ACTUARIAL BASIS FOR PREMIUMS AND RESERVES IN PERSONAL ACCIDENT AND HEALTH INSURANCE*

BY

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The "Report of the Committee of Five on Statistics to the Bureau of Personal Accident and Health Underwriters," dated November 15, 1929, shows the combined health experience on commercial policies during the policy years 1921 to 1926 inclusive combined.   Page 12 of this Report shows the average number of weeks of disability, both total and partial, per year of exposure, under a policy known as "Policy Form 02-52 Weeks Limit, Total Disability with Full Weekly Indemnity Irrespective of House Confinement, and Partial Disability."   The data of this report are used for illustrative purposes only.   The technique applies with equal force to any form of disability insurance,—health and/or accident insurance for individuals.

For ages under 50 years, the average number of weeks of disability per year of exposure was .69 for total disability and .19 for partial disability.   It will be noted that benefits are payable in the event of either total or partial disability.   Disability as used here implies incapacity resulting from disease.   Certain types of policies provide for the payment of benefits only in the event of disability due to accident.   Other policies include disability arising from sickness and accident.

COMPUTATION OF NET ONE YEAR TERM PREMIUMS, BENEFIT LIMITED TO 52 WEEKS

Suppose now that a benefit of one dollar a week, up to a limit of fifty-two weeks, is to be paid for disability incurred during any

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*Editor's Note: This paper was prepared as a chapter in a textbook intended for use by non-actuarial students. Many of the explanations and references are obvious to members of the Society but are left in to make the paper complete.

given year prior to age 50.   Then, assuming the foregoing rates of disability to hold, $.69 must on the average be paid during the year to each person on account of total disability, and $.19 must be so paid on account of partial disability.   If half benefit is payable in event of partial disability, then the average amount payable to each person during the year is $.69, plus $.10, or $.79. The net premium to be collected on account of each person is, therefore, the sum that will provide these payments.   Premiums are ordinarily payable in advance and are improved with interest until the funds are needed for the payment of claims.   For practical purposes, it is assumed that if there be a large number of lives under consideration, the payments on account of disability will be evenly distributed throughout the year.   They may therefore be considered as all occurring in the middle of the year. In actual fact, however, the payments are made in varying amounts and at varying times during the year.   The premium to be collected from each person insured is then the sum which, improved at interest for half a year, will amount to $.79.   From a strictly theoretical viewpoint, we should further reduce this sum by multiplying by the probability of living half a year; for a person must be alive at the middle of the year in order to receive his disability benefit.   This probability, however, is very nearly unity, and its use in this simple illustration is further prohibited by the fact that we are not considering individual ages.   We can, therefore, neglect it for the time being, although in the illustrations further on it will be considered.

The interest on $1.00 at $3\frac{1}{2}\%$ for six months is $.0175, so that the present value of $.79 payable in six months is $.79 divided by 1.0175, or $.776.   Such a value is known as the *net one year term premium*, and is defined as the sum which, paid at the beginning of a year by each person insured, will exactly pay the claims during the year if the rates of disability are experienced exactly in accordance with the expected rates, and if the interest earned is at exactly the rate assumed in the calculation.   The assumption is also implicitly made that there are enough cases under consideration to provide a stable, representative experience.   No allowance is made in a net premium for contingencies or expenses.

The report quoted further shows the average number of weeks of disability according to quinquennial age groups.   These figures are as follows:

AVERAGE NUMBER OF WEEKS OF DISABILITY PER YEAR OF
EXPOSURE FOR POLICY FORM 02

Age	Weeks of disability	
	Total	Partial
Up to 20.........	.68	.09
20 to 24.........	.83	.13
25 to 29.........	.74	.16
30 to 34.........	.74	.18
35 to 39.........	.66	.19
40 to 44.........	.66	.19
45 to 49.........	.66	.20
50 to 54.........	.83	.24
55 to 59.........	.94	.28
60 to 64.........	1.20	.36

The net premium for each age group is obtained exactly as
explained for the total of ages under 50. The following table
then shows these premiums:

NET ONE YEAR TERM PREMIUM TO PROVIDE ONE DOLLAR
A WEEK IN EVENT OF TOTAL DISABILITY, WITH A LIMIT
OF FIFTY-TWO WEEKS, AND A BENEFIT OF ONE-HALF
DOLLAR A WEEK IN EVENT OF PARTIAL DISABILITY

Age	Weeks of sickness			Net premium Column (3) discounted for half a year
	Total disability	One-half of partial disability	Total	
	(1)	(2)	(3)	(4)
Up to 20.....	.68	.045	.725	$0.713
20 to 24......	.83	.065	.895	.880
25 to 29......	.74	.080	.820	.806
30 to 34......	.74	.090	.830	.816
35 to 39......	.66	.095	.755	.742
40 to 44......	.66	.095	.755	.742
45 to 49......	.66	.100	.760	.747
50 to 54......	.83	.120	.950	.934
55 to 59......	.94	.140	1.080	1.061
60 to 64......	1.20	.180	1.380	1.356

Under this plan the policy is issued for a term of one year, and
at the end of that period must be reissued or renewed for another
year. Renewal is optional with both the company and the
insured. According to the table, the net premiums for the age
group 30 to 34 years would be .816 for a weekly benefit consisting
of $1.00 in the event of total disability and $.50 in the event of
partial disability. When the insured attains an age between
40 and 44 years inclusive, the premium would be $.742, and as
each successively higher age group was attained the premium
would be increased until $1.356 would be charged at ages 60-64.

