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## PROCEEDINGS

## OF THE

# Casualty Actuarial Society

Organized 1914



## 2003

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#### FOREWORD

Actuarial science originated in England in 1792 in the early days of life insurance. Because of the technical nature of the business, the first actuaries were mathematicians. Eventually, their numerical growth resulted in the formation of the Institute of Actuaries in England in 1848. Eight years later, in Scotland, the Faculty of Actuaries was formed. In the United States, the Actuarial Society of America was formed in 1889 and the American Institute of Actuaries in 1909. These two American organizations merged in 1949 to become the Society of Actuaries.

In the early years of the 20th century in the United States, problems requiring actuarial treatment were emerging in sickness, disability, and casualty insurance—particularly in workers compensation, which was introduced in 1911. The differences between the new problems and those of traditional life insurance led to the organization of the Casualty Actuarial and Statistical Society of America in 1914. Dr. I.M. Rubinow, who was responsible for the Society's formation, became its first president. At the time of its formation, the Casualty Actuarial and Statistical Society of America had 97 charter members of the grade of Fellow. The Society adopted its present name, the Casualty Actuarial Society, on May 14, 1921.

The purposes of the Society are to advance the body of knowledge of actuarial science applied to property, casualty, and similar risk exposures, to establish and maintain standards of qualification for membership, to promote and maintain high standards of conduct and competence for the members, and to increase the awareness of actuarial science. The Society's activities in support of this purpose include communication with those affected by insurance, presentation and discussion of papers, attendance at seminars and workshops, collection of a library, research, and other means.

Since the problems of workers compensation were the most urgent at the time of the Society's formation, many of the Society's original members played a leading part in developing the scientific basis for that line of insurance. From the beginning, however, the Society has grown constantly, not only in membership, but also in range of interest and in scientific and related contributions to all lines of insurance other than life, including automobile, liability other than automobile, fire, homeowners, commercial multiple peril, and others. These contributions are found principally in original papers prepared by members of the Society and published annually in the *Proceedings of the Casualty Actuarial Society*. The presidential addresses, also published in the *Proceedings*, have called attention to the most pressing actuarial problems, some of them still unsolved, that the industry has faced over the years.

The membership of the Society includes actuaries employed by insurance companies, industry advisory organizations, national brokers, accounting firms, educational institutions, state insurance departments, and the federal government. It also includes independent consultants. The Society has three classes of members—Fellows, Associates, and Affiliates. Both Fellows and Associates require successful completion of examinations, held in the spring and fall of each year in various cities of the United States, Canada, Bermuda, and selected overseas sites. In addition, Associateship requires completion of the CAS Course on Professionalism. Affiliates are qualified actuaries who practice in the general insurance field and wish to be active in the CAS but do not meet the qualifications to become a Fellow or an Associate.

The publications of the Society and their respective prices are listed in the Society's Yearbook. The Syllabus of Examinations outlines the course of study recommended for the examinations. Both the Yearbook, at a charge of \$40 (U.S. funds), and the Syllabus of Examinations, without charge, may be obtained from the Casualty Actuarial Society, 1100 North Glebe Road, Suite 600, Arlington, Virginia 22201.

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#### NOTICE

Papers submitted to the *Proceedings* of the Casualty Actuarial Society are subject to review by the members of the Committee on Review of Papers and, where appropriate, additional individuals with expertise in the relevant topics. In order to qualify for publication, a paper must be relevant to casualty actuarial science, include original research ideas and/or techniques, or have special educational value, and must not have been previously copyrighted or published or be concurrently considered for publication elsewhere. Specific instructions for preparation and submission of papers are included in the *Yearbook* of the Casualty Actuarial Society.

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## PROCEEDINGS May 18, 19, 20, 21, 2003

#### SOURCE OF EARNINGS ANALYSIS FOR PROPERTY-CASUALTY INSURERS

#### SHOLOM FELDBLUM

#### Abstract

Source of earnings analysis has long been a staple of life insurance policy pricing and profitability monitoring. It has grown in importance with the advent of universal life insurance and similar contracts with nonguaranteed benefits or charges. Statement of Financial Accounting Standard (SFAS) 97 requires insurers to use source of earnings analysis for Generally Accepted Accounting Practice (GAAP) reporting of universal lifetype contracts.

Source of earnings analysis is not a specific ratemaking method, like the loss ratio method or the pure premium method. Rather, source of earnings analysis is a reporting structure that reveals the sources of gain and loss on a block of business, highlighting errors in the pricing parameters, as well as the sensitivity of profit and loss to various pricing factors, and enabling more accurate selection of new parameters and factors. This paper applies source of earnings analysis to workers compensation and personal automobile insurance. The uncertainty in many casualty insurance pricing factors, such as loss development factors and loss trend factors, makes source of earnings analysis particularly important for casualty products.

The paper shows how to use the source of earnings exhibits to better analyze insurance profitability. The private passenger auto illustration divides the difference between actual and expected results between estimation error, which is within the purview of the pricing actuary, and random errors, which result from stochastic fluctuations in loss occurrences, inflation rates, or interest rates.

The workers compensation illustration focuses on the spread between the earned and credited interest rates, the solicitation costs for not-taken business,<sup>1</sup> and the amortization of initial expense and loss costs by policy year.

Analysis of the variances from previous years' predictions is a means of improving next year's predictions. Sources of earnings analysis provides the needed postmortem to judge the accuracy of the pricing assumptions.<sup>2</sup>

#### 1. INTRODUCTION

This paper illustrates source of earnings analysis for propertycasualty insurance. Source of earnings analysis is a staple of life insurance policy pricing. It is mandated by National Association of Insurance Commissioners (NAIC) regulations for par-

<sup>&</sup>lt;sup>1</sup>"Not-taken" business is business that is underwritten and for which an insurance offer is made but not accepted. The importance of not-taken business for determining fixed expense provisions by classification is discussed in Feldblum [1996], which deals with policy pricing. This paper shows the methods to test for variance of actual results from the pricing assumptions.

<sup>&</sup>lt;sup>2</sup>I am indebted to Jill Petker, Ruy Cardozo, and John Conners, for extensive comments on an earlier draft of this paper.

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ticipating policies issued by mutual life insurance companies, and it is required by SFAS 97 for amortization of deferred policy acquisition expenses on universal life policies and policies with non-guaranteed benefits or charges.

We discuss source of earnings analysis for private passenger automobile and workers compensation ratemaking. Personal auto ratemaking is well suited to source of earnings analysis, since the volume of business is large enough for the effects of estimation error and random error to be distinguished. In addition, private passenger automobile has high retention rates and different acquisition costs for new policies vs. renewal policies, making profitability highly sensitive to persistency patterns.

Workers compensation retrospectively rated policies are analogous to universal life insurance contracts in that expected profits stem from margins in the pricing assumptions. The casualty actuary prices the components of the retrospectively rated policy, such as the insurance charge and the excess loss charge, even as the life actuary prices the components of the universal life policy.

Large commercial policies have high not-taken rates, various premium payment plans, and much investment income, all of which require pricing expertise. Comparing total premiums with total costs may not yield the information needed to improve the pricing process. Source of earnings analysis is better suited to identifying the causes of superior and inferior performance.

#### Structure of This Paper

Section 2 provides a description of source of earnings analysis as applied to life insurance products, with specific reference to (i) the calculation of policyholder dividends by means of the contribution principle for mutual life insurance companies and (ii) the SFAS 97 accounting for universal life-type products. This section is background; it may be skipped by readers who are already familiar with source of earnings analysis or those who wish to focus on only the casualty applications. Section 3 applies source of earnings analysis to private passenger automobile ratemaking. This section explains the difference between estimation error and process error; the handling of credibility; and the difference between implicit and explicit profit margins.

Section 4 applies source of earnings analysis to workers compensation ratemaking for retrospectively rated contracts. This section discusses static versus dynamic amortization of deferred policy acquisition costs, and the source of earnings exhibits showing charged, expected, and actual results.

Section 5 summarizes the implications of the paper for pricing paradigms and the effects of random variations.

#### 2. CLASSICAL SOURCE OF EARNINGS ANALYSIS

Source of earnings analysis was first used to set policyholder dividends for participating life insurance sold by mutual insurance companies. Source of earnings analysis is also needed to amortize the GAAP deferred policy acquisition expenses for universal life-type contracts (SFAS 97) and for participating policies sold by mutual life insurance companies (SFAS 120).

#### Policyholder Dividends

The contribution principle, which is required by the NAIC model act on policyholder dividends and by the American Academy of Actuaries *Standards of Practice*, mandates that the amount of divisible surplus used to pay policyholder dividends on a block of business reflect the contribution of that block to company earnings.<sup>3</sup> Although simple and elegant, this principle

<sup>&</sup>lt;sup>3</sup>See particularly Actuarial Standard of Practice #15, "Dividend Determination and Illustration for Participating Individual Life Insurance Policies and Annuity Contracts," and Actuarial Standard of Practice #24, "Compliance with the NAIC Life Insurance Illustrations Model Regulation."

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is difficult to apply rigorously, since it requires the actuary to quantify the long-term contribution to profit from variations in the pricing assumptions.

The major elements affecting life insurance profitability and used in source of earnings analysis are persistency rates (or withdrawal rates), interest earnings, and mortality ratios. Each of these is also applicable to property-casualty business.

#### Illustration—Persistency Rates

Suppose the expected withdrawal rates were 10% for the second year of a cohort of permanent life insurance policies, but the actual withdrawal rates are 15%. The surrender charges and the takedown of conservative statutory reserves cause an increase in statutory profits in the second year. But the smaller block of persisting business leads to lower profits in succeeding years. These lower profits offset the statutory gain from the second year. If the initial acquisition costs are not fully recovered by the surrender charges, policyholder dividends may have to be reduced. Source of earnings analysis helps quantify the equitable change in the dividend rate.

For casualty products, we use a simpler adjustment for persistency changes. Solicitation costs on not-taken business, as well as high first year acquisition expenses, are amortized over the expected policy lifetimes. If withdrawal rates increase, the amortization period is reduced and profitability declines.<sup>4</sup>

#### Illustration—Interest Earnings

Suppose that the expected Treasury bill yield for the second year of a cohort of permanent life policies was 6% but the actual yield is 5% per annum. The change in statutory investment earnings during this year may be slight, since (i) the coupons on existing bonds have not changed, (ii) bonds are valued at amor-

<sup>&</sup>lt;sup>4</sup>Casualty products do not show the temporary increase in statutory profitability from higher terminations stemming from surrender charges and the release of policy reserves, so decreased persistency shows a drop in both immediate and long-term profits.

tized cost in statutory statements, and (iii) invested assets are still small in the second year of a cohort of permanent life insurance policies. The change in long-term profitability depends on the duration and inflation sensitivity of the liabilities. For a guaranteed cost block of traditional whole life business, the expected long-term profitability might drop (since liability durations are generally longer than asset durations), possibly causing a decrease in policyholder dividends.

The effects of changing interest rates are more complex for casualty products, since inflation affects loss payments and interest rates affect asset returns.<sup>5</sup> A full source of earnings exhibit shows the effects of variation in loss cost trends side-by-side with the effects of variation in the investment yield. The difference is the net effect on profitability.

#### Mortality

Variations in mortality ratios highlight the importance of distinguishing estimation error from process error. Suppose the ratio of actual-to-expected mortality in the second year of a cohort of business is 150%. If the higher than expected mortality reflects random deaths, policyholder dividends paid to the remaining insureds should not be changed. If the higher than expected mortality reflects a poor quality book of business, the policyholder dividends may have to be reduced.

For casualty business, loss frequency and severity are similar to life insurance mortality rates. Higher than expected loss frequency or severity may reflect either random loss occurrences or estimation error. Severe estimation errors call for re-examination of the pricing assumptions.

#### Amortization of the Deferred Policy Acquisition Cost (DPAC)

In statutory statements, acquisition costs are written off when they are incurred. In GAAP statements for traditional life insur-

<sup>&</sup>lt;sup>5</sup>See Feldblum ["Investment Strategy," forthcoming].

ance policies, deferred policy acquisition costs (DPAC) are expensed as the premium is earned. For universal life-type policies, there is no set premium, so one cannot amortize the DPAC asset in relation to premiums. SFAS 97 mandates that the DPAC asset be amortized as a proportion of future expected gross profits.<sup>6</sup>

To illustrate the use of source of earnings analysis in FAS 97 accounting, consider an unexpected increase in the withdrawal rate from 10% to 15% in the second year of a cohort of policies. If this cohort consists of universal life-type policies, the DPAC asset would be amortized in relation to future expected gross profits. Suppose that originally the second year profits were expected to be 10% of all future profits. After the withdrawal rate increase, the actual second year profits increase and the future expected profits decrease. The second year profits are now higher than 10% of all profits, and a correspondingly larger amount of deferred policy acquisition costs is amortized in the second year.<sup>7</sup>

#### Extension to Casualty Products

Source of earnings analysis is applicable to any insurance product whose returns depend on conditions subsequent to policy pricing. This is true of all property-casualty products, since their returns depend on random loss occurrences, interest rates, and inflation rates.

Profitability also depends on the persistency of the business, particularly for direct writing insurers (D'Arcy and Doherty [1989]). Prospective pricing of products whose profitability depends on persistency patterns relies on asset share models; see Feldblum [1996]. Subsequent monitoring of product perfor-

<sup>&</sup>lt;sup>6</sup>The term "universal life-type" is the GAAP term for policies with benefits or charges that are not fixed. Gross profits are profits before the amortization of deferred policy acquisition costs; net profits are profits after the amortization of deferred policy acquisition costs. The amortization of these costs in relation to expected gross profits, rather than in relation to premiums, makes sense for all policies, not just universal life. The Financial Accounting Standards Board (FASB) did not wish to change accounting practice for existing policies, so the new rules apply only to universal life-type policies.

<sup>&</sup>lt;sup>7</sup>For a thorough analysis of SFAS 97, along with illustrations of the source of earnings exhibits, see Tan [1989] and Eckman [1990].

mance uses dynamic amortization of the deferred policy acquisition costs by means of multi-year source of earnings exhibits. We examine the dynamic amortization of solicitation costs for not-taken business in retrospectively rated workers compensation policies.

Workers compensation retrospectively rated policies have premiums based on the total exposure, but they provide insurance coverage for only certain layers of loss. The cost of the coverage is based on an insurance charge calculation that considers premium bounds, loss limits, the risk size, and hazard group. Profitability depends on implicit margins in the insurance charge and on the investment income from the underwriting cash flows. Source of earnings analysis allows the actuary to monitor the performance of the business in terms of the pricing assumptions.

As these illustrations show, source of earnings exhibits can deal even with gains and losses that are not generally reflected in profitability monitoring. But the primary benefits of source of earnings analysis are more general. Source of earnings analysis serves as a postmortem of previous reviews, evaluating the accuracy of the assumptions, and uncovering the causes of poor performance.

#### 3. PRIVATE PASSENGER AUTOMOBILE

The structure of the source of earnings analysis depends on the factors affecting the rates for each line of business. Most life insurance products use a four-factor analysis, focusing on withdrawal rates, mortality ratios, interest rates, and expense ratios. For property-casualty products, mortality ratios are replaced by loss assumptions, such as loss development and loss trend, or loss frequency and loss severity.

There are three levels of the source of earnings analysis: individual factor, policy year, and policy cohort:

• The individual factor level shows the application of source of earnings analysis to each earnings factor. For private passen-

ger automobile, we examine loss severity trends in this paper, differentiating between estimation error and process error. For workers compensation, we examine several earnings factors: non-ratable losses, acquisition costs, and interest earnings.

- The source of earnings exhibits for a single policy or a single policy year combine the earnings factors but do not consider policy persistency (retention rates). These exhibits are appropriate for blocks of business with (i) low persistency rates, (ii) little difference between first year and renewal year loss and expense costs, and (iii) low solicitation costs for not-taken business.
- The source of earnings exhibits for a cohort of policies considers both the new writings and all the renewals. These are the standard exhibits required for universal life-type policies and for participating policies issued by mutual life insurance companies.

Maintenance expenses are not discussed in this paper. Maintenance expenses are generally stable, and they are more easily analyzed by direct examination than by source of earnings exhibits.

#### Individual Factor Level: Estimation Error and Process Error

We illustrate source of earnings analysis with loss cost trend adjustments. For private passenger automobile, whose exposure base (car-years) is not inflation sensitive, trend factors are critical for rate adequacy.

Actual results frequently differ from expected results. Source of earnings analysis relates this difference to the underlying earnings factors (or "sources"). For each factor, there are two potential reasons for the difference: estimation error and process error.

• Estimation error is the difference between the forecast and the true expectation.

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- Process error is the difference between the true expected result and the actual realization.

These errors emerge over time, from the date of the rate review to the final settlement of claims. Estimation error can often be controlled by the pricing actuary, whereas process error is an unavoidable element of insurance operations.<sup>8</sup>

The personal auto trend illustration here uses an experience period of accident year 20X4 to set rates for annual policies written in 20X6. Thus the trend period is 2.5 years (7/1/20X4 to 1/1/20X7). Suppose the projected trend rates estimated from countrywide fast track data are +7% severity and +1% frequency.<sup>9</sup>

Errors may result from three sources:

- 1. predicting future countrywide loss trends based on historical fast track experience,
- 2. applying countrywide trends to a particular state, and
- 3. using loss trend estimates to predict the changes in actual losses incurred.

*Estimation Error:* Suppose that several months after the policy year expires, the source of earnings analysis shows that the actual fast track trend rates were +8% for severity and +2% for frequency. The fast track estimates, which we used as a proxy for the actual loss trends, were too low. This is estimation error.<sup>10</sup>

<sup>&</sup>lt;sup>8</sup>Separating estimation error from process error is not always easy; see the comments in footnote 9.

<sup>&</sup>lt;sup>9</sup>Numerous data sources are available for trend estimates. The illustration in the text assumes that the pricing actuary uses countrywide fast track data to estimate trend factors, since this allows a clear demarcation between estimation error and process error. The same two sources of error exist when one extrapolates future trend factors from the company's historical statewide experience, though it is harder to separate the two sources of error.

<sup>&</sup>lt;sup>10</sup>The concepts are important, not the mechanics. Conceive of this illustration as an initial derivation of a 7% annual trend by fitting an exponential curve to 1996–1999 experience. Two and a half years later we retrospectively find that the actual fit was an 8% annual trend.

#### TABLE 1

	Estimated Fast Track	Actual Fast Track	Loss Cost Change	
Loss Severity	+7%	+8%	+5%	
Loss Frequency	+1%	+2%	+4%	

#### ESTIMATION ERROR AND PROCESS ERROR

*State Differences:* Differences between countrywide and statewide trends are not easily discerned. When there is no change in state compensation systems or other exogenous factors, no difference would be expected. When there is a change in compensation systems or in other exogenous factors (such as attorney involvement in insurance claims), trend differences can be significant. To simplify the presentation, we do not analyze countrywide-statewide differences.<sup>11</sup>

We examine the average loss severities and frequencies in the experience period and in the new policy period. Our initial numbers are estimates, since (i) the figures for the new policy year are immature, and (ii) even for the experience period the loss severities are not yet final. We won't have actual loss severity and loss figures for the new policy period until all the policies have expired, and these figures will change further as the losses are settled. For the first source of earnings exhibit, we use a mix of actual data and revised estimates. For subsequent source of earnings exhibits, the actual data are more complete.

Suppose the new loss severity and frequency figures show a change of +5% for severity and +4% for frequency, as shown in Table 1.<sup>12</sup>

<sup>&</sup>lt;sup>11</sup>The 1991 compensation system changes in Massachusetts showed the effect of structural changes on expected loss frequency and loss severity; see Marter and Weisberg [1992]. On the importance of these regional differences as private passenger automobile cost drivers, see Conners and Feldblum [1998].

<sup>&</sup>lt;sup>12</sup>Table 1 refers to the observed change as the "loss cost change" expressed as an annual trend. An observed change in the statewide average loss cost per claim of +12.97% over the  $2\frac{1}{2}$  period is shown as a +5.0% actual annual change (1.050<sup>2.5</sup> = 1.1297).

We underestimated loss severity by 1 percentage point (+7% versus +8%), and we underestimated loss frequency by 1 percentage point (+1% versus +2%). For a 2.5 year trend period, this causes the rates to be inadequate by 4.9% [(( $1.08 \times 1.02$ )/( $1.07 \times 1.01$ ))<sup>2.5</sup>]. This is the estimation error.

The actual loss severity change was +5% per annum, and the actual loss frequency change was +4% per annum. We do not call this the actual trend, since it may be influenced by random losses. Lacking other information, we presume that the true severity trend is +8% per annum, and the true frequency trend is +2% per annum. The low observed severity trend may stem from unusually large claims in the experience period or a lack of large claims in the new policy period. Similar random effects may account for the large change in claim frequency.

If compensation system changes and structural changes are not explicitly considered, they are subsumed under the process risk component of the source of earnings exhibits. For instance, there may be an influx of nuisance claims in the new policy period that are settled for small amounts.<sup>13</sup>

We group the various explanations of the difference between the observed patterns in the state and the "hindsight" trend observed in the fast track data as the process error in the trend estimate. This term is not ideal, since not all of the causes of the observed difference result from process error. We simply mean that the observed difference does not stem from misestimating the expected trend.

As the new policy year develops and actual data replaces estimates, the observed loss trends may change. The changes can be large until the new policy year is fully earned, followed by smaller changes as losses are settled. For a single policy year, the first few years of the source of earnings exhibits are most

<sup>&</sup>lt;sup>13</sup>The phenomenon has plagued private passenger automobile insurance for the past twenty years, and it must always be considered when the frequency change is large and the severity change is small.

#### TABLE 2

Valuation Date	n Proje	ction	Estimation		Process	Total	
	Initial Projection	Implicit Profit	Revised Estimate	Estimation Error	Actual Change (Annualized)	Process Error	Total Variance
12/20X7	+7%	\$0	+8%	-\$250K	+5%	+\$750K	+\$500K

#### PRIVATE PASSENGER AUTO LOSS SEVERITY (ONE YEAR)

valuable. For a cohort of business whose profitability depends (in part) on persistency, the year-by-year source of earnings exhibits are more important.

#### Extending the Exhibits

To analyze the sensitivity of profits to trend errors, we convert the estimation and process errors into dollar amounts. Assuming \$10 million of annual losses and using the figures above, we begin the source of earnings exhibits, as shown in Table 2.

The figures are simplified for ease of presentation. We assume a 2.5 year trend period, so a 1% understatement of the trend causes a loss of \$250,000 on a \$10 million book of losses.<sup>14</sup> Some estimation error is unavoidable; some estimation error reflects poor work and can be corrected by better pricing techniques. The conscientious actuary examines past estimation errors to check for biases in the rate review.

The \$0 profit in the initial projection of +7% severity trends means there is no implicit profit margin in this pricing assump-

<sup>&</sup>lt;sup>14</sup>For clarity's sake, we use rough numbers. "Book of losses" is not an ideal measure of volume, since the size of the losses depends on the trend factors. The gain or loss is the difference in profits under the two trend assumptions. In this analysis, we use nominal losses for the trend figures, and we separately quantify the gain or loss from investment earnings. When an increase in trend stems from higher inflation that is associated with higher interest rates, the loss from trend may be offset in part by a gain from interest; see the discussion below in the text.

#### TABLE 3

PRIVATE PASSENGER AUTO LOSS SEVERITY (MULTIPLE YEARS)

Valuation Date	n Projec	ction	Estimation		Process	Total	
	Initial	Implicit	Revised	Estimation	Actual Change	Process	Total
	Projection	Profit	Estimate	Error	(Annualized)	Error	Variance
12/20X7	+7%	\$0	+8%	-\$250K	+5%	+\$750K	+\$500K
12/20X8	+7%	\$0	+8%	-\$250K	+6%	+\$500K	+\$250K

tion; contrast the workers compensation source of earnings exhibits in the second half of this paper.

The analysis of process error is important for two purposes:

- 1. The management of an insurance company must know whether differences of actual results from expected arise from misestimation of future costs or random loss fluctuations. Random differences may mean the business is unstable, but systematic differences indicate possible ratemaking biases.
- 2. Analysis of process error may uncover effects of exogenous factors, such as changes in compensation systems and in attorney involvement.

Full source of earnings exhibits use a multi-year format. Suppose that by 12/31/20X8, the actual severity increase is +6%, stemming from adverse development on reported claims. A second line would be added to the source of earnings exhibit as shown in Table 3.

Estimation error is the difference between projected and revised; process error is the difference between revised and actual. The projection is the original pricing assumption. Since the trend assumption has no implicit margin, the original "gain or loss" is \$0. The projection columns do not change as additional years are added. The revised estimate shows the actual trend rate in fast track data. The estimation error is the difference between the actual trend rate and the projected trend rate translated into dollars of gain or loss. In this example, the actual fast track trend is 1 percentage point per annum greater than the projected trend rate. For a trend period of 2.5 years and a \$10 million book of losses, the estimation error is a loss of \$250,000.

To keep the exposition simple, the actual fast track trend does not change from 20X7 to 20X8.<sup>15</sup> When the first row of the exhibit is completed before final fast track data are available (as is true in this example), the estimation error may change between the first and second rows.

The "actual change (annualized)" shows the actual severity change in the company's ratemaking data for that state. If no exogenous changes affect loss severity trends in this state, the difference between the fast track trend and the actual severity change stems from random loss occurrences in either the experience period or the policy period. The average severity in both the experience period and the policy period may change as the losses mature, so the difference stemming from process error changes as years are added to the exhibit.

Revisions stem from both actual data and revised estimates of the future. Consider the first row in Table 3. The "projection" column shows the estimated trend for 7/1/20X4 through 1/1/20X7 at the time of the rate analysis. The fast track trend is a mix of actual and expected figures: if the rate analysis is done in the middle of 20X5, the fast track trend for 7/1/20X4 through 12/31/20X4 may be actual and the remaining trend is an estimate. A revised analysis at a valuation date of December 31, 20X6, might use actual data for 7/1/20X4 through 6/30/20X6 and a revised estimate for 7/1/20X6 through 12/31/20X7.

The source of earnings exhibits trace the replacement of prior assumptions by actual data and revised assumptions. We need

<sup>&</sup>lt;sup>15</sup>December 20X7 and December 20X8 are the valuation dates; at each valuation date, the fast track trend refers to the same period (July 1, 20X4 to January 1, 20X7).

not wait for final data to form the exhibits. For instance, if the actual fast track trend is higher than the projected trend for the first half of the trend period, we might expect that it will be higher for the second half of the trend period as well.

#### Distinguishing Sources of Error

Distinguishing estimation error from process error is not easy. For personal auto, with high frequency low severity losses, the actual fast track trend is a reasonable estimate of true loss trend. Other insurance coverages are more complex. When estimating hurricane loss costs for Homeowners, we may never know the true expected losses, since hurricane frequency and severity are difficult to predict.

The postmortem analysis used in source of earnings analysis works best for lines with high claim frequency and little variability in the size of loss distribution. Examples are life insurance, medical insurance, private passenger automobile, and workers compensation. It is more difficult when loss are large and highly variable, as is true for excess of loss reinsurance, commercial property, and catastrophe coverages.<sup>16</sup>

#### Credibility

Unlike casualty ratemaking, life insurance pricing does not use credibility adjustments. Source of earnings exhibits are more complex when credibility is used.

For other pricing assumptions, actual values are known after the policy expires and the experience is mature. For credibility, there is no actual value. The source of earnings analysis does not compare the initial credibility assumption with a subsequent (revised) value. Rather, the credibility value is used to adjust the initial assumptions.

<sup>&</sup>lt;sup>16</sup>Even for the more stable lines, separation of estimation error from process error relies somewhat on actuarial judgment.

We focus here on statewide credibility factors. The credibility factors adjust the past experience to be a better proxy for the true expected losses in the experience period.<sup>17</sup>

*Illustration*: Suppose the underlying pure premium during the experience period of accident year 20X4 was \$500 per car, based on a rate filing effective January 1, 20X3, and intended to be in effect for one year. The current rate review has an effective date of January 1, 20X6, and is intended for policies written in 20X6. Because of administrative problems, no rate changes were filed between January 1, 20X3, and January 1, 20X6.

Suppose the pure premium trend is 10% per annum, the experience (indicated) pure premium during accident year 20X4 is \$600, and the credibility for the experience pure premium is 50%. The pure premium used in ratemaking is an equal weighting of the trended experience pure premium and the trended underlying pure premium. We adjust the source of earnings exhibits to reflect the 50% credibility factor.

The trend factor is the same whether it is applied to the experience pure premium or to the underlying pure premium. The credibility factor implies that the true expected loss during accident year 20X4 is a 50:50 average of the information from the accident year 20X4 experience and the rates underlying the accident year 20X4 writings.

Since the \$500 rate was intended to be adequate for 20X3, the adequate rates underlying the accident year 20X4 losses are  $$500 \times 1.10^{0.5} = $524.40$ .

<sup>&</sup>lt;sup>17</sup>Statewide credibility factors are traditionally applied to the developed and trended experience loss ratios, perhaps giving the impression that credibility adjusts the development factors, the trend factors, or the future expected values. This is not correct. Separate credibility factors may be applied to trend and development factors. The statewide credibility factors adjust the actual data to be a better proxy of the expected experience in the past.

The discussion here is based on the "greatest accuracy" justification for credibility. Venter [1992] argues that the justification for classical credibility is to limit rate fluctuation and that the Bayesian-Bühlmann credibility procedure is designed to optimize rate accuracy; Mahler [1997] argues that even traditional credibility procedures improve rate accuracy; see also Mahler and Dean [2001].

The credibility weighted average experience rates are (\$600 + \$524.40)/2 = \$562.20.<sup>18</sup> On the source of earnings exhibits, this is reflected in the actual loss cost change. The initial trend assumption is 10% per annum. The actual trend rate based on hind-sight is whatever the trend index reveals. The actual loss cost change is the change between \$562.20 and the observed pure premium during the new policy year (20X4).

In sum, the source of earnings analysis accepts the credibility adjustment and tests the loss cost change; it does not test the credibility value itself.<sup>19</sup>

#### Implicit and Explicit Profit Margins

Actuaries may use implicit or explicit profit margins.

- For explicit profit margins, best-estimate assumptions (for development, trend, investment income) are used in the ratemaking process and a full profit margin is included in the rates.
- For implicit profit margins, conservative assumptions are used in the ratemaking process and a lower explicit profit margin is included in the rates.

To illustrate the difference, we contrast trend factors with discount factors.

• *Trend Factors:* Suppose that fast track data imply a loss severity trend of +5% per annum. This estimate is uncertain, not only because it is a future projection but also because the fast track data may not be comparable to the ratemaking data (dif-

<sup>&</sup>lt;sup>18</sup>The \$500 rates were intended for policies written in the 12-month period from January 1, 1998, through December 31, 1998. The losses on these policies extend from January 1, 1998, through December 31, 1999, for an average loss date of January 1, 1999. The average loss date in the experience period of accident year 1999 is July 1, 1999, or half a year later than the average loss date expected in the filing. For a more complete exposition, see Feldblum [1998: discussion of "The Complement of Credibility"].

<sup>&</sup>lt;sup>19</sup>This is not to imply that credibility procedures are impervious to empirical testing. Mahler [1990] gives three methods for testing the accuracy of credibility estimators. However, Mahler tests the accuracy of the credibility estimator; one cannot test the accuracy of a particular credibility factor. There is no such thing as the variance between the actual credibility and the assumed credibility.

ferent companies, different states, accident year versus calendar year, closed claims versus incurred claims, and so forth). We presume the trend rate is between 4% and 6% per annum.

The explicit profit method would use a +5% trend and a full explicit profit margin. The implicit profit method might use a +6% trend and a lower profit margin. Some actuaries prefer the use of explicit profit margins to better monitor the adequacy of the rates; other actuaries prefer the use of implicit profit margins to prevent overly aggressive pricing.<sup>20</sup> Rate filing exigencies sometimes compel companies to use lower explicit profit margins offset by conservative assumptions.

• *Discount Factors:* Suppose that losses are discounted to present value at the expected risk-free interest rate in a discounted cash-flow pricing model. The estimate of future interest rates, based on an analysis of the current yield curve and of any mean-reverting tendencies in the assumed interest rate paths, is 5% per annum. This estimate is uncertain because we are projecting a future rate and because the interest rate model may itself be flawed. We might presume that the future interest rate will probably be between 4% and 6% per annum.

The explicit method would use a 5% assumed interest rate with a full explicit profit margin. The implicit method might use a 4% assumed interest rate with a lower profit margin.<sup>21</sup>

<sup>&</sup>lt;sup>20</sup>See Benjamin [1976], page 238: "The explicit method seems natural and right in contrast to the implicit method which appears to have no good or credible foundation. But in life insurance actuaries have come down very strongly in favor of the implicit margin method." See also Anderson [1959], page 368: With the inclusion of specific contingency margins and profit objectives, it is proposed that other assumptions necessary to calculate gross premiums be introduced on the basis of "best estimates" rather than "conservative estimates."

<sup>&</sup>lt;sup>21</sup>The use of an implicit profit margin in the interest rate is not the same as a risk adjustment to the discount rate. For example, Myers and Cohn [1987] use a CAPM-based risk-adjusted loss discount rate that reflects the covariance of loss returns with market returns, following procedures used by Fairley [1979] and Hill [1979]. The CAPM-based risk adjustment is intended to reflect the true present value of the loss payments, not "conservatism" or an implicit profit margin. Similarly, Butsic [1988] uses a risk adjustment to the loss reserve discount rate to estimate the true economic value of the loss reserves.

#### Investment Income

The expected investment income on the assets supporting the book of business is an important component in pricing. Banks and life insurance companies often model the interest rate spread on assets versus liabilities.

- The pricing of universal life and variable life products uses the spread between the earned rate on invested assets and the credited rate in the policy.
- Annuity writers model the spread between interest earned on the policyholder's account balance and the accrual rates stipulated in the contract.
- Depository institutions (commercial banks, savings and loans, credit unions, thrifts) monitor the spread between the yield on loans and the interest paid on deposits.

The source of earnings analysis considers the difference between the spread assumed in the pricing analysis and the spread that is actually achieved.

*Illustration*: Suppose the benchmark investment yield (the casualty equivalent of the credited interest rate) used in policy pricing is 7%, and the company expects to earn 7.5% per annum on its invested assets (the projected earned rate). The actual investment yield varies with market interest rates and capital gains or losses.

The source of earnings exhibits use three sets of figures:

- 1. the investment yield originally assumed for the future pricing period (assumed earned interest rate), or  $IY_0$ ;
- 2. the credited interest rate (CR), or the investment yield used in the pricing model; and
- 3. the actual investment yield during the period that reserves are held, or  $IY_t$ .

The actual investment yield includes dividends, interest, rents, and capital gains and losses. For investment management purposes, the source of earnings exhibits differentiate market yields from realized plus unrealized capital gains and losses.

The interest spread is most important for the long-tailed lines of business. We estimate the invested funds for each year  $(IF_t)$ .<sup>22</sup>

The source of earnings analysis quantifies the implicit profit margin in the investment yield assumptions and the subsequent unfolding of the actual profit margin. Each year's implicit expected profit margin is the invested funds times the difference between the expected investment yield and the investment yield used in pricing, or  $IF_t \times (IY_0 - CR)$ . The total profit is the sum of the annual profits discounted at the cost of capital.<sup>23</sup>

*Illustration*: Suppose we are analyzing a \$10 million book of business, with average invested funds of \$3 million during the policy year, \$4 million the next year, and declining by \$1 million a year until all losses are settled.<sup>24</sup> The company expects an investment yield of 8% per annum, and it prices the business assuming an investment yield of 7% per annum and a 12% cost

<sup>&</sup>lt;sup>22</sup>Most casualty pricing models estimate the invested funds by projecting premium collection patterns, loss payment patterns, and expense payment patterns. Life actuaries use the term "account balance" instead of invested funds. In life insurance and annuities, the account balance belongs to the policyholder and may be withdrawn on demand, sometimes with a surrender charge deducted or a market value adjustment. In casualty products, the policyholder does not own the funds used to support the reserves. The term invested funds refers to the assets supporting the unearned premium and loss reserves.

<sup>&</sup>lt;sup>23</sup>This formula assumes that  $IY_0$  is the pricing assumption for all future years; that is, the actuary assumes a constant future investment yield.

<sup>&</sup>lt;sup>24</sup>This progression of the invested funds reflects a policy year of writings. With a prepaid acquisition expense ratio of 20%, a net premium of \$8 million collected up-front on some policies and by installment plans on others, and some losses paid during the policy year, the average invested funds during the policy year are about \$3 million. The invested funds peak about 12 months after inception of the policy year, since premiums have been collected but losses remain in reserves. During the next 12 months, the invested assets remain relatively constant, as the remaining premium is collected and some losses are paid. The invested funds decline to zero as losses are settled. To keep the illustration simple, we use an expected policy lifetime of four years; actual lifetimes for long-tailed lines of business are longer.

#### TABLE 4

Source of Earnings Analysis for Interest Spread at Policy Inception

Year	Invested Funds	Expected Invest Yield	Credited Interest Rate	Interest Rate Spread	Interest Rate Margin	PV of Margin
0	3,000,000	8%	7%	1%	30,000	30,000.00
1	4,000,000	8%	7%	1%	40,000	35,714.29
2	3,000,000	8%	7%	1%	30,000	23,915.82
3	2,000,000	8%	7%	1%	20,000	14,235.60
4	1,000,000	8%	7%	1%	10,000	6,355.18
Total						110,220.89

of capital.<sup>25</sup> The implicit profit margin in the investment yield assumption is shown below. The present values are taken to the middle of the initial policy year (year 0) as shown in Table 4.

*Illustration*: Average investable funds in year 3 are \$2,000,000. With a 1 point spread, the interest margin is \$20,000. Discounting to the middle of year 0 at the 12% cost of capital gives  $20,000/1.120^3 = 14,235.60$ .

Between initial pricing and final settlement of claims, several items may change.

- 1. Interest rates may change, causing immediate (unrealized) capital gains or losses in GAAP statements and market values (though not in statutory accounting) and revised investment yields in future years.
- 2. The amount of invested funds may differ from the initial assumption.

<sup>&</sup>lt;sup>25</sup>To keep the arithmetic simple, we ignore federal income taxes in this paper. In practice, they must be considered, particularly since different investments have different tax rates (see Feldblum and Thandi [2003]). For prospective pricing, one often assumes that the present value of future investment income does not depend on the type of investment; see Derrig [1994]. In contrast, the source of earnings analysis focuses on the defaults and market value changes of risky investments.

#### TABLE 5

Source of Earnings Analysis for Interest Spread after One Year

Year	Invested Funds	Investment Yield	Credited Interest	Interest Spread	Interest Margin	Capital Gain/Loss	PV of Margin
0	\$2,500,000	9.5%	7%	2.5%	\$62,500	-\$50,000	\$12,500.00
1	\$3,500,000	10%	7%	3.0%	\$105,000	\$0	\$93,750.00
2	\$3,000,000	10%	7%	3.0%	\$90,000	\$0	\$71,747.45
3	\$2,000,000	10%	7%	3.0%	\$60,000	\$0	\$42,706.81
4	\$1,000,000	10%	7%	3.0%	\$30,000	\$0	\$19,065.54
Total							\$239,769.81

## 3. There may be unexpected capital gains or losses for reasons other than interest rate changes.

The new entries in the source of earnings exhibits are a mix of actual figures and revised estimates.

*Illustration*: In Table 5, the investment yield rises to 10% per annum between the rate review and the end of the policy year. Year 0 shows a 9.5% average actual yield, and years 1 through 4 show 10% as the revised (estimated) yield. More insureds used installment payment plans; the actual investable assets in year 0 and the estimated investable assets in year 1 are reduced.

The investment yield increase from 8% at the rate review date to 10% by the end of the policy year causes the \$50,000 capital loss in year 0. Since most of the investment yield increase occurred before assets were bought, the capital loss is small and the greater future investment income more than offsets it.<sup>26</sup>

#### Inflation Rates and Interest Rates

The full effect of interest rate changes requires a combined analysis of assets and liabilities. If inflation rates rise along with interest rates, loss severity increases. The revised expected loss

<sup>&</sup>lt;sup>26</sup>If the investment yield increase occurs after fixed income assets are bought, the capital loss may more than offset the higher reinvestment rate for coupon payments.

ratio exceeds the target loss ratio, but this loss may be offset by the rise in the investment yield (see Butsic [1981]).<sup>27</sup>

Inflation rates and interest rates are correlated, but they do not move in lock step. The source of earnings exhibits provide a year-by-year analysis of the gains and losses from inflation and interest, allowing a better analysis of net profitability.

*Illustration*: Interest rates and inflation rates rise shortly before the inception of the policy year. Losses are larger than initially projected, but investment income is greater than initially projected as well; the net profit variance shows the combined effects of both. This analysis is particularly important for retrospectively rated workers compensation policies and large dollar deductible policies, since inflation has a leveraged effect on losses above the deductible. Equal increases in interest rates and inflation rates generally reduce the net profits on this business.

#### Persistency

Of the four life insurance earnings factors—mortality, maintenance expenses, interest, and persistency—persistency is the least well understood but often the most important. Mortality rates change slowly over time; maintenance expenses are equally stable. Interest earnings come from the spread between earned rates and credited rates. Although the earned rates may vary from year to year, many companies try to keep the spreads stable.

Persistency rates can only be estimated. Differences of actual from expected persistency can be large, and they strongly affect profitability; see Tan [1989] and Eckman [1990].

Persistency patterns greatly affect property-casualty profitability as well. For a variety of reasons, casualty actuaries have not always given persistency patterns the attention they deserve.

<sup>&</sup>lt;sup>27</sup>Traditional profitability measures of loss ratios and combined ratios can be misleading. Statutory measures of total profitability, as reflected in the investment income allocation procedure in the Insurance Expense Exhibit, are distorted by the use of portfolio investment yields and amortized values of fixed income securities; see Feldblum [1997].

- Acquisition expense differences between new business and renewal business are not as great for property-casualty insurance as for life insurance. First-year commissions for permanent life insurance may exceed the annual premium; for casualty products, commissions are rarely more than 25% of the premium.
- Life and health insurers must renew their permanent policies as long as the policyholder pays the premium. They have more incentive to quantify the effects of persistency on product profitability. A property-casualty insurer may cancel the policy or decline to renew it.
- For companies using the independent agency distribution system, the agent owns the renewals, and commissions are level from year to year. Persistency patterns are not under the control of the company, and they have less effect on expense ratios.
- Rating bureaus, which set the traditional workers compensation ratemaking procedures in the twentieth century, have less interest in persistency patterns than competitive insurers have. Life and health insurers do not use rating bureaus.

Ideally, persistency patterns are incorporated in prospective ratemaking by asset share pricing models. The source of earnings analysis evaluates the profits achieved from a cohort of policies.

*Illustration*: A personal auto direct writer has had a 90% retention rate in past years. The retention rate drops to 80% for the new policy year. Acquisition expenses are 20% for new business and 5% for renewal business. The expected loss ratio is 80% for new business and 70% for renewal business. The total spread between new and renewal business is 25% of premium.

The drop in the retention rate reduces profitability. The decline in profitability may be estimated as the reduction in renewal business times the spread between new and renewal business, or  $(90\% - 80\%) \times 25\% = 2.5\%$  of premium each year.

The traditional premium, loss, and expense exhibits show higher than expected loss and expense ratios. But neither expense costs nor loss costs have changed.<sup>28</sup> If the pricing actuary did not consider persistency effects, the source of earnings analysis is all the more necessary to tease apart the underlying sources of profit or loss.

We illustrate below one way of amortizing acquisition expenses. Traditional property-casualty ratemaking combines acquisition expenses with on-going maintenance expenses and treats the sum as either an additive factor (fixed expenses) or a multiplicative factor (variable expenses). This obscures the effects of expense items. In the illustration here, acquisition costs and solicitation costs on not-taken business are treated separately and amortized over the expected lifetimes of the insurance policies.

#### 4. RETROSPECTIVELY RATED POLICIES

#### Policy Economics

Pricing and accounting should reflect the underlying economics of the insurance product. The FASB introduced SFAS 97 to make the accounting for universal life and variable life contracts consistent with their economic structure. This section applies the FASB's distinction between traditional whole life and universal life policies to prospectively priced private passenger automobile versus retrospectively rated workers compensation.<sup>29</sup>

For a traditional whole life policy (SFAS 60), the premium due is an income statement revenue and the increase in the policy reserve plus any death benefit in excess of reserves is an income statement expense. For casualty products, earned premium is the revenue and incurred losses are the expense. Greater earned premium reflects additional profits and greater losses reflect decreased profits. The pricing actuary sets the premium rate (the

<sup>&</sup>lt;sup>28</sup>The business growth illustration in Feldblum [1996, "Personal Automobile"] analyzes these profitability effects.

<sup>&</sup>lt;sup>29</sup>Over the past decade, many insurers have shifted much of their retrospectively rated workers compensation business to large dollar deductible policies. The money paid by the employer to cover losses below the deductible is termed an assessment, not a premium, and it is generally paid shortly before or after the benefits are paid. In most states, premium taxes and involuntary market burdens are not levied on assessments. The discussion in the text applies to both retrospectively rated contracts and to large dollar deductible contracts.

revenues) based on estimates of the ultimate losses and expenses (the income statement expenses).

When a universal life policyholder pays premiums, the money belongs to the policyholder, not to the life insurance company. The insurance company is a financial intermediary, investing the policyholder's money. It deducts a management fee for investment services and specified charges for underwriting protection, such as the mortality charge and the maintenance expense charge. Premiums are a deposit, not a revenue.

- The policy charges plus the investment income earned on the account value are revenues.
- Benefit payments in excess of the account value, interest credited to the account value, and expenses paid are expenditures.

A workers compensation retrospectively rated policy is similar in substance. The insurer uses the policy premium to pay losses and to cover the various charges, such as the insurance charge and the other components of the basic premium. If the losses do not materialize, the insurer returns part of the premium to the insured. If actual losses exceed the original expectations, the insurer collects additional premium.<sup>30</sup>

For retrospectively rated policies, additional incurred losses lead to additional retrospective premiums, with the net effect depending on the premium sensitivity (Teng and Perkins [1996], Feldblum [1997], Bender [1994], Mahler [1994]). A change in losses or in premiums does not by itself signal higher or lower profitability. Traditional exhibits of premiums and losses are not always an appropriate means of monitoring the profitability of this business.

<sup>&</sup>lt;sup>30</sup>The various charges in a universal life policy, such as the mortality charge, asset management charge, surrender charge, and expense charge, are noted in the policy and in periodic reports to the policyholder, particularly if the asset accumulation rate is tied to external investment indices. For the retrospectively rated workers compensation policy, the pricing actuary knows the individual charges, but the insured may not be aware of them.

#### Retro Policies vs. Universal Life

The source of earnings analysis for workers compensation retrospectively rated policies has two differences from the analysis for universal life-type policies.

- 1. The *insurance charge* takes the place of the *mortality charge*, and *non-ratable losses* takes the place of *policy*-*holder benefits in excess of the account value*. The mortality charge in a universal life policy pays for policyholder benefits in excess of the account value; the insurance charge in the retrospectively rated workers compensation policy pays for non-ratable losses.<sup>31</sup>
- 2. SFAS 97 amortizes deferred acquisition costs in relation to expected gross profits, with a year-by-year unlocking of assumptions as actual experience emerges. We use a simpler amortization procedure here but the amortization schedule is still dynamic, so that persistency is reflected in the source of earnings exhibits.

#### Evaluation of Results

Pricing for retrospectively rated policies depends on four sources of earnings: (a) investment income, (b) non-ratable losses, (c) expense levels, and (d) retention rates.

Standard reports of premiums and losses do not show the expected profits on retrospectively rated policies stemming from these earnings factors or the variations in profit caused by changes in these factors. The reports do not show if the ratemaking assumptions accurately reflect the expected experience on the book of business.

If profits are unexpectedly low, we do not know if the cause is (i) higher than anticipated non-ratable losses, (ii) lower than expected investment income, (iii) excessive expenses, or (iv) higher than anticipated lapse rates or not-taken rates.

 $<sup>^{31}</sup>$ Non-ratable losses are losses above the loss limit or losses that would cause premium above the maximum.

#### Amortization of Deferred Acquisition Costs

For two reasons, the amortization of deferred policy acquisition costs is essential for monitoring universal life profitability.

- Deferred acquisition costs are as much as 50%–60% of gross profits for many universal life contracts.<sup>32</sup> In the first one or two policy years these products show large statutory losses from acquisition costs and low investment income, since (i) first year agents' commissions are high (often 100% of the annual premium), and (ii) invested assets from policyholder funds are zero in the initial policy year and low in the first renewal year. There is little profit from the interest spread in these years.
- Retention rates greatly affect long-term profitability. Statutory accounting distorts the effects, since only the surrender charge (a gain) is shown for the current calendar year. Dynamic amortization of deferred policy acquisition costs reveals the effects of retention rates on long-term profitability.

The capitalization and amortization of acquisition and issue costs is also important for retrospectively rated policies. First year agents' compensation, initial underwriting, workplace inspection, loss engineering, and policy issue costs are the major expenses for retrospectively rated policies.<sup>33</sup>

For large account retrospectively rated business, not-taken rates can be high. There are a limited number of large workers compensation accounts, with \$2 million or more of annual premium. The risk manager of each insured might put the account out to bid every five years or so. Developing the bids is costly, but each bid may have only a 10% to 20% chance of being accepted, leading to an 80% to 90% not taken rate.

<sup>&</sup>lt;sup>32</sup>"Gross profits" are the present value of lifetime profits from the block of business before consideration of prepaid acquisition costs; see SFAS 97.

<sup>&</sup>lt;sup>33</sup>This is especially true for direct writers, with large first year commissions and low renewal commissions.
The costs of not-taken policies must be included with acquisition costs. Some companies spread these costs over related books of business, thereby raising the apparent profitability of the book being priced and lowering the profitability of the related books. For instance, some companies spread the costs of not-taken business over the entire workers compensation line of business.<sup>34</sup>

The high acquisition expense costs—including the cost of nottaken policies—must be amortized over the policy lifetimes. It is tempting to overestimate persistency rates and underestimate not-taken rates. Source of earnings analysis with dynamic amortization of policy acquisition costs counteracts this temptation.

#### Static vs. Dynamic Amortization

Static amortization schedules, like static depreciation schedules, do not change with the passage of time. The rate of amortization or depreciation may vary over time, as with double declining balance depreciation schedules, but the amortization schedule is not re-estimated as more is learned about the business.

Static amortization schedules distort profitability analyses if actual persistency rates or investment yields differ from those assumed in pricing. Dynamic amortization allows for revision of the schedule as actual experience becomes known and as future expectations change.<sup>35</sup>

DPAC amortization schedules use an implicit interest rate, so that the present value of the expenses amortized equals the deferred expenses incurred. To simplify the illustrations here, we use pro rata amortization with a 0% amortization interest rate.

<sup>&</sup>lt;sup>34</sup>This leads to incorrect pricing and marketing decisions. There may be strategic reasons for this practice, such as a desire to break into the large account market. More often this practice stems from data limitations that hamper the allocation of expense costs.

<sup>&</sup>lt;sup>35</sup>For the universal life-type policies covered by SFAS 97, the deferred policy acquisition costs are amortized in proportion to future expected gross profits. The amortization schedule is revised whenever actual experience or future expectations differ from assumptions for any of three items: persistency rates, investment yield, and expected or actual gross profits.

*Illustration—Static Amortization*: If all policies are expected to persist eight years, one eighth of a policy's deferred policy acquisition costs is amortized each year (assuming a zero interest rate for amortization). If after two years of experience, the average policy lifetime is expected to differ from eight years, the amortization schedule is not changed.

*Illustration—Dynamic Amortization*: Suppose the excess of first year over renewal acquisition costs on a \$100 million block of business is 20% of premium, and the solicitation cost for not-taken business is 10% of the not-taken premium. The pricing actuary assumes an 8 year average policy lifetime and a 20% not-taken rate.

*Computation*: The excess acquisition costs in the first year are \$20 million. The not-taken rate is 20%, so the premium solicited but not taken is \$100 million  $\times (20\%/(1-20\%)) =$  \$25 million. The solicitation costs for not-taken business are \$25 million  $\times 20\% \times 50\% =$  \$2.5 million. The total acquisition expenses are \$22.5 million. Since policies last an average of 8 years, the annual cost is \$22.5 million/8 = \$2,812,500.

The assumptions used for the amortization schedule are uncertain, though they become known with the passage of time. The not-taken rates and the solicitation costs for not-taken business are known once the new policies are written. The average policy lifetime is re-estimated two or three years after the expiration of the initial policy year (by projecting from early retention rates).

If these figures are revised after the policies are written to an average lifetime of 5 years and a not-taken rate of 60%, the annual acquisition cost is revised as well as shown in Table  $6.^{36}$ 

<sup>&</sup>lt;sup>36</sup>Table 6 is simplified. If the anticipated not-taken rate is 20% and the actual rate is 60%, the insurer has written about \$125 million  $\times (1 - 60\%) = $50$  million of premium. The dollar amortization figure in the exhibit is overstated, but the ratio of the amortization amount to the premium is correct.

# DPAC DYNAMIC AMORTIZATION SCHEDULE: SOLICITATION COSTS FOR NOT-TAKEN BUSINESS

	Assumptions		
	Initial	Revised	
A. Premium	\$100,000,000	\$100,000,000	
B. Excess acquisition costs	\$20,000,000	\$20,000,000	
C. Not-taken rate	20%	60%	
D. Not-taken premium $[= A \times C/(1 - C)]$	\$25,000,000	\$150,000,000	
E. Not-taken acquisition costs $\left[=\frac{1}{2} \times D \times 20\%\right]$	\$2,500,000	\$15,000,000	
F. Total acquisition costs $[= B + E]$	\$22,500,000	\$35,000,000	
G. Average policy lifetime	8 years	5 years	
H. Annual amortization $[= F/G]$	\$2,810,000	\$7,000,000	

#### Invested Capital

The SFAS 97 source of earnings exhibits for universal life policies do not consider invested capital. Before the advent of risk-based capital requirements, this approach was reasonable, at least for GAAP statements.

- Little capital is embedded in the policy reserves, which do not much exceed the account balance.
- Deferred policy acquisition costs are amortized on GAAP statements, so the initial underwriting loss is small.
- Little surplus was needed to satisfy regulatory requirements. Even with the advent of risk-based capital requirements, the surplus requirements for life insurance products are lower than for casualty products.

The capital contributed by investors is much smaller than the policyholder premium.

In contrast, the capital invested for workers compensation is large. Much investors' capital is embedded in undiscounted loss reserves and gross unearned premium reserves. Additional capital is needed to meet the NAIC's risk-based capital requirements or rating agency capital formulas. The investment spread in the source of earnings analysis applies to the investment income on both policyholder-supplied funds and investors' funds.<sup>37</sup>

#### Charged, Expected, and Actual

For the private passenger automobile source of earnings analysis, we showed three values for the loss severity trend factors:

- 1. initial (ex ante) trend factors,
- 2. revised (ex post) trend factors, and
- 3. the actual loss cost change.

The change from estimated trend to actual trend is estimation error; the change from actual trend to actual loss cost change is process error. The same three-level analysis applies to loss development factors, loss frequency trends, and other ratemaking items.

Judging the adequacy of the insurance charge is more difficult. The insurance charge is based on size of loss distributions developed from a large volume of industry experience. The actual policy-year experience tells us the actual non-ratable losses, not the proper insurance charge. The credibility of the excess loss experience for a given block of business is hard to measure.

Personal auto policies are sold for a single premium. The underwriter does not assemble a policy for a given insured with separate charges for development, trend, and expenses. In contrast, a retrospectively rated policy is assembled by the underwriter or sales agent, given values for the insurance charge, the excess loss charge, and other plan parameters. For each earnings

<sup>&</sup>lt;sup>37</sup>For source of earnings analysis applied to financial pricing models, see E. Schirmacher and S. Feldblum [forthcoming].

# TABLE 7A

RETROSPECTIVE RATING COSTS—CHARGED, EXPECTED, ACTUAL

		Expected		Actual	Variance	
	Insurance	Non-Ratable	Expected	Non-Ratable	(Actual from	Actual
Date	Charge	losses	Gain	losses	Expected)	Gain
1/20X1	\$500,000	\$450,000	+\$50,000	_	—	—

element there are three values:<sup>38</sup>

- 1. the amount charged in the pricing analysis,
- 2. the expected cost at policy inception, and
- 3. the actual (realized) cost.

*Illustration*: A policy is issued on January 1, 20X1, with an insurance charge (including the excess loss charge) of \$500,000, and with expected non-ratable losses of \$450,000.<sup>39</sup> The initial report at policy inception is shown in Table 7A.

On December 31, 20X1, at the expiration of the policy, the estimated non-ratable losses (including bulk reserves) are 470,000. The variance of actual from expected is -220,000,

<sup>&</sup>lt;sup>38</sup>The charges for the various pricing components do not sum to the policy premium, since much of the policy premium serves as a deposit to pay ratable losses. Compare universal life policies, much of whose premium is an investment designed for tax-deferred accumulation, not for insurance protection.

<sup>&</sup>lt;sup>39</sup>Some actuaries use an insurance charge equal to the expected non-ratable losses along with a separate profit provision. Other actuaries use a more conservative insurance charge. The insurance charge minus the expected non-ratable losses is an implicit profit margin. Life insurance pricing often uses implicit mortality and interest margins, or conservative mortality tables and a spread between the earned interest rate and the credited interest rate. Similarly, the exhibits here use conservative assumptions and implicit profit margins. A company that uses explicit profit margins with no spreads in the pricing components would show zeroes as the initial profit from each source. The gains and losses are shown here as dollar amounts. In pricing the policies, many of these items—such as the insurance charge—are shown as percentages of standard earned premium.

### TABLE 7B

RETROSPECTIVE RATING COSTS—CHARGED, EXPECTED, ACTUAL

		Expected		Actual		
Data	Insurance	Non-Ratable	Expected	Non-Ratable	Variance	Actual
Date	Charge	losses	Gain	losses	variance	Gain
12/20X1	\$500,000	\$450,000	+\$50,000	\$470,000	-\$20,000	+\$30,000

### TABLE 7C

RETROSPECTIVE RATING COSTS-CHARGED, EXPECTED, ACTUAL

Date	Insurance Charge	Expected Non-Ratable losses	Expected Gain	Actual Non-Ratable losses	Variance	Actual Gain
1/20X1 12/20X1 12/20X2	\$500,000 \$500,000 \$500,000	\$450,000 \$450,000 \$500,000	+\$50,000 +\$50,000 +\$50,000	\$470,000 \$515,000	 _\$20,000 _\$65,000	+\$30,000 -\$15,000

and the actual gain is +\$30,000.<sup>40</sup> Table 7B shows the entries for December 20X1.

Actual non-ratable losses increase to \$515,000 by December 31, 20X2, and Table 7C shows the updated figures.

We comment on each source of earnings in this table.

#### Insurance Charge

The insurance charge illustrates the difficulty in assigning gains and losses to sources. Ideally, we should separate the difference between (i) actual and expected excess losses and (ii) the earnings from interest. But the insurance charge is stated in nominal dollar terms, not in present value terms, whereas the actual excess losses are paid many years after the premium is collected. A zero dollar initial variance is an implicit profit margin.

 $<sup>^{40}</sup>$ The term *variance* is used in the accounting sense, meaning the difference between expected and actual.

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Disentangling the insurance charge from the time value of money is a general problem in retrospective rating. Retrospectively rated policies can be priced in several ways:

- 1. In theory, the insurance charge should reflect the present value of excess losses, though since the loss limit and the maximum and minimum premiums are stated in nominal dollars, present values are rarely used.
- 2. The insurance charge is based on the ultimate values of losses, but it is reduced for the expected investment income on the excess losses. Some actuaries presume that this is done implicitly, since the insurance charge is a percentage of standard premium, whose profit provision considers the expected investment income. The resultant insurance charge may be less than the expected (nominal) excess losses. But this assumes that the loss payment pattern for excess losses is similar to that for ratable losses. In fact, excess losses have slower payment patterns, leading to an implicit profit margin in the insurance charge.<sup>41</sup>
- 3. The insurance charge is based on ultimate losses, and a separate investment income factor calculated from all insurance cash flows reduces the basic premium.

For simplicity, this illustration uses a single policy. Actual source of earnings analyses use blocks of policies, such as all large account business written by a particular sales office in policy year 20XX. Since non-ratable losses have great random fluctuation, a report showing variances is more meaningful on a block of business basis. The subsequent examples are for policy year blocks of business.

<sup>&</sup>lt;sup>41</sup>The explanation in the text is simplistic: the consideration of investment income in the underwriting profit provision has no mathematical relation to the lag between collection of the insurance charge and the payment of excess losses.

# Source of Earning Analysis for Retrospectively Rated Policies (\$000)

Valuation	Non-Ratable	Interest	Persistency	Maintenance	Explicit	Total
Date	Losses	Earned		Expenses	Profit	Profit
1/1/20X1	\$2,000	\$2,500	-\$1,500	\$750	\$1,250	\$5,000
12/31/20X1	\$1,400	\$3,400	-\$2,500	\$750	\$1,100	\$4,150
12/31/20X2	\$2,100	\$3,600	-\$2,900	\$750	\$1,100	\$4,650

#### Expenses

Expenses are divided into two components:

- 1. underwriting and acquisition expenses, including solicitation costs for not-taken business, and
- 2. policy maintenance expenses, including unallocated loss adjustment expenses.

The effect of acquisition and underwriting expenses on profitability depends on the difference between expected and actual (i) not-taken rates and (ii) renewal rates. We speak of these as earnings from persistency. Maintenance expenses are rarely a material source of gain or loss, and they are not discussed further here.

#### Combining the Earnings Factors

The first row in Table 8 shows the profit from each factor in the pricing assumptions. Subsequent rows show the variance resulting from actual data and revised estimates.

### Pricing Assumptions

At January 1, 20X1, the inception of the policy year, the figures show the implicit and explicit profit margins. Most of the expected profit (\$3.75 million out of \$5 million) is embedded in the pricing assumptions.

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- The insurance charges exceed the expected non-ratable losses by \$2 million.
- The company expects an average lag of about one year between premium collection and loss payment, with a small spread between the interest earned and the interest credited to policyholders in the pricing assumptions.<sup>42</sup> The actual investment income is expected to exceed the investment income assumed in pricing by \$2,500,000.
- The company expects actual maintenance expenses (including unallocated loss adjustment expenses) to be \$750,000 below the amount assumed in pricing.<sup>43</sup>
- The company loses money from solicitation costs on not-taken business. Some of this money is recouped from acquisition expense charges in the basic premium. The amount that is not recouped is a negative implicit profit margin of \$1,500,000.<sup>44</sup>
- The company builds an explicit profit component of \$1,250,000 into the rates.

## Underwriting

The first row shows the pricing assumptions at the inception of the policy year. Rarely are all pricing assumptions realized. The second row shows the revised values at the end of the policy year. The variances from expected profits stem from two causes:

• If the sales price differs from the actuarial indications, the charges embedded in the policy components may differ from those anticipated by the actuary. For instance, the indicated

<sup>&</sup>lt;sup>42</sup>Incurred loss retros may have long lags between premium collection and loss payment; paid loss retros and large dollar deductible policies have short lags. The one-year lag is an average.

 $<sup>^{43}</sup>$ We include unallocated loss adjustment expenses (ULAE) with underwriting expenses because both reflect operating efficiency.

<sup>&</sup>lt;sup>44</sup>It is hard to persuade policyholders that they should reimburse the costs of soliciting other business, and the company does not expect to recover all the costs from expense charges in the premium.

insurance charge may be \$25,000, but the company may use only a \$15,000 charge.

• Fluctuations in losses or interest rate changes affect the costs. Even if the company uses the \$25,000 insurance charge, a large loss may eliminate the expected profit.

In the illustration, interest rates have risen and the marketplace has softened, but the underwriters have adhered closely to the pricing recommendations.

- The rising interest rates lead to greater excess losses, since inflation has a leveraged effect on higher layers of loss, reducing the implicit profit from non-ratable losses by \$600,000.
- A few insureds are given premium credits, reducing the explicit profit margin by \$150,000.
- Because of the soft market, not-taken rates increase, leading to an additional \$1 million loss from unfulfilled solicitation costs.
- Interest rates rise before the company invests the premiums, leading to \$900,000 additional implicit profit from the interest spread.<sup>45</sup>

#### Actual Experience

Subsequent revisions arise from random loss occurrences and from interest rate changes. For instance, the 12/31/20X2 row shows an increase in the expected profits from non-ratable losses. By December 31, 20X2, all policies have run their course, and there have been fewer large losses than expected. This may result from stringent underwriting or random loss fluctuations.<sup>46</sup>

<sup>&</sup>lt;sup>45</sup>The pricing actuary must take care to reflect the higher interest rate, and the potentially higher inflation rates, in the insurance charge. If this is not done, the implicit profit margin from non-ratable losses may be overstated.

<sup>&</sup>lt;sup>46</sup>Because the claim severity distribution is highly skewed, most years show fewer large losses than expected, offset by a few years with more large losses than average.

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The December 31, 20X2, figures are a combination of actual figures and estimates:

- The investment yield in 20X1 and 20X2 is known.
- The effect of acquisition costs on policy profitability still depends somewhat on future persistency rates.
- The ultimate amount of large losses may remain uncertain for years.

The source of earnings exhibits are updated until most of the losses have been settled or until subsequent changes in estimated earnings are not material.

*Non-ratable losses*: When pricing retrospectively rated contracts, some actuaries rely on aggregate industry figures, such as NCCI Table M data. Individual company data may not be considered sufficiently credible for revising Table M figures, and the needed adjustments for inflation and for changes in the size of loss distribution are complex.

Ideally, Table M charges should be reviewed periodically to ensure their adequacy. The source of earnings analysis provides a hindsight view of the adequacy of the insurance charges that can be especially valuable for the pricing actuary. The challenge for the pricing actuary is to discern from the emerging experience how much of the variance stems from estimation error and how much stems from process error.

*Interest*: The earnings from interest depend on the investment yield received versus that used to price the policy and the collection dates for premium and losses. Large accounts often want customized cash flow plans to retain more of the investment income. For these accounts, the expected earnings from interest may be determined on a plan-by-plan basis.

The interest earnings factor troubles some practicing actuaries, who say: This analysis presupposes an investment yield assumption in the rate analysis. But that is not how we develop rates. We price to a target combined ratio, or a target underwriting profit provision. This target is set by company management, not by the pricing actuary doing the rate review. The target combined ratio may have been set by an internal rate of return model or a discounted cash flow model. Even in these models, there may be no simple interest assumption. Our pricing procedure does not fit into the source of earnings mold.

This criticism is dismaying. It has been more than twenty years since actuaries began using financial pricing models for casualty insurance products. The parameters of these models such as the assumed investment yield, the target return on capital, the surplus requirements, and the implied equity flows—greatly affect the final premiums. Yet some actuaries who are expert in other pricing issues cannot figure out what their pricing model says. They can tell you the effect of a one-point increase in the trend factor, but they can't tell you the effect of a one point increase in the investment yield.

The source of earnings analysis compares the investment income actually received with the investment income assumed in pricing. The analysis of this difference, along with related interest rate changes and capital gains, helps the practicing actuary understand the implications of the financial pricing model.

*Persistency*: For large account retrospectively rated business, the solicitation costs for not-taken business and the persistency of insured business greatly affect overall profitability.<sup>47</sup> The source

<sup>&</sup>lt;sup>47</sup>The full effects of interest rate changes and persistency changes take several years to play out. Some pricing actuaries disclaim responsibility for interest rate changes, not-taken rates, and persistency rates, since traditional ratemaking procedures do not deal with these items. The common disclaimer is that "the investment yield is the responsibility of the Investment Department; we simply use the projections that they provide us." Similarly one hears that "the persistency rate, or the not-taken rate, is the responsibility of the sales force; we simply use the projections that they provide us." This retort is disingenuous.

of earnings analysis ensures that pricing actuaries incorporate these effects in the ratemaking formulas.

# Combined Effects

Implementing source of earnings analysis requires some means of dealing with combined (non-linear) effects.

*Illustration*: Suppose the developed and trended losses are \$100 million. The source of earnings analysis shows that the loss development factor should have been 10% higher and the loss trend factor should have been 10% higher. A simple source of earnings exhibit might show a (negative) gain of -\$10 million from development and a similar -\$10 million from trend. But the total variance is -\$21 million, not -\$20 million.

The allocation of the extra \$1 million to earnings sources is problematic. When there are multiple non-linear factors, the problem is more complex. We may use three types of solutions:

- 1. Assign the linear component of the variance to the individual factors, and assign the non-linear components to a "combined" bucket.
- 2. Compute the variances by the order of application of the ratemaking factors. This solution is arbitrary, since there is no inherent order to the calculations. For example, either loss trending or loss development may precede the other.
- 3. Spread the non-linear components over the individual factors on a formula basis. This method is the most sophisticated, but it is the most complex.

The mathematics of source of earnings analysis is not as simple as one might infer from the example in this paper, partic-

The source of earnings analysis does not bring investment policy or marketing philosophy under the purview of the actuary. Nevertheless, just as the reserving actuary does not rely solely on the claims department's loss estimates, the pricing actuary cannot rely solely on others' estimates for the pricing assumptions.

ularly when multi-year persistency effects are considered. The appendix shows more realistic source of earnings exhibits for retrospectively rated workers compensation business. When the total variance is small, the non-linear components (or the "second order" components) are small enough that they do not affect the analysis. When the total variance is large, one of the above procedures may be used for the non-linear components.

#### 5. CONCLUSIONS

Two topics run through this paper: the unbundling of the insurance contract, and the differentiation between estimation error and random fluctuation. We summarize the two topics below and their implications for practicing actuaries.

### Pricing Paradigms

A premium-loss pricing paradigm now dominates casualty actuarial ratemaking. The actuary determines policy premiums to cover expected losses and expenses.

With the life insurance policy revolution of the 1980s, life actuaries moved to a credit-charge paradigm. The new interestsensitive policies were unbundled into their components. The actuary determines charges and credits for the policy components, which may be rearranged into full policies to meet customer needs.

The flexibility of the credit-charge paradigm makes it ideal for large account workers compensation pricing. The employer purchases a customized policy with specialized components: deductibles, premium payment plans, retrospective rating, loss engineering services, claims handling services, self-insured retentions, excess coverage, and so forth.

The actuary prices the components, which are assembled by the underwriter into the policy. For instance, the actuary determines the appropriate insurance charge for a set of plan parameters, or the appropriate interest credit for a given plan type and premium payment pattern. Source of earnings analysis enables the actuary to monitor the adequacy of the charges and credits.<sup>48</sup>

The shift from a premium-loss pricing paradigm to a creditcharge pricing paradigm brought "universal" contracts to the life insurance industry. We may conceive of universal policies as retrospectively rated contracts where the premium adjustment depends on the investment yield achieved, not on the loss experience.<sup>49</sup>

By unbundling the policy into its components, the casualty insurer can offer varied product designs, such as universal policies for lines with long term claim payments. The actuary sets the investment spread; the actual premium for the coverage varies with the investment income actually earned. Such policies may be particularly attractive to large accounts seeking aggressive investment returns and reluctant to pay the premium before the losses come due.

#### Random Variations

Actuaries often attribute differences between expected and actual results to random loss fluctuations, to unforeseeable changes in inflation, or to unanticipated market pressures on underwriters and agents. The work pressures on actuaries are so great, and the potential causes of adverse results are so diverse, that many pricing actuaries never examine the variances in past re-

<sup>&</sup>lt;sup>48</sup>An analogy with computer manufacturing is instructive. IBM once built machines in pre-set models. Dell builds machines to consumer desires, with the price based on the components that are included. Insurers used to offer pre-determined policies to all insureds. Now insurers offer flexible policy design to large commercial accounts. Actuarial pricing must be equally flexible, so that the customized policies are priced by sound economic principles.

<sup>&</sup>lt;sup>49</sup>There are differences, of course. Universal life policies allow more management discretion in setting the credited interest rate; workers compensation retrospectively rated policies have contractually determined premium adjustments. Universal life contracts depends on the insurer's investment yield or on an external interest index; retrospectively rated policies depend on the individual insured's loss experience.

sults. Some actuaries believe that their time is too valuable to be spent re-examining their past analyses.

