VOLUME LXXXVIII NUMBERS 168 AND 169

PROCEEDINGS

OF THE

Casualty Actuarial Society

ORGANIZED 1914

2001

VOLUME LXXXVIII

Number 168—May 2001

Number 169—November 2001

Library of Congress Catalog No. HG9956.C3
ISSN 0893-2980

Printed for the Society by
United Book Press
Baltimore, Maryland

Typesetting Services by
Minnesota Technical Typography, Inc.
St. Paul, Minnesota

FOREWORD

Actuarial science originated in England in 1792 in the early days of life insurance. Because of the technical nature of the business, the first actuaries were mathematicians. Eventually, their numerical growth resulted in the formation of the Institute of Actuaries in England in 1848. Eight years later, in Scotland, the Faculty of Actuaries was formed. In the United States, the Actuarial Society of America was formed in 1889 and the American Institute of Actuaries in 1909. These two American organizations merged in 1949 to become the Society of Actuaries.

In the early years of the 20th century in the United States, problems requiring actuarial treatment were emerging in sickness, disability, and casualty insurance—particularly in workers compensation, which was introduced in 1911. The differences between the new problems and those of traditional life insurance led to the organization of the Casualty Actuarial and Statistical Society of America in 1914. Dr. I. M. Rubinow, who was responsible for the Society's formation, became its first president. At the time of its formation, the Casualty Actuarial and Statistical Society of America had 97 charter members of the grade of Fellow. The Society adopted its present name, the Casualty Actuarial Society, on May 14, 1921.

The purposes of the Society are to advance the body of knowledge of actuarial science applied to property, casualty, and similar risk exposures, to establish and maintain standards of qualification for membership, to promote and maintain high standards of conduct and competence for the members, and to increase the awareness of actuarial science. The Society's activities in support of this purpose include communication with those affected by insurance, presentation and discussion of papers, attendance at seminars and workshops, collection of a library, research, and other means.

Since the problems of workers compensation were the most urgent at the time of the Society's formation, many of the Society's original members played a leading part in developing the scientific basis for that line of insurance. From the beginning, however, the Society has grown constantly, not only in membership, but also in range of interest and in scientific and related contributions to all lines of insurance other than life, including automobile, liability other than automobile, fire, homeowners, commercial multiple peril, and others. These contributions are found principally in original papers prepared by members of the Society and published annually in the *Proceedings of the Casualty Actuarial Society*. The presidential addresses, also published in the *Proceedings*, have called attention to the most pressing actuarial problems, some of them still unsolved, that have faced the industry over the years.

The membership of the Society includes actuaries employed by insurance companies, industry advisory organizations, national brokers, accounting firms, educational institutions, state insurance departments, and the federal government. It also includes independent consultants. The Society has three classes of members—Fellows, Associates, and Affiliates. Both Fellowship and Associateship require successful completion of examinations, held in the spring and fall of each year in various cities of the United States, Canada, Bermuda, and selected overseas sites. In addition, Associateship requires completion of the CAS Course on Professionalism. Affiliates are qualified actuaries who practice in the general insurance field and wish to be active in the CAS but do not meet the qualifications to become a Fellow or Associate.

The publications of the Society and their respective prices are listed in the Society's *Yearbook*. The *Syllabus of Examinations* outlines the course of study recommended for the examinations. Both the *Yearbook*, at a charge of $40 (U.S. funds), and the *Syllabus of Examinations*, without charge, may be obtained from the Casualty Actuarial Society, 1100 North Glebe Road, Suite 600, Arlington, Virginia 22201.

JANUARY 1, 2001
EXECUTIVE COUNCIL*

Patrick J. Grannan . . . . *President*
Robert F. Conger . . . . *President-Elect*
Sheldon Rosenberg . . . . *Vice President–Administration*
Mary Frances Miller . . . . *Vice President–Admissions*
Abbe S. Bensimon . . . . *Vice President–Continuing Education*
LeRoy A. Boison . . . . *Vice President–International*
David R. Chernick . . . *Vice President–Programs & Communications*
Gary R. Josephson . . . . . *Vice President–Research & Development*

THE BOARD OF DIRECTORS

*Officers**

Patrick J. Grannan . . . . *President*
Robert F. Conger . . . . *President-Elect*

Immediate Past President†

Alice H. Gannon 2001

Elected Directors†

Charles A. Bryan 2001
John J. Kollar 2001
Gail M. Ross 2001
Michael L. Toothman 2001
Amy S. Bouska 2002
Stephen P. D'Arcy 2002
Frederick O. Kist 2002
Susan E. Witcraft 2002
Ralph S. Blanchard III 2003
Janet L. Fagan 2003
Michael J. Miller 2003
Deborah M. Rosenberg 2003

*Term expires at the 2001 Annual Meeting. All members of the Executive Council are Officers. The Vice President–Administration also serves as the Secretary and Treasurer.

† Term expires at Annual Meeting of year given.

2001 PROCEEDINGS
CONTENTS OF VOLUME LXXXVIII

Page

2001 PROCEEDINGS
CONTENTS OF VOLUME LXXXVIII

NOTICE

Papers submitted to the *Proceedings* of the Casualty Actuarial Society are subject to review by the members of the Committee on Review of Papers and, where appropriate, additional individuals with expertise in the relevant topics. In order to qualify for publication, a paper must be relevant to casualty actuarial science, include original research ideas and/or techniques, or have special educational value, and must not have been previously copyrighted or published or be concurrently considered for publication elsewhere. Specific instructions for preparation and submission of papers are included in the *Yearbook* of the Casualty Actuarial Society.

The Society is not responsible for statements of opinion expressed in the articles, criticisms, and discussions published in these *Proceedings*.

Editorial Committee, *Proceedings* Editors

Volume LXXXVIII, Part 1 **No. 168**

PROCEEDINGS
May 6, 7, 8, 9, 2001

A FLEXIBLE FRAMEWORK FOR STOCHASTIC CLAIMS RESERVING

PETER D. ENGLAND AND RICHARD J. VERRALL

Abstract

In this paper, a flexible framework for stochastic claims reserving is considered which includes several models proposed to date as special cases. The methodology is embedded within the generalized additive class of models (Hastie and Tibshirani [7]). The methodology is particularly useful since it allows smoothing of chain ladder development factors and estimation of tail factors automatically and easily as part of the model-fitting process, traditionally performed as an additional stage in the claims reserving process. The framework also provides estimates of reserve variability, which could prove useful in formulating and calibrating dynamic financial analysis (DFA) models.

1. INTRODUCTION

The setting and monitoring of claims reserves is a vital task required of the general insurance actuary. To aid in the setting of reserves, the actuary can make use of a variety of techniques, the most familiar of which is the chain ladder model or variation thereof (e.g., inflation-adjusted chain ladder, *n*-year average volume-weighted chain ladder, etc.). The principal aim of a reserving exercise is to provide an *estimate* of the amount of money a company should set aside now to meet claims arising in the future on the policies already written. The actuary cannot predict with certainty and knows that there is a distribution of possible outcomes, but uses the techniques at his or her disposal to arrive at the best estimate of the reserve (even if the *best* estimate is not that which is carried in the accounts). Knowledge of the *precision* of that estimate is also desirable. Traditional reserving techniques can help provide a best estimate (a measure of *location* in the distribution of possible outcomes), but cannot help with measures of precision. Of course, the actuary knows that the reserve estimate associated with a well-behaved class of business will be more precise than that of a poorly-behaved class, and that the reserve estimate associated with a short-tailed class is likely to be more precise than that of a long-tailed class, but measuring that precision is difficult.

Stochastic claims reserving models aim to provide measures of location (best estimates) and measures of precision (measures of variability) by treating the reserving process as a data analysis exercise and building a reserving model within a statistical framework. Once within a statistical framework, diagnostic checks of the fitted models are possible, such as goodness-of-fit tests and analysis of residuals (which highlight systematic and isolated departures from the fitted model). Various stochastic reserving models have been proposed over the last two decades, and work progresses as new techniques in the field of statistical modeling become available.

Considerable attention has been given to the relationship between various stochastic models and the chain ladder technique. Stochastic models have been constructed with the aim of producing exactly the same reserve estimates as the traditional deterministic chain ladder model. This might seem like a futile exercise, but has the advantages that measures of precision are readily available, and the assumptions underlying the chain ladder model are clarified. More importantly, it provides a bridge between traditional methods and stochastic methods, which is useful for the practitioner who is familiar with traditional methods and needs a starting point for exploring stochastic methods.

Other stochastic reserving models which have been proposed attempt to overcome shortcomings of the chain ladder model by incorporating smoothing, or a parametric form which reduces the number of underlying parameters used to fit the model. The aim of this paper is to present a flexible framework for stochastic claims reserving which allows the practitioner to choose whether to use the basic chain ladder model, or to apply some smoothing, or in the limit to use a parametric curve for the runoff. Several of the models proposed to date fit within this framework, and further extensions are possible which have not yet been tried.

For technical reasons, we consider the modeling of paid losses only. Furthermore, information regarding claim numbers is not taken into account; we consider the modeling of claim amounts only. In this respect we take the basic chain ladder model with paid losses as our starting point. Typically data provided for a simple reserving exercise is in the form of a triangle of paid losses (see Section 6) in which the rows i denote accident years and the columns j delay or development years. Although we consider annual development here only, the methods can be extended easily to semiannual, quarterly or monthly development. The triangle is augmented each period by the addition of a new diagonal. The aim in reserving is to predict likely claim amounts in the missing southeast corner of the claims rectangle, the total reserve (ignoring the tail for the moment) being the sum of these amounts. For monitoring purposes, we might also be interested in the reserve for each accident year.

A review of some existing stochastic reserving models appears in Section 2. This is not exhaustive but provides the necessary background from which the flexible framework in Section 3 can be derived. In Section 4, variability of reserve estimates is considered, and formal goodness-of-fit is considered briefly in Section 5. A worked example is then provided, considering the systematic structure of the model in Section 6 and the error structure in Section 7, before concluding in Section 8.

2. A BRIEF REVIEW OF EXISTING STOCHASTIC CLAIMS RESERVING MODELS

Let C_{ij} denote the (incremental) claims amount arising from accident year i paid in development year j. Early work in this field focused on the logarithm of the incremental claims amounts $Y_{ij} = \ln(C_{ij})$ and the lognormal class of models $Y_{ij} = m_{ij} + \varepsilon_{ij}$ with

$$\varepsilon_{ij} \sim IN(0,\sigma^2) \qquad \text{and} \qquad Y_{ij} \sim IN(m_{ij},\sigma^2), \tag{2.1}$$

where the expression "$\sim IN(\mu,\sigma^2)$" is interpreted as "distributed as independent normal with mean μ and variance σ^2."

The use of the logarithmic transform immediately imposes a limitation on this class of models in that claim amounts must be positive. The normal responses Y_{ij} are assumed to decompose (additively) into a deterministic nonrandom component with mean $m_{ij} = \eta_{ij}$ and homoscedastic normally distributed random error components about a zero mean. Two model structures are of specific interest:

CASE 1

$$\eta_{ij} = c + \alpha_i + \beta_j; \tag{2.2}$$

CASE 2

$$\eta_{ij} = c + \alpha_i + \beta_i \ln(j) + \gamma_i j \qquad (j > 0). \tag{2.3}$$

A third case, which is a mixture of Cases 1 and 2, uses Equation 2.2 for $j \le q$ and Equation 2.3 for $j > q$ for some integer q specified by the modeler.

Equations 2.1 and 2.2 define the model introduced by Kremer [8] and used by Renshaw [13], Verrall [21], Zehnwirth [27] and Christofides [2], amongst others. Accident year and development year are treated as factors, with a parameter α for each accident year i and a parameter β for each development year j. This representation is analogous to the chain ladder model, which implies the same development pattern for all accident years, where that pattern is defined by the parameters β_j. Use of this model produces predicted values close, but not identical, to those from the simple chain ladder technique.

Equations 2.1 and 2.3 broadly define the model used by Zehnwirth [28]. A special case is created by setting $\beta_i = \beta$ for all i and $\gamma_i = \gamma$ for all i, where the decay pattern is the same for all accident years and represented by only two parameters. Unlike Case 1, this imposes a strict parametric form on the shape of the runoff. Although this sacrifices goodness-of-fit, it has the advantage that payments can be predicted by extrapolation beyond the range of j observed. This representation is known as the Hoerl curve.

Parameters in the predictor structure η_{ij} are estimated by maximum likelihood, which in the case of normally distributed data is equivalent to minimizing the residual sum of squares. Obtaining this "least squares" solution is straightforward, and is a major reason for the importance of log-linear models in the history of stochastic claims reserving. Although it was possible to use other error distributions (using generalized linear models) at the time these models were propounded, their use was not common and suitable statistical software was in its infancy. De Jong and Zehnwirth [4] adopted the Kalman filter to pass information between accident years and provide smoothed estimates of the parameters β_i and γ_i in Equation 2.3. This idea was adopted by Verrall [21] who used the Kalman filter to smooth over the parameters α_i and β_j in Equation 2.2.

The unknown variance σ^2 is estimated by the residual sum of squares divided by the degrees of freedom (the number of observations minus the number of parameters estimated). Zehnwirth [29] also considers allowing a different variance estimator for each development period.

Given the parameter estimates, predicted values on a log scale can be obtained by introducing those estimates back into the appropriate equation. Exponentiating then provides an estimate of the median on the untransformed scale, and an estimate of the mean is given by incorporating a variance component to give predicted values on the untransformed scale. Specific details can be found in Verrall [22].

Significant advances were made in stochastic claims reserving with the publication of a paper by Wright [26], which was interesting in two main respects:

- The systematic and random components of the underlying model for the data are based on a risk theoretic model of the claims generating process;
- The error distribution implied by the model is no longer (log) normal.

Wright considered the incremental paid claims C_{ij} to be the sum of N_{ij} (independent) claims of amount X_{ij}. Standard results from risk theory give:

$$\mathrm{E}[C_{ij}] = \mathrm{E}[N_{ij}]\mathrm{E}[X_{ij}], \tag{2.4}$$

and

$$\mathrm{Var}[C_{ij}] = \mathrm{E}[N_{ij}]\mathrm{Var}[X_{ij}] + \{\mathrm{E}[X_{ij}]\}^2\mathrm{Var}[N_{ij}]. \tag{2.5}$$

The formulation is completed by specifying a model for each of $\mathrm{E}[N_{ij}]$ and $\mathrm{E}[X_{ij}]$, a relationship between the mean and variance of the claim numbers N_{ij}, and a relationship between the mean and variance of the claim severities X_{ij}.

Wright considered the claim numbers N_{ij} to be Poisson random variables where

$$\mathrm{E}[N_{ij}] = e_i a_j \kappa_i j^{A_i} e^{-b_i j}, \tag{2.6}$$

and

$$\mathrm{Var}[N_{ij}] = \mathrm{E}[N_{ij}] \tag{2.7}$$

where κ, A and b are unknown constants to be estimated, e_i is a measure of exposure, and a is a *known* adjustment term needed on technical grounds. The values a are specified in Appendix 1 of Wright [26] for each value of j. (Note: Wright also recommended a technical adjustment to development time j, which has been ignored here for simplicity.)

Claim amounts X_{ij} were considered to be Gamma type random variables where

$$\mathrm{E}[X_{ij}] = e^{\delta t} k j^{\lambda}, \tag{2.8}$$

and

$$\mathrm{Var}[X_{ij}] = \upsilon \{\mathrm{E}[X_{ij}]\}^2, \tag{2.9}$$

where k and λ are unknown constants. The optional term $e^{\delta t}$ is included to allow for possible claims inflation, where $t = i + j$ represents calendar time and δ is the estimated constant force of claims inflation. Wright chose not to assume that the claim amounts are actually Gamma distributed, only that the variance exists and is proportional to the mean squared with constant of proportionality υ. This is a subtle technicality which makes no practical difference when claim amounts are all positive.

Equations 2.6 and 2.8 are designed to model the mean claim numbers and mean individual claim severities as functions of delay j.

This formulation is interesting because it uses the same model specification in the claims reserving context as in pricing; that is, claim numbers are modeled as Poisson random variables and claim severities are modeled as Gamma random variables.

Combining Equations 2.4 to 2.9 gives

$$\mathrm{E}[C_{ij}] = m_{ij} = e_i a_j \kappa_i j^{A_i} e^{-b_i j} e^{\delta t} k j^{\lambda}, \tag{2.10}$$

and

$$\mathrm{Var}[C_{ij}] = (1 + \upsilon)kj^{\lambda}e^{\delta t}\mathrm{E}[C_{ij}]. \qquad (2.11)$$

Wright showed that with a suitable reparameterization, Equations 2.10 and 2.11 represent a generalized linear model (GLM). Standard statistical methods can be used to estimate the parameters involved.

This model formulation can be viewed as a way of allowing the incremental paid claims C_{ij} to be modeled directly, without the necessity of modeling claim numbers and claim severities separately and then combining. The only information needed to fit the model is the standard triangle of incremental paid claims.

Wright went on to use the Kalman filter to pass information between accident years to produce smoothed parameter estimates, thus avoiding problems associated with the excessive parameterization.

The formulation of the problem as a GLM and the fitting method adopted by Wright are not easy to follow, so the simpler derivation by Renshaw [14] is presented here. Writing:

$$\begin{aligned}
u_{ij} &= \ln(e_i a_j),\\
c &= \ln(k),\\
\alpha_i &= \ln(\kappa_i) \qquad \text{with} \quad \kappa_1 = 1,\\
\beta_i &= \lambda + A_i, \qquad \text{and}\\
\gamma_i &= -b_i,
\end{aligned}$$

gives

$$\mathrm{E}[C_{ij}] = e^{(u_{ij}+c+\alpha_i+\beta_i\ln(j)+\gamma_i j+\delta t)}.$$

We can then write

$$\eta_{ij} = u_{ij} + c + \alpha_i + \beta_i\ln(j) + \gamma_i j + \delta t, \qquad (2.12)$$

giving

$$\mathrm{E}[C_{ij}] = m_{ij}, \qquad (2.13)$$

where

$$\ln(m_{ij}) = \eta_{ij}. \tag{2.14}$$

Ignoring the known offset (u_{ij}) and the optional term for claims inflation (δt), Equation 2.12 represents the familiar Hoerl curve which appeared in Equation 2.3.

Using Equations 2.7 and 2.9 in 2.5 gives:

$$\mathrm{Var}[C_{ij}] = \mathrm{E}[N_{ij}]\ \upsilon\{\mathrm{E}[X_{ij}]\}^2 + \{\mathrm{E}[X_{ij}]\}^2\mathrm{E}[N_{ij}];$$

then using Equation 2.4 gives

$$\mathrm{Var}[C_{ij}] = (1+\upsilon)\mathrm{E}[X_{ij}]\mathrm{E}[C_{ij}].$$

Writing

$$\phi_{ij} = (1+\upsilon)\mathrm{E}[X_{ij}]$$

gives

$$\mathrm{Var}[C_{ij}] = \phi_{ij}\mathrm{E}[C_{ij}] = \phi_{ij}m_{ij}. \tag{2.15}$$

Equations 2.13, 2.14 and 2.15 define a GLM (see Section 3) in which the response C_{ij} is modeled with a logarithmic link function, the variance is proportional to the mean, and the linear predictor is given by Equation 2.12. The ϕ_{ij} are unknown scale parameters to be estimated by the model.

With GLMs, the unknown scale parameter is usually constant for all observations (i.e., $\phi_{ij} = \phi$ for all i, j) and is estimated by the deviance (or alternatively the Pearson χ^2 statistic) divided by the degrees of freedom. However, in this formulation, it is possible to estimate the scale parameters as part of an extended fitting procedure, known as joint modeling (see Renshaw [14]).

It should be noted that in Renshaw's formulation, the assumption that claim numbers are Poisson distributed was relaxed slightly, the only requirement being that the variance of the number of claims exists and is proportional to the mean.

Therefore

$$\mathrm{Var}[N_{ij}] = \varphi \mathrm{E}[N_{ij}]. \tag{2.16}$$

This is in the spirit of the relaxed assumptions made by Wright [26] about the distribution of claim severities. Claim numbers are said to be distributed as "overdispersed" Poisson random variables. Using Equation 2.16 instead of 2.7 gives:

$$\phi_{ij} = (\varphi + \upsilon)\mathrm{E}[X_{ij}]$$

without changing the specification as a GLM.

Comparing Equation 2.12 with 2.3, it can be seen that Wright is effectively using the same linear predictor as Zehnwirth [28], with the inclusion of an optional term to model possible claims inflation. The u_{ij} terms are known and represent small technical adjustments. They are declared as *offsets* when fitting the model using standard statistical software packages. The important differences between the model used by Zehnwirth and the model proposed by Wright are that:

- Zehnwirth uses the logarithm of the incremental claims as the response, and links the predictor (2.3) to the expected value of the response through the identity link function, therefore requiring the introduction of a variance component when focusing on the mean on the untransformed scale. Wright treats the incremental claims themselves as the responses, and links (essentially) the same predictor to the expected value of the response through the logarithmic link function, thereby avoiding the necessity of the inclusion of a variance component when focusing on the predicted mean.
- In the model proposed by Zehnwirth, the variance is constant for all observations (or constant for each development period), whereas in the model proposed by Wright, the variance is proportional to the mean. A critique of these assumptions can be found in Appendix 4 of Wright [26].
- The log transformation used by Zehnwirth excludes the mass point at zero (although it is possible to make minor adjustments

to zero payments in the data), which Wright's model includes naturally. In fact Wright's model can also be used for data sets that include some negative payments.

It should be noted that in the software package ICRFS, Zehnwirth [29] includes a range of predictor structures, not just the one alluded to above, which could provide an improved fit to the data. However, all are based on log-incremental claims in his "Probabilistic Trend Family."

Equations 2.12 to 2.15 define the model proposed by Wright, and suggest possible alternatives. For example, Renshaw and Verrall [16, 17] replace the linear predictor used by Wright (Equation 2.12) by the linear predictor suggested by Kremer [8], and use a constant scale parameter by setting $\phi_{ij} = \phi$ for all i, j. Therefore,

$$\mathrm{E}[C_{ij}] = m_{ij} \qquad \text{and} \qquad \mathrm{Var}[C_{ij}] = \phi m_{ij}, \tag{2.17}$$

where

$$\ln(m_{ij}) = \eta_{ij} = c + \alpha_i + \beta_j. \tag{2.18}$$

Equations 2.17 and 2.18 define a GLM in which incremental claims are modeled as overdispersed Poisson random variables. This model is particularly interesting since the predicted values given by the model are *exactly* the same as those given by the simple chain ladder model, thus providing a stochastic version of the chain ladder model.

Renshaw and Verrall were not the first to notice the link between the chain ladder model and the Poisson distribution, but were the first to implement the model using standard methodology in statistical modeling and to provide a link with the analysis of contingency tables. Wright [26] also describes a similar model, including a term to model claims inflation, but did not consider the model in detail. Mack [9] also points out that the chain ladder estimates can be obtained by maximizing a Poisson likelihood by appealing to the so-called "method of marginal totals."

Mack [9] suggested using the same linear predictor as Kremer [8] (and therefore the same as Renshaw and Verrall [17]) but proposed using a Gamma distribution for claim amounts. However, Mack developed his own fitting procedure for obtaining maximum likelihood parameter estimates. As Renshaw and Verrall [17] note, the same model can be fitted using the GLM described by Equations 2.17 and 2.18, but replacing $\mathrm{Var}[C_{ij}] = \phi m_{ij}$ by $\mathrm{Var}[C_{ij}] = \phi m_{ij}^2$. Standard statistical software packages can then be used to obtain maximum likelihood parameter estimates.

In Verrall [24], the stochastic chain ladder model of Renshaw and Verrall [16] was extended to incorporate smoothing of parameter estimates over accident years (the α_is in Equation 2.18), while leaving the model describing the runoff pattern (the β_js) alone. Nonparametric smoothers were used and fitted using generalized *additive* models (GAMs). GAMs differ from GLMs in the way in which the relationship between the response variable and the covariates is modeled. In GLMs the relationship is parametric; in GAMs the response is assumed to vary smoothly with the covariates through the introduction of a smoothing procedure. In this paper, the idea is extended to allow smoothing over development years, which is of considerable practical benefit and provides a flexible framework for stochastic claims reserving.

3. A FLEXIBLE FRAMEWORK FOR STOCHASTIC CLAIMS RESERVING

A GLM is defined by focusing on a set of independent response variables $\{Y_u : u = 1,2,\ldots,n\}$. The objective is to model the expected value of the response as a function of one or more covariates. We assume that the Y_u are distributed according to a member of the one-parameter exponential family of distributions, which includes the normal, Poisson and Gamma distributions, amongst others. Denoting the expected value of Y_u by m_u, the first two moments take the general form

$$\mathrm{E}[Y_u] = m_u \qquad \text{and} \qquad \mathrm{Var}[Y_u] = \frac{\phi V(m_u)}{w_u},$$

TABLE 3.1

SCALE PARAMETERS AND VARIANCE FUNCTIONS FOR SOME STANDARD DISTRIBUTIONS

Distribution	Scale Parameter ϕ	Variance function $V(m_u)$
Normal	σ^2	1
Poisson	1	m_u
Gamma	>0	m_u^2
Inverse Gaussian	>0	m_u^3

where ϕ denotes a scale parameter, w_u are prior weights (often set to 1 for all observations), and $V()$ is the so-called *variance function* (a function of the mean). The choice of distribution dictates the values of ϕ and $V()$. The values of the scale parameter and variance function for various standard distributions are shown in Table 3.1. The definition of a GLM is completed by specifying the deterministic structure, which is achieved through a linear predictor η_u where

$$\eta_u = \sum_{v=1}^{p} x_{uv}\beta_v \tag{3.1}$$

with known covariates x_v associated with each observation u, and unknown parameters β_v. The expected value of the response is linked to the linear predictor through a link function $g()$ such that

$$g(m_u) = \eta_u.$$

It is helpful to think of GAMs as extensions of GLMs. A GAM is defined by replacing Equation 3.1 by

$$\eta_u = \sum_{v=1}^{p} s_v(x_u),$$

where $s(x)$ represents a nonparametric smoother on x. It is possible to choose from several different types of smoothers, such as locally weighted regression smoothers (loess), cubic smoothing splines and kernel smoothers. Other features of GAMs, such as the choice of error distribution, link function, goodness-of-fit measures and residual definitions are common to GLMs with the main difference between GAMs and GLMs being the specification of the predictor η.

A complete exposition of the statistical background of generalized linear models and generalized additive models can be found in McCullagh and Nelder [12] and Hastie and Tibshirani [7] respectively.

It should be noted in passing that we are not restricted to using a smoother for all covariates; the predictor may comprise a mixture of parametric and nonparametric components. The predictor then becomes

$$\eta_u \sum_{v=1}^{p-r} x_{uv}\beta_v + \sum_{v=p-r+1}^{p} s_v(x_u).$$

In claims reserving, the cubic smoothing spline has been found to be particularly useful. When data are normally distributed, the (univariate) cubic smoothing spline $s(x)$ is found by minimizing the penalized residual sum of squares

$$\sum_{u=1}^{n} (y_u - s(x_u))^2 + \theta \int (s''(t))^2 dt. \tag{3.2}$$

The second part of Equation 3.2 defines a smoothness penalty based on curvature of the spline function $s(x)$. The level of smoothing is controlled by the single parameter $\theta(> 0)$. When θ tends to zero, there is no smoothness penalty and the model provides a perfect fit: the fitted values are the data points themselves. When θ is large (tends to infinity), the fit is perfectly smooth and the fitted values fall along a straight line, effectively forcing the relationship to be linear in x. The parameter θ is set between these extremes to produce the desired level of smoothness, and controls the trade-off between goodness-of-fit and smoothness. Although the cubic smoothing spline has received considerable attention recently in statistical modeling, it is usually attributed with appearing first in the actuarial literature in a paper on graduating mortality rates by Whittaker [25]. In fact Whittaker graduation is used widely for graduating mortality rates in the US.

Within the context of non-normal error distributions from the exponential family, a weighted version of Equation 3.2 is fitted by inserting an extra iterative algorithm within the optimization procedure. Details of this can be found in Hastie and Tibshirani [7], and Green and Silverman [6].

To construct a flexible framework for stochastic claims reserving, within which several of the models described in Section 2 can be regarded as special cases, we focus on the incremental paid claims C_{ij} and define

$$\mathrm{E}[C_{ij}] = m_{ij}, \tag{3.3}$$

$$\mathrm{Var}[C_{ij}] = \phi m_{ij}^{\rho}, \tag{3.4}$$

and

$$\ln(m_{ij}) = \eta_{ij} = u_{ij} + \delta t + c + s_{\theta_i}(i) + s_{\theta_j}(j) + s_{\theta_j}(\ln(j)). \tag{3.5}$$

Equations 3.3, 3.4 and 3.5 specify a generalized additive model with power variance function and constant scale parameter. The power ρ dictates the choice of error distribution, with normal, Poisson, Gamma and Inverse Gaussian specified by $\rho = 0$, 1, 2, and 3, respectively. The predictor is linked to the expected value of the response through the logarithmic link function. The offsets u_{ij} and inflation term δt are optional (where $t = i + j$), and may be suggested by a particular context. The function $s(i)$ represents a smooth of accident year i, obtained using a smoothing spline with smoothing parameter θ_i. Similarly, the functions $s(j)$ and $s(\ln(j))$ represent smoothing splines specifying the shape of the runoff pattern, with smoothing parameter θ_j chosen (for simplicity) to be the same for both functions. In practice, it may not be necessary to include smooths in both j and $\ln(j)$. It should be noted that both accident year i and development year j are considered as continuous covariates. It can

TABLE 3.2

GENERALIZED ADDITIVE MODEL REPRESENTATION OF SOME PUBLISHED STOCHASTIC RESERVING MODELS

	Variance power ρ	Row smoothing parameter θ_i	Column smoothing parameter θ_j
Wright (1990)*	1	0	∞
Mack (1991)	2	0	0
Renshaw and Verrall (1994, 1998)	1	0	0
Renshaw (1994)	1,2	0	0
Verrall (1996)	1	>0	0

*We consider here only the special case in which the same runoff pattern is used for all accident years, the Kalman filter is not used, and the scale parameter is constant.

be seen that use of Equation 3.5 implicitly assumes the same runoff pattern for all accident years, although the model can be extended using carefully chosen interaction terms. It is trivial to extend Equation 3.5 further, for example, to allow for a step change in a particular calendar year introduced by a change in legislation.

The extremes of the smoothing parameters are interesting and provide the link between Equation 3.5 and Equations 2.12 and 2.18 (ignoring the optional terms u_{ij} and δt). When θ_i is zero, there is no smoothing and the model is forced to pass through each value of i, which treats accident year i as though it is a factor (as in 2.12 and 2.18). The same is true of θ_j; when θ_j is zero, the model is forced to pass through each value of j, and development time is treated as though it is a factor (as in 2.18). When θ_j tends to infinity, the part of the model relating to development time is linear in j and $\ln(j)$, giving the Hoerl curve (as in 2.12 and 2.3). It is also necessary to choose the power function ρ to complete the model specification.

Table 3.2 shows how several previous stochastic reserving models can be seen as special cases of the model specified by

Equations 3.3, 3.4 and 3.5. The optional terms u_{ij} and δt are ignored without loss of generality.

The early log-linear models do not fit so neatly into the same framework because those models used log-incremental claims as the response, and required incorporation of a variance component in the mean of the predicted values. However, the framework could easily be extended to allow for this.

Notice that we consider only models in which the scale parameter in Equation 3.4 is assumed constant. This is for ease of exposition, although the model can be generalized further by relaxing this assumption and estimating the unknown scale parameters by joint modeling.

Having chosen the model specification, the model can be fitted using maximum quasi likelihood to obtain parameter estimates (and their approximate standard errors). At this point we make use of standard statistical software packages which have the facility to fit generalized additive models. Currently the choice is limited, although greater choice is likely in the future as the popularity of generalized additive models increases. The authors used S-PLUS [19] for the example (see also Chambers and Hastie [1]).

Having fitted the model, we obtained reserve estimates by summing the appropriate predicted values in the southeast region of the claims rectangle. All that remains is the estimation of variability in the reserve estimates, considered in the next section.

4. PRECISION OF RESERVE ESTIMATES

One of the principal advantages of stochastic reserving models is the availability of estimates of precision. Commonly used in prediction problems (as we have here) is the standard error of prediction, also known as the prediction error, or root mean square error of prediction. For claim payments in development year j for accident year i (yet to be observed), the mean square error of prediction is given by

$$\mathrm{E}[(C_{ij} - \hat{C}_{ij})^2] \approx \mathrm{Var}[C_{ij}] + \mathrm{Var}[\hat{C}_{ij}]. \qquad (4.1)$$

Note that the mean square error of prediction can be considered as the sum of two components: variability in the data (process variance) and variability due to estimation (estimation variance). The precise form of the two components of variance is dictated by the specification of the model fitted. For a detailed justification of Equation 4.1, see Renshaw [15].

For the general model defined above, the process variance is given by Equation 3.4. For the estimation variance, we note that

$$\hat{C}_{ij} = \hat{m}_{ij} = e^{\hat{\eta}_{ij}}.$$

Then, using a Taylor series expansion,

$$\mathrm{Var}[\hat{C}_{ij}] \approx \left| \frac{\partial m_{ij}}{\partial \eta_{ij}} \right|^2 \mathrm{Var}[\hat{\eta}_{ij}],$$

giving

$$\mathrm{E}[(C_{ij} - \hat{C}_{ij})^2] \approx \phi \hat{m}_{ij}^{\rho} + \hat{m}_{ij}^2 \mathrm{Var}[\hat{\eta}_{ij}]. \qquad (4.2)$$

The final component of Equation 4.2, the variance of the (linear) predictor, is usually available directly from statistical software packages, enabling the mean square error to be calculated without difficulty. The standard error of prediction is the square root of the mean square error of prediction.

The standard error of prediction for origin year reserve estimates and the total reserve estimates can also be calculated. Denoting the missing southeast region of the claims rectangle by Δ, then the reserve estimate in origin year i is given by summing the predicted values in row i of Δ; that is,

$$\hat{C}_{i+} = \sum_{j \in \Delta_i} \hat{C}_{ij}.$$

The mean square error of prediction of the origin year reserve is given by

$$\mathrm{E}[(C_{i+} - \hat{C}_{i+})^2] = \sum_{j\in\Delta_i} \phi\hat{m}_{ij}^{\rho} + \sum_{j\in\Delta_i} \hat{m}_{ij}^2 \mathrm{Var}[\hat{\eta}_{ij}] + 2 \sum_{\substack{j_1,j_2\in\Delta_i \\ j_2>j_1}} \hat{m_{ij_1}} \hat{m}_{ij_2} \mathrm{Cov}. \tag{4.3}$$

The total reserve estimate is given by

$$\hat{C}_{++} = \sum_{i,j\in\Delta} \hat{C}_{ij},$$

and the mean square error of prediction of the total reserve is given by

$$\mathrm{E}[(C_{++} - \hat{C}_{++})^2] = \sum_{i,j\in\Delta} \phi\hat{m}_{ij}^{\rho} + \sum_{i,j\in\Delta} \hat{m}_{ij}^2 \mathrm{Var}[\hat{\eta}_{ij}] + 2 \sum_{\substack{i_1j_1\in\Delta \\ i_2j_2\in\Delta \\ i_1j_1\neq i_2j_2}} \hat{m}_{i_1j_1} \hat{m}_{i_2j_2} \mathrm{Cov}[\hat{\eta}_{i_1j_1}, \hat{\eta}_{i_2j_2}]. \tag{4.4}$$

Although Equations 4.3 and 4.4 look fairly complex, they are relatively easy to calculate by summing the appropriate elements. The only components not readily available from statistical software packages are the covariance terms. Provided the *design matrix* and *variance-covariance matrix* of the parameter estimates can be extracted from the statistical software package used, a full matrix of the covariance terms can be calculated without difficulty for any specification of the predictor η. Indeed, the variances of the (linear) predictors are simply the diagonal of such a matrix.

It is also possible to obtain estimates of payments to be made in future settlement years by summing over diagonals in Δ, and also to obtain the associated standard error of prediction. Further details of this and a detailed derivation of Equations 4.3 and 4.4 can be found in Renshaw [15].

5. ASSESSING THE GOODNESS-OF-FIT

For a given error distribution (chosen by the power ρ), specific models are chosen by the smoothing parameters θ_i and θ_j, and different models are fitted by varying the smoothing parameters until a satisfactory fit is achieved. Assessing whether a model is satisfactory in practice is part art and part science. Usually, informal checks will suffice in practice, although model comparison can proceed formally in the usual way by comparing the difference in *deviances* of the fitted models (for fixed ρ) to the appropriate percentage point on the χ^2 or F distributions. However, because the smoothers are nonparametric, it is not obvious how many degrees of freedom should be used in the model comparison. According to the theory of cubic smoothing splines, it is possible to assess the equivalent degrees of freedom used in fitting the spline. This has an inverse relationship to the smoothing parameter: as the smoothing parameter increases, the equivalent degrees of freedom decrease. After fitting a cubic smoothing spline, statistical software packages provide the equivalent degrees of freedom as part of the model output. One problem is that the smoothing parameter is a continuous measure, which can result in noninteger degrees of freedom. For this reason, software packages tend to allow the amount of smoothness to be defined alternatively by the equivalent degrees of freedom, which is provided by the user. The smoothness parameter to be used is then calculated from the given degrees of freedom.

The choice of error distribution is not easy to justify but may be suggested on theoretical grounds. Formally, given identical specifications of the predictor, the optimum value of ρ (which specifies the choice of error distribution) is that which produces the highest likelihood.

Residual plots are also used to assess the adequacy of any fitted model. Two types of residual used commonly are the Pearson and deviance residuals. The scaled Pearson residuals are defined by

$$r_{ij} = \frac{C_{ij} - \hat{m}_{ij}}{\sqrt{\phi \hat{m}_{ij}^{\rho}}},$$

and the scaled deviance residuals are defined by

$$\sim r_{ij} = \operatorname{sign}(C_{ij} - \hat{m}_{ij})\sqrt{\frac{d_{ij}}{\phi}},$$

where d_{ij} is the contribution to the deviance made by observation C_{ij}.

For a reasonable model, a histogram of scaled residuals is expected to be approximately normal (i.e., bell shaped) with 95% of the residuals between the values plus two and minus two. Residuals can also be plotted against the predictor, against origin year and against development year. The plots are expected to be pattern free, where an obvious pattern in the residuals would indicate a systematic departure from the fitted model. Isolated departures from the model would be indicated by residuals whose values are far from zero. Other residual plots are also possible. It is usual to assess residual plots visually, any serious model deficiencies being immediately obvious.