The premium rates show a drop in the middle age groups, but for ages up to 50 they are reasonably uniform. From age 50 upward, however, the premium for each age group shows a substantial increase. Thus there is a justification for the practice generally followed by companies issuing insurance under this type of policy; namely to charge the same net premium, $.80 say, for all ages up to about 50, then to increase the premium for renewal as well as for new entrants and often to further increase it at age 55, the policy being finally cancelled by expiry at age 60 or 65. As an alternative, a company may charge a premium slightly higher than $.80 which would remain level throughout the duration of the policy. Such a premium would start at $.90 say, and remain level, or constant, until the insured reached age 65 and the policy then expired.

With the rates of incapacity varying with such factors as occupation, underwriting and claim adjustment practices, with duration of insurance and amount of benefit, it naturally follows that there are no standard tables of sickness or accident rates. Companies generally keep records, however, of their own sickness and accident experience and construct rates for their own underwriting purposes. The Manchester Unity tables, showing the sickness rates experienced by the members of the Independent Order of Odd Fellows, Manchester Unity, of Great Britain and Ireland, have been used with certain modifications as a basis for the calculation of premium rates in the United States.* The A. H. J. group of these tables have generally been assumed to be a safe estimate of the sickness and accident experience in this country. No data on disability from sickness and accident, suitable for insurance purposes, have as yet been published for the general population of the United States. Another reason for the lack of published tables is the fact that neither the sickness nor the accident rates have seemed to vary greatly in insurance experience below age 50. Likewise, there have been no standard tables adopted for the principal sum benefits which are payable in event of accidental death or dismember-

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*For further details of the Manchester Unity Tables, *see*: "An Account of an Investigation of the Sickness and Mortality Experience of the I. O. O. F. Manchester Unity, During the Five Years 1893-1897" by Alfred W. Watson, published by the Grand Master and Board of Directors of the I. O. O. F. M. U., Spectator Company, New York, or C. and E. Layton, London.

ment.   Premiums, therefore, have largely been based on the experience of some individual companies or combination of companies.   The data used herein are an example of the latter, showing premiums based upon the experience of those companies contributing to the Bureau of Personal Accident and Health Underwriters.

The most important element in the determination of accident insurance rates is the occupation of the insured.   A "Standard Manual" giving occupational classifications has been formulated by a committee of the Bureau of Personal Accident and Health Underwriters.   This Manual has, as a matter of economy and good judgment, been adopted by practically all companies.

At the 1928 annual meeting of the Bureau, the following gross premium rates for each $1,500 of principal sum and $5.00 of weekly indemnity were recommended for the various occupational classes as indicated:

New class*	Rate	Old class*
(A)	$5.00	Select and preferred
(B)	6.00	
(C)	7.00	Extra preferred
(D)	8.50	Ordinary

*The new Manual designates the classes in ascending order of hazard as A, B, C, etc., while in former Manuals these classes were called Select, Preferred, Extra Preferred, etc.   Class B, however, is a new class and was not included in the old classification.

About 60% of the above rates covers the weekly indemnity and 40% the principal sum benefits.

For classes other than the four given above, several companies have been charging the following gross premium rates for the same $1,500 of principal sum and $5.00 of weekly benefit.

New class	Rate	Old class
E	$10.00	Medium
F	12.50	Special
G	15.00	Hazardous
H	20.00	Extra Hazardous
I	25.00	Perilous
J	30.00	Extra perilous

In 1927 the Bureau reviewed by age groups the available commercial accident experience of two of the larger member companies writing this insurance, the accident experience prepared by the Health and Accident Underwriters Conference under date of June 1927, and the accidental death rate by age groups as shown by the Group Life experience of six of the larger companies writing this form of insurance. After a careful review of the above statistics the Committee for the Bureau concluded that the following additional gross premiums for each $1,000 of original accidental death insurance should be charged for ages sixty and over: Preferred Class $1.40, Extra Preferred Class $2.40. Additional premium in the Ordinary and Medium classes should start at age fifty-five and should be: Ordinary Class $2.50, Medium Class $3.00.

A review of the accident weekly indemnity experience showed a sharp increase in the average number of days of disability per claim as the age increased. On the other hand, there was a decided decrease in the number of claims per one thousand years of exposure as the age increased. As a result there was only a slight increase in the weekly indemnity loss ratio at the older ages as compared with the loss ratio for all ages. This Committee, therefore, came to the conclusion that it was not necessary to charge an additional premium for weekly indemnity insurance at the older ages.

In general, the base rate is tabulated for those occupations with little, if any, special hazard, after which other occupations are grouped so that those of approximately equal extra hazard are included together. In order that this may be more readily accomplished, the extra hazards by occupations are subdivided according to classes. Experience is also kept separately for death, dismemberment claims, weekly benefits and other benefits, while the same applies to such special features as surgical operations, medical attendance fees, hospital benefits, nursing benefits, identification and similar minor items in order that the variations therein can be known and the cost of each more accurately determined.

The occupation of the insured has less effect on the sickness rate than on the accident rate in the insurance experience available. The general practice is to use standard rates for Select, Preferred, Extra Preferred and Ordinary classes and a rate 30% higher for the more hazardous classes.

COMPUTATION OF RESERVES

The next problem to be considered is that of reserves.

Unearned Premium Reserves, One Year Term Policies

When a policy is issued on the basis of a one year term premium, the net premium of any policy year is calculated so as to be sufficient to pay the claims of that year, and no balance is required to be on hand after the claims are paid. At any time during the policy year only the unearned portion of the year's premium is required to be held as a fund to meet future claims that may arise. *The unearned premium as of any date during the policy year may be defined as that portion of the year's premium which properly applies to the remaining portion of the year.**

Claim Reserves

Furthermore, it naturally happens that when the year has expired certain disabilities have occurred but have not terminated, and payments will be continued into the next and possibly subsequent years. Thus there are two separate funds or reserves which must be held during the year by the company—first, the unearned premium reserve, which will provide for incapacity commencing during the remainder of the time for which premiums have been paid, and second the claim reserve, which will provide for future payments on claims already incurred. In actual practice, the claim reserves are sometimes made a matter of individual judgment, but they should more properly be determined on the basis of the sickness expected to be incurred according to the table used in the calculation of the premium, or upon such other table as subsequent experience may dictate.

