VALUATION OF PENSION FUNDS, WITH SPECIAL REFERENCE TO THE WORK OF THE NEW YORK CITY PENSION COMMISSION.

BY

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The City of New York is facing the problem of reorganizing its pension systems. Its methods of meeting this problem may be of general interest in that the inherent defects of its system, which have forced a reorganization, are commonly found in municipal systems throughout the country.

According to the Commission's records, the first law relating to pensions for municipal employees in this country was passed in 1857. This law was passed by the New York Legislature and applied to policemen of New York City. In the next year $30 were expended in pension payments by the city; in the first thirty years about $1,342,000 were paid out; but last year over $5,000,000 were disbursed. In other words, in one year the city is now disbursing over three times as much as it did in the entire first thirty years of its pension experience, which is at a rate over one hundred times as great as the average of the first thirty years. But this is not all. Pension laws did not formerly include all city employees in their scope—the pension payments are therefore small to what may be expected. Within the last five years legislation has been broadened to cover over 35,000 additional active employees. The pension load from this source has not yet made itself evident.

But what about the assets of the funds? The pension payments are enormous but they do not represent an extravagant pension policy in themselves when contrasted with the present payroll of $99,000,000, if it were not for the fact that the payments are increasing by leaps and bounds and no adequate methods have been provided for meeting them. Each of the funds is in the position of an insolvent insurance company that has no reserve and which is constantly adding new policyholders and using their premiums to pay maturing claims. Fortunately the insurance laws will not permit this in the case of companies, but thus far pension funds are

not restricted. New York City has set aside certain excise and other indirect taxes to cover the liability but these funds are wholly inadequate to meet the future demands. So far the systems have been financed on the assessment basis, as it were, by continually adding new sources of revenue as the others become inadequate; but this method is becoming more and more onerous, so that some reorganization of the general system of financing seems imperative.

Practically all of the 228 cities in the United States with more than 25,000 inhabitants have pension systems of some kind. The eighteen cities with a population of over 300,000 pension their firemen, policemen and teachers; and seven of these cities have additional funds for other branches of the municipal service. The majority of these cities, like the City of New York, have failed to exhibit forethought in providing for the actuarial soundness of their system. The time is approaching when either faith cannot be kept with their employees or the cities themselves will be overburdened by a financial strain for which they have not made adequate preparation. The size of the New York system and the longer period of its establishment have resulted in a more imperative need for reorganization than elsewhere. The fact that New York is a pioneer in this field gives peculiar value to the results of its experience.

The Commission on Pensions of New York City was appointed by Mayor Gaynor in the summer of 1913, to inquire into the present situation of the city as regards pensions and to recommend such changes as might be considered desirable. In 1914 a staff was organized to carry on the necessary statistical and actuarial work. The first question which was asked of the staff was: "What about the existing funds, do they meet the requirement of properly relieving the superannuation problem and what will they cost, if continued?" The preparation of the answer to the second half of this question was assigned to the author, who fortunately had the advice and counsel of a distinguished committee of actuaries appointed by the Actuarial Society of America, to act as an advisory committee to the Commission in its work. This committee consisted of Mr. William A. Hutcheson, Actuary of the Mutual Life Insurance Company; Mr. Robert B. Henderson, Actuary of the Equitable Life Assurance Society, and Mr. Henry Moir, Actuary of the Home Life Insurance Company.

With the exception of certain tables for school teachers and for

police, the former prepared by Messrs. Hutcheson and Thompson, and the latter by the author, there were no basic tables on which calculations of prospective cost or pension liability could be made. The task was that of valuing the assets and liabilities of one of the largest insurance and annuity carriers in this country and there was not available even a list of policyholders, much less the proper experience tables upon which reserves or any calculations of prospective liability might be made.

The first step of the Commission was to obtain adequate data both in regard to employees covered and in regard to employees who had separated from the service, during a period of six years, which was adopted as a proper basis for the experience. There had been no centralized administration of these funds and no adequate records had been kept from which the Commission could directly obtain its data. Uniform schedules bearing necessary data were secured, therefore, for all employees actively employed on June 30, 1914, the date set for the valuation. Another schedule was obtained from each pensioner. From such sources as were available, records were collected in regard to out of service employees and pensioners who had been separated from the roll in the period of experience.