A further visual check which is useful when comparing models is to plot that part of the predictor that explains the runoff pattern against development time. From Equation 3.5, this translates into plotting $c + s_{\theta_j}(j) + s_{\theta_j}(\ln(j))$ against j for various values of θ_j. The constant c is needed to ensure the plots start at equivalent levels. A plot such as this might result in the choice of a model which is not optimal in the statistical sense, but which may have convenient properties (for example, the way it behaves when extrapolating into the tail).

TABLE 6.1

INCREMENTAL PAID LOSSES FORMED BY AGGREGATING ACROSS DIFFERENT CLASSES

	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	$j=6$	$j=7$	$j=8$	$j=9$	$j=10$
$i=1$	45630	23350	2924	1798	2007	1204	1298	563	777	621
$i=2$	53025	26466	2829	1748	732	1424	399	537	340	
$i=3$	67318	42333	−1854	3178	3045	3281	2909	2613		
$i=4$	93489	37473	7431	6648	4207	5762	1890			
$i=5$	80517	33061	6863	4328	4003	2350				
$i=6$	68690	33931	5645	6178	3479					
$i=7$	63091	32198	8938	6879						
$i=8$	64430	32491	8414							
$i=9$	68548	35366								
$i=10$	76013									

6. EXAMPLE: PART 1—A COMPARISON OF PREDICTOR STRUCTURES

Incremental paid losses from an aggregation of classes of business are shown in Table 6.1 and are used to illustrate the methodology. The incremental claims fall fairly rapidly, but are not completely runoff by the end of the tenth development year, implying the necessity for a tail factor greater than 1 when using the traditional chain ladder model. Notice the negative incremental claim at position (3,3), which is not a problem when implementing the models.

Initially, to illustrate the methodology, we fit three models, using an overdispersed Poisson model ($\rho = 1$ in Equation 3.4) with a logarithmic link function. For all three models

$$E[C_{ij}] = m_{ij}, \qquad \mathrm{Var}[C_{ij}] = \phi m_{ij}, \qquad \text{and} \qquad \ln(m_{ij}) = \eta_{ij}.$$

The models differ only in the choice of the predictor. The predictor structures are:

- Model 1: The stochastic model of Renshaw and Verrall [17], which gives the same reserve estimates as the chain ladder model:

$$\eta_{ij} = c + \alpha_i + \beta_j.$$

This model can be specified as a generalized additive model with $\theta_i = 0$ and $\theta_j = 0$ (no smoothing), giving

$$\eta_{ij} = c + s_0(i) + s_0(j) + s_0(\ln(j)).$$

- Model 2: The Hoerl curve, ignoring inflation:

$$\eta_{ij} = u_j + c + \alpha_i + \beta \ln(j) + \gamma j.$$

This is in the spirit of the model proposed by Wright [26]. Here we adopt the technical adjustments to development time recommended by Wright, and the associated offset (ignoring exposure information). However, we are using the same runoff pattern for each accident year (since β and γ do not depend on i), we ignore the Kalman filter, and we are using a constant scale parameter.

Again, this model can be specified as a generalized additive model with $\theta_i = 0$ and $\theta_j = \infty$, giving

$$\eta_{ij} = u_j + c + s_0(i) + s_\infty(j) + s_\infty(\ln(j)).$$

- Model 3: A generalized additive model with a parameter for each accident year, but with the pattern over development year represented by a smooth in log development time. We have chosen not to include additionally a smooth in development time, which in this case is unnecessary. Therefore $\theta_i = 0$ and θ_j is chosen to provide a suitable level of smoothing, giving

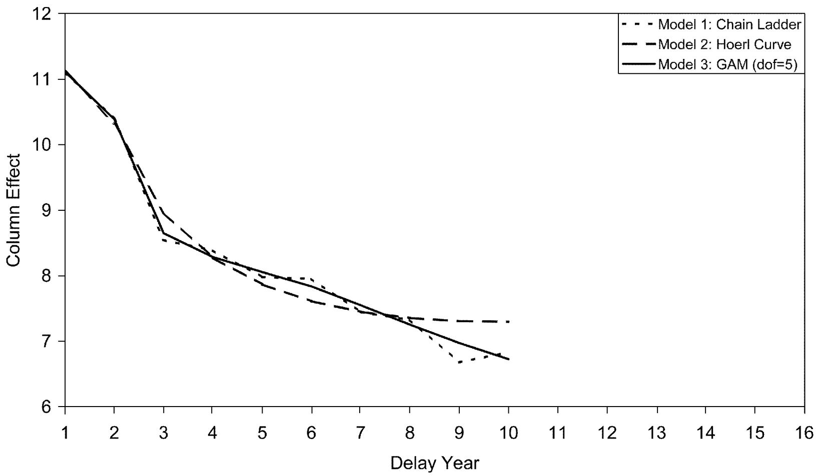
$$\eta_{ij} = u_j + c + s_0(i) + s_{\theta_j}(\ln(j)),$$

or equivalently

$$\eta_{ij} = u_j + c + \alpha_i + s_{\theta_j}(\ln(j)).$$

FIGURE 6.1

COLUMN EFFECTS VS. DELAY YEAR

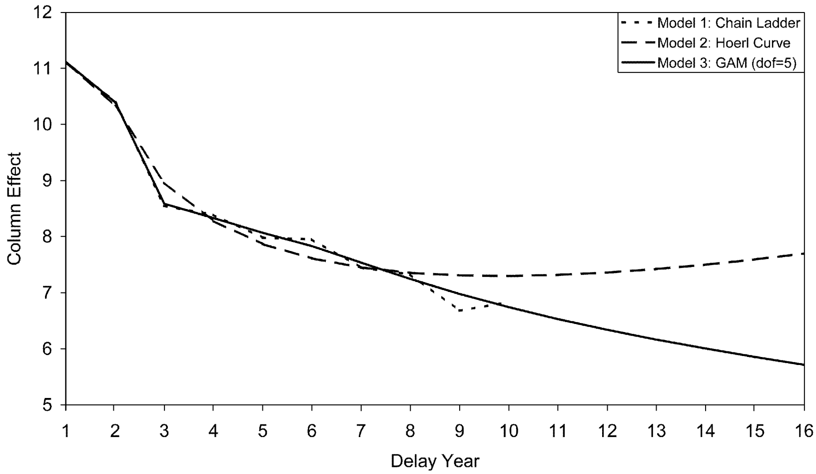


This can be seen as a smooth model in between the chain ladder and Hoerl curve models. For this example, the smoothing parameter was dictated by setting the equivalent degrees of freedom (dof) used in the fit (in this case dof = 5).

First, consider the part of each predictor that describes the shape of the decay of the incremental claims (the sum of the components not dependent on i). We shall call this the "column effects." Figure 6.1 shows the column effects for all three models, and there we can see the jagged shape of the decay in the incremental claims assumed by the chain ladder model and the smooth shape of the model using the Hoerl curve. The Hoerl curve passes through the chain ladder model, fitting closely in the early stages of development (where we have the most data) but fails to fall rapidly enough in the later stages of development. This is the result of the strict parametric form imposed by the Hoerl curve. (A practitioner would probably reject the model at this point, but we will continue to highlight the characteristics

FIGURE 6.2

COLUMN EFFECTS (EXTRAPOLATED) VS. DELAY YEAR



of the Hoerl curve and to enable a comparison with the generalized additive model methodology.) Model 3 is in between the extremes of Models 1 and 2, and exhibits a satisfactory mix of smoothness and adherence to the data. If the smoothing parameter of Model 3 is reduced, it will tend towards Model 1. Conversely, if the smoothing parameter of Model 3 is increased, it will tend towards Model 2.

In Figure 6.2, Models 2 and 3 have been extrapolated a further six years. With this example, an inherent danger of extrapolating using rigid parametric curves like the Hoerl curve is highlighted since the curve bends upwards beyond the range of data observed. One advantage of Model 3 is that it continues in a more desirable direction when extrapolating.

Although natural in stochastic claims reserving, it is unusual to focus on the shape of the decay of incremental claims using traditional actuarial methods, in which it is common to focus on the relative increase in cumulative claims through *development factors*, the traditional "parameters" in a standard chain ladder exercise. After fitting a stochastic claims reserving model, it is straightforward to obtain *equivalent development factors* by applying the standard chain ladder model to the fitted values of the stochastic model. If the model is fully parametric, it may be possible to obtain a relationship between the model parameters and the chain ladder development factors (e.g., Verrall [23]).

Equivalent development factors are shown in Table 6.2 for Models 1 to 3, together with the actual development factors obtained by applying the standard chain ladder model to the data in Table 6.1. It can be seen that the development factors implied by the stochastic chain ladder model (Model 1) are identical to those obtained using standard chain ladder methodology (therefore reserve estimates obtained using the two models will also be identical). A comparison of the development factors implied by the Hoerl curve (Model 2) and the chain ladder models reveals where these two models differ. In particular, the Hoerl curve does not fully capture the fall in the development factors in the later stages of development. The development factors implied by Model 3 can be seen as a smoothed version of the chain ladder development factors.

Also shown in Table 6.2 are the equivalent development factors obtained when extrapolating beyond development year 10. It can be seen clearly that the development factors implied by the Hoerl curve increase in value, whereas the development factors implied by Model 3 continue to decrease.

The reserve estimates implied by Models 1, 2 and 3 are shown in Table 6.3, together with their prediction errors (as a percentage of the reserves). For ease of comparison with the chain ladder model, we have not extrapolated into the tail. The reserve estimates given by the Hoerl curve are higher for the older years than those given by the chain ladder model, reflecting the higher development factors at the later stages of development. The

TABLE 6.2

EQUIVALENT DEVELOPMENT FACTORS: OVERDISPERSED POISSON MODEL

Delay Year	Standard Chain Ladder	Model 1 Stochastic Chain Ladder	Model 2 Hoerl Curve	Model 3 GAM (dof = 5)
2	1.4906	1.4906	1.4496	1.4891
3	1.0516	1.0516	1.0796	1.0537
4	1.0419	1.0419	1.0372	1.0395
5	1.0268	1.0268	1.0238	1.0292
6	1.0254	1.0254	1.0180	1.0224
7	1.0149	1.0149	1.0150	1.0163
8	1.0130	1.0130	1.0135	1.0120
9	1.0067	1.0067	1.0127	1.0091
10	1.0078	1.0078	1.0124	1.0071
11			1.0125	1.0057
12			1.0129	1.0047
13			1.0135	1.0039
14			1.0144	1.0033
15			1.0156	1.0029
16			1.0171	1.0025

reserve estimates given by Model 3 are close to those provided by the chain ladder model for all years individually and in total, with any differences arising due to the amount of smoothing.

The reduced number of parameters in the Hoerl curve compared to the stochastic chain ladder model should drive down the prediction error, but this is offset by the increased variability imposed by the poor fit, resulting in prediction errors for the Hoerl curve which are close to those provided by the stochastic chain ladder model. The equivalent degrees of freedom used up in fitting Model 3 is lower than the degrees of freedom used up in fitting the stochastic chain ladder model, which will drive down the prediction errors. Furthermore, the fit is good relative to the chain ladder model, which has the desirable effect of lower prediction errors for Model 3 compared to the stochastic chain ladder model.

TABLE 6.3

RESERVE ESTIMATES AND PREDICTION ERRORS: OVERDISPERSED POISSON MODEL

	Reserve Estimates			Prediction Error		
Accident Year	Model 1 Stochastic Chain Ladder	Model 2 Hoerl Curve	Model 3 GAM (dof = 5)	Model 1 Stochastic Chain Ladder	Model 2 Hoerl Curve	Model 3 GAM (dof = 5)
1	0	0	0	—	—	—
2	683	1,085	622	159%	95%	110%
3	1,792	3,101	1,998	100%	61%	62%
4	4,363	6,129	4,470	63%	46%	43%
5	5,657	7,173	5,940	50%	43%	38%
6	8,209	8,689	8,106	40%	39%	33%
7	10,914	11,031	11,106	34%	34%	29%
8	15,199	14,765	15,112	28%	30%	25%
9	21,135	24,002	21,293	24%	23%	22%
10	60,335	59,625	60,377	17%	17%	16%
Total	128,286	135,600	129,024	15%	15%	12%

Models 1 and 2 can be fitted in any statistical software package that fits generalized linear models. Model 3 can only be fitted in statistical software packages that fit generalized additive models.

The comparison of Model 3 with Models 1 and 2 begins to show how our modeling framework can be considered generic, since the chain ladder model and Hoerl curve model can be fitted as special cases, using extremes of the smoothing parameters. A model that has the desirable characteristic of being able to smooth development factors can be fitted by choosing smoothing parameters between these extremes.

7. EXAMPLE: PART 2—A COMPARISON OF ERROR STRUCTURES

Continuing the example, the same three model predictors are used, but with a Gamma error structure ($\rho = 2$) giving:

$$E[C_{ij}] = m_{ij}, \qquad \mathrm{Var}[C_{ij}] = \phi m_{ij}^2, \qquad \text{and} \qquad \ln(m_{ij}) = \eta_{ij},$$

TABLE 7.1

EQUIVALENT DEVELOPMENT FACTORS: GAMMA MODEL

Delay Year	Standard Chain Ladder	Model 4 Stochastic Chain Ladder	Model 5 Hoerl Curve	Model 6 GAM (dof = 5)
2	1.4906	1.4969	1.4515	1.4771
3	1.0516	1.0470	1.0799	1.0512
4	1.0419	1.0381	1.0372	1.0357
5	1.0268	1.0259	1.0237	1.0280
6	1.0254	1.0251	1.0178	1.0221
7	1.0149	1.0154	1.0148	1.0165
8	1.0130	1.0131	1.0131	1.0125
9	1.0067	1.0084	1.0123	1.0098
10	1.0078	1.0086	1.0119	1.0079
11			1.0119	1.0066
12			1.0122	1.0055
13			1.0127	1.0048
14			1.0135	1.0041
15			1.0145	1.0036
16			1.0157	1.0032

and the following three models:

- Model 4:

$$\eta_{ij} = c + \alpha_i + \beta_j;$$

- Model 5:

$$\eta_{ij} = u_j + c + \alpha_i + \beta \ln(j) + \gamma j;$$

- Model 6:

$$\eta_{ij} = u_j + c + \alpha_i + s_{\theta_j}(\ln(j)).$$

Equivalent development factors are shown in Table 7.1, and reserve estimates and prediction errors are shown in Table 7.2 (ignoring tail factors).

Comparison of the equivalent development factors from the Gamma model with those from the overdispersed Poisson model is uninformative on the whole. It is perhaps surprising at first

TABLE 7.2

RESERVE ESTIMATES AND PREDICTION ERRORS: GAMMA MODEL

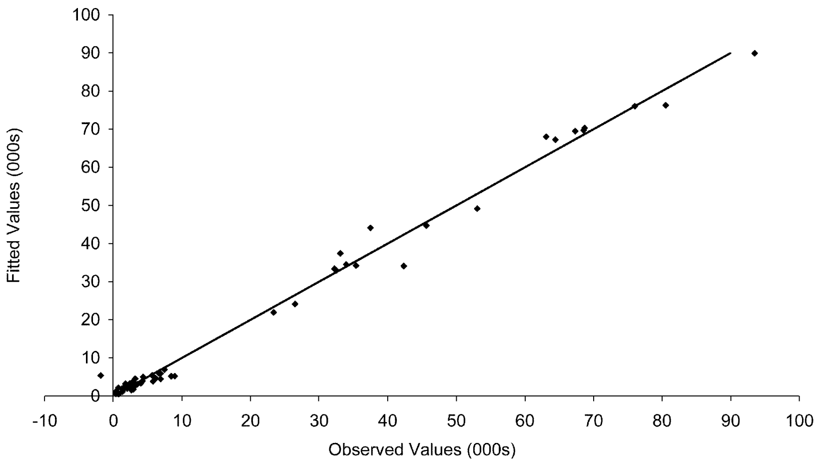
	Reserve Estimates			Prediction Error		
Accident Year	Model 4 Stochastic Chain Ladder	Model 5 Hoerl Curve	Model 6 GAM (dof = 5)	Model 4 Stochastic Chain Ladder	Model 5 Hoerl Curve	Model 6 GAM (dof = 5)
1	0	0	0	—	—	—
2	488	675	450	62%	46%	43%
3	2,086	3,296	2,205	43%	36%	33%
4	5,240	6,818	5,300	36%	32%	29%
5	6,169	7,061	6,313	32%	30%	28%
6	9,750	9,305	9,427	31%	29%	28%
7	15,080	13,029	15,097	31%	29%	29%
8	18,498	15,069	17,671	32%	30%	31%
9	20,470	24,400	20,896	36%	35%	35%
10	60,043	59,576	58,519	52%	48%	48%
Total	137,824	139,229	135,878	25%	23%	24%

sight that the final development factor for the Gamma "chain ladder" model (1.0086) is greater than the equivalent factor from the overdispersed Poisson model (1.0078), but at the same time the reserve estimate is lower (488 vs. 683). This is because the cumulative fitted values for the final observed diagonal of the two models are not the same, resulting in the observed effect. In fact, the cumulative fitted values for the final observed diagonal are identical to the cumulative paid to date for the overdispersed Poisson chain ladder model only.

The main difference between the overdispersed Poisson and Gamma models in this example is in the prediction errors as a percentage of the total reserve estimates, which for the Gamma model are around twice those of the Poisson model. Inspection of the prediction errors of the row reserves gives a hint as to why this is so. For the Gamma model, the prediction errors for the earlier years are lower than those for the Poisson model.

FIGURE 7.1

FITTED VALUES (POISSON MODEL) VS. OBSERVED VALUES

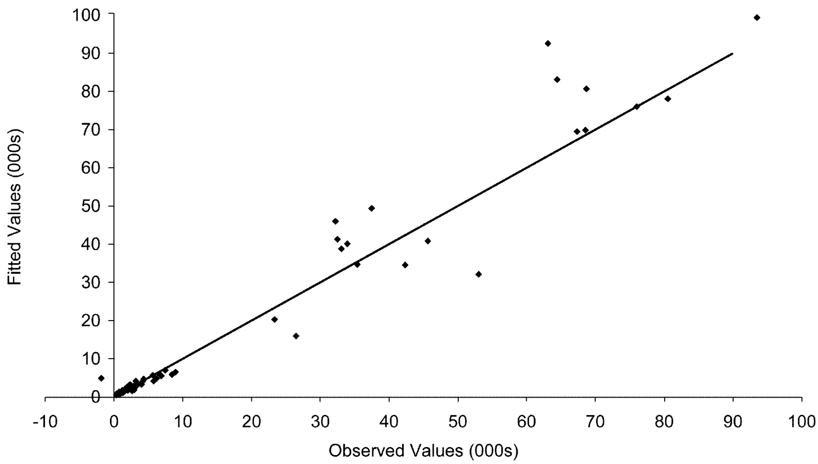


However, the pattern is reversed in the later years, particularly for year 10. The later years contribute by far the largest proportion of the total reserves, which is reflected in the high prediction error of the total.

The fit of the Gamma model is in fact poor in this example, particularly in the early stages of development, where the large incremental observed values are given less weight in the model fitting than in the Poisson model. This is not apparent from an inspection of residual plots (not shown), which look satisfactory for both error structures, but becomes apparent when plotting fitted values against observed values (Figures 7.1 and 7.2), which show clearly the superiority of the Poisson model in this example. This is not always the case, however, and care must be taken in making inferences from these results. For a further example in which prediction errors of claims reserves are compared using different error structures and different methodologies, see England and Verrall [5].

FIGURE 7.2

FITTED VALUES (GAMMA MODEL) VS. OBSERVED VALUES



8. DISCUSSION AND CONCLUSIONS

Given a triangle of data, a simple reserving exercise might proceed by fitting a chain ladder model (usually a 3, 4, or 5 year volume-weighted average chain ladder) and looking at the resultant development factors. It would then be common to smooth the factors manually and consider the necessity of a tail factor for projecting beyond the range of data observed. Judgment is used to smooth the factors with the aim of smoothing out random variations, particularly in the later stages of development, while leaving the systematic trend intact. A tail factor might be chosen by calculating the ratio of cumulative incurred claims to cumulative paid claims for the oldest accident year, or by fitting a curve to the later development factors and extrapolating (see, for example, Craighead [3] and Sherman [18]). Advantages of this procedure are that it is extremely flexible, and it forces the actuary to look at the data. Disadvantages are that it is time consuming, statistically inefficient, and it is not always easy to be consistent over the level of smoothing (or confident in the results).

The main strength of the method presented in this paper is that both the smoothing and extrapolating can be performed at the same time in the same model. The actuary simply has to choose one parameter for smoothing across the whole range of development time, choose an error distribution, and choose how far to extrapolate (an additional parameter is necessary if smoothing over accident years). Further advantages are that it is also possible to obtain measures of precision of the reserve estimates, and investigate where the data deviate from the fitted model by viewing residual plots. The fact that standard models can be fitted by choosing smoothing parameters at the extremes is a useful additional feature, if only for clarity of understanding, since at one extreme the model can be considered overparameterized, and at the other that the structure is too rigid. However, we do not consider the method to be a panacea. A thorough reserving exercise will involve an in depth investigation of the data, an understanding of the class of business under review, and a comparison of the results of several reserving methods relying on complementary sets of data. We believe the method proposed here is simply an extremely useful additional tool for the reserving specialist.

Incremental data are used for the method put forward in this paper: this is both an advantage and a disadvantage. It is advantageous since the method can be used when the data history is incomplete. If incremental data were recorded by accident year only after a certain date, accident years prior to that date will have incomplete runoff information, and a section of the claims triangle in the northwest corner will be missing (this is a reasonably common occurrence). This presents difficulties using standard deterministic techniques that rely on cumulative data, but is not a problem for stochastic techniques which treat the unobserved data as "missing" and estimate the data as part of the fitting procedure. The disadvantage is that negative incremental values sometimes occur in data based on paid losses, and frequently occur in data based on incurred losses where case estimates are often set on a conservative basis and overestimated. The method proposed is robust to a small number of negative incremental claims (as in the example), but will always produce positive fitted values (due to the use of the logarithmic link function) and hence will always produce development factors greater than one. For this reason, the techniques are often not suitable for use with incurred data which often include a series of negative incremental losses in the later stages of development requiring development factors less than one.

In the framework proposed in this paper, a constant scale parameter has been used. This is for ease of exposition; the assumption can be relaxed to allow the scale parameter to be modeled as part of an extended procedure. The difference between the prediction errors of the overdispersed Poisson and Gamma models in Section 7 is partly due to the use of a constant scale parameter, and further research is needed to evaluate how much of the difference can be ameliorated by joint modeling.

The main use of stochastic reserving methods is in the provision of estimates of reserve variability, not in the reserve estimates themselves. Until recently, measures of variability have been of little interest to most general insurance actuaries, but interest is likely to increase as the need to parameterize and calibrate dynamic financial analysis (DFA) models becomes routine. Part of a DFA exercise is quantifying reserving risk, and to do this, it is necessary to have a model that simulates the likely payments of outstanding liabilities. Stochastic reserving techniques provide a model structure and a way of calibrating the model to real data, from which payments can be simulated (taking care to allow for process and estimation error).

As outlined in Section 2, there is a wide variety of methods available for stochastic claims reserving. If the use of these methods increases, it is important that the similarities and differences of the models are understood, and their properties examined. By presenting some of the models within the same framework, and extending to allow flexibility between the extremes of two well-known models, it is hoped that this paper has contributed to the process.

REFERENCES

[1] Chambers, John M. and Trevor J. Hastie, *Statistical Models in S*, Chapman and Hall, London, 1992.

[2] Christofides, Stavros, "Regression Models Based on Log-Incremental Payments," *Claims Reserving Manual* 2, 1990, Institute of Actuaries, London.

[3] Craighead, David H., "Some Aspects of the London Reinsurance Market in World-Wide Short-Term Business," *Journal of the Institute of Actuaries* 106, 1979, pp. 227–287.

[4] De Jong, Piet and Ben Zehnwirth, "Claims Reserving, State-Space Models and the Kalman Filter," *Journal of the Institute of Actuaries* 110, 1983, pp. 157–182.

[5] England, Peter D. and Richard J. Verrall, "Analytic and Bootstrap Estimates of Prediction Errors in Claims Reserving," *Insurance: Mathematics and Economics* 25, 1999, pp. 281–293.

[6] Green, Peter J. and Bernard W. Silverman, *Nonparametric Regression and Generalized Linear Models*, Chapman and Hall, London, 1994.

[7] Hastie, Trevor J. and Robert J. Tibshirani, *Generalized Additive Models*, Chapman and Hall, London, 1990.

[8] Kremer, Erhard, "IBNR Claims and the Two Way Model of ANOVA," *Scandinavian Actuarial Journal*, 1982, pp. 47–55.

[9] Mack, Thomas, "A Simple Parametric Model for Rating Automobile Insurance or Estimating IBNR Claims Reserves," *ASTIN Bulletin* 22, 1, 1991, pp. 93–109.

[10] Mack, Thomas, "Distribution Free Calculation of the Standard Error of Chain Ladder Reserve Estimates," *ASTIN Bulletin* 23, 2, 1993, pp. 213–225.

[11] Mack, Thomas, "Which Stochastic Model is Underlying the Chain Ladder Model?" *Insurance: Mathematics and Economics* 15, 1994, pp. 133–138.

[12] McCullagh, Peter and John Nelder, Generalized Linear Models, 2nd Edition, Chapman and Hall, 1989.

[13] Renshaw, Arthur E., "Chain Ladder and Interactive Modelling (Claims Reserving and GLIM)," *Journal of the Institute of Actuaries* 116, III, 1989, pp. 559–587.

[14] Renshaw, Arthur E., "Claims Reserving by Joint Modelling," *Actuarial Research Paper No. 72*, 1994, Department of Actuarial Science and Statistics, City University, London, EC1V 0HB.

[15] Renshaw, Arthur E., "On the Second Moment Properties and the Implementation of Certain GLIM Based Stochastic Claims Reserving Models," *Actuarial Research Paper No. 65*, 1994, Department of Actuarial Science and Statistics, City University, London, EC1V 0HB.

[16] Renshaw, Arthur E. and Richard J. Verrall, "A Stochastic Model Underlying the Chain Ladder Technique," *Proceedings XXV ASTIN Colloquium*, Cannes, 1994.

[17] Renshaw, Arthur E. and Richard J. Verrall, "A Stochastic Model Underlying the Chain Ladder Technique," *British Actuarial Journal* 4, IV, 1998, pp. 903–923.

[18] Sherman, Richard E., "Extrapolating, Smoothing and Interpolating Development Factors," *PCAS* LXXI, 1984, pp.122–192.

[19] *S-PLUS User's Guide*, Data Analysis Products Division, Mathsoft, Seattle, WA, 1997.

[20] Taylor, Greg C. and F. R. Ashe, "Second Moments of Estimates of Outstanding Claims," *Journal of Econometrics* 23, 1983, pp. 37–61.

[21] Verrall, Richard J., "A State Space Representation of the Chain Ladder Linear Model," *Journal of the Institute of Actuaries* 116, III, 1989, pp. 589–609.

[22] Verrall, Richard J., "On the Unbiased Estimation of Reserves From Log-Linear Models," *Insurance: Mathematics and Economics* 10, 1991, pp. 75–80.

[23] Verrall, Richard J., "Chain Ladder and Maximum Likelihood," *Journal of the Institute of Actuaries* 118, III, 1991, pp. 489–499.

[24] Verrall, Richard J., "Claims Reserving and Generalised Additive Models," *Insurance: Mathematics and Economics* 19, 1996, pp. 31–43.

[25] Whittaker, Edmund T., "On a New Method of Graduation," *Proceedings of the Edinburgh Mathematical Society* 41, 1923, pp. 63–75.

[26] Wright, Tom S., "A Stochastic Method for Claims Reserving in General Insurance," *Journal of the Institute of Actuaries* 117, 1990, pp. 677–731.

[27] Zehnwirth, Ben, "The Chain Ladder Technique—A Stochastic Model," *Claims Reserving Manual* 2, 1989, Institute of Actuaries, London.

[28] Zehnwirth, Ben, *ICRFS Version 4 Manual and Users Guide*, Benhar Nominees Pty. Ltd., Turramurra, NSW, Australia, 1985.

[29] Zehnwirth, Ben, *Interactive Claims Reserving Forecasting System (ICRFS)*, Insureware P/L, E. St. Kilda, Victoria 3183, Australia, 1991.

THE *n*-MOMENT INSURANCE CAPM

THOMAS J. KOZIK AND AARON M. LARSON

Abstract

Until recently, the importance of skewness in the rate of return distribution has been largely unrecognized in financial journals. The re-emergence of skewness in financial literature is particularly relevant to catastrophe insurance products where some of the most extremely skewed distributions occur. This paper presents an argument for including a provision in the equilibrium premium to cover the cost of skewness. It also generalizes the insurance CAPM to n *moments. This extension permits explicitly determining the impact that skewness and other higher moments have on the needed premium.*

1. ASYMMETRY AND ITS IMPLICATIONS

In much of modern finance theory, it is presumed that the standard deviation of the rate of return is the appropriate measure of risk to the investor. The Capital Asset Pricing Model (CAPM), for example, assumes this to be true. It is then a mathematical consequence of this and a few other assumptions that only the systematic component (beta) of this risk is rewarded in financial markets. This seems quite reasonable for returns that are symmetrically distributed. It does not seem so reasonable, however, for returns that are asymmetrically distributed. Consider that, although investors dislike unexpected large losses, they like unexpected large gains. It seems reasonable then that investors place different values on two different securities that promise the same expected return and the same standard deviation of return but differ in that the return on one is symmetrically distributed while the return on the other is positively skewed.[1] In fact, there are reasons to believe, and evidence which corroborates, that the latter security is preferred to the former.

For example, Arditti (1967, p. 21) argues that it is reasonable to expect risk aversion to decrease with wealth. He gives an example of a bet with two equally likely outcomes: either a loss of \$10,000 or a gain of \$20,000. Since both outcomes are equally likely the expected value is \$5,000. He then asks if a wealthy man or a poor man would more likely pay a higher price for this bet. Arditti concludes that it is reasonable to expect a wealthy man to pay more for this bet since in his words "a loss of \$10,000 to him would be trivial while a similar loss to the poor man would render him assetless." Arditti goes on to show that whenever risk aversion decreases with wealth, it necessarily follows that positive skewness is preferred. That is, investors are willing to pay a premium, or give up expected return, in exchange for positive skewness.

One does not have to go any farther than to consider all of the various state-run lotteries as corroborating examples. Lottery players face an almost certain loss of a trivial amount in exchange for a trivial probability of a very large gain. The expected return on lottery tickets is, of course, negative since government extracts a significant portion of the revenues. Lottery players, thus, pay a premium in exchange for positive skewness.

Others have reached the same conclusions for opportunities similar to the lottery. In a discussion trying to explain Internet stock price increases, Alan Greenspan (1999, p. C1) described this "lottery premium" in the *Wall Street Journal*:

> What lottery managers have known for centuries is that you could get somebody to pay for a one-in-a-

[1] For purposes of this paper, we are using William Sharpe's (1985) definition of security, i.e., a security is "a legal representation of the right to receive prospective future benefits under stated conditions."

> million shot more than the [pure economic] value of that chance.

Consider, for a moment, the lottery as a relevant analogy to understand the skewness associated with catastrophes. Catastrophe insurance can be thought of from the policyholder's perspective as a conditional lottery. This provides a concrete example of the cost of skewness. With this lottery, if the catastrophe occurs then there is a huge payoff. Of course, there is also a large loss that offsets the payoff. But the loss is there regardless of insurance. Thus, if the loss is going to happen, it is preferable to have insurance.

Imagine a security that trades in financial markets and promises a large payoff in the event of a catastrophe somewhere else in the world. The details don't really matter for this example, as long as the payoff is triggered by a rare, random event. Since the cash flows are similar to those of a lottery, we can expect that the purchasers, as is true with a lottery, would pay a skewness premium. One implication of the Capital Asset Pricing Model is that all investors hold the same portfolio of risky assets, the market portfolio, even if it might include lottery tickets. Since investors are holding the market portfolio, the skewness premium would reflect only systematic components of skewness, i.e., that portion of skewness that cannot be diversified away. But the cash flows on this security are also similar to those of catastrophe insurance. Hence, the free market price of this security, which includes the cost of skewness, must also equal the equilibrium price for a perfectly corresponding catastrophe insurance contract, i.e., a contract with the same expected cash flows, the same systematic risk of receiving those cash flows, the same systematic skewness, etc.

One might argue that this analogy is inappropriate, since there is a fundamental difference between the demand for lottery tickets and catastrophe insurance. The cost of skewness, however, is unaffected. Consider that a person might be willing to buy a lottery ticket for a dollar, but unwilling to buy

1,000,000 of them. Clearly a person's willingness to buy tickets depends on his overall wealth as well as his preference for skewness and other factors. Certainly he would be more willing to buy one lottery ticket rather than say 200 (the price of the catastrophe insurance). With a single lottery ticket there is only one dollar at risk. With 200 tickets, there are 200 dollars at risk. What motivates people to buy the catastrophe insurance, though, is that the lottery is contingent on an otherwise bad event. It is offsetting the risk of that bad event that motivates them to buy catastrophe insurance. Accordingly, we can expect that people are more willing to buy 200 dollars worth of catastrophe insurance than 200 dollars worth of lottery tickets. But the cash flows in the catastrophe insurance are identical to the cash flows in the lottery, so the cost of skewness must be the same for both. Preference for skewness varies from individual to individual in a complex and unknown way. It is certainly multi-variate, with wealth being one of the variables. But in the aggregate, the market determines the price for skewness in such a way that the markets clear. Demand is also a variable that depends upon price, and so supply and demand are in balance at the equilibrium price.

Hence, the equilibrium returns implied by the CAPM may be inadequate for securities with heavily skewed returns. Accordingly, to adequately charge for an insurance policy covering hurricane and other catastrophic risks, a provision covering the cost of skewness must be added to the otherwise needed premium to compensate investors for the extremely skewed loss distributions of catastrophes.

Others have also recognized this shortcoming of the CAPM. For example, Yehuda Kahane (1979) notes the need for analyzing higher moments of profit distributions for certain utility assumptions in his paper deriving the insurance CAPM. He states on page 237:

> All distributions were assumed to be characterized by the first two moments. This makes the model ac-

> ceptable only for certain utility assumptions. … Thus, measures of asymmetry, like the skewness and semi-variance, may be needed in a loading formula (especially for risks with catastrophic nature—which are represented by extremely skewed distributions).

Alan Kraus and Robert Litzenberger (1976) go even further by stating on page 1086 that:

> The evidence suggests that prior empirical findings that are interpreted as inconsistent with the traditional theory can be attributed to misspecification of the capital asset pricing model by omission of systematic (non-diversifiable) skewness.

Campbell Harvey and Akhtar Siddique (2000) define systematic skewness, or coskewness on page 1265:

> [Coskewness is] the component of an asset's skewness related to the market portfolio's skewness.

In order to capture the contribution of the cost of skewness to the equilibrium return, it is necessary to generalize the CAPM. Section 2 presents the three-moment CAPM derived by Rubinstein (1973) and Kraus and Litzenberger (1976). Section 3 derives the three-moment insurance CAPM. Section 4 derives the n-moment insurance CAPM. This derivation depends on the n-moment CAPM that is derived in the Appendix. Section 5 presents conclusions and implications.

2. THE THREE-MOMENT CAPM

2.1. The Model

Kraus–Litzenberger (1976) follow Rubinstein's lead (1973) in their development of a three-moment capital asset pricing model that incorporates the coskewness of an asset. (See the Appendix for a formal derivation of the model.) Their model of equilibrium returns, assuming the rate of return on the market portfolio is nonsymmetrically distributed, is given below:

$$E(R_i) - R_f = b_1\beta_i + b_2\gamma_i \tag{2.1}$$

where

$$R_f = 1 + r_f = \text{one plus the risk-free rate of return,}$$

$$R_i = 1 + r_i = \text{one plus the rate of return on } i\text{th asset,}$$

$$R_M = 1 + r_M = \text{one plus the rate of return on market portfolio,}$$

$$\beta_i = \frac{\sigma_{R_i R_M}}{\sigma^2_{R_M}} = \frac{E([R_i - E(R_i)][R_M - E(R_M)])}{E([R_M - E(R_M)]^2)},$$

$$\gamma_i = \frac{\tau_{R_i R_M R_M}}{\tau^3_{R_M}} = \frac{E([R_i - E(R_i)][R_M - E(R_M)]^2)}{E([R_M - E(R_M)]^3)},$$

$$\tau_{R_M} = (E[(R_M - E(R_M))^3])^{1/3},$$

$$b_1 = \text{market risk premium, and}$$

$$b_2 = \text{market skewness premium.}$$

Simplifying (2.1) leads to:

$$E(r_i) - r_f = b_1\beta_i + b_2\gamma_i. \tag{2.2}$$

One final simplification leads to the intercept form of the equation:

$$E(r_i) = r_f + b_1\beta_i + b_2\gamma_i. \tag{2.3}$$

Kraus and Litzenberger's derivation assumes that all investors have the same probability beliefs, and further, that each investor's risk tolerance is a linear function of wealth, $(a_i + bW_i)$, with the same cautiousness, b, for all investors. These assumptions are required to ensure that each investor's optimal risk asset portfolio is the same, that is, the market portfolio. These assumptions are very strong and arguably unreasonable. However, if one's purpose is to estimate equilibrium returns, then it is not essential that all investors have the same optimal risk asset portfolio. In the case of disagreement, b_1 and b_2 may still be interpreted as the market price of risk and the market price of skewness, respectively, as will be shown in a later section of this paper.

Kraus and Litzenberger empirically tested the three-moment model using monthly, deflated excess rates of return. That is, their measure of the rate of return for the ith security is $(R_i - R_f)/R_f$, where the returns are measured over a monthly holding period. They state on page 1098:

> Empirical evidence is presented that is consistent with a three moment valuation model. Investors are found to have an aversion to variance and a preference for positive skewness.

Specifically, they found the values of b_1 (the market risk premium) and b_2 (the market skewness premium) to be 1.119 and -0.212, respectively. Moreover, both were significant. As Arditti shows, whenever risk aversion decreases with wealth, it follows that positive skewness is preferred. This further implies that b_2 and τ_{R_M} are of opposite sign. For example, if the market is positively skewed, or τ_{R_M} is positive, then investors will give up return, which implies a negative b_2, in exchange for this positive skewness. Kraus and Litzenberger's results confirm this expectation. Since β and γ for the market portfolio are both equal to one, a negative value for b_2 and a positive value for τ_{R_M} necessarily increases the market risk premium, and thus, the significance of risk.

