	90,706	807,185	4.292,537	247,45	1.243,36	5.158,67	41
42	10.215,34	71.177,443	87,468	716,479	3.485,352	202,06	995,91	3.915,31	42
43	9.512,998	60.962,103	84,701	629,011	2.768,873	166,84	793,85	2.919,40	43
44	8.874,280	51.449,105	82,440	544,310	2.139,862	133,23	627,01	2.125,55	44
45	8.304,062	42.571,825	80,374	461,870	1.595,552	116,34	488,78	1.498,54	45
46	7.765,510	34.270,763	78,680	381,496	1.133,682	98,168	372,438	1.009,762	46
47	7.275,879	26.505,253	77,259	302,816	752,186	84,057	274,270	637,324	47
48	6.821,590	19.229,374	76,083	225,557	449,370	72,129	190,213	363,054	48
49	6.399,825	12.407,784	75,135	149,474	223,813	63,327	118,084	172,841	49
50	6.007,959	6.007,959	74,339	74,339	74,339	54,757	54,757	54,757	50

NOTE: In Table IIIa the French system of decimal notation has been retained. In that system periods are used to separate large whole numbers in sets of three figures, and a comma is used where the English system uses a decimal point.

TABLE IVa

SINGLE PREMIUM FOR AN AMOUNT OF $1,000 PAYABLE AT AGE 35 UNDER SPECIFIED CONDITIONS

Age x	Deferred Amount Payable Age 35. Basis: Life Table R. F. and 4¼% Interest	MALES		FEMALES	
		If Living and Single at Age 35	At Marriage if Before 35 Years of Age	If Living and Single at Age 35	At Marriage if Before 35 Years of Age
	$\frac{D_{35}}{D_x}$	$\frac{{}^cD_{35}}{{}^cD_x}$	$\frac{{}^mM_x-{}^mM_{35}}{{}^cD_x}$	$\frac{{}^cD_{35}}{{}^cD_x}$	$\frac{{}^mM_x-{}^mM_{35}}{{}^cD_x}$
0	173.70	22.18	230.46	18.57	281.03
5	239.50	30.59	317.87	25.61	387.61
10	303.90	38.80	403.19	32.48	491.65
15	381.80	48.75	506.55	40.81	617.68
20	484.50	62.45	639.03	63.07	714.52
25	617.10	107.19	716.21	191.15	603.53
30	784.60	399.32	449.83	532.51	291.64
35	1000.00	1000.00		1000.00	

TABLE IVb

SINGLE PREMIUM FOR AN AMOUNT OF $1,000 PAYABLE IF INSURED DIES BEFORE AGE 35

Age x	Temporary Insurance Basis: Life Table R. F. and 4¼% Interest	MALES		FEMALES	
		In Case of Death Single	In Case of Death After Marriage	In Case of Death Single	In Case of Death After Marriage
	$\frac{M_x-M_{35}}{D_x}$	$\frac{{}^cM_x-{}^cM_{35}}{{}^cD_x}$	$\frac{M_x-M_{35}}{D_x}-\frac{{}^cM_x-{}^cM_{35}}{{}^cD_x}$	$\frac{{}^cM_x-{}^cM_{35}}{{}^cD_x}$	$\frac{M_x-M_{35}}{D_x}-\frac{{}^cM_x-{}^cM_{35}}{{}^cD_x}$
0	166.56	154.99	11.57	148.44	18.12
5	91.96	77.00	15.96	67.96	25.00
10	83.84	63.60	20.24	52.13	31.71
15	82.89	57.46	25.43	43.05	39.84
20	71.26	39.44	31.82	28.84	43.42
25	52.49	21.74	30.75	26.12	26.37
30	30.59	20.03	10.56	23.99	6.60

TABLE V

TABLE OF REPRODUCTIVE RATES μ_x

Father's Age	Males	Mother's Age	Females
15 years		15 years	0.4092
16 —		16 —	0.3061
17 —		17 —	0.2731
18 —	0.4471	18 —	0.2633
19 —	0.3292	19 —	0.2641
20 —	0.2932	20 —	0.2690
21 —	0.2843	21 —	0.2736
22 —	0.2872	22 —	0.2746
23 —	0.2946	23 —	0.2700
24 —	0.3013	24 —	0.2595
25 —	0.3031	25 —	0.2445
26 —	0.2977	26 —	0.2270
27 —	0.2855	27 —	0.2088
28 —	0.2677	28 —	0.1916
29 —	0.2472	29 —	0.1759
30 —	0.2263	30 —	0.1620
31 —	0.2065	31 —	0.1498
32 —	0.1885	32 —	0.1389
33 —	0.1727	33 —	0.1289
34 —	0.1588	34 —	0.1192
35 —	0.1464	35 —	0.1095
36 —	0.1353	36 —	0.0993
37 —	0.1250	37 —	0.0886
38 —	0.1151	38 —	0.0774
39 —	0.1055	39 —	0.0661
40 —	0.0958	40 —	0.0554
41 —	0.0861	41 —	0.0455
42 —	0.0765	42 —	0.0368
43 —	0.0672	43 —	0.0249
44 —	0.0584	44 —	0.0164
45 —	0.0503	45 —	0.0107
46 —	0.0430	46 —	0.0071
47 —	0.0365	47 —	0.0048
48 —	0.0306	48 —	0.0033
49 —	0.0263	49 —	0.0023
50 —	0.0223	50 —	0.0017

