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FOREWORD

Actuarial science originated in England in 1792 in the early days of life insurance. Because of the technical nature of the business, the first actuaries were mathematicians. Eventually, their numerical growth resulted in the formation of the Institute of Actuaries in England in 1848. Eight years later, in Scotland, the Faculty of Actuaries was formed. In the United States, the Actuarial Society of America was formed in 1889 and the American Institute of Actuaries in 1909. These two American organizations merged in 1949 to become the Society of Actuaries.

In the early years of the 20th century in the United States, problems requiring actuarial treatment were emerging in sickness, disability, and casualty insurance, particularly in workers compensation, which was introduced in 1911. The differences between the new problems and those of traditional life insurance led to the organization of the Casualty Actuarial and Statistical Society of America in 1914. Dr. I. M. Rubinow, who was responsible for the Society's formation, became its first president. At the time of its formation, the Casualty Actuarial and Statistical Society of America had 97 charter members of the grade of Fellow. The Society adopted its present name, the Casualty Actuarial Society, on May 14, 1921.

The purposes of the Society are to advance the body of knowledge of actuarial science applied to property, casualty, and similar risk exposures, to establish and maintain standards of qualification for membership, to promote and maintain high standards of conduct and competence for the members, and to increase the awareness of actuarial science. The Society's activities in support of this purpose include communication with those affected by insurance, presentation and discussion of papers, attendance at seminars and workshops, collection of a library, research, and other means.

Since the problems of workers compensation were the most urgent at the time of the Society's formation, many of the Society's original members played a leading part in developing the scientific basis for that line of insurance. From the beginning, however, the Society has grown constantly, not only in membership, but also in range of interest and in scientific and related contributions to all lines of insurance other than life, including automobile, liability other than automobile, fire, homeowners, commercial multiple peril, and others. These contributions are found principally in original papers prepared by members of the Society and published annually in the *Proceedings of the Casualty Actuarial Society*. The presidential addresses, also published in the *Proceedings*, have called attention to the most pressing actuarial problems, some of them still unsolved, that the industry has faced over the years.

The membership of the Society includes actuaries employed by insurance companies, industry advisory organizations, national brokers, accounting firms, educational institutions, state insurance departments, and the federal government. It also includes independent consultants. The Society has three classes of members: Fellows, Associates, and Affiliates. Both Fellows and Associates require successful completion of examinations, held in the spring and fall of each year in various cities of the United States, Canada, Bermuda, and selected overseas sites. In addition, Associateship requires completion of the CAS Course on Professionalism.

Affiliates are qualified actuaries who practice in the general insurance field and wish to be active in the CAS but do not meet the qualifications to become a Fellow or an Associate.

The publications of the Society and their respective prices are listed in the Society's Yearbook. The *Syllabus of Examinations* outlines the course of study recommended for the examinations. Both the *Yearbook*, at a charge of $40 (U.S. funds), and the *Syllabus of Examinations,* without charge, may be obtained from the Casualty Actuarial Society, 1100 North Glebe Road, Suite 600, Arlington, Virginia 22201.

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NOTICE

Papers submitted to the *Proceedings of the Casualty Actuarial Society* are subject to review by the members of the Committee On Review of Papers and, where appropriate, additional individuals with expertise in the relevant topics. In order to qualify for publication, a paper must be relevant to casualty actuarial science, include original research ideas and/or techniques, or have special educational value, and must not have been previously copyrighted or published or be concurrently considered for publication elsewhere.

The Society is not responsible for statements of opinion expressed in the articles, criticisms, and discussions published in these *Proceedings*.

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AN EXAMINATION OF THE INFLUENCE OF LEADING ACTUARIAL JOURNALS

L. LEE COLQUITT

Abstract

The relative significance of research published in eight actuarial journals is evaluated by examining the frequency of citations in 16 risk, insurance, and actuarial journals during the years 1996 through 2000. First, the frequency with which each sample journal cites itself and the other journals is provided so as to communicate the degree to which each journal's published research has had an influence on the other sample journals. Then the 16 journals are divided into 1) the actuarial journal group and 2) the risk and insurance journal group. The actuarial journals are then ranked based on their total number of citations including and excluding self-citations. Also, a ranking of journals within the actuarial journal group is provided based on the journals' influence on a per article published basis. Finally, the most frequently cited articles from the actuarial journals are observed and reported.

ACKNOWLEDGEMENT

Portions of "An Analysis of Risk, Insurance, and Actuarial Research: Citations From 1996 to 2000" (*Journal of Risk and Insurance*, 70:315–338) were reproduced in this article with the expressed written permission of Richard MacMinn (Editor of the *Journal of Risk and Insurance*) on behalf of the *Journal of Risk and Insurance*. The author gratefully acknowledges information provided by Colin Ramsey (editor of the *Journal of Actuarial Practice*), Steve D'Arcy (President-Elect of the Casualty Actuarial Society), the Casualty Actuarial Society Committee on Review of Papers, and David Sommer.

1. INTRODUCTION

The importance of evaluating journal quality is noted in the finance literature (see Alexander and Mabry [1], Zivney and Reichenstein [12], McNulty and Boekeloo [10], Borokhovich, Bricker, and Simkins [4], Chung, Cox, and Mitchell [5], and Arnold, Butler, Crack, and Altintig [2]). In the risk, insurance, and actuarial literature, a number of studies have been conducted to provide information on the relative quality of the journals and articles in this field, including Outreville and Malouin [11], McNamara and Kolbe [9], Baur, Zivney, and Wells [3], Hollman and Zeitz [8], and two studies by Colquitt [6], [7].

The purpose of the first and second Colquitt studies was to determine the impact that various risk, insurance, and actuarial journals and articles have had on research in that field by examining citations found in the leading risk, insurance, actuarial, and finance journals over the periods 1991–1995 and 1996–2000, respectively.[1] According to Colquitt [6], [7], reasons for assessing

[1]While citation studies are more common in other disciplines and thought to be the most comprehensive method in which to evaluate journal quality [see Alexander and Mabry [1], Zivney and Reichenstein [12], Borokhovich, Bricker, and Simkins [4], Chung, Cox, and Mitchell [5], and Arnold, Butler, Crack, and Altintig [2]], presumably the reason why citation analysis had not been used to evaluate journal quality in the insurance and actuarial literature up until Colquitt [6], [7] is that very few of the risk, insurance, and actuarial journals are tracked by the Social Sciences Citation Index (SSCI). Currently, only the *Geneva Papers on Risk and Insurance Issues and Practice, Geneva Papers on Risk and Insurance Theory*, *Insurance: Mathematics and Economics*, the *Journal of Risk and*

journal quality include its significance to 1) those conducting research; 2) faculty and administrators who are charged with evaluating the work of those conducting this research; 3) the editors and sponsoring organizations of the journals being evaluated, and; 4) the institutions that are making purchasing decisions.

The primary purpose for this update of the Colquitt studies is to provide the members of the Casualty Actuarial Society and others interested in actuarial research more specific information about the influence of the leading actuarial journals as well as information about how the Casualty Actuarial Society's two publications, the *Casualty Actuarial Society Forum* (*CASF*) and the *Proceedings of the Casualty Actuarial Society* (*PCAS*), contribute to the overall landscape of actuarial research. For those subscribing or contributing research to the Casualty Actuarial Society's publications, the study will provide information on the connection that these journals have with other risk, insurance, and actuarial journals and offer ideas as to other journals in which to subscribe or submit research. For those involved with the publication and dissemination of the two publications, the study will provide an idea as to the sphere of influence these journals have within the actuarial research community and perhaps shed light on how widely read and known these publications are among those conducting actuarial research.

2. RESEARCH METHODOLOGY AND DATA

The study is based on citations found *in* the 16 sample risk, insurance, and actuarial journals *only to* articles published in these same 16 journals (see the following chart for a list of the sample journals). As a result, this study only assesses the significance of the research published in these 16 risk, insurance, and actuarial journals. The difference in the journals analyzed in the Colquitt (2003) study and this one is the exclusion in this study of *Benefits Quarterly* and the *Journal of Financial Services Professionals*

Insurance, and the *Journal of Risk and Uncertainty* are included in the journals tracked by the SSCI. Collection of the data needed to conduct a citation analysis without the use of the SSCI is tedious and time consuming.

as well as the finance journals and the inclusion of the *Casualty Actuarial Society Forum* and the *Proceedings of the Casualty Actuarial Society*.[2] The data include the total number of citations in the 16 sample journals during the years 1996 through 2000.

For the purposes of evaluating the 16 risk, insurance, and actuarial journals, the journals are separated into two groups; the actuarial journal group and the risk and insurance journal group.

Sample Journals

ASTIN Bulletin (AB)
British Actuarial Journal (BAJ)
Casualty Actuarial Society Forum (CASF)
Insurance: Mathematics and Economics (IME)
Journal of Actuarial Practice (JAP)
North American Actuarial Journal (NAAJ)
Proceedings of the Casualty Actuarial Society (PCAS)
Scandinavian Actuarial Journal (SAJ)

Risk and Insurance Journals

CPCU Journal (CPCU)
Geneva Papers on Risk and Insurance Issues and Practice (GPIP)
Geneva Papers on Risk and Insurance Theory (GPT)
Journal of Insurance Issues (JII)
Journal of Insurance Regulation (JIR)
Journal of Risk and Insurance (JRI)
Journal of Risk and Uncertainty (JRU)
Risk Management and Insurance Review (RMIR)

While the Colquitt studies focused primarily on the risk and insurance journal group (with a particular focus on the *JRI*), this paper focuses primarily on the findings of the actuarial journal group (with a particular focus on the *CASF* and the *PCAS*).

[2] In the Colquitt study [7], *Benefits Quarterly* and the *Journal of Financial Services Professionals* produced no citations to any of the sample actuarial journals. In addition, of the approximately 70,000 citations found in the finance journals evaluated, only 24 were to the sample actuarial journals (17 of which were to the *British Actuarial Journal*).

Given that the Social Sciences Citation Index (SSCI) does not include all of the risk, insurance, and actuarial journals relevant to this study, the citation data are gathered by reviewing the bibliographies of each of the sample journals for references to the risk, insurance, and actuarial journals included in the study. Unless a paper was stated as being "forthcoming" in one of the sample journals, citations to working papers that were published in one of these journals subsequent to the citation are not recorded. Data gathered include the author, journal edition, and page numbers of the cited article as well as the journal edition and page number of the citing article. Only citations from feature articles, short articles, discussions, and notes and communications regarding research are included in the data. Opinion pieces and regular columns like those found in the *CPCU Journal* are not reviewed for citations.

The citation data collected are used to evaluate the citation patterns of the sample journals and the relative impact that each journal is having on risk, insurance, and actuarial research in total and on a per article published basis. In addition, the data are used to provide information on which of the articles published in the sample actuarial journals have been the most influential in recent years.

3. DISCUSSION OF RESULTS

Table 1 provides the distribution of citations by the year in which the cited article was published for each journal group, and for the two journal groups combined. The unavoidable lag that exists between the time period evaluated and data collection that was described in the Colquitt studies is again supported by the results found in Table 1. There is a difference in the distribution of citations found in the actuarial journal group and those found in the risk and insurance journal group. While over 50% of the citations recorded from the risk and insurance journal group were to articles published between the years 1992 and 1997, only 41.53% of the citations from the actuarial journal group were to

TABLE 1

DISTRIBUTION OF CITATIONS BY YEAR OF CITED ARTICLE

	Actuarial Journals		Risk and Insurance Journals		Total	
Year	Percentage	Cumulative Percentage	Percentage	Cumulative Percentage	Percentage	Cumulative Percentage
2000	0.83	0.83	0.75	0.75	0.80	0.80
1999	2.68	3.51	2.94	3.68	2.78	3.57
1998	5.78	9.28	3.73	7.41	5.02	8.59
1997	7.40	16.68	8.42	15.83	7.78	16.37
1996	7.63	24.32	7.50	23.32	7.58	23.95
1995	5.85	30.17	10.00	33.32	7.39	31.34
1994	8.82	38.99	8.37	41.69	8.65	39.99
1993	5.67	44.66	7.10	48.79	6.20	46.19
1992	6.16	50.83	8.94	57.74	7.19	53.39
1991	5.11	55.93	6.14	63.88	5.49	58.87
1990	4.85	60.78	5.30	69.18	5.02	63.89
1989	3.51	64.29	3.55	72.73	3.52	67.41
1988	3.95	68.23	3.68	76.41	3.85	71.26
1987	2.97	71.20	3.90	80.32	3.31	74.57
1986	2.35	73.54	2.24	82.55	2.31	76.88
pre-1986	26.46	100.00	17.45	100.00	23.12	100.00

articles from the same period.[3] A large portion of this difference comes from the two groups' citations to articles published in the years before 1986. This suggests that many of the articles cited by the actuarial journal group (presumably actuarial articles) have a more lasting influence than do the articles cited by the risk and insurance journal group (presumably the risk and insurance articles).

[3]Three to four years appears to be the time lag between when an article is published and when it is read, incorporated into future research, and referenced in a published article. This would explain the apparent significance of the articles published between 1992 and 1997 when reviewing articles published from 1996 to 2000.

3.1. Journal Results

Table 2 provides the citation patterns for all of the sample journals. Table 3 provides the same citation pattern information on a normalized basis (per one thousand citations). Essentially, these tables allow one to view the frequency with which each sample journal cites the other risk, insurance, and actuarial journals. In addition, the total source articles and the number of references to sources other than the sample journals are provided.

The first column on the far left of Table 2 contains the journals that were reviewed for citations. By reading across each row, you can see the journals that were cited by the journal listed in the first column. For example, the first journal listed at the top of the first column is the *AB*. There were 92 articles during the years 1996–2000 from the *AB* that were reviewed for citations. These 92 articles cited the *AB* 177 times, the *BAJ* 21 times, the *CASF* three times, and so on. The *AB* cited sources other than the sample risk, insurance, and actuarial journals 942 times for a total of 1,327 citations. The two shaded numbers across each row denote the two most frequently cited journals by the journal reviewed. As can been seen in Table 2, the *AB* (177) and the *IME* (105) were the two journals most frequently cited by the *AB*.

As was observed in the Colquitt studies, the journal most frequently cited by the majority of the citing journals is the citing journal itself. This can be seen by observing that most of the cells starting from the top left corner of the grid and proceeding down to the right bottom corner are shaded (indicating that the journal cited was either the first or second most frequently cited journal). The exception to this was the *CASF*, the *GPT*, the *JAP*, the *JII*, the *NAAJ*, and *RMIR*.[4] Among those, the *GPT*, the *JII*, and *RMIR* all cited the *JRI* with the most frequency. The most frequently cited journal by the *CASF* was the *PCAS*, the most frequently cited journal by the *JAP* was the *BAJ*, and

[4]Given that the *NAAJ* and *RMIR* both began publication in 1997, it is not surprising that these two journals cite themselves with relative infrequency.

TABLE 2

JOURNAL CITATIONS PATTERNS

Citations From (# of source articles)*	Citations to the Sample Risk, Insurance, and Actuarial Journals																Total Outside the Sample Journals	Overall Total
	AB	*BAJ*	*CASF*	*IME*	*JAP*	*NAAJ*	*PCAS*	*SAJ*	*CPCU*	*GPIP*	*GPT*	*JII*	*JIR*	*JRI*	*JRU*	*RMIR*		
AB (92)	**177****	21	3	**105**	0	6	6	55	1	1	1	0	0	9	0	0	942	1327
BAJ (115)	11	**229**	5	**15**	0	5	7	11	0	3	0	0	0	1	1	0	1689	1977
CASF (174)	30	8	**102**	12	1	7	**272**	0	1	0	2	0	6	43	1	0	946	1431
IME (202)	**255**	53	3	**441**	0	39	15	188	1	1	12	0	4	58	4	0	2380	3454
JAP (50)	18	**44**	8	**32**	15	14	24	23	0	0	0	0	1	17	3	0	685	884
NAAJ (126)	50	33	8	**103**	6	47	17	**51**	1	2	5	0	3	50	4	0	2781	3161
PCAS (114)	20	7	**59**	7	1	4	**262**	4	3	0	2	1	15	51	1	0	723	116
SAJ (63)	70	6	0	**93**	0	3	4	**96**	0	0	2	0	0	1	2	0	721	998
CPCU (81)	0	0	0	0	0	1	0	0	**57**	2	0	0	6	**13**	1	4	1100	1184
GPIP (186)	2	5	3	5	1	1	0	2	1	**85**	26	0	3	**56**	9	0	2917	3116

GPT (51)	7	1	0	10	0	0	0	12	0	5	19	0	2	**35**	**20**	0	936	1047
JII (50)	0	1	0	3	0	3	4	0	11	7	0	25	**38**	**192**	5	3	756	1048
JIR (123)	0	1	2	1	1	3	5	0	13	8	0	11	**99**	**168**	4	2	1536	1854
JRI (141)	18	1	1	25	1	15	9	9	11	6	29	12	**37**	**504**	29	2	2965	3674
JRU (140)	0	0	0	1	0	0	1	2	0	0	9	1	3	**30**	**313**	1	3728	4089
RMIR (59)	0	0	0	1	0	0	0	0	**18**	10	0	4	17	**94**	4	7	857	1012
TOTAL	658	410	194	854	26	148	626	453	118	130	107	54	234	1322	401	19		
Self-citation rate	.1334	.1158	.0713	.1277	.0170	.0149	.2259	.0962	.0481	.0273	.0181	.0239	.0534	.1372	.0765	.0069		

AB = *ASTIN Bulletin*; *BAJ* = *British Actuarial Journal*; *CASF* = *Casualty Actuarial Society Forum*; *CPCU* = *CPCU Journal*; *GPIP* = *Geneva Papers on Risk and Insurance Issues and Practice*; *GPT* = *Geneva Papers on Risk and Insurance Theory*; *IME* = *Insurance: Mathematics and Economics*; *JAP* = *Journal of Actuarial Practice*; *JII* = *Journal of Insurance Issues*; *JIR* = *Journal of Insurance Regulation*; *JRI* = *Journal of Risk and Insurance*; *JRU* = *Journal of Risk and Uncertainty*; *NAAJ* = *North American Actuarial Journal*; *PCAS* = *Proceedings of the Casualty Actuarial Society*; *RMIR* = *Risk Management and Insurance Review*; *SAJ* = *Scandinavian Actuarial Journal*; Total Outside the Sample Journals = the number of citations in the journal that are to articles not published in one of the 16 sample risk, insurance, or actuarial journals; Self-Citation Rate = the percentage of a journal = s citations attributable to its own articles. Citations to the *Geneva Papers* prior to 1990 (the year that the *Geneva papers* were split into two journals, the *GPIP* and the *GPT*) are attributed to the *GPIP* and the *GPT* in the proportion that the *GPIP* and the *GPT* received their own citations from that journal during 1990 and beyond.

*The source articles reviewed for citations were from 1996–2000.

**The two shaded numbers across each row denote the two most frequently cited journals by the jornal reviewed.

TABLE 3

NORMALIZED JOURNAL CITATIONS

Citations From (# of source articles)*	Citations to the Sample Risk, Insurance, and Actuarial Journals																Total Outside the Sample Journals	Overall Total
	AB	*BAJ*	*CASF*	*IME*	*JAP*	*NAAJ*	*PCAS*	*SAJ*	*CPCU*	*GPIP*	*GPT*	*JII*	*JIR*	*JRI*	*JRU*	*RMIR*		
AB (92)	133	16	2	79	0	5	5	41	1	1	1	0	0	70	0	0	710	1000
BAJ (115)	6	116	3	8	0	3	4	6	0	2	0	0	0	10	1	0	854	1000
CASF (174)	21	6	71	8	1	5	190	0	1	0	1	0	4	300	1	0	661	1000
IME (202)	74	15	1	128	0	11	4	54	0	0	3	0	1	170	1	0	689	1000
JAP (50)	20	50	9	36	17	16	27	26	0	0	0	0	1	190	3	0	775	1000
NAAJ (126)	16	10	3	33	2	15	5	16	0	1	2	0	1	160	1	0	880	1000
PCAS (114)	17	6	51	6	1	3	226	3	3	0	2	1	13	440	1	0	623	1000
SAJ (63)	70	6	0	93	0	3	4	96	0	0	2	0	0	10	2	0	722	1000
CPCU (81)	0	0	0	0	0	1	0	0	48	2	0	0	5	110	1	3	929	1000
GPIP (186)	1	2	1	2	0	0	0	1	0	27	8	0	1	180	3	0	936	1000

GPT (51)	7	1	0	10	0	0	0	11	0	5	18	0	2	330	19	0	894	1000
JII (50)	0	1	0	3	0	3	4	0	10	7	0	24	36	1830	5	3	725	1000
JIR (123)	0	1	1	1	1	2	3	0	7	4	0	6	53	910	2	1	828	1000
JRI (141)	5	0	0	7	0	4	2	2	3	2	8	3	10	1370	8	1	807	1000
JRU (140)	0	0	0	0	0	0	0	0	0	0	2	0	1	70	77	0	912	1000
RMIR (59)	0	0	0	1	0	0	0	0	18	10	0	4	17	930	4	7	847	1000
Avg over journals	23.13	14.38	8.88	25.94	1.38	4.44	29.63	16.00	5.69	3.81	2.94	2.38	9.06	44.25	8.06	0.94		
Avg with no self-citation	15.80	7.60	4.73	19.13	0.33	3.73	16.53	10.67	2.87	2.27	1.93	0.93	6.13	38.07	3.47	0.53		
Self-citation index	0.84	1.52	1.51	0.67	5.15	0.40	1.37	0.90	1.68	1.20	0.94	2.57	0.87	0.36	2.20	1.30		

AB = *ASTIN Bulletin*; *BAJ* = *British Actuarial Journal*; *CASF* = *Casualty Actuarial Society Forum*; *CPCU* = *CPCU Journal*; *GPIP* = *Geneva Papers on Risk and Insurance Issues and Practice*; *GPT* = *Geneva Papers on Risk and Insurance Theory*; *IME* = *Insurance: Mathematics and Economics*; *JAP* = *Journal of Actuarial Practice*; *JII* = *Journal of Insurance Issues*; *JIR* = *Journal of Insurance Regulation*; *JRI* = *Journal of Risk and Insurance*; *JRU* = *Journal of Risk and Uncertainty*; *NAAJ* = *North American Actuarial Journal*; *PCAS* = *Proceedings of the Casualty Actuarial Society*; *RMIR* = *Risk Management and Insurance Review*; *SAJ* = *Scandinavian Actuarial Journal*; Total Outside the Sample Journals = the number of citations in the journal that are to articles not published in one of the 16 sample risk, insurance, or actuarial journals; Self-Citation Index = the self-citation rate × 100/normalized average citation rate excluding self-citations (per thousand citations). Totals may not add due to rounding.

the most frequently cited journal by the *NAAJ* was *IME*. In addition, the actuarial journals and the risk and insurance journals tend to cite the journals within their same group with the most frequency, with the only meaningful overlap being the frequency with which the *JRI* is cited by the *CASF*, *IME*, the *NAAJ*, and the *PCAS*. Tables 2 and 3 also show the influence of *IME*. *IME* was either the first or second most frequently cited journal of six of the eight actuarial journals. The only two actuarial journals where the *IME* was not the first or second most frequently cited journal were the *CASF* and the *PCAS*.

Table 2 also provides each journal's self-citation rate and each journal's self-citation index is found in Table 3.[5] The higher the self-citation index, the higher a journal's frequency of self-citations relative to the frequency with which it is cited by the other sample journals. The lower the self-citation index, the more influential the journal is presumed to be. While a high self-citation index could suggest that a journal is guilty of self-promotion, it also could be that a journal with a high self-citation index publishes research on topics that are of a specialized nature and, as a result, is most frequently referenced by other articles within that same journal (see Colquitt [6]). Among the actuarial journals, the *NAAJ* (0.40) has the lowest self-citation index, with *IME* (0.67), the *AB* (0.84), and the *SAJ* (0.92) following close behind. The remaining four actuarial journals and their self-citation indices are the *PCAS* (1.39), the *CASF* (1.51), the *BAJ* (1.52) and the *JAP* (5.15).

Table 4 provides a ranking of the sample actuarial journals based on total citations, including and excluding self-citations. When looking at total citations, *IME* is the most frequently cited actuarial journal with 854, followed by the *AB* (658), the *PCAS* (626), the *SAJ* (453), and the *BAJ* (410). The remaining three

[5]The calculation of both the self-citation rate and the self-citation index follows that of Borokhovich, Bricker, and Simkins [4] and Colquitt [6], [7]. The self-citation rate is the number of self-citations from a journal divided by the total number of citations found in that journal. The self-citation index is the self-citation rate $\times$100/normalized average citation rate excluding self-citations (per thousand citations).

TABLE 4

ACTUARIAL JOURNALS RANKED BY TOTAL NUMBER OF CITATIONS BY THE SAMPLE JOURNALS DURING THE YEARS 1996 THROUGH 2000

Rank	Actuarial Journals	Total Citations	Self-Citations	Non-Self-Citations	Adj Rank[1]
1	*Insurance: Mathematics and Economics*	854	441	413	2
2	*ASTIN Bulletin*	658	177	481	1
3	*Proceedings of the Casualty Actuarial Society*	626	262[2]	364	3
4	*Scandinavian Actuarial Journal*	453	96	357	4
5	*British Actuarial Journal*	410	229	181	5
6	*Casualty Actuarial Society Forum*	194	102[3]	92	7
7	*North American Actuarial Journal*	148	47	101	6
8	*Journal of Actuarial Practice*	26	15	11	8

[1]Ranking based upon total number of non-self-citations.
[2]If the *CASF* citations (272) are included as self-citations to the *PCAS*, then the number of *PCAS* non-self-citations falls to 92 and its adjusted rank is just below that of the *NAAJ*.
[3]If the *PCAS* citations (59) are included as self-citations to the *CASF*, then the number of *CASF* non-self-citations falls to 33 and its adjusted rank is just above that of the *JAP*.

actuarial journals were the *CASF* (194), the *NAAJ* (148), and the *JAP* (26). One reason for the low citation totals for the *NAAJ* and the *JAP* is likely the relative newness of these journals. In addition, the pedagogical nature of some of the articles in the *JAP* and the relatively low number of *JAP* subscribers are also likely reasons for its low number of citations.[6]

When excluding self-citations, the only changes in the order is a switch in the first and second positions between *IME* (413) and the *AB* (481) and the switch in the sixth and seventh positions between the *CASF* (92) and the *NAAJ* (101). Interestingly, when the *CASF* citations to the *PCAS* are considered to be self-citations to the *PCAS*, then the number of non-self-citations to the *PCAS* falls to 92 and its adjusted rank falls to just below that

[6]Baur, Zivney, and Wells [3] report that (at the time of their study) only 2% (5 out of 265) of all AACSB schools and only 3% (1 out of 30) of schools with a major in actuarial sciences subscribed to the *JAP*.

of the *NAAJ* (seventh position). Also, when the *PCAS* citations to the *CASF* are considered to be self-citations to the *CASF*, then the number of non-self-citations to the *CASF* falls to 33 and its adjusted rank falls to just above the *JAP* (again, seventh position). This is likely due to the fact that these two journals have an actuarial focus that is of primary interest to the members of the Casualty Actuarial Society.

While the total number of citations for the sample journals provides a measure of the total impact that each journal has on risk, insurance, and actuarial research, the total number of citations is greatly affected by the number of citable articles published by the sample journals. Table 5 provides the insurance impact factor (IIF) for the sample actuarial journals. The IIF follows Colquitt and captures the relative research impact of a journal on a per article basis.[7]

When evaluating the research impact of a journal on a per article basis, the *AB* is ranked first among actuarial journals with an IIF of 2.0175. This essentially means that the *AB* articles published during the period between 1991 and 2000 were cited an average of 2.0175 times per article by the sample risk, insurance, and actuarial journals analyzed. Following the *AB* is the *PCAS* (1.9825), *IME* (1.6336), the *SAJ* (1.5656), the *BAJ* (1.3892), the *NAAJ* (1.1746), the *CASF* (0.6078), and the *JAP* (0.2766). When looking at the adjusted insurance impact factor[8] (AIIF) for the actuarial journal group, there is a considerable difference in the rankings. The *AB* (1.4561) has the highest AIIF, followed by the *SAJ* (1.1475), the *PCAS* (1.1404), the *NAAJ* (0.8016), *IME* (0.7466), the *BAJ* (0.4162), the *CASF* (0.2778), and the *JAP* (0.1170). As was the case when evaluating the IIF, when the *CASF* citations are subtracted when calculating the *PCAS*'s AIIF, the AIIF falls to 0.2719 and the *PCAS*'s ranking falls to seventh.

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[7]The IIF equals citations to a journal's articles published in a certain period divided by the number of citable articles during the same period. The period used for all of the journals except the *JAP* and the *NAAJ* is 1991 through 2000. The *JAP* was established in 1993 and the period used for the *JAP* is 1993 through 2000. The *NAAJ* was established in 1997 and the period used for this journal is 1997 through 2000.

[8]The AIIF is the IIF calculated excluding self-citations.

TABLE 5

RELATIVE IMPACT OF ACTUARIAL JOURNALS
(INSURANCE IMPACT FACTOR—PERIOD FROM 1991–2000)

Actuarial Journals	All Citations Insurance Impact Factor[1]	Rank	No Self-Citations Adj Insurance Impact Factor[2]	Adj Rank
ASTIN Bulletin	2.0175	1	1.4561	1
Proceedings of the Casualty Actuarial Society	1.9825	2	1.1404[3]	3
Insurance: Mathematics and Economics	1.6336	3	0.7466	5
Scandinavian Actuarial Journal	1.5656	4	1.1475	2
British Actuarial Journal	1.3892	5	0.4162	6
North American Actuarial Journal	1.1746	6	0.8016	4
Casualty Actuarial Society Forum	0.6078	7	0.2778[4]	7
Journal of Actuarial Practice	0.2766	8	0.1170	8

[1]Insurance Impact Factor (IIF) = citations to a journal = s articles published in a certain period divided by the number of citable articles published during the same period. The period used for all of the journals except the *JAP* and the *NAAJ* is 1991 through 2000. The *JAP* was established in 1993 and the period used for this journal is between 1993 through 2000. The *NAAJ* was established in 1997 and the period used for this journal is between 1997 through 2000.
[2]Adj Insurance Impact Factor (AIIF) = the IIF calculated using only the non-self-citations.
[3]If the *CASF* citations are subtracted when creating the *PCAS*' AIIF, the *PCAS*' AIIF falls to 0.2719 (ranked 7th).
[4]If the *PCAS* citations are subtracted when creating the *CASF*'s AIIF, the *CASF*'s AIIF falls to 0.1046 (ranked 8th).

Also, when the *PCAS* citations are subtracted when calculating the *CASF*'s AIIF, the AIIF falls to 0.1046 and the *CASF*'s ranking falls to eighth.

3.2. Article Results

In addition to knowing the relative impact of the actuarial journals, it also is helpful to know which of the articles published in the past have been the most influential in recent years. Reasons provided by Colquitt include the importance of this knowledge to 1) researchers who can use this information to determine the subjects, methodology, style, and the like that have been a part of the most influential research; 2) editors who use this information to form opinions on the value of future research submitted for publication; and 3) those responsible for developing reading lists for graduate-level seminar courses in actuarial science. In addition, it is important for actuarial societies that administer professional examinations to have knowledge of the most influential actuarial articles so that syllabus committees can consider the incorporation of these articles in the examination process.

When highlighting the most frequently cited articles published in the sample actuarial journals, it is important to remind the readers of a significant point. There are, perhaps, influential actuarial articles that have been published in journals not included in the sample journals in this study. As a result, it should be recognized that the articles listed here are the most influential among those published in the sample journals and not necessarily in the entire universe of actuarial literature.

Similar to loss reserve development, it takes time for published articles to be fully recognized by other researchers and incorporated into future research. As a result, it is appropriate to make comparisons between articles that were published during the same year. The most frequently cited *CASF* articles published in each year, 1990 through 1999 are found in Table 6.[9] Among the most frequently cited *CASF* articles, authors appearing on more than one article (not including committee participation) include Butsic (1990 and 1999), D'Arcy (1997 and 1998), Feldblum (two articles in 1996), Gorvett (1997 and 1998), Hettinger (1997 and 1998), and Hodes (two articles in 1996). Also, articles that were the most frequently cited for the years 1992, 1993, and 1995 were authored by committees. Finally, only three of the 13 articles listed in Table 6 are by single authors and six of the 13 were either written by a committee or by four or more authors.

[9]No articles published in the *CASF* during the year 2000 or prior to 1990 were cited by the sample journals more than once.

The most frequently cited *PCAS* articles or discussions published in each year, 1985 through 1999 are found in Table 7.[10] Interestingly, D'Arcy (1989 and 1997) and Feldblum (1990 and 1996) are the only authors credited with two of the most frequently cited *PCAS* articles for a particular year. Another interesting finding for the top *PCAS* articles is that of the 18 articles listed in Table 7, all but three are single-authored papers. In addition, the three that were co-authored only have two co-authors. This is distinctly different from what was found in the list of top *CASF* articles.

Table 8 lists the *CASF* and *PCAS* articles that are the most frequently cited by the sample journals regardless of the year in which they were published. Of the 16 articles on the list, 13 of them were published in the *PCAS* and three in the *CASF*. With regard to the age of the articles, there is a fair distribution scattered over the last 40 years. Seven of the top *CASF* and *PCAS* articles were published in the 1990s, four were published in the 1980s, three were published in the 1970s, and two were published in the 1960s, with the oldest article being the Longley-Cook article that was published in the *PCAS* in 1962. This is in stark contrast to the distribution of the most frequently cited *JRI* articles found in Colquitt [7]. Of the 15 top *JRI* articles, ten of them were published between 1992 and 1996. In addition, only three of the top *JRI* articles were published in the 1980s and the oldest article was from 1986. The difference in the distribution of the most frequently cited *PCAS* and *CASF* articles and the most frequently cited *JRI* articles is evidence of the more lasting influence that actuarial articles have on future research as compared to risk and insurance articles.

Table 9 provides a listing of the most frequently cited articles published in each of the sample actuarial journals. All but two of the most frequently cited articles for each of the journals listed were published during the 1990s. The only exceptions are

[10]No articles published in the *PCAS* during the year 2000 were cited by the sample journals more than once.

TABLE 6

The CASF Articles or Discussions Published During Each Year, 1990 Through 1999, That are the Most Frequently Cited by the Sample Journals During the Years 1996 Through 2000*

Year of Publication	Title	Pages	Author(s)	Citations
1999	Capital Allocation for Property-Liability Insurers: A Catastrophe Reinsurance Application	1–70	Butsic	3
1998	Using the Public Access DFA Model: A Case Study	53–118	D'Arcy, Gorvett, Hettinger, and Walling	4
1997	Building a Public Access PC-Based DFA Model	1–40	D'Arcy, Gorvett, Herbers, Hettinger, Lehmann, and Miller	4
1996	Workers Compensation Reserve Uncertainty	61–150	Hodes, Feldblum, and Blumsohn	5
	The Financial Modeling of Property/Casualty Insurance Companies	3–88	Hodes, Neghaiwi, Cummings, Philips, and Feldblum	5
	An Integrated Dynamic Financial Analysis and Decision Support System for a Property Catastrophe Reinsurer	89–118	Lowe and Stanard	5

1995	Dynamic Financial Models of Property/Casualty Insurers	93–127	Subcommittee on Dynamic Financial Models of the CAS Committee on Valuation and Financial Analysis	2
	A Simulation Procedure for Comparing Different Claims Reserving Methods	128–156	Pentikainen and Rantala	2
1994	Accounting for Risk Margins	1–90	Philbrick	12
1993	Report on Reserve and Underwriting Risk Factors	105–171	American Academy of Actuaries Property/Casualty Risk-Based Capital Task Force	6
1992	Property-Casualty Risk-Based Capital Requirement—A Conceptual Framework	211–280	Actuarial Advisory Committee to the NAIC Property & Casualty Risk-Based Capital Working Group	6
1991	The Development of Property-Liability Insurance Pricing Models in the United States	19–46	Derrig	3
1990	An Illustrated Guide to the Use of the Risk-Compensated Discounted Cash Flow Method	303–348	Butsic and Lerwick	4

TABLE 7

PCAS Articles or Discussions Published During Each Year, 1985 Through 1999, That are the Most Frequently Cited by the Sample Journals During the Years 1996 Through 2000*

Year of Publication	Title	Pages	Author(s)	Citations
1999	A Systematic Relationship Between Minimum Bias and Generalized Linear Models	393–487	Mildenhall	4
1998	Aggregation of Correlated Risk Portfolios: Models and Algorithms	848–939	Wang	6
1997	Ratemaking: A Financial Economics Approach	301–390	D'Arcy and Dyer	5
1996	NAIC Property/Casualty Insurance Company Risk-Based Capital Requirements	297–435	Feldblum	8
1995	Discussion of "Risk Loads for Insurers"	78–96	Bault	6
1994	Unbiased Loss Development Factors	154–222	Murphy	6
1993	Surplus-Concepts, Measures of Return, and Determination	55–109	Bingham	7

1992	Discussion of "Reinsurer Risk Loads From Marginal Surplus Requirements"	362–366	Gogol	7
1991	The Competitive Market Equilibrium Risk Load Formula for Increased Limits Ratemaking	163–200	Meyers	10
1990	Risk Loads for Insurers	160–195	Feldblum	13
	Reinsurer Risk Loads from Marginal Surplus Requirements	196–203	Kreps	13
1989	The Aging Phenomenon and Insurance Prices	24–44	D'Arcy and Doherty	4
1988	Federal Income Taxes Provisions Affecting Property-Casualty Insurers	95–161	Almagro and Ghezzi	9
1987	Discussion of "An Analysis of Experience Rating"	119–189	Mahler	3
	Reserving Long Term Medical Claims	322–353	Snader	3
	Regression Models in Claims Analysis I: Theory	354–383	Taylor	3
1986	A Formal Approach to Catastrophe Risk Assessment in Management	69–92	Clark	3
1985	A Simulation Test of Prediction Errors of Loss Reserve Estimation Techniques	124–148	Stanard	16

TABLE 8

PCAS AND CASF ARTICLES MOST FREQUENTLY CITED BY THE SAMPLE JOURNALS DURING THE YEARS 1996 THROUGH 2000, REGARDLESS OF THE YEAR PUBLISHED

Rank	Title	Journal/Year	Pages	Author(s)	Citations
1	The Calculation of Aggregate Loss Distributions from Claim Severity and Claim Count Distributions	*PCAS*/1983	22–71	Heckman and Meyers	18
2	A Simulation Test of Prediction Errors of Loss Reserve Estimation Techniques	*PCAS*/1985	124–148	Stanard	16
T3	Risk Loads for Insurers	PCAS/1990	160–195	Feldblum	13
T3	Reinsurer Risk Loads from Marginal Surplus Requirements	*PCAS*/1990	196–203	Kreps	13
T5	Accounting for Risk Margins	*CASF*/1994	1–90	Philbrick	12
T5	On the Theory of Increased Limits and Excess of Loss Pricing	*PCAS*/1977	27–59	Miccolis	12
T5	Loss Reserve Adequacy Testing: A Comprehensive, Systematic Approach	*PCAS*/1977	123–184	Berquist and Sherman	12

T8	Measuring the Variability of Chain Ladder Reserve Estimates	*CASF*/1994	101–182	Mack	10
T8	The Competitive Market Equilibrium Risk Load Formula for Increased Limits Ratemaking	*PCAS*/1991	163–200	Meyers	10
T8	Extrapolating, Smoothing, and Interpolating Development Factors	*PCAS*/1984	122–155	Sherman	10
T8	An Introduction to Credibility Theory	*PCAS*/1962	194–221	Longley-Cook	10
T12	Federal Income Taxes Provisions Affecting Property-Casualty Insurers	*PCAS*/1988	95–161	Almagro and Ghezzi	9
T12	The Actuary and the IBNR	*PCAS*/1972	181–195	Bornhuetter and Ferguson	9
T12	A Bayesian View of Credibility	*PCAS*/1964	85–104	Mayerson	9
T15	NAIC Property/Casualty Insurance Company Risk-Based Capital Requirements	*PCAS*/1996	297–435	Feldblum	8
T15	Probabilistic Development Factor Models with Applications to Loss Reserve Variability, Prediction Intervals, and Risk Based Capital	*CASF*/1994	447–605	Zehnwirth	8

TABLE 9

The Article from Each Actuarial Journal Most Frequently Cited by the Sample Journals During the Years 1996 Through 2000

Journal	Title of Most Frequently Cited Article	Year/Pages	Author(s)	Citations
ASTIN Bulletin	Recursive Evaluation of a Family of Compound Distributions	1981/22–26	Panjer	33
British Actuarial Journal	More on a Stochastic Asset Model for Actuarial Use	1995/777–964	Wilkie	34
Casualty Actuarial Society Forum	Accounting for Risk Margins	1994/1–90	Philbrick	12
Insurance: Mathematics and Economics	Insurance Pricing and Increased Limits Ratemaking by Proportional Hazards Transforms	1995/43–54	Wang	18
Journal of Actuarial Practice	A Critique of Defined Contribution Plans Using a Simulation Approach	1993/49–68	Knox	4
North American Actuarial Journal	On the Time Value of Ruin	1998/48–72	Gerber and Shiu	9
Proceedings of the Casualty Actuarial Society	The Calculation of Aggregate Loss Distributions from Claim Severity and Claim Count Distributions	1983/22–71	Heckman and Meyers	18
Scandinavian Actuarial Journal	The Distribution of a Perpetuity, with Applications to Risk Theory and Pension Funding	1990/39–79	Dufresne	14

Panjer's 1981 *AB* article and Heckman and Meyers' 1983 *PCAS* article. As was the case with the most frequently cited *PCAS* articles (Table 7), most (six of eight) of the articles are single-authored and the two articles not single-authored only have two co-authors. Finally, while not shown in Table 9, the author of the most frequently cited *JRI* article ("Solvency Measurement for Property-Liability Risk-Based Capital Applications;" 22 citations) is Butsic, who also coauthored the most frequently cited *CASF* article in 1990 and authored the most frequently cited *CASF* article in 1999.

The most frequently cited articles in any of the actuarial journals are found in Table 10. All actuarial journals except the *CASF*, the *JAP*, and the *NAAJ* are represented on this list. The *AB* and *IME* lead the list with five articles each. Close behind the *AB* and *IME* is the *PCAS* with four of the top actuarial articles and the *BAJ* (including the *Journal of the Institute of Actuaries* article from 1992) and *SAJ* have two and one on the list, respectively. All but one of the articles on the list are from the 1980s and 1990s. Interestingly, the only article on the list that was not published in these two decades is Bühlman's *AB* article, "Experience Rating and Credibility," published more than 35 years ago in 1967. It places fourth on the list with 19 citations. Wilkie's 1995 *BAJ* article, "More on a Stochastic Asset Model for Actuarial Use" leads all actuarial articles with 33 citations. The authors with multiple articles on the list of the most frequently cited actuarial articles are Panjer (with two) and Wang and Goovaerts (both with three). Finally, there are two themes that are common among several of the 17 most influential articles published in the sample actuarial journals in recent years: pricing and financial distress are the subjects of over a third of the articles.

4. CONCLUSION

The bibliographies of articles from 16 risk, insurance, and actuarial journals during the years 1996 through 2000 were

TABLE 10

MOST FREQUENTLY CITED ARTICLES PUBLISHED IN ANY OF THE ACTUARIAL JOURNALS

Rank	Title	Journal/Year	Pages	Author(s)	Citations
1	More on a Stochastic Asset Model for Actuarial Use	*BAJ*/95	777–964	Wilkie	33
2	Recursive Evaluation of a Family of Compound Distributions	*AB*/81	22–26	Panjer	29
3	Premium Calculation by Transforming the Layer Premium Density	*AB*/96	71–92	Wang	21
4	Experience Rating and Credibility	*AB*/67	199–207	Buhlman	19
5	The Calculation of Aggregate Loss Distributions from Claim Severity and Claim Count Distributions	*PCAS*/83	22–71	Heckman and Meyers	18
6	Dependency of Risks and Stop-Loss Order	*AB*/96	201–212	Dhaene and Goovaerts	17
7	A Simulation Test of Prediction Errors of Loss Reserve Estimation Techniques	*PCAS*/85	124–148	Stanard	16
8	Axiomatic Characterization of Insurance Prices	*IME*/97	173–183	Wang, Young and Panjer	15

T9	Insurance Pricing and Increased Limits Rate-Making by Proportional Hazards Transformed	*IME*/95	43–54	Wang	14
T9	The Distribution of a Perpetuity, with Applications to Risk Theory and Pension Funding	*SAJ*/90	39–79	Dufresne	14
T9	Recursive Calculation of Finite-Time Ruin Probabilities	*IME*/88	1–8	DeVylder and Goovaerts	14
T12	Report on the Wilkie Stochastic Investment Model	*JIA*/92	173–228	Geoghegan, et al.	13
T12	On the Distribution of the Surplus Prior to Ruin	*IME*/92	191–207	Dickson	13
T12	Risk Loads for Insurers	*PCAS*/90	160–195	Feldblum	13
T12	Reinsurer Risk Loads from Marginal Surplus Requirements	*PCAS*/90	196–203	Kreps	13
T12	On the Probability and Severity of Ruin	*AB*/87	151–163	Gerber, Goovaerts and Kaas	13
T12	Estimates for the Probability of Ruin with Special Emphasis on the Possibility of Large Claims	*IME*/82	55–72	Embrechts and Veraverbeke	13

reviewed and recorded. After observing the citation patterns of the sample journals, the journals were put into two separate groups; 1) the actuarial journal group and 2) the risk and insurance journal group. Then the actuarial journals were ranked based on the total number of citations and their research impact on a per article basis.

The most frequently cited journal for ten of the 16 sample journals was the citing journal itself. For the actuarial journals, *IME* was the first or second most frequently cited journal for six of the eight journals evaluated, with the *CASF* and the *PCAS* being the only actuarial journals not having *IME* among their top two. The *PCAS* was the most frequently cited journal and the *CASF* was the second most frequently cited journal by both the *CASF* and the *PCAS*. For the sample risk and insurance journals, the *JRI* was the first or second most frequently cited journal by all journals.

The top actuarial journal based on the total number of citations from the sample journals including self-citations is *IME* with the *AB* and the *PCAS* having the second and third most citations, respectively. These journals remain the top three when excluding self-citations, but the positions of the *IME* and *AB* are reversed. Using the per article impact measure to rank the actuarial journals, the *AB* is the highest ranked journal with the *PCAS* and the *SAJ* ranking second when including and excluding self-citations, respectively.

The most frequently cited articles are also reported. The list of the most frequently cited *CASF* and *PCAS* articles includes 13 *PCAS* articles and three *CASF* articles. Heckman and Meyers' 1983 *PCAS* article "The Calculation of Aggregate Loss Distributions from Claim Severity and Claim Count Distributions" is the most frequently cited. The list of the most frequently cited articles published in all of the sample actuarial journals includes five articles from both the *AB* and *IME*, four from the *PCAS*, two from the *BAJ* (including a *JIA* article from 1992), and one from the *SAJ*.

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RISKINESS LEVERAGE MODELS

RODNEY KREPS

Abstract

A general formulation of risk load for total cash flows is presented. It allows completely additive co-measures[1] *at any level of detail for any dependency structure between random variables constituting the total. It is founded on the intuition that some total outcomes are more risky per dollar than others, and the measure of that is a "riskiness leverage ratio." This riskiness leverage function is an essentially arbitrary choice, enabling an infinite variety of management attitudes toward risk to be expressed.*

The complete additivity makes these models useful. What makes them interesting is that attention can be turned toward asking "What is a plausible risk measure for the whole, while being prepared to use the indicated allocation technique for the pieces?" The usual measures are special cases of this form, as shown in some examples.

While the author does not particularly advocate allocating capital to do pricing, this class of models does allow pricing at the individual policy clause level, if so desired.

Further, the desirability of reinsurance or other hedges can be quantitatively evaluated from the cedant's point of view by comparing the increase in the mean cost of underwriting with the decrease in capital cost from reduction of capital required.

[1] Gary Venter coined this term, in parallel with variance and covariance.

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1. INTRODUCTION

The generic problem is that there are a number of random liabilities and assets for a company and a single pool of shared capital to support them. Their mean is usually meant to be supported by the reserves and their variability supported by the surplus, with the total assets of the company being the sum. Frequently, it is desired that the supporting capital be allocated in considerable detail—for example, to underwriter within line of business within state. This is not an end in itself, but is usually meant to help to understand profitability (or lack of it) in a business unit by associating a target rate of return with the allocated surplus and comparing to the actual profit return distribution. Sometimes the allocation is meant to be used for creating a pricing risk load as the allocated surplus times a target rate of return. Really, it is the cost of capital that is being allocated.[2]

One would like to have a methodology that would allow allocation of an essentially arbitrary form for the total capital required, and would also like to have an interpretation of the form in terms of statistical decision theory. The total capital including surplus will usually be represented as the sum of a risk load and a mean outcome. These can be calculated for a given distribution of total results. No attempt to connect risk load to a theory of pricing will be made here, although given the shape of the distribution in the context of a given theory such a connection could be made. It is simply assumed that some appropriate mean return is needed to attract and retain capital for the total risk.

[2]Gary Venter, private communication.

There are several desirable qualities for an allocatable risk load formulation: (1) it should be able to be allocated to any desired level of definition; (2) the risk load allocated for any sum of random variables should be the sum of the risk load amounts allocated individually; (3) the same additive formula is used to calculate the risk load for any subgroup or group of groups.

This means that senior management can allocate capital to regions, and then regional management can allocate their capital to lines of business, and the allocations will add back up to the original. Further, it also means that the lines of business will add to the allocations for total lines of business as seen at the senior management level.

Ultimately, the choice of the riskiness leverage function will reflect management attitudes toward risk. The intention of this paper is to provide an interpretable framework for infinitely many choices, all of which can be appropriately allocated. It will be argued that the risk load must be considered in the context of the capital to support the risk.

Once management has experimented with various riskiness leverage functions and found a formulation with which they are comfortable, then it can be used to evaluate potential management decisions quantitatively. For example, buying reinsurance or choosing between reinsurance programs can be framed by including the variables representing the reinsurance cash flows. The general effects from a well-designed program will be to increase the mean cost—because the reinsurer needs to make a profit, on average—and to decrease the risk load and its associated cost—because the reinsurance is a good hedge against severe outcomes. If there is a net reduction in total cost, then there is an advantage to the alternative. It is worth noting that no financial information except the price is needed from the reinsurer. In particular, whatever return the reinsurer may think he will get from the contract is irrelevant to the cedant's decision to buy or not.

Section 2 introduces the framework and some practical notes; Section 3 is the development of the form and some of its properties; Section 4 is various examples, including some of the usual suspects for risk measures; Section 5 talks about what general properties might be desirable; and Section 6 is a numerical example with an accompanying spreadsheet.

2. THE FRAMEWORK

Assume n random financial variables X_k, $k = 1$ to n; and let $X = \sum_{k=1}^{n} X_k$ be their sum, the net result to the company. These variables may be from assets and/or liabilities but we will think of them for the initial exposition as liabilities. The convention used here is the actuarial view that liabilities are positive and assets are negative. This is an odd point of view for financial reports, and so in the accompanying exemplar spreadsheet, to be discussed at length in Section 6, the formulas are rephrased with the variables being net income streams and positive income being positive numbers.

Denote by μ the mean of X, C the total capital to support X, and R the risk load for X. Their relationship is

$$C = \mu + R \tag{2.1}$$

In more familiar terms, for balance sheet variables the capital would be the total assets, the mean the booked net liabilities, and the risk load the surplus.

Correspondingly, let μ_k be the mean of X_k, C_k be the capital allocated to X_k and R_k be the risk load for X_k. These satisfy the equation analogous to Equation (2.1):

$$C_k = \mu_k + R_k. \tag{2.2}$$

Using the abbreviation

$$\overline{dF} \equiv f(x_1, x_2, \ldots, x_n) dx_1 dx_2 \ldots dx_n, \tag{2.3}$$

where $f(x_1, x_2, \ldots, x_n)$ is the joint probability density function of all the variables, the individual means are defined by

$$\mu_k \equiv \int x_k \overline{dF}, \tag{2.4}$$

and the overall mean is

$$\mu \equiv \int \left[\sum_{k=1}^{n} x_k\right] \overline{dF} = \sum_{k=1}^{n} \mu_k. \tag{2.5}$$

Riskiness leverage models have the form

$$R_k \equiv \int \overline{dF}(x_k - \mu_k)L(x) \qquad \text{with} \quad x \equiv \sum_{k=1}^{n} x_k. \tag{2.6}$$

Then

$$R = \int \overline{dF}(x - \mu)L(x) = \int f(x)(x - \mu)L(x)dx. \tag{2.7}$$

The essential key to this formulation is that *the riskiness leverage L depends only on the sum of the individual variables*. In the second form of Equation (2.7), $f(x)$ is the density function for X, the sum of random variables.

It follows directly from their definitions that $R = \sum_{k=1}^{n} R_k$ and $C = \sum_{k=1}^{n} C_k$, no matter what the joint dependence of the variables may be.

In analogy with the relation of covariance to variance, the R_k will be referred to as co-measures of risk for the measure R. On occasion, the C_k will also be referred to as co-measures when the context is clear. Since additivity is automatic with these co-measures, what remains is to *find appropriate forms for the riskiness leverage* $L(x)$.

The form can be thought of as the risk load being a probability-weighted average of risk loads over outcomes of the total net loss:

$$R = \int dx\, f(x)r(x) \qquad \text{where} \quad r(x) = (x - \mu)L(x). \tag{2.8}$$

Again, the riskiness leverage reflects that not all dollars are equally risky, especially dollars that trigger analyst or regulatory tests.

Equation (2.8) is a standard decision-theoretic formulation for R. It could have been written down immediately, except that the special form for the risk load for outcomes is needed so that the co-measures have good properties. Another version of Equation (2.8) is to represent the risk load as an integral over risk load density:

$$R = \int rld(x)dx \qquad \text{where} \quad rld(x) = f(x)(x-\mu)L(x). \tag{2.9}$$

This has the advantage of showing which outcomes most contribute to the risk load. Another formulation, of note to theorists, is to say that the riskiness leverage modifies the joint density function and that the allocations are statistical expectations on a risk-adjusted density function. However, the support of L needs to be the same as the support of f to make this really work.

$$R = \int dx\, f^*(x)(x-\mu) \qquad \text{with} \quad f^*(x) = f(x)L(x). \tag{2.10}$$

A closely related useful form for thinking about the risk loads is that they are conditional expectations of a variable less its mean on the risk-adjusted measure, and that the conditions refer to the overall total variable. A typical condition might be that the total loss is greater than some specified value.

If we just want to think about co-measures without the explicit breakout into mean and risk load, we can use the generalization

$$R_k \equiv \int \overline{dF}(x_k - a\mu_k)L(x) \qquad \text{with} \quad x \equiv \sum_{k=1}^{n} x_k, \tag{2.11}$$

where any constant value can be used for a. A prime candidate is $a = 0$, and in the exemplar spreadsheet in Section 6 this is done because the variables considered there are net income variables.

It is also clear from Equation (2.6) that some variables may have negative risk loads, if they happen to be below their mean when the riskiness leverage on the total is large. This is a desirable feature, not a bug, as software developers say. Hedges in general and reinsurance variables in particular should exhibit this behavior, since when losses are large they have negative values (ceded loss) greater than their mean costs.

Practical Notes

Actual calculation of Equations (2.6) and (2.7) cannot be done analytically, except in relatively simple cases. However, in a true Monte Carlo simulation environment they are trivially evaluated. All one has to do is to accumulate the values of X_k, $L(X)$, and $X_k L(X)$ at each simulation. At the end, divide by the number of simulations and you have the building blocks[3] for a numerical evaluation of the integrals. As usual, the more simulations that are done the more accurate the evaluation will be. For companies that are already modeling with some DFA model it is easy to try out various forms for the riskiness leverage.

This numerical procedure is followed in the spreadsheet of Section 6, which has assets and two correlated lines of business. All the formulas are lognormal so that the exact calculations for moments could be done. However, the spreadsheet is set up to do simulation in parallel with the treatment on a much more complex model. It is also easy to expand the scope. If one starts at a very high level and does allocations, these allocations will not change if one later expands one variable (e.g., countrywide results) into many (results by state) so long as the total does not change.

[3]The mean for X_k is just the average over simulations, and it might be advantageous to calculate this first. The risk load is just the average over simulations of $X_k L(X)$ minus the mean of X_k times the average over simulations of $L(X)$.

Fundamentally, a risk measure should arise from economic requirements and management attitudes toward risk as part of the management business model. In this paper's class of models the risk attitude information is in the riskiness leverage function.

Gedanken[4] experiments indicate that to get the riskiness leverage it is probably desirable to start with plausible relativities between outcomes. After that is done, set the overall scale by some criterion such as probability of ruin (Value At Risk), mean policyholder deficit, Tail Value At Risk (TVAR)[5] or anything else that references the total capital and suits management's predilections. It is best if the overall level can be framed in the same terms as the relativities. In the Section 6 spreadsheet, TVAR is used.

In general, it might be good to start with simple representations, say with two parameters, and then see what consequences emerge during the course of testing. More remarks will be made later on specific forms. It will also be shown that the usual forms of risk measure can be easily framed and the differences between them interpreted in terms of different riskiness leverages.

A warning: there is no sign of time dependence in this formulation so far. Presumably the variables refer to the present or future value of future stochastic cash flows, but there is considerable work to be done to flesh this out.[6]

3. FORM DEVELOPMENT

Here we will start from a covariance formulation and proceed to the framework above by a detailed mathematical derivation.

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[4]That is, thought experiments, as contrasted with the real thing. The term is from the early days of relativity.

[5]TVAR is the average value of a variable, given that it is past some defined point in the tail. For example, one could ask for the average loss size given that the loss is excess of $10M.

[6]The work of Leigh Halliwell "The Valuation of Stochastic Cash Flows" may provide a way of looking at this problem.

Various proposed schemes[7] have utilized the fact that an allocation formula of the form

$$C_k = \alpha\mu_k + \beta\mathrm{Cov}(X_k, X) \tag{3.1}$$

will always be additive no matter what the dependency between the X_k may be. That is,

$$\begin{aligned} C &\equiv \alpha\mu + \beta\mathrm{Var}(X) \\ &= \alpha\mathrm{E}(X) + \beta\mathrm{Cov}(X, X) \\ &= \alpha\sum_{k=1}^{n}\mu_k + \beta\sum_{k=1}^{n}\mathrm{Cov}(X_k, X) \\ &= \sum_{k=1}^{n} C_k. \end{aligned} \tag{3.2}$$

A similar result will hold for the sum of any subset of the variables, thus ensuring the desired properties of the allocation. The sum of covariances of the individual variables with the total is the covariance of the total with itself. This paper generalizes this notion.

This form can be pushed further by imposing the reasonable requirement[8] that if a variable has no variation, then the capital to support it is simply its mean value with no additional capital requirement. This requires $\alpha = 1$. Then, with capital being the sum of the mean and the risk load,

$$R_k = \beta\mathrm{Cov}(X_k, X) \tag{3.3}$$

and

$$R = \beta\mathrm{Var}(X) \tag{3.4}$$

and so finally

$$R_k = R\frac{\mathrm{Cov}(X_k, X)}{\mathrm{Var}(X)}. \tag{3.5}$$

[7]For a sampling, try [6], [2], and [4]. There are no doubt others.

[8]In [6], since the company can default, a constant value carries a negative risk load. We are assuming an ongoing company.

This form is familiar from CAPM.

However, it is clear that there are many independent linearly additive statistics. Back up a little to the definitions of mean and covariance, expressed as integrals over the joint density function:

$$\begin{aligned}\mu_k \equiv \mathrm{E}(X_k) &= \int x_k f(x_1,\ldots,x_n)dx_1\ldots dx_n \\ &\equiv \int x_k \overline{dF}.\end{aligned} \tag{3.6}$$

The additivity of the mean then comes from

$$\mu \equiv \mathrm{E}(X) = \int \sum_{k=1}^{n} x_k = \sum_{k=1}^{n} \int x_k = \sum_{k=1}^{n} \mu_k. \tag{3.7}$$

The covariance of one variable with the total is defined as

$$\mathrm{Cov}(X_k, X) \equiv \int \overline{dF}(x_k - \mu_k)(x - \mu), \tag{3.8}$$

where $x \equiv \sum_{k=1}^{n} x_k$. The additivity of the covariance is from

$$\begin{aligned}\mathrm{Cov}(X,X) &= \int \overline{dF}(x-\mu)^2 \\ &= \int \overline{dF}\left[\sum_{k=1}^{n}(x_k - \mu_k)\right](x-\mu) \\ &= \sum_{k=1}^{n}\int \overline{dF}(x_k-\mu_k)(x-\mu) \\ &= \sum_{k=1}^{n}\mathrm{Cov}(X_k, X).\end{aligned} \tag{3.9}$$

We want to generalize this result, and to do so we need more independent statistics that are linear functionals in X_k. Define the moment expectations

$$\mathrm{E}_m(X_k) \equiv \int \overline{dF}[(x_k - \mu_k)(x-\mu)^m]. \tag{3.10}$$

Then, following the same argument as in Equation (3.9), for any m

$$\mathrm{E}_m(X) = \sum_{k=1}^{n} \mathrm{E}_m(X_k). \tag{3.11}$$

Notice that the moment expectation for $m = 1$ is just the covariance of X_k with the total.

The individual risk load may now be formulated as

$$R_k = \sum_{m=1}^{\infty} \beta_m \mathrm{E}_m(X_k), \tag{3.12}$$

and there are now an infinite number of arbitrary constants to play with. Since there are so many independent constants, essentially any form can be approximated arbitrarily well.

For any choice of the constants β_m, the total risk load is the sum of the individual risk loads:

$$R = \sum_{m=1}^{\infty} \beta_m \mathrm{E}_m(X) = \sum_{m=1}^{\infty} \beta_m \sum_{k=1}^{n} \mathrm{E}_m(X_k) = \sum_{k=1}^{n} R_k. \tag{3.13}$$

This risk load can be put into a more transparent form by writing it as

$$R_k = \sum_{m=1}^{\infty} \beta_m \mathrm{E}_m(X_k) = \int \overline{dF}(x_k - \mu_k) \sum_{m=1}^{\infty} \beta_m (x-\mu)^m. \tag{3.14}$$

Since the term with $m = 0$ integrates to 0 (that being the definition of the mean), what is present is a Taylor series expansion of a function of the total losses about μ. Thus, Equation (3.14) may be written as

$$R_k = \int \overline{dF}(x_k - \mu_k) L(x). \tag{3.15}$$

This is the framework described earlier.

Properties

Clearly, the allocation properties are all satisfied for any choice of $L(x)$. The risk load has no risk for constant variable

$$R(c) = 0.$$

It also will scale with a currency change

$$R(\lambda X) = \lambda R(X),$$

provided $L(x)$ is homogeneous of order zero:

$$L(\lambda x) = L(x).$$

The reason this is required is that there is already a currency dimension in the term multiplying L. This can be made to happen, for example, by making L a function of ratios of currencies such as x/μ or x/σ, where σ is the standard deviation of X.

However, a more interesting possibility is to make L also be a function of x/S, where again S is the total surplus of the company. Since asset variability is in principle included in the random variables, S should be a guaranteed-to-be-available, easily liquefiable capital. This could come, for example, by having it in risk-free instruments or by buying a put option on investments with a strike price equal to what a risk-free investment would bring, or any other means with a sure result.

It is intuitively clear that S must come into the picture. Consider the case where loss is normally distributed with mean 100 and standard deviation 5. Is this risky for ruin, from a business point of view? If the surplus is 105, it is—but if it is 200 it is not. The natural interpretation is that the riskiness leverage should be a function of the ratio of the difference of the outcome from the mean to the surplus. Since the riskiness leverage could be used (with a pre-determined leverage) to give the surplus, there is a certain recursive quality present.

This formulation of risk load may or may not produce a coherent risk measure.[9] The major reason is that subadditivity[10] $[R(X+Y) \leq R(X)+R(Y)]$ depends on the form of $L(x)$. It might be remarked that superadditivity $[R(X+Y) > R(X)+R(Y)]$ is well known in drug response interactions, where two drugs taken separately are harmless but taken together are dangerous. While axiomatic treatments may prefer one form or another, *it would seem plausible that the risk measure should emerge from the fundamental economics of the business* and the mathematical properties should emerge from the risk measure, rather than vice versa.

A riskiness leverage formulation clearly allows the entire distribution to influence the risk load, and does not prescribe any particular functional form for the risk measure. In addition, many familiar measures of risk can be obtained from simple forms for the riskiness leverage ratio.

4. EXAMPLES

Risk-Neutral

Take the riskiness leverage to be a constant; the risk load is zero.

The positive risk load balances the negative risk load. This would be appropriate for risk of ruin if the range of x where $f(x)$ is significant is small compared to the available capital, or if the capital is infinite. It would be appropriate for risk of not meeting plan if you don't care whether you meet it or not.

Variance

Take

$$L(x) = \frac{\beta}{S}(x-\mu). \tag{4.1}$$

This riskiness leverage says that the whole distribution is relevant; that there is risk associated with good outcomes as much as

[9]In the sense of [1] the actual risk measure is mean $+R$.

[10]A requirement for coherence. See [5] or [1].

bad; and that the outcome risk load increases quadratically out to infinity.

This gives the usual

$$R = \frac{\beta}{S}\int_0^\infty dx\, f(x)(x-\mu)^2 \tag{4.2}$$

and

$$R_k = \frac{\beta}{S}\int \overline{dF}(x_k - \mu_k)\left(\sum_{j=1}^{n} x_j - \mu\right). \tag{4.3}$$

Note that Equation (4.1) is suggestively framed so that β is a dimensionless constant available for overall scaling. The total capital then satisfies

$$C = \mu + S, \tag{4.4}$$

and the solution for $S = R$ is proportional to the standard deviation of the total:

$$S = \sqrt{\beta \operatorname{Var}(X)}. \tag{4.5}$$

It is perfectly possible, of course, to use some other formulation of the constant, say β/μ, which would then give a different measure. Such a measure would imply that the riskiness leverage does not depend on the amount of surplus available unless it was hidden in the scaling factor β.

TVAR (Tail Value At Risk)

Take the riskiness leverage

$$L(x) = \frac{\theta(x - x_q)}{1-q}. \tag{4.6}$$

The value q is a management-chosen percentage; for example, $q = 99\%$. The quantile x_q is the value of x where the cumulative distribution of X, the total, is equal to q. That is, $F(x_q) = q$. $\theta(x)$ is the step function: zero for negative argument and 1 for positive. See Appendix A for mathematical asides on this function.

This riskiness leverage ratio is zero up to a point, and then constant. Here the constant is chosen so as to exactly recreate TVAR, but clearly any constant will give a similar result. In fact, a riskiness leverage ratio that is constant up to a point and then jumps to another constant will give a similar result.

$$\begin{aligned} C &= \mu + \int dx\, f(x)(x-\mu)\frac{\theta(x-x_q)}{1-q} \\ &= \mu + \int_{x_q}^{\infty} dx\, f(x)\frac{x-\mu}{1-q} \\ &= \mu - \frac{\mu}{1-q}(1-q) + \frac{1}{1-q}\int_{x_q}^{\infty} dx\, f(x)x \\ &= \frac{1}{1-q}\int_{x_q}^{\infty} dx\, f(x)x. \end{aligned} \tag{4.7}$$

This is the definition of TVAR, well known to be coherent.[11]

We see shortly that the allocated capital is just the average value of the variable of interest in the situations where the total is greater than x_q. This is one example of the conditional expectation referred to earlier.

$$\begin{aligned} C_k &= \mu_k + \int \overline{dF}(x_k-\mu_k)\frac{\theta(x-x_q)}{1-q} \\ &= \mu_k - \frac{\mu_k}{1-q}\int \overline{dF}\theta(x-x_q) + \frac{\int \overline{dF} x_k \theta(x-x_q)}{1-q} \\ &= \frac{\int \overline{dF} x_k \theta(x-x_q)}{1-q}. \end{aligned} \tag{4.8}$$

This measure says that only the part of the distribution at the high end is relevant.

[11] [5], Op. cit.

VAR (Value At Risk)

Take the riskiness leverage

$$L(x) = \frac{\delta(x - x_q)}{f(x_q)}. \tag{4.9}$$

In Equation (4.9) $\delta(x)$ is the Dirac delta function.[12] Its salient features are that it is zero everywhere except at (well, arbitrarily close to) zero and integrates to one.[13] See Appendix A for remarks about this very useful function. Here the riskiness leverage ratio is all concentrated at one point. The constant factor has been chosen to reproduce VAR exactly, but clearly could have been anything.

$$\begin{aligned} C &= \mu + \int dx\, f(x)(x - \mu)\frac{\delta(x - x_q)}{f(x_q)} \\ &= \mu + x_q - \mu \\ &= x_q. \end{aligned} \tag{4.10}$$

This gives value at risk, known not to be coherent.[14] This measure says that only the value x_q is relevant; the shape of the loss distribution does not matter except to determine that value.

The capital co-measure is the mean of the variable over the hyperplane where the total is constant at x_q:

$$\begin{aligned} C_k &= \mu_i + \int \overline{dF}(x_k - \mu_k)\frac{\delta(x - x_q)}{f(x_q)} \\ &= \frac{1}{f(x_q)} \int \overline{dF} x_k \delta\left(\sum_{j=1}^{n} x_j - x_q\right). \end{aligned} \tag{4.11}$$

In a simulation environment one would have to take a small region rather than a plane. This could most easily be done as the

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[12] Introduced in 1926.

[13] This implies that $\int dx\, f(x)\delta(x - a) = f(a)$. See Appendix 1.

[14] [5], Op. cit.

difference of two closely neighboring TVAR regions. This was done using the formulation of the exemplar spreadsheet and a 1% width of the region.

SVAR (Semi-Variance)

Take the riskiness leverage

$$L(x) = \frac{\beta}{S}(x-\mu)\theta(x-\mu). \tag{4.12}$$

The risk load is the semi-variance—the "downside" of the variance:

$$R = \frac{\beta}{S}\int_{\mu}^{\infty} dx\, f(x)(x-\mu)^2, \tag{4.13}$$

and

$$R_k = \frac{\beta}{S}\int \overline{dF}(x_k-\mu_k)(x-\mu)\theta(x-\mu). \tag{4.14}$$

This measure says that risk loads are only non-zero for results worse (greater) than the mean. This accords with the usual accountant's view that risk is only relevant for bad results, not for good ones. Further, this says the load should be quadratic to infinity.

Mean Downside Deviation

Take the riskiness leverage

$$L(x) = \beta\frac{\theta(x-\mu)}{1-F(\mu)}. \tag{4.15}$$

$F(x)$ is the cumulative distribution function for X, the total. This risk load is a multiple of the mean downside deviation, which is also TVAR with $x_q = \mu$. This riskiness leverage ratio is zero below the mean, and constant above it. Then

$$R(X) = \frac{\beta}{1-F(\mu)}\int_{\mu}^{\infty} dx\, f(x)(x-\mu), \tag{4.16}$$

and

$$R_k = \frac{\beta}{1-F(\mu)} \int \overline{dF}(x_k - \mu_k)\theta(x-\mu). \tag{4.17}$$

In some sense this may be the most natural naive measure, as it simply assigns capital for bad outcomes in proportion to how bad they are. Both this measure and the preceding one could be used for risks such as not achieving plan, even though ruin is not in question.

In fact, there is a heuristic argument suggesting that $\beta \approx 2$. It runs as follows: suppose the underlying distribution is uniform in the interval $\mu - \Delta \leq x \leq \mu + \Delta$. Then in the cases where the half-width Δ is small compared to μ, the natural risk load is Δ. For example, if the liability is \$95M to \$105M, then the natural risk load is \$5M. So from Equation (4.17)

$$\Delta = R(X) = \frac{\beta}{0.5} \int_{\mu}^{\mu+\Delta} \frac{dx}{2\Delta}(x-\mu) = \frac{\beta\Delta}{2}. \tag{4.18}$$

However, for a distribution that is not uniform or tightly gathered around the mean, if one decided to use this measure, the multiplier would probably be chosen by some other test such as the probability of seriously weakening surplus.

Proportional Excess[15]

Take the riskiness leverage

$$L(x) = \frac{h(x)\theta[x-(\mu+\Delta)]}{x-\mu}, \tag{4.19}$$

where to maintain the integrability of R_k either $h(\mu) = 0$ or $\Delta > 0$. Then

$$R = \int f(x)h(x)\theta[x-(\mu+\Delta)]dx, \tag{4.20}$$

and

$$R_k = \int \overline{dF} \frac{x_k - \mu_k}{x-\mu} h(x)\theta[x-(\mu+\Delta)]. \tag{4.21}$$

[15] Another contribution from Gary Venter.

The last form has the simple interpretation that the individual allocation for any given outcome is pro-rata on its contribution to the excess over the mean.

5. GENERIC MANAGEMENT RISK LOAD

Most of the world lives in a situation of finite capital. Frame the question as "given the characteristics of the business, what is an appropriate measure of risk to the business, which generates a needed surplus S?" In the spreadsheet example this is done with a simplistic riskiness leverage function.

Clearly, the question at the heart of the matter is what an appropriate measure of riskiness might be. There are many sources of risk among which are the risk of not making plan, the risk of serious deviation from plan, the risk of not meeting investor analysts' expectations, the risk of a downgrade from the rating agencies, the risk of triggering regulatory notice, the risk of going into receivership, the risk of not getting a bonus, etc.

Given the above, it seems plausible that company management's list for the properties of the riskiness leverage ratio should be that it:

1. be a downside measure (the accountant's point of view);
2. be more or less constant for excess that is small compared to capital (risk of not making plan, but also not a disaster);
3. become much larger for excess significantly impacting capital; and
4. go to zero (or at least not increase) for excess significantly exceeding capital—once you are buried, it doesn't matter how much dirt is on top.

With respect to (3), the risk function probably has steps in it, especially as regulatory triggers are hit. For (4), a regulator might want to give more attention to the extreme areas. In fact, a regulator's list of properties for the riskiness leverage might include that it

1. be zero until capital is seriously impacted, and
2. not decrease, because of the risk to the state guaranty fund.

TVAR could be used as such a risk measure if the quantile is chosen to correspond to an appropriate fraction α of surplus. This would be

$$L_{\text{Regulator}}(x) = \frac{\theta(x-\alpha S)}{1-F(\alpha S)}. \tag{5.1}$$

However, everyone recognizes that at some level of probability management will have to bet the whole company. There is always business risk.

Management may more typically formulate its risk appetite in forms such as "For next year, we want not more than a 0.1% chance of losing all our capital, and not more than a 10% chance of losing 20% of capital." This is basically two separate VAR requirements, and can be satisfied by using the larger of the two required capital amounts. Or, as in the spreadsheet, management may choose to say something like, "We want our surplus to be $1\frac{1}{2}$ times the average bad result in the worst 2% of cases."

A (much too) simple example approximately satisfying (1) to (3) on management's list consists of linear downside riskiness leverage:

$$L(x) = \begin{cases} 0 & \text{for} \quad x < \mu \\ \beta\left[1+\alpha\dfrac{(x-\mu)}{S}\right] & \text{for} \quad x > \mu \end{cases}. \tag{5.2}$$

The value of α is essentially the relative riskiness at the mean and at excess over mean equal to surplus. The value of β is again an overall scale factor. In the spreadsheet the allocations are nearly independent of the value of α, and TVAR is used for the example. The suggested use is to get the riskiness leverage function, and then to evaluate the effects of reinsurance (approximated by an increase in the mean and a decrease in the coefficient of variation) by seeing how the capital requirement changes for the same leverage function.

6. EXEMPLAR SPREADSHEET

The Excel workbook “Mini DFA.xls” has two lines of business with a correlation between the lines and investment income. The example is meant to be oversimplified but plausible, and takes the underwriting result for each line as a fixed premium less random draw on loss and expense. There is investment income on the surplus but no explicit consideration of it within the reserves. On the other hand, the lines of business are priced to a net positive underwriting result, so we could say that we are looking at future values including all investment income.

Cells with a blue background are input cells, and the reader is invited to change them and see how the results change. All the formulas are lognormal so that the exact calculations could be done. However, there is a “Simulate” button on the spreadsheet that will give statistics and cumulative distribution functions for whatever set of cells is selected. Simulation is used to get the overall results and the allocation ratios for different risk measures.

The sheets in the workbook are of two types: the data sheets (e.g., “basics”) and the simulations done on them (“Sim basics”). The different sheets are generally different business alternatives. We start with “basics,” which gives the basic setup of the business, and continue on: “TVARS” calculates various TVAR measures, “change volume” changes the volumes of the lines, and “reinsurance” and “reinsurance (2)” explore the effects of reinsurance. We will walk through them in detail, with commentary.

In all of them, the layout is the same. The two lines of business and the investment on surplus are laid out in columns, with blue background for user input. The financial variables are the two net underwriting results and the investment result, all of which vary randomly. F9 will recalculate to a new set of results. Below the income variables are the starting and ending surplus, and calculated mean and current (random) return. Interesting simulation results such as allocation percentages are displayed to the right of the surplus calculation.

Starting with "basics," Line A has a mean surplus of 10,000,000 and a standard deviation of 1,000,000 and Line B has a mean surplus of 8,000,000 and a standard deviation of 2,000,000. There is a correlation of about 25% between the lines (if the functions were normal rather than lognormal, it would be exactly 25%). Each line is written with a premium equal to the mean loss plus 5%. We interpret this calculation as our estimate at time zero of the value at time 1 of the underwriting cash flows, including all investment returns on reserves and premiums.

The investment income on the surplus is taken directly. The investment is at a mean rate of 4% with a standard deviation of 10%. The total of the results, on which we will define our leverage functions, is then added to the beginning surplus of 9,000,000 to get the ending surplus. As a consequence of the input values, the mean return on surplus is 14%. We would all be happy to have such a company, provided it is not too risky.

The simulation ("Sim basics") shows the actual correlation of the lines and the coefficient of variation on the return, as well as the distribution of total ending surplus and return. From the "Sim basics" sheet we can see that the probability of ruin is less than one in a thousand, and the coefficient of variation on the return is better than on the investment, which is good. We can also see from comparing the simulated means and standard deviations of the income variables to their known underlying values that the simulation is running correctly.

Management has decided that it wants to consider not just ruin, but on-going risk measures. In particular, it wants to get the TVAR values at various percentiles. It wants to formulate its risk appetite as *"For the x percent of possibilities of net income that are less than $(income corresponding to x%), we want the surplus to be a prudent multiple of the average value so that we can go on in business."* What we do not know yet is what is *x*%, and what is the "prudent multiple." Gary Venter has suggested that the prudent multiple could be such that the renewal book can still be serviced after an average bad hit.

The sheet "TVARs" has the calculations needed for TVAR simulation in cells G36:N42. Column G contains the percentage values from 10% to 0.1%, and Column H the values of the total net income corresponding to those percentages. These values come from the sheet "Sim basics." Column I answers the question if whether the income is less than the value in Column H. Columns J through M are either "FALSE" if Column I is FALSE, or contain respectively the total income, the Line A income, the Line B income, and the investment income. Column N is a variable that is 1 if Column I is TRUE, and zero if it is FALSE. Upon selecting these cells and simulating, the mean value of Column N (for each row) will be the percentage of the time that the condition was satisfied. This should be close to the percentage in Column G. During simulation, non-numeric values in the selected cells are ignored. The mean values of cells in Columns J through M are the conditional means of the income variables for different threshold values, as desired.

The result of simulation is:

%	Income is Below	Mean Value of TVAR and Allocation Percentages			
		Total	Line A	Line B	Investment
0.1	(8,892,260)	(10,197,682)	12.30%	85.99%	1.71%
0.2	(7,967,851)	(9,326,936)	12.49%	85.73%	1.78%
0.4	(7,024,056)	(8,380,265)	12.89%	85.09%	2.02%
1	(5,749,362)	(7,129,796)	13.38%	84.67%	1.95%
2	*(4,732,795)*	*(6,159,564)*	*13.60%*	*84.30%*	*2.10%*
5	(3,309,641)	(4,811,947)	13.60%	84.20%	2.20%
10	(2,143,327)	(3,734,177)	13.26%	84.94%	1.80%

The allocation percentages are just the ratios of the means for the pieces to the mean for the total; they automatically will add to 100%. What is noticeable here is that the allocation percentages change very little with the TVAR level, and that Line B needs some six times the surplus of Line A. That it needs more is not surprising; that it needs so much more perhaps is. What these allocations say is that when the total result is in the worst 10% of cases, about 5/6 of it is from Line B.

Management decides to adopt the rule *"We want our surplus to be* $1\frac{1}{2}$ *times the average negative income in the cases where it is below the 2% level."* That row is in *italic*, and this rule means that the 9,000,000 surplus is sufficient.

Using those allocation percentages, the mean returns on allocated surplus are Total: 14%; Line A: 40.9%; Line B: 5.3%; Investment: 190.6%. The total is a weighted average of the pieces. One needs to be careful in interpreting these return numbers, because they are dependent on both the relative volume of the lines and on the allocation method. But in any case, because Line B needs so much of the surplus, its return is depressed and the other returns are enhanced.

The next sheet, "change volume," looks at the case where we can change the underwriting volumes of Lines A and B. Clearly we want to reduce Line B and increase Column A, so the example has Column A increased by 60% and Column B decreased by 75%. This keeps the same mean net income. The standard deviations have been taken as proportional to volume, thinking of each line as a sum of independent policies.

Running the simulations, the allocations for Line A, Line B, and Investments now are respectively 32.8%, 60.9%, and 6.4%. Their implied returns change to 27.1%, 1.8%, and 62.8%. Line B is still bad, but because there is less of it, there is not such a large contribution at the 2% level. The 2% level, which was (4,732,795), is now better at about (3,250,000).

We also see that according to the management rule, we can release surplus of about 2,500,000. Alternatively, we can keep the same surplus and have a more conservative rule, with the prudent ratio being 2 instead of $1\frac{1}{2}$.

However, it may not be possible to change line volume, for various reasons. For example, these may be two parts of an indivisible policy, like property and liability from homeowners. Regulatory requirements may make it difficult to exit Line B. In addition, it takes time to switch the portfolio and requires a major underwriting effort. Management may decide to look at the possibility of buying reinsurance to improve the picture, since that is a decision that can be implemented quickly and easily changed next year.

The sheet "reinsurance" has an excess reinsurance contract on Line B, with a limit of 5,000,000 and an attachment of 10,000,000. It is priced with a load of 25% of its standard deviation. Once again, note that in the spreadsheet the results are calculated because we used easy forms, but that we could have complex forms and just simulate. The reinsurance results flow into the total net income.

Running the simulations, the allocations for Lines A and B, Investments, and now Reinsurance are respectively 36.3%, 73.9%, 14.2%, and −24.4%. The negative value for the reinsurance allocation reflects that the hedge is working, effectively supplying capital in these events. However, because of the positive net average cost of reinsurance, the return on the total is reduced to 12.1%. The implied returns on the pieces are 15.3%, 6.0%, 28.3%, and 7.9%. Line B is still bad, but because of the reinsurance there is not such a large contribution at the 2% level. Again, the 2% level has gone from (4,732,795) to (3,300,000). If we were to combine the reinsurance into Line B the combined allocation would be 49.5% and the return would be 5.1%.

There is also some 3,000,000 in surplus that the management rule would allow to be released. In the sheet "reinsurance (2)" the starting surplus has been reduced to 7,250,000 in order to bring the mean return on the total back up to 14%. Running the simulations, the 2% level on income is actually (3,237,000) but we ran the TVAR at (3,300,000). The essential point is that the results look reasonable, and the rule would allow release of still more surplus.

What is omitted in the calculation is the value of the 1,750,000 already released from the original 9,000,000 surplus. What this is worth depends on how the released surplus is going to be used. At the very least, this should be worth the risk-free income from it. Classical financial theory would suggest that it should be evaluated at the firm's cost of borrowing.

Measures other than TVAR were also run on the same basic situation, but are not shown in the spreadsheet. They were of two types. One was VAR measures, using a 1% interval around the VAR values. This measure says, given that the total loss is *at* a particular level, how much of it is from the different contributions. The other class of measures is the power measures, as in Equation (3.10). Each measure is a power of $(\mu - x)$ for $\mu > x$, and zero otherwise. In other words, these are downside measures.[16] The powers 0 and 1 are respectively the mean downside deviation and the semivariance. The others could be called "semiskewness," "semikurtosis," and so on—but why bother?

The results for VAR are quite similar to TVAR, except at the 10% level. This is because of the particular conditions we have for variability and correlation, and will not be true in general.

[16]Note that in contrast to the earlier discussion on losses where the downside is outcomes greater than the mean, here on return to surplus the downside is outcomes less than the mean.

	Mean Value and Allocation Percentages			
%	Total	Line A	Line B	Investment
0.1	(8,892,557)	13.51%	84.01%	2.48%
0.2	(7,969,738)	13.41%	84.74%	1.85%
0.4	(7,021,936)	15.32%	83.22%	1.46%
1	(5,746,279)	13.94%	84.18%	1.88%
2	(4,731,425)	14.20%	83.43%	2.38%
5	(3,308,824)	13.38%	83.64%	2.98%
10	(2,143,340)	11.16%	88.07%	0.76%

The downside power measure simulation results are:

	Mean Values ^(1/(N + 1)) and Allocations from Simulation			
Power	Total	Line A	Line B	Investment
0	2,183,834	22.44%	65.52%	12.04%
1	2,839,130	20.63%	69.79%	9.58%
2	3,424,465	19.42%	72.30%	8.28%
3	3,985,058	18.35%	74.30%	7.35%
4	4,510,337	17.43%	75.97%	6.60%
5	5,018,663	16.55%	77.45%	6.00%
6	5,514,616	15.69%	78.79%	5.51%

As the power increases and the measure is increasingly sensitive to the extreme values, the allocations move toward the TVAR allocations. This is probably not surprising.

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APPENDIX A

Some Mathematical Asides

$\theta(x)$ is the step function: zero for negative argument and 1 for positive. It is also referred to as the index function.

$\delta(x)$ is the Dirac delta function. It can be heuristically thought of as the density function of a normal distribution with mean zero and standard deviation arbitrarily small compared to anything else in the problem. This makes it essentially zero everywhere except at zero but it still integrates to 1.

The index function can also be thought of as the cumulative distribution function of the same normal distribution, and it is in this sense that the delta function can be thought of as the derivative of the index function. All the usual calculus rules about derivatives apply without modification.

Always, we are implicitly taking the limit as the standard deviation of this distribution goes to zero. This whole usage can be justified in the theory of linear functionals, but the author has no idea where.

These notions lead to some fundamental properties of the delta function. For any continuous function $f(x)$

$$f(a) = \int f(x)\delta(x-a)dx, \qquad \text{(A.1)}$$

and for $c > b$

$$\int_b^c f(x)\delta(x-a)dx = \theta(c-a)\theta(a-b)f(a). \qquad \text{(A.2)}$$

If $h(a) = 0$ then

$$\int f(x)\delta(h(x))dx = \frac{f(a)}{|h'(a)|}. \qquad \text{(A.3)}$$

The density function $f(x)$ for the total sum of variables can most easily be written as

$$f(x) = \int \overline{dF}\,\delta\left(x - \sum_{k=1}^{n} x_k\right)$$
$$\equiv \int dx_1 \ldots dx_n f(x_1,\ldots,x_n)\delta\left(x - \sum_{k=1}^{n} x_k\right). \qquad \text{(A.4)}$$

For calculation this is often a convenient form, as in the derivation of Equation (2.7):

$$\int \overline{dF}\left(\sum_{k=1}^{n} x_i - \mu\right) g\left(\sum_{k=1}^{n} x_k\right)$$
$$= \int dx \int \overline{dF}\,\delta\left(x - \sum_{k=1}^{n} x_k\right)(x-\mu)g(x)$$
$$= \int f(x)(x-\mu)g(x)dx. \qquad \text{(A.5)}$$

Similarly, the marginal density for any variable can be written

$$f_k(y) = \int \overline{dF}\,\delta(y - x_k). \qquad \text{(A.6)}$$

The cumulative distribution function for the total is

$$F(x) = \int \overline{dF}\,\theta\left(x - \sum_{k=1}^{n} x_k\right)$$
$$\equiv \int dx_1 \ldots dx_n f(x_1,\ldots,x_n)\theta\left(x - \sum_{k=1}^{n} x_k\right), \qquad \text{(A.7)}$$

and

$$f(x) = \frac{d}{dx}F(x) \qquad \text{(A.8)}$$

emerges from simple differentiation rules.

DISCUSSION OF A PAPER PRESENTED AT CAS SPRING 2005 MEETING

RISKINESS LEVERAGE MODELS

RODNEY KREPS

DISCUSSION BY ROBERT A. BEAR

Abstract

Rodney Kreps has written a paper that is a major contribution to the CAS literature on the central topics of risk load and capital allocation for profitability measurement, which is a core component of an enterprise risk management system. He has given us a rich class of mathematical models that satisfy two very desirable properties for a risk-load or surplus-allocation method: They can allocate risk down to any desired level of definition and they satisfy the additivity property. Tail Value at Risk and Excess Tail Value at Risk reasonably satisfy the properties that management would likely want of such a model, while still satisfying the properties of a riskiness leverage model and the properties of coherent measures of risk.

Donald Mango's ground-breaking work in developing the concepts of insurance capital as a shared asset and Economic Value Added [2] are discussed. A Risk Return on Capital model is suggested as an integration of the approaches presented by Kreps and Mango. This method measures returns on capital after reflecting the mean rental cost of rating agency capital. Reinsurance alternatives are compared using both the Return on Risk Adjusted Capital approach presented by Kreps and this integrated approach.

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1. INTRODUCTION

Rodney Kreps begins his paper by describing the generic problem as a situation where a company holds a single pool of shared capital to support a number of random liabilities and assets. The reserves are ordinarily meant to support their mean value, while the surplus is meant to support their variability around their means. Kreps, and by reference Gary Venter [5], first allay actuarial concerns about allocation of capital (discussed in [3]) by pointing out that return on equity (ROE) methods of computing pricing risk loads are really allocating the return on capital. If a line of business is returning 10% on allocated capital, one should ask whether this is a sufficient return to compensate the providers of that capital. Kreps then enumerates two desirable qualities for allocable risk load (the product of allocated surplus and a target rate of return):

1. It should be allocable down to any desired level.
2. It should be additive, in that risk load or capital allocated to components of the portfolio sum to the total risk load or capital need for the portfolio. This would be true for subsets of the portfolio as well.

Kreps does not insist that a risk load or capital allocation method satisfy all the requirements for a coherent risk measure [1], as he believes the risk measure should emerge from the fundamental economics of the business rather than the desired mathematical properties. Thus, Value at Risk (VAR) and Tail Value at Risk (TVAR) are both examples of riskiness leverage models, while VAR is not a coherent risk measure [1] and TVAR is well known to be a coherent risk measure according to Kreps. The fact that TVAR satisfies the subadditivity requirement of a coherent risk measure (the risk of a combination of exposures should not exceed the sum of the risks of the components) may increase the confidence of many actuaries that TVAR is measuring insurance risk appropriately.

Kreps develops the framework for a rich class of models for determining risk loads and allocating capital that possess the above desirable qualities. He then selects a particular example, TVAR, and demonstrates through a spreadsheet model how management can use such a model (once comfortable with the parameterization) to quantitatively evaluate alternative decisions, such as selecting among alternative reinsurance programs to enhance the risk-reward characteristics of a portfolio.

2. SUMMARY WITH COMMENTS

2.1. The Framework

This section summarizes the framework for the riskiness leverage models. Let X_k, $k = 1,\ldots,n$, represent losses associated with n risks or portfolio segments, whose sum represents the total loss to the company:

$$X = \sum X_k \qquad \text{where} \quad k = 1,\ldots,n.$$

If μ represents the mean of X, A is the total premium collected for this portfolio of risks, and R is the total risk load collected, then $A = \mu + R$. Alternatively, A may be interpreted as the total assets and R would represent the capital or surplus supporting this portfolio. (I am using A instead of C which was used in the paper, so as to avoid confusion of assets with capital. I use the terms capital, surplus, and equity interchangeably.)

Correspondingly, let μ_k represent the mean of X_k, let A_k represent the assets (or premium collected), and let R_k represent the surplus allocated to X_k (or risk load collected). Then

$A_k = \mu_k + R_k$ and $\mu = \sum \mu_k$ (where $k = 1,\ldots,n$) because expectations are additive. Riskiness leverage models have the form $R_k = E[(X_k - \mu_k)L(X)]$, where the riskiness leverage $L(X)$ is a function that depends only on the sum X of the individual variables and the expectation is taken with respect to that sum. Similarly, $R = E[(X - \mu)L(X)] = E[r(X)]$. Allocated capital and risk loads are probability-weighted averages of risk loads over outcomes of the total net loss. Riskiness leverage models can reflect the fact that not all loss outcomes are equally risky.

From their definitions, $R = \sum R_k$ and $A = \sum A_k$ where $k = 1,\ldots,n$, no matter what the joint dependence of the variables may be. Analogous to the relation of covariance to variance, the R_k will be referred to as co-measures of risk for the measure R. Since additivity follows automatically for these co-measures, Kreps searched for appropriate forms for the riskiness leverage $L(X)$.

Kreps points out that the capital allocations for risk loads may be efficiently computed through Monte-Carlo simulation. One simply simulates the quantity for which we want the expectation for a large number of years, and then averages the results from these scenarios. Kreps generalizes the covariance concepts to suggest the mathematical form of riskiness leverage models.

2.2. *Properties*

The desirable allocation properties for risk load or surplus allocation listed in the Introduction (allocable down to any desired level and additivity) are clearly satisfied for any choice of $L(X)$.

Risk load or surplus allocated will scale with a currency change if $L(X)$ is independent of a currency change: $R(\lambda X) = \lambda R(X)$ if $L(\lambda X) = L(X)$. This will be true if L is a function of ratios of currencies such as x/μ, x/σ (where σ is the standard deviation of X), or x/S (where S is the total surplus of the company). It is intuitively appealing to select the riskiness leverage to be a function of the ratio of the difference of the outcome from the mean to the surplus.

However, the general formulation of risk load or surplus allocation may not yield a coherent risk load or surplus allocation [1]. The major reason is the subadditivity requirement that the risk load for the portfolio not exceed the sum of the risk loads for the components.

2.3. Examples of Riskiness Leverage Models

Risk-Neutral: If the riskiness leverage $L(X)$ is a constant, then the risk load is zero. This would be appropriate for risk of ruin where potential losses are small relative to capital, or for risk of not meeting plan if you are indifferent to the consequences of not meeting plan.

Variance: If $L(x) = (\beta/S)(x-\mu)$, then it can be shown that required surplus or risk load is a multiple of the standard deviation of the aggregate loss distribution. This model suggests that there is risk associated with favorable outcomes as much as there is with unfavorable outcomes, and that the risk load or surplus need increases quadratically with deviations of the loss from its mean.

Tail Value at Risk: Let $L(x) = [\theta(x-x_q)]/(1-q)$, where the quantile x_q is the value of x where the cumulative distribution of X is q and $\theta(x)$ is the step function (1 when the argument is positive, 0 otherwise). Kreps shows that the assets needed to support the portfolio would be the average portfolio loss X when it exceeds x_q (the definition of TVAR).

He reminds us that this is a coherent risk measure [1] and states that only the part of the distribution at the high end is relevant for this measure. Kreps calculates the assets needed to support a line of business k as the average loss in line k in those years where the portfolio loss X exceeds x_q. This quantity is referred to by Kreps as a co-measure, and is defined by Venter [5] as co-Tail VaR (co-TVAR). (I further refer to the Venter paper below because it is very helpful in clarifying concepts in the Kreps paper.)

Venter also discusses Excess Tail Value at Risk (XTVAR) defined to be the average value of $X - \mu$ when $X > x_q$. The same properties that Kreps proved for TVAR and co-TVAR can be shown to hold for XTVAR and co-XTVAR.

Venter notes that if capital is set by XTVAR, it would cover average losses in excess of expected losses for those years where the portfolio losses X exceed the qth quantile x_q. It is assumed that expected losses have been fully reflected in pricing and in loss reserves. The capital allocated by co-XTVAR to a line would be the line's average losses above its mean losses in those same adverse years. Venter notes that there should be some probability level q for which XTVAR or a multiple of it makes sense as a capital standard. He points out that co-XTVAR may not allocate capital to a line that didn't contribute significantly to adverse outcomes. That is, the deviations from the mean for a line of business may average to approximately zero when total losses exceeded the qth quantile. Venter believes this makes sense if capital is being held for adverse outcomes only.

Value at Risk (VAR): Kreps defines a riskiness leverage model that produces the quantile x_q as the assets needed to support a portfolio of risks. This measure says that the shape of the loss distribution does not matter except to determine the one relevant value x_q. The VAR measure is known not to be coherent [1].

Semi-Variance: Kreps defines a riskiness leverage model that yields needed surplus or risk load as a multiple of the semi-deviation of the aggregate loss distribution. This is the standard deviation with all favorable deviations from the mean ignored (treated as zero). This measure implies that only outcomes worse (greater) than the mean should contribute to required risk load or surplus. This measure is consistent with the usual accounting view that risk is only relevant for adverse outcomes and further implies that the risk load or surplus required increases quadratically with adverse deviations of the loss from its mean.

Mean Downside Deviation: Kreps defines another riskiness leverage model that produces a multiple of mean downside deviation as the risk load. This is really XTVAR with $x_q = \mu$. Kreps notes that this measure could be used for risks such as not meeting a plan, even though ruin is not in question.

Proportional Excess: Finally, Kreps defines a riskiness leverage model that produces a capital allocation for a line that is pro rata on its average contribution to the excess over the mean.

The wide range of risk loads that can be produced by these riskiness leverage models suggests that this is a very flexible, rich class of models from which one should be able to select a measure that not only reflects one's risk preferences but also satisfies the very desirable additivity property.

2.4. Generic Management of Risk Load

Kreps points out that there are many sources of risk, such as the risk of not making plan, the risk of serious deviation from plan, the risk of not meeting investor analysts' expectations, the risk of a rating agency downgrade, the risk of regulatory notice, the risk of going into receivership, the risk of not getting a bonus, etc. Given these risks, he states that it seems plausible that company management's list of desirable properties of the riskiness leverage ratio should be as follows:

1. It should be a down-side measure (the accountant's point of view).
2. It should be more or less constant for excess that is small compared to capital (risk of not making plan, but also not a disaster).
3. It should become much larger for excess significantly impacting capital.

4. It should go to zero (or at least not increase) for excess significantly exceeding capital (once you are buried, it doesn't matter how deep).

Concerning (4), he notes that a regulator might want more attention paid to the extreme areas and might list desirable properties for the riskiness leverage ratio as follows:

1. It should be zero unless capital is seriously affected.
2. It should not decrease with loss significantly exceeding capital, because of the risk to the state guaranty fund.

Kreps points out that TVAR could be such a risk measure if the quantile is chosen to correspond to an appropriate fraction of surplus. However, he notes that at some level of probability, management will have to bet the whole company.

I also believe that rating agencies would not look favorably on the fourth item in management's hypothetical list of desirable properties of a riskiness leverage ratio. While I believe that management would have to take into account the regulatory and rating agency views, I believe they might well not prefer the Variance or Semi-Variance models, which increase quadratically to infinity. It would seem that TVAR and XTVAR reasonably satisfy the properties that management would likely want of such a model, while still satisfying the properties of a riskiness leverage model (additivity, allocable down to any desired level) and the requirements for a coherent measure of risk (including the subadditivity property for portfolio risk).

He states that management might typically formulate its risk appetite as satisfaction of two VAR requirements (limit chance of losing all capital to 0.1%, while limit chance of losing 20% of capital to 10%). In this case, one would take the larger of the two required capital amounts.

For his simulation examples, the author selects the criteria that "we want our surplus to be a prudent multiple of the average bad result in the worst 2% of cases." He notes that Gary Venter has suggested that the prudent multiple could be such that the renewal book could still be serviced after a bad year. Thus, Kreps selects TVAR with a prudent multiple of 150%.

2.5. Simulation Application

As he includes investments as a separate line in his model, TVAR is calculated for net income rather than portfolio losses. He has two insurance lines, one low risk and the other high risk. He shows that surplus can be released by writing less of the risky line, but this may not be possible if one is writing indivisible policies or if one is constrained by regulations. He demonstrates that an excess-of-loss reinsurance treaty can reduce required capital significantly and improve the portfolio's return on allocated surplus. Note that expected profit has decreased due to the cost of reinsurance, but capital needed to support the portfolio has decreased by a larger percentage.

In his simulation example, Kreps notes that the percentage allocation of surplus to line based on the co-TVAR measures is consistent for a wide range of quantiles x_q. That is, when the tail probability varies between 0.1% and 10%, the capital allocation percentage for a given line doesn't change very much. Kreps also tested his simulation model on VAR and power measures, such as mean downside deviation and semi-variance. He discovered that as the power increases, the measure is more sensitive to extreme values and the allocation to line of business moves toward the TVAR allocations.

3. INSURANCE CAPITAL AS A SHARED ASSET

In a paper submitted to the *ASTIN Bulletin* [2], Donald Mango treats insurance capital as a shared asset, with the insurance contracts having simultaneous rights to access potentially all of that shared capital. Shared assets can be scarce and essential public entities (e.g., reservoirs, fisheries, national forests) or desirable private entities (e.g., hotels, golf courses, beach houses). The access to and use of the assets is controlled and regulated by their owners; this control and regulation is essential to preserving the asset for future use. The aggregation risk is a common characteristic of shared asset usage, since shared assets typically have more members who could potentially use the asset than the asset can safely bear.

He differentiates between consumptive and non-consumptive use of an asset. A consumptive use involves the transfer of a portion or share of the asset from the communal asset to an individual, such as in the reservoir water usage and fishery examples. Non-consumptive use involves temporary, limited transfer of control that is intended to be non-depletive in that it is left intact for subsequent users. Examples of non-consumptive use include boating on a reservoir, playing on a golf course or renting a car or hotel room.

While shared assets are typically used in only one of the two manners, some shared assets can be used in either a consumptive or non-consumptive manner, depending on the situation. Mango gives the example of renting a hotel room. While the intended use is benign occupancy (non-consumptive), there is the risk that a guest may fall asleep with a lit cigarette and burn down a wing of the hotel (clearly consumptive).

Mango notes that rating agencies use different approaches in establishing ratings, but the key variable is the capital-adequacy ratio (CAR) that is the ratio of actual capital to required capital. Typically, the rating agency formulas generate required capital from three sources: premiums, reserves, and assets. Current year underwriting activity will generate required premium capital. As that premium ages, reserves will be established that will generate the required reserve capital. As the reserves are run off, the amount of required reserve capital will diminish and eventually reach zero when all claims are settled. As there are usually minimum CAR levels associated with each rating level, Mango points out that a given amount of actual capital corresponds to a maximum amount of rating agency required capital. For given reserve levels, this implies a limit to premium capital and thus to how much business can be written. Mango summarizes by stating that an insurer's actual capital creates underwriting capacity, while underwriting activity (either past or present) uses up underwriting capacity.

Mango notes that the generation of required capital, whether by premiums or reserves, temporarily reduces the amount of capacity available for other underwriting. Being temporary, it is similar to capacity occupancy, a non-consumptive use of the shared asset. Capacity consumption occurs when reserves must be increased beyond planned levels. Mango points out that this involves a transfer of funds from the capital account to the reserve account, and eventually out of the firm. Mango recaps by stating that the two distinct impacts of underwriting an insurance portfolio are as follows:

1. Certain occupation of underwriting capacity for a period of time.
2. Possible consumption of capital.

He notes that this "bi-polar" capital usage is structurally similar to a bank issuing a letter of credit (LOC). The dual impacts of a bank issuing a LOC are as follows:

1. Certain occupation of capacity to issue LOCs, for the term of the LOC.
2. Possible loan to the LOC holder.

Mango notes that banks receive income for the issuance of LOCs in two ways:

1. An access fee (i.e., option fee) for the right to draw upon the credit line.
2. Loan payback with interest.

Mango notes that every insurance contract receives a parental guarantee: should it be unable to pay for its own claims, the contract can draw upon the available funds of the company. He states that the cost of this guarantee has two pieces:

1. A Capacity-Occupation Cost, similar to the LOC access fee.
2. A Capital-Call Cost, similar to the payback costs of accessing an LOC, but adjusted for the facts that the call is not for a loan but for a permanet transfer and that the call destroys future underwriting capacity.

Mango states that there is an opportunity cost to capacity occupation, and thinks of it as a minimum risk-adjusted hurdle rate. He computes it as the product of an opportunity cost rate and the amount of required rating agency capital generated over the active life of the contract. Mango also develops a formula for computing capital-call costs, which are his true risk loads, and defines the expected capital-usage cost to be the sum of the capacity-occupation cost and the expected capital-call cost. He defines his key decision metric Economic Value Added (EVA) to be the NPV Return minus the expected capital usage cost:

$$\text{EVA} = \text{NPV Return} - \text{Capacity-Occupation Cost} \\ - \text{Capital-Call Cost.}$$

Mango's shared-asset view eliminates the need for allocating capital in evaluating whether the expected profit for a contract is sufficient to compensate for the risks assumed. He also shows how this approach can be used to evaluate portfolio mixes. His approach permits stakeholders great flexibility in expressing risk reward preferences. As Mango, Kreps, and David Ruhm jointly contributed to the development of the RMK (Rhum, Mango, and Kreps) algorithm, which is a conditional risk allocation method [4], it is no surprise that the Capital-Call Costs satisfy the key properties of a riskiness-leverage model (additivity, allocable down to any desired level).

4. INTEGRATION OF APPROACHES

This reviewer sees a limitation in the return on risk-adjusted capital (RORAC) approach as applied by Kreps that can easily be corrected by borrowing a concept from EVA. RORAC based upon riskiness leverage models does not reflect rating agency capital requirements, particularly the requirement to hold capital to support reserves until all claims are settled. This is especially important for long-tailed casualty lines. In the RORAC calculation as applied by Kreps, Expected Total Underwriting Return is computed by adding the mean NPV of interest on reserves from the simulation, interest on allocated capital, and expected underwriting return (profit and overhead). RORAC is computed as the ratio of Expected Total Underwriting Return to allocated risk capital and represents the expected return for both benign and potentially consumptive usage of capital.

As an alternative, I have developed a modified RORAC approach, which I call a risk-return on capital (RROC) model. A mean rating agency capital is computed by averaging rating agency required capital from the simulation (capital needed to support premium writings is added to the NPV of the capital needed to support reserves on each iteration of the simulation). The mean rental cost for rating agency capital is calculated by multiplying the mean rating agency capital by the selected rental cost percentage, which serves the same function as Mango's opportunity cost rate. Expected underwriting return is computed by adding the mean NPV of interest on reserves and interest on mean rating agency capital to expected underwriting return (profit and overhead). The expected underwriting return after rental cost of capital is computed by subtracting the mean rental cost of rating agency capital.

In my comparisons of RORAC and RROC, risk capital is a selected multiple of XTVAR. Capital is allocated to line of business based upon Co-XTVAR. RROC is computed as the ratio of expected underwriting return after rental cost of capital to allocated risk capital. It is assumed that expense items like overhead and taxes, as well as returns from any capital excess of the rating agency required capital or from riskier investments that would require additional rating agency capital, would be handled within corporate planning.

RROC represents the expected return for exposing capital to risk of loss, as the cost of benign rental of capital has already been reflected. It is analogous to the Capital-Call Cost in the EVA approach, here expressed as a return on capital rather than applied as a cost. In the discussion of Tail Value at Risk, it was observed that Venter has noted that co-XTVAR may not allocate capital to a line of business that didn't contribute significantly to adverse outcomes. In such a situation, the RORAC calculation based upon riskiness leverage models may show the line to be highly profitable, whereas RROC may show that the line is unprofitable because it did not cover the mean rental cost of rating agency capital.

In the EVA approach, risk preferences are reflected in the function selected and parameterized in computing the Capital-Call Cost. In the RROC approach, risk preferences are specified in the selection of the riskiness leverage model used to measure risk. This riskiness leverage model in practice would be parameterized to equal the total capital of the company, which would be maintained to at least cover rating agency capital required to maintain the desired rating. Both approaches utilize the RMK algorithm for allocating risk (measured as a Capital Call Cost in EVA and as risk capital in RROC) to line of business.

5. SIMULATION EXAMPLE

The RORAC and RROC approaches were tested and the results are summarized in the attached exhibits. Exhibit 1.1 summarizes the examples tested, including underlying assumptions, while Exhibit 1.2 summarizes the technical differences between the two approaches. In the base case, Example 1, the lines 1 and 2 are 50% correlated while being uncorrelated to line 3, and no reinsurance is purchased. Equal amounts of premium are written in the three lines, and pricing is assumed to be accurate with the plan loss ratio equaling the true Expected Loss Ratio (ELR) of 80% for each line. Aggregate losses are assumed to be modeled accurately by lognormal distributions with coefficients of variation of 80%, 20% and 40% for lines of business (LOB) 1–3, respectively. In Example 2, a stop-loss reinsurance treaty is purchased for line 1 covering a 30% excess of 90% loss ratio layer for a 10% rate. In Example 3, a 50% quota share is purchased for line 1 with commissions just covering variable costs.

Payout Patterns were generated based on an exponential settlement lag distribution with mean lag to settlement of one year, five years and ten years for LOB 1–3, respectively. Thus, the payout patterns for LOB 1–3 can be characterized as fast, average, and slow, respectively. Interest is credited on supporting surplus using risk-free rates for bonds of duration equal to the average payment lag in each line of business. In this example, interest rates of 3%, 4% and 5% for LOB 1–3, respectively, were assumed. These are the same rates that are used to calculate NPV reserves, interest on supporting surplus, and the NPV Reserves-Capital component of Required Rating Agency Capital. For simplicity, interest rates and payment patterns are assumed to be deterministic.

For both RORAC and RROC models, capital needed to support the portfolio risk is calculated as 150% of XTVAR. That is, the company wants 50% more capital than needed to support 1-in-50-year or worse deviations from plan. Capital needed to support the portfolio risk is allocated to the lines of business based upon Co-XTVAR.

Exhibit 2 summarizes the test results. Recall that in the base case no reinsurance is purchased. In Example 2, a stop-loss reinsurance treaty is purchased for line 1 that modestly improves both RORAC and RROC measures. (RORAC increases from 17.50% to 17.88%, while RROC increases from 9.95% to 10.05%.) However, in Example 3, a 50% quota share for line 1 improves the portfolio RORAC measure by 47% (from 17.50% to 25.74%), RROC improves by 54% (from 9.95% to 15.36%), and risk capital needed to support the portfolio decreases by over 40% (from $5.71 million to $3.39 million).

Line 1 and the reinsurance line 4 were combined in calculating returns by line of business. It is interesting that the expected returns for lines 1 and 2 did not change very much with the purchase of reinsurance, while the highly profitable returns for line 3 declined because it is now contributing to more of the 1-in-50 year adverse deviations. The portfolio returns with reinsurance improved because a smaller share of capital is now allocated to the marginally profitable line 1 and greater shares of capital are allocated to the highly profitable lines 2 and 3 (this can be seen by reviewing the change in the distributions of allocated capital displayed for the reinsurance examples at the bottom of Exhibit 2). It is also interesting that returns for line 2 improve a little because of its correlation with line 1 and because it has not been allocated any of the cost of reinsurance.

For the portfolio, Exhibit 2 also displays the Cost of Capital Released for the two reinsurance examples, which is the ratio of the cost of the reinsurance (decrease in expected profitability due to reinsurer's profit margin) to the decrease in capital needed to support the portfolio. The Cost of Capital Released was modestly lower than the company's net returns for the stop-loss example (12.6% versus 17.9% for RORAC, and 8.6% versus 10.1% for RROC), but dramatically lower for the quota-share example (5.6% versus 25.7% for RORAC, and 2.1% versus 15.4% for RROC). Thus, the company's cost to release over 40% of its capital for other purposes was a small fraction of its net returns for both metrics in the quota-share example.

However, the net capital allocated to the portfolio based on the 150% of XTVAR standard is less than the mean rating agency required capital computed for the RROC metric. It was determined that a 200% of XTVAR capital standard is consistent with the rating agency required capital, providing sufficient capital, beyond the amounts required to support premium written and loss reserves, to also cover rating agency capital required to cover investments.

The model output is displayed as Exhibit 3 for the quota-share example with a 200% of XTVAR capital standard. Net RORAC declines from 25.74% to 20.22%, while net RROC declines from 15.36% to 11.52%. However, note that RROC has been computed after applying a 10% Rental Cost Percentage to the Mean Rating Agency Capital from the simulation. Net capital required to support the 200% of XTVAR standard is now more than 40% lower than a larger gross requirement, while the Cost of Capital Released has declined for both metrics.

6. CONCLUSIONS

Rodney Kreps has written an important paper on the central topics of risk load and capital allocation. He has given us a class of mathematical models that satisfy two highly desirable properties for a risk load procedure, additivity and allocable down to any desired level. Tail Value at Risk and Excess Tail Value at Risk reasonably satisfy the properties that management would likely want of such a model, while still being coherent measures of risk.

Donald Mango's very innovative work in developing the concepts of insurance capital as a shared asset and Economic Value Added contribute significantly to understanding the way capital supports an insurance enterprise. A Risk Return on Capital model is suggested as a way to integrate desirable properties of the approaches presented by Kreps and Mango. This method measures returns on capital after reflecting the mean rental cost of rating agency capital. Thus, returns for exposing capital to risk are measured after reflecting the cost of carrying capital to support both premium written and loss reserves, which is especially important for long-tailed casualty lines.

While actuarial literature frequently refers to risk preferences of the capital provider, little mention is made of the risk-measurement preferences of the actuary. Good arguments can be made for both approaches to measuring exposure to risk of loss from insured events: The choice is either to allocate costs or to allocate capital. The Return on Risk Adjusted Capital approach based upon riskiness leverage models can be modified to reflect the opportunity cost of holding capital to support written premium and loss reserves, while still providing a metric that is understandable to financially oriented non-actuaries.

REFERENCES

[1] Kaye, Paul, "A Guide to Risk Measurement, Capital Allocation and Related Decision Support Issues," *Casualty Actuarial Society 2005 Discussion Paper Program*.

[2] Mango, Donald F., "Insurance Capital as a Shared Asset," *ASTIN Bulletin*, November 2005.

[3] McClenahan, Charles L., "Risk Theory and Profit Loads—Remarks," CAS 1990 Spring Forum, pp. 145–162.

[4] Ruhm, David, Donald Mango, and Rodney Kreps, "A General Additive Method for Portfolio Risk Analysis," accepted for publication in *ASTIN Bulletin.*

[5] Venter, Gary G., "Capital Allocation Survey with Commentary," *North American Actuarial Journal*, 8, 2, 2004, p. 96.

SUPPLEMENTARY MATERIAL

1. Seminar notes from the 2005 Seminar on Reinsurance on "Risk Load, Profitability Measures, and Enterprise Risk Management" may be downloaded from the CAS Web Site.

2. Abbreviations and Notation

CAR, Capital Adequacy Ratio

Co-TVAR, Co-Tail Value at Risk

Co-XTVAR, Co-Excess Tail Value at Risk

ELR, Expected Loss Ratio

EVA, Economic Value Added

LOB, Line of Business

LOC, Letter of Credit

RMK algorithm, a conditional risk allocation method

ROE, Return on Equity

RORAC, Return on Risk-Adjusted Capital

RROC, Risk Return on Capital After Rental Cost of Capital

TVAR, Tail Value at Risk

VAR, Value at Risk

XTVAR, Excess Tail Value at Risk

EXHIBIT 1.1

Summary of Model Assumptions

1. Payout Patterns were generated based upon an exponential settlement lag distribution with mean lags to settlement of one year, five years, and ten years for LOB 1-3, respectively. Thus, the payout patterns for LOB 1-3 can be characterized as Fast, Average, and Slow, respectively. Payments are assumed to be made in the middle of each year.
2. Interest is credited on supporting surplus using risk free rates for bonds of duration equal to the average payment lag in each line of business. In this example, interest rates of 3%, 4% and 5% for LOB 1-3, respectively, were assumed. These are the same rates that are used to calculate Net Present Value (NPV) reserves, interest on supporting surplus, and the NPV Reserves Capital component of Required Rating Agency Capital.
3. For simplicity, interest rates and payment patterns are assumed to be deterministic.
4. Profitability measures are computed before taxes, overhead, and returns on capital excess of the rating agency required capital.

Example	Key Assumptions	Purpose of Example
1	Write equal amounts of premium in three lines of business. Pricing is accurate, as the Plan Loss Ratios equal the true ELR's. The ELR's are equal to 80% for all three lines. No reinsurance is purchased. Aggregate losses are assumed to be modeled accurately by lognormal distributions with coefficients of variation of 80%, 20% and 40% for LOB 1-3, respectively. The correlation between LOB 1 and LOB 2 losses is 50%.	Base example with no reinsurance.
2	Same assumptions as in Example 1, except a 30% xs 90% Loss Ratio Stop Loss reinsurance program is purchased for LOB 1 at a 10% rate.	Test impact of stop loss reinsurance program for LOB 1.
3	Same assumptions as in Example 1, except a 50% Quota Share is purchased for LOB 1 with commission just covering variable costs.	Test impact of quota share reinsurance program for LOB 1.

EXHIBIT 1.2

MODEL SUMMARIES

1. For both models, capital needed to support the portfolio risk is calculated as 150% of Excess Tail Value at Risk (XTVAR). That is, the Company wants 50% more capital than needed to support 1 in 50 year or worse deviations from plan. Capital needed to support the portfolio risk is allocated to line of business based upon Co-Excess Tail Values at Risk (Co-XTVAR).
2. Returns on Risk Adjusted Capital Model (RORAC):
   Expected Total Underwriting Return is computed by adding the mean NPV of interest on reserves from the simulation, interest on allocated capital, and expected underwriting return (profit and overhead). RORAC is computed as the ratio of Expected Total Underwriting Return to allocated risk capital, and represents the expected return for both benign and potentially consumptive usage of capital.
3. Risk Returns on Capital Model (RROC):
   a. Risk Returns on Capital (RROC) may be thought of as a composite of the EVA and RORAC approaches to measuring profitability. The Mean Rental Cost of Rating Agency Capital (an EVA Concept) is subtracted as a cost before applying RORAC concepts to compute the return on allocated capital for exposing capital to potential loss.
   b. Required Rating Agency Capital is computed based upon rating agency premium and reserves capital charge factors assumed appropriate for the Company's desired rating. Somewhat smaller factors were selected for the reinsurance line (LOB 4) under the assumption that the Company would not receive full credit for ceded premium and reserves because a charge for potential uncollectibility would be applied.

      Capital needed to support reserves for a calendar year is the product of the reserves factors and the previous year-end reserves.

      Capital needed to support reserves must be calculated for all future calendar years until reserves run off.

      Required capital to support reserves is the NPV of these capital amounts.
   c. The Mean Rental Cost of Rating Agency Capital is calculated by multiplying the Mean Rating Agency Capital from the simulation by the selected Rental Cost Percentage, an opportunity cost of capacity.
   d. Expected Underwriting Return is computed by adding the mean NPV of interest on reserves and interest on mean rating agency capital to expected underwriting return (profit and overhead). The Expected Underwriting Return After Rental Cost of Capital is computed by subtracting the Mean Rental Cost of Rating Agency Capital. As for RORAC, risk capital is 150% of XTVAR. Capital is allocated to line of business based upon Co-XTVAR. RROC is computed as the ratio of the Expected Underwriting Return After Rental Cost of Capital to allocated risk capital. RROC represents the expected return for exposing capital to risk of loss, as the cost of benign rental of capital has already been reflected.

EXHIBIT 2

Model Comparisons for All Lines Combined

	Returns on Risk Adjusted Capital (RORAC)				Risk Returns on Capital After Rental Cost of Capital (RROC)			
Example	Gross *	Net	Difference	Cost of Capital Released	Gross *	Net	Difference	Cost of Capital Released
1	17.50%	17.50%			9.95%	9.95%		
2	17.50%	17.88%	0.4%	12.6%	9.94%	10.05%	0.1%	8.6%
3	17.55%	25.74%	8.2%	5.6%	9.97%	15.36%	5.4%	2.1%

Model Comparisons LOB 1 and LOB 4 (Reinsurance) Combined

	Returns on Risk Adjusted Capital (RORAC)			Risk Returns on Capital After Rental Cost of Capital (RROC)		
Example	Gross *	Net	Difference	Gross *	Net	Difference
1	5.84%	5.84%		1.80%	1.80%	
2	5.85%	5.21%	−0.6%	1.81%	1.17%	−0.6%
3	5.86%	6.44%	0.6%	1.82%	2.00%	0.2%

Model Comparisons LOB 2

	Returns on Risk Adjusted Capital (RORAC)			Risk Returns on Capital After Rental Cost of Capital (RROC)		
Example	Gross *	Net	Difference	Gross *	Net	Difference
1	61.41%	61.41%		37.52%	37.52%	
2	62.09%	62.54%	0.4%	37.96%	38.25%	0.3%
3	63.45%	63.83%	0.4%	38.85%	39.10%	0.3%

*Note that Gross simulated returns differ somewhat between Examples for lines allocated a small share of the total capital due to differences in simulation results.

EXHIBIT 2

Continued

Model Comparisons LOB 3

	Returns on Risk Adjusted Capital (RORAC)			Risk Returns on Capital After Rental Cost of Capital (RROC)		
Example	Gross *	Net	Difference	Gross *	Net	Difference
1	131.06%	131.06%		93.60%	93.60%	
2	122.28%	112.67%	−9.6%	87.09%	79.95%	−7.1%
3	114.39%	50.28%	−64.1%	81.22%	33.62%	−47.6%

Stop Loss Example 2: Comparison of Capital Requirements						Quota Share Example 3: Comparison of Capital Requirements					
Line	Gross *	Gross Weight	Net	Net Combining Lines 1 and 4	Net Weight	Line	Gross	Gross Weight	Net	Net Combining Lines 1 and 4	Net Weight
1	4,919,918	85.90%	4,892,514	4,468,187	84.24%	1	4,886,073	85.59%	4,069,551	2,034,775	59.99%
2	453,766	7.92%	450,304	450,304	8.49%	2	443,376	7.77%	440,592	440,592	12.99%
3	353,859	6.18%	385,446	385,446	7.27%	3	379,403	6.65%	916,596	916,596	27.02%
4			(424,327)			4			(2,034,776)		
	5,727,543	100.00%	5,303,937	5,303,937	100.00%		5,708,852	100.00%	3,391,963	3,391,963	100.00%
Average RORAC:	17.50%		17.88%			Average RORAC:	17.55%		25.74%		
Average RROC:	9.94%		10.05%			Average RROC:	9.97%		15.36%		

*Note that Gross simulated returns differ somewhat between Examples for lines allocated a small share of the total capital due to differences in simulation results.

EXHIBIT 3.1

QUOTA-SHARE REINSURANCE EXAMPLE COMPARING RETURNS ON RISK-ADJUSTED CAPITAL WITH RETURNS ON CAPITAL AFTER RENTAL COST OF CAPITAL

1) Loss Generator	Fast Pay LOB 1	Average Pay LOB 2	Slow Pay LOB 3	Reinsurance LOB 4	Net Total	Gross Total
1A) True Expected Loss: Copy and Paste-Special from LOB 4 of 3H).	1,000,000	1,000,000	1,000,000	(500,000)	2,500,000	3,000,000
1B) Coefficient of Variation of Assumed Lognormal Loss Distribution	80.0%	20.0%	40.0%			
1C) Standard Deviation	800,000	200,000	400,000			
1D) Profit and Overhead Margin (includes Brokerage on Reinsurance)	9.0%	8.0%	7.0%	9.0%	7.8%	8.0%
1E) Variable Expense Ratio	11.0%	12.0%	13.0%	11.0%	12.2%	12.0%
1F) Plan Premium	1,250,000	1,250,000	1,250,000	(625,000)	3,125,000	3,750,000
1G) Expected Loss Ratio = (1A)/(1F)	80.0%	80.0%	80.0%	80.0%	80.0%	80.0%
1H) Expected Underwriting Return (Profit & Overhead)	112,500	100,000	87,500	(56,250)	243,750	300,000
1I) Plan Loss Ratio	80.0%	80.0%	80.0%	80.0%	80.0%	80.0%
1J) Plan Expected Loss	1,000,000	1,000,000	1,000,000	(500,000)	2,500,000	3,000,000
1K) Pricing Error = ((1J) – (1A))/(1A)	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%

2) Capital Usage Calculation	LOB 1	LOB 2	LOB 3	LOB 4	Net Total	Gross Total
2A) Required Capital Charge on Premium	40.0%	40.0%	40.0%	35.0%	41.0%	40.0%
2B) Required Capital Charge on Reserves	25.0%	25.0%	25.0%	20.0%	25.2%	25.0%
2C) Rental Fee	10.0%					
2D) Required Premium Capital = (1F) × (2A)	500,000	500,000	500,000	(218,750)	1,281,250	1,500,000
2E) Simulated Required NPV Reserves Capital = (2B) × (NPV Future Reserves)	229,011	1,022,318	1,637,097	(91,604)	2,796,821	2,888,425
2F) Simulated Total Required Rating Agency Capital = (2D) + (2E)	729,011	1,522,318	2,137,097	(310,354)	4,078,071	4,388,425

EXHIBIT 3.1

Continued

3) Annual Simulation	LOB 1	LOB 2	LOB 3	LOB 4	Net Total	Gross Total
3A) Simulated Losses	1,000,000	1,000,000	1,000,000	(500,000)	2,500,000	3,000,000
3B) Deviations From Plan = (1J) – (3A)	—	—	—	—	—	—
3C) Deviation from Plan at 2nd Percentile: Copy and Paste-Special from (3K), re-run simulation to calculate XTVAR.	(2,310,809)	(472,747)	(1,048,430)	1,155,288	(1,679,627)	(2,671,871)
3D) Deviation from Plan when Exceed 1 in 50 Year Result	—	—	—	—	—	—
3E) Flag to Count Number of Simulations in Excess of 1 in 50 Year Result	—	—	—	—	—	—
3F) Contribution to Gross 1 in 50 Year Result	—	—	—			—
3G) Contribution to Net 1 in 50 Year Result	—	—	—	—	—	

Number of Simulations: 100,000

Loss Simulation Statistics	LOB 1	LOB 2	LOB 3	LOB 4	Net Total	Gross Total
3H) Expected Loss	1,000,009	1,000,003	999,998	(500,005)	2,500,006	3,000,010
3I) Standard Deviation	800,103	200,028	399,974	400,052	657,760	992,642
3J) Coefficient of Variation	80.0%	20.0%	40.0%	-80.0%	26.3%	33.1%
3K) Percentiles of Deviations from Plan (Negatives are Values at Risk)						
0.1 Percentile (1 in 1000)	(5,870,875)	(808,528)	(2,056,781)	2,928,441	(3,538,483)	(6,282,611)
1st Percentile (1 in 100)	(3,010,960)	(554,496)	(1,275,329)	1,505,136	(2,063,235)	(3,412,871)
2nd Percentile (1 in 50)	(2,310,938)	(472,759)	(1,048,428)	1,155,244	(1,692,567)	(2,681,451)
5th Percentile (1 in 20)	(1,483,359)	(358,194)	(749,783)	741,605	(1,210,214)	(1,819,725)
10th Percentile (1 in 10)	(923,290)	(263,894)	(521,248)	461,604	(837,389)	(1,204,122)
50th Percentile (1 in 2)	219,129	19,415	71,515	(109,568)	104,896	195,304
90th Percentile	682,957	239,216	433,300	(341,484)	717,829	996,299

EXHIBIT 3.2

4) Returns on Risk Adjusted Capital (RORAC)	LOB 1	Risk Capital Standard (Multiple K of XTVAR): 200% LOB 2	LOB 3	LOB 4	Net Total	Gross Total
4A) Plan Premium	1,250,000	1,250,000	1,250,000	(625,000)	3,125,000	3,750,000
4B) Expected Underwriting Return (Profit & Overhead)	112,500	100,000	87,500	(56,250)	243,750	300,000
4C) Average Deviation from Plan When Exceed 1 in 50 Year Result (XTVAR)	(3,423,226)	(588,435)	(1,381,853)	1,711,613	(2,260,925)	(3,805,255)
4D) Gross Risk Capital K% of XTVAR, Allocated to Line Based Upon Co-XTVARs	6,514,764	591,168	505,871			7,611,804
4E) Interest Rate Assumed	3.0%	4.0%	5.0%	3.0%		
4F) Interest Earned on Gross Allocated Capital = (4D) × (4E)	195,443	23,647	25,294			244,383
4G) Mean Net Present Value of Interest Earned on Reserves	27,485	163,603	327,516	(13,742)	504,861	518,603
4H) Gross Expected Total Underwriting Return = (4B) + (4F) + (4G)	335,427	287,250	440,310			1,062,987
4I) Gross Return on Risk Adjusted Capital = GRORAC = (4H)/(4D)	5.15%	48.59%	87.04%			13.96%
4J) Net Risk Capital K% of XTVAR, Allocated to Line Based Upon Co-XTVARs	5,426,069	587,457	1,222,128	(2,713,034)	4,522,619	
4K) Interest Earned on Net Allocated Capital = (4E) × (4J)	162,782	23,498	61,106	(81,391)	165,996	
4L) Net Expected Total Underwriting Return = (4B) + (4G) + (4K)	302,767	287,101	476,123	(151,383)	914,607	
4M) Net Return on Risk Adjusted Capital = NRORAC = (4L/(4J)	5.58%	48.87%	38.96%	5.58%	20.22%	
4N) Change in Return Due to Reinsurance = (4L – Net Total) – (4H – Gross Total)	(148,380)					
4O) Change in Allocated Capital = (4J – Net Total) – (4D – Gross Total)	(3,089,184)	4P) Cost of Additional XTVAR Capital = (4N)/(4O)				4.8%

EXHIBIT 3.2

Continued

5) Risk Returns on Capital (RROC) After Rental Cost of Capital	Risk Capital Standard (Multiple K of XTVAR): 200% LOB 1	LOB 2	LOB 3	LOB 4	Net Total	Gross Total
5A) Mean Rating Agency Capital = Mean of (2F)	729,013	1,522,321	2,137,093	(310,355)	4,078,072	4,388,427
5B) Mean Rental Cost of Rating Agency Capital = (5A) × (2C)	72,901	152,232	213,709	(31,036)	407,807	438,843
5C) Mean Interest Earned on Rating Agency Capital = (5A) × (4E)	21,870	60,893	106,855	(9,311)	180,307	189,618
5D) Expected Underwriting Return After Rental Cost of Capital = (4B) + (4G) + (5C) − (5B)	88,954	172,264	308,161	(48,267)	521,111	569,379
5E) Gross Risk Capital K% of XTVAR, Allocated to Line Based Upon Co-XTVARs	6,514,764	591,168	505,871			7,611,804
5F) Gross Risk Return on Capital = GRROC = (5D)/(5E)	1.37%	29.14%	60.92%			7.48%
5G) Net Risk Capital K% of XTVAR, Allocated to Line Based Upon Co-XTVARs	5,426,069	587,457	1,222,128	(2,713,034)	4,522,619	
5H) Net Risk Return on Capital = NRROC = (5D)/(5G)	1.64%	29.32%	25.22%	1.78%	11.52%	
5I) Change in Return Due to Reinsurance = (5D for LOB 4)	(48,267)					
5J) Change in Allocated Capital = (5G − Net Total) − (5E − Gross Total)	(3,089,184)	5K) Cost of Additional XTVAR Capital=(5I)/(5J)				1.6%

WHY LARGER RISKS HAVE SMALLER INSURANCE CHARGES

IRA ROBBIN

Abstract

The insurance-charge function is defined as the excess ratio (the ratio of expected loss excess of an attachment point to the expected total loss) and is expressed as a function of the entry ratio (the ratio of the attachment to the total loss expectation). Actuaries use insurance-charge algorithms to price retrospective rating maximums and excess of aggregate coverages. Many of these algorithms are based on models that can be viewed as particular applications of the Collective Risk Model (CRM) developed by Heckman and Meyers [4]. If we examine the insurance-charge functions for risks of different sizes produced by these models, we will find invariably that the insurance charge for a large risk is less than or equal to the charge for a small risk at every entry ratio. The specific purpose of this paper is to prove that this must be so. In other words, we will show the assumptions of the CRM force charge functions to decline by size of risk. We will take a fairly general approach to the problem, develop some theory, and prove several results along the way that apply beyond the CRM.

We will first prove that the charge for a sum of two non-negative random variables is less than or equal to the weighted average of their charges. We will extend that result to show that under certain conditions, the charge for a sum of identically distributed, but not necessarily independent, samples declines with the sample size. The extension is not entirely straightforward, as the desired result cannot be directly derived using simple in-

duction or straightforward analysis of the coefficient of variation (CV).

To explore size and charge in some generality, we will define the construct of a risk-size model. A risk-size model may initially be viewed as a collection of non-negative random variables whose sizes are defined by their expectation values. Given an appropriate measure on the risks of a particular size, we will be able to regard the cumulative distribution and the charge as well-defined functions of risk size. In a complete and continuous model, there are risks of every size and the cumulative distribution is a continuous function of risk size. We will first show that the charge declines with size if any risk can be decomposed into the independent sum of smaller risks in the model. Then we will employ the usual Bayesian construction to introduce parameter risk and extend the result to models that are not decomposable. This is an important extension, because actuaries have long known from study of Table M that a large risk is not the independent sum of smaller ones. In particular, our result implies that charges decrease with size in the standard contagion model of the Negative Binomial used in the CRM. Finally, we will introduce severity, prove our result assuming a fixed severity distribution, and then extend it to cover the type of parameter uncertainty in risk severity modeled in the CRM. Thus we will arrive at the conclusion that the assumptions of the CRM force charges to decline by size of risk.

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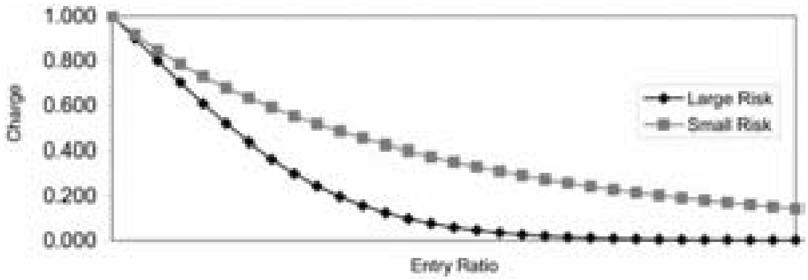
FIGURE 1

ACTUARIAL INTUITION ABOUT CHARGES AND RISK SIZE



is a business analyst who merits honorable mention as a closet actuary whose examples, thought experiments, and practical observations helped motivate this work.

1. INTRODUCTION

Our broad objective is to study the dependence of insurance-charge functions on risk size. We would like to arrive at some sufficiently general conditions that will force charges to obey the actuarial intuition that larger risks ought to have smaller insurance charges. More precisely, we would like to show that the assumptions of the Collective Risk Model (CRM) [4] lead to decreasing charges by size of risk. In keeping with standard actuarial terminology, the insurance charge refers to the excess ratio, not the absolute dollar amount, and the charge is viewed as a function of the entry ratio. When we say the charge is smaller, we mean that the excess ratio is less than or equal to its initial value at every entry ratio (see Figure 1).

Before proving this holds under certain conditions, we should note that no one has published any article disputing it. Neither does the literature contain any example with actual data for which it fails to hold. In practice, it is implicitly assumed to be true or turns out to be true under the assumptions made for a particular model. Under the procedure promulgated by the National Council on Compensation Insurance (NCCI) [6], a column of insurance charges is selected for a given risk based on its expected losses. The columns of charges have been constructed to effectively guarantee that a large risk will always be assigned smaller insurance charge values than a small one [3].

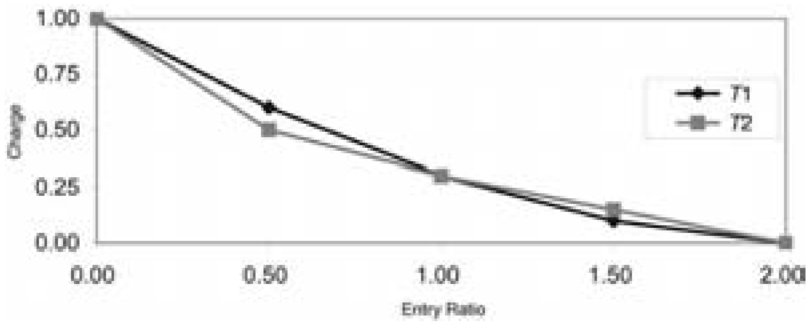
But why should this property hold? The basic intuition is that the excess ratio is related to the propensity of a distribution to take on relatively extreme values. When a large risk can be viewed as the independent sum of smaller risks, the Law of Large Numbers will apply and the likelihood of relatively extreme outcomes will decline.

Looking at the coefficient of variation (CV), the ratio of the standard deviation to the mean, supports these intuitions. When a large risk is the sum of independent, identically distributed smaller risks, the CV declines as risk size increases. Since the square of the CV is directly related to the integral of the insurance charge [8], these arguments suggest the insurance charge should also decline with risk size. However, this does not constitute a strict proof. The counterexample in Exhibit 1 shows that the CV does not uniquely determine the charge at every entry ratio. In that example, the risk with the smaller CV has a larger charge at some entry ratios (see Figure 2). In conclusion, because the arrow goes the wrong way, we cannot use a CV argument to arrive at a relatively trivial proof.

Instead we will use some numeric tricks and inequalities relating limited expected values to rigorously show that insurance charges do indeed decline as risk size increases when a large risk can be decomposed into a sum of independent smaller ones. This is a useful result, but alone it is insufficient for our larger purpose. Actuaries have long known that a large risk in practice has a distribution different than that resulting from the independent summation of smaller risks [5]. So we will go further and

FIGURE 2

RANDOM VARIABLE WITH SMALLER CV HAS LARGER CHARGE AT SOME ENTRY RATIOS



extend our result to models in which independent decomposability is only conditionally true. To do this, we will follow the usual Bayesian construction and view the true mean of a risk as a random variable having a prior distribution. A family of priors will then be used to define a family of unconditional distributions. This introduces parameter risk. Assuming an arbitrarily decomposable conditional model and priors having charges that decline with size, we will show the charges for the unconditional model decline with the unconditional risk size. However, the unconditional risk-size model will not be decomposable, and, due to the parameter uncertainty introduced via the priors, the CV for an unconditional risk will not tend towards zero as risk size becomes infinite.

We then turn to aggregate loss distributions that are generated by sampling claim counts and sampling claim severities in the manner described by the CRM. We will first show that an aggregate loss model inherits decomposability from its underlying claim count model, assuming severities are independently sampled from a fixed severity distribution. This leads to the conclusion that decomposable counts and independent fixed severities produce a model in which charges decrease by size of risk. We will then extend this result to cover the type of parameter risk in severity and frequency that is modeled in the CRM.

A reader versed in stochastic-process theory might observe the concept of decomposability is quite similar to the idea of "infinite divisibility" used in connection with Levy processes ([1], [7]). However, after some thought, we decided not to employ the terminology or results of stochastic theory. Though there is an analogy between increasing the risk size in a size-of-risk model and increasing the time in a stochastic process, we wish to maintain relevant distinctions between the two operations. In stochastic processes, the major concern is how a random variable changes over time [9] and the cumulative effect of possible jumps over a time interval. In short, it is the study of sample paths. For example, $N(t)$ might be the number of times a particular event has occurred as of time t and we might assume $N(t)$ is a right continuous function of t. The distribution of $N(t)$ would be a probabilistic summary of the number of events that have occurred as of time t, averaged over the space of sample paths (equipped with appropriate measure). In risk-size models, we are concerned with how risks of different size relate to one another; but there is no real analogue to the space of sample paths. This is not to say that many of the results to be presented here could not have been proven by applying stochastic-process theory after appropriately accounting for the distinction between time dependent paths and risk size. However, we will leave that work as a task for others who are more knowledgeable about stochastic-process theory. Also, we will not adopt the terminology of stochastic process theory since this might confuse the discussion of risk size. As well, in keeping with our actuarial focus, we will tend to make whatever reasonable assumptions we need, even though some of these could possibly be proved from previous assumptions or from more minimalist hypotheses. For instance, some of the assumptions that will be made about differentiability of our models with respect to risk size might be replaceable with more general and less restrictive statements or perhaps could be derived from the decomposability property. Since none of the assumptions put us outside the CRM, we leave the more abstract development along these lines as a topic of future research.

To maintain focus in the main exposition, many basic definitions and important foundational results have been relegated to the Appendices. The reader may be well advised to review these before proceeding much further.

In the end, most actuaries will find nothing surprising in what we will prove. But we will have rigorously established that actuarial intuition about insurance charges does indeed hold true for some fairly general classes of risk-size models, including the CRM.

2. THE INSURANCE CHARGE FOR A SUM OF RANDOM VARIABLES

We start by studying inequalities for insurance charges of sums. Our first result is that the insurance charge for the sum of two non-negative random variables is bounded by the weighted average of their insurance charges.

2.1. Charge for Sum Bounded by Weighted Average of Charges

Suppose T_1 and T_2 are non-negative random variables with means, μ_1 and μ_2, which are positive. Then, it follows that:

$$\varphi_{T_1+T_2}(r) \le \frac{\mu_1}{\mu_1+\mu_2}\varphi_{T_1}(r) + \frac{\mu_2}{\mu_1+\mu_2}\varphi_{T_2}(r). \qquad (2.1)$$

Proof Applying definition A.1 from the Appendix, we write

$$\begin{aligned}\varphi_{T_1+T_2}(r) &= \frac{\mathrm{E}[\max(0,(T_1+T_2)-r(\mu_1+\mu_2))]}{\mu_1+\mu_2}\\ &= \frac{\mathrm{E}[\max(0,(T_1-r\mu_1)+(T_2-r\mu_2))]}{\mu_1+\mu_2}.\end{aligned}$$

Next, we use the subadditivity property of the "max" operator to get $\max(0, A+B) \leq \max(0, A) + \max(0, B)$. We apply this to the previous equation and do some simple algebra to find:

$$\begin{aligned}\varphi_{T_1+T_2}(r) &\leq \frac{\mu_1}{\mu_1+\mu_2}\frac{\mathrm{E}[\max(0,(T_1-r\mu_1)]}{\mu_1}\\ &\quad+\frac{\mu_2}{\mu_1+\mu_2}\frac{\mathrm{E}[\max(0,(T_2-r\mu_2)]}{\mu_2}\\ &=\frac{\mu_1}{\mu_1+\mu_2}\varphi_{T_1}(r)+\frac{\mu_2}{\mu_1+\mu_2}\varphi_{T_2}(r).\end{aligned}$$

Note this result applies even if T_1 and T_2 are not independent, as it follows from subadditivity of the max operator and the linearity of the expectation operator. This result leads directly to the proof that the summation of two identically distributed risks leads to a smaller insurance charge.

2.2. Summation of Two Identically Distributed Variables Reduces the Charge

Suppose T_1 and T_2 are identically distributed and let T denote a random variable with their common distribution. Then

$$\varphi_{T_1+T_2}(r) \leq \varphi_T(r). \tag{2.2}$$

Proof From 2.1 it follows that

$$\varphi_{T_1+T_2}(r) \leq \tfrac{1}{2}\varphi_{T_1}(r) + \tfrac{1}{2}\varphi_{T_2}(r) = \varphi_T(r). \tag{2.3}$$

Note that if T_1 and T_2 were perfectly correlated, then A.6 would apply and summing would be equivalent to doubling and this would not change the charge. In Exhibit 2, we show a discrete example with two cases: one in which the two variables are independent and the other in which they are correlated. Not surprisingly, the sum of independent variables does have a lower charge than the charge for the sum when the variables are correlated. Yet, even when correlation exists, the charge for the sum is less than or equal to the charge for T.

To generalize Equation (2.2), suppose we take samples $(T_1, T_2, \ldots)$ of a random variable, T. We define $S(1,2,\ldots,n) = T_1 + T_2 + \cdots + T_n$. We make the assumption that such sums are *sample selection independent* by which we mean that the distribution of $S(1,2,\ldots,n)$ is the same as the distribution of $S(i_1, i_2, \ldots, i_n)$ where $(i_1, i_2, \ldots, i_n)$ is any ordered n-tuple of distinct positive integers. Note this does not require the T_i to be independent of one another, but it does force the distribution of $T_1 + T_2$, for example, to be the same as $T_1 + T_3$, $T_2 + T_3$, $T_{21} + T_{225}$, or the sum of any pair of distinct variables in our original sample. This would imply that there is a common correlation between any two of our samples. Under the assumption of sample selection independence, we will show the insurance charge for S_n declines as n increases. While it might seem that there ought to be some simple induction proof based on Equation (2.1), the only quick extension is that the charge for S_{nm} is less than or equal to the charge for S_n. To arrive at the general proof, we will use a numeric grouping trick and properties of the "min" operator.

2.3. Insurance Charge for Sample Selection Independent Sums Declines with Sample Size

$$\varphi_{S_{n+1}}(r) \leq \varphi_{S_n}(r). \qquad (2.4)$$

Proof For $k = 1, 2, \ldots, n+1$, define

$$S(\sim k/n+1) = T_1 + T_2 + \cdots + T_{k-1} + T_{k+1} + \cdots + T_{n+1}.$$

In other words, $S(\sim k/n+1)$ is the sum of the n out of the first $n+1$ trials obtained by excluding the kth trial. For example, $S(\sim 2/3) = T_1 + T_3$. With this notation, we may write the following formula

$$n \cdot S_{n+1} = \sum_{k=1}^{n+1} S(\sim k/n+1).$$

When $n = 2$, this formula says

$$\begin{aligned} 2 \cdot S_3 &= 2(T_1 + T_2 + T_3) \\ &= (T_2 + T_3) + (T_1 + T_3) + (T_1 + T_2) \\ &= S(\sim 1/3) + S(\sim 2/3) + S(\sim 3/3). \end{aligned}$$

The formula implies

$$\begin{aligned} n(n+1)\mathrm{E}\left[\frac{S_{n+1}}{n+1};r\right] &= \mathrm{E}[n \cdot S_{n+1}; n(n+1)r] \\ &= \mathrm{E}\left[\sum_{k=1}^{n+1} S(\sim k/n+1); n(n+1)r\right]. \end{aligned}$$

Next, we apply the inequality $\min(A+B, C+D) \geq \min(A,B) + \min(C,D)$ repeatedly to get

$$\mathrm{E}\left[\sum_{k=1}^{n+1} S(\sim k/n+1); n(n+1)r\right] \geq \sum_{k=1}^{n+1} \mathrm{E}[S(\sim k/n+1); nr].$$

Since the T_i are identically distributed and sample selection independent, it follows that

$$\mathrm{E}[S(\sim k/n+1); nr] = \mathrm{E}[S_n; nr]$$

and thus that

$$\mathrm{E}\left[\sum_{k=1}^{n+1} S(\sim k/n+1); n(n+1)r\right] \geq (n+1)\mathrm{E}[S_n; nr].$$

Connecting inequalities and factoring out n from the right hand expectation, we obtain

$$n(n+1)\mathrm{E}\left[\frac{S_{n+1}}{n+1};r\right] \geq (n+1)n\mathrm{E}\left[\frac{S_n}{n};r\right].$$

Assuming without loss of generality for the purpose at hand that $\mathrm{E}[T] = 1$, this inequality implies $(1 - \varphi_{S_{n+1}}(r)) \geq (1 - \varphi_{S_n}(r))$. The result then follows.

While the proof is rather abstract and the algebra of our numeric trick can be confusing, it is easy to see how it all works in any simple example.

EXAMPLE 1:

$$\varphi_{S_3}(r) \le \varphi_{S_2}(r).$$

Proof Consider

$$\begin{aligned} 6\mathrm{E}\left[\frac{S_3}{3};r\right] &= \mathrm{E}[2\cdot S_3;6r] = \mathrm{E}[(T_1+T_2+T_3)+(T_1+T_2+T_3);6r] \\ &= \mathrm{E}[(T_1+T_2)+(T_1+T_3)+(T_2+T_3);6r] \\ &\ge \mathrm{E}[T_1+T_2;2r]+\mathrm{E}[T_1+T_3;2r]+\mathrm{E}[T_2+T_3;2r] \\ &= 3\mathrm{E}[S_2;2r]. \end{aligned}$$

Thus we have

$$\mathrm{E}\left[\frac{S_3}{3};r\right] \ge \mathrm{E}\left[\frac{S_2}{2};r\right]$$

and the desired conclusion follows.

It is important to understand that we have *not* proved that any way of adding risks together reduces the charge. For example, if we had a portfolio of independent risks with small charges, and then added another risk with a large charge function, the addition of that risk could well result in a new larger portfolio with a larger charge. However, that would violate our assumption that the risks were identically distributed. Also, if we had two identically distributed risks, initially independent, and then added a third risk, but while doing so combined their operations so that all the risks were now strongly correlated, the charge might well increase. This is not a counterexample to our result, because our construction does not allow one to change correlations in the middle of the example.

3. RISK-SIZE MODELS

We need to introduce some precision in our discussion to at least guarantee that there is a well-defined notion of the insurance charge for a particular risk size. To start, we initially ignore parameter risk so that we can unambiguously identify the size of a risk with its expectation value. We then define a risk-size model, **M**, as a collection of non-negative random variables each having a finite non-negative mean. We index a random variable within such a model by its mean. We then use the risks of a particular size in the model to define the cumulative distribution, limited expected value, and insurance charge at that size. We let $\mathbf{M}_\mu$ be the set of risks in **M** of size μ and we suppose there is a measure Σ_μ on $\mathbf{M}_\mu$. We then define the cumulative distribution as a function of risk size via: $F_\mathbf{M}(t \mid \mu) = \mathrm{E}[F_T(t) \mid T \in \mathbf{M}_\mu]$ where the expectation is taken with respect to Σ_μ. Similarly, we define limited expected values and insurance charges as functions of risk size. We say **M** is well-defined if the measures give rise to a well-defined cumulative distribution for every $\mathbf{M}_\mu$ that is non-empty. We say a well-defined model is complete if there is a cumulative distribution for the model at every size. Unless otherwise noted, we henceforth assume all models are well-defined and complete. We define **M** to be size continuous at $t > 0$ if $F_\mathbf{M}(t \mid \mu)$ is a continuous function of μ and nth order size differentiable at $t > 0$ if $F_\mathbf{M}(t \mid \mu)$ has a nth order partial derivative with respect to μ, for $\mu > 0$. Note that **M** could be size continuous and differentiable even if all the random variables in **M** are discrete.

In the simplest case, each $\mathbf{M}_\mu$ consists of a single random variable that we denote as T_μ, and the measure, Σ_μ, assigns a mass point of 100% to this random variable. We say this is a unique size model and we use $F_{T_\mu}(t)$, the cumulative distribution function for the unique risk of size, μ, to define $F_\mathbf{M}(t \mid \mu)$, the cumulative distribution function at t for the model at size μ. Similarly we use the limited expected value and charge function of T_μ. to define the limited expected value and charge for the model **M** at size μ. To simplify notation when working with a unique size model, we may sometimes write $F_{T_\mu}(t)$ in place of $F_{\mathbf{M}}(t \mid \mu)$.

Next we define the notions of closure under independent summation, and decomposability in a unique size model.

3.1. Definitions of Independence, Closure and Decomposability in a Unique Size Model

Given a unique size model, **M**, and assuming $\mu_1 > 0$, $\mu_2 > 0$, we say

> M is *closed under independent summation* if $T_{\mu_1} \in \mathbf{M}$ and $T_{\mu_2} \in \mathbf{M}$ implies their independent sum, $T_{\mu_1} + T_{\mu_2}$, is also in **M**. Note these could well be independent samples of the same random variable. (3.1a)

> M is *arbitrarily decomposable* if for any positive μ, μ_1, and μ_2 with $\mu = \mu_1 + \mu_2$, there exist $T_\mu \in \mathbf{M}$, $T_{\mu_1} \in \mathbf{M}$, and $T_{\mu_2} \in \mathbf{M}$ such that the independent sum, $T_{\mu_1} + T_{\mu_2}$ has the same distribution as T_μ. (3.1b)

Unless there is need for greater specificity, we will usually say "closed" instead of "closed under independent summation." In a closed complete model, we can add identically distributed random samples of any given risk in the model and still stay in the model.

Arbitrary decomposability is a strong condition. It says that any way of splitting the mean of a risk into a sum leads to a decomposition of that risk into the independent sum of smaller risks in the model. To simplify terminology when no confusion should ensue, we may hereafter refer to "arbitrarily decomposable" models as simply "decomposable." We will show that charges decrease with size in a decomposable unique model.

First, we observe:

3.2. Decomposability Equivalence to Closure in Unique Size Model

$$\mathbf{M} \text{ is decomposable} \Leftrightarrow \mathbf{M} \text{ is closed.} \qquad (3.2)$$

Proof We prove one direction and leave the other as an exercise.

"$\Rightarrow$" Omitted.

"$\Leftarrow$" Since **M** is complete, there exist $T_\mu \in M$, $T_{\mu_1} \in M$, and $T_{\mu_2} \in M$. By assumption, **M** is closed under independent summation. So the independent sum, $T_{\mu_1} + T_{\mu_2}$, is in **M**. Taking expectations one has $E[T_{\mu_1} + T_{\mu_2}] = \mu_1 + \mu_2$. In a unique size model, we know $T_{\mu_1+\mu_2}$ has the unique distribution in **M** with $E[T_{\mu_1+\mu_2}] = \mu_1 + \mu_2$.

If we assume size differentiability in a decomposable model, we can obtain some results constraining the behavior of the cumulative distribution and the limited expected value function when these are viewed as functions of risk size.

3.3. Inequalities for Risk Size Partials in Decomposable Models

If **M** is a continuously differentiable decomposable risk-size model, then

$$\frac{\partial F_{T_\mu}}{\partial \mu} \leq 0 \qquad (3.3a)$$

$$1 \geq \frac{\partial E[T_\mu; t]}{\partial \mu} \geq 0 \qquad (3.3b)$$

$$\frac{\partial^2 E[T_\mu; t]}{\partial \mu^2} \leq 0. \qquad (3.3c)$$

Proof Applying the definition of decomposability, we derive

$$F_{T_\mu}(t) = \Pr(T_\mu \leq t) \geq \Pr(T_\mu + T_{\Delta\mu} \leq t) = F_{T_\mu + T_{\Delta\mu}}(t) = F_{T_{\mu+\Delta\mu}}(t).$$

F is thus a decreasing function of the risk size and Equation (3.3a) follows.

To prove the partial exceeds zero in Equation (3.3b), we use the decomposability property to derive

$$\mathrm{E}[T_{\mu+\Delta\mu};t] - \mathrm{E}[T_\mu;t] = \mathrm{E}[T_\mu + T_{\Delta\mu};t] - \mathrm{E}[T_\mu;t] \geq 0.$$

To prove the partial is less than unity, we similarly derive

$$\begin{aligned}\mathrm{E}[T_{\mu+\Delta\mu};t] - \mathrm{E}[T_\mu;t] &= \mathrm{E}[T_\mu + T_{\Delta\mu};t] - \mathrm{E}[T_\mu;t] \\ &\leq \mathrm{E}[T_\mu;t] + \mathrm{E}[T_{\Delta\mu};t] - \mathrm{E}[T_\mu;t] = \mathrm{E}[T_{\Delta\mu};t] \leq \Delta\mu.\end{aligned}$$

It follows that

$$\frac{\mathrm{E}[T_{\mu+\Delta\mu};t] - \mathrm{E}[T_\mu;t]}{\Delta\mu} \leq 1$$

and this leads immediately to our result.

