# STATISTICS NECESSARY FOR COMPUTING NET COMPENSATION RATES.

BY

EDWARD OLIFIERS.

The main object of this paper is to determine the abstract actuarial formula of the net cost of workmen's compensation and to indicate on what lines the statistical data must be collected to find such cost. The general form in which the subject has been treated will trace the way to a uniform keeping of statistical data for the different states. The subject is of paramount importance and it justly occupies the attention of committees composed of public representatives.

Let us consider three classes of benefits granted by compensation laws.

I. Death benefits.

II. Permanent partial and total disability benefits.

III. Temporary partial and total disability benefits.

The net premium for each of these benefits is fixed upon by the simple rule of expectation. Mr. W. A. Whitworth writes in the treatise "Choice and Chance," Rule V, page 142, "The expectation from any event is obtained by multiplying the sum to be realized on the event happening by the chance that the result will happen." I neglect to consider the discounting factor due to the deferment in the payment of the benefits, *i. e.*, $V^{\frac{1}{2}}$; an extra safety loading is so introduced.

The following elements will have to enter into this expression of the expectation:

(*a*) The accident frequency;

(*b*) the effect of the social surroundings such as the distribution by ages of the workmen, the amount of salaries paid, the values of the annuities etc.;

(*c*) the legal compensation factors as fixed by the compensation laws.

## I. Death Benefits.

To fix our ideas let us denote by

$q_x^d$ the probability of a workman of age $x$ dying from the results of an accident.

$q^d$ the probability of the death of a workman being caused by an accident.

$F_x$ the number of full workers of age $x$ exposed to the risk of death.

$F$ the number of full workers exposed to the risk of death.

$s_x$ the average salary of a workman at age $x$.

$s_a$ the average salary of a workman.

$B_x^d$ the mean value of the benefits payable to the claimants of the the deceased workman of age $x$. The legal compensation factor is assumed to be included in this symbol.

The net premium for the death benefits may thus be written:

$$\sum_x^w q_x^d \frac{s_x \cdot F_x}{s_a \cdot F} \cdot B_x^d = \pi_d.$$

If we multiply and divide the expression by $q^d$ we have

$$\sum_x^w q^d \frac{s_x \cdot q_x^d \cdot F_x}{s_a \cdot q^d \cdot F} \cdot B_x^d,$$

putting $q_x^d . F_x = N_x^d$, i. e., the number of workmen killed by an accident at age $x$; and $q^d . F = N^d$, i. e., the number of workmen killed by an accident, we have:

$$\frac{q^d}{s_a \cdot N^d} \sum_x^w s_x \cdot N_x^d \cdot B_x^d. \qquad (1)$$

Having started with the acknowledgment of a variation of the probability of death according to ages, I have been led to the conclusion that there is no need for such an investigation and that instead all that need be known is the distribution by ages of the workmen killed by an accident.

The statistical data required according to formula (1) are:

(*a*) The number of full workers.

(*b*) The payroll on which the premium is based which enables us to find the value of $s_a$.

(*c*) The number of workmen killed by an accident at each age.

(*d*) The salary at each age on which the compensation benefits have been computed.

(*e*) The age of the workman killed and his civil status, the ages of

the annuitants (dependents of the workman fatally injured) and their degree of dependency, longevity, etc.

II. PERMANENT DISABILITY, PARTIAL AND TOTAL.

The formula for the pure premium of this benefit is:

$$\frac{\sum_{a=0}^{a=1} \delta^{(p,a)} \left[ \sum_{x}^{w} s_x \cdot P_x^a \cdot F_x \cdot B_x^{(p,a)} \right]}{F \cdot s_a} = \pi_p. \qquad (2)$$

$P_x^d$ denotes the probability that a workman of age $x$ be the victim of an accident causing a permanent partial disability of the degree $a$. When $a=1$ the disability is permanent and total.

$P^a$ denotes the probability that a workman be the victim of an accident causing permanent disability of the degree $a$.

$\delta^{(p,a)}$ denotes the coefficient of reduction of the salary to obtain the value of the legal allowance.

$B_x^{(p,a)}$ denotes the value of an indemnity of 1 payable to a workman of age $x$ for an accident causing a permanent disability of degree $a$.

If we write $\delta^{(p,a)} = \delta.a$ where $\delta$ represents the legal coefficient of reduction as fixed by the law and if the indemnity is paid as a continuous life annuity the above expression (2) becomes:

$$\frac{\sum_{a=0}^{a=1} \delta \cdot \alpha \left[ \sum_{x}^{w} s_x \cdot P_x^a \cdot F_x \cdot \bar{a}_x^{(p,a)} \right]}{F \cdot s_a} = \pi_p.$$

If we multiply and divide the above expression by $P$, i. e., the probability of a workman being permanently disabled, we have denoting $P.F$ by $N^p$ and $P_x^a.F_x$ by $N_x^{(p,a)}$

$$\frac{P}{s_a \cdot N^p} \sum_{a=0}^{a=1} \delta \cdot \alpha \left[ \sum_{x}^{w} s_x \cdot N_x^{(p,a)} \cdot \bar{a}_x^{(p,a)} \right].$$

I have used the indice $a$ in the notation, $B_x^{(p,a)}$ and $\bar{a}_x^{(p,a)}$, to indicate that the degree of disability may have an influence on the value of the annuity.