TABLE Va
Birth Insurance
Commutation Table
Males 4.25%

X	$V_x - W_x$ 1	$H_x = V_x - W_x + \frac{m_x}{4}$ 2	$n_x = H_x \mu_x$ 3	E_x 4	U_x 5
18 years....	0	564	252	116.7	514,957.9
19 —.......	2,240	3,626	1,194	530.3	514,841.2
20 —.......	7,665	10,992	3,223	1,373.1	514,310.9
21 —.......	20,871	27,192	7,731	3,159.4	512,937.8
22 —.......	45,842	55,947	16,068	6,298.7	509,778.4
23 —.......	85,651	99,398	29,283	11,011.0	503,479.7
24 —.......	139,696	157,874	47,567	17,157.0	492,468.7
25 —.......	210,989	233,264	70,702	24,462.0	475,311.7
26 —.......	298,277	322,140	95,901	31,827.9	450,849.7
27 —.......	391,289	412,609	117,800	37,501.9	419,021.8
28 —.......	473,543	489,943	131,158	40,052.3	381,519.9
29 —.......	535,656	547,211	135,271	39,624.2	341,467.6
30 —.......	578,066	586,087	132,631	37,267.1	301,843.4
31 —.......	606,116	611,812	126,339	34,051.9	264,576.3
32 —.......	624,639	628,820	118,533	30,645.5	230,524.4
33 —.......	636,910	640,085	110,543	27,414.7	199,878.9
34 —.......	645,069	647,547	102,830	21,462.2	172,464.2
35 —.......	650,307	652,283	95,494	21,790.9	148,002.0
36 —.......	653,408	655,011	88,623	19,398.6	126,211.1
37 —.......	654,920	656,236	82,030	17,223.5	106,812.5
38 —.......	655,155	656,249	75,534	15,213.0	89,589.0
39 —.......	654,342	655,244	69,128	13,355.2	74,376.0
40 —.......	652,634	653,386	62,594	11,599.8	61,020.8
41 —.......	650,105	650,734	56,028	9,959.8	49,421.0
42 —.......	647,038	647,563	49,539	8,447.2	39,461.2
43 —.......	643,360	643,800	43,263	7,076.3	31,014.0
44 —.......	639,133	639,504	37,347	5,859.6	23,937.7
45 —.......	634,419	634,732	31,927	4,805.0	18,078.1
46 —.......	629,342	629,609	27,073	3,908.4	13,273.1
47 —.......	623,754	623,982	22,775	3,153.9	9,364.7
48 —.......	617,841	618,037	18,912	2,512.2	6,210.8
49 —.......	611,519	611,689	16,087	2,049.8	3,698.6
50 —.......	604,768	604,916	13,490	1,648.8	1,648.8