The extent of the data that was collected is indicated by the fact that over 76,000 employees on a payroll of $99,000,000 were accounted for. The out of service cards numbered 29,000. The number of dependents considered was over 60,000. The combined exposure of active employees in the period amounted to nearly 395,000 years.

New York City covers all of its employees by means of nine distinct funds and as one of these funds covers such dissimilar classes of employees that it had to be subdivided before experience tables were prepared, twelve distinct services had to be investigated. For these twelve services, altogether some twelve active service tables were prepared and as many pensioners' mortality tables were constructed or adopted for use. A complete account of the methods of dealing with the data and the procedure followed in deriving all basic tables is given in the Actuarial Report of the Commission, which is now in the hands of the printer.

Perhaps the tables of greatest general interest to the actuary or statistician in the Actuarial Report are those giving data to be used in the valuation of benefits to widows and children. The

Commission in its study of municipal and industrial pension funds in this country was unable to locate a single source from which such data could be obtained in a form suitable for use in a valuation. Actuaries in this country have used data compiled abroad for this purpose. The work of the Commission makes available for the first time some American data based on tabulations covering about 84,000 persons in all.

Although a detailed description of such data is given in the report, it might be well to draw to your attention the special features covered. It was found that the family history data tabulated showed little correlation with the occupation of the employee. Therefore, all experience was combined to form a single basis for a single set of tables. The tables presented cover the marital condition of employees by age and the average age of wife to that of husband for each age of husband.

Full tabulations are presented of the total number of children per family, classified by age, and of all youngest children per family, classified by age. The work of the Commission required the average age of the youngest child of the father dying and the proportion of fathers leaving a youngest child under age of eighteen. These adjusted values are given, in which, moreover, a distinction is made between children of husbands and those of widowers and divorcees.

General methods of valuing pensions have been already expounded by Messrs. Manly, King, M'Laughlan and others in their admirable papers before the Institute of Actuaries and the Faculty of Actuaries. To develop the fundamental principles at this time would be a repetition of what has already been most ably done, consequently it seems advisable to bring forward certain methods of valuing which have been found of service in appraising funds which do not admit of valuation by any of the methods which have thus far found their way into the chief repositories of such information, the publication of the actuarial societies. These methods refer to the valuation of those funds to which aggregate tables are not directly applicable.

The use of select service tables has been generally recognized as an ideal method of reflecting the peculiarities of certain services. Mr. George King, in a discussion of this point before the Institute of Actuaries, has remarked that "Some day actuaries would be driven to use select tables," that he was "quite sure it made a

great deal of difference at certain ages of entry if the subject were treated by the select method." The theoretical nicety of results to be obtained by select methods has generally been overbalanced by the attendant practical difficulties, the thought of which, as Mr. King relates, was sufficient, when he predicted the use of select tables to the Institute of Actuaries, to "cause a shiver to go round the hall." Probably everyone who has had experience in pension fund valuation enjoys the aforesaid "shiver" when select tables are mentioned. But the sensation is highly intensified when such a one finds himself for the first time confronted with the valuation of a fund which will not admit of handling on an aggregate basis. In such a case the mechanical short cuts hereafter described may be of service.

It was found in certain of the services covered by the Commission's investigation, and similar conditions are frequently encountered in industrial pension funds, that the rates of resignation and dismissal, when developed on the aggregate basis, averaged in the neighborhood of from 10 per cent. to 15 per cent. for the ages 20 to 40, which made the use of an aggregate table impracticable, because, no matter what radix was assumed at age 20, the employment of such a high rate in the construction of a table reduced it almost to a decimal within a very few years, thus rendering the table unfit for valuation purposes. The reason for such an excessive rate of withdrawal was because of the wholesale resignations and dismissals which occurred among employees with less than four or five years' service. Various methods were tested for overcoming this difficulty but the results did not commend them for use. Finally the following method was devised, which is best described by a consideration of one of the actual services demanding select development.

