VOLUME XCI NUMBERS 174 and 175

PROCEEDINGS

OF THE

Casualty Actuarial Society

ORGANIZED 1914

2004

VOLUME XCI

Number 174—May 2004

Number 175—November 2004

Library of Congress Catalog No. HG9956.C3
ISSN 0893-2980

Printed for the Society by
United Book Press
Baltimore, Maryland

Typesetting Services by
Minnesota Technical Typography, Inc.
St. Paul, Minnesota

FOREWORD

Actuarial science originated in England in 1792 in the early days of life insurance. Because of the technical nature of the business, the first actuaries were mathematicians. Eventually, their numerical growth resulted in the formation of the Institute of Actuaries in England in 1848. Eight years later, in Scotland, the Faculty of Actuaries was formed. In the United States, the Actuarial Society of America was formed in 1889 and the American Institute of Actuaries in 1909. These two American organizations merged in 1949 to become the Society of Actuaries.

In the early years of the 20th century in the United States, problems requiring actuarial treatment were emerging in sickness, disability, and casualty insurance, particularly in workers compensation, which was introduced in 1911. The differences between the new problems and those of traditional life insurance led to the organization of the Casualty Actuarial and Statistical Society of America in 1914. Dr. I. M. Rubinow, who was responsible for the Society's formation, became its first president. At the time of its formation, the Casualty Actuarial and Statistical Society of America had 97 charter members of the grade of Fellow. The Society adopted its present name, the Casualty Actuarial Society, on May 14, 1921.

The purposes of the Society are to advance the body of knowledge of actuarial science applied to property, casualty, and similar risk exposures, to establish and maintain standards of qualification for membership, to promote and maintain high standards of conduct and competence for the members, and to increase the awareness of actuarial science. The Society's activities in support of this purpose include communication with those affected by insurance, presentation and discussion of papers, attendance at seminars and workshops, collection of a library, research, and other means.

Since the problems of workers compensation were the most urgent at the time of the Society's formation, many of the Society's original members played a leading part in developing the scientific basis for that line of insurance. From the beginning, however, the Society has grown constantly, not only in membership, but also in range of interest and in scientific and related contributions to all lines of insurance other than life, including automobile, liability other than automobile, fire, homeowners, commercial multiple peril, and others. These contributions are found principally in original papers prepared by members of the Society and published annually in the *Proceedings of the Casualty Actuarial Society*. The presidential addresses, also published in the *Proceedings*, have called attention to the most pressing actuarial problems, some of them still unsolved, that the industry has faced over the years.

The membership of the Society includes actuaries employed by insurance companies, industry advisory organizations, national brokers, accounting firms, educational institutions, state insurance departments, and the federal government. It also includes independent consultants. The Society has three classes of members: Fellows, Associates, and Affiliates. Both Fellows and Associates require successful completion of examinations, held in the spring and fall of each year in various cities of the United States, Canada, Bermuda, and selected overseas sites. In addition, Associateship requires completion of the CAS Course on Professionalism.

Affiliates are qualified actuaries who practice in the general insurance field and wish to be active in the CAS but do not meet the qualifications to become a Fellow or an Associate.

The publications of the Society and their respective prices are listed in the Society's Yearbook. The *Syllabus of Examinations* outlines the course of study recommended for the examinations. Both the *Yearbook*, at a charge of $40 (U.S. funds), and the *Syllabus of Examinations,* without charge, may be obtained from the Casualty Actuarial Society, 1100 North Glebe Road, Suite 600, Arlington, Virginia 22201.

JANUARY 1, 2004

EXECUTIVE COUNCIL*

MARY FRANCES MILLER . . . . *President*
STEPHEN P. D'ARCY . . . . *President-Elect*
DEBORAH M. ROSENBERG . . . . *Vice President–Administration*
THOMAS G. MYERS . . . . *Vice President–Admissions*
JOHN C. NARVELL . . . . *Vice President–International*
JOANNE S. SPALLA. . . . *Vice President–Marketing & Communications*
CHRISTOPHER S. CARLSON . . *Vice President–Professional Education*
DONALD F. MANGO . . . . . . *Vice President–Research & Development*

THE BOARD OF DIRECTORS

*Officers**

MARY FRANCES MILLER . . . . *President*
STEPHEN P. D'ARCY . . . . *President-Elect*

Immediate Past President†

GAIL M. ROSS. 2004

Elected Directors†

PHILLIP N. BEN-ZVI 2004
CURTIS GARY DEAN 2004
DAVID G. HARTMAN 2004
JANET R. NELSON 2004
GARY R. JOSEPHSON 2005
DAVID J. OAKDEN 2005
PATRICIA A. TEUFEL 2005
OAKLEY E. VAN SLYKE. 2005
ROBERT V. DEUTSCH 2006
SHOLOM FELDBLUM 2006
ANDREW E. KUDERA. 2006
ROBERT F. WOLF. 2006

*Term expires at the 2004 Annual Meeting. All members of the Executive Council are officers. The vice president-administration also serves as the secretary and treasurer.

† Term expires at the Annual Meeting of year given.

2004 PROCEEDINGS
CONTENTS OF VOLUME XCI

2004 PROCEEDINGS
CONTENTS OF VOLUME XCI

Page

NOTICE

Papers submitted to the *Proceedings of the Casualty Actuarial Society* are subject to review by the members of the Committee On Review of Papers and, where appropriate, additional individuals with expertise in the relevant topics. In order to qualify for publication, a paper must be relevant to casualty actuarial science, include original research ideas and/or techniques, or have special educational value, and must not have been previously copyrighted or published or be concurrently considered for publication elsewhere. Specific instructions for preparation and submission of papers are included in the *Yearbook* of the Casualty Actuarial Society.

The Society is not responsible for statements of opinion expressed in the articles, criticisms, and discussions published in these *Proceedings*.

Volume XCI, Part 1 **No. 174**

PROCEEDINGS
May 16, 17, 18, 19, 2004

THE "MODIFIED BORNHUETTER-FERGUSON" APPROACH TO IBNR ALLOCATION

TRENT R. VAUGHN AND PHOEBE TINNEY

Abstract

This paper presents a "Modified Bornhuetter-Ferguson" approach to allocating IBNR. Essentially, this approach involves a credibility-weighted average of the earned premium and case-incurred loss (or loss adjustment expense) allocation bases. This combined allocation provides a more reasonable and stable result than methods based solely on either earned premium or case-incurred loss. Moreover, the method is easy to automate, explainable in intuitive terms, and does not require the use of an "off-balance" adjustment factor.

ACKNOWLEDGEMENTS

The authors would like to thank Kirk Fleming, Rich Sieger, and Regina Berens for providing helpful comments regarding an earlier draft of this paper. The authors would also like to thank the reviewers of this paper, especially Glenn Walker, for several helpful suggestions.

1. INTRODUCTION

In property/casualty loss reserving, the definition of the relevant groupings (or "reserve segments") results from a trade-off between the conflicting goals of obtaining homogenous groupings and achieving a sufficient volume of data. For instance, the Casualty Actuarial Society's *Statement of Principles Regarding Property and Casualty Loss and Loss Adjustment Expense Reserves* [2] states the following:

> Credibility is a measure of the predictive value that the actuary attaches to a body of data. The degree to which consideration is given to homogeneity is related to the consideration of credibility. Credibility is increased by making groupings more homogenous or by increasing the number of claims analyzed within each group. A group of claims should be large enough to be statistically reliable. Obtaining homogenous groupings requires refinement and partitioning of the total database. There is a point at which partitioning divides data into cells too small to provide credible development patterns. Each situation requires a balancing of the homogeneity and amount of data in each grouping.

In consideration of this principle, reserving actuaries often combine many accounts, programs, and/or Annual Statement lines of business into a single reserve segment. By doing so, the reserving actuary is able to achieve a proper combination of volume and homogeneity. These reserve segment definitions are then utilized to produce a reasonable estimate of the incurred but not reported (IBNR) loss and allocated loss adjustment expense.

These resulting IBNR estimates, however, may need to be allocated down to a more detailed level. For instance, the Annual Statement may require IBNR estimates at a finer level of detail than the reserve segment definitions. In addition, company management may require accident year results (including IBNR) at the individual program or account level in order to properly manage the business. These accident year results by program or account can then be compared with the corresponding estimates produced by the company pricing actuaries. This comparison between reserving and pricing can serve as a useful process of “checks and balances” within the actuarial department.

Actuaries should be aware, however, of the possible pitfalls of allocating IBNR down to an extremely fine level of detail. For instance, such allocations may incorrectly imply a degree of precision that does not exist. The actuary must be aware of this risk and communicate any related concerns to the end user.

Given that such a breakdown is appropriate, there are two common—and simple—methods for allocating IBNR: the earned premium method and the case-incurred loss method. Both of these methods are subject to serious weaknesses. For instance, the earned premium method essentially allocates IBNR for each reserve segment and accident year in proportion to the calendar year earned premium for each program or account. This method ignores the fact that certain programs may have experienced a much greater claim frequency, paid loss ratio, and case-incurred loss ratio, and thereby merit a larger proportion of the indicated IBNR. On the other hand, the case-incurred loss ratio method allocates IBNR in proportion to the underlying case-incurred loss (or ALAE) amount. Essentially, this method is equivalent to applying an identical cumulative loss development factor to the case-incurred losses for each component program or account. Unfortunately, this method often results in very unstable and unreliable allocations, especially for recent accident years and long-tailed reserve segments.

The following section describes the “Modified Bornhuetter-Ferguson”[1] allocation method, which provides a simple alternative to the earned premium and case-incurred methods.

[1]The original Bornhuetter-Ferguson methodology [1] pertained to the establishment of reserves for an entire reserve segment. From this point forward, we will abbreviate “Bornhuetter-Ferguson” as “BF.”

2. THE MODIFIED BF ALLOCATION

The Modified BF method essentially offers a compromise between allocating by earned premium and allocating by case-incurred loss or ALAE. The relative weights assigned to each of the two methods vary by accident year. For the most recent accident year, most (but not all) of the weight will be given to the earned premium allocation. As an accident year matures, more weight is assigned to the case-incurred allocation.

This section describes the method by means of a specific reserving example. Let’s assume that the reserve review has been completed for a given reserve segment. The results of this hypothetical review are shown in Table 1.

In this table, let’s assume that the projected ultimate loss amounts (and the corresponding selected loss IBNR) have been determined by some reasonable loss reserving methodology. The specific methodology utilized is not relevant to the IBNR allocation procedure. The table only displays projections for the latest three accident years; the Modified BF procedure, however, will work for any desired number of years.

In addition, let’s assume that this reserve segment is comprised of three specific programs. Table 2 provides the breakdown of earned premium and case-incurred loss by program, for each of the three calendar/accident years.

The first step in the Modified BF procedure is to allocate the IBNR in proportion to the calendar year earned premium for each program. Table 3 displays the results of this calculation.

The second step involves allocating the IBNR in proportion to the case-incurred loss amount for each program,[2] as shown in

[2]For lines of business that are subject to very large claims, or “shock losses,” the actuary may choose to utilize limited (for example, “basic limits”) losses for the case-incurred allocation.

TABLE 1

Calendar/ Accident Year	Earned Premium	Cumulative Case-Incurred Losses	Projected Ultimate Losses	Loss IBNR	Projected Ultimate Loss Ratio
2000	1,200	700	900	200	75.0%
2001	1,200	650	900	250	75.0%
2002	1,200	200	900	700	75.0%

TABLE 2

Calendar/ Accident Year	Program	Earned Premium	Cumulative Case-Incurred Losses
2000	A	500	400
2000	B	400	200
2000	C	300	100
2001	A	500	350
2001	B	400	200
2001	C	300	100
2002	A	500	185
2002	B	400	10
2002	C	300	5

TABLE 3

Calendar/ Accident Year	Program	Earned Premium	Pro Rata Earned Premium	Allocated IBNR
2000	A	500	0.417	83.33
2000	B	400	0.333	66.67
2000	C	300	0.250	50.00
2001	A	500	0.417	104.17
2001	B	400	0.333	83.33
2001	C	300	0.250	62.50
2002	A	500	0.417	291.67
2002	B	400	0.333	233.33
2002	C	300	0.250	175.00

TABLE 4

Calendar/ Accident Year	Program	Cumulative Case-Incurred Losses	Pro Rata Case-Incurred Losses	Allocated IBNR
2000	A	400	0.571	114.29
2000	B	200	0.286	57.14
2000	C	100	0.143	28.57
2001	A	350	0.538	134.62
2001	B	200	0.308	76.92
2001	C	100	0.154	38.46
2002	A	185	0.925	647.50
2002	B	10	0.050	35.00
2002	C	5	0.025	17.50

Table 4. For very immature accident years, claims may emerge sporadically, and allocating IBNR according to case-incurred losses will generally produce very unreliable and unstable results. Yet, we don't want to completely ignore the information contained in early case-incurred loss tallies. On the other hand, for older accident years, case-incurred loss ratios tend to provide a more accurate indication of the relative profitability of the underlying programs. Even so, we still may want to "smooth out" the projected ultimate loss ratios to some degree by considering an earned premium allocation.

In order to determine the proper weighting between the earned premium and case-incurred allocations, the Modified BF approach calculates an "implied loss development factor (LDF)" for each accident year. This implied LDF serves as a proxy for the maturity of the accident year, and is simply defined as the ratio of projected ultimate losses to case-incurred losses. As an alternative, we could select the LDF for each accident year by examining the underlying case-incurred loss triangle, making link ratio selections, then taking the product of the relevant link ratios—just like in traditional chain-ladder reserving methods.

TABLE 5

Calendar/ Accident Year	Cumulative Case-Incurred Losses	Projected Ultimate Losses	Implied LDF	Weight to Case-Incurred Method	Weight to Premium Method
2000	700	900	1.286	0.778	0.222
2001	650	900	1.385	0.722	0.278
2002	200	900	4.500	0.222	0.778

TABLE 6

Calendar/ Accident Year	Program	Case-Incurred Based Allocation	Premium Based Allocation	Weighted Average Allocation
2000	A	114.29	83.33	107.41
2000	B	57.14	66.67	59.26
2000	C	28.57	50.00	33.33
2001	A	134.62	104.17	126.16
2001	B	76.92	83.33	78.70
2001	C	38.46	62.50	45.14
2002	A	647.50	291.67	370.74
2002	B	35.00	233.33	189.26
2002	C	17.50	175.00	140.00

The advantages of the implied LDF calculation are that it is easy to automate and that it reflects the method actually utilized to select the ultimate losses (which may be much different from the case-incurred chain-ladder method).

For each accident year, the weight given to the case-incurred allocation is equal to the reciprocal of the implied LDF; the weight given to the earned premium allocation is then equal to the complement (relative to unity) of the case-incurred weight. In this manner, the weights are assigned according to the traditional Bornhuetter-Ferguson formula (subject to the implied LDF), which provides the rationale for describing the method as a Modified BF approach. Using the data in our example,

TABLE 7

SUMMARY

Program	Calendar/ Accident Year	Earned Premium	Cumulative Case-Incurred Losses	Projected Ultimate Losses	Projected Ultimate Loss Ratio
A	2000	500	400	507	101.5%
	2001	500	350	476	95.2%
	2002	500	185	556	111.1%
	2000–2002	1,500	935	1,539	102.6%
B	2000	400	200	259	64.8%
	2001	400	200	279	69.7%
	2002	400	10	199	49.8%
	2000–2002	1,200	410	737	61.4%
C	2000	300	100	133	44.4%
	2001	300	100	145	48.4%
	2002	300	5	145	48.3%
	2000–2002	900	205	423	47.1%
All Programs	2000	1,200	700	900	75.0%
	2001	1,200	650	900	75.0%
	2002	1,200	200	900	75.0%
Total	2000–2002	3,600	1,550	2,700	75.0%

Table 5 calculates the implied LDF and the respective weights, for each of the accident years.

Once these relative weights are determined, the Modified BF method calculates a weighted-average IBNR allocation for each accident year. Table 6 displays the calculation of this weighted-average allocation.

As a final step, the method can be used to produce management reports that display the projected ultimate loss ratio by accident year for each underlying program. An example of a final, end-user management report is provided in Table 7. This table essentially combines the results of our illustrative example into a useful summary exhibit.

This Modified BF approach offers several theoretical advantages over allocations done solely on the basis of either earned premium or case-incurred loss. For instance, the Modified BF approach combines both elements of information in the underlying allocation; that is, the allocation method considers both the size of each underlying program (via the earned premium allocation) and the relative underwriting results to date (via the case-incurred loss allocation). As a result, the Modified BF method should produce more reasonable and stable allocations than either simpler method in isolation. Furthermore, this combined estimate is produced by a familiar weighting technique—namely, the BF weighting—that has proven over many years of use to be a reasonable method for combining an experience-based estimate with an a priori estimate.

In addition, the Modified BF approach offers three practical advantages. First, the Modified BF approach is easily automated in an Excel/Access environment, which allows for a quick turnaround on the resulting management reports. Second, the resulting allocations always sum to the total IBNR, eliminating the need for any "off-balance" adjustment factors. Third, the method is easily explained and understood in intuitive terms, which results in greater acceptance of the results.

3. AN ALTERNATIVE TO EARNED PREMIUM ALLOCATION

The Modified BF approach, as presented above, does not offer the flexibility of adjusting the a priori loss ratio by program. This lack of flexibility may cause problems in certain circumstances. For instance, let's assume that we are dealing with the most recent accident year for a very long-tailed reserve segment, and that we have selected an ultimate loss ratio of 75%. Since the case-incurred loss amount for this accident year would be very low, the Modified BF method would allocate IBNR largely in proportion to earned premium. Thus, each of the programs in this reserve segment would show a loss ratio of roughly 75%.

TABLE 8

Calendar/ Accident Year	Program	Earned Premium	Cumulative Case-Incurred Losses	Expected Loss Ratio	Expected Ultimate Losses
2002	A	400	50	65.0%	260
2002	B	400	75	75.0%	300
2002	C	400	25	85.0%	340

In contrast, let’s assume that there are three equally sized programs in this reserve segment with very different expected levels of profitability. Specifically, Program A has historically been priced at a 65% expected loss ratio, Program B at a 75% expected loss ratio, and Program C at an 85% ratio. In this case, if the management reports project a roughly equal (at 75%) loss ratio for the most recent accident year for each of the programs, the accuracy of these reports will be challenged.

The solution to this problem would be to replace the earned premium portion of the allocation with an “expected loss” allocation. As an example, Table 8 provides some hypothetical data for calendar/accident year 2002; assume that the total projected ultimate loss ratio for this accident year is 75%. In addition, let’s assume for this year that the earned premium is evenly spread between three programs, and that the programs have been priced as described above.

In this case, the earned premium allocation in the Modified BF method is replaced with an expected loss allocation, as shown in Table 9. By comparison, the earned premium allocation would have assigned $250 of IBNR to each of the three accident programs. As a final step, the Modified BF procedure would then combine the expected loss allocation with the case-incurred allocation, in a manner similar to that described in the previous section.

TABLE 9

Calendar/ Accident Year	Program	Expected Ultimate Losses	Pro-Rata Expected Losses	Allocated IBNR
2002	A	260	0.289	216.67
2002	B	300	0.333	250.00
2002	C	340	0.378	283.33

4. MATURE ACCIDENT YEARS AND NEGATIVE IBNR

For very mature accident years (for example, accident years that are developed to 84 months or more), the Modified BF procedure for allocating IBNR may not work as well as another, simpler method. In particular, for older accident years, the Modified BF method allocates IBNR largely in accordance with case-incurred losses; moreover, for these accident years paid losses will tend to be very close to case-incurred losses. In fact, for many specific programs or accounts in the detailed allocation, all of the accident year claims will be closed, and paid losses will equal case-incurred losses. Even so, the Modified BF method may allocate a large proportion of the remaining IBNR to these programs. For reserve segments that are subject to very late-reported claims, or reopened claims, this allocation may be appropriate. For other segments, however, the actuary may consider replacing the Modified BF approach with an allocation in proportion to either open claim counts or case reserves.

In addition, the Modified BF method may be inappropriate for accident years with negative IBNR amounts. In this case, the "implied LDF" that is utilized in the Modified BF weighting procedure is less than unity. As a result, the weight given to the case-incurred allocation is greater than unity, and the weight given to the earned premium allocation is less than zero.[3] Thus, in situa-

[3]In the more-common case where IBNR is positive, the weights assigned to both the case-incurred and the earned premium allocation are between zero and unity.

tions with negative IBNR, another allocation method (such as a straight case-incurred allocation) may be more appropriate.

5. SUMMARY

The Modified BF allocation procedure presented in this paper provides a simple and reliable method for allocating IBNR down to a finer level. The resulting IBNR allocation can then be utilized to create validated accident year management information reports.

The Modified BF methodology can also be utilized to perform the IBNR allocations that are required for statutory or GAAP reporting—either on a net, direct/assumed, or ceded basis. For example, the approach can be used to allocate ceded IBNR to individual reinsurer for Schedule F purposes.

In addition, the procedure can easily be modified to handle loss, loss adjustment expense, or even salvage/subrogation. The obvious modification would be to replace the case-incurred loss with the relevant component—for example, paid or case-incurred ALAE (depending on whether case reserves are established for ALAE), or salvage/subrogation received.

REFERENCES

[1] Bornhuetter, Ronald L., and Ronald E. Ferguson, "The Actuary and IBNR," *PCAS* LIX, 1972, pp. 181–195.

[2] Casualty Actuarial Society, *Statement of Principles Regarding Property and Casualty Loss and Loss Adjustment Expense Reserves*, May 1988.

DISCUSSION OF PAPER PUBLISHED IN VOLUME XC

DISTRIBUTION-BASED PRICING FORMULAS ARE NOT ARBITRAGE-FREE

DAVID L. RUHM

DISCUSSION BY MICHAEL G. WACEK

Abstract

David Ruhm's paper is a welcome addition to the actuarial literature. It illustrates some difficult concepts in a refreshing way. As actuaries are increasingly faced with the need to price non-traditional risks, it is important that they understand how to do so.

One of the paper's main points is to emphasize the important finding from financial economics that the probability distribution of risk outcomes does not always contain enough information to produce arbitrage-free prices for that risk. However, the probability distribution of outcomes can, and indeed must, be used to determine the expected cost of that risk. This discussion uses Ruhm's examples to underscore the distinction between price and cost, and the potential implications for the seller of a derivative.

Ruhm's paper also seeks to generalize about the arbitrage-free prices of calls and puts compared to their expected value payoff. Ruhm concludes that calls are priced at a discount and puts at a premium, at least when the underlying security has an expected return E that is greater than the risk-free rate r. He then seeks to explain why investors would buy puts, given that they are priced at a premium to expected value. He concludes that some risks have a qualitative nature as either insurance or investment. This pattern of discounted calls

and surcharged puts is true ONLY if $E > r$. Under the condition $E < r$, which Ruhm did not discuss, calls are surcharged and puts are discounted. As a result investor behavior can be accounted for in a simpler way than by appealing to investor risk aversion or the "qualitative nature of a risk."

1. INTRODUCTION

David Ruhm's paper is a fascinating attempt to make the paradox of Black-Scholes risk-neutral pricing more comprehensible to actuaries. His mapping of the distribution of stock prices onto a roulette wheel is a brilliant construct that makes plain just how bizarre the arbitrage-free prices that emerge from the risk-neutral framework are.

I have no quarrel with much of the paper. I do have a minor quibble with the title. It is clearly too categorical, and should be something like "Distribution-Based Pricing Formulas Are Not *Always* Arbitrage-Free." In the paper the author himself points out that insurance prices that are based on the probability distribution of outcomes can be arbitrage-free.

I also found it surprising that Ruhm focuses on the derivative buyer's perspective and virtually ignores the seller's perspective. Since actuaries are usually concerned with the pricing problem from the seller's point of view, and particularly since the author does not address it, I am going to weigh in with a discussion of the latter.

In addition, I will show that some of Ruhm's conclusions about his "risk discount" function and the buyer's motivation, which have the *appearance* of generality, depend on certain of his assumptions. He notes these assumptions but does not explore their importance to his conclusions. As a result, some readers might not realize that his conclusions do not hold under some realistic conditions that the author does not discuss.

2. DISTINCTION BETWEEN PRICE AND COST—SELLER'S PERSPECTIVE

Ruhm is correct in saying that the seller of an insurance policy or financial derivative cannot necessarily rely on the probability distribution of outcomes to correctly *price* the risk at the market clearing level unless certain conditions are present (or absent, depending on one's perspective). However, unless the seller takes certain actions that effectively change the applicable probability distribution (about which more later), he *must* use the probability distribution of outcomes to accurately assess the expected *cost* of the risk.

Suppose I decide to open a casino. I acquire the Ruhm Roulette Wheel together with a set of instructions that includes the set of correct arbitrage-free prices (from Exhibit 3 of the paper) to charge for bets on each number from 00 to 36. These prices seem counterintuitive, since they call for varying prices for equally likely outcomes, but I am new to this business, so who am I to question them? There is a section on hedging, but it looks complicated and I ignore it.

I open my casino and charge the prices given in the instructions. For example, for a \$100 payoff on number 30 I charge a "premium" of \$2.08. I monitor the profit and loss on each number, of course, and after some time I notice that my average payoff cost on number 30 is actually \$2.63. One of the features of the Ruhm Roulette Wheel is that I don't have to make the payoff for a year, which allows me to earn 8 cents interest on the premium, but \$2.16 is still 47 cents short of \$2.63. Analyzing the results for the other numbers, I find that, except for number 16, the payoff costs do not match the interest-adjusted premiums. The reason for the mismatch is that while the premiums were determined correctly from the risk-neutral pricing framework, the payoffs continue to be governed by the real world probabilities. The results for each of the numbers 00 through 36 are summarized in Exhibit 1 of this discussion, the core of which is excerpted from Exhibit 3 of Ruhm's paper.

The good news for me as the casino owner is that if bets had been placed in equal proportions over all 38 numbers, the total premiums and interest would match the total payoff costs. The bad news is that the players know that the Ruhm Roulette Wheel is fair, meaning each number is equally likely to come up. Since I charge less for the high numbers than for the low ones, I get more high number bets than low number bets. My casino business is a big loser!

There is a way around this. Along with the arbitrage-free price list, Ruhm's instructions also tell me how to hedge the risk for each number. If I follow that hedging procedure, the sum of my payoff cost for any given number and the associated hedging gain or loss will match the arbitrage-free premiums I collect for that number. For the low numbers the hedging will produce losses. For the high numbers it will produce gains. For the number 30 example, hedging will produce an average gain of 47 cents, which reduces the total expected payoff cost from \$2.63 to \$2.16. The hedging effectively transforms my payoff cost to what it would be if the underlying stock had an expected return equal to the risk-free rate.

If I hedge the bets against each number, then it won't matter whether customers prefer high numbers or low ones.

3. VALUE FOR MONEY—BUYER'S PERSPECTIVE

Ruhm speculates why anyone would place bets on the low numbers, since under the conditions he assumes,[1] they include a surcharge over expected value. He extends the same question to put options generally. He claims a put buyer must be motivated by a desire to hedge the risk, since a speculator would not make an investment with such poor prospects. I don't find his argument

[1] Namely, that the expected return on the stock E exceeds the risk-free rate r.

compelling. There is a simpler explanation that doesn't depend on investor psychology.

Ruhm appears to have overlooked the importance of his assumptions that the expected annual returns on the underlying stock and Treasuries are $E = 10\%$ and $r = 4\%$, respectively. If the expected annual return on the stock is less than the risk-free rate, i.e., $E < r$, then the pattern reverses, and arbitrage-free prices for the put-like low numbers are discounted and the prices for the call-like high numbers are surcharged.

To see this, let's start by grouping the low numbers 00 through 17 and the high numbers 18 through 36. There is a 50% probability associated with each group. The low number group corresponds to a "binary put option" having a fixed payoff (in this case \$100) if the stock price is less than the median of the stock price distribution. The high number group corresponds to a "binary call option" that has a payoff of \$100 if the stock price closes above the median.

In Exhibit 1, which is based on an expected annual stock return of 10%, the sum of the arbitrage-free premiums for the high number group is \$40.95. The sum of the premiums for the low number group is \$55.21. With interest these amounts are \$42.58 and \$57.42, respectively. The puts are priced at a premium to expected cost. The calls are priced at a discount.

Suppose the expected annual stock return is really 0%, in which case $E < r$. Then the median of the stock price distribution is \$95.60. From the price formula for a ray included in the paper, the arbitrage-free price for the binary call option with a strike price of \$95.60 is \$53.08. The corresponding price for the binary put option is \$43.08. With interest these amounts become \$55.20 and \$44.80, respectively. The puts are priced at a discount to expected cost. The calls are priced at a premium.

Remember that no one knows the true parameters of the stock price distribution. If an investor believes that the true expected annual stock return E exceeds the risk-free rate r, then arbitrage-free calls will look attractively priced and puts will not. Such an investor might buy the calls but will shun the puts. On the other hand, if an investor believes that the true stock return parameter is less than the risk-free rate, calls will look expensive and puts will look attractive. That investor will shun the calls and might buy the puts. This is logical profit-maximizing behavior. It is not necessary to appeal to differences in risk aversion or "the qualitative nature of the risk" to explain the behavior.

Meanwhile, the seller of puts and calls can be indifferent to the true stock return parameter, provided he hedges the puts and calls that he sells.

4. THE $w(s)$ FUNCTION

Attempting to generalize his findings from the roulette wheel, Ruhm introduces his $w(s)$ function as a measure of the risk discount for betting on the event $X = s$. Like the roulette wheel concept, this function neatly captures important information about a complex relationship, in this case between the risk-neutral pricing framework and the perceived real world probabilities.

However, as in his analysis of the roulette wheel, the author again overlooks the scenario in which the underlying stock return is less than the risk-free rate. If $E < r$, then the slope of $w(s)$ is positive. Small values of s (i.e., low strike prices) yield large discounts to the expected payoff. High strike prices yield large surcharges. This is the opposite of the behavior of $w(s)$ sketched in the paper, which addressed only the scenario of $E > r$. We give an example of this below. It can be generalized, but it should be clear enough from the example.

The $w(s)$ function is the ratio of the "risk neutral" pdf to the "real world" pdf pertaining to the underlying stock. The following formula is equivalent to Ruhm's. It is not a function of merely s, but of a number of parameters, the most important of which for our purposes is E

$$w(s,E) = \frac{g(d_2)}{g(d_2 + ((\ln(1+E) - \ln(1+r))/\sigma)\sqrt{t})}, \qquad \text{where}$$
$$d_2 = \frac{\ln(P_0/s) + (\ln(1+r) - 0.5\sigma^2)t}{\sigma\sqrt{t}} \tag{1}$$

where $g(x)$ is the standard normal pdf evaluated at x. (Note that d_2 is the well-known Black-Scholes parameter.)

For example, using formula (1) for the scenario involving a strike price s of 90, an initial stock price P_0 of 100, with E = 10%, r = 4%, σ = 30% and t = 1 year, which are the parameters Ruhm used in his main example, we obtain the following value of $w(90, 10\%)$:

$$w(90,10\%) = \frac{0.3776}{0.3487} = 1.083$$

Contrast this with the value of $w(90,0\%)$ that we obtain when we change E to zero, leaving all of the other parameters unchanged:

$$w(90,0\%) = \frac{0.3776}{0.3909} = 0.966$$

Exhibit 2 shows $w(s,10\%)$ and $w(s,0\%)$ for the parameter set given above and strike prices ranging from \$10 to \$200 in \$10 increments.

Because $w(s)$ is a function of E, and E is inherently unknowable, $w(s)$ is not unique. Ruhm has a belief about the value of E that might be the same as mine, but it might be different. It is possible to talk about Ruhm's $w(s)$ function or mine, but unless he knows my $w(s)$, he cannot make claims about whether my put or call buying behavior is motivated by investment or insurance considerations. For example, if he believes a particular stock will go up (implying a negatively sloped $w(s)$), then my buying what looks like an expensive put on that stock will strike him as evidence of extremely risk averse behavior, suggesting an insurance orientation on my part. However, if I expect the stock to trade sideways or go down (implying a positively sloped $w(s)$), then my behavior in buying what looks to me to be a cheap put is actually consistent with a profit maximizing investment strategy.

Consequently, Ruhm overreaches in his conclusion about how $w(s)$ can be used. There is no unique value of $w(s)$ independent of E that can tell us whether a risk is viewed as an investment or as insurance. Only if a put buyer is known to believe that $E > r$ could we correctly say that he is acting to "insure" the risk (which the author sees as synonymous with a willingness to pay a surcharge to the risk's expected value). If, on the other hand, he believes that $E < r$, he is "investing" in the risk. I have a hunch that most investors who buy puts believe $E < r$.

5. SUMMARY

There is much to like in Ruhm's paper. His roulette wheel is an excellent metaphor that makes the implications of the Black-Scholes framework more tangible. Likewise, his invention and use of the $w(s)$ function is a laudable attempt to distill important information into a simple measure. While I believe his interpretation of $w(s)$ is flawed, I appreciate his attempt.

Against these positives, I have sought to clarify three points the author chose not to emphasize.

First, from the seller's perspective it is critical to make a distinction between price and cost. While the risk-neutral pricing framework produces arbitrage-free prices in markets where hedging is available, the prices are not necessarily adequate to cover the seller's expected value cost. As we saw in the roulette wheel example with an underlying stock expected return of $E > r$, the seller can expect to lose money on the high numbers if he does not hedge.

Second, it is not correct to say that call options priced in the risk-neutral framework are priced at a discount to their expected value cost and that puts are priced at a premium, without being clear that this is true only if $E > r$. The author noted in passing that his result is true if $E > r$, but did not point out that the opposite is true if $E < r$, in which case calls are priced at a premium and puts are priced at a discount.

Third, I have pointed out that the second point extends to the behavior of the author's $w(s)$ function, which makes it largely useless as a means of categorizing individual behavior as investment or insurance oriented, as Ruhm had hoped.

EXHIBIT 1

PREMIUM, COST, AND PROFIT OR LOSS BY NUMBER

		(1)	(2)	(3) (1)+(2)	(4)	(5) (3)-(4)
Roulette Wheel Number		Arbitrage-Free Premium	Interest on Premium	Premium with Interest	Expected Payoff ("Cost")	Casino Profit or (Loss)
00		$3.84	$0.15	$3.99	$2.63	$1.36
0		$3.46	$0.14	$3.60	$2.63	$0.97
1		$3.30	$0.13	$3.43	$2.63	$0.80
2		$3.19	$0.13	$3.32	$2.63	$0.69
3		$3.10	$0.12	$3.22	$2.63	$0.59
4		$3.03	$0.12	$3.15	$2.63	$0.52
5		$2.97	$0.12	$3.09	$2.63	$0.46
6		$2.92	$0.12	$3.04	$2.63	$0.41
7		$2.87	$0.11	$2.98	$2.63	$0.35
8		$2.82	$0.11	$2.93	$2.63	$0.30
9		$2.78	$0.11	$2.89	$2.63	$0.26
10		$2.74	$0.11	$2.85	$2.63	$0.22
11		$2.70	$0.11	$2.81	$2.63	$0.18
12		$2.67	$0.11	$2.78	$2.63	$0.15
13		$2.63	$0.11	$2.74	$2.63	$0.10
14		$2.60	$0.10	$2.70	$2.63	$0.07
15		$2.56	$0.10	$2.66	$2.63	$0.03
16		$2.53	$0.10	$2.63	$2.63	($0.00)
17		$2.50	$0.10	$2.60	$2.63	($0.03)
18		$2.47	$0.10	$2.57	$2.63	($0.06)
19		$2.44	$0.10	$2.54	$2.63	($0.09)
20		$2.41	$0.10	$2.51	$2.63	($0.13)
21		$2.38	$0.10	$2.48	$2.63	($0.16)
22		$2.35	$0.09	$2.44	$2.63	($0.19)
23		$2.32	$0.09	$2.41	$2.63	($0.22)
24		$2.29	$0.09	$2.38	$2.63	($0.25)
25		$2.26	$0.09	$2.35	$2.63	($0.28)
26		$2.23	$0.09	$2.32	$2.63	($0.31)
27		$2.19	$0.09	$2.28	$2.63	($0.35)
28		$2.16	$0.09	$2.25	$2.63	($0.39)
29		$2.12	$0.08	$2.20	$2.63	($0.43)
30		$2.08	$0.08	$2.16	$2.63	($0.47)
31		$2.04	$0.08	$2.12	$2.63	($0.51)
32		$1.99	$0.08	$2.07	$2.63	($0.56)
33		$1.94	$0.08	$2.02	$2.63	($0.61)
34		$1.87	$0.07	$1.94	$2.63	($0.69)
35		$1.79	$0.07	$1.86	$2.63	($0.77)
36		$1.62	$0.06	$1.68	$2.63	($0.95)
Total		$96.16	$3.85	$100.00	$100.00	$0.00
00–17	"Put"	$55.21	$2.21	$57.42	$50.00	$7.42
18–36	"Call"	$40.95	$1.64	$42.58	$50.00	($7.42)

EXHIBIT 2

COMPARISON OF $w(s, 10\%)$ AND $w(s, 0\%)$

	Value of $w(s,E)$	
Strike Price, s	$E = 10\%$	$E = 0\%$
10	4.2584	0.3707
20	2.7646	0.5014
30	2.1473	0.5983
40	1.7949	0.6782
50	1.5618	0.7475
60	1.3941	0.8093
70	1.2664	0.8656
80	1.1653	0.9174
90	1.0828	0.9658
100	1.0140	1.0111
110	0.9555	1.0540
120	0.9051	1.0947
130	0.8610	1.1336
140	0.8222	1.1708
150	0.7876	1.2065
160	0.7565	1.2410
170	0.7285	1.2742
180	0.7030	1.3063
190	0.6797	1.3375
200	0.6583	1.3677
$P_0 = 100$, $r = 4\%$, $\sigma = 30\%$, $t = 1$ year		

DISCUSSION OF PAPER PUBLISHED IN VOLUME XC

DISTRIBUTION-BASED PRICING FORMULAS ARE NOT ARBITRAGE-FREE

DAVID RUHM

DISCUSSION BY GARY G. VENTER

Abstract

David Ruhm is entirely correct that risk load formulas based on transforming probability distributions of contract outcomes cannot guarantee arbitrage-free prices. This is what he illustrates by a clever and entertaining example. But the title of the paper seems to assert that no method of transforming distributions is arbitrage-free. This is not the case, as transforms of the probabilities of the underlying events that generate the outcomes are well known to produce arbitrage-free prices. In fact, Ruhm illustrates this by showing that the Black-Scholes formula arises from such a transform. He also shows that this formula builds in risk-adjustments to prices, thus addressing the misapprehension that since the options prices come from a risk-neutral valuation they do not incorporate risk adjustments.

To illustrate the application of probability transforms to fundamental events in insurance, this discussion provides an example of using an alternative transform of underlying frequency and severity distributions to price loss layers.

Arbitrage pricing theory is often described as showing that prices are arbitrage-free if and only if they are based on transformed probabilities. This is an over-simplification. As David Ruhm's paper shows, it is possible to create examples where no transformations of the probabilities of the outcomes produce arbitrage-free prices. What arbitrage-pricing theory actually requires is transforms of the probabilities of the underlying events that generate the outcomes. This is the fundamental result of arbitrage pricing theory, but is often stated more abstractly. For example, see Brigo and Mercurio (2001) page 25. Also Panjer (1998) page 180 makes it clear that probabilities are applied to states of nature, and the prices of securities are functions of the states. Thus any probability transform will apply to the underlying states. Furthermore, the impossible events (those with zero probability) have to be the same under the original and transformed probabilities. Two probability measures with the same set of possible outcomes are sometimes called "equivalent."

Ruhm's paper illustrates this effect for the pricing of stock options. Although he does not present it this way, his results show that applying transforms to options prices will not be arbitrage-free, but applying them to the prices of the underlying securities will be. For a stock that has a lognormal distribution in μ and σ with expected return E and risk-free rate r Ruhm states:

"The Black-Scholes price of an option is equal to the option's discounted expected value, under a risk-neutral lognormal density function that is parameterized by μ^* and σ^*:

$$\mu^* = \mu - \ln[(1+E)/(1+r)]$$
$$\sigma^* = \sigma."$$

This makes it clear that it is the probability distribution of the stock price that is transformed in the Black-Scholes model. The expected value of an option's outcome under the transformed probability distribution is the option price, and these prices are known to be arbitrage-free in this model. Ruhm's roulette wheel example shows further that transforming the probabilities of the outcomes of the options themselves will not give the same answer. Ruhm does not claim that Black-Scholes prices contain arbitrage, as one might think from the paper's title. Thus the paper effectively distinguishes between transforming event probabilities and transforming the probabilities of contract outcomes, even though it does not strongly emphasize this distinction.

For insurance pricing, the comparable underlying events are the primary insurance claim counts and loss sizes. Pricing based on transforming these frequency and severity probabilities (keeping the same zero probability events—negative losses, perhaps) is what is required by arbitrage-pricing theory. Ruhm comes close to this conclusion when he states: "The value of the insurance is determined by the stochastic process of the covered perils; the value of the derivative is driven by the stochastic process of the asset's market price. If insurance could be thought of as a derivative at all, it would be as a derivative of hurricane occurrence and severity, auto accident occurrences and severities, etc."

What will not work is transforming the probabilities of outcomes of contracts—such as aggregate losses, reinsurance layers, etc.—which would be like applying transforms to option prices. This is not entirely new to the CAS literature. Arbitrage-free pricing provides a completely additive allocation of the overall company risk load to line and contract. Wang (1998) gives examples where transforming the probabilities of the results of aggregate covers produces strictly sub-additive allocations, which are thus not arbitrage-free.

However, this does not automatically mean the sub-additive allocations are wrong. For one thing, there is a tension in the pricing literature between calculating actual market prices and the prices a company would ideally like to achieve, which might contain arbitrage possibilities. For another, there are issues of incompleteness in insurance markets that some observers feel permit a degree of theoretical arbitrage possibilities that can never in fact be realized. The arbitrage possibilities that can actually exist in the insurance market is not a settled issue. However some lines of business are very competitive, and if a company has pricing structures that would allow arbitrage against it in a complete market, it could end up with competitive disadvantages.

As an example, suppose a company would like to make the profit load on a fleet of 100 cars 10 times the load on a single car, so it is 10% of the load on a per car basis. It wants to do this because the fleet is more stable. In a complete market, an arbitrageur might sell 100 individual policies and cede them bundled to the company at the fleet price, thus ending up with 90% of the profit and no risk. But barriers to entry, etc. might prevent this from taking place in the real market. Nonetheless, a competitor could decide that since it is doing the diversification internally, it can sell the individual policies at the fleet price. If the fleet price has the right risk and return characteristics, then the competitor ends up with this risk profile on its book of individual policies, and the first company loses a book of business that as a whole it would find desirable. Thus arbitrage opportunities can be competed away, even in an incomplete insurance market without arbitrageurs.

Supposing that a company does want to set arbitrage-free prices, what transforms would be appropriate for claim frequency and severity distributions? The basic result for arbitrage-free pricing is that prices are expected values from a transformed process that is a martingale. This criterion requires that there is no expected upward or downward trend in the transformed process. For insurance this means that the aggregate transformed frequency and severity processes have a mean equal to that of the overall loaded premium. Then premium minus transformed losses has an expected value of zero, although perhaps a great deal of volatility. Unlike the Black-Scholes case, however, there is not a unique transform in the insurance market. This is typical of incomplete markets—that is markets where not every instrument can be subdivided and hedged at will.

A recent paper, Møller (2003), summarizes much of the literature on probability-transform pricing for the compound Poisson process with risk-loaded premium. The fundamental result he presents, based on Girsanov's Theorem (a basic element of arbitrage-pricing theory), describes a procedure for producing arbitrage-free transforms of frequency and severity distributions.

The starting point is selecting a function $\phi(y)$, where the loss size variable is Y, with the only restriction being that $\phi(y) > -1$ for all positive losses y. The frequency parameter λ is transformed to $\lambda[1 + E\phi(Y)]$. The severity density $g(y)$ gets transformed to $g(y)[1 + \phi(y)]/[1 + E\phi(Y)]$.

Møller introduces a ranking order for such transforms, based on specific pricing impacts. He provides several examples, three of which are reasonable in terms of being in the middle of the ranking order. The transforms are calibrated by a parameter $\theta = E[Y\phi(Y)]/EY$ which is the loading in the primary rates, so that the primary loaded pure premium is $(1 + \theta)\lambda E(Y)$.

The first example, from Delbaen and Haezendonck (1989), sets $\phi(y) = \theta(y - EY)EY/\mathrm{Var}(Y)$. This is > -1 $\forall y$ as long as $\theta < \mathrm{CV}^2$, where CV is the severity coefficient of variation—its ratio of standard deviation to mean. Since $E\phi(Y) = 0$, the transformed frequency parameter is just the actual parameter λ and the transformed severity density is $g(y)[1 + \phi(y)]$.

An example Møller introduces, which he calls the minimum martingale measure, takes $\phi(y) = (y/EY)\theta/[1 + \mathrm{CV}^2]$, with $E\phi(Y) = \theta/[1 + \mathrm{CV}^2]$. The transformed frequency is $\lambda[1 + \theta/(1 + \mathrm{CV}^2)]$, and the transformed severity density is $g(y)[1 + \mathrm{CV}^2 + \theta y/EY]/[1 + \mathrm{CV}^2 + \theta]$. In this and the previous transform, severity probabilities are reduced for losses below the mean, and increased for losses above it. This is minimal in its squared distance from the actual probability measure.

The third example Møller calls the minimum entropy martingale measure. It starts with $\phi(y) = e^{\eta y} - 1$. Then the transformed frequency is $\lambda E e^{\eta Y}$ and the severity is $g(y)e^{\eta y}/Ee^{\eta Y}$. If you want this to match a pre- existing premium load θ, you need to find $\eta > 0$ so that $E[Ye^{\eta Y}] = (1 + \theta)E(Y)$. This is not possible for some severity distributions, but if the severity has policy limits or is light tailed, like a mixed exponential, the expectation will exist. Usually η will be quite small, like maybe 10^{-10}.

The relative entropy between two measures P and Q is $EP[dQ/dP\log(dQ/dP)]$. This is a distance of a sort, as it is zero if $P = Q$ and is otherwise positive. However it is not symmetric in P and Q. Minimizing the relative entropy is a popular fitting method and is related to optimizing a fit given the information available, according to principles of information theory. In the insurance pricing case, P is the real-world measure and Møller shows that the transform above gives the martingale Q that minimizes the relative entropy. Q is then the martingale closest to the actual probability measure P in the sense of relative entropy. The minimum entropy is usually realized by the Esscher transform. In fact Ballotta (2004) shows that the minimum entropy transform above is the Esscher transform applied to frequency and severity combined. However, Møller shows that applying the Esscher transform to severity alone, which would be the above transform for severity but with no change to frequency, gives less satisfactory results by his criteria. This transform uses $\phi(y) = e^{\eta y}/Ee^{\eta Y} - 1$. Then the transformed frequency is just λ and the severity is still $g(y)e^{\eta y}/Ee^{\eta Y}$.

For an example of the minimum martingale measure, consider a book of business with 2,500 expected claims, a Pareto severity $G(y) = 1 - (1 + y/10{,}000)^{-1.2}$, a policy limit of 10,000,000, and a loading of $\theta = 20\%$. The severity mean and CV^2 are about 37,443 and 43.11. This makes the λ load factor $(1 + 0.2/44.11) = 1.00453$. The factor on $g(y)$ is $(44.11 + y/187{,}215)/44.31$. This can be applied numerically to a discretization of the severity distribution. The maximum severity has to stay at 10,000,000 in order to keep the zero-probability events the same. The original probability mass at 10,000,000 is 0.025% which gets transformed to 0.055%. The severity mean is increased by 19.46%, which together with the frequency transform gets the 20% load.

The transformed probabilities can be used to price any type of contract on this business as the expected value of the contract using the transformed probabilities. In this case that has to be done numerically with the discretized severity. The risk load for any contract is its expected value from the transformed probabilities less its expected value from the actual probabilities. For instance, a 4,000,000 xs. 1,000,000 contract ends up with a risk load of 62.3%, and a 5,000,000 xs. 5,000,000 gets 112.8%. The total amount of those loads is 13,730,500, which is the risk load for the layer 9,000,000 xs. 1,000,000 calculated separately. This is 73.3% of the entire loading on the primary business—as most of the risk is attributed to the higher layers by this method.

It is also interesting to apply this example to a difficult test case for pricing methods attributed to Thomas Mack, which is to price a buy-back of a franchise deductible. For example, for a deductible of 1,000, this contract would pay the full loss if it is less than or equal to 1,000, but nothing if it is greater. Venter (1998) tries a number of pricing transforms on such contracts, and they all give negative risk loads in some cases. For the book of business outlined above and a range of deductibles, minimum martingale pricing gives a (barely) positive risk load. In fact the severity-only risk loads are negative in all cases tested, but the frequency load, small as it is, is enough to compensate and make the total load positive. The combined frequency-severity increment $\lambda g(y)$ can be seen to transform by a factor of $1 + \theta y/[(1+\mathrm{CV}^2)EY]$, which is > 1 for any positive y. Thus any combination of losses will get a positive risk load. This will hold for the minimum entropy martingale as well.

The reason Mack's example is difficult is that transforms of severity have to produce a density that integrates to 1, so giving more probability to large losses must take it away from small losses. Thus contracts that cover only small losses tend to get negative loads. But as these examples show, that problem can be alleviated by making the percentage load on frequency greater than the largest reduction in severity probability. The transforms that do not do this, such as the one of Delbaen and Haezendonck and the Esscher transform of severity only, would be subject to Mack's problem.

The minimum entropy transform, with its exponential moment, gives higher loads to higher layers. In some reinsurance contracts tested, this was better than the minimum martingale transform at pricing low-mean high-variance layers, like top layers of cat programs. For example, see Venter, Barnett, and Owen (2004). Quadratic transforms, like the minimal martingale measure, appear to be less capable of matching market pricing of higher cat layers. The minimum entropy transform also has more theoretical strength, in that it is the closest martingale to the actual probabilities in the quasi-distance measure from information theory.

The minimum entropy and minimum martingale measures provide reasonable candidates for probability transforms for pricing insurance and reinsurance contracts. The key is that the price for a contract is the expected value of the contract outcomes under the transformed primary frequency and severity probabilities, and not, as Ruhm emphasizes, the mean of any transformation of the probabilities of the possible outcomes of the contract itself.

REFERENCES

[1] Ballotta, L., "Alternative Framework for the Fair Value of Participating Life Insurance Contracts," International AFIR Colloquium, http://afir2004.soa.org/afir_papers.htm.

[2] Brigo, D., and F. Mercurio, *Interest Rate Models: Theory and Practice* (2nd edition), Springer-Verlag, 2001.

[3] Delbaen, F., and J. Haezendonck, "A Martingale Approach to Premium Calculation Principles in an Arbitrage Free Market," *Insurance: Mathematics and Economics*, 8, 1989, pp. 269–277.

[4] Møller, T., "Stochastic Orders in Dynamic Reinsurance Markets," *ASTIN Colloquium Papers*, 2003, http://www.astin2003.de/img/papers/moeller.pdf (Updates to appear at http://www.math.ku.dk/ tmoller/).

[5] Panjer, H. (Ed.), *Financial Economics: with Applications to Investments, Insurance, and Pensions*, 1998, The Actuarial Foundation, Schaumberg, IL.

[6] Venter, G., Discussion of "Implementation of Proportional Hazards Transforms in Ratemaking," *PCAS* LXXXV, 1998, pp. 980–990.

[7] Venter, G., J. Barnett, and M. Owen, "Market Value of Risk Transfer: Catastrophe Reinsurance Case," International AFIR Colloquium, 2004, http://afir2004.soa.org/afir_papers.htm.

[8] Wang, S., "Implementation of Proportional Hazards Transforms in Ratemaking," *PCAS* LXXXV, 1998, pp. 940–979.

AUTHOR'S RESPONSE TO DISCUSSION OF PAPER PUBLISHED IN VOLUME XCI

DISTRIBUTION-BASED PRICING FORMULAS ARE NOT ARBITRAGE-FREE

DAVID L. RUHM

1. INTRODUCTION

I am honored that the paper has drawn interest from my colleagues and that Mr. Wacek has written a discussion of it. This reply addresses some matters that were raised in the discussion.

2. THE PAPER'S TITLE

The discussion criticizes the paper's title, claiming it is "clearly too categorical," but provides no support for this claim other than:

"In the paper, the author himself points out that insurance prices that are based on the probability distribution of outcomes can be arbitrage-free."

The author believes this is a misinterpretation of some part of the paper, since the paper makes the opposite statement in several places, supported by a proof. Perhaps the discussion's claim is referring to Section 6.3, which states that insurance can be priced with distribution-based formulas but does *not* state that those prices are arbitrage-free:

"Insurance almost never covers asset-event combinations that are traded in a liquid market.... As vulnerability to arbitrage does not exist for insurance, formulas that are theoretically not arbitrage-free can be used to price insurance risks without consequent economic penalty."

The paper's formal mathematical proof that distribution-based pricing formulas are not arbitrage-free covers the general case, which supports the title's accuracy.

3. PRICE VERSUS EXPECTED COST AND PRACTICAL RISK PRICING

The discussion highlighted the paper's point that a risk's price and expected value are generally distinct. In practice, risk pricing methods often deal with expected value directly, by charging the expected cost plus a risk load that is based on the risk's distribution so as to provide a margin of safety and an expected gain opportunity.

This is a reasonable, effective approach to risk pricing. The fact that the resulting prices are often technically not arbitrage-free is usually of no practical consequence. Even derivatives could be selectively bought or sold in this way with successful results. Furthermore, such a strategy can be implemented using simple risk pricing methods, without employing exotic formulas. There is interpretive value gained from using simple, transparent formulas, and there is usually not much (if any) benefit to be gained from using more complex formulas, even if they have some theoretical appeal.

With recent advances in risk pricing theory, an actuary could be tempted to use a complex pricing formula that has theoretical connections with arbitrage-free pricing, believing that the resulting prices will be arbitrage-free and, therefore, more economically accurate than prices derived by simpler methods. The point of the paper is that this is generally not true if the complex formula uses only outcome probabilities to calculate the risk load. The pricing formula generally has to incorporate covariation with underlying events in order to produce genuine arbitrage-free prices.

For example, the single-parameter Wang transform can produce arbitrage-free options prices if the formula's parameter is set to the right value for a particular stock, but the specific parameter value varies widely among stocks and might not even be defined for most insurance risks. Different values of the parameter will give different prices, and it can be unclear how to set the parameter in order to obtain a sensible risk load.

On the other hand, the Wang transform can be used to calibrate a set of interrelated insurance prices to a set of options prices, providing some consistency to the insurance pricing structure. For example, various loss layers for a particular risk can be assigned prices corresponding to analogous option spreads on a stock. The prices for the loss layers would then be additive and generally free of internal inconsistencies, like arbitrage-free option prices.

4. THE ROULETTE WHEEL WITH PAYOFFS THAT VARY BY SPACE

Regarding the example involving the "Ruhm Roulette Wheel" (the name used in the discussion for the metaphorical roulette wheel described in the paper), note that the paper does not claim that a *physical* roulette wheel should have the varying payoffs described. The paper explains in Sections 5.2 and 6.1 that these peculiar roulette-like bets, which all have equal odds but varying payoffs, exist in markets with Black-Scholes pricing. The roulette wheel with varying payoffs is effectively embedded in any such market. One places a bet by buying and selling derivatives in a combination designed to create the particular bet desired, as explained in the paper. (Whether the position is achieved by buying or selling is not relevant, since only the net resulting position determines the economics.)

These surprising bets also exist in other markets. As mentioned above, the paper proves the result in general for markets with arbitrage-free pricing. Black-Scholes pricing is a case that is particularly useful for demonstration, since it is probably a well-known arbitrage-free pricing formula. (The roulette-like bets can also exist in markets that are not arbitrage-free.)

The question that the discussion's example begs is, "If these bets exist in actual financial markets, then why doesn't every participant make only the highest-payoff bets?" The short answer is that probability and risk are distinct concepts, meaning that two events can have the same probability but still differ in risk because of existing risk aggregation. High-payoff bets in markets require assuming risk on just those possibilities for which many parties are already exposed to capital loss (such as a catastrophe). Some market participants can't afford to bet on them because of their existing exposure, even though such a bet offers a positive expected value, since it would expose them to loss when they could least afford it. The high-payoff bets are less attractive to the market as a whole than their simple expected value would suggest because of broad existing risk exposure to the underlying events.

By contrast, a physical roulette wheel offers bets on trivial physical events, all of which have no connection with a participant's existing capital and exposure to risk. Therefore, the spaces are equally preferable, so, logically, payoffs for the spaces are equal.

This key concept—the distinction between probability and risk—is the crucial point underlying the results described in the paper. The difference between probability and risk becomes clearer in the insurance examples presented below, which also make the varying-payoff bets more apparent.

The discussion proposed a different answer to the question, based on the idea that people's opinions differ as to whether the expected return on a stock, E, is higher or lower than the risk-free rate, r. While people do have a variety of opinions on stocks, that explanation does not actually answer the question, because the Black-Scholes pricing theory still works even when the exact value of E is fixed and known by all market participants. (Other pricing theories also work under this condition.) The roulette-like bets with varying payoffs would still exist in such a market, where all participants' opinions about E are the same. The unusual roulette-like bets do not depend on differences of opinion regarding expected value; they depend on exposure of existing capital to loss from potential future events.

The probability/risk distinction is an economic concept based on existing capital and risk exposure that does not rely upon any assumed psychological causes, in contrast to the discussion's characterization of it. By contrast, the "variation of opinion" conjecture offered in the discussion seems more psychologically based than economic.

5. THE DISTINCTION BETWEEN PROBABILITY AND RISK

The insurance market demonstrates the probability/risk concept more clearly than the options market. Differences in exposure, rather than differences in opinion, drive demand for insurance from both personal and commercial customers. Insurance buyers generally do not expect to profit from purchasing insurance, and they do not undertake it as an investment with the prospect of a gain based on expected loss costs. (Those who do might comprise the moral hazard element in the insured population.) Insurance is commonly understood as the cost for hedging risk on assets.

Reinsurance is a clear example. Insurers often accept an expected net cost when buying reinsurance and expect that the reinsurer has an expected profit built into the price. The ceding insurer pays this net cost in order to hedge and manage risk on its book. Insurers are not ignorant in regard to insurance and expected value, yet they often pay more than expected value for reinsurance. They make rational, risk-hedging bets that have negative expected outcomes.

In summary, people and companies that buy insurance effectively make bets having negative expected values in order to obtain reduction of risk, just as some stockholders buy put options on their stocks to reduce risk.

The roulette wheel described in the paper is tied to financial events. The low-numbered spaces come up when a specified asset (such as a stock or a property) suffers a loss of value, exactly when the owners of the asset would require financial relief from the loss. Betting on a low-numbered space is analogous to buying a risk-hedge on the asset, like insurance. The event-driven roulette wheel is a model for representing risk transfer transactions that occur in a variety of forms, such as put options and insurance, but that are all similar in nature: they are wagers on events that impact assets that are valuable to their owners.

6. HURRICANE INSURANCE

Taking another example, the total capital exposed to risk from a hurricane in Florida appears to drive the market price of risk transfer. The more property that is exposed, the greater the demand for this type of coverage. (Variation in people's opinions regarding a hurricane's expected loss cost probably doesn't create most of the demand for coverage.)

While coastal property assets are exposed to the risk of a hurricane's occurrence, parties who do not own coastal property may be economically unaffected by the event, so it poses no risk to them. Although the event's probability remains constant, they have no capital exposure and, therefore, no risk from the event. The noncoastal parties could be in a position to profit by selling insurance to coastal property owners.

Some parties, such as owners of building materials, might actually stand to obtain an economic benefit from a hurricane's occurrence. They would be in a better position to make the positive-value bet of writing insurance, since they own natural hedges to the risk. In insurance parlance, they have more "capacity" to assume risk on such an event.

In summary, insurance buyers are making a negative-expected-value bet on hurricane occurrence during the year, while insurance writers are making a positive expected-value bet on this being a lighter year for hurricane losses. The roulette wheel exists in this market as well, and there are plenty of players willing to bet on the low-payoff spaces because of existing risk exposure.

When risk from hurricane is summed across all parties worldwide—including those who are exposed to loss, those who could gain from such an event, and those not exposed—the net total result is positive risk exposure, because the net economic result of a hurricane is destruction of existing capital. (After a hurricane, there is less total capital than before.)

This positive net risk exposure means that the total demand for insurance risk transfer from those exposed to loss should be stronger than the total supply of insurance risk-hedging capacity, in the absence of a risk load. In other words, if regulations stipulated that coastal hurricane insurance could only be offered at expected value pricing, it's likely that there would be more demand for insurance than supply. This conclusion of the theory coincides with what one would reasonably expect in actual insurance markets.

Risk charges in premiums bring supply and demand into balance. Even if there were a perfect, liquid worldwide market for hurricane coverage, the risk load for hurricane risk transfer would have to be positive, based on these economic forces. This dynamic of profit incentive versus risk reduction on capital makes risk transfer markets possible.

7. NET CAPITAL AT RISK DRIVES THE RISK CHARGE FOR THE EVENT

The hurricane example demonstrates the nature of markets in risk transfer; they are driven by potential loss of current capital. In the hurricane insurance example, the capital assets are coastal properties. In the put options example, the market value of business equity is at stake. In all such cases, there is net capital at risk. The risk of loss to capital carries a net risk charge in a liquid market; the other side of that bet is insurance, which carries a compensating premium. The net risk charge and the compensating premium are mirror images of the same quantity. In practical terms, the risk charge that can be included in insurance sold to a property owner is driven by risk to the owner's property, not just by probability.

8. INTERPRETATION OF THE RISK DISCOUNT FUNCTION $w(s)$

On a technical point raised in the discussion, it is true that the risk discount function $w(s)$ is parameterized by the variable E, as was shown in the paper's derivation of the formula for $w(s)$. The paper's formula for $w(s)$ appears to be simpler than the discussion's.

The discussion states that each person has his own opinion of E's value and challenges the paper's conclusions on that basis. After considering that argument, the author believes that the paper's main conclusions in regard to $w(s)$ still stand: if a person's estimate of E gives $w(s) < 1$, then the price is discounted for risk in the person's estimation and represents an investment with net expected gain. If $w(s) > 1$, the participant is paying a surcharge (in the participant's estimation) and is doing so for insurance (i.e., to hedge risk) or possibly for the entertainment value of a gamble.

9. CONCLUSION

In summary, it is the risk of an event to net current capital that determines whether there is a risk charge and how much it will be in a liquid market. A loss distribution alone is generally not sufficient to establish risk load; risk load also depends on the impacts that the events have on the specific capital base against which risk is to be assumed. The existing exposure of that capital base is central to evaluating the contemplated risk assumption. Distribution-based pricing formulas, by definition, measure potential events but do not measure any relationship between those events and the assuming capital's existing exposure. The nature of the capital base is the reference for defining risk.

DISCUSSION OF PAPER PUBLISHED IN VOLUME LXXXIV

RATEMAKING: A FINANCIAL ECONOMICS APPROACH

STEPHEN P. D'ARCY AND MICHAEL A. DYER

DISCUSSION BY MICHAEL G. WACEK

1. INTRODUCTION

In their 1997 *Proceedings* paper, Stephen D'Arcy and Michael Dyer survey the property-casualty ratemaking landscape from a financial economics perspective. While they summarize several approaches, including target total return, CAPM, and discounted cash flow, this discussion is devoted entirely to Section 8 of their paper, which deals with a method that draws upon option pricing theory (OPT).