As for Equation (3.3c), we claim that with our continuous differentiability assumption, it suffices to show that $\mathrm{E}[T_{\mu+\Delta\mu};t] - \mathrm{E}[T_\mu;t]$ is a decreasing function of μ for any $\Delta\mu > 0$. This is sufficient because, if it is true, we can then use an argument based on the Mean Value Theorem to show that the first partial derivative with respect to risk size is decreasing. A decreasing first partial derivative forces the second partial to be less than or equal to zero.

To show $\mathrm{E}[T_{\mu+\Delta\mu};t] - \mathrm{E}[T_\mu;t]$ is decreasing, we first use the additivity and independence assumptions to write the convolution formula:

$$F_{T_{\mu+\Delta\mu}}(t) = F_{T_\mu + T_{\Delta\mu}}(t) = \int_0^t dF_{T_\mu}(s) \cdot F_{T_{\Delta\mu}}(t-s).$$

This implies

$$G_{T_{\mu+\Delta\mu}}(t) - G_{T_\mu}(t) = \int_0^t dF_{T_\mu}(s) \cdot G_{T_{\Delta\mu}}(t-s).$$

Using

$$\mathrm{E}[T;t] = \int_0^t ds\, G_T(s)$$

we derive

$$\mathrm{E}[T_{\mu+\Delta\mu};t] - \mathrm{E}[T_\mu;t] = \int_0^t dx \int_0^x dF_{T_\mu}(y) \cdot G_{T_{\Delta\mu}}(x-y).$$

Switching orders of integration, we have

$$\begin{aligned}\mathrm{E}[T_{\mu+\Delta\mu};t] - \mathrm{E}[T_\mu;t] &= \int_0^t dF_{T_\mu}(y) \int_y^t dx\, G_{T_{\Delta\mu}}(x-y)\\ &= \int_0^t dF_{T_\mu}(y)\mathrm{E}[T_{\Delta\mu};t-y].\end{aligned}$$

Next, we integrate by parts and evaluate terms to obtain

$$\begin{aligned}\mathrm{E}[T_{\mu+\Delta\mu};t] - \mathrm{E}[T_\mu;t] &= F_{T_\mu}(0) \cdot \mathrm{E}[T_{\Delta\mu};t]\\ &\quad + \int_0^t dy\, F_{T_\mu}(y) \cdot G_{T_{\Delta\mu}}(t-y).\end{aligned}$$

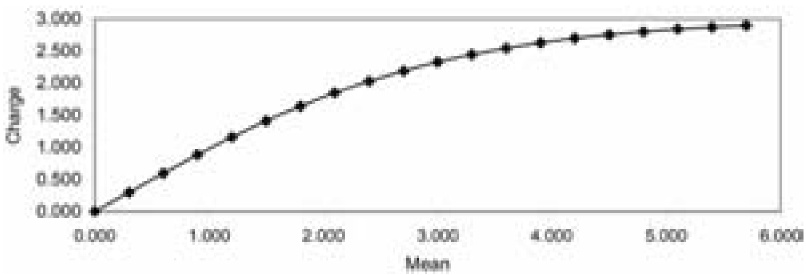
Applying Equation (3.3a), we can conclude that this decreases with μ, thus proving our result.

Note that these results apply to discrete as well as continuous distributions and that the proof does not require $F_{T_\mu}(0)$ to equal zero. Exhibit 3 shows how Poisson, Negative Binomial, and Gamma limited expected values vary as the mean changes. See Figure 3 for a graph of Poisson limited expected values at the limit 3.0 that vary as a function of risk size.

Now inequality 2.4 will be used to show that decomposable models must have charges that decrease with risk size.

FIGURE 3

POISSON LIMITED EXPECTED VALUES $E[T_\mu;3]$



3.4. Charges Decrease with Risk Size in Decomposable Models

Suppose **M** is a decomposable model. Then:

$$\mu_1 < \mu_2 \Rightarrow \varphi_{T_{\mu_1}} \geq \varphi_{T_{\mu_2}}. \tag{3.4}$$

Proof Since the charge function is a continuous function of risk size, it suffices to prove the result when μ_1 and μ_2 are rational. Assuming rationality, there exists θ such that $\mu_1 = m_1\theta$ and $\mu_2 = m_2\theta$ where m_1 and m_2 are integers with $m_1 < m_2$. Using completeness and decomposability, it follows that there exists T_θ in **M** and that

$$T_{\mu_1} = (T_\theta)_1 + (T_\theta)_2 + \cdots + (T_\theta)_{m_1}$$

$$T_{\mu_2} = (T_\theta)_1 + (T_\theta)_2 + \cdots + (T_\theta)_{m_1} + \cdots + (T_\theta)_{m_2}$$

where the sums are of independent samples of T_θ. Then, via inequality 2.4, the charge for T_{μ_1} exceeds the charge for T_{μ_2}.

We now apply Equation (3.4) to prove that insurance charges decrease with risk size for several families of distributions commonly used in insurance models.

3.5. Charges Decrease with Size in Poisson, Negative Binomial, and Gamma Models

The insurance charge decreases by size of risk in each of the following models:

Poisson:

$$\mathbf{M} = \{N \sim \text{Poisson}(\mu) \mid \mu > 0\}. \tag{3.5a}$$

Negative Binomial with common q:

$$\mathbf{M} = \{N \sim \text{Negative Binomial}(\alpha, q) \mid q \text{ is fixed and } \alpha > 0\}. \tag{3.5b}$$

Gamma with common scale parameter λ:

$$\mathbf{M} = \{T \sim \text{Gamma}(\alpha, \lambda) \mid \lambda \text{ is fixed and } \alpha > 0\}. \tag{3.5c}$$

Proof With the given restrictions, it can be easily shown that each of the families is a unique size model that has well-defined charges. It is also readily seen that each is decomposable. The results then follow from Equation (3.4).

Exhibit 4 shows columns of charges for risks of different sizes for Poisson random variables, Negative Binomials with common failure rate parameter, and Gammas with common scale parameter.

In a decomposable model, the charge decreases to the smallest possible charge as risk size goes to infinity.

3.6. Charge for an Infinitely Large Risk Equals Smallest Possible Charge in Decomposable Model

Suppose **M** is a differentiable decomposable model. Then

$$\varphi_{T_\mu} \to \varphi_0 \quad \text{as} \quad \mu \to \infty \quad \text{where} \quad \varphi_0(r) = \max(0, 1-r). \tag{3.6}$$

Proof It suffices to show $\varphi_{T_{n\mu}} \to \varphi_0$ as n approaches ∞ for arbitrary fixed μ. We use a CV argument. Consider $\mathrm{Var}(T_{n\mu}) = n\mathrm{Var}(T_\mu)$ for a decomposable model. Thus,

$$\mathrm{CV}^2(T_{n\mu}) = \mathrm{Var}(R_{n\mu}) = \frac{\mathrm{Var}(T_\mu)}{n\mu^2} \to 0 \qquad \text{as} \quad n \to \infty.$$

By Equation (A.11), this implies

$$\int_0^\infty dr\, \varphi_{R_{n\mu}}(r) \to \frac{1}{2}.$$

Using $\int_0^\infty dr\, \varphi_0(r) = \frac{1}{2}$, the result follows.

It is not difficult to construct a risk-size model in which charges decrease with size, even though it is not decomposable.

EXAMPLE 2: Charges Decrease by Size in a Non-Decomposable Model

Define T_μ to be the distribution having probability mass $p = \mu/(\mu+1)$, at $t = \mu + 1$ and mass $1 - p$, at $t = 0$. It follows that $\mathrm{E}[T_\mu] = \mu$ and

$$\varphi_{T_\mu}(r) = 1 - \frac{r}{r_\mu} \qquad \text{for} \quad 0 \le r < r_\mu \quad \text{where} \quad r_\mu = \frac{\mu+1}{\mu}.$$

Since r_μ declines as μ increases, the charge function declines with risk size. Yet, the independent sum of two members of this family always yields a distribution with three mass points and thus a sum that is not even in the family.

4. PARAMETER UNCERTAINTY IN RISK-SIZE MODELS

To introduce parameter uncertainty, we now suppose a model in which there may be many random variables sharing a common *expected* mean, but whose *actual* true means are uncertain. Because we do not know in advance the true mean of a risk in such a model, we use the a priori expected mean to define risk size. If we let θ represent the true mean of a risk, and μ the a priori mean, then M_μ consists of all the risks with prior mean equal to μ. While M_μ can therefore contain many risks each having a true mean, θ, which is not equal to μ, we do insist that Σ_μ, the measure, is defined so that μ is the average mean over all risks in M_μ. Following the usual Bayesian construction and the CRM, we will restrict our attention to models in which we may represent Σ_μ using a prior distribution, $H(\theta \mid \mu)$.

Before going further, it is instructive to see how such a construction can be used to model the combined effects of population parameter uncertainty and population heterogeneity.

EXAMPLE 3: Population Uncertainty and Heterogeneity
Consider a model in which the group of risks of size 100 actually consisted of risks whose true means were 90, 100, and 110. Assume we have no way of determining the true mean of any risk in advance of an experiment. Suppose there are two possible states of the world, "L" and "H," each of which has an equal random chance of occurring. If "L" applies, then we will sample from a subgroup of low risks, half of which have a true mean of 90 and half which have a true mean of 100. If "H" applies, the sampling subgroup will consist of an even split of high risks with true means of 100 and 110. We then take independent samples with replacement. In this analogy, the two possible states of the world correspond to population parameter uncertainty and the mix of risks in each state corresponds to heterogeneity of the population. To carry the analogy further, suppose the expected losses excess of 120 are 4, 10, and 18 for risks whose true means are 90, 100, and 110 respectively. If we are in state "L," our sampling will produce an average excess loss of 7, while the average excess loss will be 14 if we are in state "H." The average over all replications of this sampling process over all states will be 10.5. With a prior distribution of 25%, 50%, and 25% for risks with true means of 90, 100, and 110 respectively, we will duplicate this result. Note the correct average charge for the population

exceeds that charge for a risk whose mean equals the average mean of the population.

We will now study a specific class of risk-size models with parameter uncertainty that are constructed by applying priors to the risk sizes in a conditionally decomposable model. Under the usual Bayesian construction, the (prior probability) weighted average of the conditional distributions generates the unconditional distribution. If the family of priors itself forms a well-defined risk-size model, the family of resulting unconditional distributions will also be a well-defined risk-size model. If the risk model of priors is sufficiently well-behaved, we will be able to derive conclusions about the insurance charges of the unconditional distributions. Suppose the priors have charges that decrease, not necessarily strictly, with the unconditional risk size. We will then show the resulting unconditional distributions must also have charges that decrease with unconditional risk size.

To begin the mathematical development of this construction, let $T(\theta)$ be a non-negative random variable parametrized by θ such that $\mathrm{E}[T(\theta)] = \theta$. Suppose the family $\{T(\theta) \mid \theta > 0\}$ is differentiable with respect to θ and that it has insurance charges that decrease with risk size. Now we view the parameter θ as a random variable Θ and let $H(\theta) = H(\theta \mid \mu)$ denote its cumulative distribution. Assume $H(0) = 0$ and that Θ has density $h(\theta) = H'(\theta)$, which is continuously differentiable. Let $\mathrm{E}[\Theta]$ be finite. Since $\mathrm{E}[T(\theta)] = \theta$, it follows that the unconditional risk size, $\mathrm{E}[T(\Theta)]$, is equal to the mean, $\mathrm{E}[\Theta]$, of the parameter distribution. We often will use $\mu = \mathrm{E}[\Theta]$ to simplify notation. Finally, we let $\varphi_{T(\Theta)}$ denote the unconditional insurance charge. Given these definitions, the usual Bayesian construction leads to:

4.1. Unconditional Insurance Charge Formula

$$\varphi_{T(\Theta)}(r) = \frac{1}{\mu}\int d\theta\, h(\theta)\theta\varphi_{T(\theta)}\left(\frac{r\mu}{\theta}\right). \tag{4.1}$$

Proof Omitted.

The Bayesian construction implies there is some parameter uncertainty about the true mean of any risk. Under one interpretation, the final charge value is the (prior) probability-weighted average of the dollar charge over all values of the true mean, divided by the expected value of the true mean. Under another interpretation, we are dealing with a population of risks whose true overall mean we know, even though there is some parameter uncertainty regarding the mean of any particular risk in the population. The prior then represents the spread in the population and the formula arrives at the correct average charge for the population. It is also important to note that, as in our example, the charge for an average risk is not the same as (and is usually lower than) the weighted average charge for the population of risks. These two interpretations correspond to two types of parameter risk. The first expresses our uncertainty about the overall mean of a population, while the second expresses our uncertainty about the parameter dispersion or heterogeneity of a population. While a hierarchical, "double-integral" model could be used to separately delineate their effects, Example 3 shows that with respect to insurance charges, Equation (4.1) can be used to model both types of parameter risk together. For other applications such as in credibility theory, it may be important to maintain a distinction between these sources of parameter risk.

Now let $\mathbf{Q}$ be a family of Θ random variables and $\mathbf{M}(\mathbf{Q})$ the associated set of unconditional random variables. One quick, but important, result can be obtained assuming all the priors are scaled versions of a single distribution. Thus all the priors have the same insurance-charge function. Assume we have a conditional model in which charges decline with size. Then we show that applying the scaled priors to this model generates an unconditional model in which the insurance charge declines as a function of the unconditional risk size.

4.2. Unconditional Charge Declines with Risk Size in Scaled Priors Model

If

$$\frac{\partial \varphi_{T(\theta)}}{\partial \theta} \leq 0 \qquad \text{and} \qquad \Theta_2 = (1+c)\Theta_1 \quad \text{for} \quad c > 0,$$

then

$$\varphi_{T(\Theta_2)}(r) \leq \varphi_{T(\Theta_1)}(r). \tag{4.2}$$

Proof We start by using Equation (4.1) to write

$$\varphi_{T(\Theta_2)}(r) = \frac{1}{\mu_2}\int_0^\infty d\theta\, h_2(\theta)\cdot\theta\cdot\varphi_{T(\theta)}\left(\frac{r\mu_2}{\theta}\right)$$

where

$$\mu_2 = \mathrm{E}[\Theta_2] = \int_0^\infty d\theta\, h_2(\theta)\cdot\theta.$$

Then consider

$$\begin{aligned}
\mu_2 &= (1+c)\mu_1 \\
H_2(\theta) &= \Pr(\Theta_2 \leq \theta) = \Pr((1+c)\Theta_1 \leq \theta) \\
&= \Pr\left(\Theta_1 \leq \frac{\theta}{1+c}\right) = H_1\left(\frac{\theta}{1+c}\right) \\
h_2(\theta) &= h_1\left(\frac{\theta}{1+c}\right)\frac{1}{1+c}.
\end{aligned}$$

Substituting, rewrite the integral as

$$\varphi_{T(\Theta_2)}(r) = \frac{1}{\mu_1(1+c)}\int_0^\infty d\theta\, h_1\left(\frac{\theta}{1+c}\right)\frac{1}{1+c} \cdot\theta\cdot\varphi_{T(\theta)}\left(\frac{r\mu_1(1+c)}{\theta}\right).$$

Then change variables using $\eta = (\theta/1 + c)$ to get

$$\varphi_{T(\Theta_2)}(r) = \frac{1}{\mu_1}\int_0^{\infty} d\eta\, h_1(\eta)\cdot\eta\cdot\varphi_{(T(\eta(1+c))}\left(\frac{r\mu_1}{\eta}\right).$$

Because we assumed the conditional insurance charges were decreasing with risk size, it follows that

$$\varphi_{T(\eta(1+c))} \leq \varphi_{T(\eta)}.$$

This leads to

$$\varphi_{T(\Theta_2)}(r) \leq \frac{1}{\mu_1}\int_0^{\infty} d\eta\, h_1(\eta)\cdot\eta\cdot\varphi_{T(\eta)}\left(\frac{r\mu_1}{\eta}\right) = \varphi_{T(\Theta_1)}(r).$$

Why does this result make intuitive sense, despite the fact that scaling up the prior doesn't change its insurance charge? The answer is that scaling up does raise the mean of the prior so that larger conditional risks have more weight in the weighted average signified by the integral. Since the large conditional risks have smaller charges, the net effect of scaling up the prior is to reduce the charge of the unconditional distribution. Note that this result did not depend on decomposability of the conditional model, merely that the conditional risk-size model had charges that decrease with size.

For an example, consider the following:

EXAMPLE 4: Gamma Contagion on Conditional Poissons
Let $T(\theta)$ be conditionally Poisson with parameter, θ. Suppose $\theta = \mu\cdot\nu$ where ν is Gamma distributed with shape parameter α, and scale parameter $\lambda = \alpha$, so that $\mathrm{E}[\nu] = 1$. The variable, ν, introduces parameter uncertainty and $\mathrm{Var}(\nu) = (1/\alpha) = c$ is called the contagion [4] parameter for claim counts. It follows that Θ_μ, the random variable for θ, is Gamma distributed with shape parameter α, and scale parameter $\lambda = \alpha/\mu$, so that $\mathrm{E}[\Theta_\mu] = \mu$. The unconditional distribution $T(\Theta_\mu)$, is Negative Binomial with failure rate probability parameter $q = (1+\lambda)^{-1} = \mu/(\mu+\alpha)$. If $\mathbf{Q} = \{\Theta_\mu \mid \mu > 0\}$ and $\mathbf{M(Q)}$ is the resulting set of Negative Bino-

mials. It follows as a consequence of Equation (4.2) that charges decrease with risk size in **M**(**Q**).

Note **M**(**Q**) consists of Negative Binomials with a common shape but different failure rate parameters. This is different from our previous decomposable Negative Binomial risk-size model, from Equation (3.5b), in which all the variables had a common failure rate parameter. Our result is that charges decline with size in the standard Gamma-Poisson claim-count contagion model. Note this model is not closed under independent summation. Further, observe that the square of the coefficient of variation, $CV^2 = Var(\Theta_\mu)/\mu^2 = 1/(\alpha q) = c + (1/\mu)$, decreases toward the contagion, and not zero, as the risk size grows infinite. See Exhibit 5 for tables of charges for Negative Binomials as defined in Example 3.

Next we will consider two priors with the same mean. Assume these priors are acting on a continuously differentiable decomposable conditional risk-size model. We will show that, under certain conditions, if one prior has a smaller insurance charge, then its resulting unconditional random variable also has a smaller insurance charge. In order to prove this, we will first use integration by parts to express the unconditional insurance charge in terms of an integral of a product of the risk partials of the limited expected values of the conditional model and the prior.

4.3. Unconditional Charge Formula

$$\varphi_T(\Theta)(r) = 1 - \frac{1}{\mu}\int_0^\infty d\theta \frac{\partial \mathrm{E}[\Theta;\theta]}{\partial\theta} \cdot \frac{\partial \mathrm{E}[T(\theta);r\mu]}{\partial\theta}. \qquad (4.3)$$

Proof We write

$$\varphi_{T(\Theta)}(r) = 1 - \frac{1}{\mu}\int_0^\infty d\theta\, h(\theta)\cdot \mathrm{E}[T(\theta);r\mu].$$

Then we perform integration by parts as follows to derive

$$\int_0^\infty d\theta\, h(\theta)\mathrm{E}[T(\theta);r\mu] = -(1-H(\theta))\mathrm{E}[T(\theta);r\mu]\Big|_{\theta=0}^{\theta=\infty}$$

$$+\int_0^\infty d\theta(1-H(\theta))\frac{\partial \mathrm{E}[T(\theta);r\mu]}{\partial\theta}$$

$$=\int_0^\infty d\theta(1-H(\theta))\frac{\partial \mathrm{E}[(T(\theta);r\mu]}{\partial\theta}.$$

The result follows since

$$1-H(\theta)=\frac{\partial \mathrm{E}[\Theta;\theta]}{\partial\theta}.$$

With this we can now show a smaller charge for the prior leads to a smaller charge for the unconditional distribution at a common risk size.

4.4. Smaller Charges for the Prior Lead to Smaller Unconditional Charges—Size Fixed

If

$$\mu=\mathrm{E}[\Theta_1]=\mathrm{E}[\Theta_2] \qquad \text{and} \qquad \frac{\partial^2\mathrm{E}[T(\theta);r\mu]}{\partial\theta^2}\le 0$$

then

$$\varphi_{\Theta_2}\le\varphi_{\Theta_1} \quad \text{implies} \quad \varphi_{T(\Theta_2)}\le\varphi_{T(\Theta_1)}. \tag{4.4}$$

Proof We use Equation (4.5) to obtain

$$\varphi_{T(\Theta_1)}(r)-\varphi_{T(\Theta_2)}(r)$$

$$=-\frac{1}{\mu}\int_0^\infty d\theta\left(\frac{\partial\mathrm{E}[\Theta_1;\theta]}{\partial\theta}-\frac{\partial\mathrm{E}[\Theta^2;\theta]}{\partial\theta}\right)\frac{\partial\mathrm{E}[T(\theta);r\mu]}{\partial\theta}.$$

We integrate by parts to obtain

$$\mu(\varphi_{T(\Theta_1)}(r) - \varphi_{T(\Theta_2)}(r))$$

$$= -(\mathrm{E}[\Theta_1;\theta] - \mathrm{E}[\Theta_2;\theta]) \left.\frac{\partial \mathrm{E}[T(\theta); r\mu]}{\partial\theta}\right|_{\theta=0}^{\theta=\infty}$$

$$+ \int_0^\infty d\theta(\mathrm{E}[\Theta_1;\theta] - \mathrm{E}[\Theta_2;\theta])\frac{\partial^2 \mathrm{E}[T(\theta); r\mu]}{\partial\theta^2}.$$

Since $\mathrm{E}[\Theta_1] = \mathrm{E}[\Theta_2] = \mu$ and the first partial of the limited expected value is bounded by unity as per Equation (3.3b), it follows that the first term vanishes and we have

$$\mu \cdot (\varphi_{T(\Theta_1)}(r) - \varphi_{T(\Theta_1)}(r))$$

$$= \int_0^\infty d\theta(\mathrm{E}[\Theta_1;\theta] - \mathrm{E}[\Theta_2;\theta])\frac{\partial^2 \mathrm{E}[T(\theta); r\mu]}{\partial\theta^2}.$$

We use the expectation formula for the insurance charge, A1, to arrive at the formula

$$\mathrm{E}[\Theta;\theta] = \mu\left(1 - \varphi_{T(\Theta)}\left(\frac{\theta}{\mu}\right)\right).$$

We then use this to substitute into the previous integral to yield

$$\varphi_{T(\Theta_1)}(r) - \varphi_{T(\Theta_2)}(r)$$

$$= \int_0^\infty d\theta \left(\varphi_{\Theta_2}\left(\frac{\theta}{\mu}\right) - \varphi_{\Theta_1}\left(\frac{\theta}{\mu}\right)\right)\frac{\partial^2 \mathrm{E}[T(\theta); r\mu]}{\partial\theta^2}.$$

The result then follows immediately from the assumptions of the proposition.

To gain a better understanding of the formulas, consider the following example.

EXAMPLE 5: Poisson Conditionals and Exponential Priors
Let $T(\theta)$ be Poisson. We leave it as an exercise for the reader to

show

$$\frac{\partial F(n \mid \theta)}{\partial \theta} = -f(n \mid \theta)$$

$$\frac{\partial \mathrm{E}[T(\theta);n]}{\partial \theta} = F(n-1 \mid \theta)$$

$$\frac{\partial^2 \mathrm{E}[T(\theta);n]}{\partial \theta^2} = -f(n-1 \mid \theta).$$

Suppose the prior on θ is an exponential with mean μ, so that $1 - H(\theta) = \exp(-\theta/\mu)$. Applying Equation (4.3), we derive

$$\varphi_{T(\Theta)}\left(\frac{n}{\mu}\right) = \int_0^\infty d\theta\, e^{-\theta/\mu} \cdot e^{-\theta} \frac{\theta^{n-1}}{(n-1)!} = \left(\frac{\mu}{\mu+1}\right)^n.$$

To see this is correct, we apply the prior to the conditional density and integrate to obtain the unconditional density

$$f_{T(\Theta)}(n) = \frac{1}{\mu+1}\left(\frac{\mu}{\mu+1}\right)^n.$$

We recognize this as a Geometric density. It is an exercise in summation formulas to then verify the insurance charge associated with this density is in fact the same as the one just derived using Equation (4.3).

We may now put the results from Equations (4.2) and (4.4) together to show that decreasing charges by risk size for the priors acting on a differentiable decomposable conditional family lead to unconditional charges that decrease with risk size.

4.5. Charges Decrease by Size for Model Based on Decomposable Conditionals with Priors that Decrease by Size

Suppose $\mathbf{M} = \{T(\theta) \mid \theta > 0\}$ is a differentiable decomposable risk-size model and let $\mathbf{Q}$ be a risk-size model with unique random variables, $\{\Theta_\mu\}$ such that $\mathrm{E}[\Theta_\mu] = \mu$.

If $\varphi_{\Theta_2} \leq \varphi_{\Theta_1}$ when $\mu_1 < \mu_2$, then

$$\varphi_{T(\Theta_2)} \leq \varphi_{T(\Theta_1)}. \tag{4.6}$$

Proof Let $\mu_2 = (1+c)\mu_1$ where $c > 0$ since $\mu_2 > \mu_1$. Via Equation (4.2) we have

$$\varphi_{T((1+c)\Theta_1)} \leq \varphi_{T(\Theta_1)}.$$

Since $\varphi_{\Theta_2} \leq \varphi_{\Theta_1} = \varphi_{(1+c)\Theta_1}$, we can use Equation (4.4) to show

$$\varphi_{T(\Theta_2)} \leq \varphi_{T((1+c)\Theta_1)}.$$

Note it is valid to apply Equation (4.4) since the second partial is negative for differentiable decomposable models via Equation (3.3c). Connecting the two inequalities leads to the desired result.

Next we extend these results to aggregate loss distributions.

5. INSURANCE CHARGES FOR LOSSES

We define an aggregate loss random variable as the compound distribution generated by selecting a claim count from a claim count distribution and then summing up that number of severities, where each claim severity is drawn from a severity distribution. We associate a risk with a particular count distribution and a particular severity distribution. When we talk about the aggregate losses for a risk, we mean the aggregate losses generated by samples appropriately drawn from its count and severity distributions according to our protocols. Suppose we have a collection of risks whose claim count distributions form a risk-size model. If we make appropriate assumptions about the severities of our risks, the aggregate-loss random variables for these risks will also constitute a risk-size model. We will show under certain conditions that, if the claim counts have charge functions that decrease by size of risk, then so do the charge functions for the aggregate losses.

Beginning the mathematical exposition, let N be the random variable representing the number of claims for a particular risk and let $p_N(n) = \Pr(N = n)$. Use a non-negative random variable X, with finite mean μ_X, to represent claim severity. Assume X

has finite variance and write $\tau^2 = \text{Var}(X)$. Let X_i be the ith in a sequence of trials of X. Define the aggregate loss random variable for the risk via $T(N,X) = X_1 + X_2 + \cdots + X_N$. Note in this process of generating results, we are generating losses for a particular risk. When no confusion should result, we will write T instead of $T(N,X)$. Using Equation (A.6), we know the random variable, (T/μ_X), has the same insurance charge as T. This means we can assume $\mu_X = 1$ for the purpose at hand without loss of generality. When there are exactly n claims, define $T(n,X) = X_1 + X_2 + \cdots + X_n$. Assuming $\mu_X = 1$, it follows that $\text{E}[T(n,X)] = n$. To simplify notation, we may write $\varphi_{n/X}$ or even φ_n in place of $\varphi_{T(n,X)}$.

The insurance charge for $T(N,X)$ can be decomposed as a weighted sum of the insurance charges for $T(n,X)$, each evaluated at an appropriately scaled entry ratio.

5.1. Count Decomposition of the Insurance Charge for Aggregate Loss

$$\varphi_{T(N,X)}(r) = \frac{1}{\mu_N}\sum_{n=1} p_N(n)\cdot n\cdot \varphi_{T(n,X)}\left(\frac{r\mu_N}{n}\right). \qquad (5.1)$$

Proof Left as an exercise for the reader.

While one could get some general results by working with the claim count decomposition and using discrete distribution analogues of integration by parts, the proofs are a bit messy. Instead, we will employ the simpler strategy of deriving properties of compound distributions from their claim count models. Then we can use the results of Chapters 3 and 4 to arrive at relatively painless conclusions about charges for the aggregate loss models. We start by proving that if the claim count model is decomposable, then so is the resulting compound distribution model. To do this, we will first make the following severity assumptions:

5.2. *Fixed Independent Severity*

A compound risk-size model has independent fixed severity if:

i) all risks share a common severity distribution, X.

ii) $\{X_1, X_2, \ldots, X_N\}$ is an independent set.

iii) X_i is independent of N. (5.2)

iv) X_i is independent of θ, where θ is the true mean of N for a risk.

Given these severity assumptions and a decomposable claim count model, we can show the aggregate loss model is also decomposable.

5.3. *Aggregate Loss Model Inherits Decomposability from Claim Count Model Assuming Fixed Independent Severity*

If $\mathbf{M}_N$ is a decomposable claim count model and the compound model, $\mathbf{M}_{T(N,X)} = \{T(N,X) \mid N \in \mathbf{M}_N\}$ has fixed independent severity, then $\mathbf{M}_{T(N,X)} = \{T(N,X) \mid N \in \mathbf{M}_N\}$ is also decomposable. (5.3)

Proof Recall we have assumed without loss of generality that $\mathrm{E}[X] = 1$. Thus $\mathrm{E}[T(N,X)] = \mathrm{E}[N]\mathrm{E}[X] = \mathrm{E}[N]$. Given $\theta > 0$, completeness of $\mathbf{M}_N$ implies there exists a unique $N(\theta) \in \mathbf{M}_N$ such that $\mathrm{E}[N(\theta)] = \theta$. It follows that $\mathrm{E}[T(N(\theta),X)] = \theta$. Thus, $\mathbf{M}_{T(N,X)}$ is complete. Now let $T(N(\theta_1),X)$ and $T(N(\theta_2),X)$ be in $\mathbf{M}_{T(N,X)}$. Then using our severity assumptions we can show $T(N(\theta_1),X) + T(N(\theta_2),X) = T(N(\theta_1) + N(\theta_2),X)$. Since $\mathbf{M}_N$ is closed under independent summation, it follows that $N(\theta_1) + N(\theta_2) = N(\theta_1 + \theta_2)$ and $N(\theta_1 + \theta_2) \in \mathbf{M}_N$. Therefore, $T(N(\theta_1) + N(\theta_2),X) \in \mathbf{M}_{T(N,X)}$, proving that $\mathbf{M}_{T(N,X)}$. is closed under independent summation. Now we apply Equation (3.2) to finish the proof.

Using this, it follows as a direct application of Equation (3.4) that the aggregate loss model has charges that decrease with risk size.

5.4. Decomposable Claim Counts Imply Aggregate Loss Model Has Charges that Decrease with Risk Size Assuming Fixed Independent Severity

> If $\mathbf{M}_N$ is a differentiable decomposable risk-size model for claim counts and the compound model, $\mathbf{M}_T(\mathbf{N},X) = \{T(N,X) \mid N \in \mathbf{M}_N\}$ has fixed independent severity, then $\mathbf{M}_T(\mathbf{N},X)$ has charges that decrease with risk size. (5.4)

Proof Via Equation (5.2), $\mathbf{M}_{T(N,X)}$ is decomposable and the introduction of fixed independent severity does not affect differentiability with respect to risk size. The result then follows from Equation (3.4).

Note in this result that all risks share a common severity distribution and there is no parameter uncertainty regarding this severity distribution. While adding severity to the model does lead to larger charges for all risks, the result says that under these assumptions charges for aggregate loss still decline by size of risk.

We now apply Statement 5.4 to prove that insurance charges decrease with risk size for several classes of distributions commonly used in insurance models.

5.5. Charges for Aggregate Loss Decrease with Risk Size When Counts Are Poisson or Negative Binomial (Fixed Failure Rate)

Assuming claim sizes are independently and identically distributed and independent of the claim count, the insurance charge decreases as the size of a risk is increased in each of the following models, $\mathbf{M}_{T(N,X)}$, where $\mathbf{M}_N$ is:

Poisson:

$$\mathbf{M} = \{N \in \text{Poisson}(\mu) \mid \mu \geq 0\} \tag{5.5a}$$

Negative Binomial with common q:

$$\mathbf{M} = \{N \in \text{Negative Binomial}(\alpha, q) \mid q \text{ is fixed and } \alpha \geq 0\}. \tag{5.5b}$$

Proof Apply Statement 5.4.

We now introduce parameter uncertainty regarding the mean severity for a risk. We follow the CRM [4] as shown in Appendix B and assume severity may vary from risk to risk only due to a scale factor. Let β be a positive continuous random variable with density, $w(\beta)$, such that $E[1/\beta] = 1$ and $\text{Var}(1/\beta) = b$. The parameter b is called the mixing parameter.

We let X be a fixed severity distribution that does not change by risk. The severity distribution for a particular risk Y is obtained by first randomly selecting a β and then using the formula $Y = X/\beta$. Under this construction, each risk has a particular β that does not change from claim to claim. We will prove that when the selection of β is independent of θ, it follows that the compound model on decomposable counts has charges that decrease with size. To ensure clarity, we first define the notion of independent severity with scale parameter uncertainty by risk. This provides with us a terminology for describing the severity model just presented.

5.6. Independent Severity with Scale Parameter Uncertainty

A compound risk-size model has independent severity with scale parameter uncertainty if

i) each particular risk has a particular β and associated severity distribution, $Y = X/\beta$, where X is fixed for all risks and b is a positive continuous random variable with $\mathrm{E}[1/\beta] = 1$ and $\mathrm{Var}(1/\beta) = b$.

ii) $\{X_1, X_2, \ldots, X_N\}$ is an independent set.

iii) X_i is independent of N.

iv) The selection of β for a risk is independent of θ, where θ is the true mean of N for a risk.

v) The selection of β for a risk is independent of μ, where μ is the a priori mean of N for a risk. (5.6)

We now show

5.7. Decomposable Claim Counts Imply Aggregate Loss Model Has Charges that Decrease with Risk Size Assuming Independent Severity with Scale Parameter Uncertainty

If $\mathbf{M}_N$ is a differentiable decomposable risk-size model for claim counts and the compound model, $\mathbf{M}_{T(N,Y)} = \{T(N,Y) \mid N \in \mathbf{M}_N, Y = X/\beta\}$, has independent severity with scale parameter uncertainty, then $\mathbf{M}_{T(N,Y)}$ has charges that decrease with risk size. (5.7)

Proof By the usual Bayesian conditioning and using the independent severity assumptions, we can show the charge for the model at risk size θ is given via:

$$\varphi_{T(N(\theta),Y)}(r) = \int_0^\infty d\beta\, w(\beta)(1/\beta)\varphi_{T(N(\theta),X)}(r\beta).$$

Using Statement 5.5, we know the integrands decrease by size of risk, and the result follows using the independence of β and θ.

Even though the aggregate model in Statement 5.7 is based on decomposable counts, it will not be decomposable. Indeed, the aggregate model is not even a unique model as the introduction of the scale parameter leads to an infinite number of risks with the same a priori expected aggregate loss and thus the same size. We also need to be careful in interpreting the order in which the scaling parameter is averaged over the population. Suppose each risk in a decomposable count model has an exponential severity distribution and the prior on severity is a Gamma. It follows that the unconditional severity over all risks is Pareto distributed. If we then construct a model where each risk has this Pareto as its severity, we will have a decomposable model that is different and has different charges than the one in Statement 5.7.

Next we reprise the work done in Chapter 4 and extend our result to aggregate loss models in which the counts are subject to parameter uncertainty. We start with decomposable counts and then introduce a family of prior distributions on the mean claim counts, such that the priors constitute a risk-size model. If the priors have charges that decrease, not necessarily strictly, with risk size and if the compound model has independent severity with scaling parameter uncertainty, then the aggregate model has charges that decline with risk size. This is the key result of the paper.

5.8. Unconditional Aggregate Loss Model Charges Decrease with Risk Size Assuming Counts Based on Decomposable Conditionals with Priors that Decrease by Size and Independent Severity with Scaling Parameter Uncertainty

> Assume $\mathbf{M}_N$ is a differentiable decomposable claim-count model parameterized by θ such that $E[N(\theta)] = \theta$. Let $\mathbf{Q} = \{\Theta_\mu\}$ be a complete set of priors on θ having charges that decrease with risk size. Let Y denote risk severity and suppose the aggregate loss model, $\mathbf{M}_T$, has independent severity with scale parameter uncertainty. Then $\mathbf{M}_T$ has charges that decrease with risk size. (5.8)

Proof The charge in the model for risks of size μ is given by

$$\varphi_{M_T(\mu)}(r) = \int_0^\infty d\beta\, w(\beta)(1/\beta)\frac{1}{\mu}\int_0^\infty d\theta\, h(\theta \mid \mu) \cdot \theta \cdot \varphi_{T(N(\theta),X)}(r\beta\mu/\theta).$$

Using the same integration by parts argument made in proving Statement 4.5, we can show the integral

$$\frac{1}{\mu}\int_0^\infty d\theta\, h(\theta \mid \mu) \cdot \theta \cdot \varphi_{T(N(\theta),X)}(r\beta\mu/\theta)$$

declines as a function of μ for any fixed β. The result then follows directly using the independence of β with μ.

Note the assumption of independence between β and θ allows us to integrate over the priors for severity and the priors for claim counts in any order. Thus we need our assumption that β is independent of θ as well as μ.

6. CONCLUSION

We started by proving some general inequalities for the insurance charge of a sum of identically distributed random variables. We used some numeric grouping techniques to show the key basic result that the charge for such a sum declines with the sample size. We then introduced the construct of a risk-size model. We showed that charges decline with size in a decomposable model. We introduced parameter risk with a family of Bayesian priors. We demonstrated that if the priors had decreasing charges by size and they acted on a decomposable conditional model, the resulting unconditional model has charges that decline by size of risk. We showed this to be true, even though the resulting unconditional models were not decomposable. Then we extended our results to aggregate loss models by adding severity and making some independence assumptions. We finally extended our result to aggregate loss models in which severity is subject to scale parameter risk. Though our final model is based on conditionally decomposable claim counts, the parameter risk on both counts and severity produce a model that is not decomposable.

The CRM is based on conditional Poisson counts and has severities that satisfy our independence assumptions. The parameter scale uncertainty in the CRM is the same as in our model. Thus the CRM satisfies the assumptions of our key result in Statement 5.8 and therefore it will generate charges that decline by size of risk. This is what we set out to prove.

The latest NCCI Table M was produced with the Gamma-Poisson claim-count model, where, to fit the data, the contagion declines with risk size [3]. This does not imply that the latest Table M is based on a decomposable model, but rather that the straight CRM model with constant contagion by size may lead to an overstatement of the charges for large risks.

In our unconditional model, we found charges were not forced to asymptotically approach the lowest possible charge function, $\varphi_0(r) = \max(0, 1 - r)$, as risk size tends to infinity. While we have shown $\varphi_0(r)$ is indeed the charge function for "an infinitely large risk" in a decomposable model, in our unconditional count model the charge for a very large risk approaches the charge for the prior of that risk.

Though severity increases insurance charges, the introduction of severity did not cause our size versus charge relation to fail. Intuitively this is because risk size is driven by the expected claim count. In short, severity does increase the insurance charge, but it does not change the relation between charge and size in the models we have developed here. While in the actual derivation of the latest Table M, severity did vary a bit by size in order to reconcile against the expected losses and fitted frequencies [3], the variation was not sufficient to cause inversions of the declining charge by size of risk relation. It is a topic of future research to understand how far severity assumptions may be relaxed before such inversions would occur.

So we conclude, having proved there is a fairly large class of risk-size models in which charges decline with size of risk. This class includes widely used actuarial models such as the CRM and Table M. Also, we have developed some theoretical constructs that should also provide a solid foundation for future research.

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APPENDIX A

BASIC INSURANCE CHARGE THEORY

Let T be a non-negative random variable having finite positive mean, μ, cumulative distribution function F, and tail probability function $G = 1 - F$. Define the normalized random variable R associated with T via $R = T/\mu$.

Perhaps the most compact mathematical definitions of the charge and saving can be given by taking the expected values of "min" and "max" operators.

A.1. Charge and Saving Functions Defined using Min and Max Expectations

Charge:

$$\varphi(r) = \frac{\mathrm{E}[\max(0, T - r\mu)]}{\mu} = \frac{\mathrm{E}[T - \min(T, r\mu)]}{\mu} = 1 - \frac{\mathrm{E}[T; r\mu]}{\mu} \tag{A.1a}$$

Saving:

$$\psi(r) = \frac{\mathrm{E}[\max(0, r\mu - T)]}{\mu} = r - \frac{\mathrm{E}[T; r\mu]}{\mu}. \tag{A.1b}$$

The definitions can be simplified even further by using the normalized random variable.

A.2. Charge and Saving Definitions using the Normalized Random Variable

Charge:

$$\varphi(r) = \mathrm{E}[\max(0, R - r)] = \mathrm{E}[R - \min(R, r)] = 1 - \mathrm{E}[R; r] \tag{A.2a}$$

Saving:

$$\psi(r) = \mathrm{E}[\max(0, r - R)] = r - \mathrm{E}[R; r]. \tag{A.2b}$$

The charge and saving can also be expressed in terms of integrals.

A.3. Insurance Charge and Saving Functions Defined using Integrals

$$\varphi(r) = \frac{1}{\mu}\int_{r\mu}^{\infty} dF_T(t)(t - r\mu) = \int_{r}^{\infty} dF_R(s)(s - r)$$

$$= \frac{1}{\mu}\int_{r\mu}^{\infty} dt\, G_T(t) = \int_{r}^{\infty} ds\, G_R(s) \qquad \text{(A.3a)}$$

$$\psi(r) = \frac{1}{\mu}\int_{0}^{r\mu} dF_T(t)(r\mu - t) = \int_{0}^{r} dF_R(s)(r - s)$$

$$= \frac{1}{\mu}\int_{0}^{r\mu} dt\, F_T(t) = \int_{0}^{r} ds\, F_R(s). \qquad \text{(A.3b)}$$

When the random variable is discrete, these are viewed as Reimann integrals and are interpreted as sums.

Many basic properties can be proved directly from the definitions using simple properties of integrals, minimum operators, and expectations.

A.4. Insurance Charge: Basic Properties

φ is a continuous function of r. (A.4a)

φ is a decreasing function of r, which is strictly decreasing when $\varphi(r) > 0$. (A.4b)

$\varphi(0) = 1$ and, as $r \to \infty$, $\varphi(r) \to 0$. (A.4c)

$\varphi_0(r) \le \varphi(r) \le 1$, where $\varphi_0(r) = \max(0, 1 - r)$. (A.4d)

With the definitions in A.2, one can show the charge and saving are related by a simple formula.

A.5. Relation Between Charge and Saving

$$\varphi(r) - \psi(r) = 1 - r. \tag{A.5}$$

Proof Use Equations (A.2a) and (A.2b) to write $\varphi(r) - \psi(r) = 1 - \mathrm{E}[R;r] - (r - \mathrm{E}[R;r]) = 1 - r$.

It is straightforward to see that multiplication of the underlying random variable by a scalar does not change its insurance charge.

A.6. Scaling a Random Variable Does Not Change Its Charge

For $c > 0$,

$$\varphi_T(r) = \varphi_{cT}(r). \tag{A.6}$$

Proof Left as an exercise for the reader.

The charge can never decline too rapidly between any two points.

A.7. Insurance Charge Slope Between Two Points Greater Than -1

If $s > r$ then

$$\frac{\varphi(s) - \varphi(r)}{s - r} \geq -1. \tag{A.7}$$

Proof Using Equation (A.4), write

$$\varphi(s) - \varphi(r) = -\int_r^s du\, G_R(u) = \int_r^s du(-1 + F_R(u))$$
$$= -(s - r) + \int_r^s du\, F_R(u).$$

Therefore,

$$\frac{\varphi(s) - \varphi(r)}{s - r} \geq -1 + \frac{1}{s - r}\int_r^s du\, F_R(u) \geq -1.$$

Further, the insurance charge must always be concave up. This means the insurance charge curve never goes above a straight line drawn between any two points on the curve.

A.8. Insurance Charge is Concave Up

$$\varphi(wr + (1-w)s) \leq w\varphi(r) + (1-w)\varphi(s) \qquad \text{for} \quad 0 \leq w \leq 1. \tag{A.8}$$

Proof Using the general property of the "min" operator

$$\min(A+B, C+D) \geq \min(A,C) + \min(B,D)$$

we derive

$$\min(R, wr + (1-w)s) \geq \min(wR, wr) + \min((1-w)R, (1-w)s).$$

Factoring out w and $(1-w)$ respectively, yields

$$\min(R, wr + (1-w)s) \geq w \cdot \min(R,r) + (1-w)\min(R,s).$$

Using Equation (A.2) repeatedly, we find

$$\begin{aligned}
\varphi(wr + (1-w)s) &= 1 - \mathrm{E}[R; wr + (1-w)s] \\
&\leq w + (1-w) - w\mathrm{E}[R,r] - (1-w)\mathrm{E}[R;s] \\
&= w(1 - \mathrm{E}[R;r]) + (1-w)(1-\mathrm{E}[R;s]) \\
&= w\varphi(r) + (1-w)\varphi(s).
\end{aligned}$$

Though the charge and saving functions are continuous, they need not always be differentiable. However, when they are, the following formulas hold.

A.9. Insurance Charge and Saving First Derivatives

If φ or ψ is known to be differentiable at r, then

$$\frac{d\varphi}{dr}(r) = -G_R(r) \qquad \text{and} \qquad \frac{d\left(\varphi\left(\frac{t}{\mu}\right)\right)}{dt} = -\frac{G_T(t)}{\mu} \tag{A.9a}$$

$$\frac{d\psi}{dr}(r) = F_R(r) \qquad \text{and} \qquad \frac{d\left(\psi\left(\frac{t}{\mu}\right)\right)}{dt} = \frac{F_T(t)}{\mu}. \tag{A.9b}$$

Proof Apply Equation (A.4) and the Fundamental Theorem of Calculus.

If R has a density function at a point r, one can take second derivatives.

A.10. Insurance Charge and Saving Second Derivatives

If R has a well-defined density f_R at a point r, then

$$\frac{d^2\varphi}{dr^2}(r) = f_R(r) \tag{A.10a}$$

$$\frac{d^2\psi}{dr^2}(r) = f_R(r). \tag{A.10b}$$

Proof Take derivatives of the first derivatives shown in Equation (A.9a) and (A.9b).

The variance can be expressed in terms of an integral of the insurance charge.

A.11. Variance Formula using the Integral of the Insurance Charge

If $r^2 G_R(r) \to 0$ as $r \to \infty$, then

$$\mathrm{Var}(T) = \mu^2 \mathrm{Var}(R) = \mu^2 \left(2 \int_0^\infty dr\, \varphi(r) - 1 \right). \qquad \text{(A.11)}$$

Proof It suffices to prove the result when T is a continuous random variable. Integrate by parts twice and use the assumptions to derive

$$\int_0^\infty dr\, \varphi(r) = -rG_R(r)\Big|_{r=0}^{r=\infty} + \int_0^\infty dr\, rG_R(r)$$

$$= 0 + \frac{r^2}{2} G_R(r)\Big|_{r=0}^{r=\infty} + \int_0^\infty dr \frac{r^2}{2} f_R(r) = \frac{1}{2}\mathrm{E}[R^2].$$

Therefore, one has $2\int_0^\infty dr\, \varphi(r) = \mathrm{E}[R^2]$.

Then, using the definition of the variance along with the fact that $\mathrm{E}[R] = 1$, one can write

$$\mathrm{Var}(R) = \mathrm{E}[R^2] - (\mathrm{E}[R])^2 = 2\int_0^\infty dr\, \varphi(r) - 1.$$

Note the coefficient of variation, CV, is given as the square root of $\mathrm{Var}(R)$.

The result, Equation (A.11), is intuitive since to have a large insurance charge a random variable must take on extreme values with some significant probability. This means it has a relatively large CV. The converse is not true. If $\mathrm{CV}(R_1) \geq \mathrm{CV}(R_2)$, then $\varphi_1(r)$ must exceed $\varphi_2(r)$ on average, but not necessarily at every entry ratio. Exhibit 1 shows a discrete counterexample in which one random variable has a larger charge at some entry ratios, even though it has a smaller CV than another random variable.

APPENDIX B

COLLECTIVE RISK MODEL SUMMARY

The quick summary uses notation that is equivalent to, but not always identical with, the notation used by Heckman and Meyers [4].

We start with the claim count model and define the number of claims as a counting random variable, N. Let θ be the conditional expected number of claims so that $\mathrm{E}[N|_{\theta}] = \theta$. We also write $N(\theta)$ to denote the conditional claim count distribution.

Let μ be the unconditional mean claim count. To introduce parameter uncertainty, we let ν be a non-negative random variable with $\mathrm{E}[\nu] = 1$ and $\mathrm{Var}(\nu) = c$. The parameter c is called the contagion. To model unconditional claim counts, we first select a value of ν at random and then randomly select a claim count N from the distribution $N(\theta)$ where $\theta = \nu\mu$. Heckman and Meyers assume N is conditionally Poisson, so that it follows that

B.1. Unconditional Claim Count Mean and Variance

$$\mathrm{E}[N] = \mathrm{E}[\mathrm{E}[N(\theta) \mid \theta = \nu\mu]] = \mathrm{E}[\nu\mu] = \mu\mathrm{E}[\nu] = \mu. \quad \text{(B.1a)}$$

$$\begin{aligned}\mathrm{Var}(N) &= \mathrm{E}[\mathrm{Var}(N(\theta) \mid \theta = \nu\mu)] + \mathrm{Var}(\mathrm{E}[N(\theta) \mid \theta = \nu\mu]) \\ &= \mathrm{E}[\mu\nu] + \mathrm{Var}(\mu\nu) = \mu\mathrm{E}[\nu] + \mu^2\mathrm{Var}(\nu) = \mu + \mu^2 c.\end{aligned} \quad \text{(B.1b)}$$

If ν is Gamma distributed, the unconditional claim count distribution is Negative Binomial.

We now add severity to the model. We let $X(\lambda)$ be the conditional claim severity random variable defined so that $\mathrm{E}[X(\lambda)] = \lambda$. Heckman and Meyers model severity parameter uncertainty by assuming the shape of the severity distribution is known but there is uncertainty about its scale. Let β be a positive random variable such that $\mathrm{E}[1/\beta] = 1$ and $\mathrm{Var}(1/\beta) = b$. We call b the mixing parameter. Suppose γ is the unconditional expected severity and let $\mathrm{Var}(X(\gamma)) = \tau^2$ To model severity, we first take a sample from β and then sample the scaled severity distribution $Y = X(\gamma)/\beta$.

To generate aggregate losses T, we first independently sample the number of claims N, and the scaling parameter, β. Then we independently draw N samples from the severity random variable $Y = X(\gamma)/\beta$. The aggregate loss T is the sum of these N severity samples.

Formulas for the unconditional mean and variance of the aggregate loss are derived as follows

B.2. Unconditional Aggregate Loss Mean and Variance

$$\mathrm{E}[T] = \mathrm{E}[N]\mathrm{E}[Y] = \mathrm{E}[\nu\mu]\mathrm{E}[X(\gamma)/\beta] = \mu\mathrm{E}[\nu]\gamma\mathrm{E}[1/\beta] = \mu\gamma. \tag{B.2a}$$

$$\begin{aligned}
\mathrm{Var}(T) &= \mathrm{E}[\mathrm{Var}(T|_{\nu},\beta)] + \mathrm{Var}(\mathrm{E}[T \mid \nu,\beta]) \\
&= \mathrm{E}[\mathrm{E}[N \mid \mu\nu]\mathrm{Var}(Y \mid \beta) + \mathrm{Var}(N \mid \mu\nu)\mathrm{E}[Y \mid \beta]^2] \\
&\quad + \mathrm{Var}(\mu\nu\gamma/\beta) \\
&= \mu\mathrm{E}[\nu]\tau^2\mathrm{E}[(1/\beta)^2] + \mu\mathrm{E}[\nu]\gamma^2\mathrm{E}[(1/\beta)^2] \\
&\quad + (\mu\gamma)^2\mathrm{Var}(\nu/\beta) \\
&= \mu\tau^2(1+b) + \mu\gamma^2(1+b) + (\mu\gamma)^2\mathrm{Var}(\nu/\beta) \\
&= \mu(\tau^2+\gamma^2)(1+b) + \mu^2\gamma^2(b+c+bc).
\end{aligned} \tag{B.2b}$$

Heckman and Meyers assume that β has a Gamma distribution.

EXHIBIT 1

DISCRETE EXAMPLE
SMALLER CV DOES NOT IMPLY SMALLER CHARGE

Statistics	$T1$	$T2$
Mean	4.00	4.00
Variance	8.00	7.20
Relative Variance = Var(R)	0.50	0.45
CV	0.71	0.67

Random Variable $T1$

Index i	Point t_i	Density $f(t_i)$	Square t_i^2	Ratio r_i	CDF $F(r_i)$	Savings $\psi(r_i)$	Tail $G(r_i)$	Charge $\phi(r_i)$
1	0.00	20.0%	0.00	0.00	20.0%	0.0%	80.0%	100.0%
2	2.00	20.0%	4.00	0.50	40.0%	10.0%	60.0%	60.0%
3	4.00	20.0%	16.00	1.00	60.0%	30.0%	40.0%	30.0%
4	6.00	20.0%	36.00	1.50	80.0%	60.0%	20.0%	10.0%
5	8.00	20.0%	64.00	2.00	100.0%	100.0%	0.0%	0.0%
Mean	4.00		24.00	1.00				

Random Variable $T2$

Index i	Point t_i	Density $f(t_i)$	Square t_i^2	Ratio r_i	CDF $F(r_i)$	Savings $\psi(r_i)$	Tail $G(r_i)$	Charge $\phi(r_i)$
1	0.00	0.0%	0.00	0.00	0.0%	0.0%	100.0%	100.0%
2	2.00	60.0%	4.00	0.50	60.0%	0.0%	40.0%	50.0%
3	4.00	10.0%	16.00	1.00	70.0%	30.0%	30.0%	30.0%
4	6.00	0.0%	36.00	1.50	70.0%	65.0%	30.0%	15.0%
5	8.00	30.0%	64.00	2.00	100.0%	100.0%	0.0%	0.0%
Mean	4.00		23.20	1.00				

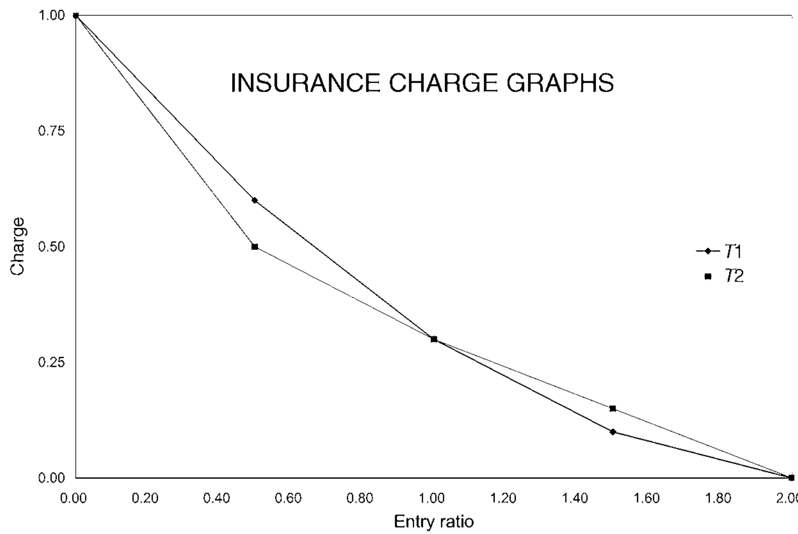


EXHIBIT 2

Sheet 1
Charge for Independent Sum: Discrete Example

Statistics	$T1$	$T2$	$T1 + T2$
Mean	10.00	10.00	20.00
Variance	63.20	63.20	126.40
Relative Variance = Var(R)	0.63	0.63	0.32
CV	0.79	0.79	0.56

	$T1,T2$
Covariance	0.00
Correlation	0.00

Random Variable $T1$

Index i	$T1$ Value t_i	Density $f(t_i)$	Square t_i^2	Ratio r_i	CDF $F(r_i)$	Savings $\psi(r_i)$	Tail $G(r_i)$	Charge $\phi(r_i)$
1	0.00	20.0%	0.00	0.00	20.0%	0.0%	80.0%	100.0%
2	8.00	50.0%	64.00	0.80	70.0%	16.0%	30.0%	36.0%
3	16.00	20.0%	256.00	1.60	90.0%	72.0%	10.0%	12.0%
4	24.00	5.0%	576.00	2.40	95.0%	144.0%	5.0%	4.0%
5	32.00	5.0%	1,024.00	3.20	100.0%	220.0%	0.0%	0.0%
Mean	10.00	100.0%	163.20	1.00				

Random Variable $T2$

Index i	$T2$ Value t_i	Density $f(t_i)$	Square t_i^2	Ratio r_i	CDF $F(r_i)$	Savings $\psi(r_i)$	Tail $G(r_i)$	Charge $\phi(r_i)$
1	0.00	20.0%	0.00	0.00	20.0%	0.0%	80.0%	100.0%
2	8.00	50.0%	64.00	0.80	70.0%	16.0%	30.0%	36.0%
3	16.00	20.0%	256.00	1.60	90.0%	72.0%	10.0%	12.0%
4	24.00	5.0%	576.00	2.40	95.0%	144.0%	5.0%	4.0%
5	32.00	5.0%	1,024.00	3.20	100.0%	220.0%	0.0%	0.0%
Mean	10.00	100.0%	163.20	1.00				

Joint Density of $T1$ and $T2$

$T1$ Value	$T2$ Value 0.00	8.00	16.00	24.00	32.00	$T1$ Marginal
0.00	4.0%	10.0%	4.0%	1.0%	1.0%	20.0%
8.00	10.0%	25.0%	10.0%	2.5%	2.5%	50.0%
16.00	4.0%	10.0%	4.0%	1.0%	1.0%	20.0%
24.00	1.0%	2.5%	1.0%	0.3%	0.3%	5.0%
32.00	1.0%	2.5%	1.0%	0.3%	0.3%	5.0%
$T2$ Marginal	20.0%	50.0%	20.0%	5.0%	5.0%	100.0%

Random Variable $T1 + T2$

Index i	$T1 + T2$ t_i	Density $f(t_i)$	Square t_i^2	Ratio r_i	CDF $F(r_i)$	Savings $\psi(r_i)$	Tail $G(r_i)$	Charge $\phi(r_i)$
1	0.00	4.0%	0.00	0.00	4.0%	0.0%	96.0%	100.0%
2	8.00	20.0%	64.00	0.40	24.0%	1.6%	76.0%	61.6%
3	16.00	33.0%	256.00	0.80	57.0%	11.2%	43.0%	31.2%
4	24.00	22.0%	576.00	1.20	79.0%	34.0%	21.0%	14.0%
5	32.00	11.0%	1,024.00	1.60	90.0%	65.6%	10.0%	5.6%
6	40.00	7.0%	1,600.00	2.00	97.0%	101.6%	3.0%	1.6%
7	48.00	2.3%	2,304.00	2.40	99.3%	140.4%	0.8%	0.4%
8	56.00	0.5%	3,136.00	2.80	99.8%	180.1%	0.3%	0.1%
9	64.00	0.3%	4,096.00	3.20	100.0%	220.0%	0.0%	0.0%
Mean	20.00	100.0%	526.40	1.00				

EXHIBIT 2

SHEET 2
CHARGE FOR CORRELATED SUM: DISCRETE EXAMPLE

Statistics	$T1$	$T2$	$T1+T2$
Mean	10.00	10.00	20.00
Variance	63.20	63.20	201.60
Relative Variance = Var(R)	0.63	0.63	0.50
CV	0.79	0.79	0.71

	$T1,T2$
Covariance	37.60
Correlation	0.59

Random Variable $T1$

Index i	$T1$ Value t_i	Density $f(t_i)$	Square t_i^2	Ratio r_i	CDF $F(r_i)$	Savings $\psi(r_i)$	Tail $G(r_i)$	Charge $\phi(r_i)$
1	0.00	20.0%	0.00	0.00	20.0%	0.0%	80.0%	100.0%
2	8.00	50.0%	64.00	0.80	70.0%	16.0%	30.0%	36.0%
3	16.00	20.0%	256.00	1.60	90.0%	72.0%	10.0%	12.0%
4	24.00	5.0%	576.00	2.40	95.0%	144.0%	5.0%	4.0%
5	32.00	5.0%	1,024.00	3.20	100.0%	220.0%	0.0%	0.0%
Mean	10.00	100.0%	163.20	1.00				

Random Variable $T2$

Index i	$T2$ Value t_i	Density $f(t_i)$	Square t_i^2	Ratio r_i	CDF $F(r_i)$	Savings $\psi(r_i)$	Tail $G(r_i)$	Charge $\phi(r_i)$
1	0.00	20.0%	0.00	0.00	20.0%	0.0%	80.0%	100.0%
2	8.00	50.0%	64.00	0.80	70.0%	16.0%	30.0%	36.0%
3	16.00	20.0%	256.00	1.60	90.0%	72.0%	10.0%	12.0%
4	24.00	5.0%	576.00	2.40	95.0%	144.0%	5.0%	4.0%
5	32.00	5.0%	1,024.00	3.20	100.0%	220.0%	0.0%	0.0%
Mean	10.00	100.0%	163.20	1.00				

Joint Density of $T1$ and $T2$

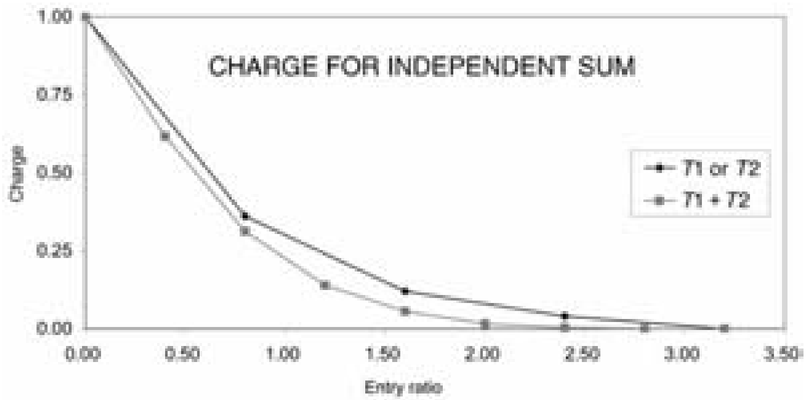
	$T2$ Value					
$T1$ Value	0.00	8.00	16.00	24.00	32.00	$T1$ Marginal
0.00	10.0%	7.0%	3.0%	0.0%	0.0%	20.0%
8.00	5.0%	37.0%	8.0%	0.0%	0.0%	50.0%
16.00	5.0%	5.0%	7.0%	2.0%	1.0%	20.0%
24.00	0.0%	0.0%	1.0%	2.0%	2.0%	5.0%
32.00	0.0%	1.0%	1.0%	1.0%	2.0%	5.0%
$T2$ Marginal	20.0%	50.0%	20.0%	5.0%	5.0%	100.0%

Random Variable $T1+T2$

Index i	$T1+T2$ t_i	Density $f(t_i)$	Square t_i^2	Ratio r_i	CDF $F(r_i)$	Savings $\psi(r_i)$	Tail $G(r_i)$	Charge $\phi(r_i)$
1	0.00	10.0%	0.00	0.00	10.0%	0.0%	90.0%	100.0%
2	8.00	12.0%	64.00	0.40	22.0%	4.0%	78.0%	64.0%
3	16.00	45.0%	256.00	0.80	67.0%	12.8%	33.0%	32.8%
4	24.00	13.0%	576.00	1.20	80.0%	39.6%	20.0%	19.6%
5	32.00	7.0%	1,024.00	1.60	87.0%	71.6%	13.0%	11.6%
6	40.00	4.0%	1,600.00	2.00	91.0%	106.4%	9.0%	6.4%
7	48.00	4.0%	2,304.00	2.40	95.0%	142.8%	5.0%	2.8%
8	56.00	3.0%	3,136.00	2.80	98.0%	180.8%	2.0%	0.8%
9	64.00	2.0%	4,096.00	3.20	100.0%	220.0%	0.0%	0.0%
Mean	20.00	100.0%	601.60	1.00				

EXHIBIT 2

SHEET 3



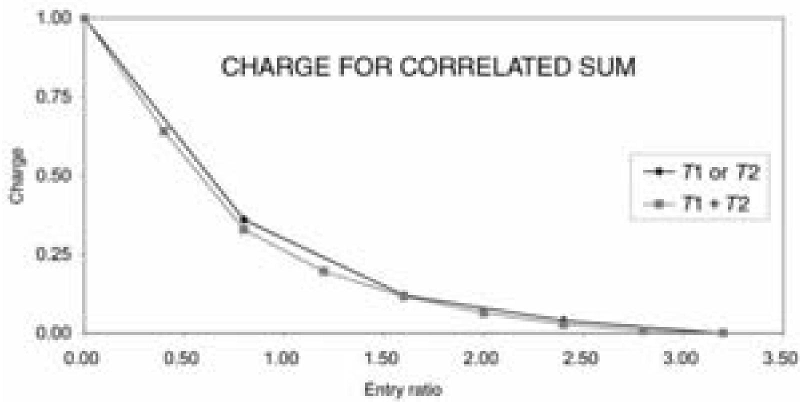


EXHIBIT 3

Sheet 1
Poisson Limited Expected Values and Partials

Mean	Limit = 3.000			
	LEV	Numerical 1st Partial of LEV	Numerical 2nd Partial of LEV	Theoretical 2nd Partial of LEV
0.300	0.300	0.988		
0.600	0.596	0.959	−0.099	−0.099
0.900	0.884	0.910	−0.163	−0.165
1.200	1.157	0.845	−0.215	−0.217
1.500	1.410	0.770	−0.250	−0.251
1.800	1.641	0.690	−0.267	−0.268
2.100	1.848	0.609	−0.269	−0.270
2.400	2.031	0.531	−0.261	−0.261
2.700	2.191	0.458	−0.245	−0.245
3.000	2.328	0.391	−0.224	−0.224
3.300	2.445	0.330	−0.201	−0.201
3.600	2.544	0.277	−0.177	−0.177
3.900	2.627	0.231	−0.154	−0.154
4.200	2.697	0.191	−0.132	−0.132
4.500	2.754	0.158	−0.113	−0.112
4.800	2.801	0.129	−0.095	−0.095
5.100	2.840	0.105	−0.079	−0.079
5.400	2.872	0.085	−0.066	−0.066
5.700	2.897	0.069	−0.055	−0.054
6.000	2.918			

EXHIBIT 3

SHEET 2
NEGATIVE BINOMIAL LIMITED EXPECTED VALUES AND PARTIALS

Fixed $q = 0.750$

	Limit = 3.000		
Mean	LEV	Numerical 1st Partial of LEV	Numerical 2nd Partial of LEV
0.300	0.231	0.724	
0.600	0.448	0.679	−0.150
0.900	0.651	0.635	−0.146
1.200	0.842	0.592	−0.142
1.500	1.020	0.551	−0.136
1.800	1.185	0.512	−0.131
2.100	1.339	0.475	−0.125
2.400	1.481	0.439	−0.118
2.700	1.613	0.406	−0.112
3.000	1.734	0.374	−0.105
3.300	1.847	0.344	−0.099
3.600	1.950	0.317	−0.093
3.900	2.045	0.291	−0.087
4.200	2.132	0.266	−0.081
4.500	2.212	0.244	−0.075
4.800	2.285	0.223	−0.070
5.100	2.352	0.204	−0.064
5.400	2.413	0.186	−0.060
5.700	2.469	0.169	−0.055
6.000	2.520		

EXHIBIT 3

SHEET 3
GAMMA LIMITED EXPECTED VALUES AND PARTIALS

Scale = 1.000

	Limit = 3.000		
Mean	LEV	Numerical 1st Partial of LEV	Numerical 2nd Partial of LEV
0.300	0.294	0.981	
0.600	0.582	0.959	−0.074
0.900	0.860	0.927	−0.107
1.200	1.126	0.885	−0.141
1.500	1.376	0.833	−0.173
1.800	1.607	0.773	−0.200
2.100	1.819	0.707	−0.221
2.400	2.010	0.637	−0.234
2.700	2.180	0.565	−0.240
3.000	2.328	0.493	−0.237
3.300	2.455	0.425	−0.229
3.600	2.564	0.361	−0.214
3.900	2.654	0.302	−0.196
4.200	2.729	0.249	−0.176
4.500	2.790	0.203	−0.154
4.800	2.839	0.163	−0.133
5.100	2.877	0.129	−0.112
5.400	2.908	0.101	−0.093
5.700	2.931	0.079	−0.076
6.000	2.949	0.060	

EXHIBIT 4

Sheet 1
Poisson Insurance Charges

Entry Ratio	Mean					
	0.500	1.000	1.500	2.000	2.500	3.000
0.000	1.000	1.000	1.000	1.000	1.000	1.000
0.100	0.961	0.937	0.922	0.914	0.908	0.905
0.200	0.921	0.874	0.845	0.827	0.816	0.810
0.300	0.882	0.810	0.767	0.741	0.725	0.715
0.400	0.843	0.747	0.689	0.654	0.633	0.630
0.500	0.803	0.684	0.612	0.568	0.562	0.550
0.600	0.764	0.621	0.534	0.508	0.490	0.470
0.700	0.725	0.558	0.467	0.449	0.419	0.397
0.800	0.685	0.494	0.423	0.389	0.348	0.339
0.900	0.646	0.431	0.379	0.330	0.302	0.282
1.000	0.607	0.368	0.335	0.271	0.257	0.224
1.100	0.567	0.341	0.290	0.238	0.211	0.189
1.200	0.528	0.315	0.246	0.206	0.165	0.153
1.300	0.488	0.289	0.202	0.174	0.141	0.118
1.400	0.449	0.262	0.175	0.141	0.117	0.094
1.500	0.410	0.236	0.155	0.109	0.093	0.076
1.600	0.370	0.209	0.136	0.095	0.068	0.057
1.700	0.331	0.183	0.117	0.080	0.057	0.042
1.800	0.292	0.156	0.098	0.066	0.047	0.034
1.900	0.252	0.130	0.079	0.052	0.036	0.025
2.000	0.213	0.104	0.060	0.038	0.025	0.017
2.100	0.204	0.096	0.053	0.032	0.021	0.014
2.200	0.195	0.088	0.047	0.027	0.016	0.010
2.300	0.186	0.080	0.040	0.022	0.012	0.007
2.400	0.177	0.072	0.034	0.017	0.008	0.005
2.500	0.168	0.063	0.027	0.011	0.007	0.004
2.600	0.159	0.055	0.020	0.010	0.005	0.003
2.700	0.150	0.047	0.015	0.008	0.004	0.002
2.800	0.141	0.039	0.014	0.006	0.002	0.001
2.900	0.132	0.031	0.012	0.005	0.002	0.001
3.000	0.123	0.023	0.010	0.003	0.001	0.000

EXHIBIT 4

SHEET 2
NEGATIVE BINOMIAL INSURANCE CHARGES

Fixed $q = 0.750$

Entry Ratio	Mean 0.500	1.000	1.500	2.000	2.500	3.000
0.000	1.000	1.000	1.000	1.000	1.000	1.000
0.100	0.979	0.963	0.950	0.940	0.931	0.925
0.200	0.959	0.926	0.900	0.879	0.863	0.850
0.300	0.938	0.889	0.850	0.819	0.794	0.775
0.400	0.917	0.852	0.800	0.759	0.726	0.713
0.500	0.897	0.815	0.750	0.698	0.677	0.656
0.600	0.876	0.778	0.700	0.658	0.628	0.600
0.700	0.856	0.741	0.656	0.617	0.580	0.548
0.800	0.835	0.704	0.625	0.577	0.531	0.506
0.900	0.814	0.667	0.594	0.537	0.495	0.464
1.000	0.794	0.630	0.562	0.496	0.460	0.422
1.100	0.773	0.609	0.531	0.468	0.425	0.390
1.200	0.752	0.587	0.500	0.440	0.390	0.359
1.300	0.732	0.566	0.469	0.412	0.364	0.327
1.400	0.711	0.545	0.445	0.384	0.338	0.301
1.500	0.691	0.524	0.424	0.356	0.313	0.277
1.600	0.670	0.502	0.403	0.336	0.287	0.253
1.700	0.649	0.481	0.382	0.316	0.268	0.231
1.800	0.629	0.460	0.362	0.296	0.249	0.214
1.900	0.608	0.439	0.341	0.276	0.230	0.196
2.000	0.587	0.417	0.320	0.257	0.212	0.178
2.100	0.577	0.404	0.306	0.243	0.198	0.165
2.200	0.566	0.391	0.292	0.228	0.184	0.151
2.300	0.555	0.377	0.278	0.214	0.170	0.138
2.400	0.545	0.364	0.264	0.200	0.156	0.127
2.500	0.534	0.351	0.250	0.186	0.146	0.117
2.600	0.523	0.337	0.236	0.176	0.136	0.107
2.700	0.512	0.324	0.223	0.166	0.126	0.098
2.800	0.502	0.310	0.213	0.156	0.116	0.090
2.900	0.491	0.297	0.203	0.146	0.108	0.083
3.000	0.480	0.284	0.194	0.135	0.101	0.075

EXHIBIT 4

Sheet 3
Gamma Insurance Charges

Scale = 1.000

Entry Ratio	Mean					
	0.500	1.000	1.500	2.000	2.500	3.000
0.000	1.000	1.000	1.000	1.000	1.000	1.000
0.100	0.917	0.905	0.902	0.901	0.900	0.900
0.200	0.847	0.819	0.809	0.804	0.802	0.801
0.300	0.785	0.741	0.723	0.713	0.708	0.705
0.400	0.729	0.670	0.644	0.629	0.620	0.614
0.500	0.679	0.607	0.572	0.552	0.539	0.530
0.600	0.633	0.549	0.507	0.482	0.465	0.453
0.700	0.591	0.497	0.449	0.419	0.399	0.384
0.800	0.553	0.449	0.397	0.363	0.340	0.323
0.900	0.517	0.407	0.350	0.314	0.289	0.270
1.000	0.484	0.368	0.308	0.271	0.244	0.224
1.100	0.453	0.333	0.271	0.233	0.205	0.185
1.200	0.425	0.301	0.239	0.200	0.172	0.152
1.300	0.399	0.273	0.210	0.171	0.144	0.124
1.400	0.374	0.247	0.184	0.146	0.120	0.101
1.500	0.351	0.223	0.161	0.124	0.100	0.082
1.600	0.330	0.202	0.142	0.106	0.083	0.066
1.700	0.310	0.183	0.124	0.090	0.068	0.053
1.800	0.291	0.165	0.108	0.077	0.056	0.043
1.900	0.274	0.150	0.095	0.065	0.046	0.034
2.000	0.258	0.135	0.083	0.055	0.038	0.027
2.100	0.243	0.122	0.073	0.046	0.031	0.022
2.200	0.228	0.111	0.063	0.039	0.026	0.017
2.300	0.215	0.100	0.055	0.033	0.021	0.014
2.400	0.202	0.091	0.048	0.028	0.017	0.011
2.500	0.191	0.082	0.042	0.024	0.014	0.009
2.600	0.180	0.074	0.037	0.020	0.011	0.007
2.700	0.169	0.067	0.032	0.017	0.009	0.005
2.800	0.160	0.061	0.028	0.014	0.007	0.004
2.900	0.150	0.055	0.024	0.012	0.006	0.003
3.000	0.142	0.050	0.021	0.010	0.005	0.003

EXHIBIT 5

Charges for Gamma-Poisson Contagion Model

$q = \mu/(\mu + \alpha)$, $\alpha = 2.00$, Contagion = .50

Entry Ratio	Mean					
	0.500	1.000	1.500	2.000	2.500	3.000
0.000	1.000	1.000	1.000	1.000	1.000	1.000
0.100	0.964	0.944	0.933	0.925	0.920	0.916
0.200	0.928	0.889	0.865	0.850	0.840	0.832
0.300	0.892	0.833	0.798	0.775	0.759	0.748
0.400	0.856	0.778	0.731	0.700	0.679	0.677
0.500	0.820	0.722	0.663	0.625	0.621	0.612
0.600	0.784	0.667	0.596	0.575	0.562	0.547
0.700	0.748	0.611	0.538	0.525	0.504	0.488
0.800	0.712	0.556	0.499	0.475	0.446	0.441
0.900	0.676	0.500	0.459	0.425	0.406	0.393
1.000	0.640	0.444	0.420	0.375	0.366	0.346
1.100	0.604	0.419	0.380	0.344	0.326	0.312
1.200	0.568	0.393	0.341	0.313	0.286	0.278
1.300	0.532	0.367	0.302	0.281	0.259	0.245
1.400	0.496	0.341	0.274	0.250	0.233	0.218
1.500	0.460	0.315	0.253	0.219	0.206	0.194
1.600	0.424	0.289	0.232	0.200	0.180	0.171
1.700	0.388	0.263	0.210	0.181	0.163	0.150
1.800	0.352	0.237	0.189	0.163	0.146	0.134
1.900	0.316	0.211	0.168	0.144	0.129	0.119
2.000	0.280	0.185	0.146	0.125	0.112	0.103
2.100	0.270	0.174	0.135	0.114	0.101	0.092
2.200	0.259	0.163	0.124	0.103	0.090	0.081
2.300	0.249	0.152	0.113	0.092	0.079	0.071
2.400	0.238	0.141	0.102	0.081	0.069	0.062
2.500	0.228	0.130	0.091	0.070	0.062	0.055
2.600	0.218	0.119	0.080	0.064	0.055	0.048
2.700	0.207	0.107	0.070	0.058	0.048	0.042
2.800	0.197	0.096	0.065	0.052	0.042	0.037
2.900	0.186	0.085	0.059	0.045	0.038	0.033
3.000	0.176	0.074	0.054	0.039	0.033	0.028

ADDRESS TO NEW MEMBERS—MAY 16, 2005

THE MEANING OF LIFE, THE UNIVERSE, AND ALL THAT

ALLAN M. KAUFMAN

I am supposed to congratulate you, tell you what a great profession you have joined, and tell you about the responsibilities you will have as new members of the CAS. I will do that in due course.

In preparing for that I thought back to the words of wisdom I had heard in previous new member addresses. I remember David Hartman describing the careers of the seven members of his Fellowship class. I remember Roy Simon explaining the significance of the invention of mass marketed deodorants. But what else did Roy say? And what about all of the other talks? What about the talks I was forced (oops, privileged) to hear when I was a new Fellow and Associate. I drew a blank. That concerned me. Was this a major senior moment?

Before I get further into trouble, let me quickly say something to past presidents like Stan Khury and Irene Bass who spoke in the past five years. I'm sure I would have remembered your talks, but I was based in London and hadn't been to those CAS meetings. But before that?

Fortunately, the *Proceedings* includes new member addresses. I joined the select group of people, past presidents giving new member addresses, I imagine, who have read through the complete collection of those addresses. The collection is smaller than I feared. The *Proceedings* have no such speeches prior to 1985—but was that a gap in the *Proceedings*? No, back in 1996 Mike Toothman observed in his new member speech that the new member address was only about a dozen years old at that time. One mystery solved—there were no addresses at my Associateship and Fellowship graduations, which go back to the early 1970s.

Still, that search highlights the fact that there is a high threshold if I expect either you or me to remember this speech. For you, this is a unique day, so you'll remember the day if not the speech. For me, I choose to talk about something familiar so I can remember what I said, if not when I said it.

My career has been about training actuaries—from student to Associate and Fellow and beyond. I'll use this time to share with you ten rules that help me and which I have used to advise my younger colleagues. You'll need to decide if they apply to you.

Rule #1: 80 Percent of Life is Showing Up

Something close to this is attributed to Woody Allen. The key point is deciding to show up. I'm an introvert, and I feel much more comfortable thinking about it than doing it. I draw comfort from this rule. If I agree to show up, and I'm sure I can, that gives me eighty points. So "let's do it," I say to myself.

Having decided to show up, I inevitably prepare so that I can score additional points. I have discovered that others do the same. A corollary to this is that "If a project is stalled, call a meeting." Usually, if forced to "show up," everyone will prepare.

Other corollaries are:

- No one has ever passed an exam without showing up. (New Associates take note.)
- In the consulting world I live in, getting out to see your clients is central. You communicate just by taking the effort to show up.
- For the highly analytical among you, make sure that you "show up" and talk to the rest of the world, even if you think the analysis was not yet done.

Rule #2: Remember, You are an Important Person Now

I discovered at some point in my career that I had become an important person. This was a surprise to me. People cared about what I thought, said, and did. They found my advice helpful. They considered my approval, or lack of approval, important.

Then I discovered that other people also failed to recognize when they had become important. Every few years, particularly when you are young, you seem to need to be reminded.

This is a very good time to remember that YOU ARE AN IMPORTANT PERSON. You are successful. You are members of the Casualty Actuarial Society—Fellows or Associates.

The responsibilities of being an important person include:

- Speak up.
- Be a good role model.
- Treat your colleagues with respect.
- Be confident in yourself. You are a qualified actuary—What is your professional opinion?

You are a part of a profession that has become increasingly important. Rating agencies are holding the actuarial profession responsible for the reserve shortfalls in the United States. Regulators are holding the actuarial profession responsible for major insurance company failures in the United Kingdom. We may not think that this is completely correct, but it is not all bad. There is no doubt that they think our profession is far more important now than it was when I sat in your seat. As an important profession, we must be able to say what we can do, what we cannot do, and how we will go about doing what we can.

Rule #3: No Good Deed Goes Unpunished

This is one of my favorite rules.

You should volunteer. Don't be surprised when good work is rewarded by more assignments. Good work is how we build "capital" with our colleagues, clients, and bosses.

I have found that every one of my good deeds in volunteering has been punished threefold by requests for more volunteer effort.

Please, go forth and be punished.

Rule #4: Take Notes

There are two reasons for this:

1) If you are under 50, it shows that you care what the person is saying. You are an important person. Taking notes shows your concern for the people you are dealing with.

2) If you are over 50, it is because you have to or you will not remember anything.

Rule #5: Every Silver Lining has a Dark Cloud

This complements the better-known principle that every dark cloud has a silver lining. Actuaries are usually good at the dark clouds. But if you also remember that every dark cloud has a silver lining it will help you keep a balanced view.

You are an important person now and you need to keep your head when people around you are too exuberant or depressed to remember.

Rule #6: Treat Your Children With Love and Respect; Never Compare Them

You might think this has something to do with raising a family. Actually, it's not bad advice for that also. No, this is about your subordinates, clients, and bosses.

Like children, they will behave badly—demanding, whining, and disobeying. But, like children, it is probably because they have concerns that are not visible to you. They might be hungry, tired, or dirty. With enough love and respect and patience you can, together, solve their problems.

Second, never compare your employees or clients, at least not while they are listening. Everyone is unique. Everyone should be your best or favorite of that type. If you want them to understand what you are telling them, it needs to be presented to them as individuals, not related to how they differ from their colleagues or competitors.

Rule #7: Let Them Speak—They'll Think You're Brilliant

It sounds backwards, but it is true.

The best way to engage an audience, of one or many, is to let them speak. The best way to run a conference is to be sure that the audience is encouraged, and has plenty of time, to speak.

I learned this rule from CAS program planners and then I tried it in meetings that I have run. The best parts of this meeting for you will probably be when you have a chance to speak up. If there are enough chances for you to participate, you will think the program was brilliant.

There are dangers in overusing this principle. First, the introverts among you might abuse it as an excuse to simply be silent. Second, you are an important person now and sometimes you simply need to just tell others what to do and have them get on with it.

Rule #8: Know Yourself—Even if Learning is Painful

You are the only tool you have for life.

You are all good at acquiring technical knowledge—that's how you got here. But do you know yourself? You might think of yourself as having strengths and weaknesses. Actually, we simply are who we are. Your traits might be strengths or weaknesses depending on how and when you use those traits. You need to know when your normal ways of doing things are a strength or a weakness. You need to know when to act out of character.

That knowledge comes only from seeing yourself through other people's eyes. Even your friends will have trouble telling you enough about yourself. Even if they do tell you, you probably won't hear what they are telling you.

One of the best rules in determining whether you have heard useful information is whether it hurts. If it doesn't hurt at all it's probably not useful. Remember that during your next performance review.

Rule #9: Build a Cathedral

Perhaps you've heard the story of the two masons in the Middle Ages. When asked what they were doing, the first mason said he was laying bricks and he couldn't wait to finish for the day to have a pint with his friends. The second mason said that he was building a cathedral and could barely wait to begin work every day.

When applying for my first consulting job, my potential future boss said, "You know we do a lot of reserving here. Won't you find that boring" I was naïve, but fortunately more correct than I imagined. I told him if the work was important to the clients, then I would find it interesting.

Actuaries are doing work that is very important to their employer's or client's business. If you think your work is boring you might not understand how it is being used.

On the other hand, sometimes the work is boring and it's time to move on.

Rule #10: There is a Game Called Life

Yes, really. It's a board game that is at least 45-years-old. At the beginning you set your goals. You allocate 60 points between fame, fortune, and love—think of that as relationships. You go around the board and see whether you can achieve your goals.

By deciding on an actuarial career you have positioned yourself well on the fortune points.

If you volunteer for the CAS and in your companies, you are well positioned to achieve some fame. If want to achieve more fame points, you'll need to enter politics and maybe the American Academy can help.

You will also be able to build many longstanding friendships within the CAS. But don't forget Rule #1 and make sure you also show up where your core relationships are—at home.

So in conclusion, you have been around the game board a few times and now you have completed a major milestone. Congratulations on that.

I'm glad to have had this chance to share my thoughts about what has helped me go around the board.

Thank you.

MINUTES OF THE 2005 CAS SPRING MEETING

May 15–18, 2005

POINTE SOUTH MOUNTAIN RESORT

PHOENIX, ARIZONA

Sunday, May 15, 2005

The CAS board of directors met from 8:00 a.m. to 5:00 p.m.

Registration was held from 4:00 p.m. to 6:30 p.m.

An officers' reception for new Associates and accompanying persons was held from 5:30 p.m. to 6:30 p.m.

A welcome reception for all attendees was held from 6:30 p.m. to 7:30 p.m.

Monday, May 16, 2005

Registration continued from 7:00 a.m. to 5:00 p.m.

CAS President Stephen D'Arcy opened the business session at 8:00 a.m., welcoming all to the CAS Spring Meeting and announcing that the morning's events would be Webcast over the CAS Web Site. President D'Arcy introduced the current members of the CAS Executive Council (EC) and the CAS Board of Directors.

Mr. D'Arcy introduced special guests and the past presidents of the CAS attending the meeting including Robert A. Anker (1996), Irene K. Bass (1993), Ronald L. Bornhuetter (1975), David G. Hartman (1987), Allan M. Kaufman (1994), C. K. "Stan" Khury (1984), Steven G. Lehmann (1998), Mary Frances Miller (2004), and Gail M. Ross (2002).

Mr. D'Arcy asked that all Fellows and Associates who have been CAS members for 25 years or more and all CAS volunteers stand to be recognized. Mr. D'Arcy also recognized those individuals who have worked on the American Academy of Actuaries' committees or committees of other actuarial organizations.

Mr. D'Arcy commented that "seasoned" members have much to offer the profession, and that the CAS could do more to get them (or keep them) involved in the organization as they move toward and into retirement. He then told the audience about the possible formation of a special interest section to serve the needs of retired members, and asked that anyone interested in volunteering or learning more about the Retirees Special Interest Section to contact either Melanie Pennington, chairperson of the Regional Affiliates Committee, or Todd Rogers at the CAS office.

Vice President–Professional Education Beth Fitzgerald then gave an overview of the meeting. Ms. Fitzgerald first recognized meeting supporters D. W. Simpson and Company Select Actuarial Services as well as meeting exhibitors Milliman, Towers Perrin, and the Actuarial Foundation. She encouraged attendees to visit their displays in the registration area.

Ms. Fitzgerald noted that there will be two *Proceedings* papers presented at this meeting, as well as two discussion papers on the topic "Primer on Enterprise Risk Management."

In concluding her presentation, Ms. Fitzgerald complimented and thanked chairperson Patrick Woods and the Program Planning Committee, along with CAS Meeting Planner Kathy Spicer and other members of the CAS office staff for coordinating the meeting.

Following these announcements, the new CAS Fellows and Associates in attendance were honored in a special ceremony. The CAS admitted 43 new Fellows and 22 new Associates in May 2005. Thomas G. Myers, Vice President–Admissions, announced the new Associates and Paul Braithwaite, President-Elect, announced the new Fellows. The names of the members of the Spring 2005 class follow.

NEW FELLOWS

John Leslie Baldan
Christopher M. Bilski
Kirk David Bitu
Amber L. Butek
James Chang
Hung Francis Cheung
Matthew Peter Collins
Keith William Curley
David A. DeNicola
Nicholas John De Palma
Ryan Michael Diehl
Melanie S. Dihora
Brian Michael Donlan
Ellen Donahue Fitzsimmons
William J. Fogarty
Sébastien Fortin
Marie LeStourgeon
Charles R. Grilliot
James Donald Heidt
Eric David Huls
Scott Robert Hurt
Young Yong Kim
Brandon Earl Kubitz
Hoi Keung Law
Amanda Marie Levinson
Gavin Xavier Lienemann
John Thomas Maher
Laura Suzanne Martin
James Paul McCoy
Shawn Allan McKenzie
James Lewis Norris
Bruce George Pendergast
Matthew James Perkins
Michele S. Raeihle
Robert Allan Rowe
Quan Shen
Summer Lynn Sipes
James M. Smieszkal
Liana St-Laurent
Keith Jeremy Sunvold
Erica W. Szeto
Malgorzata Timberg
Nicholas Jaime Williamson

NEW ASSOCIATES

Richard J. Bell III
Darryl Robert Benjamin
Stacey Jo Bitler
Karen Beth Buchbinder
Simon Castonguay
Denise L. Cheung
Melissa Diane Elliott
Solomon Carlos Feinberg
John S. Flattum
Jonathan W. Fox
Edward Lionberger
Brent Layne McGill
Thomas Edward Meyer
Alan E. Morris
Catherine Ann Morse
Lisa M. Nield
Frank W. Shermoen
Shannon Whalen
Stephen C. Williams
Stephen K. Woodard
Navid Zarinejad
Robert John Zehr

Mr. D'Arcy then introduced Allan Kaufman, a past president of the Society, who gave the address to new members. Following the address, Mr. D'Arcy announced Alex Jin as the recipient of the Harold W. Schloss Memorial Scholarship Fund, which is awarded to benefit a deserving and academically outstanding student in the University of Iowa's actuarial program, offered through the Department of Statistics and Actuarial Science. The student recipient is selected by the Trustees of the CAS Trust, based on the recommendation of the department chair at the University. Mr. Jin received $500 in scholarship funds.

Mr. D'Arcy then introduced Robert Anker and Mary D. Miller, who spoke about the work of the Actuarial Foundation and the American Academy of Actuaries' Casualty Practice Council, respectively.

After making a few announcements, Mr. D'Arcy concluded the business session.

Following a refreshment break, the first General Session was held from 10:00 a.m. to 11:30 a.m.

Actuarial Accountability—In a Changing World

Moderator/ Panelist:	Mary Frances Miller Past President, Casualty Actuarial Society
Panelists:	Lauren Bloom General Counsel American Academy of Actuaries
	Karen Terry Member Actuarial Standards Board

Following a short break, keynote speaker Ron Pressman addressed the meeting attendees during a luncheon held from 11:45 a.m. to 1:15 p.m. Mr. Pressman is president and CEO of GE Insurance and chairman, president, and CEO of GE Insurance Solutions, one of the world's largest reinsurance and insurance organizations.

After the luncheon, the afternoon was devoted to presentations of concurrent sessions. The panel presentations from 1:30 p.m. to 3:00 p.m. covered the following topics:

1. Actuaries Embrace Operational Risk

Moderator/ Panelist:	Donald F. Mango Director of Research and Development GE Insurance Solutions

Panelist: Dr. Ali Samad-Khan
President
OpRisk Advisory

Mark Verheyen
Vice President
Carvill America

2. Aviation Pricing and Modeling

Moderator: Michael A. Falcone
Chief Actuary
Global Aerospace

Panelists: Matthew Maddocks
Underwriter
ACE Tempest Re, London

Kurt Maureder
Actuary
GE Insurance Solutions

3. Discussion Draft on Reserving Principles

Moderator/Panelist: Bertram A. Horowitz
Chairperson
Task Force on Reserving Principles
President
Bertram Horowitz, Inc.

Panelists: Aaron M. Halpert
Principal
KPMG LLP

Jon Michelson
Owner
Expert Actuarial Services LLC

Thomas A. Ryan
Consulting Actuary
Milliman Inc.

Deborah M. Rosenberg
Deputy Chief Casualty Actuary
New York State Insurance Department

4. Estimating the Effect of Tort Reform on Medical Malpractice Costs

Moderator/ Panelist: Robert J. Walling III
Principal and Consulting Actuary
Pinnacle Actuarial Resources Inc.

Panelists: Kevin M. Bingham
Senior Manager
Deloitte Consulting LLP

Richard B. Lord
Principal and Consulting Actuary
Milliman Inc.

5. From the Actuary's "Best Estimate Range" to "Management's Best Estimate:" Which "X" Marks the Spot?

Moderator/ Panelist: Robert F. Wolf
Principal
Mercer Oliver Wyman

Panelists: Roger M. Hayne
Consulting Actuary
Milliman USA

C. K. "Stan" Khury
Principal
Bass & Khury

6. Privacy of Information

Moderator: Patrick B. Woods
Assistant Vice President and Actuary
ISO

Panelists: Jon Neiditz
Of Counsel
Lord, Bissell & Brook LLP

John B. Storey
CISSP, Director and Chief Information Security Official
ISO

7. Reinsurance: Recycled or Reinvented?

Moderator: Nolan Asch
Principal
ISO

Panelists: Craig Johnson
President
Signet Star Reinsurance Company

Michael G. Wacek
President
Odyssey America Reinsurance Corporation

After a refreshment break, the following concurrent sessions were presented from 3:30 p.m. to 5:00 p.m.:

1. Are D&O Rates Really Softening?

Moderator: Elissa M. Sirovatka
Consultant and Principal
Towers Perrin

Panelists: David K. Bradford
Executive Vice President
Advisen Ltd.

Kraig Paul Peterson
AVP and Actuary
Chubb Group of Insurance Companies

2. Auto Injury Claims: The What, Why, and How of It All

Moderator/ Panelist: Richard A. Derrig
President
OPAL Consulting LLC

Panelist: Adam Carmichael
Senior Research Associate
Insurance Research Council

3. Beyond Indications

Moderator: Jonathan White
Assistant Vice President and Actuary
The Hartford

Panelists: Peter Orlay
Director
Optimal Decisions Group

Geoffrey Werner
Senior Consultant
EMB America LLC

4. Enterprise Risk Management—The Present and the Future
CAS

Moderator: Donald F. Mango
Director, Research & Development
GE Insurance Solutions

Panelists: James E. Rech
Vice President
GPW and Associates Inc.

John Kollar
Vice President
ISO

5. Future of TRIA 2004

Moderator: Benoit Carrier
Second Vice President
GE Insurance Solutions

Panelists: Lloyd Dixon
Senior Economist
RAND Corporation

Glenn Pomeroy
Associate General Counsel-Government Relations
GE Insurance Solutions

6. Predictive Modeling—Panacea or Placebo?

Moderator: Glenn G. Meyers
Chief Actuary
ISO Innovative Analytics

Panelists: Daniel Finnegan
President
ISO Innovative Analytics

Cheng-Sheng Peter Wu
Director
Deloitte & Touche LLP

7. Presenting Dynamic Financial Analysis Results to Decision Makers

Moderator: Mark R. Shapland
Actuary
Milliman USA

Panelists: Raju Bohra
Vice President–Client Modeling
American Re-Insurance Company

Michael R. Larsen
Working Party Chair, Property Consultant
The Hartford

Aleksey Popelyukhin
Vice President, Information Systems
2 Wings Risk Services

An officers' reception for New Fellows and accompanying persons was held from 5:30 p.m. to 6:30 p.m.

A general reception for all attendees followed from 6:30 p.m. to 7:30 p.m.

Tuesday, May 17, 2005

Registration continued from 7:00 a.m. to 8:00 a.m.

The following General Sessions were held from 8:00 a.m. to 9:30 a.m.

The Industry's Ability to Attract Capital Given Historically Low ROE

Moderator: Gail M. Ross
Manager and Senior Consultant
Milliman USA

Panelists: Jeffrey Cohen
Principal
MMC Capital Inc.

Rajat Duggal
Managing Director
Friedman, Fleischer & Lowe LLC

Joan Lamm-Tennant
Senior Vice President
General Reinsurance Corporation

The Actuarial Role in Mergers and Acquisitions (PR)

Moderator/ Panelist: Alan Hines
Principal Consultant
PricewaterhouseCoopers LLP

Panelists: John Butler
Senior Vice President
Houlihan, Lokey, Howard & Zukin

Hugh T. McCormick
Executive Vice President, Corporate Development
Scottish Re Group Limited

James Toole
Managing Director, Life and Health
MBA Actuaries

After a break, the following concurrent sessions were held from 10:00 a.m. to 11:30 a.m.

1. Interaction with Underwriters

Moderator: Christopher M. Norman
Actuary
United Services Automobile Association

Panelists: Brian Evans
Chief Underwriter, Individual Risk E & S
GE Insurance Solutions

John Herder
Senior Actuary
GE Insurance Solutions

2. Managing the Insurance Cycle

Moderator: Benoit Carrier
Second Vice President
GE Insurance Solutions

Panelists: Joseph A. Boor
Reserving Actuary
Florida Department of Financial Services

Harry Shuford
Chief Economist
NCCI

3. Update on a Global Risk-Based Capital Standard

Moderator: Robert F. Wolf
Principal
Mercer Oliver Wyman

Panelists: Glenn G. Meyers
Chief Actuary
ISO Innovative Analytics

Nino Savelli
Associate Professor
Catholic University of the Sacred Heart

4. Why Do Specialty/Niche Companies Outperform Their Peers?

Moderator: David C. Snow
Managing Actuary
GE Insurance Solutions

Panelist: Gary R. Josephson
Consulting Actuary
Milliman, Inc.

Michael F. McManus
Senior Vice President and Chief Actuary
Chubb Group of Insurance Companies

Kenneth Quintilian
Vice President and Chief Actuary
Medical Liability Mutual Insurance Company

After a break for lunch, a golf tournament commenced at 1:00 p.m. Certain sessions were repeated and the following concurrent sessions presented from 12:30 p.m. to 2:00 p.m.:

1. Insurance Accounting for Actuaries

Moderator: Michael C. Dubin
Director
PricewaterhouseCoopers LLP

Panelists: Roger M. Hayne
Consulting Actuary
Milliman USA

Kevin L. Wick
Principal Consultant
PricewaterhouseCoopers LLP

2. What is the Next Asbestos?

Moderator: James Larkin
Chief Actuary, Broker Market
American Re-Insurance Company

Panelists: Bonnie L. Boccitto
Senior Vice President
American Re-Insurance Company

John P. Yonkunas
Principal
Towers Perrin

The following discussion papers were presented from 12:30 p.m. to 2:00 p.m.:

1. "Risk Measurement in Insurance: A Guide to Risk Measurement, Capital Allocation, and Related Decision Support Issues"

Author: Paul Kaye
Benfield Group

2. "Modeling the Solvency Impact of TRIA on the Workers Compensation Insurance Industry"

Authors: Harry Shuford
Chief Economist
National Council on Compensation Insurance

Jonathan Evans
Actuary
National Council on Compensation Insurance

The day concluded with a western barbeque and entertainment for all attendees at Rustler's Rooste Barn from 6:30 p.m. to 9:30 p.m.

Wednesday, May 18, 2005

From 8:00 a.m. to 9:30 a.m. the following concurrent sessions were held:

1. 2004 U.S. Hurricanes and 2005 Reinsurance Market

Moderator/ Panelist: Thomas E. Hettinger
Managing Director
EMB America LLC

Panelist: Randall E. Brubaker
Senior Vice President
Aon Corporation

2. Investment Principles—Session with the CAS Investment Committee

Moderator/ Panelist: François Morin
Principal
Towers Perrin

Panelists: Curtis Gary Dean
Distinguished Professor of Actuarial Science
Ball State University

Todd Rogers
Director of Finance and Operations
Casualty Actuarial Society

3. The State of Construction Defects

Moderator/ Panelist: Ronald T. Kozlowski
Principal
Towers Perrin

Panelist: Paul Swank
Senior Claims Consultant
Towers Perrin

4. Terrorism Modeling

Moderator: Rhonda K. Aikens
Second Vice President
GE Insurance Solutions

Panelists: François Dagneau
Senior Vice President
AON Re Canada Inc.

Timothy Tetlow
Senior Vice President Global Reinsurance
Axis Specialty Limited, Bermuda

From 8:00 a.m. to 9:30 a.m., the following *Proceedings* papers were presented:

1. "An Examination of the Influence of Leading Actuarial Journals"

Author: L. Lee Colquitt

2. "Riskiness Leverage Models"

Author: Rodney E. Kreps

After a break, the final General Session was held from 10:00 a.m. to 11:30 a.m.

The Future of Finite Insurance

Moderator: Marc F. Oberholtzer
Director
PricewaterhouseCoopers LLP

Panelists: Keith Buckley
Fitch Ratings

Christopher E. Hall
Vice President, Senior Accounting Analyst
Moody's Investors Service

Kenneth Kruger
Senior Vice President
Willis Re., Inc.

Daniel Malloy
Executive Vice President
Benfield

At the conclusion of this session, Mr. D'Arcy announced future meetings and seminars and adjourned the meeting.

Attendees of the 2005 CAS Spring Meeting

The 2005 CAS Spring Meeting was attended by 415 Fellows, 118 Associates, two Affiliate members, and 60 guests. The names of the Fellows, Associates, and Affiliates in attendance follow:

FELLOWS

Jeffrey R. Adcock
Barbara J. Addie
Mark A. Addiego
Rhonda K. Aikens
Terry J. Alfuth
Ethan D. Allen
Timothy Paul Aman
Paul D. Anderson
Robert A. Anker
Deborah Herman Ardern
Nolan E. Asch
Martha E. Ashman
Peter Attanasio
Richard J. Babel
John L. Baldan
Phillip W. Banet
D. Lee Barclay
Irene K. Bass
Edward J. Baum
Nicolas Beaupre
Andrew Steven Becker
Jody J. Bembenek
Abbe Sohne Bensimon
Regina M. Berens
Raji Bhagavatula
Kirk D. Bitu
Ralph S. Blanchard
Carol Blomstrom
Bonnie L. Boccitto
Neil M. Bodoff
Raju Bohra
Ann M. Bok
Joseph A. Boor
Ronald L. Bornhuetter
Peter T. Bothwell
Amy S. Bouska
Jerelyn S. Boysia
Nancy A. Braithwaite

Paul Braithwaite
Kelly A. Bramwell
Michael D. Brannon
Rebecca Bredehoeft
Mark D. Brissman
Linda K. Brobeck
Dale L. Brooks
Randall E. Brubaker
Stephanie Anne Bruno
Ron Brusky
George Burger
Hayden Heschel Burrus
Michelle L. Busch
James E. Calton
Douglas A. Carlone
Christopher S. Carlson
Allison Faith Carp
William Brent Carr
Benoit Carrier
Bethany L. Cass
R. Scott Cederburg
Dennis K. Chan
Hung Francis Cheung
Thomas Joseph Chisholm
Kin Lun (Victor) Choi
Stephan L. Christiansen
Mark M. Cis
Susan M. Cleaver
J. Paul Cochran
William Brian Cody
Jeffrey R. Cole
Matthew P. Collins
Larry Kevin Conlee
Hugo Corbeil
Brian C. Cornelison
Francis X. Corr
Michael J. Covert
Brian K. Cox
Richard R. Crabb
Alan M. Crowe
M. Elizabeth Cunningham
Keith W. Curley
Diana M. Currie
Ross A. Currie
François Dagneau
Stephen P. D'Arcy
Lawrence S. Davis
John Dawson
John D. Deacon
Curtis Gary Dean
Kris D. DeFrain
Jeffrey F. Deigl
Camley A. Delach
David A. DeNicola
Nicholas J. DePalma
Robert V. Deutsch
John T. Devereux
Chris Diamantoukos
Melanie Sue Dihora
Mark A. Doepke
Andrew J. Doll
Robert B. Downer
Michael C. Dubin
Judith E. Dukatz
Tammi B. Dulberger
Dennis Herman Dunham
Gregory L. Dunn
Kenneth Easlon
Jeffrey Eddinger
Grover M. Edie
Dale R. Edlefson
David M. Elkins
Julia L. Evanello
Jonathan Palmer Evans
Philip A. Evensen
Michael A. Falcone
Vicki A. Fendley
Mark E. Fiebrink
Beth E. Fitzgerald
Ellen D. Fitzsimmons
Chauncey Edwin Fleetwood
Daniel J. Flick
William J. Fogarty
David A. Foley
Sean Paul Forbes
Feifei Ford
Hugo Fortin
Sebastien Fortin
Louise A. Francis
Dana R. Frantz
Michelle L. Freitag
Bruce F. Friedberg
Michael Fusco
Jean-Pierre Gagnon
James E. Gant
Louis Gariepy
Roberta J. Garland
John F. Gibson
Isabelle Gingras
Gregory S. Girard
John T. Gleba
Joel D. Glockler
Andrew Samuel Golfin
Matthew R. Gorrell

Ann E. Green
John E. Green
Steven A. Green
Ann V. Griffith
Charles R. Grilliot
Stacie R. W. Grindstaff
Linda M. Groh
Jacqueline Lewis Gronski
Nasser Hadidi
James A. Hall
Robert C. Hallstrom
Alexander Hammett
George M. Hansen
David G. Hartman
Eric Christian Hassel
Roger M. Hayne
Scott E. Henck
John Herder
Jay T. Hieb
Alan M. Hines
Todd H. Hoivik
Mark J. Homan
Allen J. Hope
David J. Horn
Bertram A. Horowitz
Mary T. Hosford
Marie-Josée Huard
Scott R. Hurt
Paul R. Hussian
Jamison Joel Ihrke
Philip M. Imm
Aguedo M. Ingco
Craig D. Isaacs
Paul Ivanovskis
Christopher D. Jacks
Charles B. Jin
Eric J. Johnson
Erik A. Johnson
Warren H. Johnson
Thomas S. Johnston
Steven M. Jokerst
Jeffrey R. Jordan
Julie A. Jordan
Gary R. Josephson
Erin Hye-Sook Kang
Kyewook Gary Kang
Robert B. Katzman
Allan M. Kaufman
Robert J. Kelley
Brian Danforth Kemp
David R. Kennerud
C. K. “Stan” Khury
Young Y. Kim
Ziv Kimmel
Joseph E. Kirsits
Craig W. Kliethermes
Joan M. Klucarich
Timothy F. Koester
John J. Kollar
Ronald T. Kozlowski
Israel Krakowski
Rodney E. Kreps
John R. Kryczka
Brandon E. Kubitz
Andrew E. Kudera
Howard A. Kunst
Steven M. Lacke
Paul E. Lacko
Blair W. Laddusaw
Salvatore T. LaDuca
D. Scott Lamb
Dean K. Lamb
Robin M. LaPrete
James W. Larkin
Michael R. Larsen
Aaron M. Larson
Dawn M. Lawson
Thomas V. Le
Borwen Lee
P. Claude Lefébvre
Scott J. Lefkowitz
Steven G. Lehmann
Yuxiang Reng Lin Lei
Bradley H. Lemons
Paul B. LeStourgeon
Kenneth A. Levine
Amanda M. Levinson
Sally Margaret Levy
Peter M. Licht
Gavin X. Lienemann
Jia (Judy) Liu
Jan A. Lommele
Richard Borge Lord
Robert G. Lowery
Robb W. Luck
John T. Maher
Donald F. Mango
Jason Aaron Martin
Jason N. Masch
Stuart B. Mathewson
Robert W. Matthews
Jeffrey H. Mayer
Michael G. McCarter
James P. McCoy
Heather L. McIntosh
Shawn Allan McKenzie
Michael F. McManus

John D. McMichael
Timothy Messier
Glenn G. Meyers
Ryan A. Michel
Jon W. Michelson
Mary Frances Miller
Ronald R. Miller
William J. Miller
Paul W. Mills
Claudine H. Modlin
F. James Mohl
Christopher J. Monsour
David Patrick Moore
Matthew Kevin Moran
François Morin
Robert Joseph Moser
Timothy C. Mosler
Roosevelt C. Mosley Jr.
James C. Murphy
Jarow G. Myers
Karen E. Myers
Thomas G. Myers
Chris E. Nelson
Kenneth J. Nemlick
Aaron West Newhoff
Lynn Nielsen
John Nissenbaum
Darci Z. Noonan
Christopher M. Norman
James L. Norris
Corine Nutting
James L. Nutting
David J. Oakden
Steven Brian Oakley
Richard Alan Olsen
Mary L. O'Neil
Todd F. Orrett
Joanne M. Ottone
Michael Guerin Owen
Richard D. Pagnozzi
Ajay Pahwa
Robert G. Palm
Joseph M. Palmer
Cosimo Pantaleo
Curtis M. Parker
Julie Parsons
M. Charles Parsons
John R. Pedrick
Bruce G. Pendergast
Matthew J. Perkins
Kraig Paul Peterson
Mark W. Phillips
Daniel C. Pickens
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MODELING FINANCIAL SCENARIOS: A FRAMEWORK FOR THE ACTUARIAL PROFESSION

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Abstract

This paper summarizes the research project on Modeling of Economic Series Coordinated with Interest Rate Scenarios initiated by the joint request for proposals by the Casualty Actuarial Society and the Society of Actuaries. The project involved the construction of a financial scenario model that simulates a variety of economic variables over a 50-year period. The variables projected by this model include interest rates, inflation, equity returns, dividend yields, real estate returns, and unemployment rates. This paper contains a description of the key issues involved in modeling these series, a review of the primary literature in this area, an explanation of parameter selection issues, and an illustration of the model's output. The paper is intended to serve as a practical guide to understanding the financial scenario model in order to facilitate the use of this model for such actuarial applications as dynamic financial analysis, development

of solvency margins, cash flow testing, operational planning, and other financial analyses of insurer operations.

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1. INTRODUCTION

The insurance industry is increasingly relying on financial models. Financial models are an integral part of any dynamic financial analysis (DFA) approach and are frequently used for solvency regulation, capital allocation, and pricing insurance policies. Financial models can also be used to determine the economic value of loss reserves. As financial models become a widely used tool, actuaries have a greater need to understand current models and to develop improvements.

A considerable amount of research suggests that sophisticated tools are needed to accurately evaluate the financial condition of insurers. Santomero and Babbel [37] review the financial risk management practices of both the life and property-liability insurers and find that significant improvements are necessary. They state that even the most advanced insurers are not effectively managing their financial risks. Research also shows that the potential consequences of the lack of risk measurement cannot be ignored. A study by the Casualty Actuarial Society Financial Analysis Committee [9] discusses the potential impact of interest rate risk for property-liability insurers. Hodes and Feldblum [26] examine the effects of interest rate risk on the assets and liabilities of a property-liability insurer and conclude that "casualty actuaries must understand interest rate risk thoroughly if they wish to participate in the industry discussions and to influence the coming professional and regulatory guidelines." Staking and

Babbel [41] find that significant work is needed to better understand the interest rate sensitivity of an insurer's surplus. D'Arcy and Gorvett [14] and Ahlgrim, D'Arcy, and Gorvett [2] apply more advanced measures to determine the interest rate sensitivity of loss reserves and illustrate how these measures depend on the interest rate model chosen. All of these articles focus on the need for a better understanding of the financial risks facing an insurance company.

Many actuaries are now familiar with the traditional techniques that form the basis of asset-liability management (ALM), including the measures of duration, convexity, and the term structure of interest rates (Hull [27], Chapter 5). Duration and convexity help insurers understand their interest rate sensitivity and assist portfolio managers in reducing surplus volatility. However, the calculations for duration and convexity rely heavily on underlying assumptions about the level and potential movements of interest rates, and these issues have not been thoroughly evaluated by the actuarial community.

In order to enhance actuaries' understanding of financial models, the Casualty Actuarial Society (CAS) and the Society of Actuaries (SOA) jointly issued a request for proposals on the research topic "Modeling of Economic Series Coordinated with Interest Rate Scenarios." There were several specific objectives of the request:

- Review the previous literature in the area of economic scenario modeling;
- Determine appropriate data sources and methodologies to enhance economic modeling efforts relevant to the actuarial profession; and
- Produce a working model of economic series, coordinated with interest rates, that could be made public and used by actuaries via the CAS and SOA Web Sites to project future economic scenarios.

The economic series to be included in the model were interest rates, equity returns, inflation rates, unemployment rates, and real estate returns. An important consideration in this project is the recognition of the interdependencies between the various economic and financial series—for example, between interest rates and inflation and between equity returns and interest rate movements.

This paper provides a summary of the development of a scenario generation model, which is now available for public use. This work represents an initial step in the process of helping actuaries develop a better understanding of financial risk. The complete project, available on the CAS Web Site (http://casact.org/research/econ/), includes a literature review, the model, sample results, and a user's guide.

This paper is organized as follows. Section 2 reviews some of the key issues from the financial modeling literature, including term structure and equity return development. Previous actuarial models are also discussed. Section 3 describes the underlying variables of the model, illustrates how each process is simulated, discusses how the default parameters of the process were selected, and provides sources of data that are used to select the appropriate parameters. Section 4 briefly explains how to use the financial scenario model and discusses how to incorporate the model into other actuarial applications. Section 5 illustrates the use of the model, summarizes the output produced in one simulation, and includes a number of tabular and graphical displays of the output. Section 6 provides a conclusion that encourages actuaries to advance the work in this area.

2. ISSUES AND LITERATURE REVIEW

There are many issues involved in building an integrated financial scenario model for actuarial use. This section reviews the literature in the modeling of the term structure and equity returns. In addition, the financial models in the actuarial literature are reviewed.

Term Structure Modeling

Insurance companies have large investments in fixed income securities, and their liabilities often have significant interest rate sensitivities. Therefore, any financial model of insurance operations must include an interest rate model at its core. This section describes some of the relevant research issues involved in term structure modeling. (For an overview of fixed income markets, see Tuckman [42].)

The role of the financial scenario generator is not to explain past movements in interest rates, nor is the model attempting to perfectly predict interest rates in any future period in order to exploit potential trading profits.[1] Rather, the model purports to depict plausible interest rate scenarios that may be observed at some point in the future. Ideally, the model should allow for a wide variety of interest rate environments to which an insurer might be exposed.

The literature in the area of interest rate modeling is voluminous. One strand of the literature looks to explore the possibility of predictive power in the term structure. Fama [17] uses forward rates in an attempt to forecast future spot rates. He finds evidence that very short-term (one-month) forward rates can forecast spot rates one month ahead. Fama and Bliss [19] examine expected returns on U.S. Treasury securities with maturities of up to five years. They find that the one-year interest rate has a mean-reverting tendency, which results in one-year forward rates having some long-term forecasting power.

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[1] It might be noted that trying to develop a model that mimics past rate movements may be a futile exercise since, despite the volume of research in the area, no tractable model has yet been shown to be satisfactory in accurately explaining history.

Historical Interest Rate Movements

Other research reviews historical interest rate movements in an attempt to determine general characteristics of plausible interest rate scenarios. Ahlgrim, D'Arcy, and Gorvett [3] review historical interest rate movements from 1953 to 1999, summarizing the key elements of these movements. Chapman and Pearson [12] provide a similar review of history in an attempt to assess what is known about interest rate movements (or at least what is commonly accepted) and what is unknown (or unknowable). Litterman and Scheinkman [32] use principal component analysis to isolate the most important factors driving movements of the entire term structure. Some of the findings of these studies include

- Short-term interest rates are more volatile than long-term rates. Ahlgrim, D'Arcy, and Gorvett [3] use statistics (such as standard deviation) to show that long-term rates tend to be somewhat tethered, while short-term rates tend to be much more dispersed. (A graphical presentation of historical interest rate movements is available at http://www.business.uiuc.edu/∼sdarcy/present/casdfa3/GraphShow.exe.)
- Interest rates appear to revert to some "average" level. For example, when interest rates are high, there is a tendency for rates to subsequently fall. Similarly, when rates are low, they later tend to increase. While economically plausible, Chapman and Pearson [12] point out that due to a relatively short history of data, there is only weak support for mean reversion. If anything, evidence suggests that mean reversion is strong only in extreme interest rate environments (see also Chapman and Pearson [11]).
- While interest rate movements are complex, 99% of the total variation in the term structure can be explained by three basic shifts. Litterman and Scheinkman [32] show that over 90% of the movement in the term structure can be explained by simple parallel shifts (called the *level* component). Adding a shift in

the *slope* of the term structure improves explanatory power to over 95%. Finally, including U-shaped shifts (called *curvature*) explains over 99% of the variation observed in historical term structure movements. Chapman and Pearson [12] confirm that these three factors are persistent over different time periods.

- Volatility of interest rates is related to the level of the short-term interest rate. Chapman and Pearson [12] further point out that the appropriate measure for volatility depends on whether the period from 1979 to 1982—when the Federal Reserve shifted policy from focusing on interest rates to controlling inflation, resulting in a rapid increase in interest rates—is treated as an aberration or included in the sample period.

Equilibrium and Arbitrage Free Models

Several popular models have been proposed to incorporate some of the characteristics of historical interest rate movements. Often these continuous time models are based on only one stochastic factor, movements (changes) in the short-term interest rate (the instantaneous rate). A generic form of a one-factor term structure model is

$$dr_t = \kappa(\theta - r_t)dt + \sigma rt^{\gamma} dB_t. \tag{2.1}$$

Formula (2.1) incorporates mean reversion. To see this, consider the case where the current level of the short-term rate (r_t) is above the mean reversion level θ. The change in the interest rate is then expected to be negative—interest rates are expected to fall. The speed of the reversion is determined by the parameter κ. The last term in (2.1) incorporates the unknown, volatile component of interest rate changes over the next instant. The last term, dB_t, is the change in a Brownian motion—it has mean zero and variance equal to dt. This uncertainty is scaled by the volatility parameter σ. If $\gamma > 0$, then interest rate volatility is related to the level of the interest rate. When $\gamma = 0$, this model is equivalent to the formulation of Vasicek [43]; when $\gamma = 0.5$, the model is the process proposed by Cox, Ingersoll, Ross [13] (hereafter CIR). Chan et al. [10] estimate this class of interest rate models and determine that based on monthly data from 1964 through 1989 the value of γ is approximately 1.5.

Models of the type shown in Formula (2.1) are called "equilibrium models" since investors price bonds by responding to the known expectations of future interest rates. Using the assumed process for short-term rates, one can determine the yield on longer-term bonds by looking at the expected path of interest rates until the bond's maturity. To determine the full-term structure, one can price bonds of any maturity based on the expected evolution in short-term rates over the life of the bond:[2]

$$P(t,T) = E\left[\exp\left(-\int_t^T r_u du\right)\right] \tag{2.2}$$

where $P(t,T)$ is the time t price of a bond that pays \$1 in $(T-t)$ years. One of the primary advantages of equilibrium models is that bond prices and many other interest rate contingent claims have closed-form analytic solutions. Vasicek and CIR evaluate Formula (2.2) to find bond prices:

$$P(t,T) = A(t,T)e^{-r_t B(t,T)}, \tag{2.3}$$

where $A(t,T)$ and $B(t,T)$ are functions of the known process parameters κ, θ, and σ. Therefore, given a realized value for r_t, rates of all maturities can be obtained.

One immediate problem with equilibrium models of the term structure is that the resulting term structure is inconsistent with observed market prices, even if the parameters of the model are chosen carefully; while internally consistent, equilibrium models are at odds with the way the market is actually pricing bonds. Where equilibrium models generate the term structure as an output, "arbitrage-free models" take the term structure as an input. All future interest rate paths are projected from the existing yield curve.

[2]It should be noted that the expectations in Formula (2.2) are evaluated under the risk neutral measure. See chapter 9 of Tuckman [42] for an introduction to risk neutral valuation of bonds.

Ho and Lee [25] discuss a discrete time model of the no-arbitrage approach and include a time-dependent drift so that observed market prices of all bonds can be replicated. The continuous-time equivalent of the Ho-Lee model is

$$dr_t = \theta(t)dt + \sigma dB_t. \tag{2.4}$$

The time-dependent drift ($\theta(t)$) of the Ho and Lee model is selected so that expected future interest rates agree with market expectations as reflected in the existing term structure. This drift is closely related to implied forward rates. Hull and White [28] use Ho and Lee's [25] time-dependent drift to extend the equilibrium models of Vasicek and CIR. The one-factor Hull-White model is

$$dr_t = \kappa(\theta(t) - r_t)dt + \sigma dB_t. \tag{2.5}$$

Heath, Jarrow, and Morton [23] (hereafter HJM) generalize the arbitrage-free approach by allowing movements across the entire term structure rather than a single process for the short rate. HJM posit a family of forward rate processes, $f(t,T)$. In this family

$$df(t,T) = \mu(t,T,f(t,T))dt + \sigma(t,T,f(t,T))dB_t, \tag{2.6}$$

where

$$f(t,T) = -\frac{\partial \ln P(t,T)}{\partial T}. \tag{2.7}$$

Choosing between an arbitrage-free term structure model and an equilibrium model often depends on the specific application. Despite their initial appeal, arbitrage-free approaches often have disadvantages.[3] Some of these include the following:

- Arbitrage-free models are most useful for pricing purposes, especially interest rate derivatives. Since derivatives are priced against the underlying assets, a model that explicitly captures the market prices of those underlying assets is superior to models that more or less ignore market values. Hull [27] comments

[3]In addition to the references in this section, Tuckman [42] provides an excellent overview of the advantages and disadvantages of equilibrium models vs. arbitrage-free models.

that equilibrium models are judged to be inferior since traders will have little confidence in the price of an option if the model cannot accurately price the underlying asset. Research supports this argument. Jegadeesh [31] looks at the pricing of interest rate caps and determines that arbitrage-free models price interest rate caps more accurately than equilibrium models. Unfortunately, the pricing accuracy of arbitrage-free term structure models is based on short pricing horizons; there have been no formal comparative tests of the pricing accuracy using long-term assets.

- Fitton and McNatt [21] comment that arbitrage-free models are most useful for short-term pricing applications when similar market data are readily available. Arbitrage-free models are intractable over long periods of time. With many arbitrage-free models, the forward rate plays a central role in the expected path of interest rates. Forward rates are related to the slope of the term structure and may exhibit strange behavior that significantly impacts projections of interest rate paths in arbitrage-free term structure models. For steeply sloped yield curves, the forward rate may become very large. For parts of the term structure that are downward sloping, the forward rate may even become negative. Especially for long-term projections, simulation paths may become extreme since the effects of small fluctuations in the term structure are magnified in long-term forward rates. For long-term analysis, equilibrium models are more appropriate.

- Arbitrage-free models also suffer from inconsistency across time (see Wilmott [46] and Tuckman [42]). As mentioned above, many arbitrage-free term structure models assume that the risk-free rate is closely related to the forward rate curve. The forward curve is often quite dissimilar at different points in time. For example, at time 0, the model uses the existing term structure to determine forward rates for years into the future. If the model were correct, we should be able to restart the simulation at some subsequent time t using the forward rates for longer maturities that were implied from the earlier projection. Clearly the actual path of interest rates will differ from the implied forward rate curve, as well as the volatilities of these rates. This requires the model to be refit to available market data each time it is used, which means that future projections make different assumptions about future spot rates and volatilities. Equilibrium models provide more consistent statements about projected interest rates over time.

- Determining the input into an arbitrage-free model is not straightforward. One usually considers the term structure implied by risk-free securities such as U.S. Treasuries. There are several difficulties in looking at U.S. Treasury data. First, market data gathered from zero coupon securities data, such as STRIPs (separate trading of registered interest and principal securities), are noisy, especially at long maturities. An alternative source for long-term interest rate data is to look at yields on long-term U.S. Treasury bonds. However, the liquidity of these long-term coupon bonds is suspect, and since on-the-run (the most current issue of a particular bond) Treasury securities typically have higher liquidity (and higher prices), yields of the longest maturity bonds are forced down. The forward rate curve initially reflects interest rate information for short-term rates, but for longer maturities, liquidity issues dominate. The result is a strangely shaped forward rate curve that can have significant undulations stemming from illiquidity. In addition, the future of 30-year bonds is uncertain given the Treasury's curtailment of 30-year bond issues. Fewer points on the term structure make arbitrage-free models very sensitive to the market data and particularly vulnerable to market inefficiencies. Equilibrium models do not suffer from these "dirty" data issues.

- Depending on the specific arbitrage-free model, one may have to resort to numerical techniques such as simulation or interest rate trees to value contingent claims. Equilibrium models often have closed-form solutions for common interest rate dependent securities.

Single- vs. Multifactor Models

The models presented above are all one-factor term structure models since there is only a single variable generating stochastic movements in interest rates. One problem with one-factor models is that the single source of uncertainty drives all term structure movements. As a result, yields of all maturities are perfectly correlated to the one stochastic factor and the range of potential yield curves is limited. The effects of multi-dimensional moves in the term structure can have serious consequences on a portfolio's value. Reitano [35] demonstrates that even small nonparallel shifts in the yield curve can cause extreme changes in asset values.

Introducing additional sources of uncertainty (such as allowing the long end of the curve to fluctuate or introducing stochastic volatility or both) provide for a fuller range of yield curve movements and shapes. The downside is that introducing multiple dimensions of yield curve movements quickly increases the complexity and tractability of the model. Choosing the number of stochastic factors for a term structure model represents an important balance between accuracy and simplicity.

To illustrate an example of a multifactor term structure model, Hull and White [29] extend the one-factor Hull-White model [28] to include a stochastic mean reversion level:

$$\begin{aligned} dr_t &= (\theta(t) + u_t - ar_t)dt + \sigma_1 dB_{1t} \\ du_t &= -bu_t dt + \sigma_2 dB_{2t}. \end{aligned} \tag{2.8}$$

Similar to the one-factor Hull-White model, the instantaneous short-term rate (r_t) reverts to some time-dependent reversion level ($\theta(t) + u_t$). The introduction of a stochastic process for u_t in the second equation in Formula (2.8) shows that the mean reversion level is also variable. The effect of introducing this second stochastic factor is to allow movements at opposite ends of the yield curve. Any correlation between short and long rates is accounted for in the correlation of the Brownian motion components of Formula (2.8).

Summary of Term Structure Issues

The final choice of term structure model is a decision that frequently elicits passionate debate. Decisions are needed to select among the various kinds of assumptions including matching the existing term structure (equilibrium vs. arbitrage-free model), the number of parameters employed, and so on. In making these decisions, it is vital to bear in mind the application of the model. The choice of a term structure model is likely to be different for short-term applications that require precision and comparability to traded securities than for long-term strategic planning exercises.

For this research, it is *not* intended that our model will be used for trading purposes. Rather, it is meant to give insurers a range of potential interest rate scenarios that are possible in the future. In selecting a term structure model for the financial scenario generator, we attempted to balance three important (and often opposing) goals: (1) mimicking the key historical characteristics of term structure movements, (2) generating the entire term structure for any future projection date, and (3) recognizing the desire for parsimony.

The first concern led us to a multifactor model that allows for some flexibility in yield curve shapes. While single-factor models are often easier to describe and use, their restricted yield dynamics are too important for insurers to ignore. The second issue highlights the importance of interest rates of all time horizons, not of any specific key rates on the curve. Based on the realizations of the limited number of stochastic factors, we preferred term structure models that have closed-form solutions for bond prices so that the entire term structure can be quickly and easily retrieved. When closed-form solutions for bond yields are available, this allows users of the term structure model to track all interest rates on the yield curve during a simulation, not a limited few. For example, users of a term structure model who are interested in mortgage prepayment rates will be interested in the refinancing rate, which may be closely related to bond yields of specific maturities (such as 10 years). Other users may be concerned about crediting rates that are a function of historical 5-year interest rates. Without some explicit closed-form solution, the modeler has no foundation to imply yields of different maturities from a limited set of stochastic factors. The two-factor equilibrium model selected for the financial scenario model is described in the third section of this paper.

Equity Returns

Similar to interest rates, there have been many studies that have looked at the behavior of equity returns. Shiller [39] and Siegel [38] analyze long-term patterns in stock returns and provide helpful analyses of long-term trends. Sornette [40] examines the behavior of stock markets, investigating why complex systems such as stock markets crash.

Often equity returns are assumed to follow a normal distribution. For example, in the development of their famous option pricing formula, Black and Scholes [7] assume that (continuously compounded) returns for stocks are normally distributed. However, historical observation of equity returns reveals that the distribution has "fatter tails" than predicted by the assumption of normality (Campbell, Lo, and MacKinlay [8]).

A number of alternative assumptions have been proposed for stock movements. Alexander [4] summarizes a variety of substitutes, including generalized autoregressive conditional heteroskedasticity (GARCH) processes and principal component analysis. Hardy [22] uses a regime-switching model for stock returns and concludes that the performance of the regime-switching model is favorable relative to competing models. To understand the rationale for Hardy's model, consider the severe decline of the stock market in October 1987. This single observation may appear to be too "extreme" and very unlikely given a single-variance assumption. Instead, suppose that equity returns at any point in time are generated from two distinct distributions, a "high volatility" regime or a "low volatility" regime. The chance of switching from one regime to the other over the next time step is dictated by transition probabilities. During times of economic instability, the returns on equities may be more uncertain, representing a transition to the high volatility regime. Thus, the observation from October 1987 may simply be a draw from the high volatility regime.

We use Hardy's approach for equity returns but apply the regime-switching process to excess returns over and above the nominal risk-free rate. At any point in time, the excess return of stocks is a draw from a normal distribution that is conditional on the current regime.[4] For each period, there is a matrix of probabilities that dictate the movement between regimes. While there is no limit to the number of regimes that can be embedded in the model, Hardy finds only marginal improvement in fit when extending the equity return model to more than two regimes.

Given two regimes (i.e., i and j), Hardy uses these transition probabilities to determine the unconditional probability π_i of being in state i at any point in time:

$$\pi_i = \frac{p_{j,i}}{p_{i,j} + p_{j,i}}, \qquad \pi_j = 1 - \pi_i. \tag{2.9}$$

Actuarial Models

Redington [34] pioneered the work in modeling insurers. This early work introduced the concept of immunization against interest rate risk and introduced the "funnel of doubt" terminology to convey uncertainty in outcomes. Modern approaches to modeling (including this research) focus first on assumptions of the

[4] Ahlgrim and D'Arcy [1] extend this regime-switching approach to international equities.

external economic and financial environment before incorporating the impact of these variables on the operations of the insurer.

Wilkie's [44] model proposes inflation as the independent variable, using a first-order autoregressive model to simulate inflation. Wilkie links the realization of inflation with other variables using a cascade approach. Wilkie's original model [44] includes (1) dividends, (2) dividend yields, and (3) interest rates.

Wilkie [45] updates his earlier work by expanding on the structural form of the processes used to represent key variables in his "stochastic investment model." The paper includes several appendices that fully develop the time-series tools used throughout the presentation, including cointegration, simultaneity, vector autoregression (VAR), autoregressive conditional heteroskedasticity (ARCH), and forecasting. Wilkie [45] also estimates parameters for each equation of the model by looking at data from 1923 through 1994 and performs tests on competing models for fit. As in the 1986 model, Wilkie's updated model simulates inflation as an autoregressive process that drives all of the other economic variables, including dividend yields, long-term interest rates, short-term interest rates, real estate returns, wages, and foreign exchange rates. One shortfall of the Wilkie model is the inconsistent relationships generated among inflation and short-term vs. long-term interest rates. In addition, the equity returns are based on an autoregressive process that leads to a distribution of returns that is much more compact than history indicates.

Hibbert, Mowbray, and Turnbull [24] describe a model using modern financial technology that generates values for the term structure of interest rates (both real and nominal interest rates), inflation, equity returns, and dividend payouts. They use a two-factor model for both interest rates and inflation, a regime-switching model for equities, and a one-factor autoregressive dividend yield model. The paper discusses issues related to parameter selection and also illustrates a simulation under alternate parameters, comparing results with the Wilkie model.

Dynamic financial analysis (DFA) has become the label under which these financial models are combined with an insurer's operations when performing a variety of applications including pricing, reserve adequacy, and cash flow testing. D'Arcy et al. [15, 16] walk through the development of a public-access DFA model and illustrate the use of the model in a case study.

3. DESCRIPTIONS OF THE FINANCIAL SCENARIO GENERATOR AND DATA

In this section, detailed descriptions are provided for each of the economic time series included in our model. Embedded in these descriptions are references to the sources of historical time-series data used to select the parameters of the model.

Inflation

Inflation (denoted by q) is assumed to follow an Ornstein-Uhlenbeck process[5] of the form (in continuous time):

$$dq_t = \kappa_q(\mu_q - q_t)dt + \sigma_q dB_q. \tag{3.1}$$

The simulation model samples the discrete form equivalent of this process as

$$\begin{aligned} \Delta q_t &= q_{t+1} - q_t = \kappa_q(\mu_q - q_t)\Delta t + \varepsilon_q\sigma_q\sqrt{\Delta t} \\ q_{t+1} &= q_t + \kappa_q(\mu_q - q_t)\Delta t + \varepsilon_q\sigma_q\sqrt{\Delta t} \\ &= \kappa_q\Delta t \cdot \mu_q + (1 - \kappa_q\Delta t)\cdot q_t + \varepsilon_q\sigma_q\sqrt{\Delta t}. \end{aligned} \tag{3.2}$$

From this last equation, we can see that the expected level of future inflation is a weighted average of the most recent value of inflation (q_t) and a mean reversion level of inflation, μ_q. The speed of reversion is determined by the parameter κ_q. In the continuous model, mean reversion can be seen by considering the first term on the right-hand side of Formula (3.1) (which is called the drift of the process). If the current level of inflation q_t is

[5] The Vasicek process discussed in Section 2 is also an Ornstein-Uhlenbeck process.

above the mean reversion level, the first term is negative. Therefore, Formula (3.1) predicts that the expected change in inflation will be negative; that is, inflation is expected to fall. The second term on the right-hand side of Formula (3.1) represents the uncertainty in the process. The change in Brownian motion (dB_t) can be likened to a draw from a standardized normal random variable (represented by ε_q in the discrete form of the model). The uncertainty is scaled by the parameter σ_q, which affects the magnitude of the volatility associated with the inflation process.

We can rearrange the last equation above to show that the Orstein-Uhlenbeck process is a continuous time version of a first-order autoregressive process:

$$\begin{aligned} q_{t+1} - \mu_q &= \mu_q \kappa_q \Delta t - \mu_q + (1 - \kappa_q \Delta t) \cdot q_t + \varepsilon_q \sigma_q \sqrt{\Delta t} \\ &= (1 - \kappa_q \Delta t) \cdot q_t - (1 - \kappa_q \Delta t) \cdot \mu_q + \varepsilon_q \sigma_q \sqrt{\Delta t} \\ &= (1 - \kappa_q \Delta t) \cdot (q_t - \mu_q) + \varepsilon_q \sigma_q \sqrt{\Delta t}. \end{aligned} \tag{3.3}$$

Using the last equation in (3.2), we can estimate the parameters of the inflation model using the following time-series regression:

$$q_{t+1} = \alpha + \beta q_t + \varepsilon'_{qt}. \tag{3.4}$$

Note that we have not run the regression using the change in inflation as the dependent variable since this would not allow us to simultaneously derive the mean reversion speed (κ_q) and the mean reversion level (μ_q). To derive the parameters of the inflation process, we transform the regression coefficients in (3.4):

$$\beta = 1 - \kappa_q \Delta t, \qquad \kappa_q = \frac{1 - \beta}{\Delta t} \tag{3.5}$$

$$\alpha = \kappa_q \mu_q \Delta t = \frac{1 - \beta}{\Delta t} \mu_q \Delta t, \qquad \mu_q = \frac{\alpha}{1 - \beta}. \tag{3.6}$$

We gathered inflation data from the Consumer Price Index (CPI) data collected by the Bureau of Labor statistics (http://www.bls.gov) and ran several regressions of this type to estimate κ_q and μ_q. One specific concern of this data was that individual *monthly* CPI levels might contain self-correcting errors that would bias the regression coefficients. For example, if the CPI of September 2004 was overstated and then corrected in the following month, then inflation in September would temporarily appear "high" while the subsequent inference of monthly inflation would appear "low." If the time series of CPI contained any errors of this type, the resulting mean reversion strength and volatility parameters may be overstated. Given the noisy fluctuations in monthly data, we selected the parameters for the inflation process by looking at annual regressions. By calculating the change in CPI over the course of a year, the inflation rate would appear less volatile.

The often-cited time series of CPI uses a base period (i.e., resets the index value at 100) between the years 1982 and 1984. Given the fact that the CPI level is reported only to the first decimal place, using the current base does not lend itself to capturing minor changes in inflation in the first half of the 20th century; a small change in CPI may lead to large swings in inflation when the level of the index is low. The only other publicly available series reported on the old base level (1967 = 100) is the one that is not seasonally adjusted, U.S. city averages, all items.[6]

The annual rate of inflation was measured as

$$q_t = \ln \frac{CPI_t}{CPI_{t-1}}, \tag{3.7}$$

where CPI_t is the reported index value for year t and CPI_{t-1} is the prior year's reported index value of the same month. We ran two annual regressions: (1) all available data and (2) the years after World War II.

[6]Often in economic data, seasonal adjustments are required to remove persistent cyclical factors that may affect raw (unadjusted) values. Examples of seasonal factors that may have an impact on CPI include effects from climatic changes, holidays, and production cycles.

Time Period	κ_q	μ_q	σ_q
1913–2001	0.37	3.3%	4.0%
1946–2001	0.47	4.8%	3.0%

We selected the default mean reversion speed (κ_q) to be 0.4 and the mean reversion level (μ_q) to be 4.8% to capture the post-war economic period. Although it might appear that the speed of mean reversion over the second half of the 20th century has increased, it should be noted that the standard error of the estimate of κ_q is higher (which undoubtedly is due to fewer data points in the shorter period).

Instead of being concerned with the annualized, instantaneous level of inflation, bond investors are more concerned with the expected level of inflation over the life of their investment. Given the existing level of inflation (q_t) and the parameters of the assumed process in Formula (3.1), we can derive expectations of future inflation over various horizons. Our process for inflation follows the same Ornstein-Uhlenbeck process as in Vasicek [43], so we can develop a "term structure" of inflation analogous to Formula (2.3). This term structure posits an expected inflation rate over various horizons. A term structure of inflation is needed to generate nominal interest rates, since investors are concerned about not only the time value of money, but also the erosion of purchasing power expected over the life of their investment.

Real Interest Rates

To derive real interest rates, we selected a simple case of the two-factor Hull-White model (Formula (2.8)). In this model, the short-term rate (denoted by r) reverts to a long-term rate (denoted by l) that is itself stochastic. The long rate reverts to an average mean reversion level μ_r

$$\begin{aligned} dr_t &= \kappa_r(l_t - r_t)dt + \sigma_r dB_r, \\ dl_t &= \kappa_l(\mu_r - l_t)dt + \sigma_l dB_l. \end{aligned} \tag{3.8}$$

In order to estimate the parameters of the model, we look at the discrete analog of the model:

$$\begin{aligned} \Delta r_t &= \kappa_r(l_t - r_t)\Delta t + \sigma_r \varepsilon_{rt}\sqrt{\Delta t}, \\ \Delta l_t &= \kappa_l(\mu_r - l_t)\Delta t + \sigma_l \varepsilon_{lt}\sqrt{\Delta t}. \end{aligned} \tag{3.9}$$

$$\begin{aligned} r_{t+1} - r_t &= \kappa_r(l_t - r_t)\Delta t + \sigma_r \varepsilon_{rt}\sqrt{\Delta t} \\ &= (\kappa_r l_t - \kappa_r r_t)\Delta t + \sigma_r \varepsilon_{rt}\sqrt{\Delta t}, \\ l_{t+1} - l_t &= \kappa_l(\mu_r - l_t)\Delta t + \sigma_l \varepsilon_{lt}\sqrt{\Delta t} \\ &= (\kappa_l \mu_r - \kappa_l l_t)\Delta t + \sigma_l \varepsilon_{lt}\sqrt{\Delta t}. \end{aligned} \tag{3.10}$$

$$\begin{aligned} r_{t+1} &= r_t + (\kappa_r l_t - \kappa_r r_t)\Delta t + \sigma_r \varepsilon_{rt}\sqrt{\Delta t} \\ &= \kappa_r \Delta t \cdot l_t + (1 - \kappa_r \Delta t) \cdot r_t + \sigma_r \varepsilon_{rt}\sqrt{\Delta t}, \\ l_{t+1} &= l_t + (\kappa_l \mu_r - \kappa_l l_t)\Delta t + \sigma_l \varepsilon_{lt}\sqrt{\Delta t} \\ &= \kappa_l \Delta t \cdot \mu_r + (1 - \kappa_l \Delta t) \cdot l_t + \sigma_l \varepsilon_{lt}\sqrt{\Delta t}. \end{aligned} \tag{3.11}$$

From these equations, we can see that the short rate is again a weighted average of the current levels of r_t and the mean reversion factor l_t. The mean reversion factor is itself a weighted average of its long-term mean (μ_r) and its current value (l_t).

Hibbert, Mowbray, and Turnbull [24] (hereafter HMT) also use this process for real interest rates. They derive closed-form solutions for bond prices (and therefore yields), which are slightly more complicated than the one-factor Ornstein-Uhlenbeck process for inflation:

$$P^r(t,T) = A^r(t,T)e^{-r_t B_1(t,T) - l_t B_2(t,T)} \tag{3.12}$$

where r_t and l_t are the values for the short and long real interest rates and A^r, B_1, and B_2 are functions of underlying parameters in the two-factor Hull-White specification for real interest rates.

Estimating the parameters in Formula (3.11) is a difficult procedure since real interest rates are not directly observable in the market. We compute ex post real interest rates based on the difference between nominal rates observed in the market less the monthly (annualized) inflation rate. We use the three-month Constant Maturity Treasury (CMT) as a proxy for the instantaneous short rate and the 10-year CMT yield as a proxy for the long rate. (We also looked at longer Treasury yields as a proxy for the long rate. Results were not sensitive to the choice of maturity.) Nominal interest rates are from the Federal Reserve's historical database. (See http://www.federalreserve.gov/releases/.)

There are several issues related to the Federal Reserve's interest rate data. First, at the long end of the yield curve, there are significant gaps in many of the time series. For example, the 20-year CMT was discontinued in 1987; yields on 20-year securities after 1987 would have to be interpolated from other yields. Also, the future of 30-year rate data is uncertain, given the decision of the Treasury to stop issuing 30-year bonds (in fact, the Fed stops reporting 30-year CMT data in early 2002). At the short end of the yield curve, there are several choices for a proxy of the short rate. Ideally, one would want an interest rate that most closely resembles a default-free instantaneous rate. While the one-month CMT is reported back only to 2001, the three-month rate is available beginning in 1982. While we could have reverted to a private, proprietary source of data to create a longer time series, we restricted ourselves to only publicly available data sources that would be available to any user of the model.

Based on Formula (3.11), we use the following regressions on monthly data from 1982 to 2001:

$$\begin{aligned} r_{t+1} &= \alpha_1 l_t + \alpha_2 r_t + \varepsilon'_{rt}, \\ l_{t+1} &= \beta_1 + \beta_2 l_t + \varepsilon'_{lt}. \end{aligned} \tag{3.13}$$

Traditional ordinary least squares (OLS) regressions are not possible given the dependence of the short-rate process on the long rate. To estimate these simultaneous equations, we use two-stage least squares estimation. In order to estimate the short-rate equation in stage 2, we first obtain fitted estimates for the long rate $\hat{l}_t$, based on the parameter estimates from stage 1:

$$\begin{aligned} \text{Stage 1:} \quad & l_{t+1} = \beta_1 + \beta_2 l_t + \varepsilon'_{lt}, \\ \text{Stage 2:} \quad & r_{t+1} = \alpha_1 \hat{l}_t + \alpha_2 r_t + \varepsilon'_{rt}. \end{aligned} \tag{3.14}$$

The resulting parameters were generated from the regression results.

REAL INTEREST RATE PROCESS ESTIMATED FROM 1982 TO 2001

κ_r	μ_r	σ_r	κ_l	σ_l
6.1	2.8%	10.0%	5.1	10.0%

These parameters indicate a very high level of volatility that is tempered by strong levels of mean reversion. See the discussion of the nominal interest rates below for the parameters that are used in the simulation illustration in section five.

Nominal Interest Rates

Fisher [20] provides a thorough presentation of the interaction of real interest rates and inflation and their effects on nominal interest rates. He argues that nominal interest rates compensate investors not only for the time value of money but also for the erosion of purchasing power that results from inflation. In the model presented here, the underlying movements in inflation and real interest rates generate the process for nominal interest rates. If bonds are priced using expectations of inflation and real interest rates until the bond's maturity, then nominal interest rates are implied by combining the term structure of inflation and the term structure of real interest rates. Therefore,

$$P^i(t,T) = P^r(t,T) \cdot P^q(t,T), \tag{3.15}$$

where i refers to nominal interest rates and the superscripts on the bond prices correspond to the underlying stochastic variables.

Unfortunately, the parameters for the real interest rate process shown above generate a distribution that severely restricts the range of potential future nominal interest rates. For example, using the regression results from Formulas (3.13) and (3.14), the 1st percentile of the distribution for the 20-year nominal rate is 5.9% and the 99th percentile is 8.2%. There are several candidates for problems with real interest rates that may lead to this seemingly unrealistic distribution of future nominal rates: (1) the use of ex post real interest rate measures is unsuitable, (2) because of potential errors in monthly reporting of CPI mentioned above, monthly measurements of real interest rates produce self-correcting errors that exaggerate mean reversion speed, or (3) the time period used to measure real interest rates is too short.

As a result, the parameters for real interest rates were altered to allow nominal interest rates to better reflect historical volatility. Specifically, mean reversion speed was dramatically reduced. Given that mean reversion speed and volatility work together to affect the range of interest rate projections, volatility was also reduced. The following parameters are used as the "base case" in the model. These parameters are in line with what was used in Hull [27].

κ_r	μ_r	σ_r	κ_l	σ_l
1.0	2.8%	1.00%	0.1	1.65%

An important consideration in the model is the correlation between interest rates and inflation. Risa [36] reviews the literature on the relationship between inflation and interest rates. Pennacchi [33] finds evidence that the instantaneous real interest rates and expected inflation are significantly negatively correlated. Ang and Bekaert [5] develop a regime-switching model for inflation and real interest rates. They find that inflation is negatively correlated with the short-term real interest rate. Fama [18] examines how one-year spot interest rates can be used to forecast its components: the one-year inflation rate and the real return on one-year bonds. It is found that the expected values of those two components move opposite to one another. As a result, the financial scenario model includes a negative correlation between real interest rates and inflation.

Equity Returns

Equity returns are equal to the risk-free nominal interest rate ($q + r$) and a risk premium or excess equity return attributable to capital appreciation (x):

$$s_t = q_t + r_t + x_t. \tag{3.16}$$

In her model, Hardy [22] assumes that stock prices are lognormally distributed under each regime. But while Hardy looks at total equity returns, including dividends and the underlying compensation from the risk-free rate, we use the excess equity returns from capital appreciation x. To estimate the parameters of the regime-switching equity return model, we follow the procedure outlined in Hardy [22], maximizing the likelihood function implied from the regime-switching process.

We estimate the process for the returns of small stocks and large stocks separately. Numerous web sites are available to capture the time series of capital appreciation of these indices (see, for example, http://finance.yahoo.com). The large stocks are based on the Standard and Poor's (S&P) 500 (or a sample chosen to behave similarly for the years prior to the construction of the S&P 500). The data are available online at a Web Site generated by Robert Shiller, author of *Irrational Exuberance* (http://www.econ.yale.edu/~shiller/data/ie_data.htm). The small stock values are based on Ibbotson's *Stocks, Bonds and Bills* [30]. As expected, the risk and return of small stocks appear higher than large stocks under both regimes. The following parameter estimates were developed:

EXCESS MONTHLY RETURNS

	Large Stocks (1871–2002)		Small Stocks (1926–1999)	
	Low Volatility Regime	High Volatility Regime	Low Volatility Regime	High Volatility Regime
Mean	0.8%	−1.1%	1.0%	0.3%
Standard Deviation	3.9%	11.3%	5.2%	16.6%
Probability of Switching	1.1%	5.9%	2.4%	10.0%

Note that while the expected return in the high volatility regime is lower, it is more likely that if the high volatility regime is ever reached, the equity market will revert back to the low volatility regime since the probability of switching is higher. The regime switches are correlated, so if large stocks are in the low volatility regime, then small stocks are more likely to be in the low volatility regime as well.

Equity Dividend Yields

Similar to the process used by HMT and Wilkie [44], we assume that the log of the dividend yield follows an autoregressive process:

$$d(\ln y_t) = \kappa_y(\mu_y - \ln y_t)dt + \sigma_y dB_{yt}. \tag{3.17}$$

One source of difficulty associated with estimating the dividend yield process involves obtaining data. There is no long time series of dividend yields that is publicly available for equity indices. To obtain this information, we used a proprietary source of financial data (Telerate). However, one may be able to estimate the dividend yield of indices that contained a limited number of stocks (such as the Dow Jones industrial average). It should be noted that the process for dividend yields is clearly time-dependent. Average dividend yields have fallen dramatically over the last 50 years given the recognition of double taxation effects. Recent tax changes that levy lower taxes on dividends may (or may not) reverse the long-term trends of lower dividends.

Estimation of this process is analogous to the inflation process described above. The mean reversion speed of the series is not significantly different from zero. Given the long-term changes in historical dividend patterns, the log of dividends appears to be a random walk around its starting value.

Real Estate (Property)

Given that the real estate portfolios of insurers are dominated by commercial properties, we use the National Council of Real Estate Investment Fiduciaries (NCREIF) pricing index to capture the quarterly returns on commercial properties (see http://www.ncreif.com). The NCREIF data are generated from market appraisals of various property types, including apartment, industrial, office, and retail. While the use of appraisal data may only approximate sharp fluctuations in market valuation, publicly obtainable transaction-based real estate data were not available.

Using *quarterly* return data from NCREIF from 1978 to 2001 (http://www.ncreif.com/indices/), we estimated the following Ornstein-Uhlenbeck model for real estate:

$$d(re)_t = \kappa_{re}(\mu_{re} - (re)_t)dt + \sigma_{re}dB_{re}. \tag{3.18}$$

We estimated two separate models including the levels of inflation. While we expected inflation to provide additional explanatory power for real estate returns, the results were not significant. The following parameters were used to project quarterly real estate returns:

κ_{re}	μ_{re}	σ_{re}
1.20	2.3%	1.3%

Unemployment

There are many plausible ways to link unemployment rates to other economic variables. One approach to estimating unemployment is based on the well-known Phillips curve. The Phillips curve illustrates a common inverse relationship between unemployment and inflation. The approach taken by Phillips seems plausible: As the economy picks up, inflation increases to help temper the demand-driven economy. At the same time, unemployment falls as firms hire to meet the increasing demand. When the economy slows down, unemployment rises, and inflationary pressures subside.

We also include a first-order autoregressive process in the unemployment process, in addition to the relationship suggested by the Phillips curve:

$$du_t = \kappa_u(\mu_u - u_t)dt + \alpha_u dq_t + \sigma_u dB_{ut}. \tag{3.19}$$

It is expected that when inflation increases ($dq_t > 0$), unemployment decreases (i.e., $\alpha_u < 0$). One may argue that there is a lag between inflation and unemployment. To keep the model simple, we did not pursue any distributed lag approach.

The discrete form of the unemployment model is shown as

$$\begin{aligned} u_{t+1} &= u_t + \kappa_u\mu_u - \kappa_u\Delta t \cdot u_t + \alpha_u(q_{t+1} - q_t) + \sigma_u\varepsilon_{ut}\sqrt{\Delta t} \\ &= \kappa_u\mu_u + (1 - \kappa_u\Delta t) \cdot u_t + \alpha_u(q_{t+1} - q_t) + \sigma_u\varepsilon_{ut}\sqrt{\Delta t}. \end{aligned} \tag{3.20}$$

This suggests the following regression:

$$u_{t+1} = \beta_1 + \beta_2 u_t + \beta_3(q_{t+1} - q_t) + \sigma_u\varepsilon'_{ut}. \tag{3.21}$$

We use inflation data as described above and retrieve monthly unemployment data from the Bureau of Labor Statistics (http://www.bls.gov). Using data from 1948 to 2001 and transforming the regression coefficients as in Formulas (3.5) and (3.6), we get

$$du_t = 0.13 \cdot (6.1\% - u_t)dt - 0.72dq_t + 0.76\% \cdot dB_{ut}. \tag{3.22}$$

Comments on Selecting Parameters of the Model

Some have argued that the performance of any model should be measured by comparing projected results against history. It is not our intent to perfectly match the distribution of historical values for interest rates, equity returns, and so on. To do so would naively predict a future based on random draws from the past. If perfect fit is desired, history already provides the set of economic scenarios that may be used for actuarial applications and the development of an integrated financial scenario model is completely unnecessary. Instead, the model presented here provides an alternative: an integrated approach to creating alternative scenarios that are tractable and realistic. While history is used to gain important insights into the characteristics of relevant variables, it would be impossible to build tractable models that yield a perfect fit to historical distributions. In general, we believe our theoretical framework provides a parsimonious approach to closed-form solutions of particular variables of interest.

4. USING THE FINANCIAL SCENARIO MODEL

The financial scenario model is an Excel spreadsheet that benefits from the use of a simulation software package called @RISK, available through Palisade Corporation (http://www.palisade.com). @RISK leverages the simplicity of spreadsheets and integrates powerful analysis tools that are used to help randomly select future scenarios and examine risk in a stochastic financial environment. The software package allows users to define uncertain variables as a distribution, take numerous draws from these inputs, and then capture each iteration's impact on a user-defined output variable of interest, such as profits, sales, or an insurer's surplus.

Excluding Negative Nominal Interest Rates

There has been significant debate over the proper way to deal with negative nominal interest rates in interest rate models. Some modelers have set boundary conditions that prevent nominal interest rates from becoming negative. Other modelers have not been concerned over negative interest rates, either because the mathematical characteristics of the model are more important than the practical applications or because the incidence of negative nominal interest rates is too infrequent to require significant attention.

While it depends on the specific application, the occurrence of negative nominal interest rates can be problematic. Economically, certain variables have natural limits. For example, while theory may not reject negative interest rates, reality suggests that it is unlikely that investors would ever accept negative nominal interest rates when lending money. Therefore, the model provides users with two options:

- *Placing lower bounds on the levels of inflation and real interest rates*. The model simulates these processes as if there were no lower bound, but then it chooses the maximum of the lower bound and the simulated value.
- *Eliminating the potential for negative nominal interest rates.* In this case, the model uses the standard inflation simulation, but effectively places a lower bound on the real interest rate such that the resulting nominal interest rate is non-negative.