The following hypothetical example demonstrates the impact of coskewness on the traditional CAPM estimate. In the traditional two-moment CAPM, the excess of the expected return on the market portfolio over the risk-free rate is the market risk premium, but in the three-moment model this excess amount is the sum of the market risk premium and the market skewness premium. By definition, the beta and gamma of the market portfolio are one. Hence, from Equation (2.2) for the market portfolio we have:

$$\mathrm{E}(r_m) - r_f = b_1 + b_2.$$

As mentioned earlier, Kraus and Litzenberger estimated b_1 and b_2 to be 1.119% per month and –0.212% per month, respectively. Using the sum of these values of the risk premium and the skewness premium, respectively, to estimate the excess of the expected return on the market portfolio over the risk-free rate, we get:

$$\mathrm{E}(r_m) - r_f = 1.119\% - 0.212\% = 0.91\% \text{ per month.}$$

The excess of the expected return on the market portfolio over the risk-free rate must be the same for both the traditional two-moment CAPM and the three-moment CAPM. In the two-moment model, however, this quantity is simply the market risk premium:

$$\mathrm{E}(r_m) - r_f = b_1' = 0.91\% \text{ per month.}$$

Hence, the failure to include skewness in the two-moment CAPM results in understating the market risk premium by 19% (i.e., 1.0 – .91/1.119).

There are two implications of this theoretical example for a negatively skewed market such as the market for catastrophe insurance. First, the market risk premium is understated in the traditional two-moment CAPM. Second, additional return is required to compensate insurers and their investors for the negative skewness of catastrophe insurance products. Therefore, the three-moment CAPM is of particular significance to the insurance industry.

In an exercise on pages 1276–1278, Harvey and Siddique (2000) estimate the risk premium for coskewness. They rank stocks based on their past coskewness and create three value-weighted portfolios using 60 months of returns: 30 percent with the most negative skewness, 40 percent with medium values of skewness, and 30 percent with the highest skewness. Harvey and Siddique conclude on page 1263 that "Systematic skewness is economically significant and commands a risk premium, on average, of 3.60 percent per year." They estimate a skewness premium for coskewness of 3.60 percent by taking the difference in annual excess returns between the portfolio with the most negative coskewness and the portfolio with the highest coskewness.

Moreover, Harvey and Siddique (2000) conclude (pp. 1287–1288) that systematic skewness is not only statistically significant but also economically significant. They reached this conclusion by analyzing pricing errors with the model containing coskewness as a variable relative to the traditional CAPM and by measuring the expected return implied by a change in coskewness.

Friend and Westerfield (1980) also found evidence that investors prefer skewness; however, they did not find that evidence to be compelling. They state on page 913:

> Our analysis provides some but not conclusive evidence… suggesting that investors may be willing to pay a premium for positive skewness in their portfolios.

Kian-Guan Lim (1989), though, found strong evidence that confirms Kraus and Litzenberger's earlier conclusions. Lim divided the fifty-year period from January 1933 through December 1982 into ten consecutive five-year periods. The model was then tested using data from each of the sub-periods as well as for the entire period. Lim concluded that investors prefer coskewness when market returns are positively skewed, and dislike coskewness when market returns are negatively skewed. Moreover, in all of the subperiods in which the model was not rejected at the one percent level of significance, the skewness premium and the skewness of the market return were of opposite sign. Further, Lim found the evidence to be particularly strong when data from the entire period was used.

2.2. Properties of Covariance and Coskewness

As is the case with the traditional two-moment CAPM, beta in the three-moment CAPM is the measure of systematic risk. As a measure of risk, beta is linear in the sense that the beta of a linear combination of securities is the linear combination of the betas of the securities themselves. Specifically, the beta of a portfolio is equal to the weighted average of the betas of the securities in the portfolio.

Let

$$
\begin{aligned}
Z &= \text{a portfolio of } n \text{ securities,} \\
S_i &= \text{the dollars invested in the } i\text{th security,} \\
r_i &= \text{the rate of return on the } i\text{th security,} \\
r_Z &= \text{the rate of return on the portfolio,} \\
r_M &= \text{the return on the market portfolio, and} \\
S &= \sum_i S_i;
\end{aligned}
$$

then

$$
\begin{aligned}
\beta_Z = \frac{\sigma_{R_Z R_M}}{\sigma^2_{R_M}} = \frac{\mathrm{Cov}(r_Z, r_M)}{\mathrm{Var}(r_M)} &= \frac{\mathrm{Cov}\left(\left(\frac{\sum S_i r_i}{S}\right), r_M\right)}{\mathrm{Var}(r_M)} \\
&= \frac{(\sum S_i \mathrm{Cov}(r_i, r_M))}{S\,\mathrm{Var}(r_M)} \\
&= \frac{\sum S_i \beta_i}{S}.
\end{aligned}
$$

For Z equal to the market portfolio, the covariance of the rate of return on the market portfolio with itself is equal to the variance of the rate of return on the market portfolio. Therefore, the weighted sum of covariances of the rates of return on all of the securities in the market portfolio is equal to the variance of the rate of return on the market portfolio.

Similarly, the gamma of a portfolio is the weighted average of the gammas of the individual securities.

$$\gamma_Z = \frac{\tau_{r_Z r_M r_M}}{\tau^3_{r_M}} = \frac{\mathrm{E}((r_Z - \mathrm{E}(r_Z))(r_M - \mathrm{E}(r_M))^2)}{\mathrm{E}((r_M - \mathrm{E}(r_M))^3)}$$

$$= \frac{\mathrm{E}\left(\left[\left(\sum \frac{S_i r_i}{S}\right) - \mathrm{E}\left(\sum \frac{S_i r_i}{S}\right)\right][r_M - \mathrm{E}(r_M)]^2\right)}{\mathrm{E}((r_M - \mathrm{E}(r_M))^3)}$$

$$= \frac{\sum \left(\frac{S_i}{S}\right)\mathrm{E}([r_i - \mathrm{E}(r_i)][r_M - \mathrm{E}(r_M)]^2)}{\mathrm{E}([r_M - \mathrm{E}(r_M)]^3)}$$

$$= \sum \frac{S_i \gamma_i}{S}.$$

The coskewness of the return on the market portfolio with itself is equal to the skewness of the return on the market portfolio. Hence, the weighted sum of the coskewnesses of the returns on all of the securities in the market portfolio is equal to the skewness of the return on the market portfolio.

2.3. Disagreement

As noted earlier, under the assumptions of complete agreement on the part of investors about expected returns and identical risk tolerance functions, the optimal combination of risky assets is the same for each investor. It necessarily follows that the optimal portfolio is the market portfolio. These are very strong assumptions. But they are not intrinsic to the three-moment CAPM. Rather, they also apply to the traditional two-moment CAPM. Sharpe relaxes these assumptions in Appendix D of his book.

He concludes on page 291:

> [T]he equilibrium relationships derived for a world of complete agreement can be said to apply to a world in which there is disagreement, if certain values are considered to be averages.

In this section, we will relax these assumptions and investigate the implications.

In the case of disagreement, each investor has his own optimal risk asset portfolio, which depends entirely on his expectations. Different investors do not necessarily have the same optimal risk asset portfolios. For simplicity, assume that there are only two investors. The arguments presented here can be extended to any finite number of investors.

Suppose that M_1 and M_2 are the optimal risk asset portfolios of the two investors. Let M be the market portfolio.

Then

$$M = M_1 + M_2.$$

Let

r_{ij} = the rate of return for security i that is expected by the jth investor,

S_{ij} = the dollars invested in security i by the jth investor,

$$S_1 = \sum_i S_{i1},$$

$$S_2 = \sum_i S_{i2},$$

$$r_{M_1} = \frac{\sum_i S_{i1} r_{i1}}{S_1}, \text{ and}$$

$$r_{M_2} = \frac{\sum_i S_{i2} r_{i2}}{S_2}.$$

Then, the average expected returns are given by:

$$r_i = \frac{(S_{i1}r_{i1} + S_{i2}r_{i2})}{(S_{i1} + S_{i2})},$$

and

$$r_M = \frac{(S_1 r_{M_1} + S_2 r_{M_2})}{(S_1 + S_2)}.$$

Thus,

$$\begin{aligned}\mathrm{Cov}(r_i, r_M) &= \mathrm{Cov}\left(\frac{(S_{i1}r_{i1} + S_{i2}r_{i2})}{(S_{i1} + S_{i2})}, r_M\right)\\ &= \left(\frac{S_{i1}}{S_{i1} + S_{i2}}\right)\mathrm{Cov}(r_{i1}, r_M) + \left(\frac{S_{i2}}{S_{i1} + S_{i2}}\right)\mathrm{Cov}(r_{i2}, r_M).\end{aligned}$$

Hence, recalling that

$$\beta_i = \frac{\mathrm{Cov}(r_i, r_M)}{\mathrm{Var}(r_M)} \qquad \text{implies that:}$$

$$\beta_i = \left(\frac{S_{i1}}{S_{i1} + S_{i2}}\right)\beta_{i1} + \left(\frac{S_{i2}}{S_{i1} + S_{i2}}\right)\beta_{i2}.$$

Note that β_{i1} and β_{i2} are computed with respect to the total market portfolio, rather than with respect to each investor's optimal portfolio. Thus, in a world of agreement everybody has the same estimate of β, and in a world of disagreement, β turns out to be a weighted average over all investors.

The same relationship holds true for coskewness and gamma. Let the coskewness be denoted by:

$$\tau_{abb} = \mathrm{Cosk}(a, b, b) = \mathrm{E}([a - \mathrm{E}(a)][b - \mathrm{E}(b)]^2).$$

Assume again that there are only two investors who disagree. Then for any security:

$$\mathrm{Cosk}(r_i, r_M, r_M) = \mathrm{Cosk}\left(\frac{(S_{i1}r_{i1} + S_{i2}r_{i2})}{(S_{i1} + S_{i2})}, r_M, r_M\right).$$

It can be shown using the results from Section 2.2 and the linearity of the expected value operator that for any three random variables, x, y, and z, and any two constants, a and b, that:

$$\mathrm{Cosk}(ax+by,z,z) = a\,\mathrm{Cosk}(x,z,z) + b\,\mathrm{Cosk}(y,z,z).$$

Hence,

$$\mathrm{Cosk}(r_i,r_M,r_M) = \left(\frac{S_{i1}}{S_{i1}+S_{i2}}\right)\mathrm{Cosk}(r_{i1},r_M,r_M) + \left(\frac{S_{i2}}{S_{i1}+S_{i2}}\right)\mathrm{Cosk}(r_{i2},r_M,r_M).$$

And since,

$$\gamma_i = \frac{\mathrm{Cosk}(r_i,r_M,r_M)}{\tau^3_{R_M}},$$

it follows that:

$$\gamma_i = \left(\frac{S_{i1}}{S_{i1}+S_{i2}}\right)\gamma_{i1} + \frac{S_{i2}}{(S_{i1}+S_{i2})}\gamma_{i2},$$

where γ_{i1} and γ_{i2} are computed with respect to the total market rather than with respect to each investor's optimal portfolio. Hence, in a world of agreement everybody has the same estimate of γ, and in a world of disagreement, γ turns out to be a weighted average over all investors.

3. THE THREE-MOMENT INSURANCE CAPM

Following D'Arcy and Doherty's (1988) derivation of the insurance CAPM, the rate of return to the insurer, r_e, is composed of a linear combination of both an underwriting rate of return, r_u, and an investment rate of return, r_i.

$$r_e = \frac{r_uP(1-t_u)}{S} + \frac{r_i(S+kP)(1-t_i)}{S}, \tag{3.1}$$

where

r_e = rate of return on equity,

P = premiums in a given year,

S = shareholders' equity,

r_u = underwriting return per dollar of premium,

t_u = tax rate on underwriting income,

k = funds generating coefficient,[2]

r_i = investment return per dollar invested, and

t_i = tax rate on investment income.

At equilibrium based on Equation (2.3) and assuming that shareholders' equity, S, is valued at its expected market value, rather than at its statutory accounting or GAAP accounting value:

$$\mathrm{E}(r_e) = r_f + b_1\beta_e + b_2\gamma_e. \tag{3.2}$$

Further,

$$\mathrm{E}(r_i) = r_f + b_1\beta_i + b_2\gamma_i. \tag{3.3}$$

Moreover, the equity beta (gamma) can be expressed as a linear combination of an underwriting beta (gamma) and an investment beta (gamma) as follows:

$$\beta_e = \frac{P\beta_u(1-t_u)}{S} + \frac{(S+kP)\beta_i(1-t_i)}{S}, \text{ and} \tag{3.4}$$

$$\gamma_e = \frac{P\gamma_u(1-t_u)}{S} + \frac{(S+kP)\gamma_i(1-t_i)}{S}. \tag{3.5}$$

Setting Equation (3.1) equal to Equation (3.2) results, at equilibrium, in:

$$\frac{\mathrm{E}(r_u)P(1-t_u)}{S} + \frac{\mathrm{E}(r_i)(S+kP)(1-t_i)}{S} = r_f + b_1\beta_e + b_2\gamma_e.$$

[2]This is sometimes estimated by the ratio of the invested portion of reserves to premiums.

Substituting with the above three expressions for $E(r_i)$, β_e and γ_e from Equations (3.3), (3.4) and (3.5) gives:

$$\frac{E(r_u)P(1-t_u)}{S} + \frac{(S+kP)(r_f + b_1\beta_i + b_2\gamma_i)(1-t_i)}{S}$$
$$= r_f + \frac{P(1-t_u)(b_1\beta_u + b_2\gamma_u)}{S}$$
$$+ \frac{(S+kP)(1-t_i)(b_1\beta_i + b_2\gamma_i)}{S}.$$

Simplifying and solving for the after-tax equilibrium underwriting return yields:

$$E(r_u)(1-t_u) = -kr_f(1-t_i) + \frac{t_i r_f S}{P} + (1-t_u)b_1\beta_u + (1-t_u)b_2\gamma_u. \tag{3.6}$$

Thus the equilibrium after-tax underwriting return consists of four components: the first effectively represents interest paid to policyholders for the use of their funds; the second is to recapture the tax penalty of being an insurer;[3] the third component is a provision to compensate for risk; and the fourth component is a provision to compensate for skewness.

4. THE n-MOMENT INSURANCE CAPM

There is strong evidence as reported in this paper that including the third moment significantly improves the CAPM and the insurance CAPM. Any benefits of including moments beyond the third are unclear now and await further research. Nevertheless, generalizing the model to n moments is simple and straightforward and is presented here.

[3]The tax penalty is the double taxation of investment income—once at the corporate level and once at the personal level—on underlying equity. Mutual funds, in contrast, are not subject to corporate income taxes. Accordingly, investors will not invest in an insurance company unless the underwriting operation is expected to at least recover the tax penalty.

At equilibrium based on Equation (A.6) and assuming that shareholder's equity, S, is valued at its expected market value, rather than at its statutory accounting or GAAP accounting value:

$$\mathrm{E}(r_e) = r_f + \sum_{n=2}^{\infty} b_{(n-1)} \nu_{n_e}. \tag{4.1}$$

Further,

$$\mathrm{E}(r_i) = r_f + \sum_{n=2}^{\infty} b_{(n-1)} \nu_{n_i}. \tag{4.2}$$

Moreover, for $n = 2, \ldots, \infty$,

$$\nu_{n_e} = \frac{P\nu_{n_u}(1 - t_u)}{S} + \frac{(S + kP)\nu_{n_i}(1 - t_i)}{S}. \tag{4.3}$$

Setting Equation (3.1) equal to Equation (4.1) results, at equilibrium, in:

$$\frac{\mathrm{E}(r_u)P(1 - t_u)}{S} + \frac{\mathrm{E}(r_i)(S + kP)(1 - t_i)}{S} = r_f + \sum_{n=2}^{\infty} b_{(n-1)} \nu_{n_e}.$$

Substituting with the above expressions for $\mathrm{E}(r_i)$ and ν_{n_e}, for $n = 2, \ldots, \infty$ from Equations (4.2) and (4.3) gives:

$$\frac{\mathrm{E}(r_u)P(1 - t_u)}{S} + \frac{(S + kP)(r_f + \sum_{n=2}^{\infty} b_{(n-1)} \nu_{n_i})(1 - t_i)}{S}$$
$$= r_f + \sum_{n=2}^{\infty} b_{(n-1)} \left[\frac{P\nu_{n_u}(1 - t_u)}{S} + \frac{(S + kP)\nu_{n_i}(1 - t_i)}{S} \right].$$

Simplifying and solving for the after-tax equilibrium underwriting return yields:

$$\mathrm{E}(r_u)(1 - t_u) = -kr_f(1 - t_i) + \frac{t_i r_f S}{P} + \sum_{n=2}^{\infty} b_{(n-1)} \nu_{n_u}(1 - t_u).$$

5. CONCLUSIONS

Until recently the importance of skewness in the rate of return distribution has largely been unrecognized in financial journals. But it is in the actuarial realm that some of the most extremely skewed return distributions occur, particularly those for catastrophe insurance products. Because some of those distributions are so overwhelmingly skewed, it is essential to assess systematic skewness when determining equilibrium returns and needed premiums.

This paper presents an argument for including a provision in the equilibrium premium to cover the cost of skewness. It also generalizes the insurance CAPM to include the cost of skewness. This permits an explicit determination of the impact that skewness has on the equilibrium premium, at least theoretically. Practical application awaits further empirical studies that measure the amount of systematic skewness in the insurance industry as well as further investigation into the magnitude of the market skewness premium and the market risk premium in the context of a three-moment model.

REFERENCES

[1] Arditti, Fred D., "Risk and the Required Return on Equity," *The Journal of Finance*, 1967, pp. 19–36.

[2] D'Arcy, Stephen P., and Neil A. Doherty, *The Financial Theory of Pricing Property-Liability Insurance Contracts*, Heubner Foundation, 1988.

[3] Friend, Irwin and Randolph Westerfield, "Co-Skewness and Capital Asset Pricing," *The Journal of Finance*, 1980, pp. 897–913.

[4] Greenspan, Alan, as quoted in Wessel, David, "Internet Mania Is Like Lottery, Fed Chief Says," *Wall Street Journal*, January 29, 1999, p. C1.

[5] Harvey, Campbell R. and Akhtar Siddique, "Conditional Skewness in Asset Pricing Tests," *The Journal of Finance*, 2000, pp. 1263–1295.

[6] Kahane, Yehuda, "The Theory of Insurance Risk Premiums," *The ASTIN Bulletin* 10, 1979, pp. 223–239.

[7] Kraus, Alan and Robert Litzenberger, "Skewness Preference and the Valuation of Risk Assets," *The Journal of Finance*, 1976, pp. 1085–1100.

[8] Lim, Kian-Guan, "A New Test of the Three-Moment Capital Asset Pricing Model," *Journal of Financial and Quantitative Analysis*, 1989, pp. 205–216.

[9] Rubinstein, Mark E., "The Fundamental Theorem of Parameter-Preference Security Valuation," *Journal of Financial and Quantitative Analysis*, 1973, pp. 61–69.

[10] Sharpe, William F., *Investments*, Third Edition, Prentence-Hall, 1985.

[11] Sharpe, William F., *Portfolio Theory and Capital Markets*, First Edition, McGraw-Hill, 1970.

APPENDIX

DERIVATION OF THE n-MOMENT CAPM

This appendix presents Rubinstein's derivation of the n-moment CAPM and extends it to derive the market risk premium and the market skewness premium.

Let W_i be the initial wealth of the ith individual. Assume that every dollar of that wealth is invested in one of j securities. Let S_{ij} be the amount that the ith individual has invested in the jth security. Then

$$W_i = \sum_j S_{ij},$$

and the wealth at the end of the year is:

$$\tilde{W}_i = \sum_j S_{ij} R_j,$$

where $R_j = (1 + r_j) =$ one plus the rate of return on the jth security.

Let U_i be the continuously differentiable utility of wealth function for the ith individual. Assume that every individual maximizes $\mathrm{E}_i(U_i(\tilde{W}_i))$ subject to the constraint $W_i = \sum_j S_{ij}$.

Taking the expected value of the Taylor series expansion of $U_i(\tilde{W}_i)$ around $\mathrm{E}_i(\tilde{W}_i)$ gives:

$$\mathrm{E}_i(U_i(\tilde{W}_i)) = \sum_{n=0}^{\infty} \frac{U_i^{(n)} \mu_{in}}{n!},$$

where $U_i^{(n)}$ is the nth derivative of U_i evaluated at $\mathrm{E}_i(\tilde{W}_i)$, and $\mu_{in} = \mathrm{E}_i(\tilde{W}_i - \mathrm{E}_i(\tilde{W}_i))^n$ is the nth central moment of $\tilde{W}_i$. Forming the Lagrangian, the individual's problem is to maximize Z, where

$$Z = \sum_{n=0}^{\infty} \frac{U_i^{(n)} \mu_{in}}{n!} + L_i \left(W_i - \sum_j S_{ij} \right).$$

Dropping the subscript i for simplicity and differentiating gives:

$$\frac{\partial Z}{\partial S_j} = \sum_n \left\{ \frac{\partial}{\partial S_j}\left(\frac{U^{(n)}}{n!}\right)(\mu_n) + \left(\frac{U^{(n)}}{n!}\right)\left(\frac{\partial \mu_n}{\partial S_j}\right)\right\} - L = 0,$$

and

$$W = \sum_j S_j.$$

Let

$$\bar{W} = \mathrm{E}(\tilde{W}) = \sum_j S_j \mathrm{E}(R_j) \Rightarrow \frac{\partial \bar{W}}{\partial S_j} = \mathrm{E}(R_j).$$

So

$$\frac{\partial}{\partial S_j}\left(\frac{U^{(n)}}{n!}\right) = \frac{U^{(n+1)}}{n!}\left(\frac{\partial \bar{W}}{\partial S_j}\right) = \frac{U^{(n+1)}}{n!}\mathrm{E}(R_j).$$

Thus,

$$\frac{\partial Z}{\partial S_j} = \mathrm{E}(R_j)\left(\sum_n \frac{U^{(n+1)}\mu_n}{n!}\right) + \sum_n \frac{U^{(n)}}{n!}\left(\frac{\partial \mu_n}{\partial S_j}\right) - L = 0.$$

But the term $\sum_n U^{(n+1)}\mu_n/n!$ is the Taylor series expansion of $U^{(1)}$ around $\bar{W}$. And,

$$\begin{aligned}
\frac{\partial}{\partial S_j}(\mu_n) &= \frac{\partial}{\partial S_j}\left(\mathrm{E}\left(\sum_j S_j R_j - \sum_j S_j \mathrm{E}(R_j)\right)^n\right) \\
&= n\mathrm{E}\left\{(\tilde{W} - \bar{W})^{n-1}\left(\frac{\partial}{\partial S_j}\right)\left(\sum_j S_j R_j - \sum_j S_j \mathrm{E}(R_j)\right)\right\} \\
&= n\mathrm{E}\{(\tilde{W} - \bar{W})^{n-1}(R_j - \mathrm{E}(R_j))\}.
\end{aligned}$$

Hence,

$$\mathrm{E}(R_j)U^{(1)} + \sum_{n=2}^{\infty} \frac{U^{(n)}\mathrm{E}[(R_j - \mathrm{E}(R_j))(\tilde{W} - \bar{W})^{n-1}]}{(n-1)!} = L. \tag{A.1}$$

Since $\mu_0 = 1$ and $\mu_1 = 0 \Rightarrow \partial/\partial S_j(\mu_0) = \partial/\partial S_j(\mu_1) = 0$.

The expression in (A.1) is true for all j. Subtracting the expression for the kth security from the expression for the jth security gives:

$$\mathrm{E}(R_j - R_k)U^{(1)} + \sum_{n=2}^{\infty} \frac{U^{(n)}\mathrm{E}[(R_j - R_k - (\mathrm{E}(R_j) - \mathrm{E}(R_k)))(\tilde{W} - \bar{W})^{n-1}]}{(n-1)!} = 0.$$

Hence,

$$\mathrm{E}(R_j) = \mathrm{E}(R_k) - \sum_{n=2}^{\infty} \frac{U^{(n)}\mathrm{E}[(R_j - R_k - (\mathrm{E}(R_j) - \mathrm{E}(R_k)))(\tilde{W} - \bar{W})^{n-1}]}{U^{(1)}(n-1)!}.$$

Let $\theta_n = -U^{(n)}/U^{(1)}(n-1)!$. Then,

$$\mathrm{E}(R_j) = \mathrm{E}(R_k) + \sum_{n=2}^{\infty} \theta_n \mathrm{E}[(R_j - R_k - (\mathrm{E}(R_j) - \mathrm{E}(R_k)))(\tilde{W} - \bar{W})^{n-1}]. \tag{A.2}$$

Assume that a risk-free security exists. Let R_f be one plus the rate of return on the risk-free security.

Equation (A.2) applies to all securities, so substituting R_f for R_k gives:

$$\mathrm{E}(R_j) = R_f + \sum_{n=2}^{\infty} \theta_n \mathrm{E}[(R_j - \mathrm{E}(R_j))(\tilde{W} - \bar{W})^{n-1}]. \tag{A.3}$$

Let S_f denote the amount that the individual has invested in the risk-free security,

$P = W - S_f$ denote the amount that the individual has invested in his portfolio of risky securities, and

R_p = one plus the rate of return on the portfolio of risky securities.

Then, $\tilde{W} = PR_p + S_f R_f$, and

$$\mathrm{E}(\tilde{W}) = P\mathrm{E}(R_p) + S_f R_f.$$

Thus,

$$\mathrm{E}(R_j) = R_f + \sum_{n=2}^{\infty} \theta_n P^{n-1} \mathrm{E}[(R_j - \mathrm{E}(R_j))(R_p - \mathrm{E}(R_p))^{n-1}].$$

Under the assumptions of complete agreement among individuals and identical risk tolerance functions, it follows that every individual has the same optimal portfolio of risky assets. Moreover, that portfolio is the market portfolio. Hence,

$$\mathrm{E}(R_j) = R_f + \sum_{n=2}^{\infty} \theta_n P^{n-1} \mathrm{E}[(R_j - \mathrm{E}(R_j))(R_M - \mathrm{E}(R_M))^{n-1}], \tag{A.4}$$

where R_M = one plus the rate of return on the market portfolio. Let

$$\nu_{n_j} = \frac{\mathrm{E}[(R_j - \mathrm{E}(R_j))(R_M - \mathrm{E}(R_M))^{(n-1)}]}{\mathrm{E}[(R_M - \mathrm{E}(R_M))^n]} \qquad \text{for} \quad n = 2, \ldots, \infty,$$

and

$$b_{(n-1)} = \theta_n P^{(n-1)} \mathrm{E}(R_M - \mathrm{E}(R_M))^n.$$

Then, the n-moment CAPM is:

$$\mathrm{E}(R_j) = R_f + \sum_{n=2}^{\infty} b_{(n-1)} \nu_{n_j}. \tag{A.5}$$

Equivalently,

$$\mathrm{E}(r_j) = r_f + \sum_{n=2}^{\infty} b_{(n-1)} \nu_{n_j}. \tag{A.6}$$

For the three-moment CAPM, the traditional notation is given by:

$$\beta_j = \nu_{2_j}, \text{ and}$$

$$\gamma_j = \nu_{3_j}.$$

Then the three-moment CAPM is:

$$\mathrm{E}(R_j) = R_f + b_1 \beta_j + b_2 \gamma_j. \tag{A.7}$$

Additional insight into the coefficients b_1 and b_2 can be gained as follows.

Let R_W denote one plus the rate of return on the individual's entire portfolio, and let σ_{R_W} and τ_{R_W} denote the standard deviation and the skewness, respectively, of the rate of return on the individual's entire portfolio.

Then, in conjunction with the results from Section 2.2,

$$\sigma_{R_W} = \sum_j \frac{S_j \beta_j \sigma_{R_M}}{W}, \text{ and}$$

$$\tau_{R_W} = \sum_j \frac{S_j \gamma_j \tau_{R_M}}{W}.$$

Let β_W and γ_W denote the beta and the gamma of the individual's entire portfolio. It follows that

$$\beta_W = \frac{\sigma_{R_W}}{\sigma_{R_M}} \qquad \text{and} \qquad \gamma_W = \frac{\tau_{R_W}}{\tau_{R_M}}.$$

Moreover,

$$\sigma_W = W \sigma_{R_W} \qquad \text{and} \qquad \tau_W = W \tau_{R_W}.$$

Consider that

$$\begin{aligned}\bar{W} &= W\mathrm{E}(R_W) = WR_f + Wb_1\beta_W + Wb_2\gamma_W \\ &= W(R_f) + \frac{b_1\sigma_W}{\sigma_{R_M}} + \frac{b_2\tau_W}{\tau_{R_M}}.\end{aligned}$$

Since the market portfolio is unchanging, σ_{R_M} and τ_{R_M} are constants. It follows that

$$b_1 = \frac{\partial\bar{W}}{\partial\sigma_W}(\sigma_{R_M}), \text{ and}$$

$$b_2 = \frac{\partial\bar{W}}{\partial\tau_W}(\tau_{R_M}).$$

Thus, the coefficients are the additional required returns per unit of risk and skewness, respectively, times the units of risk and skewness, respectively.

USING CLAIM DEPARTMENT WORK MEASUREMENT SYSTEMS TO DETERMINE CLAIM ADJUSTMENT EXPENSE RESERVES

JOANNE S. SPALLA

Abstract

This paper discusses a methodology for establishing reserves for the portion of loss adjustment expense associated with the cost of claim adjusters. The actuarial literature contains very little material on how to estimate unallocated loss adjustment expense (ULAE) reserves. The literature briefly mentions "transaction-based" methods that require claim department time studies. However, many feel that the improvement in estimating ULAE reserves does not justify the high cost of performing such a study. Fortunately, most claim departments of major insurance carriers and third party administrators now utilize sophisticated automated work measurement tools that may capture the type of data that can be used to perform an automated time study.

The first section describes a process that can be used to perform the work-study, including a discussion of the technical and practical issues in conducting such a study. The second section shows how the results of the study can be utilized to determine claim adjuster expense reserves. Other potential applications of the claim standards will be discussed, including pricing unbundled claim service, allocating claim department expense to line of business for statutory and management reporting purposes, and monitoring claim department expenses. Changes in the NAIC definition of loss adjustment expense are also discussed in the paper.

1. INTRODUCTION

This paper will illustrate a methodology for establishing the estimated liability for the portion of loss adjustment expense associated with the cost of claim adjusters. Common techniques that are used to determine these liabilities will be discussed at the beginning of this paper. The paper will then describe an alternative method of estimating these costs, using a claim department work-study. The study utilized an automated work measurement system to determine a standard cost of handling different types of claims. The paper will then describe how these claim standard costs can be used to determine outstanding liabilities for claim adjuster expense. Other applications of the study will be described in the final section.

A. *Definition of Loss Adjustment Expense*

Before discussing how to determine a reserve for claim adjuster expenses, it is first necessary to review changes in the definition of loss adjustment expenses. Claim adjuster expenses have been included in the traditional definition of unallocated loss adjustment expense (ULAE). In the past, there had been some inconsistency in the distinction between allocated and unallocated loss adjustment expenses. Part of the confusion resulted from the common assumption that the term "allocated" refers to expenses that could be identified with a specific claim file. Companies utilizing different business procedures to settle claims may thus have had different definitions for unallocated and allocated loss adjustment expense. This issue was further complicated because different definitions were used for statistical reporting.

To increase the consistency of reporting between insurers, the Casualty Actuarial (Technical) Task Force (CATF) recommended to the National Association of Insurance Commissioners (NAIC) Accounting Practices and Procedures (EX4) Task Force that a revised loss adjustment expense (LAE) definition be adopted [1]. The Accounting Practices and Procedures Task Force adopted the change effective January 1, 1998. The task force's objective was consistent reporting of expenses related to defense, litigation, and medical cost containment regardless of whether a company uses its own employees or hires outside firms. To eliminate any confusion arising from the association of the term "allocated" with the ability to assign expenses to a specific claim, the NAIC approved a Blanks Proposal to change the titles effective with the 1999 Annual Statement.

Under the revised rules, the ability of an insurer to assign expenses to a specific claim no longer determines how it is classified. Defense, litigation, and medical cost containment expenses—both internal and external—are now assigned to "Defense & Cost Containment" (DCC); the remaining expenses associated with adjusting and recording claims are assigned to "Adjusting & Other."

Specifically, DCC now includes:

(i) surveillance expenses;

(ii) fixed amounts for medical cost containment expenses;

(iii) litigation management expenses;

(iv) loss adjustment expenses for participation in voluntary and involuntary market pools, if reported by accident year;

(v) fees or salaries for appraisers, private investigators, hearing representatives, reinspectors, and fraud investigators, if working in defense of a claim, and fees or salaries for rehabilitation nurses, if such cost is not included in losses;

(vi) attorney fees incurred owing to a duty to defend, even when other coverage does not exist; and

(vii) the cost of engaging experts.

Adjusting & Other is now defined as those loss adjustment expenses other than the DCC expenses as defined above. Adjusting & Other expenses include the following items:

(i) fees of adjusters and settling agents;

(ii) loss adjustment expenses for participation in voluntary and involuntary market pools, if reported by calendar year;

(iii) attorney fees incurred in the determination of coverage, including litigation between the insurer and the policyholder; and

(iv) fees or salaries for appraisers, private investigators, hearing representatives, reinspectors, and fraud investigators, if working in the capacity of an adjuster.

The claim department expense study discussed in this paper will focus on the first item in the above definition of Adjusting & Other expenses. These costs, which compose the largest portion of Adjusting & Other, will be referred to as "claim adjuster expenses" throughout the paper. Provisions for the other items included in the definition of Adjusting & Other must be calculated independently and added to the adjuster reserves determined by the methodology discussed in this paper.

B. Summary of Common Reserving Methods

The actuarial literature contains very few techniques for determining the outstanding liabilities for what has traditionally been called unallocated loss adjustment expense, or ULAE. The existing techniques fall into three categories:

- paid-to-paid methods
- methods based on claim reporting and closing patterns (the Johnson Method [2])
- transaction-based methods.

The paid-to-paid method—as well as its shortcomings—has been described in detail in the actuarial literature. Under this method, the historical ratio of calendar year ULAE payments to calendar year paid losses is calculated. The ULAE reserve is then determined by applying 100% of this ratio to the incurred but not reported (IBNR) reserve and 50% of this ratio to the case reserve. This methodology is based on the assumption that 50% of the ULAE is paid when a claim is opened and the remaining 50% of the ULAE is paid as losses are paid. It also assumes that the IBNR reserve only provides for pure IBNR claims.

Several authors (Kittel [3, p. 311] and Johnson [2]) have pointed out the shortcomings of the assumptions underlying this method. In particular, the use of a calendar year ratio will either understate or overstate the ULAE reserve in a changing claims environment. For example, if a line of business is growing, this method will understate the reserve. Similarly, if there is a change in the claim reporting and settlement pattern, this method will fail to produce the correct reserve. In addition, this method assumes that ULAE will inflate at the same rate as losses. Finally, this method assumes that the underlying loss reserves are adequate. It should be noted that the distortions in this method would be magnified for long-tailed lines of business.

The Johnson Method overcomes many of the problems associated with the traditional paid-to-paid methodology. The first step in this method is to calculate historical average ULAE expense per weighted open claim. The number of claims open at future year-end points is then projected based on claim reporting and settlement patterns. Finally, the ULAE reserve is calculated by multiplying the number of open claims by the trended average expense.

By relating calendar year ULAE to claim counts, Johnson recognizes that ULAE payments are not necessarily tied to loss payments. The ULAE reserve calculated by the Johnson Method is also independent of the adequacy of the underlying loss reserves.

In addition, the method is responsive to changes in exposures and inflation.

While the Johnson Method overcomes many of the shortcomings of the classical paid-to-paid methods, it has a major limitation: the technique is dependent on the allocation of ULAE to line of business. As Johnson [2, pp. 113–114] noted:

> One of the problems with unallocated loss adjustment expenses is that it is difficult to test one's assumptions about them because the expenses by definition are generally hard to allocate and therefore hard to track. The only real way that comes to mind to test assumptions would be to conduct a claim expense study, such as a time and motion study, which establishes artificial expense allocation procedures for a temporary time period.

The allocation of calendar year ULAE to line may not be an issue for a company writing only a single line of business or for a company that has fully dedicated claim staff for each line. However, it can be a significant issue for insurance companies that utilize multi-line claim offices. Any distortions from a misallocation of calendar ULAE will, in turn, distort the average ULAE used to estimate the reserve. In her paper, Johnson [2] uses a growing book of medical malpractice business in a single state as an example. She notes that the dramatic annual 17.4% trend in the calendar year average ULAE was surprising. Johnson does not describe the company that generated the data in the example or the methodology used to allocate calendar year ULAE to line of business and state. It is, therefore, not possible to determine if the increase in calendar year ULAE was due to the calendar year allocation methodology. However, this example illustrates the reliance of Johnson's technique on the calendar year ULAE allocation methodology.

Johnson, among other authors, has acknowledged that the only way to accurately determine the true cost of handling various types of claims is to conduct a claim department work-study. However, all of these authors recognize that such a study would have been very time-consuming. It would have involved literally standing over a claim adjuster's desk armed with a stopwatch or requiring claims adjusters to track every minute of their time. Improving the measurement of ULAE liabilities would probably not suffice to justify the high cost of performing such a study. Fortunately, today's modern technology offers a more efficient and accurate way of conducting such a study.

2. OVERVIEW OF CLAIM DEPARTMENT EXPENSE STUDY

A. *Big Brother Is Watching*

Most modern claim departments utilize automated claim systems. Claim representatives use these systems to perform the various functions involved in the claim process, such as opening claims and making payments, as well as adding notes and composing correspondence. In fact, much of an inside claim representative's day is spent at the computer. Many of these systems capture the individual transaction detail, along with the duration of time spent on each type of activity. This data will often identify the claim staff position performing the task, as well as the claim generating the activity. Multiplying the duration of activity for each transaction times the average hourly cost of the claim position performing the task yields the cost of performing the transaction. Dividing the sum of all the transaction costs by the number of claims yields the average cost of handling a claim.

This paper describes an actual claim study utilizing an automated work measurement system and its application to determining the reserve for unallocated loss adjustment expense. While the use of an automated work measurement system greatly simplified the effort of performing a claim study, the project involved an investment of significant resources. The cost of such an investment goes beyond the benefit that would be derived by merely improving the accuracy of the estimation of ULAE liabilities. In fact, the main justification for this study was an improvement in the allocation of claim costs to product and profit center, which, in turn, would enhance the accuracy of product pricing.