The foregoing discussion is intended to give a general idea of the fundamental problems involved in determining net premiums and reserves for one year term contracts. Net premiums, once computed, must then be increased by an amount sufficient to provide for expenses and exceptional fluctuations in loss experience. Such an increase is considered as loading and, when added to the net premium, produces the gross premium charged.

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*In actual accounting practice, earned and unearned premiums are computed on the basis of gross premiums. *See*: Section 86, New York Insurance Law, subdivision 4, paragraph 2.

ACTUARIAL FORMULAE AND PROCEDURE

These results have all been obtained by a process of general reasoning, without reference to any formulae or commutation columns.† It is now proposed to develop, from first principles, commutation columns and formulae for deriving the usual types of net premiums and reserves. Actuarial formulae can deal only with the rate as indicated by the experience table assumed, but it would be well at the outset to emphasize the fact that the rate of sickness actually experienced by a company is largely a matter of managerial control. When a claim for life insurance is received, there is ordinarily little difficulty in determining whether or not the insured actually is dead; however, when a claim is made under accident insurance it becomes necessary to determine whether the death or disability was in fact due to accidental injury. When a claim for sickness is received it is often difficult to determine whether or not the insured actually is sick. The rate of sickness also varies somewhat with factors such as the form of policy issued, amount of weekly indemnity, age, habitat, sex, race and physical condition of the insured; it also varies for the different periods of sickness to be considered, such as the first three months of sickness, the second three months, the second six months, the second year, or sickness after two years.

In the developments which follow it will not be necessary for the reader to have a previous knowledge of the fundamental principles of compound interest and life contingencies. However, these are quite complicated subjects and some previous knowledge would be helpful in obtaining a broader understanding of this chapter.‡

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†*Commutation columns* are columnar aids to computation which have no meaning in themselves, but which are constructed from the rates of interest, mortality, and sickness in such a manner as to bear a workable relation among themselves and so permit the ready calculation of premiums, reserves and other related values. In this chapter we shall be concerned with values of D_x, N_x, H_x, and K_x, each of which will be explained in its proper place.

‡If the reader desires to do further work along these lines, the following references will be helpful:
Moir, Henry, "Life Assurance Primer,"—New York, The Spectator Co.—1921; Maclean, Joseph D., "Life Insurance"—New York, McGraw-Hill Book Co.—1924; Rietz, Crathorne & Rietz, "Mathematics of Finance"—New York, Henry Holt—1921; Dawson, M. M., "Practical Lessons in Actuarial Science"—New York, The Spectator Company—1905.

The Rate of Sickness

If there be L_x people between the ages x and $x + 1$, and if among them they experience an aggregate of r weeks of sickness in the following year, then the average number of weeks of sickness per person during the year is $\dfrac{r}{L_x}$. This ratio is defined as the rate of sickness at age x, and is denoted by s_x. We therefore have the relation:

$$s_x = \frac{r}{L_x}$$

The word "sickness" as used hereafter in this paper will apply to disability from either disease or accident, or both, it being understood that the basic rates used include only the hazard insured against.

If each sick person is to receive \$1 for each week that he is sick, then, following the previous line of reasoning, the value of the sickness benefits for a person exactly aged x for that year is $v^{\frac{1}{2}}\,{}_{\frac{1}{2}}p_x\,s_x$, where $v^{\frac{1}{2}}$ represents the value of \$1 due in half a year and ${}_{\frac{1}{2}}p_x$ represents the probability of a person exactly age x surviving half a year. Similarly, the present value to a person exactly aged x of a benefit of \$1 per week of sickness experienced during the year of age $(x + n)$ to $(x + n + 1)$ is $v^{n+\frac{1}{2}}\,{}_{n+\frac{1}{2}}p_x\,s_{x+n}$, where $v^{n+\frac{1}{2}}$ represents the present value of \$1 due in $n + \dfrac{1}{2}$ years, ${}_{n+\frac{1}{2}}p_x$ represents the probability of a person now exactly aged x surviving to the middle of the $(n + 1)$th year. Clearly, if there are a given number of persons alive, all exactly aged x, all of them will not be alive at age $x + n$. If L_x is the number of persons alive according to the mortality table between ages x and $x + 1$, and l_x is the number alive at exact age x, then, assuming an even distribution of deaths throughout the year,

$$L_x = l_{x+\frac{1}{2}}.$$

$l_{x+n+\frac{1}{2}}$ may then be considered as the number alive in the middle of the year of age $x + n$ to $x + n + 1$.

$${}_{n+\frac{1}{2}}p_x \text{ is therefore defined as } \frac{l_{x+n+\frac{1}{2}}}{l_x}.$$

$v^{n+\frac{1}{2}} \,_{n+\frac{1}{2}}p_x \, s_{x+n}$ then becomes $\dfrac{v^{n+\frac{1}{2}} \, l_{x+n+\frac{1}{2}} \, s_{x+n}}{l_x}$.