User-Defined Scenarios

The financial scenario model provides for stochastic simulation of future economic variables, based upon user-specified parameters for the assumed processes. However, there are instances where it may be desirable to allow the user to input specific scenarios for the future values of certain processes. For example, regulations may require sensitivity testing based on specific equity return patterns over the next decade. The financial scenario model allows users to specify scenarios for three economic variables in the model; nominal interest rates, inflation, and equity returns. For example, with respect to nominal interest rates, each of the "New York 7"[7] regulatory interest rate tests are preprogrammed into the model and may be selected by the user; the user may also specify a scenario of her or his own creation for any of the three economic processes.

Employing the Financial Scenario Model

It is expected that the financial scenario model will be implemented in a variety of different analyses. The model can be used as the underlying engine for creating many financial scenarios and can be tailored for a user's specific purposes. For example, Ahlgrim and D'Arcy [1] use the model as the underlying asset return generator to assess the risk inherent in pension obligation bonds issued by the state of Illinois. In this case, the model was extended to include international equities and to compute yields on coupon bonds from the nominal interest rates.

5. ILLUSTRATIVE SIMULATION RESULTS

Regardless of the mathematical sophistication of the variables incorporated in a model, the accuracy of the procedures used to determine the parameters, and the timeliness of the values on which the calibration is based, the most important test of the validity of any model is the reasonability of the results. This section will examine the results of a representative run of the financial scenario model and compare the output with historical values. It should be reiterated that the goal of choosing the parameters for the model was *not* to replicate history. Correspondingly, we do not include measures of fit when comparing the sample results

[7] The "New York 7" are seven different interest rate scenarios originally specified by NY Regulation 126 for use in asset adequacy testing and actuarial opinions for life insurers. Each scenario is based on deviations from the current term structure.

to history. This section uses history to review results of an illustrative simulation to subjectively assess the model's plausibility.

A simulation is performed generating 5,000 iterations (sample paths) using the base parameters described in Section 3, disallowing negative nominal interest rates.[8] The results are presented in several different ways (these results are discussed in the following section).

- Table 1 provides key statistics for key variables in the simulation. Mean values of the output are shown for the first and last (50th) projection years. The 1st and 99th percentiles of the distribution of results are indicated for an intermediate projection year (year 10).
- Tables 2 and 3 show the correlation matrices, comparing the simulation correlations (Table 2) and historical correlations (Table 3).
- Some of the Figures (1–6, 8–10, 14–15, 18, 20, and 22) show "funnel of doubt" plots, indicating the level of uncertainty surrounding each output variable *over time*.[9] The x-axis indicates the time period and the y-axis indicates the value(s) assumed by the variable of interest. The "funnel of doubt" graphs show the mean value for the 5,000 iterations (solid line), the 25th and 75th percentile values (dark shaded section), and the 1st and 99th percentile values (lighter shaded section). Expanding funnels indicate that the values become more uncertain over the projection period. Narrowing funnels indicate that the variables become more predictable when making long-term forecasts.
- Figures 7, 11–13, 16–17, 19, 21, and 23 are histograms, illustrating the full probability distribution of the values for a par-

[8]The output of this illustration has been saved in a file and is posted at http://casact.org/research/econ. The American Academy of Actuaries uses a similar prepackaged scenario approach in looking at C-3 risk of life insurers.

[9]These "funnel of doubt" graphs are referred to as "summary graphs" in @RISK.

TABLE 1

KEY VARIABLES FROM FINANCIAL SCENARIO MODEL RUN

Date	7/17/2004			
Iterations	5,000			
Parameters	Base			
	Nominal Interest Rates Not Allowed to Be Negative			

	Mean		Range at Year 10 (25)	
	First Year (16)	Last (30)	1%	99%
Output Interest Rates				
Real Interest Rates				
1-month (B)	0.009	0.030	−0.053	0.100
1-year (D)	0.009	0.029	−0.051	0.097
10-year (G)	0.011	0.026	−0.033	0.076
Inflation Rate				
1-month (J)	0.023	0.048	−0.053	0.145
1-year (L)	0.027	0.048	−0.037	0.129
10-year (O)	0.039	0.045	0.020	0.069
Nominal Interest Rates				
1-month (R)	0.032	0.078	0.000	0.194
1-year (T)	0.036	0.077	0.000	0.183
10-year (W)	0.051	0.071	0.006	0.127
Other Output				
Large Stocks (B)	0.087	0.116	−0.159	0.296
Small Stocks (C)	0.134	0.136	−0.159	0.397
Dividend Yield (D)	0.015	0.023	0.006	0.039
Unemployment (E)	0.060	0.061	0.035	0.087
Real Estate (F)	0.081	0.094	0.030	0.161

The letters in the first column indicate the columns, and the numbers in the headings indicate the rows, of the cells where the values are located in the @RISK output files.

ticular variable *at one point in time* (a single projection year). For comparative purposes, the distribution of historical values, where appropriate, is also plotted in these histograms.

Real Interest Rates

We start by looking at the one-month real interest rate in Table 1. The mean value for the first projection month is 0%. By the end of the 50-year projection period, this value has moved to 3.0%. This result is entirely in line with the specifications of the model. The one-month value would be closely aligned with the initial short-term real interest rate (rinit1). To estimate this rate, we backed out an estimate of inflation from the observed risk-free, short-term interest rate. During the summer of 2004, the resulting value of the real interest rate was near 0%. Under the projections, the initial value would begin to revert to the long-term mean after one month. The mean of the final value in the results, after 50 years, is around the mean reversion level for the long rate (rm2), which is 2.8%.

To provide an idea about the range of values for the one-month real interest rate, columns 3 and 4 of Table 1 display the 1st and 99th percentiles of the distribution in the tenth projection year. In 1 percent of the iterations, the one-month real interest rate, on an annualized basis, is less than −5.3%. On first observation, this result seems nonsensical. Why would an investor be willing to lose money, in real terms, by investing at a negative real interest rate? Instead, an investor would just hold cash rather than lose 5.3% a year, after adjusting for inflation. However, this may not be as unrealistic as it seems. First, this result is the annualized rate as opposed to the one-month real rate of only −0.4%. Second, this return may represent the best return available. If inflation is high, then holding cash would generate an even larger loss. In times of high inflation, the best real return an investor can receive may be negative. Finally, real interest rates are not observable. The true real interest rate is the return required, over and above expected inflation, for the specific interval. However, the precise expected inflation rate is unobservable in the financial markets.

In practice, two approaches have been used for estimating the expected inflation rate. First, one can use economists' forecasts of inflation. Economists, though, do not represent investors. By training and occupation, the economists included in the surveys are not at all representative of the general financial market participants. Investors may consider some economists' forecasts in making their own determination of what to expect regarding future economic conditions, but many other factors, including their own experience, the counsel of other participants, and recent historical experience, are used to determine their inflation expectations. There is no survey of representative market participants to determine what they truly anticipate for the inflation rate.

The second approach has been to examine actual inflation rates that have occurred, and then subtract those from prior interest rates (ex post analysis). This approach is also flawed for several reasons. First, there is no reason to believe that the market is prescient regarding inflation expectations. Especially in the case of an unexpected shock to the system, such as oil price increases during the 1970s, the market does not know what will happen in the future. It cannot even be assumed that errors in forecasting will cancel out over time, since the market could be biased to underestimate or overestimate future inflation. Second, actual inflation cannot be accurately measured. The Consumer Price Index and other values commonly used to determine inflation are widely recognized as being imperfect. These indices track the prices of specific goods and services that are not completely representative of the entire economy. These indices cannot recognize the substitution effect in which consumers continually engage, such as buying more chicken than usual when beef prices rise, or driving less when gasoline prices soar. Due to these problems, it is not possible to claim that real interest rates cannot be negative, so a small negative value over a short time interval does not necessarily represent a problem.

On the opposite side of the distribution, the 99th percentile value for one-month real interest rates after 10 years is 10%. The same limitations described above also apply to this value.

Going further out on the term structure, the mean value of the one-year real interest rate after the first projection year is 0.9%. This reflects reversion from the initial value of 0% to the long-term mean of 2.8%. The mean of the last value, after 50 years, which is in line with these parameters, is 2.9%. The

1st–99th percentile range after 10 years is −5.1% to +9.7%, reflecting a similar distribution for the full year as was observed for the monthly values. For the 10-year real interest rates, the mean after the first projection month is 1.1%, and in the last projection month, the mean is 2.6%, reflecting the strength of the mean reversion over this long a period of time. The 1st–99th percentile range after 10 years is −3.3% to +7.6%, reflecting the more compact distribution for long-term (10-year) real interest rates, compared to shorter time horizons.

Figures 1 through 3 depict the funnel of doubt graphs of one-month, one-year and 10-year real interest rates. All reflect the same shape, although the scaling differs. The "kink" in the early portion of the graph occurs because the first 12 points represent monthly intervals, which have small changes in values, and the latter steps are larger intervals, which lead to correspondingly larger changes. The level of uncertainty increases over the entire 50-year time frame, but the shifts toward the end of the simulation period are less pronounced. This shape occurs because of the structure and parameterization of the model. The uncertainty inherent in the real interest rate process generates the initial spread of the distribution, but the impact of mean reversion offsets this tendency, keeping the "funnel of doubt" from expanding further.

Inflation

The next variable of interest is the inflation rate. As shown in Table 1, the mean value of the (annualized) one-month inflation rate is 2.3% after the first projection year and 4.8% after 50 years. Note that the initial inflation rate (qinit1) is set at 1.0% and 4.8% is the long-term mean (qm2). The 1st–99th percentile range after 10 years is −5.3% to +14.5%, which is wider than the distribution for real interest rates since the mean reversion speed for inflation is lower (0.4 compared to 1.0). Negative inflation (or deflation) is not objectionable since small negative monthly values have occurred in recent years. Also, monthly inflation values in excess of 14.5% did occur during the 1970s.

FIGURE 1

DISTRIBUTION OF 1-MONTH REAL INTEREST RATE PROJECTION
PERIOD: 1 MONTH TO 50 YEARS

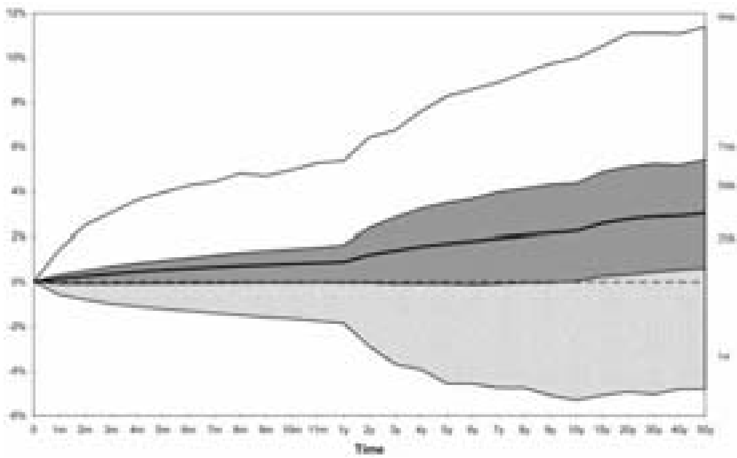


FIGURE 2

DISTRIBUTION OF 1-YEAR REAL INTEREST RATE PROJECTION
PERIOD: 1 MONTH TO 50 YEARS

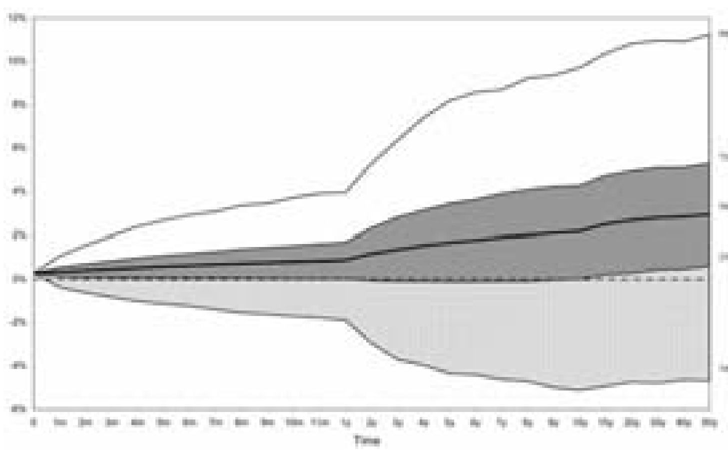
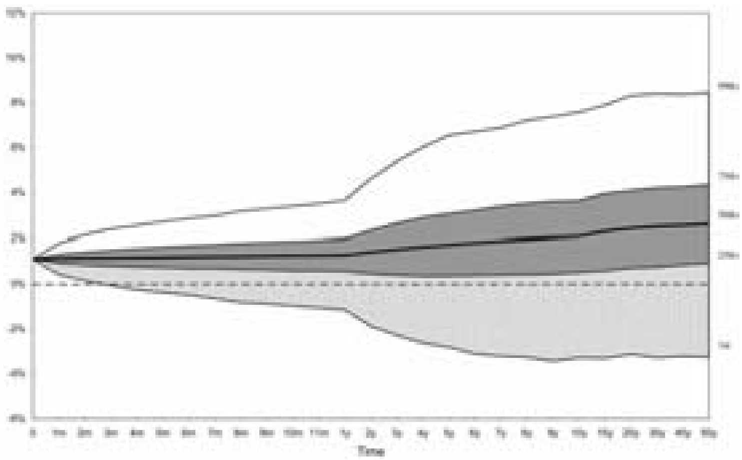


FIGURE 3

DISTRIBUTION OF 10-YEAR REAL INTEREST RATE PROJECTION
PERIOD: 1 MONTH TO 50 YEARS



The mean one-year inflation rate begins at 1.6% and moves to 4.8% by the end of 50 years, both in line with the model parameters. The 1st–99th percentile range of the one-year inflation rate after 10 years is −3.7% to +12.9%. Although the United States has not experienced deflation over an entire year since 1954, it seems quite appropriate to assign positive probability to this event.

From the description in Section 3, recall that the 10-year inflation rate is derived from the expected path of inflation over the next ten years. Given the assumption of mean reversion of inflation, it is expected that there is less uncertainty inherent in predicting longer-term inflation rates. The simulation confirms this—the mean 10-year inflation rate begins at 3.6% and moves to 4.5% by the end of 50 years, closer to the long-term mean parameter of 4.8%. Also, the 1st–99th percentile range of the 10-year inflation rate after 10 years is 2.0% to 6.9%, demonstrating that, over longer time horizons, the (geometric) average rate of inflation is less variable.

The funnel of doubt graphs of one-month, one-year, and 10-year inflation rates are shown in Figures 4 through 6. The uncertainty of the 10-year inflation rate is much smaller than it is for one-month and one-year rates, reflecting the strength of the mean reversion term for this single-factor model. Although inflation varies widely over shorter time horizons, in this model the long-term inflation rate is much less variable. This pattern can be altered by increasing the volatility of the inflation process (σ_q) or reducing the mean reversion speed (κ_q).

The histograms for the one-year projected inflation rates and of actual one-year inflation rates from 1913 through 2003 (from January to January) are shown on Figure 7. It is readily apparent that the modeled inflation rates generate a nice bell-shaped curve, whereas the actual inflation rates are much less smooth. One reason for this difference is that the model results are based on 5,000 iterations, while the actual data contain only 90 data points. More importantly, though, the projected values are derived from a concise mathematical expression that will produce a smooth distribution of results, but the actual inflation rates depend on the interactions of an almost unlimited number of variables. The key question is whether the model adequately expresses the probability distribution of potential inflation rates. The actual inflation rates are more leptokurtic (fatter in the tails than a normal distribution) than the modeled values, but they reflect the central portion of the graph fairly well. All of the large negative inflation rates occurred prior to 1950. Many of the positive outliers are from years prior to 1980, when monetary policy was less focused on controlling inflation.

Nominal Interest Rates

Nominal interest rates reflect the combination of the real interest rate and inflation. The mean values for one-month nominal interest rates were 1.1% for the first month and 7.8% for the

FIGURE 4

DISTRIBUTION OF 1-MONTH INFLATION RATE PROJECTION
PERIOD: 1 MONTH TO 50 YEARS

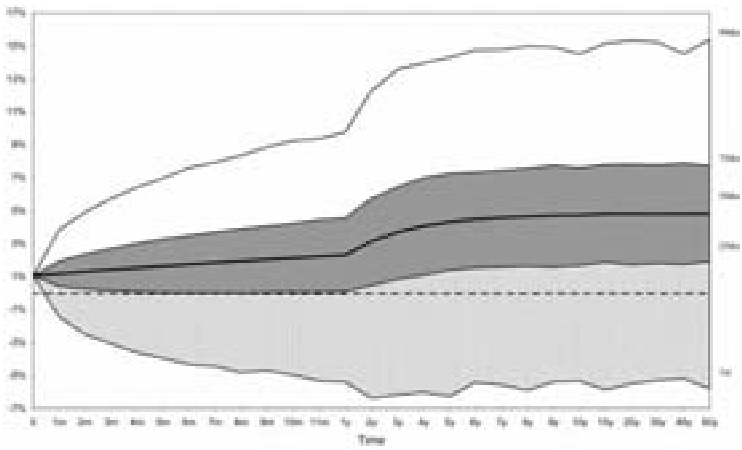


FIGURE 5

DISTRIBUTION OF 1-YEAR INFLATION RATE PROJECTION
PERIOD: 1 MONTH TO 50 YEARS

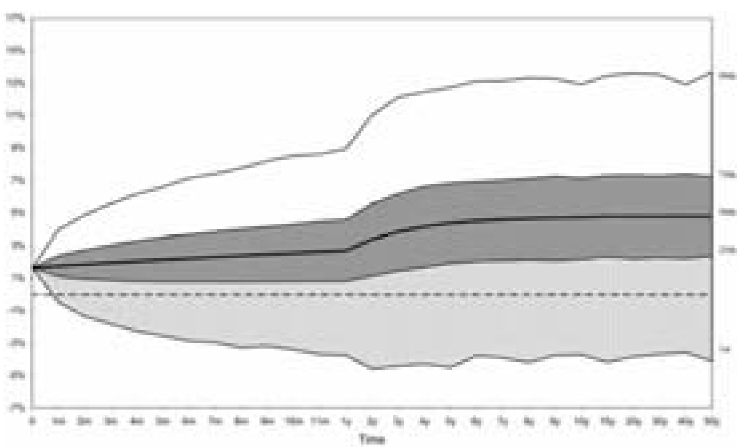


FIGURE 6

DISTRIBUTION OF 10-YEAR INFLATION RATE PROJECTION
PERIOD: 1 MONTH TO 50 YEARS

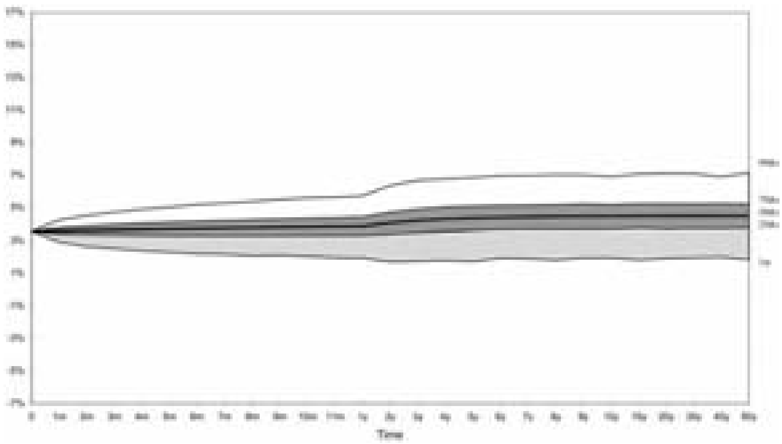
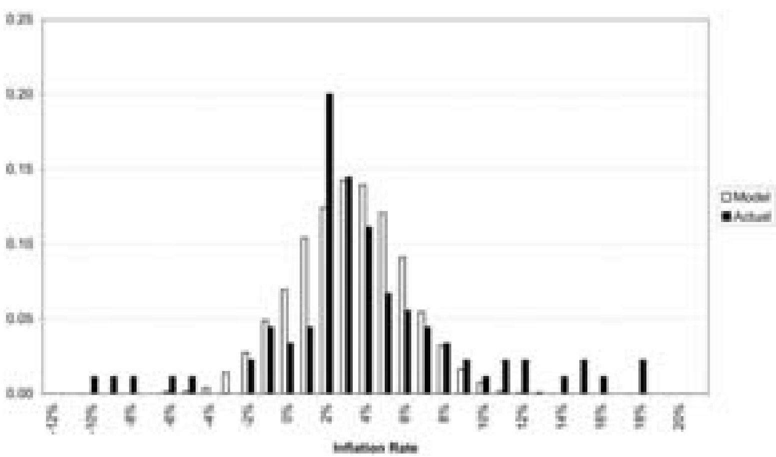


FIGURE 7

ACTUAL INFLATION RATES (1913–2003) VS. MODEL VALUES
(5,000 ITERATIONS)



50th year. The initial nominal interest rate indicated in the model (1.1%) is in line with the user-defined starting level (June 2004) of 1.1%. The 1st–99th percentile range for the one-month nominal interest rate after 10 years is 0.0% to 19.4%.

The mean one-year nominal interest begins at 1.9% and moves to 7.7% by the end of 50 years. The initial value is again in line with the current level of interest rates. The 1st–99th percentile range of the one-year nominal interest rate after 10 years is 0.0% to 18.3%.

The mean 10-year nominal interest begins at 4.6% and moves to 7.1% by the end of 50 years. The initial value is in line with the current level of interest rates for long-term bonds, given the June 2004 10-year U.S. Treasury yield of 4.4%. The 1st–99th percentile range of the one-year nominal interest rate after 10 years is 0.6% to 12.7%.

The funnel of doubt graphs of one-month, one-year, and 10-year nominal interest rates, Figures 8 through 10, are similar to the real interest rate and inflation graphs, but have a barrier at zero since the restriction that nominal interest rates not be negative is applied in this case. This restriction affects the 1st percentile line on Figures 8 and 9, but not the 25th percentile line. The effect of the restriction is not apparent for the 10-year nominal interest rates. The level of uncertainty increases over the 50-year time period used in the forecast. Since the nominal interest rate is determined by adding the real interest rate to the inflation rate, the increasing uncertainty reflected by real interest rates and the inflation rate generates the same behavior for nominal interest rates.

The histograms for the three-month, one-year, and 10-year model nominal interest rates and the actual three-month, one-year, and 10-year nominal interest rates are displayed in Figures 11 through 13. (The one-month values are not consistently available for historical data over a long enough time period to be re-

FIGURE 8

Distribution of 1-Month Nominal Interest Rate
Projection Period: 1 Month to 50 Years

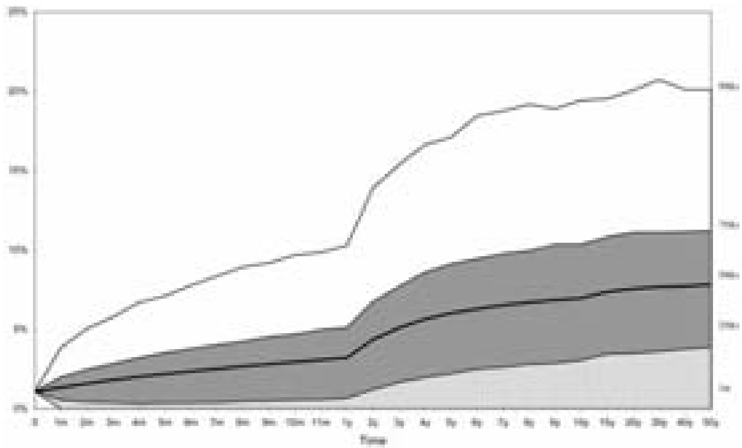


FIGURE 9

Distribution of 1-Year Nominal Interest Rate
Projection Period: 1 Month to 50 Years

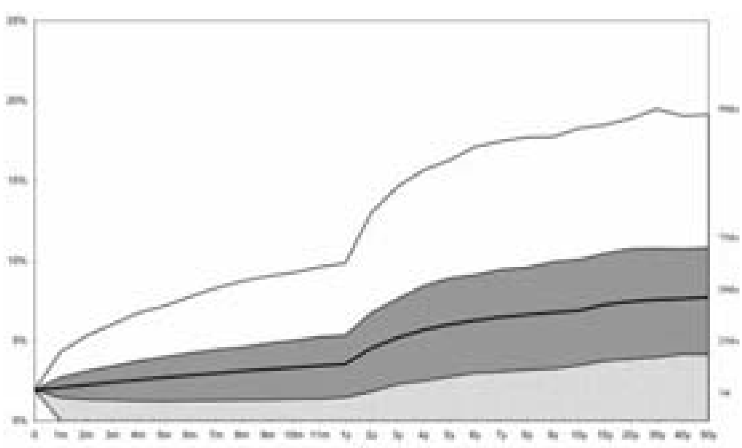


FIGURE 10

DISTRIBUTION OF 10-YEAR NOMINAL INTEREST RATE
PROJECTION PERIOD: 1 MONTH TO 50 YEARS

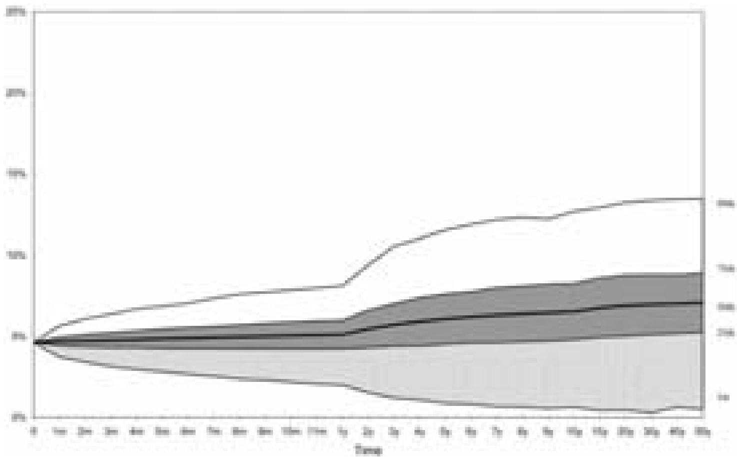


FIGURE 11

ACTUAL 3-MONTH NOMINAL INTEREST RATES (JANUARY 1934–MAY 2004) VS. MODEL VALUES (5,000 ITERATIONS)

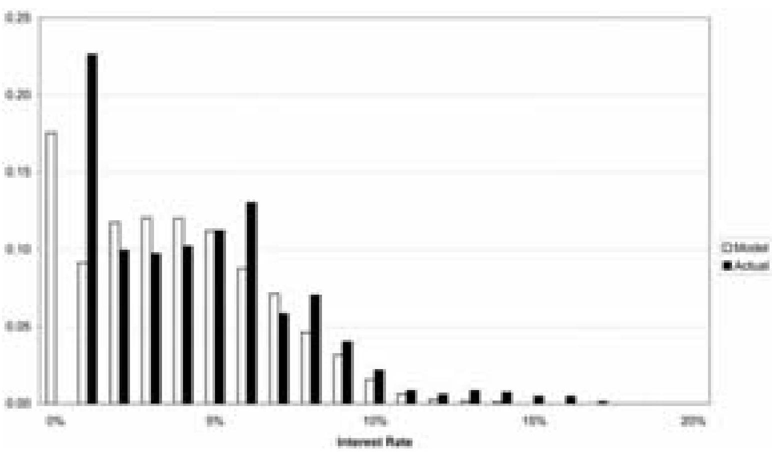


FIGURE 12

ACTUAL 1-YEAR NOMINAL INTEREST RATES (APRIL 1953–MAY 2004) VS. MODEL VALUES (5,000 ITERATIONS)

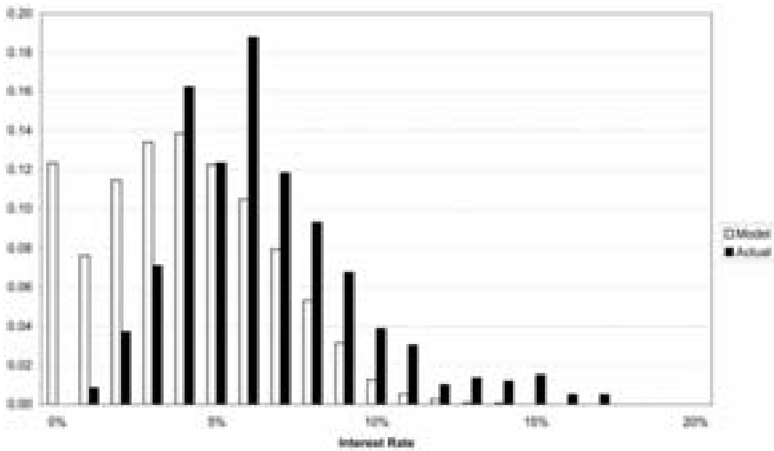
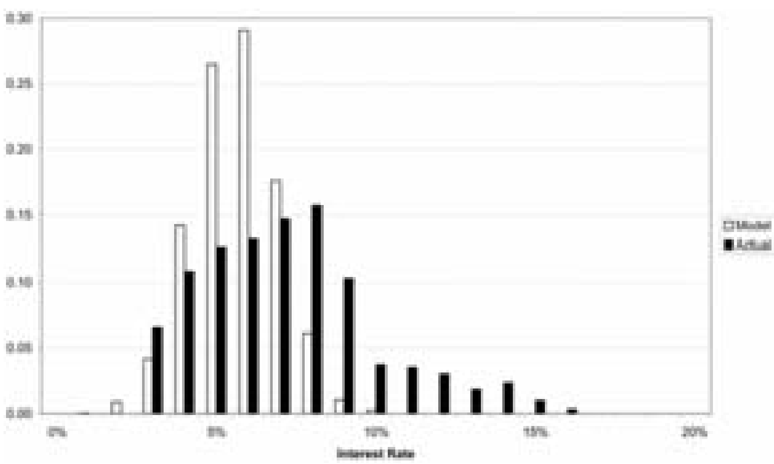


FIGURE 13

ACTUAL 10-YEAR NOMINAL INTEREST RATES (APRIL 1954–MAY 2004) VS. MODEL VALUES (5,000 ITERATIONS)



levant. Therefore, three-month interest rates are used for in Figure 11.) Figures 11 through 13 show the distribution of nominal interest rates one year into the projection period.

Significant differences exist between the modeled and historical distributions for interest rates. In Figure 11, the modeled three-month nominal interest rates are 0.0% in almost 20% of the cases, whereas actual three-month interest rates have never been below 0.5 percent (the column reflecting the 1% bin represents values between 0.5 and 1.5 percent). However, combining the model values for 0 and 1 percent indicates a total in line with actual values. In addition, the model distributions are smoother than the actual values, which is natural since the model results are based on 5,000 iterations, whereas the actual results, even though derived from 845 (monthly) or 614 (one- and 10-year) observations, are not at nearly as smooth, indicating that the system that generates interest rates is not as straightforward as the model.

At first glance, modeled interest rates are generally lower than the historical rates. It is important to note that the modeled interest rates are influenced by the starting values for the initial real interest rate (rinit1), the initial mean reversion level for the real interest rate (rinit2), and the initial inflation level (qinit1), which are lower than historical averages.

The comparison between the 10-year modeled rates and the 10-year historical rates, Figure 13, indicates a few differences. The modeled interest rates are more compact than actual 10-year interest rates have been. If the user feels that the variance of the model values should be closer to the historical distribution, then the strength of the mean reversion factor in the interest rate model can be reduced, but this would increase the incidence of negative interest rates unless the user selects to avoid negative nominal interest rates. The other significant difference is the skewness. The historical rates exhibit positive skewness, but the modeled rates have a slight negative skewness. Finally, the model rates are lower than historical values, again due to starting with the current low levels of interest rates.

Stock Returns and Dividends

The values for large and small stock returns indicate, as expected, higher average returns and greater variability for small stocks than for large stocks. As shown in Table 1, the mean of the initial values (after one year) of large stocks is 8.7% and of small stocks is 13.4%. The mean of the large stock values increases to 11.6% at the end of 50 years and for small stocks increases to 13.6%. The 1st–99th percentile range after 10 years is −15.9% to 29.6% for large stocks and −15.9% to 39.7% for small stocks.

The funnel of doubt graphs (Figures 14 and 15) indicate an inverted funnel, compared to the displays of interest rates and inflation. This means that uncertainty reduces over time and is due to the way the values are calculated. The projected values shown are *geometric average* returns for large and small stocks over the projection period. For example, the one-year values are returns over a one-year period, the 10-year values are average annual returns over a 10-year period, and so on. Thus, Figures 14 and 15 show that the average annual returns expected over a 50-year period are much more predictable than those for a one-year period.

Histograms of the one-year returns for the large (Figure 16) and small (Figure 17) stock returns as generated by the model are displayed, along with actual one-year returns for 500 large stocks for 1871 through 2004 and small stock returns over the period 1926 through 2003. The graphs for large stocks (Figure 16) are relatively similar, although, as would be expected, the results of the 5,000 iterations of the model produce a smoother distribution. The histograms for small stocks (Figure 17) show that historical values have been more variable, with a notable outlier at 190% return, which represents a single observation. The model values also have single observations around that level,

FIGURE 14

DISTRIBUTION OF COMPOUND AVERAGE LARGE STOCK RETURNS PROJECTION PERIOD: 1 YEAR TO 50 YEARS

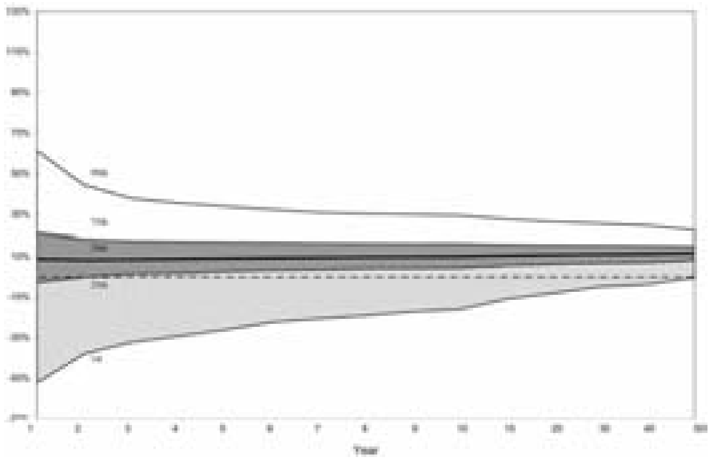


FIGURE 15

DISTRIBUTION OF COMPOUND AVERAGE SMALL STOCK RETURNS PROJECTION PERIOD: 1 YEAR TO 50 YEARS

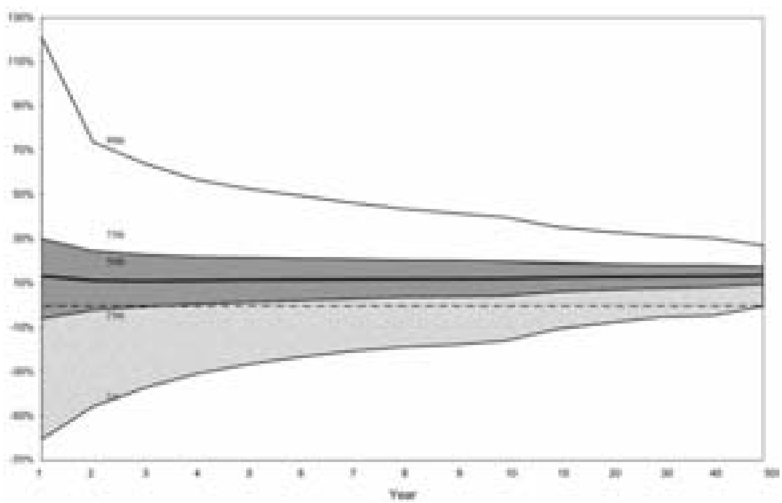


FIGURE 16

ACTUAL LARGE STOCK RETURNS (1871–2004) VS. MODEL VALUES (5,000 ITERATIONS)

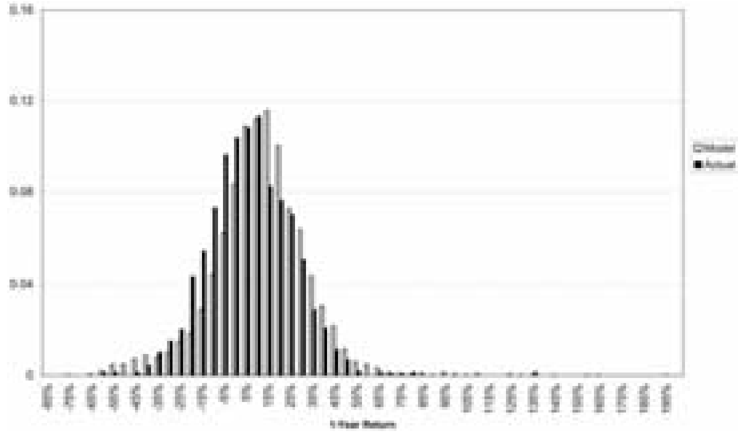
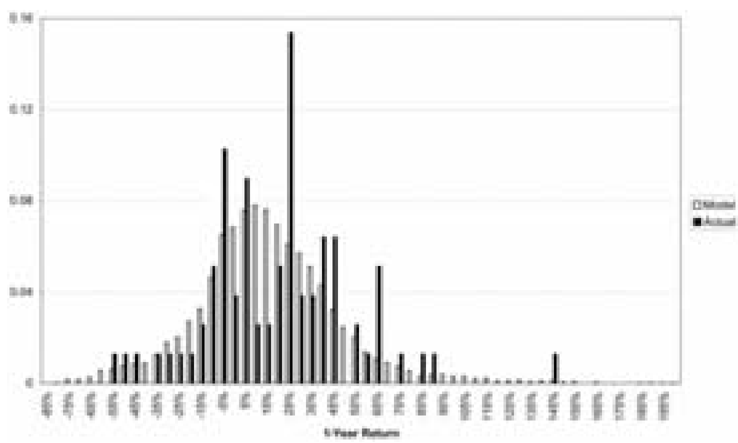


FIGURE 17

ACTUAL SMALL STOCK RETURNS (1926–2003) VS. MODEL VALUES (5,000 ITERATIONS)



but no one bin produces as large a proportion of the outcomes as the one occurrence out of 78 years of the historical experience to be as obvious on the graph.

The mean dividend yield for equities is 1.5% for the first year and 2.3% for the 50th-year values. The 1st–99th percentile range after 10 years is 0.6% to 3.9%. The funnel of doubt graph of the dividend yield (Figure 18) increases over time as interest rates and inflation do. Figure 19 displays the histogram of the modeled dividend yields and the actual dividend yields over the period 1871 through 2003, based on data available from Robert Shiller [39]. Historically, dividend yields have varied more widely than the model predicts and have been centered at a higher level. This may be a result, in part, of a structural shift in the dividend payment history in the United States. Bernstein [6] notes that prior to the late 1950s, stock dividends tended to be higher than interest rates on corporate bonds. This was based on the understanding that stocks were riskier than bonds and therefore should pay a higher return. Since 1959 though, dividend yields have tended to be lower than interest rates, ranging from 1.1% to 5.4%, which is in line with the simulation results.

Unemployment and Real Estate Returns

The mean value of the unemployment rate, as shown in Table 1, begins at 6.0% and increases to 6.1% (which is the long-run mean value) for the end of 50 years. The 1st–99th percentile range after 10 years is 3.5% to 8.7%. Figure 20 shows that the funnel of doubt graph neither increases over time (as interest rates and inflation do) nor decreases (as stock returns do). The histogram of modeled unemployment rates along with the distribution of historical values over the period from 1948 through 2003 are shown in Figure 21. By selecting only a single unemployment rate from each year (January), the frequency of the historical values corresponds with that of the model values, which are the unemployment rates indicated after the first year of the

FIGURE 18

DISTRIBUTION OF DIVIDEND YIELD PROJECTION PERIOD: 1 YEAR TO 50 YEARS

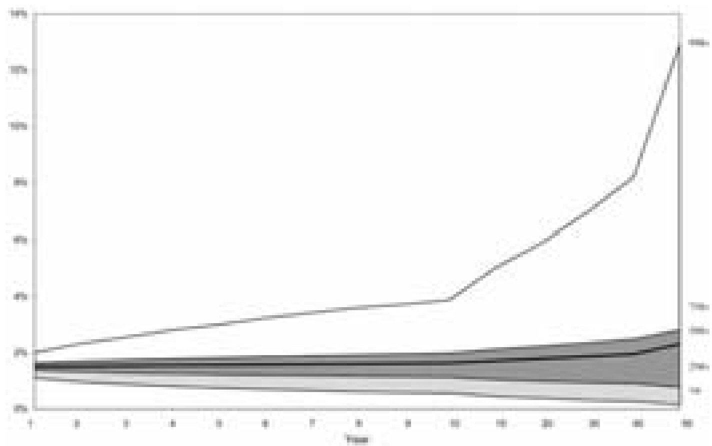


FIGURE 19

ACTUAL DIVIDEND YIELDS ON S&P 500 (1871–2003) VS. MODEL VALUES (5,000 ITERATIONS)

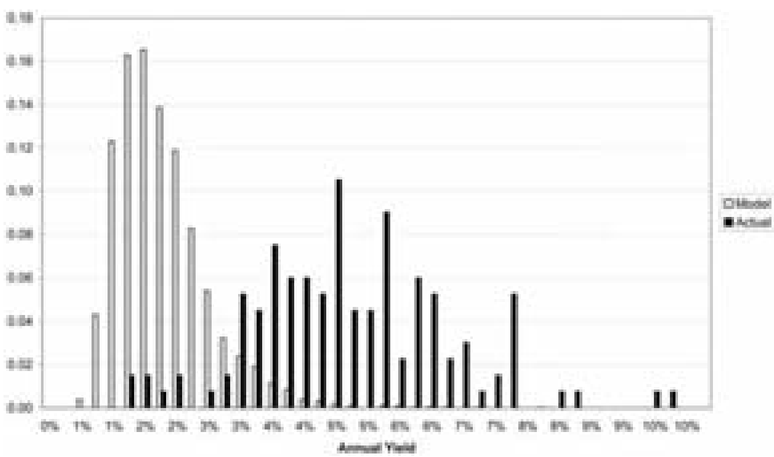


FIGURE 20

DISTRIBUTION OF UNEMPLOYMENT RATE PROJECTION PERIOD: 1 YEAR TO 50 YEARS

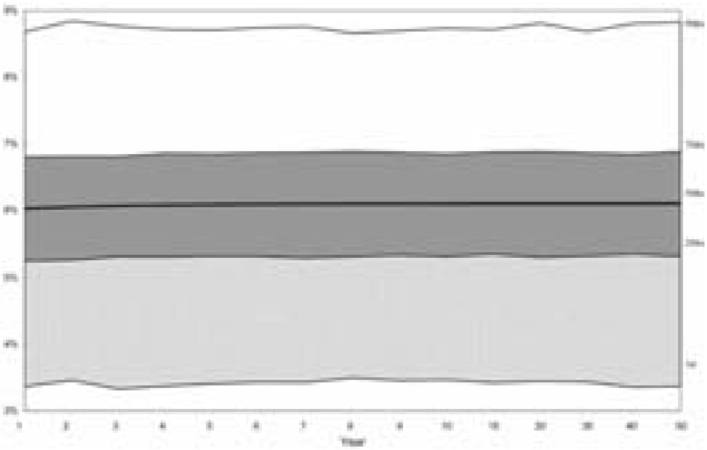


FIGURE 21

ACTUAL UNEMPLOYMENT RATE (1948–2003) VS. MODEL VALUES (5,000 ITERATIONS)

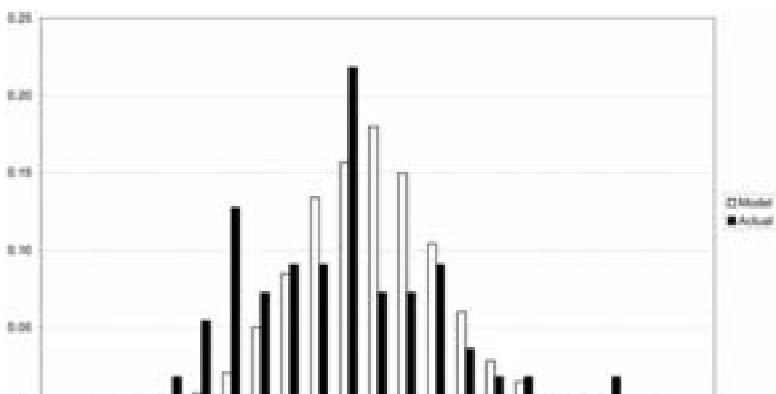
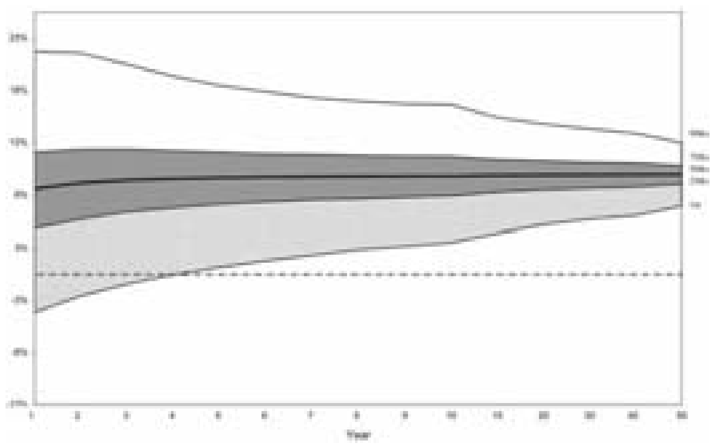


FIGURE 22

DISTRIBUTION OF COMPOUND AVERAGE REAL ESTATE RETURNS PROJECTION PERIOD: 1 YEAR TO 50 YEARS

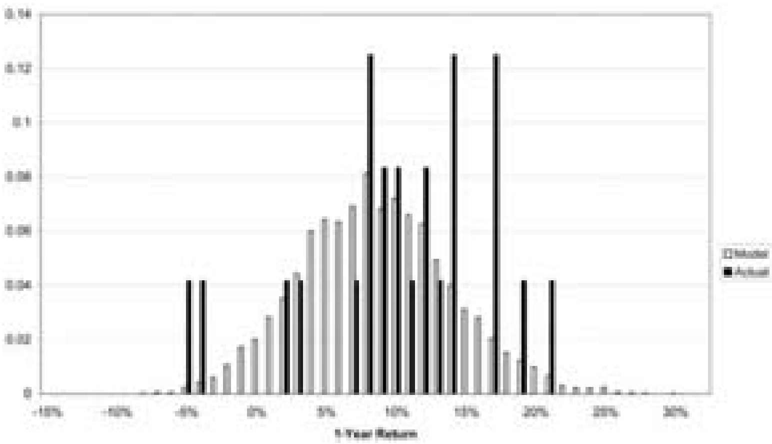


model run. Although the actual unemployment rates have varied a bit more than the model results do, the distributions are quite similar.

Real estate returns are the final variable included in the financial scenario model. From Table 1, the mean value of real estate returns is 8.1% in the first year and 9.4% after 50 years. The 1st–99th percentile range after 10 years is 3.0% to 16.1%. The funnel of doubt graph (Figure 22) is similar to the returns on stocks, for the same reasons. The histograms of modeled results and the historical returns based on the National Index from the National Council of Real Estate Investment Fiduciaries for 1978 through 2003 are shown on Figure 23. The model values show a smooth distribution centered about the historical returns. Unfortunately, only 26 years of annual returns are available, so it is difficult to draw any conclusions on the fit.

FIGURE 23

ACTUAL REAL ESTATE RETURNS (1978–2004) VS. MODEL VALUES (5,000 ITERATIONS)



Correlations

Table 2 displays the correlation matrix for all the output variables at the end of the first projection year (row 16 of the spreadsheet). Table 3 displays the corresponding matrix from history over the period from April 1953 through December 2001. Stock data are based on Ibbotson [30] and interest rates and inflation are from St. Louis Federal Reserve Data (http://research.stlouisfed.org/fred2/).

Several comments can be made when comparing the two correlation matrices. First, the historical correlation between large and small stocks is 0.744. The correlation between the model values of large and small stocks is 0.699, which looks quite reasonable. The correlation between inflation and Treasury bills (T-bills) has been 0.593 historically. This correlation is also clearly reflected in the model values, with a correlation of 0.906 between the one-month inflation rate and the one-month nominal interest rate, 0.892 between the one-year inflation rate and

TABLE 2

MODEL CORRELATIONS AMONG VARIABLES

	1-Month Inflation Rate	1-Month Nominal Interest Rate	1-Month Real Interest Rate	1-Year Inflation Rate	1-Year Nominal Interest Rate	1-Year Real Interest Rate	10-Year Inflation Rate	10-Year Nominal Interest Rate	10-Year Real Interest Rate	Large Stock Returns	Small Stock Returns	Dividend Yield	Unemploy-ment Rate	Real Estate Returns
1-Month Inflation Rate	1.000													
1-Month Nominal Interest Rate	0.906	1.000												
1-Month Real Interest Rate	−0.571	−0.170	1.000											
1-Year Inflation Rate	1.000	0.906	−0.571	1.000										
1-Year Nominal Interest Rate	0.892	0.987	−0.161	0.892	1.000									
1-Year Real Interest Rate	−0.363	0.040	0.923	−0.363	0.099	1.000								
10-Year Inflation Rate	1.000	0.906	−0.571	1.000	0.892	−0.363	1.000							
10-Year Nominal Interest Rate	0.617	0.779	0.076	0.617	0.855	0.405	0.617	1.000						
10-Year Real Interest Rate	−0.016	0.266	0.552	−0.016	0.374	0.804	−0.016	0.777	1.000					
Large Stock Returns	0.089	0.098	−0.018	0.089	0.099	0.008	0.089	0.085	0.037	1.000				
Small Stock Returns	0.076	0.084	−0.015	0.076	0.087	0.011	0.076	0.079	0.039	0.699	1.000			
Dividend Yield	−0.502	−0.483	0.232	−0.502	−0.469	0.137	−0.502	−0.318	−0.004	−0.248	−0.030	1.000		
Unemployment Rate	−0.194	−0.166	0.129	−0.194	−0.166	0.085	−0.194	−0.110	0.016	−0.026	−0.014	0.083	1.000	
Real Estate Returns	−0.019	−0.017	0.013	−0.019	−0.019	0.003	−0.019	−0.021	−0.012	−0.021	−0.021	−0.001	0.005	1.000

TABLE 3

HISTORICAL CORRELATIONS (APRIL 1953–DECEMBER 2001)

	Large Stocks	Small Stocks	3-Month T-Bills	1-Year Treasuries	10-Year Treasuries	Inflation Rate
Large Stocks	1.000					
Small Stocks	0.744	1.000				
3-Month T-Bills	−0.078	−0.065	1.000			
1-Year Treasuries	−0.074	−0.066	0.991	1.000		
10-Year Treasuries	−0.030	−0.025	0.912	0.942	1.000	
Inflation Rate	−0.138	−0.100	0.593	0.576	0.478	1.000

the one-year nominal interest rate, and 0.617 between the 10-year inflation rate and the 10-year nominal interest rate. Since nominal interest is the sum of the real interest rate and the inflation rate, and the real interest rate is constrained to be no less than the negative of the inflation rate, this correlation is built into the model.

Historically, T-bill rates and stock returns have been negatively correlated (−0.078 for large stocks and −0.065 for small stocks). In the model, there was a slight positive correlation between the one-year nominal interest rate and stock returns (0.099 for large stocks and 0.087 for small stocks). Also, the historical correlation between inflation and stock returns has been negative (−0.138 for large stocks and −0.100 for small stocks). The correlations in the model values between the one-year inflation rate and large stocks and small stocks were 0.089 and 0.076, respectively.

Alternate Parameters

The base parameters provide one feasible set of values to use in modeling future economic conditions. These should be viewed as a starting point in these applications. However, users should develop an understanding of the impact of the different parameters and then adjust these parameters as necessary to generate distributions that they feel may be more suitable for a particular application. For example, from the results shown above, a user may feel that the default parameters for equity returns, while consistent with historical experience through 2003, produce very high equity risk premiums that may not be expected to continue in the future. When testing long-term insurer solvency, an actuary might change the regime-switching parameters to look at the effects of lower stock returns over the next 50 years.

6. CONCLUSION

Historically, actuaries tended to use deterministic calculations to value financial products. As technology improved, actuaries began to incorporate different assumptions about insurance and economic variables that would lead to several distinct scenarios to better measure financial risk. The explosion of computing power now gives actuaries and other financial analysts tremendous tools for more refined risk analyses. Modern approaches to financial modeling begin by specifying the underlying economic and financial environments based on sophisticated mathematical equations, and then incorporate product-specific features that are commonly related to those external conditions. This approach yields a much richer understanding of the risks associated with financial products.

The financial scenario model and its underlying mathematical structure presented in this paper provide an integrated framework for sampling from a wide range of future financial scenarios. The model produces output values for interest rates, inflation, stock and real estate returns, dividends, and unemployment. The model can be incorporated into a variety of insurance applications, including dynamic financial analysis, cash flow testing, solvency testing, and operational planning. It is hoped that this work will facilitate the use of recent advances in economic and financial modeling in the actuarial profession.

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WHEN CAN ACCIDENT YEARS BE REGARDED AS DEVELOPMENT YEARS?

GLEN BARNETT, BEN ZEHNWIRTH AND EUGENE DUBOSSARSKY

Abstract

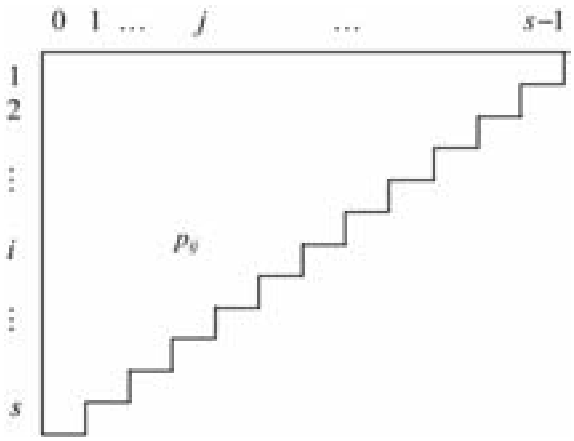
The chain ladder (volume-weighted average development factor) is perhaps the most widely used of the link ratio (age-to-age development factor) techniques, being popular among actuaries in many countries. The chain ladder technique has a number of interesting properties. We present one such property, which indicates that the chain ladder doesn't distinguish between accident years and development years. While we have not seen a proof of this property in English language journals, it appears in Dannenburg, Kaas and Usman [2]. The result is also discussed in Kaas et al. [3]. We give a simple proof that the chain ladder possesses this property and discuss its implications for the chain ladder technique. It becomes clear that the chain ladder does not capture the structure of real triangles.

1. INTRODUCTION

Link ratio (loss development factor) methods are widely used for reserving. The chain ladder technique is one such method applied to cumulative paid loss (or sometimes case incurred loss). The development factor is an average of the individual link ratios, weighted by the previous cumulative loss (volume-weighted average). The chain ladder is normally applied to cumulated paid loss arrays, incurred loss arrays, or sometimes to cumulated claim numbers, such as claims incurred, claims notified or claims closed. This "formal" chain ladder is described by Mack in [5], but we give a detailed description of it below. We present the chain ladder for a paid loss array with annual data; the expo-

FIGURE 1

INCREMENTAL PAID LOSS ARRAY



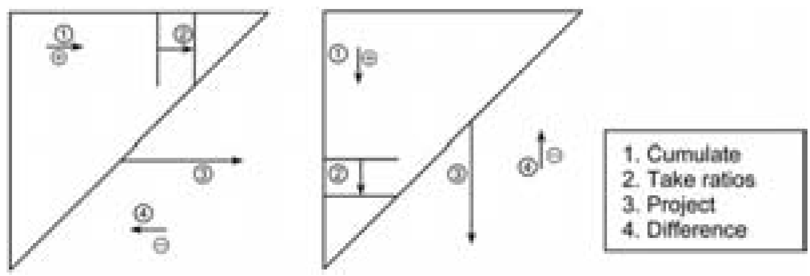
sition is essentially the same for other kinds of data. We assume that the reader is familiar with the usual triangular development layout.

Consider the incremental array $P = [p_{ij}]$, $i = 1,\ldots,s$; $j = 0,\ldots,s-1$; $i + j \leq s$ (the array of incremental payments—the actual amounts paid in each development year in respect of each accident year—contains the fundamental observed quantities).

The chain ladder is usually presented in something like the following fashion. Let us take an array of paid losses (incremental amounts paid), p_{ij}, and cumulate along the accident years, $c_{ij} = p_{i0} + p_{i1} + \cdots + p_{ij}$, so that $c_{ij} = \sum_{k=0}^{j} p_{ik}$ are the corresponding cumulative paid loss amounts. Then compute ratios $r_j = \sum_i c_{ij} / \sum_i c_{i,j-1}$, where the sum is over all available terms that are present in both the jth and $(j-1)$th columns. Forecasts are produced by projecting elements on the last diagonal $c_{i,s-i}$ to the next development by multiplying by the development ra-

FIGURE 2

TWO INCREMENTAL ARRAYS TO WHICH AN "ACROSS" (THE STANDARD CHAIN LADDER) AND A "DOWN" VERSION OF THE CHAIN LADDER ARE APPLIED



tio r_{s-i+1}, and recursively projecting those forecasts in turn by multiplying by the next ratio.

Now imagine a version of the chain ladder working in the other direction ("down" rather than "across")—where you cumulate downward, take ratios running down (accident-year-to-accident-year ratios), project down into the future, and difference back to incrementals, as in Figure 2. It turns out that the incremental forecasts for both the usual chain ladder (the version that runs across) and this new "down" version of the chain ladder are the same.

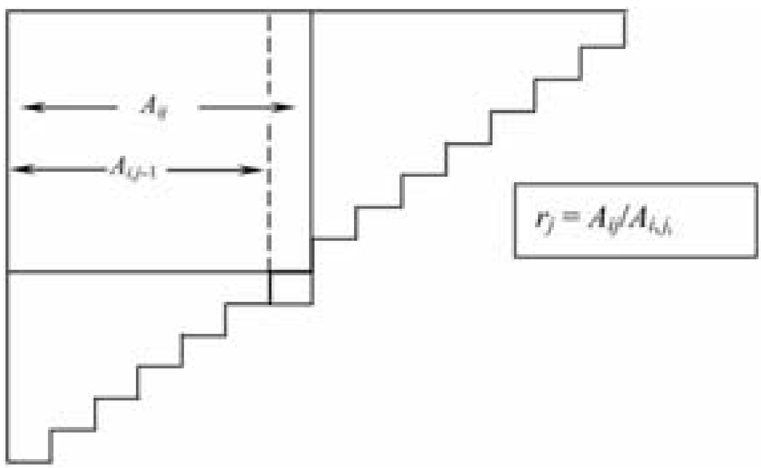
2. THE INCREMENTAL CHAIN LADDER

To see that the "across" and the "down" versions of the chain ladder are the same, we will first write the chain ladder purely in terms of incrementals (which we call *the incremental chain ladder*).

Consider that we are attempting to forecast a cumulative paid loss amount, c_{ij}, in the next calendar year. Let $A_{ij} = \sum_k c_{kj}$, that is, A_{ij} is the sum of all the cumulatives in the column above

FIGURE 3

DEPICTION OF THE INCREMENTALS INVOLVED IN THE CALCULATION OF THE RATIO

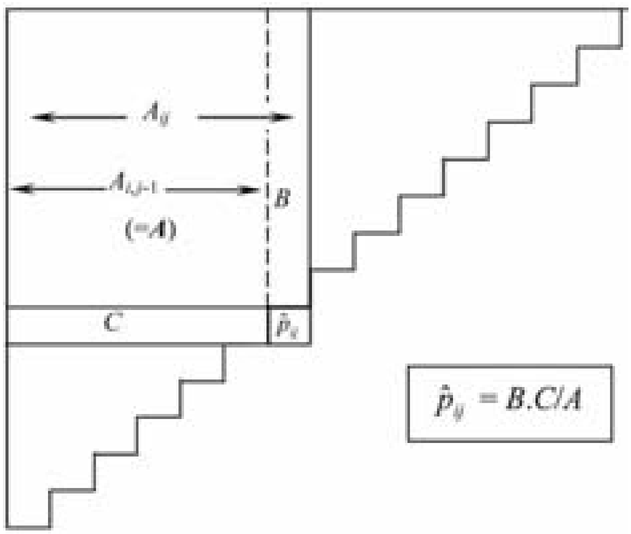


c_{ij}. Then the jth ratio is $r_j = A_{ij}/A_{i,j-1}$. Note that A_{ij} is also the sum of all the incremental loss amounts above the (i,j) cell (p_{kj}, $k = 1,2,\ldots,i-1$) plus all the incremental loss amounts to the left of those. That is, A_{ij} and $A_{i,j-1}$ are the sums of all the incremental loss amounts in the regions shown in Figure 3. If the values of the incrementals are represented by heights of square prisms in each (i,j) cell, values represented by A, B and C in Figure 4, may be thought of as the "total volume" in the marked regions.

Note further that (since the forecasts of the cumulative paid loss are in the same ratio) the formula $r_j = A_{ij}/A_{i,j-1}$ as a ratio of sums of incrementals as defined above applies to observations in later (further into the future) calendar years as well, as long as any unobserved incremental loss amounts in the sum are replaced with their predicted values.

FIGURE 4

CALCULATION OF CUMULATIVE AND INCREMENTAL FORECASTS IN TERMS OF INCREMENTALS (LABELS REPRESENT THE SUMS OF THE INCREMENTALS IN THEIR REGION)



In the usual form of the chain ladder, you compute forecasts $\hat{c}_{ij} = r_j \times c_{i,j-1}$ (where $c_{i,j-1}$ is again replaced by its forecast when it is unavailable). That is, compute the forecasts $\hat{c}_{ij} = A_{ij}/A_{i,j-1} \times c_{i,j-1}$. Predicted incremental paid loss amounts may be formed by taking first differences of predicted cumulative paid amounts. Computation of incremental paid loss forecasts is essential for incorporating future inflation and discounting, (where relevant) and for computation of annual claim cash flows.

Now let $b_{ij} = A_{ij} - A_{i,j-1}$, which is the sum of the incrementals above p_{ij}. For simplicity, in Figure 4 this is just called B. Similarly, let $C = c_{i,j-1}$, and let $A = A_{i,j-1}$.

Then the forecast may be written

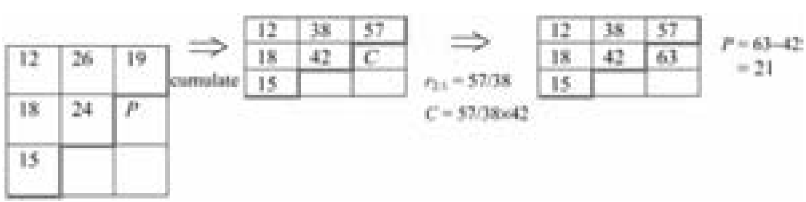
$$\begin{aligned}\hat{c}_{ij} &= [A_{ij}/A_{i,j-1}]c_{i,j-1}\\ &= [(A_{i,j-1} + b_{ij})/A_{i,j-1}]c_{i,j-1}\\ &= [(A + B)/A]C\\ &= [1 + B/A]C.\end{aligned}$$

Similarly, the incremental forecast is

$$\begin{aligned}\hat{p}_{ij} &= \hat{c}_{ij} - c_{i,j-1}\\ &= [1 + B/A]C - c_{i,j-1}\\ &= [1 + B/A]C - C\\ &= B.C/A.\end{aligned}$$

That is, the forecast of the incremental observation is the product of (the sum of the incrementals above it) and (the sum of the incrementals to its left) divided by (the sum of all the incrementals that are both above and to the left). Note that this is symmetric in B and C (and also A)—interchanging i and j merely changes the role of B and C. Thus we see that the chain ladder may be neatly defined directly in terms of the incremental paid loss amounts. See the appendix for a more formal proof of the above symmetry.

It may help to give an example. Imagine we have an incremental paid loss array as follows, and we wish to predict the incremental paid loss cell labeled P:



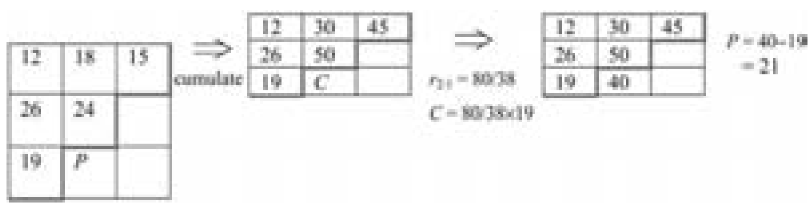
Note what the cumulative forecast, C, consists of in terms of the incrementals:

$$C = 57 \times 42/38 = (12+26+19) \times (18+24)/(12+26).$$

Hence, we have

$$\begin{aligned} P &= 57 \times 42/38 - 42 \\ &= (12+26+19) \times (18+24)/(12+26) - (18+24) \\ &= (12+26+19) \times (18+24)/(12+26) - (18+24) \\ &\quad \times (12+26)/(12+26) \\ &= (12+26+19-12-26) \times (18+24)/(12+26) \\ &= 19 \times (18+24)/(12+26) = B.C/A. \end{aligned}$$

Every incremental forecast turns out to work the same way (recall that you must replace unobserved values in A, B and C by their forecasts in this formulation). Consequently, when we interchange (i.e., transpose) accident and development years in the original array and apply the chain ladder, note that the same value is obtained:



Considered in terms of cumulative paid loss, it is not immediately clear that the chain ladder incremental prediction, P, will not change as a result of the transposition. However, if you consider it in the incremental paid loss form, while B and C have interchanged, their product is obviously the same. Further, A is unchanged, so the forecast is unchanged. In each case, we have $P = 19 \times (18+24)/(12+26)$.

This applies generally when interchanging accident and development years in the incremental paid loss array and applying the chain ladder. When considered in terms of the incremental paid loss formula, the transposition merely interchanges the values of B and C, and leaves A unchanged, so the incremental paid loss forecasts are unchanged.

One advantage of this incremental paid loss version of the chain ladder is that it is often more convenient to implement in a spreadsheet. This is because it can be implemented in terms of formulas that can be successfully cut and pasted without the effort involved in computing the ratios first. The usual ratios (and cumulative paid loss forecasts if needed) are then easily computed from the completed array.

3. A BRIEF DISCUSSION OF SOME RELATED WORK

Kremer [4] recognizes the connection between a ratio model (which he calls a multiplicative model) and two-way analysis of variance with missing values, computed on the logarithms. He uses this to derive an approach to forecasting outstanding claims. Kremer points out the connection to the chain ladder method in detail.

Mack [5] derives standard error calculations (including process and parameter error) for a mean-variance model whose forecasts reproduce the standard chain ladder technique in a recursive fashion. Mack makes use of the Gauss-Markov theorem to avoid specific distributional assumptions for the losses. He compares results for a particular case study with results from similar models for which computation of exact or approximate standard errors are available.

In his later paper, Mack [6] argues that while several different stochastic ratio models had previously been referred to as the stochastic chain ladder, the model he discusses in [5] reproduces the classical chain ladder forecasts and that models that don't do so should not be referred to as chain ladder models.

Murphy [7] explicitly writes several loss development factor methods as stochastic models and derives forecast variances, working in a least-squares framework. He argues that it is often necessary to extend ratio models to include intercepts.

Barnett and Zehnwirth [1] develop a statistical framework extending Murphy's approach to include some adjustment for common accident and calendar period trends as a general diagnostic tool for testing the suitability of ratio models to data. Multiple examples point toward some common deficiencies of ratio models, including the need for an intercept and the lack of predictive power of ratios after incorporating obvious predictors.

Renshaw and Verrall [8] derive another model that reproduces the chain ladder. Their formulation makes the number of parameters describing the mean process in the chain ladder explicit. The model is initially presented as a Poisson model, which extends to a quasi-likelihood framework as a model with variance proportional to the mean.

Even though we started with a form of the chain ladder that looked something like the stochastic form presented by Mack [5, 6] and Murphy [7], by the end of Section 2 there are strong similarities to the stochastic form presented by Renshaw and Verrall [8]. Despite arguments in the literature, the two approaches differ mainly in the data on which they appear to condition when describing the past, and in the number of variance parameters they employ. They are identical in the way they describe the mean predictions for the future, which is why they both reproduce the chain ladder forecasts. Given a quasi-likelihood approach, differences in forecast standard errors appear to be largely due to two factors—the number of variance parameters, and the number of degrees of freedom to fit the data (i.e., parameters) for which the parameter uncertainty is ignored.

Note that Kremer [4] describes the relationship between the ratios of the chain ladder and the column parameters in the two-way loglinear model. This is akin to the relationship between the parameters in the Mack formulation and those of Renshaw and Verrall.

4. INTERCHANGEABILITY OF ACCIDENT AND DEVELOPMENT YEARS

We immediately see from the previously mentioned symmetry that the incremental predictions from the chain ladder of the incremental array with accident years and development years interchanged (with the array transposed) are simply the corresponding predictions from the original array, with accident years and development years interchanged (transposed). That is, the chain ladder has the property that its incremental forecasts are the *same* whether the chain ladder is applied to an incremental array running across (as is usual) or down—where you cumulate down, take accident-year to accident-year ratios, project down into the future, and difference back to incrementals!

Note that this property must hold for the forecasts of all models that reproduce the chain ladder forecasts. Such a property might most accurately be called the "transpose-forecast commutativity property of the chain ladder." However, in the interest of brevity we simply call it *transpose-invariance*.

This property implies that any fact that applies to the accident years applies to development years, and vice versa, and that any asymmetry of directions in our description of the chain ladder is an artifact of our description, and is not an inherent part of the chain ladder itself. That is, the chain ladder doesn't differentiate between accident and development periods. It treats them in identical fashion, even though the actual structure in the two directions is completely different. This result obviously applies to forecasts for all the stochastic chain-ladder-reproducing models as well.

Consider the issue of accident years being treated like development years. Imagine you have homogeneous accident years (a not uncommon occurrence, especially after you adjust for changes in exposure and inflation, assuming no superimposed inflation). You wouldn't predict the level of the next accident year using ratios—it would be far more sensible and informative to take some kind of average. But as we have seen, the chain ladder *does* use ratios in both directions.

If this way of looking at the chain ladder seems a little nonsensical, it is because we are inferring additional meaning in the usual form of the chain ladder that it doesn't really possess. The two descriptions (the across version and the down version) are in reality the same description of the data.

Note also that we can now see that there are in fact parameters in both directions in the chain ladder. This is not a consequence of any particular formulation of the chain ladder—every chain-ladder-reproducing model has degrees of freedom to fit the data (i.e., parameters) that run both across and down. Some formulations make the existence of both kinds of parameters explicit (as in Renshaw and Verrall [8]); some other formulations do not (such as Mack [6])—the row parameters become hidden by the fact that the model is conditioned on the first column. The chain ladder itself still unavoidably has degrees of freedom to fit changes in accident level, so the parameters remain, even where not explicitly represented in the formulation. All formulations of the chain ladder have $2s-1$ parameters for the mean, though the number of variance parameters and distributional assumptions may vary.

We note that so many parameters make the forecasts quite sensitive to relatively small changes in a few values, making the chain ladder unsuitable for forecasting. Yet even with so many parameters the chain ladder is still unable to model changing superimposed inflation.

A further important consequence of this property is that parameters in the two directions can take the roles of both a level and a ratio.

We know within ourselves that the two directions are fundamentally different, both in general appearance of their trends and in spirit. The development year direction tends to have a smooth run-off shape, where the incremental losses tend to increase initially to a peak somewhere in the first few developments and then smoothly decrease in the tail, while the accident years tend to have quite a different pattern. Yet the model itself makes no such distinction—it does not contain important information we already know about claims payments (i.e., the structure in loss data).

Indeed, the chain ladder model, being a two-way cross classification model (as has been recognized by numerous authors), not only fails to distinguish between accident years and development years, it ignores the relationships between years within either category.

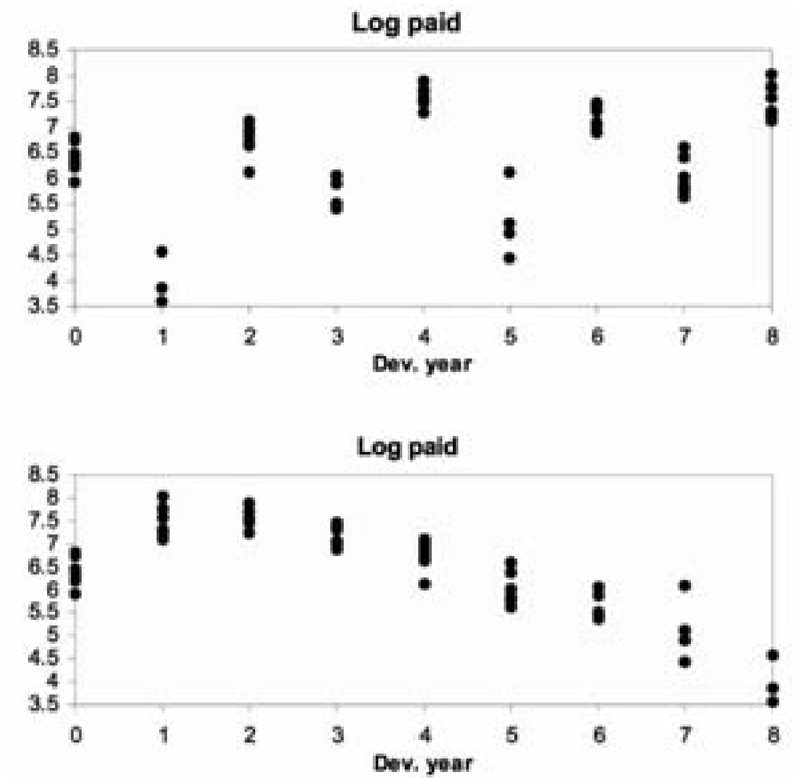
Consider the two plots of logs of paid data against development year in Figure 5. Can you tell which one is the real data?

Most practitioners will instantly (and correctly) guess that the lower plot is the real one. We *know* that paid data often has a strong pattern to its runoff—that nearby development periods tend to be more alike than ones further away, and further, we usually observe smooth trends relating them.

Clearly the accident and development labels mean something, and you can't arbitrarily relabel them without affecting the information in the data. The observations in development year 3 do not just tend to be closer to each other than to observations from other columns, they also tend to be more like the observations in development years 2 and 4 than they are like observations from columns further away. In a two-way cross-classification model, we can arbitrarily rearrange the group labels within both factors, and even interchange the factors *without changing the fit.* If the

FIGURE 5

PLOT OF EXPOSURE-ADJUSTED LOG PAID INCREMENTAL DATA AGAINST DEVELOPMENT YEAR (ONE OF THE PLOTS HAS HAD ITS DEVELOPMENT PERIOD LABELS RANDOMLY ALLOCATED)



labels do carry information over and above being arbitrary identifiers of a category into which the observation falls (as they do in claims runoff), the chain ladder model is inappropriate.

With the lower plot of Figure 5, one could omit *all* of the data for any development period between 1 and 8 (i.e., replace the observations with missing values) and still be able to get a good estimate of the values in that development period. Nearby periods carry a great deal of information about each development period's level. If we consider only the first plot, and we omit a development period, what do we know about it? The information was there, but we threw it away when we threw away the ordering in the development year. There is also information in the accident year direction—nearby accident years often tend to be more alike than ones further away. The chain ladder ignores that information in both directions. This loss of information causes the predictive distributions of chain ladder forecasts to be very wide, much wider than they should be if the model used what we know about losses.

The top plot of Figure 5 actually looks a bit more like a plot against accident years (though nearby accident years in practice are often closer together than those further away, and so they tend to be smoother than the top plot, even though they don't normally exhibit the smooth curves of the development direction).

That is not to say that a plot against development years looking something like the top one could *never* arise, but it is quite rare—and if it *does* arise, an ANOVA-style model is not very helpful in forming good forecasts, particularly in the tail and for future developments. It has parameters where it has little data, and that makes for poor forecast prediction errors. An underparameterized model is often substantially better for forecasting in that circumstance. If we were in the rare circumstance that the means for each development didn't have any strong trend to them, we'd want to quantify the extent to which the means tend to shift around their overall average, and use as much information as possible in identifying what little trend there might be. When there is less information in the data, it is even more crucial not to waste it.

Many of the problems discussed with respect to the chain ladder apply to other link ratio methods. The exact transpose-invariance property no longer applies (since different weights are involved), but basic link ratio methods are still two-way cross classification models (with different assumptions about variance), so they generally share the problem of overparameterization in the development and accident year directions, and ignore the relationships between adjacent year levels. Further, although the correspondence isn't exact, there is generally a strong similarity between forecasts (on the incremental scale) and the transposed-forecasted-transposed forecasts. This is hardly surprising, since other ratios may be written as weighted versions of the chain ladder; the transposing merely results in a differently weighted version of a method that *is* transpose-invariant.

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APPENDIX

Derivation of the Incremental Chain Ladder

In the following, when an unavailable term appears on the right hand side of an equation, it is replaced by its predicted value. The usual form of the chain ladder predictions is given by:

$$\begin{aligned}
\hat{c}_{ij} &= \hat{\beta}_j c_{i,j-1} \\
&= \left[\sum_{h=1}^{s-j} c_{hj}\right] \bigg/ \left[\sum_{h=1}^{s-j} c_{h,j-1}\right] . c_{i,j-1} \\
&= \left(1 + c_{s-j,j} \bigg/ \left[\sum_{h=1}^{s-j} c_{h,j-1}\right]\right) . c_{i,j-1} \\
&= \left(1 + \left[\sum_{h=1}^{s-j} p_{hj}\right] \bigg/ \left[\sum_{h=1}^{s-j}\sum_{k=0}^{j-1} p_{hk}\right]\right) \cdot \sum_{k=0}^{j-1} p_{ik}.
\end{aligned}$$

Hence

$$\begin{aligned}
\hat{p}_{ij} &= \left(1 + \left[\sum_{h=1}^{s-j} p_{hj}\right] \bigg/ \left[\sum_{h=1}^{s-j}\sum_{k=0}^{j-1} p_{hk}\right]\right) \cdot \sum_{k=0}^{j-1} p_{ik} - \sum_{k=0}^{j-1} p_{ik} \\
&= \left(\left[\sum_{h=1}^{s-j} p_{hj}\right] \bigg/ \left[\sum_{h=1}^{s-j}\sum_{k=0}^{j-1} p_{hk}\right]\right) \cdot \sum_{k=0}^{j-1} p_{ik} \\
&= (B/A).C, \qquad \text{(see Figure 4)} \\
&= B.C/A
\end{aligned}$$

where $A = \sum_{h=1}^{s-j}\sum_{k=0}^{j-1} p_{hk}$, $B = \sum_{h=1}^{s-j} p_{hj}$ and $C = \sum_{k=0}^{j-1} p_{ik}$. Note the symmetry in the subscripts.

The Transpose Invariance Property

The symmetry immediately establishes the transpose invariance property. Equivalently, refer to Figure 4, and note that the numerator of the equation for $\hat{p}_{ij}$ is the product of the total of the values above it and the total of the values to its left. Consequently, if the array were transposed (rows and columns interchanged), the numerator for $\hat{p}_{ji}$ would be unchanged (and of course the denominator is also unchanged) from that for $\hat{p}_{ij}$.

THE APPLICATION OF FUNDAMENTAL VALUATION PRINCIPLES TO PROPERTY/CASUALTY INSURANCE COMPANIES

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Abstract

This paper explores the concepts underlying the valuation of an insurance company in the context of how other (noninsurance) companies are valued. Among actuaries, the value of an insurance company is often calculated as (i) adjusted net worth, plus (ii) the present value of future earnings, less (iii) the cost of capital. Among other financial professionals (e.g., chief financial officers, investment bankers, economists), value is often calculated as the present value of future cash flows. This paper will discuss both methods and explain under what circumstances the two methodologies derive equivalent value and under what circumstances the results of the two methods diverge. This paper also addresses recent developments in the insurance industry that could affect valuation, including the NAIC's codification of statutory accounting principles, fair value accounting, and the Gramm-Leach-Bliley Act of 1999.

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1. INTRODUCTION

Valuation of a property/casualty insurance company is an important feature of actuarial work. Much of the work arises from merger, acquisition, and divestiture activity, although the need for valuation arises from other sources. An insurance company valuation might be prepared for lending institutions or rating agencies. It might be performed as part of a taxable liquidation of an insurance company, reflecting the value of existing insurance policies in force. A valuation might also be prepared for the corporate management of insurance companies in order to provide the clearest picture of value and changes in value of the company over a given time period.

The assumptions underlying the valuation, and therefore the computed value, may differ for different uses.[1] As such, the purpose of the valuation and the source of the assumptions should be clearly identified.

Before discussing valuation methodologies, we introduce some basic principles.

1. The value of any business has two determining factors:
   (a) The future earnings stream generated by a company's assets and liabilities, and
   (b) The risk of the stream of earnings. This risk is reflected in the cost to the entity of acquiring capital, measured by the investors' required rate of return (i.e., the "hurdle rate").
2. For a given level of future risk, the greater the expected profits,[2] the greater the value of the business.
3. For a given level of future profitability, the greater the volatility (and therefore the higher the hurdle rate), the lower the value of the business.

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[1]For example, in an acquisition, the purchaser may be able to lower expenses, grow a business faster because of the purchaser's current business, reduce the effective tax rate, or reduce the cost of capital for the acquired or target entity. These assumptions would serve to increase the value of the target entity. These same assumptions may not be valid for valuing the target entity as a stand-alone business unit.

[2]Expected profits refer to the present value of the expected earnings stream.

4. A company has value in excess of its invested capital only when future returns are in excess of the hurdle rate.

5. When a company is expected to produce an earnings stream that yields a return on invested capital that is less than the hurdle rate, the economic value of the required capital is less than its face value. In this case, the logical action would be to liquidate assets.

2. VALUATION METHODOLOGIES

There are two methodologies prevalent in valuation literature that form the basis of our discussion of insurance company valuation:

1. Discounted cash flow (DCF)
2. Economic value added (EVA)

A DCF model discounts free cash flows to the equity holders at the hurdle rate. The starting capital of the entity is *not* a direct element in the valuation formula.[3]

An EVA model begins with the starting capital of the entity and defines value as the following:

Value = Initial capital invested
+ the present value (PV) of expected "excess returns" to equity investors.

Sturgis [20] refers to two methods in his paper on valuation:

1. The discounted value of maximum stockholder dividends.

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[3]If the starting capital of the entity is higher (or lower) than capital required, it will generate a positive (or negative) cash flow to the investor at "time zero."

2. Current net worth[4] plus the discounted value of future earnings less cost of capital.

The first method corresponds to DCF methodology. The second method is also discussed by Miccolis [17] and in other actuarial literature as

$$\text{ANW} + \text{PVFE} - \text{COC},$$

where

ANW = adjusted net worth (statutory capital and surplus with a series of modifications);

PVFE = present value (PV) of future earnings attributable to in-force business and new business; and

COC = cost of capital.

= PV of [(hurdle rate $\times$ required starting capital for each period) – (investment earnings on capital excluded from future earnings)].[5]

This second method is a form of the EVA model, in which PVFE – COC equals the present value of expected excess returns.

2.1. Discounted Cash Flow

A company's value may be determined by discounting free cash flows to the equity owners of the company[6] at the cost of equity, or the hurdle rate. Free cash flow is often defined as the after-tax operating earnings of the company, decreased by earnings that will be retained in the company, or increased

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[4]Throughout this paper, we use the terms capital, equity, net worth, and surplus interchangeably.

[5]If future earnings include investment income on capital, the cost of capital calculation will be modified to be equal to the present value of (hurdle rate $\times$ starting capital each period).

[6]Free cash flows are released in the form of dividends or other capital releases to the equity owners.

by capital releases to maintain an appropriate level of capital to support the ongoing business of the company.

After-tax operating earnings usually constitute changes in capital during a period other than capital infusions or distributions. For property/casualty insurance companies, however, there are gains and losses in surplus due to "below the line" adjustments[7] that do not flow through statutory earnings. Capital changes associated with the change in unrealized capital gains or losses, the change in nonadmitted assets, the change in statutory reinsurance penalties, the change in foreign exchange adjustment, and the change in deferred income tax must be considered along with after-tax operating earnings when evaluating free cash flows. For the valuation formulas discussed throughout this paper, after-tax operating earnings include these direct charges and credits to statutory surplus.

A company creates value for its shareholders only when it earns a rate of return on invested capital (ROIC) that exceeds its cost of capital or hurdle rate. ROIC and the proportion of after-tax operating earnings that the company invests for growth drive free cash flow, which in turn drives value. For some industries, regulatory or statutory restrictions create an additional consideration that limits dividendable free cash flow.

The DCF value of the business is often projected as two separate components: (i) the value of an explicit forecast period, and (ii) the value of all years subsequent to the explicit forecast period (the "terminal value"). Projections for the forecast period, which is usually five to 10 years,[8] typically include detailed annual earnings projections that reflect revenue projections, loss and

[7] "Below the line" refers to the Underwriting and Investment Exhibit in the statutory Annual Statement prescribed by the NAIC. Direct charges and credits to surplus are shown below the line for Net Income, which is the starting point for regular taxable income.

[8] Five to 10 years is typical because beyond that period it is usually too speculative to project detailed financials. A long-term earnings growth rate and a corresponding capital growth rate are selected to derive value beyond the forecast period.

expense projections, investment income projections, tax liabilities, after-tax operating earnings, assets, liabilities, initial capital, and the marginal capital that needs to be invested in the company to grow the company at the expected annual growth rate.[9]

The DCF value of the forecast period cash flow is

$$\mathrm{FC}_0 + \sum_{x=1}^{n} \frac{\mathrm{OE}_x - (C_{x-1} \times g_x)}{(1+h)^x},$$

where

n = the number of years in the forecast period (usually five to 10 years);

OE_x = after-tax operating earnings in year x (including gains and losses in capital that do not flow through earnings);

g_x = expected growth rate of capital in year x;

C_{x-1} = capital at the end of year $x-1$ (this equals capital at the beginning of year x);

$C_{x-1} \times g_x$ = incremental capital required to fund future growth;

h = hurdle rate; and

FC_0 = free capital at time zero; which represents capital that may be either released from the company at the valuation date if the company is overcapitalized or infused into the company at the valuation date if the company is undercapitalized

= $\mathrm{SC}_0 - C_0$, the difference between SC_0, the starting capital of the entity, and C_0, the capital *needed* at the end of year zero/beginning of year 1.

The value of the second component of DCF value is often referred to as the *terminal value*. The terminal value can be devel-

[9]Appendix A addresses these earnings forecasts in detail and provides an example.

oped using a simplified formula based on (i) projected after-tax net operating profits in the first year *after* the forecast period, (ii) the perpetual growth rate, and (iii) the hurdle rate.

$$\text{Terminal value} = \sum_{x=n+1}^{\infty} \frac{\text{OE}_x - (C_{x-1} \times g)}{(1+h)^x}$$
$$= \frac{\text{OE}_{n+1} - (C_n \times g)}{(h-g)(1+h)^n},$$

where

n = the number of periods in the forecast period;

C_n = the capital at the end of the last period of the forecast period;

g = the expected perpetual growth rate of capital and of after-tax operating earnings;

h = the hurdle rate;

OE_{n+1} = after-tax operating earnings in the period after the forecast period; and

$\text{OE}_{n+1} - (C_n \times g)$ = free earnings, equal to after-tax earnings less amounts needed to be retained in the company to grow the capital at rate g.

This terminal value calculation gives credit for earnings into the future in perpetuity. Sometimes a higher hurdle rate is used for the terminal value than for the forecast period to reflect the increased uncertainty associated with operating earnings many years in the future. A discussion of considerations related to the selection of the hurdle rate is provided in Section 4.

The terminal value can be thought of as the present value of the free earnings (in the period after the forecast period) multiplied by a price to earnings (P/E) ratio. The P/E ratio is determined by the hurdle rate, h, and the growth rate, g, and is equal to $1/(h - g)$.[10] If the hurdle rate is 15% and the growth rate is 5%, then the P/E ratio = 1/(.15 – .05) = 10.

In practice, the P/E ratio underlying the terminal value calculation can be selected by reviewing sale prices of recent insurance company transactions relative to earnings. Relating that P/E factor to an implicit growth rate and hurdle rate may make the price-to-earnings ratio more intuitive.

2.2. *Economic Value Added*

The value of a company can be written as the sum of the equity invested and the expected *excess returns* to investors from these and future investments.

$$\begin{aligned}\text{Value} &= \text{Initial capital invested}\\ &\quad + \text{PV of expected "excess returns" to equity investors.}\end{aligned}$$

The expected "excess returns" in each period are defined as

$$\begin{aligned}&(\text{rate of return on capital invested} - \text{hurdle rate}) \times \text{capital invested}\\ &\quad = \text{after-tax operating earnings} - (\text{hurdle rate} \times \text{capital invested}).\end{aligned}$$

The general expression of EVA is

$$\text{Value} = \text{SC}_0 + \sum_{x=1}^{\infty}[\text{OE}_x - (h \times C_{x-1})] \times (1 + h)^{-x},$$

where

SC_0 = Starting capital, which is equal to the sum of free capital and required capital at time 0 (FC_0 and C_0, respectively, as defined in the DCF discussion); and

OE_x, C_x, and h have the same definitions as in the DCF discussion.

[10]The expected growth rate will typically be between 0% and the selected hurdle rate. If, however, the growth rate g were less than 0%, the resulting P/E ratio would decrease (as $h - g$ increases).

This formula represents the required capital at the valuation date (time = 0) plus the present value of future economic profits. Economic profits for time period x are defined as after-tax operating earnings (OE_x) reduced by the cost of capital, which is the product of the hurdle rate and the required capital at the beginning of each period ($h \times C_x$).

To calculate EVA, we need three basic inputs:

1. The level of capital needed for each period to support the investment, both initial capital invested and additional capital to support growth.
2. The actual rate of return earned on the invested capital for each period, that is, ROIC.
3. The selected hurdle rate.

These are the same inputs required for the DCF model.

To determine initial capital invested, we start with the book value of a company. The book value of an insurance company is an amount that reflects the accounting decisions made over time on how to depreciate assets, whether reserves are discounted, and conservatism in estimating unrecoverable reinsurance, among other factors. As such, the book value of the company may be modified in the valuation formula to adjust for some of the accounting influence on assets and liabilities.

In valuing an insurance company, the initial capital invested is represented by the statutory capital and surplus[11] at the valuation date, modified with a series of adjustments discussed later in this paper. The surplus after modifications is often referred to as adjusted net worth (ANW). The capital needed to support growth is funded by retained earnings for the DCF model and reflected through the cost of capital calculation for the EVA model.

[11]The reasons for using statutory accounting values instead of generally accepted accounting principles (GAAP) or other accounting values are discussed in Section 4.

To evaluate the ROIC an estimate of after-tax income earned by the firm in each period is needed. Again, the accounting measure of operating income has to be considered. For an insurance company valuation, this component represents the projection of future statutory earnings of the insurance entity, modified in consideration of initial valuation adjustments made to statutory capital, and inclusive of all direct charges and credits to statutory surplus. These earnings will include the runoff of the existing balance sheet assets and liabilities along with the earnings contributions from new and renewal business written. This component may also include investment income on the capital base.[12]

The earnings will reflect a specific growth rate (which could be positive, flat, or negative) that must also be reflected in growth in capital needed to support the business. The ROIC represents the after-tax operating earnings in each period (including any "below the line" changes to capital during the period) as a ratio to the starting capital for the period.

The third and final component needed to estimate the EVA is the hurdle rate. Considerations in the determination of the hurdle rate are discussed in Section 4.

For the EVA model, "excess returns" are represented by the excess of (i) the operating earnings in each period over (ii) the product of the starting capital for each period and the hurdle rate.[13] Recall that a company has value in excess of its invested capital only when ROIC exceeds the hurdle rate for the company. Therefore, a company has positive "excess returns" in a period only when the after-tax operating earnings for that period exceed the product of the hurdle rate and the required capital at the beginning of the period.

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[12]If investment income on the capital base is excluded from earnings, the cost of capital calculation will be modified accordingly. This is discussed further in Section 3.

[13]If operating earnings exclude investment income on capital, then the investment income on capital will be subtracted from term (ii).

In the valuation formula ANW + PVFE – COC, the term PVFE – COC represents these “excess returns.”

Excess returns have positive value only when the future earnings exceed the cost of capital. In this case, the cost of capital represents the present value of the product of the hurdle rate and the starting capital for each period for which earnings are projected. If investment earnings on the capital are excluded from future earnings, then the cost of capital calculation will be the present value of the product of the hurdle rate and the starting capital less the investment earnings on the capital.

While the two calculations of excess returns should be mathematically equivalent, there are numerous practical advantages to including earnings on the capital in future earnings. First, the earnings projections will be more in line with historical earnings so one can review the reasonableness of the projections relative to past experience. Second, allocation of assets between capital and liabilities is unnecessary. Third, one does not need to allocate taxes, tax loss carryforwards, and other factors between investment earnings on capital and all other earnings.

In Appendix A, this paper will demonstrate that the two methodologies, DCF and EVA, produce equivalent values when specific conditions hold [7]. These conditions are the following:

1. The starting capital and the after-tax operating income that is used to estimate free cash flows to the firm for a DCF valuation should be equal to the starting capital and the after-tax operating income used to compute EVA. (For insurance company valuations, after-tax operating income should include “below the line” gains and losses in capital that do not flow through earnings.)

2. The capital invested that is used to compute excess returns in future periods should be the capital invested at the *beginning* of the period:

$$\text{Excess return}_t = \text{after-tax operating income}_t - (\text{hurdle rate} \times \text{capital invested}_{t-1}).$$

3. Consistent assumptions about the value of the company after the explicit forecast period are required. That means that for both models, capital required, earnings growth rate, and the hurdle rate must be consistent in computing the terminal value.
4. The hurdle rate for the explicit forecast period must be the same as the hurdle rate after the explicit forecast period.[14]

2.3. Relative or Market Multiple Valuation

While the value of a company may be derived from the DCF or EVA valuation methodologies, other more simplistic methods are often used to corroborate or supplement more sophisticated models. In relative valuation, one estimates the value of a company by looking at how similar companies are priced. Relative valuation methods are typically based on market-based multiples of balance sheet or income statement values such as earnings, revenues, or book value.

Comparable Companies

The first step in the market multiple approach is to identify a peer group for the subject company. To select insurers for the peer group, it is common to rely on data for publicly traded insurers that meet certain criteria based on premium volume, mix of business, asset size, statutory or GAAP equity, and regulatory environment. These criteria are intended to assure that the peer group is reasonably comparable to the subject company. In selecting the criteria, however, it is important to balance pre-

[14]While it is not uncommon for a higher hurdle rate to apply to earnings at a later date to account for the uncertainty, it is also common to apply one hurdle rate for all periods reflecting the expected cost of acquiring capital to perform an acquisition of such an entity, that is, the required rate of return to investors.

cision and sample size. While the analysis could be restricted to only those insurers that were virtually identical to the subject company, the sample size would likely be too small to yield meaningful results.

Valuation Bases

The market multiple valuation method estimates the "market price" of the subject company by reference to the multiples of its peer group. For example, if the average ratio of price to earnings per share is 15.0 for the peer group, and the subject company's most recent annual earnings are $10 million, then the estimated market value of the subject company is $150 million. Typically, several alternative ratios will be used in performing a market multiple valuation. In most instances, the ratios employed include an operating multiple (such as the price-to-earnings ratio), a revenue multiple (such as price-to-premium or price-to-total revenues), and a balance sheet multiple (such as the price-to-book value ratio).

A relative valuation is more likely to reflect the current mood of the market because it is a measure of relative value, not intrinsic value [7]. While these methods serve a valuable purpose in the formulation of an opinion on the price the market may be willing to pay, they provide little guidance on the returns that will be achievable and the extent to which capital outlaid now can be repaid.

3. VALUATION RESULTS: EVA VERSUS DCF

3.1. Introduction

The following examples illustrate the DCF and EVA valuation methodologies and derive relevant conclusions related to the use of the two methods. This section focuses on the mechanics and properties of the DCF and EVA valuation calculations. Appendix A will provide a property/casualty insurance company example.

We will demonstrate two equivalent forms of the EVA model. The first form, EVA(a), will follow the basic EVA formula structure in which

$$\text{Excess returns} = \text{after-tax operating income} - (\text{hurdle rate} \times \text{capital invested}).$$

The second form, EVA(b), will use the following definition:

$$\begin{aligned}\text{Excess returns} = {} & \text{after-tax earnings on insurance operations} \\ & \text{excluding investment income on capital} \\ & - [(\text{hurdle rate} - \text{average investment rate} \\ & \quad \text{for capital}) \times \text{capital invested}].\end{aligned}$$

Excess returns for EVA(a) and EVA(b) are equivalent in theory. However, while EVA(b) is discussed in actuarial literature on company valuation [20], there are a number of advantages to using the EVA(a) model in practice. The advantages, previously disclosed, are these:

1. The earnings projections will be more in line with historical earnings so one can review the reasonableness of the projections relative to past experience.
2. It is not necessary to allocate assets between capital and liabilities.
3. It is not necessary to allocate taxes, tax carryforwards, and other factors between investment earnings on capital and all other earnings.

3.2. Basic Model Assumptions

We will use the following assumptions to demonstrate the basic calculations for the DCF and EVA models applied to a property/casualty insurer.

- The capital at time 0, just prior to projected year 1, is \$100. For a property/casualty insurance company, this amount is the surplus.
- Expected growth rate values of $g = 0\%$ and $g = 3\%$ were used.

- Investment income return on capital is 4% per annum.
- The hurdle rate is 15% per annum.
- Capital is determined based on a premium-to-capital ratio of 2 : 1.
- Total earnings are identified separately as investment income on capital and earnings from insurance operations.[15]
- The investment income on the capital component equals the product of the investment income rate and the capital at the beginning of the year.
- The insurance operation earnings component is a percentage of premiums earned for the year. Premium-related earnings encompass underwriting profits and investment earnings associated with all noncapital assets.

For projection scenarios in which the hurdle rate is exactly achieved, earnings are 5.5% of the earned premium.[16] For projection scenarios in which the hurdle rate is not achieved, earnings are 5% of the premium. When earnings exceed the hurdle rate requirement, this percentage is 6%.

We compiled projection scenarios using two time horizons. First, we estimated the company's value using a 10-year forecast period. We also estimated the continuing value using the present value of earnings beyond 10 years using the same model assumptions.

This time horizon is important in valuing an actual company. The 10-year forecast period value will be based on detailed financial projections by line of business as shown in Appendix A. The terminal value will be based on the simplified assumptions

[15] A number of judgments regarding asset allocation and tax allocation must be made to do this in practice.

[16] That is, 5.5%=15% hurdle rate less 4% investment income on capital, yielding 11%, which is divided by the premium-to-surplus ratio of 2.

with respect to (i) expected growth in earnings by future period and (ii) expected changes in capital required by future period.

3.3. Total Earnings Equal Hurdle Rate and the Company Is Not Growing

Table 1 displays the company value results for the three models in which the annual total earnings relative to capital equal the hurdle rate, and neither the company's capital nor its business is growing.[17] Exhibits 1A, 1B, and 1C show the calculations leading to these results.

The *In Perpetuity* results are 100.00, equal to the starting capital of the company.

For the DCF model, the value calculation simplifies to

$$\frac{\mathrm{OE}_1 - 0}{h - 0} = (100 \times 15\%) \div 15\% = 100.$$

For the EVA(a) model, Exhibit 1A shows that for each forecasted year the total earnings are exactly offset by the cost of capital. This result, of course, follows because both earnings and cost of capital are 15% of each year's starting capital of 100. The same progression is demonstrated by the EVA(b) model except earnings are only $100 \times 11\%$ (earnings on insurance operations only) offset by cost of capital of $100 \times (15\% - 4\%)$.

As noted in Section 2.2, a company has value in excess of its capital invested or hurdle rate only when future returns are in excess of the hurdle rate requirement. In the DCF model, the present value of the perpetual cash flow is equal to the starting capital because annual earnings of 15% of capital, discounted at 15% annually, yield the starting capital. In the EVA models, excess returns are always 0 and, therefore, the only contribution to value is the capital.

Looking at the modeled time periods (10-year forecast period and terminal value) reveals a fundamental difference in the

[17]Excess earnings are 0, so value for the EVA methods is equal to the starting capital.

TABLE 1

VALUATION RESULTS WHEN TOTAL EARNINGS EQUAL HURDLE RATE AND THERE IS NO GROWTH

Model	10-Year Forecast Period	Terminal Value	In Perpetuity (Total)
DCF	75.28	24.72	100.00
EVA(a)	100.00	0.00	100.00
EVA(b)	100.00	0.00	100.00

DCF and EVA models. The DCF model must be computed *in perpetuity* (forecast period plus terminal value) to capture the capital value in the company. The EVA models, however, recognize the value of the capital "immediately" as it incorporates the capital amount directly in the value computation. *Therefore, the EVA model will produce higher estimates of value than DCF when earnings are not valued in perpetuity.*

3.4. Total Earnings Equal Hurdle Rate and the Company Is Growing

Table 2 displays the company value results for the three models in which the annual total earnings relative to capital equals the hurdle rate and the company's capital and earnings are growing by 3% per annum. Exhibits 2A, 2B, and 2C show the calculations leading to these results.

The results in Table 2 are nearly identical to the value results shown in Table 1 in which no business growth was modeled. Basically, the two EVA models behave exactly the same—the earnings each year are exactly offset by the cost of capital. Incorporating growth into the model only changes the earnings and cost of capital amounts for each year, not the difference between the two values. However, this basic demonstration still emphasizes the relationship of earnings to hurdle rate as the determinant of value, positive or negative, in conjunction with starting capital.

TABLE 2

VALUATION RESULTS WHEN
TOTAL EARNINGS EQUAL HURDLE RATE AND
EARNINGS (AND CAPITAL) ARE GROWING @ 3% PER ANNUM

Model	10-Year Forecast Period	Terminal Value	In Perpetuity (Total)
DCF	66.78	33.22	100.00
EVA(a)	100.00	0.00	100.00
EVA(b)	100.00	0.00	100.00

The components of the results for the DCF model do change between the no-growth and growth scenarios. The value amount for the 10-year forecast decreases and is exactly offset by an increase in the terminal value. The "total" *in perpetuity* amount, however, is not affected by growth because annual earnings are still equivalent to the hurdle rate. Growth, however, shifts more of the company's value to later projected years at the expense of earlier projected years. This "value shift" occurs because the DCF model accounts for capital growth via a reinvestment of a portion of annual earnings, thereby reducing free cash flows.

3.5. Funding Capital Growth: Comparing the DCF and EVA Models

The DCF and EVA models have different treatments of the costs associated with growing the capital base of the company. We can think of the DCF model as a reinvestment for growth process and the EVA model as a capital borrowing process.

Exhibit 2A, Column (8), shows the annual capital reinvestment amount necessary for the DCF model to account for the 3% growth in capital. The capital reinvestment amount is taken from current year earnings to fund the following year starting capital—Column (2) equals Column (8) shifted one year. The DCF model fully funds capital growth, thereby reducing "free cash flows" for valuation.

In the EVA models, the cost for growing the capital is a part of the cost of capital calculation. For the EVA(a) model, Exhibit 2B, Columns (10a) and (10b), show the components of the cost of capital related to the initial capital and additional capital for growth, respectively. The reduction in growth-related earnings equals the product of the hurdle rate and the cumulative additional capital amount beyond the initial capital. This increment can be thought of as the interest payment on "borrowed" capital used to fund business growth.

Although the negative cash flows necessary to support capital growth are different for the DCF and EVA models, the present values of the cash flows are identical when considered *in perpetuity*. The DCF model reinvestment to grow the capital is a larger offset to earnings in early forecasted years than the EVA model required return on additional capital amounts. By the ninth forecasted year, however, the EVA model capital growth cost (Exhibit 2B, Column 10b) overtakes the reinvestment amount in the DCF model (Exhibit 2A, Column 8).

3.6. Total Earnings Are Not Equal to the Hurdle Rate and the Company is Not Growing

Table 3 displays the company value results for the three models in the scenario in which the annual total earnings relative to capital *do not* equal the hurdle rate and the company is not growing. Exhibits 3A, 3B, 3C, 4A, 4B, and 4C show the calculations leading to these results.

Table 3 reaffirms the *in perpetuity* equivalence of the DCF and EVA models. Like the previous examples, the 10-year and terminal values are different between the DCF and EVA valuations but the *in perpetuity* valuations are equal. The equivalency of the DCF and EVA models in perpetuity will be shown on an algebraic basis in Appendix B.

When the earnings are not equal to the hurdle rate there is a marginal value (positive or negative) in addition to the initial

TABLE 3

VALUATION RESULTS WHEN
TOTAL EARNINGS ARE NOT EQUAL TO HURDLE RATE
AND THERE IS NO GROWTH

Model	10-Year Forecast Period	Terminal Value	In Perpetuity (Total)
Earnings Less Than Hurdle Rate			
DCF	70.26	23.07	93.33
EVA(a)	94.98	(1.65)	93.33
EVA(b)	94.98	(1.65)	93.33
Earnings Greater Than Hurdle Rate			
DCF	80.30	26.37	106.67
EVA(a)	105.02	1.65	106.67
EVA(b)	105.02	1.65	106.67

capital. As expected, when the hurdle rate requirement exceeds earnings, the value of the company drops below the value of the starting capital ($100 in this example). Likewise, when earnings exceed the hurdle rate, there is additional value created. In Exhibits 3A, 3B, and 3C, total annual created earnings are 16% and the cost of capital is dictated by the hurdle rate, 15%, leaving an excess return on capital of 1% for each year in the future. The present value of the 1% marginal profit in return on capital of 100 is 6.67 *in perpetuity*. Referring to Exhibits 4A, 4B, and 4C, a 1% marginal loss in return on capital of 100 leads to a value decrease of 6.67.

3.7. Total Earnings Not Equal to Hurdle Rate and the Company Is Growing

Table 4 displays the company value results for the three models in the scenarios in which the annual total earnings relative to capital *do not* equal the hurdle rate and the company's capital and earnings are growing by 3% per annum. Exhibits 5A, 5B, 5C, 6A, 6B, and 6C show the calculations leading to these results.

TABLE 4

VALUATION RESULTS WHEN
TOTAL EARNINGS ARE NOT EQUAL TO HURDLE RATE AND
EARNINGS AND CAPITAL ARE GROWING @ 3% PER ANNUM

Model	10-Year Forecast Period	Terminal Value	In Perpetuity (Total)
Earnings Less Than Hurdle Rate			
DCF	61.22	30.45	91.67
EVA(a)	94.43	(2.76)	91.67
EVA(b)	94.43	(2.76)	91.67
Earnings Greater Than Hurdle Rate			
DCF	72.35	35.99	108.33
EVA(a)	105.57	2.76	108.33
EVA(b)	105.57	2.76	108.33

The impact of growth on the company's value is to increase the portion of value contributed in the future. If the company's earnings are not achieving the hurdle rate, growing the business further lowers value. When earnings exceed the hurdle rate, growth produces increased value.

The DCF model results show that capital growth, necessary to support business and earnings growth, reduces free cash flow in the short term in return for an increase in future earnings. Looking at the *Earnings Greater Than Hurdle Rate* scenario, the 10-year forecast period value with no growth is 80.30, dropping to 72.35 with 3% annual growth. However, the comparable terminal values increase from 26.37 to 35.99, yielding an *in perpetuity* gain in total value of 1.66 with growth (108.33 with 3% growth versus 106.67 with 0% growth). In the early projection years, the reinvestment earnings to grow the capital (thereby reducing free cash flows) exceed the marginal increase in earnings on the additional capital. This reverses itself in later projection years, resulting in higher terminal values.

3.8. Comparison of DCF and EVA Models

The parameterization of the DCF and EVA models presented in the paper cause the models to produce equal value if considered in perpetuity. The parameters selected to populate the models should be equivalent as they are independent of which model is used. For example, the appropriate hurdle rate does not depend on the model selected. Appendix B discusses the formula assumptions necessary to ensure the equivalence property. The equivalence of these valuation methodologies is expected because each model is measuring the same value contributors, just using different formula structures.

In the DCF model, the starting capital is used only to determine free cash flow at time 0. The principle of a DCF valuation is that an investment, a company for our discussion, is worth the value of its future earnings. If the capital leads to future earnings (by investment and supporting profitable business), then value will emerge. If future earnings are less than the hurdle rate, then the capital invested in this entity is less than its face value.[18]

The EVA model (both forms, EVA(a) and EVA(b)) includes the full starting capital for its determination of value, but at a cost represented by the cost of capital calculation. Column 10a in the EVA model calculations (Exhibits 1B, 2B, 3B, 4B, 5B, and 6B) shows the cost of the initial capital. The present value of this negative cash flow in perpetuity exactly offsets the value contributed by immediate recognition of the capital in the EVA formula. If the capital does not provide earnings equal to or greater than the hurdle rate in the form of excess profits, then the capital does not substantiate its value and is worth less than 100 cents on the dollar.

[18]The value of capital is worth 100 cents on the dollar if you can release the capital at time zero. Otherwise, the capital is worth the present value of the distributable earnings generated by the capital. If distributable earnings represent a return lower than the hurdle rate, then capital is worth less than 100 cents on the dollar.

TABLE 5

VALUATION RESULTS WHEN
EARNINGS ON OPERATIONS = 0.0%
TOTAL EARNINGS = 4.0% (INVESTMENT ONLY)
AND THERE IS NO GROWTH

Model	10-Year Forecast Period	Terminal Value	In Perpetuity (Total)
DCF	20.08	6.59	26.67
EVA(a)	44.79	(18.13)	26.67
EVA(b)	44.79	(18.13)	26.67

That the EVA model counts the initial capital amount as value and the DCF model does not leads to significant differences in value contributors between the forecast period value and the terminal value. Tables 1, 2, 3, and 4 all show that the results for the 10-year forecast period for the EVA model are close (and sometimes equal) to the *in perpetuity* time frame results. In the EVA model, therefore, excluding earnings beyond a certain time period does not have a material effect on value. In contrast, a significant portion of the value indicated by the DCF model is captured as terminal value. In these examples in which the total earnings of the company are set close or equal to the hurdle rate, the EVA model approaches the *in perpetuity* value faster.

Table 5 shows model value results in which earnings related to operations are 0.0%.

For a scenario in which the company's earnings potential is low, the DCF model produces a value closer to the *in perpetuity* value in the 10-year period than the EVA model. The DCF model is not "fooled" by the value of the stated initial capital in the short term. The DCF model considers only the earnings potential of the capital, not the capital itself. The result is further exaggerated when growth is incorporated as shown in Table 6.

TABLE 6

VALUATION RESULTS WHEN
EARNINGS ON OPERATIONS = 0.0%
AND TOTAL EARNINGS = 4.0% (INVESTMENT ONLY) AND
EARNINGS AND CAPITAL ARE GROWING @ 3% PER ANNUM

Model	10-Year Forecast Period	Terminal Value	In Perpetuity (Total)
DCF	5.57	2.77	8.33
EVA(a)	38.78	(30.45)	8.33
EVA(b)	38.78	(30.45)	8.33

3.9. Comparison of EVA(a) and EVA(b)

We present two versions of the EVA model: EVA(a) and EVA(b). The EVA(a) version defines excess earnings as the difference in after-tax operating income and the cost of invested capital. After-tax operating income is recognized for the company as a whole; the amount is not segregated into investment versus operational earnings. Likewise, the cost of capital relies on the product of the "full" hurdle rate and the amount of capital.

The EVA(b) model formula defines earnings and cost of capital differently. The EVA(b) model formula does not include investment earnings related to the capital as earnings. In the context of a property/casualty insurer, earnings are only underwriting earnings from premium written and investment income on assets supporting the liabilities ensuing from writing insurance policies. Under EVA(b), earnings are lower, but so is the cost of capital. The cost of capital is the hurdle rate less the investment income rate the company will earn on its capital—in a sense, the shortfall in investment earnings relative to the hurdle rate.

From the basic valuation examples presented in this section, the two forms of the EVA produce identical results. EVA(a) follows from financial valuation fundamentals [15]. EVA(b) is often regarded as the "actuarial valuation method." Sturgis [20] describes the economic value of a property/casualty insurance company as composed of three parts: (i) current net worth, plus (ii) the discounted value of future earnings, less (iii) the cost of capital, where future earnings and cost of capital are defined per our EVA(b) model.

Miccolis [17] describes a computation similar to the one by Sturgis to determine an insurer's economic value: (i) adjusted surplus, plus (ii) discounted value of future earnings, less (iii) cost of capital. Miccolis, however, is unclear regarding the computation for the cost of capital.

We consider the EVA(a) model to be the preferred structure for applying the economic value added model. EVA(a) is more straightforward to apply and avoids potential complications. It relies on financial estimates of earnings that are comparable to actual financial projections for a property/casualty insurer. To use the EVA(b) model, one must attempt to isolate the source of earnings between amounts earned from premium written and investment income on the capital. This approach further necessitates an allocation of invested assets between those supporting the liabilities and assets underlying the capital and surplus. In addition, splitting earnings into its "component" parts raises potential tax application questions that complicate the valuation process.

4. PARAMETERIZING THE VALUATION MODEL

4.1. Accounting

Insurance companies in the United States use multiple forms of accounting. Statutory Accounting Principles (SAP) are used for reporting to state regulatory authorities and Generally Accepted Accounting Principles (GAAP) are used for reporting to the Securities and Exchange Commission and the public. Tax accounting underlies the computation of taxable income. SAP focuses on the current solvency of an insurance company and its ability to meet its obligations. Due to this focus on protection of policyholders, assets and liabilities are generally valued conservatively on the statutory balance sheet, although the result is dependent on specific company or financial conditions.

Historically, noteworthy differences between GAAP and SAP for property/casualty insurance companies are related to

1. Deferred acquisition costs (DAC),
2. Deferred tax assets (DTA) and liabilities (DTL),
3. Premium deficiency reserve (PDR), and
4. Valuation of bonds.

1. Deferred acquisition costs

The asset associated with DAC recognizes that the unearned premium reserve (UEPR) may be overstated because it funds expenses (e.g., agents' commissions) that are typically paid at the beginning of the policy period and have already been incurred on the income statement. As the unearned premium reserve is earned, this overstatement disappears.[19] Statutory accounting does not permit recognition of the value of this asset until it materializes in future statutory earnings. In isolation, this difference in the treatment of the DAC asset would cause GAAP equity always to be greater than or equal to SAP equity.

2. Deferred tax assets and liabilities

Deferred tax assets and liabilities are created primarily from taxes resulting from discounted loss reserves and unrealized gains and losses. For a growing company, the tax calculation results in an "overpayment" of taxes initially related to discounted incurred losses, offset by a lower payment in subsequent years when claims are paid. This difference is solely a

[19]For a going concern, we acknowledge that it is replaced by equity in the unearned premium reserves for the following year's business.

timing issue, as the total amount of taxes that will be paid for profits associated with a block of business or block of assets does not change. The prepayment of taxes (or tax credit for unrealized losses) is corrected as the business runs off or the assets are sold.

With the introduction of DTA and DTL for statutory accounting, these assets and liabilities are now recognized on the balance sheet before the business runs off or the assets are sold. For many companies, this change increases their statutory capital.

3. *Premium deficiency reserves*

   The PDR is required when the unearned premium reserve is expected to be insufficient to fund the future loss and expense payments originating from those policies. This reserve will reduce statutory capital.

4. *Valuation of bonds*

   In general, SAP requires bonds to be held at amortized cost (although bonds that are not "in good standing" are carried at market value). GAAP, on the other hand, uses amortized cost for only "held-to-maturity" bonds, which the company has both the intent and ability to hold to maturity. For those bonds in the company's active trading portfolio, GAAP requires market value treatment on the balance sheet.

With the codification of statutory accounting principles, which became effective January 1, 2001, deferred tax assets, deferred tax liabilities, and premium deficiency reserves were recognized on the statutory balance sheet. The most significant difference that remains relates to deferred acquisition costs.

As stated in Actuarial Standard of Practice (ASOP) No. 19 [1], for insurance companies, statutory (or regulatory) earnings form the basis for determining distributable earnings, since the availability of dividends to equity owners is constrained by the amount of accumulated earnings and minimum capital and surplus requirements. Both of these amounts must be determined on a statutory accounting basis. Distributable earnings consist of statutory earnings, adjusted as appropriate in recognition of minimum capital and surplus levels necessary to support existing business. Therefore, statutory accounting determines the earnings available to the equity owners.

While future earnings calculated according to GAAP or another basis will often be of interest to the user of an actuarial appraisal, the free cash flow calculations contemplated within the definition of actuarial appraisal in ASOP No. 19 should be developed in consideration of statutory earnings, rather than some other basis.

GAAP earnings and GAAP net worth, however, are often the basis of the relative valuation methods involving market multiples.

As the major difference between GAAP and SAP accounting is DAC, which may be recognized as an asset on the GAAP balance sheet immediately instead of through future earnings, GAAP net worth is typically higher than SAP net worth. SAP net worth may be greater, however, when the amortized value of bonds in the SAP asset portfolio is higher than the market value of bonds in the GAAP asset portfolio.

4.2. Estimating Free Cash Flows or Value Added

Estimating free cash flows for a DCF valuation or changes in value of the company in each period for an EVA valuation requires the use of after-tax operating earnings from accounting statements. However, accounting earnings may not represent true earnings because of limitations in accounting rules and the firms' own actions.

For a property/casualty insurance company, changes in the equity of the firm derive not only from (i) after-tax operating earnings (net income in the statutory income statement) and (ii) capital infusions or distributions, but also from (iii) "below the line" adjustments to capital. These adjustments represent items that do not flow through the statutory income statement for changes in unrealized capital gains or losses, changes in nonadmitted assets, changes in provisions for reinsurance, change in foreign exchange adjustment, or changes in deferred income taxes. To the extent that these adjustments increase (decrease) the equity of the firm, they also increase (decrease) free cash flows for the DCF valuation methodology and increase (decrease) excess returns for the EVA valuation methodology.

For a property/casualty insurer, estimating after-tax operating earnings (including "below the line" statutory adjustments to capital) typically requires rigorous analysis. For the purpose of analysis, the sources of future earnings can be subdivided into two broad categories: the runoff of the existing balance sheet and future written business.

4.3. Runoff of the Existing Balance Sheet

The runoff of the existing balance sheet produces earnings associated with (i) underwriting profit embedded in the UEPR[20]; (ii) investment income on the assets supporting (a) the loss reserves (inclusive of all loss, allocated loss adjustment expense, and unallocated loss adjustment expense reserves) and (b) UEPR liabilities until all the associated claims are paid; and (iii) investment income on the capital base supporting the runoff of the business.[21]

The earnings associated with new (or renewal) business derives from (i) the underwriting profit generated by the business; (ii) the investment income on the assets generated by the pre-

[20]Profit embedded in the UEPR represents underwriting profit and profit associated with the prepaid expenses (corresponding to the deferred acquisition cost asset established for GAAP accounting).

[21]For an EVA valuation, if one projects earnings with a capital base of zero (an EVA(b) scenario), this component will be zero.

mium, supporting loss reserves and UEPRs until all of the claims are paid, and (iii) investment income on the capital base supporting the writing of the new business.

Developing financial projections (income statements, balance sheets, and cash flows) related to running off the existing balance sheet liabilities, assuming no new or renewal business is written, will provide the basic elements for valuing the company in runoff. The key factors involved are (i) the payout of the loss reserves, (ii) the ultimate losses and expenses associated with the unearned premium reserve, (iii) the payout of the losses and expenses associated with the unearned premium reserve, (iv) the capital needed each year to support the company in runoff, and (v) the investment yield earned on assets until all claims are paid and all capital is released. In practice, when running off a company that writes personal lines business, renewals may be mandated for several years by the regulatory authorities. In those instances, running off the company might also reflect the writing of some renewal business.

When it is important to understand the value associated with the runoff of the business separate from value associated with the writing of new (or renewal) business, we recommend the following approach. Value the company in runoff reflecting the level of capital required to run off the company. Then, value the company reflecting earnings and capital needs associated with maintaining the company as a going concern. That is, earnings projections and capital needs are developed for the combination of running off the existing balance sheet and writing new business. The value of solely writing new business should be computed as the difference between the two valuations.

The suggested approach is beneficial on both a practical and a theoretical basis. On a theoretical basis, the valuation of the runoff company relative to the going concern improves the determination of capital required for new business. On a practical basis, both valuations will use the same starting balance sheet.

4.4. Future Written Business

For property/casualty insurance companies, in contrast with life insurance companies, the distinction between new and renewal business is often not meaningful for developing financial projections for future written business. For direct writers of personal lines business, however, for whom the initial cost of acquiring new business and the associated expected loss ratio differs substantially from the expenses and loss ratios associated with renewal business, the distinction between new and renewal business may be very important for developing financial projections.

Financial projections are usually developed by line of business or business segment corresponding to the detail in which the company being valued provides its premium forecasts. The key elements to be estimated by year and line of business are

- Gross written premium;
- Net written premium;
- Accident year gross and ceded loss and loss expense ratios;
- Gross commissions and ceding commissions;
- Other overhead expenses (premium taxes, general and administrative expenses, other acquisition costs);
- Collection schedules for premium;
- Payment schedules for commissions and other overhead expenses;
- Payment pattern for gross and ceded accident year loss and loss adjustment expense; and
- Collection pattern for ceded reinsurance recoveries.

For the book of business in total, the key elements to be estimated are

- Investment yield on investible assets,

- Capital needed to support the entire book of business, and
- Federal income taxes applicable to earnings.

The primary contributors to investment earnings are the timing differences between the collection of the premium and the payment of claims and loss adjustment expense. For most lines of business, there is little delay in premium payment by the policyholder. When premiums are paid in installments, however, or when audit premiums represent a significant portion of the ultimate collected premium, it is important to evaluate the lag because of the resulting impact on the investment income calculation. Reinsurance recoveries may need to be projected on a contract-by-contract basis if the indemnification terms vary significantly.

In determining the future earnings from new and renewal business, projected loss and expense ratios are the most important components to be modeled. As Miccolis [17] and Ryan and Larner [19] note in their papers on valuation, issues to be considered in the projection of future loss and expense ratios include

- Changes in price levels;
- Trends in loss severity, claim frequency, and exposure base;
- Historical industry results;
- Underwriting cycles;
- Target rates of return;
- Expected future growth rates;
- Degree of competition in market;
- Regulatory environment;
- Exposure to catastrophes; and
- Changes in ceded reinsurance (coverage, terms, pricing).

4.5. Present Value of Future Earnings

Once the future earnings stream (including gains and losses in capital that do not flow through earnings) from running off the existing balance sheet and future written business has been estimated, it is discounted to present value at the selected hurdle rate. For an EVA valuation, the future earnings stream is used directly without consideration of capital infusions or distributions. For a DCF valuation, the future earnings stream (i) less earnings retained for capital growth or (ii) plus additional capital released represents free cash flows.

4.6. Adjusted Net Worth

In valuing a company, it is common practice to adjust the equity of the firm at time zero to consider value (positive or negative) associated with reserve deficiencies or redundancies, market value of assets, nonadmitted assets, and statutory provisions for reinsurance, among other factors.

The adjustments to statutory equity in the computation of ANW for an EVA valuation (and free cash flow at time 0, FC_0, for a DCF valuation) represent an effort to adjust the starting statutory balance sheet to its true market value. These adjustments described by Miccolis [17] and Ryan and Larner [19] and summarized below represent an attempt to recognize the market value of some items on the statutory balance sheet. For example, common adjustments include reflecting assets at market value and eliminating goodwill. In contrast, there are usually no comparable adjustments for liabilities. For loss reserves and unearned premium reserves, market value would reflect future investment income plus a provision for risk. Instead of market value adjustments, any value associated with the liabilities (other than adjusting reserves to their actuarially indicated amount) is recognized through the present value of future earnings.

Since statutory accounting determines free cash flows to investors, one could support the position that adjustments to the equity of the firm at time zero should be limited to tax-affected reserve adjustments (to bring carried reserves to the actuarial indicated level) and other changes that "true up" the statutory balance sheet. Adjustments to statutory capital to compute ANW that are not permitted under statutory accounting will not change statutory capital and therefore will not affect free cash flows. Many financial experts, however, insist that assets be adjusted to their market value at the date of valuation. Further, goodwill carried on the balance sheet is almost always eliminated for valuation, even though it is now a statutory asset. Experts continue to disagree on how these adjustments should be handled for valuation.

Either way, if the net worth or the equity of the firm is adjusted to recognize nonadmitted assets, or reflect the market value of all assets, then the firm's future earnings or changes in capital must be adjusted to prevent double counting this value. For example, if all assets are marked to market for the valuation, then future earnings of the firm must not reflect any realized gains or losses associated with assets unless the market values change. Further, if nonadmitted assets are added back to the starting net worth of the firm, then any capital increases associated with the recognition of nonadmitted assets must be eliminated from future financial projections.

Any adjustments to the starting capital to determine ANW will cause the EVA and DCF valuation results to diverge unless the same adjustments are made for both valuation methodologies. Otherwise, for DCF, these values will be recognized on a discounted basis through future earnings or "below the line" adjustments to equity. For EVA, they will be recognized at time zero, thereby reflecting no present value discount in the computation of value.

The common adjustments to the starting capital (SC_0) for valuation are listed below. Only items 1 and 6 are consistent with statutory accounting principles and therefore will have the same effect on EVA and DCF valuations. The other adjustments to

ANW, unless also assumed to impact SC_0 for DCF, thereby affecting FC_0, will cause the EVA and DCF valuation results to diverge. The direction (positive or negative) of the difference between the EVA and DCF valuation result will be dependent on the direction (positive or negative) of the tax-affected adjustments for items 2, 3, 4, and 5.

1. Loss reserve adequacy

For a property/casualty insurance company, policyholder claim obligations are usually the largest liability on the statutory balance sheet. As a result, it is critical to assess the reasonableness of the carried loss and loss adjustment expense (LAE) reserves as of the valuation date to meet unpaid claim obligations.

Adjustments for the loss reserve position should be made directly against statutory equity as of the valuation date for both DCF and EVA valuations. Adjustments to the carried loss reserves will impact ANW for an EVA valuation and FC_0 for a DCF valuation.

2. Market value of assets

Traditionally, the majority of the investment portfolios for property/casualty insurance companies have been placed in bonds, especially U.S. Treasury or other federal agency instruments. SAP requires bonds "in good standing" to be valued at amortized cost. For the purpose of a valuation, however, bonds should be valued at market value in order to reflect what an independent buyer would actually pay to purchase the securities.

Common and preferred stocks, which represent the next largest portion of most property/casualty insurance companies' portfolios, are recorded at values provided by the National Association of Insurance Commissioners' (NAIC) Securities Valuation Office. These values are typically equal to market value and thus are less likely to require an additional adjustment. Other investable assets should also be adjusted to market value, but are a much smaller component of the total portfolio and thus the adjustments are likely to have a smaller impact on the adjusted net worth.

3. Inclusion of nonadmitted assets

Some states do not admit certain assets on the statutory balance sheet because they either do not conform to the laws and regulations of the state or are not readily convertible to liquid assets. Exclusion from the balance sheet results in a charge to statutory equity. For the purpose of a valuation, however, one should include any portion of nonadmitted assets that has financial value and may be convertible to cash.

Examples of nonadmitted assets include

- Agents' balances overdue by 90 days or longer;
- Bills receivable that have not been taken for premium;
- Furniture, equipment (other than electronic data processing (EDP) equipment and software), and supplies; and
- Leasehold improvements.

In some cases, there may be overlap with the adjustment of assets to market value. For example, when the market value of real estate is below its net book value, the excess of book over market value is recorded as a nonadmitted asset while the admitted asset, which underlies the amount of statutory surplus, is equal to the market value. Care should be taken to ensure that there is no double counting.

4. Accounting goodwill

SAP for purchases defines *goodwill* as the difference between the cost of acquiring a subsidiary, controlled, or affiliated entity and the purchaser's share of the book value of the acquired entity. Positive goodwill exists when the cost of the acquired entity is greater than the purchaser's share of the book value. According to codified SAP, however, positive goodwill from all sources is limited in the aggregate to 10% of the parent's capital and surplus (adjusted to exclude any net positive goodwill, EDP equipment, and software).

Assets for goodwill are generally assumed to have zero value until such value emerges through future earnings.

5. Provision for reinsurance

SAP produces a "provision for reinsurance" that is calculated in Schedule F of the NAIC Annual Statement and is carried forward to the statutory balance sheet as a liability. This provision is intended to be a measure of conservatism to reflect unsecured reinsurance placed with unauthorized companies and collectibility issues with all reinsurers.

In a valuation, a more detailed review of collectibility issues is worthwhile in order to estimate any additions (or further reductions) to equity to reflect a more rigorous estimate of reinsurance recoverables.

6. Tax issues regarding all of the above

Any adjustments to the statutory balance sheet may also have a corresponding impact on the company's federal income tax liability. The federal tax liability, or deferrable tax asset, is based on statutory net income and a series of adjustments. Any adjustments made to statutory equity for valuation should be tax-affected.

In mergers or acquisitions, taxes are particularly difficult to address because one must consider the tax position of both parties.

4.7. Hurdle Rate

The hurdle rate used in a valuation should reflect the cost to the firm of acquiring the capital necessary to make the acquisition or perform the transaction in question. Typically, this value will be provided by management based on its appraisal of the acquisition's relative risk and required return. When not provided by management, the hurdle rate can be estimated using a variety of security valuation methods.[22] In either case, when establishing the hurdle rate, it is important for the analyst to consider several issues, including the following:

1. *Risks attributable to business activities of the acquisition*
   The risk attributable to the business activities of the acquisition determines the cost of the capital required to make the acquisition. This risk measure should not be confused with the risk associated with the acquiring entity, which may be different. The risk of a firm, in total, reflects an interaction of the risks of its underlying business activities. The cost of capital of any particular activity may differ from that of the firm as a whole.

2. *Consideration of multiple hurdle rates*
   If the target acquisition is engaged in several activities (e.g., different lines of business) of varying risk, it may be appropriate to consider projecting several streams of free cash flow and discounting them at different rates. An alternative to this approach may be to allocate capital to business activity in such a way as to equalize risk across lines. If this approach is used, then a single discount rate for all cash flows may be appropriate.

One reason to consider the latter approach is that one can generally observe the hurdle rate only for the firm as a whole, and not for its component parts. Thus, the hurdle rates reflect the average risk of the firm's activities and are not necessarily appropriate for any single business. If there were large samples of publicly traded firms specializing in particular lines of business, then it would be possible in theory to observe the hurdle rate for those specific activities. In practice, however, there are a limited

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[22]The most prominent models in widespread use are the capital asset pricing model (CAPM) and the dividend valuation model (sometimes known as the DCF or Gordon growth model). Both models are described in numerous sources, including Damadaran [7].

number of publicly traded insurers and they tend to be multiline firms involved in a wide variety of businesses (many of which have substantially different risk profiles). These considerations support using a single hurdle rate reflecting average risk activities and then adjusting the amount of required capital so that the risk of the acquisition is equivalent to the average risk of the firm.

3. *Method of financing the acquisition*

If the acquisition is to be financed with a mix of debt and preferred and common equity, then the appropriate hurdle rate should reflect the weighted average after-tax costs to the firm of acquiring capital through these vehicles. The capital structure underlying the acquisition, and not necessarily the existing capital structure of the acquiring entity, is the relevant issue. For example, if a firm is currently financed with a mix of debt and equity, but intends to pursue an acquisition financed solely by equity, then the relevant hurdle rate is the equity cost of capital.

4. *Consistency with other assumptions*

The discount rate depends on relative risk, which in turn depends on several factors that may be related to other aspects of the valuation. For example, in addition to the intrinsic risk of its specific business activities, the cost of capital for a firm will depend, among other things, on the firm's leverage and mix of assets. Both of these factors, however, will have an impact on the projected free cash flow that forms the foundation of the valuation. There must be consistency between the assumptions used to develop the cash flows and those used to develop the discount rate.[23]

4.8. Capital Needs

The capital required to support an insurance company is a key assumption in the valuation process.

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[23]The discount rate is often viewed as the sum of a risk-free rate and a market risk premium as in the CAPM. The value of the market risk premium is a topic of debate among financial economists.

For the DCF methodology, capital requirements dictate the amount of capital to be retained in the company to support ongoing operations, thereby determining distributable earnings and associated value. For the EVA methodology, capital requirements dictate the capital that underlies the cost of capital calculation. The higher the capital requirement, the higher the cost of the capital element of the valuation formula.

Property/casualty insurance companies are subject to statutory capital requirements. Statutory capital requirements are determinable through the property/casualty insurance industry's risk-based capital (RBC) requirements. The results can be viewed as minimum capital requirements. Often, larger capital investments are required to satisfy the financial rating agencies such as A. M. Best, Standard and Poor's, and Moody's in order to maintain desirable financial ratings. All of these factors are considerations in determining capital requirements for valuation.

Premium-to-surplus ratios, loss reserves-to-surplus ratios, and multiples of RBC have been used in valuation to determine capital needs. These are typically based on comparable ratios for "peer companies," which are companies with premium volume and lines of business comparable to the subject company. In these instances, it is essential that the selected capital match or exceed RBC requirements.

In actuarial and finance literature, there are many articles and papers related to capital requirements and capital allocation for insurers. Theories about capital requirements range from simplistic rules of thumb (e.g., maintenance of a premium-to-surplus ratio of 2.0) to intricate risk models. In practice, it is common for insurance companies to maintain a level of capital that is sufficient for a desired financial rating.

4.9. Cost of Capital

We defined the cost of capital (COC) as the product of the present value of each period's starting capital and the hurdle rate.

The COC is used to measure excess returns in each period for the EVA valuation methodology. Excess returns are computed as the difference between operating earnings in each period (inclusive of gains and losses in capital that do not flow through earnings) and the COC. This concept is more thoroughly discussed in Sections 2 and 3.

Economists and other financial professionals equate the term cost of capital with the hurdle rate. Care should be taken in using and understanding the meaning of the term in a particular context.

5. RECENT CHANGES AND OTHER CONSIDERATIONS

A variety of changes have occurred over the past 15 years that may affect the valuation of a property/casualty insurer. While many of these changes may not affect valuation methodology, they are relatively new developments that require consideration in the determination of value.

5.1. Accounting[24]

Codification of Statutory Accounting Principles

The starting point for valuation based on EVA and DCF methodologies is the statutory balance sheet. One significant change with respect to the determination of statutory surplus is the 2001 codification of statutory accounting principles (SAP).

With the introduction of codified SAP, there are at least two key changes that affect statutory surplus for many companies: (i) the treatment of deferred taxes, and (ii) the requirement to establish a premium deficiency reserve. Both of these changes mitigate the differences between statutory and GAAP accounting.

[24]One might question why accounting changes should affect value. As statutory earnings and statutory capital influence free cash flows (when either capital can be released from a company or additional capital contributions are required), accounting changes that affect statutory income or statutory surplus influence value.

Codified SAP now requires the accrual of a deferred tax asset (DTA) or liability (DTL). Consider a company that purchases one share of stock on January 1, 2001, for $100. If the company holds the stock and it appreciates to $1,000 as of December 31, 2001, the company will be required to accrue a DTL for the unrealized capital gain. (The DTL is calculated as $t \times (1{,}000 - 100)$, where t is the corporate tax rate.) Conversely, the determination of federal taxes using discounted loss reserves results in the accrual of a DTA. As a result, a company's statutory surplus is affected by necessary adjustments for DTAs and DTLs.

A premium deficiency reserve (PDR) is required to supplement the unearned premium reserve (UEPR) when the UEPR is inadequate to fund for future liabilities related to the unearned exposure.

Each of these changes resulting from codification affects the starting statutory surplus in a valuation and, as a result, the entity's future earnings. Prior to codification, a shortfall in the UEPR or the value of a DTL or DTA would have been recognized in future earnings as losses are incurred or assets are sold. Codified SAP reflects the associated value immediately on the balance sheet. In computing value prior to codification, the value associated with the PDR, DTA, or DTL would have been recognized on a discounted basis through the present value of future earnings component of the DCF or EVA valuation methods. After codification, value associated with the PDR, DTA, or DTL is as recorded in the statutory balance sheet.

Fair Value Accounting

Financial assets and liabilities are accounted for in numerous ways under current U.S. accounting rules. For property/casualty insurance companies there is GAAP accounting, statutory accounting, and tax accounting. Each of the various measuring approaches has its advantages and disadvantages. In general, GAAP accounting for property/casualty insurance companies is accounting for a "going concern." It reflects adjustments that make insurance financials comparable to other industries. Statutory accounting is a more conservative form of accounting to meet regulatory requirements targeted at protecting policyholders. Tax accounting is the basis of the tax calculation.

Historically, many financial assets were accounted for at cost or amortized cost. These values are readily available and verifiable. Many financial liabilities were at ultimate settlement value, which is a value that in many cases is contractually set and thus readily available and auditable.

The adoption of Financial Accounting Standard (FAS) 115 [12], which requires market value accounting for assets held in a "trading portfolio," led to the discussion of fair value accounting for financial assets and liabilities. With the adoption of FAS 115, several parties raised concerns about requiring assets to be held at market value when the liabilities were not reported at market values. Since then, the Financial Accounting Standards Board has stated a vision of having all financial assets and liabilities reported at fair value, which is considered an economic value.

The *fair value* of an asset or liability could be defined as the estimated market value or as the actual market value when a sufficiently active market exists. If no sufficiently similar assets or liabilities exist by which to estimate a market value, the estimated market value is based on present value of future cash flows adjusted for risks.

Fair value accounting is most commonly an issue for financial assets or liabilities. Financial assets are generally either cash or contractual rights to receive cash or other financial assets. Financial liabilities are generally obligations to provide financial assets.

Fair value accounting may have an important influence in valuing property/casualty insurance companies. If a fair value accounting approach is adopted for statutory accounting, recognition of many flows will be accelerated relative to statutory accounting. As such, the introduction of fair value accounting will change the value estimates derived from the methods described in this paper, with value estimates increasing if accelerated revenues are higher than accelerated expenses and value estimates decreasing when the reverse is true.[25]

For example, any embedded value associated with investment income on the loss and LAE reserves or profit in the unearned premium reserve would be reflected in fair value accounting at the time the loss or unearned premium reserve is reported. However, fair value accounting, at least initially, may not consider cash flows and associated profits from policy renewals or new business. Therefore, the fair value accounting net worth of an insurance company, initially, may approximate its runoff value.

5.2. Regulatory Changes

Risk-Based Capital Requirements

In 1993, the NAIC adopted RBC standards for property/casualty insurers. These standards are used by regulators to help to identify insurers that require regulatory attention and, as a result, the standards may be viewed as minimum capital requirements. As such, these requirements affect valuation because they can form a key determinant in the amount of capital a company must hold. Further changes in RBC could affect insurance company valuations if there are changes in required capital levels.

Gramm-Leach-Bliley Act

The Financial Services Modernization Act of 1999 (Gramm-Leach-Bliley Act or GLBA) enabled closer alignment of insurance companies and other financial institutions such as banks and securities firms. A primary feature of GLBA is that a bank holding company or foreign bank that meets certain eligibility criteria may become a financial holding company (FHC). FHCs

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[25]The impact on value is relevant whether these accelerated revenues and expenses are recognized in the income statement or solely as a direct adjustment to surplus. As both after-tax operating income and amount of capital affect free cash flows, either change could influence value.

are authorized to engage in a range of financial activities such as insurance agency and underwriting activities, merchant banking activities, and securities underwriting and dealing.

To date, GLBA has not had a significant impact on the property/casualty insurance industry because there are very few affiliations of insurance companies with other financial institutions. The 1998 merger of Citicorp and Travelers Group to form Citigroup was the first merger between an insurer and a bank since such mergers were prohibited in 1933. (In August 2002, however, Citicorp spun off the property/casualty operations of Travelers to end the affiliation of the banking institution and life insurance operation with the property/casualty insurance operation.) There has been no subsequent merger activity between property/casualty insurers and other financial institutions since the Citicorp merger.

Nonetheless, if a property/casualty insurer were affiliated with an FHC, the affiliation might affect certain assumptions related to the valuation of the insurer. The Federal Reserve Board, which regulates FHCs, is prohibited from directly imposing capital requirements on insurance affiliates, but it does establish capital requirements for FHCs. These FHC capital requirements may have an implicit influence on the capital level of an insurance subsidiary.

5.3. Stochastic Analysis of Insurance Company Financial Results

A unique feature of property/casualty insurance is the stochastic nature of claim emergence and settlement. In general, it is difficult to predict the timing of cash flows related to policyholder claims. While almost every line of business has the potential to generate unexpected claim experience, catastrophic insured events are particularly difficult to estimate because of the low frequency and high severity of these events. These events may have a severe and adverse impact on the operating earnings of an insurer and thus should be considered during the financial projection process. There are two broad approaches to modeling future financial projections: scenario testing and stochastic modeling.

Scenario testing is a deterministic approach in which results are projected from a specific set of conditions and assumptions. With this static approach, the user defines a scenario that reflects assumptions about various components of the company. The user is able to define the specific interrelationships of components and evaluate the impact of changes in different factors on the financial projections. This approach produces results that are easy to explain and easy to modify by incorporating one or more alternative assumptions.

Stochastic modeling has become increasingly popular in recent years for the property/casualty industry via dynamic financial analysis (DFA). Underlying stochastic models are probability distributions for each of the stochastic variables reflected in the model. Based on the probability distributions and a random number generator, the stochastic model produces a range of outcomes from which probabilities may be determined for the results. Its flexibility and ability to test the impact of a wide range of variables simultaneously make it an appealing approach. With respect to the implementation of stochastic modeling, however, the probability distributions for the stochastic variables and the correlations between components are critical to a meaningful model.

Over the past 10 to 15 years considerable emphasis has been placed on the DFA of financial results for insurance companies to evaluate capital needs, capital allocation, ceded reinsurance structures, and the risk associated with specific business initiatives. Since valuation formulas include the present value of future earnings, stochastic modeling of insurance financial results would seem like a natural adjunct to valuation.

In practice, valuing an insurance company is often undertaken in a limited timeframe. Valuation is usually based on expected value results for earnings with sensitivity tests related to changes in premium growth rates, changes in loss ratios, changes in hurdle rates, and changes in annual investment yields.

The contribution from stochastic modeling for valuation is that it would provide better definition of risk (the distribution of possible outcomes around the expected value) and could be used to derive better estimates of the cost of capital.

5.4. Exposure to Natural Catastrophes

As noted by Gorvett et al. [5], exposure to natural catastrophes has had a very significant impact on the performance of the property/casualty insurance industry worldwide. As a result, the major catastrophic events during the past 15 years have accelerated the evolution of the modeling of natural catastrophes and also led to a recent proposal to create a prefunded catastrophe reserve on the statutory balance sheet.

Though the range of sophistication of catastrophe models varies widely, there are three essential elements of most models regardless of whether the model is deterministic or stochastic. First, there must be an estimate of the intensity of the underlying peril. This estimate is often simulated based on historical information about catastrophes related to the particular peril. Second, for the underlying peril, the model requires an estimate of the total damage caused by the peril. For a given peril, the damage estimate primarily depends on the geographical location of the risk and the value and construction type of the structure affected by the peril. The final key element is an estimate of the loss to the insurer, based directly on the location of policies written and limits provided.

For the purpose of insurer valuation, the primary benefit of catastrophe modeling is related to scenario testing. While it is beneficial to understand the expected average severity of natural catastrophes, catastrophe models are unable to help identify the future timing of these events. As a result, the future earnings stream of an insurer with significant insurance exposure to natural catastrophes is much more difficult to predict.

Due to the immediate and extremely adverse impact catastrophes may have on the balance sheets of property/casualty insurers and reinsurers, there has been a recent NAIC proposal to establish a tax-deferred prefunded catastrophe reserve. The intent of this proposal is to establish a simple mechanism by which insurers and reinsurers can prudently manage risk created by exposure to natural catastrophes. This mechanism is intended to reduce the uncertainty related to the future earnings stream of insurers with significant exposure to natural catastrophes. The focus of the current proposal is on exposure of property insurance coverages to natural mega-catastrophes (e.g., Hurricane Andrew in 1992) that are expected to occur in the future.

As currently proposed, this "reserve" can be more appropriately viewed as segregated surplus. For the purpose of solvency regulation, the pre-funded nature of this reserve is also expected to come with restrictions on how it may be taken down over time.

This reserve and its funding mechanism will lead to additional considerations related to the determination of starting capital and future earnings for the purpose of a valuation. If the catastrophe reserve is immediately funded out of existing capital and as a liability, the entity's starting capital for the purpose of valuation will be reduced. If, however, the reserve is considered to be segregated surplus, the value of the company will not change. An alternative pre-funding approach is to contribute a percentage of premiums to the catastrophe reserve fund. This would have no impact on starting capital, but would affect future earnings. The direction of the change, however, is uncertain.

6. CONCLUSION

The valuation of property/casualty insurance companies is an important feature of actuarial work. Much of the actuarial literature on valuation focuses on the method referred to throughout this paper as economic value added. Other financial service professionals, however, often rely on a discounted cash flow approach to valuation. One of the principal intentions of this paper is to demonstrate that, with a common set of assumptions, the EVA and DCF modeling approaches will produce equivalent values. For both methods, the key factors underlying value are (i) the projection of future income, (ii) the required capital, and (iii) the hurdle rate. Developing future income estimates, appropriate growth assumptions (and the resultant capital needs), and the appropriate hurdle rate for the entity requires sophisticated analysis. Furthermore, there are aspects of valuation, such as the determination of adjustments to the starting capital of the entity, for which experts have varying points of view. Recent changes such as the development of fair value accounting principles will provide further ideas on the valuation of assets and liabilities of a property/casualty insurance company. We hope that this paper will help actuaries and other financial professionals to explain the valuation process for property/casualty insurance.

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APPENDIX A

SAMPLE COMPANY VALUATION

This section presents a detailed example of valuing a property/casualty insurance company. The modeled valuation will focus on

- Modeling aspects of a property/casualty insurer given current financial statements, investment assumptions, underwriting assumptions for current and future business, and loss and expense payment assumptions;
- Determination of future earnings from projected financial statements based on selected surplus and business volume constraints;
- Application of DCF and EVA valuation approaches using an existing balance sheet and projected financial statement amounts (balance sheet, income statement, and cash flow exhibits);
- Testing the sensitivity of indicated value to changes in key assumptions (risk-based capital-to-surplus requirement, loss ratios, investment yield, hurdle rate, and growth rate).

Our objective is to provide a thorough and functional discussion of the valuation of a property/casualty insurance company and a basic discussion of the development of earnings projections. The actuary or other professional preparing the valuation will, of course, undertake extensive analysis to develop premium, loss, and expense assumptions, investment yields, and other factors to project earnings. We present many assumptions "as given" without further explanation.

A.1. Valuation Estimates Based on Financial Model Results

The valuation results for the sample company, Primary Stock Insurance Company (PSIC), rely on two basic assumption sets:

1. Financial modeling assumptions underlying financial statement projections; and
2. Valuation assumptions underlying the application of the DCF and EVA methodologies yielding value estimates of PSIC based on the financial statement projections.

Exhibit 7 shows the value estimates for PSIC for each method and the principal components for applying the valuation formulae. The fundamental financial amounts entering the valuation calculations are current and future year-end surplus estimates and future total income estimates. Basic financial modeling assumptions will be discussed later in this section; the primary focus is the application of the valuation methodologies with the modeled surplus and income amounts given specific valuation assumptions.

The valuation assumptions are the following:

1. A valuation date of December 31, 2001.
2. PSIC's risk-based capital (RBC) indication at each year-end dictates the statutory surplus at the respective year-end. The example uses a surplus-to-RBC relationship of 2-to-1 where the RBC indication is the Company Action Level (100% of the RBC calculation) [11].
3. A hurdle rate of 15% per annum for all future years.
4. After the explicit forecast period ending December 31, 2011, we assume the surplus and total company income will increase at 2% per annum indefinitely.

For each valuation methodology, future valuation amounts are modeled in two distinct time periods: the explicit forecast period (10 years for the example, 2002 through 2011), and all subsequent years (2012 and later). For our sample company valuation, the explicit forecast period income and surplus estimates (via the RBC calculation) rely on financial modeling procedures. Valuation indications for all subsequent years were estimated using the respective method's value formulas starting one year after the explicit forecast period. For the DCF method, this calculation develops to the terminal value. For the EVA method this calculation develops the "continuing value added" after the explicit forecast period.

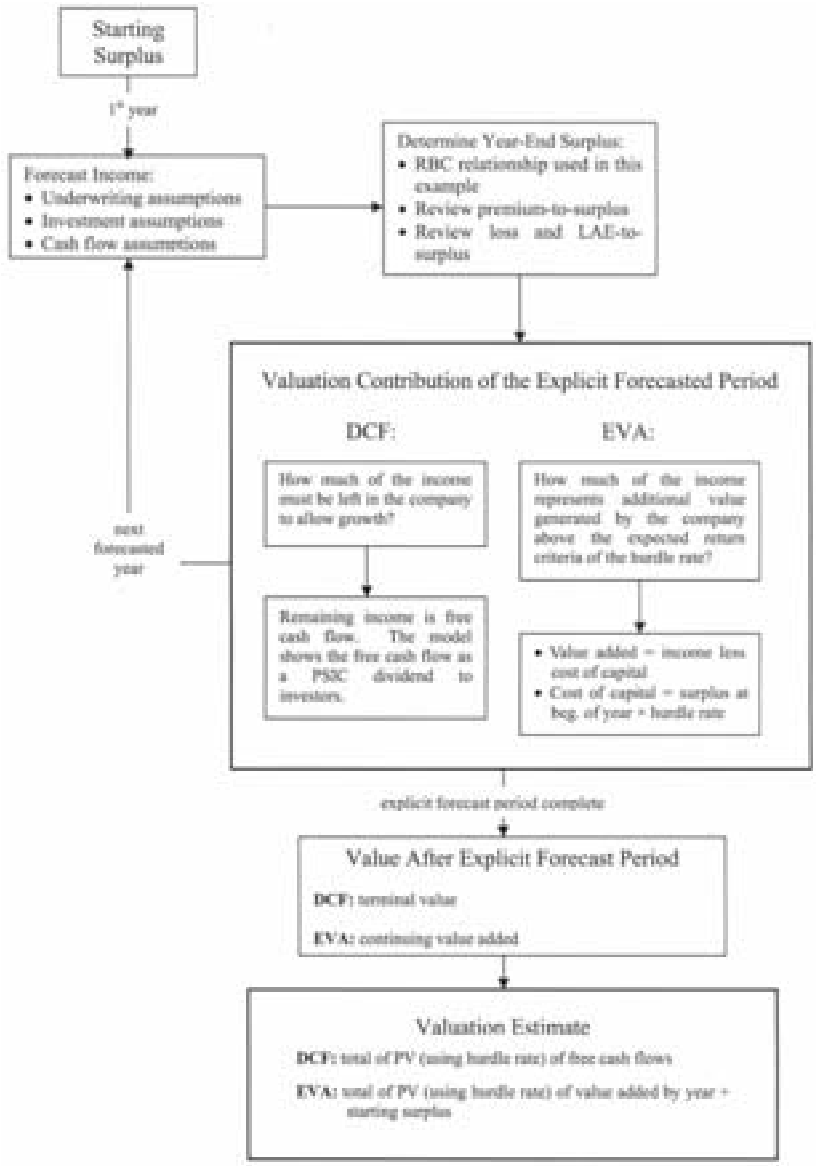
Both models yield value of approximately $88 million as of December 31, 2001. The comparison of the value components for the two methodologies parallels observations made in Section 3 about the scenario in which a company achieves more than the hurdle rate and is growing.

- The EVA method recognizes value amounts in the forecast process faster than the DCF method. As of the end of the explicit forecast period, through 2011, the EVA method value estimate is $73.9 million ($42.1 million surplus plus $31.8 million as the present value of future value added in years 2001 through 2011). The DCF method value estimate is $54.7 million representing the present value of free cash flow for years 2002 through 2011.
- The present value of the reinvestment cost (retained earnings) of $21.9 million (for all years) for DCF equals the present value of the cost of growth capital for EVA. The DCF reinvestment cost over the 10-year explicit forecast period ($18.97 million) is greater than the EVA cost of growth capital during the same period ($10.14 million). The difference is offset in modeled amounts for 2012 and subsequent years, $2.96 million for DCF and $11.79 million for EVA.[26]

The following diagram shows the steps in the development of value presented in Exhibit 7.

The recorded statutory surplus for PSIC as of December 31, 2001 is $45.00 million. However, this amount exceeds the

[26]DCF (18.97 + 2.96) = EVA (10.14 + 11.79) = 21.93.


Starting Surplus
1st year
Forecast Income:
• Underwriting assumptions
• Investment assumptions
• Cash flow assumptions
Determine Year-End Surplus:
• RBC relationship used in this example
• Review premium-to-surplus
• Review loss and LAE-to-surplus
Valuation Contribution of the Explicit Forecasted Period
DCF:
EVA:
How much of the income must be left in the company to allow growth?
Remaining income is free cash flow. The model shows the free cash flow as a PSIC dividend to investors.
How much of the income represents additional value generated by the company above the expected return criteria of the hurdle rate?
• Value added = income less cost of capital
• Cost of capital = surplus at beg. of year × hurdle rate
next forecasted year
explicit forecast period complete
Value After Explicit Forecast Period
DCF: terminal value
EVA: continuing value added
Valuation Estimate
DCF: total of PV (using hurdle rate) of free cash flows
EVA: total of PV (using hurdle rate) of value added by year + starting surplus

selected capitalization standard result of $2.0 \times \$21.07$ million (the RBC indication at December 31, 2001) or $42.13 million. The "excess" surplus is recognized as free cash flow/value added for both DCF and EVA at December 31, 2001 (time 0), and our valuation models begin with a statutory surplus of $42.13 million. For the EVA model, the surplus of $42.13 million is recognized immediately as value. It is also the basis of the cost of capital calculation in the first period. For the DCF model, the surplus of $42.13 million contributes to value only through the investment income it earns in subsequent periods.

No other adjustments were made to the starting surplus for valuation. Carried reserves were assumed to be at the actuarially indicated amount. There was no difference between market value and book value of investments and no other adjustments were deemed warranted.

After establishing PSIC's adjusted net worth, the valuation process requires the total statutory income and RBC amounts for the first future projection year, 2002, from the financial model constructed for PSIC. Exhibit 8, Changes in Statutory Surplus, shows the estimated future income for PSIC during 2002 to be $10.44 million. The PSIC valuation model includes income from two categories: statutory net income and changes in unrealized capital gains. Exhibit 9 shows the computation of statutory net income. Unrealized capital gains stem from increases in market value for preferred and common stock investments.

The projected RBC for year-end 2002 is $23.25 million, leading to a December 31, 2002, required surplus of $46.50 million. Exhibit 12 shows PSIC's RBC calculation. During 2002, the required surplus increases by $4.37 million, from $42.13 million to $46.51 million.

The DCF methodology determines value from free cash flow estimates; for 2002 free cash equals $10.44 million of income less earnings retained to fund a surplus growth of \$4.37 million. Exhibit 8 shows the \$6.07 million free cash flow (\$10.44 – \$4.37 = \$6.07) as a stockholder dividend. The contribution to value of the 2002 free cash flow is the PV of \$6.07 million using the 15% hurdle rate.

The EVA methodology values returns in excess of the cost of capital. For 2002, excess returns equal \$10.44 million of income less the cost of capital of \$6.32 million, or \$4.12 million. The cost of capital equals the surplus as of the end of the prior year, \$42.13 million, multiplied by the hurdle rate of 15%. The contribution to value of the 2002 excess returns is the PV of \$4.12 million using the 15% selected hurdle rate, or \$3.59 million.