B. Claim Data Utilized in the Study

The data that is available in the claim system varies by company. Hence, the design of the claim department study will be governed by the data captured in the system. The data elements used for the claim study in this paper are discussed below.

Claim Data identifies the individual claim that generated the activity:

- Claim Symbol—identifies the coverage that generated the claim.
- Claim Office—identifies the branch office that is handling the claim.
- Age of Claim—the automated work measurement system utilized in our claim study classified claims into four age classifications:
  - (i) Intake—this category represents the work that is performed in the first 30 days of the claim.
  - (ii) Outstanding 31–90 Days (OS1)—this category represents the work that is done in the next 60 days in the life of the claim.
  - (iii) Outstanding > 90 Days (OS2)—this category represents the work that is done on claims that are over 90 days old. For workers compensation, this category excludes claims that are older than 60 months, which were handled separately.
  - (iv) Outstanding > 60 Months (OS3)—this category, which was utilized only for workers compensation, represents the work that is done on claims that are more than five

years old. For the sake of simplicity, the calculation for this category is not illustrated in this paper.

The choice of these claim categories was governed by the claim system that we used to perform the study. Other classifications could be used. For example, we considered adding a category for claim settlement to reflect the work to close a claim. However, we decided not to do so when we learned that the claim file might not be officially closed in the month in which the claim actually settled. Claims may be kept open until all the final bills have been paid and any recoveries (such as salvage, subrogation and second injury funds) have been collected. For this reason, the work in the final month the claim is open may not accurately reflect the work associated with settling a claim.

When undertaking a claim work-study, it is important to understand how the claim system counts claims. Some claim systems count all the claimants from an occurrence as a single claim, while other systems create individual claim files for each claimant and coverage. For example, an automobile accident may generate one or more bodily injury claims, a property damage liability claim and a physical damage claim. Another consideration is how reopened claims are handled—some systems utilize the original claim number, while others create a new claim.

Policy Data identifies the business unit that wrote the policy that generated the claim. Depending on the business needs of the organization, the following level of detail may be included in the claim study:

- Regional Office
- Risk State
- Market Segment

In a multi-line insurance company, the claim study may distinguish between personal and commercial business. A company writing commercial lines may wish to further distinguish between small commercial, middle market, and large national account policies if it is felt that the costs of handling these claims are different. For the same reason, the company may wish to separately identify assigned risk claims. In our study, we found that large national account claims required less handling time than standard business. It was believed that this was because large accounts normally have a large volume of claims. These accounts typically have a risk management department with defined claim reporting procedures that assists in the claim process by gathering the necessary information and providing it to the claim adjuster. Smaller accounts have very few claims, and therefore are less experienced in handling claims. Assigned risk claims were found to have the highest claim adjuster costs.

Work Measurement Data is the basis for the cost of handling the claim. We utilized the following information from our claim system:

- Type of Transaction—this data element identifies the type of activity on the claim file. Examples of transactions include creating a claim, making a payment, and adding notes to the file.

- Claim Position—this data element is the job classification of the claim representative that performed the activity on the claim. Examples of job classifications include claim representative (inside and outside), clerical and supervisor.

- Duration of Transaction—this item measures the length of time expended performing a task.

Claim Expense Data is required to determine the cost of handling each transaction. To estimate these costs, it was necessary to collect salary data by claim position, as well as other expenses such as benefits, rent, automobile, travel, etc.

3. STEPS IN PERFORMING A CLAIM DEPARTMENT STUDY

The steps involved in performing a claim department study are summarized below:

1. Determine average hourly cost for each claim position
2. Collect duration of claim transactions by claim position
3. Determine raw costs by multiplying durations by average costs for each claim position
4. Load standards for unrecorded time
5. Divide costs by claim volumes to determine average cost
6. Load standards for other field office claim overhead not captured in the work-study
7. Load standards for home office claim adjustment expense overhead

Each of the above steps will be discussed in more depth in subsequent sections using workers compensation lost-time claims as an example. Because workers compensation claims with lost work time have very different characteristics than medical-only claims, we chose to calculate separate standards for each category. It should be noted that the data in the exhibits have been disguised to preserve confidentiality.

Step 1: Determine Average Hourly Cost for Each Claim Position

The claim-handling costs underlying the work-study are determined by multiplying the time every claim position spent handling a claim times the average hourly cost for that claim position. The first step in the study is to determine the average hourly cost for each position in the claim study. Exhibit 1 shows the calculation of the average hourly cost. For our study, we used annualized countrywide average salary levels for each position, loaded for benefits and other expenses. The hourly cost is based on 50 weeks per year at $36\frac{1}{4}$ hours per week for each staff position. The use of countrywide salary levels reduces the bias from using a sampling of claim offices. Benefits are loaded as a flat 30% of salary. Other expense categories, such as rent and furniture and equipment, are allocated to position. Certain categories, such as automobile expense, should be allocated only to the job categories that generate those expenses. Depending on the nature of the expense categories, the allocations may be based on salaries, headcount or any other reasonable basis for allocation.

When we performed our study, we found that the system captured a sufficient proportion of time at the individual claim level for only five positions (inside claim representative, outside claim representative, clerical, supervisor and claim processor). We chose to include only those positions in the work-study, accounting for 64.3% of the total claim field costs. The costs for the remaining positions will be reflected in a Field Office Overhead Factor, discussed later.

Step 2: Collect Duration of Claim Transactions by Claim Position

Exhibits 2, 3, and 5 are each divided into three sections representing the three age categories in the study: Intake, OS1 and OS2. Exhibit 2 displays the number of hours recorded in the claim system for each of the job positions that handled workers compensation lost-time claims during the study. The number of claims handled in each category is shown at the bottom of each section. For example, in Office #1, inside claim representatives spent a total of 387.5 hours handling the intake of 585 lost-time claims. Other positions, including outside claim representatives, clerical, claim processors, and supervisors, also worked on these claims. The system recorded a total of 825.8 hours of staff time handling lost-time claim intake in this office. During the same time period, there were 996 open claims that were between 31 and 90 days old in Office #1. The system recorded a total of 554.6 hours handling these claims. Finally, 1,879.3 hours were captured for the 4,600 claims that were between 90 days and 60 months old.

It should be noted that several positions—such as supervisor, claim representative, and clerical staff—perform activities on a single claim. At the same time, there are many claims that do not have any activity on them in the month. The standard that we are calculating represents the average monthly cost of handling an open claim.

Step 3: Determine Raw Recorded Costs by Multiplying Durations by Average Salaries

In Exhibit 3, the average hourly cost of the position handling the claim is multiplied by the duration of the task to determine the total cost. For example, the average hourly cost of an inside claim representative is $29.95. This hourly cost is multiplied times the 387.5 hours spent handling intake claims to get a cost of $11,607 for Office #1. The costs are calculated similarly for the other job categories.

Step 4: Load Costs for Unrecorded Time

The average costs determined above must be adjusted to reflect the fact that 100% of work time is not recorded in the claim system for the positions in the study. Exhibit 4 shows the time that was captured in the claim system for each position in Office #1 during the four-month study period. It should be noted that this exhibit reflects the total time recorded for each position during the study period and includes work on all types of claims. For this reason, the number of recorded hours exceeds the hours shown in the sample for workers compensation lost-time claims. The number of available hours is equal to the staff count times the number of work hours during the study period. The number of hours recorded at the claim level reflects the time that is spent working on a specific claim. Examples of time that is not recorded at the claim file level include absence and vacation time, training, and customer service. Note that the percent of time recorded at the claim level varies significantly by the type of position. The system captures the largest proportion of time for claim processors. On the other hand, only 28.9% of clerical time can be recorded to specific claims. The proportion of time recorded is summarized for each position and office at the bottom of Exhibit 4.

To adjust for the time that cannot be allocated to specific claims, the costs determined in Exhibit 3 are grossed up by dividing the cost by the percent of time recorded for each position in each office (see Exhibit 5). For example, the $11,607 of costs for inside claim representatives in Office #1 is divided by the 66.6% time recorded to obtain a grossed-up cost of $17,428. In using a factor to gross up the costs, we are allocating unrecorded time for each claim in the same proportion as the recorded time.

Step 5: Determine Average Costs by Claim Category

The calculation of the average monthly costs for workers compensation lost-time claims for each of the claim categories is displayed at the bottom of each section in Exhibit 5. For each of the age categories, the average cost was calculated by dividing the grossed-up costs for all offices by the number of claims that were handled in the age category during the study period. Dividing the total intake costs of $273,505 by the claim intake of 2,645 yields a preliminary standard of $103.40 for handling a lost-time claim intake. Since the claim intake includes all claims that were reported during the four-month study period, the resulting standard represents the average monthly cost that is incurred on a lost-time claim in the first month that it is reported to the company.

In determining the number of outstanding claims in the averages for the OS1 and OS2 categories, every claim in the office is counted once for each month that it is open during the study period, regardless of whether there was any activity on the claim. For example, if a claim were open for the first three months of the study and then closed, it would produce a count of three. As a result, the calculated standards represent the average monthly cost of handling outstanding claims.

At this point, it may be appropriate to apply judgment in selecting the final standards. Unusual results for any office and category should be reviewed. For example, the costs for Office #3 consistently fall below the costs in the other offices. The statistics for this office should be validated to make sure that all the data were collected properly. Given the data are correct, the reasons for the lower cost should be explored. One possible explanation for the lower cost may be that the workers compensation laws in the jurisdictions served by the office make it easier to adjudicate claims. If it is felt that the data for this office is anomalous, it may be appropriate to exclude it from the final selection of the standards.

Step 6: Load Standards for Other Field Office Claim Overhead

As mentioned above, not all the staff in a field claim office actually handles claims in the system. For example, the claim office staff may include an office manager, system administrator, and quality assurance and training resources, as well as clerical and mailroom staff. These field costs must also be factored into the claim standards. In our study, these costs were added using a percentage factor. Since the positions included in the work-study accounted for 64.3% of total claim expenses, the standards were multiplied by 1.555 (1/.643) in Exhibit 6. In making this adjustment, we are again allocating field office overhead to claim in the same proportion as the staff handling time captured in the system at the claim level.

Step 7: Load Standards for Home Office Claim Adjuster Expense Overhead

In addition to the field overhead discussed above, claim adjuster expense also includes home office claim department costs, as well as general overhead. Examples of the types of expenses included in overhead are shown in Exhibit 7. The general overhead factor was calculated by dividing the annual overhead cost of $66,976 by the total claim field expenses of $174,933 from Exhibit 1. General Overhead was reflected by multiplying the standards in Exhibit 6 by a factor of 1.383.

4. USING THE CLAIM STANDARDS TO CALCULATE THE CLAIM ADJUSTER EXPENSE RESERVE

The standards developed in the claim work-study can be used as the basis for the calculation of the claim adjuster expense reserve. Kay Rahardjo described a technique for doing so in her paper, "A Methodology for Pricing and Reserving for Claim Expenses in Workers Compensation" [4].

The major steps in Rahardjo's paper are:

(i) project ultimate claim counts using triangles of open and reported claims,

(ii) determine the number of claims open at various development ages, and

(iii) calculate the reserve by multiplying the number of open claims by the cost per outstanding claim.

The remaining exhibits in the paper illustrate the application of this methodology using a simplified example to calculate the required ULAE reserve as of 12/31/97. Modifications to Rahardjo's methodology will also be discussed.

In the reserve evaluation described below, we have elected to calculate the reserves for reported claims and IBNR claims separately. When claim service is sold on an unbundled basis, the revenue for claim service is typically collected when the claim is *reported*, and the claim administrator has no obligation to handle the claims that have not yet been reported. In such situations, including only reported claims in the claim adjuster expense reserve is appropriate. However, when the revenue for claim service is included in the insurance premium and the insurance carrier has the obligation to handle all claims that are reported, the claim adjuster expense reserve must include a provision for pure IBNR claims. Calculating the reserve separately for reported and IBNR claims provides the flexibility to address both situations.

A. Projection of Ultimate Claim Counts

Exhibit 8 shows a report year triangle of reported claim counts that forms the basis of the projection of ultimate claim counts. At the bottom of the exhibit, development factors are calculated using standard methodologies to project the claim counts to ultimate. For the sake of simplicity, it is assumed that there is no development in the report year claim counts after 12 months.

B. Projection of Outstanding Claim Counts

A report year triangle of outstanding claim counts is displayed in the top portion of Exhibit 9. It is important to emphasize that the definition of claim counts used in the reserving triangles must be consistent with the definition used to generate the average costs in the claim study. Exhibit 9 illustrates the method described in Rahardjo's paper to project outstanding claim counts. The number of outstanding claims at future development intervals is projected by calculating the ratio of outstanding claims to ultimate claims at historical points. These ratios are selected for each development age and are used to calculate the number of outstanding claims at future year-end development points. More sophisticated assumptions about the claim closing patterns during the development period could be used. For example, link ratios could be used to project the number of outstanding claims at each development age. Alternatively, the number of claims closed at each age could be estimated by using ratios of closed claims to the number of claims open at the beginning of the interval.

C. Projection of Claim Adjuster Expenses

Exhibit 10 illustrates how the total claim adjuster expenses are calculated by multiplying the number of claims at each development interval times the cost of handling a claim. The average number of outstanding claims shown at the top of the exhibit is calculated by averaging the number of claims outstanding at the beginning and end of the development interval in Exhibit 9. Use of the average number of outstanding claims reflects the fact that some of the claims that are open at the beginning of the interval will be closed.

The monthly standard claim costs developed in the claim study are the starting point for the estimates of future claim adjuster expenses. For the sake of simplicity, the reserve evaluation utilizes triangles with annual development points. Consequently, the monthly standard costs from the claim study must be converted into annual costs so that they are on a comparable basis. Since our standard varies with the age of the outstanding claim, the monthly standard costs must be weighted to reflect the mix by age of outstanding claims. Exhibit 11 shows how this conversion can be made.

Recall that in our claim study, the intake standard reflects the work that takes place in the month in which the claim is reported. A claim that remains open incurs the 31–90 day (OS1) cost for the next two months and then incurs the OS2 cost for months 4 through 60. Report year claims that are open between 12 and 24 months incur 3/12 months of the OS1 cost and 141/12 months of the OS2 cost, or \$613.04. After 24 months, open claims incur 12 months of OS2 costs (\$597.60) for each year that they are open up to 60 months.

Between 60 and 72 months, it is again necessary to adjust the standard to reflect the mix by age. The bottom section of Exhibit 11 shows that between 61 and 72 months open claims incur 66/12 months of the OS2 cost and 78/12 months of the OS3 cost, or \$384.40 per year. After 72 months, all open claims incur the OS3 cost of $17 per month, or $204.00 per year. The average costs calculated in the claim study are at 1997 cost levels. To reflect future costs, the 1997 standards are trended using an inflation assumption of 3% per year in the middle section of Exhibit 10.

D. Determination of Claim Adjuster Expense Reserve for Reported Claims

Once the future claim costs are estimated, calculating the claim adjuster expense reserve is simply a matter of summing the claim adjuster expenses for future development ages. If we make the simplifying assumption that a claim incurs the intake cost on the day it is reported, it is not necessary to include this cost in the reserve for reported claims. This calculation is illustrated in the bottom section of Exhibit 10 for a 12/31/97 reserve evaluation date.

For a long-tailed line such as workers compensation, it is necessary to include a provision for expenses incurred beyond ten years. Rahardjo [4] describes a methodology for determining a tail for workers compensation tabular claims that uses mortality assumptions. The tail reserve must include appropriate inflation assumptions.

E. Determination of Reserve for IBNR Claims

The top section of Exhibit 12 shows projected IBNR claims by accident year and development period. Any standard method for calculating the number of pure IBNR claims could be utilized. To select the expected ultimate cost per IBNR claim, we examine historical average ultimate claim adjuster expense per claim. The historical average costs are calculated in Exhibit 13 by dividing the total ultimate adjuster costs by the ultimate number of claims from Exhibit 8.

Ultimate costs are calculated by report year in Exhibit 14 through Exhibit 16. The ultimate adjuster costs must include the intake cost of handling a claim in the first month that it is reported to the company, as well as the cost of handling the claim for each month that it is outstanding. The average cost per outstanding claim for development ages beyond the first 12 months can be calculated in the same manner as in Exhibit 10. Since we are calculating ultimate costs, the costs for all development ages must be included. The 1997 average costs from the claim study have been adjusted in Exhibit 15 to reflect both historical and future cost levels. Future costs are calculated by applying an inflation factor of 3% per year to the 1997 standards. Historical costs are similarly calculated by detrending the 1997 standards at a rate of 3% per year. If actual historical average claim costs are available for prior years, they can be substituted for the detrended costs.

The calculation of costs for the first 12 months in the life of a claim, which is illustrated in Exhibit 17 and Exhibit 18, is more complicated. The calculation must include the intake cost for every claim that is reported to the company. It must also reflect the claims that are settled before the end of the year. In the claim study, all costs for the first month that the claim is open are reflected in the average intake cost, which was developed in Exhibit 6. The costs for the second and subsequent development months of a report year are calculated by multiplying the appropriate standard times the percentage of claims that remain open. The monthly costs are then accumulated for each report month. The final cost for the first year, $444.17, is calculated by averaging the total costs for each report month. Again, this must be adjusted to historical cost levels. The costs for the first 12 months are then combined with the costs for subsequent development periods in Exhibit 16 to get the total ultimate claim adjuster costs.

The ultimate report year cost per claim is calculated in Exhibit 13 by dividing the total ultimate cost by the ultimate number of claims. Since IBNR claims for the 12/31/97 reserve will emerge in 1998 and subsequent report years, the historical average costs in Exhibit 13 are then brought to 1998 cost levels using an inflation assumption of 3%. An expected average cost per claim is then selected.

The number of IBNR claims can then be multiplied by an expected ultimate cost per claim to derive the claim adjuster expense reserve for IBNR claims shown in Exhibit 12. The expected ultimate cost per IBNR claim selected in Exhibit 13 should be trended to reflect cost levels in the year that the IBNR claim emerges. By using the historical report year ultimate cost per claim, we assume that the cost of handling an IBNR claim is the same as the cost of handling a claim that has already been reported.

Finally, in Exhibit 19, the total claim adjuster expense reserve is the sum of the reserves for reported and IBNR claims. It should be noted that the total Adjusting and Other (A&O) reserve must include a provision for the other components of A&O that are not reflected in the claim expense study.

5. PRACTICAL CONSIDERATIONS IN CONDUCTING THE CLAIM STUDY

A. *Scope of the Study*

When setting up the study, one important consideration is its scope. One of the first decisions that must be made is whether to include the entire population of claim offices in the study. Because the volume of data that is collected at the transaction level is so extensive, it may not be possible to include all the claim offices. Instead, it may be more practical to include a sample of claim offices. If the decision is made to only sample claim offices, it is important to select offices that provide a representative sample of the company's geographical mix. The use of countrywide salary levels when calculating average costs can mitigate geographic differences in cost of living. However, variations in state claim adjudication requirements for certain lines, such as workers compensation, can significantly impact claim costs. It is also important to make sure that the mix of claims by age in the sample offices reflects the mix for the total claim population.

Our study included five sample offices that handled workers compensation claims, accounting for approximately 20% of our claim volume. It should be noted that an automated work measurement study allows a much larger sample size than would be practical under a traditional time and motion study.

B. Duration of Study

Another consideration is the time period for the study. Our claim study spanned four months. When conducting a work-study, it is important to select a representative time period. It makes sense to avoid unusual times, such as holidays. In addition, it is important to avoid periods when the office is handling a heavy volume of catastrophe claims. Even with these caveats, it may be necessary to adjust the data for months that have fewer workdays.

C. Credibility

There may not be sufficient volume in every claim category to select valid standards. In our study, we selected different standards by market segment. However, certain claim categories such as auto uninsured motorist did not have a sufficient volume of claim data. For these categories, we selected data for all market segments combined.

D. Adjusting the Data for Anomalies in Claim Transaction Durations

We found several data issues that required adjustment. A significant issue was unusually long durations for individual transactions compared to the norm. We learned that these anomalies typically occurred when the claim representative was interrupted in the middle of a transaction. In order to address this issue, we elected to cap any value for a transaction that exceeded the mean by more than three standard deviations.

E. Participation of Claim Department Personnel

Active participation by the claim department is essential to a successful claim work-study. Before undertaking the study, it is important to thoroughly understand the claim system and how adjusters utilize it. In enlisting the cooperation of the claim office staff, it is useful to explain the purpose of the study and to provide appropriate assurances that the goal of the study is not to reduce staff. Cooperation from the claim office staff—particularly the manager and systems administrator—during the data collection phase is crucial. To ensure all the data are collected, it is important to make sure that the system is fully operational and that all the data files are retained. The study team should be notified of any outages during work hours; data for days with outages may need to be excluded from the study, and appropriate adjustments must be made. Adjustments may also be necessary if there is a significant departure from the typical workload, such as an all-day training session.

After preliminary results are tabulated, it is useful to review them with a cross section of claim staff. While the staff may not be able to validate the actual average dollar cost of each type of claim, they may provide valuable insights into the cost differentials among different types of claims or the cost of handling similar claims for different market segments.

F. Other Participants in the Study

A cross-functional team was critical to the success of the claim study. Since the project was originally designed as a cost allocation study, controllers played a central role in the design and execution of the study. The study team included several staff members from both claims financial and cost accounting areas, as well as two actuaries from the claims actuarial area. A representative from the claims work measurement unit also served on the team. It was also helpful to have a systems analyst and programmer dedicated to the project. In addition, actuaries and controllers from the market segments and the corporate actuarial unit peer-reviewed the results of the study.

G. Final Validation of the Claim Standards

As a final validation of the claim standards, the study team tried to replicate actual claim adjustment expense spending levels using the standards. The standards (loaded for claim office overhead) were multiplied times the number of claims processed within each category in a given quarter and the results were summed. The fact that the total was within 2% of the actual claim adjuster expense spent during that time period helped demonstrate that the standards were reasonable.

H. Adjusting the Standards for Inflation and Trend in Claim Department Costs

Since conducting this type of claim study requires a significant resource investment, it is not practical to update it frequently. For this reason, it is necessary to adjust the standards for inflation in claim department costs. The simplest solution is to multiply the standards times an inflation factor. An alternative method is to update the salary and other expense data used in calculating the average hourly cost in Exhibit 1. However, neither of these methods recognizes productivity changes resulting from the claim department handling a higher or lower volume of claims with the same amount of staff.

A more refined approach can be used to adjust the standards. Each quarter, the actual spending in each claim office can be compared to the indicated claim adjuster expense that results from applying the standards to the claim volume. This is similar to the exercise described in the previous section that was used to validate the standards. The ratio of actual expenses to indicated expenses could be used to adjust the claim standards for inflation and productivity changes. This ratio can also be calculated at a claim office level and applied to the countrywide claim standards to customize the standards by claim office. Of course, it is important to note that this approach assumes that all types of claims in the office will inflate at the same rate. It also implies that the relativity among the standards for different types of claims will remain constant over time and across claim offices.

I. Adjusting the Standards for Changes in Claim Department Work Flow

While the above adjustment makes it unnecessary to update the standards every year to reflect inflation, it is necessary to modify the standards when there is a material change in claim department workflow. Examples of changes that may impact the standards are the creation of a centralized 800 number for claim reporting, changes in the process for reviewing and paying medical bills, and other managed care initiatives. In addition, outsourcing certain claim functions (such as case management, appraisals or fraud management) may require adjustments to the standards.

J. Workers Compensation Claims Greater Than 60 Months Old

The treatment of claims in the tail is an important consideration, particularly in a long-tailed line such as workers compensation. In workers compensation, claims that are open beyond a certain age require much less attention. Typically, when workers compensation claims reach this age, the investigation of the claim has been concluded. Weekly indemnity payments, and occasional medical payments, are processed with little intervention from a claim representative. For this reason, the claim adjuster expenses associated with these claim files are considerably lower. Accordingly, we established a separate OS3 cost for workers compensation "maintenance claim files" open longer than five years and excluded claims open more than five years from the OS2 age category. For the sake of simplicity, the calculation of the OS3 cost for workers compensation claims older than five years is not illustrated in this paper.

6. OTHER APPLICATIONS FOR CLAIM STUDY

In addition to calculating the claim adjuster expense reserve, the standards have several other practical applications: allocating claim adjuster expense to line of business for statutory and management reporting, pricing unbundled claim service, and monitoring claim department expenses.

A. Allocation of Claim Adjuster Expense

In many companies, internal claim adjuster expense is not typically assigned to a specific claim. For this reason, it is often impossible to identify these expenses by claim type and line of business. This becomes a particularly difficult issue when a single claim unit handles several different types of claims or the same type of claims for different market segments. The standards that are determined in this study could form the basis of an expense allocation system. As mentioned above, the original purpose of our claim study was to develop a new claim expense allocation system.

In our allocation methodology, the system tabulates the number of claims reported to the office and the number of claims in each age category. The monthly claim counts are then multiplied by the appropriate standard for the claim type and age category. The results are then summed by claim office to determine the indicated claim expense for each office. The indicated claim expense is compared to the actual claim expense in the office and the standards are adjusted to balance to the actual spending. Depending on individual company data reporting needs, the results can be summarized at various levels of detail. For internal management reporting, the data may be summarized by market segment and subline, branch office, and state. For Annual Statement reporting, the data may be tabulated by statutory line and state. In addition, the data may be further summarized by accident year.

B. Allocation of Adjusting and Other Expense Payments in Schedule P

The above method provides a methodology to allocate Adjusting and Other Expense to accident year in Schedule P. Prior to the 1997 Blank, the instructions to the Annual Statement prescribed a methodology—commonly referred to as the "45/5 Rule"—to allocate ULAE payments and reserves to accident year. The rule allocates calendar year ULAE payments as follows: (1) 45% to the most recent accident year, (2) 5% to the next most recent year, and (3) the balance in proportion to the amount of loss payments for each accident year during the most recent calendar year. This allocation method is based on the assumption that half of the ULAE is incurred when the loss is reported and the other half is incurred as loss payments are made. In addition, the method assumes that 90% of claims are reported in the same year as the accident year and the remaining 10% are reported in the following year. Of course, these assumptions do not apply to most lines of business typically written by today's insurers. The old Annual Statement rule was repealed effective with the 1997 Blank. The revised rule states that insurers should now apportion Adjusting and Other Expense payments and reserves by year based on claim counts using any appropriate method. The claim department standards described in this paper can be multiplied by accident year claim counts for each annual statement line to form the basis of the allocation of Adjusting and Other Expense payments in Schedule P.

C. Pricing Claim Service

Another important application of the claim standards is the pricing of claim service. The ultimate claim costs estimated above can form the basis of a handle-to-conclusion charge for insurance companies and third party administrators. In addition, assigned risk servicing carriers for workers compensation and automobile insurance can use these claim standards to reflect the cost of handling claims in the servicing carrier allowance in their bids. As an in-depth discussion of pricing is beyond the scope of this paper, the reader should refer to Rahardjo's paper [4, pp. 164–167] for more details.

D. Claim Department Expense Planning, Monitoring and Control

In addition to the applications discussed above, the claim study provides a set of tools to plan and monitor claim department costs. Future claim adjuster expenses can be forecasted using a projection of future adjuster costs similar to the triangles displayed in Exhibit 10 for reported claims and Exhibit 12 for incurred but not reported claims. Such a forecast can form the foundation of claim department budgets.

The work-study also produces useful monitoring statistics. As Exhibit 5 shows, the cost of handling each type of claim varies substantially by office. These average costs can be used to benchmark claim office productivity. Since the length of time that a claim remains open directly influences the cost of handling the claim, it is also important to monitor claim closing patterns. The triangle of ratios shown in Exhibit 9 provides a useful tool to monitor the proportion of claims remaining open.

7. SUMMARY

While the claim work-study described in this paper is simpler to conduct than the traditional time and motion study, it still involves a considerable amount of work. However, a claim work-study approach offers many advantages. The work-study more closely reflects the actual work involved in creating and handling different types of claims. The method is responsive to changes in claim volumes and is independent of loss payment patterns and the adequacy of loss reserves. The standards can be adjusted to explicitly reflect trends in claim department costs due to inflation and productivity changes. Finally, the work products resulting from the study provide useful operational tools for monitoring claim department expenses. The amount of work involved in conducting such a study is a worthwhile tradeoff for improvement in the accuracy of reserving, pricing, and monitoring claim adjustment expense.

REFERENCES

[1] Casualty Actuarial (Technical) Task Force, "Clarification of Revised ALAE Definition," 6/24/97 Draft.

[2] Johnson, Wendy A., "Determination of Outstanding Liabilities for Unallocated Loss Adjustment Expenses," *PCAS* LXXVI, 1989, pp. 111–125.

[3] Kittel, John, "Unallocated Loss Adjustment Expense Reserves in an Inflationary Economic Environment," Casualty Actuarial Society Discussion Paper Program, May 1981, pp. 311–331.

[4] Rahardjo, Kay Kellogg, "A Methodology for Pricing and Reserving for Claim Expenses in Workers Compensation," Casualty Actuarial Society *Forum*, Summer 1996, pp. 151–184.

EXHIBIT 1

TOTAL COUNTRYWIDE FIELD CLAIM EXPENSES

	$(000)						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Position	Staff	Salary & Benefits	Auto	Travel	Other	Total Field Expenses	Field Cost per Hour#
Trainee	24	940	—	—	329	1,268	29.16
Systems Administrator	57	2,538	—	—	776	3,314	32.08
Manager	80	8,174	—	333	1,088	9,596	66.18
*Inside Claim Representative	513	20,827	—	—	7,021	27,848	29.95
*Outside Claim Representative	265	12,855	1,930	1,109	3,621	19,515	40.63
*Clerical	904	24,640	—	—	12,374	37,014	22.59
Clerical Supervisor	31	1,184	—	—	424	1,608	28.62
Health Service Representative	67	3,453	209	122	425	4,209	34.66
Claim Processing Supervisor	57	2,748	—	—	780	3,528	34.15
*Claim Processor	195	6,539	—	—	2,672	9,211	26.06
Compensation Processor	41	1,302	—	—	557	1,859	25.01
Auto Service Rep./Supervisor	112	5,616	686	154	649	7,105	35.00
Claim Assistant	99	2,725	—	—	1,355	4,080	22.74
General Adjuster	26	1,747	105	127	136	2,115	44.88
Hearing Representative	14	887	102	59	192	1,240	48.85
File Supervisor	156	9,583	—	—	2,129	11,712	41.42
Assistant Manager	125	9,116	—	—	1,716	10,832	47.81
*Supervisor	261	15,305	—	—	3,575	18,880	39.91
Total Field	3,027	130,178	3,033	1,904	39,818	174,933	
Sum of Expenses Included in Study	2,138	80,166	1,930	1,109	29,263	112,467	
% of Total Field Expenses Included in Study	70.6%	61.6%				64.3%	

Field Overhead Factor (1/.643) = 1.555

*positions included in study
#based on 50 weeks at 36.25 hours per week

EXHIBIT 2

Summary of Claim Transaction Durations

Age Category: Intake

Avg. Hourly Cost	Position Name	Number of Hours				
		Office #1	Office #2	Office #3	Office #4	Office #5
$29.95	Inside Claim Representative	387.5	148.7	252.9	783.5	347.4
$40.63	Outside Claim Representative	74.2	243.7	68.6	38.9	49.6
$22.59	Clerical	129.9	120.9	52.7	398.1	91.0
$39.91	Supervisor	112.1	91.6	75.8	274.4	686.5
$26.06	Claim Processor	122.0	7.0	233.8	171.3	154.4
	Total Hours	825.8	612.0	683.8	1,666.1	1,329.0
	Number of Claims	585	304	654	650	452

Age Category: Outstanding 31–90 Days

Avg. Hourly Cost	Position Name	Number of Hours				
		Office #1	Office #2	Office #3	Office #4	Office #5
$29.95	Inside Claim Representative	241.2	69.6	175.9	735.1	241.9
$40.63	Outside Claim Representative	32.9	146.4	30.3	17.6	42.7
$22.59	Clerical	86.1	132.1	65.7	366.3	263.2
$39.91	Supervisor	134.9	79.6	122.4	363.8	227.7
$26.06	Claim Processor	59.5	4.4	142.6	158.1	106.7
	Total Hours	554.6	432.1	536.9	1,640.8	882.2
	Number of Claims	996	518	948	1,176	667

Age Category: Outstanding > 90 Days

Avg. Hourly Cost	Position Name	Number of Hours				
		Office #1	Office #2	Office #3	Office #4	Office #5
$29.95	Inside Claim Representative	736.0	157.6	712.5	1,928.7	971.9
$40.63	Outside Claim Representative	68.7	378.3	93.6	182.3	93.7
$22.59	Clerical	294.4	336.3	251.9	1,381.8	234.2
$39.91	Supervisor	662.4	483.4	914.0	978.8	187.4
$26.06	Claim Processor	117.8	10.5	453.4	374.2	181.5
	Total Hours	1,879.3	1,366.1	2,425.3	4,845.8	1,668.7
	Number of Claims	4,600	3,284	6,747	8,996	5,489

EXHIBIT 3

Development of Raw Recorded Costs

Age Category: Intake

Avg. Hourly Cost	Position Name	Total Recorded Costs Office #1	Office #2	Office #3	Office #4	Office #5	Total
$29.95	Inside Claim Representative	11,607	4,454	7,575	23,465	10,405	57,506
$40.63	Outside Claim Representative	3,016	9,900	2,788	1,579	2,016	19,299
$22.59	Clerical	2,934	2,731	1,190	8,993	2,055	17,903
$39.91	Supervisor	4,472	3,656	3,025	10,950	27,400	49,503
$26.06	Claim Processor	3,180	184	6,092	4,464	4,024	17,944
	Total	25,209	20,925	20,670	49,451	45,900	162,156
	Number of Claims	585	304	654	650	452	2,645

Age Category: Outstanding 31–90 Days

Avg. Hourly Cost	Position Name	Total Recorded Costs Office #1	Office #2	Office #3	Office #4	Office #5	Total
$29.95	Inside Claim Representative	7,223	2,085	5,270	22,016	7,245	43,838
$40.63	Outside Claim Representative	1,338	5,949	1,233	714	1,734	10,968
$22.59	Clerical	1,944	2,983	1,485	8,274	5,947	20,633
$39.91	Supervisor	5,385	3,175	4,883	14,518	9,086	37,048
$26.06	Claim Processor	1,550	115	3,716	4,119	2,781	12,281
	Total	17,440	14,308	16,586	49,641	26,793	124,768
	Number of Claims	996	518	948	1,176	667	4,305

Age Category: Outstanding > 90 Days

Avg. Hourly Cost	Position Name	Total Recorded Costs Office #1	Office #2	Office #3	Office #4	Office #5	Total
$29.95	Inside Claim Representative	22,043	4,721	21,339	57,766	29,109	134,978
$40.63	Outside Claim Representative	2,791	15,371	3,801	7,408	3,806	33,177
$22.59	Clerical	6,650	7,597	5,690	31,215	5,291	56,442
$39.91	Supervisor	26,436	19,293	36,477	39,063	7,477	128,747
$26.06	Claim Processor	3,069	274	11,816	9,753	4,730	29,641
	Total	60,990	47,255	79,123	145,203	50,413	382,985
	Number of Claims	4,600	3,284	6,747	8,996	5,489	29,116

EXHIBIT 4

SUMMARY OF HOURS IN CLAIM STUDY—ALL CLAIM TYPES

Office #1

Position Name	Available Monthly Hours	Recorded at Claim Level	Not Recorded at Claim Level: Customer Service	Not Recorded at Claim Level: Non-Functional	Not Recorded at Claim Level: Absence/ Vacation	Total Recorded
Inside Claim Representative	5,817	3,875	199	350	698	5,122
Outside Claim Representative	3,424	2,204	113	250	223	2,790
Clerical	11,709	3,389	405	470	735	4,999
Supervisor	4,425	3,129	154	541	491	4,315
Claim Processor	2,380	1,790	80	96	145	2,110
Total	27,755	14,387	951	1,707	2,292	19,336

Position Name	Recorded at Claim Level	Not Recorded at Claim Level: Customer Service	Not Recorded at Claim Level: Non-Functional	Not Recorded at Claim Level: Absence/ Vacation	Total Recorded
Inside Claim Representative	66.6%	3.4%	6.0%	12.0%	88.0%
Outside Claim Representative	64.4%	3.3%	7.3%	6.5%	81.5%
Clerical	28.9%	3.5%	4.0%	6.3%	42.7%
Supervisor	70.7%	3.5%	12.2%	11.1%	97.5%
Claim Processor	75.2%	3.4%	4.0%	6.1%	88.7%
Total	51.8%	3.4%	6.1%	8.3%	69.6%

Position Name	Percent of Total Time Recorded at Claim Level: Office #1	Office #2	Office #3	Office #4	Office #5
Inside Claim Representative	66.6%	64.6%	67.2%	68.2%	65.5%
Outside Claim Representative	64.4%	63.2%	65.6%	66.1%	63.9%
Clerical	28.9%	24.4%	18.6%	31.7%	31.2%
Supervisor	70.7%	69.1%	71.2%	71.9%	68.7%
Claim Processor	75.2%	74.3%	75.9%	76.1%	74.1%

EXHIBIT 5

Summary of Grossed-Up Costs

Age Category: Intake

Position Name	Office #1	Office #2	Office #3	Office #4	Office #5	Total
	Total Costs Grossed Up for Unrecorded Time					
Inside Claim Representative	17,428	6,895	11,273	34,407	15,885	85,887
Outside Claim Representative	4,683	15,665	4,250	2,388	3,156	30,142
Clerical	10,153	11,194	6,396	28,368	6,588	62,699
Supervisor	6,325	5,291	4,249	15,230	39,883	70,978
Claim Processor	4,229	247	8,027	5,866	5,430	23,799
Total	42,818	39,292	34,194	86,259	70,942	273,505
Number of Claims	585	304	654	650	452	2,645
Average Cost per Claim	$73.19	$129.25	$52.28	$132.71	$156.95	$103.40

Age Category: Outstanding 31–90 Days

Position Name	Office #1	Office #2	Office #3	Office #4	Office #5	Total
	Total Costs Grossed Up for Unrecorded Time					
Inside Claim Representative	10,845	3,228	7,842	32,281	11,061	65,256
Outside Claim Representative	2,078	9,413	1,879	1,079	2,714	17,164
Clerical	6,727	12,226	7,983	26,102	19,060	72,097
Supervisor	7,616	4,595	6,858	20,192	13,226	52,489
Claim Processor	2,062	155	4,895	5,412	3,753	16,278
Total	29,328	29,617	29,457	85,067	49,814	223,284
Number of Claims	996	518	948	1,176	667	4,305
Average Cost per Claim	$29.45	$57.18	$31.07	$72.34	$74.68	$51.87