In the New York City Street Cleaning Department the rates of resignation and dismissal were found to be comparatively high in the first three years of service, whereas the rates of death and disability did not show a material variation with years of service. The experience was prepared on a select basis for the first three years of service and the experience for the remaining years of service carried on an ultimate basis. Rates were then graduated from the select experience only for those contingencies which showed a direct variation with years of service; that is, the contingencies of resignation and dismissal. From the ultimate expe-

rience, rates of death, disability and service retirement were derived in addition to ultimate rates of resignation and dismissal.

A service table was constructed as follows: The ultimate column of the table was run according to the ordinary method of constructing an active service table. The values for the number living for the select periods of service were built up on this, not by the ordinary method of dividing the number living in the ultimate column by the probability of living for the next earlier age and year of service, but by adding the numerical value of the ultimate decrements caused by death and disability to the ultimate number living and then by dividing the result by the complement of the combined select rates of dismissal and resignation.

The method is more clearly described by the use of symbols.

Let the rates graduated be represented as follows:

${}^{dw}q_{[x]}$; ${}^{dw}q_{[x-1]+1}$; ${}^{dw}q_{[x-2]+2}$ = rates of dismissal at age x in the first, second and third years of service respectively.

${}^{rw}q_{[x]}$; ${}^{rw}q_{[x-1]+1}$; ${}^{rw}q_{[x-2]+2}$ = rates of resignation at age x in the first, second and third years of service respectively.

${}^{dw}q_x$ = ultimate rate of dismissal.

${}^{rw}q_x$ = ultimate rate of resignation.

${}^{d}q_x$ = ultimate rate of death.

${}^{ir}q_x$ = ultimate rate of disability.

${}^{or}q_x$ = ultimate rate of retirement.

The first step—the construction of the ultimate table—is represented by

$$l_x = l_{x-1} \cdot [1 - ({}^{dw}q_{x-1} + {}^{rw}q_{x-1} + {}^{d}q_{x-1} + {}^{ir}q_{x-1} + {}^{or}q_{x-1})].$$

The ultimate decrements are obtained:

$$\begin{aligned} &\text{Ultimate deaths} &&= d_x = l_x \cdot {}^{d}q_x, \\ &\text{Disability cases} &&= {}^{i}r_x = l_x \cdot {}^{ir}q_x, \\ &\text{Regular retirements} &&= {}^{o}r_x = l_x \cdot {}^{or}q_x, \text{ etc.} \end{aligned}$$

Then the column representing those living in the third year of service is obtained as follows:

$$l_{[x-2]+1} = \frac{l_x + d_{x-1} + {}^{i}r_{x-1} + {}^{o}r_{x-1}}{1 - ({}^{dw}q_{[x-2]+1} + {}^{rw}q_{[x-2]+1})}$$

and accordingly the decrements in the third year of service are, for resignation and dismissal, obtained by multiplying the $l_{[x-2]+1}$ value by

$${}^{rw}q_{[x-2]+1} \quad \text{and} \quad {}^{dw}q_{[x-2]+1} \text{ respectively.}$$

All other decrements are numerically equivalent to the decrements at the same age in the ultimate column, and the result is that the rates for death and disability in the second year of service are represented by the ratios:

$$\frac{d_{x-1}}{l_{[x-2]+1}} \quad \text{and} \quad \frac{{}^{i}r_{x-1}}{l_{[x-2]+1}}.$$

By continuing this process the select table may be carried back to the first year of service.

The service table is of the form shown on page 430.

That this method automatically causes a reduction in the death and disability rates in the select years of experience is apparent. Therefore, it can be applied only when the service warrants the assumption that there is a slight increase in the rate of death and disability with increase in years of service. The assumption is generally a fair one even where no medical examination of appointees may be made, as there is a certain selection which takes place at appointment, since it is not to be supposed that an appointing officer will employ a man who is not apparently in good health.

This form of service table effects a very great saving of time over what is required if a valuation be made on a select table developed in the customary manner. Of course, select commutation columns involving the number living are necessary, but where no benefits are payable upon resignation or dismissal all benefits can be valued on single sets of commutation columns based on the ultimate numerical values common to all periods of service. In some services which have distinct benefits payable upon death in performance of duty, death from other causes, disability in performance of duty, disability from other causes, and service retirement, there is sometimes a very great saving in the amount of labor which would ordinarily be required in the preparation of commutation columns and in the actuarial valuation work.