As shown on Exhibit 7, the application of the DCF and EVA methodologies given the total income, RBC, surplus projections, and valuation assumptions is repeated for each year in the 10-year explicit forecast period. The PV of free cash flow for the DCF method during the 10-year period is \$54.69 million. The PV of excess returns for the EVA method through the 10-year period is \$31.81. The PV of excess returns plus the starting surplus of \$42.13 million yields the EVA indicated value through year 10 of \$73.94 million.

The All Years value of PSIC under both valuation methods includes the PV contribution of value amounts beyond the explicit forecast period. The amounts shown in the "Total '12 to ∞" column in Exhibit 7 rely on perpetuity formula calculations rather than annual detailed financial projections for 2012 and subsequent years. Appendix B and Section 2 show these formulas for both methods and the algebraic derivation. The key assumptions for these calculations are the following:

- The expected annual growth rate of surplus and total income after 2011 is 2%. Thus, the implicitly projected surplus for 2012 is \$77.86 million $\times 1.02$ = \$79.42 million and the income for 2012 is \$18.71 million $\times 1.02$ = \$19.08 million.

- The hurdle rate is 15% for calculating the cost of capital for the EVA method and for determining the PV of 2012 and subsequent value amounts.

Both methods produce a valuation result of $88.03 million.

DCF		
(1)	Present value of free cash flow during the explicit forecast period	$54.69
(2)	Terminal value (present value of free cash flow subsequent to the explicit forecast period)	$33.33
	Total	$88.03
EVA		
(1)	Adjusted net worth (starting surplus)	$42.13
(2)	Present value of value added amounts during the explicit forecast period	$31.81
(3)	Present value of continuing value added subsequent to the explicit forecast period	$14.08
	Total	$88.03

A.2. Overview of the Financial Model

The property/casualty insurer financial model for the PSIC valuation performs all of the necessary computations to produce prospective statutory and GAAP financial statements. The major functions of the model are (i) runoff of loss and LAE reserves, (ii) payout of loss and loss adjustment expenses stemming from the earning of the unearned premium reserve, (iii) estimation of the level of future written premium and associated earned premium and application of the loss and expense ratio assumptions, (iv) calculation of investment income, and (v) calculation of federal income tax due.

There are two items of note before discussing the details of PSIC financial model projections. First, the model does not reflect all the changes resulting from the NAIC's codification of statutory accounting principles. An example is the recognition of a statutory asset or liability for deferred taxes. Even without these items, the financial model results provide significant insight into the considerations and calculations for valuing a property/casualty insurance company. Second, the GAAP balance sheet and income statements are provided for the interested reader. The GAAP results are not discussed in the text because the valuation estimate relies exclusively on amounts computed using statutory accounting.

Exhibit 11 is the detailed statutory balance sheet for PSIC. The "Actual 2001" column shows amounts from PSIC's December 31, 2001, statutory Annual Statement. Balance sheet items are either the sum of amounts from individual lines of business or for PSIC in total. Investment and cash amounts, items (1a) through (1g) and the Total Investments and Cash subtotal, are not segregated by line; neither are capital and surplus.

The remaining assets (receivables) and liabilities (payables and loss, LAE, and unearned premium reserves) are the sums of individual line of business amounts. In this example, PSIC wrote and continues to write three lines of business: workers compensation, auto liability, and general liability, all on a primary basis. Exhibits 18, 19, and 20 show the December 31, 2001, balance sheet amounts and business assumptions for the workers compensation, auto liability, and general liability books of business, respectively.

The largest single balance sheet item from the line of business data is the net loss and ALAE reserve. Sheet 6 for Exhibits 18, 19, and 20 show the loss and LAE reserves as of December 31, 2001, for accident years 2001 and prior for each line of business. Sheet 5 for each line of business shows the payment patterns for the respective 2001 balance sheet reserve amounts.

Sheet 4 for Exhibits 18, 19, and 20 shows the other balance sheet items associated with each line of business as of December 31, 2001.

Exhibit 9 is PSIC's Statutory Income Statement. Exhibit 8, Change in Statutory Surplus, uses net income from Exhibit 9. The annual change in statutory surplus equals net income plus change in unrealized capital gains. Net income has three basic components: underwriting income plus investment income less federal income taxes. (The PSIC model does not include any "other income" amounts.) PSIC's underwriting income equals the sum of income amounts for individual line of business underwriting. Investment income and federal income taxes are computed for PSIC in total. Investment income includes investment income on the capital along with the assets generated by line of business.

Sheet 1 for Exhibits 18, 19, and 20 provides the underwriting income by line of business. Sheet 2 provides the calculation notes for the components of the line of business underwriting income. The principal assumptions are as follows:

Net Earned Premium

- Direct written premium (DWP) annual growth is 4%.
- 50% of DWP is earned in the year written, 50% in the following year.
- Workers compensation and general liability have excess reinsurance (10% of the DWP is ceded).

Net Incurred Loss and LAE

- As shown in Sheet 4 of Exhibits 18, 19, and 20, the selected loss and LAE ratios for each line of business are as follows:

	Direct Loss Ratio	ALAE to Loss Ratio	ULAE to Loss Ratio	Ceded Loss Ratio
Workers Comp	70.0%	8.0%	8.5%	100.0%
Auto Liability	64.0%	8.5%	7.5%	N/A
General Liability	68.0%	15.0%	8.5%	100.0%

- These gross loss, gross LAE, and ceded ratios are applied to the December 31, 2001, unearned premium reserve and earned premium generated by the forecasted written premium.

Total Underwriting Year Expenses

- As shown in Sheet 4 of Exhibits 18, 19, and 20, the underwriting expense ratios for each line of business are as follows (DEP = direct earned premium, CWP = ceded written premium):

	Agents' Commission	Premium Tax	Other Underwriting Expenses		Reinsurance Commissions
	(% DWP)	(% DWP)	(% DEP)	(% DWP)	(% CWP)
Workers Comp	10.0%	3.0%	3.0%	2.25%	0.0%
Auto Liability	15.0%	2.0%	2.25%	3.25%	N/A
General Liability	12.5%	2.0%	4.0%	1.0%	0.0%

Investment income is shown in row (5) of the Statutory Income Statement (Exhibit 9). The sources of investment income are realized capital gains, interest income, and dividends. The annual yield rates (pretax) for each asset type are shown below:

Realized Capital Gains

Preferred Stocks	2.5%
Common Stocks	4.0%
Real Estate	4.0%

Interest Income

Taxable Bonds	6.0%
Non-taxable Bonds	4.0%
Cash	3.0%
Real Estate	4.0%
Other	2.0%

Dividends

Preferred Stocks	5.0%
Common Stocks	2.0%

Invested Asset and Cash Distribution

Taxable Bonds	42.0%
Non-taxable Bonds	24.0%
Preferred Stocks	1.0%
Common Stocks	25.0%
Cash	5.0%
Real Estate	1.0%
Other	2.0%
Total	100.0%

Invested assets held at the beginning of a forecasted year will earn a full year of investment income based on the assumed yield percentages. Investment income is also earned on new cash generated by PSIC's insurance operations. The financial model assumes that cash from operations is collected and invested at the midpoint of each forecasted year. The collected cash is invested according to the distribution of invested assets and cash shown above. Thus, the distribution is constant for all forecasted years.

Cash flows from operations are shown in Exhibit 13. Premium collections, loss and LAE payments, and underwriting expense payments are modeled for each line of business. Sheet 3 of Exhibits 18, 19, and 20 shows the cash flow from underwriting for each line of business, respectively. In addition to the premium, loss, LAE, and underwriting expense assumptions, the line of business underwriting cash flow relies on the following assumptions:

- Loss and LAE payment patterns for each line of business shown in Sheet 5 of Exhibits 18, 19, and 20, respectively. The payment patterns apply to reserves carried as of December

31, 2001, and loss and LAE incurred in 2002 and subsequent accident years.

- Lag of one month in collection of direct premium.
- Lag of three months in paying ceded premium.
- Lag of one month in collection of ceded loss recovery.

Federal income tax is the final component for computing net statutory income. The PSIC model followed the 2001 instructions for computing federal income tax for U.S. property/casualty insurance companies.

Total income for valuation equals net statutory income plus unrealized capital gains as shown in Exhibit 8. Unrealized capital gains are computed as total annual capital gains in equity investments less realized capital gains. The capital gain percentages are the following:

Preferred Stocks	11.0%
Common Stocks	9.5%

A.3. Sensitivity Testing

Table 7 shows the sensitivity of DCF and EVA value estimates to changes in underlying assumptions. Exhibit 21 shows additional detail related to each of these alternative scenarios.

For ease of reference, the assumptions underlying the base case follow:

- Starting capital as of December 31, 2001 = $42.13 million.
- Surplus/RBC ratio = 2.0.
- Workers compensation loss ratio = 70%.
- Auto liability loss ratio = 64%.
- General liability loss ratio = 68%.

- Average investment yield = 4.26% (weighted average of yields by asset type).
- Premium growth = 3%.
- Hurdle rate = 15% for explicit forecast period and subsequent years.

TABLE 7

SENSITIVITY TESTING OF ALTERNATIVE ASSUMPTIONS

	DCF Model			EVA Model		
	2001–2011	2012 to ∞	Total	2001–2011	2012 to ∞	Total
Base Case	54.7	33.3	88.0	73.9	14.1	88.0
Change in Assumption						
Surplus/RBC ratio = 2.5	43.1	34.7	77.7	67.3	10.4	77.7
Base loss ratios + 2% points	46.0	30.4	76.4	66.0	10.4	76.4
Base loss ratios −2% points	63.3	36.2	99.5	81.8	17.7	99.5
Investment yield +100 basis pts	67.6	39.8	107.5	86.9	20.6	107.5
Investment yield −100 basis pts	41.6	26.8	68.4	60.9	7.5	68.4
Premium growth = 0%	58.1	26.3	84.4	72.5	11.9	84.4
Premium growth = 6%	52.4	37.3	89.8	74.6	15.1	89.8
Hurdle rate +3% points	48.3	20.9	69.3	63.2	6.1	69.3
Hurdle rate −3% points	62.5	56.4	118.9	87.5	31.4	118.9

Table 8 shows the changes in value implied by the alternative assumptions. Section 3 discusses the similarities and differences of the models' structure and results using varying assumptions.

These tables show that company value is very sensitive to changes in the assumptions underlying the valuation. Every sensitivity test alters value by at least 10%, except for the premium growth assumptions. Large changes in premium growth assumptions had a small impact on value because the underwriting prof-

TABLE 8

CHANGES FROM BASE CASE IN VALUATION ESTIMATES

	DCF Model			EVA Model		
	2001–2011	2012 to ∞	Total	2001–2011	2012 to ∞	Total
Surplus/RBC ratio = 2.5	(11.6)	1.3	(10.3)	(6.7)	(3.7)	(10.3)
Base loss ratios +2% points	(8.7)	(2.9)	(11.7)	(8.0)	(3.7)	(11.7)
Base loss ratios −2% points	8.6	2.9	11.5	7.9	3.6	11.5
Investment yield +100 basis pts	12.9	6.5	19.4	12.9	6.5	19.4
Investment yield −100 basis pts	(13.1)	(6.6)	(19.6)	(13.1)	(6.6)	(19.6)
Premium growth = 0%	3.4	(7.0)	(3.6)	(1.4)	(2.2)	(3.6)
Premium growth = 6%	(2.3)	4.0	1.7	0.7	1.1	1.7
Hurdle rate +3% points	(6.4)	(12.4)	(18.8)	(10.7)	(8.0)	(18.8)
Hurdle rate −3% points	7.8	23.1	30.9	13.6	17.3	30.9

its of the insurance company are modest. This is apparent in Exhibit 9, which shows the underwriting income contribution to pretax operating income for 2002 through 2011.

The hurdle rate for the entire valuation period is also a key assumption. Decreasing the hurdle rate from 15% to 12% for all projection periods increases value by 35%.

An increase in the required surplus (raising the surplus-to-RBC ratio from 2.0 to 2.5) lowers value. This result is logical in that the higher the capital required, the lower the free cash flows for DCF and the higher the cost of capital for EVA.

Value is also very sensitive to changes in the investment yield for the asset portfolio. This result is logical for this company since over 95% of the pretax operating income is related to investment income (as shown in Exhibit 9).

Valuation results will always be sensitive to small changes in loss ratios as shown in Tables 7 and 8. A reduction in loss ratio of 2% for each line of business results in an increase in value of 13%.

Since the value of any company is a function of the assumptions used, as noted in Section 1, a valuation report should clearly identify the source of every assumption. The report should specify whether the assumption was provided by the subject company, derived from historical experience, provided by a potential investor, or developed from other sources. The source of an assumption may be an indication of whether the assumption is conservative, optimistic, or unbiased.

APPENDIX B

DEMONSTRATION OF ALGEBRAIC EQUIVALENCE OF EVA AND DCF

The general expression for value based on the discounted cash flow (DCF) approach is

$$\text{Value} = \text{FC}_0 + \sum_{x=1}^{\infty}[\text{OE}_x - \Delta C_x] \times (1 + h)^{-x}, \quad \text{(DCF-1)}$$

where

FC_0 = Free cash available at time 0 to be released to shareholders;

OE_x = After-tax operating earnings generated in time period x;

ΔC_x = Change in required capital over time period $x = C_x - C_{x-1}$, where C_x = required capital at the end of time period x (this is equivalent to the required capital at the beginning of time period $x + 1$); and

h = Hurdle rate (required return on capital).

Equation DCF-1 represents the sum of the free cash available at time 0 and the present value of future free cash flows, where future free cash flows ($\text{OE}_x - \Delta C_x$) are defined as after-tax operating earnings less the amount of required capital reinvestment. For ease of illustration, we have made the simplifying assumption that all cash flows occur at the end of the period.

Distributing and separating Equation DCF-1 into two separate sums, we produce

$$\text{Value} = \text{FC}_0 + \sum_{x=1}^{\infty}\text{OE}_x \times (1 + h)^{-x} - \sum_{x=1}^{\infty}\Delta C_x \times (1 + h)^{-x}. \quad \text{(DCF-2)}$$

If we assume that both operating earnings and capital grow at constant rate g, then

$$\mathrm{OE}_x = \mathrm{OE}_{x-1} \times (1+g) = \mathrm{OE}_1 \times (1+g)^{x-1}$$

and

$$C_x = C_{x-1} \times (1+g) = C_0 \times (1+g)^x$$

so

$$\Delta C_x = C_x - C_{x-1} = C_{x-1} \times g = C_0 \times (1+g)^{x-1} \times g.$$

Substituting into Equation DCF-2, the DCF value becomes

$$\text{Value} = \mathrm{FC}_0 + \sum_{x=1}^{\infty} \mathrm{OE}_1 \times (1+g)^{x-1} \times (1+h)^{-x} - \sum_{x=1}^{\infty} C_0 \times g \times (1+g)^{x-1} \times (1+h)^{-x}. \quad \text{(DCF-3)}$$

By factoring out the constants, this equation is rewritten as

$$\text{Value} = \mathrm{FC}_0 + \frac{\mathrm{OE}_1}{(1+h)} \sum_{x=1}^{\infty} \left[\frac{(1+g)}{(1+h)}\right]^{x-1} - \frac{C_0 \times g}{(1+h)} \sum_{x=1}^{\infty} \left[\frac{(1+g)}{(1+h)}\right]^{x-1}. \quad \text{(DCF-4)}$$

Note that g, the growth rate, will always be less than h, the hurdle rate. As a result, the sum of the infinite geometric series can be solved easily as $A \div (1-R)$, where A is the first term in the series and R is the multiplicative factor used to generate the next term in the series. The sum converges to

$$\frac{1}{1 - \dfrac{(1+g)}{(1+h)}} = \frac{1+h}{h-g}.$$

When we substitute this into Equation DCF-4, the $(1+h)$ terms cancel, so the formula for value based on a DCF approach becomes

$$\text{Value} = \text{FC}_0 + \frac{\text{OE}_1}{(h-g)} - \frac{C_0 \times g}{(h-g)}. \tag{DCF-5}$$

This is appropriately viewed as the sum of all free cash flows, or initial capital plus the present value of future earnings, minus the present value of future required capital reinvestments.

The general expression of EVA is

$$\text{Value} = \text{SC}_0 + \sum_{x=1}^{\infty}[\text{OE}_x - (h \times C_{x-1})] \times (1+h)^{-x}, \tag{EVA-1}$$

where

SC_0 = Starting capital, which is equal to the sum of free capital and required capital at time 0 (FC_0 and C_0, respectively, as defined in the DCF discussion); and

OE_x, C_x, and h have the same definitions as in the DCF discussion.

Formula EVA-1 represents the required capital at the valuation date (time = 0) plus the present value of future economic profits. Economic profits for time period x are defined as after-tax operating earnings (OE_x) reduced by the cost of capital, which is the product of the hurdle rate and the required capital at the beginning of each period ($h \times C_x$).

Distributing and separating Equation EVA-1 into two separate sums, we produce

$$\text{Value} = \text{SC}_0 + \sum_{x=1}^{\infty}\text{OE}_x \times (1+h)^{-x} - \sum_{x=1}^{\infty}(h \times C_{x-1}) \times (1+h)^{-x}. \tag{EVA-2}$$

Based on a constant growth rate g for both after-tax operating earnings and capital and the identities defined above in the DCF discussion, the formula for EVA value is restated as

$$\text{Value} = \text{SC}_0 + \sum_{x=1}^{\infty} \text{OE}_1 \times (1+g)^{x-1} \times (1+h)^{-x} - \sum_{x=1}^{\infty} h \times C_0 \times (1+g)^{x-1} \times (1+h)^{-x}. \quad \text{(EVA-3)}$$

By factoring out the constants, this may be rewritten as

$$\text{Value} = \text{SC}_0 + \frac{\text{OE}_1}{(1+h)} \sum_{x=1}^{\infty} \left[\frac{(1+g)}{(1+h)}\right]^{x-1} - \frac{(h \times C_0)}{(1+h)} \sum_{x=1}^{\infty} \left[\frac{(1+g)}{(1+h)}\right]^{x-1}. \quad \text{(EVA-4)}$$

Again, we use identities defined in the DCF discussion to simplify Equation EVA-4 to the following:

$$\text{Value} = \text{SC}_0 + \frac{\text{OE}_1}{(h-g)} - \frac{h \times C_0}{(h-g)}. \quad \text{(EVA-5)}$$

Formula EVA-5 can also be expressed as

$$\text{Value} = \text{SC}_0 + \frac{\text{OE}_1}{(h-g)} - \frac{(h-g+g) \times C_0}{(h-g)}, \quad \text{(EVA-6)}$$

or

$$\text{Value} = \text{SC}_0 + \frac{\text{OE}_1}{(h-g)} - \frac{(h-g) \times C_0}{(h-g)} - \frac{g \times C_0}{(h-g)}, \quad \text{(EVA-7)}$$

or

$$\text{Value} = \text{SC}_0 + \frac{\text{OE}_1}{(h-g)} - C_0 - \frac{g \times C_0}{(h-g)}, \quad \text{(EVA-8)}$$

or

$$\text{Value} = \text{FC}_0 + C_0 + \frac{\text{OE}_1}{(h-g)} - C_0 - \frac{g \times C_0}{(h-g)}, \quad \text{(EVA-9)}$$

or

$$\text{Value} = \text{FC}_0 + \frac{\text{OE}_1}{(h-g)} - \frac{g \times C_0}{(h-g)}. \qquad \text{(EVA-10)}$$

This is the same result as for the DCF model, as shown in Equation DCF-5.

EXHIBIT 1A

BASIC VALUATION EXAMPLE
DISCOUNTED CASH FLOW MODEL

Scenario Assumptions:
Total Earnings: hurdle rate exactly achieved
Annual Growth: 0%

	(1)	(2)	(3)	(4)	(5)	(6)	(6a)	(6b)	(7)	(8)	(9)	(10)
Projected Year	Initial Capital	Required Growth in Capital	Total Required Capital at Beginning of Year	Discount Factor at 15%	Premium Earned During Year	After-tax Investment Income on Capital	After-tax Investment Income on Original Capital	After-tax Investment Income on Additional Capital	After-tax Total Income From Insurance Operations	Required Reinv.	Available After-tax Net Income	Indicated Value
1	100	0	100	0.870	200	4	4	0	11	0	15	
2		0	100	0.756	200	4	4	0	11	0	15	
3		0	100	0.658	200	4	4	0	11	0	15	
4		0	100	0.572	200	4	4	0	11	0	15	
5		0	100	0.497	200	4	4	0	11	0	15	
6		0	100	0.432	200	4	4	0	11	0	15	
7		0	100	0.376	200	4	4	0	11	0	15	
8		0	100	0.327	200	4	4	0	11	0	15	
9		0	100	0.284	200	4	4	0	11	0	15	
10		0	100	0.247	200	4	4	0	11	0	15	

EXHIBIT 1A

Continued

Projected Year	(1) Initial Capital	(2) Required Growth in Capital	(3) Total Required Capital at Beginning of Year	(4) Discount Factor at 15%	(5) Premium Earned During Year	(6) After-tax Investment Income on Capital	(6a) After-tax Investment Income on Original Capital	(6b) After-tax Investment Income on Additional Capital	(7) After-tax Total Income From Insurance Operations	(8) Required Reinv.	(9) Available After-tax Net Income	(10) Indicated Value
Discounted Totals												
(11) Yrs. 1–10	100					20.08	20.08	0.00	55.21	0.00	75.28	75.28
(12) Terminal Value	0					6.59	6.59	0.00	18.13	0.00	24.72	24.72
(13) All Yrs.	100					26.67	26.67	0.00	73.33	0.00	100.00	100.00

(1)—selected judgmentally for illustration purposes
(2) = (3) − previous year's (3); for year 1, (3) − (1)
(3) = (5) ÷ 2.0, where 2.0 represents the target premium-to-surplus ratio
(4)—factor to discount from the end of the projected year to the beginning of year 1 at the hurdle rate (15%)
(5) = 200 for first projected year; subsequent years increased by the selected growth rate
(6) = (3) × selected investment yield of 4%; (6a) = (6) multiplied by the ratio of initial capital to (3); (6b) = (6) − (6a)
(7) = 5.5% of (5), selected so that earnings achieve the hurdle rate exactly; 5.5% = [hurdle rate − investment yield] ÷ premium-to-surplus ratio
(8) = (3) − following year's (3); difference between required capital at the beginning of year and required capital at beginning of following year
(9) = (6) + (7) + (8)
(10) = (9)
(11) = Explicit forecast period value
(12) = Terminal value
(13) = (11) + (12) = Value in perpetuity

EXHIBIT 1B

BASIC VALUATION EXAMPLE
ECONOMIC VALUE ADDED (a) MODEL

Scenario Assumptions:
Total Earnings: hurdle rate exactly achieved
Annual Growth: 0%

Projected Year	(1) Initial Capital	(2) Required Growth in Capital	(3) Total Required Capital at Beginning of Year	(4) Discount Factor at 15%	(5) Premium Earned During Year	(6) After-tax Investment Income on Capital	(6a) After-tax Investment Income on Original Capital	(6b) After-tax Investment Income on Additional Capital	(7) After-tax Total Income From Insurance Operations
1	100	0	100	0.870	200	4	4	0	11
2		0	100	0.756	200	4	4	0	11
3		0	100	0.658	200	4	4	0	11
4		0	100	0.572	200	4	4	0	11
5		0	100	0.497	200	4	4	0	11
6		0	100	0.432	200	4	4	0	11
7		0	100	0.376	200	4	4	0	11
8		0	100	0.327	200	4	4	0	11
9		0	100	0.284	200	4	4	0	11
10		0	100	0.247	200	4	4	0	11
Discounted Totals									
(13) Yrs. 1–10	100					20.08	20.08	0.00	55.21
(14) Terminal Value	0					6.59	6.59	0.00	18.13
(15) All Yrs.	100					26.67	26.67	0.00	73.33

EXHIBIT 1B

Continued

Projected Year	(8) Required Reinv.	(9) Available After-tax Net Income	(10) Hurdle Required Return on Capital	(10a) Hurdle Required Return on Original Capital	(10b) Hurdle Required Return on Additional Capital	(11) Total Cost of Capital	(11a) Cost of Original Capital	(11b) Cost of Additional Capital	(12) Indicated Value
1	0	15	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00	
2	11	0	15	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00
3	11	0	15	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00
4	11	0	15	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00
5	11	0	15	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00
6	11	0	15	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00
7	11	0	15	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00
8	11	0	15	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00
9	11	0	15	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00
10	11	0	15	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00
Discounted Totals									
(13) Yrs. 1–10	0.00	75.28	(75.28)	(75.28)	0.00	(55.21)	(55.21)	0.00	100.00
(14) Terminal Value	0.00	24.72	(24.72)	(24.72)	0.00	(18.13)	(18.13)	0.00	0.00
(15) All Yrs.	0.00	100.00	(100.00)	(100.00)	0.00	(73.33)	(73.33)	0.00	100.00

(1)—selected judgmentally for illustration purposes
(2) = (3) – previous year's (3); for year 1, (3) – (1)
(3) = (5) ÷ 2.0, where 2.0 represents the target premium-to-surplus ratio
(4)—factor to discount from the end of the projected year to the beginning of year 1 at the hurdle rate (15%)
(5) = 200 for first projected year; subsequent years increased by the selected growth rate
(6) = (3) × selected investment yield of 4%; (6a) = (6) multiplied by the ratio of initial capital to (3); (6b) = (6) – (6a)
(7) = 5.5% of (5), selected so that earnings achieve the hurdle rate exactly; 5.5% = [hurdle rate – investment yield] ÷ premium-to-surplus ratio
(8) = (3) – following year's (3); difference between required capital at the beginning of year and required capital at beginning of following year
(9) = (6) + (7) + (8)
(10) = (3) × −15% (hurdle rate); (10a) = (10) multiplied by the ratio of initial capital to (3); (10b) = (10) – (10a)
(11) = (6) + (10); (11*a*) = (6*a*) + (10*a*); (11*b*) = (6*b*) + (10*b*)
(12) = (1) + [(6) + (7)] + (10); EVA (a) does not reduce the cost of capital to reflect investment income earned on capital
(13) Forecast period value
(14) Terminal value
(15) = (13) + (14) Value in perpetuity

EXHIBIT 1C

BASIC VALUATION EXAMPLE
ECONOMIC VALUE ADDED (b) MODEL

Scenario Assumptions:
Total Earnings: hurdle rate exactly achieved
Annual Growth: 0%

Projected Year	(1) Initial Capital	(2) Required Growth in Capital	(3) Total Required Capital at Beginning of Year	(4) Discount Factor at 15%	(5) Premium Earned During Year	(6) After-tax Investment Income on Capital	(6a) After-tax Investment Income on Original Capital	(6b) After-tax Investment Income on Additional Capital	(7) After-tax Total Income From Insurance Operations
1	100	0	100	0.870	200	4	4	0	11
2		0	100	0.756	200	4	4	0	11
3		0	100	0.658	200	4	4	0	11
4		0	100	0.572	200	4	4	0	11
5		0	100	0.497	200	4	4	0	11
6		0	100	0.432	200	4	4	0	11
7		0	100	0.376	200	4	4	0	11
8		0	100	0.327	200	4	4	0	11
9		0	100	0.284	200	4	4	0	11
10		0	100	0.247	200	4	4	0	11
Discounted Totals									
(13) Yrs. 1–10	100					20.08	20.08	0.00	55.21
(14) Terminal Value	0					6.59	6.59	0.00	18.13
(15) All Yrs.	100					26.67	26.67	0.00	73.33

EXHIBIT 1C

Continued

Projected Year	(8) Required Reinv.	(9) Available After-tax Net Income	(10) Hurdle Required Return on Capital	(10a) Hurdle Required Return on Original Capital	(10b) Hurdle Required Return on Additional Capital	(11) Total Cost of Capital	(11a) Cost of Original Capital	(11b) Cost of Additional Capital	(12) Indicated Value
1	0	15	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00	
2	0	15	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00	
3	0	15	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00	
4	0	15	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00	
5	0	15	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00	
6	0	15	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00	
7	0	15	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00	
8	0	15	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00	
9	0	15	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00	
10	0	15	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00	
Discounted Totals									
(13) Yrs. 1–10	0.00	75.28	(75.28)	(75.28)	0.00	(55.21)	(55.21)	0.00	100.00
(14) Terminal Value	0.00	24.72	(24.72)	(24.72)	0.00	(18.13)	(18.13)	0.00	0.00
(15) All Yrs.	0.00	100.00	(100.00)	(100.00)	0.00	(73.33)	(73.33)	0.00	100.00

EXHIBIT 1C

Continued

(1)—selected judgmentally for illustration purposes
(2) = (3) – previous year's (3); for year 1, (3) – (1)
(3) = (5) ÷ 2.0, where 2.0 represents the target premium-to-surplus ratio
(4)—factor to discount from the end of the projected year to the beginning of year 1 at the hurdle rate (15%)
(5) = 200 for first projected year; subsequent years increased by the selected growth rate
(6) = (3) × selected investment yield of 4%; (6a) = (6) multiplied by the ratio of initial capital to (3); (6b) = (6) – (6a)
(7) = 5.5% of (5), selected so that earnings achieve the hurdle rate exactly; 5.5% = [hurdle rate – investment yield] ÷ premium-to-surplus ratio
(8) = (3) – following year's (3); difference between required capital at the beginning of year and required capital at beginning of following year
(9) = (6) + (7) + (8)
(10) = (3) × −15% (hurdle rate); (10a) = (10) multiplied by the ratio of initial capital to (3); (10b) = (10) – (10a)
(11) = (6) + (10); (11a) = (6a) + (10a); (11b) = (6b) + (10b)
(12) = (1) + (7) + (11); EVA (b) reduces the cost of capital component to reflect investment income earned on capital
(13) Forecast period value
(14) Terminal value
(15) = (13) + (14) = Value in perpetuity

EXHIBIT 2A

BASIC VALUATION EXAMPLE
DISCOUNTED CASH FLOW MODEL

Scenario Assumptions:
Total Earnings: hurdle rate exactly achieved
Annual Growth: 3%

Projected Year	(1) Initial Capital	(2) Required Growth in Capital	(3) Total Required Capital at Beginning of Year	(4) Discount Factor at 15%	(5) Premium Earned During Year	(6) After-tax Investment Income on Capital	(6a) After-tax Investment Income on Original Capital	(6b) After-tax Investment Income on Additional Capital	(7) After-tax Total Income From Insurance Operations	(8) Required Reinv.	(9) Available After-tax Net Income	(10) Indicated Value
1	100	0	100	0.870	200	4	4	0	11	(3)	12	
2		3	103	0.756	206	4.12	4	0.12	11.33	(3.09)	12.36	
3		3.09	106.09	0.658	212.18	4.24	4	0.24	11.67	(3.18)	12.73	
4		3.18	109.27	0.572	218.55	4.37	4	0.37	12.02	(3.28)	13.11	
5		3.28	112.55	0.497	225.10	4.50	4	0.50	12.38	(3.38)	13.51	
6		3.38	115.93	0.432	231.85	4.64	4	0.64	12.75	(3.48)	13.91	
7		3.48	119.41	0.376	238.81	4.78	4	0.78	13.13	(3.58)	14.33	
8		3.58	122.99	0.327	245.97	4.92	4	0.92	13.53	(3.69)	14.76	
9		3.69	126.68	0.284	253.35	5.07	4	1.07	13.93	(3.80)	15.20	
10		3.80	130.48	0.247	260.95	5.22	4	1.22	14.35	(3.91)	15.66	
Discounted Totals												
(11) Yrs. 1–10	100					22.26	20.08	2.19	61.22	(16.70)	66.78	66.78
(12) Terminal Value	0					11.07	6.59	4.48	30.45	(8.30)	33.22	33.22
(13) All Yrs.	100					33.33	26.67	6.67	91.67	(25.00)	100.00	100.00

EXHIBIT 2A

Continued

(1)—selected judgmentally for illustration purposes
(2) = (3) – previous year's (3); for year 1, (3) - (1)
(3) = (5)/2, where 2.0 represents the target premium-to-surplus ratio
(4)—factor to discount from the end of the projected year to the beginning of year 1 at the hurdle rate (15%)
(5) = 200 for first projected year, growing by growth rate
(6) = (3) × selected investment yield of 4%; (6a) = (6) multiplied by the ratio of initial capital to (3); (6b) = (6) – (6a)
(7) = 5.5% of (5), selected so that earnings achieve the hurdle rate exactly; 5.5% = [hurdle rate – investment yield] ÷ premium-to-surplus ratio
(8) = (3) – following year's (3); difference between required capital at the beginning of year and required capital at beginning of following year
(9) = (6) + (7) + (8)
(10) = (9)
(11) = Explicit forecast period value
(12) = Terminal value
(13) = (11) + (12) = Value in perpetuity

EXHIBIT 2B

Basic Valuation Example
Economic Value Added (a) Model

Scenario Assumptions:
Total Earnings: hurdle rate exactly achieved
Annual Growth: 3%

Projected Year	(1) Initial Capital	(2) Required Growth in Capital	(3) Total Required Capital at Beginning of Year	(4) Discount Factor at 15%	(5) Premium Earned During Year	(6) After-tax Investment Income on Capital	(6a) After-tax Investment Income on Original Capital	(6b) After-tax Investment Income on Additional Capital	(7) After-tax Total Income From Insurance Operations
1	100	0	100	0.870	200	4	4	0	11
2		3	103	0.756	206	4.12	4	0.12	11.33
3		3.09	106.09	0.658	212.18	4.24	4	0.24	11.67
4		3.18	109.27	0.572	218.55	4.37	4	0.37	12.02
5		3.28	112.55	0.497	225.10	4.50	4	0.50	12.38
6		3.38	115.93	0.432	231.85	4.64	4	0.64	12.75
7		3.48	119.41	0.376	238.81	4.78	4	0.78	13.13
8		3.58	122.99	0.327	245.97	4.92	4	0.92	13.53
9		3.69	126.68	0.284	253.35	5.07	4	1.07	13.93
10		3.80	130.48	0.247	260.95	5.22	4	1.22	14.35
Discounted Totals									
(13) Yrs. 1–10	100					22.26	20.08	2.19	61.22
(14) Terminal Value	0					11.07	6.59	4.48	30.45
(15) All Yrs.	100					33.33	26.67	6.67	91.67

EXHIBIT 2B

Continued

Projected Year	(8) Required Reinv.	(9) Available After-tax Net Income	(10) Hurdle Required Return on Capital	(10a) Hurdle Required Return on Original Capital	(10b) Hurdle Required Return on Additional Capital	(11) Total Cost of Capital	(11a) Cost of Original Capital	(11b) Cost of Additional Capital	(12) Indicated Value
1	(3)	12	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00	
2	(3.09)	12.36	(15.45)	(15.00)	(0.45)	(11.33)	(11.00)	(0.33)	
3	(3.18)	12.73	(15.91)	(15.00)	(0.91)	(11.67)	(11.00)	(0.67)	
4	(3.28)	13.11	(16.39)	(15.00)	(1.39)	(12.02)	(11.00)	(1.02)	
5	(3.38)	13.51	(16.88)	(15.00)	(1.88)	(12.38)	(11.00)	(1.38)	
6	(3.48)	13.91	(17.39)	(15.00)	(2.39)	(12.75)	(11.00)	(1.75)	
7	(3.58)	14.33	(17.91)	(15.00)	(2.91)	(13.13)	(11.00)	(2.13)	
8	(3.69)	14.76	(18.45)	(15.00)	(3.45)	(13.53)	(11.00)	(2.53)	
9	(3.80)	15.20	(19.00)	(15.00)	(4.00)	(13.93)	(11.00)	(2.93)	
10	(3.91)	15.66	(19.57)	(15.00)	(4.57)	(14.35)	(11.00)	(3.35)	
Discounted Totals									
(13) Yrs. 1–10	(16.70)	66.78	(83.48)	(75.28)	(8.19)	(61.22)	(55.21)	(6.01)	100.00
(14) Terminal Value	(8.30)	33.22	(41.52)	(24.72)	(16.81)	(30.45)	(18.13)	(12.32)	0.00
(15) All Yrs.	(25.00)	100.00	(125.00)	(100.00)	(25.00)	(91.67)	(73.33)	(18.33)	100.00

(1)—selected judgmentally for illustration purposes
(2) = (3) – previous year's (3); for year 1, (3) – (1)
(3) = (5) ÷ 2.0, where 2.0 represents the target premium-to-surplus ratio
(4)—factor to discount from the end of the projected year to the beginning of year 1 at the hurdle rate (15%)
(5) = 200 for first projected year; subsequent years increased by the selected growth rate
(6) = (3) × selected investment yield of 4%; (6a) = (6) multiplied by the ratio of initial capital to (3); (6b) = (6) – (6a)
(7) = 5.5% of (5), selected so that earnings achieve the hurdle rate exactly; 5.5% = [hurdle rate – investment yield] ÷ premium-to-surplus ratio
(8) = (3) – following year's (3); difference between required capital at the beginning of year and required capital at beginning of following year
(9) = (6) + (7) + (8)
(10) = (3) × –15% (hurdle rate); (10a) = (10) multiplied by the ratio of initial capital to (3); (10b) = (10) – (10a)
(11) = (6) + (10); (11a) = (6a) + (10a); (11b) = (6b) + (10b)
(12) = (1) + [(6) + (7)] + (10); EVA (a) does not reduce the cost of capital to reflect investment income earned on capital
(13)—Forecast period value
(14)—Terminal value

EXHIBIT 2C

Basic Valuation Example
Economic Value Added (b) Model

Scenario Assumptions:
Total Earnings: hurdle rate exactly achieved
Annual Growth: 3%

Projected Year	(1) Initial Capital	(2) Required Growth in Capital	(3) Total Required Capital at Beginning of Year	(4) Discount Factor at 15%	(5) Premium Earned During Year	(6) After-tax Investment Income on Capital	(6a) After-tax Investment Income on Original Capital	(6b) After-tax Investment Income on Additional Capital	(7) After-tax Total Income From Insurance Operations
1	100	0	100	0.870	200	4	4	0	11
2		3	103	0.756	206	4.12	4	0.12	11.33
3		3.09	106.09	0.658	212.18	4.24	4	0.24	11.67
4		3.18	109.27	0.572	218.55	4.37	4	0.37	12.02
5		3.28	112.55	0.497	225.10	4.50	4	0.50	12.38
6		3.38	115.93	0.432	231.85	4.64	4	0.64	12.75
7		3.48	119.41	0.376	238.81	4.78	4	0.78	13.13
8		3.58	122.99	0.327	245.97	4.92	4	0.92	13.53
9		3.69	126.68	0.284	253.35	5.07	4	1.07	13.93
10		3.80	130.48	0.247	260.95	5.22	4	1.22	14.35
Discounted Totals									
(13) Yrs. 1–10	100					22.26	20.08	2.19	61.22
(14) Terminal Value	0					11.07	6.59	4.48	30.45
(15) All Yrs.	100					33.33	26.67	6.67	91.67

EXHIBIT 2C

Continued

Projected Year	(8) Required Reinv.	(9) Available After-tax Net Income	(10) Hurdle Required Return on Capital	(10a) Hurdle Required Return on Original Capital	(10b) Hurdle Required Return on Additional Capital	(11) Total Cost of Capital	(11a) Cost of Original Capital	(11b) Cost of Additional Capital	(12) Indicated Value
1	(3)	12	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00	
2	(3.09)	12.36	(15.45)	(15.00)	(0.45)	(11.33)	(11.00)	(0.33)	
3	(3.18)	12.73	(15.91)	(15.00)	(0.91)	(11.67)	(11.00)	(0.67)	
4	(3.28)	13.11	(16.39)	(15.00)	(1.39)	(12.02)	(11.00)	(1.02)	
5	(3.38)	13.51	(16.88)	(15.00)	(1.88)	(12.38)	(11.00)	(1.38)	
6	(3.48)	13.91	(17.39)	(15.00)	(2.39)	(12.75)	(11.00)	(1.75)	
7	(3.58)	14.33	(17.91)	(15.00)	(2.91)	(13.13)	(11.00)	(2.13)	
8	(3.69)	14.76	(18.45)	(15.00)	(3.45)	(13.53)	(11.00)	(2.53)	
9	(3.80)	15.20	(19.00)	(15.00)	(4.00)	(13.93)	(11.00)	(2.93)	
10	(3.91)	15.66	(19.57)	(15.00)	(4.57)	(14.35)	(11.00)	(3.35)	
Discounted Totals									
(13) Yrs. 1–10	(16.70)	66.78	(83.48)	(75.28)	(8.19)	(61.22)	(55.21)	(6.01)	100.00
(14) Terminal Value	(8.30)	33.22	(41.52)	(24.72)	(16.81)	(30.45)	(18.13)	(12.32)	0.00
(15) All Yrs.	(25.00)	100.00	(125.00)	(100.00)	(25.00)	(91.67)	(73.33)	(18.33)	100.00

EXHIBIT 2C

Continued

(1)—selected judgmentally for illustration purposes
(2) = (3) – previous year's (3); for year 1, (3) – (1)
(3) = (5) ÷ 2.0, where 2.0 represents the target premium-to-surplus ratio
(4)—factor to discount from the end of the projected year to the beginning of year 1 at the hurdle rate (15%)
(5) = 200 for first projected year; subsequent years increased by the selected growth rate
(6) = (3) × selected investment yield of 4%; (6a) = (6) multiplied by the ratio of initial capital to (3); (6b) = (6) – (6a)
(7) = 5.5% of (5), selected so that earnings achieve the hurdle rate exactly; 5.5% = [hurdle rate – investment yield] ÷ premium-to-surplus ratio
(8) = (3) – following year's (3); difference between required capital at the beginning of year and required capital at beginning of following year
(9) = (6) + (7) + (8)
(10) = (3) × –15% (hurdle rate); (10a) = (10) multiplied by the ratio of initial capital to (3); (10b) = (10) – (10a)
(11) = (6) + (10); (11a) = (6a) + (10a); (11b) = (6b) + (10b)
(12) = (1) + (7) + (11); EVA (b) reduces the cost of capital component to reflect investment income earned on capital
(13)—Forecast period value
(14)—Terminal value
(15) = (13) + (14) = Value in perpetuity

EXHIBIT 3A

Basic Valuation Example
Discounted Cash Flow Model

Scenario Assumptions:
Total Earnings: hurdle rate more than achieved
Annual Growth: 0%

Projected Year	(1) Initial Capital	(2) Required Growth in Capital	(3) Total Required Capital at Beginning of Year	(4) Discount Factor at 15%	(5) Premium Earned During Year	(6) After-tax Investment Income on Capital	(6a) After-tax Investment Income on Original Capital	(6b) After-tax Investment Income on Additional Capital	(7) After-tax Total Income From Insurance Operations	(8) Required Reinv.	(9) Available After-tax Net Income	(10) Indicated Value
1	100	0	100	0.870	200	4	4	0	12	0	16	
2		0	100	0.756	200	4	4	0	12	0	16	
3		0	100	0.658	200	4	4	0	12	0	16	
4		0	100	0.572	200	4	4	0	12	0	16	
5		0	100	0.497	200	4	4	0	12	0	16	
6		0	100	0.432	200	4	4	0	12	0	16	
7		0	100	0.376	200	4	4	0	12	0	16	
8		0	100	0.327	200	4	4	0	12	0	16	
9		0	100	0.284	200	4	4	0	12	0	16	
10		0	100	0.247	200	4	4	0	12	0	16	
Discounted Totals												
(11) Yrs. 1–10	100					20.08	20.08	0.00	60.23	0.00	80.30	80.30
(12) Terminal Value	0					6.59	6.59	0.00	19.77	0.00	26.37	26.37
(13) All Yrs.	100					26.67	26.67	0.00	80.00	0.00	106.67	106.67

EXHIBIT 3A

Continued

(1)—selected judgmentally for illustration purposes
(2) = (3) – previous year's (3); for year 1, (3) – (1)
(3) = (5) ÷ 2.0, where 2.0 represents the target premium-to-surplus ratio
(4)—factor to discount from the end of the projected year to the beginning of year 1 at the hurdle rate (15%)
(5) = 200 for first projected year; subsequent years increased by the selected growth rate
(6) = (3) × selected investment yield of 4%; (6a) = (6) multiplied by the ratio of initial capital to (3); (6b) = (6) – (6a)
(7) = 6% of (5), selected so that earnings exceed the hurdle rate requirement
(8) = (3) – following year's (3); difference between required capital at the beginning of year and required capital at beginning of following year
(9) = (6) + (7) + (8)
(10) = (9)
(11) = Explicit forecast period value
(12) = Terminal value
(13) = (11) + (12) = Value in perpetuity

EXHIBIT 3B

Basic Valuation Example
Economic Value Added (a) Model

Scenario Assumptions:
Total Earnings: hurdle rate more than achieved
Annual Growth: 0%

Projected Year	(1) Initial Capital	(2) Required Growth in Capital	(3) Total Required Capital at Beginning of Year	(4) Discount Factor at 15%	(5) Premium Earned During Year	(6) After-tax Investment Income on Capital	(6a) After-tax Investment Income on Original Capital	(6b) After-tax Investment Income on Additional Capital	(7) After-tax Total Income From Insurance Operations
1	100	0	100	0.870	200	4	4	0	12
2		0	100	0.756	200	4	4	0	12
3		0	100	0.658	200	4	4	0	12
4		0	100	0.572	200	4	4	0	12
5		0	100	0.497	200	4	4	0	12
6		0	100	0.432	200	4	4	0	12
7		0	100	0.376	200	4	4	0	12
8		0	100	0.327	200	4	4	0	12
9		0	100	0.284	200	4	4	0	12
10		0	100	0.247	200	4	4	0	12
Discounted Totals									
(13) Yrs. 1–10	100					20.08	20.08	0.00	60.23
(14) Terminal Value	0					6.59	6.59	0.00	19.77
(15) All Yrs.	100					26.67	26.67	0.00	80.00

EXHIBIT 3B

Continued

Projected Year	(8) Required Reinv.	(9) Available After-tax Net Income	(10) Hurdle Required Return on Capital	(10a) Hurdle Required Return on Original Capital	(10b) Hurdle Required Return on Additional Capital	(11) Total Cost of Capital	(11a) Cost of Original Capital	(11b) Cost of Additional Capital	(12) Indicated Value
1	0	16	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00	
2	0	16	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00	
3	0	16	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00	
4	0	16	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00	
5	0	16	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00	
6	0	16	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00	
7	0	16	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00	
8	0	16	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00	
9	0	16	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00	
10	0	16	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00	
Discounted Totals									
(13) Yrs. 1–10	0.00	80.30	(75.28)	(75.28)	0.00	(55.21)	(55.21)	0.00	105.02
(14) Terminal Value	0.00	26.37	(24.72)	(24.72)	0.00	(18.13)	(18.13)	0.00	1.65
(15) All Yrs.	0.00	106.67	(100.00)	(100.00)	0.00	(73.33)	(73.33)	0.00	106.67

(1)—selected judgmentally for illustration purposes
(2) = (3) – previous year's (3); for year 1, (3) – (1)
(3) = (5) ÷ 2.0, where 2.0 represents the target premium-to-surplus ratio
(4)—factor to discount from the end of the projected year to the beginning of year 1 at the hurdle rate (15%)
(5) = 200 for first projected year; subsequent years increased by the selected growth rate
(6) = (3) × selected investment yield of 4%; (6a) = (6) multiplied by the ratio of initial capital to (3); (6b) = (6) – (6a)
(7) = 6% of (5), selected so that earnings exceed the hurdle rate requirement
(8) = (3) – following year's (3); difference between required capital at the beginning of year and required capital at beginning of following year
(9) = (6) + (7) + (8)
(10) = (3) × –15% (hurdle rate); (10a) = (10) multiplied by the ratio of initial capital to (3); (10b) = (10) – (10a)
(11) = (6) + (10); (11a) = (6a) + (10a); (11b) = (6b) + (10b)
(12) = (1) + [(6) + (7)] + (10); EVA (a) does not reduce the cost of capital to reflect investment income earned on capital
(13)—Forecast period value
(14)—Terminal value
(15) = (13) + (14) = Value in perpetuity

EXHIBIT 3C

Basic Valuation Example
Economic Value Added (b) Model

Scenario Assumptions:
Total Earnings: hurdle rate more than achieved
Annual Growth: 0%

Projected Year	(1) Initial Capital	(2) Required Growth in Capital	(3) Total Required Capital at Beginning of Year	(4) Discount Factor at 15%	(5) Premium Earned During Year	(6) After-tax Investment Income on Capital	(6a) After-tax Investment Income on Original Capital	(6b) After-tax Investment Income on Additional Capital	(7) After-tax Total Income From Insurance Operations
1	100	0	100	0.870	200	4	4	0	12
2		0	100	0.756	200	4	4	0	12
3		0	100	0.658	200	4	4	0	12
4		0	100	0.572	200	4	4	0	12
5		0	100	0.497	200	4	4	0	12
6		0	100	0.432	200	4	4	0	12
7		0	100	0.376	200	4	4	0	12
8		0	100	0.327	200	4	4	0	12
9		0	100	0.284	200	4	4	0	12
10		0	100	0.247	200	4	4	0	12
Discounted Totals									
(13) Yrs. 1–10	100					20.08	20.08	0.00	60.23
(14) Terminal Value	0					6.59	6.59	0.00	19.77
(15) All Yrs.	100					26.67	26.67	0.00	80.00

EXHIBIT 3C

Continued

Projected Year	(8) Required Reinv.	(9) Available After-tax Net Income	(10) Hurdle Required Return on Capital	(10a) Hurdle Required Return on Original Capital	(10b) Hurdle Required Return on Additional Capital	(11) Total Cost of Capital	(11a) Cost of Original Capital	(11b) Cost of Additional Capital	(12) Indicated Value
1	0	16	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00	
2	0	16	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00	
3	0	16	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00	
4	0	16	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00	
5	0	16	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00	
6	0	16	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00	
7	0	16	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00	
8	0	16	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00	
9	0	16	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00	
10	0	16	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00	
Discounted Totals									
(13) Yrs. 1–10	0.00	80.30	(75.28)	(75.28)	0.00	(55.21)	(55.21)	0.00	105.02
(14) Terminal Value	0.00	26.37	(24.72)	(24.72)	0.00	(18.13)	(18.13)	0.00	1.65
(15) All Yrs.	0.00	106.67	(100.00)	(100.00)	0.00	(73.33)	(73.33)	0.00	106.67

EXHIBIT 3C

Continued

(1)—selected judgmentally for illustration purposes
(2) = (3) − previous year's (3); for year 1, (3) − (1)
(3) = (5) ÷ 2.0, where 2.0 represents the target premium-to-surplus ratio
(4)—factor to discount from the end of the projected year to the beginning of year 1 at the hurdle rate (15%)
(5) = 200 for first projected year; subsequent years increased by the selected growth rate
(6) = (3) × selected investment yield of 4%; (6a) = (6) multiplied by the ratio of initial capital to (3); (6b) = (6) − (6a)
(7) = 6% of (5), selected so that earnings exceed the hurdle rate requirement
(8) = (3) − following year's (3); difference between required capital at the beginning of year and required capital at beginning of following year
(9) = (6) + (7) + (8)
(10) = (3) × −15% (hurdle rate); (10a) = (10) multiplied by the ratio of initial capital to (3); (10b) = (10) − (10a)
(11) = (6) + (10); (11a) = (6a) + (10a); (11b) = (6b) + (10b)
(12) = (1) + (7) + (11); EVA (b) reduces the cost of capital component to reflect investment income earned on capital
(13)—Forecast period value
(14)—Terminal value
(15) = (13) + (14) = Value in perpetuity

EXHIBIT 4A

Basic Valuation Example
Discounted Cash Flow Model

Scenario Assumptions:
Total Earnings: hurdle rate not achieved
Annual Growth: 0%

Projected Year	(1) Initial Capital	(2) Required Growth in Capital	(3) Total Required Capital at Beginning of Year	(4) Discount Factor at 15%	(5) Premium Earned During Year	(6) After-tax Investment Income on Capital	(6a) After-tax Investment Income on Original Capital	(6b) After-tax Investment Income on Additional Capital	(7) After-tax Total Income From Insurance Operations	(8) Required Reinv.	(9) Available After-tax Net Income	(10) Indicated Value
1	100	0	100	0.870	200	4	4	0	10	0	14	
2		0	100	0.756	200	4	4	0	10	0	14	
3		0	100	0.658	200	4	4	0	10	0	14	
4		0	100	0.572	200	4	4	0	10	0	14	
5		0	100	0.497	200	4	4	0	10	0	14	
6		0	100	0.432	200	4	4	0	10	0	14	
7		0	100	0.376	200	4	4	0	10	0	14	
8		0	100	0.327	200	4	4	0	10	0	14	
9		0	100	0.284	200	4	4	0	10	0	14	
10		0	100	0.247	200	4	4	0	10	0	14	
Discounted Totals												
(11) Yrs. 1–10	100					20.08	20.08	0.00	50.19	0.00	70.26	70.26
(12) Terminal Value	0					6.59	6.59	0.00	16.48	0.00	23.07	23.07
(13) All Yrs.	100					26.67	26.67	0.00	66.67	0.00	93.33	93.33

EXHIBIT 4A

Continued

(1)—assumed
(2) = (3) – previous year's (3); for year 1, (3) – (1)
(3) = (5) ÷ 2.0, where 2.0 represents the target premium-to-surplus ratio
(4)—factor to discount from the end of the projected year to the beginning of year 1 at the hurdle rate (15%)
(5) = 200 for first projected year; subsequent years increased by the selected growth rate
(6) = (3) × selected investment yield of 4%; (6a) = (6) multiplied by the ratio of initial capital to (3); (6b) = (6) – (6a)
(7) = 5% of (5), selected so that earnings are less than the hurdle rate requirement
(8) = (3) – following year's (3); difference between required capital at the beginning of year and required capital at beginning of following year
(9) = (6) + (7) + (8)
(10) = (9)
(11)—Forecast period value
(12)—Terminal value
(13) = (11) + (12) = Value in perpetuity

EXHIBIT 4B

BASIC VALUATION EXAMPLE
ECONOMIC VALUE ADDED (a) MODEL

Scenario Assumptions:
Total Earnings: hurdle rate not achieved
Annual Growth: 0%

Projected Year	(1) Initial Capital	(2) Required Growth in Capital	(3) Total Required Capital at Beginning of Year	(4) Discount Factor at 15%	(5) Premium Earned During Year	(6) After-tax Investment Income on Capital	(6a) After-tax Investment Income on Original Capital	(6b) After-tax Investment Income on Additional Capital	(7) After-tax Total Income From Insurance Operations
1	100	0	100	0.870	200	4	4	0	10
2		0	100	0.756	200	4	4	0	10
3		0	100	0.658	200	4	4	0	10
4		0	100	0.572	200	4	4	0	10
5		0	100	0.497	200	4	4	0	10
6		0	100	0.432	200	4	4	0	10
7		0	100	0.376	200	4	4	0	10
8		0	100	0.327	200	4	4	0	10
9		0	100	0.284	200	4	4	0	10
10		0	100	0.247	200	4	4	0	10
Discounted Totals									
(13) Yrs. 1–10	100					20.08	20.08	0.00	50.19
(14) Terminal Value	0					6.59	6.59	0.00	16.48
(15) All Yrs.	100					26.67	26.67	0.00	66.67

EXHIBIT 4B

Continued

Projected Year	(8) Required Reinv.	(9) Available After-tax Net Income	(10) Hurdle Required Return on Capital	(10a) Hurdle Required Return on Original Capital	(10b) Hurdle Required Return on Additional Capital	(11) Total Cost of Capital	(11a) Cost of Original Capital	(11b) Cost of Additional Capital	(12) Indicated Value
1	0	14	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00	
2	0	14	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00	
3	0	14	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00	
4	0	14	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00	
5	0	14	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00	
6	0	14	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00	
7	0	14	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00	
8	0	14	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00	
9	0	14	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00	
10	0	14	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00	
Discounted Totals									
(13) Yrs. 1–10	0.00	70.26	(75.28)	(75.28)	0.00	(55.21)	(55.21)	0.00	94.98
(14) Terminal Value	0.00	23.07	(24.72)	(24.72)	0.00	(18.13)	(18.13)	0.00	−1.65
(15) All Yrs.	0.00	93.33	(100.00)	(100.00)	0.00	(73.33)	(73.33)	0.00	93.33

(1)—selected judgmentally for illustration purposes
(2) = (3) – previous year's (3); for year 1, (3) – (1)
(3) = (5) ÷ 2.0, where 2.0 represents the target premium-to-surplus ratio
(4)—factor to discount from the end of the projected year to the beginning of year 1 at the hurdle rate (15%)
(5) = 200 for first projected year; subsequent years increased by the selected growth rate
(6) = (3) × selected investment yield of 4%; (6a) = (6) multiplied by the ratio of initial capital to (3); (6b) = (6) – (6a)
(7) = 5% of (5), selected so that earnings are less than the hurdle rate requirement
(8) = (3) – following year's (3); difference between required capital at the beginning of year and required capital at beginning of following year
(9) = (6) + (7) + (8)
(10) = (3) × –15% (hurdle rate); (10a) = (10) multiplied by the ratio of initial capital to (3); (10b) = (10) – (10a)
(11) = (6) + (10); (11a) = (6a) + (10a); (11b) = (6b) + (10b)
(12) = (1) + (6) + (7) + (10)
(13)—Forecast period value
(14)—Terminal value
(15) = (13) + (14) Value in perpetuity

EXHIBIT 4C

Basic Valuation Example
Economic Value Added (b) Model

Scenario Assumptions:
Total Earnings: hurdle rate not achieved
Annual Growth: 0%

Projected Year	(1) Initial Capital	(2) Required Growth in Capital	(3) Total Required Capital at Beginning of Year	(4) Discount Factor at 15%	(5) Premium Earned During Year	(6) After-tax Investment Income on Capital	(6a) After-tax Investment Income on Original Capital	(6b) After-tax Investment Income on Additional Capital	(7) After-tax Total Income From Insurance Operations
1	100	0	100	0.870	200	4	4	0	10
2		0	100	0.756	200	4	4	0	10
3		0	100	0.658	200	4	4	0	10
4		0	100	0.572	200	4	4	0	10
5		0	100	0.497	200	4	4	0	10
6		0	100	0.432	200	4	4	0	10
7		0	100	0.376	200	4	4	0	10
8		0	100	0.327	200	4	4	0	10
9		0	100	0.284	200	4	4	0	10
10		0	100	0.247	200	4	4	0	10
Discounted Totals									
(13) Yrs. 1–10	100					20.08	20.08	0.00	50.19
(14) Terminal Value	0					6.59	6.59	0.00	16.48
(15) All Yrs.	100					26.67	26.67	0.00	66.67

EXHIBIT 4C

Continued

Projected Year	(8) Required Reinv.	(9) Available After-tax Net Income	(10) Hurdle Required Return on Capital	(10a) Hurdle Required Return on Original Capital	(10b) Hurdle Required Return on Additional Capital	(11) Total Cost of Capital	(11a) Cost of Original Capital	(11b) Cost of Additional Capital	(12) Indicated Value
1	0	14	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00	
2	0	14	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00	
3	0	14	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00	
4	0	14	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00	
5	0	14	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00	
6	0	14	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00	
7	0	14	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00	
8	0	14	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00	
9	0	14	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00	
10	0	14	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00	
Discounted Totals									
(13) Yrs. 1–10	0.00	70.26	(75.28)	(75.28)	0.00	(55.21)	(55.21)	0.00	94.98
(14) Terminal Value	0.00	23.07	(24.72)	(24.72)	0.00	(18.13)	(18.13)	0.00	−1.65
(15) All Yrs.	0.00	93.33	(100.00)	(100.00)	0.00	(73.33)	(73.33)	0.00	93.33

EXHIBIT 4C

Continued

(1)—selected judgmentally for illustration purposes
(2) = (3) – previous year's (3); for year 1, (3) – (1)
(3) = (5) ÷ 2.0, where 2.0 represents the target premium-to-surplus ratio
(4)—factor to discount from the end of the projected year to the beginning of year 1 at the hurdle rate (15%)
(5) = 200 for first projected year; subsequent years increased by the selected growth rate
(6) = (3) × selected investment yield of 4%; (6a) = (6) multiplied by the ratio of initial capital to (3); (6b) = (6) – (6a)
(7) = 5% of (5), selected so that earnings are less than the hurdle rate requirement
(8) = (3) – following year's (3); difference between required capital at the beginning of year and required capital at beginning of following year
(9) = (6) + (7) + (8)
(10) = (3) × –15% (hurdle rate); (10a) = (10) multiplied by the ratio of initial capital to (3); (10b) = (10) – (10a)
(11) = (6) + (10); (11a) = (6a) + (10a); (11b) = (6b) + (10b)
(12) = (1) + (7) + (11); EVA (b) reduces the cost of capital component to reflect investment income earned on capital
(13)—Forecast period value
(14)—Terminal value
(15) = (13) + (14)Value in perpetuity

EXHIBIT 5A

BASIC VALUATION EXAMPLE
DISCOUNTED CASH FLOW MODEL

Scenario Assumptions:
Total Earnings: hurdle rate more than achieved
Annual Growth: 3%

Projected Year	(1) Initial Capital	(2) Required Growth in Capital	(3) Total Required Capital at Beginning of Year	(4) Discount Factor at 15%	(5) Premium Earned During Year	(6) After-tax Investment Income on Capital	(6a) After-tax Investment Income on Original Capital	(6b) After-tax Investment Income on Additional Capital	(7) After-tax Total Income From Insurance Operations	(8) Required Reinv.	(9) Available After-tax Net Income	(10) Indicated Value
1	100	0	100	0.870	200	4	4	0	12	(3)	13	
2		3	103	0.756	206	4.12	4	0.12	12.36	(3.09)	13.39	
3		3.09	106.09	0.658	212.18	4.24	4	0.24	12.73	(3.18)	13.79	
4		3.18	109.27	0.572	218.55	4.37	4	0.37	13.11	(3.28)	14.21	
5		3.28	112.55	0.497	225.10	4.50	4	0.50	13.51	(3.38)	14.63	
6		3.38	115.93	0.432	231.85	4.64	4	0.64	13.91	(3.48)	15.07	
7		3.48	119.41	0.376	238.81	4.78	4	0.78	14.33	(3.58)	15.52	
8		3.58	122.99	0.327	245.97	4.92	4	0.92	14.76	(3.69)	15.99	
9		3.69	126.68	0.284	253.35	5.07	4	1.07	15.20	(3.80)	16.47	
10		3.80	130.48	0.247	260.95	5.22	4	1.22	15.66	(3.91)	16.96	
Discounted Totals												
(11) Yrs. 1–10	100					22.26	20.08	2.19	66.78	(16.70)	72.35	72.35
(12) Terminal Value	0					11.07	6.59	4.48	33.22	(8.30)	35.99	35.99
(13) All Yrs.	100					33.33	26.67	6.67	100.00	(25.00)	108.33	108.33

EXHIBIT 5A

Continued

(1)—selected judgmentally for illustration purposes
(2) = (3) – previous year's (3); for year 1, (3) – (1)
(3) = (5) ÷ 2.0, where 2.0 represents the target premium-to-surplus ratio
(4)—factor to discount from the end of the projected year to the beginning of year 1 at the hurdle rate (15%)
(5) = 200 for first projected year; subsequent years increased by the selected growth rate
(6) = (3) × selected investment yield of 4%; (6a) = (6) multiplied by the ratio of initial capital to (3); (6b) = (6) – (6a)
(7) = 6% of (5), selected so that earnings exceed the hurdle rate requirement
(8) = (3) – following year's (3); difference between required capital at the beginning of year and required capital at beginning of following year
(9) = (6) + (7) + (8)
(10) = (9)
(11) = Explicit forecast period value
(12) = Terminal value
(13) = (11) + (12) = Value in perpetuity

EXHIBIT 5B

Basic Valuation Example
Economic Value Added (a) Model

Scenario Assumptions:
Total Earnings: hurdle rate more than achieved
Annual Growth: 3%

Projected Year	(1) Initial Capital	(2) Required Growth in Capital	(3) Total Required Capital at Beginning of Year	(4) Discount Factor at 15%	(5) Premium Earned During Year	(6) After-tax Investment Income on Capital	(6a) After-tax Investment Income on Original Capital	(6b) After-tax Investment Income on Additional Capital	(7) After-tax Total Income From Insurance Operations
1	100	0	100	0.870	200	4	4	0	12
2		3	103	0.756	206	4.12	4	0.12	12.36
3		3.09	106.09	0.658	212.18	4.24	4	0.24	12.73
4		3.18	109.27	0.572	218.55	4.37	4	0.37	13.11
5		3.28	112.55	0.497	225.10	4.50	4	0.50	13.51
6		3.38	115.93	0.432	231.85	4.64	4	0.64	13.91
7		3.48	119.41	0.376	238.81	4.78	4	0.78	14.33
8		3.58	122.99	0.327	245.97	4.92	4	0.92	14.76
9		3.69	126.68	0.284	253.35	5.07	4	1.07	15.20
10		3.80	130.48	0.247	260.95	5.22	4	1.22	15.66
Discounted Totals									
(13) Yrs. 1–10	100					22.26	20.08	2.19	66.78
(14) Terminal Value	0					11.07	6.59	4.48	33.22
(15) All Yrs.	100					33.33	26.67	6.67	100.00

EXHIBIT 5B

Continued

Projected Year	(8) Required Reinv.	(9) Available After-tax Net Income	(10) Hurdle Required Return on Capital	(10a) Hurdle Required Return on Original Capital	(10b) Hurdle Required Return on Additional Capital	(11) Total Cost of Capital	(11a) Cost of Original Capital	(11b) Cost of Additional Capital	(12) Indicated Value
1	(3)	13	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00	
2	(3.09)	13.39	(15.45)	(15.00)	(0.45)	(11.33)	(11.00)	(0.33)	
3	(3.18)	13.79	(15.91)	(15.00)	(0.91)	(11.67)	(11.00)	(0.67)	
4	(3.28)	14.21	(16.39)	(15.00)	(1.39)	(12.02)	(11.00)	(1.02)	
5	(3.38)	14.63	(16.88)	(15.00)	(1.88)	(12.38)	(11.00)	(1.38)	
6	(3.48)	15.07	(17.39)	(15.00)	(2.39)	(12.75)	(11.00)	(1.75)	
7	(3.58)	15.52	(17.91)	(15.00)	(2.91)	(13.13)	(11.00)	(2.13)	
8	(3.69)	15.99	(18.45)	(15.00)	(3.45)	(13.53)	(11.00)	(2.53)	
9	(3.80)	16.47	(19.00)	(15.00)	(4.00)	(13.93)	(11.00)	(2.93)	
10	(3.91)	16.96	(19.57)	(15.00)	(4.57)	(14.35)	(11.00)	(3.35)	
Discounted Totals									
(13) Yrs. 1–10	(16.70)	72.35	(83.48)	(75.28)	(8.19)	(61.22)	(55.21)	(6.01)	105.57
(14) Terminal Value	(8.30)	35.99	(41.52)	(24.72)	(16.81)	(30.45)	(18.13)	(12.32)	2.77
(15) All Yrs.	(25.00)	108.33	(125.00)	(100.00)	(25.00)	(91.67)	(73.33)	(18.33)	108.33

(1)—selected judgmentally for illustration purposes
(2) = (3) – previous year's (3); for year 1, (3) – (1)
(3) = (5) ÷ 2.0, where 2.0 represents the target premium-to-surplus ratio
(4)—factor to discount from the end of the projected year to the beginning of year 1 at the hurdle rate (15%)
(5) = 200 for first projected year; subsequent years increased by the selected growth rate
(6) = (3) × selected investment yield of 4%; (6a) = (6) multiplied by the ratio of initial capital to (3); (6b) = (6) – (6*a*)
(7) = 6% of (5), selected so that earnings exceed the hurdle rate requirement
(8) = (3) – following year's (3); difference between required capital at the beginning of year and required capital at beginning of following year
(9) = (6) + (7) + (8)
(10) = (3) × –15% (hurdle rate); (10a) = (10) multiplied by the ratio of initial capital to (3); (10b) = (10) – (10a)
(11) = (6) + (10); (11a) = (6a) + (10a); (11b) = (6b) + (10b)
(12) = (1) + [(6) + (7)] + (10); EVA (a) does not reduce the cost of capital to reflect investment income earned on capital
(13)—Forecast period value
(14)—Terminal value
(15) = (13) + (14) = Value in perpetuity

EXHIBIT 5C

Basic Valuation Example
Economic Value Added (b) Model

Scenario Assumptions:
Total Earnings: hurdle rate more than achieved
Annual Growth: 3%

Projected Year	(1) Initial Capital	(2) Required Growth in Capital	(3) Total Required Capital at Beginning of Year	(4) Discount Factor at 15%	(5) Premium Earned During Year	(6) After-tax Investment Income on Capital	(6a) After-tax Investment Income on Original Capital	(6b) After-tax Investment Income on Additional Capital	(7) After-tax Total Income From Insurance Operations
1	100	0	100	0.870	200	4	4	0	12
2		3	103	0.756	206	4.12	4	0.12	12.36
3		3.09	106.09	0.658	212.18	4.24	4	0.24	12.73
4		3.18	109.27	0.572	218.55	4.37	4	0.37	13.11
5		3.28	112.55	0.497	225.10	4.50	4	0.50	13.51
6		3.38	115.93	0.432	231.85	4.64	4	0.64	13.91
7		3.48	119.41	0.376	238.81	4.78	4	0.78	14.33
8		3.58	122.99	0.327	245.97	4.92	4	0.92	14.76
9		3.69	126.68	0.284	253.35	5.07	4	1.07	15.20
10		3.80	130.48	0.247	260.95	5.22	4	1.22	15.66
Discounted Totals									
(13) Yrs. 1–10	100					22.26	20.08	2.19	66.78
(14) Terminal Value	0					11.07	6.59	4.48	33.22
(15) All Yrs.	100					33.33	26.67	6.67	100.00

EXHIBIT 5C

Continued

Projected Year	(8) Required Reinv.	(9) Available After-tax Net Income	(10) Hurdle Required Return on Capital	(10a) Hurdle Required Return on Original Capital	(10b) Hurdle Required Return on Additional Capital	(11) Total Cost of Capital	(11a) Cost of Original Capital	(11b) Cost of Additional Capital	(12) Indicated Value
1	(3)	13	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00	
2	(3.09)	13.39	(15.45)	(15.00)	(0.45)	(11.33)	(11.00)	(0.33)	
3	(3.18)	13.79	(15.91)	(15.00)	(0.91)	(11.67)	(11.00)	(0.67)	
4	(3.28)	14.21	(16.39)	(15.00)	(1.39)	(12.02)	(11.00)	(1.02)	
5	(3.38)	14.63	(16.88)	(15.00)	(1.88)	(12.38)	(11.00)	(1.38)	
6	(3.48)	15.07	(17.39)	(15.00)	(2.39)	(12.75)	(11.00)	(1.75)	
7	(3.58)	15.52	(17.91)	(15.00)	(2.91)	(13.13)	(11.00)	(2.13)	
8	(3.69)	15.99	(18.45)	(15.00)	(3.45)	(13.53)	(11.00)	(2.53)	
9	(3.80)	16.47	(19.00)	(15.00)	(4.00)	(13.93)	(11.00)	(2.93)	
10	(3.91)	16.96	(19.57)	(15.00)	(4.57)	(14.35)	(11.00)	(3.35)	
Discounted Totals									
(13) Yrs. 1–10	(16.70)	72.35	(83.48)	(75.28)	(8.19)	(61.22)	(55.21)	(6.01)	105.57
(14) Terminal Value	(8.30)	35.99	(41.52)	(24.72)	(16.81)	(30.45)	(18.13)	(12.32)	2.77
(15) All Yrs.	(25.00)	108.33	(125.00)	(100.00)	(25.00)	(91.67)	(73.33)	(18.33)	108.33

EXHIBIT 5C

Continued

(1)—selected judgmentally for illustration purposes
(2) = (3) – previous year's (3); for year 1, (3) – (1)
(3) = (5) ÷ 2.0, where 2.0 represents the target premium-to-surplus ratio
(4)—factor to discount from the end of the projected year to the beginning of year 1 at the hurdle rate (15%)
(5) = 200 for first projected year; subsequent years increased by the selected growth rate
(6) = (3) × selected investment yield of 4%; (6a) = (6) multiplied by the ratio of initial capital to (3); (6b) = (6) – (6a)
(7) = 6% of (5), selected so that earnings exceed the hurdle rate requirement
(8) = (3) – following year's (3); difference between required capital at the beginning of year and required capital at beginning of following year
(9) = (6) + (7) + (8)
(10) = (3) × –15% (hurdle rate); (10a) = (10) multiplied by the ratio of initial capital to (3); (10b) = (10) – (10a)
(11) = (6) + (10); (11a) = (6a) + (10a); (11b) = (6b) + (10b)
(12) = (1) + (7) + (11); EVA (b) reduces the cost of capital component to reflect investment income earned on capital
(13)—Forecast period value
(14)—Terminal value
(15) = (13) + (14) = Value in perpetuity

EXHIBIT 6A

Basic Valuation Example
Discounted Cash Flow Model

Scenario Assumptions:
Total Earnings: hurdle rate not achieved
Annual Growth: 3%

Projected Year	(1) Initial Capital	(2) Required Growth in Capital	(3) Total Required Capital at Beginning of Year	(4) Discount Factor at 15%	(5) Premium Earned During Year	(6) After-tax Investment Income on Capital	(6a) After-tax Investment Income on Original Capital	(6b) After-tax Investment Income on Additional Capital	(7) After-tax Total Income From Insurance Operations	(8) Required Reinv.	(9) Available After-tax Net Income	(10) Indicated Value
1	100	0	100	0.870	200	4	4	0	10	(3)	11	
2		3	103	0.756	206	4.12	4	0.12	10.3	(3.09)	11.33	
3		3.09	106.09	0.658	212.18	4.24	4	0.24	10.61	(3.18)	11.67	
4		3.18	109.27	0.572	218.55	4.37	4	0.37	10.93	(3.28)	12.02	
5		3.28	112.55	0.497	225.10	4.50	4	0.50	11.26	(3.38)	12.38	
6		3.38	115.93	0.432	231.85	4.64	4	0.64	11.59	(3.48)	12.75	
7		3.48	119.41	0.376	238.81	4.78	4	0.78	11.94	(3.58)	13.13	
8		3.58	122.99	0.327	245.97	4.92	4	0.92	12.30	(3.69)	13.53	
9		3.69	126.68	0.284	253.35	5.07	4	1.07	12.67	(3.80)	13.93	
10		3.80	130.48	0.247	260.95	5.22	4	1.22	13.05	(3.91)	14.35	
Discounted Totals												
(11) Yrs. 1–10	100					22.26	20.08	2.19	55.65	(16.70)	61.22	61.22
(12) Terminal Value	0					11.07	6.59	4.48	27.68	(8.30)	30.45	30.45
(13) All Yrs.	100					33.33	26.67	6.67	83.33	(25.00)	91.67	91.67

EXHIBIT 6A

Continued

(1)—selected judgmentally for illustration purposes
(2) = (3) – previous year's (3); for year 1, (3) – (1)
(3) = (5) ÷ 2.0, where 2.0 represents the target premium-to-surplus ratio
(4)—factor to discount from the end of the projected year to the beginning of year 1 at the hurdle rate (15%)
(5) = 200 for first projected year; subsequent years increased by the selected growth rate
(6) = (3) × selected investment yield of 4%; (6a) = (6) multiplied by the ratio of initial capital to (3); (6b) = (6) – (6a)
(7) = 5% of (5), selected to be lower than the hurdle rate
(8) = (3) – following year's (3); difference between required capital at the beginning of year and required capital at beginning of following year
(9) = (6) + (7) + (8)
(10) = (9)
(11) = Explicit forecast period value
(12) = Terminal value
(13) = (11) + (12) = Value in perpetuity

EXHIBIT 6B

Basic Valuation Example
Economic Value Added (a) Model

Scenario Assumptions:
Total Earnings: hurdle rate not achieved
Annual Growth: 3%

Projected Year	(1) Initial Capital	(2) Required Growth in Capital	(3) Total Required Capital at Beginning of Year	(4) Discount Factor at 15%	(5) Premium Earned During Year	(6) After-tax Investment Income on Capital	(6a) After-tax Investment Income on Original Capital	(6b) After-tax Investment Income on Additional Capital	(7) After-tax Total Income From Insurance Operations
1	100	0	100	0.870	200	4	4	0	10
2		3	103	0.756	206	4.12	4	0.12	10.3
3		3.09	106.09	0.658	212.18	4.24	4	0.24	10.61
4		3.18	109.27	0.572	218.55	4.37	4	0.37	10.93
5		3.28	112.55	0.497	225.10	4.50	4	0.50	11.26
6		3.38	115.93	0.432	231.85	4.64	4	0.64	11.59
7		3.48	119.41	0.376	238.81	4.78	4	0.78	11.94
8		3.58	122.99	0.327	245.97	4.92	4	0.92	12.30
9		3.69	126.68	0.284	253.35	5.07	4	1.07	12.67
10		3.80	130.48	0.247	260.95	5.22	4	1.22	13.05
Discounted Totals									
(13) Yrs. 1–10	100					22.26	20.08	2.19	55.65
(14) Terminal Value	0					11.07	6.59	4.48	27.68
(15) All Yrs.	100					33.33	26.67	6.67	83.33

EXHIBIT 6B

Continued

Projected Year	(8) Required Reinv.	(9) Available After-tax Net Income	(10) Hurdle Required Return on Capital	(10a) Hurdle Required Return on Original Capital	(10b) Hurdle Required Return on Additional Capital	(11) Total Cost of Capital	(11a) Cost of Original Capital	(11b) Cost of Additional Capital	(12) Indicated Value
1	(3)	11	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00	
2	(3.09)	11.33	(15.45)	(15.00)	(0.45)	(11.33)	(11.00)	(0.33)	
3	(3.18)	11.67	(15.91)	(15.00)	(0.91)	(11.67)	(11.00)	(0.67)	
4	(3.28)	12.02	(16.39)	(15.00)	(1.39)	(12.02)	(11.00)	(1.02)	
5	(3.38)	12.38	(16.88)	(15.00)	(1.88)	(12.38)	(11.00)	(1.38)	
6	(3.48)	12.75	(17.39)	(15.00)	(2.39)	(12.75)	(11.00)	(1.75)	
7	(3.58)	13.13	(17.91)	(15.00)	(2.91)	(13.13)	(11.00)	(2.13)	
8	(3.69)	13.53	(18.45)	(15.00)	(3.45)	(13.53)	(11.00)	(2.53)	
9	(3.80)	13.93	(19.00)	(15.00)	(4.00)	(13.93)	(11.00)	(2.93)	
10	(3.91)	14.35	(19.57)	(15.00)	(4.57)	(14.35)	(11.00)	(3.35)	
Discounted Totals									
(13) Yrs. 1–10	(16.70)	61.22	(83.48)	(75.28)	(8.19)	(61.22)	(55.21)	(6.01)	94.43
(14) Terminal Value	(8.30)	30.45	(41.52)	(24.72)	(16.81)	(30.45)	(18.13)	(12.32)	-2.77
(15) All Yrs.	(25.00)	91.67	(125.00)	(100.00)	(25.00)	(91.67)	(73.33)	(18.33)	91.67

(1)—selected judgmentally for illustration purposes
(2) = (3) – previous year's (3); for year 1, (3) – (1)
(3) = (5) ÷ 2.0, where 2.0 represents the target premium-to-surplus ratio
(4)—factor to discount from the end of the projected year to the beginning of year 1 at the hurdle rate (15%)
(5) = 200 for first projected year; subsequent years increased by the selected growth rate
(6) = (3) × selected investment yield of 4%; (6a) = (6) multiplied by the ratio of initial capital to (3); (6b) = (6) – (6a)
(7) = 5% of (5), selected so that earnings are less than the hurdle rate requirement
(8) = (3) – following year's (3); difference between required capital at the beginning of year and required capital at beginning of following year
(9) = (6) + (7) + (8)
(10) = (3) × −15% (hurdle rate); (10a) = (10) multiplied by the ratio of initial capital to (3); (10b) = (10) – (10a)
(11) = (6) + (10); (11a) = (6a) + (10a); (11b) = (6b) + (10b)
(12) = (1) + [(6) + (7)] + (10); EVA (a) does not reduce the cost of capital to reflect investment income earned on capital
(13)—Forecast period value
(14)—Terminal value
(15) = (13) + (14) = Value in perpetuity

EXHIBIT 6C

Basic Valuation Example
Economic Value Added (b) Model

Scenario Assumptions:
Total Earnings: hurdle rate not achieved
Annual Growth: 3%

Projected Year	(1) Initial Capital	(2) Required Growth in Capital	(3) Total Required Capital at Beginning of Year	(4) Discount Factor at 15%	(5) Premium Earned During Year	(6) After-tax Investment Income on Capital	(6a) After-tax Investment Income on Original Capital	(6b) After-tax Investment Income on Additional Capital	(7) After-tax Total Income From Insurance Operations
1	100	0	100	0.870	200	4	4	0	10
2		3	103	0.756	206	4.12	4	0.12	10.3
3		3.09	106.09	0.658	212.18	4.24	4	0.24	10.61
4		3.18	109.27	0.572	218.55	4.37	4	0.37	10.93
5		3.28	112.55	0.497	225.10	4.50	4	0.50	11.26
6		3.38	115.93	0.432	231.85	4.64	4	0.64	11.59
7		3.48	119.41	0.376	238.81	4.78	4	0.78	11.94
8		3.58	122.99	0.327	245.97	4.92	4	0.92	12.30
9		3.69	126.68	0.284	253.35	5.07	4	1.07	12.67
10		3.80	130.48	0.247	260.95	5.22	4	1.22	13.05
Discounted Totals									
(13) Yrs. 1–10	100					22.26	20.08	2.19	55.65
(14) Terminal Value	0					11.07	6.59	4.48	27.68
(15) All Yrs.	100					33.33	26.67	6.67	83.33

EXHIBIT 6C

Continued

Projected Year	(8) Required Reinv.	(9) Available After-tax Net Income	(10) Hurdle Required Return on Capital	(10a) Hurdle Required Return on Original Capital	(10b) Hurdle Required Return on Additional Capital	(11) Total Cost of Capital	(11a) Cost of Original Capital	(11b) Cost of Additional Capital	(12) Indicated Value
1	(3)	11	(15.00)	(15.00)	0.00	(11.00)	(11.00)	0.00	
2	(3.09)	11.33	(15.45)	(15.00)	(0.45)	(11.33)	(11.00)	(0.33)	
3	(3.18)	11.67	(15.91)	(15.00)	(0.91)	(11.67)	(11.00)	(0.67)	
4	(3.28)	12.02	(16.39)	(15.00)	(1.39)	(12.02)	(11.00)	(1.02)	
5	(3.38)	12.38	(16.88)	(15.00)	(1.88)	(12.38)	(11.00)	(1.38)	
6	(3.48)	12.75	(17.39)	(15.00)	(2.39)	(12.75)	(11.00)	(1.75)	
7	(3.58)	13.13	(17.91)	(15.00)	(2.91)	(13.13)	(11.00)	(2.13)	
8	(3.69)	13.53	(18.45)	(15.00)	(3.45)	(13.53)	(11.00)	(2.53)	
9	(3.80)	13.93	(19.00)	(15.00)	(4.00)	(13.93)	(11.00)	(2.93)	
10	(3.91)	14.35	(19.57)	(15.00)	(4.57)	(14.35)	(11.00)	(3.35)	
Discounted Totals									
(13) Yrs. 1–10	(16.70)	61.22	(83.48)	(75.28)	(8.19)	(61.22)	(55.21)	(6.01)	94.43
(14) Terminal Value	(8.30)	30.45	(41.52)	(24.72)	(16.81)	(30.45)	(18.13)	(12.32)	−2.77
(15) All Yrs.	(25.00)	91.67	(125.00)	(100.00)	(25.00)	(91.67)	(73.33)	(18.33)	91.67

EXHIBIT 6C

Continued

(1)—selected judgmentally for illustration purposes
(2) = (3) – previous year's (3); for year 1, (3) – (1)
(3) = (5) ÷ 2.0, where 2.0 represents the target premium-to-surplus ratio
(4)—factor to discount from the end of the projected year to the beginning of year 1 at the hurdle rate (15%)
(5) = 200 for first projected year; subsequent years increased by the selected growth rate
(6) = (3) × selected investment yield of 4%; (6a) = (6) multiplied by the ratio of initial capital to (3); (6b) = (6) – (6a)
(7) = 5% of (5), selected so that earnings are less than the hurdle rate requirement
(8) = (3) – following year's (3); difference between required capital at the beginning of year and required capital at beginning of following year
(9) = (6) + (7) + (8)
(10) = (3) × –15% (hurdle rate); (10a) = (10) multiplied by the ratio of initial capital to (3); (10b) = (10) – (10a)
(11) = (6) + (10); (11a) = (6a) + (10a); (11b) = (6b) + (10b)
(12) = (1) + (7) + (11); EVA (b) reduces the cost of capital component to reflect investment income earned on capital
(13)—Forecast period value
(14)—Terminal value
(15) = (13) + (14) Value in perpetuity

EXHIBIT 7
SHEET 1

PRIMARY STOCK INSURANCE COMPANY
VALUATION ESTIMATES AS OF DECEMBER 31, 2001

Establishing Starting Surplus											
(1) Booked Statutory Surplus @ 12/31/01	$45,000										
(2) Selected Starting Surplus for Future Income Projections (based on 12/31/01 RBC level)	$42,131										
(3) Income Recognized @ 12/31/01	$ 2,869										
Monitoring and Selecting Surplus						As of 12/31 of:					
	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
(4) Selected Surplus	$42,131	$46,506	$51,328	$55,362	$58,890	$62,150	$65,288	$68,388	$71,500	$74,652	$77,864
(5) Indicated Risk-Based Capital Company Action Level	21,069	23,253	25,664	27,681	29,445	31,075	32,644	34,194	35,750	37,326	38,932
(6) Surplus to RBC Ratio	200.0%	200.0%	200.0%	200.0%	200.0%	200.0%	200.0%	200.0%	200.0%	200.0%	200.0%
(7) Surplus Growth		4,375	9,197	13,231	16,759	20,019	23,157	26,257	29,369	32,521	35,733
(8) Net Written Premium-to-Surplus Ratio	1.99	1.88	1.77	1.71	1.67	1.64	1.63	1.62	1.61	1.60	1.60
(9) Loss and LAE Reserves-to-Surplus Ratio	2.54	2.71	2.74	2.75	2.75	2.75	2.74	2.74	2.73	2.73	2.72
Estimated Future Income							During:				
		2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
(10) Total Company Net Income		10,445	12,054	13,187	14,153	14,977	15,752	16,507	17,248	17,976	18,713
Hurdle Rate	@ 12/31/										
	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
(11) Selected Hurdle Rate		15.0%	15.0%	15.0%	15.0%	15.0%	15.0%	15.0%	15.0%	15.0%	15.0%
(12) Present Value Factor for 12/31/01 (assuming year-end income recognition)	1.000	0.870	0.756	0.658	0.572	0.497	0.432	0.376	0.327	0.284	0.247

EXHIBIT 7
SHEET 1
Continued

Expected Annual Perpetual Growth Rate of Capital & Income After 2011	2.0%													

Application of DCF Method	@ 12/31/ 2001	2002	2003	2004	2005	During: 2006	2007	2008	2009	2010	2011	Total '01 to '11	Total '12 to ∞	All Years
(13) Total Company Net Income	2,869	10,445	12,054	13,187	14,153	14,977	15,752	16,507	17,248	17,976	18,713			
(13pv) PV of Total Company Net Income	2,869	9,083	9,115	8,671	8,092	7,446	6,810	6,206	5,638	5,110	4,626	73,665	36,293	109,957
(14) Reinvestment Cost to Grow Company		4,375	4,822	4,034	3,528	3,260	3,138	3,100	3,112	3,152	3,212			
(14pv) PV of Reinv. Cost to Grow Company		3,804	3,646	2,652	2,017	1,621	1,357	1,165	1,017	896	794	18,970	2,961	21,931
(15) Free Cash Flow	2,869	6,070	7,232	9,153	10,625	11,717	12,614	13,407	14,136	14,824	15,501			
(15pv) PV of Free Cash Flow	2,869	5,278	5,468	6,018	6,075	5,825	5,453	5,040	4,621	4,214	3,832	54,694	33,332	88,026

Application of EVA Method	@ 12/31/ 2001	2002	2003	2004	2005	During: 2006	2007	2008	2009	2010	2011	Total '01 to '11	Total '12 to ∞	All Years
(16) Statutory Surplus @ 12/31/01	42,131													
(17) Total Company Net Income	2,869	10,445	12,054	13,187	14,153	14,977	15,752	16,507	17,248	17,976	18,713			
(17pv) PV of Total Company Net Income	2,869	9,083	9,115	8,671	8,092	7,446	6,810	6,206	5,638	5,110	4,626	73,665	36,293	109,957
(18) Cost of Capital		6,320	6,976	7,699	8,304	8,834	9,323	9,793	10,258	10,725	11,198			
(18pv) PV of Cost of Capital		5,495	5,275	5,062	4,748	4,392	4,030	3,682	3,353	3,049	2,768	41,854	22,208	64,062
(19) Cost of Starting Capital		6,320	6,320	6,320	6,320	6,320	6,320	6,320	6,320	6,320	6,320			
(19pv) PV of Cost of Starting Capital		5,495	4,779	4,155	3,613	3,142	2,732	2,376	2,066	1,796	1,562	31,717	10,414	42,131
(20) Cost of Growth Capital			656	1,380	1,985	2,514	3,003	3,474	3,939	4,405	4,878			
(20pv) PV of Cost of Growth Capital			496	907	1,135	1,250	1,298	1,306	1,288	1,252	1,206	10,137	11,794	21,931
(21) Excess Returns in Year	2,869	4,125	5,078	5,488	5,849	6,144	6,430	6,714	6,990	7,251	7,515			
(21pv) PV of Excess Returns in Year	2,869	3,587	3,840	3,608	3,344	3,054	2,780	2,524	2,285	2,061	1,858	31,810	14,085	45,895
(22) Total Indicated Value														88,026

EXHIBIT 7
SHEET 2

PRIMARY STOCK INSURANCE COMPANY
CALCULATION NOTES FOR VALUATION ESTIMATES AS OF DECEMBER 31, 2001

(1) from Primary Stock Insurance Company's 12/31/01 booked balance sheet
(2) selected starting surplus for future income projections based on the selected 200.0% surplus to indicated RBC ratio
(3) = (1) – (2)
(4) selected surplus based on the selected 200.0% surplus to indicated RBC ratio
(5) calculated RBC at the Company Action Level—based on Exhibit 12 for 2002 and subsequent.
(6) selection of 200.0% surplus to RBC ratio for determining required surplus at each year end
(7) cumulative increase in (4) from starting surplus, (4) – (2)
(8) NPW for all lines ÷ (4)
(9) net loss and LAE reserves for all lines ÷ (4)
(10) from Exhibit 8, line (11)
(11) is selected hurdle rate of 15.0% used for determining Cost of Capital in EVA method and the present value of future earnings and value added
(12) = 1.000 at 12/31/01; for future years = (1.0 + 15.0%) raised to (2001 – year) exponent
(13) = (10)
(13pv) = (13) × (12) for each year; Total '01 to '11 is the total of the estimates by year; Total '12 to ∞ = $(13)_{2011}$ × (1 + Growth Rate) ÷ (Hurdle Rate – Growth Rate) × $(12)_{2011}$; All Years = Total '01 to '11 + Total '12 to ∞
(14) annual change in (4)
(14pv) = (14) × (12) for each year; Total '01 to '11 is the total of the estimates by year; Total '12 to ∞ = $(4)_{2011}$ × Growth Rate ÷ (Hurdle Rate – Growth Rate) × $(12)_{2011}$ All Years = Total '01 to '11 + Total '12 to ∞

EXHIBIT 7
SHEET 2
Continued

(15) = (13) – (14)
(15pv) = (13pv) – (14pv)
(16) = (2)
(17) = (10)
(17pv) = (17) × (12) for each year; Total '01 to '11 is the total of the estimates by year; Total '12 to ∞ = $(17)_{2011}$ × (1 + Growth Rate) ÷ (Hurdle Rate – Growth Rate) × $(12)_{2011}$; All Years = Total '01 to '11 + Total '12 to ∞
(18) = (19) + (20)
(18pv) = (19pv) + (20pv)
(19) = (2) × (11)
(19pv) = (19) × (12) for each year; Total '01 to '11 is the total of the estimates by year; Total '12 to ∞ = (16) × $(12)_{2011}$; All Years = Total '01 to '11 + Total '12 to ∞
(20) = $(7)_{\text{prior year}}$ × (11)
(20pv) = (20) × (12) for each year; Total '01 to '11 is the total of the estimates by year; Total '12 to ∞ = [$(4)_{2011}$ × Hurdle Rate ÷ (Hurdle Rate – Growth Rate) – (16)] × $(12)_{2011}$; All Years = Total '01 to '11 + Total '12 to ∞
(21) = (17) – (18)
(21pv) = (17pv) – (18pv)
(22) = (16) + $(21\text{pv})_{\text{All Years}}$

EXHIBIT 8

PRIMARY STOCK INSURANCE COMPANY

	Actual 2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
Changes in Statutory Surplus											
(1) Net Income	$8,531	$6,933	$8,111	$8,909	$9,592	$10,160	$10,697	$11,218	$11,726	$12,218	$12,715
(2) Changes in Unrealized Cap. Gains	0	3,512	3,943	4,278	4,563	4,816	5,055	5,288	5,522	5,758	5,998
(3) Changes in Nonadmitted Assets	0	0	0	0	0	0	0	0	0	0	0
(4) Capital Paid In	0	0	0	0	0	0	0	0	0	0	0
(5) Increase in Surplus Notes	0	0	0	0	0	0	0	0	0	0	0
(6) Principal Repayment	0	0	0	0	0	0	0	0	0	0	0
(7) Other Surplus Adjustments	0	0	0	0	0	0	0	0	0	0	0
(8) Contributions to Meet P/S Target		0	0	0	0	0	0	0	0	0	0
(9) Contributions to Meet R/S Target		0	0	0	0	0	0	0	0	0	0
(10) Change in Statutory Reserve	0	0	0	0	(2)	1	0	1	0	0	0
(11) Subtotal: Company Income	8,531	10,445	12,054	13,187	14,153	14,977	15,752	16,507	17,248	17,976	18,713
(12) Dividends to Stockholders	0	(6,070)	(7,232)	(9,153)	(10,625)	(11,717)	(12,614)	(13,407)	(14,136)	(14,824)	(15,501)
(12) Total Surplus Adjustments	$8,531	$4,375	$4,822	$4,034	$3,528	$3,260	$3,138	$3,100	$3,112	$3,152	$3,212

Calculation Notes:
(1) = After-tax net income from the statutory income statement (Exhibit 9)
(3), (4), (5), (6), (7) are set to $0.
(11) = sum of lines (1) through (10)
(12) is the maximum dividend that satisfies required surplus based on risk-based capital multiple of 2.0
(13) = (11) + (12)

EXHIBIT 9

PRIMARY STOCK INSURANCE COMPANY
STATUTORY INCOME STATEMENT

	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
(1) Net Earned Premium	$82,385	$85,680	$89,108	$92,671	$96,379	$100,234	$104,244	$108,412	$112,749	$117,259	$121,952
(2) Net Incurred Losses & LAE	63,988	66,547	69,208	71,974	74,856	77,850	80,963	84,200	87,571	91,074	94,719
(3) Underwriting Expenses	18,022	18,743	19,492	20,271	21,083	21,927	22,803	23,716	24,664	25,650	26,676
(4) Underwriting Income	$375	$390	$408	$426	$440	$457	$478	$496	$514	$535	$557
(5) Investment Income	11,000	10,410	11,685	12,683	13,525	14,277	14,984	15,677	16,368	17,068	17,780
(6) Other Income	0	0	0	0	0	0	0	0	0	0	0
(7) Policyholder Dividends	0	0	0	0	0	0	0	0	0	0	0
(8) Pre-Tax Operating Income	$11,375	$10,800	$12,093	$13,109	$13,965	$14,734	$15,462	$16,173	$16,882	$17,603	$18,337
(9) Federal Income Tax	2,844	3,867	3,982	4,200	4,373	4,574	4,765	4,955	5,156	5,385	5,622
(10) Net Income	$8,531	$6,933	$8,111	$8,909	$9,592	$10,160	$10,697	$11,218	$11,726	$12,218	$12,715

Calculation Notes:
(1) = sum of net earned premium for all lines of business
(2) = sum of net incurred losses and LAE for all lines of business
(3) = sum of underwriting expenses for all lines of business
(4) = (1) – (2) – (3)
(6) = set to $0 for the company
(7) = set to $0 for the company
(8) = (4) + (5) + (6) – (7)
(10) = (8) – (9)

EXHIBIT 10

Primary Stock Insurance Company
Summary Statutory Balance Sheet

	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
(1) Invested Assets	$185,216	$213,090	$233,984	$251,262	$266,218	$279,994	$293,292	$306,473	$319,754	$333,230	$346,972
(2) Other Assets	7,500	6,828	7,127	7,431	7,744	8,065	8,397	8,741	9,099	9,470	9,854
(3) Total Assets	$192,716	$219,918	$241,111	$258,693	$273,962	$288,059	$301,689	$315,214	$328,853	$342,700	$356,826
(4) Net Loss & LAE Reserves	$107,085	$126,175	$140,678	$152,257	$161,965	$170,680	$178,971	$187,110	$195,258	$203,474	$211,810
(5) Net Unearned Premium Reserve	42,000	43,680	45,426	47,244	49,134	51,100	53,143	55,269	57,479	59,778	62,169
(6) Other Liabilities	1,500	3,557	3,679	3,830	3,973	4,129	4,287	4,447	4,616	4,796	4,983
(7) Total Liabilities	$150,585	$173,412	$189,783	$203,331	$215,072	$225,909	$236,401	$246,826	$257,353	$268,048	$278,962
(8) Statutory Surplus	$42,131	$46,506	$51,328	$55,362	$58,890	$62,150	$65,288	$68,388	$71,500	$74,652	$77,864
(9) Total Liabilities & Surplus	$192,716	$219,918	$241,111	$258,693	$273,962	$288,059	$301,689	$315,214	$328,853	$342,700	$356,826

Calculation Notes:
(1) Exhibit 11, Line (1)
(2) = (Premium Receivable + Receivables from Reinsurers + Other Assets) from Exhibit 11
(3) = (1) + (2)
(4) Exhibit 11, Line (8)
(5) Exhibit 11, Line (9)
(6) = [(10) + (11) + (12) + (13) + (14)] from Exhibit 11
(7) = (4) + (5) + (6)
(8) = (3) − (7)
(9) = (7) + (8)

EXHIBIT 11
SHEET 1

PRIMARY STOCK INSURANCE COMPANY
DETAILED STATUTORY BALANCE SHEET

	Actual 2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
(1a) Taxable Bonds	$77,791	$89,496	$98,273	$105,529	$111,812	$117,596	$123,182	$128,718	$134,295	$139,956	$145,728
(1b) Non-taxable Bonds	44,452	51,142	56,156	60,303	63,892	67,199	70,390	73,554	76,741	79,975	83,273
(1c) Stocks—Preferred	1,852	2,131	2,340	2,513	2,662	2,800	2,933	3,065	3,198	3,332	3,470
(1d) Stocks—Common	46,304	53,273	58,496	62,816	66,555	69,999	73,323	76,618	79,939	83,308	86,743
(1e) Cash	9,261	10,655	11,699	12,563	13,311	14,000	14,665	15,324	15,988	16,662	17,349
(1f) Real Estate	1,852	2,131	2,340	2,513	2,662	2,800	2,933	3,065	3,198	3,332	3,470
(1g) Other Income Producing Assets	3,704	4,262	4,680	5,025	5,324	5,600	5,866	6,129	6,395	6,665	6,939
(1) Total Investments & Cash	$185,216	$213,090	$233,984	$251,262	$266,218	$279,994	$293,292	$306,473	$319,754	$333,230	$346,972
(2) Premium Receivable	$7,500	$6,500	$6,760	$7,030	$7,310	$7,602	$7,907	$8,222	$8,552	$8,894	$9,249
(3) Receivables from Reinsurers	0	328	367	401	434	463	490	519	547	576	605
(4) Other Assets	0	0	0	0	0	0	0	0	0	0	0
(5) Total Assets	$192,716	$219,918	$241,111	$258,693	$273,962	$288,059	$301,689	$315,214	$328,853	$342,700	$356,826
(6) Net Loss Reserve	$79,567	$98,763	$113,298	$124,727	$134,151	$142,471	$150,267	$157,814	$165,275	$172,717	$180,202
(7) Net LAE Reserve	27,518	27,412	27,380	27,530	27,812	28,208	28,703	29,296	29,983	30,757	31,608
(8) Net Loss & LAE Reserve	107,085	126,175	140,678	152,257	161,965	170,680	178,971	187,110	195,258	203,474	211,810
(9) Net Unearned Premium Reserve	42,000	43,680	45,426	47,244	49,134	51,100	53,143	55,269	57,479	59,778	62,169
(10) Expenses Payable	1,000	1,030	1,061	1,093	1,126	1,162	1,198	1,236	1,275	1,316	1,359
(11) Income Taxes Payable	0	967	996	1,050	1,093	1,143	1,191	1,238	1,288	1,345	1,404
(12) Dividends Declared and Unpaid											
(12a) Policyholders	0	0	0	0	0	0	0	0	0	0	0
(12b) Stockholders	0	0	0	0	0	0	0	0	0	0	0
(13) Balances due Reinsurers	500	1,560	1,622	1,687	1,754	1,824	1,898	1,973	2,053	2,135	2,220
(14) Other Liabilities	0	0	0	0	0	0	0	0	0	0	0

	Actual 2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
(15) Total Liabilities	$150,585	$173,412	$189,783	$203,331	$215,072	$225,909	$236,401	$246,826	$257,353	$268,048	$278,962
(16) Capital	$40,000	$40,000	$40,000	$40,000	$40,000	$40,000	$40,000	$40,000	$40,000	$40,000	$40,000
(17) Surplus Notes	0	0	0	0	0	0	0	0	0	0	0
(18) Unassigned Funds	2,131	6,506	11,328	15,362	18,890	22,150	25,288	28,388	31,500	34,652	37,864
(19) Policyholder Surplus	$42,131	$46,506	$51,328	$55,362	$58,890	$62,150	$65,288	$68,388	$71,500	$74,652	$77,864
(20) Total Liabilities and Surplus	$192,716	$219,918	$241,111	$258,693	$273,962	$288,059	$301,689	$315,214	$328,853	$342,700	$356,826

EXHIBIT 11
SHEET 2

PRIMARY STOCK INSURANCE COMPANY
CALCULATION NOTES FOR DETAILED STATUTORY BALANCE SHEET

(1a) = 42.0% of (1) Total Investments & Cash
(1b) = 24.0% of (1) Total Investments & Cash
(1c) = 1.0% of (1) Total Investments & Cash
(1d) = 25.0% of (1) Total Investments & Cash
(1e) = 5.0% of (1) Total Investments & Cash
(1f) = 1.0% of (1) Total Investments & Cash
(1g) = 2.0% of (1) Total Investments & Cash
(1) 2001 value is input, subsequent values = prior year value + net cash in* + changes in unrealized capital gains**
(2) 2001 value is input, subsequent values = prior year value + direct written premium for all lines – direct premium collected for all lines
(3) 2001 value is input, subsequent values = prior year value + ceded loss & LAE paid – ceded loss & LAE received
(4) value set to $0 for all years
(5) = (1) + (2) + (3) + (4)
(6), (7): For 2001, starting net loss and LAE reserve for three modeled lines of business. For 2002–2011, sum of (20) – (21) in Sheet 1 of Exhibits 18, 19, and 20
(8) = (6) + (7)
(9) 2001 value is input, subsequent values = prior year value + net written premium for all lines – net earned premium for all lines
(10) 2001 value is input, subsequent values = prior year value + (agents' commissions + other underwriting expenses + premium taxes) for all lines – underwriting expenses paid*
(11) 2001 value is input, subsequent values = prior year value + federal income tax – federal income tax paid*
(12) values set to $0 for all years
(13) 2001 value is input, subsequent values = prior year value + (ceded written premium – premium ceded + reinsurance commission – reinsurance commission paid) for all lines
(14) value set to $0 for all years
(15) = (8) + (9) + (10) + (11) + (12a) + (12b) + (13) + (14)
(16) 2001 value is input, subsequent values = prior year value + capital paid in**
(17) 2001 value is input, subsequent values = prior year value + Increase in Surplus Notes** – Principal Repayment**
(18) = (19) – (16) – (17)
(19) = (5) – (15)
(20) = (15) + (19)

Notes:
*Value is from Exhibit 13.
**Value is from Exhibit 8.

EXHIBIT 12

Primary Stock Insurance Company
Risk-Based Capital

	Factor	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
INVESTED ASSET RISK											
Bonds	0.3%	$140,638	$154,429	$165,832	$175,704	$184,795	$193,572	$202,272	$211,036	$219,931	$229,001
Common Stocks	15.0%	53,273	58,496	62,816	66,555	69,999	73,323	76,618	79,939	83,308	86,743
Preferred Stocks	2.3%	2,131	2,340	2,513	2,662	2,800	2,933	3,065	3,198	3,332	3,470
Cash	0.3%	10,655	11,699	12,563	13,311	14,000	14,665	15,324	15,988	16,662	17,349
Real Estate	10.0%	2,131	2,340	2,513	2,662	2,800	2,933	3,065	3,198	3,332	3,470
Short-Term Investments	0.3%	4,262	4,680	5,025	5,324	5,600	5,866	6,129	6,395	6,665	6,939
Fixed-Income RBC		$467	$512	$550	$583	$613	$642	$671	$700	$730	$760
Equity-Asset RBC		$8,253	$9,062	$9,731	$10,311	$10,844	$11,359	$11,870	$12,384	$12,906	$13,438
CREDIT RISK											
Reinsurance Ceded (excl. US affiliates, pools)	10.0%	$29,359	$32,092	$34,694	$37,198	$39,653	$42,090	$44,523	$46,959	$49,410	$51,880
All Other Receivables	1.0%	0	0	0	0	0	0	0	0	0	0
Credit RBC		$2,936	$3,209	$3,469	$3,720	$3,965	$4,209	$4,452	$4,696	$4,941	$5,188
PREMIUM RISK											
Total of By-Line RBC		$5,111	$5,537	$5,987	$6,465	$6,972	$7,510	$8,078	$8,681	$9,318	$9,994
Premium Concentration Factor		0.357	0.357	0.357	0.357	0.357	0.357	0.357	0.357	0.357	0.357
Premium RBC		$4,125	$4,469	$4,832	$5,218	$5,627	$6,061	$6,520	$7,006	$7,521	$8,066

EXHIBIT 12

Continued

	Factor	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
RESERVE RISK											
Total of By-Line RBC		$23,909	$26,565	$28,728	$30,561	$32,218	$33,795	$35,341	$36,886	$38,444	$40,025
Reserve Concentration Factor		0.430	0.418	0.412	0.410	0.410	0.410	0.410	0.410	0.410	0.410
Reserve RBC		$19,821	$21,927	$23,660	$25,152	$26,515	$27,813	$29,086	$30,357	$31,639	$32,941
GROWTH RISK											
Three-Year Average Growth		4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%
Net Written Premium	22.5%	$87,360	$90,854	$94,489	$98,269	$102,200	$106,287	$110,537	$114,960	$119,558	$124,343
Net Loss & LAE Reserves	45.0%	126,175	140,678	152,257	161,963	170,679	178,970	187,110	195,258	203,474	211,810
Growth RBC		$0	$0	$0	$0	$0	$0	$0	$0	$0	$0
INDICATED RBC (Upper end of Company Action Level)		$23,253	$25,664	$27,681	$29,445	$31,075	$32,644	$34,194	$35,750	$37,326	$38,932
ADJUSTED SURPLUS		$46,506	$51,328	$55,362	$58,890	$62,150	$65,288	$68,388	$71,500	$74,652	$77,864
Ratio to Authorized Control Level		400.00%	400.00%	400.00%	400.00%	400.00%	400.00%	400.00%	400.00%	400.00%	400.00%

EXHIBIT 13
SHEET 1

PRIMARY STOCK INSURANCE COMPANY
CASH FLOW FROM OPERATIONS

	Actual 2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
(1) Direct Premium Collected	$90,962	$94,600	$97,084	$100,968	$105,008	$109,208	$113,574	$118,118	$122,841	$127,756	$132,869
(2) Premium Ceded	4,981	5,180	6,428	6,684	6,952	7,230	7,518	7,820	8,132	8,458	8,796
(3) Net Premium Collected	85,981	89,420	90,656	94,284	98,056	101,978	106,056	110,298	114,709	119,298	124,073
(4) Reinsurance Commissions Paid	0	0	0	0	0	0	0	0	0	0	0
(5) Interest & Dividends	9,400	8,344	9,366	10,165	10,840	11,443	12,010	12,565	13,119	13,680	14,251
(6) Realized Capital Gains	1,600	2,066	2,319	2,518	2,685	2,834	2,974	3,112	3,249	3,388	3,529
(7) Capital Received	0	0	0	0	0	0	0	0	0	0	0
(8) Capital Contributions	0	0	0	0	0	0	0	0	0	0	0
(9) Increase in Surplus Notes	0	0	0	0	0	0	0	0	0	0	0
(10) Other Income	0	0	0	0	0	0	0	0	0	0	0
(11) Total Collected	$96,981	$99,830	$102,341	$106,967	$111,581	$116,255	$121,040	$125,975	$131,077	$136,366	$141,853
(12) Gross Losses Paid	$38,806	$40,358	$47,610	$53,360	$58,086	$61,983	$65,395	$68,639	$71,837	$75,083	$78,397
(13) Loss Recoveries Received	3,404	3,540	4,195	4,526	4,838	5,127	5,410	5,695	5,986	6,281	6,586
(14) Net Losses Paid	35,402	36,818	43,415	48,834	53,248	56,856	59,985	62,944	65,851	68,802	71,811
(15) Gross LAE Paid	10,603	11,027	11,496	11,849	12,269	12,707	13,169	13,652	14,158	14,692	15,256
(16) LAE Recoveries Received	0	60	167	254	334	400	455	507	558	607	655
(17) Net LAE Paid	10,603	10,967	11,329	11,595	11,935	12,307	12,714	13,145	13,600	14,085	14,601
(18) Non-admitted Assets Purchased		0	0	0	0	0	0	0	0	0	0
(19) Total Underwriting Expenses Paid	17,993	18,713	19,461	20,239	21,050	21,891	22,767	23,678	24,625	25,609	26,633
(20) Federal Income Tax Paid	2,133	2,900	3,953	4,146	4,330	4,524	4,717	4,908	5,106	5,328	5,563
(21) Stockholder Dividends Paid	0	6,070	7,232	9,153	10,625	11,717	12,614	13,407	14,136	14,824	15,501
(22) Policyholder Dividends Paid	0	0	0	0	0	0	0	0	0	0	0
(23) Principal Repayment	0	0	0	0	0	0	0	0	0	0	0
(24) Interest Expense	0	0	0	0	0	0	0	0	0	0	0
(25) Total Paid	$66,131	$75,468	$85,390	$93,967	$101,188	$107,295	$112,797	$118,082	$123,318	$128,648	$134,109
(26) Net Cash from Operations	$30,850	$24,362	$16,951	$13,000	$10,393	$8,960	$8,243	$7,893	$7,759	$7,718	$7,744
NET CHANGE IN CASH	$30,850	$1,394	$1,044	$864	$748	$689	$665	$659	$664	$674	$687

EXHIBIT 13
SHEET 2

CALCULATION NOTES FOR CASH FLOW FROM OPERATIONS

(1) = Direct Premium Collected for all lines
(2) = Premium Ceded for all lines
(3) = (1) – (2)
(4) = Reinsurance Commissions Paid for all lines
(7) value set to \$0 for all years
(8) value set to \$0 for all years
(9) value set to \$0 for all years
(10) value set to \$0 for all years
(11) = (3) + (4) + (5) + (6) + (7) + (8) + (9) + (10)
(12) = Gross Losses Paid for all lines
(13) = Loss Recoveries Received for all lines
(14) = (12) – (13)
(15) = Gross ALAE Paid for all lines + ULAE Paid for all lines
(16) = ALAE Recoveries Received for all lines
(17) = (15) – (16)
(18) value set to \$0 for all years
(19) Underwriting Expense Paid for all lines
(20) = 75% × Federal Income Tax + prior year Federal Income Tax Payable
(21) is the maximum dividend that satisfies required surplus based on risk-based capital multiple of 2.0
(22) value set to \$0 for all years
(25) = (14) + (17) + (18) + (19) + (20) + (21) + (22) + (23) + (24)
(26) = (11) – (25)

EXHIBIT 14

PRIMARY STOCK INSURANCE COMPANY
GAAP INCOME STATEMENT

	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
(1) Net Earned Premium	$82,385	$85,680	$89,108	$92,671	$96,379	$100,234	$104,244	$108,412	$112,749	$117,259	$121,952
(2) Net Incurred Losses & LAE	63,988	66,547	69,208	71,974	74,856	77,850	80,963	84,200	87,571	91,074	94,719
(3) Net Underwriting Expenses	17,693	18,401	19,214	19,983	20,782	21,615	22,480	23,377	24,311	25,286	26,296
(4) Underwriting Income	$704	$732	$686	$714	$741	$769	$801	$835	$867	$899	$937
(5) Investment Income	$11,000	$10,410	$11,685	$12,683	$13,525	$14,277	$14,984	$15,677	$16,368	$17,068	$17,780
(6) Other Income	0	0	0	0	0	0	0	0	0	0	0
(7) Policyholder Dividends Incurred	0	0	0	0	0	0	0	0	0	0	0
(8) Pre-Tax Operating Income	$11,704	$11,142	$12,371	$13,397	$14,266	$15,046	$15,785	$16,512	$17,235	$17,967	$18,717
(9) Federal Income Tax	2,926	3,101	3,452	3,736	3,977	4,193	4,399	4,601	4,802	5,006	5,215
(10) Net Income	$8,778	$8,041	$8,919	$9,661	$10,289	$10,853	$11,386	$11,911	$12,433	$12,961	$13,502

Calculation Notes:
(1) = sum of net earned premium for all lines of business
(2) = sum of net incurred losses and LAE for all lines of business
(3) = sum of GAAP underwriting expenses for all lines of business
(4) = (1) − (2) − (3)
(6) = set to $0 for the company
(7) = set to $0 for the company
(8) = (4) + (5) + (6) − (7)
(10) = (8) − (9)

EXHIBIT 15

PRIMARY STOCK INSURANCE COMPANY
SUMMARY GAAP BALANCE SHEET

	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
(1) Invested Assets	$185,216	$213,090	$233,984	$251,262	$266,218	$279,994	$293,292	$306,473	$319,754	$333,230	$346,972
(2) Ceded Loss & LAE Reserves	26,134	29,030	31,725	34,293	36,763	39,190	41,600	44,004	46,412	48,834	51,275
(3) Ceded Unearned Premium Reserves	3,000	3,120	3,246	3,375	3,510	3,650	3,796	3,947	4,105	4,269	4,440
(4) Other Assets	14,100	13,770	14,347	14,939	15,553	16,186	16,841	17,524	18,235	18,970	19,734
(5) Total Assets	$228,450	$259,010	$283,302	$303,869	$322,044	$339,020	$355,529	$371,948	$388,506	$405,303	$422,421
(6) Gross Loss & LAE Reserves	$133,219	$155,205	$172,403	$186,550	$198,726	$209,869	$220,570	$231,114	$241,670	$252,308	$263,085
(7) Gross Unearned Premium Reserves	45,000	46,800	48,672	50,619	52,644	54,750	56,939	59,216	61,584	64,047	66,609
(8) Other Liabilities	1,500	2,791	2,383	2,070	1,817	1,592	1,384	1,190	1,005	806	586
(9) Total Liabilities	$179,719	$204,796	$223,458	$239,239	$253,187	$266,211	$278,893	$291,520	$304,259	$317,161	$330,280
(10) Total Capital & Surplus	48,731	54,214	59,844	64,630	68,857	72,809	76,636	80,428	84,247	88,142	92,141
(11) Total Liabilities & Surplus	$228,450	$259,010	$283,302	$303,869	$322,044	$339,020	$355,529	$371,948	$388,506	$405,303	$422,421

Calculation Notes:
(1) Exhibit 16, Line (1)
(2) Exhibit 16, Line (2) + Line (3)
(3) Exhibit 16, Line (4)
(4) = (Premium Receivable + Receivables from Reinsurers + Other Assets) from Exhibit 16
(5) = (1) + (2) + (3) + (4)
(6) Exhibit 16, Line (10) + Line (11)
(7) Exhibit 16, Line (12)
(8) = [(10) + (11) + (12) + (13) + (14)] from Exhibit 16
(9) = (6) + (7) + (8)
(10) = (5) − (9)
(11) = (9) + (10)

EXHIBIT 16
SHEET 1

PRIMARY STOCK INSURANCE COMPANY
DETAILED GAAP BALANCE SHEET

	Actual 2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
(1) Investments & Cash	$185,216	$213,090	$233,984	$251,262	$266,218	$279,994	$293,292	$306,473	$319,754	$333,230	$346,972
(2) Ceded Unpaid Losses	26,134	28,392	30,531	32,598	34,618	36,628	38,641	40,666	42,710	44,780	46,878
(3) Ceded Unpaid LAE	0	638	1,194	1,695	2,145	2,562	2,959	3,338	3,702	4,054	4,397
(4) Ceded Unearned Premium Reserves	3,000	3,120	3,246	3,375	3,510	3,650	3,796	3,947	4,105	4,269	4,440
(5) Premiums Receivable	7,500	6,500	6,760	7,030	7,310	7,602	7,907	8,222	8,552	8,894	9,249
(6) Deferred Acquisition Costs	6,600	6,942	7,220	7,508	7,809	8,121	8,444	8,783	9,136	9,500	9,880
(7) Receivables from Reinsurers	0	328	367	401	434	463	490	519	547	576	605
(8) Other Assets	0	0	0	0	0	0	0	0	0	0	0
(9) Total Assets	$228,450	$259,010	$283,302	$303,869	$322,044	$339,020	$355,529	$371,948	$388,506	$405,303	$422,421
(10) Gross Unpaid Losses	$105,701	$127,155	$143,829	$157,325	$168,769	$179,099	$188,908	$198,480	$207,985	$217,497	$227,080
(11) Gross Unpaid LAE	27,518	28,050	28,574	29,225	29,957	30,770	31,662	32,634	33,685	34,811	36,005
(12) Gross Unearned Premium Reserve	45,000	46,800	48,672	50,619	52,644	54,750	56,939	59,216	61,584	64,047	66,609
(13) Premium Deficiency Reserve	0	0	0	0	0	0	0	0	0	0	0
(14) Expenses Payable	1,000	1,030	1,061	1,093	1,126	1,162	1,198	1,236	1,275	1,316	1,359
(15) Balances Due Reinsurers	500	1,560	1,622	1,687	1,754	1,824	1,898	1,973	2,053	2,135	2,220
(16) Dividends Payable											
(16a) Policyholders	0	0	0	0	0	0	0	0	0	0	0
(16b) Stockholders	0	0	0	0	0	0	0	0	0	0	0
(17) Federal Income Taxes Payable											
(17a) Current	0	967	996	1,050	1,093	1,143	1,191	1,238	1,288	1,345	1,404
(17b) Deferred	0	(766)	(1,296)	(1,760)	(2,156)	(2,537)	(2,903)	(3,257)	(3,611)	(3,990)	(4,397)
(18) Surplus Notes	0	0	0	0	0	0	0	0	0	0	0
(19) Accrued Interest	0	0	0	0	0	0	0	0	0	0	0
(20) Other Liabilities	0	0	0	0	0	0	0	0	0	0	0
(21) Total Liabilities	$179,719	$204,796	$223,458	$239,239	$253,187	$266,211	$278,893	$291,520	$304,259	$317,161	$330,280

EXHIBIT 16
Sheet 1
Continued

	Actual 2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
(22) Capital	$40,000	$40,000	$40,000	$40,000	$40,000	$40,000	$40,000	$40,000	$40,000	$40,000	$40,000
(23) Unrealized Capital Gains	0	3,512	7,455	11,733	16,296	21,112	26,167	31,455	36,977	42,735	48,733
(24) Retained Earnings	8,731	10,702	12,389	12,897	12,561	11,697	10,469	8,973	7,270	5,407	3,408
(25) Total Capital & Surplus	$48,731	$54,214	$59,844	$64,630	$68,857	$72,809	$76,636	$80,428	$84,247	$88,142	$92,141
(26) Total Liabilities & Surplus	$228,450	$259,010	$283,302	$303,869	$322,044	$339,020	$355,529	$371,948	$388,506	$405,303	$422,421

EXHIBIT 16
SHEET 2

PRIMARY STOCK INSURANCE COMPANY
CALCULATION NOTES FOR DETAILED GAAP BALANCE SHEET

(1) shown in detail on Exhibit 11
(4) 2001 value is input, subsequent values = prior year value + (GWP – NWP + GEP – NEP) for all lines
(5) 2001 value is input, subsequent values = prior year value + DWP for all lines – Direct Premium Collected for all lines
(6) 2001 value is input, subsequent
values = prior year value + [underwriting expenses (statutory) – underwriting expenses (GAAP) + reinsurance commission (GAAP)] for all lines
(7) 2001 value is input, subsequent values = prior year value + ceded loss & LAE paid – ceded loss & LAE received
(8) value set to $0 for all years
(9) = (1) + (2) + (3) + (4) + (5) + (6) + (7) + (8)
(12) 2001 value is input, subsequent values = prior year value + GWP for all lines – GEP for all lines
(14) 2001 value is input, subsequent
values = prior year value + (agents' commissions + other underwriting expenses + premium taxes) for all lines – Underwriting Expenses Paid*
(15) 2001 value is input, subsequent
values = prior year value + (ceded written premium – premium ceded + reinsurance commission – reinsurance commission paid) for all lines
(16), (16a), (16b) values set to $0 for all years
(17a) 2001 value is input, subsequent values = prior year value + Federal Income Tax – Federal Income Tax Paid*
(18) 2001 value is input, subsequent values = prior year value + Increase in Surplus Notes** – Principal Repayment**
(19) unpaid principal and interest associated with (18)
(20) value set to $0 for all years
(21) = (10) + (11) + (12) + (13) + (14) + (15) + (16a) + (16b) + (17a) + (17b) + (18) + (19) + (20)
(22) 2001 value is input, subsequent values = prior year value + capital paid in**
(24) = (25) – (22) – (23)
(25) = (9) – (21)
(26) = (21) + (25)

Notes:
*Value is from Exhibit 13.
**Value is from Exhibit 8.

EXHIBIT 17

Primary Stock Insurance Company
Changes in GAAP Net Worth

	Actual 2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
(1) Net Income	$8,778	$8,041	$8,919	$9,661	10,289	$10,853	$11,386	$11,911	$12,433	$12,961	$13,502
(2) Stockholder Dividends Incurred	0	6,070	7,232	9,153	10,625	11,717	12,614	13,407	14,136	14,824	15,501
(3) Other Surplus Adjustments	0	0	0	0	0	0	0	0	0	0	0
(4) Change in Retained Earnings	$8,778	$1,971	$1,687	$508	($336)	($864)	($1,228)	($1,496)	($1,703)	($1,863)	($1,999)
(5) Capital Contributions	0	0	0	0	0	0	0	0	0	0	0
(6) Change in Unrealized Capital Gains	0	3,512	3,943	4,278	4,563	4,816	5,055	5,288	5,522	5,758	5,998
(7) Change in Net Worth	$8,778	$5,483	$5,630	$4,786	$4,227	$3,952	$3,827	$3,792	$3,819	$3,895	$3,999

Calculation Notes:
(1) = GAAP Net Income
(3) value set to $0 for all years
(4) = (1) – (2) + (3)
(5) value set to $0 for all years
(7) = (4) + (5) + (6)

EXHIBIT 18
Sheet 1

Primary Stock Insurance Company
Workers Compensation

	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
Statutory Underwriting Income for Line of Business										
(A) Net Earned Premium	$27,540	$28,642	$29,787	$30,979	$32,218	$33,507	$34,847	$36,241	$37,690	$39,199
(B) Net Incurred Loss and LAE	21,650	22,516	23,415	24,354	25,328	26,340	27,393	28,490	29,629	30,815
(C) Total Underwriting Expenses	5,676	5,903	6,139	6,385	6,640	6,906	7,182	7,470	7,768	8,079
(D) Underwriting Income	$214	$223	$233	$240	$250	$261	$272	$281	$293	$305
Modeled Amounts										
(1) Direct Written Premium	$31,200	$32,448	$33,746	$35,096	$36,500	$37,960	$39,478	$41,057	$42,699	$44,408
(2) Direct Earned Premium	30,600	31,824	33,097	34,421	35,798	37,230	38,719	40,268	41,878	43,554
(3) Ceded Written Premium	3,120	3,245	3,375	3,510	3,650	3,796	3,948	4,106	4,270	4,441
(4) Ceded Earned Premium	3,060	3,182	3,310	3,442	3,580	3,723	3,872	4,027	4,188	4,355
(5) Net Written Premium	28,080	29,203	30,371	31,586	32,850	34,164	35,530	36,951	38,429	39,967
(6) Net Earned Premium	27,540	28,642	29,787	30,979	32,218	33,507	34,847	36,241	37,690	39,199
(7) Direct Incurred Losses	21,420	22,277	23,168	24,095	25,059	26,061	27,103	28,188	29,315	30,488
(8) Ceded Incurred Losses	3,060	3,182	3,310	3,442	3,580	3,723	3,872	4,027	4,188	4,355
(9) Net Incurred Losses	18,360	19,095	19,858	20,653	21,479	22,338	23,231	24,161	25,127	26,133
(10) Direct Incurred ALAE	1,714	1,782	1,853	1,928	2,005	2,085	2,168	2,255	2,345	2,439
(11) Ceded ALAE	245	255	265	275	286	298	310	322	335	348
(12) Net Incurred ALAE	1,469	1,527	1,588	1,653	1,719	1,787	1,858	1,933	2,010	2,091
(13) Gross Incurred ULAE	1,821	1,894	1,969	2,048	2,130	2,215	2,304	2,396	2,492	2,591
(14) Net Incurred Loss & LAE	21,650	22,516	23,415	24,354	25,328	26,340	27,393	28,490	29,629	30,815

EXHIBIT 18
Sheet 1
Continued

	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
(15) Agents' Commissions	3,120	3,245	3,375	3,510	3,650	3,796	3,948	4,106	4,270	4,441
(16) Other Underwriting Expenses	1,620	1,685	1,752	1,822	1,895	1,971	2,050	2,132	2,217	2,306
(17) Premium Taxes	936	973	1,012	1,053	1,095	1,139	1,184	1,232	1,281	1,332
(18) Reinsurance Commissions	0	0	0	0	0	0	0	0	0	0
(19) Total Underwriting Expenses	5,676	5,903	6,139	6,385	6,640	6,906	7,182	7,470	7,768	8,079
Modeled GAAP Amounts										
(20) Gross Reserves	$62,716	$66,999	$70,848	$74,575	$78,303	$82,088	$85,937	$89,864	$93,879	$97,978
(21) Ceded Reserves	17,172	18,539	19,870	21,175	22,486	23,811	25,156	26,520	27,902	29,303
(22) Agents' Commissions	3,060	3,182	3,310	3,442	3,580	3,723	3,872	4,027	4,188	4,355
(23) Underwriting Expenses	1,620	1,685	1,752	1,822	1,895	1,971	2,050	2,132	2,217	2,306
(24) Premium Tax	873	955	993	1,033	1,074	1,117	1,162	1,208	1,256	1,307
(25) Reinsurance Commissions	0	0	0	0	0	0	0	0	0	0
(26) Total Underwriting Expenses	5,553	5,822	6,055	6,297	6,549	6,811	7,084	7,367	7,661	7,968

EXHIBIT 18

Sheet 2

Primary Stock Insurance Company, Workers Compensation Calculation Notes Statutory Underwriting Income, Modeled Amounts, and Modeled GAAP Amounts

(A) = (6); (B) = (14); (C) = (19); (D) = (A) − (B) − (C)
(1) = 2001 DWP × (annual growth rate)$^{(\text{year}\ \ 2001)}$
(2) = (prior year UEPR) + (earned % × DWP)
(3) = excess ceded % × DWP
(4) = excess ceded % × DEP
(5) = (1) − (3)
(6) = (2) − (4)

(7) = GEP × expected loss ratio
(8) = (4) × ceded loss ratio
(9) = (7) − (8)
(10) = (7) × Gross ALAE to loss %
(11) = [(8) ÷ (7)] × (10)
(12) = (10) − (11)
(13) = (7) × Gross ULAE to loss %
(14) = (9) + (12) + (13)

(15) = DWP × Agents' commission %
(16) = Amount of fixed underwriting + (% of DEP × DEP) + (% of DWP × DWP)
(17) = DWP × Premium tax %
(18) = Reinsurance commission % × (3)
(19) = (15) + (16) + (17) + (18)

(22) = Business earned in 1st year % × DWP × Agents' commission % + 2001 GAAP Deferred Commission in 2002
= Agents' commission % × DEP in other years
(23) = (16) × [1 − Deferrable %] + (16) × Business earned in 1st year % × Deferrable % + 2001 GAAP Deferred U/W Expense in 2002
= (16) × [1 − Deferrable %] + Deferrable % × [(16) × Business earned in 1st year % + (1 − Business earned in 1st year %) × prior yr (16)] in other years
(24) = Business earned in 1st year % × DWP × Premium Tax % + 2001 GAAP Deferred Premium Tax in 2002
= Premium Tax % × DEP in other years
(25) = (4) × Reinsurance Commission %
(26) = (22) + (23) + (24) + (25)

EXHIBIT 18
SHEET 3

PRIMARY STOCK INSURANCE COMPANY
WORKERS COMPENSATION

	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
Underwriting Cash Flow For Lines of Business										
(A) Total Collected	$28,510	$29,130	$30,296	$31,508	$32,768	$34,079	$35,442	$36,859	$38,333	$39,868
(B) Net Loss and LAE Payments	18,114	19,617	20,911	21,945	22,924	23,892	24,901	25,939	27,010	28,131
(C) Underwriting Expense Paid	5,666	5,893	6,128	6,374	6,628	6,894	7,169	7,457	7,754	8,065
(D) Cash Flow from Underwriting	$4,730	$3,620	$3,257	$3,189	$3,216	$3,293	$3,372	$3,463	$3,569	$3,672
Modeled Amounts										
(1) Gross Premium Collected	$31,100	$32,344	$33,638	$34,984	$36,383	$37,838	$39,352	$40,925	$42,562	$44,266
(2) Premium Ceded	2,590	3,214	3,342	3,476	3,615	3,759	3,910	4,066	4,229	4,398
(3) Net Premium Collected	28,510	29,130	30,296	31,508	32,768	34,079	35,442	36,859	38,333	39,868
(4) Reinsurance Commissions	0	0	0	0	0	0	0	0	0	0
(5) Total Collected	28,510	29,130	30,296	31,508	32,768	34,079	35,442	36,859	38,333	39,868
(6) Gross Losses Paid	15,990	17,834	19,277	20,405	21,421	22,411	23,430	24,477	25,554	26,677
(7) Loss Recoveries Received	1,686	1,992	2,143	2,284	2,413	2,540	2,670	2,806	2,947	3,095
(8) Net Loss Paid	14,304	15,842	17,134	18,121	19,008	19,871	20,760	21,671	22,607	23,582
(9) Gross ALAE Paid	1,279	1,427	1,542	1,632	1,714	1,793	1,874	1,958	2,044	2,134
(10) ALAE Recoveries Received	18	61	87	114	130	144	155	167	180	194
(11) Net ALAE Paid	1,261	1,366	1,455	1,518	1,584	1,649	1,719	1,791	1,864	1,940
(12) ULAE Paid	2,549	2,409	2,322	2,306	2,332	2,372	2,422	2,477	2,539	2,609
(13) Net Loss & LAE Payments	18,114	19,617	20,911	21,945	22,924	23,892	24,901	25,939	27,010	28,131
(14) Agents' Commissions	3,110	3,235	3,364	3,499	3,638	3,784	3,935	4,093	4,256	4,427
(15) Other Underwriting Expenses	1,620	1,685	1,752	1,822	1,895	1,971	2,050	2,132	2,217	2,306
(16) Premium Taxes Paid	936	973	1,012	1,053	1,095	1,139	1,184	1,232	1,281	1,332
(17) Underwriting Expense Paid	5,666	5,893	6,128	6,374	6,628	6,894	7,169	7,457	7,754	8,065

Calculation Notes:
(A) = (5); (B) = (13); (C) = (17); (D) = (A) – (B) – (C)
(1) = DWP × (1 – Monthly Premium Collection Lag / 12) + 2001 Direct Premium Uncollected in 2002
    = DWP × (1 – Monthly Premium Collection Lag / 12) + prior year DWP × Monthly Premium Collection Lag / 12 in other years
(2) = Ceded WP × (1 – Monthly Ceding Premium Lag / 12) + 2001 Ceded Unearned Premium in 2002
    = Ceded WP × (1 – Monthly Ceding Premium Lag / 12) + prior year Ceded WP × Premium Collection Lag / 12 in other years
(3) = (1) – (2)
(4) = Reinsurance commission % × (2)
(5) = (3) + (4)
(8) = (6) – (7)
(11) = (9) – (10)
(13) = (8) + (11) + (12)
(14) = Agents' commission from cash flow × (1 – Monthly Premium Collection Lag / 12) + 2001 Unpaid Agents' Commission in 2002
    = Agents' commission* × (1 – Monthly Premium Collection Lag / 12) + Prior year Agents' commission* × Monthly Premium Collection Lag / 12 in other years
(15) = Fixed underwriting$ + (% of DEP × DEP) + (% of DWP × DWP)
(16) = DWP × Premium tax %
(17) = (14) + (15) + (16)

EXHIBIT 18
SHEET 4

FINANCIAL MODELING ASSUMPTIONS, WORKERS COMPENSATION
MODELING ASSUMPTIONS FOR CURRENT AND FUTURE BUSINESS

Amounts as of 12/31/01		Assumptions for Future Business	
Gross Unearned Premium	$15,000	Gross Written Premium in 2002	$31,200
Net Unearned Premium	$13,500	Annual Growth Rate of Business	4.00%
Ceded Unearned Premium	1,500		
Gross Premium Uncollected	$2,500	Percent of Business Earned in 1st Year	50.00%
Unpaid Agent's Commission	250	Premium Collection Lag (in months)	1
Ceded Premium Not Yet Remitted	250		
Ceded Paid Losses Not Yet Coll.	0	Expected Loss Ratio	70.00%
Ceded Paid ALAE Not Yet Coll.	0	Gross ALAE as a % of Loss	8.00%
Reinsurance Comm. Not Yet Coll.	0	Gross ULAE as a % of Loss	8.50%
GAAP Deferred Commission	1,500		
GAAP Deferred U/W Expense	0	Agents' Commission as % of DWP	10.00%
GAAP Deferred Premium Tax	405	Premium Tax as % of DWP	3.00%
GAAP Deferred Reins. Commission	0	Other Underwriting Expenses	
		Fixed	$0
		Variable (% of DEP)	3.00%
		Variable (% of DWP)	2.25%
		% Deferrable	0.00%
		Reinsurance (per occ Excess)	
		Percent of premium ceded	10.00%
		Lag in Ceding Premium (in months)	3
		Ceded Loss Ratio	100.00%
		Ceded Loss Collection Lag (in months)	1

Loss and LAE reserves are carried at nominal (undiscounted) value for both SAP and GAAP.
"Reserve strengthening" adjustments are not needed; ultimate loss and LAE amounts do not deteriorate or improve over time.

EXHIBIT 18
SHEET 5

FINANCIAL MODELING ASSUMPTIONS
WORKERS COMPENSATION
LOSS PAYMENT AND DISCOUNTING

Payment Patterns for Loss and LAE				Interest Rate for Discounted Tax Reserves	
Accident Year +	Gross	Ceded	IRS	Accident Year	Rate
0	21.00%	8.00%	25.00%	1982	7.20%
1	30.00%	18.00%	33.00%	1983	7.20%
2	14.00%	9.00%	16.00%	1984	7.20%
3	10.00%	9.75%	12.00%	1985	7.20%
4	4.00%	4.25%	4.00%	1986	7.20%
5	3.00%	3.25%	2.00%	1987	7.20%
6	2.00%	2.25%	1.50%	1988	7.77%
7	2.00%	2.50%	0.75%	1989	8.16%
8	1.75%	2.50%	0.75%	1990	8.37%
9	1.50%	2.50%	0.75%	1991	7.00%
10	1.50%	2.75%	0.75%	1992	7.00%
11	1.25%	2.50%	0.75%	1993	7.00%
12	1.00%	2.25%	0.75%	1994	7.00%
13	1.00%	2.50%	0.50%	1995	7.00%
14	0.75%	2.00%	0.50%	1996	7.00%
15	0.75%	2.25%	1.00%	1997	7.00%
16	0.50%	1.75%		1998	7.00%
17	0.50%	2.00%		1999	7.00%
18	0.50%	2.25%		2000	7.00%
19	0.25%	1.25%		2001	7.00%
20	0.25%	1.50%		2002	7.00%
21	0.25%	1.50%		2003	7.00%
22	0.25%	1.50%		2004	7.00%
23	0.25%	1.50%		2005	7.00%
24	0.25%	1.50%		2006	7.00%
25	0.25%	1.50%		2007	7.00%
26	0.25%	1.50%		2008	7.00%
27	0.25%	1.50%		2009	7.00%
28	0.25%	1.50%		2010	7.00%
29	0.25%	1.50%		2011	7.00%
30	0.25%	1.50%			
Total	100.00%	100.00%	100.00%		

EXHIBIT 18
SHEET 6

FINANCIAL MODELING ASSUMPTIONS
WORKERS COMPENSATION
PRIOR YEARS' INFORMATION

	Estimates of Ultimate Loss and LAE @ 12/31/01							Paid Loss and LAE @ 12/31/01						
Accident Year	Gross Loss	Ceded Loss	Net Loss	Gross ALAE	Ceded ALAE	Net ALAE	ULAE	Gross Loss	Ceded Loss	Net Loss	Gross ALAE	Ceded ALAE	Net ALAE	ULAE
1982	8,971	1,282	7,689	718	0	718	763	8,746	1,089	7,657	700	0	700	740
1983	9,330	1,333	7,997	746	0	746	793	9,073	1,113	7,960	726	0	726	761
1984	9,703	1,386	8,317	776	0	776	825	9,412	1,140	8,272	753	0	753	783
1985	10,091	1,442	8,649	807	0	807	858	9,738	1,153	8,584	779	0	779	806
1986	10,494	1,499	8,995	840	0	840	892	10,075	1,169	8,905	806	0	806	830
1987	10,914	1,559	9,355	873	0	873	928	10,423	1,189	9,234	834	0	834	853
1988	11,351	1,622	9,729	908	0	908	965	10,755	1,200	9,555	860	0	860	868
1989	11,805	1,686	10,118	944	0	944	1,003	11,097	1,214	9,882	888	0	888	873
1990	12,277	1,754	10,523	982	0	982	1,044	11,418	1,219	10,199	913	0	913	866
1991	12,768	1,824	10,944	1,021	0	1,021	1,085	11,747	1,227	10,520	940	0	940	857
1992	13,279	1,897	11,382	1,062	0	1,062	1,129	12,051	1,228	10,822	964	0	964	835
1993	13,810	1,973	11,837	1,105	0	1,105	1,174	12,325	1,223	11,102	986	0	986	810
1994	14,362	2,052	12,311	1,149	0	1,149	1,221	12,603	1,221	11,382	1,008	0	1,008	769
1995	14,937	2,134	12,803	1,195	0	1,195	1,270	12,846	1,216	11,629	1,028	0	1,028	736
1996	15,534	2,219	13,315	1,243	0	1,243	1,320	13,049	1,209	11,839	1,044	0	1,044	713
1997	16,156	2,308	13,848	1,292	0	1,292	1,373	13,248	1,206	12,042	1,060	0	1,060	687
1998	16,802	2,400	14,402	1,344	0	1,344	1,428	13,274	1,176	12,097	1,062	0	1,062	657
1999	17,474	2,496	14,978	1,398	0	1,398	1,485	13,106	1,117	11,988	1,048	0	1,048	624
2000	18,173	2,596	15,577	1,454	0	1,454	1,545	11,813	909	10,904	945	0	945	587
2001	18,900	2,700	16,200	1,512	0	1,512	1,607	3,969	216	3,753	318	0	318	546

Loss and LAE Reserves @ 12/31/01

Net Accident Year	Gross Loss	Ceded Loss	Net Loss	Gross ALAE	Ceded ALAE	Net ALAE	ULAE	Earned Premium
1982	224	192	32	18	0	18	23	12,815
1983	257	220	37	21	0	21	32	13,328
1984	291	246	45	23	0	23	41	13,861
1985	353	288	65	28	0	28	51	14,416
1986	420	330	90	34	0	34	62	14,992
1987	491	370	121	39	0	39	74	15,592
1988	596	422	174	48	0	48	96	16,216
1989	708	472	236	57	0	57	130	16,864
1990	859	535	324	69	0	69	177	17,539
1991	1,021	597	424	82	0	82	228	18,240
1992	1,228	669	560	98	0	98	293	18,970
1993	1,485	750	735	119	0	119	364	19,729
1994	1,759	831	928	141	0	141	452	20,518
1995	2,091	918	1,174	167	0	167	533	21,338
1996	2,486	1,010	1,476	199	0	199	607	22,192
1997	2,908	1,102	1,806	233	0	233	687	23,080
1998	3,528	1,224	2,304	282	0	282	771	24,003
1999	4,369	1,379	2,989	349	0	349	861	24,963
2000	6,361	1,688	4,673	509	0	509	958	25,962
2001	14,931	2,484	12,447	1,194	0	1,194	1,060	27,000
Total	46,367	15,726	30,640	3,709	0	3,709	7,503	

EXHIBIT 19
Sheet 1

Primary Stock Insurance Company
Auto Liability

	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
Statutory Underwriting Income for Line of Business										
(A) Net Earned Premium	$30,600	$31,824	$33,097	$34,421	$35,798	$37,230	$38,719	$40,268	$41,879	$43,554
(B) Net Incurred Loss and LAE	22,718	23,626	24,571	25,553	26,576	27,639	28,745	29,896	31,091	32,335
(C) Total Underwriting Expenses	7,007	7,287	7,578	7,881	8,197	8,524	8,866	9,220	9,589	9,972
(D) Underwriting Income	$875	$911	$948	$987	$1,025	$1,067	$1,108	$1,152	$1,199	$1,247
Modeled Amounts										
(1) Direct Written Premium	$31,200	$32,448	$33,746	$35,096	$36,500	$37,960	$39,478	$41,057	$42,700	$44,408
(2) Direct Earned Premium	30,600	31,824	33,097	34,421	35,798	37,230	38,719	40,268	41,879	43,554
(3) Ceded Written Premium	0	0	0	0	0	0	0	0	0	0
(4) Ceded Earned Premium	0	0	0	0	0	0	0	0	0	0
(5) Net Written Premium	31,200	32,448	33,746	35,096	36,500	37,960	39,478	41,057	42,700	44,408
(6) Net Earned Premium	30,600	31,824	33,097	34,421	35,798	37,230	38,719	40,268	41,879	43,554
(7) Direct Incurred Losses	19,584	20,367	21,182	22,029	22,911	23,827	24,780	25,772	26,803	27,875
(8) Ceded Incurred Losses	0	0	0	0	0	0	0	0	0	0
(9) Net Incurred Losses	19,584	20,367	21,182	22,029	22,911	23,827	24,780	25,772	26,803	27,875
(10) Direct Incurred ALAE	1,665	1,731	1,800	1,872	1,947	2,025	2,106	2,191	2,278	2,369
(11) Ceded ALAE	0	0	0	0	0	0	0	0	0	0
(12) Net Incurred ALAE	1,665	1,731	1,800	1,872	1,947	2,025	2,106	2,191	2,278	2,369
(13) Gross Incurred ULAE	1,469	1,528	1,589	1,652	1,718	1,787	1,859	1,933	2,010	2,091
(14) Net Incurred Loss & LAE	22,718	23,626	24,571	25,553	26,576	27,639	28,745	29,896	31,091	32,335

	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
(15) Agents' Commissions	4,680	4,867	5,062	5,264	5,475	5,694	5,922	6,159	6,405	6,661
(16) Other Underwriting Expenses	1,703	1,771	1,841	1,915	1,992	2,071	2,154	2,240	2,330	2,423
(17) Premium Taxes	624	649	675	702	730	759	790	821	854	888
(18) Reinsurance Commissions	0	0	0	0	0	0	0	0	0	0
(19) Total Underwriting Expenses	7,007	7,287	7,578	7,881	8,197	8,524	8,866	9,220	9,589	9,972
Modeled GAAP Amounts										
(20) Gross Reserves	$26,347	$33,470	$38,521	$42,146	$44,953	$47,388	$49,672	$51,904	$54,083	$56,246
(21) Ceded Reserves	0	0	0	0	0	0	0	0	0	0
(22) Agents' Commissions	4,590	4,774	4,965	5,163	5,370	5,585	5,808	6,040	6,282	6,533
(23) Underwriting Expenses	1,703	1,771	1,841	1,915	1,992	2,071	2,154	2,240	2,330	2,423
(24) Premium Tax	612	636	662	688	716	745	774	805	838	871
(25) Reinsurance Commissions	0	0	0	0	0	0	0	0	0	0
(26) Total Underwriting Expenses	6,905	7,181	7,468	7,766	8,078	8,401	8,736	9,085	9,450	9,827

EXHIBIT 19
SHEET 2

PRIMARY STOCK INSURANCE COMPANY
AUTO LIABILITY
CALCULATION NOTES STATUTORY UNDERWRITING INCOME, MODELED AMOUNTS, AND MODELED GAAP AMOUNTS

(A) = (6); (B) = (14); (C) = (19); (D) = (A) − (B) − (C)
(1) = 2001DWP × (annual growth rate)$^{(\text{year}\ \ 2001)}$
(2) = (prior year UEPR) + (earned % × DWP)
(3) = excess ceded % × DWP
(4) = excess ceded % × DEP
(5) = (1) − (3)
(6) = (2) − (4)
(7) = GEP × expected loss ratio
(8) = (4) × ceded loss ratio
(9) = (7) − (8)
(10) = (7) × Gross ALAE to loss %
(11) = [(8) ÷ (7)] × (10)
(12) = (10) − (11)
(13) = (7) × Gross ULAE to loss %
(14) = (9) + (12) + (13)
(15) = DWP × Agents' commission %

(16) = Fixed underwriting \$ + (% of DEP × DEP) + (% of DWP × DWP)
(17) = DWP × Premium tax %
(18) = Reinsurance commission % × (3)
(19) = (15) + (16) + (17) + (18)
(22) = Business earned in 1st year % × DWP × Agents' commission % + 2001 GAAP Deferred Commission in 2002
= Agents' commission % × DEP in other years
(23) = (16) × [1 − Deferrable %] + (16) × Business earned in 1st year % × Deferrable % + 2001 GAAP Deferred U/W Expense in 2002
= (16) × [1 − Deferrable %] + Deferrable % × [(16) × Business earned in 1st year % + (1 − Business earned in 1st year %)× prior yr (16)] in other years
(24) = Business earned in 1st year % × DWP × Premium Tax % + 2001 GAAP Deferred Premium Tax in 2002
= Premium Tax % × DEP in other years
(25) = (4) × Reinsurance Commission %
(26) = (22) + (23) + (24) + (25)

EXHIBIT 19
SHEET 3

PRIMARY STOCK INSURANCE COMPANY
AUTO LIABILITY

	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
Underwriting Cash Flow For Lines of Business										
(A) Total Collected	$32,400	$32,396	$33,692	$35,040	$36,442	$37,899	$39,415	$40,991	$42,632	$44,337
(B) Net Loss and LAE Payments	12,526	16,503	19,519	21,928	23,770	25,205	26,460	27,664	28,913	30,172
(C) Underwriting Expense Paid	7,062	7,279	7,570	7,873	8,188	8,515	8,857	9,210	9,579	9,961
(D) Cash Flow from Underwriting	$12,812	$8,614	$6,603	$5,239	$4,484	$4,179	$4,098	$4,117	$4,140	$4,204
Modeled Amounts										
(1) Gross Premium Collected	$32,400	$32,396	$33,692	$35,040	$36,442	$37,899	$39,415	$40,991	$42,632	$44,337
(2) Premium Ceded	0	0	0	0	0	0	0	0	0	0
(3) Net Premium Collected	32,400	32,396	33,692	35,040	36,442	37,899	39,415	40,991	42,632	44,337
(4) Reinsurance Commissions	0	0	0	0	0	0	0	0	0	0
(5) Total Collected	32,400	32,396	33,692	35,040	36,442	37,899	39,415	40,991	42,632	44,337
(6) Gross Losses Paid	10,006	13,674	16,472	18,684	20,352	21,629	22,730	23,777	24,861	25,951
(7) Loss Recoveries Received	0	0	0	0	0	0	0	0	0	0
(8) Net Loss Paid	10,006	13,674	16,472	18,684	20,352	21,629	22,730	23,777	24,861	25,951
(9) Gross ALAE Paid	851	1,162	1,400	1,588	1,730	1,838	1,932	2,021	2,113	2,206
(10) ALAE Recoveries Received	0	0	0	0	0	0	0	0	0	0
(11) Net ALAE Paid	851	1,162	1,400	1,588	1,730	1,838	1,932	2,021	2,113	2,206
(12) ULAE Paid	1,669	1,667	1,647	1,656	1,688	1,738	1,798	1,866	1,939	2,015
(13) Net Loss & LAE Payments	12,526	16,503	19,519	21,928	23,770	25,205	26,460	27,664	28,913	30,172
(14) Agents' Commissions	4,735	4,859	5,054	5,256	5,466	5,685	5,913	6,149	6,395	6,650
(15) Other Underwriting Expenses	1,703	1,771	1,841	1,915	1,992	2,071	2,154	2,240	2,330	2,423
(16) Premium Taxes Paid	624	649	675	702	730	759	790	821	854	888
(17) Underwriting Expense Paid	7,062	7,279	7,570	7,873	8,188	8,515	8,857	9,210	9,579	9,961

Calculation Notes:
(A) = (5); (B) = (13); (C) = (17); (D) = (A) – (B) – (C)
(1) = DWP × (1 – Monthly Premium Collection Lag / 12) + 2001 Direct Premium Uncollected in 2002
= DWP × (1 – Monthly Premium Collection Lag / 12) + prior year DWP × Monthly Premium Collection Lag / 12 in other years
(2) = Ceded WP × (1 – Monthly Ceding Premium Lag / 12) + 2001 Ceded Unearned Premium in 2002
= Ceded WP × (1 – Monthly Ceding Premium Lag / 12) + prior year Ceded WP × Premium Collection Lag / 12 in other years
(3) = (1) – (2)
(4) = Reinsurance commission % × (2)
(5) = (3) + (4)
(8) = (6) – (7)
(11) = (9) – (10)
(13) = (8) + (11) + (12)
(14) = Agents' commission from cash flow × (1 – Monthly Premium Collection Lag / 12) + 2001 Unpaid Agents' Commission in 2002
= Agents' commission × (1 – Monthly Premium Collection Lag / 12) + Prior year Agents' commission × Monthly Premium Collection Lag / 12 in other years
(15) = Fixed underwriting $ + (% of DEP × DEP) + (% of DWP × DWP)
(16) = DWP × Premium tax %
(17) = (14) + (15) + (16)

EXHIBIT 19
SHEET 4

FINANCIAL MODELING ASSUMPTIONS, AUTO LIABILITY
MODELING ASSUMPTIONS FOR CURRENT AND FUTURE BUSINESS

Amounts as of 12/31/01		Assumptions for Future Business	
Gross Unearned Premium	$15,000	Gross Written Premium in 2002	$31,200
Net Unearned Premium	$15,000	Annual Growth Rate of Business	4.00%
Ceded Unearned Premium	0		
Gross Premium Uncollected	$2,500	Percent of Business Earned in 1st Year	50.00%
Unpaid Agent's Commission	250	Premium Collection Lag (in months)	0.5
Ceded Premium Not Yet Remitted	0		
Ceded Paid Losses Not Yet Coll.	0	Expected Loss Ratio	64.00%
Ceded Paid ALAE Not Yet Coll.	0	Gross ALAE as a % of Loss	8.50%
Reinsurance Comm. Not Yet Coll.	0	Gross ULAE as a % of Loss	7.50%
GAAP Deferred Commission	2,250		
GAAP Deferred U/W Expense	0	Agents' Commission as % of DWP	15.00%
GAAP Deferred Premium Tax	300	Premium Tax as % of DWP	2.00%
GAAP Deferred Reins. Commission	0	Other Underwriting Expenses	
		Fixed	$0
		Variable (% of DEP)	2.25%
		Variable (% of DWP)	3.25%
		Reinsurance (per occ Excess)	
		Percent of premium ceded	0.00%
		Lag in Ceding Premium (in months)	0
		Ceded Loss Ratio	N/A
		Ceded Loss Collection Lag (in months)	1

Loss and LAE reserves are carried at nominal (undiscounted) value for both SAP and GAAP.
"Reserve strengthening" adjustments are not needed; ultimate loss and LAE amounts do not deteriorate or improve over time.