In truth, efficient examination of past results is a requisite for accurate prospective pricing. The source of earnings exhibits enable the actuary to quantify the contribution of each earnings factor to changes in profitability and to differentiate between estimation errors and process errors within the earnings factors. This "policy postmortem" may reveal biases in earnings factors or unstable pricing procedures.

#### Actuarial Productivity and Alice's Rabbit

Practicing actuaries are busy, busier than Alice's White Rabbit. These busy actuaries are forever computing things, crunching numbers, forming exhibits. There is never time to review previous work, since current tasks are pressing.

All too often, actuaries are computing numbers that do not get used, because they do not accurately reflect the values that they purport to measure. The busy actuaries do not realize this, because they do not have time to evaluate the accuracy of their work.

This is the actuary's destiny: the incessant computation of complex exhibits that bewilder the audience and sometimes entrap even the actuary, so that when errors creep in and lead the results astray, no one can distinguish right from wrong.

Source of earnings analysis is crucial to good actuarial work. Source of earnings analysis asks whether the assumptions are borne out by actual results. Some assumptions, like trend factors, development factors, credibility factors, seem trivial. One wonders: "How can one get these factors wrong?" But as actuarial procedures get more sophisticated, the work on trend factors, development factors, and credibility factors may lead to erroneous results, unbeknownst to the actuaries. Source of earnings analysis enables the practicing actuary to examine the accuracy of the efforts. Other assumptions are more elusive. The pricing actuary's rate indications rely on investment income assumptions, persistency patterns, acquisition cost assumptions, and loss discount rates. Sometimes the assumptions are explicitly worked into the underwriting profit margin or the underwriting expense ratio; sometimes the assumptions are implicit in the actuary's target loss ratio or target combined ratio. Year after year these implicit assumptions are repeated in the rate reviews. Rarely—if ever—does the actuary examine the validity of the assumptions.<sup>50</sup>

The practicing actuary may object that it is difficult to implement the source of earnings analysis for a particular factor, such as the interest earnings factor or the persistency factor. What the actuary is saying is that it is hard to determine whether the factors being used are correct. Let us rephrase this: if it is hard to determine whether the factors are correct, then it is quite possible that the factors are not correct. If the factors are not correct, then not only has the actuary wasted time computing these factors, but the actuary has wasted more time performing the analyses that rely on these factors. Source of earnings analysis is not an impediment to productivity; it is crucial to making the actuarial time become more productive.

#### Data Availability

A common complaint about source of earnings analysis is that the data are not available. Regarding retrospectively rated policies, the pricing actuary might say:

"We don't have the data needed for the analysis of expenses. We don't keep track of our not-taken rates,

<sup>&</sup>lt;sup>50</sup>Two examples illustrate the questionable paths along which actuarial science has progressed. (i) Casualty actuaries have produced a plethora of financial pricing models, some of which are at odds with financial theory. With no way of checking their validity, rate makers use these models over and over again. (ii) Auto pricing actuaries churn out rate indications in state after state, repeating the cycle year after year. Yet the incessant work may miss the true cost drivers of auto insurance losses; see. Conners and Feldblum [1998]. Source of earnings analysis forces the actuary to rethink the pricing assumptions.

we don't quantify the solicitation costs for the nottaken business, we don't separately evaluate the first year acquisition costs, and we don't keep records of policy persistency."

One wonders: "If you don't know your expenses, how do you price the business?"

The pricing actuary adds:

"We don't have the data needed for the analysis of the interest factor. We track loss cash flows, but not premium cash flows. We have incurred loss retros and paid loss retros, and we have all sorts of premium payment patterns; we don't know when the average premium comes in. We don't know when the expenses are paid; all we have are aggregate calendar year figures. We estimate our new money rates, but we don't know how much we actually earn on a given book of business. We don't have the data to quantify the interest we actually earn."

One wonders: "If you don't know your interest earnings, how do you price the business?"

The answer to these questions is straightforward: "We price the business as well as we can, using estimates and guesses when we don't have data."

If an assumption is not material, then it can be ignored in the source of earnings exhibits. An example is maintenance expenses, which are ignored in this paper.

If an assumption is critical to the pricing analysis, such as the acquisition expense assumption or the interest earnings assumption, then it cannot be ignored in the source of earnings analysis. But it cannot be ignored in the original pricing analysis either. The source of earnings analysis tells the actuary the work that must be done. One wonders: "Why do some pricing actuaries credibility weight loss development link ratios that are computed to three decimal places while they are oblivious of the acquisition expenses or the interest earnings on their book of business?"

## Actuarial Rates and Market Prices

Some readers have commented on an earlier draft of this paper that the actuarial indications are not the only problem. An additional problem is that the sales force or the underwriters cut the prices below the indications, either to meet peer company competition or to retain valued customers.

The source of earnings analysis explicitly incorporates such price adjustments. A market decision to change the price is an adjustment to the explicit profit provision.

*Illustration*: If the underwriting profit margin, after incorporation of investment income, is 8% of premium, and the underwriter grants a 10% premium reduction, the revised explicit profit provision is a negative 2.2% [= 1 - (1 - 8%)/(1 - 10%)].<sup>51</sup>

One critique of this analysis is that price-cutting is not done arbitrarily. The 10% rate reduction may have been offered to retain market share or to keep a valued customer who may turn more profitable in subsequent years. The source of earnings analysis does not tell us if the 10% rate reduction is justified.

This is correct. A single policy year perspective is not sufficient. Both pricing and profitability measurement must be done using "lifetime" methods. This does not mean that we must wait several years to measure profitability. On the contrary, source of earnings analysis enables us to examine long-term profitability reasonably quickly, since we can examine whether original pricing assumptions are validated by experience.

<sup>&</sup>lt;sup>51</sup>We should adjust for expenses that vary directly with premium. If the variable expense ratio is 15%, expenses are reduced by 1.5% of the original premium, and the new underwriting profit margin is  $-2.2\% + 1.5\% \times 10/9 = -0.5\%$ .

Ratemaking is prospective; we price next year's business, not last year's business. The pricing actuary succeeds by peering into the future, not by looking back.

Yet our ratemaking procedures are not infallible. Sometimes our methods are defective and our predictions are erroneous. Ever afraid of looking back, we try to outrun the errors.

We cannot outrun our errors. If we never look back, we never know the causes of our errors. We never learn if a variance of actual from expected results from random loss fluctuations or from improper ratemaking assumptions.

Our actuarial expertise is built on our past efforts. By examining our past efforts, we strengthen our current work.

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#### APPENDIX

### Source of Earnings Illustration

## Prepared by

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#### INTRODUCTION

This appendix focuses on three aspects of the source of earnings exhibits.

- It presents the source of earnings exhibits in sufficient detail that the practicing actuary can implement the procedure.
- It outlines the deferral and amortization of acquisition costs over the life of the business, in contrast with the standard GAAP amortization for property-casualty contracts over one year.
- It shows the effects of renewal rates in the book of business.

### ASSUMPTIONS

The model assumptions are summarized in Table 9, Table 10 and Table 11.

- All policies have January 1 effective dates.
- At the end of each year, some policies lapse and some policies renew.
- Acquisition costs are amortized over the lifetime of the policies. The profitability of the business depends on the acquisition costs. The income reported in each accounting period depends on the amortization schedule for these costs.
- Loss costs are higher on new business than on renewal business, but they do not vary by renewal year.

		Expenses		Loss Adi	
Time	Premium	Acq. + UW	Not-Taken	Expenses	Losses
0					
1	1500	-200	-50		
2				-12.50	-100
3				-18.75	-150
4				-31.25	-250
5				-43.75	-350
6				-18.75	-150

# NEW BUSINESS ASSUMPTIONS

### TABLE 10

RENEWAL	BUSINESS	ASSUMPTIONS
---------	----------	-------------

		Expenses		Loss Adi.	
Time	Premium	Acq. + UW	Not-Taken	Expenses	Losses
+1	1500	-50	0		
+2				-10.00	-80
+3				-15.00	-120
+4				-25.00	-200
+5				-35.00	-280
+6				-15.00	-120

Investment income is 8% of the assets required at the beginning of the year. The required assets are the discounted value of the reserves at year-end, using the investment yield of 8% as the discount rate.

Some of the policies lapse each year. The lapse rate assumptions are summarized in Table 11. The lapse rate times the inforce number of policies is the number of policies that leave the cohort at the end of the year.

*Illustration*: The cohort contains 100 policies in year one, with lapse rates of 1/10, 1/9, 1/8, 1/7, and 1/1 in years 1 through

Time	1	2	3	4	5+
Lapse rate	1/10	1/9	1/8	1/7	1

LAPSE RATE ASSUMPTIONS

5. Ten policies lapse at the end of year 1, and 90 continue into year 2. Ten more policies lapse at the end of year 2, and 80 continue into year 3. At the end of year 5, the remaining 60 policies all lapse. The abbreviated amortization schedule simplifies the exhibits in this appendix. In practice, a 15 or 20 year amortization schedule would be used.

#### DEFERRED ACQUISITION COSTS (DAC)

The deferrable first-year acquisition costs are 16.6% of the first-year premium. We amortize these costs over the five year expected lifetime of the cohort of policies.

#### The Expected DAC Schedule

The illustration in the text of this paper amortizes the acquisition costs over a fixed number of years with a 0% valuation rate. The actual GAAP amortization schedule for deferred policy acquisition costs differentiates between FAS 60 policies and FAS 97 policies.

- FAS 60: Deferrable expenses for long-duration contracts are amortized at a constant percentage of premium income.
- FAS 97: Deferrable expenses for universal life type contracts are amortized against gross profits (see below). The amount amortized each year is a constant percentage of book profits.

Two concepts underlie the DAC schedule.

- 1. The amortization percentage equals the ratio of the present value of deferrable expenses to the present value of the amortizing stream.
- 2. The amortization schedule takes into account the time value of money by considering the present value of the expenses written off each year.

For these exhibits, the DAC valuation rate, or the discount rate for computing the present value of book profits and deferrable expenses, is the 8% investment yield. Book profits are premium plus investment income less expenses, paid losses, and the increase in loss reserves.

## Mechanics of The DAC Schedule For One Policy Year

We determine the DAC schedule for one policy year. The DAC schedule for a book of business is the sum of the DAC schedules over all policy years.

First, we determine which expenses are deferrable and separate them from other expenses. We then project book profits for each policy year.

Next, we compute the present value of book profits (PVBP) and the present value of deferrable expenses (PVDE). The ratio PVDE/PVBP = k is the percentage of book profits that we use to amortize the DAC in each year. A *k* value larger than one implies that book profits are not sufficient to pay for the deferred acquisition costs.

The DAC amortization proceeds in three steps.

- 1. Deferrable expenses in the current year are added to the DAC balance at the end of the previous year.
- 2. The new DAC balance is accumulated for interest for one year.

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3. The accumulated DAC balance is reduced by the product of *k* and the book profits for the year.

Algebraically,

 $DAC_t = \{DAC_{t-1} + DE_t\} \times (1 + r) - k \times BP_t \text{ for } t = 1, 2, 3, \dots,$  where

 $DAC_t$  is the deferred acquisition cost asset balance at the end of year *t*,

 $DE_t$  is the deferrable expenses in year t,

 $BP_t$  is the book profit for year *t*.

The DAC balance at year zero is defined to be zero.

### THE INCOME STATEMENT

The income statement has two components: book profits and the charge due to the amortization of the DAC.

- 1. Book profits equal premium plus investment income less expenses, paid losses, and the increase in nominal reserves.
- 2. The charge due to amortization of the DAC is the difference in the DAC balance at two adjacent valuation dates. Table 12 shows the income statement, along with symbols that we use further below.

#### SOURCES OF EARNINGS

We track five sources of earnings: premium, investment income, expenses, incurred losses, and persistency.<sup>52</sup> A reduction in the persistency rate, or a higher than expected lapse rate, reduces the profitability of the business by forcing the initial acquisition costs to be spread over a smaller number of policies

<sup>&</sup>lt;sup>52</sup>The text of the paper does not include premiums as a source of earnings, since most of the illustrations in the text do not include variances in the lapse rate.

# INCOME STATEMENT

	Income Statement Item	Symbols
Profit =	Premium + Investment Income – Acquisition & UW expenses – Not-taken costs – Loss Adjustment expenses – Losses – Change in Loss Reserve – Amortization of DAC	Premium( $t$ ) + Investment Income( $t$ ) – Expenses Acq & UW( $t$ ) – Expenses NT( $t$ ) – Expenses Loss Adj( $t$ ) – Paid Losses( $t$ ) – [Nominal Reserve( $t$ ) – Nominal Reserve( $t$ – 1)] – [DAC( $t$ – 1) – DAC( $t$ )]

### TABLE 13

# ANALYSIS OF SOURCES

		Variation in
Actual profit( <i>t</i> ) =	Expected profit( <i>t</i> ) + [actual premium – expected premium] + [actual investment income	Premium
	– expected investment income] + [actual expenses – expected expenses] + [actual losses – expected losses] + [actual change in reserves]	Investment income Expenses Losses
	- expected change in reserves] + [actual DAC amortization	Change in Reserves
	- expected DAC amoruzation]	DAC amortization

or policy years. This is true even if other pricing assumptions remain valid.

The source of earnings exhibits measure the deviation between actual results and initial expectations, as shown in Table 13.

We divide the variation between expected and actual results into two components.

- 1. The variation between expected and "accumulated past experience"
- 2. The variation between "accumulated past experience" and actual results.

We focus on two aspects of the source of earnings analysis:

- 1. The revision of the pricing assumptions for future years based on information gathered up to now.
- 2. The division of the variances into those stemming from past year events and those arising from current year events.

### TRACKING ACTUAL EXPERIENCE

We record experience as it emerges and adjust the DAC schedule based on the new information.

At inception of the cohort, we project expected results for all future years based on the pricing assumptions. This benchmark projection does not change as actual experience comes in.

Analysis of sources of earnings is a continuous process with the following steps:

- 1. Projection of results for the current year, taking into account all past events.
- 2. Analysis of deviations between the initial benchmark projection and the projection from step (1). We call these deviations "variation due to past accumulated experience."
- 3. Capturing actual experience over the current year.
- 4. Recalculating the DAC schedule.
- 5. Analysis of deviations between the projection from step (1) and the actual results. These deviations are called "variation due to current year experience."

As a final step, we project the results for the upcoming valuation date, incorporating all available information from accumulated past experience and any new estimates for future years. This projection is the best estimate of actual experience over the next valuation period. Since this projection incorporates more information, it may differ substantially from the pricing benchmark. These deviations are "deviations due to past accumulated experience."

As the new year's experience emerges, there may be additional "deviations stemming from current experience." These are deviations between the projected experience at the beginning of the year and the actual experience that emerges.

We separate these two sources of deviation to better understand their causes. The sum of the two sets of deviations gives the total deviation between the pricing benchmark and the actual results.

The deviations show the dollar differences between pricing assumptions and actual experience. Analysis of the deviations enables the pricing actuary to refine the ratemaking procedure and the pricing assumptions.

The recalculation of the DAC schedule is the most complex part of the analysis. The DAC is amortized in proportion to book profits in each year. As actual experience emerges, the book profits change, and the percentage of book profits used to amortize the DAC changes as well.

At each valuation date, we recalculate the remaining DAC schedule. The calculation for the DAC ratio is the same as in the first year except that we have a non-zero previous DAC balance. The DAC ratio is equal to the ratio of:

- 1. the previous DAC balance plus present value of remaining deferrable expenses, to
- 2. the present value of remaining book profits.

Time	Book Profit	DAC Balance	
1 2	88.9 81.5	120.2 75.0	
3 4 5 6	66.7 44.4 14.8 0.0	36.2 9.2 0.0 0.0	

### INITIAL DAC SCHEDULE AND BOOK PROFITS

#### TABLE 15

### **REVISED BOOK PROFITS**

Time	Book Profit	DAC Balance
1 2	88.9 81.5	120.2 75.0
3 4 5 6	56.7 44.4 14.8 0.0	

*Illustration*: The book of business has the stream of book profits and the DAC schedule shown in Table 14. The first two periods reflect actual results. The remaining periods are projected results based on all information available at the end of period two.

The expected book profit in period three is 66.7. The expected DAC balance at the end of period three is 36.2. There are no additional deferrable expenses in period 3 through 6.

Adverse loss experience in period 3 alters the book profit from 66.7 to 56.7, as shown in Table 15.

The new DAC ratio equals 75.0/102.31 = 73.31%

Time	Book Profit	DAC Balance	
1 2	88.9 81.5	120.2 75.0	
3	56.7	39.4	
4 5 6	44.4 14.8 0.0	10.0 0.0 0.0	

# REVISED DAC SCHEDULE

- The numerator is the sum of the previous DAC balance of 75.0 and the present value of remaining deferrable expenses, which are zero.
- The denominator is the present value of remaining book profits (i.e., 56.7, 44.4, 14.8). The present values are computed at an 8% discount rate.

The DAC balance at the end of period three equals  $75.0 \times 1.08 - 73.31\% \times 56.7 = 39.43$ . The remaining amortization schedule is shown in Table 16. The depressed book profits in period 3 increases the DAC ratio from the original 67.25% to 73.31% and changes all remaining values.

### CHANGES IN INVESTMENT INCOME AND INCURRED LOSSES

We track the evolution of a hypothetical example with exhibits and commentary. The initial assumptions are the same as those described above. Table 17 shows the pricing actuary's projection for the block of business.

Table 9, Table 10 and Table 11 show the pricing assumptions for a cohort of business. Premium, expense, and expected loss ratio assumptions are provided by the pricing actuary. Table 17 shows the new business plus four renewal years. To simplify the

		Investment	Expe	inses	l oss		A (Nominal	Book	A (DAC	
Time	Premium	Income	Acq. + UW	Not-taken	Adj. Exp	Losses	Reserve)	Profit	Balance)	Profit
0										
1	1,500.0	83.3	-200.0	-50.0	0.0	0.0	1,125.0	208.3	119.4	327.8
2	1,350.0	135.0	-45.0	0.0	-12.5	-100.0	697.5	630.0	-36.1	593.9
З	1,200.0	169.8	-40.0	0.0	-27.8	-222.0	470.3	609.8	-34.8	575.1
4	1,050.0	181.3	-35.0	0.0	-52.8	-422.0	155.3	566.3	-26.3	540.1
5	900.0	164.5	-30.0	0.0	-85.3	-682.0	-227.3	494.5	-9.6	484.9
9	0.0	106.7	0.0	0.0	-86.8	-694.0	-780.8	106.7	-6.7	100.0
7	0.0	61.3	0.0	0.0	-68.0	-544.0	-612.0	61.3	-3.9	57.4
8	0.0	27.0	0.0	0.0	-51.5	-412.0	-463.5	27.0	-1.8	25.2
6	0.0	6.0	0.0	0.0	-31.5	-252.0	-283.5	6.0	-0.4	5.6
10	0.0	0.0	0.0	0.0	-9.0	-72.0	-81.0	0.0	0.0	0.0

PRICING EXPECTATIONS FOR BLOCK OF BUSINESS

Time	First Year	First Renewal	Second Renewal	Third Renewal	Fourth Renewal	Total Losses		
0								
1	1500					1500.0		
2		1500				1350.0		
3			1500			1200.0		
4				1500		1050.0		
5					1500	900.0		
6						0.0		
7						0.0		
8						0.0		
9						0.0		
10						0.0		
Persistency factors (based on lapse assumptions)								
	1	0.9	0.8	0.7	0.6			

#### PREMIUM CALCULATION

illustration, we assume that all remaining policies lapse at the end of the fifth year. Years 6 through 10 show the run-off of the remaining reserves. In practice, the table would show policy renewals and run-off of reserves until the figures were not material. For a book of workers compensation business, this would be about 30 years.

Before proceeding with the analysis of deviations, we document the procedures used to create the projection for the block of business. The projection includes the initial year of production plus four years of renewals. Of the original number of policies, only a fraction renew into the first year. Similarly, of those policies in-force during the second calendar year, only a fraction renew into the third year. Table 18 below shows the premium per policy that is collected for the first year of production and each renewal year. The last column of Table 18 shows the total premium collected for each calendar year. The bottom row shows the fraction of the *original* polices that are
Time	First Year	First Renewal	Second Renewal	Third Renewal	Fourth Renewal	Total Losses
0						
1	0					0.0
2	-100	0				-100.0
3	-150	-80	0			-222.0
4	-250	-120	-80	0		-422.0
5	-350	-200	-120	-80	0	-682.0
6	-150	-280	-200	-120	-80	-694.0
7		-120	-280	-200	-120	-544.0
8			-120	-280	-200	-412.0
9				-120	-280	-252.0
10					-120	-72.0
	Persist	ency factors	s (based on ]	apse assumj	otions)	
	1	0.9	0.8	0.7	0.6	

# TOTAL LOSS CALCULATION

in-force through the various renewal years. The total premium is equal to the sum of each row times the appropriate persistency factor. For example, the total premium at time 3 equals  $1 \times 0 + 0.9 \times 0 + 0.8 \times 1500 + 0.7 \times 0 + 0.6 \times 0 = 1,200$ .

The same procedure is applied to expenses, loss adjustment expenses, and losses. Table 19 shows the total losses, in each calendar year, for this block of business. The losses shown is the first 5 columns are on a per policy basis. For the first year, the entries come from Table 9. For the renewal years they come from Table 10. The total loss of -682 at time 5 is equal to

$$(-350) \times 1 + (-200) \times 0.9 + (-120) \times 0.8$$
  
+  $(-80) \times 0.7 + 0 \times 0.6 = -682.$ 

The calculations necessary to obtain the total nominal reserves are more complex. The total nominal reserve is equal to the to-

Time	First Year	First Renewal	Second Renewal	Third Renewal	Fourth Renewal	Total Loss Reserve			
0									
1	1000					1000.0			
2	900	720				1620.0			
3	750	648	640			2038.0			
4	500	540	576	560		2176.0			
5	150	360	480	504	480	1974.0			
6	0	108	320	420	432	1280.0			
7	0	0	96	280	360	736.0			
8	0	0	0	84	240	324.0			
9	0	0	0	0	72	72.0			
10	0	0	0	0	0	0.0			
	Persistency factors (based on lapse assumptions)								
	1	0.9	0.8	0.7	0.6				

# TOTAL LOSS RESERVE

tal loss reserve plus the total loss adjustment expense reserve. To calculate the total loss reserve we first compute the nominal reserves for the first year of production and each renewal year. Table 20 shows the nominal reserves for each year and the grand total.

For example, the nominal reserve of 750 for the first year (see the second column of Table 20) at time 3 is equal to the persistency factor for the first year times the sum of losses (from Table 19) at time 4, 5, 6, and so forth. That is,

$$750 = 1 \times (250 + 350 + 150).$$

The nominal reserve for the second renewal year at time 5 is equal to

$$480 = 0.8 \times (200 + 280 + 120).$$

The total loss reserve for the book of business is the sum of the rows. For instance, at time 7 the total loss reserve equals 96 + 280 + 360 = 736.

Time	First Year	First Renewal	Second Renewal	Third Renewal	Fourth Renewal	Total Loss Adj Exp
0						
1	0.0					0.0
2	-12.5	0.0				-12.5
3	-18.8	-10.0	0.0			-27.8
4	-31.3	-15.0	-10.0	0.0		-52.8
5	-43.8	-25.0	-15.0	-10.0	0.0	-85.3
6	-18.8	-35.0	-25.0	-15.0	-10.0	-86.8
7		-15.0	-35.0	-25.0	-15.0	-68.0
8			-15.0	-35.0	-25.0	-51.5
9				-15.0	-35.0	-31.5
10					-15.0	-9.0
	Persist	ency factors	s (based on ]	lapse assumj	ptions)	
	1	0.9	0.8	0.7	0.6	

# TOTAL LOSS ADJUSTMENT EXPENSES

The second component of the nominal reserve is the loss adjustment expenses reserve. Table 21 and Table 22 show the derivation of the loss adjustment expense reserve.

The total loss reserve (Table 20) plus the total loss adjustment expense reserve (Table 22) equals the total nominal reserve. Table 23 shows the nominal reserve and the change in the reserve. This change in nominal reserve is also shown in Table 17.

We now begin the analysis. First year expenses are assumed to be 200/1,500 = 13.3% of premium. Solicitation costs for not-taken business are 25% [= 50/200] of first year expenses.

Loss adjustment expenses are assumed to be 12.5% of paid losses. Since the payout schedule for loss adjustment expenses differs between allocated and unallocated expenses, a more refined schedule would be used in practice.

For simplicity, we assume that the loss cost trend is 0% per annum and that no premium changes are expected over the fiveyear span of the table. In practice, the appropriate trend rates

Time	First Year	First Renewal	Second Renewal	Third Renewal	Fourth Renewal	Total Loss Adj. Reserve
0						
1	125					125.0
2	113	90				202.5
3	94	81	80			254.8
4	63	68	72	70		272.0
5	19	45	60	63	60	246.8
6	0	14	40	53	54	160.0
7	0	0	12	35	45	92.0
8	0	0	0	11	30	40.5
9	0	0	0	0	9	9.0
10	0	0	0	0	0	0.0
	Persist	ency factors	s (based on l	apse assump	otions)	
	1	0.9	0.8	0.7	0.6	

# TOTAL LOSS ADJUSTMENT EXPENSE RESERVE

for premiums, losses, and expenses should be included. For the heuristic purposes of this illustration, the simplified model shows the workings of the exhibits without excessive refinements.

The nominal reserves are the sum of future loss and loss adjustment expense payments on this block of policies. The expected loss and LAE ratio is 1,125/1,500 = 75%. Since the previous year reserve is zero, the change in the reserve equals the reserve at the end of the year.

Investment income equals the investment yield of 8% times the required assets at the start of the year. Required assets are defined as the discounted value, at the investment rate of return, of the year-end nominal reserves. For year 1, investment income equals  $83.3 = 8\% \times 1,125/1.08.^{53}$ 

<sup>&</sup>lt;sup>53</sup>We use the present value of the year end reserve to illustrate the standard life actuarial use of these exhibits. The traditional property-casualty perspective would use the nominal value of the year end reserve.

Time	Total Nominal Reserve	Change in Reserve
0		
1	1125.0	1125.0
2	1822.5	697.5
3	2292.8	470.3
4	2448.0	155.3
5	2220.8	-227.3
6	1440.0	-780.8
7	828.0	-612.0
8	364.5	-463.5
9	81.0	-283.5
10	0.0	-81.0

TOTAL NOMINAL RESERVE AND THE CHANGE IN RESERVE

Book profit is equal to premium plus investment income less expenses, paid losses, and the change in nominal reserve. For year 1,

$$208.3 = 1,500 + 83.3 - 200 - 50 - 1,125.$$

At inception of the cohort of policies, the DAC ratio is the present value of deferrable expenses divided by the present value of all future book profits.

- The present value of the deferrable expenses is 250.
- At inception, the projected book profits are 208.3, 75.0, 62.5, 41.7, and 12.5. Their present value at an 8% investment yield is 345.9.
- The DAC ratio = 72.27% = 250/345.9.

The DAC balance accumulated to the end of year one equals  $250 \times 1.08 = 270$ . The DAC expenses amortized in year 1 are  $72.27\% \times 208.3 = 150.5$ . The DAC balance at the end of year

one equals

$$250 \times (1 + 8\%) - 72.27\% \times 208.3 = 119.5.$$

Since the previous DAC balance is zero, the change in DAC equals 119.5. The net profit for year one equals the sum of the book profit and the change in the DAC balance

$$327.8 = 208.3 + 119.5$$
.

Years 2 through 5 show the combination of new and renewal business and the use of changes in the nominal reserves. We document the entries for year 3. The first year premium is 1,500, and the lapse rates for year one and two are 1/10 and 1/9. We expect  $(1 - 1/10) \times (1 - 1/9) = 80\%$  of the policyholders to renew into year three. The expected premium in year 3 is

$$1,200 = 1,500 \times 80\%$$
.

Underwriting and acquisition costs are  $40 = 50 \times 80\%$ . The loss adjustment expenses of 27.8 stem from policies written in years 1 and 2 (see Table 21 time 3 row):

$$27.8 = 18.8 \times 1 + 10 \times (1 - 1/10).$$

The first term on the right hand side reflects the loss adjustment expenses from the first year of writings and the second term reflects the loss adjustment expenses from the first renewal year.

Similarly, the losses of 222 in row 3 are the sum of paid losses from two underwriting years (see Table 19 time 3 row):

$$222 = 150 \times 1 + 80 \times (1 - 1/10).$$

The nominal reserve at the end of year three is the sum of all future loss and loss adjustment expense payments from the first three years of writings. Table 20, Table 22, and Table 23 show the calculation of the total nominal reserve

$$2,292.8 = \{250 + 31.25 + 350 + 43.75 + 150 + 18.75\} \\ \times 1 + \{120 + 15 + 200 + 25 + 280 + 35 + 120 + 15\} \\ \times (1 - 1/10) + \{80 + 10 + 120 + 15 + 200 + 25 + 280 \\ + 35 + 120 + 15\} \\ \times (1 - 1/10) \times (1 - 1/9).$$

The nominal reserve at the end of year two of 1,822.5 is calculated in the same fashion. The change in the nominal reserve at the end of year three is equal to

$$470.3 = 2,292.8 - 1,822.5.$$

The investment income of 169.8 equals  $8\% \times 2,292.8/1.08$ . The book profit is

$$609.7 = 1,200 + 169.8 - 40 - 27.8 - 222 - 470.3$$

The DAC balance at the end of year two equals

$$\begin{split} 83.3 &= \{ 119.4 \times 1.08 - 72.27\%75.0 \} \\ &+ \{ 45 \times 1.08 - 7.211\%^{54} \times 555.0 \}. \end{split}$$

The DAC balance at the end of year three equals

$$\begin{split} 48.6 &= \{74.8 \times 1.08 - 72.27\% \times 62.5\} \\ &+ \{8.6 \times 1.08 - 7.211\% \times 54.0\} \\ &+ \{40 \times 1.08 - 7.211\% \times 493.3\} \end{split}$$

• The DAC ratio, for renewal years, equals 7.211% = 45/624.1.

<sup>&</sup>lt;sup>54</sup>For each renewal year we calculate the appropriate DAC ratio. Since all of the renewal years are identical, up to a proportionality factor, the DAC ratios are equal for all the years. The calculation of the renewal DAC ratio is performed as follows:

<sup>•</sup> The present value of deferrable renewal expenses for the first renewal year is 45.

<sup>•</sup> The projected book profits for the first renewal year are 555.0, 54.0, 45.0, 30.0, and 9.0. Their present value at a discount rate of 8% is 624.1.

The entries of 45 and 555.0 in the second summand of the equation above represent the deferrable expenses and the book profit for the first renewal year at time two.

Here the entries of 8.6 and 54.0 in the second summand represent the DAC balance at time 2 and the book profit at time 3 for the first renewal year. In the third summand, the entries of 40 and 493.3 are the deferrable expenses and book profit, respectively, for the second renewal year.

The change in the DAC balance is -34.7 = 48.6 - 83.3. The profit for year three equals the book profit of 609.8 plus the change in DAC balance of -34.7; hence, the profit is 575.1.

At the inception of the block of business we project the results for the upcoming year (year one). In this illustration, we assume there is no new information between the pricing of the block and the actual issuing of policies.<sup>55</sup> Table 24 presents the projection based on past accumulated experience.

Over the course of the year we tabulate actual experience. This illustration assumes that actual first year experience exactly matches the initial projections, and all variations are zero (see Table 25, Table 26 and Table 27).

As the final step of the first evaluation, we project the results for the upcoming year, taking into consideration all available information. Table 28 shows the projections for year two.

In year two, actual results do not exactly match expectations (see Table 29). The pricing assumptions project year two loss payments of 100 units; actual year two loss payments are 120 units. This change necessitates a recalculation of the DAC schedule as well.

The variation due to past accumulated experience is zero (see Table 30) because at the start of year two there are no past variances.

<sup>&</sup>lt;sup>55</sup>The illustration assumes all policies are written at the start of the year. In practice, policy year writings are spread over the year. As the first policies are issued, we might learn more about the expected experience and thereby alter the projection.

			Profit	327.8				Profit	327.8	
		A MAC	Balance)	119.4			A (DAC	Balance)	119.4	
	AR ONE	Book	Profit	208.3			Book	Profit	208.3	
	rt of Ye∕	A (Nominal	Reserve)	1,125.0		YEAR ONE	A (Nominal	Reserve)	1,125.0	
<del></del>	ατ Sta		Losses	0.0	2	ND OF		Losses	0.0	
ABLE 24	ERIENCE	I.oss	Adj. Exp	0.0	ABLE 2:	NCE AT E	Loss	Adj. Exp	0.0	
Η	ted Exp	Ises	Not-taken	-50.0	T	EXPERIEN	Ises	Not-taken	-50.0	
	CCUMULA	Expei	Acq. + UW	-200.0		Actual I	Expei	Acq. + UW	-200.0	
	Past A	Investment	Income	83.3			Investment	Income	83.3	
			Premium	1,500.0				Premium	1,500.0	
			Time	0 1				Time	0 1	

VARIATION DUE TO PAST ACCUMULATED EXPERIENCE AT END OF YEAR ONE

	ofit	0				ofit	0
	Prc	0.				Prc	o l
A (DAC	Balance)	0.0		R ONE	A (DAC	Balance)	0.0
Book	Profit	0.0		of Yea	Book	Profit	0.0
Δ (Nominal	Reserve)	0.0		e at End	∆ (Nominal	Reserve)	0.0
	Losses	0.0	7	ERIENCI		Losses	0.0
Loss	Adj. Exp	0.0	ABLE 27	ear Exp	Loss	Adj. Exp	0.0
suses	Not-taken	0.0	L	JRRENT	suses	Not-taken	0.0
Expe	Acq. + UW	0.0		ue to Cl	Expe	Acq. + UW	0.0
Investment	Income	0.0		ATION DI	Investment	Income	0.0
	Premium	0.0		Vari		Premium	0.0
	Time	0 1				Time	0 1

IABLE 28	Past Accumulated Experience at Year Two	$\begin{array}{c c} Expenses \\ Investment \\ \hline \\ \hline \\ \hline \\ \hline \\ \\ \hline \\ \\ \hline \\ \\ \hline \\ \\ \\ \hline \\ \\ \\ \\ \hline \\ \\ \\ \\ \\ \hline \\$	Income Acq. + UW Not-taken Adj. Exp Losses Reserve) Profit Balance) Profit	83.3 -200.0 -50.0 0.0 0.0 1.125.0 208.3 119.4 327.8	135.0 -45.0 0.0 -12.5 -100.0 697.5 630.0 -36.1 593.9	TABLE 29	ACTUAL RESULTS AT END OF YEAR TWO	Expenses Loss A (Nominal Rook A (DAC	Income Acq. + UW Not-taken Adj. Exp Losses Reserve) Profit Balance) Profit		83.3 –200.0 –50.0 0.0 0.0 1,125.0 208.3 119.4 327.8	135.0 -45.0 0.0 -12.5 -120.0 697.5 610.0 -26.6 583.4
	PAST AC	Investment	Income Acq. + I	83.3 -200	135.0 -45		Асти	Investment	Income Acq. + I		83.3 –200	135.0 -45
			Premium	1.500.0	1,350.0				Premium		1,500.0	1,350.0
			Time	0 -	5				Time	0	-	7

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Variation due to current year experience (Table 31) shows an increase in paid losses of 20 units and a variation in the DAC balance of 9.4 units. The variation in paid losses is the 20 unit difference between projected and actual loss payments. The change in the DAC balance arises because the book profits change.

The additional 20 units of paid losses depress the book profits in year 2. This increases the DAC ratio, which is the ratio of the present value of total deferred expenses to the present value of total book profits. Since the projected book profits in subsequent years have not changed, the relative book profits in year 2 decline as a percentage of total book profits. Similarly, the amount of DAC amortized in year 2 declines as a percentage of the total DAC as well as in dollar terms.

The general principle is that a change in book profits in a single year is partially offset by a change in DAC amortization. This principle is not applicable to changes in book profits that affect multiple years, as is true for changes stemming from investment yields or retention rates.

Year three shows no deviations from experience expected at the beginning of the year, though there are deviations stemming from past experience. Table 32 shows the projection at the beginning of the year, taking into account all previous deviations.

The actual experience for year three in Table 33 is identical to the projected experience at the beginning of the year. The variances based on past experience and current experience are shown in Table 34 and Table 35.

The DAC schedule changes from the initial projections because of the additional paid losses in year two. 9.4 units less of DAC are amortized in year two and 4.9 units more of DAC are amortized in year three.

For year four, we assume that actual experience equals the projected experience (see Tables 36 and 37). During year five, three events occur.

			Profit	0.0	0.0			Profit	00	-10.6
ear Two	ar Two	A (DAC	Balance)	0.0	0.0	ar Two	A (DAC	Balance)	00	9.4
	d of Ye	Book	Profit	0.0	0.0	OF YE.	Book	Profit	00	-20.0
	ICE AT ENI	A (Nominal	Reserve)	0.0	0.0	T THE ENI	∆ (Nominal	Reserve)	00	0.0
_	XPERIEN		Losses	0.0	0.0	ENCE A		Losses	00	-20.0
ABLE 30	LATED E	l oss	Adj. Exp	0.0	0.0	ABLE 31 r Experi	Loss	Adj. Exp	00	0.0
Τ	Accumu	nses	Not-taken	0.0	0.0	T ENT YEA	nses	Not-taken	00	0.0
	to Past	Expe	Acq. + UW	0.0	0.0	to Curr	Expe	Acq. + UW	00	0.0
	on Due	Investment	Income	0.0	0.0	ON DUE	Investment	Income	00	0.0
	VARIATI		Premium	0.0	0.0	Variati		Premium	00	0.0
			Time	0 1	2			Time	0 -	5 7

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				1	ABLE 34	+				
	VARIATIC	DUE T	o Past A	ACCUMUL	ATED EX	(PERIEN(	ce at End	OF YE	ar Three	
		Investment	Expe	nses	Loss		A (Nominal	Book	A (DAC	
Time	Premium	Income	Acq. + UW	Not-taken	Adj. Exp	Losses	Reserve)	Profit	Balance)	Profit
0										
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-4.9	-4.9
				E	ABLE 3;	10				
	Varia	TION DU	e to Cur	rrent Ye	ar Expe	RIENCE	AT END OI	f Year	THREE	
		Investment	Expe	nses	Loss		Δ (Nominal	Book	A (DAC	
Time	Premium	Income	Acq. + UW	Not-taken	Adj. Exp	Losses	Reserve)	Profit	Balance)	Profit
0										
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0	0.0	-20.0	0.0	-20.0	9.4	-10.6
ŝ	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

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PROJECTION BASED ON PAST ACCUMULATED EXPERIENCE

		Investment	Expe	nses	Loss		∆ (Nominal	Book	A (DAC	
Time	Premium	Income	Acq. + UW	Not-taken	Adj. Exp	Losses	Reserve)	Profit	Balance)	Profit
0										
1	1,500.0	83.3	-200.0	-50.0	0.0	0.0	1,125.0	208.3	119.4	327.8
2	1,350.0	135.0	-45.0	0.0	-12.5	-100.0	697.5	630.0	-36.1	593.9
3	1,200.0	169.8	-40.0	0.0	-27.8	-222.0	470.3	609.8	-39.7	570.1
4	1,050.0	181.3	-35.0	0.0	-52.8	-422.0	155.3	566.3	-29.7	536.6
5	900.0	164.5	-30.0	0.0	-85.3	-682.0	-227.3	494.5	-10.7	483.8
9	0.0	130.9	0.0	0.0	-86.8	-744.0	-830.8	130.9	-7.0	123.9
7	0.0	75.3	0.0	0.0	-68.0	-544.0	-612.0	75.3	-4.2	71.0
8	0.0	33.1	0.0	0.0	-51.5	-412.0	-463.5	33.1	-2.0	31.1
6	0.0	7.4	0.0	0.0	-31.5	-252.0	-283.5	7.4	-0.5	6.9
10	0.0	0.0	0.0	0.0	-9.0	-72.0	-81.0	0.0	0.0	0.0

		Investment	Expe	suses	Loss		A (Nominal	Book	A (DAC	
Time	Premium	Income	Acq. + UW	Not-taken	Adj. Exp	Losses	Reserve)	Profit	Balance)	Profit
0										
1	1,500.0	83.3	-200.0	-50.0	0.0	0.0	1,125.0	208.3	119.4	327.8
2	1,350.0	135.0	-45.0	0.0	-12.5	-120.0	697.5	610.0	-26.6	583.4
33	1,200.0	169.8	-40.0	0.0	-27.8	-222.0	470.3	609.8	-39.7	570.1
4	1,050.0	181.3	-35.0	0.0	-52.8	-422.0	155.3	566.3	-29.7	536.6
5	900.0	206.4	-30.0	0.0	-85.3	-657.0	-177.3	511.4	-9.8	501.7
9	0.0	130.9	0.0	0.0	-86.8	-744.0	-830.8	130.9	-7.0	123.9
7	0.0	75.3	0.0	0.0	-68.0	-544.0	-612.0	75.3	-4.2	71.0
8	0.0	33.1	0.0	0.0	-51.5	-412.0	-463.5	33.1	-2.0	31.1
6	0.0	7.4	0.0	0.0	-31.5	-252.0	-283.5	7.4	-0.5	6.9
10	0.0	0.0	0.0	0.0	-9.0	-72.0	-81.0	0.0	0.0	0.0

TABLE 37 Actual Results of Our Block of Business

- 1. Paid losses are less than expected.
- 2. The investment department reports that invested assets will generate more investment income than had been anticipated for year five and for subsequent years (see Table 38).
- 3. A revised loss reserve analysis raises the estimate of unpaid losses.

These three events have partially offsetting effects on total profitability. The source of earnings exhibits enable us to tease apart the effects of each event.

Analysis of Table 38 and Table 39 leads to the following conclusions:

- 1. Changes from projected experience occur only in years 2 and 5 (see Table 39).
- 2. The column for "Investment Income" has non-zero entries for year 5 in Table 39 (variation stemming from current experience) and for years 6 through 9 in Table 38 (variation stemming from past accumulated experience). The changed investment yield in year 5 causes increased investment income in that year and the four subsequent years.
- 3. The column " $\Delta$  (Nominal Reserve)" in the current year variation table (Table 39) shows the revised reserve estimate of +50 in year 5.
- 4. The DAC balance changes for year 5 from current experience Table 39 and in years 6 through 9 from past accumulated experience Table 38. Events that change book profits or deferrable expenses change the DAC schedule for the current year and all subsequent years.<sup>56</sup>

<sup>&</sup>lt;sup>56</sup>To fully separate the effects of the three events, one could attribute the non-zero entries in the DAC balance column to the various sources (premium, investment income, expenses, paid losses, and change in reserves).

ΞX μ	enses						
		Loss		Δ (Nominal	Book	∆ (DAC	
+ UW	Not-taken	Adj. Exp	Losses	Reserve)	Profit	Balance)	Profit
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	-4.9	-4.9
0.0	0.0	0.0	0.0	0.0	0.0	-3.4	-3.4
0.0	0.0	0.0	0.0	0.0	0.0	-1.1	-1.1
0.0	0.0	0.0	-50.0	-50.0	24.2	-0.3	23.9
0.0	0.0	0.0	0.0	0.0	13.9	-0.3	13.6
0.0	0.0	0.0	0.0	0.0	6.1	-0.2	5.9
0.0	0.0	0.0	0.0	0.0	1.4	-0.1	1.3
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

VARIATION DUE TO PAST ACCUMULATED EXPERIENCE

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VARIATION DUE TO CURRENT YEAR EXPERIENCE

	Profit		0.0	-10.6	0.0	0.0	17.8	0.0	0.0	0.0	0.0	0.0
A (DAC	Balance)		0.0	9.4	0.0	0.0	0.9	0.0	0.0	0.0	0.0	0.0
Book	Profit		0.0	-20.0	0.0	0.0	16.9	0.0	0.0	0.0	0.0	0.0
A (Nominal	Reserve)		0.0	0.0	0.0	0.0	50.0	0.0	0.0	0.0	0.0	0.0
	Losses		0.0	-20.0	0.0	0.0	25.0	0.0	0.0	0.0	0.0	0.0
I.oss	Adj. Exp		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
nses	Not-taken		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Expe	Acq. + UW		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Investment	Income		0.0	0.0	0.0	0.0	41.9	0.0	0.0	0.0	0.0	0.0
	Premium		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	Time	0	1	7	б	4	5	9	7	8	6	10

# CHANGES IN RETENTION RATES

The previous example has no variance of actual retention rates from projected retention rates. The following example (Table 40) shows the effects of changes in retention rates.<sup>57</sup>

We start with the same block of business as in the previous example. At inception of the cohort, we assume that 10% of the policies will lapse at the first renewal date. Actual experience at the end of year two shows the following:

Variances occur in several of the columns, as shown in Table 41.

The corresponding variation in premium and first year expenses suggests a change in the lapse rate.

- The year 1 premium is 1,500 and the projected lapse rate at the end of year 1 is 10%, giving the year 2 projected premium of 1,350.
- The actual year 2 premium is 1,335, implying a lapse rate of 11%.

The variance of -15 implies an excess lapse rate of 1% [= 15/1,500].

The expenses show the same effect. The underwriting and acquisition cost expense ratio in renewal years is  $3\frac{1}{3}\%$  of premium.

- For a premium of 1,500, the expenses equal  $3\frac{1}{3}\% \times 1,500 = 50.0$ .
- For a premium of 1,335, the expenses equal  $3\frac{1}{3}\% \times 1,335 = 44.5$ .

The half unit variance (Table 41) in the expense column reflects the 1% excess lapse rate.

<sup>&</sup>lt;sup>57</sup>Life actuaries and casualty actuaries use a variety of terms: retention rates or persistency rates for the percentage of policies that renew and lapse rates or termination rates for the percentage of policies that do not renew.

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# ACTUAL RESULTS AT END OF SECOND YEAR

I		I		I		I		l
	Profit	327.8	587.7				Profit	0.0
A (DAC	Balance)	119.4	-36.2			A (DAC	Balance)	0.0
Book	Profit	208.3	623.8		E	Book	Profit	0.0
A (Nominal	Reserve)	1.125.0	688.5		Experienc	A (Nominal	Reserve)	0.0
	Losses	0.0	-100.0		r Year		Losses	0.0
l oss	Adj. Exp	0.0	-12.5	ABLE 4	CURRENT	l aso	Adj. Exp	0.0
nses	Not-taken	-50.0	0.0		DUE TO	nses	Not-taken	0.0
Expe	Acq. + UW	-200.0	-44.5		RIATION	Expe	Acq. + UW	0.0
Investment	Income	83.3	134.3		VA	Investment	Income	0.0
	Premium	1.500.0	1,335.0				Premium	0.0
	Time	0 1	2				Time	0 1
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$								

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-6.3

-0.1

-6.2

-9.0

0.0

0.0

0.0

0.5

-0.7

-15.0

2

The calculations shown above for premiums and expenses are more complex when applied to investment income and nominal reserves. A lapse rate deviation in one year affects the number of policies in-force for all future years, thereby changing the reserves and the dollars of investment income (see Tables 42 and 43).

Table 45, "variation due to current year experience," shows variations in year four as well.

Both premiums and expenses have similar deviations, suggesting a change in retentions. Table 42 indicates that expected premium collections were 1,038.3 but actual collections were only 1,008.7, for a variance of 29.7 (see Table 45). The benchmark pricing lapse rate for year three is 12.5%. If everyone had renewed, the collected premium would have been 1,038.3/0.875 = 1,186.7, so the excess lapse rate is 29.7/1,186.7 = 2.5%.

The underwriting and acquisition cost expense deviation is 1.0. The projection based on past accumulated experience indicates that expenses should have been 34.6 (see Table 42) for a renewal rate of 87.5%. If everyone had renewed, the expenses would have been 39.5 (= 34.6/0.875)). The indicated excess lapse rate is 1.0/39.5 = 2.5%.

The total profit deviation, or the sum of all entries in the last columns of Table 44 and Table 45, is -53.9.

# NOTATION AND FORMULAE

The formulae underlying the exhibits in this appendix are listed below. Policy years are denoted as superscripts and calendar years as subscripts. For example,

# $DAC_{CY}^{PY}$

represents the DAC balance at the end of calendar year *CY* for policy year *PY*.

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EXPERIENCE
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		Investment	Expe	uses	Loss		A (Nominal	Book	A (DAC	
Time	Premium	Income	Acq. + UW	Not-taken	Adj. Exp	Losses	Reserve)	Profit	Balance)	Profit
0										
1	1,500.0	83.3	-200.0	-50.0	0.0	0.0	1,125.0	208.3	119.4	327.8
2	1,350.0	135.0	-45.0	0.0	-12.5	-100.0	697.5	630.0	-36.1	593.9
ŝ	1,186.7	168.6	-39.6	0.0	-27.7	-221.2	463.2	603.8	-34.8	568.9
4	1,038.3	179.8	-34.6	0.0	-52.5	-420.1	150.4	560.5	-26.3	534.2
5	864.6	160.5	-28.8	0.0	-84.6	-676.7	-242.6	477.5	-9.7	467.8
9	0.0	103.5	0.0	0.0	-85.5	-684.2	-769.8	103.5	-6.5	97.0
7	0.0	59.1	0.0	0.0	-66.5	-532.0	-598.5	59.1	-3.8	55.4
8	0.0	25.9	0.0	0.0	-49.8	-398.5	-448.3	25.9	-1.7	24.2
6	0.0	5.8	0.0	0.0	-30.3	-242.1	-272.3	5.8	-0.4	5.4
10	0.0	0.0	0.0	0.0	-8.6	-69.2	-77.8	0.0	0.0	0.0

		Investment	Expei	uses	Loss		A (Nominal	Book	A (DAC	
Time	Premium	Income	Acq. + UW	Not-taken	Adj. Exp	Losses	Reserve)	Profit	Balance)	Profit
0										
1	1,500.0	83.3	-200.0	-50.0	0.0	0.0	1,125.0	208.3	119.4	327.8
2	1,335.0	134.3	-44.5	0.0	-12.5	-100.0	688.5	623.8	-36.2	587.7
3	1,186.7	168.6	-39.6	0.0	-27.7	-221.2	463.2	603.8	-34.8	568.9
4	1,008.7	178.5	-33.6	0.0	-52.5	-420.1	132.6	548.3	-26.5	521.9
5	864.6	160.5	-28.8	0.0	-84.6	-676.7	-242.6	477.5	-9.7	467.8
9	0.0	103.5	0.0	0.0	-85.5	-684.2	-769.8	103.5	-6.5	97.0
7	0.0	59.1	0.0	0.0	-66.5	-532.0	-598.5	59.1	-3.8	55.4
8	0.0	25.9	0.0	0.0	-49.8	-398.5	-448.3	25.9	-1.7	24.2
6	0.0	5.8	0.0	0.0	-30.3	-242.1	-272.3	5.8	-0.4	5.4
10	0.0	0.0	0.0	0.0	-8.6	-69.2	-77.8	0.0	0.0	0.0

TABLE 43 Actual Experience

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VARIATION DUE TO PAST ACCUMULATED EXPERIENCE

		Investmen		Expe	nses	Loss		Δ (Nominal	Book	A (DAC	
Time	Premium	Income	Acq. + l	MU	Not-taken	Adj. Exp	Losses	Reserve)	Profit	Balance)	Profit
0											
1	0.0	0.0		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	0.0	0.0		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	-13.3	-1.2		0.4	0.0	0.1	0.8	-7.1	-6.1	0.0	-6.1
4	-11.7	-1.6		0.4	0.0	0.2	1.9	-4.8	-5.8	0.0	-5.8
5	-35.4	-4.0		1.2	0.0	0.7	5.3	-15.3	-17.0	-0.1	-17.1
9	0.0	-3.2		0.0	0.0	1.2	9.8	11.0	-3.2	0.2	-3.0
7	0.0	-2.2		0.0	0.0	1.5	12.0	13.5	-2.2	0.1	-2.0
×	0.0	-1.1		0.0	0.0	1.7	13.5	15.2	-1.1	0.1	-1.0
6	0.0	-0.2		0.0	0.0	1.2	9.6	11.2	-0.2	0.0	-0.2
10	0.0	0.0		0.0	0.0	0.4	2.8	3.2	0.0	0.0	0.0

		Profit		0.0	-6.3	0.0	-12.4	0.0	0.0	0.0	0.0	0.0	0.0
	A (DAC	Balance)		0.0	-0.1	0.0	-0.2	0.0	0.0	0.0	0.0	0.0	0.0
	Book	Profit		0.0	-6.2	0.0	-12.2	0.0	0.0	0.0	0.0	0.0	0.0
	A (Nominal	Reserve)		0.0	-9.0	0.0	-17.8	0.0	0.0	0.0	0.0	0.0	0.0
		Losses		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	Loss	Adj. Exp		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	Expenses	Not-taken		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
		Acq. + UW		0.0	0.5	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0
	Investment .	Income		0.0	-0.7	0.0	-1.3	0.0	0.0	0.0	0.0	0.0	0.0
		Premium		0.0	-15.0	0.0	-29.7	0.0	0.0	0.0	0.0	0.0	0.0
		Time	0	1	7	ŝ	4	5	9	7	8	6	10

VARIATION DUE TO CURRENT YEAR EXPERIENCE

**TABLE 45** 

The total calendar year value is the sum over all policy years. We denote this total by omitting the policy year superscript. For example, the DAC balance at the end of calendar year *CY* for all policy years is given by

$$DAC_{CY} = \sum_{PY=1}^{\infty} DAC_{CY}^{PY}.$$

# Premium

For simplicity, premium is assumed to be paid at the beginning of the year, and there are no subsequent audits or retrospective adjustments. Premium<sub>CY</sub> represents the premium collected. If *PY* does not equal *CY*, the premium is zero. When PY = CY, the premium is an input parameter from the pricing actuary.

The total premium for the book of business depends on the number policies in-force in each year. Policies In  $Force_{CY}$  denotes the number of policies in-force *at the beginning* of calendar year *CY*. The total premium at the start of a calendar year equals

# Expenses

Expenses are paid at the start of the year. Expenses are classified as:

- Underwriting and acquisition expenses: Expenses Acq & UW<sup>PY</sup><sub>CY</sub>
- 2. Solicitation costs for not-taken business: Expenses  $NT_{CY}^{PY}$
- 3. Loss adjustment expenses: Expenses Loss  $\operatorname{Adj}_{CY}^{PY}$

Expenses Acq &  $UW_{CY}^{PY}$  and Expenses  $NT_{CY}^{PY}$  are zero if *PY* does not equal *CY*. For simplicity, we have assumed that no losses or loss adjustment expenses are paid until the policy term expires. Expenses Loss  $Adj_{CY}^{PY}$  is zero when PY = CY and non-

zero when CY > PY up to a certain point in time. Once all losses are paid, it is again zero. These quantities are per policy and are an input from the pricing actuary.

Expense  $_{CY}^{PY}$  is the sum of the three expense categories. The total expenses in calendar year CY is

$$\text{Expense}_{CY} = \sum_{PY=1}^{\infty} \text{Expense}_{CY}^{PY} \cdot \text{Policies In Force}_{PY}.$$

Losses

Paid Losses $_{CY}^{PY}$  denotes the amount of losses paid per policy for a given policy year PY and calendar year CY. We sum over all policy years and multiply by the number of policies in-force to get the total losses paid in the calendar year

Paid Losses<sub>CY</sub> = 
$$\sum_{PY=1}^{\infty}$$
 Paid Losses<sub>CY</sub> · Policies In Force<sub>PY</sub>.

# $\Delta$ (Nominal Reserve)

The nominal reserve is the sum of future losses and loss adjustment expenses. For a given policy year PY and calendar year CY the reserve equals

Nominal Reserve<sub>CY</sub><sup>PY</sup> =  $\sum_{i=CY+1}^{\infty}$  (Paid Losses<sub>i</sub><sup>PY</sup> + Expense Re<sub>i</sub><sup>PY</sup>)

· Policies In Force<sub>PY</sub>.

The total reserve for calendar year *CY* is the sum over all policy years:

Nominal Reserve<sub>CY</sub> = 
$$\sum_{PY=1}^{\infty}$$
 Nominal Reserve<sub>CY</sub>.

The change in the nominal reserve equals

 $\Delta$ (Nominal Reserve<sub>CY</sub>) = Nominal Reserve<sub>CY</sub>

- Nominal Reserve<sub>CY-1</sub>.

# Investment Income

The investment income is the product of the investable assets at the start of the calendar year and the investment yield. The investable assets at the start of the year is the nominal reserve at the end of the year discounted to the beginning of the year.<sup>58</sup> This assumption is consistent with traditional source of earnings exhibits for permanent life insurance products. For propertycasualty products, a more refined calculation based on the loss payment pattern would be used in practice. The investment income is

Investment Income<sub>CY</sub><sup>PY</sup> = Nominal Reserve<sub>CY</sub><sup>PY</sup> · Discount Factor<sub>CY</sub> · Investment Return<sub>CY</sub>.

The total investment income for calendar year CY is the sum over all policy years

Investment Income<sub>CY</sub> = 
$$\sum_{PY=1}^{\infty}$$
 Investment Income<sub>CY</sub>.

Book Profit

The book profit for a given calendar year CY equals

Book  $\operatorname{Profit}_{CY} = \operatorname{Premium}_{CY} + \operatorname{Investment Income}_{CY}$ +  $\operatorname{Expense}_{CY} + \operatorname{Paid Losses}_{CY}$ -  $\Delta(\operatorname{Nominal Reserve}_{CY}).$ 

 $\Delta$  (DAC Balance)

The deferred acquisition cost (DAC) balance is calculated for each policy year.  $DAC_{CY}^{PY}$  denotes the balance at the end of calen-

<sup>&</sup>lt;sup>58</sup>For simplicity, we do not estimate the amount of non-investable assets supporting the nominal reserves. For workers compensation, these include premiums receivable, expected audits, accrued retrospective premiums, and deferred tax assets. For most companies, the amounts are material.

dar year *CY* for policy year *PY*. The DAC balance for the entire book of business is the sum over all policy years

$$DAC_{CY} = \sum_{PY=1}^{\infty} DAC_{CY}^{PY}.$$

The recursive formula for  $DAC_{PY+i}^{PY}$  is given by

$$DAC_{PY+i}^{PY} = (DAC_{PY+i-1}^{PY} + Deferrable Expense_{PY+i}^{PY})$$
$$\cdot (1 + Interest Rate_{PY+i}) - k \cdot Book Profit_{PY+i}^{PY}$$

for *i* greater than or equal to zero. We define  $DAC_{PY-1}^{PY} = 0$ . The change in DAC is

$$\Delta(\text{DAC}_{CY}) = \text{DAC}_{CY} - \text{DAC}_{CY-1}.$$

Profit

The net profit for the book of business takes into account the amortization of the deferred acquisition cost asset. It is given by

$$Profit_{CY}^{PY} = Book Profit_{CY}^{PY} + \Delta(DAC_{CY}^{PY})$$

and

$$\operatorname{Profit}_{CY} = \sum_{PY=1}^{\infty} \operatorname{Profit}_{CY}^{PY}.$$

# DISTRIBUTION-BASED PRICING FORMULAS ARE NOT ARBITRAGE-FREE

#### DAVID L. RUHM

# Abstract

A number of actuarial risk-pricing methods calculate risk-adjusted price from the probability distribution of future outcomes. Such methods implicitly assume that the probability distribution of outcomes contains enough information to determine an economically accurate risk adjustment.

In this paper, it will be shown that distinct risks having identical distributions of outcomes generally have different arbitrage-free prices. This is true even when the outcomes are completely determined by the same underlying contingent events. Risk-load formulas that use only the risk's outcome distribution cannot produce arbitragefree prices and, in that sense, are not economically accurate for risks traded in markets where arbitrage is possible. In practice, most insurance underwriting risks are not traded in such markets. Distribution-based pricing usually does not carry a direct arbitrage penalty for insurance and can reflect an insurer's risk preferences.

A ratio is used to measure the implicit discount or surcharge for risk that is present in a price: the ratio of price density to discounted probability density. This ratio can be used to identify the qualitative nature of a risk as investment or insurance: a risk discount factor less than unity indicates investment, whereas a risk surcharge factor above unity indicates insurance.

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# 1. INTRODUCTION

Risk, in a financial context, can be considered exposure to potential financial loss. Pricing for risk is a central problem in casualty actuarial science. Casualty actuaries have developed several mathematical pricing methods intended to compensate for risk equitably and adequately.

Recently, a number of authors have sought to make actuarial pricing methods consistent with Black-Scholes options pricing theory, or (more generally) arbitrage-free pricing theory, notably Wang [4] and Venter [3]. This is an appealing goal, since such methods would price a variety of risks in a consistent, "universal" way, regardless of whether the risk arose from insurance or from a financial market. For both insurance risk and capital risk, such methods would produce the correct charge, based on the philosophy that "risk is risk," irrespective of context.

A second benefit of actuarial pricing that is consistent with arbitrage-free pricing is integrity. In an ideal market, an option's price would have to be the arbitrage-free price, otherwise an arbitrage opportunity would exist. In such a market, arbitrage opportunities would not exist for any appreciable length of time, so market forces would actually drive the market price to the arbitrage-free price. This lends an integrity to arbitrage-free pricing that is not found in other risk-pricing methods. In this sense, arbitrage-free pricing is the "natural" risk-adjusted price. Furthermore, no additional assumptions about the cost of risk are needed, other than the market's implicit pricing of risk in the security.

Several actuarial methods for pricing risk use only the probability distribution of the risk's economic outcomes to determine the risk charge. In this paper, the Black-Scholes options pricing formula is used to price some derivatives that have simple outcome distributions. A surprising result of the analysis is that two risks with identical outcome distributions generally have different Black-Scholes prices, even when they are both derivatives of the same security. The reasons that underlie this phenomenon are discussed. The result is shown to be true in general: arbitragefree pricing cannot be produced by any formula that uses only the distribution of economic outcomes.

# 2. DEFINITIONS AND TERMINOLOGY

Throughout this paper, the term "security" will be used to refer to a hypothetical ideal stock that satisfies the hypotheses of the Black-Scholes model. This security will be the basis for most of the theoretical development that follows.

Also, the term "derivative" refers to any financial instrument whose value at some fixed time in the future is a function of the security's price at that time. The term "option" refers to a European call or put option. All options discussed in this paper have the same expiration date, which can be any date. "Price," as applied to an option or portfolio of options, refers to the Black-Scholes formula price.

The variable *p* represents the current price of the security, and *X* denotes the future price of the security on the options expiration date. From the Black-Scholes hypotheses, *X* is a lognormal random variable. The positive number line  $(0, \infty)$  contains *X* and can be thought of as the space of possible expiration prices.

To facilitate the analysis, special derivatives will be constructed from call options. These derivatives, which will be referred to as "binary risks," are worth one unit at expiration if Xis in a specified price range and zero if X is outside of the range. Note that the expected value of a binary risk at expiration equals the probability that X will be within the specified price range.

### 3. BACKGROUND

Define R, the return on the security at the options expiration date, by the formula:

$$R = (X/p) - 1.$$

Then, since X is lognormal and p is a constant, (1 + R) also has a lognormal distribution. A lognormal variable can be parameterized in terms of its expectation and sigma parameter, so the distribution of R is completely determined by the expected return E = E[R], and the return volatility  $\sigma$ :

$$\ln(1+R) \sim N(\mu_R, \sigma^2), \quad \text{and}$$
$$\mu_R = \ln(1+E) - \sigma^2/2.$$

Note the absence of a time variable. Many options pricing formulas define the volatility parameter with respect to an annual time horizon, and then apply a square-root time factor to adjust for the time to expiration. In this formulation, the time factor is implicitly included within the volatility parameter for notational convenience.

The distribution of *X* can be expressed in terms of current price, expected return, and volatility of return:

$$X = p(1 + R),$$
  

$$\ln(X) = \ln(p) + \ln(1 + R), \text{ and}$$
  

$$\ln(X) \sim N(\mu, \sigma^2), \text{ where}$$
  

$$\mu = \ln(p) + \mu_R = \ln(p) + \ln(1 + E) - \sigma^2/2$$

The Black-Scholes price of an option is equal to the option's discounted expected value, under a risk-neutral lognormal density function that is parameterized by  $\mu^*$  and  $\sigma^*$ :

$$\label{eq:multiplicative} \begin{split} \mu^* &= \mu - \ln[(1+E)/(1+r)], \qquad \text{and} \\ \sigma^* &= \sigma, \end{split}$$

where  $\mu$  and  $\sigma$  are the parameters given above in the probability density function of X, E is defined as above, and r is the risk-free return for the time period to expiration. This risk-neutral density will be referred to as the "price density" function. Arbitrage-free pricing implies price additivity, which means the price density function can be used to price derivatives that are equivalent to combinations of options.

Using the formula given above for  $\mu$ , the risk-neutral distribution's  $\mu^*$  parameter is given by:

$$\mu^* = \ln(p) + \ln(1+r) - \sigma^2/2.$$

The graph of a call option's value at expiration is shown in Exhibit 1. The call's value is zero when the security's expiration price is below the strike price. Above the strike, the call's value increases linearly with a slope of one.

By buying one call and selling another call, an investor can create what is commonly known as a "spread." For example, the investor could buy a call option with a December expiration and a strike price of 50 and sell a call option with a December expiration date is the same for both calls, and the purchased call has a lower strike price than the sold call. The graph of this spread's value at expiration prices at or below 50, then increases dollar-for-dollar from 50 to 60, and finally remains constant at 10 for expiration prices above 60. With a few arithmetic calculations, the reader can verify that the graph accurately represents the spread's value at expiration as a function of X, the security's expiration price.

This spread would commonly be referred to as a "bull spread," because the spread's value at expiration is positively related to the underlying security's price. Bull spreads can be constructed from either call or put options; a consequence of arbitrage-free pricing is that the price of the spread is the same under either construction. A bull spread is a combination of options (one long
and one short), so its price is found by calculating its discounted expected value at expiration using the price density function.

#### 4. THEORETICAL DEVELOPMENT

In the following sections, pricing theory is developed for particular derivatives. This will provide the basis for comparisons between derivatives and games of chance.

#### 4.1. Rays

As defined earlier, X represents the future price of the security on a specific options expiration date. For the purpose of the construction that follows, it is assumed that call and put options are available for all strike prices and in any amount, including fractional amounts. Let A(s,e) denote a position consisting of (1/e) bull spreads from expiration prices s to (s + e). For example, A(50, 10) represents 0.10 bull spreads on the expiration price range [50,60]. Then the value at expiration of A(s,e) is:

$$Value[A(s,e)] = \begin{cases} 0, & \text{when } X < s; \\ (X-s)/e, & \text{when } X \in [s,s+e]; \\ 1, & \text{when } X > s+e. \end{cases}$$

As the variable *e* approaches zero, A(s,e) converges pointwise to a limiting bull spread, denoted by  $A^*(s)$ , which has a binary payoff at expiration:

$$Value[A^*(s)] = \begin{cases} 0, & \text{when } X \leq s, & \text{and} \\ 1, & \text{when } X > s. \end{cases}$$

In other words,  $A^*(s)$  pays one unit if  $X \in (s, \infty)$ , and zero otherwise. In this way,  $A^*(s)$  can be viewed as corresponding to the set  $(s, \infty)$ . The graph of  $(s, \infty)$  on a number line is a ray with open endpoint at *s*, extending to the right. In this paper, the limiting bull spread  $A^*(s)$  will be referred to as a "ray," after its geometric representation.

A ray can be thought of as a gamble.  $A^*(s)$  is effectively a bet on whether the future expiration price will be higher than the strike price, X > s. If  $X \le s$ , then  $A^*(s)$  is worth nothing and the purchase price is lost, just as a wager can be lost in a bet. The price of  $A^*(s)$  is the amount wagered. If X > s, then  $A^*(s)$  is worth one unit, which has present value v = 1/(1 + r). This amount can be decomposed into two parts: the return of the purchase price (the wager), plus a "payoff" of v minus the purchase price:

$$v =$$
 (return of purchase price) + (payoff of  $v -$  purchase price).

This wager perspective makes it possible to compare probabilities and payoffs for rays with those for games of chance, which creates some surprising results discussed below.

 $A^*(s)$  is worth either 0 or 1 at expiration, so it is a binary risk. The expected value of  $A^*(s)$  at expiration equals the probability that X > s (the "Value" operator on  $A^*(s)$  is omitted for notational convenience):

$$\mathbf{E}[A^*(s)] = \mathbf{P}(X > s).$$

Since *X* is lognormal, the "payoff probability" P(X > s) can be expressed in terms of the cumulative normal distribution function:

$$P(X > s) = P(\ln(X) > \ln(s))$$
  
= 1 -  $\Phi[(\ln(s) - \mu)/\sigma] = \Phi[(\mu - \ln(s))/\sigma].$ 

The  $\mu$  parameter was given previously in the density function for *X*:

$$\mu = \ln(p) + \ln(1+E) - \sigma^2/2.$$

Substituting yields the formula for payoff probability and  $E[A^*(s)]$ :

$$E[A^*(s)] = P(X > s) = \Phi[\ln(p(1 + E)/s)/\sigma - \sigma/2].$$

As discussed above, the price of  $A^*(s)$  equals the discounted expected value under the price density function. The price density has the same formula as the probability density with the  $\mu^*$  parameter in place of  $\mu$ . With v = 1/(1 + r), the risk-free discount factor:

Price[
$$A^*(s)$$
] =  $v\Phi[(\mu^* - \ln(s))/\sigma]$ ,  
 $\mu^* = \ln(p) + \ln(1+r) - \sigma^2/2$ , and  
Price[ $A^*(s)$ ] =  $v\Phi[\ln(p(1+r)/s)/\sigma - \sigma/2]$ .

An example will illustrate how these formulas can be used. Suppose the following:

p = Current price of underlying security = 100, R = Expected return on underlying security = 10%,  $\sigma$  = Volatility of return = 30%, r = Risk-free rate = 4%,  $v = 1/(1 + r) = (1.04)^{-1}$ , and s = Strike price = 120.

Then, the price of  $A^*(120)$  and the payoff probability can be calculated:

```
Price = v\Phi[\ln(p(1+r)/s)/\sigma - \sigma/2];

Price = (1.04)^{-1}\Phi[\ln(104/120)/0.30 - 0.15];

Price = 0.2551.

Probability = \Phi[\ln(p(1+E)/s)/\sigma - \sigma/2];

Probability = \Phi[\ln(110/120)/0.30 - 0.15];

Probability = 0.3300.
```

 $A^*(120)$  has about a 1/3 chance of paying one unit, and about a 2/3 chance of zero payment. Suppose a gambler purchases 100 units of  $A^*(120)$  and views it as a bet. Then the amount wagered is \$25.51 (the purchase price, which is the amount placed at risk), the odds of winning are 33%, and the payoff at present value is:

Payoff = \$100(v) - Wager = \$96.15 - \$25.51 = \$70.64.

This is a gamble with better-than-breakeven prospects. The amount of the advantage can be quantified by calculating expected return. Since the value of  $A^*(120)$  at expiration is one or zero, the expected return equals the ratio of the probability to the price, minus unity:

Expected Return[ $A^*(120)$ ]

= [(\$100)(.3300) + (\$0)(.6700)]/\$25.51 - 100%;

Expected Return[ $A^*(120)$ ]

= 0.3300/0.2551 - 100% = 29.33%.

An interpretation of this result is that the  $A^*(120)$  derivative is riskier than the underlying security, and therefore commands a higher expected return: 29.33% versus 10%.

The high expected return for  $A^*(120)$  implies that the price of  $A^*(120)$  contains a large discount beyond risk-free discounting. The implicit discount factor is the price divided by the risk-freediscounted expected value (which equals the discounted probability):

> Discount Factor = Price/Discounted Probability = 0.2551/0.3300v, Discount Factor = 80.41%.

This factor can be interpreted as a "risk discount" within the price of 80.41%. For an even gamble with no statistical advantage, the price would be equal to the discounted expected value of the payoff, so that the expected net outcome at present value would be zero. This 80.41% risk discount factor is the extent to which  $A^*(120)$  deviates from an even gamble—the derivative  $A^*(120)$  sells for 80.41% of the even-gamble price.

As it turns out,  $A^*(s)$  is never an even gamble, except for one particular value of *s*. By calculating prices and probabilities for various values of *s*, one finds a wide range of gambles, with both positive and negative expected net outcomes.