Age Category: Outstanding > 90 Days

Position Name	Office #1	Office #2	Office #3	Office #4	Office #5	Total
	Total Costs Grossed Up for Unrecorded Time					
Inside Claim Representative	33,098	7,308	31,754	84,701	44,441	201,302
Outside Claim Representative	4,334	24,321	5,795	11,207	5,956	51,613
Clerical	23,012	31,134	30,592	98,469	16,957	200,163
Supervisor	37,392	27,920	51,232	54,329	10,884	181,758
Claim Processor	4,081	369	15,567	12,815	6,383	39,215
Total	101,917	91,052	134,941	261,520	84,622	674,052
Number of Claims	4,600	3,284	6,747	8,996	5,489	29,116
Average Cost per Claim	$22.16	$27.73	$20.00	$29.07	$15.42	$23.15

EXHIBIT 6

WORKERS COMPENSATION LOST-TIME CLAIMS DEVELOPMENT OF FINAL STANDARD COSTS

		Cost per Outstanding Claim	
	Intake	31–90 Days	> 90 Days
Claim Study Costs Excl. Field Office Overhead (from Exhibit 5)	$103.40	$51.87	$23.15
Field Office Overhead (from Exhibit 1)	1.555	1.555	1.555
Standards Including Field Overhead	$160.84	$80.67	$36.01
Home Office Overhead (from Exhibit 7)	1.383	1.383	1.383
Fully Loaded Standard Costs	$222.42	$111.56	$49.80

EXHIBIT 7

Calculation of General Overhead Factor

General Overhead Categories	$(000) Total Expense	% of Field Claim
Actuarial	1,835	1.0%
Claim Headquarters	8,922	5.1%
Commercial Lines Field	11,572	6.6%
Commercial Lines Home Office	512	0.3%
Controllers	6,789	3.9%
Corporate Finance	640	0.4%
Corporate Relations	175	0.1%
Executive	5,015	2.9%
General	20,557	11.8%
Government Affairs	0	0.0%
Human Resources	3,151	1.8%
Information Management	1,168	0.7%
Legal	3,319	1.9%
Operations	3,319	1.9%
Total Overhead	66,976	38.3%
Total Field Expenses (from Exhibit 1, Column (6) Total)	174,933	

EXHIBIT 8

PROJECTION OF ULTIMATE REPORT YEAR CLAIM COUNTS

Workers Comp.—Lost-Time

Report Year	12	24	36	48	60	72	84	96	108	120	Ultimate Claims
	Reported Number of Claims as of Elapsed Months										
1988	15,189	15,189	15,189	15,189	15,189	15,189	15,189	15,189	15,189	15,189	15,189
1989	17,426	17,426	17,426	17,426	17,426	17,426	17,426	17,426	17,426	17,426	17,426
1990	16,918	16,918	16,918	16,918	16,918	16,918	16,918	16,918	16,918	16,918	16,918
1991	16,923	16,923	16,923	16,923	16,923	16,923	16,923	16,923	16,923	16,923	16,923
1992	18,602	18,602	18,602	18,602	18,602	18,602	18,602	18,602	18,602	18,602	18,602
1993	17,001	17,001	17,001	17,001	17,001	17,001	17,001	17,001	17,001	17,001	17,001
1994	19,333	19,333	19,333	19,333	19,333	19,333	19,333	19,333	19,333	19,333	19,333
1995	17,693	17,693	17,693	17,693	17,693	17,693	17,693	17,693	17,693	17,693	17,693
1996	15,386	15,386	15,386	15,386	15,386	15,386	15,386	15,386	15,386	15,386	15,386
1997	15,025	15,025	15,025	15,025	15,025	15,025	15,025	15,025	15,025	15,025	15,025
	Development Factors										
1988	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000		
1989	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000			
1990	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000				
1991	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000					
1992	1.0000	1.0000	1.0000	1.0000	1.0000						
1993	1.0000	1.0000	1.0000	1.0000							
1994	1.0000	1.0000	1.0000								
1995	1.0000	1.0000									
1996	1.0000										
Avg.	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000		
Wtd. Avg.	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000		
3 Yr. Avg.	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000		
3 Yr. Wtd. Avg.	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000		
Selected	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000		
Cumulative	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000		
Percentage	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%		

EXHIBIT 9

PROJECTION OF OUTSTANDING REPORT YEAR CLAIM COUNTS

Workers Comp.—Lost-Time

	Outstanding Number of Claims as of Elapsed Months										
Report Year	12	24	36	48	60	72	84	96	108	120	Ultimate Claims
1988	7,083	3,250	1,855	1,324	981	753	547	366	287	228	15,189
1989	8,196	3,632	2,283	1,507	1,104	770	576	391	303	262	17,426
1990	8,463	4,181	2,638	1,866	1,352	938	702	566	307	254	16,918
1991	8,803	4,367	2,848	1,925	1,229	793	560	451	307	254	16,923
1992	9,961	5,287	3,429	1,988	1,246	837	667	496	337	279	18,602
1993	9,408	4,239	2,414	1,421	953	835	610	453	308	255	17,001
1994	10,365	4,667	2,744	1,616	1,261	949	694	515	351	290	19,333
1995	8,879	4,136	2,312	1,616	1,154	869	635	472	321	266	17,693
1996	7,596	3,785	2,126	1,405	1,003	755	552	410	279	231	15,386
1997	8,107	3,612	2,076	1,372	980	738	539	401	273	226	15,025
	Portion of Ultimate Claims Outstanding										
1988	0.4663	0.2140	0.1221	0.0872	0.0646	0.0496	0.0360	0.0241	0.0189	0.0150	
1989	0.4703	0.2084	0.1310	0.0865	0.0634	0.0442	0.0331	0.0224	0.0174		
1990	0.5002	0.2471	0.1559	0.1103	0.0799	0.0554	0.0415	0.0335			
1991	0.5202	0.2581	0.1683	0.1138	0.0726	0.0469	0.0331				
1992	0.5355	0.2842	0.1843	0.1069	0.0670	0.0450					
1993	0.5534	0.2493	0.1420	0.0836	0.0561						
1994	0.5361	0.2414	0.1419	0.0836							
1995	0.5018	0.2338	0.1307								
1996	0.4937	0.2460									
1997	0.5396										
Avg.	0.5117	0.2425	0.1470	0.0960	0.0673	0.0482	0.0359	0.0267	0.0181	0.0150	
Wtd. Avg.	0.5125	0.2430	0.1476	0.0959	0.0673	0.0481	0.0359	0.0267	0.0181	0.0150	
3 Yr. Avg.	0.5117	0.2404	0.1382	0.0913	0.0652	0.0491	0.0359	0.0267	0.0181	0.0150	
Selected	0.5117	0.2404	0.1382	0.0913	0.0652	0.0491	0.0359	0.0267	0.0181	0.0150	

EXHIBIT 10

Determination of Claim Adjuster Expense Reserve for Outstanding Claims

Workers Comp.—Lost-Time

	Average Outstanding Number of Claims								
Report Year	12–24 Mos	24–36 Mos	36–48 Mos	48–60 Mos	60–72 Mos	72–84 Mos	84–96 Mos	96–108 Mos	108–120 Mos
1988									
1989									282
1990								436	280
1991							506	379	281
1992						752	582	417	308
1993					894	722	532	381	282
1994				1,438	1,105	821	605	433	320
1995			1,964	1,385	1,011	752	553	396	293
1996		2,956	1,766	1,204	879	654	481	345	255
1997	5,859	2,844	1,724	1,176	859	638	470	337	249

Future Inflation Assumption 1.03

	Future Annual Claim Adjuster Expense per Outstanding Claim								
Report Year	12–24 Mos	24–36 Mos	36–48 Mos	48–60 Mos	60–72 Mos	72–84 Mos	84–96 Mos	96–108 Mos	108–120 Mos
1988									
1989									$210.12
1990								$210.12	$216.42
1991							$210.12	$216.42	$222.92
1992						$210.12	$216.42	$222.92	$229.60
1993					$395.93	$216.42	$222.92	$229.60	$236.49
1994				$615.53	$407.81	$222.92	$229.60	$236.49	$243.59
1995			$615.53	$633.99	$420.04	$229.60	$236.49	$243.59	$250.89
1996		$615.53	$633.99	$653.01	$432.65	$236.49	$243.59	$250.89	$258.42
1997	$631.43	$633.99	$653.01	$672.60	$445.62	$243.59	$250.89	$258.42	$266.17

EXHIBIT 10

(Continued)

	Future Claim Adjuster Expense for Outstanding Claims ($)									Claim Adjuster Reserve for Reported Claims
Report Year	12–24 Mos	24–36 Mos	36–48 Mos	48–60 Mos	60–72 Mos	72–84 Mos	84–96 Mos	96–108 Mos	108–120 Mos	
1988										0
1989									59,315	59,315
1990								91,709	60,693	152,402
1991							106,239	82,049	62,532	250,821
1992						158,056	125,896	92,896	70,798	447,646
1993					353,912	156,337	118,513	87,448	66,646	782,855
1994				885,407	450,659	183,115	138,812	102,426	78,062	1,838,480
1995			1,208,960	878,127	424,803	172,609	130,848	96,549	73,583	2,985,479
1996		1,819,297	1,119,570	786,537	380,495	154,605	117,200	86,479	65,908	4,530,091
1997	3,699,824	1,803,173	1,126,101	791,125	382,715	155,507	117,884	86,983	66,293	8,229,605
										19,276,694

EXHIBIT 11

Conversion of Monthly Standards to Annual Costs

Average Cost per Outstanding Claim			
1–2 Months	2–60 Months	> 60 Months	
OS1	OS2	OS3	
\$111.56	\$49.80	\$17.00	(from Exhibit 6)

Report Mo.	13	14	15	16	17	18	19	20	21	22	23	24	Total
	Development Month												
1	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	
2	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	
3	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	
4	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	
5	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	
6	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	
7	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	
8	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	
9	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	
10	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	
11	111.56	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	
12	111.56	111.56	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	
Total	721.12	659.36	597.60	597.60	597.60	597.60	597.60	597.60	597.60	597.60	597.60	597.60	7,356

Total Cost per Outstanding Claim in Months 12–24 of Report Year $= ((141\ \text{Mos.} * 49.80) + (3\ \text{Mos.} * 111.56))/12$ $= 613.04$ — \$613.04

EXHIBIT 11

(Continued)

Report	Development Month												
Mo.	61	62	63	64	65	66	67	68	69	70	71	72	Total
1	17.00	17.00	17.00	17.00	17.00	17.00	17.00	17.00	17.00	17.00	17.00	17.00	
2	49.80	17.00	17.00	17.00	17.00	17.00	17.00	17.00	17.00	17.00	17.00	17.00	
3	49.80	49.80	17.00	17.00	17.00	17.00	17.00	17.00	17.00	17.00	17.00	17.00	
4	49.80	49.80	49.80	17.00	17.00	17.00	17.00	17.00	17.00	17.00	17.00	17.00	
5	49.80	49.80	49.80	49.80	17.00	17.00	17.00	17.00	17.00	17.00	17.00	17.00	
6	49.80	49.80	49.80	49.80	49.80	17.00	17.00	17.00	17.00	17.00	17.00	17.00	
7	49.80	49.80	49.80	49.80	49.80	49.80	17.00	17.00	17.00	17.00	17.00	17.00	
8	49.80	49.80	49.80	49.80	49.80	49.80	49.80	17.00	17.00	17.00	17.00	17.00	
9	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	17.00	17.00	17.00	17.00	
10	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	17.00	17.00	17.00	
11	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	17.00	17.00	
12	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	49.80	17.00	
Total	564.80	532.00	499.20	466.40	433.60	400.80	368.00	335.20	302.40	269.60	236.80	204.00	4,613

Total Cost per Outstanding Claim in Months 60–72 of Report Year $384.40

$= ((66 \text{ Mos.} * 49.80) + (78 \text{ Mos.} * 17.00))/12$

$= 384.40$

EXHIBIT 12

Determination of Claim Adjuster Expense for IBNR Claims

Workers Comp.—Lost-Time

Accident Year	Expected Number of IBNR Claim as of Elapsed Months: 24	36	48	60	72	84	96	108	120	Number of IBNR Claims
1988										0
1989									0	0
1990								0	0	0
1991							0	0	0	0
1992						0	0	0	0	0
1993					0	0	0	0	0	0
1994				50	0	0	0	0	0	50
1995			296	0	0	0	0	0	0	296
1996		597	92	0	0	0	0	0	0	689
1997	584	218	34	0	0	0	0	0	0	836
										1,871

Accident Year	Cost per IBNR Claim as of Elapsed Months: 24	36	48	60	72	84	96	108	120
1988									
1989									1,015
1990								1,015	1,045
1991							1,015	1,045	1,077
1992						1,015	1,045	1,077	1,109
1993					1,015	1,045	1,077	1,109	1,142
1994				1,015	1,045	1,077	1,109	1,142	1,177
1995			1,015	1,045	1,077	1,109	1,142	1,177	1,212
1996		1,015	1,045	1,077	1,109	1,142	1,177	1,212	1,248
1997	1,015	1,045	1,077	1,109	1,142	1,177	1,212	1,248	1,286

EXHIBIT 12

(Continued)

	Total Cost of IBNR Claim as of Elapsed Months									Total IBNR
Accident Year	24	36	48	60	72	84	96	108	120	Cost
1988										0
1989									0	0
1990								0	0	0
1991							0	0	0	0
1992						0	0	0	0	0
1993					0	0	0	0	0	0
1994				50,750	0	0	0	0	0	50,750
1995			300,440	0	0	0	0	0	0	300,440
1996		605,934	96,481	0	0	0	0	0	0	702,415
1997	592,958	228,082	36,317	0	0	0	0	0	0	857,357
										1,910,962

EXHIBIT 13

DETERMINATION OF ULTIMATE CLAIM ADJUSTER EXPENSE PER CLAIM

Report Year	(1) Ultimate Claims	(2) Total Cost	(3) Cost per Claim	(4) Trended Cost per Claim
1988	15,189	10,923,237	719	966
1989	17,426	12,893,052	740	965
1990	16,918	14,047,986	830	1,052
1991	16,923	14,652,296	866	1,065
1992	18,602	16,974,995	913	1,090
1993	17,001	15,015,915	883	1,024
1994	19,333	17,492,749	905	1,018
1995	17,693	16,192,833	915	1,000
1996	15,386	14,653,500	952	1,010
1997	15,025	14,903,259	992	1,022
		All Year Average		1,021
		Latest 3 Years		1,011
		Latest 5 Years		1,015
		Latest 5 Years Excl. High/Low		1,017
		Selected		1,015

(1) from Exhibit 8
(2) from Exhibit 16
(3) = (2)/(1)
(4) Costs in Column (3) are trended to 1998 levels using inflation factor of 3%

EXHIBIT 14

DETERMINATION OF CLAIM ADJUSTER EXPENSE COSTS FOR REPORTED CLAIMS

Workers Comp.—Lost-Time

	Average Number of Outstanding Claims								
Report Year	12–24 Mos	24–36 Mos	36–48 Mos	48–60 Mos	60–72 Mos	72–84 Mos	84–96 Mos	96–108 Mos	108–120 Mos
1988	5,167	2,553	1,590	1,153	867	650	457	327	258
1989	5,914	2,958	1,895	1,306	937	673	484	347	282
1990	6,322	3,410	2,252	1,609	1,145	820	634	436	280
1991	6,585	3,608	2,387	1,577	1,011	677	506	379	281
1992	7,624	4,358	2,709	1,617	1,042	752	582	417	308
1993	6,824	3,327	1,918	1,187	894	722	532	381	282
1994	7,516	3,706	2,180	1,438	1,105	821	605	433	320
1995	6,508	3,224	1,964	1,385	1,011	752	553	396	293
1996	5,691	2,956	1,766	1,204	879	654	481	345	255
1997	5,859	2,844	1,724	1,176	859	638	470	337	249

EXHIBIT 15

ANNUAL CLAIM ADJUSTER EXPENSE PER OUTSTANDING CLAIM

Workers Comp.—Lost-Time

	Future Inflation Assumption			1.03	Historical Inflation Assumption				1.03
Report Year	12–24 Mos	24–36 Mos	36–48 Mos	48–60 Mos	60–72 Mos	72–84 Mos	84–96 Mos	96–108 Mos	108–120 Mos
1988	483.94	485.90	500.48	515.50	341.53	186.69	192.29	198.06	204.00
1989	498.46	500.48	515.50	530.96	351.78	192.29	198.06	204.00	210.12
1990	513.41	515.50	530.96	546.89	362.33	198.06	204.00	210.12	216.42
1991	528.81	530.96	546.89	563.30	373.20	204.00	210.12	216.42	222.92
1992	544.68	546.89	563.30	580.19	384.40	210.12	216.42	222.92	229.60
1993	561.02	563.30	580.19	597.60	395.93	216.42	222.92	229.60	236.49
1994	577.85	580.19	597.60	615.53	407.81	222.92	229.60	236.49	243.59
1995	595.18	597.60	615.53	633.99	420.04	229.60	236.49	243.59	250.89
1996	613.04	615.53	633.99	653.01	432.65	236.49	243.59	250.89	258.42
1997	631.43	633.99	653.01	672.60	445.62	243.59	250.89	258.42	266.17

EXHIBIT 16

Determination of Total Adjuster Expense for Reported Claims

Workers Comp.—Lost-Time

Report Year	0–12 Mos	12–24 Mos	24–36 Mos	36–48 Mos	48–60 Mos	60–72 Mos	72–84 Mos	84–96 Mos	96–108 Mos	108–120 Mos	Total Cost
1988	5,170,639	2,500,273	1,240,269	795,514	594,108	296,110	121,348	87,780	64,666	52,530	10,923,237
1989	6,110,078	2,947,878	1,480,171	976,863	693,168	329,618	129,411	95,761	70,788	59,315	12,893,052
1990	6,109,936	3,245,787	1,757,580	1,195,722	879,944	414,872	162,408	129,336	91,709	60,693	14,047,986
1991	6,295,018	3,482,238	1,915,438	1,305,150	888,317	377,309	138,006	106,239	82,049	62,532	14,652,296
1992	7,127,170	4,152,626	2,383,341	1,525,685	938,174	400,353	158,056	125,896	92,896	70,798	16,974,995
1993	6,709,275	3,828,109	1,873,802	1,112,522	709,351	353,912	156,337	118,513	87,448	66,646	15,015,915
1994	7,858,478	4,343,113	2,149,910	1,302,768	885,407	450,659	183,115	138,812	102,426	78,062	17,492,749
1995	7,407,528	3,873,163	1,926,662	1,208,960	878,127	424,803	172,609	130,848	96,549	73,583	16,192,833
1996	6,634,905	3,488,504	1,819,297	1,119,570	786,537	380,495	154,605	117,200	86,479	65,908	14,653,500
1997	6,673,654	3,699,824	1,803,173	1,126,101	791,125	382,715	155,507	117,884	86,983	66,293	14,903,259

0–12 Months: from Exhibit 17

12–108 Months: Average Number of Outstanding Claims (Exhibit 14) × Annual Cost per Adjuster Expense per Outstanding Claim (Exhibit 15)

EXHIBIT 17

Determination of Adjuster Costs Between 12 and 24 Months

Workers Comp.—Lost-Time

Report Year	(1) Ultimate Claims	(2) 0–12 Month Cost per Reported Claim	(3) Total 0–12 Month Cost
1988	15,189	340.42	5,170,639
1989	17,426	350.63	6,110,078
1990	16,918	361.15	6,109,936
1991	16,923	371.98	6,295,018
1992	18,602	383.14	7,127,170
1993	17,001	394.64	6,709,275
1994	19,333	406.48	7,858,478
1995	17,693	418.67	7,407,528
1996	15,386	431.23	6,634,905
1997	15,025	444.17	6,673,654

(1) from Exhibit 8
(2) 1997 average cost from Exhibit 18; 1996 and prior costs are calculated by detrending 1997 cost using an inflation factor of 3%
(3) = (1) × (2)

EXHIBIT 18

Development of Claim Adjuster Cost per Claim for Months 0–12

Average Cost Per Intake Claim	Average Cost per Outstanding Claim: 1–2 Months OS1	2–60 Months OS2	> 60 Months OS3	
$222.42	$111.56	$49.80	$17.00	(from Exhibit 6)

% of Claims	1	2	3	4	5	6	7	8	9	10	11	12
Outstanding	73.2%	68.9%	64.6%	60.3%	56.0%	51.7%	47.4%	43.1%	38.8%	34.5%	30.2%	25.8%

Report Mo.	Development Month 1	2	3	4	5	6	7	8	9	10	11	12	Total
1	222.42	76.90	72.09	30.04	27.89	25.75	23.60	21.45	19.31	17.16	15.02	12.87	
2		222.42	76.90	72.09	30.04	27.89	25.75	23.60	21.45	19.31	17.16	15.02	
3			222.42	76.90	72.09	30.04	27.89	25.75	23.60	21.45	19.31	17.16	
4				222.42	76.90	72.09	30.04	27.89	25.75	23.60	21.45	19.31	
5					222.42	76.90	72.09	30.04	27.89	25.75	23.60	21.45	
6						222.42	76.90	72.09	30.04	27.89	25.75	23.60	
7							222.42	76.90	72.09	30.04	27.89	25.75	
8								222.42	76.90	72.09	30.04	27.89	
9									222.42	76.90	72.09	30.04	
10										222.42	76.90	72.09	
11											222.42	76.90	
12												222.42	
Total	222.42	299.32	371.41	401.45	429.34	455.08	478.68	500.14	519.45	536.61	551.63	564.50	5,330
Total Cost per Claim in Months 1–12 of Report Year (including Intake)													$444.17

EXHIBIT 19

Determination of Total Claim Adjuster Reserve Including IBNR

(1) Reserve for IBNR Claims	(2) Reserve for Reported Claims	(3) Total Reserve
1,910,962	19,276,694	21,187,656

(1) from Exhibit 12
(2) from Exhibit 10
(3) = (1) + (2)

ADDRESS TO NEW MEMBERS—MAY 7, 2001

AN UNCENSORED VIEW OF RECENT HISTORY

JEROME A. SCHEIBL

A few moments ago, we witnessed a ceremony that has been repeated over 150 times in one form or another since the Casualty Actuarial Society was formed. It is a fairly long ceremony at times when there are a large number of new Fellows and Associates (such as in some recent meetings), and it has the potential of being dull and repetitious. Yet it is anything but that and it continues to be a highlight of our program—meeting after meeting.

Why is this?

The answer should be obvious for you new Associates who were just introduced and for you new Fellows who were just awarded your diplomas. You have worked long and hard towards reaching this goal and you have finally achieved it. With it come the rewards of recognition, an increased potential for financial gain and, hopefully, a satisfying career. You have reason to celebrate and I am happy to join with those who have already congratulated you on your achievements.

As for you spouses, companions, family members, and close friends who have witnessed these candidates prepare for these examinations and have shared in the sacrifices they have made, your satisfaction comes more in the form of pride and relief. Your role in these achievements is often overlooked despite its importance.

We have just applauded the new members. Now it is your turn. I would like all spouses, companions, family members, and close friends of the new Fellows and Associates to stand at this time and be recognized.

Thank you.

We "older" members also experience warm feelings as names of new members are called. Continued growth of our Society assures us that we have chosen a vibrant profession that attracts new minds and energies. We are thrilled to be associated with the enthusiasm of youth and we are impressed with the high level of scholarship and sacrifice demonstrated by our new colleagues.

We also feel some nostalgia as we recall when we were first introduced to other members and received our diplomas.

I hope that the custom of individually recognizing each new Associate and Fellow at meetings where they attain such levels of membership continues as long as there is a Casualty Actuarial Society—no matter how long the list may be or how much time it may consume.

If you will permit me to digress at this point, I cannot help but recall my own introduction as an Associate just 40 years ago this November. The meeting was at the Palmer House in Chicago. There were 22 new Associates (a large class for those days) and seven new Fellows. One hundred twenty-four members constituting one-third of the total membership attended the meeting.

A personal highlight of that meeting was meeting authors of papers that I had read in preparation for my exams. Suddenly these names took on personalities and as I met these people face-to-face, I inwardly apologized for having less than complimentary thoughts about them as I struggled with my studies.

Examinations were given just once a year in May, so if you didn't pass you had to wait a full year to try again. Fees were $6 for Associate exams and $10 for Fellowship exams. I understand that they are now more like $450 for each part.

In those days, passing examinations and a cursory acquaintance with the "Guides to Professional Conduct" was all that was needed to be fully accepted as a casualty actuary. If there was any indication of an additional obligation, it came in the form of a gentle urging in most presidential welcoming speeches to write papers and serve on committees.

After we had the letters after our names, we usually achieved a more lofty status with our employers, with our fellow employees, and with others who had become members of our Society.

Was this enough to claim status as a professional? History now tells us that it was not.

The American Heritage Dictionary definition of "professional" closest to that which might describe an actuary is, "One who has an assured competence in a particular field or occupation."

This definition begs the meaning of the term "competence." Of course, it means an advanced level of education and training. But is that enough? Today's society says, "no." Our contemporary world calls for a definition of "competence" that includes "public trust." Such trust may be imposed by laws such as licensing or franchise, or may simply be by general acceptance in some other form of structured adherence to standards.

The key point is that in today's society, a field of specialty is a profession only if the general public considers it to be so.

A few leaders in the various actuarial societies in the United States[1] engaged in a great debate over 40 years ago as to what might be done to strengthen the professional posture and public image of actuaries. While debates centered on the number of actuarial organizations there should be and how each should function, no serious efforts were made to reach any agreement.

The real impetus for some action came in the form of a federal requirement for the licensing of actuaries who would be certify-

[1]The organizations that founded the American Academy of Actuaries were the Casualty Actuarial Society, the Conference of Actuaries in Public Practice (now known as the Conference of Consulting Actuaries), the Fraternal Actuarial Society (now defunct), and the Society of Actuaries.

ing the compliance of pension plans with government regulations under the Employee Retirement Income Security Act (ERISA). The concern of a government takeover of our profession was no longer a threat—it was a reality—at least in the pension area of practice. Under ERISA, the government prepared and administered the examinations, awarded a designation (Enrolled Actuary), prescribed practice standards, and provided for discipline of errant practitioners.

Discussions among the actuarial organizations began again in earnest—picking up where they had left off. This time there was a greater sense of urgency and when the smoke cleared, the American Academy of Actuaries was formed to address public policy issues by speaking for the entire actuarial profession in public forums with a single voice as a defense against further governmental intrusion. That continues to be the mission of the Academy today—going strong 36 years after its founding.

A synergistic effect of the founding of the Academy was the creation of a forum whereby the founding organizations could better communicate with each other on matters of mutual concern. This lead to discussions on the need for uniformity in Practice Standards, Professional Codes of Conduct, and a device to police the compliance of members with such codes.

I'm sure that you have been briefed on these matters in your professionalism courses that are required of all new members. So I won't dwell on the mission and operations of the Actuarial Standards Board (ASB) and the Actuarial Board for Conduct and Discipline (ABCD) or on the content of the uniform Codes of Professional Conduct. However, I would like to briefly comment on my own experiences with these three initiatives.

I chaired the joint committee that was charged with finalizing the 1990 version of the Professional Codes of Conduct. (You will note that I refer to them in a plural sense, despite the fact that they may be almost identical, in that none of the cooperating actuarial organizations[2] had relinquished or delegated any of their responsibilities for the conduct of its members.) That committee inherited the work of two previous committees that had suggested an outline for common codes.

After completing our initial draft, we took to presenting our proposal to every actuarial gathering that would grant us the time. We were concerned not only with selling actuaries on the draft but for feedback on how the draft might be improved.

I recall one visit that I made to an actuarial club. The president told me that there would be a brief business meeting before I made my presentation. As it turned out, the business meeting was to ask for a vote to raise the dues so that they could get better speakers in the future. I don't think they were kidding but at least they could have given me the chance to speak first. Fortunately, the business portion of today's meeting has already taken place.

As you probably know, the 1990 Codes were recently updated, completing the initial plan for a review after a ten-year period.

The establishment of an Actuarial Standards Board and Actuarial Board for Counseling and Discipline presented a challenge to the drafters of these concepts as these boards needed to be multidisciplined boards but yet not part of any one organization, including the American Academy of Actuaries. This was accomplished by creating them as completely independent bodies with members appointed by presidents of the cooperating actuarial organizations. Administratively, they look to the Academy for support but are in no way considered as part of that organization.

The Actuarial Standards Board got a fast start on its assignment to codify Standards of Practice once it was appointed. I

[2]The American Academy of Actuaries, the American Society of Pension Actuaries, the Casualty Actuarial Society, the Conference of Actuaries in Public Practice (now known as the Conference of Consulting Actuaries), and the Society of Actuaries.

served as chairman of the Editorial Advisory Committee of that Board. Unfortunately, the decision to have such a committee came after a number of Practice Standards had been promulgated. This effectively made the Editorial Committee somewhat moot.

Its major project was the development of a Glossary of Actuarial Terms. After about two years of work and several meetings, such a glossary was published but only on an advisory basis. Part of the problem was that by this time, the committee had discovered that different definitions for similar terms had already found their way into the Practice Standards and it was impractical to revise them. I believe the Glossary no longer exists or if it does, it exists only in the archives.

Perhaps it is time to resurrect this project—this time with the idea of giving it the status of being a “stand-alone” Practice Standard so all actuaries and regulators can better understand each other.

I was also privileged to be one of the original members of the Actuarial Board for Counseling and Discipline. The concept of a joint effort for enforcing Practice and Qualification Standards of separate independent actuarial organizations presented a real challenge to those who drafted the concept of the ABCD. Perhaps the most glaring mistake they made was in choosing the name for that body. I hate acronyms. Some people got caught up in the “cute” ABCD acronym and flew with it. Since then, it has been a difficult sell to convince people that the ABCD does not have any authority to discipline members of actuarial organizations despite the use of the word “discipline” in its name. Each organization retains this exclusive right—as it should. The ABCD investigates possible violations of the Codes of Conduct much as a grand jury would. However, the individual actuarial organizations have delegated the additional authority of privately counseling actuaries when circumstances seem to warrant such counseling and to provide guidance for compliance with such codes when requested to do so by individuals. No authority for public action of any kind has been delegated to the ABCD.

I sat on the ABCD for its first seven years (the ABCD will be 10 years old this year) and considered over 200 cases in that time, including requests for guidance. Very few cases went beyond the private counseling or guidance stage.

I am aware that from time to time, there are some that feel that the ABCD should be more open. I see no reason for this. As a matter of fact, I see open hearings as an unnecessary intrusion into the privacy of an investigated actuary which, in itself, could be interpreted by some as a public suggestion of wrongdoing. They serve no purpose except to embarrass the actuary and possibly discredit the delicately balanced enforcement process. If indeed there is some justification for satisfying public curiosity in a particular case (and I can think of none), the logical time and place would seem to be when a case reaches the disciplinary forum of the membership organization (in our case, the CAS Discipline Committee). To do otherwise would be an imposition on the responsibility each organization has over its own members—something that is well beyond the scope of authority that has been assigned to the ABCD.

As you can tell from this brief visit to the more recent history of the actuarial profession, progress did not come smoothly and not without considerable effort. However, I think you will agree that a lot of hard work and talent—mostly volunteer—has gone into meeting the challenges of the last several years, thus making the profession a better world for you to inherit. It is your world now. And it is up to you to care for it and, where necessary, improve on it.

The admonitions to participate in committees and panel discussions and to write papers that were given in welcoming speeches for these many years hold true today. In addition, there are other matters that need monitoring and perhaps modification to keep abreast with the times. Currently, they include adapting the profession to a global environment, preserving the identity of the Casualty Actuarial Society as the authority on matters falling in the statistical realms of the mathematical discipline and risk management and expanding on a variety of high-level comprehensive continuing education opportunities, to name a few.

The opportunities are there. Grab them.

When I made my presidential address twenty years ago, I closed with a quotation from Francis Bacon that seems to be just as appropriate today as it was then. Bacon wrote, "I hold every man to be a debtor to his profession." Except for the politically inappropriate gender reference, that quote should remain in your minds and serve as your guide as you journey though the marvelous world of a casualty actuary. New Fellows, it is time to pay your debt. New Associates, keep plugging away; your turn will come soon.

MINUTES OF THE 2001 SPRING MEETING

May 6–9, 2001

FONTAINEBLEAU HILTON RESORT

MIAMI BEACH, FLORIDA

Saturday, May 5, 2001

The Board of Directors held their regular quarterly meeting from 12:00 p.m. to 6:00 p.m.

Sunday, May 6, 2001

The Board of Directors continued their regular quarterly meeting from 9:00 a.m. to 5:00 p.m.

Registration was held from 4:00 p.m. to 6:00 p.m.

New Associates and their guests were honored with a special presentation from 5:30 p.m. to 6:30 p.m. Members of the 2001 Executive Council discussed their roles in the Society with the new members. In addition, Steven G. Lehmann, who is a past president of the CAS, gave a short talk on the American Academy of Actuaries' (AAA) Casualty Practice Council.

A reception for all meeting attendees followed the new Associates reception and was held from 6:30 p.m. to 7:30 p.m.

Monday, May 7, 2001

Registration continued from 7:00 a.m. to 8:00 a.m.

The 2001 Business Session, which was held from 8:00 a.m. to 9:15 a.m., started off the first full day of activities for the 2001 Spring Meeting. CAS President Patrick J. Grannan introduced the CAS Executive Council, the Board of Directors, and CAS past presidents who were in attendance, including Robert A. Anker (1996), Phillip N. Ben-Zvi (1985), Ronald L. Bornhuetter (1975), Charles A. Bryan (1990), Michael Fusco (1989), Alice H. Gannon

(1999), David G. Hartman (1987), Steven G. Lehmann (1998), Jerome A. Scheibl (1980), and Michael L. Toothman (1991).

Mr. Grannan also recognized special guests in the audience: Morris W. Chambers, President, International Actuarial Association (IAA) and Christopher Daykin, Past President of the IAA and Institute of Actuaries.

Mary Frances Miller announced the 33 new Associates, and Robert F. Conger announced the 30 new Fellows. The names of these individuals follow.

NEW FELLOWS

David Matthew Biewer
David R. Border
Conni Jean Brown
Stephanie T. Carlson
Jeffrey Alan Courchene
Laura Ann Esboldt
Joseph Gerard Evleth
Emily C. Gilde
Bryan Hartigan
Kurt D. Hines
Randall Allen Jacobson
Michael G. Kerner
Kimberly J. Kurban
James P. Lynch
Daniel Patrick Maguire
Atul Malhotra
Julie Martineau
Eric Millaire-Morin
Scott Allan Miller
Michael A. Pauletti
John M. Pergrossi
Jordan J. Pitz
Sean Evans Porreca
Joseph John Sacala
Gary Frederick Scherer
Annmarie Schuster
Alastair Charles Shore
Mark Alan Verheyen
Shaun S. Wang
Mark Lee Woods

NEW ASSOCIATES

Afrouz Assadian
Sara T. Broadrick
Stephanie Anne Bruno
Hugo Corbeil
David Francis Dahl
Feifei Ford
Edward Kofi Gyampo
James Anthony Heer
Suzanne Barry Holohan
Christopher Wayne Hurst
Jamison Joel Ihrke
Shantelle Adrienne Johnson
Tricia Lynne Johnson
William Russell Johnson
Joseph E. Kirsits
Matthew Allen Lillegard
Timothy James McCarthy
Sharon D. Mott
Michael A. Onofrietti
Matthew R. Ostiguy
Chad Michael Ott

Michael Robert Petrarca	Ellen Marie Tierney	Stephanie C. Young
Jayne L. Plunkett	Jennifer Anne Vezza	Michael R. Zarember
Gregory T. Preble	Cameron Jason Vogt	Xiangfei Zeng
Jennifer L. Richard	Scott Michael Woomer	
	Jimmy L. Wright	

Mr. Grannan then introduced Jerome A. Scheibl, a past president of the Society, who presented the Address to New Members.

David R. Chernick, CAS vice president-programs and communications, spoke to the meeting participants about the highlights of this meeting and what was planned in the program.

Richard I. Fein, chairperson of the Committee On Review of Papers, announced that three *Proceedings* papers would be presented at this meeting. All three papers were accepted for publication in the 2001 *Proceedings of the Casualty Actuarial Society*.

Abbe S. Bensimon, vice president–continuing education, gave a brief description of this year's Call Paper Program on Financial and Accounting Systems and Issues Associated with the Globalization of Insurance. She announced that all of the call papers would be presented at this meeting. (The papers are published in the 2001 CAS *Discussion Paper Program* and can be found on the CAS Web Site.)

Mr. Grannan then began the presentation of awards. He explained that the CAS Harold W. Schloss Memorial Scholarship Fund benefits deserving and academically outstanding students in the actuarial program of the Department of Statistics and Actuarial Science at the University of Iowa. The student recipient is selected by the Trustees of the CAS Trust, based on the recommendation of the department chair at the University of Iowa. Mr. Grannan announced that Ms. Hongyan Hao is the recipient of the 2001 CAS Harold W. Schloss Memorial Scholarship Fund. Hao will be presented with a $500 scholarship.

Mr. Grannan then presented the CAS Online Services Award. This award was established as the result of the 2001 Call for Contributions to the CAS Web Site. The purposes of the call are to promote the use of Internet-based technology in the actuarial profession, to encourage the CAS membership to be actively involved in the CAS Web Site, and to establish the CAS Web Site as a primary forum for the sharing of actuarial related news, ideas, and products among CAS members. Mr. Grannan announced Stephen J. Mildenhall as the winner of the CAS Online Services Award for "The Mildenhall Aggregate Loss Tools Site," which is an interactive site that provides an introduction to using fast Fourier transform (FFT) methods to compute aggregate loss distributions.

Mr. Grannan then concluded the business session of the Spring Meeting.

Mr. Grannan next introduced the featured speaker, Mike Jensen, an Emmy-Award-winning chief financial correspondent for NBC News.

The first General Session was held from 10:45 a.m. to 12:15 p.m.