PENSION FUND ACTIVE SERVICE TABLE IN SELECT FORM.

Age.	Living.				Withdrawals.								Total Ultimate.	Deaths.	Separations by Disability.			Service Retirement.
					Resignations.				Dismissals.						In Performance of Duty.	Other Causes.	Total.	
	$l_{[x]}$	$l_{[x-1]+1}$	$l_{[x-2]+2}$	l_x	$r_{w_{[x]}}$	$r_{w_{[x-1]+1}}$	$r_{w_{[x-2]+2}}$	r_{w_x}	$d_{w_{[x]}}$	$d_{w_{[x-1]+1}}$	$d_{w_{[x-2]+2}}$	d_{w_x}	w_x	$\frac{d_{[x]}}{d_{[x-1]+1}}$ $\frac{}{d_{[x-2]+2}}$ d_x	$a i'_{r_{[x]}}$ $a i'_{r_{[x-1]+1}}$ $a i'_{r_{[x-2]+2}}$ $a i'_{r_x}$	$o i'_{r_{[x]}}$ $o i'_{r_{[x-1]+1}}$ $o i'_{r_{[x-2]+2}}$ $o i'_{r_x}$	$i'_{r_{[x]}}$ $i'_{r_{[x-1]+1}}$ $i'_{r_{[x-2]+2}}$ i'_{r_x}	o_{r_x}
20	1,000,000	....	....	....	47,000	....	....	....	62,200	....	....	....	....	3,849	46	....	46	....
21	975,932	886,905	....	....	44,990	33,968	....	....	84,516	41,507	....	....	....	3,870	49	....	49	....
22	959,536	842,507	807,511	....	43,659	30,920	24,952	....	106,029	49,624	37,550	....	....	3,896	53	....	53	....
23	922,621	805,899	758,014	741,060	41,334	28,287	21,755	17,155	110,622	57,219	38,810	26,456	43,611	3,935	58	....	58	....
24	871,486	766,672	716,400	693,456	38,258	25,760	18,985	14,043	104,840	58,574	38,112	20,110	34,153	3,994	60	....	60	....
25	812,098	724,334	678,284	655,249	34,920	23,179	16,618	11,827	91,605	54,325	35,543	17,299	29,126	4,076	69	....	69	....
26	752,352	681,428	642,685	621,978	31,712	20,988	14,653	10,013	76,138	46,473	31,492	15,425	25,438	4,136	75	....	75	....
27	697,917	640,291	609,756	592,329	28,859	18,696	12,805	8,411	61,137	37,778	27,012	13,970	22,390	4,223	84	11	95	....
28	651,691	603,603	579,499	565,621	26,393	16,901	11,242	7,183	49,073	30,301	22,659	12,840	20,023	4,293	94	19	113	....
29	614,111	571,819	551,995	541,192	24,441	15,325	9,936	6,213	40,900	24,988	18,878	11,798	18,011	4,345	107	56	163	....
30	583,825	544,262	526,998	518,673	22,828	14,042	8,906	5,447	35,905	21,226	15,810	10,944	16,391	4,372	119	192	311	....
31	557,404	520,409	504,311	497,599	21,349	13,010	7,918	4,822	32,273	18,839	13,717	10,101	14,923	4,369	134	413	547	....
32	533,873	498,866	483,644	477,760	20,020	12,023	7,162	4,300	29,737	17,460	12,338	9,316	13,616	4,362	153	707	860	....
33	512,246	478,894	464,161	458,922	18,953	11,206	6,498	3,910	27,508	16,522	11,233	8,582	12,492	4,323	172	883	1,055	....
34	491,739	460,407	445,788	441,052	17,899	10,497	5,884	3,551	25,718	15,792	10,254	7,851	11,402	4,247	189	1,090	1,279	....
35	472,683	442,596	428,592	424,124	16,922	9,781	5,357	3,236	24,532	15,049	9,558	7,211	10,447	4,186	203	1,281	1,484	....
36	453,232	425,559	412,096	408,007	15,954	9,150	4,904	2,962	22,571	14,383	8,736	6,589	9,551	4,125	211	1,421	1,632	....
37	435,411	408,950	396,269	392,699	15,065	8,588	4,478	2,726	21,466	13,495	7,846	6,028	8,754	4,064	216	1,669	1,885	....
38	417,885	392,931	380,918	377,996	14,250	8,094	4,114	2,499	20,309	12,378	6,856	5,549	8,048	4,003	218	1,785	2,003	....
39	401,294	377,320	366,453	363,942	13,544	7,622	3,774	2,307	19,222	11,055	6,157	5,113	7,420	3,938	217	2,040	2,257	....
40	385,369	362,333	352,448	350,327	12,756	7,210	3,524	2,119	18,266	9,783	5,463	4,747	6,866	3,872	213	2,239	2,452	....
41	....	348,023	339,016	337,137	....	6,786	3,322	1,949	.....	8,701	4,916	4,410	6,359	3,799	206	2,322	2,528	....
42	....	....	326,209	324,451	....	....	3,099	1,794	.....	....	4,567	4,114	5,908	3,718	198	2,463	2,661	....
43	....	....	....	312,164	....	....	....	1,648	.....	....	....	3,846	5,494	3,630	188	2,653	2,841	....
44	....	....	....	300,199	....	....	....	-1,504	.....	....	....	3,611	5,115	3,537	177	2,825	3,002	....
45	....	....	....	288,545	....	....	....	1,379	.....	....	....	3,399	4,778	3,434	164	2,895	3,059	....