The risk discount factor is inversely proportional to the expected return:

Risk Discount Factor = (1 + r)/(1 + Expected Return).

The right-hand-side expression is the inverse of the risk premium in the derivative's expected return, expressed as a ratio.

The risk discount factor will be used frequently in the discussion below. For convenience, it will be denoted by *w*:

w =Risk Discount Factor

= Price/Discounted Expected Value at Expiration.

When applied to binary risks, this reduces to:

w = Price/Discounted Payoff Probability.

#### 4.2. Segments

Just as  $A^*(s)$  represents a derivative that corresponds to a geometric ray, we can construct a derivative that corresponds to a line segment. As before, let A(s,e) represent a portfolio consisting of (1/e) bull spreads from *s* to (s + e). Then, define a "segment" derivative D(s,t) as the limit of a long position in A(s,e) and a short position in A(t,e), where s < t, as *e* approaches zero. Informally, a segment is the difference of two rays:

$$D(s,t) = A^*(s) - A^*(t)$$
, when  $s < t$ .

It is easily verified that the value at expiration of a segment is binary:

Value[
$$D(s,t)$$
] =   

$$\begin{cases}
1, & \text{when } X \in (s,t], & \text{and} \\
0, & \text{when } X \notin (s,t].
\end{cases}$$

In other words, D(s,t) pays one unit if the expiration price of the security is contained in the segment (s,t], and zero otherwise. Geometrically, D(s,t) corresponds to the half-open line segment

(s,t]. Like a ray, a segment is a binary risk with expected value equal to its probability.

Just as the ray  $A^*(s)$  can be thought of as a bet on the set of events X > s, the segment D(s,t) is effectively a bet on  $X \in (s,t]$ . If  $X \le s$  or X > t, then D(s,t) is worth zero and the purchase price is lost. If  $X \in (s,t]$  then D(s,t) is worth one unit at expiration.

Each segment has a payoff probability and a price. In the section above, the price and probability were calculated for the ray  $A^*(120)$ :

Price
$$[A^*(120)] = 0.2551$$
, and  
Probability $[A^*(120)] = 0.3300$ .

These values implied a risk discount factor for the ray:

 $w[A^*(120)] = Price/Discounted Probability = 0.2551/0.3300v;$  $w[A^*(120)] = 80.41\%.$ 

Continuing with this example, we can calculate the price and probability for the ray  $A^*(150)$ , and then subtract the results to calculate values for the segment D(120, 150):

Price[ $A^*(150)$ ] =  $v\Phi[\ln(p(1+r)/s)/\sigma - \sigma/2]$ ; Price[ $A^*(150)$ ] =  $(1.04)^{-1}\Phi[\ln(104/150)/0.30 - 0.15]$ ; Price[ $A^*(150)$ ] = 0.0819. Probability[ $A^*(150)$ ] =  $\Phi[\ln(p(1+E)/s)/\sigma - \sigma/2]$ ; Probability[ $A^*(150)$ ] =  $\Phi[\ln(110/150)/0.30 - 0.15]$ ; Probability[ $A^*(150)$ ] = 0.1182.

Values for the segment can now be found by subtracting values for the rays:

Price[D(120, 150)] = 0.2551 - 0.0819 = 0.1732;Probability[D(120, 150)] = 0.3300 - 0.1182 = 0.2117. D(120, 150) has about a 21% chance of paying one unit, and about a 79% chance of expiring worthless. If a gambler purchased 100 units of D(120, 150) and viewed it as a bet, the amount wagered would be \$17.32 (the purchase price), the odds of winning would be 21.17%, and the payoff at present value would be:

Payoff = \$100(v) - Wager = \$96.15 - \$17.32 = \$78.83.

In comparison with  $A^*(120)$ , this gamble has lower odds of winning, but also has a lower wagered amount and a higher payoff.

As with a ray, the segment's statistical advantage can be quantified by calculating expected return. Since the expected value at expiration equals the payoff probability:

Expected Return[D(120, 150)] = 0.2117/0.1732 – 1 = 22.25%.

The risk discount factor for the segment can also be calculated:

$$w[D(120, 150)] = 0.1732/0.2117v = 85.07\%.$$

This segment's calculated price is 85.07% of the price that would offer an even gamble. It is not as strongly discounted as the ray  $A^*(120)$ , so it has a lower expected return.

## 5. CONSTRUCTING A ROULETTE WHEEL FROM SEGMENTS AND RAYS

Using the pricing theory developed in the previous section, it is now possible to directly compare the performance of derivatives with the results from a roulette gamble.

#### 5.1. The Game of Roulette

Roulette is a casino game that uses a wheel with a ring around its perimeter. The ring is evenly divided into 38 spaces, numbered "00" and 0 through 36. Players can bet on any numbered space except 0 and 00, or on several spaces at once. After bets are placed, the wheel is spun and a winning number is determined. The payoff for betting on the winning number is 35:1, meaning that the bettor's wager is returned plus a payoff from the casino of 35 times the wager. Wagers on losing numbers are lost to the casino.

Roulette does not offer an even gamble—the casino has a constant advantage. For the gambler, \$1 bet on a number returns \$36 total in the event of a win, which has probability 1/38, or zero in the event of a loss, which has probability 37/38. The gambler's expected net outcome is therefore -2/38:

Expected Net Outcome = E[Outcome] – Wager  
= 
$$36(1/38) + 0(37/38) - 1 = -2/38$$

The term "binary risk" was defined earlier as a financial instrument that pays either zero or one at expiration. Rays and segments were shown to be examples of binary risks. A roulette wager of 1/36 on a numbered space is also a binary risk. The player is effectively paying the casino a "price" of 1/36 in the form of a bet, and receives either zero if the wager is lost, or one (return of amount bet, plus payoff) if the wager is won.

The "expiration date" for a roulette spin is the time at which the outcome is finalized, the moment at which the wheel's spin is completed and the winning number is determined. Since this occurs only seconds after the wager, a time value factor of one can be used with no significant loss of accuracy.

The risk discount factor for a roulette gamble from the gambler's perspective can be calculated:

Risk Discount Factor = Price/Discounted Probability

= (1/36)/(1/38) = 105.56%.

Since this factor is greater than one, it is actually not a discount but rather a 5.56% surcharge to the gambler for the entertainment value of assuming the gambling risk.

The risk discount factor can also be calculated from the casino's perspective. The "price" is the amount that the casino might have to pay the gambler, which is 35/36. The probability of the casino winning is 37/38. When the gambler bets 1/36, it is the same as if the casino pays the gambler 35/36 up front, spins the wheel, and collects 1 from the gambler if he loses, or nothing if he wins. The casino's "price" is thus 35/36 and the success probability is 37/38. Then,

Risk Discount Factor = Price/Discounted Probability

$$=(35/36)/(37/38)=99.85\%.$$

This discount represents a small edge for the casino, but more than sufficient given the frequency of play. It is interesting that the casino's discount is far smaller than the gambler's surcharge, even though these are just opposite sides of the same gamble. The reason is that the casino's expected return on the amount it places at risk is small but positive; the gambler's expected loss is a much larger percentage of the amount risked.

All numbered spaces on the wheel have the same probability (1/38), and the same payoff ratio (35:1). In regard to probability and payoff the spaces are completely identical to each other. The bettor's expected net outcome has the same value (-2/38) for every space; the gambler's prospects are the same no matter which space he chooses to bet.

## 5.2. Mapping Expiration Prices onto the Roulette Wheel

Recall that a ray  $A^*(s)$  is equivalent to a bet on whether  $X \in (s, \infty)$ , and a segment D(s,t) is equivalent to a bet on whether  $X \in (s,t]$ . As such, rays and segments are similar to bets on roulette numbers, just with different probabilities and payoffs. A

correspondence between these derivatives and the roulette wheel can be constructed as follows:

The space of all possible expiration prices is the positive number line  $(0, \infty)$ . Segments and rays will be referred to collectively as "sections." Partition this space into 38 sections, consisting of 37 segments and 1 ray:

$$(0, s_1] (s_1, s_2] (s_2, s_3] ... (s_{37}, +\infty),$$

where the  $s_n$  are specifically chosen so that each section has a 1/38 probability of containing the expiration price for the security. Then, each of the 38 sections has the same probability distribution as a space on a roulette wheel: 1/38 probability of payoff, 37/38 probability of no payoff. Taken as a group, these sections cover the entire space of expiration prices  $(0,\infty)$  with no overlap, like the 38 spaces that cover the roulette wheel. The space of expiration prices  $(0,\infty)$  can thus be viewed as a roulette wheel, divided into these 38 section-spaces. Just as a roulette spin produces a single winning number, exactly one of these sections will contain the expiration price and will have a value of one on the expiration date. In summary, purchasing one of these section derivatives is almost exactly like making a bet on a roulette number.

The major difference between this roulette-like partitioning of  $(0, \infty)$  and an actual roulette wheel is the payoff ratio. A roulette bet pays 35:1 regardless of which space is selected. A bet of 1/36 produces a payoff of 1 for winning. The 38 sections on the space  $(0, \infty)$  have equal probabilities (1/38) and equal payoffs (1 unit), but they have varying prices. This means that one can "bet" at a discount or a surcharge, depending on which "space" one selects.

To demonstrate this tangibly, let's consider the example used earlier. As before, the parameters are:

p = Current price of underlying security = 100, R = Expected return on underlying security = 10%,  $\sigma$  = Volatility of return = 30%, r = Risk-free rate = 4%, and  $v = 1/(1 + r) = (1.04)^{-1}$ .

Exhibit 3 shows roulette spaces mapped to sections on  $(0, \infty)$  for this example. The mapping starts with the "00" space, which corresponds to the segment D(0,58.80). Next is the "0" space representing the adjacent segment D(58.80,64.68), and so on up to the "36" space representing the ray  $A^*(188.08)$ .

Probabilities and prices for each space are shown in the exhibit. These are calculated by using the probability and price formulas for rays and segments derived above. Each space has the same 1/38 probability of a one-unit payoff at expiration. The prices vary by space, decreasing for the higher numbered spaces that correspond to higher prices for the security. Expected returns and corresponding risk factors (both discounts < 100% and surcharges > 100%) are also shown.

As the exhibit indicates, some section-spaces are more favorable than others. On a wheel of derivatives such as this, each number has a different payoff ratio even though all numbers are equally likely. The right-hand column, "Ratio to Roulette Payoff," shows what the payoffs are compared to a real roulette wheel. For example, the "00" space has an equivalent roulette payoff of 69%, meaning that buying the "00" segment is like placing a bet on a roulette number and receiving just 69% of the usual payoff in the event of a win. The "36" space is the best choice on this wheel of derivatives, since it pays 165% of the standard roulette payoff if you win. Two of the spaces are of particular interest. Space "12" has a risk factor of 105.33%, which is approximately equal to the 105.56% factor for an actual roulette wager. Purchasing the "12" segment D(93.08,95.08) is about equivalent to placing an actual roulette bet. This is also shown in the "Ratio to Roulette Payoff" column, where a value of 100% appears for space 12.

The other interesting space is "16," which is the segment D(101.07, 103.10). The risk factor for "16" is 100.11%, meaning that this segment is approximately an even gamble. Note that all spaces numbered less than "16" are disadvantageous, while those numbered higher than "16" offer an advantage. Space "16" is located just above the current security price (100) on a nominal basis, but since strike prices represent future values at expiration, it is actually just below the current price at future value (104).

In summary, all 38 sections are identically distributed, and their outcomes are determined by the same underlying event, but they have different risk factors. Higher spaces have stronger risk discounts, meaning that there is a positive risk load paid to the purchaser for accepting the distribution of outcomes (the "risk"). Lower spaces have risk surcharges, meaning that the purchaser actually pays a charge to assume the risk.

# 6. PROBABILITY DISTRIBUTION, RISK LOAD, AND EXPECTED RETURN

The results of the roulette mapping are somewhat counterintuitive, but some conclusions can be drawn about distributions, risk load, and return from the roulette analysis.

# 6.1. Probability Distribution Does Not Imply Risk Load in an Arbitrage-Free Environment

The surprising result of the roulette wheel mapping is that the 38 risks have different risk loads under arbitrage-free Black-Scholes pricing, even though their distributions are identical and their outcomes are completely determined by the same underlying event (the expiration price of the underlying security). It is therefore not possible to price these risks with any pricing formula that uses only the probability distribution of outcomes. This result can be generalized to other derivatives of a security in an arbitrage-free market. The principal result is that, if the risk load for any of a security's derivatives can be calculated from the probability distribution of the derivative's outcomes, then it must be the trivial risk load and the price must be the derivative's discounted expected value. The formal reasoning is as follows.

Let *F* be the cumulative distribution function of the underlying security at expiration, with F(x) = 0 for  $x \le 0$  and F(x) smooth on  $[0,\infty)$ . For  $E \subset [0,\infty)$  define  $\mu(E) = \int_E dF(x)$ . Given any measurable function  $g: [0,\infty) \to (-\infty,\infty)$ , let p(g) = the price of the derivative with payoff function *g*. We assume that  $p(\bullet)$  is a linear operator, and that  $h(x) \ge j(x)$  for all *x* implies  $p(h) \ge p(j)$  (the arbitrage-free condition). Let v = p(h), where h = 1 on  $[0,\infty)$ . (The constant *v* represents the present value of one unit certain at expiration.) If p(g) is completely determined by the probability distribution of the payoff values  $\{g(x)\}$  for any *g*, then  $p(g) = v \int g dF$ .

The proof is in three parts:

A) If  $g(x) = X_E(x)$ , where  $E \subset [0, \infty)$  is measurable and  $X_E$  is the indicator for *E*, then  $p(g) = v\mu(E) \equiv v \int g \, dF$ .

B) If g(x) is a linear combination of indicator functions (i.e., if g is a simple function), then  $p(g) = v \int g dF$ .

C)  $p(g) = v \int g \, dF$  for measurable g(x).

*Proof of "A"* First, the result is shown for  $\mu(E) = 2^{-n}$ ,  $n \ge 0$ . If n = 0, then by the definition of v,  $p(g) = v = v\mu(E)$ . By induction on n, assume that the result is true for n = k. If  $\mu(E) = 2^{-(k+1)}$  then  $\mu(E^c) = 1 - 2^{-(k+1)} \ge 2^{-(k+1)}$ . Since F is smooth and therefore continuous, there exists x such that  $\mu(E^c \cap [0, x)) = 2^{-(k+1)}$ (with  $x = \infty$  if k = 0). Let  $E_x = E^c \cap [0, x)$ . Then,  $\mu(E) = 2^{-(k+1)}$ and  $\mu(E \cup E_x) = 2^{-k}$ , since *E* and  $E_x$  are disjoint. Since the values of  $X_E$  and  $X_{E_x}$  have the same distribution function (0 with probability  $1 - 2^{-(k+1)}$  and 1 with probability  $2^{-(k+1)}$ ),  $p(X_E) = p(X_{E_x})$ . Let  $G = E \cup E_x$ . Then, by linearity of  $p(\bullet)$ ,  $p(X_G) = p(X_E) + p(X_{E_x}) = 2p(X_E)$ . By the induction hypothesis,  $p(X_G) = v2^{-k}$ , so  $p(X_E) = (1/2)(v2^{-k}) = v2^{-(k+1)} = v\mu(E)$ , completing the induction.

Next, if  $\mu(E) = 0$ , then letting  $F = E^c$ ,  $p(X_E) + p(X_F) = p(X_E + X_F) = p(X_{E \cup F}) = v$ . Since  $\mu(F) = 1 - \mu(E) = 1$ ,  $p(X_F) = v$  and  $p(X_E) = 0 = v\mu(E)$ .

If  $0 < \mu(E) < 1$ , then for any  $\varepsilon > 0$  there exist positive integers k,n such that  $2^{-n} < \varepsilon/\nu$  and  $k2^{-n} \le \mu(E) < (k+1)2^{-n} < 1$ . By continuity of F(x), there exist  $\{x_i\}, 1 \le i \le k$ , such that  $\mu([0, x_i) \cap$ E) =  $i2^{-n}$  (with  $x_k = \infty$  if  $\mu(E) = k2^{-n}$ ), and y such that  $\mu([0, y) \cap$  $E^{c}$ ) =  $(k + 1)2^{-n} - \mu(E)$ . Defining  $x_0 = 0$ , let  $D_{i,\varepsilon} = [x_{i-1}, x_i) \cap E$ for each *i*. Then, the  $D_{i,\varepsilon}$  are mutually disjoint subsets of *E*, with  $\mu(D_{i,\varepsilon}) = 2^{-n}$  for all *i*. Define  $D_{\varepsilon} = \bigcup D_{i,\varepsilon}$  and let  $F_{\varepsilon} = (E \setminus D_{\varepsilon}) \cup$  $([0,y) \cap E^c)$ . Then,  $\mu(D_s) = k2^{-n}$ ,  $\mu(F_s) = (\mu(E) - k2^{-n}) + (k + 1)^{-n}$  $1)2^{-n} - \mu(E) = 2^{-n}$ , and  $D_{\varepsilon} \cap F_{\varepsilon} = \emptyset$ . Therefore,  $\mu(D_{\varepsilon} \cup F_{\varepsilon}) = \emptyset$  $\mu(D_{\varepsilon}) + \mu(F_{\varepsilon}) = (k+1)2^{-n}$ . Also,  $D_{\varepsilon} \subseteq E \subset (D_{\varepsilon} \cup F_{\varepsilon})$ , so by inclusion,  $X_{D_{c}}(x) \leq X_{E}(x) \leq X_{(D_{c} \cup F_{c})}(x)$  for all x. Because this price functional is arbitrage-free, we also know that  $p(X_{D_{e}}) \leq p(X_{E})$  $\leq p(X_{(D_{\varepsilon} \cup F_{\varepsilon})}) \text{ or } vk2^{-n} \leq p(X_E) \leq v(k+1)2^{-n}. \text{ From above,} \\ k2^{-n} \leq \mu(E) < (k+1)2^{-n} \text{ and } vk2^{-n} \leq v\mu(E) < v(k+1)2^{-n}, \end{cases}$ so  $|p(X_E) - v\mu(E)| \le v2^{-n}$ , the length of the interval containing both quantities. As  $2^{-n} < \varepsilon/\nu$ ,  $|p(X_E) - \nu\mu(E)| < \varepsilon$ , and since  $\varepsilon$ is arbitrarily small,  $p(X_E) = v\mu(E) \equiv v \int g \, dF$ .

*Proof of "B"* This follows immediately from "A," as  $p(\bullet)$  is a linear operator.

*Proof of "C"* For any  $\varepsilon > 0$ , let  $h_{\varepsilon}(x)$  be a simple function such that  $|g(x) - h_{\varepsilon}(x)| < \varepsilon$  for all x. (For example, let

$$\begin{split} h_{\varepsilon}(x) &= \sum_{i \in \mathbb{Z}} (i\varepsilon) X_{i,\varepsilon}(x), \text{ where } X_{i,\varepsilon}(x) \text{ is the indicator function for the set } g^{-1}\{[i\varepsilon,(i+1)\varepsilon)\}.) \text{ Then, } -\varepsilon < g(x) - h_{\varepsilon}(x) < \varepsilon \text{ for all } x. \\ \text{By the absence of arbitrage, } p(-\varepsilon) \leq p(g-h_{\varepsilon}) \leq p(\varepsilon) \text{ and, equivalently, } -v\varepsilon \leq p(g) - p(h_{\varepsilon}) \leq v\varepsilon. \\ \text{Therefore, } |p(g) - p(h_{\varepsilon})| \leq v\varepsilon. \\ \text{As } h_{\varepsilon} \text{ is simple, } p(h_{\varepsilon}) = v \int h_{\varepsilon} dF, \text{ so } |p(g) - v \int h_{\varepsilon} dF| \leq v\varepsilon. \\ \text{Also, } |v \int g \, dF - v \int h_{\varepsilon} dF| = |v \int (g-h_{\varepsilon}) dF| \leq v \int |(g-h_{\varepsilon})| dF \leq v \int \varepsilon \, dF = v\varepsilon. \\ \text{Applying the triangle inequality, } |p(g) - v \int g \, dF| = |[p(g) - v \int h_{\varepsilon} dF] - [v \int g \, dF - v \int h_{\varepsilon} dF]| \leq 2v\varepsilon. \\ \text{Since } \varepsilon \text{ is arbitrary, } p(g) = v \int g \, dF. \end{split}$$

This proof relies on the continuity of F(x). A counterexample for discontinuous F(x) is as follows: F(x) = 0 on  $(-\infty, 1)$ , F(x) = 0.2 on [1,2), F(x) = 1.0 on  $[2,\infty)$ . In other words, the underlying security's future value is 1 with probability 20% and 2 with probability 80%. Then, suppose v = 1,  $p(X_{\{1\}}) = 0.30$  and  $p(X_{\{2\}}) = 0.70$ . For this example, the price-integral equality does not hold; yet the probability distribution of any derivative's outcomes uniquely determines its price, and there is no arbitrage possibility.

In general, methods that calculate risk loads based only on the probability distribution of outcomes will produce prices that are not arbitrage-free. Offering such prices could produce economically disadvantageous transactions in an ideal arbitrage-free market, through a process akin to adverse selection.

# 6.2. The Insurance Risk Load in the Form of Negative Expected Real Return

In the "roulette wheel" construction above, the surcharges are most punitive for the bets on low expiration prices, and the most advantageous discounts are available for bets on high expiration prices. It can be shown that this is the case for any security with positive expected real return, and it applies to options on stocks as well: high-strike calls sell at a discount to expected present value, while low-strike puts sell at a surcharge to expected present value. This suggests a question: Why would anyone purchase one of the lower-numbered spaces or lower-strike put options that offers a negative expected real return?

High-numbered spaces and high-strike calls, which have strong risk discounts and corresponding high expected returns, are speculative purchases. With their high expected returns, they would be attractive to speculators, even some who are mildly risk-averse.

The lower-numbered spaces and low-strike puts, which carry a charge for their purchase and have expected returns below the risk-free rate, are hedges that function like insurance. A buyer would purchase such a derivative only in order to hedge risk, not to speculate. Even the most extreme risk-seeking gambler would never pay a premium for a lower-numbered space, when a higher-numbered space offers identical odds and a better payoff ratio.

Consider investors in a company. Shareholders own both the expected positive return and the risk of loss. They have two possible transactions with external parties to reduce risk: 1) Purchase some form of insurance against the possibility of an adverse outcome, and 2) Sell participation in any or all favorable outcome possibilities.

For either transaction, any risk-averse external party will require a fee for engaging in the transaction and assuming risk to their own capital. For insurance, the shareholders will have to pay a surcharge above discounted expected value for the derivatives that pay off in the event of an adverse outcome. They might be willing to do so in order to reduce their loss exposure. On the investment side, the owners will have to offer participation in the favorable outcomes at a discount to expected present value. They might be willing to, since funds from the derivatives' sale will offset loss in the event of an adverse outcome, again reducing risk. In summary, gambles in the direction of capital growth will be priced at a discount by the owners of the capital, and by the potential investors. Gambles in the direction of capital loss will be priced at a surcharge by the same parties. In general, instruments with E[R] < r, which is a risk surcharge, are insurance. Instruments with E[R] > r, which is a risk discount, are investments in the broad sense of the word (some are very speculative).

This explains why the distribution of a derivative's outcomes does not imply the risk load for the derivative. The distribution contains information only about variations in future value for that derivative. It provides no information about the relationship between that particular derivative's outcomes and the risk to investments in the underlying security. The correspondence between the future value of the derivative and the future value of participants' total capital could be the most significant factor in determining the risk load that will be set by the parties to the risk transfer.

# 6.3. Insurance Can Still be Priced with Distribution-Based Risk Loads

These results still do not invalidate distribution-based pricing of insurance risks, for the following reasons.

Insurance protects assets against loss from specific destructive perils. Insurance does not generally respond to a decline in the value of the insured asset, unless the decline is specifically attributable to a covered peril. In particular, it does not respond to a decline in the market value of an asset the way that a put option does.

In general, the value of an insurance policy on an asset is very different than the value of a derivative on the asset's market price. The value of the insurance is determined by the stochastic process of the covered perils; the value of the derivative is driven by the stochastic process of the asset's market price. If insurance could be thought of as a derivative at all, it would be as a derivative of hurricane occurrence and severity, auto accident occurrences and severities, etc.

Insurance almost never covers asset-event combinations that are traded in a liquid market. Those insured assets that are somewhat liquid (such as property and vehicles) do not usually have traded derivatives. Even if they did, the derivatives would correlate with insurance only on losses due to covered perils, providing a very incomplete hedge. As vulnerability to arbitrage does not exist for insurance, formulas that are theoretically not arbitrage-free can be used to price insurance risks without consequent economic penalty.

While a formula that uses only the outcome distribution cannot produce arbitrage-free prices, such a formula can accurately represent the risk evaluation of a market participant, such as an insurer in an insurance market. The potential for correlations among risks within the insurer's portfolio would still have to be handled, possibly through limiting the aggregate accumulation of particular risks or by using an additional covariance load.

### 7. THE RISK DISCOUNT FUNCTION

We can follow the construction used in the roulette example above to partition the space of expiration prices into more than 38 sections. In the limit, this leads to a continuous function that shows the variation in the implicit risk discount for equally probable events.

Choose a large positive integer M and partition the positive number line  $(0,\infty)$  into M adjacent sections (M-1) half-open line segments and one ray), so that the probability of any given section containing the expiration price is 1/M. Each of the Msections has a corresponding binary derivative with expiration value of one, if the section contains the expiration price, and zero otherwise. Each of these M section-derivatives has a risk discount or surcharge factor, which can be calculated using the formulas derived earlier:

Risk Factor = Price/Discounted Probability.

For a segment D(s,t), the price is given by:

$$Price[D(s,t)] = Price[A^*(s)] - Price[A^*(t)];$$

$$Price[A^*(s)] - Price[A^*(t)] = v\Phi[\ln(p(1+r)/s)/\sigma - \sigma/2]$$

$$-v\Phi[\ln(p(1+r))/\sigma - \sigma/2];$$

$$Price[D(s,t)] = v\{\Phi[\ln(p(1+r))/\sigma - \ln(s)/\sigma - \sigma/2]\}.$$

For sufficiently large M, the difference in cumulative probabilities (shown in braces) is approximated by the normal density function times the interval width:

Price[
$$D(s,t)$$
]  $\approx vg[\ln(p(1+r))/\sigma - \ln(s)/\sigma - \sigma/2]$   
  $\cdot [\ln(t) - \ln(s)]/\sigma,$ 

where g represents the standard normal density. Similarly,

Probability[
$$D(s,t)$$
] =  $\Phi[\ln(p(1+E))/\sigma - \ln(s)/\sigma - \sigma/2]$   
-  $\Phi[\ln(p(1+E))/\sigma - \ln(t)/\sigma - \sigma/2]$ , and

Discounted Probability[D(s,t)]

$$\approx vg[\ln(p(1+E))/\sigma - \ln(s)/\sigma - \sigma/2] \cdot [\ln(t) - \ln(s)]/\sigma.$$

Then,

Risk Factor 
$$\approx g[\ln(p(1+r))/\sigma - \ln(s)/\sigma - \sigma/2]/g[\ln(p(1+E))/\sigma - \ln(s)/\sigma - \sigma/2].$$

As *M* becomes infinitely large, the risk factor approaches this ratio of the normal densities for price and probability, the "risk

discount function" w(s):

$$w(s) = g[\ln(p(1+r))/\sigma - \ln(s)/\sigma - \sigma/2]/$$
$$g[\ln(p(1+E))/\sigma - \ln(s)/\sigma - \sigma/2].$$

Substituting the standard normal density for g(x), this expression can be simplified greatly, so that w(s) is found to be a monomial (derivation given in Appendix A):

$$w(s) = (s/s_n)^{-k}$$
, where  
 $k = [\ln(1+E) - \ln(1+r)]/\sigma^2$ , and  
 $s_n$  = the "neutral" strike price satisfying  $w(s_n) = 100\%$ .

This function w(s) represents the risk discount for betting on the event X = s. The risk discount function is equivalent to Bühlmann's "pricing density" [1]. The graph of w(s) is shown in Exhibit 4 for the parameters used in the examples. As the graph indicates, the risk surcharge factor increases without bound as the strike price approaches zero. As the strike price increases, the risk discount factor decreases toward zero but at a very gradual rate: for a strike price of 212, the risk discount is still relatively mild, at 63%. This strike price is just above the 99th percentile of the future price distribution for the security, where one might expect a more substantial discount.

The graph of w(s) (Exhibit 4) also shows that the continuum of expiration price events splits into discount and surcharge zones. Strike prices below  $s_n$  (which is 102.25 in this example) can be considered within the "insurance zone" of potential outcomes, while prices above 102.25 are in the "speculation zone."

Any derivative's risk discount factor can be calculated by averaging w(s) against probability density times expiration value:

Derivative's Risk Discount Factor

$$= \int w(s)p(s)x(s)ds \Big/ \int p(s)x(s)ds,$$

where x(s) is the expiration value of the derivative as a function of the underlying security's price, and p(s) is the probability density of the expiration price.

For almost any finite payoff density function, one can construct an unlimited number of distinct derivatives having the same specified payoff density and different prices.

#### 8. CONCLUSION

The probability distribution of a risk's outcomes does not imply the price of the risk under arbitrage-free pricing in an ideal market. Distinct, identically distributed risks generally have different arbitrage-free prices. In practice, this result does not invalidate distribution-based pricing for most insurance risks, but it should be considered when insurance pricing is related to financial theory involving arbitrage-free prices.

It is possible for a formula to produce arbitrage-free prices from a risk's probability distribution, if the formula contains an adjustment parameter that varies by risk. In one recent theory, this adjustment parameter indicates the correlation (in the general sense) between the underlying security's risk and overall market risk [5].

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#### APPENDIX A

#### DERIVATION OF THE w(s) FORMULA

From the text,

$$w(s) = g[\ln(p(1+r))/\sigma - \ln(s)/\sigma - \sigma/2]/$$
$$g[\ln(p(1+E))/\sigma - \ln(s)/\sigma - \sigma/2],$$

where  $g(x) = (2\pi)^{-1/2} \exp(-x^2/2)$ , and E = E[R], the expected return on the security. Then,

$$w(s) = (2\pi)^{-1/2} \exp(-b^2/2)/(2\pi)^{-1/2} \exp(-c^2/2),$$

where b and c are the respective arguments of g in the w(s) formula above:

$$b = [\ln(p) + \ln(1+r) - \ln(s) - \sigma^2/2]/\sigma, \text{ and}$$
  
$$c = [\ln(p) + \ln(1+E) - \ln(s) - \sigma^2/2]/\sigma.$$

Simplifying:

$$w(s) = \exp[(c^2 - b^2)/2],$$
 and  
 $w(s) = \exp[(c - b)(c + b)/2].$ 

Evaluating,

$$c-b = [\ln(1+E) - \ln(1+r)]/\sigma = \Delta E/\sigma,$$

where  $\Delta E = \ln(1 + E) - \ln(1 + r)$  is a measure of the risk premium in the security's expected return. Next,

$$\begin{aligned} (c+b)/2 \\ &= [\ln(p) + (1/2)\ln(1+E) + (1/2)\ln(1+r) - \ln(s) - \sigma^2/2]/\sigma. \\ \text{Define } s_n &= p[(1+E)(1+r)]^{1/2} / \exp(\sigma^2/2). \text{ Then,} \\ &\quad (c+b)/2 = [\ln(s_n) - \ln(s)]/\sigma. \end{aligned}$$

Substituting yields:

$$w(s) = \exp\{(\Delta E/\sigma)[\ln(s_n) - \ln(s)]/\sigma\}.$$

Note that  $w(s_n) = 1$ . Finally,

$$w(s) = \exp\{(-\Delta E/\sigma^2)[\ln(s/s_n)]\};$$
$$w(s) = (s/s_n)^{(-\Delta E/\sigma^2)}.$$





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# EXHIBIT 3

# DERIVATIVES CORRESPONDING TO ROULETTE WHEEL SPACES

Roulette Wheel Space	Derivative Type	Lower Strike (s)	Upper Strike (t)	Payoff Probability	Price	Expected Return	Risk Factor	Roulette Risk Surcharge	Ratio to Roulette Payoff
00	Segment	0.00	58.80	0.0263	0.0384	-31.54%	151.91%	105.56%	69%
0	Segment	58.80	64.68	0.0263	0.0346	-23.91%	136.68%	105.56%	77%
1	Segment	64.68	68.84	0.0263	0.0330	-20.21%	130.35%	105.56%	81%
2	Segment	68.84	72.23	0.0263	0.0319	-17.46%	126.00%	105.56%	84%
3	Segment	72.23	75.17	0.0263	0.0310	-15.18%	122.61%	105.56%	86%
4	Segment	75.17	77.83	0.0263	0.0303	-13.19%	119.81%	105.56%	88%
5	Segment	77.83	80.29	0.0263	0.0297	-11.40%	117.38%	105.56%	90%
6	Segment	80.29	82.61	0.0263	0.0292	-9.74%	115.23%	105.56%	92%
7	Segment	82.61	84.82	0.0263	0.0287	-8.19%	113.27%	105.56%	93%
8	Segment	84.82	86.95	0.0263	0.0282	-6.71%	111.48%	105.56%	95%
9	Segment	86.95	89.03	0.0263	0.0278	-5.29%	109.81%	105.56%	96%
10	Segment	89.03	91.07	0.0263	0.0274	-3.92%	108.24%	105.56%	98%
11	Segment	91.07	93.08	0.0263	0.0270	-2.58%	106.75%	105.56%	99%
12	Segment	93.08	95.08	0.0263	0.0267	-1.26%	105.33%	105.56%	100%
13	Segment	95.08	97.07	0.0263	0.0263	0.03%	103.96%	105.56%	102%
14	Segment	97.07	99.06	0.0263	0.0260	1.32%	102.64%	105.56%	103%
15	Segment	99.06	101.07	0.0263	0.0256	2.60%	101.36%	105.56%	104%
16	Segment	101.07	103.10	0.0263	0.0253	3.89%	100.11%	105.56%	105%
17	Segment	103.10	105.16	0.0263	0.0250	5.18%	98.87%	105.56%	107%
18	Segment	105.16	107.26	0.0263	0.0247	6.49%	97.66%	105.56%	108%
19	Segment	107.26	109.42	0.0263	0.0244	7.81%	96.47%	105.56%	109%
20	Segment	109.42	111.64	0.0263	0.0241	9.16%	95.27%	105.56%	111%
21	Segment	111.64	113.93	0.0263	0.0238	10.55%	94.08%	105.56%	112%
22	Segment	113.93	116.31	0.0263	0.0235	11.97%	92.88%	105.56%	114%
23	Segment	116.31	118.81	0.0263	0.0232	13.44%	91.68%	105.56%	115%
24	Segment	118.81	121.43	0.0263	0.0229	14.97%	90.46%	105.56%	117%
25	Segment	121.43	124.21	0.0263	0.0226	16.57%	89.21%	105.56%	118%
26	Segment	124.21	127.18	0.0263	0.0223	18.26%	87.94%	105.56%	120%
27	Segment	127.18	130.38	0.0263	0.0219	20.06%	86.62%	105.56%	122%
28	Segment	130.38	133.87	0.0263	0.0216	21.99%	85.25%	105.56%	124%
29	Segment	133.87	137.73	0.0263	0.0212	24.09%	83.81%	105.56%	126%
30	Segment	137.73	142.08	0.0263	0.0208	26.41%	82.27%	105.56%	128%
31	Segment	142.08	147.11	0.0263	0.0204	29.03%	80.60%	105.56%	131%
32	Segment	147.11	153.10	0.0263	0.0199	32.05%	78.76%	105.56%	134%
33	Segment	153.10	160.64	0.0263	0.0194	35.69%	76.65%	105.56%	138%
34	Segment	160.64	170.96	0.0263	0.0187	40.36%	74.09%	105.56%	142%
35	Segment	170.96	188.08	0.0263	0.0179	47.16%	70.67%	105.56%	149%
36	Ray	188.08	Infinity	0.0263	0.0162	62.93%	63.83%	105.56%	165%



#### ADDRESS TO NEW MEMBERS—MAY 19, 2003

#### ALBERT J. BEER

Thank you, Gail, for that most gracious introduction.

But most importantly, thank you for giving me an opportunity to give a little something back to the CAS today.

It's hard to believe, but it has been twenty-five years since I sat out there...pumped up with the confidence borne out of my new CAS designation and frightened out of my mind contemplating what the rest of my career would bring!

And if you can stand my ramblings for a few minutes, I'd like to share with you what I have learned about life during this last quarter of a century.

But, first, let me begin by personally congratulating all our new Fellows for their significant accomplishment.

That said, however, I would be terribly remiss if I didn't take a moment to acknowledge those people who have provided the tireless, behind-the-scenes support that has proven to be invaluable in your quest for the FCAS designation. Please join me in a raucous round of applause honoring the spouses, children, and parents of our New Fellows.

Now...indulge me for a few moments while I share with you some personal and professional "secrets" of success.

In this era of Enron-like scandals, insatiable greed and corporate misgovernance, it is easy to question the very existence of ethics and morality in the business world. To me, these and other recent aberrant behaviors are clear signs of a culture unsure of its definition of success. Therefore, this a perfect moment in your career, as you are setting future goals, to spend some time establishing parameters you can use to identify "success." Looking back over my career, I'm reminded of the metaphor "Success is a journey, not a destination." Misdirected goals mistakenly measured by the amount of your compensation, the loftiness of your title, or the number of people reporting to you should never be confused with success. Rather, the manner in which these things were attained and how they are put to use are much better indications of a productive life.

For example, I've been very fortunate to have experienced quite a bit during my career and have had the privilege of meeting some real "movers and shakers" around the world. However, you might find it interesting to learn that, without any doubt, the most successful person I have ever known was not a "Titan of Industry." In fact, my role model worked for the New York City Transit Authority for thirty-five years, never earned more than \$40,000 a year and yet still was able to support a family and inspire a very incorrigible son. My father could never be accused of being "rich," but he left an invaluable inheritance of honesty, laughter, and love. The "richness" in his quality of life was evident every day to everyone with whom he came in contact and that richness could never be measured by the IRS. I learned very early that, whether your career is associated with white collars, blue collars, or any color in between, you should never be ashamed of living a life that cherishes integrity, hard work, and dedication to family. External recognition is gratifying but, at the end of the day, self-respect and self-satisfaction are the most important metrics of a successful life. So my first piece of advice to you would be to set absolute ethical standards and have them guide you without compromise throughout your personal and professional lives.

Now please don't infer that I believe ambition and career development are evil. Coasting through life without really testing the boundaries of your talents will lead to a very unfulfilling existence. I was blessed early in life to learn that your personal life and your career do not have to be two diametrically opposite forces, repelling each other and forcing daily "either/or" decisions. I have clearly found that, in general, people who are able to enjoy their personal life tend to also perform extraordinarily well in their business life. Of course, there will always be pushes and pulls between the two. Forgive the mathematical reference, but anyone who has studied the solution of simultaneous equations realizes that an optimal solution is rarely the one associated with setting one variable to its maximum and letting all the others go to zero. I often feel as though learning from experience is simply nothing more than living through these iterative attempts at finding the right balance along this optimal "Frontier." God knows it's not easy. Sometimes it means taking the red-eye to make it home for that parent/teacher conference or rescheduling the dinner with that important client just to be there for that special anniversary. You should never be ashamed or embarrassed to do the right thing. Remember, no one has ever seen a gravestone that said, "I wish I had spent more time in the office!"

I realize that it may be an unfair characterization, but it seems to me that professionals, particularly those in financial services, are unusually prone to stressing monetary rewards. Clearly you have invested thousands of hours of your life to get to this point today and it would be wasteful not to reap the benefits accorded those with your skills and training. However, remember that, while promotions and titles may describe "*What*" you are, it is your adherence to ethical behavior that will ultimately determine "*Who*" you are.

In many ways, I envy this new class of Fellows. You find yourself at a point in time that offers unprecedented opportunities to effect an enormously positive influence on your industry and your profession. It is belaboring the obvious to say that the recent financial performance of the property/casualty industry has been abysmal. And don't think this is our little secret! My son was finishing up his MBA two years ago and the class was reviewing "Profitability" among various industry segments. He approached me uncomfortably one day to innocently ask why, over the decade of the nineties, the average annual return on equity for diversified financial services was 17.4 percent while, over the same period, the ROE for property/casualty insurers was 7.5 percent. The only thing more embarrassing to me than answering that question was that I felt obligated to begin, "Kevin, you first have to understand that the 7.5 percent for the P/C industry was overstated!"

That story doesn't end there. A year later Kevin called to tell me that he had just passed his second CPCU exam. After congratulating him, I reminded him of the ROE discussion and asked, in light of the obvious differences, why had he chosen to enter the insurance business. His answer provided me with a perspective on our industry that I have since shared with every actuary, underwriter, agent, and client that will listen. Kevin looked sheepishly at me and said, "Dad, I hope you're not offended…but I was looking for the best career opportunities and it just doesn't look like the sharpest knives in the drawer are in the insurance industry."

Now, actuaries can timidly hide behind vacuous arguments such as "We couldn't control what the underwriters charged" or "Management made all the decisions regarding what was held in reserves" but the bottom line is the actuarial profession must accept a significant share of the responsibility for the horrendous financial results of the past few years.

Today, our *Industry* is in desperate need of discipline, courage and leadership. Our *Profession* is ideally suited to drive sound financial management and yet we still hide under the blanket of "providing information" instead of driving change and forcing objective decision-making. I challenge each of you in the CAS to help stop our industry from mindlessly accepting the inevitability of "the Cycle." Seize this moment to fight the lemming mentality of "we are only as smart as our dumbest competitor." We must influence senior management to develop strategies, create sound long-term business plans, and implement realistic metrics to assure that we ultimately achieve results of which we can all be proud. Your Fellowship is merely a license to practice. Your behavior will determine whether you are really a professional. Your generation has the chance to lead this change...don't pass up this important opportunity.

Finally, I urge you to give back to your profession. You are now members of one of the most elite organizations in the world. Although you have personally worked hard, a large part of the esteem in which you are now held is derived from the accomplishments of all those actuaries who came before you. They gave generously of their time to help create a profession, which is arguably the most respected in our industry. You have now been given the very serious responsibility of passing that legacy untarnished on to succeeding generations.

I honestly don't think I have told you anything this morning that you didn't already know. Sometimes you just need to be reminded of what you really feel. To prove that point, I'd like to leave you with three simple questions:

If you were told you had only five minutes left in your life and you could make only one phone call:

#1. "Who would you call?"

(By the way, if anyone out there answered, "Check my e-mail," I'm afraid I've been wasting my breath on *you* for the past ten minutes!)

So, #1. "Who would you call?"

#2. "What would you say?" and,

#3. "What are you waiting for?"—Speak to that person today and tell them how you feel!

See, you've got your priorities straight. Now make sure you live them!

Good luck and thanks for your attention.

### MINUTES OF THE 2003 SPRING MEETING

### May 18–21, 2003

#### MARCO ISLAND MARRIOTT RESORT, GOLF CLUB & SPA

#### MARCO ISLAND, FLORIDA

## Sunday, May 18, 2003

Registration was held from 4:00 p.m. to 6:30 p.m.

New Associates and their guests were honored with a special presentation from 5:30 p.m. to 6:30 p.m. Members of the 2003 Executive Council discussed their roles in the Society with the new members.

A reception for all meeting attendees followed the new Associates reception and was held from 6:30 p.m. to 7:30 p.m.

Monday, May 19, 2003

Registration continued from 7:00 a.m. to 8:00 a.m.

The 2003 business session, which was held from 8:00 a.m. to 9:15 a.m., started off the first full day of activities for the 2003 Spring Meeting. Gail M. Ross introduced the CAS Executive Council, the Board of Directors, and CAS past presidents who were in attendance, including Albert J. Beer (1995), Ronald L. Bornhuetter (1975), Charles C. Hewitt Jr. (1972), M. Stanley Hughey (1974), Steven G. Lehmann (1998), and Michael A. Walters (1986).

Ms. Ross also recognized a special guest in the audience: Mareb del Rosario, President, Academia de Actuarios de Puerto Rico and John P. Ryan, Representative, Institute of Actuaries and General Insurance Board of the U.K. Actuarial Profession. Thomas G. Myers announced the 38 new Associates and Christopher S. Carlson announced the 30 new Fellows. The names of these individuals follow.

#### NEW FELLOWS

Robert D. Bachler	Patrick P. Gallagher	Lester Pun		
Andrew W. Bernstein	Matthew R. Gorrell	Jeremy D. Shoemaker		
Raju Bohra	Christopher J. Grasso	Anthony A. Solak		
Jennifer A. Charlonne	Serhat Guven	Michael William		
Robert E. Davis	Ung Min Kim	Starke		
Michael Devine	Laurie A. Knoke	Matthew D. Trone		
Kyle A. Falconbury	Jonathan D. Levy	Brian A. Viscusi		
John D. Ferraro	Matthew Kevin Moran	Bethany R. Webb		
Kevin M. Finn	Matthew P. Nimchek	Patti West		
William M. Finn	Michael Robert	Yingjie Zhang		
Greg Frankowiak	Petrarca			
	NEW ASSOCIATES			
Danielle L.	David S. Futterleib	Kee Heng Ng		
Bartosiewicz	Joel D. Glockler	Wade H. Oshiro		
Lisa K. Buege	Melanie T. Goodman	Robert A. Painter		
Suejeudi Buehler	Ann E. Green	Christopher A. Pett		
James Chang	Jonathan M. Guy	Monica L. Ransom		

Joseph Hebert

Scott R. Jean

William Brian Johnson

Kyewook Gary Kang

Jonathan David Koch

Christopher A. Najim

John J. Karwath

Damon T. Lay

Joseph L. Rizzo

Paul Silberbush

Douglas E. Smith

Beth M. Sweeney

Joseph S. Tripodi

Natalie Vishnevsky

Nicholas J. Williamson

Julie Clarisse Russell

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Christian J. Coleianne

Matthew P. Collins

Keith R. Cummings Nicholas J. De Palma

Timothy M. Devine

Jeffrey J. Fratantaro

Robert C. Fox

David C. Coplan

Ms. Ross then introduced Albert J. Beer, a past president of the Society, who presented the Address to New Members.

Mr. Carlson spoke to the meeting participants about the highlights of this meeting and what was planned in the program.

Ms. Ross then began the presentation of awards. She explained that the Charles A. Hachemeister Award was established in 1993 in recognition of Charles A. Hachemeister's many contributions to Actuarial Studies in Non-Life Insurance (ASTIN) and his efforts to establish a closer relationship between the CAS and ASTIN. Ms. Ross presented the 2002 Charles A. Hachemeister Award to Nicholas E. Frangos and Spyridon D. Vrontos for their paper, "Design of Optimal Bonus-Malus Systems with a Frequency and a Severity Component on an Individual Basis in Automobile Insurance."

Ms. Ross then presented the CAS Harold W. Schloss Memorial Scholarship Fund. This award benefits deserving and academically outstanding students in the actuarial program of the Department of Statistics and Actuarial Science at the University of Iowa. The student recipient is selected by the Trustees of the CAS Trust, based on the recommendation of the department chair at the University of Iowa. Ms. Ross announced that Biou Xu is the recipient of the 2003 CAS Harold W. Schloss Memorial Scholarship Fund. Xu will be presented with a \$500 scholarship.

Ms. Ross, Mr. Donald F. Mango, and Mr. Michael J. Miller then held a panel discussion and update on Mutual Recognition.

Ms. Ross then concluded the business session of the Spring Meeting.

Ms. Ross next introduced the featured speaker, Mike Abrashoff, who, at 36, became the youngest commanding officer in the Pacific Fleet.

After a refreshment break, the first general session was held from 10:45 a.m. to 12:15 p.m.
"How Can the Ins	urance Industry Make Money?"
Moderator:	Jeanne M. Hollister
	Consulting Actuary
	Tillinghast-Towers Perrin
Panelists:	Jay Cohen
	First Vice President of Research
	Merrill Lynch
	Leonard R. Goldberg
	President
	Alea North America Insurance Company
	Gary K. Ransom
	Managing Director
	Fox-Pitt, Kelton
	Mark D. Shapiro
	Director
	McKinsey & Co., Inc.

After a luncheon, the afternoon was devoted to presentations of concurrent sessions. The concurrent sessions presented from 1:30 p.m. to 3:00 p.m. were:

1. Determining Reserve Ranges and Variability of Loss Reserves

	Moderator/	Roger M. Hayne
	Panelist:	Consulting Actuary
		Milliman USA
	Panelist:	Rodney E. Kreps
		Managing Director
		Guy Carpenter & Company, Inc.
2.	An Overview of Florida's Workers Compensation Marke	
	Moderator:	Thomas L. Gallagher
		Consulting Actuary
		Tillinghast-Towers Perrin

Panelists:	Anthony M. DiDonato
	Vice President and Actuary
	National Council on Compensation
	Insurance
	Tom Stewart
	Senior Vice President Commercial Lines
	Accordia
	James D. Watford
	Actuary
	Florida Insurance Department

3. Pay As You Drive

4.

Moderator:	Gregory L. Hayward Actuary State Farm Mutual Automobile Insurance Company
Panelists:	Patrick J. Crowe Vice President and Actuary, Market Research Kentucky Farm Bureau Mutual Insurance Company
	Edmund M. Coe
	Program Analyst
	United States Environmental Protection
	Agency
	Randall Guensler
	Associate Professor
	Georgia Institute of Technology
Risk-Based Capital Developments	
Moderator/	Glenn G. Meyers
Panelist:	Chief of Actuarial Research and Assistant
	Vice President
	ISO

	Panelist:	Robert F. Wolf
		Principal
		Mercer Risk, Finance & Insurance
		Consulting
5.	Volunteerism-	-The Essential Energy Driving the CAS
	Moderator:	Regina M. Berens
		CAS Committee on Volunteer Resources
		Chairperson
		Actuary
		Employers Reinsurance Corporation
	Panelists:	Carol Blomstrom
		Actuary
		Liberty Mutual Group
		James B. Rowland
		State Manager
		Allstate Insurance Company
		Louise Francis
		Consulting Principal
		Francis Analytics and Data Mining, Inc.
6.	Actuaries' Responsibility to Users of Their Work Products	
	Moderator:	Chad C. Wischmeyer
		Managing Director and Southeast Practice
		Mercer Risk Finance & Insurance
		Consulting
	Panelists:	Cara M. Blank
		Vice President and Regional Actuary
		OneBeacon Insurance Companies
		Mary D. Miller
		Actuary
		Ohio Department of Insurance
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Thomas M. Mount Managing Senior Financial Analyst/Actuary A. M. Best Company

7. An Overview of Asbestos Claims Liabilities

Moderator/	Michael E. Angelina
Panelist:	Consulting Actuary Tillinghast-Towers Perrin
Panelists:	Stephen J. Carroll Senior Economist RAND Institute for Civil Justice
	Trish Henry Vice President ACE/INA Government Affairs

After a refreshment break, presentations of concurrent sessions continued from 3:30 p.m. to 5:00 p.m. Certain concurrent sessions that had been presented earlier were repeated. Additional concurrent sessions presented were:

1. Geographical Spatial Analysis in Personal Lines Territorial Ratemaking

Moderator:	Claudine H. Modlin Senior Consultant Watson Wyatt Insurance & Financial Services
Panelists:	Duncan Anderson Partner Watson Wyatt LLP Geoffrey T. Werner Executive Director United Services Automobile Association

2. Extreme Events

Moderator:	John J. Kollar
	Vice President
	ISO

	Panelists:	David A. Lalonde Senior Vice President AIR Worldwide
		Thomas A. Weidman Senior Vice President and Chief Actuary XL America
3.	Do Exclusive Remedies Work?	
	Moderator:	David F. Mohrman Consulting Actuary Tillinghast-Towers Perrin
	Panelists:	Richard A. Hofmann Principal and Consulting Actuary Epic Consulting, LLC
		Anne E. Kelly Chief Casualty Actuary New York State Insurance Department
		Donald D. Palmer Manager-Actuarial Services Manitoba Public Insurance Company
4.	Financial Pricing and Performance Measurement	
	Moderator/ Panelist:	Sholom Feldblum Vice President Liberty Mutual Group
	Panelist:	Neeza Thandi Actuary Liberty Mutual Group
5.	The Changing Philosophy of Underwriting—Increased Emphasis on Exposure Accumulation	
	Moderator:	Gregory M. Wacker Vice President and Actuary Fireman's Fund Insurance Companies
	Panelists:	John G. Aquino Executive Vice President Aon Re Services

Bill Tuttle Vice President Product Marketing Risk Management Solutions

A reception for new Fellows and their guests was held from 5:30 p.m. to 6:30 p.m., and the general reception for all members and their guests was held from 6:30 p.m. to 7:30 p.m.

Tuesday, May 20, 2003

Registration continued from 7:00 a.m. to 8:00 a.m.

The general sessions presented from 8:00 a.m. to 9:30 a.m. were:

"Government Involvement and Uninsurable Exposures"

Moderator:	Steven F. Goldberg Senior Vice President Benfield Group
Panelists:	Karen M. Clark President AIR Worldwide
	Andrew J. Kaiser Managing Director Goldman, Sachs & Co.
	Tim Richison Chief Financial Officer California Earthquake Authority
	Terrie Troxel President and CEO American Institute of CPCU and Insurance Institute of America
"Use of Credit Scor	ring"
Moderator:	Jonathan White Assistant Vice President and Actuary The Hartford

Panelists. Wesley Bissett Vice President of State Relations and State Government Affairs Independent Insurance Agents and Brokers of America John Fitts Deputy General Counsel Progressive Casualty Insurance Company J. Robert Hunter Jr. **Director of Insurance** Consumer Federation of America Michael J. Miller Consultant EPIC Consulting, LLC Lisa M. Smego Senior Policy Analyst - Policy and Legislative Affairs Washington Insurance Department

After a refreshment break, presentation of concurrent sessions continued from 10:00 a.m. to 11:30 a.m. A concurrent session presented earlier was repeated. Additional concurrent sessions presented were:

1. Finite Reinsurance Discussion and Demonstration

Moderator:	James M. Kelly Vice President and Actuary American Re-Insurance Company
Panelists:	John G. Aquino Executive Vice President Aon Re Services
	Douglas A. Carlone Senior Vice President Swiss Re Financial Services Corporation

- 3. CAS Examination Process

	Moderator/ Panelist:	Thomas G. Myers CAS Vice President–Admissions Vice President Auto Product Management Prudential Property & Casualty Insurance Company
	Panelists:	Mary D. Miller CAS Examination Committee Member Actuary Ohio Department of Insurance
		Julia C. Stenberg CAS Exam Part Chair Actuary Travelers Insurance
		Edward C. Stone CAS Syllabus Committee Chairperson Principal Mercer Risk, Finance & Insurance Consulting
4.	ARIA Prize Paper, "Capital Allocation for Insurance Companies"	
	Authors:	Stewart C. Myers MIT Sloan School of Management The Brattle Group

		James A. Read Jr. The Brattle Group		
	Moderator:	Stephen P. D'Arcy Professor University of Illinois		
	Presenter:	Stewart C. Myers MIT Sloan School of Management The Brattle Group		
5.	Building Comr	nunication Skills through Improvisation		
	Moderator/ Panelist:	Robert C. Morand Partner D. W. Simpson and Company		
6.	Changes in the Florida Market			
	Moderator:	Sean R. Devlin Chief Pricing Actuary-Property Employers Reinsurance Corporation		
	Panelists:	Joe Mawhinney Senior Vice President of Actuarial Services First Professional Insurance Company		
		John W. Rollins Chief Actuary Florida Farm Bureau Insurance Companies		
		James C. Santo Actuary First Floridian Auto & Home		

Various CAS committees met from 12:00 p.m. to 5:00 p.m.

Concurrent sessions presented from 2:00 p.m. to 3:30 p.m. were:

1.	Do's and Don'ts in Dealing with the Media			
	Moderator:	Robert F. Wolf		
		Principal		
Mercer Risk, H Consulting		Mercer Risk, Finance & Insurance		
		Consulting		
	Panelist:	John W. Rollins		
		Chief Actuary		
		Florida Farm Bureau Insurance		
		Companies		

2. Hachemeister Prize Paper, "Design of Optimal Bonus-Malus Systems with a Frequency and a Severity Component on an Individual Basis in Automobile Insurance"

Authors:	Nicholas E. Frangos Athens University of Economics and Business
	Spyridon D. Vrontos Athens University of Economics and Business
Moderator:	Paula L. Elliott Principal Mercer Risk, Finance & Insurance Consulting
ARIA Prize Pa	aper—Discussion Call

3. ap

Moderator:	Philip E. Heckman Senior Consultant and Actuary Aon Risk Consultants, Inc.
Discussion Call Respondents:	Paul Brehm and Kyle Vrieze Paul Kneuer Donald Mango and David Ruhm Glenn Meyers Stephen Mildenhall Gary Venter

4.	Pricing for Terrorism		
	Moderator:	Samir Shah Consultant Tillinghast-Towers Perrin	
	Panelists:	George Burger Assistant Vice President ISO	
		Jeffrey Eddinger Actuary National Council on Compensation Insurance	

All members and guests enjoyed dinner and entertainment from 6:00 p.m. to 9:00 p.m.

Wednesday, May 21, 2003

Certain concurrent sessions presented earlier during the meeting were repeated this morning from 8:00 a.m. to 9:30 a.m. Additional concurrent sessions presented were:

1. Toward a CAS Centennial, or Maybe Not

Moderator:	Michael A. Walters Consulting Actuary
	Tillinghast-Towers Perrin
Panelists:	Clive L. Keatinge
	Associate Actuary
	ISO
	Steven G. Lehmann
	Principal
	Pinnacle Actuarial Resources, Inc.
	David J. Oakden
	Principal
	Tillinghast-Towers Perrin

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2.	Enterprise Da	ta Strategies
	Moderator:	Peter A. Marotta Principal
		ISO
	Panelists:	Beth Grossman Assistant Vice President ACORD
		Gary Knoble Vice President The Hartford
		Randy Molnar Senior Analyst NCCI Holdings

The *Proceedings* papers presented during this time were:

1.	"Source of l	Earnings Analysis for Property-Casualty
	Insurers"	
	Author:	Sholom Feldblum

or: Sholom Feldblum Liberty Mutual Group

2. "Distribution-Based Pricing Formulas Are Not Arbitrage-Free"

Author: David L. Ruhm The Hartford

After a refreshment break, the final general session was held from 10:00 a.m. to 11:30 a.m.:

"The State of the Property/Casualty Insurance Market— A Consumer Town Hall Meeting"

Moderator:	Michael G. McCarter Vice President, Industry and Regulatory Affairs AIG
Panelists:	Frederick Cripe Vice President Allstate Insurance Company

J. Robert Hunter Jr. Director of Insurance Consumer Federation of America

Gail M. Ross officially adjourned the 2003 CAS Spring Meeting at 11:45 a.m. after closing remarks and an announcement of future CAS meetings.

# Attendees of the 2003 CAS Spring Meeting

The 2003 CAS Spring Meeting was attended by 290 Fellows, 110 Associates, and 52 Guests. The names of the Fellows and Associates in attendance follow:

#### FELLOWS

Mark A. Addiego	Carol Blomstrom	Bethany L. Cass
Ethan D. Allen	Michael J. Bluzer	Michael J. Caulfield
Brian M. Ancharski	Neil M. Bodoff	Todd D. Cheema
Mark B. Anderson	Raju Bohra	Michael Joseph
Paul D. Anderson	Mark E. Bohrer	Christian
Lawrence J. Artes	LeRoy A. Boison	Michael A. Coca
Peter Attanasio	Steven W. Book	JoEllen Cockley
Guy A. Avagliano	Ronald L. Bornhuetter	Hugo Corbeil
Craig Victor Avitabile	Peter T. Bothwell	Francis X. Corr
Richard J. Babel	Pierre Bourassa	Frederick F. Cripe
Timothy J. Banick	Erik R. Bouvin	Alan M. Crowe
W. Brian Barnes	Douglas J. Bradac	Patrick J. Crowe
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# PROCEEDINGS November 9, 10, 11, 12, 2003

# THE STANARD-BÜHLMANN RESERVING PROCEDURE: A PRACTITIONER'S GUIDE

#### SHOLOM FELDBLUM

#### Abstract

The Stanard-Bühlmann reserving method, commonly used by reinsurance actuaries, combines the stability of expected loss methods with the adherence to empirical data that is characteristic of the chain ladder method. This paper is a teaching guide for the reserving actuary: it explains the intuition for the Stanard-Bühlmann reserving method, shows an algebraic derivation from the Bornhuetter-Ferguson reserving method, uses a series of illustrations to explain the needed premium adjustments, and compares the Stanard-Bühlmann reserving method with the Bornhuetter-Ferguson and chain ladder methods.

#### 1. INTRODUCTION

An ideal loss reserving method would rely primarily on observed data but not be subject to random loss fluctuations. The chain ladder reserving method relies entirely on historical loss triangles, but it is sensitive to random loss fluctuations in the most recent years. The Bornhuetter-Ferguson expected loss reserving method sometimes provides more stable reserve indications, but it requires an a priori estimate of the expected losses. The Stanard-Bühlmann reserving method has the stability of an expected loss method, yet it draws all the needed information from the observed experience.

The Stanard-Bühlmann procedure has been a major advance in actuarial loss reserving methods. It has proved especially useful for reinsurers lacking the pricing data to use other expected loss methods. Primary companies may benefit equally from this technique, particularly if the reserving actuary does not have a good estimate of the expected loss ratio.<sup>1</sup>

This paper explains the intuition for the Stanard-Bühlmann reserving method. We begin with the assumption underlying most reserving techniques—that historical patterns may be repeated in the future—and we differentiate among the patterns that chain ladder and expected loss methods use. We provide two derivations of the Stanard-Bühlmann method: an algebraic derivation from the Bornhuetter-Ferguson expected loss method and an intuitive derivation based on the speed of "processing" premiums. Finally, we explain the premium adjustments made in the Stanard-Bühlmann method and we give illustrations for several scenarios.

# 1.1. Patterns of Stability

Most actuarial reserving techniques assume that certain loss reporting patterns or loss settlement patterns remain relatively stable over time. The observed patterns, adjusted (if necessary) for changes in the insurance environment and company claims practices, are a reasonable predictor of future experience.

<sup>&</sup>lt;sup>1</sup>The Stanard-Bühlmann technique is also called the "adjustment to total known losses" (Stanard [1985]). Patrik [2002] provides a general introduction to this method.

TABLE 1.1

Development Months	0–12	12–24	24–36	36–48	48–60	
Incremental Paid Losses	\$100,000	\$200,000	\$150,000	\$75,000	\$25,000	

Illustration: Most formulations of the stability principle are chain ladder relationships, such as

- Reported losses as of 24 months since inception of the accident year are expected to be 50% higher than reported losses as of 12 months for that accident year.
- Cumulative paid losses as of 48 months since inception of the accident year are expected to be 20% higher than cumulative paid losses as of 36 months for that accident year.

The format of the two statements above is that the cumulative losses (of whatever type) as of development period i + 1 are X% greater or lower than the same cumulative losses as of development period i. This is the chain ladder format; the format differs for expected loss reserving methods.

Suppose \$550,000 of accident year losses were paid over a five-year period as shown in Table 1.1.

We formulate the observed pattern:

A. Incremental Development: Losses paid between 12 and 24 months are twice the losses paid between 0 and 12 months. Losses paid between 24 and 36 months are three quarters of the losses paid between 12 and 24 months.

B. Cumulative Development: The cumulative losses paid from 0 to 24 months are three times the cumulative losses paid from 0 to 12 months. The cumulative losses paid from 0 to 36 months are one and a half times the cumulative losses paid from 0 to 24 months.

Development Months	0–12	12–24	24–36	36–48	48-60
Incremental Paid Losses	\$100,000	\$200,000	\$150,000	\$75,000	\$25,000
Cumulative Paid Losses	\$100,000	\$300,000	\$450,000	\$525,000	\$550,000
Incremental Ratio		2.000	0.750	0.500	0.333
Cumulative Ratio		3.000	1.500	1.167	1.048
Loss Development Factor		5.500	1.833	1.222	1.048
Incremental Portion of Ultimate	0.182	0.364	0.273	0.136	0.045
Cumulative Portion of Ultimate	0.182	0.546	0.819	0.955	1.000
Future Portion of Ultimate	0.818	0.454	0.181	0.045	0.000

TABLE 1.2

C. Percentages of Ultimate: Of the \$550,000 total paid losses, 18.2% are paid in the first 12 months, and 36.4% are paid in the next 12 months.

These patterns differ in their measurement bases. The patterns are shown in Table 1.2.

For the expected losses, the different bases can be converted into one another. Given the incremental ratio pattern, we can derive the cumulative ratio and the percent of ultimate.

The chain ladder method uses cumulative ratios. The paid loss link ratio from 36 to 48 months is the 1.167 in the 36 to 48 months column of the Cumulative Ratio row. The cumulative product of the link ratios from a given development date to ultimate is the ultimate loss development factor. The loss development factor from 36 months to ultimate is  $1.167 \times 1.048 = 1.222.^2$ 

The Incremental Portion of Ultimate row is used for expected loss methods. The Bornhuetter-Ferguson factor is the sum of the portion of ultimate figures from a given development date forward. For instance, the factor from 36 months to ultimate is 0.136 + 0.045 = 0.181. The Bornhuetter-Ferguson factor equals 1 - (1/the link ratio). In this example, 0.181 = 1 - (1/1.222).

<sup>&</sup>lt;sup>2</sup>The chain ladder and expected loss reserving methods are described in Wiser [2001], Salzmann [1984], Peterson [1981], and Feldblum [2002].

The portion of ultimate factor used in the Stanard-Bühlmann method is the complement of the Bornhuetter-Ferguson factor. The portion of ultimate factor at 36 months is 1 - 0.181 = 0.819.

### 1.2. Estimated Ultimate Losses

The Bornhuetter-Ferguson method needs an estimate of the ultimate losses. For primary companies, the pricing actuary estimates ultimate losses to set premium rates. The reserving actuary can use the estimate provided by the pricing actuary.

The estimate of ultimate losses is the premium times the expected loss ratio. This estimate is suitable when the premium charged is the indicated premium. It must be adjusted if the manual premium differs from the indicated premium or if underwriters provide schedule credits and debits to individual insureds. These adjustments demand business acumen, but a knowledgeable actuary can often make a reasonable estimate of the ultimate losses.

The reinsurer's reserving actuary may not have this underwriting information. The reinsurer's reserving book of business may consist of disparate pieces with different expected loss ratios. The reinsurer may not have the information to adjust for the adequacy level of the primary premiums or for schedule credits and debits provided by the primary underwriters. This is also true for primary insurers if the reserving actuary does not have access to the pricing actuary's estimates, to manual deviations from indicated rates, or to the underwriters discretionary price modifications. This is often the case for large commercial lines insurers.

James Stanard and Hans Bühlmann provided a solution to this quandary. If we have sufficient past experience, they argued, we do not need to know the expected loss ratio. We simply adjust all premiums in the historical period to the same level of adequacy, so the expected loss ratios are the same in each year. We first provide the intuition underlying their method, and then we show the premium adjustments.

Development	Percent	Development	Percent
Date	Reported	Date	Reported
12 months	30%	72 months	85%
24 months	50%	84 months	90%
36 months	65%	96 months	94%
48 months	75%	108 months	97%
60 months	80%	120 months	99%

TABLE 1.3

#### 1.3. Derivation

We first derive the Stanard-Bühlmann method from the Bornhuetter-Ferguson method, which is better known to many readers; we then proceed to the intuition for the method. The Bornhuetter-Ferguson method defines the bulk reserve<sup>3</sup> as

adequate premium × expected loss ratio × percentage unreported.

Illustration: Tables 1.3 and 1.4 show the expected loss reporting percentages from inception of the accident year and the premiums and losses by accident year at year-end 20X9. A slow reporting pattern is common for casualty excess-of-loss reinsurance, products liability, and professional liability.

We explain the derivation of the Adjusted Premiums after explaining the reserving method.

If the premiums are at the same adequacy level, then the multiplicative factor needed to arrive at the expected losses is the same for all accident years. For instance, if the premiums are all 20% inadequate, then the expected losses in each accident year equal

premium  $\times 1.200 \times$  expected loss ratio.<sup>4</sup>

<sup>&</sup>lt;sup>3</sup>The bulk reserve, or the actuarial reserve, covers the emergence on unreported claims and adverse development on reported claims.

<sup>&</sup>lt;sup>4</sup>The terms "premium adequacy" and "expected loss ratio" have numerous interpretations. When used in a pricing context, premium adequacy generally has an economic meaning: premiums are adequate if they provide a reasonable return to the insurance enter-

### TABLE 1.4

Accident	Adjusted	Reported	Accident	Adjusted	Reported
Year	Premiums	Losses	Year	Premiums	Losses
20X0	\$200,000,000	\$150,000,000	20X5	\$300,000,000	\$185,000,000
20X1	\$220,000,000	\$155,000,000	20X6	\$320,000,000	\$205,000,000
20X2	\$240,000,000	\$200,000,000	20X7	\$340,000,000	\$155,000,000
20X3	\$260,000,000	\$175,000,000	20X8	\$375,000,000	\$185,000,000
20X4	\$280,000,000	\$215,000,000	20X9	\$400,000,000	\$75,000,000

Let Z = the expected loss ratio times the factor needed to bring premiums to adequate levels.

Let  $Y_i$  = the bulk reserves for year *i*.

Let *Y* = the total bulk reserve; that is, *Y* =  $\sum Y_i$ .

The index *i* ranges from 0 to 9, corresponding to accident years 20X0 through 20X9.

The Bornhuetter-Ferguson expected loss method defines the bulk loss reserves as

adequate premium × expected loss ratio × percentage unreported.

For year 20X0, the expected percentage already reported is 99%, so the Bornhuetter-Ferguson estimate of the bulk reserves is \$200,000,000 × Z × (1 – 99%) =  $Y_0$ . Similarly, for the 20X9 accident year, the estimate is \$400,000,000 × Z × (1 – 30%) =  $Y_9$ . We sum all 10 equations to get Z × [\$200,000,000 × (1 – 30%)] =  $\sum Y_i = Y$ .

If the expected loss ratio is accurate, the total reported losses plus the total bulk reserves should be close to the total expected

prise. Statutory reserving uses undiscounted losses. By "premium adequacy" and "expected loss ratio" in this paper we mean figures such that ultimate (undiscounted) losses equal adequate premiums times the expected loss ratio.

losses. We write the equation for this statement as

$$[\$150,000,000 + \dots + \$75,000,000] + Y$$
$$= Z \times [\$200,000,000 + \dots + \$400,000,000].$$

We have a pair of simultaneous linear equations. We compute the sums in these equations.

- The sum of the adjusted premiums is \$2,935,000,000.
- The sum of the reported losses is \$1,700,000,000.
- The sum of the adjusted premiums × (1 percentage reported) is \$817,500,000. Then

 $Z \times$ \$817,500,000 = *Y*.

$$1,700,000,000 + Y = Z \times 2,935,000,000.$$

We need to find *Y*, the total bulk reserve. We eliminate *Z* by substituting Z = Y/\$817,500,000.

$$1,700,000,000 + Y = Y \times 2,935,000,000/817,500,000.$$

 $1,700,000,000 \times 817,500,000 = Y \times 2,117,500,000.$ 

$$Y = \$1,700,000,000 \times \$817,500,000/\$2,117,500,000$$

= \$656,320,000.

### 1.4. Intuition

First, we explain the intuition for the chain ladder reserving method versus the Stanard-Bühlmann reserving method. Consider year 20X9. The adjusted premium is \$400,000,000. By 12 months from the inception of the accident year, 30% of the adjusted premium, or \$120,000,000, has been processed into reported losses. The other 70% of the adjusted premium, or \$280,000,000, has not yet been processed into reported losses.

The word *processed* warrants explanation. The adjusted premium does not become reported losses. Rather, think of the verb *process* as connoting emergence or development or settlement. It

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denotes the relationship between the premium collected and the loss activity.

There is some relationship between the \$400,000,000 of premium and the ultimate reported losses. At 12 months of development, 30% of the losses should have been reported. \$120,000,000 of premium has the same relationship to the losses that have already been reported as the other \$280,000,000 of premium has to the losses that are yet to be reported.