Multiplying the numerator and denominator by v^x, we have

$$\frac{v^{x+n+\frac{1}{2}} \, l_{x+n+\frac{1}{2}} \, s_{x+n}}{v^x \, l_x},$$

or since $v^x \, l_x$ is denoted by D_x, we have

$$\frac{v^{x+n+\frac{1}{2}} \, l_{x+n+\frac{1}{2}} \, s_{x+n}}{D_x}.$$

To find the value of a sickness allowance of \$1 per week of sickness throughout life, every integral value from 0 to $\omega - x$ must be given to n in the last expression, where ω is the age at which the mortality table ceases and $l_{x+n+\frac{1}{2}}$ then vanishes. The required value is then

$$\frac{1}{D_x} \left[v^{x+\frac{1}{2}} \, l_{x+\frac{1}{2}} \, s_x + v^{x+\frac{3}{2}} \, l_{x+\frac{3}{2}} \, s_{x+1} + v^{x+\frac{5}{2}} \, l_{x+\frac{5}{2}} \, s_{x+2} \right.$$

$$\left. + \, \ldots \ldots \text{ to end of table} \right].$$

If we denote $v^{x+\frac{1}{2}} \, l_{x+\frac{1}{2}} \, s_x$ by H_x, $v^{x+\frac{3}{2}} \, l_{x+\frac{3}{2}} \, s_{x+1}$ by H_{x+1}, etc., and the required value by $(A\,S)_x$, we have

$$(A\,S)_x = \frac{1}{D_x} \left[H_x + H_{x+1} + H_{x+2} + \ldots \ldots \text{ to end of table} \right]$$

$$= \frac{K_x}{D_x}, \text{ where } K_x = \sum_{t=0}^{t=\omega-x} H_{x+t}.$$

Sickness Originating Prior to or at Age x

A change in concept introduces a very important variation which affects the fundamental principles. The change permits of two different series of calculations. At age x (age x is used in this sense to imply the year of age between x and $x + 1$) there may be s_x weeks of sickness experienced, a certain proportion of which resulted from sickness that had originated prior to age x, and a certain proportion from sickness that originates at age x. In like manner, sickness will originate at age x, continue for several years and be a part of the sickness experienced at age $x + 1, x + 2$, etc.

Certain premium rates, as for instance those on the Manchester Unity Tables, are based upon the assumption that all sickness occurring at age x will be compensated for irrespective of the

age at which such sickness originated. In America the general practice is to base rates for any age not upon all sickness that occurs, but upon all sickness which originates at age x and after. It, therefore, becomes necessary to know the weeks of sickness experienced at age x which originate at age x; the weeks of sickness experienced at $x + 1$ that originated at age x; the weeks of sickness experienced at age $x + 2$ that originated at age x; and so on. The rate of sickness will, therefore, vary (1) according as the sickness in any period is the total sickness for that period, or (2) whether it is only the sickness originating within that period. One rate of sickness may, therefore, include all of the sickness experienced at age x, while another may include only that sickness experienced at age x which originated at age x, while still another may include that sickness originating at age x which is experienced at age x, $(x + 1)$, $(x + 2)$, etc.

If it is desired to find the value for a person aged x of a benefit of \$1.00 a week for each week of sickness originating at age x and continuing thereafter, the rate of sickness s_x must include such sickness only. One must consider the rate of sickness experienced at age x which originated at that age, the rate of sickness at age $(x + 1)$ where the sickness originated age x, the rate of sickness at age $(x + 2)$ where the sickness originated at age x, and so on.

Where the published rates of sickness s_x include all sickness experienced at age x, the rates for each age must be subdivided into that part which originated at that age, that part which originated at an age one year younger, two years younger, etc., after which the rate of first year's sickness at age x, second year's sickness at age $(x + 1)$, third year's sickness at age $(x + 2)$ etc., can all be considered as covering sickness originating at age x. The rates of sickness originating at age x would, therefore, be $s_x^{0/1}; s_x^{1/1}; s_x^{2/1}$ . . . . . ., where the number before the vertical line indicates the period of years of sickness excluded, and the number after the line the period of years covered after the exclusion period. The symbol $s_x^{0/1}$, therefore, includes one year after the beginning of sickness originating at age x, or the first year of sickness; $s_x^{1/1}$ includes one year after the first year of sickness, or the second year of sickness, and so on.

The total sickness originating at age x would then be the sum of the first year's sickness at age x, the second year's sickness

included at age $(x + 1)$ etc., and the present value to a person now aged x of an allowance of \$1.00 for each week of sickness originating during the year of age x to $(x + 1)$ and continuing thereafter as long as that sickness lasts is, therefore, represented by

$$\frac{H_x^{0/all}}{D_x} = \frac{1}{D_x}\left[v^{x+\frac{1}{2}}\,l_{x+\frac{1}{2}}\,s_x^{0/1} + v^{x+\frac{3}{2}}\,l_{x+\frac{3}{2}}\,s_x^{1/1} + v^{x+\frac{5}{2}}\,l_{x+\frac{5}{2}}\,s_x^{2/1}\right.$$

$$\left. + \ldots \ldots \text{ to end of table}\right]$$

$$= \frac{1}{D_x}\left[H_x^{0/1} + H_x^{1/1} + H_x^{2/1} + \ldots\right], \text{ where } v^{x+\frac{1}{2}}\,l_{x+\frac{1}{2}}\,s_x^{0/1}$$

$$= H_x^{0/1}, \text{ etc.}$$

The value of a sickness benefit of \$1 for each week of sickness originating at any time throughout the balance of life is, therefore, the value for all sickness originating at age x, at age $(x + 1)$ etc., and, therefore, by analogy we have $K_x^{0/all} = H_x^{0/all} + H_{x+1}^{0/all} + H_{x+2}^{0/all} + \ldots$ to end of table, where "o" shows no period of exclusion and indicates that the benefit was payable from the first day of sickness, while "all" shows that the entire duration of sickness is covered and that there is no limit to the period of payment.