In valuing it is necessary to multiply the number of persons or their salaries, at any age or length of service, by certain multipliers or reserve values which are derived from the commutation columns. These values are usually expressed as fractions, in which the denominator is ordinarily some form of the symbol D_x. If this be found the case in a valuation, then the number or salaries of persons at age x, with less than one year of service may be divided by the proper $D_{[x]}$ value; those at age x with less than two years' service by the proper $D_{[x-1]+1}$ value; and so on through the ultimate value D_x, after which all the quotients thus obtained for persons of age x may be added together. The result may then be treated for all values as if we were using an aggregate table and the proper reserve for all persons at that age, regardless of the length of service, may be obtained directly by the application, by way of multiplication, of the appropriate ultimate numerator, if the benefit is not dependent on the employee's length of service. In this case neither the values for the select years nor the values for the ultimate years can be grouped together unless we resort to some arbitrary grouping for the sake of reducing the amount of labor required in the valuation work.

The next method which is presented has to do with the valuation of one of such benefits limited by length of service; namely, a method for valuing regular or service pensions allowed after some definite period of service, as 30 or 35 years, where no definite age limitation is included. To be sure, such benefits may be valued simply as deferred annuities, which assumes that each employee will retire on the completion of the span of service required for eligibility. The deferred annuity basis is safe. If everyone should elect to go out immediately after reaching eligibility there would be sufficient money to grant and pay all annuities. But as a matter of fact everyone does not elect to retire at this time; in fact some would never retire of their own volition. If we value on this basis we would accumulate a fund which in all probability would be very much more than sufficient to pay the annuities actually claimed (since the average length of service will be several years more than the minimum period required), and the rates of contribution required to support the fund would be very much larger than those actually needed to pay the pensions necessary to relieve the service superannuation. Not only is it undesirable from the social viewpoint to have excessive premiums or contributions paid by an em-

ployee between the ages of 20 and 50, but the practical difficulty of reorganizing insolvent pension funds, because of the contributions required of employee or employer, requires that the transition from a period of hopeless insolvency to one of absolute solvency be made as easily as possible.

We must therefore resort to a rate of service retirement just as we have a rate of disability retirement; however, such a rate cannot be handled in the same manner as is the rate of disability. Disabilities may be expected to occur without regard to the pension allowances, as they are dependent on vital conditions which we may assume would have been prevented if possible and which will occur whether the pension be granted or not. But service retirement is quite different; obviously there could be no service retirement if there were no pension, and consequently no rate of retirement until the employee enters the period of eligibility for pension. This again brings up the question of the necessity of treatment on a select basis.