The chain ladder reserving method uses this relationship for each accident year. Let X be the bulk reserve. Then

$$\frac{\$75,000,000}{\$120,000,000} = \frac{X}{\$280,000,000}$$

We solve for the bulk reserve:

 $X = \$75,000,000 \times \$280,000,000/\$120,000,000,$ or X = \$175,000,000.

This is the chain ladder reserving method. The bulk reserve for the chain ladder technique is directly dependent on the losses that have been reported so far. If the reported losses at 12 months were twice as high, \$150,000,000 instead of \$75,000,000, the bulk reserve would be twice as large. We verify this by writing

120,000,000/150,000,000 = 280,000,000/X, or X = 350,000,000.

If LDF is the loss development factor, the bulk reserve in the chain ladder technique is the reported losses times (LDF - 1). In the equation above, the bulk reserve equals the reported losses times (1 – portion of ultimate)/(portion of ultimate). The loss lag is the reciprocal of the loss development factor. We rewrite the expression above:

(1 - portion of ultimate)/(portion of ultimate)= (1 - 1/LDF)/(1/LDF) = LDF - 1. The Stanard-Bühlmann reserving method argues that losses are volatile, and that we may not want to give too much credence to the \$75,000,000 of losses that have been reported as of 12 months for accident year 20X9. Instead, we combine the processed premium from each year, and we combine the reported losses from each year. The total processed premium is \$2,117,500,000. The total reported losses are \$1,700,000,000. The total processed is \$817,500,000. We form the equation

\$1,700,000,000	_ X
\$2,117,500,000	- \$817,500,000

We solve for X, the total bulk reserve, as

$$X = \$1,700,000,000 \times \$817,500,000/\$2,117,5000,000$$
  
= \\$656,300,000.

#### 2. ADJUSTED PREMIUMS

The premium adjustments differ for dollars of loss versus number of claims. We explain the premium adjustments by means of a series of illustrations. The experience period consists of two accident years, 20X1 and 20X2, with premium of \$100,000,000 in 20X1 and \$120,000,000 in 20X2. For simplicity, all policies are effective on January 1, and all rate changes occur on January 1; we relax these assumptions at the end of Section 2.1. In each illustration, we adjust the earned premiums to the same adequate level. Unless specified otherwise, text and formulas apply to dollars of loss, not the number of claims.

## Illustration 1: Rate Change

Earned premium is 100,000,000 in 20X1 and 120,000,000 in 20X2. On January 1, 20X2, there was a +10% rate change. The exposure base is not inflation-sensitive. There is no loss trend: neither a loss severity trend nor a loss frequency trend.

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- A. If the 20X1 premiums are exactly adequate, the 20X2 premiums are 10% redundant. To bring the premiums to the same adequacy level, we divide the 20X2 premiums by 1.100.
- B. If the 20X2 premiums are exactly adequate, the 20X1 premiums are deficient by a factor of 1/1.100. To bring the premiums to the same adequacy level, we multiply the 20X1 premiums by 1.100. These two scenarios give the same result in the Stanard-Bühlmann technique. Multiplying the numerator of a ratio by a constant has the same effect as dividing the denominator of the ratio by the same constant.
- C. There are a variety of other possibilities. The 20X1 premiums might be deficient by 5% or they might be redundant by 5%. They all lead to the same Stanard-Bühlmann result.

Given the various possibilities, which should we choose? The actuarial convention is to leave the most recent year unadjusted and to adjust prior years to the level of the most recent year.<sup>5</sup> We multiply the 20X1 premium by unity plus the January 1, 20X2, rate change amount.

These various scenarios give the same result in the Stanard-Bühlmann technique. If all premiums are at the same adequacy level, we can multiply all premiums by a constant Z to convert premiums into expected losses. Suppose the expected loss ratio is 70%, the 20X1 premiums are exactly adequate, and the 20X2 premiums are 10% redundant.

1. If we multiply the 20X1 premium by 1.100, the premiums in both years are 10% redundant. The value of Z

<sup>&</sup>lt;sup>5</sup>This is a general actuarial convention. The readers of the reserving actuary's report may not understand the Stanard-Bühlmann technique. In most situations, other company personnel believe that the current year is "correct." It is easier to explain an adjustment of prior years to the adequacy level of the current year than to explain an adjustment of the current year to the adequacy level of past years.

is 70%/1.100. In combination, we have multiplied the 20X1 premium by  $1.100 \times 70\%/1.100 = 70\%$ . We have multiplied the 20X2 premium by 70%/1.100.

2. If we divide the 20X2 premium by 1.100, the premiums in both years are exactly adequate. The value of Z is 70%. In combination, we have multiplied 20X1 premium by 70%. We have multiplied the 20X2 premium by 70%/1.100.

### Illustration 2: Loss Trends

The loss severity trend is +10% per annum. There have been no rate changes, and the exposure base is not inflation-sensitive.

- A. If the 20X1 premium is adequate, the 20X2 premium is deficient by 10%, since losses increased by 10% per exposure unit in 20X2 and there was no rate change. We multiply the 20X2 premiums by 1.100 to bring them to an adequate level.
- B. If the 20X2 premium is adequate, the 20X1 premium was redundant, since the 20X2 losses were 10% higher per exposure unit and there was no rate change. We divide the 20X1 premiums by 1.100 to bring them to an adequate level.
- C. The absolute premium adequacy level does not affect the result. By convention, we adjust prior years' premiums to the adequacy level of the most recent year.

In general, we determine the loss cost trend factors to bring prior years' losses to the level of the most recent year, and we divide the prior years' premiums by the trend factors.

## Illustration 3: Rate Changes and Loss Trends

The loss severity trend is +10% per annum. A rate change of +10% was effective on January 1, 20X2. The exposure base is not inflation-sensitive.

Both premium rates and losses increased by 10% between the two years. The premiums are at the same adequacy level. Using the general rules, we multiply the 20X1 premium by 1.100 for the rate change, and we divide the 20X1 premium by 1.100 for the loss trend. The net adjustment is no change.

#### Illustration 4: Exposure Trends

The loss severity trend is +10% per annum. The exposure base is inflation-sensitive, and the exposure trend is 10% per annum. No rate changes were taken.

The exposure trend of +10% offsets the loss cost trend of +10%. We conceive of an exposure trend as the reciprocal of a loss cost trend. The net trend is 0% per annum.

#### 2.1. General Rules

Premiums: The illustrations above assume January 1 effective dates for rate changes and policies. That is not necessary. Rather, we determine calendar year on-level factors to bring the earned premium in each calendar year to the current rate level.<sup>6</sup>

The loss severity trend is 0%. Policies are written evenly through the year. We took a rate change of +10% on July 1, 20X1. The exposure base is not inflation-sensitive.

The calendar year on-level factors are 1.075 for 20X1 and 1.025 for 20X2. We multiply the 20X1 premium by 1.075 and the 20X2 premium by 1.025.

Losses: We trend all losses to a common date with the net trend factors. The net trend equals the loss frequency trend times

<sup>&</sup>lt;sup>6</sup>The Stanard-Bühlmann technique is commonly used by reinsurance actuaries. Most excess-of-loss reinsurance treaties are effective on January 1, and reinsurance rate changes are effective on January 1 as well. The underlying policies written by the ceding company may be written evenly during the year, and the ceding company's rate changes may have occurred during the year. The on-level factors are taken into account to determine the reinsurance rate changes; they need not be recomputed for the reserve estimate.

the loss severity trend divided by the exposure trend. However, we adjust the premiums, not the losses, so we divide the premiums by the net trend factors.

## 2.2. Claim Counts

The Stanard-Bühlmann technique can be used with claim counts in place of dollars of loss. Suppose claims are reported quickly but claim severities are highly variable and may remain uncertain for many years. The reserving actuary may project ultimate claims by a development procedure and the average claim severity by a trend procedure.

Illustration: Workers compensation permanent disability claims are reported quickly, though it may take years before the severity of the injury is clear. The claims are paid over the remaining lifetime of the injured worker. Both the indemnity (loss of income) benefits and the medical benefits extend over decades, and they are difficult to estimate.

The reserving actuary may project ultimate claim counts by a development year procedure and ultimate claim severities by an accident year trend. Suppose we are estimating accident year 20X9 workers compensation reserves for permanent disability claims. Within a year or two after the expiration of the 20X9 accident year, we have a preliminary estimate of the ultimate claim count. Since we have only a year or two of payments on these claims—each of which may extend for 20 or 30 years—we cannot estimate claim severities from the 20X9 data.

Instead, we estimate ultimate claim severities for the more mature accident years, such as 20X0 through 20X7. We use the workers compensation loss cost trend factors derived from shorter-term injuries to extend the claim severity trend through 20X9. This procedure is suited for excess-of-loss reinsurance reserving, since most of the claims are permanent injuries.

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#### Illustration 5: Claim Frequencies

When we deal with reported losses, the ratio of reported losses to unreported losses is set equal to the ratio of processed premium to unprocessed premium. The unreported losses are the bulk reserve. When we deal with reported claims, the ratio of reported claims to unreported claims is set equal to the ratio of processed premium to unprocessed premium.

Premium is \$100,000,000 in 20X1 and \$120,000,000 in 20X2. Policies are effective on January 1, there have been no rate changes, and the exposure base is not inflation-sensitive. The loss severity trend is  $\pm 10\%$  per annum. We use the Stanard-Bühlmann technique to estimate ultimate claim counts.

We comment on the meaning of premium adequacy with respect to claim counts. If the expected claim frequency is 100 claims for each \$1,000,000 of premium in 20X1, then 20X2 has the same level of premium adequacy if the expected claim frequency is still 100 claims for each \$1,000,000 of premium.

In Illustration 5, there were no rate changes in 20X1 or 20X2. If there were no changes in expected claim frequency, the premiums in 20X1 and 20X2 are at the same level of adequacy with respect to claim frequency.

If the average loss severity rose by 10% from 20X1 to 20X2, the premiums in the two years are not at the same level of adequacy with respect to dollars of losses. For the Stanard-Bühlmann method, we use a premium adjustment if we are dealing with reported losses. We make no premium adjustment in this case if we are dealing with reported claims.

# Illustration 6: Frequency and Severity Trends

The loss cost trend is +10% per annum, consisting of 7.8% claim severity trend and a 2.0% claim frequency trend. There have been no rate changes, and the exposure base is not inflation-sensitive.

To estimate ultimate losses, we use the total loss cost trend of +10% per annum. To estimate ultimate claim counts, we use the claim frequency trend of 2.0% per annum.

Pricing actuaries have learned to be wary of claim frequency trends. In most lines of business, claim frequency does not follow simple exponential growth patterns. Econometric modeling of claim frequency has generally been disappointing. One might wonder how useful the claim frequency trends would be for the Stanard-Bühlmann reserving technique.

The pricing actuary and the reserving actuary use the trend factors for different purposes. The pricing actuary is projecting future claim frequency; most trend estimates have been poor predictors. The reserving actuary is quantifying the change between two past years. The claim frequency is a historical figure; it is not better or worse than the historical loss cost trend.

- 1. If we are given both claim frequency trends and claim severity trends, we use the product of these trends when we deal with dollars of loss. When we deal with claim counts, we use only the claim frequency trends.
- 2. If we have a single loss cost trend, we use the claim frequency portion of the trend. If we do not know the claim frequency portion, we might estimate the claim severity portion from other indices and "back out" the claim severity portion to derive the claim frequency portion.
- 3. The loss frequency trends in the historical data may reflect shifts in the mix of business, not real changes in claim frequency. Such trends may not be used in pricing, though they may be appropriate for aggregate reserving analyses.
- 4. For some lines of business, the exposure trends largely offset the loss severity trends, and the net trend is not material. When we are dealing with claim counts, we

ignore loss severity trends but we still include exposure trends to calculate the premium adjustments.

Illustration: Payroll in 20X1 is \$100,000,000. The workers compensation premium rate is 2% of payroll, giving a premium of \$2,000,000. Employment stays the same for 20X2. Wage inflation is 10% per annum, so payroll is \$110,000,000 and premium is \$2,200,000. If nothing has changed in the physical plant, we expect the same number of claims. We increase the 20X1 premiums by +10% to bring them to the adequacy level of the 20X2 premiums.

#### 3. CLAIM COUNTS VS. LOSS DOLLARS

We illustrate the Stanard-Bühlmann method's premium adjustments by calculating first an IBNR claim count and then an IBNR loss reserve. In Table 3.1, all policies have effective dates of January 1, and all rate changes occur on January 1. We consider two separate problems: one using the column Reported Claims (Scenario A) and the other using the column Reported Losses (Scenario B).

For clarity, the loss dollars are \$1,000 times the claim count in each year; only the premium adjustments differ between the two scenarios. The processed premium differs for each year because only the losses have a trend, not the claim counts. These are "either-or" columns for two different scenarios; they are not the claim counts and losses in a single scenario.

Annual loss trends and rate changes are shown in Table 3.2 (Scenario A). There is no exposure trend.

There are two premium adjustments: one for rate changes and another for trend. We bring all premiums to the same rate level, and we divide by the appropriate trend factors.

Because all policies are effective on January 1, the rate change on January 1, 20X1 affects all years equally. In Table 3.3 we set

Cal./Acc. Year	Pure Premium	Reporting Percentage	Reported Claims at 12/31/20X5	Reported Losses at 12/31/20X5
20X1	\$40,000	38%	9	\$9,000
20X2	\$44,000	28%	8	\$8,000
20X3	\$40,000	18%	8	\$8,000
20X4	\$45,000	9%	5	\$5,000
20X5	\$50,000	2%	1	\$1,000

TABLE 3.1

TABLE 3.2

Loss Severity Trends		Rate Changes		
20X0 to 20X1	15.0%	1/1/20X1	30.0%	
20X1 to 20X2	12.5%	1/1/20X2	10.0%	
20X2 to 20X3	10.0%	1/1/20X3	-10.0%	
20X3 to 20X4	10.0%	1/1/20X4	0.0%	
20X4 to 20X5	10.0%	1/1/20X5	5.0%	

TABLE 3.3

Date	Rate Change	Rate Level Index	On-Level Factor
1/1/20X1	30.0%	1.0000	1.0395
1/1/20X2	10.0%	1.1000	0.9450
1/1/20X3	-10.0%	0.9900	1.0500
1/1/20X4	0.0%	0.9900	1.0500
1/1/20X5	5.0%	1.0395	1.0000

the 20X1 rate level to a rate level of 1.000, and we use the other rate changes to bring premiums to the current rate level.

Scenario A—Number of Claims: The rate level index is the cumulative downward product of the rate changes. (When the policy effective dates are distributed through the year and the rate changes occur on different dates, the rate level index is the average rate level during the year.) The on-level factor is the current rate level index divided by the rate level index for the

		I	

Cal./Acc. Year	Pure Premium	On-Level Factor	Adjusted Premium	Reporting Percentage	Processed Premium at 12/31/20X5	Unprocessed Premium at 12/31/20X5
20X1 20X2 20X3 20X4 20X5	\$40,000 \$44,000 \$40,000 \$45,000 \$50,000	1.0395 0.9450 1.0500 1.0500 1.0000	\$41,580 \$41,580 \$42,000 \$47,250 \$50,000	38.0% 28.0% 18.0% 9.0% 2.0%	\$15,800.40 \$11,642.40 \$7,560.00 \$4,252.50 \$1,000.00	\$25,779.60 \$29,937.60 \$34,440.00 \$42,997.50 \$49,000.00
Total					\$40,255.30	\$182,154.70

TABLE 3.4

accident year under consideration. We multiply the premiums by the on-level factors to put all premiums on the same adequacy level. The loss trend is a severity trend; we assume that the claim frequency trend is zero. We use the severity trend adjustment when dealing with loss dollars, not claim counts.

In the claim amount scenario in Table 3.4, 31 claims are reported by December 31, 20X5. We determine the total processed premium and the total unprocessed premium.

The claims expected to emerge in the future are  $31 \times$  \$182,154.70/\$40,255.30 = 140.

The reserve indication has great uncertainty. From 31 claims that have been reported so far, we are estimating future emergence of 140 claims. The volatility of the reported claim counts can be seen by a comparison of accident years 20X1 and 20X3. As of December 31, 20X5, the processed adjusted premium for 20X1 is \$15,800 and 9 claims have been reported, while the processed adjusted premium for 20X3 is \$7,560 and 8 claims have been reported.<sup>7</sup>

Scenario B—Dollars of Loss: In Table 3.5 we adjust for the loss severity trend by forming an index of relative loss costs,

<sup>&</sup>lt;sup>7</sup>The reserve indication is for five accident years only. For the oldest year in the experience period, only 38% of claims have been reported so far. If the company had business in preceding years, we would still expect much claim emergence for these older years.
Period	Loss Trend	Index Value	Trend Factor
20X0 to 20X1	15.0%	1.0000	1.497
20X1 to 20X2	12.5%	1.1250	1.331
20X2 to 20X3	10.0%	1.2375	1.210
20X3 to 20X4	10.0%	1.3613	1.100
20X4 to 20X5	10.0%	1.4974	1.000

TABLE 3.5

TABLE 3.6

Cal./Acc. Year	Pure Premium	On-Level Factor	Trend Factor	Adjusted Premium	Reporting Percentage	Processed Premium at 12/31/20X5	Unprocessed Premium at 12/31/20X5
20X1 20X2 20X3 20X4 20X5	\$40,000 \$44,000 \$40,000 \$45,000 \$50,000	$\begin{array}{c} 1.0395\\ 0.9450\\ 1.0500\\ 1.0500\\ 1.0000\end{array}$	1.497 1.331 1.210 1.100 1.000	\$27,776 \$31,240 \$34,711 \$42,955 \$50,000	38% 28% 18% 9% 2%	\$10,554.71 \$8,747.11 \$6,247.93 \$3,865.91 \$1,000.00	\$17,220.84 \$22,492.56 \$28,462.81 \$39,088.64 \$49,000.00
Total						\$30,427.66	\$156,276.85

using 20X1 as the base year. The loss trend from 20X0 to 20X1 affects all years equally. The index value for 20X1 is unity, the index value for 20X2 is 1.125, and so forth. The trend factor is the index value for the most recent year divided by the index value for the year under consideration. If we adjust losses to the current level, we multiply by these trend factors. Since we are adjusting premiums here, we divide by the trend factors.

In the claim count scenario, \$31,000 of losses are reported by December 31, 20X5. The premium adjustment factors are the on-level factors calculated for rate changes divided by the trend factors. We determine the total processed premium and total unprocessed premium in Table 3.6.

The bulk loss reserve is  $31,000 \times 156,276.85/30,427.66 = 159,216.40$ .

### 3.1. Calendar Year Emergence

So far we have examined the future emergence of losses, both new claims + adverse development on known claims = bulk reserve, and the future payment of losses, or the total (case + bulk) reserve. The reserving actuary may be asked to show the expected emergence and payment of losses by development period (i.e., by calendar period) subsequent to the valuation date. The emergence and payment patterns have several uses.

- 1. Reserving: The loss emergence and loss payment in the next calendar period provide a check on the accuracy of the reserve indication. It is difficult to judge the loss reserve indication itself, since the losses may not emerge or settle for many years. By comparing the actual emergence or settlement in the next calendar period with the estimates implied by the reserve indication, one gets a better feel for the accuracy or bias in the indication.
- 2. Investments: The expected emergence and settlement of claims is necessary for asset/liability management. The insurer's investment department seeks expected liability cash flows in the coming months to optimize its investment strategy. Many insurers structure their investment portfolio in accordance with their insurance liabilities, selecting security types, fixed-income durations, and investment quality to best manage their overall risk. The reserving actuary provides the settlement patterns for the loss reserve portfolio.
- The bulk reserve as of December 31, 20XX, equals the losses expected to emerge in calendar years 20XX+1 and subsequent for accident years 20XX and prior.
- The expected emergence in 20XX+1 equals the losses expected to emerge in calendar years 20XX+1 for only accident years 20XX and prior.

Cal./Acc. Year	Adjusted Earned	Difference in	Premium
	Premium	Percent Reported	Processed in 20X6
20X1	\$41,580	10.0%	\$4,158
20X2	\$41,580	10.0%	\$4,158
20X3	\$42,000	10.0%	\$4,200
20X4	\$47,250	9.0%	\$4,253
20X5 20X1–20X5	\$50,000	7.0%	\$3,500 \$20,269

TABLE 3.7

We illustrate the method using Table 3.7. We calculate the number of claims expected to emerge for accident years 20X1 through 20X5 during calendar year 20X6.

We estimate the adjusted premium that will be processed in 20X6. For any accident year, the adjusted premium that will be processed in 20X6 is the adjusted premium for that accident year times the difference in the report lag between that accident year and the previous year. For example, the 20X2 adjusted premium processed in 20X6 is  $41,580 \times (38.0\% - 28.0\%) = 4,158$ .

We do not know the difference between the reporting percentage as of 60 months and 72 months. For the other figures in Table 3.7, we estimate this difference as 10%.

The adjusted premium for 20X1 through 20X5 processed in 20X6 is \$20,269. The estimated claim emergence in 20X6 is  $31 \times \frac{20,269}{40,255} = 16$  claims.

#### 4. RESERVE ASSUMPTIONS: CHAIN LADDER VS. EXPECTED LOSS

Chain ladder reserving methods work better in some situations and expected loss methods work better in others. We examine the perspective of each type of method, so that we may judge when a Stanard-Bühlmann reserving method is most appropriate. Brosius, following Hugh White's discussion of the BornhuetterFerguson paper, explains the differing philosophy of the chain ladder versus the expected loss reserving methods.<sup>8</sup>

- The chain ladder method assumes that unusually high or low cumulative paid losses to date are indicative of similar high or low paid losses in future development periods.
- The Bornhuetter-Ferguson method assumes that unusually high or loss cumulative paid losses to date reflect random loss fluctuations. They are not indicative of unusually high or low paid losses in the remaining development periods.

As Brosius points out, the truth is generally in between these two alternatives. Yet the extreme cases interest us, because certain attributes of the insurance scenario argue for one or the other of these cases.

- When losses are very immature, or when loss severity is large but loss frequency is low, or when the variability of losses is unusually great, the Bornhuetter-Ferguson expected loss method may be favored.
- When losses are mature, or when loss severity is low but loss frequency is high, or when the variability of losses is small, the chain ladder method may be favored.

Excess of loss reinsurance has the former attributes, so many reinsurance actuaries are inclined to use expected loss reserving procedures. Since the reinsurance actuary may not have a good sense of the expected loss ratio, the Stanard-Bühlmann method is often used.

### 4.1. Accident Year Weights

James Stanard [1985] provides another perspective on the Stanard-Bühlmann method (which he refers to as the "Adjustment to Total Known Losses Method" or the "Cape Cod

<sup>&</sup>lt;sup>8</sup>See Bornhuetter-Ferguson [1972] and Brosius [1993]. Brosius presents a statistical procedure for selecting the base that allows for multiple bases, such as 60% of one base plus 40% of another base, and he determines the optimal percent of each.

Method"). The Stanard-Bühlmann method estimates the expected losses from the historical data. There is a simpler method of doing this, but a comparison with the Stanard-Bühlmann method shows one of the latter's advantages.

Given the historical data, the chain ladder method estimates the total losses for each accident year. If there is no trend or exposure change from year to year, we can estimate the expected losses as the simple average of the estimated incurred losses for each accident year and then use a Bornhuetter-Ferguson loss reserving method. The Stanard-Bühlmann reserving method does the same, except that it uses a weighted average, where the weights are the reporting percentage for each accident year. This gives more weight to older accident years, for which the total incurred loss is more certain.<sup>9</sup>

We use two illustrations. Scenario A has actual losses equal to expected losses in each year; the chain ladder, Bornhuetter-Ferguson, and Stanard-Bühlmann reserving methods give the same reserve indication. Scenario B switches the actual losses between the oldest accident year and the most recent accident year. The total actual losses remain the same, but more than expected occur in the most recent accident year and fewer than expected occur in the oldest accident year.

- The chain ladder method treats each accident year independently. The most recent accident year has the largest loss development factor, so shifting more of the actual losses in that year increases the reserve indication.
- If we use a straight average of the indicated reserves by accident year as the expected losses for the Bornhuetter-Ferguson method, the expected loss for each accident year is higher than in Scenario A. But because the Bornhuetter-Ferguson gives

<sup>&</sup>lt;sup>9</sup>If the exposures differ by accident year or if there is a loss trend, the accident years must be put on equal cost and exposure bases before taking an average. Exposures are known for each accident year. Placing accident years on the same cost level is discussed in Section 2. For simplicity we do not repeat these adjustments here.

Chain Ladder	Adjusted Pure Premium	Reporting Percentage	Reported Losses	LDF – 1	Bulk Reserve
20X1 20X2 20X3	\$100,000 \$100,000 \$100,000	80% 50% 20%	\$80,000 \$50,000 \$20,000	0.25 1.00 4.00	\$20,000 \$50,000 \$80,000
Total	\$300,000		\$150,000		\$150,000

DETERMINATION OF BULK RESERVE-CHAIN LADDER METHOD

less leverage to the actual losses in determining the reserve, the indicated reserve is lower than the chain ladder indication.

• The Stanard-Bühlmann reserving method weights the chain ladder reserve indications by the reporting percentage in each accident year. If reported losses do not change, the estimated ultimate losses do not change.<sup>10</sup>

Illustration—Scenario A: In Table 4.1 suppose the adjusted pure premiums are \$100,000 in each of three accident years. The reporting percentages are 20%, 50%, and 80% as of 12, 24, and 36 months from inception of the accident year. The reported claims in the past three accident years are \$80,000, \$50,000, and \$20,000 at the end of the most recent accident year.

There are no loss trend or exposure changes from year to year. The chain ladder, Bornhuetter-Ferguson, and Stanard-Bühlmann methods give the same reserve indication of \$150,000.

For the chain ladder method, the loss development factor equals the reciprocal of the reporting percentage. The incurred loss is the reported loss times the loss development factor, and the bulk reserve is the reported loss times (LDF - 1).

<sup>&</sup>lt;sup>10</sup>One Review team member of the CAS Committee on Review of Papers noted that "in SB the exact relationship by year of reported losses to adjusted premiums does not matter. That may be considered an advantage or disadvantage of the method."

# DETERMINATION OF BULK RESERVE-BORNHUETTER-FERGUSON

Bornhuetter-	Adjusted Pure	Reporting	Reported	Expected	Bulk Reserve
Ferguson	Premium	Percentage	Losses	Loss	
20X1	\$100,000	80%	\$80,000	\$100,000	\$20,000
20X2	\$100,000	50%	\$50,000	\$100,000	\$50,000
20X3	\$100,000	20%	\$20,000	\$100,000	\$80,000
Total	\$300,000		\$150,000		\$150,000

### TABLE 4.3

DETERMINATION OF BULK RESERVE-STANARD-BÜHLMANN

Stanard-	Adjusted Pure	Reporting	Reported	Processed	Unprocessed
Bühlmann	Premium	Percentage	Losses	Premium	Premium
20X1	\$100,000	80%	\$80,000	\$80,000	\$20,000
20X2	\$100,000	50%	\$50,000	\$50,000	\$50,000
20X3	\$100,000	20%	\$20,000	\$20,000	\$80,000
Total	\$300,000		\$150,000	\$150,000	\$150,000

The chain ladder method estimates the total incurred loss for all three years as 150,000 (reported loss) + 150,000 (bulk reserves) = 300,000.

The Bornhuetter-Ferguson bulk reserve is the expected loss times the complement of the reporting percentage (see Table 4.2).

The Stanard-Bühlmann indicated reserve is  $150,000 \times 150,000 = 150,000$  (see Table 4.3).

Illustration—Scenario B: In Table 4.4 we switch the reported losses in 20X1 and 20X3. The chain ladder method bases the reserve for each accident year directly on the reported losses in that year. Since the most recent year has the highest loss devel-

### DETERMINATION OF BULK RESERVE-CHAIN LADDER METHOD

Chain Ladder	Adjusted Pure Premium	Reporting Percentage	Reported Losses	LDF – 1	Bulk Reserve
20X1 20X2 20X3	\$100,000 \$100,000 \$100,000	80% 50% 20%	\$20,000 \$50,000 \$80,000	0.25 1.00 4.00	\$5,000 \$50,000 \$320,000
Total	\$300,000		\$150,000		\$375,000

### TABLE 4.5

# DETERMINATION OF BULK RESERVE-BORNHUETTER-FERGUSON

Bornhuetter-	Adjusted Pure	Reporting	Reported	Expected	Bulk Reserve
Ferguson	Premium	Percentage	Losses	Loss	
20X1	\$100,000	80%	\$20,000	\$175,000	\$35,000
20X2	\$100,000	50%	\$50,000	\$175,000	\$87,500
20X3	\$100,000	20%	\$80,000	\$175,000	\$140,000
Total	\$300,000		\$150,000		\$262,500

opment factor, it has the most leverage, and the reserve indication is higher.

The Bornhuetter-Ferguson method smooths the effects of random loss fluctuations among the years. If we use a straight average of the chain ladder estimates of ultimate losses as the estimate of the expected losses, the Bornhuetter-Ferguson reserve indication increases, though not as much as the chain ladder indication does (see Table 4.5).

The Stanard-Bühlmann reserving method weights the estimated losses in each accident year by the reporting percentage in that year to get its estimate of expected losses. The chain ladder estimated losses above are \$25,000 for 20X1, \$100,000 for 20X2, and \$400,000 for 20X3. Weighting these estimates by the

Stanard-	Adjusted Pure	Reporting	Reported	Processed	Unprocessed
Bühlmann	Premium	Percentage	Losses	Premium	Premium
20X1	\$100,000	80%	\$20,000	\$80,000	\$20,000
20X2	\$100,000	50%	\$50,000	\$50,000	\$50,000
20X3	\$100,000	20%	\$80,000	\$20,000	\$80,000
Total	\$300,000		\$150,000	\$150,000	\$150,000

DETERMINATION OF BULK RESERVE-STANARD-BÜHLMANN

percentage reported (80%, 50%, and 20%) gives estimated expected losses of \$100,000. Changes to reported losses by year are divided by the percentage reported to estimate the change in ultimate losses. The ultimate losses are weighted by the percentage reported, so, if the changes to reported losses in different years offset one another, there is no change to the estimated expected losses (see Table 4.6).

### 5. SUMMARY

The Stanard-Bühlmann reserving technique is a simple, intuitive procedure that works well even in situations that don't lend themselves to easy estimates, such as reserving for high layers of loss. It is most useful for recent accident years in lines of business where random loss fluctuations preclude the use of chain ladder reserving methods but uncertainty about the excess loss ratio precludes the use of a Bornhuetter-Ferguson expected loss reserving method.

Illustration: Losses may not be reported on casualty excessof-loss reinsurance treaties until several years after the accident date, so a chain ladder reserving method is not appropriate for the most recent two or three accident years. In a chain ladder method, we apply the loss development factors to the reported losses for that accident year. If the loss development factors are very high (say, more than 10.000) and the expected reported losses at a early maturity are very low, a change of \$1,000,000 in the reported losses causes a change greater than \$10,000,000 in the estimate of incurred losses.

But the Bornhuetter-Ferguson expected loss reserving method may also be inappropriate. If the reinsurer is not sufficiently familiar with the book of business and the underwriting practices of the ceding company, the reserving actuary may not know the expected loss ratio to determine expected losses.

Reinsurance actuaries often use a Stanard-Bühlmann reserving method in this situation. If the reinsurer has enough years of historical data, the Stanard-Bühlmann method derives the expected loss ratio from the actual experience. This practitioner's guide should encourage the use of the Stanard-Bühlmann reserving method by primary company actuaries for volatile lines of business, in addition to its current use by reinsurance actuaries.

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#### APPENDIX A

### PATTERNS AND PROJECTIONS

This appendix discusses patterns of stability in the historical data. It is geared to the student, though even the experienced reserving actuary may find the review useful.

#### A.1. Weighted vs. Unweighted Averages

The prospective future pattern is based on the observed patterns. One may use either unweighted or weighted averages of historical observations. There are two reasons for using weighted averages: shifting risk parameters and credibility considerations.

Shifting Risk Parameters: If a more recent year is a better predictor of future experience than an older year, more recent accident years should receive more weight than older accident years. This approach is most important when trends appear in the columns of age-to-age factors.

The consideration of shifting risk parameters is particularly applicable to loss reserving, since the covariance matrix can be estimated from the experience; see Mahler [1990, 1998]. In many scenarios, a broad range of credibility values is close to optimal. A weighting system may be selected one year and used for subsequent reviews as well.

Credibility: The experience years should be weighted in proportion to the real volume of business. The loss amounts in each year differ for two reasons: (i) the real dollar amount of losses may differ, and (ii) inflation causes the nominal amount of losses to differ.

Ideally, one should use weighted averages of the observed link ratios, where the weights are the deflated dollars of loss. If deflated triangles are used in the reserve analysis, weighted averages based on dollars of loss at the earlier of the two development periods should be used. If nominal loss triangles are used, the

Accident Year	At 12 months	At 24 months	Link Ratio
20X1	\$10,000,000	\$15,000,000	1.500
20X2	\$20,000,000	\$25,000,000	1.250
20X3	\$30,000,000	\$60,000,000	2.000
Total	\$60,000,000	\$100,000,000	(see text)

TABLE A.1

following rule is a reasonable guide. When the dollar amount of losses is consistent with monetary inflation, one should use unweighted averages. When the dollar amount of losses is considerably different from monetary inflation, one should use weighted averages, with an adjustment to offset monetary inflation.

When the weights are the same as the measurement base (e.g., the weights are the losses at the start of the period for the chain ladder link ratios) the weighted average may be computed by summing the losses for several years as the numerators and denominators of the link ratios.

Illustration: Cumulative paid losses in deflated dollars are shown in Table A.1.

The unweighted average of the link ratios is (1.500 + 1.250 + 2.000)/3 = 1.583. The weighted average is most easily determined as the sum of losses at 24 months divided by the sum of losses at 12 months: 100,000,000/\$60,000,000 = 1.667.

Suppose that the covariance matrix to determine optimal weights based on shifting risk parameters gives weights of 20%–30%–50%. If there were no changes in the volume of business by year, the weighted link ratio would be  $20\% \times 1.500 + 30\% \times 1.250 + 50\% \times 2.000 = 1.675$ . In the scenario given above, where the deflated losses are increasing, the weighted link ratio is  $(20\% \times 15 + 30\% \times 25 + 50\% \times 60)/(20\% \times 10 + 30\% \times 20 + 50\% \times 30) = 1.761$ .

Nominal Dollars: If the dollars in the table above are not adjusted for inflation, and the loss cost trend is 10% per annum,

the weighted average (not adjusted for shifting risk parameters) is  $(10 \times 1.500 + 20/1.100 \times 1.250 + 30/1.100^2 \times 2.000)/(10 + 20/1.100 + 30/1.100^2) = 1.648.$ 

#### A.2. Inflation

Changing inflation rates may bias the projected pattern. The effects of inflation are most significant for the long-tailed commercial casualty lines of business.

Illustration: We are using a paid loss development method for workers compensation (WC) medical benefits. The workers compensation medical benefits severity trend has been 8% per annum during the experience period. Medical inflation has recently risen, and we expect the future workers compensation medical benefits severity trend to be 12% per annum.

In the company's book of business, WC medical benefits are paid about three years after the accident date (on average). However, many medical cases close early. The time until payment for the medical loss reserves is five years, on average. We assume that medical benefits are affected by inflation through the payment date; see Butsic [1981].

The change in the medical severity trend raises the reserve indication by  $(1.120/1.080)^5 = 1.199$ , or about 20%. If no adjustment to the reserving procedure is made to account for the change in the inflation rate, the reserve indication may be severely understated.

To correct for changes in the inflation rate, one may deflate the historical triangle for past inflation, perform the actuarial analysis on "real dollar" figures, and project forward with future expected inflation or stochastic inflation rate paths.<sup>11</sup>

<sup>&</sup>lt;sup>11</sup>Hodes, Feldblum, and Blumsohn [1999] use an interest rate generator and a stochastic inflation model with a probability distribution of loss realizations in future calendar years. Feldblum [2002] summarizes the procedure, with an application to Schedule P loss reserve monitoring.

### A.3. Trend, Outliers, and Credibility

Trend: When the insurance environment is changing, one might trend the historical figures. Examples are changing attorney involvement in private passenger automobile claims and changing claims management practices in workers compensation insurance.<sup>12</sup>

Outliers: To eliminate outliers, one might use averages that discard the high value and the low value. When discarding outliers, one must be careful not to introduce additional bias.

- If the distribution of link ratios is skewed, eliminating outliers gives a biased average.
- The chain ladder reserving method may have an inherent bias; see Stanard [1985]. The elimination of outliers may offset some of this bias.

Credibility: For small insurers, one might weight company averages with industry averages, or state averages with country-wide averages; see Graves and Castillo [1990].

### A.4. Stability Patterns: Derivation vs. Application

The derivation of the stability pattern is similar for all reserving methods. Once we determine any one pattern, we have determined the other patterns as well. One sometimes hears that chain ladder methods and expected loss methods both start with the observed link ratios. We could equally well say that the methods start with the observed percentages of ultimate.

It is in the application of the patterns that the reserving methods differ.

• Chain ladder methods apply the factors to the cumulative paid or reported losses for each experience year. We do not use the estimated ultimate losses.

<sup>&</sup>lt;sup>12</sup>Feldblum [2005] discusses trends in workers compensation loss development factors in the 1980s and 1990s.

• Expected loss methods apply the factors to the estimated ultimate losses. We do not use the cumulative paid or reported losses for each experience year.

In the example in Section 1 (Table 1.2), the paid losses in the first 12 months equal 18.2% of the estimated ultimate paid losses. Suppose we are using this historical pattern to estimate the needed reserves for a more recent accident year. What if the paid losses in the first 12 months of this accident year equal 25% of the estimated ultimate losses, not 18.2% of the ultimate losses?

- A chain ladder method says: "Use the cumulative paid losses in the first 12 months; ignore the estimated ultimate losses."
- An expected loss method says: "Use the estimated ultimate losses; ignore the cumulative paid losses in the first 12 months."

### A.5. Determining the Pattern

If we determine the incremental ratios or the cumulative ratios, we know the percentages of ultimate. Conversely, if we determine the percentages of ultimate, we know the incremental ratios and the cumulative ratios. We ask: "Which is the easiest pattern to determine?" not "Which pattern do we want to use?"

If we try to determine the percentages of ultimate, we can't use all the data at our disposal. In particular, we can't use the most current data. If we try to determine the incremental ratios or the cumulative ratios, we use all the historical data, including the most recent data.

If we try to determine the percentages of ultimate directly, we can use only mature accident years that have developed to ultimate. The patterns may have changed in the intervening years, as the social, economic, and insurance environments changed.

If we use incremental ratios or cumulative ratios, we can use all accident years, including even the most recent calendar year information in each accident year. This was the advance in casualty loss reserving theory that gave rise to the chain ladder method.<sup>13</sup>

We still must choose between the incremental ratios and the cumulative ratios. At early development periods, neither method is clearly superior. At later development periods, the incremental losses are relatively small. Small figures in the numerators of the link ratios do not distort the estimation procedure. But small figures in the denominator cause ratios that may be unrealistically large, reducing the accuracy of the results and adding significant bias.

Illustration: Table A.2 shows reported loss development in thousands of dollars from ten years to twelve years. Table A.2, Part 1 has five accident years and five columns, showing

- cumulative reported losses at ten years of development,
- incremental reported losses in year eleven,
- cumulative reported losses at eleven years of development,
- incremental reported losses in year twelve, and
- cumulative reported losses at twelve years of development.

The age-to-age link ratio from year eleven to year twelve is stable when using cumulative reported losses but is not stable when using incremental reported losses (Table A.2, Part 2).

This is the rationale for the method of determining the pattern. All three reserving procedures discussed in the text—chain ladder, Bornhuetter-Ferguson, and Stanard-Bühlmann—begin by estimating link ratios (or cumulative age-to-age factors). Loss development factors are determined as the cumulative products of the link ratios.

<sup>&</sup>lt;sup>13</sup>Health actuaries often use "claim completion percentages," which are chain ladder paid loss development factors that rely on mature years only. Since medical claims are settled quickly, the reliance on mature experience periods is not onerous; see Bluhm [2000], chapter 30. For a typology of reserving procedures, see Salzmann [1984].

Accident Year (1)	Reported Losses at Ten Years (2)	Incremental Losses in Year Eleven (3)	Reported Losses at Eleven Yrs. (4)	Incremental Losses in Year Twelve (5)	Reported Losses at Twelve Yrs. (6)
20X0	100,000	$100 \\ 1,100 \\ 0 \\ -100 \\ 1$	100,100	1,100	101,200
20X1	110,000		111,100	0	111,100
20X2	120,000		120,000	1	120,001
20X3	130,000		129,900	1,100	131,000
20X4	140,000		140,001	100	140,101

# TABLE A.2

### Part 1

# TABLE A.2

# $Part \ 2$

Accident	Age-to-Age Factor using	Age-to-Age Factor using
Year	Cumulative Reported Losses	Incremental Reported Losses
(1)	(7) = (6)/(4)	(8) = (5)/(3)
20X0 20X1 20X2 20X3 20X4	1.011 1.000 1.000 1.008 1.001	$11.000 \\ 0.000 \\ \infty \\ -11.000 \\ 100.000$

- The reporting percentage is the percent of ultimate losses that are expected to have been reported by the development date.
- The paid loss percentage is the percent of ultimate losses that are expected to have been paid by the development date.
- The percentage of ultimate equals the reciprocal of the loss development factor.
- The Bornhuetter-Ferguson factor is the complement of the percentage of ultimate.

Development Months	12–24	24–36	36–48	48-60	60–Ult.
Link Ratio	1.500	1.250	1.100	1.050	1.020

TABLE A.3

TABLE A.4

Development Months	12	24	36	48	60
Link Ratio Loss Development Factor	1.500 2.209	1.250 1.473	1.100 1.178	1.050 1.071	1.020 1.020
Loss Lag B-F Factor	0.453	0.679	0.849	0.934	0.980
B-F Factor	0.547	0.521	0.151	0.000	0.020

Illustration: Reported loss link ratios for a block of business are shown in Table A.3. We compute the loss development factors, percentages of ultimate, and Bornhuetter-Ferguson factors.

The loss development factors are the cumulative products of the link ratios. The loss development factor from 12 months to ultimate equals

 $1.500 \times 1.250 \times 1.100 \times 1.050 \times 1.020 = 2.209.$ 

The percent of ultimate at 12 months equals 1/2.209 = 0.453. The Bornhuetter-Ferguson factor at 12 months equals 1 - 0.453 = 0.547 (see Table A.4).

### A.6. Actuarial Present Values

The Stanard-Bühlmann reserving method adds premiums and losses from different accident years. Adding dollars from two different years is problematic. In an inflationary economy, a dollar from year X is worth more than a dollar from year X + 1.

In theory, we ought to use present values. We can add present values of dollars that have been discounted or accumulated to the same date. If we know the present value of 20X1 premiums as of a given date and the present value of 20X2 premiums as of

the same date, we can add them to determine the present value of the combined premiums as of that date.

Calculating present values is not always easy, particularly for reported loss reserving methods. Case reserves are ultimate values, not present values. The reported losses in an accident year may be paid over a dozen years. In some lines of business, such as workers compensation and private passenger automobile nofault, even individual claim benefits are paid periodically over months or years. Similarly, the premiums may be collected over the policy term, and audit premiums may be collected several months later.

If the Stanard-Bühlmann reserving method were dependent on calculating present values, it would not be practical. But we don't always need the present values. We are comparing premiums to losses. We require only that the change in premiums from year to year should equal the change in expected losses from year to year. Two conditions suffice for this:

- i. The expense ratio stays constant from year to year, and
- ii. The premiums are at the same level of adequacy from year to year.

The adjustments to premium ensure the adequacy level remains constant from year to year. The constancy of the expense ratio is rarely an issue. Expense ratios don't change much from year to year, and we may assume that they stay constant. A significant change in expense ratios would necessitate additional premium adjustments, but such changes are not common.

We said above that "we don't always need present values." We might rephrase this to say that

since we are comparing premiums to losses, we can get away with adding nominal amounts from different years. We are not adding apples and oranges; we are

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adding golden delicious apples with McIntosh apples. It's not perfect, but it's a practical solution. The cost of getting present values is greater than the improved accuracy we may obtain.

# THE MINIMUM BIAS PROCEDURE: A PRACTITIONER'S GUIDE

#### SHOLOM FELDBLUM AND J. ERIC BROSIUS

# Abstract

The minimum bias classification ratemaking procedure, introduced by Robert Bailey and LeRoy Simon in 1960, determines rate relativities simultaneously for two or more classification dimensions. This paper summarizes the minimum bias procedure for the practicing actuary and provides the intuition for several bias functions: balance principle, least squares,  $\chi$ -squared, and maximum likelihood. The exposition is structured around a series of illustrations using a two-dimensional private passenger automobile classification system: male/female and urban/rural.

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### 1. THE MINIMUM BIAS PROCEDURE

### Introduction

This paper is geared to the practicing actuary or actuarial student seeking to optimize classification relativities. It provides the intuition underlying the minimum bias procedure along with simple illustrations to show the computations required for each method.

### Background

The minimum bias procedure was first introduced in a 1960 *Proceedings*<sup>1</sup> paper by Robert Bailey and LeRoy Simon, "Two Studies in Automobile Insurance." Bailey and Simon examined models with two types of arithmetic functions (multiplicative and additive), two types of bias functions (balance principle and  $\chi$ -squared), and two data types (loss costs and loss ratios).

Bailey and Simon used their procedure (i) to judge the merits of an additive versus a multiplicative classification model for Canadian private passenger automobile business and (ii) to choose optimal rate relativities.<sup>2</sup> They discuss the rationale for the minimum bias procedure, the characteristics of a suitable rating model, and the rating scenarios that fit the various types of models. The authors concluded that: (i) the additive model fits the Canada private passenger automobile data better than the multiplicative model, and (ii) the  $\chi$ -squared function is the optimal bias function. The first conclusion was based on a goodnessof-fit test; the second conclusion was based on the credibility assigned by the  $\chi$ -squared function.

In a 1963 *Proceedings* paper, "Insurance Rates with Minimum Bias," Robert Bailey summarized the minimum bias theory, outlining the considerations that support the use of the balance principle as the bias function and explaining when loss ratios serve better than loss costs. This paper was on the CAS examination syllabus for many years.

In a 1988 *Proceedings* paper, "Minimum Bias with Generalized Linear Models," Robert Brown expanded the minimum bias

<sup>&</sup>lt;sup>1</sup>References to the *Proceedings* are to the *Proceedings of the Casualty Actuarial Society*. <sup>2</sup>The minimum bias procedure deals with loss cost relativities, which we refer to here as pure premium relativities. In practice, actuaries determine rate relativities. The two types

of relativities may differ if expenses are not a fixed percentage of premiums.

method to use two additional types of bias function. Brown retained the balance principle and  $\chi$ -squared functions from the Bailey and Simon papers. He added a least squares function (similar to the  $\chi$ -squared function) and a maximum likelihood function, which assumes certain distributions of claim frequency or claim severity in the insured population. Brown also examined generalized linear models (GLM), which have potential statistical advantages and may accomplish the same objectives as the minimum bias procedures, though he did not find that they produced more accurate results.<sup>3</sup> For the Canadian private passenger automobile business, Brown found the multiplicative model superior to the additive model.

In 1990, Gary Venter introduced several extensions of the existing procedures in a discussion of Brown's paper, along with an analysis of credibility consideration and other modeling issues. Brown's *Proceedings* paper, along with Venter's discussion, was placed on the CAS actuarial syllabus in the mid-1990s.

These papers have proved difficult for practicing actuaries to understand and for actuarial candidates to master. The authors wrote for experienced actuaries who were familiar with the ratemaking issues and proficient with the statistical models.

This paper combines the theory of the original actuarial papers with the teaching material prepared by the authors and used to teach the minimum bias procedure to several hundred actuarial candidates since 1995. It explains the rationale for the procedure and shows its applications. It presents the method to new actuaries and gives them the background to read the original *Proceedings* papers.

The title of this paper is the "Minimum Bias Procedure," since that name is now common in the U.S. actuarial profession. The subject of this paper should more properly be described as the development of multidimensional classification systems. This sub-

<sup>&</sup>lt;sup>3</sup>On generalized linear models, see Feldblum et al. [forthcoming].

ject is broad. The paper covers part of this subject, of which one component is the minimum bias procedure and the alternative methods discussed here.

This paper does not cover generalized linear models, which are commonly used in the United Kingdom and in continental Europe for multidimensional classification ratemaking; see instead the companion paper by S. Feldblum, D. Anderson, E. Schirmacher, D. Schirmacher, and N. Thandi [forthcoming], "Generalized Linear Models: A Practitioner's Guide."

### 2. CLASSIFICATION MODELS

### Introduction

Before Bailey and Simon introduced the minimum bias procedure, classification relativities were determined one dimension at a time. This is suitable for a single-dimensional classification system. Workers compensation, for example, uses industry as the only classification dimension within each state. Insurers are now examining other classification dimensions for workers compensation; the minimum bias procedure and generalized linear models may prove valuable in this analysis.

The minimum bias procedure becomes useful when the classification system has multiple dimensions. In this paper, we use examples with two dimensions; the extension to three or more dimensions is straightforward, but the arithmetic and display are cumbersome.

We define the minimum bias terms, explain the statistical procedures, and review the intuition underlying each method. It is hard to grasp the intuition until one has a working knowledge of the methods. We provide the explanations alongside a series of heuristic illustrations.

The illustrations form the backbone of this paper. The basic illustration has two dimensions with two values in each dimension. This prevents the intuition from getting submerged under tedious mathematics. In practice, the minimum bias procedure is most useful for multidimensional classification systems with many entries in each dimension.

We show the computations for one iteration in each illustration, followed by the series of values until convergence. The illustrations here converge in a few steps. In practice, more iterations are needed for convergence of larger models. The work is tedious by hand but elementary with current spreadsheet applications. Some spreadsheets have built-in iterative functions, such as "goal-seek" and "solver" in Excel. Some software packages, such as SAS, have built-in routines for GLMs. Once the intuition is clear, the programming is not difficult.

# The Multiplicative Model

We are setting pure premiums; we do not deal with expenses or profit by classification or with gross premiums. We base the pure premiums upon the empirical observations in each cell of an array. For a two-dimensional classification system, this means each cell in a matrix. The observations can be average loss costs, loss frequencies, or loss ratios. In practice, the data would consist of losses and exposures (for loss costs), claim counts and exposures (for loss frequencies), or losses and premiums (for loss ratios).

*Illustration 1:* A classification system for private passenger automobile insurance has two dimensions: (i) urban versus rural and (ii) male versus female. A company insures exactly four drivers, one in each cell, with the following observed loss costs:<sup>4</sup>

	Urban	Rural
Male	\$600	\$200
Female	\$300	\$100

We determine pure premium relativities. We first compare all males with all females, or \$800 for two exposures compared to

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<sup>&</sup>lt;sup>4</sup>We deal with unequal cell populations later in the paper.

\$400 for two exposures. This gives a pure premium relativity of male to female = 2 to 1.

We do the same for urban versus rural, and we get a relativity of 3 to 1. We arbitrarily choose "rural female" as the base class; by convention, the lowest cost class or the class with the largest number of exposures is often chosen as the base class. We get the following relativities:

Male:	$2.00 = s_1$	Urban:	$3.00 = t_1$
Female:	$1.00 = s_2$	Rural:	$1.00 = t_2$ .

The indicated pure premium for a male urban driver is the base pure premium times the urban relativity times the male relativity, or  $100 \times 2.00 \times 3.00 = 600$ . More generally, the pure premium in cell (*i*, *j*) is  $100 \times s_i \times t_j$ .

In this illustration, the indicated pure premiums exactly match the observed loss costs. The minimum bias method is not needed for this case.

### The Additive Model

The indicated pure premiums may differ from the observed loss costs because the model structure may be incorrect or because random loss fluctuation may affect the observed loss costs. We treat the first reason, the model structure, in this section.

*Illustration 2:* The observed loss costs for four drivers are shown below.

	Urban	Rural
Male	\$700	\$500
Female	\$400	\$200

We begin in the same fashion as before, using rural females as the base class. We compare all males to all females, giving a pure premium relativity of \$1,200 to \$600, or 2 to 1. We compare all urban to all rural, giving a pure premium relativity of \$1,100 to \$700, or 1.571 to 1.

The indicated pure premium relativities no longer match the observed loss costs. The indicated pure premium for rural males is  $200 \times 2.000 = 400$ , but the observed loss cost is 500. The indicated pure premium for urban females is  $200 \times 1.571 = 314$ , but the observed loss cost is 400. The differences are significant.

No multiplicative factors work perfectly. In urban territories, the relationship of male to female is \$700 to \$400, or 1.75 to 1. In rural territories, the relationship of male to female is \$500 to \$200, or 2.50 to 1. A male-to-female relativity appropriate for the urban territories is not optimal for the rural territories.

Similarly, the urban-to-rural relativity is \$700 to \$500, or 1.4 to 1, for male drivers, and \$400 to \$200, or 2 to 1, for female drivers.

The discussion in the paragraphs above assumes that the rating model is multiplicative; in this illustration, an additive model works better. We add or subtract a dollar amount for each class instead of multiplying by a factor. We choose rural females as the base class, and we use the relativities below:

Male:	+\$300	Urban:	+\$200
Female:	+\$0	Rural:	+\$0.

The pure premium for any cell is the base pure premium plus the male/female relativity plus the urban/rural relativity. The indicated pure premiums now match the observed loss costs. Rural male = \$200 + \$300 + \$0 = \$500; urban male = \$200 + \$300 + \$200 = \$700; urban female = \$200 + \$0 + \$200 = \$400. The additive method provides an exact match to the observed loss costs because the dollar differences are the same in each row (\$200) and in each column (\$300).

### Additive and Multiplicative Intuition

Some actuaries implicitly assume that pure premium relativities should be multiplicative, not additive. If urban male drivers have twice the accident frequency that rural male drivers have, urban female drivers should have twice the accident frequency that rural female drivers have. This assumption is most persuasive when class dimensions are independent, that is, when the high accident frequency of urban drivers is not correlated with the high accident frequency of male drivers.<sup>5</sup> Most multidimensional class systems for the casualty lines of business use multiplicative factors.

Regulators sometimes harshly criticize insurers for using multiplicative factors that compound increases in the rates for highrisk insureds. This criticism is often—but not always—political. When two or more dimensions of the classification system are correlated, multiplicative systems are often biased. For some types of insurance, multiplicative systems may be biased even when classification dimensions are not correlated.<sup>6</sup>

*Illustration 3:* The 1960 Bailey and Simon paper discusses two rating dimensions: (i) class group and (ii) merit rating class.

- 1. Class group refers to the driver characteristics, such as age, sex, and marital status, and use of the vehicle, such as pleasure use or business use.
- 2. Merit rating class refers to the number of immediately preceding accident-free years, ranging from 0 to 3 or more.

<sup>&</sup>lt;sup>5</sup>This assumption is rarely tested, and the independence of classification dimensions does not necessarily imply a multiplicative model. The authors' impressions from private passenger automobile loss costs is that neither the additive nor the multiplicative model is perfect, but the multiplicative model is usually better.

<sup>&</sup>lt;sup>6</sup>Life insurance rating systems provide an example. If smokers have twice the mortality of non-smokers, and persons with high-blood pressure have twice the mortality of persons with average blood pressure, should high-blood pressure smokers have four times the mortality of average blood pressure non-smokers? Life insurance underwriters employ judgment to assess the rating for applicants with multiple causes of high mortality. A pure multiplicative rating system would not be appropriate.

These two rating dimensions are correlated. For example, young, unmarried male drivers have a high average class relativity. Because these drivers either are new drivers or (if not new) are more likely to have had an accident in the past year, they have relatively few accident-free years, and a multiplicative model would penalize many young male drivers twice for the same risk factor.

#### 3. BIAS FUNCTIONS

In practice, the indicated pure premiums do not perfectly match the observed loss costs for either an additive model or a multiplicative model. We illustrate with the same 2-by-2 classification system. The observed loss costs are shown in the table below:

	Urban	Rural
Male	\$800	\$500
Female	\$400	\$200

Neither an additive model nor a multiplicative model provides a perfect match. If we use a model that does not perfectly match the observed data, we must determine how to minimize the mismatch between the observed loss costs and the indicated pure premiums. A "bias function" is a means of comparing two or more models to see which fits the data with the smallest degree of mismatch.<sup>7</sup> To choose the optimal model, we proceed along three steps:

1. We choose a rating method, such as an additive model or a multiplicative model.

<sup>&</sup>lt;sup>7</sup>The bias function is not a standard statistical term, and the balance principle is not a standard principle. As used here, the bias function determines how "close" the indicated pure premiums are to the observed loss costs or how great the mismatch is between these two sets of data. The sum of the squared deviations and the  $\chi$ -squared deviation are common statistical bias functions. The balance principle, introduced by Bailey and Simon in 1960 and endorsed again by Bailey in 1963, minimizes the bias along the dimensions of the classification system, thereby leading to the term "minimum bias."

- 2. We select a bias function and use it to optimize the rating method. This paper discusses the balance principle, least squares,  $\chi$ -squared, and maximum likelihood bias functions. For models using a maximum likelihood bias function, we must also choose a probability density function for losses within each cell.
- 3. For each optimized rating method, we examine the goodness-of-fit of the indicated pure premiums to the observed loss costs.

We begin with the balance principle, since it is the bias function most commonly used.

### The Balance Principle

The balance principle requires that (after optimizing the relativities) the sum of the indicated pure premiums equals the sum of the observed loss costs along every row and every column.

*Illustration 4:* We examine the balance principle for both the additive and the multiplicative models. There is one exposure in each cell. On the left are the observed loss costs; on the right are the indicated pure premiums. We begin with the multiplicative model.<sup>8</sup>

	Urban	Rural		terr <sub>1</sub>	terr <sub>2</sub>
Male Female	\$800 \$400	\$500 \$200	sex <sub>1</sub> sex <sub>2</sub>	$\begin{array}{c} 200 \times s_1 \times t_1 \\ 200 \times s_2 \times t_1 \end{array}$	$\begin{array}{c} 200 \times s_1 \times t_2 \\ 200 \times s_2 \times t_2 \end{array}$

To balance along the first row (the "male" row), we must have  $800 + 500 = 200 \times s_1 \times t_1 + 200 \times s_1 \times t_2.$ 

<sup>&</sup>lt;sup>8</sup>To keep the notation simple, we use rating dimensions of male versus female and urban versus rural throughout this paper. For the formulas in the illustrations, we use  $\sec_1 = s_1 = \text{male}$ ,  $\sec_2 = s_2 = \text{female}$ ,  $\operatorname{terr}_1 = t_1 = \text{urban}$ , and  $\operatorname{terr}_2 = t_2 = \text{rural}$ . The recursive equations use variable names of *x*, *y*, and *z*, and rating dimensions of *i* and *j*.

To balance along the second row (the "female" row), we must have

$$400 + 200 = 200 \times s_2 \times t_1 + 200 \times s_2 \times t_2.$$

To balance along the first column (the "urban" column), we must have

$$800 + 400 = 200 \times s_1 \times t_1 + 200 \times s_2 \times t_1.$$

To balance along the second column (the "rural" column), we must have

$$500 + 200 = 200 \times s_1 \times t_2 + 200 \times s_2 \times t_2.$$

Although we have four equations in four unknowns, we do not have a unique solution for the classification relativities. There are two special considerations we must be aware of. These two considerations offset each other so as to yield a unique set of indicated pure premiums for each cell of the matrix (see below).

Dependence among the equations: These equations are related by a totality constraint—using any three equations we can derive the fourth, since the sum of the rows equals the sum of the columns. For instance, the fourth equation equals the first equation plus the second equation minus the third equation.

More generally, the equation for any column equals the sum of the equations for the rows minus the sum of the equations for the other columns, and likewise for the equation in any row.

Invariance under reciprocal scalar multiplication: We can set one of the variables arbitrarily, and we can still solve the system of equations. To see this most clearly, suppose that we have solved these equations for values of the four variables  $s_1$ ,  $s_2$ ,  $t_1$ , and  $t_2$ . Another solution is  $2s_1$ ,  $2s_2$ ,  $\frac{1}{2}t_1$ , and  $\frac{1}{2}t_2$ . We could use any constant in place of 2. But no matter which set of relativities we pick, the values in the cells remain the same. The values in the cells are the product of an *s* relativity and a *t* relativity, so the additional constant cancels out.

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We have an additional variable. The pure premium in each cell depends on the base pure premium. If the relativities  $s_1$ ,  $s_2$ ,  $t_1$ , and  $t_2$  optimize the rating model for a base pure premium of \$200, the relativities  $2s_1$ ,  $2s_2$ ,  $t_1$ , and  $t_2$  optimize the rating model for a base pure premium of \$100.<sup>9</sup>

The minimum bias procedure makes the relationship of the rating variables along each dimension of the classification system constant. If  $s_1 = 2 \times s_2$  for a given base pure premium and a given set of territorial relativities, then  $s_1 = 2 \times s_2$  for any other base pure premium and for any constant multiple of the territorial relativities.

We choose a base class in each classification dimension. This is often the largest class or the lowest-cost class, though any class may be used. The base class in each classification dimension is given a relativity of 1. This determines the values of the base pure premium and of all other rating variables.

### Solving the Equations

The equations are not linear, so there is no closed-form solution. We begin with an arbitrary (but reasonable) set of relativities for one dimension, and we solve the equations iteratively.

*Illustration 5:* We choose an urban relativity of 2.00 and rural relativity of 1.00; this choice does not affect the final pure premiums.

	Urban	Rural		$terr_1 = 2$	$terr_2 = 1$
Male Female	\$800 \$400	\$500 \$200	sex <sub>1</sub> sex <sub>2</sub>	$\begin{array}{c} 200 \times s_1 \times 2 \\ 200 \times s_2 \times 2 \end{array}$	$\begin{array}{c} 200 \times s_1 \times 1 \\ 200 \times s_2 \times 1 \end{array}$

<sup>&</sup>lt;sup>9</sup>With so much leeway in choosing the classification relativities, one might ask what we are "optimizing." We are optimizing the indicated pure premiums. Each set of classification relativities give the same indicated pure premiums. The optimization is relative to the bias function. For example, the optimal pure premiums have the least bias or the least  $\chi$ -squared value.

The balance equation for the first row (the "male" row) says that

$$800 + 500 = 200 \times s_1 \times 2 + 200 \times s_1 \times 1,$$
  
or  $s_1 = 1,300/600 = 13/6.$ 

Balancing along the second row (the "female" row) gives

$$400 + 200 = 200 \times s_2 \times 2 + 200 \times s_2 \times 1$$
  
or  $s_2 = 600/600 = 1$ .

We now have intermediate values for the male and female relativities of 13/6 and 1. We discard the initial values for the urban and rural relativities of 2.00 and 1.00, and we solve for new intermediate values by balancing along the columns. The balance equation for the first column (the "urban" column) says that

$$800 + 400 = 200 \times (13/6) \times t_1 + 200 \times 1 \times t_1,$$
  
or  $t_1 = 1,200/633.33 = 1.895.$ 

Balancing along the second column (the "rural" column) gives

$$500 + 200 = 200 \times (13/6) \times t_2 + 200 \times 1 \times t_2$$
,  
or  $t_2 = 1.105$ .

We continue in this fashion. We discard the previous male and female relativities, and we solve for new ones. Balancing along the first row (the "male" row) gives

$$800 + 500 = 200 \times s_1 \times 1.895 + 200 \times s_1 \times 1.105,$$

and balancing along the second row (the "female" row) gives

$$400 + 200 = 200 \times s_2 \times 1.895 + 200 \times s_2 \times 1.105$$

We solve these two equations for new values of the male and female relativities, we discard the previous values of the urban and rural relativities, and we balance along the columns for new values of the urban and rural relativities.

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We continue in this fashion until the relativities converge, i.e., the change in the relativities from an additional iteration is not material. Calculating minimum bias relativities is tedious by hand but easy with a spreadsheet. In this case, convergence is rapid, since there are only four cells. Once the series converges, common practice is to normalize the base class relativities to unity and change the base pure premium (to \$221.05), as we do below:

Iteration	Urban	Rural	Male	Female
Initial	2.0000	1.0000		
1-a			2.1667	1.0000
1-b	1.8947	1.1053		
2-а			2.1667	1.0000
Final	1.8947	1.1053		
Normalized	1.7143	1.0000	2.1667	1.0000
Normalized Ba	ase Pure Premium:	$$200 \times 1.10$	53 = \$221.05	

The initial territorial relativities of 2.00 and 1.00 were arbitrary; we generally begin with starting values determined by a one-way relativities procedure. The starting values have no effect on the final rates in each cell, though better starting values reduce the iterations required to reach convergence. In this illustration, the urban to rural relativity is 12 to 7. If we choose a pure premium relativity of 1.000 as the starting value for the rural class, we would choose a starting value of  $12 \div 7 = 1.714$  for the urban class. With a starting value of  $t_1 = 1.714$ , the series converges immediately. We used a different starting value to show the steps of the procedure.

#### The Additive Model

There are several equivalent formulas for the additive model. The pure premium in cell (i, j), or row *i* and column *j*, is

A. Base pure premium +  $x_i$  +  $y_i$ ,

- B. Base pure premium  $\times (1 + u_i + v_j)$ , or
- C. Base pure premium  $\times (p_i + q_j)$ .
To see the equivalence of these formulas, suppose the base pure premium in formula A is \$10.

- In formula B, the base pure premium is also \$10, each *u* value is one tenth the corresponding *x* value in formula A, and each *v* value in formula B is one tenth the corresponding *y* value in formula A:  $u_i = 0.1 \times x_i$  and  $v_i = 0.1 \times y_i$ .
- Formula C is equivalent to formula B, except that either the *p* values are all increased by 1, the *q* values are all increased by 1, or the *p* values are increased by a constant (*c*) and the *q* values are increased by the complement of that constant (1-c):  $p_i = 1 + u_i$  or  $q_j = 1 + v_j$  (but not both) or  $p_i = c + u_i$  and  $q_j = 1 c + v_j$ .

We use the first form—formula A—for our example, since it shows the method most clearly.<sup>10</sup>

*Illustration 6:* We choose initial values for urban and rural relativities of \$250 and \$0. These initial values are based on the traditional pure premium relativities procedure; the average differential between the urban and rural observed loss costs is  $\frac{1}{2} \times [(800 - 500) + (400 - 200)] = $250.$ 

	Urban	Rural		$terr_1 = 250$	$terr_2 = 0$
Male Female	\$800 \$400	\$500 \$200	sex <sub>1</sub> sex <sub>2</sub>	$200 + s_1 + 250 200 + s_2 + 250$	$200 + s_1 + 0 200 + s_2 + 0$

Balancing along the first row (the "male" row) gives  $800 + 500 = 200 + s_1 + 250 + 200 + s_1 + 0$ , or  $s_1 = 650/2 = 325$ .

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<sup>&</sup>lt;sup>10</sup>In practice, formulas B or C might be preferred, since only the base pure premium need be increased for inflation. In formula A, the base pure premium and all the relativities must be increased for inflation.

Balancing along the second row (the "female" row) gives

$$400 + 200 = 200 + s_2 + 250 + 200 + s_2 + 0,$$
  
or  $s_2 = -50/2 = -25.$ 

We discard the initial values for the urban and rural relativities, and we balance along the columns. We use the intermediate values of the male and female relativities to get new values for the urban and rural relativities. We continue this iterative process until the series converges.

The relativity of -\$25 for females seems odd at first. In truth, the relativity for female drivers is not inherently negative; this is an artifact of the base pure premium and the starting values. We could make the relativity for females positive by adding a constant to the male and female relativities and subtracting the same constant from the rural and urban relativities. For instance, we could add \$75 to the male and female relativities to get relativities of \$400 and \$50, and we would subtract \$75 from the rural and urban relativities.

Iteration	Urban	Rural	Male	Female
Initial 1-a	\$250.00	\$0.00	\$325.00	(\$25.00)
1-b	\$250.00	\$0.00		
Normalized Normalized Ba	\$250.00 ase Pure Premium:	\$0.00 \$200.00 - \$	\$350.00 25.00 = \$175.00	\$0.00

We can even make all the relativities negative by adjusting the base pure premium. For instance, by choosing a base pure premium of \$1,000, we obtain negative relativities for all classes.<sup>11</sup> In this illustration, we added dollar amounts to make the base class relativities equal to zero.

<sup>&</sup>lt;sup>11</sup>Companies may do this for marketing reasons. All drivers get discounts from the base pure premium, so all drivers feel they are gaining from the classification system.

# Exposures

The illustrations above assume one driver in each cell or the same number of drivers in each cell. In practice, there are generally different numbers of risks in each cell. Two adjustments are needed, one to the bias function and another for credibility:

- We adjust the bias function for the relative volume of business in each cell.
- We may make a credibility adjustment based on the absolute volume of business in a cell.

*Illustration* 7—*Credibility:* We note the credibility issue, but we defer the possible adjustments until later. Suppose insurer A has 100 exposures per cell, and insurer B has 10,000 exposures per cell. Insurer A may rely more heavily on the minimum bias procedure. Insurer B may give greater weight to the empirical observations.

We deal here with the adjustment to the bias function. The balance principle requires that the sum of the observed loss costs in each row or column equal the sum of the indicated pure premiums in the corresponding row or column. If there are two drivers in a cell, we double both the observed loss cost and the indicated pure premium in that cell. If there are n drivers in a cell, we multiply both the observed loss cost and the indicated pure premium by n. When the number of drivers varies by cell, we need an additional matrix of the number of drivers in each cell.

*Illustration 8:* For the multiplicative model, suppose that the number of drivers is as follows:

Male urban:	1,200	Male rural:	600
Female urban:	1,000	Female rural:	800.

We include the number of drivers in the equations.

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	Urban	Rural		terr <sub>1</sub>	terr <sub>2</sub>
Male	$1200 \times \$800$	$600 \times \$500$	sex <sub>1</sub>	$1200 \times 200 \times s_1 \times t_1$	$600 \times 200 \times s_1 \times t_2$
Female	$1000\times\$400$	$800\times\$200$	sex <sub>2</sub>	$1000 \times 200 \times s_2 \times t_1$	$800 \times 200 \times s_2 \times t_2$

To balance along the first row (the "male" row), we must have

$$\begin{split} 1200 \times 800 + 600 \times 500 \\ = 1200 \times 200 \times s_1 \times t_1 + 600 \times 200 \times s_1 \times t_2. \end{split}$$

To balance along the second row (the "female" row), we must have

$$1000 \times 400 + 800 \times 200$$
  
= 1000 \times 200 \times s\_2 \times t\_1 + 800 \times 200 \times s\_2 \times t\_2.

To balance along the first column (the "urban" column), we must have

$$\begin{split} 1200 \times 800 + 1000 \times 400 \\ = 1200 \times 200 \times s_1 \times t_1 + 1000 \times 200 \times s_2 \times t_1. \end{split}$$

To balance along the second column (the "rural" column), we must have

$$600 \times 500 + 800 \times 200$$
  
= 600 \times 200 \times s\_1 \times t\_2 + 800 \times 200 \times s\_2 \times t\_2.

### Empirical Observations versus Modeled Pure Premiums

One might wonder: Why not use the observed loss costs, appropriately developed and trended, as the indicated pure premiums for the coming policy period? Instead of fitting either multiplicative or additive models to the observed data, let us use \$800 as the indicated pure premium for urban male drivers, \$400 for urban female drivers, \$500 for rural male drivers, and \$200 for rural female drivers. The common answer is that the individual cells are "not fully credible." This answer is correct, though the terminology is not ideal. The term "credible" is vague. To understand the situation, we must be more precise.

Credibility is a relative concept. No cell is inherently credible or not credible. A cell's credibility depends on the reliability of its own experience in comparison with information in other cells. Consider our basic illustration with the following observed loss costs:

	Urban	Rural
Male	\$800	\$500
Female	\$400	\$200

The urban male observed pure premium of \$800 represents a mixture of expected losses and random loss fluctuations. How might we judge whether this figure is higher or lower than the true expected loss costs?

Suppose that the rating values combine additively to generate the expected losses. The observed loss cost for urban males of \$800 is \$300 more than the observed loss cost of rural males of \$500. This suggests that the urban attribute of the vehicle's location adds about \$300 to the expected loss costs.