EXHIBIT 19
SHEET 5

FINANCIAL MODELING ASSUMPTIONS
AUTO LIABILITY
LOSS PAYMENT AND DISCOUNTING

Payment Patterns for Loss and LAE				Interest Rate for Discounted Tax Reserves	
Accident Year +	Gross	Ceded	IRS	Accident Year	Rate
0	26.00%	22.00%	30.00%	1982	7.20%
1	26.00%	23.00%	29.00%	1983	7.20%
2	18.00%	16.00%	19.00%	1984	7.20%
3	13.00%	18.00%	10.00%	1985	7.20%
4	8.00%	10.00%	6.00%	1986	7.20%
5	4.00%	4.00%	3.00%	1987	7.20%
6	2.00%	3.00%	1.00%	1988	7.77%
7	1.00%	2.00%	1.00%	1989	8.16%
8	1.00%	1.00%	0.50%	1990	8.37%
9	1.00%	1.00%	0.50%	1991	7.00%
10	0.00%	0.00%	0.00%	1992	7.00%
11	0.00%	0.00%	0.00%	1993	7.00%
12	0.00%	0.00%	0.00%	1994	7.00%
13	0.00%	0.00%	0.00%	1995	7.00%
14	0.00%	0.00%	0.00%	1996	7.00%
15	0.00%	0.00%	0.00%	1997	7.00%
16	0.00%	0.00%		1998	7.00%
17	0.00%	0.00%		1999	7.00%
18	0.00%	0.00%		2000	7.00%
19	0.00%	0.00%		2001	7.00%
20	0.00%	0.00%		2002	7.00%
21	0.00%	0.00%		2003	7.00%
22	0.00%	0.00%		2004	7.00%
23	0.00%	0.00%		2005	7.00%
24	0.00%	0.00%		2006	7.00%
25	0.00%	0.00%		2007	7.00%
26	0.00%	0.00%		2008	7.00%
27	0.00%	0.00%		2009	7.00%
28	0.00%	0.00%		2010	7.00%
29	0.00%	0.00%		2011	7.00%
30	0.00%	0.00%			
Total	100.00%	100.00%	100.00%		

EXHIBIT 19
SHEET 6

FINANCIAL MODELING ASSUMPTIONS
AUTO LIABILITY
PRIOR YEARS' INFORMATION

	Estimates of Ultimate Loss and LAE @ 12/31/01							Paid Loss and LAE @ 12/31/01						
Accident Year	Gross Loss	Ceded Loss	Net Loss	Gross ALAE	Ceded ALAE	Net ALAE	ULAE	Gross Loss	Ceded Loss	Net Loss	Gross ALAE	Ceded ALAE	Net ALAE	ULAE
1982	4,557	0	4,557	387	0	387	342	4,557	0	4,557	387	0	387	342
1983	4,739	0	4,739	403	0	403	355	4,739	0	4,739	403	0	403	355
1984	4,928	0	4,928	419	0	419	370	4,928	0	4,928	419	0	419	370
1985	5,126	0	5,126	436	0	436	384	5,126	0	5,126	436	0	436	384
1986	5,331	0	5,331	453	0	453	400	5,331	0	5,331	453	0	453	400
1987	5,544	0	5,544	471	0	471	416	5,544	0	5,544	471	0	471	416
1988	5,766	0	5,766	490	0	490	432	5,766	0	5,766	490	0	490	432
1989	5,996	0	5,996	510	0	510	450	5,996	0	5,996	510	0	510	450
1990	6,236	0	6,236	530	0	530	468	6,236	0	6,236	530	0	530	468
1991	6,485	0	6,485	551	0	551	486	6,485	0	6,485	551	0	551	486
1992	6,745	0	6,745	573	0	573	506	6,745	0	6,745	573	0	573	506
1993	7,015	0	7,015	596	0	596	526	7,015	0	7,015	596	0	596	500
1994	7,295	0	7,295	620	0	620	547	7,222	0	7,222	614	0	614	492
1995	7,587	0	7,587	645	0	645	569	7,435	0	7,435	632	0	632	484
1996	7,891	0	7,891	671	0	671	592	7,654	0	7,654	651	0	651	473
1997	8,206	0	8,206	698	0	698	615	7,796	0	7,796	663	0	663	462
1998	8,534	0	8,534	725	0	725	640	7,766	0	7,766	660	0	660	416
1999	8,876	0	8,876	754	0	754	666	7,367	0	7,367	626	0	626	366
2000	9,231	0	9,231	785	0	785	692	6,462	0	6,462	549	0	549	277
2001	9,600	0	9,600	816	0	816	720	2,496	0	2,496	212	0	212	72

Loss and LAE Reserves @ 12/31/01

Net Accident Year	Gross Loss	Ceded Loss	Net Loss	Gross ALAE	Ceded ALAE	Net ALAE	ULAE	Earned Premium
1982	0	0	0	0	0	0	0	7,120
1983	0	0	0	0	0	0	0	7,404
1984	0	0	0	0	0	0	0	7,701
1985	0	0	0	0	0	0	0	8,009
1986	0	0	0	0	0	0	0	8,329
1987	0	0	0	0	0	0	0	8,662
1988	0	0	0	0	0	0	0	9,009
1989	0	0	0	0	0	0	0	9,369
1990	0	0	0	0	0	0	0	9,744
1991	0	0	0	0	0	0	0	10,133
1992	0	0	0	0	0	0	0	10,539
1993	0	0	0	0	0	0	26	10,960
1994	73	0	73	6	0	6	55	11,399
1995	152	0	152	13	0	13	85	11,855
1996	237	0	237	20	0	20	118	12,329
1997	410	0	410	35	0	35	154	12,822
1998	768	0	768	65	0	65	224	13,335
1999	1,509	0	1,509	128	0	128	300	13,868
2000	2,769	0	2,769	235	0	235	415	14,423
2001	7,104	0	7,104	604	0	604	648	15,000
Total	13,022	0	13,022	1,107	0	1,107	2,026	

EXHIBIT 20
SHEET 1

PRIMARY STOCK INSURANCE COMPANY
GENERAL LIABILITY

	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
Statutory Underwriting Income for Line of Business										
(A) Net Earned Premium	$27,540	$28,642	$29,787	$30,979	$32,218	$33,507	$34,846	$36,240	$37,690	$39,199
(B) Net Incurred Loss and LAE	22,179	23,066	23,988	24,949	25,946	26,984	28,062	29,185	30,354	31,569
(C) Total Underwriting Expenses	6,060	6,302	6,554	6,817	7,090	7,373	7,668	7,974	8,293	8,625
(D) Underwriting Income	($699)	($726)	($755)	($787)	($818)	($850)	($884)	($919)	($957)	($995)
Modeled Amounts										
(1) Direct Written Premium	$31,200	$32,448	$33,746	$35,096	$36,500	$37,959	$39,477	$41,057	$42,699	$44,408
(2) Direct Earned Premium	30,600	31,824	33,097	34,421	35,798	37,230	38,718	40,267	41,878	43,554
(3) Ceded Written Premium	3,120	3,245	3,375	3,510	3,650	3,796	3,948	4,106	4,270	4,441
(4) Ceded Earned Premium	3,060	3,182	3,310	3,442	3,580	3,723	3,872	4,027	4,188	4,355
(5) Net Written Premium	28,080	29,203	30,371	31,586	32,850	34,163	35,529	36,951	38,429	39,967
(6) Net Earned Premium	27,540	28,642	29,787	30,979	32,218	33,507	34,846	36,240	37,690	39,199
(7) Direct Incurred Losses	20,808	21,640	22,506	23,406	24,343	25,316	26,328	27,382	28,477	29,617
(8) Ceded Incurred Losses	3,060	3,182	3,310	3,442	3,580	3,723	3,872	4,027	4,188	4,355
(9) Net Incurred Losses	17,748	18,458	19,196	19,964	20,763	21,593	22,456	23,355	24,289	25,262
(10) Direct Incurred ALAE	3,121	3,246	3,376	3,511	3,651	3,797	3,949	4,107	4,272	4,443
(11) Ceded ALAE	459	477	497	516	537	558	581	604	628	653
(12) Net Incurred ALAE	2,662	2,769	2,879	2,995	3,114	3,239	3,368	3,503	3,644	3,790
(13) Gross Incurred ULAE	1,769	1,839	1,913	1,990	2,069	2,152	2,238	2,327	2,421	2,517
(14) Net Incurred Loss & LAE	22,179	23,066	23,988	24,949	25,946	26,984	28,062	29,185	30,354	31,569

	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
(15) Agents' Commissions	3,900	4,056	4,218	4,387	4,563	4,745	4,935	5,132	5,337	5,551
(16) Other Underwriting Expenses	1,536	1,597	1,661	1,728	1,797	1,869	1,943	2,021	2,102	2,186
(17) Premium Taxes	624	649	675	702	730	759	790	821	854	888
(18) Reinsurance Commissions	0	0	0	0	0	0	0	0	0	0
(19) Total Underwriting Expenses	6,060	6,302	6,554	6,817	7,090	7,373	7,668	7,974	8,293	8,625
Modeled GAAP Amounts										
(20) Gross Reserves	$66,142	$71,933	$77,179	$82,003	$86,613	$91,095	$95,505	$99,902	$104,346	$108,862
(21) Ceded Reserves	11,858	13,186	14,423	15,589	16,704	17,789	18,848	19,893	20,932	21,972
(22) Agents' Commissions	3,825	3,978	4,137	4,303	4,475	4,654	4,840	5,033	5,235	5,444
(23) Underwriting Expenses	1,536	1,597	1,661	1,728	1,797	1,869	1,943	2,021	2,102	2,186
(24) Premium Tax	582	636	662	688	716	745	774	805	838	871
(25) Reinsurance Commissions	0	0	0	0	0	0	0	0	0	0
(26) Total Underwriting Expenses	5,943	6,211	6,460	6,719	6,988	7,268	7,557	7,859	8,175	8,501

EXHIBIT 20
SHEET 2

PRIMARY STOCK INSURANCE COMPANY
GENERAL LIABILITY
CALCULATION NOTES STATUTORY UNDERWRITING INCOME, MODELED AMOUNTS, AND MODELED GAAP AMOUNTS

(A) = (6); (B) = (14); (C) = (19); (D) = (A) – (B) – (C)
(1) = 2001DWP × (annual growth rate)$^{(\text{year} - 2001)}$
(2) = (prior year UEPR) + (earned % × DWP)
(3) = excess ceded % × DWP
(4) = excess ceded % × DEP
(5) = (1) – (3)
(6) = (2) – (4)
(7) = GEP × expected loss ratio
(8) = (4) × ceded loss ratio
(9) = (7) – (8)
(10) = (7) × Gross ALAE to loss %
(11) = [(8) ÷ (7)] × (10)
(12) = (10) – (11)
(13) = (7) × Gross ULAE to loss %
(14) = (9) + (12) + (13)

(15) = DWP × Agents' commission %
(16) = Fixed underwriting$ + (% of DEP × DEP) + (% of DWP × DWP)
(17) = DWP × Premium tax %
(18) = Reinsurance commission % × (3)
(19) = (15) + (16) + (17) + (18)
(22) = Business earned in 1st year % × DWP × Agents' commission % + 2001 GAAP Deferred Commission in 2002
= Agents' commission % × DEP in other years
(23) = (16) × [1 − Deferrable %] + (16) × Business earned in 1st year % × Deferrable % + 2001 GAAP Deferred U/W Expense in 2002
= (16) × [1 − Deferrable %] + Deferrable % × [(16) × Business earned in 1st year % + (1 − Business earned in 1st year %)× prior yr (16)] in other years
(24) = Business earned in 1st year % × DWP × Premium Tax % + 2001 GAAP Deferred Premium Tax in 2002
= Premium Tax % × DEP in other years
(25) = (4) × Reinsurance Commission %
(26) = (22) + (23) + (24) + (25)

EXHIBIT 20
SHEET 3

PRIMARY STOCK INSURANCE COMPANY
GENERAL LIABILITY

	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
Underwriting Cash Flow For Lines of Business										
(A) Total Collected	$28,510	$29,130	$30,296	$31,508	$32,768	$34,078	$35,441	$36,859	$38,333	$39,868
(B) Net Loss and LAE Payments	17,145	18,624	19,999	21,310	22,469	23,602	24,728	25,848	26,964	28,109
(C) Underwriting Expense Paid	5,985	6,289	6,541	6,803	7,075	7,358	7,652	7,958	8,276	8,607
(D) Cash Flow from Underwriting	$5,380	$4,217	$3,756	$3,395	$3,224	$3,118	$3,061	$3,053	$3,093	$3,152
Modeled Amounts										
(1) Gross Premium Collected	$31,100	$32,344	$33,638	$34,984	$36,383	$37,837	$39,351	$40,925	$42,562	$44,266
(2) Premium Ceded	2,590	3,214	3,342	3,476	3,615	3,759	3,910	4,066	4,229	4,398
(3) Net Premium Collected	28,510	29,130	30,296	31,508	32,768	34,078	35,441	36,859	38,333	39,868
(4) Reinsurance Commissions	0	0	0	0	0	0	0	0	0	0
(5) Total Collected	28,510	29,130	30,296	31,508	32,768	34,078	35,441	36,859	38,333	39,868
(6) Gross Losses Paid	14,362	16,102	17,611	18,997	20,210	21,355	22,479	23,583	24,668	25,769
(7) Loss Recoveries Received	1,854	2,203	2,383	2,554	2,714	2,870	3,025	3,180	3,334	3,491
(8) Net Loss Paid	12,508	13,899	15,228	16,443	17,496	18,485	19,454	20,403	21,334	22,278
(9) Gross ALAE Paid	2,154	2,415	2,642	2,850	3,031	3,203	3,372	3,537	3,700	3,865
(10) ALAE Recoveries Received	42	106	167	220	270	311	352	391	427	461
(11) Net ALAE Paid	2,112	2,309	2,475	2,630	2,761	2,892	3,020	3,146	3,273	3,404
(12) ULAE Paid	2,525	2,416	2,296	2,237	2,212	2,225	2,254	2,299	2,357	2,427
(13) Net Loss & LAE Payments	17,145	18,624	19,999	21,310	22,469	23,602	24,728	25,848	26,964	28,109
(14) Agents' Commissions	3,825	4,043	4,205	4,373	4,548	4,730	4,919	5,116	5,320	5,533
(15) Other Underwriting Expenses	1,536	1,597	1,661	1,728	1,797	1,869	1,943	2,021	2,102	2,186
(16) Premium Taxes Paid	624	649	675	702	730	759	790	821	854	888
(17) Underwriting Expense Paid	5,985	6,289	6,541	6,803	7,075	7,358	7,652	7,958	8,276	8,607

Calculation Notes:
(A) = (5); (B) = (13); (C) = (17); (D) = (A) – (B) – (C)
(1) = DWP × (1 – Monthly Premium Collection Lag / 12) + 2001 Direct Premium Uncollected in 2002
= DWP × (1 – Monthly Premium Collection Lag / 12) + prior year DWP × Monthly Premium Collection Lag / 12 in other years
(2) = Ceded WP × (1 – Monthly Ceding Premium Lag / 12) + 2001 Ceded Unearned Premium in 2002
= Ceded WP × (1 – Monthly Ceding Premium Lag / 12) + prior year Ceded WP × Premium Collection Lag / 12 in other years
(3) = (1) – (2)
(4) = Reinsurance commission % × (2)
(5) = (3) + (4)
(8) = (6) – (7)
(11) = (9) – (10)
(13) = (8) + (11) + (12)
(14) = Agents' commission from cash flow × (1 – Monthly Premium Collection Lag / 12) + 2001 Unpaid Agents' Commission in 2002
= Agents' commission × (1 – Monthly Premium Collection Lag / 12) + Prior year Agents' commission × Monthly Premium Collection Lag / 12 in other years
(15) = Fixed underwriting $ + (% of DEP × DEP) + (% of DWP × DWP)
(16) = DWP × Premium tax %
(17) = (14) + (15) + (16)

EXHIBIT 20
SHEET 4

FINANCIAL MODELING ASSUMPTIONS, GENERAL LIABILITY
MODELING ASSUMPTIONS FOR CURRENT AND FUTURE BUSINESS

Amounts as of 12/31/01		Assumptions for Future Business	
Gross Unearned Premium	$15,000	Gross Written Premium in 2002	$31,200
Net Unearned Premium	$13,500	Annual Growth Rate of Business	4.00%
Ceded Unearned Premium	1,500		
Gross Premium Uncollected	$2,500	Percent of Business Earned in 1st Year	50.00%
Unpaid Agent's Commission	250	Premium Collection Lag (in months)	1
Ceded Premium Not Yet Remitted	250		
Ceded Paid Losses Not Yet Coll.	0	Expected Loss Ratio	68.00%
Ceded Paid ALAE Not Yet Coll.	0	Gross ALAE as a % of Loss	15.00%
Reinsurance Comm. Not Yet Coll.	0	Gross ULAE as a % of Loss	8.50%
GAAP Deferred Commission	1,875		
GAAP Deferred U/W Expense	0	Agents' Commission as % of DWP	12.50%
GAAP Deferred Premium Tax	270	Premium Tax as % of DWP	2.00%
GAAP Deferred Reins. Commission	0	Other Underwriting Expenses	
		Fixed	$0
		Variable (% of DEP)	4.00%
		Variable (% of DWP)	1.00%
		Reinsurance (per occ Excess)	
		Percent of premium ceded	10.00%
		Lag in Ceding Premium (in months)	3
		Ceded Loss Ratio	100.00%
		Ceded Loss Collection Lag (in months)	1

Loss and LAE reserves are carried at nominal (undiscounted) value for both SAP and GAAP.
"Reserve strengthening" adjustments are not needed; ultimate loss and LAE amounts do not deteriorate or improve over time.

EXHIBIT 20
Sheet 5

Financial Modeling Assumptions
General Liability
Loss Payment and Discounting

Payment Patterns for Loss and LAE				Interest Rate for Discounted Tax Reserves	
Accident Year +	Gross	Ceded	IRS	Accident Year	Rate
0	15.00%	10.00%	17.00%	1982	7.20%
1	19.00%	14.00%	21.00%	1983	7.20%
2	17.00%	12.00%	19.00%	1984	7.20%
3	12.00%	10.00%	11.00%	1985	7.20%
4	10.00%	9.00%	9.00%	1986	7.20%
5	6.00%	6.25%	5.70%	1987	7.20%
6	5.00%	6.25%	4.00%	1988	7.77%
7	4.00%	5.25%	3.50%	1989	8.16%
8	3.00%	4.50%	2.50%	1990	8.37%
9	2.00%	3.50%	2.00%	1991	7.00%
10	1.75%	3.50%	1.75%	1992	7.00%
11	1.50%	3.50%	1.50%	1993	7.00%
12	1.25%	3.50%	1.00%	1994	7.00%
13	1.00%	3.00%	0.50%	1995	7.00%
14	0.75%	2.50%	0.50%	1996	7.00%
15	0.50%	2.00%	0.00%	1997	7.00%
16	0.25%	1.25%		1998	7.00%
17	0.00%	0.00%		1999	7.00%
18	0.00%	0.00%		2000	7.00%
19	0.00%	0.00%		2001	7.00%
20	0.00%	0.00%		2002	7.00%
21	0.00%	0.00%		2003	7.00%
22	0.00%	0.00%		2004	7.00%
23	0.00%	0.00%		2005	7.00%
24	0.00%	0.00%		2006	7.00%
25	0.00%	0.00%		2007	7.00%
26	0.00%	0.00%		2008	7.00%
27	0.00%	0.00%		2009	7.00%
28	0.00%	0.00%		2010	7.00%
29	0.00%	0.00%		2011	7.00%
30	0.00%	0.00%			
Total	100.00%	100.00%	99.95%		

EXHIBIT 20
SHEET 6

FINANCIAL MODELING ASSUMPTIONS
GENERAL LIABILITY
PRIOR YEARS' INFORMATION

	Estimates of Ultimate Loss and LAE @ 12/31/01							Paid Loss and LAE @ 12/31/01						
Accident Year	Gross Loss	Ceded Loss	Net Loss	Gross ALAE	Ceded ALAE	Net ALAE	ULAE	Gross Loss	Ceded Loss	Net Loss	Gross ALAE	Ceded ALAE	Net ALAE	ULAE
1982	8,714	1,282	7,433	1,307	0	1,307	741	8,714	1,282	7,433	1,307	0	1,307	733
1983	9,063	1,333	7,730	1,359	0	1,359	770	9,063	1,333	7,730	1,359	0	1,359	755
1984	9,426	1,386	8,039	1,414	0	1,414	801	9,426	1,386	8,039	1,414	0	1,414	777
1985	9,803	1,442	8,361	1,470	0	1,470	833	9,803	1,442	8,361	1,470	0	1,470	800
1986	10,195	1,499	8,695	1,529	0	1,529	867	10,195	1,499	8,695	1,529	0	1,529	823
1987	10,602	1,559	9,043	1,590	0	1,590	901	10,576	1,540	9,036	1,586	0	1,586	847
1988	11,027	1,622	9,405	1,654	0	1,654	937	10,944	1,569	9,375	1,642	0	1,642	862
1989	11,468	1,686	9,781	1,720	0	1,720	975	11,296	1,589	9,706	1,694	0	1,694	887
1990	11,926	1,754	10,172	1,789	0	1,789	1,014	11,628	1,600	10,028	1,744	0	1,744	912
1991	12,403	1,824	10,579	1,861	0	1,861	1,054	11,938	1,601	10,338	1,791	0	1,791	928
1992	12,899	1,897	11,003	1,935	0	1,935	1,096	12,222	1,598	10,624	1,833	0	1,833	943
1993	13,415	1,973	11,443	2,012	0	2,012	1,140	12,476	1,593	10,883	1,871	0	1,871	958
1994	13,952	2,052	11,900	2,093	0	2,093	1,186	12,696	1,585	11,111	1,904	0	1,904	949
1995	14,510	2,134	12,376	2,177	0	2,177	1,233	12,769	1,552	11,217	1,915	0	1,915	925
1996	15,091	2,219	12,871	2,264	0	2,264	1,283	12,676	1,498	11,178	1,901	0	1,901	898
1997	15,694	2,308	13,386	2,354	0	2,354	1,334	12,398	1,414	10,985	1,860	0	1,860	800
1998	16,322	2,400	13,922	2,448	0	2,448	1,387	11,915	1,320	10,595	1,787	0	1,787	694
1999	16,975	2,496	14,479	2,546	0	2,546	1,443	10,694	1,148	9,546	1,604	0	1,604	577
2000	17,654	2,596	15,058	2,648	0	2,648	1,501	9,003	935	8,069	1,351	0	1,351	450
2001	18,360	2,700	15,660	2,754	0	2,754	1,561	2,754	270	2,484	413	0	413	312

Loss and LAE Reserves @ 12/31/01

Net Accident Year	Gross Loss	Ceded Loss	Net Loss	Gross ALAE	Ceded ALAE	Net ALAE	ULAE	Earned Premium
1982	0	0	0	0	0	0	7	12,815
1983	0	0	0	0	0	0	15	13,328
1984	0	0	0	0	0	0	24	13,861
1985	0	0	0	0	0	0	33	14,416
1986	0	0	0	0	0	0	43	14,992
1987	27	19	7	4	0	4	54	15,592
1988	83	53	30	12	0	12	75	16,216
1989	172	97	75	26	0	26	88	16,864
1990	298	153	145	45	0	45	101	17,539
1991	465	223	242	70	0	70	127	18,240
1992	677	299	378	102	0	102	154	18,970
1993	939	380	559	141	0	141	182	19,729
1994	1,256	467	789	188	0	188	237	20,518
1995	1,741	581	1,160	261	0	261	308	21,338
1996	2,414	721	1,693	362	0	362	385	22,192
1997	3,296	894	2,401	494	0	494	534	23,080
1998	4,407	1,080	3,327	661	0	661	694	24,003
1999	6,281	1,348	4,933	942	0	942	866	24,963
2000	8,650	1,662	6,989	1,298	0	1,298	1,050	25,962
2001	15,606	2,430	13,176	2,341	0	2,341	1,248	27,000
Total	46,312	10,408	35,904	6,947	0	6,947	6,226	

EXHIBIT 21
SHEET 1

PRIMARY STOCK INSURANCE COMPANY
VALUATION ESTIMATES AS OF DECEMBER 31, 2001 SURPLUS-TO-RBC RATIO OF 2.5

Establishing Starting Surplus											
(1) Booked Statutory Surplus @ 12/31/01	45,000										
(2) Selected Starting Surplus for Future Income Projections (based on 12/31/01 RBC level)	52,671										
(3) Income Recognized @ 12/31/01	(7,671)										
Monitoring and Selecting Surplus	As of 12/31 of:										
	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
(4) Selected Surplus	$52,671	$58,560	$64,630	$69,708	$74,145	$78,251	$82,202	$86,104	$90,027	$94,003	$98,067
(5) Indicated Risk-Based Capital Company Action Level	21,069	23,424	25,852	27,883	29,658	31,300	32,880	34,440	36,007	37,593	39,211
(6) Surplus-to-RBC Ratio	250.0%	250.0%	250.0%	250.0%	250.0%	250.0%	250.0%	250.0%	250.0%	250.1%	250.1%
(7) Surplus Growth		5,889	11,959	17,036	21,474	25,580	29,531	33,433	37,355	41,331	45,395
(8) Net Written Premium-to-Surplus Ratio	1.59	1.49	1.41	1.36	1.33	1.31	1.29	1.28	1.28	1.27	1.27
(9) Loss and LAE Reserves-to-Surplus Ratio	2.03	2.15	2.18	2.18	2.18	2.18	2.18	2.17	2.17	2.16	2.16
Estimated Future Income		During:									
		2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
(10) Total Company Net Income		11,074	12,761	13,958	14,978	15,852	16,672	17,467	18,236	19,010	19,793
Hurdle Rate	@ 12/31/ 2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
(11) Selected Hurdle Rate		15.0%	15.0%	15.0%	15.0%	15.0%	15.0%	15.0%	15.0%	15.0%	15.0%
(12) Present Value Factor for 12/31/01 (assuming year end income recognition)	1.000	0.870	0.756	0.658	0.572	0.497	0.432	0.376	0.327	0.284	0.247
Expected Annual Perpetual Growth Rate of: Capital & Income After 2011		2.0%									

Application of DCF Method	@ 12/31/ 2001	During: 2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	Total '01 to '11	Total '12 to ∞	All Years
(13) Total Company Net Income	(7,671)	11,074	12,761	13,958	14,978	15,852	16,672	17,467	18,236	19,010	19,793			
(13pv) PV of Total Company Net Income	(7,671)	9,630	9,649	9,178	8,564	7,881	7,208	6,566	5,961	5,404	4,893	67,262	38,388	105,650
(14) Reinvestment Cost to Grow Company		5,889	6,070	5,078	4,438	4,106	3,951	3,902	3,923	3,976	4,064			
(14pv) PV of Reinv. Cost to Grow Company		5,121	4,590	3,339	2,537	2,041	1,708	1,467	1,282	1,130	1,005	24,220	3,729	27,949
(15) Free Cash Flow	(7,671)	5,185	6,691	8,881	10,541	11,746	12,721	13,565	14,314	15,034	15,729			
(15pv) PV of Free Cash Flow	(7,671)	4,509	5,059	5,839	6,027	5,840	5,500	5,100	4,679	4,274	3,888	43,042	34,658	77,701

Application of EVA Method	@ 12/31/ 2001	During: 2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	Total '01 to '11	Total '12 to ∞	All Years
(16) Statutory Surplus @ 12/31/01	52,671													
(17) Total Company Net Income	(7,671)	11,074	12,761	13,958	14,978	15,852	16,672	17,467	18,236	19,010	19,793			
(17pv) PV of Total Company Net Income	(7,671)	9,630	9,649	9,178	8,564	7,881	7,208	6,566	5,961	5,404	4,893	67,262	38,388	105,650
(18) Cost of Capital		7,901	8,784	9,695	10,456	11,122	11,738	12,330	12,916	13,504	14,100			
(18pv) PV of Cost of Capital		6,870	6,642	6,374	5,978	5,529	5,075	4,635	4,222	3,839	3,485	52,650	27,970	80,620
(19) Cost of Starting Capital		7,901	7,901	7,901	7,901	7,901	7,901	7,901	7,901	7,901	7,901			
(19pv) PV of Cost of Starting Capital		6,870	5,974	5,195	4,517	3,928	3,416	2,970	2,583	2,246	1,953	39,652	13,020	52,671
(20) Cost of Growth Capital			883	1,794	2,555	3,221	3,837	4,430	5,015	5,603	6,200			
(20pv) PV of Cost of Growth Capital			668	1,179	1,461	1,601	1,659	1,665	1,639	1,593	1,532	12,999	14,950	27,949
(21) Excess Returns in Year	(7,671)	3,173	3,977	4,264	4,522	4,730	4,934	5,137	5,320	5,506	5,693			
(21pv) PV of Excess Returns in Year	(7,671)	2,759	3,007	2,803	2,585	2,352	2,133	1,931	1,739	1,565	1,407	14,612	10,418	25,029
(22) Total Indicated Value														77,701

EXHIBIT 21
SHEET 2

PRIMARY STOCK INSURANCE COMPANY
CALCULATION NOTES FOR VALUATION ESTIMATES AS OF DECEMBER 31, 2001

(1) from Primary Stock Insurance Company's 12/31/01 booked balance sheet
(2) selected starting surplus for future income projections based on the selected 250.0% surplus to indicated RBC ratio
(3) = (1) – (2)
(4) selected surplus based on the selected 250.0% surplus to indicated RBC ratio
(5) calculated RBC at the Company Action Level
(6) selection of 250.0% surplus to RBC ratio for determining required surplus at each year end
(7) cumulative increase in (4) from starting surplus, (4) – (2)
(8) NPW for all lines ÷ (4)
(9) net loss and LAE reserves for all lines ÷ (4)
(10) = (11) Subtotal: Company Income from Change in Statutory Surplus
(11) is selected hurdle of 15.0% used for determining Cost of Capital in EVA method and the present value of future earnings and value added
(12) = 1.000 at 12/31/01; for future years = (1.0 + 15.0%) raised to (year – 2001) exponent
(13) = (10)
(13pv) = (13) × (12) for each year; Total '01 to '11 is the total of the by year estimates; Total '12 to ∞ = $(13)_{2011}$ × (1 + Growth Rate) ÷ (Hurdle Rate – Growth Rate) × $(12)_{2011}$;
All Years = Total '01 to '11 + Total '12 to ∞
(14) annual change in (4)
(14pv) = (14) × (12) for each year; Total '01 to '11 is the total of the by year estimates; Total '12 to ∞ = $(4)_{2011}$ × Growth Rate ÷ (Hurdle Rate – Growth Rate) × $(12)_{2011}$;
All Years = Total '01 to '11 + Total '12 to ∞

(15) = (13) – (14)
(15pv) = (13pv) – (14pv)
(16) = (2)
(17) = (10)
(17pv) = (17) × (12) for each year; Total '01 to '11 is the total of the by year estimates; Total '12 to ∞ = $(17)_{2011}$ × (1 + Growth Rate) ÷ (Hurdle Rate – Growth Rate) × $(12)_{2011}$;
All Years = Total '01 to '11 + Total '12 to ∞
(18) = (19) + (20)
(18pv) = (19pv) + (20pv)
(19) = (2) × (11)
(19pv) = (19) × (12) for each year; Total '01 to '11 is the total of the by year estimates; Total '12 to ∞ = (16) × $(12)_{2011}$;
All Years = Total '01 to '11 + Total '12 to ∞
(20) = $(7)_{\text{prior year}}$ × (11)
(20pv) = (20) × (12) for each year; Total '01 to '11 is the total of the by year estimates; Total '12 to ∞ = [$(4)_{2011}$ × Hurdle Rate ÷ (Hurdle Rate – Growth Rate) – (16)] × $(12)_{2011}$;
All Years = Total '01 to '11 + Total '12 to ∞
(21) = (17) – (18)4
(21pv) = (17pv) – (18pv)
(22) = (16) + $(21pv)_{\text{All Years}}$

EXHIBIT 21
SHEET 3

PRIMARY STOCK INSURANCE COMPANY
VALUATION ESTIMATES AS OF DECEMBER 31, 2001 USING BASE LOSS RATIOS +2%

Establishing Starting Surplus											
(1) Booked Statutory Surplus @ 12/31/01	45,000										
(2) Selected Starting Surplus for Future Income Projections (based on 12/31/01 RBC level)	42,131										
(3) Income Recognized @ 12/31/01	2,869										
Monitoring and Selecting Surplus	As of 12/31 of:										
	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
(4) Selected Surplus	$42,131	$47,054	$52,310	$56,700	$60,506	$64,018	$67,386	$70,708	$74,038	$77,410	$80,870
(5) Indicated Risk-Based Capital Company Action Level	21,069	23,527	26,155	28,350	30,253	32,009	33,693	35,354	37,019	38,705	40,435
(6) Surplus-to-RBC Ratio	200.0%	200.0%	200.0%	200.0%	200.0%	200.0%	200.0%	200.0%	200.0%	200.0%	200.0%
(7) Surplus Growth		4,923	10,179	14,569	18,375	21,887	25,255	28,577	31,907	35,279	38,739
(8) Net Written Premium-to-Surplus Ratio	1.99	1.86	1.74	1.67	1.62	1.60	1.58	1.56	1.55	1.54	1.54
(9) Loss and LAE Reserves-to-Surplus Ratio	2.54	2.72	2.74	2.75	2.75	2.75	2.74	2.73	2.73	2.72	2.71
Estimated Future Income		During:									
		2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
(10) Total Company Net Income		8,998	10,675	11,848	12,828	13,636	14,393	15,122	15,835	16,544	17,257
Hurdle Rate	@ 12/31/ 2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
(11) Selected Hurdle Rate		15.0%	15.0%	15.0%	15.0%	15.0%	15.0%	15.0%	15.0%	15.0%	15.0%
(12) Present Value Factor for 12/31/01 (assuming year end income recognition)	1.000	0.870	0.756	0.658	0.572	0.497	0.432	0.376	0.327	0.284	0.247
Expected Annual Perpetual Growth Rate of: Capital & Income After 2011		2.0%									

Application of DCF Method	@ 12/31/ 2001	During: 2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	Total '01 to '11	Total '12 to ∞	All Years
(13) Total Company Net Income	2,869	8,998	10,675	11,848	12,828	13,636	14,393	15,122	15,835	16,544	17,257			
(13pv) PV of Total Company Net Income	2,869	7,824	8,072	7,790	7,334	6,780	6,222	5,685	5,176	4,703	4,266	66,722	33,469	100,191
(14) Reinvestment Cost to Grow Company		4,923	5,256	4,390	3,806	3,512	3,368	3,322	3,330	3,372	3,460			
(14pv) PV of Reinv. Cost to Grow Company		4,281	3,974	2,886	2,176	1,746	1,456	1,249	1,089	959	855	20,671	3,075	23,746
(15) Free Cash Flow	2,869	4,075	5,419	7,458	9,022	10,124	11,025	11,800	12,505	13,172	13,797			
(15pv) PV of Free Cash Flow	2,869	3,544	4,098	4,904	5,158	5,033	4,766	4,436	4,088	3,744	3,410	46,051	30,394	76,444

Application of EVA Method	@ 12/31/ 2001	During: 2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	Total '01 to '11	Total '12 to ∞	All Years
(16) Statutory Surplus @ 12/31/01	42,131													
(17) Total Company Net Income	2,869	8,998	10,675	11,848	12,828	13,636	14,393	15,122	15,835	16,544	17,257			
(17pv) PV of Total Company Net Income	2,869	7,824	8,072	7,790	7,334	6,780	6,222	5,685	5,176	4,703	4,266	66,722	33,469	100,191
(18) Cost of Capital		6,320	7,058	7,847	8,505	9,076	9,603	10,108	10,606	11,106	11,612			
(18pv) PV of Cost of Capital		5,495	5,337	5,159	4,863	4,512	4,152	3,800	3,467	3,157	2,870	42,812	23,065	65,878
(19) Cost of Starting Capital		6,320	6,320	6,320	6,320	6,320	6,320	6,320	6,320	6,320	6,320			
(19pv) PV of Cost of Starting Capital		5,495	4,779	4,155	3,613	3,142	2,732	2,376	2,066	1,796	1,562	31,717	10,414	42,131
(20) Cost of Growth Capital			738	1,527	2,185	2,756	3,283	3,788	4,287	4,786	5,292			
(20pv) PV of Cost of Growth Capital			558	1,004	1,249	1,370	1,419	1,424	1,401	1,360	1,308	11,095	12,651	23,746
(21) Excess Returns in Year	2,869	2,678	3,617	4,002	4,323	4,560	4,790	5,014	5,229	5,438	5,646			
(21pv) PV of Excess Returns in Year	2,869	2,329	2,735	2,631	2,472	2,267	2,071	1,885	1,709	1,546	1,395	23,909	10,404	34,313
(22) Total Indicated Value														76,444

EXHIBIT 21
SHEET 4

PRIMARY STOCK INSURANCE COMPANY
CALCULATION NOTES FOR VALUATION ESTIMATES AS OF DECEMBER 31, 2001

(1) from Primary Stock Insurance Company's 12/31/01 booked balance sheet
(2) selected starting surplus for future income projections based on the selected 200.0% surplus to indicated RBC ratio
(3) = (1) – (2)
(4) selected surplus based on the selected 200.0% surplus to indicated RBC ratio
(5) calculated RBC at the Company Action Level
(6) selection of 200.0% surplus to RBC ratio for determining required surplus at each year end
(7) cumulative increase in (4) from starting surplus, (4) – (2)
(8) NPW for all lines ÷ (4)
(9) net loss and LAE reserves for all lines ÷ (4)
(10) = (11) Subtotal: Company Income from Change in Statutory Surplus
(11) is selected hurdle of 15.0% used for determining Cost of Capital in EVA method and the present value of future earnings and value added
(12) = 1.000 at 12/31/01; for future years = (1.0 + 15.0%) raised to (year – 2001) exponent
(13) = (10)
(13pv) = (13) × (12) for each year; Total '01 to '11 is the total of the by year estimates; Total '12 to ∞ = $(13)_{2011}$ × (1 + Growth Rate) ÷ (Hurdle Rate – Growth Rate) × $(12)_{2011}$;
All Years = Total '01 to '11 + Total '12 to ∞
(14) annual change in (4)
(14pv) = (14) × (12) for each year; Total '01 to '11 is the total of the by year estimates; Total '12 to ∞ = $(4)_{2011}$ × Growth Rate ÷ (Hurdle Rate – Growth Rate) × $(12)_{2011}$;
All Years = Total '01 to '11 + Total '12 to ∞

(15) = (13) – (14)
(15pv) = (13pv) – (14pv)
(16) = (2)
(17) = (10)
(17pv) = (17) × (12) for each year; Total '01 to '11 is the total of the by year estimates; Total '12 to ∞ = $(17)_{2011}$ × (1 + Growth Rate) ÷ (Hurdle Rate – Growth Rate) × $(12)_{2011}$;
All Years = Total '01 to '11 + Total '12 to ∞
(18) = (19) + (20)
(18pv) = (19pv) + (20pv)
(19) = (2) × (11)
(19pv) = (19) × (12) for each year; Total '01 to '11 is the total of the by year estimates; Total '12 to ∞ = (16) × $(12)_{2011}$;
All Years = Total '01 to '11 + Total '12 to ∞
(20) = $(7)_{\text{prior year}}$ × (11)
(20pv) = (20) × (12) for each year; Total '01 to '11 is the total of the by year estimates; Total '12 to ∞ = [$(4)_{2011}$ × Hurdle Rate ÷ (Hurdle Rate – Growth Rate) – (16)] × $(12)_{2011}$;
All Years = Total '01 to '11 + Total '12 to ∞
(21) = (17) – (18)
(21pv) = (17pv) – (18pv)
(22) = (16) + $(21pv)_{\text{All Years}}$

EXHIBIT 21
Sheet 5

Primary Stock Insurance Company
Valuation Estimates as of December 31, 2001 Using Base Loss Ratios −2%

Establishing Starting Surplus											
(1) Booked Statutory Surplus @ 12/31/01	45,000										
(2) Selected Starting Surplus for Future Income Projections (based on 12/31/01 RBC level)	42,131										
(3) Income Recognized @ 12/31/01	2,869										
Monitoring and Selecting Surplus	As of 12/31 of:										
	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
(4) Selected Surplus	$42,131	$45,978	$50,376	$54,094	$57,276	$60,280	$63,192	$66,078	$68,980	$71,924	$74,946
(5) Indicated Risk-Based Capital Company Action Level	21,069	22,989	25,188	27,047	28,638	30,140	31,596	33,039	34,490	35,962	37,473
(6) Surplus-to-RBC Ratio	200.0%	200.0%	200.0%	200.0%	200.0%	200.0%	200.0%	200.0%	200.0%	200.0%	200.0%
(7) Surplus Growth		3,847	8,245	11,963	15,145	18,149	21,061	23,947	26,849	29,793	32,815
(8) Net Written Premium-to-Surplus Ratio	1.99	1.90	1.80	1.75	1.72	1.70	1.68	1.67	1.67	1.66	1.66
(9) Loss and LAE Reserves-to-Surplus Ratio	2.54	2.71	2.74	2.74	2.75	2.75	2.74	2.74	2.74	2.73	2.73
Estimated Future Income		During:									
		2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
(10) Total Company Net Income		11,198	12,752	13,866	17,761	16,299	17,077	17,837	18,595	19,357	20,149
Hurdle Rate	@ 12/31/ 2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
(11) Selected Hurdle Rate		15.0%	15.0%	15.0%	15.0%	15.0%	15.0%	15.0%	15.0%	15.0%	15.0%
(12) Present Value Factor for 12/31/01 (assuming year end income recognition)	1.000	0.870	0.756	0.658	0.572	0.497	0.432	0.376	0.327	0.284	0.247
Expected Annual Perpetual Growth Rate of: Capital & Income After 2011		2.0%									

Application of DCF Method	@ 12/31/ 2001	During: 2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	Total '01 to '11	Total '12 to ∞	All Years
(13) Total Company Net Income	2,869	11,198	12,752	13,866	17,761	16,299	17,077	17,837	18,595	19,357	20,149			
(13pv) PV of Total Company Net Income	2,869	9,737	9,642	9,117	10,155	8,103	7,383	6,706	6,079	5,502	4,981	80,274	39,078	119,352
(14) Reinvestment Cost to Grow Company		3,847	4,398	3,718	3,182	3,004	2,912	2,886	2,902	2,944	3,022			
(14pv) PV of Reinv. Cost to Grow Company		3,345	3,326	2,445	1,819	1,494	1,259	1,085	949	837	747	17,305	2,850	20,155
(15) Free Cash Flow	2,869	7,351	8,354	10,148	14,579	13,295	14,165	14,951	15,693	16,413	17,127			
(15pv) PV of Free Cash Flow	2,869	6,392	6,317	6,672	8,336	6,610	6,124	5,621	5,130	4,666	4,234	62,970	36,228	99,198

Application of EVA Method	@ 12/31/ 2001	During: 2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	Total '01 to '11	Total '12 to ∞	All Years
(16) Statutory Surplus @ 12/31/01	42,131													
(17) Total Company Net Income	2,869	11,198	12,752	13,866	17,761	16,299	17,077	17,837	18,595	19,357	20,149			
(17pv) PV of Total Company Net Income	2,869	9,737	9,642	9,117	10,155	8,103	7,383	6,706	6,079	5,502	4,981	80,274	39,078	119,352
(18) Cost of Capital		6,320	6,897	7,556	8,114	8,591	9,042	9,479	9,912	10,347	10,789			
(18pv) PV of Cost of Capital		5,495	5,215	4,968	4,639	4,271	3,909	3,563	3,240	2,941	2,667	40,910	21,376	62,286
(19) Cost of Starting Capital		6,320	6,320	6,320	6,320	6,320	6,320	6,320	6,320	6,320	6,320			
(19pv) PV of Cost of Starting Capital		5,495	4,779	4,155	3,613	3,142	2,732	2,376	2,066	1,796	1,562	31,717	10,414	42,131
(20) Cost of Growth Capital			577	1,237	1,794	2,272	2,722	3,159	3,592	4,027	4,469			
(20pv) PV of Cost of Growth Capital			436	813	1,026	1,129	1,177	1,188	1,174	1,145	1,105	9,193	10,961	20,155
(21) Excess Returns in Year	2,869	4,878	5,855	6,310	9,647	7,708	8,035	8,358	8,683	9,010	9,360			
(21pv) PV of Excess Returns in Year	2,869	4,242	4,427	4,149	5,516	3,832	3,474	3,142	2,839	2,561	2,314	39,364	17,702	57,067
(22) Total Indicated Value														99,198

EXHIBIT 21
SHEET 6

PRIMARY STOCK INSURANCE COMPANY
CALCULATION NOTES FOR VALUATION ESTIMATES AS OF DECEMBER 31, 2001

(1) from Primary Stock Insurance Company's 12/31/01 booked balance sheet
(2) selected starting surplus for future income projections based on the selected 200.0% surplus to indicated RBC ratio
(3) = (1) − (2)
(4) selected surplus based on the selected 200.0% surplus to indicated RBC ratio
(5) calculated RBC at the Company Action Level
(6) selection of 200.0% surplus to RBC ratio for determining required surplus at each year end
(7) cumulative increase in (4) from starting surplus, (4) − (2)
(8) NPW for all lines ÷ (4)
(9) net loss and LAE reserves for all lines ÷ (4)
(10) = (11) Subtotal: Company Income from Change in Statutory Surplus
(11) is selected hurdle of 15.0% used for determining Cost of Capital in EVA method and the present value of future earnings and value added
(12) = 1.000 at 12/31/01; for future years = (1.0 + 15.0%) raised to (year − 2001) exponent
(13) = (10)
(13pv) = (13) × (12) for each year; Total '01 to '11 is the total of the by year estimates; Total '12 to ∞ = $(13)_{2011}$ × (1 + Growth Rate) ÷ (Hurdle Rate − Growth Rate) × $(12)_{2011}$;
All Years = Total '01 to '11 + Total '12 to ∞
(14) annual change in (4)
(14pv) = (14) × (12) for each year; Total '01 to '11 is the total of the by year estimates; Total '12 to ∞ = $(4)_{2011}$ × Growth Rate ÷ (Hurdle Rate − Growth Rate) × $(12)_{2011}$;
All Years = Total '01 to '11 + Total '12 to ∞

(15) = (13) – (14)
(15pv) = (13pv) – (14pv)
(16) = (2)
(17) = (10)
(17pv) = (17) × (12) for each year; Total '01 to '11 is the total of the by year estimates; Total '12 to ∞ = $(17)_{2011}$ × (1 + Growth Rate) ÷ (Hurdle Rate – Growth Rate) × $(12)_{2011}$;
All Years = Total '01 to '11 + Total '12 to ∞
(18) = (19) + (20)
(18pv) = (19pv) + (20pv)
(19) = (2) × (11)
(19pv) = (19) × (12) for each year; Total '01 to '11 is the total of the by year estimates; Total '12 to ∞ = (16) × $(12)_{2011}$;
All Years = Total '01 to '11 + Total '12 to ∞
(20) = $(7)_{\text{prior year}}$ × (11)
(20pv) = (20) × (12) for each year; Total '01 to '11 is the total of the by year estimates; Total '12 to ∞ = [$(4)_{2011}$ × Hurdle Rate ÷ (Hurdle Rate – Growth Rate) – (16)] × $(12)_{2011}$;
All Years = Total '01 to '11 + Total '12 to ∞
(21) = (17) – (18)
(21pv) = (17pv) – (18pv)
(22) = (16) + $(21pv)_{\text{All Years}}$

EXHIBIT 21
Sheet 7

Primary Stock Insurance Company
Valuation Estimates as of December 31, 2001 Using Base Investment Yields +100bp

Establishing Starting Surplus											
(1) Booked Statutory Surplus @ 12/31/01	45,000										
(2) Selected Starting Surplus for Future Income Projections (based on 12/31/01 RBC level)	42,131										
(3) Income Recognized @ 12/31/01	2,869										
Monitoring and Selecting Surplus	As of 12/31 of:										
	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
(4) Selected Surplus	$42,131	$46,514	$51,334	$55,368	$58,896	$62,156	$65,296	$68,396	$71,508	$74,660	$77,872
(5) Indicated Risk-Based Capital Company Action Level	21,069	23,257	25,667	27,684	29,448	31,078	32,648	34,198	35,754	37,330	38,936
(6) Surplus-to-RBC Ratio	200.0%	200.0%	200.0%	200.0%	200.0%	200.0%	200.0%	200.0%	200.0%	200.0%	200.0%
(7) Surplus Growth	4,383	9,203	13,237	16,765	20,025	23,165	26,265	29,377	32,529	35,741	
(8) Net Written Premium-to-Surplus Ratio	1.99	1.88	1.77	1.71	1.67	1.64	1.63	1.62	1.61	1.60	1.60
(9) Loss and LAE Reserves-to-Surplus Ratio	2.54	2.71	2.74	2.75	2.75	2.75	2.74	2.74	2.73	2.73	2.72
Estimated Future Income		During:									
		2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
(10) Total Company Net Income		12,816	14,093	15,400	16,511	17,445	18,325	19,180	20,023	20,875	21,754
Hurdle Rate	@ 12/31/ 2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
(11) Selected Hurdle Rate		15.0%	15.0%	15.0%	15.0%	15.0%	15.0%	15.0%	15.0%	15.0%	15.0%
(12) Present Value Factor for 12/31/01 (assuming year-end income recognition)	1.000	0.870	0.756	0.658	0.572	0.497	0.432	0.376	0.327	0.284	0.247
Expected Annual Perpetual Growth Rate of: Capital & Income After 2011		2.0%									

Application of DCF Method	@ 12/31/ 2001	During: 2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	Total '01 to '11	Total '12 to ∞	All Years
(13) Total Company Net Income	2,869	12,816	14,093	15,400	16,511	17,445	18,325	19,180	20,023	20,875	21,754			
(13pv) PV of Total Company Net Income	2,869	11,144	10,656	10,126	9,440	8,673	7,922	7,210	6,546	5,934	5,377	85,899	42,191	128,089
(14) Reinvestment Cost to Grow Company		4,383	4,820	4,034	3,528	3,260	3,140	3,100	3,112	3,152	3,212			
(14pv) PV of Reinv. Cost to Grow Company		3,811	3,645	2,652	2,017	1,621	1,358	1,165	1,017	896	794	18,976	2,961	21,938
(15) Free Cash Flow	2,869	8,433	9,273	11,366	12,983	14,185	15,185	16,080	16,911	17,723	18,542			
(15pv) PV of Free Cash Flow	2,869	7,333	7,012	7,473	7,423	7,052	6,565	6,045	5,528	5,038	4,583	66,922	39,229	106,152

Application of EVA Method	@ 12/31/ 2001	During: 2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	Total '01 to '11	Total '12 to ∞	All Years
(16) Statutory Surplus @ 12/31/01	42,131													
(17) Total Company Net Income	2,869	12,816	14,093	15,400	16,511	17,445	18,325	19,180	20,023	20,875	21,754			
(17pv) PV of Total Company Net Income	2,869	11,144	10,656	10,126	9,440	8,673	7,922	7,210	6,546	5,934	5,377	85,899	42,191	128,089
(18) Cost of Capital		6,320	6,977	7,700	8,305	8,834	9,323	9,794	10,259	10,726	11,199			
(18pv) PV of Cost of Capital		5,495	5,276	5,063	4,749	4,392	4,031	3,682	3,354	3,049	2,768	41,859	22,210	64,069
(19) Cost of Starting Capital		6,320	6,320	6,320	6,320	6,320	6,320	6,320	6,320	6,320	6,320			
(19pv) PV of Cost of Starting Capital		5,495	4,779	4,155	3,613	3,142	2,732	2,376	2,066	1,796	1,562	31,717	10,414	42,131
(20) Cost of Growth Capital			657	1,380	1,986	2,515	3,004	3,475	3,940	4,407	4,879			
(20pv) PV of Cost of Growth Capital			497	908	1,135	1,250	1,299	1,306	1,288	1,253	1,206	10,142	11,796	21,938
(21) Excess Returns in Year	2,869	6,496	7,116	7,700	8,206	8,611	9,002	9,386	9,764	10,149	10,555			
(21pv) PV of Excess Returns in Year	2,869	5,649	5,381	5,063	4,692	4,281	3,892	3,528	3,192	2,885	2,609	44,040	19,981	64,020
(22) Total Indicated Value														106,152

EXHIBIT 21
SHEET 8

PRIMARY STOCK INSURANCE COMPANY
CALCULATION NOTES FOR VALUATION ESTIMATES AS OF DECEMBER 31, 2001

(1) from Primary Stock Insurance Company's 12/31/01 booked balance sheet
(2) selected starting surplus for future income projections based on the selected 200.0% surplus to indicated RBC ratio
(3) = (1) – (2)
(4) selected surplus based on the selected 200.0% surplus to indicated RBC ratio
(5) calculated RBC at the Company Action Level
(6) selection of 200.0% surplus to RBC ratio for determining required surplus at each year end
(7) cumulative increase in (4) from starting surplus, (4) – (2)
(8) NPW for all lines ÷ (4)
(9) net loss and LAE reserves for all lines ÷ (4)
(10) = (11) Subtotal: Company Income from Change in Statutory Surplus
(11) is selected hurdle of 15.0% used for determining Cost of Capital in EVA method and the present value of future earnings and value added
(12) = 1.000 at 12/31/01; for future years = (1.0 + 15.0%) raised to (year – 2001) exponent
(13) = (10)
(13pv) = (13) × (12) for each year; Total '01 to '11 is the total of the by-year estimates; Total '12 to ∞ = $(13)_{2011}$ × (1 + Growth Rate) ÷
(Hurdle Rate – Growth Rate) × $(12)_{2011}$;
All Years = Total '01 to '11 + Total '12 to ∞
(14) annual change in (4)
(14pv) = (14) × (12) for each year; Total '01 to '11 is the total of the by-year estimates; Total '12 to ∞ = $(4)_{2011}$ × Growth Rate ÷
(Hurdle Rate – Growth Rate) × $(12)_{2011}$;
All Years = Total '01 to '11 + Total '12 to ∞

(15) = (13) – (14)
(15pv) = (13pv) – (14pv)
(16) = (2)
(17) = (10)
(17pv) = (17) × (12) for each year; Total '01 to '11 is the total of the by-year estimates; Total '12 to ∞ = $(17)_{2011}$ × (1 + Growth Rate) ÷ (Hurdle Rate – Growth Rate) × $(12)_{2011}$;
All Years = Total '01 to '11 + Total '12 to ∞
(18) = (19) + (20)
(18pv) = (19pv) + (20pv)
(19) = (2) × (11)
(19pv) = (19) × (12) for each year; Total '01 to '11 is the total of the by-year estimates; Total '12 to ∞ = (16) × $(12)_{2011}$;
All Years = Total '01 to '11 + Total '12 to ∞
(20) = $(7)_{\text{prior year}}$ × (11)
(20pv) = (20) × (12) for each year; Total '01 to '11 is the total of the by-year estimates; Total '12 to ∞ = [$(4)_{2011}$ × Hurdle Rate ÷ (Hurdle Rate – Growth Rate) – (16)] × $(12)_{2011}$;
All Years = Total '01 to '11 + Total '12 to ∞
(21) = (17) – (18)
(21pv) = (17pv) – (18pv)
(22) = (16) + $(21pv)_{\text{All Years}}$

EXHIBIT 21
SHEET 9

PRIMARY STOCK INSURANCE COMPANY
VALUATION ESTIMATES AS OF DECEMBER 31, 2001 USING BASE INVESTMENT YIELDS −100BP

Establishing Starting Surplus											
(1) Booked Statutory Surplus @ 12/31/01	45,000										
(2) Selected Starting Surplus for Future Income Projections (based on 12/31/01 RBC level)	42,131										
(3) Income Recognized @ 12/31/01	2,869										
Monitoring and Selecting Surplus	As of 12/31 of:										
	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
(4) Selected Surplus	$42,131	$46,504	$51,324	$55,356	$58,884	$62,144	$65,284	$68,382	$71,494	$74,644	$77,856
(5) Indicated Risk-Based Capital Company Action Level	21,069	23,252	25,662	27,678	29,442	31,072	32,642	34,191	35,747	37,322	38,928
(6) Surplus-to-RBC Ratio	200.0%	200.0%	200.0%	200.0%	200.0%	200.0%	200.0%	200.0%	200.0%	200.0%	200.0%
(7) Surplus Growth		4,373	9,193	13,225	16,753	20,013	23,153	26,251	29,363	32,513	35,725
(8) Net Written Premium-to-Surplus Ratio	1.99	1.88	1.77	1.71	1.67	1.64	1.63	1.62	1.61	1.60	1.60
(9) Loss and LAE Reserves-to-Surplus Ratio	2.54	2.71	2.74	2.75	2.75	2.75	2.74	2.74	2.73	2.73	2.72
Estimated Future Income		During:									
		2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
(10) Total Company Net Income		9,173	10,006	10,968	11,786	12,478	13,129	13,762	14,383	15,007	15,638
Hurdle Rate	@ 12/31/ 2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
(11) Selected Hurdle Rate		15.0%	15.0%	15.0%	15.0%	15.0%	15.0%	15.0%	15.0%	15.0%	15.0%
(12) Present Value Factor for 12/31/01 (assuming year-end income recognition)	1.000	0.870	0.756	0.658	0.572	0.497	0.432	0.376	0.327	0.284	0.247
Expected Annual Perpetual Growth Rate of: Capital & Income After 2011		2.0%									

Application of DCF Method	@ 12/31/ 2001	During: 2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	Total '01 to '11	Total '12 to ∞	All Years
(13) Total Company Net Income	2,869	9,173	10,006	10,968	11,786	12,478	13,129	13,762	14,383	15,007	15,638			
(13pv) PV of Total Company Net Income	2,869	7,977	7,566	7,212	6,739	6,204	5,676	5,174	4,702	4,266	3,865	62,248	30,329	92,578
(14) Reinvestment Cost to Grow Company		4,373	4,820	4,032	3,528	3,260	3,140	3,098	3,112	3,150	3,212			
(14pv) PV of Reinv. Cost to Grow Company		3,803	3,645	2,651	2,017	1,621	1,358	1,165	1,017	895	794	18,965	2,961	21,926
(15) Free Cash Flow	2,869	4,800	5,186	6,936	8,258	9,218	9,989	10,664	11,271	11,857	12,426			
(15pv) PV of Free Cash Flow	2,869	4,174	3,921	4,561	4,722	4,583	4,319	4,009	3,685	3,370	3,072	43,283	27,368	70,652

Application of EVA Method	@ 12/31/ 2001	During: 2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	Total '01 to '11	Total '12 to ∞	All Years
(16) Statutory Surplus @ 12/31/01	42,131													
(17) Total Company Net Income	2,869	9,173	10,006	10,968	11,786	12,478	13,129	13,762	14,383	15,007	15,638			
(17pv) PV of Total Company Net Income	2,869	7,977	7,566	7,212	6,739	6,204	5,676	5,174	4,702	4,266	3,865	62,248	30,329	92,578
(18) Cost of Capital		6,320	6,976	7,699	8,303	8,833	9,322	9,793	10,257	10,724	11,197			
(18pv) PV of Cost of Capital		5,495	5,275	5,062	4,747	4,391	4,030	3,681	3,353	3,048	2,768	41,851	22,206	64,057
(19) Cost of Starting Capital		6,320	6,320	6,320	6,320	6,320	6,320	6,320	6,320	6,320	6,320			
(19pv) PV of Cost of Starting Capital		5,495	4,779	4,155	3,613	3,142	2,732	2,376	2,066	1,796	1,562	31,717	10,414	42,131
(20) Cost of Growth Capital			656	1,379	1,984	2,513	3,002	3,473	3,938	4,404	4,877			
(20pv) PV of Cost of Growth Capital			496	907	1,134	1,249	1,298	1,306	1,287	1,252	1,206	10,134	11,791	21,926
(21) Excess Returns in Year	2,869	2,853	3,030	3,269	3,483	3,645	3,807	3,969	4,126	4,283	4,441			
(21pv) PV of Excess Returns in Year	2,869	2,481	2,291	2,150	1,991	1,812	1,646	1,492	1,349	1,217	1,098	20,397	8,124	28,521
(22) Total Indicated Value														70,652

EXHIBIT 21
SHEET 10

PRIMARY STOCK INSURANCE COMPANY
CALCULATION NOTES FOR VALUATION ESTIMATES AS OF DECEMBER 31, 2001

(1) from Primary Stock Insurance Company's 12/31/01 booked balance sheet
(2) selected starting surplus for future income projections based on the selected 200.0% surplus to indicated RBC ratio
(3) = (1) – (2)
(4) selected surplus based on the selected 200.0% surplus to indicated RBC ratio
(5) calculated RBC at the Company Action Level
(6) selection of 200.0% surplus to RBC ratio for determining required surplus at each year end
(7) cumulative increase in (4) from starting surplus, (4) – (2)
(8) NPW for all lines ÷ (4)
(9) net loss and LAE reserves for all lines ÷ (4)
(10) = (11) Subtotal: Company Income from Change in Statutory Surplus
(11) is selected hurdle of 15.0% used for determining Cost of Capital in EVA method and the present value of future earnings and value added
(12) = 1.000 at 12/31/01; for future years = (1.0 + 15.0%) raised to (year – 2001) exponent
(13) = (10)
(13pv) = (13) × (12) for each year; Total '01 to '11 is the total of the by-year estimates; Total '12 to ∞ = $(13)_{2011}$ × (1 + Growth Rate) ÷ (Hurdle Rate – Growth Rate) × $(12)_{2011}$;
All Years = Total '01 to '11 + Total '12 to ∞
(14) annual change in (4)
(14pv) = (14) × (12) for each year; Total '01 to '11 is the total of the by-year estimates; Total '12 to ∞ = $(4)_{2011}$ × Growth Rate ÷ (Hurdle Rate – Growth Rate) × $(12)_{2011}$;
All Years = Total '01 to '11 + Total '12 to ∞

(15) = (13) – (14)
(15pv) = (13pv) – (14pv)
(16) = (2)
(17) = (10)
(17pv) = (17) × (12) for each year; Total '01 to '11 is the total of the by-year estimates; Total '12 to ∞ = $(17)_{2011}$ × (1 + Growth Rate) ÷ (Hurdle Rate – Growth Rate) × $(12)_{2011}$;
All Years = Total '01 to '11 + Total '12 to ∞
(18) = (19) + (20)
(18pv) = (19pv) + (20pv)
(19) = (2) × (11)
(19pv) = (19) × (12) for each year; Total '01 to '11 is the total of the by-year estimates; Total '12 to ∞ = (16) × $(12)_{2011}$;
All Years = Total '01 to '11 + Total '12 to ∞
(20) = $(7)_{\text{prior year}}$ × (11)
(20pv) = (20) × (12) for each year; Total '01 to '11 is the total of the by-year estimates; Total '12 to ∞ = [$(4)_{2011}$ × Hurdle Rate ÷ (Hurdle Rate – Growth Rate) – (16)] × $(12)_{2011}$;
All Years = Total '01 to '11 + Total '12 to ∞
(21) = (17) – (18)
(21pv) = (17pv) – (18pv)
(22) = (16) + $(21pv)_{\text{All Years}}$

EXHIBIT 21
SHEET 11

PRIMARY STOCK INSURANCE COMPANY
VALUATION ESTIMATES AS OF DECEMBER 31, 2001 USING BASE HURDLE RATE +3%

Establishing Starting Surplus											
(1) Booked Statutory Surplus @ 12/31/01	45,000										
(2) Selected Starting Surplus for Future Income Projections (based on 12/31/01 RBC level)	42,131										
(3) Income Recognized @ 12/31/01	2,869										
Monitoring and Selecting Surplus	As of 12/31 of:										
	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
(4) Selected Surplus	$42,131	$46,506	$51,328	$55,362	$58,890	$62,150	$65,288	$68,388	$71,500	$74,652	$77,864
(5) Indicated Risk-Based Capital Company Action Level	21,069	23,253	25,664	27,681	29,445	31,075	32,644	34,194	35,750	37,326	38,932
(6) Surplus-to-RBC Ratio	200.0%	200.0%	200.0%	200.0%	200.0%	200.0%	200.0%	200.0%	200.0%	200.0%	200.0%
(7) Surplus Growth		4,375	9,197	13,231	16,759	20,019	23,157	26,257	29,369	32,521	35,733
(8) Net Written Premium-to-Surplus Ratio	1.99	1.88	1.77	1.71	1.67	1.64	1.63	1.62	1.61	1.60	1.60
(9) Loss and LAE Reserves-to-Surplus Ratio	2.54	2.71	2.74	2.75	2.75	2.75	2.74	2.74	2.73	2.73	2.72
Estimated Future Income		During:									
		2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
(10) Total Company Net Income		10,445	12,054	13,187	14,153	14,977	15,752	16,507	17,248	17,976	18,713
Hurdle Rate	@ 12/31/ 2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
(11) Selected Hurdle Rate		18.0%	18.0%	18.0%	18.0%	18.0%	18.0%	18.0%	18.0%	18.0%	18.0%
(12) Present Value Factor for 12/31/01 (assuming year-end income recognition)	1.000	0.847	0.718	0.609	0.516	0.437	0.370	0.314	0.266	0.225	0.191
Expected Annual Perpetual Growth Rate of: Capital & Income After 2011		2.0%									

Application of DCF Method	@ 12/31/ 2001	During: 2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	Total '01 to '11	Total '12 to ∞	All Years
(13) Total Company Net Income	2,869	10,445	12,054	13,187	14,153	14,977	15,752	16,507	17,248	17,976	18,713			
(13pv) PV of Total Company Net Income	2,869	8,852	8,657	8,026	7,300	6,547	5,835	5,182	4,589	4,053	3,575	65,484	22,793	88,277
(14) Reinvestment Cost to Grow Company		4,375	4,822	4,034	3,528	3,260	3,138	3,100	3,112	3,152	3,212			
(14pv) PV of Reinv. Cost to Grow Company		3,708	3,463	2,455	1,820	1,425	1,162	973	828	711	614	17,158	1,860	19,018
(15) Free Cash Flow	2,869	6,070	7,232	9,153	10,625	11,717	12,614	13,407	14,136	14,824	15,501			
(15pv) PV of Free Cash Flow	2,869	5,144	5,194	5,571	5,480	5,122	4,673	4,209	3,761	3,342	2,962	48,326	20,933	69,259

Application of EVA Method	@ 12/31/ 2001	During: 2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	Total '01 to '11	Total '12 to ∞	All Years
(16) Statutory Surplus @ 12/31/01	42,131													
(17) Total Company Net Income	2,869	10,445	12,054	13,187	14,153	14,977	15,752	16,507	17,248	17,976	18,713			
(17pv) PV of Total Company Net Income	2,869	8,852	8,657	8,026	7,300	6,547	5,835	5,182	4,589	4,053	3,575	65,484	22,793	88,277
(18) Cost of Capital		7,584	8,371	9,239	9,965	10,600	11,187	11,752	12,310	12,870	13,437			
(18pv) PV of Cost of Capital		6,427	6,012	5,623	5,140	4,633	4,144	3,689	3,275	2,902	2,567	44,412	16,737	61,149
(19) Cost of Starting Capital		7,584	7,584	7,584	7,584	7,584	7,584	7,584	7,584	7,584	7,584			
(19pv) PV of Cost of Starting Capital		6,427	5,446	4,616	3,912	3,315	2,809	2,381	2,018	1,710	1,449	34,081	8,050	42,131
(20) Cost of Growth Capital		787	1,655	2,382	3,017	3,603	4,168	4,726	5,286	5,854				
(20pv) PV of Cost of Growth Capital		566	1,008	1,228	1,319	1,335	1,309	1,257	1,192	1,118	10,331	8,687	19,018	
(21) Excess Returns in Year	2,869	2,861	3,683	3,948	4,188	4,377	4,565	4,755	4,938	5,106	5,276			
(21pv) PV of Excess Returns in Year	2,869	2,425	2,645	2,403	2,160	1,913	1,691	1,493	1,314	1,151	1,008	21,072	6,056	27,128
(22) Total Indicated Value														69,259

EXHIBIT 21
SHEET 12

PRIMARY STOCK INSURANCE COMPANY
CALCULATION NOTES FOR VALUATION ESTIMATES AS OF DECEMBER 31, 2001

(1) from Primary Stock Insurance Company's 12/31/01 booked balance sheet
(2) selected starting surplus for future income projections based on the selected 200.0% surplus to indicated RBC ratio
(3) = (1) − (2)
(4) selected surplus based on the selected 200.0% surplus to indicated RBC ratio
(5) calculated RBC at the Company Action Level
(6) selection of 200.0% surplus to RBC ratio for determining required surplus at each year end
(7) cumulative increase in (4) from starting surplus, (4) − (2)
(8) NPW for all lines ÷ (4)
(9) net loss and LAE reserves for all lines ÷ (4)
(10) = (11) Subtotal: Company Income from Change in Statutory Surplus
(11) is selected hurdle of 18.0% used for determining Cost of Capital in EVA method and the present value of future earnings and value added
(12) = 1.000 at 12/31/01; for future years = (1.0 + 18.0%) raised to (year − 2001) exponent
(13) = (10)
(13pv) = (13) × (12) for each year; Total '01 to '11 is the total of the by-year estimates; Total '12 to ∞ = $(13)_{2011}$ × (1 + Growth Rate) ÷ (Hurdle Rate − Growth Rate) × $(12)_{2011}$;
All Years = Total '01 to '11 + Total '12 to ∞
(14) annual change in (4)
(14pv) = (14) × (12) for each year; Total '01 to '11 is the total of the by-year estimates; Total '12 to ∞ = $(4)_{2011}$ × Growth Rate ÷ (Hurdle Rate − Growth Rate) × $(12)_{2011}$;
All Years = Total '01 to '11 + Total '12 to ∞

(15) = (13) – (14)
(15pv) = (13pv) – (14pv)
(16) = (2)
(17) = (10)
(17pv) = (17) × (12) for each year; Total '01 to '11 is the total of the by-year estimates; Total '12 to ∞ = $(17)_{2011}$ × (1 + Growth Rate) ÷ (Hurdle Rate – Growth Rate) × $(12)_{2011}$;
All Years = Total '01 to '11 + Total '12 to ∞
(18) = (19) + (20)
(18pv) = (19pv) + (20pv)
(19) = (2) × (11)
(19pv) = (19) × (12) for each year; Total '01 to '11 is the total of the by-year estimates; Total '12 to ∞ = (16) × $(12)_{2011}$;
All Years = Total '01 to '11 + Total '12 to ∞
(20) = $(7)_{\text{prior year}}$ × (11)
(20pv) = (20) × (12) for each year; Total '01 to '11 is the total of the by-year estimates; Total '12 to ∞ = [$(4)_{2011}$ × Hurdle Rate ÷ (Hurdle Rate – Growth Rate) – (16)] × $(12)_{2011}$;
All Years = Total '01 to '11 + Total '12 to ∞
(21) = (17) – (18)
(21pv) = (17pv) – (18pv)
(22) = (16) + $(21pv)_{\text{All Years}}$

EXHIBIT 21
SHEET 13

PRIMARY STOCK INSURANCE COMPANY
VALUATION ESTIMATES AS OF DECEMBER 31, 2001 USING BASE HURDLE RATE −3%

Establishing Starting Surplus											
(1) Booked Statutory Surplus @ 12/31/01	45,000										
(2) Selected Starting Surplus for Future Income Projections (based on 12/31/01 RBC level)	42,131										
(3) Income Recognized @ 12/31/01	2,869										
Monitoring and Selecting Surplus	As of 12/31 of:										
	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
(4) Selected Surplus	$42,131	$46,506	$51,328	$55,362	$58,890	$62,150	$65,288	$68,388	$71,500	$74,652	$77,864
(5) Indicated Risk-Based Capital Company Action Level	21,069	23,253	25,664	27,681	29,445	31,075	32,644	34,194	35,750	37,326	38,932
(6) Surplus-to-RBC Ratio	200.0%	200.0%	200.0%	200.0%	200.0%	200.0%	200.0%	200.0%	200.0%	200.0%	200.0%
(7) Surplus Growth		4,375	9,197	13,231	16,759	20,019	23,157	26,257	29,369	32,521	35,733
(8) Net Written Premium-to-Surplus Ratio	1.99	1.88	1.77	1.71	1.67	1.64	1.63	1.62	1.61	1.60	1.60
(9) Loss and LAE Reserves-to-Surplus Ratio	2.54	2.71	2.74	2.75	2.75	2.75	2.74	2.74	2.73	2.73	2.72
Estimated Future Income		During:									
		2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
(10) Total Company Net Income		10,445	12,054	13,187	14,153	14,977	15,752	16,507	17,248	17,976	18,713
Hurdle Rate	@ 12/31/ 2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
(11) Selected Hurdle Rate		12.0%	12.0%	12.0%	12.0%	12.0%	12.0%	12.0%	12.0%	12.0%	12.0%
(12) Present Value Factor for 12/31/01 (assuming year-end income recognition)	1.000	0.893	0.797	0.712	0.636	0.567	0.507	0.452	0.404	0.361	0.322
Expected Annual Perpetual Growth Rate of: Capital & Income After 2011		2.0%									

Application of DCF Method	@ 12/31/ 2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	Total '01 to '11	Total '12 to ∞	All Years
		During:												
(13) Total Company Net Income	2,869	10,445	12,054	13,187	14,153	14,977	15,752	16,507	17,248	17,976	18,713			
(13pv) PV of Total Company Net Income	2,869	9,326	9,609	9,386	8,994	8,498	7,980	7,467	6,966	6,482	6,025	83,604	61,456	145,060
(14) Reinvestment Cost to Grow Company		4,375	4,822	4,034	3,528	3,260	3,138	3,100	3,112	3,152	3,212			
(14pv) PV of Reinv. Cost to Grow Company		3,906	3,844	2,871	2,242	1,850	1,590	1,402	1,257	1,137	1,034	21,133	5,014	26,147
(15) Free Cash Flow	2,869	6,070	7,232	9,153	10,625	11,717	12,614	13,407	14,136	14,824	15,501			
(15pv) PV of Free Cash Flow	2,869	5,420	5,765	6,515	6,752	6,649	6,391	6,065	5,709	5,346	4,991	62,471	56,442	118,913

Application of EVA Method	@ 12/31/ 2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	Total '01 to '11	Total '12 to ∞	All Years
		During:												
(16) Statutory Surplus @ 12/31/01	42,131													
(17) Total Company Net Income	2,869	10,445	12,054	13,187	14,153	14,977	15,752	16,507	17,248	17,976	18,713			
(17pv) PV of Total Company Net Income	2,869	9,326	9,609	9,386	8,994	8,498	7,980	7,467	6,966	6,482	6,025	83,604	61,456	145,060
(18) Cost of Capital		5,056	5,581	6,159	6,643	7,067	7,458	7,835	8,207	8,580	8,958			
(18pv) PV of Cost of Capital		4,514	4,449	4,384	4,222	4,010	3,778	3,544	3,314	3,094	2,884	38,194	30,084	68,278
(19) Cost of Starting Capital		5,056	5,056	5,056	5,056	5,056	5,056	5,056	5,056	5,056	5,056			
(19pv) PV of Cost of Starting Capital		4,514	4,030	3,599	3,213	2,869	2,561	2,287	2,042	1,823	1,628	28,566	13,565	42,131
(20) Cost of Growth Capital			525	1,104	1,588	2,011	2,402	2,779	3,151	3,524	3,903			
(20pv) PV of Cost of Growth Capital			419	786	1,009	1,141	1,217	1,257	1,273	1,271	1,257	9,628	16,519	26,147
(21) Excess Returns in Year	2,869	5,389	6,473	7,028	7,510	7,910	8,294	8,672	9,041	9,396	9,755			
(21pv) PV of Excess Returns in Year	2,869	4,812	5,160	5,002	4,772	4,488	4,202	3,923	3,652	3,388	3,141	45,410	31,372	76,782
(22) Total Indicated Value														118,913

EXHIBIT 21
SHEET 14

PRIMARY STOCK INSURANCE COMPANY
CALCULATION NOTES FOR VALUATION ESTIMATES AS OF DECEMBER 31, 2001

(1) from Primary Stock Insurance Company's 12/31/01 booked balance sheet
(2) selected starting surplus for future income projections based on the selected 200.0% surplus to indicated RBC ratio
(3) = (1) − (2)
(4) selected surplus based on the selected 200.0% surplus to indicated RBC ratio
(5) calculated RBC at the Company Action Level
(6) selection of 200.0% surplus to RBC ratio for determining required surplus at each year end
(7) cumulative increase in (4) from starting surplus, (4) − (2)
(8) NPW for all lines ÷ (4)
(9) net loss and LAE reserves for all lines ÷ (4)
(10) = (11) Subtotal: Company Income from Change in Statutory Surplus
(11) is selected hurdle of 12.0% used for determining Cost of Capital in EVA method and the present value of future earnings and value added
(12) = 1.000 at 12/31/01; for future years = (1.0 + 12.0%) raised to (year − 2001) exponent
(13) = (10)
(13pv) = (13) × (12) for each year; Total '01 to '11 is the total of the by-year estimates; Total '12 to ∞ = $(13)_{2011}$ × (1 + Growth Rate) ÷ (Hurdle Rate − Growth Rate) × $(12)_{2011}$;
All Years = Total '01 to '11 + Total '12 to ∞
(14) annual change in (4)
(14pv) = (14) × (12) for each year; Total '01 to '11 is the total of the by-year estimates; Total '12 to ∞ = $(4)_{2011}$ × Growth Rate ÷ (Hurdle Rate − Growth Rate) × $(12)_{2011}$;
All Years = Total '01 to '11 + Total '12 to ∞

(15) = (13) – (14)
(15pv) = (13pv) – (14pv)
(16) = (2)
(17) = (10)
(17pv) = (17) × (12) for each year; Total '01 to '11 is the total of the by-year estimates; Total '12 to ∞ = $(17)_{2011}$ × (1 + Growth Rate) ÷ (Hurdle Rate – Growth Rate) × $(12)_{2011}$;
All Years = Total '01 to '11 + Total '12 to ∞
(18) = (19) + (20)
(18pv) = (19pv) + (20pv)
(19) = (2) × (11)
(19pv) = (19) × (12) for each year; Total '01 to '11 is the total of the by-year estimates; Total '12 to ∞ = (16) × $(12)_{2011}$;
All Years = Total '01 to '11 + Total '12 to ∞
(20) = $(7)_{\text{prior year}}$ × (11)
(20pv) = (20) × (12) for each year; Total '01 to '11 is the total of the by-year estimates; Total '12 to ∞ = [$(4)_{2011}$ × Hurdle Rate ÷ (Hurdle Rate – Growth Rate) – (16)] × $(12)_{2011}$;
All Years = Total '01 to '11 + Total '12 to ∞
(21) = (17) – (18)
(21pv) = (17pv) – (18pv)
(22) = (16) + $(21\text{pv})_{\text{All Years}}$

EXHIBIT 21
SHEET 15

PRIMARY STOCK INSURANCE COMPANY
VALUATION ESTIMATES AS OF DECEMBER 31, 2001 NO PREMIUM GROWTH IN EXPLICIT FORECAST PERIOD

Establishing Starting Surplus											
(1) Booked Statutory Surplus @ 12/31/01	45,000										
(2) Selected Starting Surplus for Future Income Projections (based on 12/31/01 RBC level)	42,131										
(3) Income Recognized @ 12/31/01	2,869										
Monitoring and Selecting Surplus	As of 12/31 of:										
	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
(4) Selected Surplus	$42,131	$46,096	$49,934	$52,574	$54,362	$55,628	$56,550	$57,240	$57,760	$58,154	$58,444
(5) Indicated Risk-Based Capital Company Action Level	21,069	23,048	24,967	26,287	27,181	27,814	28,275	28,620	28,880	29,077	29,222
(6) Surplus-to-RBC Ratio	200.0%	200.0%	200.0%	200.0%	200.0%	200.0%	200.0%	200.0%	200.0%	200.0%	200.0%
(7) Surplus Growth		3,965	7,803	10,443	12,231	13,497	14,419	15,109	15,629	16,023	16,313
(8) Net Written Premium-to-Surplus Ratio	1.99	1.82	1.68	1.60	1.55	1.51	1.49	1.47	1.45	1.44	1.44
(9) Loss and LAE Reserves-to-Surplus Ratio	2.54	2.72	2.74	2.75	2.75	2.75	2.74	2.74	2.73	2.73	2.72
Estimated Future Income		During:									
		2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
(10) Total Company Net Income		10,718	12,174	13,025	13,611	13,977	14,240	14,431	14,568	14,665	14,730
Hurdle Rate	@ 12/31/ 2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
(11) Selected Hurdle Rate		15.0%	15.0%	15.0%	15.0%	15.0%	15.0%	15.0%	15.0%	15.0%	15.0%
(12) Present Value Factor for 12/31/01 (assuming year-end income recognition)	1.000	0.870	0.756	0.658	0.572	0.497	0.432	0.376	0.327	0.284	0.247
Expected Annual Perpetual Growth Rate of: Capital & Income After 2011		2.0%									

Application of DCF Method	@ 12/31/ 2001	During: 2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	Total '01 to '11	Total '12 to ∞	All Years
(13) Total Company Net Income	2,869	10,718	12,174	13,025	13,611	13,977	14,240	14,431	14,568	14,665	14,730			
(13pv) PV of Total Company Net Income	2,869	9,320	9,205	8,564	7,782	6,949	6,156	5,425	4,762	4,169	3,641	68,843	28,568	97,411
(14) Reinvestment Cost to Grow Company		3,965	3,838	2,640	1,788	1,266	922	690	520	394	290			
(14pv) PV of Reinv. Cost to Grow Company		3,448	2,902	1,736	1,022	629	399	259	170	112	72	10,749	2,223	12,972
(15) Free Cash Flow	2,869	6,753	8,336	10,385	11,823	12,711	13,318	13,741	14,048	14,271	14,440			
(15pv) PV of Free Cash Flow	2,869	5,872	6,303	6,828	6,760	6,320	5,758	5,166	4,592	4,057	3,569	58,094	26,346	84,440

Application of EVA Method	@ 12/31/ 2001	During: 2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	Total '01 to '11	Total '12 to ∞	All Years
(16) Statutory Surplus @ 12/31/01	42,131													
(17) Total Company Net Income	2,869	10,718	12,174	13,025	13,611	13,977	14,240	14,431	14,568	14,665	14,730			
(17pv) PV of Total Company Net Income	2,869	9,320	9,205	8,564	7,782	6,949	6,156	5,425	4,762	4,169	3,641	68,843	28,568	97,411
(18) Cost of Capital		6,320	6,914	7,490	7,886	8,154	8,344	8,483	8,586	8,664	8,723			
(18pv) PV of Cost of Capital		5,495	5,228	4,925	4,509	4,054	3,607	3,189	2,807	2,463	2,156	38,434	16,669	55,103
(19) Cost of Starting Capital		6,320	6,320	6,320	6,320	6,320	6,320	6,320	6,320	6,320	6,320			
(19pv) PV of Cost of Starting Capital		5,495	4,779	4,155	3,613	3,142	2,732	2,376	2,066	1,796	1,562	31,717	10,414	42,131
(20) Cost of Growth Capital			595	1,170	1,566	1,835	2,025	2,163	2,266	2,344	2,403			
(20pv) PV of Cost of Growth Capital			450	770	896	912	875	813	741	666	594	6,717	6,255	12,972
(21) Excess Returns in Year	2,869	4,398	5,260	5,535	5,725	5,823	5,896	5,949	5,982	6,001	6,007			
(21pv) PV of Excess Returns in Year	2,869	3,825	3,977	3,639	3,273	2,895	2,549	2,236	1,956	1,706	1,485	30,409	11,899	42,309
(22) Total Indicated Value														84,440

EXHIBIT 21
SHEET 16

PRIMARY STOCK INSURANCE COMPANY
CALCULATION NOTES FOR VALUATION ESTIMATES AS OF DECEMBER 31, 2001

(1) from Primary Stock Insurance Company's 12/31/01 booked balance sheet
(2) selected starting surplus for future income projections based on the selected 200.0% surplus to indicated RBC ratio
(3) = (1) – (2)
(4) selected surplus based on the selected 200.0% surplus to indicated RBC ratio
(5) calculated RBC at the Company Action Level
(6) selection of 200.0% surplus to RBC ratio for determining required surplus at each year end
(7) cumulative increase in (4) from starting surplus, (4) – (2)
(8) NPW for all lines ÷ (4)
(9) net loss and LAE reserves for all lines ÷ (4)
(10) = (11) Subtotal: Company Income from Change in Statutory Surplus
(11) is selected hurdle of 15.0% used for determining Cost of Capital in EVA method and the present value of future earnings and value added
(12) = 1.000 at 12/31/01; for future years = (1.0 + 15.0%) raised to (year – 2001) exponent
(13) = (10)
(13pv) = (13) × (12) for each year; Total '01 to '11 is the total of the by-year estimates; Total '12 to ∞ = (13)2011 × (1 + Growth Rate) ÷ (Hurdle Rate – Growth Rate) × $(12)_{2011}$;
All Years = Total '01 to '11 + Total '12 to ∞
(14) annual change in (4)
(14pv) = (14) × (12) for each year; Total '01 to '11 is the total of the by-year estimates; Total '12 to ∞ = $(4)_{2011}$ × Growth Rate ÷ (Hurdle Rate – Growth Rate) × $(12)_{2011}$;
All Years = Total '01 to '11 + Total '12 to ∞

(15) = (13) – (14)
(15pv) = (13pv) – (14pv)
(16) = (2)
(17) = (10)
(17pv) = (17) × (12) for each year; Total '01 to '11 is the total of the by-year estimates; Total '12 to ∞ = $(17)_{2011}$ × (1 + Growth Rate) ÷ (Hurdle Rate – Growth Rate) × $(12)_{2011}$;
All Years = Total '01 to '11 + Total '12 to ∞
(18) = (19) + (20)
(18pv) = (19pv) + (20pv)
(19) = (2) × (11)
(19pv) = (19) × (12) for each year; Total '01 to '11 is the total of the by-year estimates; Total '12 to ∞ = (16) × $(12)_{2011}$;
All Years = Total '01 to '11 + Total '12 to ∞
(20) = $(7)_{\text{prior year}}$ × (11)
(20pv) = (20) × (12) for each year; Total '01 to '11 is the total of the by-year estimates; Total '12 to ∞ = [$(4)_{2011}$ × Hurdle Rate ÷ (Hurdle Rate – Growth Rate) – (16)] × $(12)_{2011}$;
All Years = Total '01 to '11 + Total '12 to ∞
(21) = (17) – (18)
(21pv) = (17pv) – (18pv)
(22) = (16) + $(21pv)_{\text{All Years}}$

EXHIBIT 21
Sheet 17

Primary Stock Insurance Company
Valuation Estimates as of December 31, 2001 6% Premium Growth in Explicit Forecast Period

Establishing Starting Surplus											
(1) Booked Statutory Surplus @ 12/31/01	45,000										
(2) Selected Starting Surplus for Future Income Projections (based on 12/31/01 RBC level)	42,131										
(3) Income Recognized @ 12/31/01	2,869										
Monitoring and Selecting Surplus	As of 12/31 of:										
	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
(4) Selected Surplus	$42,131	$46,692	$52,006	$56,786	$61,266	$65,656	$70,110	$74,730	$79,520	$84,538	$89,806
(5) Indicated Risk-Based Capital Company Action Level	21,069	23,346	26,003	28,393	30,633	32,828	35,055	37,365	39,760	42,269	44,903
(6) Surplus-to-RBC Ratio	200.0%	200.0%	200.0%	200.0%	200.0%	200.0%	200.0%	200.0%	200.0%	200.0%	200.0%
(7) Surplus Growth		4,561	9,875	14,655	19,135	23,525	27,979	32,599	37,389	42,407	47,675
(8) Net Written Premium-to-Surplus Ratio	1.99	1.91	1.81	1.76	1.73	1.71	1.70	1.69	1.68	1.68	1.68
(9) Loss and LAE Reserves-to-Surplus Ratio	2.54	2.71	2.74	2.75	2.75	2.75	2.74	2.74	2.73	2.73	2.72
Estimated Future Income		During:									
		2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
(10) Total Company Net Income		10,311	11,984	13,256	14,417	15,482	16,531	17,588	18,674	19,803	21,018
Hurdle Rate	@ 12/31/ 2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
(11) Selected Hurdle Rate		15.0%	15.0%	15.0%	15.0%	15.0%	15.0%	15.0%	15.0%	15.0%	15.0%
(12) Present Value Factor for 12/31/01 (assuming year-end income recognition)	1.000	0.870	0.756	0.658	0.572	0.497	0.432	0.376	0.327	0.284	0.247
Expected Annual Perpetual Growth Rate of: Capital & Income After 2011		2.0%									

Application of DCF Method	@ 12/31/ 2001	During: 2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	Total '01 to '11	Total '12 to ∞	All Years
(13) Total Company Net Income	2,869	10,311	11,984	13,256	14,417	15,482	16,531	17,588	18,674	19,803	21,018			
(13pv) PV of Total Company Net Income	2,869	8,966	9,062	8,716	8,243	7,697	7,147	6,612	6,105	5,629	5,195	76,241	40,763	117,004
(14) Reinvestment Cost to Grow Company		4,561	5,314	4,780	4,480	4,390	4,454	4,620	4,790	5,018	5,268			
(14pv) PV of Reinv. Cost to Grow Company		3,966	4,018	3,143	2,561	2,183	1,926	1,737	1,566	1,426	1,302	23,828	3,415	27,243
(15) Free Cash Flow	2,869	5,750	6,670	8,476	9,937	11,092	12,077	12,968	13,884	14,785	15,750			
(15pv) PV of Free Cash Flow	2,869	5,000	5,043	5,573	5,682	5,515	5,221	4,875	4,539	4,203	3,893	52,413	37,348	89,761

Application of EVA Method	@ 12/31/ 2001	During: 2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	Total '01 to '11	Total '12 to ∞	All Years
(16) Statutory Surplus @ 12/31/01	42,131													
(17) Total Company Net Income	2,869	10,311	11,984	13,256	14,417	15,482	16,531	17,588	18,674	19,803	21,018			
(17pv) PV of Total Company Net Income	2,869	8,966	9,062	8,716	8,243	7,697	7,147	6,612	6,105	5,629	5,195	76,241	40,763	117,004
(18) Cost of Capital		6,320	7,004	7,801	8,518	9,190	9,848	10,517	11,210	11,928	12,681			
(18pv) PV of Cost of Capital		5,495	5,296	5,129	4,870	4,569	4,258	3,954	3,664	3,391	3,134	43,760	25,614	69,374
(19) Cost of Starting Capital		6,320	6,320	6,320	6,320	6,320	6,320	6,320	6,320	6,320	6,320			
(19pv) PV of Cost of Starting Capital		5,495	4,779	4,155	3,613	3,142	2,732	2,376	2,066	1,796	1,562	31,717	10,414	42,131
(20) Cost of Growth Capital			684	1,481	2,198	2,870	3,529	4,197	4,890	5,608	6,361			
(20pv) PV of Cost of Growth Capital			517	974	1,257	1,427	1,526	1,578	1,598	1,594	1,572	12,044	15,200	27,243
(21) Excess Returns in Year	2,869	3,991	4,980	5,455	5,899	6,292	6,683	7,072	7,465	7,875	8,337			
(21pv) PV of Excess Returns in Year	2,869	3,471	3,766	3,587	3,373	3,128	2,889	2,658	2,440	2,239	2,061	32,480	15,149	47,630
(22) Total Indicated Value														89,761

EXHIBIT 21
SHEET 18

PRIMARY STOCK INSURANCE COMPANY
CALCULATION NOTES FOR VALUATION ESTIMATES AS OF DECEMBER 31, 2001

(1) from Primary Stock Insurance Company's 12/31/01 booked balance sheet
(2) selected starting surplus for future income projections based on the selected 200.0% surplus to indicated RBC ratio
(3) = (1) – (2)
(4) selected surplus based on the selected 200.0% surplus to indicated RBC ratio
(5) calculated RBC at the Company Action Level
(6) selection of 200.0% surplus to RBC ratio for determining required surplus at each year end
(7) cumulative increase in (4) from starting surplus, (4) – (2)
(8) NPW for all lines ÷ (4)
(9) net loss and LAE reserves for all lines ÷ (4)
(10) = (11) Subtotal: Company Income from Change in Statutory Surplus
(11) is selected hurdle of 15.0% used for determining Cost of Capital in EVA method and the present value of future earnings and value added
(12) = 1.000 at 12/31/01; for future years = (1.0 + 15.0%) raised to (year – 2001) exponent
(13) = (10)
(13pv) = (13) × (12) for each year; Total '01 to '11 is the total of the by-year estimates; Total '12 to ∞ = $(13)_{2011}$ × (1 + Growth Rate) ÷ (Hurdle Rate – Growth Rate) × $(12)_{2011}$;
All Years = Total '01 to '11 + Total '12 to ∞
(14) annual change in (4)
(14pv) = (14) × (12) for each year; Total '01 to '11 is the total of the by-year estimates; Total '12 to ∞ = $(4)_{2011}$ × Growth Rate ÷ (Hurdle Rate – Growth Rate) × $(12)_{2011}$;
All Years = Total '01 to '11 + Total '12 to ∞

(15) = (13) – (14)
(15pv) = (13pv) – (14pv)
(16) = (2)
(17) = (10)
(17pv) = (17) × (12) for each year; Total '01 to '11 is the total of the by-year estimates; Total '12 to ∞ = $(17)_{2011}$ × (1 + Growth Rate) ÷ (Hurdle Rate – Growth Rate) × $(12)_{2011}$;
All Years = Total '01 to '11 + Total '12 to ∞
(18) = (19) + (20)
(18pv) = (19pv) + (20pv)
(19) = (2) × (11)
(19pv) = (19) × (12) for each year; Total '01 to '11 is the total of the by-year estimates; Total '12 to ∞ = (16) × $(12)_{2011}$;
All Years = Total '01 to '11 + Total '12 to ∞
(20) = $(7)_{\text{prior year}}$ × (11)
(20pv) = (20) × (12) for each year; Total '01 to '11 is the total of the by-year estimates; Total '12 to ∞ = [$(4)_{2011}$ × Hurdle Rate ÷ (Hurdle Rate – Growth Rate) – (16)] × $(12)_{2011}$;
All Years = Total '01 to '11 + Total '12 to ∞
(21) = (17) – (18)
(21pv) = (17pv) – (18pv)
(22) = (16) + $(21pv)_{\text{All Years}}$

A NEW METHOD OF ESTIMATING LOSS RESERVES

COLIN M. RAMSAY

Abstract

This paper introduces a new method for estimating loss reserves. The method is fundamentally different from other loss reserving methods because it explicitly assumes that the evolution of the incremental incurred loss for an accident year is the result of a random split of the ultimate loss for that accident year into separate pieces that are observed in each development year over the claim settlement period. The nature of the random split and the pattern of the evolution of incremental incurred loss must be specified by the reserving actuary, thus giving the method tremendous flexibility. A key feature of this method is that it provides loss development factors without any knowledge of the distribution of the ultimate loss and without the actual cumulative incurred loss. Thus this method is suitable for calculating reserves for new lines of business where there is little or no loss settlement data.

1. INTRODUCTION

The loss reserving problem can be briefly described as follows. Let S_i denote the unknown ultimate incurred loss[1] for accident year i (excluding expected income from salvage and subrogation) and C_{ij} denote the best estimate of cumulative incurred loss amounts for accident year i and development year[2] j. The data used to estimate loss reserves are usually presented in the

[1]The loss reserving problem can also be described in terms of cumulative paid losses or incurred but not reported (IBNR) losses.

[2]The development year refers to the number of calendar years as measured from the accident year so that $j = 0$ refers to the accident year.

TABLE 1

LOSS DEVELOPMENT TRIANGLE

Accident Year (i)	Development Year (j)					
	0	1	2	$\cdots$	$k-1$	k
1	C_{10}	C_{11}	C_{12}	$\cdots$	$C_{1,k-1}$	C_{1k}
2	C_{20}	C_{21}	C_{22}	$\cdots$	$C_{2,k-1}$	
$\vdots$	$\vdots$	$\vdots$	$\vdots$			
$k-1$	$C_{k-1,0}$	$C_{k-1,1}$				
k	$C_{k,0}$					

form of a loss development triangle as shown in Table 1. A basic assumption in loss reserving is that the data in the rows of Table 1 are mutually independent, i.e., C_{ij} and C_{rm} are independent if $i \neq r$. In other words, losses from different accident years evolve independently. Another assumption is that all losses are settled within a certain number of calendar years, N years, say, from their date of occurrence, regardless of the year of occurrence. This means that $C_{ij} = C_{iN}$ for $j \geq N$ and $i = 1,2,\ldots$. Sometimes, however, the data in the loss development triangle consist of incremental incurred losses, c_{ij}, where

$$c_{ij} = \begin{cases} C_{ij} - C_{i,j-1} & j = 1,2,\ldots \\ C_{i0} & j = 0. \end{cases}$$

The decision to use either incremental or cumulative values depends on the loss reserving method used.

Given C_{ij}, the ultimate incurred loss for accident year i, S_i, is estimated as:

$$S_i = C_{ij} \times \mathrm{LDF}_j \qquad (1)$$

where LDF_j is the incurred loss development factor for development year j to ultimate. When the total paid loss for occurrence year i at the end of development year j (TPL_{ij}) is known, the loss reserve at that point in time (LR_{ij}) is then given by

$$\mathrm{LR}_{ij} = C_{ij} \times \mathrm{LDF}_j - \mathrm{TPL}_{ij}. \qquad (2)$$

There are numerous methods for estimating loss reserves. These include the chain ladder method and its many modifications, separation methods, probabilistic methods such as Bühlmann et al. [5], Bayesian methods (see De Alba [7] and references therein), and many ad hoc methods such as the Bornhuetter-Ferguson method [3]. For a detailed discussion of the practical issues involved in developing loss reserves, see Berquist and Sherman [1], Salzmann [16], Wiser [19], or Booth et al. [2, Chapter 16]. For an overview of many older actuarial loss reserving methods, see Van Eeghen [18]. A more modern treatment of loss reserves is given in Taylor [17] and England and Verrall [8].

The important common characteristic of established loss reserving methods is their reliance on the existence of a sufficiently long loss run-off triangle. This makes many of them unsuitable for estimating loss reserves for new lines of business, especially in the early years where the loss development process is immature.[3]

For new lines of business, practical approach to loss reserving may be as follows:

1. The actuary tries to get an understanding of the business by talking to the underwriters and claims-handlers; then
2. The actuary makes his/her best a priori guess of the reserve based on this knowledge.

The actuary's guess may be based on a simple loss ratio reserving method together with a rough conservative guess as to the development pattern (possibly based on the experience from some other similar business).

[3]One method that is suited for the early years is the Bornhuetter-Ferguson method.

The objective of this paper is to provide reserving actuaries with a method or process to assist them with their "best guess" in the early years of development and with loss reserving in general. The method introduced fundamentally is different from other loss reserving methods because it explicitly assumes the evolution of the incremental incurred loss for an accident year is the result of a random split of the ultimate loss for that accident year into separate pieces of losses that are observed in each development year over the claim settlement period. The nature of the random split and the pattern of the evolution of incremental incurred loss must be specified by the reserving actuary, thus giving the method tremendous flexibility. As this method provides loss development factors without any knowledge of the distribution of the ultimate loss or of the actual cumulative incurred loss, it is suitable for calculating loss reserves for new lines of business, where there is little or no loss development data. This method is suitable for paid and incurred loss, and can also be used in conjunction with the Bornhuetter-Ferguson method by providing the necessary loss development factors.

2. THE BASIC MODEL

As is common in many models of the property/casualty loss reserving process, we assume:[4]

1. The maximum number of years it takes for incurred losses to be completely paid and settled is fixed and known to be N, i.e., a claim occurring in accident year i is settled by the end of accident year $i+N$;
2. The incremental loss development processes from different accident years are mutually independent, i.e., c_{ij} and c_{kl} are independent if $i \neq k$; and
3. The incremental incurred loss in each accident year forms a non-negative decreasing sequence, i.e.,

[4]This model can also be described in terms of paid losses.

> $c_{ij} > c_{i,j+1}$ for $j = 0, 1, \ldots, N-1$. (The case where the incremental incurred losses form an arbitrary sequence is considered later in Section 5.2.)

Clearly, from the definitions of N, S_i, and the c_{ij}'s,

$$S_i = c_{i0} + c_{i1} + \cdots + c_{iN}. \tag{3}$$

where $N = 1, 2, \ldots.$ is known. Equation (3) shows that S_i can be viewed as being split at random into $N+1$ pieces of loss $c_{i0}, c_{i1}, \ldots, c_{iN}$ with the jth piece of loss being revealed (i.e., made known) at the end of the jth development year. On the other hand, Assumption 3 implies that the sequence $c_{i0}, c_{i1}, \ldots, c_{iN}$ is an ordered sequence. It is unlikely that a purely random split will lead to an ordered sequence. Thus the precise nature of the split must be specified.

Suppose the total unknown incurred S_i is split at random under a uniform distribution into $N+1$ pieces of loss labeled $X_{i1}, X_{i2}, \ldots, X_{i,N+1}$ such that

$$S_i = X_{i1} + X_{i2} + \cdots + X_{i,N+1}. \tag{4}$$

We further assume that these pieces of loss are ordered and relabeled so that

$$X_{i(1)} \le X_{i(2)} \le \cdots \le X_{i(N+1)}.$$

By Assumption 3 the incremental incurred loss is a realization of the ordered pieces of loss, i.e.,

$$c_{ij} = X_{i(N+1-j)} \qquad \text{and} \qquad C_{ij} = \sum_{k=0}^{j} X_{i(N+1-k)} \tag{5}$$

for $j = 0, 1, \ldots, N$.

At this point, it is important to clarify what is meant by the statement "S_i is split at random under a uniform distribution." Suppose we have N independent and identically distributed random variables, $U_1, U_2, \ldots, U_N$, which are uniformly distributed on $(0,1)$. The $U_{(j)}$'s are ordered and relabeled as

$$0 < U_{(1)} \leq U_{(2)} \leq \cdots \leq U_{(N)} < 1 \tag{6}$$

and the end points are then defined as $U_{(0)} = 0$ and $U_{(N+1)} = 1$. Next we define the spacings[5] between the consecutive ordered U_j's as

$$Y_j = U_{(j)} - U_{(j-1)} \tag{7}$$

for $j = 1, 2, \ldots, N+1$. Then a random split of S_i into $N+1$ pieces of loss $X_{i1}, X_{i2}, \ldots, X_{i,N+1}$ means

$$X_{ij} = S_i \times Y_j \qquad \text{for} \quad j = 1, 2, \ldots, N+1. \tag{8}$$

Ordering the Y_j's as $Y_{(1)} \leq Y_{(2)} \leq \cdots \leq Y_{(N+1)}$ and an application of Assumption 3 immediately yields

$$c_{ij} = S_i Y_{(N+1-j)} \qquad \text{and} \tag{9}$$

$$C_{ij} = S_i \sum_{k=0}^{j} Y_{(N+1-k)} \tag{10}$$

for $j = 0, 1, \ldots, N$. However, as the the cumulative incurred tends to be more stable than the incremental incurred, Equation (9) is not used to estimate S_i. Instead, we use

$$S_i = \frac{C_{ij}}{\sum_{k=0}^{j} Y_{(N+1-k)}}. \tag{11}$$

3. ESTIMATING ULTIMATE LOSS

Recall that the $Y_{(j)}$'s are not known until S_i is known, hence they must be estimated. The obvious estimator of $Y_{(j)}$ is its mean,

[5]For comprehensive treatment of the distribution of the spacings between successive ordered random variables, see Pyke [14].

which leads to the first estimate of the total incurred for accident year i given the incurred losses through development year j:

$$\hat{S}_i^{(1)} = \frac{C_{ij}}{\sum_{r=0}^{j} \mathbf{E}[Y_{(N+1-r)}]} \tag{12}$$

i.e., the loss development factor from j to ultimate is

$$\mathrm{LDF}_{j,N}^{(1)} = \frac{1}{\sum_{r=0}^{j} \mathbf{E}[Y_{(N+1-r)}]} \tag{13}$$

for $j = 0,1,\ldots,N$. Alternatively, we may use

$$\hat{S}_i^{(2)} = C_{ij}\mathbf{E}\left[\frac{1}{\sum_{r=0}^{j} Y_{(N+1-r)}}\right] \tag{14}$$

which yields the alternative loss development factor from j to ultimate

$$\mathrm{LDF}_{j,N}^{(2)} = \mathbf{E}\left[\frac{1}{\sum_{r=0}^{j} Y_{(N+1-r)}}\right]. \tag{15}$$

From Jensen's inequality, $\mathrm{LDF}_{j,N}^{(2)} \geq \mathrm{LDF}_{j,N}^{(1)}$ for every j. Before calculating the values of $\mathrm{LDF}_{j,N}^{(1)}$ and $\mathrm{LDF}_{j,N}^{(2)}$ for various values of j and N, the distribution of the $Y_{(j)}$'s will be provided.

From the theory of the random division of an interval of unit length (for example, David [6, chapter 5.4] or Feller [9, chapter 1]), the variables $Y_1, Y_2, \ldots, Y_{N+1}$ form an exchangeable sequence of dependent random variables with joint pdf:

$$f(y_1, y_2, \ldots, y_{N+1}) = \begin{cases} (N+1)! & \text{if} \quad y_j \geq 0 \quad \text{and} \quad \displaystyle\sum_{k=1}^{N+1} y_k = 1. \\ 0 & \text{otherwise.} \end{cases}$$

The marginal distribution of $Y_{(j)}$ is given by Maurer and Margolin [12, equation (4.4)] as

$$\Pr[Y_{(j)} > y] = \sum_{m=N+2-j}^{N+1} (-1)^{m-(N+2-j)} \binom{m-1}{N+1-j} \times \binom{N+1}{m} (1-my)^N I\left\{y < \frac{1}{m}\right\}$$

where $I\{A\}$ is an indicator of the occurrence of the event A. In addition, the moments of $Y_{(j)}$ satisfy the recursion

$$(n-j)\mu_{j:n}^{(k)} + j\mu_{j+1:n}^{(k)} = n\mu_{j:n-1}^{(k)} \tag{16}$$

where $\mu_{j:n}^{(k)} = E[Y_{(j)}^k]$ for sample size n.

Due to the difficulties in deriving the inverse moments needed in $\mathrm{LDF}_{j,N}^{(2)}$, however, Monte Carlo simulations[6] are used to determine both sets of loss development factors. Table A1 in the Appendix shows the loss development factors $\mathrm{LDF}_{j,N}^{(1)}$ and $\mathrm{LDF}_{j,N}^{(2)}$, respectively.

4. NUMERICAL EXAMPLES

Example 1: A Simple Data Set

Suppose a new product was introduced on January 1, 2000 and the loss development data as of December 31, 2002 are given in Table 2.

The total paid loss to date is thus 1,719 + 2,573 + 1,761 = 6,053. To estimate the total ultimate loss, we must first specify N. If we assume $N = 3$, then the total estimated ultimate loss for

[6]The uniform (0,1) random number generator run in Press et al. [13, chapter B7, page 1142] is used to perform all simulations.

TABLE 2

HYPOTHETICAL CUMULATIVE INCURRED AND PAID LOSSES (IN 000S)

Accident Year (i)	Earned Premiums (in 000s)	Development Year (j)					
		Incurred Loss (C_{ij})			Paid Loss		
		0	1	2	0	1	2
2000	4,500	1,447	1,976	2,454	401	1,166	1,761
2001	8,500	3,578	3,911		906	2,573	
2002	16,000	4,754			1,719		

2000–2002, as of December 31, 2002, is:

$$\hat{S}^{(1)} = 4{,}754 \times 1.9195 + 3{,}911 \times 1.2627 + 2{,}454 \times 1.0662$$
$$= 16{,}680.18$$
$$\hat{S}^{(2)} = 4{,}754 \times 2.0379 + 3{,}911 \times 1.2826 + 2{,}454 \times 1.0691$$
$$= 17{,}328.00$$

using the $N = 3$ rows of Table A1. The corresponding reserve estimates are 10,627.18 = 16,680.18 – 6,053 and 11,275.00 = 17,328.00 – 6,053, respectively.

If, on the other hand, we assume $N = 5$, then the estimated ultimate loss for 2000–2002, as of December 31, 2002, is:

$$\hat{S}^{(1)} = 4{,}754 \times 2.4564 + 3{,}911 \times 1.5412 + 2{,}454 \times 1.2384$$
$$= 20{,}744.39$$
$$\hat{S}^{(2)} = 4{,}754 \times 2.6221 + 3{,}911 \times 1.5800 + 2{,}454 \times 1.2505$$
$$= 21{,}713.57$$

using the $N = 5$ row of Table A1. These ultimates lead to reserve estimates of 14,691.39 = 20,744.39 – 6,053, and 15,660.57 = 21,713.57 – 6,053, respectively.

TABLE 3

HYPOTHETICAL PREMIUM AND LOSS DEVELOPMENT DATA (IN 000S)

Accident Year	Earned Premium	Development Year (j)					
		0	1	2	3	4	5
1997	5,000	2,500	3,650	4,200	4,325	4,335	4,330
1998	5,500	2,150	3,225	3,775	3,965	3,960	
1999	6,000	3,250	4,500	5,050	5,150		
2000	7,000	3,700	5,200	5,775			
2001	7,500	3,300	4,800				
2002	8,000	4,250					

Source: Based on the data in Bornhuetter and Ferguson [3, page 193, Exhibit A] with "Year of Origin" changed from 1966–1971 to 1997–2002.

TABLE 4

ANNUAL LOSS DEVELOPMENT FACTORS FOR TABLE 3

Accident Year	Earned Premium	Development Year (j)				
		0	1	2	3	4
1997	5,000	1.460	1.151	1.030	1.002	0.999
1998	5,500	1.500	1.171	1.050	0.999	
1999	6,000	1.385	1.122	1.102		
2000	7,000	1.405	1.111			
2001	7,500	1.455				
2002	8,000					

Example 2: A Modified Bornhuetter-Ferguson Method

This method can be used to provide the ultimate loss development factors needed in applications of the Bornhuetter-Ferguson (B-F) method. For example, using the data in Table 3, the IBNR reserves are estimated using the traditional B-F method and a modified B-F method based on the $\text{LDF}^{(1)}_{j,N}$ given in Table A1. In deriving their estimates, Bornhuetter and Ferguson [3] assume losses in the three most recent calendar years are settled in 3 years. Table 4 shows the annual loss development factors. Table 5 provides a summary of the results.

TABLE 5

HYPOTHETICAL IBNR RESERVE COMPUTATION AS OF DECEMBER 31, 2002

Accident Year	(1) Expected Losses	LDFs (2) $LDF_j^{(BF)}$	LDFs (3) $LDF_{j,3}^{(1)}$	IBNR Factor (4) $1-1/LDF_j^{(BF)}$	IBNR Factor (5) $1-1/LDF_{j,3}^{(1)}$	Indicated IBNR (6) B-F	Indicated IBNR (7) Mod. B-F
1999	5,700	1.000	1.0000	0.000	0.0000	0	0
2000	6,650	1.032	1.0662	0.031	0.0621	206	413
2001	7,125	1.166	1.2627	0.142	0.2080	1,012	1,482
2002	7,600	1.650	1.9195	0.394	0.4790	2,994	3,640
						4,212	5,535

Notes: Expected Losses are 95% of the earned premium. The information in Columns (2), (4) and (6) are provided by Bornhuetter and Ferguson [3, page 194, Exhibit B]. The information in Columns (3) and (5) are derived from Table A1 with $N = 3$. Column (7) = Column (1) × Column (5).

5. GENERALIZATIONS AND PRACTICAL CONSIDERATIONS

The loss reserving method introduced above is flexible and can be generalized in at least two ways. For example, one can consider a non-uniform random split and/or consider an arbitrary ordered sequence of random spacings to reflect the evolution of the incremental incurred loss.

5.1. A Non-Uniform Random Split

One can observe in Appendix Tables A1 and A2 that, under the uniform random split, $C_{i,0}$ is a relatively small percentage of S_i then there is a fairly rapid development of incurred loss. For example, in Table A1, $1/LDF_{j,N}^{(1)}$ and $1/LDF_{j,N}^{(2)}$ are small for $j = 0, 1$ or 2, while Table A2 shows that the loss development factors for years 1 and 2 are high suggesting the rapid development of incurred losses.

If the actuary has loss development factors that are not similar to the quantities in Tables A1 and A2, another distribution defined on $(0,1)$ must be used to form the basis of the split. Unfortunately, there is no obvious alternative distribution, especially one that is intuitively appealing. It is up to the reserving actuary to specify a continuous distribution with support on $(0,1)$. Some alternative distributions with support on $(0,1)$ include the beta, the truncated gamma, and the truncated Pareto distributions. One strategy that can be used is to have on hand tables similar to Tables A1 and A2 for each potential alternative random split distribution. The actuary can then use the table (i.e., distribution) that best matches the observed loss development factors.

Suppose the actuary chooses a specific cumulative distribution function $F_U(u)$ with continuous support on $(0,1)$. As the length of the claims settlement period is $N+1$ years, we sample N independent and identically distributed random variables, $U_1,\ldots,U_j,\ldots,U_N$ from $F_U(u)$.[7] The sampled U_j's are then ordered and relabeled as before. The resulting spacings, $Y_j = U_{(j)} - U_{(j-1)}$, with $U_{(0)} = 0$ and $U_{(N+1)} = 1$, are then used to define the random split. As before, simulations are then used to determine the expectations needed to determine the loss development factors. As an example, Tables A3 and A4 provide the loss development factors to ultimate and the annual loss development factors in the case of the truncated exponential pdf of U_j defined by

$$f_U(u) = \frac{\lambda e^{-\lambda u}}{1 - e^{-\lambda u}}, \tag{17}$$

for $0 \le u \le 1$ and $\lambda > 0$.

5.2. *An Arbitrary Ordered Sequence*

Recall equation (9) in which we defined the sequence of incremental incurred loss as $c_{i0} \ge c_{i1} \ge \cdots \ge c_{iN}$. If the actuary be-

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[7]For more on techniques for generating random variables from continuous distributions see Bratley, Fox, and Shrage [4, chapters 5 and 6]; Kalos and Whitlock [11, chapter 3]; Fishman [10, chapter 3]; and Ross [15, chapters 3 to 5].

lieves, however, that the pattern of incremental incurred loss is different, then tables of loss development factors to ultimate and annual loss development factors that are consistent with the specified pattern must be derived.

To be precise, for $j = 0,1,\ldots,N$ let θ_j denote the order of the set of order statistics $Y_{(1)},Y_{(2)},\ldots,Y_{(N+1)}$ that is used to define c_{ij}. Note that $\theta_0,\theta_1,\ldots,\theta_N$ is a permutation of the elements of the set $\{1,2,\ldots,N+1\}$. (For example, equation (9) implies $\theta_j = N+1-j$. As another example, the actuary may specifically believe that $\theta_0 = N-1,\theta_1 = N,\theta_2 = N+1,\ \theta_j = N+1-j$ for $j = 3,\ldots,N$, which implies $c_{i0} \le c_{i1} \le c_{i2} \ge c_{i3} \ge \cdots \ge c_{iN}$.) It follows that c_{ij} and C_{ij} are defined as

$$c_{ij} = S_i Y_{(\theta_j)}, \qquad \text{and} \tag{18}$$

$$S_i = \frac{C_{ij}}{\sum_{k=0}^{j} Y_{(\theta_k)}}. \tag{19}$$

for $j = 0,1,\ldots,N$.

In general, the loss development factors can be obtained via a simulation of sample size M as follows:

STEP 1. For given settlement period of $N+1$ years, set $\text{TEMP}^{(1)}_{j,N} = 0$ and $\text{TEMP}^{(2)}_{j,N} = 0$ for $j = 0,1,2,\ldots,N$.

STEP 2. Create an $(N+1) = 2$ dimensional permutation vector $\theta = (\theta_0,\theta_1,\ldots,\theta_N)$ containing the actuary's specified pattern of incremental incurred losses.

STEP 3. Generate N random variables $U_1,\ldots,U_j,\ldots,U_N$ from the actuary's specified random splitting distribution, $F_U(u)$.

STEP 4. Order the sampled U_j's as $U_{(1)} \le U_{(2)} \ldots \le U_{(N)}$.

STEP 5. For $j = 1,2,\ldots,N+1$, define $Y_j = U_{(j)} - U_{(j-1)}$, with $U_{(0)} = 0$ and $U_{(N+1)} = 1$.

STEP 6. Order the $N+1$ Y_j's as $Y_{(1)} \le Y_{(2)} \cdots \le Y_{(N+1)}$.

STEP 7. For $j = 0,1,2,\ldots,N$, set

$$\mathrm{TEMP}^{(1)}_{j,N} = \mathrm{TEMP}^{(1)}_{j,N} + \sum_{r=0}^{j} \mathbf{E}[Y_{(\theta_r)}]$$

$$\mathrm{TEMP}^{(2)}_{j,N} = \mathrm{TEMP}^{(2)}_{j,N} + \frac{1}{\sum_{r=0}^{j} Y_{(\theta_r)}}.$$

STEP 8. Repeat Steps 3 to 7 a total of M times.

STEP 9. For $j = 0,1,2,\ldots,N$, the loss development factors are estimated as:

$$\widehat{\mathrm{LDF}}^{(1)}_{j,N} = \frac{M}{\mathrm{TEMP}^{(1)}_{j,N}} \tag{20}$$

$$\widehat{\mathrm{LDF}}^{(2)}_{j,N} = \frac{\mathrm{TEMP}^{(2)}_{j,N}}{M}. \tag{21}$$

5.3. Other Practical Considerations

In practice, other potential problems may occur such as different accident years having different claim settlement periods. For example, N depends on i, or the existence of negative incremental incurred loss amounts. Tables 6 and 7 display two hypothetical data sets with several problems. In Table 6, one can assume that losses are settled in three years, i.e., $N = 3$. However, the losses do not exhibit the pattern assumed by Tables A1 and A2. In fact, even though the losses are settled in 3 years, the total incurred loss changes only slightly after development year 1, and for year 2001 the cumulative incurred loss is decreasing. Our method is not ideally suited to the data in Table 6 because of the negative incremental losses. Further research is needed in this area.

TABLE 6

SECOND HYPOTHETICAL CUMULATIVE INCURRED LOSS DATA (IN $000S)

Accident Year (i)	Development Year (j)						
	0	1	2	3	4	5	6
1997	2,237	2,369	2,376	2,376	2,376	2,376	
1998	2,899	2,942	2,936	2,934	2,934		
1999	2,225	2,330	2,322	2,325			
2000	2,145	2,205	2,207				
2001	1,513	1,499					
2002	1,168						

TABLE 7

THIRD HYPOTHETICAL CUMULATIVE INCURRED LOSS DATA (IN $000S)

Accident Year (i)	Development Year (j)						
	0	1	2	3	4	5	6
1996	1,076	927	927	951	960	1,087	1,087
1997	957	1,193	1,312	1,295	1,220	1,392	
1998	1,421	1,788	2,086	2,236	2,252		
1999	1,473	1,910	2,235	2,192			
2000	1,447	1,976	2,454				
2001	3,578	3,911					
2002	4,754						

Table 7 presents similar challenges as some incremental incurred loss amounts are zero and some are negative. The sequence of incurred losses generated in 1996 and 1997 appear to have a pattern distinct from those in subsequent years. In addition, it may be incorrect to assume the accident years all have the same settlement period, i.e., N depends on i. In such cases where there are non-positive incremental incurred losses and/or different claim settlement periods, the ultimate incurred loss for accident

TABLE 8

LOSS DEVELOPMENT FACTORS ($C_{i,j+1}/C_{i,j}$) FROM TABLE 7

Accident Year	Development Year ($j/j+1$)					
	0/1	1/2	2/3	3/4	4/5	5/6
1996	0.8615	1.0000	1.0259	1.0095	1.1323	1.0000
1997	1.2466	1.0997	0.9870	0.9421	1.1410	
1998	1.2583	1.1667	1.0719	1.0072		
1999	1.2967	1.1702	0.9808			
2000	1.3656	1.2419				
2001	1.0931					
2002						

year i, S_i, can still be estimated as

$$S_i = \frac{C_{ij}}{\sum_{k=0}^{j} Y_{(N_i+1-k)}}. \tag{22}$$

Note, the length of the claim settlement period can be approximated by observing cumulative loss development factors. If the assumptions of this model (as stated in Section 2) hold, then the jth annual cumulative loss development factor for accident year i, $C_{i,j+1}/C_{i,j}$, should satisfy

$$\frac{C_{i,j+1}}{C_{i,j}} \approx \frac{\mathrm{LDF}_{j,N}^{(k)}}{\mathrm{LDF}_{j+1,N}^{(k)}} \tag{23}$$

for $k = 1,2$ and $j = 0,1,\ldots N-1$. Table A2 shows the values $\mathrm{LDF}_{j,N}^{(k)}/\mathrm{LDF}_{j+1,N}^{(k)}$ for $k = 1,2$, $j = 0,1,\ldots,9$ and $N = 1,2,\ldots,9$. The annual cumulative loss development factors should then be compared with those in Table A2. Table 8 shows the actual cumulative loss development factors generated by Table 7. Comparing the first two columns of Table 8 with those expected in Table A2 show that patterns of actual cumulative loss development factors for years 1997 to 2001 are too low, making the data in Table 7 inconsistent with the assumption of a uniform random split. Notice that the results of Tables A3 and A4 for $\lambda = 5$ seem to better fit the data from the later years in Tables 7 and 8 than the uniform random split.

6. SUMMARY AND CLOSING COMMENTS

For new lines of business, the practical approach to loss reserving requires the actuary to make his/her best guess of the reserve level based on prior knowledge. The actuary's guess may be based on a simple loss ratio reserving method together with a rough conservative guess as to the development pattern (possibly based on the experience from some other similar business). This paper provides reserving actuaries with a tool to assist them with their "best guess" of the reserves, especially in the early years of development. The method essentially uses an a priori pattern in the table of expected loss development factors to determine the loss reserves. The pattern of expected loss development factors is independent of the distribution of the cumulative incurred loss in the accident year and can be varied depending on the actuary's estimate of the length of the claim settlement period, and the random split used.

When there is a sufficiently large amount of data in the loss development triangle, the actuary can use the method of this paper to generate tables of expected loss development factors to see which ones match the observed loss development factors. The best matched tables can be used to estimate the loss reserves.

In closing, there are several important attributes of this method:

1. It can be used for new and old business.
2. It can be used in conjunction with other methods such as the Bornhuetter-Ferguson method.
3. It makes no assumptions about the underlying distribution of the ultimate losses.

4. The ultimate losses are estimated using only the most recent cumulative loss data.
5. The method can be used if the length of the settlement period varies by year of origin.
6. The factors $\mathrm{LDF}_{j,N}^{(1)}$, $\mathrm{LDF}_{j,N}^{(2)}$ and their ratios $\mathrm{LDF}_{j,N}^{(1)}/\mathrm{LDF}_{j+1,N}^{(1)}$ and $\mathrm{LDF}_{j,N}^{(2)}/\mathrm{LDF}_{j+1,N}^{(2)}$ do not depend on the actual loss development pattern.
7. Tables of factors and ratios can be created and saved for each combination of development year j and settlement period N, and for various types of random splits such as the uniform and beta distributions. The appropriate table can be chosen to:
   a. Match the observed pattern of loss development factors or to
   b. Match the actuary's or underwriter's best guess of what the pattern of loss development factors should be.

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APPENDIX

TABLE A1

LOSS DEVELOPMENT FACTOR FROM j TO ULTIMATE FOR VARIOUS DEVELOPMENT YEARS AND SETTLEMENT PERIODS (N)

	Development Year j									
N	0	1	2	3	4	5	6	7	8	9
	Results for $LDF^{(1)}_{j,N}$:									
0	1.0000									
1	1.3345	1.0000								
2	1.6353	1.1253	1.0000							
3	1.9195	1.2627	1.0662	1.0000						
4	2.1903	1.4015	1.1495	1.0416	1.0000					
5	2.4564	1.5412	1.2384	1.0980	1.0288	1.0000				
6	2.6959	1.6715	1.3259	1.1590	1.0689	1.0208	1.0000			
7	2.9402	1.8007	1.4137	1.2232	1.1152	1.0516	1.0159	1.0000		
8	3.1853	1.9333	1.5038	1.2894	1.1650	1.0881	1.0403	1.0126	1.0000	
9	3.4169	2.0584	1.5905	1.3542	1.2153	1.1270	1.0691	1.0321	1.0101	1.0000
	Results for $LDF^{(2)}_{j,N}$:									
0	1.0000									
1	1.3871	1.0000								
2	1.7247	1.1347	1.0000							
3	2.0379	1.2826	1.0691	1.0000						
4	2.3333	1.4312	1.1569	1.0428	1.0000					
5	2.6221	1.5800	1.2505	1.1015	1.0294	1.0000				
6	2.8804	1.7182	1.3422	1.1649	1.0707	1.0211	1.0000			
7	3.1417	1.8547	1.4343	1.2316	1.1185	1.0526	1.0161	1.0000		
8	3.4033	1.9938	1.5281	1.3003	1.1698	1.0901	1.0409	1.0127	1.0000	
9	3.6511	2.1259	1.6188	1.3676	1.2219	1.1301	1.0704	1.0325	1.0102	1.0000

Notes: Development year 0 refers to the year in which the claim was incurred. N is the number of calendar years it takes to settle all claims occurring in the same calendar year. Thus $N = 0$ implies claims are settled in the calendar year of their occurrence.

TABLE A2

THE RATIO LOSS DEVELOPMENT FACTORS FOR VARIOUS DEVELOPMENT YEARS AND SETTLEMENT PERIODS

	Development Year $j/j+1$									
N	0/1	1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/10
	Results for $LDF^{(1)}_{j,N}/LDF^{(1)}_{j+1,N}$:									
1	1.3345									
2	1.4531	1.1253								
3	1.5201	1.1843	1.0662							
4	1.5628	1.2192	1.1036	1.0416						
5	1.5938	1.2445	1.1279	1.0673	1.0288					
6	1.6129	1.2606	1.1440	1.0843	1.0472	1.0208				
7	1.6328	1.2737	1.1558	1.0968	1.0605	1.0351	1.0159			
8	1.6476	1.2856	1.1663	1.1068	1.0707	1.0460	1.0273	1.0126		
9	1.6600	1.2941	1.1745	1.1143	1.0784	1.0541	1.0359	1.0218	1.0101	
	Results for $LDF^{(2)}_{j,N}/LDF^{(2)}_{j+1,N}$:									
1	1.3871									
2	1.5200	1.1347								
3	1.5889	1.1997	1.0691							
4	1.6303	1.2371	1.1094	1.0428						
5	1.6595	1.2635	1.1353	1.0700	1.0294					
6	1.6764	1.2801	1.1522	1.0880	1.0486	1.0211				
7	1.6939	1.2931	1.1646	1.1011	1.0626	1.0359	1.0161			
8	1.7070	1.3047	1.1753	1.1115	1.0732	1.0472	1.0278	1.0127		
9	1.7175	1.3133	1.1836	1.1193	1.0813	1.0557	1.0367	1.0221	1.0102	

Notes: Development year 0 refers to the year in which the claim was incurred. N is the number of calendar years it takes to settle all claims occurring in the same calendar year. Thus $N = 0$ implies claims are settled in the calendar year of their occurrence.

TABLE A3

$LDF^{(1)}_{j,N}$ FOR THE TRUNCATED EXPONENTIAL DISTRIBUTION WITH PARAMETER λ

	Development Year j									
N	0	1	2	3	4	5	6	7	8	9
	$\lambda = 1$:									
0	1.0000									
1	1.3261	1.0000								
2	1.6130	1.1209	1.0000							
3	1.8811	1.2514	1.0634	1.0000						
4	2.1304	1.3823	1.1424	1.0396	1.0000					
5	2.3776	1.5139	1.2264	1.0931	1.0273	1.0000				
6	2.6049	1.6376	1.3101	1.1513	1.0655	1.0197	1.0000			
7	2.8284	1.7576	1.3920	1.2117	1.1093	1.0487	1.0150	1.0000		
8	3.0482	1.8786	1.4756	1.2737	1.1562	1.0834	1.0381	1.0119	1.0000	
9	3.2585	1.9959	1.5570	1.3349	1.2036	1.1200	1.0654	1.0303	1.0095	1.0000
	$\lambda = 5$:									
0	1.0000									
1	1.2041	1.0000								
2	1.3468	1.0672	1.0000							
3	1.4597	1.1323	1.0331	1.0000						
4	1.5554	1.1912	1.0716	1.0200	1.0000					
5	1.6469	1.2461	1.1103	1.0456	1.0133	1.0000				
6	1.7291	1.2954	1.1467	1.0725	1.0315	1.0094	1.0000			
7	1.7950	1.3388	1.1802	1.0989	1.0515	1.0231	1.0070	1.0000		
8	1.8582	1.3814	1.2124	1.1244	1.0717	1.0385	1.0176	1.0054	1.0000	
9	1.9291	1.4239	1.2447	1.1508	1.0932	1.0555	1.0304	1.0141	1.0044	1.0000
	$\lambda = 10$:									
0	1.0000									
1	1.1094	1.0000								
2	1.1747	1.0346	1.0000							
3	1.2211	1.0656	1.0169	1.0000						
4	1.2589	1.0923	1.0357	1.0100	1.0000					
5	1.2895	1.1155	1.0536	1.0226	1.0066	1.0000				
6	1.3190	1.1360	1.0702	1.0355	1.0157	1.0047	1.0000			
7	1.3454	1.1551	1.0861	1.0486	1.0258	1.0117	1.0036	1.0000		
8	1.3649	1.1711	1.0997	1.0603	1.0355	1.0193	1.0089	1.0028	1.0000	
9	1.3904	1.1879	1.1136	1.0722	1.0456	1.0275	1.0152	1.0071	1.0022	1.0000

TABLE A4

$\text{LDF}_{j,N}^{(2)}$ FOR THE TRUNCATED EXPONENTIAL DISTRIBUTION WITH PARAMETER λ

	Development Year j									
N	0	1	2	3	4	5	6	7	8	9
	$\lambda = 1$:									
0	1.0000									
1	1.3791	1.0000								
2	1.7023	1.1300	1.0000							
3	2.0004	1.2710	1.0662	1.0000						
4	2.2764	1.4115	1.1494	1.0408	1.0000					
5	2.5492	1.5530	1.2382	1.0964	1.0279	1.0000				
6	2.7976	1.6850	1.3263	1.1571	1.0673	1.0200	1.0000			
7	3.0423	1.8129	1.4124	1.2199	1.1124	1.0498	1.0152	1.0000		
8	3.2778	1.9405	1.5000	1.2843	1.1609	1.0853	1.0387	1.0120	1.0000	
9	3.5078	2.0654	1.5854	1.3480	1.2099	1.1229	1.0666	1.0307	1.0096	1.0000
	$\lambda = 5$:									
0	1.0000									
1	1.2424	1.0000								
2	1.4106	1.0717	1.0000							
3	1.5424	1.1419	1.0343	1.0000						
4	1.6542	1.2056	1.0744	1.0205	1.0000					
5	1.7630	1.2651	1.1150	1.0468	1.0135	1.0000				
6	1.8570	1.3179	1.1531	1.0745	1.0320	1.0095	1.0000			
7	1.9343	1.3650	1.1882	1.1018	1.0525	1.0234	1.0071	1.0000		
8	2.0093	1.4113	1.2220	1.1282	1.0732	1.0390	1.0177	1.0054	1.0000	
9	2.0895	1.4569	1.2559	1.1555	1.0953	1.0564	1.0307	1.0142	1.0044	1.0000
	$\lambda = 10$:									
0	1.0000									
1	1.1246	1.0000								
2	1.1978	1.0360	1.0000							
3	1.2486	1.0682	1.0172	1.0000						
4	1.2896	1.0959	1.0364	1.0101	1.0000					
5	1.3235	1.1199	1.0547	1.0229	1.0067	1.0000				
6	1.3546	1.1409	1.0716	1.0360	1.0158	1.0048	1.0000			
7	1.3847	1.1608	1.0879	1.0493	1.0260	1.0118	1.0036	1.0000		
8	1.4054	1.1773	1.1018	1.0612	1.0359	1.0194	1.0090	1.0028	1.0000	
9	1.4342	1.1947	1.1160	1.0733	1.0460	1.0277	1.0153	1.0071	1.0022	1.0000

A MODERN ARCHITECTURE FOR RESIDENTIAL PROPERTY INSURANCE RATEMAKING

JOHN W. ROLLINS

Abstract

This paper argues that obsolete rating architecture is a cause of decades of documented poor financial performance of residential property insurance products. Improving rating efficiency and equity through modernization of rating and statistical plans is critical to the continued viability of the products. In particular:

- *The overall rate level should reflect an appropriate provision for the cost of capital held for catastrophic events, and the cost of capital should be allocated appropriately in development of rating factors.*
- *The indivisible premium concept should be replaced with peril-based rating, and rating factors developed or adjusted to apply to peril-specific partial base rates.*
- *Catastrophe simulation and geographic coding technology, incorporating non-historical experimental data sets, should be applied to the development of base rates, territory boundaries and factors, and classification plans.*
- *Rating for the numerous miscellaneous exposures and coverage options, as well as maintenance of statistical plans, should be aligned with the peril rating concept.*

The author develops an architecture and techniques for ratemaking that satisfy the above precepts for the homeowners product in a hurricane-prone state. The transition from indivisible to divisible base premium facilitated by this architecture is illustrated in case study fashion, with practical implementation challenges and solutions discussed. Many ideas are transferable to

ratemaking for other residential and commercial property products.

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1. MOTIVATIONS: THEORETICAL

The insurance industry has earned a chronically inadequate rate of return on its chief residential property line, the homeowners product, since the 1980s. Catastrophic ("cat") events, the toxic mold phenomenon, the growing popularity of vicious dogs, outbreaks of sinkholes, and other root causes of loss have repeatedly surprised insurers in recent years. To date, responses have been almost exclusively reactive: underwriting restrictions, coverage limitations and sharp corrections in overall price level.

The classical "indivisible premium" rating plan in common use for residential property products is a significant obstacle limiting the industry to reactive responses of questionable economic *efficiency* and actuarial *equity*. Actuaries can show strategic leadership by engineering a proactive response—the development of a modern architecture for ratemaking that improves overall rate level efficiency as well as risk classification equity. The classical plan is demonstrably obsolete, particularly in catastrophe-prone areas, and greatly hinders the ability of insurers to identify, segregate and monitor the component drivers of loss costs. Harmonious advances in technology and actuarial science now allow us to overcome the obstacles to modern rating architecture.

Specifically, these structural changes to rating plans are overdue:

- Actuarially sound prices should reflect an explicit provision for cost of capital, in addition to actual non-cat loss costs, expected cat loss costs from simulation tools ("cat models"), and underwriting expenses.

- Indivisible base premium should be replaced with several partial base premiums by peril; for example, hurricane, other wind, fire, liability/medical, and all other perils (AOP).
- Partial base premiums should be modified by distinct class and territory (geographic location) rating plans for several reasons. Property attributes affecting equitable risk classification vary significantly by peril, and the cost of capital is not generated (and should not be allocated) uniformly among perils.
- Rating for base premium adjustments and miscellaneous endorsements should be recalibrated to take advantage of the unbundling of base premiums.

Why does the classical rating plan doom insurers to poor long-run underwriting results? Recalling fundamental principles of actuarial science [4], improper insurance prices can result from two distinct ratemaking failures:

1. Failure to recover all costs associated with risk transfer in the final premium;
2. Failure to differentiate rates for identifiable classes of risks with demonstrable differences in expected cost of risk.

Indivisible base premium in residential property insurance facilitates both failures. Two components represent the bulk of the premium for the product—expected loss costs and cost of capital. In turn, these components are generated by an aggregation of individual perils insured against—the largest contributors often fire, liability, and windstorm. In recent years, it has become apparent that the loss costs and costs of capital for distinct perils are distributed in an extremely lopsided fashion or "maldistributed"[1]

[1]The use of this term is with a respectful nod to Bailey and Simon's seminal 1959 paper on class ratemaking.

with respect to many classification attributes, particularly territory. As an architectural matter, indivisible base premium precludes the proper allocation of costs to class and territory, disallowing development of distinct and non-interacting class plans and for each peril based on construction features, not to mention distinct territory boundaries and rating factors. A corollary is that the recognition of the full cost of capital in overall rate level is discouraged due to an inability to spread it fairly among risks.

Now consider the consequences of failure to fairly allocate costs to class and territory by peril. Even if overall premium level generates adequate revenue to fund the losses of the diverse book of business, rating factors must apply to multiple and perhaps unrelated perils, generating unavoidable and perhaps severe premium subsidies. As some insurers improve rating plans to target the risks who are overpaying for certain perils, adverse selection by the affected risks will leave the insurers who fail to modernize with underpriced segments of the market, which generate poor underwriting results until overall rate level is raised. Raising the overall rate level without improving the distribution of premiums by rating factor amplifies the adverse selection, perpetuates the cycle and leads to a "death spiral" for the insurer. Cummins [7] contains a formal development of the economics of adverse selection.

The indivisible premium is a remnant of historical technology and marketing architecture. When agents were expected to quote policies in the field with a pencil and rating manual, simplicity of rating logic was paramount. The days of hand-rating are long gone in standardized personal lines products, but the rating plan based on that limitation persists. Technology is an enabler of modern rating architecture in the form of rating engines accessible from the field, as well as in geographic information systems and simulation modeling applications. Regardless of theoretical appeal, implementation of peril-specific rating would have been difficult even twenty years ago.

In addition, residential property insurance was historically marketed as "complete" coverage for the hazards inherent in the lifestyle of the typical homeowner. Today's consumers are increasingly demographically diverse and willing to choose products to fit their needs. Policy forms have evolved in response to these trends, and it is imperative that the pricing of personal lines products also evolve with the spectrum of exposures insured.

Many actuarial concepts discussed in this paper are venerable. Its contribution is to synthesize them in new ways in response to a specific challenge—the *transition* from a classical to a modern component rating architecture. Once this transition is accomplished, maintenance of some aspects of the rating plan using classical actuarial methods (such as the "loss ratio" method of determining rating factors) may still be optimal. The goal is to get the actuary across a sort of river Styx of property insurance ratemaking while ensuring that each part of the transition withstands review of, and is consistent with, canonical principles. Indeed, it is critical to undertake such a modernization in order to remain true to many of the Actuarial Standards of Practice, particularly those of more recent vintage.[2] When standing on the other side, one hopefully can look back and recognize some of the architecture as embodying potentially long-lasting innovations in actuarial techniques.

2. MOTIVATIONS: PUBLIC POLICY AND PRODUCT CHANGES

Notwithstanding theoretical motivations, necessity is the mother of invention. This paper may also be read as a case study of the actuarial response to a pricing challenge manifested in public policy. The State of Florida passed into law a statewide Unified Florida Building Code (FBC) that supersedes all local

[2]A thoughtful review of ASOPs 12, 23, 29, 30, and particularly 38 and 39 is helpful before and after reading this paper. Such a review should go a long way to convince the practicing actuary that the motivations are consistent with upholding these Standards of Practice.

codes for buildings permitted after March 1, 2002. One goal of the FBC is to improve the resistance of new construction to windstorm losses by specifying robust features and construction techniques to be used, in accordance with the recommendations of scientists and engineers. The code is heavily geography-dependent, differentiating among many elements based on the wind speed "zone" in which the site is located. In particular, properties located in the "120-mph" (in a 100-year event) and above wind zones must be built with significant levels of resistance to wind. Zones are (generally) concentric boundaries defined by the standards of ASCE 7-98 (see Figure 1).[3]

The insurance industry strongly supported the FBC, and its enabling legislation contained a quid pro quo—that insurers would develop class plans to provide rate differentials for devices that demonstrably mitigate windstorm losses, whether such devices were included on new construction *or* extant on, or retrofitted onto, existing structures. The Florida Office of Insurance Regulation (OIR) commissioned a public domain study from Applied Research Associates, Inc. (ARA) that developed a mitigation class plan containing benchmark class factors for various combinations of construction features and techniques [2], and an analogous study was conducted by Applied Insurance Research, Inc. (AIR) in support of a mitigation class plan filed by Insurance Services Office, Inc. (ISO). The deadline for individual companies to make rate filings to implement a mitigation class plan was February 28, 2003.

This mandate is the death knell for indivisible premium rating plans in Florida. This paper shows that the maldistribution of loss and capital costs by territory and peril makes such a lack of rating resolution intolerable in the presence of a windstorm

[3]The exception to this statement is along parts of the Florida Panhandle, where the political influence of home-builder associations caused a "one mile from the coast" rule to delineate areas where 120-mph standards for building materials are to apply. Examining Figure 1, the areas excepted comprise most of several west Florida counties.

FIGURE 1

mitigation class plan. The hurricane share of all-perils loss costs varies between 20% and 75% by county, and the fire share varies between 5% and 35%. No compact set of actuarially sound class factors would be workable against such a variable premium base. In parallel, it is also a great improvement to target the existing Public Protection Class (PPC) factors to the proper fire premium base.

The classical design has been tolerated by the insurance industry and its advisory organizations for too long, probably because the magnitude of the errors is manageable in areas of the United States where the distribution of loss costs is more geographically consistent and the contribution of catastrophic events to the aggregate loss costs is moderate. In Florida, the mitigation class plan is a catalyst for the development of base rates and rating factors by peril, as well as the redefinition of territories using GIS mapping software and the extension of catastrophe modeling technology into windstorm class and territory ratemaking. However, the concepts are applicable to homeowners pricing in other geographic areas, and more generally to other property insurance pricing exercises.

Public policy also influences emerging non-catastrophic causes of loss. Statutory coverage mandates and resistance to coverage restrictions in states with "prior approval" policy form regulation have contributed to the skyrocketing portion of policy loss costs associated with sinkhole claims in Florida and toxic mold claims in Texas and elsewhere. Florida statutes require sinkhole coverage and severely restrict claim settlement options, resulting in frequent total or near-total losses after moderate settling and cracking occurs in the residential structure. While mold is only a cause of loss as a result of another covered peril, the loss adjustment expenses and risk of large judgments have been well documented.

This paper is not about specific emerging perils; the point is that having the ability to segregate base premiums allows a "quarantine" of the loss costs associated with these perils at an earlier statistical stage. Pricing must keep up with the ever-expanding coverage in the property insurance product.

3. MOTIVATIONS: TECHNOLOGY AND CATASTROPHE MODELING

It is now settled actuarial science that actual losses from cat events over short experience periods should be replaced in ratemaking data by long-run expected cat losses derived from a simulation tool and the insurer's expected exposures. See Clark [6] for an excellent fundamental justification for building cat models to replace historical cat losses. In addition, several authors have tackled aspects of the problem of incorporating simulated cat losses into the overall rate level and rating factor calculations, notably Walters and Morin [18] and Burger et al. [3].[4]

A few features of cat models[5] are particularly relevant and are exploited in populating the new rating architecture:

1. They are peril-specific—one model may be used for hurricanes, another for severe thunderstorms (including tornado and hailstorm), and yet another for earthquake analysis. It is thus natural to segregate covered perils for ratemaking in such a way that the cat model can be used to build the rates for each peril separately and adequately.
2. Cat models are fundamentally exposure rating tools—loss costs can be generated from any set of relevant data, whether actual or experimental. Scenarios can be contrived and tested to develop rating factors, reducing the need for complex normalizations of experience data.
3. Some vendors offer models that output the complete empirical distribution of event losses. From this, annual losses are easily aggregated. Therefore, in addition to expected losses, we can generate moments, percentiles, and more sophisticated risk metrics for any modeled property (whether real or experimental) or aggregation thereof. These metrics are critical in deriving proxies for cost of capital and allocating risky expected losses to class and territory.

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[4]The reader unfamiliar with cat models should thoroughly peruse these references, as neither their descriptions of the design and operation of cat models nor their justifications for the use of modeled losses in ratemaking are repeated here. However, the treatment of base rate and rating factor development here is generally consistent with previous literature and often builds upon concepts formalized by these authors.

[5]All simulated cat losses used in this paper were derived from the CLASIC/2™ catastrophe models for Atlantic Hurricane and U.S. Severe Thunderstorm ("other wind"), products of AIR Worldwide Corp.

4. Every property is "geo-coded" with exact longitude and latitude, allowing us to place "pins" on maps and analyze statistics from any geographic aggregation we wish. Optimal territory boundaries with respect to gradients in loss and capital costs can be quickly identified with GIS software and scenario testing. This contrasts with the limitations of ZIP code experienced by some earlier authors.

The following case study leverages each of these key attributes of the simulation tools. The advancement of modeling science and related technology is the enabler of much of the work to follow.

4. OVERALL RATE LEVEL CHANGES

Following is a complete study of ratemaking for residential property lines, not simply a description of modern enhancements. Accordingly, first comes a discussion of the development of overall rate level changes. Components that will be targeted by our detailed rating architecture are highlighted.

A comprehensive description of classical overall rate level indications for homeowners insurance in a pre-catastrophe modeling environment is contained in Homan [10], and a concise, thorough review of basic techniques in McClenahan [14]. The following data is used to develop the indicated change in rate levels:

- Five accident years of direct paid and case-incurred losses and "defense and cost containment" (D&CC) expenses, organized by calendar year (development age), with cat losses identified;
- Five calendar years of direct written and earned premiums, and the historical rate tables necessary to bring them to present rate levels using the extension-of-exposures technique;
- Five calendar years of direct earned exposures (house-years and total values insured or "TVI");

- Three calendar years of underwriting expenses, including "adjusting & other" expenses (A&OE) associated with claim administration, allocated to category and line of business by the accounting function;
- Modeled expected cat losses by line of business, produced by a simulation tool from exposures in-force as of a given date;
- The latest calendar year's ceded catastrophe reinsurance premiums;
- Various Annual Statement data required to generate a regulated profit provision.

Exhibit 1 shows the development of the indicated overall rate level change. The formula[6] is:

$$\Delta = \frac{x + f + f_R}{1 - v} - 1 \tag{1}$$

where:

x = the weighted average experience ratio;

f = the fixed (not varying directly with premium) underwriting expense ratio to direct premium;[7]

f_R = the fixed non-loss reinsurance costs (premium in excess of modeled expected losses), expressed as a ratio to direct premium;

v = the variable expense rate per dollar of direct premium, which includes the profit provision calculated in accordance with regulations[8] and treated as a percent of premium.

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[6]The general convention here is to let capital letters represent quantities in dollars (or dollars per policy) and lowercase letters represent factors or ratios to premium. Greek letters represent relativities or constants. Carets (ˆ) represent modeled amounts.

[7]Some actuaries include a trend adjustment in the expected future fixed expense ratio to reflect inflation of underwriting expense elements.

[8]Florida statutes prescribe a profit load calculation very similar to the Calendar Year Investment Income Offset Method described by Robbin [9], with the assumed "fair" profit at 5% of premium. An economically "fair" profit provision would compensate the insurer for a variety of business risks, well documented in actuarial literature. The statutory load considers only time value of money on reserves held; we load the catastrophe cost of

The derivation of each component of the overall rate level change will be discussed in turn, illustrated for the hypothetical A-Florida Insurance Company.[9]

Average Experience Ratio

The weighted average experience ratio is the inner product of the vector of experience ratios for each calendar/accident year and a vector of selected weights. In Exhibit 1, we reflect typical judgments about the relationship of credibility to age of experience period in our weight selections.

The annual experience ratios themselves are developed as:

$$x_i = \frac{L \times l \times t_L \times (1+u) + \hat{C} \times (1+u_C) \times t_C}{P \times \delta \times t_P} \tag{2}$$

where:

L = losses plus D&CC, excluding cat losses for modeled perils;

l = loss development factor to ultimate;

t_L = selected loss cost (pure premium) trend factor;

u = loading for A&OE as a proportion to losses;

$\hat{C}$ = modeled expected annual cat losses;

u_C = the expected ratio of loss adjustment expenses (LAE, which includes D&CC *and* A&OE) to losses for catastrophic claims;

t_C = exposure de-trend factor for modeled cat losses;

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capital elsewhere in the fair premium. Other valid risks potentially compensated by the profit load are not treated in this paper.

[9]As the reader follows through several A-Florida exhibits, note that numbers generally "tie" within and across exhibits as much as possible, for tutorial purposes. However, the numbers used are not necessarily representative of actual data or benchmarks for individual companies nor the industry as a whole.

P = direct collected earned premiums including any expense fees;

δ = on-level factor to restate premiums as if earned at present rate levels;

t_P = selected premium (per earned house-year) trend factor.

Losses might be paid or case-incurred, as long as the development factor is estimated on the same basis. The estimation of development factors is not reviewed.

The loss cost trend factor and premium trend factor reflect expected changes in economic conditions making the expected losses and premiums *per exposure unit* in the prospective period different than that observed in the experience period. To estimate these factors, compiled are calendar quarter earned house-years, earned TVI, earned premiums, and paid losses plus D&CC for twenty or more consecutive quarters as shown in Exhibit 2. Time indices for each quarter are aligned as the regressor variable. The earned rate (premium per house-year) is the basis for premium trend and the loss cost (paid losses per house-year) is the basis for loss trend. The trend in earned house-years itself will be used in the de-trending of cat losses later.

An exponential regression line is fitted to each response variable. The exponential coefficient in each equation is the least-squares best fit annual change. Two-, three-, four-, and five-year domains are fitted and examined, and a representative annual change selected for each series. The trend period is the power to which the annual change is raised to derive the final trend factor for each experience period, and is determined as the number of years between the midpoint of the experience period and the midpoint of the anticipated effective period of the proposed rates. Exponentiating the annual change for the trend period provides the final trend factor, which is carried to Exhibit 1.

The volume of exposures underlying every item in formula (2) (for an individual experience year) should be the same. This is why loss *cost* (per policy) trends and earned rate trends are used to adjust the losses and premiums in each experience ratio. Accordingly, the de-trend factor for modeled cat losses is necessary to state the expected cat losses on the same volume of exposures as is underlying the approximate midpoint of each experience period. By nature, cat models produce losses given in-force exposures as of a predetermined (but presumably recent) date. Due to run-time, data storage, and labor costs, it is impractical to repeatedly simulate losses using in-force exposures from several historical years. As an alternative, the selected annual change in earned house-years from Exhibit 2 is raised to a negative power representing the trend period between the in-force date used in the cat model and the midpoint of each experience period to derive a de-trend factor. The factor is applied to the single modeled expected loss estimate to get the cat losses that are loaded into each period shown on Exhibit 1.[10]

Note that the match between the attributes of excluded actual cat losses and modeled expected cat losses should be as close as possible for actuarially efficient ratemaking. Claims departments are often responsible for coding individual claims as "catastrophic," and there is generally no mandate for consistency with the basis used for simulated cat losses. For example, if modeled hurricane losses reflect only landfalling hurricanes, but the claims unit designates weak, bypassing tropical storms as the basis for many "cat" claims received during a season, the excluded losses are broader than the simulated losses that replace them, making overall rate level indications inadequate. Actuaries should be vigilant and proactive in setting parameters for cat loss coding with respect to:

1. Event definitions (example: hurricane versus tropical storm)

[10]The de-trended cat losses are *not* then trended forward to the midpoint of the effective period because the modeled loss per exposure unit for cat perils is not inherently inflationary. Annual updates to models reflect the latest meteorological and scientific knowledge but not cost trends per dollar of value insured.

2. Time periods (example: 72 hours during which losses are eligible for "cat" treatment)
3. Geographic areas affected (example: areas subject to government warnings)
4. Lines of business (example: exclusion of liability losses from "cat" eligibility)

It is wise to consider the associated definitions in company reinsurance treaties as an example when developing parameters for cat loss reporting.

The expected ratio of LAE to losses for catastrophes will depend heavily on how the insurer's claims department handles these events. Use of mobile claims centers and contracting of outside adjusters may affect the assumed ratio. Historical data on specific past events can be used as a guide in some cases; sometimes the ratio is very low (because the losses in the denominator are high, not because adjusting catastrophic claims is cheap) when the insurer's own claims personnel are the bulk of the adjusting corps. Hence, a provision for catastrophic LAE is omitted from the example for simplicity.

The collected premium can be placed on-level by either the parallelogram method or the extension-of-exposures method discussed in Homan [10], though the latter is of great help when it comes to estimation of rating factors. If the parallelogram method is used, the factor for each experience period will be derived explicitly from knowledge of overall rate changes and effective dates thereof, as detailed in McClenahan [14]; if the extension method is used, the raw premium data must be linked to all necessary categorical variables (class, territory, etc.) used in ratemaking and complete sets of historical rate tables or "ratebooks" must be available to compute the premium for each policy as if it were written on the current ratebook. Then the factor in formula (2) is implicitly the ratio of on-level to collected direct earned premium. Neither method will be discussed in detail here.

Underwriting and Adjusting Expense Ratios

Once the experience ratios are determined with formula (2), one must consider underwriting and adjusting expenses, reinsurance costs, and profit. Exhibit 3 shows an analysis of expense provisions. Recent calendar year underwriting expense ratios for each component:

- Commissions and brokerage;
- Other acquisition expenses;
- General (overhead) expenses;
- Premium taxes (which must be shown separately in some states);
- Other taxes, licenses and fees;

are used to estimate future expected expense ratios. It is tempting to select the multi-year average, but trends in expense ratios often reflect structural changes in finance or operations and must be given some credence in the selection of future ratios.

An assumption must also be made about the proportion of each component that varies directly with the premium charged. Commissions and most taxes and fees are assessed as percentages of net premium and thus treated as 100% variable. General expenses are almost exclusively allocated amounts of fixed overhead amounts and a 100% fixed assumption is usually appropriate. Other acquisition expenses include some fixed administrative costs, but also the cost of field inspections and policy-specific costs that may vary with premium size. This study assumes 50% of expenses in this category are fixed. Underwriting expense ratios are usually expressed to direct written premium, as they are almost fully incurred prior to policy inception.

Fixed underwriting expenses reported by line actually reflect accounting department allocations of companywide expenses to line of business. Actuaries are strongly encouraged to review the allocation procedures and judge whether the allocation basis accurately captures the true expenses associated with the line, especially in the presence of historical cat events. Often a premium-based allocation (the preference of many accountants) will be sufficient. For catastrophe reinsurance costs, this method will *not* be accurate, as discussed below.

A caution is in order about bulk assessments from residential property residual markets and guaranty funds, usually found in the "other taxes, licenses and fees" category. In some states, these assessments can be recouped over a given time period from policyholders via a premium surcharge. If the company chooses to surcharge, assessments should be removed from the expenses used for ratemaking to avoid redundant recovery of the cost. If recoupment is not allowed, a different problem arises—in the absence of a cat event in the historical three-year period, a tax provision that includes no residual market deficits will be inadequate in the long term. The annual expected value of assessments is material to the expense ratio despite a "lucky" zero over a short term. The same arguments that urge consideration of expected direct catastrophe losses in ratemaking should convince the actuary that the company's share of long-term expected residual market deficits should be considered in expenses in ratemaking. Failure to do so will harm profitability in the same fashion as would ignorance of the long-term impact of cat losses.

Adjusting and other expenses are usually related to the sum of paid losses and D&CC, rather than premiums. Since these amounts are generally line-specific rather than allocated companywide amounts, the calendar year ratios fluctuate more than those for underwriting expenses and the long-term average should influence the selection unless emerging causes of loss (such as mold, which involves extensive pre-settlement scientific testing) are driving a structural change in claim adjustment expenses.

Cost of Capital

The cost of capital held to protect the insurer against infrequent catastrophic events that produce losses far in excess of the long-term average for the peril must be considered in ratemaking. The held capital may be internally generated, borrowed from investors, or "rented" from reinsurers. Most insurers capitalize their cat risk using a combination of sources, with the largest often being reinsurance. Reinsurance may be available from private sources, which include a market-determined cost of capital in their premium, and/or public sources, which generally do not. Musulin and Rollins [15] contains a description and comparison of private and public property cat reinsurance options in Florida and a breakdown of the reinsurance premium as follows:

$$P_R = \hat{C} - R(\hat{C}) + \theta + T \quad (3)$$

where:

$\hat{C}$ = expected direct cat losses (i.e. modeled gross annual losses);

$R(\hat{C})$ = expected net retained cat losses (determined by reinsurance program design);

θ = charge for cost of capital (a.k.a. reinsurance risk load);

T = transaction costs (such as brokerage and reinsurer administrative expenses).

A spectrum of approaches exists for efficiently reflecting the cost of catastrophic events in ratemaking, such as:

1. Treatment of the entire reinsurance premium (appropriately allocated to line) as a fixed expense in ratemaking and consideration of only non-cat and retained cat losses in the loss portion of the experience ratio. This method would be most appropriate for a heavily reinsured com-

pany to which differentiation of other rating factors according to modeled losses was not important. It has the advantage of not requiring detailed cat model output.

2. Loading of simulated expected direct cat losses in place of actual cat losses in the numerator of the experience ratio, and adjustment of those losses for a cost of capital charge calculated directly from assumptions, with no tie to the empirical market-determined cost of capital. This method might be required for an entity that has no benchmarks, such as an insurer that funds catastrophes solely from internal capital, a residual market, or a rating advisory organization.

3. A blended method, where the loss portion of simulated catastrophe costs is reflected directly in the experience ratio, and the cost of capital portion is treated as a fixed expense reflecting the market charge indicated by the non-loss portion of reinsurance costs. This is the method used here, so that $f_R = (\theta + T)/P$.

Homan [11] uses the first approach in his treatment of reinsurance costs in property ratemaking, and Rollins [17] has contrasted the relative strengths and weaknesses of the three approaches.