D'Arcy and Dyer base their exposition on the approach first presented by Doherty and Garven [2] in 1986 and updated by Garven [3] in 1988. Unfortunately, their presentation falls short as a primer on the OPT approach to ratemaking for several reasons. The first problem is that they make some mystifying mistakes with the options, mischaracterizing both the policyholders' claim (wrongly calling it a call option) and the government's tax claim (correctly calling it a call option but wrongly parameterizing it). This might stem from the cumbersome notation they employ (largely borrowed from Doherty-Garven), which they do not use consistently. Second, while the discourse is ostensibly about ratemaking, the authors provide neither a formula for nor an example of the calculation of the fair premium that is the objective of the ratemaking exercise. Finally, most of their discussion treats the aggregate amount of insurance claims as a fixed and known quantity, essentially as a loan. They merely note that allowing claims to vary stochastically "complicates the calculation" and provide a formula but no example. Unfortunately, the formula they give (which is wrongly attributed to Doherty-Garven) does not adequately address the stochastic claims scenario.

Despite these and other shortcomings, the D'Arcy-Dyer paper is still a useful springboard for discussing the OPT approach. The purpose of this discussion is to correct, clarify, and extend their work. We will 1) point out and correct what we see as the paper's shortcomings, 2) rework and extend the examples, and 3) expand the exposition to allow for a more complete treatment of taxes and claims that vary stochastically.

2. THE OPTION PRICING THEORY RATEMAKING FRAMEWORK

Essentially, the Doherty-Garven idea is that, given some simplifying assumptions, the pre-tax value of an insurance company can be represented by a call option on the company's assets. This call option has a variable strike price equal to the aggregate amount of claims. The government's tax claim can also be represented as a call option. The implication for ratemaking is that the appropriate rate level is the one that results in equality between beginning policyholders' surplus and the value of the call option representing the shareholders' interest, net of tax.

Doherty and Garven assume the insurer's assets consist of tradable investments on which it is possible to price an option. They entertain asset-value distributions in both the normal and lognormal family. They also allow for the possibility of tax-advantaged investments within the company's investment portfolio, though they do not allow for tax-loss carryforwards or carrybacks. In addition, their analysis considers the possibility that premium funds might be held for more than one year while claims are being negotiated.

D'Arcy and Dyer simplified some assumptions in order to make their illustration easier to follow:

1. All policies are written for a one-year term at a common date.

2. Claims totaling L are paid exactly one year from policy inception.
3. Premium funds (net of expenses) are received at policy inception.
4. Premium receipts, P_0, and initial surplus, S_0, are invested solely in taxable assets initially valued at Y_0.

3. NOTATION

In a bid to clarify key concepts, this discussion uses the basic D'Arcy-Dyer notation with some refinements. For example, we will use numerical subscripts to refer to time only: 0 meaning inception, 1 meaning one year after inception. Variables l and y refer to the random variables representing aggregate claims and invested assets, respectively, one year from inception. The equation $\text{call}_1(y \mid Y_1, L)$ refers to the expiry value of a one-year European call option on y, given a price at expiry of $y = Y_1$ and an exercise price of $l = L$. The equation $\text{call}_0(y \mid Y_0, L)$ refers to the value of the same option at inception, when the value of invested assets is Y_0. We use a similar notation for European puts. Other notation will be introduced and defined as needed. In Appendix A, we restate (and, where necessary, correct) the D'Arcy-Dyer formulas (8.3) through (8.9) in our notation.

4. FINDING THE OPTIONS

As D'Arcy and Dyer point out, Y_1, which represents the value of the investment portfolio after one year, is the amount the insurer has available to pay the claims, L. If $Y_1 \geq L$, the insurer will pay the policyholder claims in full. If $Y_1 < L$, the insurer will pay the policyholder claims up to the extent of its available assets, i.e., Y_1. If Y_1 is limited to a minimum value of zero, this policyholders' interest can be summarized as:

$$H_1 = \max[\min(Y_1, L), 0]. \tag{8.3}$$

While (8.3) is correct, D'Arcy and Dyer *incorrectly* describe H_1 as equivalent to the payoff at expiry from a European call option on the invested assets with a strike price of L. Formula (8.3) does not define a call option. The call option of the authors' description belongs to the *shareholders*, not the policyholders. The sale of the insurance policies in exchange for premiums is equivalent to the sale of the company's assets (including the premiums) to the policyholders in exchange for a call option to reacquire the assets at a price of L. If $Y_1 \geq L$, the insurer will exercise the option and reacquire the assets. This results in a gain of $Y_1 - L$. If $Y_1 < L$, the insurer will not exercise the option. The gain is 0. The pre-tax shareholders' interest can be summarized as:

$$\begin{aligned} C_1 &= \max(Y_1 - L, 0) \\ &= \text{call}_1(y \mid Y_1, L). \end{aligned} \tag{1}$$

C_1 matches the payoff value at expiry of a European call option on y, given invested assets at expiry of Y_1 and an option strike price of L.[1]

The policyholders' interest at expiry, H_1, can be characterized as a long position in y and a short position in the call or, alternatively, a long position in y net of the pre-tax shareholders' interest:

$$H_1 = Y_1 - \text{call}_1(y \mid Y_1, L) = Y_1 - C_1. \tag{2}$$

Formula (2) is equivalent to formula (8.3).

Neither D'Arcy-Dyer nor Doherty-Garven mention the formulation of the policyholders' interest in terms of a put option. Since put-call parity implies $Y_1 - \text{call}_1(y \mid Y_1, L) = L - \text{put}_1(y \mid Y_1, L)$, the policyholders' interest at the end of the period can also be characterized as:

$$H_1 = L - \text{put}_1(y \mid Y_1, L), \tag{3}$$

[1]Note that we use C_1 to denote the value of the pre-tax shareholders' interest at time 1, whereas D'Arcy and Dyer confusingly use C_1 to denote its value at time 0.

where $\text{put}_1(y \mid Y_1, L)$ denotes the payoff value at expiry of a European put option on y, given invested assets at expiry of Y_1 and an option strike price of L. Clearly, it is in the policyholders' best interest to minimize the value of the put option, since they are paying for the full recovery of L. Failing that, it seems fair that they should receive a premium discount to reflect the fact they are not receiving full coverage. A fair premium would then be:

$$P_0 = H_0 = Le^{-rt} - \text{put}_0(y \mid Y_0, L). \tag{4}$$

The fair premium is equal to the present value of L less the value of the default put option. Since $\text{put}_0(y \mid Y_0, L)$ is a function of P_0, formula (4) must be solved by numerical methods.

The value of the pre-tax shareholders' interest, C_0, at policy inception is:[2]

$$C_0 = Y_0 - H_0 = Y_0 - Le^{-rt} + \text{put}_0(y \mid Y_0, L). \tag{5}$$

Since $C_0 = \text{call}_0(y \mid Y_0, L)$ and y represents an asset whose behavior is consistent with the conditions required by the Black-Scholes call option pricing formula, the value of C_0 can easily be calculated. D'Arcy and Dyer illustrate its calculation with an example. Given S_0 = \$100 million, P_0 = \$160 million, L = \$150 million, risk-free rate $r = 4\%$, $k = t = 1$ year, and asset volatility $\sigma = 50\%$ (reflecting an *extremely* aggressive investment strategy!), they show the Black-Scholes value of the pre-tax shareholders' interest at inception is \$121.41 million. (This discussion calculates it at \$121.42 million, but we will use their number.) D'Arcy and Dyer note that this is surprisingly high, since "adding the initial equity to the underwriting profit totals \$110 million." They attribute the difference to the "default option" considered by the option methodology. Qualitatively, this is correct, but it is wrong to compare the \$110 to the \$121.41, since the first number is valued at the end of the period (but without interest) while the latter is valued at the

[2]Note that this discussion uses C_0 instead of D'Arcy-Dyer's C_1 to denote the pre-tax shareholders' interest.

beginning. From formula (5) it is easy to see that, if the value of the put is zero, the pre-tax shareholders' interest is equal to $Y_0 - Le^{-rt} = (\$100 + \$160) - \$150e^{-0.04} = \$115.88$. The amount attributable to the default option arising from the investment in risky securities is the difference, $C_0 - (Y_0 - Le^{-rt}) = \text{put}_0(y \mid Y_0, L) = \$121.41 - \$115.88 = \$5.53$. This is a substantial amount, but not nearly as large as D'Arcy and Dyer's wording suggests.

Doherty and Garven observe that in equilibrium the present value of the shareholders' interest must equal the initial surplus, so in the pre-tax case this implies:

$$C_0 = S_0. \tag{6}$$

Combining (8.1), which is $Y_0 = S_0 + P_0$, with (5) and (6), we have

$$(S_0 + P_0) - Le^{-rt} + \text{put}_0(Y_0, L) = S_0$$

$$P_0 = Le^{-rt} - \text{put}_0(Y_0, L), \quad \text{which is formula (4).}$$

Solving (4) for P_0 in the authors' example results in $P_0 =$ \$136.44 at equilibrium.

In this example, there is no underwriting risk to the insurer, since L is fixed at \$150. To pay the \$150 at the end of the year, the insurer needs $\$150e^{-0.04} = \$144.12$ at inception to meet that obligation, reflecting an interest credit of \$5.88. The difference \$144.12 – \$136.44 = \$7.68, the value of the put, represents a credit to the policyholders to reflect the risk the insurer will default on claim payments.

This illustrates one of the interesting aspects of the option-based approach, namely, that it automatically incorporates the claim default risk into the insurance rate. It highlights the solvency implications of investment strategy and underwriting leverage. Other ratemaking methods implicitly assume the default risk is immaterial. The Doherty-Garven equilibrium is premised on the idea that an insurer's shareholders should not receive the windfall benefit of the default option that arises from a pursuit of a risky investment strategy and/or high underwriting leverage. Instead, the policyholder premium should be reduced. However, that has the paradoxical implication that insurers most at risk of insolvency are *required* to charge premiums that are less than the expected value of their claim obligations, which clearly can only hasten their demise. That hardly seems like the right recipe for rehabilitation of a financially weak or poorly managed insurer! It would seem to be better public policy for regulators to establish investment and underwriting leverage standards that avoid anything beyond a negligible risk of insolvency. Where the risk of insolvency is found to be material, the remedy should be to correct the insurer's financial or strategy weaknesses, rather than to require it to reduce its rates. In that light, it seems entirely appropriate that other ratemaking methods ignore the risk of insolvency, since effective regulation should make it remote.

Note that D'Arcy and Dyer chose an unrealistic investment volatility parameter, σ, for their example. The standard deviation of U.S. stock market returns from 1900–2000 was 20.2% [1]. The authors' choice of $\sigma = 50\%$ implies an investment strategy much riskier than investing 100% of assets in a diversified portfolio of U.S. equities, which itself is a strategy that an insurer and regulators would find far too risky. The example of $\sigma = 50\%$ was undoubtedly chosen in order to illustrate a material default-risk credit.

If we rework the authors' example using $\sigma = 20\%$, which is consistent with 100% of assets in U.S. equities and thus still very aggressive for an insurer, a premium of \$160 implies $C_0 = \$115.90$. Since the value of the underwriting profit alone indicates a value of \$115.88, the default put option is worth only \$0.02! For more realistic and prudent investment strategies with $\sigma < 20\%$, the value of the default put option is essentially zero.

If we solve equation (4) to find the equilibrium value of P_0, given $\sigma = 20\%$, we obtain $P_0 = \$144.07$. Since the present value of L is \$144.12, this implies the value of the default option is \$0.05. The risk of default is slightly higher with a premium of

\$144.07 than it is with a premium of \$160, resulting in an increase in the default-risk credit from \$0.02 to \$0.05.

5. EFFECT OF TAXES

Let's now consider the effect of taxes. If taxes apply only to income and no tax credits arise from losses, then the government's tax interest in the insurer's income can also be characterized as a call option. D'Arcy and Dyer correctly describe the payoff value of this tax interest as:

$$T_1 = \max\{\text{tax} \cdot [i \cdot (Y_1 - Y_0) + P_0 - L], 0\}. \tag{8.4}$$

Setting the tax rate = 35% and the proportion of taxable assets $i = 100\%$ in their example with $P_0 = \$160$, they claim this corresponds to the payoff value of 0.35 European call options with a total value of \$16.05. Unfortunately, the parameters they use in the Black-Scholes formula do not make sense. Their parameters and the \$16.05 correspond to $0.35 \cdot \text{call}_0(y \mid Y_1 - Y_0 + P_0, L)$, implying the value of invested assets at time zero is $Y_1 - Y_0 + P_0$, when clearly it must be Y_0. The correct value of the tax call is \$20.96, as we show below.

Consistent with (8.4) with $i = 100\%$, the value of the insurer's income at the end of the period is:

$$\begin{aligned} I_1 &= (Y_1 - Y_0) + (P_0 - L) \\ &= Y_1 - (S_0 + P_0) + (P_0 - L) \\ &= Y_1 - (S_0 + L). \end{aligned} \tag{8}$$

If we focus only on positive outcomes, $\max(I_1, 0)$ is the payoff profile at expiry of a call option on invested assets, y, with a strike price of $S_0 + L$. Assuming the investment return is 100% taxable, the present value of the government's positive tax interest is equal to the tax rate times this call option:

$$T_0 = \text{tax} \cdot \text{call}_0(y \mid Y_0, S_0 + L). \tag{9}$$

The correct value of the tax call in the authors' example is

$$\begin{aligned}T_0 &= 0.35 \cdot \mathrm{call}_0(y \mid Y_0, S_0 + L) = 0.35 \cdot \mathrm{call}_0(y \mid 260, 250)\\ &= (0.35)(\$59.89) = \$20.96.\end{aligned}$$

Then the shareholders' interest, net of tax, is

$$C_0 - T_0 = \$121.41 - \$20.96 = \$100.45$$

instead of the \$105.36 given by D'Arcy and Dyer.

To find the fair premium in these circumstances, we solve for the value of P_0 that meets the condition $C_0 - T_0 = S_0$. Since $C_0 = Y_0 - [Le^{-rt} - \mathrm{put}_0(y \mid Y_0, L)]$ and $Y_0 = S_0 + P_0$, then

$$C_0 - T_0 = S_0 \quad \text{implies}$$

$$(S_0 + P_0) - [Le^{-rt} - \mathrm{put}_0(y \mid Y_0, L)] - \mathrm{tax} \cdot \mathrm{call}_0(y \mid Y_0, S_0 + L) = S_0$$

$$\text{and} \qquad P_0 = Le^{-rt} - \mathrm{put}_0(y \mid Y_0, L) + \mathrm{tax} \cdot \mathrm{call}_0(y \mid Y_0, S_0 + L). \tag{10}$$

This implies a fair premium $P_0 = \$159.33$ in the authors' after-tax example.

The Doherty-Garven model deliberately ignored tax-loss carryforward and carryback provisions. However, they are actually easy to deal with within the simple framework presented by D'Arcy and Dyer.

Assume the tax code allows for tax-loss carryforwards and carrybacks. If $Y_1 < S_0 + L$, which implies a loss, the insurer earns a tax credit of $\mathrm{tax} \cdot (S_0 + L - Y_1)$. If $Y_1 \geq S_0 + L$, which implies a profit, the insurer earns a tax credit of zero. That tax-credit pattern matches the payoff profile of tax European-put options on y having a strike price of $S_0 + L$. Thus the tax credit equates to a long put-option position owned by the insurer, and a short put-option position on the part of the government. If the insurer becomes insolvent ($Y_1 < L$), then it won't be in a position to use its tax credit. Therefore, the portion of the credit arising from insolvency scenarios must be removed.

The government's net tax-option position in this symmetrical tax scenario is:

$$T_0^* =$$
$$\text{tax} \cdot \{\text{call}_0(y \mid Y_0, S_0 + L) - [\text{put}_0(y \mid Y_0, S_0 + L) - \text{put}_0(y \mid Y_0, L)]\}.$$

In the authors' $P_0 = \$160$ example with tax = 35%, the tax put has a value of

$$\begin{aligned} 0.35 \cdot [\text{put}_0(y \mid Y_0, S_0 + L) &- \text{put}_0(y \mid Y_0, S_0 + L)] \\ &= 0.35 \cdot (\$40.08 - \$5.54) = \$12.09; \end{aligned}$$

and the symmetrical after-tax value of the shareholders' interest, $V_0^*(P_0 \mid L)$, is

$$\begin{aligned} V_0^*(160 \mid 150) = C_0 - T_0^* &= \$121.41 - \$20.96 + \$14.03 - \$1.93 \\ &= \$112.55. \end{aligned}$$

The formula for T_0^* can be simplified. Since put-call parity implies $\text{call}_0(y \mid Y_0, S_0 + L) - \text{put}_0(y \mid Y_0, S_0 + L) = Y_0 - (S_0 + L)e^{-rt}$, the symmetrical tax obligation can be expressed as

$$T_0^* = \text{tax} \cdot [Y_0 - (S_0 + L)e^{-rt} + \text{put}_0(y \mid Y_0, L)]. \tag{11}$$

If the tax treatment of profits and losses is symmetrical, we can determine the fair premium by substituting T_0^* for the tax term in (10):

$$\begin{aligned} P_0 &= Le^{-rt} - \text{put}_0(y \mid Y_0, L) \\ &\qquad + \text{tax} \cdot [Y_0 - (S_0 + L)e^{-rt} + \text{put}_0(y \mid Y_0, L)] \\ &= Le^{-rt} - \text{put}_0(y \mid Y_0, L) + \frac{\text{tax} \cdot (1 - e^{-rt})}{1 - \text{tax}} \cdot S_0. \end{aligned} \tag{12}$$

Formula (12) implies a fair premium of $P_0 = \$138.80$, given the non-tax parameters used in the authors' example combined with symmetrical taxation.

6. STOCHASTIC CLAIMS

D'Arcy and Dyer discuss the application of the Doherty-Garven approach to the real insurance world in which claims vary stochastically only very briefly. They focus mainly on the scenario where the aggregate claim amount, L, is an amount certain; in effect treating the transaction as a loan. In fact, for realistic insurance-ratemaking applications the claim amount is a random variable, which we will denote l.

If we know $f(l)$, we can determine the unconditional expected value of the shareholders' interest. Assuming symmetrical tax treatment of profits and losses, the expected value of the after-tax shareholders' interest, $\mathrm{E}[V_0^*(P_0)]$, is given by:

$$\begin{aligned}
\mathrm{E}[V_0^*(P_0)] &= \int_0^\infty (C_0 - T_0^*)\cdot f(l)dl \\
&= \int_0^\infty \{Y_0 - [le^{-rt} - \mathrm{put}_0(y \mid Y_0, l)] \\
&\quad - \mathrm{tax}\cdot[Y_0 - (S_0 + l)e^{-rt} + \mathrm{put}_0(y \mid Y_0, l)]\}\cdot f(l)dl \\
&= (1 - \mathrm{tax})\cdot[Y_0 - \mathrm{E}(l)e^{-rt}] + \mathrm{tax}\cdot S_0 e^{-rt} \\
&\quad + (1 - \mathrm{tax})\int_0^\infty \mathrm{put}_0(y \mid Y_0, l)\cdot f(l)dl. \qquad (13)
\end{aligned}$$

While we are most interested in the symmetrical taxation scenario embodied in (13), before we explore that case further, we will discuss the treatment of stochastic claims in the D'Arcy-Dyer world in which tax-loss carryforwards and carrybacks are not allowed. In that case, the expected value of the after-tax shareholders' interest, $\mathrm{E}[V_0(P_0)]$, is given by:

$$\begin{aligned}
\mathrm{E}[V_0(P_0)] &= \int_0^\infty (C_0 - T_0)\cdot f(l)dl \\
&= \int_0^\infty \mathrm{call}_0(y \mid Y_0, l)\cdot f(l)dl \\
&\quad - \int_0^\infty \mathrm{tax}\cdot \mathrm{call}_0(y \mid Y_0, S_0 + l)\cdot f(l)dl. \qquad (14)
\end{aligned}$$

Let's compare formula (14) to the authors' (8.9), which they describe as applicable when losses are assumed to vary

$$V_e = C[Y_1(P^*);\mathrm{E}(L)] - t \cdot C\{i \cdot [Y_1(P^*) - Y_0(P^*)] + P^*;\mathrm{E}(L)\}, \tag{8.9}$$

where P^* is chosen so that $V_e = S_0$.[3] They attribute (8.9) to Doherty-Garven. As discussed earlier, the second term of (8.9) representing the tax call is wrong. Correcting for that and restating the formula in our notation with $i = 100\%$, the formula becomes:

$$\mathrm{E}[V_0(P_0)] = \mathrm{call}_0[y \mid Y_0, \mathrm{E}(l)] - \mathrm{tax} \cdot \mathrm{call}_0[y \mid Y_0, S_0 + \mathrm{E}(l)]. \tag{8.9*}$$

Formula (8.9*) is equivalent to (14) only in the special case where

$$\int_0^\infty \mathrm{call}_0(y \mid Y_0, l) \cdot f(l)dl = \mathrm{call}_0[y \mid Y_0, \mathrm{E}(l)] \qquad \text{and}$$

$$\int_0^\infty \mathrm{call}_0(y \mid Y_0, S_0 + l) \cdot f(l)dl = \mathrm{call}_0[y \mid Y_0, S_0 + \mathrm{E}(l)].$$

Clearly, formula (8.9), in either its original or corrected (8.9*) form, cannot represent the value of the shareholders' interest in the stochastic claims case. Both of the call terms depend only on the first moment of the claim distribution, which is a constant, rather than on the whole distribution.

For the sake of illustration, let's assume $f(l)$ is log-normally distributed with $\sigma_l = 11\%$ (a choice inspired by Van Kampen [4]) and $\mathrm{E}(l) = e^{\mu + 0.5\sigma^2} = \$150$. Let all other premium and investment parameters match the authors' original assumptions. Using these parameters, formula (14) with the constraint $\mathrm{E}[V_0(P_0)] = S_0$ indicates a fair premium of \$158.89. Alternatively, if we maintain $\mathrm{E}(l) = \$150$ but let $\sigma_l = 15\%$, the indicated fair premium from (14) with the same constraint is \$158.50. In comparison, the fair

[3]Note that here V_e denotes a time 0 value, while in (8.5) it denotes a time 1 value.

premium indicated by the similarly constrained formula (8.9*) is always \$159.33, irrespective of the value of σ_l. Note also that the indicated premiums arising from the stochastic claims scenarios are lower than the premium indicated by the constant claim scenario because there is a slightly higher risk of insolvency and default when claims can vary.

From D'Arcy and Dyer's discussion about (8.9) it is clear that they believe that formula is faithful to Doherty-Garven, not only in reflecting stochastic variation in claims, but also in reflecting an underwriting-risk charge. The fact is their formula reflects neither. We suspect their confusion arises from the fact that Doherty and Garven presented a similar but not identical formula (their formula (7)):

$$V_e = C[\tilde{Y}_1(P^*);\tilde{L}] - \tau \cdot C\{\theta \cdot [\tilde{Y}_1(P^*) - \tilde{Y}_0(P^*)] + P^*;\tilde{L}\}. \tag{DG.7}$$

Note the difference in the strike prices of $\mathrm{E}(L)$ in (8.9) and $\tilde{L}$ in (DG.7). The former is a constant, while the latter is defined by Doherty-Garven to be a random variable. For that reason alone, the two formulas are clearly different. Since the call options used in DG.7 have variable strike prices *and* embedded underwriting-risk charges, they are not the usual kind of European options that have fixed exercise prices. In contrast, the D'Arcy-Dyer formula (8.9) uses standard European calls and reflects no underwriting-risk charge.

Returning to our formula (13) for the expected value of the after-tax shareholders' interest in the symmetrical taxation case, note that if we solve $\mathrm{E}[V_0^*(P_0)] = S_0$ for P_0, we obtain the following stochastic claims analogue to formula (12):

$$P_0 = \mathrm{E}(l)e^{-rt} - \int_0^\infty \mathrm{put}_0(y \mid Y_0, l) \cdot f(l)dl + \frac{\mathrm{tax} \cdot (1 - e^{-rt})}{1 - \mathrm{tax}} \cdot S_0, \tag{15}$$

which, for the example we have been following, implies a premium $P_0 = \$138.22$ (vs. \$138.80 for the fixed $L = \$150$ case) that reflects both symmetrical tax effects and a credit to policyholders to compensate them for the risk of default by the insurer. However, $P_0 = \$138.22$ does not reflect a risk charge to reflect the stochastic nature of l.

To reflect such a risk charge formula, we need to solve for the value of P_0 that satisfies $\mathrm{E}[V_0^*(P_0)] = S_0 + \lambda$, where λ is the after-tax charge for pure underwriting risk. That implies a fair premium in the stochastic claims case with symmetrical tax treatment of

$$P_0 = \mathrm{E}(l)e^{-rt} - \int_0^{\infty} \mathrm{put}_0(y \mid Y_0, l) \cdot f(l)dl + \lambda + \mathrm{tax} \cdot \frac{(1 - e^{-rt}) \cdot S_0 + \lambda}{1 - \mathrm{tax}}. \tag{16}$$

We see that the only option in formula (16) is the put option representing the credit for insurer insolvency. If that put option has a value of zero, as it should under any effective regulatory regime, formula (16) reduces to the standard actuarial ratemaking formula.

Note that the approach this discussion has taken with respect to the underwriting-risk charge is slightly different from that of Doherty-Garven. They incorporate the risk charge for underwriting risk into the non-standard call-option formula they derived for calculating the value of the options in (DG.7). We prefer to treat the risk charge for underwriting risk explicitly.

Using the D'Arcy-Dyer parameters (except substituting the more realistic investment volatility value $\sigma = 10\%$ for their 50%), and setting $\sigma_l = 11\%$ and $\lambda = 0.0325P_0$, formula (16) indicates $P_0 = \$153.92$ as the appropriate premium reflecting symmetrical taxation, policyholder credit for insurer default risk, and a risk charge for stochastic claims. In this example with a more realistic investment policy assumption, the value of the default option is zero. Table 1 shows the composition of premium.

TABLE 1

COMPOSITION OF FAIR PREMIUM
STOCHASTIC CLAIMS, DEFAULT CREDIT, SYMMETRICAL TAX

		% of P_0
Losses	\$150.00	97.45%
– PV of Interest on Losses	(\$5.88)	–3.82%
= PV of Losses	\$144.12	93.63%
+ PV of Default Option	\$0.00	0.00%
= PV Pure Premium	\$144.12	93.63%
+ PV Taxes T_0	\$4.80	3.12%
= PV Tax-Adj Pure Premium	\$148.92	96.75%
+ PV U/W Risk Charge	\$5.00	3.25%
= Premium (P_0)	\$153.92	100.00%

7. CONCLUSION

D'Arcy and Dyer concluded that the OPT approach is more complex than the CAPM or Discounted Cash Flow approaches, but that it avoids some of the problems associated with CAPM (such as estimating betas). This discussion aims to make it clear that if taxation is symmetrical, which seems more realistic than assuming it is not, and default risk is zero, then the OPT premium formula (16) is the same as the D'Arcy-Dyer Discounted Cash Flow (DCF) premium formula (6.1) with the underwriting-risk charge broken out explicitly. The only real difference from conventional DCF ratemaking in the OPT framework is in the Doherty-Garven approach to the underwriting-risk charge, which they base on the correlation between insurance claims and the stock market, making it similar to the CAPM approach described in Section 4 of the D'Arcy-Dyer paper. Far from avoiding the problems associated with estimating betas, etc., the Doherty-Garven approach to quantifying underwriting risk has *exactly* the same problems as CAPM.

The Doherty-Garven approach is an interesting application of option theory, but it is also much less exotic than it first appears. If insurance regulations aimed at avoiding insolvencies are formulated and executed effectively, then the insolvency put embedded in the fair premium will be zero. There is no need to resort to the option approach. Options can be used, at least conceptually, to describe the effect of a tax law that does not treat profits and losses symmetrically. However, this discussion has shown, if taxation is symmetrical, those options disappear too, and the ratemaking formula reduces to the conventional one, where the remaining debate is about how to calculate the underwriting-risk charge.

REFERENCES

[1] Dimson, Elroy, Paul Marsh, and Mike Staunton, "Long Run Global Capital Market Returns and Risk Premia," *Social Sciences Research Network Electronic Paper Collection, Working Paper Series* (http://papers.ssrn.com), February 2002.

[2] Doherty, Neil A., and James R. Garven, "Price Regulation in Property-Liability Insurance: A Contingent Claims Approach," *Journal of Finance*, Vol. 41, 1986, pp. 1031–1050.

[3] Garven, James R., "Investment Income, Underwriting Profit and Contingencies: Future Developments," Casualty Actuarial Society *Forum*, Fall 1988, pp. 177–203.

[4] Van Kampen, Charles E., "Estimating the Parameter Risk of a Loss Ratio Distribution," Casualty Actuarial Society *Forum*, Spring 2003, pp. 177–213.

APPENDIX A

D'ARCY-DYER FORMULAS RESTATED IN NOTATION OF THIS DISCUSSION

(8.3) $H_1 = \max\{\min[L,Y_1],0\}$	$H_1 = \max\{\min[L,Y_1],0\}$
(8.4) $T_1 = \max\{t[i(Y_1 - Y_0) + P_0 - L],0\}$	$T_1 = \max\{\text{tax}[i(Y_1 - Y_0) + P_0 - L],0\}$
(8.5) $V_e = Y_1 - H_1 - T_1$	$C_1 - T_1 = Y_1 - H_1 - T_1$
(8.6) $H_0 = V(Y_1) - C[Y_0;\mathrm{E}(L)]$	$H_0 = Y_0 - C_0$
	$= Y_0 - \text{call}_0(y \mid Y_0, L)$
(8.7) $T_0 = tC[i(Y_1 - Y_0) + P_0;\mathrm{E}(L)]$	$T_0 = Y_0 - \text{tax} \cdot [\text{call}_0(y \mid Y_0, S_0 + L)]^\dagger$
(8.8) $V_e = V(Y_1) - H_0 - T_0$	$C_0 - T_0 = Y_0 - H_0 - T_0$
$= C[Y_0;\mathrm{E}(L)]$	$= \text{call}_0(y \mid Y_0, L)$
$- tC[i(Y_1 - Y_0) + P_0;\mathrm{E}(L)]$	$- \text{tax} \cdot \text{call}_0(y \mid Y_0, S_0 + L)^\dagger$
$= C_1 - tC_2$	$= C_0 - T_0$
(8.9) $V_e = C[Y_1(P^*);\mathrm{E}(L)]$	$\mathrm{E}[V_0(P_0)] = \text{call}_0[y \mid Y_0, \mathrm{E}(l)]$
$- tC[i(Y_1(P^*) - Y_0(P^*)) + P^*;\mathrm{E}(L)]$	$- \text{tax} \cdot \text{call}_0[y \mid Y_0, S_0 + \mathrm{E}(l)]$
$= C_1^* - tC_2^* = S_0$	$= C_0 - T_0^* = S_0{}^\dagger$

†for $i = 1$.

ADDRESS TO NEW MEMBERS—MAY 17, 2004

MAVIS A. WALTERS

It is an honor to be here today to address this class of Fellows and Associates. Let me begin by congratulating all of you for finally achieving your goal. I know that it was not easy. It takes enormous dedication, commitment, and personal sacrifice to get to where you are today. You can be very proud of what you have accomplished.

And your family members also deserve to be congratulated. I hope you fully appreciate all the sacrifices they have had to make to enable you to get to this point in your lives.

The tradition of having a past president address the new members originated in 1985 when Stan Khury was president. Until 1994 there was only one address each year. But then we began having large numbers of new Fellows and Associates being admitted at both the May and November meetings. For the past 10 years then, there have been two such talks annually. That makes almost 30 talks to date!

So when Mary Frances Miller asked me to take on this role today, I was flattered but somewhat intimidated. I wasn't sure how to begin, so I called my brother, Mike, for advice. Having done this twice, he suggested that I read the previous addresses for ideas. While that made sense, my initial reaction was not to do that. I was afraid I would be unduly influenced and might inadvertently plagiarize some of the ideas from other past presidents. But my first attempt to draft these remarks was pretty lame. Then I decided maybe I should read what others have said on this occasion and I am very glad I did. Almost nothing that follows is original with me. I freely admit I am plagiarizing from my predecessors, but their advice is well worth repeating. Here is my selection of the top six pearls of wisdom from other past presidents for your consideration.

1. This first one is easy and it is for the Associates. There is a unanimous recommendation that all of you continue to pursue your Fellowship.

2. For you Fellows, although your formal exams are finished, your education is just beginning. Carl Honebein opined on the difference between taking college exams and CAS exams: "You must pass the college exams to get out but CAS exams to get in." As Chuck Bryan put it, "The attainment of FCAS is merely the end of the beginning of your career." As a CAS member, it is your duty and responsibility to continue your education. Dave Flynn said simply, "You must continue the learning process."

LeRoy Simon used the phrase that, "It is easier to become an actuary than to be one." He explained that it is easy to apply a familiar tool to a problem but perhaps a new technique would be better. He warned against letting our actuarial expertise stagnate at the level when we attained Fellowship. Ruth Salzmann advised that we should ask of any methodology, "Why might I want to do it differently." This is how we learn and expand our techniques. Mike Walters warned that many more skills will need to be developed in the years ahead to solve the new problems that are constantly arising. So it is extremely important that all of us continue to grow and expand our actuarial skills throughout our careers.

3. "Actuaries have a terrible time making themselves understood by non-actuaries." This is a quote from Charlie Hewitt, but this same theme was repeated by several past presidents. We need to recognize that we, as actuaries, provide a service, and so effective communication is essential. We are not of much help to a client or employer if we can't explain our reasoning and conclusions to them. Avoid actuarial jargon so that others can understand what you are saying.

4. Almost all of the past presidents agreed with Jim MacGinnitie that it is the responsibility of today's actuaries to contribute to the growth and development of our profession. This can be done

in a variety of ways. One way might be to do as Jerry Scheibl and others have said: volunteer for committees, write papers, and appear on panels. We all know that the Exam Committee is always looking for new Fellows and we strongly urge you to consider such an assignment. Jerry quoted Francis Bacon's declaration: "I hold every man to be a debtor to his profession." It would be well for all of us to live accordingly.

Another way to give back would be to do as Ron Bornheutter, among others, urged: expand our horizons beyond what has been considered "traditional actuarial work." Adger Williams noted that our actuarial training provides a versatile base for us to choose a variety of paths to pursue. These paths can lead to an enhanced reputation for our profession.

Stan Hughey said that we must forge ahead because there are lots of new frontiers that actuaries can explore. Irene Bass noted that "actuaries do have a level of creativity, insight, and knowledge that goes well beyond the rote application of actuarial arithmetic." She went on to say that we look to the younger members of our Society to find new approaches, to use new tools, and find innovative ways to test our work. Charlie Hewitt quoted a Greek proverb that I found thought-provoking and challenging for all of us: "A society grows great when old men plant trees in whose shade they will never sit." That certainly does speak to the responsibility to give back to the profession and make a contribution to the future.

5. Steve Newman's advice to the class of 1990 is still relevant today: broaden your business vision to figure out how what you do fits into the whole of your employer's or client's business. He added that it is important to develop a sense of humility in your life because there is always a great deal more to learn about our business. Phil Ben-Zvi echoed Steve's comments when he warned the class of 1993 not to believe you know everything about the insurance business. He went on to say that a lot can be learned from other insurance professionals, i.e., non-actuaries. In a CEO interview I did several years ago on behalf of the CAS

Board of Directors, Ramani Ayer of The Hartford referred to the "arrogance of the actuary." While he is a big fan of actuaries, he has observed that there is a tendency for actuaries to believe that they have all the answers and have no need of any other opinions or advice. So clearly, Phil and Steve are on target here with their admonition that we can and should learn a lot from others.

6. Advice from both Dave Hartman and Mike Toothman that was echoed by almost all the other past presidents was to always conduct yourselves under the highest ethical standards. Remember that all our reputations are at stake. The code of conduct and standards of practice need to be taken very seriously and followed by all members of our profession.

7. Finally, my own closing piece of advice for Associates and Fellows: have fun! While you have chosen a difficult and demanding profession it offers many rewards. Make sure you enjoy them. Recognize that the life is short. Take time for yourselves and your families so that 30 or more years from now you will not look back with any regrets over time wasted or opportunities lost.

Good luck and God bless you all.

MINUTES OF THE 2004 CAS SPRING MEETING

May 16–19, 2004

THE BROADMOOR

COLORADO SPRINGS, COLORADO

Sunday, May 16, 2004

The CAS Board of Directors met from 8:30 a.m. to 4:30 p.m.

Registration was held from 4:00 p.m. to 6:30 p.m.

An officers' reception for New Associates and Accompanying Persons was held from 5:30 p.m. to 6:30 p.m.

A welcome reception for all attendees was held from 6:30 p.m. to 7:30 p.m.

Monday, May 17, 2004

Registration continued from 7:00 a.m. to 8:00 a.m.

CAS President Mary Frances Miller opened the business session at 8:00 a.m., welcoming all to the CAS Spring Meeting and announcing that the morning's events would be Web-cast over the CAS Web Site. President Miller introduced the current members of the Executive Council (EC) and the CAS Board of Directors.

Ms. Miller recognized past presidents of the CAS attending the meeting: Ronald L. Bornhuetter (1975), Charles A. Bryan (1990), Robert F. Conger (2001), Michael Fusco (1989), David G. Hartman (1987), C.K. "Stan" Khury (1984), Steven G. Lehmann (1998), Gail M. Ross (2002), Mavis A. Walters (1997), and Michael A. Walters (1986).

Ms. Miller also recognized a special guest in the audience, Margaret Tiller Sherwood, president of the Conference of Consulting Actuaries and CAS Fellow.

Ms. Miller asked all CAS volunteers to stand and be recognized, including committee chairpersons; board members and officers of the executive council; committee members; individuals who have worked on the AAA committees or committees of other actuarial organizations; Regional Affiliate officers; authors of papers; and moderators and panelists of this Annual Meeting or any previous CAS meeting. Ms. Miller asked the audience to applaud the efforts of these volunteers.

Christopher Carlson, vice president–marketing & communications, gave an overview of this meeting's highlights, including the addition of a new exhibit hall and corporate supporter program, and the presentation of the work of one the newly created CAS Research Working Parties. Carlson announced that two discussions by Michael Wacek of *Proceedings* papers would be presented. The discussions of the papers are David Ruhm's "Distribution-Based Pricing Formulas are not Arbitrage-Free" and Stephen P. D'Arcy and Michael Dyers' "Ratemaking: A Financial Economic Approach."

Following these announcements, the new CAS Fellows and Associates in attendance were honored in a special ceremony. The CAS admitted 41 new Fellows and 28 new Associates in May 2004. Thomas G. Myers, vice president–admissions, announced the new Associates and Stephen P. D'Arcy, CAS president-elect, announced the new Fellows. The names of the members of this class follow.

NEW FELLOWS

Afrouz Assadian	Joel D. Glockler	Matthew E. Morin
Kevin J. Atinsky	Ann E. Green	Kyle S. Mrotek
Stephanie Anne Bruno	David J. Horn Jr.	Lester M. Y. Ng
Phyllis B. Chan	Jesse T. Jacobs	Tom E. Norwood
Brian Kenneth Ciferri	Jonathan David Koch	Faith M. Pipitone
Christian J. Coleianne	Kristine Kuzora	Jayne L. Plunkett
Richard R. Crabb	James A. Landgrebe	John T. Raeihle
Paul B. Deemer	Jia Liu	Laura D. Rinker
Kiera Elizabeth Doster	John R. McCollough	Paul Silberbush
Robin V. Fitzgerald	Jeffrey B. McDonald	Christopher J. Styrsky
David S. Futterleib	Martin Menard	Shantelle Adrienne Thomas
Keith R. Gentile	Ryan A. Michel	

Peggy J. Urness	Keith A. Walsh	Carolyn D. Yau
Gaetan R. Veilleux	Matthew J. Wasta	Eric Zlochevsky

NEW ASSOCIATES

Keith P. Allen	Luke G. C. Johnston	Timothy K. Pollis
Kris Bagchi	John B. Kelly	Keith A. Rogers
Amber L. Butek	John E. Kollar	Robert J. Schutte
Scott W. Carpinteri	Twiggy Lemercier	Mark Sturm
Melanie Sue Dihora	Eric F. Liland	Patrick Thorpe
Stephen E. Dupon	Lynn C. Malloney	Jeffrey J. Voss
Alexander R. George	Meagan S. Mirkovich	Christopher M. White
Chun Hua Hoo	John A. Nauss	Arthur S. Whitson
Eric David Huls	Robert Anthony Peterson	Joshua C. Worsham
Joseph M. Izzo		

Ms. Miller then introduced past CAS President Mavis A. Walters, who gave the address to new members.

Vice President Carlson then continued with presentations of awards.

The Michelbacher Prize commemorates the work of Gustav F. Michelbacher and is presented to the author of the best paper submitted in response to a call for discussion papers. The papers are judged by a specially appointed committee on the basis of originality, research, readability, and completeness. Recipients of the 2004 Michelbacher Prize are Greg Taylor and Gráinne McGuire for their paper, "Loss Reserving with GLMs: A Case Study," published in the 2004 *Discussion Paper Program*.

The Harold W. Schloss Memorial Scholarship Fund benefits a deserving and academically outstanding student in the actuarial program of the Department of Statistics and Actuarial Science at the University of Iowa. The student recipient is selected by the Trustees of the CAS Trust, based on the recommendation of the Department Chair at the University of Iowa. Tony Van Berkel received this year's $500 scholarship.

Ms. Miller then introduced Gail Ross who spoke about the work of The Actuarial Foundation. Ms. Ross introduced Ken Levine, CAS Fellow and volunteer actuarial mentor at the Arrowhead Elementary School in Phoenix, Arizona. Mr. Levine spoke about his experience volunteering for the Foundation's program, Advancing Student Achievement.

Ms. Miller concluded the business session and then introduced the featured speaker, Terry "Moose" Millard, a Southwest Airlines executive who spoke about building and maintaining high performance corporate cultures, nurturing gutsy leadership, and dealing with adversity through the concept of Realistic Optimism.

Following a refreshment break, the first General Session was held from 10:30 a.m. to 12:00 p.m.

The Truth About Loss Reserve Adequacy

Moderator:	C. K. "Stan" Khury Principal Bass & Khury
Panelists:	Chuck Bryan President CAB Consulting LLC
	Joseph P. Dailey Partner Dailey & Selznick
	Mary D. Miller Actuary Ohio Department of Insurance

After the luncheon, the afternoon was devoted to presentations of concurrent sessions. The panel presentations from 1:30 p.m. to 3:00 p.m. covered the following topics:

1. ARIA Prize Paper: "Fraud Classification Using Principal Component Analysis of RIDITs"

Moderator:	Louise A. Francis Consulting Principal Francis Analytics & Actuarial Data Mining Inc.

Panelists:	Patrick L. Brockett University of Texas
	Mark Alpert University of Texas
	Richard A. Derrig Automobile Insurers Bureau of Massachusetts
	Linda L. Golden University of Texas
	Arnold Levine Tulane University

2. California Workers Compensation

Moderator/ Panelist:	David M. Bellusci Senior Vice President and Chief Actuary Workers Compensation Insurance Rating Bureau
Panelists:	Alex Swedlow Executive Vice President California Workers Compensation Institute
	Lawrence White Workers Compensation Insurance Policy Advisor California Department of Insurance

3. Capping Noneconomic Damages in Medical Malpractice Claims

Moderator/ Panelist:	Leon R. Gottlieb Principal Mercer Oliver Wyman Inc.
Panelists:	William Bell General Counsel Florida Hospital Association
	Richard S. Biondi Principal and Consulting Actuary Milliman USA

4. Correlation: An Update

Moderator/ Panelist:	Glenn G. Meyers Chief of Actuarial Research and Assistant Vice President ISO
Panelists:	Youngju Lee Vice President–Quantitative Investment Research Allianz Hedge Fund Partners
	Stephen J. Mildenhall Senior Vice President Aon Re Services

5. Estimated Reserve Ranges–How Do We Determine "Reasonableness?"

Moderator/ Panelist:	Mark R. Shapland Actuary Milliman USA
Panelist:	Lee Van Slyke President Capital Management Technology

6. How To Get The Biggest Bang For Your Reinsurance Buck

Moderator/ Panelist:	Karen Pachyn Senior Vice President and Chief Actuary GE Reinsurance Corporation
Panelists:	Michele P. Bernal Vice President and Actuary American Re-Insurance Company
	Daniel Carberry Vice President Benfield Inc.

Discussion papers presented during this time were:

1. "A Practitioner's Approach to Marine Liability Pricing Using Generalised Linear Models"

   Authors: Brian Gedalla
   Milliman U.K.

   Denise Jackson
   Milliman U.K.

   David Sanders
   Milliman U.K.

2. "Loss Reserving with GLMs: A Case Study"

   Authors: Greg Taylor
   Taylor Fry Consulting Actuaries and University of Melbourne

   Gráinne McGuire
   Taylor Fry Consulting Actuaries

After a refreshment break, presentations of concurrent sessions continued from 3:30 p.m. to 5:00 p.m.

1. Building Communication Skills Through Improvisation

   Moderator/Panelist: Robert C. Morand
   Partner
   D. W. Simpson and Company

2. Loss Reserve Discounting

   Moderator/Panelist: David J. Oakden
   Consulting Actuary
   Towers Perrin

   Panelists: Ralph S. Blanchard III
   Second Vice President and Actuary
   Travelers Property Casualty Insurance Company

   Joseph A. Herbers
   Principal and Consulting Actuary
   Pinnacle Actuarial Resources Inc.

3. Movie Ticket, \$8; Popcorn, \$5; Pay-As-You Drive Insurance, \$1.10

Moderator/ Panelist:	Chester J. Szczepanski Chief Actuary Pennsylvania Insurance Department
Panelists:	Robin A. Harbage General Manager, Direct Product Progressive Insurance Company
	Todd Litman Director Victoria Transport Policy Institute

4. Patents—Can We Share?

Moderator:	Donald F. Mango Director of Research and Development GE Employers Reinsurance Corporation
Panelists:	Tom Bakos President Tom Bakos Consulting Inc.
	Phil Hargrove Vice President Intellectual Asset Management GE Employers Reinsurance Corporation
	Mark Nowotarski President Markets, Patents & Alliances L.L.C.

5. Research Update—The Risk Premium Project

Moderator:	Louise A. Francis Consulting Principal Francis Analytics & Actuarial Data Mining Inc.
Panelists:	J. David Cummins Wharton School University of Pennsylvania

Richard A. Derrig
Automobile Insurers Bureau of Massachusetts

Richard D. Phillips
Georgia State University

Discussion papers presented during this time were:

1. "Estimating Claim Settlement Values Using GLM"

   Author: Roosevelt C. Mosley Jr.
   Pinnacle Actuarial Resources Inc.

2. "The Case of the Medical Malpractice Crisis: A Classic Who Dunnit"

   Author: Robert J. Walling III
   Pinnacle Actuarial Resources Inc.

An officers' reception for New Fellows and Accompanying Persons was held from 5:30 p.m. to 6:30 p.m.

A general reception for all attendees followed from 6:30 p.m. to 7:30 p.m.

Tuesday, May 18, 2004

Registration continued from 7:00 a.m. to 8:00 a.m.

The following General Sessions were held from 8:00 a.m. to 9:30 a.m.

Professionalism Standards in Practice

Moderator: Richard J. Currie
Vice President and Actuary
American Re-Insurance Company

Panelists: John T. Gleba
Consulting Actuary
Madison Consulting Group Inc.

Roosevelt C. Mosley Jr.
Consulting Actuary
Pinnacle Actuarial Resources Inc.

Proposed Global Framework for Insurer Solvency Assessment

Moderator/ Panelist:	Stuart Wason Director Mercer Oliver Wyman Inc.
Panelists:	Glenn G. Meyers Chief of Actuarial Research and Assistant Vice President ISO
	Allan Brender Senior Director Actuarial Division Office of the Superintendent of Financial Institutions Canada

After a break, the following concurrent sessions were held from 10:00 a.m. to 11:30 a.m.

1. Actuaries in Nontraditional Roles

Moderator:	Robert C. Morand Partner D. W. Simpson & Company
Panelists:	Todd R. Bault Analyst-Institutional Research Sanford C. Bernstein & Company Inc.
	Eric Lemieux Principal The Black Diamond Group LLC
	John V. Mulhall Insurance Practice Consultant McKinsey & Company

2. Personal Auto Insurance—Surprised the Car Makes a Difference?

Moderator:	Patrick Woods Assistant Vice President & Actuary ISO

Panelists:	Daniel Charbonneau Actuary Allstate Insurance Company
	Thomas Rau Director, Personal Lines Pricing Support Nationwide Insurance Company

3. Presenting Dynamic Financial Analysis Results to Decision Makers

Moderator:	Mark R. Shapland Actuary Milliman USA
Panelists:	Michael R. Larsen Working Party Chair, Property Consultant The Hartford
	Raju Bohra Vice President–Client Modeling American Re-Insurance Company
	Patrick Crowe Vice President and Actuary, Market Research Kentucky Farm Bureau
	Aleksey Popelyukhin Vice President, Information Systems Commercial Risk Re

4. The Price is Right (a.k.a. Optimal Pricing)

Moderator:	Floyd M. Yager Senior Actuary Allstate
Panelists:	Peter Orlay Director Optimal Decisions Group
	Chester J. Szczepanski Chief Actuary Pennsylvania Insurance Department

5. A Research Update on Techniques in the Valuation of Insurance Companies

Moderator: Robert F. Wolf
Principal
Mercer Oliver Wyman Inc.

Panelists: Wayne E. Blackburn
Principal and Consulting Actuary
Milliman USA

Petr Sosik
University of Economics of Prague

Also during this time, certain discussion papers were repeated and the following discussion papers were presented:

1. "Severity Distributions for GLMs: Gamma or Lognormal? Evidence from Monte Carlo Simulations"

Authors: Luyang Fu
Grange Insurance Company

Richard B. Moncher
Bristol West Insurance Group

2. "A Primer on the Exponential Family of Distributions"

Authors: David R. Clark
American Re-Insurance Company

Charles A. Thayer
American Re-Insurance Company

After a break for lunch, CAS committee meetings were held from noon to 5:00 p.m. and a golf tournament commenced at 1:00 p.m. The concurrent sessions presented from 12:30 p.m. to 2:00 p.m. repeated certain discussion papers and featured the following discussion papers:

1. "Multivariate Spatial Analysis of the Territory Rating Variable"

Author: Serhat Guven
United Services Automobile Association

2. “A Practitioner’s Guide to Generalized Linear Models”

Authors:	Duncan Anderson Watson Wyatt LLP
	Sholom Feldblum Liberty Mutual Group
	Claudine Modlin Watson Wyatt Insurance & Financial Services
	Doris Schirmacher Liberty Mutual Group
	Ernesto Schirmacher Liberty Mutual Group
	Neeza Thandi Liberty Mutual Group

The day concluded with a Western barbeque celebration for all attendees at the Cheyenne Lodge from 6:00 p.m. to 9:00 p.m.

Wednesday, May 19, 2004

From 8:00 a.m. to 9:30 a.m., certain discussion paper presentations were repeated and the following concurrent sessions were held:

1. The Actuary and Management

Moderator:	Vincent M. Senia Vice President and Chief Reserving Actuary American Re-Insurance Company
Panelists:	Roger A. Atkinson III Actuary Employers Reinsurance Corporation
	Dr. John C. Burville

2. CAS 2003 Membership Survey

Moderator/ Panelist:	Joanne S. Spalla Senior Vice President and Corporate Actuary Converium Consulting

3. Data Quality and Standards

Moderator:	Marc F. Oberholtzer Principal Consultant PricewaterhouseCoopers LLP
Panelists:	Mark W. Littmann Principal PricewaterhouseCoopers LLP
	John McCauley Partner PricewaterhouseCoopers LLP

4. The NAIC Risk-Based Capital Formula Revisited

Moderator:	Robert F. Wolf Principal Mercer Oliver Wyman
Panelists:	G. Chris Nyce Manager KPMG LLP
	Anthony G. Phillips Vice President, Actuarial Accident Fund Insurance Company of America

5. Professionalism and Actuarial Limits of Liability

Moderator:	Richard Currie Vice President and Actuary America Re-Insurance Company
Panelists:	Lauren M. Bloom General Counsel American Academy of Actuaries

Amy S. Bouska
Consulting Actuary
Towers Perrin

John Gleba
Consulting Actuary
Madison Consulting Group

Proceedings papers presented at this time were:

1. Discussion of Stephen P. D'Arcy and Michael A. Dyer's "Ratemaking: A Financial Economic Approach"

   Author: Michael G. Wacek

2. Discussion of David Ruhm's "Distribution-Based Pricing Formulas are Not Arbitrage-Free"

   Author: Michael G. Wacek

After a refreshment break, the final General Session was presented from 10:00 a.m. to 11:30 a.m.:

Fair Value Accounting—Can It Work?

Moderator: Ralph S. Blanchard III
Second Vice President and Actuary
Travelers Property Casualty Insurance Company

Panelists: Mark W. Littmann
Principal
PricewaterhouseCoopers LLP

Stephen P. Lowe
Principal and Consulting Actuary
Towers Perrin

At the conclusion of this session, CAS President Mary Frances Miller made closing remarks and adjourned the meeting.

Attendees of the 2004 CAS Spring Meeting

The 2004 CAS Spring Meeting was attended by 373 Fellows, and 116 Associates. The names of the Fellows and Associates in attendance follow:

FELLOWS

Barbara J. Addie
Mark A. Addiego
Rhonda K. Aikens
Gregory N. Alff
Ethan D. Allen
Paul D. Anderson
Charles M. Angell
Lawrence J. Artes
Nolan E. Asch
Afrouz Assadian
Kevin J. Atinsky
Roger A. Atkinson
William M. Atkinson
Phillip W. Banet
Todd R. Bault
Patrick Beaudoin
Gregory S. Beaulieu
David M. Bellusci
Phillip N. Ben-Zvi
Douglas S. Benedict
Cynthia A. Bentley
Michele P. Bernal
François Bertrand
Eric D. Besman
Kristen M. Bessette
Richard S. Biondi
Linda Jean Bjork
Wayne E. Blackburn
Ralph S. Blanchard
Barry E. Blodgett
Nathan L. Bluhm
Michael J. Bluzer
Raju Bohra
Mark E. Bohrer
LeRoy A. Boison
David R. Border
Sherri Lynn Border
Ronald L. Bornhuetter
Theresa W. Bourdon
Amy S. Bouska
Erik R. Bouvin
Margaret A. Brinkmann
Dale L. Brooks
Lisa J. Brubaker
Kirsten Brumley Saunders
Charles A. Bryan
James E. Buck
Peter Vincent Burchett
Christopher S. Carlson
Kenneth E. Carlton
William Brent Carr
Hao Chai
Dennis K. Chan
Richard M. Chiarini
Michael Joseph Christian
Kuei-Hsia Ruth Chu
Brian Kenneth Ciferri
David R. Clark
Michael A. Coca
J. Paul Cochran
William Brian Cody
Robert F. Conger
Larry Kevin Conlee
Christopher L. Cooksey
Jeffrey Alan Courchene
Richard R. Crabb
Stephen P. D'Arcy
Ronald A. Dahlquist
Kenneth S. Dailey
Guy Rollin Danielson
Peter R. DeMallie
Curtis Gary Dean
Paul B. Deemer
Brian Harris Deephouse
Jeffrey F. Deigl
Linda A. Dembiec
Patricia A. Deo-Campo Vuong
Robert V. Deutsch
Patrick K. Devlin
Kiera Elizabeth Doster
Tammi B. Dulberger
Dennis Herman Dunham
M. L. "Butch" Dye
Grover M. Edie
Dale R. Edlefson
Mark Kelly Edmunds

David M. Elkins
Paul E. Ericksen
Kathleen Marie Farrell
Randall A. Farwell
Dennis D. Fasking
Vicki Agerton Fendley
Mark E. Fiebrink
Kevin M. Finn
Beth E. Fitzgerald
Robin V. Fitzgerald
James E. Fletcher
Ron Fowler
Louise A. Francis
Michelle L. Freitag
Michael Fusco
David S. Futterleib
David B. Gelinne
Keith R. Gentile
John F. Gibson
Susan I. Gildea
Gregory S. Girard
Todd B. Glassman
John T. Gleba
Sanjay Godhwani
Annette J. Goodreau
Leon R. Gottlieb
Ann E. Green
John E. Green
Steven A. Green
Eric L. Greenhill
Daniel Cyrus Greer
Steven J. Groeschen
Linda M. Groh
James Christopher Guszcza
Serhat Guven
Edward Kofi Gyampo
Nasser Hadidi
Robert C. Hallstrom
George M. Hansen
Robin A. Harbage
Michelle Lynne Harnick
David G. Hartman
Jeffery Tim Hay
Matthew T. Hayden
Lisa A. Hays
Qing He
Christopher Ross Heim
Amy L. Hoffman
Wayne Hommes
Carlton W. Honebein
David J. Horn
Mary T. Hosford
Derek Reid Hoyme
George A. Hroziencik
Jesse T. Jacobs
Katherine Jacques
Christian Jobidon
Andrew P. Johnson
Daniel Keith Johnson
Eric J. Johnson
Laura A. Johnson
Thomas S. Johnston
Julie A. Jordan
Gary R. Josephson
John J. Joyce
Jeremy M. Jump
Tony J. Kellner
Susanlisa Kessler
C. K. “Stan” Khury
Frederick W. Kilbourne
Gerald S. Kirschner
Brandelyn C. Klenner
Terry A. Knull
Timothy F. Koester
Richard F. Kohan
John J. Kollar
Henry J. Konstanty
Eleni Kourou
Rodney E. Kreps
Richard Scott Krivo
Jane Jasper Krumrie
John R. Kryczka
Andrew E. Kudera
Jason Anthony Kundrot
David R. Kunze
Scott C. Kurban
Edward M. Kuss
Kristine Kuzora
Salvatore T. LaDuca
Michael A. LaMonica
Steven M. Lacke
Blair W. Laddusaw
Julie-Linda Laforce
Dean K. Lamb
James A. Landgrebe
James W. Larkin
Michael R. Larsen
Aaron M. Larson
Michael D. Larson
Jason A. Lauterbach
Paul W. Lavrey
Thomas V. Le
Nicholas M. Leccese
Guy Lecours
Borwen Lee
Lewis Y. Lee

P. Claude Lefébvre
Steven G. Lehmann
Eric F. Lemieux
Bradley H. Lemons
Kenneth A. Levine
Martin A. Lewis
Mark W. Littmann
Jia Liu
Jan A. Lommele
Stephen P. Lowe
Robert G. Lowery
Donald F. Mango
Nick Manolache
Jason Aaron Martin
Stuart B. Mathewson
Robert W. Matthews
Michael G. McCarter
John R. McCollough
James B. McCreesh
Gary P. McDonald
Jeffrey B. McDonald
Liam Michael McFarlane
William T. Mech
Brian James Melas
Martin Menard
David L. Menning
Stephen V. Merkey
Timothy Messier
Robert J. Meyer
Glenn G. Meyers
Stephen J. Mildenhall
Mary D. Miller
Mary Frances Miller
Michael J. Miller
Ronald R. Miller
Paul W. Mills
Claudine H. Modlin
Richard B. Moncher
Matthew Kevin Moran
Matthew E. Morin
Roosevelt C. Mosley Jr.
Kyle S. Mrotek
Joseph J. Muccio
Robert T. Muleski
Mark W. Mulvaney
Jarow G. Myers
Seth Wayne Myers
Thomas G. Myers
John C. Narvell
Kenneth J. Nemlick
Ray E. Niswander
John E. Noble
Tom E. Norwood
G. Chris Nyce
David J. Oakden
Marc F. Oberholtzer
Denise R. Olson
Todd F. Orrett
Rebecca Ruth Orsi
Joanne M. Ottone
Karen Alice Pachyn
Richard D. Pagnozzi
Jennifer J. Palo
Cosimo Pantaleo
Jacqueline Edith Pasley
Michael A. Pauletti
Lisa Michelle Pawlowski
Kathleen M. Pechan
Daniel Berenson Perry
Kraig Paul Peterson
Anne Marlene Petrides
Faith M. Pipitone
Jordan J. Pitz
Arthur C. Placek
Jayne L. Plunkett
Igor Pogrebinsky
Daniel P. Post
Stuart Powers
Ronald D. Pridgeon
Alan K. Putney
Kenneth Quintilian
Christine E. Radau
William Dwayne Rader
John T. Raeihle
Kiran Rasaretnam
Mario Richard
Stephen Daniel Riihimaki
Laura D. Rinker
Delia E. Roberts
Rebecca L. Roever
Scott Romito
Jay Andrew Rosen
Deborah M. Rosenberg
Gail M. Ross
Tracy A. Ryan
Michael B. Schenk
Doris Y. Schirmacher
Parr T. Schoolman
Timothy D. Schutz
Peter R. Schwanke
Stuart A. Schweidel
Kim A. Scott
William Harold Scully
Steven George Searle

Terry Michael Seckel
Vincent M. Senia
Mark R. Shapland
Bonnie C. Shek
Harvey A. Sherman
Margaret Tiller Sherwood
Jeffrey Shirazi
Edward C. Shoop
Bret Charles Shroyer
Jill C. Sidney
Jerome J. Siewert
Paul Silberbush
Scott G. Sobel
Joanne S. Spalla
Angela Kaye Sparks
Michael William Starke
Grant D. Steer
Ilene G. Stone
Christopher J. Styrsky
Lisa M. Sukow
Jeanne E. Swanson
Christopher C. Swetonic
John A. Swift
Chester John Szczepanski
Susan T. Szkoda
Catherine Harwood Taylor
Patricia A. Teufel
Mary A. Theilen
Shantelle Adrienne Thomas
Michael J. Toth
Jeffrey S. Trichon
Everett J. Truttmann
Warren B. Tucker
Brian K. Turner
Peggy J. Urness
Jennifer L. Vadney
Richard Alan Van Dyke
Jeffrey A. Van Kley
Oakley E. Van Slyke
Gaetan R. Veilleux
Kyle Jay Vrieze
Michael G. Wacek
Gregory M. Wacker
Josephine M. Waldman
Christopher P. Walker
Robert J. Walling
Keith A. Walsh
Mavis A. Walters
Michael A. Walters
David W. Warren
Thomas V. Warthen
Matthew J. Wasta
Robert S. Weishaar
L. Nicholas Weltmann
Geoffrey Todd Werner
David C. Westerholm
P. Cheryl White
Mark Whitman
William Robert Wilkins
Gregory S. Wilson
John J. Winkleman
Martha A. Winslow
Dean M. Winters
Robert F. Wolf
Tad E. Womack
Patrick B. Woods
Floyd M. Yager
Run Yan
Linda Yang
Carolyn D. Yau
Charles J. Yesker
Jeffery Michael Zacek
Michael R. Zarember
Doug A. Zearfoss
Xiangfei Zeng
Ralph T. Zimmer
Eric Zlochevsky

ASSOCIATES

Christopher R. Allan
Keith P. Allen
Kris Bagchi
Thomas C. Bates
Esther Becker
Brian K. Bell
Thomas S. Boardman
Mary Denise Boarman
Thomas Leininger Boyer
Donald R. Brockmeier
Karen Ann Brostrom
Amber L. Butek
Kenrick A. Campbell
Scott W. Carpinteri
Daniel George Charbonneau
Hung Francis Cheung

Hall D. Crowder
Richard J. Currie
Krikor Derderian
Melanie Sue Dihora
Ross C. Fonticella
Mauricio Freyre
Mary B. Gaillard
Alexander R. George
Stewart H. Gleason
Paul E. Green
Dawson T. Grubbs
Aaron M. Halpert
Thomas L. Hayes
Philip E. Heckman
Joseph A. Herbers
Thomas E. Hettinger
Eric David Huls
Joseph M. Izzo
Jean-Claude Joseph Jacob
Dana F. Joseph
Susan M. Keaveny
Cheryl R. Kellogg
John Hun Kim
Martin T. King
Omar A. Kitchlew
John E. Kollar
David W. Lacefield
Bobb J. Lackey
William Scott Lennox
Karen N. Levine
Sharon Xiaoyin Li
Eric F. Liland
Ronald P. Lowe
David J. Macesic
Vahan A. Mahdasian
Lynn C. Malloney
Richard J. Manship
Janice L. Marks
Joseph Marracello
William R. McClintock
D. Michael McConnell
James P. McCoy
Phillip E. McKneely
John D. McMichael
Sylwia S. McMichael
Meagan S. Mirkovich
Stephen A. Moffett
Michael W. Morro
Sharon D. Mott
Thomas M. Mount
Prakash Narayan
Richard U. Newell
Kwok C. Ng
James D. O'Malley
Leo Martin Orth
Wade H. Oshiro
Rosemary Catherine Peck
Robert Anthony Peterson
Richard A. Plano
Timothy K. Pollis
Richard B. Puchalski
Patricia Ann Pyle
Michele S. Raeihle
Thomas O. Rau
James E. Rech
Marn Rivelle
Keith A. Rogers
Scott J. Roth
George A. Rudduck
Julie Clarisse Russell
Shama S. Sabade
David Garrett Shafer
Janel M. Sinacori
Steven A. Skov
G. Dennis Sparks
Calvin C. Spence
Esperanza Stephens
Mark Sturm
Gary A. Sudbeck
Craig P. Taylor
Wendy Artecona Thompson
Joseph O. Thorne
Patrick Thorpe
Dovid C. Tkatch
Dominic A. Tocci
Steven J. Vercellini
John E. Wade
David G. Walker
Felicia Wang
Denise R. Webb
Lynne K. Wehmueller
Jo Dee Westbrook
Christopher M. White
Thomas J. White
Arthur S. Whitson
Bonnie S. Wittman
Karin H. Wohlgemuth
Jeffrey S. Wood
Andrew F. Yashar
Robert S. Yenke

Volume XCI, Part 2 No. 175

PROCEEDINGS
November 14, 15, 16, 17, 2004

VALUE CREATION IN INSURANCE—A FINANCE PERSPECTIVE

RUSSELL E. BINGHAM

Abstract

The ultimate challenge for the management of an insurance company, as for any business, lies in understanding the components of the value creation process and in controlling and influencing these components in order to enhance the long-run value of the firm. The definition of value and its measurement involve important financial concepts extending beyond those traditionally employed by actuarial and accounting professionals.