However, the urban female observed loss cost of \$400 is only \$200 more than the rural female observed loss cost of \$200. This suggests that the extra cost associated with the urban attribute is only \$200, not \$300, and it implies that the observed urban male loss cost of \$800 might be too high.

We perform a similar analysis for male versus female. Comparing urban drivers suggests that the male attribute adds about 400 to the expected loss costs, since male/urban = 800 and female/urban = 400. However, comparing rural drivers suggests that the extra cost associated with the male attribute is only 300, not 400, since male/rural = 500 and female/rural = 200. In other words, the urban male loss cost of 800 might be too high. The \$800 observed loss cost in the urban male cell does not tell us what part of this observed loss cost is expected and what part is distorted by random loss fluctuations. If we know the mathematical function linking the cells—that is, if the pure premiums of the driver and the vehicle have some additive or multiplicative relationship—we can use additional cells to provide information about the true expected costs for urban male drivers, as we have here.

If we assume that the cells are linked multiplicatively, our inferences change. The urban male observed value of \$800 is 160% of the rural male observed value of \$500. This suggests that the urban attribute adds about 60% to the expected loss costs.

The urban female observed loss cost of \$400 is twice the rural female observed loss cost of \$200. This suggests that the extra cost associated with the urban attribute is +100%, not +60%. The urban male loss cost of \$800 might be too low.

Using a similar analysis for male versus female using the urban column suggests that the male attribute adds about 100% to the expected loss costs. The rural column suggests that the extra cost associated with the male attribute is +150%, not +100%. The urban male loss cost of \$800 might be too low.

If the cells are linked additively, we infer that the urban male observed loss costs of \$800 might be too high. If the cells are linked multiplicatively, we infer that the urban male observed loss costs of \$800 might be too low.<sup>12</sup>

If the exposures in a 2 by 2 matrix are evenly distributed among the cells, each cell has 25% of the total exposures, whether there is 1 car or 10,000 cars in each cell. We give much

<sup>&</sup>lt;sup>12</sup>In most cases, the direction of the bias does not depend on the type of rating model. The more common scenario might show an observed loss cost of \$600, an additive model indicated pure premium of \$550, and a multiplicative model indicated pure premium of \$530. We might infer that the random loss fluctuations underlying these cell values have had a net positive effect. For very high rated or very low rated classifications, the multiplicative and additive models often give opposite results, as is the case here.

credence to the observed value in that cell compared to our inferences from other cells. With a larger array, such as a 10 by 10 by 10 array, there are many more cells. The average cell contains only 0.1% of the total exposures. We give less credence to the observed loss costs in that cell compared to our inferences from other cells.

This is the intuition for the minimum bias procedure. The rating model—such as additive, multiplicative, or combined—tells us the relationship joining the cells. The bias function—such as balance principle,  $\chi$ -squared, least squared error, or maximum likelihood—provides a method of drawing inferences for one cell using the information in the other cells.

### Credibility—Original Papers

The original papers on the minimum bias procedure differ regarding credibility.

The 1960 Bailey and Simon paper uses credibility considerations to pick a bias function. The authors' view that credibility should be inversely proportional to the variance of the observations led them to choose the  $\chi$ -squared bias function over the balance principle.

The 1963 Bailey paper, which advocates the balance principle, has no explicit discussion of credibility. The balance principle has an implicit credibility component, since it weights the observed loss costs and pure premiums in each cell by the number of exposures in the cell.

This implicit credibility examines the relative weights of different cells in the minimum bias procedure. Venter looks at credibility from a different angle—the relative weight given to the indicated pure premium from the minimum bias procedure versus the observed loss costs from the experience. We said above that the \$800 observed loss cost for urban male drivers might be overstated under an additive model or understated under a multiplicative model. The over- or understatement stems from random loss fluctuations. If there is a single exposure in each cell, an over- or understatement is likely. If there are 10,000 exposures in each cell, the degree of over- or understatement is likely to be smaller.

## Iterative Formulas

We have so far presented simple illustrations. To program more complicated versions of this procedure, we need general formulas.

We derive the iterative formulas for the multiplicative balance principle model. We designate the base pure premium by b, the number of exposures in row i and column j by  $n_{ij}$ , and the observed pure premiums in row i and column j by  $r_{ij}$ . With a multiplicative model, the balancing equation for row i is

$$\sum_{j} (n_{ij}r_{ij}) = \sum_{j} (bn_{ij}x_iy_j).$$

Similarly, the balancing equation for column j is

$$\sum_{i} (n_{ij}r_{ij}) = \sum_{i} (bn_{ij}x_iy_j).$$

In these equations, x is a row relativity and y is a column relativity.<sup>13</sup> We solve these equations to get the indicated x and y relativities in each row and column:<sup>14</sup>

$$x_i = \frac{\sum_j n_{ij} r_{ij}}{\sum_j b n_{ij} y_j}$$
 and  $y_i = \frac{\sum_i n_{ij} r_{ij}}{\sum_i b n_{ij} x_i}$ .

When the series converges, we set the relativity for the base class in each classification dimension to unity, and we adjust the base pure premium to offset this.

<sup>&</sup>lt;sup>13</sup>In the illustrations, we use *s* for the row relativity and *t* for the column relativity as abbreviations for the classification dimensions (sex and territory). The variables *x* and *y* are commonly used in the literature.

<sup>&</sup>lt;sup>14</sup>We sum over the *j* subscript when we balance along the rows (the *i* subscripts). We do this separately for each *i*. When we balance along the columns, we sum over the *i* subscripts separately for each *j*.

We used two dimensions in this formula. One might assume that the two dimensions correspond to the two variables x and y. That is not correct. The two dimensions correspond to the two subscripts i and j. The x and y variables correspond to two sets of relativities. A model can have two or even more sets of relativities in a single dimension.

*Illustration 9:* The classification system has two dimensions: male versus female and territory A versus territory B. Territory A has more attorneys than territory B has, resulting in a higher propensity to sue and higher loss costs per claim. Territory B has several blind intersections, leading to more accidents. We might presume that the higher attorney involvement in territory A increases the cost of all claims, so a multiplicative factor is appropriate, whereas the blind intersections in territory B add additional hazards, so an additive factor is appropriate. The rating model might take the form

indicated pure premium relativity =  $x_i \times y_i + z_i$ ,

where *i* represents the male/female classification dimension and *j* represents the territory dimension. The variable *x* is the relativity for the male/female dimension, and the variables *y* and *z* are the relativities for the urban/rural dimension. In this model, *x* and *y* are multiplicative factors, and *z* is an additive factor.<sup>15</sup>

The arithmetic is similar for any number of dimensions. The multiplicative model has one set of relativities for each dimension. With three dimensions, for example, the iterative formula for the i dimension is

$$x_i = \frac{\sum_{j,k} n_{ijk} r_{ijk}}{\sum_{j,k} b n_{ijk} y_j z_k}.$$

<sup>&</sup>lt;sup>15</sup>To optimize this rating model, the balance principle is not sufficient; we would have to employ one of the other bias functions. The balance principle provides i + j equations, but we have i + 2j variables. The other bias functions discussed in this paper provide i + 2j equations.

We develop the general formula for the balance principle additive model by assuming a base pure premium of \$0. The balance principle equation is

$$\sum_{j} (n_{ij}r_{ij}) = \sum_{j} n_{ij}(x_i + y_j),$$

and the iterative formula is

$$x_i = \frac{\sum_j n_{ij} (r_{ij} - y_j)}{\sum_j n_{ij}}.$$

## Exercise: Multiplicative Model

*Illustration 10:* We use the balance principle to optimize a multiplicative rating model with two dimensions and two classes in each dimension. The observed loss costs and exposures in each class are shown below:

	Loss	Loss Costs		sures
	$y_1$	<i>y</i> <sub>2</sub>	$y_1$	<i>y</i> <sub>2</sub>
<i>x</i> <sub>1</sub>	300	300	100	150
<i>x</i> <sub>2</sub>	200	400	100	100

We assume a base pure premium of \$100, so the indicated pure premiums are  $100x_iy_j$ . To simplify the mathematics, we compute all values in units of \$100. The indicated pure premiums are  $x_i \times y_j$ , and the observed loss costs are \$3, \$3, \$2, and \$4.

We form a matrix of observed loss costs and indicated pure premiums:

	<i>y</i> <sub>1</sub>	<i>y</i> <sub>2</sub>		<i>y</i> <sub>1</sub>	<i>y</i> <sub>2</sub>
$x_1$ $x_2$	3 2	3 4	$x_1$ $x_2$	$\begin{array}{c} x_1 \times y_1 \\ x_2 \times y_1 \end{array}$	$\begin{array}{c} x_1 \times y_2 \\ x_2 \times y_2 \end{array}$

We multiply each figure by the exposures in the cell:

	<i>y</i> <sub>1</sub>	<i>y</i> <sub>2</sub>		<i>y</i> <sub>1</sub>	<i>y</i> <sub>2</sub>
$x_1$ $x_2$	$\begin{array}{c} 100\times 3\\ 100\times 2 \end{array}$	$\begin{array}{c} 150\times 3\\ 100\times 4 \end{array}$	$x_1$ $x_2$	$\begin{array}{c} 100 \times x_1 \times y_1 \\ 100 \times x_2 \times y_1 \end{array}$	$150 \times x_1 \times y_2 \\ 100 \times x_2 \times y_2$

We choose 1.00 and 1.50 as the starting values for  $y_1$  and  $y_2$ . We use the balance principle to obtain intermediate values for  $x_1$  and  $x_2$ :

$$100 \times 3 + 150 \times 3 = 100 \times x_1 \times 1.00 + 150 \times x_1 \times 1.50,$$
  
or  $300 + 450 = 100 \times x_1 + 225 \times x_1,$   
or  $x_1 = 2.308;$ 

and

$$100 \times 2 + 100 \times 4 = 100 \times x_2 \times 1.00 + 100 \times x_2 \times 1.50$$
  
or  $200 + 400 = 100 \times x_2 + 150 \times x_2$ ,  
or  $x_2 = 2.400$ .

We now discard the initial values for  $y_1$  and  $y_2$ , and we balance along the columns:

$$100 \times 3 + 100 \times 2 = 100 \times 2.308 \times y_1 + 100 \times 2.400 \times y_1,$$
  
or  $300 + 200 = 230.8 \times y_1 + 240 \times y_1,$   
or  $y_1 = 1.062;$ 

and

$$150 \times 3 + 100 \times 4 = 150 \times 2.308 \times y_2 + 100 \times 2.400 \times y_2,$$
  
or  $450 + 400 = 346.2 \times y_2 + 240 \times y_2,$   
or  $y_2 = 1.450.$ 

This completes one iteration. To solve for the optimal relativities, we continue in this fashion until convergence. We comment on several items in this exercise.

### Data and Assumptions

The number of exposures in each cell is a credibility measure. We give 50% more credence to the observed loss costs in the  $x_1y_2$  cell than to the loss costs in the other cells.

- The observed loss costs in the  $x_1$  row indicate that there is no difference between  $y_1$  and  $y_2$ . The observed loss costs in the  $x_2$  row indicate that the  $y_2$  class should have twice the pure premium that  $y_1$  has. We give more credence to the first of these two relationships.
- The observed loss costs in the  $y_1$  column indicate that the  $x_2$  class should have a pure premium 33% lower than the  $x_1$  class. The observed loss costs in the  $y_2$  column indicate that the  $x_2$  class should have a pure premium 33% higher than the  $x_1$  class. We give more credence to the second of these two relationships, so the  $x_2$  relativity is slightly higher than the  $x_1$  relativity.

### Exercise: Additive Model

*Illustration 11:* An additive model with two dimensions has the observed loss costs shown below. Each cell has 1,000 exposures. The base loss cost is \$100. The formula for loss costs by cell is Loss  $Cost_{ij} = (Base Loss Cost) \times (x_i + y_i)$ . We use the starting values shown below to compute intermediate values for  $y_1$  and  $y_2$ .

	Average Loss C y <sub>1</sub>	bosts per Exposure $y_2$	Starting Values
$x_1$	500	750	4.500
$x_2$	250	475	3.000
$x_3$	150	400	2.000

Since the number of exposures is the same in each cell, we may assume that there is a single exposure in each cell; the 1,000 cancels out of every equation.

The base pure premium is \$100. To simplify, we use units of \$100 and a base pure premium of unity. The matrix of observed loss costs and indicated pure premiums is shown below:

	Observe	d Values	Indicate	d Values
	<i>y</i> <sub>1</sub>	<i>y</i> <sub>2</sub>	<i>y</i> <sub>1</sub>	<i>y</i> <sub>2</sub>
<i>x</i> <sub>1</sub>	5	7.5	$x_1 + y_1$	$x_1 + y_2$
$x_2$	2.5	4.75	$x_2 + y_1$	$x_2 + y_2$
<i>x</i> <sub>3</sub>	1.5	4	$x_3 + y_1$	$x_3 + y_2$

We balance along the columns. For the first column, we have

$$5.00 + 2.50 + 1.50 = (x_1 + y_1) + (x_2 + y_1) + (x_3 + y_1),$$
  

$$5.00 + 2.50 + 1.50 = (4.50 + y_1) + (3.00 + y_1) + (2.00 + y_1),$$
  
or  $3y_1 = 9.00 - 9.50,$  or  $y_1 = -0.167.$ 

For the second column, we have

7.50 + 4.75 + 4.00 = 
$$(x_1 + y_2) + (x_2 + y_2) + (x_3 + y_2)$$
,  
7.50 + 4.75 + 4.00 =  $(4.50 + y_2) + (3.00 + y_2) + (2.00 + y_2)$ ,  
or  $3y_2 = 16.25 - 9.50$ , or  $y_2 = 2.25$ .

We have finished balancing along the columns. The next step is to balance along the rows. We take the new y values,  $y_1 = -0.167$  and  $y_2 = +2.25$ , and we compute new values for  $x_1$  and  $x_2$  by balancing along each row. We continue this process—alternately balancing along rows and columns—until we reach convergence.

During the iterative process, the plan is alternately balanced along the rows or along the columns, but not along both. We have just balanced along the columns. To see that we are not yet balanced along the rows, we examine the first row:

$$5.00 + 7.50 = (x_1 + y_1) + (x_1 + y_2).$$

Substituting the starting values of the *x*s and the first iterative values of the *y*s, we get

$$12.50 = 4.50 + (-0.167) + 4.50 + 2.25 = 11.083.$$

The equality does not hold, since the plan is not yet balanced. The final values, after convergence, are shown below.

Iteration	<i>x</i> <sub>1</sub>	<i>x</i> <sub>2</sub>	<i>x</i> <sub>3</sub>	<i>y</i> <sub>1</sub>	<i>y</i> <sub>2</sub>
Initial	4.50000	3.00000	2.00000		
1-a				-0.16667	2.25000
1-b	5.20833	2.58333	1.70833		
2-а				-0.16667	2.25000
Final	5.20833	2.58333	1.70833	-0.16667	2.25000

### 4. OTHER CLASSIFICATION DIMENSIONS

The basic illustrations use the minimum bias procedure to set pure premium relativities simultaneously for the male/female dimension and the urban/rural dimension. There may be other dimensions to the classification plan as well, such as age of driver, marital status, type of vehicle, use of the car, driver education, prior accident history, and so forth.

Suppose that we analyze the male/female dimension and the urban/rural dimension on a statewide basis, and we set relativities for other classification dimensions on a countrywide basis. We use a minimum bias method for the statewide analysis.

If all the classification dimensions are independent, the analysis should work well. If one or more of the other classification dimensions is correlated with the male/female or urban/rural dimensions, the rating analysis may be distorted.

*Illustration 12:* Suppose that young people migrate to urban areas for university education, work opportunities, and the

glamor of urban social activities. Older people move to the suburbs and rural areas to buy homes and raise families away from the vices of urban areas. The age and marital status of the driver are correlated with the urban/rural garaging location.

The statewide analysis may indicate an urban to rural relativity of 2 to 1. The countrywide analysis, summing over all territories, may indicate a relativity for young unmarried male drivers of 3 to 1 when compared to adult drivers. The relativity for young unmarried urban male drivers is not 6 to 1, even if a multiplicative model is appropriate for automobile insurance.

### Multiple Dimensions

Ideally, we would use a multidimensional minimum bias procedure to set all classification relativities simultaneously. In practice, this may not be possible. Some relativities may be analyzed each year, whereas other relativities may be analyzed every several years. Some relativities, such as territory, must be set on a statewide basis. Certain driver characteristics and vehicle characteristics may be analyzed on a countrywide basis, for two reasons:<sup>16</sup>

- 1. The relativities are not expected to vary by state, as long as the states use the same insurance compensation system.
- 2. Some classification cells would have few exposures in a state analysis, and the results may be distorted by random loss fluctuations. The countrywide analysis uses more data, providing more credible results. For example, we may wish to analyze driver age in yearly increments: age 17, age 18, age 19, and so forth. Single-state data may be too sparse to give credible results.

<sup>&</sup>lt;sup>16</sup>The countrywide analysis may actually be done on all tort liability states or all no-fault states, since the bodily injury rate relativities may be higher for SUVs (sports utility vehicles) than for sedans in tort liability states, whereas the reverse may be true in no-fault states.

Some classification dimensions, such as driver education, have a minor effect on overall loss costs. We may analyze these classification dimensions every five years or so, not every year.

# Loss Ratios

One method of dealing with an uneven distribution of business along other classification dimensions is to use loss ratios instead of loss costs in the minimum bias procedure.<sup>17,18</sup>

Suppose the empirical experience consists of the following loss ratios by classification.

	Urban	Rural
Male	75%	85%
Female	90%	80%

We could take either of two approaches:

*First Approach:* We treat the unadjusted loss ratios as though they were loss costs. Instead of using pure premium relativities, we develop loss ratio relativities. These relativities are adjustments to whatever pure premium relativities are embedded in these loss ratios.

In this scenario, the minimum bias procedure will indicate a loss ratio relativity close to 1.000 for urban versus rural and a relativity slightly higher than 1.000 for females versus males. This does not mean that urban risks are similar to rural risks, or that female drivers have more accidents than male drivers have. If the current rate relativities are reasonable, we would expect the loss ratios in all cells to be about equal. Suppose that

<sup>&</sup>lt;sup>17</sup>In practice, we use loss ratios adjusted to the base rates for the classification dimensions included in the minimum bias analysis, though this is not shown in the illustration.

<sup>&</sup>lt;sup>18</sup>This section assumes that the pure premium relativities are the same as the rate relativities.

the current male-to-female rate relativity is 2.4 to 1. Since the average female loss ratio of 85% is higher than the average male loss ratio of 80%, the loss ratio relativities would indicate that we should slightly reduce the male-to-female rate relativity.

*Second Approach:* We convert the raw loss ratios to base class loss ratios. Suppose the current rate relativities are 2.4 to 1 for male to female and 1.8 to 1 for urban to rural. We must divide the male premiums by 2.4 and the urban premiums by 1.8. This is equivalent to multiplying the male loss ratios by 2.4 and the urban loss ratios by 1.8. We multiply the raw loss ratios by the current classification relativities, as shown in the table below.

	Urban	Rural
Male	$75\% \times 2.4 \times 1.8 = 324\%$	$85\% \times 2.4 \times 1.0 = 204\%$
Female	$90\% \times 1.0 \times 1.8 = 162\%$	$80\% \times 1.0 \times 1.0 = 80\%$

We apply the minimum bias procedure to the adjusted loss ratios. The resulting loss ratio relativities would be the same as the indicated rate relativities.

To see this, suppose that the base rate is \$100. For the male urban cell, the premium is  $100 \times 2.4 \times 1.8 = 432$ . The observed loss ratio is 75%, so the loss cost is  $75\% \times 432 = 324$ . We may verify this for the other three cells in the same fashion.

To set the rate relativity to unity for the base class in each dimension, we divide each adjusted loss ratio in the matrix by the adjusted loss ratio for the base class.

	Urban	Rural	
Male	324%/80% = 405.0%	204%/80% = 255.0%	
Female	162%/80% = 202.5%	80.0%/80% = 100.0%	

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## Loss Ratio Intuition

We have shown how to convert loss ratios to reflect the loss costs in each cell. This might be useful if the observed data were loss ratios and we wanted to use loss costs for the minimum bias procedure. But if observed data are loss costs, we must convert the observed loss costs to loss ratios before converting back to loss costs. The purpose of this conversion from loss costs to loss ratios and then back to loss costs is to eliminate the potentially distorting effects of other classification dimensions that are not being analyzed in the minimum bias procedure.

*Illustration 13:* We explain by illustration. We have average observed bodily injury loss costs for four groups of drivers, with 1,000 drivers in each cell.

	Urban	Rural
Male	\$800	\$500
Female	\$400	\$200

Other dimensions in the classification system are correlated with the two dimensions above.

*Type of Vehicle:* For bodily injury rating, cars are subdivided between (a) large cars, such as sports utility vehicles (SUVs), station wagons, and light trucks, and (b) small cars, such as sedans. The large vehicles provide better protection for their occupants, but they cause greater damage to others. Smaller vehicles cause less damage to others, but they provide less protection for their occupants. Sedans and other small cars are more common in urban areas; SUVs and light trucks are more common in rural areas. The distribution of vehicle types between urban and rural areas, along with the appropriate surcharge or discount for each type of vehicle, affect the observed loss costs.

Suppose that SUVs and other large vehicles receive a 20% surcharge for bodily injury. In this state, SUVs comprise 40%

of the rural vehicles and 10% of the urban vehicles. The pricing actuary may not actually have this distribution for the state under review. This is not necessary; the use of loss ratios instead of loss costs corrects for the effects of vehicle type.

Age of Driver: The male/female rate relativity applies to all male and female drivers. Unmarried male drivers under the age of 21 receive additional surcharges, ranging from 25% for 20-year-old drivers to 125% for 16-year-old drivers. In this state, 10% of male drivers are unmarried and under the age of 21. The average surcharge for these drivers is 50%. (For this illustration, there is no corresponding surcharge for unmarried female drivers under the age of 21.) The pricing actuary may not actually have a distribution of male drivers by age and marital status. Again, this is not necessary; the loss ratios are sufficient.

# Double Counting and Offsetting

If we do not take vehicle type and driver age into account, we overcharge male drivers and rural drivers.

*Male Drivers:* The male/female relativity is based on the statewide analysis. The surcharges for young unmarried male drivers are determined from a separate countrywide analysis. The poor driving experience of young unmarried male drivers is counted twice: once at the countrywide level for the surcharges and once at the state level for the male/female relativity. To determine accurately the male/female relativity, we must remove the hazardous effects of being young and unmarried from the male driver classification.

*Rural Drivers:* Rural drivers are less hazardous than urban drivers, but they drive vehicles more dangerous to others. The vehicle surcharge is determined in a countrywide analysis. To properly determine the urban/rural relativity, we must remove the effects of vehicle type from the statewide experience.

To remove the effects of vehicle type and driver age from the statewide analysis, we assume that the countrywide relativities are accurate. We examine each risk in the minimum bias procedure. We divide the actual loss costs by the vehicle type relativity and by the driver age relativity. This gives the relative loss costs that we would have expected to see were the vehicle types and driver ages evenly distributed over all other rating dimensions.

*Illustration 13, continued:* A four-door sedan is the base vehicle type and age 21+ is the base age. A two-door compact has a bodily injury discount of 10%, and an unmarried 20-year-old male driver has a surcharge of 25%. Suppose the observed loss costs for a 20-year-old unmarried male driver of a two-door compact car are \$450. The loss costs adjusted for driver age and vehicle type are  $\frac{450}{0.90 \times 1.25} = \frac{400}{0.90}$ .

It is not practical to make these adjustments car by car. Using loss ratios adjusts for all classification dimensions simultaneously. Using observed loss ratios instead of observed loss costs adjusts for driver age, driver sex, territory, vehicle types, and all other rating dimensions. We then restore the current rating relativities for the classification dimensions that we are analyzing male/female and urban/rural in this illustration.

The average observed loss costs for the 1,000 drivers in each of four classes are displayed in the table after illustration 13. The current relativities are 2.4 to 1 for male to female and 1.8 to 1 for urban to rural. The average SUV-to-sedan relativity is 1.2 to 1. SUVs account for 40% of rural cars and 10% of urban cars. Unmarried males under the age of 21 make up 10% of male drivers, and their average surcharge is 50%. Ideally, we would convert the observed loss costs to adjusted loss costs for the minimum bias analysis in the following manner.

• *Rural female:* SUVs are 40% of rural cars, increasing the loss costs by a factor of  $1 + (20\% \times 40\%) = 1.08$ . Were the cars all sedans, the observed loss costs would be reduced by a factor of 1/1.08 = 92.59%.

- Urban female: The vehicle type factor is  $1 + (20\% \times 10\%) = 1.02$ . Were the cars all sedans, the observed loss costs would be reduced by a factor of 1/1.02 = 98.04%.
- *Rural male:* The vehicle type factor is  $1 + (20\% \times 40\%) = 1.08$  and the driver age factor is  $1 + (10\% \times 50\%) = 1.05$ . Were the cars all sedans driven by adult drivers, the observed loss costs would be reduced by a factor of  $1/(1.08 \times 1.05) = 88.18\%$ .
- Urban male: The vehicle type factor is  $1 + (20\% \times 10\%) =$ 1.02 and the driver age factor is  $1 + (10\% \times 50\%) = 1.05$ . Were the cars all sedans driven by adult drivers, the observed loss costs would be reduced by a factor of  $1/(1.02 \times 1.05) =$ 93.37%.

We have made all the adjustments by our knowledge of the distribution of other classification dimensions in the four cells of the matrix. This information is generally not available, and the procedure is complex when there are several classification dimensions. A simple alternative is to divide the losses by the premium charged in each cell, and then multiply by the base rate times the current relativities for the two classification dimensions which we are examining.

For each vehicle, we divide the losses by the premium, which is the base rate times the classification relativities for all classification dimensions. We multiply the result by the base rate times the classification relativities for male/female and urban/rural. This is equivalent to dividing by the classification relativities for the remaining dimensions.

# Exercise: Loss Ratio Method

The incurred losses and earned premium in each cell are shown in the following table.

	Incurred Losses		Earned I	Premium	
	Urban	Rural	Urban	Rural	
Male	\$2,700	\$2,000	\$3,000	\$4,000	
Female	\$1,500	\$1,200	\$2,400	\$1,600	

The current relativities by sex of driver and by garaging location are

Male:	1.50	Urban:	1.20
Female:	1.00	Rural:	1.00.

## Causes of Unequal Loss Ratios

To correct for potential distortions caused by an uneven distribution of insureds by other classification dimensions, we use loss ratios instead of loss costs. If the rate relativities match the loss cost differences, the loss ratios should be equal in all cells, except for random loss fluctuations. In this example, the loss ratios are not all equal.

	Loss Ratios		
	Urban	Rural	
Male	90.00%	50.00%	
Female	62.50%	75.00%	

There are several possible causes for the unequal loss ratios.

*Cause 1—Random loss fluctuations:* Random loss fluctuations are a credibility issue. This paper assumes either that the data are fully credible or that the pricing actuary has already made (or will make) whatever adjustments are warranted by credibility considerations. Credibility adjustments for sparse data are an important actuarial issue, though they are beyond the scope of this paper.

*Cause 2—Improper rate relativities in other classification dimensions combined with an uneven distribution of insureds by these*  *other classification dimensions:* For example, perhaps the rates for a certain type of vehicle are too low, and the proportion of urban males driving that type of vehicle is greater than the proportions of the insureds in the other cells driving that type of vehicle.

If this is the cause of the differences, there is no perfect solution.<sup>19</sup> But if the distribution of insureds by the other classification dimension is not too uneven, an inaccuracy in the rates will not distort our analysis too much. We may restate our assumption as follows:

For other classification dimensions, either the current rate relativities are accurate or the mix of insureds is relatively even across these other dimensions.

In many instances, this assumption is not perfect. Nevertheless, even if the use of loss ratios does not perfectly correct for distortions caused by an uneven distribution of insureds along other classification dimensions, it provides a partial correction.

*Cause 3—Inaccuracies in the rate relativities for the two classification dimensions that we are examining (sex and territory):* This is corrected by the minimum bias procedure, since the loss ratios by cell times the current relativities by cell equal the relative loss costs by cell.

*Illustration 14:* Suppose the loss ratio for male drivers is 90% and the loss ratio for female drivers is 62.5%. If the current male-to-female rate relativity is 1.5 to 1, the male-to-female loss cost relativity is  $1.5 \times 90\%$  to  $1 \times 62.5\% = 2.16$  to 1.

For the illustration in this section, we form a matrix of relativities by sex and territory:

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<sup>&</sup>lt;sup>19</sup>Without information about the other classification dimensions, we cannot optimize the class system.

	Current Rate Relativities		
	Urban Rural		
Male	1.80	1.50	
Female	1.20	1.00	

The relative loss costs by sex and territory are the product of the relativities and the loss ratios:

	Loss Cost Relativities		
	Urban	Rural	
Male	1.62	0.75	
Female	0.75	0.75	

We can now determine optimal rate relativities by any of the minimum bias models discussed in this paper.

*Cause 4—Improper model specification:* We may be using a multiplicative model when an additive model would be more proper (or vice versa). Sometimes neither a multiplicative nor an additive model is ideal. We discuss the choice of model further below.

### 5. THE SQUARED ERROR BIAS FUNCTION

In this section, we examine other bias functions, beginning with the squared error function and the  $\chi$ -squared function. We continue with our simple 2 by 2 illustration for both additive and multiplicative models using these bias functions. We review arguments for and against specific bias functions. We examine two goodness-of-fit tests—average absolute error and  $\chi$ -squared and we consider the relationship between the bias function chosen and the goodness-of-fit test.

We review also the maximum likelihood bias function and the distributions commonly used with it. We discuss the potential advantages and drawbacks of the more sophisticated bias functions compared to the balance principle. *Illustration 15:* We return to the simple illustration with which we began, as reproduced below.

	Urban	Rural		terr <sub>1</sub>	terr <sub>2</sub>
Male Female	\$800 \$400	\$500 \$200	sex <sub>1</sub> sex <sub>2</sub>	$\begin{array}{c} 200 \times s_1 \times t_1 \\ 200 \times s_2 \times t_1 \end{array}$	$\begin{array}{c} 200 \times s_1 \times t_2 \\ 200 \times s_2 \times t_2 \end{array}$

The left-hand side of the matrix shows the observed loss costs; the right-hand side shows the indicated pure premiums. Our objective is to pick classification relativities such that the indicated pure premiums are "as close as possible" to the observed loss costs.

Statisticians would fit the classification relativities using one of the methods below:

- 1. Minimize the average absolute error between the indicated and observed figures.
- 2. Minimize the sum of the squared errors between the indicated and observed figures (i.e., the least squares bias function).
- 3. Minimize the sum of the relative squared errors between the indicated and observed figures (i.e., minimize the  $\chi$ -squared error).
- 4. Maximize the likelihood of obtaining the observations given the classification relativities.

Although minimizing the average absolute error makes sense to practitioners, it is rarely used in statistics, because it is less mathematically tractable.<sup>20</sup> We use the average absolute error

<sup>&</sup>lt;sup>20</sup>See, however, Cook [1967], p. 200: "Why then do we use the method of least squares? Simply because absolute values are alleged to be mathematically inconvenient." Cook provides an algorithm for minimizing the average absolute error, which is simple to compute and even easier to program.

as one of the goodness-of-fit tests. Given a set of classification relativities, it is easy to calculate the average absolute error. (It is less easy to determine the set of classification relativities that minimize the average absolute error.)

The three other methods result in relatively simple iterative equations for the minimum bias procedure. We first show the procedures, and then we discuss the intuition for each.

The squared error for each cell is the square of the difference between the observed loss costs and the indicated pure premium. For urban male drivers, this is  $(\$800 - \$200 \times s_1 \times t_1)^2$ .

We sum the squared errors for the four cells to get the sum of squared errors (SSE):

SSE = 
$$(\$800 - \$200 \times s_1 \times t_1)^2$$
 urban male  
+  $(\$500 - \$200 \times s_1 \times t_2)^2$  rural male  
+  $(\$400 - \$200 \times s_2 \times t_1)^2$  urban female  
+  $(\$200 - \$200 \times s_2 \times t_2)^2$  rural female.

To minimize the sum of the squared errors, we set the partial derivatives with respect to each variable equal to zero. For the "male" classification relativity  $(s_1)$ , we have

$$0 = \partial SSE / \partial s_1 = 2 \times (\$800 - \$200 \times s_1 \times t_1) \times (-\$200 \times t_1)$$
  
+ 2 \times (\\$500 - \\$200 \times s\_1 \times t\_2) \times (-\\$200 \times t\_2).

We need to consider the cells only in the male  $(s_1)$  row. The other cells do not have an  $s_1$  term in the squared error, so the partial derivative with respect to  $s_1$  is zero.

Taking partial derivatives with respect to each of the classification relativities gives four equations in four unknowns. The equations are not linear, so we solve them iteratively. Let us choose the same starting values for the squared error bias function as we chose for the balance principle (namely  $t_1 = 2$  and  $t_2 = 1$ ):

	Urban	Rural		$terr_1 = 2$	$terr_2 = 1$
Male Female	\$800 \$400	\$500 \$200	sex <sub>1</sub> sex <sub>2</sub>	$\begin{array}{c} 200 \times s_1 \times 2 \\ 200 \times s_2 \times 2 \end{array}$	$\begin{array}{c} 200 \times s_1 \times 1 \\ 200 \times s_2 \times 1 \end{array}$

Using the squared error bias function, we solve for the male relativity  $s_1$ . To avoid dealing with multiples of 100, we choose a base pure premium of \$2 and we evaluate the observed pure premiums in multiples of \$100.

$$0 = \partial SSE / \partial s_1 = 2 \times (\$8 - \$2 \times s_1 \times 2) \times (-\$2 \times 2)$$
  
+ 2 × (\\$5 - \\$2 × s\_1 × 1) × (-\\$2 × 1)  
-64 + 32s\_1 - 20 + 8s\_1 = 0  
40s\_1 = 84  
s\_1 = 2.1.

Similarly, we solve for the female relativity  $(s_2)$ :

$$0 = \partial SSE / \partial s_2 = 2 \times (\$4 - \$2 \times s_2 \times 2) \times (-\$2 \times 2)$$
  
+ 2 × (\\$2 - \\$2 × s\_2 × 1) × (-\\$2 × 1)  
-32 + 32s\_2 - 8 + 8s\_2 = 0  
-40 + 40s\_2 = 0  
s\_2 = 1.

We now discard the starting values of  $t_1 = 2$  and  $t_2 = 1$ . Using the intermediate values of  $s_1 = 2.1$  and  $s_2 = 1$ , we set the partial derivatives of the sum of the squared errors with respect to  $t_1$ and  $t_2$  equal to zero and we solve for new values of  $t_1$  and  $t_2$ . We continue in this fashion until the series converges.

Iteration	Urban	Rural	Male	Female
Initial	2.0000	1.0000		
1-a			2.1000	1.0000
1-b	1.9224	1.1553		
2-a			2.1029	0.9940
2-b	1.9223	1.1555		
Normalized	1.6636	1.0000	2.1155	1.0000
Normalized Ba	se Pure Premium	\$200.00 × 1.1	$1555 \times 0.9940 = $2$	229.71

## Squared Error and $\chi$ -Squared Intuition

The properties of squared-error minimization in the minimum bias procedure are unlike the properties of squared-error minimization in other statistical problems, as explained below. We note first that the bias function makes a difference, even in this simple illustration.

- A. The balance principle bias function gives  $s_1 = 2.1667$ and  $s_2 = 1$ .
- B. The squared error bias function gives  $s_1 = 2.1155$  and  $s_2 = 1$ .

The balance principle ensures that the total error in each classification dimension is zero. The squared-error bias function minimizes the aggregate squared error, and the  $\chi$ -squared bias function minimizes the aggregate squared error as percentages of the expected values. The squared-error and  $\chi$ -squared bias functions place more weight on outlying cells, where the squares of the errors are large. The balance principle and the squared-error bias function place more weight on the cells with large dollar values.

*Illustration 16:* A classification system with two dimensions has male versus female in one dimension and territories 1, 2, and 3 in the other dimension. The starting relativities are 1.00, 2.00, and 3.00 for territories 1, 2, and 3. The observed loss costs for the three territories in the male row are \$2, \$4, and \$12, with

equal exposures in each cell. We assume a base pure premium of \$1.00.

	Territory 1 (1.00)	Territory 2 (2.00)	Territory 3 (3.00)
Male	\$2.00	\$4.00	\$12.00
Female	—	—	—

We want to determine the indicated relativity for males. Our concern here is not to solve this problem but to understand the effects of the different bias functions.

- If the male relativity is 2.00, the indicated pure premiums are \$2, \$4, and \$6. The first two cells have a perfect fit, and the third cell is too low by \$6.
- If the male relativity is 4.00, the indicated pure premiums are \$4, \$8, and \$12. The first two cells are too high by a total of \$6, and the third cell has a perfect fit.

The balance principle considers the first power of the errors. The average observed loss cost is (\$2 + \$4 + \$12)/3 = \$6.00. The average territory relativity is 2.00. To achieve balance, we choose a male relativity of 3.00. The indicated pure premiums are \$3, \$6, and \$9. The first two cells are too high by a total of \$3, and the third cell is too low by \$3. The indicated male/female relativity is \$6/\$2 = 3.00.

If we optimize with the balance principle, the sum of the squared errors is  $(3-2)^2 + (6-4)^2 + (9-12)^2 = 14$ . We compare this figure with the result of the least squares bias function.

The squared error bias function is more concerned with the large error in territory 3 than with the small errors in territories 1 and 2. To minimize the sum of squared errors, we increase the male relativity slightly, reducing the error in territory 3 and increasing the errors in territories 1 and 2.

### Squared Error Minimization

To solve the problem using a squared error bias function, we minimize the sum of squared errors:

$$SSE = (2 - x)^2 + (4 - 2x)^2 + (12 - 3x)^2.$$

Taking the partial derivative with respect to x and setting it equal to zero gives

 $\partial SSE / \partial x = 2(2 - x)(-1) + 2(4 - 2x)(-2) + 2(12 - 3x)(-3) = 0$  4 + 16 + 72 = 2x + 8x + 18x 92 = 28xx = 92/28 = 3.286.

The sum of the squared errors is  $(3.286 - 2)^2 + (6.571 - 4)^2 + (9.857 - 12)^2 = 12.857$ , which is less than the squared error of 14 under the balance principle. Minimizing the sum of the squared errors yields 3.286, not the average, which is 3.00.

## Squared Error Minimization

The illustration above seems odd to some statisticians. We are choosing a value to minimize the squared error among a series of observations. An elementary statistical theorem is that the average minimizes the sum of the squared errors. This seems inconsistent with the comments above.

When we set rates in a single dimension class system, minimizing the squared error produces the arithmetic average. The following illustration explains this.

*Illustration 17:* We are measuring a patient's fever with an old, imprecise thermometer. The thermometer is unbiased, but the observed readings are distorted by sampling error. We perform nine trials, and we observe readings of (100.1,100.2,...,100.9). (The readings were not in this order, so there is no trend; we have simply arranged them in ascending numerical order.) Using the least squared error function, we determine the best estimate of the patient's temperature.

We rephrase the illustration mathematically. We have observed values of  $z_1, z_2, ..., z_n$ , and we must choose a single value—call it  $z^*$ —to minimize the squared error.

The sum of the squared errors is  $\sum (z_i - z^*)^2$ . The partial derivative of this sum with respect to  $z^*$  is  $\sum 2(z_i - z^*)(-1)$ . Setting this equal to zero gives  $z^* = (\sum z_i)/n$ . The indicated  $z^*$  is the average of the  $z_i$ s.

In the temperature measurement illustration, the average of the nine observations is 100.5. This is the solution using the squared error bias function.

If we had chosen instead some other value, such as 100.3, we could correct this estimate by the average of the errors. The error in each observation is the observation minus 100.3. This is the series  $(-0.02, -0.01, 0, +0.01, \dots, +0.06)$ . The average is +0.02. The corrected estimate is 100.3 + 0.02 = 100.5.

This is not true for multidimensional systems. In a multiplicative model with two dimensions, the  $z_i$ s are the observed values. The  $z^*$  is the indicated relativity for one of the two dimensions. The other dimension has relativities of  $y_1, y_2, \ldots, y_m$ .

The sum of the squared errors is  $\sum (z_i - y_j \times z^*)^2$ . The partial derivative of this sum with respect to  $z^*$  is  $\sum (z_i - y_j) \times z^*(-y_j)$ . Setting this equal to zero gives  $z^* = (\sum z_i) / \sum y_j^2$ .

The indicated  $z^*$  is no longer the average of the  $z_i$ s. Rather, this result is the solution to the minimum bias procedure using the squared error bias function, as we show next.

## Balance Principle Optimization

When we seek a pure premium for one dimension, minimizing the squared error produces the arithmetic average. With two or more dimensions, the balance principle selects the multidimensional equivalent to the mean of each class across the other dimension(s).

The balance principle provides an unbiased solution; Bailey [1963] considers it the only unbiased solution (see below). Some actuaries believe that an unbiased solution is more likely to maximize the firm's profitability than a biased solution.<sup>21</sup>

#### General Squared Error Minimization, Multiplicative Model

We consider a more general two dimensional classification system. The base pure premium is B. We again assume one exposure per cell (or the same number of exposures per cell) to keep the equations simple. In practice, one must multiply all terms by the number of exposures.

Suppose we have two dimensions, age of driver and territory, with *n* age classes and *m* territories. The observed loss cost in the *i*th age class and the *j*th territory is  $r_{ij}$ . The indicated pure premium in the *i*th age class and the *j*th territory is  $B \times x_i \times y_i$ .

The squared error in any cell is  $(r_{ij} - Bx_iy_j)^2$ . The sum of the squared errors is

$$Q = \sum_{i=1}^{n} \sum_{j=1}^{m} (r_{ij} - Bx_i y_j)^2.$$

We take partial derivatives with respect to each variable and set them equal to zero. The (n + m) equations are not linear, so we must search for a solution by numerical methods. We choose

<sup>&</sup>lt;sup>21</sup>There are exceptional scenarios when a different bias function may be better. In a jurisdiction that places restrictions on risk classification, the bias function may have to be changed to accommodate these restrictions. If the insurer seeks to expand in certain classifications for competitive or marketing reasons, the minimum bias procedure may not accommodate the insurer's strategy. In most scenarios, however, the balance principle serves the economic interests of the firm.

starting values for one dimension—say, the  $y_j$ . To solve for the value of  $x_i$ , we take the partial derivative with respect to  $x_i$  and set it equal to zero:

$$\sum_{j=1}^{m} 2(r_{ij} - Bx_i y_j)(-By_j) = 0.$$

This gives

$$x_i = \sum_{j=1}^m (r_{ij} \times y_j) \Big/ \sum_{j=1}^m B y_j^2.$$

The  $x_i$  is a variable. The y values are fixed; they are not variables once we have assigned starting values to the y values.

We use this procedure to solve for  $x_1, x_2, ..., x_n$ . We then discard the starting *y* values and solve for new values of the *y* variables using the same procedure as we used for the *x* variables.

We have (n + m) variables, and we have (n + m) equations. The constraints for least squares minimization are the same as the constraints for the balance principle. There is one totality constraint, since taking the sum of the squared errors along the rows is the same as taking the sum of the squared errors along the columns. This means that we have only (n + m - 1) independent equations. In addition, we could multiply all the relativities along any dimension by a constant and divide the base pure premium by the same constant.

### Squared Error Minimization, Additive Model

We can also use an additive model with the least squares bias function. We first show the results for the elementary 2 by 2 illustration. Below are the same observed loss costs and indicated pure premiums we have been using.

	Urban	Rural		terr <sub>1</sub>	terr <sub>2</sub>
Male Female	\$800 \$400	\$500 \$200	sex <sub>1</sub> sex <sub>2</sub>	$200 + s_1 + t_1 \\ 200 + s_2 + t_1$	$200 + s_1 + t_2 200 + s_2 + t_2$

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As mentioned earlier, there are three mathematically equivalent ways of defining the additive model; the solution method is the same for each of them. The pure premium in cell  $x_iy_j$  is

- A. Base pure premium +  $x_i$  +  $y_i$ ,
- B. Base pure premium  $\times (1 + u_i + v_j)$ , or
- C. Base pure premium  $\times (p_i + q_i)$ .

We use the first of these three equations here for its intuitive simplicity. Note that a multiplicative relationship between the base pure premium and the relativities does not make the model multiplicative. If the relationship among the factors is additive, the model is additive. A combined multiplicative and additive model has relationships among the relativities that are both multiplicative and additive.

The method used here is the same as the method used for the multiplicative model above. For the male urban cell, the squared error is  $(\$800 - \$200 - s_1 - t_1)^2$ . The sum of the squared errors for all four cells is

$$Q = (\$800 - \$200 - s_1 - t_1)^2 + (\$500 - \$200 - s_1 - t_2)^2 + (\$400 - \$200 - s_2 - t_1)^2 + (\$200 - \$200 - s_2 - t_2)^2.$$

We take partial derivatives with respect to each variable and set them equal to zero. The partial derivative with respect to  $s_1$  is

$$\partial Q / \partial s_1 = 2(\$800 - \$200 - s_1 - t_1)(-1) + 2(\$500 - \$200 - s_1 - t_2)(-1) = 0,$$

or

$$s_1 = (\$900 - t_1 - t_2)/2.$$

For the additive model with the least squares bias function, the simultaneous equations are linear, and we can solve them directly. Nevertheless, it is easier to program the solution using numerical methods. If we choose starting values of  $t_1 = $250$ 

and  $t_2 = \$0$ , we get  $s_1 = \$325$ . We leave it to the reader to verify that the relativities converge to the same figures as the additive model with the balance principle.

# General Squared Error Minimization, Additive Model

For the general formula, we let B = the base pure premium. The sum of the squared errors is

SSE = 
$$\sum_{i=1}^{n} \sum_{j=1}^{m} (r_{ij} - B - x_i - y_j)^2$$
.

We take the partial derivative with respect to  $x_i$  and set it equal to zero:

$$\frac{\partial \text{SSE}}{\partial x_i} = \sum_{j=1}^m 2(r_{ij} - B - x_i - y_j)(-1) = 0,$$

or

$$x_i = \sum_{j=1}^m (r_{ij} - y_j)/m - B.$$

### 6. THE $\chi$ -SQUARED BIAS FUNCTION

The  $\chi$ -squared bias function is similar to the squared error bias function, except that each squared error is divided by the expected value in that cell. We define the  $\chi$ -squared bias function, and then we apply it to the minimum bias procedure.

*Illustration 18:* Suppose the expected distribution in two cells is 40%–60% and the observed distribution is 30%–70%. The squared error is  $(40\% - 30\%)^2 + (60\% - 70\%)^2 = 2.00\%$ ; the  $\chi$ -squared error is  $(40\% - 30\%)^2/40\% + (60\% - 70\%)^2/60\% = 4.17\%$ .

We show the application of the  $\chi$ -squared bias function to the multiplicative illustration.

	Urban	Rural		terr <sub>1</sub>	terr <sub>2</sub>
Male Female	\$800 \$400	\$500 \$200	sex <sub>1</sub> sex <sub>2</sub>	$\begin{array}{c} 200 \times s_1 \times t_1 \\ 200 \times s_2 \times t_1 \end{array}$	$\begin{array}{c} 200 \times s_1 \times t_2 \\ 200 \times s_2 \times t_2 \end{array}$

The  $\chi$ -squared value for each cell is (observed value – expected value)<sup>2</sup>/expected value. For urban male drivers in our basic illustration, this number is

$$(\$800 - \$200 \times s_1 \times t_1)^2 / (\$200 \times s_1 \times t_1).$$

We sum the squared errors for the four cells to get the sum of  $\chi$ -squared values:

SSE = 
$$(\$800 - \$200 \times s_1 \times t_1)^2 / (\$200 \times s_1 \times t_1)$$
 urban male  
+  $(\$500 - \$200 \times s_1 \times t_2)^2 / (\$200 \times s_1 \times t_2)$  rural male  
+  $(\$400 - \$200 \times s_2 \times t_1)^2 / (\$200 \times s_2 \times t_1)$  urban female  
+  $(\$200 - \$200 \times s_2 \times t_2)^2 / (\$200 \times s_2 \times t_2)$  rural female.

To minimize the sum of the squared errors, we take partial derivatives with respect to each variable and set them to zero. For the male classification relativity  $(s_1)$ , we have<sup>22</sup>

$$\begin{split} 0 &= \partial SSE/\partial s_1 \\ &= [(\$200 \times s_1 \times t_1) \times 2 \times (\$800 - \$200 \times s_1 \times t_1) \times (-\$200 \times t_1) \\ &- (\$800 - \$200 \times s_1 \times t_1)^2 \times (\$200 \times t_1)]/(\$200 \times s_1 \times t_1)^2 \\ &+ [(\$200 \times s_1 \times t_2) \times 2 \times (\$500 - \$200 \times s_1 \times t_2) \times (-\$200 \times t_2) \\ &- (\$500 - \$200 \times s_1 \times t_2)^2 \times (\$200 \times t_2)]/(\$200 \times s_1 \times t_2)^2 \\ &+ 0 + 0. \end{split}$$

Although the arithmetic looks cumbersome, the equation can be simplified. To avoid needless arithmetic, we derive the general solution, and we then resume the illustration.

<sup>&</sup>lt;sup>22</sup>We use the quotient rule that if y(x) = f(x)/g(x), then  $dy/dx = [g(x) \times df/dx - f(x) \times dg/dx]/g^2(x)$ .
## $\chi$ -Squared Recursive Equations

We show the general recursive equations for the  $\chi$ -squared bias function with two classes in each of two dimensions; the extension to three or more dimensions is straightforward. To save space, we include the number of exposures and derive the final recursive equations.

	<i>y</i> <sub>1</sub>	<i>Y</i> <sub>2</sub>		<i>y</i> <sub>1</sub>	<i>y</i> <sub>2</sub>	
$\begin{array}{c} x_1 \\ x_2 \end{array}$	$n_{11} \\ n_{21}$	$n_{12} \\ n_{22}$	$\begin{array}{c} x_1 \\ x_2 \end{array}$	$B \times x_1 \times y_1 B \times x_2 \times y_1$	$B \times x_1 \times y_2 B \times x_2 \times y_2$	

We form the  $\chi$ -squared bias function as a double summation covering all the cells in the array.

SSE = 
$$\sum_{i=1}^{n} \sum_{j=1}^{m} \frac{(n_{ij}r_{ij} - n_{ij}Bx_iy_j)^2}{n_{ij}Bx_iy_j}$$
.

We factor out the number of exposures in each cell from the summand to give

SSE = 
$$\sum_{i=1}^{n} \sum_{j=1}^{m} \frac{n_{ij}(r_{ij} - Bx_iy_j)^2}{n_{ij}Bx_iy_j}$$
.

We seek to minimize the  $\chi$ -squared value. To simplify the mathematics, we use a base pure premium of \$1.00. In practice, we would choose the base pure premium at the end of the procedure to set the base relativities in each dimension to unity.

As before, given starting values for either dimension, we determine the intermediate values for the other dimension. Assume we have chosen starting values for the y relativities and we are solving for an intermediate value of  $x_i$ . Only the cells in the *i*th row have terms with  $x_i$  in them. We take the partial derivative of this row with respect to  $x_i$ , and we set it equal to 0.

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In the equation below, we have differentiated with respect to  $x_i$ , and the summation is over the *j* dimension. The value of *i* is fixed.

$$\sum_{j=1}^{m} \frac{n_{ij} [x_i y_j 2(r_{ij} - x_i y_j)(-y_j) - (r_{ij} - x_i y_j)^2 y_j]}{(x_i y_j)^2} = 0.$$

The value  $x_i = 0$  does not minimize the equation (or all the indicated pure premiums would be zero), so we multiply both sides of the equation by  $(x_i)^2$ . Simplifying further,

$$\sum_{j=1}^{m} \left[ -2n_{ij}x_i(r_{ij} - x_iy_j) - \frac{n_{ij}}{y_j}(r_{ij} - x_iy_j)^2 \right] = 0.$$

We expand the square and combine like terms:

$$\begin{split} &\sum_{j=1}^{m} \left[ -2n_{ij}x_ir_{ij} + 2n_{ij}x_i^2y_j - \left(\frac{n_{ij}}{y_j}\right)r_{ij}^2 + 2n_{ij}x_ir_{ij} - n_{ij}x_i^2y_j \right] = 0, \\ &\sum_{j=1}^{m} \left[ n_{ij}x_i^2y_j - \left(\frac{n_{ij}}{y_j}\right)r_{ij}^2 \right] = 0. \end{split}$$

This gives a relatively simple expression for each  $x_i$  in terms of the  $y_i$  values:

$$x_i = \left[ \sum_{j=1}^m \left( \frac{n_{ij} r_{ij}^2}{y_j} \right) \middle/ \sum_{j=1}^m n_{ij} y_j \right]^{0.5}$$

In the illustration used here, there is one exposure in each cell. The starting values are  $t_1 = 2$  and  $t_2 = 1$ . We use a base pure premium of \$200, and we divide all cells by \$200.

	Urban	Rural		$terr_1 = 2$	$terr_2 = 1$
Male Female	\$4 \$2	\$2.5 \$1	sex <sub>1</sub> sex <sub>2</sub>	$\begin{array}{c} s_1 \times 2 \\ s_2 \times 2 \end{array}$	$\begin{array}{c} s_1 \times 1 \\ s_2 \times 1 \end{array}$

Using the  $\chi$ -squared bias function along the first row, we get

$$s_1$$
(male relativity) =  $[(4^2/2 + 2.5^2/1)/(2+1)]^{0.5} = 2.179.$ 

Using the  $\chi$ -squared bias function along the second row, we get

 $s_2$ (female relativity) =  $[(2^2/2 + 1^2/1)/(2 + 1)]^{0.5} = 1.000.$ 

The male-to-female relativity is 2.179 to 1. The series converges.

Iteration	Urban	Rural	Male	Female
Initial	2.0000	1.0000		
1-a			2.1794	1.0000
1-b	1.8887	1.1029		
2-a			2.1739	1.0055
2-b	1.8884	1.1032		
3-а			2.1739	1.0055
Normalized	1.7118	1.0000	2.1620	1.0000
Normalized B	ase Pure Premium	$200.00 \times 1.1$	$1032 \times 1.0055 = $	21.85

The final relativities are 2.1620 for the  $\chi$ -squared bias function and 2.1155 for the least squares bias function. The dollar values in the urban male cell are larger than the dollar values in the rural male cell, so the least squares bias function gives more weight to the urban male cell as compared to the rural male cell than the  $\chi$ -squared bias function gives.

# Additive Model with $\chi$ -Squared

The  $\chi$ -squared bias function can be used with any type of model, whether multiplicative, additive, or combined. If an additive model is used, we minimize the following expression:

SSE = 
$$\sum_{i=1}^{n} \sum_{j=1}^{m} n_{ij} (r_{ij} - x_i - y_j)^2 / (x_i + y_j).$$

We set the partial derivative with respect to each relativity equal to zero. It is easiest to solve the resulting set of simultaneous equations by iteration, solving for  $\Delta x_i$  rather than for  $x_i$ . Bailey and Simon [1960], followed by Brown [1988], give the recursive equations as

$$\Delta x_i = \frac{\sum_j n_{ij} \left(\frac{r_{ij}}{x_i + y_j}\right)^2 - \sum_j n_{ij}}{2\sum_j n_{ij} \left(\frac{r_{ij}}{x_i + y_j}\right)^2 \left(\frac{1}{x_i + y_j}\right)}.$$

The series converges along the following path.

Iteration	Urban	Rural	Male	Female
Initial	\$250.00	\$0.00		
1			\$96.81	(\$15.53)
2	\$334.23	\$63.18		
3			\$193.97	(\$115.14)
4	\$349.61	\$97.35		
5			\$220.32	(\$112.08)
6	\$336.84	\$103.42		
7	\$224.64	(\$113.40)		
Final	\$233.43	\$0.00	\$338.04	\$0.00
Base Pure Prem	nium			\$190.02

## 7. MAXIMUM LIKELIHOOD

Some statisticians prefer a maximum likelihood bias function to either a  $\chi$ -squared or a least squares bias function when fitting a distribution to observed data. In his 1988 *Proceedings* paper, Rob Brown used a maximum likelihood bias function to optimize classification relativities. The maximum likelihood bias function is rarely used in practical work, and not all actuaries are familiar with it.

The maximum likelihood bias function requires an assumption about the distribution of values in each class. The appropriate distribution for loss costs is not evident. It probably is not a simple parametric distribution, such as an exponential distribution or a Poisson distribution. If there is not support for a specific distribution, the merits of a maximum likelihood bias function are less clear.

# Likelihood and Probability

We use the term likelihood, not probability. For a continuous distribution, the probability of observing a specific value is zero. If the exponential distribution function has  $\lambda$  of 0.0001, the likelihood of a loss of size \$20,000 is  $0.0001 \times e^{-2}$ .

*Illustration 19:* We are fitting an exponential curve to a set of insurance losses. For the exponential distribution function, the likelihood of a loss of size *x* is  $\lambda e^{-\lambda x}$ . We use integration by parts to solve for the mean of the exponential distribution function:

$$\int_0^\infty x\lambda e^{-\lambda x} dx = \left(-x\lambda e^{-\lambda x} - \frac{1}{\lambda}e^{-\lambda x}\right)\Big|_0^\infty = \frac{1}{\lambda}$$

To fit an exponential curve to a set of insurance losses, we must determine  $\lambda$ . After discussing two methods outlined above (for comparison), we show the maximum likelihood method.

# Method of Moments

The mean of the exponential distribution is  $1/\lambda$ . We take the average of the observations, and we set  $\lambda$  equal to the reciprocal of this average.

## Least Squares

We divide the loss sizes into ranges, such as \$0 to \$5,000, \$5,001 to \$25,000, \$25,001 to \$100,000, and so forth. We calculate the percentage of observed losses that fall into each range. For any given  $\lambda$ , we determine the percentage of theoretical losses that would fall into each range.

For each range, we calculate the squared difference between the observed percentage and the theoretical percentage. We sum the squared differences over all the ranges. The result is a function of  $\lambda$ . To minimize this squared difference, we set the derivative with respect to  $\lambda$  equal to zero.

# $\chi$ -Squared

The  $\chi$ -squared procedure is similar to the least squares procedure, but instead of taking the squared difference we take the  $\chi$ -squared difference. For each range, we divide the squared difference by the expected value.

## Maximum Likelihood

We explain the method by means of an illustration. Suppose we have observed five losses with sizes of \$3,000, \$5,000, \$15,000, \$20,000, and \$80,000. For a given value of  $\lambda$ , the likelihood of a loss equal to \$3,000 is  $\lambda e^{-\lambda \times 3,000}$ . The likelihood of five losses for the values listed above is the product of the likelihoods of each individual loss, or

$$L = \lambda e^{-\lambda \times 3,000} \times \lambda e^{-\lambda \times 5,000} \times \lambda e^{-\lambda \times 15,000} \times \lambda e^{-\lambda \times 20,000} \times \lambda e^{-\lambda \times 80,000}.$$

We simplify the likelihood to  $\lambda^5 e^{-\lambda \times 123,000}$  (\$123,000 is the sum of the losses). To find the  $\lambda$  that gives the greatest likelihood, we set the derivative with respect to  $\lambda$  equal to zero.

Before taking the derivative, we make one simplification. Maximizing a strictly increasing function, like the likelihood function, is the same as maximizing its logarithm. The logarithm of the likelihood (the log-likelihood, or LL) is

LL = 
$$\ln L = 5 \ln \lambda - 123,000 \times \lambda$$
  
 $d(\ln L)/d\lambda = 5/\lambda - 123,000 = 0$ , or  $\lambda = 5/123,000$ .

# Maximum Likelihood and Minimum Bias Procedure

The rating model uses the classification relativities to determine the expected loss in each cell. The maximum likelihood test is most practicable as a bias function when a single parameter distribution is used and the mean of the distribution equals the parameter itself or some simple function of the parameter, such as its reciprocal. It is most valuable when the distribution is a reasonable reflection of the insurance process.

The exponential and Poisson distributions have these properties. We illustrate a multiplicative model with the exponential distribution function, using the same illustration as before.

	Urban	Rural		terr <sub>1</sub>	terr <sub>2</sub>
Male Female	\$800 \$400	\$500 \$200	sex <sub>1</sub> sex <sub>2</sub>	$\begin{array}{c} 200 \times s_1 \times t_1 \\ 200 \times s_2 \times t_1 \end{array}$	$\begin{array}{c} 200 \times s_1 \times t_2 \\ 200 \times s_2 \times t_2 \end{array}$

Each class has an assumed exponential distribution of loss costs. If the indicated pure premium is \$200, we expect the observed losses to follow an exponential distribution with a mean of \$200. The  $\lambda$  differs by cell. The indicated pure premium in each cell is  $1/\lambda$ .

*Illustration 20:* For the urban male cell, the loss costs have an exponential distribution with the parameter  $\lambda$  equal to  $1/(\$200 \times s_1 \times t_1)$ .

We choose starting values for  $t_1 = 2.00$  and  $t_2 = 1.00$ . We determine the likelihood of the observed loss costs. The value of  $\lambda$  for the urban male cell is  $1/(200 \times s_1 \times t_1) = 1/(400 \times s_1)$ . The likelihood of the \$800 loss cost in the urban male cell is

$$\frac{1}{400s_1}e^{-800/400s_1} = \frac{1}{400s_1}e^{-2/s_1}$$

The likelihoods of the observed values in the other cells are determined in the same manner. To maximize the likelihood, we maximize the log-likelihood. To repeat,

• The likelihood of four observed values is the product of the four individual likelihoods.

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• The log-likelihood of four observed values is the sum of the four individual log-likelihoods.

The partial derivative of the log-likelihood with respect to  $s_1$  depends on the log-likelihoods in the male row only. This is the same simplification that we used for the least squares method and the  $\chi$ -squared method.

The log-likelihood of the values in the male row is

$$LL = -\ln(400s_1) - (800/400) \times 1/s_1 - \ln(200s_1)$$
$$- (500/200) \times 1/s_1$$
$$\partial LL/\partial s_1 = -1/s_1 + 2s_1^{-2} - 1/s_1 + 2.5s_1^{-2} = 0$$
$$-s_1 + 2 - s_1 + 2.5 = 0, \text{ because } s_1 \neq 0$$
$$s_1 = 2.25.$$

The log-likelihood of the values in the female row is

$$LL = -\ln(400s_2) - (400/400) \times 1/s_2 - \ln(200s_2)$$
$$- (200/200) \times 1/s_2$$
$$\partial LL/\partial s_2 = -1/s_2 + 1s_2^{-2} - 1/s_2 + 1s_2^{-2} = 0$$
$$-s_2 + 1 - s_2 + 1 = 0$$
$$s_2 = 1.00.$$

The series converges to the following relativities.

Iteration	Urban	Rural	Male	Female
Initial	2.0000	1.0000		
1-a			2.2500	1.0000
1-b	1.8889	1.0556		
2-a			2.2430	1.0031
2-b	1.8886	1.0557		
3-a			2.2430	1.0031
Normalized	1.7889	1.0000	2.2361	1.0000
Normalized Ba	se Pure Premium	$200.00 \times 1.0$	$557 \times 1.0031 = $	211.80

# **Justification**

If the distribution of loss costs is a simple mathematical function, such as a Poisson distribution, a normal distribution, a lognormal distribution, or an exponential distribution, we can derive simple recursive equations; see Brown [1988]. In practice, we don't know the proper distributions. The distributions that have been suggested for use in the minimum bias procedure are not necessarily correct. They are simply tractable.

The Poisson distribution is a reasonable model for loss frequency distributions, though not for loss severity distributions. The normal, lognormal, and exponential distributions may not be ideal fits to the loss costs distribution. However, maximum likelihood estimation is particularly useful when examining loss frequency and loss severity with generalized linear models; see Feldblum et al. [forthcoming].

# The Bias Function

The optimal class relativities for a given data set depend on the choice of the bias function. The choice of bias function can be viewed from three perspectives:

- 1. Mathematical tractability,
- 2. Social equity, or
- 3. Economic optimization.

Mathematical tractability was of concern when computational capacity was limited and some bias functions gave simple relationships while other bias functions gave intractable equations. The minimum bias procedure gives simple equations for the bias functions discussed in this paper. With modern spreadsheets, however, even the average absolute error does not pose tractability issues. Just as the solution for the balance principle is the mean, the solution for the average absolute error is the median. It is not uncommon for actuaries to use the median instead of the mean in practical problems. Social equity is subjective, though it is vital to the success of a highly regulated industry like insurance. The balance principle sometimes results in large errors for outlying cells. The errors may be particularly large for high-rated cells. If a multiplicative model is used when an additive model is more appropriate, the errors for outlying cells are frequently overcharges.

Of the bias functions that we consider in this paper, the squared-error bias function is the best at reducing large overcharges for individual cells. Ferreira's critique of insurance industry classification systems in Massachusetts illustrates this social position.<sup>23</sup>

Economic optimization drives the behavior of firms in free markets. Firms seek to maximize profits and to minimize losses (among other firm objectives). Suppose an insurer issues three policies. It must choose between two rating systems:

- A. Under rating system A, it expects to lose \$1.00 each on the first two policies and to break even on the third policy.
- B. Under rating system B, it expects to break even on the first two policies and to lose \$1.50 on the third policy.

Rating system A is off by \$2.00 using the balance principle while rating system B is off by \$1.50. Using the squared error bias function, rating system A is off by 2.00 dollars-squared while rating system B is off by 2.25 dollars-squared. The balance principle says we should choose rating system B, and the squared error bias function says we should choose rating system A.

To maximize profits (or minimize losses), we would probably prefer rating system B, as the balance principle says. In practice,

 $<sup>^{23}</sup>$ See Ferreira [1978], as well as Cummins et al., [1983] chapter 4. We are not endorsing Ferreira's views, which are inconsistent with competitive insurance markets; see the discussion in the text of this paper.

economic forces are more complex than short-term profit maximization. There are many reasons for avoiding errors, including consumer dissatisfaction, consumer switching, and public relations. In democratic systems where social opinion and political pressures are strong, firms may sacrifice short-term profit maximization to achieve other ends; objectives such as workforce diversity and environmental protection are examples. Furthermore, manager incentives may encourage the pursuit of other goals, such as corporate growth instead of profit maximization. Nevertheless, profit maximization remains the dominant corporate goal. The pricing actuary should keep these social and economic considerations in mind when choosing a bias function for the minimum bias procedure.

## 8. COMBINED MODELS

Throughout this paper, we have used simple multiplicative and additive models, not combined models. This reflects current insurance practice.

In truth, business practice reflects ratemaking capabilities. Actuaries have not had simple methods to optimize combined models, so these models have not gained wide acceptance.

The rationale for combined models is strong. Since the least squares and  $\chi$ -squared bias functions provide simple recursive equations for many combined models, these models may become more popular in the future.<sup>24</sup>

*Illustration 21:* Rating territory may have a variety of effects on insurance loss costs.

1. High-crime areas may have a greater incidence of car theft and claim fraud. Thefts would raise comprehensive pure premiums, and fraud would raise liability pure premiums.

<sup>&</sup>lt;sup>24</sup>Generalized linear models allow the optimization of even more complex rating models; see Feldblum et al. [forthcoming].

- 2. Areas with more sophisticated medical facilities may have higher loss costs for bodily injury claims.
- 3. Territories with a higher number of attorneys per capita may experience a higher incidence of bodily injury claims per physical accident.<sup>25</sup>

The first effect argues for an additive model; the third effect suggests a multiplicative model; and the second effect may have both additive and multiplicative components. The greater incidence of theft may be unrelated to other hazards, whereas a higher proportion of attorneys may affect claim filing for all hazards.<sup>26</sup> Intuition alone, though, is rarely sufficient to optimize a rating model. The minimum bias method allows the pricing actuary to determine the optimal rating structure from the observed loss costs.

*Illustration 22: Combined Model:* We keep the same male/ female and urban/rural classification system. We assume now that the male/female rating dimension has a multiplicative effect on loss costs, and the rating territory dimension has both a multiplicative and an additive effect on loss costs. We show the structure of this rating model, and we explain how to optimize it.

For the male/female classification dimension, we use pure premium relativities of  $s_1$  and  $s_2$ . For the urban/rural dimension, each class has two relativities: a multiplicative relativity denoted by  $t_1$  and  $t_2$ , and an additive relativity denoted by  $z_1$  and  $z_2$ . We denote the base pure premium as B.

The indicated pure premium for any class is  $B \times (s_i \times t_j + z_j)$ . The subscripts *i* and *j* denote the classification dimension. The indicated pure premiums are shown in the following table.

<sup>&</sup>lt;sup>25</sup>See Conners and Feldblum [1998] for the effects of territory on private passenger automobile claim frequency.

<sup>&</sup>lt;sup>26</sup>On reviewing this paper, Ginda Fisher suggested that rating variables such as fire protection, theft protection devices, and age of dwelling may be additively related for Home-owners insurance, though there may also be some multiplicative relationships among them.

Observed Loss Costs				Indicated Pure Premiums		
	Urban	Rural		Urban $(t_1, z_1)$	Rural $(t_2, z_2)$	
Male	$r_{11} = \$800$	$r_{12} = \$500$	Male $(s_1)$	$B\times (s_1\times t_1+z_1)$	$B\times (s_1\times t_2+z_2)$	
Female	$r_{21} = \$400$	$r_{22} = \$200$	Female $(s_2)$	$B\times (s_2\times t_1+z_1)$	$B\times (s_2\times t_2+z_2)$	

If we use the balance principle as the bias function, we balance along the two rows and the two columns. This gives four equations, of which only three are independent, since there is a totality constraint. We must solve for six classification relativities.

When there are more unknowns than equations, the iterations will not necessarily converge. If they do converge, the convergence is generally not unique. If the balance principle is used with a multidimensional combined multiplicative and additive model, there are more relativities than there are equations.

If we use a least squares or a  $\chi$ -squared bias function, the combined model is not conceptually different from a simple multiplicative or additive model. We set the partial derivative with respect to each rating variable equal to zero. This guarantees the same number of equations as rating variables.

The use of the minimum bias procedure with combined models is a powerful rating tool. But as the rating models grow more complex, there are more classification dimensions, more cells, and fewer exposures in each cell. The potential rating errors become more serious as the effect of random loss fluctuations grows.

# Outliers

The least squares and  $\chi$ -squared bias functions are particularly sensitive to outliers. Outliers are observed values that differ substantially from their expected values because of random loss fluctuations. Distortions stemming from random loss fluctuations can be controlled in several ways:

- Losses can be capped at basic limits or similar retentions.
- Low-volume classes can be assigned limited credibility.
- The data in each cell can be examined for unusual values.

The use of low retentions or low credibility conflicts with the objective of basing rates on observed experience as much as possible. The examination of the observed data for unusual values is sometimes too time-consuming for practical work. In any case, one should choose a bias function that is not too sensitive to outliers.

Illustration 23: A classification system has two dimensions: male/female along one dimension and ten territories along the other dimension. The current driver relativities are 1.00 for female and 2.00 for male. The current territorial relativities are 1.00, 2.00, ..., 10.00 for the ten territories, labeled (01, 02, ..., 10). The base pure premium is \$100, and a multiplicative model is used.

*Scenario A:* The observed loss costs are shown below, in units of 100 dollars.

Territory:	01	02	03	04	05	06	07	08	09	10
Male	\$2	\$4	\$6	\$8	\$10	\$12	\$14	\$16	\$18	\$20
Female	\$1	\$2	\$3	\$4	\$5	\$6	\$7	\$8	\$9	\$10

The observed loss costs exactly match the indicated pure premiums in the current rating system. No change to the current relativities is indicated.

*Scenario B:* Because of a random large loss, the observed loss costs for the males in territory 10 are \$10,000 instead of \$2,000. The "territory 10 male" cell shows \$100 instead of \$20. This type of random loss fluctuation is common in classifica-

tion analysis for small populations. We have starting values of (1.000, 2.000, ..., 10.000) for the ten territories. We determine the intermediate value for the male relativity.