"Issues in Enterprise Risk Management"

Moderator:	Janet R. Nelson Chief Risk Officer and Senior Vice President St. Paul Companies
Panelists:	Christine Jones Business Development Manager Internet Security Systems
	Brian M. Kawamoto Director Swiss Re New Markets
	Gary Taylor Manager–Weather Risk Management ENRON

After a luncheon, the afternoon was devoted to presentations of concurrent sessions, *Proceedings* papers, and call papers. The call paper presented from 1:30 p.m. to 3:00 p.m. was:

1. "Foreign Exchange Rate Risk: Institutional Issues and Stochastic Modeling"

   Author: Richard W. Gorvett
   University of Illinois

The concurrent sessions presented from 1:30 p.m. to 3:00 p.m. were:

1. Personal Auto Classification Issues

   Moderator: Roosevelt C. Mosley
   Consulting Actuary
   MHL/Paratus

   Panelists: Howard M. Eagelfeld
   Actuary
   Florida Department of Insurance

   Alice H. Gannon
   Vice President
   United Services Automobile Association

   Gregory L. Hayward
   Actuary
   State Farm Mutual Automobile
   Insurance Company

2. Statistical Distribution of Losses—Evolution Over Time

   Moderator: Philip E. Heckman
   Vice President and Actuary
   Aon Risk Consultants

   Panelist: Greg Taylor, Ph.D.
   Director
   Taylor Fry Consulting Actuaries

3. Project Finance and Credit Enhancement

Moderator/ Panelist:	Paul Kazmierczak Managing Director Gerling Global Financial Products
Panelists:	Christine Hazen Vice President American Re-Insurance Company
	Paul R. Hussian Director Gerling Global Financial Products

4. Homeowners Classification Issues

Moderator:	Steven G. Lehmann Principal and Consulting Actuary MHL/Paratus
Panelists:	John Bargagallo Agent Product Development Manager Progressive
	Jeffrey L. Kucera Consulting Actuary MHL/Paratus
	Chester J. Szczepanski Chief Actuary Pennsylvania Insurance Department

5. Workers Compensation Insurers in Transition

Moderator:	Robert F. Conger Consulting Actuary Tillinghast-Towers Perrin
Panelists:	Douglas D. Dirks President and CEO Employers Insurance Company of Nevada
	Roger J. Fries President and CEO Kentucky Employers Mutual Insurance

Wayne Johnson
Bureau Chief, Bureau of Property and Casualty Insurer Solvency
Florida Department of Insurance

Fred R. Lowe
Chief Executive Officer
AmCOMP Incorporated

The *Proceedings* paper presented during this time was:

1. "Measuring the Interest Rate Sensitivity of Loss Reserves"

   Authors: Richard W. Gorvett
   University of Illinois

   Stephen P. D'Arcy
   University of Illinois

After a refreshment break, presentations of concurrent sessions continued from 3:30 p.m. to 5:00 p.m. Certain concurrent sessions that had been presented earlier were repeated. Additional concurrent sessions presented were:

1. The Many Pitfalls of Capital Allocation

   Moderator: John G. Aquino
   Executive Vice President
   Aon Re Services

   Panelists: Russell E. Bingham
   Vice President and Director, Corporate Research
   The Hartford

   Donald F. Mango
   Vice President
   American Re-Insurance Company

2. Space and Aviation Insurance

   Moderator/Panelist: R. Justyn Harding
   Berkshire Hathaway/CGNU/Resolute Management

Panelist:	Orin M. Linden Partner Ernst & Young LLP

3. The Education Process

Moderator:	Robert F. Conger President-Elect CAS
Panelists:	Stephen P. D'Arcy Member CAS Board of Directors
	Mary Frances Miller Vice President–Admissions CAS

4. The Changing Nature of Workers Compensation

Moderator:	Matthew T. Hayden Vice President Liberty Mutual Group
Panelists:	Susan C. Fisch Senior Vice President E.W. Blanch Co., Inc.
	Richard A. Hofmann President SIGMA Consulting Group, Inc.
	Oakley E. Van Slyke President Capital Management Technology

A reception for new Fellows and their guests was held from 5:30 p.m. to 6:30 p.m., and the general reception for all members and their guests was held from 6:30 p.m. to 7:30 p.m.

Tuesday, May 8, 2001

Registration continued from 7:00 a.m. to 8:00 a.m.

The General Sessions presented from 8:00 a.m. to 9:30 a.m. were:

"What Will 2001 Bring? A Worldwide Look at Catastrophes"

Moderator: Douglas J. Collins
Consulting Actuary and Principal
Tillinghast-Towers Perrin

Panelists: Jean-Paul Conoscente
Manager
Benfield Greig Paris

James B. Elsner, Ph.D.
Associate Professor
Florida State University

Rade T. Musulin
Vice President and Actuary
Florida Farm Bureau Insurance Companies

"Expert Systems, Technology and Fraud"

Moderator: J. Parker Boone
President
Chesapeake Consulting Group

Panelists: Thomas Boehning
Vice President
ADP Integrated Medical Solutions

Daniel J. Johnston
President
Automobile Insurers Bureau of Massachusetts

James P. Streff
President
Streff Insurance Services

A limited attendance workshop, "Executive Presentation Skills," was held from 8:00 a.m. to 12:00 p.m.

Certain concurrent sessions that had been presented earlier during the meeting were repeated this morning from 10:00 a.m. to 11:30 a.m. Additional concurrent sessions presented at this time were:

1. Building a More Diverse Actuarial Staff

Moderator:	Alice H. Gannon Vice President United Services Automobile Association
Panelists:	Edwin Felice Director, Actuarial Resources Allstate Insurance Company
	Harold L. Gray Sr. Director of Professional Development Howard University
	Edward M. Kuss Chairperson, Joint CAS/SOA Committee on Minority Recruiting Assistant Vice President Ohio Casualty Group

2. Securitization Update

Moderator:	Frederick O. Kist Senior Vice President and Chief Actuary Kemper Insurance Companies
Panelists:	Nicholas P. Giuntini Director Swiss Re New Markets
	William M. Wilt Vice President–Senior Analyst Moody's Investors Service

3. Update on Lloyd's

Moderator:	Todd J. Hess Managing Director Alleghany Underwriting Ltd.
Panelists:	Douglas J. Morton Chief Analyst Lloyd's of London
	Julian G. Ross Group Actuary Alleghany Underwriting Ltd.

4. The Martin Frankel Case: Can It Happen Again?

Moderator:	Ralph S. Blanchard Second Vice President and Actuary Travelers Property & Casualty Corporation
Panelists:	James R. Black Senior Evaluator U.S. General Accounting Office
	Michael J. Moriarty Director, Capital Markets Bureau New York State Insurance Department
	Brady Kelley Director of Financial Services NAIC

Proceedings papers presented during this time were:

1. "The *n*-Moment Insurance CAPM"

Authors:	Thomas J. Kozik
	Aaron M. Larson Allied Insurance

2. "Using Claim Department Work Measurement Systems to Determine Claim Adjustment Expense Reserves"

Author:	Joanne S. Spalla The Hartford

Various CAS committees met from 12:00 p.m. to 5:00 p.m. A limited attendance workshop, "Executive Presentation Skills," was held from 1:00 p.m. to 5:00 p.m. Certain concurrent sessions presented earlier were repeated from 1:00 p.m. to 2:30 p.m. The call papers presented during this time were:

1. "The Makings of Imminent Insurance Markets in Asia"

   Author: Julia F. Chu
   Milliman & Robertson, Inc.

2. "Conversion of European Reporting Systems to U.S. Generally Accepted Accounting Principles—A Claims Reserve Perspective"

   Authors: Chandu C. Patel
   KPMG LLP

   Leslie R. Marlo
   KPMG LLP

All members and guests enjoyed a Caribbean festival from 7:00 p.m. to 9:30 p.m.

Wednesday, May 9, 2001

Certain call papers and concurrent sessions that had been presented earlier during the meeting were repeated this morning from 8:00 a.m. to 9:30 a.m. Additional concurrent sessions presented were:

1. Nursing Home Professional Liability Issues

   Moderator: Jennifer K. Price
   Principal
   MMC Enterprise Risk

   Panelists: Susan J. Amster
   Corporate Director of Risk Management
   Avante Group

   Keith P. Becker
   Vice President
   Marsh USA

	Theresa W. Bourdon Managing Director Aon Risk Consultants, Inc.

2. Offshore Perspectives

Moderator/ Panelist:	David Y. Na Manager Deloitte & Touche, LLP
Panelists:	Michael McKnight Consulting Actuary Milliman & Robertson, Inc.
	Lisa Marie Walsh Senior Vice President London Life and General Reinsurance Company

After a refreshment break, the final General Session was held from 10:00 a.m. to 11:30 a.m.:

"The Global Village: Casualty Actuaries Meeting the Challenges of International Markets"

Moderator:	Christopher Daykin Government Actuary Government Actuary's Department, U.K.
Panelists:	Robert A. Anker Quay Quest
	Luis Huerta Director General Seguros Genesis, S.A.
	Jay B. Morrow Vice President and Actuary American International Underwriters

Patrick J. Grannan officially adjourned the 2001 CAS Spring Meeting at 11:45 a.m. after closing remarks and an announcement of future CAS meetings.

Attendees of the 2001 CAS Spring Meeting

The 2001 CAS Spring Meeting was attended by 272 Fellows, 133 Associates, and 51 Guests. The names of the Fellows and Associates in attendance follow:

FELLOWS

Jean-Luc E. Allard
Ethan D. Allen
Scott C. Anderson
Susan Gozzo Andrews
Robert A. Anker
John G. Aquino
Steven D. Armstrong
Martin S. Arnold
Lawrence J. Artes
David Steen Atkinson
Nicolas Beaupre
Douglas L. Beck
Stephen A. Belden
Abbe Sohne Bensimon
Phillip N. Ben-Zvi
Regina M. Berens
Everett G. Bishop
Lisa A. Bjorkman
Ralph S. Blanchard
Robert G. Blanco
Barry E. Blodgett
Carol Blomstrom
LeRoy A. Boison
Paul Boisvert
James Parker Boone
Joseph A. Boor
David R. Border
Sherri Lynn Border
Ronald L. Bornhuetter
Theresa W. Bourdon
Amy S. Bouska
Erik R. Bouvin
Wallis A. Boyd
Jerelyn S. Boysia
George P. Bradley
Mark D. Brissman
Conni Jean Brown
Kirsten R. Brumley
Charles A. Bryan
Peter Vincent Burchett
Julie Burdick
Mark J. Cain
Christopher S. Carlson
Stephanie T. Carlson
Kenneth E. Carlton
Michael J. Caulfield
Joseph Gerald Cerreta
David R. Chernick
Douglas J. Collins
Robert F. Conger
Francis X. Corr
Jeffrey Alan Courchene
Catherine Cresswell
Mary Elizabeth Frances Cunningham
Stephen P. D'Arcy
Ronald A. Dahlquist
Kenneth S. Dailey
Guy Rollin Danielson
Lawrence S. Davis
Curtis Gary Dean
Jeffrey F. Deigl
Marie-Julie Demers
Patrick K. Devlin
Sean R. Devlin
Behram M. Dinshaw
Michael C. Dolan
James L. Dornfeld
Michael C. Dubin
Tammi B. Dulberger
Howard M. Eagelfeld
Thomas J. Ellefson
Charles C. Emma
Laura Ann Esboldt
Philip A. Evensen
Joseph Gerard Evleth
John S. Ewert
Janet L. Fagan
Kendra M. Felisky-Watson
Wayne H. Fisher
Beth E. Fitzgerald
Richard L. Fox
Bruce F. Friedberg
Michael Fusco
Scott F. Galiardo
Alice H. Gannon

David B. Gelinne
William R. Gillam
Nicholas P. Giuntini
Todd B. Glassman
Sanjay Godhwani
Charles T. Goldie
Richard W. Gorvett
Leon R. Gottlieb
Patrick J. Grannan
Linda M. Groh
Marshall J. Grossack
David N. Hafling
Greg M. Haft
Leigh Joseph Halliwell
Robert C. Hallstrom
Alexander Archibold Hammett
George M. Hansen
Steven Thomas Harr
David C. Harrison
Bryan Hartigan
David G. Hartman
Matthew T. Hayden
Gregory L. Hayward
Qing He
Todd J. Hess
Amy Louise Hicks
Kurt D. Hines
Robert J. Hopper
Mary T. Hosford
George A. Hroziencik
Jeffrey R. Hughes
Paul R. Hussian
Aguedo M. Ingco
Randall Allen Jacobson
Richard M. Jaeger
Christian Jobidon
Daniel Johnson
Eric J. Johnson
Thomas S. Johnston
Jeffrey R. Jordan
Gary R. Josephson
Jeremy M. Jump
Janet S. Katz
Mark J. Kaufman
Steven A. Kelner
Michael G. Kerner
Frederick O. Kist
Joel M. Kleinman
Brandelyn C. Klenner
John J. Kollar
Gary I. Koupf
Thomas J. Kozik
John R. Kryczka
Jeffrey L. Kucera
David R. Kunze
Kimberly J. Kurban
Edward M. Kuss
Blair W. Laddusaw
Salvatore T. LaDuca
Robin M. LaPrete
James W. Larkin
Michael D. Larson
Guy Lecours
Robert H. Lee
Steven G. Lehmann
Charles Letourneau
Orin M. Linden
Richard W. Lo
Michelle Luneau
Aileen Conlon Lyle
James P. Lynch
Daniel Patrick Maguire
Barbara S. Mahoney
Gary P. Maile
Atul Malhotra
Donald F. Mango
Leslie R. Marlo
Julie Martineau
Kelly J. Mathson
Robert W. Matthews
Michael G. McCarter
Richard Timmins McDonald
Liam Michael McFarlane
Allison Michelle McManus
Dennis T. McNeese
Eric Millaire-Morin
David L. Miller
Mary Frances Miller
Michael J. Miller
Scott Allan Miller
Paul David Miotke
Jay B. Morrow
Roosevelt C. Mosley
Robert V. Mucci
Seth Wayne Myers
David Y. Na
Jennifer A. Na
Janet R. Nelson
Kenneth J. Nemlick
Hiep T. Nguyen
Mindy Y. Nguyen
Paul G. O'Connell
Mihaela Luminita S. O'Leary

Christopher Edward Olson
Joseph M. Palmer
M. Charles Parsons
Chandrakant C. Patel
Michael A. Pauletti
Anthony A. Peraine
John M. Pergrossi
Luba O. Pesis
George N. Phillips
Richard Matthew Pilotte
Jordan J. Pitz
Igor Pogrebinsky
Sean Evans Porreca
Jennifer K. Price
Mark R. Proska
David S. Pugel
Alan K. Putney
Kathleen Mary Quinn
Donald K. Rainey
Hany Rifai
Brad M. Ritter
Rebecca L. Roever
Deborah M. Rosenberg
Richard A. Rosengarten
Kevin D. Rosenstein
Gail M. Ross
Bradley H. Rowe
James B. Rowland
David L. Ruhm
Joseph J. Sacala
Romel G. Salam
Jerome A. Scheibl
Gary Frederick Scherer
Neal J. Schmidt
Annmarie Schuster
Kim A. Scott
Terry Michael Seckel
Ernest C. Segal
Harvey A. Sherman
Jerome J. Siewert
Christopher M. Smerald
Richard A. Smith
Joanne S. Spalla
David Spiegler
Thomas N. Stanford
Grant D. Steer
Lawrence J. Steinert
Carol A. Stevenson
Michael J. Steward
Ilene G. Stone
James P. Streff
James Surrago
Susan T. Szkoda
Kathleen W. Terrill
Richard D. Thomas
Kevin B. Thompson
Michael Toledano
Michael L. Toothman
Linda Kay Torkelson
Theresa Ann Turnacioglu
James F. Tygh
Oakley E. VanSlyke
Ricardo Verges
Mark Alan Verheyen
William J. VonSeggern
Claude A. Wagner
Robert H. Wainscott
Patrick M. Walton
Shaun S. Wang
Bryan C. Ware
Thomas V. Warthen
Geoffrey Todd Werner
William B. Westrate
Mark Whitman
William M. Wilt
Chad C. Wischmeyer
Susan E. Witcraft
Richard G. Woll
Tad E. Womack
Mark Lee Woods
Yuhong Yang
Roger Allan Yard
Gerald Thomas Yeung
Heather E. Yow

ASSOCIATES

Nancy S. Allen
Paul D. Anderson
Afrouz Assadian
Craig Victor Avitabile
David James Belany
Sara T. Broadrick
Lisa A. Brown
Stephanie Anne Bruno
Kenrick A. Campbell
Paul A. Chabarek
Philip S. Chou
Julia Feng-Ming Chu
Jeffrey A. Clements
Vincent P. Connor
Sean O. Cooper
Hugo Corbeil
David Ernest Corsi
Richard J. Currie
David Francis Dahl
Frank H. Douglas
Stephen C. Dugan
Dominick A. Elia
Anders Ericson
John D. Ferraro
William P. Fisanick
Natalie Shayer Fisher
Sean Paul Forbes
Feifei Ford
Sarah J. Fore
Timothy J. Friers
Dustin Wayne Gary
Lynn A. Gehant
Nathan Terry Godbold
Steven B. Goldberg
Terry L. Goldberg
John W. Gradwell
Edward Kofi Gyampo
Nasser Hadidi
Aaron Halpert
Thomas L. Hayes
Pamela Barlow Heard
Philip E. Heckman
James Anthony Heer
Joseph A. Herbers
Richard A. Hofmann
Jane W. Hughes
Christopher Wayne Hurst
Jamison Joel Ihrke
Philip M. Imm
Shantelle Adrienne Johnson
Tricia Lynne Johnson
William Russell Johnson
Daniel J. Johnston
William Rosco Jones
Anthony N. Katz
John Hun Kim
Martin T. King
Diane L. Kinner
Joseph E. Kirsits
Richard Kollmar
Bobb J. Lackey
Marc LaPalme
Aaron Michael Larson
Isabelle Lemay
Sharon Xiaoyin Li
Matthew Allen Lillegard
Jing Liu
Erik Frank Livingston
William F. Loyd
Robb W. Luck
Christopher J. Luker
David J. Macesic
Kevin M. Madigan
Betsy F. Maniloff
Albert Maroun
Timothy J. McCarthy
Michael B. McKnight
Gregory A. Moore
Michael James Moss
Sharon D. Mott
Thomas M. Mount
Rade T. Musulin
Prakash Narayan
Rodrick Raymond Osborn
Matthew R. Ostiguy
Chad Michael Ott
Wendy Wei-Chi Peng
Jill E. Peppers
Michael Robert Petrarca
Robert C. Phifer
Jayne L. Plunkett
Thomas LeRoy Poklen
Ruth Youngner Poutanen
Gregory T. Preble
Richard B. Puchalski

James E. Rech
Jennifer L. Richard
David C. Riek
Brad E. Rigotty
Stephen Daniel Riihimaki
Karen Lynn Rivara
Joseph Francis Rosta
Maureen S. Ruth
John P. Ryan
Shama S. Sabade
James C. Sandor
Michael Sansevero
Jennifer Arlene Scher
Arlyn G. Shapiro
Donald P. Skrodenis
David C. Snow
Klayton N. Southwood
Dennis G. Sparks
Gary A. Sudbeck
Brian K. Sullivan
Diane R. Thurston
Ellen Marie Tierney
Brian K. Turner
Jennifer Anne Vezza
Phillip C. Vigliaturo
Gregory S. Wanner
Stephen D. Warfel
Denise R. Webb
Petra Lynn Wegerich
Robert G. Weinberg
Bonnie S. Wittman
Robert F. Wolf
Scott Michael Woomer
Jimmy L. Wright
Stephanie C. Young
Michael R. Zarember
Xiangfei Zeng
Steven Bradley Zielke

Volume LXXXVIII, Part 2 **No. 169**

PROCEEDINGS
November 11, 12, 13, 14, 2001

SMOOTHED NPML ESTIMATION OF THE RISK DISTRIBUTION UNDERLYING BONUS-MALUS SYSTEMS

MICHEL DENUIT AND PHILIPPE LAMBERT

Abstract

Mixed Poisson distributions are widely used for modeling claim counts when the portfolio is thought to be heterogeneous. The risk (or mixing) distribution then represents a measure of this heterogeneity. The aim of this paper is to use a variant of the Patilea and Rolin [15] smoothed version of the Simar [20] Non-Parametric Maximum Likelihood Estimator of the risk distribution in the mixed Poisson model. Empirical results based on two data sets from automobile third-party liability insurance demonstrate the relevance of this approach. The design of merit-rating schemes is discussed in the second part of the paper.

ACKNOWLEDGEMENT

The authors would like to thank the CAS review team for helpful comments and suggestions. The support of the Belgian Government under "Projet d'Actions de Recherche Concertées" (No. 98/03-217) is gratefully acknowledged.

1. INTRODUCTION AND MOTIVATION

In most developed countries, third-party liability automobile insurance represents a considerable share of the yearly non-life premium collection (for instance, in Belgium, 26% during the year 1998). Therefore, many attempts have been made in the actuarial literature to find a probabilistic model for the distribution of the number of automobile accidents; for a review of the existing literature, we refer the interested reader, e.g., to Lemaire [12] or to Denuit [7]. Most of these models are parametric (i.e., an analytical expression is assumed for the probabilities that a policyholder reports k claims during an insurance period, depending on one or several parameters to be estimated on the basis of the observations).

In order to see if there exists a universal model for claims distributions in automobile portfolios, Gossiaux and Lemaire [10] examined six observed data sets. Those came from five countries and were studied before by other researchers. Gossiaux and Lemaire [10] fitted the Poisson distribution, the Generalized Geometric distribution, the Negative Binomial distribution and a two-point mixed Poisson distribution to each of the data sets by the Maximum Likelihood method and the method of moments. They concluded that no single probability law seems to emerge as providing a good fit to all of them. Moreover, there was at least one example where each model got rejected by a chi-square test (at the level 10%). Seal [18] supplemented the paper by Gossiaux and Lemaire [10] with an analysis of some automobile accident data from California. This author concluded that his analyses supported the mixed Poisson hypothesis for the distribution of the number of claims.

In this paper, we will work in the mixed Poisson model, but no assumption will be made about the risk (or mixing) distribution.

Following Walhin and Paris [23], we first recall the basic features about the Non-Parametric Maximum Likelihood Estimator (NPMLE, in short) of the risk distribution. As pointed out by these authors, the NPMLE suffers from some serious drawbacks in the design of Bonus-Malus systems. The problems are mainly due to its purely discrete nature. Therefore, we will propose a smoothed version of the NPMLE. In the second part of this paper, we focus on "Bonus-Malus Systems" (BMS, in short). A BMS is a particular form of experience rating. It penalizes insureds responsible for one or more accidents by premium surcharges, or *maluses*, and rewards claim-free policyholders by awarding them discounts, or *bonuses*. An excellent account of these systems can be found in Lemaire [12].

Let us consider a portfolio consisting of n policies, numbered 1 to n. Denote as K_{ij} the number of claims incurred by the ith policyholder during the jth year that the policy is in force. We adopt the assumptions usually made in credibility theory (e.g., claim frequencies vary from policy to policy, claim numbers for different policyholders are independent, and claim numbers for one policyholder in different periods are conditionally independent). Formally, it is assumed that, for fixed i, the K_{ij}s are conditionally independent and identically distributed given a random risk parameter Θ_i that represents unknown risk characteristics of the policy. After t years, the available data are $(K_{i1}, K_{i2}, \ldots, K_{it})$ and the insurance company wants to use these data to adjust the premium for year $t+1$; the premium for year $t+1$ is thus a function $\Psi(K_{i1}, K_{i2}, \ldots, K_{it})$ of the past claims. Actuaries have traditionally applied minimization of the expected quadratic loss in order to determine Ψ; that is, Ψ minimizes $\mathrm{E}[\Psi(K_{i1}, K_{i2}, \ldots, K_{it}) - \Theta_i]^2$, which is interpreted as the expected difference between the "true" premium Θ_i and the credibility premium $\Psi(K_{i1}, K_{i2}, \ldots, K_{it})$. Henceforth, we assume that the sequences $\{\Theta_i, K_{i1}, K_{i2}, K_{i3}, \ldots\}$ are independent and identically distributed; for ease of explanation, we drop the policyholder's index i.

Considering the last paragraph, the very basic elements of a BMS are as follows:

1. an appropriate premium calculation principle;
2. a conditional distribution for the number of claims, that is, for the $[K_j \mid \Theta = \theta]$s;
3. a distribution for the risk parameter Θ to describe how the conditional distributions vary across the portfolio.

Let us give some details on these aspects. Considering the premium calculation principle, we use the expected value principle. This principle requires the insured to pay the pure premium plus a safety loading proportional to the pure premium. The pure premium will be the individual claim frequency per year multiplied by the average cost of a claim and can be scaled so that it will be equal to the claim frequency. The problem of the insurer is to predict, at the renewal of the policy, the claim frequency of the insured for this new year, given the observations of the reported accidents in the preceding periods.

Let us now turn to the conditional distribution of the annual claim numbers. In automobile third-party liability insurance portfolios, the Poisson distribution provides a good description of the number of claims incurred by an individual policyholder during a given reference period (one year, say). The assumptions underlying the Poisson counting model indeed provide a good approximation to the accident generating mechanism; see, e.g., Lemaire [12]. Therefore, in the remainder of the paper, we consider that the number of claims incurred by a given policyholder during a reference period conforms to a Poisson distribution.

Now, individual driving abilities vary from individual to individual. Consequently, the portfolio is heterogeneous and policyholders will have different Poisson parameters. This is indicated by the rejection of the homogeneous Poisson model when it is applied to fit data sets from automobile insurance portfolios; for empirical evidence supporting this assertion, see Gossiaux and

Lemaire [10]. In order to reflect the different underlying risk profiles, each policyholder is characterized by the value of his mean claim frequency θ, and θ is considered to be a realization of a non-observable random variable Θ, whose support is contained in the half-positive real line $\mathbb{R}^+ \equiv [0, +\infty)$. In other words, the conditional probability that a driver with annual mean claim frequency θ is involved in k accidents during the ith year is

$$\mathrm{P}[K_j = k \mid \Theta = \theta] = p(k \mid \theta) = \exp(-\theta)\frac{\theta^k}{k!},$$
$$k \in \mathbb{N} \equiv \{0, 1, 2, \ldots\}. \qquad (1.1)$$

The annual number of accidents caused by a randomly selected policyholder of the portfolio during the jth year is then distributed according to a mixed Poisson law, that is,

$$\mathrm{P}[K_j = k] = p(k \mid \Theta) = \int_{\theta \in \mathbb{R}^+} p(k \mid \theta) dF_\Theta(\theta),$$
$$k \in \mathbb{N}, \qquad (1.2)$$

where F_Θ denotes the cumulative distribution function (cdf, in short) of Θ, assumed to fulfill $F_\Theta(0) = 0$. The mixing distribution described by F_Θ represents the heterogeneity of the portfolio of interest; F_Θ is often called the structure function. It is worth mentioning that the mixed Poisson model (1.2) is an accident-proneness model: it assumes that a policyholder's mean claim frequency does not change over time but allows some insured persons to have higher mean claim frequencies than others.

Sometimes, (1.2) is taken to be a finite mixture model, that is, the mixing distribution is discrete and puts positive masses $\pi_1, \pi_2, \ldots, \pi_q$ on only a finite number q of positive real atoms $0 < \theta_1 < \theta_2 < \cdots < \theta_q$. Then,

$$p(k \mid \Theta) = \sum_{\ell=1}^{q} p(k \mid \theta_\ell)\pi_\ell, \qquad k \in \mathbb{N}. \qquad (1.3)$$

The fact that Θ has a distribution with q support points means that the portfolio of interest consists of only q categories of policyholders. The special case $q = 2$ gives the classical "good risk/bad risk" model considered in Gossiaux and Lemaire [10]. Note that the actual reality of the insurance business is a finite mixture model (by taking q to be the number of policyholders in the portfolio). In risk theory, the finite mixture model (1.3) was first proposed by Grenander [8]; see also Grenander [9].

Let us now consider the choice of F_Θ. Traditionally, actuaries have assumed that the distribution of θ values among all drivers is well approximated by a two-parameter Gamma distribution. This choice is particularly desirable because the class of the Gamma distributions is the natural conjugate family for the Poisson and facilitates a Bayesian approach towards updating mean frequency estimates. The resulting probability distribution for the number of claims is Negative Binomial. Other classical choices for F_Θ include the Inverse-Gaussian (which results in the Poisson-Inverse-Gaussian law for the number of claims; see, e.g., Willmot [24] and Tremblay [21]) and Hoffman's distributions (see Kestemont and Paris [11] and Walhin and Paris [23]). However, there is no particular reason to believe that F_Θ belongs to some specified parametric family of distributions. Therefore, we would like to resort to a nonparametric estimator for F_Θ. This will thus lead to BMS relying on fewer assumptions than the usual ones.

More precisely, after having recalled some key features of the model (1.2) in Section 2, we apply the Simar [20] NPMLE of F_Θ in Section 3. The Maximum Likelihood approach results in a finite mixture model (1.3) with relatively few support points (see (3.1)). As pointed out by Walhin and Paris [23], this model is undesirable for constructing BMS. Therefore, we propose in Section 4 to use a variant of the Patilea and Rolin [15] Empirical Nonparametric Bayesian estimator for F_Θ: this estimator is a finite mixture of Gamma distributions and can be intuitively considered as a smoothed version of the NPMLE, with the Gamma distribution playing the role of a kernel. In Section 5, we examine the BMS obtained with this model.

The present paper expands on several previous works. Albrecht [1] gave a first account of statistical methods connected with model (1.2), mainly in a maximum likelihood approach. More recently, Walhin and Paris [23] compared BMS obtained with Hofmann's parametric family and Simar's NPMLE for F_Θ. These authors showed that, although the NPMLE is powerful to evaluate functionals of claim counts, it is not suitable for building BMS, because it is purely discrete. Our approach consists in smoothing Simar's estimator with a Gamma kernel and is thus comparable with Carrière's [4] study that smoothed the Tucker-Lindsay moment estimator with a Log-Normal kernel.

Let us now detail some of the notations used throughout this paper. We denote as K_θ (resp. K_Θ) a random variable with probability distribution $\{p(k \mid \theta),\ k \in \mathbb{N}\}$ in (1.1) (resp. $\{p(k \mid \Theta),\ k \in \mathbb{N}\}$ in (1.2)). We denote by μ_k, $k = 1, 2, \ldots$, the moments $\mathrm{E}K_\Theta^k$ of K_Θ. Those of Θ are the ν_ks, $k = 1, 2, \ldots$, that is, $\nu_k = \mathrm{E}\Theta^k$. By convention, $\mu_0 = \nu_0 \equiv 1$. Henceforth, we assume that we have observed an insurance collective consisting of n independent policies. The data that we have at our disposal are as follows: we know that n_k policies caused k claims during the reference period, $k = 0, 1, \ldots, k_{\max}$; $k_{\max}$ is the maximal number of claims observed for a policy. The empirical claim frequencies are

$$\begin{cases} \hat{p}(k) = \dfrac{n_k}{n}, & k = 0, 1, \ldots, k_{\max}, \\ \hat{p}(k) = 0, & k \geq k_{\max} + 1. \end{cases}$$

These unconstrained estimations reproduce exactly what is observed in the data. Thus, the moments μ_k are estimated with the help of their sample analogs $\hat{\mu}_k$, given by

$$\hat{\mu}_k = \frac{1}{n} \sum_{j=1}^{k_{\max}} j^k \hat{p}(j), \qquad k \in \mathbb{N}.$$

For the numerical illustrations, we used the two data sets presented in Appendix A. Portfolio 1 relates to Belgium and has been observed in 1958; it can be found in Gossiaux and Lemaire

[10]. Portfolio 2 has been kindly provided to us by a large insurance company operating in the Benelux; it has been observed in 1995.

2. BASIC PROPERTIES OF THE MIXED POISSON MODEL

2.1. Estimation of Mixing Functionals

According to Carrière [3], given a function $\phi : \mathbb{R}^+ \to \mathbb{R}$, the quantity $\mathrm{E}\phi(\Theta)$ is estimable if there exists a function $\psi : \mathbb{N} \to \mathbb{R}$ such that

$$\mathrm{E}\phi(\Theta) = \mathrm{E}\psi(K_\Theta). \tag{2.1}$$

Of course, such a function ψ theoretically always exists. It suffices to take $\psi(K_\Theta) = \mathrm{E}[\phi(\Theta) \mid K_\Theta]$ so that (2.1) holds, provided ϕ is integrable. The actual meaning of (2.1) is that we desire an explicit expression for ψ. If ϕ possesses some desirable property, ψ can be obtained explicitly. This is, for instance, the case when ϕ is an absolutely monotone function, i.e., that all the derivatives $\phi^{(1)}, \phi^{(2)}, \phi^{(3)}, \ldots$ of ϕ exist and are non-negative. Carrière [3] proved that the function ψ involved in (2.1) is then given by

$$\psi(\ell) = \sum_{k=0}^{\ell} \binom{\ell}{k} \phi^{(k)}(0), \qquad \ell \in \mathbb{N}.$$

In practice, in order to estimate a quantity $\mathrm{E}\phi(\Theta)$, we use

$$\widehat{\mathrm{E}\phi(\Theta)} = \sum_{k=0}^{k_{\max}} \psi(k)\hat{p}(k).$$

Carrière [3] proved the asymptotic normality for such estimators.

Let us now examine two simple examples.

EXAMPLE 2.1 Take $\phi(\theta) = \exp(t\theta)$; then

$$\psi(\ell) = \sum_{k=0}^{\ell} \binom{\ell}{k} t^k = (1+t)^\ell.$$

As a consequence, the moment generating function of Θ is estimable. The knowledge of $\{p(k \mid \Theta),\ k \in \mathbb{N}\}$ is thus equivalent to the knowledge of F_Θ.

EXAMPLE 2.2 For $\phi(\theta) = \theta^k$, we get

$$\psi(\ell) = \ell(\ell-1)\dots(\ell-k+1) \qquad \text{for} \quad \ell = k, k+1, \dots.$$

The moments ν_k of Θ are thus estimable. More precisely, the ν_ks are estimated by

$$\begin{cases} \hat{\nu}_k = \displaystyle\sum_{j=k}^{k_{\max}} j(j-1)\dots(j-k+1)\hat{p}(j), & k = 1, 2, \dots, k_{\max}, \\ \hat{\nu}_k = 0, \quad k \geq k_{\max}+1. \end{cases}$$

The estimator $\hat{\nu}_k$ is unbiased and almost surely consistent for ν_k.

The fact that the first moments of Θ can be estimated from realizations of K_Θ will be used at several occasions in the remainder of this paper.

2.2. Testing the Mixed Poisson Hypothesis

The present work focuses on the model (1.2). Considering the possibility of misspecification, there is a need for a statistical test to decide whether the model (1.2) is reasonable to fit the data. To this end, let us present the non-parametric test proposed by Carrière [3]. The reasoning behind this test is as follows. For any positive integer k, let $\mu_{[k]}$ be the kth descending factorial moment of K_Θ, i.e.,

$$\mu_{[k]} = \mathrm{E}[K_\Theta(K_\Theta - 1)\dots(K_\Theta - k + 1)],$$

and let $\hat{\mu}_{[k]}$ be the sample analogs, i.e.,

$$\hat{\mu}_{[k]} = \sum_{j=k}^{k_{\max}} j(j-1)\dots(j-k+1)\hat{p}(j), \qquad k = 1, 2, \dots, k_{\max},$$

and $\hat{\mu}_{[k]} = 0$ for $k \geq k_{\max}+1$. If K_Θ has a mixed Poisson distribution then $\mu_{[k]} = \nu_k = \mathrm{E}\Theta^k$ by virtue of Example 2.2. Conse-

TABLE 2.1

EMPIRICAL FACTORIAL MOMENTS RELATING TO PORTFOLIOS 1 AND 2

Factorial Moments	Portfolio 1	Portfolio 2
$\hat{\mu}_{[1]}$	0.2144	0.0936
$\hat{\mu}_{[2]}$	0.1205	0.0177
$\hat{\mu}_{[3]}$	0.1605	0.0066
$\hat{\mu}_{[4]}$	0.3272	0.0036

quently, $\hat{\mu}_{[k]}$ estimates ν_k. From Jensen inequality, we find that

$$\mu_{[k]} \geq (\mathrm{E}\Theta)^k = (\mu_{[1]})^k$$

must hold for $k = 2, 3, \ldots$, whenever K_Θ is mixed Poisson. Therefore, if $\mu_{[k]} < (\mu_{[1]})^k$ for some k, then the underlying distribution cannot be of mixed Poisson type. Based on this fact, Carrière [3] suggested the test statistic $\sqrt{n}\{(\hat{\mu}_{[1]}, \hat{\mu}_{[k]}) - (\mu_{[1]}, \mu_{[k]})\}$, that weakly converges to a bivariate Normal distribution as $n \to +\infty$. The factorial moments used in the test statistic for Portfolios 1 and 2 in Appendix A are given in Table 2.1.

Carrière [3] constructed a Bonferroni multiple comparison test. In its simplest form, this statistical procedure is as follows. In order to decide whether the number of claims caused by a policyholder of the portfolio can conform to a mixed Poisson distribution (i.e., to test the null hypothesis H_0 that the underlying distribution is of the form (1.2)), it suffices to compute the value T_{obs} of the test statistic

$$T = \frac{\sqrt{n}(\hat{\mu}_{[1]}^2 - \hat{\mu}_{[2]})}{\sqrt{4(1-\hat{\mu}_{[1]})(\hat{\mu}_{[1]}^3 - 2\hat{\mu}_{[2]}\hat{\mu}_{[1]} + \hat{\mu}_{[3]}) + \hat{\mu}_{[4]} + 2\hat{\mu}_{[2]} - \hat{\mu}_{[2]}^2}}$$

and to reject H_0 if $T_{\text{obs}} > z_\alpha$, where z_α is such that

$$\frac{1}{\sqrt{2\pi}} \int_{t=-\infty}^{z_\alpha} \exp(-t^2/2)dt = 1 - \alpha.$$

Note that this test relies on the asymptotic properties of T so that n has to be large enough.

On each of the two data sets presented in Appendix A, the model (1.2) was never rejected on the basis of Carrière's test. In both cases, $\hat{\mu}_{[1]}^2 < \hat{\mu}_{[2]}$ so that $T_{\text{obs}} < 0$ and the null assumption is not rejected.

2.3. Poisson vs. Poisson Mixture

Let us now recall some basic facts about the model (1.2). First of all, it makes sense to study the mixed Poisson model through F_Θ. As noticed in Example 2.1, there is indeed a one-to-one correspondence between the mixing distribution and the resulting mixed distribution, that is, if K_{Θ_1} and K_{Θ_2} are identically distributed, then Θ_1 and Θ_2 also are.

To each of the two data sets presented in Appendix A, we fitted a homogeneous Poisson distribution to the observations. These fits, given in column A, were clearly rejected (p-values smaller than 10^{-3}). This indicates that the two portfolios are heterogeneous.