An approximation to the above expression may be written

$$\frac{P}{s_a \cdot N^p} \left( \frac{\sum_{a=0}^{a=1} \delta \cdot \alpha \cdot N^{(p,a)}}{N^p} \right) \left( \sum_{x}^{w} s_x \cdot N_x^p \cdot \bar{a}_x \right). \qquad (2_a)$$

The indemnity paid may thus be said to be made as if the $N^p$ cases of permanent invalidity, total and partial, were

$$\sum_{a=0}^{a=1} \alpha \cdot N^{(p,a)}$$

cases of total permanent disability. It is assumed in $(2_a)$ that the distribution by ages of the persons permanently disabled is uniform whatever the degree of disability.

The statistical data required to allow of a concrete application of the abstract formula (2) for permanent disability, partial and total, are:

(*a*) The number of full workers.

(*b*) The payroll on which the premium is collected so as to admit of determining $s_a$.

(*c*) The number of workmen at each age incapacitated by an accident causing a permanent disability of degree $a$.

(*d*) The salary at each age on which the benefits have been computed.

(*e*) The rates of mortality among permanently disabled lives, consideration being taken of the degree of disability.

### III. Temporary Disability, Partial and Total.

Mutatis mutandis the formula (2) of permanent disability becomes when applied to the temporary disability

$$\frac{1}{52} \cdot \frac{\sum_{a=1}^{a=1} \delta^{(t,a)} \left[ \sum_{x}^{w} s_x \cdot T_x^a \cdot F_x \cdot t_x^a \right]}{F \cdot s_a} = \pi_t, \qquad (3)$$

where $T_x^a$ denotes the probability that a workman of age $x$ will receive an allowance for a temporary disability.

$t_x^a$ denotes the mean duration in weeks of the temporary disability of the degree $a$.

Multiplying and dividing the expression (3) by $T$ and replacing $T_x^a \cdot F_x$ by $N_x^{(t,a)}$ and $F \cdot T$ by $N^{t,a}$ to designate the number of workance for a temporary disability of the degree $a$ and all degrees we have:

$$\frac{T}{52 \cdot s_a \cdot N^t} \sum_{a=0}^{a=1} \delta^{(t,a)} \left[ \sum_{x}^{w} s_x \cdot N_x^{(t,a)} \cdot t_x^a \right].$$

If we assume that the distribution by ages of the persons disabled temporarily, be it partial or total, is the same whatever be their degree of disability,

$$\frac{T}{52 \cdot s_a \cdot N^t}\left(\sum_{\alpha=0}^{\alpha=1}(\delta \cdot \alpha)\frac{N^{(t,\alpha)}}{N^t}\right)\left(\sum_{x}^{w} s_x \cdot N_x^t \cdot t_x\right). \qquad (3_a)$$

If the ages and the degree of disability of the persons temporarily disabled are ignored we have

$$\delta \frac{T}{52} \cdot \frac{s_n}{s_a} \cdot t. \qquad (3_b)$$

The statistical information required according to the expression (3) are:

(*a*) The number of full workers.

(*b*) The payroll on which the premium is based which enables us to find the value of $s_a$.

(*c*) The number of workmen at each age incapacitated by an accident causing a permanent disability of degree $\alpha$.

(*d*) The salary at each age on which the benefits have been computed.

(*e*) The mean duration of the temporary disability by ages and degree of disability.

In the light of what precedes, it would seem that the statistics have to give the following data to find the cost of the three benefits above mentioned:

(*a*) The number of full workers.

(*b*) The payroll on which the premium is collected which will allow of determining $s_a$.

(*c*) The number of workmen at each age killed and disabled, be it permanently or temporarily, by an accident and the degree of disability.

(*d*) The salary at each age on which the benefits have been computed.

(*e*) The age of the workman and the ages of the annuitants (dependents of the workman fatally injured) and their degree of dependency, longevity, etc.

(*f*) The rates of mortality among permanently disabled lives, consideration being taken of the degree of disability.

(*g*) The mean duration of the temporary disability by ages and degrees of disability. The statistical data to find the med-

ical, the pharmaceutical cost and the cost of other benefits are to be gathered on lines which can be traced without great difficulty.

It must be clearly understood that it is not my contention that nothing less in the form of required statistical data would not give as good results; the simplified formulas for permanent and temporary disability ($2_a$), ($3_a$) and ($3_b$) previously given would still more restrict the number of different statistical data given above, if found to give in the light of actual experience a good approximation to the formulas (2) and (3). Neither is it my contention that more detailed statistical data as, e. g., giving the number of full workers assured at each age, would not be of further assistance in detecting those laws or the causes and nature of the accidents for schedule rating purposes. My contention is simply that the above given formulas admit of a rational determination of the net premium without necessitating an excessive compilation of statistical data. At this point, it may be interesting to draw certain analogies of the above described method of determining the net premium, which may be described as the "prospective method," with what may be termed the "retrospective method" and which consists in finding the net cost by dividing the losses by the payroll. The two methods would give identical results in the purely theoretical case of there being no accidental deviations. But in the light of the study of the workmen's compensation statistics one can but expect these deviations to be appreciable, considering the smallness of the probabilities of accidents, for it is well known that the probability of the deviation is larger when the probabilities of the events considered are smaller.

The ungraduated statistical data will have to be intelligently interpreted and the sagacity of the observer must be directed towards detecting the laws of frequency and correlation in what may seem uncoordinated results. The introduction of these errors of observation in the net premium as found by the retrospective method is objectionable when a more accurate estimation may be made by using the general laws of the calculus of probability. This is more true the smaller the number of observations. Other reasons which make for the superiority of the prospective method over the retrospective method are the observed increasing coefficients of risk, the uncertainty of the estimation of deferred risks, etc.