The balance principle selects the male relativity  $s_1$  such that (on the first iteration)

$$(s_1 \times t_1) + (s_1 \times t_2) + \dots + (s_1 \times t_{10}) = r_{1,1} + r_{1,2} + \dots + r_{1,10}$$
$$s_1 \times \$55 = \$190$$
$$s_1 = 3.455.$$

The least squares bias function selects the male relativity to minimize the squared error:

$$SSE = \sum_{j=1}^{m} (r_{1j} - s_1 \times t_j)^2$$
$$\partial SSE / \partial s_1 = \sum_{j=1}^{m} 2(r_{1j} - s_1 \times t_j) \times (-t_j) = 0$$
$$s_1 = \frac{\sum_{j=1}^{m} (r_{1j} \times t_j)}{\sum_{j=1}^{m} t_j^2}$$

 $[(1 \times 2) + (2 \times 4) + (3 \times 6) + \dots + (9 \times 18) + (10 \times 100)]/$  $[1^2 + 2^2 + 3^2 + \dots + 9^2 + 10^2] = 4.078.$ 

Compared with the balance principle, the least squares bias function exacerbates the distortion caused by random loss fluctuations. In this instance, the  $\chi$ -squared bias function magnifies the distortion less than the least squares bias function does. This is not always the case; in other instances, the  $\chi$ -squared bias function magnifies the distortion more than the least squares bias function does. Since combined models are more sensitive to random loss fluctuations than simple models are, and since the least-squares or  $\chi$ -squared bias function must be used, the pricing actuary must be particularly careful to exclude outliers from the data.

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## 9. GOODNESS-OF-FIT

For a given rating model and bias function, the minimum bias procedure optimizes the relativities. We now wish to optimize the rating system by choosing the best rating model and bias function. The choice of rating model, such as multiplicative, additive, or combined, depends on the characteristics of the observed loss costs. The choice of the bias function depends on the objective:

- The statistician seeking the best fit might use a maximum likelihood function if a tractable distribution function is appropriate for this coverage, or a  $\chi$ -squared function if the probability distribution function is not known or not tractable.
- The regulator seeking to avoid large dollar mismatches between observed loss costs and indicated pure premiums might use a least squares function.
- The insurer seeking to avoid monetary losses might use the balance principle.

The preferences listed above are examples; other preferences are also possible. In particular, a regulator might prefer the balance principle to provide the most efficient rating system.

## Empirical Tests

We can test the choice of rating model empirically.

*Illustration 24:* We are using a  $\chi$ -squared bias function to optimize classification relativities. We do not know whether a multiplicative model or an additive model is more appropriate.

We perform the minimum bias procedure twice with a  $\chi$ -squared bias function: once with the multiplicative model and once with an additive model. After optimizing the relativities for each model, we compare the final  $\chi$ -squared difference between the observed loss costs and the indicated pure premiums for each model. The model with the lower  $\chi$ -squared is preferred.

*Illustration 25:* We are using the balance principle to optimize classification relativities. We do not know whether a multiplicative model or an additive model is more appropriate.

We perform the minimum bias procedure twice with a balance principle bias function: once with the multiplicative model and once with an additive model. After optimizing the relativities for each model, we compare the average absolute difference between the observed loss costs and the indicated pure premiums for each model. The model with the lower average absolute difference is preferred.

We cannot empirically test the suitability of the bias function, as explained below.

*Illustration 26:* We are using a multiplicative model, and we are deciding between the balance principle and the  $\chi$ -squared function.

We perform the minimum bias procedure twice: once with the multiplicative model and the balance principle and once with the multiplicative model and a  $\chi$ -squared bias function.

If we test the performance of the models by using a  $\chi$ -squared test to measure the difference between the observed loss costs and the indicated pure premiums, the  $\chi$ -squared bias function does better. This result is tautological, since the  $\chi$ -squared bias function minimized the  $\chi$ -squared difference between the observed loss costs and the indicated pure premiums.

If we test the performance of the two models by using the average absolute difference between the observed loss costs and the indicated pure premiums, the balance principle generally does better. The  $\chi$ -squared bias function minimizes large percentage errors. The balance principle and the average absolute difference minimize dollar differences.

The choice of bias function is a qualitative choice, depending on the objectives of the rating system. It is not subject to a quantitative test of suitability.

## Squared Error versus $\chi$ -Squared

The squared error bias function is similar to the  $\chi$ -squared bias function, but whereas the squared error test looks at absolute differences, the  $\chi$ -squared test looks at percentage differences. Some statisticians prefer the  $\chi$ -squared test to a least squares test.

*Illustration 27:* We are fitting a distribution to two empirical data points:

- Point A has an observed value of \$101 and a fitted value of \$100.
- Point B has an observed value of \$1.50 and a fitted value of \$1.00.

We examine the errors for each point:

- The squared error is  $(101 100)^2 = 1.00$  for point A and  $(1.50 1.00)^2 = 0.25$  for point B. This distribution fits point B better.
- The  $\chi$ -squared value is  $(101 100)^2/100 = 0.01$  for point A and  $(1.50 1.00)^2/1.00 = 0.25$  for point B. This distribution fits point A better.

The statistician might prefer the  $\chi$ -squared test to the squared error test. The practical businessperson might argue that the insurance enterprise is not concerned with optimizing a statistical fit. It is concerned with optimizing net income. At point A, the insurer has a gain or loss of \$1.00. At point B, the gain or loss is \$0.50. The squared error test is preferred.

This argument does not fully reflect the purpose of the minimum bias procedure. The argument would be correct if we fully believed the observed values—that is, if the observed values were fully credible. But if the observed values were fully credible, we would have no need to use the minimum bias procedure; we would just use the rates indicated by the observed loss costs in each cell.

We are using the minimum bias procedure because the individual observed values are not fully credible, and we believe that the relationships among all the cells in the observed matrix provide useful information for choosing the true expected values. When we say that a particular fit *X* has less of an error than another fit *Y*, we do not mean that we know the true values and that model *X* is closer to these true values. We generally do not know the true values, but we presume that these true values might be represented by a mathematical function. When we say that fit *X* is better, we mean that model *X* is more likely to be a better model. The  $\chi$ -squared bias function perhaps does a better job of choosing the better model. If so, the businessperson might also prefer the  $\chi$ -squared bias function.

# Balance Principle versus $\chi$ -Squared

The 1960 Bailey and Simon paper prefers the  $\chi$ -squared bias function to the balance principle, whereas the 1963 Bailey paper argues for the balance principle. In defense of the  $\chi$ -squared bias function, the 1960 Bailey and Simon paper says (p. 10):

... the indication of each group should be given a weight inversely proportional to the standard deviation of the indication.

This is a traditional justification for classical credibility, as Bailey and Simon continue:

The standard deviation of the indication is inversely proportional to the square root of the expected number of losses for the group.<sup>27</sup>

<sup>&</sup>lt;sup>27</sup>Bailey and Simon [1960] assume that if all claims are independent, the variance is proportional to the number of claims, so the standard deviation is proportional to the square root of the number of claims (see also Longley-Cook [1962]). After the writings of Hans Bühlmann, Gary Venter, Howard Mahler, and others, this assumption is no longer the standard rationale for credibility.

The 1963 Bailey paper prefers the balance principle because it is unbiased, whereas the  $\chi$ -squared bias function may be biased. By "unbiased," Bailey means that the balance principle constrains the relativities so that the total indicated pure premiums along any dimension equal the total observed loss costs along that dimension.

The balance principle uses the first-order departure, which is generally preferred by firms seeking to maximize profits. This is perhaps the strongest argument for the balance principle.

Common practice among casualty actuaries is to use the balance principle. One might presume that since more effective procedures drive out less effective procedures in a competitive market, the balance principle is perhaps the most effective bias function.

In truth, many ratemaking procedures are selected for ease of implementation, not necessarily for accuracy. The balance principle was easier to implement before the widespread use of desktop computers. Few actuaries have tried the  $\chi$ -squared bias function or the least squares bias function. No conclusions should be drawn from the current practice.

## 10. CREDIBILITY

Many practitioners combine the minimum bias procedure with credibility weighting of the indicated pure premiums, either with the observed loss costs or with the underlying pure premiums. We show illustrations of each method.

The minimum bias procedure gives the indicated pure premiums for each class in an array. One may choose the pure premiums used for the final rates as a weighted average of the indicated pure premiums and the observed loss costs for that class. The credibility for the observed loss costs is a function of the volume of business in the class. Classes with greater volume place more weight on the observed loss costs; see Venter [1992]. Classical credibility formulas are the most commonly used. Classes with a certain volume of claims or of exposures are given full credibility. The square root rule is used for classes with lower volume of claims or exposures.

*Illustration 28:* Suppose that classes with exposure of 10,000 or more car-years are accorded full credibility. A class with 3,600 car-years of exposure has an \$800 observed loss cost. The minimum bias indicated pure premium for this class is \$700. The credibility assigned to the class is  $(3,600/10,000)^{0.5} = 60\%$  credibility. The credibility weighted pure premium is

 $60\% \times \$800 + (1 - 60\%) \times \$700 = \$760.$ 

*Illustration 29:* For premises and operations ratemaking, Insurance Services Office (ISO) uses a balance principle minimum bias procedure with observed loss ratios to determine the indicated changes to class group and type of policy relativities.<sup>28</sup>

- An indicated relativity change of 1.08 for type of policy 12 means that the existing relativity for type of policy 12 should be increased by 8%.
- The full credibility standard is based on the number of claims in the class during the experience period. These standards are 2,500 claims for OL&T BI, 3,000 claims for M&C BI, and 7,500 claims for M&C PD.
- Partial credibility is based on the square root rule. For example, 1,080 claims in M&C BI gives  $(1,080/3,000)^{0.5} = 60\%$  credibility.
- The indicated relativity change for the class is raised to the power of the credibility. If the indicated relativity change is 1.08 and the credibility is 60%, the credibility weighted relativity change is  $1.080^{0.6} = 1.047$ .

<sup>&</sup>lt;sup>28</sup>Type of policy refers to monoline versus multiline policies (and type of multiline policy). See Graves and Castillo [1990] for a more complete discussion of the ISO procedure.

These two illustration show different uses of credibility. ISO credibility weights the indicated classification relativities with the current classification relativities to dampen the changes from year to year. The observed loss costs in the first illustration are credibility weighted with the indicated pure premiums to increase the accuracy of the final pure premiums.<sup>29</sup>

## Embedded Credibility

The minimum bias procedure has credibility embedded in the calculations, since each cell is weighted by the number of exposures in that cell.

A comparison with the single-dimensional classification ratemaking procedure should clarify this. Suppose there are three territories in a state with the experience shown below. The exposures are car-years, and the dollar figures are in thousands.

	Exposures	Claims	Premium	Losses	Loss Ratio	Indication
Terr 01	5,000	500	5,000	3,500	70.0%	0.972
Terr 02	10,000	1,000	15,000	10,800	72.0%	1.000
Terr 03	2,000	200	4,000	2,980	74.5%	1.035
Total	17,000	1,700	24,000	17,280	72.0%	

The unadjusted observed data suggest that

- Territory 01 should have a reduction of 2.8% in its base rate.
- Territory 02 should have no change in its base rate.
- Territory 03 should have an increase of 3.5% in its base rate.

The indications in the table do not consider the number of exposures or claims in each territory. Since territory 03 has only 200 claims in the experience period, the +3.5% indication may

<sup>&</sup>lt;sup>29</sup>See Venter's [1992] distinction between classical credibility used to minimize rate fluctuations from year to year and Bayesian-Bühlmann credibility used to increase the accuracy of the estimate.

be distorted by random loss fluctuations. To adjust for the volume of business in each territory, the raw indications may be credibility weighted with the overall average of unity, where the credibility depends on the number of exposures or claims.

In the minimum bias procedure, the number of exposures in each cell affects the computation. The weight accorded to the observed loss costs in the cell is proportional to the number of exposures in the cell. From this perspective, credibility weighting the observed loss costs by the number of exposures would be applying credibility twice.

Nevertheless, some justification remains for a credibility adjustment. To determine the indicated pure premium for a cell, the minimum bias procedure uses the type of rating model along with all the cells in the array. The credibility embedded in the minimum bias procedure deals with random loss fluctuations. A second credibility adjustment deals with model specification risk. We explain these concepts with an illustration.

*Illustration 30:* The observed loss cost for young unmarried urban male drivers is \$2,500 per car. After applying a minimum bias procedure, the indicated pure premium for these drivers is \$3,000 per car. There are two explanations for the difference.

- 1. Random loss fluctuations account for the difference. The credibility embedded in the minimum bias procedure is sufficient. No additional credibility adjustment should be used.
- 2. The rating model is not correct. For example, the minimum bias procedure may be using a multiplicative model when an additive model is proper. This is model specification risk, and a second credibility adjustment is warranted.

Classical credibility procedures are not an ideal compensation for model specification risk. The ideal approach is to use several models, such as multiplicative, additive, and combined models, and to test the goodness-of-fit for each model. Time constraints preclude this ideal approach in many cases, and a credibility adjustment may be a practical alternative.

## Rate Fluctuations

When rating bureaus made advisory rates, they had more incentive to temper rate fluctuations from year to year than private insurers have. ISO's credibility procedure may not have firm statistical justification, but it fulfills the objective of tempering the requested rate changes.

The use of credibility to temper rate fluctuations from year to year is a dubious practice. In practice, most actuaries conceive of credibility as a means to price more accurately. Although Venter correctly notes that the stated rationale for classical credibility deals with tempering rate fluctuations, even classical credibility does serve the objective of increasing the accuracy of the rate indications.<sup>30</sup>

## 11. SUMMARY

For each model discussed in this paper, there are simple iterative functions. The task of the pricing actuary is to determine a rating function—such as multiplicative, additive, or combined and a bias function (balance principle, least squares,  $\chi$ -squared, or maximum likelihood). If the maximum likelihood bias function is used, the actuary must also select a probability distribution function for the loss costs (or other values) in each cell.

The type of data in each cell will generally be either loss costs or loss ratios. If the pricing actuary is using all the dimensions of the classification system in the minimum bias analysis, it is easier to use loss costs. If there are significant classification dimensions that are not included, and if there may be an uneven

<sup>&</sup>lt;sup>30</sup>See Venter's chapter on "Credibility" in any of the first three editions of the CAS textbook, *Foundations of Casualty Actuarial Science*, and Mahler [1986]. As Mahler points out, tempering rate changes and aiming for rate accuracy are different purposes, but they usually have a similar result.

distribution of exposures along these other classification dimensions, the pricing actuary may prefer to use loss ratios.

We list here the models that have been proposed for insurance use, along with their recursive equations.

Multiplicative model, balance principle:

$$x_i = \frac{\sum_j n_{ij} r_{ij}}{\sum_j n_{ij} y_j}.$$

Additive model, balance principle:

$$x_i = \frac{\sum_j n_{ij} (r_{ij} - y_j)}{\sum_j n_{ij}}.$$

Multiplicative model, least squares:

$$x_i = \sum (n_{ij} \times r_{ij} \times y_j) \bigg/ \sum (n_{ij} \times y_j^2).$$

Additive model, least squares:

$$x_i = \frac{\sum n_{ij} \times (r_{ij} - y_j)}{\sum n_{ij}} - B.$$

Multiplicative model,  $\chi$ -squared:

$$x_i = \left[ \sum (n_{ij} \times r_{ij}^2 / y_j) \middle/ \sum n_{ij} y_j \right]^{0.5}$$

~ ~

Additive model,  $\chi$ -squared:

$$\Delta x_i = \frac{\sum_j n_{ij} \left(\frac{r_{ij}}{x_i + y_j}\right)^2 - \sum_j n_{ij}}{2\sum_j n_{ij} \left(\frac{r_{ij}}{x_i + y_j}\right)^2 \left(\frac{1}{x_i + y_j}\right)}$$

Multiplicative model, maximum likelihood, normal density function:

$$x_i = \frac{\sum_j n_{ij} r_{ij} y_j}{\sum_j n_{ij} y_j^2}.$$

Additive model, maximum likelihood, normal density function:

$$x_i = \frac{\sum_j n_{ij} (r_{ij} - y_j)}{\sum_j n_{ij}}.$$

Multiplicative model, maximum likelihood, exponential density function:

$$x_i = \frac{\sum_j \frac{\gamma_{ij}}{y_j}}{k},$$

where k is the number of classes in the j dimension.

The recursive functions for a multiplicative model, maximum likelihood, Poisson distribution function are the same as those for the multiplicative model, balance principle.

Derivations of the formulas for the maximum likelihood models may be found in Brown [1988].

Accurate classification systems are the bedrock of insurance pricing. Accurate and unbiased rating systems enable insurers to attain competitive advantages over their peer companies. Inaccurate rating systems lead to unsatisfactory profits and to loss of market share.

As competition increases in the insurance industry, and as companies are forced to rely on their own pricing instead of bureau rates, the need for more accurate ratemaking increases. The minimum bias procedure can be used to optimize a variety of rating models.

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# INSURANCE APPLICATIONS OF BIVARIATE DISTRIBUTIONS

## DAVID L. HOMER AND DAVID R. CLARK

## Abstract

A technique is demonstrated for aggregating bivariate claim size distributions using a two-dimensional Fast Fourier Transform. Three insurance applications are described in detail relating to: 1) individual risk rating, 2) loss and allocated expenses, and 3) Dynamic Financial Analysis.

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## 1. INTRODUCTION

# 1.1. The Basic Problem

When pricing insurance contracts it is useful to estimate not only the average insured loss but also the insured loss distribution. Although an initial approach may include only an estimate of the mean, risk measures generally require an estimate of the distribution. This problem is often solved by modeling losses as a sum of individual claims. A frequency distribution describes the number of claims N; a severity distribution describes the size of each claim  $X_k$ . The individual claim sizes are usually assumed to be independent and identically distributed (iid) as well as independent from the claim counts. This model is known as the *Collective Risk Model* [3]. The aggregate loss dollars Z are the sum of the individual claim sizes

$$Z = X_1 + \dots + X_N. \tag{1.1}$$

The expectation and variance of Z are easily expressed in terms of the frequency and severity components

$$E(Z) = E(X)E(N).$$
(1.2)

$$\operatorname{Var}(Z) = \operatorname{Var}(X) \operatorname{E}(N) + \operatorname{E}(X)^2 \operatorname{Var}(N).$$
(1.3)

Estimating the aggregate loss distribution requires more work, but there are numerous techniques available: simulation, Fast Fourier Transform, continuous Fourier Transform [1], recursion [4, 8], and moment matching [5, 9]. In this paper, the Fast Fourier Transform (FFT) will be used. The FFT has been described in detail by Robertson [7] and Wang [10], and an overview is also included here as Appendix C.

# *1.2. A Problem That Includes Dependencies between Loss Components*

The collective risk model as outlined above is sufficient to describe most insurance policies. One example in which this model is not sufficient arises in individual risk rating. A policy may provide specific excess coverage above a per-occurrence retention, and may also provide coverage in excess of an aggregate amount for the retained losses. The excess of aggregate cover is commonly called a *stop loss* cover.

The distributions for either the specific excess or stop loss covers can be estimated using the collective risk model. However, it is more difficult to estimate the distribution for the sum of the two covers because there is a dependence between the pieces. One trivial element of the dependence is easily seen—if there are no retained losses then there are no losses in excess of the retention.

Section 2 provides a more detailed description of this problem.

# 1.3. Aggregating with the FFT—A Brief Review

Before introducing the complication of the dependence between two coverages, we will briefly review the Fast Fourier Transform (FFT) technique for evaluating a standard collective risk model. Appendix C provides a more detailed review.

In order to compute the aggregate loss distribution using the FFT, the severity distribution is expressed as a probability vector<sup>1</sup>  $x = (x_0, x_1, \dots, x_{n-1})$ . Each element  $x_k$  is the probability of a claim having size ck, where c is a scaling constant.

The distribution of the claim counts N is incorporated with the use of its Probability Generating Function (PGF)

$$PGF(t) = E(t^{N}).$$
(1.4)

The frequency and severity components are put together using a standard FFT technique. Denoting the FFT and its inverse as FFT(x) and IFFT(x), respectively, the probability vector for the aggregate losses is computed as

$$z = (z_0, z_1, \dots, z_{n-1}) = \text{IFFT}(\text{PGF}(\text{FFT}(x))).$$
 (1.5)

The PGF is applied *elementwise*, i.e., with some abuse of notation,

$$PGF((t_0, t_1, \dots, t_{n-1})) = (PGF(t_0), PGF(t_1), \dots, PGF(t_{n-1})).$$
(1.6)

The vector size *n* must be large enough that the probability of aggregate losses greater than *cn* is negligible. Any probability mass for losses greater than *cn* will wrap around, i.e., mass for losses greater than *cn* will be treated as though it is mass for the available claim sizes (0, c, 2c, ..., nc). The wrap-around problem is typically avoided by padding the vector with zeros as discussed in Robertson [7] and Wang [10].

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 $<sup>^{1}</sup>x$  is indexed starting at zero.  $x_{0}$  is the probability of a claim of size zero.

## 1.4. Building a Bivariate Loss Distribution

The goal is to obtain a bivariate distribution of aggregate retained losses and aggregate excess losses. This will be represented as a probability matrix<sup>2</sup>  $M_z$  where  $M_z(j,k)$  is the probability that aggregate retained losses are  $c_1j$  and aggregate excess losses are  $c_2k$ . As before,  $c_1$  and  $c_2$  are constant scale factors.

For a single claim this matrix is easily constructed. Suppose x = (.4, .3, .3) and c = 1,000. Then for a 1,000 deductible, with  $c_1 = c_2 = c = 1,000$ ,

$$M_{x} = \begin{bmatrix} .4 & 0 & 0 \\ .3 & .3 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$
 (1.7)

The matrix  $M_x$  fully specifies the probabilities and dependencies of losses in the retained and excess layers. The sum across rows (.4, .6, 0) produces the distribution of the retained losses; the sum down the columns (.7, .3, 0) produces the distribution of the excess losses.

The advantage at this point is that the same FFT technique can be used to calculate aggregate losses for  $M_x$  that we used to calculate aggregate losses for x. With FFT() and IFFT() now representing the two-dimensional FFT and its inverse, and with PGF() as before, we compute the aggregate loss matrix  $M_z$ 

$$M_{\tau} = \text{IFFT}(\text{PGF}(\text{FFT}(M_{r}))). \tag{1.8}$$

As in the one-dimensional treatment, the PGF is applied elementwise and the matrix  $M_x$  must have sufficient padding so that  $M_z$ can hold the significant mass. Appendix A provides an example of the two-dimensional FFT using publicly available software.

The FFT technique is not the only way to aggregate  $M_x$ . Sundt [8] shows that  $M_x$  can be aggregated using a recursive technique.

 $<sup>^{2}</sup>M_{z}$  indices start from zero.

The aggregation of bivariate severity matrices can be applied to other problems as well. In what follows, three specific examples will be explored. In the first, the combined distribution of losses on specific excess and aggregate excess is considered. In the second, bivariate loss and ALAE distributions are computed, and in the third example, a problem with a simulation technique often used in DFA analysis is reviewed and corrected.

# 2. PER-OCCURRENCE AND EXCESS-OF-AGGREGATE COVERS IN INDIVIDUAL RISK RATING

The first problem that we will review is common in individual risk rating.

A fictional large insured, Dietrichson Drilling, is interested in retaining the majority of their "predictable" workers compensation losses, and mainly seeks to purchase insurance to cover individual large claims. For example, they may choose to retain the first 600,000 of each loss occurrence. At the same time, they may have a concern that the number of occurrences could also be higher than expected, and therefore seek protection on the total dollars of retained loss.

Our company, Pacific All Risk Insurance Company, has been asked to provide coverage on a per-occurrence basis of 400,000 excess of 600,000, and then also a stop loss cover to pay in the event that their total retained loss exceeds 3,000,000. The underwriter at Pacific All Risk has proposed the structure shown in Table 2.1.

As the Pacific All Risk actuary, you have selected frequency and severity distributions, and have estimated the expected losses for each of these coverages. In order to calculate the needed risk load on the program, however, you need to estimate the distribution of the sum of the two coverages.

The company's Fast Fourier Transform (FFT) model allows you to estimate a distribution for either the per-occurrence or the

# TABLE 2.1

# POLICY STRUCTURE FOR DIETRICHSON DRILLING

Named Insured:	Dietrichson Drilling				
Insurance Company:	Pacific All Risk Insurance Co.				
Per-Occurrence Layer:	400,000 xs 600,000				
Stop Loss Layer:	5,000,000 xs 3,000,000				
Allocated expenses included in the definition of "loss"					

## TABLE 2.2

# SEVERITY DISTRIBUTION FOR DIETRICHSON DRILLING

Probability	Loss Amount	Excess Loss	
0.00%	0	0	
37.80%	200,000	0	
23.50%	400,000	0	
14.60%	600,000	0	
9.10%	800,000	200,000	
15.00%	1,000,000	400,000	
Average	480,000	78,200	

stop loss layer with no problem, but you recognize that there is likely to be a strong dependence between the results of the two covers and you want to reflect this in your pricing.

We will consider a simplified version of this problem. First, we will assume that the loss distribution can be reasonably approximated using only a five-point discretized severity distribution. In practice, a curve of more than a hundred points would be needed in order to accurately capture the true shape. For our example, the simpler distribution shown in Table 2.2 will be used.

Consistent with this loss distribution, our average severity is estimated to be 480,000 and the average in the 400,000 xs 600,000 layer is 78,200.

# TABLE 2.3

# SINGLE CLAIM PRIMARY & EXCESS LOSS BIVARIATE DISTRIBUTION

	Loss Excess of 600,000							
Loss Capped at 600,000	0	200,000	400,000	600,000	800,000	1,000,000		
0	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%		
200,000	37.80%	0.00%	0.00%	0.00%	0.00%	0.00%		
400,000	23.50%	0.00%	0.00%	0.00%	0.00%	0.00%		
600,000	14.60%	9.10%	15.00%	0.00%	0.00%	0.00%		
800,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%		
1,000,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%		
1,200,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%		
1,400,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%		
1,600,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%		
1,800,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%		
2,000,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%		

We have also estimated that the expected number of claims is 5, with a variance of 6, and the frequency will be modeled using a Negative Binomial<sup>3</sup> distribution. The overall loss pick is therefore 2,400,000 ( $5 \times 480,000$ ). Our aggregate model calculates expected losses of 123,529 in the proposed stop loss layer above 3,000,000.

The first step in calculating the overall loss distribution is to create a bivariate severity distribution of primary and excess losses. This is shown in Table 2.3.

From Table 2.3, we can observe a strong dependence structure between the primary and excess losses: we can have an excess loss only if the primary 600,000 retention is hit.

This bivariate severity matrix becomes the input for the FFT model, and may be denoted  $M_x$ . The matrix of the aggregate distribution may be denoted  $M_z$  and is produced using the two-

<sup>&</sup>lt;sup>3</sup>See Appendix D for details on the Negative Binomial distribution and its Probability Generating Function.

dimensional Fast Fourier Transform calculation:

$$M_7 = \text{IFFT}(\text{PGF}(\text{FFT}(M_x))), \text{ and } (2.1)$$

$$PGF(t) = (1.2 - .2t)^{-25}.$$
 (2.2)

For the bivariate matrix  $M_x$  shown in Table 2.3, the resulting  $M_z$  is given in Table 2.4.

An additional step is needed in order to calculate the estimated results in the stop loss layer above 3,000,000. For that calculation, the rows of Table 2.4 for all amounts 3,000,000 or less are summed to compute the probabilities of no excess-of-aggregate losses. The remaining rows are intact but the row labels are reduced by 3,000,000. The result is Table 2.5.

From Table 2.5, several statistics of interest can be calculated.<sup>4</sup> The expected loss to the stop loss layer is 123,529 and the probability that the stop loss is hit is 15.08%. The average loss amount conditional upon the stop loss being hit is 819,210.

More dramatic from a risk management perspective is the dependence between the per-occurrence and stop loss covers. The expected loss to the per-occurrence layer is 391,000 (5  $\times$  78,200), but this increases to 830,334 when we include only the scenarios in which the stop loss is also hit. This dependence needs to be considered in the decision to write the contract: *on average, when the stop loss is hit we will also be paying about twice the expected amount in the per-occurrence layer.* 

The two-dimensional matrix shown in Table 2.5 can be used to verify the expected loss pricing for either coverage individually. The probabilities associated with the stop loss program are found by summing across rows; the probabilities associated with the per-occurrence excess layer are found by summing down

<sup>&</sup>lt;sup>4</sup>The probabilities for the aggregate distribution extend beyond the rows and columns actually displayed.
## TABLE 2.4

## Aggregate Primary & Aggregate Excess Loss **BIVARIATE DISTRIBUTION**

	Loss Excess of 600,000						
Loss Capped	0	200.000	400.000	(00.000	800.000	1 000 000	1 200 000
at 600,000	0	200,000	400,000	600,000	800,000	1,000,000	1,200,000
0	1.05%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
200,000	1.65%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
400,000	2.38%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
600,000	3.09%	0.40%	0.66%	0.00%	0.00%	0.00%	0.00%
800,000	3.34%	0.65%	1.07%	0.00%	0.00%	0.00%	0.00%
1,000,000	3.39%	0.96%	1.58%	0.00%	0.00%	0.00%	0.00%
1,200,000	3.22%	1.27%	2.16%	0.26%	0.21%	0.00%	0.00%
1,400,000	2.86%	1.40%	2.44%	0.44%	0.36%	0.00%	0.00%
1,600,000	2.43%	1.45%	2.59%	0.66%	0.54%	0.00%	0.00%
1,800,000	1.97%	1.40%	2.57%	0.90%	0.78%	0.09%	0.05%
2,000,000	1.54%	1.26%	2.38%	1.02%	0.92%	0.15%	0.08%
2,200,000	1.17%	1.09%	2.12%	1.08%	1.01%	0.24%	0.13%
2,400,000	0.86%	0.90%	1.80%	1.08%	1.05%	0.33%	0.20%
2,600,000	0.62%	0.72%	1.47%	1.00%	1.01%	0.38%	0.24%
2,800,000	0.43%	0.55%	1.16%	0.88%	0.93%	0.42%	0.27%
3,000,000	0.29%	0.41%	0.89%	0.75%	0.82%	0.43%	0.29%
3,200,000	0.20%	0.30%	0.66%	0.61%	0.70%	0.41%	0.29%
3,400,000	0.13%	0.21%	0.48%	0.48%	0.57%	0.37%	0.28%
3,600,000	0.08%	0.15%	0.34%	0.37%	0.45%	0.32%	0.25%
3,800,000	0.05%	0.10%	0.24%	0.27%	0.35%	0.27%	0.22%
4,000,000	0.03%	0.07%	0.16%	0.20%	0.26%	0.22%	0.19%
4,200,000	0.02%	0.04%	0.11%	0.14%	0.19%	0.17%	0.15%
4,400,000	0.01%	0.03%	0.07%	0.10%	0.14%	0.13%	0.12%
4,600,000	0.01%	0.02%	0.05%	0.07%	0.10%	0.10%	0.09%
4,800,000	0.00%	0.01%	0.03%	0.04%	0.07%	0.07%	0.07%
5,000,000	0.00%	0.01%	0.02%	0.03%	0.04%	0.05%	0.05%
5,200,000	0.00%	0.00%	0.01%	0.02%	0.03%	0.03%	0.04%
5,400,000	0.00%	0.00%	0.01%	0.01%	0.02%	0.02%	0.03%
5,600,000	0.00%	0.00%	0.00%	0.01%	0.01%	0.02%	0.02%
5,800,000	0.00%	0.00%	0.00%	0.00%	0.01%	0.01%	0.01%
6,000,000	0.00%	0.00%	0.00%	0.00%	0.01%	0.01%	0.01%
6,200,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%
6,400,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
6,600,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
6,800,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
7,000,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
7,200,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
7,400,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
7,600,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
7,800,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
8,000,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

## TABLE 2.5

## Aggregate Primary Excess & Aggregate Excess Loss Bivariate Distribution

	Loss Excess of 600,000						
Stop							
Loss	0	200,000	400,000	600,000	800,000	1,000,000	1,200,000
0	30.28%	12.45%	22.89%	8.07%	7.65%	2.04%	1.26%
200,000	0.20%	0.30%	0.66%	0.61%	0.70%	0.41%	0.29%
400,000	0.13%	0.21%	0.48%	0.48%	0.57%	0.37%	0.28%
600,000	0.08%	0.15%	0.34%	0.37%	0.45%	0.32%	0.25%
800,000	0.05%	0.10%	0.24%	0.27%	0.35%	0.27%	0.22%
1,000,000	0.03%	0.07%	0.16%	0.20%	0.26%	0.22%	0.19%
1,200,000	0.02%	0.04%	0.11%	0.14%	0.19%	0.17%	0.15%
1,400,000	0.01%	0.03%	0.07%	0.10%	0.14%	0.13%	0.12%
1,600,000	0.01%	0.02%	0.05%	0.07%	0.10%	0.10%	0.09%
1,800,000	0.00%	0.01%	0.03%	0.04%	0.07%	0.07%	0.07%
2,000,000	0.00%	0.01%	0.02%	0.03%	0.04%	0.05%	0.05%
2,200,000	0.00%	0.00%	0.01%	0.02%	0.03%	0.03%	0.04%
2,400,000	0.00%	0.00%	0.01%	0.01%	0.02%	0.02%	0.03%
2,600,000	0.00%	0.00%	0.00%	0.01%	0.01%	0.02%	0.02%
2,800,000	0.00%	0.00%	0.00%	0.00%	0.01%	0.01%	0.01%
3,000,000	0.00%	0.00%	0.00%	0.00%	0.01%	0.01%	0.01%
3,200,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%
3,400,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
3,600,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
3,800,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
4,000,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
4,200,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
4,400,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
4,600,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
4,800,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
5,000,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

columns. By summing across rows or down columns, we calculate the marginal distributions.

In order to calculate the distribution of the sum of the two coverages combined, we sum the probabilities along each lower-left to upper-right diagonal. Table 2.6 shows this calculation.

## TABLE 2.6

PROBABILITIES FOR AGGREGATE PRIMARY EXCESS PLUS AGGREGATE EXCESS LOSS

Loss & ALAE	Probability	Calculation
0 200,000 400,000 600,000 800,000 :	30.28% 12.64% 23.31% 9.02% 8.94%	= 30.28% = 0.20% + 12.45% = 0.13% + 0.30% + 22.89% = 0.08% + 0.21% + 0.66% + 8.07% = 0.05% + 0.15% + 0.48% + 0.61% + 7.65% :

## 3. DISTRIBUTION FOR LOSS ONLY SUBJECT TO AGGREGATE LIMIT PLUS UNLIMITED ALLOCATED LOSS ADJUSTMENT EXPENSE (ALAE)

Our insured, Dietrichson Drilling, requests a general liability policy on a traditional guaranteed cost basis. Our company, Pacific All Risk Insurance Company, is willing to offer a standard policy form with a 1,000,000 per-occurrence limit and a 2,000,000 general policy aggregate.

Both the per-occurrence limit and the general aggregate limit apply to the indemnity loss only. All defense costs and associated expenses (allocated loss adjustment expense—ALAE) are covered in addition to these limits. The Pacific All Risk policy is summarized in Table 3.1. The loss distribution is approximated in Table 3.2.

As the Pacific All Risk actuary, you have been asked to estimate the aggregate distribution of the sum of the loss and ALAE combined. The first step in calculating the overall loss distribution is to assemble the bivariate severity distribution of loss and ALAE. This is shown in Table 3.3.

For Dietrichson Drilling, we believe that there will be a strong dependence between loss and ALAE; larger losses are generally

## POLICY STRUCTURE FOR DIETRICHSON DRILLING

Named Insured:	Dietrichson Drilling				
Insurance Company:	Pacific All Risk Insurance Co.				
Per-Occurrence Limit:	1,000,000				
General Aggregate Limit:	2,000,000				
Allocated expenses paid in addition to loss					

## TABLE 3.2

## SEVERITY DISTRIBUTION FOR DIETRICHSON DRILLING

Probability	Loss Amount	
10.00%	0	
45.00%	200,000	
9.00%	400,000	
9.00%	600,000	
9.00%	800,000	
18.00%	1,000,000	
Average	432,000	
Average ALAE %	37.29%	

assumed to have larger dollars of associated expenses. The numbers in Table 3.3 are for illustration only, but were selected to demonstrate such a dependence.

The table is constructed such that the loss severity curve does not extend beyond the 1,000,000 per-occurrence limit, whereas the ALAE curve does not have an explicit cap. By convention, we are also including closed-without-pay claims in this analysis, at least to the extent that they contribute ALAE.

This bivariate severity matrix becomes the input for the FFT model, and will again be denoted as  $M_r$ . The matrix of aggregate

SINGLE CLAIM LOSS & ALAE BIVARIATE DISTRIBUTION

				ALAE			
Loss Amount	0	200,000	400,000	600,000	800,000	1,000,000	1,200,000
0	8.39%	1.47%	0.13%	0.01%	0.00%	0.00%	0.00%
200,000	27.98%	13.29%	3.16%	0.50%	0.06%	0.01%	0.00%
400,000	4.15%	3.21%	1.25%	0.32%	0.06%	0.01%	0.00%
600,000	3.07%	3.30%	1.77%	0.64%	0.17%	0.04%	0.01%
800,000	2.28%	3.13%	2.15%	0.99%	0.34%	0.09%	0.02%
1,000,000	3.37%	5.65%	4.73%	2.64%	1.11%	0.37%	0.10%
1,200,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
1,400,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
1,600,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
1,800,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
2,000,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

distributions  $M_{\tau}$  is again given by the formula:

$$M_{z} = \text{IFFT}(\text{PGF}(\text{FFT}(M_{x}))), \text{ and } (3.1)$$

$$PGF(t) = (2-t)^{-4}.$$
(3.2)

The frequency distribution is assumed to be Negative Binomial, with a mean of 4 and a variance of 8.

The final matrix of aggregate distributions is shown in Table 3.4. In order to cap the loss-only exposure at the 2,000,000 general aggregate, we sum the probabilities for losses above 2,000,000 into a single row. The result is Table 3.5. Finally, we can create a single distribution from this matrix by summing along each lower-left to upper-right diagonal to obtain Table 3.6.

It is also instructive to show a graph of the distribution of the combined loss and ALAE both before and after the general aggregate cap. In Graph 3.1 we can see that the "tail" of the cumulative distribution is greatly reduced by imposing a 2,000,000 general aggregate. However, we note that there is still a non-

## Aggregate Loss & Aggregate Allocated Loss Adjustment Expense Joint Distribution

				ALAE			
Loss	0	200.000	100.000	(00.000	800.000	1 000 000	1 200 000
Amount	0	200,000	400,000	600,000	800,000	1,000,000	1,200,000
0	7.42%	0.23%	0.02%	0.00%	0.00%	0.00%	0.00%
200,000	4.33%	2.23%	0.59%	0.11%	0.02%	0.00%	0.00%
400,000	2.22%	2.10%	1.01%	0.33%	0.08%	0.02%	0.00%
600,000	1.41%	1.82%	1.20%	0.54%	0.18%	0.05%	0.01%
800,000	1.06%	1.71%	1.40%	0.77%	0.33%	0.11%	0.03%
1,000,000	1.11%	2.09%	1.98%	1.27%	0.62%	0.25%	0.08%
1,200,000	0.69%	1.59%	1.84%	1.42%	0.83%	0.39%	0.16%
1,400,000	0.40%	1.07%	1.45%	1.32%	0.90%	0.50%	0.23%
1,600,000	0.25%	0.75%	1.15%	1.19%	0.92%	0.57%	0.30%
1,800,000	0.16%	0.56%	0.95%	1.08%	0.93%	0.64%	0.37%
2,000,000	0.12%	0.44%	0.82%	1.02%	0.96%	0.72%	0.46%
2,200,000	0.07%	0.30%	0.61%	0.84%	0.87%	0.72%	0.50%
2,400,000	0.04%	0.19%	0.43%	0.65%	0.73%	0.66%	0.50%
2,600,000	0.03%	0.13%	0.31%	0.50%	0.60%	0.59%	0.48%
2,800,000	0.02%	0.09%	0.22%	0.38%	0.50%	0.52%	0.45%
3,000,000	0.01%	0.06%	0.16%	0.30%	0.41%	0.46%	0.42%
3,200,000	0.01%	0.04%	0.11%	0.22%	0.32%	0.38%	0.37%
3,400,000	0.00%	0.02%	0.07%	0.15%	0.24%	0.30%	0.32%
3,600,000	0.00%	0.01%	0.05%	0.11%	0.18%	0.24%	0.27%
3,800,000	0.00%	0.01%	0.03%	0.08%	0.13%	0.19%	0.22%
4,000,000	0.00%	0.01%	0.02%	0.05%	0.10%	0.14%	0.18%

remote probability of loss even above 3,000,000, due to the inclusion of ALAE on an unlimited basis.

#### 4. DYNAMIC FINANCIAL ANALYSIS

As the actuary for Pacific All Risk, you have now completed your pricing work for individual insurance contracts. As a reward for your hard work, you have been rotated to the actuarial team that runs the company's Dynamic Financial Analysis (DFA) model, called Pacific Enterprise Risk Model (PERM).

## Aggregate Loss Capped at 2,000,000 & Aggregate Allocated Loss Adjustment Expense Joint Distribution

				ALAE			
Loss Amount	0	200,000	400,000	600,000	800,000	1,000,000	1,200,000
0	7.42%	0.23%	0.02%	0.00%	0.00%	0.00%	0.00%
200,000	4.33%	2.23%	0.59%	0.11%	0.02%	0.00%	0.00%
400,000	2.22%	2.10%	1.01%	0.33%	0.08%	0.02%	0.00%
600,000	1.41%	1.82%	1.20%	0.54%	0.18%	0.05%	0.01%
800,000	1.06%	1.71%	1.40%	0.77%	0.33%	0.11%	0.03%
1,000,000	1.11%	2.09%	1.98%	1.27%	0.62%	0.25%	0.08%
1,200,000	0.69%	1.59%	1.84%	1.42%	0.83%	0.39%	0.16%
1,400,000	0.40%	1.07%	1.45%	1.32%	0.90%	0.50%	0.23%
1,600,000	0.25%	0.75%	1.15%	1.19%	0.92%	0.57%	0.30%
1,800,000	0.16%	0.56%	0.95%	1.08%	0.93%	0.64%	0.37%
2,000,000	0.31%	1.31%	2.88%	4.41%	5.28%	5.31%	4.73%
2,200,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

## TABLE 3.6

PROBABILITIES FOR LIMITED LOSS PLUS ALAE

Combined Loss + ALAE	Probability	Calculation
0 200,000 400,000 600,000 800,000 :	7.42% 4.56% 4.47% 4.09% 3.99%	= 7.42% = 4.33% + 0.23% = 2.22% + 2.23% + 0.02% = 1.41% + 2.10% + 0.59% + 0.00% = 1.06% + 1.82% + 1.01% + 0.11% + 0.00%

The goal of the PERM team is to model the distribution of results for Pacific All Risk Insurance Company as a whole. Included in this analysis is sensitivity testing for interest rates and various complex reinsurance structures. The PERM is a giant simulation model that needs to be parameterized for the business actually written.

## GRAPH 3.1



CUMULATIVE DISTRIBUTION FUNCTIONS FOR CAPPED AND UNCAPPED LOSS & ALAE

A simplification made in the PERM is that the model separately simulates an aggregate value for all "small" losses and then simulates individual "large" losses. A truncation point of 1,000,000 has been selected for segregating large from small losses.

An early version of the PERM made the assumption that the small and large losses are independent. That is, the small and large losses were simulated separately and then the results were summed. However, this independence assumption was found to be false, resulting in understated variability and unrealistically low probabilities in the tail of the combined distribution.

In fact, the aggregate distributions of the small and large losses are generally not independent. If a single frequency distribution is used to generate the overall number of losses, N, then the covariance<sup>5</sup> can be written explicitly

$$Cov(S,L) = p\mu_S(1-p)\mu_L(\sigma_N^2 - \mu_N),$$
 (4.1)

where

S = aggregate small losses,

L = aggregate large losses,

 $\mu_{\rm S}$  = conditional mean of small claim size,

 $\mu_L$  = conditional mean of large claim size,

p = probability that a given claim is small,

 $\sigma_N^2$  = variance of the claim counts, and

 $\mu_N$  = mean of the claim counts.

The sign of the covariance term is driven by the claim count distribution. For the commonly used Negative Binomial this is positive; for the Poisson it is zero.<sup>6</sup> Equation (4.1) is derived in Appendix B.

In order to model the losses for Pacific All Risk, we begin by approximating the total loss distribution with a few discrete points (Table 4.1). As in the previous examples, a five-point distribution is used here, but would need to be expanded to a greater number of points in a more realistic application.

This single severity curve is then reconfigured into Table 4.2, a bivariate matrix  $M_x$ . The first column defines the severity of the "small" loss distribution. The first row is a single point containing the probability of a "large" loss.

This format is a bit different than the previous examples, since the vertical and horizontal axes are in different units: the vertical

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<sup>&</sup>lt;sup>5</sup>Sundt shows a more general formula in [8].

<sup>&</sup>lt;sup>6</sup>In the case of the Poisson it can be shown that the large and small claims are actually independent.

## TABLE 4.1

## SEVERITY DISTRIBUTION

Probability	Loss Amount
0.00%	0
43.80%	200,000
24.60%	400,000
13.80%	600,000
7.80%	800,000
10.00%	1,000,000
Average	431,200

#### TABLE 4.2

# SINGLE CLAIM SMALL LOSS & LARGE COUNTS JOINT DISTRIBUTION

			Large Lo	ss Counts		
Small Loss	0	1	2	3	4	5
0	0.00%	10.00%	0.00%	0.00%	0.00%	0.00%
200,000	43.80%	0.00%	0.00%	0.00%	0.00%	0.00%
400,000	24.60%	0.00%	0.00%	0.00%	0.00%	0.00%
600,000	13.80%	0.00%	0.00%	0.00%	0.00%	0.00%
800,000	7.80%	0.00%	0.00%	0.00%	0.00%	0.00%
1,000,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
1,200,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
1,400,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
1,600,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
1,800,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
2,000,000	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

in dollars and the horizontal in counts. This illustrates the flexibility in the FFT technique to allow for different scale factors for the two dimensions.

For a frequency distribution, we use a Negative Binomial with mean 10 and variance 20. For an actual insurance company, the overall frequency is likely to be much higher, but we continue with this simplified assumption for clarity. The aggregate distribution matrix  $M_7$  is again given by the expression:

$$M_z = \text{IFFT}(\text{PGF}(\text{FFT}(M_x))), \text{ and } (4.2)$$

$$PGF(t) = (2-t)^{-10}.$$
(4.3)

The resulting aggregate distribution matrix  $M_z$  is in Table 4.3. Like the original bivariate severity, this matrix has units in dollars for the "small" losses, and counts for the "large" losses. The marginal distribution for the aggregate small losses is found by summing the probabilities in each row.

The simulation procedure first simulates an aggregate amount for the "small" losses, and then finds a conditional frequency distribution for the "large" loss counts. The conditional large loss frequency distributions are created by rescaling each row of  $M_{\tau}$  to total 100%. This is shown in Table 4.4.

The conditional matrix shown in Table 4.4 is also instructive in itself, because it clearly shows the dependence between large and small losses. Simply put, an increase in frequency means more losses in both the large and small categories.

The final simulation procedure for the PERM is then:

- simulate the aggregate dollars of small losses out of its marginal distribution;
- simulate the number of large losses from the corresponding conditional frequency distribution;
- simulate a severity amount for each of the large losses.

This procedure allows us to efficiently simulate losses without the need to individually simulate every small loss, and at the same time preserves the dependence structure between the large and small losses.

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## TABLE 4.3

## Aggregate Claim Small Loss & Large Counts Bivariate Distribution

	Large Loss Counts							
Small								
Loss	0	1	2	3	4	5	6	
0	0.10%	0.05%	0.01%	0.00%	0.00%	0.00%	0.00%	
200,000	0.21%	0.12%	0.04%	0.01%	0.00%	0.00%	0.00%	
400,000	0.38%	0.22%	0.07%	0.02%	0.00%	0.00%	0.00%	
600,000	0.58%	0.36%	0.12%	0.03%	0.01%	0.00%	0.00%	
800,000	0.82%	0.53%	0.18%	0.05%	0.01%	0.00%	0.00%	
1,000,000	1.07%	0.71%	0.26%	0.07%	0.01%	0.00%	0.00%	
1,200,000	1.31%	0.91%	0.34%	0.09%	0.02%	0.00%	0.00%	
1,400,000	1.54%	1.11%	0.43%	0.12%	0.03%	0.00%	0.00%	
1,600,000	1.74%	1.30%	0.52%	0.15%	0.03%	0.01%	0.00%	
1,800,000	1.90%	1.46%	0.60%	0.18%	0.04%	0.01%	0.00%	
2,000,000	2.02%	1.60%	0.68%	0.20%	0.05%	0.01%	0.00%	
2,200,000	2.09%	1.71%	0.75%	0.23%	0.06%	0.01%	0.00%	
2,400,000	2.12%	1.78%	0.80%	0.25%	0.06%	0.01%	0.00%	
2,600,000	2.11%	1.82%	0.84%	0.27%	0.07%	0.02%	0.00%	
2,800,000	2.06%	1.83%	0.86%	0.29%	0.08%	0.02%	0.00%	
3,000,000	1.98%	1.81%	0.87%	0.30%	0.08%	0.02%	0.00%	
3,200,000	1.88%	1.75%	0.87%	0.30%	0.08%	0.02%	0.00%	
3,400,000	1.76%	1.68%	0.85%	0.30%	0.09%	0.02%	0.00%	
3,600,000	1.62%	1.59%	0.82%	0.30%	0.09%	0.02%	0.00%	
3,800,000	1.48%	1.49%	0.79%	0.29%	0.09%	0.02%	0.00%	
4,000,000	1.34%	1.38%	0.74%	0.28%	0.08%	0.02%	0.00%	
4,200,000	1.20%	1.26%	0.70%	0.27%	0.08%	0.02%	0.00%	
4,400,000	1.06%	1.14%	0.64%	0.25%	0.08%	0.02%	0.00%	
4,600,000	0.94%	1.03%	0.59%	0.24%	0.08%	0.02%	0.00%	
4,800,000	0.82%	0.91%	0.54%	0.22%	0.07%	0.02%	0.00%	
5,000,000	0.71%	0.81%	0.48%	0.20%	0.07%	0.02%	0.00%	

#### 5. CONCLUSION

Aggregating a bivariate severity distribution is a useful technique. Two severity components are separately aggregated while preserving their dependence structure. This technique can be applied when pricing a policy with a per-occurrence retention and a stop loss on the aggregate retention. It can also be applied more

## TABLE 4.4

CONDITIONAL DISTRIBUTIONS OF LARGE COUNTS GIVEN AGGREGATE SMALL LOSSES

	Large Loss Counts						
Small							
Loss	0	1	2	3	4	5	6
0	59.87%	29.94%	8.23%	1.65%	0.27%	0.04%	0.00%
200,000	56.88%	31.28%	9.39%	2.03%	0.36%	0.05%	0.01%
400,000	54.91%	32.07%	10.18%	2.33%	0.43%	0.07%	0.01%
600,000	53.26%	32.68%	10.87%	2.60%	0.50%	0.08%	0.01%
800,000	51.80%	33.17%	11.49%	2.85%	0.57%	0.10%	0.01%
1,000,000	50.37%	33.62%	12.11%	3.12%	0.64%	0.11%	0.02%
1,200,000	49.03%	34.01%	12.70%	3.39%	0.72%	0.13%	0.02%
1,400,000	47.77%	34.34%	13.26%	3.65%	0.80%	0.15%	0.02%
1,600,000	46.55%	34.63%	13.81%	3.92%	0.89%	0.17%	0.03%
1,800,000	45.38%	34.87%	14.34%	4.19%	0.97%	0.19%	0.03%
2,000,000	44.26%	35.09%	14.86%	4.47%	1.07%	0.22%	0.04%
2,200,000	43.17%	35.26%	15.37%	4.74%	1.16%	0.24%	0.04%
2,400,000	42.12%	35.41%	15.86%	5.02%	1.26%	0.27%	0.05%
2,600,000	41.10%	35.53%	16.34%	5.31%	1.37%	0.30%	0.06%
2,800,000	40.11%	35.62%	16.80%	5.59%	1.47%	0.33%	0.06%
3,000,000	39.16%	35.69%	17.25%	5.88%	1.58%	0.36%	0.07%
3,200,000	38.22%	35.73%	17.69%	6.17%	1.70%	0.39%	0.08%
3,400,000	37.32%	35.75%	18.12%	6.46%	1.82%	0.43%	0.09%
3,600,000	36.44%	35.75%	18.54%	6.75%	1.94%	0.47%	0.10%
3,800,000	35.58%	35.73%	18.94%	7.05%	2.07%	0.51%	0.11%
4,000,000	34.74%	35.69%	19.33%	7.34%	2.19%	0.55%	0.12%
4,200,000	33.93%	35.63%	19.71%	7.64%	2.33%	0.59%	0.13%
4,400,000	33.14%	35.56%	20.08%	7.94%	2.46%	0.64%	0.14%
4,600,000	32.37%	35.47%	20.44%	8.24%	2.61%	0.69%	0.16%
4,800,000	31.61%	35.36%	20.78%	8.54%	2.75%	0.74%	0.17%
5,000,000	30.88%	35.25%	21.12%	8.83%	2.90%	0.79%	0.19%

generally. The two random variables can be different items such as dollars and counts.

In this paper we aggregate the bivariate distribution using the FFT, but it is possible to do this with the continuous Fourier Transform or simulation. Sundt [8] shows that this can be done

with recursive techniques. It may sometimes be preferable to utilize a mix of techniques.

This technique can be extended to *n* dimensions by developing a multivariate distribution  $M_x$ . With the claim count PGF and an *n*-dimensional FFT, the aggregate multivariate array  $M_z$  is obtained as

$$M_{z} = \text{IFFT}(\text{PGF}(\text{FFT}(M_{x}))). \tag{5.1}$$

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#### APPENDIX A

## SAMPLE TWO-DIMENSIONAL FAST FOURIER TRANSFORM USING R

It is convenient to compute FFTs using preprogrammed software. An excellent piece of software that includes FFT functions is based on the S language and is publicly available for free. It is called "R" [2]. Versions of R for various operating systems can be found by following 'http://cran.r-project.org/'. R is copyrighted software made publicly available under the GNU General Public License which is available at 'http://www.gnu.org/copyleft/gpl.html'. The FFT function is also available in commercial software packages, e.g., MATLAB and S-Plus.

A listing from a session with R shows how easy it is to compute two-dimensional FFTs. Lines typed by the user begin with ">". The inverse of a matrix M is obtained with "fft(M,T)/n," where n is the number of elements in the matrix.

```
>ms<-matrix(c(.4,0,0,.3,.3,0,0,0,0),3,3,byrow=T)</pre>
>ms
[,1] [,2] [,3]
[1,] 0.4 0.0 0
[2,] 0.3 0.3 0
[3, ] 0.00.00
>f<-fft(ms)
>f
[,1] [,2] [,3]
[1,] 1.0+0.0000000i 0.55-0.2598076i 0.55+0.2598076i
[2,] 0.1-0.5196152i 0.10+0.0000000i 0.55-0.2598076i
[3,] 0.1+0.5196152i 0.55+0.2598076i 0.10+0.00000000i
>f*f
[,1] [,2] [,3]
[1,] 1.00+0.0000000i 0.235-0.2857884i 0.235+0.2857884i
[2,] -0.26-0.1039230i 0.010+0.0000000i 0.235-0.2857884i
[3,] -0.26+0.1039230i 0.235+0.2857884i 0.010+0.0000000i
>ma<-fft(f*f,T)/9
>ma
```

[,1] [,2] [,3] [1,] 0.16+0i 1.652685e-18+0i 2.301894e-17+0i [2,] 0.24+0i 2.400000e-01+0i 2.467162e-17+0i [3,] 0.09+0i 1.800000e-01+0i 9.000000e-02+0i

For those wishing to program their own algorithms, see [6]. Note that, when the object to be transformed consists only of real numbers, there are symmetries that can be used to decrease the amount of computing required. Also note that many software packages, including R, define the FFT as  $FFT(x)_k = \sum_{j=0}^{n-1} \exp(-2\pi i j k/n)$ , using a negative exponent instead of a positive one as we have in Equation (C.1). The corresponding inverse is  $IFFT(\tilde{x})_k = (1/n) \sum_{j=0}^{n-1} \exp(2\pi i j k/n)$ . The reader wishing to verify his code with a package like R should use the "negative" sign convention.

#### APPENDIX B

#### CORRELATION OF LARGE AND SMALL LOSSES

Consider the Collective Risk Model with aggregate losses represented by the sum of individual claims

$$Z = X_1 + \dots + X_N. \tag{B.1}$$

The  $X_i$  are independent and identically distributed (iid) random variables denoting claim sizes. Claim counts are denoted by the random variable N, which is independent from each  $X_i$ . It is further assumed that the first moment of  $X_i$  is finite and that the second moment of N is finite.

Let *T* denote the threshold for distinguishing between small claims and large claims; i.e.,  $X_i$  is small if  $X_i \leq T$ . Define a small loss indicator,  $I_i = 1$  for  $X_i \leq T$  and 0 otherwise. Then we have small aggregate losses

$$Z_S = X_1 I_1 + \dots + X_N I_N, \tag{B.2}$$

and large aggregate losses

$$Z_L = X_1(1 - I_1) + \dots + X_N(1 - I_N).$$
(B.3)

Let *p* be the probability that  $X_i \leq T$ . Denote the conditional means for small and large claim sizes with

$$\mu_S = \mathbb{E}[X_i | X_i \le T], \quad \text{and} \quad (B.4)$$

$$\mu_L = \mathbf{E}[X_i | X_i > T]. \tag{B.5}$$

Denote the claim count mean and variance with

$$\mu_N = \mathbf{E}[N], \quad \text{and} \quad (B.6)$$

$$\sigma_N^2 = \operatorname{Var}[N]. \tag{B.7}$$

PROPOSITION

$$Cov[Z_S, Z_L] = p\mu_S(1-p)\mu_L(\sigma_N^2 - \mu_N).$$
 (B.8)

$$Proof$$

$$E[Z_S Z_L] = E\left[\left(\sum_{i=1}^{N} X_i I_i\right) \left(\sum_{i=1}^{N} X_i (1-I_i)\right)\right]$$

$$= E_N E_X \left[\left(\sum_{i=1}^{N} X_i I_i\right) \left(\sum_{i=1}^{N} X_i (1-I_i)\right)\right]$$

$$= E_N E_X \left[\left(\sum_{i=j} X_i I_i X_j (1-I_j)\right) + \left(\sum_{j \neq i} X_i I_i X_j (1-I_j)\right)\right]$$

$$= E_N [N(N-1) E_X [XI] E_X [X(1-I)]],$$
since  $I_i (1-I_j) = 0$  for  $i = j$ 

$$= E_N [N(N-1) \mu_S p \mu_L (1-p)]$$

$$= (E(N^2) - \mu_N) \mu_S p \mu_L (1-p).$$
(B.9)

$$E[Z_L]E[Z_S] = (\mu_N \mu_S p)(\mu_N \mu_L (1-p)) = \mu_N^2 \mu_S p \mu_L (1-p).$$
(B.10)

These yield Equation (B.8), since

$$\operatorname{Cov}[Z_S, Z_L] = \operatorname{E}[Z_S Z_L] - \operatorname{E}[Z_S] \operatorname{E}[Z_L].$$
(B.11)

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#### APPENDIX C

## THE DISCRETE FOURIER TRANSFORM AND THE PROBABILITY GENERATING FUNCTION

In many insurance applications, we need to calculate an aggregate distribution in which the claim size X and the number of claims N are independent random variables. The aggregate losses are  $Z = X_1 + \cdots + X_N$ , where the  $X_k$  are independent and identically distributed as X. This is known as the *Collective Risk Model*.

The Fast Fourier Transform<sup>7</sup> (FFT) together with the Probability Generating Function (PGF) of the claim count distribution provide a convenient technique for computing the distribution of the aggregate losses.

This Appendix lists the key definitions and theorems underlying this technique. The authors recommend that the reader interested in a more comprehensive review refer to Robertson [7] and Wang [10].

## C.1. Definition of FFT

We assume the claim size random variable *X* is discrete and describe it with an *n* element probability vector  $x = (x_0, ..., x_{n-1})$ , where  $\operatorname{Prob}(X = ck) = x_k$ , and *c* is a scalar constant. For the claim count *N* we know the probability of each possible number of claims  $\operatorname{Prob}(N = j)$  (j = 0, 1, ...). Let  $\operatorname{FFT}(x)$  denote the FFT of *x*. FFT(*x*) is a vector with elements,

$$\tilde{x}_k = \text{FFT}(x)_k = \sum_{j=0}^{n-1} x_j \exp(2\pi i j k/n),$$
 (C.1)

<sup>&</sup>lt;sup>7</sup>The Fast Fourier Transform is a specific implementation of the Discrete Fourier Transform. Following Wang [10] we use the term FFT for both.

where  $i = \sqrt{-1}$ . The FFT is also invertible; let IFFT denote its inverse.

$$x_k = \text{IFFT}(\tilde{x})_k = \frac{1}{n} \sum_{j=0}^{n-1} \tilde{x}_j \exp(-2\pi i j k/n).$$
 (C.2)

#### C.2. Convolution

If *Z* is the sum of random variables *X* and *Y*, i.e., Z = X + Y, then its probability vector *z* is known as the *convolution* of *x* and *y* and is denoted x \* y, where

$$(x*y)_k = \sum_{l=0}^k x_l y_{k-l}.$$
 (C.3)

Similarly, if Z is the sum of j independent random variables identically distributed as X, then its probability vector z is known as the jth fold convolution of x and is denoted  $x^{*j} = (x_0^{*j}, \dots, x_{n-1}^{*j})$ .

It is convenient to define

$$x^{*0} = (1, 0, \dots, 0).$$
 (C.4)

The *j*th fold convolution can then be computed recursively for  $j \ge 1$ .

$$x_k^{*j} = \sum_{l=0}^k x_l x_{k-l}^{*j-1}.$$
 (C.5)

#### C.3. Convolution Theorem for the Discrete Fourier Transform

THEOREM Let x and y denote the probability vectors of random variables X and Y with n elements. If  $x_k = y_k = 0$  for all  $k \ge n/2$  when n is even and for all  $k \ge (n + 1)/2$  when n is odd, then

$$FFT(x*y)_k = FFT(x)_k \cdot FFT(y)_k.$$
(C.6)

For convenience we write FFT(x \* y) = FFT(x)FFT(y), with the understanding that the multiplication is applied elementwise.

Extending this convention to powers, we also write  $(FFT(x))^j$  for  $((FFT(x)_0)^j, \dots, (FFT(x)_{n-1})^j)$ .

Applying the inverse transform to both sides of Equation (C.6) we obtain a method for computing the convolution of two variables

$$x * y = \text{IFFT}(\text{FFT}(x)\text{FFT}(y)). \tag{C.7}$$

Similarly, for the *j*th fold convolution of *x* 

$$FFT(x^{*j}) = (FFT(x))^j, \qquad (C.8)$$

and

$$x^{*j} = \text{IFFT}(\text{FFT}(x)^j). \tag{C.9}$$

## C.4. Wrapping

Consider the *j*th fold convolution of *x* and note that  $x_0^{*j} + \cdots + x_{n-1}^{*j}$  is not necessarily equal to 1, because *n* is finite. For example, suppose we have n = 3 and x = (0,1,0). Then  $x^{*3} = (0,0,0)$ , since

$$\begin{aligned} x^{*1} &= (0, 1, 0) \\ x^{*2} &= (0 \times 0, 1 \times 0 + 0 \times 1, 0 \times 0 + 1 \times 1 + 0 \times 0) = (0, 0, 1) \\ x^{*3} &= (0 \times 0, 1 \times 0 + 0 \times 0, 0 \times 0 + 1 \times 0 + 0 \times 1) = (0, 0, 0). \end{aligned}$$

With n = 4 we would have x = (0, 1, 0, 0) and  $x^{*3} = (0, 0, 0, 1)$ , since

$$\begin{aligned} x^{*1} &= (0, 1, 0, 0) \\ x^{*2} &= (0 \times 0, 1 \times 0 + 0 \times 1, 0 \times 0 + 1 \times 1 + 0 \times 0, \\ 0 \times 0 + 0 \times 1 + 1 \times 0 + 0 \times 0) \\ &= (0, 0, 1, 0) \\ x^{*3} &= (0 \times 0, 1 \times 0 + 0 \times 0, 0 \times 0 + 1 \times 0 + 0 \times 1, \\ 0 \times 0 + 0 \times 0 + 1 \times 1 + 0 \times 0) \\ &= (0, 0, 0, 1). \end{aligned}$$

Robertson [7] describes the convolution in (C.3) as an *unwrapped* convolution. Equation (C.9) returns an unwrapped convolution if x is properly padded with zeros.

When x is not properly padded (C.9) returns a *wrapped*<sup>8</sup> convolution. Let  $(x \cdot y)$  denote a wrapped convolution; then

$$(x \tilde{*} y)_k = \sum_{j=0}^{n-1} x_j y_{(k-j) \bmod n},$$
 (C.10)

and

$$x_k^{\tilde{*}j} = \sum_{l=0}^{n-1} x_l x_{(k-l) \bmod n}^{\tilde{*}j-1}$$
(C.11)

The wrapped convolution  $x^{\tilde{*}3}$  for n = 3 is computed as

$$\begin{aligned} x^{\tilde{*}1} &= (0,1,0) \\ x^{\tilde{*}2} &= ((0,1,0) \cdot (0,0,1), (0,1,0) \cdot (1,0,0), (0,1,0) \cdot (0,1,0)) \\ &= (0 \times 0 + 1 \times 0 + 0 \times 1, 0 \times 1 + 1 \times 0 + 0 \times 0, \\ &\quad 0 \times 0 + 1 \times 1 + 0 \times 0) \\ &= (0,0,1) \\ x^{\tilde{*}3} &= ((0,1,0) \cdot (0,1,0), (0,1,0) \cdot (0,0,1), (0,1,0) \cdot (1,0,0)) \\ &= (1,0,0). \end{aligned}$$

The probability mass that is truncated with the un-wrapped convolution *wraps* with the wrapped convolution. Equation (C.9) *always* produces a wrapped convolution, but wrapped and unwrapped convolutions are equal when x is properly padded with zeros.

<sup>&</sup>lt;sup>8</sup>Robertson [7] calls this a *regular* convolution.

#### C.5. Definition of the Probability Generating Function

For a random variable *N* the Probability Generating Function (PGF) is

$$\mathrm{PGF}_{N}(t) = \sum_{j=0}^{\infty} t^{j} \mathrm{Prob}(N=j) = \mathrm{E}(t^{N}). \tag{C.12}$$

The PGF for the Negative Binomial distribution is given in Appendix D.

## C.6. Collective Risk Theorem for the Discrete Fourier Transform

We now show how to compute the aggregate probability vector z for the collective risk model using the FFT and the PGF of the claims count. This technique has an error term R due to wrapping which can be made arbitrarily small with sufficient zero padding of the claim size vector.

THEOREM Suppose we have a collective risk model with claim size probability vector  $x = (x_0, ..., x_{n-1})$ . Let PGF<sub>N</sub> be the Probability Generating Function for the claim counts N. Let M be the largest integer such that  $x_0^{*M} + \cdots + x_{n-1}^{*M} = 1$ . That is, M is the largest number of times one can convolute x and still have room for all the probability mass. Then

$$z = \text{IFFT}(\text{PGF}_N(\text{FFT}(x))) + R, \quad where \quad (C.13)$$

$$|R_k| \le \sum_{j=M+1}^{\infty} \operatorname{Prob}(N=j).$$
(C.14)

Proof

$$z = \sum_{j=0}^{\infty} x^{*j} \operatorname{Prob}(N=j).$$
 (C.15)

Define

$$d(j) = \text{IFFT}((\text{FFT}(x))^j). \tag{C.16}$$

Then

$$z = \sum_{j=0}^{\infty} d(j) \operatorname{Prob}(N = j) + \sum_{j=0}^{\infty} (x^{*j} - d(j)) \operatorname{Prob}(N = j).$$
(C.17)

Let R denote the second sum. Then

$$z = \sum_{j=0}^{\infty} d(j) \operatorname{Prob}(N = j) + R$$
  
= 
$$\sum_{j=0}^{\infty} \operatorname{IFFT}((\operatorname{FFT}(x))^{j}) \operatorname{Prob}(N = j) + R$$
  
= 
$$\sum_{j=0}^{\infty} \operatorname{IFFT}((\operatorname{FFT}(x))^{j} \operatorname{Prob}(N = j)) + R. \quad (C.18)$$

Because IFFT is linear and continuous, we can bring it outside the summation. So,

$$z = \text{IFFT}\left(\sum_{j=0}^{\infty} (\text{FFT}(x))^{j} \text{Prob}(N = j)\right) + R$$
$$= \text{IFFT}(\text{PGF}_{N}(\text{FFT}(x))) + R. \tag{C.19}$$

Now,

$$R = \sum_{j=M+1}^{\infty} (x^{*j} - d(j)) \operatorname{Prob}(N = j), \qquad (C.20)$$

since  $d(j) = x^{*j}$  for  $j \le M$ . Also,  $|x_k^{*j} - d(j)_k| < 1$ , since each  $d(j)_k, x_k^{*j} \in [0, 1]$ .

Thus,

$$|R_k| \le \sum_{j=M+1}^{\infty} \operatorname{Prob}(N=j).$$
 (C.21)

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#### APPENDIX D

## THE NEGATIVE BINOMIAL DISTRIBUTION

The Negative Binomial distribution with parameters p and k has

$$\operatorname{Prob}(N=j) = \frac{\Gamma(k+j)}{\Gamma(k)j!} p^k (1-p)^j, \quad \text{and} \quad (D.1)$$

PGF(t) = 
$$p^{k}(1 - (1 - p)t)^{-k}$$
. (D.2)

The Negative Binomial mean and variance are

Mean = 
$$M = \frac{k(1-p)}{p}$$
, and (D.3)

Variance = 
$$V = \frac{k(1-p)}{p^2}$$
. (D.4)

In terms of the mean and variance the PGF is

PGF(t) = 
$$(V/M - (V/M - 1)t)^{M^2/(V-M)}$$
. (D.5)

For example, a Negative Binomial with a mean of 5 and a variance of 6 has the Probability Generating Function

PGF(t) = 
$$(6/5 - (6/5 - 1)t)^{5^2/(6-5)} = (1.2 - .2t)^{-25}$$
. (D.6)

## DISCUSSION OF PAPER PUBLISHED IN VOLUME LXXXIX

## TESTING THE REASONABLENESS OF LOSS RESERVES: RESERVE RATIOS

#### C. K. KHURY

#### DISCUSSION BY CHARLES A. BRYAN

#### Abstract

Mr. Khury's paper advocates using various reserve ratios to test the reasonableness of loss reserve estimates. This review expands upon these ideas by discussing the practitioners for whom these techniques will be most useful, the practical decisions required to apply Mr. Khury's concepts, and a statistical technique to evaluate whether ratios derived from the loss reserve estimates are reasonable relative to other available data.

Mr. Khury has provided us a paper whose basic idea is "compilations of histories of reserve ratios are likely to reveal stable patterns that can be useful in testing loss reserves for reasonableness." Actuaries have long used reserve ratios as part of the process of constructing loss development factors, but have not generally used the ratios discussed in this paper to evaluate retrospectively the reasonableness of the reserves in the context of many years' experience and many development points. The paper is useful in that it adds to the actuary's collection of tools and techniques available to reach conclusions on the adequacy of loss reserves and the judgments made in setting those reserves. Mr. Khury's paper emphasizes that loss reserves should behave in a stable manner and that it is the job of the actuary to determine what that behavior is and to understand and explain any variance from the expected behavior. The purpose of this review is to provide some historical context, comment on where this technique might be most useful, provide an approach that will make these techniques easier to apply, and evaluate some of the statements made in the paper.