When the rate of sickness is expressed on the basis of the age at which it was experienced, and divided, as in the case of the Manchester Unity Table into sickness for the first three months of sickness, second three months of sickness, second six months of sickness, second year of sickness, and sickness after two years, it can be subdivided according to the year in which sickness originated.[*]   The rates of sickness on the A. H. J. Group of the Manchester Unity were so subdivided and a sample of these rates of sickness appears below:

Age	First 6 mos.	Second 6 mos.	Second year	Third year	Fourth year	Fifth year	Sixth year	Seventh year
x	$s_x^{0/\frac{1}{2}}$	$s_x^{\frac{1}{2}/\frac{1}{2}}$	$s_{x-1}^{1/1}$	$s_{x-2}^{2/1}$	$s_{x-3}^{3/1}$	$s_{x-4}^{4/1}$	$s_{x-5}^{5/1}$	$s_{x-6}^{6/1}$
20	.763	.035	.013	.002	.001	.001		
21	.741	.041	.019	.005	.002	.001	.001	
22	.726	.046	.026	.008	.004	.002	.001	.001

[*]The details of this calculation appear in a paper "Health Insurance From a Theoretical and Practical Aspect" published in Volume 15 of the *Transactions* of the Actuarial Society, and in a paper by E. E. Cammack, before this Society entitled "Premiums and Reserves on Non-Cancellable Accident and Health Policies," *Proceedings*, Vol. VII, page 267).

For each person there are, then, assumed to be .763 weeks of first six months' sickness and .035 weeks of second six months' sickness, or a total of .798 weeks experienced at age 20 which originated at age 20; .019 weeks experienced at age 21 which originated at age 20; .008 weeks experienced at age 22 which originated at age 20; and so on. In assuming that the sickness of the first six months all originated at age x a slight error is admitted, as some of this sickness originated at $(x - 1)$, but by analogy some of the first six months' sickness at age $(x + 1)$ originated at age x and the error is only the difference between that sickness included in $(x + 1)$ which originated at age x, and that sickness included at age x which originated at age $(x - 1)$. This difference must of necessity be small and is generally ignored. The same applies to the rates of sickness for other periods.

Sickness Rates with Limited Periods of Benefit

The payment of sickness benefits for any one sickness is usually limited to a certain number of weeks, even though the insured may not have recovered at the expiration of that period. If a benefit is granted with a limit of fifty-two weeks for any one sickness the rate of sickness in the first year after it originates constitutes the required rate, or in symbols $s_x^{0/1}$. If the limit is 104 weeks, the rates of sickness occurring in both the first and second year after sickness originates constitutes the required rate, or in symbols $s_x^{0/1}$ and $s_x^{1/1}$. In the first case values of $H_x^{0/1}$ and $K_x^{0/1}$ are required; in the second case values of $H_x^{0/2}$ (or $H_x^{0/1} + H_x^{1/1}$) and $K_x^{0/2}$ (or $H_x^{0/2} + H_{x+1}^{0/2} + H_{x+2}^{0/2} +$ etc.). When benefits are payable for more than fifty-two weeks it is necessary to determine the amount of sickness originating at age x which continues beyond age x. This sickness must be shown separately for each year thereafter in order to apply the interest factor and the probability of survivorship. From these values can be obtained the values of $H_x^{0/m}$ and $K_x^{0/m}$, where m is the period to which the payment of benefits is limited. The calculation of these values for each age becomes rather lengthy. A good approximation is obtained by preparing the values of $H_x^{0/m}$ for each tenth age, then dividing these values by D_x at the respective ages in order to get the one year term premium at those ages.

By interpolation the one year term premium for the intervening ages can be obtained and this can in turn be multiplied by D_x at the respective ages to get the corresponding values of $H_x^{0/m}$.

Then, as before $K_x^{0/m} = \sum_{t=0}^{t=\omega-x} H_{x+t}^{0/m}$ and with columns of

$K_x^{0/m}$, for various desired values of m, premiums can be calculated for the corresponding benefits.

Net Single Premiums

The net single premium at age x for a benefit with a limit of m years, payable for sickness originating after age x and prior to the expiration of n years then becomes

$$\frac{H_x^{0/m} + H_{x+1}^{0/m} + H_{x+2}^{0/m} + \ldots H_{x+n-1}^{0/m}}{D_x}, \text{ or } \frac{K_x^{0/m} - K_{x+n}^{0/m}}{D_x}$$

and is denoted by $(A\,S)_{x:\overline{n}|}^{0/m}$. If the benefit is for sickness

originating after age x and prior to age 60, the net single premium

becomes $\dfrac{K_x^{0/m} - K_{60}^{0/m}}{D_x}$ which is denoted by $(A\,S)_{x:\overline{60-x}|}^{0/m}$.

Net Level Annual Premiums

The corresponding net level annual premiums are

$$\frac{K_x^{0/m} - K_{x+n}^{0/m}}{N_x - N_{x+n}} \text{ and } \frac{K_x^{0/m} - K_{60}^{0/m}}{N_x - N_{60}},$$

where $N_x = \sum_{t=0}^{t=\omega-x} D_{x+t}$ and are denoted by $(PS)_{x:\overline{n}|}^{0/m}$ and $(PS)_{x:\overline{60-x}|}^{0/m}$

respectively.