Already included in formula (2) are the total direct expected cat losses by removing actual cat losses and adding modeled gross annual losses to each year's experience ratio. A provision for non-loss reinsurance costs in formula (1), in order to provide for all costs associated with risk transfer, should consist of the reinsurance premium, less expected ceded cat losses, as a ratio to direct premium, or

$$f_R = \frac{P_R - (\hat{C} - R(\hat{C}))}{P}. \qquad (4)$$

Since θ and T in formula (3) are not observed directly, this is the practical formula for the total non-loss portion of reinsurance costs.[11]

The fixed reinsurance cost provision from typical data is derived in Exhibit 4. Direct earned premiums for the line, the portion subject to the cat reinsurance program, modeled gross annual losses, and actual cat reinsurance premiums ceded to various sources are compiled. The reinsured portion of modeled losses is derived by subtracting the retention (often based on subject premium) and the losses not covered due to coinsurance features of the treaty (typically 5% of losses above the retention). The actual ceded premium is normally significantly larger than this amount, and the difference represents cost of capital and transaction costs. For the overall rate level indication, the fixed reinsurance costs are expressed as a ratio to direct earned premium. In addition, it is useful later to think of these costs as a load to the gross ceded losses or "capacity charge" per dollar of expected loss. The fixed cost provision is carried to Exhibit 1.

Note that the ceded premiums are specific to the line of business under review. In practice, ceded cat reinsurance premiums are rarely specified by line, only in aggregate. The actuary must assist accountants in allocating the ceded premiums to line of business. Exhibit 5 provides an example. Direct earned premiums by line are compiled, with the property portion extracted for (currently) "indivisible" premium lines of business. This becomes the subject premium for most cat reinsurance programs. The portion due to property perils must be estimated from loss cost data. The actual all-lines ceded premium is allocated to line based on the modeled gross annual losses, separable by line from

[11] The astute reader will note that the "blended" formula is actually incomplete. There is no cost of capital levied on the internal capital held for retained catastrophic losses. In a heavily reinsured company, we can ignore this part of the capital charge for simplicity of presentation. Obviously, the formula cost of capital for an insurer which retained all losses and built a risk load into rates directly would not be zero. This presentation assumes that the bulk of cat losses are ceded and that the associated cost of capital is revealed by the market.

catastrophe simulation output, rather than a direct premium measure. The direct premium, subject premium, and allocated ceded premium for the line under review are carried to Exhibit 4.[12]

5. STRUCTURE OF FAIR PREMIUM

Derivation of Fair Premium Components

Given the components of overall rate level, our next task is to design a rate structure that collects a fair premium through a combination of charges. When partial base rates vary by peril, yet some fixed expenses (the reinsurance provision) are not allocated equally to peril, the classic ratemaking formulas need some careful modification.

The overall rate level change is developed using the loss ratio ratemaking method. In contrast, the new base rates and rating factors are developed from loss costs. This is necessary because each base rate and relativity is new and peril-specific, and cannot be expressed as a change to a previous factor, yet conversion to premium rates and rating factors is necessary for pricing. Note that "loss ratio" ratemaking (which produces the indicated *changes* to existing base rates needed to reconcile the indicated overall rate level change with the expected rate level impact of the rate and rating factor changes) is not incompatible with divisible premiums once the modern plan is in place and divisible premium statistics are used to do periodic rate reviews. It is, however, incompatible with the *transition* from indivisible to divisible premium.

In the proposed rating plan, premiums are levied in three parts:

- *Base rates* by peril, which cover raw loss costs (and fixed reinsurance costs where necessary), "loaded" for variable underwriting expenses and profit;

[12] In Florida, the public reinsurer develops participating primary insurers' ceded premiums directly from exposure rather than in aggregate. Therefore, there is no need to allocate the public cat reinsurance premium.

- *Rating factors* by peril, which adjust each base rate for risk class and territory differences in expected costs;
- A single policy *expense fee* (to cover all fixed underwriting expenses other than reinsurance).

Recall the classic formula for policy-level fair premium, expanded to separate non-loss reinsurance costs from other expenses not varying with direct premium:

$$P = X + F + F_R + vP \tag{5}$$

where P represents fair premium dollars, X is the expected loss cost, F represents the fixed underwriting expense dollars associated with the policy, and F_R represents the associated fixed reinsurance cost dollars. Given the choice to structure our rating plan so that the fair premium is collected via a combination of base rate (B) and expense fee (E):

$$P = B + E \tag{6}$$

solving for P in formula (5) and setting it equal to (6) yields:

$$\frac{X + F_R}{1 - v} + \frac{F}{1 - v} = B + E$$

This formula suggests a natural decomposition, designating base rates to cover losses and fixed reinsurance costs, and expense fee to cover only fixed underwriting expenses, so that:

$$B = \frac{X + F_R}{1 - v} \tag{7}$$

$$E = \frac{F}{1 - v}. \tag{8}$$

In developing base rates for non-modeled perils, X is determined directly from experience data. For modeled perils, it is determined from the model output. Later, a choice will be made and justified to recover all fixed reinsurance costs in the base rate for the hurricane peril. F_R is determined from the reinsurance data described above. Finally, recovery of all fixed underwriting expenses is in the policy expense fee, using:

$$E = f \times \bar{P} \tag{9}$$

where $\bar{P}$ is the average premium per policy from experience data.

The decomposition of fair premium may affect rating factors as well, depending upon actuarial assumptions. Class rating factors by peril are derived from class loss cost relativities, which in turn are determined from experience data or model output. Assume a loss cost relativity is α, so that the class loss cost is:

$$X' = \alpha X.$$

If fixed reinsurance costs are included in the base rate and *not* increased or reduced in proportion to the expected loss cost for the class, then the indicated class rate is:

$$B' = \frac{X' + F_R}{1 - v}$$

per formula (7). Substituting for X', the ratio of the class to base rate (a.k.a. the correct class factor) is:

$$\rho = \frac{B'}{B} = \frac{\alpha X + F_R}{X + F_R} \tag{10}$$

as the variable expense ratio cancels out of the quotient.

Note that in cases where:

1. One chooses not to recover a portion of fixed reinsurance costs in the base rate for the peril, or;
2. One assumes that fixed reinsurance costs allocated to the class vary in proportion to class loss costs;

the formula reduces to α and the loss cost relativity is the correct premium relativity. Though non-loss reinsurance costs were assumed "fixed" for the overall rate level calculation, the assumption about whether they should be treated as fixed by class or territory is crucial for derivation of the proper rating factors. The choice to recover all non-loss reinsurance costs in the hurricane base rate means that the classical formula for rating factors will apply for non-hurricane perils. For the hurricane peril, an example is shown of class (mitigation) factors calculated using the non-proportional assumption for these costs, and territory factors calculated using a modified proportional assumption for these costs. Also note that the formula can be expressed using an expected loss *ratio* and fixed reinsurance cost *ratio* to unmodified premiums (for the peril in question), since the premium dollars cancel out in formula (10).

Basic rating logic for the proposed structure is outlined in Exhibit 17. The rates and factors shown are for purposes of example only and do not have any particular significance. The derivation of base rates and various rating factors follows in later sections for each peril:

- Fire
- Hurricane
- Other Wind (non-hurricane windstorm, including tornado and hail)
- Liability/Medical
- All other perils (AOP)

Total "key" premium and total base premium are the sum of the key and base premiums, respectively, for each of the five components. Key premium (retaining the terminology used in the classical plan) represents the actuarially sound rate for first-dollar coverage on a risk of a base amount of insurance. Base premium is key premium adjusted for:

- The total value insured relative to the base amount, and
- The chosen deductible.

The choice of base amounts and deductibles is discussed later. Each component is rated for territory and most for class,[13] and non-liability components are rated for value insured and deductible as well. The general total base premium formula reflecting N different perils is:

$$P = \left(\sum_{i=1}^{N} B_i \times \rho_i \times \tau_i \times k_i \times d_i\right) + E \qquad (11)$$

where

B = base rate;

ρ = class factor;

τ = territory factor;

k = key factor (for non-liability perils);

d = deductible factor (for non-liability perils);

E = expense fee.

Once total base premium is determined, the application of various charges and credits (primarily for coverage modifications) results in "adjusted base premium" that is comparable to that in the classical rating plan. However, the existence of component partial base premiums allows credits and charges to apply to only the components of base premium judged actuarially relevant, with appropriate modifications to the percentage charges and credits. Adjustments to base premium will be discussed further below.

Implications of Fair Premium Structure

Let us review some actuarial advantages and note some practical benefits of peril-specific base premiums, all of which contribute to a more sustainably competitive pricing of individual risks in a 21st-century property insurance environment.

[13] This is a general term, encompassing the construction/protection factor (Fire), increased limits factor (Liability), and mitigation factor (Hurricane).

1. Fixed (non-loss) reinsurance costs can be allocated appropriately by peril to specific base rates and rating factors.

2. The share of actuarially sound base premiums by peril may be highly geography-dependent. Class and territory rating factors should be calibrated to the expected experience differentials for individual perils and applied by peril.

3. Territory boundaries should reflect loss cost gradients, which are heterogeneous by peril—distance to coast drives those for hurricane, other geographic features drive those for non-hurricane windstorm, and political boundaries may drive those for other perils. Peril-specific development of territory boundaries allows more accurate rating factors by peril.

4. The existing construction types used in rating are primarily designed for differentiating fire danger, and the relative wind damageability inherent in these classes overlaps with an explicit windstorm mitigation class plan. Base premium separation allows targeting of classification features to the perils they affect.

5. It is shown later that amount of insurance (“key factor”) curves and the loss distributions for deductible factors differ greatly by peril. Peril-specific rating allows proper differentiation of base premiums by value insured and deductible amount.

6. Percentage deductibles are (at this time) specific to the hurricane peril in Florida, due largely to statutory mandates to offer flat deductibles. The current rating plan must adjust for flat dollar/percent deductible combinations through a complex set of tables a problem removed in the unbundled rating plan.

7. The hurricane portion of premium must be reported separately by territory per regulatory instruction in Florida.[14] Currently, this is typically done via a complex set of extraction factors by territory a complexity removed in the unbundled rating plan.

8. Actuarially sound hurricane rates must be determined with the help of catastrophe simulation models, facilitated by separation of this peril in the rating plan.

9. Proposed mitigation credits in all industrial/engineering studies done to date are calculated as a percentage of windstorm premium. A crucial assumption about the wind portion of base rates would be necessary to convert them for usage with the current rating plan.

10. Experience data on "other wind" (tornado, hail, straight-line wind) events is sparse and of low credibility for ratemaking, but a catastrophe simulation model can assist in determining the peril-specific rates.

11. Liability peril-specific rates allow the application of benchmark increased limits factors (which assume liability-only premium) rather than the dollar charges used in some current rating plans.

12. Liability premium should be separated for any loss reserving, as well as ratemaking and most management reporting exercises a task facilitated by the unbundled rating plan.

13. Many endorsements and some base premium adjustments change peril-specific exposures, and the charges or credits for these should be calibrated to the appropriate portion of the base rates.

In summary, key and base premiums will be determined by peril and added together to determine the total key and base

[14] Rule 4-170.014(12) of the Florida Administrative Code.

premium. Each partial base premium will be calculated with a peril-specific partial base rate, territory factor, class factor, key (amount of insurance) factor, and deductible factor. This modernization of the rating plan streamlines many aspects of property insurance ratemaking.

6. DEFINITIONS FOR RATEMAKING

Territory Boundary Definitions

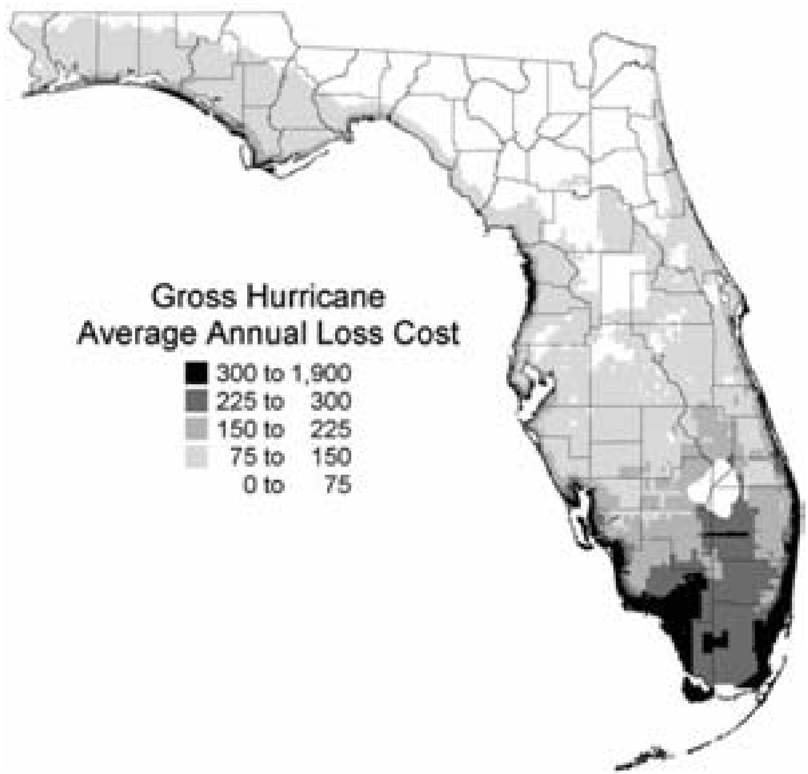
Appropriate territory definitions are a critical companion to peril-specific rating. Given a Cartesian surface or geographic map where loss costs are expressed as a function of latitude and longitude, risk classification principles [1] imply that territory definitions should correspond to loss cost gradients (contours on the map). Previous authors have explored the use of loss cost gradients and GIS software to define territories, but their approaches have generally been based on data organized at the ZIP code level [5, 13].[15] Unfortunately, the public purpose of ZIP codes is such that they do not represent a sufficiently granular starting point for the analysis of hurricane loss potential.[16] The basic problem in property insurance is that loss cost gradients may vary widely by peril, and in fact the direction of the gradient for one peril may frequently be opposite that for another. In plain English, the contour maps by peril may not "match up" very well.

In Florida, there is significant conflict among meteorological indications, as well as conflict between meteorological and political boundaries. Modeled hurricane loss cost gradients largely reflect proximity to the coastline, meaning the optimal set of territories would make a contour map of the state look somewhat like an onion, with concentric closed polygons. In addition, the

[15]To be fair, Kozlowski and Mathewson advocated the use of square-mile loss densities given that the data is available.

[16]ZIP codes are based on urban demographics and tend to be convex polygons rather than thin "strips" parallel to coastlines, which is the general pattern of hurricane loss cost gradients.

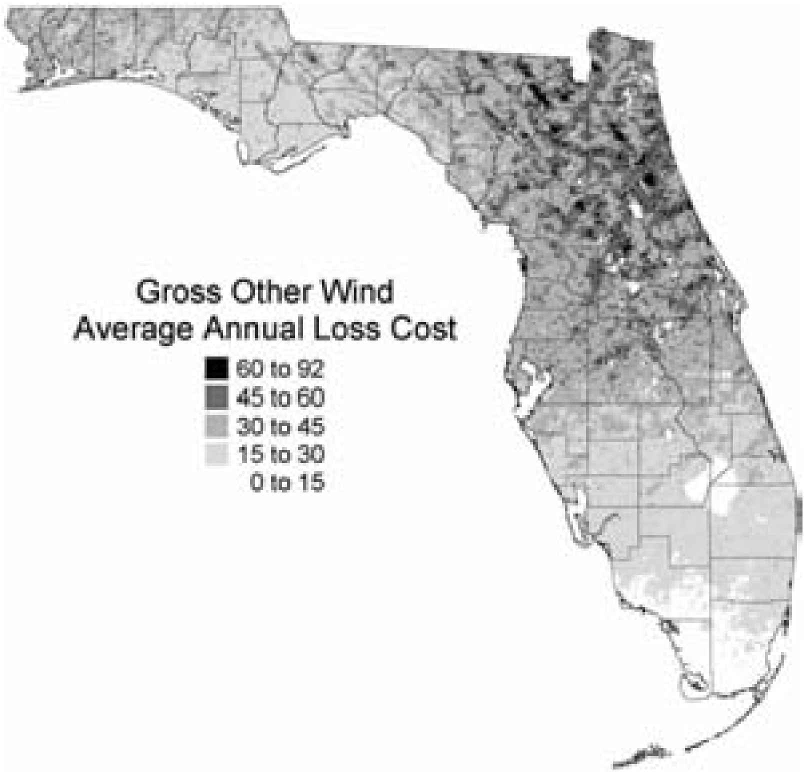
FIGURE 2



loss costs in the southern latitudes are much higher given the same distance to coast. Figure 2 maps modeled expected annual hurricane loss costs by land survey "section" defined by the Public Land Survey System—a unit of one square mile. The data set is weighted equally in each section, as explained below.

In contrast, modeled loss costs for other wind do not follow the geographical pattern of those for hurricane. In fact, they tend to be negatively correlated with the hurricane indications in similar groups of sections. Figure 3 maps the modeled other wind loss costs by section on the same data set.

FIGURE 3



Loss cost gradients for AOP and liability rating largely reflect demographics, though meteorological and geological phenomena significantly impact sinkhole and lightning losses. On the liability side, urban areas tend to be more litigious, and on the property side, urban areas may be more prone to theft and vandalism losses. Traditional (and ISO) territory boundaries for the classical rating plan were derived primarily from municipality and county lines. These lines may serve as sales territories as well. In short, for actuarial and practical reasons, political lines may still have a place in a modern territory rating system.

For the fire peril, two sets of geographic factors may apply—a construction/PPC factor and a territory rating factor. A solid argument can be made for the elimination of territory factors in fire once construction/PPC factors are redefined to apply to the fire-only partial base rate. On the other hand, regional or demographic differences in loss costs may persist even after adjusting for the level of fire protection by area. This case study found enough variation in fire hazards among territories to justify continued use of a territory rating factor in addition to the construction/PPC factor.

All peril-specific actuarial considerations must be weighed in promulgating revised territory definitions. In addition, practical considerations favor having a single set of boundaries, and those boundaries being determined by landmarks (such as major roads and bodies of water) that are recognizable to salespeople and consumers. In summary, territory boundaries are based on the intersection of all of the following geographic data sets:

1. Actuarially defined contours reflecting loss cost gradients by peril and convenient landmarks that are located close to the modeled contours;
2. Classical (existing) company territory definitions, which segregate barrier islands and some coastal areas;
3. ISO territory definitions (which generally follow county lines, with some municipalities and a few barrier islands separated);
4. Existing county lines.

Consideration of existing company territory boundaries is extremely important for the processes of transitioning individual policy/location records to the new lines. A one-to-many relationship between each existing territory and the new territories allows a simple "lookup" rather than a geo-coding exercise in policy management systems. Likewise, consideration of existing

FIGURE 4

advisory organization lines is valuable for implementing statistical reporting under the modern rating architecture.

Overlaying all of the listed data sets geographically produced 187 new distinct territories.[17] Figure 4 maps this set of proposed territory definitions.

Analysis of loss cost gradients for non-hurricane perils indicated that no significant actuarial advantage (reflecting steep

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[17]To facilitate coding and statistical conversion, existing three-digit codes are redefined as the county code (01-67) plus a third digit of 1 for the territory closest to the coast, followed by 2, 3,...further inland, or 0 for a county which contains only one territory.

gradients in loss costs) was to be gained by rating at a level more resolved than county. Therefore, the remainder of the paper shows rating factors that vary at the county level for non-hurricane perils and the territory level for the hurricane peril.

Definition of Base Structure

Any base rate for property insurance reflects an assumption about the "base" structure insured. For non-modeled perils, this is important because all rating factors are keyed to the base house. For modeled perils, the definition helps incorporate public classification studies and build territory factors.

First, base values insured by policy form for the calculation of key factors are:

Form	Base Value	Coverage
HO-2	$100,000	A (building)
HO-3	$100,000	A
HO-4	$10,000	C (contents)
HO-6	$10,000	C
HO-9	$100,000	A
MH-2	$20,000	A
MH-3	$20,000	A

These values are generally consistent with those used by advisory organizations such as ISO.

Other base attributes, most following industry norms, are as follows:

- Base deductible is a flat $500 for all perils other than hurricane.
- Base deductible for hurricane is 2% of the coverage A amount, in keeping with the Florida practice of percent rather than flat dollar deductibles for hurricane. This also aligns the proposed rating plan with the public domain studies promulgating class factors for mitigation attributes.

- Base liability limit is $100,000 and medical payments limit is $1,000.
- Base fire protection class (PPC) is 9 (on the ISO scale of 1 to 10).[18]
- Base construction type (for fire premium) is frame.
- There is no base territory—the territory factors for hurricane, other wind, liability, and AOP are expressed and balanced relative to the statewide average of 1.00.

In the rating factor analyses for fire, AOP, and liability discussed below, actual experience data is used to develop indicated factors and various adjustments are made when it is necessary to bring experience to a "base class" level for a particular attribute.

In contrast, the rating factors for modeled perils are determined from an experimental data set consisting of hypothetical "base" structures placed around the state. An input data set was built containing one base structure in the geographic land centroid of every square mile section of the state—55,930 modeled locations in all. This is similar to, but much more extensive than, the approach taken by ARA in their public domain study. Base house attributes are as follows:

- HO-3 policy form insured for all perils;
- $100,000 coverage A, coverages B/C/D at 10%/50%/20% of A (respectively);
- A $500 other wind and a 2% hurricane deductible;
- Frame construction type;
- Gable roof attached with clips and covered by standard shingles;

[18]Most companies and ISO use 3 as the base; our departure reflects the predominantly rural demographic profile of our policyholders.

- A garage with unreinforced door and no other opening protection (i.e. storm shutters).

In other words, the base house is of base rating values and "unmitigated" with respect to hurricane damage, roughly as defined in the ARA study.[19]

It is advantageous to use experimental data sets for rating factor development for modeled perils for several reasons. Actual exposure data generally reflects vastly different property profiles by region. These maldistributions extend to nearly every rating variable—average total value insured, average windstorm mitigation and fire protection level, average deductible amount, and others. Hurricane or other wind modeled relative loss costs generated from these lopsided exposure profiles would be so biased as to be nearly useless.

A related problem is that of "missing" exposure. In the extreme case, the lack of exposure in a new, more refined coastal territory could result in an indication of a zero rating factor as a zero loss cost for the region is produced by the model. Alternatively, much of the existing exposure in coastal territories could be written on an "ex-wind" basis, whereby the hurricane peril is excluded from the policy. If the exclusions are noted in the data supplied to the model, the same problem will result. In short, when the territory boundaries are redefined, it is essential to consider the full spectrum of possible exposures in geographical rating factors. This is possible only with a contrived data set.

7. BASE RATES AND EXPENSE FEES

Recalling formulae (7) and (8), base rates and expense fees are built from loss costs, fixed (non-loss) reinsurance costs, and

[19] In a parallel study for mobile-homeowners, an analogous experimental data set was built for a mobile home with MH-3 policy form, $20,000 coverage A and associated standard relationships for coverages B/C/D, and a "mobile home" construction type with no mitigation devices.

fixed underwriting expenses, all expressed in dollars per policy, then loaded for variable expenses and profit. In turn, these components are determined from cat models (for hurricane and other wind perils), historical loss and exposure data and distributions (for non-modeled perils), and the breakdowns of underwriting and reinsurance expenses used in the overall rate level change calculations.

Exhibit 6 shows how base rates are constructed for modeled perils. First, the fixed reinsurance costs for the homeowners line of business are allocated to policy form on the basis of the product of the latest year's actual distribution of exposure (earned house-years) by policy and the known base coverage amount, or "earned TVI at base value insured." The indicated loading in the base rate is just the ratio of allocated fixed reinsurance costs to earned house-years (policies).

To obtain the loss portion of the base rate, the cat model is run against the experimental data sets and the simulated expected gross annual losses are recorded for every location. Location results are aggregated statewide to obtain the overall average loss for the base structure in a season.[20] The final base rate for hurricane, by policy form, is the loaded sum of the loss cost and fixed reinsurance cost. Recall we have chosen to allocate all non-loss reinsurance costs to the hurricane peril, so the other wind base rate by form is just the loaded loss cost.

The analogous base rates for non-modeled perils are based on historical data and developed on Exhibit 7. When using the loss

[20]Model results are less credible for HO-4 (renters) and HO-6 (condominium unit-owners) policy forms. The choice was made to reduce modeled loss costs for the site-built homeowners forms, based on the ratio of the sum of base coverage A/B/C/D amounts for the forms, to derive a reasonable hurricane loss cost for HO-4 and HO-6 forms. Specifically, the HO-4 policy provides a $10,000 base for contents coverage, no coverage for structures, and "loss of use" coverage of 20% of the contents coverage, while the HO-3 provides a $100,000 base amount for dwelling coverage, 10% of the dwelling amount for other structures, 50% of the dwelling amount for contents, and 20% of the dwelling amount for loss of use. The ratio of total modeled coverage between these two forms is therefore $(10 + 2)/(100 + 10 + 50 + 20)$, or about 6.7%. This assumes the same average damageability ratios over all coverages.

cost ratemaking method along with historical property exposure data, several distributional adjustments may be necessary. For the fire peril, the average underlying key factor (a function of TVI) and average underlying construction/PPC factor are likely highly divergent by policy form. The exposure base, the denominator of the loss cost, is multiplied by the average underlying factor in the *proposed* rate structure (for each maldistributed rating factor) to restate it at a "base class" level for determining the base rate. Similar adjustments apply for average underlying limits in the base rate for liability and average underlying TVI in the AOP base rate. The adjusted loss cost must still be loaded for variable expenses and profit, of course.

The need for distributional adjustments to the loss cost based on proposed rating factors means that these rating factors must be determined before the final base rates are. This is necessary for an efficient and equitable rate structure when rates are developed from the ground up. Later, it is shown that we achieve adequate revenue under the modern rating plan by "solving for" the base rate that matches indicated overall rate level to estimated rate impact.

Expense fees by policy form are developed on Exhibit 8. The ratio of the latest year's earned premiums (including such fees) to earned house-years represents an average premium per policy. The fixed expense ratio is applied to this value, and loaded to obtain the indicated fee. In practice, round numbers are often selected for expense fees and they are often set equal for similar policy forms.

8. TERRITORY AND CLASS RATING FACTORS

In the basic rating logic, territory factors apply to every peril. In addition, class factors apply to fire (construction/protection) and hurricane (mitigation), and increased limits factors adjust the liability premium. Base premiums for each non-liability peril reflect coverage adjustments for amount of insurance and amount of (or percentage) deductible.

Territory Factors—Modeled Perils

Exhibit 9 presents one method of determining hurricane territory rating factors that incorporate an allocation of fixed reinsurance costs. Most actuarial techniques for the development of rating factors use only the mean loss cost to modify the base rate. This method uses the modeled mean loss costs by territory to modify the loss portion of the base rate, and the standard deviation of these loss costs to modify the fixed reinsurance cost portion of the base rate.

Recalling formula (7), F_R denotes the fixed (non-loss) reinsurance expense dollars per policy. The bulk of non-loss reinsurance costs reflect some measure of risk as perceived by the reinsurer. Many risk metrics (as functions of the possible loss outcomes on a portfolio of policies) exist, and it is beyond the scope of this paper to capture the essence of the (considerable) actuarial debate over the best metric for reinsurance premium development. The assumption used here is simple and squares with observations of the global reinsurance market:

$$F_R = K \times S_L \tag{12}$$

where

S_L = the standard deviation of the modeled annual losses—readily available by location or in geographical aggregate from the cat model;

K = an empirical scale factor that relates the volatility in modeled losses to the actual non-loss ceded reinsurance premium.

In other words, assume that reinsurers charge for cost of capital in proportion to the standard deviation of annual losses. While reinsurance pricing models tend to be proprietary, there is longstanding support in both actuarial literature [8] and market practice to brand this assumption reasonable.

We choose the scale K_i identically for each territory so that the exposure-weighted F_R by territory, based on S_L, balances to the aggregate F_R derived on Exhibit 4 (expressed per unit of losses). The success of the technique does not require this choice—the scale factor could be set lower in some territories and higher in others to reflect second-order assumptions about the capacity charges levied by reinsurers in different areas.

The modeled number of exposures (essentially a land-area weight given the construction of the experimental data sets), modeled (mean) loss cost, and modeled standard deviation of losses are collected for locations falling in each proposed territory. Note that the overall modeled loss cost is the exposure-weighted average by territory, but the aggregate standard deviation is *not* additive—it must be collected directly from the model output. By design, the allocated fixed reinsurance costs, reflecting the scale factor, do average (exposure-weighted) to the aggregate fixed costs derived in the overall rate level indication.

The sum of the modeled loss cost and fixed reinsurance cost for each territory is the basis for the cost relativity to the statewide average. This relativity is the theoretical territory factor. In practice, allowance is made for a tempering of the indicated rating factor toward unity due to competitive or regulatory pressure. This is not "credibility weighting" because the modeled loss costs are fully credible in a convergent hurricane model.[21] The tempering is a non-actuarial exercise. If it is present, the resulting factors must be rebalanced to unity.

The techniques may be applied in an identical fashion to experimental data sets for both homeowners and mobile home forms. This study found that the statewide range of territory factors was slightly wider for mobile homes.

[21]The Florida Commission on Hurricane Loss Projection Methodology, an agency charged with certifying the validity of catastrophe models used in rate filings in the state, uses a standard by which modeled mean loss costs must "converge" within a certain tolerance at the ZIP code level. The simulation size required for convergence can be very large (50,000 years in the case of at least one model).

Though loss and fixed reinsurance costs must both be considered in all rating factors for the hurricane peril, formula (10) does not apply directly when this technique is used because:

- The rates are balanced to the statewide average of unity, so there is no base territory relative to which credits and debits are expressed;
- The fixed reinsurance costs are allocated directly in the calculation of the territory factors using the standard deviation of modeled losses.

An adjustment to loss cost relativities may be necessary when mitigation class factors are developed later.

The territory factors for the other wind peril are developed using identical experimental data sets, with the exception of lowering the base deductible to $500. The same basic technique is applied to the model output, with the deletion of the allocation of fixed reinsurance costs—the modeled mean loss cost relativities are the sole basis for the (possibly tempered and rebalanced) territory factors. The advent of simulation models for other wind offers the opportunity to exorcise the last vestiges of the classical ISO "excess wind procedure" and its brethren from ratemaking for infrequent catastrophic events.[22]

Territory Factors—Non-Modeled Perils

Standard one-way actuarial techniques are applied to the problem of setting territory rating factors for AOP, liability, and fire (if desired) from historical experience. Exhibit 10 shows an analysis of AOP territory factors for completeness.

In a loss ratio ratemaking approach, the actuarially correct inner product used to balance the average statewide factor to

[22] See Burger et al. [3] for an excellent contrasting description of the use of cat models for hurricane and an excess wind procedure for other wind.

unity would be that of:

- Adjusted relative loss ratios (losses divided by premiums stated at present level and adjusted to "base" or statewide average territory level) by territory, and
- A weighting vector of earned premiums (on present base territory level) by territory.

In the loss cost approach used in our study, the appropriate weight becomes whatever exposure base is used to calculate relative loss costs. Earned total value insured is the base (one unit of earned TVI is equal to one house insured for $1,000 for one year), so it is used both to calculate relative loss costs and to balance the statewide average territory factors to unity.

Classical (limited-fluctuation) credibility is applied to the relative loss costs, again using earned TVI as the base, to obtain final indicated factors. Many other credibility techniques could be applied, but a survey of them is beyond the scope of this paper. The full credibility standard V_f is chosen by judgment, and the credibility for a single territory is

$$Z_i = \sqrt{\frac{V_i}{V_f}}. \tag{13}$$

Again, it is possible that selected territory factors may differ from indications for non-actuarial reasons. The selected territory factors are rebalanced to a statewide average of unity using the weighting discussed above.

Depending on claim volume, territory factors for the liability peril may be set using regional aggregations of territories. Alternatively, regional loss cost relativities might serve as the complement of credibility for territory-level relativities. In fact, these regions do not have to be geographically contiguous if liability trends tend to follow city and suburban demographics. In any case, the same techniques are applicable except that loss data should also be converted to basic limits to avoid demographic bias.

Class Factors—Hurricane (Mitigation)

Property insurance has always been rated by type of construction, but construction rating attributes were historically designed to rate the predominant peril of fire. The blunt distinction between frame and masonry wall construction was often deemed sufficient. As hurricane has replaced fire as the cause of loss underlying the plurality of the base premium in some states, construction class plans should evolve accordingly. The modern rating architecture should include class plans based on distinct construction attributes for both fire and hurricane perils.

In hurricane, a “mitigation” class plan focuses on features, techniques and devices specifically designed (and often retrofitted to the home after initial construction) to reduce such losses. As discussed earlier, Florida statutes now enumerate several devices that must be considered in the development of the class plan. The public domain ARA study is also required reading for those seeking to understand the rationale for the choice of devices that serve as elements of the class plan. The study found that the following devices significantly reduce hurricane losses and should be treated as “primary rating factors”:

- *Roof shape* (gable, hip, flat, and others)
- *Roof covering* (shingles compliant with FBC, shingles not compliant with FBC, tile, metal, and others)
- *Secondary water resistance* of roof (present in the form of taped or sprayed sealant, or not)
- *Roof-to-wall* connection (toe nails, clips, hurricane wraps of single or double layers)
- *Roof deck attachment* method (four categories based on nail size and spacing)
- *Opening protection* (engineered storm shutters, non-engineered attachments such as anchored plywood, or none at all)

The study noted several additional attributes that reduce hurricane losses enough to be treated as "secondary rating factors":

- Opening protection *coverage* (windows only or all openings including doors and garage doors)
- *Gable end bracing* (present or not)
- *Wall construction* (the traditional fire class variable, frame or masonry)
- *Wall-to-foundation restraints* (present or not)

An actuarially interesting result of the study is that the reductions in expected loss cost for various combinations of devices turn out to be highly interactive, meaning that the class factors cannot be set for individual devices and multiplied or added across all devices present to determine the appropriate comprehensive class factor. Instead, a multi-dimensional table of modeled primary rating factors for each combination is needed, such as the one shown in Exhibit 11.[23]

The indicated reductions in loss costs for the various combinations also depend upon the terrain category (flat, swampy, hilly) associated with the property location. ARA divided the state into two basic terrain categories that they denoted "B" and "C." A reasonable choice is to map the terrain category definitions shown in the study to the proposed territory structure, designating each entire territory as one category to facilitate the determination of class factors from the tables without additional geo-coding.

Given the raw loss cost relativities, the final class factors must still embody a key actuarial assumption. When the mean loss cost is reduced (relative to the unmitigated base structure) for a house by application of mitigation devices, should its allocated portion

[23]Exhibit 11 shows the actual factors promulgated in the ARA study, relative to a base structure which is largely unmitigated and carries a 2% hurricane deductible.

of fixed reinsurance costs be reduced as well? If so, should it be reduced in proportion to the mean or should the reduction be tempered? Recall that formula (12) assumes that fixed reinsurance costs are proportional to the standard deviation of modeled losses. Even if one believes this assumption is valid at the individual risk level, it is entirely possible that a reduction in mean losses could decrease S_L less than proportionally, or even increase it. Alternatively, other seemingly intuitive assumptions—for example, that the coefficient of variation of modeled losses would remain constant when a mitigation regime were applied—would lead to a fully proportional reduction in fixed reinsurance costs (and therefore class factors that are identical to the raw loss cost relativities).

Under the assumption that non-loss reinsurance costs are truly "fixed" even in the presence of mitigation, the class factors may be derived from the loss cost relativities using formula (10), where α is the relative loss cost, X is the permissible loss ratio from Exhibit 1, and F_R is the fixed reinsurance cost ratio from the same exhibit. For example, a loss cost reduction of 20% for a device, along with a permissible loss ratio of 65% and a fixed reinsurance cost ratio of 10% would lead to a class factor of

$$\rho = \frac{(1 - 20\%) \times 65\% + 10\%}{65\% + 10\%} = .827.$$

In addition to the key issue of reductions in fixed reinsurance costs, the public domain studies have been silent on several important issues for ratemaking:

- Should this mitigation class plan apply to losses from other wind (non-hurricane storms containing tornadoes, hail, and severe straight-line wind)? If not this plan, what about a modified alternative? Other wind causes of loss were not considered.
- Should this mitigation class plan apply equally to owners, renters, and condominium policy forms? It stands to reason that the factors should be modified when contents coverage is

the predominant exposure under the policy. Yet little guidance was provided in the studies.

- How should mitigation class plans be modified for commercial construction exposures?
- How should mitigation experience data from actual catastrophic events, as they are occasionally experienced, be assimilated into the class factors? It would be hubristic indeed to assume that mitigation devices and combinations thereof will perform exactly as modeled when we observe the effects of a real hurricane. To the extent they do not, what is the actuarially appropriate credibility for the vital data from actual events in future class factors?

In summary, actuaries and their scientific partners have a long way to go in developing comprehensive mitigation class plans for the relevant perils. To the extent we do not ask all the right questions, unpalatable answers may be forced upon the insurance industry.[24]

Class Factors—Fire (Construction/PPC)

In the classical rating plan, class factors are targeted at the fire peril and two attributes of residential structures: the resistance of the structure to fire damage, and the level of fire protection afforded by the community in which the structure is located. These two attributes are highly interactive—masonry construction, which is more fire resistive, is more common in suburban environments where fire hydrants are prevalent and fire stations plentiful. Therefore, rating factors are developed using "two-way" actuarial analysis, as detailed by many contributors to actuarial literature.

This study breaks no new technical ground here—fire peril experience data is used along with a two-way "minimum bias" procedure to develop sound construction/PPC factors for the modern

[24]Regulators in Florida have already encouraged blanket application of the class factors for residential structures to HO-4 and HO-6 policies and other wind base rates.

rating plan. No exhibits on this topic are included, but following are some empirical results associated with a peril-specific analysis:

- The "spread" of construction/protection class factors is much wider when only the fire peril experience is considered in the analysis, as losses for other perils are not part of the experience base. Non-fire losses, which do not vary significantly by fire rating attributes, serve as ballast dampening the construction/protection class factors toward unity in the classical plan. This result confirms one of the stated advantages of the modern rating plan—greater rating resolution for non-catastrophic perils.
- Significant differentials in loss experience are found for individual (ISO) protection classes 4, 5, and 6, prompting development of separate factors for these classes. Most insurers combine classes 1–5 or 1–6 and use the same rating factor in classical rating plans.
- Fire experience for hybrid construction types such as brick veneer (over frame) and "hardi-plank" siding varies significantly from that for either full frame or full masonry construction. Expansion of the classical "frame vs. masonry" construction class distinction to include an intermediate rating class for these hybrid types is advised.

9. KEY, DEDUCTIBLE, AND LIMIT FACTORS

As Exhibit 17 shows, the modification of base rates for territory and class leads to partial key premiums by peril. The key premiums are further modified for attributes reflecting the volume of coverage provided, via key factors, deductible factors, and increased limit factors (for liability), to obtain partial base premiums. It turns out that the incongruities in the loss distributions for fire, AOP, and modeled perils are significant enough to warrant separate development of key and deductible factors for each peril. In addition, the presence of percentage deductibles for hurricane requires a separate set of deductible factors.

Key Factors—Non-Modeled Perils

The reflection of value insured is probably the single most important rating factor in pricing property insurance. Given its critical importance, it is one of the most under-represented topics in the actuarial literature. Background research for this study was frustrated by little existing guidance on techniques for developing key factors from experience data for even an indivisible premium, and absolutely none on key factor relationships for distinct perils when rated separately. Most papers on homeowners pricing do not treat the subject at all. Homan [10] provides a clever frequency/severity approach for an all-perils development, but his reliance on industrywide loss cost distributions for the complement of credibility is not helpful when no analogous complement is available by peril.[25] In summary, one is caught between the "rock" of low credibility of experience data by peril within small ranges of insured value, and the "hard place" of no suitable complement of credibility in the form of larger-scale studies.

In response, an approach is developed for AOP and fire perils based on accumulations of experience data at successive levels of value insured. It reflects the value of experience data while facilitating smoothing of the indicated loss costs to produce tables that square with actuarial theory.

Exhibit 12 shows the development for the fire peril. Five calendar years of experience is segregated by $5,000 ranges of (coverage A only) TVI. First, the average classical all-perils key factor for the midpoint of the range is shown for reference, along with the earned house-years and paid fire losses (with D&CC). Second, the exposure and losses for all TVI ranges up to and including the current range is accumulated, and the cumulative loss cost calculated.

[25] Homan also includes a treatment of fixed expenses, which is not necessary when an explicit expense fee is charged—as it is in the fair premium structure developed here.

Why accumulate? Theoretically, the key factor represents the loss cost at a given (incremental) TVI range relative to the loss cost at the base value, but the loss cost series for individual TVI ranges is simply too volatile to use directly. Instead, use the more stable cumulative loss cost series to mark selected cumulative loss costs at "target" points (generally every \$25,000 of TVI), and calculate the implied incremental loss cost in the target range by decomposing the cumulative value as follows.

The known cumulative losses can be represented as the sum of a series of incremental loss costs times incremental exposure in each TVI range up to the current one:

$$L_k = \bar{\lambda}_k \bar{W}_k = \lambda_1 W_1 + \cdots + \lambda_k W_k$$

where:

λ_i = the incremental loss cost in each range ($i = 1,2,\ldots,k$);

W_i = the exposure weight in each range;

and bars above indicate cumulative totals for ranges "up through" an amount. Then solve for the incremental loss cost for the current range (denoted by k) from the cumulative totals and the exposure in the current range:

$$\lambda_k = \frac{\bar{\lambda}_k \bar{W}_k - \bar{\lambda}_{(k-1)} \bar{W}_{(k-1)}}{W_k}. \qquad (14)$$

Once the implied key factors are found for each of the target ranges, interpolate linearly between every two target points to find the key factor for the \$5,000 ranges in between.

When selecting cumulative loss costs at target points, one must be careful to keep the implied marginal key factor (difference between key factors for successive \$5,000 ranges) between the theoretical lower and upper bounds of:

- Zero (meaning no additional losses are expected despite the increase in policy limit), and

- .05 (meaning all losses are total and will "burn through" the additional policy limit of 5% of the basic limit in a linear fashion).[26]

This is a non-trivial exercise requiring some trial and error. Exhibit 12 shows a reasonable curve given the credibility of some actual data and the theoretical limitations. Also, the factor for "each additional $5,000" beyond $250,000 primarily reflects the marginal factor in the last target interval.

The effects of fire protection, construction, and average TVI overlap severely in the rating plan. Higher-valued homes tend to be of masonry construction and located in well-protected suburban areas. Accordingly, the fire peril exposure amounts may be adjusted to ameliorate this distortion to the raw incremental and cumulative loss costs. Specifically, divide out the proposed construction/PPC rating factor from each exposure record in the statistical data to get a loss cost stated "on base class."

Key Factors—Modeled Perils

In catastrophe simulation models, the result for each simulated event at each location is typically the "mean damage ratio," a value representing the damage as a proportion of the value of the structure(s) insured. The value of the structure is given as a parameter by the user of the model and the mean damage ratio is applied to it to generate the modeled losses. Put another way, in the models there is an assumption of independence between the mean damage ratio for the structure and its insured value, all other attribute held constant. Most insurers make blanket (as opposed to policy-level) assumptions about insurance-to-value when populating an exposure data set for simulation, which proportionally affect the modeled cat loss costs for pricing purposes.

Assuming the values insured reported to the model are reflective of sufficient insurance to value, this attribute of cat models

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[26]Recall that the base structure is defined as one of $100,000 TVI.

implies that the key factor table is linear with respect to TVI for the modeled perils. The hurricane base premium for a given \$200,000 house is twice that for an identical \$100,000 house. A further discussion of the appropriateness of this assumption appears in the ISO [12] filing to partition wind base premiums, as part of their statutory compliance filing of the Florida mitigation class plan. The note the ISO key premiums are nearly linear for the wind peril.

To some, the assumption may appear to be an unacceptable oversimplification and a weakness of using simulated catastrophe losses in pricing. For a heavily reinsured company, the argument over whether the key factor table should be driven by the linearity of modeled loss costs is largely academic. Market reinsurance costs are increasingly driven by the distribution of modeled losses, and the retailer of insurance must reflect its "wholesale" cost for each risk, as charged by the reinsurer, to avoid economically irrational underwriting.

A linear scale of key factors for both hurricane and other wind perils is thus reasonable. The key factors vary by policy form only because the base value insured differs by form. In Florida, one practical effect of the separation of key factors by peril is higher hurricane rates for high-valued homes. These homes were significantly subsidized by application of sub-linear key factors to indivisible premium, of which a plurality (if not a majority) is typically hurricane premium.

Deductible Factors—Non-Modeled Perils

Unlike key factors based on the aggregate loss cost distribution, deductible factors depend solely upon the loss severity distribution. An excellent review of general deductible pricing theory appears in Hogg and Klugman [9], and familiarity with the "loss elimination ratio" (LER) as the kernel of the deductible rating factor is assumed. This study provides strong evidence that the LER profile varies greatly by peril. In addition, one might expect that the LERs should vary significantly across many other rating factors, such as value insured, territory, and class. In order to maintain manageable rating logic, flat dollar deductible factors are allowed to vary by peril and TVI range, and by territory only for modeled perils.

The deductible factors for non-modeled perils are developed directly from five years of individual claim data. Flat dollar deductibles ($500, $1,000, and $2,500) are the only options for non-modeled perils, in contrast with the percent (of coverage A TVI) deductibles offered for the hurricane peril and discussed below.[27] Each existing claim is stated on a "ground-up" basis by adding back the deductible amount associated with the claim.[28] The net of deductible claim amount is determined for each claim under each flat deductible option. The sum of all claims valued at each deductible option is compared to the ground-up losses to determine the empirical LER for each deductible amount. Then the deductible rating factor for each non-base deductible is calculated as

$$d_i = \frac{1 - \mathrm{LER}_d}{1 - \mathrm{LER}_{\mathrm{Base}}} \tag{15}$$

or the ratio of the losses retained (not eliminated) at the target deductible to those retained at the base deductible (of $500 in this study).

These factors depend heavily on the underlying exposure (TVI) distribution of the empirical data, since the amounts of total losses vary by claim but the flat amount does not.[29] Ac-

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[27]There is no theoretical reason percent deductibles by peril cannot be priced from experience data. In fact, one could argue that percent deductibles are actuarially superior for all perils because they "inflate" with the value insured and therefore with the corresponding loss severity distribution, a big help in preserving the loss elimination ratios underlying the rating factors. The resulting factors become obsolete over time much more slowly. Though state statutes tend to restrict deductible options depending upon TVI, at least one Florida insurer has recently introduced an all-perils percent deductible.

[28]This does not solve the "missing claims" problem of losses not exceeding the actual deductible which "would have been filed" if the deductible were smaller. This distortion is ignored here.

[29]The data was divided into TVI ranges which produced a credible and approximately equal amount of earned house-years in each range.

tuarial theory states that the LER for the same deductible option and the same underlying (unlimited) loss distribution will be smaller as the average TVI (policy limit) increases. Further, the relationship between the LERs for two (small amount) deductible options should be dampened as both options represent an ever-smaller portion of increasing TVI. This implies a two-way consistency test for deductible factors:

1. The selected factor for a given TVI range should (obviously) decrease as the deductible increases, and
2. The selected factors for a given deductible should converge toward unity as the TVI range increases.[30]

When this process is compared for multiple perils, one expects the loss distribution for perils which tend to result in more total losses (such as fire) to imply smaller LERs at all deductibles, and therefore deductible factors closer to unity, than those implied by a peril producing more partial losses (such as AOP). Therefore, across multiple perils a third consistency test applies:

3. The selected factor for a given TVI range and deductible option should be closer to unity for the more "severe" peril (the one with the more right-skewed distribution of loss amounts).

Exhibit 13 shows representative LERs and selected deductible factors that reflect all three tests.

Deductible Factors—Modeled Perils

Percent deductibles applicable only to the hurricane peril are the rule in Florida. They were originally introduced as an innovative way to reduce loss exposure without nonrenewals in the market turbulence following Hurricane Andrew in 1992. In lieu of experience data, this study uses the cat model to deter-

[30] Whether they start above or below unity is determined by the base deductible.

mine hurricane deductible factors by scenario testing over several model runs on the same experimental data sets, with only the deductible option changed in each scenario. Specifically, replacing the base 2% deductible with each of the other deductible options (in our study, 0.5%, 1% and 5%), the model is repeatedly run to determine the simulated loss elimination ratio.

Catastrophe simulation science indicates that the shape, as well as the scale, of hurricane loss distributions varies widely by territory. In fact, areas with high average hurricane loss costs also tend to have a greater frequency of severe storms that produce more near-total property losses. Ideal hurricane deductible factors should therefore vary by territory. In consideration of maintaining manageable rating logic, the study examines the scale (expected annual loss costs by territory) of the hurricane loss distribution by territory from the experimental base data set and divides the territory set into Low (less than $400 per year), Medium ($400–$599), High ($600–$1,099), and Extreme ($1,100 and over) hurricane intensity zones. The boundaries are determined by judgment, and intended to include a reasonable number of modeled locations in each zone—though most modeled points are in the Low zone, the higher-intensity zones must be segregated to produce reasonably accurate factors. The modeled losses are aggregated under each scenario in each zone, the relativities to the modeled losses at the base deductible are computed, and deductible factors selected. Exhibit 14 shows the results.

When using the model to price flat dollar deductibles as a modification to the base rate for a percent deductible, the problem of exogenous values insured pops up again, in a different disguise. Any flat amount represents a constant percentage of a single experimental base value insured, no matter what the choice. For example, the modeled losses, and therefore the loss elimination ratio, for a $500 deductible scenario will be identical to those for a 0.5% deductible scenario when the base value is

\$100,000.[31] The actual deductible factor charged in rating, even for the hurricane peril, should depend upon the empirical TVI distribution of the insurer's book, and indeed the TVI of each property. By design, this is not considered in the experimental data set.

Rather than resolving "the" proper way to differentiate flat dollar hurricane deductible factors by TVI range, the study settles on an adjustment to a base scenario (that for the 0.5% deductible, which is equivalent a flat \$500 deductible for the majority of units in the experimental data set). The implied relative loss cost for *AOP perils* by value range, shown on Exhibit 13, is the ratio of the complement of the loss elimination ratios in each range; the calculation is analogous to formula (15), but relates TVI ranges rather than deductible amounts. Select a relativity, then apply it to the modeled 0.5% deductible factors by zone to produce \$500 flat deductible factors that vary by both TVI range and zone. For example:

Low zone, under \$75,000:

$$1.17 \approx \frac{(1 - 25.0\%)}{(1 - 20.4\%)} \times 1.23$$

Medium zone, \$225,000 and over:

$$1.26 \approx \frac{(1 - 14.7\%)}{(1 - 20.4\%)} \times 1.18$$

and so on. The end result is a reasonable consideration of both value insured and territory loss distributions in the pricing of hurricane flat dollar deductibles. The calculation could be repeated for other flat deductible options.

The deductible factors for other wind, where only flat dollar deductibles are offered, are calculated using exactly the same procedure and modeled scenario testing, except that factors are

[31] This is true assuming that the model contains a "static" event set which is applied to every location. Some models build a "secondary uncertainty" randomization component into the analysis, which means the modeled losses for the same scenario on the same event set will still differ somewhat every time the model is run.

not differentiated by zone. This simplifies the process by removing one dimension from the matrix of rating factors. Catastrophe simulation science indicates that other wind aggregate loss costs are driven by expected event frequency and that the shape of the severity distribution of individual severe thunderstorm events is not as critically different by territory. Further, other wind is a much smaller portion of overall base premium in Florida, leading to the decision to waive this adjustment.

Limit Factors—Liability/Medical

This paper breaks no ground with respect to the actuarial techniques for calculating limit factors for the liability peril (coverage E), but there are still advantages to divisible base premium. Limit factors are often based on benchmarks obtained from the voluminous databases and advanced loss distribution analysis provided by advisory organizations such as ISO. With distinct liability base premium, there is an opportunity to move away from the cumbersome additive charges commonly used in residential property insurance and develop multiplicative limit factors for liability base rates with appropriate reference to industrywide data. The modern rating logic includes a liability base rate modified by a multiplicative factor.

Medical payments coverage (coverage F) is such a small part of the overall base premium that one may simply add the base rate to that for liability (after modification by the limit factor) and allow for the existing additive medical limit factors. Application of multiplicative factors to medical might even result in premium changes of less than one dollar, which is not practically desirable in most policy administration systems.

10. ADJUSTMENTS TO BASE PREMIUM

Many adjustments (charges and credits) are made to the base premium to determine a final homeowners policy premium, even without the presence of specific endorsements. The modern rating architecture allows several improvements to these adjustments:

- Some adjustments may be recalculated as a modification to an appropriate subset of the total base premium rather than a blanket modification of premium for possibly impertinent perils;
- Some adjustments for excluded perils may be accomplished by partial or total elimination of a portion of the base premium, simplifying the rating logic.

Exhibit 15 shows how charges and credits are recalibrated to a smaller premium base when changes must be revenue-neutral in aggregate. One may tabulate the statewide distribution of base premium by peril and policy form, then simply divide the current credit or charge by the proportion of the proposed premium base represented by the components to which the credit or charge is targeted, to make the modifier appropriate for the smaller base. Of course, the actuary may determine that larger or smaller revenue effects are indicated and use experience data to adjust the charges and credits in line with indications, provided the expected revenue gain or loss is acknowledged as an off-balance in the determination of overall rate level impact.

Some examples of actuarially sensible changes to adjustments to base premium are:

- *Wind and hail exclusion* may be accomplished by simply eliminating the base premium for hurricane and other wind in the total base premium calculation. Tabular factors formerly used for this purpose may be eliminated, streamlining rating logic.
- *Superior construction and storm shutter credits* may be eliminated, as they are superseded by the comprehensive windstorm mitigation class plan.
- The *seasonal occupancy charge* may be adjusted to apply to the (AOP + fire + liability) base premium, if it is believed that the wind resistance of the structure does not depend on occupancy.

- The *protective devices credit* for smoke and burglar alarm combinations may be adjusted to apply to (AOP + fire) base premium.
- The *age of home credit* may be adjusted to apply to the (AOP + fire) base premium, or eliminated with the advent of fire and hurricane class plans.
- The *town/row house charge* may be adjusted to apply to the (AOP + fire) base premium.
- The *replacement cost provisions charge* for "guaranteed replacement cost" endorsements may be adjusted to apply to the non-liability base premium.

11. IMPLEMENTATION ISSUES

The move to a modern rating architecture for residential property insurance affects many non-actuarial functional areas within an insurer, including:

- Operations (programming, policy management, statistical reporting)
- External affairs (filings, regulatory relations)
- Marketing (sales force, customer service training, competitive analysis)

Several specific items and issues with actuarial overtones and cross-functional impacts are discussed below.

Measurement of Overall Rate Level Impact

Most rate reviews proceed in three major steps:

1. Examine the indicated overall rate level change;
2. Determine base rates and rating factors (and rating logic as necessary);

3. Assess the overall rate level impact of the selected rate structure and reconcile it with the indicated overall change.

Step 3 is extremely important to both internal and external stakeholders in the insurance economy as well as to the actuary charged with maintaining profitability. It may be accomplished at several levels of granularity. When only a few base rates and rating factors are changing and there are no significant changes to the rating logic, aggregate estimates of the overall impact may be sufficient. The extreme case would be a single change to a base rate that applies to all policyholders, in which case the actuary could state with certainty the overall impact without analyzing the effect at the policy level. When the rating logic and territory definitions are completely redesigned and each base rate, class and territory rate table is developed from first principles, the other extreme applies. The overall rate level impact must be measured by re-rating every existing policy on the proposed rate structure.

The actuary must be prepared to build tools that compare "before and after" premiums for each existing policyholder and that can be run iteratively in a timely fashion. Again, technology is the enabler allowing the extraction of high-quality data and execution of rating logic quickly to measure rate impacts in this fashion. As the impacts are compared against the indications, the most efficient technique for iterative adjustment is a flat factor applied to the base rates by policy form.[32] This study does not vary the flat factor by peril, which has the effect of preserving the overall distribution of base premium.

Competitive and Residual Market Analysis

Even a policy-level measurement of static overall rate level impact is still insufficient to indicate the likely second-order or

[32] As a regulatory matter, some states require rate indications developed by policy form—in this milieu, the flat factor applied to the indicated base rates to achieve the overall indication should also vary by form.

dynamic effect on overall premium and policy volume (as prices incent consumer actions) and distributions by policy form, territory and class. Yet this is actually the effect of greater magnitude to the profitability and growth of the insurer in the long run. When all the insurers competing in a market have similar rate structures and the market is relatively stable, the effect of an overall rate level change that does not displace many existing customers differently than the overall average may perhaps be measured with ignorance of dynamic competitive effects. When an insurer makes a market-leading change to a modern rating architecture, the likely competitive effects must be examined in advance and monitored closely as the architecture is rolled out. Returning to Cummins [7] will remind the reader of how critically certain market attributes can affect the possibility of adverse selection against the insurer.

On the flip side, a modern rating plan is one of the few ways to gain a sustainable competitive advantage in the market without a significant investment in operational scale and surplus capacity. Further, marketing and underwriting restrictions should be comprehensively reviewed and aligned with the rating plan once it is implemented. Historical restrictions that reflected rate adequacy considerations in particular territories and classes may be rethought as the marketing plan is revisited. In summary, a more refined rating plan should facilitate some additional growth given constant surplus.

The regulators (and possibly private sources) in many states collect proposed premiums for standard rating examples (a.k.a. “risk profiles”), which are most often publicly available. These rate comparisons may also include the residual market rate from the insurer of last resort if there is one. The actuary can compile such comparisons as a leading indicator of changes in competitive position, at least for “typical” risks. Regulators may be interested in the proposed position of the insurer against public (residual market) as well as private competitors, depending on the level of political pressure against raising residual market rates to maintain minimal competition with the private market.[33] Exhibit 16 shows an example of a rate comparison that might be useful. The actuary should encourage all stakeholders to keep in mind several distortions inherent in rate comparisons:

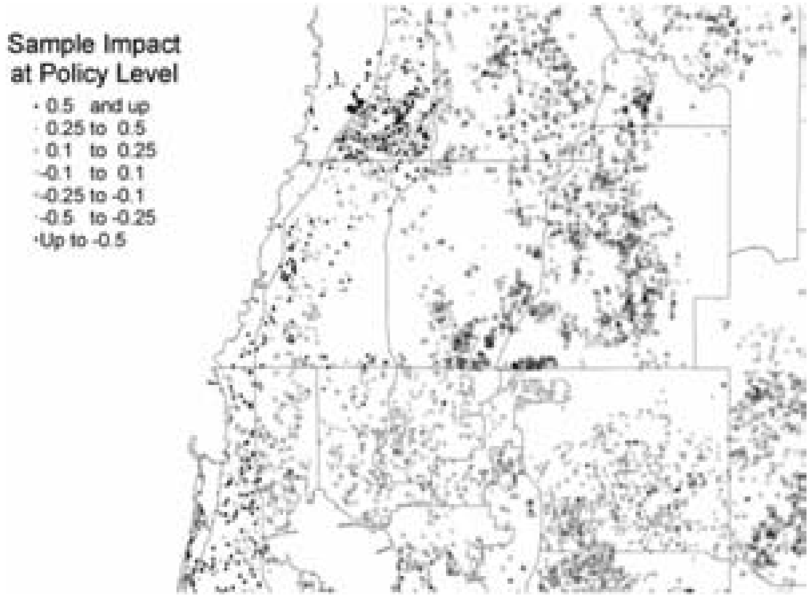
- Comparing an individual insurer's proposed rates to the competition's current rates may produce a false sense of competitive position when rate levels are rapidly rising or falling industry-wide, due to the natural time lag between successive filings. Emerging causes of loss, capacity problems affecting reinsurance prices, and other phenomena may not yet be reflected in the current (more accurately, the last filed) rates of competitors or the residual market.
- Comparisons are often based on the "average" rate for a particular county or wider geographic region. The average may be weighted by an exposure distribution that does reflect that of the insurer, or it may not be weighted at all—a simple arithmetic average using one rating example for each territory within the area. The insurer implementing more refined territory definitions than its competitors produces an average for coastal areas that is most likely skewed upward in this case, because of its removal of inland subsidies to coastal business in a more refined hurricane territory structure. The example for a small coastal territory, perhaps even one in which the insurer has no current business, gets equal weight with the inland example from a much wider land area and more populated area letting the high coastal rate drive the average.

Rate Dislocations and Transition Planning

As critical as it is to understand the proposed rating plan's competitive impact on the ability to write new business in each

[33]In Florida, some residual market rates are set based on the highest premium reported by the top twenty private insurers (as ranked by premium market share) for a given rating example in each county, which focuses regulatory attention more directly on the differential between an insurer's proposed rates and those for the residual market in the same geographic area.

FIGURE 5



territory, it is just as important to manage customer retention when many existing insureds likely face significant rate changes. First, the actuary can inform the marketing and sales force by geographic area in a comprehensive fashion. Figure 5 shows an example of a "pin map" that delineates the proposed territory boundaries and contains a color-coded pin for each existing insured location. The shades indicate the spectrum of rate changes that will be experienced by each location.

Second, serious consideration should be given to a transition plan that caps annual swings in premium to a maximum and minimum percent value, phasing in the premium change for those subject to severe rate dislocations. There is a legitimate debate as to whether such plans are inherently unfairly discriminatory, as new business and renewals would be charged different rates for an identical risk. A complete discussion of the economics and public policy associated with such plans is beyond the scope of this paper, though it is noted that "swing limits," capping changes in rating factors in spite of credibility-weighted indications, are used throughout many accepted rating plans in most lines of insurance. In any case, the practical business advantages of a phasing-in of premium changes for existing insureds cannot be overlooked.

It sounds simple to implement such a plan, but the devil is in the details of how the premium subject to transition is calculated and carried forward from year to year. Basic logic for a plan that caps annual premium increases might be as follows:

1. Calculate P_0, the premium on current rates at the current TVI. P includes premium for miscellaneous coverages and endorsements, but does *not* include expense fees. Premium for endorsements added during the current term is restated as full-term premium on current rates.

2. Calculate P_1, the premium on *proposed* rates at the *current* TVI, for the standard policy coverages and only endorsements that are effective *before* the renewal date (in other words, on an "apples to apples" basis whereby premium for new additional coverages is not compared against current premium totals). P_1 also excludes expense fees.

3. The premium change factor is the ratio of the premium on proposed rates to premium on current rates less unity:
$$H = \frac{P_1}{P_0} - 1. \tag{16}$$

4. If the premium change factor exceeds M, the selected maximum premium increase, let transition factor
$$T_0 = \frac{M}{H}. \tag{17}$$

5. Multiply each peril partial base premium by T_0 in development of final policy premium. Store T_0 with policy

statistics. At the next renewal, update the transition factor by multiplying by the maximum premium increase, limiting it to unity:

$$T_1 = \mathrm{Min}(T_0 \times M, 1.00). \qquad (18)$$

6. Repeat the adjustment of base premium and storage of T_i for as many periods as necessary until it is 1.00.

It is straightforward to modify this algorithm to accommodate a transition plan that limits both premium increases and decreases for individual policyholders.

Steps 1 and 2 reflect the fact that there are many exposures such as endorsements and "inflation guard" (which provides automatic annual increases in TVI to keep pace with replacement cost inflation) of which the treatment should be carefully specified in designing any transition logic. Just as important is a cost-benefit analysis of the revenue loss expected from the transition plan, at least in the first year. Figure 5 should be reproduced to show the rate impacts net of the transition plan. A granular analysis of premiums on proposed rates, by policy, with and without the transition plan should be conducted to aggregate the revenue impact companywide and by territory. This is the only reliable way to assess the plan's impact.

Miscellaneous Rates, Endorsements, and Operational Impacts

Most miscellaneous coverages are rated using key premium as the base. Recall that this is the fair premium for the class and territory, but reflecting a given base coverage amount and deductible. Simply changing "key premium" to "total key premium" (the sum of the key premiums by peril) will allow migration of much of the rating logic for endorsements in a sound manner. However, rates per $1,000 of coverage and flat dollar charges should be thoroughly reviewed to assess their adequacy as the overall rate level and its distribution by peril shift under the modern rating plan.

The basic rating logic may be of primary concern to the actuary, but the policy services, programming, statistical reporting, and manual writing personnel will spend most of their time dealing with its effect on the adjustments to base premium and the miscellaneous rules for and endorsements available in the residential property program. The actuary should be prepared to invest significant time and effort in assisting these vital stakeholders in modifying the other processes downstream that are affected by the changes in basic rating logic.

12. CONCLUSION

Whether due to necessity or strategy, insurers can improve the stability and adequacy of overall rate level as well as the actuarial equity of individual policy rates by investing in a modern rating architecture for residential property insurance. Elements of the modern rating plan may include:

- Proper use of simulated losses for catastrophic perils in overall rate level, territory and class rating;
- A fair premium structure that is aligned with the need for appropriate consideration of expected losses, fixed and variable underwriting expenses, and costs of capital by peril;
- Base premiums divisible by peril and subject to distinct classification and territory rating plans;
- Refinement of corresponding territory definitions;
- Introduction of new class plans targeted to individual perils formerly not class rated;
- Coverage modification (amount of insurance, deductible and limit) factors that reflect differing loss distributions by peril and appropriate assumptions about the loss cost distribution for catastrophic events;

- Rating logic for adjustments to base premium and miscellaneous endorsement premiums that is targeted to the perils affected by such modifications of the policy and consistent with the logic for base premium determination.

In addition, many practical considerations apply as the modern rating architecture progresses from actuarial theory to operational reality within the organization and competitive reality in the outside market. The actuary should take an active role in addressing each issue.

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EXHIBIT 1

A-Florida Insurance Company
Homeowners Rates Effective 1/1/2004
Overall Rate Level Change

Accident Year	[1] Paid Loss +D&CC	[2] Loss Devel. Factor	[3] Ultimate Loss + D&CC	[4] Trend Factor	[5] Trended Loss + LAE Excl. CAT	[5a] De-trended Modeled CAT Loss
1998	10,754,022	1.014	10,904,579	1.461	17,748,250	9,198,579
1999	12,023,961	1.028	12,362,989	1.378	18,982,974	9,474,536
2000	13,774,286	1.043	14,360,943	1.300	20,799,291	9,759,563
2001	16,043,017	1.090	17,478,985	1.227	23,882,287	10,052,349
2002	11,999,482	1.454	17,453,154	1.157	22,497,164	10,353,920
Total	64,594,768		72,560,648		103,909,965	48,838,947

Calendar Year	[6] Direct Earned Premium	[7] On-Level Earned Premium	[8] Trend Factor	[9] Trended Earned Premium	[10] Experience Ratio	[11] Experience Weight
1998	40,100,324	37,606,910	1.000	37,606,910	71.7%	10%
1999	39,949,945	39,038,560	1.000	39,038,560	72.9%	15%
2000	41,122,889	41,122,889	1.000	41,122,889	74.3%	20%
2001	43,280,161	43,280,161	1.000	43,280,161	78.4%	25%
2002	46,105,811	46,105,811	1.000	46,105,811	71.3%	30%
Total	210,559,132	207,154,332		207,154,332	73.9%	100%

EXHIBIT 1

Continued

[A]	Weighted Average Experience Ratio	73.9%
[B]	Adjusting & Other Expense Load	11.4%
[C]	Fixed Underwriting Expenses	6.0%
[D]	Fixed Reinsurance Cost Provision	15.7%
[E]	Variable Underwriting Expenses	17.8%
[F]	(Variable) Profit and Contingency Factor	3.9%
[G]	Indicated Overall Rate Level Change	22.1%
[H]	Permissible Loss + LAE Ratio	56.6%

[1], [6], [7] from company data
[2] from loss devel. analysis (not shown)
[3] = [1] × [2]
[4], [8] from Exhibit 2
[5] = [3] × [4] × (1 + [B])
[5a] latest year from Exhibit 5; prior years detrended with factors from Exhibit 2
[9] = [7] × [8]
[10] = ([5] + [5a])/[9]
[11] selected by actuarial judgment

[A] = average of [10] weighted on [11]
[B], [C], [E] from Exhibit 3
[D] from Exhibit 4
[F] derived per regulatory rule (not shown)
[G] = ([A] + [C] + [D])/(1 – [E] – [F]) – 1
[H] = 1 – ([C] + [D] + [E] + [F])

EXHIBIT 2

A-Florida Insurance Company
Homeowners Rates Effective 1/1/2004
Trend Factors

						Annual Moving Averages		
Calendar Quarter	[1] Earned House-Years	[2] Earned TVI ($000)	[3] Earned Premium excl. Fees	[4] Earned Expense Fees	[5] Paid Losses and D&CC	[6] Earned Rate	[7] Earned Exposure	[8] Loss Cost*
1997Q1	12,835	2,165,308	8,427,043	721,389	2,245,436			
1997Q2	13,120	2,226,806	8,798,473	755,941	2,699,437			
1997Q3	13,426	2,290,692	8,921,938	776,210	3,279,226			
1997Q4	13,604	2,333,822	8,965,026	786,669	2,598,940	720	52,985	204
1998Q1	13,926	2,403,000	9,100,200	802,749	2,287,484	719	54,076	201
1998Q2	14,033	2,447,804	9,157,679	811,602	2,626,653	715	54,989	196
1998Q3	14,276	2,514,409	9,285,824	825,854	2,918,068	712	55,839	187
1998Q4	14,430	2,561,319	9,279,669	833,661	2,518,275	708	56,665	183
1999Q1	14,536	2,599,823	9,210,946	840,193	2,602,927	703	57,274	186
1999Q2	14,654	2,632,956	9,132,242	848,042	3,073,424	695	57,895	192
1999Q3	14,767	2,671,008	9,078,897	856,347	4,293,516	686	58,386	214
1999Q4	14,937	2,721,972	9,115,905	864,260	2,270,183	678	58,893	208
2000Q1	15,048	2,765,971	9,202,260	869,833	2,522,362	673	59,406	205
2000Q2	15,143	2,809,975	9,314,807	876,294	3,089,474	671	59,895	203
2000Q3	15,298	2,866,477	9,477,229	886,071	3,596,407	672	60,426	190
2000Q4	15,395	2,916,663	9,602,072	891,242	3,327,800	675	60,883	206
2001Q1	15,493	2,967,175	9,714,748	895,251	3,007,986	679	61,328	212
2001Q2	15,503	3,007,891	9,829,334	897,285	3,412,061	684	61,688	216
2001Q3	15,609	3,068,204	10,004,985	904,487	4,860,321	689	62,000	236
2001Q4	15,679	3,111,541	10,122,235	908,852	3,570,750	695	62,284	238

EXHIBIT 2

Continued

Calendar Quarter	[1] Earned House-Years	[2] Earned TVI ($000)	[3] Earned Premium excl. Fees	[4] Earned Expense Fees	[5] Paid Losses and D&CC	Annual Moving Averages [6] Earned Rate	[7] Earned Exposure	[8] Loss Cost*
2002Q1	15,778	3,168,506	10,261,147	913,362	4,103,676	701	62,569	255
2002Q2	15,852	3,229,286	10,462,594	920,065	4,045,259	707	62,918	264
2002Q3	15,967	3,310,911	10,707,226	928,750	6,391,492	715	63,277	286

Calendar Year	[9] Trend Period	[10] Premium Factor	[11] Loss Cost Factor	[12] Cat Loss De-Trend
1998	6.507	1.000	1.461	0.888
1999	5.507	1.000	1.378	0.915
2000	4.504	1.000	1.300	0.943
2001	3.504	1.000	1.227	0.971
2002	2.504	1.000	1.157	1.000

	Fitted Annual Changes:		
5 years	−0.56%	3.57%	7.00%
4 years	0.39%	2.95%	10.33%
3 years	2.21%	2.58%	13.31%
2 years	3.28%	2.14%	20.35%
Selected	0.0%	3.0%	6.0%
	[A]	[B]	[C]

*Loss costs are paid losses and D&CC per earned house-year.
[1]...[5] from company data.
[9] = # of years between one year after effective date and midpoint of experience year.
[10] = (1 + [A])^[9].
[11] = (1 + [C])^[9].
[12] = (1 + [B])^ − (2002-Year)

EXHIBIT 3

A-Florida Insurance Company
Homeowners Rates Effective 1/1/2004
Underwriting and Loss Adjusting Expenses

	Calendar Year				
Item	2000	2001	2002	3-year Total	Selected
Direct Written Premium	56,788,429	57,553,000	62,936,260	177,277,689	
Direct Paid Loss + D&CC ex-Cat	17,158,203	21,167,830	24,114,351	62,440,384	
Commission & Brokerage	7,140,183	7,534,182	7,936,740	22,611,105	
Ratio to DWP	12.6%	13.1%	12.6%	12.8%	12.8%
General Expenses	1,392,835	1,317,561	1,453,641	4,164,037	
Ratio to DWP	2.5%	2.3%	2.3%	2.3%	2.4%
Other Acquisition	4,141,156	4,587,674	4,157,582	12,886,413	
Ratio to DWP	7.3%	8.0%	6.6%	7.3%	7.2%
Premium Taxes	663,829	547,425	614,602	1,825,856	
Ratio to DWP	1.2%	1.0%	1.0%	1.0%	1.0%
Other Taxes, Licenses, & Fees	438,182	233,723	211,106	883,011	
Ratio to DWP	0.8%	0.4%	0.3%	0.5%	0.4%
Paid A&OE ex-Cat	2,050,848	2,386,143	2,756,391	7,193,381	
Ratio to Paid Loss + D&CC	12.0%	11.3%	11.4%	11.5%	11.4%

Item	Selected	Fixed %	[A] Fixed	[B] Variable
Commission & Brokerage	12.8%	0.0%	0.0%	12.8%
General Expenses	2.4%	100.0%	2.4%	0.0%
Other Acquisition	7.2%	50.0%	3.6%	3.6%
Premium Taxes	1.0%	0.0%	0.0%	1.0%
Other Taxes, Licenses, & Fees	0.4%	0.0%	0.0%	0.4%
Total Underwriting Expenses			6.0%	17.8%

EXHIBIT 4

A-Florida Insurance Company
Homeowners Rates Effective 1/1/2004
Fixed Reinsurance Cost Provision

Item	Source	Description	Amount
[1]	Exhibit 5	Direct Earned Premium	46,105,811
[2]	Exhibit 5	Private Cat Subject Premium	40,573,114
[3]	Exhibit 5	Modeled Hurricane Gross Annual Losses	10,353,920
[4]	Exhibit 5	Private Cat Reinsurance Premium	9,385,801
[5]	Exhibit 5	Public Cat Reinsurance Premium	3,820,128
[6]	accounting	Private Cat Retention % SMP	10%
[7]	accounting	Private Cat Layer Coverage Level	95%
[8]	$([3]-[6]\times[2])\times[7]$	Reinsured Portion of Loss Cost	5,981,778
[9]	$[4]+[5]-[8]$	Implied Reinsurance Expenses	7,224,150
[10]	[9]/[1]	Provision for Fixed Reinsurance Costs	15.7%
[11]	[9]/[3]	Risk Load as % of Gross Loss Cost	69.8%

EXHIBIT 5

A-Florida Insurance Company
Homeowners Rates Effective 1/1/2004
Allocation of Ceded Cat Reinsurance Premiums

Program	[1] Direct Earned Premium	[2] Property Portion	[3] [1] × [2] Subject Earned Premium	[4] Modeled Expected Hurr. Loss	[5] [5T] × [4]/[4T] Allocated Private Cat Premium	[6] Public Cat Premium
Homeowners	46,105,811	88%	40,573,114	10,353,920	9,385,801	3,820,128
Mobile Homeowners	6,978,546	88%	6,141,120	1,063,477	964,039	201,902
Dwelling EC	2,104,790	100%	2,104,790	1,037,755	940,722	246,155
Businessowners	1,302,042	80%	1,041,634	68,180	61,805	8,058
Inland Marine	81,759	100%	81,759	2,122	1,924	0
Total [T]	56,572,949		49,942,418	12,525,454	11,354,291	4,276,243

[1], [5T], [6] from accounting data
[2] convention assumed in reinsurance contracts
[4] from catastrophe simulation model

EXHIBIT 6

A-FLORIDA INSURANCE COMPANY
HOMEOWNERS RATES EFFECTIVE 1/1/2004
BASE RATES FOR MODELED PERILS

[A] Var. U/W Expense Ratio: 21.7%

Allocation of Reinsurance Costs to Policy Form—Hurricane

Policy Form	[1] Base Value Insured	[2] CY 2002 House-Yrs.	[3] 2002 Base Earned TVI	[4] 2002 Alloc. Re. Expense	[5] Indicated Reins. Load
HO2/3/9	100,000	72,765	7,276,499	7,191,638	98.83
HO4/6	10,000	3,290	32,896	32,512	9.88
Total		76,055	7,309,394	7,224,150	

Modeled Base Rates for Hurricane and Other Wind

Form	[6] Hurricane Loss Cost	[7] Reinsurance Fixed Load	[8] Indicated Base Rate	[9] Other Wind Loss Cost	[10] Indicated Base Rate
HO2/3/9	135.24	98.83	298.95	33.64	42.96
HO4/6*	9.02	9.88	24.14	2.24	2.86

[A] from Exhibit 1, includes profit load
[1], [2] from company data
[3] = [1] × [2]
[4] total = [9] from Exhibit 4, then allocated on [3]
[5] = [4]/[2]
[6], [9] from cat model for HO 2,3,9; scaled by ratio of base coverage amounts for HO 4,6
[7] = [5]
[8] = ([6] + [7])/(1 − [A])
[10] = [9]/(1 − [A])
*Ratio of base coverage amounts reflects Cov. A + B + C + D

EXHIBIT 7

A-FLORIDA INSURANCE COMPANY
HOMEOWNERS RATES EFFECTIVE 1/1/2004
BASE RATES FOR NON-MODELED PERILS

[A] Var. U/W Expense Ratio: 21.7%

	Fire						
Form	[1] Average Underlying Covg. A/C	[2] Average Underlying Key Factor	[3] Average Underlying Const/Prot	[4] Earned House-Yrs.	[5] 5 CY Paid Loss + D&CC	[6] Loss Cost	[7] Indicated Base Rate
HO2	56,704	0.819	0.735	3,477	194,354	55.89	118.53
HO3	103,536	1.008	0.548	315,958	20,807,086	65.85	152.22
HO4	29,256	1.963	0.488	12,060	159,155	13.20	17.61
HO6	37,341	2.367	0.342	7,811	86,899	11.12	17.55
HO9	137,183	1.103	0.371	64,341	2,955,761	45.94	143.40

[A] from Exhibit 1, includes profit load
[1]…[5] from company data
[6] = [5]/[4]
[7] = [6]/((1 − [A]) × [2] × [3])

EXHIBIT 8

A-FLORIDA INSURANCE COMPANY
HOMEOWNERS RATES EFFECTIVE 1/1/2004
PROPOSED EXPENSE FEES

[A] Variable Expense Ratio: 21.7%
[B] Fixed Expense Ratio: 6.0%

Form	[1] CY 2002 EP incl. Fees	[2] CY 2002 House-Years	[3] Indicated Expense Fee
HO2,3	38,277,064	60,174	48.74
HO9	6,867,503	12,591	41.79
HO4	506,870	1,895	20.50
HO6	454,375	1,395	24.96

[A], [B] from Exhibit 1, includes profit load
[1], [2] from company data
[3] = [1]/[2] × [B]/(1 − [A])

EXHIBIT 9

A-Florida Insurance Company
Homeowners Rates Effective 1/1/2004
Territory Factors—Hurricane

Ki: 0.103

Proposed Territory	[1] Exposure Weight	[2] Modeled Hurricane Loss Cost	[3] Modeled Standard Deviation	[4] Allocated Risk Load	[5] Total Cost Relative to State Avg.	[6] Tempering Ratio	[7] Tempered Territory Factor	[8] Balanced Tempered Factor
011	418	72.43	441.17	45.52	0.723	1.000	0.723	0.744
012	533	62.97	383.20	39.54	0.628	1.000	0.628	0.646
020	611	47.46	309.94	31.98	0.487	1.000	0.487	0.501
031	184	300.03	2,256.26	232.81	3.266	0.800	2.813	2.893
032	138	162.63	1,182.12	121.98	1.744	1.000	1.744	1.794
033	520	121.66	860.37	88.78	1.290	1.000	1.290	1.326
671	551	95.16	610.25	62.97	0.969	1.000	0.969	0.997
672	61	60.03	326.68	33.71	0.575	1.000	0.575	0.591
[T] All	3,016	96.10	750.00	67.05	1.000	1.000	0.972	1.000

[A] Risk Load as % of Modeled Loss Cost:	69.8%
[B] Indicated Dollar Risk Load:	67.05
Difference [4T]–[B]:	0

[1], [2], [3] from cat model
[2T] = avg [2] wtd on [1]
[4] = [Ki] × [3]
[4T] = avg [4] wtd on [1]
[5] = ([2] + [4])/([2T] + [4T])
[6] selected

[7] = [5] × [6] + (1 − [6])
[7T] = avg of [7] wtd on [1]
[8] = [7]/[7T]
[A] from Exhibit 4
[B] = [A] × [2T]
[Ki] chosen to make [4T] equal to [B]

EXHIBIT 10

A-Florida Insurance Company
Homeowners Rates Effective 1/1/2004
Territory Factors—All Other Perils

[Z] Full Credibility ETVI: 5,000,000

Column:	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
Source:	data	data	data	[3]/[2]	[4]/[4T]	([2]/[Z])^.5	[5] × [6] + (1 − [6])	selected	[8]/[8T]
County	Earned House-Years	Earned TVI ($000)	Paid Loss + D&CC	Paid Loss Cost per $TVI	Relative Loss Cost to State Avg	Credibility	Z-Wtd Relative Loss Cost	Selected Territory Factor	Balanced Territory Factor
Alachua	20,317	3,538,662	2,067,554	0.584	1.106	84.1%	1.089	1.08	1.05
Baker	4,903	694,043	208,870	0.301	0.570	37.3%	0.840	0.84	0.82
Bay	7,469	1,093,334	571,762	0.523	0.990	46.8%	0.995	1.00	0.97
Washington	3,400	427,166	190,492	0.446	0.844	29.2%	0.954	0.95	0.93
Total [T]	36,089	5,753,205	3,038,677	0.528	1.000	100.0%	1.000	1.03	1.00

[8T] = avg of [8] wtd on [2]

EXHIBIT 11

A-Florida Insurance Company
Homeowners Rates Effective 1/1/2004
Class Factors—Hurricane Mitigation

				Terrain B				Terrain C			
				Roof Shape				Roof Shape			
				Other		Hip		Other		Hip	
Roof Cover	Roof Deck Attachment	Roof-Wall Connection	Opening Protection	No SWR	SWR	No SWR	SWR	No SWR	SWR	No SWR	SWR
Non-FBC Equivalent	A (6d@6″/12″)	Toe Nails	None	1.00	0.97	0.77	0.75	1.00	0.97	0.86	0.84
			Basic—Windows or All	0.82	0.79	0.69	0.68	0.85	0.81	0.72	0.69
			Hurricane—Windows or All	0.78	0.74	0.67	0.65	0.81	0.76	0.68	0.64
		Clips	None	0.83	0.79	0.69	0.67	0.91	0.87	0.78	0.75
			Basic—Windows or All	0.77	0.73	0.66	0.64	0.81	0.76	0.68	0.64
			Hurricane—Windows or All	0.75	0.71	0.65	0.63	0.78	0.73	0.66	0.62
		Single Wraps	None	0.82	0.78	0.69	0.67	0.90	0.86	0.78	0.74
			Basic—Windows or All	0.76	0.73	0.66	0.64	0.80	0.75	0.68	0.64
			Hurricane—Windows or All	0.75	0.71	0.65	0.63	0.78	0.73	0.66	0.62
		Double Wraps	None	0.82	0.78	0.69	0.67	0.90	0.86	0.78	0.74
			Basic—Windows or All	0.76	0.73	0.66	0.64	0.80	0.75	0.68	0.64
			Hurricane—Windows or All	0.75	0.71	0.65	0.63	0.78	0.73	0.66	0.62

EXHIBIT 11

Continued

				Terrain B				Terrain C			
				Roof Shape				Roof Shape			
				Other		Hip		Other		Hip	
Roof Cover	Roof Deck Attachment	Roof-Wall Connection	Opening Protection	No SWR	SWR	No SWR	SWR	No SWR	SWR	No SWR	SWR
Non-FBC Equivalent	B (8d@6″/12″)	Toe Nails	None	0.96	0.93	0.76	0.74	0.96	0.93	0.85	0.83
			Basic—Windows or All	0.77	0.75	0.69	0.67	0.78	0.75	0.70	0.68
			Hurricane—Windows or All	0.72	0.69	0.66	0.64	0.73	0.69	0.66	0.63
		Clips	None	0.71	0.68	0.66	0.64	0.81	0.78	0.72	0.68
			Basic—Windows or All	0.68	0.65	0.64	0.62	0.69	0.65	0.63	0.60
			Hurricane—Windows or All	0.67	0.64	0.63	0.62	0.65	0.61	0.62	0.59
		Single Wraps	None	0.70	0.66	0.66	0.64	0.76	0.71	0.70	0.65
			Basic—Windows or All	0.67	0.64	0.64	0.62	0.67	0.62	0.63	0.59
			Hurricane—Windows or All	0.66	0.63	0.63	0.62	0.65	0.60	0.62	0.59
		Double Wraps	None	0.70	0.66	0.66	0.64	0.75	0.68	0.70	0.64
			Basic—Windows or All	0.67	0.63	0.64	0.62	0.66	0.61	0.63	0.59
			Hurricane—Windows or All	0.66	0.63	0.63	0.62	0.65	0.60	0.62	0.58

Non-FBC Equivalent	C (8d@6″/6″) and D (8d@6″/6″) Dimensional Lumber Deck	Toe Nails	None	0.95	0.93	0.76	0.74	0.95	0.93	0.85	0.83
			Basic—Windows or All	0.77	0.74	0.69	0.67	0.78	0.75	0.70	0.68
			Hurricane—Windows or All	0.72	0.69	0.66	0.64	0.72	0.69	0.66	0.63
		Clips	None	0.71	0.67	0.66	0.64	0.81	0.78	0.72	0.68
			Basic—Windows or All	0.67	0.65	0.64	0.62	0.68	0.64	0.63	0.60
			Hurricane—Windows or All	0.66	0.64	0.63	0.62	0.64	0.61	0.62	0.59
		Single Wraps	None	0.69	0.65	0.66	0.63	0.75	0.70	0.70	0.64
			Basic—Windows or All	0.66	0.63	0.64	0.62	0.65	0.61	0.63	0.59
			Hurricane—Windows or All	0.66	0.63	0.63	0.62	0.63	0.59	0.62	0.58
		Double Wraps	None	0.69	0.65	0.66	0.63	0.73	0.65	0.69	0.63
			Basic—Windows or All	0.66	0.63	0.64	0.62	0.64	0.59	0.62	0.58
			Hurricane—Windows or All	0.66	0.63	0.63	0.61	0.63	0.59	0.62	0.58
FBC Equivalent	A (6d@6″/12″)	Toe Nails	None	0.95	0.93	0.73	0.72	0.97	0.95	0.83	0.82
			Basic—Windows or All	0.77	0.76	0.65	0.65	0.80	0.79	0.68	0.68
			Hurricane—Windows or All	0.72	0.71	0.62	0.62	0.75	0.74	0.63	0.63
		Clips	None	0.76	0.75	0.64	0.64	0.86	0.85	0.73	0.73
			Basic—Windows or All	0.70	0.69	0.61	0.61	0.75	0.74	0.63	0.62
			Hurricane—Windows or All	0.69	0.68	0.60	0.60	0.72	0.71	0.61	0.60
		Single Wraps	None	0.76	0.75	0.64	0.64	0.85	0.84	0.73	0.73
			Basic—Windows or All	0.70	0.69	0.61	0.61	0.75	0.73	0.63	0.62
			Hurricane—Windows or All	0.68	0.68	0.60	0.60	0.72	0.71	0.61	0.60
		Double Wraps	None	0.76	0.75	0.64	0.64	0.85	0.84	0.73	0.73
			Basic—Windows or All	0.70	0.69	0.61	0.61	0.74	0.73	0.63	0.62
			Hurricane—Windows or All	0.68	0.68	0.60	0.60	0.72	0.71	0.61	0.60

EXHIBIT 11

Continued

				Terrain B				Terrain C			
				Roof Shape				Roof Shape			
				Other		Hip		Other		Hip	
Roof Cover	Roof Deck Attachment	Roof-Wall Connection	Opening Protection	No SWR	SWR	No SWR	SWR	No SWR	SWR	No SWR	SWR
FBC Equivalent	B (8d@6″/12″)	Toe Nails	None	0.91	0.90	0.72	0.71	0.93	0.91	0.83	0.82
			Basic—Windows or All	0.72	0.72	0.65	0.64	0.74	0.73	0.67	0.67
			Hurricane—Windows or All	0.67	0.66	0.62	0.62	0.69	0.68	0.62	0.61
		Clips	None	0.65	0.65	0.61	0.61	0.77	0.76	0.67	0.66
			Basic—Windows or All	0.62	0.62	0.59	0.59	0.64	0.63	0.60	0.59
			Hurricane—Windows or All	0.61	0.61	0.59	0.59	0.61	0.59	0.58	0.57
		Single Wraps	None	0.64	0.63	0.61	0.61	0.71	0.70	0.64	0.63
			Basic—Windows or All	0.61	0.60	0.59	0.59	0.62	0.60	0.59	0.58
			Hurricane—Windows or All	0.61	0.60	0.59	0.59	0.60	0.58	0.58	0.57
		Double Wraps	None	0.64	0.63	0.61	0.61	0.69	0.67	0.64	0.62
			Basic—Windows or All	0.61	0.60	0.59	0.59	0.61	0.59	0.58	0.57
			Hurricane—Windows or All	0.61	0.60	0.59	0.59	0.60	0.58	0.58	0.57

FBC Equivalent	C (8d@6″/6″) and D (8d@6″/6″) Dimensional Lumber Deck	Toe Nails	None	0.91	0.90	0.72	0.71	0.93	0.91	0.83	0.82
			Basic—Windows or All	0.72	0.72	0.65	0.64	0.74	0.73	0.67	0.67
			Hurricane—Windows or All	0.67	0.66	0.62	0.62	0.68	0.68	0.62	0.61
		Clips	None	0.65	0.64	0.61	0.61	0.77	0.76	0.67	0.66
			Basic—Windows or All	0.62	0.62	0.59	0.59	0.64	0.63	0.59	0.58
			Hurricane—Windows or All	0.61	0.61	0.59	0.59	0.60	0.59	0.58	0.57
		Single Wraps	None	0.63	0.62	0.61	0.60	0.70	0.68	0.64	0.62
			Basic—Windows or All	0.61	0.60	0.59	0.59	0.61	0.60	0.58	0.57
			Hurricane—Windows or All	0.60	0.60	0.59	0.59	0.59	0.58	0.58	0.57
		Double Wraps	None	0.63	0.62	0.61	0.60	0.66	0.63	0.63	0.61
			Basic—Windows or All	0.61	0.60	0.59	0.59	0.59	0.58	0.58	0.57
			Hurricane—Windows or All	0.60	0.60	0.59	0.59	0.59	0.57	0.58	0.57
Reinforced Concrete Roof Deck			None				0.59				0.60
			Basic—Windows or All				0.58				0.56
			Hurricane—Windows or All				0.58				0.56

EXHIBIT 12

A-Florida Insurance Company
Homeowners Rates Effective 1/1/2004
Key Factors—Fire, Form HO-3

Break: 5,000

	Covg A Range		[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]
Group	Low	High	Current Avg Factor	Adj. Earned House-Yrs	Paid Loss Fire	Exposure	Cumulative Paid Loss	Loss Cost	Selected Loss Cost	Implied Inc. Loss Cost	Interp. Key Factor	Marginal Key Factor
1	5,000	9,999	n/a	0	0	0	0	n/a				
2	10,000	14,999	0.363	1	0	1	0	0			0.630	
3	15,000	19,999	0.388	3	0	4	0	0			0.650	0.020
4	20,000	24,999	0.415	11	0	16	0	0			0.670	0.020
5	25,000	29,999	0.445	48	1,250	63	1,250	19.71	80.00	80.000	0.690	0.020
6	30,000	34,999	0.475	217	8,521	280	9,771	34.85			0.706	0.016
7	35,000	39,999	0.505	1,140	113,072	1,420	122,843	86.49			0.721	0.015
8	40,000	44,999	0.545	3,734	254,721	5,154	377,564	73.25			0.737	0.016
9	45,000	49,999	0.595	4,625	337,706	9,779	715,270	73.14			0.752	0.015
10	50,000	54,999	0.643	6,681	754,930	16,460	1,470,200	89.32	89.00	89.035	0.768	0.016
11	55,000	59,999	0.688	6,264	750,172	22,724	2,220,372	97.71			0.806	0.038
12	60,000	64,999	0.733	8,377	1,256,725	31,101	3,477,097	111.80			0.844	0.038
13	65,000	69,999	0.778	8,775	712,445	39,877	4,189,542	105.06			0.883	0.039
14	70,000	74,999	0.815	9,397	1,691,670	49,274	5,881,212	119.36			0.921	0.038
15	75,000	79,999	0.845	9,607	995,686	58,880	6,876,898	116.79	105.00	111.208	0.959	0.038
16	80,000	84,999	0.875	10,477	762,228	69,358	7,639,126	110.14			0.967	0.008
17	85,000	89,999	0.905	10,063	1,601,942	79,421	9,241,068	116.36			0.975	0.008
18	90,000	94,999	0.940	10,439	1,311,610	89,860	10,552,678	117.44			0.984	0.009
19	95,000	99,999	0.980	8,997	676,689	98,856	11,229,367	113.59			0.992	0.008
20	100,000	104,999	1.024	9,587	866,301	108,444	12,095,668	111.54	110.00	115.940	1.000	0.008
21	105,000	109,999	1.072	7,529	388,984	115,973	12,484,652	107.65			1.012	0.012

22	110,000	114,999	1.119	7,107	698,291	123,080	13,182,943	107.11			1.024	0.012
23	115,000	119,999	1.167	6,185	1,213,427	129,265	14,396,370	111.37			1.037	0.013
24	120,000	124,999	1.214	6,031	734,987	135,296	15,131,357	111.84			1.049	0.012
25	125,000	129,999	1.262	5,680	754,843	140,976	15,886,200	112.69	113.00	123.000	1.061	0.012
26	130,000	134,999	1.309	5,081	438,875	146,057	16,325,075	111.77			1.078	0.017
27	135,000	139,999	1.357	4,236	410,158	150,293	16,735,234	111.35			1.095	0.017
28	140,000	144,999	1.404	3,895	185,363	154,188	16,920,597	109.74			1.113	0.018
29	145,000	149,999	1.452	3,158	421,333	157,346	17,341,930	110.22			1.130	0.017
30	150,000	154,999	1.499	3,772	748,756	161,118	18,090,686	112.28	115.50	132.998	1.147	0.017
31	155,000	159,999	1.547	2,693	849,466	163,811	18,940,152	115.62			1.173	0.026
32	160,000	164,999	1.589	2,463	676,881	166,274	19,617,033	117.98			1.199	0.026
33	165,000	169,999	1.627	1,996	431,865	168,270	20,048,897	119.15			1.226	0.027
34	170,000	174,999	1.669	1,747	393,824	170,017	20,442,721	120.24			1.252	0.026
35	175,000	179,999	1.713	1,594	87,091	171,610	20,529,812	119.63	117.50	148.212	1.278	0.026
36	180,000	184,999	1.757	1,456	328,610	173,066	20,858,422	120.52			1.305	0.027
37	185,000	189,999	1.801	1,168	676,463	174,234	21,534,885	123.60			1.332	0.027
38	190,000	194,999	1.846	1,057	58,931	175,291	21,593,816	123.19			1.360	0.028
39	195,000	199,999	1.890	853	27,860	176,144	21,621,675	122.75			1.387	0.027
40	200,000	204,999	1.935	1,199	53,960	177,342	21,675,636	122.22	119.00	163.910	1.414	0.027
41	205,000	209,999	1.979	837	359,670	178,179	22,035,306	123.67			1.433	0.019
42	210,000	214,999	2.023	743	9,286	178,923	22,044,592	123.21			1.451	0.018
43	215,000	219,999	2.067	587	195,540	179,510	22,240,133	123.89			1.470	0.019
44	220,000	224,999	2.112	504	0	180,014	22,240,133	123.55			1.488	0.018
45	225,000	229,999	2.156	567	15,870	180,581	22,256,003	123.25	120.00	174.760	1.507	0.019
46	230,000	234,999	2.201	450	39,775	181,031	22,295,778	123.16			1.538	0.031
47	235,000	239,999	2.245	380	2,250	181,411	22,298,028	122.91			1.569	0.031
48	240,000	244,999	2.290	337	21,253	181,747	22,319,281	122.80			1.599	0.030
49	245,000	249,999	2.334	287	0	182,034	22,319,281	122.61			1.630	0.031
50	250,000		2.378	433	40,848	182,467	22,360,130	122.54	120.75	192.542	1.661	0.031
								Each Additional $5,000:			0.031	

[1], [3] from company data
[2] from data, adjusted to base class (PPC 9, Frame)
[4] = accumulation of [2] up to current TVI group
[5] = accumulation of [3] up to current TVI group
[6] = [5]/[4]
[7] selected from [6] with smoothing
[8] = (([4] × [7]current group) − ([4] × [7]previous group))/(sum[2]current group)
[9] = [8]/([8] at base limit) or linear interpolation
[10] = [9] − ([9] at previous limit)

EXHIBIT 13

A-Florida Insurance Company
Homeowners Rates Effective 1/1/2004
Deductible Factors—Non-Modeled Perils

All Other Perils—All Forms

	[1] Loss Elimination Ratios				[2] Indicated Factors ($500 Base)				[3] Selected Deductible Factors			
Cov. A/C Amount	$250	$500	$1,000	$2,500	$250	$500	$1,000	$2,500	$250	$500	$1,000	$2,500
Less than $75,000	12.6%	25.0%	40.4%	64.7%	1.165	1.000	0.795	0.470	1.17	1.00	0.80	0.47
$75,000 to $149,999	10.3%	20.4%	33.9%	55.8%	1.128	1.000	0.831	0.556	1.13	1.00	0.83	0.56
$150,000 to $224,999	9.0%	17.8%	30.2%	52.1%	1.108	1.000	0.849	0.583	1.11	1.00	0.85	0.58
$225,000 and Over	7.4%	14.7%	25.3%	45.1%	1.086	1.000	0.875	0.643	1.09	1.00	0.88	0.64

Fire—All Forms

	[1] Loss Elimination Ratios				[2] Indicated Factors ($500 Base)				[3] Selected Deductible Factors			
Cov. A/C Amount	$250	$500	$1,000	$2,500	$250	$500	$1,000	$2,500	$250	$500	$1,000	$2,500
Less than $75,000	1.4%	2.7%	5.0%	10.9%	1.014	1.000	0.976	0.916	1.01	1.00	0.98	0.92
$75,000 to $149,999	1.3%	2.7%	5.0%	10.8%	1.014	1.000	0.976	0.916	1.01	1.00	0.98	0.92
$150,000 to $224,999	0.7%	1.4%	2.6%	5.6%	1.007	1.000	0.987	0.957	1.01	1.00	0.99	0.96
$225,000 and Over	0.8%	1.6%	2.9%	6.3%	1.008	1.000	0.987	0.952	1.01	1.00	0.99	0.96

[1] from company data, losses grossed up by actual policy deductible then truncated at each deductible amount.
[2] = (1 – [1])/(1 – [1] for $500)
[3] selected with smoothing

EXHIBIT 14

A-Florida Insurance Company
Homeowners Rates Effective 1/1/2004
Deductible Factors—Hurricane

Hurricane Loss Cost Group	[1] Modeled Losses at $100,000 Cov A 0.5%	1%	2% [B]	5%	Modeled # of Units
Low	7,774,635	7,165,936	6,333,953	5,091,801	54,005
Medium	420,339	394,885	356,583	288,122	856
High	825,763	784,311	719,737	594,485	1,031
Xtreme	47,001	45,049	42,006	35,780	38

Hurricane Loss Cost Group	[2] Relative Modeled Losses 0.5%	1%	2% [B]	5%
Low	1.227	1.131	1.000	0.804
Medium	1.179	1.107	1.000	0.808
High	1.147	1.090	1.000	0.826
Xtreme	1.119	1.072	1.000	0.852

Hurricane Loss Cost Group	[3] Selected Deductible Factors 0.5%	1%	2% [B]	5%
Low	1.23	1.13	1.00	0.80
Medium	1.18	1.11	1.00	0.81
High	1.15	1.09	1.00	0.83
Xtreme	1.12	1.07	1.00	0.85

EXHIBIT 14

Continued

AOP $500 Deductible Data

Cov. A/C Range	[4] Relative Loss Cost	[5] Selected Adjustment	[6] Adjusted $500 Deductible Factors by Group			
			Low	Medium	High	Xtreme
Less than $75,000	0.943	0.95	1.17	1.12	1.09	1.09
$75,000 to $149,999	1.000	1.00	1.23	1.18	1.15	1.12
$150,000 to $224,999	1.033	1.03	1.27	1.22	1.18	1.15
$225,000 and Over	1.072	1.07	1.32	1.26	1.23	1.20

[1] from AIR CLASIC/2™ v. 5.1 and experimental data
[2] = [1]/[1B]
[3] selected on [2]
[4] = (1 − [1] from Exh. 13@TVI)/(1 − [1] from Exh. 13@75–149K)
[5] selected on [4]
[6] = ([3] at 0.5%)∗[5]

EXHIBIT 15

A-Florida Insurance Company
Homeowners Rates Effective 1/1/2004
Adjusted Base Premium Charges and Credits

Form	Earned House-Yrs.	Base Rate Distribution by Peril AOP	Fire	Liability	Medical	Hurricane	Wind	Total
HO2	3,477	136	119	28	2	299	43	627
HO3	315,958	151	152	31	2	299	43	678
HO4	12,060	14	18	3	2	24	3	64
HO6	7,811	11	18	2	2	24	3	60
HO9	64,341	120	143	29	2	299	43	636
HO Avg.	403,648	139	144	29	2	285	41	641

Protective Devices (AOP + Fire base)
Premium Base: 44.2%

Code	Current	Implied	Selected
1	−5.0%	−11.3%	−11.0%
2	−5.0%	−11.3%	−11.0%
3	−5.0%	−11.3%	−11.0%
4	−5.0%	−11.3%	−11.0%
5	−5.0%	−11.3%	−11.0%
6	−2.0%	−4.5%	−4.0%
7	−2.0%	−4.5%	−4.0%
8	−10.0%	−22.6%	−22.0%
9	−10.0%	−22.6%	−22.0%
10	−4.0%	−9.1%	−9.0%
11	−7.0%	−15.8%	−15.0%
12	−7.0%	−15.8%	−15.0%
13	−7.0%	−15.8%	−15.0%

EXHIBIT 16

A-Florida Insurance Company
Homeowners Rates Effective 1/1/2004
Competitive Analysis

Preferred HO-3, $75,000 Frame Risk

County	A-Florida Current Rate	A-Florida Proposed Rate	Top 20 Competitor Current Avg.	Residual Market Rate	A-Florida Change	Difference from Competition
Alachua	429	492	466	737	14.7%	5.7%
Baker	459	435	517	750	–5.2%	–15.9%
Bay	530	791	724	1,097	49.2%	9.3%
Washington	483	515	552	753	6.6%	–6.7%

Preferred HO-3, $150,000 Masonry Risk

County	A-Florida Current Rate	A-Florida Proposed Rate	Top 20 Competitor Current Avg.	Residual Market Rate	A-Florida Change	Difference from Competition
Alachua	527	716	673	1,031	35.9%	6.5%
Baker	617	617	750	1,048	0.0%	–17.7%
Bay	718	1,182	1,054	1,535	64.7%	12.2%
Washington	653	741	798	1,053	13.5%	–7.2%

EXHIBIT 17

A-Florida Insurance Company
Homeowners Rates Effective 1/1/2004
Rating Logic for Calculation of Adjusted Base Premium

Calculation of Total Base Premium

Op.	Value	Premium	Description
	163	163	Fire Base Rate (by Form)
×	1.00	0	Fire Territory Factor
×	1.00	0	Fire Construction/Protection Class Factor (by Form)
=		163	Fire Key Premium
×	1.006	1	Fire Key (amount of insurance) Factor (by Form)
×	1.00	0	Fire Deductible Factor (by AOI)
=		164	Fire Base Premium
	282	282	Hurricane Base Rate (by Form)
×	0.57	(121)	Hurricane Territory Factor
×	0.73	(76)	Hurricane Mitigation Factor
=		85	Hurricane Key Premium
×	1.025	2	Hurricane Key Factor (by Form)
×	1.00	0	Hurricane Deductible Factor (by Zone, & AOI if flat $500)
=		87	Hurricane Base Premium
	46	46	Other Wind Base Rate (by Form)
×	1.09	4	Other Wind Territory Factor
=		50	Other Wind Key Premium
×	1.025	1	Other Wind Key Factor (by Form)
×	1.00	0	Other Wind Deductible Factor (by AOI)
=		51	Other Wind Base Premium
	31	31	Liability Base Rate (by Form)
×	1.00	0	Liability Increased Limits Factor
×	0.92	(2)	Liability Territory (group) Factor
+	2	2	Medical Payments Base Rate
+	0	0	Medical Limit Charge/Credit
=		31	Liability/Medical Base Premium
	151	151	All Other Perils Base Rate (by Form)
×	1.01	2	AOP Territory Factor
=		153	AOP Key Premium
×	1.022	3	AOP Key Factor (by Form)
×	1.00	0	AOP Deductible Factor (by AOI)
=		156	AOP Base Premium
		488	Total Base Premium

EXHIBIT 17

Continued

Calculation of Adjusted Base Premium

Op.	Value	$ Impact	Description
–	(0.05)	(24)	Claim Free Credit (to Total)
–	(0.11)	(35)	Protective Device Credit (to AOP + Fire)
+	0	0	Seasonal Occupancy modifier (to AOP + Fire + Liab)
–	0	0	Wind Exclusion Credit (to Hurr + Other Wind)
+	0	0	Screen Enclosure Charge (flat charge)
–	(0.04)	(13)	Age of Home Credit (to AOP + Fire)—HO
+	0	0	Multi-Unit or Town/Rowhouse mod (to AOP + Fire)—HO
+	0.16	73	Replacement Cost Provisions mod (to non-Liab)—HO
–	0	0	Law/Ordinance Exclusion Credit (to non-Liab)—HO
–	0	0	In-Construction Credit (to Total)—HO
–	(0.06)	(8)	BCEGS Credit (to Hurr + Other Wind)—HO
+/–	0	0	Loss Settlement Options mod (to non-Liab)—MH
–	0	0	ANSI/ASCE Credit (to non-Liab)—MH
=		481	Adjusted Base Premium
+		55	Expense Fee
=		536	Total Policy Premium

ESTIMATING THE WORKERS COMPENSATION TAIL

RICHARD E. SHERMAN AND GORDON F. DISS

Abstract

The workers compensation tail largely consists of the medical component of permanent disability claims (MPD). Yet the nature of MPD payments is not widely understood and is counter to that presumed in common actuarial methods.

This paper presents an analysis of medical payments based on 160,000 permanently disabled claimants over 77 accident years. It introduces a method for utilizing incremental payment data prior to the standard triangle to extend development factors beyond the end of the triangle (for any casualty line).

A model is presented that explicitly reflects the opposing effects of medical cost escalation and the force of mortality. It demonstrates that

- *paid loss development factors (PLDFs) tend to increase over many successive, "mature" years of development,*
- *PLDFs and tails will trend upward over time due to expected future improvement in mortality—that is, people will be living longer, and*
- *average medical costs for elderly claimants are substantially higher than for younger claimants.*

The paper also demonstrates that case reserves based on inflating payments until the expected year of death are significantly less than the expected value of such reserves. A method is introduced for realistically simulating the high expected value and variability of MPD reserves. It is based on a Markov chain model of annual payments on individual claims.

1. SUMMARY AND INTRODUCTION

Historically, the ability of workers compensation insurers to reasonably estimate tail factors has been hampered by a dearth of available development experience at maturities beyond 10 to 20 years. Substantive advances in workers compensation tail estimation depend on the availability of a substantial database extending to 50 or more years of development.

This paper presents the results of a thorough analysis of the extensive paid loss development database of the SAIF Corporation, Oregon's state fund. This database extends out to 77 years of development separately for medical and indemnity, and separately *by injury type* (i.e., permanent total, permanent partial, fatal, temporary total, temporary partial, and medical only).

This paper predominantly focuses on the behavior of *medical payments for permanently disabled claimants* (MPD) on an unlimited basis. Some of the key findings from this analysis of MPD payments include the following:

1. MPD tail factors calculated empirically are significantly greater than those derived from extrapolation techniques. This occurs because MPD paid loss development factors (PLDFs) do not decrease monotonically for many later development years (DYs).
2. There is an effective, systematic way (the Mueller Incremental Tail method) to utilize incremental payment data prior to the standard triangle to extend PLDFs beyond the end of the triangle for any casualty line.
3. Medical cost escalation rates have generally been much higher than annual changes in the medical component of the Consumer Price Index (CPI). Medical cost escalation rates include increases in utilization rates of different services and the effects of shifts in the mix of services toward more expensive care alternatives.

4. Medical cost escalation rates and the force of mortality are the key drivers of MPD tail factors. Unfortunately, the paid loss development method is not designed to treat these two influences separately. A method (incremental paid to prior open claim) is presented that provides for the separate, explicit treatment of the effects of these two drivers.

5. In the early stages of the MPD tail, medical cost escalation overpowers the force of mortality, leading to increases in incremental paid losses and PLDFs.

6. Assuming recent mortality rates, the incremental paid to prior open claim method fits the empirical data very well out to DY 40, but then tends to understate losses for the next 15 DYs. This understatement is due to the added costs of caring for the elderly, who make up a rapidly increasing percentage of surviving claimants.

7. The common actuarial assumption that the incremental medical severities for each claimant (at current cost level) during each future DY will remain constant is not valid. Such current level severities tend to increase noticeably as each surviving claimant becomes elderly.

8. Declining mortality rates have a substantial effect on medical tail factors. Mortality improvement will also cause individual PLDFs to trend upward slowly for any given DY.

9. The common method of estimating the tail by applying the ratio of incurred to paid for the most mature accident years will underestimate reserves, unless case reserves adequately reflect the implications of points 3, 7, and 8. This is rarely the case.

10. The most significant factor affecting the indications in this paper is the applicable retention. Tail factors and PLDFs at more mature years of development should

be expected to be significantly less at relatively low retentions.

11. The expected value of an MPD case reserve is much greater than cumulative inflated payments through the expected year of death. This is similar to the situation that occurs when reinsurance contracts are commuted, where using the life expectancy of the claimant produces an estimate well below the weighted average of outcomes based on a mortality table [2].

12. The variability of total MPD reserves can be gauged realistically by a Markov chain simulation model that separately estimates payments for each future DY by claimant.

13. The potential for common actuarial methods to understate the MPD reserve, and consequently the entire workers compensation reserve, is significant. This is also true regarding common methods for estimating the degree of variability in the workers compensation reserve.

14. The MPD loss reserve is a high percentage of the total workers compensation loss reserve for maturities of 10 years or more. And that percentage increases noticeably at higher maturities.

It is important to note that the applicability of the above findings depends not only on the retention level, but also the presence (or absence) of permanent disability (PD) claimants with ongoing medical costs and on the specific provisions of state workers compensation laws.

Statutory indemnity benefits differ by state. For example, some states allow for escalation of PD benefits while others do not. Medical benefit structures are much more uniform across states. This paper focuses on MPD payments, which generally do not vary significantly between states.

Organization of Paper

This paper is divided into 10 sections:

1. Summary and Introduction
2. Using Prior Incremental Paid Data to Extend the PLDF Triangle
3. Incorporating the Static Mortality Model into the Incremental Paid to Prior Open Claim Method
4. Mortality Improvement
5. The Trended Mortality Model
6. A Comparison of Indicated Tail Factors
7. Sensitivity Considerations
8. Estimating the Expected Value of MPD Reserves
9. Estimating the Variability of the MPD Reserve with a Markov Chain Simulation
10. Concluding Remarks

The paper also includes five appendices:

A. The Mueller Incremental Tail Method

B. Historical PLDFs for All Other Workers Compensation

C. Incorporating the Static Mortality Model into the Incremental Paid to Prior Open Claim Method

D. Incorporating the Trended Mortality Model into the Incremental Paid to Prior Open Claim Method

E. Quantifying the Elder Care Cost Bulge

Introduction

The workers compensation tail behaves quite differently from that of any other casualty line. For other lines, it is virtually axiomatic that PLDFs will decrease monotonically to 1.0 for later DYs. In sharp contrast, PLDFs for MPD payments quite often increase for later DYs.

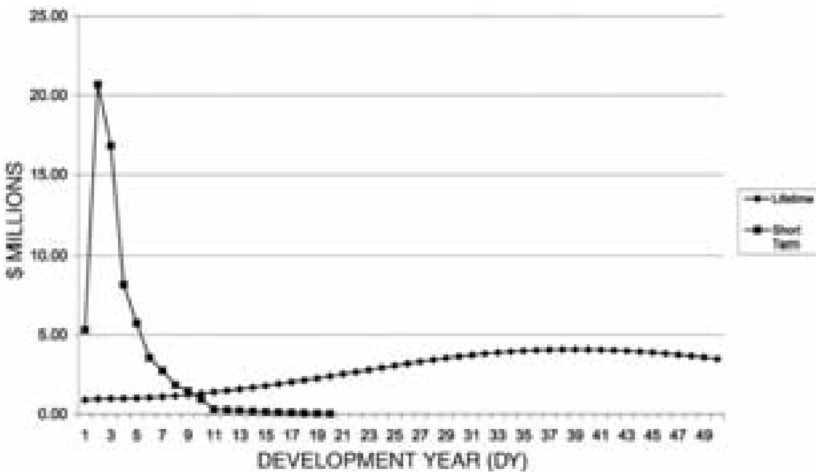
The payout pattern for MPD losses is a composite of two radically different types of payments: short-term and lifetime. What separates these two types is how long work-related medical payments continue. Short-term payments cease well before the claimant dies, either because the need for periodic medical treatments ceases or because the claimant returns to work. Lifetime payments, on the other hand, persist until the claimant dies. Figure 1.1 contrasts these payout patterns. These two categories are conceptual, to help in understanding the behavior of workers compensation payments over time, rather than practical, since MPD payments cannot be precisely separated into these two categories until all claimants die. As such, precise categorization requires hindsight on an ultimate basis.

From Figure 1.1, we see that short-term payments overshadow lifetime payments during the first 10 or so DYs, and lifetime payments dominate soon after that. PLDFs for successive DYs during DYs 3 through 15 tend to drop, largely because of the cessation of short-term payments for a significant percentage of claimants during each DY. For later DYs, the predominant influence affecting whether PLDFs increase or decrease is the relative magnitude of the force of medical cost escalation versus that of claimant mortality, since death is virtually the sole reason for the closure of claims.

An MPD payment history is the result of the sum of the above two payout patterns. As is evident, this will be a bimodal pattern, peaking during DY 2 and around DY 40. If total medical or total workers compensation paid experience is all that is available, the second peak will be much less evident, to the point where the

FIGURE 1.1

PAYOUT PATTERNS—LIFETIME VERSUS SHORT-TERM MPD PAYMENTS FOR A SINGLE ACCIDENT YEAR



tendency of later PLDFs to refuse to decline could easily be seen as an anomaly, when in reality it is to be expected.

The payout pattern for lifetime payments does not end at DY 50. A severely injured worker in his or her late teens or early 20s could require work-related medical payments for up to 90 years after the accident. As a result, the total area under the lifetime payout pattern (i.e., ultimate payments) can easily be three to four times that under the short-term payout pattern.

Often the reserving actuary will have paid losses only for the first 15 (or fewer) DYs. Consequently, the only paid loss experience available consists primarily of short-term payments, and yet the bulk of the loss reserve will be due to lifetime payments. Since the two types of payments are radically different, the risk of underestimating the loss reserve is significant. Frequently the actuary will rely to some degree on the ratio of incurred loss to paid loss for the most mature accident years (AYs) as a guide in selecting a tail factor. Since this typically indicates a larger tail (when there are open permanent disability claims), the actuary may feel that reliance on this latter method will produce a safely conservative reserve estimate. However, such an estimate is only as unbiased as the MPD case reserves are. As will be shown later, MPD case reserves are particularly susceptible to underestimation.

Table 1.1 illustrates the hazards of attempting to extrapolate medical paid loss development factors beyond DY 15 using a common method (exponential decay), as applied to historical PLDFs for DYs 10–15 (highlighted by a box) in Oregon, Washington and California.

In Table 1.1, as well as throughout this paper, a PLDF for a given DY is denoted by the maturity at the end of that year. For example, the factors in the row headed by "2" are for development from 1 to 2 years of age, since this is the second year of development.

In the lower portion of Table 1.1 these extrapolated factors are directly compared with known historical factors. In each state, the extrapolated factors increasingly fall below the historical ones for later DYs. These persistent shortfalls are compounded when tail factors are calculated, such as those shown in the bottom row of the table.

Table 1.1 provides these comparisons for SAIF, the Washington Department of Labor and Industries (WA LNI) and the California Workers Compensation Insurance Rating Bureau (WCIRB), respectively. The SAIF factors are for MPD only, while for the other two states, the factors are for total medical. So, everything else being equal, SAIF's PLDFs will tend to be greater for later DYs.

The problem of persistent shortfalls in the extrapolated factors can be reduced, but not eliminated, by applying inverse power [5] fits to the PLDFs for DYs 10–15. Such fits also assume that PLDFs will decrease monotonically for increasing DYs. The

TABLE 1.1

A COMPARISON OF PLDFs EXTRAPOLATED FROM HISTORICAL FACTORS FOR DYs 10–15 WITH KNOWN HISTORICAL PLDFs FOR LATER DYs [MPD LOSSES (SAIF) AND MEDICAL LOSSES (WA LNI AND WCIRB)]

	(1)	(2)	(3)	(4)	(5)	(6)
Development Year (DY)	Historical SAIF MPD PLDFs	Fitted/ Extrapolated SAIF MPD PLDFs	Historical WA LNI Medical PLDFs	Fitted/ Extrapolated WA LNI Medical PLDFs	Historical WCIRB Medical PLDFs	Fitted/ Extrapolated WCIRB Medical PLDFs
2	6.624		1.914		1.740	
3	1.525		1.175		1.296	
4	1.140		1.090		1.152	
5	1.072		1.060		1.104	
6	1.041		1.045		1.069	
7	1.027		1.036		1.058	
8	1.019		1.027		1.030	
9	1.020		1.023		1.022	
10	1.015	1.015	1.020	1.019	1.015	1.015
11	1.013	1.014	1.017	1.018	1.012	1.012
12	1.012	1.013	1.016	1.016	1.009	1.009
13	1.013	1.012	1.015	1.015	1.007	1.007
14	1.012	1.011	1.013	1.014	1.006	1.006
15	1.010	1.011	1.013	1.012	1.005	1.005

TABLE 1.1

Continued

16	1.011	*1.010*	1.012	*1.011*	1.006	*1.004*
17	1.013	*1.009*	1.010	*1.010*	1.005	*1.003*
18	1.011	*1.009*	1.010	*1.010*	1.005	*1.002*
19	1.011	*1.008*	1.009	*1.009*	1.005	*1.002*
20	1.012	*1.008*	1.009	*1.008*	1.008	*1.002*
21	1.012	*1.007*	1.008	*1.007*	1.006	*1.001*
22	1.014	*1.007*	1.009	*1.007*	1.007	*1.001*
23	1.012	*1.006*	1.009	*1.006*	1.007	*1.001*
24	1.015	*1.006*	1.009	*1.006*	1.007	*1.001*
25	1.015	*1.006*	1.009	*1.005*	1.009	*1.001*
26	1.016	*1.005*	1.008	*1.005*	1.010	*1.000*
27	1.020	*1.005*	1.009	*1.004*	1.008	*1.000*
28	1.023	*1.005*	1.009	*1.004*	1.009	*1.000*
29	1.027	*1.004*	1.011	*1.004*		
30	1.026	*1.004*	1.009	*1.003*		
31	1.022	*1.004*	1.010	*1.003*		
32	1.018	*1.004*	1.013	*1.003*		
33	1.015	*1.003*	1.013	*1.003*		
34	1.017	*1.003*	1.015	*1.002*		
35	1.018	*1.003*	1.010	*1.002*		
36	1.029	*1.003*				
37	1.033	*1.003*				
Tail @ 15	1.471	1.130	1.221	1.120	1.096	1.018

Notes: (1) The italicized factors in columns (2), (4) and (6) were extrapolated on the basis of an exponential curve fit to the boxed historical factors (less 1.0) for DYs 10–15 for each respective state's experience.
(2) The DYs shown for the WCIRB are off by half a year (e.g., DY 10.5 is shown as DY 10).

reality is that the historical PLDFs in all three Western states *often increase* for later DYs. The shortfalls produced by inverse power fits are smaller because the ratios of the projected factors (less 1.0) rise asymptotically to 1.0, while the decay ratios for the exponential curve fits remain constant at a value well below 1.0.

In addressing the problem of extrapolating paid development when the most mature PLDFs are increasing, some insurers or self-insureds may have data for longer periods of time than the latest 20 years. However, because of system changes or acquisitions, *cumulative* loss development data for old accident years are frequently lacking. In these cases *incremental* calendar year data for old accident years may be available because payments are still being made on the old open claims. Section 2 and Appendix A present the Mueller Incremental Tail method for making full use of the incremental data to calculate empirical tail factors. We have used this method to derive empirically based PLDFs out to 65 years of development based on SAIF's actual MPD loss experience.

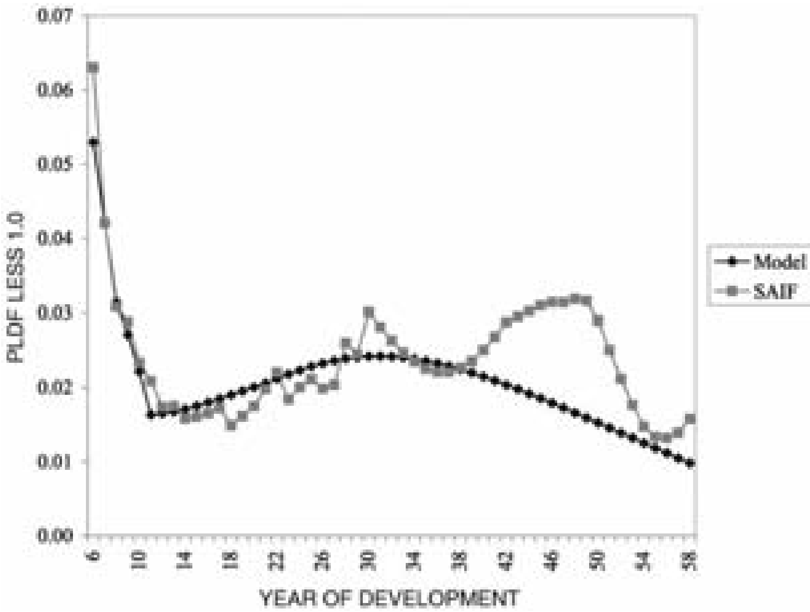
The PLDF model is not designed to reasonably predict the behavior of lifetime payments during later DYs. An alternative approach using the incremental paid to prior open claim method is well suited to this purpose. It separately treats changes in incremental severities (due to annual rates of medical cost escalation) and the slow decline in the number of open claims (due to mortality). A version of it using a recent mortality is presented in Section 3. It will be referred to as the *static mortality model*.

When the rate of medical cost escalation clearly exceeds the percentage of remaining claimants who die during a given DY, then incremental MPD payments will increase from one DY to the next. Such increases should be quite common during DYs 15 through 40.

In Figure 1.2, the PLDFs indicated by the static mortality model are compared with SAIF's empirical PLDFs. The static

FIGURE 1.2

STATIC MORTALITY MODEL AND ACTUAL SAIF PLDFS LESS 1.0



mortality model PLDFs are shown in the last column of Table 3.2. The empirical PLDFs for the first 29 DYs are the averages of the latest 15 historical factors. For DYs 30–58, the PLDFs appear in Tables A.1, A.2 and A.3, where the Mueller Incremental Tail method is applied.

As Figure 1.2 shows, SAIF's actual development experience for DYs 40 through 54 is consistently worse than the model predicts. The bulge in adverse paid development evident for DYs 40 through 54 is attributable to the rapidly increasing percentage of surviving claimants who are elderly. Not uncommonly, elderly PD claimants simply require more extensive and expensive medical care than younger claimants. And as PD claimants

TABLE 1.2

TWO INDICATORS OF AN INCREASING PROPORTION OF THE ELDERLY AMONG SURVIVING CLAIMANTS

DY	Portion 80 Years of Age or Older	Portion Who Will Die Within Five Years
0	0.0%	4.4%
10	0.9%	9.4%
20	10.9%	18.3%
30	36.5%	30.1%
40	51.2%	39.0%
50	64.7%	47.2%
60	100.0%	60.3%

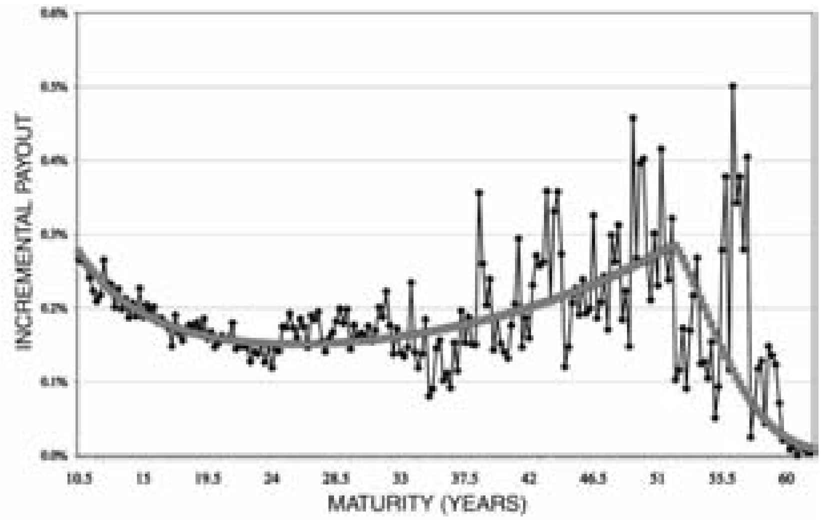
The percentages in Table 1.2 are based on 2000 mortality tables published by the Social Security Administration (SSA), assuming 75% of the claimants are male, and a census of SAIF's permanent total disability claimants by age-at-injury.

age, so do their spouses. Often spouses reach an age where they can no longer provide as much care as previously, and insurers then pay for the increased cost of hiring outside assistants. Table 1.2 indicates the percentage of surviving claimants who will be 80 or older at the beginning of various years of development. It also shows the percentage of surviving claimants expected to die within the succeeding five years. It has also been observed that incremental severities tend to undergo an increase during the last years before a claimant's death that exceeds normal rates of medical cost escalation.

Table 1.2 indicates that for DYs 40 and higher, over half of the surviving claimants will be 80 or more years old. Clearly, this fact could have been anticipated on an a priori basis. After all, if the average claimant were age 40 when injured, it should be expected that 40 years after the injury year the average surviving claimant would be about 80 years old. However, the above table underscores a reality that casualty actuaries may not have heretofore given much consideration. The behavior of loss development for later DYs may well be more adverse than what

FIGURE 1.3

WASHINGTON STATE FUND
MEDICAL TAIL



Solid points = actual data; shaded = fitted data.

would be expected on the basis of earlier DYs, because of the increasing infirmities of surviving claimants and their spouses.

The adverse pattern evident in Figure 1.2 is also quite pronounced in the medical PLDFs for the Washington State Fund, as shown in Figure 1.3. This graph was provided by William Vasek, FCAS.

Table 1.3 provides a direct comparison of the tail factors (to ultimate) at 15 years produced by various extrapolation techniques with that based on SAIF's historical experience.

Clearly, the extrapolated MPD loss reserves at 15 years of maturity are only a small fraction of the MPD reserve indicated by SAIF's development history.

TABLE 1.3

A COMPARISON OF SAIF'S EMPIRICAL TAIL FACTOR WITH EXTRAPOLATED TAIL FACTORS AT 15 YEARS (BASED ON A FIT TO HISTORICAL PLDFS FOR DYS 10–15)

Extrapolation Method	Indicated Tail Factor at 15 Years	Extrapolated Reserve as a Portion of the Reserve Indicated by SAIF's History
Linear Decay	1.046	3.5%
Exponential Decay	1.175	13.4%
Inverse Power Curve	1.234	17.9%
SAIF's Historical Factors	2.309	100.0%

As high as SAIF's paid tail factor at 15 years is (2.309), it is understated because it implicitly assumes that past mortality rates will continue indefinitely into the future. As noted in Section 4, mortality rates have been declining steadily for at least the past four decades, and the Social Security Administration (SSA) reasonably expects such declines to continue throughout the next century.

A second reserving model that explicitly accounts for the compounding effects of *downward trends in future mortality rates* and persistently high rates of future medical cost escalation will be referred to as the *trended mortality model*. It will be described in Section 5.

The indications of the trended mortality model for MPD are significant and troubling:

- Paid tail factors at the end of any selected year of development should be expected to increase slowly but steadily over successive accident years.
- Incremental PLDFs for any selected year of development will also trend upward slowly but inexorably for successive AYs.

- The above effects on MPD will cause corresponding upward trends in paid tails and incremental PLDFs *for all workers compensation losses in the aggregate.*

Unless the effects of downward trends in mortality rates are incorporated into a workers compensation reserve analysis, the resulting reserve estimates will be low when numerous AYs are involved and the retention is very high.

We believe that the most appropriate approach to estimating gross workers compensation loss reserves is to separately evaluate MPD loss reserves by one or more of the methods presented in this paper. Lacking separate MPD loss experience, the static mortality and trended mortality models and the Mueller Incremental Tail method can be applied satisfactorily to total medical loss experience for DYs 20 and higher, since virtually all medical payments are MPD payments at such maturities.

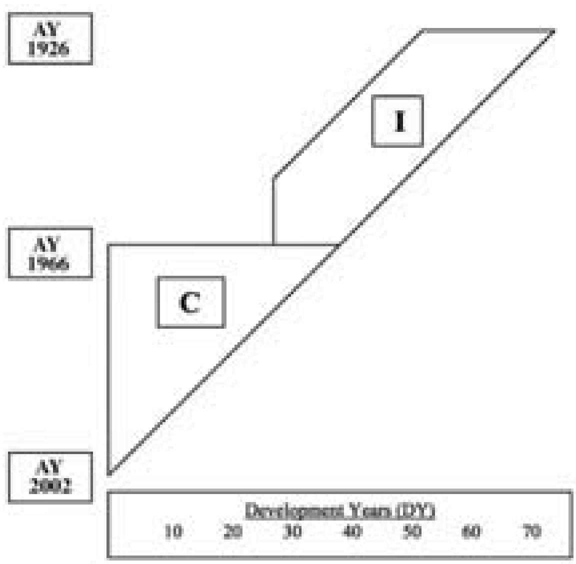
There is an additional reason to utilize the methods presented in this paper instead of the standard PLDF method. In general, legislated benefit changes tend to have a much greater impact on the magnitude and duration of short-term payments than on lifetime payments. When a PLDF method is used, it assumes that the relative magnitude of short-term and lifetime payments for each AY is relatively constant. Benefit changes can significantly change this mix, causing distortions in projections of remaining lifetime payments based on PLDFs. In contrast, projections of future lifetime payments based on the incremental paid to prior open claim method should be comparatively independent of shifts in the relative magnitude of short-term payments.

2. USING PRIOR INCREMENTAL PAID DATA TO EXTEND THE PLDF TRIANGLE

Figure 2.1 provides a graphic summary of the available portions of the incremental MPD payments experience of the SAIF Corporation. A complete triangle of MPD payments exists for

FIGURE 2.1

CONFIGURATION OF SAIF'S MPD PAID LOSS DATA



AYs 1966 through 2002. This region is the triangle labeled "C" to designate that *cumulative* paid losses are available for all of these AYs. In addition, since calendar year 1985, incremental MPD payments have been captured for AYs 1926 through 1965 for DYs 29 and higher. This region is the diagonally shaped area labeled "I" to designate that only incremental payments are available.

2.1. The Mueller Incremental Tail Method

Given the availability of the incremental paid data for DYs well beyond the standard triangle of cumulative paid losses, and the value of such information in more accurately estimating the tail, a method was devised to utilize this data. It was designed by Conrad Mueller, ACAS, and is based on decay ratios of incremental payments. We will use SAIF experience as an example. This section describes the Mueller Incremental Tail (MIT) method and provides the formulas and key results. The actual calculations are included in Appendix A.

The MIT method was used to calculate empirical 37 to ultimate tail factors using the incremental data on old accident years. The empirical data ended at 65 years of development, which for purposes of this section will be considered to be ultimate. We describe the method in three stages:

1. Incremental age-to-age decay ratios.
2. Anchored decay factors.
3. Tail factors.

Notation:

Let S_n = Cumulative payments through n years of development

p_n = Incremental payments made in year n; and

$S_n = \sum p_i$ (i = 1 to n).

Let PLDF_n = Age $n-1$ to n paid loss development factor.

$\text{PLDF}_n = S_n/S_{n-1} = (S_{n-1} + p_n)/S_{n-1} = 1 + p_n/S_{n-1}$.

Let $f_n = p_n/S_{n-1}$, then

$\text{PLDF}_n = 1 + f_n$.

1. *Incremental age-to-age decay ratios.* The first step is to calculate incremental age-to-age decay ratios: p_{n+1}/p_n, p_{n+2}/p_{n+1}, p_{n+3}/p_{n+2}, and so on. With the SAIF data, we are able to calculate ratios of incremental paid loss at age $(n+1)$ to incremental paid at age (n), for n ranging from 29 to 65, using 20-year weighted averages. Because of the sparseness of claims of this age, the empirical decay ratios needed to be smoothed before they could be used. The smoothing was done using five-year centered

TABLE 2.1

INDICATED DECAY FACTORS RELATIVE TO ANCHOR YEAR 37 INCREMENTAL PAYMENTS

Year of Development	Decay Factor
55	.962
50	1.880
45	1.724
40	1.211
Anchor Year 37	1.000

moving averages. These calculations are shown in Appendix A, Tables A.1 through A.4.

2. *Anchored decay factors.* After calculating incremental age-to-age decay ratios, we then anchor them to a base year. We illustrate this using development year n as our anchor year. These anchored decay factors are calculated as the cumulative product from the last column on Table A.4.

We call the anchored age-to-age factor d_n, where $d_n = p_n/p_n = 1$, $d_{n+1} = p_{n+1}/p_n$, $d_{n+2} = p_{n+2}/p_n \ldots$, all relative to p_n.

In general,

$$p_{n+r}/p_n = p_{n+1}/p_n * p_{n+2}/p_{n+1} * \cdots p_{n+r}/p_{n+r-1}.$$

The anchored decay factors are cumulative products of the age-to-age decay ratios and represent payments made in year $n+r$ relative to payments made in the anchor year n.

Table 2.1 shows the anchored decay factors for payments made in accident years of age 40, 45, 50, and 55 relative to payments made in an accident year of age 37 (our anchor year).

TABLE 2.2

Cumulative Decay Factors Relative to Incremental Payments During Different Anchor Years

Anchor Year	Cumulative Decay Factor
37	30.071
36	30.115
35	29.508
34	28.280
33	26.961

For example, payments made in DY 50 are, on average, almost double (88.0% greater) the payments made in DY 37.

By summing the anchored decay factors from 38 to ultimate, we get the payments made in ages 38 to 65 relative to payments made in year 37. We will refer to each of these as anchored cumulative decay factors D_n, where

$$D_{n+1} = p_{n+1}/p_n + p_{n+2}/p_n + \cdots = \sum d_i.$$

The sums of the decay factors are similar to tail factors, but instead of being relative to cumulative payments they are relative to the incremental payments made in the anchor year.

The process can be repeated using a different anchor year. In addition to anchor year 37, the calculations were also performed using anchor years 36, 35, 34, and 33. In each case, the payments from 38 to ultimate were compared to the payments made in the selected anchor year. Table 2.2 shows the cumulative decay factors for each of these anchor years.

The cumulative decay factors can be interpreted as follows: Payments made from ages 38 to ultimate are 30.071 times the payments made in age 37. Similarly, payments made in ages 38 to ultimate are 30.115 times the payments made in age 36, and so on.

3. *Tail factors.* To convert these cumulative decay factors into tail factors, we make use of the selected cumulative loss development factors from the customary cumulative paid loss development triangle.

   The tail factor from n to ultimate

$$\begin{aligned}
&= S_8/S_n \\
&= (S_n + \textstyle\sum p_i)/S_n \\
&= 1 + \textstyle\sum p_i/S_n \\
&= 1 + p_{n+1}/S_n + p_{n+2}/S_n + \cdots \\
&= 1 + p_n/S_n(p_{n+1}/p_n + p_{n+2}/p_n + \cdots).
\end{aligned}$$

   But $p_n/S_n = (p_n/S_{n-1})/(S_n/S_{n-1}) = f_n/(1+f_n)$, so the tail factor is $1 + [f_n/(1+f_n)] \times D_{n+1}$.

   The general formula for the tail factor at age n is

$$\text{Tail factor}_n = f_n D_{n+1}/[1 + f_n],$$

   where f_n is the PLDF, less one, for the nth year of development, and D_{n+1} is the cumulative decay factor for payments made during years $n+1$ to ultimate relative to payments made in anchor year n.

In a similar way, an age-to-age loss development factor (less 1.0) extending beyond the cumulative triangle is

$$f_{n+1} = f_n d_{n+1}/[1 + f_n],$$

where d_{n+1} is the decay factor for payments made in year $n+1$ relative to payments made in anchor year n.

This method is sensitive to f_n, the 37:36 PLDF less 1. For this reason the analysis can be repeated using the 36, 35, 34, or 33 anchor years. Table 2.3 shows the 37 to ultimate tail factor calculated using each of these anchor years.

TABLE 2.3

37 TO ULTIMATE MPD TAIL FACTORS BASED ON DIFFERENT ANCHOR YEARS

Anchor Year	37 to Ultimate MPD Tail Factor
37	1.964
36	1.808
35	1.496
34	1.439
33	1.369
Selected	1.581*

*Average excluding the high and low.

The empirically calculated 37 to ultimate MPD tail factors range from a low of 1.369 to a high of 1.964. The value is sensitive to relatively small changes either in incremental age-to-age factors in the tail or in the cumulative age-to-age factors at the end of the cumulative triangle.

Another approach for reducing the high level of volatility of the tail factors shown in Table 2.3 is presented in Table A.6 of Appendix A. Each of the average PLDFs for ages 30 through 36 is adjusted to what it would be for age 37 using the appropriate products of incremental decay factors from AYs 1965 and prior. A weighted average of all of these adjusted PLDFs (1.022) is then used to replace the actual PLDF for DY 37 (1.033). The final selected tail factor from age 37 to ultimate is then 1.0 plus the product of the cumulative decay factor of 30.071 and .022/1.022 (1.647).

2.2. *SAIF's Indicated Paid Tail Factors*

When the indications from SAIF's incremental paid estimation of the tail from 37 years to ultimate are combined with those of a standard paid loss development approach up to 37 years of maturity, the MPD tails shown in the left column of Table 2.4 at different maturities were derived. Some readers may be interested in the Total Workers Compensation tail factor (medical and

TABLE 2.4

SAIF'S INDICATED PAID TAIL FACTORS

Maturity (Years)	MPD	Other Workers Compensation	Total Workers Compensation
10	2.469	1.263	1.671
15	2.328	1.234	1.613
25	2.054	1.129	1.457
35	1.680	1.052	1.294

indemnity combined). These are shown in Table 2.4 assuming an ultimate mix of MPD and Other Workers Compensation of 50% for each. We selected 50% for ease of presentation because in practice the mix would vary by state and over time.

In addition to MPD tail factors, Table 2.4 also displays indicated paid tail factors for all other types of workers compensation losses as well as for workers compensation in total. Most of the Other Workers Compensation tail factors reflect paid development for indemnity losses of permanently disabled claimants. A small portion is also due to paid development on fatal cases. The above table puts the impact of MPD paid tails in perspective relative to the indicated paid tail for all WC losses (i.e., for all injury types and for medical and indemnity combined).

Appendix B provides a comparison of SAIF's historical PLDFs for MPD, all other workers compensation and total workers compensation by DY. MPD is the primary reason why PLDFs for total workers compensation decline much more slowly than generally expected.

To gain an appreciation for the relative contribution to the total loss reserves for a given AY of MPD versus all other workers compensation at each of the above years of maturities, Table 2.5 provides a comparison of what the reserve would be, assuming that total ultimate losses for that AY were $100 million and assuming that 50% of ultimate losses are MPD.

TABLE 2.5

INDICATED LOSS RESERVE AT DIFFERENT MATURITIES (DOLLARS IN MILLIONS)

Maturity (Years)	MPD Reserve	Other Workers Compensation Reserve	MPD Reserve as a Percentage of Total Workers Compensation Reserve
10	$29.8	$10.4	74
15	28.5	9.5	75
25	25.7	5.7	82
35	20.2	2.5	89

TABLE 2.6

WCIRB'S INDICATED CALIFORNIA PAID TAIL FACTORS

Maturity (Years)	Medical Tail	Indemnity Tail	Total Workers Compensation Loss Tail
10	1.276	1.064	1.168
15	1.217	1.041	1.129
25	1.143	1.025	1.086

Source: WCIRB Bulletin No. 2003-24, pp. 8–9 [7].

The MPD reserve makes up an increasing percentage of the total WC loss reserve at later maturities.

It should be borne in mind that Tables 2.4 and 2.5 provide MPD and other workers compensation indications specific to SAIF's loss experience in the state of Oregon, and not that of workers compensation insurers in general.

Table 2.6 provides a comparison of indicated tails at different maturities for California workers compensation experience, as projected by the Workers Compensation Insurance Rating Bureau (WCIRB).

Although the California tails are consistently smaller than SAIF's, it is again true that the medical tails are decidedly greater

TABLE 2.7

WCIRB INDICATED LOSS RESERVE BY LOSS TYPE AT DIFFERENT MATURITIES (DOLLARS IN MILLIONS)

Maturity (Years)	Medical Loss Reserve	Indemnity Loss Reserve	Medical Reserve as a Percentage of Total Reserve
10	$11.7	$2.7	81%
15	9.6	1.8	84%
25	6.8	1.1	86%

than the indemnity tails. Table 2.7 provides a comparison of the size of the medical and indemnity loss reserves at different maturities, again assuming an AY with $100 million of ultimate losses.

In California, medical loss reserves make up an increasing percentage of the total workers compensation loss reserve at later maturities.

3. INCORPORATING THE STATIC MORTALITY MODEL INTO THE INCREMENTAL PAID TO PRIOR OPEN CLAIM METHOD

This section presents the incremental paid to prior open claim method of reserve estimation. The basics of this method bear much resemblance to the structural methods developed by Fisher and Lange [3] and Adler and Kline [1]. In essence, incremental payments for every development year are estimated by taking the product of the number of open claims at the end of the prior development year and an estimated claim severity.

While this method is of limited value for less mature DYs, its merit relative to other reserving methods is substantial in estimating reserves for future MPD payments for more mature DYs. For such mature DYs, future incremental payments are essentially a function of how many claims are still open and the average size of incremental payments per open claim. In contrast, future incremental MPD payments have almost no causal link to payments for rapidly settled claims during early DYs.

Table 3.1 provides a specific example of how this method is applied. The specific steps to be taken in applying the incremental paid to prior open claim method are as follows:

1. Incremental paid losses (A) and open counts (B) are compiled by AY and DY.
2. Historical averages of incremental paid to prior open claim (C) are computed as to (A) divided by claim (B).
3. Each historical average is trended to the expected severity level for the first calendar year (CY) (2003) after the evaluation date (12/31/2002), and a representative average is selected for each DY [last row of (D)]. A trend factor of 9% per year was assumed in this example.
4. Ratios of open counts at successive year-ends are computed (E).
5. The selected ratios from (E) by DY are used to project the number of open claims for each future DY of each AY, thereby completing (B).
6. Future values of incremental paid to prior open claim (C) are projected on the basis of the representative averages in the last row of (D).
7. Projections of incremental paid losses for future DYs for each AY (A) are determined as the product of the projected open counts from the lower right portion of (B) and the projected values of incremental paid to prior open claim from (C).

The descriptions in the lower right portion of sections (A), (B) and (C) of Table 3.1 also detail how the estimates in each portion are derived.

TABLE 3.1

Sample Application of the Incremental Paid to Prior Open Claim Method

(A) Incremental Paid Losses ($000s)

AY	12	24	36	48	60	72
1997	2,822.8	15,936.1	9,182.3	4,281.6	2,063.8	1,411.4
1998	2,638.0	14,249.9	9,096.4	2,935.8	3,214.7	
1999	3,331.3	15,805.8	9,734.9	4,308.9		
2000	3,170.4	18,602.1	12,462.0			
2001	3,143.1	20,305.9		*Product of Projected (B)*		
2002	4,263.1			*and Projected (C)*		

(B) Open Counts

AY	12	24	36	48	60	72
1997	362	1,112	793	490	375	324
1998	338	888	628	431	352	
1999	343	840	664	492		
2000	268	867	731			
2001	276	897		*Use Ratios from (D) to*		
2002	333			*Project Future Open Counts*		

(C) Incremental Paid to Prior Open Claim

AY	24	36	48	60	72
1997	44,022	8,257	5,399	4,212	3,764
1998	42,159	10,244	4,675	7,459	
1999	46,081	11,589	6,489		
2000	69,411	14,374			
2001	73,572			*Selected Average at CY 2003*	
2002				*Level (E) Adjusted for 9% Inflation*	

(D) Incremental Paid to Prior Open Claim Trended to CY 2003 at 9%/Yr.

AY	24	36	48	60	72
1997	67,734	11,656	6,992	5,004	4,102
1998	59,511	13,266	5,554	8,130	
1999	59,676	13,769	7,073		
2000	82,467	15,667			
2001	80,194				
Avg. Latest 3	74,112	14,234	6,540	6,567	4,102

(E) Ratio of Open Counts at Successive Year-Ends

AY	24	36	48	60	72
1997	3.072	0.713	0.618	0.765	0.864
1998	2.627	0.707	0.686	0.817	
1999	2.449	0.790	0.741		
2000	3.235	0.843			
2001	3.250				
Avg. Latest 3	2.978	0.780	0.682	0.791	0.864

Table 3.2 presents a sample application of this method in estimating incremental payments for accident year 2002, assuming 5,000 ultimate PD claims and a series of additional assumptions derived from SAIF's historical loss experience (as described in Appendix B).

The following observations can be made about the phenomena exhibited in Table 3.2:

- Incremental payments consistently increase for every DY from the 11th through the 40th, a counterintuitive pattern.
- The PLDFs consistently increase for every DY from the 11th through the 31st.
- This method produces projected PLDFs out to 85 years of development. Such development is possible because a worker could be injured at age 16 and live to be over 100.
- Incremental payments do not decrease below the local minimum of $1.7 million during the 11th year of development until the 65th year of development.

To understand why incremental payments, as well as PLDFs, tend to increase during many "mature" years of development, it is helpful to examine how the two key components of the incremental paid to prior open claim method change over successive development years.

This section illustrates how a static mortality model has been incorporated into the incremental paid to prior open claim method. It describes the main framework of the method, while Appendix C covers the derivation of various assumptions that involve a complex analysis.

As is evident from Column (4) in Table 3.3, it was assumed that incremental payments per prior open claim would increase by 9% per year for every DY beyond the seventh, except for the 11th DY. This was based on an analysis of SAIF's historical

TABLE 3.2

Estimation of Incremental MPD Payments for AY 2002 by Static Mortality Model

Development Year	# Prior Open	Paid to Prior Open ($000s)	Incremental Paid Loss ($000,000s)	Cumulative Paid Loss ($000,000s)	PLDF	Paid Factor to Ultimate
1	460*	13.5	6.2	6.2		44.579
2	460	78.4	36.1	42.3	6.8187	6.538
3	1,531	16.6	25.4	67.7	1.6014	4.082
4	1,366	8.4	11.5	79.2	1.1692	3.492
5	949	7.9	7.5	86.7	1.0948	3.189
6	677	6.8	4.6	91.2	1.0530	3.029
7	554	6.9	3.8	95.1	1.0420	2.907
8	396	7.5	3.0	98.1	1.0314	2.818
9	323	8.2	2.7	100.7	1.0271	2.744
10	249	9.0	2.2	103.0	1.0222	2.684
11	209	8.0	1.7	104.6	1.0163	2.641
12	197	8.8	1.7	106.4	1.0165	2.598
13	187	9.5	1.8	108.1	1.0167	2.556
14	178	10.4	1.8	110.0	1.0171	2.513
15	170	11.3	1.9	111.9	1.0175	2.469
16	163	12.4	2.0	113.9	1.0180	2.426
17	156	13.5	2.1	116.0	1.0185	2.382
18	150	14.7	2.2	118.2	1.0190	2.337
19	144	16.0	2.3	120.6	1.0195	2.293
20	139	17.5	2.4	123.0	1.0201	2.248
21	133	19.0	2.5	125.5	1.0205	2.202
22	128	20.7	2.7	128.2	1.0212	2.157
23	124	22.6	2.8	130.9	1.0218	2.111
24	119	24.6	2.9	133.9	1.0223	2.065
25	114	26.9	3.1	136.9	1.0228	2.018
26	109	29.3	3.2	140.1	1.0232	1.973
27	104	31.9	3.3	143.4	1.0236	1.927
28	98	34.8	3.4	146.8	1.0239	1.882
29	93	37.9	3.5	150.4	1.0241	1.838
30	88	41.3	3.6	154.0	1.0242	1.795
31	83	45.0	3.7	157.7	1.0242	1.752
32	78	49.1	3.8	161.5	1.0242	1.711
33	73	53.5	3.9	165.4	1.0240	1.671
34	68	58.3	3.9	169.4	1.0238	1.632
35	63	63.6	4.0	173.4	1.0236	1.594

40	*42*	*97.8*	*4.1*	*193.7*	*1.0215*	*1.427*
45	*26*	*150.5*	*3.9*	*213.6*	*1.0185*	*1.294*
50	*15*	*231.6*	*3.5*	*231.8*	*1.0152*	*1.192*
55	*8.1*	*356.3*	*2.9*	*247.6*	*1.0118*	*1.116*
60	*4.0*	*548.2*	*2.2*	*260.0*	*1.0085*	*1.063*
65	*1.7*	*843.5*	*1.4*	*268.6*	*1.0053*	*1.029*
70	*0.56*	*1,297.8*	*0.73*	*273.5*	*1.0027*	*1.010*
75	*0.13*	*1,996.8*	*0.26*	*275.7*	*1.0009*	*1.003*
80	*0.019*	*3,072.3*	*0.06*	*276.3*	*1.0002*	*1.0004*
85	*0.002*	*4,727.2*	*0.01*	*276.4*	*1.0000*	*1.0000*

For the first DY only, the number of claims open at the end of the year is shown.
After DY 35, the italicized amounts are shown only for each fifth DY.
The PLDFs in this table closely fit SAIF's 10-year historical average factors.

TABLE 3.3

Estimation of Incremental Payments by Static Mortality Model

Development Year (DY)	(1) # Open at End of Prior DY	(2) % Decline in Prior Open Counts	(3) Increm. Pd. to Prior Open ($000s)	(4) % Severity Change
1	0.0		13.478	
2	460.0		78.425	481.9
3	1,531.0		16.607	–78.8
4	1,366.0	10.8	8.388	–49.5
5	949.0	30.5	7.903	–5.8
6	677.0	28.7	6.781	–14.2
7	554.0	18.2	6.924	2.1
8	396.0	28.5	7.547	9.0
9	323.0	18.4	8.226	9.0
10	249.0	22.9	8.967	9.0
11	209.0	16.1	8.036	–10.4
12	196.9	5.8	8.759	9.0
13	186.5	5.3	9.548	9.0
14	177.5	4.8	10.407	9.0
15	169.7	4.4	11.343	9.0
20	138.5	3.8	17.453	9.0
25	113.8	4.2	26.854	9.0
30	88.0	5.6	41.318	9.0
35	62.8	7.1	63.574	9.0
40	41.6	8.4	97.816	9.0
45	25.8	9.6	150.502	9.0

incremental severities for these DYs (see Section C.3 of Appendix C). The fact that SAIF's historical PLDFs for DYs 40–54 are noticeably higher than those predicted by this model (see Figure 1.1) is evidence that there are additional costs associated with caring for elderly claimants, who comprise the majority of claimants during these DYs.

The basis for our selection of 9% as the long-term rate of medical cost escalation is presented in Section C.3 of Appendix C. This assumed annual rate of change in the total cost per claim should be expected to be noticeably greater than the change in the medical component of the CPI. Key reasons for this are

1. *Larger increases in unit costs.* The types of services provided to permanently disabled claimants will likely inflate at a greater rate than that of overall medical services. Examples of these include prosthetic devices, new drugs, surgeries and so on.
2. *Increasing utilization.* The rate at which claimants utilize given services has tended to increase over time.
3. *Shifting mix of services.* There has been a trend toward the greater utilization of more expensive alternatives of care.

Because of these three factors, SAIF's historical rate of medical cost escalation for PD claims has consistently exceeded the change in the medical CPI by a discernable margin. As shown in Table C.4.1, the average rate of MPD cost escalation from 1966 to 2003 was 9.2%, while the average annual change in the medical CPI was 6.8%. Therefore, the average annual change in utilization and mix for 1966–2003 was 2.4%. For 1998–2003, the average utilization and mix change was much larger (i.e., 7.4%, per Table C.4.3).

In Table 3.2 incremental payments continue to increase until age 40 because the impact of claims inflation is greater than the force of mortality in closing existing claims.

The percentage declines in prior open counts reflect the composite effects of three factors affecting the number of open claims: (1) increases due to newly reported claims; (2) decreases due to the death of a few claimants; and (3) net changes due to other reasons (including increases due to reopened claims). After 20 years of development newly reported claims become negligible, as do net claim closures. Thus, after 20 years of development, virtually all claim closures are attributable to the death of claimants. Consequently, changes in the number of open claims at the end of each development year beyond 20 years can be predicted entirely on the basis of mortality rates. And changes in the number of open claims can be estimated beyond 15 years via mortality rates and inclusion of the small number of newly reported claims and net closures for other reasons. This is subject to fine-tuning due to the possibility that the mortality rates of disabled claimants might be higher than those of the general populace, although recent improvements in medical technology have reduced the influence of medical impairment on mortality rates.

Table 3.4 presents an accounting of how each of the above factors affects the number of open MPD claims during the development of a typical accident year. Derivation of these assumptions is disclosed in Appendix C.

SAIF's historical database includes the total number of closed claims. The number of claimant deaths was estimated based on SSA mortality tables and any additional claim closures are presumed to be for other reasons. The breakdown was derived by estimating the number of claim closures due to death from the SSA mortality tables for 2000.