The actuary has played the primary role in setting and evaluating loss reserves since the late 1980s when many of the requirements for actuarial opinions were implemented. The most significant impetus was the NAIC solvency agenda, which in turn was motivated by the United States House of Representatives Failed Promises report (A Report of the Subcommittee on Oversight and Investigations of the Committee on Energy and Commerce, February, 1990) and the A. M. Best Insolvency Study: Property/Casualty Insurers 1969–1990. Both these reports helped convince the NAIC and its members that strengthening the analysis of loss reserves is critical to improving the regulatory approach to solvency. The A. M. Best report noted, "Deficient loss reserves (intrinsically linked with inadequate product pricing) and rapid growth were the most dominant causes of insolvencies." To address this issue, the NAIC adopted requirements for an actuarial opinion backed up by an actuarial report. The Casualty Actuarial Society strengthened the Casualty Loss Reserve Seminar to respond to training needs in support of the primary role of the actuary. The Actuarial Standards Board adopted documentation standards that specified the type of information that must be in the supporting report.

It is helpful to remember that the setting of reasonable reserves for life contingency products is fundamentally different than setting reasonable reserves for property and casualty products. The methods for life contingency products emphasize standardized reserve requirements such as those embedded in the Commissioner's Reserve Tables. The methods for property and casualty products emphasize the training and judgment of individual actuaries and, in fact, there are no standardized tables against which P&C reserve adequacy is measured. Therefore, the first question the reader should ask is: "Does the Khury paper provide some evidence that there may be a more standardized approach to judging the reasonableness of loss reserves?" My view is that the paper does point the way to introducing a useful set of standardized ratios that can flag where the reviewer of loss reserves should spend his or her time. Therefore, I classify these tools as important in an efficiency sense.

The second question, then, that the reader should ask is, "In what context and for what situations can these ratio tools be most useful?" The paper emphasizes that these ratio techniques are useful when reviewing reserves for reasonableness and not in setting reserves. This reviewer agrees with the author's conclusions and sees four key reasons why use of these techniques is not appropriate for setting reserves:

- 1. The historical information embedded in the ratios may not be the same as the most current information available, and reserve setting should use the most current information.
- 2. The process of setting the reserves must not be bound by what has happened in the past. The essence of a thorough setting of reserves requires the actuary to have no preconceived notion of what a reasonable reserve level will be and so not be influenced in judgments by an expected result.
- 3. The use of these techniques in setting reserves would encourage too high-level a view on the reserves and provide an excuse for inadequate analysis of the data.
- 4. P&C companies are sufficiently different from one another and even different over time that the use of historical ratios in setting reserves may not be accurate.

Even with these four limitations, it is appealing to consider situations where we know enough about a line of business or a company to at least put bounds around what reasonable results are. I see seven persons/situations where it is most likely that these tests for reasonableness will be helpful:

- 1. The chief actuary responsible for the overall adequacy of the reserves: Typically, a large company has several independent business units and each of these business units has its own actuary who sets the reserves. The chief actuary receives reports, backup material, and reserve valuations from each business unit. The chief actuary can use these ratio techniques to provide a reasonability check on the valuations from the business units before accepting the estimates as reasonable.
- 2. Regulatory authorities: These authorities currently receive actuarial opinions and actuarial reports for each company but often have insufficient time to delve deeply into any particular reserve valuation until long after the end of the reporting period. The IRIS ratios are available to red flag any unusual historical development, but these ratios contain misleading information if the current reserve valuation is inadequate. I can envision a situation in which a company would submit a reserve ratio review report with its reserve valuation and would provide a mandatory explanation of any significant departures from the historical reserve ratios. This report would allow the regulatory actuary to focus on red-flagged situations first.
- 3. Rating agency analysts: It may be useful to request a reserve ratio analysis and an explanation of any outliers as part of the analytical information collected by the rating agency.
- 4. A legal proceeding, in which a court or an arbitrator must determine if there was a reasonable basis for questioning the adequacy of a prior reserve level: The factual situation might include a very inadequate reserve, appropriate data and correct calculations at the time the reserve was

set, no significant change in court decisions affecting the coverages provided subsequent to the reserve setting process, and no other substantial reason identified for the reserve deficiency. The ratio techniques can provide a view on the question "did the resulting reserve seem so unlike the expected reserve that additional analysis or skepticism was warranted?"

- 5. Internal Revenue Service agents: If the historical reserves are out of line with the ratios, then the agent may have a basis for beginning discussions with the company. In fact, it may be possible to automate the ratio tests and so help the agent focus on companies that may have overstated reserves.
- 6. External auditors assessing the reasonableness of the reserves: While the auditor generally has a specialist, usually an actuary, available to help in the reserve evaluation, it is beneficial for the audit partner himself or herself to have some techniques available to reach a view on the reserves. The calculation of the ratios is straightforward and requires no complicated mathematical formulas.
- 7. A company that has seen large reserve developments: This is the "crashed airplane situation" where we are interested in seeing if we could have detected problems prior to the severe financial distress.

With these contexts in mind, it is important to consider in detail the approach the paper suggests and how the author has selected historical data and the tests applied. My emphasis in this discussion is whether or not the selections the author recommends are available to and useful to the seven groups delineated above. This discussion requires digging deeper into the following two areas: 1) data selection for establishing benchmarks, and 2) using benchmarks to identify outliers.

## 1) Data Selection

The first area to examine is the question of what data and segmentation of data should be used for establishing a set of benchmark ratios. The choices to be made include (a) industry data or company data, (b) all lines combined or specific lines, (c) direct or net data, and (d) which ultimate estimates to use.

## (a) Industry data or company data

While the company data may be more relevant to the company's particular situation, my election would be the use of industry data. Industry data has the following advantages:

- 1. The larger database should be more stable.
- 2. The ratio comparisons will provide insight into what may be different about the company, if anything.

Historical industry data normally requires using Schedule P information because of its standard format and availability for all licensed U.S. companies. Data can be obtained directly from the Annual Statements or from compilations such as those prepared by A. M. Best and other data service providers.

For most lines of business, the use of Schedule P data is perfectly acceptable. However, there have been changes in the Schedule P definitions of lines of business and expenses over time, and any analysis should consider these changes and comment on them. For example, the older Schedule P data used allocated loss adjustment expense. The newer Schedule Ps use the "defense and cost containment" categorization instead of "allocated loss adjustment expense." By-line applications will generally be much more useful than all lines, because it is easier to fit a company's situation with a by-line analysis than it is with an all lines analysis.

There are many situations in which ratios developed from Schedule P will not be useful. For example, many loss reserve problems arise after the 10-year time period displayed in Schedule P. We are all familiar with environmental, asbestos, and other mass tort claims that arise more than 10 years after the policy was written. We also encounter this issue with long tailed workers compensation and unlimited PIP.

## (b) All lines versus specific lines

Benchmark ratios can be easily established for Schedule P lines of business. Schedule P now contains detailed loss reserve development detail for many lines of business, and even a cursory review for reasonableness will be much more informative if done by line of business. By-line applications will generally be much more useful because it is easier to fit a company's situation with a by-line analysis. I suggest reviewing the largest lines of business to cover a pre-determined portion, say 80%, of the total business. The threshold can be established based on the importance of the review. For some companies, selecting one line of business may fulfill the 80% requirement. For other companies, fulfillment may require selecting and checking several lines of business.

#### (c) Direct versus net

Ratios should be calculated separately for direct, assumed, and net business. Such a calculation will reveal any unusual effects of reinsurance on the results. For example, if direct and assumed data provided values consistent with the model, but net data produced outliers, then one area of inquiry is the effect of reinsurance.

#### (d) Selection of ultimates

Where applicable, the most useful benchmarks would be ratios restated using the most recent estimate of ultimate losses, which should be the most accurate estimate. The latest estimate is available from the most recent year Schedule P. While the restatement will not normally affect the premium, paid losses, or reported losses, it will affect the IBNR.

## 2) Using Benchmarks for Identifying Outliers

How should stability and acceptable ratio values be evaluated, and which values should be flagged as outliers? In layman's terms, "stable" can mean all the numbers look similar in a visual inspection. Such a definition may be satisfactory for some contexts, such as for a review by a chief actuary prior to signing off on reserves set by other actuaries.

However, other contexts require a more rigorous definition. Future papers can address the most appropriate way to define "stable" and "reasonable values." For purposes of this review, I will use a Normal distribution to identify outliers.

For example, Table C of the paper provides historical ratios of IBNR to premium for the Reinsurance industry. For 2 years of development and 10 sample years, we have the sample values 29, 27, 30, 28, 25, 27, 27, 28, 25, and 23. Visually, this appears to be a tight distribution of numbers. If we calculate a value of 20 for the most recent year, should that flag the reserve for further analysis? Although visually, the observer can see that the 20 falls outside the range of historical results, it is useful to have a mathematical test to apply so that we will have the maximum consistency on the decision. The test would allow us to set a tolerance level in advance of calculating the current year result and so avoid any bias.

I select the situation in which the 24-month ratios of IBNR to premium are distributed normally. The mean value for the sequence of numbers in Table C as stated in the above paragraph is 26.90. The corresponding sample standard deviation is 1.97. A common convention is to define outliers as those values that are more than two standard deviations from the mean. In this case, we would define as outliers any values less than 22.96 or greater

than 30.84. Therefore, a value of 20 is an outlier and we would red flag the most recent year as a year needing more analysis and explanation. Depending upon the line of business, and the context, other decision rules can be selected. For example, the decision rules should vary by the size of the database used in the calculation.

Of course, other distributions may be more appropriate than the Normal distribution. The user may elect a different distribution but the Normal distribution will usually be satisfactory for identifying outliers in a consistent manner.

I hope that the CAS or one of the paper's readers will take on the task of developing calculations and cut off points for selected distributions. It would be helpful and feasible to use the last 10 years for each Schedule P line that has 10 years of industry data to construct a table each year to be used in flagging outliers. The availability of such a table would advance the science of evaluating the reasonableness of loss reserves by setting scientific standards for the expected ratios and assuring that ratios outside the specified range would be investigated.

In conclusion, Mr. Khury has made a valuable addition to the loss reserve literature. These techniques and tools should provide some important benchmarks on the way to more accurate reserves. The real usefulness of this paper is in pointing the way to a different approach to the reasonableness review process. If we follow the lead provided by the paper, we can look forward to significant improvements in the ability to detect loss reserve accuracy problems in an organized and systematic way.

#### ADDRESS TO NEW MEMBERS—NOVEMBER 10, 2003

#### OUR CORE VALUES

## C. K. "STAN" KHURY

First of all I would like to congratulate you, the new Associates of the Casualty Actuarial Society and the new Fellows of the Casualty Actuarial Society, on your achievement. Indeed it is a special milestone of your professional life. It is also a special milestone generally, in your and your family's life.

And given the pain that has been endured by your friends, colleagues, and members of your family while you were dedicating yourself to passing the exams, I would like to acknowledge their support and high tolerance for pain while you were going through your exams.

I also would like to welcome the new Associates to the community we know as the CAS and the new Fellows to their new status within this community. Members of this community, of which you are now a part, are, in effect, your new professional life-long partners.

Today I would like to talk to you about this community we know as the CAS, and introduce it to you from an unconventional perspective—the perspective of the core values of the CAS.

It has been a special privilege for me to serve on the CAS Long Range Planning Committee during the past two years. During this period, under the very capable leadership of Chairperson Stephen D'Arcy, our incoming president-elect, the Long Range Planning Committee went through a comprehensive process of analysis and study, ultimately yielding the CAS Centennial Goal.

One of the steps in this process was the identification of what the CAS would consider its core values.
Today I will focus on these core values: describing them and making a couple of observations about what the Long Range Planning Committee concluded.

Core values are generally defined as "the essential and enduring principles that guide an organization and its members." Looking back over the past 90 years, the Long Range Planning Committee identified five central values: Learning, Innovation, Volunteerism, Community, and Professionalism.

**Learning.** This is the belief that the continuing effectiveness of a casualty actuary is built upon dedication to the idea of lifelong learning. This, as you well know, is most evident in the extensive learning associated with qualifying for membership in the CAS as well as by the numerous continuing education activities of the CAS.

**Innovation.** This is the belief that the continuing vitality of the CAS is best served when creative thinking and research are fostered and new ideas are openly entertained. This is evident in the numerous ways in which the CAS encourages the generation of ideas, and the sharing and discussion of these ideas.

**Volunteerism.** This is the belief that the core purpose of the CAS is best served when every member is directly involved in the affairs of the CAS—and volunteers to serve other members. This is most evident in the remarkably high rates of participation of CAS members in all types of professional activities. The most remarkable aspect of CAS volunteerism is that the rate of participation of members has continued to increase as the overall size of the membership has increased. This is a long term-trend. Today roughly three out of ten members are active volunteers. And when Fellows are considered separately, the rate of participation is in the range of four out of ten. This clearly illustrates the depth of belief in the idea of volunteerism.

**Community.** This is the belief that members of the CAS are best served when the open sharing and exchange of ideas characterize the activities of the CAS. Indeed, one only has to go to any meeting of the CAS to get a full appreciation of the scope of sharing and exchange of ideas—not only in the regular sessions, but also in the hallways, at social gatherings, and through follow-up contacts after a meeting has ended.

**Professionalism.** This is the belief that the professionalism of casualty actuaries is best realized when the CAS, as an organization as well as its members individually, are committed to the idea of adhering to the highest professional and ethical standards of education, qualification, and practice. This is demonstrated through the myriad activities of the CAS and its members, in association with other actuarial organizations, that serve to articulate principles and guidance, codify standards of practice, provide professional advice in specific situations as needed, and, ultimately, when all fails, provide the means of administering disciplinary action.

I would only add two observations:

First, I would note that none of the five core values is specific to the CAS and its members' area of expertise. In other words, while the Long Range Planning Committee views these values as relating to the CAS and its members, there is nothing about them that makes them necessary for the CAS to adopt. These core values are ennobling by their very definition and in the various ways that the CAS and its members choose to make them our own.

Second, it is noteworthy that these core values emerged by a process of observation, not by the promulgation of some board-approved statement of core values. For nearly ninety years, the CAS and its members have conducted themselves without an explicitly stated set of core values. However, in retrospect, viewing the success and progression of the CAS over nearly a century, these are the values that turn out to have been operating all along. I believe that this condition gives them special gravity; as they are not things that we just talk about, but things that we live and put into practice.

Setting aside the subject matter concentration of the CAS, putting all of this together yields an interesting working definition of the CAS:

The CAS is an association of individual professionals, each of whom is dedicated to the idea of life-long learning, each of whom is actively involved in advancing the practice, both in scope and in depth of subject matter, each of whom believes that the governance of the association is best achieved when large segments of the membership are directly involved in the affairs of the association, and each of whom adheres to the highest professional and ethical standards of conduct.

This is a pretty formidable idea. I would suggest to you that you contemplate this definition and identify exactly how it manifests itself in your professional life. I promise you an interesting and revealing experience.

Once again, let me, on behalf of the entire membership, welcome you to this community we know as the CAS. I wish you every success and remind you that now you are very much a part of those who carry the torch forward. May God bless you and give you the strength to endure and serve with distinction.

## PRESIDENTIAL ADDRESS—NOVEMBER 10, 2003

## THE RACE

#### GAIL M. ROSS

Last year, when Bob Conger passed the CAS Presidential Gavel to me, I remember thinking—along with the thrill and excitement over the prestigious honor of becoming the president of the CAS—that the moment felt like the start of a marathon. Now mind you, I'm not really a runner. Periodically, I'll get into my exercise mode when I feel like I've gained a few pounds and I'll run a couple of miles a day at a turtle's pace, for a few weeks. But my wonderful husband, Steve, is a runner and I've watched him run enough races to probably circle the globe nearly two times with the amount of miles he's run in our 15 years of marriage. So I feel as if I understand the mind and spirit of the runner. Trust me when I tell you that, standing here today, I feel as if I've just completed my first marathon.

However, I didn't prepare for, nor run, this marathon alone. I've been fortunate to have worked this year with an enthusiastic executive council (EC), an outstanding board, the most dedicated group of volunteer committee members, and the hardest working CAS office staff who should be the envy of professional associations around the world. We've run this race together and as a group and we should all be proud of this marathon year of accomplishments for the CAS.

Please allow me a few minutes to recap the major accomplishments that we've achieved during this year, as well as highlight some of the initiatives that are in progress.

Over the past year our board approved our Centennial Goal our vision for the Casualty Actuarial Society in the year 2014 when we celebrate our 100th anniversary. The goal is this: "The CAS will be globally recognized as the preeminent resource in educating casualty actuaries and conducting research in casualty actuarial science. CAS members will be recognized as the leading experts in the evaluation of hazard risk and the integration of hazard risk with strategic, financial and operational risk."

We've set the bar high for ourselves with this goal, but I know we're up to the challenge.

This summer we took a huge first step toward achieving part of our vision. In order to enhance our participation in the global community of actuaries, our membership approved a constitutional amendment that will allow the CAS to enter into mutual recognition agreements with other actuarial organizations that have specialty exam tracks in property/casualty insurance. Constitutional amendments don't get presented often; and, when one is necessary, it takes a tremendous effort to educate our members in order for them to vote responsibly for the change. While this educational process was a huge endeavor on the part of the board, EC, and CAS staff, I must especially thank two people for their hard work—our next president of the CAS, Mary Frances Miller, and our immediate past president, Bob Conger.

Mary Frances invested considerable time with me, visiting Regional Affiliates and large employers, explaining the mutual recognition concept. Bob, as chairperson of the board, wrote a number of articles and taped a video to help our members understand how mutual recognition would benefit the CAS as we strive to achieve our Centennial Goal. Mary Frances and Bob, thank you both for your help in this important initiative.

I have also had the privilege to work with a fantastic group of vice presidents, who set an unbelievable pace during this year's marathon, and who often don't get recognized for all of their hard work.

Tom Myers is vice president–admissions. Tom and his committees have worked diligently to improve our examination process, and they have done an outstanding job this year in the following areas:

• They completed working with The Chauncey Group, an educational consulting firm that the CAS engaged a few years ago. The Chauncey Group worked with our Syllabus Committee to develop and publish appropriate learning objectives for our examinations. They worked with our Exam Committee to add pass mark panels as a routine part of the exam process, to conduct item writer training courses for all exam writers, and finally to introduce the use of open-book testing (with the rating manuals on Exams 5 and 9).

• On very short notice, Tom's committees also developed a new CAS Exam 3 and accompanying syllabus, which was just offered during this past exam sitting.

• In addition, Tom oversaw the completion of the work of the Future Education Task Force and is working to initiate the next steps.

Thank you, Tom, for all of your hard work and for participating with me in this year's race.

Don Mango is vice president–research & development, in addition to being an outstanding award-winning contributor to CAS research literature. Don and his committees are working on a complete overhaul of the CAS research activities.

• They recently introduced the working party approach to research. Working parties are group call paper task forces with the specific charge to produce a single research product over the next year. This research concept was introduced at this year's Risk and Capital Management Seminar and Casualty Loss Reserve Seminar, and four working party projects are underway.

• Don was instrumental in working with the CAS office in the hiring of Erin Clougherty, our new CAS librarian. Together

they've implemented the taxonomy that we are using for our online index of literature published in CAS publications.

• Don and his committees have also worked closely with the Society of Actuaries, as we move toward our Centennial Goal in the area of enterprise risk management.

Don, thank you for being such an energetic member of the EC. It's been fun running the race together this year.

Chris Carlson is our vice president–professional education. His committees have done an outstanding job of producing consistently high-caliber programs for our members. Some of the innovative activities Chris and his committees have implemented this year include:

• Partnering with Georgia State University to present the 2003 Bowles Symposium. This partnership resulted in the largest attendance in the history of the Symposium and received great acclaim for its scientific contributions.

• Leading the first jointly sponsored CAS/SOA Enterprise Risk Management Seminar, thus opening the doors of cooperation between the CAS and the SOA in this emerging area of practice that is one of the cornerstones of our Centennial Goal.

• Beginning to review CAS continuing education opportunities to ensure that we are meeting the needs of our membership.

Thanks to you, Chris, for your outstanding work this year and for also participating with me in the marathon.

As I announced earlier, Roger Schultz will have to step down as vice president–marketing & communications. But during this year, Roger and his committees brought many new ideas to the CAS including:

• The creation of an integrated marketing and communication plan that has been widely and effectively used to improve our communication actions. The success of the mutual recognition vote and our efforts to raise the awareness of the members regarding our Centennial Goal are evidence of this.

• Roger's committees have devoted time to assuring that the CAS remains focused on providing value to our members. They have introduced the on-line committee want ad process, solicited member participation in the development of special interest seminars, and created the Member Advisory Panel, which will provide ongoing feedback on behalf of our members.

Roger, I hope you're watching the Webcast right now, because I'd like to thank you for all of your great ideas. Best of luck as you begin your next race.

John Narvell is our vice president–international and, unfortunately, couldn't be with us at this meeting, because he's in Japan for the month of November.

• Since John took over as vice president, he has helped to reorganize our international committee structure to more effectively introduce the CAS as a resource in Europe, Latin America, and Asia.

• CAS leadership and members have been represented this year at meetings in India, Australia, Germany, the Philippines, Wales, and Brazil.

• We also hosted actuaries from the newly emerging country of Kazakhstan. They turned to the CAS as the preeminent resource to help them educate future non-life actuaries in their country. This is exactly the vision of the role of the CAS that is set forth in our Centennial Goal.

• We have distributed approximately 25 copies of our *Foundations of Casualty Actuarial Science* textbook to newly formed actuarial associations around the world. Our Web site has become a tremendous resource to these associations as well. Although it's nighttime in Japan, I hope you are watching the Webcast, John. Thanks for helping the CAS work towards achieving our global vision and for taking part with me in the race this year.

Our final vice president is Shelly Rosenberg, vice president– administration. Shelly and his committees have had a very active year including:

• Overseeing the successful electronic voting process that was introduced this year.

• Working closely with our Finance Committee to help the board more clearly define our surplus needs and the uses of surplus to meet emergencies and to help fund new initiatives for our members.

• And, finally, creating a disaster recovery plan for the CAS office.

Shelly wasn't able to be here, but I hope you're watching the Webcast as well. Thank you for being the sage of the EC and a wonderful member of our marathon team.

In addition to running this race with all of our dedicated volunteers, I would have never been able to complete this marathon had it not been for our outstanding team at the CAS office. Much of what gets done throughout the year is due to the behind the scenes efforts of our staff. My "From the President" column in this month's *Actuarial Review* is focused on the CAS staff and all they do. Thanks to Mike, Todd, Kathleen, Elizabeth, Tom, Jane, Tiffany, Josh, Jen, Erin, Kathy, Carrie, Patsy, Bob, Suellen, Sybil, Randy, and Joe.

In particular, I'd like to thank our executive director, Cynthia Ziegler, who joined us two years ago with a wealth of business, association, and insurance industry experience. In addition to providing almost daily support to me in my presidential activities, I consider her a valued friend. By the way, Cynthia was the one that bought me my own personalized yellow step stool, decorated with flowers and butterflies, so that I can see over the lectern! Cynthia, thanks for running this marathon right next to me this year.

As with life in general, a marathon course is never perfectly flat—there are uphills and downhills all along the way. Two major uphill challenges came upon me during this year's marathon.

In March of this year, my father passed away after a two-year battle with lymphoma. My dad was the most upbeat, positive, and determined person I've ever known. Nike must have invented the "Just Do It" saying with my dad in mind. Sadly, he was not able to see me complete this marathon, but I am grateful that he got to see me begin the race. I consider myself the luckiest person in the world to be blessed with a wonderful family who has provided me with tremendous love and support throughout my life and, especially, this year. In addition, I am grateful to Mary Frances Miller for stepping in for me on many occasions in February and March, so I was able to spend quality time with my dad. Thank you for being there for me, Mary Frances.

The other uphill challenge was on the employment front. I began my presidential term with Am-Re Consultants, a subsidiary of American Re. In April, American Re made a strategic decision to focus solely on their core business of reinsurance, and they announced that they would terminate activities in all of their nonreinsurance subsidiaries, including Am-Re Consultants. I know that many of our members have faced this same challenge in recent years. Thankfully, our core team of consultants was able to remain together, and we formed the Princeton office of Milliman USA—a great outcome! Thanks to the entire Milliman family for welcoming our team.

So now I've reached the end of my first marathon. But there are plenty more races for the CAS that are already underway and many more to begin.

• First, the Centennial Goal Implementation Task Force is working with each committee to facilitate the adoption of significant, attainable, and measurable goals to keep us moving toward our Centennial Goal.

• We continue to work on ways of reducing travel time the time from entering the actuarial profession to completing the examinations. These activities include introducing computerbased testing, beginning with Exam 1 in Spring 2005; exploring the idea of validation by college experience for topics like economics, finance, and mathematical statistics; and considering hiring professionals to write the study notes for our exams.

• On another front, and one that is likely to generate a lot of CASNET discussion, the board has just appointed two task forces: one focused on the role of different classes of membership and the other to consider an ACAS right to vote.

• Finally, recognizing the goal of continuously searching for ways to serve our members, we're exploring the formation of two special interest sections: one focused on the special needs of our members operating in small companies or consultancies, and the other for our ever-growing retired or near-retired members.

So, as you can see, there are plenty of races yet to run. I encourage you all to find an area of interest and continue to participate in the growth and vision of the CAS.

As I cross the finish line of my marathon year, I'm feeling a flood of emotions. Aside from the normal exhaustion that one feels at the end of a marathon, I'm also basking in a great feeling of accomplishment and I am thrilled to be greeted at the finish line by a wonderful support team to whom I'm ever indebted.

• First, to all of my colleagues in the Princeton office of Milliman, especially Bill Azzara, Urb Leimkuhler, and my administrative assistant, Carol Pollock: Thanks for carrying the weight for the last year. I'm ready to return to being a billable member of the team again. • Next, to my loving family—many of whom are here, including my mother, my mother-in-law and her companion, and my three younger sisters—and to my three brothers-in-law, six nephews, and two nieces who could not be here: Your love and support mean everything to me, especially in this very difficult year for our family.

• And last, but certainly not least, to my wonderful husband, Steve: You've been with me every step of the way throughout this marathon—I truly don't know if I could have gotten to this day without you. Thank you for the many sacrifices you've made this year, your constant words of encouragement, your ability to make me laugh, your unconditional love, and for being my best friend. I love you.

Standing here at the finish line, I also have tremendous feelings of pride. I'm proud to be a member of the CAS, proud to have been elected to lead our association for the past year, and proud to have helped this prestigious organization set the vision for ourselves as we move towards our 100th anniversary.

Finally, now that I've completed my first marathon, like the typical runner, I'm already thinking about my second one. I am pleased to announce that I've agreed to serve as the chair of the 2014 CAS Centennial Planning Committee. I look forward to celebrating at that finish line with all of you and future CAS members to come.

Before I pass the gavel to Mary Frances Miller, I'd like to close with some inspirational words of George Sheehan—doctor, writer, and a runner until his death in 1993. My husband introduced me to his essays, which often appeared in *Runner's World Magazine*. Dr. Sheehan wrote:

"No matter how old I get, the race remains one of life's most rewarding experiences. My times become slower and slower, but the experience of the race is unchanged: each race a drama, each race a challenge, each race stretching me in one way or another and each race giving meaning to my life."

I wish you all many rewarding races in the future. Thank you.

## MINUTES OF THE 2003 ANNUAL MEETING

### November 9–12, 2003

#### NEW ORLEANS MARRIOTT

#### NEW ORLEANS, LOUISIANA

## Sunday, November 9, 2003

The Board of Directors held their regular quarterly meeting from 12:00 noon to 4:30 p.m.

Registration was held from 4:00 p.m. to 6:30 p.m.

An officers' reception for New Associates and Accompanying Persons was held from 5:30 p.m. to 6:30 p.m.

Monday, November 10, 2003

Registration continued from 7:00 a.m. to 8:00 a.m.

CAS President Gail M. Ross opened the business session at 8:00 a.m., welcoming all to the CAS Annual Meeting and announcing that once again the CAS would be broadcasting the morning's events via a Webcast over the CAS Web Site. President Ross introduced the current members of the Executive Council (EC) and thanked EC members Sheldon Rosenberg and Roger Schultz, who were retiring from their positions.

Ms. Ross announced that the board appointed new vice presidents to succeed the outgoing Rosenberg and Schultz. Deborah M. Rosenberg was appointed vice president-administration and Joanne S. Spalla vice president-marketing & communications. Ms. Ross then introduced members of the CAS Board of Directors and thanked outgoing board members Ralph S. Blanchard, Janet L. Fagan, Michael J. Miller, and Deborah M. Rosenberg. CAS Immediate Past President Robert F. Conger was presented a plaque on his retirement as chairperson of the board. Ms. Ross then commented on the CAS election process and results, stating that voter turnout for the 2003 elections was outstanding with 51 percent of Fellows voting. Online voting, which was allowed for the first time, was very successful, with 34 percent of the 2,508 eligible voters, registering to vote online. Postage costs were reduced by not needing to send so many paper ballot packages through the mail. Of the 1,273 Fellows who submitted ballots, 55 percent were submitted online. Ms. Ross said the CAS is planning to offer online voting again in 2004 and encouraged CAS Fellows to vote online in the future.

Ms. Ross then announced the results of the CAS elections. Mary Frances Miller, who was elected president-elect in 2002, would become CAS President at the conclusion of the Annual Business Meeting. CAS Fellows elected Stephen P. D'Arcy as 2003 president-elect. Joining Rosenberg and Spalla as members of the CAS Executive Council for 2003–2004 will be Thomas G. Myers, vice president–admissions; John C. Narvell, vice president–international; Christopher S. Carlson, vice president–professional education; and Donald F. Mango, vice president–research & development. New members of the CAS Board of Directors are Robert V. Deutsch, Sholom Feldblum, Andrew E. Kudera, and Robert F. Wolf.

Ms. Ross also recognized past presidents of the CAS attending the meeting: Robert A. Anker (1996), Ronald L. Bornhuetter (1975), Michael Fusco (1989), David G. Hartman (1987), C. K. "Stan" Khury (1984), Steven G. Lehmann (1998), and W. James MacGinnitie (1979).

Ms. Ross also recognized special guests in the audience: Mareb del Rosario, president of the Academia de Actuarios de Puerto Rico; W. James MacGinnitie, president of the International Actuarial Association (IAA); and John P. Ryan, representative of the Institute and General Insurance Board of the U.K. Actuarial Profession.

Ms. Ross recognized CAS Fellows and Associates who have been members for 25 years or more. She also asked all CAS volunteers to stand and be recognized, including committee chairpersons; board members and officers of the executive council; committee members; individuals who have worked on the AAA committees or committees of other actuarial organizations; Regional Affiliate officers; authors of papers; and moderators and panelists of this Annual Meeting or any previous CAS meeting. Ms. Ross acknowledged with pride the Society's phenomenal participation rate and asked the audience to applaud the efforts of these volunteers.

New Fellows and Associates were honored in a special ceremony. Vice Presidents Myers and Carlson announced the 108 new Associates and President-Elect Miller announced the 128 new Fellows. The names of these individuals follow.

#### NEW FELLOWS

Jeffrey R. Adcock	Michelle L. Busch	Christopher P.
Denise M. Ambrogio	Tara E. Bush	DiMartino
Kevin L. Anderson	James E. Calton	Scott H. Drab
Deborah Herman	Kristi Irene	Gregory L. Dunn
Ardern	Carpine-Taber	Ruchira Dutta
Silvia J. Bach	William Brent Carr	Ramakrishna Duvvuri
Stevan S. Baloski	Jennifer L. Caulder	Dana R. Frantz
Patrick Barbeau	Hao Chai	Isabelle Gingras
Jack Barnett	Nathalie Charbonneau	Isabelle Girard
Rick D. Beam	Thomas Joseph	Andrew Samuel Golfin
Jonathan P. Berenbom	Chisholm	Natasha C. Gonzalez
Jason E. Berkey	Wai Yip Chow	Christopher David
Frank J. Bilotti	Cameron A. Cook	Goodwin
Linda Jean Bjork	Richard Jason Cook	Donald B. Grimm
Tony Francis Bloemer	Aaron T. Cushing	Robert A. Grocock
Nathan L. Bluhm	David W. Dahlen	Jacqueline Lewis
Nebojsa Bojer	Patricia A. Deo-Campo	Gronski
Erich A. Brandt	Vuong	James Christopher
Claude B. Bunick	Mark Richard	Guszcza
Angela D. Burgess	Desrochers	David Lee Handschke

Guo Harrison Hans Heldner Allen J. Hope Eric J. Hornick Christopher Wayne Hurst Tina Tuyet Huynh Philip M. Imm Ali Ishaq Julie A. Jordan Daniel R. Kamen Erin Hye-Sook Kang John J. Karwath Douglas H. Kemppainen Stacey M. Kidd Jeff A. Kluck Henry Joseph Konstanty Charles B. Kullmann Thomas P. Langer Jason A. Lauterbach Stephen E. Lehecka Shangjing Li Jenn Y. Lian Jing Liu Andrew M. Lloyd Jason K. Machtinger Stephen P. Marsden

Anthony G. Martella Michael E. Mielzynski Ethan Charles Mowry Karen E. Myers Kee Heng Ng Khanh K. Nguyen Michael Douglas Nielsen John E Noble Jason M. Nonis Darci Z. Noonan Gerard J. Palisi Michael Thomas Patterson Tracie L. Pencak Sylvain Perrier Isabelle Perron Kevin Thomas Peterson Mitchell S Pollack Warren T. Printz William Dwavne Rader Michelle L Rockafellow Robert C. Roddy Scott I Rosenthal Michael R. Rozema Brian P Rucci

Laura Beth Sachs Jeremy N. Scharnick Thomas Schneider Larry J. Seymour Vladimir Shander Michelle L. Sheppard Douglas E. Smith Scott G Sobel Sharon L. Sowka Karine St-Onge Mark Richard Strona Wei Hua Su Lisa Liqin Sun Stephen James Talley Rick C. H. Tzeng Jennifer L. Vadney Richard Alan Van Dyke Paul A. Vendetti Marie-Eve J Vesel Karen E Watson Robert S. Weishaar Joseph C. Wenc Linda Yang Stephanie C. Young Xiangfei Zeng Larry Xu Zhang Lianmin Zhou

#### NEW ASSOCIATES

Fernando Alberto Alvarado Brian C. Alvers Jonathan L. Ankney Melissa J. Appenzeller Nicki C. Austin Thomas C. Bates Patrick Beaulieu Chris M. Bilski Kirk D. Bitu Matthew E. Butler Christine Cadieux Kevin K. W. Chan Yves Charbonneau Hung Francis Cheung

Philip A. Clancey Jason T Clarke Thomas Marie Cordier Richard R. Crabb Richard S. Crandall Keith W. Curley Kelly K. Cusick Willie L. Davis Amy L. DeHart Jeffrey A. Dvinoff Tomer Eilam Jessica L. Elsinger Matthew B. Feldman Peter L. Forester Susan J. Forrav Robert W. Geist David A. Gelberg William J. Gerhardt Gregory Evan Gilbert John S Giles William G Golush Lori A. Gordon Jennifer Graunas Stacie R W Grindstaff Simon Guenette Brian O Haaseth Faisal O Hamid Kimberly Baker Hand Trevor C. Handley Jason C. Harland Robert D. Harrington Eric A. Hatch Kathryn E. Herzog

Joseph S. Highbarger Ryan Yin-kei Ho Scott R Hurt Victoria K. Imperato Young Y. Kim Anne Marie Klein-Lee Raymond J. Kluesner Steven T. Knight James J. Konstanty Bradley S. Kove Hooi Lee Lai ZhenZhen Lai Heather D. Lake Anh Tu Le James J. Leonard Hayden Anthony Lewis Gavin X. Lienemann Hazel Joynson Luckey John T Maher Steven Manilov Robert B. McCleish Laurence R. McClure James P. McCoy Camilo Mohipp Timothy C. Mosler James C. Murphy Jacqueline Lee Neal Richard U Newell Melissa A. Ogden Eva M. Paxhia Michele S. Raeihle William C. Reddington

Stuart C. Rowe Jin Shao Peter M. Shelley Janel M. Sinacori James M Smieszkal Michael D. Sowka Laura T. Sprouse Christine Seung Steer Esperanza Stephens Erik J. Steuernagel Liana St-Laurent Ju-Young Suh Keith Jeremy Sunvold Jonas F. Thisner Malgorzata Timberg Dovid C. Tkatch Dominic A. Tocci David A. Traugott Jean-François Tremblav Lien K Tu-Chalmers Susan B. Van Horn Kevin K. Vesel Hanny C. Wai Gary C. Wang Paul D Wilbert Micah Grant Woolstenhulme Andrew F Yashar Jonathan K Yu Yi Zhang

Ms. Ross then introduced past CAS President C. K. "Stan" Khury, who gave the address to new members.

At the beginning of the awards ceremony, Ms. Ross spoke briefly about the Matthew S. Rodermund Service Award. This award was established in 1990 in honor of Matt Rodermund's years of volunteer service to the Casualty Actuarial Society. The Award recognizes a CAS member or members, who have made significant volunteer contributions to the actuarial profession such as committee involvement, participation in CAS meetings and seminars, volunteer efforts for Regional Affiliates or special interest sections, and involvement with non-CAS actuarial professional organizations such as the American Academy of Actuaries or the Canadian Institute of Actuaries.

Ms. Ross announced that this year's winner of the Matthew S. Rodermund Service Award was Charles Walter Stewart. Mr. Stewart was unable to attend but Ms. Ross read part of a letter he sent:

"I was totally amazed and pleased...I knew Matt from his participation in the Philadelphia Actuarial Club...initially in the late 1960's. Matt was the first actuary who made me truly appreciate that there were two masters for an actuary to attempt to provide excellent service to: one's employer, and the actuarial profession, most notably the Casualty Actuarial Society. So, at this point in my career, I feel a special connection to Matt. Thank you and the Board very much for this award."

Ms. Ross then presented the "Above & Beyond" Achievement Awards, a new CAS honor designed to celebrate the spirit of volunteerism. The award honors individuals who perform with exceptional merit but whose efforts may not be apparent or widely known to the vast majority of CAS members. Ms. Ross announced the winners for 2003: Sholom Feldblum, Aaron Halpert, and Tom Struppeck.

The president remarked that Mr. Feldblum has worked tirelessly to improve the actuarial profession. In the last year, he authored or co-authored three papers to be published in the 2003 *Proceedings*. In addition, he wrote or co-wrote three papers and one discussion that are currently before the Committee on Review of papers.

Ms. Ross cited Mr. Halpert for leading the charge in crafting and synthesizing two "Big Audacious Goals" into a single integrated, very lucid Centennial Goal. The quality of his original work and high degree of leadership shepherded the goal through to its approval by the board.

Mr. Struppeck was recognized for his work as the Exam 3 part chair. On short notice, he prepared a plan for staffing the Exam 3 Committee, helped train the new members and took the lead role in assembling questions for the exam. His work significantly exceeded the amount of work normally expected for an exam chair.

Vice President Mango then announced the winners of three awards given to authors for their contributions to actuarial literature.

Mr. Mango announced that the Woodward-Fondiller Prize would go to David Ruhm for his paper, "Distribution-Based Pricing Formulas Are Not Arbitrage-Free." The award, which commemorates the work of Joseph J. Woodward and Richard Fondiller, and is awarded for the best paper submitted each year by an Associate or Fellow who attained his or her designation within the last five years. Mr. Ruhm was unable to attend the meeting.

Mr. Mango announced Gary Venter as the winner of the year's Dorweiler Prize for his paper, "Tails of Copulas." The award commemorates the work of Paul Dorweiler and is awarded for the best paper submitted each year by an Associate or Fellow who attained his or her designation more than five years ago. Mr. Venter was also unable to attend.

Mr. Mango then presented the Charles A. Hachemeister Award to Shaun Wang for his paper, "A Universal Framework for Pricing

Financial and Insurance Risks." The Hachemeister Prize was established in 1993 in recognition of Charles A. Hachemeister's many contributions to Actuarial Studies in Non-Life Insurance (ASTIN) and his efforts to establish a closer relationship between the CAS and ASTIN.

After the award presentation, Ms. Ross asked the audience to pause for a moment of silence to acknowledge CAS members who died since November 2002: Clyde B. Fulton Jr.; William J. Rowland; Gary P. Hobart; Cindy R. Schauer; David J. Kretsch; James Surrago; and Jack Moseley.

Ms. Ross then spoke on behalf of the Trustees for the CAS Trust (CAST) to recognize D. W. Simpson & Company and its funding for the advancement of actuarial science. The company donated \$10,000 this year to the Trust and has cumulatively donated \$80,000 to the CAST. The CAST was established in 1979 as a non-profit 501(c) (3) organization to afford members and others an income tax deduction for contributions of funds to be used for scientific, literary, research, or educational purposes.

Ms. Ross announced that two *Proceedings* papers would be presented at this meeting. The titles and authors of these papers are "The Minimum Bias Procedure" by Sholom Feldblum and Eric Brosius, and "Insurance Applications of Bivariate Distributions" by David Homer and David Clark.

Jim MacGinnitie, president of the International Actuarial Association (IAA) and past president of the CAS, followed with an update on IAA activities.

After a brief introduction by Ms. Miller, Ms. Ross gave the presidential address.

At the conclusion of the address, Ms. Miller presented Ms. Ross with the presidential plaque. Outgoing CAS President Ross then passed on the presidential gavel to incoming CAS President Miller, who then closed the business session. After a refreshment break, the first General Session was held from 10:15 a.m. to 11:45 a.m.

"Underwriting Cycles—Are They Inevitable?"

Moderator:	Robert F. Conger Consulting Actuary Tillinghast-Towers Perrin
Panelists:	Robert P. Hartwig Ph.D. Senior Vice President and Chief Economist Insurance Information Institute
	Michael J. Miller Principal and Consulting Actuary EPIC Actuaries, LLC
	Mary A. Weiss Deaver Professor, Risk and Insurance Temple University
	Henry C. Wurts Ph.D. Director of Analytics Gallagher Financial Products

After the General Session, a luncheon was held where featured speaker, Dr. Rushworth M. Kidder, gave his presentation on ethical decision-making.

After the luncheon and featured speaker, the afternoon was devoted to presentations of concurrent sessions. The panel presentations from 1:30 p.m. to 3:00 p.m. covered the following topics:

1. Cyber Hurricane

Moderator/	John L. Tedeschi
Panelist:	Managing Director
	Guy Carpenter and Company, Inc.
Panelists:	Harrison D. Oellrich
	Managing Director
	Guy Carpenter & Company, Inc.

Tracey A. Vispoli Vice President, Department of Financial Institutions Chubb & Son

2. Investigation and Settlement Negotiation of Auto Bodily Injury Liability Claims in the Presence of Fraud

Moderator/	Richard A. Derrig
Panelist:	Automobile Insurers Bureau of
	Massachusetts
Panelist:	Judith L. Dickson

## Liberty Mutual Insurance Company 3 The Asbestos Explosion

Moderator:	Michael E. Angelina	
	Consulting Actuary	
	Tillinghast-Towers Perrin	
Panelist:	Glenn A. Pomeroy	
	Associate Concred Counsel	

Associate General Counsel GE Employers Reinsurance Corporation

- 4. People Have Birthdays—What Can We Do About It?
  - Moderator/Jonathan WhitePanelist:Assistant Vice President and Actuary<br/>The Hartford
  - Panelists: Susan Ferguson Ph.D. Senior Vice President, Research Insurance Institute for Highway Safety Roosevelt C. Mosely Consulting Actuary Pinnacle Actuarial Resources, Inc.
- 5. Professionalism and Actuarial Limits of Liability

Moderator:	Chad C. Wischmeyer
	Managing Director
	Mercer Risk, Finance & Insurance
	Consulting

Panelists:	Lauren M. Bloom General Counsel American Academy of Actuaries
	Thomas L. Ghezzi Consulting Actuary Tillinghast-Towers Perrin
	David J. Otto Actuary The Kilbourne Company

6. Update on Medical Malpractice

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Moderator:	Carl X. Ashenbrenner	
	Actuary	
	Milliman USA, Inc.	
Panelists:	Sarah M. Dore	
	Principal	
	SM Dore Consulting	
	Kathy Pinkham	
	President, Healthcare First	
	Arthur J. Gallagher & Company	
What's Next: Federal Terrorism Legislation		
M. 1	Christen hen Teit	

Moderator:	Christopher Tait
	Principal and Consulting Actuary
	Milliman USA, Inc.
Panelists:	David Lalonde
	Senior Vice President
	Applied Insurance Research
	Christopher H. Yaure
	Risk Manager, Terrorism and Emerging
	Issues
	GE Employers Reinsurance Corporation

After a refreshment break, presentations of concurrent sessions continued from 3:30 p.m. to 5:00 p.m.

1.	Models: Do You Trust Them?	
	Moderator/ Panelist:	Stuart B. Mathewson Lead Pricing Actuary GE Employers Reinsurance Corporation
	Panelists:	Louise A. Francis Consulting Principal Francis Analytics & Actuarial Data Mining Inc.
		Jonathan B. Hayes Senior Vice President Guy Carpenter & Company, Inc.
2.	Only a Dummy	Would Want to Miss This
	Moderator/ Panelist:	Patrick B. Woods Assistant Vice President ISO
	Panelist:	Susan Ferguson Ph.D. Senior Vice President, Research Insurance Institute for Highway Safety
3.	Underwriting Risk Models	
	Moderator/ Panelist:	John J. Kollar Vice President ISO
	Panelist:	Urban E. Leimkuhler Consulting Actuary Milliman USA, Inc.
4.	Capital Movement: Over to Under—How?	
	Moderator/ Panelist:	W. James MacGinnitie Actuary and Consultant
	Panelist:	Matthew C. Mosher Vice President

A. M. Best Company

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5.	The CAS Centennial Goal	
	Moderator:	Aaron M. Halpert Principal KPMG LLP
	Panelists:	David G. Hartman Senior Vice President and Chief Actuary Chubb Group of Insurance Companies
		C. K. "Stan" Khury Principal Bass & Khury

6. Changes to Construction Defects Coverage, Claims, and Reserving

Moderator:	Jennifer M. Levine	
	Regional Actuary	
	Zurich North America	
Panelists:	Michael D. Green Principal Deloitte & Touche LLP	
	Thomas I. Ghezzi	
	Consulting Actuary	
	Tillinghast-Towers Perrin	
	Paul B. Swank	
	Claims Consultant	
	Tillinghast-Towers Perrin	
Current State and Future of the Surety Industry		
Moderator:	Robert J. Meyer	
	Principal and Consulting Actuary	
	Milliman USA, Inc.	

7.

Panelists: Athula Alwis Vice President Willis Re Richard M. Young Managing Director Guy Carpenter & Company An officers' reception for New Fellows and Accompanying Persons was held from 5:30 p.m. to 6:30 p.m.

A general reception for all attendees followed from 6:30 p.m. to 7:30 p.m.

Tuesday, November 11, 2003

Registration continued from 7:00 a.m. to 8:00 a.m.

The following General Sessions were held from 8:00 a.m. to 9:30 a.m.

"Communicating with Wall Street"

Moderator:	Gayle E. Haskell
	Chief Financial Officer
	Aspen Specialty Insurance Company
Panelist:	Alice Schroeder
	Consultant
	Wall Street GS

"Sarbanes-Oxley Act and the Actuary"

Moderator:	Robert A. Anker Quay Quest
Panelists:	Peter M. Licht Senior Manager PriceWaterhouseCoopers LLP
	Howard L. Mosbacher Senior Vice President, General Auditor, and Chief Information Security Officer The Hartford
	Patricia A. Teufel Principal KPMG LLP

After a break, the following concurrent sessions were held from 10:00 a.m. to 11:30 a.m.

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1. Homeowners Profitability—Is the P/C Industry Taking the Right Actions and Have We Learned Anything?

Moderator:	Jeffrey L. Kucera Consulting Actuary Pinnacle Actuarial Resources, Inc.
Panelists:	Randall E. Brubaker Senior Vice President Aon Re Services
	Dan J. Davis Senior Actuary Louisiana Department of Insurance

2. Captives—Alternatives or Obstacles?

3.

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Moderator:	François Morin Consulting Actuary Tillinghast-Towers Perrin
Panelists:	Marc-Andre Lefebvre Credit & Market Risk Executive Royal & SunAlliance
	Charles R. Woodman Senior Vice President Marsh Inc.
	John Yonkunas Principal Tillinghast-Towers Perrin
Claims Issues	
Madamatam	Timethy E. Vesster

Moderator:	Timothy F. Koester
	Claims Actuary
	GE Employers Reinsurance Corporation
Panelist:	Chris K. Carpenter
	Claims Specialist
	GE Employers Reinsurance Corporation

- 4. Education in the 21st Century Moderator/ Jeanne E. Swanson Panelist: Associate Actuary Liberty Mutual Group
  Panelists: Mary Frances Miller Select Actuarial Services Thomas G. Myers Vice President Prudential Property & Casualty Insurance Company
- 5. Small Commercial Lines Underwriting

Moderator/ Panelist:	Beth E. Fitzgerald Assistant Vice President ISO
Panelist:	Robert J. Walling III Principal and Consulting Actuary Pinnacle Actuarial Resources, Inc.

# 6. Liquidity—How to Fix It?

Moderator:	Chester J. Szczepanski
	Chief Actuary
	Pennsylvania Insurance Department
Panelist:	Matthew C. Mosher
	Group Vice President–P/C
	A. M. Best Company

7. No-Fault—Then and Now

Moderator/ Panelist:	Gavin C. Blair Vice President and Actuary Liberty Mutual Group
Panelists:	Steven G. Lehmann Principal and Consulting Actuary Pinnacle Actuarial Resources, Inc.

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Elizabeth A. Sprinkel Senior Vice President Insurance Research Council

After a break for lunch, the following concurrent sessions continued from 12:30 p.m. to 2:00 p.m.

1. Fair Value Accounting and the Actuary

Moderator:	Robert F. Wolf
	Principal
	Mercer Risk, Finance & Insurance
	Consulting
Panelists:	Philip E. Heckman
	Senior Consultant and Actuary
	Aon Risk Consultants, Inc.
	Robert S. Miccolis
	Director
	Deloitte & Touche LLP

2. California Workers Compensation-An Update

Moderator/ Panelist:	David M. Bellusci Senior Vice President and Chief Actuary Workers Compensation Insurance Rating Bureau of California
Panelists:	Alex Swedlow Executive Vice President California Workers Compensation Institute
	Lawrence White Workers Compensation Insurance Policy Advisor California Department of Insurance

3. Texas Homeowners

Moderator:	Jeffrey L. Kucera
	Consulting Actuary
	Pinnacle Actuarial Resources, Inc.

Panelists:	Ron Cobb
	Director
	American Insurance Association,
	Southwestern Office
	David W. Lacefield
	President
	Texas Select Lloyds Insurance Company
	Philip O. Presley
	Chief Actuary
	Texas Department of Insurance

4. The Matrix Inverted—A Primer in GLM Theory and Practical Issues

Moderator:	Kevin Anderson Actuary Winterthur North America
Panelists:	Gilbert Korthals Assistant Vice President, Actuary GuideOne Insurance Group
	Claudine H. Modlin Senior Consultant Watson Wyatt Insurance & Financial Services

The Hachemeister Prize Paper was presented from 12:30 p.m. to 2:00 p.m.

"A Universal Framework for Pricing Financial and Insurance Risks"

Author:	Shaun Wang	
	SCOR Reinsurance	

Two *Proceedings* papers presented from 12:30 p.m. to 2:00 p.m. time were:

1. "The Minimum Bias Procedure"

Authors:	Sholom Feldblum	
	Liberty Mutual Group	

# J. Eric Brosius Liberty Mutual Group

2. "Insurance Applications of Bivariate Distributions"

Authors: David L. Homer Towers Perrin David R. Clark American Re-Insurance Company

The first session of the General Business Skills Workshop, "Effective Communication and Behavior Styles," was held from 12:30 p.m. to 2:30 p.m. Certain concurrent sessions were repeated from 2:30 p.m. to 4:00 p.m. Additional concurrent sessions presented at this time were:

1. Actuarial Techniques for Nontraditional Projects

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Moderator/	Michael A. McMurray
Panelist:	Official Scorer
	Yakima Bears Professional Baseball
Panelists:	Harry T. Garland Ph.D.
	Principal
	Garland Actuarial LLC
	Michael W. Shackleford
	President
	Wizard of Odds
DFA Success S	tory—Not a Fairy Tale
Moderator:	François Morin
	Consulting Actuary
	Tillinghast-Towers Perrin
Panelists:	Charles T. Longua
	Vice President, Corporate Financial
	Planning
	Erie Insurance Group
	Douglas E. Smith
	Associate Actuary, Pricing & Financial
	Analysis

Erie Insurance Group

2.

Christopher M. Suchar Principal DFA Capital Management, Inc.

3. Legislative Reform: Reform or Be Reformed—How Can You Make a Difference?

Moderator/ Panelist:	Steven G. Lehmann Principal and Consulting Actuary Pinnacle Actuarial Resources, Inc.
Panelists:	Jeffrey A. Skelton Assistant Vice President, Personal Insurance Legislative Affairs ChoicePoint
	Kevin T. Sullivan Deputy General Counsel Allstate Insurance Company

The second session of the General Business Skills Workshop, "Leading and Managing Effective Teams," was held from 12:30 p.m. to 2:30 p.m. Certain concurrent sessions were repeated from 3:00 p.m. to 5:30 p.m.

A Mardi Gras celebration and entertainment for all attendees was held from 6:30 p.m. to 9:30 p.m.

Wednesday, November 12, 2003

Concurrent sessions presented from 8:00 a.m. to 9:30 a.m. were:

1. Private Mortgage Insurance

Moderator:	Michael C. Schmitz
	Consulting Actuary
	Milliman USA
Panelists:	John E. Gaines
	Vice President–Structured Products
	United Guaranty Corporation

John F. Gibson Principal PricewaterhouseCoopers LLP Kyle S. Mrotek Associate Actuary Milliman USA, Inc.

2. CAS Examination Process

3.

Moderator:	Richard P. Yocius Senior Actuary
	Allstate Insurance Company
Panelists:	Steven D. Armstrong Senior Actuary Allstate Insurance Company
	Christopher S. Throckmorton Chief Actuary Fairmont Specialty Group
Nursing Home	Professional Liability Crisis-An Update
Moderator:	Jennifer K. Price Principal Mercer Oliver Wyman
Panelists:	Keith P. Becker Southeast Long Term Care Practice Leader Marsh USA Inc. Theresa W. Bourdon Managing Director Aon Risk Consultants
	Sherry L. Brunner Vice President, Risk and Insurance Services

Bon Secours Health System, Inc.

4. Product Development

Moderator/	Justin M. Van Opdorp
Panelist:	Corporate Actuary
	GE Employers Reinsurance Corporation
Panelist:	Philip O. Presley
	Chief Actuary
	Texas Department of Insurance

5. Extreme Events

Moderator/ Panelist:	David A. Lalonde Senior Vice President AIR Worldwide
Panelists:	Kay A. Cleary Vice President–Actuarial Aon Re Services
	Matthew C. Mosher Vice President A. M. Best Company

After a refreshment break, the final General Session was held from 10:00 a.m. to 11:30 a.m.

"Insurer Failures-Does the Past Teach Anything?"

Moderator:	David G. Hartman Senior Vice President and Chief Actuary Chubb Group of Insurance Companies
Panelists:	Laline Carvalho Director, Financial Services Ratings Group Standard & Poor's
	Michael A. Coutu Chief Executive Officer and Chairman of the Board Kenning Financial Advisors, LLC

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Vincent L. Laurenzano Insurance Financial Consultant Stroock & Stroock & Lavan LLP

Gail M. Ross officially adjourned the 2003 CAS Annual Meeting at 11:45 a.m., after closing remarks and an announcement of future CAS meetings.

## Attendees of the 2003 CAS Annual Meeting

The 2003 CAS Annual Meeting was attended by 403 Fellows, 177 Associates, 1 Affiliate and 116 Guests. The names of the Fellows, Associates and Affiliate in attendance follow:

#### FELLOWS

Jeffrey R. Adcock	Andrew Steven Becker	Conni Jean Brown
Barbara J. Addie	Michael J. Belfatti	Randall E. Brubaker
Denise M. Ambrogio	David M. Bellusci	James E. Buck
Kevin L. Anderson	Jonathan P. Berenbom	Russell J. Buckley
Robert A. Anker	Jason E. Berkey	Gary S. Bujaucius
Deborah Herman	Neil A. Bethel	Claude B. Bunick
Ardern	Terry J. Biscoglia	Angela D. Burgess
Steven D. Armstrong	Everett G. Bishop	Michelle L. Busch
Nolan E. Asch	Gavin C. Blair	James E. Calton
Carl Xavier	Tony Francis Bloemer	Christopher S. Carlson
Ashenbrenner	Nathan L. Bluhm	Kristi Irene Carpine-
Richard V. Atkinson	Nebojsa Bojer	Taber
Peter Attanasio	Ann M. Bok	Martin Carrier
Karen F. Ayres	David R. Border	Jennifer L. Caulder
Barry Luke Bablin	Ronald L. Bornhuetter	Francis D. Cerasoli
Silvia J. Bach	Lesley R. Bosniack	Nathalie Charbonneau
Stevan S. Baloski	Theresa W. Bourdon	Thomas Joseph
Phil W. Banet	Erich A. Brandt	Chisholm
Patrick Barbeau	Sara T. Broadrick	Wai Yip Chow
Jack Barnett	Linda K. Brobeck	David R. Clark
Andrea C. Bautista	Ward Brooks	Kay A. Cleary
Rick D. Beam	J. Eric Brosius	Susan M. Cleaver
J. Paul Cochran Robert F. Conger Larry Kevin Conlee Eugene C. Connell John B. Conners Cameron A Cook **Richard Jason Cook** Michael J Covert M Elizabeth Cunningham Aaron T. Cushing David W. Dahlen Karen Barrett Daley Stephen P. D'Arcy Dan J Davis John D Deacon Kris D DeFrain Mark Richard Desrochers Robert V. Deutsch Christopher P. DiMartino Andrew J Doll Scott H Drab Sara P. Drexler Timothy B. Duffy Gregory L. Dunn Ruchira Dutta Ramakrishna Duvvuri Richard D Easton Grover M. Edie Dale R. Edlefson Gary J. Egnasko Valere M. Egnasko Warren S. Ehrlich

Nancy R. Einck Douglas D. Eland Thomas J. Ellefson Charles C. Emma Paul E Ericksen Julia L. Evanello Glenn A. Evans John S Ewert Doreen S. Faga Richard I. Fein Sholom Feldblum Judith M. Feldmeier Beth E. Fitzgerald Chauncey Edwin Fleetwood James E. Fletcher Daniel J Flick Feifei Ford Claudia S. Forde Hugo Fortin Louise A Francis Dana R. Frantz Michelle L. Freitag Bruce F. Friedberg Michael Fusco John E Gaines Roberta J Garland **Richard Gauthier** Thomas L. Ghezzi John F Gibson Bruce R. Gifford Patrick John Gilhool William R. Gillam Bryan C. Gillespie Isabelle Gingras

Isabelle Girard Bradley J. Gleason Steven A Glicksman Andrew Samuel Golfin Jr Christopher David Goodwin Patrick J Grannan Robert A Grocock Linda M. Groh Jacqueline Lewis Gronski Carleton R. Grose Nasser Hadidi Allen A Hall Marc S. Hall Leigh Joseph Halliwell Steven Thomas Harr David C. Harrison Guo Harrison Bryan Hartigan David G. Hartman Matthew T. Hayden David H. Hays Kevin B. Held Hans Heldner John Herder Laura Esboldt Heyne Mark D. Heyne David L. Homer Allen J. Hope Eric J. Hornick Jeffrey R. Hughes Christopher Wayne Hurst

Tina Tuyet Huynh Jamison Joel Ihrke Philip M. Imm Craig D. Isaacs Ali Ishaq Katherine Jacques Richard M. Jaeger Christian Jobidon Eric J. Johnson Erik A. Johnson Mark Robert Johnson Bryon Robert Jones Jeffrey R. Jordan Julie A. Jordan Alan E Kaliski Daniel R. Kamen Erin Hye-Sook Kang John J. Karwath Hsien-Ming Keh Brandon Daniel Keller Scott Andrew Kelly Douglas H. Kemppainen Rebecca Anne Kennedy C. K. "Stan" Khury Stacey M. Kidd Richard O. Kirste Frederick O Kist Brandelyn C. Klenner Craig W. Kliethermes Jeff A. Kluck Leon W. Koch Timothy F. Koester John J. Kollar

Gilbert M. Korthals Israel Krakowski Gustave A Krause Rodney E. Kreps Jeffrey L. Kucera Andrew E. Kudera Anand S. Kulkarni David R Kunze Pamela G. Kurtz Mylene J. Labelle Blair W. Laddusaw Julie-Linda Laforce Stephane Lalancette David A. Lalonde Chingyee Teresa Lam Alan E. Lange Dennis L. Lange Matthew G. Lange Jason A. Lauterbach Kevin A Lee Marc-Andre Lefebvre P. Claude Lefebvre Stephen E. Lehecka Steven G Lehmann Todd William Lehmann Neal Marey Leibowitz Urban E. Leimkuhler Ir James P. Leise Christian Lemay Winsome Leong Alain Lessard Jennifer McCullough Levine

Kenneth A. Levine Shangjing Li Xiaoying Liang Peter M. Licht Matthew Allen Lillegard Barry Lipton Andrew M. Lloyd W. James MacGinnitie Jason K. Machtinger Maria Mahon Donald F. Mango Stephen P. Marsden Anthony G. Martella Jr Stuart B Mathewson Robert W. Matthews Kevin Paul McClanahan Ian John McCracken James B McCreesh **Richard Timmins** McDonald Michael F McManus Michael A. McMurray Dennis C. Mealy Brian James Melas Robert E. Meyer Robert J. Meyer Stephen J. Meyer Vadim Y. Mezhebovsky Michael E. Mielzynski Mary Frances Miller Michael J. Miller

Ronald R. Miller Claudine H Modlin Richard B Moncher David Patrick Moore Lisa J. Moorev François Morin Robert Joseph Moser Matthew C Mosher Roosevelt C. Mosley Ethan Charles Mowry Nancy Diane Mueller Robert T. Muleski Karen E. Myers Thomas G. Myers Patrick R. Newlin Khanh K. Nguyen Mindy Y. Nguyen Lynn Nielsen Michael Douglas Nielsen John Nissenbaum Ray E. Niswander Jr. John E. Noble Corine Nutting David J. Oakden Christopher Edward Olson Rebecca Ruth Orsi David J Otto Timothy A. Paddock Teresa K. Paffenback Richard D. Pagnozzi Rudy A. Palenik Gerard J. Palisi Cosimo Pantaleo

Julie Parsons M Charles Parsons Jacqueline Edith Paslev Michael Thomas Patterson Harry Todd Pearce John R Pedrick Tracie L. Pencak Sylvain Perrier Isabelle Perron Kevin Thomas Peterson Kraig Paul Peterson **Richard Matthew** Pilotte Kristin Sarah Piltzecker Dylan P. Place Arthur C. Placek Richard C. Plunkett Igor Pogrebinsky Philip O. Presley Deborah W Price Jennifer K. Price Warren T Printz Arlie J Proctor Kenneth Quintilian **Ricardo** Anthony Ramotar John J. Reynolds III Michelle L. Rockafellow Robert C. Roddy William P. Roland

Steven Carl Rominske Scott I Rosenthal Gail M Ross Jean-Denis Rov Michael R. Rozema Brian P Rucci Laura Beth Sachs Thomas E Schadler Jeremy N. Scharnick Timothy L. Schilling Michael C. Schmitz Thomas Schneider Susan C. Schoenberger Timothy D. Schutz Larry J. Seymour Vladimir Shander Michelle L. Sheppard Jeremy D. Shoemaker Christy L. Simon Lisa A. Slotznick Christopher M. Smerald Douglas E. Smith Scott G Sobel Sharon L. Sowka Joanne S. Spalla Daniel L. Splitt Anya K. Sri-Skanda-Rajah Thomas N. Stanford Curt A. Stewart Deborah L. Stone Karine St-Onge Douglas N. Strommen Mark Richard Strona

Thomas Struppeck Wei Hua Su Christopher M. Suchar Christie L. Sullivan Lisa Ligin Sun Brian Tohru Suzuki Roman Svirsky Scott J. Swanay Jeanne E Swanson Chester John Szczepanski Christopher Tait Stephen James Talley Michael Joseph Tempesta Karen F. Terry Patricia A Teufel Chris S Throckmorton Charles F. Toney II Nancy R. Treitel

Brian K. Turner Laura M Turner William R Van Ark Oakley E. Van Slyke Paul A. Vendetti Geraldine Marie L Verano Marie-Eve J Vesel Jennifer Anne Vezza Jennifer S. Vincent Gerald R. Visintine Steven M. Visner Christopher P. Walker Robert J. Walling III Shaun S. Wang Kelly A. Wargo Karen E. Watson Peter A. Weisenberger Christopher John Westermeyer

### ASSOCIATES

Danny M. Allen Nancy S. Allen Fernando Alberto Alvarado Athula Alwis Michael E. Angelina Jonathan L. Ankney Melissa J. Appenzeller Afrouz Assadian Nicki C. Austin Paul C. Barone Patrick Beaulieu Chris M. Bilski John T. Binder Kirk D. Bitu Richard S. Brutto Christine Cadieux Kevin K. W. Chan Yves Charbonneau Joyce Chen Philip A. Clancey Jr. Thomas P. Conway Thomas Marie Cordier Richard R. Crabb Richard S. Crandall Hall D. Crowder

Jonathan White Mark Whitman William Robert Wilkins Gregory S. Wilson John J. Winkleman Jr. Dean M. Winters Chad C. Wischmeyer Robert F Wolf Tad E. Womack Jeffrev F. Woodcock Patrick B. Woods Richard P. Yocius John P Yonkunas Stephanie C. Young Xiangfei Zeng Larry Xu Zhang Yin Zhang Lianmin Zhou John D Zicarelli

Keith W. Curley Kelly K. Cusick Thomas V. Daley James R. Davis Willie L. Davis Jr. Catherine L. DePolo Timothy M. Devine Jeffrey A. Dvinoff Jessica L. Elsinger James C. Epstein Brian A. Evans Denise M. Farnan Thomas R. Fauerbach

Matthew B. Feldman William P Fisanick Susan J. Forray Jeffrey J. Fratantaro Timothy J. Friers Robert W. Geist David A. Gelberg Justin Gordon Gensler William J. Gerhardt Gregory Evan Gilbert Lori A. Gordon Jennifer Graunas Michael D. Green Stacie R. W. Grindstaff Christopher Gerald Gross Simon Guenette Brian O. Haaseth Aaron M. Halpert Faisal O. Hamid Trevor C. Handley Jason C. Harland Robert D. Harrington Gary M. Harvey Eric A. Hatch Jonathan B. Hayes Philip E. Heckman Joseph A. Herbers Kathryn E. Herzog Joseph S. Highbarger Robert C. Hill Rvan Yin-kei Ho Melissa S. Holt Scott R. Hurt Jeffrey R. Ill

Victoria K. Imperato Jean-Claude Joseph Jacob Brian B. Johnson Pamela A. Kaplan David L. Kaufman Chester T. Kido Martin T. King Jill E. Kirby Anne Marie Klein-Lee Linda S. Klenk Raymond J. Kluesner Steven T. Knight James J. Konstanty David W. Lacefield Hooi Lee Lai ZhenZhen Lai Heather D. Lake Anh Tu Le Charles R Lenz James J. Leonard Sharon Xiaovin Li Gavin X. Lienemann Hazel Joynson Luckey Eric A. Madia John T Maher Steven Manilov Rosemary Marks-Samuelson Scott A Martin Robert B. McCleish IV Laurence R. McClure II Jennifer Ann McCurry Leonard L. Millar

Douglas H. Min Stanley K. Miyao Camilo Mohipp Timothy C. Mosler Sharon D. Mott Kvle S. Mrotek James C. Murphy Jr. Rade T Musulin Jacqueline Lee Neal Charles P. Neeson Anthony J. Nerone W. H. Odell Melissa A. Ogden Charles P Orlowicz Eva M. Paxhia Wendy Wei-Chi Peng Claude Penland Anthony J. Pipia Peter Victor Polanskyj Ruth Youngner Poutanen Richard B. Puchalski John T. Raeihle Michele S Raeihle James E. Rech William C. Reddington John Dale Reynolds Brad E. Rigotty Christopher R. Ritter Joseph Francis Rosta Jr. Stuart C. Rowe Frederick Douglas Ryan Frances G Sarrel

Ronald J. Schuler David Garrett Shafer Jin Shao Peter M. Shelley Ben Silberstein Summer L. Sipes Donald P. Skrodenis James M. Smieszkal Lee Oliver Smith David C. Snow Michael D. Sowka Calvin C. Spence Jr. Laura T. Sprouse Erik J. Steuernagel Liana St-Laurent Christopher J. Styrsky Ju-Young Suh Brian K. Sullivan Keith Jeremy Sunvold John L. Tedeschi Jonas F. Thisner Diane R. Thurston Malgorzata Timberg Dovid C. Tkatch David A. Traugott Jean-François Tremblay Lien K. Tu-Chalmers Susan B. Van Horn Justin M. Van Opdorp Kevin K. Vesel John E. Wade David G. Walker Gary C. Wang Monty James Washburn David L. Whitley Miroslaw (Mirek) Wieczorek Paul D. Wilbert Joel F. Witt Micah Grant Woolstenhulme Jonathan K. Yu Yi Zhang

AFFILIATES

Bradford S. Gile

### REPORT OF THE VICE PRESIDENT-ADMINISTRATION

This report provides a one-year summary of CAS activities since the 2002 CAS Annual Meeting. I will first comment on these activities as they relate to the following purposes of the Casualty Actuarial Society, as stated in our Constitution:

- 1. Advance the body of knowledge of actuarial science applied to property, casualty, and similar risk exposures;
- 2. Establish and maintain standards of qualifications for membership;
- 3. Promote and maintain high standards of conduct and competence for the members; and
- 4. Increase the awareness of actuarial science.

I will then provide a summary of other activities that may not relate to a specific purpose, but are critical to the ongoing vitality of the CAS. Finally, I will summarize the current status of our finances and key membership statistics.

### CAS ACTIVITIES

The *Forum* and *Proceedings* contribute to the attainment of purpose #1. The winter, spring, summer, and fall volumes of the *Forum* focus on topics in data management, quality, and technology; ratemaking; reinsurance; enterprise risk management and dynamic financial analysis modeling; and reserves. The *Proceedings* include papers addressing topics on source of earning analysis, distribution-based pricing formulas, reserving methods, minimum bias procedure, insurance applications of bivariate distributions, as well as a discussion of a previous *Proceedings* paper on testing the reasonableness of loss reserves.

With the introduction of research working parties in 2003, CAS Research & Development (R&D) committees did much to contribute to purpose #1. These working parties function as call paper task forces, with specific charges to produce a

single research product over the course of a year. The working party concept was first introduced to CAS members at the 2003 Risk and Capital Management Seminar and Loss Reserve Seminar. Four working parties commenced projects this year.

CAS R&D and Professional Education committees also worked closely with other organizations to expand actuarial knowledge. The CAS teamed up with the Society of Actuaries (SOA) to conduct the first jointly sponsored CAS/SOA Enterprise Risk Management Seminar. CAS also partnered with Georgia State University to present the 2003 Bowles Symposium, an enthusiastically attended and much acclaimed scientific meeting.

In 2003 the CAS continued to use the Internet to foster research in pursuit of purpose #1. In January the CAS helped launch the Actuarial Research Exchange, an online service designed to link academic researchers with practicing actuaries for collaborative work on practical business problems. The Actuarial Research Exchange lists research opportunities posted by organizations and faculty members interested in conducting research. The Committee on Academic Relations, a joint committee of the Casualty Actuarial Society, Canadian Institute of Actuaries, and the SOA, established this service.

Another contribution to purpose #1 was adding an information specialist to the CAS staff. The CAS librarian and R&D committees developed and implemented a taxonomy for the online index of CAS literature. The taxonomy is a categorization schema for casualty actuarial science literature that will improve the ability to identify research articles using standardized terminology.

In regards to purpose #2, to establish and maintain standards of qualifications for membership, a number of developments occurred in the CAS education and examination system during 2003. CAS Admissions committees completed their work with the educational consulting firm The Chauncey Group. With the firm's guidance, the Syllabus Committee developed and published appropriate learning objectives for CAS examinations. The Chauncey Group also worked with the CAS Exam Committee to establish pass mark panels as a routine procedure of the exam process, to conduct item writer training courses for all exam writers, and to introduce open-book testing for parts of Exams 5 and 9.