Thus we have $(A\ S)_{x:\overline{n}|}^{0/m} = \dfrac{K_x^{\ 0/m} - K_{x+n}^{\ 0/m}}{D_x}$

$\qquad\qquad (P\ S)_{x:\overline{n}|}^{0/m} = \dfrac{K_x^{\ 0/m} - K_{x+n}^{\ 0/m}}{N_x - N_{x+n}}$

$\qquad\qquad (A\ S)_{x:\overline{60-x}|}^{0/m} = \dfrac{K_x^{\ 0/m} - K_{60}^{\ 0/m}}{D_x}$

$\qquad\qquad (P\ S)_{x:\overline{60-x}|}^{0/m} = \dfrac{K_x^{\ 0/m} - K_{60}^{\ 0/m}}{N_x - N_{60}}$

If, for example, the payment of the benefit is limited to fifty-two weeks in any one sickness, m becomes 1, and in the usual case of a policy providing benefits for sickness originating prior to age 60, we have

$$(A\ S)_{x:\overline{60-x}|}^{0/1} = \frac{K_x^{\ 0/1} - K_{60}^{\ 0/1}}{D_x}$$

$$(P\ S)_{x:\overline{60-x}|}^{0/1} = \frac{K_x^{\ 0/1} - K_{60}^{\ 0/1}}{N_x - N_{60}}$$

With Period of Exclusion

In actual practice policies are often issued with a so-called "period of exclusion." That is to say, the payment of benefits does not commence until the insured has been sick for a given period. These exclusion periods are generally one, two or four weeks, and sometimes as high as thirteen weeks. Thus, if policies are to be issued with k weeks' exclusion and m years' limit, values of $H_x^{\frac{k}{52}/m}$ and $K_x^{\frac{k}{52}/m}$ are required, where

$$H_x^{\frac{k}{52}/m} = H_x^{\frac{k}{52}/1-\frac{k}{52}} + H_x^{1/1} + H_x^{2/1} + \ldots + H_x^{m-1/1} + H_x^{m/\frac{k}{52}}$$

and $H_x^{\frac{k}{52}/1-\frac{k}{52}} = v^{x+\frac{1}{2}}\ l_{x+\frac{1}{2}}\ s_x^{\frac{k}{52}/1-\frac{k}{52}}$, $\quad H_x^{1/1} = v^{x+\frac{3}{2}}\ l_{x+\frac{3}{2}}\ s_x^{1/1}$,

$H_x^{2/1} = v^{x+\frac{5}{2}}\ l_{x+\frac{5}{2}}\ s_x^{2/1}$, etc., and $K_x^{\frac{k}{52}/m} = \displaystyle\sum_{t=0}^{t=\omega-x} H_{x+t}^{\frac{k}{52}/m}$

If $k = 0$, the formulae reduce to those already established for benefits with no exclusion period. Thus, for a policy issued at age x, renewable up to age 60, with a one week exclusion period and fifty-two weeks' limit, we have, for single and annual premiums

$$(A\ S)^{\frac{1}{52}/1}_{x:\overline{60-x}|} = \frac{K_x^{\frac{1}{52}/1} - K_{60}^{\frac{1}{52}/1}}{D_x}$$

$$(P\ S)^{\frac{1}{52}/1}_{x:\overline{60-x}|} = \frac{K_x^{\frac{1}{52}/1} - K_{60}^{\frac{1}{52}/1}}{N_x - N_{60}}$$

The one year term premium for a benefit with a one week exclusion period and fifty-two weeks' limit then becomes

$$(A\ S)^{\frac{1}{52}/1}_{x:\overline{1}|} = \frac{H_x^{\frac{1}{52}/1}}{D_x}$$

Since in practice benefits are seldom, if ever, payable for incapacity originating beyond age 60, the values of $H_x^{\frac{k}{52}/m}$ are generally not computed beyond age 59, and the values of $K_x^{\frac{k}{52}/m}$ are computed by summing the H column upward from 59; that is, assuming age 59 to be the limiting age of the table. We then

have

$$(A\ S)^{\frac{k}{52}/m}_{x:\overline{60-x}|} = \frac{K_x^{\frac{k}{52}/m}}{D_x}$$

$$(P\ S)^{\frac{k}{52}/m}_{x:\overline{60-x}|} = \frac{K_x^{\frac{k}{52}/m}}{N_x - N_{60}}$$

ACTUARIAL BASIS FOR PREMIUM AND CLAIM RESERVES

Reserves, as already stated, are of two kinds: first the premium reserves or reserves on active lives, and second, the claim reserves or reserves on incapacitated lives. For non-cancellable policies issued on the level premium plan, an additional reserve on active lives is required which is the accumulated excess of premiums

collected in the early years over the one year term premiums for such years.   While the convention blank does not require it, this level premium reserve should also be set up on cancellable business where premiums are computed on a level premium basis. The convention statement blank provides for setting up as un-earned premium reserve one-half of the gross premiums current as of the date of the statement "upon all unexpired risks running one year or less from date of policy," and a pro-rata part of the premium as unearned "upon all unexpired risks running more than one year from date of policy."   For instance, on a three year policy $\dfrac{2\frac{1}{2}}{3}$ times the premium would be set up as un-earned at the end of the calendar year in which the policy was issued and at the end of the subsequent calendar year $\dfrac{1\frac{1}{2}}{3}$ times the premium would be set up.   This assumes that the premium due dates are equally distributed throughout the year.   Such an assumption is not always correct.   A more accurate result is sometimes obtained by classifying the premium by months of renewal and determining unearned pro rata premiums on the assumption that the renewal dates in each month are equally distributed throughout that month; that is, that the due date comes, on the average, in the middle of the month.   This latter method is used by many companies with the idea that it gives a more accurate estimate of the liability.