The problem is admirably stated by Mr. E. C. Thomas, of the Institute of Actuaries, in a discussion before the members of that body in the meeting of June, 1908. He said, in part, "Assume that a pension was given according to scale or retirement after fifteen years' service . . . and it was required to know the rate of contribution for an entrant aged 50. The aggregate service table on which calculations were based would show a very large number of retirements between the ages of 50 and 65, so that the number of survivors at 65 debited with a pension would be comparatively small, but in the commutation column for obtaining the value of the benefit for the entrants at age 50 one would ignore all the retirements under 65, for the rules said that those should have no benefit, and the column ${}^{ra}C_x$ would commence at that age. Thus, full credit would be taken for the diminution in the number of survivors caused by the early retirements, without allowing for any compensating benefits. The probability was that the rate of retirement in such a case would be practically nil, and in order to get a proper estimate of the cost the retirements assumed in the aggregate service table ought to be put back into the l_x column for that purpose, so that those of them who survived the mortality risk would be assumed to take out, at 65, the pension to which they were then entitled." (*J. I. A.*, Vol. XLII, p. 63.)

In valuing service pensions based on length of service the Com-

mission on Pensions used a method which apparently performs what Mr. Thomas suggested. Had his writing been at hand when the formulae were prepared, the author probably would have asked for his criticism of the method, but unfortunately it did not come up in this connection until recently, when one of the members of the staff went through the books in the office to see if the methods herein described had been suggested or covered in papers before any of the societies.

The method of accomplishing the desired result is as follows: A supplementary column is constructed in connection with the column showing the number living of the active service table, which carries forward those who are shown by the table to retire. In bringing them forward they are assumed to be subject to the risk of death only. The group at age x in the supplementary column is denoted by the symbol

$$^{or}l_x = \left({}^{or}l_{x-1} + \frac{{}^{o}r_{x-1}}{2} \right) p_{x-1} + \frac{{}^{o}r_{x-1}}{2}.$$

The rate of retirement in the first year of eligibility by this method is

$$\frac{{}^{o}r_x + {}^{or}l_x}{l_x + {}^{or}l_x}$$

and in the ages of eligibility after the first year it is $\frac{{}^{o}r_x}{l_x}$.

Commutation columns were developed on the basis of this column as follows:

$$^{or_l}D_x = {}^{or}l_x \cdot v^x,$$

$$^{or_l}N_x = \sum_{x=x+1}^{x=\omega} {}^{or_l}D_x,$$

$$^{or_l}C_x = {}^{or}l_x \cdot v^x \cdot \bar{a}_x.$$

The symbol "ω" is used to indicate the highest age in the table under consideration.

The present value at entrance of a pension of one to the employee described by Mr. Thomas, entering at age 50 and being required to serve fifteen years before retirement, becomes in columnar values

$$\frac{{}^{or}\overline{M}_{65} + {}^{or_l}C_{65}}{D_{50} + {}^{or_l}D_{50}},$$

where

$$ {}^{or}\overline{M}_{65} = \sum_{x=65}^{x=\omega} {}^{or}\overline{C}_x, $$

$$ {}^{or}\overline{C}_x = {}^{o}r_x \cdot v^{x+\frac{1}{2}} \cdot \tilde{a}_{x+\frac{1}{2}}, $$

$$ D_{50} = l_{50} \cdot v^{50}, $$

and the contribution rate per year would be

$$ \frac{{}^{or}\overline{M}_{65} + {}^{or_1}C_{65}}{N_{50-\frac{1}{2}} + {}^{or_1}N_{50-\frac{1}{2}} - {}^{or_1}N_{65-\frac{1}{2}}}. $$

Without some sort of an adjustment of this nature in the use of an aggregate retirement rate, the cost of the retirement benefit will be greatly underestimated. Undoubtedly, the most advisable method of valuing benefits under these conditions would be to use select tables, but this would require a table for each age at entrance. This would mean from ten to twenty or more tables which, because of the many commutation columns, reserve values, etc., required, would be almost impracticable; while on the other hand, the possibility of obtaining an experience large enough to give an actual basis for such a set of tables is very remote. It is now generally recognized that every pension fund is a law unto itself and that no general active service or valuation tables or rules of procedure can be formulated, which are applicable to all funds. How to handle the retirement rate depends, of course, upon the service under investigation. In services where the minimum and maximum ages of entrance are close together and very few enter the service at ages much in advance of the average entrance age, the method just explained will be found probably to apply very satisfactorily.