While the many approaches and models applied to the analysis of insurance company financial data differ in their specific purposes and levels of application, they all should share a common objective: the assessment of profitability, performance, and, ultimately, value creation. The potential value of these analyses is enhanced if they present a sufficiently broad and complete financial perspective. This value is enhanced further if the results are presented in a language that management can understand and relate to familiar standards.

A broad financial perspective of the essential elements of the value-creation process in insurance is presented here to demonstrate a more conceptually inclusive framework for insurance financial analysis. External capital costs, often dealt with separately or as an afterthought, are introduced and integrated into the framework alongside internal costs. Understanding the economics of insurance, particularly the important financial concepts and linkages among variables can only help practitioners, such as actuaries and accountants, to become more relevant in a converging financial marketplace. Incorporating these concepts into models currently used in ratemaking and financial analysis can enhance their effectiveness.

1. SUMMARY

The insurance industry spends much time analyzing all the data that it both generates and acquires. However, most of the analysis performed is focused on internal information (such as company revenues and expenses), often at the exclusion of external factors important to long-run company success (such as capital flows and their costs). The management of an insurance company, striving to create value, must consider all factors that affect the financial performance of the company, both internal and external. Understanding of the broader financial concepts of value creation in insurance, and the subsequent deployment of models that incorporate all the costs and contributors to value, is important for the many disciplines and practitioners involved in insurance financial analysis.

For example, although actuarial principles require that capital costs be included as an element of the ratemaking process, debates continue regarding how, or even if, capital should be included and what rate of return should apply [10, 11]. Some still refuse to speak the total return (i.e., ROE) language of management [12, 13]. Consequently, far too many disjointed approaches exist, causing unnecessary confusion and making comparisons of their results difficult. Many models lack financial integrity: they are incomplete and not clear in specifying their underlying conceptual and/or financial assumptions [5, 6, 10, 11]. This lack of financial discipline opens the regulatory process to abuse by constituents with social or other non-financial agendas [14].

Many actuaries and accountants realize the shortcomings inherent in the calendar-period orientation of accounting, specifically the lack of a full economic accounting. Both professions have come to realize the need to broaden their traditional areas of analysis to incorporate all aspects of insurer financial performance, on both reported and economic bases. For example, the respective evolutions of Dynamic Financial Analysis (DFA) and Enterprise Risk Management (ERM) within the CAS are part of an explicit movement by the actuarial profession to expand its analytical role beyond the liability focus of the past, to include items such as capital and invested assets and the firm in its entirety.

The following is intended to further this effort by presenting a framework that reflects vital financial concepts and elements of value creation in an integrated manner. This extends beyond the more traditional internal cost focus to include external costs of capital and valuation principles. First, it is worth reviewing a few essentials that should be incorporated in any model framework.

2. BASIC BUILDING BLOCKS FOR MEASUREMENT OF VALUE

Three essential building blocks provide a critical foundation for portraying and measuring the value-creation process:

1. A complete and tightly linked package of balance sheet, income, and cash-flow statements provide the basis for the financial analysis to follow.

2. Utilization of policy (or accident) period as the basis of analysis, with calendar-period financial statements derived from the contributions of current and prior policy (or accident) periods. This is analogous to loss triangles, applied more broadly to all financial statement items.

3. The joint presentation of items viewed under both conventionally based (GAAP or statutory) accounting rules as well as under economic accounting rules.

While additional effort may be required to create them, experience has shown that failure to include the three basic complements of balance sheet, income, and cash flow eventually will lead to modeling mistakes or inconsistencies, including inability to assess value accurately.

While actuaries often must analyze insurance profitability and risk at the policy-period level, regulators and accountants are more accustomed to a calendar-period orientation. However, many are not aware that calendar results are a mixture of many contributing policy periods and are thus an amalgamation of many mismatched bits of premium and expense data. Quite simply, calendar financials are the *end* result of numerous actions, such as pricing, which are managed by policy period. Thus, analysis should never begin with calendar-period financials, when policy-period financials are available. The focus of key decisions, centered on actions oriented to the sale of insurance policies, should align with financial analysis.

By providing additional information beyond that under conventional accounting, such as systems based on GAAP and statutory rules, an economic perspective is broader and presents a more complete valuation picture. While the focus of conventional accounting is necessarily restricted to a calendar-period activity basis, the focus of economic accounting is on present and future cash flows, market value, and the time value of money, not restricted by calendar-period. To better measure value and understand the linkages between conventional and economic accounting, both views should be available.

In addition to the three fundamental building blocks, the analytical framework should also possess the following attributes:

4. An ability to separate the contributions from the underwriting, investment, and finance functions.
5. A structured discipline for risk/return-based decision-making.

Underwriting, investment, and finance are different activities that each contribute to the overall performance of the company. Each function is accountable for decisions related to the relationship between the risks and returns that can be realized by that activity. In order to maintain balance and financial discipline throughout the organization, and not expose the company to unnecessary risks in any one area, it is important that there be overall consistency in the decision-making process among them. The contributions that each makes to the overall return of the company, and the risks associated with generating those returns, should be judged similarly. In order to understand the distinct contributions to value creation and the corresponding risks from the three functions, the analytical framework must be capable of separately measuring each of them as part of a unified framework.

A model framework that reflects these five important features will provide the key economic measures that are needed to assess value creation.

3. BASIC COMPONENTS OF VALUE CREATION

To add economic value, the cost of insurance company funds acquired must simply be less than the value derived from their investment. Insurance companies derive funds from equity, debt, and policyholder funds that support net insurance liabilities. If the income on invested assets is less than the cost of those funds, then economic value added is negative (i.e., value is being lost). While this simple view does not fully reflect the role of underwriting, particularly with respect to the dimension of risk in the pricing process, the fact is that, from a purely financial perspective, the underwriting process serves simply as a source of funds and value is created primarily from the investment of those and other (capital) funds. These principles will be explored more deeply, beginning with an explanation of the essential elements that together create value in insurance. The important variables of the *value matrix* are:

Item	Amount	Funds Rate	Net Cost/Value	Functional Accounting
Source/Cost of Funds				
Underwriting Equity	S_u	$-C_u$	$-C_uS_u$	Underwriting
Investment Equity	S_i	$-C_i$	$-C_iS_i$	Investment
Debt	S_d	$-C_d$	$-C_dS_d$	Finance
UW Liabilities	L	$-C_L$	$-C_LL$	Underwriting
Use/Value of Funds				
Underwriting Funds	L	R_i	R_iL	Underwriting
Underwriting Equity	S_u	R_i	R_iS_u	Underwriting
Investment Lift on Underwriting	$L + S_u$	$R_a - R_i$	$(R_a - R_i)(L + S_u)$	Investment
Investment Equity	S_i	R_a	R_aS_i	Investment
Debt	S_d	R_a	R_aS_d	Finance
Total	A	R	V	

These variables are defined as

S_u: Surplus (equity) supporting underwriting risk
C_u: Cost of underwriting surplus
S_i: Surplus (equity) supporting investment risk
C_i: Cost of investment surplus
S_d: Debt

C_d: Cost of debt
L: Net insurance liabilities
C_L: Cost of liabilities
R_i: Low-risk investment rate, after-tax
R_a: Actual investment rate, after-tax
A: Total invested assets
R: Total value return on assets
V: Net value added

Note that the total funds from underwriting, $(L + S_u)$, are a source of funds to investment whose cost is R_i, the low-risk base earnings commitment to underwriting for its use. The value of those funds to investment is determined by the spread it earns above this base, $(R_a - R_i)$.

While equity (and its cost) may be viewed as under the functional control of finance, it is considered part of either underwriting or investment for the purpose of determining rates of return and value creation. This is because those areas are responsible for earning a return on the risk-based equity supporting their respective operations and their financial performance is thus connected to it.

Debt considerations within the finance function are much different from the underwriting and investment risk/return considerations. Typically the effect of debt will indicate a net cost, which reflects the fact that borrowing rates are generally greater than the rate at which these funds can be invested. However, indirect benefits of debt include a reduction in insurance premium leverage and a likely improvement in financial ratings. Furthermore, equity costs are likely to decline and the potential exists for greater, more profitable business growth.

The value created, V, is the net sum of the products of the amounts of funds and the applicable funds rates, whether costs or value contributors. The amount of value created can be determined by any of the following five alternative formulae, presented simply to emphasize different perspectives. (Note that the calculation of an *economic value created* requires that this figure be discounted to present time using the cost of capital as a discount rate.) The most basic view of value created is total investment income less the total cost of funds:

Basic View:

$$V = R_a(S_u + S_i + S_d + L) - (C_u S_u + C_i S_i + C_d S_d + C_L L). \quad (1)$$

Funds are derived from three basic sources: surplus (equity), debt, and policyholder funds that support net liabilities. Equity and debt together represent total capital. The net of premium, loss, and expense that remains in the company to support net future liability obligations is also an important source of funds. This source may have an associated cost if business is written at an underwriting loss. However, profitably written business (i.e., under 100 combined ratio) has a negative cost, in effect producing value directly. In such instances, policyholders are effectively paying insurers to hold their money. This may be a necessity if interest rates are low and also to reflect the uncertainty and risk that the insurer is assuming from the policyholder. (The risk dimension will be discussed later; but it should be noted here that, in some cases where risks are significant, a combined ratio below 100 is absolutely necessary to provide adequate profits and create value.)

Distinguishing between underwriting and investment risk equity is optional, but it is strongly suggested in order to permit the separate assessment of the underwriting and investment functional contributions to the creation of value. This more traditional "insurance" view, which reflects these functional contributions separately, is total underwriting return plus total investment return plus finance return:

Functional Total Return View:

$$\begin{aligned}\text{Let}\quad V &= [\text{Underwriting Return}] + [\text{Investment Return}]\\ &\quad + [\text{Finance Return}],\\ \text{then}\quad V &= [(R_i - C_L)L + R_iS_u] + [(R_a - R_i)(L + S_u) + R_aS_i]\\ &\quad + [(R_a - C_d)S_d - C_uS_u - C_iS_i]. \qquad (2)\end{aligned}$$

The return from underwriting operations represents the spread between the cost of liability funds and what they can earn when invested at a low risk yield, together with the low-risk investment earnings on the supporting underwriting risk surplus (the first bracketed part of (2)). Underwriting returns are judged on a "benchmark," low-risk investment standard basis.

Similarly, return from investment is the margin earned from the spread of actual yield over the low-risk yield on the underwriting related funds (liabilities plus underwriting surplus), plus the investment earnings on the supporting investment-risk surplus (the second bracketed part of (2)). The net financing costs calculation reflects that the net cost of debt is the difference between the borrowing rate and what those assets earn while invested (along with other company assets). To reflect taxes, all items are expressed at their after-tax values. Since equity costs (C_u, C_i) are not tax deductible to the company, the pre-tax basis is equal to the post-tax basis.

A slight repackaging of (2) leads to the following form that better reflects the net value creation contribution from each of the underwriting, investment and finance functions:

Value Creation View:

$$\begin{aligned}V &= [V_u] + [V_i] + [V_d],\\ V &= [(R_i - C_L)L + R_iS_u - C_uS_u]\\ &\quad + [(R_a - R_i)(L + S_u) + R_aS_i - C_iS_i] + [(R_a - C_d)S_d],\end{aligned} \qquad (3)$$

where V_u, V_i, and V_d represent value added from underwriting, investment and finance, respectively.

Note that *economic rates of return* can be defined as $E = V/(S_u + S_i + S_d)$, $E_u = V_u/S_u$, $E_i = V_i/S_i$, and $E_d = V_d/S_d$, corresponding to the total, underwriting, investment and debt value-creation components, respectively. Therefore, the value created by each function equals the return less the cost for each, with the rate of return being the return amount in ratio to the amount of equity or debt, respectively.

A "purer" operationally focused view on underwriting and investment, without the implicit allocation of equity to underwriting and investment, is value created as the sum of operating return from underwriting, operating return from investment, and net finance return (usually a net *cost* of capital):

Pure Operations View:

$$\begin{aligned} V &= [\text{Underwriting Operating Return}] \\ &\quad + [\text{Investment Operating Return}] \\ &\quad + [\text{Finance Net Capital Return}] \\ V &= [(R_i - C_L)L] + [(R_a - R_i)L] \\ &\quad + [(R_a - C_u)S_u + (R_a - C_i)S_i + (R_a - C_d)S_d]. \end{aligned} \tag{4}$$

Here the net cost/value of all capital is combined, and the return from underwriting and investment is viewed with respect to liability funds only. Since the weighted average cost of capital (WACC) is $(C_uS_u + C_iS_i + C_dS_d)/(S_u + S_i + S_d)$, the net cost of capital in this view is $(\text{WACC} - R_a)\times$ total capital. This is the WACC excess over the actual rate capital earns when invested. It should be noted that capital in insurance represents a financial cushion that exists as an invested asset. It differs from the non-earning investment of capital in plant and equipment in manufacturing.

Insurance risk is largely dominated by the uncertainty and volatility of losses and reserves over time. The role of surplus to act as a financial buffer against this risk is often addressed in modeling by controlling the initial needed level of underwriting surplus and the subsequent timing of its release by a linkage to liabilities, primarily driven by the runoff of reserves as claims are paid. Similarly, investment surplus is often maintained through a linkage to invested assets. In other words, insurance equity is largely proportional to reserves, and investment equity proportional to invested assets.

Given an underwriting leverage factor $F_u = L/S_u$, investment leverage factor $F_i = (L + S_u)/S_i$, debt/equity factor $F_d = S_d/(S_u + S_i)$, and since typically $C_u = C_i = C$, then (3) can be restated as follows:

Fundamental Factors View:

$$\begin{aligned} V &= L[(R_a - C_L) - K(C - R_a) - KF_d(C_d - R_a)], \\ \text{where} \quad K &= 1/F_u + (1 + 1/F_u)/F_i. \end{aligned} \tag{5}$$

This shows that the key drivers of return (and risk) in insurance are liabilities (L), the cost of liabilities (C_L), investment returns (R_a), and leverage, in conjunction with the costs of equity (C) and debt (C_d). This is a mathematical expression of the basic fact that insurance consists fundamentally of underwriting, investment and leverage and that value is created in relation to capital costs.

Operating return represents the spread between the return earned on funds held, less the cost of those funds. The total return (essentially the traditional ROE) represents the operating return leveraged in relation to supporting risk equity, plus the investment return on the equity itself. The operating return (O_u) and total return (T_u) for underwriting are defined, respectively, by the following:

$$O_u = R_i - C_L \tag{6}$$

and

$$T_u = O_u(L/S_u) + R_i. \tag{7}$$

Equation (7) is the first bracketed expression for underwriting return in (2) divided by S_u. The operating return (O_i) and total return (T_i) for investment are defined, respectively, by the following:

$$O_i = R_a - R_i \tag{8}$$

and

$$T_i = O_u[(L + S_u)/S_i] + R_a. \tag{9}$$

Equation (9) is the second bracketed expression for investment return in (2) divided by S_i.

Note that underwriting leverage is in relation to liabilities, whereas investment leverage is in relation to the invested-asset sum of liabilities and underwriting equity, since this is the investment base that is being managed to higher risk investments by the investment function (i.e., the investment lift). By this division, it is possible to quantify the total return contribution separately for the underwriting and investment functions.

The "traditional" total return (on equity) (T) is the composite of the underwriting and investment total returns. This is expressed as follows:

$$T = [T_u S_u + T_i S_i]/[S_u + S_i]. \tag{10}$$

The total return on total capital (T_c) is determined as:

$$T_c = [T_u S_u + T_i S_i + R_a S_d]/[S_u + S_i + S_d]. \tag{11}$$

Note that the total economic rate of return and those for underwriting, investment, and finance can be expressed simply as: $E = T_c - \text{WACC}$, $E_u = T_u - C_u$, $E_i = T_i - C_i$, and $E_d = R_a - C_d$.

The analysis of insurance must reflect the multi-year nature of the cash flows that generally follow well after the initial policy sale. When policies are sold, premium collections and expense payments occur relatively quickly. However, the key determining cost of insurance is claims payments that can span many years subsequent to the policy period in which the insurance coverage was in force and the original claim-generating incidents occurred. This means that an economic perspective is important in order to properly reflect the amount and timing of value that is generated. The specific meanings and calculations of the funds rates noted above will be discussed in the next section.

First, the following brief thoughts are offered as a point of discussion and, perhaps, to provide a basis for further risk/return model development. Most of the variables in the value matrix are subject to variability. Liabilities (L) and their cost (C_L), for example, are both very volatile and have a significant effect on value. (Note that this is the composite of the amounts of premium, loss, and expense and the timing of their cash flows.) Focusing on value (V) via equations (1) through (5) provides a basis for dealing with risk and return consistently across all sources of cost and value. Given assumptions as to the distributions of each of the underlying variables, the resultant distribution of V provides a single unifying basis for assessing risk and return of each contributor to cost or value creation. This allows for judging the underwriting, investment, and finance functions by a common performance standard, such as in equation (3). If one were looking for a relatively simple, tight package containing the essential value drivers of insurance, then this framework might be worth considering.

4. THE FINANCIAL MODEL FOR VALUATION

As noted earlier, the multi-year dimension of the insurance financial transaction requires that the time value of money and other economic principles be considered in the determination of economic-value creation. An example will be used to demonstrate the key concepts and show how the key funds rates, measured economically, provide the information needed to support the calculations presented in (1) through (5).

The following example shows how a single policy year emerges over its financial lifetime and contributes to future calendar-period results and value. To achieve a full calendar accounting, all current and prior policy periods must be modeled and calendar contributions from all of them properly aggregated.

This example provides high-level balance sheet, income, and cash flow statements. Various rate of return calculations are also shown to demonstrate the equivalence of conventionally reported rates of return, IRR, and net present value rates of return, assuming certain risk-based, economic rules are followed to control the flow of surplus and to distribute profits. Basically, surplus contributions are controlled over time to maintain a three-to-one liability to surplus relationship and profits are released (as dividends) proportionally to liability exposure and settlement over time. Conventional net income is not the basis for the determination of dividends. A most important result is the development of the economically based measures of the funds rates that determine value created. These financial assumptions form the basis for the example presented:

For underwriting function activities:

- 103.1% Combined ratio
- $9,700 Premium, collected without delay when written
- $10,000 Loss, single payment at end of year 3
- $0 Expense
- 35% Income tax rate, no delay in payment
- 6.0% Low-risk investment interest rate before tax, 3.9% after tax
- No loss discount tax or unearned premium tax
- 3.0 Liability/surplus ratio
- 15.0% Cost of underwriting equity

For investment function activities:

- 6.2% Investment interest rate before tax, 4.65% after tax, assuming a 25% tax rate
- 20% Investment equity/underwriting equity ratio, equivalent to using a 20:1 (liability plus underwriting equity)/investment equity ratio
- 15.0% Cost of investment equity

For finance function activities:

- 6.2% Investment interest rate before tax, 4.65% after tax, assuming a 25% tax rate
- 25% Debt/total equity ratio
- 8.0% Cost of debt before-tax, 5.2% after tax.

Simplified balance sheet, income, and cash-flow statements for this example are shown in Exhibit 1. The rules governing the flow of surplus follow: (1) the level of surplus is maintained at a 1/3 ratio to loss reserves, (2) after-tax investment income on all capital (surplus and debt) is paid to the shareholder as earned, and (3) operating earnings from underwriting and investment of underwriting funds are distributed in proportion to the level of insurance exposure in each year, measured by loss reserve level, relative to the total exposure over the policy year's financial lifetime. Since loss reserves are level at $10,000 in each of the three years, operating earnings are distributed to the shareholders equally in each year. The cost of debt is paid as it is incurred.

Three "levels" of return exist within an insurance company with respect to the underwriting function. The first is the underwriting rate of return, which is how much the company "earns" (a cost when writing above a combined ratio of 100) on pure underwriting cash flows, before reflecting investment income on the float. If negative, this is the company's cost of policyholder-supplied liability funds. The second, operating return, reflects what the company earns on underwriting when investment income on the float is netted against the cost of funds. This is the "risk charge" to the policyholder for the transfer of risk to the company. The third, the total (levered) return, is the net result of underwriting and investment income from operations, together with investment income on underwriting risk surplus.

Investment returns can be viewed similarly at several levels. The investment function generates a yield lift on the funds provided by underwriting (liabilities and underwriting equity). The base "cost" of these funds to the investment function is the low risk yield already credited to the underwriting function. The operating return earned by investment reflects what is earned on actual investments netted against this cost. The total (leveraged) investment return is the net result of investment income generated from investment operations together with the total investment income on supporting investment risk surplus.

These rates of return can be determined by either a cash-flow-based internal rate-of-return (IRR) calculation or by relating income earned to the amount invested (or asset equivalent liability). It is important to note that IRR calculations are meaningful for cash flows other than just at the shareholder level. The income versus investment (i.e., "ROE-like") approach relates the income over the full three-year aggregate financial life of the business to the investment base over this same period. This calculation can use either nominal (i.e., undiscounted) or present value (discounted, but without risk-adjustment) dollars. All three approaches should produce the same result, assuming risk-based economic rules are used to control capital flows and to distribute profits. In addition, the total return realized at the shareholder level via dividends is identical in each year. This attribute follows from the fact that the rules used to control the flow of surplus are the same as those used to distribute profits. Note that if a risk-adjusted discount rate were used in the present-value calculation of income, the present-value-based total shareholder returns thus generated would equal the risk-free rate.

The cost of liabilities, or underwriting-generated funds, is based on the net of premium, loss, and expense cash flows, including the funding of liabilities to nominal levels via an internal transfer from the shareholder to the policyholder account. The IRR for this series of cash flows is −0.7%. Equivalently, this can be derived as the ratio of the present value of underwriting income of −$195 to the present value of underwriting liabilities of $27,804. On a nominal basis this is −$210 divided by $30,000, where the −$210 includes −$15 due to the loss of investment income on negative retained earnings. (To fully fund liabilities and reconcile with accounting earnings on a nominal basis, special attention must be paid to what is traditionally referred to as "retained earnings." This is a critical balancing item that reflects the amount of undistributed accounting profits that remain after the dividend of profits.)

The operating return is equal to the after-tax investment rate of 3.9% less the 0.7% funds cost, or 3.2%. This can be calculated in three alternative ways. (1) The net cash flow inclusive of underwriting and investment income generates an IRR of 3.2%. (2) The present-valued operating income of $889 is a 3.2% return on the $27,804 present-valued liabilities. (3) The nominal operating income of $960 is a 3.2% return on the $30,000 total balance sheet policyholder-supplied float upon which these earnings were generated.

The total underwriting return on underwriting-risk equity, which includes underwriting income and investment income on both float and equity, is also derivable in three ways. First, the net shareholder flows produce an IRR of 13.5%. The "ROE-like" calculation of income in ratio to equity is $1,251 divided by $9,268 on a present value basis and $1,350 divided by $10,000 on a nominal basis, both 13.5%. It should be noted that the rate of return based on the dividend of underwriting-based profits is also 13.5% in each period. (It can't be buttoned up any tighter than this!)

If investment yields vary over time, as opposed to the simple flat yield curve assumed in this example, refinements in the discount rates and the dividend rule are necessary to maintain the tight linkages shown here, and some corresponding variations in return over time will emerge, particularly in the period dividend return.

The 0.75% operating rate of return on investment is simply the difference between the actual earnings rate of 4.65% and the low-risk rate credited to underwriting of 3.9%. The total investment return on investment-risk equity, which includes the investment lift on underwriting funds and underwriting equity, and the investment income on investment equity, is also derivable in three ways. First, the net shareholder flows from investment operations produces an IRR of 19.7%. The "ROE-like" calculation of income in ratio to equity is also 19.7% on both a present-value and on a nominal basis. The rate of return in each period based on the dividend of investment profits is also 19.7% in each period.

The total return on total capital is also derivable in three similar ways. The IRR, present value, and nominal value ratios of total income to total capital all produce a rate of return of 11.5% including the cost of debt, and 12.5% excluding it. The "dividend" returns in each period match these as well.

5. MEASURING VALUE CREATION

The measurement of value created can proceed using the rates of return provided by the financial model. The following is a recap of the value matrix for the example presented above. While the preference would be to use net present-value figures, those that are presented use the "Total All Periods" nominal policy-lifetime values for ease of presentation. This is what would be observed in a calendar-period accounting of a firm that was at "steady state" (i.e., identical successive policy period performance without growth).

Item	Amount	Funds Rate	Net Cost/Value	Functional Accounting
Source/Cost of Funds				
Underwriting Equity	10,000	−15.00%	−1,500	Underwriting
Investment Equity	2,000	−15.00	−300	Investment
Debt	3,000	−5.20	−156	Finance
Underwriting Liabilities	30,000	−0.70	−210	Underwriting
Use/Value of Funds				
Underwriting Liabilities	30,000	3.90%	1,170	Underwriting
Underwriting Equity	10,000	3.90	390	Underwriting
Investment Lift on Underwriting	40,000	0.75	300	Investment
Investment Equity	2,000	4.65	93	Investment
Debt	3,000	4.65	140	Finance
Total	45,000	− 0.16%	−74	

The net value created is a negative −74, which represents a failure to earn the cost of capital. (To calculate the *economic* value created, the cash flows underlying this figure must be discounted at the cost of capital rate of 13.0%, which results in a value of negative −59.) The various returns of interest are recapped below.

Key Rates of Return

Un-leveraged "pure" returns

C_L: −0.70% Underwriting liability return (cost of policyholder-supplied funds)

R_i: 3.90% Investment return on underwriting funds

O_u: 3.20% Operating return from underwriting operations (risk charge)

O_i: 0.75% Investment lift on benchmark underwriting assets

R_a: 4.65% Investment return on invested assets

Leveraged returns

T_u:	13.50%	Underwriting total return on underwriting equity
T_u:	19.65%	Investment total return on investment equity
T:	14.52%	Total insurance return on total equity
T_c:	11.51%	Total return on total capital including debt cost
T_c:	12.55%	Total return on total capital excluding debt cost

Economic returns

E:	–0.49%	Economic total return on total capital (12.55% – 13.04%)
R:	–0.16%	Economic total return on invested assets
E_u:	–1.50%	Economic underwriting return on underwriting equity (13.50% – 15.00%)
E_i:	4.65%	Economic investment return on investment equity (19.65% – 15.00%)
E_d:	–0.55%	Economic debt return on debt capital (4.65% – 5.20%)

The value-creation components can be viewed graphically (Figure 1) to get a better sense of their relative degrees of influence. The x-axis represents the funds rates, either a cost (left side) or a value contributor (right side). The y-axis scale represents the amounts of funds to which the rates are eventually applied (i.e., multiplied). This should be viewed as a seesaw with the fulcrum to be determined as the point along the x-axis that causes costs and value to be in balance. Both the weights sitting on top of the seesaw (the amounts of funds) and the distance from the to-be-determined fulcrum (the funds rates) are determining factors. In this example with a negative total created value, the point of balance is a negative return on assets of –0.2%, the point at which "net value created" sits.

The net impact of the amounts of funds and funds rates are shown in Figure 2. The net costs and value contributions are the products of the amount of funds and funds rates shown in the value matrix. From this view, it is easy to judge the most significant drivers of cost and value. Clearly, the cost of equity is

FIGURE 1

COSTS AND CONTRIBUTIORS TO VALUE CREATION

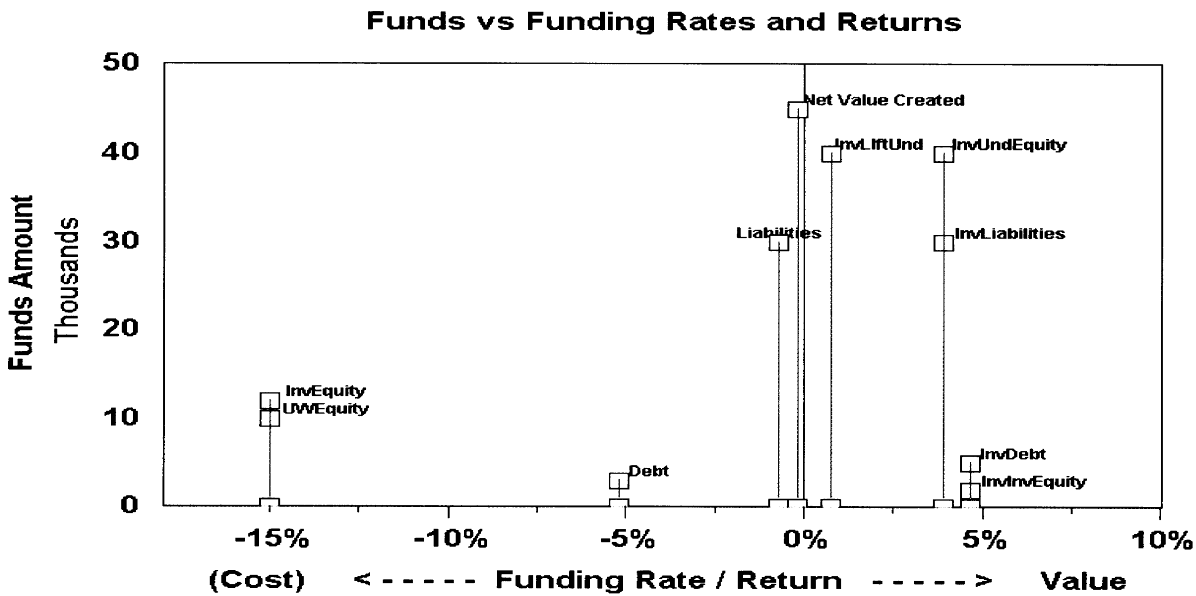
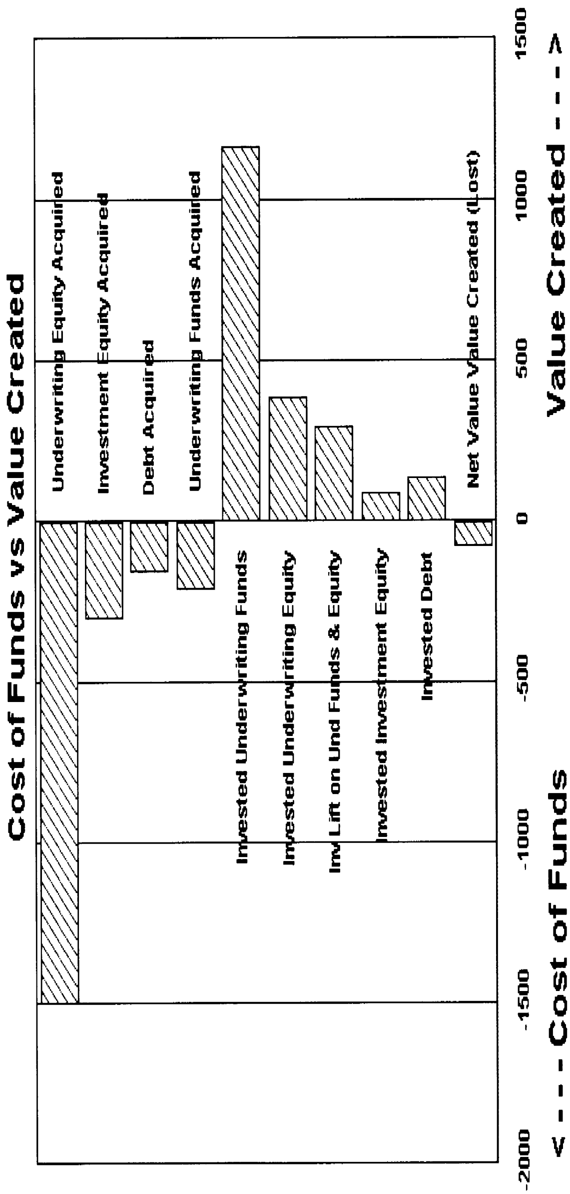


FIGURE 2

THE ECONOMIC COST AND VALUE CREATION COMPONENTS
(THE PRODUCT OF FUNDS AND FUNDS RATE)



the major cost contributor, while the investment value of underwriting funds acquired from the policyholder is the major value contributor.

The significant amount of capital costs and their impact on net value created serves to highlight the concern as to if and how ratemaking models deal with issues of capital and capital costs. Many models do not explicitly integrate capital costs into the ratemaking equation. Often this is addressed simply by comparison of the resultant rate of return generated by a particular rate to a "reasonable" cost-of-capital figure. The magnitude of the impact of capital cost and its connection to value creation is perhaps not as widely recognized as it should be. By incorporating this aspect into ratemaking and other activities, actuaries enhance their position as key role-players in the converging financial marketplace, where knowledge of a broader and more complete financial perspective is critical.

The model framework presented here has intentionally provided delineation between the underwriting, investment, and finance functions and their respective performance judged against individual benchmark standards. From using equation (3) and summing the appropriate items in the value matrix based on functional accountability, and also as shown in Exhibit 1, Underwriting *lost* value of 150, Investment *added* value of 93, and Finance *lost* value of 17 in this example. When discounted at the cost of capital, the respective contributions to economic value created are -114, 71, and -15 which total -59.

These amounts represent the "benchmark value created" (BVC) for underwriting, investment, and finance. By comparison, the traditional method of determining *economic value added*, or EVA, only provides a total, firm-level view, and no attempt is made to judge the separate contributions of underwriting, investment, and finance against their own individual capital cost benchmarks. The standard application of EVA to insurance is also problematic in that it uses calendar year net income as its starting point. Appropriate insurance valuation requires a policy-period orientation coupled with economic accounting. The -59 in this example is EVA when created under an overall framework of benchmark (i.e., economic, risk-based) rules and valuation.

It is worth noting that the embedded-value approach that is emerging on the life side of the insurance industry differs in the form of presentation, but is consistent with and reconcilable in result to the valuation material presented here (assuming a sufficiently complete model that can do it, of course). As noted at the bottom of Exhibit 1, the beginning embedded value is the sum of BVC and capital contributed at the time of policy inception. Embedded value demonstrates the *remaining economic value created* that exists at each point in time, based on the remaining capital and future profits to be distributed, adjusted to the time of inception by discounting at the cost of capital. The typical format of presentation (not shown here) provides a breakdown of the embedded value into two components: (1) contributed nominally valued capital and (2) the remainder, which is referred to as the value of "in-force" business. In this way, embedded value is linked to the levels of published capital (usually statutory) that remain on the balance sheet at each interval in time.

6. RATEMAKING

The portion of ratemaking practices that deals with such issues as profit margins and the cost of capital is a collection of many diverse approaches that often do not provide as complete a financial perspective or as helpful a linkage to overall company financials as they could. The historical focus on internal cost drivers, while understandable, can be supplemented to more formally address external capital costs and other financial market considerations.

Furthermore, modelers, including actuarial ratemakers, tend to talk like priests speaking Latin—elegant, complex, and appropriate to the situation, but not understood by anyone else.

By developing more complete and integrated financial models, ratemaking can also be reworked in the language of management, which is total return and economic value, keeping score using things such as return on equity and return on capital.

Ratemaking models that do not explicitly address capital-cost issues are at a disadvantage, since this absence makes justification for rate action more difficult to demonstrate. The direct integration of both capital costs and a more complete financial perspective into ratemaking models could avoid some of the confusion that clouds discussions of rate adequacy in regulatory applications. Certainly, this would address the concern that significant costs of capital are not being properly reflected in rates.

To ensure that capital costs are reflected and to speak the language that management and financial markets employ, it is clear that development of more financially complete ratemaking models that reflect all elements of value creation would be beneficial. Coupled with the basic building blocks and attributes discussed previously, ratemaking models that are able to accomplish this would be able to better meet the wide range of demands of regulators, insurance company management, and financial markets, and all in a consistent manner.

There is an important opportunity to be gained by doing so: to bridge the gap between the somewhat disjointed regulatory activities of ratemaking and solvency. Solvency is guarded by fair returns. It is imperative that the connection be made between rates, return, and the resultant growth in surplus that is necessary to maintain adequate solvency margins. This can be accomplished better if the models contain all elements of cost and value and present results in a language that can be understood by all.

7. CONCLUSION

Insurance financial analysts of all disciplines can benefit from better understanding the finance perspective on value creation, especially as the financial marketplace converges and previous industry boundaries blur. Practitioners who expand beyond more narrowly focused analytical methods, to a broader and more integrated company level of application, enhance their own value significantly. Understanding the key costs and contributors to value creation and how to measure and influence them is an essential part of this process.

In addition to understanding the value-creation process, those who develop and apply analytical models need to make models more complete and incorporate the key building blocks suggested. Results need to be relevant and expressed in a language that management can understand. The breakdown of value creation into the key cost and value contributors presented spans the underwriting, investment, and finance activities of the insurance company and the more specific operating activities embedded within each of them. This structure provides the capability to measure consistently the contributions of each activity to total company performance and judge them by the same risk/return standards.

The cost of capital is too often viewed as beyond the scope of the financial analysis that occurs within an insurance company. The value formulation presented here provides the ability to integrate these costs more directly with the internal financials. Many financial activities can benefit from the use of this broader and more complete finance perspective, including ratemaking, risk analysis, and capital allocation, since the decisions in these areas are ultimately all related to value creation in the whole.

REFERENCES

[1] Bault, Todd R., "Discussion of Risk Loads for Insurers," *PCAS* LXXXII, 1995, pp. 78–96.

[2] Bender, Robert K., "Discussion of Surplus—Concepts, Measures of Return, and Determination," by Russell E. Bingham, *PCAS* LXXXIV, 1997, pp. 44–105.

[3] Bingham, Russell E., "Cash Flow Models in Ratemaking: A Reformulation of Myers-Cohn NPV and IRR Models for Equivalency," *Actuarial Considerations Regarding Risk and Return in Property-Casualty Insurance Pricing*, Casualty Actuarial Society, 1999, pp. 27–60.

[4] Bingham, Russell E., "Fundamental Building Blocks of Insurance Profitability Measurement," *Actuarial Considerations Regarding Risk and Return in Property-Casualty Insurance Pricing*, Casualty Actuarial Society, 1999, pp. 13–18.

[5] Bingham, Russell E., "Rate of Return—Policyholder, Company, and Shareholder Perspectives," *PCAS* LXXX, 1993, pp. 110–147.

[6] Bingham, Russell E., "Risk and Return: Underwriting, Investment and Leverage—The Probability of Surplus Drawdown and Pricing for Underwriting and Investment Risk," *PCAS* LXXXVII, 2000, pp. 31–78.

[7] Bingham, Russell E., "Surplus—Concepts, Measures of Return, and Determination," *PCAS* LXXX, 1993, pp. 55–109.

[8] Butsic, Robert P., "Determining the Proper Interest Rate for Loss Reserve Discounting," Casualty Actuarial Society *Discussion Paper Program*, 1988, pp. 147–188.

[9] Casualty Actuarial Society, "Statement of Principles Regarding Property and Casualty Insurance Ratemaking," CAS *Yearbook*, 2002.

[10] D'Arcy, Stephen P. and Michael A. Dyer, "Ratemaking: A Financial Economics Approach," *PCAS* LXXXIV, 1997, pp. 301–390.

[11] D'Arcy, Stephen P. and Richard W. Gorvett, "A Comparison of Property/Casualty Insurance Financial Pricing Models," *PCAS* LXXXV, 1998, pp. 1–88.

[12] Halliwell, Leigh J., "ROE, Utility and the Pricing of Risk," Casualty Actuarial Society *Forum*, Spring 1999, pp. 71–136.

[13] Halliwell, Leigh J., "The Valuation of Stochastic Cash Flows," Casualty Actuarial Society *Forum*, Spring 2003, pp. 1–68.

[14] Hunter, J. Robert, "Excerpts from Proposition 103 Testimony (Declaration)," Casualty Actuarial Society *Forum*, Spring 1990, pp. 375–408.

[15] Kreps, Rodney E., "Investment-Equivalent Reinsurance Pricing," *PCAS* LXXXV, 1998, pp. 101–131.

[16] Kreps, Rodney E., "Reinsurer Risk Loads from Marginal Surplus Requirements," *PCAS* LXXVII, 1990, pp. 196–203.

[17] Mango, Donald F., "Risk Load and the Default Rate of Surplus," *Securitization of Risk* Casualty Actuarial Society *Discussion Paper Program*, 1999, pp. 175–222.

[18] Mildenhall, Stephen J., "Discussion of Application of the Option Market Paradigm to the Solution of Insurance Problems," *PCAS* LXXXVII, 2000, pp. 162–187.

[19] Philbrick, Stephen W., "Accounting for Risk Margins," Casualty Actuarial Society *Forum*, Spring 1994, pp. 1–90.

[20] Roth, Richard J., Jr., "Analysis of Surplus and Rate of Return Without Using Leverage Ratios," Casualty Actuarial Society *Discussion Paper Program*, May 1992, pp. 439–464.

[21] Venter, Gary G., "Premium Calculation Implications of Reinsurance without Arbitrage," *ASTIN Bulletin* 21, 1991, pp. 223–230.

[22] Venter, Gary G., “Profit Contingency Loadings and Surplus: Ruin and Return Implications,” Casualty Actuarial Society *Discussion Paper Program*, May 1979, pp. 352–366.

[23] Wang, Shaun S., “Premium Calculation by Transforming the Layer Premium Density,” *ASTIN Bulletin* 26, 1996, pp. 71–92.

EXHIBIT 1

Three Period Demonstration Example
Balance Sheet, Income, Cash Flow, and Rates of Return

	Calendar Period 1	2	3	4	Total All Periods	Benchmark L/S	NPV
BALANCE SHEET (Beginning of Period)							
Invested Assets	14,805	14,868	14,932	0	44,605		41,705
Net Ultimate Ins Liabilities	10,000	10,000	10,000	0	30,000	3.00	27,804
Retained Earnings	−195	−132	−68	0	−395		
Underwriting Risk Surplus	3,333	3,333	3,333	0	10,000		9,268
Investment Risk Surplus	667	667	667	0	2,000	15.00	1,854
Additional Capital	1,000	1,000	1,000	0	3,000	10.00	2,780
Total Capital	5,000	5,000	5,000	0	15,000	2.00	13,902
INCOME BEFORE TAX (During Period)							
Earned Premium	9,700	0	0	0	9,700		9,700
Loss & Loss Expense	10,000	0	0	0	10,000		10,000
Underwriting Income	−300	0	0	0	−300		−300
Total Investment Income	918	922	926	0	2,766	6.2% Inv Yield	
Additional Capital Cost	−80	−80	−80	0	−240		
Total Net Income	538	842	846	0	2,226		

EXHIBIT 1

(CONTINUED)

	Calendar Period 1	2	3	4	Total All Periods	IRR	NPV
INCOME AFTER TAX (During Period)							
Underwriting	−195	0	0	0	−195		−195
Inv Inc Net Liabilities	390	390	390	0	1,170		1,084
Inv Inc Ret. Earns.	−8	−5	−3	0	−15		
Inv Inc Surplus	130	130	130	0	390		361
Underwriting Net Income	317	515	517	0	1,350		1,251
Inv Inc Lift On PH related assets	100	100	100	0	300		278
Inv Inc on Inv Surplus	31	31	31	0	93		86
Investment Net Income	131	131	131	0	393		364
Inv Inc on Additional Capital	47	47	47	0	140		129
Cost of Additional Capital	−52	−52	−52	0	−156		−145
Net Income on Additional Capital	−6	−6	−6	0	−17		−15
Total Net Income	443	640	643	0	1,726		1,600
CASH FLOW (Beginning of Period)						IRR	
Premium Receipts	9,700	0	0	0	9,700		9,700
Loss Payments	0	0	0	−10,000	−10,000		−8,916
Underwriting Tax Payment	105	0	0	0	105		105
Net Underwriting Cash Flow	9,805	0	0	−10,000	−195	−0.7%	
Net U/W incl Liab Funding & Ret Earns Inv Inc	10,000	−70	−70	−10,070	−210	−0.7%	−195
Net Operating Cash Flow inc Inv of Float	10,000	320	320	−9,680	960	3.2%	889
Net Shareholder incl Inv on Surp and Lift	4,000	−581	−581	−4,581	−1,743		−1,615
Additional Capital Flow	1,000	6	6	−995	17		
Net Cash Flow	14,805	63	65	−14,932	0		

EXHIBIT 1

(CONTINUED)

	Calendar Period 1	2	3	4	Total All Periods	IRR	
SURPLUS FLOW (Beginning of Period)							
Underwriting Net Flow	3,333	−450	−450	−3,783	−1,350	13.5%	
Investment Net Flow	667	−131	−131	−798	−393	19.7%	
Net Shareholder Flow	4,000	−581	−581	−4,581	−1,743	14.5%	
Net Total Capital Flows	5,000	−575	−575	−5,575	−1,726	11.5%	
Net Cash Flow excluding Debt Cost	5,000	−627	−627	−5,627	−1,882	12.5%	
RATES OF RETURN					Nominal Return	IRR	NPV Return
Underwriting Return		−0.7%	−0.7%	−0.7%	−0.70%	−0.70%	−0.70%
Investment on Return on Float		3.9%	3.9%	3.9%	3.90%	3.90%	3.90%
Und Operating Return		3.2%	3.2%	3.2%	3.20%	3.20%	3.20%
Total Underwriting Return on Underwriting Surplus		13.5%	13.5%	13.5%	13.50%	13.50%	13.50%
Inv Operating Return (Lift on PH Related Assets)		0.8%	0.8%	0.8%	0.76%	Inv Spread	
Total Investment Return on Investment Surplus		19.7%	19.7%	19.7%	19.65%	19.65%	19.65%
Total Return on Surplus		14.5%	14.5%	14.5%	14.52%	14.52%	14.52%
Total Return on Total Capital		11.5%	11.5%	11.5%	11.51%	11.51%	11.51%
Total Return on Total Capital excl Debt Cost		12.5%	12.5%	12.5%	12.55%	12.55%	12.55%

EXHIBIT 1

(CONTINUED)

ECONOMIC VALUE CALCULATIONS		Calendar Period				Total All		
Return Achieved vs Cost	Avg Wtd Cost	1	2	3	4	Periods		NPV
Underwriting	15.0%		−1.5%	−1.5%	−1.5%	−1.5%		−1.5%
Investment	15.0%		4.7%	4.7%	4.7%	4.7%		4.7%
Finance (Additional Capital)	5.2%		−0.6%	−0.6%	−0.6%	−0.6%		−0.6%
Total	13.0%		−0.5%	−0.5%	−0.5%	−0.5%		−0.5%
Value Created						Created Value	Discount Rate	BVC
Underwriting			−50	−50	−50	−150	15.0%	−114
Investment			31	31	31	93	15.0%	71
Finance (Additional Capital)			−6	−6	−6	−17	5.2%	−15
Total Value Created			−25	−25	−25	−74	13.0%	−59
Embedded Value	Initial Contribution							
Underwriting	3,333	3,219	2,828	2,488	0	Note: Initial period Embedded		
Investment	667	737	624	524	0	Value is equal to the sum of		
Finance (Additional Capital)	1,000	985	941	899	0	initial contributed equity/		
Total Embedded Value	5,000	4,941	4,392	3,911	0	capital and BVC		

Benchmark Net Present Value discounted at the low-risk underwriting investment rate. BVC discounted at equity/debt cost rate.

CLASSIFICATION RATEMAKING—FURTHER DISCUSSION

ROBERT L. BROWN

Abstract

Classification ratemaking is one of the most important elements in the process of a property-casualty rate calculation. It is here that the pricing actuary moves from a rate change that is appropriate for an entire portfolio of policyholders, to prices that attempt to be fair and equitable for each policyholder in the portfolio.

Classification ratemaking is so important that is has its own chapter in the textbook Foundations of Casualty Actuarial Science (Chapter 6, authored by R. Finger). Other sources of P&C study material also present lengthy analysis of this topic [e.g., Introduction to Ratemaking and Loss Reserving for Property and Casualty Insurance (2nd edition) by Brown and Gottlieb (2001)].

This paper illustrates that these two important references do not arrive at exactly the same results for a classification ratemaking situation where some cells have less than full credibility. The paper then goes on to attempt to isolate the reason for the differences, and in so doing, sheds new light on the process itself.

ACKNOWLEDGEMENT

The author would like to acknowledge the assistance of David Oakden in making substantive improvements to the first version of this paper.

1. INTRODUCTION

For more than a decade now, students of the CAS syllabus have learned classification ratemaking from R. Finger's chapter, "Risk Classification" in the textbook *Foundations of Casualty Actuarial Science*, currently Chapter 6 in the 4th edition.

However, this is not the only source of study material on this topic. The Society of Actuaries also introduces their students to some P&C topics through their Part 5 course, and they use the textbook *Introduction to Ratemaking and Loss Reserving for Property and Casualty Insurance*, authored by Brown and Gottlieb.

Interestingly, it will be shown that these two text references do not arrive at exactly the same solution for a classification ratemaking question where some classes in the analysis do not have full credibility.

By analyzing the reason for the differences in the two answers, this paper attempts to elucidate the entire process of classification ratemaking.

2. THE PROBLEM BY ILLUSTRATION

In *Introduction to Ratemaking and Loss Reserving for Property and Casualty Insurance*, authors Brown and Gottlieb present an algebraic proof that the two classical methods to calculate class differentials (namely, the loss ratio method and the loss cost method) are equivalent. This example, however, only covers the case where all risk classes being analyzed have credibility equal to one [see Brown and Gottlieb, pp. 173–175].

It is also the case that for a portfolio of risks where every class has full credibility, the class relativities produced by Finger are equal to the relativities produced by Brown and Gottlieb.

This will now be illustrated with a simple example.

Example, Part I

The pricing actuary has decided upon a statewide-adopted rate level increase of +6%. Given the following data, show the new adopted rates for Classes 1, 2, and 3.

The existing base rate is $100 in Class 1.

All classes have full credibility ($Z = 1$).

You also have the following data by class:

TABLE 1

Class	Existing Relativity	Exposure Units	Earned Premium	$ Loss	Loss Cost	Loss Ratio
1	1.00	500	$50,000	$30,000	$60.00	0.6000
2	1.25	150	18,750	12,750	85.00	0.6800
3	1.50	200	30,000	15,900	79.50	0.5300
Total		850	98,750	58,650	69.00	0.5939

Method I

We will use the loss cost method using Class 1 as the base rate for the calculation. Remember that $Z = 1$ throughout. We will use seven decimal accuracy in all calculations, even if fewer decimal place accuracy is displayed.

TABLE 2

Class	Existing Relativity	Loss Cost	Indicated Relativity
1	1.00	60.00	1.000
2	1.25	85.00	1.416
3	1.50	79.50	1.325

Since $Z = 1$ in all cells, the existing relativity does not have any impact on the answer and could be ignored (as it is in some examples below).

We have set the class relativity for Class 1 equal to 1.000. This means that our overall rate change may not balance to +6%. So, we need to balance back, as follows:

$$\begin{aligned}
\text{Old Average Relativity} &= [500(1.00) + 150(1.25) + 200(1.50)]/850 \\
&= 1.1617647 \\
\text{New Average Relativity} &= [500(1.00) + 150(1.416) + 200(1.325)]/850 \\
&= 1.15 \\
\text{Balance-Back Factor} &= 1.1617647/1.15 = 1.0102302,
\end{aligned}$$

giving us:

TABLE 3

Class	New Rate	Exposure Units	Premium Income
1	$107.08	500	$53,542
2	151.70	150	22,755
3	141.89	200	28,377
Total		850	104,675

Now, $\$104,675 = \$98,750 \times (1.06)$, so, everything is as it should be.

Method II

We will use the loss cost method but the base class will be the state (loss cost).

TABLE 4

Class	Loss Cost	Indicated Relativity	Relativity with Class 1 = 1.000
1	60.00	0.8695652	1.000
2	85.00	1.2318841	1.416
3	79.50	1.1521739	1.325
State	69.00	1.0000000	

This gives us the same relativities as does Method I, and there is no reason to go further (i.e., there is no reason to do the balance-back calculation).

Method III

This method follows the loss ratio approach with the base class being Class 1.

TABLE 5

Class	Loss Ratio	Existing Relativity	Indicated Change $[LR_i/LR_1]$	Indicated Relativity
1	0.6000	1.00	1.000	1.000
2	0.6800	1.25	1.133	1.416
3	0.5300	1.50	0.883	1.325

Again, the same answer. Thus, it has been shown that under the set conditions, the loss ratio method and the loss cost method do provide the same answer (as proven algebraically by Brown and Gottlieb).

Method IV

We will follow the loss ratio approach again, but now the base class will be the state (loss ratio).

TABLE 6

Class	Loss Ratio	Existing Relativity	Indicated Change $[LR_i/LR_S]$	Indicated Relativity	Indicated Relativity with Class 1 = 1.000
1	0.6000	1.00	1.0102302	1.0102302	1.000
2	0.6800	1.25	1.1449275	1.4311594	1.416
3	0.5300	1.50	0.8923700	1.3385550	1.325
State	0.5939		1.0000000		

Again, the same answer.

Method V

Finally, we follow the template presented in Chapter 6 of the *Foundations* textbook (Finger). Remember that the overall rate change is +6%.

TABLE 7

Class (1)	Existing Relativity (2)	Adjusted Exposures [(2)× Given Exp] (3)	Adjusted Loss Costs [$ Loss/(3)] (4)	Indicated Adjustment [(4)/(4) Total] (5)	Extension [(5) × Old Rate × 1.06] (6)	Adopted Relativity† (7)
1	1.00	500	60.00	1.0102302	107.08	1.000
2	1.25	187.5	68.00	1.1449275	151.70	1.416
3	1.50	300	53.00	0.8923700	141.89	1.325
		987.5	59.39			

†This is not produced by Finger, but is clearly consistent.

Obviously, we had identical answers for the adopted relativities and the new rates from all the approaches attempted. This should be gratifying and should create a level of comfort among users. Further, because all Classes have full credibility, in Methods I and III, we could have chosen Class 2 or Class 3 as our base and the results would have been the same.

Example, Part II

We now stir the pot somewhat by stipulating credibility factors for the different classes where only Class 1 has full credibility.

We will use the following data in this illustration:

TABLE 8

Class	Existing Relativity	Exposure Units	Earned Premium	$ Loss	Loss Cost	Loss Ratio	Credibility Z
1	1.00	500	$50,000	$30,000	60.00	0.6000	1.000
2	1.25	150	18,750	12,750	85.00	0.6800	0.500
3	1.50	200	30,000	15,900	79.50	0.5300	0.600
State		850	98,750	58,650	69.00	0.5939	1.000

Again, we will find the new Class 1, 2, and 3 (base) rates with an overall +6% rate increase.

We will now repeat the original five methods of calculation to see if they again produce identical answers.

*Method I**

Remember that this is the loss cost method with the base class being Class 1.

TABLE 9

Class (1)	Existing Relativity (2)	Loss Cost (3)	Indicated Relativity (4)	Z (5)	Adopted Relativity $[Z(4)+(1-Z)(2)]$ (6)
1	1.00	60.00	1.000	1.000	1.000
2	1.25	85.00	1.416	0.500	1.333
3	1.50	79.50	1.325	0.600	1.395

Again we have created off-balance, so we balance back:

$$\text{Old Average Relativity} = [500(1.00) + 150(1.25) + 200(1.50)]/850$$
$$= 1.1617647$$
$$\text{New Average Relativity} = [500(1.000) + 150(1.333) + 200(1.395)]/850$$
$$= 1.1517647$$
$$\text{Balance-Back Factor} = 1.1617647/1.1517647 = 1.0086823.$$

This produces the following new "base" rates:

TABLE 10

Class	New Rate	Exposure Units	Premium Income
1	$106.92	500	$53,460.16
2	142.56	150	21,384.06
3	149.15	200	29,830.77
Total		850	104,674.99

or, $104,675, which is what we want.

*Refers to methods with credibility-weighted relativities.

*Method II**

This is the loss cost method, but with the "base" being the state (loss cost).

TABLE 11

Class (1)	Existing Relativity (2)	Loss Cost (3)	Indicated Relativity [(3)/(3) Total] (4)	Ind. Rel. Class 1 = 1.00 $[(4)/(4)_1]$ (5)	Z (6)	Adopted Relativity $[Z \times (5) + (1 - Z) \times (2)]$ (7)
1	1.00	60.00	0.8695652	1.000	1.000	1.000
2	1.25	85.00	1.2318841	1.416	0.500	1.333
3	1.50	79.50	1.1521739	1.325	0.600	1.395
		69.00				

This is the same answer as *Method I**.

However, it is possible to get an incorrect answer by changing the order of the arithmetic operations. For example, one might do the following erroneous calculation:

TABLE 12

Class (1)	Existing Relativity (2)	Loss Cost (3)	Indicated Relativity [(3)/(3) Total] (4)	Z (5)	Adopted Relativity $[Z \times (4) + (1 - Z) \times (2)]$ (6)	Rate Manual Relativity (Class 1 = 1.00) (7)
1	1.00	60.00	0.8695652	1.000	0.8695652	1.0000000
2	1.25	85.00	1.2318841	0.500	1.2409421	1.4270834
3	1.50	79.50	1.1521739	0.600	1.2913043	1.4850000
Total		69.00				

This answer is different than those found in the previous two calculations, and it is wrong.

It is wrong because in the formula for the adopted relativity $[Z \times (4) + (1 - Z) \times (2)]$, you do not have the relativities on the same basis. Column (2) has the relativities "normalized" such that the relativity for Class 1 equals 1.000, but in Column (4) the data have not been "normalized." Thus, in the formula for the adopted relativity, we are taking the weighted average of "apples" from Column (2) and "oranges" from Column (4). One could extend the analogy to consider one vector as degrees Fahrenheit and the other, degrees Celsius. These should not be commingled in a weighted average. Obviously, this would lead to an incorrect result.

*Method III**

This is the classical loss ratio method with Class 1 being the base.

TABLE 13

Class (1)	Loss Ratio (2)	Existing Relativity (3)	Indicated Change $[LR_i/LR_1]$ (4)	Indicated Relativity (5)	Z (6)	Adopted Relativity $[Z \times (5) + (1 - Z) \times (3)]$ (7)
1	0.6000	1.00	1.000	1.000	1.000	1.000
2	0.6800	1.25	1.133	1.416	0.500	1.333
3	0.5300	1.50	0.883	1.325	0.600	1.395

This agrees nicely with all of our previous work.

*Method IV**

Again, this is the loss ratio method with the base "class" being the state (loss ratio).

TABLE 14

Class (1)	Loss Ratio (2)	Existing Relativity (3)	Indicated Change $[LR_i/LR_S]$ (4)	Indicated Relativity $[(4) \times (3)]$ (5)	Indicated Relativity Class 1 = 1.00 (6)	Z (7)	Adopted Relativity $[Z \times (6) + (1 - Z) \times (3)]$ (8)
1	0.6000	1.00	1.0102302	1.0102302	1.000	1.000	1.000
2	0.6800	1.25	1.1449275	1.4311594	1.416	0.500	1.333
3	0.5300	1.50	0.8923700	1.3385550	1.325	0.600	1.395
State	0.5939						

Obviously, this is an acceptable answer. But again, the order of calculation and the use of factors that are "normalized" to the same base are of the essence. For example, we could erroneously do the following:

TABLE 15

Class (1)	Loss Ratio (2)	Existing Relativity (3)	Indicated Change $[LR_i/LR_S]$ (4)	Indicated Relativity $[(4) \times (3)]$ (5)	Z (6)	Adopted Relativity $[Z(5) + (1 - Z) \times (3)]$ (7)	Rate Manual Relativity (Class 1 = 1.000) (8)
1	0.6000	1.00	1.0102302	1.0102302	1.000	1.0102302	1.000
2	0.5280	1.25	1.1449275	1.4311594	0.500	1.3405797	1.327
3	0.5400	1.50	0.8923700	1.3385550	0.600	1.4031330	1.389
State	0.5676						

This is an incorrect answer because in our adopted relativity calculation, Column (3) has been "normalized" so that Class 1 has a relativity equal to 1.00, but Column (5) has not. Thus, we are attempting to do a weighted average of "apples" and "oranges."

*Method V**

This uses the template found in Chapter 6 of the *Foundations* text as authored by Finger (2001).

TABLE 16

Class (1)	Existing Relativities (2)	Exposure Units (3)	Earned Premiums (4)	Adjusted Exposures [(3) × (2)] (5)	Adjusted Loss Costs [$ Loss/(5)] (6)	Indicated Adjustment [(6)/(6) Total] (7)	Z (8)
1	1.00	500	50,000	500	60.00	1.0102302	1.000
2	1.25	150	18,750	187.5	68.00	1.1449275	0.500
3	1.50	200	30,000	300	53.00	0.8923700	0.600
Total		850	98,750	987.5	59.39		

Continuing with the template:

TABLE 17

Class	Credibility Weighted Adjustment [$Z \times (7) + (1 - Z)$] (9)	Extension [(9) × (4)] (10)	Balanced Adjustment [(9)/(9) Total] (11)	New Rates [(11) × Old × 1.06] (12)	Extension [(12) × (3)] (13)
1	1.0102302	50,511.51	1.0109174	107.16	53,578.62
2	1.0724638	20,108.70	1.0731934	142.20	21,329.72
3	0.9354220	28,062.66	0.9360583	148.83	29,766.65
Total	0.9993202†	98,682.87			104,674.99

†0.9993202 = 98,682.87/98,750.