The SSA tables were not modified by a disabled lives scale factor because key values predicted by the model either (1) closely fit SAIF's actual experience; or (2) underestimated actual development (e.g., DYs 40–54). Furthermore, prior actuarial inquiries into this question have been mixed regarding whether such a factor is justified. This is discussed in two papers in the Winter 1991 edition of the CAS *Forum* ("Injured Worker Mortal-

TABLE 3.4

FACTORS AFFECTING THE NUMBER OF OPEN MPD CLAIMS FOR A SINGLE ACCIDENT YEAR

Development Year (DY)	(1) # Open at End of Prior DY [(5) of Prior DY End]	(2) Newly Reported Claims	(3) Estimated # of Claimant Deaths	(4) Estimated Claims Closed for Other Reasons	(5) # Open at End of Current DY [(1) + (2)− (3) − (4)]
1		926	3.5	462.5	460.0
2	460.0	2,790	15.0	1,704.0	1531.0
3	1,531.0	866	17.3	1,013.7	1366.0
4	1,366.0	215	14.1	617.9	949.0
5	949.0	91	10.3	352.7	677.0
6	677.0	47	7.9	162.1	554.0
7	554.0	19	6.9	170.1	396.0
8	396.0	11	5.3	78.7	323.0
9	323.0	8	4.7	77.3	249.0
10	249.0	5	3.9	41.1	209.0
11	209.0	4	3.5	12.5	196.9
12	196.9	3	3.6	9.8	186.5
13	186.5	3	3.6	8.4	177.5
14	177.5	3	3.7	7.1	169.7
15	169.7	3	3.8	5.9	162.9
16	162.9	2	3.9	4.9	156.1
17	156.1	2	4.0	3.9	150.2
18	150.2	1	4.2	3.0	144.0
19	144.0	1	4.3	2.2	138.5
20	138.5	0	4.4	1.4	132.8
21	132.8	0	4.5	0.0	128.2
22	128.2	0	4.7	0.0	123.6
23	123.6	0	4.8	0.0	118.7
24	118.7	0	4.9	0.0	113.8
25	113.8	0	5.1	0.0	108.8

ity" by William R. Gillam [6] and "Review of Report of Committee on Mortality for Disabled Lives" by Gary G. Venter, Barbara Schill, and Jack Barnett [7]). It is quite possible that permanently disabled workers receive better medical care, on average, than nondisabled people, helping to close a gap in mortality rates that would otherwise exist.

TABLE 3.5

INDICATED PAID FACTORS TO ULTIMATE

End of Year of Development	With 9% Inflation	Without Inflation	Ratio of 9% Inflation Reserve to Zero Inflation Reserve
10	2.684	1.152	11.1
15	2.469	1.110	13.4
25	2.019	1.054	18.9
35	1.594	1.022	27.0
50	1.192	1.003	64.0

The paid factors to ultimate in the last column of Table 3.2 above are exceptionally sensitive to future rates of claim inflation. Table 3.5 provides a comparison of the indicated tail factors with and without inflation at various representative ages of development.

An example will put the implications of Table 3.5 into practical terms. Suppose a claims adjuster reviews all PD claims open at the end of 25 years of development. For each PD claim, he estimates the medical portion by multiplying current medical payments by an annuity factor that is the life expectancy of the claimant at his or her current age. The ratio of 18.9 in the right column of Table 3.5 is saying is that future medical payments will be 18.9 times the case reserve derived by this method. One might think that the error would decrease the more mature the accident year became, but in actuality the percentage of error dramatically increases at high maturities. In addition, the mortality table used by the claims adjuster may be out of date.

Just as we have modeled the expected PLDF patterns for MPD losses, analogous incurred loss development factor (ILDF) patterns can be estimated if we define total case reserves as the product of the latest year's incremental payments times the average annuity factor for all living PD claimants. This is presented in Table 3.6.

TABLE 3.6

EXPECTED ILDFs IF CASE RESERVES ARE BASED ON ZERO INFLATION ANNUITY FACTORS

DY	# Prior Open	Upward Sum of # Prior Open	Zero Inflation Annuity Factor	Increm. Pd. to Prior Open	Zero Inflation Case Reserve	Cum. Paid	Zero Inflation Case Incurred	ILDF	Incurred Tail
5	949	6,912.6	6.28	7.9	47.1	86.7	133.8	0.9756	2.066
6	677	5,963.6	7.81	6.8	35.8	91.2	127.1	0.9500	2.175
7	554	5,286.6	8.54	6.9	32.8	95.1	127.9	1.0059	2.162
8	396	4,732.6	10.95	7.5	32.7	98.1	130.8	1.0231	2.113
9	323	4,336.6	12.43	8.2	33.0	100.7	133.7	1.0225	2.066
10	249	4,013.6	15.12	9.0	33.8	103.0	136.7	1.0222	2.022
11	209	3,764.6	17.01	8.0	28.6	104.6	133.2	0.9744	2.075
12	196.9	3,555.6	17.05	8.8	29.4	106.4	135.8	1.0193	2.035
13	186.5	3,358.7	17.01	9.5	30.3	108.1	138.4	1.0195	1.996
14	177.5	3,172.1	16.87	10.4	31.2	110.0	141.2	1.0197	1.958
15	169.7	2,994.6	16.65	11.3	32.0	111.9	144.0	1.0199	1.920
16	162.9	2,824.9	16.34	12.4	32.9	113.9	146.8	1.0200	1.882
17	156.1	2,662.0	16.05	13.5	33.8	116.0	149.8	1.0202	1.845
18	150.2	2,505.9	15.69	14.7	34.6	118.2	152.9	1.0203	1.808
19	144.0	2,355.8	15.36	16.0	35.4	120.6	156.0	1.0204	1.772
20	138.5	2,211.8	14.96	17.5	36.2	123.0	159.2	1.0204	1.737
21	132.8	2,073.2	14.62	19.0	36.9	125.5	162.4	1.0205	1.702
22	128.2	1,940.5	14.13	20.7	37.6	128.2	165.7	1.0205	1.668
23	123.6	1,812.2	13.67	22.6	38.2	130.9	169.1	1.0204	1.634
24	118.7	1,688.7	13.22	24.6	38.7	133.9	172.6	1.0203	1.602
25	113.8	1,569.9	12.80	26.9	39.1	136.9	176.0	1.0202	1.570
26	108.8	1,456.1	12.39	29.3	39.4	140.1	179.6	1.0200	1.539
27	103.6	1,347.4	12.00	31.9	39.7	143.4	183.1	1.0198	1.509
28	98.4	1,243.8	11.64	34.8	39.8	146.8	186.7	1.0195	1.481
29	93.2	1,145.4	11.29	37.9	39.9	150.4	190.3	1.0192	1.453
30	88.0	1,052.2	10.96	41.3	39.8	154.0	193.8	1.0189	1.426
31	82.8	964.2	10.65	45.0	39.7	157.7	197.4	1.0185	1.400
32	77.6	881.5	10.36	49.1	39.5	161.5	201.0	1.0181	1.375
33	72.5	803.9	10.08	53.5	39.1	165.4	204.6	1.0177	1.351
34	67.6	731.3	9.82	58.3	38.7	169.4	208.1	1.0172	1.328
35	62.8	663.7	9.57	63.6	38.2	173.4	211.6	1.0167	1.306
36	58.2	600.9	9.33	69.3	37.6	177.4	215.0	1.0163	1.286
37	53.7	542.8	9.11	75.5	36.9	181.4	218.4	1.0157	1.266
38	49.5	489.0	8.89	82.3	36.2	185.5	221.7	1.0152	1.247
39	45.4	439.6	8.68	89.7	35.4	189.6	225.0	1.0147	1.229
40	41.6	394.2	8.48	97.8	34.5	193.7	228.1	1.0142	1.211

41	38.0	352.6	8.28	106.6	33.5	197.7	231.3	1.0136	1.195
42	34.6	314.6	8.08	116.2	32.5	201.7	234.3	1.0131	1.180
43	31.5	279.9	7.89	126.7	31.5	205.7	237.2	1.0125	1.165
44	28.5	248.5	7.71	138.1	30.4	209.7	240.0	1.0119	1.151
45	25.8	219.9	7.52	150.5	29.2	213.6	242.8	1.0114	1.138
46	23.3	194.1	7.33	164.0	28.0	217.4	245.4	1.0108	1.126
47	21.0	170.8	7.15	178.8	26.8	221.1	247.9	1.0103	1.115
48	18.8	149.9	6.97	194.9	25.5	224.8	250.3	1.0097	1.104
49	16.8	131.0	6.78	212.4	24.3	228.4	252.6	1.0092	1.094
50	15.0	114.2	6.60	231.6	23.0	231.8	254.8	1.0086	1.085

A review of this table reveals the following:

- Although there are ILDFs less than 1.0 for the fifth, sixth, and 11th development years, subsequent factors become noticeably greater than 1.0, even up through the 50th year of development, and beyond.
- Incurred loss development factors are expected to increase during each development year from the 12th through the 21st years.
- The rate of decrease in ILDFs after the 21st development year is surprisingly small, resulting in very large incurred tails for nearly all ages.

This example raises major concerns about the practice of estimating the paid tail by taking the ratio of incurred (perhaps with some modest upward adjustment) to paid at the most mature development year. If case reserves do not include any provision for future medical inflation, then reported incurred at each given DY should be multiplied by the corresponding incurred tail factor shown in the last column of Table 3.6 before the ratio of incurred to paid is applied to paid losses for the most mature years. At DY 10, the incurred tail factor is 2.022. Even at DY 30, an incurred factor of 1.426 is needed. Obviously, to the extent that case reserves include a realistic provision for escalation of future medical costs, the above indicated incurred tail factors would be reduced.

TABLE 4.1

LIFE EXPECTANCIES AT DIFFERENT AGES FOR MALES BASED ON SOCIAL SECURITY ADMINISTRATION MORTALITY TABLES

Current Age	1960	1980	2000	*2020*	*2040*	*2060*	*2080*
20	49.7	51.7	54.7	*56.8*	*58.7*	*60.3*	*61.8*
40	31.3	33.5	36.2	*38.1*	*39.8*	*41.4*	*42.7*
60	15.9	17.3	19.3	*20.8*	*22.2*	*23.4*	*24.6*
80	6.0	6.8	7.2	*7.8*	*8.6*	*9.4*	*10.1*

Note: Projections are in italics.

TABLE 4.2

PERCENTAGE INCREASE IN MALE LIFE EXPECTANCIES BASED ON SOCIAL SECURITY ADMINISTRATION MORTALITY TABLES

Current Age	1980 1960	2000 1980	*2020 2000*	*2040 2020*	*2060 2040*	*2080 2060*
20	4.2	5.8	*3.7*	*3.3*	*2.8*	*2.5*
40	7.0	8.2	*5.2*	*4.5*	*3.8*	*3.3*
60	9.1	11.7	*7.6*	*6.6*	*5.6*	*4.9*
80	11.9	6.5	*8.7*	*10.0*	*8.6*	*7.6*

4. MORTALITY IMPROVEMENT

Life expectancies have been increasing steadily and noticeably for at least the past several decades and are expected to continue to increase throughout the next century, if not beyond.

Consider these trends in life expectancies that have occurred over past decades, and those projected by the SSA. Table 4.1 presents male life expectancies, since a high percentage of permanently disabled claimants are male. Table 4.2 displays the percentage increases in life expectancy corresponding to the estimates in Table 4.1.

Typically, PD claimants receive a percentage of replacement wages until their retirement age, and coverage for their medical expenses related to their work injuries is paid until they die. Since medical expenses are expected to continue rising at high rates of inflation, coverage of such expenses significantly compounds the effects of expected increases in life expectancies.

Consequently, the difference between MPD reserves calculated using constant recent mortality rates and those calculated with trended mortality rates is substantial. The latter calculations are unusually complex. They can best be measured and understood with the aid of a heuristic model.

While the effects of declining mortality rates on gross MPD reserves are almost undetectable over the short run, their magnitude over future decades is quite substantial. However, the extent of these effects is negligible on net MPD when retentions are relatively low. The effect is also fairly small for indemnity loss reserves for permanently disabled claimants.

5. THE TRENDED MORTALITY MODEL

This method is similar to the static mortality model adaptation of the incremental paid to prior open claim method described in Section 3 and Appendix C. The key difference is that the change in the number of open claims for every future development year of every AY is determined by applying mortality tables forecasted by the SSA for the appropriate future development year. The rest of the method is essentially unchanged. A sample of these differences is provided in Table 5.1 for every fifth DY of AY 2002.

As is evident in Table 5.1, small improvements in the annual survival rate of remaining claimants result in major differences in the number of claims still open at higher development years. Given that the greatest differences occur during development years in the distant future, when the effects of medical inflation have had an opportunity to compound over decades, the total

TABLE 5.1

COMPARISON OF MORTALITY RATES AND CLAIMS OPEN AT DIFFERENT DEVELOPMENT YEARS FOR ACCIDENT YEAR 2002

	Mortality Table Assumed		Group Survival Rate		Claims Open at Prior Year-End		% Greater Open
DY	Static	Trended	Static	Trended	Static	Trended	Claims
30	2000	2031	0.941	0.946	88.0	91.5	4.0
35	2000	2036	0.926	0.933	62.8	67.4	7.3
40	2000	2041	0.914	0.922	41.6	46.5	11.7
45	2000	2046	0.902	0.912	25.8	30.3	17.3
50	2000	2051	0.890	0.902	15.0	18.7	24.2
55	2000	2056	0.875	0.889	8.1	10.8	33.3
60	2000	2061	0.853	0.872	3.99	5.82	46.1
65	2000	2066	0.821	0.846	1.68	2.78	65.4
70	2000	2071	0.772	0.811	0.560	1.11	98.9
75	2000	2076	0.709	0.767	0.131	0.351	167.9
80	2000	2081	0.637	0.719	0.019	0.082	329.8
85	2000	2086	0.545	0.716	0.002	0.018	1086.8

reserve indicated by the trended mortality method is decidedly greater than that indicated by the static mortality method.

To fully present the projections of the trended mortality model would require the display of arrays consisting of 37 rows and about 90 columns, with the rows representing accident years and the columns years of development. Since this would be unwieldy, summary arrays will be presented in which data for every fifth accident year are shown at the end of every fifth development year. An example is given in Table 5.2.

Table 5.2 shows the calendar year mortality table that should be used in determining the probability of continuation of a claim for each AY-DY combination. If a current table (e.g., 2000) is used, differences between the static and trended mortality rates will increase the further the year of the appropriate mortality table is from CY 2000.

What effects will the above trends in mortality have on MPD loss reserves? It is not hard to foresee the general effects. Per-

TABLE 5.2

SAMPLE LAYOUT OF SUMMARIZED RESULTS
CALENDAR YEAR OF PAYMENTS—FOR EVERY FIFTH ACCIDENT YEAR AT EVERY FIFTH DEVELOPMENT YEAR

	Development Year															
AY	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80
1970	1974	1979	1984	1989	1994	1999	2004	2009	2014	2019	2024	2029	2034	2039	2044	2049
1975	1979	1984	1989	1994	1999	2004	2009	2014	2019	2024	2029	2034	2039	2044	2049	2054
1980	1984	1989	1994	1999	2004	2009	2014	2019	2024	2029	2034	2039	2044	2049	2054	2059
1985	1989	1994	1999	2004	2009	2014	2019	2024	2029	2034	2039	2044	2049	2054	2059	2064
1990	1994	1999	2004	2009	2014	2019	2024	2029	2034	2039	2044	2049	2054	2059	2064	2069
1995	1999	2004	2009	2014	2019	2024	2029	2034	2039	2044	2049	2054	2059	2064	2069	2074
2000	2004	2009	2014	2019	2024	2029	2034	2039	2044	2049	2054	2059	2064	2069	2074	2079

manent disability claimants for more recent accident years are expected to live longer than their counterparts from old accident years. This is a direct consequence of declining mortality rates. As a result, a higher percentage of PD claimants will still be alive at any given age of development. Therefore, the percentage of claims closed will decline at any given age, and thus simple paid loss development projections will need to be adjusted upward to reflect these declines in claims disposal ratios. Hence, tail factors that reflect the effects of declining mortality rates must increase over successive accident years *for every possible development age*.

While the general effects of anticipated future mortality trends are easy to grasp, the best way to quantify these effects is to construct a heuristic model designed to isolate the effects of mortality trends on PLDFs and paid tails. The trended mortality model we have constructed is such that

- The only thing that changes over time is mortality rates, as historically compiled and as officially forecasted by the SSA.
- Medical inflation is a constant 9% per year, both historically and prospectively. Support for this assumption is provided in Section C.4 of Appendix C.

- The number of ultimate reported claims for every accident year, from 1966 through 2002, is held at a constant level of 5,000 per year.
- Claim reporting and closure patterns for SAIF's PD claimants over the past 10 calendar years served as the basis for these key assumptions in order to make the model as realistic as possible.

By designing a model where claimant mortality rates are the only thing that changes from accident year to accident year, the effects of mortality trends can be clearly seen. Details of the model are presented in Appendices C and D.

Projections of the number of open claims were derived from the heuristic model for each accident year from 1966 through 2002 at the end of every development year from the first to the 80th. As noted above, each accident year was assumed to have 5,000 ultimate reported claims. Claim closure patterns, for reasons other than death of the claimant, were held constant for all accident years. The only thing that varied from accident year to accident year in the model was the number of claims closed due to death. In this way the effects of mortality trends on the number of open claims at the end of each development year for each accident year can be isolated.

What is evident from the summarized results presented in Table 5.3 is that the expected number of open claims at any given year of development will slowly increase as one moves from the oldest accident years to the most recent.

For example, at the end of 35 years of development, the number of open claims is expected to increase from 50 for accident year 1970 to 62 for accident year 2000. This is an increase of 24% in the number of open claims. And at the end of 60 years of development, the number of open claims is expected to increase from 3.5 to 5.0, an increase of 42.9%. The percentage rate of increase in the number of open claims for each given column

TABLE 5.3

NUMBER OF OPEN CLAIMS FOR REPRESENTATIVE ACCIDENT YEARS AT FIVE-YEAR INTERVALS OF DEVELOPMENT

	End of Development Year															
AY	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80
1970	653	196	149	119	95	71	50	33	21	12	6.9	3.5	1.5	0.5	0.1	0.02
1975	655	197	150	120	97	73	52	34	22	13	7.2	3.7	1.6	0.6	0.1	0.03
1980	659	200	153	123	100	76	54	36	23	14	7.7	3.9	1.7	0.6	0.2	0.03
1985	662	202	156	126	103	79	56	38	24	14	8.1	4.2	1.9	0.7	0.2	0.04
1990	665	204	158	128	105	81	58	39	25	15	8.5	4.4	2.0	0.7	0.2	0.04
1995	668	206	160	130	108	83	60	41	26	16	9.0	4.7	2.1	0.8	0.2	0.05
2000	670	207	161	132	110	86	62	42	27	17	9.5	5.0	2.3	0.9	0.3	0.06

TABLE 5.4

PERCENTAGE INCREASES IN THE NUMBER OF OPEN CLAIMS AT THE END OF REPRESENTATIVE DEVELOPMENT YEARS—FROM ACCIDENT YEAR 1970 TO ACCIDENT YEAR 2000

End of Development Year															
5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80
2.6	5.6	8.3	11.6	15.6	19.8	23.9	27.4	30.3	33.5	37.5	43.7	54.3	73.2	106.8	164.5

increases as one moves from the earlier development years on the left to the later development years on the right. This is due to the compounding effect of expected declines in future mortality rates. Table 5.4 displays the total percentage increase for each development year column.

Since the number of open claims at any given development year will be increasing steadily over successive accident years, the total proportion of ultimate losses paid through that development year will decline slightly over time. Because of this we would naturally expect that the appropriate tail factors at any given development year will also increase steadily over time. The projected results are displayed in Table 5.5.

TABLE 5.5

INDICATED TAIL FACTORS

	End of Development Year															
AY	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80
1970	3.037	2.570	2.375	2.177	1.973	1.773	1.592	1.438	1.311	1.210	1.132	1.075	1.037	1.015	1.004	1.001
1975	3.108	2.628	2.428	2.223	2.012	1.805	1.617	1.456	1.325	1.220	1.139	1.080	1.040	1.016	1.005	1.001
1980	3.197	2.701	2.492	2.279	2.058	1.842	1.645	1.477	1.340	1.231	1.146	1.085	1.043	1.018	1.006	1.001
1985	3.286	2.774	2.558	2.336	2.106	1.879	1.674	1.499	1.356	1.242	1.154	1.090	1.046	1.020	1.007	1.002
1990	3.376	2.848	2.624	2.393	2.154	1.918	1.704	1.521	1.372	1.253	1.162	1.095	1.049	1.021	1.007	1.002
1995	3.466	2.921	2.690	2.451	2.203	1.957	1.733	1.543	1.388	1.265	1.170	1.101	1.053	1.023	1.008	1.002
2000	3.549	2.990	2.752	2.505	2.248	1.993	1.761	1.563	1.402	1.275	1.177	1.105	1.054	1.023	1.008	1.002

TABLE 5.6

INDICATED PERCENTAGE UNDERSTATEMENT IN AY 2000 LOSS RESERVES (IF BASED ON AY 1970 TAIL FACTORS)

End of Development Year															
5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80
25	27	27	28	28	28	28	29	29	31	34	39	47	59	85	102

Table 5.6 displays the percentage understatement in AY 2000 loss reserves at different development ages, if such reserves were based on AY 1970 tail factors. It clearly indicates that the use of constant tail factors will result in material inadequacies in the indicated loss reserves.

The trended mortality model also indicates that incremental PLDFs at any given maturity will trend upward over time. In Table 5.7, five-year paid loss development factors, each of which are the cumulative products of five successive one-year paid loss development factors, inch upward over time within any given development column.

Table 5.7 rebuts the conjecture that the paid loss development factors for earlier (as well as middle) development years will hold constant over successive accident years. However, it is also evident that the rate of increase in these paid development factors

TABLE 5.7

TRENDS IN FIVE-YEAR PAID LOSS DEVELOPMENT FACTORS

	Development Years															
AY	10/5	15/10	20/15	25/20	30/25	35/30	40/35	45/40	50/45	55/50	60/55	65/60	70/65	75/70	80/75	85/80
1970	1.182	1.082	1.091	1.103	1.113	1.114	1.107	1.097	1.084	1.069	1.053	1.037	1.022	1.010	1.004	1.001
1975	1.183	1.083	1.092	1.105	1.115	1.116	1.110	1.099	1.086	1.071	1.055	1.039	1.023	1.011	1.004	1.001
1980	1.184	1.084	1.094	1.107	1.118	1.119	1.114	1.103	1.089	1.073	1.057	1.040	1.024	1.012	1.004	1.001
1985	1.185	1.084	1.095	1.109	1.120	1.123	1.117	1.106	1.092	1.076	1.059	1.042	1.026	1.013	1.005	1.002
1990	1.186	1.085	1.096	1.111	1.123	1.126	1.120	1.109	1.094	1.078	1.061	1.044	1.027	1.014	1.005	1.002
1995	1.186	1.086	1.097	1.113	1.126	1.129	1.123	1.112	1.097	1.081	1.063	1.046	1.029	1.015	1.006	1.002
2000	1.187	1.087	1.098	1.114	1.128	1.132	1.126	1.115	1.100	1.083	1.065	1.048	1.030	1.015	1.006	1.002

TABLE 5.8

PLDFS FACTORS INDICATED BY THE TRENDED MORTALITY MODEL DURING EARLY YEARS OF DEVELOPMENT

	Years of Development										
AY	2	3	4	5	6	7	8	9	10	11	12
1990	6.81875	1.59471	1.16775	1.09383	1.05240	1.04154	1.03101	1.02670	1.02182	1.01604	1.01618
1991	6.81875	1.59488	1.16781	1.09387	1.05243	1.04157	1.03104	1.02673	1.02185	1.01607	1.01621
1992	6.81875	1.59505	1.16786	1.09392	1.05246	1.04160	1.03107	1.02676	1.02187	1.01609	1.01623
1993	6.81875	1.59522	1.16792	1.09396	1.05250	1.04163	1.03110	1.02679	1.02190	1.01611	1.01625
1994	6.81875	1.59539	1.16797	1.09400	1.05253	1.04166	1.03113	1.02681	1.02192	1.01613	1.01628
1995	6.81875	1.59557	1.16803	1.09405	1.05256	1.04169	1.03115	1.02684	1.02195	1.01615	1.01630
1996	6.81875	1.59571	1.16807	1.09408	1.05259	1.04172	1.03118	1.02686	1.02197	1.01617	1.01632
1997	6.81875	1.59586	1.16812	1.09412	1.05261	1.04174	1.03120	1.02688	1.02199	1.01618	1.01634
1998	6.81875	1.59601	1.16816	1.09415	1.05263	1.04176	1.03122	1.02691	1.02201	1.01620	1.01636
1999	6.81875	1.59616	1.16821	1.09419	1.05266	1.04179	1.03124	1.02693	1.02203	1.01622	1.01638
2000	6.81875	1.59631	1.16825	1.09422	1.05268	1.04181	1.03126	1.02695	1.02205	1.01623	1.01639
2001	6.81875	1.59647	1.16830	1.09426	1.05271	1.04184	1.03129	1.02697	1.02208	1.01625	1.01642
2002	6.81875	1.59662	1.16835	1.09430	1.05273	1.04186	1.03131	1.02699	1.02210	1.01627	1.01644

is small. It is small enough that it would not be detectable to an experienced actuary reviewing historical PLDFs. This becomes even more evident if we look at different sections of the typical triangle of paid loss development factors that are generated by the trended mortality model.

In Table 5.8 the individual PLDFs generated by the model are displayed for AYs 1990–2002 for the earliest development years.

TABLE 5.9

PLDFs Indicated by the Trended Mortality Model During Later Years of Development

	Year of Development										
AY	27	28	29	30	31	32	33	34	35	36	37
1966	1.02103	1.02124	1.02139	1.02147	1.02149	1.02146	1.02136	1.02121	1.02101	1.02077	1.02049
1967	1.02112	1.02134	1.02149	1.02157	1.02160	1.02156	1.02147	1.02132	1.02113	1.02088	1.02060
1968	1.02121	1.02143	1.02159	1.02168	1.02170	1.02167	1.02158	1.02143	1.02124	1.02100	1.02072
1969	1.02130	1.02153	1.02168	1.02178	1.02181	1.02178	1.02169	1.02154	1.02135	1.02111	1.02083
1970	1.02140	1.02163	1.02179	1.02189	1.02192	1.02189	1.02180	**1.02166**	1.02147	1.02123	1.02095
1971	1.02148	1.02171	1.02187	1.02198	1.02201	1.02199	1.02190	1.02176	1.02157	1.02133	1.02106
1972	1.02155	1.02179	1.02196	1.02207	1.02211	1.02209	1.02200	1.02187	1.02168	1.02144	1.02116
1973	1.02163	1.02187	1.02205	1.02216	1.02220	1.02218	1.02211	1.02197	1.02178	1.02155	1.02127
1974	1.02170	1.02195	1.02213	1.02225	1.02230	1.02228	1.02221	1.02208	1.02189	1.02165	1.02138
1975	1.02178	1.02203	1.02222	1.02234	1.02239	1.02238	1.02231	**1.02218**	1.02200	1.02176	1.02148
1976	1.02188	1.02214	1.02233	1.02245	1.02250	1.02250	1.02243	1.02230	1.02211	1.02188	1.02160
1977	1.02199	1.02225	1.02244	1.02256	1.02262	1.02261	1.02254	1.02241	1.02223	1.02200	1.02172

The constant PLDFs in the column for DY 2 merely reflect a simplifying assumption in the model.

In Table 5.9 individual PLDFs generated by the model are displayed for accident years 1966–1977 for the most mature historical development years. Projected PLDFs for the short-term future are also shown below the diagonal.

Table 5.10 provides an example of the kinds of errors in estimating future incremental payments that can occur when it is assumed that PLDFs for each year of development hold constant. First, a PLDF of 1.02138 is selected as the average of the latest four historical factors during the 34th year of development (the boxed items in Table 5.9). By comparing this selection with the true underlying trended PLDF, the percentage error in incremental payments for that development year is shown for every fifth AY. These errors assume, however, that other similar errors did not occur during preceding development years.

Though all of the errors above are small, these errors compound significantly in the calculation of tail factors, which are the product of numerous individual PLDFs.

TABLE 5.10

Errors in PLDFs During 34th Year of Development Due to Selecting a Constant Historical Average PLDF

Accident Year	Selected PLDF	True Underlying PLDF	% Error in Incremental Payments
1970	1.02138	**1.02166**	−1.3
1975	1.02138	**1.02218**	−3.6
1980	1.02138	1.02276	−6.1
1985	1.02138	1.02336	−8.5
1990	1.02138	1.02395	−10.7
1995	1.02138	1.02452	−12.8
2000	1.02138	1.02507	−14.7

TABLE 6.1

A Comparison of Indicated MPD Tail Factors

Maturity (Years)	Based on SAIF's Experience	Based on Static Mortality Model	Based on Trended Mortality Model
10	2.469	2.684	3.025
15	2.328	2.469	2.783
25	2.054	2.019	2.271
35	1.680	1.594	1.776

Even though it is true that past declines in mortality rates are implicitly embedded in historical PLDFs, the above example clearly illustrates that it would be incorrect to assume that the selection of historical factors as estimates of future PLDFs would implicitly incorporate the effects of future declines in mortality rates. What would be more appropriate would be to select representative PLDFs for each development year based on recent historical factors and then to trend these upward in a manner parallel to the PLDFs indicated by a realistic model.

6. A COMPARISON OF INDICATED TAIL FACTORS

Table 6.1 provides a comparison of the MPD tails indicated by SAIF's own loss experience with those indicated by the static

TABLE 6.2

INDICATED LOSS RESERVE AT DIFFERENT MATURITIES (DOLLARS IN MILLIONS)

Maturity (Years)	MPD Reserve	Other Workers Compensation Reserve	MPD Reserve as a Percentage of Total Workers Compensation Reserve
10	$41.3	$10.4	80
15	39.6	9.5	81
25	34.6	5.7	86
35	27.0	2.5	92

and trended mortality methods. This table repeats the MPD tails indicated by SAIF's experience in Table 2.4.

As noted earlier, the indications of the static mortality model reasonably fit those from SAIF's historical loss experience, except that the model somewhat understates development for DYs 40–54.

The relative contribution of MPD versus all other workers compensation to the total loss reserves for a given AY is much greater if the trended mortality model is assumed. Those percentages at various maturities are shown in the last column of Table 6.2.

The above table is analogous to Table 2.5, which shows results based on SAIF's historical loss experience. In deriving these estimates, total AY ultimate losses of $100 million were assumed, together with a 50–50 split between MPD and other workers compensation. However, the $50 million figure for ultimate MPD was changed to the product of cumulative paid MPD at 10 years of development and the 10 to ultimate tail factor from the trended mortality model. That increased ultimate MPD to $61.75 million.

Table 6.3 provides a side-by-side comparison of the percentages of the total workers compensation loss reserve attributable to MPD, as estimated using historical PLDFs and PLDFs indicated by the trended mortality model.

TABLE 6.3

COMPARISON OF MPD LOSS RESERVE AS A PERCENTAGE OF THE TOTAL WORKERS COMPENSATION LOSS RESERVE (BASED ON DIFFERENT PLDF ASSUMPTIONS; DOLLARS IN MILLIONS)

Maturity (Years)	Indicated by Historical PLDFs	Indicated by Trended Mortality PLDFs	Percentage Increase in MPD Reserve Due to Using Trended Mortality Rates
10	$29.6	$41.3	+39.7
15	28.3	39.6	+39.6
25	25.5	34.6	+35.8
35	20.0	27.0	+34.9

Clearly, the trended mortality model indicates MPD loss reserves that are significantly larger than straight historical experience would indicate.

7. SENSITIVITY CONSIDERATIONS

The most significant factor affecting the indications in this paper is the applicable retention. This paper presents indications on an unlimited basis. Tail factors and PLDFs at more mature years of development should be expected to be significantly less at relatively low retentions. This is evident on an a priori basis.

Consider a hypothetical PD claimant injured on December 15, 2003, at age 35.9 years, with a life expectancy of 40 years. His medical costs are $5,000 during 2004, and future medical inflation is 9% per year. Indemnity losses are a flat $25,000 per year, beginning in 2004. Table 7.1 indicates the total cumulative loss payments at the end of each of the first 41 years of development.

For this hypothetical PD claimant, net paid losses would top out by the end of the ninth year of development with a retention of $250,000; after 16 years with a $500,000 retention; after 26 years with a $1 million retention; and after 37 years with a $2 million retention.

TABLE 7.1

Cumulative Loss Payments for Hypothetical PD Claimant

(A) Age of Claimant	(B) DY	(C) Incremental Medical Payments	(D) Cumulative Medical Payments	(E) Cumulative Indemnity Payments	(F) Cumulative Loss Payments	
35	1	0.0	0.0	0.0	0.0	
36	2	5.0	5.0	25.0	30.0	
37	3	5.5	10.5	50.0	60.5	
38	4	5.9	16.4	75.0	91.4	
39	5	6.5	22.9	100.0	122.9	
40	6	7.1	29.9	125.0	154.9	
41	7	7.7	37.6	150.0	187.6	
42	8	8.4	46.0	175.0	221.0	
43	9	9.1	55.1	200.0	255.1	
44	10	10.0	65.1	225.0	290.1	(a)
45	11	10.9	76.0	250.0	326.0	
46	12	11.8	87.8	275.0	362.8	
47	13	12.9	100.7	300.0	400.7	
48	14	14.1	114.8	325.0	439.8	
49	15	15.3	130.1	350.0	480.1	
50	16	16.7	146.8	375.0	521.8	(b)
51	17	18.2	165.0	400.0	565.0	
52	18	19.9	184.9	425.0	609.9	
53	19	21.6	206.5	450.0	656.5	
54	20	23.6	230.1	475.0	705.1	
55	21	25.7	255.8	500.0	755.8	
56	22	28.0	283.8	525.0	808.8	
57	23	30.5	314.4	550.0	864.4	
58	24	33.3	347.7	575.0	922.7	
59	25	36.3	383.9	600.0	983.9	
60	26	39.6	423.5	625.0	1,048.5	(c)
61	27	43.1	466.6	650.0	1,116.6	
62	28	47.0	513.6	675.0	1,188.6	
63	29	51.2	564.8	700.0	1,264.8	
64	30	55.8	620.7	725.0	1,345.7	
65	31	60.9	681.5	750.0	1,431.5	
66	32	66.3	747.9	775.0	1,522.9	
67	33	72.3	820.2	800.0	1,620.2	
68	34	78.8	899.0	825.0	1,724.0	
69	35	85.9	984.9	850.0	1,834.9	
70	36	93.6	1,078.6	875.0	1,953.6	
71	37	102.1	1,180.6	900.0	2,080.6	(d)

Note: (a) Development stops if return is \$250K. (b) Development stops if return is \$500K. (c) Development stops if return is \$1M. (d) Development stops if return is \$2M.

While this dampening effect of retentions can obviously serve to greatly mitigate the magnitude of the applicable tail factors for different insurers and self-insureds, this effect can rapidly dissipate when retentions rise significantly from year to year. It is quite common for insurers as well as self-insureds to significantly increase retentions when faced with costs for excess coverage that have risen substantially as the market has hardened. The effect of recognizing the upward impact greater retentions will have on assumed tails can be sizable.

Other factors that can have a material impact on MPD tail factors are the following:

- The assumed future rate of medical cost escalation.
- The observed tendency of medical losses to step up noticeably as an increasing proportion of claimants become elderly.
- The possibility that actual mortality rates of PD claimants might be higher (or lower) than those for the general populace.
- Variations in the gender mix and age-at-injury mix of PD claimants.

An entire paper could be devoted to quantifying the effects that changes in any or all of the above factors would have on indicated tail factors. Of the above factors, the first two are the most significant. While some believe that the long-term future rate of medical cost escalation will be less than the historical rate of 9%, others believe a constant 9% assumption is reasonable. Arguably, the differential between medical inflation and general inflation may lessen over future decades. However, workers compensation medical costs are a very small portion of total health costs, so a workers compensation medical escalation rate of 9% could continue for a very long period of time without having much effect on the overall medical CPI or GNP. Furthermore, long-term general inflation may move upward as a result of shortages in critical commodities (such as petroleum) and their ubiquitous derivative products (e.g., plastics and synthetics).

We note that SAIF's actual age-at-injury distribution is weighted heavily toward the middle-age groups. If a much younger distribution were assumed, this would dramatically increase the survival probabilities during each year of development, and the resulting tails would be considerably greater than those presented in this paper. The age-at-injury distribution can vary significantly depending on statutory provisions for qualification for a permanent disability award and the nature of the risks insured or self-insured.

In the static mortality model, we started with the assumption of a beginning gender mix of 75% male and 25% female. Because of the higher mortality rates of males at all ages, by the 50th year of development, the percentage of surviving claimants that are male is expected to drop to 64.5%. By the 72nd year of development, a 50–50 gender split is expected.

The magnitude of the elder care cost bulge is quite significant. It fully accounts for the large degree to which SAIF's actual MPD PLDFs exceed those indicated by the static mortality model during later DYs (see Figure 1.2).

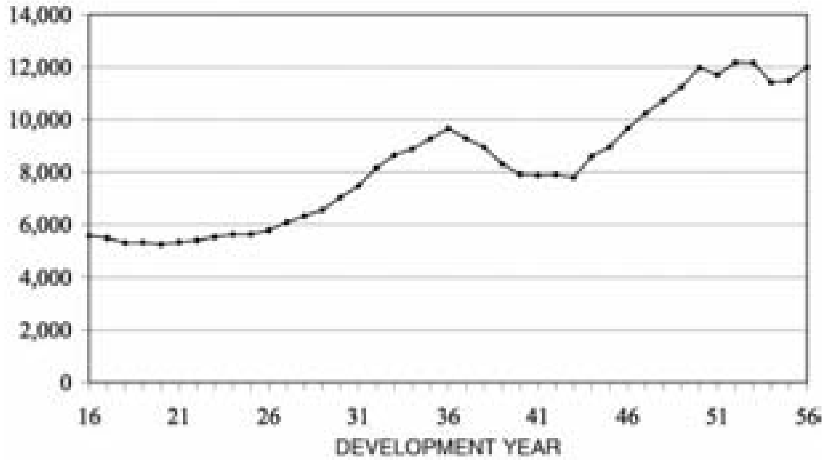
Figure 7.1 provides documentation of the extent of increases in SAIF's incremental paid medical costs per open claim at a constant 2003 cost level for DYs 16–56. If the common actuarial assumption that incremental medical severities are independent of the age of the claimant were true, then the graph line in Figure 7.1 would be essentially flat, since all severities have been placed on a constant 2003 cost level.

The above average costs at 2003 cost level were for AY 1945 and subsequent accident years during CYs 1991–2003. Table E.1, Appendix E provides a summary of the detailed data supporting Figure 7.1.

The implications of Figure 7.1 are serious with respect to the reasonableness of the practice of estimating MPD reserves by

FIGURE 7.1

INCREMENTAL PAID SEVERITY (AT 2003 COST LEVEL)



inflating current annual medical costs for each claim at normal rates of medical cost escalation until the expected year of death. In doing so, the actuary would be assuming, on average for all claims open during DYs 10–20, that an annual severity at a 2003 cost level of approximately $6,000 per year would be appropriate for all future years, regardless of how old the claimant becomes. Figure 7.1 indicates that as each claimant advances into his or her 70s or 80s, a significantly higher assumed severity at a 2003 cost level would be more appropriate.

8. ESTIMATING THE EXPECTED VALUE OF MPD RESERVES

In Tables 8.1A and 8.1B cumulative loss payments for a hypothetical PD claimant are displayed. This might be a profile of paid losses for a male claimant injured on December 15, the reserve evaluation date. At age 35.9, the claimant is expected to live another 40 years. Two different methods of estimating the *medical case reserve* for this claimant at the end of the first year of development are common. They are the following:

1. *Zero Inflation Case Reserve Based on Projected Payments Through Expected Year of Death.* Estimated annual medical expenses of $5,000 per year (during the first full year of development) are multiplied by the life expectancy of 40 years to obtain a case reserve of $200,000.
2. *Inflation Case Reserve (9%) Based on Projected Payments Through Expected Year of Death.* Escalating medical expenses are cumulated up through age 75, yielding a total incurred amount of $1,689,000.

Two additional methods may also be applied. Each of these produces much higher, and more accurate, estimates of the expected value of the case reserve:

3. *Expected Total Payout over Scenarios of All Possible Years of Death.* This method, described below, yields an expected reserve of $2,879,000.
4. *Expected Value of Trials from a Markov Chain Simulation.* This method, described in Section 9, yields an expected reserve of $2,854,000.

In applying the third method, cumulative payments are calculated through each possible future year of death. Each of these estimates represents the scenario of the claimant's death during a specific *n*th year of development. The probability of occurrence of the *n*th scenario is the product of the probability the claimant will live through all prior years of development and then die during the *n*th year of development. The expected value of the case reserve is then the weighted average of all of these estimates of final cumulative payments, weighted by their associated probability of occurrence. In this example, the expected value of total incurred is $2,879,000, which is 70.5% higher than the second estimate. This kind of estimate is often not calculated by self-insureds or insurers who have only a few PD claimants. Yet it is

TABLE 8.1A

Calculation of Case Reserve by Second and Third Methods (Age 36 through 88)

Male Claimant, Age 35.9 at Reserve Date, 9% Future Inflation Assumed

Age	(A) $l(x)$	(B) $d(x)$	(C) Probability of Dying at Age x (B)/(A)@36	(D) Cumulative Probability of Dying at Age x	(E) Incremental Medical Paid (in millions of dollars)	(F) Cumulative Medical Paid (in millions of dollars)	(G) Expected Total Payout (C) × (F) (in millions of dollars)	(H) Cumulative Expected Total Payout (in millions of dollars)	(I) % of Expected Total Payout
36	96023	198	0.00206	0.206%	5	5	0.01	0.01	0.000
37	95825	209	0.00218	0.424%	5	10	0.02	0.03	0.001
38	95616	225	0.00234	0.658%	6	16	0.04	0.07	0.003
39	95391	241	0.00251	0.909%	6	23	0.06	0.13	0.005
40	95150	260	0.00271	1.180%	7	30	0.08	0.21	0.007
41	94890	279	0.00291	1.470%	8	38	0.11	0.32	0.011
42	94611	300	0.00312	1.783%	8	46	0.14	0.46	0.016
43	94311	321	0.00334	2.117%	9	55	0.18	0.65	0.023
44	93990	343	0.00357	2.474%	10	65	0.23	0.9	0.031
45	93647	368	0.00383	2.858%	11	76	0.29	1.2	0.041
46	93279	395	0.00411	3.269%	12	88	0.36	1.5	0.054
47	92884	422	0.00439	3.708%	13	101	0.44	2.0	0.069
48	92462	448	0.00467	4.175%	14	115	0.54	2.5	0.088
49	92014	477	0.00497	4.672%	15	130	0.65	3.2	0.111
50	91537	508	0.00529	5.201%	17	147	0.78	3.9	0.138
51	91029	544	0.00567	5.767%	18	165	0.9	4.9	0.171
52	90485	583	0.00607	6.375%	20	185	1.1	6.0	0.210
53	89902	629	0.00655	7.030%	22	207	1.4	7.3	0.257
54	89273	679	0.00707	7.737%	24	230	1.6	9	0.314
55	88594	735	0.00765	8.502%	26	256	2.0	11	0.383

56	87859	797	0.00830	9.332%	28	284	2.4	13	0.466
57	87062	865	0.00901	10.233%	31	314	2.8	16	0.565
58	86197	936	0.00975	11.208%	33	348	3.4	20	0.684
59	85261	1014	0.01056	12.264%	36	384	4.1	24	0.826
60	84247	1096	0.01141	13.405%	40	424	4.8	28	0.995
61	83151	1184	0.01233	14.638%	43	467	5.8	34	1.197
62	81967	1287	0.01340	15.978%	47	514	6.9	41	1.438
63	80680	1405	0.01463	17.442%	51	565	8.3	49	1.728
64	79275	1532	0.01595	19.037%	56	621	10	59	2.075
65	77743	1669	0.01738	20.775%	61	682	12	71	2.490
66	76074	1803	0.01878	22.653%	66	748	14	85	2.982
67	74271	1923	0.02003	24.656%	72	820	16	102	3.558
68	72348	2023	0.02107	26.762%	79	899	19	120	4.222
69	70325	2109	0.02196	28.959%	86	985	22	142	4.980
70	68216	2203	0.02294	31.253%	94	1,079	25	167	5.847
71	66013	2305	0.02400	33.653%	102	1,181	28	195	6.840
72	63708	2407	0.02507	36.160%	111	1,292	32	228	7.903
73	61301	2504	0.02608	38.768%	121	1,413	37	264	9.182
74	58797	2603	0.02711	41.479%	132	1,545	42	306	10.637
75	56194	2704	0.02816	44.295%	144	1,689	48	354	12.289
76	53490	2808	0.02924	47.219%	157	1,846	54	408	14.164
77	50682	2915	0.03036	50.255%	171	2,018	61	469	16.292
78	47767	3021	0.03146	53.401%	187	2,204	69	538	18.700
79	44746	3119	0.03248	56.649%	203	2,408	78	617	21.416
80	41627	3199	0.03331	59.980%	222	2,629	88	704	24.458
81	38428	3253	0.03388	63.368%	242	2,871	97	802	27.836
82	35175	3281	0.03417	66.785%	263	3,134	107	909	31.555
83	31894	3276	0.03412	70.197%	287	3,421	117	1,025	35.609
84	28618	3232	0.03366	73.563%	313	3,734	126	1,151	39.974
85	25386	3147	0.03277	76.840%	341	4,075	134	1,285	44.612
86	22239	3020	0.03145	79.985%	372	4,447	140	1,424	49.470
87	19219	2852	0.02970	82.955%	405	4,852	144	1,569	54.475
88	16367	2649	0.02759	85.714%	442	5,294	146	1,715	59.547

TABLE 8.1B

CALCULATION OF CASE RESERVE BY SECOND AND THIRD METHODS (AGE 89 THROUGH 109)

Male Claimant, Age 35.9 at Reserve Date, 9% Future Inflation Assumed

Age	(A) $l(x)$	(B) $d(x)$	(C) Probability of Dying at Age x (B)/(A)@36	(D) Cumulative Probability of Dying at Age x	(E) Incremental Medical Paid (in millions of dollars)	(F) Cumulative Medical Paid (in millions of dollars)	(G) Expected Total Payout (C) × (F) (in millions of dollars)	(H) Cumulative Expected Total Payout (in millions of dollars)	(I) % of Expected Total Payout
89	13718	2414	0.02514	88.228%	481	5,776	145	1,860	64.589
90	11304	2159	0.02248	90.476%	525	6,300	142	2,002	69.509
91	9145	1890	0.01968	92.445%	572	6,873	135	2,137	74.207
92	7255	1619	0.01686	94.131%	624	7,496	126	2,263	78.596
93	5636	1355	0.01411	95.542%	680	8,176	115	2,379	82.603
94	4281	1106	0.01152	96.694%	741	8,916	103	2,481	86.169
95	3175	878	0.00914	97.608%	807	9,724	89	2,570	89.257
96	2297	676	0.00704	98.312%	880	10,604	75	2,645	91.850
97	1621	506	0.00527	98.839%	959	11,563	61	2,706	93.966
98	1115	367	0.00382	99.221%	1,046	12,609	48	2,754	95.639
99	748	258	0.00269	99.490%	1,140	13,749	37	2,791	96.922
100	490	178	0.00185	99.675%	1,242	14,991	28	2,819	97.888
101	312	119	0.00124	99.799%	1,354	16,346	20	2,839	98.591
102	193	77	0.00080	99.879%	1,476	17,822	14	2,853	99.087
103	116	49	0.00051	99.930%	1,609	19,431	10	2,863	99.432
104	67	29	0.00030	99.960%	1,754	21,185	6	2,870	99.654
105	38	18	0.00019	99.979%	1,912	23,096	4	2,874	99.804
106	20	10	0.00010	99.990%	2,084	25,180	3	2,876	99.895
107	10	5	0.00005	99.995%	2,271	27,451	1	2,878	99.945
108	5	4	0.00004	99.999%	2,476	29,927	1	2,879	99.988
109	1	1	0.00001	100.000%	2,698	32,625	0	2,879	100.00

in keeping with the standard definition of the expected value of total incurred.

The total case reserve based on this third approach is dramatically higher than that derived from the second approach because the cumulative paid amounts associated with death at ages beyond the claimant's expected year of death are given more weight, due to the compounding effects of medical cost escalation.

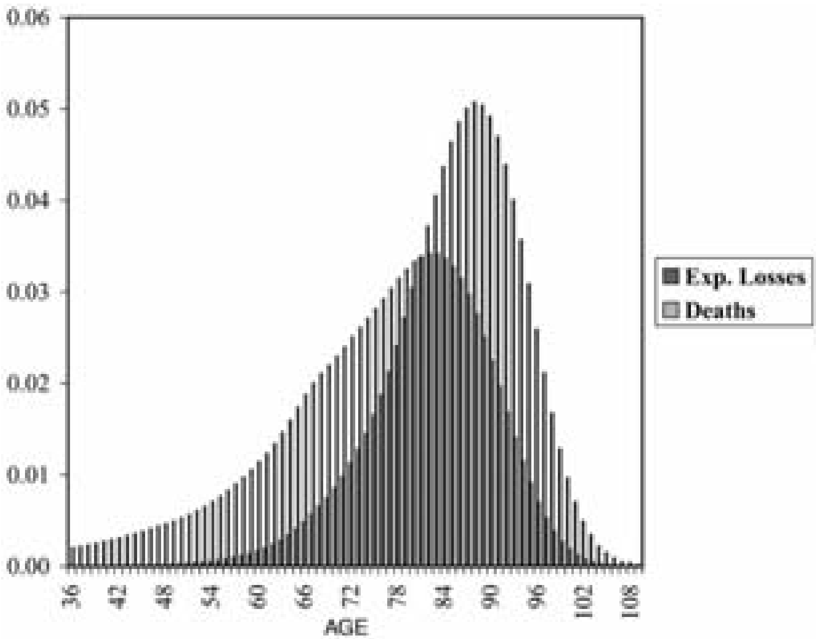
In Tables 8.1A and 8.1B the medical case reserve for the hypothetical PD claimant is calculated for the second and third methods. For the second method, Projected Payments Through Expected Year of Death, the cumulative payments from Column (F) at the end of the expected year of death (at age 75) yields the estimate of $1,689,000.

For the third method, each row is treated as a different scenario, with its probability of occurrence shown in Column (C). These probabilities are the weights applied to the estimates of cumulative medical payments in Column (F) to obtain the components of the expected total payout in Column (G) that are cumulated in Column (H). Hence, the expected value of the case reserve is the bottom number in Column (H) in Table 8.1B ($2,879,000).

The distribution of deaths by age of death (Column (C)) would be the same as the distribution of the different scenarios for the *indemnity* case reserve, since incremental indemnity payments are not generally subject to inflation. Figure 8.1 illustrates the shift in the distribution of the different scenarios for the *medical* case reserve [Column (I) decumulated, or Column (G) divided by Total in Column (H)], due to the effects of compounding medical cost escalation in giving more dollar weight to scenarios where the claimant lives beyond his expected year of death.

FIGURE 8.1

DEATHS AND EXPECTED PAYOUTS BY AGE



The impact of medical cost escalation causes the age corresponding to the median of the distribution of medical payments (87) to exceed the age corresponding to the median of the distribution of the indemnity payments (77) by 10 years. This can be seen by comparing the age corresponding to a cumulative probability of 50% in Column (D) to the age when Column (I) reaches 50% . To further appreciate the significance of this shift, consider the following observations drawn from Table 8.1B:

- While 83% of such claimants die before they reach the age of 87, medical payments to claimants who live beyond 86 years of age account for over half of total expected future medical payments.

- While 90% of such claimants die before they reach the age of 90, medical payments to claimants who live beyond 89 years of age account for over 30% of total expected medical losses.

The ratio of the estimated case reserve based on the second method to that from the first method varies dramatically with the age of the claimant at the reserve date. It is also dependent on gender. This is also true, though to a lesser degree, for the ratio of the third method case reserve to the second method reserve. These ratios are displayed in Table 8.2.

There are a number of reasons to believe that the reserve estimates produced by the static mortality model presented in Section 3 are analogous to estimates produced by the second method. If that is true, then it would be necessary to multiply reserve estimates based on the static mortality model by some weighted average of the ratios in Column (E) of Table 8.2 to arrive at an estimated reserve at the expected level. Whether that ratio is 1.25 or 1.40 or 1.55, it represents a substantial add-on to a reserve estimate that is likely higher than what would be obtained using more traditional methods.

Why are reserve estimates based on the static mortality model similar to those produced by the second method? A fundamental assumption of the model is that all claimants die according to a schedule dictated by current mortality tables. When an expected value of the reserve is calculated, it is based on a weighted average of a full range of scenarios, including those where many claimants die earlier than planned and others die later. Total future payments for those claimants that die later will be given more dollar weight. Hence, the expected value of the reserve will be correspondingly greater than that projected by the static mortality model.

All of the methods presented in this section are based on the common assumption that the current level incremental severities

TABLE 8.2

COMPARISON OF DIFFERENT TYPES OF MPD RESERVE ESTIMATES

Assuming SSA 2000 Male & Female Mortality Tables and 9% Medical Cost Escalation					
	(A)	(B)	(C)	(D)	(E)
	Reserve ($000s) at Eval. Date				
Age at Reserve Date	First Method (Zero Inflation Case Reserve)	Second Method (9% Inflation Case Reserve)	Third Method (Total Expected Future Payout)	Ratio of Second Method Reserve to First Method Reserve	Ratio of Third Method Reserve to Second Method Reserve
	Male Claimants				
20	$273.7	$7,333.9	$11,318.1	26.795	1.543
30	227.3	2,989.5	4,816.3	13.155	1.611
40	181.2	1,321.0	2,042.3	7.290	1.546
50	137.3	590.0	864.0	4.298	1.464
60	96.7	265.3	362.9	2.744	1.368
70	62.9	123.5	153.2	1.965	1.240
80	36.0	57.1	63.4	1.587	1.110
	Female Claimants				
20	$301.0	$10,796.0	$16,724.2	35.867	1.549
30	252.4	4,641.7	7,069.1	18.390	1.523
40	204.7	2,005.7	2,983.6	9.800	1.488
50	158.4	873.8	1,254.5	5.516	1.436
60	115.1	384.3	524.0	3.341	1.363
70	77.0	165.0	217.3	2.144	1.317
80	45.2	76.3	87.2	1.690	1.142

do not increase with the age of the claimant. This was done to simplify the presentation of methods that are already complex. If the tendency of incremental medical severities to increase with age were incorporated into these methods, the differences between the reserves projected by these methods would expand noticeably.

9. ESTIMATING THE VARIABILITY OF THE MPD RESERVE WITH A MARKOV CHAIN SIMULATION

The size of loss distribution for the medical component of a single PD claim is far more skewed to the right than can be modeled by distributions commonly used by casualty actuaries. This distribution can be described by the ultimate costs in Column (F) of Tables 8.1A and 8.1B, with the associated confidence levels taken from Column (D). In attempting to find a distribution that produced a reasonable fit, it was necessary to first transform the ultimate cost amounts by taking the natural log of the natural log of the natural log and then taking the *n*th root before a common distribution could be found. Taking the fifth root of the triple natural log appears to produce a distribution of ultimate costs that conforms well with an extreme value distribution. The fact that such intense transformations were needed suggests that a totally different approach than fitting commonly used distributions should be used.

As is indicated from Table 8.2, the ratio of the expected value of the individual case reserve to the projected payments through expected year of death estimate varies dramatically according to the gender and current age of each claimant. This suggests that the variability of the total MPD reserve can best be modeled by simulating the variability of the future payout for each claim separately. Table 9.1 provides a sample framework for this type of simulation. The sample insurer has 10 open PD claims.

An individual row in Table 9.1 is devoted to each open claim. Census data on the gender and current age of each living PD claimants appears in two columns on the left side of the table. Consider claim number 1 in the top row. Actual medical payments in 2003 were $3,000. A random number between 0 and 1 is generated. If that number is between 0 and q_{75}, the claimant dies during 2004. Recall that q_x denotes the probability of death at age x, given survival to that age. If the random number is greater than q_{75}, the claimant lives throughout 2004.

TABLE 9.1

LAYOUT FOR SIMULATION OF VARIABILITY OF TOTAL MPD RESERVE AT YEAR-END 2003

Claim Number	Gender	Current Age	Projected Annual Medical Costs ($000s) During: 2004	2005	2006	…	2038	2039	2040	…	2068	2069	2070	Total Future Medical Payments
1	F	75	3.2	3.5	3.8		0	0	0		0	0	0	10.5
2	F	47	5.6	6.1	6.7		105	114	125		0	0	0	2,354.8
3	M	22	1.9	2.1	2.3		35.6	38.8	42.2		472	515	0	6,211.4
4	M	46	0.7	0.8	0.8		13.1	14.3	15.6		0	0	0	312.1
5	M	55	12.7	13.8	15.1		0	0	0		0	0	0	181.4
6	F	82	6.3	6.9	7.5		0	0	0		0	0	0	55.2
7	M	66	8.1	8.8	9.6		0	0	0		0	0	0	99.7
8	M	34	1.2	1.3	1.4		22.5	24.5	26.7		0	0	0	443.8
9	F	57	4.4	4.8	5.2		82.4	0	0		0	0	0	949.1
10	M	71	3.6	3.9	4.3		67.4	73.5	80.1		0	0	0	1,468.3
													Total:	**12,086.3**

In effect, in Table 9.1, projected annual medical costs for each future year are estimated via a Markov chain simulation model. The state space consists of two outcomes from each trial: (1) the claimant does not die during a given future DY, or (2) the claimant dies during that DY. The transition probabilities in this model are simply the $(1 - q_x)$ and q_x values from a mortality table. The outcome of any trial depends at most upon the outcome of the immediately preceding trial and not upon any other previous outcome. Death is an "absorbing" state, since one cannot transition out of it.

An assumed rate of medical cost escalation of 9% per year is applied to the prior year's payments if the claimant lives throughout the year. Otherwise, if the claimant dies during the year, projected medical payments for the year are still shown, after which medical losses drop to zero for every future year of development. While projected medical payments may arguably be only for half a year, assuming the average claimant dies in the middle of the final year of development, in reality medical costs are often higher during the year of death. Thus assuming a full year's worth of medical payments is a reasonable assumption.

For each trial, total projected future payments from the cell at the bottom right are recorded and confidence levels for the reserve can be derived from a ranking of all of the simulated total reserve estimates. If this is done for a single claim, the resulting probability distribution closely conforms to that described in the first paragraph of this section.

Simulating the variability of the MPD reserve for unreported claims is naturally more complicated. First, the total number of IBNR claims should be represented by a Poisson (or similar) distribution. Then census data of the age at injury of recent claimants can be used to randomly generate these ages for unreported claimants. Then additional rows can be added to Table 9.1 to further simulate future payments for each unreported claimant. The degree of variability of the MPD reserve for unreported claimants is exceptionally high, because some of those claimants may have been quite young when injured, and because the total expected future payment for workers injured at a young age is dramatically higher than for those injured at an older age. An appreciation for this can be gained by reviewing either Columns (B) or (C) of Table 8.2. For example, the total expected future payout for a female who is 20 at the accident date is $16.7 million, while it is only $3.0 million if she is 40.

The Markov chain method presented in this section is based on the common assumption that the current level incremental severities of each claimant remain constant regardless of the age of the claimant during each future year. Clearly, if the tendency of incremental medical severities to increase with age were incorporated into this method, future medical payments for each trial of the simulation would be higher.

10. CONCLUDING REMARKS

In this paper we have seen that common actuarial methods will tend to underestimate the true MPD loss reserve. This is also the case for typical methods of estimating MPD reserves at higher confidence levels based on commonly used size-of-loss distributions. The need to develop and apply new methods that directly reflect the characteristics of MPD payments is substantial.

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APPENDIX A

THE MUELLER INCREMENTAL TAIL METHOD

The Mueller Incremental Tail method calculates tail factors based on cumulative paid loss development triangles augmented by incremental calendar year payments from older accident years.

The method was described in Section 2 of the paper as consisting of three stages:

1. Incremental age-to-age decay ratios.
2. Anchored decay factors.
3. Tail factors.

This Appendix provides more specifics regarding these stages.

1. *Incremental age-to-age decay ratios*. The first step is to calculate incremental age-to-age decay ratios. With the SAIF data, we can calculate incremental paid at age $n + 1$ to incremental paid at age n ratios for n ranging from 29 to 65 years, using 20-year weighted averages.

   Tables A.1 through A.3 display incremental MPD payments for DYs 29 through 40, 40 through 50, and 50 through 60, respectively.

   Because the underlying data for any individual accident year are volatile, the age-to-age factors were smoothed using centered moving averages. The empirical age-to-age decay factors and smoothed factors are shown in Table A.4.

   The empirical factors are calculated directly from the raw data. The centered average is a simple five-year average based on the empirical factor averaged with the two factors above and the two below. When it was not possible to calculate a five-year average, shorter term centered averages were used.

The weighted average is similar but uses corresponding paid losses as weights. The geometric mean provides another level of smoothing. It is also a five-year centered average, but it is the fifth root of the product of the five weighted average factors.

2. *Anchored decay factors.* After selecting the geometric mean incremental age-to-age factors, they are then anchored to a base year. Table A.5 shows the anchored decay factors using five different anchor years. The anchored decay factors represent incremental payments made in year $n + r$ relative to payments made in the anchor year. These anchored decay factors are calculated as the cumulative product starting with the anchor year and moving up the last column on Table A.4. For example, as shown in Table A.5, payments made in development year 50 are 88.0% greater than the payments made in year 37. The main reason that payments rise over time is because the force of medical cost escalation exceeds the force of mortality until most of the claimants are fairly advanced in age, when the force of mortality becomes stronger than the force of medical cost escalation.

By summing the decay factors from 38 to 65, we get the payments made in age 38 to 65 relative to the payments made in the selected anchor year. The sums of the decay factors are similar to tail factors, but instead of being relative to cumulative payments they are relative to the incremental payments made in a given anchor year.

The cumulative decay factors can be interpreted as follows: Payments made in ages 38 to 65 are 30.071 times the payments made in age 37. Similarly, payments made in ages 38 to 65 are 26.961 times the payments made in age 33.

TABLE A.1

Derivation of Incremental Age-to-Age Decay Ratios for DYs 30 to 40

	Incremental Payments ($000s) During DY X:											
AY	29	30	31	32	33	34	35	36	37	38	39	40
1943												4
1944											1	3
1945										16	14	20
1946									1	11	19	5
1947								0	0	3	1	5
1948							7	3	9	16	6	17
1949						49	27	52	29	12	15	48
1950					16	28	20	26	8	30	16	22
1951				17	16	5	7	6	16	6	2	11
1952			4	3	16	11	9	26	32	16	62	9
1953		32	21	14	17	16	28	11	11	9	31	17
1954	54	43	48	59	80	52	109	44	65	81	59	63
1955	25	16	20	14	13	26	33	8	41	22	12	14
1956	45	66	35	68	44	48	24	68	17	40	36	13
1957	53	57	51	35	21	20	39	60	38	36	79	51
1958	26	30	29	33	23	24	30	14	9	75	7	4
1959	110	138	75	81	81	195	122	161	127	148	116	84
1960	47	89	56	71	107	94	69	46	30	26	89	203
1961	97	97	146	118	140	105	101	109	91	121	81	95
1962	96	80	60	46	55	114	57	23	85	108	64	118

1963	82	239	84	81	82	101	85	47	48	36	56	46
1964	465	177	210	178	28	65	106	34	36	55	168	
1965	143	123	107	191	150	153	53	75	69	93		
(A) Sum × 1st:		1,155	943	994	874	1,056	919	812	762	942	933	847
(B) Sum Prior:		1,241	1,187	947	1,011	890	1,105	926	812	763	865	766
(C) Indicated Decay Ratios:		0.931	0.794	1.050	0.864	1.187	0.832	0.877	0.938	1.235	1.079	1.106
(D) Selected Decay Ratios:		0.930	0.933	0.937	0.937	0.953	0.958	0.980	1.001	1.048	1.063	1.088
PLDF–1.0:	0.025	0.030	0.028	0.026	0.025	0.023	0.023	0.022	0.022	0.023	0.022	0.021
PLDF:	1.025	1.030	1.028	1.026	1.025	1.023	1.023	1.022	1.022	1.023	1.022	1.021
Model:	1.024	1.024	1.024	1.024	1.024	1.024	1.024	1.023	1.023	1.022	1.022	1.021

Notes:
(1) The selected decay ratios were derived in Table A.4. See last column.
(2) The PLDFs for DYs 29–37 were derived in Table A.6. (See Row I).
(3) The (PLDF–1.0)s for ages 38 through 40 were computed as the product of the previous (PLDF–1.0) and the current decay ratio, divided by the prior PLDF.

TABLE A.2

Derivation of Incremental Age-to-Age Decay Ratios for DYs 40 to 50

	Incremental Payments ($000s) During DY X:										
AY	40	41	42	43	44	45	46	47	48	49	50
1933											2
1934										2	0
1935									1	4	14
1936								15	0	3	5
1937							0	4	0	1	0
1938						0	0	1	1	0	2
1939					0	3	0	1	0	0	0
1940				1	2	1	3	0	0	4	0
1941			4	1	1	0	0	0	0	0	1
1942		0	7	8	3	1	16	2	0	2	0
1943	4	1	11	4	3	2	4	1	6	7	8
1944	3	3	6	2	1	3	1	1	0	0	0
1945	20	24	17	14	6	15	(1)	50	73	75	63
1946	5	5	(5)	4	9	4	2	30	31	29	31
1947	5	(2)	0	4	0	32	0	0	0	3	5
1948	17	7	2	1	1	12	0	6	7	14	3
1949	48	42	17	9	39	7	20	41	83	225	116
1950	22	18	43	24	11	165	71	9	2	4	1
1951	11	32	12	13	6	4	23	26	19	10	18
1952	9	48	7	7	170	44	12	1	1	22	1
1953	17	10	7	23	13	18	37	15	43	70	68
1954	63	83	49	67	70	142	67	62	96	101	

1955	14	21	26	28	26	21	67	15	13		
1956	13	9	15	21	35	17	8	33			
1957	51	66	367	116	51	28	94				
1958	4	10	19	32	41	21					
1959	84	93	88	83	87						
1960	203	133	181	230							
1961	95	74	158								
1962	118	105									
1963	46										
Sum × 1st:		782	1,027	691	575	540	424	298	375	574	336
Sum Prior × Last:		806	677	873	462	488	519	330	280	363	475
Indicated Decay Ratios:		0.970	1.517	0.792	1.245	1.107	0.817	0.903	1.339	1.581	0.707
Selected Decay Ratios:		1.098	1.101	1.056	1.054	1.058	1.044	1.031	1.047	1.023	0.946
PLDF–1.0:	0.025	0.027	0.029	0.029	0.030	0.031	0.031	0.031	0.032	0.032	0.029
PLDF:	1.025	1.027	1.029	1.029	1.030	1.031	1.031	1.031	1.032	1.032	1.029
Model:	1.021	1.021	1.020	1.020	1.019	1.019	1.018	1.017	1.017	1.016	1.015

Notes:
(1) The selected decay ratios were derived in Table A.4. See last column.
2) The (PLDF–1.0)s for ages 40 through 50 were computed as the product of the previous (PLDF–1.0) and the current decay ratio, divided by the prior PLDF.

TABLE A.3

Derivation of Incremental Age-to-Age Decay Ratios for DYs 50 to 60

	Incremental Payments During DY X:										
AY	50	51	52	53	54	55	56	57	58	59	60
1926								0	3	0	0
1927							0	0	2	0	0
1928						0	0	0	0	0	15
1929					9	5	1	4	0	0	0
1930				0	0	0	0	0	0	0	0
1931			0	0	0	0	0	0	0	0	0
1932		0	0	0	0	0	0	0	0	0	0
1933	2	0	0	0	0	0	0	0	0	0	0
1934	0	0	0	0	0	0	0	0	0	0	0
1935	14	4	0	1	0	0	0	9	0	0	1
1936	5	2	7	1	0	0	0	0	0	0	4
1937	0	0	15	13	0	0	0	0	0	0	0
1938	2	10	0	3	4	0	0	1	0	0	0
1939	0	0	0	0	0	0	0	0	0	0	0
1940	0	0	0	0	0	0	0	0	0	0	0
1941	1	1	0	1	5	4	10	37	9	0	0
1942	0	0	1	1	0	0	0	0	0	0	0
1943	8	2	7	8	3	1	0	0	10	0	0
1944	0	0	0	0	0	1	3	1	4	2	
1945	63	63	48	43	35	34	71	11	6		
1946	31	32	7	14	23	6	1	2			

1947	5	0	0	4	0	1	0				
1948	3	0	4	0	35	5					
1949	116	5	8	1	0						
1950	1	3		2							
1951	18	44	32								
1952	1	1									
1953	68										
Sum × 1st:		167	129	92	105	57	86	65	34	2	20
Sum Prior × Last:		270	166	97	90	114	52	86	63	28	0
Indicated Decay Ratios:		0.619	0.777	0.948	1.167	0.500	1.654	0.756	0.540	0.071	
Selected Decay Ratios:		0.888	0.868	0.850	0.851	0.919	1.002	1.067	1.151		
PLDF–1.0:	0.029	0.025	0.021	0.018	0.015	0.013	0.013	0.014	0.016		
PLDF:	1.029	1.025	1.021	1.018	1.015	1.013	1.013	1.014	1.016		
Model:	1.015	1.015	1.014	1.013	1.013	1.012	1.011	1.010	1.010		

Notes:
(1) The selected decay ratios were derived in Table A.4. See last column.
(2) The (PLDF–1.0)s for ages 50 through 58 were computed as the product of the previous (PLDF–1.0) and the current decay ratio, divided by the prior PLDF.

TABLE A.4

Calculation of Age-to-Age Decay Factors

Age to Age	Empirical	Centered Average	Weighted Average	Geometric Mean
58+	1.151	1.151	1.151	1.151
57/56	0.744	1.186	1.108	1.067
56/55	1.661	1.046	0.952	1.002
55/54	0.502	1.001	0.918	0.919
54/53	1.171	1.011	0.907	0.851
53/52	0.928	0.801	0.745	0.850
52/51	0.792	0.843	0.756	0.868
51/50	0.610	0.924	0.946	0.888
50/49	0.712	1.008	1.019	0.946
49/48	1.579	1.028	1.016	1.023
48/47	1.345	1.070	1.022	1.047
47/46	0.892	1.149	1.117	1.031
46/45	0.824	1.081	1.063	1.044
45/44	1.107	0.971	0.946	1.058
44/43	1.237	1.096	1.080	1.054
43/42	0.793	1.125	1.093	1.056
42/41	1.516	1.125	1.094	1.101
41/40	0.970	1.093	1.074	1.098
40/39	1.108	1.182	1.169	1.088
39/38	1.079	1.066	1.064	1.063
38/37	1.235	1.047	1.040	1.048
37/36	0.939	0.992	0.977	1.001
36/35	0.877	1.014	0.999	0.980
35/34	0.832	0.940	0.932	0.958
34/33	1.186	0.962	0.954	0.953
33/32	0.864	0.945	0.931	0.937
32/31	1.049	0.965	0.952	0.937
31/30	0.795	0.925	0.916	0.933
30/29	0.930	0.930	0.930	0.930

TABLE A.5

ANCHORED DECAY FACTORS

Year of Development	Anchor Year 37	36	35	34	33
> 57	1.184	1.186	1.162	1.113	1.062
57	1.028	1.030	1.009	0.967	0.922
56	0.964	0.966	0.946	0.907	0.864
55	0.962	0.964	0.944	0.905	0.863
54	1.047	1.049	1.028	0.985	0.939
53	1.231	1.233	1.208	1.158	1.104
52	1.448	1.450	1.421	1.362	1.298
51	1.669	1.671	1.637	1.569	1.496
50	1.880	1.882	1.844	1.768	1.685
49	1.987	1.990	1.950	1.869	1.782
48	1.943	1.946	1.907	1.827	1.742
47	1.856	1.859	1.821	1.746	1.664
46	1.800	1.803	1.766	1.693	1.614
45	1.724	1.727	1.692	1.622	1.546
44	1.630	1.633	1.600	1.533	1.462
43	1.547	1.550	1.518	1.455	1.387
42	1.466	1.468	1.438	1.378	1.314
41	1.331	1.332	1.306	1.251	1.193
40	1.211	1.213	1.189	1.139	1.086
39	1.114	1.116	1.093	1.048	0.999
38	1.048	1.049	1.028	0.985	0.939
37	1.000	1.001	0.981	0.940	0.897
36		1.000	0.980	0.939	0.895
35			1.000	0.958	0.914
34				1.000	0.953
33					1.000
Totals (38 to ultimate)	30.071	30.115	29.508	28.280	26.961
Relative to Anchor Year	37	36	35	34	33

Because this approach produces volatile indicated tail factors, Table A.6 presents an approach for stabilizing those indications (see Table 2.6). Each of the average PLDFs for ages 30 through 36 are adjusted to what they would be for age 37 using the appropriate products of incremental decay factors from AY 1965 and prior years. A weighted average of all of these adjusted PLDFs is then used to replace the actual PLDF for DY 37. In this way, the PLDF for DY 37 is changed from being entirely determined by only one historical PLDF for one AY, to being an indication based on all 36 PLDFs for DYs 30 through 37. This results in a reduction of the PLDF for anchor year 37 from 1.033 to 1.022. The final selected tail factor from age 37 to 65 is then 1 plus the product of 0.022/1.022, the cumulative decay factor of 30.071 and 1/1.022 (= 1.634).

Once the best estimate of the PLDF for the anchor year (DY 37) is selected, then all of the subsequent PLDFs can be easily generated using the iterative formula:

$$f_{n+1} = f_n d_{n+1}/[1 + f_n],$$

where f_n is the paid loss development factor, less one, for the nth year of development, and d_{n+1} is the decay ratio between incremental paid during year $(n+1)$ and year (n). See Section 2 for a derivation of this formula.

3. *Tail factors*. Tail factors can be calculated either by cumulating the age-to-age PLDFs calculated above or directly from the cumulative decay factors D_{n+1} linked to an age-to-age factor f_n from the cumulative triangle using the formula

$$\text{Tail factor}_n = f_n D_{n+1}/[1 + f_n],$$

where D_{n+1} is the cumulative decay factor calculated from the incremental data, and f_n is derived from the normal cumulative triangle. See Section 2.

TABLE A.6

USING THE MUELLER INCREMENTAL TAIL METHOD TO PRODUCE A MORE STABLE ESTIMATE OF THE PLDF FOR ANCHOR YEAR 37

AY	29	30	31	32	33	34	35	36	37
1966	1.015	1.025	1.020	1.017	1.021	1.017	1.026	1.027	1.033
1967	1.019	1.030	1.026	1.026	1.023	1.025	1.025	1.030	
1968	1.013	1.009	1.006	1.004	1.003	1.003	1.004		
1969	1.018	1.017	1.019	1.021	1.013	1.023			
1970	1.017	1.016	1.030	1.013	1.017				
1971	1.014	1.040	1.040	1.026					
1972	1.036	1.021	1.015						
1973	1.042	1.037							
1974	1.025								
(A) Average	1.022	1.024	1.023	1.018	1.015	1.017	1.018	1.029	1.033
(B) Avg. − 1.0	0.022	0.024	0.023	0.018	0.015	0.017	0.018	0.029	0.033
(C) Decay Ratios	0.930	0.933	0.937	0.937	0.953	0.958	0.980	1.001	
(D) Adjustment Factor to Age 37	0.734	0.787	0.840	0.897	0.940	0.981	1.001	1.000	
(E) Row (B) Adjusted to Age 37	0.018	0.018	0.015	0.014	0.016	0.018	0.029	0.033	
(F) Weights for (E)	5%	5%	10%	10%	15%	15%	20%	20%	
(G) Weighted Avg. of (E)								0.022	
(H) Revised (B)	0.030	0.028	0.026	0.025	0.023	0.023	0.022	0.022	
(I) Revised PLDFs	1.030	1.028	1.026	1.025	1.023	1.023	1.022	1.022	

Notes:
(C) From Table A.4, last column.
(D) Product of all decay ratios to the right of given age.
(E) (B) × (D). (H) (G)/(D).

APPENDIX B

HISTORICAL PLDFS FOR ALL OTHER WORKERS COMPENSATION

This section presents SAIF's historical PLDFs for MPD losses as well as workers compensation losses other than MPD. The averages of the latest five PLDFs are shown for each development year in Table B.1. These factors are counterparts to the MPD PLDFs shown in Table 1.1.

The 37 to ultimate tail factor indicated for other workers compensation is 1.039. In Oregon, escalation of indemnity benefits is paid out of a second injury fund. The Other Workers Compensation development factors do not include the escalation of indemnity benefits. The Other than MPD tail factor of 1.039 can be compared to the MPD tail factor of 1.581. These tail factors can be derived from Table B.1 by cumulating backwards.

It is medical losses that contribute significantly to the tail factor and it is the medical cost escalation component of the medical tail factor that contributes significantly to the medical tail factor. Without medical cost escalation, the medical factor drops from 1.581 to 1.030 when put on a current cost basis.

The above PLDFs serve as the basis for the tail factors presented in Table 2.4.

TABLE B.1

A COMPARISON OF HISTORICAL AGE-TO-AGE PAID LOSS DEVELOPMENT FACTORS (BY YEAR OF DEVELOPMENT)

	Year of Development													
	2	3	4	5	6	7	8	9	10	11	12	13	14	15
MPD	6.624	1.525	1.140	1.072	1.041	1.027	1.019	1.020	1.015	1.013	1.012	1.013	1.012	1.010
Other Workers Comp.	1.843	1.131	1.043	1.023	1.018	1.013	1.009	1.006	1.005	1.004	1.004	1.004	1.005	1.006
Total Workers Comp.	2.168	1.213	1.069	1.036	1.025	1.017	1.012	1.010	1.008	1.007	1.006	1.007	1.007	1.007

	Year of Development										
	16	17	18	19	20	21	22	23	24	25	26
MPD	1.011	1.013	1.011	1.011	1.012	1.012	1.014	1.012	1.015	1.015	1.016
Other Workers Comp.	1.006	1.008	1.010	1.009	1.009	1.009	1.008	1.009	1.010	1.010	1.010
Total Workers Comp.	1.008	1.010	1.010	1.010	1.010	1.010	1.010	1.010	1.012	1.011	1.012

	Year of Development										
	27	28	29	30	31	32	33	34	35	36	37
MPD	1.020	1.023	1.027	1.026	1.022	1.018	1.015	1.017	1.018	1.029	1.033
Other Workers Comp.	1.009	1.008	1.008	1.007	1.007	1.006	1.006	1.005	1.004	1.006	1.006
Total Workers Comp.	1.012	1.012	1.013	1.013	1.011	1.010	1.009	1.009	1.009	1.013	1.014

APPENDIX C

INCORPORATING THE STATIC MORTALITY MODEL INTO THE INCREMENTAL PAID TO PRIOR OPEN CLAIM METHOD

SECTION C.1. OVERVIEW

Given the complexity of this method, Table C.1 provides a road map to the key steps involved in the application of the method and the location of tables and narrative describing those steps. The method was originally introduced in Section 3 by presenting Step 7 since this step is easily understood.

Table C.1 lists the key steps of this method in the order in which they were applied, which is not necessarily the order in which they are presented in this appendix.

This Appendix consists of five sections: (1) Overview; (2) Derivation of Number of Open Claims at the End of Each Development Year; (3) Selection of Representative Values of Incremental Paid to Prior Open Claim; (4) Basis of 9% Assumption for Future Rate of Medical Cost Escalation; and (5) Derivation of Assumed Claim Reporting and Closure Patterns.

SECTION C.2. DERIVATION OF NUMBER OF OPEN CLAIMS AT THE END OF EACH DEVELOPMENT YEAR

The first part of this Appendix describes the derivation of the estimated number of PD claimant deaths shown in Column (3) of Table 3.4. Such estimates also directly become the number by which the total number of open claims declines for each development year after the 20th year. After that year, it is assumed that no new claims will be reported and that the number of claim closures for reasons other than death will be cancelled out by the number of reopened claims for each development year.

The survival probabilities for each development year were derived from a claimant mortality model and these were compared

TABLE C.1

GUIDE TO LOCATION OF DESCRIPTION AND DISPLAY OF KEY STEPS OF METHOD

	Step	Appendix C	Section 3 of Main Text
(1)	Select representative historical claim reporting pattern	Section C.5	
(2)	Select representative historical claim closing pattern	Section C.5	
(3)	Derive historical open count pattern by subtracting (2) from (1)	Section C.5	
(4)	Derive projections of number of claims closed due to death	Section C.2	Table 3.4
(5)	Derive assumptions regarding percentage of claims closed for other reasons	Section C.5	Table 3.4
(6)	Synchronize open count estimates of historical experience and mortality model	Section C.5	Table 3.4
(7)	Select appropriate medical inflation assumption	Section C.4	
(8)	Trend historical incremental paid to prior open claim averages to current level	Section C.3	
(9)	Select representative paid severities	Section C.3	
(10)	Trend paid severities to year of payment	Section C.3	Table 3.2
(11)	Estimate incremental payments as the product of trended paid severities and projections of the number of prior open claims		Table 3.2

with the actual probabilities of a claim remaining open throughout each given development year. For each development year under 10, the probability of a claim remaining open during a given development year was substantially less than the survival probability, since most (or many) claims will close for reasons other than death of the claimant. However, these two sets of probabilities converge for increasing development years until they are virtually identical for development years 20 and higher.

Mortality rates were used to derive a claims closure pattern (due to death) by development year in the following way. A two-

TABLE C.2.1

NUMBER OF LIVING MALE CLAIMANTS FOR ACCIDENT YEAR 2002 AT SUCCESSIVE YEAR-ENDS ASSUMING A 2000 MORTALITY TABLE

Age-at-	Beginning of Development Year						
Injury	1	2	3	4	5	10	20
40	12.99	12.96	12.92	12.88	12.83	12.56	11.50
41	14.71	14.66	14.62	14.57	14.51	14.19	12.89
42	16.09	16.04	15.99	15.93	15.87	15.48	13.94
43	16.03	15.97	15.91	15.85	15.78	15.38	13.71
44	17.48	17.41	17.34	17.27	17.19	16.72	14.74
45	18.86	18.79	18.71	18.62	18.53	17.98	15.66
46	20.12	20.03	19.94	19.84	19.74	19.10	16.41
47	21.43	21.34	21.23	21.12	21.01	20.27	17.14
48	22.69	22.58	22.46	22.34	22.20	21.36	17.75
49	23.02	22.90	22.77	22.63	22.49	21.56	17.59
40–49	**183.41**	**182.68**	**181.89**	**181.06**	**180.16**	**174.61**	**154.38**

dimensional array was created, with the age-at-injury down the leftmost column and the development years as column headings.

Table C.2.1 presents a small portion of the array, including only ages-at-injury from 40 through 49 shown at the beginning of the first five development years and at the beginning of the 10th and 20th development years.

Appendix D provides a more detailed description of the array structure. The arrays described in these two appendices differ only in the applicable mortality tables. For the static method, the 2000 mortality table is assumed for all future years. In the trended method (Appendix D), projections of future mortality tables are used.

Table C.2.1 is a segment of the male lives array. We assumed that the initial PD claimant population consisted of 750 males and 250 females. A corresponding array was used for the female claimants.

The first column to the right of the age-at-injury values is a portion of the distribution of 750 male PD claimants by age, based on individual permanent total disability (PTD) claimant data from SAIF for accident years 1975 through 1990. We assumed that the age-at-injury distribution for PD claims would be the same as for PTD claims. The actual census data were smoothed among different age-at-injury categories to derive the numbers in Column (1).

Consider the row for the age-at-injury of 40. Suppose that 12.99 of the 1,000 total claimants were injured at age 40. The probability of living from age 40 to age 41 from the male 2000 SSA mortality table is used to calculate the expected number of male claimants still alive one year after the accident, and so forth for each subsequent age and year of development out to development year 90. In this way each age-at-injury row is filled out in the array. For each development year column, the expected total number of surviving claimants is simply the sum of the expected number of surviving claimants for each age-at-injury ranging from 40 through 49.

The same calculations were performed for all possible ages-at-injury and all development years from 1 through 90. The resulting estimates of the number of surviving male claimants is summarized in Table C.2.2 for different age-at-injury groupings at different selected years of development. The totals derived in Table C.2.1 are displayed in the 40–49 age-at-injury row in Table C.2.2.

In Table C.2.2, the expected number of surviving claimants at the beginning of development year 5 is 722.1 and at development year 10 is 674.4. Hence the probability of survival during the fifth through ninth development years for all male claimants is 93.4%. It is evident from a review of the bottom row of Table C.2.2 that the survival probabilities steadily decline as the claimant population ages.

TABLE C.2.2

NUMBER OF SURVIVING MALE CLAIMANTS AT THE BEGINNING OF VARIOUS DEVELOPMENT YEARS FOR ACCIDENT YEAR 2002

Age-at-	Number of Surviving Male Claimants at the Beginning of Development Year											
Injury	1	5	10	15	20	25	30	40	50	60	70	80
16–29	30.7	30.5	30.2	29.9	29.4	28.8	27.9	24.6	18.0	8.7	1.5	0.0
30–39	78.9	78.2	77.0	75.4	73.0	69.5	64.3	47.3	22.8	4.0	0.1	0.0
40–49	183.4	180.2	174.6	166.5	154.4	137.0	114.3	56.2	10.2	0.3	0.0	0.0
50–59	321.3	309.0	286.9	255.0	213.4	162.7	106.1	19.7	0.6	0.0	0.0	0.0
60+	135.7	124.2	105.6	83.2	58.0	33.0	13.8	0.6	0.0	0.0	0.0	0.0
TOTAL	750.0	722.1	674.4	609.9	528.1	431.0	326.4	148.3	51.7	13.0	1.6	0.0
Survival Probability*		96.3	93.4	90.4	86.6	81.6	75.7	45.4	34.8	25.1	12.3	2.6

*In %.

TABLE C.2.3

INDICATED MALE CLAIMANT SURVIVAL PROBABILITIES

Age-at-	Beginning of Development Year										
Injury	5	10	15	20	25	30	40	50	60	70	80
16–29	99.4%	99.2%	98.9%	98.5%	97.8%	96.9%	88.1%	73.4%	48.0%	17.0%	2.8%
30–39	99.1%	98.5%	97.8%	96.9%	95.2%	92.5%	73.5%	48.2%	17.5%	3.0%	0.2%
40–49	98.2%	96.9%	95.4%	92.7%	88.7%	83.5%	49.1%	18.2%	3.1%	0.2%	0.0%
50–59	96.2%	92.8%	88.9%	83.7%	76.3%	65.2%	18.6%	3.1%	0.2%	0.0%	
60+	91.6%	85.0%	78.8%	69.7%	56.9%	41.8%	4.1%	0.2%	0.0%		

Table C.2.3 displays the survival probabilities for each age-at-injury grouping during each grouping of development years.

Given that survival probabilities vary significantly for different age-at-injury groups, it is clear that the group survival probabilities will be highly sensitive to the distribution of claimants by age-at-injury. The greater the proportion of younger claimants, the bigger the MPD tail.

TABLE C.3.1

INCREMENTAL PAID TO PRIOR OPEN CLAIM AVERAGES TRENDED TO 2003 COST LEVEL (YEARS 1 THROUGH 13)

AY	1	2	3	4	5	6	7	8	9	10	11	12	13
1979													8.43
1980												6.12	6.14
1981											7.30	3.40	2.71
1982										5.62	5.81	2.21	2.81
1983									9.25	4.41	2.81	2.99	2.81
1984								6.15	4.86	3.48	3.44	2.72	2.23
1985							5.77	4.16	4.27	2.47	2.44	3.40	2.07
1986						9.07	5.56	4.84	2.52	4.13	3.27	2.95	2.95
1987					7.54	5.42	3.32	2.09	2.48	1.77	2.44	2.25	2.96
1988				10.27	5.47	5.32	2.58	2.01	2.43	3.12	2.87	2.99	4.31
1989			17.75	8.43	6.35	4.37	2.78	2.70	3.06	3.47	3.30	3.90	5.28
1990		74.81	16.65	7.26	5.41	3.39	2.68	3.42	4.64	2.21	2.64	4.47	4.66
1991		76.98	14.65	4.59	5.24	3.02	1.92	1.74	2.33	2.38	2.82	3.03	
1992		63.73	13.44	5.24	2.94	3.36	3.16	3.03	3.52	5.88	4.61		
1993		70.06	12.53	6.19	3.76	4.44	3.79	3.97	6.94	5.67			
1994		63.18	11.46	3.36	4.03	3.93	2.30	2.24	4.02				
1995		61.45	9.60	7.14	4.56	4.10	5.48	5.39					
1996		63.62	12.44	5.00	4.65	5.39	4.44						
1997		69.86	12.13	7.35	5.31	4.39							
1998		61.95	13.94	5.89	8.70								
1999		62.69	14.60	7.57									
2000		87.44	16.77										
2001		85.81											
2002													
Average		70.13	13.83	6.52	5.33	4.68	3.65	3.48	4.19	3.72	3.64	3.37	3.95
X Hi/Lo		69.27	13.86	6.47	5.23	4.41	3.61	3.39	3.87	3.70	3.40	3.21	3.69
Avg. Last 3		78.65	15.10	6.94	6.22	4.63	4.08	3.87	4.83	4.65	3.35	3.80	4.75
Selected		78.42	15.24	7.06	6.10	4.80	4.50	4.50	4.50	4.50	3.70	3.70	3.70

TABLE C.3.2

INCREMENTAL PAID TO PRIOR OPEN CLAIM AVERAGES TRENDED TO 2003 COST LEVEL (YEARS 14 THROUGH 27)

AY	14	15	16	17	18	19	20	21	22	23	24	25	26	27
1966													3.27	4.29
1967												6.80	5.81	6.96
1968											10.50	11.88	8.88	2.71
1969										4.71	6.75	4.72	5.20	3.18
1970									3.85	2.89	3.50	3.20	2.34	2.70
1971								5.47	4.36	3.03	2.37	2.37	1.95	2.02
1972							3.95	2.89	2.76	2.43	2.70	1.44	1.22	1.79
1973						6.55	4.95	3.84	3.64	3.83	4.23	4.28	4.91	6.26
1974					4.74	3.84	4.04	2.69	3.85	2.14	2.28	3.78	0.95	2.70
1975				5.05	3.84	4.09	4.24	5.63	3.63	3.61	3.96	3.29	3.81	5.18
1976			7.97	4.73	3.12	3.44	2.68	2.68	3.20	2.58	2.36	2.03	2.38	1.74
1977		6.72	5.40	3.22	2.55	2.51	3.23	2.84	2.62	2.88	3.24	2.78	3.90	
1978	5.73	4.12	5.11	2.94	3.50	2.64	3.12	2.89	4.34	4.23	3.68	3.67		
1979	6.66	4.05	4.14	5.43	4.26	3.88	4.62	3.79	4.56	4.06	4.03			
1980	3.18	3.08	2.62	3.22	3.28	2.82	2.38	3.40	3.06	2.18				
1981	3.10	4.62	2.72	3.05	3.19	3.27	3.16	2.72	4.46					
1982	2.42	2.78	2.82	2.84	2.73	3.04	3.94	3.49						
1983	1.91	4.26	2.96	3.52	3.78	4.84	3.21							
1984	2.73	2.94	3.39	4.04	3.19	2.82								
1985	2.25	3.58	3.51	5.73	4.68									
1986	3.39	3.18	4.15	4.60										
1987	3.52	3.25	4.20											
1988	3.50	3.74												
1989	5.76													
Average	3.68	3.86	4.08	4.03	3.57	3.65	3.63	3.53	3.69	3.21	4.13	4.19	3.72	3.60
X Hi/Lo	3.56	3.68	3.84	3.98	3.56	3.47	3.62	3.40	3.72	3.17	3.68	3.69	3.48	3.43
Avg. Last 3	4.26	3.39	3.95	4.79	3.89	3.57	3.44	3.20	4.03	3.49	3.65	2.83	3.36	3.21
Selected	3.70	3.70	3.70	3.70	3.70	3.70	3.70	3.70	3.70	3.70	3.70	3.70	3.70	3.70

SECTION C.3. SELECTION OF REPRESENTATIVE VALUES OF INCREMENTAL PAID TO PRIOR OPEN CLAIM

Historical incremental paid to prior open claim averages were trended to the calendar year 2003 cost level using an assumed annual medical inflation rate of 9% per year. The resultant trended averages are displayed in Tables C.3.1 and C.3.2.

SECTION C.4. BASIS OF 9% ASSUMPTION FOR FUTURE RATE OF MEDICAL COST ESCALATION

Forecasts of future rates of medical cost escalation are based on an analysis of actual medical severity since 1966. Future medical severity is expected to grow on average at the same rate observed over this 38-year period. Internal studies have shown that the best predictor of long-term medical cost escalation is the long-term historical average itself. Short-term medical cost escalation rates are more accurately predicted using shorter-term historical averages.

In this paper we use an expected 9% future medical cost escalation rate. Intuitively, this rate might seem high, especially when compared to the medical component of the CPI (Bureau of Labor Statistics). Table C.4.1 provides a historical comparison of these two measures of change in average medical costs.

SAIF's average rate of medical cost escalation for 1983–1993 was depressed by the effects of significant reform legislation enacted in 1990 and the introduction of managed care into workers compensation. Absent these reforms, SAIF's average difference for 1983–1993 would have been similar in magnitude to the other multiyear periods.

It should be expected that a workers compensation insurer's average rate of medical cost escalation would exceed the aver-

TABLE C.4.1

COMPARISON OF SAIF'S HISTORICAL RATE OF MEDICAL COST ESCALATION WITH AVERAGE CHANGES IN THE MEDICAL COMPONENT OF THE CONSUMER PRICE INDEX

Accident Years	Average Rate of Medical Cost Escalation for Time Loss Claims	Average Rate of Change in Medical Component of the CPI	Average Difference
1966–1973	10.5%	5.7%	4.8%
1973–1983	12.2%	10.0%	2.2%
1983–1993	7.2%	7.2%	0.0%
1993–2003	7.3%	4.0%	3.3%
1966–2003	9.2%	6.8%	2.4%

age rate of change in the medical component of the CPI. The latter measures changes in household expenditures for health insurance premiums, as well as for out-of-pocket medical expenses, whereas the workers compensation medical costs include all medical expenses.

SAIF's rate of medical cost escalation measures the rate of change in all occupational medical costs. The medical cost of workers compensation claims is more difficult for an insurer to control because there are no patient co-pays or deductibles. Workers compensation insurers find it difficult to deny medical benefits when the attending physician deems the service necessary.

As Table C.4.1 shows, the average difference between the rate of change in occupational medical costs and that for consumer medical expenses measured by the medical component of the CPI has been 2.4% per year. That differential for SAIF increased during the most recent years to 7.4%, as documented in Table C.4.2.

TABLE C.4.2

COMPARISON OF SAIF'S RECENT RATES OF MEDICAL COST ESCALATION WITH AVERAGE CHANGES IN THE MEDICAL COMPONENT OF THE CONSUMER PRICE INDEX

Accident Year	Average Rate of Medical Cost Escalation for Time Loss Claims	Average Rate of Change in Medical Component of the CPI	Average Change in Mix and Utilization
1998	9.2%	3.2%	6.0%
1999	5.3%	3.5%	1.8%
2000	18.6%	4.1%	14.5%
2001	13.6%	4.6%	9.0%
2002	12.7%	4.7%	8.0%
2003	9.1%	4.0%	5.0%
1998–2003	11.4%	4.0%	7.4%

Escalation rates for workers compensation medical costs are driven by unit cost inflation, changes in the utilization of services, changes in the relative mix of services across service categories, as well as the substitution of more expensive services for less expensive services within a service category.

The medical cost escalation rate is the change in the cost per claim. The following formulae show one way to decompose the cost per claim into utilization, unit cost, and mix.

Payments are first combined into service categories. Examples of service categories are office visits, pharmacy, physical medicine, surgery, radiology, and so on. *For a particular service category*, the cost per claim can be decomposed into utilization, unit cost and mix.

$$\text{Cost per claim} = \text{Utilization} \times \text{Unit cost} \times \text{Mix},$$

where

$$\text{Utilization} = \frac{\text{Number of services in the service category}}{\begin{array}{c}\text{Number of claims receiving services}\\ \text{in that category}\end{array}}.$$

Utilization measures the number of services per claim for those claims receiving services in that category.

$$\text{Unit cost} = \frac{\text{Paid losses in the service category}}{\text{Number of services in the service category}}.$$

Unit cost measures the average paid loss per service in that service category.

$$\text{Mix} = \frac{\text{Number of claims receiving services in that category}}{\text{Total number of claims receiving any kind of service}}.$$

Mix measures the proportion of claims receiving that service.

If you multiply these three components together you get

$$\text{Cost per claim} = \frac{\text{Paid losses for the service category}}{\begin{array}{c}\text{Total number of claims receiving any kind}\\ \text{of service in that category}\end{array}}.$$

The total cost per claim is then the sum of the cost per claim over all service categories. The 9% medical cost escalation referred to in this paper is the combined effect of utilization, unit cost, and mix on the average cost per claim over time.

SECTION C.5. DERIVATION OF ASSUMED CLAIM REPORTING AND CLOSURE PATTERNS

Tables C.5.1 and C.5.2 disclose the specific assumptions (from SAIF experience) that form the basis for the PLDF static and trended mortality model estimates.

The following assumptions are held constant for all accident years in the model:

- There are 5,000 ultimately reported PD claims.
- A claim-reporting pattern is based on recent historical experience.

TABLE C.5.1

Derivation of Key Assumptions of the Static and Trended Models Accident Year 2002 MPD Losses

	Development Year						
	1	2	3	4	5	6	7
(1) Percentage of Claims Reported	18.52%	74.33%	91.64%	95.93%	97.77%	98.69%	99.08%
(2) Selected Reported Counts	926	3,716	4,582	4,797	4,888	4,935	4,954
(3) Percentage of Reported Still Open	49.65%	41.20%	29.82%	19.78%	13.85%	11.23%	8.00%
(4) Selected Open Counts	460	1,531	1,366	949	677	554	396
(5) Group Survival Probability	0.993	0.992	0.991	0.990	0.990	0.989	0.988
(6) Number Closed by Death	3.46	15.05	17.30	14.09	10.33	7.89	6.88
(7) Total Closed Counts	466	2185	3216	3848	4211	4381	4558
(8) Closed for Other Causes	462.54	1703.95	1013.70	617.91	352.67	162.11	170.12
(9) Newly Reported Counts	926	2,790	866	215	91	47	19
(10) Open+Newly Reported	926	3,250	2,397	1,581	1,040	724	573
(11) Indicated Percentage Closed (Other)	99.26%	52.43%	42.29%	39.08%	33.91%	22.39%	29.69%
(12) Selected Percentage Closed (Other)	99.26%	52.43%	42.29%	39.08%	33.91%	22.39%	29.69%

Notes:
(1) Based on average reported count development factors for the latest 10 CYs.
(2) $5,000 \times (1)$. The constant ultimate claim count of 5,000 was assumed for all years.
(3) Based on the average percentage open for the most recent CYs.
(4) $(2) \times (3)$, for DYs 1–10; [Prior $(4) + (9) - (6) - (8)$], for later DYs.
(5) See Section C.1 of Appendix C.
(6) $[(4) + 0.5 \times (9)] \times [(1) - (5)]$.
(7) $(2) - (4)$.
(8) [Change in (7)] $- (6)$.
(9) Change in (2).
(10) $(4) + (9)$.
(11) $(8)/(10)$.
(12) Selected on the basis of (11). Actual percentages were selected for the first 10 DYs.

TABLE C.5.2

Derivation of Key Assumptions of the Static and Trended Models Accident Year 2002 MPD Losses

	Development Year						
	8	9	10	11	12	13	14
(1) Percentage of Claims Reported	99.30%	99.46%	99.56%	99.64%	99.69%	99.76%	99.81%
(2) Selected Reported Counts	4,965	4,973	4,978	4,982	4,985	4,988	4,991
(3) Percentage of Reported Still Open	6.50%	5.00%	4.20%	3.95%	3.74%	3.56%	3.40%
(4) Selected Open Counts	323	249	209	197	187	178	170
(5) Group Survival Probability	0.987	0.986	0.985	0.983	0.982	0.981	0.979
(6) Number Closed by Death	5.31	4.68	3.89	3.52	3.56	3.63	3.72
(7) Total Closed Counts	4,642	4,724	4,769	4,785	4,798	4,810	4,821
(8) Closed for Other Causes	78.69	77.32	41.11	12.54	9.85	8.39	7.10
(9) Newly Reported Counts	11	8	5	4	3	3	3
(10) Open+Newly Reported	407	331	254	213	200	190	181
(11) Indicated Percentage Closed (Other)	19.33%	23.36%	16.19%	5.89%	4.92%	4.43%	3.93%
(12) Selected Percentage Closed (Other)	19.33%	23.36%	16.19%	6.00%	5.00%	4.50%	4.00%

(See notes on Table C.5.1.)

- Percentages of cumulative reported claims still open at the end of each DY are based on recent historical experience.
- Estimates of PD claims closed by death are based on SSA mortality tables.
- Estimates of PD claims closed for reasons other than death are calculated as total claim closures less expected deaths.

From the above, the percentage of claims available for closure that closed for reasons other than death was derived from AY 2002 for the static mortality model. These percentages were also assumed for the trended mortality model. Consequently, the only thing different between the two models is the expected number of claimant deaths during each DY.

APPENDIX D

INCORPORATING THE TRENDED MORTALITY MODEL INTO THE INCREMENTAL PAID TO PRIOR OPEN CLAIM METHOD

Table C.1 displays each of the steps taken in incorporating the static mortality model into the incremental paid to prior open claim method. The trended mortality method is the same as the static mortality method, except for Step (4), where projections of the number of claims closed due to death are derived. In the trended method, mortality tables forecasted by the SSA for the appropriate future development year are used instead of some fixed historical mortality table. The differences between these tables grows exponentially for development years that are decades into the future. A sample of these differences is disclosed in Table 5.1 of Section 5. These differences are compounded by medical costs that have risen dramatically due to expected high future rates of medical inflation.

The focus of this Appendix is to disclose the specific manner by which a series of 90 different mortality tables were derived and applied to the expected number of surviving claimants by age-at-injury for every future development year. The final result is a slowly evolving and elongating series of claims closure patterns for each accident year out to 90 years of development.

Standard mortality tables for each decade since 1970 and projected tables for each decade through 2080 were obtained from the actuarial publications section of the SSA Web Site www.ssa.gov/OACT/NOTES/pdf_studies/.

The separate male and female tables were combined into one using an assumed 75%/25% male/female mix, the proportion indicated from SAIF's PD claimant census data. The resulting weighted mortality rates were then compiled into an array of expected mortality rates for each age at each future calendar year.

TABLE D.1

SAMPLE $q(x)$ VALUES

	Calendar Year						
Age	1970	1980	1990	2000	2020	2040	2060
20	.00175	.00156	.00130	.00110	.00091	.00078	.00066
35	.00239	.00187	.00217	.00172	.00154	.00130	.00110
50	.00861	.00685	.00556	.00496	.00397	.00330	.00278
65	.02961	.02524	.02206	.01938	.01615	.01371	.01182
80	.09386	.08308	.07604	.07028	.05929	.04976	.04261

Six models of the number of PD claimants who would still be alive at the end of each future development year were derived separately for accident years 1975, 1980, 1985, 1990, 1995, and 2000. Each of these models consists of a separate two-dimensional array, such as presented in Tables C.2.1 of Appendix C.

The first step in deriving these arrays was to compile mortality rates from the SSA tables. Table D.1 displays a sampling of these $q(x)$, or probability of death, values.

Each of the one-year $q(x)$ values was converted into survival rates, denoted $l(x)$, by taking the complement, yielding the ratios in Table D.2.

The entire array of resulting one-year $l(x)$s was then shifted so that the rows of the original array became the diagonals of a new array, that is, each successive column was shifted up one row. After the shift, the $l(x)$s were arranged as shown in Table D.3.

Each row thus has a structure similar to an accident year reporting format, as displayed in Table D.4.

This shift facilitated multiplication of the survival ratios times the preceding number of surviving claimants for each age-at-injury row, working successively from left to right within each age-at-injury row.

TABLE D.2

Sample One-Year $l(x)$ Values

	Calendar Year						
Age	1970	1980	1990	2000	2020	2040	2060
20	.99825	.99844	.99870	.99890	.99909	.99922	.99934
35	.99761	.99813	.99783	.99828	.99846	.99870	.99890
50	.99139	.99315	.99444	.99504	.99603	.9967	.99722
65	.97039	.97476	.97794	.98062	.98385	.98629	.98818
80	.90614	.91692	.92396	.92972	.94071	.95024	.95739

TABLE D.3

Shifted $l(x)$ Array: Age

	Year of Development						
Age at Injury	1	2	3	4	5	6	7
20	21	22	23	24	25	26	27
21	22	23	24	25	26	27	28
22	23	24	25	26	27	28	29
23	24	25	26	27	28	29	30
24	25	26	27	28	29	30	31

Table D.5 provides a side-by-side comparison of parallel calculations of the expected number of surviving claimants at the end of each calendar year for the static and trended mortality methods. The example presented is for claimants who were 50 years old when they were injured (during AY 2002).

In Table D.5 we started with the same number of surviving claimants at the beginning of CY 2031 (100.00). Nevertheless, at the beginning of CY 2035, we would be expecting 73.42 such claimants to still be alive using a 2000 mortality table while 79.30 claimants would be alive using a series of mortality tables corresponding to CYs 2031 through 2034. In this example, we would be expecting 8% more claimants to still be alive at the

TABLE D.4

Calendar Year of Payments and Applicable Mortality Table for Each Accident Year and Development Year

	Year of Development								
AY	1	2	3	4	5	6	7	8	9
1996	1996	1997	1998	1999	2000	2001	2002	2003	2004
1997	1997	1998	1999	2000	2001	2002	2003	2004	2005
1998	1998	1999	2000	2001	2002	2003	2004	2005	2006
1999	1999	2000	2001	2002	2003	2004	2005	2006	2007
2000	2000	2001	2002	2003	2004	2005	2006	2007	2008
2001	2001	2002	2003	2004	2005	2006	2007	2008	2009
2002	2002	2003	2004	2005	2006	2007	2008	2009	2010

TABLE D.5

Comparison of the Estimation of the Number of Living Claimants with Age-at-Injury of 50 for Accident Year 2002 at Successive Year-Ends Under the Static and Trended Mortality Methods

Static Mortality Method					
	Calendar Year				
	2031	2032	2033	2034	2035
Number of Surviving Claimants	100.00	93.63	87.05	80.30	73.42
CY of Mortality Table	2000	2000	2000	2000	2000
Survival Probability	.93633	.92972	.92242	.91439	.90562
Trended Mortality Method					
	Calendar Year				
	2031	2032	2033	2034	2035
Number of Surviving Claimants	100.00	95.12	90.05	84.79	79.30
CY of Mortality Table	2031	2032	2033	2034	2035
Survival Probability	.95121	.94671	.94152	.93526	.92769

beginning of CY 2035 assuming the trended mortality method (versus the static method). Although there is little difference in the survival probabilities shown in Table D.5, these differences become fairly significant during future decades. This can be seen by comparing these rates to those shown in the Group Survival Rate columns of Table 5.1.

APPENDIX E

QUANTIFYING THE ELDER CARE COST BULGE

Table E.1 discloses summarized data behind Figure 7.1. The incremental paid amounts in the second column of Table E.1 have been adjusted to a 2003 cost level assuming a constant 9% per year rate of medical cost escalation. The incremental amounts included in these totals were for accident years from 1945 on, during calendar years 1991 through 2003. These have been totaled for groupings of five successive development years.

The claim counts in the third column of Table E.1 are on a different basis than in the rest of this paper in order to focus only on severity changes for claims where ongoing medical payments are being made. Consequently, these counts only include claims where some medical payment was made during the given calendar year.

The magnitude of the increases in on-level incremental severities for later DYs shown in Figure 1.2 is greater than if the number of prior open counts was used. This is because the percentage of MPD claims for which payment activity occurs tends to decline somewhat for later DYs. This decline indicates that

TABLE E.1

INCREMENTAL PAID SEVERITIES AT 2003 LEVEL

Development Years (DYs)	Incremental Paid ($000s)	Claims with Payment Activity	Incremental Paid Severity
16–20	537,626	99,417	5,408
21–25	406,047	73,876	5,496
26–30	318,881	50,646	6,296
31–35	243,062	29,068	8,362
36–40	129,420	14,486	8,934
41–45	60,487	7,429	8,142
46–50	38,960	3,674	10,604
51–55	22,674	1,919	11,816

mortality rates are higher for those MPD claimants with ongoing covered medical costs. However, the disabled life factors indicated by SAIF's total open counts for later DYs are in the range of 70% to 80%, leaving some room for the actual mortality rates of claimants with ongoing covered medical costs to be close to those of the general populace.

INCORPORATION OF FIXED EXPENSES

GEOFF WERNER

Abstract

When setting rates, actuaries must include all of the costs of doing business, including underwriting expenses. Actuaries generally divide the underwriting expenses into two groups: fixed and variable. This paper addresses the incorporation of fixed expenses in the calculation of the actuarial indication. More specifically, the paper describes how the generally accepted method for including fixed expenses overstates or understates the actuarial indication. The materiality of the distortion depends on the magnitude of past rate changes, premium trend, and variations in average premiums for multistate companies. Finally, the paper suggests an alternative procedure that addresses the distortions.

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1. INTRODUCTION

The role of a pricing actuary is to set rates that provide for the expected future amount of all costs associated with the transfer of risk [2]. Historically, actuarial literature has focused either on the larger costs of doing business (e.g., losses) or the more complex topics (e.g., profit provisions). Thus, there is relatively little literature dealing with the treatment of underwriting expenses.

Actuaries generally divide underwriting expenses into two groups: fixed and variable. Fixed expenses are those expenses that are assumed to be the same for each exposure, regardless of the size of the premium (i.e., the expense is a constant dollar amount for each risk). Typically, overhead costs associated with the home office are considered a fixed expense.[1] Variable expenses are those expenses that vary directly with premium; in other words, the expense is a constant percentage of the premium. Premium taxes and commissions are two good examples of variable expenses.

This paper discusses the often-overlooked portion of the premium, the fixed expenses. Specifically, the paper addresses:

- The generally accepted method for calculating a fixed expense provision and including it within the overall statewide rate level indication;
- Potential distortions that may make the current methodology misstate the actuarial indication; and
- An alternative procedure for calculating and incorporating a fixed expense provision.

2. CURRENT METHOD

Calculation of Projected Fixed and Variable Expense Provision

A review of filings from several insurers of property/casualty personal lines confirms that most actuaries use a method similar to the one outlined by Schofield [4] to calculate a fixed expense provision and expense fee. Basically, the procedure assumes that historical expense ratios (i.e., historical expenses divided by historical premiums) are the best estimate of projected expenses.

The first step of Schofield's procedure is to determine the percentage of premium attributable to expenses for each of

[1]It is likely that some of these expenses bear some relationship to risk and may vary at least slightly with premium. Activity-based cost studies may be able to verify the true relationship, and appropriate adjustments can be made.

the expense categories. To accomplish this, actuaries generally relate historical expenses to either written or earned premium for that same historical experience period. The choice of premium depends on whether the actuary believes the expenses are generally incurred at the onset of the policy or throughout the policy. Written premium is used in the former case and earned premium is used in the latter case. Once the appropriate ratios are determined for each type of expense, the ratios are then split into a fixed expense ratio and a variable expense ratio based on all available expense data, regulations, and judgment.

Exhibit 1 demonstrates this using homeowners data adjusted so that the three-year historical expense ratios (expenses divided by premiums) are approximately equal to the three-year industry historical expense ratios.

Exhibit 1-A displays three years of historical expense ratios. The underlying data can be obtained from the Insurance Expense Exhibit (IEE) and Statutory Page 14, although they may not have the finest level of detail desired. For example, the homeowners data include data on renters and mobile homes. Ideally, the actuary can access and use the source expense data to get the data corresponding to the product being priced. Of course, the actuary should always balance the additional cost of obtaining such data against the additional accuracy gained.

In this case, the company assumes that all expenses, except general expenses, are incurred at the onset of the policy and divides them by the written premium. General expenses are assumed to be incurred throughout the policy period and thus are divided by the earned premium.

Typically, the data used (countrywide or state) also vary by type of expense. Other acquisition and general expenses are usually assumed to be uniform across all locations and hence can be handled using countrywide figures that can be found in the IEE. Handling of commission and brokerage expenses varies from carrier to carrier, with some carriers using state-specific data and others using countrywide figures. The treatment should be based on the variation of the company's commission plans by location. Taxes, licenses, and fees vary by state, therefore they are typically based on state data from the applicable Statutory Page 14. Ideally, the company can break the category into taxes, which are a variable expense, and licenses and fees, which are typically treated as fixed expenses.[2]

The following chart summarizes these expense characteristics:

Type of Expense	Data Used	Divided By
General Expense	Countrywide	Earned Premium
Other Acquisition	Countrywide	Written Premium
Commission and Brokerage	Countrywide/State	Written Premium
Taxes	State	Written Premium
Licenses and Fees	State	Written Premium

Once the historical ratios are calculated, the actuary chooses a selected provision for each expense type. Generally, the selection is based on either the latest year or a multiyear average; however, there are several things that may affect the selection:

- If the actuary is aware of a future change in the expenses, the new figure should be used. For example, if the commission structure is changing, the actuary should use the expected commission percentage, not the historical percentage.
- If there was a one-time shift in expense levels during the experience period, the expected future expense level should be used. For example, if productivity gains led to a significant reduction in necessary staffing levels during the historical experience period, then the selected ratios should be based on the ratios after the reduction rather than the all-year average.

[2]Licenses and fees tend to be a smaller portion of the overall taxes, licenses, and fees category. Thus, if a company is unable to separate them, the inclusion of these with variable expenses will not have a material effect.

- If there were nonrecurring expense items during the historical period, the actuary should examine the materiality and nature of the expense to determine how to best incorporate the expense in the rates—if at all. If the aggregate dollars spent are consistent with dollars spent on similar non-recurring projects in other years, the expense ratios should be similar and no adjustment is warranted. If, however, the expense item represents an extraordinary expense, then the actuary must decide to what extent it should be included. Assume, for example, the extraordinary expense is from a major systems project to improve the policy issuance process. That project clearly benefits future policyholders and should be included in the rates. Assuming the new system will be used for a significant length of time, it may be appropriate to dampen the impact of the item and spread the expense over a period of several years. If the actuary consistently selects the three-year average, the expense will automatically be spread over three years, assuming rates are revised annually.[3] On the other hand, the actuary may determine that it is inappropriate to charge future policyholders for a given nonrecurring expense. If so, the actuary should exclude the expense from the ratemaking data altogether. In that case, the expense is basically funded by existing surplus.

- Finally, a few states place restrictions on which expenses can be included for the purpose of determining rates. For example, Texas does not allow insurers to include charitable contributions or lobbying expenses. These expenses must be excluded from the calculation of the historical expense ratios when performing the analysis for that state. If such expenses are recurring, overall future income will be reduced by that state's proportion of the expenses.

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[3]This assumes all of the expense is booked in that year. Statutory accounting guidelines allow some expenses to be amortized over several years. If the extraordinary expense is amortized over three years, then the use of a three-year average will actually spread the expense over five years. The three-year average expense ratio will increase for the first three years and decrease for the last two years.

In the example in Exhibit 1-A, the data are fairly stable and there are no extraordinary expenses; therefore, a three-year straight average is selected.

Once the expense ratios are selected, they are divided into fixed and variable ratios. Ideally, the company has detailed expense data and can do this directly or has activity-based cost studies that help split the expenses appropriately. In the absence of any such data, the actuary should consult with other insurance professionals within the company to arrive at the best possible assumptions given the company's allocation of expenses. In this example, the company assumes that 75% of the general expenses and other acquisition costs and 100% of the licenses and fees are fixed. All other expenses are assumed to be variable. Some sensitivity testing was performed on these selections. For the example included, the difference in the indications between assuming that the aforementioned percentage of the general expenses, other acquisition costs, and licenses and fees were fixed and assuming that 100% of those expenses were fixed is not material. The exact impact will vary and depend on the magnitude of the expense ratios.

The fixed expense ratio represents the fixed expenses for the historical time period divided by the premium written or earned during that same time period. Often, companies trend this ratio to account for expected growth in fixed expenses. Some companies use internal expense data to select an appropriate trend. Given the volatility of internal data, many companies use government indices (e.g., Consumer Price Index, Employment Cost Index, etc.) and knowledge of anticipated changes in internal company practices to estimate an appropriate trend. Exhibit 1-B displays one such methodology. Basically, the indicated trend is a weighting of the Employment Cost Index and the Consumer Price Index. The weights are based on the percentage that salaries represent of the total expenditures for the two largest fixed expense categories, other acquisition and general expenses. These weights can be determined directly from data contained in the IEE. The selected fixed expense ratio will be trended from the average date expenses were incurred in the historical expense period to the average date expenses will be incurred in the period the rates are assumed to be in effect (see Appendix A).[4] After making this adjustment, the ratio is often called the projected (or trended) fixed expense provision.

Variable expenses and profit are a constant percentage of the premium. This selected percentage will apply to the premiums from policies written during the time the rates will be in effect. Thus, there is no need to trend this ratio, called the variable expense provision.

Calculation of Statewide Indicated Rate Change

Exhibit 1-C shows the most commonly found method of incorporating the fixed expense provision within the calculation of the indicated statewide rate level change. The general formula for the statewide (SW) indicated rate change based on the loss ratio method is as follows:

$$\text{SW Indicated Rate Change} = \frac{\text{Projected Loss Ratio} + \text{Projected Fixed Expense Provision}}{1.00 - \text{Variable Expense Provision} - \text{Profit \& Contingency Provision}} - 1.00.$$

The projected fixed expense provision and the variable expense provision are calculated as discussed in the prior section. Much literature is dedicated to the determination of loss ratios

[4]When multiyear historical ratios are used, there is often no trending to bring each year's ratio to the same expense and premium levels before making a selection. If the expenses and average premiums are changing at the same rate, then the two would offset each other and the ratios would remain constant. However, if the expense trend exceeds the change in average premium (or the change in average premium exceeds the expense trend), this would tend to result in increasing (decreasing) ratios over the historical period. The materiality of this distortion depends on the magnitude of the difference between the trends.

and profit and contingency provisions; thus, they will not be discussed further here.

3. POTENTIAL DISTORTIONS

There are a few items that can cause the preceding methodology to create inaccurate and inequitable indicated rate changes.

First, rate changes[5] can impact the historical expense ratios and lead to an excessive or inadequate overall rate indication. The historical fixed expense ratios are based on written and earned premiums during the historical time period. To the extent that there are rate increases (or decreases) that impact only a portion of the premium in the historical time period or were implemented after the historical period entirely, the current procedure will tend to overstate (or understate) the expected fixed expenses. The materiality of the distortion depends on the magnitude of rate changes not fully reflected in the historical countrywide premiums. Also, utilizing three-year historical expense ratios increases the historical experience period, thereby increasing the chances of rate changes not being fully reflected in the historical premiums. One potential solution for the distortion caused by rate changes is to restate the historical written or earned premiums at current rates i.e., premiums level.

Second, a significant premium trend between the historical experience period and the projected period can lead to an excessive or inadequate overall rate indication.[6] Again, the historical

[5]The term "rate changes" (or premium level changes) is intended to refer to changes resulting from an increase or decrease in the premiums. The term is not intended to be used interchangeably with "rate level changes," which can be caused by premium changes, coverage changes, or both. If a rate level change is caused solely by a change in coverage, it may or may not impact the appropriateness of the historical expense ratios. If the actuary adjusts the losses to account for coverage level changes, there will not be a distortion. If, however, the actuary adjusts premiums to account for such changes, the distortion will still exist.

[6]This assumes that the premium trend is due to changes that do not proportionately increase (or decrease) the fixed expenses. While this is the most common scenario, there may be situations that deviate from this assumption. For example, assume a company is

expenses are divided by the written and earned premium during the historical time period. To the extent that there have been distributional shifts that have increased the average premium (e.g., higher amounts of insurance) or decreased the average premium (e.g., higher deductibles), this methodology will tend to overstate or understate the estimated fixed expenses, respectively. The magnitude of overstatement or understatement depends on the magnitude of the premium trend. Utilizing three-year historical expense ratios increases the impact of a premium trend by increasing the amount of time between the historical and projected periods. A potential solution for this is to trend the historical premiums to prospective levels.

Third, this methodology can create inequitable rates for regional or nationwide carriers because it uses countrywide expense ratios[7] and applies them to state projected premiums to determine the expected fixed expenses. In other words, fixed expenses are allocated to each state based on premium. The average premium level in states can vary because of overall loss cost differences (e.g., coastal states tend to have higher overall homeowners loss costs) as well as distributional differences (e.g., some states have significantly higher average amounts of insurance than other states). If there exists significant variation in average rates across the states, a disproportionate share of projected fixed expenses will be allocated to the higher-than-average premium states. Thus, the estimated fixed expenses will be overstated in higher-than-average premium states and understated in lower-than-average premium states. If a company tracks fixed expenses by state and calculates fixed expense ratios for each state, then this distortion will not exist.

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pursuing an insurance-to-value (ITV) effort with an external inspection company. Presumably, the additional expenses incurred will lead to better ITV and higher average premiums. Thus, both average premiums and average expenses would be increasing. In a case like this, the actuary should determine the impact, decide if this is a one-time shift, and adjust the selections accordingly.

[7] State-specific data are usually used for taxes, licenses, and fees. However, these expenses are relatively small compared with the expenses that are generally evaluated on a countrywide level.

4. PROPOSED METHODOLOGY

By assumption, fixed expenses are assumed to be constant for each exposure and are not assumed to vary with the premium. The proposed methodology uses the concepts outlined by Childs and Currie [1]. In essence, historical fixed expenses are divided by historical exposures rather than by premium. Exhibit 2 displays this procedure.

Calculation of Projected Fixed and Variable Expense Provisions

Exhibit 2-A, Sheet 1 shows the development of the fixed and variable expenses for the general expense category. The total expenses for the category can be taken directly from the IEE. The total expenses are split into variable and fixed expenses. Ideally, the expenses are maintained at a level of detail that allows an accurate allocation between the variable and fixed expense categories. Typically, the total expenses are split using percentages based on internal company data and actuarial judgment. This example uses the same percentages assumed in the current procedure (75% of general expenses and other acquisition costs and 100% of licenses and fees are fixed, and all other expenses are variable).[8]

The total fixed expenses are then divided by the exposures[9] for the same time period. As general expenses are assumed to be incurred throughout the policy, the expense dollars are divided by earned exposures, rather than written, to determine an average expense per exposure for the historical period. The average expense figures are trended using the same approach discussed earlier in the paper (see Exhibit 1-B). All of the

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[8] If premiums and expenses are changing at different rates, then the ratio of fixed expenses to total expenses will change over time, but that does not result in a material distortion. See Appendix B for more discussion on this issue.

[9] House-years were used as the exposure unit for the example in the paper. Using amount-of-insurance years as an exposure base will lead to distortions similar to those caused by the current procedure if there are significant differences in amounts of insurance over time and among various locations.

average expense amounts are trended from the average date they were incurred in the historical period to the average date expenses will be incurred in the period the rates will be in effect.[10] Once the projected expenses per exposures are determined, the actuary then must select an appropriate figure.

As with the current procedure, the selection will generally be based on either the latest year or a multiyear average. Consistent values for the projected average expense per exposure imply that expenses are increasing or decreasing proportionately to exposures. This makes intuitive sense for many expense categories (e.g., full-time employee costs), but may not be accurate for all fixed expenses because of economies of scale. If the company is growing and the projected average expense per exposure is declining steadily each year, the selected expense trend may be too high or expenses may not be increasing as quickly as exposures because of economies of scale. If the decline is significant and the actuary believes it is because of economies of scale, then the selection should be adjusted to include the impact of economies of scale corresponding to expected growth in the book.[11] As mentioned earlier, nonrecurring expense items, one-time changes in expense levels, or anticipated changes in expenses should be considered in making the selection. In the example shown the figures are stable and the three-year average is selected to facilitate comparisons with the results of the current procedure.

Exhibit 2-A, Sheets 1–4 show the calculations for each of the major expense categories. The following chart summarizes the

[10]In the example, the same trend period is used for all expense categories to maintain consistency with the current procedure. See Appendix A for more discussion on this issue.

[11]If the selected expense trend is based on historical internal expense data (e.g., historical changes in average expense per exposure) rather than external indices, then the trend would implicitly include the impact of economies of scale in the past. Assuming the impact of economies of scale will be the same as in the past, the projected average expense per exposure should be consistent and no further adjustment would be necessary.

characteristics of the data used:

Expense	Data Used	Divided By Fixed	Divided By Variable
General Expense	Countrywide	Earned Exposures	Earned Premium
Other Acquisition	Countrywide	Written Exposures	Written Premium
Commissions and Brokerage	Countrywide/State	—	Written Premium
Taxes	State	—	Written Premium
Licenses and Fees	State	Written Exposures	—

Exhibit 2-B summarizes the results of the analysis of the fixed and variable portions of each major expense group.

Calculation of Statewide Indicated Rate Change

The most straightforward way to calculate the indicated rate change is displayed on Exhibit 2-C. The statewide required projected average premium is calculated as follows:

$$\text{SW Projected Average Required Premium} = \frac{\text{SW Projected Average Loss \& LAE Per Exposure} + \text{Projected Average Fixed Expense Per Exposure}}{1.00 - \text{Variable Expense Provision} - \text{Profit \& Contingency Provision}}$$

This figure is compared to the statewide projected average premium at present rates to determine the statewide indicated rate change:

$$\text{SW Indicated Rate Change} = \frac{\text{SW Projected Average Required Premium}}{\text{SW Projected Average Premium at Present Rates}} - 1.00.$$

Alternatively, the projected average fixed expense per exposure can be converted to a projected fixed expense provision by dividing the projected average fixed expense per exposure by the statewide projected average premium at present rates. This figure can then be used within the same formula to indicate loss ratio provided earlier:

$$\text{SW Indicated Rate Change} = \frac{\text{Projected Loss Ratio} + \text{Projected Fixed Expense Provision}}{1.00 - \text{Variable Expense Provision} - \text{Profit \& Contingency Provision}} - 1.00.$$

Calculation of Expense Fees

Some insurers may have expense fees or minimum premiums. If that is the case, this procedure directly lends itself to the determination of such values.

Exhibit 2-D displays the necessary calculations for an expense fee. The projected average fixed expense per exposure has already been calculated. To calculate an expense fee, that figure needs to be increased to cover the variable items (variable expenses and profit) associated with the fixed portion of the premium. This is accomplished simply by dividing the fixed expense per exposure by the variable permissible loss ratio (i.e., 1.00 minus variable expense provision minus profit provision).

To determine a minimum premium, the expense fee should be combined with a minimum provision for losses.

5. CURRENT METHODOLOGY VERSUS PROPOSED METHODOLOGY

This section algebraically shows the difference in the projected fixed expense dollars calculated under the two different methodologies. The formula for calculating the total dollars of projected statewide fixed expenses using the current methodology is as follows:[12]

$$\text{Proj SW Fixed Expenses}_{\text{Curr}} = \frac{\text{Historical CW Fixed Expenses}}{\text{Historical CW Premium}} * \text{Expense Trend Factor} * \text{Proj SW Premium}.$$

The formula for calculating the projected statewide fixed expenses collected using the proposed methodology is as follows:

$$\text{Proj SW Fixed Expenses}_{\text{Prop}} = \frac{\text{Historical CW Fixed Expenses}}{\text{Historical CW Exposures}} * \text{Expense Trend Factor} * \text{Proj SW Exposures}.$$

Dividing the first formula by the second highlights the relative difference between the fixed expenses produced by the two procedures:

$$\frac{\text{Proj SW Fixed Expenses}_{\text{Curr}}}{\text{Proj SW Fixed Expenses}_{\text{Prop}}} = \frac{\text{Historical CW Exposures}}{\text{Historical CW Premium}} * \frac{\text{Proj SW Premium}}{\text{Proj SW Exposures}}.$$

Equivalently,

$$\frac{\text{Proj SW Fixed Expenses}_{\text{Curr}}}{\text{Proj SW Fixed Expenses}_{\text{Prop}}} = \frac{\text{Proj SW Avg Premium}}{\text{Historical CW Avg Premium}}.$$

Multiplying by unity (i.e., Proj CW Avg Premium/Proj CW Avg Premium),

$$\frac{\text{Proj SW Fixed Expenses}_{\text{Curr}}}{\text{Proj SW Fixed Expenses}_{\text{Prop}}} = \frac{\text{Proj SW Avg Premium}}{\text{Historical CW Avg Premium}} * \frac{\text{Proj CW Avg Premium}}{\text{Proj CW Avg Premium}}.$$

[12]The following section only deals with the categories of expenses that use the countrywide (CW) expenses. Taxes, licenses, and fees are not addressed. Those expenses represent a relatively small portion of the total expense dollars.

Rearranging the terms,

$$\frac{\text{Proj SW Fixed Expenses}_{\text{Curr}}}{\text{Proj SW Fixed Expenses}_{\text{Prop}}} = \frac{\text{Proj CW Avg Premium}}{\text{Historical CW Avg Premium}} * \frac{\text{Proj SW Avg Premium}}{\text{Proj CW Avg Premium}}.$$

Since

$$\text{Proj CW Avg Premium} = \text{Historical CW Avg Premium} * \text{Premium Trend Factor} * \text{On-Level Factor},$$

we have

$$\frac{\text{Proj SW Fixed Expenses}_{\text{Curr}}}{\text{Proj SW Fixed Expenses}_{\text{Prop}}} = \text{Premium Trend Factor} * \text{On-Level Factor} * \frac{\text{Proj SW Avg Premium}}{\text{Proj CW Avg Premium}}.$$

The difference between the fixed expenses produced by the two methodologies is driven by premium trend, on-level factors, and the relationship of the statewide average premium to the countrywide average premium. These are the three distortions in the current methodology mentioned earlier. Thus, the proposed methodology is not affected by these three distortions.

Exhibit 3 shows the impact on the overall indication by location for the two methodologies (Exhibit 3-A lists the information in table form and Exhibit 3-B displays the data graphically). This information is included to show two items: the total amount the current procedure overstates (understates) the overall indication relative to the proposed procedure and the variation of the overstatement (understatement) by location. The former tells us about the impact on the accuracy of the overall countrywide indication, while the latter is more indicative the potential for inequity between states.

An examination of the "countrywide" line on Exhibit 3-A shows the current procedure overstates the premium needed to cover projected fixed expenses by +1.8 percentage points relative to the proposed procedure. During the historical period used, homeowners insurance rates were being increased and the overall premium trend was slightly positive. For these two reasons, the proposed procedure results in a fixed expense provision that is less than that produced by the current procedure.

A survey of the impact by location shows significant variation (from a high of +10.6 percentage points to a low of −8.3 percentage points). The location-specific differences are driven by the differences in average projected premiums at present rates (PPR). The average projected PPR can vary significantly from location to location due to the overall cost of doing business in the states as well as to differing distributions of insureds with high and low risk in the states. The relationship of each state's average projected PPR to the countrywide average projected PPR is included. In general, the higher the average projected PPR, the more the current procedure overstates the indication relative to the proposed procedure.

As mentioned earlier, the expense ratios in the example approximate the homeowners industry three-year expense ratios. The impacts will be larger (or smaller) for an individual company that has greater (or lesser) fixed expenses than the industry average. Additionally, the results depend on the rate changes, premium trends, and statewide rate relativities underlying the data.

6. OTHER CONSIDERATIONS AND FUTURE ENHANCEMENTS

While the procedure does correct for the three distortions mentioned, there are still some concerns that are not addressed.

First, the proposed procedure, like the current procedure, requires the actuary to separate the expenses into fixed and variable categories. Today, this is generally done judgmentally. Perhaps future activity-based cost studies will more accurately segregate expenses. As mentioned earlier, sensitivity testing revealed that the overall indication is not materially impacted by moderate swings in the categorization of expenses.

Second, the proposed procedure essentially allocates countrywide fixed expenses to each state based on the by-state exposure distribution (as it assumes fixed expenses do not vary by exposure). In reality, average fixed expense levels may vary by location (e.g., advertising costs may be higher in some locations than others). If a regional or national carrier feels the variation is material, the company should collect data at a finer level and make the appropriate adjustments. Once again, the cost of the data collection should be balanced against the additional accuracy gained.

Third, some expenses considered fixed probably vary slightly with premium. For example, policies for coastal homes may be more costly to service than other homes. Further studies may uncover a more accurate quantification of this relationship. However, assuming the expenses are "nearly" fixed, the resulting inequity is not material.

Fourth, some expenses considered fixed vary by other characteristics. For example, fixed expenses may vary between new and renewal business. This only affects the overall statewide indication if the distribution of risks for a given characteristic is changing dramatically or varies significantly by state. Even if there is no impact on the overall indication, any material fixed expense cost difference not reflected in the rates will have an impact on the equity of the two groups. To make rates equitable for the example of new versus renewal business, material differences in new and renewal provisions should be reflected with consideration given to varying persistency levels as described by Feldblum [3].

Finally, the existence of economies of scale in a changing book will lead to increasing or decreasing figures for projected average expense per exposure. Further studies may reveal techniques for better approximating the relationship between changes in exposures and expenses and capturing the impact of economies of scale. Until then, internal expense trend data and actuarial judgment should suffice for incorporating the impact of economies of scale.

7. CONCLUSION

The prevailing methodology for incorporating fixed expenses in the statewide indication has some methodological flaws. These flaws can lead to overstated or understated actuarial indications. While this paper describes a simple alternative that corrects the three weaknesses discussed, there are still improvements that can be made.

REFERENCES

[1] Childs, Dianna, and Ross Currie, "Expense Allocation in Insurance Ratemaking," Casualty Actuarial Society *Discussion Paper Program*, May 1980, pp. 32–60.

[2] Committee on Ratemaking Principles, Statement of Principles Regarding Property and Casualty Insurance Ratemaking, Casualty Actuarial Society, May 1988.

[3] Feldblum, Sholom, "Asset Share Pricing for Property and Casualty Insurance," Casualty Actuarial Society *Forum*, Special Edition, 1993, pp. 241–312.

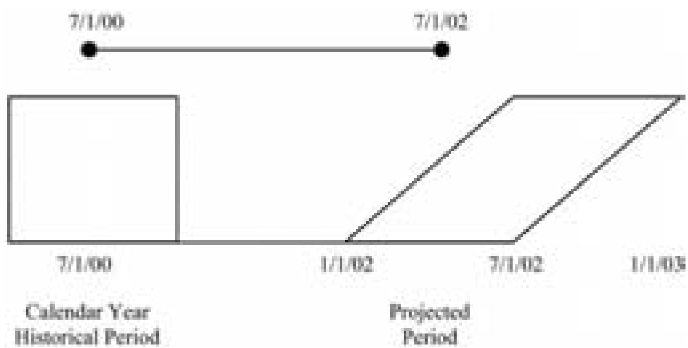
[4] Schofield, David, "Going from a Pure Premium to a Rate," Casualty Actuarial Society Study Note, 1998.

APPENDIX A

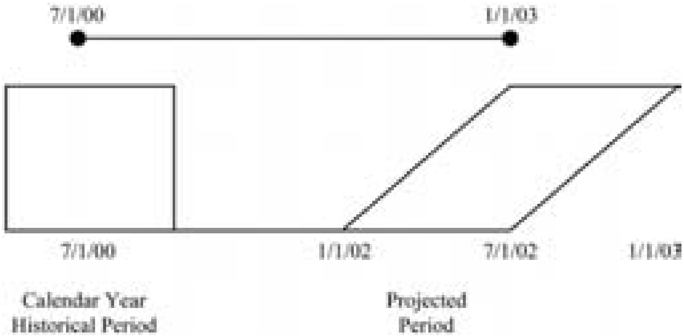
TRENDING PERIODS

Expenses should be trended from the average date they were incurred in the historical period to the average date they will be incurred in the projected period. Actuaries generally make the simplifying assumption that expenses are either incurred at the inception of the policy or are incurred evenly throughout the policy period. When using calendar year historical expense data, the trend periods should be different for the two different types of expenses.

First, expenses that are incurred at the inception of the policy should be trended from the average written date in the historical period to the average written date in the projection period. The following figure shows the resulting trend period assuming annual policies, a steady book of business, and that the projected rates will be in effect for one year:



Second, expenses that are incurred evenly throughout the policy period should be trended from the average earned date in the historical period to the average earned date in the projection period. The following figure shows the resulting trend period assuming annual policies, a steady book of business, and that the projected rates will be in effect for one year:



As can be seen by the figures, under our assumptions, expenses incurred throughout the policy are trended 6 months longer than expenses incurred at inception. Indications do not generally include different trend periods for the different expenses. Presumably, a common trend period is used for simplicity, as this distinction does not have a material impact. The exhibits in the paper use a common trend period.

APPENDIX B

DOES THE PERCENTAGE THAT FIXED EXPENSES REPRESENT OF TOTAL EXPENSES VARY OVER TIME?

In both the current and proposed procedures, the actuary must separate the expenses into fixed and variable expenses. Since detailed expense data may not be available, the actuary may have to use a judgmentally selected percentage to split the expenses into these two categories.

Generally, that same percentage is applied to the expenses for each of the years in the historical period. If the change in the average premium does not equal the fixed expense trend, then fixed and variable expenses will grow at different rates. Thus, the percentages that fixed expenses and variable expenses represent of total expenses will change over time.

A sensitivity analysis was performed to determine the impact on the indications of a change in the distribution of fixed and variable expenses. For the sensitivity analysis, the same example as given in the text of the paper was used with the assumption the percentage was accurately determined in year 1. Even with the very unlikely assumption that average premiums subsequently changed at a rate in excess of +10 percentage points differently than expenses, the indicated rate change increased by only about +0.2 percentage points. In reality, premiums and expenses would likely be changing at a more equivalent rate. Thus, it is reasonable to assume that fixed expenses will remain a constant percentage of total expenses throughout a three-year period.

EXHIBIT 1-A

COMPANY
STATE XX HOMEOWNERS
PROJECTED FIXED AND VARIABLE EXPENSE PROVISIONS: CURRENT METHOD

	Year 1	Year 2	Year 3	3-Year Average	Selected
(1) General Expenses					
a. Countrywide Expenses	\$26,531,974	\$28,702,771	\$31,195,169		
b. Countrywide Earned Premium	\$450,000,000	\$490,950,000	\$545,250,000		
c. Ratio [(*a*)/(*b*)]	5.9%	5.8%	5.7%	5.8%	5.8%
d. % Assumed Fixed					75.0%
e. Fixed Expense % [(*c*) × (*d*)]					4.4%
f. Variable Expense % [(*c*) × (1.0 − (*d*))]					1.5%
(2) Other Acquisition Expenses					
a. Countrywide Expenses	\$41,758,296	\$45,612,462	\$49,582,543		
b. Countrywide Written Premium	\$468,850,000	\$515,550,000	\$577,900,000		
c. Ratio [(*a*)/(*b*)]	8.9%	8.8%	8.6%	8.8%	8.8%
d. % Assumed Fixed					75.0%
e. Fixed Expense % [(*c*) × (*d*)]					6.6%
f. Variable Expense % [(*c*) × (1.0 − (*d*))]					2.2%
(3) Licenses and Fees					
a. State Expenses*	\$1,157,006	\$1,210,200	\$1,321,419		
b. State Written Premium*	\$468,850,000	\$515,550,000	\$577,900,000		
c. Ratio [(a)/(b)]	0.2%	0.2%	0.2%	0.2%	0.2%
d. % Assumed Fixed					100.0%
e. Fixed Expense % [(*c*) × (*d*)]					0.2%
f. Variable Expense % [(*c*) × (1.0 − (*d*))]					0.0%

EXHIBIT 1-A

Continued

	Year 1	Year 2	Year 3	3-Year Average	Selected
(4) Commission and Brokerage Expenses					
a. Countrywide Expenses	$63,507,320	$69,832,993	$78,278,512		
b. Countrywide Written Premium	$468,850,000	$515,550,000	$577,900,000		
c. Ratio [(a)/(b)]	13.5%	13.5%	13.5%	13.5%	13.5%
d. % Assumed Fixed					0.0%
e. Fixed Expense % $[(c) \times (d)]$					0.0%
f. Variable Expense % $[(c) \times (1.0 - (d))]$					13.5%
(5) Taxes					
a. State Expenses*	$10,607,226	$9,917,093	$11,580,187		
b. State Written Premium*	$468,850,000	$515,550,000	$577,900,000		
c. Ratio $[(a)/(b)]$	2.3%	1.9%	2.0%	2.1%	2.1%
d. % Assumed Fixed					0.0%
e. Fixed Expense % $[(c) \times (d)]$					0.0%
f. Variable Expense % $[(c) \times (1.0 - (d))]$					2.1%
(6) Subtotal Fixed Expenses	$[(1e) + (2e) + (3e) + (4e) + (5e)]$				11.2%
(7) Fixed Expense Trend	[Exhibit 1-B]				3.4%
(8) Trend Period					3.00
(9) Fixed Expense Trend Factor	$[(1.00 + (7))^{(8)}]$				1.1055
(10) Projected Fixed Expense Provision	$[(6) \times (9)]$				12.4%
(11) Variable Expense Provision	$[(1f) + (2f) + (3f) + (4f) + (5f)]$				19.3%

*Countrywide data were used for the example.

EXHIBIT 1-B

COMPANY
COUNTRYWIDE HOMEOWNERS
CALCULATION OF ANNUAL EXPENSE TREND

(1)	Employment Cost Index—Finance, Insurance, and Real Estate, excluding Sales Occupations— (annual change over latest 2 years) U.S. Department of Labor	4.8%
(2)	% of Other Acquisition and General Expenses used for Salaries and Employee Relations and Welfare— Insurance Expense Exhibit, Year 3	50.0%
(3)	Consumer Price Index, All Items— (annual change over latest 2 years)	1.9%
(4)	Annual Expense Trend— $\{(1) * (2)\} + \{(3) * [100\% - (2)]\}$	3.4%
	Selected Annual Expense Trend	3.4%

EXHIBIT 1-C

COMPANY
STATE XX HOMEOWNERS
CALCULATION OF INDICATED RATE CHANGE: CURRENT METHOD

(1)	Projected Loss and LAE Ratio	64.7%
(2)	Projected Fixed Expense Provision	12.4%
(3)	Variable Expense Provision	19.3%
(4)	Profit and Contingencies Provision	5.0%
(5)	Variable Permissible Loss Ratio $[100\% - (3) - (4)]$	75.7%
(6)	Indicated Rate Change $\{[(1) + (2)]/(5) - 100\%\}$	1.8%

EXHIBIT 2-A
SHEET 1

COMPANY
STATE XX HOMEOWNERS
GENERAL EXPENSES:
PROPOSED METHOD

	Year 1	Year 2	Year 3	3-Year Straight Average	Selected
(1) Total Countrywide General Expenses (IEE)	$26,531,974	$28,702,771	$31,195,169		
FIXED					
(2) Fixed General Expense as % of Total General Expense	75.0%	75.0%	75.0%		
(3) Fixed General Expense $	$19,898,981	$21,527,078	$23,396,377		
(4) Total Countrywide Earned Exposures	625,500	666,500	696,000		
(5) Average Fixed General Expense Per Exposure	$31.81	$32.30	$33.62		
(6) Expense Trend	3.4%	3.4%	3.4%		
(7) Trend Period	4.00	3.00	2.00		
(8) Expense Trend Factor	1.1431	1.1055	1.0692		
(9) Projected Average Fixed General Expense Per Exposure	$36.36	$35.71	$35.95	$36.01	$36.01

VARIABLE					
(10) Variable General Expense as % of Total General Expense	25.0%	25.0%	25.0%		
(11) Variable General Expense $	$6,632,994	$7,175,693	$7,798,792		
(12) Countrywide Earned Premium	$450,000,000	$490,950,000	$545,250,000		
(13) Variable General Expense %	1.5%	1.5%	1.4%	1.5%	1.5%

Notes:
(3) = (1) × (2).
(5) = (3)/(4).
(6) Exhibit 1-B.
(7) 6/30/XX to midpoint of anticipated coverage period.
(8) = $[1.00 + (6)]^{(7)}$.
(9) = (5) × (8).
(10) = 100% − (2).
(11) = (1) × (10).
(13) = (11)/(12).

EXHIBIT 2-A
Sheet 2

Company
State XX Homeowners
Other Acquisition (Acq.) Expenses:
Proposed Method

	Year 1	Year 2	Year 3	3-Year Straight Average	Selected
(1) Total Countrywide Other Acq. Expenses (IEE)	$41,758,296	$45,612,462	$49,582,543		
FIXED					
(2) Fixed Other Acq. Expense as % of Total Other Acq. Expense	75.0%	75.0%	75.0%		
(3) Fixed Other Acq. Expense $	$31,318,722	$34,209,347	$37,186,907		
(4) Total Countrywide Written Exposures	646,500	687,000	717,500		
(5) Average Fixed Other Acq. Expense Per Exposure	$48.44	$49.80	$51.83		
(6) Expense Trend	3.4%	3.4%	3.4%		
(7) Trend Period	4.00	3.00	2.00		
(8) Expense Trend Factor	1.1431	1.1055	1.0692		
(9) Projected Average Fixed Other Acq. Expense Per Exposure	$55.37	$55.05	$55.42	$55.28	$55.28

VARIABLE					
(10) Variable Other Acq. Expense as % of Total Other Acq. Expense	25.0%	25.0%	25.0%		
(11) Variable Other Acq. Expense $	$10,439,574	$11,403,116	$12,395,636		
(12) Countrywide Written Premium	$468,850,000	$515,550,000	$577,900,000		
(13) Variable Other Acq. Expense %	2.2%	2.2%	2.1%	2.2%	2.2%

Notes:
(3) = (1) × (2).
(5) = (3)/(4).
(6) Exhibit 1-B.
(7) 6/30/XX to midpoint of anticipated coverage period.
(8) = $[1.00 + (6)]^{(7)}$.
(9) = (5) × (8).
(10) = 100% − (2).
(11) = (1) × (10).
(13) = (11)/(12).

EXHIBIT 2-A
SHEET 3

COMPANY
STATE XX HOMEOWNERS
TAXES, LICENSES AND FEES:
PROPOSED METHOD

	Year 1	Year 2	Year 3	3-Year Straight Average	Selected
(1) Total State Taxes, Licenses and Fees (AS, Page 15)*	$11,764,232	$11,127,293	$12,901,606		
FIXED					
(2) Fixed Licenses & Fees Expense $	$1,157,006	$1,210,200	$1,321,419		
(3) Total State Written Exposures*	646,750	687,000	717,650		
(4) Average Fixed Licenses and Fees Expense Per Exposure	$1.79	$1.76	$1.84		
(5) Expense Trend	3.4%	3.4%	3.4%		
(6) Trend Period	4.00	3.00	2.00		
(7) Expense Trend Factor	1.1431	1.1055	1.0692		
(8) Projected Average Fixed Licenses & Fees Expense Per Exposure	$2.05	$1.95	$1.97	$1.99	$1.99

VARIABLE					
(9) Variable Premium Tax Expense $	$10,607,226	$9,917,093	$11,580,187		
(10) State Written Premium*	$468,850,000	$515,550,000	$577,900,000		
(11) Variable Premium Tax Expense %	2.3%	1.9%	2.0%	2.1%	2.1%

Notes:
(4) = (2)/(3).
(5) Exhibit 1-B.
(6) 6/30/XX to midpoint of anticipated coverage period.
(7) = $[1.00 + (5)]^{(6)}$.
(8) = (4) × (7).
(11) = (9)/(10).

*Countrywide data were used for the example.

EXHIBIT 2-A
SHEET 4

COMPANY
STATE XX HOMEOWNERS
COMMISSION AND BROKERAGE EXPENSES: PROPOSED METHOD

	Year 1	Year 2	Year 3	3-Year Straight Average	Selected
(1) Total Countrywide Commission and Brokerage Expenses (IEE)	$63,507,320	$69,832,993	$78,278,512		
FIXED					
(2) Fixed Commission & Brokerage Expense as % of Total Commission and Brokerage Expense	0.0%	0.0%	0.0%		
(3) Fixed Commission and Brokerage Expense $	$—	$—	$—		
(4) Total Countrywide Written Exposures	646,500	687,000	717,500		
(5) Average Fixed Commission and Brokerage Expense Per Exposure	$—	$—	$—		
(6) Expense Trend	3.4%	3.4%	3.4%		
(7) Trend Period	4.00	3.00	2.00		
(8) Expense Trend Factor	1.1431	1.1055	1.0692		
(9) Projected Average Fixed Commission and Brokerage Expense Per Exposure	$—	$—	$—	$—	$—

VARIABLE					
(10) Variable Commission and Brokerage Expense as % of Total Commission and Brokerage Expense	100.0%	100.0%	100.0%		
(11) Variable Commission and Brokerage Expense $	$63,507,320	$69,832,993	$78,278,512		
(12) Countrywide Written Premium	$468,850,000	$515,550,000	$577,900,000		
(13) Variable Commission and Brokerage Expense %	13.5%	13.5%	13.5%	13.5%	13.5%

Notes:
(3) = (1) × (2).
(5) = (3)/(4).
(6) Exhibit 1-B.
(7) 6/30/XX to midpoint of anticipated coverage period.
(8) = $[1.00 + (6)]^{(7)}$.
(9) = (5) × (8).
(10) = 100% − (2).
(11) = (1) × (10).
(13) = (11)/(12).

EXHIBIT 2-B

Company
State XX Homeowners
Projected Fixed and Variable Expense Provisions: Proposed Method

	Fixed	Variable
(1) General Expenses	$36.01	1.5%
(2) Other Acquisition Expenses	$55.28	2.2%
(3) Taxes, Licenses and Fees	$1.99	2.1%
(4) Commission and Brokerage Expenses	$—	13.5%
(5) Total	$93.28	19.3%

EXHIBIT 2-C

Company
State XX Homeowners
Calculation of Indicated Rate Change: Proposed Method

(1) Statewide Projected Average Premium at Present Rates*	$850.59
(2) Statewide Projected Loss and LAE Ratio	64.7%
(3) Statewide Projected Average Loss and LAE [(1) × (2)]	$550.33
(4) Projected Average Fixed Expense Per Exposure	$93.28
(5) Variable Expense Provision	19.3%
(6) Profit and Contingencies Provision	5.0%
(7) Variable Permissible Loss Ratio [100% – (5) – (6)]	75.7%
(8) Statewide Projected Average Required Premium [(3) + (4)]/(7)	$850.21
(9) Indicated Rate Change (8)/(1) – 100%	0.0%

*Countrywide data were used in the example.

EXHIBIT 2-D

Company
State XX Homeowners
Calculation of Proposed Expense Fee: Proposed Method

(1) Total Projected Average Fixed Expense Per Exposure	$93.28
(2) Variable Expense Provision	19.3%
(3) Profit and Contingencies Provision	5.0%
(4) Proposed Expense Fee [(1)]/[100% – (2) – (3)]	$123.22

EXHIBIT 3-A

Comparison of Results

Location	(1) Average Projected Premium at Present Rates Relativity	(2) Indication* Current Methodology	(3) Indication* Proposed Methodology	(4) Current Indication Proposed (2)–(3)
1	2.53	1.7%	–8.9%	10.6%
2	1.44	1.3%	–4.9%	6.2%
3	1.44	1.7%	–4.5%	6.2%
4	1.31	1.8%	–3.4%	5.2%
5	1.31	1.3%	–3.8%	5.1%
6	1.27	1.7%	–3.2%	4.9%
7	1.23	1.7%	–2.8%	4.5%
8	1.22	1.2%	–3.2%	4.4%
9	1.13	1.8%	–1.7%	3.5%
10	1.12	1.3%	–2.0%	3.3%
11	1.11	1.5%	–1.7%	3.2%
12	1.06	1.7%	–0.9%	2.6%
13	1.05	1.6%	–0.9%	2.5%
14	1.03	1.7%	–0.7%	2.4%
15	1.01	1.7%	–0.3%	2.0%
16	0.95	0.5%	–0.5%	1.0%
17	0.91	1.2%	0.8%	0.4%
18	0.91	1.2%	0.8%	0.4%
19	0.87	0.7%	1.1%	–0.4%
20	0.86	1.7%	2.2%	–0.5%
21	0.85	1.7%	2.4%	–0.7%
22	0.85	1.7%	2.4%	–0.7%
23	0.85	1.3%	2.1%	–0.8%
24	0.82	2.1%	3.3%	–1.2%
25	0.82	1.7%	2.9%	–1.2%
26	0.81	1.5%	2.9%	–1.4%
27	0.80	0.5%	2.4%	–1.9%
28	0.80	1.1%	2.9%	–1.8%
29	0.80	1.7%	3.6%	–1.9%
30	0.75	1.5%	4.4%	–2.9%

EXHIBIT 3-A

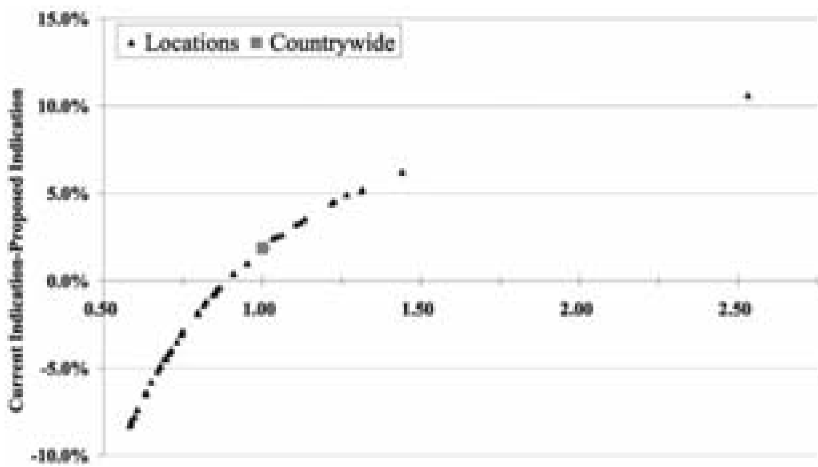
Comparison of Results

Location	(1) Average Projected Premium at Present Rates Relativity	(2) Indication* Current Methodology	(3) Indication* Proposed Methodology	(4) Current Indication Proposed (2)–(3)
31	0.75	1.7%	4.8%	−3.1%
32	0.75	1.5%	4.5%	−3.0%
33	0.75	1.6%	4.6%	−3.0%
34	0.74	1.7%	4.8%	−3.1%
35	0.73	1.7%	5.2%	−3.5%
36	0.71	1.3%	5.3%	−4.0%
37	0.71	1.8%	5.9%	−4.1%
38	0.70	1.6%	5.8%	−4.2%
39	0.70	0.8%	5.3%	−4.5%
40	0.69	1.7%	6.2%	−4.5%
41	0.68	1.6%	6.5%	−4.9%
42	0.67	1.2%	6.2%	−5.0%
43	0.67	1.7%	6.9%	−5.2%
44	0.65	1.7%	7.5%	−5.8%
45	0.63	1.7%	8.2%	−6.5%
46	0.63	1.8%	8.2%	−6.4%
47	0.61	1.6%	9.0%	−7.4%
48	0.60	1.2%	9.0%	−7.8%
49	0.59	2.0%	10.0%	−8.0%
50	0.59	1.7%	9.9%	−8.2%
51	0.58	1.6%	9.9%	−8.3%
Countrywide	1.00	1.8%	0.0%	1.8%

*Loss ratio set at 64.7% to make countrywide indication equal 0% for proposed methodology.