Level Premium Reserves

The level premium reserve for an active life at the end of any policy year is also the difference between the then value of the benefit and the value of the future contributions to be received. Thus, if the benefit is payable for all durations of sickness with no exclusion period, and for sickness originating throughout the balance of life the reserve at the end of the year immediately before the next premium is paid is $_n(V\,S)_x^{0/all} = (A\,S)_{x+n}^{0/all} - (P\,S)_x^{0/all}\, a_{x+n}$ where $_n(V\,S)_x^{0/all}$ represents the desired value at the end of n years and a_{x+n} represents the then present value of

one dollar payable throughout life (first payment due immediately). If the benefit provides only for sickness originating before age 60 the formula becomes

$$_n(V\ S)^{0/all}_{x:\overline{60-x}|} = (A\ S)^{0/all}_{x+n:\overline{60-(x+n)}|} - (P\ S)^{0/all}_{x:\overline{60-x}|}\ a_{x+n:\overline{60-(x+n)}|}$$

In the practical case of a benefit with k weeks' exclusion and m years' limit, payable for sickness originating prior to age 60, the formula is:

$$_n(V\ S)^{\frac{k}{52}/m}_{x:\overline{60-x}|} = (A\ S)^{\frac{k}{52}/m}_{x+n:\overline{60-(x+n)}|} - (P\ S)^{\frac{k}{52}/m}_{x:\overline{60-x}|}\ a_{x+n:\overline{60-(x+n)}|}$$

Or, in terms of commutation symbols with the H_x column ceasing at age 59,

$$_n(V\ S)^{\frac{k}{52}/m}_{x:\overline{60-x}|} = \frac{K^{\frac{k}{52}/m}_{x+n}}{D_{x+n}} - \frac{K^{\frac{k}{52}/m}_{x}}{N_x - N_{60}} \cdot \frac{N_{x+n} - N_{60}}{D_{x+n}}$$

For the "level premium reserve" in the annual statement, it is customary to assume that the business is evenly distributed throughout the year. The convention blank calls for a reserve of the unearned premium for the current year. On this basis the additional level premium reserve necessary as of December 31st, assuming annual premium business, would be the mean of the reserve at the end of the policy year preceding and at the end of the policy year subsequent to that December 31st. When premiums are payable other than annually, the necessary adjustment should be made.

Claim Reserves

The reserve on incapacitated lives is the value of an annuity under which payments are made for the period of incapacity provided for. Those who become sick in any year produce sickness in that year, and in each year following, until all have either recovered or died. The present value of all sickness originating at age x is expressed in commutation form as $\dfrac{H_x^{0/all}}{D_x}$ where

$D_x = v^x l_x$ and the l_x used represents the total number of persons living at age x according to the mortality table. If instead of this total number of persons living, the total number becoming incapacitated were used, a new table with D_x^i equal to $v^x l_x^i$ could be constructed, where l_x^i represents the total number becoming incapacitated at age x. The value of future sickness payments

on a life just becoming sick then becomes $\dfrac{H_x^{0/all}}{D_x^i}$. In like

manner the value of the future payments on lives who had been

sick for one year would be $\dfrac{H_x^{1/all}}{D_{x+1}^{i\,1/all}}$ where $D_{x+1}^{i\,1/all} = v^{x+1} l_{x+1}^{i\,1/all}$

and $l_{x+1}^{i\,1/all}$ represents the number who become sick at age x and are still incapacitated at the end of the year. These values represent the general case; for the practical cases of benefits with k weeks' exclusion and m years' limit the values can be found by suitable modification of the general case.

In some cases the number of persons who become incapacitated at age x is not readily available, and a close approximation may be

obtained by dividing the sickness rate for ages $\left[x - \dfrac{1}{2} \right]$ by 52

in order to obtain the mean annual sickness rate expressed as a fraction of a year and then multiplying by l_x. This method is described in detail in Appendix VI of the Report for 1912-13 on the Administration of the National Insurance Act of Great Britain.

As stated at the opening of this paper, sickness as here used is construed to be either sickness in the more restricted sense or sickness in the sense of any incapacity resulting from sickness or accident. In any particular case s_x must be taken to include only the hazard insured against. For example, if benefits are to be paid only in the event of incapacity resulting from accident, s_x must be taken to include weeks of incapacity resulting from accident only. If benefits payable for incapacity on account of sickness are limited to m years while benefits for incapacity on account of accident are unlimited, then for the sickness an s_x

must be ascertained and commutation columns prepared in the form of $K_x^{\frac{k}{52}/m}$; while for the accident benefits an s_x must be ascertained and commutation columns prepared in the form of $K_x^{\frac{k}{52}/all}$. The premiums for each must then be computed separately. The formula is perfectly general, although the sickness rate s_x required can be enlarged to mean the incapacity rate for sickness, accident, or other contingency, or for benefits reducing in any proportion after any period of time.

The experience under Policy Form 02, as shown in the first part of this paper, is practically constant from ages 20 to 50. In general sickness rates for policies renewable annually at the option of the company have been found to follow this tendency, and when such is the case $(A\ S)^{\frac{k}{52}/1}_{x:\overline{1}|}$ remains practically constant for all values of x from 20 to 50. This is the reason why when benefits are limited to a short period, companies have developed the practice of issuing cancellable health policies on the basis of one year term contracts, charging the same premiums from ages 20 to 50, and then increasing the premium for the remaining ages up to 55 or 60. The only object of the company in reserving the right to cancel at the end of any year is to keep the sickness rate reasonably within the limits assumed. Under non-cancellable business, however, the sickness rate shows a gradual increase by age for ages 30 and over. It is, therefore, necessary either to use an experience which is based on conditions similar to those for the policies for which premiums are being determined or to make due allowance for any variations from these conditions.