This all seems to check out just fine. The final answer ($104,675) is a +6% rate increase as requested. However, the "New Rates" are different than what we got in the other four methods.

One can also see that the new class relativities created by Finger (but never actually displayed) are as follows and differ from those calculated by Methods I* to IV*.

TABLE 18

Class	Relativity with Class 1 = 1.00
1	1.000
2	1.327
3	1.389

Why is this?

With a little bit of work (and some insight) the differences are easily reconciled.

In Methods I* to IV*, we calculated all relativities using a base relativity of 1.000 for Class 1. What Finger does is to calculate all relativities using a base relativity of 1.000 for the state. We can show that this is true by recalculating Methods I* to IV* using a relativity of 1.000 for the state.

In our existing examples, the following hold:

TABLE 19

Class	Relativity
1	1.000
2	1.250
3	1.500
State	1.1617647

Switch these values to equivalent values with the state relativity equal to 1.000 and you get:

TABLE 20

Class	Relativity
1	0.8607595
2	1.0759494
3	1.2911392
State	1.0000000

Now, calculate your credibility-weighted new relativities using the above as starting points. You will get the following:

TABLE 21

Class (1)	Existing Relativity (2)	Loss Cost (3)	Indicated Relativity (4)	Z (5)	Adopted Relativity (6)
1	0.8607595	$60.00	0.8695652	1.00	0.8695652
2	1.0759494	85.00	1.2318841	0.50	1.1539167
3	1.2911392	79.50	1.1521739	0.60	1.2077600
Total		69.00			

With this change in format, you will arrive at the premiums and relativities derived by Finger. Just recreate the adopted relativities in Table 21 with Class 1 = 1.000, and you get:

TABLE 22

Class	Adopted Relativity with Class 1 = 1.000
1	1.000
2	1.327
3	1.389

Thus, one cannot conclude that one methodology is correct and the other incorrect. They are merely two versions of the same analysis that happen to result in slightly different answers. However, there are some implications to these findings, including:

- Regulators cannot guarantee that two actuaries will arrive at the same answer given the same data without prescribing the methodology in extreme detail.
- Educators, like me, who have to create questions for term tests and final examinations, cannot guarantee a uniquely correct answer to a question unless the method of solution is defined in extreme detail.

- The pricing actuary who is aware of these differences might then be able to use them to his or her advantage.

For example, assume you have two large classes (A and B) that are fully credible and a few smaller classes with little credibility. If we assume that A increases by 10% and B declines by 10%, then the choice of A or B as the base class will drive the rates of the classes with little credibility. If we choose A, their rates will go up and if we choose B their rates will go down. If we choose the statewide average, their rates will change little (all else being equal).

3. CONCLUSION

As stated in the introduction, classification ratemaking is one of the most important steps in arriving at new rate manual rates.

This topic has been presented in a variety of forms, templates, and methodologies over the years. Unfortunately, the different methods presented to students do not necessarily produce the same unique result.

It is the belief and hope of this author that a full understanding of the consequences as presented in this paper will bring the level of knowledge of future students to a new high in this important area.

REFERENCES

[1] Brown, R. L., and L. R. Gottlieb, *Introduction to Ratemaking and Loss Reserving for Property and Casualty Insurance*, 2nd ed., (Winstead, Ct.: ACTEX Publications Inc., 2001).

[2] Finger, R. J., "Risk Classification," Chapter 6 in *Foundations of Casualty Actuarial Science*, 4th ed., (Arlington, Va.: Casualty Actuarial Society, 2001).

SIMPSON'S PARADOX, CONFOUNDING VARIABLES, AND INSURANCE RATEMAKING

JOHN A. STENMARK AND CHENG-SHENG PETER WU

Abstract

The insurance process is complex, with numerous factors combining to produce both premiums and losses. When compiling rates, actuaries often aggregate data from more than one source, while at the same time stratifying the data to achieve homogeneity. Such exercises may lead to biased and sometimes even surprising results, called Simpson's paradox, because the variables involved in the aggregation process or the stratification process are confounded by the presence of other variables. In this paper, we will describe Simpson's paradox and confounding and the statistical underpinning associated with those phenomena. We will further discuss how such bias may exist in P&C actuarial rating applications and solutions that can resolve the bias.

1. INTRODUCTION

An actuary is asked by the CEO for a small insurance company to examine the good student discount that the company offers. The discount is currently fifteen percent, but several competitors offer a twenty percent discount for qualifying youthful operators. As usual, the CEO is in a hurry, so the actuary compiles the experience and develops a relativity based on the pure premiums for all youthful operators (Age 15 to 25). Imagine the actuary's shock when the experience indicates, not the twenty percent discount for which the CEO had been hoping, but a twenty percent surcharge. The loss experience appears in Table 1.

TABLE 1

WITHOUT GOOD STUDENT DISCOUNT			
Exposures	Distribution	Losses	Pure Premium
18,980	86.3%	$44,210,062	$2,329

WITH GOOD STUDENT DISCOUNT				
Exposures	Distribution	Losses	Pure Premium	Relativity
3,020	13.7%	$8,475,292	$2,806	20%

TABLE 2

WITHOUT GOOD STUDENT DISCOUNT				
Age	Exposures	Distribution Within Age	Losses	Pure Premium
15–18	5,500	68.8%	$21,661,344	$3,938
19–21	5,580	93.0%	$12,488,608	$2,238
22–25	7,900	98.8%	$10,060,110	$1,273
Total	18,980		$44,210,062	$2,329

WITH GOOD STUDENT DISCOUNT					
Age	Exposures	Distribution Within Age	Losses	Pure Premium	Relativity
15–18	2,500	31.3%	$7,653,680	$3,061	−22%
19–21	420	7.0%	$705,002	$1,679	−25%
22–25	100	1.3%	$116,610	$1,166	−8%
Total	3,020		$8,475,292	$2,806	20%

The actuary knows of the problems incumbent with pure premiums, but certainly they can't cause this magnitude of a disparity. The actuary decides to review the experience by driver age that is available from the company's class plan. Table 2 displays that experience.

The relativities by class appear more reasonable, but the actuary still has a concern. How can the "average" of these three

TABLE 3

WITHOUT GOOD STUDENT DISCOUNT				
Age	Exposures	Distribution Within Age	Losses	Pure Premium
15	1,300	65.0%	$6,500,000	$5,000
16	1,300	65.0%	$5,525,000	$4,250
17	1,350	67.5%	$4,876,875	$3,613
18	1,550	77.5%	$4,759,469	$3,071
19	1,860	93.0%	$4,854,658	$2,610
20	1,860	93.0%	$4,126,459	$2,219
21	1,860	93.0%	$3,507,490	$1,886
22	1,920	96.0%	$3,077,540	$1,603
23	1,980	99.0%	$2,697,656	$1,362
24	2,000	100.0%	$2,316,169	$1,158
25	2,000	100.0%	$1,968,744	$984
Total	18,980		$44,210,062	$2,329

WITH GOOD STUDENT DISCOUNT					
Age	Exposures	Distribution Within Age	Losses	Pure Premium	Relativity
15	700	35.0%	$2,625,000	$3,750	−25%
16	700	35.0%	$2,231,250	$3,187	−25%
17	650	32.5%	$1,761,094	$2,709	−25%
18	450	22.5%	$1,036,336	$2,303	−25%
19	140	7.0%	$274,053	$1,958	−25%
20	140	7.0%	$232,945	$1,664	−25%
21	140	7.0%	$198,003	$1,414	−25%
22	80	4.0%	$96,173	$1,202	−25%
23	20	1.0%	$20,437	$1,022	−25%
24	—	0.0%	—	—	0%
25	—	—	—	—	0%
Total	3,020		$8,475,292	$2,806	20%

discounts produce a surcharge? The actuary is also concerned about the variation in the indicated relativities. The actuary requests data by driver age from the company's IS department and reviews the experience, which is displayed in Table 3.

By further stratifying the data, even more precision appears to be achieved and it appears that an even higher discount is

TABLE 4

	Male			Female		
School	Applying	Accepted	Acceptance Ratio	Applying	Accepted	Acceptance Ratio
Engineering	1000	400	40%	200	100	50%
Arts	200	20	10%	1000	125	13%
Total	1200	420	35%	1200	225	19%

justified. In addition, the same discount seems to be supported for all driver ages. Nevertheless, the question remains: "How does the accumulation of all these discounts produce a surcharge?" The answer is Simpson's paradox.

2. SIMPSON'S PARADOX

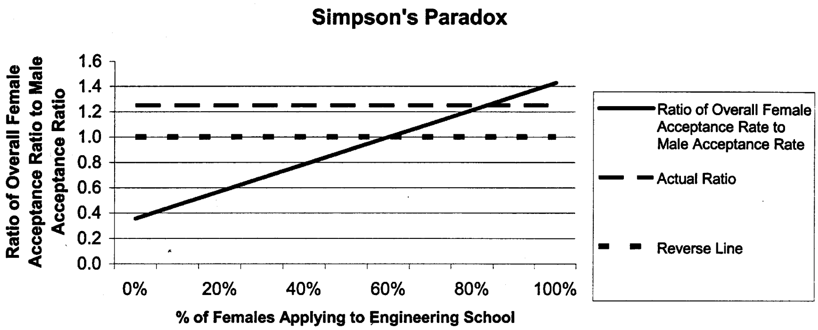
E. H. Simpson first described the paradox in 1951 in a paper titled "The Interpretation of Interaction in Contingency Tables" [14]. It is an interesting statistical phenomenon that causes a potential bias in certain data analyses. The paradox occurs when a relationship or association between two variables reverses when a third factor, called a confounding variable, is introduced. The paradox also occurs when a relationship/association reverses when the data is aggregated over a confounding variable.

2.1. The College Admissions Example

The classic illustration of the paradox involves college admissions by gender, which can be illustrated in the example in Table 4 [3].

In Table 4, the overall acceptance ratio for female applicants, 19%, is lower than the ratio for the male applicants, 35%. However, this relationship reverses when the factor of the school to which they apply is introduced. When this variable is considered, the acceptance ratio for female applicants is 25% higher

FIGURE 1



than male applicants for both the engineering school (50% to 40%) and the art school (13% to 10%).

The reason why Simpson's paradox occurs is that more female applicants apply to the art school, which has an overall lower acceptance rate than the engineering school. The engineering school has a 40% to 50% acceptance rate, while the art school has a 10% to 13% acceptance rate. In the above example, about 83% of female applicants apply to the art school, while 83% of male applicants apply to the engineering school.

Let's vary the percentage of the female applicants applying to the art school and assume all the other parameters in the example remain the same. Then, calculate the ratio of the overall female applicants to the male applicants.

In Figure 1, the solid line represents the ratio of the overall female acceptance rate to the overall male acceptance rate by varying the percentage of females applying to the engineering school. We know that the underlying ratio is 1.25 when we analyze the acceptance by school, and the dashed line represents the actual ratio of 1.25.

We can see that only when the percentage of female students applying to the engineering school is 83% is the overall ratio the same as the true ratio. This 83% is the same percentage as the male students applying to the engineering school. For all the other percentages, the overall ratio is different from the true ratio.

Another interesting point indicated in Figure 1 is that when the percentage of female students applying to the engineering school is less than 60%, the ratio of the overall acceptance rate of female to male is less than 1.00, represented by the dotted line, suggesting that the overall female acceptance rate is lower. This is a reversal of the fact that the female acceptance rate is higher than the male acceptance rate, which is Simpson's paradox [12].

From the example above, we can see that Simpson's paradox occurs when the distributions of the sample population are not uniform across the two predictive variables. When this takes place, the variable of "school" is confounding the acceptance rate and is confusing the relationship between the acceptance rate and applicants' gender. We will discuss the concept of confounding variables in detail later.

2.2. The Simple Math of Simpson's Paradox

Simpson's paradox arises from one simple mathematical truth. Given eight real numbers: a, b, c, d, A, B, C, D with the following properties: $a/A > b/B$ and $c/C > d/D$, then it is not necessarily true that $(a+c)/(A+C) > (b+d)/(B+D)$. In fact, it may be true that: $(a+c)/(A+C) < (b+d)/(B+D)$. This is Simpson's paradox. This is an obvious math reality, yet it has significant ramifications in Bayesian analysis, medical research, science and engineering studies, societal statistical analysis and yes, insurance ratemaking. It is of concern for any statistical activity involving the calculation and analysis of ratios of two measurements. This activity is prevalent in insurance; loss ratios, pure premium, frequency, severity and loss development factors are just some of the statistics involving the ratio of two measures.

3. CONFOUNDING VARIABLES

A variable can confound the results of a statistical analysis only if it is related (non-independent) to both the dependent variable and at least one of the other (independent) variables in the analysis. More specifically, a variable can confound the results of an insurance rate structure analysis only if it is related (non-independent) to both the experience measure (loss ratio, pure premium, etc.) and at least one of the other rating variables in the analysis.

3.1. Experimental Design

Confounding and Simpson's paradox are of great concern in the design of research studies. For example, in a typical design of medical research, researchers would like to know the impact of an intervention measure. Using the notation introduced in Section 2.2, assume that A and C are the number of observations where the intervention has taken place. B and D are the number of observations in the group where the intervention has not been executed (the control group). The distinction between the A and C (and also B and D) observations is the potential confounding variable. For example, in Cates [5], A and C would represent smokers attempting to quit with nurse intervention (the intervention) from two different studies (the potential confounding variable).[1] Also, in our previous college admission example, A and C might represent the number of females (the intervention) applying to the art and engineering schools (the potential confounding variable) respectively, as displayed in Table 5.

Further, the number of events is represented by a, b, c and d and the ratio a/A is the proportion of events per number of

[1] Cates [5] described the meta-analysis of smokers attempting to quit with and without high intensity nurse intervention. Cates illustrated several methods of combining studies from independent sources. Methods included Maentel-Hensel fixed effects method and a random effects methodology. Both of these methodologies produced weights that were used to combine the risk differences, rather than the underlying data. Cates showed that a reversal (Simpson's paradox) occurred when the raw data were combined.

TABLE 5

			Variables Under Study	
		Example	1 Females	2 Males
Confounding Variable Value 1	Number of Events	Art School	a	b
	Number of Observations		A	B
Confounding Variable Value 2	Number of Events	Engineering School	c	d
	Number of Observations		C	D

observations; e.g., the percentage of females being admitted to art school or the proportion of smokers in Study #1 (of the Cates paper) who quit with the aid of a nurse.

While both the college admission example and the smoking intervention example involve studies where existing data are observed and analyzed, assume for a moment that this is not the case—that we can design an experiment in such manner as to minimize the bias of any potential confounding variable. Ultimately, we find the bias is eliminated if the confounding variable and the variable under study are independent. The bias is also eliminated if either the groups are balanced (possess an equal number of observations) or are proportionally distributed (there is the same ratio of observations of the variable under study for each value of the confounding variable).[2] It is possible to illustrate this using the following argument.

Consider an experiment with groups A, B, C, D as described above. Also assume that the ratio differences are known and are equal to some K: $(a/A) - (b/B) = K = (c/C) - (d/D)$.

[2]Of course, the balance condition is a special case of the proportional condition. The balance condition is especially important in experiment design.

How can the experiment be designed so that $(a+c)/(A+C) - (b+d)/(B+D) = K$?

First assume that the potential confounding variable is independent of the variable under study, i.e., that $a/A = c/C$ and $b/B = d/D$. Therefore $A = aC/c$ and $B = bD/d$ and

$$\frac{a+c}{\frac{aC}{c}+C} - \frac{b+d}{\frac{bD}{d}+D} = \frac{a+c}{\frac{C}{c}(a+c)} - \frac{b+d}{\frac{D}{d}(b+d)} = \frac{c}{C} - \frac{d}{D} = K.$$

Therefore, if the potential confounding variable and the variable under study are independent then there is no confounding.

Now instead of assuming independence, assume that the experiment has a balanced distribution; i.e., there is the same number of observations in each group relative to the variable under study (the same number of females applying to the art school and the engineering school and the same number of males applying to both schools). Then $A = C$ and $B = D$. And

$$\begin{aligned}
\frac{a+c}{A+C} - \frac{b+d}{B+D} &= \frac{a}{A+C} + \frac{c}{A+C} - \frac{b}{B+D} - \frac{d}{B+D} \\
&= \frac{a}{A+A} + \frac{c}{C+C} - \frac{b}{B+B} - \frac{d}{D+D} \\
&= \frac{1}{2}\left[\frac{a}{A} + \frac{c}{C} - \frac{b}{B} - \frac{d}{D}\right] \\
&= \frac{1}{2}\left[\frac{a}{A} - \frac{b}{B} + \frac{c}{C} - \frac{d}{D}\right] \\
&= \frac{1}{2}[K+K] = K.
\end{aligned}$$

So there is no confounding if the observations possess a balanced distribution.

Now assume that the experiment is proportionally distributed; i.e., there is the same ratio of observations of the variable under study for each value of the confounding variable $(A/B = C/D)$.

That is, the ratio of females applying to the art school to the number of males applying to the art school is the same as the ratio of females applying to the engineering school to the number of males applying to the engineering school. If $A/B = C/D$, then define $A/C = B/D = K'$. Then $A = CK'$, $B = DK'$. Therefore

$$
\begin{aligned}
\frac{a+c}{A+C} - \frac{b+d}{B+D} &= \frac{a+c}{CK'+C} - \frac{b+d}{DK'+D} \\
&= \frac{1}{K'+1}\left(\frac{a+c}{C} - \frac{b+d}{D}\right) \\
&= \frac{1}{K'+1}\left(\frac{a}{C} + \frac{c}{C} - \frac{b}{D} - \frac{d}{D}\right) \\
&= \frac{1}{K'+1}\left(\frac{a}{C} - \frac{b}{D} + K\right) \\
&= \frac{1}{K'+1}\left(\frac{a}{\frac{A}{K'}} - \frac{b}{\frac{B}{K'}} + K\right) \\
&= \frac{1}{K'+1}\left[K'\left(\frac{a}{A} - \frac{b}{B}\right) + K\right] \\
&= \frac{1}{K'+1}(K'K + K) = \frac{K'+1}{K'+1}K = K.
\end{aligned}
$$

Therefore, if the observations are proportionally distributed, there is no confounding.

In the example detailed in the introduction of the paper, the good student pure premiums and ultimately the indicated good student discount were confounded by driver age. It is not surprising that there is the observed relationship between the distribution of drivers by age and those with the good student discount. As driver age approaches 25, fewer are students, much less good students. The reversal occurs since there is a higher distribution of young drivers with good student discount and young drivers have higher pure premiums.

Important Principle: If there is independence between the potential confounding variable and the variable under study, or if the study is balanced or proportionally distributed, then there is no confounding.

Insurance ratemaking differs from most statistical studies in a number of ways:

1. It is generally not possible to design the makeup of groups of insureds so that classifications are balanced.
2. Generally there are far more values for each variable and probably more variables in insurance than in research analysis.
3. In most statistical studies, the objective is to accept or reject a hypothesis. The primary concern in insurance ratemaking is to properly calculate a rate, which requires a continuous rather than binary output.

In the next four sections, we will further examine and extend the Important Principle of confounding to more than two variables using general statistical models and experimental design theories. The two statistical models that we will use are the simple additive and the multiplicative models, both without an interaction term. Such additive and multiplicative multivariable models are the ideal models, and are similar to many insurance rating and class plan structures [1]. For illustrative purposes, we will use a 2-by-2 rating example with age of driver (youthful drivers vs. adult drivers) and territory (urban territories vs. suburban territories) throughout the sections. For more details of the additive and multiplicative statistical models and experimental design theories, please see Montgomery [9] and Neter, et al. [10].

3.2. The Confounding Effect on an Additive Model with No Interaction Term

Let's start with a 2-by-2 additive model. Assume that the observation or exposure distribution of each cell is $w_{i_1 i_2}$. Later we will extend the models to more dimensions and values.

Define: $w_{(i_1),1} = w_{i_1,1}/\sum_{i_2} w_{i_1,i_2}$; e.g.,

$$w_{(1),1} = \frac{w_{1,1}}{w_{1,1} + w_{1,2}}, \qquad w_{(1),2} = \frac{w_{1,2}}{w_{1,1} + w_{1,2}},$$

$$w_{(2),1} = \frac{w_{2,1}}{w_{2,1} + w_{2,2}}, \qquad w_{(2),2} = \frac{w_{2,2}}{w_{2,1} + w_{2,2}}.$$

Note: While this notation may be unfamiliar, please accept this verbal interpretation. If w_{i_1,i_2} represents the exposures in cell i_1, i_2, then $w_{(i_1),1}$ represents the marginal exposure distribution of cell i_1, i_2 for cells with $i_2 = 1$.

For a linearly independent additive model, the mean value (underlying rate) for each of the 2-by-2 cells can be represented as follows: $\mu_{i_1 i_2} = \mu + \mu_{i_1,\bullet} + \mu_{\bullet,i_2}$: $i_1 = 1,2$, $i_2 = 1,2$, where a dot ($\bullet$) index indicates the mean across that index.

By linearly independent we mean that there are no interaction terms. If the model were not linearly independent, the mean value (underlying rate) for each of the 2-by-2 cells would be represented as: $\mu_{i_1 i_2} = \mu + \mu_{i_1,\bullet} + \mu_{\bullet,i_2} + \varepsilon_{i_1,i_2}$: $i_1 = 1,2$, $i_2 = 1,2$, where ε_{i_1,i_2} is the interaction term.

More specifically, we define the following for the 2-by-2 age of driver and territory example: $\mu_{i_1 i_2} = \mu + \mu_{i_1,\bullet}$(Age of Driver) + $\mu_{\bullet,i_2}$(Vehicle Territory).

Now we want to compare the difference in the aggregate rate between adult and youthful drivers:

Then the aggregate rate for each i_1 is $m_{i_1,\bullet} = \mu_{i_1,1} w_{(i_1),1} + \mu_{i_1,2} w_{(i_1),2}$.

And

$$m_{1,\bullet} - m_{2,\bullet} = \mu_{1,1} w_{(1),1} + \mu_{1,2} w_{(1),2} - \mu_{2,1} w_{(2),1} - \mu_{2,2} w_{(2),2}.$$

Then

$$\begin{aligned} m_{1,\bullet} - m_{2,\bullet} = {} & w_{(1),1}(\mu + \mu_{1,\bullet} + \mu_{\bullet,1}) + w_{(1),2}(\mu + \mu_{1,\bullet} + \mu_{\bullet,2}) \\ & - w_{(2),1}(\mu + \mu_{2,\bullet} + \mu_{\bullet,1}) - w_{(2),2}(\mu + \mu_{2,\bullet} + \mu_{\bullet,2}). \end{aligned}$$

If $w_{(1),1} = w_{(2),1}$ and $w_{(1),2} = w_{(2),2}$, then

$$\begin{aligned} m_{1,\bullet} - m_{2,\bullet} &= w_{(1),1}(\mu + \mu_{1,\bullet} + \mu_{\bullet,1}) + w_{(1),2}(\mu + \mu_{1,\bullet} + \mu_{\bullet,2}) \\ &\quad - w_{(1),1}(\mu + \mu_{2,\bullet} + \mu_{\bullet,1}) - w_{(1),2}(\mu + \mu_{2,\bullet} + \mu_{\bullet,2}) \\ &= w_{(1),1}(\mu_{1,\bullet} - \mu_{2,\bullet}) + w_{(1),2}(\mu_{1,\bullet} - \mu_{2,\bullet}) = \mu_{1,\bullet} - \mu_{2,\bullet}. \end{aligned}$$

Since for the 2-by-2 case: $w_{(1),1} + w_{(1),2} = 1$, we can derive the same results for the other factor, the vehicle territory.

If $w_{1,(1)} = w_{2,(1)}$ and $w_{2,(1)} = w_{2,(2)}$, then $m_{\bullet,2} - m_{\bullet,1} = \mu_{\bullet,2} - \mu_{\bullet,1}$.

Therefore, territory does not confound the age of driver relativities for this 2-by-2 linearly independent additive model if territorial distribution of exposures is independent of the age of driver distribution of exposures. That is, if $w_{(1),1} = w_{(2),1}$, $w_{(1),2} = w_{(2),2}$, $w_{1,(1)} = w_{2,(1)}$ and $w_{1,(2)} = w_{2,(2)}$. This is a *proportional* distribution. Of course, a special case for such a distribution occurs when each cell has the same number of data points, $w_{1,1} = w_{1,2} = w_{2,1} = w_{2,2}$. This is a *balanced* distribution.

The following is a numerical example that illustrates such an additive model. The statistics for the example are as follows:

$$\mu_{i_1 i_2} = \mu + \mu_{i_1,\bullet}(\text{Age of Driver}) + \mu_{\bullet,i_2}(\text{Vehicle Territory})$$

Let $\mu = \$400$,

$$\begin{aligned} \mu_{1,\bullet} &= +\$100 \text{ for youthful drivers} \\ \mu_{2,\bullet} &= -\$100 \text{ for adult drivers,} \\ \mu_{\bullet,1} &= +\$100 \text{ for urban drivers,} \\ \mu_{\bullet,2} &= -\$100 \text{ for suburban drivers.} \end{aligned}$$

Therefore, the pure premiums for each of the four combinations are:

$$\begin{aligned} \mu_{1,1} &= \$600, \qquad \mu_{1,2} = \$400, \\ \mu_{2,1} &= \$400, \qquad \mu_{2,2} = \$200. \end{aligned}$$

TABLE 6

		Urban	Suburban	Total
Youthful	Total Loss	\$3,000	\$6,000	\$9,000
	Exposures	5	15	20
	Distribution	12.5%	37.5%	50.0%
	Pure Premium	\$600	\$400	\$450
Adult	Total Loss	\$2,000	\$3,000	\$5,000
	Exposures	5	15	20
	Distribution	12.5%	37.5%	50.0%
	Pure Premium	\$400	\$200	\$250
Total	Total Loss	\$5,000	\$9,000	\$14,000
	Exposures	10	30	40
	Distribution	25.0%	75.0%	100.0%
	Pure Premium	\$500	\$300	\$350

Also assume that

$$w_{1,1} = 12.5\%, \qquad w_{1,2} = 37.5\%,$$
$$w_{2,1} = 12.5\%, \qquad w_{2,2} = 37.5\%.$$

If we study Table 6, we can see that the difference between youthful driver underlying rate and the adult driver underlying rate is: $\mu_{1,\bullet} - \mu_{2,\bullet} = \$200$, which is the same as the difference between the aggregate rates, \$450 – \$250 = \$200. Therefore, in this case, confounding does not occur.

The data for the other factor, vehicle territory, yield the same result. The difference between underlying rates for the urban territory and the suburban territory is: $\mu_{\bullet,1} - \mu_{\bullet,2} = \$200$, which is the same as if we use the aggregate rates, \$500 – \$300 = \$200. Therefore, in this case as well, confounding does not occur.

Now consider a different distribution:

$$w_{1,1} = 12.5\%, \qquad w_{1,2} = 37.5\%,$$
$$w_{2,1} = 37.5\%, \qquad w_{2,2} = 12.5\%.$$

TABLE 7

		Urban	Suburban	Total
Youthful	Total Loss	\$3,000	\$6,000	\$9,000
	Exposures	5	15	20
	Distribution	12.5%	37.5%	50.0%
	Pure Premium	\$600	\$400	\$450
Adult	Total Loss	\$6,000	\$1,000	\$7,000
	Exposures	15	5	20
	Distribution	37.5%	12.5%	50.0%
	Pure Premium	\$400	\$200	\$350
Total	Total Loss	\$9,000	\$7,000	\$16,000
	Exposures	20	20	40
	Distribution	50.0%	50.0%	100.0%
	Pure Premium	\$450	\$350	\$400

This distribution is neither balanced nor proportional. The confounding effect of territory on class (and vice versa) becomes apparent. Table 7 displays that in this case for the age of the driver factor, we can see that the difference between the underlying rate for youthful drivers and adult drivers is: $\mu_{1,\bullet} - \mu_{2,\bullet} = \$200.00$, as before. However the aggregate rate difference is \$450 – \$350 = \$100.

3.3. The Confounding Effect for an n-Dimensional Additive Model with No Interaction Term

Now we want to extend the linearly additive model from two dimensions to n dimensions. Also we will extend the number of values for each variable to more than two, that is, m values. This is because a typical insurance rating structure has many variables with multiple values. It is understood that the lower bound of the summation is equal to unity. Again, assume that the sample distribution of each cell is $w_{i_1 i_2 i_3 i_4 \ldots}$.

Define:

$$w_{(i_1),i_2,i_3,\ldots,i_n} = \frac{w_{i_1,i_2,i_3,\ldots,i_n}}{\sum_{i_2}^{m_2}\sum_{i_3}^{m_3}\sum_{i_4}^{m_4}\cdots\sum_{i_n}^{m_n} w_{i_1,i_2,i_3,\ldots,i_n}}.$$

For a linearly additive model, the mean value for each of the $\prod_i^n m_i$ cells can be represented as follows:

$$\mu_{i_1 i_2 i_3 \ldots i_n} = \mu + \mu_{i_1,\bullet,\bullet,\ldots,\bullet} + \mu_{\bullet,i_2,\bullet,\ldots,\bullet} + \cdots + \mu_{\bullet,\bullet,\bullet,\ldots,i_n} :$$
$$i_1 = 1,2,3,\ldots,m_1, \quad i_2 = 1,2,3,\ldots,m_2,$$
$$i_3 = 1,2,3,\ldots,m_3, \quad i_n = 1,2,3,\ldots,m_n,$$

where a dot ($\bullet$) index indicates the mean across that index.

Again, we want to compare the difference in the aggregate rate and the underlying rate between any two values for the first factor, i_1.

Then the expected rate for each i_1 is

$$m_{i_1,\bullet,\bullet,\ldots,\bullet} = \sum_{i_2}^{m_2}\sum_{i_3}^{m_3}\sum_{i_4}^{m_4}\cdots\sum_{i_n}^{m_n} \mu_{i_1,i_2,i_3,\ldots,i_n} w_{(i_1),i_2,i_3,\ldots,i_n}$$

and

$$\begin{aligned} m_{i_1,\bullet,\bullet,\ldots,\bullet} - m_{1,\bullet,\bullet,\ldots,\bullet} &= \sum_{i_2}^{m_2}\sum_{i_3}^{m_3}\sum_{i_4}^{m_4}\cdots\sum_{i_n}^{m_n} \mu_{i_1,i_2,i_3,\ldots,i_n} w_{(i_1),i_2,i_3,\ldots,i_n} \\ &\quad - \sum_{i_2}^{m_2}\sum_{i_3}^{m_3}\sum_{i_4}^{m_4}\cdots\sum_{i_n}^{m_n} \mu_{1,i_2,i_3,\ldots,i_n} w_{(1),i_2,i_3,\ldots,i_n}. \end{aligned}$$

Then

$$\begin{aligned} &m_{i_1,\bullet,\bullet,\ldots,\bullet} - m_{1,\bullet,\bullet,\ldots,\bullet} \\ &\quad = \sum_{i_2}^{m_2}\sum_{i_3}^{m_3}\sum_{i_4}^{m_4}\cdots\sum_{i_n}^{m_n} w_{(i_1),i_2,i_3,\ldots,i_n} \\ &\qquad \times (\mu + \mu_{i_1,\bullet,\bullet,\ldots,\bullet} + \mu_{\bullet,i_2,\bullet,\ldots,\bullet} + \cdots + \mu_{\bullet,\bullet,\bullet,\ldots,i_n}) \\ &\qquad - \sum_{i_2}^{m_2}\sum_{i_3}^{m_3}\sum_{i_4}^{m_4}\cdots\sum_{i_n}^{m_n} w_{(1),i_2,i_3,\ldots,i_n} \\ &\qquad \times (\mu + \mu_{1,\bullet,\bullet,\ldots,\bullet} + \mu_{\bullet,i_2,\bullet,\ldots,\bullet} + \cdots + \mu_{\bullet,\bullet,\bullet,\ldots,i_n}). \end{aligned}$$

If $w_{(i_1),i_2,i_3,\ldots,i_n} = w_{(1),i_2,i_3,\ldots,i_n}$ then

$$
\begin{aligned}
m_{i_1,\bullet,\bullet,\ldots,\bullet} - m_{1,\bullet,\bullet,\ldots,\bullet}
&= \sum_{i_2}^{m_2}\sum_{i_3}^{m_3}\sum_{i_4}^{m_4}\cdots\sum_{i_n}^{m_n} w_{(1),i_2,i_3,\ldots,i_n} \\
&\quad \times (\mu + \mu_{i_1,\bullet,\bullet,\ldots,\bullet} + \mu_{\bullet,i_2,\bullet,\ldots,\bullet} + \cdots + \mu_{\bullet,\bullet,\bullet,\ldots,i_n}) \\
&\quad - \sum_{i_2}^{m_2}\sum_{i_3}^{m_3}\sum_{i_4}^{m_4}\cdots\sum_{i_n}^{m_n} w_{(1),i_2,i_3,\ldots,i_n} \\
&\quad \times (\mu + \mu_{1,\bullet,\bullet,\ldots,\bullet} + \mu_{\bullet,i_2,\bullet,\ldots,\bullet} + \cdots + \mu_{\bullet,\bullet,\bullet,\ldots,i_n}) \\
&= \sum_{i_2}^{m_2}\sum_{i_3}^{m_3}\sum_{i_4}^{m_4}\cdots\sum_{i_n}^{m_n} w_{(1),i_2,i_3,\ldots,i_n} \\
&\quad \times [(\mu + \mu_{i_1,\bullet,\bullet,\ldots,\bullet} + \mu_{\bullet,i_2,\bullet,\ldots,\bullet} + \cdots + \mu_{\bullet,\bullet,\bullet,\ldots,i_n}) \\
&\qquad - (\mu + \mu_{1,\bullet,\bullet,\ldots,\bullet} + \mu_{\bullet,i_2,\bullet,\ldots,\bullet} + \cdots + \mu_{\bullet,\bullet,\bullet,\ldots,i_n})] \\
&= \sum_{i_2}^{m_2}\sum_{i_3}^{m_3}\sum_{i_4}^{m_4}\cdots\sum_{i_n}^{m_n} w_{(1),i_2,i_3,\ldots,i_n}(\mu_{i_1,\bullet,\bullet,\ldots,\bullet} - \mu_{1,\bullet,\bullet,\ldots,\bullet}) \\
&= (\mu_{i_1,\bullet,\bullet,\ldots,\bullet} - \mu_{1,\bullet,\bullet,\ldots,\bullet})\sum_{i_2}^{m_2}\sum_{i_3}^{m_3}\sum_{i_4}^{m_4}\cdots\sum_{i_n}^{m_n} w_{(1),i_2,i_3,\ldots,i_n}.
\end{aligned}
$$

But

$$\sum_{i_2}^{m_2}\sum_{i_3}^{m_3}\sum_{i_4}^{m_4}\cdots\sum_{i_n}^{m_n} w_{(1),i_2,i_3,\ldots,i_n} = 1,$$

so $m_{i_1,\bullet,\bullet,\ldots,\bullet} - m_{1,\bullet,\bullet,\ldots,\bullet} = \mu_{i_1,\bullet,\bullet,\ldots,\bullet} - \mu_{1,\bullet,\bullet,\ldots,\bullet}$.

The *distribution* of the sample population is defined as proportional when:

$$
\begin{aligned}
w_{(i_1),i_2,i_3,\ldots,i_n} &= w_{(1),i_2,i_3,\ldots,i_n} && \text{for all} \quad i_1,\\
w_{i_1,(i_2),i_3,\ldots,i_n} &= w_{i_1,(1),i_3,\ldots,i_n} && \text{for all} \quad i_2,\\
w_{i_1,i_2,(i_3),\ldots,i_n} &= w_{i_1,i_2,(1),\ldots,i_n} && \text{for all} \quad i_3,\\
&\cdots\\
w_{i_1,i_2,i_3,\ldots,(i_n)} &= w_{i_1,i_2,i_3,\ldots,(1)} && \text{for all} \quad i_n.
\end{aligned}
$$

Confounding will not occur for the n-dimensional linearly additive model if the sample distribution is proportional.

3.4. The Confounding Effect on a Multiplicative Model with No Interaction Term

Let's start with a 2-by-2 multiplicative model without an interaction term. Assume that the sample distribution of each cell is $w_{i_1 i_2}$ as before.

Again define: $w_{(i_1),1} = w_{i_1,1}/\sum_{i_2} w_{i_1,i_2}$; e.g.,

$$
w_{(1),1} = \frac{w_{1,1}}{w_{1,1}+w_{1,2}}, \qquad w_{(1),2} = \frac{w_{1,2}}{w_{1,1}+w_{1,2}},
$$
$$
w_{(2),1} = \frac{w_{2,1}}{w_{2,1}+w_{2,2}}, \qquad w_{(2),2} = \frac{w_{2,2}}{w_{2,1}+w_{2,2}}.
$$

For a multiplicative model with no interaction term, the mean value for each of the 2-by-2 cells can be represented as follows:

$$
\mu_{i_1 i_2} = \mu \times \mu_{i_1,\bullet} \times \mu_{\bullet,i_2} : \qquad i_1 = 1,2, \quad i_2 = 1,2.
$$

More specifically, we define the following for the 2-by-2 age of driver and territory example:

$$
\mu_{i_1 i_2} = \mu \times \mu_{i_1,\bullet}(\text{Age of Driver}) \times \mu_{\bullet,i_2}(\text{Vehicle Territory}),
$$

where a dot ($\bullet$) index indicates the mean across that index.

Now we want to compare the difference in the aggregate rate and the underlying rate between adult and youthful drivers.

Then the expected rate for each i_1 is $m_{i_1,\bullet} = \mu_{i_1,1} w_{(i_1),1} + \mu_{i_1,2} w_{(i_1),2}$ and

$$\frac{m_{1,\bullet}}{m_{2,\bullet}} = \frac{\mu_{1,1} w_{(1),1} + \mu_{1,2} w_{(1),2}}{\mu_{2,1} w_{(2),1} + \mu_{2,2} w_{(2),2}}.$$

Then

$$\frac{m_{1,\bullet}}{m_{2,\bullet}} = \frac{w_{(1),1}(\mu\mu_{1,\bullet}\mu_{\bullet,1}) + w_{(1),2}(\mu\mu_{1,\bullet}\mu_{\bullet,2})}{w_{(2),1}(\mu\mu_{2,\bullet}\mu_{\bullet,1}) + w_{(2),2}(\mu\mu_{2,\bullet}\mu_{\bullet,2})}.$$

If $w_{(1),1} = w_{(2),1}$ and $w_{(1),2} = w_{(2),2}$, then

$$\frac{m_{1,\bullet}}{m_{2,\bullet}} = \frac{w_{(1),1}(\mu\mu_{1,\bullet}\mu_{\bullet,1}) + w_{(1),2}(\mu\mu_{1,\bullet}\mu_{\bullet,2})}{w_{(1),1}(\mu\mu_{2,\bullet}\mu_{\bullet,1}) + w_{(1),2}(\mu\mu_{2,\bullet}\mu_{\bullet,2})}$$

and

$$\frac{m_{1,\bullet}}{m_{2,\bullet}} = \frac{\mu_{1,\bullet}(w_{(1),1}\mu_{\bullet,1} + w_{(1),2}\mu_{\bullet,2})}{\mu_{2,\bullet}(w_{(1),1}\mu_{\bullet,1} + w_{(1),2}\mu_{\bullet,2})} = \frac{\mu_{1,\bullet}}{\mu_{2,\bullet}}.$$

Therefore, territory does not confound the age of driver relativities for this 2-by-2 multiplicative model if the territorial distribution of exposures is independent of the age of driver distribution of exposures. That is, if $w_{(1),1} = w_{(2),1}$, $w_{(1),2} = w_{(2),2}$, $w_{1,(1)} = w_{2,(1)}$ and $w_{1,(2)} = w_{2,(2)}$.

This occurs when the distributions among the predictive variables are independent and proportional to each other. Of course, a special case for such independent distributions is when each cell has the same number of data points; i.e., $w_{1,1} = w_{1,2} = w_{2,1} = w_{2,2}$. Again, this is a balanced distribution.

The following is a numerical example that illustrates such a multiplicative model. The statistics for the example are as follows:

$$\mu_{i_1 i_2} = \mu \times \mu_{i_1,\bullet}(\text{Age of Driver}) \times \mu_{\bullet,i_2}(\text{Vehicle Territory})$$

TABLE 8

		Urban	Suburban	Total
Youthful	Total Loss	\$3,750	\$3,750	\$7,500
	Exposures	5	15	20
	Distribution	12.5%	37.5%	50.0%
	Pure Premium	\$750	\$250	\$375
Adult	Total Loss	\$2,250	\$2,250	\$4,500
	Exposures	5	15	20
	Distribution	12.5%	37.5%	50.0%
	Pure Premium	\$450	\$150	\$225
Total	Total Loss	\$6,000	\$6,000	\$12,000
	Exposures	10	30	40
	Distribution	25.0%	75.0%	100.0%
	Pure Premium	\$600	\$200	\$300

Let $\mu = \$400$,

$$\mu_{1,\bullet} = 1.25 \text{ for youthful drivers}$$

$$\mu_{2,\bullet} = 0.75 \text{ for adult drivers,}$$

$$\mu_{\bullet,1} = 1.50 \text{ for urban drivers}$$

$$\mu_{\bullet,2} = 0.50 \text{ for suburban drivers.}$$

Therefore, the pure premiums for the four combinations are:

$$\mu_{1,1} = \$750, \qquad \mu_{1,2} = \$250,$$

$$\mu_{2,1} = \$450, \qquad \mu_{2,2} = \$150.$$

Also assume that $w_{1,1} = 12.5\%$, $w_{1,2} = 37.5\%$, $w_{2,1} = 12.5\%$, $w_{2,2} = 37.5\%$.

If we study Table 8 for the age of the driver factor, we can see that the underlying rate for the difference between youthful drivers and adult drivers is $\mu_{1,\bullet}/\mu_{2,\bullet} = 1.25/0.75 = 1.67$, which is the same as if we use the aggregate rate, $\$375/\$225 = 1.67$. Therefore, in this case, confounding does not occur.

TABLE 9

		Urban	Suburban	Total
Youthful	Total Loss	\$3,750	\$3,750	\$7,500
	Exposures	5	15	20
	Distribution	12.5%	37.5%	50.0%
	Pure Premium	\$750	\$250	\$375
Adult	Total Loss	\$6,750	\$750	\$5,000
	Exposures	15	5	20
	Distribution	37.5%	12.5%	50.0%
	Pure Premium	\$450	\$150	\$250
Total	Total Loss	\$10,500	\$4,500	\$15,000
	Exposures	20	20	40
	Distribution	50.0%	50.0%	100.0%
	Pure Premium	\$525	\$225	\$375

Now assume a different distribution:

$$w_{1,1} = 12.5\%, \qquad w_{1,2} = 37.5\%,$$
$$w_{2,1} = 37.5\%, \qquad w_{2,2} = 12.5\%.$$

This distribution is neither balanced nor proportional, and the confounding effect of territory on class (and vice versa) is again obvious. Table 9 shows that in this case for the age of the driver factor, the relationship between the underlying rates for youthful drivers and the adult drivers is $\mu_{1,\bullet}/\mu_{2,\bullet} = 1.25/0.75 = 1.67$, as before. However the aggregate rate is biased; \$375/\$250 = 1.50.

3.5. The Confounding Effect on an n-Dimensional Multiplicative Model with No Interaction Term

Now, we want to extend the multiplicative model from two dimensions to n dimensions. In addition, for each variable, we will extend the number of values to more than two, that is, m values. Again, assume that the sample distribution of each cell is $w_{i_1,i_2,i_3,i_4,\ldots}$.

Define:

$$w_{(i_1),i_2,i_3,\ldots,i_n} = \frac{w_{i_1,i_2,i_3,\ldots,i_n}}{\sum_{i_2}^{m_2}\sum_{i_3}^{m_3}\sum_{i_4}^{m_4}\cdots\sum_{i_n}^{m_n} w_{i_1,i_2,i_3,\ldots,i_n}}.$$

For a multiplicative model, the mean value for each of the $\prod_1^n m_i$ cells can be represented as follows:

$$\mu_{i_1,i_2,i_3\ldots i_n} = \mu\mu_{i_1,\bullet,\bullet,\ldots,\bullet}\mu_{\bullet,i_2,\bullet,\ldots,\bullet}\cdots\mu_{\bullet,\bullet,\bullet,\ldots,i_n}:$$

$$i_1 = 1,2,3,\ldots,m_1, \qquad i_2 = 1,2,3,\ldots,m_2,$$

$$i_3 = 1,2,3,\ldots,m_3, \ \ldots\ , i_n = 1,2,3,\ldots,m_n,$$

where a dot index indicates the mean across that index. Again we want to compare the difference in the aggregate rate and the underlying rate between any two values for the first factor, i_1.

Then the expected rate for each i_1 is

$$m_{i_1,\bullet,\bullet,\ldots,\bullet} = \sum_{i_2}^{m_2}\sum_{i_3}^{m_3}\sum_{i_4}^{m_4}\cdots\sum_{i_n}^{m_n}\mu_{i_1,i_2,i_3,\ldots,i_n}w_{(i_1),i_2,i_3,\ldots,i_n}$$

and

$$\frac{m_{i_1,\bullet,\bullet,\ldots,\bullet}}{m_{1,\bullet,\bullet,\ldots,\bullet}} = \frac{\sum_{i_2}^{m_2}\sum_{i_3}^{m_3}\sum_{i_4}^{m_4}\cdots\sum_{i_n}^{m_n}\mu_{i_1,i_2,i_3,\ldots,i_n}w_{(i_1),i_2,i_3,\ldots,i_n}}{\sum_{i_2}^{m_2}\sum_{i_3}^{m_3}\sum_{i_4}^{m_4}\cdots\sum_{i_n}^{m_n}\mu_{1,i_2,i_3,\ldots,i_n}w_{(1),i_2,i_3,\ldots,i_n}}.$$

Then

$$\frac{m_{i_1,\bullet,\bullet,\ldots,\bullet}}{m_{1,\bullet,\bullet,\ldots,\bullet}}$$

$$= \frac{\sum_{i_2}^{m_2}\sum_{i_3}^{m_3}\sum_{i_4}^{m_4}\cdots\sum_{i_n}^{m_n} w_{(i_1),i_2,i_3,\ldots,i_n}(\mu\mu_{i_1,\bullet,\bullet,\ldots,\bullet}\mu_{\bullet,i_2,\bullet,\ldots,\bullet}\cdots\mu_{\bullet,\bullet,\bullet,\ldots,i_n})}{\sum_{i_2}^{m_2}\sum_{i_3}^{m_3}\sum_{i_4}^{m_4}\cdots\sum_{i_n}^{m_n} w_{(1),i_2,i_3,\ldots,i_n}(\mu\mu_{1,\bullet,\bullet,\ldots,\bullet} + \mu_{\bullet,i_2,\bullet,\ldots,\bullet}\cdots\mu_{\bullet,\bullet,\bullet,\ldots,i_n})}.$$

If $w_{(i_1),i_2,i_3,\ldots,i_n} = w_{(1),i_2,i_3,\ldots,i_n}$ for all i_1 then

$$\frac{m_{i_1,\bullet,\bullet,\ldots,\bullet}}{m_{1,\bullet,\bullet,\ldots,\bullet}}$$

$$= \frac{\sum_{i_2}^{m_2}\sum_{i_3}^{m_3}\sum_{i_4}^{m_4}\cdots\sum_{i_n}^{m_n} w_{(1),i_2,i_3,\ldots,i_n}(\mu\mu_{i_1,\bullet,\bullet,\ldots,\bullet}\mu_{\bullet,i_2,\bullet,\ldots,\bullet}\cdots\mu_{\bullet,\bullet,\bullet,\ldots,i_n})}{\sum_{i_2}^{m_2}\sum_{i_3}^{m_3}\sum_{i_4}^{m_4}\cdots\sum_{i_n}^{m_n} w_{(1),i_2,i_3,\ldots,i_n}(\mu\mu_{1,\bullet,\bullet,\ldots,\bullet}\mu_{\bullet,i_2,\bullet,\ldots,\bullet}\cdots\mu_{\bullet,\bullet,\bullet,\ldots,i_n})}$$

$$= \frac{(\mu+\mu_{\bullet,i_2,\bullet,\ldots,\bullet}+\cdots+\mu_{\bullet,\bullet,\bullet,\ldots,i_n})\sum_{i_2}^{m_2}\sum_{i_3}^{m_3}\sum_{i_4}^{m_4}\cdots\sum_{i_n}^{m_n} w_{(1),i_2,i_3,\ldots,i_n}\mu_{i_1,\bullet,\bullet,\ldots,\bullet}}{(\mu+\mu_{\bullet,i_2,\bullet,\ldots,\bullet}+\cdots+\mu_{\bullet,\bullet,\bullet,\ldots,i_n})\sum_{i_2}^{m_2}\sum_{i_3}^{m_3}\sum_{i_4}^{m_4}\cdots\sum_{i_n}^{m_n} w_{(1),i_2,i_3,\ldots,i_n}\mu_{1,\bullet,\bullet,\ldots,\bullet}}$$

$$= \frac{\mu_{i_1,\bullet,\bullet,\ldots,\bullet}}{\mu_{1,\bullet,\bullet,\ldots,\bullet}}.$$

Confounding will not occur for the n-dimensional multiplicative model if the sample distribution meets the above independence or proportionality conditions.

4. TYPES OF CONFOUNDING VARIABLES

A variable that confounds the results of a study does so in essentially the same way regardless of the nature of the variable under study or the confounding variable itself. However, the nature of the variable may affect its identification and treatment. For the purpose of this paper, confounding variables will be categorized as one of three types: stratification confounding variable, aggregation confounding variable or lurking confounding variable.

4.1. Stratification Confounding Variable

In order to properly price a pool of risks, it may be necessary to stratify those risks into smaller, more homogeneous groups. Often a structure is stratified using more than one criterion. An example that has already been discussed is personal automobile, which is usually rated by territory and by classification. Each of these rating variables is customarily analyzed separately and rating factors developed reflecting past loss experience. If territory is independent of classification, then the rates developed will be appropriate. If the distribution by classification varies between territories (that is classification is not independent of territory), then such a simple analysis will yield biased rates. For example, if there is a disproportionately high number of youthful operators in a particular territory and youthful operators are underpriced, a univariate analysis of each rating variable will yield rates that are too high for the youthful drivers in that territory. If the rating variable under analysis is territory, then classification is a potential stratification confounding variable.

4.2. Aggregation Confounding Variable

> "It's a well accepted rule of thumb that the larger the data set, the more reliable the conclusions drawn. Simpson' [sic] paradox, however, slams a hammer down on the rule and the result is a good deal worse than a sore thumb. Unfortunately, Simpson's paradox demonstrates that a great deal of care has to be taken when combining small data sets into a large one. Sometimes conclusions from the large data set are exactly the opposite of conclusion from the smaller sets. Unfortunately, the conclusions from the large set are also usually wrong." [6]

In order to stratify data into smaller and more homogeneous classes, actuaries gather data from as many sources as possible. Adding states, companies and years of experience are three ways that an actuary may maintain class homogeneity while increasing class size (and thus credibility). If the variable along which data is aggregated is correlated with one or more rating variables, then that variable may confound the results of any analysis of those rating variables. For example, assume that state B's loss experience is to be combined with state A's loss experience to increase the volume of data available for a class relativity analysis. Also assume that state B has a higher proportion of youthful operators as well as worse loss experience overall. While an analysis of each state's youthful operator experience alone might yield the same appropriate relativity, when combined the analysis will produce an indicated youthful operator relativity that is inappropriately high.

Exhibit 1 [15] illustrates the effect of two aggregation confounding variables. In this scenario both loss ratio and exposure distribution by class are related to both year and state. The loss experience displayed in Exhibit 1 (second page) arises from the required factors of 1.00 for class 01 and 2.10 for class 02. The derivation of the indicated class 02 relativity is displayed on the first page of Exhibit 1. The indicated relativities are 2.30 using the loss ratio method, 2.68 using the pure premium method and 2.30 using the modified loss ratio method. Although each of the indicated relativities are biased, the pure premium method is more susceptible to bias than either of the other two methods. Aggregation confounding variables (though not identified as such) were discussed at length by Stenmark [15]. The example of aggregation confounding variables given in Exhibit 1 will be discussed further in Section 5.5.

4.3. Lurking Confounding Variable

As displayed in the introduction to this paper, it is possible that a confounding variable may not be under examination. While many references use the terms lurking variable and confounding variable interchangeably, a more narrow definition of lurking confounding variable is being used here. A lurking confounding variable, then, is a variable that has not yet been uncovered as a stratification confounding variable or an aggregation confounding variable. A lurking confounding variable may exist outside of an actuary's ratemaking data, possibly to be detected using one of the many data mining techniques available. A lurking confounding variable may be a data element that is available only through demographic data, not captured through a company's processing system. Most discouraging of all, the piece of information may not exist anywhere as data.

Insurance companies have been collecting more and more information, and underwriters and actuaries have become sensitive to criteria that might affect the loss process. Hopefully, then, there are not too many undiscovered confounding variables lurking in our data that will significantly distort our rates. Regardless, one only needs to point at the use of credit scores to recognize an important lurking confounding variable that has only recently been utilized to its full potential.

There are two issues relative to the discussion of confounding in previously unused rating variables. First, prior to its use as a rating variable, the failure to segment insureds according to any credit measure may have caused confounding of those rating variables actually in use. For example, assume that a certain class of insureds often displays a poor credit rating and, as a result, that class manifests poor loss experience. The rates for insureds in that class with a better credit score would be inappropriately high.

Second, once credit score has been established as a rating variable, proper methods must be undertaken to prevent the continued confounding of the class rates through the use of one of the treatments described in the next section. For example, a company that provides a discount in automobile for insureds with a homeowner's policy might find that, after introducing a discount for good credit, the rates for automobile risks with an accompanying homeowner's policy are too low. This challenge is discussed at length by Guszcza and Wu [16].

5. TREATMENT OF POTENTIAL CONFOUNDING VARIABLES

We have presented the empirical and theoretical evidence for the existence of Simpson's paradox and confounding variables. In this section, we present several alternatives for the treatment of this phenomenon.

5.1. No Treatment

Pearl [12] concludes that there is no test for confounding. Much of Pearl's writing concerns the principle of causality [11], presumably because confounding is of such great concern in medical research and, in that research, causality is of prime importance. Since, in insurance, we are more concerned with statistical correlation than causality we allow a more liberal test for confounding. Therefore, we say that if a variable is unrelated to the variable under study or to the loss measure, then confounding will not result and no treatment is necessary. However, it is ill-advised for an actuary to assume that there is no confounding without extensive examination of the relationship of all the variables affecting the loss process.

5.2. Controlling Confounding Through Experiment Design

As discussed in Section 3.1, if we can carefully design an analysis and then collect the data accordingly, then we can control confounding. Whether we have prior knowledge of the relation between the potential confounding variable and the target information or not, we can control its effect if the confounding variable is unrelated to the variable(s) under study. This can be achieved through completely balanced design or proportional design of the experiment. That is, for each combination of the confounding variable and the variable(s) of interest, the same or proportional amount of data is collected. This concept is commonly used in many research areas such as medical, engineering, and scientific research projects.

When an actuary analyzes insurance data, the actuary typically cannot "design" the analysis. The actuary has to analyze whatever he or she is given. The data are mostly determined by the company's book of business, which is largely determined by the market segments that the company serves. Moreover, since there are multiple rating variables, and for each rating variable there exist many different values, it is possible that many combinations of the variables will have missing or very little data. In other words, insurance data is highly non-ideal for the confounding issue, and it is difficult, if not impossible, for us to control the bias through the experimental design approach.

5.3. Controlling Confounding Through Multivariate Analysis

If the insurance data is highly non-ideal and we cannot control confounding through standard experimental design, we can control it by using multivariate analysis. That is, we can perform the Bailey's [1] minimum bias analysis or GLM analysis [4, 8] by including the confounding variable along with the variable(s) of interest. By doing so, the confounding variable's relation with the target variable and the variable(s) in interest will be properly determined and be allocated through the multivariate approach. Therefore, the true relationship of the variable(s) under study can be revealed.

While multivariate analysis may be an ideal solution to deal with the confounding issue, there may exist practical issues against using the approach within insurance applications. One issue is that insurance applications constantly aggregate or stratify data with regards to states, years, and companies. In theory, we can include these potential confounding variables in the analysis, but the inclusion of these non-rating variables in the multivariate analysis may lead to other issues such as credibility of the data for analysis and reasonability and interpretation of the analysis result for the variables. Therefore, later we propose an alternative approach, using scaling factors, for actuaries to consider for addressing confounding. The alternative approach will be discussed in detail in Section 5.5.

5.4. Controlling Confounding Through the Use of Meta-Analysis

Researchers are often faced with situations that compel the use of data from several studies. In insurance we strive to increase the volume of our data to increase credibility, and medical researchers attempt to do the same through compilations of more than one study called meta-analyses [5].

A research study typically includes observations from two groups: an intervention group (N_i) and a control group (N_c). From these observations four pieces of data are derived: an intervention with an event (n_i), intervention without an event ($N_i - n_i$), control with an event (n_c) and control without an event ($N_c - n_c$). From these a statistic, generally called a "size effect," is calculated. The two size effects in general use are termed the "risk difference" and the "odds ratio." The risk difference is the difference between the ratio of the number of interventions with an event to the total observations of all interventions and the ratio of the number of control subjects with an event to the total observations of the control group. Risk difference = $(n_i/N_i) - (n_c/N_c)$. The reciprocal of the risk difference is termed the "number needed to treat" (or "harm") and represents the number of interventions required to achieve one event. The "odds ratio" is the ratio of the ratio of the number of interventions with an event to the number of interventions without an event to the ratio of the number of control subjects with an event to the number of control subjects without an event. Odds ratio = $(n_i/(N_i - n_i)) \div (n_c/(N_c - n_c))$.

If an analyst naively combines all of the observations, confounding can result and lead to biased findings because there is a different distribution of observations between studies. For example, in Cates [5] seven out of ten studies resulted in a positive number needed to treat, and the three that did not represented only 839 of the 6,121 observations in the meta-analysis. Regardless, combining the raw data produced a number needed to harm in contrast to the number needed to treat in the majority of studies.

A discipline has risen centered around the optimum method to be used to combine such studies. In general, methodologies focus on calculating a variance for each study. The reciprocal of this variance is used to weight the size effects themselves rather than the raw observations.

This treatment is analogous to calculating class relativities for each year and state and weighting those relativities to arrive at a composite relativity for each class. As such, it has some similarities to credibility weighting. However, one major difference between typical medical research and insurance ratemaking is that medical research results are binary outputs and insurance ratemaking results are relativities or rates on a continuous scale. Therefore, although meta-analysis provides an interesting example of the effect and treatment of confounding in medical research, it does not appear to have any direct application to insurance pricing.

5.5. Controlling Confounding Through the Use of Scaling Factors

In this section, we introduce a practical approach, called "scaling factors," to treat the confounding effect that may commonly exist in insurance rating applications. We believe that this approach was first proposed by Stenmark in his 1990 paper [15], and we are revisiting the approach from the perspective of confounding variables and Simpson's paradox. This approach is important because there are some confounding variables that are not optimally addressed using any of the treatments mentioned above.

It is not usually desirable or practical to include a multivariate analysis of most aggregation confounding variables as described in Section 4.2, since if data from several states are included, a multiplier by state is probably not a necessary rating model output. This is because each state's overall rate change requirements will be calculated through a statewide indication, possibly at some indeterminate time in the future. In addition, a multiplier for each experience year has no direct application or interpretation. Regardless, recognition of such variables in multivariate analysis, through the use of dummy variables, is an accepted and effective practice as will be discussed in Section 5.6. An alternative to that approach will be discussed in this section.

Is there a way that data from several experience years and several states can be aggregated to increase data volume without possibly confounding the results of the study and without the necessity of inclusion of the confounding variable in the analysis?

As stated previously, there are two conditions necessary for a variable to confound the results of an analysis:

1. There must be a relationship between that variable and the experience variable.
2. There must be a relationship between that variable and at least one of the rating variables under analysis.

If either of those two conditions is not met then there is no confounding of the results.

This leads to the question: if both conditions are met, can the data be modified so that one of the conditions is no longer met, eliminating the confounding? This must be done in such a manner that the important underlying relationships in the data are not disturbed. In the following sections, we will show the scaling factors approach using a class plan analysis example with two potential confounding variables—states and years.

5.5.1. The Loss Experience Model

To eliminate the confounding effect, it is first necessary to quantify that effect on a classification loss model. The model need not be complex and is composed, at the atomic level, of exposures, base rate, current rating factors, required rating factors and base class loss ratio. Appendix A outlines this model and the quantification of the impact of confounding. For example, the total earned premiums for class i on present rates $= P_i = \sum_y \sum_s e_{iys} B_{ys} c_i$ and the total incurred losses for class $i = L_i = \sum_y \sum_s e_{iys} r_{ys} B_{ys} f_i$. The notation introduced in Appendix A will be used throughout the remainder of Section 5.

The impact on indicated class relativities due to the confounding effect of aggregation of experience by year and by state is displayed for three classification ratemaking methods: the loss ratio method, the pure premium method and the modified loss ratio method. The modified loss ratio method bears some description. The premiums are calculated at base class rates so that the output of the method is the class relativity, not the indicated change to that relativity.

In addition to the three methods presented there is another subtle variation in methodology. It is possible to develop each class relativity as a ratio of the selected statistic (e.g., loss ratio) to that of a base class (special case) or to the statistic of the all-class experience (general case). The words "special" or "general" are used to identify each method. For example, in Personal Automobile Insurance it is common to divide the class loss ratio by the loss ratio for adult driver (pleasure use). This is the special case. It is not always the case that the base class has a large portion of the business, so the all-class loss ratio may provide a more stable base. This is the general case. The class relativities can be normalized back to the base class after the indicated relativities have been credibility weighted and selections have been made from those credibility-weighted relativities. The model introduced by Stenmark [15] was for the special case only. Including the general case adds further flexibility.

The bias produced by confounding is derived in Appendix A and is reproduced below:

Bias arising from confounding using pure premium method (special case):

$$\frac{g_i}{f_i} - 1 = \frac{\sum_y \sum_s e_{iys} r_{ys} B_{ys}}{\sum_y \sum_s e_{iys}} \bullet \frac{\sum_y \sum_s e_{bys}}{\sum_y \sum_s e_{bys} r_{ys} B_{ys} f_i} - 1$$

$$= \frac{\sum_y \sum_s e_{iys} r_{ys} B_{ys}}{E_i} \bullet \frac{E_b}{L_b} - 1. \tag{5.1}$$

Bias arising from confounding using loss ratio method (special case):

$$\begin{aligned}\frac{g_i}{f_i}-1&=\frac{\sum_y\sum_s e_{iys}r_{ys}B_{ys}}{\sum_y\sum_s e_{iys}B_{ys}c_i}\bullet\frac{\sum_y\sum_s e_{bys}B_{ys}}{\sum_y\sum_s e_{bys}r_{ys}B_{ys}}-1\\&=\frac{\sum_y\sum_s e_{iys}r_{ys}B_{ys}}{P_i}\bullet\frac{P_b}{L_b}-1.\end{aligned}\tag{5.2}$$

Bias arising from confounding using modified loss ratio method (special case)

$$\begin{aligned}\frac{g_i}{f_i}-1&=\frac{\sum_y\sum_s e_{iys}r_{ys}B_{ys}}{\sum_y\sum_s e_{iys}B_{ys}c_b}\bullet\frac{\sum_y\sum_s e_{bys}B_{ys}}{\sum_y\sum_s e_{bys}r_{ys}B_{ys}}-1\\&=\frac{\sum_y\sum_s e_{iys}r_{ys}B_{ys}}{\sum_y\sum_s e_{iys}B_{ys}c_b}\bullet\frac{P_b}{L_b}-1.\end{aligned}\tag{5.3}$$

Each of the above utilizes the base class experience as the base. If the relativity is calculated utilizing, instead, the all-class experience (general case), then the bias for the modified loss ratio method is shown in Equation (5.4).

$$\frac{g_i}{f_i}-1=\frac{\sum_y\sum_s e_{iys}r_{ys}B_{ys}}{\sum_y\sum_s e_{iys}B_{ys}c_b}\bullet\frac{\sum_y\sum_s e_{bys}B_{ys}}{\sum_y\sum_s e_{bys}r_{ys}B_{ys}}-1.\tag{5.4}$$

5.5.2. Derivation of the Scaling Factor

Is it possible to scale the premiums or losses (or both) in such a manner that the bias is removed when the data from one state and/or year are combined with that of another state or year? What characteristics should such a scaling factor possess? Two criteria must be met by any scaling factor candidate:

Criterion 1: The scaling factors should maintain the relationship between class loss ratios by year and state (the scaling factors should not change the underlying relativities).