On the other hand, in services where there are no limitations on the age at entrance and the dispersion around the average age at entrance is very great, then the method explained above will probably somewhat understate the liability.

The Pension Commission, in showing the liability of the city as regards pensions, preferred rather to understate than overstate the liability, since the purpose of its report was to show, not what the now insolvent systems might cost if continued, but rather what the probable minimum amount would be; that is, what the city and employees would actually have to provide as a minimum in order to prevent the collapse of the systems. If the problem involves the

valuation of a fund in a healthy condition, or the calculation of rates for a reorganized fund, where the desire would be to keep the fund on the side of safety, then the rate of retirement must be arbitrarily increased to what might be considered a proper ultimate rate of retirement, or might better be prepared from the experience from which all persons who have not been eligible for pension for at least one year have been excluded. This will give an ultimate rate of retirement which might be used in place of the aggregate rate in the construction of the aggregate service table. The rate for the first year of eligibility will be supplied automatically by the mechanical action of the formula suggested.

If an ultimate rate which is higher than the aggregate be employed as suggested, the results obtained will ordinarily tend to overstate the liability and be entirely on the side of safety. If the experience will permit, the method may be tested before being used and the extent of the overstatement will be known. If necessary, an adjustment may be made in the ${}^{or}{}^{1}C_x$ value which will so modify the action of the formula as to make it reproduce the experience at hand; however, this is generally not possible in small funds. In small funds it may not seem advisable to tabulate the data in a form which will furnish more than a set of aggregate rates. If so, an ultimate rate may be adopted as reasonable by estimating from the age and service distribution of the active service about what proportion of the exposure in the older ages is on account of persons who have reached the period of eligibility and then modifying the aggregate rate accordingly. If this rate be used the method just described will probably give satisfactory results, even if no modification be made in the ${}^{or}{}^{1}C_x$ value, because it will not produce reserves or contribution rates as high as the deferred annuity method, and yet will safely overcome the danger which has been pointed out by Mr. Thomas.

The method is set forth simply as one of several mechanical methods of dealing with this type of benefit. This benefit presents many difficulties which are not found in valuing funds having age limitations on pensions, since where the regular pension is limited by service it cannot be considered apart from the other benefits of the fund. For example, if the period of service necessary for eligibility be changed, the values or contribution rates for disability pensioners are thrown out as the change will affect the number of disability cases, and so on; a reduction in the disability allowances

will in turn throw out the service pension rates, and changes in either will affect the death benefit premiums and may change the whole form of the formula employed to calculate the cost of return contributions premiums, if such are included.

Because of these practical difficulties the application of the following method of valuing service pensions does not always seem advisable. However, it is believed that this method affords a more scientific means of valuing pensions of this type than any other that has been presented. It is as follows:

The active service experience of the fund is divided; all the experience relating to years of service prior to the period of eligibility is considered in one group, and all the experience of the employees after reaching the period of eligibility is considered in another. From the experience of the first group a service table is prepared in the customary manner. This table will of course not show a service retirement column, since no rate of service retirement can be developed from the experience of employees prior to eligibility. From the remainder of the experience, that is, for the experience of employees after reaching the age of eligibility, another table is prepared similar in form to the active service table. The entrants into the experience used as a basis for this table will not be entrants into service but entrants into the period of pension eligibility, and if there be a very high rate of retirements in the first few years of eligibility then this table may be prepared on a select basis. If this be done the method of construction may be similar to the method suggested earlier in this paper. Upon this second table commutation columns may be prepared, as follows:

l'_x = number living at age x in second table,

${}^{o}r'_x$ = number of retirements between ages of x and $x+1$,

$$D'_x = l'_x \cdot v^x,$$

$$N'_x = \sum_{x=x+1}^{x=\infty} D'_x,$$

$${}^{or}\bar{C}'_x = {}^{o}r'_x \cdot v^{x+\frac{1}{2}} \cdot \bar{a}_{x+\frac{1}{2}},$$

$${}^{or}\bar{M}'_x = \sum_{x=x}^{x=\omega} {}^{or}\bar{C}'_x.$$

Of course, if the table be prepared in select form these values will be based on the select table. If these commutation columns

be prepared they may be used in connection with the first active service table, by the combination:

$$^{or}\bar{C}_x = l_x \cdot v^x \cdot (^{r}\bar{M}'_x \div D'_x),$$
$$D_x = l_x \cdot v^x.$$