Accident and health policies usually provide, in addition to the weekly indemnities, specific benefits for loss of life, limb, or sight, resulting from accidental injury, and sometimes certain payments for hospital confinements and surgical operations. The value of these can be obtained by adjusting the s_x value, or by using a rate for the benefit payable in the event of death or dismemberment corresponding to the s_x and a special s_x rate prepared for the other benefit.

AUXILIARY FACTORS IN RATING FOR PERSONAL ACCIDENT AND HEALTH INSURANCE

Certain psychological conditions have a decided influence on the sickness rate actually experienced and must be considered in determining premium rates. If total disability benefits are supplemented by partial disability benefits there may be a decrease in the amount of claim paid rather than an increase. When premiums are found to be inadequate a reduction in benefits with a pro rata reduction in premiums is sometimes more likely to improve the experience than an increase in premium rates. If no benefits are payable during, say, the first four weeks of any sickness the sickness rate of more than four weeks probably will differ from the sickness rate for the same period when benefits are payable from the first day of sickness. Where the benefits granted are a relatively high proportion of the earned income of the insured, the sickness rate will be higher than where it is a smaller proportion. It, therefore, has been found necessary to limit the benefit to about 75% of the earnings. Even where such a limit is imposed the sickness rate will be higher for relatively or even actually higher amounts of benefit than where the amounts are low. In the experience of the Bureau of Personal Accident and Health Underwriters already referred to, the sickness rate for benefits of $100 a week and over is about 20% to 65% higher than for benefits of less than $100 a week. Part of this is probably due to the higher proportion of the earnings paid as benefit, but the same tendency of higher sickness rates accompanying larger benefits is shown for practically every classification of amount of benefit.

LOADING

The net premiums calculated by the methods of the foregoing formulas must be loaded for expenses and contingencies in order to produce the gross premium charged by the company and paid by the insured. If, for example, taxes, commissions, other expenses and contingencies are estimated to be 50% of the gross premium, then the net premium must be doubled in order to determine the premium to be charged. If it is assumed that the expenses and contingencies will be 33⅓% of the gross premium, then 50% must be added to the net premium in order to obtain

the gross premium to be charged. The amount of loading in all cases is clearly determined by the expense rate expected to be incurred and by whatever is to be allowed for contingencies.

The theoretical premiums on both the One Year Term basis and the Level Annual Premium basis follow from the sickness rates assumed. An illustration of the computation is given at the end of this paper. There are certain practical considerations which make it necessary to vary the final premiums from the theoretical premiums as computed. In the illustration, the One Year Term rate at age 20 is higher than at age 30, which in turn is higher than the rate at age 40. After age 40 the One Year Term rates increase up to the end of the table. In practice it is desirable to have gross premium rates which are either constant by age or which grade smoothly upward with the age. On the One Year Term basis a constant premium would probably be charged for all ages from 20 to 50 at a rate of about $.82 a year.

The desirability of having the gross level premium rates constant by ages or grading upward with true ages bears some explanation. If a higher rate were charged at age 20, then at age 25 a policyholder who was insured at age 20 could discontinue his policy at age 25 and obtain another policy at a lower rate. Where the level premium decreases it also indicates that the premium charged in the early years does not cover the benefit granted. The practical solution would be to charge the rate necessary at age 20 for all ages at entry from 20 up to the age where premium was the same as that at age 20. Gross premiums on the One Year Term and on the Level premium basis, calculated according to the corresponding formulae described herein and on the basis of the s_x column quoted at the beginning of the paper, are shown in the last two columns of the schedule.

ILLUSTRATIONS OF PREMIUM CALCULATION FOR A BENEFIT OF \$1 A WEEK
No Exclusion Period, Fifty-two Weeks Limit

Age x	$l_{x+\frac{1}{2}}$ Cammack's Clerical	$v^{x+\frac{1}{2}}$ 3½%	Total Dis. Rate $+\frac{1}{2}$ Part Dis. Rate (Pol. Form 02) $S_x^{0/1}$	$H_x^{0/1}$ $v^{x+\frac{1}{2}} l_x$ $+\frac{1}{2} S_x^{0/1}$	$K_x^{0/1}$ $= \Sigma\, H_x^{0/1}$ All ages from x to 59	D_x Cammack's Clerical	$N_x - N_{60}$ Cammack's Clerical	Net Premium 1 yr. Term $(PS)_{x:\overline{1}|}^{0/1}$ $= \dfrac{H_x^{0/1}}{D_x}$	Net Premium Annual to age 60 $(PS)_{x:\overline{60-x}|}^{0/1}$ $= \dfrac{K_x^{0/1} - K_{60}^{0/1}}{N_x - N_{60}}$	Net Premium Adjusted 1 yr. Term	Net Premium Adjusted Level	Gross Loading 33% of gross 1 yr. Term	Gross Loading 33% of gross Level
	(1)	(2)	(3)	(4) $1\times2\times3$	(5)	(6)	(7)	(8) $4\div6$	(9) $5\div7$	(10)	(11)	(12)	(13)
20	98,948	.493995	.881	43062.9	845076.6	49,778	1,046,184	.86510	.80777	.82	.83	1.23	1.25
30	96,799	.350202	.770	26102.4	487399.9	34,529	621,493	.75596	.78424	.82	.83	1.23	1.25
40	93,736	.248265	.715	16638.9	274566.4	23,727	327,712	.70126	.83783	.82	.84	1.23	1.26
50	87,385	.175999	.835	12842.0	129497.0	15,733	128,207	.81625	1.01006	.82	1.01	1.23	1.52
60	74,126	.124769	1.270	11745.7	0	9,527	..	1.23289		1.23	..	1.85	..

NOTE: K_x is shown for ages 20, 30, 40 and 50 only; however this is the result of summing the H_x column for all ages from 59 to x.