Criterion 2: The scaling factors should reduce the bias due to confounding to zero.

Any scaling factor that is applied uniformly to each class within a specific state for a particular year *or* is applied to both premiums and losses for a specific class will fulfill the requirements of Criterion 1. When either the exposure distribution or the base class loss ratio remains constant, the distortion is not present and any scaling factor that stabilizes either the base or total class loss ratio (in the special case or general case, respectively), or the exposure distribution should fulfill the requirements of Criterion 2.

A clue as to how to approach the derivation of a scaling factor was discussed in the section on experiment design. If the experience is balanced or there is no relationship between the experience and the confounding variable, then confounding does not occur. If a scaling factor candidate can promote either characteristic, then confounding should be mitigated.

Appendix B displays the evaluation of four types of scaling factors that meet the needs of both criteria. These scaling factors can be broken into two categories. One category applies to the special case and the other applies to the general case. Each category has one scaling factor that is used to address the non-independence of the confounding variable and the loss statistic (loss ratio, pure premium, etc.). The other scaling factor addresses the non-independence of the confounding variable and the rating element(s) under study (balance). *Only one type of scaling factor need be used in a rate analysis.* The type of factor to use is the choice of the actuary.

Please note that these factors were arrived at by inspection. This was not a trivial process, but the authors believe that a mathematical derivation of the factor is not possible. The factors are tested within Appendix B to display that the bias from confounding is eliminated.

The first scaling factor that is considered is the reciprocal of the base class loss ratio for each state and year. By applying this factor uniformly to the losses for each class, the relationship between each of the class loss ratios is maintained (Criterion 1) while the method error is reduced to zero (Criterion 2). This is shown in Appendix B.

The example given in Exhibits 1–5 is used to examine the scaling factors. Exhibit 2 displays the effect of scaling losses with the reciprocal of the base class loss ratio. Both Exhibits 2 and 3 utilize input parameters that were set forth in Exhibit 1. The modified loss ratio method is utilized in Exhibit 2. The premium is modified to the base class rate level by dividing by the class factor prior to calculating the loss ratio. For each class, the losses are scaled by the base class adjusted loss ratio for that year and state. For example, the incurred losses for state 01, year 1 ($500,000) are multiplied by the reciprocal of class 01 loss ratio (1.00/0.50 = 2.00) to yield the scaled losses of $1,000,000. The class 02 incurred losses ($525,000) are also multiplied by this factor to yield the scaled losses for that class of $1,050,000. These scaled losses maintain the relationship between the class loss ratios, but lose any information regarding the actual base class loss ratio. It is possible to apply a scaling factor (the base class loss ratio in this case rather than its reciprocal) to the premium rather than the losses. This method should be used only for larger, more stable lines of business. In cases where even the base class loss ratio can fluctuate wildly, it is more appropriate to scale the losses. The reason is that scaled losses are equal, in total, to premium. If the scaling factor were applied to premium, the result would be equal to (the more volatile) losses.

The second scaling factor derived in Appendix B addresses the different exposure distribution by year and state. The ratio of the total exposures for each class to the total exposures for the base class is multiplied by the ratio of the base class exposures in each state and year to the class exposures in each state and year to provide the scaling factor (algebraically, $S_{iys} = (\sum_y \sum_s e_{iys} / \sum_y \sum_s e_{bys}) \bullet (e_{bys}/e_{iys})$). As opposed to the first scaling factor, the second scaling factor is unique for each class, year and state. However, since the factor is applied to both premiums and losses, this scaling factor also satisfies the requirements of Criterion 1. When e'_{iys} replaces e_{iys} in the equation for the error developed in Appendix A, error is reduced to zero, thus satisfying the requirements of Criterion 2. Exhibit 3 displays the effect of utilizing the second scaling factor.

The third scaling factor is for the general case and it addresses the non-independence of the confounding variable and the loss experience, as did Scaling Factor 1. Appendix B displays the derivation of this factor. The reciprocal of the loss ratio for the state and year is shown to eliminate the bias in the loss experience.

The fourth scaling factor is similarly tested in Appendix B. This factor is applied in the general case and addresses balance. As displayed in the appendix this scaling factor is $S_{iys} = (\sum_y \sum_s e_{iys} / \sum_i \sum_y \sum_s e_{iys}) \bullet (\sum_i e_{iys} / e_{iys})$.

The advantages of Scaling Factors 1 and 3 are:

1. Ease of use: The base class and statewide loss ratios are directly obtainable from the data already necessary for the modified loss ratio method.

2. Since the scaling factor is applied uniformly for each class, the premium distribution by class for each year and state is left unaltered.

3. Many of the traditional adjustments to premium and loss data are no longer necessary. Any adjustment that applies uniformly to the premiums or losses of all classes is nullified by the application of that scaling factor. These adjustments would include present level adjustments for overall rate changes, development factors and trend factors. If, however, an adjustment is not applied uniformly by class, it will still be necessary. For example, if trend factors are applied by cause of loss, these factors will need to be applied prior to the scaling process.

TABLE 10

RESULT OF MINIMUM BIAS
USING DUMMY VARIABLES FOR STATE AND YEAR

Raw Output			
	1	2	Number of Iterations
State	1.1544	1.3853	11
Year	0.7432	1.1148	
Class	0.5828	1.2238	
Normalized			
	1	2	
State	0.5000	1.8900	
Year	1.0000	1.5000	
Class	1.0000	2.1000	

The advantage of Scaling Factors 2 and 4 is that if the exposure distribution is more stable than the loss ratios from year to year, then Scaling Factors 2 and 4 will result in less abrupt adjustments for most classes than will Scaling Factors 1 and 3.

5.6. Comparison of Multivariate Analysis vs. Scaling

It is common practice to include dummy variables for potential aggregation confounding variables in a multivariate analysis. Inclusion of a dummy variable for both year and state, for example, would allow the non-independence of those variables with the dependent variable (e.g., loss ratio) to be reflected in the dummy variables. Does this methodology compensate for the confounding observed previously? If it does, is this method more or less effective than the use of one of the scaling factors discussed in the previous section? Table 10 displays the results of such a computation. The resulting factors for States 1 and 2, Years 1 and 2 and Classes 01 and 02 are shown. In eleven iterations the minimum bias equations converged to the raw output displayed in Table 10.

The raw output was then normalized to base class (year and class) and the state factors were adjusted to correct for the normalization. The normalized class 02 factor is equal to the correct value, 2.10. It appears that both the scaling factors discussed in Section 5.5 and the multivariate analysis discussed above yield the correct factor in this deterministic scenario.

The world in which we live is hardly deterministic. It is necessary to test each method in a stochastic model. The deterministic model was used to parameterize such a model. Separate frequency and severity averages were derived assuming a frequency of 0.01 adjusted by the class and year loss ratio. The state loss ratio was reflected in the severity. The frequency distribution was assumed to be Poisson and the severity distribution was assumed to be Lognormal. Exhibit 4 displays the model output. One thousand iterations were simulated. Within each iteration for each exposure for each year, state, and class a number of claims was derived from the Poisson. For each of these claims, a claim size was determined from the Lognormal distribution. The loss ratio for each year, state, and class was determined and from these the Class 02 relativity was derived using the univariate (traditional) method, each of the four scaling factor methods, as well as Bailey's minimum bias. The authors acknowledge that the use of a linear model based on the Lognormal might have been more appropriate.

The values that emerged from the deterministic model are displayed as the expected values. Below these are the average values from all one thousand iterations. Finally, the next row displays the mean square error (MSE) for each column. The value used to calculate this error for the univariate method was the correct class relativity rather than the relativity emerging from the deterministic model (i.e., 2.10 rather than 2.2958).

The presence or absence of a loss limit might affect the sensitivity of each method to variability in losses. Therefore, the model was repeated, but this time losses were limited to $25,000. Of course, the Lognormal parameters had to be adjusted upward to compensate for the excluded losses at the top end of the distribution.

The mean square error for the univariate method was somewhat higher than that for the other methods with or without the loss limitation. This was expected since the method possessed a relatively large bias in the first place. On the other hand, there was no significant difference between the errors for Bailey's minimum bias and the four scaling methods. It appears that while use of an iterative bias reducing methodology does, in fact, reduce bias, so do each of the scaling factors described earlier in this paper.

6. CONFOUNDING VARIABLES AND CURRENT ACTUARIAL PRACTICE

6.1. Areas Where Confounding Variables Have Been Recognized

Bailey and Simon [2] first recognized the potential for bias from confounding in 1960, though they did not identify it as such. Are there other areas where actuaries have recognized this bias and compensated for it?

One answer is in the trending process that actuaries frequently employ in their rating and reserving applications to adjust premium and loss data. It is customary when preparing a rate indication to trend losses to recognize the increase in severity and changing frequency. It is also necessary to trend premium to recognize that some loss trend is from factors that will increase the premium over time. These inflation and coverage-sensitive rating factors confound the loss trends necessitating an adjustment. Since deductible, for example, is both the trend measure, pure premium, or frequency and severity, as well as time (deductibles tend to increase over time), deductible is a confounding variable for trend data. Other confounding variables for trend might be symbol, model year, limit of liability, and amount of insurance, to name just a few.

6.2. Areas Where Confounding May Be an Unrecognized Problem

Confounding is a frequent and serious problem in ratemaking. Obviously, almost all the rating variables can confound each other because their distributions are hardly independent. As discussed above, the premium and loss on-leveling and trending is a process that actuaries employ to deal with such confounding to the best we can. However, the process may not be able to remove all the potential confounding relationships between the variables.

Moreover, there are other potential confounding variables that exist outside the rating variables that may not be fully recognized and explored, i.e., lurking confounding variables. The following are a few examples, some of which have been discussed previously:

• *Geographic Information*: While a rating plan may include geographic rating variables, such as state and territory, these variables may not be enough to fully explain the confounding relationship in the rating data. The real underlying drivers for such geographic factors include the underlying demographic, consumer, economic, traffic, and weather information. This information includes, but is not limited to, information such as education, employment, credit, lifestyle, consumer spending, traffic volume, crime, cold, heat, hail, storms, etc. Especially for commercial lines of business, such geographic information is usually under-represented in the rating process.

• *Market Segment*: The distribution of rating variables is significantly influenced by market segments. For example, a non-standard book of business might be expected to have a much higher distribution of younger drivers, more risks with prior claims and violations, and insurance with lower coverages. Therefore, it might be prudent to aggregate or stratify data along different market segments. In many instances, companies or tiering will be used to separate different market segments. It is highly likely that classification experience will be confounded by rating tier or company. Variables used in company placement or tiers typically include both rating variables and non-rating variables. Company or rating tier can be used as a variable with classification in a linear model, or the experience should be treated with one of scaling factors introduced in Section 5.5.

• *Distribution Channels*: Our experience indicates that distribution channels will also affect the composition of and the information gathered for a book of business. This issue has become even more significant as many companies are marketing online in addition to the two traditional channels of direct writers and independent agents. We have found that business flowing through different channels may be of very different quality and contain differing amounts of information.

• *External Environment*: The insurance industry is not operating within an isolated world, and its performance is a part of the increasingly more integrated national or even worldwide economies. Therefore, in this fast-changing world, issues such as technological development, economic cycles, and recent terrorist activity will affect the insurance industry. The current hard market condition is clear evidence of how the insurance underwriting cycle is influenced by the external world. Therefore, combining multiple years with possible year-to-year changes and insurance cycles requires special care. Additional care must be rendered when projecting historical information into the future.

7. CONCLUSIONS

E. H. Simpson introduced the concept now known as Simpson's paradox. It is the extreme case of a phenomenon known as confounding. While such extreme cases may not occur frequently in actuarial calculations, the change in relationship due to confounding does.

A variable can confound the results of an insurance rate structure analysis only if it is related (non-independent) to both the experience measure (loss ratio, pure premium, etc.) and at least one of the other rating variables in the analysis. Confounding variables can be categorized as either a stratification confounding variable, an aggregation confounding variable, or a lurking confounding variable.

Several methods for the treatment of confounding were discussed including no treatment, experimental design, multivariate analysis, meta-analysis, and use of scaling factors.

The combination of data from more than one year may cause distortion in traditional classification ratemaking techniques if each set of data represents a different base rate adequacy and different exposure distribution by class. The combination of data from more than one state may cause distortion in the traditional pure premium method if the base rate from each state is different and possesses different exposure distributions by class. The combination of data from more than one state may cause distortion in both of the traditional methods if the base rate from each state is different, the base class loss ratio is different, and the state/year data exhibit a different exposure distribution by class. It is more than likely that these conditions will exist within most sets of ratemaking data. These distortions may be remedied by the application of a scaling factor to the data from each year and each state. This scaling factor may address either the exposure distribution or the base rate adequacy. An investigation of the effectiveness of multivariate analysis in comparison with the use of scaling factors reveals that both methodologies reduce the effect of confounding, probably to the same degree.

The authors have encountered the confounding experience numerous times in their work, and it is with this motivation that we introduce Simpson's paradox and the concept of confounding to the actuarial community. We believe that understanding these concepts is a key for actuaries in understanding the "correlation" issue that exists frequently in our actuarial work, and the impact of such "correlation" on analysis results.

REFERENCES

[1] Bailey, Robert A., "Insurance Rates with Minimum Bias," *PCAS* L, 1963, pp. 4–11.

[2] Bailey, Robert A., and LeRoy J. Simon, "Two Studies in Automobile Insurance Ratemaking," *PCAS* XLVII, 1960, pp. 1–19.

[3] Bickel, P. J., E. A. Hammel, and J. W. O'Connell, "Sex Bias in Graduate Admissions: Data from Berkeley," *Science*, Vol. 187, 1975, pp. 398–404.

[4] Brockman, M. J., and T. S. Wright, "Statistical Motor Rating: Making Effective Use of Your Data," *Journal of the Institute of Actuaries*, Vol. 119, 1992, Part III.

[5] Cates, Christopher J., "Simpson's Paradox and Calculation of Number Needed to Treat From Meta-Analysis," *BMC Medical Research Methodology*, 2002, 2:1.

[6] Guszcza, James C., and Cheng-Sheng Peter Wu, "Does Credit Score Really Explain Insurance Losses? Multivariate Analysis from a Data Mining Point of View," Casualty Actuarial Society *Forum*, Winter 2003, pp. 113–138.

[7] Intuiter.com, "Simpson's Paradox—When Big Data Sets Go Bad," http://www.intuitor.com/statistics/SimpsonsParadox.html.

[8] Mildenhall, S. J., "A Systematic Relationship Between Minimum Bias and Generalized Linear Models," *PCAS* LXXXVI, 1999, pp. 393–487.

[9] Montgomery, D. C., *Design and Analysis of Experiments*, John Wiley & Sons, 3rd Version, 1991.

[10] Neter, J., W. Wasserman, and M. H. Kutner, *Applied Linear Statistical Models*, Richard D. Irwin, Inc., 2nd Edition, 1985.

[11] Pearl, Judea, "Simpson's Paradox: An Anatomy," *Causality: Models, Reasoning and Inference*, Chapter 6, Cambridge, 2000.

[12] Pearl, Judea, "Why There Is No Statistical Test For Confounding, Why Many Think There Is, And Why They Are Almost Right," *Technical Report R-256*, Cognitive System Laboratory, UCLA, 1998.

[13] Schield, Milo, "Simpson's Paradox and Cornfield's Conditions," Dept. of Business & MIS, Augsburg College, 2211 Riverside Drive, Minneapolis, MN 55454.

[14] Simpson, E. H., "The Interpretation of Interaction in Contingency Tables," *Journal of the Royal Statistical Society* B, 1951, 2:238–41.

[15] Stenmark, John A., "Sources of Distortion in Classification Ratemaking Data and their Treatment," Casualty Actuarial Society *Discussion Paper Program*, 1990, pp. 85–112.

EXHIBIT 1

PART 1

MULTIPLE STATE—MULTIPLE YEAR SITUATION DIFFERENT LOSS RATIOS—DIFFERENT DISTRIBUTION

Assumptions

Class Factors Underlying Experience			Class 01 Loss Ratio		
Class	Current Factor	Required Factor	State	Loss Ratios Year 1	Year 2
01	1.00	1.00	1	50%	75%
02	2.00	2.10	2	60%	90%

Distribution of Exposures

	State 1		State 2		
Class	Year 1	Year 2	Year 1	Year 2	Total
01	10,000	15,000	10,000	15,000	50,000
02	5,000	15,000	15,000	45,000	80,000
Total	15,000	30,000	25,000	60,000	130,000

State 1 Base Rate = \$100
State 2 Base Rate = \$200

(The Derived Loss Experience is shown on the next page.)

Indicated Class 02 Relativity

Loss Ratio Method: $(84.56\%/73.67\%) \times 2.00 = 2.30$
Pure Premium Method: $295.97/110.50 = 2.68$
Modified Loss Ratio Method: $(169.13\%/73.67\%) = 2.30$

EXHIBIT 1

PART 2

MULTIPLE STATE—MULTIPLE YEAR SITUATION
DIFFERENT LOSS RATIOS—DIFFERENT DISTRIBUTION

Derived Loss Experience

	Class	Exposures	Earned Premium	Class Factor	Modified Earned Premium	Incurred Losses	Loss Ratio	Pure Premium	Modified Loss Ratio
State 1									
Year 1	01	10,000	$1,000,000	1.00	$1,000,000	$500,000	50.00%	$50.00	50.00%
	02	5,000	$1,000,000	2.00	$500,000	$525,000	52.50%	$105.00	105.00%
	Total	15,000	$2,000,000		$1,500,000	$1,025,000	51.25%	$68.33	68.33%
Year 2	01	15,000	$1,500,000	1.00	$1,500,000	$1,125,000	75.00%	$75.00	75.00%
	02	15,000	$3,000,000	2.00	$1,500,000	$2,362,500	78.75%	$157.50	157.50%
	Total	30,000	$4,500,000		$3,000,000	$3,487,500	77.50%	$116.25	116.25%
All Years	01	25,000	$2,500,000		$2,500,000	$1,625,000	65.00%	$65.00	65.00%
	02	20,000	$4,000,000		$2,000,000	$2,887,500	72.19%	$144.38	144.38%
	Total	45,000	$6,500,000		$4,500,000	$4,512,500	69.42%	$100.28	100.28%

State 2									
Year 1	01	10,000	$2,000,000	1.00	$2,000,000	$1,200,000	60.00%	$120.00	60.00%
	02	15,000	$6,000,000	2.00	$3,000,000	$3,780,000	63.00%	$252.00	126.00%
	Total	25,000	$8,000,000		$5,000,000	$4,980,000	62.25%	$199.20	99.60%
Year 2	01	15,000	$3,000,000	1.00	$3,000,000	$2,700,000	90.00%	$180.00	90.00%
	02	45,000	$18,000,000	2.00	$9,000,000	$17,010,000	94.50%	$378.00	189.00%
	Total	60,000	$21,000,000		$12,000,000	$19,710,000	93.86%	$328.50	164.25%
All Years	01	25,000	$5,000,000		$5,000,000	$3,900,000	78.00%	$156.00	78.00%
	02	60,000	$24,000,000		$12,000,000	$20,790,000	86.63%	$346.50	173.25%
	Total	85,000	$29,000,000		$17,000,000	$24,690,000	85.14%	$290.47	145.24%

EXHIBIT 1

Part 2

Multiple State—Multiple Year Situation
Different Loss Ratios—Different Distribution, *Continued*

Derived Loss Experience

	Class	Exposures	Earned Premium	Class Factor	Modified Earned Premium	Incurred Losses	Loss Ratio	Pure Premium	Modified Loss Ratio
All States									
Year 1	01	20,000	$3,000,000		$3,000,000	$1,700,000	56.67%	$85.00	56.67%
	02	20,000	$7,000,000		$3,500,000	$4,305,000	61.50%	$215.25	123.00%
	Total	40,000	$10,000,000		$6,500,000	$6,005,000	60.05%	$150.13	92.38%
Year 2	01	30,000	$4,500,000		$4,500,000	$3,825,000	85.00%	$127.50	85.00%
	02	60,000	$21,000,000		$10,500,000	$19,372,500	92.25%	$322.88	184.50%
	Total	90,000	$25,500,000		$15,000,000	$23,197,500	90.97%	$257.75	154.65%
All Years	01	50,000	$7,500,000		$7,500,000	$5,525,000	73.67%	$110.50	73.67%
	02	80,000	$28,000,000		$14,000,000	$23,677,500	84.56%	$295.97	169.13%
	Total	130,000	$35,500,000		$21,500,000	$29,202,500	82.26%	$224.63	135.83%

EXHIBIT 2

MULTIPLE STATE—MULTIPLE YEAR SITUATION
DIFFERENT LOSS RATIOS—DIFFERENT DISTRIBUTION

Derived Loss Experience

	Class	Exposures	Earned Premium	Class Factor	Adjusted Premium	Incurred Losses	Adjusted Loss Ratio	Scaled Losses	Modified Loss Ratio
State 1									
Year 1	01	10,000	\$1,000,000	1.00	\$1,000,000	\$500,000	50.00%	\$1,000,000	100.00%
	02	5,000	\$1,000,000	2.00	\$500,000	\$525,000	105.00%	\$1,050,000	210.00%
	Total	15,000	\$2,000,000		\$1,500,000	\$1,025,000	68.33%	\$2,050,000	136.67%
Year 2	01	15,000	\$1,500,000	1.00	\$1,500,000	\$1,125,000	75.00%	\$1,500,000	100.00%
	02	15,000	\$3,000,000	2.00	\$1,500,000	\$2,362,500	157.50%	\$3,150,000	210.00%
	Total	30,000	\$4,500,000		\$3,000,000	\$3,487,500	116.25%	\$4,650,000	155.00%
All Years	01	25,000	\$2,500,000		\$2,500,000	\$1,625,000	65.00%	\$2,500,000	100.00%
	02	20,000	\$4,000,000		\$2,000,000	\$2,887,500	144.38%	\$4,200,000	210.00%
	Total	45,000	\$6,500,000		\$4,500,000	\$4,512,500	100.28%	\$6,700,000	148.89%

EXHIBIT 2

Multiple State—Multiple Year Situation
Different Loss Ratios—Different Distribution, *Continued*

					Derived Loss Experience				
	Class	Exposures	Earned Premium	Class Factor	Adjusted Premium	Incurred Losses	Adjusted Loss Ratio	Scaled Losses	Modified Loss Ratio
State 2									
Year 1	01	10,000	$2,000,000	1.00	$2,000,000	$1,200,000	60.00%	$2,000,000	100.00%
	02	15,000	$6,000,000	2.00	$3,000,000	$3,780,000	126.00%	$6,300,000	210.00%
	Total	25,000	$8,000,000		$5,000,000	$4,980,000	99.60%	$8,300,000	166.00%
Year 2	01	15,000	$3,000,000	1.00	$3,000,000	$2,700,000	90.00%	$3,000,000	100.00%
	02	45,000	$18,000,000	2.00	$9,000,000	$17,010,000	189.00%	$18,900,000	210.00%
	Total	60,000	$21,000,000		$12,000,000	$19,710,000	164.25%	$21,900,000	182.50%
All Years	01	25,000	$5,000,000		$5,000,000	$3,900,000	78.00%	$5,000,000	100.00%
	02	60,000	$24,000,000		$12,000,000	$20,790,000	173.25%	$25,200,000	210.00%
	Total	85,000	$29,000,000		$17,000,000	$24,690,000	145.24%	$30,200,000	177.65%

All States								
Year 1	01	20,000	$3,000,000	$3,000,000	$1,700,000	56.67%	$3,000,000	100.00%
	02	20,000	$7,000,000	$3,500,000	$4,305,000	123.00%	$7,350,000	210.00%
	Total	40,000	$10,000,000	$6,500,000	$6,005,000	92.38%	$10,350,000	159.23%
Year 2	01	30,000	$4,500,000	$4,500,000	$3,825,000	85.00%	$4,500,000	100.00%
	02	60,000	$21,000,000	$10,500,000	$19,372,500	184.50%	$22,050,000	210.00%
	Total	90,000	$25,500,000	$15,000,000	$23,197,500	154.65%	$26,550,000	177.00%
All Years	01	50,000	$7,500,000	$7,500,000	$5,525,000	73.67%	$7,500,000	100.00%
	02	80,000	$28,000,000	$14,000,000	$23,677,500	169.13%	$29,400,000	210.00%
	Total	130,000	$35,500,000	$21,500,000	$29,202,500	135.83%	$36,900,000	171.63%

Indicated Class 02 Relativity

Modified Loss Ratio Method: $(210.00\%/100.00\%)x = 2.10$

EXHIBIT 3

Multiple State—Multiple Year Situation
Different Loss Ratios—Different Distribution

Derived Loss Experience									
	Class	Exposures	Earned Premium	Class Factor	Incurred Losses	Scaling Factor	Scaled Premium	Scaled Losses	Modified Loss Ratio
State 1									
Year 1	01	10,000	\$1,000,000	1.00	\$500,000	1.000	\$1,000,000	\$500,000	50.00%
	02	5,000	\$1,000,000	2.00	\$525,000	3.200	\$1,600,000	\$1,680,000	105.00%
	Total	15,000	\$2,000,000		\$1,025,000		\$2,600,000	\$2,180,000	83.85%
Year 2	01	15,000	\$1,500,000	1.00	\$1,125,000	1.000	\$1,500,000	\$1,125,000	75.00%
	02	15,000	\$3,000,000	2.00	\$2,362,500	1.600	\$2,400,000	\$3,780,000	157.50%
	Total	30,000	\$4,500,000		\$3,487,500		\$3,900,000	\$4,905,000	125.77%
All Years	01	25,000	\$2,500,000		\$1,625,000		\$2,500,000	\$1,625,000	65.00%
	02	20,000	\$4,000,000		\$2,887,500		\$4,000,000	\$5,460,000	136.50%
	Total	45,000	\$6,500,000		\$4,512,500		\$6,500,000	\$7,085,000	109.00%

State 2									
Year 1	01	10,000	$2,000,000	1.00	$1,200,000	1.000	$2,000,000	$1,200,000	60.00%
	02	15,000	$6,000,000	2.00	$3,780,000	1.067	$3,200,000	$4,032,000	126.00%
	Total	25,000	$8,000,000		$4,980,000		$5,200,000	$5,232,000	100.62%
Year 2	01	15,000	$3,000,000	1.00	$2,700,000	1.000	$3,000,000	$2,700,000	90.00%
	02	45,000	$18,000,000	2.00	$17,010,000	0.533	$4,800,000	$9,072,000	189.00%
	Total	60,000	$21,000,000		$19,710,000		$7,800,000	$11,772,000	150.92%
All Years	01	25,000	$5,000,000		$3,900,000		$5,000,000	$3,900,000	78.00%
	02	60,000	$24,000,000		$20,790,000		$8,000,000	$13,104,000	163.80%
	Total	85,000	$29,000,000		$24,690,000		$13,000,000	$17,004,000	130.80%

EXHIBIT 3

Multiple State—Multiple Year Situation
Different Loss Ratios—Different Distribution, *Continued*

	Derived Loss Experience								
	Class	Exposures	Earned Premium	Class Factor	Incurred Losses	Scaling Factor	Scaled Premium	Scaled Losses	Modified Loss Ratio
All States									
Year 1	01	20,000	\$3,000,000		\$1,700,000		\$3,000,000	\$1,700,000	56.67%
	02	20,000	\$7,000,000		\$4,305,000		\$4,800,000	\$5,712,000	119.00%
	Total	40,000	\$10,000,000		\$6,005,000		\$7,800,000	\$7,412,000	95.03%
Year 2	01	30,000	\$4,500,000		\$3,825,000		\$4,500,000	\$3,825,000	85.00%
	02	60,000	\$21,000,000		\$19,372,500		\$7,200,000	\$12,852,000	178.50%
	Total	90,000	\$25,500,000		\$23,197,500		\$11,700,000	\$16,677,000	142.54%
All Years	01	50,000	\$7,500,000		\$5,525,000		\$7,500,000	\$5,525,000	73.67%
	02	80,000	\$28,000,000		\$23,677,500		\$12,000,000	\$18,564,000	154.70%
	Total	130,000	\$35,500,000		\$29,202,500		\$19,500,000	\$24,089,000	123.53%

Indicated Class 02 Relativity
Modified Loss Ratio Method: (154.70%/73.67%) = 2.10

EXHIBIT 4

PART 1

STOCHASTIC MODEL WITH LOGNORMALLY DISTRIBUTED LOSSES (UNLIMITED)

Iterations: 1000

Class 02 Factor

Iteration	Univariate Method	Scaling Factor 1	Scaling Factor 2	Scaling Factor 3	Scaling Factor 4	Minimum Bias State	Minimum Bias Year	Minimum Bias Class
Expected	2.2657	2.1000	2.1000	2.1000	2.1000	1.1478	1.4403	2.1016
Observed	2.3129	2.1573	2.1149	2.1330	2.1183	1.2071	1.5166	2.1128
MSE	0.1020	0.0643	0.0474	0.0465	0.0554	0.0168	0.0276	0.0461
1	2.3935	2.1936	2.1631	2.1932	2.1736	1.2049	1.5474	2.1476
2	1.8113	1.7371	1.6567	1.6791	1.7141	1.0902	1.6097	1.6514
3	2.2538	2.0191	2.0735	2.0988	1.9792	1.2186	1.5926	2.1182
4	2.4408	2.4355	2.1858	2.1875	2.3471	1.2026	1.4451	2.1536
5	2.1421	2.0335	1.9766	2.0229	1.9670	1.1834	1.4379	1.9186
6	2.4667	2.3157	2.3354	2.3564	2.3083	1.0514	1.4017	2.3349
7	2.5048	2.4576	2.2818	2.3519	2.2324	1.2738	1.5613	2.1804
8	2.3571	2.2147	2.1329	2.1659	2.1591	1.3534	1.3752	2.0514
9	2.1105	1.9180	1.8851	1.8830	1.9294	1.1960	1.7415	1.8736
10	2.4410	2.3869	2.1701	2.1827	2.3078	1.2407	1.4736	2.0896
11	2.1020	1.9224	1.9825	1.9872	1.9249	1.1422	1.3841	2.0751
12	2.0084	1.8736	1.8714	1.9208	1.7754	1.1715	1.5789	1.8671
13	2.2959	2.2104	2.0896	2.1382	2.1279	1.1512	1.6218	1.9925
14	2.1490	1.9574	1.9076	1.9281	1.9514	1.4529	1.3877	1.9140
15	2.2361	1.9854	2.0147	2.0346	1.9838	1.3287	1.5561	2.0263
16	2.1911	2.0321	1.9674	2.0141	1.9646	1.2217	1.7060	1.9186
17	2.2988	2.2033	2.0942	2.0757	2.1698	1.0465	1.7953	2.1426
18	2.5449	2.5903	2.3678	2.4434	2.4114	1.0333	1.5893	2.2433
19	2.3596	2.1615	2.1741	2.1971	2.1472	1.1520	1.6126	2.1583
20	2.2831	2.0866	2.0896	2.1006	2.0688	1.1301	1.5666	2.1466
21	1.9180	1.6937	1.7110	1.7552	1.6477	1.3537	1.6972	1.7397
22	2.1284	2.0116	2.0323	2.0510	2.0127	1.0356	1.3674	2.0325
23	1.9500	1.7895	1.7802	1.7939	1.7911	1.1713	1.4976	1.8193
24	2.5896	2.4719	2.3317	2.3935	2.2911	1.2323	1.6382	2.2866
25	2.5466	2.4214	2.4049	2.4214	2.3956	1.1283	1.3022	2.3517
26	2.2363	2.0273	2.0281	2.0448	2.0267	1.2561	1.5132	2.0196
27	2.4753	2.3979	2.2668	2.3078	2.2603	1.2619	1.3369	2.1870
28	2.3489	2.1869	2.1419	2.1741	2.1485	1.1248	1.6669	2.1070
29	2.1399	2.0637	1.9157	1.9197	2.0193	1.1871	1.5602	1.9278
30	2.4339	2.2187	2.1744	2.1927	2.1946	1.2553	1.7265	2.1142
31	2.6216	2.4300	2.4392	2.4183	2.4331	1.1500	1.4939	2.4109

EXHIBIT 4

Part 2

Stochastic Model with Lognormally Distributed Losses (unlimited)

Loss Ratios								
	State 1				State 2			
	Year 1		Year 2		Year 1		Year 2	
	Class 01	Class 02	Class 01	Class 02	Class 01	Class 02	Class 01	Class 02
Expected	0.5000	0.5250	0.7500	0.7875	0.6000	0.6300	0.9000	0.9450
Observed	0.4985	0.5240	0.7530	0.7865	0.5956	0.6283	0.9019	0.9445
MSE	0.0140	0.0130	0.0150	0.0070	0.0159	0.0057	0.0168	0.0032
1	0.6534	0.5833	0.6450	0.7317	0.4710	0.5502	0.8950	0.9649
2	0.5139	0.4857	1.0362	0.7646	0.7784	0.5104	0.9056	0.8667
3	0.4053	0.4337	0.6989	0.8902	0.5492	0.6797	1.0675	0.9460
4	0.5763	0.4172	0.7978	0.7673	0.6987	0.6395	0.6964	0.9671
5	0.5296	0.5573	0.8452	0.6703	0.4758	0.6336	0.9148	0.8664
6	0.5477	0.6310	0.6757	0.8165	0.4827	0.6095	0.7831	0.8729
7	0.2809	0.5142	0.9039	0.6999	0.3743	0.7287	0.9168	0.9460
8	0.3796	0.4685	0.8075	0.5984	0.5284	0.7062	0.8134	0.8825
9	0.5170	0.5689	0.8034	0.6887	0.5964	0.4380	0.8943	0.9327
10	0.5652	0.3443	0.9224	0.7652	0.5930	0.7358	0.7854	0.9980
11	0.4327	0.4029	0.6272	0.8902	0.7467	0.6580	0.9062	0.8275
12	0.4173	0.4809	0.8150	0.7818	0.4637	0.6358	1.0940	0.8493
13	0.4549	0.6291	0.9469	0.6585	0.4334	0.5548	0.8385	0.9222
14	0.4469	0.3350	0.6642	0.6383	0.7410	0.6557	0.8636	0.8857
15	0.4699	0.5356	0.7201	0.7646	0.6318	0.6411	1.0427	1.0069
16	0.4701	0.4849	0.9358	0.7629	0.4819	0.6214	1.0280	0.9842
17	0.3373	0.3505	0.8378	0.9307	0.7147	0.5039	0.7264	0.8976
18	0.3690	0.5303	1.0123	0.8015	0.3737	0.6576	0.7376	0.8971
19	0.3948	0.6219	0.8550	0.8306	0.5561	0.6195	0.9290	0.9851
20	0.6618	0.4523	0.6090	0.9025	0.5637	0.5814	0.9532	0.9476
21	0.6081	0.4859	0.6171	0.7104	0.5322	0.5148	1.2337	0.9360
22	0.5790	0.6386	0.8475	0.8545	0.6246	0.6556	0.8901	0.8771
23	0.6768	0.4714	0.7298	0.8152	0.7343	0.5626	1.0024	0.9137
24	0.4986	0.4616	0.6781	0.7542	0.3279	0.6220	0.9056	0.9541
25	0.5320	0.7422	0.7481	0.7485	0.5069	0.6984	0.7892	0.9311
26	0.5480	0.5858	0.7793	0.7667	0.6260	0.6374	0.9919	1.0059
27	0.5829	0.5300	0.7134	0.6655	0.4030	0.7254	0.8618	0.9127
28	0.4914	0.5183	0.9033	0.8609	0.4984	0.6278	0.9344	0.9938
29	0.5140	0.3776	0.8174	0.7764	0.7685	0.5537	0.7845	0.9145
30	0.4255	0.6517	0.8581	0.7072	0.4974	0.5526	0.9187	1.0407
31	0.4136	0.8439	0.7312	0.7344	0.5475	0.5574	0.7777	0.9875

EXHIBIT 5

PART 1

STOCHASTIC MODEL WITH TRUNCATED LOGNORMALLY DISTRIBUTED LOSSES ($25,000 LIMIT)

Iterations: 1000

Class 02 Factor								
	Univariate	Scaling	Scaling	Scaling	Scaling	Minimum Bias		
Iteration	Method	Factor 1	Factor 2	Factor 3	Factor 4	State	Year	Class
Expected	2.2657	2.1000	2.1000	2.1000	2.1000	1.1478	1.4403	2.1016
Observed	2.3053	2.1271	2.1088	2.1271	2.1095	1.2030	1.5084	2.1083
MSE	0.0658	0.0249	0.0207	0.0206	0.0228	0.0099	0.0149	0.0213
1	2.3802	2.1588	2.1801	2.2083	2.1394	1.2469	1.4534	2.1771
2	2.0924	1.9451	1.9030	1.9097	1.9427	1.1631	1.5583	1.9315
3	2.1894	1.9774	2.0200	2.0371	1.9742	1.1944	1.5059	2.0514
4	2.2927	2.1755	2.0755	2.0928	2.1518	1.2505	1.3676	2.0510
5	2.3338	2.1908	2.1298	2.1655	2.1426	1.1665	1.5365	2.0773
6	2.3500	2.2127	2.2156	2.2302	2.2132	1.0679	1.3904	2.2182
7	2.4182	2.1886	2.1941	2.2289	2.1514	1.3159	1.5362	2.1689
8	2.2514	2.0719	2.0278	2.0596	2.0368	1.3684	1.4208	1.9732
9	2.0880	1.9078	1.9158	1.9279	1.9166	1.1744	1.5555	1.9314
10	2.3873	2.2889	2.1538	2.1701	2.2455	1.1996	1.4365	2.1102
11	2.2490	2.0481	2.0324	2.0433	2.0486	1.2785	1.4612	2.0641
12	2.3228	2.1178	2.1601	2.1825	2.1109	1.1420	1.5201	2.1736
13	2.3093	2.2093	2.0827	2.1137	2.1556	1.2011	1.5610	2.0053
14	2.0638	1.8534	1.8354	1.8600	1.8543	1.3891	1.4913	1.8448
15	2.1417	1.9027	1.9514	1.9829	1.8830	1.2770	1.6096	1.9699
16	2.0675	1.9028	1.8807	1.9202	1.8672	1.2018	1.5927	1.8508
17	2.2452	2.0472	2.0482	2.0599	2.0556	1.1755	1.5753	2.0882
18	2.4198	2.3042	2.2645	2.3026	2.2484	1.0750	1.5073	2.2190
19	2.3490	2.1635	2.1565	2.1792	2.1507	1.1715	1.5553	2.1383
20	2.3289	2.1199	2.1324	2.1408	2.1210	1.1385	1.5696	2.1678
21	2.1259	1.8874	1.9175	1.9436	1.8794	1.2672	1.6252	1.9396
22	2.1688	2.0283	2.0268	2.0478	2.0267	1.1261	1.3799	2.0264
23	2.2252	2.0744	2.0476	2.0590	2.0720	1.1920	1.4154	2.0694
24	2.4252	2.2689	2.1816	2.2237	2.1666	1.2822	1.5327	2.1346
25	2.3196	2.1454	2.1753	2.1842	2.1531	1.1397	1.3929	2.1718
26	2.1485	1.9262	1.9610	1.9834	1.9253	1.1609	1.7169	1.9640
27	2.4424	2.2630	2.2040	2.2237	2.2470	1.2571	1.4515	2.1820
28	2.1618	2.0127	1.9780	2.0022	1.9958	1.1605	1.5399	1.9492
29	2.2255	2.1499	1.9920	1.9877	2.0999	1.2164	1.5028	2.0116
30	2.4324	2.2201	2.1840	2.1841	2.2225	1.2327	1.6465	2.1610
31	2.7787	2.5628	2.5558	2.5526	2.5620	1.1933	1.4861	2.5268

EXHIBIT 5

Part 2

Stochastic Model with Truncated Lognormally Distributed Losses ($25,000 Limit)

	Loss Ratios							
	State 1				State 2			
	Year 1		Year 2		Year 1		Year 2	
	Class 01	Class 02	Class 01	Class 02	Class 01	Class 02	Class 01	Class 02
Expected	0.5000	0.5250	0.7500	0.7875	0.6000	0.6300	0.9000	0.9450
Observed	0.4973	0.5229	0.7530	0.7890	0.5969	0.6309	0.9005	0.9448
MSE	0.0077	0.0077	0.0083	0.0043	0.0065	0.0024	0.0065	0.0013
1	0.5754	0.6351	0.6206	0.7198	0.5008	0.6076	0.9345	0.9533
2	0.5050	0.4458	0.7667	0.7910	0.7212	0.5417	0.8576	0.8932
3	0.4305	0.4164	0.7654	0.8759	0.6392	0.6960	0.9926	0.9237
4	0.6062	0.4894	0.7510	0.7166	0.6680	0.6652	0.8156	0.9430
5	0.5885	0.6086	0.8329	0.7549	0.4763	0.6185	0.9199	0.9774
6	0.5641	0.6105	0.7519	0.8485	0.5819	0.6475	0.8220	0.9058
7	0.3439	0.5371	0.7703	0.7359	0.5369	0.6851	0.9492	0.9785
8	0.4277	0.4732	0.7943	0.6271	0.5626	0.6907	0.9059	0.9142
9	0.4561	0.5476	0.7536	0.7320	0.6524	0.5204	0.8835	0.8741
10	0.5993	0.3873	0.8301	0.7960	0.5989	0.7115	0.8115	0.9646
11	0.5112	0.4113	0.6639	0.7816	0.6937	0.6421	0.8956	0.9346
12	0.4277	0.5407	0.7578	0.8461	0.5572	0.6457	0.9186	0.9228
13	0.4600	0.5396	0.9565	0.7053	0.5594	0.6241	0.8322	0.9640
14	0.4575	0.3483	0.6963	0.6661	0.6889	0.6096	0.9341	0.8825
15	0.4153	0.5242	0.7279	0.7735	0.5746	0.6100	1.0758	0.9454
16	0.5335	0.5326	0.8806	0.7403	0.5319	0.6038	1.0331	0.9360
17	0.3894	0.3630	0.6886	0.7945	0.6294	0.5661	0.8089	0.8480
18	0.4358	0.5631	0.8784	0.8425	0.4735	0.6724	0.8431	0.9165
19	0.4207	0.5548	0.8551	0.8276	0.5852	0.6575	0.9104	0.9774
20	0.6114	0.5239	0.6711	0.8798	0.5741	0.5867	0.9348	0.9741
21	0.5512	0.5160	0.6732	0.7589	0.5653	0.5557	1.0586	0.9532
22	0.5676	0.5579	0.7885	0.8012	0.6365	0.6578	0.8857	0.8938
23	0.4954	0.5022	0.7312	0.7877	0.7174	0.6319	0.8392	0.9097
24	0.5059	0.4322	0.7337	0.7261	0.4233	0.6761	0.9291	0.9475
25	0.5268	0.7172	0.7390	0.7731	0.6009	0.6251	0.8721	0.9348
26	0.4864	0.5932	0.8073	0.8001	0.5366	0.5285	1.0234	0.9589
27	0.5085	0.4261	0.6857	0.7220	0.5505	0.6438	0.7971	0.9153
28	0.5388	0.5572	0.9476	0.8214	0.6261	0.6596	0.9785	0.9905
29	0.5729	0.4374	0.7486	0.7783	0.7889	0.5675	0.7865	0.9569
30	0.4480	0.5968	0.7583	0.7247	0.5706	0.5350	0.8382	0.9898
31	0.4251	0.7158	0.6981	0.7751	0.5259	0.6301	0.7727	1.0163

APPENDIX A

The symbolic representation of the impact of confounding on class relativity analysis due to the aggregation of more than one year and more than one state.

The Loss Experience Model

Let

e_{iys} = Earned exposures for class i, year y, state s

r_{ys} = Base class loss ratio for year y, state s

B_{ys} = Base Rate for year y, state s

b = Base class subscript

c_i = Current class factor for class i ($c_b = 1$)

f_i = Required factor for class i ($f_b = 1$)

g_i = Factor yielded by method for class i

E_i = Total earned exposures for class $i = \sum_y \sum_s e_{iys}$

P_i = Total earned premiums for class i on present rates

$$= \sum_y \sum_s e_{iys} B_{ys} c_i$$

L_i = Total incurred losses for class $i = \sum_y \sum_s e_{iys} r_{ys} B_{ys} f_i$

An "O" superscript indicates that the variable is relative to overall (all class) rather than the base class. For example:

r_{ys}^O = Overall class loss ratio for year y, state s

f_{iys}^O = Required factor for class i where overall class factor for year y, state s is unity

Special Case

If each class's loss ratio is related to the base class loss ratio, use the Special Case below to determine relativities. The bias resulting from the Loss Ratio Method, the Pure Premium Method, and the Modified Loss Ratio Method have been derived.

Pure Premium Method

$$g_i = \frac{\dfrac{\sum_y \sum_s e_{iys} r_{ys} B_{ys} f_i}{\sum_y \sum_s e_{iys}}}{\dfrac{\sum_y \sum_s e_{bys} r_{ys} B_{ys} f_b}{\sum_y \sum_s e_{bys}}}$$

The bias in the method is

$$\begin{aligned}\frac{g_i}{f_i} - 1 &= \frac{\sum_y \sum_s e_{iys} r_{ys} B_{ys}}{\sum_y \sum_s e_{iys}} \bullet \frac{\sum_y \sum_s e_{bys}}{\sum_y \sum_s e_{bys} r_{ys} B_{ys} f_i} - 1 \\ &= \frac{\sum_y \sum_s e_{iys} r_{ys} B_{ys}}{E_i} \bullet \frac{E_b}{L_b} - 1.\end{aligned}$$

Loss Ratio Method

$$g_i = \frac{\dfrac{\sum_y \sum_s e_{iys} r_{ys} B_{ys} f_i}{\sum_y \sum_s e_{iys} B_{ys} c_i}}{\dfrac{\sum_y \sum_s e_{bys} r_{ys} B_{ys} f_b}{\sum_y \sum_s e_{bys} B_{ys} c_b}}$$

The bias in the method is

$$\begin{aligned}\frac{g_i}{f_i} - 1 &= \frac{\sum_y \sum_s e_{iys} r_{ys} B_{ys}}{\sum_y \sum_s e_{iys} B_{ys} c_i} \bullet \frac{\sum_y \sum_s e_{bys} B_{ys}}{\sum_y \sum_s e_{bys} r_{ys} B_{ys}} - 1 \\ &= \frac{\sum_y \sum_s e_{iys} r_{ys} B_{ys}}{P_i} \bullet \frac{P_b}{L_b} - 1.\end{aligned}$$

Modified Loss Ratio Method

$$g_i = \frac{\dfrac{\sum_y \sum_s e_{iys} r_{ys} B_{ys} f_i}{\sum_y \sum_s e_{iys} B_{ys} c_b}}{\dfrac{\sum_y \sum_s e_{bys} r_{ys} B_{ys} f_b}{\sum_y \sum_s e_{bys} B_{ys} c_b}}$$

The bias in the method is

$$\begin{aligned}\frac{g_i}{f_i} - 1 &= \frac{\sum_y \sum_s e_{iys} r_{ys} B_{ys}}{\sum_y \sum_s e_{iys} B_{ys} c_b} \bullet \frac{\sum_y \sum_s e_{bys} B_{ys}}{\sum_y \sum_s e_{bys} r_{ys} B_{ys}} - 1 \\ &= \frac{\sum_y \sum_s e_{iys} r_{ys} B_{ys}}{\sum_y \sum_s e_{iys} B_{ys} c_b} \bullet \frac{P_b}{L_b} - 1. \qquad \text{(A.1)}\end{aligned}$$

General Case

If each class's loss ratio is related to the overall loss ratio rather than the base class loss ratio, to determine relativities use the General Case below. Only the error resulting from the modified loss ratio method has been derived.

Modified Loss Ratio Method

$$g_i^O = \frac{\dfrac{\sum_y \sum_s e_{iys} r_{ys} B_{ys} f_i}{\sum_y \sum_s e_{iys} B_{ys} c_b}}{\dfrac{\sum_i \sum_y \sum_s e_{iys} r_{ys} B_{ys} f_i}{\sum_i \sum_y \sum_s e_{iys} B_{ys} c_b}}$$

The bias in the method is

$$\frac{g_i^O}{f_{iys}^O} - 1 = \frac{1}{f_{iys}^O} \frac{\sum_y \sum_s e_{iys} r_{ys} B_{ys} f_i}{\sum_y \sum_s e_{iys} B_{ys} c_b} \bullet \frac{\sum_i \sum_y \sum_s e_{iys} B_{ys} c_b}{\sum_i \sum_y \sum_s e_{iys} r_{ys} B_{ys} f_i} - 1.$$

APPENDIX B

Derivation of Scaling Factors

Criterion 1: The scaling factor should maintain the relationship between class loss ratios by year and state.

Criterion 2: The scaling factor should reduce the method error to zero.

Let primed variables indicate variables after the application of a scaling factor (e.g., g_i' is the factor yielded by a method after the application of a scaling factor).

First Special Scaling Factor—Scaling Factor 1

Consider Equation (A.1) (from Appendix A):

The bias in the method is

$$\begin{aligned}\frac{g_i}{f_i} - 1 &= \frac{\sum_y \sum_s e_{iys} r_{ys} B_{ys}}{\sum_y \sum_s e_{iys} B_{ys} c_b} \bullet \frac{\sum_y \sum_s e_{bys} B_{ys}}{\sum_y \sum_s e_{bys} r_{ys} B_{ys}} - 1 \\ &= \frac{\sum_y \sum_s s_{ys} e_{iys} r_{ys} B_{ys}}{\sum_y \sum_s e_{iys} B_{ys} c_b} \bullet \frac{\sum_y \sum_s e_{bys} B_{ys}}{\sum_y \sum_s s_{ys} e_{bys} r_{ys} B_{ys}} - 1 = 0.\end{aligned}$$

If each $r_{ys}' = 1$, then error in method $= 0$; therefore $1/r_{ys}$ is a scaling factor and $S_{ys} = 1/r_{ys}$ and will be applied to each loss.

Second Special Scaling Factor—Scaling Factor 2

Consider a scaling factor S, to be applied to premiums and losses:

$$\text{Let} \quad S_{iys} = \frac{\sum_y \sum_s e_{iys}}{\sum_y \sum_s e_{bys}} \bullet \frac{e_{bys}}{e_{iys}}.$$

Also (for convenience)

$$\text{Let} \quad e_{iys}' = \frac{\sum_y \sum_s e_{iys}}{\sum_y \sum_s e_{bys}} \bullet e_{bys}.$$

The bias in the method is

$$\begin{aligned}
\frac{g_i'}{f_i} - 1 &= \frac{\sum_y \sum_s S_{iys} e_{iys} r_{ys} B_{ys}}{\sum_y \sum_s S_{iys} e_{iys} B_{ys} c_b} \bullet \frac{\sum_y \sum_s S_{iys} e_{iys} B_{ys}}{\sum_y \sum_s S_{iys} e_{iys} r_{ys} B_{ys}} - 1 \\
&= \frac{\sum_y \sum_s e_{iys}' r_{ys} B_{ys}}{\sum_y \sum_s e_{iys}' B_{ys} c_b} \bullet \frac{\sum_y \sum_s e_{bys}' B_{ys}}{\sum_y \sum_s e_{bys}' r_{ys} B_{ys}} - 1 \\
&= \frac{e_i' \sum_y \sum_s e_{bys} r_{ys} B_{ys}}{e_i' \sum_y \sum_s e_{bys} B_{ys} c_b} \bullet \frac{e_b' \sum_y \sum_s e_{bys} B_{ys}}{e_b' \sum_y \sum_s e_{bys} r_{ys} B_{ys}} - 1 = 0,
\end{aligned}$$

where $e_i' = \sum_y \sum_s e_{iys} / \sum_y \sum_s e_{bys}$.

So this scaling factor satisfies Criterion 2.

Since this scaling factor is applied to premiums *and* losses by class, each class loss ratio remains unchanged satisfying Criterion 1.

Second Special Scaling Factor:

$$S_i = \frac{\sum_y \sum_s e_{iys}}{\sum_y \sum_s e_{bys}} \bullet \frac{e_{bys}}{e_{iys}}.$$

General Scaling Factors

If each class's loss ratio related to the overall loss ratio is used rather than the base class loss ratio, another set of scaling factors (generalized scaling factors) is used. First it is necessary to establish some relationships:

Define

$$f_{iys}^O = \frac{f_i \sum_i e_{iys}}{\sum_i e_{iys} f_i}.$$

Then

$$r_{ys}^O = \frac{\sum_i e_{iys} r_{ys} B_{ys} f_i}{\sum_i e_{iys} B_{ys} c_b} = \frac{r_{ys} \sum_i e_{iys} f_i}{\sum_i e_{iys}} = \frac{r_{ys} f_i}{f_{iys}^O}$$

and $r^O_{ys} f^O_{iys} = r_{ys} f_i$. Also

$$r^O_{ys} = \frac{\sum_i e_{iys} r_{ys} B_{ys} f_i}{\sum_i e_{iys} B_{ys} c_b} = \frac{\sum_i e_{iys} r^O_{ys} B_{ys} f^O_{iys}}{\sum_i e_{iys} B_{ys} c_b}$$

$$= \frac{r^O_{ys} B_{ys} \sum_i e_{iys} f^O_{iys}}{B_{ys} \sum_i e_{iys} c_b} = \frac{r^O_{ys} \sum_i e_{iys} f^O_{iys}}{\sum_i e_{iys}},$$

therefore $\sum_i e_{iys} f^O_{iys} = \sum_i e_{iys}$.

First General Scaling Factor—Scaling Factor 3

Consider a scaling factor, to be applied to losses only.

$$S_{ys} = \frac{\dfrac{\sum_i e_{iys} c_b B_{ys}}{\sum_i e_{iys}}}{\dfrac{\sum_i e_{iys} r_{ys} f_i B_{ys}}{\sum_i e_{iys}}} = \frac{\sum_i e_{iys} c_b}{\sum_i e_{iys} r_{ys} f_i} = \frac{\sum_i e_{iys}}{r_{ys} \sum_i e_{iys} f_i} = \frac{f^O_{iys}}{r_{ys} f_i} = \frac{1}{r^O_{ys}}.$$

$$\frac{g_i}{f_i} - 1 = \frac{\dfrac{\sum_y \sum_s e_{iys} r_{ys} B_{ys} f_i}{\sum_y \sum_s e_{iys} B_{ys} c_b} \bullet \dfrac{\sum_i \sum_y \sum_s e_{iys} B_{ys} c_b}{\sum_i \sum_y \sum_s e_{iys} r_{ys} B_{ys} f_i}}{f^O_{iys}} - 1$$

$$= \frac{1}{f^O_{iys}} \bullet \frac{\sum_y \sum_s e_{iys} r_{ys} B_{ys} f_i}{\sum_y \sum_s e_{iys} B_{ys}} \bullet \frac{\sum_i \sum_y \sum_s e_{iys} B_{ys}}{\sum_i \sum_y \sum_s e_{iys} r_{ys} B_{ys} f_i} - 1.$$

$$\frac{g'_i}{f_i} - 1 = \frac{1}{f^O_{iys}} \bullet \frac{\sum_y \sum_s e_{iys} S_{ys} r_{ys} B_{ys} f_i}{\sum_y \sum_s e_{iys} B_{ys}} \bullet \frac{\sum_i \sum_y \sum_s e_{iys} B_{ys}}{\sum_i \sum_y \sum_s e_{iys} S_{ys} r_{ys} B_{ys} f_i} - 1.$$

$$\frac{g'_i}{f_i} - 1 = \frac{1}{f^O_{iys}} \bullet \frac{\sum_y \sum_s e_{iys} \left(\dfrac{f^O_{iys}}{r_{ys} f_i} \right) r_{ys} B_{ys} f_i}{\sum_y \sum_s e_{iys} B_{ys}}$$

$$\bullet \frac{\sum_i \sum_y \sum_s e_{iys} B_{ys}}{\sum_i \sum_y \sum_s e_{iys} \left(\dfrac{f^O_{iys}}{r_{ys} f_i} \right) r_{ys} B_{ys} f_i} - 1.$$

$$\frac{g_i'}{f_i} - 1 = \frac{1}{f_{iys}^O} \bullet \frac{\sum_y \sum_s e_{iys} f_{iys}^O B_{ys}}{\sum_y \sum_s e_{iys} B_{ys}} \bullet \frac{\sum_i \sum_y \sum_s e_{iys} B_{ys}}{\sum_i \sum_y \sum_s e_{iys} f_{iys}^O B_{ys}} - 1.$$

$$\frac{g_i'}{f_i} - 1 = \frac{1}{f_{iys}^O} \bullet \frac{f_{iys}^O \sum_y \sum_s e_{iys} B_{ys}}{\sum_y \sum_s e_{iys} B_{ys}} \bullet \frac{\sum_i \sum_y \sum_s e_{iys} B_{ys}}{\sum_y \sum_s B_{ys} \sum_i e_{iys} f_{iys}^O} - 1.$$

$$\frac{g_i'}{f_i} - 1 = \frac{\sum_y \sum_s e_{iys} B_{ys}}{\sum_y \sum_s e_{iys} B_{ys}} \bullet \frac{\sum_i \sum_y \sum_s e_{iys} B_{ys}}{\sum_y \sum_s B_{ys} \sum_i e_{iys}} - 1.$$

$$\frac{g_i'}{f_i} - 1 = \frac{\sum_y \sum_s e_{iys} B_{ys}}{\sum_y \sum_s e_{iys} B_{ys}} \bullet \frac{\sum_i \sum_y \sum_s e_{iys} B_{ys}}{\sum_i \sum_y \sum_s e_{iys} B_{ys}} - 1 = 0.$$

Second General Scaling Factor—Scaling Factor 4

Consider a scaling factor, to be applied to premiums and losses.

$$S_{iys} = \frac{\sum_y \sum_s e_{iys}}{\sum_i \sum_y \sum_s e_{iys}} \bullet \frac{\sum_i e_{iys}}{e_{iys}}.$$

$$\text{Let} \quad e_{iys}' = \frac{\sum_y \sum_s e_{iys}}{\sum_i \sum_y \sum_s e_{iys}} \bullet \sum_i e_{iys}.$$

The bias in the method is

$$\frac{g_i'^O}{f_{iys}^O} - 1 = \frac{1}{f_{iys}^O} \bullet \frac{\sum_y \sum_s e'_{iys} r_{ys} B_{ys} f_i}{\sum_y \sum_s e'_{iys} B_{ys} c_b} \bullet \frac{\sum_i \sum_y \sum_s e'_{iys} B_{ys}}{\sum_i \sum_y \sum_s e'_{iys} r_{ys} B_{ys} f_i} - 1$$

$$= \frac{1}{f_{iys}^O} \bullet \frac{\sum_y \sum_s \left(\frac{\sum_y \sum_s e_{iys}}{\sum_i \sum_y \sum_s e_{iys}} \bullet \sum_i e_{iys} \right) r_{ys}^O B_{ys} f_{iys}^O}{\sum_y \sum_s \left(\frac{\sum_y \sum_s e_{iys}}{\sum_i \sum_y \sum_s e_{iys}} \bullet \sum_i e_{iys} \right) B_{ys} c_b}$$

$$\bullet \frac{\sum_i \sum_y \sum_s \left(\frac{\sum_y \sum_s e_{iys}}{\sum_i \sum_y \sum_s e_{iys}} \bullet \sum_i e_{iys} \right) B_{ys}}{\sum_i \sum_y \sum_s \left(\frac{\sum_y \sum_s e_{iys}}{\sum_i \sum_y \sum_s e_{iys}} \bullet \sum_i e_{iys} \right) r_{ys}^O B_{ys} f_{iys}^O} - 1$$

$$
= \frac{\left(\frac{\sum_y \sum_s e_{iys}}{\sum_i \sum_y \sum_s e_{iys}}\right) \sum_y \sum_s \left(\sum_i e_{iys}\right) r^O_{ys} B_{ys}}{\left(\frac{\sum_y \sum_s e_{iys}}{\sum_i \sum_y \sum_s e_{iys}}\right) \sum_y \sum_s \left(\sum_i e_{iys}\right) B_{ys} c_b}
$$

$$
\bullet \frac{\left(\frac{1}{\sum_i \sum_y \sum_s e_{iys}}\right) \sum_i \left(\sum_y \sum_s e_{iys}\right) \sum_y \sum_s \left(\sum_i e_{iys}\right) B_{ys}}{\left(\frac{1}{\sum_i \sum_y \sum_s e_{iys}}\right) \sum_i \left(\sum_y \sum_s e_{iys}\right) \sum_y \sum_s \left(\sum_i e_{iys} f^O_{iys}\right) r^O_{ys} B_{ys}} - 1
$$

$$
= \frac{\left(\frac{\sum_y \sum_s e_{iys}}{\sum_i \sum_y \sum_s e_{iys}}\right) \sum_i \sum_y \sum_s e_{iys} r_{ys} B_{ys}}{\left(\frac{\sum_y \sum_s e_{iys}}{\sum_i \sum_y \sum_s e_{iys}}\right) \sum_i \sum_y \sum_s e_{iys} B_{ys} c_b}
$$

$$
\bullet \frac{\left(\frac{\sum_y \sum_s e_{iys}}{\sum_i \sum_y \sum_s e_{iys}}\right) \sum_i \sum_y \sum_s e_{iys} B_{ys}}{\left(\frac{\sum_y \sum_s e_{iys}}{\sum_i \sum_y \sum_s e_{iys}}\right) \sum_i \sum_y \sum_s e_{iys} r_{ys} B_{ys}} - 1 = 0.
$$

So this scaling factor satisfies Criterion 2.

Since this scaling factor is applied to premiums *and* losses by class, each class loss ratio remains unchanged, satisfying Criterion 1.

Second General Scaling Factor:

$$
S_{iys} = \frac{\sum_y \sum_s e_{iys}}{\sum_i \sum_y \sum_s e_{iys}} \bullet \frac{\sum_i e_{iys}}{e_{iys}}.
$$

DISCUSSION OF PAPER PUBLISHED IN VOLUME LXXX

MINIMUM DISTANCE ESTIMATION OF LOSS DISTRIBUTIONS

STUART A. KLUGMAN AND A. RAHULJI PARSA

DISCUSSION BY CLIVE L. KEATINGE

1. INTRODUCTION

Klugman and Parsa have introduced the theory underlying minimum distance estimation with parametric distributions. In this review, I develop their ideas further to provide a more complete view of the characteristics of minimum distance estimation. I conclude that minimum distance estimation can be more efficient than the authors imply—but that there is little basis for using it in place of maximum likelihood estimation.