EXHIBIT 3-B

Comparison of Current and Proposed Methods



DISCUSSION OF PAPER PUBLISHED IN VOLUME LXXXIV

APPLICATION OF THE OPTION MARKET PARADIGM TO THE SOLUTION OF INSURANCE PROBLEMS

MICHAEL G. WACEK

DISCUSSION BY STEPHEN J. MILDENHALL

RESPONSE BY THE AUTHOR

In his 2000 discussion [1] of my 1997 paper [2], Stephen Mildenhall chided me for overstating the similarity between options and insurance. He accepted the main point of the paper; namely, that the close resemblance between call option and excess of loss concepts can lead to insights about insurance and reinsurance risk management and product development. However, at a detailed level he dismissed my assertion that "the pricing mathematics is basically the same" for options and insurance, politely describing it as "inappropriate." He was correct in doing so. Unfortunately, in emphasizing the difference in the details of the pricing of call options and excess insurance, he missed the opportunity to show how these differences *can* be explained within a single pricing framework, though different from the one I originally presented. The purpose of this response is first to acknowledge my error at the formula level, but then to move beyond it to illustrate how Black-Scholes and excess insurance pricing *are* consistent, even if the pricing formula details are different.

Mea Culpa

I recognize that I overreached in claiming that my Formula (1.3) is a general formula for European call option pricing, which, I said, reduces to the Black-Scholes Formula (1.1) under the right conditions. It does reduce to Formula (1.1) when the underlying asset's price distribution at expiry is lognormal and the expected annualized continuous rate of return on the asset, μ, equals r, the annualized continuous risk-free rate. But that hardly represents the general case. Most of the time Formula (1.1) produces a different value from that produced by Formula (1.3).

To illustrate this point, consider a call option on a stock currently priced at $P_0 = \$100$ that gives the holder the right to buy the stock at a price of $S = \$100$ at option expiry in 20 days ($t = 20/365$). Assume the stock's expected annualized return and volatility are $\mu = 13\%$ and $\sigma = 25\%$, respectively. If the stock price movements follow geometric Brownian motion through time, the stock price distribution at option expiry is lognormal with a mean of $P_0 e^{\mu t} = (100)(e^{(.13)(20/365)}) = \$100.7149$ and a coefficient of variation (c.v.) of 5.857%. The expected expiry value of the option, given correctly by Formula (1.2), is \$2.7174. If all the Black-Scholes conditions are present, and the annualized risk-free rate $r = 5\%$, then the correct price of the option is given by Formula (1.1) as \$2.4705. In contrast, the Formula (1.3) *pure* premium is \$2.7100.[1] Clearly, my contention that Formula (1.3) is a "general formula for European call option pricing" is not only "inappropriate," it is wrong.

Bear in mind that while (1.3) is not a *general* formula for pricing a European call option, it *is* correct in some circumstances. For example, suppose the same lognormal distribution we just used to describe the stock price distribution at option expiry describes a distribution of aggregate insurance claims. Since the Black-Scholes conditions are not present, Formula (1.1) cannot be used to price a call option (more commonly called an "aggregate excess" or "stop-loss" cover in insurance circles) on the

[1]Generally, a risk charge needs to be added to convert the (1.3) value to a premium. The Black-Scholes value does not require an additional risk charge.

aggregate claims. Instead, actuarial ratemaking theory tells us to use Formula (1.3).

Same Paradigm, Different Details

That the same liability at expiry can give rise to different premiums, each of which is appropriate in its own context, is a paradox. It is clear that the premium is not a function solely of the liability. Mildenhall attributes the pricing difference to the different risk management paradigms operative in the financial and insurance markets: financial risks are hedged, whereas insurance risks are diversified. Yet it *is* possible to bring these two apparently distinct pricing paradigms together within a single framework. While it is possible to do so by reference to martingale measures and incomplete markets theory (see, for example, Moller [3], [4]), my aim is to make this subject as accessible as possible to practicing actuaries who may not be familiar with those concepts. Accordingly, I present the common framework as the more tangible and familiar one of asset-liability matching. *Within that framework the price for the transfer of a liability is a function of both the liability and its optimal matching assets.*

Before we search for the optimal asset strategy, let us explore the nature of the option liability. If the stock price at expiry is represented by a lognormally distributed[2] random variable, x, the expected value at expiry of the payoff obligation of a European call option is given by

$$\begin{aligned} \mathrm{E}(\mathrm{call}_t) &= \int_S^\infty (x-S)f(x)dx \\ &= \mathrm{E}(x)\cdot N(d_1^{(\mu)}) - S\cdot N(d_2^{(\mu)}) \\ &= P_0 e^{\mu t}\cdot N(d_1^{(\mu)}) - S\cdot N(d_2^{(\mu)}), \qquad (1) \end{aligned}$$

[2]If the stock price moves through time in accordance with geometric Brownian motion, the distribution of prices at expiry is lognormal. Note, however, that while Brownian motion is sufficient for lognormality, it is not necessary.

where $N(z)$ is the cumulative distribution function (c.d.f.) of the standard normal distribution, and

$$d_1^{(\mu)} = \frac{\ln(P_0/S) + (\mu + 0.5\sigma^2)t}{\sigma\sqrt{t}} \qquad \text{and}$$

$$d_2^{(\mu)} = \frac{\ln(P_0/S) + (\mu - 0.5\sigma^2)t}{\sigma\sqrt{t}} = d_1^{(\mu)} - \sigma\sqrt{t}.$$

The first term in Formula (1) is the expected market value of the assets to be sold by the call option grantor to the option holder at expiry. The second term is the expected value of the sale proceeds from that transaction.

The variance of the call payoff obligation at expiry is given by

$$\begin{aligned}\text{Var}(\text{call}_t) &= \int_S^{\infty} (x - S)^2 f(x)dx - \text{E}(\text{call}_t)^2 \\ &= \text{E}(x^2) \cdot N(d_0^{(\mu)}) - 2S \cdot \text{E}(x) \cdot N(d_1^{(\mu)}) \\ &\quad + S^2 \cdot N(d_2^{(\mu)}) - \text{E}(\text{call}_t)^2, \end{aligned} \qquad (2)$$

where $N(z)$ is the c.d.f. of the standard normal distribution, and $d_1^{(\mu)}$ and $d_2^{(\mu)}$ are defined as in Formula (1) and $d_0^{(\mu)} = d_1^{(\mu)} + \sigma\sqrt{t}$.

Returning to the example of the 20-day call option with $P_0 = S = \$100$, $\mu = 13\%$, $r = 5\%$ and $\sigma = 25\%$, the expected payoff liability at expiry associated with that option is \$2.7174. That amount is the difference between the expected market value of the stock the grantor of the option will sell to the option holder (\$56.4009), given by the first term of Formula (1), and the expected value of his sale proceeds (\$53.6835), which is given by the second term of Formula (1). The variance, given by Formula (2), is \$14.4456, implying a standard deviation of \$3.8007.

We will illustrate the pricing of this expected payoff liability of \$2.7174 in various available asset scenarios. The premium that the market can be expected to ask for assuming this liability depends on the optimal strategy available for investment of the premium to fund the liability. We will assume that enough investors or traders will find and execute the optimal strategy to force the asking price[3] in the market to be no greater than the level indicated by this strategy. (This is the standard "no arbitrage" requirement.)

This market premium is equal to the minimum expected present value cost of acquiring sufficient assets to fund the expected value liability at expiry and a risk charge related to the undiversifiable variability of the net result. If the variance of the net result can be forced to zero, as it can be when Black-Scholes conditions are present, then the risk charge is zero and the premium is simply equal to the minimum cost of acquiring the assets to fund the liability.

Case A—Underlying Asset is Tradable

The traditional actuarial approach to valuing the liability, embodied in Formula (1.3), is to assume the matching assets are invested in risk-free Treasuries.[4] However, where the liability arises from an option on a traded stock, it is easy to improve on this approach. Since the expected value of the stock to be transferred to the option holder at expiry is $P_0 e^{\mu t} \cdot N(d_1^{(\mu)})$, the option seller can match this expected liability by buying $N(d_1^{(\mu)})$ shares of stock at inception and holding them to expiry. He can fund most of the cost of the purchase, $P_0 \cdot N(d_1^{(\mu)})$, by borrowing against his expected sale proceeds at expiry of $S \cdot N(d_2^{(\mu)})$. Assuming he can borrow at the risk-free rate, he can raise $Se^{-rt} \cdot N(d_2^{(\mu)})$ in this way. That leaves him short of the $P_0 \cdot N(d_1^{(\mu)})$ he needs to buy the shares by $P_0 \cdot N(d_1^{(\mu)}) - Se^{-rt} \cdot N(d_2^{(\mu)})$, which is the amount he should ask for the option, before consideration of a risk charge. This indicates a formula for the premium before

[3]We will focus on the seller's asking price. The question of whether there are buyers at this asking price is beyond the scope of this discussion.

[4]Throughout this paper Treasuries are treated as risk-free assets and their yield as the risk free rate. If other assets meet that definition, they may be substituted for Treasuries.

risk charge (λ) of

$$\text{call}_0 - \lambda = P_0 \cdot N(d_1^{(\mu)}) - Se^{-rt} \cdot N(d_2^{(\mu)}). \qquad (3)$$

In the case of the 20-day option we have been following, he would buy 0.560005 shares at a total cost of \$56.0005, borrow \$53.5366, and charge an option premium before risk charge of \$2.4639. This is a much lower pure premium than the \$2.7100 given by the traditional actuarial Formula (1.3). Moreover, despite the investment of assets in the stock, an ostensibly riskier strategy, the option seller faces less risk (as measured by the standard deviation of the net result) than he would if he invested in risk-free Treasuries. The standard deviation of the option seller's net result is \$1.7527, which is much lower than the \$3.8007 that arises from the Treasuries investment strategy.[5] (For the details of the standard deviation calculation, see Appendix A.) Clearly, this strategy of investing the assets in the stock underlying the option is superior to investing them in Treasuries, since it produces a lower pure premium and a lower standard deviation, which together imply a lower risk-adjusted price.

However, as Black and Scholes proved, this strategy, while better than Treasuries, does not represent the optimal one. Assume the option is on the stock of a publicly traded company whose shares trade in accordance with the Black-Scholes assumptions; i.e., the price follows geometric Brownian motion through time, the shares are continuously tradable at zero transaction costs, etc. Black and Scholes showed that, under these conditions, the optimal investment strategy is one of dynamic asset-liability matching conducted in continuous time.

To execute this strategy, at inception the option seller buys n_0 shares of the underlying stock,[6] financed by a loan of L_0 and call premium proceeds of $P_0 \cdot n_0 - L_0$. Then, an instant later, he ad-

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[5]Since the true values of μ and σ are unknown, there is parameter as well as process risk that needs to be taken into account in setting the risk charge for both asset strategies.

[6]Where n_0 is the first derivative of the call option price with respect to the stock price.

justs the number of shares he holds (to n_1) to reflect any change in the stock price and the infinitesimal passage of time. He adjusts the loan accordingly (to L_1). If n_0 and L_0 have been chosen correctly and the time interval is short enough, the gain or loss in his net position (i.e., the value of the net stock position less the value of the option) is effectively zero. The mean and variance of his net result is also zero. He repeats this adjustment procedure continuously until the option expires. In this way he ends up with exactly the right amount of stock at expiry to generate the funds to meet the option liability and repay the outstanding loan. Provided the sequences of n_i and L_i have been chosen correctly, the cumulative net result and its variance are both zero. Since the variance is zero, there is no justification for a risk charge. Black and Scholes proved that $n_0 = N(d_1)$ and $L_0 = Se^{-rt} \cdot N(d_2)$ and thus that

$$\text{call}_0 = P_0 \cdot N(d_1) - Se^{-rt} \cdot N(d_2), \tag{1.1}$$

where $N(z)$ is the c.d.f. of the standard normal distribution and

$$d_1 = \frac{\ln(P_0/S) + (r + 0.5\sigma^2)t}{\sigma\sqrt{t}} \qquad \text{and}$$

$$d_2 = \frac{\ln(P_0/S) + (r - 0.5\sigma^2)t}{\sigma\sqrt{t}} = d_1 - \sigma\sqrt{t}.$$

Since (1.1) does not depend on μ, the option seller engaging in the hedging strategy underlying the formula not only faces no process risk but also no μ-related parameter risk. (There is still parameter risk associated with σ.) In our example, Formula (1.1) indicates a call premium of \$2.4705.

In the highly liquid, efficient market in which execution of this dynamic hedging strategy is possible, arbitrageurs will force the market's "ask" price of the option to \$2.4705. If the option seller seeks a higher price, he will find no buyers, since another trader can and will undercut him without assuming any additional risk, simply by executing the hedging strategy. Note, however, that the option seller cannot afford to sell the option for \$2.4705 without assuming risk, unless he engages in the Black-Scholes hedging strategy that underpins this price.

Clearly, in order to engage in the kind of hedging activity described above, it is necessary that the stock be continuously tradable at zero transaction costs. The less liquid the market for the stock and the greater the trading costs, the less accurate Formula (1.1) will be in predicting the market asking price of the call. This is because the option seller will have to assume either residual volatility exposure requiring a risk charge (see Esipov and Guo [5]) or expenses not contemplated by Formula (1.1).

For example, if the mix of assets held by the option seller to hedge the 20-day option is adjusted on a daily basis, then the expected present value funding cost (excluding trading costs) is \$2.4708. The standard deviation is \$0.4405. Daily rebalancing is not sufficient to force the funding cost to the Black-Scholes predicted value of \$2.4705 and the standard deviation to zero. In the real world, where transaction costs are not zero, the trade-off between further reducing residual volatility and the cost of doing so will be valued by the market, often resulting in some deviation from the price predicted by Black-Scholes.

Case B—Underlying Asset is not Tradable

Suppose the call option is on the stock of a private company that will go public in 20 days time. Assume there is no "when issued" or forward market for this stock prior to the IPO. The stock is valued today at P_0 = \$100. The other parameters are the same as in Case A: S = \$100, $\mu = 13\%$, $r = 5\%$ and $\sigma = 25\%$. How should an option seller price this option? The key question is how to invest the call premium to fund the expected payoff obligation at option expiry. Since the option seller cannot invest in the underlying stock, it seems a good strategy would be to invest in Treasuries, which has the virtue of not increasing the variance of the net result.[7] If he does so, he needs to collect an option premium of \$2.7100 to fund the expected obligation of \$2.7174, plus a risk charge to compensate for the variability of the net result. The standard deviation of the net result, given $\mu = 13\%$, is \$3.8007. Note that the option seller does not know the true value of μ, 13% being merely an estimate. This means that there is parameter risk in addition to the process risk of \$3.8007.

Note that this scenario is identical to that faced by the excess insurer writing a stop-loss cover attaching at \$100 on an insurance portfolio in which aggregate claims notified and payable in 20 days time are lognormally distributed with mean \$100.7149 and coefficient of variation 5.857%. Conventional ratemaking theory prescribes investment in Treasuries, which indicates a premium of \$2.7100 to fund the expected claims of \$2.7174, plus a risk charge.

As plausible as this Treasury oriented investment strategy is, it is not necessarily the optimal one. If there are no assets available for investment that are correlated with the liability, then the conventional Treasury strategy is optimal. Otherwise, other strategies produce lower prices, lower risk, or both.

Case C—Underlying Asset is Not Tradable, but Tradable Proxy Exists[8]

Taking the stock option example first, suppose there is a publicly traded competitor of our soon to be public company that shares the same characteristics of $P_0 = \$100$, $\mu = 13\%$ and $\sigma = 25\%$. In addition, assume the two stocks' price movements in continuous time are believed to be correlated with $\rho = 60\%$. Under these circumstances it is possible to use the competitor's stock to partially hedge the call option on the non-public company's stock at a lower cost than that implied by investing in

[7]Investing in risky assets uncorrelated with the stock would increase variability.

[8]My thanks to Stephen Mildenhall for suggesting this scenario.

Treasuries. The option seller employs exactly the same procedure that he would use if he were hedging the target company's stock directly, except that he invests in the competitor's stock.

For example, at the moment he sells the call option, he buys \$53.0321 of the competitor's stock (0.530321 shares at \$100 a share), financing the purchase with a loan of \$50.5616 and proceeds from the sale of the call. By pursuing the same dynamic hedging procedure that he would use if he were able to buy and sell the target company's stock directly, the option seller will accumulate the assets that match the option payoff liability at an expected present value cost of \$2.4705. The difference from the scenario in which he can invest in the stock directly is that in that case the \$2.4705 is exact, whereas here it is an expected value.

This scenario involves risk. For example, if the hedge is adjusted on a daily basis, we found from a Monte Carlo simulation consisting of 10,000 trials that the standard deviation of the net result was \$3.6870. While this implies much more risk than that associated with hedging the option directly with the underlying stock (where we found the standard deviation associated with daily rebalancing to be \$0.4405), it is less than the \$3.8007 standard deviation of the net result arising from investing the call proceeds in Treasuries. Clearly, since the call option can be funded at an expected cost of $\$2.4705 < \$2.7100$ with an associated standard deviation of $\$3.6870 < \$3.8007$ by investing in a correlated asset rather than in Treasuries, investment in Treasuries in this scenario must be dismissed as a suboptimal asset strategy.

The same must be said of the analogous excess insurance example. Suppose the natural logarithms of the aggregate insurance claims covered by the stop-loss contract are known to be correlated ($\rho = 60\%$) with the natural logarithms of the values of the consumer price index (CPI-U). In this situation, the insurer can reduce the variance of its net result by investing in the index-linked Treasury notes known as TIPS (Treasury Inflation Protected Securities) rather than in conventional Treasuries.

TIPS pay a fixed rate of interest on a principal amount that is adjusted twice a year based on the change in the CPI-U index.

To illustrate this, assume the expected annualized return on the TIPS is 5%, comprising a fixed interest rate of 2% and expected inflation adjustment of 3%, the same expected total return as the fixed $r = 5\%$ that is available from standard Treasuries. While we usually think of an excess of loss claim as being the amount by which a claim exceeds the retention, we can also think of it as a total limits claim net of reimbursement for the retention. This characterization is useful here. The expected total limits claim is \$56.4009. To fund this payment, the insurer invests $\$56.4009 \cdot e^{-(.05)(20/365)} = \$56.2466$ in TIPS. To finance the purchase of the TIPS, the insurer borrows the present value of the retention reimbursement, $\$53.6835 \cdot e^{-(.05)(20/365)} = \$53.5366$. The remainder, \$2.7100, the insurer collects from the insured. This is the same amount the insurer would collect as a premium before risk charge if the insurer had simply invested in ordinary Treasuries. The benefit of investing in TIPS, which are correlated with the aggregate claim costs, is that the insurer can reduce the variability of the net underwriting result.

The standard deviation associated with this strategy was measured in a Monte Carlo simulation of 10,000 trials. Given a CPI-U index value at inception of 100, the value of the index 20 days later was assumed to be lognormally distributed with mean 100.1645 and c.v. 2.341%, which is consistent with the assumption that the inflation rate is 3% per annum, continuously compounded. The simulation indicated a standard deviation of \$3.2946, which is about 13% less than the standard deviation associated with the otherwise comparable investment in uncorrelated Treasuries.

We saw in the stock option example that hedging with the competitor's stock resulted in a much lower funding cost with less risk than investing in risk-free Treasuries, even with imperfect correlation. This raises the intriguing question of whether an insurer could similarly lower both its risk *and* its required pricing by identifying and investing in higher return securities that are partially correlated with its liabilities. This is food for thought.

Analysis

In all of these scenarios the expected value of the payoff obligation at expiry is the same: \$2.7174. The only differences are the type and tradability of assets available for investment. The characteristics of the *asset* side of the asset-liability equation determine the optimal asking price! Thus, pricing is a function of both the liability *and* the nature of the assets needed to fund it. In insurance applications, where there are usually no suitable assets other than Treasuries available, the liability alone appears to drive the price. This is only because historically, actuaries have assumed that investing in Treasuries is the only reasonable choice. However, as we have seen, when other assets are available, investing in Treasuries is not *always* the only reasonable choice and, in the case of tradable assets, it is not the optimal one.

If the pricing of a given option liability is driven by the optimal asset strategy, then it is critical that the seller of the option actually invests consistently with the pricing assumptions. For example, if an option trader believes that $\mu = 13\%$, and sells the call option described in Case A for the Black-Scholes price of \$2.4705, then it would be a mistake for him to simply invest the option proceeds in Treasuries. If he does that, he faces an expected loss of $\$2.7174 - \$2.4705e^{rt} = \$0.2402$. Beyond that, he is also assuming a sizeable amount of risk, since the standard deviation of his net result is \$3.8007 (plus parameter risk) instead of the zero promised by Black-Scholes. The lesson here is that while the Black-Scholes price is based on assumptions that remove all risk of loss and variability of outcomes, the option seller is not automatically protected. He must actively *manage* his risk.

Summary

My main aim in this response to Mildenhall's review of my paper has been to acknowledge that my Formula (1.3) does not have the generality I originally claimed for it, but then to press on with my contention that, even if the pricing formulas are not identical, call options and excess insurance are still governed by the same pricing paradigm; in particular, one that rests on optimal asset-liability matching.

There is another point I hope I have made clear. The dynamic asset-liability matching regimen that underlies the Black-Scholes Formula (1.1) imposes a different burden on the seller of a call option than the more passive asset-liability matching seen in Case B and in insurance applications. As we saw in our discussion of Case A, it is foolhardy to sell a hedgable call for the Black-Scholes price and then fail to dynamically hedge it. There are other situations where hedging is not possible, because the asset is either not traded or extremely illiquid. In such cases, it is also a mistake to sell the call option for the Black-Scholes price, since it *cannot* be dynamically hedged. In the case of liquid tradable assets, arbitrageurs will drive the option price to the Black-Scholes level. In illiquid or non-traded markets, there will be no such arbitrage activity and in these markets, the pricing formulas used in Case B are applicable.

In closing, I would like to thank Stephen Mildenhall for his excellent discussion, which not only corrected shortcomings in my paper but also added greatly to the understanding (including my own) of option concepts among actuaries.

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APPENDIX A

MEAN AND VARIANCE OF A SIMPLE BUY-AND-HOLD OPTION HEDGE

Let $h = bx - y$ define a random variable for the value at expiry of a hedged portfolio comprising b shares and one short call (i.e., sold short). Here x is a lognormal random variable representing the stock price distribution at option expiry, and y is the random variable representing the value at expiry of the call option on the stock. The option strike price is denoted S.

Mean of Hedged Portfolio at Expiry

$$\begin{aligned} \mathrm{E}(h) &= \mathrm{E}(bx - y) \\ &= b\mathrm{E}(x) - \mathrm{E}(y) \\ &= b\mathrm{E}(x) - (\mathrm{E}(x) \cdot N(d_1^{(\mu)}) - S \cdot N(d_2^{(\mu)})) \\ &= \mathrm{E}(x) \cdot (b - N(d_1^{(\mu)})) + S \cdot N(d_2^{(\mu)}). \end{aligned} \tag{A.1}$$

For the special case of $b = N(d_1^{(\mu)})$,

$$\mathrm{E}(h) = S \cdot N(d_2^{(\mu)}). \tag{A.1a}$$

Second Moment of Hedged Portfolio at Expiry

$$\begin{aligned} \mathrm{E}(h^2) &= \mathrm{E}((bx - y)^2) \\ &= \mathrm{E}(b^2x^2 - 2bxy + y^2) \\ &= b^2\mathrm{E}(x^2) - 2b\mathrm{E}(xy) + \mathrm{E}(y^2) \\ &= b^2\mathrm{E}(x^2) - 2b(\mathrm{E}(x^2) \cdot N(d_0^{(\mu)}) - S\mathrm{E}(x) \cdot N(d_1^{(\mu)})) \\ &\quad + (\mathrm{E}(x^2) \cdot N(d_0^{(\mu)}) - 2 \cdot S\mathrm{E}(x) \cdot N(d_1^{(\mu)}) + S^2 \cdot N(d_2^{(\mu)})). \\ &= (b^2 + N(d_0^{(\mu)}) \cdot (1 - 2b)) \cdot \mathrm{E}(x^2) \\ &\quad - 2S \cdot N(d_1^{(\mu)}) \cdot (1 - b) \cdot \mathrm{E}(x) + S^2 \cdot N(d_2^{(\mu)}). \end{aligned} \tag{A.2}$$

Variance of Hedged Portfolio at Expiry

$$
\begin{aligned}
\sigma_h^2 &= \mathrm{E}(h^2) - \mathrm{E}(h)^2 \\
&= (b^2 + N(d_0^{(\mu)}) \cdot (1 - 2b)) \cdot \mathrm{E}(x^2) \\
&\quad - 2S \cdot N(d_1^{(\mu)}) \cdot (1 - b) \cdot \mathrm{E}(x) \\
&\quad + S^2 \cdot N(d_2^{(\mu)}) - (\mathrm{E}(x) \cdot (b - N(d_1^{(\mu)})) \\
&\quad + S \cdot N(d_2^{(\mu)}))^2 \\
&= (b^2 + N(d_0^{(\mu)}) \cdot (1 - 2b)) \cdot \mathrm{E}(x^2) \\
&\quad - 2S \cdot N(d_1^{(\mu)}) \cdot (1 - b) \cdot \mathrm{E}(x) \\
&\quad + S^2 \cdot N(d_2^{(\mu)}) - \mathrm{E}(x)^2 \cdot (b - N(d_1^{(\mu)}))^2 \\
&\quad - 2S\mathrm{E}(x) \cdot (b - N(d_1^{(\mu)})) \cdot N(d_2^{(\mu)}) \\
&\quad - S^2 \cdot N(d_2^{(\mu)})^2 \\
&= \mathrm{E}(x^2) \cdot (b^2 + N(d_0^{(\mu)}) \cdot (1 - 2b) - \mathrm{E}(x)^2 \cdot (b - N(d_1^{(\mu)}))^2 \\
&\quad - \mathrm{E}(x) \cdot 2S \cdot (N(d_1^{(\mu)}) \cdot (1 - b) + (b - N(d_1^{(\mu)})) \cdot N(d_2^{(\mu)})) \\
&\quad + S^2 \cdot N(d_2^{(\mu)}) \cdot (1 - N(d_2^{(\mu)})). \qquad \text{(A.3)}
\end{aligned}
$$

For the special case of $b = N(d_1^{(\mu)})$,

$$
\begin{aligned}
\sigma_h^2 &= \mathrm{E}(x^2) \cdot ((N(d_1^{(\mu)})^2 + N(d_0^{(\mu)}) \cdot (1 - 2N(d_1^{(\mu)}))) \\
&\quad - \mathrm{E}(x) \cdot 2S \cdot (N(d_1^{(\mu)}) \cdot (1 - N(d_1^{(\mu)})) \\
&\quad + S^2 \cdot N(d_2^{(\mu)}) \cdot (1 - N(d_2^{(\mu)})). \qquad \text{(A.3a)}
\end{aligned}
$$

In the example used in the paper,

$$\mathrm{E}(x^2) = 10178.28293, \qquad N(d_0^{(\mu)}) = 0.58297$$

$$\mathrm{E}(x) = 100.71487, \qquad N(d_1^{(\mu)}) = 0.56001$$

$$
\begin{aligned}
S &= 100, \qquad N(d_2^{(\mu)}) = 0.53683 \\
\sigma_h^2 &= 10,178.28293 \cdot 0.24364 - 100.71487 \cdot 49.27987 \\
&\quad + 10,000 \cdot 0.24864 \\
&= 2,479.86796 - 4,963.2156 + 2,486.432 \\
&= 3.80433 \\
\sigma_h &= 1.75623.
\end{aligned}
$$

DISCUSSION OF A PAPER PUBLISHED IN VOLUME XCI

THE "MODIFIED BORNHUETTER-FERGUSON" APPROACH TO IBNR ALLOCATION

TRENT VAUGHN AND PHOEBE TINNEY

DISCUSSION BY GLENN WALKER

Abstract

Trent Vaughn and Phoebe Tinney have presented a valuable methodology for allocating IBNR to allocation units that do not always warrant separate IBNR analyses. Vaughn and Tinney properly warn that:

> *Actuaries should be aware, however, of the possible pitfalls of allocating IBNR down to an extremely fine level of detail. For instance, such allocations may incorrectly imply a degree of precision that does not exist. The actuary must be aware of this risk and communicate any concerns to the end user.*

But as we are quite painfully aware, there are times when the actuary has no choice. For instance, someone must allocate ceded IBNR to individual reinsurers in order to complete Schedule F—preferably someone who appreciates the implications of a misallocation. Though Vaughn and Tinney have suggested that their methodology may apply, the analysis process should not end there.

This discussion highlights an area that is typically a pro forma accounting function, virtually ignored by actuaries, yet it has profound actuarial implications that deserve significant attention not yet addressed in actuarial literature.

TABLE 1

REINSURANCE RECOVERABLE

Reinsurance Recoverable On:	
Paid Losses	$1,000
Paid LAE	200
Known Case Loss Reserves	1,100
Known Case LAE Reserves	200
IBNR Loss Reserves	500
IBNR LAE Reserves	0
Unearned Premiums	2,000
Contingent Commissions	0
Total	$5,000
Ceded Balances Payable	0
Other Amounts Due Reinsurer	0
Net Amount Recoverable	$5,000
Funds Held by Company Under Reinsurance Treaty	$5,000

1. THE PROBLEM OF ALLOCATING CEDED IBNR TO INDIVIDUAL INSURERS

The actuary makes one last review of his assumptions before signing his name to the loss reserve opinion. He reviews Schedule F to satisfy himself that he has no reason to question the collectibility of the client’s reinsurance recoverable. A particular unauthorized reinsurer catches his attention. The line for that reinsurer is shown on Table 1.

Just before he signs his name, he asks himself, “What happens if the ceded IBNR was incorrectly allocated to the unauthorized reinsurer?”

He has reviewed his analysis thoroughly, and is satisfied that the aggregate direct IBNR loss reserve of $2,000 is reasonable. He is even satisfied that the aggregate ceded IBNR of $1,500 is reasonable. The actuary used Vaughn and Tinney’s Modified Bornhuetter-Ferguson approach, and has allocated $500 of the $1,500 ceded IBNR to this particular reinsurer. For Vaughn and Tinney have advised: "The approach can be used to allocate ceded IBNR to individual reinsurers for Schedule F purposes."

The client acted on the actuary's preliminary advice and withheld $5,000 to assure collectibility.

What happens if the $1,500 of ceded IBNR emerges in such a manner that the unauthorized reinsurer is liable—not for $500—but for $1,000 of the IBNR recoverable?

Assuming that the client is unable to secure the additional $500 in time for statement filing, that the actuary has no means of confirming the unauthorized reinsurer's financial strength, and that $500 of upward reserve development is judged material, the ceded IBNR becomes of more questionable collectibility than Schedule F has made it appear.

Vaughn and Tinney concede: "The Annual Statement may require IBNR estimates at a finer level of detail than the reserve segment definitions."

And as already noted, they wisely caution us:

> Actuaries should be aware, however, of the possible pitfalls of allocating IBNR down to an extremely fine level of detail. For instance, such allocations may incorrectly imply a degree of precision that does not exist. The actuary must be aware of this risk and communicate any concerns to the end user.

Schedule F indeed requires an allocation of ceded IBNR to individual reinsurers whether or not the actuary believes the resulting allocation is meaningful. In some cases, the allocation to reinsurer may not be a problem. For instance, the company may list only a single reinsurer, and no allocation of ceded IBNR to reinsurers is needed. The company may have separate reinsurers that align nicely with the actuary's level of detail for analysis, such as line of business or accident year. Heavy use of quota share treaties can also facilitate the allocation of ceded IBNR.

Finally, in rare cases, there may be sufficiently credible loss experience by reinsurer that such separate analyses become feasible.

But for most companies with a comprehensive reinsurance program, the entries onto Schedule F result from some manner of allocation process. The fact that the company involved the actuary in that allocation process is likely a step up from most companies. Actuarial discussion on the process of allocating ceded IBNR to reinsurer seems terribly sparse, leading me to believe that in most cases, actuaries are not involved in the process at all. Even worse is the likely case that those who do allocate ceded IBNR to reinsurers are not necessarily trained to appreciate the relationship between the result of the allocation and collectibility from any individual reinsurer.

2. REQUIRED SECURITY TO GUARANTEE COLLECTIBILITY

In cases where the company is reinsured by a collection of authorized reinsurers whose financial security raises no concerns, allocation may not be a concern. But even when unauthorized reinsurers are involved, ceded IBNR continues to be allocated without the influence of the actuary. This could have adverse implications for the actuary who will eventually opine on the collectibility of the ceded IBNR, however it was allocated. Multiple unauthorized reinsurers can seriously add to the risks of improper allocation.

Vaughn and Tinney deserve credit for noting that the allocation method deserves more careful attention than it normally gets. However, it seems that Vaughn and Tinney's Modified BF allocation method merely replaces one mechanical methodology with another. In cases where security is required to guarantee collectibility, whether arising from lack of jurisdiction or from financial concerns, the amount of IBNR to be allocated to that particular reinsurer should receive more careful attention than a passing remark in a *Proceedings* paper on IBNR allocation.

The amount of security a company holds should not be a function of any particular ceded IBNR allocation methodology, but rather should be carefully considered as a function of the probable maximum amount ceded to a reinsurer. Consider the following much simplified example.

A company insures two large buildings (A and B) for $1,000 each. Separate unauthorized reinsurers fully reinsure each of the two policies that protect the buildings. During the reserve review, the company and the actuary become aware that one of the buildings was destroyed just prior to year end, though there is no information on which building. The actuary adds a $1,000 IBNR reserve, and a $1,000 ceded IBNR reserve, with no effect on the net reserves. These entries are made on Schedule P and on all Annual Statement pages that depend on Schedule P.

On Schedule F, the $1,000 ceded IBNR reserve must be assigned to specific reinsurers. If building A was the one destroyed, then reinsurer A should be assigned the $1,000 ceded IBNR, and to the extent that security is required, it should be demanded. The same scenario applies to building B if that was the one destroyed. In practice, the $1,000 ceded IBNR reserve will almost certainly be allocated between the two reinsurers, either fifty-fifty, or through some other methodology that guarantees that one of the reinsurers will be inadequately secured. A probable maximum ceded methodology would require the company to secure a full $1,000 from both reinsurers.

A ceded IBNR allocation will balance to the correct total, but will expose the company to inadequate security for one reinsurer. A probable maximum ceded allocation will generate adequate security from each reinsurer, but will result in an excessive ceded IBNR total for Schedule F, and an inadequate, perhaps even a negative, net IBNR. The only way to assure adequate security would be for each reinsurer to place the full $1,000 in trust.

Perhaps this discussion concerns the rationale of Schedule F security, whether funds withheld, letters of credit, ceded balances payable or any other form of collateral, more than it does what method is used to allocate ceded IBNR to reinsurer. For in the case of a single reinsurer, 100% of the ceded IBNR is allocated to that single reinsurer. If that single reinsurer were unauthorized, then collateral would be demanded and received. But even at 100% allocation, the security can prove inadequate. Reinsurance collectibility, though fully secured, is not assured, and the methodology of ceded IBNR allocation is not at fault. At this level, it seems more prudent to base the collateral on a probable maximum ceded methodology.

Realistic applications of the probable maximum ceded method are generally not as extreme as the simplistic example set forth for illustration. But where security is required for at least one reinsurer, the aggregate probable maximum ceded IBNR can easily become significantly and materially higher than the aggregate ceded IBNR. It may be a challenge for a company to obtain the security beyond that implied by a ceded IBNR allocation. But the difference between the aggregate probable maximum ceded IBNR and the aggregate ceded IBNR is simply the amount of adverse reserve development to which the company is exposed, arising from possible ceded IBNR misallocation.

There are several difficulties associated with a probable maximum ceded methodology. Among these difficulties are the burden on the actuary to calculate it, the burden on the company to obtain higher security, and the burden on the reinsurer to commit additional funds to this new standard. Of course the burden of adverse reserve development arising from a ceded IBNR misallocation to a troubled reinsurer can be significant as well.

I do not anticipate that Schedule F will be transformed to a probable maximum ceded basis of security. Just as primary insurers cannot possibly reserve for probable maximum loss risk by risk, few reinsurers, even the financially healthiest, can possibly post collateral at probable maximum loss by risk. If a reinsurer could secure its liabilities on a probable maximum basis by risk, its financial condition would appear better. There is hope that primary companies can look beyond the Schedule F criteria in demanding security, and there is hope that opining actuaries can be more conscious of the implications of misallocating ceded IBNR to a reinsurer.

ADDRESS TO NEW MEMBERS—NOVEMBER 14, 2005

HORIZONS

RONALD L. BORNHUETTER

Before I begin my formal remarks, I would like to share a brief anecdote that occurred during my presidency. Some of you have heard it before, but many have not.

For the Annual Meeting in November 1976, which was to be held in San Diego, California, it was decided to have a paid luncheon speaker for the first time. I asked Carl Honebein to see what he could develop in the way of possibilities. After some time, there were several speakers to choose from with a variety of fees.

Our choices included Senator Ted Kennedy (who charged $10,000 plus expenses for himself and two bodyguards), Governor Jerry Brown, and a retired part-time journalist and radio commentator (who wanted $5,000 although he lived twenty minutes from San Diego). We finally decided on the then senior United States Senator from California, Alan Cranston, who only charged a fee of $2,000 and no expense money.

Our choice was a good one as Senator Cranston had just returned from Plains, Georgia, where he met with the newly elected president Jimmy Carter. His comments were very interesting as he gave his insight on our next president. Cranston was quite a good choice for our first venture into paid speakers.

(As a footnote, the retired part-time journalist and radio commentator who lived close by, whom I turned down, was future president Ronald Reagan.)

Before I begin my formal remarks about your horizons, let me congratulate each of you on your hard work that culminated with achieving Fellowship in our Society. It is a culmination of a considerable amount of hard work and effort and you should be very proud. Also, my compliments and congratulations to all of the spouses who gave much needed support throughout this journey. You were integral parts of your partners' successes.

Let's spend a few minutes on the subject of looking forward. Where do you go next? What is your horizon—both inward outward?

You are a member of a wonderful, dynamic society that has helped prepare you for where you are today. I am sure you know that by now. Part of your inward horizon will always be the CAS—it deserves your support and participation. You will continue to learn from your involvement. As the years unfold, you will realize you have a debt to repay. Our Society needs your help and it is up to you and others to continue its success.

On the inward side, I would be remiss if I did not add that you should be aware of involvement in our sister organizations. For example, the American Academy of Actuaries and the Canadian Institute of Actuaries accomplish many good things on our behalf. I am talking about practice councils, principles developed by the Actuarial Standards Board, and congressional hearings, to mention a few. Now add Sarbanes-Oxley and other external public involvement and you can see that the AAA and CIA are important keys to your future actuarial life.

Another semi-internal horizon that might be of interest to you at one point in your career is ASTIN and the International Actuarial Association (IAA). You will find actuaries from outside North America to be quite interesting and they sometimes do have a different perspective on actuarial matters. Your officers and board of directors are pursuing the globalization of the casualty actuarial profession in many ways and this is certainly a growth area for our profession.

Perhaps it would take too many words to tell you our Society and sister organizations do need everyone's help. Without it they will not survive. It is up to you.

Now to an area I would refer to as your outward horizon. Again, a short anecdote: Many years ago I had a brief discussion with the chief executive officer of General Re, Harold Hudson. We talked about why no officer had the title of "actuary" in the corporation. His response was that the title was "too limiting." I did not necessarily agree, but I understood the point. It's certainly food for thought.

I know most of you look forward to pursuing your career as an actuary. It is a great profession. J.D. Powers & Associates always ranked it very high or number one on several occasions when compared to other professions. Now let me offer a few outward horizons that may arise along the way.

Again, a brief anecdote: At one point in time, Ace, General Re, American Re, NAC-Re, Everest Re, Employers Re, Underwriters Re, and Renaissance Re all had one thing in common—all their chief executive officers were actuaries. Outside of the Swiss and Germans, this was most of the United States reinsurance market. All were members of our Society except one, and he was a member of the Danish Actuarial Society. Many of them got there by different routes such as actuarial, finance, underwriting, and accounting, just to name a few.

Think about it. Your actuarial background prepares you well to take on other assignments that may come your way in your horizon.

For example, today I see a large group of casualty actuaries in the room—I would wager that less than 25 in the audience spend one quarter of their time working on the asset side of the balance sheet, yet investment analysis and management is natural for an actuarial mind. Did you know that, at one time, and perhaps still today, a substantial percentage of the British Institute of Actuaries were employed in the investment industry rather than the insurance arena? Certainly, this is an appropriate area when you are thinking about risk theory.

Also, I know one of the last things you want to hear is that there are other designations that can be helpful to your careers such as CFA, MBA, and CPA, to name a few. Some day you might want to pursue one of these areas. Several of you have already added MBA to your achievements and it will be very helpful.

Please don't get me wrong, the actuarial profession is a proud and meaningful one. It will do you well. Pursue it to the fullest. It has been ranked nationally as one of the best professions, if not the best. What it will be to you is the catalyst to provide you with opportunities that will appear on your horizon. Give them consideration. You never know where it will lead.

I would be remiss if I did not briefly mention today's world of Sarbanes-Oxley and the political environment we live in. Acts you may perform today according to a "normal course of business environment" may be received differently in years to come by others under a different set of rules or interpretations. Even an outside auditor's sign off may not be good enough. In any event, you may be involved in one or more transactions in the future. So be careful and be confident that you are participating in a proper transaction. Even if you say this area is not really applicable to what you currently do, it is in your best interest to follow current events and educate yourselves in order to be prepared if and when an applicable instance occurs.

Lastly, do me a favor. Enjoy the day. You earned it. And, when you return home, take an underwriter or two out to lunch. They most certainly will learn something.

Thank you for your patience and attention and enjoy your horizon, whatever it may be.

PRESIDENTIAL ADDRESS—NOVEMBER 14, 2005

ON BECOMING AN ACTUARY OF THE FOURTH KIND

STEPHEN P. D'ARCY

A presidential address is an opportunity for outgoing presidents to thank the many people who have helped them over the course of the year, to wax philosophical about things actuarial, and to enlighten the membership about relevant issues facing our profession. As a teacher, though, I cannot pass up the opportunity of a few minutes in front of an audience to try to provide a useful learning experience for the class, I mean the membership. Thus, today's lecture is, "On Becoming an Actuary of the Fourth Kind."

Hans Bühlmann first offered this classification of actuaries in an *ASTIN Bulletin* editorial (1987) entitled, "Actuaries of the Third Kind?" Actuaries of the first kind, who emerged in the 17th century, focused on life insurance issues and tended to use deterministic methods. In the early 20th century, actuaries of the second kind developed—casualty actuaries who used probabilistic approaches in dealing with workers compensation, automobile insurance, property insurance and similar risks. The actuaries of the third kind, who were the object of Bühlmann's editorial, were the investment actuaries applying stochastic processes, contingent claims and derivatives to assets and liabilities. This specialty developed in the 1980s as financial risk became more important and tools to manage financial risk were created. For advice on learning the tools and techniques of this type of actuary, your assignment is to read, "On Becoming an Actuary of the Third Kind," which I presented at the 75th anniversary of the CAS and is published in the 1989 *Proceedings of the Casualty Actuarial Society*.

At a recent ASTIN meeting in Zurich, Professor Paul Embrechts of the Swiss Federal Institute of Technology (ETH) referred to those actuaries working in enterprise risk management

(ERM) as actuaries of the fourth kind. Change has certainly sped up in the actuarial profession, as it took 250 years for the actuaries of the second kind to emerge, 70 more years for actuaries of the third kind to develop, but less than three decades for the newest type of actuary to arise. I would like to provide some guidance on becoming an actuary of the fourth kind.

Risk is present whenever the outcome is uncertain, whether favorable or unfavorable. Risk exists whenever there is uncertainty. ERM is the systematic evaluation of all the significant risks facing an organization and how they affect the organization in aggregate. A variety of classifications of risk have been proposed, but I find that categorizing risks as hazard, financial, operational, or strategic to be most useful. Hazard risks are the risks actuaries have most commonly considered. These are the pure risks, the loss/no loss situations that may injure people, damage property, or create a liability. Traditional actuarial mathematics work best on hazard risks, as they are generally independent and discontinuous. Actuaries and other risk professionals have generally done a remarkably good job assessing and evaluating hazard risks. Organizations rarely become insolvent due to failure to manage hazard risks, and insurers can generally withstand major losses of this type, even when they exceed all prior incidences of such losses by a significant amount. If only this were true for similar occurrences of the other types of risk. Daily we learn of companies going into bankruptcy because of mismanaging other risks.

Financial risks are those that affect assets, including interest rates, inflation, equity values, and foreign exchange rates. These risks are correlated, continuous, and require an understanding of stochastic calculus to be measured appropriately. Unlike hazard risks, financial risks provide the possibility of a gain, not just a loss. The techniques for managing financial risks—financial derivatives such as forwards, futures, options, and swaps—are relatively new, developed only over the last several decades. Misuse of these techniques and the resulting financial debacles they caused have actually led to the need for ERM.

Operating risks represent the failure of people, processes, or systems. One recent example of operating risk is the announcement (*The Wall Street Journal*, November 9, 2005) by Freddie Mac, the large mortgage finance company, that it discovered a computer error that, since 2001, has been overvaluing accrued interest on variable rate home equity loans. The effect of this error is estimated to be at least $220 million. The next day General Motors announced that it had incorrectly booked credits from suppliers in 2001 (*WSJ*, November 10, 2005). The overstated earnings are estimated to be as high as $400 million, or 50 percent of its reported profit during that year. To clarify the distinction between hazard and operational risk, if an employee steals from an employer, that is a hazard risk and can be covered by typical insurance policies. However, if that employee inflates earnings in order to "qualify" for a bonus, that is not considered hazard risk and is not covered by insurance. This would be operational risk.

Strategic risk reflects the business decisions of an organization or the impact of competition or regulation. An organization that adapts (or fails to adapt) to new markets, whose activities lead to new forms of regulation that either help or hinder future operation, or whose business plan proves either successful or unsuccessful—all are examples of strategic risk. Examples of strategic risk for insurance are the benefits produced for those first to use credit scoring as a rating variable, and the market share losses of those companies that were slow to adopt this approach.

ERM originally focused on loss prevention, controlling negative surprises, and reducing downside risk. That was the initial reaction of both regulators and boards to the failures at Barings Bank, Enron, WorldCom, Arthur Andersen, and other corporations. ERM evolved into accepting risk, but measuring the risk associated with the expected returns from different business strategies. When organizations began to use ERM approaches for capital allocation and tied compensation to the resulting risk adjusted returns, it became serious for many managers. Cases are now told of dueling modelers, each with their own capital allocation process favoring their sponsoring area, who vie to have their model adopted by the organization. ERM is now evolving into risk optimization and the efficient deployment of capital. When an organization accepts risks where it has a comparative advantage, and transfers or avoids risks where it does not, the system is adding value by efficient risk treatment. ERM deals with the entire range of potential outcomes, not just downside risk.

So, how to become an ERM actuary? Step one in ERM, as in traditional risk management, is risk identification—to identify all significant risks an organization faces. Although actuaries are good at quantifying risks, other specialties, such as traditional risk managers, have greater expertise in the identification of risk, particularly hazard risks. Traditional risk managers, just as most actuaries, also tended to ignore financial risks. A first step in becoming an actuary of the fourth kind is to master the skills of the risk managers in the identification of risk and then to expand this identification process to financial, operational, and strategic risks, as well as hazard risks. The risks an organization faces, though, are myriad. The advice of one ERM pioneer, James Lam, is instructive. His admonition is, "Don't boil the ocean." Instead focus on the most significant risks an organization faces. Deal with those first, then in future iterations expand the focus to the next level of risk elements.

Step two in ERM, as in traditional risk management, is to quantify the risks. Actuaries are well skilled in this area, at least for hazard risks, but ERM also requires the quantification of the correlations among different risks. ERM is concerned with risk in aggregate and to the extent that one risk offsets other risks, then the organization benefits. To the extent that different risks combine to increase the negative impact, the organization is at risk. Measuring the correlations is also more complicated than just looking at the correlation coefficient, or how two variables tend to move in relation to each other. Two risks can be generally uncorrelated, but, if an extreme event were to occur, then they could be highly correlated. Techniques for evaluating these forms of correlations, filters, tail dependency, copulas, and other numerical techniques must be incorporated.

Much needs to be done to be able to quantify operational and strategic risk to the standards common in hazard and financial risk, but progress is being made. The Basel Accord proposes several methods for determining capital charges for operational risk. These methods were devised to be applicable to banks, but regulatory consolidation is expanding the application to insurers and other financial institutions. Other techniques include measuring change in the market value of publicly traded companies when operational risks are revealed, such as accounting problems, product recalls, or the legal troubles of closely identified executives. Much more needs to be done in this area. Actuaries have the skill set that can improve the quantification process, but other specialties are moving into this area as well, such as the former financial engineers (yes, the same ones responsible for Long Term Capital Management, Enron, and others), accountants, and risk managers. If we want this done correctly, we need to step up to the plate now or other groups will claim this area. Our unique advantages—the combination of math skills, practice in explaining complex mathematics to nonmathematical managers, and a professional code of conduct—make actuaries the ideal professionals to assume a leadership role in ERM. The opportunity for staking a claim will not exist for long, however. If you are interested in becoming an actuary of the fourth kind, start now.

Step three of the risk management process involves evaluating the different methods for handling risk. Risks can be assumed, transferred, or reduced. A variety of methods exist for transferring or reducing risk. Risk can be transferred by subcontracting, by insurance, or by securitization. Risk can be reduced by loss control, contract, or reinsurance. These techniques range from engineering to legal to financial to actuarial, requiring the actuary of the fourth kind to be conversant in each area.

Step four is to select the best method for handling the risk, which in most cases will involve a combination of different techniques. ERM aims to establish a consistent approach to dealing with risk. This means that the organization wants to make consistent choices about all of the risks it faces, how much risk it will accept, and what return it would require for accepting a particular level of risk.

Step five is to monitor the risk management approach selected. ERM is an ongoing process that must be monitored, adjusted, and revised as new information and new techniques become available. Thus, as soon as one round of an ERM process is completed, the next round begins. It is an iterative process that entails identifying additional significant risks, quantifying those risks, and improving the quantification of previously identified risks based on additional information and improved mathematical techniques. It also entails reevaluating the different approaches to handle risk, implementing an improved strategy, and then, once more, monitoring the results. It sounds like rate filings or loss reserve analysis—do it and then do it again. This is full employment for actuaries, perhaps?

Actuaries need to become the ERM specialists of the insurance industry. For one reason, no one else understands the mathematics underlying this industry as well as actuaries do, so no one else can do as good a job. For another reason, if actuaries fail to grow into insurers' ERM positions, someone else will. After they master that position, they will feel qualified, and perhaps even be qualified, to assume the roles actuaries now fill in the hazard risk area. Move up or move out. That is only fair. Academics have long lived with the publish or perish dictum. Now the actuarial profession has its equivalent challenge: master ERM or face extinction. It doesn't have quite the right flow, but the consequences are just as dire.

The quantification of risks in an ERM process involves combining the actuarial tools and techniques with those of financial economists. Become an actuary of the third kind on your way to becoming an actuary of the fourth kind. This path is already well laid out. Start with "On Becoming an Actuary of the Third Kind," add readings from Cairns (2004), Hull (2003), Jorion (2001), and Das (1997). Then expand into ERM by reading Lam (2003), Samad-Kahn (2005), and Standard and Poor's (2005).

Not all actuaries need to become actuaries of the fourth kind, although I hope that many of you will. There will likely be plenty of work for actuaries of the first, second, and third kinds. But the actuary of the fourth kind represents a new frontier, one that we are well suited for by training and temperament, and one that I think we can fulfill better than any other group.

For an example of ERM in action applied to the insurance industry, let's look to Hurricane Katrina, which hit the coast of Louisiana the end of August 2005. This hurricane caused an estimated $125 billion in economic losses, of which the insurance industry is expected to pay approximately $35 billion. This would make Katrina twice as costly as the previous largest natural disaster, Hurricane Andrew in 1992. Simultaneously, oil prices surged to record levels as a significant portion of the oil producing capacity of the United States was damaged by this storm. It is clear, in retrospect, that a perfect natural hedge existed for insurers exposed to property losses on the Louisiana coastline—derivatives on oil prices. An insurer would purchase oil futures if it were willing to incur losses as energy prices declined, or options if it only wanted protection against oil price increases. Any storm wreaking havoc on the coastline had to affect the oil wells situated in the Gulf and therefore the price of oil, at least temporarily. Rather than reducing exposure to property damage in Louisiana, as at least one major insurer has already announced it would do, insurers could hedge their property exposure with financial derivatives. Now, if we could only ring the coast of Florida with a valuable natural resource, the industry might be able to cope with hurricane losses there as well.

There is another hedge for property insurers that is related to the spike in oil prices caused by Hurricane Katrina. High gas prices led to a reduction in driving, which reduced the number of automobile accidents, and lowered collision and liability losses for insurers. On the other hand, drivers encumbered with gas guzzling SUVs could be tempted to generate additional comprehensive claims as these vehicles are "stolen" or "burned." Moral hazard also affects ERM. Start thinking in this manner and you are on your way to becoming an actuary of the fourth kind. Welcome aboard!

Despite my proclivity to turn this into a teaching opportunity, I do not want to miss the chance to thank those many people who helped me during my term in office. To you, the members of the CAS who elected me, thank you for putting your confidence in me. I hope I have met your expectations. To all the volunteers, the committee and task force chairs, and especially the vice presidents and board members who worked so closely with me over the year, thanks for helping me achieve what we did accomplish. Progress would not have been possible without you. Thank you to the wonderful CAS staff members, here and back at the office, and to Cynthia Ziegler for motivating and leading them, for you truly do the essential work of our organization. To the leaders of the other actuarial organizations, some of whom are with us today, others unable to be here, thank you for your openness and cooperation as we worked together to enhance the actuarial profession throughout the world.

Thanks to my family for supporting me during my term. My daughter, Meriden, who is not here, accompanied me to a North American Actuarial Council meeting in Hawaii and helped out at the Spring Meeting in Phoenix. My son, Grant, who was here until this morning, helped me explore Norway and Zurich during ASTIN and AFIR meetings. Most of all, I want to thank my best friend, my wife, Cleo, whose leadership experience helped me keep my equilibrium during this sometimes hectic year and whose love of teaching has inspired mine.

Now it is with mixed emotions—some sadness and some relief—that I pass on the presidential responsibilities to Paul Braithwaite. It has been a wonderful year, thanks to all of you. Good luck, Paul!

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MINUTES OF THE 2005 CAS ANNUAL MEETING

November 13–16, 2005

RENAISSANCE HARBORPLACE HOTEL

BALTIMORE, MARYLAND

Sunday, November 13, 2005

The board of directors held their regular quarterly meeting from 8:00 a.m. to 4:30 p.m.

Registration was held from 4:00 p.m. to 7:30 p.m.

From 5:30 p.m. to 6:30 p.m. there was a presentation to new Associates and their guests. CAS President Steve D'Arcy made a short presentation to new Associates with a brief overview of the CAS organization.

A welcome reception for all members and guests was held from 6:30 p.m. to 7:30 p.m.

Monday, November 14, 2005

Registration continued from 7:00 a.m. to 5:00 p.m. and a continental breakfast was served from 7:00 a.m. to 9:00 a.m.

Mr. D'Arcy opened the business session, which was held from 8:00 a.m. to 10:00 a.m., and announced that the meeting was being broadcast on the CAS Web Site. He then introduced the members of the executive council and the CAS Board of Directors. He thanked exiting EC members Tom Myers and Don Mango, who would be retiring from the positions of Vice President–Admissions and the Vice President–Research & Development, respectively, and outgoing board members Gary R. Josephson, David J. Oakden, Patricia A. Teufel, Oakley E. Van Slyke, and Mary Frances Miller for their contributions.

Mr. D'Arcy then announced the results of the CAS elections. The next president will be Paul Braithwaite and the president-elect will be Thomas G. Myers. New board members will be

Irene K. Bass, Glenn Meyers, Donald F. Mango, and Roosevelt C. Mosley Jr.

Following these announcements, Mr. D'Arcy introduced the presidents from other actuarial organizations who were in attendance including Ana Maria Ramirez Lozano, Asociación Mexicana de Actuarios (AMA); Charles McLeod, Canadian Institute of Actuaries (CIA); Fred Kilbourne, Conference of Consulting Actuaries (CCA); Jean Kwon, Asia-Pacific Risk and Insurance Association (APRIA); and Shigeru Taguchi, Institute of Actuaries of Japan (IAJ). Mr. D'Arcy also recognized all CAS volunteers.

Vice President–Professional Education Beth Fitzgerald informed the audience that the meeting will offer over 30 different concurrent sessions, as well as four general sessions. She also announced the 2005 CAS Annual Meeting Exhibitors, which included the American Academy of Actuaries; EQECAT, Inc.; Insureware Pty. Ltd.; ISO; Milliman; and Northstar International Insurance Recruiters, Inc. She also noted the financial contributions of Pryor Associates and Towers Perrin, who are 2005 CAS Annual Meeting Supporters.

Ms. Fitzgerald and Mr. Myers announced the new Associates, and Mr. D'Arcy announced that Matthew W. Kunish of Crum & Forster recently became a Fellow via Mutual Recognition. Paul Braithwaite announced the names of the new Fellows.

NEW FELLOWS

Fernando A. Alvarado
Brian C. Alvers
Maura Curran Baker
Rose D. Barrett
Derek Dennis Berget
Brian Jeffrey Biggs
Corey J. Bilot
Rebekah Susan Biondo
Tapio Nikolai Boles
James L. Bresnahan
John R. Broadrick
Michele Lee Brooks
Elaine K. Brunner
Matthew Daniel Buchalter
Anthony Robert Bustillo
Hsiu-Mei Chang
Alan M. Chow
Jason Travis Clarke
Kevin A. Cormier
Justin B. Cruz
David F. Dahl
Chantal Delisle
Laura S. Doherty
Tomer Eilam
Bruce Joseph Fatz
Dale Albert Fethke
William John Gerhardt
John S. Giles
Kristen Marie Gill
David Barry Gordon
Jeffrey Robert Grimmer

Megan Taylor Harder
Robin Andrew Haworth
Brandon L. Heutmaker
Joseph Suhr Highbarger
Bo Huang
Richard Clay Jenkins
Philip J. Jennings
Shiwen Jiang
Yi Jing
Dana F. Joseph
Omar A. Kitchlew
Scott Michael Klabacha
Andrew Mark Koren
Bradley Scott Kove
Matthew W. Kunish
Terry Thomas Kuruvilla
François Lacroix
Kahshin Leow
Xin Li
Erik Frank Livingston
Jonathan LaVerne Matthews
Brent Layne McGill
Christopher Charles McKenna
Sylwia S. McMichael
Meagan Sue Mirkovich
Rodney Scott Morris
Leonidas V. Nguyen
Miodrag Novakovic
Timothy J. O'Connor
Kathleen C. Odomirok
Jeremy Parker Pecora
Gregory T. Preble
Damon Joshua Raben
Dale M. Riemer
Brad E. Rigotty
Bryant Edward Russell
Frances Ginette Sarrel
Jeffery Wayne Scholl
Genine Darrough Schwartz
Justin Nicholas Smith
Mark Stephen Struck
Zongli Sun
Dovid Tkatch
Jennifer Marie Tornquist
Joel Andrew Vaag
Daniel Jacob VanderPloeg
Kevin K. Vesel
Mo Wang
Kevin Earl Weathers
Grace Hueywen Yang
Yuanhe Yao
Sung G. Yim
Ronald Joseph Zaleski Jr.
Lijuan Zhang
Hongbo Zhou
Steven Bradley Zielke

NEW ASSOCIATES

Avraham Adler
Vera Afanassieva
Amit Agarwal
Sajjad Ahmad
Kelleen D. Arquette
Yanfei Zhu Atwell
Gregory S. Babushkin
Kristi Spencer Badgerow
Gregory Keith Bangs
Tiffany Jean Baron
Angelo Edward Bastianpillai
Mark Alex Belasco
Jeffrey Donald Bellmont
Matthew Craig Berasi
Sonal Bhargava
Jonathan Bilbul
Brad Stephen Billerman
Jon Bloom
Peter George Blouin
Nicolas Boivin
Randall Todd Buda
Morgan Haire Bugbee
Jessica Y. Cao
Jeffrey McKinley Casaday
Paul Andrew Ceaser
Matthew Scott Chamberlain
Bernard Lee Chan
Michael Tsz-Kin Chan
Joung-Ju Chung
Raul Cisneros
Glenn Anthony Colby
Kirk Allen Conrad
Lawrence Glenn Cranor

Tighe Christian Crovetti
Walter Casimir Dabrowski
Jonathan Everett DeVilbiss
Brent Pollock Donaldson
Michael Dana Ersevim
Choya Aisha Everett
Marc Olivier Faulkner
Jason Alan Flick
Mark Allen Florenz
Kyle Patrick Freeman
Derek Freihaut
Timothy M. Garcia
Nina Vladimirovna Gau
Stuart G. Gelbwasser
Maxime Gélinas
Simon Girard
Gregory Paul Goddu
Rebecca Joan Gordon
Wesley John Griffiths
Isabelle Guérin
Gary S. Haase
William Joseph Hackman
Brian P. Hall
James Warren Harmon
Megann Elizabeth Hess
Nathan Jaymes Hubbell
Yu Shan Hwang
Alison Susanne Jennings
Ziyi Jiao
Jeremiah David Johnson
Ross Evan Johnson
Amy A. Juknelis
Jennifer Ge Kang
Brian Martin Karl
Jean-Philippé Keable
Sarah M. Kemp
David J. Klemish
Rachel Marie Klingler
Christine Kelly Kogut
Thomas Ryan Kolde
Leland Kraemer
Michael Alexander Lardis
Catherine Marie Larson
Annie Latouche
Kak Lau
Jeremy Matthew Lehmann
Sean Maxmillian Leonard
Jean-François Lessard
Mingyue Miriam Li
Andrew H. Liao
Cunbo Liu
Jin Liu
Nannan Liu
Todd L. Livergood
Andrew Loach
Laura Joann Lothschutz
Neelam Patel Mankoff
Minchong Mao
Angela Garrett McGhee
Albert-Michael Micozzi
Daniel Evan Mikesh
Aaron G. Mills
Richard James Mills
Lori A. Moore
Allison Lynn Morabito
Mundia Mubyana
Daniel G. Myers
Marc Lawrence Nerenberg
Benjamin Reiter Newton
Tho D. Ngo
Stephanie Jo Odell
Christopher J. Olsen
Alejandro Antonio Ortega
Keith William Palmer
Joy-Ann Cecile Payne
Joseph Gregory Pietraszewski
Jean-Philippe Plante
Lynellen McDonnell Ramirez
Arthur Roosevelt Randolph II
Zia Rehman
Zoë F. S. Rico
Arnie W. Rippener
Randall David Ross
John C. Ruth
Anita A. Sathe
Lawrence Michael Schober
Erika Helen Schurr
Ronald S. Scott

Sheri Lee Scott
Clista Elizabeth Sheker
Robert K. Smith
Patrick Shiu-Fai So
Joanna M. Solarz
Richard Cambran Soulsby
Michael Patrick Speedling
Paul Quinn Stahlschmidt
Mindy Marie Steichen
Yuchen Su
Feixue Tang
Luc Tanguay
Aaron Asher Temples
Robert Bradley Tiger
Phoebe Alexis Tinney
Levente Thomas Tolnai
Rachel Katrina Tritz
Benjamin Joel Turner
Jonathan Kowalczyk Turnes
Allan Stone Voltz III
Todd Patrick Walker
Xuelian Wan
Jingtao Wang
Amanda Jane White
Andrew Thomas Wiest
Martin Ernest Wietfeldt
Ronald Harris Wilkins
Shauna Suzanne Williams
Benjamin Todd Witkowski
Dorothy A. Woodrum
Yi-chuang Sylvia Yang
Min Yao
Yanjun Yao
Hui Yu Zhang
Wei Zhao
Michael V. Ziniti

Mr. D'Arcy then introduced Ronald L. Bornhuetter, a past president of the Society (1975), who presented the address to new members.

A short award program followed the address. Mr. D'Arcy presented the Matthew S. Rodermund Service Award to Anne E. Kelly. Ms. Kelly, the chief casualty actuary for the New York State Insurance Department, served as the chairperson of the Committee on Volunteer Resources. Mr. D'Arcy also presented the "Above & Beyond" achievement award to Kristine Kuzora, David L. Menning, Michael G. Wacek, and Jerome F. Vogel. During the past two years Mr. Menning has been highly effective in leading the CAS's joint efforts with the SOA and CIA to implement computer-based testing (CBT) for Exam 1. Ms. Kuzora volunteered on the Research Paper Classifier Project Committee where she classified over 600 abstracts with the new CAS Research Taxonomy in just 25 days. The CAS also honored Mr. Vogel for his work on the Research Paper Classifier Project Committee. At the time the award was given, he had classified over 500 abstracts, or 13 percent of the 4,200 abstracts needing classification. Mr. Wacek was selected for his work in chairing the CAS Working Party on Risk Transfer

Testing where he produced a thorough, high-quality research paper in just one month's time.

Mr. D'Arcy announced that the Charles A. Hachemeister Award recipient was Jon Holtan for his paper entitled "Pragmatic Insurance Option Pricing." Roger Hayne presented the Dorweiler Prize to Rodney E. Kreps Ph.D. for his paper "Riskiness Leverage Models."

After the awards presentation Mr. D'Arcy requested a moment of silence for members who had died in the past year: Robert G. Espie, Sidney M. Hammer, Richard L. Johe, J. Gary LaRose, Edward Merrill Smith, and Leo M. Stankus.

Mr. D'Arcy said the Trustees for the CAS Trust (CAST) were pleased to recognize D.W. Simpson & Company, which has donated $10,000 to the Trust. Their cumulative donations to the CAS Trust have now reached the milestone of $100,000.

Brian Johnson, committee representative from the Joint CAS/SOA Committee on Minority Recruiting, discussed their work and presented the college scholarship to recipient Chris Martin. Other recipients of the scholarship included Denita Hill, Kayun Ng, Diandra Daniel, Tyre Wise, Desmond Cooper, Jacob Mallol, and Nahathai Srivali. Mr. D'Arcy also announced that two high school teachers were in attendance accompanying their students: Joshua Bessicks of Baltimore Polytechnical High School and Victoria Stephenson of Western High School.

Mary Weiss of Temple University gave the ARIA presentation. Ms. Weiss currently is the Vice President of ARIA and also the editor of *Risk Management and Insurance Review*, a journal that ARIA publishes specifically to encourage a research interface between academe and industry.

In conclusion, Mr. D'Arcy gave his presidential address and officially passed on the CAS presidential gavel to new CAS President Paul Braithwaite. Mr. Braithwaite gave a brief overview of the future year before closing the business session.

After a refreshment break, the first General Session was held from 10:30 a.m. to 12:00 p.m.:

Our Credibility at Risk? Loss Reserves—Facts and Perceptions

Moderator: Allan M. Kaufman
Actuary and Consultant
AMK Consulting

Panelists: Steve Dreyer
Practice Leader
North America Insurance Ratings,
Standard & Poor's

Mary D. Miller
Actuary
Ohio Department of Insurance

Patrica A. Teufel
Principal
KPMG LLP

Following the general session, Roger Lowenstein was the featured speaker during a luncheon from 12:00 p.m. to 1:30 p.m. Lowenstein has authored several books including *Buffett: The Making of an American Capitalist* (1995), *When Genius Failed: The Rise and Fall of Long-Term Capital Management* (2000), and *Origins of the Crash* (2004). He has also written for many other publications, including *Smart Money*, *The Wall Street Journal*, and *The New York Times*.

After the luncheon, the afternoon was devoted to concurrent sessions. The panel presentations covered the following topics:

1. What's Happening in the Reinsurance Market?

Moderator: Richard A. Lino
Consulting Actuary
Pinnacle Actuarial Resources Inc.

Panelists: Yves Provencher
Senior Vice President
Willis Re Inc.

Joy Y. Takahashi
Senior Vice President-Corporate
American Re-Insurance Company

2. Sarbanes 404—Risks and Controls for Actuarial Processes

Moderator: Alan M. Hines
Director
PricewaterhouseCoopers LLP

Panelists: John Brabazon
Vice President and Assistant Corporate Controller
Allmerica Financial

Heidi M. Hoeller
Senior Manager
PricewaterhouseCoopers LLP

3. Enterprise Risk Management—The Present and the Future CAS

Moderator/Panelist: James E. Rech
Actuary
GPW & Associates

Panelists: Christopher David Bohn
Assistant Director and Actuary
Aon Risk Consultants Inc.

Gary G. Venter
Managing Director
Guy Carpenter & Company Inc.

4. Discussion of the Claim Liability Estimation Proposed Standard

Moderator: Raji Bhagavatula
Principal
Milliman USA
Chair Subcommittee on Reserving of the Casualty Committee of the Actuarial Standards Board

Panelists: Ralph S. Blanchard III
Second Vice President and Actuary
Travelers Property Casualty Insurance Company

Christopher S. Carlson
Consultant
Pinnacle Actuarial Resources Inc.

Jason L. Russ
Consulting Actuary
Milliman Inc.

5. Presidents' Forum

Moderator: Stephen P. D'Arcy
President
Casualty Actuarial Society

Panelists: Charles C. McLeod
President
Canadian Institute of Actuaries

Ana Maria Ramirez
President
Asociación Mexicana de Actuarios

6. Accounting Implications of Actuarial Decisions

Panelist: Alison T. Spivey
Associate Chief Accountant and Office of the Chief Accountant
U.S. Securities and Exchange Commission

7. Actuaries Embrace Operational Risk

Moderator: Mark Alan Verheyen
Vice President
Carvill America

Panelists: Ali Samad-Khan
President
OpRisk Advisory LLC

Samir Shah
Principal
Towers Perrin

The following Proceedings papers were presented:

1. "The Application of Fundamental Valuation Principles to Property/Casualty Insurance Companies"

Authors: Wayne E. Blackburn
Milliman Inc.

Derek A. Jones
Milliman Inc.

Joy A. Schwartzman
Milliman Inc.

Dov A. Siegman
Milliman Inc.

2. "When Can Accident Years Be Regarded As Development Years?"

Authors: Glen Barnett
Insureware Pty. Ltd.

Ben Zehnwirth
Insureware Pty. Ltd.

Eugene Dubossarsky
Ernst & Young ABC Pty. Ltd.

After a refreshment break from 3:00 p.m. to 3:30 p.m., concurrent sessions continued. Certain concurrent sessions presented earlier were repeated. Additional concurrent sessions presented from 3:30 p.m. to 5:00 p.m. were:

1. Enhancing the Reputation of the Actuary Task Force and the Potential Changes to ASOP 36

Moderator: Christopher S. Carlson
Pinnacle Actuarial Resources, Chairperson
Casualty Committee of the Actuarial
Standards Board

Panelist: Patricia Teufel
KPMG LLP, Chairperson
CAS Enhancing the Reputation of the Actuary Task Force

2. Actuarial Opinions and Risk of Material Adverse Deviation

Moderator: Robert F. Wolf
Principal
Mercer Oliver Wyman

Panelists: Charles F. Cook
Consulting Actuary
MBA Inc.

James Votta
Partner
Ernst & Young LLP

3. International Actuarial Practice Guidelines: The Impact on the U.S. Actuary

Moderator: Amy Bouska
Consulting Actuary
Towers Perrin

Panelist: Robert Miccolis
Director
Deloitte Consulting LLP
Drafting Member
International Actuarial Association
Subcommittee on Actuarial Standards and
CAS Representative

4. CAS Myth Busters, Demystifying Volunteering

Moderator/Panelists: Robert J. Walling
Principal and Consulting Actuary
Pinnacle Actuarial Resources Inc.

Jeremy Brigham
Consultant
Towers Perrin

5. CAS Examination Process

Moderator: Daniel G. Roth
Vice President and Actuary
CNA Insurance Companies

Panelists: Steven D. Armstrong
Senior Actuary
Allstate Insurance Company

Manalur S. Sandilya
Corporate Actuary
Max RE Europe Ltd.

Thomas Struppeck
Director
CIFG

6. General Trends in Tort Litigation

Moderators/Panelists: Claire M. Louis
Director
PricewaterhouseCoopers LLP

Stephen P. Lowe
Managing Director
Towers Perrin

A reception for new Fellows and guests was held from 5:30 p.m. to 6:30 p.m. The general reception for all members and their guests was held from 6:30 p.m. to 7:30 p.m.

Mr. D'Arcy and his wife Cleo hosted an evening gathering in their hospitality suite from 9:00 p.m. to 11:00 p.m.

Tuesday, November 15, 2005

Registration and a continental breakfast took place from 7:00 a.m. to 8:00 a.m.

Two General Sessions were held simultaneously from 8:00 a.m. to 9:30 a.m.:

Rating Agencies

Moderator: Thomas Conway
Partner
Ernst & Young LLP

Panelists: Keith M. Buckley
Group Managing Director-Insurance/Financial Guarantors
Fitch Ratings

Matthew Mosher
Group Vice President–P/C
A.M. Best Company

Risk Transfer

Moderator: Peter M. Licht
Managing Director
PricewaterhouseCoopers LLP

Panelists: Donald Doran
Partner
PricewaterhouseCoopers LLP

John M. Purple
Chief Actuary
State of Connecticut Insurance Department

Michael G. Wacek
President
Odyssey America Reinsurance Company
Chairman of the CAS Working Party on Risk Transfer Testing

After a refreshment break, concurrent sessions were held from 10:00 a.m. to 11:30 a.m. In addition to concurrent sessions that were presented the previous day, the following sessions were presented.

1. Current Loss Reserving Developments

Moderator/Panelist: John J. Kollar
Vice President
ISO

Panelists: Charles C. Emma
Principal
Pinnacle Actuarial Resources Inc.

Thomas A. Ryan
Consulting Actuary
Milliman USA

2. What's the Future of Asbestos Legislation?

Moderator: Sandra C. Santomenno
Senior Actuary
GE Insurance Solutions

Panelists: Jennifer L. Biggs
Consulting Actuary
Towers Perrin

Philip Goldberg
Shook Hardy & Bacon

Mary Z. Seidel
Vice President and Director of Federal Affairs
Reinsurance Association of America

3. Actuarial Techniques in Banking Operational Risk

Moderator/Panelist: Ali Samad-Khan
President
OpRisk Advisory LLC

4. Predictive Modeling—What Is Out There?

Moderator/Panelist: Serhat Guven
Consultant
EMB America LLC

Panelist: Louise A. Francis
Consulting Principal
Francis Analytics & Actuarial Data Mining Inc.

5. General Business Skills I: Strategic Thinking Presentation

Moderator/Panelist: Eli Harari
The Thinking Coach

6. Town Hall Meeting: Statements of Actuarial Opinions—Today and in the Year 2014

Moderator: Robert F. Wolf
Principal
Mercer Oliver Wyman

Panelists: Chester John Szczepanski
Vice President and Chief Actuary
Donegal Insurance Group

Patrica A. Teufel
Principal
KPMG LLP

Mary D. Miller
Actuary
Ohio Department of Insurance

7. Enterprise Risk Management—The Present and the Future CAS

Moderator/Panelist: James E. Rech
Actuary
GPW & Associates

Panelists: Christopher David Bohn
Assistant Director and Actuary
Aon Risk Consultants, Inc.

Gary G. Venter
Managing Director
Guy Carpenter & Company, Inc.

An ARIA Prize Paper and a *Proceedings* paper were also presented during this time:

1. "Bonus-Malus Scales in Segmented Tariffs With Stochastic Migration Between Segments" (ARIA Prize Paper)

Authors: Natacha Brouhns

Montserrat Guillén

Michel Denuit

Jean Pinquet

Moderator/ Panelist: Jean Pinquet
Professor
University of Paris

2. "Modeling Financial Scenarios: A Framework for the Actuarial Profession"

Authors: Kevin C. Ahlgrim
Illinois State University

Stephen P. D'Arcy
University of Illinois

Richard W. Gorvett
University of Illinois

Attendees enjoyed a boxed lunch from 11:30 a.m. to 12:30 p.m. before concurrent sessions continued from 12:30 p.m. to 2:00 p.m.

1. What's the Value in Value-Added Reserving?

Moderator/ Panelist: Martha Winslow
Senior Consultant
Towers Perrin

Panelists: Sean Duffy
Second Vice President, Specialty Claims
St. Paul Travelers

Thomas Ghezzi
Principal
Towers Perrin

Kevin Rehnberg
Senior Vice President
OneBeacon Insurance Companies

2. Progress Reports: The Dynamic Risk Modeling Handbook Working Party and the Public-Access DFA Model Working Party

Moderator: Mark R. Shapland
Actuary
Milliman Inc.

Panelists: James E. Rech
Vice President
GPW and Associates Inc.

Patrick J. Crowe
Vice President and Actuary, Market Research
Kentucky Farm Bureau

3. COTOR Challenge: Round 3

Moderator: Steven M. Visner
Principal
Deloitte Consulting LLP

Panelists: Winners of COTOR Challenge: Round 3

4. General Business Skills II: Strategic Thinking Presentation

Moderator/Panelist: Eli Harari
The Thinking Coach

5. Predictive Modeling—Pitfalls and Potentials

Moderator/Panelist: Jeffrey L. Kucera
Senior Consultant
EMB America LLC

Panelists: Michael R. Larsen
Property Consultant
The Hartford

John R. Pedrick
Assistant Director
Ohio Department of Insurance

The following *Proceedings* papers were presented:

1. "Incorporation of Fixed Expenses"

Author: Geoffrey Todd Werner
EMB America LLC

2. "A Modern Architecture for Residential Property Insurance Ratemaking"

Author: John W. Rollins
Watson Wyatt Insurance & Financial Services Inc.

All meeting participants and their guests enjoyed dinner and entertainment at the Maryland Science Center from 6:30 p.m. to 9:30 p.m.

Wednesday, November 16, 2005

A continental breakfast was held from 7:00 a.m. to 9:00 a.m.

In addition to concurrent sessions that had been given previously and which were repeated, the following concurrent sessions were presented from 8:00 a.m. to 9:30 a.m.

1. The Road to 2014: The Centennial Goal/Long Range Planning Committee Report

Moderator: Aaron M. Halpert
Principal
KPMG LLP

Panelist: Larry A. Haefner
Vice President and Actuary
St. Paul Travelers Inc.

2. Progress Reports: The Tail Factor Working Party and the Bornhuetter-Ferguson-Initial Expected Losses Working Party

Moderator: Thomas A. Ryan
Consulting Actuary
Milliman Inc.
Chairperson of the Committee on Reserves

Panelists: F. Douglas Ryan
Consulting Actuary
MBA Actuaries Inc.

Mark R. Shapland
Actuary
Milliman Inc.

Members of the Tail Factor Working Party and the Bornhuetter-Ferguson-Initial Expected Losses
Working Party

3. Advancements in Hurricane Risk Management

Moderator: Michael A. Walters
Consulting Actuary
Towers Perrin

Panelists: Richard R. Anderson
Chief Actuary
Risk Management Solutions Inc.

Sean R. Devlin
Chief Pricing Actuary–P&C Reinsurance
GE Insurance Solutions

Alice H. Gannon
Senior Vice President
United Services Automobile Association

4. Progress Report: Elicitation and Elucidation of Risk Preferences Working Party

Moderator/Panelists: David L. Ruhm
Portfolio Risk Manager
Hartford Investment Management

Parr T. Schoolman
Manager
Ernst & Young LLP

The following *Proceedings* papers were presented:

1. Discussion of "The 'Modified Bornhuetter-Ferguson' Approach To IBNR Allocation"

Author: Glenn M. Walker
G. M. Walker Actuarial Services

2. Author Response to a Discussion of "Application of the Option Market Paradigm to the Solution of Insurance Problems"

Author: Michael G. Wacek
Odyssey America Reinsurance Corporation

3. Discussion of "Distribution-Based Pricing Formulas are not Arbitrage-Free"

Panelist: Gary G. Venter
Guy Carpenter & Company, Inc.

A final General Session was held from 10:00 a.m. to 11:30 a.m. after a 30-minute refreshment break.

Developments in Regulatory Capital Models Around the World

Moderator: Elise C. Liebers
Insurance Specialist
Federal Reserve Bank of New York

Panelists: Lou Felice
Assistant Chief Examiner
New York Insurance Department

Mary Frances Monroe
Manager, Supervisory & Risk Policy-Division of Banking Supervision and Regulation
Federal Reserve Board

Gary Wells
Principal
Milliman Inc.

Mr. Braithwaite officially adjourned the 2005 CAS Annual meeting at 11:45 a.m. after closing remarks and an announcement of future CAS meetings.

Attendees of the 2005 CAS Annual Meeting

The 2005 CAS Annual Meeting was attended by 318 Fellows, 186 Associates, and 53 guests. The names of the Fellows and Associates in attendance follow:

FELLOWS

Martin Adler
Richard R. Anderson
Steven D. Armstrong
Lawrence J. Artes
Carl Xavier Ashenbrenner
Richard V. Atkinson
Craig Victor Avitabile
Maura Curran Baker
W. Brian Barnes
Rose D. Barrett
Patrick Beaudoin
Robert S. Bennett
Regina M. Berens
Derek Dennis Berget
Wayne F. Berner
Ellen A. Berning
Raji Bhagavatula

Brian J. Biggs
Jennifer L. Biggs
Corey J. Bilot
Rebekah Susan Biondo
Wayne E. Blackburn
Jonathan Everett Blake
Ralph S. Blanchard
Tapio N. Boles
Theresa W. Bourdon
Amy S. Bouska
Alicia E. Bowen
Paul Braithwaite
Yaakov B. Brauner
James L. Bresnahan
Jeremy James Brigham
Karen E. Brinster
John R. Broadrick
Sara T. Broadrick
Michele L. Brooks
Elaine K. Brunner
Matthew D. Buchalter
Mark E. Burgess
Kevin D. Burns
Anthony Robert Bustillo
John F. Butcher
Amber L. Butek
Christopher S. Carlson
Hsiu-Mei Chang
Hong Chen
Yvonne W. Y. Cheng
Alan M. Chow
Jason T. Clarke
Eric John Clymer
Michael A. Coca
Eugene C. Connell
Cameron A. Cook
Charles F. Cook
Christopher William Cooney
Kevin A. Cormier
Patrick J. Crowe
Justin B. Cruz
Jonathan Scott Curlee
Loren Rainard Danielson
Stephen P. D'Arcy
Jeffrey F. Deigl
Chantal Delisle
Peter R. DeMallie
Sean R. Devlin
Laura S. Doherty
Michael Edward Doyle
Grover M. Edie
Tomer Eilam
Thomas J. Ellefson
Charles C. Emma
Paul E. Ericksen
Bruce Fatz
Vicki A. Fendley
Dale A. Fethke
Beth E. Fitzgerald
Louise A. Francis
Michael Fusco
Alice H. Gannon
James J. Gebhard
William John Gerhardt
Thomas L. Ghezzi
John F. Gibson
Patrick John Gilhool
Kristen Marie Gill
Isabelle Girard
Bradley J. Gleason
Spencer M. Gluck
James F. Golz
Annette J. Goodreau
Christopher David Goodwin
David B. Gordon
Eric L. Greenhill
Jeffrey Robert Grimmer
Victoria Grossack
Serhat Guven
Edward Kofi Gyampo
Nasser Hadidi
Larry A. Haefner
Marc S. Hall
Robert C. Hallstrom
Brian D. Haney
Jonathan M. Harbus
Megan Taylor Harder
Allison Michelle Harris
David G. Hartman
Robin A. Haworth
Gordon K. Hay
Brandon L. Heutmaker
Laura Esboldt Heyne
Mark D. Heyne
Joseph S. Highbarger
Anthony D. Hill
Alan M. Hines
Amy L. Hoffman
Ruth A. Howald
Thomas A. Huberty
Christopher Wayne Hurst
Richard M. Jaeger
John F. Janssen

Richard Clay Jenkins
Shiwen Jiang
Yi Jing
Eric J. Johnson
Mark Robert Johnson
Thomas S. Johnston
Derek A. Jones
Gary R. Josephson
Stephen H. Kantor
Allan M. Kaufman
Clive L. Keatinge
Susan M. Keaveny
Frederick W. Kilbourne
Omar A. Kitchlew
Scott M. Klabacha
Brandelyn Klenner
Fredrick L. Klinker
Jeff A. Kluck
Raymond J. Kluesner
Leon W. Koch
John J. Kollar
Henry Joseph Konstanty
Andrew M. Koren
Gustave A. Krause
Rodney E. Kreps
Kenneth R. Krissinger
Jeffrey L. Kucera
Andrew E. Kudera
Kay E. Kufera
Kristine Kuzora
Dennis L. Lange
Gregory D. Larcher
Michael R. Larsen
Francis A. Laterza
Jason A. Lauterbach
Todd Lehmann
Kahshin Leow
Xin Li
Peter M. Licht
Elise C. Liebers
Matthew Allen Lillegard
Dengxing Lin
Shu C. Lin
Richard A. Lino
Erik Frank Livingston
Richard W. Lo
Edward P. Lotkowski
Stephen P. Lowe
Rimma Maasbach
Brett A. MacKinnon
Eric A. Madia
Donald F. Mango
Donald E. Manis
Laura S. Martin
Jonathan L. Matthews
Michael G. McCarter
Jeffrey F. McCarty
Kevin Paul McClanahan
Brent L. McGill
Christopher C. McKenna
David L. Menning
Stephen V. Merkey
Robert E. Meyer
Stephen J. Meyer
Glenn G. Meyers
Robert S. Miccolis
Mary D. Miller
Meagan S. Mirkovich
John H. Mize
Rodney S. Morris
Matthew C. Mosher
Roosevelt C. Mosley
Evelyn Toni Mulder
Raymond D. Muller
Thomas G. Myers
Leonidas V. Nguyen
William A. Niemczyk
James R. Nikstad
Tom E. Norwood
Miodrag Novakovic
David J. Oakden
Timothy James O'Connor
Teresa K. Paffenback
Donald W. Palmer
Jacqueline Edith Pasley
Michael Thomas Patterson
Harry Todd Pearce
Kathleen M. Pechan
Jeremy Parker Pecora
John R. Pedrick
Brian G. Pelly
Jeffrey J. Pfluger
Marian R. Piet
Jordan J. Pitz
Kristine E. Plickys
Jayne L. Plunkett
Gregory T. Preble
Virginia R. Prevosto
John M. Purple
Mark S. Quigley
Kenneth Quintilian
Michele S. Raeihle

Rajagopalan K. Raman
Scott E. Reddig
Elizabeth M. Riczko
Dale M. Riemer
Brad E. Rigotty
Laura D. Rinker
Robert S. Roesch
Rebecca L. Roever
John W. Rollins
Sheldon Rosenberg
Christine R. Ross
Gail M. Ross
Daniel G. Roth
Jean Aviva Roy
Jean-Denis Roy
David L. Ruhm
Jason L. Russ
Bryant Edward Russell
Frederick Douglas Ryan
Thomas A. Ryan
Manalur S. Sandilya
Frances G. Sarrel
Jeffery Scholl
Parr T. Schoolman
Genine Schwartz
Joy A. Schwartzman
Jin Shao
Mark R. Shapland
Junning Shi
Jeremy D. Shoemaker
Lisa A. Slotznick
Justin Nicholas Smith
Richard H. Snader
Joanne S. Spalla
David Spiegler
Daniel L. Splitt
Michael William Starke
Karine St-Onge
Mark Stephen Struck
Thomas Struppeck
Zongli Sun
Keith Jeremy Sunvold
Scott J. Swanay
Chester Szczepanski
Varsha A. Tantri
Karen F. Terry
Patricia A. Teufel
Kevin B. Thompson
Chris S. Throckmorton
Jennifer L. Throm
Dovid C. Tkatch
Jennifer M. Tornquist
Joel A. Vaag
Eric Vaith
William R. Van Ark
Daniel Jacob VanderPloeg
Oakley E. Van Slyke
Gary G. Venter
Mark Alan Verheyen
Kevin K. Vesel
Marie-Eve J. Vesel
Jennifer S. Vincent
Steven M. Visner
William J. VonSeggern
James C. Votta
Mary Elizabeth Waak
Michael G. Wacek
Christopher P. Walker
Glenn M. Walker
Joseph W. Wallen
Robert J. Walling
Michael A. Walters
Mo Wang
Bryan C. Ware
Kevin E. Weathers
Peter A. Weisenberger
Joseph C. Wenc
Geoffrey Todd Werner
William B. Westrate
Mark Whitman
Kevin L. Wick
Kendall P. Williams
Martha A. Winslow
Dean M. Winters
Robert F. Wolf
Patrick B. Woods
Micah G. Woolstenhulme
Yuanhe Yao
Gerald T. Yeung
Sung G. Yim
Richard P. Yocius
Ronald J. Zaleski Jr.
Ronald J. Zaleski Sr.
Doug A. Zearfoss
Lijuan Zhang
Hongbo Zhou

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Jodie Marie Agan
Amit Alea Agarwal
Sajjad Ahmad
Gwendolyn L. Anderson
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Kelleen D. Arquette
Yanfei Z. Atwell
Nathan J. Babcock
Gregory S. Babushkin
Kristi Spencer Badgerow
Tiffany Jean Baron
Angelo Edward Bastianpillai
Jeffrey Donald Bellmont
Matthew C. Berasi
Jonathan Bilbul
Brad Stephen Billerman
Jon Paul Bloom
Peter George Blouin
Sharon D. Blumer
Christopher David Bohn
Nicolas Boivin
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David A. Christhilf
Raul Cisneros
Donald L. Closter
Glenn A. Colby
Kirk Allen Conrad
Thomas P. Conway
Chad J. Covelli
Daniel A. Crifo
Tighe Christian Crovetti
Walter C. Dabrowski
Raymond V. Debs
Krikor Derderian
Jonathan E. Devilbiss
Brent P. Donaldson
Alice H. Edmondson
Michael D. Ersevim
Juan Espadas
Brian A. Evans
Stephen Charles Fiete
Jason A. Flick
Mark A. Florenz
Kyle P. Freeman
Derek W. Freihaut
Stuart G. Gelbwasser
Maxime Gelinas
Simon Girard
Gregory P. Goddu
Rebecca J. Gordon
Dawson T. Grubbs
Gerald S. Haase
William Joseph Hackman
Brian P. Hall
Aaron M. Halpert
James W. Harmon
Philip E. Heckman
Kathryn E. Herzog
Megann Elizabeth Hess
Nathan Jaymes Hubbell
Jeffrey R. Ill
John J. Javaruski
Min Jiang
Ziyi Jiao
Brian E. Johnson
Jeremiah D. Johnson
Ross Evan Johnson
Amy Ann Juknelis
David L. Kaufman
Jean-Philippé Keable
Sarah M. Kemp
Martin T. King
Diane L. Kinner
David J. Klemish
Rachel M. Klingler
Christine K. Kogut
Thomas R. Kolde
Leland S. Kraemer
Annie Latouche
Kak Lau
Khanh M. Le
Sean M. Leonard
Jean-François Lessard
Mingyue Li
Sharon Xiaoyin Li
Andy Hankuang Liao

Cunbo Liu
Jin Liu
Nannan Liu
Todd L. Livergood
Andrew F. Loach
Laura J. Lothschutz
Neelam P. Mankoff
Minchong Mao
Albert-Michael Micozzi
Daniel E. Mikesh
Aaron G. Mills
Richard James Mills
Lori A. Moore
Allison L. Morabito
Mundia Mubyana
Daniel G. Myers
Marc L. Nerenberg
Anthony J. Nerone
Stephanie Jo Odell
Christopher John Olsen
Alejandro Antonio Ortega
Wade H. Oshiro
Keith William Palmer
Joy-Ann C. Payne
Willard W. Peacock
Robert B. Penwick
Robert C. Phifer
Joseph G. Pietraszewski
Susan R. Pino
Jean-Philippe Plante
Ruth Youngner Poutanen
Yves Provencher
Arthur R. Randolph
James E. Rech
Cynthia L. Rice
Zoe F. Rico
Christopher R. Ritter
Benjamin G. Rosenblum
Randall D. Ross
John C. Ruth
Sandra C. Santomenno
Anita A. Sathe
Lawrence M. Schober
Erika Helen Schurr
Ronald S. Scott
Sheri Lee Scott
Ben Silberstein
Charles Sizer
Jeffery J. Smith
Katherine R.S. Smith
Robert K. Smith
Patrick Shiu-Fai So
Joanna Solarz
Jessica Elsinger Somerfeld
Richard Cambran Soulsby
Michael P. Speedling
Paul Quinn Stahlschmidt
Mindy M. Steichen
Yuchen Su
Beth M. Sweeney
Joy Y. Takahashi
Luc Tanguay
Aaron A. Temples
Robert Bradley Tiger
Phoebe A. Tinney
Thomas A. Trocchia
Jonathan Turnes
Susan B. VanHorn
John E. Wade
David G. Walker
Todd Patrick Walker
Xuelian Wan
Felicia Wang
Jingtao Wang
Scott Werfel
Amanda J. White
Andrew T. Wiest
Martin E. Wietfeldt
Ronald Harris Wilkins
Shauna S. Williams
Benjamin T. Witkowski
Dorothy A. Woodrum
Donald S. Wroe
Xinxin Xu
Yi-Chuang Yang
Min Yao
Yanjun Yao
Joshua A. Youdovin
Wei Zhao
Michael V. Ziniti

REPORT OF THE VICE PRESIDENT–ADMINISTRATION

This report provides a one-year summary of Casualty Actuarial Society activities since the 2004 CAS Annual Meeting. I will first comment on these activities as they relate to the organization's purposes as stated in the CAS constitution, which are to:

1. Advance the body of knowledge of actuarial science applied to property, casualty, and similar risk exposures;

2. Establish and maintain standards of qualifications for membership;

3. Promote and maintain high standards of conduct and competence for the members; and

4. Increase the awareness of actuarial science.

I will then provide a summary of other activities that may not relate to a specific purpose, but are critical to the ongoing vitality of the CAS. Finally, I will summarize the current status of our finances and key membership statistics.

CAS ACTIVITIES

1. Advance the body of knowledge of actuarial science applied to property, casualty, and similar risk exposures.

A significant step in achieving this goal came in 2005 when the CAS announced plans to launch a new peer-reviewed journal. The journal will publish practical research that can be applied by the practicing actuary. Curtis Gary Dean was appointed editor in chief and Richard Fein, Dale Edlefson, and Gary Venter were named associate editors in charge of peer review, copy editing, and development, respectively. Following is the journal's adopted mission statement:

> The Journal (which is not yet named) is a peer-reviewed journal published by the Casualty Actuarial Society to disseminate work of interest to casualty actuaries worldwide. The Journal's focus is original practical and theoretical research in casualty actuarial science. Significant survey or similar articles are also considered for publication. Membership in the Casualty Actuarial Society is not a prerequisite for submitting papers to the Journal and submission by non-CAS members is encouraged.

Publication of the *Forum* and the *Proceedings of the Casualty Actuarial Society* provides significant means for the advancement of the body of actuarial science. The 2005 winter, spring, and fall volumes of the *Forum* focus on ratemaking, reinsurance, and research working party papers, respectively. The *Proceedings* include papers exploring many different subjects—from modeling financial scenarios to estimating the workers compensation tail.

The CAS also approved $9,500 in funding from the CAS Research Fund for a Society of Actuaries Committee on Knowledge Extension Research (CKER) grant proposal. Vytaras Brazauskas of the University of Wisconsin in Milwaukee proposed the study, "Robust and Efficient Methods for Credibility." The study will develop reliable credibility estimators that are designed to perform well under realistic model assumptions. The CAS Committee on Theory of Risk (COTOR) reviewed the project and concluded that the research represents groundbreaking work of great interest to CAS members. COTOR will also oversee the project.

With an eye on promoting actuarial science concepts to a wider business audience, the Enterprise Risk Management Task Force was charged with implementing an ERM process for the CAS as an example of ERM best practices. Once the ERM study is completed, it could be made the basis of an article in the association management press. This study would include quantifying certain types of risks facing the CAS, as well as identifying opportunities.

2. Establish and maintain standards of qualifications for membership.

CAS Admissions Committees and Task Forces pursued a number of developments. The Task Force on Study Materials recommended that the CAS issue an RFP to develop more effective study materials for basic education. In response to the recommendation, the Syllabus Committee issued an RFP for an integrated study note on catastrophe ratemaking in July 2005 and contracted with a vendor in November 2005. The committee liaison will continue to work with the author to finalize the paper in 2007. This pilot project will help to determine whether the RFP process is appropriate for creating integrated study notes.

The CAS also made strides in mutual recognition, executing agreements with the United Kingdom's Institute of Actuaries and the Faculty of Actuaries. The mutual recognition agreements give Fellows of the CAS the opportunity to become Fellows of either the Institute of Actuaries or the Faculty of Actuaries. Additionally, members of the U.K. associations also have the opportunity to join the CAS. Following those agreements, the Society of Actuaries in Ireland (SAI) determined that CAS Fellows were eligible for Fellowship in the SAI, subject to having at least three years of recent and appropriate practical experience, with at least one of those years working within the Republic of Ireland.

In considering a move to a single class of membership, the board instructed the executive council to establish the Task Force on FCAS Education. The task force was charged with proposing a set of learning objectives by which the FCAS designation can be obtained. These learning objectives should result in the ability to attain the FCAS designation with less material than the current exam, (i.e., with fewer than the current 9 exams), while meeting the requirements of the IAA and being consistent with the CAS Centennial Goal. The task force concluded that it required additional education policy guidance in order to complete this charge and requested that the board of directors articulate a specific policy with respect to the capabilities of its future members. The task force listed ten specific areas to be addressed. A separate task force, the FCAS Education Board Task Force, was created in order to provide the desired guidance and address the concerns.

In November 2005 the board decided to move ahead with the issues of giving the ACAS the right to vote and moving to one class of membership, despite the fact that the education issue had not yet been resolved. The board instructed the CAS Executive Director and the VP–Administration to delineate a process whereby the awarding of ACAS could be phased out within five years, and the vote could be given to ACAS immediately (subject to the constraints previously prescribed by board resolutions). The proposition for the ACAS vote is scheduled to be placed on the membership election ballot in summer 2006. Proposed language for the ballot would be considered by the board at its March 2006 meeting.

3. Promote and maintain high standards of conduct and competence of members.

Throughout the years, the CAS quality programs of continuing education opportunities and the Code of Professional Conduct have successfully fulfilled this purpose. The CAS provides education opportunities through the publication of actuarial materials and the sponsorship of meetings and seminars. This year's meetings and seminars included the following:

Meetings:

	Location	CAS Members
Spring	Phoenix	527
Annual	Baltimore	512

Seminars:

Topic	Location	CAS Members
Ratemaking	New Orleans	284
Symposium on Enterprise Risk Management	Chicago	72
Reinsurance	Bermuda	422
Casualty Loss Reserves	Boston	429
Special Interest Seminar on Predictive Modeling	Chicago	171
Course on Professionalism, December 2004	2 locations	111
Course on Professionalism, June 2005	2 locations	120

Limited attendance seminars included two sessions on "Reinsurance" and one session each on "Practical Applications of Loss Distributions" and "Asset Liability Management Principles of Finance." the CAS responded to the growing number of candidates in Asia when Hong Kong hosted a CAS Course on Professionalism (COP)—the first COP held outside the United States and Canada.

The executive council also approved creating audio recordings of a select number of sessions from the 2005 CAS Annual Meeting, which were made available through the CAS Web Site. Four online courses were also conducted on the Web site.

4. Increase the awareness of actuarial science.

In 2005 the CAS, along with SOA, AAA, CIA, and CCA launched a public relations campaign to promote establishing the chief risk officer position in U.S. and Canadian businesses. An Image Advisory Group representing the entire profession guided the campaign. The image campaign's goals are to create a more dynamic and relevant image in the minds of the employers, build a vibrant new image within the profession itself, and create a sustainable program that builds on each success.

The initial focus of the program includes CEOs and CFOs in insurance, reinsurance, and consulting firms, and actuaries, recruiters, and decision makers in the "broader financial services" sector (e.g., investment banks and mutual fund companies).

Immediate plans for the campaign include introducing an advertising program to build grassroots understanding and involvement within the profession, developing a resource kit enabling actuaries to communicate their value more effectively, and implementing a public relations program targeting key decision makers on enterprise risk management.

Another effort to promote the actuarial profession involves reaching out to young people with interests in math and science. The CAS, in conjunction with the Minority Recruitment Committee, awarded four Baltimore high school students with $500 college scholarships at the 2005 CAS Annual Meeting. The students, who were honored for their achievements in mathematics, were Nahathai Srivali, Desmond Cooper, Tyre Wise, and Denita Hill.

OTHER CAS ACTIVITIES

Several other CAS activities contributed to the ongoing vitality to the organization during 2005. The Publications Implementation Task Force (PITF) was created to implement the recommendations of the Publications Task Force. Chief among the PITF's tasks is to launch the new CAS journal and to integrate it with existing CAS publications. The PITF recommended that the *Proceedings* continue to be published with CAS specifics such as meeting minutes, financial reports, and obituaries, and that all peer-reviewed papers be published in the new journal. The contents of the *Yearbook*, minus the membership directory, will also be included combined with the *Proceedings* to create a new CAS publication. A yearly hard copy of the membership directory will be available to members who request it, though it is expected that many members will opt to use the more up-to-date online directory.

The Society achieves quite a lot through the volunteer efforts of members. To acknowledge the vital participants in CAS activities, CAS established the Above and Beyond Achievement Award as a way to recognize one-time accomplishments occurring over the past two years by volunteers. The CAS awarded the 2005 ABAA to David Menning, Kristine Kuzora, Jerome Vogel, and Michael Wacek.

David Menning has been highly effective in leading the CAS's joint efforts with the SOA and CIA to implement computer-based testing (CBT) for Exam 1. Kristine Kuzora volunteered on the Research Paper Classifier Project where she classified over 600 abstracts with the new CAS Research Taxonomy—in just 25 days. The CAS honored Jerome Vogel for working on the Research Paper Classifier Project Committee. At the time the award was given, he had classified over 500 abstracts, or 13 percent of the 4,200 abstracts needing classification. Michael Wacek was selected for his work in chairing the CAS Working Party on Risk Transfer Testing where he produced a thorough, high-quality research paper in just one month's time.

In December the CAS office moved to a new location in Arlington, Virginia. The larger office space allows for more staff growth and is in closer proximity to the Washington, D.C. Metrorail system.

MEMBERSHIP STATISTICS

Membership growth continued with 180 new Associates and 132 new Fellows. The total number of members as of November 2005 was 4,150, up four percent from the previous year. The CAS reached the 4,000 member milestone with the addition of 22 new members on February 16, 2005.

GOVERNANCE

With the aim of expanding governance to include an emerging field of actuarial science, the board approved a new executive council position of vice president–risk integration. The function of the new position is to coordinate all CAS activities relating to risk integration, with specific attention to integrating hazard risk with financial, strategic, and operational risk. John J. Kollar was appointed the first Vice President–Risk Integration in fall 2005.

Thomas G. Myers was elected president-elect for 2005–2006. CAS Fellows also elected Irene K. Bass, Glenn Meyers, Donald F. Mango, and Roosevelt C. Mosley Jr. to the CAS Board of Directors. Paul Braithwaite assumed the presidency.

The CAS Board of Directors elected the following vice presidents for the coming year: Deborah M. Rosenberg, Vice President–Administration; James K. Christie, Vice President–Admissions; Amy S. Bouska, Vice President–International; Joanne Spalla, Vice President–Marketing and Communications; Beth E. Fitzgerald, Vice President–Professional Education; Roger M. Hayne, Vice President–Research and Development; and John J. Kollar, Vice President–Risk Integration. The CAS Executive Council met either by telephone or in person at least once a month during the year (except September) to discuss day-to-day and long-range operations.

FINANCIAL STATUS

The CPA firm Langan Associates PC examined the CAS books for fiscal year 2005 and the CAS Audit Committee reported the firm's findings to the CAS Board of Directors in March 2006. The fiscal year ended with an audited net gain of $413,509 compared to a budgeted net loss of $114,363.

The CAS Surplus now stands at $3,725,552. This represents an increase in surplus of $413,509 over the amount reported last year. In addition to the net gain from operations of $151,823, there was interest and dividend revenue of $155,411, an unrealized gain of $109,210, and a realized loss of $2,936. There was also a total net increase of $145,461 in various research, prize, and scholarship accounts (including the CAS Trust) arising from the difference between incoming funds and interest earned less expenditures, and a favorable adjustment to the CAS pension liability. These amounts are not reflected in the net revenue from operations. Total Members' Equity (CAS Surplus plus nonsurplus accounts) now stands at $4,155,268, an overall increase of $558,970 over last year.

For 2005–2006, the CAS Board of Directors has approved a budget of approximately $5.9 million. Members' dues for next year will be $365, an increase of $10, and fees for the Subscriber Program will increase $10 to $435. A $45 discount is available to members and subscribers who elect to receive the *Forum* and *Discussion Paper Program* in electronic format from the CAS Web Site.

Respectfully submitted,

Deborah M. Rosenberg
Vice President–Administration

FINANCIAL REPORT
FISCAL YEAR ENDED 9/30/2005

OPERATING RESULTS BY FUNCTION

FUNCTION	*REVENUE*	*EXPENSE*	*DIFFERENCE*
Membership Services	$ 1,360,269	$ 1,946,324	$ (586,056)
Seminars	1,526,321	1,258,511	267,810
Meetings	789,795	801,071	(11,276)
Exams	3,993,726 (a)	3,499,641 (a)	494,085
Publications	22,136	34,877	(12,741)
TOTALS FROM OPERATIONS	$ 7,692,246	$ 7,540,423	$ 151,823
Interest and Dividend Revenue			155,411
Realized Gain/(Loss) on Marketable Securities			(2,936)
Unrealized Gain/(Loss) on Marketable Securities			109,210
TOTAL NET INCOME (LOSS)			$ 413,509

NOTE: (a) Includes $2,119,313 of Volunteer Services for income and expense (SFAS 116).

BALANCE SHEET

ASSETS	*9/30/2004*	*9/30/2005*	*DIFFERENCE*
Cash and Cash Equivalents	$ 1,293,453	$ 755,704	$ (537,749)
T-Bills/Notes, Marketable Securities	3,634,448	4,783,051	1,148,603
Accrued Interest	24,211	23,937	(274)
Prepaid Expenses/Deposits	88,261	131,631	43,370
Prepaid Insurance	30,338	35,192	4,854
Accounts Receivable	51,482	52,000	518
Intangible Pension Asset	7,860	5,742	(2,118)
Textbook Inventory	12,369	9,523	(2,846)
Computers, Furniture	467,516	448,450	(19,066)
Less: Accumulated Depreciation	(377,124)	(376,273)	851
TOTAL ASSETS	$ 5,232,814	$ 5,868,958	$ 636,143

LIABILITIES	*9/30/2004*	*9/30/2005*	*DIFFERENCE*
Exam Fees Deferred	$ 714,605	$ 792,000	$ 77,395
Annual Meeting Fees Deferred	70,070	81,155	11,085
Seminar Fees Deferred	181,060	33,000	(148,060)
Accounts Payable and Accrued Expenses	478,481	624,782	146,301
Accrued Pension	192,301	182,754	(9,547)
TOTAL LIABILITIES	$ 1,636,516	$ 1,713,691	$ 77,174

MEMBERS' EQUITY

Unrestricted	*9/30/2004*	*9/30/2005*	*DIFFERENCE*
CAS Surplus	$ 3,312,044	$ 3,725,552	$ 413,509
Pension minimum liability (net of unamortized service cost of $5,742–2005 and $7,860–2004)	(80,318)	(38,065)	42,253
Michelbacher Fund	129,160	134,322	5,162
CAS Trust-Operating Fund	107,825	135,466	27,641
Centennial Fund	23,944	49,742	25,798
ASTIN Fund	0	10,400	10,400
Research Fund	62,482	99,576	37,094
Subtotal Unrestricted	$ 3,555,137	$ 4,116,994	$ 561,857

Temporarily Restricted	*9/30/2004*	*9/30/2005*	*DIFFERENCE*
Scholarship Fund	$ 5,728	$ 5,457	$ (271)
Rodermund Fund	7,391	6,686	(705)
CAS Trust-Ronald Ferguson Fund	28,042	26,131	(1,911)
Subtotal Temporarily Restricted	41,161	38,274	(2,887)
TOTAL MEMBERS' EQUITY	$ 3,596,297	$ 4,155,268	$ 558,970

Deborah Rosenberg, Vice President–Administration

This is to certify that the assets and accounts shown in the above financial statement have been audited and found to be correct.

CAS Audit Committee: Bob Deutsch, Chairperson;
Regina M. Berens, Michael P. Blivess, and Natalie Vishnevsky

2005 EXAMINATIONS—SUCCESSFUL CANDIDATES

Examinations for Exams 3, 5, 7-Canada, 7-United States, and 8 of the Casualty Actuarial Society were held May 3–6, 2005. Transitional Validation by Education Experience (VEE) Examinations for Economics, Corporate Finance, and Applied Statistical Methods were held August 10, 2005. Examinations for Exams 3, 6, and 9 of the Casualty Actuarial Society were held November 1–3, 2005.

Examinations for Exams 1, 2, and 4 are jointly sponsored by the Canadian Institute of Actuaries, Casualty Actuarial Society, and the Society of Actuaries and were held in May and November 2005. Candidates successful on these examinations were listed in joint releases of the Societies.

The following candidates were admitted as Fellows and Associates at the 2005 CAS Spring Meeting in May. By passing Fall 2004 CAS examinations, these candidates successfully fulfilled the Society requirements for Fellowship or Associateship designation.

NEW FELLOWS

John Leslie Baldan
Christopher M. Bilski
Kirk David Bitu
Amber L. Butek
James Chang
Hung Francis Cheung
Matthew Peter Collins
Keith William Curley
David A. De Nicola
Nicholas John De Palma
Ryan Michael Diehl
Melanie S. Dihora
Brian Michael Donlan
Ellen Donahue Fitzsimmons
William J. Fogarty
Sébastien Fortin
Charles R. Grilliot
James Donald Heidt
Eric David Huls
Scott Robert Hurt
Marie LeStourgeon
Young Yong Kim
Brandon Earl Kubitz
Hoi Keung Law
Amanda Marie Levinson
Gavin Xavier Lienemann
John Thomas Maher
Laura Suzanne Martin
James Paul McCoy
Shawn Allan McKenzie
James Lewis Norris
Bruce George Pendergast
Matthew James Perkins
Michele S. Raeihle
Robert Allan Rowe
Quan Shen
Summer Lynn Sipes
James M. Smieszkal
Liana St-Laurent
Keith Jeremy Sunvold
Erica W. Szeto
Malgorzata Timberg
Nicholas Jaime Williamson

NEW ASSOCIATES

Richard J. Bell III
Darryl Robert Benjamin
Stacey Jo Bitler
Karen Beth Buchbinder
Simon Castonguay
Denise L. Cheung
Melissa Diane Elliott
Solomon Carlos Feinberg
John S. Flattum
Jonathan W. Fox
Edward Lionberger
Brent Layne McGill
Thomas Edward Meyer
Alan E. Morris
Catherine Ann Morse
Lisa M. Nield
Frank W. Shermoen
Shannon Whalen
Stephen C. Williams
Stephen K. Woodard
Navid Zarinejad
Robert John Zehr

The following candidates successfully completed the following Spring 2005 CAS examinations.

The following candidates were admitted as Fellows and Associates at the 2005 CAS Annual Meeting in November. By passing Spring 2005 CAS examinations, these candidates successfully fulfilled the Society requirements for Fellowship or Associateship designation.

NEW FELLOWS

Fernando Alberto Alvarado
Brian C. Alvers
Maura Curran Baker
Rose D. Barrett
Derek Dennis Berget
Brian J. Biggs
Corey J. Bilot
Rebekah Susan Biondo
Tapio N. Boles
James L. Bresnahan
John R. Broadrick
Michele L. Brooks
Elaine K. Brunner
Matthew D. Buchalter
Anthony Robert Bustillo
Hsiu-Mei Chang
Alan M. Chow
Jason T. Clarke
Kevin A. Cormier
Justin B. Cruz
David F. Dahl
Chantal Delisle
Laura S. Doherty
Tomer Eilam
Bruce Fatz
Dale A. Fethke
William John Gerhardt
John S. Giles
Kristen Marie Gill
David B. Gordon
Jeffrey Robert Grimmer
Megan Taylor Harder
Robin A. Haworth
Brandon L. Heutmaker
Joseph S. Highbarger
Bo Huang
Richard Clay Jenkins
Philip J. Jennings
Shiwen Jiang
Yi Jing
Dana F. Joseph
Omar A. Kitchlew
Scott M. Klabacha
Andrew M. Koren
Bradley S. Kove
Terry T. Kuruvilla
Francois Lacroix
Kahshin Leow
Xin Li

Erik Frank Livingston
Jonathan L. Matthews
Brent L. McGill
Christopher Charles McKenna
Sylwia S. McMichael
Meagan S. Mirkovich
Rodney S. Morris
Leonidas V. Nguyen
Miodrag Novakovic
Timothy James O'Connor
Kathleen C. Odomirok
Jeremy Parker Pecora
Gregory T. Preble
Damon Joshua Raben
Dale M. Riemer
Brad E. Rigotty
Bryant Edward Russell
Frances G. Sarrel
Jeffery Wayne Scholl
Genine Darrough Schwartz
Justin Nicholas Smith
Mark Stephen Struck
Zongli Sun
Dovid C. Tkatch
Jennifer M. Tornquist
Joel A. Vaag
Daniel Jacob VanderPloeg
Kevin K. Vesel
Mo Wang
Kevin E. Weathers
Huey Wen Yang
Yuanhe Yao
Sung G. Yim
Ronald Joseph Zaleski Jr.
Lijuan Zhang
Hongbo Zhou
Steven Bradley Zielke

NEW ASSOCIATES

Avraham Adler
Vera E. Afanassieva
Amit Agarwal
Sajjad Ahmad
Kelleen D. Arquette
Yanfei Z. Atwell
Gregory S. Babushkin
Kristi Spencer Badgerow
Gregory K. Bangs
Tiffany Jean Baron
Angelo Edward Bastianpillai
Mark Belasco
Jeffrey Donald Bellmont
Matthew C. Berasi
Sonal Bhargava
Jonathan Bilbul
Brad Stephen Billerman
Jon Paul Bloom
Peter George Blouin
Nicolas Boivin
Randall T. Buda
Morgan Haire Bugbee
Jessica Yiqing Cao
Jeffrey M. Casaday
Paul A. Ceaser
Matthew S. Chamberlain
Bernard L. Chan
Tsz Kin Chan
Joung-Ju Chung
Raul Cisneros
Glenn A. Colby
Kirk Allen Conrad
Lawrence G. Cranor
Tighe Christian Crovetti
Walter C. Dabrowski
Jonathan E. DeVilbiss
Brent Pollock Donaldson
Michael D. Ersevim
Choya A. Everett
Marc-Olivier Faulkner
Jason A. Flick
Mark A. Florenz
Kyle P. Freeman
Derek W. Freihaut
Timothy M. Garcia
Nina Vladimirovna Gau
Stuart G. Gelbwasser
Maxime Gelinas
Simon Girard
Gregory P. Goddu
Rebecca J. Gordon
Wesley John Griffiths
Isabelle Guerin
Gerald S. Haase

William Joseph Hackman
Brian P. Hall
James W. Harmon
Megann Elizabeth Hess
Nathan Jaymes Hubbell
Yu Shan Hwang
Alison Susanne Jennings
Ziyi Jiao
Jeremiah D. Johnson
Ross Evan Johnson
Amy Ann Juknelis
Jennifer Ge Kang
Brian M. Karl
Jean-Philippe Keable
Sarah M. Kemp
David J. Klemish
Rachel M. Klingler
Christine K. Kogut
Thomas R. Kolde
Leland S. Kraemer
Michael A. Lardis
Catherine M. Larson
Annie Latouche
Kak Lau
Jeremy M. Lehmann
Sean M. Leonard
Jean-Francois Lessard
Mingyue Li
Andy Hankuang Liao
Cunbo Liu
Jin Liu
Nannan Liu
Todd L. Livergood
Andrew F. Loach
Laura J. Lothschutz
Neelam P. Mankoff
Minchong Mao
Angela Garrett McGhee
Albert-Michael Micozzi
Daniel E. Mikesh
Aaron G. Mills
Richard James Mills
Lori A. Moore
Allison L. Morabito
Mundia Mubyana
Daniel G. Myers
Marc L. Nerenberg
Benjamin R. Newton
Tho D. Ngo
Stephanie Jo Odell
Christopher John Olsen
Alejandro Antonio Ortega Jr.
Keith William Palmer
Joy-Ann C. Payne
Joseph G. Pietraszewski
Jean-Philippe Plante
Lynellen M. Ramirez
Arthur R. Randolph
Zia Rehman
Zoe F. Rico
Arnie W. Rippener
Randall D. Ross
John C. Ruth
Anita A. Sathe
Lawrence M. Schober
Erika Helen Schurr
Ronald S. Scott
Sheri Lee Scott
Clista E. Sheker
Robert K. Smith
Patrick Shiu-Fai So
Joanna Solarz
Richard Cambran Soulsby
Michael P. Speedling
Paul Quinn Stahlschmidt
Mindy M. Steichen
Yuchen Su
Feixue Tang
Luc Tanguay
Aaron A. Temples
Robert Bradley Tiger
Phoebe A. Tinney
Levente Tolnai
Rachel Katrina Tritz
Benjamin Joel Turner
Jonathan Kowalczyk Turnes
Allan S. Voltz
Todd Patrick Walker
Xuelian Wan
Jingtao Wang
Amanda J. White
Andrew T. Wiest
Martin E. Wietfeldt
Ronald Harris Wilkins
Shauna S. Williams
Benjamin T. Witkowski
Dorothy A. Woodrum
Yi-Chuang Yang
Min Yao
Yanjun Yao
Hui Yu Zhang
Wei Zhao
Michael V. Ziniti

The following candidates successfully completed the following Spring 2005 CAS examinations.

Exam 3

Roselyn M. Abbiw-Jackson
Victoria L. Adamczyk
Bilal Ahmed Alam
Christopher Robert Allard
John E. Amundsen
Amel Arhab
Elizabeth Asher
Shobhit Awasthi
Nathan H. Beaven
Aaron J. Beharelle
Jean-Michel Belanger
Kevin D. Bell
Zhihui Bian
Genevieve Boivin
Stephanie Elizabeth Booth
Jason Braun
Anthony Dane Bredel
Natalie L. Brown
Courtney L. Burch
Duoduo Cai
Li Cao
Ryan V. Capponi
Jeffrey H. Carter
Elizabeth Cashman
Bryan D. Chapman
Nicole K. Chizek
Wasim Chowdhury
Steven C. Coakley
Ryan Crabtree
Lawrence G. Cranor
Caitlin M. Cunningham
Andrew John Dalgaard
Mark K. Damron
Scott C. Davidson
Joseph T. Degeneffe
Jennifer Lynn Dempster
Amanda Emily Diegel
Blake W. Eastman
Shannon Erdmann
John Daniel Fanning
Kevin L. Feltz
Gregory Andrew Finestine
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Yu Xia
Ting Xu
Xuan Yang
Koh Han Yeap
Chi Yung Yeung
Hua Zhai
Buyi Zhang
Jialin Zhang
Jing Zhang
Li Zhang
Yan Zheng
Bei Zhu

The following candidates successfully completed the following Fall 2005 CAS examinations.

Exam 3

Jennifer Lynn Abel
Rocklyn Tee Altshuler
Desmond D. Andrews
Scott Nelson Applequist
Daryl S. Atkinson
Daniel P. Barker
Emily C. Barker
Danny Baxter
Yvan Berthou
Jesse David Bollinger
Samantha E. Bonde
John P. Booher
Abderrahmane Boumehdi
Kelli Ann Broin
Elizabeth Buhro
Jinghua Cao
Frank H. Chang
Young Ho Cho
Emily Daters Cilek
Lindsey Nicole Clark
Joshua J. Crumley
Auntara De
Rachel Caryn Dein
Kenneth Wayne Doss
Donald W. Doty
Daniel Joseph Eisel
Vladimir V. Eremine
Gregory Matthew Fanoe
Anusar Farooqui
Denise D. Fast
Jon R. Fredrickson
Shenzhao S. Fu
Scott A. Gibson
Kristen Marie Gilpin
Kristen M. Goodrich
Marco J. Gorgonio
Legare W. Gresham
Erin Ashley Groark
Kathleen J. Gunnery
Nicole A. Hackett
Jennifer L. Hagemo
Michael Bentley Howell
Lihu Huang
Kevin Hughes
Kapil Jain
Neha Jain
Nitesh Jain

Yin Jiang
Nancy A. Kelley
Kevin Dennis Kelly
Thomas Patrick King Jr.
Anton M. Klemme
Dionne L. Knight
Alex Gerald Kranz
Rachel Lynne Krefft
David Joseph LaRocque
Yvonne Y. Lai
Clifton J. Lancaster
Susan J. Lear
Ping Hsin Lee
Kerjern Lim
Lay Choo Lim
Alex Lin
Chin Ho Lin
Wladyslaw Derek Lorek
Brian Michael Lubeck
Kevin Arnold Lynde
Guodong Ma
Evan P. Mackey
Saikat Maitra
Steven Russell Marlow
Richard A. McCleary
Mary K. McKinnon
Eliade M. Micu
Christopher G. Moore
Timothy D. Morant
Douglas Franklin Moses
Chelsea C. Myers
Todd M. Nagy
Easter Namkung
Dorothy Elizabeth Nixon
Chanyuth Norachaipeerapat
Kathleen S. Ores Walsh
Nathan V. Owens
Elisa Pagan
Michelle Anne Pederson
Jennifer Kew Petit
Douglas E. Pirtle
Anwesha Prabhu
Matthew R. Purdy
David Adam Ring
Shaun A. Roach
Peter K. Robson
Ashley Carver Roya
Michael R. Sadowski
Mark A. Schiebel
Jennifer L. Scull
Satadru Sengupta
Andra Catalina Serban
Hsien-Ying Shen
Jean N. Spaltenstein
Mark R. Spinozzi
Eileen P. Toth
Timothy J. Vincze
Ryan N. Voge
Hui Wang
Jin Wang
Jinchuan Wang
Thomas S. Wang
Amber M. Wilson
Chad P. Wilson
Jade Woodford
XiJia Xu
Iva Yuan
Arthur J. Zaremba
Cyril Max Zormelo

Exam 6

Yazeed F. Abu-Sa’a
Eve Ingrid Adamson
Justin L. Albert
Xin Allen
David Michael Andrist
William M. Arthur II
Genevieve Aubin
Melissa Ann Baro
Aaron Thomas Basler
Anthony O. Beirne
Christine Beland
Guillaume Benoit
Chandler P. Benson
Yan Bergeron
Keith R. Berman
Davina Bhandari
Sarah Bhanji
Jennifer L. Blackmore
Stephanie Boivin
Elizabeth Bomboy
Kimberly A. Borgelt
James T. Botelho
Stephen A. Bowen
Ishmealin Boye

Edward G. Bradford
Justin J. Brenden
Sara Lynn Buchheim
Ali Ahmed Bukhari
John Lee Butel
Laura N. Cali
Sandra J. Callanan
Julianne A. Callaway
Samuel C. Cargnel
Laura M. Carstensen
Lauren Jill Cavanaugh
Matthew J. Cavanaugh
Sai Fan Chan
Mei-Hsuan Chao
Mingjen Chen
Houston Hau-Shing Cheng
Nitin Chhabra
Max Chiao
Shawn T. Chrisman
Gareth John Christopher
Melissa Chung
Meng-Fang Chung
Jeffrey J. Clair
David Garvin Clark
Jennifer Elizabeth Clark
Matthew D. Clark
Jason A. Clay
Brenda Clemens
Peter M. Corrigan
Ryan J. Crawford
Jason J. Culp
Alexandre Cung-Rousseau
Randi M. Dahl
Dawne Laurenne Davenport
Keri P. Davenport
Andrew G. Davies
Craig C. Davis
Paige M. Demeter
Hussain Dhalla
Scott A. Donoho
Kirt Michael Dooley
Yiping Dou
Mathew David Eberhardt
Malika El Kacemi-Grande
Isaac R. Espinoza
Muhammad Ali Fahad
Horng-Jiun K. Fann
Jeffrey N. Farr
Jacob C. Fetzer
Danielle J. Fiorello
Tricia D. Floyd
Jill A. Frackenpohl
Vincent M. Franz
Yan L. Fung
Joseph A. Gage
Heidi Marie Garand
Carol Ann Garney
Steve G. Gentle
Daniel J. Gieske
Evan W. Glisson
Kasi Joelle Golden
Rui Gong
Linda Grand
Joshua S. Grunin
Neng Gu
Todd N. Gunnell
Tao Guo
Jeannette Marie Haines
John C. Hanna Jr.
Tyree Harris
Ryan D. Hartman
Eric M. Herman
Roberto A. Hernandez
Wade R. Hess
Scott P. Higginbotham
Sheri L. Holzman
Hugh D. Hopper
Scott D. Hornyak
Yongtao Hu
Ying Huang
Zhigang Huang
Caleb Enders Huntington
Jed Nathaniel Isaman
Shira Lisa Jacobson-Rogers
Pierre-Alexandre Jalbert
John R. Jasinski
Xiang Ji
Bei Li Jiang
Yong Jiang
Mark C. Jones
Karine Julien
Yongwoon Kang
David M. Kaye
Erin E. Kelly
Jacob J. Kelly
Andrew P. Kempen
Eric J. Kendig
Tatyana Kerbel

Jim Klann
Jonathan M. Knotwell
Richard A. Knudson Jr.
Brian Yoonsyok Ko
Ebo Koranteng
Lucas James Koury
Nikolai S. Kovtunenko
Nadya Kuzkina
Lan See Lam
Alexander Jonathan Laurie
Vanessa Leblanc
Sara Leclerc
David F. Lee
Jennifer Marie Lehman
Jean-Francois Lessard
Ronald S. Lettofsky
Adrienne Jeanette Lewis
Kelly Carmody Lewis
Chunsu Li
Hongmei Li
Kexin Li
Yuan Li
Zhe Li
Manjuan Liang
Nanci B. Light
Chia-Ching Lin
Hua Lin
Katherine Yukyue Lin
Liming Lin
Paul T. Lintner
Millie S. Lo
Siew-Won Loh
John David Lower
Jie Lu
Amanda Lubking
Keyang Luo
Eric Lussier
Thomas J. Macintyre
Dorothy Lentz Magnuson
Derek Michael Martisus
Kelli R. McGinty
Kenneth James Meluch
Daniel John Messner
Jennifer L. Meyer
Rui Min
Kazuko Minagawa
Erick E. Mortenson
Randall K. Motchan
Brian J. Mullen
Eric L. Murray
Randy J. Murray
Treva A. Myers
Claude Nadeau
Marie-Eve Nadeau
Nora Kathleen Newman
Rosalie Nolet
Pablo F. Nunez
Ginette Pacansky
John Francis Pagano
Suyash G. Paliwal
Kristin Marie Palm
Michael W. Payne
Samuel Robert Peters
Sergey V. Pflyuk
Mathieu Picard
Etienne Plante-Dube
Daniel James Plasterer
Ricky R. Poulin
Donald S. Priest
Junhua Qin
Vincent Quirion
Rachel Radoff
William Steve Randolph
Scott J. Rasmussen
Andrew D. Reid
Michel Rivet
Jacob Roe
Bradley M. Rolling
Jason M. Rosin
Adam J. Rosowicz
William Paige Rudolph
Idir Saidani
Frederic Saillant
Mitra Sanandajifar
Eric L. Savage
Chad R. Schlippert
Jeffrey M. Schroeder
Daniel Owen Schwanke
Shayan Sen
Linda Sew
Vikas P. Shah
Xiaoyu Sheng
David J. Sheridan
Yevgeniy V. Shevchuk
Jacqueline W. Y. Shoong
Raymond Bond Shum
Sergey S. Sidorov
Syntheia W.H. Sin
Joel M. Smerchek
James M. Smith
Jared G. Smollik

Karl K. Song
Darryl R. Sorenson
Aleta J. Stack
Stephen R. Sten
Ian P. Sterling
Darin Stojanovic
Christopher James Stoll
Qiang Su
Zhongmei Su
Taher I. Suratwala
Martin Surovy
Jessica R. Sweets
Ann M. Sydnor
Wei-Chyin Tan
Josy-Anne Tanguay
Anne M. Taylor
Anne M. Thomas
Colin A. Thompson
Kathy M. Thompson
Lori R. Thompson
Kris Todoroski
Peter Tomopoulos
Helen L. Trainor
Tang-Hung Trang
Jaya Trivedi
Alice H. Tsai
Humberto M. Valdes
Tony Alan Van Berkel
Daniel M. Van der Zee
Chris John Van Kooten
Marina Vaninsky
Melinda K. Vasecka
Thomas W. Vasey
Kanika Vats
Daniel Viau
Anping Wang
Guixiang Wang
HongTao Wang
Min Wang
Ning Wang
Stream S. Wang
Tom C. Wang
Yang Wang
Gabriel Matthew Ware
David J. Watson
Kristen A. Weisensee
Timothy R. Wengerd
Melvyn R. Windham Jr.
Cari Bergen Winebrenner
Michael J. Wittmann
Andrea Wong
Chung Yin Wong
Aaron A. Wright
Terrence D. Wright
Xiaohui Wu
Xueming Grace Wu
Zhikun Wu
Hui Xia
Zhijian Xiong
Dehong Xu
Tong Xu
Marcus M. Yamashiro
Meng Yan
Hao Yang
Yulai Yang
Jenny Man Yan Yiu
Vanessa Ann Yost
Guanrong You
Yuan-Hung Yu
Anton Zalesky
Jin Zhang
Junya Zhang
Kun Zhang
Yan Zhang
Zhenyong Zhang
Kan Zhong
Yuanli Zhu

Exam 9

Amit Agarwal
Xin Allen
Brian D. Archdeacon
George N. Argesanu
Kelleen D. Arquette
Yanfei Z. Atwell
Gregory S. Babushkin
Kristi Spencer Badgerow
Tiffany Jean Baron
Michael Alan Bean
Jeffrey Donald Bellmont
Carolyn J. Bergh
Brad Stephen Billerman
Francois Blais
Michael J. Blasko
Jon Paul Bloom
Peter G. Blouin
James M. Boland III
Thomas Leininger Boyer II
Yisheng Bu
Morgan Haire Bugbee
Christine Cadieux

Chuan Cao
Jessica Yiqing Cao
Jennifer L. Carrick
Jeffrey M. Casaday
Thomas L. Cawley
Annie Chang
Zhijian Chen
Agnes H. Cheung
Chun Kit Cheung
Denise L. Cheung
Leong Yeong Chew
Joung-Ju Chung
Philip Arthur Clancey Jr.
Eric R. Clark
Christopher J. Cleveland
Kirk Allen Conrad
Peter J. Cooper
Thomas Marie Cordier
Keith Richard Cummings
David J. Curtis
Melisa L. Darnieder
Lucia De Carvalho
Amy L. DeHart
Jonathan E. DeVilbiss
Kenneth M. Decker
Stephen P. Decoteau
Christopher A. Donahue
Brent P. Donaldson
Joseph P. Drennan
Christopher G. Fanslau
Marc-Olivier Faulkner
Louise Frankland
Luyang Fu Ph.D.
Chong Gao
Nina Vladimirovna Gau
Maxime Gelinas
Gregory P. Goddu
Victoria A. Gomez
Jennifer Graunas
Neil A. Greiner
Joshua Rolf Harold Griffin
Liang Guo
RenBin Guo
Amit K. Gupta
Gary S. Haase
William Joseph Hackman
Faisal O. Hamid
Bobby Earl Hancock Jr.
Joseph Patrick Hasday
Joseph Hebert
James Anthony Heer
Christopher M. Holt
Chun Hua Hoo
Alison Therese Hover
YinYin Huang
Nathan Jaymes Hubbell
Mohammad A. T. Hussain
Joseph M. Izzo
William T. Jarman
Ziyi Jiao
Jeremiah D. Johnson
Luke G. C. Johnston
Burt D. Jones
Julie M. Joyce
Scott A. Kaminski
Hyeji Kang
Inga Kasatkina
Kayne M. Kirby
David J. Klemish
Rachel M. Klingler
Perry A. Klingman
Christine K. Kogut
Thomas R. Kolde
Leland S. Kraemer
Douglas H. Lacoss
Francois Langevin
Hugues Laquerre
Catherine M. Larson
Nathalie M. Lavigne
Henry T. Lee
Catherine Lemay
Kenneth L. Leonard
Hoi Fai Leung
Hayden Anthony Lewis
Mingyue Li
Sharon Xiaoyin Li
Joshua Yuri Ligosky
Edward P. Lionberger
Weichen Liu
Millie S. Lo
Lynn C. Malloney
Minchong Mao
Raul Gabriel Martin
Laurence R. McClure II
Isaac Merchant Jr.
Daniel E. Mikesh
Travis J. Miller
Aaron G. Mills
Allison L. Morabito
Mundia Mubyana
Christopher A. Najim

Tho D. Ngo
Norman Niami
Stephanie Jo Odell
Helen S. Oliveto
Christopher John Olsen
Keith William Palmer
Ying Pan
Jean-Pierre Paquet
Christopher A. Pett
Andrea L. Phillips
Christopher James Platania
Michael J. Quigley
Justin Radick
Arthur R. Randolph II
Eric W. L. Ratti
Nicholas J. Reed
Stephane Renaud
Ira L. Robbin Ph.D.
Beth A. Robison
Dave H. Rodriguez
Juan Carlos Rodriguez
Keith A. Rogers
Benjamin G. Rosenblum
Richard R. Ross
Brian Craig Ryder
Suzanne A. M. Scott
Peter Abraham Scourtis
Clista E. Sheker
Yipei Shen
Seth Shenghit
Yiping Shi
Annemarie Sinclair
Lleweilun Smith
Jiyang Song
Paul Quinn Stahlschmidt
Erik J. Steuernagel
Natalie St-Jean
Jayme Peter Stubitz
Yuchen Su
Feixue Tang
Jonas F. Thisner
Levente Tolnai
Rachel Katrina Tritz
Jonathan Kowalczyk Turnes
Kevin John Van Prooyen
Allan S. Voltz III
Xuelian Wan
Jingtao Wang
Chang-Hsien Wei
Lei Wei
Minwei Wei
Timothy G. Wheeler
Martin E. Wietfeldt
Ronald Harris Wilkins
Jianlu Xu
Xinxin Xu
Yanjun Yao
Shuk Han Lisa Yeung
Jiwei Yu
Navid Zarinejad
Robert J. Zehr
Juemin Zhang
Lang Zhang
Zhenyong Zhang
Wei Zhao
Yue Zhao
Weina Zhou
Xi Zhu
Michael V. Ziniti

NEW FELLOWS ADMITTED IN MAY 2005

First row, from left: Summer Lynn Sipes, Amanda Marie Levinson, Malgorzata Timberg, **CAS President Stephen P. D'Arcy**, Lien K. Tu-Chalmers, John Thomas Maher, Young Yong Kim. **Second row, from left:** Charles R. Grilliot, Liana St-Laurent, Melanie S. Dihora, Erica W. Szeto, Bruce George Pendergast, Yuxing Lei, Robert B. Katzman, Kirk David Bitu. **Third row, from left:** Jimmy L. Wright, Scott Robert Hurt, James M. Smieszkal, Ellen Marie Tierney, James Lewis Norris. **Fourth row, from left:** Matthew Peter Collins, Gavin Xavier Lienemann, Keith William Curley.

New Fellows Admitted in May 2005

First row, from left: John D. McMichael, Nicholas John De Palma, **CAS President Stephen P. D'Arcy**, Ellen Donahue Fitzsimmons, Matthew James Perkins, Sébastien Fortin. **Second row, from left:** Brandon Earl Kubitz, Brian Michael Donlan, Shawn Allan McKenzie, John Leslie Baldan, James Paul McCoy. **Third row, from left:** Robert Allan Rowe, Ziv Kimmel, Quan Shen, Timothy C. Mosler. **Fourth row, from left:** David A. De Nicola, Nicholas Jaime Williamson, William J. Fogarty. **New Fellows not pictured:** Christopher M. Bilski, Amber L. Butek, James Chang, Hung Francis Cheung, Ryan Michael Diehl, James Donald Heidt, Eric David Huls, Hoi Keung Law, Marie LeStourgeon, Laura Suzanne Martin, Michele S. Raeihle, Keith Jeremy Sunvold.

New Associates Admitted in May 2005

First row, from left: Karen Beth Buchbinder, Stacey Jo Bitler, Melissa Diane Elliott, **CAS President Stephen P. D'Arcy**, Wei Li, Lijuan Zhang, Simon Castonguay. **Second row, from left:** John S. Flattum, Brent Layne McGill, Lisa M. Nield, Shannon Whalen, Lorie A. Pate. **Third row, from left:** Sébastien Fortin, Thomas Edward Meyer, Navid Zarinejad. **Fourth row, from left:** Jonathan W. Fox, Stephen C. Williams, Edward Lionberger. **Fifth row, from left:** Robert John Zehr, Frank W. Shermoen, Stephen K. Woodard. **New Associates not pictured:** Richard J. Bell III, Darryl Robert Benjamin, Denise L. Cheung, Solomon Carlos Feinberg, Alan E. Morris, Catherine Ann Morse.

NEW FELLOWS ADMITTED IN NOVEMBER 2005

First row from left: Hsiu-Mei Chang, Michele Raeihle, Michele Brooks, Elaine Brunner, Shiwen Jiang, **CAS President Stephen P. D'Arcy**, Matthew Buchalter, Lijuan Zhang, Maura Curran Baker, Genine D. Schwartz, Xin Li, Bill Gerhardt. **Second row from left:** Andrew M. Koren, Scott M. Klabacha, Tomer Eilam, Jason T. Clarke, Laura S. Doherty, Hongbo Zhou, Kristen Marie Gill, Keith Jeremy Sunvold, Brent L. McGill, Joseph S. Highbarger, Zongli Sun. **Third row from left:** Leonidas V. Nguyen, Frances G. Sarrel, Mark Stephen Struck, Gregory T. Preble, Brandon L. Heutmaker, Jonathan L. Matthews, Chantal Delisle, Dale M. Riemer, Omar A. Kitchlew, Mo Wang.

New Fellows Admitted in November 2005

First row from left: Anthony Robert Bustillo, Yi Jing, Dovid C. Tkatch, Brad E. Rigotty, Joel A. Vaag, Jennifer M. Tornquist, **CAS President Stephen P. D'Arcy**, Sung G. Yim, Christopher Charles McKenna, Rodney S. Morris, Rose D. Barrett, Megan Taylor Harder. **Second row from left:** Derek Dennis Berget, Erik Frank Livingston, Bryant Edward Russell, Corey J. Bilot, David B. Gordon, Brian J. Biggs, Robin A. Haworth, Eric David Huls, Rebekah Susan Biondo, Bruce Fatz, Timothy James O'Connor. **Third row from left:** Jeffrey Robert Grimmer, John R. Broadrick, Ronald Joseph Zaleski Jr., Justin B. Cruz, Miodrag Novakovic, Daniel Jacob VanderPloeg, Meagan S. Mirkovich, Kevin E. Weathers, Jeffery Wayne Scholl, Richard Clay Jenkins.

New Fellows Admitted in November 2005

First row from left: Justin Nicholas Smith, Dale A. Fethke, **CAS President Stephen P. D'Arcy**, Kevin A. Cormier. **Second row from left:** Yuanhe Yao, Tapio N. Boles, Jeremy Parker Pecora, Kahshin Leow. **New Fellows not pictured:** Fernando Alberto Alvarado, Brian C. Alvers, James L. Bresnahan, Elaine K. Brunner, Alan M. Chow, David F. Dahl, John S. Giles, Bo Huang, Philip J. Jennings, Dana F. Joseph, Bradley S. Kove, Terry T. Kuruvilla, Francois Lacroix, Erik Frank Livingston, Sylwia S. McMichael, Kathleen C. Odomirok, Damon Joshua Raben, Kevin K. Vesel, Huey Wen Yang, Steven Bradley Zielke.

NEW ASSOCIATES ADMITTED IN NOVEMBER 2005

First row from left: Joy-Ann C. Payne, Robert Bradley Tiger, Randall T. Buda, Lori A. Moore, Aaron A. Temples, **CAS President Stephen P. D'Arcy**, Anita A. Sathe, Amit Agarwal, Dorothy A. Woodrum, Rebecca J. Gordon, Erika H. Schurr, Christine K. Kogut. **Second row from left:** Brian P. Hall, David J. Klemish, Gregory S. Babushkin, Kyle P. Freeman, Gerald S. Haase, Jon Paul Bloom, Andrew F. Loach, Simon Girard, Nicolas Boivin, Cunbo Liu, Avraham Adler.
Third row from left: Patrick Shiu-Fai So, William J. Hackman, Jean-Philippe Plante, Luc Tanguay, Maxime Gelinas, Jonathan E. DeVilbiss, Jonathan K. Turnes, John C. Ruth, Leland S. Kraemer.

NEW ASSOCIATES ADMITTED IN NOVEMBER 2005

First row from left: Wei Zhao, Xuelian Wan, Allison L. Morabito, Megann E. Hess, Brian M. Karl, Michael P. Speedling, **CAS President Stephen P. D'Arcy**, Mundia Mubyana, Richard James Mills, Jason A. Flick, James W. Harmon, Amy Ann Juknelis. **Second row from left:** Raul Cisneros, Mindy M. Steichen, Amanda J. White, Michael D. Ersevim, Robert K. Smith, Mark A. Florenz, Laura J. Lothschutz, Arthur R. Randolph II, Tighe Christian Crovetti, Allan Voltz, Sarah M. Kemp. **Third row from left:** Ronald S. Scott, Marc L. Nerenberg, Albert-Michael Micozzi, Jonathan Bilbul, Daniel G. Myers, Bernard L. Chan, Jeremiah D. Johnson, Aaron G. Mills, Benjamin T. Witkowski, Marc-Olivier Faulkner.

New Associates Admitted in November 2005

First row from left: Paul Q. Stahlschmidt, Yanfei Z. Atwell, Angelo Edward Bastianpillai, Alejandro Antonio Ortega Jr., Phoebe A. Tinney, Daniel E. Mikesh, **President Stephen P. D'Arcy**, Yanjun Yao, Joanna Solarz, Min Yao, Kak Lau, Clista E. Sheker. **Second row from left:** Stephanie Jo Odell, Brad Stephen Billerman, Michael V. Ziniti, Andy Hankuang Liao, Christopher John Olsen, Gregory P. Goddu, Kelleen D. Arquette, Walter C. Dabrowski, Paul A. Ceaser, Gregory K. Bangs, Thomas R. Kolde. **Third row from left:** Jingtao Wang, Joseph G. Pietraszewski, Kristi Spencer Badgerow, Matthew C. Berasi, Peter George Blouin, Nannan Liu, Sajjad Ahmad, Glenn A. Colby, Randall D. Ross, Zoe F. Rico, Jin Liu.

New Associates Admitted in November 2005

First row from left: Morgan Haire Bugbee, Shauna S. Williams, Yi-Chuang Sylvia Yang, Tiffany Jean Baron, Minchung Mao, Sheri Lee Scott, **President Stephen P. D'Arcy**, Jessica Yiqing Cao, Mingyue Miriam Li, Rachel M. Klingler, Andrew T. Weist, Sean M. Leonard. **Second row from left:** Jean-Philippe Keable, Annie Latouche, Jean-Francois Lessard,Vera E. Afanassieva, Richard C. Soulsby, Yuchen Su, Ziyi Jiao, Stuart G. Gelbwasser, Ronald Harris Wilkins, Brent Pollock Donaldson, Jeffrey M. Casaday. **Third row from left:** Keith William Palmer, Martin E. Wietfeldt, Derek W. Freihaut, Matthew S. Chamberlain, Nathan Jaymes Hubbell, Kirk Allen Conrad, Jeffrey Donald Bellmont, Todd L. Livergood, Lawrence M. Schober. **New Associates not pictured:** Mark Belasco, Sonal Bhargava, Tsz Kin Chan, Joung-Ju Chung, Lawrence G. Cranor, Choya A. Everett, Timothy M. Garcia, Nina Vladimirovna Gau, Wesley John Griffiths, Isabelle Guerin, Yu Shan Hwang, Alison Susanne Jennings, Ross Evan Johnson, Jennifer Ge Kang, Michael A. Lardis, Catherine M. Larson, Jeremy M. Lehmann, Neelam P. Mankoff, Angela Garrett McGhee, Benjamin R. Newton, Tho D. Ngo, Lynellen M. Ramirez, Zia Rehman, Arnie W. Rippener, Feixue Tang, Levente Tolnai, Rachel Katrina Tritz, Benjamin Joel Turner, Todd Patrick Walker.

OBITUARIES

ROBERT GRANT ESPIE
CLYDE H. GRAVES
SIDNEY M. HAMMER
RICHARD L. JOHE
J. GARY LAROSE
HERBERT J. PHILLIPS
PAUL J. SCHEEL SR.
EDWARD MERRILL SMITH
LEO M. STANKUS
JOHN A. W. TRIST

ROBERT GRANT ESPIE
1913–2005

Robert Espie passed away on October 16, 2005. He was born on December 15, 1913 in Toronto, Canada to Robert J. and Carrie Espie. At the age of 16, he enrolled in the Toronto School of Commerce and Finance and was a member of Phi Sigma Kappa. After graduating in 1934, he was stationed in London while he served in the U.S. Army Air Corps.

He became a CAS Fellow in 1958 and worked as an actuary at Aetna Life for over 35 years. He served on the Committee on Annual Statement from 1964–1969 and published papers in the *Proceedings* including "Some Observations Concerning Fire and Casualty Insurance Company Financial Statements" (1966) and "Insurance Investment Regulations" (1969). He retired as the vice president corporate comptroller in 1980. He enjoyed playing bridge and traveling. He is survived by his wife of 67 years, Jeanne Struthers Espie, two daughters, and four grandchildren.

CLYDE H. GRAVES
1908–2005

Clyde Graves died December 3, 2005 at the age of 97. He was born October 14, 1908 in Jackson, Mississippi. In 1928 he received his doctorate in mathematics from the University of Chicago. He then taught at Pennsylvania State College and, in World War II, worked as a statistician in the Office of Price Administration.

He became a CAS Associate in 1951 and a CAS Fellow in 1953. He worked as an actuary at the American Mutual Insurance Alliance in Chicago, Illinois for over 25 years, retiring as a vice president in 1973. He served as the chairman of the Committee on Informal Publications from 1954–1956 and as a member of the Publications Committee from 1954–1958, the Committee on Professional Status from 1960–1963, and the Advisory Committee to Department of Transportation from 1969–1970. He wrote the *Proceedings* papers "The Uniform Statistical Plan for Fire and Allied Lines" (1953), "Implications of Sampling Theory for Package Policy Ratemaking" (1967), and "Insurance Investment Regulations" (1969).

In retirement, Graves joined the International Executive Service Corp and taught actuarial science in the Philippines and Nigeria. He later taught at the University of Hartford in the 1990s.)

Graves is survived by five children, 14 grandchildren, and five great-grandchildren. Graves was preceded in death by his 10 older brothers and sisters, his wife, Jane McMaster, wife, Marie Reimer, and oldest son, John Kirk Graves.

SIDNEY M. HAMMER
1931–2005

Sidney Hammer passed away on February 17, 2005. He was born on February 8, 1931 in Brooklyn, New York. Hammer graduated from City College on June 18, 1952 with a degree in mathematics. He pursued additional studies at Hunter College and the University of Michigan at Ann Arbor. He served in the U.S. Army in the early 1950s when he was stationed in France and northern Italy.

He began his career at the Mutual Insurance Rating Bureau (MIRB) and, later, at the National Bureau of Casualty Underwriters (NBCU). Both are predecessor organizations of the Insurance Services Office. While at MIRB, he came to know his wife via telephone conversations as she was the supervisor of the clerical staff at NBCU. Once he joined NBCU, they began dating. They married on January 6, 1963.

In 1963, he earned his Associate designation and began working as the assistant manager for the actuarial department of the Home Insurance Company in New York City. During his tenure at the Home Insurance Company, he left his mark on every kind of actuarial project, including ratemaking for commercial lines, creating and managing the first reserve test protocols for the company, and reporting on regulatory issues. He retired in June 1995 as the lead actuary in the statistical department.

Home Insurance coworker Karl Moller (FCAS 1990) said of Hammer, “All the actuaries who worked with Sid will remember his passion for detail and his elephant-like memory. Those who knew him well will remember also his deep humanity.” Hammer is survived by his wife, Doris, three children (John, Anne, and Jean), and five grandchildren.

RICHARD L. JOHE
1927–2005

Former CAS President Richard L. Johe died Sunday, Aug. 14, 2005. He was born on May 2, 1927, in Hornell, N.Y., to Ilah Lee and Lorenzo Frank Johe.

In 1945 he was appointed to the U.S. Naval Academy. He graduated from Alfred University with a B.S. in mathematics in 1949 and also attended Duke, Syracuse, and Dartmouth Universities. He became a CAS Associate in 1951 and a CAS Fellow in 1954. He served as senior vice president for USF&G Insurance for over 25 years and then worked for Michigan Mutual for 14 years. He was a council member from 1959–1962, and served on several committees including the Education and Examination Committee—Education (1964–1967), the Committee on Review of Papers (1965–1966), the Program Committee (1969–1970), and the Nominating Committee (1971–1975). In addition to his Presidential Address, "A Look Ahead," which was published in the *Proceedings* in 1971, he also published "Underwriting Profit from Investments" (1967).

In 1971 he was ordained to the Permanent Diaconate of the Roman Catholic Church. He also enjoyed golf, military novels, and history. He is survived by his wife, Angelina Margaret Maranto Johe; daughter and son-in-law Kathleen A. and David A. Bartholow; son Gary L. Johe; brother Donald F. Lee; grandchildren Jennifer, Shannon, Mylinda, Chris, and Tommy; and great-granddaughter Taylor.

J. GARY LaROSE
1947–2005

Gary LaRose passed away on April 11, 2005. Born on April 15, 1947, he attended Andrews University in Berrien Springs, Michigan, graduating in 1969 with a B.A. in mathematics. He went on to study at Notre Dame and earned a masters in mathematics in 1974. He became an Associate in 1979 and a Fellow in 1981. Early in his career he worked in medical malpractice insurance with the Medical Protective Company. Subsequent career moves were to Employers Re and Ernst & Young.

In his involvement with CAS, he volunteered on several committees including the Education and Examination Committee—Examination (1981–1983), the Examination Committee, (1984–1985), The Syllabus Committee (1984–1989), and the Education Policy Committee (1989, 1990, and 1993). He also served as a representative for the Actuarial Education and Research Foundation from 1983 to 1990. LaRose also volunteered as an instructor in a graduate-level actuarial science program sponsored by the Society of Actuaries at universities in Moscow and Warsaw. Upon returning to the United States, he worked as a sole practitioner consulting actuary and as director of curriculum for the American Institute of CPCUs, and finally for the Nevada Insurance Department. In 1982 he published "A Note on Loss Distributions" in the *Proceedings*.

Jim Hall (FCAS, 1973) remembers LaRose using aspects of the Bailey-Simon paper to develop a system of credibility values for loss-free workers compensation risks. This system was used successfully for a number of years by a specialty underwriter. Hall was so amazed by the results that he asked author LeRoy Simon to peer review the work and Simon found no corrections were required.

HERBERT J. PHILLIPS
1923–2005

Born April 10, 1923, to Herbert and Agnes Phillips, Herbert Phillips joined the U.S. Marine Corps in 1941 and served with Third Marine Division in the South Pacific. When he returned home, he attended Boston College and graduated in 1949 with a B.S. in mathematics. He taught mathematics in the Boston high school system until he decided to pursue a career as a property and casualty actuary.

He became a CAS Associate in 1956 and a CAS Fellow in 1959. He later became vice president and senior actuary for Commercial Union in Boston, Massachusetts. In 1976, he moved to Toronto to work for the Insurers Advisory Organization. He retired in 1982. While a member of CAS, he served as the CAS Treasurer in 1982 and as the vice president-administration from 1983–1984. He also volunteered in the Committee on Sites (1980–1982), the Finance Committee (1982–1985), and the Program Planning Committee (1987–1989).

He passed away on October 6, 2005. He is survived by his wife, Anne Carr Phillips; daughter, Caren Houston (Kerri); grandchildren, Caitlin and Connor; and sister, Doris Phillips. He is predeceased by his parents and by his sisters Anne Grant and Agnes Phillips.

PAUL J. SCHEEL SR.
1933–2005

Born on November 15, 1933, Paul Scheel graduated from Loyola University in 1959. Scheel became a CAS Fellow in 1970 and worked for the United States Fidelity and Guaranty Company in Baltimore, Maryland for many years. He volunteered on the Publicity Committee (1967–68), the Public Relations Committee (1969–71), Vice Chairperson, 1971. Scheel had a keen interest in advancing the actuarial profession and mathematics appreciation. In 1971 he served on the CAS Special Task Force to Study Recruitment of New Candidates to the Profession and as a Representative to Mathematical Association of America. In 1970, he published the review "Trend and Loss Development Factors" in the *Proceedings*.

Scheel passed away on November 17, 2005. He is survived by his wife Beverly; his two children, Paul J. Jr. and Mary Schmidt; four grandchildren; and his brother and sister.

EDWARD MERRILL SMITH
1925–2005

Born August 27, 1925, in Marblehead, Massachusetts, Edward Smith received a B.S. degree from the University of Massachusetts and a master's degree in forestry from Yale University. He served in the U.S. Navy in World War II. He earned his CAS Associate designation in 1956 and became a CAS Fellow in 1958.

He worked as an actuary in the property-casualty department of the Travelers Insurance Company in Hartford, Connecticut. Smith wrote two *Proceedings* papers: "Economic Factors in Liability and Property Insurance Claim Costs, 1935–1967" (1968) and a review of "Actuarial Note on Workmens Compensation Loss Reserves" (1972).

An avid outdoorsman, he enjoyed activities such as hunting, fishing, canoeing, and hiking. Smith died on September 11, 2005 at the age of 80. He is survived by his wife, Attrude Lewis Smith, their two children, and six grandchildren.

LEO M. STANKUS
Circa 1925–2004

Leo Stankus passed away on September 30, 2004 at the age of 79. He became a CAS Associate in 1958 and a Fellow in 1962. Stankus worked for several years for the Allstate Insurance Company in Skokie, Illinois. He served on the CAS Committee on Automobile Insurance Research and published the Seminar Report "Guaranteed Renewable Automobile Insurance" in the *Proceedings* in 1960.

Stankus is survived by his sons, Randall and Robert Stankus; Robert's wife Kimberly; and his grandchildren, Thomas Joseph "TJ," Michael, Nicholas, Christopher, and Karli. His funeral was held in Glenview, Illinois where he had lived.

JOHN A. W. TRIST
1921–2005

John Trist died on November 8, 2005. He was born on January 10, 1921. He attended the University of Manitoba, in Winnipeg, Manitoba, Canada and graduated with a degree in actuarial science in 1949. Trist became an Associate in 1950 and a Fellow in 1953. He worked at the Insurance Company of North America in Philadelphia, Pennsylvania for a number of years before becoming an associate actuary at CIGNA Property and Casualty Group, also in Philadelphia, in 1984. In 1985 he became the director and worked at CIGNA until his retirement in 1992. In 1973 he wrote "How Adequate are Loss and Loss Expense Liabilities," which was published in the *Proceedings*. After retirement, Trist remained active, traveling to Canada often, working out at the CIGNA gym, and golfing regularly.

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