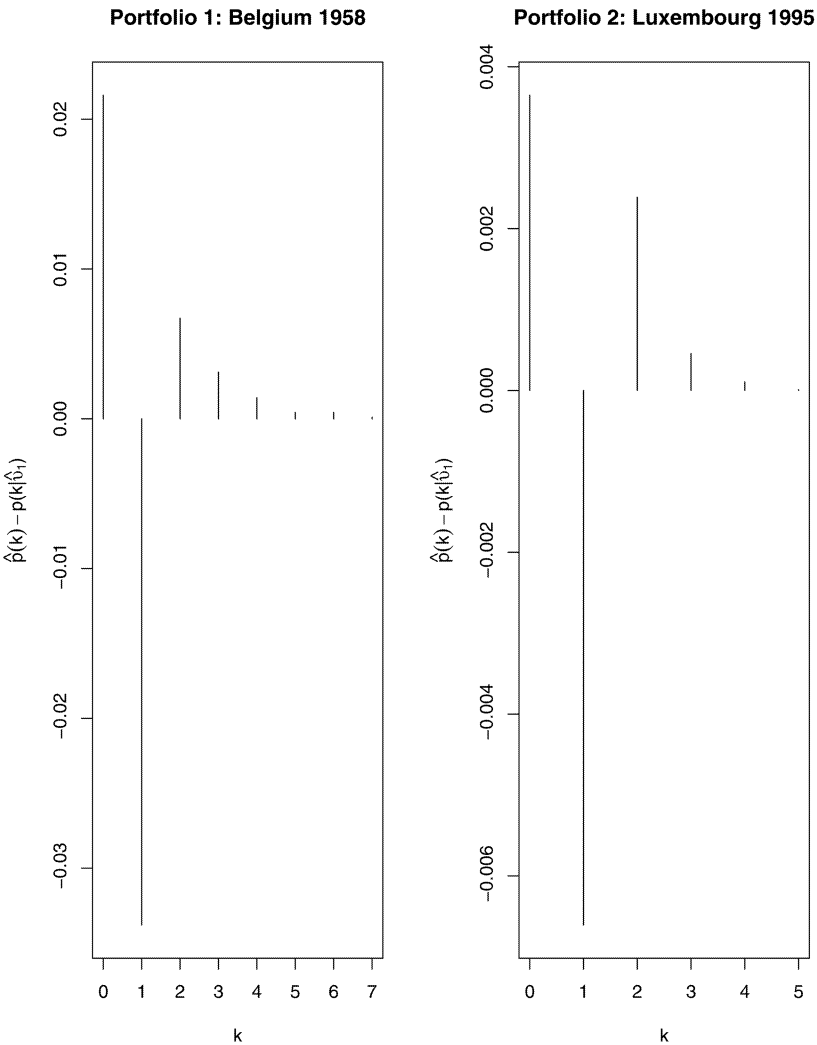
Another technique to check for the heterogeneity of the portfolio is described next. Therefore, let us recall that the model (1.2) enjoys the following nice property. Let $p(k \mid \Theta)$ be as given in (1.2) and $\{p(k \mid \nu_1),\ k \in \mathbb{N}\}$ be the discrete probability density function of the Poisson distribution with mean $\nu_1 = \mathrm{E}\Theta$, i.e.,

$$p(k \mid \nu_1) = \exp(-\nu_1)\frac{\nu_1^k}{k!}, \qquad k \in \mathbb{N}.$$

For any Θ such that $\mathrm{Var}[\Theta] > 0$, the number of sign changes of the sequence $\{p(k \mid \Theta) - p(k \mid \nu_1),\ k \in \mathbb{N}\}$ equals 2 (the first sign being a plus). This result has been established by Shaked [19]. For the data sets presented in Appendix A, we plot in Figure 1 the sequence $\{\hat{p}(k) - p(k \mid \hat{\nu}_1),\ k = 0, 1, \ldots, k_{\max}\}$. We expect to observe two sign changes if the data come from a Poisson mixture (1.2). The actual values are $\{0.0216; -0.0338; 0.0067; 0.0031; 0.0014; 0.0004; 0.0004; 0.0001\}$ for Portfolio 1

FIGURE 1

SEQUENCE $\{\hat{p}(k) - p(k \mid \hat{\nu}_1),\ k = 0, 1, \ldots, k_{\text{MAX}}\}$ FOR THE DATA SETS PRESENTED IN APPENDIX A



and $\{0.0037; -0.0066; 0.0024; 0.0005; 0.0001; 9 \times 10^{-6}\}$ for Portfolio 2. We notice that the difference between the observed data and its Poisson fit exhibits two sign changes, as it is bound to do when the underlying distribution is a mixture of Poisson distributions. This indicates that the Poisson parameter varies from individual to individual.

3. NON-PARAMETRIC ESTIMATION OF THE RISK DISTRIBUTION

3.1. NPMLE

In a seminal paper, Simar [20] gave a detailed description of the NPMLE of F_Θ, as well as an algorithm for its computation. The NPMLE is a discrete distribution, so that the resulting model is of the form (1.3). Simar [20] obtained an upper bound for the size of the support of the NPMLE. This upper bound uses the quantity κ defined to be the number of observed distinct values, i.e.,

$$\kappa = \#\{k \in \mathbb{N} \text{ such that } \hat{p}(k) > 0\}.$$

In most cases, $\kappa = k_{\max} + 1$. To be specific, Simar [20] showed that the NPMLE $\hat{F}_\Theta$ of F_Θ exists and is unique. The number of support points of the NPMLE is less than or equal to

$$\hat{q} = \min\left\{\left[\frac{k_{\max} + 1}{2}\right], \kappa\right\}, \tag{3.1}$$

where $[x]$ denotes the integer part of the real x; for the data sets in Appendix A, $\hat{q} = 4$ for Portfolio 1 and $\hat{q} = 3$ for Portfolio 2. The solution $\hat{F}_\Theta$ puts probability masses $\hat{\pi}_1, \hat{\pi}_1, \ldots, \hat{\pi}_{\hat{q}}$ at the atoms $\hat{\theta}_1, \hat{\theta}_1, \ldots, \hat{\theta}_{\hat{q}}$. In order to get a first approximation of $\hat{F}_\Theta$, we resort to the moment estimator for F_Θ proposed by Tucker [22] and suitably made precise by Lindsay [13], [14]. The moments of Θ were estimated as described in Example 2.2. MLE's were obtained with the help of the numerical optimization procedure nlm in the software R (S-plus clone; see Ross and Gentleman [16]). The algorithms implemented in nlm are given in Dennis and Schnabel [6] and Schnabel, Koontz and Weiss [17].

The NPMLE fits to each of the data sets can be found in Appendix A, together with the corresponding observed values of the χ^2-statistics. When $\hat{q} \geq 3$, we fitted a model with 2 and 3 components. The results can be summarized as follows:

1. For Portfolio 1, the NPMLE of F_Θ has at most 4 support points. It appeared that a 3-point NPMLE gave a satisfactory fit (displayed in Column B), reflected in a p-value of 51%. The 3-point $\hat{F}_\Theta$ is thus preferred by virtue of the statistical principle of parsimony. The NPMLE creates 3 categories of policyholders: the best ones (with a claim frequency of about 0) representing 41.8% of the portfolio, the standard ones (with a claim frequency of 33.6%) representing 57.3% of the portfolio, and the bad ones (with a claim frequency of 254.4%) representing 0.1% of the portfolio. The fit provided by a 2-point $\hat{F}_\Theta$ (displayed in Column C) is rejected since the p-value is equal to 0.5%.
2. For Portfolio 2, we have $\hat{q} = 3$ and we fitted the data with a 3-point (Column B) and a 2-point (Column C) NPMLE. Since the quality of the two fits is similar (p-values of 26% and 29%, respectively), we prefer the 2-point $\hat{F}_\Theta$. We thus have a good risk/bad risk model, with 93.3% of good drivers whose claim frequency is 6.8% and 6.7% of bad drivers with a claim frequency of 44.6%.

3.2. Smoothed NPMLE

The purely discrete nature of the NPMLE of the risk distribution sometimes causes problems in ratemaking (as shown in Section 4). For this reason, a smoothed version of it is desirable; it is the aim of this section to propose such an estimator.

In order to estimate F_Θ, Patilea and Rolin [15] suggested resorting to a finite mixture of natural conjugate priors of the Poisson distribution; they call this estimator an Empirical Non-

Parametric Bayesian estimator (ENBE, in short). These authors proved that the ENBE is an asymptotic Maximum Likelihood estimator. In other words, it is an estimator that almost maximizes the likelihood in the sense that the difference between the maximal value of the likelihood (as a function of F_Θ) and the value of the likelihood corresponding to the ENBE tends to zero as the sample size grows to $+\infty$. This ensures the consistency of the ENBE. We propose here a slightly modified version of the Patilea-Rolin estimator. In order to smooth the NPMLE of F_Θ, we let the family of natural conjugate priors play the role of a kernel. This technique is somewhat similar to the approach followed by Carrière [4], who proposed to smooth the Tucker-Lindsay moment estimator with a Log-Normal kernel.

The natural way to smooth the NPMLE $\hat{F}_\Theta$ consists in using

$$\sum_{k=1}^{\hat{q}} \hat{\pi}_k \Gamma(\theta \mid n\hat{\pi}_k\hat{\theta}_k, n\hat{\pi}_k), \qquad \theta \in \mathbb{R}^+,$$

where $\Gamma(. \mid \alpha, \beta)$ denotes the cumulative distribution function corresponding to a two-parameter Gamma law with mean α/β and variance α/β^2, $\hat{q}$ is Simar's upper bound (3.1) for the support size of the NPMLE, and $\hat{\pi}_k$s and $\hat{\theta}_k$s are the corresponding masses and atoms. It is easily seen that the kth component of the mixture is centered at $\hat{\theta}_k$. This corresponds to the intuitive idea that the NPMLE indicates the number and the locations of policyholder classes in the portfolio. Then the distribution of the risk parameter in these classes is represented by a two-parameter Gamma distribution, resulting in a mixture of Gammas. However, the variance of each component equals $\hat{\theta}_k/n\hat{\pi}_k$, which is virtually 0 since the number n of policies is usually very large. As a consequence, the smoothed estimator is more or less indistinguishable from the NPMLE. In order to avoid this phenomenon, we resort on an estimator of the form

$$\tilde{F}(\theta) = \sum_{k=1}^{\tilde{q}} \tilde{\pi}_k \Gamma(\theta \mid n^{\tilde{\lambda}}\tilde{\pi}_k\tilde{\theta}_k, n^{\tilde{\lambda}}\tilde{\pi}_k), \qquad \theta \in \mathbb{R}^+, \tag{3.2}$$

where $\tilde{q}$ is taken as small as possible and, in any case, smaller than Simar's upper bound (3.1) for the support size of the NPMLE, where the $\tilde{\pi}_k$s, $\tilde{\theta}_k$s and $\tilde{\lambda}$ are maximum likelihood estimators. The only difference with Patilea and Rolin's work is thus the introduction of the parameter λ in order to avoid the variance of each component of the mixture defining $\tilde{F}_\Theta$ to be virtually zero.

With (3.2), (1.2) reduces to a mixture of Negative Binomial distributions, i.e.,

$$\tilde{p}(k \mid \Theta) = \sum_{j=1}^{\tilde{q}} \tilde{\pi}_j \binom{n^{\tilde{\lambda}}\tilde{\pi}_j\tilde{\theta}_j + k - 1}{k} \left(\frac{n^{\tilde{\lambda}}\tilde{\pi}_j}{1 + n^{\tilde{\lambda}}\tilde{\pi}_j}\right)^{n^{\tilde{\lambda}}\tilde{\pi}_j\tilde{\theta}_j} \times \left(\frac{1}{1 + n^{\tilde{\lambda}}\tilde{\pi}_j}\right)^k, \qquad k \in \mathbb{N}. \tag{3.3}$$

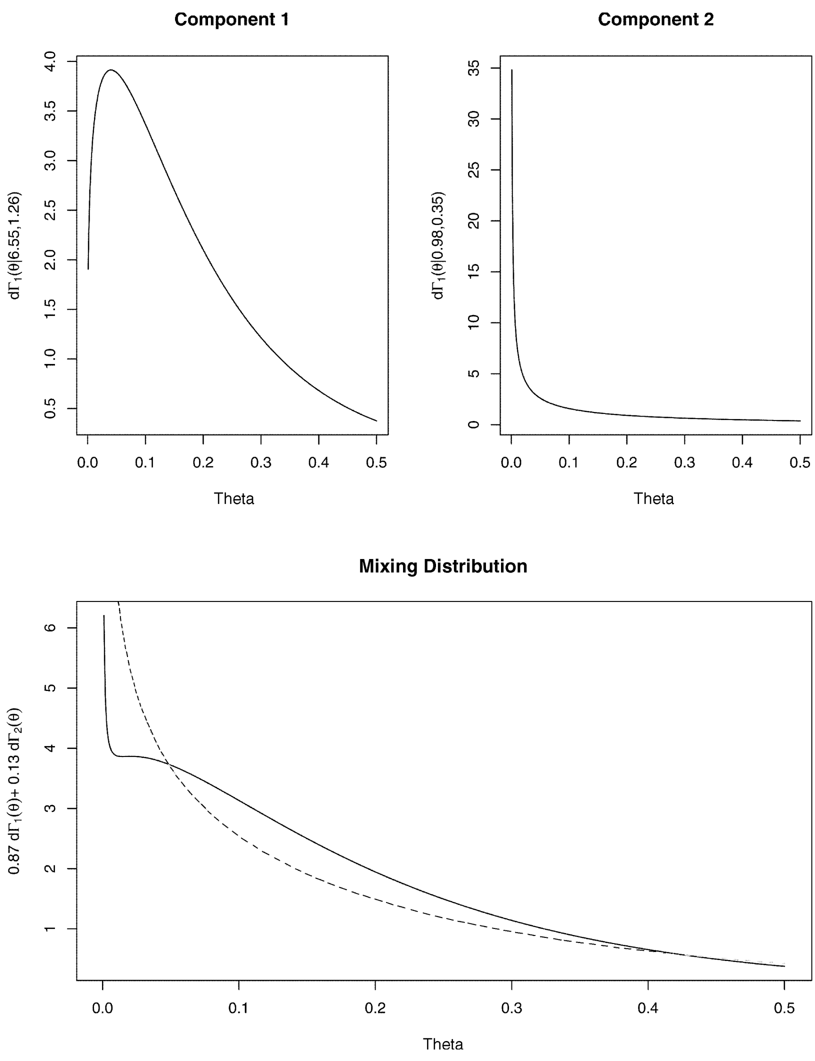
Let us now apply this method to the data sets of Appendix A. In both cases, we took $\tilde{q} = 2$ in order to avoid overparameterization. In Figures 2 and 3, one can find the densities corresponding to the different components involved in the mixture $\tilde{F}_\Theta$, as well as the resulting risk distribution (the continuous line represents $d\tilde{F}_\Theta$ and the dotted line the classical two-parameter Gamma mixing with parameters estimated via maximum likelihood). The model proposed is a slight generalization of the good risk/bad risk model: the portfolio is split into two populations, each one having its own two-parameter Gamma structure function.

Let us now examine the fits obtained with the 2-component $\tilde{F}_\Theta$:

1. For Portfolio 1, $\tilde{\lambda} = 0.22$. The fit is given in Column E; it is very accurate and is regarded as satisfactory on the basis of the χ^2-criterion (p-value of 36%). It is worth mentioning that the Negative Binomial fit displayed in Column D is clearly rejected. Considering Figure 2, we

FIGURE 2

COMPONENTS OF (3.2) AND RESULTING $\tilde{F}_{\Theta}$ FOR PORTFOLIO 1 IN APPENDIX A



see that $\tilde{F}_\Theta$ puts more mass on large values than the classical two-parameter Gamma.

2. For Portfolio 2, we get $\tilde{\lambda} = 0.28$. Again, the fit is satisfactory, and better than the Negative Binomial one. Figure 3 illustrates the difference between the Gamma mixing and $\tilde{F}_\Theta$.

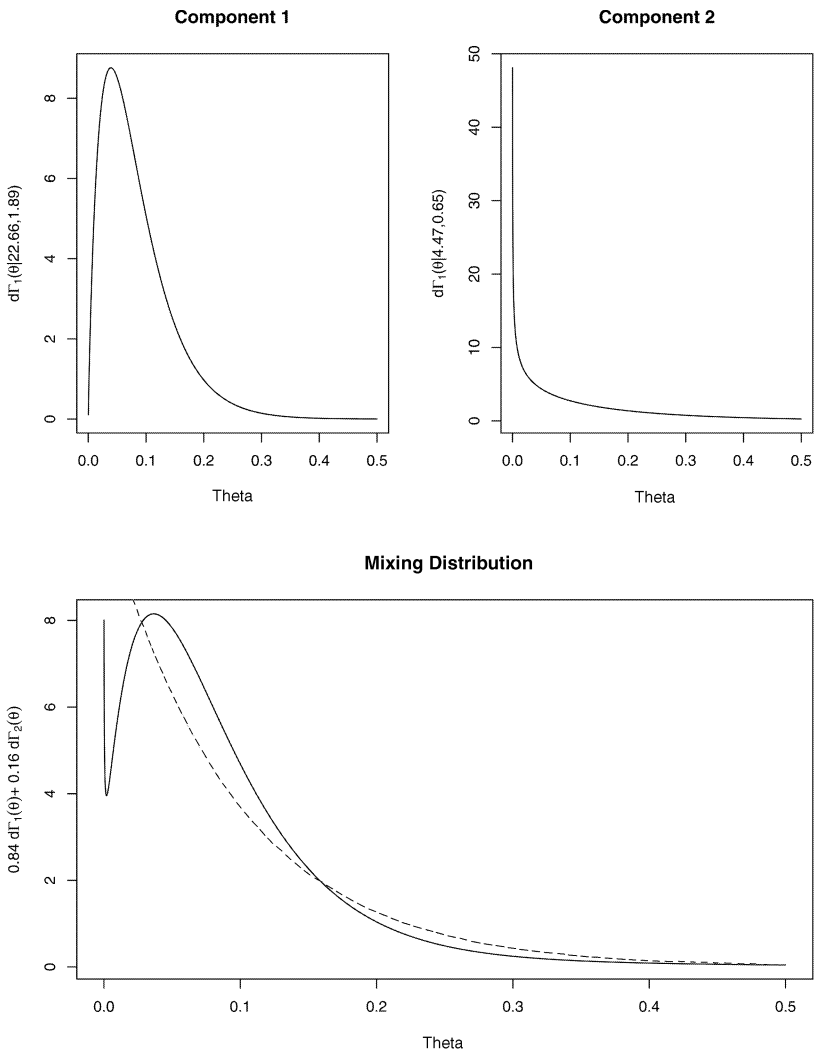
4. RESULTING BMS

Let us now examine the merit-rating schemes obtained in the mixed Poisson model (1.2) using a quadratic loss function and the structure function $\tilde{F}_\Theta$ defined in (3.2). The net premium for a new insured is given by $P_1 = \mathrm{E}[K_1] = \mathrm{E}[\Theta]$. After t years of coverage, the amount of premium for the $(t+1)$th period is $P_{t+1}(K_1, K_2, \ldots, K_t)$. It is determined so as to minimize the expected squared difference between the true premium Θ and the premium P_{t+1} charged to the policyholder, i.e., to minimize $\mathrm{E}[P_{t+1}(K_1, K_2, \ldots, K_t) - \Theta]^2$. The solution of this optimization problem is the posterior mean $P_{t+1}(K_1, K_2, \ldots, K_t) = \mathrm{E}[\Theta \mid K_1, K_2, \ldots, K_t]$. Given $K_1 = k_1,\ K_2 = k_2, \ldots, K_t = k_t$, denote $k = \sum_{j=1}^{t} k_j$. We then get

$$
\begin{aligned}
&P_{t+1}(k_1, k_2, \ldots, k_t) \\
&\quad = \int_{\theta \in \mathbb{R}^+} \theta d\mathrm{P}[\Theta \le \theta \mid K_1 = k_1,\ K_2 = k_2, \ldots, K_t = k_t] \\
&\quad = \frac{\displaystyle\int_{\theta \in \mathbb{R}^+} \theta \left\{ \prod_{i=1}^{t} \mathrm{P}[K_i = k_i \mid \Theta = \theta] \right\} dF_\Theta(\theta)}{\displaystyle\int_{\eta \in \mathbb{R}^+} \left\{ \prod_{i=1}^{t} \mathrm{P}[K_i = k_i \mid \Theta = \eta] \right\} dF_\Theta(\eta)} \\
&\quad = \frac{\displaystyle\int_{\theta \in \mathbb{R}^+} \exp(-t\theta)\theta^{k+1} dF_\Theta(\theta)}{\displaystyle\int_{\eta \in \mathbb{R}^+} \exp(-t\eta)\eta^{k} dF_\Theta(\eta)} \equiv P_{t+1}(k).
\end{aligned}
$$

FIGURE 3

COMPONENTS OF (3.2) AND RESULTING $\tilde{F}_{\Theta}$ FOR PORTFOLIO 2 IN APPENDIX A



$P_{t+1}(k)$ appears as the ratio of two Mellin transforms, as expected from Albrecht [2]. It is interesting to note that the premium P_{t+1} depends only on the total number k of accidents caused in the past t years of insurance, and not on the history of these claims. This is a characteristic of the theoretical Bonus-Malus scales (with an infinite number of levels). In practice, since the Bonus-Malus scale is upper bounded, policyholders always take an advantage of concentrating all the claims during a single period.

Assume that the first premium paid is 100 and that a given policyholder reported k claims at fault during t years of coverage. The Bonus-Malus coefficient is then computed with the help of the formula

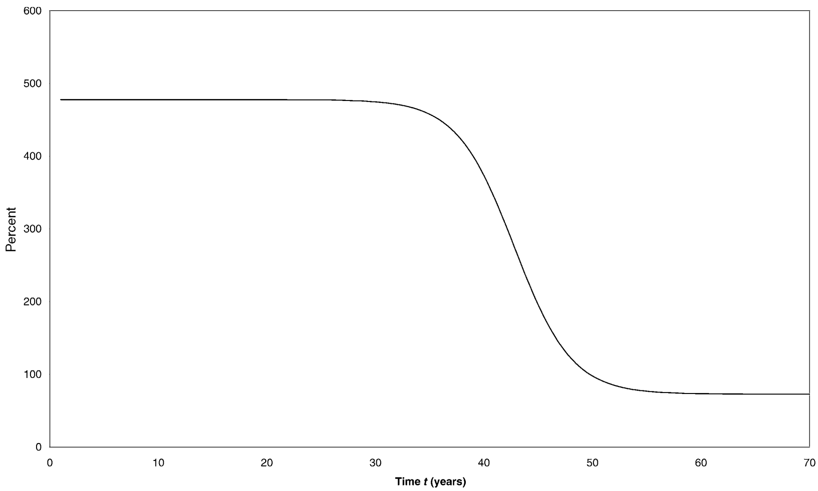
$$\beta(k,t) = 100 \times \frac{P_{t+1}(k)}{P_1}\%.$$

In words, $\beta(k,t)$ is the relative level of premium for the $(t+1)$th year of coverage for an insured person who caused k accidents during the first t years.

In Appendix B, we considered Portfolio 2 (two support points for $\hat{F}_\Theta$ and two components for $\tilde{F}_\Theta$). We first built a BMS with the NPMLE $\hat{F}_\Theta$. The $\beta(k,t)$s so obtained are given in Table B.1. A "block" structure is clearly apparent, each block with almost constant $\beta(k,t)$ corresponding to one support point of $\hat{F}_\Theta$. In Figure 4, the evolution of the premium for a driver who caused 10 claims during $[0,t]$ is depicted as a function of $t \in \mathbb{N}$. A step behavior is clearly apparent. The policyholder is first put in the category $\hat{\theta}_2 = 0.446$. Then, the BMS needs several claim-free years to decide that this individual belongs to the category $\hat{\theta}_1 = 0.068$. Broadly speaking, there is only one discount, the premium being constant before and after. At first, $\beta(10,1)$ equals 477.8946% (whereas it equals 477.8947% if we know that the driver is a bad risk), and after that, the premium decreases to $\beta(10,70) = 72.8767\%$ (it equals 72.8629% for good risks). Such a behavior, which is a byproduct of the purely discrete nature of the NPMLE, is undesirable. In order to avoid this, we need a

FIGURE 4

EVOLUTION OF $\beta(10,t)$ AS A FUNCTION OF $t = 1,2,\ldots,70$ WITH $\hat{F}_\Theta$ FOR PORTFOLIO 2



smooth risk distribution, as (3.2). The $\beta(k,t)$s derived from the estimator $\tilde{F}_\Theta$ of the structure function F_Θ are given in Table B.2, while Figure 5 is the counterpart of Figure 4. See Appendix B for the details of the computations. The BMS is now "smooth," with continuous variations of the $\beta(10,t)$s; this can be regarded as commercially desirable.

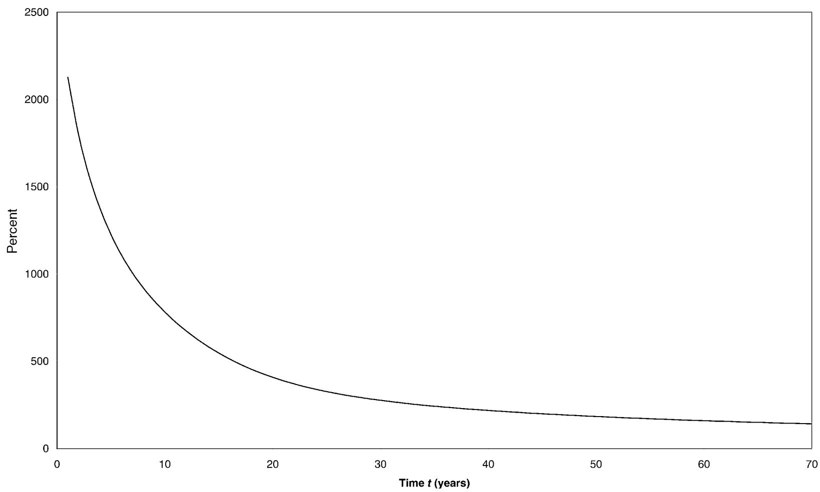
To end with, let us mention that the $\beta(k,t)$s of Table B.2 can be transformed in a standard table following the method proposed by Coene and Doray [5].

5. CONCLUSIONS

In this paper, we demonstrated that an adequately smoothed version of the NPMLE is a good candidate for estimating the risk distribution in a mixed Poisson model for the claim count. This estimator is nonparametric; no assumption is thus made on

FIGURE 5

EVOLUTION OF $\beta(10,t)$ AS A FUNCTION OF $t = 1,2,\ldots,70$ WITH $\tilde{F}_\Theta$ FOR PORTFOLIO 2



the mixing distribution. Moreover, as a mixture of Gamma distributions, it is mathematically tractable to elaborate BMS. In that respect, it performs better than the NPMLE, which is purely discrete and results in "discontinuous" experience rating plans. Of course, the smoothed NPMLE does not provide accurate fits in all the cases. For instance, both NPMLE and smoothed NPMLE yielded poor fits for the data set relating to Belgium 1975–1976 provided in Gossiaux and Lemaire [10].

In a forthcoming paper, the same problem will be considered when a priori risk classification is enforced. Specifically, we will examine how to design merit rating plans in accordance with a priori ratemaking structure of the insurance company.

REFERENCES

[1] Albrecht, P., "On Some Statistical Methods Connected with the Mixed Poisson Process," *Scandinavian Actuarial Journal*, 1982, pp. 1–14.

[2] Albrecht, P., "Laplace Transforms, Mellin Tranforms and Mixed Poisson Processes," *Scandinavian Actuarial Journal*, 1984, pp. 58–64.

[3] Carrière, J., "Nonparametric Tests for Mixed Poisson Distributions," *Insurance: Mathematics & Economics* 12, 1993, pp. 3–8.

[4] Carrière, J., "A Semiparametric Estimator of a Risk Distribution," *Insurance: Mathematics & Economics* 13, 1993, pp. 75–81.

[5] Coene, G., and L. G. Doray, "A Financially Balanced Bonus-Malus System," *ASTIN Bulletin* 26, 1996, pp. 107–116.

[6] Dennis, J. E., and R. B. Schnabel, *Numerical Methods for Unconstrained Optimization and Nonlinear Equations*, Englewood Cliffs, NJ: Prentice-Hall, 1983.

[7] Denuit, M., "A New Distribution of Poisson-Type for the Number of Claims," *ASTIN Bulletin* 27, 1997, pp. 229–242.

[8] Grenander, U. "On the Heterogeneity in Non-Life Insurance," *Scandinavian Actuarial Journal*, 1957, pp. 153–179.

[9] Grenander, U. "Some Remarks on Bonus Systems in Automobile Insurance," *Scandinavian Actuarial Journal*, 1957, pp. 180–197.

[10] Gossiaux, A.-M., and J. Lemaire, "Méthodes D'ajustement de Distributions de Sinistres," *Bulletin of the Swiss Association of Actuaries*, 1981, pp. 87–95.

[11] Kestemont, R.-M., and J. Paris, "Sur l'ajustement du Nombre de Sinistres," *Bulletin of the Swiss Association of Actuaries*, 1985, pp. 157–163.

[12] Lemaire, J., *Bonus-Malus Systems in Automobile Insurance*, Boston: Kluwer Academic Publisher, 1995.

[13] Lindsay, B. G., "On the Determinants of Moment Matrices," *Annals of Statistics* 17, 1989, pp. 711–721.

[14] Lindsay, B. G., "Moment Matrices: Applications in Mixtures," *Annals of Statistics* 17, 1989, pp. 722–740.

[15] Patilea, V., and J.-M. Rolin, "Smooth vs. Likelihood Estimation for a Class of Discrete Distributions," *Discussion Paper* 9516, Insitut de Statistique, Université Catholique de Louvain, Louvain-la-Neuve, Belgium.

[16] Ross, I., and R. Gentleman, "R: A Language for Data Analysis and Graphics," *Journal of Computational and Graphical Statistics* 5, 1996, pp. 299–314.

[17] Schnabel, R. B., J. E. Koontz, and B. E. Weiss, "A Modular System of Algorithms for Unconstrained Minimization," *ACM Transactions Mathematical Software* 11, 1985, pp. 419–440.

[18] Seal, H. L., "Mixed Poisson—An Ideal Distribution of Claim Number?" *Bulletin of the Swiss Association of Actuaries*, 1982, pp. 293–295.

[19] Shaked, M., "On Mixtures from Exponential Families," *Journal of the Royal Statistical Society—Series B* 42, 1980, pp. 192–198.

[20] Simar, L., "Maximum Likelihood Estimation of a Compound Poisson Process," *The Annals of Statistics* 4, 1976, pp. 1200–1209.

[21] Tremblay, L., "Using the Poisson-Inverse Gaussian distribution in Bonus-Malus Systems," *ASTIN Bulletin* 22, 1963, pp. 97–106.

[22] Tucker, H. G., "An Estimate of the Compounding Distribution of a Compound Poisson Distribution," *Theory of Probability and Its Applications* 8, 1963, pp. 195–200.

[23] Walhin, J.-F., and J. Paris, “Using Mixed Poisson Distributions in Connection with Bonus-Malus Systems,” *ASTIN Bulletin* 29, 1999, pp. 81–99.

[24] Willmot, G. E. “The Poisson-Inverse Gaussian Distribution as an Alternative to the Negative Binomial,” *Scandinavian Actuarial Journal*, 1987, pp. 113–127.

APPENDIX A

DATA SETS

The reader will find herein two data sets from Benelux countries, together with all the fits considered in the present paper. To measure the goodness-of-fit, standard χ^2-statistics are used, with the following calculation procedure:

$$\chi^2_{\text{obs}} = -2\sum_{k=0}^{k_{\max}} n_k \ln\left(\frac{\widehat{p(k \mid \Theta)}}{\widehat{p(k)}}\right).$$

TABLE A.1

FITS TO PORTFOLIO 1

		Fitting Technique				
k	n_k	A	B	C	D	E
0	7,840	7,636	7,840	7,832	7,847	7,839
1	1,317	1,637	1,317	1,337	1,288	1,322
2	239	175	239	213	257	231
3	42	13	42	57	54	48
4	14	1	13	17	12	13
5	4	0	6	4	3	5
6	4	0	2	1	1	2
7	1	0	1	0	0	1
≥ 8	0	0	0	0	0	0
χ^2_{obs}		302.48	2.33	16.85	17.00	4.36
d.f.		6	3	5	6	4
p-value		$< 10^{-3}$	0.51	0.005	0.009	0.36

Column A: expected frequency with homogeneous Poisson
Column B: expected frequency with 3-point NPMLE $\hat{F}_\Theta$
$\hat{\theta}_1 = 0.336$, $\hat{\theta}_2 \approx 0.000$, $\hat{\theta}_3 = 2.545$
$\hat{\pi}_1 = 0.573$, $\hat{\pi}_2 = 0.418$, and $\hat{\pi}_3 = 0.001$
Column C: expected frequency with 2-point NPMLE $\hat{F}_\Theta$
$\hat{\theta}_1 = 0.147$, $\hat{\theta}_2 = 1.231$, $\hat{\pi}_1 = 0.938$, and $\hat{\pi}_2 = 0.062$
Column D: expected frequency with Negative Binomial
Column E: expected frequency with 2-component $\tilde{F}_\Theta$
$\tilde{\lambda} = 0.22$, $\tilde{\theta}_1 = 0.193$, $\tilde{\theta}_2 = 0.355$, $\tilde{\pi}_1 = 0.869$, and $\tilde{\pi}_2 = 0.131$

TABLE A.2

Fits to Portfolio 2

		Fitting Technique				
k	n_k	A	B	C	D	E
0	102,435	102,026	102,435	102,435	102,442	102,435
1	8,804	9,544	8,805	8,811	8,774	8,806
2	714	446	712	703	746	710
3	65	14	68	76	63	70
4	12	0	10	8	5	9
5	1	0	2	1	0	1
≥ 6	0	0	0	0	0	0
χ^2_{obs}		365.67	1.25	3.78	8.18	1.94
d.f.		5	1	3	4	2
p-value		$< 10^{-3}$	0.26	0.29	0.09	0.38

Column A: expected frequency with homogeneous Poisson
Column B: expected frequency with 3-point NPMLE $\hat{F}_\Theta$
$\hat{\theta}_1 = 0.132$, $\hat{\theta}_2 = 0.829$, $\hat{\theta}_3 \approx 0.000$
$\hat{\pi}_1 = 0.651$, $\hat{\pi}_2 = 0.009$, and $\hat{\pi}_3 = 0.340$
Column C: expected frequency with 2-point NPMLE $\hat{F}_\Theta$
$\hat{\theta}_1 = 0.068$, $\hat{\theta}_2 = 0.446$, $\hat{\pi}_1 = 0.933$, and $\hat{\pi}_2 = 0.067$
Column D: expected frequency with Negative Binomial
Column E: expected frequency with 2-component $\tilde{F}_\Theta$
$\tilde{\lambda} = 0.28$, $\tilde{\theta}_1 = 0.083$, $\tilde{\theta}_2 = 0.145$, $\tilde{\pi}_1 = 0.835$, and $\tilde{\pi}_2 = 0.165$

APPENDIX B

THEORETICAL BMS

Table B.1 contains the Bonus-Malus coefficients $\beta(k,t)$ computed with the NPMLE $\hat{F}_\Theta$ of F_Θ. Its counterpart B.2 is based on the smoothed NPMLE $\tilde{F}_\Theta$. These quantities are computed on the basis of Portfolio 2, 2-point $\hat{F}_\Theta$ and 2-component $\tilde{F}_\Theta$. In Table B.1,

$$\beta(k,t) = \frac{\sum_{j=1}^{\hat{q}} \exp(-t\hat{\theta}_j)\hat{\theta}_j^{k+1}\hat{\pi}_j}{\sum_{j=1}^{\hat{q}} \exp(-t\hat{\theta}_j)\hat{\theta}_j^{k}\hat{\pi}_j} \times \frac{100}{\sum_{j=1}^{\hat{q}} \hat{\theta}_j \hat{\pi}_j}.$$

Let us briefly detail the computational aspects of Table B.2. When the risk distribution is a Gamma mixture, i.e.,

$$F_\Theta(\theta) = \sum_{j=1}^{q} \alpha_j \Gamma(\theta \mid a_j, \tau_j), \qquad \theta \in \mathbb{R}^+, \tag{B.1}$$

we get

$$\begin{aligned}
&dF_\Theta(\theta \mid K_1 = k_1,\ K_2 = k_2, \ldots, K_t = k_t) \\
&\quad = \frac{\sum_{j=1}^{q} \alpha_j \exp(-t\theta)\theta^k d\Gamma(\theta \mid a_j, \tau_j)}{\sum_{i=1}^{q} \alpha_i \int_{\eta \in \mathbb{R}^+} \exp(-t\eta)\eta^k d\Gamma(\eta \mid a_i, \tau_i)} \\
&\quad = \sum_{j=1}^{q} A(j,k) d\Gamma(\theta \mid a_j + k, \tau_j + t),
\end{aligned}$$

where

$$A(j,k) = \alpha_j \frac{\displaystyle\int_{\eta\in\mathbb{R}^+} \exp(-t\eta)\eta^k d\Gamma(\eta \mid a_j,\tau_j)}{\displaystyle\sum_{i=1}^{q} \alpha_i \int_{\eta\in\mathbb{R}^+} \exp(-t\eta)\eta^k d\Gamma(\eta \mid a_i,\tau_i)}.$$

This yields

$$P_{t+1}(k_1,k_2,\ldots,k_t) = \sum_{j=1}^{q} A(j,k)\frac{a_j+k}{\tau_j+t}.$$

The coefficients $A(j,k)$s are easy to compute. Indeed, they can be cast into

$$A(j,k) = \alpha_j \frac{\epsilon(j,k)}{\displaystyle\sum_{i=1}^{q} \alpha_i \epsilon(i,k)},$$

where

$$\begin{aligned}
\epsilon(j,k) &= \int_{\eta\in\mathbb{R}^+} \frac{\exp(-t\eta)(t\eta)^k}{k!} d\Gamma(\eta \mid a_j,\tau_j) \\
&= \int_{\eta\in\mathbb{R}^+} \frac{\exp(-\eta)\eta^k}{k!} d\Gamma(\eta \mid a_j,\tau_j/t) \\
&= \binom{a_j+k-1}{k} \left(\frac{\tau_j}{\tau_j+t}\right)^{a_j} \left(\frac{t}{\tau_j+t}\right)^{k}.
\end{aligned}$$