2. THEORY

The objective function that Klugman and Parsa consider is

$$Q(\boldsymbol{\theta}) = \sum_{i=1}^{k} w_i[G(c_i;\boldsymbol{\theta}) - G_n(c_i)]^2, \tag{2.1}$$

where G is the model functional, G_n is the corresponding empirical functional, $c_1 < c_2 < \cdots < c_k$ are arbitrarily selected values, and $w_1, w_2, \ldots, w_k > 0$ are arbitrarily selected weights. The functionals that Klugman and Parsa consider are the limited expected value function and the cumulative distribution function. The minimum distance estimate is the value of $\boldsymbol{\theta}$ that minimizes $Q(\boldsymbol{\theta})$. From here on, I will follow the authors' convention of writing $G(c_i;\boldsymbol{\theta})$ as G_i and $G_n(c_i)$ as $G_{n,i}$.

A necessary condition for $Q(\boldsymbol{\theta})$ to be at a minimum is for the p functions

$$\partial Q/\partial\theta_j = 2\sum_{i=1}^{k} w_i[G_i - G_{n,i}]G_i^{(j)} \tag{2.2}$$

to be equal to zero, where $G_i^{(j)}$ is the partial derivative of the model functional with respect to θ_j evaluated at c_i, and p is the number of elements in the parameter vector $\boldsymbol{\theta}$. Another necessary condition for $Q(\boldsymbol{\theta})$ to be at a minimum is for the $p \times p$ matrix with jlth element

$$\partial^2 Q/\partial\theta_j\partial\theta_l = 2\sum_{i=1}^{k} w_i G_i^{(j)}G_i^{(l)} + 2\sum_{i=1}^{k} w_i[G_i - G_{n,i}]G_i^{(j,l)} \tag{2.3}$$

to be positive semidefinite (which includes the positive definite case).

As the sample size goes to infinity, there will be a solution that satisfies these two conditions if and only if the $p \times p$ matrix with jlth element

$$E[\partial^2 Q/\partial\theta_j\partial\theta_l] = 2\sum_{i=1}^{k} w_i G_i^{(j)}G_i^{(l)} \tag{2.4}$$

is positive semidefinite, where the derivatives are evaluated at the true parameter values. If all the weights are positive, this matrix must be positive semidefinite. Though having some negative weights is counterintuitive, the theory does not rule them out as long as the matrix is positive semidefinite. For smaller sample sizes, the more negative weights there are, and the larger they are in magnitude, the less likely it is that a solution that satisfies the two necessary conditions above will exist.

Klugman and Parsa state that the minimum distance estimator is consistent and asymptotically unbiased with asymptotic covariance matrix $n^{-1}\mathbf{A}^{-1}\mathbf{B}\boldsymbol{\Sigma}\mathbf{B}'\mathbf{A}^{-1}$ if $\mathbf{A}$ is positive definite, where

$\mathbf{A}$ is the matrix defined by Equation (2.3), $\mathbf{B}$ is the $p \times k$ matrix with jlth element $\partial^2 Q/\partial\theta_j \partial G_{n,l} = -2w_l G_l^{(j)}$, and $n^{-1}\mathbf{\Sigma}$ is the asymptotic covariance matrix of the empirical functional. This is correct, except that, as noted in Benichou and Gail [2], one should use the asymptotic expectation of the empirical functional instead of the observed value in Equation (2.3). Making this correction causes the second term of Equation (2.3) to vanish, thus yielding Equation (2.4). Luong and Thompson [4] show this result in a more general setting.

An issue that Klugman and Parsa do not address is identifying the sets of weights that will produce the minimum asymptotic variance for the estimators of the parameters or of functions of the parameters. A set of weights $w_1, w_2, \ldots, w_k$ will produce the minimum asymptotic variance for the estimator of a function $h(\boldsymbol{\theta})$ if $\mathbf{A}$ is positive definite and

$$w_i = \frac{(\mathbf{\Sigma}^{-1}\mathbf{D}'(\mathbf{D}\mathbf{\Sigma}^{-1}\mathbf{D}')^{-1}\mathbf{d})_i}{(\mathbf{D}'\mathbf{v})_i}, \tag{2.5}$$

where $\mathbf{D}$ is the $p \times k$ matrix with jlth element $G_l^{(j)}$, $\mathbf{d}$ is the vector of length p with jth element $\partial h/\partial\theta_j$, and $\mathbf{v}$ is an arbitrary nonzero vector of length p. The minimum asymptotic variance is $n^{-1}\mathbf{d}'(\mathbf{D}\mathbf{\Sigma}^{-1}\mathbf{D}')^{-1}\mathbf{d}$. The proof is in the appendix. Defining $h(\boldsymbol{\theta})$ to be θ_j yields weights that produce the minimum asymptotic variance for the estimator of the parameter θ_j itself. The main diagonal of $n^{-1}(\mathbf{D}\mathbf{\Sigma}^{-1}\mathbf{D}')^{-1}$ gives the minimum asymptotic variances for the estimators of the θ_js.

In general, the asymptotic variances cannot be minimal for all of the parameters at the same time. However, the asymptotic variances can be minimal simultaneously if the definition of the objective function is expanded to

$$Q^*(\boldsymbol{\theta}) = \sum_{i=1}^{k}\sum_{j=1}^{k} w_{ij}[G_i - G_{n,i}][G_j - G_{n,j}]. \tag{2.6}$$

The equation uses an entire matrix of weights instead of one weight for each c_i. The appendix gives the minimum asymptotic variance condition. The most obvious matrix of weights that satisfies this condition is $\mathbf{\Sigma}^{-1}$. Luong and Thompson [4] show this result in a more general setting.

When estimating parameters, it is not possible to find an optimal set of weights, since the true values of the parameters are unknown. A reasonable requirement is that the weights used to estimate the parameters be asymptotically optimal, or at least close to asymptotically optimal, under the assumption that the estimated parameter values are the true parameter values. Finding an acceptable set of weights by trial and error is one option. Alternatively, a systematic procedure that often works, hereafter called Procedure 1, is to estimate the parameters using any reasonable set of weights, optimize the weights using the estimated parameter values, estimate the parameters again using the new set of weights, and so on, until the process converges. Yet it is possible that the process will not converge.

With Equation (2.1), finding the optimal set of weights at each iteration of the process is problematical, since minimum asymptotic variance can be achieved for only one parameter at a time. One possible solution is to consider the sets of weights defined by Equation (2.5) with $h(\boldsymbol{\theta})$ defined to be one particular parameter θ_j, look at the ratios of the diagonal elements of $n^{-1}\mathbf{A}^{-1}\mathbf{B}\mathbf{\Sigma}\mathbf{B}'\mathbf{A}^{-1}$ to the corresponding diagonal elements of the minimum asymptotic covariance matrix $n^{-1}(\mathbf{D}\mathbf{\Sigma}^{-1}\mathbf{D}')^{-1}$, and then search for a $\mathbf{v}$ that minimizes the sum of these ratios. There is no easy way to do this, since, as the components of $\mathbf{v}$ vary, there are many local minima for the sum. The best one can practically do is to systematically try a number of values for the components of $\mathbf{v}$ and use those that yield the smallest values for the sum as starting values in an optimization routine. An additional potential problem is that the minimum for the sum could occur at a point where $\mathbf{A}$ is not positive definite. In that case, modifications to the procedure would be necessary.

With Equation (2.6), the easiest matrix of weights to use at each iteration of the process is $\boldsymbol{\Sigma}^{-1}$. If the functional is the cumulative distribution function, then the process, if it converges, yields the grouped maximum likelihood estimate. The appendix shows this result. Of course, it would be much easier just to find the grouped maximum likelihood estimate directly.

Another possible procedure, hereafter called Procedure 2, is to use $\boldsymbol{\Sigma}^{-1}$ as the matrix of weights in Equation (2.6) and to treat it as a function of the parameters, instead of fixed, when minimizing the objective function. This procedure produces an estimate for each of the parameters directly, instead of a series of estimates that might or might not converge. If the functional is the cumulative distribution function, the result is the minimum chi-square estimate. The appendix shows this result.

Moore [5] shows that the asymptotic covariance matrix of both the grouped maximum likelihood estimator and the minimum chi-square estimator is $n^{-1}(\mathbf{D}\boldsymbol{\Sigma}^{-1}\mathbf{D}')^{-1}$, with the cumulative distribution function and the true parameter values used to evaluate the expression. If one uses Procedure 1 with Equation (2.6), or if one uses Procedure 2, with a functional other than the cumulative distribution function, similar reasoning reveals that the asymptotic covariance matrix will also be $n^{-1}(\mathbf{D}\boldsymbol{\Sigma}^{-1}\mathbf{D}')^{-1}$, with the selected functional and the true parameter values used to evaluate the expression. If one uses Procedure 1 with Equation (2.1), the asymptotic covariance matrix will be $n^{-1}\mathbf{A}^{-1}\mathbf{B}\boldsymbol{\Sigma}\mathbf{B}'\mathbf{A}^{-1}$, with weights optimized using the selected functional and the true parameter values.

In the last section of their paper, Klugman and Parsa provide results of a simulation study they conducted with samples of size 500 to investigate how well the asymptotic estimates perform. They make the point that averaging the values in estimated asymptotic covariance matrices considerably overstates the values in the true asymptotic covariance matrix when using minimum distance estimation with the Pareto distribution.

This phenomenon occurs because the distribution of estimates is skewed to the right. The overstatement is not a feature specific to minimum distance estimation. It also occurs with other estimation methods, including maximum likelihood estimation.

Right skewness also causes the sample covariance matrix of parameter estimates to tend to be larger than the asymptotic covariance matrix, though Klugman and Parsa did not note this because of an errant asymptotic covariance matrix. They showed this matrix to be

$$\begin{bmatrix} 0.6640 & 120.3 \\ 120.3 & 21{,}830 \end{bmatrix}$$

when it should have been

$$\begin{bmatrix} 0.3595 & 75.68 \\ 75.68 & 16{,}794 \end{bmatrix}.$$

Their sample covariance matrix of parameter estimates was

$$\begin{bmatrix} 0.5133 & 108.8 \\ 108.8 & 24{,}150 \end{bmatrix}.$$

This right skewness of estimates is a feature of the Pareto distribution. Other distributions may exhibit different behavior.

3. EXAMPLES

I will now illustrate results from the previous section using examples that Klugman and Parsa use. I will also discuss each of the examples. Where I show numerical values that differ slightly from what Klugman and Parsa show in their paper, I have used values that I believe are more accurate.

Example One—Improving the Efficiency of the Minimum Distance Estimator

The first example involves a Pareto distribution fit to 6,656 general liability claims. Klugman and Parsa show 10,000,000 as the largest c_i. However, they actually used 100,000,000. I will use that value here. Grouped maximum likelihood estimation yields parameter estimates of $\hat{\alpha} = 1.4826$ and $\hat{\lambda} = 705.79$. The asymptotic covariance matrix for these parameter values is

$$\begin{bmatrix} 0.0020472 & 1.3679 \\ 1.3679 & 1{,}090.4 \end{bmatrix}.$$

Klugman and Parsa then use minimum distance estimation with the limited expected value function and weights of 1 at all c_is. This yields parameter estimates of $\hat{\alpha} = 1.3388$ and $\hat{\lambda} =$ 590.33. The authors give the asymptotic covariance matrix for these parameter values as

$$\begin{bmatrix} 0.034751 & 33.571 \\ 33.571 & 32{,}765 \end{bmatrix}.$$

Making the correction to the matrix $\mathbf{A}$ noted in the previous section yields a corrected asymptotic covariance matrix of

$$\begin{bmatrix} 0.036691 & 35.603 \\ 35.603 & 34{,}880 \end{bmatrix}.$$

These values are substantially higher than the maximum likelihood values. A fairer comparison would be to compare the asymptotic covariance matrices at the same parameter values. The minimum distance asymptotic covariance matrix at the maximum likelihood parameter estimates is

$$\begin{bmatrix} 0.023518 & 21.819 \\ 21.819 & 20{,}579 \end{bmatrix}.$$

These values are still substantially higher than the maximum likelihood values.

The procedures described in the previous section can do better. Using Procedure 1 and Equation (2.1), we can start with the authors' estimates of $\hat{\alpha} = 1.3388$ and $\hat{\lambda} = 590.33$. We define $h(\boldsymbol{\theta})$ to be the parameter α and proceed with the iterative process until it converges to estimates of $\hat{\alpha} = 1.4753$ and $\hat{\lambda} = 704.94$, quite similar to the maximum likelihood estimates.

Using Procedure 1 and Equation (2.6), we can also start with the authors' estimates of $\hat{\alpha} = 1.3388$ and $\hat{\lambda} = 590.33$. With $\boldsymbol{\Sigma}^{-1}$ as the matrix of weights, the process converges to estimates of $\hat{\alpha} = 1.4752$ and $\hat{\lambda} = 705.51$, again quite similar to the maximum likelihood estimates.

Using Procedure 2 produces estimates of $\hat{\alpha} = 1.4431$ and $\hat{\lambda} = 684.63$, somewhat removed from the maximum likelihood estimates, but still much closer to them than to the authors' minimum distance estimates.

To compare the different estimators directly, we will examine the asymptotic covariance matrices for each of the estimators at the maximum likelihood parameter estimates. Both Procedure 1 with Equation (2.6) and Procedure 2 have asymptotic covariance matrices of

$$\begin{bmatrix} 0.0020356 & 1.3594 \\ 1.3594 & 1{,}083.6 \end{bmatrix}.$$

These are slightly smaller than the maximum likelihood values. Procedure 1 with Equation (2.1) has an asymptotic covariance matrix of

$$\begin{bmatrix} 0.0020356 & 1.3594 \\ 1.3594 & 1{,}083.7 \end{bmatrix}.$$

The asymptotic variances of $\hat{\alpha}$ and the asymptotic covariances of $\hat{\alpha}$ with $\hat{\lambda}$ are identical in the preceding two matrices. This must be true, as the appendix shows. The asymptotic variance of $\hat{\lambda}$ is only very slightly higher in the second matrix than in the first.

Table 1 shows the weights that underlie the second matrix, generated by a $\mathbf{v}$ of

$$\begin{bmatrix} 0.000001 \\ 0.0007093 \end{bmatrix}.$$

One could multiply this vector by a nonzero factor without affecting the variances or covariance. However, the factor has to be positive to keep $\mathbf{A}$ positive definite. Note that several of the weights are negative, but that is not a problem here. Adjacent positive weights offset the two largest negative weights. Of course, assigning the weight within each pair to just one of the two adjacent values would yield virtually the same result.

The optimized weights decrease substantially as the c_is increase, in contrast to the uniform weights that Klugman and Parsa use. Clearly, the poor performance of the uniform weights results from an excessive amount of weight in the tail of the distribution. If one were simply to remove the weight at 100,000,000, the asymptotic covariance matrix at the maximum likelihood parameter estimates would improve to

$$\begin{bmatrix} 0.0069132 & 5.8391 \\ 5.8391 & 5{,}199.8 \end{bmatrix}.$$

Example One—Discussion

Table 2 shows the empirical limited expected values along with the fitted limited expected values for the maximum likelihood parameter estimates and the original minimum distance estimates with uniform weights. I confine attention to these here, since the other minimum distance estimates obtained are fairly close to the maximum likelihood estimates.

I suspect that most modelers would prefer the original minimum distance parameter estimates, since they provide a much closer fit in the tail at a modest cost in terms of the fit low in

TABLE 1

EXAMPLE 1 WEIGHTS

Limit	Original	Optimized
50	1	5509
100	1	1279
150	1	537
200	1	300
250	1	145
300	1	183
400	1	130
500	1	71
600	1	49
700	1	32
800	1	30
900	1	−4
1,000	1	52
1,500	1	36
2,000	1	18
2,500	1	−4
3,000	1	2.0
3,500	1	1.7
4,000	1	1.4
4,500	1	1.1
4,999	1	−76
5,000	1	77
6,000	1	1.3
7,500	1	1.3
9,999	1	160
10,000	1	−159
12,000	1	0.48
15,000	1	0.51
20,000	1	0.31
25,000	1	0.25
35,000	1	0.21
50,000	1	0.15
75,000	1	0.05
100,000	1	0.11
250,000	1	0.054
500,000	1	0.0011
1,000,000	1	0.0236
100,000,000	1	0.00087

TABLE 2

EXAMPLE 1 LIMITED EXPECTED VALUES

		Pareto		Mixed Exponential	
Limit	Empirical	Maximum Likelihood	Min Dist Unif Wts	Maximum Likelihood	Min Dist Unif Wts
50	48	48	47	48	48
100	92	91	90	91	91
150	133	130	129	131	131
200	170	166	164	168	168
250	203	199	196	202	202
300	235	230	226	233	233
400	291	285	280	288	289
500	338	333	327	336	337
600	379	376	368	378	379
700	415	414	406	415	416
800	448	448	439	447	449
900	477	479	469	477	478
1,000	504	507	497	503	505
1,500	610	619	607	607	610
2,000	686	698	687	682	686
2,500	745	758	748	740	744
3,000	792	806	797	787	792
3,500	831	844	838	826	831
4,000	864	877	873	860	865
4,500	893	905	903	889	894
4,999	919	929	929	914	919
5,000	920	929	929	914	919
6,000	962	969	973	957	963
7,500	1,014	1,015	1,025	1,008	1,014
9,999	1,079	1,069	1,087	1,071	1,078
10,000	1,079	1,069	1,087	1,071	1,078
12,000	1,117	1,100	1,124	1,110	1,118
15,000	1,163	1,135	1,168	1,156	1,164
20,000	1,222	1,176	1,219	1,213	1,221
25,000	1,264	1,204	1,256	1,254	1,263
35,000	1,318	1,242	1,308	1,308	1,319
50,000	1,367	1,277	1,357	1,353	1,367
75,000	1,408	1,309	1,406	1,393	1,408
100,000	1,433	1,329	1,437	1,416	1,430
250,000	1,511	1,377	1,518	1,481	1,514
500,000	1,587	1,401	1,565	1,540	1,592
1,000,000	1,662	1,418	1,602	1,592	1,650
100,000,000	1,662	1,458	1,713	1,618	1,669

the distribution. This is in spite of the fact that the original minimum distance estimator has much greater asymptotic variances than the maximum likelihood estimator. If one makes this judgment, then one is implicitly acknowledging that the assumption that the data comes from a Pareto distribution is not appropriate here. Otherwise, one would prefer the estimator with the smaller asymptotic variances.

This situation is quite common with parametric distributions. They often are not flexible enough to provide a high quality fit over the entire range of the data. In this case, there are alternatives to using minimum distance estimation with weights selected to trade off the quality of fit in one part of the distribution for another. One option would be to fit a parametric distribution to the upper section of the data only and to use the empirical distribution below that. Another option would be to use the semiparametric mixed exponential distribution, which is more flexible and thus better able to provide a good fit over the entire distribution in many situations. The mixed exponential distribution generally works very well with loss distributions, as I discussed in detail in Keatinge [3].

Table 2 shows the fitted limited expected values for mixed exponential distributions fit using maximum likelihood estimation and minimum distance estimation with uniform weights. The means and weights of the exponential distributions in each mixture are as follows:

Maximum Likelihood		Minimum Distance Uniform Weights	
Mean	Weight	Mean	Weight
398	0.659077	394	0.648798
1,326	0.215884	1,405	0.259393
3,097	0.088849	4,446	0.067363
12,285	0.030721	18,513	0.023568
36,128	0.004935	356,076	0.000878
445,785	0.000535		

Each of these provides an excellent fit over the entire range of the data. In this comparison of limited expected values, the minimum distance estimator provides a closer fit than the maximum likelihood estimator because it uses the empirical limited expected values directly, whereas the maximum likelihood estimator uses the number of losses that fall in each interval. Since the mixed exponential distribution is flexible enough to provide a good fit over the entire distribution, it is not very sensitive to the choice of weights.

Example Two—Improving the Efficiency of the Minimum Distance Estimator

The second example involves 463 medical malpractice claim report lags truncated from above and fit to a Burr distribution. Grouped maximum likelihood estimation yields parameter estimates of $\hat{\alpha} = 0.40274$, $\hat{\lambda} = 34.224$, and $\hat{\tau} = 3.1181$. The asymptotic covariance matrix for these parameter values is

$$\begin{bmatrix} 0.017336 & 0.57436 & -0.035566 \\ 0.57436 & 20.6558 & -1.21351 \\ -0.035566 & -1.21351 & 0.10703 \end{bmatrix}.$$

Klugman and Parsa then use minimum distance estimation with the cumulative distribution function. They use weights of 4 where the empirical cumulative distribution function is less than 0.5, and the reciprocal of the empirical variance where the cumulative distribution function is greater than 0.5. (The weight at lag 162 is set equal to the weight at lag 156, since the empirical cumulative distribution function at lag 162 is 1.) This might or might not produce good results, but there is no particular theoretical justification for it, since it does not take into account the correlation among the values of the empirical functional. These weights yield parameter estimates of $\hat{\alpha} = 0.48800$, $\hat{\lambda} = 36.989$, and $\hat{\tau} = 2.9495$. The authors give the asymptotic covariance matrix for these parameter values as

$$\begin{bmatrix} 0.081077 & 2.6655 & -0.16625 \\ 2.6655 & 89.507 & -5.5313 \\ -0.16625 & -5.5313 & 0.33525 \end{bmatrix}.$$

Making the correction to the matrix $\mathbf{A}$ noted in the previous section yields a corrected asymptotic covariance matrix of

$$\begin{bmatrix} 0.055717 & 1.8069 & -0.10330 \\ 1.8069 & 60.474 & -3.3933 \\ -0.10330 & -3.3933 & 0.22450 \end{bmatrix}.$$

These values are substantially higher than the maximum likelihood values. A fairer comparison would be to compare the asymptotic covariance matrices at the same parameter values. The minimum distance asymptotic covariance matrix at the maximum likelihood parameter estimates is

$$\begin{bmatrix} 0.039702 & 1.3327 & -0.10037 \\ 1.3327 & 46.371 & -3.4111 \\ -0.10037 & -3.4111 & 0.29501 \end{bmatrix}.$$

These values are still significantly higher than the maximum likelihood values.

We now try the procedures described in the previous section. Using Procedure 1 and Equation (2.1), we can start with the authors' estimates of $\hat{\alpha} = 0.48800$, $\hat{\lambda} = 36.989$, and $\hat{\tau} = 2.9495$. We define $h(\boldsymbol{\theta})$ to be the parameter α and proceed with the iterative process until it converges to estimates of $\hat{\alpha} = 0.40253$, $\hat{\lambda} = 34.205$, and $\hat{\tau} = 3.1270$, quite similar to the maximum likelihood estimates.

Procedure 1, with Equation (2.6) and $\boldsymbol{\Sigma}^{-1}$ as the matrix of weights, does not converge. If it did, it would yield the maximum likelihood estimates. Procedure 2 produces minimum chi-square estimates of $\hat{\alpha} = 0.36995$, $\hat{\lambda} = 33.702$, and $\hat{\tau} = 2.8685$.

Procedure 1 with Equation (2.1), at the maximum likelihood parameter estimates, has an asymptotic covariance matrix of

$$\begin{bmatrix} 0.017336 & 0.57436 & -0.035566 \\ 0.57436 & 20.6564 & -1.21355 \\ -0.035566 & -1.21355 & 0.10704 \end{bmatrix}.$$

As must be true, the asymptotic variance of $\hat{\alpha}$ and the asymptotic covariances of $\hat{\alpha}$ with $\hat{\lambda}$ and $\hat{\tau}$ are identical to the maximum likelihood (and minimum chi-square) values. The other entries are only very slightly higher than the maximum likelihood (and minimum chi-square) values. Table 3 shows the weights that underlie the matrix, generated by a **v** of

$$\begin{bmatrix} 1 \\ 34.25 \\ -2.229 \end{bmatrix}.$$

Example Two—Discussion

Table 4 shows the empirical cumulative distribution function along with the fitted cumulative distribution function for the maximum likelihood parameter estimates, the original minimum distance estimates, and the minimum chi-square estimates. If one believes that a Burr distribution is appropriate, then one should prefer the maximum likelihood or minimum chi-square estimators, since they have smaller asymptotic variances.

None of the distributions provides a particularly good fit very low in the distribution. If one does not believe that a Burr distribution is appropriate over the entire range of the data, one could fit that distribution only above a certain point and use an empirical distribution below that. The mixed exponential distribution always has a mode at zero, and since the data clearly shows a mode significantly greater than zero, the mixed exponential would not fit well over the entire range of the data. However, one could fit the mixed exponential to the section of the distribution to the right of the mode.

TABLE 3

EXAMPLE 2 WEIGHTS

Lag	Original	Optimized
6	4.0	2195
12	4.0	309
18	4.0	112
24	4.0	59.4
30	4.0	42.0
36	4.0	31.1
42	4.0	24.7
48	4.0	20.5
54	4.1	17.6
60	4.2	13.6
66	4.5	15.2
72	4.9	14.0
78	5.5	13.3
84	6.2	13.0
90	6.8	12.8
96	7.4	12.8
102	8.1	13.1
108	9.7	13.5
114	10.6	14.2
120	11.0	15.2
126	12.7	16.6
132	21.2	18.6
138	26.8	21.5
144	47.3	25.9
150	58.9	33.4
156	232.5	48.6
162	232.5	94.5

At the conclusion of the second example, Klugman and Parsa show numbers implying that an approximate 95% confidence interval for the number of claims that will be reported after Lag 168 is $72 + / - 15$ for the maximum likelihood estimator, and $59 + / - 20$ for the minimum distance estimator with their set of weights. These are incorrect. The actual confidence intervals should be $72 + / - 57$ and $59 + / - 61$, respectively. For the minimum chi-square estimator, the confidence interval is $102 + / - 89$. The lengths of these confidence intervals indicate

TABLE 4

EXAMPLE 2 CUMULATIVE DISTRIBUTION FUNCTIONS

		Burr			Weibull
Lag	Empirical	Maximum Likelihood	Min Dist Original Wts	Minimum Chi-square	Maximum Likelihood
6	0.0086	0.0020	0.0026	0.0032	0.0159
12	0.0216	0.0173	0.0194	0.0226	0.0513
18	0.0389	0.0574	0.0604	0.0672	0.1000
24	0.1210	0.1257	0.1276	0.1365	0.1585
30	0.2181	0.2142	0.2139	0.2212	0.2235
36	0.2959	0.3101	0.3079	0.3102	0.2924
42	0.4298	0.4025	0.3998	0.3955	0.3628
48	0.5011	0.4860	0.4838	0.4729	0.4327
54	0.5637	0.5585	0.5576	0.5411	0.5005
60	0.6156	0.6207	0.6212	0.6006	0.5649
66	0.6631	0.6736	0.6754	0.6521	0.6250
72	0.7149	0.7188	0.7216	0.6968	0.6802
78	0.7603	0.7574	0.7611	0.7357	0.7301
84	0.7970	0.7907	0.7949	0.7698	0.7746
90	0.8207	0.8195	0.8241	0.7998	0.8138
96	0.8402	0.8447	0.8493	0.8263	0.8478
102	0.8553	0.8668	0.8714	0.8498	0.8770
108	0.8834	0.8863	0.8907	0.8709	0.9019
114	0.8942	0.9036	0.9077	0.8898	0.9227
120	0.8985	0.9190	0.9229	0.9069	0.9401
126	0.9136	0.9329	0.9363	0.9224	0.9544
132	0.9503	0.9454	0.9484	0.9365	0.9660
138	0.9611	0.9567	0.9592	0.9494	0.9754
144	0.9784	0.9669	0.9690	0.9612	0.9829
150	0.9827	0.9763	0.9778	0.9721	0.9889
156	0.9957	0.9849	0.9859	0.9821	0.9936
162	1.0000	0.9927	0.9933	0.9914	0.9972
168	1.0000	1.0000	1.0000	1.0000	1.0000

that the volume of data is not sufficient to provide a reliable estimate of the number of claims that will be reported after Lag 168, even if one accepts the assumption that a Burr distribution is appropriate for this data.

Accomando and Weissner [1] suggest using a Weibull distribution for this data. Maximum likelihood estimation yields parameter estimates of $\hat{\theta} = 67.3$ and $\hat{\tau} = 1.71$, with the cumulative distribution function expressed as $F(x) = 1 - e^{-(x/\theta)^{\tau}}$. Table 4 shows the fitted cumulative distribution function. The approximate 95% confidence interval for the number of claims that will be reported after Lag 168 is $4 + / - 3$. The reason that this is so different from the Burr confidence intervals is that the confidence intervals depend on the assumption that a particular distribution is appropriate over the entire range of the distribution, including the portion for which we do not yet have data. There is no way to tell whether a Burr distribution, a Weibull distribution, or some other distribution is most appropriate beyond the range of the data. Attempting to extrapolate from the data to obtain the number of unreported claims, without reference to other experience for which claims after Lag 168 have been observed, is likely to lead to a very unreliable estimate.

The data in this example is truncated at a single point, and though that makes the data of limited use for estimation beyond the truncation point, adjusting for the truncation in the minimum distance estimation procedure is straightforward. Likewise, data with a single censorship point does not present difficulties. However, with data that contains multiple truncation or censorship points on the left or the right, constructing the empirical distribution becomes more complicated. The most logical approach is to use the Kaplan-Meier Product-Limit estimator.

4. GOODNESS-OF-FIT TESTS

Klugman and Parsa propose a goodness-of-fit test using the statistic

$$(\mathbf{G}_n - \mathbf{G})'\mathbf{W}^{1/2}\{n^{-1}\mathbf{W}^{1/2}[\mathbf{I} - \mathbf{D}'(\mathbf{DWD}')^{-1}\mathbf{DW}] \\ \times \boldsymbol{\Sigma}[\mathbf{I} - \mathbf{WD}'(\mathbf{DWD}')^{-1}\mathbf{D}]\mathbf{W}^{1/2}\}^{-}\mathbf{W}^{1/2}(\mathbf{G}_n - \mathbf{G}),$$

where $\mathbf{W}$ is a matrix of the weights and "$^{-}$" indicates a generalized inverse. If the distribution being fit is the correct one, this statistic has an asymptotic chi-square distribution with $k-p$ degrees of freedom. The statistic

$$(\mathbf{G}_n - \mathbf{G})'\{n^{-1}[\mathbf{I} - \mathbf{D}'(\mathbf{DWD}')^{-1}\mathbf{DW}] \\ \times \boldsymbol{\Sigma}[\mathbf{I} - \mathbf{WD}'(\mathbf{DWD}')^{-1}\mathbf{D}]\}^{-}(\mathbf{G}_n - \mathbf{G})$$

also has an asymptotic chi-square distribution with $k-p$ degrees of freedom. This statistic does not contain the square root of $\mathbf{W}$, which could be messy when $\mathbf{W}$ is not a diagonal matrix. In their proof, Klugman and Parsa use a vector $\mathbf{V}_n = \mathbf{W}^{1/2}(\mathbf{G}_n - \mathbf{G})$ and a matrix $\mathbf{R} = \mathbf{W}^{1/2}\mathbf{D}'$. By leaving off the $\mathbf{W}^{1/2}$ in this vector and matrix, one can use the same reasoning to obtain the alternate statistic.

If one uses Procedure 1 with Equation (2.6) and $\boldsymbol{\Sigma}^{-1}$ as the matrix of weights, or if one uses Procedure 2, $n(\mathbf{G}_n - \mathbf{G})' \cdot \boldsymbol{\Sigma}^{-1}(\mathbf{G}_n - \mathbf{G})$ has an asymptotic chi-square distribution with $k-p$ degrees of freedom. This follows either from using $n\mathbf{I}$ as the generalized inverse in the authors' statistic or $n\boldsymbol{\Sigma}^{-1}$ as the generalized inverse in the alternate statistic. If G is the cumulative distribution function, this is the standard chi-square goodness-of-fit statistic.

With $n\mathbf{W}^{-1/2}\boldsymbol{\Sigma}^{-1}[\mathbf{I} - \mathbf{D}'(\mathbf{D}\boldsymbol{\Sigma}^{-1}\mathbf{D}')^{-1}\mathbf{D}\boldsymbol{\Sigma}^{-1}]\mathbf{W}^{-1/2}$ as the generalized inverse in the authors' statistic (the Moore-Penrose inverse) or $n\boldsymbol{\Sigma}^{-1}[\mathbf{I} - \mathbf{D}'(\mathbf{D}\boldsymbol{\Sigma}^{-1}\mathbf{D}')^{-1}\mathbf{D}\boldsymbol{\Sigma}^{-1}]$ as the generalized inverse in the alternate statistic, one finds that

$$n(\mathbf{G}_n - \mathbf{G})'\{\boldsymbol{\Sigma}^{-1}[\mathbf{I} - \mathbf{D}'(\mathbf{D}\boldsymbol{\Sigma}^{-1}\mathbf{D}')^{-1}\mathbf{D}\boldsymbol{\Sigma}^{-1}]\}(\mathbf{G}_n - \mathbf{G})$$

has an asymptotic chi-square distribution with $k-p$ degrees of freedom, which is independent of the weights used. If G is the cumulative distribution function, this is the statistic given by Moore [5, p. 90] as applicable with the maximum likelihood estimator and the minimum chi-square estimator, among others.

Regardless of which generalized inverse one uses, the tests in this section are valid as long as the weights in the test statistic are consistent with the weights used in fitting. One may fix the weights beforehand, or derive them as in Procedure 1 or Procedure 2. The tests are valid even if the weights are suboptimal. If the test statistic exceeds its critical value, that indicates a problem with the selected distribution, not, as the authors imply, with the weights. The weights may indeed be poorly chosen and thus give large asymptotic variances, but that does not affect the validity of the test.

5. CONCLUSION

Minimum distance estimation has some interesting properties, but as a practical matter, I see little reason to prefer it to maximum likelihood estimation. The main purported advantage of minimum distance estimation is that, through adjustment of the weights, it can provide a closer fit to the parts of the distribution that are of the most interest. This leads to an estimator with a larger variance than the maximum likelihood estimator, however. And, if one believes that the model one is using is appropriate, one should prefer the estimator with the smaller variance.

Minimum distance estimation is a clumsy remedy for a model that is not flexible enough. Instead of resorting to minimum distance estimation, I believe one would be better off addressing the inadequacies of the model itself. One possible option is to fit a parametric distribution to the upper section of the data only and to use the empirical distribution below that. Another possible option is to use the semiparametric mixed exponential distribution.

Minimum distance estimation performed with weights selected to achieve high efficiency generally produces parameter estimates close to the maximum likelihood estimates. But maximum likelihood estimation is usually somewhat easier to implement than minimum distance (or minimum chi-square) estimation, especially if the data contains multiple truncation or censorship points. Minimum distance estimation would be most useful in situations where maximum likelihood estimation is not feasible, such as when limited expected values are the only data available.

REFERENCES

[1] Accomando, Frank W., and Edward W. Weissner, "Report Lag Distributions: Estimation and Application to IBNR Counts," *Transcripts of the 1988 Casualty Loss Reserve Seminar*, 1988, pp. 1038–1133.

[2] Benichou, Jacques, and Mitchell H. Gail, "A Delta Method for Implicitly Defined Random Variables," *The American Statistician* 43, no. 1 (February 1989), pp. 41–44.

[3] Keatinge, Clive L., "Modeling Losses with the Mixed Exponential Distribution," *PCAS* LXXXVI, 1999, pp. 654–698.

[4] Luong, A., and M. E. Thompson, "Minimum-distance Methods based on Quadratic Distances for Transforms," *The Canadian Journal of Statistics* 15, no. 3 (September 1987), pp. 239–251.

[5] Moore, David S., "Chi-square Tests," *Studies in Statistics*, editor Robert V. Hogg, The Mathematical Association of America, 1978, pp. 66–106.

APPENDIX

Here I find the sets of weights that minimize the asymptotic variance of $h(\hat{\boldsymbol{\theta}})$, given by

$$n^{-1}\mathbf{d}'\mathbf{A}^{-1}\mathbf{B\Sigma B}'\mathbf{A}^{-1}\mathbf{d} = n^{-1}\mathbf{d}'(\mathbf{DWD}')^{-1}\mathbf{DW\Sigma WD}'(\mathbf{DWD}')^{-1}\mathbf{d}. \tag{A.1}$$

The matrix $\mathbf{W}$ is a symmetric matrix of weights, which may or may not be a diagonal matrix. One can express any set of weights as a symmetric matrix by allocating the weight assigned to each off-diagonal term equally to both sides of the diagonal. I assume that $\boldsymbol{\Sigma}$ has rank k, $\mathbf{D}$ has rank p, and $\mathbf{d}$ has at least one nonzero element.

The first step is to take the derivative with respect to each entry in $\mathbf{W}$. The derivative of $(\mathbf{DWD}')(\mathbf{DWD}')^{-1}$ is zero. Therefore, by the product rule for differentiation, $(\mathbf{DWD}')$ times the derivative with respect to a particular entry in $\mathbf{W}$ within $(\mathbf{DWD}')^{-1}$ must be equal to the negative of the derivative with respect to that entry within $(\mathbf{DWD}')$ times $(\mathbf{DWD}')^{-1}$. Thus, using the product rule and the symmetry of (A.1), the derivative with respect to the ijth entry in $\mathbf{W}$ is

$$n^{-1}\mathbf{d}'(\mathbf{DWD}')^{-1}\mathbf{D}(\mathbf{1}_{ij} + \mathbf{1}_{ji}) \times [\mathbf{I} - \mathbf{D}'(\mathbf{DWD}')^{-1}\mathbf{DW}]\mathbf{\Sigma WD}'(\mathbf{DWD}')^{-1}\mathbf{d},$$

where $\mathbf{1}_{ij}$ indicates a $k \times k$ matrix with the ijth entry equal to $\mathbf{1}$ and the remaining entries equal to 0.

Since the derivative with respect to all entries in $\mathbf{W}$ must be 0 for (A.1) to be at a minimum, and since the expression in brackets is idempotent with a nullspace consisting of the p columns of $\mathbf{D}'$, $\mathbf{\Sigma WD}'(\mathbf{DWD}')^{-1}\mathbf{d}$ must be in the column space of $\mathbf{D}'$ or $\mathbf{\Sigma WD}'(\mathbf{DWD}')^{-1}\mathbf{d} = \mathbf{D}'\mathbf{u}$, where $\mathbf{u}$ is a vector of length p. Multiplying both sides by $(\mathbf{D\Sigma}^{-1}\mathbf{D}')^{-1}\mathbf{D\Sigma}^{-1}$ yields $\mathbf{u} = (\mathbf{D\Sigma}^{-1}\mathbf{D}')^{-1}\mathbf{d}$. Thus,

$$\mathbf{WD}'(\mathbf{DWD}')^{-1}\mathbf{d} = \mathbf{\Sigma}^{-1}\mathbf{D}'(\mathbf{D\Sigma}^{-1}\mathbf{D}')^{-1}\mathbf{d}. \tag{A.2}$$

Substituting (A.2) into (A.1) shows that the minimum asymptotic variance of $h(\hat{\boldsymbol{\theta}})$ is $n^{-1}\mathbf{d}'(\mathbf{D\Sigma}^{-1}\mathbf{D}')^{-1}\mathbf{d}$.

This also shows that if the asymptotic variance of $h(\hat{\boldsymbol{\theta}})$ is at its minimum value, then the asymptotic covariance of $h(\hat{\boldsymbol{\theta}})$ with any other function $h^*(\hat{\boldsymbol{\theta}})$ is $n^{-1}\mathbf{d}^{*\prime}(\mathbf{D\Sigma}^{-1}\mathbf{D}')^{-1}\mathbf{d}$, where $\mathbf{d}^*$ is the vector of length p with jth element $\partial h^*/\partial\theta_j$. However, this does not imply that the asymptotic variance of $h^*(\hat{\boldsymbol{\theta}})$ is $n^{-1}\mathbf{d}^{*\prime}(\mathbf{D\Sigma}^{-1}\mathbf{D}')^{-1}\mathbf{d}^*$.

Multiplying both sides of (A.2) by $(\mathbf{WD}')^-$, a left-inverse of $\mathbf{WD}'$, yields $(\mathbf{DWD}')^{-1}\mathbf{d} = (\mathbf{WD}')^-\mathbf{\Sigma}^{-1}\mathbf{D}'(\mathbf{D\Sigma}^{-1}\mathbf{D}')^{-1}\mathbf{d}$. Substituting this into (A.2) yields $[\mathbf{I} - \mathbf{WD}'(\mathbf{WD}')^-]\mathbf{\Sigma}^{-1}\mathbf{D}'(\mathbf{D\Sigma}^{-1}\mathbf{D}')^{-1}\mathbf{d} = \mathbf{0}$. The expression in brackets is idempotent with a nullspace consisting of the p columns of $\mathbf{WD}'$, so $\mathbf{\Sigma}^{-1}\mathbf{D}'(\mathbf{D\Sigma}^{-1}\mathbf{D}')^{-1}\mathbf{d}$ must be in the column space of $\mathbf{WD}'$ or $\mathbf{\Sigma}^{-1}\mathbf{D}'(\mathbf{D\Sigma}^{-1}\mathbf{D}')^{-1}\mathbf{d} = \mathbf{WD}'\mathbf{v}$, where $\mathbf{v}$ is a vector of length p.

If $\mathbf{W}$ must be a diagonal matrix, then

$$w_i = (\mathbf{\Sigma}^{-1}\mathbf{D}'(\mathbf{D\Sigma}^{-1}\mathbf{D}')^{-1}\mathbf{d})_i/(\mathbf{D}'\mathbf{v})_i,$$

where $\mathbf{v}$ is an arbitrary nonzero vector. Thus, the equation can be satisfied for weights associated with a space of dimension p. In general, a set of weights cannot satisfy this equation for all functions $h(\boldsymbol{\theta})$, unless $k = p + 1$. It is possible in this case because the intersection of p spaces of dimension p within a space of dimension $p + 1$ has a dimension of at least 1.

If $\mathbf{W}$ may be a full symmetric matrix, then there are many $\mathbf{W}$s that will satisfy this equation. If $\mathbf{WD}'$ has the same column space as $\mathbf{\Sigma}^{-1}\mathbf{D}'(\mathbf{D\Sigma}^{-1}\mathbf{D}')^{-1}$, then $\mathbf{W}$ can satisfy this equation for all functions $h(\boldsymbol{\theta})$. The most obvious choice for $\mathbf{W}$ with this property is $\mathbf{\Sigma}^{-1}$.

For a set of weights to produce minimum asymptotic variance, $\mathbf{A} = \mathbf{DWD}'$ must be positive definite, whether $\mathbf{W}$ is a diagonal or a full symmetric matrix. If $\mathbf{W} = \mathbf{\Sigma}^{-1}$, then since $\mathbf{D\Sigma}^{-1}\mathbf{D}'$ is positive definite, minimum asymptotic variance is achieved for all functions $h(\boldsymbol{\theta})$.

Results specific to the cumulative distribution function

If G is the cumulative distribution function, then $\boldsymbol{\Sigma}^{-1}$ is a tridiagonal matrix with

$$\boldsymbol{\Sigma}_{ii}^{-1} = \frac{G_{i+1} - G_{i-1}}{(G_i - G_{i-1})(G_{i+1} - G_i)} \qquad \text{and}$$

$$\boldsymbol{\Sigma}_{i,i-1}^{-1} = \boldsymbol{\Sigma}_{i-1,i}^{-1} = \frac{-1}{G_i - G_{i-1}},$$

where $G_0 = 0$ and $G_{k+1} = 1$.

* * * * *

If the matrix of weights is $\boldsymbol{\Sigma}^{-1*}$, based on a given cumulative distribution function, then the objective function is proportional to

$$\begin{aligned}
& n(\mathbf{G}_n - \mathbf{G})'\boldsymbol{\Sigma}^{-1*}(\mathbf{G}_n - \mathbf{G}) \\
&= n\left[\sum_{i=1}^{k} \frac{(G_{i+1}^* - G_{i-1}^*)(G_{n,i} - G_i)^2}{(G_i^* - G_{i-1}^*)(G_{i+1}^* - G_i^*)}\right. \\
&\qquad \left. -2\sum_{i=2}^{k} \frac{(G_{n,i} - G_i)(G_{n,i-1} - G_{i-1})}{G_i^* - G_{i-1}^*}\right] \\
&= n\left[\sum_{i=1}^{k} \frac{(G_{n,i} - G_i)^2}{G_i^* - G_{i-1}^*} + \sum_{i=2}^{k+1} \frac{(G_{n,i-1} - G_{i-1})^2}{G_i^* - G_{i-1}^*}\right. \\
&\qquad \left. -2\sum_{i=2}^{k} \frac{(G_{n,i} - G_i)(G_{n,i-1} - G_{i-1})}{G_i^* - G_{i-1}^*}\right] \\
&= n\sum_{i=1}^{k+1} \frac{[(G_{n,i} - G_{n,i-1}) - (G_i - G_{i-1})]^2}{G_i^* - G_{i-1}^*}.
\end{aligned}$$

If one treats the matrix of weights as a function of the parameters as in Procedure 2 from Section 2, then $\mathbf{G}$ and $\mathbf{G}^*$ are identical, and the objective function is proportional to the chi-square function.

If the matrix of weights is fixed, then one finds the minimum of the objective function by taking the derivative of the numerator of each of the terms with respect to each of the parameters and finding the point at which all of the derivatives are equal to zero. The derivative with respect to the jth parameter is

$$-2n\sum_{i=1}^{k+1}\frac{[(G_{n,i}-G_{n,i-1})-(G_i-G_{i-1})](G_i^{(j)}-G_{i-1}^{(j)})}{G_i^*-G_{i-1}^*}.$$

With Procedure 1 from Section 2, $\mathbf{G}$ and $\mathbf{G}^*$ must be identical at the final parameter estimates. At that point, the expression reduces to

$$-2n\sum_{i=1}^{k+1}(G_{n,i}-G_{n,i-1})\frac{(G_i^{(j)}-G_{i-1}^{(j)})}{G_i-G_{i-1}},$$

which is proportional to the derivative of the grouped log-likelihood function

$$n\sum_{i=1}^{k+1}(G_{n,i}-G_{n,i-1})\ln(G_i-G_{i-1})$$

with respect to the jth parameter.

* * * * *

The jlth entry of the inverse of the minimum asymptotic covariance matrix is

$$(n\mathbf{D}\boldsymbol{\Sigma}^{-1}\mathbf{D}')_{jl}$$
$$= n\left[\sum_{i=1}^{k}\frac{(G_{i+1}-G_{i-1})G_i^{(j)}G_i^{(l)}}{(G_i-G_{i-1})(G_{i+1}-G_i)}-\sum_{i=2}^{k}\frac{G_i^{(j)}G_{i-1}^{(l)}+G_i^{(l)}G_{i-1}^{(j)}}{G_i-G_{i-1}}\right]$$

$$= n\left[\sum_{i=1}^{k}\frac{G_i^{(j)}G_i^{(l)}}{G_i - G_{i-1}} + \sum_{i=2}^{k+1}\frac{G_{i-1}^{(j)}G_{i-1}^{(l)}}{G_i - G_{i-1}} - \sum_{i=2}^{k}\frac{G_i^{(j)}G_{i-1}^{(l)} + G_i^{(l)}G_{i-1}^{(j)}}{G_i - G_{i-1}}\right]$$

$$= n\sum_{i=1}^{k+1}\frac{(G_i^{(j)} - G_{i-1}^{(j)})(G_i^{(l)} - G_{i-1}^{(l)})}{G_i - G_{i-1}}.$$

Then, to confirm that this is identical to the grouped maximum likelihood value, the second derivative with respect to the jth and lth parameters of the grouped loglikelihood function $n\sum_{i=1}^{k+1}(G_{n,i} - G_{n,i-1})\ln(G_i - G_{i-1})$ is

$$n\sum_{i=1}^{k+1}(G_{n,i} - G_{n,i-1})\left[\frac{G_i^{(j,l)} - G_{i-1}^{(j,l)}}{G_i - G_{i-1}} - \frac{(G_i^{(j)} - G_{i-1}^{(j)})(G_i^{(l)} - G_{i-1}^{(l)})}{(G_i - G_{i-1})^2}\right],$$

and its negative expectation is

$$n\sum_{i=1}^{k+1}\frac{(G_i^{(j)} - G_{i-1}^{(j)})(G_i^{(l)} - G_{i-1}^{(l)})}{G_i - G_{i-1}}.$$

ADDRESS TO NEW MEMBERS—NOVEMBER 15, 2004

ALICE H. GANNON

It is a pleasure and honor to be here this morning representing the members of the CAS in welcoming our 126 new members and congratulating all 221 new designees. Each of you have earned your place in a society of professionals that is greatly admired and respected throughout the world, maybe not by hundreds of millions of people, but certainly by those whose own professional work gives them the opportunity to be familiar with the work of the CAS and its members. I am very proud to be a member of the CAS and I hope you are as well.

You achieved your new designation through a great deal of hard work, focused application of your natural ability, and for most of you, a lot of support from long-suffering family members and friends. Your top priority for this meeting should be to savor with true joy your accomplishment. Right now you are surrounded by hundreds of others who fully appreciate what you have been through and how significant your accomplishment is. In fact, this is probably the first time since passing your last exam that you have been in a room with so many others who can fully appreciate your accomplishment. Take advantage of that! Now is the perfect time to celebrate your success!

It is also a good time to reflect a bit on the fact that you did not achieve your designation only because of your efforts and support of family and friends. There were many others who developed the body of knowledge that you studied and learned; who wrote the papers and books that communicated that knowledge to you; who developed the learning objectives for becoming a good casualty actuary; who wrote, administered, and graded the exams through which you gained your designation. It is because of all their work as well as your own that you are here today.

You have received a valuable gift from them. It is now your privilege to use it and your obligation to use it well. To paraphrase Spiderman: With great knowledge comes great responsibility. You have demonstrated that your knowledge is great, so it is now your responsibility to use that knowledge with integrity and honor.

Though I just noted that your knowledge is great, I also know that it is not complete. The learning process is never over; we always have new things to learn. So I thought I would spend the rest of my allotted time this morning sharing with you just five of the lessons I have learned about the CAS and the actuarial profession that I did not know when I first became a member.

1. The members of the Casualty Actuarial Society Examination Committee are not evil. In fact the members of the CAS Exam Committee are very nice human beings and remarkably dedicated to providing the best possible way to determine qualified casualty actuaries. The fact that the exams remain an imperfect way to qualify actuaries is not because the CAS members and CAS staff involved in the exam process *want* to torment actuarial students.

2. The best way to learn for yourself that #1 is true, as well as to get over any bitterness you might still have about the exam process, is to join the Exam Committee. I assure you that serving on the Exam Committee will change your perception of the exam process for the better. Not only will it help you overcome any ill will you may still be harboring toward the members of the Exam Committee, but you might be the one who can help make a fundamental breakthrough in finding better ways to accurately qualify casualty actuaries in the future.

3. Life after exams is great! However, there still isn't enough time to do all the things your really want to do. You will find that the exams were *not* responsible for everything you missed out on during the last few years. What's more, you will now find there are things you really don't want to do but you no longer have that

great excuse, “I have to study,” to relieve you from doing them. Remember those long-suffering family members and friends who supported you during the exams? It’s now pay-back time!

4. Adhering to the Code of Professional Conduct of the CAS is not a passive activity. It is not just about avoiding doing bad things. There will be times in your career that the Code will require from you an exceptional amount of effort and fortitude to do what is right and to do it well. LeRoy Simon said in his address to new members a few years ago: “It is easier to become an actuary than to be one.” He’s right.

But the fifth and last lesson I will share with you this morning is that Ruth Salzmann was also right when, after quoting LeRoy Simon “It is easier to become an actuary than to be one,” she added, “But being one is a lot more fun!”

Our work as casualty actuaries is intellectually challenging, dynamic, rewarding, highly respected, and meaningful. I am pretty sure that the folks who write the *Jobs Rated Almanac* have not been ranking Actuary as the number one or number two job for over 15 years based on life as an actuarial student but rather on life as a credentialed actuary. We actuaries are incredibly lucky to get to do such important, wonderful work and to be rewarded so handsomely for doing it.

So as you celebrate the successful achievement of your ACAS or FCAS designation today, you should also celebrate that you have a great future as an actuary and a member of the CAS ahead of you.

And I plan to celebrate right along with you. Congratulations and best wishes!

PRESIDENTIAL ADDRESS—NOVEMBER 15, 2004

MILESTONES AND FORKS IN THE ROAD

MARY FRANCES MILLER

Two years ago, then CAS President Bob Conger used his presidential address to challenge the CAS as a whole and each of us individually to embark on a journey into the future, into the actuarial world of the 21st century, with a little help from Dr. Seuss: "You have brains in your head. You have feet in your shoes. You can steer yourself any direction you choose."

Over the past two years we have done much. We have selected a destination for our next decade and have set out on the road. We have reached the first few milestones and identified some landmarks to expect along the way. We have also discovered that the path can be rocky at times, and the road can be steep. There will be crossroads, some perhaps sooner than we had expected. Our choices at the crossroads will determine how we approach our destination and indeed, whether we get there at all. We will not complete the journey alone, and our choices for traveling companions will also affect both the trip and the destination.

First, happy birthday, CAS! The ninetieth anniversary of the first meeting of the CAS was just eight days ago. A milestone on our journey, to be sure. Our destination, our Centennial Goal, is but ten years away. Let's review that goal. In 2014,

> The CAS will be globally recognized as the preeminent resource in educating casualty actuaries and conducting research in casualty actuarial science. CAS members will be recognized as the leading experts in the evaluation of hazard risk and the integration of hazard risk with strategic, financial, and operational risk.

GLOBALIZATION

We have made tremendous progress toward the first part of the goal. In 2003, we amended the constitution to allow for mutual recognition agreements with societies that have rigorous specialty tracks in property/casualty insurance. Constructing the agreements themselves has been a bit of a challenge, but I am happy to report that, with the anticipated approvals of our board on Wednesday and the Councils of the Institute and Faculty of Actuaries next month, we expect to execute our first mutual recognition agreements early in 2005.

Our active participation in the International Actuarial Association, in particular our contributions on international accounting issues led by Ralph Blanchard, one of our Above and Beyond Achievement Award winners, have elevated the recognition of the CAS around the world. Yesterday, leaders representing more than 80 percent of the worldwide profession met here in Montréal as guests of the CAS and the Canadian Institute of Actuaries.

Throughout the world, the number of casualty actuaries is growing rapidly, as is the interest in casualty actuarial science. Internally, when our Executive Council admits the next class of Associates in February, membership in the CAS will top 4,000. Externally, just to cite two examples, there were nearly 400 participants at the U.K.'s general insurance conference last month, and the U.K. profession has some 700 actuaries practicing in the property and casualty field. In China, the number of students taking actuarial exams is mushrooming, and more and more of those candidates are sitting the mid-level CAS exams.

With the rise of the profession in China, we may have a unique opportunity to partner with the Society of Actuaries in China. Although Chinese candidates currently sit for the Society of Actuaries (U.S.) and CAS exams, the Chinese Society—quite properly—has expressed its intent to offer its own exams as soon as possible. Rather than reinvent the wheel, we have suggested to the Chinese that they consider joint sponsorship of casualty exams with the CAS. Such a plan would require contributions from our side, to be sure, particularly translation assistance from our bilingual members, but the result would be a direct dual membership for CAS candidates who elect to write Part 7-China. This is a crossroads on our journey. Which fork shall we take?

We have had requests from around the globe to assist in the training of casualty actuaries. The common theme has been a pressing need for practical training, and Bob Conger headed up a task force this year to develop training modules from which we can pick and choose content, so that we do not have to start from scratch to respond to each request. The first rollout was in Kazakhstan in September. With some fine-tuning, we are prepared to answer the call for seminars wherever they are needed. And who could be better suited than the CAS to assist the property/casualty industry in developing economies? Our Society began with the introduction of a new insurance product, a desperate need to rate the product fairly, and very little data. The first paper in our first *Proceedings*, "Scientific Methods of Computing Compensation Rates" by our first president, I. M. Rubinow, deals directly with how to rate a product in the absence of credible data.

Seminars, however, will not meet the long-term need for casualty actuaries. The World Bank has identified a crisis in third-party liability insurance as one of the leading threats to financial services stability in the developing world. The lack of qualified regulators and actuaries, coupled with a paucity of data, are a recipe for disaster. There is an immediate, pressing need to develop an actuarial profession in the emerging nations, and with World Bank funding, the International Actuarial Association hopes to develop a training program that can be replicated around the world. Graduates of the highest levels of the program will be recognized worldwide as qualified actuaries. Is this a threat to the CAS Centennial Goal, or an opportunity to take a great leap forward? If we embrace the initiative and volunteer our resources in a leadership role in the development of the program, we can ensure that graduates, with perhaps only a bit of additional work, will meet our exacting standards for casualty actuaries and be eligible for membership in the CAS. If we take a passive role, the development of casualty actuaries around the world may suffer, and our ability to maintain the worldwide leader element of our Centennial Goal may vanish. We cannot expect that new actuaries throughout the world will sit for CAS exams. We can, however, play an active role in defining how they will be trained. This is another crossroads on our journey. Which fork in the road will we take?

These are some of our challenges on the global front. We have the opportunity and, I believe, the will to meet them.

How about our goal to be the preeminent resource? We certainly have the largest membership today, but we cannot simply assume that our research is always at the cutting edge. We must demonstrate our leadership both in publications and in practice. During the first week of my presidency, Standard & Poor's questioned both our ability and our integrity. The S&P language was deliberately over the top, and they have since backed away from some of their assertions. Nevertheless, S&P's criticism has some basis in fact. Numerous insurers have taken very large reserve increases in the past few years, often with little or no warning signs in the actuaries' opinions issued shortly before the increases. We deal on a daily basis with uncertainty, but we are not, as a profession, particularly good at expressing the uncertainty to our publics. We must find a way to communicate what our estimates mean. We cannot expect insurers and the public to rely on us, but then disclaim responsibility when things go awry.

We must improve the quality of our estimates. Yes, much of the deterioration in experience in recent years has related to new stages in the ongoing asbestos debacle. Perhaps we could not have predicted the mass tort developments, but some of the growth in loss reserves has been on recent years and reflects a pattern of poor underwriting results, particularly in workers compensation. How is it that we missed the mark on our bread and butter exposure? The standard reserving techniques taught in our curriculum and employed by most actuaries today are largely unchanged from twenty-five years ago. Where is our research on how to anticipate and reflect changing economic conditions in our pricing and reserving? What are we doing as a society to improve the accuracy of our estimates? New working parties are addressing the quality of Bornhuetter-Ferguson a priori estimates and the selection of tail factors. The topics are promising, but are they enough? The casualty profession in the U.K. has created a task force to address reserving issues, with a high priority on improving the accuracy of the estimates. Will we join that effort and devote substantial resources toward moving our science forward, or will we be passed on the road?

In addition to improving the quality of our mean estimates, we must understand and explain the uncertainty. We expend considerable effort to educate our candidates on the interaction of frequency and severity distributions in our preliminary exams, yet the syllabus for Exam 6 is devoted to estimating the mean. We must move beyond the mean and find a way to communicate all of the uncertainties. As the great modern poet, D.H. Rumsfeld, as so aptly put it:

> As we know, there are known knowns.
> There are things we know we know. We also know
> There are known unknowns. That is to say,
> We know there are some things
> We do not know.
> But there are also unknown unknowns,
> The ones we don't know
> We don't know.

We have a professional responsibility to ensure that our estimates reflect all that we know. The known knowns. At the other extreme, the public is very tolerant of our inability to predict the unknown unknowns. We must improve our ability to communicate our understanding of the middle ground, the known unknowns. We cannot decline to quantify the known unknowns because of the lurking shadow of the unknown unknowns. Our colleagues in Australia are already called upon to quantify the known unknown, and the advent of fair value accounting worldwide will force us away from the mean. By joining in the dialogue, we can do much to improve both the estimation of the distribution and the communication that there are lingering things that we cannot estimate.

Will we choose to remain at the forefront of the global profession in our research and curriculum?

BEYOND TRADITIONAL PRACTICE

Casualty actuaries are, my prior comments notwithstanding, the experts in the evaluation of hazard risk. We are rapidly, through research and practice at our more innovative employers, becoming recognized experts in the integration of hazard risk with financial, operational, and strategic risk in the insurance industry. We are committed to extending the education of CAS members in the integration of risk. To that end, we are testing a hands-on modeling seminar that we hope to make available to the membership and perhaps to include as a capstone in our qualification process.

Partnering with the SOA, we have made strides in defining the role of the enterprise risk manager in insurance. In 2003 we jointly sponsored a highly successful seminar with the SOA, and the addition of the Professional Risk Managers International Association in 2004 extended our footprint beyond the insurance world. The need for effective risk managers is rising in all sectors. Will the risk manager of the future be an actuary? Within the insurance industry, I believe the answer is yes. We are the professionals best qualified to fill that role, and we will rise to the challenge. Outside insurance, there will be a growing presence of actuaries serving as risk managers. But even if every actuary became a risk manager, the supply could not possibly meet the demand. And not every risk manager needs to be a fully qualified actuary to be effective.

Do we then abandon or scale down the second part of our Centennial Goal? By no means. Actuaries have much to bring to the table, and the actuarial profession right now has the opportunity to be instrumental in the definition of a recognized new professional. Not an actuary, but a professional with considerable skills in modeling and integration, possibly certified through an international exam process much like the CFA.

Will the CAS lead the profession in supporting the development of qualified risk managers?

THE CASUALTY ACTUARY OF TOMORROW

In the past year we have not only looked outside our traditional geography and beyond our traditional practice. We have also examined our structure and our governance, and asked ourselves what it means to be a credentialed member of the CAS. From the beginning of the Society, we have maintained two classes of membership. In 1915, we defined a "Fellow" as a manager of an actuarial or statistical department, and an "Associate" as pretty much anybody else. Over time, we introduced exams and set the Associate membership level at about halfway to Fellowship. We were silent, however, on what we meant by Associate membership, and since there were no qualification standards, we left it to the individual member's conscience to practice only where qualified. As the body of actuarial science grew, the number of exams to Fellowship grew, and for the first sixty or so years the Associateship crept up alongside, remaining at about the half-way point.

In 1965, the American Academy of Actuaries was born, and the Academy, wanting to cover as many practitioners as possible, set its single membership level at the Associate. Since one of the purposes of the American Academy was to serve as an umbrella organization for all qualified actuaries in the United

States, we now had an explicit acknowledgment that Associates of the CAS were, at least to some extent, qualified. Interestingly, the Canadian Institute of Actuaries was formed at the same time and limited its recognition to Fellows.

Enter the need for reserve opinions. In the late 1970s, the U.S. moved toward an actuarial signoff on loss reserves. There were not enough Fellows however, to meet the need. The CAS therefore made a commitment to expand the requirements for Associates so that the ACAS would have sufficient training to sign a reserve opinion. The result was a 7-exam ACAS, the first time the Associate membership had advanced beyond the half-way point. As the needs of the signing actuary have expanded, so have the requirements for ACAS. The current gap is only two exams, and there are very, very few assignments that can only be taken on by Fellows. As the gap between the ACAS and the FCAS has narrowed, the need for a governance structure that recognizes only Fellows has also come into question.

We created two task forces in late 2003: one to address the question of how many classes of credentialed membership the Society needs and one to examine whether Associates should play a role in the governance of the Society. The Task Force on Classes of Membership concluded that the future Society needs only one class of credentialed member and recommended ending the ACAS designation. The task force also came to the conclusion that the membership point should be set somewhat below the current requirements for Fellowship. Over time, we have added, and added, and added to the syllabus. Like Imelda Marcos and her shoes, we are compelled to add the newest styles to our closet, but we have a very hard time throwing even a single pair out. We have overshot the mark on the amount of material that all Fellows must master.