The solution of the present value of a pension of one to the employee entering service at age 50, and eligible to retire in fifteen years, as cited by Mr. Thomas, would be by the use of this method:

$$\frac{^{or}\bar{C}_{65}}{D_{50}},$$

and the contribution rate would be:

$$\frac{^{or}\bar{C}_{65}}{N_{50-\frac{1}{2}} - N_{65-\frac{1}{2}} + \frac{l_{65}}{l'_{65}} N'_{65-\frac{1}{2}}}.$$

These formulae are simpler than the approximate formulae involving the $^{r}l_x$ column, which were referred to above, as regards the service limitation, but their use necessitates the employment of slightly more complicated methods of valuing the various disability and death benefits which may be included in the fund. However, their use will probably give the best results where the experience is broad enough to permit of their application.

The technical problems of pensions today present their difficulties to the actuary, but these will probably be met, and as the many different systems present themselves with their varied ramifications, the methods will be modified and improved until we will have eventually reached a period where methods will be as clean cut and as general in their application as ordinary life insurance formulae.

But there are other problems of a non-technical nature to be solved by the Pension Commission. The valuation of each fund has now been completed and the total liability of the entire system determined. In round numbers such liability is $216,000,000. Of this amount $9,000,000 is provided by employees' contributions, and there remains a deficiency of $203,000,000 after deducting the funds in hand. Possibly $30,000,000 will be available from the indirect contributions of the city to cover this.

What will be done? Will the results of the Commission's work be used as a basis for building up a new system which will have the

assets adjusted to the liabilities and which will protect both the city and the employees against superannuation, or will the present funds be allowed to drag along, accumulating an ever-increasing liability until they come to the point where they cannot even meet matured obligations, as is now the situation with the teachers' pension fund?

Reorganization of the entire system will present many difficulties, not only from the technical standpoint, but in bringing the employees to realize that a solvent fund offers more advantages to them than the present one. This difficulty is aggravated by the fact that contributions from employees will probably be required or else the benefits provisions will be reduced. The systems have already accumulated such liabilities that it is difficult to see where the city can find the money to finance the systems even with the help of employees.

Probably the system can be properly reorganized for new entrants, as here the great problem of accrued liabilities need not be considered. By means of meetings and conferences with employees it may be possible to show them the conditions, and with the help of those employees who are sufficiently interested to really study the matter and obtain the facts for themselves, the Commission may be able to assist in placing all of New York City's pension systems on a rock bottom foundation.

However, the reorganization of the schemes marks but the beginning of the equally important work of keeping funds that are established on the right basis in a perpetually solvent condition. The administrative work of the funds must be properly organized, the proper record systems for the accounting work and for the accumulation of data necessary for actuarial valuations must be prepared.

The actuarial work is still to be continued. Means must be devised for valuing the experience of each fund as it grows. In the initial valuation it is often impossible to obtain a complete basis to forecast future rates. A fund changes as it grows. Changes of administration, and the fact that the fund is on an organized basis alone gives reasons for changes to be expected in the various rates used in valuation. As the forces which determine the rates of contribution can be more closely determined, the rates should be adjusted and the entire solvency of the fund tested. The fund should have as strict supervision as the law places over our life insurance companies, in order that the rights of employees, who are the bene-

ficiaries may be protected just as strongly as the rights of policy-holders are protected, and in order that the city may not contribute to an improperly constituted fund. But this is anticipating the future. The funds have not yet been reorganized on a scientific basis, and it is only by the help of the employees and the taxpayers, led by those who appreciate the significance of these conditions, that reorganization will come to pass.