CAS Admissions committees were also challenged with developing a new CAS Course 3 syllabus and examination, all within a year of the board's decision. Following its September 2002 board meeting, the CAS Board of Directors reviewed the recommendations of the Design Task Force on Exams 3 and 4 and elected to discontinue joint sponsorship of Exam 3 with the SOA. The new CAS Exam 3 syllabus reduces the coverage of life contingencies and its questions are geared toward casualty practice. The board decided to retain joint CAS/SOA sponsorship of Exam 4 because the exam continues to meet the needs of casualty actuaries. The CAS and SOA will also continue jointly sponsoring Exams 1 and 2. The new CAS Exam 3 was first administered for the Fall 2003 exam sitting.

Based on the final report of the Future Education Task Force, the board authorized the creation of four new task forces whose missions address standards of qualifications for membership: the Joint CAS/SOA Task Force on Preliminary Education, Computer-Based Testing Task Force, Task Force on Syllabus Materials, and the Modeling Workshop Task Force. The mission of the Joint CAS/SOA Task Force on Preliminary Education is to refine the learning objectives, producing a common set for a given subject where possible, and to further refine the guidelines for validation by educational experience where credit for specific topics can be obtained through university course or other educational experience rather than the traditional actuarial exam. The Computer-Based Testing Task Force is a joint effort between the CAS and SOA to implement computer-based testing for at least one exam by 2005. The Task Force on Syllabus Materials will evaluate whether study materials produced by outside organizations that purport to replace the study materials listed in the CAS Syllabus meet the preliminary education learning objectives. The Modeling Workshop Task Force will develop and pilot a workshop to determine if it is a practicable idea for future basic education, continuing education, or both.

Another notable contribution to purpose #2 was the approval of a constitutional amendment on mutual recognition. In September 2003, CAS Fellows voted overwhelmingly in support of the measure, with more than 72 percent of CAS Fellows voting to approve a constitutional amendment. The constitutional revision pertains to Article III, Section 2, entitled "Requirements for Admission to Membership." Mutual recognition agreements are reciprocal accords between two actuarial organizations whereby a member of either organization could become a member in the other, subject to the requirements in the agreement.

A quality program of continuing education opportunities and a Code of Professional Conduct promote and maintain high standards of conduct and competence for CAS members in accordance with purpose #3.

The CAS provides educational opportunities through the publication of actuarial materials and the sponsorship of meetings and seminars. This year's sessions included the following, shown with the number of CAS members in attendance:

	Location	CAS Members
Spring	San Diego, CA Boston MA	516 633
7 Annual	boston, MA	055

## **Meetings:**

Торіс	Location	CAS Members
Ratemaking	San Antonio, TX	307
Reinsurance	Philadelphia, PA	234
Risk and Capital Management	Washington, DC	165
Enterprise Risk Management	Washington, DC	192
Casualty Loss Reserves	Chicago, IL	428
Appointed Actuary—Joint CAS/CIA	Toronto, Canada	321*
Course on Professionalism—Dec '02	2 locations	75 Candidates
Course on Professionalism—June '03	2 locations	93 Candidates

### Seminars:

\*Total attendance. Separate count for CAS members is not available.

Limited attendance seminars included two sessions of "Practical Applications of Loss Distributions" and one session each of "Basic Dynamic Financial Analysis," "Advanced Dynamic Financial Analysis," and "Asset Liability Management and Principles of Finance."

In support of purpose #4, which is to increase the awareness of actuarial science, the CAS turned outward to focus on helping develop the actuarial profession worldwide. Within its own committee structure, the CAS established three new International committees to more effectively introduce the CAS as a resource in Europe, Latin America, and Asia. In 2003, CAS leaders and members were represented at meetings in India, Australia, Germany, the Philippines, Wales, and Brazil. The CAS was also honored to host a delegation of actuaries from Kazakhstan who had requested CAS assistance in educating future non-life actuaries in their emerging country. In addition, CAS has distributed copies of its textbook *Foundations of Casualty Actuarial Science* to newly formed actuarial associations around the world. The CAS Web Site has also become a tremendous resource to these associations.

The CAS Web Site is invaluable and unique as it supports all four purposes. One highlight from 2003 is the introduction of electronic elections. In 2003, Fellows were given the option of electronic or paper ballots and 853 Fellows registered to vote electronically. A total of 1,273 Fellows voted in the 2003 election, or 50.8 percent of those eligible. Fifty-five percent of the Fellows voting cast their ballots online.

The CAS also took advantage of the expediency of the Internet to post learning objectives for Fall 2003 Exams and to launch the online version of the *Index to the Literature of the Casualty Actuarial Society*. The online index is a centralized source to literature published in CAS publications and a reference to the contributions of CAS members. The online index is updated frequently and replaces the publication of the *Index*, which used to be printed every five years.

### OTHER CAS ACTIVITIES

Perhaps the Society's most sweeping action of the year 2003 was the adoption of the CAS Centennial Goal. The Centennial Goal represents the vision for the Casualty Actuarial Society in the year 2014, when the organization celebrates its 100th anniversary. The goal states,

The CAS will be globally recognized as the preeminent resource in educating casualty actuaries and conducting research in casualty actuarial science. CAS members will be recognized as the leading experts in the evaluation of hazard risk and the integration of hazard risk with strategic, financial, and operational risk.

The Centennial Goal Implementation Task Force is working with each CAS committee to facilitate the integration of significant, attainable, and measurable goals, or SAM Goals, to keep the CAS moving toward its Centennial Goal. Several other CAS activities contributed to the ongoing vitality of the organization during 2003. CAS Marketing & Communications Committees have done much to keep the Society focused on providing value to CAS members. The Committee on Volunteer Resources introduced online committee "want ads," solicited member participation to develop special interest seminars, and created the Member Advisory Panel, a focus grouptype body that will provide ongoing feedback to CAS leaders on behalf of CAS members.

In its ongoing mission to find ways to serve CAS members and improve the Society, the CAS Board conducted its five-year membership survey. The results of this survey will provide the CAS leadership with valuable input from its members, which will help to shape the short- and long-term direction of the Society. A report of the results will be available in 2004.

In addition, the board formed two new task forces. One will focus on the role of different classes of membership and the other will consider granting voting rights to Associates.

In 2003, the CAS Board focused on preparing for other future events. The board worked closely with the CAS Finance Committee to clearly define CAS surplus needs and the uses of surplus to meet emergencies and to help fund new initiatives for CAS members. The board also approved a disaster recovery plan for the CAS office.

#### MEMBERSHIP STATISTICS

Membership growth continued with 146 new Associates, 158 new Fellows, and 4 new Affiliates. The total number of members as of November 2003 was 3,847, up 3.7 percent from the previous year.

Stephen P. D'Arcy was elected president-elect for 2003–2004. CAS Fellows also elected Robert V. Deutsch, Sholom Feldblum, Andrew E. Kudera, and Robert F. Wolf to the CAS Board of Directors. Mary Frances Miller assumed the presidency.

The CAS Executive Council, with primary responsibility for day-to-day operations, met either by teleconference or in person at least once a month during the year. The CAS Board of Directors elected the following vice presidents for the coming year: Deborah M. Rosenberg, administration; Thomas G. Myers, admissions; John C. Narvell, international; Roger A. Schultz, marketing & communications; Christopher S. Carlson, professional education; and Donald F. Mango, research and development. Later in September, the board appointed Joanne S. Spalla to replace Schultz who retired as vice president–marketing & communications.

#### FINANCIAL STATUS

The CPA firm Langan Associates, PC, examined the CAS books for fiscal year 2003 and the CAS Audit Committee reported the firm's findings to the CAS Board of Directors in March 2004. The fiscal year ended with an audited Net Gain of \$17,039 compared to a budgeted Net Loss of \$238,140.

Members' equity now stands at \$3,075,800. This represents an increase in equity of \$378,402 over the amount reported last year. In addition to the net gain from operations, there was interest revenue of \$115,034 and an unrealized gain of \$197,490. There was also a total net increase of \$48,838 in various research, prize, and scholarship accounts arising from the difference between incoming funds and interest earned less expenditures, and a favorable adjustment to the CAS pension liability. These amounts are not reflected in net revenue from operations.

For 2003–2004, the CAS Board of Directors has approved a budget of approximately \$5.1 million, an increase of about \$600,000 compared to the prior fiscal year. Members' dues for next year will be \$350, an increase of \$20, while fees for the Subscriber Program will increase by \$20 to \$420. A \$35 discount is available to members and subscribers who elect to receive the *Forum* and *Discussion Paper Program* in electronic format from the CAS Web Site.

Respectfully submitted, Sheldon Rosenberg Vice President–Administration

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### FINANCIAL REPORT FISCAL YEAR ENDED 9/30/2003

#### OPERATING RESULTS BY FUNCTION

FUNCTION	REVENUE	EXPENSE	DI	FFERENCE
Membership Services	\$ 1,229,027	\$ 1,481,638	\$	(252,610)
Seminars	1,268,152	1,176,483		91,669
Meetings	749,870	715,846		34,024
Exams	3,364,905 (a)	3,225,397 (a)		139,508
Publications	52,843	48,395		4,447
TOTALS FROM OPERATIONS	\$ 6,664,798	\$ 6,647,759	\$	17,039
Interest Income				115,034
Unrealized Gain/(Loss) on Marketable	Securities			197,490
TOTAL NET INCOME (LOSS)			\$	329,564

NOTE: (a) Includes \$2,119,313 of Volunteer Services for income and expense (SFAS 116).

BALANCE SHEET

ASSETS	9/30/2002	9/30/2003	DIFFERENCE
Checking Accounts	\$ 151,821	\$ 869,659	\$ 717,838
T-Bills/Notes, Marketable Securities	3,523,655	3,423,050	(100,605)
Accrued Interest	28,458	19,327	(9,131)
Prepaid Expenses	63,034	65,094	2,060
Prepaid Insurance	23,715	29,550	5,835
Accounts Receivable	76,250	68,464	(7,786)
Intangible Pension Asset	12,721	10,019	(2,702)
Textbook Inventory	17,716	2,123	(15,593)
Computers, Furniture	394,247	436,216	41,969
Less: Accumulated Depreciation	(319,999)	(338,547)	(18,548)
TOTAL ASSETS	\$ 3,971,619	\$ 4,584,955	\$ 613,337
LIABILITIES	9/30/2002	9/30/2003	DIFFERENCE
Exam Fees Deferred	\$ 463,460	\$ 615,284	\$ 151,824
Annual Meeting Fees Deferred	149,168	169,695	20,527
Seminar Fees Deferred	50,625	3,000	(47,625)
Accounts Payable and Accrued Expenses	418,550	525,556	107,006
Accrued Pension	192,418	195,620	3,202
TOTAL LIABILITIES	\$ 1,274,221	\$ 1,509,155	\$ 234,934
MEMBERS' EQUITY			
Unrestricted	9/30/2002	9/30/2003	DIFFERENCE
CAS Surplus	\$ 2,524,858	\$ 2,854,421	\$ 329,564
Pension minimum liability (net of			
unamortized service cost of \$10,019)	(124,651)	(90,572)	34,079
Michelbacher Fund	122,057	126,329	4,272
CAS Trust-Operating Fund	85,620	98,777	13,157
Research Fund	44,418	43,668	(750)
Subtotal Unrestricted	\$ 2,652,302	\$ 3,032,623	\$ 380,322
Temporarily Restricted	9/30/2002	9/30/2003	DIFFERENCE
Scholarship Fund	\$ 6,297	\$ 6,018	\$ (280)
Rodermund Fund	8,799	8,107	(692)
CAS Trust-Ronald Ferguson Fund	30,000	29,052	(948)
Subtotal Temporarily Restricted	45,096	43,177	(1,920)
TOTAL MEMBERS' EQUITY	\$ 2,697,398	\$ 3,075,800	\$ 378,402

Sheldon Rosenberg, Vice President–Administration This is to certify that the assets and accounts shown in the above financial statement have been audited and found to be correct. CAS Audit Committee: Phillip N. Ben-Zvi, Chairperson; John F. Gibson, Frederick O. Kist, and Patricia A. Teufel

# 2003 EXAMINATIONS—SUCCESSFUL CANDIDATES

Examinations for Exams 5, 7-Canada, 7-United States, and 8 of the Casualty Actuarial Society were held March 6-8, 2003. Examinations for Exams 3, 6, and 9 of the Casualty Actuarial Society were held October 28-30, 2003.

Examinations for Exams 1, 2, and 4 are jointly sponsored by the Casualty Actuarial Society and the Society of Actuaries and were held in April and September 2002. Candidates successful on these examinations were listed in joint releases of the two Societies. (Exam 3 was originally jointly sponsored with the SOA. The exam was offered jointly for the Spring 2003 sitting and CAS began offering CAS Exam 3 in the Fall 2003 sitting.)

The following candidates were admitted as Fellows and Associates at the 2003 CAS Spring Meeting in May. By passing Fall 2002 CAS examinations, these candidates successfully fulfilled the Society requirements for Fellowship or Associateship designation.

### NEW FELLOWS

Robert D. Bachler	Patrick P. Gallagher	Lester Pun
Andrew W. Bernstein	Matthew R. Gorrell	Jeremy D. Shoemaker
Raju Bohra	Christopher J. Grasso	Anthony A. Solak
Jennifer A. Charlonne	Serhat Guven	Michael William
Robert E. Davis	Ung Min Kim	Starke
Michael Devine	Laurie A. Knoke	Matthew D. Trone
Kyle A. Falconbury	Jonathan D. Levy	Brian A. Viscusi
John D. Ferraro	Matthew Kevin Moran	Bethany R. Webb
Kevin M. Finn	Matthew P. Nimchek	Patti West
William M. Finn	Michael Robert	Yingjie Zhang
Greg Frankowiak	Petrarca	

## NEW ASSOCIATES

Danielle L.	David S. Futterleib	Kee Heng Ng
Bartosiewicz	Joel D. Glockler	Wade H. Oshiro
Lisa K. Buege	Melanie T. Goodman	Robert A. Painter
Suejeudi Buehler	Ann E. Green	Christopher A. Pett
James Chang	Jonathan M. Guy	Monica L. Ransom
Christian J. Coleianne	Joseph Hebert	Joseph L. Rizzo
Matthew P. Collins	Scott R. Jean	Julie Clarisse Russell
David C. Coplan	William Brian Johnson	Paul Silberbush
Keith R. Cummings	Kyewook Gary Kang	Douglas E. Smith
Nicholas J. De Palma	John J. Karwath	Beth M. Sweeney
Timothy M. Devine	Jonathan David Koch	Joseph S. Tripodi
Robert C. Fox	Damon T. Lay	Natalie Vishnevsky
Jeffrey J. Fratantaro	Christopher A. Najim	Nicholas J. Williamson

The following candidates successfully completed the following Spring 2003 CAS examinations.

# Exam 5

Christina Abbott	Christopher S.	David Alan Clark
Yazeed F. Abu-Sa'a	Bramstedt	Elizabeth Jill Clark
Brian C. Alvers	Michele L. Brooks	Eric Clark
Rebecca J. Armon	Matthew Buchalter	Chad J. Covelli
Kelleen D. Arquette	Morgan Haire Bugbee	Matthew K. Cremeens
Damian T. Bailey	Robert L. Bush	Tighe C. Crovetti
Michael A. Bean	Rita Bustamante	Justin B. Cruz
Mark Belasco	Heather R. Caffoe	Paul T. Cucchiara
Matthew C. Berasi	Jennifer L. Carrick	David J. Curtis
Derek Dennis Berget	Yung-Chih Chen	Jeannine Marie Danner
Brian J. Biggs	Denise L. Cheung	Keri P. Davenport
Rebekah Susan Biondo	Vivien K. Chiang	Scott C. Davidson
Michael J. Blasko	Ting Him Choi	Jesse W. Decouto
Guillaume Boily	Charles A. Cicci	Melodee S. Dixon
Tapio N. Boles	Lora Massino Ciferri	Brent P. Donaldson

Francis J. Dooley Tomer Eilam Yehoshua Y. Engelsohn Michael D. Ersevim Joyce A. Ewing Horng-Jiun K. Fann Bruce Fatz Joshua D Feldman Matthew B Feldman Alicia K. Ferraro Joshua L. Fishman Beth A. Foremsky Robert J. Foskey Vincent M Franz Katherine M Funk Joseph A. Gage Doreen Gatti Steve G. Gentle Alexander R. George Karen W. Gibbons Kristen Marie Gill Simon Girard Alla Golonesky David B. Gordon Jeffrey Robert Grimmer Isabelle Guerin Manuel S. Guerra Jr. Michael B Gunn John J. Hageman **Bobby Earl Hancock** Eric A. Hatch Kalynn D. Haubert Robin A. Haworth Eric M. Herman Joseph H. Hohman

Kimberly A. Holmes Christopher M. Holt Hugh D. Hopper Gerald K. Howard Bo Huang Queenie W. C. Huang Yehuda S. Isenberg William T Jarman Min Jiang Julie M. Joyce Brian M. Karl Sarah M. Kemp Eric J. Kendig Lisa M Kerns Roman Kimelfeld Clarence J. Kimm Jr. Scott M. Klabacha Stephen Jacob Koca Christine K. Kogut Wen Kong Jeff A. Lamy Kak Lau Henry T. Lee Jeremy M. Lehmann Yuxiang Lei James J Leonard Sean M. Leonard Kahshin Leow Ho Shan A. Leung Adrienne J. Lewis Mingyue Li Andy Hankuang Liao Yu Te Lin Edward P. Lionberger Keyang Luo Yong Feng Ma

Thomas J. Macintyre Laura S Martin Jonathan L. Matthews Rebecca R. McCarrier Brent L. McGill Christopher C. McKenna Michael E McKeon Anne A. McNair Jeffrey S. McSweeney Todd C. Meier Svlvie Menard Isaac Merchant Jr. Thomas E. Meyer Allison L. Morabito Maria M. Morrill Randy J. Murray Heather M. Nass Andre K. Nguyen Leonidas V. Nguyen Linda C Nichols Rosalie Nolet Daniela Nunnery **Timothy James** O'Connor Melanie Ostiguy Hee Kyeong Park Eva M. Paxhia Samuel Robert Peters Michael J. Peterson Joseph G. Pietraszewski Jean-Philippe Plante Feliks Podgaits Damon Joshua Raben Conni A Rader

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David P. Rafferty Nicholas J Reed Gregory S. Richardson Dale M. Riemer Susan Roberts Beth A Robison Ann-Marie Roche Mindy M. Romeo Jennifer Hany Rose Randall D. Ross Stuart C. Rowe Anthony D. Salido Derek Michael Schaff Richard T Schneider Monica S Schroeter Erika H. Schurr Daniel Owen Schwanke Genine Darrough Schwartz David J. Shaloiko Tony B. Shi Sarah J. Shine Scott D. Skansberg

### Exam 7-Canada

Patrick Beaulieu Christine Cadieux Kevin K. W. Chan Yves Charbonneau Peter J. Cooper Gregory Evan Gilbert John S. Giles Isabelle Gingras Simon Guenette Thomas R. Slader Justin N Smith Lleweilun Smith Joanna Solarz Sandra E. Starnes Esperanza Stephens Michael Daniel Stephens Zongli Sun Luc Tanguay Dawn M. Thayer Robert Bradley Tiger Malgorzata Timberg Daniel Tinoco Karen J Triebe Melissa K. Trost Alexander S. Vajda Allan Voltz Todd Patrick Walker Mo Wang Qingxian Wang Yingnian Wang Amanda J White Andrew T Wiest

Donya C. Wilson Ian G. Winograd Benjamin T. Witkowski Aaron A. Wright Lihua Wu Jie Xiao Xinxin Xu Min Yao Yuanhe Yao Andrew Yershov Ka Chun Yeung Keith Young Jiwei Yu Ronald J Zaleski Jr Navid Zarinejad Hui Yu Zhang Lijuan Zhang Haixia Zhao Gang Zhou Hongbo Zhou Yuling Zhou

Faisal O. Hamid Ryan Yin-kei Ho Luke G. C. Johnston Malika El Kacemi-Grande Anne Marie Klein-Lee Josée Lambert Anh Tu Le Isabelle Lemay

Jason K. Machtinger Camilo Mohipp Liana St-Laurent Jean-François Tremblay Mary Vacirca Hanny C. Wai

# Exam 7-United States

Karen H. Adams Keith P Allen Fernando Alberto Alvarado Jonathan L. Ankney Melissa J. Appenzeller Brian D. Archdeacon Satya M. Arya Kris Bagchi Jack Barnett Jennifer Lvnn Basanese Thomas C. Bates Richard J Bell III Corey J. Bilot Frank J. Bilotti Chris M Bilski Stacey Jo Bitler Kirk D. Bitu Thomas S. Botsko Scott T. Bruns Amber L. Butek Matthew E Butler Caryn C. Carmean Hung Francis Cheung Wai Yip Chow Philip A. Clancey Jr. Jason T. Clarke Carolyn J. Coe Thomas Marie Cordier Richard R. Crabb Richard S. Crandall Keith W. Curley Kelly K. Cusick

Willie L. Davis Jr. Amv L. DeHart Melanie Sue Dihora Stephen E. Dupon Ramakrishna Duvvuri Jeffrev A. Dvinoff Jessica L. Elsinger Donna L. Emmerling Juan Espadas Peter L. Forester Susan J. Forrav Jonathan W Fox Robert W. Geist David A. Gelberg William J. Gerhardt Natasha C. Gonzalez Lori A Gordon Jennifer Graunas Michael D. Green Stacie R. W. Grindstaff James Christopher Guszcza Brian O Haaseth Richard J. Haines Kimberly Baker Hand Trevor C. Handley David Lee Handschke Jason C. Harland Robert D. Harrington Hans Heldner Kathryn E. Herzog Joseph S. Highbarger Chun Hua Hoo Eric David Huls

Scott R. Hurt Farid Aziz Ibrahim Victoria K. Imperato Ali Ishaq Anita J. Johnson Robert B. Katzman John B. Kelly Young Y. Kim Raymond J. Kluesner Steven T. Knight John E. Kollar James J. Konstanty Bradley S. Kove Douglas H. Lacoss Hooi Lee Lai ZhenZhen Lai Hoi Keung Law Glen Alan Leibowitz Hayden Anthony Lewis Shangjing Li Gavin X. Lienemann Eric F Liland Hazel Joynson Luckey Lynn C. Malloney Steven Manilov Laura S. Martin Jason N. Masch Robert B. McCleish IV Laurence R McClure II James P. McCov Kirk Francis Menanson Jennifer Middough

Michael E. Mielzynski Stephanie A. Miller Meagan S. Mirkovich Timothy C. Mosler James C. Murphy Jr. Jacqueline Lee Neal Richard U. Newell Jason M Nonis Darci Z. Noonan Melissa A. Ogden James D. O'Malley Jeremy Parker Pecora Tracie L. Pencak Michael J Perrone Robert Anthony Peterson Timothy K. Pollis David N Prario Julie-Ann Puzzo Michele S. Raeihle Lynellen M. Ramirez William C. Reddington III Stuart C Rowe

### Exam 8

Jeffrey R. Adcock Denise M. Ambrogio Kevin L. Anderson Deborah Herman Ardern Afrouz Assadian Kevin J. Atinsky Silvia J. Bach Stevan S. Baloski

Michael R. Rozema Robert J Schutte Vladimir Shander Jin Shao Peter M. Shelley Frank W. Shermoen Janel M. Sinacori Eric K Slavich James M Smieszkal Christa Liane Sorola Michael D. Sowka Laura T. Sprouse Christine Seung Steer Erik J. Steuernagel Mark Sturm Jeffrey L. Subeck Ju-Young Suh Keith Jeremy Sunvold Ming Tang Jonas F. Thisner Patrick Thorpe Dovid C. Tkatch David A. Traugott Lien K Tu-Chalmers

Rick C.H. Tzeng Susan B Van Horn Kevin K Vesel Jeffrey J. Voss Gary C. Wang Kevin E. Weathers Shannon A. Whalen Christopher M. White Matthew M White Arthur S. Whitson Paul D. Wilbert Jill C. Willie Dana L. Winkler Micah Grant Woolstenhulme Rick A. Workman Joshua C. Worsham Andrew F. Yashar Jonathan K. Yu Robert J Zehr Yi Zhang Steven Bradley Zielke

Patrick Barbeau Jack Barnett Rick D. Beam Jonathan P. Berenbom Jason E. Berkey Linda Jean Bjork Tony Francis Bloemer Nathan L. Bluhm Nebojsa Bojer Erich A. Brandt Claude B. Bunick Angela D. Burgess Michelle L. Busch Tara E. Bush James E. Calton Kristi Irene Carpine-Taber William Brent Carr

Jennifer L. Caulder Thomas L. Cawley Hao Chai Phyllis B. Chan James Chang Nathalie Charbonneau Thomas Joseph Chisholm Julia Feng-Ming Chu Matthew P. Collins Cameron A Cook **Richard Jason Cook** Leanne M. Cornell Jose R. Couret Aaron T. Cushing David W. Dahlen Paul B. Deemer David A. DeNicola Patricia A. Deo-Campo Vuong Nicholas J De Palma Mark Richard Desrochers Christopher P. DiMartino Rvan M. Diehl Brian M Donlan Kiera Elizabeth Doster Scott H Drab Gregory L. Dunn Ruchira Dutta Ramakrishna Duvvuri Robin V. Fitzgerald William J. Fogarty Dana R. Frantz

Marie LeStourgeon Fredericks David S Futterleib Keith R. Gentile Isabelle Girard Joel D. Glockler Andrew Samuel Golfin Jr Christopher David Goodwin Ann E Green Paul E. Green Jr. Donald B. Grimm Robert A. Grocock Jacqueline Lewis Gronski Guo Harrison James D. Heidt Allen J. Hope David J Horn Jr Eric J Hornick Christopher Wayne Hurst Tina Tuyet Huynh Philip M. Imm Jesse T Jacobs Julie A Jordan Daniel R. Kamen Erin Hye-Sook Kang John J. Karwath Douglas H. Kemppainen Stacey M. Kidd Jeff A. Kluck Jonathan David Koch

Henry Joseph Konstanty Brandon E Kubitz Charles B. Kullmann Kristine Kuzora James A. Landgrebe Thomas P. Langer Jason A Lauterbach Stephen E. Lehecka Amanda M. Levinson Jenn Y Lian Jia Liu Jing Liu Andrew M. Lloyd Stephen P. Marsden Anthony G. Martella Jr. Shawn Allan McKenzie Martin Menard Rvan A. Michel Matthew E. Morin Ethan Charles Mowry Kyle S. Mrotek Sureena Binte Mustafa Karen E. Myers Kee Heng Ng Lester M. Y. Ng Khanh K. Nguyen Michael Douglas Nielsen John E. Noble Tom E. Norwood Gerard J. Palisi Michael Thomas Patterson

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Sylvain Perrier	Jeremy N. Scharnick	Peggy J. Urness
Isabelle Perron	Thomas Schneider	Jennifer L. Vadney
Kevin Thomas	Larry J. Seymour	Richard Alan Van
Peterson	Quan Shen	Dyke
Christopher A. Pett	Michelle L. Sheppard	Paul A. Vendetti
Faith M. Pipitone	Paul Silberbush	Marie-Eve J. Vesel
Jayne L. Plunkett	Summer L. Sipes	Keith A. Walsh
Mitchell S. Pollack	Douglas E. Smith	Karen E. Watson
Stephen R. Prevatt	Lleweilun Smith	Robert S. Weishaar
Warren T. Printz	Scott G. Sobel	Joseph C. Wenc
William Dwayne	Sharon L. Sowka	Mark Steven Wenger
Rader Jr.	Karine St-Onge	Nicholas J. Williamson
John T. Raeihle	Mark Richard Strona	Linda Yang
Laura D. Rinker	Christopher J. Styrsky	Carolyn D. Yau
Michelle L.	Wei Hua Su	Stephanie C. Young
Rockafellow	Lisa Liqin Sun	Xiangfei Zeng
Robert C. Roddy	Stephen James Talley	Larry Xu Zhang
Scott I. Rosenthal	Son T. Tu	Lianmin Zhou
Brian P. Rucci	Howard Raymond	Eric Zlochevsky
Laura Beth Sachs	Underwood	

The following candidates were admitted as Fellows and Associates at the 2003 CAS Annual Meeting in November. By passing Spring 2003 CAS examinations, these candidates successfully fulfilled the Society requirements for Fellowship or Associateship designation.

### NEW FELLOWS

#### 2003 EXAMINATIONS—SUCCESSFUL CANDIDATES

Tara E. Bush James E Calton Kristi Irene Carpine-Taber William Brent Carr Jennifer L. Caulder Hao Chai Nathalie Charbonneau Thomas Joseph Chisholm Wai Yip Chow Cameron A. Cook Richard Jason Cook Aaron T. Cushing David W. Dahlen Patricia A. Deo-Campo Vuong Mark Richard Desrochers Christopher P. DiMartino Scott H. Drab Gregory L. Dunn Ruchira Dutta Ramakrishna Duvvuri Dana R Frantz **Isabelle** Gingras Isabelle Girard Andrew Samuel Golfin Natasha C. Gonzalez Christopher David Goodwin Donald B. Grimm Robert A. Grocock Jacqueline Lewis Gronski

James Christopher Guszcza David Lee Handschke Guo Harrison Hans Heldner Allen J. Hope Eric J. Hornick Christopher Wayne Hurst Tina Tuyet Huynh Philip M. Imm Ali Ishaq Julie A. Jordan Daniel R Kamen Erin Hye-Sook Kang John J. Karwath Douglas H. Kemppainen Stacey M. Kidd Jeff A Kluck Henry Joseph Konstanty Charles B. Kullmann Thomas P. Langer Jason A. Lauterbach Stephen E. Lehecka Shangjing Li Jenn Y. Lian Jing Liu Andrew M. Lloyd Jason K. Machtinger Stephen P. Marsden Anthony G. Martella Michael E. Mielzynski Ethan Charles Mowry Karen E. Myers

Kee Heng Ng Khanh K. Nguyen Michael Douglas Nielsen John E Noble Jason M Nonis Darci Z. Noonan Gerard J Palisi Michael Thomas Patterson Tracie L. Pencak Svlvain Perrier Isabelle Perron Kevin Thomas Peterson Mitchell S. Pollack Warren T. Printz William Dwayne Rader Michelle L Rockafellow Robert C. Roddy Scott I Rosenthal Michael R Rozema Brian P. Rucci Laura Beth Sachs Jeremy N. Scharnick Thomas Schneider Larry J. Seymour Vladimir Shander Michelle L. Sheppard Douglas E. Smith Scott G. Sobel Sharon L. Sowka Karine St-Onge Mark Richard Strona

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# Wei Hua Su Lisa Liqin Sun

Stephen James Talley Rick C. H. Tzeng Jennifer L. Vadney Richard Alan Van Dyke Paul A. Vendetti Marie-Eve J. Vesel Karen E. Watson Robert S. Weishaar Joseph C. Wenc Linda Yang Stephanie C. Young Xiangfei Zeng Larry Xu Zhang Lianmin Zhou

#### NEW ASSOCIATES

Fernando Alberto Alvarado Brian C. Alvers Jonathan L. Ankney Melissa J. Appenzeller Nicki C Austin Thomas C. Bates Patrick Beaulieu Chris M Bilski Kirk D. Bitu Matthew E Butler Christine Cadieux Kevin K. W. Chan Yves Charbonneau Hung Francis Cheung Philip A. Clancey Jason T. Clarke Thomas Marie Cordier Richard R. Crabb Richard S. Crandall Keith W. Curley Kelly K. Cusick Willie L. Davis Amy L. DeHart Jeffrey A. Dvinoff Tomer Eilam Jessica L. Elsinger Matthew B. Feldman

Peter L. Forester Susan J. Forrav Robert W. Geist David A. Gelberg William J. Gerhardt Gregory Evan Gilbert John S. Giles William G Golush Lori A Gordon Jennifer Graunas Stacie R W Grindstaff Simon Guenette Brian O. Haaseth Faisal O. Hamid Kimberly Baker Hand Trevor C. Handley Jason C. Harland Robert D. Harrington Eric A. Hatch Kathryn E. Herzog Joseph S. Highbarger Rvan Yin-kei Ho Scott R. Hurt Victoria K. Imperato Young Y. Kim Anne Marie Klein-Lee Raymond J. Kluesner Steven T. Knight

James J. Konstanty Bradley S. Kove Hooi Lee Lai ZhenZhen Lai Heather D Lake Anh Tu Le James J. Leonard Hayden Anthony Lewis Gavin X. Lienemann Hazel Joynson Luckey John T Maher Steven Manilov Robert B McCleish Laurence R. McClure James P. McCoy Camilo Mohipp Timothy C. Mosler James C. Murphy Jacqueline Lee Neal Richard U. Newell Melissa A. Ogden Eva M. Paxhia Michele S. Raeihle William C. Reddington Stuart C. Rowe Jin Shao Peter M. Shelley

Janel M. Sinacori	Jonas F. Thisner	Hanny C. Wai
James M. Smieszkal	Malgorzata Timberg	Gary C. Wang
Michael D. Sowka	Dovid C. Tkatch	Paul D. Wilbert
Laura T. Sprouse	Dominic A. Tocci	Micah Grant
Christine Seung Steer	David A. Traugott	Woolstenhulme
Esperanza Stephens	Jean-François	Andrew F. Yashar
Erik J. Steuernagel	Tremblay	Jonathan K. Yu
Liana St-Laurent	Lien K. Tu-Chalmers	Yi Zhang
Ju-Young Suh	Susan B. Van Horn	-
Keith Jeremy Sunvold	Kevin K. Vesel	

The following candidates successfully completed the following Fall 2003 CAS examinations.

# Exam 3

Amit Agarwal	Alyce M. Chow	Joseph Gravelle
Marcus Ryan Aikin	Shawn T. Chrisman	Renbin Guo
Jasmin Alibalic	William R. Copeland	Gerald S. Haase
Robert B. Anderson	Scott A. Cosme	Megan A. Hall
David Michael Andrist	Jason J. Culp	Joseph Patrick Hasday
Yanfei Atwell	Paul B. Deemer	Chris E. Holcomb
Jonathan D. Balenzano	Anthony S. Diaz	Kimberly A. Holmes
Sarah Bhanji	Mario E. Dicaro	Jimmy Houng
Peter George Blouin	Yiping Dou	Alison Therese Hover
Ishmealin Boye	Jennifer L. Edwards	Andrew Steven
Morgan Haire Bugbee	Valerie Emond	Howard
Ali Ahmed Bukhari	Eveline Falardeau	Zhigang Huang
John L. Butel	Christoph G. Fanslau	John B. Ittner
Boni Caldeira	Christine M. Fleming	Jaehoon Jho
Robert W. Campos	Jonathan W. Fox	Litha A. John-Rose
Thomas R. Carroll	Marie LeStourgeon	Katherine Yukyue Kam
Mark S. Catron	Fredericks	Raisa Katz
Ming Chan	Chong Gao	Kayne M. Kirby
Shin-Jin Chen	Xavier Genevois	Thomas R Kolde
Jou-wen Chou	Evan W. Glisson	Corey Lang

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Micah Lenderman Ronald S. Lettofsky Yongxing Li Reed M. Loeschke Carissa T. Lorie Amanda Lubking Jonathan T. Marshall Laura A. Masi Krystal A. Mathewson Paul H. Mayfield Devyn K. McClure Charles A. Metzger Allison L. Morabito Claude Nadeau Erin M. Olson

### Exam 6

Christina Abbott Karen H Adams Keith P. Allen Rebecca J. Armon Daryl S. Atkinson Robert Joseph Azari Kris Bagchi Damian T. Bailey Angelo Bastianpillai Michael A. Bean Derek Dennis Berget Lisa M Berke Sonal Bhargava Brian J. Biggs Michael J Blasko Jon Paul Bloom Guillaume Boilv Nicolas Boivin Tapio N. Boles

Maja Osmanovic Rajesh Jairaj Pachai Bradley J. Parker Samuel Robert Peters Robert V. Phipps Joseph G. Pietraszewski Timothy R. Porter Andrew D. Reid Dolph J. Robb Anthony D. Salido Kathleen M. Schmidt Linda Sew Qi Shen David Y. Shleifer

Timothy D. Boles Steven G. Brenk Michele L. Brooks Matthew Buchalter Randall T Buda Melissa Lillian Bundt Scott G. Burke Douglas J. Busta Rita Bustamante Li Cao Scott W. Carpinteri Jennifer L Carrick Patrick J. Causgrove Zhijian Chen Chun Kit Cheung Denise L. Cheung Tsui-Hsien Joanna Chien Ting Him Choi

Scott D. Skansberg Christopher Spratt Stephen R. Sten Jessica R. Sweets Lori R. Thompson Daniel Verinder Allan Voltz Paul K. Wai Yaming Wang Xiaohui Wu Navid Zarinejad Wei Zhao Chunhua Zhi Shan Zhuge

Kevin J. Christy Charles A. Cicci Elizabeth Jill Clark Eric Clark Peter S Clarke Kirk Allen Conrad Kevin Conway Sean T. Corbett Stephen M. Couzens Chad J. Covelli Lawrence G Cranor Justin B Cruz David J. Curtis Jeannine Marie Danner Marc-Andre Desrosiers Melanie Sue Dihora Charles W Dorman Stephen E. Dupon Ponniah Elancheran

Yehoshua Y. Engelsohn Choya A. Everett Joyce A. Ewing Bruce Fatz Suzanne M. Finnegan Jason A. Flick Robert J. Foskev Mathieu Francoeur Derek W Freihaut Justin Fritz Luvang Fu Katherine M. Funk Anna Garcia Tim Garcia Maxime Gelinas Alexander R. George Kristen Marie Gill Lilian Y Giraldo Simon Girard Francois Godbout Mathieu Gravel Joshua R. Griffin Isabelle Guerin Manuel S Guerra Jr Liang Guo Kyle M. Hales Bobby Earl Hancock Shrinivas Havaldar Robin A Haworth Arie Haziza Donald F. Hendriks Bo Huang Sherry Huang Eric David Huls Yu Shan Hwang

Farid Aziz Ibrahim Chris D Izbicki Joseph M. Izzo William T. Jarman Richard C. Jenkins Alison Susanne Jennings Min Jiang Megan S. Johnson Luke G. C. Johnston Jason C. Jones Julie M. Joyce Amy Ann Juknelis Kenneth Robert Kahn Jr Jean-Philippe Keable Samir Khare Steve C. Klingemann Perry A. Klingman Stephen Jacob Koca Christine K. Kogut John E. Kollar Wen Kong Vladimir A Kremerman Ignace Y. Kuchazik Lok-Yi R Kwok Mai B. Lam Kak Lau Sik-Yu Lau Lesley-Anne Lawrence Henry T. Lee Kevin T. Lee Yuxiang Lei Twiggy Lemercier

Sean M. Leonard Kahshin Leow Jean-François Lessard Eric F. Liland Herman Lim Chiourav Lin Cunbo Liu Dong Liu Jin Liu Yong Feng Ma Lynn C. Malloney Amanda Cater Marsh Laura S. Martin Jonathan L. Matthews Sean M McAllister Brent L. McGill Todd C. Meier Meagan S. Mirkovich Maria M. Morrill Marc L. Nerenberg Andre K. Nguyen Leonidas V. Nguyen Tang-Tri Nguyen Liam F. O'Connor **Timothy James** O'Connor Helen S Oliveto Wayne A. Olivier Alejandro A. Ortega Russel W. Oslund Melanie Ostiguy Brent J. Otto Alan M. Pakula Hee Kyeong Park Lorie A. Pate

Felix Patry Joy-Ann C. Payne Robert Anthony Peterson Feliks Podgaits Damon Joshua Raben Arthur R. Randolph II Eric W L Ratti Nicholas J. Reed Jiandong Ren Peggy-Anne K. Repella Raul J. Retian Beth A Robison Keith A. Rogers Nicholas W. Saeger Robert J. Schutte Genine Darrough Schwartz Ronald S Scott Steven R. Shallcross Ouan Shen Zilan Shen Yiping Shi Sarah J. Shine Rene R Simon

### Exam 9

Fernando Alberto Alvarado Afrouz Assadian Kevin J. Atinsky Danielle L. Bartosiewicz Nicolas Marc Beaudoin

Annemarie Sinclair Heidi L. Sjoberg Justin N Smith Lleweilun Smith Patrick Shiu-Fai So Joanna Solarz Sheila R. Soulsby Michael Daniel Stephens Natalie St-Jean Mark Sturm Thomas J. Stypla Xue Su Luc Tanguay Aaron A. Temples Dawn M. Thayer Patrick Thorpe Melissa K. Trost Benjamin Joel Turner Sebastien Y. Vignola Todd Patrick Walker Kaicheng Wang Mo Wang Yingnian Wang Yongqiang Wang Timothy G. Wheeler

Matthew C. Berasi Rebekah Susan Biondo Thomas S. Botsko Elaine K. Brunner Stephanie Anne Bruno John Celidonio Phyllis B. Chan Tracy L. Child

Amanda J. White Christopher M. White Benjamin T. Witkowski Shing-Ming Wong Dorothy A. Woodrum Shawn A. Wright Jie Xiao Zhicheng Xin Xinxin Xu Min Yao Yaniun Yao Eecher Yee Shuk-Han Lisa Yeung Joshua A Youdovin Jiwei Yu Anna W. Yum Arvelle D Zacharias Ronald J. Zaleski Jr. Hui Yu Zhang Juemin Zhang Lijuan Zhang Gang Zhou Michael V Ziniti

Brian Kenneth Ciferri David Alan Clark Christian J. Coleianne Sean O. Cooper Richard R. Crabb Laura S. Doherty Brian S. Donovan Kiera Elizabeth Doster

Stephen E. Dupon Jessica L. Elsinger Juan Espadas Gina C. Ferst Robin V. Fitzgerald Robin A. Fleming Sean Paul Forbes Susan J. Forray Sebastien Fortin Matthew Timm Frank David S. Futterleib David A. Gelberg Keith R. Gentile Joel D. Glockler Olga Golod David B. Gordon Ann E. Green Veronique Grenon Jonathan M. Guy Robert D. Harrington Brandon L. Heutmaker David J. Horn Jr. Jesse T. Jacobs Gregory O. Jaynes Scott R. Jean Shiwen Jiang Yi Jing Ge Jennifer Kang Susan M. Keaveny Ziv Kimmel Scott M. Klabacha Raymond J. Kluesner Jonathan David Koch Terry T. Kuruvilla Kristine Kuzora Hooi Lee Lai

ZhenZhen Lai James A. Landgrebe John B Landkamer Annie Latouche Hoi Keung Law Anh Tu Le Jeffrey Leeds Yuxiang Lei Marc E. Levine Andy Hankuang Liao Jia Liu Nannan Liu Luis S. Marques Jason N. Masch John R. McCollough Jeffrey B. McDonald Martin Menard Rvan A. Michel Matthew E. Morin Timothy C. Mosler Kyle S. Mrotek James C. Murphy Jr. Lester M. Y. Ng Tom E Norwood William S. Ober Faith M. Pipitone Jean-Philippe Plante Jayne L. Plunkett Gregory T. Preble John T. Raeihle Gregory S. Richardson Laura D. Rinker Stuart C. Rowe William P. Rudolph Bryant Edward Russell Frederick Douglas Ryan

Erika H. Schurr Mandy M. Y. Seto Paul Silberbush Thomas M. Smith Michael D. Sowka William G. Stanfield Christopher J. Styrsky Zongli Sun Shantelle Adrienne Thomas Dovid C. Tkatch Nathalie Tremblay Matthew L. Uhoda Dennis R Unver Peggy J. Urness Daniel J. VanderPloeg Gaetan R. Veilleux Natalie Vishnevsky Jeffrey J. Voss Keith A Walsh Matthew J Walter Matthew J. Wasta Kevin E Weathers Thomas E Weist Ann Min-Sze Wong Micah Grant Woolstenhulme Jimmy L. Wright Carolyn D. Yau Andrew Yershov Bradley J. Zarn Ruth Zea Yi Zhang Haixia Zhao Hongbo Zhou Eric Zlochevsky



First row, from left: Ung Min "Robert" Kim, Patti West, Robert E. Davis, CAS President Gail M. Ross, Bethany R. Webb, Jereny D. Shoemaker, William M. Finn. Bernstein. Third row, from left: Kevin M. Finn, Matthew D. Trone, Matthew Kevin Moran, Kyle A. Falconbury, Jonathan D. Levy, Greg Frankowiak. New Fellows not pictured: Robert D. Bachler, Jennifer A. Charlonne, Michael Devine, Patrick P. Gallagher, Matthew R. Gorrell, Christopher J. Grasso, Michael Robert Petrarca, Second row, from left: Michael William Starke, Brian A. Viscusi, Matthew P. Nimchek, Raju Bohra, John D. Ferraro, Laurie A. Knoke, Serhat Guven, Andrew W. Lester Pun, Anthony A. Solak, Yingjie Zhang.



First row, from left: Melanie T. Goodman, Robert C. Fox, Joseph Hebert, CAS President Gail M. Ross, Matthew P. Collins, Christopher A. Pett, Suejeudi Buehler. Second row, from left: Joseph S. Tripodi, Scott R. Jean, Nicholas J. De Palma, Beth M. Sweeney, Natalie Vishnevsky, Keith R. Cummings. Third row, from left: Paul Siberbush, Joel D. Glockler, Nicholas J. Williamson, Jonathan M. Guy, John J. Karwath, William Brian Johnson.



First row, from left: David S. Futterleib, Jonathan David Koch, Robert A. Painter, CAS President Gail M. Ross, Joseph L. Rizzo, Lisa K. Buege, Christopher A. Najim. Second row, from left: David C. Coplan, Ann E. Green, Monica L. Ransom, Christian J. Coleianne, Danielle L. Bartosiewicz, Douglas E. Smith. Third row, from left: James Chang, Wade H. Oshiro, Damon T. Lay. New Associates not pictured: Timothy M. Devine, Jeffrey J. Fratantaro, Kyewook Gary Kang, Kee Heng Ng, Julie Clarisse Russell.



David W. Dahlen, Xiangfei Zeng, Stevan S. Baloski. Second row, from left: Christopher Wayne Hurst, Michelle L. Busch, Scott H. Drab, Mark Richard Desrochers, First row, from left: Thomas Joseph Chisholm, Nathalie Charbonneau, Gregory L. Dunn, Dana R. Frantz, CAS President Gail M. Ross, Julie A. Jordan, Ali Ishaq, Ruchira Dutta, Darci Z. Noonan, Erich A. Brandt, Rick D. Beam, Douglas H. Kemppainen. Third row, from left: Nebojsa Bojer, Jacqueline Lewis Gronski, Denise M. Ambrogio, Ramakrishna Duvvuri, Jonathan P. Berenbom, Nathan L. Bluhm, Tony Francis Bloemer, Angela D. Burgess



Kevin L. Anderson. Second row, from left: Stacey M. Kidd, Andrew Samuel Golfin, Claude B. Bunick, Aaron T. Cushing, Michelle L. Rockafellow, Stephen P. Marsden, Michael R. Rozema, Jeffery R. Adcock, Karen E. Myers. Third row, from left: Allen J. Hope, Paul A. Vendetti, Erin Hye-Sook Kang, Stephen E. Lehecka, First row, from left: Christopher P. DiMartino, Daniel R. Kamen, Eric J. Hornick, CAS President Gail M. Ross, Wai-Yip Chow, Shanjing Li, Jeremy N. Scharnick, Warren T. Printz, Jeff A. Kluck, Larry J. Seymour, Jason A. Lauterbach.


Goodwin, Jason M. Nonis, Stephanie C. Young, Kristi Irene Carpine-Taber, Gerard J. Palisi, Lianmin Zhou. Third row, from left: Hans Heldner, Thomas Schneider, First row, from left: Michelle L. Sheppard, Jennifer L. Caulder, Silvia J. Bach, CAS President Gail M. Ross, Wei Hua Su, Scott G. Sobel, Guo Harrison, Michael Thomas Patterson. Second row, from left: Vladimir Shander, Ethan Charles Mowry, Karen E. Watson, Khanh K. Nguyen, Tracie L. Pencak, Christopher David Mark Richard Strona, John E. Noble, John J. Karwath, Michael Douglas Nielsen, Larry Xu Zhang, William Dwane Rader Jr.



Perron, Robert A. Grocock, Brian P. Rucci, Anthony G. Martella, Jack Barnett, Andrew M. Lloyd, Scott I. Rosenthal, James E. Calton. Third row, from left: Richard lames Christopher Guszcza, David Lee Handschke, Tina Tuyet Huynh, Henry Joseph Konstanty, Charles B. Kullman, Thomas P. Langer, Jenn Y. Lian, Jing Liu, Kee pictured: Frank J. Bilotti, Linda Jean Bjork, Tara E. Bush, William Brent Carr, Hao Chai, Patricia A. DeoCampo Vuong, Natasha C. Gonzalez, Donald B. Grimm, First row, from left: Stephen James Talley, Laura Beth Sachs, Deborah Herman Ardem, CAS President Gail M. Ross, Marie-Eve J. Vesel, Jason K. Machtinger, lason Cook, Jason E. Berkey, Sylvain Perrier, Cameron A. Cook, Philip M. Imm, Robert C. Roddy, Douglas E. Smith, Michael E. Mielzynski. New Fellows not Lisa Liqin Sun, Sharon L. Sowka. Second row, from left: Kevin Thomas Peterson, Patrick Barbeau, Isabelle Girard, Karine St-Onge, Isabelle Gingras, Isabelle Heng Ng, Mitchell S. Pollack, Rick C.H. Tzeng, Jennifer L. Vadney, Richard Alan Van Dyke, Robert, S. Weishaar, Joseph C. Wenc, Linda Yang,



Beaulieu, Paul D. Wilbert. Second row, from left: Jin Shao, Jean-François Tremblay, Jeffery A. Dvinoff, William C. Reddington, James M. Smieszkaf, Faisal O. Hamid, Susan B. Van Horn, Robert D. Harrington, Dovid C. Tkatch, James J. Konstanty. Third row, from left: Matthew B. Feldman, Chris M. Bilski, Robert B. First row, from left: Susan J. Forray, Malgorzata Timberg, Philip A. Clancey Jr., CAS President Gail M. Ross, Lien K. Tu-Chalmers, Ju-Young Suh, Patrick McCleish, Eric A. Hatch, James J. Leonard, Kevin K. Vesel, Raymond J. Kluesner, James C. Murphy, Timothy C. Mosler.



ZhenZhenLai. Second row, from left: Keith Curley, Melissa A. Ogden, David A. Traugott, Richard R. Crabb, Richard S. Crandall, Joseph S. Highbarger, Heather D. Lake, Anne Marie Klein-Lee, Robert W. Geist. Third row, from left: Gavin X. Lienemann, Yves Charbonneau, William J. Gerhardt, Melissa J. Appenzeller, Kath-First row, from left: Laura T. Sprouse, Lauri A. Gordon, Jonas F. Thisner, CAS President Gail M. Ross, Eva M. Paxhia, Stacie R.W. Grindstaff, Willie L. Davis, eryn E. Herzog, Hooi Lee Lai, Jason C. Harland, Kevin K.W. Chan.



David A. Gelberg, Jonathan K. Yu, Garcy C. Wang, Peter M. Shelley, Jacqueline Lee Neal. Third row, from left: Michael D. Sowka, Laurence R. McClure, Scott R. Rowe, Keith Jeremy Sunvold, Kirk D. Bitu. Second row, from left: Erik J. Steuernagel, Kelly K. Cusick, Simon Guenette, Christine Cadieux, Victoria K. Imperato, Brian C. Alvers, Thomas C. Bates, Matthew E. Butler, Jason T. Clarke, Amy L. DeHart, Peter L. Forrester, Gregory Evan Gilbert, John S. Giles, William G. Golush, Hurt, Micah Grant Woolstenhulme, Nicki C. Austin, Jennifer Graunas, Anh Tu Le, Steven T. Knight. Fourth row, from left: Brian O. Haaseth, Camilo Mohipp, Yi Zhang, Trevor C. Handley, Ryan Yin-kei Ho, Thomas Marie Cordier, Hazel Joynson Luckey, Jonathan L. Ankney, Steven Manilov. New Associates not pictured: First row, from left: Fernando Alberto Alvarado, Jessica L. Elsinger, Michelle S. Raeihle, John T. Maher, CAS President Gail M. Ross, Tomer Eilam, Stuart C. Kimberly Baker Hand, Young Y. Kim, Bradley S. Kove, Hayden Anthony Lewis, James P. McCoy, Richard U. Newell, Janel M. Sinacori, Christine Seung Steer, Esperanza Stephens, Liana St. Laurent, Dominic A. Tocci, Hanny C. Wai, Andrew F. Yashar.

CLYDE BIRCH FULTON JR. JAMES BRONSON GARDINER WARD VAN BUREN HART JR. GARY PAUL HOBART DAVID JON KRETSCH MATTHEW H. MCCONNELL JR. JACK MOSELEY ROBERT F. ROACH WILLIAM J. ROWLAND CINDY RAE SCHAUER JAMES SURRAGO

#### CLYDE BIRCH FULTON JR. 1934–2002

Clyde B. Fulton died May 9, 2002, at St. Luke's Medical Center in Houston, Texas. He was 67.

Fulton retired as vice president and controller of the Travelers Insurance Company in Hartford, Connecticut, in 1995 and moved to Houston. His first actuarial job was with Travelers in 1966 as an actuarial assistant. He stayed with Travelers for the rest of his career working in various capacities, including director of tax administration and senior vice president.

Fulton was a graduate of Ouachita Parish High School and received his B.S. degree from Northwestern Louisiana State University and his masters from Florida State University in mathematics. He was honorably discharged from the United States Navy with a rank of Lieutenant JG in 1958. Fulton became an Associate of the Casualty Actuarial Society in 1966.

He was preceded in death by his parents, Clyde B. Sr. and Lillian Fulton; brother, Dan Fulton; and nephew, Jeffrey Fulton.

Survivors include his sister, Mrs. Pat Fulton Kelly and her husband Don of Natchitoches, Louisiana; and several nieces, nephews, great nieces, and great nephews.

The family requested that memorials be made to the American Lung Association.

### JAMES BRONSON GARDINER 1907–2003

James B. Gardiner died on December 11, 2003, at the age of 96.

Born January 20, 1907, Gardiner graduated in 1924 from Governor Dummer Academy and in 1928 from Yale University.

Gardiner was an actuary for more than 70 years, beginning his career with MetLife in New York City and retiring at the age of 95 from the New York State Insurance Department. He became a Fellow in the Actuarial Society of America, a predecessor to today's SOA, in 1935. He earned his ACAS in 1947 and his FCAS the following year.

In his early days with MetLife, Gardiner reportedly logged 18,000 miles in a Model A Ford while visiting 35 states in ten weeks. Few roads west of Kansas City were paved at the time; he once recalled that he went through two sets of tires on the journey.

During his career at MetLife, he helped establish the company's Immediate Participation Guarantees, which were first introduced in the 1960s.

His primary actuarial duties for the New York State Insurance Department were to review the assumptions and make recommendations for the state's pension plans for police officers, firefighters, teachers, and other government workers.

CAS President Bob Conger (FCAS 1979) and SOA President W. James MacGinnitie (FCAS 1963) presented Gardiner with a joint certificate of appreciation upon his retirement as a supervising actuary for the New York State Insurance Department in fall 2002.

On his last day of work Gardiner was interviewed for *The Actuarial Review*. *AR* staff writer Arthur Schwartz (FCAS 1998) said that Gardiner impressed him as the ideal example of an

actuary, father, and friend. "Mr. Gardiner's lifelong dedication to the actuarial profession was special, and all too different from today's climate of 'me first,'" said Schwartz. "He was cut from cloth of the old school, in which loyalty—loyalty to profession, to employer, and to family—came first."

In his spare time, Gardiner enjoyed genealogy and history, which led to his involvement in the National Society of the Sons of the American Revolution (SAR). He chaired many committees and held several leadership positions in the organization, including vice president general (1967), registrar general (1965–1966), and president general (1969–1970), the society's top post.

In tribute to his contributions to the SAR, the society flew its flag 30 days at half-staff at Fraunces Tavern in Manhattan. The historic landmark is the headquarters for the SAR New York chapter. It was one of the meeting places for the Sons of Liberty in the pre-revolutionary war years and hosted a victory banquet for General George Washington and his troops at the conclusion of the war.

Gardiner was also active in the Order of the Founders and Patriots of America, the St. Nicholas Society of New York, the National Council Arts Club of New York, and the Huguenot Society. Gardiner also loved mountain climbing and math.

His daughter Cynthia of Tucson, Arizona, and son James of Burlington, Vermont, and three grandchildren survive Gardiner.

The family requested that memorial donations be made to People for the American Way.

### WARD VAN BUREN HART JR. 1922–2003

Ward Van Buren Hart Jr. died December 31, 2003, at the age of 81.

Hart was born March 27, 1922, in Hartford, Connecticut, where he raised his family with his wife Phyllis Marlowe Hart.

Hart graduated from Kingswood School in West Hartford in 1940 and Trinity College in 1945. While at Trinity he was a member of Alpha Chi Rho fraternity.

His father, Ward Van Buren Hart Sr., became a CAS Associate and SOA Fellow in 1924. The elder Hart, whose published work was part of the CAS Syllabus until the mid-1950s, encouraged his son to pursue an actuarial career. Hart followed suit and became an Associate of the CAS in 1953 and earned his Fellowship in 1956.

Hart worked for Aetna Insurance Company for most of his actuarial career, starting in the early 1950s in the company's rating division department of compensation and liability. Hart served Aetna in several capacities including analyst-programmer, actuary, and senior actuarial assistant before retiring in 1975.

He is survived by his wife; daughters, Margaret Packard of Pennsylvania and Pauline Evans of Michigan; brother, Gilbert W. Hart of New York; and three grandchildren, Gerold Packard, and Zachary and Alaric Evans; and also two nephews and a niece. The family requested that memorial donations be made to the Remembrance Fund of Immanuel Congregational Church in Hartford.

### GARY PAUL HOBART 1951–2002

Gary P. Hobart died November 13, 2002, at his residence in Marietta, Georgia.

Born September 9, 1951, in Ironton, Ohio, Hobart was part of a large family. He attended Randolph High School in Dover, New Jersey, graduating in 1969.

Hobart got an early start on his actuarial career when he was hired by Royal Globe Insurance Company in July 1969 as part of a work-study program. Hobart worked for the company while attending the College of Insurance in New York. At Royal Globe he worked in a variety of capacities from underwriting expense accounting to private passenger auto ratemaking. The company hired him full time upon his receiving a bachelor of science degree in 1974. Hobart earned his ACAS in 1976.

Throughout his actuarial career, Hobart worked primarily for Royal Globe (later to become Royal & SunAlliance). From 1976 through 1981, Hobart worked in the company's New York City office as an actuarial associate and senior actuarial associate. He briefly joined the Massachusetts Rating Bureau in Boston from 1981 to 1985 before returning to New York and the company that give him his start. By the decade's end he would make a big change, transferring to Charlotte, North Carolina, where he served as a director for Royal.

In 1996 Hobart left his long-time employer and became an actuarial consultant for Insurance Industry Consultants in Atlanta, Georgia. By the end of 2000, Hobart had established his own consulting firm, Hobart Actuarial Services in Marietta, Georgia.

Anthony J. Pipia (ACAS 1999), a former colleague, said Hobart took great pride in being an actuary. "To a fault, being an actuary was his whole life. It was really important to him," said Pipia. Hobart's dedication to his profession was eclipsed only by his devotion to his two children, Pipia said.

Hobart loved all sports but was especially fond of football, baseball, and basketball. Well-versed on the players and statistics, Hobart ran the company pools, especially for the NCAA tournaments. Hobart was also a member of Casualty Actuaries of Greater New York and served as secretary of the College of Insurance Alumni Association.

Hobart is survived by his children, David L. and Kimberly A. Hobart, both of Stoughton, Massachusetts; his parents, Charles Neal and Mary Ruth Hobart of Sanford, North Carolina; and three brothers, four sisters, and several nieces and nephews. The family requested that memorial donations be made to the building fund of St. Stephen the First Martyr Catholic Church in Sanford, North Carolina.

### DAVID JON KRETSCH 1964–2003

David J. Kretsch died January 21, 2003, at the age of 39.

Born on June 22, 1964, the son of Herman J. Kretsch and Elizabeth V. Rewitzer Kretsch, he grew up in the Minnesota River valley city of New Ulm, which was settled by German immigrants. He attended St. John's University in Collegeville, Minnesota. After graduating in 1986 with a bachelor of science in mathematics, he went to work at St. Paul Fire and Marine Insurance Company in St. Paul, Minnesota, first in personal lines and then commercial lines insurance. While at St. Paul, Kretsch became a CAS Associate in 1989 and earned his CAS Fellowship in 1992. On becoming a Fellow, Kretsch served for a year on the CAS Examination Committee. Kretsch was senior actuarial assistant for the company until he moved to Baltimore, Maryland in 1993.

In Baltimore, Kretsch worked at Maryland Insurance Group from 1993 to 1997, serving as assistant vice president and vice president and actuary. In 1998 he made the switch to Zurich North America Small Business in Baltimore where he was senior vice president and chief actuary until 2002.

Paul Brehm (FCAS 1989) supervised Kretsch at St. Paul in the personal lines area during the late 1980s. Brehm recalled Kretsch as being a dedicated family man, a great guy to work with, and a strong person who spoke his mind. "He loved to spar with state insurance departments over rate filings—and did it very well," mused Brehm.

Kretsch is survived by his wife, Jacqueline R. Jorissen Kretsch; sons, Matthew D., Brandon N., and Jonathan R.; his mother; brothers, James and Daniel Kretsch; and inlaws, John R. Jorissen and Linda C. Roberts.

### MATTHEW H. McCONNELL JR. 1908–2003

Matthew H. "Mac" McConnell died on May 30, 2003. He was 95.

He married Jane McEwen and together they had two sons. McConnell became an Associate of the CAS in 1934 and earned his Fellowship the following year.

When he first became a CAS member, McConnell was working for the National Council on Compensation Insurance in New York. By 1939 he had moved to Philadelphia to work for Indemnity Insurance Company of North America.

From 1942 to 1944 he worked as an underwriter with Employers Mutual Liability Insurance, first in New York City and then in Philadelphia, where he eventually settled. McConnell worked with Employers Mutual until the late 1940s when he moved to General Accident Fire and Life Assurance Corporation Ltd. General Accident eventually became Aviva PLC, the world's fifthlargest insurance group and the largest in the United Kingdom. In 1956, McConnell's Philadelphia office installed the company's first computer, an Invec 650, for handling general accounts.

He would stay with the company for the rest of his career, rising to the position of the superintendent of the compensation and liability department.

After retiring in 1977, McConnell lived in Villanova, Penn Wynn, and Riddle Village, Pennsylvania.

McConnell is survived by son Alan McConnell and his wife Carolyn; son Dr. Roger H. McConnell and his wife Bette; five grandchildren; and four great grandchildren. The family requested that memorial contributions be made to Compassionate Care Hospice in Wilmington, Delaware.

### JACK MOSELEY 1931–2003

Jack Moseley died January 17, 2003, of cancer at Flowers Hospital in Dothan, Alabama. He was 71 and had lived for the past five years in Fort Gaines, Georgia.

Moseley was born June 2, 1931, in Birmingham, Alabama. He graduated in 1952 with a bachelor of science degree in mathematics and physics from Auburn University. He also studied law for two years at Auburn and had an actuarial degree, as well as honorary degrees from Loyola University and Huntington College in Montgomery, Alabama. He became a CAS Associate in 1959 and a Fellow in 1961.

After college Moseley joined USF&G in Birmingham as an actuary. His career with USF&G spanned 37 years, during which he served as executive vice president in 1971, president in 1978, and chairman and chief executive officer in 1980.

Moseley transferred from Birmingham to Baltimore in 1956. Over the years he spent in the city, Moseley became what *The Baltimore Sun* described as "a major Baltimore booster" who did much to promote development and growth in his adopted hometown. As chairman of USF&G, Moseley encouraged other Baltimore business leaders to promote the city. When Moseley retired from USF&G in 1990, a *Sun* editorial characterized him as "a driving force behind efforts to promote the Baltimore region, as well as generous corporate giving to the city's charitable and cultural institutions."

Moseley's support extended to the Baltimore Symphony Orchestra, the Baltimore Museum of Art, local public broadcasting, the Baltimore Colts and Orioles, the Center Club, and the state and national Republican Party. According to *The Sun*, although Moseley was not particularly a sports fan, he fought to keep the Colts in Baltimore. Despite Moseley's and many other people's efforts, the Colts would leave the city, much to the disappointment of the team's fans.

Moseley also served on the boards of directors of several major local companies, the Greater Baltimore Medical Center, and the national Chamber of Commerce, and was a trustee of the Maryland Institute College of Art.

Moseley is survived by his wife of 49 years, the former Patsy Blake; sons Jack Moseley Jr. of Montrose, Colorado, Glynn E. Moseley of Fort Gaines, Georgia, and Edward Moseley of Melbourne Beach, Florida; and three grandchildren.

### ROBERT F. ROACH 1950–2003

Robert F. "Bob" Roach of Newburyport, Massachusetts, died in November 2003. He was 53.

Roach (ACAS 1975) was working as an actuarial assistant for Commercial Union Assurance Companies in Boston in 1976. He would stay with the firm until 1986, serving as director and personal lines actuary, and vice president and actuary. In 1986 Roach joined American Mutual Insurance Companies in Wakefield, Massachusetts, as vice president.

By the late 1980s Roach's work interests turned to consulting. He was a consultant for Tillinghast Towers Perrin in Boston from 1989 to 1993 before accepting a position with TPA Associates Inc. in Andover, Massachusetts. Roach worked for TPA Associates for nearly 10 years before retiring in 2003.

Roach is survived by his wife, Louanne Fleury; children, Kelly Roach of Newburyport, Joshua Fleury and his wife, Allison, of Beverly, Massachusetts; brother, Martin Roach and his wife, Mary, of Danvers, Massachusetts; and many nieces and nephews. In lieu of flowers, the family requested contributions be made to the Merrimack Valley Hospice in Lawrence, Massachusetts.

### WILLIAM J. ROWLAND 1949–2003

William J. Rowland died September 27, 2003, at a Boston hospital at the age of 54.

He was born July 20, 1949, in Chicago, Illinois, to Vincent T. and Isabel (Roessler) Rowland. He had lived in Mercer Island, Washington, before moving to Bedford, Massachusetts, in 1999.

A graduate of Governor Livingston Regional High School in New Jersey, he attended Villanova University. He graduated from Muhlenberg College in Allentown, Pennsylvania, in 1971.

Rowland and his brother Vincent T. Rowland Jr. (ACAS 1982) were both drawn to the actuarial profession in large part due to their father's career as insurance company accountant. The Rowland brothers spent summers working in insurance companies and both received degrees in mathematics. Although younger, Bill Rowland completed his exams before his brother Vince, who was serving in the military. When Vince left the service, Rowland tried to get his brother a position at his firm. Company rules dictated that siblings could not work together and so the brothers worked at different companies, both specializing in ratemaking. Because their companies were direct competitors, however, the two could never talk about work.

Rowland's career spanned 32 years, beginning at Crum & Forster, which later became Talegen, then working in Chicago for The Resolution Group, which evolved into the RiverStone Group. At the time of his death, Rowland was the vice president and chief actuary of RiverStone Resources, LLC. He became a CAS Associate in 1979 and a CAS Fellow in 1980. Rowland was a member of the American Academy of Actuaries as well as being a Certified Financial Analyst.

Rowland served as a member of various CAS committees, including Examination (1981–1983), Audit (1988–1998), Financial Analysis (1990–1992), and Valuation and Financial Analysis

(1992–1996). He was also a CAS leader, chairing the CAS Finance (1988–1991) and Audit (1992–1993) Committees.

Rowland especially enjoyed playing chess, watching sports, and recording sports statistics. A devoted family man, Rowland was said to have never let a business commitment interfere with time spent with his family. "Bill was a great family man—a super nice guy and father," said brother Vincent.

Surviving family members include his wife of 30 years, Elizabeth (Wagle) Rowland of Bedford; two children, Anne P. Rowland and Daniel W. Rowland, both of Bedford; his mother, Isabel, of Southwick, Massachusetts; brother, Vincent of North Granby, Connecticut; and two nieces.

The family requested that memorial contributions be made to the Dana Farber Cancer Institute in Boston.

### CINDY RAE SCHAUER 1966–2003

Cindy Rae Schauer died August 23, 2003, at the age of 35.

Born August 27, 1966, in Story City, Iowa, to Richard and Mary Lauridsen Rosenbladt, Schauer grew up in the northern Iowa town of Lake Mills. An excellent student, Schauer graduated from Lake Mills High School in 1984 as class valedictorian.

The year 1988 was a significant one for Schauer. In that year she graduated magna cum laude from Luther College in Decorah, Iowa, with a bachelor's degree in mathematics; she started her first actuarial job with Federated Mutual Insurance Company in Owatonna, Minnesota; and she married Christopher "Chip" Schauer.

Schauer and her husband lived in the Minnesota communities of Lakeville and Medford before settling in rural Faribault, Minnesota. She worked as an actuary and he ran his own construction business while raising their children Alek, Ashley, and Joshua. She enjoyed camping and reading as well as spending time with her family.

Federated was Schauer's first and only employer out of college. In her 15 years with the company, Schauer served in several capacities including senior actuarial analyst in 1996 and actuarial manager in 2002. While at Federated Schauer priced various commercial lines of business. She developed many of the company's ratemaking applications for commercial general liability and commercial automobile. She also developed many of the specific procedures and the software applications for producing overall rate level indications, classification and territory relativity studies, as well as for other special research projects. In the latter part of her career she switched to reserving. Schauer earned her CAS Associateship in 1996.

In 2001 Schauer was diagnosed with a rare disease. She was in and out of the hospital for two years. A positive-minded person, Schauer continued to work between hospital stays and, despite her own illness, still managed to help care for her parents who were in poor health. Schauer's mother and father predeceased her.

Steven W. Judd (FCAS 1982), who was Schauer's supervisor, called her "a tribute to the actuarial profession." Judd, an Episcopal deacon, officiated at Schauer's funeral.

Richard Johnson (ACAS 1982), Schauer's former supervisor at Federated, said Schauer was "probably the most intelligent actuary we ever had here." Johnson reflected that she had a "cando" attitude often replying "Oh sure I can do that" when asked to handle a difficult project.

Schauer is survived by her husband and children; brother Gregg Rosenbladt and his wife Katherine of Mason City, Iowa; sister Sue Rosenbladt Hayungs and her husband Scott of Rochester, Minnesota; and many other relatives and friends.

### JAMES SURRAGO 1951–2003

Jim Surrago died April 25, 2003. An avid golfer, Surrago collapsed suddenly at a driving range. He was 51.

Born June 9, 1951, in Queens, New York, Surrago attended Bishop Reilly High School in Fresh Meadows, New York. He graduated from high school in 1969 and in 1973 graduated cum laude with a bachelor of science degree from Manhattan College in Riverdale, New York. He married Mary Jane McDonald and together they had two children.

For over 25 years, Surrago served as a vice president with Insurance Services Office (ISO) in New York City and then Jersey City, New Jersey. After being at ISO for several years, Surrago worked for a few different companies including a brokerage firm. He returned to ISO, taking on the company's data management department. Surrago developed data products and maintained products and services for clients. He headed the department until his death.

Dan Crifo (ACAS 1977), a colleague of Surrago's and fellow commuter, admired Surrago's wry sense of humor. "He could spot things that were slightly off kilter and make you laugh about it," said Crifo. "He enjoyed seeing things at a different angle."

Surrago was very active in CAS activities, serving as a member of the Program Planning Committee from 1986–1990. He became vice chair of the committee in 1991. From 1994 to 1995 he was a member of the Committee on Continuing Education and served as the committee's vice chair (1995–1996) and chair (1997–1998). Surrago also was an ex officio member of the CAS Education Policy Committee from 1996 to 1998.

Among his other interests, Surrago was a member of High Mountain Golf Club in Franklin Lakes, New Jersey, and St. Catharine's Roman Catholic Church in Glenrock, New Jersey.

He is survived by his wife; children, Michael and Christine Surrago of Glen Rock; and parents, James and Ethel Surrago of Ft. Lauderdale, Florida. The family requested that memorial donations be made to the American Heart Association.

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