The CAS Board overwhelmingly agreed with the task force's recommendations, but concluded that we must first get our closet in order and confine our wardrobe to the truly necessary pairs of shoes. A new task force has been formed, with the charge to return to the board with learning objectives that meet our Centennial Goal more efficiently than under the current nine exams. It is the sense of the board that, once we have implemented the revisions recommended by this task force, there will no longer be a need to create new Associates, and the ACAS class will run off over time.

Even if we transition to a single class for new members, we expect to have Associate members for some time to come however, so the recommendations of the second task force, the Task Force on ACAS Voting Rights, are still important. This task force on the governance role of Associates recommended that Associates with five years' tenure be afforded nearly all of the rights and responsibilities of Fellows of the Society. That is, experienced Associates should vote, should be allowed to serve on committees, as vice presidents and on the CAS Board of Directors. The only roles reserved to Fellows would be membership on the Discipline, Syllabus, and Fellowship Examination Committees, the vice president–admissions, and the president and president elect. Implementing this proposal will require a vote of the Fellows, and the board has elected to delay that vote until the plans for the new syllabus and probable transition to a single class of new member are more firmly established.

There are myriad other exciting things going on at the CAS. We are a society of creative thinkers, and we will never run out of new ideas and initiatives. If I were to try to list everything, we would be here long beyond my allotted time. The Society is, at ninety, nigh unto one hundred years old. But we are a young Society in our outlook, and I am confident that we will maintain our youthful vigor well beyond our centennial.

I have been privileged to serve as your president in a time of great challenge and change for the Society. I have nothing but optimism for our future, and I look forward to continued opportunities to serve with all of you in achieving the Centennial Goal.

I will very shortly turn over the gavel and the badge of office to Steve D'Arcy. I wish you the best of luck in your term, Steve, and can only hope that your year is as wonderful as mine has been. But before I go, there are a few personal milestones to mention and many thanks that need to be expressed.

It has been an eventful year. Jon and I notched twenty-five on our marital counting stick. I was clearly a child bride. That means we have only 61 years to go to break the record. The consulting practice my partners and I started on a shoestring, a prayer, and firm belief in our clients, reached its fifth birthday with bells on and doubled its original size. My mother, Pat Brennan, brilliant gem that she is, arrived at her diamond birthday. As Dr. Seuss so sagely informs us, Mom, you're only old once! Jon's dad, Jim Miller, discovered that the still he and his classmates built back in 1957 is now exhibited as a prized "antique" by the chemical engineering department at Iowa State. I guess it's a milestone to have something you made classified as antique. Did you know your grandpa had it in him, boys? And tragically, my sister, the finest attorney in the State of Colorado, came suddenly to the end of her life's journey. I miss you, Pat.

No president of the CAS can go it alone. This job is only possible with the phenomenal support provided by a whole host of people. First, and always, support comes from family:

My best friend, my soul mate and my life's partner, Jon. His constant, quiet support is the anchor in my life. With a child at home, Jon has not been able to accompany me on many of my ramblings this year, and the sacrifices he has made to hold up the home front have been tremendous. I love you, dear. Our children: Rachel, Frank, and Joe. The sunshine of our lives. The reason we do it all. We are so proud of you. My mom, my in-laws, Jon's and my siblings, your spouses and kids. Your loving support and "atta-girls" have smoothed many a rough spot along my way.

There are a few people who have played a special role in guiding me professionally.

My parents, Terry and Pat Brennan, who taught me everything there is to know about what it means to be a professional long before I even knew there was a career called "actuary." Gary Dean, a Fellow of the CAS and my first boss, who taught me how to think like an actuary, and how to take a deep breath and count to five (well, two anyway) before I stepped on everybody's toes. Bob Anker, Fellow of the CAS, who by his example taught me that my debt to the profession and the Society can never be repaid. Dick Duvall, a Ph.D. and economist, who introduced me to the world of risk management. And finally, many thanks to those who have worked with me throughout the year.

The CAS Board of Directors. We have a wonderful, energetic Board, the envy of the profession. Each and every board member takes that responsibility to heart and truly works only toward the betterment of the Society. The CAS Executive Council: Steve, Chris, John, Don, Tom, Joanne and Debbie. It has been a great pleasure to work with you. The CAS staff. Cynthia and Jane, did you have any idea what it would be like to try to back up a president who is not only scatterbrained but doesn't have an assistant? Kathy, you have made this weekend so special. And all the rest, without whom the CAS would not function, led by our team of managers: Mike, Tom, Kathleen, and Todd. You are the best you could possibly be. And, finally, my partners and our colleagues at Select Actuarial Services in Nashville. I hope you are watching on the Webcast. Your support and your confidence in me and in our partnership have made this year possible. Cheryl, Jim, Laura, Thomas, Jessica, Amanda, Sarah, David, and Linda, thank you so much for your patience. I promise I'll be back to work on Friday.

In closing, there's an Irish toast that describes my wishes for all of us in the CAS:

> May we be poor in misfortune, rich in blessings, slow to make enemies, and quick to make friends. And may we know nothing but happiness from this day forward.

Thank you for letting me serve with you this year.

MINUTES OF THE 2004 CAS ANNUAL MEETING

November 14–17, 2004

FAIRMONT THE QUEEN ELIZABETH

MONTRÉAL, QUÉBEC, CANADA

Sunday, November 14, 2004

Registration was held from 4:00 p.m. to 6:30 p.m.

An officers' reception for New Associates and Accompanying Persons was held from 5:30 p.m. to 6:30 p.m.

A welcome reception for all attendees was held from 6:30 p.m. to 7:30 p.m.

Monday, November 15, 2004

Registration continued from 7:00 a.m. to 8:00 a.m.

CAS President Mary Frances Miller opened the business session at 8:00 a.m., welcoming all to the CAS Annual Meeting and announcing that the morning's events would be Webcast over the CAS Web Site. President Miller introduced the current members of the Executive Council (EC) and thanked EC members, Christopher S. Carlson and John Narvell, who were retiring from their positions.

Ms. Miller announced that the board appointed new vice presidents to succeed the outgoing Carlson and Narvell. Beth E. Fitzgerald was appointed vice president–professional education and Amy S. Bouska vice president–international. Ms. Miller then introduced members of the CAS Board of Directors and thanked outgoing board members, Phillip N. Ben-Zvi, Curtis Gary Dean, David G. Hartman, and Janet Nelson. CAS Immediate Past President Gail M. Ross was thanked for her outstanding leadership in the past year.

Ms. Miller announced the results of the CAS elections. Stephen P. D'Arcy, who was elected president-elect in 2003, would become CAS President at the conclusion of the Annual Business

Meeting. CAS Fellows elected Paul Braithwaite as 2004 president-elect. Joining Vice Presidents Fitzgerald and Bouska as members of the CAS Executive Council for 2004–2005 will be Deborah M. Rosenberg, vice president–administration; Thomas G. Myers, vice president–admissions; Joanne S. Spalla, vice president–marketing & communications; and Donald F. Mango, vice president–research & development.

New members of the CAS Board of Directors are Regina M. Berens, Christopher S. Carlson, Allan M. Kaufman, and Karen F. Terry.

Ms. Miller also recognized past presidents of the CAS attending the meeting: Phillip N. Ben-Zvi (1985), Ronald L. Bornhuetter (1975), Robert F. Conger (2001), Michael Fusco (1989), Alice H. Gannon (1999), David G. Hartman (1987), Allan M. Kaufman (1994), W. James MacGinnitie (1979), and Gail M. Ross (2002).

Ms. Miller also recognized special guests in the audience: Harvie Brown, president of the Faculty of Actuaries; Brian FitzGerald, president of the Canadian Institute of Actuaries; Alf Guldberg, president-elect of the International Actuarial Association; Luis Huerta, president of the International Actuarial Association; Steve Kellison, president of the Society of Actuaries; W. James MacGinnitie, international secretary of the Society of Actuaries; Graham Rogers, president of the Institute of Actuaries of Australia; Tom Ross, past president of the Faculty of Actuaries; Robert Wilcox, president of the American Academy of Actuaries; and Margaret Tiller Sherwood, president of the Conference of Consulting Actuaries.

Ms. Miller also acknowledged a special delegation from the Institute of Actuaries of Japan who would be conducting the concurrent session "Insurance Market and Actuarial Profession in Japan."

Ms. Miller asked all CAS volunteers to stand and be recognized, including committee chairpersons; board members and officers of the executive council; committee members; individuals who have worked on the AAA committees or committees of other actuarial organizations; Regional Affiliate officers; authors of papers; and moderators and panelists of this Annual Meeting or any previous CAS meeting. Ms. Miller asked the audience to applaud the efforts of these volunteers.

The CAS admitted 99 new Fellows and 122 new Associates in November 2004. The new Fellows and Associates in attendance were honored in a special ceremony. Vice Presidents Myers and Carlson announced the new Associates and President-Elect D'Arcy announced the new Fellows. The names of this class follow.

NEW FELLOWS

Richard T. Arnold
Martha E. Ashman
Danielle L. Bartosiewicz
John T. Binder
Rebecca Schafer Bredehoeft
Kevin K. W. Chan
Yves Charbonneau
Julia Feng-Ming Chu
Benjamin W. Clark
David Alan Clark
Eric John Clymer
John Edward Daniel
Brian S. Donovan
Stephen E. Dupon
Jeffrey A. Dvinoff
Sean Paul Forbes
Susan J. Forray
David I. Frank
David A. Gelberg
Gregory Evan Gilbert
Olga Golod
Lori A. Gordon
Stacie R. W. Grindstaff
Isabelle Groleau
Simon Guenette
Jonathan M. Guy
Jason C. Harland
Ryan Yin-kei Ho
Jason N. Hoffman
John F. Huddleston
Li Hwan Hwang
Jason Israel
John F. Janssen
Steven M. Jokerst
Derek A. Jones
Kyewook Gary Kang
Anthony N. Katz
Robert B. Katzman
Susan M. Keaveny
Ziv Kimmel
Raymond J. Kluesner
Gregory E. Kushnir
Hooi Lee Lai
ZhenZhen Lai
Francis A. Laterza
Yuxiang Lei
Isabelle Lemay
Kenneth Lin
Nataliya A. Loboda
Eric A. Madia
Steven Manilov
Sharon L. Markowski
Jason N. Masch
Laura A. Maxwell
Jennifer A. McGrath
John D. McMichael
Sarah K. McNair-Grove
Jennifer Middough
Charles W. Mitchell
Celso M. Moreira
Timothy C. Mosler

James C. Murphy Jr.
William S. Ober
James D. O'Malley
Jeffrey J. Pfluger
Ellen K. Pierce
Gregory S. Richardson
Nancy Ross
Stuart C. Rowe
Frederick Douglas Ryan
Ronald J. Schuler
Jin Shao
Peter M. Shelley
Jimmy Shkolyar
Thomas M. Smith
Michael D. Sowka
Adam D. Swope
Robert W. Thompson
Ellen Marie Tierney
David A. Traugott
Nathalie Tremblay
Lien K. Tu-Chalmers
Stephen H. Underhill
Natalie Vishnevsky
Jeffrey J. Voss
Mary Elizabeth Waak
Amy R. Waldhauer
Kristie L. Walker
Matthew J. Walter
Gary C. Wang
Thomas E. Weist
Arthur S. Whitson
Rosemary Gabriel Wickman
Micah Grant Wollstenhulme
Joshua C. Worsham
Jimmy L. Wright
Bradley J. Zarn
Gene Q. Zhang
Yi Zhang

NEW ASSOCIATES

Karen H. Adams
Ying M. Andrew
Brian D. Archdeacon
Rebecca J. Armon
Farid Aziz Ibrahim
Dan S. Barnett
Nicolas Marc Beaudoin
Alexandra Robin Beckenstein
Derek Dennis Berget
Brian J. Biggs
Rebekah Susan Biondo
Michael J. Blasko
Tapio N. Boles
John R. Broadrick
Michele L. Brooks
Matthew Buchalter
Douglas James Busta
Jennifer L. Carrick
Patrick J. Causgrove
Tracy L. Child
Charles A. Cicci
Eric Clark
Chad J. Covelli
Spencer L. Coyle
Justin B. Cruz
Robert P. Daniel
Mari A. Davidson
Chantal Delisle
Julie A. Ekdom
Yehoshua Y. Engelsohn
Jieqiu Fan
Bruce Fatz
Gina C. Ferst
Dale A. Fethke
Suzanne M. Finnegan
Jeffrey R. Fleischer
Matthew Timm Frank
Louise Frankland
Marie LeStourgeon Fredericks
Chad J. Gambone
Kristen Marie Gill
David B. Gordon
Veronique Grenon
Jeffrey Robert Grimmer
Travis J. Grulkowski
Kyle M. Hales
Bobby Earl Hancock Jr.
Megan Taylor Harder
Robin A. Haworth
Stephen J. Higgins Jr.
Carol K. L. Ho
Bo Huang
Yehuda S. Isenberg
Kenneth Layne Israelsen
William T. Jarman
Richard Clay Jenkins
Min Jiang

Shiwen Jiang
Yi Jing
Julie M. Joyce
William J. Keros
Amy Jieseon Kim
Scott M. Klabacha
Susan L. Klein
Steve C. Klingemann
Terry T. Kuruvilla
David Matthew Lang
Nathalie M. Lavigne
Henry T. Lee
Kenneth L. Leonard
Kahshin Leow
Wei Li
Xin Li
Herman Lim
PeiQing Luo
Luis S. Marques
Raul Gabriel Martin
Laura S. Martin
Jonathan L. Matthews
Christopher Charles McKenna
Jason L. Morgan
Maria M. Morrill
Yuchun Mu
Leonidas V. Nguyen
Joshua M. Nyros
Timothy James O'Connor
Russel W. Oslund
Alan M. Pakula
Jean-Pierre Paquet
Lorie A. Pate
Michael J. Quigley
Damon Joshua Raben
Eric W. L. Ratti
Dale M. Reimer
Kevin D. Roll
Derek Michael Schaff
Genine Darrough Schwartz
Richard H. Seward IV
Steven R. Shallcross
Quan Shen
Sarah J. Shine
Anne Marie Sinclair
Thomas Richard Slader
Justin Nicholas Smith
Sheila R. Soulsby
Michael Daniel Stephens
Natalie St-Jean
Mark Stephen Struck
Zongli Sun
Michelle M. Syrotynski
Daniel Jacob VanderPloeg
Chang-Hsien Wei
Timothy P. Wiebe
Ann Min-Sze Wong
Donald S. Wroe
Xinxin Xu
Huey Wen Yang
Yuanhe Yao
Sung G. Yim
Joshua A. Youdovin
Janice Minhuei Young
Lijuan Zhang

Ms. Miller then introduced past CAS President Alice H. Gannon, who gave the address to new members.

At the beginning of the awards ceremony, Ms. Miller spoke briefly about the Matthew S. Rodermund Service Award. This award was established in 1990 in honor of Matt Rodermund's years of volunteer service to the Casualty Actuarial Society. The Award recognizes a CAS member or members, who have made significant volunteer contributions to the actuarial profession such as committee involvement, participation in CAS meetings and seminars, volunteer efforts for Regional Affiliates or special interest sections, and involvement with non-CAS actuarial professional organizations such as the American Academy of Actuaries or the Canadian Institute of Actuaries. Ms. Miller announced that Arthur R. Cadorine was this year's winner of the Matthew S. Rodermund Service Award.

Ms. Miller then presented the "Above & Beyond" Achievement Awards, a new CAS honor designed to celebrate the spirit of volunteerism. The award honors individuals who perform with exceptional merit but whose efforts may not be apparent or widely known to the vast majority of CAS members. The 2004 award recipients are Ralph S. Blanchard III, Kevin G. Dickson, and Stuart B. Suchoff.

Mr. Blanchard was recognized for leading the committee that reviews IAA statements and makes recommendations for CAS Board actions. Mr. Blanchard also organized and managed the proposal process and project oversight for the fair value accounting research project, which culminated in the CAS publication, *Fair Value of P&C Liabilities: Practical Implications*. Mr. Blanchard reviewed request for proposals, notified the winners, kept both teams on track, reviewed drafts, and arranged for presentations to the Financial Accounting Standards Board and International Accounting Standards Board.

Ms. Miller recognized Mr. Dickson for his work on the CAS Enterprise Risk Management (ERM) Committee. Mr. Dickson took the lead in setting up the program and logistics for the ERM Symposium, held in Spring 2004. This endeavor was made particularly challenging by the additional involvement of symposium co-sponsors: the Society of Actuaries, Professional Risk Managers' International Association, and the Georgia State University Bowles Symposium. In addition, the symposium schedule was moved up three months. Mr. Dickson personally recruited several speakers and resolved different organizational approaches to logistical issues. Thanks to Mr. Dickson's efforts the ERM Symposium was a tremendous success.

Mr. Suchoff, who was honored posthumously, was recognized for service as chairperson of the Program Planning Committee and the Reserves Committee. He had recently become the chairperson of the AAA Risk-Based Capital Task Force. Despite feeling he was out of his "comfort zone" on the topic, Mr. Suchoff accepted the challenge because the group needed leadership. Mr. Suchoff was also cited for his dedication, focus, and clear vision of what needed to be done and how to go about doing it, which made working with him and for him an enriching experience.

Ms. Miller then presented the Charles A. Hachemeister Award, a prize established in 1993 in recognition of Charles A. Hachemeister's many contributions to Actuarial Studies in Non-Life Insurance (ASTIN) and his efforts to establish a closer relationship between the CAS and ASTIN. Ms. Miller announced Donald F. Mango as the winner of the 2004 Hachemeister Award. The title of Mr. Mango's paper is "Capital Consumption: An Alternative Methodology for Pricing Reinsurance."

Vice President Mango then announced David L. Homer and David R. Clark as the winners of the 2004 Dorweiler Prize for their paper, "Insurance Applications of Bivariate Distributions." The award commemorates the work of Paul Dorweiler and is awarded for the best paper submitted each year by an Associate or Fellow who attained his or her designation more than five years ago.

After the award presentation, Ms. Miller asked the audience to pause for a moment of silence to acknowledge CAS members who have died since November 2003: Warren P. Cooper, Karl F. Eaton, Robert B. Foster, James B. Gardiner, William S. Gillam, Robert Anderson Miller III, Owen D. Richmond, Robert F. Roach, Stuart Suchoff, Ward Van Buren Hart Jr., and Donald M. Wood.

Ms. Miller then spoke on behalf of The Trustees for the CAS Trust (CAST) to recognize D. W. Simpson & Company and its funding for the advancement of actuarial science. The company had donated $10,000 this year to CAST and had cumulatively donated $90,000 to the Trust. CAST was established in 1979 as a non-profit 501(c) (3) organization to afford members and others an income tax deduction for contributions of funds to be used for scientific, literary, research, or educational purposes.

Ms. Miller announced that three *Proceedings* papers and one discussion of a *Proceedings* paper would be presented at this meeting.

Ms. Miller then introduced James M. Carson, president of the American Risk and Insurance Society (ARIA), who gave an update on the 2005 World Risk and Insurance Economics Congress as well as other ARIA activities. Next a representative of the Image Advisory Council gave an update on the mission, goals, and general strategy of the image campaign for the actuarial profession. Aaron Halpert, chairperson of the CAS Long Range Planning Committee, then spoke about current activities regarding the CAS Centennial Goal and the Significant, Attainable, and Measurable (SAM) Goals.

Following these information items, Mr. D'Arcy introduced Ms. Miller who gave the presidential address.

At the conclusion of the address, Mr. D'Arcy presented Ms. Miller with the presidential plaque. Outgoing CAS President Miller then passed on the presidential gavel and medallion to incoming CAS President D'Arcy, who then closed the business session.

After a refreshment break, the first General Session was held from 10:15 a.m. to 11:45 a.m.

Reinsurance Worldwide

Moderator:	Gary G. Venter Managing Director Guy Carpenter Instrat
Panelists:	Donald R. Alexander Executive Director Guy Carpenter & Company Ltd.
	James N. Stanard Chairman and CEO Renaissance Re Holdings Ltd.

Michael G. Wacek
President
Odyssey America Reinsurance
Corporation

After the General Session, a luncheon was held where featured speaker John Krubski spoke on the art and science of persuasion.

After the luncheon and featured speaker, the afternoon was devoted to concurrent sessions. The panel presentations from 1:30 p.m. to 3:00 p.m. covered the following topics:

1. AAA-Updated Research on the NAIC Risk-Based Capital Formula

   Moderator: Robert F. Wolf
   Principal
   Mercer Oliver Wyman

   Panelists: G. Chris Nyce
   Senior Manager
   KPMG

   Anthony G. Phillips
   Vice President, Actuarial
   Accident Fund Insurance Company of America

2. Enterprise Risk Management and the Actuarial Profession

   Moderator: Barry A. Franklin
   Managing Principal and Actuary
   Aon Risk Consultants Inc.

   Panelists: Donald F. Mango
   Director of Research and Development
   GE Insurance Solutions

   Mark A. Verheyen
   Vice President
   Carvill

3. Exploring New Territories

Moderator:	Christopher M. Norman Director United Services Automobile Association
Panelists:	Timothy McCarthy Actuarial Consultant ISO
	Serhat Guven Consultant EMB America LLC

4. Is It a Small World After All?

Moderator/ Panelist:	Jean Roy Regional Manager Aviva Trade
Panelists:	Robert Dalton Divisional Director Cunningham Lindsey International Ltd.
	Sean Whelan Executive Vice President Willis Re

5. Loss Reserving: Is It Broken? What Can Be Done Better?

Moderator/ Panelist:	John J. Kollar Vice President ISO
Panelists:	Charles C. Emma Principal Pinnacle Actuarial Resources
	Thomas A. Ryan Consulting Actuary Milliman USA

6. The Reinsurance Buying Process—Have the Rules Changed?

Moderator:	Brian Z. Brown Consulting Actuary Milliman USA
Panelists:	Robert C. Andrews Vice President and Manager Liberty Mutual
	Scott C. Belden Managing Director Travelers/St. Paul

Two *Proceedings* papers presented during this time were:

1. "The Modified Bornhuetter-Ferguson Approach To IBNR Allocation"

Authors:	Trent R. Vaughn Republic Group of Insurance Companies
	Phoebe Tinney Towers Perrin

2. "Value Creation In Insurance—A Finance Perspective"

Author:	Russell Bingham The Hartford

After a refreshment break, concurrent sessions continued from 3:30 p.m. to 5:00 p.m. One session was also repeated during this time.

1. Asbestos Liabilities—The Continuing Saga

Moderator/ Panelist:	Jennifer L. Biggs Principal Tillinghast-Towers Perrin

2. California Workers Compensation

Moderator/ Panelist:	David M. Bellusci Senior Vice President and Chief Actuary California Workers Compensation Insurance Rating Bureau

Panelists: Ronald A. Dahlquist
Senior Actuary
California Department of Insurance

Joanne M. Ottone
Consulting Actuary
Towers Perrin

3. CAS Examination Process

Moderator: Richard P. Yocius
Senior State Manager/Senior Actuary
Allstate Insurance Company

Panelists: Steven D. Armstrong
Senior Actuary
Allstate Insurance Company

Daniel G. Roth
Vice President and Actuary
CNA Insurance Companies

Manalur S. Sandilya
Corporate Actuary
Max Re Europe Ltd.

4. The End of Free Sharing in the Insurance Industry

Moderator: Amy S. Bouska
Consulting Actuary
Towers Perrin

Panelists: Tom Bakos
President
Tom Bakos Consulting Inc.

Mark Nowotarski
President
Markets, Patents & Alliances L.L.C.

Joseph Thomas
Supervisory Patent Examiner for
Insurance Related Inventions
United States Patent and Trademark Office

5. Identity Theft

Moderator:	John Winkleman Assistant Vice President AIPSO
Panelists:	Charles P. Orlowicz Senior Director American International Companies
	Kimberley A. Ward Chief Actuary American Association of Insurance Services

6. Sarbanes-Oxley 404—Where Do We Stand?

Moderator:	James C. Votta Partner Ernst & Young LLP
Panelists:	Lise A. Hasegawa Pricing Director MetLife Auto & Home
	Dave Perine Senior Manager Ernst & Young LLP
	Kenneth Sipora Senior Manager Deloitte

An officers' reception for New Fellows and Accompanying Persons was held from 5:30 p.m. to 6:30 p.m.

A general reception for all attendees followed from 6:30 p.m. to 7:30 p.m.

Tuesday, November 16, 2004

Registration continued from 7:00 a.m. to 8:00 a.m.

The following General Sessions were held from 8:00 a.m. to 9:30 a.m.

Workers Compensation Update

Moderator:	Robert F. Conger Principal & Consultant Towers Perrin
Panelists:	David M. Bellusci Senior Vice President and Chief Actuary California Workers Compensation Insurance Rating Bureau
	Stanton F. Long Managing Director Marsh New York
	Dennis C. Mealy Chief Actuary National Council on Compensation Insurance

Reserve Adequacy and the Underwriting Cycle

Moderator:	Stephen T. Morgan Vice President Clarendon Insurance Group
Panelists:	Michael E. Angelina Consulting Actuary Tillinghast-Towers Perrin
	Meyer Shields Analyst Legg Mason Wood Walker Inc.

After a break, the following concurrent sessions were held from 10:00 a.m. to 11:30 a.m.

Certain sessions were also repeated during this time.

1. Captives

Moderator/ Panelist:	John P. Yonkunas Principal Towers Perrin

	Panelists:	Andrew Sargeant President USA Risk Group of Vermont Inc.
		Charles R. Woodman Senior Vice President Marsh USA
2.	Copulas and Common-Shock Models—An Update	
	Panelists:	Glenn G. Meyers Chief of Actuarial Research and Assistant Vice President ISO
		Gary G. Venter Managing Director Guy Carpenter Instrat
3.	Heavy Tail Distributions: The COTOR Challenge	
	Moderator:	Steven M. Visner Principal Deloitte & Touche LLP
	Panelist:	Philip E. Heckman Consulting Actuary
4.	Insurance Market and Actuarial Profession in Japan	
	Moderator/ Panelist:	Hirohisa Mikogami Senior Consultant Milliman
	Panelists:	Hiroaki Hara Deputy Manager Mitsui Sumitomo Insurance Co. Ltd.
		Yoshitaka Hoshino Deputy Manager Tokio Marine and Nichido Fire Insurance Co. Ltd.
		Hironori Miyatake Associate Nissay Dowa General Insurance Co. Ltd.

Mitsuru Nagamori
Vice President
Sompo Japan Financial Guarantee
Insurance Co. Ltd.

Hirokazu Ogawa
Deputy Manager
The Fuji Fire & Marine Insurance Co. Ltd.

Atsuko Suto
Deputy Manager
Aioi Insurance Co. Ltd.

Shigetada Yokoi
Manager
Nipponkoa Insurance Co. Ltd.

Also during this time the following prize paper was presented:

Hachemeister Prize Paper

"Capital Consumption: An Alternative Methodology for Pricing Reinsurance"

Author: Donald F. Mango
GE Insurance Solutions

After a break for lunch, CAS committees met from noon to 5:00 p.m., and the following concurrent sessions continued from 12:30 p.m. to 2:00 p.m.

1. Catastrophe Life and Personal Accident

Moderator: Phillipe Trahan
Special Projects
Aon Re Canada Inc.

Panelists: David A. Lalonde
Senior Vice President
AIR Worldwide Corporation

Pierre-Yves Le Corre
Technical Director
SCOR Life

2. Emerging Issues in Medical Malpractice

Moderator/ Panelist:	Robert J. Walling III Principal and Consulting Actuary Pinnacle Actuarial Resources Inc.
Panelists:	Kevin M. Bingham Senior Manager Deloitte Consulting LLP
	Richard B. Lord Principal and Consulting Actuary Milliman USA

3. Security and the Reinsurer

Moderator/ Panelist:	Michael G. Kerner Chief Operations Officer Zurich North American Specialties
Panelist:	Isaac Mashitz Chief Pricing Actuary Swiss Reinsurance America Corporation

4. Tax Issues and The P/C Insurance Industry

Moderator/ Panelist:	Richard Ashley Partner PricewaterhouseCoopers LLP
Panelists:	Barry Dennis Partner PricewaterhouseCoopers LLP
	Lawrence M. Friedman Partner Lord, Bissell & Brook LLP

Two *Proceedings* papers presented from 12:30 p.m. to 2:00 p.m. time were:

1. "Simpson's Paradox: Confounding Variables and Insurance Ratemaking"

Authors:	John A. Stenmark Southern Farm Bureau Casualty Insurance Co.

Cheng-sheng Peter Wu
Deloitte & Touche LLP

2. Discussion of "Minimum Distance Estimation of Loss Distributions" by Stuart A. Klugman and A. Rahulji Parsa (*Proceedings* LXXX)

   Author: Clive L. Keatinge
   ISO

From 1:00 p.m. to 4:00 p.m. an American Academy of Actuaries- and CAS-Sponsored Limited Attendance Workshop on Media Relations was held.

A buffet dinner and entertainment for all attendees was held from 6:30 p.m. to 9:30 p.m.

Wednesday, November 17, 2004

From 8:30 a.m. to 10:00 a.m. the following joint General Session was held in conjunction with the Canadian Institute of Actuaries.

Actuaries and Actuarial Issues Around the World

Co-Moderators: Mary Frances Miller
President
Casualty Actuarial Society

Brian A. P. FitzGerald
President
Canadian Institute of Actuaries

Panelists: Harvie Brown
President
Faculty of Actuaries

Graham E. Rogers
President
Institute of Actuaries of Australia

Robert E. Wilcox
President
American Academy of Actuaries

After a refreshment break concurrent sessions presented from 10:30 a.m.–12:00 p.m. were:

1. Actuaries in Nontraditional Roles

Moderator:	Gail M. Ross Manager and Senior Consultant Milliman Inc.
Panelists:	Stephan L. Christiansen Senior Vice President Director of Research Conning Research & Consulting Inc.
	John R. Ferrara President Arrowhead General Insurance
	Duncan McCallum Managing Director RBC Capital Markets RBC Dominion Securities

2. Adjusting For Prior Experience Using Credibility Theory

Moderator:	John Have President Have Associates
Panelists:	Robert Hinrichs Vice President and Chief Actuary Workplace Safety and Insurance Board of Ontario
	Christopher J. Monsour Consulting Actuary Towers Perrin
	Gary Walters Vice President Group Insurance RGA Life Reinsurance Company of Canada

3. Automobile Insurance Reforms in Canada

Moderator/ Panelist:	Paula Elliott Principal Mercer Oliver Wyman Actuarial Consulting Ltd.
Panelists:	Shelley L. Miller QC, Partner Fraser Milner Casgrain LLP
	Alain Thibault President and CEO Meloche Monnex Incorporated

4. Fair Value Accounting—International Accounting Standards

Moderator/ Panelist:	James K. Christie Partner Ernst & Young LLP
Panelist:	Scott H. Drab Manager Ernst & Young LLP

5. The Morris Commission—and Why It Is So Relevant for Canadian Actuaries

Moderator:	Charles McLeod President-Elect Canadian Institute of Actuaries
Panelists:	Harvie Brown President Faculty of Actuaries
	Mike Lombardi Principal Towers Perrin

6. Plagues of the 21st Century

Moderator:	Pierre Saddik Vice President Group Development Optimum Reassurance Inc.

Panelists:	Emile M. Elefteriadis Assistant Vice President Actuarial Swiss Re Life and Health Canada
	Scott McGaire Assistant Vice President–Technical Consulting Manulife Financial
	Dr. Brian Ward Chief of Infectious Disease Division McGill University

7. Professionalism Standards in Practice

Moderator/ Panelist:	David J. Otto Managing Director EMB America LLC
Panelists:	Steven D. Armstrong Senior Actuary Allstate Insurance Company
	Kevin M. Dyke Vice President and Chief Actuary American Physicians Assurance Corporation

At the conclusion of these concurrent sessions, the CAS and CIA concluded their joint meeting. From 12:00 p.m. to the rest of the day, the CIA continued with their annual meeting.

From 1:00 p.m. to 6:00 p.m., the CAS Board of Directors met for their quarterly meeting. Chairperson of the board, Gail M. Ross, concluded the meeting.

Attendees of the 2004 CAS Annual Meeting

The 2004 CAS Annual Meeting was attended by 358 Fellows, 142 Associates, 1 Affiliate and 190 Guests. The names of the Fellows, Associates and Affiliate in attendance follow.

FELLOWS

Jason R. Abrams
Jeffrey R. Adcock
Christiane Allaire
Denise M. Ambrogio
Vagif Amstislavskiy
Kenneth Apfel
Deborah Herman Ardern
Steven D. Armstrong
Richard T. Arnold
Martha E. Ashman
Richard V. Atkinson
Victoria L. Bailey
Danielle L. Bartosiewicz
Andrea C. Bautista
Robert A. Bear
Nicolas Beaupre
David M. Bellusci
Phillip N. Ben-Zvi
Regina M. Berens
Jennifer L. Biggs
John T. Binder
Brad D. Birtz
Terry J. Biscoglia
Suzanne E. Black
Annie Blais
Jonathan Everett Blake
Yves Boissonnault-Francoeur
Ronald L. Bornhuetter
Amy S. Bouska
David S. Bowen
Jerelyn S. Boysia
Nancy A. Braithwaite
Paul Braithwaite
Yaakov B. Brauner
Rebecca Schafer Bredehoeft
Brian Z. Brown
George Burger
Angela D. Burgess
James E. Calton
Christopher S. Carlson
Kenneth E. Carlton
Allison Faith Carp
Jill C. Cecchini
Joseph Gerald Cerreta
Yves Charbonneau
Benjamin Christiansen
Stephan L. Christiansen
James K. Christie
Julia Feng-Ming Chu
Louise Chung-Chum-Lam
Benjamin W. Clark
Jo Ellen Cockley
Robert F. Conger
Eugene C. Connell
Cameron A. Cook
Hugo Corbeil
Michael J. Covert
Stephen P. D'Arcy
François Dagneau
Ronald A. Dahlquist
Karen Barrett Daley
John Edward Daniel
John D. Deacon
Curtis Gary Dean
Manon Debigare
Jeffrey F. Deigl
Robert V. Deutsch
Kevin G. Dickson
Brian S. Donovan
Scott H. Drab
Karl H. Driedger
Eric T. Drummond-Hay
François Dumas
Stephen E. Dupon
Jeffrey A. Dvinoff
Kevin M. Dyke
Richard D. Easton
Maribeth Ebert
Grover M. Edie
Warren S. Ehrlich
Douglas D. Eland
Thomas J. Ellefson
John W. Ellingrod
Paula L. Elliott
Dawn E. Elzinga
Charles C. Emma
Jonathan Palmer Evans
John S. Ewert
Robert G. Eyers
Kyle A. Falconbury
Richard J. Fallquist
Sylvain Fauchon
Richard I. Fein
Sholom Feldblum
John R. Ferrara
Ginda Kaplan Fisher
Sean Paul Forbes
Susan J. Forray
Christian Fournier
Louise A. Francis

David I. Frank
Barry A. Franklin
Dana R. Frantz
Noelle Christine Fries
Michael Fusco
Cecily A. Gallagher
Alice H. Gannon
Robert W. Gardner
Louis Gariepy
Anne M. Garside
James J. Gebhard
David A. Gelberg
Gregory Evan Gilbert
Isabelle Girard
Bradley J. Gleason
Olga Golod
James F. Golz
Lori A. Gordon
Eric F. Gottheim
Odile Goyer
Stacie R. W. Grindstaff
Isabelle Groleau
Simon Guenette
Elizabeth Susan Guven
Serhat Guven
Jonathan M. Guy
Nasser Hadidi
Allen A. Hall
James A. Hall
Walter J. Haner
Jason C. Harland
Guo Harrison
Bryan Hartigan
David G. Hartman
Kevin B. Held
John Herder
Thomas M. Hermes
Kirsten Costello Hernan
Laura Esboldt Heyne
Mark D. Heyne
Patricia A. Hladun
Ryan Yin-kei Ho
Jason N. Hoffman
David L. Homer
Eric J. Hornick
Marie-Josée Huard
David Dennis Hudson
Jeffrey R. Hughes
Jamison Joel Ihrke
Jason Israel
Richard M. Jaeger
Charles B. Jin
Mark Robert Johnson
Steven M. Jokerst
Derek A. Jones
Gary R. Josephson
James B. Kahn
Kyewook Gary Kang
Anthony N. Katz
Allan M. Kaufman
Clive L. Keatinge
Susan M. Keaveny
Wayne S. Keller
Scott Andrew Kelly
Rebecca Anne Kennedy
Sean M. Kennedy
Michael G. Kerner
Michael F. Klein
Craig W. Kliethermes
Jeff A. Kluck
Raymond J. Kluesner
Jonathan David Koch
Leon W. Koch
John J. Kollar
Israel Krakowski
Gary R. Kratzer
Rodney E. Kreps
Andrew E. Kudera
Ronald T. Kuehn
Charles B. Kullmann
Gregory E. Kushnir
Hooi Lee Lai
ZhenZhen Lai
David A. Lalonde
Matthew G. Lange
Robin M. La Prete
Michael D. Larson
Francis A. Laterza
Pierre Guy Laurin
Kevin A. Lee
Thomas C. Lee
Isabelle Lemay
Pierre Lepage
Alain Lessard
Jean-Marc Leveille
Joseph W. Levin
John J. Lewandowski
Peter M. Licht
Matthew Allen Lillegard
Kenneth Lin
Shu C. Lin
Barry Lipton
Andrew M. Lloyd
Nataliya A. Loboda
Richard Borge Lord
W. James MacGinnitie

Eric A. Madia
Daniel Patrick Maguire
Barbara S. Mahoney
Donald F. Mango
Steven Manilov
Sharon L. Markowski
Blaine C. Marles
Jason N. Masch
Isaac Mashitz
James J. Matusiak
David Michael Maurer
Laura A. Maxwell
Kevin C. McAllister
Michael G. McCarter
Jeffrey F. McCarty
Kelly S. McKeethan
Michael F. McManus
Dennis T. McNeese
Dennis C. Mealy
Christian Menard
Glenn G. Meyers
Ryan A. Michel
Jennifer Middough
Mary Frances Miller
Michael J. Miller
Neil B. Miner
Charles W. Mitchell
Christopher J. Monsour
David Patrick Moore
Celso M. Moreira
François L. Morissette
Todd B. Munson
Turhan E. Murguz
James C. Murphy
Thomas G. Myers
John C. Narvell
Janet R. Nelson
Khanh K. Nguyen
Gary V. Nickerson
John Nissenbaum
Sylvain Nolet
Christopher M. Norman
G. Chris Nyce
James D. O'Malley
David J. Oakden
Randall William Oja
Rodrick R. Osborn
David J. Otto
Joanne M. Ottone
Nathalie Ouellet
Teresa K. Paffenback
Donald D. Palmer
Joseph M. Palmer
John R. Pedrick
Isabelle Perigny
William Peter
Jeffrey J. Pfluger
Tony Phillips
Ellen K. Pierce
John Pierce
Dale S. Porfilio
Sean Evans Porreca
Deborah W. Price
Jennifer K. Price
Boris Privman
Ni Qin-Feng
Kenneth Quintilian
Andre Racine
Gregory S. Richardson
Gregory Riemer
John P. Robertson
Michelle L. Rockafellow
William P. Roland
Deborah M. Rosenberg
Gail M. Ross
Sandra L. Ross
Daniel G. Roth
Stuart C. Rowe
Jean Roy
Giuseppe Russo
Frederick Douglas Ryan
Thomas A. Ryan
Laura Beth Sachs
Manalur S. Sandilya
Timothy L. Schilling
Karen L. Schmitt
Vladimir Shander
Jin Shao
Michelle G. Sheng
Margaret Tiller Sherwood
Meyer Shields
Jimmy Shkolyar
Lisa A. Slotznick
Thomas M. Smith
Patricia E. Smolen
Tom A. Smolen
Michael D. Sowka
Sharon L. Sowka
Keith R. Spalding
Joanne S. Spalla
Daniel L. Splitt
James N. Stanard
John A. Stenmark
Roman Svirsky
Scott J. Swanay
Adam D. Swope
Karen F. Terry
Patricia A. Teufel

Alain Thibault
Kevin B. Thompson
Michael Toledano
David A. Traugott
Nancy R. Treitel
Nathalie Tremblay
Michel Trudeau
Stephen H. Underhill
Anne-Marie Vanier
Oakley E. Van Slyke
Trent R. Vaughn
Gary G. Venter
Mark Alan Verheyen
Marie-Eve J. Vesel
Natalie Vishnevsky
Steven M. Visner
William J. Von Seggern
James C. Votta
Mary Elizabeth Waak
Michael G. Wacek
Amy R. Waldhauer
Kristie L. Walker
Robert J. Walling
Michael C. Walsh
Matthew J. Walter
Gary C. Wang
Kimberley A. Ward
Kelly A. Wargo
Thomas E. Weist
William B. Westrate
Rosemary Gabriel Wickham
William B. Wilder
Laura M. Williams
John J. Winkleman
Robert F. Wolf
Richard G. Woll
Patrick B. Woods
Micah Grant Woolstenhulme
Cheng-sheng Peter Wu
Richard P. Yocius
John P. Yonkunas
Bradley J. Zarn
Gene Q. Zhang
Yin Zhang

ASSOCIATES

Gwendolyn L. Anderson
Ying M. Andrew
Michael E. Angelina
Brian D. Archdeacon
Farid Aziz Ibrahim
Carole J. Banfield
Daniel S. Barnett
Nicolas Marc Beaudoin
Scott C. Belden
Derek Dennis Berget
Brian J. Biggs
Kevin Michael Bingham
Rebekah Susan Biondo
Michael J. Blasko
Donna M. Bono
John R. Broadrick
Michele L. Brooks
Matthew Buchalter
Douglas James Busta
Christine Cadieux
Arthur R. Cadorine
Jennifer L. Carrick
Petra Lynn Charbonneau
Tracy L. Child
Alan M. Chow
Charles A. Cicci
Eric Clark
Donald L. Closter
Arthur I. Cohen
Chad J. Covelli
Spencer L. Coyle
Justin B. Cruz
Gregory A. Cuzzi
Thomas V. Daley
Robert P. Daniel
Mari A. Davidson
Willie L. Davis
Chantal Delisle
Jean-François Desrochers
Kevin George Donovan
François Richard Dumontet
Julie A. Ekdom
Yehoshua Yosef Engelsohn
Brian A. Evans
Wendy A. Farley
Bruce Fatz
Gina C. Ferst
Dale A. Fethke
Suzanne M. Finnegan
Jeffrey R. Fleischer
Matthew Timm Frank

Louise Frankland
Marie LeStourgeon Fredericks
Timothy J. Friers
Andre Gagnon
Bernard J. Galiley
Chad J. Gambone
Kristen Marie Gill
David B. Gordon
Jeffrey Robert Grimmer
Travis J. Grulkowski
Aaron M. Halpert
Bobby Earl Hancock
Lise A. Hasegawa
Robin A. Haworth
Philip E. Heckman
Joseph P. Henkes
Stephen J. Higgins
Robert C. Hill
Jeffrey R. Ill
Kenneth Layne Israelsen
William T. Jarman
Richard Clay Jenkins
Shiwen Jiang
Brian E. Johnson
Gregory K. Jones
James W. Jonske
Julie M. Joyce
Robert C. Kane
Mary Jo Kannon
Amy Jieseon Kim
Martin T. King
Jean-Raymond Kingsley
Scott M. Klabacha
Susan L. Klein
Steve C. Klingemann
Timothy M. Kolojay
Chung-Kuo Kuo
Terry T. Kuruvilla
David Matthew Lang
Hugues Laquerre
Nathalie M. Lavigne
Henry T. Lee
Charles R. Lenz
Kenneth L. Leonard
Kahshin Leow
Herman Lim
Luis S. Marques
Raul Gabriel Martin
Timothy C. McAuliffe
Timothy J. McCarthy
Timothy L. McCarthy
Stanley K. Miyao
Jason L. Morgan
Stephen T. Morgan
Leonidas V. Nguyen
Norman Niami
Timothy James O'Connor
Charles P. Orlowicz
Jean-François Ouellet
Jean-Pierre Paquet
Claude Penland
Michael J. Quigley
Eric W. L. Ratti
James E. Rech
Dale M. Riemer
Christopher R. Ritter
Kevin D. Roll
Joseph Francis Rosta
Sandra Samson
Sandra C. Santomenno
Genine Darrough Schwartz
Steven R. Shallcross
Sarah J. Shine
Halina H. Smosna
Sheila R. Soulsby
Mark Stephen Struck
Beth M. Sweeney
Michelle M. Syrotynski
Jonas F. Thisner
Marie-Claire Turcotte
Joel A. Vaag
Daniel Jacob VanderPloeg
David M. Vogt
Chang-Hsien Wei
Ann Min-Sze Wong
Donald S. Wroe
Huey Wen Yang
Yuanhe Yao
Sung G. Yim
Joshua A. Youdovin
Janice M. Young

AFFILIATE

Bradford S. Gile

REPORT OF THE VICE PRESIDENT–ADMINISTRATION

This report provides a one-year summary of Casualty Actuarial Society activities since the 2003 CAS Annual Meeting. I will first comment on these activities as they relate to the organization's purposes as stated in the CAS Constitution:

1. Advance the body of knowledge of actuarial science applied to property, casualty, and similar risk exposures;
2. Establish and maintain standards of qualifications for membership;
3. Promote and maintain high standards of conduct and competence for the members; and
4. Increase the awareness of actuarial science.

I will then provide a summary of other activities that may not relate to a specific purpose, but are critical to the ongoing vitality of the CAS. Finally, I will summarize the current status of our finances and key membership statistics.

CAS ACTIVITIES

1. Advance the body of knowledge of actuarial science applied to property, casualty, and similar risk exposures.

Publication of the *Forum* and *Proceedings* provide significant means for advancing the body of actuarial science. The 2004 winter and fall volumes of the *Forum* focus on ratemaking and reserves, respectively. The *Proceedings* include papers exploring a modified Bornhuetter-Ferguson approach to IBNR allocation; value creation in insurance from a financial perspective; classification ratemaking; and Simpson's Paradox, confounding variables, and insurance ratemaking. Also included are four discussions of previous *Proceedings* papers and an author's response to a discussion of his paper.

Research gains were made in various venues throughout the CAS in 2004. Introduced in 2003 as a new approach to research, CAS research working parties began their projects in 2004. The Executive Level Decision Making Using Dynamic Risk Modeling (DRM) Working Party was first to complete its charge. Co-chaired by Michael Larsen and Nathan Babcock, this group produced slide templates that practicing actuaries could use to make effective DRM presentations to senior management. The slides emphasize the financial measures that matter to the management team. The working party also produced examples of how to assemble presentations and a guideline on conducting presentations. The working party's report was published in the 2004 Fall *Forum* and the slides are available on the CAS Web Site.

CAS research funding sponsored a number of valuable projects in 2004. The CAS Committee on the Theory of Risk commissioned a team of researchers to take on the Risk Premium Project, a program supporting research on valuing property-liability risks. Robert Butsic, David Cummins, Richard Derrig, and Richard Phillips formed the team, which was tasked with thoroughly reviewing and synthesizing existing literature, paying particular attention to literature on profit loads, risk loads, and risk-adjusted discount rates. The team also conducted original research on one specific approach used to value by-line property-liability exposures. The team's research has been incorporated into an academic paper, "Estimating the Cost of Equity Capital for Property-Liability Insurers," which was submitted to the *Journal of Risk and Insurance*. Results from the team's research were presented at the 2004 Ratemaking Seminar and published in the 2004 Winter *Forum*.

Delving deeper into the workings of mergers and acquisitions, the Committee on Valuation, Finance, and Investments sponsored and funded two research projects dealing with the fundamental valuation of property/casualty insurers for merger and acquisition purposes. Research teams from the Czech Republic and the United States took on the projects. Jaroslav Danhel and Petr Sosik of the Czech Republic wrote "Acquisition Value of P&C Insurance Companies" and Wayne E. Blackburn, Derek A. Jones, Joy A. Schwartzman, and Dov Siegman of the U.S. wrote "The Application of Fundamental Valuation Principles to Property/Casualty Insurance Companies."

Seeking to voice its opinion to the International Accounting Standards Board (IASB), the CAS unveiled *Fair Value of P&C Liabilities: Practical Implications*, a new publication that was the result of two commissioned studies analyzing the impact of fair value concepts on property/casualty insurance companies. The request for proposal, which was issued in the fall of 2003, called for information on the time value of money and risk margins to reflect the market charge for uncertainty. The CAS awarded the commissions to PricewaterhouseCoopers LLP and Towers Perrin, and authors from the two firms participated in a general session at the 2004 CAS Spring Meeting.

Fair Value was targeted to the IASB and the Financial Accounting Standards Board (FASB) and their staffs, who were going to be making critical policy decisions in 2004 regarding fair value accounting for property/casualty insurance. The CAS and the research teams felt it was critical to provide the policymakers with sound research regarding possible impacts of fair value accounting decisions on property/casualty insurance companies.

Although the printed research report had been provided to IASB previously, the organization was interested in having representatives from the CAS conduct a live presentation and discussion. The IASB invited the CAS to present the fair value research, particularly material relating to discounting and risk adjustment, in London in early 2005.

Other CAS-sponsored research begun in 2004 included reports on "Modeling of Economic Series Coordinated with Interest Rate Scenarios," "Workers Compensation Ratemaking: A Textbook for the Practicing Actuary," and "Comparison of

Rating Agency Capital Models." Many of these ideas were generated by the Actuarial Foundation, which is a combined research entity co-sponsored by the CAS, Society of Actuaries (SOA), Canadian Institute of Actuaries (CIA), the American Society of Pension Professionals and Actuaries, the American Academy of Actuaries (AAA), and the Conference of Consulting Actuaries (CCA).

The CAS Web Site was enhanced and continues to be a provider of first-rate information resources to CAS members and other interested parties in 2004. A new Web site section, "Global Resources," consolidates information about actuarial activities outside of North America. The section highlights the work the CAS is doing around the world and provides a summary of CAS global initiatives, including information about the examination fee discount program for qualified countries, CAS examination centers worldwide, and mutual recognition with other actuarial organizations. Global Resources also contains links to demographic and statistical data, property/casualty insurance industry data, regulatory bodies, educational institutions, and local actuarial organizations. Other resources include a calendar of international actuarial events and links to international actuarial organizations and resources.

These research tools and projects are outstanding examples of the CAS's dedication to providing members with practical information to perform their jobs better as well as advancing actuarial science.

2. Establish and maintain standards of qualifications for membership.

CAS Admissions committees and task forces pursued a number of developments to achieve this objective. Based on the final report of the Future Education Task Force in 2003, the CAS Board authorized creating four new task forces whose missions address standards of qualifications for membership.

The first of the task forces, the Joint CAS/SOA Task Force on Preliminary Education, refined the learning objectives, produced common sets of objectives for given subjects, and defined guidelines for validation by educational experience (VEE). As opposed to the traditional actuarial exam, VEE allows candidates to receive credit for specific topics they have obtained through college courses, standardized exams, transitional VEE exams, and other educational experiences. One of the most significant changes the joint task force implemented was moving some topics off the exams to the VEE process. The new structure consists of three topics requiring VEE and four exams. Exams 1, 2, and 4 will remain joint exams with the Society of Actuaries, and Exam 3 will remain a CAS-only exam.

The second task force, the Computer-Based Testing (CBT) Task Force, explored alternative methods for candidates to take exams. The task force created an RFP, interviewed vendors, and made recommendations to the CAS Board. The work of this task force, which was made up of representatives from the CIA, SOA, and CAS, was approved in 2004 with plans to implement computer-based testing for at least one exam by 2005.

In an effort to develop more effective study materials, the third task force, the Task Force on Syllabus Materials, was charged with evaluating whether study materials produced by outside organizations could replace study materials listed in the CAS Syllabus and meet preliminary education learning objectives. The task force obtained study materials for the first four exams produced by various vendors. After reviewing the materials for content and presentations, the task force concluded that the vendors did possess the necessary skills to develop study aids for actuarial exams.

In regard to developing upper exam material, however, the task force asserted that it would be crucial for vendors to be able to produce materials that integrate purely actuarial material not covered in standard academic courses. To test for this ability, the task force issued another RFP that called for developing a trial reading in which respondents would write a chapter covering specified learning objectives in a way that demonstrates an ability to integrate actuarial material from multiple sources. For 2005, the Syllabus Committee has been tasked with identifying learning objectives that would provide an appropriate test.

The Modeling Workshop Task Force, the last of the four task forces, spearheaded the development of a new workshop on dynamic financial analysis. The task force would evaluate the pilot program for this new workshop to determine if it would be practical for future basic education, continuing education, or both. In early 2004, the task force issued an RFP to develop a DFA workshop that would educate candidates on all aspects of DFA modeling in its application to real-life business situations. After careful consideration, the task force awarded the project to General Re Capital Consultants, who designed the program and trained CAS instructors. The company also agreed to provide two years of technical support for the workshop. Pilot programs conducted in 2004 were evaluated and retooled. The task force plans to conduct another round of pilot workshops in 2005.

The activities of these four task forces in 2004 not only served to maintain standards of qualifications for membership, but also laid the groundwork for broadening the manner in which members could become qualified.

3. Promote and maintain high standards of conduct and competence for the members.

Throughout the years, CAS quality programs of continuing education opportunities and the Code of Professional Conduct have successfully fulfilled this purpose.

The CAS provides educational opportunities through the publication of actuarial materials and the sponsorship of meetings and seminars. This year’s sessions included the following, shown with the number of CAS members in attendance:

Meetings:

	Location	CAS Members
Spring	Colorado Springs, CO	484
Annual	Montréal, Canada	481

Seminars:

Topic	Location	CAS Members
Ratemaking	Philadelphia, PA	308
Symposium on Enterprise Risk Management	Chicago, IL	83
Reinsurance	Boston, MA	214
Casualty Loss Reserves	Las Vegas, NV	496
Appointed Actuary—Joint CAS/CIA	Montréal, Canada	289*
Special Interest Seminar on Predictive Modeling	Chicago, IL	187
Course on Professionalism—Dec '03	2 locations	69 Candidates
Course on Professionalism—June '04	2 locations	111 Candidates

*Total attendance. Separate count for CAS members is not available.

Limited attendance seminars included two sessions of "Practical Applications of Loss Distributions" and one session each of "Asset Liability Management and Principles of Finance," and "Reinsurance." Online courses included "Interest Rate Models," "Introduction to Financial Risk Management for Insurers," "The Building Blocks of Financial Risk Management: Forwards, Futures, Swaps, and Options," and "Financial Risk Management: Securitization."

4. Increase the awareness of actuarial science.

In pursuit of this objective, the CAS focused on the business world in North America to promote awareness of actuarial science and the actuarial profession. Near the end of 2004, the CAS teamed up with the SOA, AAA, CIA, and CCA to launch a public relations campaign to promote establishing the chief risk officer position in U.S. and Canadian businesses. The organizations contracted Ruder-Finn, an international public relations firm with strong experience in financial, corporate responsibility, and ethics issues, to handle the campaign. The campaign's goal is to educate the market about the actuarial skill set to position actuaries as the best choice for this critical role. The program not only focuses more industry attention on the importance of risk assessment but also underscores the work actuaries do and how their training, perspectives, and experience can be valuable assets as company leaders and managers.

Looking outward to the rest of the world, the CAS increased its efforts to promote the profession and actuarial science. In response to an increasing number and variety of requests to provide basic educational seminars to actuaries from countries around the world, the CAS Board approved a measure for the CAS to solicit proposals for creating a series of educational modules that could fulfill the requests. The CAS issued an RFP targeted, but not limited, to academic members and academic correspondents requesting brief proposals to create an initial set of nine modules. To oversee the process and establish funding for the program, the CAS Board formed the International Education Module Task Force.

For the very first run of the modules, CAS members Jimmy Shkolyar, George A. Rudduck, and Peter J. Murdza Jr. traveled to Kazakhstan in September to teach a weeklong educational program of actuarial practice and science to members of the Actuarial Society of Kazakhstan (ASK). The Arizona-Kazakhstan Partnership Foundation Inc., a publicly supported non-profit 501(c)(3) corporation devoted to strengthening the ties between the United States and Kazakhstan, coordinated the trip, which was partly funded by United States Agency for International Development. An educational and cultural success, the trip also built on a relationship begun in 2003 when ASK members visited the United States and the CAS Office.

In an effort to reach out to individual countries in a more personal manner, the CAS formed an Ambassador Program composed of CAS actuaries living outside the U.S. and Canada who are regular contacts for the local actuaries and universities. CAS ambassadors promote property/casualty actuarial science in the way that best suits local needs, such as organizing local CAS exam sites, speaking to university students or local societies, or identifying issues for the CAS to address. A long-time goal of the International Issues Committee, this Ambassador Program started with CAS ambassadors active in Egypt, Mexico, and Brazil in 2004. We hope for more ambassadors in the coming years.

OTHER CAS ACTIVITIES

Several other CAS activities contributed to the ongoing vitality of the organization during 2004.

Building on its strong ties with ASTIN, the CAS successfully bid to host the 50th anniversary of the ASTIN Colloquium in 2007. The colloquium will be held in Orlando, Florida in conjunction with the 2007 CAS Spring Meeting. The CAS also continues to support ASTIN by hosting the *ASTIN Bulletin* on the CAS Web Site in the Download Library.

Conducted in 2003, the CAS Membership Survey proved to be a powerful tool for enhancing services to CAS members. The report on the results of the five-year survey, presented to the Board in September 2004, generated many recommendations for the Executive Council to consider in planning goals for 2004–2005 and beyond.

The survey results revealed that several respondents were members of Association des Actuaires IARD (AAIARD), a 25-year-old casualty actuarial organization in Québec with no affiliation to other groups. After talks with the Canadian Institute of Actuaries and the Québec group, the CAS invited AAIARD to become the newest CAS Regional Affiliate. This is just one example of how member input shapes the Society. A copy of the survey's full report was posted on the CAS Web Site.

In 2004, the CAS followed through on work begun in 2003 to evaluate its membership structure. Two task forces formed by the CAS Board in 2003, Classes of Membership and the ACAS Vote, created recommendations that, if approved by the membership, would affect member credentials and voting rights.

The Task Force on Classes of Membership, chaired by Sheldon Rosenberg, recommended that the CAS move to one class of membership, that of Fellow. The CAS Board passed several motions with regard to this recommendation and instructed the Executive Council to establish a task force that would propose a set of learning objectives by which FCAS can be attained. The board stated that it supports an eventual single class of credentialed membership with no more new Associates, concurrent with the movement to a shorter syllabus.

Headed by Amy Bouska, the Task Force on the ACAS Vote was charged with investigating the advantages and disadvantages of whether current Associates should have voting rights, and whether other differences between CAS Fellows and Associates should exist, including differences in dues and in the right to hold high-level positions in the CAS. The Board accepted several task force recommendations, including granting members the unrestricted right to vote either upon attainment of Fellowship or five years after their recognition as Associates, whichever comes first.

The reports of these task forces contained several suggestions requiring a vote of CAS Fellows to make them constitutional amendments. Membership reaction to this plan will be elicited. An additional motion was passed to postpone further action on the ACAS vote pending resolution of the classes of membership issue.

Other significant activities include plans to look for a new or expanded CAS Office location in Arlington, Virginia. The change will take place in 2005.

MEMBERSHIP STATISTICS

Membership growth continued with 135 new Associates, 140 new Fellows, and three new Affiliates. The total number of members as of November 2004 was 3,988, up 3.7 percent from the previous year.

Paul Braithwaite was elected president-elect for 2004–2005. CAS Fellows also elected Regina Berens, Christopher S. Carlson, Allan Kaufman, and Karen F. Terry to the CAS Board of Directors. Stephen P. D'Arcy assumed the presidency.

The CAS Executive Council met either by teleconference or in person at least once a month during the year to discuss day-to-day and long-range operations. The CAS Board of Directors elected the following vice presidents for the coming year: Deborah M. Rosenberg, administration; Thomas G. Myers, admissions; Amy S. Bouska, international; Joanne S. Spalla, marketing and communications; Beth E. Fitzgerald, professional education; and Donald F. Mango, research and development.

FINANCIAL STATUS

The CPA firm Langan Associates PC examined the CAS books for fiscal year 2004 and the CAS Audit Committee reported the firm's findings to the CAS Board of Directors in March 2005. The fiscal year ended with an audited net gain of \$457,622 compared to a budgeted net loss of \$247,973.

Members' equity now stands at \$3,312,044. This represents an increase in equity of \$457,622 over the amount reported last year. In addition to the net gain from operations of \$216,915, there was interest revenue of \$110,544, an unrealized gain of \$164,870, and a realized loss of \$34,706. There was also a total net increase of \$62,876 in various research, prize, and scholarship accounts arising from the difference between incoming funds and interest earned less expenditures, and a favorable adjustment to the CAS pension liability. These amounts are not reflected in net revenue from operations.

For 2004–2005, the CAS Board of Directors has approved a budget of approximately $5.4 million, an increase of about $300,000 compared to the prior fiscal year. Members' dues for next year will be $355, an increase of $5, and fees for the Subscriber Program will increase by $5 to $425. A $45 discount is available to members and subscribers who elect to receive the *Forum* and *Discussion Paper Program* in electronic format from the CAS Web Site.

Respectfully submitted,
Deborah M. Rosenberg
Vice President–Administration

FINANCIAL REPORT
FISCAL YEAR ENDED 9/30/2004

OPERATING RESULTS BY FUNCTION

FUNCTION	*REVENUE*	*EXPENSE*	*DIFFERENCE*
Membership Services	$ 1,326,998	$ 1,720,421	$ (393,423)
Seminars	1,302,742	1,116,632	186,111
Meetings	824,197	809,497	14,700
Exams	3,643,053 (a)	3,227,053 (a)	416,000
Publications	30,988	37,460	(6,473)
TOTALS FROM OPERATIONS	$ 7,127,978	$ 6,911,063	$ 216,915
Interest and Dividend Revenue			110,544
Realized Gain/(Loss) on Marketable Securities			(34,706)
Unrealized Gain/(Loss) on Marketable Securities			164,870
TOTAL NET INCOME (LOSS)			$ 457,622

NOTE: (a) Includes $2,119,313 of Volunteer Services for income and expense (SFAS 116).

BALANCE SHEET

ASSETS	*9/30/2003*	*9/30/2004*	*DIFFERENCE*
Cash and Cash Equivalents	$ 869,659	$ 1,293,453	$ 423,794
T-Bills/Notes, Marketable Securities	3,423,050	3,634,448	211,398
Accrued Interest	19,327	24,211	4,884
Prepaid Expenses	65,094	88,261	23,167
Prepaid Insurance	29,550	30,338	788
Accounts Receivable	68,464	51,482	(16,982)
Intangible Pension Asset	10,019	7,860	(2,159)
Textbook Inventory	2,123	12,369	10,246
Computers, Furniture	436,216	467,516	31,300
Less: Accumulated Depreciation	(338,547)	(377,124)	(38,577)
TOTAL ASSETS	$ 4,584,955	$ 5,232,814	$ 647,859

LIABILITIES	*9/30/2003*	*9/30/2004*	*DIFFERENCE*
Exam Fees Deferred	$ 615,284	$ 714,605	$ 99,321
Annual Meeting Fees Deferred	169,695	70,070	(99,625)
Seminar Fees Deferred	3,000	181,060	178,060
Accounts Payable and Accrued Expenses	525,556	478,481	(47,075)
Accrued Pension	195,620	192,301	(3,319)
TOTAL LIABILITIES	$ 1,509,155	$ 1,636,516	$ 127,362

MEMBERS' EQUITY

Unrestricted	*9/30/2003*	*9/30/2004*	*DIFFERENCE*
CAS Surplus	$ 2,854,421	$ 3,312,044	$ 457,622
Pension minimum liability (net of unamortized service cost of $7,860–2004 and $10,019–2003)	(90,572)	(80,318)	10,254
Michelbacher Fund	126,329	129,160	2,831
CAS Trust-Operating Fund	98,777	107,825	9,048
Centennial Fund	0	23,944	23,944
Research Fund	43,668	62,482	18,814
Subtotal Unrestricted	$ 3,032,623	$ 3,555,137	$ 522,513

Temporarily Restricted	*9/30/2003*	*9/30/2004*	*DIFFERENCE*
Scholarship Fund	$ 6,018	$ 5,728	$ (291)
Rodermund Fund	8,107	7,391	(716)
CAS Trust-Ronald Ferguson Fund	29,052	28,042	(1,010)
Subtotal Temporarily Restricted	43,177	41,161	(2,016)
TOTAL MEMBERS' EQUITY	$ 3,075,800	$ 3,596,297	$ 520,498

Deborah Rosenberg, Vice President–Administration

This is to certify that the assets and accounts shown in the above financial statement have been audited and found to be correct.