TABLE Va
BIRTH INSURANCE
COMMUTATION TABLE
FEMALES

4.25%

X	$V_x - W_x$	$H_x = V_x - W_x + \frac{m_x}{4}$	$n_x = H_x \mu_x$	E_x	U_x
	1	2	3	4	5
15 years....	0	3,801	1,555	815.7	598,717.7
16 —.......	15,127	20,211	6,187	3,113.3	597,902.0
17 —.......	35,293	42,136	11,507	5,554.3	594,788.7
18 —.......	62,221	71,477	18,820	8,713.9	589,234.4
19 —.......	98,576	111,025	29,322	13,023.0	580,520.5
20 —.......	147,360	163,671	44,027	18,756.8	567,497.5
21 —.......	211,166	231,253	63,271	25,856.5	548,740.7
22 —.......	289,441	311,593	85,563	33,540.9	522,884.2
23 —.......	375,462	396,459	107,044	40,250.8	489,343.3
24 —.......	456,456	473,635	122,908	44,331.9	449,092.5
25 —.......	521,815	534,567	130,702	45,221.3	404,760.6
26 —.......	569,237	578,385	131,293	43,573.9	359,539.3
27 —.......	602,001	608,599	127,075	40,454.6	315,965.4
28 —.......	624,432	629,295	120,573	36,819.9	275,510.8
29 —.......	639,665	643,353	113,166	33,149.1	238,690.9
30 —.......	650,141	653,003	105,786	29,724.1	205,541.8
31 —.......	657,245	659,511	98,795	26,628.0	175,817.7
32 —.......	661,794	663,617	92,176	23,831.2	149,189.7
33 —.......	664,510	665,992	85,846	21,289.8	125,358.5
34 —.......	665,718	666,934	79,499	18,912.0	104,068.7
35 —.......	665,813	666,817	73,016	16,661.6	85,156.7
36 —.......	664,953	665,785	66,112	14,471.2	68,495.1
37 —.......	663,294	663,985	58,829	12,352.1	54,023.9
38 —.......	660,914	661,492	51,199	10,311.8	41,671.8
39 —.......	658,031	658,517	43,528	8,409.4	31,360.0
40 —.......	654,643	655,052	36,290	6,725.2	22,950.6
41 —.......	650,816	651,164	29,628	5,266.8	16,225.4
42 —.......	646,545	646,841	23,804	4,059.0	10,958.6
43 —.......	641,963	642,218	15,991	2,615.6	6,899.6
44 —.......	637,056	637,276	10,451	1,639.7	4,284.0
45 —.......	631,744	631,937	6,762	1,017.7	2,644.3
46 —.......	626,219	626,389	4,447	642.0	1,626.6
47 —.......	620,328	620,480	2,978	412.4	984.6
48 —.......	614,121	614,257	2,027	269.3	572.2
49 —.......	607,578	607,702	1,398	178.1	302.9
50 —.......	600,680	600,792	1,021	124.8	124.8

TABLE Vb
Birth Insurance
Complementary Commutation Table

Age at Entry x	MALES $\mu_x D_x$	MALES $\mu_x N_x$	FEMALES $\mu_x D_x$	FEMALES $\mu_x N_x$
15		1.498.308 30	186.178 90	1.583.539 30
16		1.498.308 30	132.904 20	1.397.360 40
17		1.498.308 30	113.099 80	1.264.456 20
18	176.527 40	1.498.308 30	103.958 10	1.151,356 40
19	123.870 60	1.321.780 90	99.374 91	1.047.398 26
20	105.112 30	1.197.910 30	96.436 61	948.023 35
21	97.091 86	1.092.798 00	93.437 68	851.586 74
22	93.432 82	995.706 17	89.333 75	758.149 06
23	91.306 67	902.273 35	83.682 29	668.815 31
24	88.983 17	810.966 68	76.638 34	585.133 02
25	85.315 13	721.983 51	68.820 69	508.494 68
26	79.877 82	636.668 38	60.907 85	439.673 99
27	73.018 31	556.790 56	53.401 83	378.766 14
28	65.255 97	483.772 24	46.705 43	325.364 31
29	57.429 03	418.516 28	40.864 75	278.658 88
30	50.100 29	361.087 25	35.864 99	237.794 13
31	43.561 82	310.986 96	31.600 77	201.929 14
32	37.886 16	267.425 14	27.917 18	170.328 37
33	33.066 90	229.538 98	24.680 51	142.411 19
34	28.961 90	196.472 08	21.739 66	117.730 68
35	25.429 09	167.510 18	19.019 71	95.991 02
36	22.378 57	142.081 09	16.424 18	76.971 31
37	19.684 03	119.702 52	13.952 04	60.547 13
38	17.253 05	100.018 49	11.601 97	46,595 09
39	15.050 06	82.765 44	9.429 469	34.993 1203
40	13.003 10	67.715 38	7.519 536	25.563 6513
41	11.116 53	54.712 28	5.874 587	18.044 1153
42	9.392 708	43.595 75	4.518 322	12.169 5283
43	7.843 839	34.203 04	2.906 423	7.651 2063
44	6.478 219	26.359 21	1.819 226	4.744 7833
45	5.300 664	19.880 98	1.127 577	2.925 5573
46	4.303 053	14.580 32	710 5041	1.797 9803
47	3.466 965	10.277 27	455 9297	1.087 4762
48	2.757 473	6.810 300	297 3745	631 5465
49	2.242 202	4.052 827	196 5234	334 1720
50	1.805 625	1.805 625	137 6486	137 6486

Note: In Table Vb the French notation is employed, whole numbers being separated in sets of three figures by means of a period, and the position of the decimal point indicated by the blank space.