CAS Audit Committee: Patricia A. Teufel, Chairperson;
Michael P. Blivess, Robert V. Deutsche, and Frederick O. Kist

2004 EXAMINATIONS—SUCCESSFUL CANDIDATES

Examinations for Exams 3, 5, 7-Canada, 7-United States, and 8 of the Casualty Actuarial Society were held May 4–7, 2004. Examinations for Exams 3, 6, and 9 of the Casualty Actuarial Society were held October 26–28, 2004.

Examinations for Exams 1, 2, and 4 are jointly sponsored by the Canadian Institute of Actuaries, Casualty Actuarial Society, and the Society of Actuaries and were held in April and September 2002. Candidates successful on these examinations were listed in joint releases of the Societies and the Institute.

The following candidates were admitted as Fellows and Associates at the 2004 CAS Spring Meeting in May. By passing Fall 2003 CAS examinations, these candidates successfully fulfilled the Society requirements for Fellowship or Associateship designation.

NEW FELLOWS

Afrouz Assadian	David J. Horn Jr.	Faith M. Pipitone
Kevin J. Atinsky	Jesse T. Jacobs	Jayne L. Plunkett
Stephanie Anne Bruno	Jonathan David Koch	John T. Raeihle
Phyllis B. Chan	Kristine Kuzora	Laura D. Rinker
Brian Kenneth Ciferri	James A. Landgrebe	Paul Silberbush
Christian J. Coleianne	Jia Liu	Christopher J. Styrsky
Richard R. Crabb	John R. McCollough	Shantelle Adrienne Thomas
Paul B. Deemer	Jeffrey B. McDonald	Peggy J. Urness
Kiera Elizabeth Doster	Martin Menard	Gaetan R. Veilleux
Robin V. Fitzgerald	Ryan A. Michel	Keith A. Walsh
David S. Futterleib	Matthew E. Morin	Matthew J. Wasta
Keith R. Gentile	Kyle S. Mrotek	Carolyn D. Yau
Joel D. Glockler	Lester M. Y. Ng	Eric Zlochevsky
Ann E. Green	Tom E. Norwood	

NEW ASSOCIATES

Keith P. Allen	Stephen E. Dupon	Luke G. C. Johnston
Kris Bagchi	Alexander R. George	John B. Kelly
Amber L. Butek	Chun Hua Hoo	John E. Kollar
Scott W. Carpinteri	Eric David Huls	Twiggy Lemercier
Melanie Sue Dihora	Joseph M. Izzo	Eric F. Liland

Lynn C. Malloney
Meagan S. Mirkovich
John A. Nauss
Robert Anthony Peterson
Timothy K. Pollis
Keith A. Rogers
Robert J. Schutte
Mark Sturm
Patrick Thorpe
Jeffrey J. Voss
Christopher M. White
Arthur S. Whitson
Joshua C. Worsham

The following candidates successfully completed the following Spring 2004 CAS examinations.

Exam 3

Eve Ingrid Adamson
Bradley J. Andrekus
Angelina M. Anliker
Ann Marie Bauer
Mark A. Baxter
François M. Beauchesne
Anthony O. Beirne
Duane Antony Bennett
Kelly S. Billings
Caleb A. Blakley-Cox
Elizabeth Bomboy
Kimberly A. Borgelt
Zachary L. Bowden
Sara Lynn Buchheim
Seth L. Burstein
Cui Liu Cai
Laura N. Cali
Laura M. Carstensen
Lauren Jill Cavanaugh
Luyuan Chai
Chun-Chieh Chang
Jing Chen
Sen Chen
Brenda Clemens
Sharon L. Cocconi
Peter M. Corrigan
Matthew Miller Crotts
Xiaoye Cui
Peter H. D’Orsi
Paige M. Demeter
Tehya Rose Duckworth
William R. Durrell
Curt G. Dye
Isaac R. Espinoza
Jeffrey N. Farr
Jacob C. Fetzer
Joseph A. Gage
Wei Gao
Brian P. Gill
Victoria A. Gomez
Christian Lee Goulding
Stephanie Gray
Mary Ann Grzyb
Amit K. Gupta
Julie A. Haakenson
Jeannette Marie Haines
David S. Harville
Raed J. Hasan
Peter Hennes
Wade R. Hess
Katherine Lyons Houlihan
Zhi Gang Huang
John R. Jasinski
Ya Jia
Jeffrey Axel Johnson
David M. Kaye
Johnny Siu Kei Ho
Jung-Ah Kim
Jeffrey Grant Kinsey
Jonathan M. Knotwell
John Arthur Krause
Emily J. Krebs
Steven P. Lafser
Carmen King-fung Lam
David F. Lee
Joyce Lee
Meyer Tedde Lehman
Adrienne J. Lewis
Hongmei Li
Sue Jean Liu
Kim Ho Lo
John David Lower
Kelly Mattheisz
Kelli R. McGinty
Daniel John Messner
Rui Min
Tricia C. Murphy
Nora Kathleen Newman

Liangsuo Ni
Jennifer M. Oglenski
David Robert Olson Jr.
Tad Jory Oman
Kristin Marie Palm
Sergey V. Pflyuk
Mallick Nacim Rachedi
Jason M. Rosin
Zachary K. Rutledge
Suzanna Sayre
Chad R. Schlippert
Carole B. Schumacher
Xianjing Shan
Cherie M. Simons
John J. Skowronski
Michael L. Smith
Hillel Soberman
Justin P. Sterling
Zhongmei Su
Ann M. Sydnor
Anne M. Taylor
Adrienne R. Thompson
Peter Tomopoulos
Hio-Kei Tong
William S. Turner
Pascal Vincent
Mark Russell Westmoreland
David K. Wheelock
John Spencer Wideman
Andrea Wong
Guotao Yang
Hin Kei Yeung
Pavel A. Zhardetskiy
Xu Zhu

Exam 5

Avraham Adler
Amit Agarwal
Hussain Ahmad
Rocklyn Tee Altshuler
Brian D. Archdeacon
Amel Arhab
Yanfei Z. Atwell
Kristi Spencer Badgerow
Sean Michael Bailey
Tiffany Jean Baron
Angelo Edward Bastianpillai
Jeffrey Donald Bellmont
Chandler P. Benson
Sonal Bhargava
Jonathan Bilbul
Brad Stephen Billerman
Robert C. Birmingham
Jon Paul Bloom
Peter George Blouin
Nicolas Boivin
Christie D. Bowerman
Steven G. Brenk
Peter J. Brown
Kevin S. Burke
John Lee Butel
Michael W. Buttke
Julianne A. Callaway
Daniel R. Campbell
Jessica Yiqing Cao
Li Cao
Jeffrey M. Casaday
Paul A. Ceaser
Matthew S. Chamberlain
Bernard L. Chan
Zhijian Chen
Agnes H. Cheung
Chun Kit Cheung
Max Chiao
Raul Cisneros
Jeffrey J. Clair
Jennifer Elizabeth Clark
Robert Alan Cole
Kirk Allen Conrad
Karen Cathleen Crosby
Lynn E. Cross
Andrew G. Davies
Kenneth M. Decker
Marc-Andre Desrosiers
Jonathan E. Devilbiss
Natalia Dimitrienko
Marc-Olivier Faulkner
Jason A. Flick
Mark A. Florenz
Kyle P. Freeman
Derek W. Freihaut
Jessica Morna Friedman
Justin Fritz
Nicholas A. Gammell
Timothy M. Garcia
Nina Vladimirovna Gau

Maxime Gelinas
Lilian Y. Giraldo
Barbara B. Glasbrenner
Evan W. Glisson
François Godbout
Gregory P. Goddu
Jio Young Goh
Rebecca J. Gordon
Chaim H. Gottesfeld
Melissa Lynn Greiner
Wesley John Griffiths
Samuel D. Grossman
Todd N. Gunnell
Ren Bin Guo
Gerald S. Haase
William J. Hackman
Brian P. Hall
Megan A. Hall
James W. Harmon
Joshua Rolf Harold Griffin
Michael P. Healy
Michael J. Hebenstreit
Chasity D. Hodges
Sheri L. Holzman
William A. Hossom
Candace Yolande Howell
Sheng-Fei Huang
Nathan Jaymes Hubbell
Alice H. Hung
Caleb Enders Huntington
Mohammad A. T. Hussain
Yu Shan Hwang
Lisa Michelle Hyde
Chris D. Izbicki
Ziyi Jiao
Jeremiah D. Johnson
Ross Evan Johnson
Jason C. Jones
Amy Ann Juknelis
Karine Julien
Kenneth Robert Kahn Jr.
Katherine Yukyue Kam
Jean-Philippe Keable
Steven M. Kendrick
David J. Klemish
Rachel M. Klingler
Thomas R. Kolde
Lok-Yi R. Kwok
Kayne M. Lammers
Catherine M. Larson
Annie Latouche
Vanessa Leblanc
Sara Leclerc
Seung-Won Lee
Jennifer Marie Lehman
Catherine Lemay
Nathan A. Lerman
Jean-François Lessard
Hoi Fai Leung
Xun-Yuan Liang
Paul T. Lintner
Cunbo Liu
Andrew F. Loach
Tony Lu
Neelam P. Mankoff
Minchong Mao
Michelle M. Marabella
Amanda Cater Marsh
Thomas Dudley Martin
Sean M. McAllister
Jennifer L. Meyer
Albert-Michael Micozzi
Daniel E. Mikesh
Chad M. Miller
Aaron G. Mills
Max H. Mindel
Quentin Mostoller
Mundia Mubyana
Eric L. Murray
Treva A. Myers
Marc L. Nerenberg
Benjamin R. Newton
Stephanie Jo Odell
James Patrick O'Donovan
Christopher John Olsen
Alejandro Antonio Ortega Jr.
Brent J. Otto
Ginette Pacansky
Daniel M. Padilha
Keith William Palmer
Ying Pan
Luyang Fu
Christopher James Platania
Rebecca Ann Polunas
Ricky R. Poulin
Darryl L. Raines
Arthur R. Randolph II
Zia Ur Rehman
Stephane Renaud
Zoe F. Rico

Dolph James Robb
Sophie Robichaud
Valerie Robitaille
Jeffrey N. Roth
Ray Michael Saathoff
Nicholas W. Saeger
Anita A. Sathe
Steven Michael Schafer
Ronald S. Scott
Surender S. Sekhon
Meredith A. Shadrach
Zilan Shen
Yiping Shi
Jacqueline W. Y. Shoong
Ann Marie Smith
Angela K. Sofinski
Yun Song
Richard C. Soulsby
Paul Q. Stahlschmidt
Yuchen Su
Maheswara Sudagar
Feixue Tang
Kathy M. Thompson
Lori R. Thompson
Katherine A. Tollar
Levente Tolnai
Rachel Katrina Tritz
Alice H. Tsai
Benjamin Joel Turner
Steven L. Turner
Jonathan K. Turnes
Howard Raymond Underwood
Humberto M. Valdes
Kevin John Van Prooyen
Melissa Joan Vaughn
Daniel Viau
Xuelian Wan
Hongtao Wang
Huiping Wang
Jingtao Wang
Ning Wang
Elizabeth A. Wentzien
Timothy G. Wheeler
Martin E. Wietfeldt
Ronald Harris Wilkins
Melvyn R. Windham Jr.
Andrew N. Wogman
Dorothy A. Woodrum
Shawn A. Wright
Zhikun Wu
Evelyn M. Wynen
Zhicheng Xin
Jenny Man Yan Yiu
Yi-Chuang Yang
Eecher Yee
Shuk-Han Lisa Yeung
Hidy Hiu Yin Lee
Arvelle D. Zacharias
Anton Zalesky
Robert J. Zehr
Juemin Zhang
Lang Zhang
Zhenyong Zhang
Wei Zhao
Yue Zhao
Weina Zhou
Yu Zhou
Xi Zhu
Michael V. Ziniti

Exam 7-Canada

Nicolas Marc Beaudoin
Matthew D. Buchalter
Denise L. Cheung
Chantal Delisle
Louise Frankland
Kristen Marie Gill
Veronique Grenon
Hugues Laquerre
Nathalie M. Lavigne
Raul Gabriel Martin
Jean-Pierre Paquet
Natalie St-Jean
Mark Stephen Struck
Daniel Jacob VanderPloeg
Timothy P. Wiebe

Exam 7-U.S.

Ying M. Andrew
Rebecca J. Armon
Daniel S. Barnett
Rose D. Barrett
Alexandra Robin Beckenstein
Mark Belasco
Darryl R. Benjamin
Derek Dennis Berget

Karen Lenoir Bethea
Brian J. Biggs
Rebekah Susan Biondo
Michael J. Blasko
Tapio N. Boles
John R. Broadrick
Michele L. Brooks
Karen B. Buchbinder
Melissa Lillian Bundt
Douglas James Busta
Rita Bustamante
Sandra J. Callanan
Samuel C. Cargnel
Jennifer L. Carrick
Tracy L. Child
Ting Him Choi
Gregory R. Chrin
Julia Feng-Ming Chu
Charles A. Cicci
David Alan Clark
Eric Clark
Chad J. Covelli
Spencer L. Coyle
Lawrence G. Cranor
Justin B. Cruz
Robert P. Daniel
Mari A. Davidson
Jesse W. F. De Couto
Julie A. Ekdom
Melissa Diane Elliott
Yehoshua Yosef Engelsohn
Joyce A. Ewing
Jieqiu Fan
Denise D. Fast
Bruce Fatz
Gina C. Ferst
Dale A. Fethke
Suzanne M. Finnegan
John S. Flattum
Jeffrey R. Fleischer
Matthew Timm Frank
Vincent M. Franz
Chad J. Gambone
Joseph Emmanuel Goldman
David B. Gordon
Jeffrey Robert Grimmer
Travis J. Grulkowski
Kyle M. Hales
Bobby Earl Hancock Jr.
Aaron G. Haning
Megan Taylor Harder
Robin A. Haworth
Jennifer Ann Hellmuth
Stephen J. Higgins Jr.
Carole K. L. Ho
Bo Huang
Jane W. Hughes
Li Hwan Hwang
Yehuda S. Isenberg
Jason Israel
Kenneth Layne Israelsen
William T. Jarman
Richard Clay Jenkins
Min Jiang
Shiwen Jiang
Yi Jing
Julie M. Joyce
Anthony N. Katz
William J. Keros
Amy Jieseon Kim
Scott M. Klabacha
Susan L. Klein
Steve C. Klingemann
Stephen Jacob Koca
Terry T. Kuruvilla
David Matthew Lang
Henry T. Lee
Jeffrey Leeds
Jeremy M. Lehmann
Yuxiang Lei
Kenneth L. Leonard
Kahshin Leow
Sharon Xiaoyin Li
Wei Li
Xin Li
Herman Lim
Edward P. Lionberger
Erik Frank Livingston
PeiQing Luo
Luis S. Marques
Jonathan L. Matthews
Brent L. McGill
Christopher Charles McKenna
Phillip E. McKneely
Todd C. Meier
Thomas E. Meyer
Jason L. Morgan
Maria M. Morrill
Alan E. Morris
Catherine A. Morse
Yuchun Mu
John-Giang L. Nguyen
Leonidas V. Nguyen
Lisa M. Nield
Joshua M. Nyros
Timothy James O’Connor

Russel W. Oslund
Alan M. Pakula
Lorie A. Pate
Jeffrey J. Pfluger
Edward L. Pyle
Damon Joshua Raben
Conni A. Rader
Eric W. L. Ratti
Gregory S. Richardson
Dale M. Riemer
Dave H. Rodriguez
Kevin D. Roll
William Paige Rudolph
Derek Michael Schaff
Lawrence M. Schober
Genine Darrough Schwartz
Suzanne A. M. Scott
Peter Abraham Scourtis
Richard H. Seward IV
Steven R. Shallcross
Quan Shen
Sarah J. Shine
Annemarie Sinclair
Helen A. Sirois
Heidi Leigh Sjoberg
Thomas Richard Slader
Justin Nicholas Smith
Sheila R. Soulsby
Michael Daniel Stephens
Zongli Sun
Michelle M. Syrotynski
Christian Alan Thielman
Robert W. Thompson
Benjamin Joel Turner
David J. Watson
Chang-Hsien Wei
Stephen C. Williams
Amy M. Wixon
Ann Min-Sze Wong
Donald S. Wroe
Xinxin Xu
Huey Wen Yang
Yuanhe Yao
Ka Chun Yeung
Sung G. Yim
Joshua A. Youdovin
Janice M. Young
Navid Zarinejad
Lijuan Zhang

Exam 8

Richard T. Arnold
Martha E. Ashman
John L. Baldan
Danielle L. Bartosiewicz
Michael Alan Bean
Chris M. Bilski
John T. Binder
Kirk D. Bitu
Rebecca Schafer Bredehoeft
Cheryl R. Burrows
Amber L. Butek
Matthew E. Butler
Christine Cadieux
Kevin K. W. Chan
Yves Charbonneau
Eric D. Chen
Hung Francis Cheung
Benjamin W. Clark
David Alan Clark
Eric John Clymer
Thomas Marie Cordier
Keith W. Curley
John Edward Daniel
Timothy M. Devine
Melanie Sue Dihora
Christopher A. Donahue
Brian S. Donovan
Stephen E. Dupon
Jeffrey A. Dvinoff
Ellen D. Fitzsimmons
Sean Paul Forbes
Susan J. Forray
Sebastien Fortin
Robert C. Fox
David I. Frank
Michael Anthony Garcia
David A. Gelberg
Laszlo J. Gere
Gregory Evan Gilbert
Olga Golod
Lori A. Gordon
John W. Gradwell
Stacie R. W. Grindstaff
Isabelle Groleau
Simon Guenette
Jonathan M. Guy
Jason C. Harland

Joseph Hebert
James Anthony Heer
Ryan Yin-kei Ho
Jason N. Hoffman
John F. Huddleston
Eric David Huls
Scott R. Hurt
John F. Janssen
William Brian Johnson
Luke G. C. Johnston
Steven M. Jokerst
Derek A. Jones
Kyewook Gary Kang
Robert B. Katzman
Susan M. Keaveny
Young Y. Kim
Ziv Kimmel
Raymond J. Kluesner
Gregory E. Kushnir
Hooi Lee Lai
ZhenZhen Lai
Francis A. Laterza
Hoi Keung Law
Patricia Lee
Yuxiang Lei
Isabelle Lemay
Gavin X. Lienemann
Kenneth Lin
Nataliya A. Loboda
Eric A. Madia
John T. Maher
Steven Manilov
Sharon L. Markowski
Laura S. Martin
Jason N. Masch
Laura A. Maxwell
Laurence R. McClure II
James P. McCoy
Jennifer A. McGrath
John D. McMichael
Sarah K. McNair-Grove
Jennifer Middough
Charles W. Mitchell
Celso M. Moreira
Timothy C. Mosler
James C. Murphy Jr.
Norman Niami
James L. Norris
James D. O’Malley
William S. Ober
Lowell D. Olson
Eva M. Paxhia
Bruce G. Pendergast
Matthew J. Perkins
Andrea L. Phillips
Ellen K. Pierce
Jorge Eric Pizarro
Timothy K. Pollis
Michele S. Raeihle
Brad E. Rigotty
Keith A. Rogers
Benjamin G. Rosenblum
Nancy Ross
Stuart C. Rowe
Frederick Douglas Ryan
Ronald J. Schuler
Jin Shao
Peter M. Shelley
Jimmy Shkolyar
Thomas M. Smith
Michael D. Sowka
Erik J. Steuernagel
Liana St-Laurent
Yuchen Su
Keith Jeremy Sunvold
Beth M. Sweeney
Adam D. Swope
Erica W. Szeto
Ellen Marie Tierney
Malgorzata Timberg
David A. Traugott
Nathalie Tremblay
Lien K. Tu-Chalmers
Stephen H. Underhill
Natalie Vishnevsky
Jeffrey J. Voss
Mary Elizabeth Waak
Amy R. Waldhauer
Kristie L. Walker
Matthew J. Walter
Gary C. Wang
Thomas E. Weist
Arthur S. Whitson
Rosemary Gabriel Wickham
Micah G. Woolstenhulme
Joshua C. Worsham
Jimmy L. Wright
Bradley J. Zarn
Gene Q. Zhang
Yi Zhang
Hongbo Zhou

The following candidates were admitted as Fellows and Associates at the 2004 CAS Annual Meeting in November. By passing Spring 2004 CAS examinations, these candidates successfully fulfilled the Society requirements for Fellowship or Associateship designation.

NEW FELLOWS

Richard T. Arnold
Martha E. Ashman
Danielle L. Bartosiewicz
John T. Binder
Rebecca Schafer Bredehoeft
Kevin K. W. Chan
Yves Charbonneau
Julia Feng-Ming Chu
Benjamin W. Clark
David Alan Clark
Eric John Clymer
John Edward Daniel
Brian S. Donovan
Stephen E. Dupon
Jeffrey A. Dvinoff
Sean Paul Forbes
Susan J. Forray
David I. Frank
David A. Gelberg
Gregory Evan Gilbert
Olga Golod
Lori A. Gordon
Stacie R. W. Grindstaff
Isabelle Groleau
Simon Guenette
Jonathan M. Guy
Jason C. Harland
Ryan Yin-kei Ho
Jason N. Hoffman
John F. Huddleston
Li Hwan Hwang
Jason Israel
John F. Janssen
Steven M. Jokerst
Derek A. Jones
Kyewook Gary Kang
Anthony N. Katz
Robert B. Katzman
Susan M. Keaveny
Ziv Kimmel
Raymond J. Kluesner
Gregory E. Kushnir
Hooi Lee Lai
ZhenZhen Lai
Francis A. Laterza
Yuxiang Lei
Isabelle Lemay
Kenneth Lin
Nataliya A. Loboda
Eric A. Madia
Steven Manilov
Sharon L. Markowski
Jason N. Masch
Laura A. Maxwell
Jennifer A. McGrath
John D. McMichael
Sarah K. McNair-Grove
Jennifer Middough
Charles W. Mitchell
Celso M. Moreira
Timothy C. Mosler
James C. Murphy Jr.
William S. Ober
James D. O'Malley
Jeffrey J. Pfluger
Ellen K. Pierce
Gregory S. Richardson
Nancy Ross
Stuart C. Rowe
Frederick Douglas Ryan
Ronald J. Schuler
Jin Shao
Peter M. Shelley
Jimmy Shkolyar
Thomas M. Smith
Michael D. Sowka
Adam D. Swope
Robert W. Thompson
Ellen Marie Tierney
David A. Traugott
Nathalie Tremblay
Lien K. Tu-Chalmers
Stephen H. Underhill
Natalie Vishnevsky
Jeffrey J. Voss
Mary Elizabeth Waak
Amy R. Waldhauer

Kristie L. Walker
Matthew J. Walter
Gary C. Wang
Thomas E. Weist
Arthur S. Whitson
Rosemary Gabriel Wickman
Micah Grant Wollstenhulme
Joshua C. Worsham
Jimmy L. Wright
Bradley J. Zarn
Gene Q. Zhang
Yi Zhang

NEW ASSOCIATES

Karen H. Adams
Ying M. Andrew
Brian D. Archdeacon
Rebecca J. Armon
Dan S. Barnett
Nicolas Marc Beaudoin
Alexandra Robin Beckenstein
Derek Dennis Berget
Brian J. Biggs
Rebekah Susan Biondo
Michael J. Blasko
Tapio N. Boles
John R. Broadrick
Michele L. Brooks
Matthew Buchalter
Douglas James Busta
Jennifer L. Carrick
Patrick J. Causgrove
Tracy L. Child
Charles A. Cicci
Eric Clark
Chad J. Covelli
Spencer L. Coyle
Justin B. Cruz
Robert P. Daniel
Mari A. Davidson
Chantal Delisle
Julie A. Ekdom
Yehoshua Y. Engelsohn
Jieqiu Fan
Bruce Fatz
Gina C. Ferst
Dale A. Fethke
Suzanne M. Finnegan
Jeffrey R. Fleishcer
Matthew Timm Frank
Louise Frankland
Marie LeStourgeon Fredericks
Chad J. Gambone
Kristen Marie Gill
David B. Gordon
Veronique Grenon
Jeffrey Robert Grimmer
Travis J. Grulkowski
Kyle M. Hales
Bobby Earl Hancock Jr.
Megan Taylor Harder
Robin A. Haworth
Stephen J. Higgins Jr.
Carol K. L. Ho
Bo Huang
Farid Aziz Ibrahim
Yehuda S. Isenberg
Kenneth Layne Israelsen
William T. Jarman
Richard Clay Jenkins
Min Jiang
Shiwen Jiang
Yi Jing
Julie M. Joyce
William J. Keros
Amy Jieseon Kim
Scott M. Klabacha
Susan L. Klein
Steve C. Klingemann
Terry T. Kuruvilla
David Matthew Lang
Nathalie M. Lavigne
Henry T. Lee
Kenneth L. Leonard
Kahshin Leow
Wei Li
Xin Li
Herman Lim
PeiQing Luo
Luis S. Marques
Raul Gabriel Martin
Laura S. Martin
Jonathan L. Matthews
Christopher Charles McKenna
Jason L. Morgan
Maria M. Morrill
Yuchun Mu

Leonidas V. Nguyen
Joshua M. Nyros
Timothy James O'Connor
Russel W. Oslund
Alan M. Pakula
Jean-Pierre Paquet
Lorie A. Pate
Michael J. Quigley
Damon Joshua Raben
Eric W. L. Ratti
Dale M. Reimer
Kevin D. Roll
Derek Michael Schaff
Genine Darrough Schwartz
Richard H. Seward IV
Steven R. Shallcross
Quan Shen
Sarah J. Shine
Anne Marie Sinclair
Thomas Richard Slader
Justin Nicholas Smith
Sheila R. Soulsby
Michael Daniel Stephens
Natalie St-Jean
Mark Stephen Struck
Zongli Sun
Michelle M. Syrotynski
Daniel Jacob VanderPloeg
Chang-Hsien Wei
Timothy P. Wiebe
Ann Min-Sze Wong
Donald S. Wroe
Xinxin Xu
Huey Wen Yang
Yuanhe Yao
Sung G. Yim
Joshua A. Youdovin
Janice Minhuei Young
Lijuan Zhang

The following candidates successfully completed the following Fall 2004 CAS examinations.

Exam 3

Christina Dione Abbott
Victor Manuel Alonso
Erika Lee Anderson
Waswate Pomm Ayana
Nathan James Baseman
Robert T. Bell
Guillaume Benoit
David R. Benseler
Timothy D. Boles
James T. Botelho
Steve Boudreault
Delano D. Brown
Jason A. Cabral
Keith J. Champagne
Sally H. M. Chan
Tao Chen
Gregory R. Chrin
Ryan A. Ciaccio
Jason A. Clark
David E. Colon
Jordan Paul Comacchio
Richard Joseph Cundari
Katy J. Cuthbertson
Randi M. Dahl
Jacob B. Davis
Richard Garvin Day
Laura E. Doucette
Melinda Sue Down
Alexander Dupont
Michael Keith Edison
Thomas Kent Ellingson
Christopher J. Enlund
Robert C. Enslen
Danielle J. Fiorello
Michael J. Fiorito
Adam C. Fleming
Timothy J. Fleming
David Andrew Fletcher
Demetrios Fokas
Sebastien Fortin
Augustin Gas
Stuart G. Gelbwasser
Jenifer E. Gilbert
Kasi Joelle Golden
Kristen A. Goodemote
Ruchama Graff
Joshua S. Grunin
Vincent Ha

Ryan Lyle Hansen
Christina Maria Harrington
James R. Healey
Donald F. Hendriks
Roberto A. Hernandez
Xia Huang Hu
Paul Jeffrey Hurd
Lisa Michelle Hyde
Jed Isaman
Jennifer J. Jabben
Linda Jacob
Hannah L. Janney
Robert D. Jordan
Lisa K. Juday
Anne C. Kallfisch
Jennifer Ann Kitchen
Rodney Christopher Kleve
Stephen Jacob Koca
Konstantinos Evangelos Koutsothodoros
Sz-Fan Lai
Lily K. Lam
Amie Yuen-San Lee
Christie Lai Yin Lee
Richard Brian Levy
Kelly Carmody Lewis
Kexin Li
Kenneth Eng Lim
Megan E. Link
Dustin John Loeffler
Carl Lussier
Debra Anne Maizys
Vijayakum Manghnani
Timothy F. Mankowski
Rebecca R. McCarrier
Kenneth James Meluch
Joshua D. Merck
Robert Lazar Midgette
Erick E. Mortenson
Gary J. Nelson
David Niedziela
Lisa M. Nield
Yuk N. Pang
Gena S. Park
Petya Svilenova Petrova
William J. Pitts
David N. Prario
Justin Radick
Jason M. Ramsey
Emily Ruth Reither
Bruce A. Ritter
Randall D. Ross
Kyle Nathan Roth
Ryan David Schemenauer
Kevin J. Semanick
James G. Shaheen
Daniel Sharvit
Anne Gerardene Sheehy
Adam Brian Sherwin
Steven C. Sousa
Robert Vincent Spencer
David Chan Stanek
Joshua Adam Taub
Gordon C. Thompson
Donald K. Treanor
Hung-Yu Tsai
Liza Tsai
Jon Tsou
Tony Alan Van Berkel
Marina Vaninsky
Thomas W. Vasey
Kanika Vats
Wan-Yi Wang
Zhenyu Wang
Nicole E. Warner
Timothy R. Wengerd
Chung Yin Wong
Shing-Ming Wong
Stephen Wong
Adrian Ralph Wood
Zhijian Xiong
Gang Xu
Jianlu Xu
Chunjing Zhang
Ge Zhang
Mingmin Zhang

Exam 6

Avraham Adler
Amit Agarwal
Hussain Ahmad
Rocklyn Tee Altshuler
Brandie Jean Andrews
George N. Argesanu
Kelleen D. Arquette
Yanfei Z. Atwell
Kristi Spencer Badgerow
Gregory K. Bangs
Tiffany Jean Baron

Mark Belasco
Amelie Beliveau
Richard J. Bell III
Jeffrey Donald Bellmont
Kelly Beres
Jonathan Bilbul
Brad Stephen Billerman
Charles H. Birckhead
Robert C. Birmingham
François Blais
Peter George Blouin
James M. Boland
Bernardo Bracero Jr.
Peter J. Brown
Yisheng Bu
Karen B. Buchbinder
Michael Edward Budzisz
Morgan Haire Bugbee
Kevin S. Burke
Cheryl R. Burrows
Cemal Alp Can
Chuan Cao
Jessica Yiqing Cao
Jeffrey M. Casaday
Simon Castonguay
Paul A. Ceaser
Matthew S. Chamberlain
Bernard L. Chan
Annie Chang
Yung-Chih Chen
Agnes H. Cheung
Leong Yeong Chew
Chung M. Ching
Raul Cisneros
Glenn A. Colby
Daniel G. Collins
Russell A. Creed
Karen Cathleen Crosby
Walter C. Dabrowski
Melisa L. Darnieder
Scott C. Davidson
Kenneth M. Decker
Stephen P. Decoteau
Jonathan E. Devilbiss
Matthew S. Dobrin
Melissa Diane Elliott
Nicole Elliott
David J. Engelmayer
William H. Erdman
Michael D. Ersevim
Christopher G. Fanslau
Marc-Olivier Faulkner
Solomon Carlos Feinberg
Sean W. Fisher
John S. Flattum
Mark A. Florenz
Beth A. Foremsky
Kyle P. Freeman
Chong Gao
Nina Vladimirovna Gau
Gregory P. Goddu
Priyangsha S. Godha
Jio Young Goh
Victoria A. Gomez
Jie Gong
Rebecca J. Gordon
Neil A. Greiner
Wesley John Griffiths
Ren Bin Guo
Amit K. Gupta
Gerald S. Haase
John J. Hageman
James W. Harmon
Joseph Patrick Hasday
Megann E. Hess
Keepyung Bernard Hong
William A. Hossom
Alison Therese Hover
Nai-Wen Hsu
Min Huang
Queenie W. C. Huang
YinYin Huang
Nathan Jaymes Hubbell
Alice H. Hung
Somil Jain
Kamil K. Jasinski
Ziyi Jiao
Jeremiah D. Johnson
Ross Evan Johnson
Scott A. Kaminski
Hyeji Kang
Raisa Katz
Sarah M. Kemp
Gareth L. Kennedy
Kayne M. Kirby
David J. Klemish
Rachel M. Klingler
John Karl Knapstein
Kathryn Rose Koch
Thomas R. Kolde
Matthew R. Kuczwaj
Christopher SungKu Kwon

François Langevin
Catherine M. Larson
Hidy Hiu Yin Lee
Seung-Won Lee
Jeremy M. Lehmann
Catherine Lemay
Vincent Lepage
Nathan A. Lerman
Hoi Fai Leung
Mingyue Li
Xun-Yuan Liang
Reng Lin
Edward P. Lionberger
Weichen Liu
Andrew F. Loach
Neelam P. Mankoff
Minchong Mao
Jeffrey L. Martin
Ana J. Mata
Paul H. Mayfield
George Joseph McCloskey
Angela Garrett McGhee
Jeffrey S. McSweeney
Albert-Michael Micozzi
Daniel E. Mikesh
Chad M. Miller
Travis J. Miller
Aaron G. Mills
Richard James Mills
Max H. Mindel
Mark H. Mondello
Lori A. Moore
Allison L. Morabito
Laura M. Morrison
Erica F. Morrone
Joey Douglas Moulton
Mundia Mubyana
Benjamin R. Newton
Stephanie Jo Odell
Christopher John Olsen
Erin M. Olson
Aran Jee-Yun Paik
Keith William Palmer
Ying Pan
Joseph G. Pietraszewski
James J. Pilarski
Dominique Pilote
Christopher James Platania
Rebecca Ann Polunas
Justin Radick
Moiz Rawala
Timothy O. Reed
Zia Ur Rehman
Stephane Renaud
Zoe F. Rico
Arnie W. Rippener
Dolph James Robb
Sophie Robichaud
Valerie Robitaille
Juan Carlos Rodriguez
Richard R. Ross
Brent Sallay
Anita A. Sathe
Steven Michael Schafer
Richard T. Schneider
Terri L. Schwomeyer
Brian M. Scott
Sheri Lee Scott
Daniel Silverstein
Jiyang Song
Yun Song
Richard C. Soulsby
Brooke S. Spencer
Bryan V. Spero
Paul Q. Stahlschmidt
Sandra E. Starnes
Amy Lyn Steburg
Yuchen Su
Maheswara Sudagar
Mariane Aiko Takahashi
Feixue Tang
Elissa Y. Thompson
Robby E. Thoms
Robert Bradley Tiger
Daniel Tinoco
Levente Tolnai
Rachel Katrina Tritz
Choi Nai Charlies Tu
Steven L. Turner
Jonathan K. Turnes
Kevin John Van Prooyen
Victor Cabal Victoriano
Allan Voltz
Benjamin James Walker
Christopher L. Wampole
Xuelian Wan
Chong Wang
Huiping Wang
Jingtao Wang
Jamie M. Weber
Lei Wei
Yu-Chi Wen
Shannon A. Whalen

Martin E. Wietfeldt
Ronald Harris Wilkins
Stephen C. Williams
Brant Wipperman
Stephen K. Woodard
Jianlu Xu
Yu chen Xu
Yi-Chuang Yang
Fan Ye
Robert J. Zehr
Guowen Zhang
Lang Zhang
Qin Zhao
Wei Zhao
Yue Zhao
Weina Zhou
Yu Zhou
Xi Zhu
Paul W. Zotti

Exam 9

Vera E. Afanassieva
Ying M. Andrew
Melissa J. Appenzeller
Koosh Arfa-Zanganeh
John L. Baldan
Angelo Edward Bastianpillai
Derek Dennis Berget
Sonal Bhargava
Brian J. Biggs
Corey J. Bilot
Chris M. Bilski
Kirk D. Bitu
Sharon D. Blumer
Nicolas Boivin
Tapio N. Boles
Michele L. Brooks
Matthew D. Buchalter
Suejeudi Buehler
Amber L. Butek
Li Cao
James Chang
Hung Francis Cheung
Vivien K. Chiang
Ting Him Choi
Martin P. Chouinard
Matthew P. Collins
Kevin A. Cormier
Chad J. Covelli
Tighe Christian Crovetti
Justin B. Cruz
Keith W. Curley
David A. DeNicola
Nicholas J. De Palma
Marc-Andre Desrosiers
Ryan M. Diehl
Melanie Sue Dihora
Brian M. Donlan
Tomer Eilam
Bruce Fatz
Dale A. Fethke
Ellen D. Fitzsimmons
William J. Fogarty
Peter L. Forester
Marie LeStourgeon Fredericks
Derek W. Freihaut
Michael Anthony Garcia
Timothy M. Garcia
William John Gerhardt
Kristen Marie Gill
Simon Girard
François Godbout
Charles R. Grilliot
Jeffrey Robert Grimmer
Isabelle Guerin
David Bruce Hackworth
Trevor Casey Handley
Megan Taylor Harder
Eric A. Hatch
Robin A. Haworth
James Donald Heidt
Joseph S. Highbarger
Carole K. L. Ho
Wang Yang Hu
Bo Huang
Eric David Huls
Scott R. Hurt
Yu Shan Hwang
Richard Clay Jenkins
Min Jiang
Amy Ann Juknelis
Jean-Philippe Keable
Samir Khare
Amy Jieseon Kim
Young Y. Kim
Susan L. Klein
Brandon E. Kubitz

Darjen D. Kuo
David Lawrence Larson
William Scott Lennox
Kahshin Leow
Jean-François Lessard
Amanda M. Levinson
Xin Li
Gavin X. Lienemann
Herman Lim
Hsin-Hui Grace Lin
Cunbo Liu
Jin Liu
Tony Lu
John T. Maher
Laura S. Martin
Jonathan L. Matthews
James P. McCoy
Brent L. McGill
Christopher Charles McKenna
Shawn Allan McKenzie
Mitchel Merberg
Stephanie A. Miller
Meagan S. Mirkovich
Daniel G. Myers
Leonidas V. Nguyen
James L. Norris
Timothy James O'Connor
Nancy Eugenia O'Dell-Warren
Alejandro Antonio Ortega Jr.
Bruce G. Pendergast
Matthew J. Perkins
Damon Joshua Raben
Michele S. Raeihle
Dale M. Riemer
Mindy M. Romeo
Robert Allan Rowe
Nicholas W. Saeger
Genine Darrough Schwartz
Ronald S. Scott
Quan Shen
Zilan Shen
Sarah J. Shine
Summer Lynn Sipes
James M. Smieszkal
Justin Nicholas Smith
Sheila R. Soulsby
Esperanza Stephens
Liana St-Laurent
Mark Stephen Struck
Ju-Young Suh
Keith Jeremy Sunvold
Erica W. Szeto
Luc Tanguay
Dawn M. Thayer
Malgorzata Timberg
Joel A. Vaag
Martin John Van Driel
Kevin K. Vesel
Mo Wang
Amanda J. White
Christopher Morris White
Nicholas J. Williamson
Dorothy A. Woodrum
Min Yao
Yuanhe Yao
Ka Chun Yeung
Jonathan K. Yu
Arvelle D. Zacharias
Ronald J. Zaleski Jr.
Lijuan Zhang

New Fellows Admitted in May 2004

First row, from left: Matthew J. Wasta, Faith M. Pipitone, Afrouz Assadian, Carolyn D. Yau, Kiera Elizabeth Doster, Hao Chai, **CAS President Mary Frances Miller**, James A. Landgrebe, Martin Menard, John R. McCollough, Paul Silberbush, David S. Futterleib. **Second row, from left:** William Brent Carr, Laura D. Rinker, Robin V. Fitzgerald, Keith A. Walsh, Jennifer L. Vadney, Kristine Kuzora, Jia Liu, Ann E. Green, Keith R. Gentile, Robert S. Weishaar, David J. Horn Jr., Linda Jean Bjork, Brian Kenneth Ciferri.

NEW FELLOWS ADMITTED IN MAY 2004

First row, from left: Henry Joseph Konstanty, Peggy J. Urness, Shantelle Adrienne Thomas, Gaétan R. Veilleux, **CAS President Mary Frances Miller**, Patricia Deo-Campo Vuong, Paul B. Deemer, Jeffrey B. McDonald, Matthew E. Morin, Christopher J. Styrsky. **Second row, from left:** Jesse T. Jacobs, Tom E. Norwood, Kevin J. Atinsky, Jayne L. Plunkett, Richard R. Crabb, Richard Alan Van Dyke, James Christopher Guszcza, Kyle S. Mrotek, Eric Zlochevsky, John T. Raeihle. **New Fellows not pictured:** Stephanie Anne Bruno, Phyllis B. Chan, Christian J. Coleianne, Joel D. Glockler, Ryan A. Michel, Lester M. Y. Ng, Joseph C. Wenc.

New Associates Admitted in May 2004

First row, from left: Scott W. Carpinteri, Eric F. Liland, Mark Sturm, Esperanza Stephens, Melanie Sue Dihora, **CAS President Mary Frances Miller**, Hung Francis Cheung, Kris Bagchi, Richard U. Newell, Timothy K. Pollis, Keith P. Allen, Arthur S. Whitson. **Second row, from left:** Patrick Thorpe, John E. Kollar, Joseph M. Izzo, Lynn C. Malloney, Robert Anthony Peterson, Meagan S. Mirkovich, Amber L. Butek, Eric David Huls, Christopher M. White, Alexander R. George, Keith A. Rogers, Dominic A. Tocci. **New Associates not pictured:** Stephen E. Dupon, Chun Hua Hoo, Luke G. C. Johnston, John B. Kelly, Twiggy Lemercier, John A. Nauss, Robert J. Schutte, Jeffery J. Voss, Joshua C. Worsham.

NEW FELLOWS ADMITTED IN NOVEMBER 2004

First row, from left: John Edward Daniel, Ellen K. Pierce, Bradley J. Zarn, Jeffrey J. Pfluger, **CAS President Mary Frances Miller**, Michael D. Sowka, Rosemary Gabriel Wickman, Kyewook Gary Kang, Sharon L. Markowski. **Second row, from left:** Jason N. Hoffman, Stephen E. Dupon, Ryan Yin-kei Ho, Susan M. Keaveny, Nathalie Tremblay, Stacie R. W. Grindstaff, Danielle L. Bartosiewicz, Kenneth Lin, Gene Q. Zhang, Jeffrey A. Dvinoff. **Third row, from left:** James D. O'Malley, Matthew J. Walter, Ryan Michel, David I. Frank, Eric A. Madia, Thomas M. Smith, Richard T. Arnold, James C. Murphy Jr., Raymond J. Kluesner.

NEW FELLOWS ADMITTED IN NOVEMBER 2004

First row, from left: Isabelle Groleau, Laura A. Maxwell, ZhenZhen Lai, Amy R. Waldhauer, **CAS President Mary Frances Miller**, Jennifer Middough, Stuart C. Rowe, Gary C. Wang, Martha E. Ashman. **Second row, from left:** Jonathan M. Guy, Steven M. Jokerst, Adam D. Swope, Yves Charbonneau, Lori A. Gordon, Steven Manilov, Kristie L. Walker, Natalie Vishnevsky, Mary Elizabeth Waak. **Third row, from left:** Simon Guenette, Jonathan David Koch, Gregory E. Kushnir, Celso M. Moreira, David A. Gelberg, Isabelle Lemay, Hooi Lee Lai, Stephen H. Underhill.

NEW FELLOWS ADMITTED IN NOVEMBER 2004

First row, from left: Jin Shao, Rebecca Schafer Bredehoeft, Olga Golod, **CAS President Mary Frances Miller**, Sean Paul Forbes, Jason N. Masch, Brian S. Donovan. **Second row, from left:** Susan J. Forray, Micah Grant Wollstenhulme, David A. Traugott, Frederick Douglas Ryan, Thomas E. Weist, John T. Binder. **Third row, from left:** Julia Feng-Ming Chu, Jason Israel, Gregory S. Richardson, Jason C. Harland, Benjamin W. Clark, Nataliya A. Loboda. **New Fellows not pictured:** David Alan Clark, Eric John Clymer, Gregory Evan Gilbert, John F. Huddleston, Li Hwan Hwang, John F. Janssen, Derek A. Jones, Anthony N. Katz, Francis A. Laterza, Jennifer A. McGrath, Sarah K. McNair-Grove, Charles W. Mitchell, Nancy Ross, Ronald J. Schuler, Peter M. Shelley, Jimmy Shkolyar, Robert W. Thompson, Jeffrey J. Voss, Arthur S. Whitson, Joshua C. Worsham, Yi Zhang.

NEW ASSOCIATES ADMITTED IN NOVEMBER 2004

First row, from left: Derek Dennis Berget, Dale M. Reimer, Shiwen Jiang, Raul Gabriel Martin, **CAS President Mary Frances Miller**, Sung G. Yim, Mari A. Davidson, Nicolas Marc Beaudoin, Matthew Buchalter. **Second row, from left:** Ying M. Andrew, Amy Jieseon Kim, Marie LeStourgeon Fredericks, Yehoshua Y. Engelsohn, Eric W. L. Ratti, Rebekah Susan Biondo, Jason L. Morgan, Robin A. Haworth, Susan L. Klein, Tracy L. Child, Bruce Fatz. **Third row, from left:** Justin B. Cruz, Chad J. Covelli, Spencer L. Coyle, Michael J. Blasko, Luis S. Marques, Mark Stephen Struck, Chad J. Gambone, John R. Broadrick, Michelle M. Syrotynski, Suzanne M. Finnegan, Farid Aziz Ibrahim.

New Associates Admitted in November 2004

First row, from left: Matthew Timm Frank, Chang-Hsien Wei, Jeffrey Robert Grimmer, Steve C. Klingemann, **CAS President Mary Frances Miller**, Brian D. Archdeacon, Dan S. Barnett, Bobby Earl Hancock Jr., Yuanhe Yao. **Second row, from left:** Chantal Delisle, Henry T. Lee, Leonidas V. Nguyen, Richard Clay Jenkins, Eric Clark, Jennifer L. Carrick, Michele L. Brooks, Sheila R. Soulsby, Genine Darrough Schwartz, Nathalie M. Lavigne, Donald S. Wroe. **Third row, from left:** Louise Frankland, Jean-Pierre Paquet, Julie A. Ekdom, Steven R. Shallcross, David B. Gordon, Grace H. Yang, Gina C. Ferst, William T. Jarman, Joshua A. Youdovin, Douglas James Busta, Herman Lim.

New Associates Admitted in November 2004

First row, from left: Dale A. Fethke, Kenneth Layne Israelsen, Charles A. Cicci, **CAS President Mary Frances Miller**, Kevin D. Roll, Robert P. Daniel, Timothy James O'Connor. **Second row, from left:** Janice Minhuei Young, Kristen Marie Gill, Jeffrey R. Fleischer, Michael J. Quigley, Kahshin Leow, Julie M. Joyce, Travis J. Grulkowski. **Third row, from left:** David Matthew Lang, Daniel Jacob VanderPloeg, Brian J. Biggs, Scott M. Klabacha, Terry T. Kuruvilla, Stephen J. Higgins Jr. **New Associates not pictured:** Karen H. Adams, Rebecca J. Armon, Alexandra Robin Beckenstein, Tapio N. Boles, Patrick J. Causgrove, Jieqiu Fan, Kyle M. Hales, Megan Taylor Harder, Bo Huang, Yehuda S. Isenberg, Min Jiang, Yi Jing, John B. Kelly, William J. Keros, Kenneth L. Leonard, Wei Li, Xin Li, PeiQing Luo, Laura S. Martin, Jonathan L. Matthews, Maria M. Morrill, Yuchun Mu, Joshua M. Nyros, Russel W. Oslund, Alan M. Pakula, Damon Joshua Raben, Derek Michael Schaff, Richard H. Seward IV, Quan Shen, Sarah J. Shine, Thomas Richard Slader, Anne Marie Sinclair, Natalie St-Jean, Michael Daniel Stephens, Zongli Sun, Timothy P. Wiebe, Ann Min-Sze Wong, Xinxin Xu, Huey Wen Yang.

OBITUARIES

WARREN P. COOPER
KARL F. EATON
ROBERT B. FOSTER
WILLIAM S. GILLAM
ROBERT ANDERSON MILLER III
OWEN D. RICHMOND
STUART BRIAN SUCHOFF
DONALD M. WOOD JR.

WARREN P. COOPER
1929–2004

Warren Peters Cooper died November 5, 2004 at his home in Delaware Township, New Jersey. He was 75.

Born in Bronxville, New York, Cooper graduated from Yale University in New Haven, Connecticut in 1959. A Navy veteran, Cooper served during the Korean War.

After many years of service, Cooper retired in 2000 from Ernst & Young in Philadelphia, where he was chief principal. Previously, he had worked for the Insurance Company of North America and Chubb. Earlier in his career, he was chief actuary for the state of New Jersey Department of Insurance.

Cooper earned his CAS Associateship in 1969 and was involved in CAS activities throughout the 1970s and early 1980s. He showcased his writing and analytical skills as a *Proceedings* author, discussing the paper "The Actuary and IBNR" by Bornhuetter and Ferguson in 1973. Cooper's committee service included the Committee on Financial Reporting (1974–1976), the

Committee on Loss Reserves (1977–1980), and the Committee on Reserves (1980–1983).

Cooper is survived by his companion, Jerald Stowell, and his brother, Willian Cooper.

KARL F. EATON
1925–2004

Karl F. Eaton died on April 1, 2004 in Leawood, Kansas. He was 78.

Eaton was born in Sewal, Iowa in 1925. After serving in World War II, he graduated from Kansas City University in 1949.

Eaton worked for Business Men's Assurance Company before settling at Employers Reinsurance Corp., where he retired. He was a member of Life Office Management Association and a member of the Academy of Actuaries. Eaton became an Associate of the Casualty Actuarial Society in 1954.

Eaton was very involved in his community. He was a member of the Village Presbyterian Church, scoutmaster of Troop 91, and chairman of the Kanza District. He was an honorary warrior in the Tribe of Mic-O-Say and recipient of Boy Scouts Award of Merit and Silver Beaver Award. Eaton served as presidents of Missouri Hospital Association of Auxiliaries and Truman Medical Center Auxiliary. He had recently been a volunteer at the Blue Valley Library in Johnson County, Kansas. Eaton was also chairman of Senior Net Computer User Group.

He is survived by his wife Martha; and brothers Orval Eaton of Shingle Springs, California, and Thurman Eaton of Aurora, Colorado; sister Ferne Colley of Green Valley, Arizona; and nieces, nephews, and friends. Eaton was preceded in death by his son Jay.

ROBERT B. FOSTER
1926–2004

Robert Bessom Foster of Windsor, Connecticut and North Port, Florida, died February 4, 2004 in Tampa, Florida at the age of 77.

He was born in Lynn, Massachusetts in 1926, the son of the late Robert and Josephine (Bessom) Foster. He had lived in Windsor for the past 50 years. He and his wife, Sally (Buccheri) Foster, had seven children.

Foster was a graduate of Marblehead High School in Marblehead, Massachusetts and served in the U.S. Navy during World War II. In 1948 he earned a Bachelor of Arts degree in mathematics from Dartmouth College. He was employed as an actuary at Travelers Insurance Company for 39 years, retiring in 1989.

Foster earned his ACAS designation in 1952 and his FCAS designation in 1955. An impressively active member of the Casualty Actuarial Society, Foster became the first recipient of the Matthew Rodermund Service Award, which he won in 1991. He began as a *Proceedings* author in 1954 with his paper "The Boiler and Machinery Premium Adjustment Rating Plan," and followed up in 1966 with "Budgeting—A System for Planning and Controlling Expenses." His CAS committee participation included service on the Committee on Annual Statement (1969), Program Committee (1972–1973), and Library Committee (1973–1974). He also worked as a member and consultant of CAS Examination Committee from 1978 to 1982.

Foster's CAS responsibilities increased as he took on higher-level assignments during the 1970s and 1980s. He served as assistant to the CAS secretary-treasurer in 1971 and as secretary-treasurer from 1972 to 1973. He chaired the Finance Committee from 1975 to 1978, was an Education and Examination Committee consultant from 1978 to 1982, and a member of the CAS Board of Directors from 1983 to 1985.

Foster also participated in several civic and church activities. He served on the Windsor Zoning Board of Appeals, the Windsor Board of Ethics, and the Connecticut Coalition for Organ and Tissue Donation. Foster was a communicant of St. Gabriel's Church, a member of the Good Shepherd Choir, a Eucharistic minister, and a member of the Small Christian Community at the church. He participated in bible studies and in ecumenical bible studies.

Besides his wife, Foster is survived by his children, two daughters-in-law and one son-in-law: Daniel and Gay Foster of Putney, Vermont; Donald C. Foster of Windsor; Elizabeth and Bradley Koltz of Wenham, Massachusetts, Mary L. Foster of California; Robert B. Foster Jr. of Washington State; Thomas A. Foster of Miami, Florida; and William Foster and Jana Niemeyer of California. Foster is also survived by seven grandchildren and several nieces and nephews. He was predeceased by a sister, Carol Hoitt.

WILLIAM S. GILLAM
1919–2004

William "Bill" Gillam died January 24, 2004, in Metuchen, New Jersey. He was 84.

Born May 22, 1919, Gillam had two brothers and one sister. He attended Rutgers University and earned a Bachelor of Science degree in business administration in 1940.

During college, Gillam was part of ROTC and, after graduation, went directly into training. He and his bother Edward both served on the beaches of Normandy in World War II. Gillam was permanently injured during his duty and retired from the army as a 1st Lieutenant.

He married Janet in 1947 and together they had two sons. The marriage ended in divorce. In 1968, Gillam married Helen Bremner.

Gillam earned his CAS Associateship in 1953 and Fellowship in 1957. He was involved in CAS activities throughout the 1960s and 1970s. He served as chairperson of the Publicity Committee (1964), as member of the Council (1965–1967), and as CAS Librarian (1970). He was a member of the Committee on Automobile Insurance Research (1964), Examination and Education Committee-Education (1964–1973), Financial Review Committee (1966), and a member and consultant to the Education and Examination Committee–Examination (1968).

Gillam was also a CAS author with discussions of the papers "Automobile and Casualty Insurance Ratemaking in Canada" in the 1970 *Proceedings* and "Automobile Collision Deductibles and Repair Cost Groups—The Lognormal Model" in the 1973 *Proceedings*. His other works include a seminar report on "Practical Aspects of Automobile Merit Rating," which was published in the 1960 *Proceedings*.

An expert on automobile insurance, Gillam was at the center of expanding the automobile classification system in the late 1950s and early 1960s. He worked at the National Bureau of Casualty Underwriters, which became the Insurance Rating Board and later ISO.

"My father belonged to 'the greatest generation,'" said son, Robin. "He served but never asked for thanks, never complained," said Gillam. "He lived his life with the kind of honor you see all too rarely these days. His service to the actuarial profession showed the same kind of honor he showed his country. He gave back to his profession," said Gillam.

The younger Gillam followed his father's career path and became an actuary, earning his CAS Fellowship in 1987. The elder Gillam was very proud to have a son and a daughter-in-law, Judy Gillam (FCAS 1989), in the profession.

One of Bill Gillam's greatest pleasures was getting away to his vacation home on New Jersey's Barnegat Bay where he sailed with his sons. Robin said his father's love of sailing was consistent with his character of giving and not taking. "With a sailboat, he wasn't polluting the bay with fuel or noise," said Gillam.

William S. Gillam is survived his wife, Helen; son, William Robin and daughter-in-law, Judy of Marietta, Georgia; son, Thom and daughter-in-law, Judy of Perkasie, Pennsylvania; a brother, Edward of Pennsaukin, New Jersey; a sister, Elizabeth of Ontario, Canada; and three grandchildren.

ROBERT ANDERSON MILLER III
1920–2004

Robert Anderson Miller III, of West Hartford, Connecticut, died at his home on January 8, 2004 at the age of 83. He was the beloved husband of Faith (Sutton) Miller for 56 years and the son of Robert A. Miller Jr. and Edith Hotchkiss Miller of Tarentum, Pennsylvania. He graduated from St. Paul's School in Concord, New Hampshire, where he was a Ferguson Scholar. He earned his Bachelor of Arts degree in mathematics from Yale University and was a member of Phi Beta Kappa and Sigma Psi. First commissioned as an ensign in the United States Navy in 1943, Miller served several years at the radio communications station in Oahu, Hawaii. After his honorable discharge, he joined the actuarial department of Aetna Life and Casualty as a life actuarial student in 1946. One year later, he married Faith Sutton, a former WAVE officer.

Miller became an Associate of the Casualty Actuarial Society in 1979 and achieved Fellowship in 1986. He was one of the first people to hold Fellowships in both the CAS and the Society of Actuaries. Miller served several volunteer roles in the CAS such as representative to the Joint Committee on the Valuation Actuary (1986–1989), chairperson to the committee on Valuation Principles and Techniques (1987–1989), and chairperson to the Task Force on the Appointed Actuary (1993). He also co-wrote the 1989 article, "Valuation Principles Statement" as well as the paper, "Seminar on Valuation Issues," which appeared in the Fall 1989 *Forum*. While at Aetna, he worked in the group division, becoming vice president in 1966, and vice president and corporate actuary in 1974. After retiring from Aetna in 1985, he became a consulting actuary for Milliman & Robertson Inc.

After retiring from consulting in 1993, he enjoyed traveling to national parks with his wife and daughter Betsy. Miller was a member of Westminster Presbyterian Church in West Hartford. He taught Sunday school, was a ruling elder and a trustee, and served as treasurer. He was an active member of Red Oak Swim Club and served as president. He was a life long fan of the Pittsburgh Pirates and his sense of humor brought continuing joy to his friends and family.

Miller is survived by his wife, Faith, and children: Nancy Miller Houlihan; Faith S. Miller and her husband, Robert Andrian; Robert Anderson Miller IV and his wife, Kimberlee Miller; and Elizabeth (Betsy) Hotchkiss Miller. His grandchildren and brother Richard Miller also survive him. Miller's brother Frederick H. Miller Jr., of Santa Fe, New Mexico, died in December 2002.

OWEN D. RICHMOND
1915–2004

Owen D. Richmond of Kansas City, Missouri, passed away May 1, 2004, at his home.

Richmond was born in Glasford, Illinois, and attended the public schools there. He graduated from Knox College in Galesburg, Illinois, and then attended the University of Iowa to study actuarial mathematics. After college, he worked for three years at the Metropolitan Life Insurance Company in New York City. In 1941, Richmond entered the U.S. Army, where he served as captain for five years during WWII and later during the Korean War. In 1946, he came to Kansas City and was employed by Business Men's Assurance Company for 32 years. Richmond became an Associate of the CAS in 1953.

Richmond retired in 1979 as accounting vice president. He was a member of the American Academy of Actuaries, the Civil War Round Table of Kansas City, and a charter member of the Royal Lancers. He was a mason for over 59 years and a member of Hillside Christian Church in Kansas City. He was a former adherent and volunteer for the Salvation Army.

Richmond is survived by his wife, Helen Elliot Richmond of Kansas City; his son, Dr. John Richmond, his daughter-in-law, Jill Richmond, and grandson, Jeffrey Richmond, all of Lincoln, Nebraska; and his sister, Mrs. Clarice Hamil of Clearwater, Florida.

STUART BRIAN SUCHOFF
1955–2004

Stuart B. Suchoff died August 5, 2004, after an eight-month fight against cancer. He was 48.

Suchoff was born November 11, 1955, to Joshua and Phyllis Weiss Suchoff in Newburgh, New York. He married Ann Parkhill in February 1983.

Suchoff earned a Bachelor of Science degree, cum laude, in mathematics in 1976 and a Master of Science degree in applied mathematics in 1977 from Rensselaer Polytechnic Institute in Troy, New York.

Suchoff became an Associate of the CAS in 1981 and a Fellow in 1984. He was very involved in the CAS, serving on the Program Planning Committee as a member (1988–1991), vice chair (1991–1992), and chairperson (1992–1995). He served on the Committee on Reserves as vice chairperson (1995–1996) and chairperson (1996–1999), and was a member of the External Communications Committee (1988–1991). He was also part of the CAS University Liaisons program from 2001 to 2004.

In November 2004, Stuart Suchoff was posthumously awarded the CAS Above and Beyond Achievement Award (ABAA). Suchoff was honored for his numerous contributions but specifically for his work as chairperson of the American Academy of Actuaries Risk-Based Capital Task Force. Despite his contention that the topic was out of his "comfort zone," Suchoff accepted the challenge to provide leadership to the group. One person nominating Suchoff for the ABAA wrote: "His dedication, focus, and clear vision of what needed to be done and how to go about doing it made it a pleasure to work with him and for him."

Among his other activities, Suchoff was a member of the Conference of Consulting Actuaries and the International Actuarial Association.

Suchoff last served as a principal for Milliman Inc. in Irvine, California. Although in hospice care, Suchoff continued to work from his hospital bed. In a message to Suchoff's Milliman colleagues, Ann Suchoff wrote that Stuart "put up a valiant fight for the last eight months. He has certainly enjoyed working with you."

Suchoff is survived by his wife, Ann, and daughter, Megan, of Laguna Niguel, California; a brother, Marvin G. Suchoff of Rowayton, Connecticut; and a sister, Loren Suchoff Brandman of Fairfax, Virginia.

DONALD M. WOOD JR.
1916–2004

Donald M. Wood Jr., 88, a retired insurance executive and a ubiquitous board member and volunteer in Evanston, Illinois organizations and business groups, died Sunday, March 26, in Evanston Hospital of pneumonia.

He began working in his father's insurance agency after graduating from Dartmouth College and the Amos Tuck Business School in 1933. Wood became a CAS Associate in 1937. He worked at the company, Childs & Wood, for more than half a century, except for the time he spent serving in World War II. He sold the business in 1990.

A quiet man who took his work seriously, Wood had a good sense of humor and was known for his civic activities. A resident of Evanston since 1939, Wood served as president of the city's United Way and Family Counseling, as a board member of the Evanston Historical Society, and as a trustee and usher at Evanston's First United Methodist Church. In his retirement, Wood donated his professional experience to Hispanic communities in the Chicago area, helping businesses set up insurance policies through a program run by the Executive Services Corps. "He had plenty of energy, and he was always busy," said his wife, Katherine. "He enjoyed that, and felt like he was contributing something, which is something he wanted to do."

In addition to his wife, Wood is survived by two daughters, Charlotte Wheeler and Betsy Knapp; a son, James; three grandchildren; and a great-grandson.

INDEX TO VOLUME XCI

INDEX—CONTINUED

Page

INDEX—CONTINUED

Page

INDEX—CONTINUED