The $\epsilon(j,k)$s satisfy the Panjer recurrence relations

$$\epsilon(j,k) = \frac{t}{\tau_j+t}\frac{a_j+k-1}{k}\epsilon(j,k-1), \qquad k = 1,2,\ldots,$$

starting from

$$\epsilon(j,0) = \int_{\eta\in\mathbb{R}^+} \exp(-\eta) d\Gamma(\eta \mid a_j,\tau_j/t) = \left(\frac{\tau_j}{\tau_j+t}\right)^{a_j}.$$

TABLE B.1

$\beta(k,t)$ WITH $\hat{F}_\Theta$ FOR PORTFOLIO 2 (PART 1)

	k										
t	0	1	2	3	4	5	6	7	8	9	10
0	100										
1	92	172	348	451	473	477	478	478	478	478	478
2	86	146	313	439	472	477	478	478	478	478	478
3	82	126	275	424	469	476	478	478	478	478	478
4	79	111	237	404	465	476	478	478	478	478	478
5	77	100	202	378	459	475	477	478	478	478	478
6	76	92	171	347	450	473	477	478	478	478	478
7	75	86	146	312	439	471	477	478	478	478	478
8	74	82	126	274	424	469	476	478	478	478	478
9	74	79	111	236	403	464	476	478	478	478	478
10	74	77	100	201	377	459	475	477	478	478	478
11	73	76	92	170	346	450	473	477	478	478	478
12	73	75	86	145	311	439	471	477	478	478	478
13	73	74	82	125	273	423	468	476	478	478	478
14	73	74	79	110	235	403	464	476	478	478	478
15	73	74	77	99	200	377	458	475	477	478	478
16	73	73	76	91	170	345	450	473	477	478	478
17	73	73	75	86	145	310	438	471	477	478	478
18	73	73	74	82	125	272	423	468	476	478	478
19	73	73	74	79	110	234	402	464	476	478	478
20	73	73	74	77	99	199	376	458	475	477	478
21	73	73	73	76	91	169	345	450	473	477	478
22	73	73	73	75	86	144	309	438	471	477	478
23	73	73	73	74	82	124	271	422	468	476	478
24	73	73	73	74	79	110	233	402	464	476	478
25	73	73	73	74	77	99	199	375	458	475	477
26	73	73	73	73	76	91	168	344	449	473	477
27	73	73	73	73	75	85	143	308	438	471	477
28	73	73	73	73	74	82	124	270	422	468	476
29	73	73	73	73	74	79	109	232	401	464	476
30	73	73	73	73	74	77	99	198	375	458	475
31	73	73	73	73	73	76	91	168	343	449	473
32	73	73	73	73	73	75	85	143	307	437	471
33	73	73	73	73	73	74	82	124	269	421	468
34	73	73	73	73	73	74	79	109	232	400	464
35	73	73	73	73	73	73	77	98	197	374	458
36	73	73	73	73	73	73	76	91	167	342	449
37	73	73	73	73	73	73	75	85	142	306	437
38	73	73	73	73	73	73	74	81	123	268	421
39	73	73	73	73	73	73	74	79	109	231	400
40	73	73	73	73	73	73	73	77	98	196	373

TABLE B.1

$\beta(k,t)$ WITH $\hat{F}_\Theta$ FOR PORTFOLIO 2 (PART 2)

	k										
t	0	1	2	3	4	5	6	7	8	9	10
41	73	73	73	73	73	73	73	76	91	166	341
42	73	73	73	73	73	73	73	75	85	142	305
43	73	73	73	73	73	73	73	74	81	123	267
44	73	73	73	73	73	73	73	74	79	109	230
45	73	73	73	73	73	73	73	73	77	98	195
46	73	73	73	73	73	73	73	73	76	90	166
47	73	73	73	73	73	73	73	73	75	85	141
48	73	73	73	73	73	73	73	73	74	81	122
49	73	73	73	73	73	73	73	73	74	79	108
50	73	73	73	73	73	73	73	73	73	77	98
51	73	73	73	73	73	73	73	73	73	76	90
52	73	73	73	73	73	73	73	73	73	75	85
53	73	73	73	73	73	73	73	73	73	74	81
54	73	73	73	73	73	73	73	73	73	74	79
55	73	73	73	73	73	73	73	73	73	73	77
56	73	73	73	73	73	73	73	73	73	73	76
57	73	73	73	73	73	73	73	73	73	73	75
58	73	73	73	73	73	73	73	73	73	73	74
59	73	73	73	73	73	73	73	73	73	73	74
60	73	73	73	73	73	73	73	73	73	73	73
61	73	73	73	73	73	73	73	73	73	73	73
62	73	73	73	73	73	73	73	73	73	73	73
63	73	73	73	73	73	73	73	73	73	73	73
64	73	73	73	73	73	73	73	73	73	73	73
65	73	73	73	73	73	73	73	73	73	73	73
66	73	73	73	73	73	73	73	73	73	73	73
67	73	73	73	73	73	73	73	73	73	73	73
68	73	73	73	73	73	73	73	73	73	73	73
69	73	73	73	73	73	73	73	73	73	73	73
70	73	73	73	73	73	73	73	73	73	73	73

TABLE B.2

$\beta(k,t)$ WITH $\tilde{F}_{\Theta}$ FOR PORTFOLIO 2 (PART 1)

	k										
t	0	1	2	3	4	5	6	7	8	9	10
0	100										
1	93	168	300	531	819	1084	1313	1524	1727	1928	2128
2	87	151	250	419	651	888	1096	1283	1458	1629	1799
3	82	139	219	346	531	740	932	1102	1259	1410	1558
4	78	130	197	298	445	625	802	961	1105	1241	1373
5	75	123	182	264	383	535	696	846	981	1106	1226
6	72	117	170	239	337	465	610	750	878	995	1106
7	69	112	160	220	302	411	539	669	791	902	1006
8	66	107	152	205	276	368	480	600	716	822	921
9	64	103	145	193	255	334	433	542	651	753	847
10	62	99	139	183	238	307	393	492	594	691	782
11	59	96	133	175	224	285	361	450	544	637	725
12	58	93	129	168	213	267	335	414	501	589	673
13	56	90	124	161	203	252	312	384	463	546	627
14	54	87	120	155	194	239	294	358	431	508	585
15	52	85	116	150	187	228	278	336	402	474	547
16	51	82	113	145	180	219	264	317	377	444	513
17	50	80	110	141	174	210	252	300	356	418	483
18	48	78	107	137	168	203	242	286	337	394	455
19	47	76	104	133	163	196	232	274	321	373	431
20	46	74	101	129	158	190	224	262	306	355	409
21	45	72	99	126	154	184	216	253	293	339	389
22	43	71	96	123	150	179	210	244	282	324	371
23	42	69	94	120	146	174	203	236	271	311	355
24	41	67	92	117	143	169	198	228	262	299	340
25	40	66	90	114	139	165	192	221	253	288	327
26	40	64	88	112	136	161	187	215	246	279	315
27	39	63	86	109	133	157	183	210	238	270	304
28	38	62	84	107	130	154	178	204	232	262	294
29	37	60	83	105	127	150	174	199	226	254	285
30	36	59	81	103	125	147	170	194	220	247	277
31	35	58	79	101	122	144	167	190	215	241	269
32	35	57	78	99	120	141	163	186	210	235	262
33	34	56	76	97	118	138	160	182	205	229	255
34	33	55	75	95	115	136	157	178	201	224	249
35	33	54	74	93	113	133	154	175	196	219	243
36	32	53	72	92	111	131	151	171	192	214	238
37	31	52	71	90	109	128	148	168	189	210	232
38	31	51	70	89	107	126	145	165	185	206	228
39	30	50	69	87	105	124	143	162	182	202	223
40	30	49	68	86	104	122	140	159	178	198	219

TABLE B.2

$\beta(k,t)$ WITH $\tilde{F}_\Theta$ FOR PORTFOLIO 2 (PART 2)

	k										
t	0	1	2	3	4	5	6	7	8	9	10
41	29	49	67	84	102	120	138	156	175	194	214
42	29	48	65	83	100	118	136	154	172	191	210
43	28	47	64	82	99	116	134	151	169	188	207
44	28	46	63	80	97	114	131	149	166	184	203
45	27	46	62	79	96	112	129	146	164	181	200
46	27	45	62	78	94	111	127	144	161	178	196
47	26	44	61	77	93	109	125	142	159	176	193
48	26	44	60	76	92	108	124	140	156	173	190
49	26	43	59	75	90	106	122	138	154	170	187
50	25	42	58	74	89	104	120	136	152	168	184
51	25	42	57	73	88	103	118	134	149	165	181
52	24	41	56	72	87	102	117	132	147	163	179
53	24	41	56	71	85	100	115	130	145	160	176
54	24	40	55	70	84	99	114	128	143	158	173
55	23	40	54	69	83	98	112	127	141	156	171
56	23	39	54	68	82	96	111	125	139	154	169
57	23	38	53	67	81	95	109	123	138	152	166
58	22	38	52	66	80	94	108	122	136	150	164
59	22	37	52	65	79	93	106	120	134	148	162
60	22	37	51	64	78	91	105	119	132	146	160
61	21	37	50	64	77	90	104	117	131	144	158
62	21	36	50	63	76	89	102	116	129	142	156
63	21	36	49	62	75	88	101	114	127	141	154
64	21	35	48	61	74	87	100	113	126	139	152
65	20	35	48	61	73	86	99	112	124	137	150
66	20	34	47	60	73	85	98	110	123	136	149
67	20	34	47	59	72	84	97	109	122	134	147
68	20	34	46	59	71	83	96	108	120	133	145
69	19	33	46	58	70	82	94	107	119	131	144
70	19	33	45	57	69	81	93	105	118	130	142
71	19	32	45	57	69	81	92	104	116	128	140

UNDERWRITING CYCLES AND BUSINESS STRATEGIES

SHOLOM FELDBLUM

Abstract

Underwriting cycles, with their wide and puzzling swings in premiums and profitability, challenge the pricing actuary to adapt rates to market realities. Understanding the forces behind insurance price fluctuations is a prerequisite to analyzing market prices.

Underwriting cycles have been ascribed to actuarial ratemaking procedures, to underwriting philosophy, and to interest rate volatility. These interpretations underestimate the dynamics of the insurance marketplace, and they ignore the competitive pressures that drive insurance pricing.

Underwriting cycles, like profit fluctuations in other industries, reflect the interdependence of rival firms. Strong policyholder loyalty and demand inelasticity hold the allure of large returns for incumbent firms, but the apparent ease of entry into insurance, the lack of market concentration, and the difficulty of monitoring competitors' prices preclude excessive profits. The interaction of these forces keeps the market in disequilibrium, with continuing price oscillations.

With the decline of rating bureaus and the growing competitiveness of the insurance marketplace, the proficient actuary may no longer set rates based solely on indicated costs. Insurers seek actuaries who understand the competitive forces that drive market prices and who can set future rates that are most advantageous for the firm. They seek actuaries who can price their products through the vicissitudes of the underwriting cycle.

ACKNOWLEDGEMENT

The author is indebted to Benjamin Lefkowitz, Jay Siegel, and Richard Homonoff, who suggested numerous corrections to earlier drafts of this manuscript. The remaining errors, of course, should be attributed to the author alone.

1. THE EDUCATION OF AN ACTUARY

When I began work as a pricing actuary, I was struck by the simplicity of our ratemaking procedures. Actuarial techniques are cost-based: premiums are based on anticipated losses and expenses. Marketplace pricing, however, considers supply/demand interactions, consumer desires, and competitive pressures. When I asked about this, I was told that actuaries determine the "proper" rates—those which best serve insurance companies and the public.

As the months passed, I learned that insurers do not actually set prices based on actuarial indications. Schedule rating modifications of as much as 50% are used in the Commercial Lines, and discretionary rate deviations from actuarial indications are used in the Personal Lines. So I wondered: what is the use of our ratemaking procedures?

When I asked about this, I was told that the poor, misguided folk in Underwriting and Marketing always wanted lower rates. Management was forced to cut prices below adequate levels to keep everyone happy. Rate deviations and modifications were the random effects of strong officers in the field.

Years later, I understood that these deviations are not entirely random. Underwriting cycles billow through our industry, raising and lowering the premium rates charged by insurers. The price fluctuations are not discretionary: insurers that have ignored the phases of the cycle have lost both money and market share. Most important, these are industry wide cycles, unrelated to the internal politics of individual firms. Actuaries indicate rates, but the market sets prices.

Causes of the Cycle

Some actuaries believe that rates should be based only on anticipated costs. Stable actuarial rates ensure adequate returns for insurers, and they mitigate the price variations that anger consumers. Carriers may be tempted by the marketing benefits of rate cutting, but actuaries should not encourage such follies.

However, cost-based pricing is rarely optimal. Careful consideration of the marketplace and of competitors' actions is essential for ensuring profitable operations. This aforementioned view is dangerous to the actuarial profession as well, for if actuaries ignore market realities, their companies will relegate them to technical busy-work. If actuaries wish to influence actual prices, they must address real business concerns.

The view described and deprecated above is ensconced in two prevalent convictions. First, underwriting cycles are seen as external to insurer strategies. For example, the severe downturn in Commercial Lines operating income during the early 1980s is sometimes attributed to high and fluctuating interest rates that encouraged "cash flow" underwriting. How can we price for these variations if we can not control them or even predict them?[1]

Second, underwriting cycles seem unrelated to profit cycles in other industries. Some say that insurance profits are countercyclical to general business conditions: rates are high during depressions and decline during prosperous periods. Others add that underwriting cycles vary with supply restraints, not demand pressures. Pricing techniques used in other industries are therefore inapplicable to insurance ratemaking.

To understand the relationship of insurance insolvencies to underwriting cycles, we must uncover the causes of the cycle. Four interpretations of the cycle are described in the next section,

[1] Compare Taylor [105, pg. 1]: "Individual operators in the insurance market view [the underwriting cycle] as a variable exogenous to the formation of their own plans, one whose timing and magnitude is beyond their control. This engenders a passive attitude to underwriting cycles on the part of insurers."

emphasizing (i) actuarial ratemaking, (ii) underwriting philosophy, (iii) interest rate movements, and (iv) competitive strategy. The first three imply irrational business behavior by actuaries, underwriters, or investment officers. In addition, the actuarial and underwriting interpretations do not explain the synchronized pricing of independent insurers, and the interest rate interpretation cannot account for the recurrence of cycles in more stable interest rate environments. The fourth interpretation views underwriting cycles as rational business behavior among competing firms striving to optimize long-term profits. Competition may be rough, and it may be inexact, but it tells us a rational story if we pause to listen.

2. INTERPRETATIONS OF THE CYCLE

Actuarial Ratemaking: Uncertainty and Counter-Cyclicality

Some actuaries ascribe profit cycles to the uncertainty and counter-cyclicality of loss costs:

- Property/Casualty insurance costs depend upon random loss occurrences and uncertain macroeconomic and social trends. Random losses may be unusual weather disturbances, such as windstorms, and earthquakes. Social trends may be unexpected legal changes, such as retroactive liability for pollution exposures.

- The counter-cyclicality of insurance loss costs stems from the time lag between the compilation of historical experience and the implementation of new rates. Generally, two or more years of experience are used for ratemaking, losses are developed three months beyond the end of the experience period, systems processing of the historical data requires another month or two, rate analysis and filing take six months, and the rates remain in effect for one year. Rating bureaus require an additional half year for editing and verification of insurance data and for notification to member companies of intended rate filings.

Thus, the time between the average loss date in the experience period and the midpoint of the effective period of the new rates often exceeds three or four years (Cummins and Nye [35, pp. 232–236]).

The uncertainty and counter-cyclicality of insurance loss costs contribute to underwriting cycles. During recessions, inflation is moderate, automobile travel is low, jury awards are less liberal, factories operate below capacity, industrial injuries are infrequent, and so forth.[2] The experience from this period, and the time lag between data compilation and rate implementation, ensures moderate rate revisions for several years.

The economy soon recovers, and loss costs rise rapidly. Insurers, wary of increasing their rates and losing business volume, ascribe the mounting costs to random loss occurrences. Even when the rate inadequacy is recognized, and rate revisions are requested, the time lag between data compilation and rate implementation means that the needed premiums are not earned until years later.

Historical experience continues to indicate a rate inadequacy when the economy once again slides into a recession. Insurers

[2]There are opposing influences as well. During recessions, thefts increase, leading to higher automobile comprehensive claims. Employees recently laid off are more likely to file Workers Compensation claims for minor injuries, since there is no loss of regular income while on disability. Workers Compensation claim severity also increases, since it is more difficult to find replacement jobs for injured employees (Mowbray and Black [78, pg. 425]; Greene and Roeber [51, pp. 254–255]). For a discerning discussion of the relationship between economic conditions and insurance loss costs in a depressed economy, see Tarbell [104]. For relationships by line of business, see ISO [57, pg. 2], for Personal Auto, Homeowners, and Workers Compensation, and Victor and Fleischman [113] and Victor [112] for Workers Compensation.

Unfortunately, little is known about the correlations between insurance loss costs and macro-economic conditions. Kahane [60], Hill [55], and Fairley [45] find that insurance losses have a slight negative correlation with stock returns. Since stock returns reflect economic conditions, this suggests that loss costs may be related to the economy as well. Others find no significant correlation between underwriting returns and stock prices (Cummins and Harrington [30]; D'Arcy and Garven [39]; Kozik [63]).

In general, the relationships noted in the text are based on conjecture and intuition. This explanation of underwriting cycles fails for other reasons, and the absence of facts among adherents of this theory is simply an additional flaw.

continue filing for rate increases, even though rates have returned to adequate levels. And so the cycle goes on.[3]

Awareness and Action

There are some factual problems with this interpretation. Underwriting cycles are generally not counter-cyclical to macroeconomic conditions. Further, loss cost trends are not always different in prosperous times and recessionary times. But there are more fundamental reasons why this explanation fails.

First, this interpretation presumes that pricing actuaries are unable to learn from past mistakes and are incapable of forecasting loss cost trends despite years of experience. This is not true: actuaries are proficient at estimating insurance costs and are not easily fooled by macroeconomic conditions or long-term social trends. Both actuaries and insurers are frequently aware of the true loss cost trends even as rates move in the opposite direction. For example, insurers knew that General Liability loss costs were rising rapidly in the early 1980s, but they continued cutting rates well below marginal cost.

Indications and Prices

Second, underwriting cycles are not due to actuarial rate indications. They are due to insurer reluctance to adopt actuarially recommended rate increases, to rate deviations below bureau rates, to schedule rating credits for commercial risks, and to similar "discretionary" rate reductions.[4] Underwriting cycles are as

[3]The Virginia Bureau of Insurance [114] interprets underwriting cycles in this fashion. "The insurance cycle is usually out-of-phase with the rest of the economy. When prices for general goods and services are rising, insurance rates are often stable and insurance industry profits are decreasing. By the time that the rate of increase in the price for other goods and services diminishes, data is becoming available showing that insurance rates have not kept up with underlying costs. Insurance rates then increase rapidly and profits improve. This lag between price increases in the insurance industry and the rest of the economy is in large part due to the time required for claims to be reported and settled and for claims data to be collected and evaluated."

[4]Cummins, Harrington, and Klein [32, pp. 59–60; Figure 5, pg. 59] note that "deviations below ISO advisory rates increased substantially from 1981 through the end of 1983, as the market softened" (see also Cummins, Harrington, and Klein [31, pg. 18]).

manifest in the disparity between actuarially indicated rates and marketplace prices as in the reported net income of insurers.[5]

The disparity between insurer knowledge and insurer pricing actions was particularly stark in the late 1980s, when 25% of the Workers Compensation was being written by the involuntary pools. Insurers were pricing the policies below cost, but they would not write the business that they were pricing.

If disinterested analysts, uninvolved in the economic fortunes of particular insurers, were to generate "actuarially indicated rates" to which the entire industry adhered, there might be no underwriting cycles. Ratemaking procedures have little or no influence on actual profit cycles. However, insurance premium rates are different from actuarial indications. Real-world prices are not the result of mathematical exercises, whether simple or sophisticated. And it is in the prices charged on the street that we may discern the workings of the cycle.

Underwriting Philosophy

A second interpretation of insurance underwriting cycles relies on the "mass psychology" of underwriters. During profitable years, insurers grow optimistic and compete strenuously for new business. Since capacity is limited only by financial and psychological constraints, not by physical plant and equipment, supply

---

[5] Venezian [111] presents a more sophisticated connection of underwriting cycles with ratemaking techniques: "Insurers and rating bureaus often use regression of past costs, or of loss ratios, on time as a way of estimating future rate requirements. A model of this process suggests that the rates set by such methods would create a quasi-cyclical pattern of underwriting profit margins.... Empirical data on major lines of property and liability insurance are consistent with the hypothesis that ratemaking methods contribute to the fluctuations of underwriting profit margins."

Venezian suggests only that ratemaking methods contribute to the cycles, not that they cause them. But all these "ratemaking" interpretations search for the cycle in actuarial indications where it does not exist; they ignore competitive pricing strategies, where the cycle is powerful.

Similarly, Pentikäinen et al. [88] use a statistical model of underwriting cycles to examine the influences of market prices on insurance solvency. Cummins and Outreville [34] propose a model along the same lines, though with different causal variables: (a) data collection lags, (b) regulatory lags, (c) policy renewal lags, and (d) calendar year financial reporting.

expands. Demand is inelastic, so premium growth means attracting business from other insurers. Severe competition in a mature market requires insurers to lower prices to gain market share (Stewart [101]; Bloom [16]; Berger [11]).

Profits soon decline, due to low rates and the poor quality of some risks. Underwriters become pessimistic, curtail their acceptance of marginal applicants, and file for rate increases. Profits remain low until insurers re-underwrite their business and the new rates take effect. Eventually, the rate increases and the more careful underwriting lead to increased profits, and the cycle starts anew.

This interpretation of the cycle is popular, and variations abound. Boor [17, 18] suggests numerous factors that might strengthen or weaken cycles, such as premium-to-surplus rules, reserve management, and the ease or difficulty of entry into and exit from the insurance market.

Information and Coordination

Should not the supply proffered and the quantity demanded converge on an equilibrium point, and the underwriting cycles cease? This is a central thesis of Western economics, and rapid convergence is evident in most industries with free markets. Stewart [101, pg. 293] explains the absence of such convergence:

> The cyclical process does not end for two reasons: lack of information and lack of coordination. Individual insurers do not and cannot know the precise amount of insurance to supply to reach equilibrium. They have different operating costs and, therefore, different break-even points or minimum acceptable margins of profit. Their perceptions and expectations of future profits or losses develop in different ways. In self-interest, they do not coordinate their actions. Collusion, furthermore, is illegal. Even when prior approval

and rating bureaus had more influence on prices, insurers varied supply according to their own situations.

This explanation is unusual, since the lack of strategic coordination and the imperfect information should lead to stable equilibria. If firms cannot coordinate prices and quantities, then the price mechanism effectively equates supply and demand. The competitive characteristics of the insurance industry that Stewart notes argue for a more stable equilibrium, since underwriters can quickly adjust supply to end any disparity with the quantity demanded.[6]

Uniform Psychology

The fundamental problem with this explanation is not the "lack of cooperation" or the "lack of coordination" theses. Rather it is the assumption of a uniform psychology among underwriters. An individual may be more or less optimistic in different years. But how is it that ten thousand underwriters across the United States are optimistic and pessimistic in unison?

---

[6]Stewart also cites a "cobweb" interpretation for the continuation of underwriting cycles: "Cycles that result from supply's responding to profit expectations are described in textbook economic theory by what is called a 'cobweb.' ... In agriculture, as in property-liability insurance, demand is steady and supply is variable, with the result that prices tend to move with changes in supply" [101, pg. 293].

On the contrary: standard "cobweb" explanations rely on the period to period lag in revising supply. In agriculture, supply cannot be adjusted rapidly, since it depends on the amount seeded in previous months, not just on the marketplace price. See, for instance, Ezekiel [44, pp. 426, 436–437]: "For a commodity where the production process occupies a definite interval of time, the period considered may be taken as so short that the total supply available cannot be changed within the period (as, for example, the supply of cotton or potatoes once the year's crop is harvested)," and "The cobweb theory can apply exactly only to commodities which fulfill three conditions: ... (2) where the time needed for production requires at least one full period before production can be changed ...." A six-year cycle presumes a three-year production lag. This is not the case for insurance: supply depends only on price and can be quickly adjusted.

Similarly, Cummins, Harrington, and Klein [32, pg. 63], in describing Stewart's thesis, write: "A key element in this explanation is that competition in soft markets ultimately leads to inadequate rates. Prior academic research includes little or no formal analysis of why competition could cause prices in soft markets to fall below levels needed to cover cost expected when policies are sold and to ensure insurer financial soundness."

Daykin, Pentikäinen, and Pesonen [40] illuminate the mystery of cycles. Fluctuating profits are not uncommon; even random fluctuations may look like cycles. The mystery is that while the profit patterns in each insurer seem inexplicable, these profit patterns are correlated among most of the insurers in the market.

The enigma of underwriting cycles is not that any individual underwriter accepts risks in one year that he or she would reject in another. Rather, it is that profits for insurers move in tandem.[7] In contradistinction to Stewart's explanation, this phenomenon indicates a higher level of competitive strategy than we would otherwise suspect. Insurers, no less than other firms, are sensitive to the prices charged by their competitors, and they adjust their own rates accordingly.

Stewart's thesis shows the outlines of the cycle: the stable demand, the competition among insurers, the fluctuating prices, and the relatively uniform practices among underwriters at any given time. But the connections among these phenomena remain unexamined. To flesh out these relationships, we must ask: "What additional characteristics of the insurance marketplace relate to profit cycles?" and "How do these characteristics account for the fluctuations in underwriting income?"

Cash Flow Underwriting

A third interpretation of underwriting cycles relies on interest rate volatility. Insurers pay losses well after they collect premiums, particularly in the liability and Workers Compensation lines of business. Premiums are invested in financial markets (stocks, bonds, mortgages) and earn investment income until losses are paid.

Insurance income may be divided into underwriting and investment portions. Underwriting income is the difference be-

---

[7] Daykin, Pentikäinen, and Pesonen [40] note with regard to a set of large Finnish insurers: "The cycle is effectively the same for each of the … insurers, so that we can speak about a *market cycle*" (emphasis in original).

tween (a) premium revenues and (b) loss plus expense payments. Investment income is the return on invested assets.

Interest Rates

Interest rates rose rapidly in the late 1970s, reflecting the inflationary trends in the U.S. economy. Investment income became a larger portion of insurance earnings, and underwriting income decreased. Insurers wrote policies at expected underwriting losses, since they relied on investment returns for an overall profit.

Many insurers, accustomed to underwriting profits, viewed the reliance on investment returns as a lack of "underwriting discipline." They castigated this new philosophy as "cash flow underwriting": writing policies at a loss simply to generate premium dollars for investment.

Cash flow underwriting is appropriate as long as interest rates remain high.[8] But by the mid-1980s, new money interest rates had fallen. The lack of underwriting discipline continued; insurers kept writing policies at underwriting losses. Investment income was no longer sufficient to compensate for these losses, so insurance operating returns declined. This was the underwriting cycle nadir of the mid-1980s.[9]

This argument was popular several years ago. It has lost favor recently, since the underwriting cycle has lost no force despite

[8]Compare D'Arcy and Doherty [38, pg. 86]: "While pejoratively termed 'cash flow underwriting,' this willingness to accept underwriting losses is not a symptom of temporary market insanity but is a rational economic reaction to the availability of higher interest rates."

[9]See, for instance, McGee [71, pp. 22, 25]: "Changes in interest rates are the primary force behind the recurrent swings in the industry's profitability." To explain the intensity of the 1980s cycle in the Commercial Liability lines of insurance, McGee writes: "The combined ratio for long-duration lines of insurance should move more than the ratio for short-duration lines over the interest rate cycle, and the mix of insurance by lines will affect the timing and volatility of the property/casualty cycle." He acknowledges that "workers' compensation lines are long-tailed, but their combined ratio does not behave as the increased interest-sensitivity principle would suggest," although he ascribes this anomaly to policyholder dividends and stringent rate regulation.

the present stability of interest rates. Nevertheless, it is still useful to examine the problems with this interpretation.

Underwriting and Investment Income

First, the distinction between underwriting and investment income is specious. Cash flows must be discounted to a common date to appropriately match revenues and expenses. True insurance income is the difference between (a) premium revenues and (b) discounted loss plus expense payments.[10] True investment income is the sum of (a) the return on invested surplus funds, (b) the difference between actual and expected returns on policyholder supplied funds, and perhaps (c) the difference between expected returns and the return assumed in the discount rate.[11]

[10] Although discounted cash flows may be used to measure income, the appropriate discount rate for insurance losses is unclear. Lowe [67] suggests a "negotiated rate" that is set by the senior management of the insurance company. Woll [116] recommends an after-tax "risk-free" rate, such as the Treasury Bill rate. Butsic [21] derives a "risk adjusted" discount rate based upon historical insurance experience. Fairley [45], Hill [55], and Myers and Cohn [80] use risk adjusted discount rates, based on extensions of the Capital Asset Pricing Model to insurance losses. The 1986 Federal Income Tax amendments use the federal midterm rate to discount losses; see Gleeson and Lenrow [50] or Almagro and Ghezzi [4]. Others have suggested embedded yields, as the Insurance Expense Exhibit uses, or new money market rates, as AICPA [1] recommends and which most life insurers use. The lack of agreement on the appropriate discount rate hampers consistency among insurance companies in analyzing income.

[11] Compare Woll [116] and Lowe [67]. Different means of categorizing income are possible; we do not mean to prescribe a particular method. A numerical example should help clarify the intention. Suppose the insurer has \$10 billion of funds from insurance transactions and \$4 billion of surplus. Suppose also that the expected investment income on funds from insurance transactions was 8% per annum, the actual investment income was 9% per annum, and the investment income on capital and surplus funds was 10% per annum; all investment income includes unrealized capital gains and losses.

Of the investment income, \$800 million (or 8% of \$10 billion) would be included with insurance income, as this is part of the expected return from the insurance operations. The remaining 1% return on the funds from insurance transactions plus the 10% return on capital and surplus funds would be included with investment income.

Alternatively, if the loss reserve discount rate used for internal company management reporting is 7% per annum, only \$700 million (or 7% of \$10 billion) would be included with insurance income, and the remainder would be categorized with investment income. This procedure might be used if the risk-free interest rate were 7% per annum but the expected investment yield of the company were 8% per annum.

Numerous variants of this procedure have been suggested by actuaries. They differ in the details—such as in the discount rates and the bases—but they all value cash flows as of the same time. The use of unadjusted nominal values to determine insurance profitability simply confuses performance measures and distorts patterns of profitability.

When insurance income is properly measured, it is not necessarily reduced by a rise in interest rates. Higher interest rates that are accompanied by accelerating inflation increase the nominal settlement values of insurance losses even as they raise the appropriate discount rate for loss reserves. A rise in inflation increases both investment returns and expected loss payments.

In other words, when inflation is modest, both the discount rate and expected losses are low. When inflation accelerates, both the discount rate and expected losses increase. The net effect is ambiguous.[12]

Asset-liability matching theory also implies a different outcome than that suggested by "cash flow underwriting" interpretations of the underwriting cycle. The average duration of Property/Casualty insurers' assets is longer than that of their liabilities. A drop in interest rates, as occurred in the mid-1980s, causes an increase in profits, not a decrease in profits. In fact, those insurers that bought long-term bonds at high yields in the late 1970s and early 1980s enjoyed above average investment returns in subsequent years.[13]

---

[12] For the relationship of liability losses to market interest rates, see Butsic [22]. McGee [71, pg. 23] is aware of the inflation sensitivity of liability losses: "Inflation also has an impact on the relationship between the competitive price of insurance and interest rates. If costs of settling claims are expected to rise through time, a higher premium or investment return will be necessary to cover future costs. To the extent that rising interest rates reflect anticipated inflation, they should not affect insurance premiums."

McGee hypothesizes that "uncertainty about the inflation outlook" in a competitive industry depresses market prices to those of the most optimistic insurer. Widely fluctuating interest rates lead to greater uncertainty and therefore a decline in insurer profitability. This explanation ignores McGee's own statement that as long as inflation and interest are correlated, different inflationary expectations should not affect insurance premiums.

Cummins, Harrington, and Klein [32, pg. 68], note that interest rate fluctuation is not by itself a sufficient explanation of underwriting cycles: "...prices in competitive insurance markets would reflect the interest earnings on funds held between the premium payment and loss payment dates. Thus, prices should fall when interest rates rise and rise when interest rates fall. This is not a problem unless insurers overreact to interest rate changes or unless serious pricing errors are common." (These remarks assume a positive equity duration for insurers. If liability loss payments are entirely inflation sensitive, the inverse relationship between interest rates and insurance prices does not hold.)

[13] For the effect of interest rate changes on the returns of mismatched portfolios, see Bierwag, Kaufman, and Toevs [14] or Redington [94]. For an analysis of asset and liability durations of Property/Casualty insurance portfolios, see Feldblum [46] and Panning [87].

Financial Expertise

Finally, and most fundamentally, a "cash flow underwriting" interpretation of underwriting cycles reveals a deep academic condescension towards insurance company investment managers and underwriters. It presumes either that investment managers were surprised by the fall in interest rates in the mid-1980s or that underwriters are unable to adjust rates for changes in investment income. But the investment community was not shocked by the fall in interest rates in the 1980s. On the contrary: financial analysts were surprised that interest rates stayed high even after inflation subsided. Similarly, good underwriters aim at long-term operating profits. They are not easily deceived by steady changes in investment returns.

Interpretations of the underwriting cycle abound. The majority presume that someone is erring: ratemaking methods are naive, underwriters are simplistic, regulation is rigid, or investment managers are deceived. Such explanations search for a cause where it is not to be found. Insurers are no less rational than other firms are. They exist in a highly competitive market, where the foolish firm does not long survive.

3. COMPETITION AND PROFITS

To understand the relationship of underwriting cycles to insurer solvency, we must briefly step aside from insurance and delve into economics and business theory. We ask: "What is the relationship between competition and profits?"

We consider first the economist's perspective, examining competitive, monopolistic, and oligopolistic market structures. We then analyze the insurance industry from a concrete business viewpoint, examining policy differentiation, policyholder loyalty, and the ease of entry into the insurance marketplace. We ask: "Given the structural characteristics of the insurance industry, what price-cost margin should we expect?"

Textbook Models: Competition and Monopoly

Undergraduate economics textbooks present two market models: pure competition and single firm monopolies. These models are meant only to illustrate the forces that determine prices, not to depict actual practice.

In pure competition, prices are determined by industry-wide supply and demand. No individual firm can unilaterally affect market prices. If a firm restricts supply, its competitors take up the slack. If a firm raises prices, consumers purchase the product elsewhere.

In a monopolistic industry, a single firm dominates the market. Entry of competing firms is sufficiently restricted that the monopolist can adjust the quantities supplied and the prices charged to maximize its profits.

Competition

What market price results from each model? Suppose that the price in a competitive industry exceeds the marginal cost of producing the product. Any firm could cut prices slightly, garner a greater market share, and increase its profits.

Similarly, if the market price were below marginal cost, firms would leave the industry and employ their capital elsewhere. Equilibrium is achieved when price equals marginal cost.

Equilibrium means that there is no tendency for prices to either rise or fall.[14] Economists maintain that prices generally

[14]Industrial economists, when considering firm behavior, speak of Nash equilibria (Nash [84]). A Nash equilibrium obtains when no firm has an incentive to modify its production or price strategy. If firms seek to maximize their income, this implies that no firm can obtain greater profits by raising or lowering its price or by increasing or decreasing the quantity that it supplies. Waterson, using a game-theoretic approach to industrial economics, defines a Nash non-cooperative equilibrium as the "point such that each player's strategy maximizes his expected payoff if the strategies of the others are held fixed" [115, pg. 41]. Friedman [48, pg. 49] uses a similar definition: "A [Nash] noncooperative equilibrium consists of *n* particular strategies, one for each firm, so chosen that no single firm could possibly have obtained higher profits if it, alone, had selected a different strategy." Fudenberg and Tirole [49] summarize the formal theory of Nash equilibria.

move toward equilibria in free markets. Underwriting cycles, however, seem a stark example of disequilibrium: prices continually fluctuate.

Monopoly

Under suitable conditions, the monopolist seeking to optimize its income will not price its product at marginal cost.[15] When price equals marginal cost, there are no economic profits for the firm. But if the monopolist restricts output, consumers "bid up" the price to obtain the scarce good. Price exceeds marginal cost, and the firm receives additional profits.

In a purely competitive marketplace, price equals marginal revenue which equals marginal cost. In a monopolistic market, marginal revenue generally exceeds marginal cost. Prices are higher in a monopolistic market than they would be in a competitive market.

Actual Market Structures

These market structures rarely exist in their ideal forms. Even when there are thousands of firms selling similar products, competition is seldom perfect. For instance, grocery stores exist all

---

When market conditions cause firms to have different strategies—some seek stable current income and others seek to increase sales—Nash equilibria often dissolve. This phenomenon underlies the model of underwriting cycles developed below.

[15] These conditions are that either the marginal cost rises as quantity supplied increases or the demand curve slopes downward. Marginal cost is the cost of producing an additional unit of the good. In insurance, this is the expenses and anticipated losses of writing an additional policy, not the average expenses and losses incurred on the current book of business. The demand curve is the relationship between consumer demand and the product's price. In insurance, this is the number and size of policies and endorsements desired by consumers at each premium rate.

Both conditions are satisfied in the insurance market. (1) The demand curve in many lines of business is nearly vertical, because of statutes, regulations, and business policies that mandate coverage (Sherdan [99]). (2) The marginal cost curve rises sharply, despite the preponderance of variable costs in insurance. As D'Arcy and Doherty [38, pg. 9] note: "…an insurer writing a large quantity of policies will eventually have to relax underwriting standards to increase the quantity further, and the newer policies could have a higher expected loss ratio." That is, at low quantities, insurers can "skim the cream," selecting the best risks. At higher quantities, insurers offer coverage even to mediocre and poor risks. Thus, marginal costs rise as the number of policies issued increases.

over, selling the same foods: is this not pure competition? But most individuals use the nearest corner grocery for small purchases and do not bother to price shop. In other words, the grocery store may have a near monopoly within a small neighborhood.[16]

Monopolies are equally hard to maintain. IBM dominated the market for mainframe business computers in the 1960s, and it enjoyed large price-cost margins during those years. But competitors soon entered wherever profits beckoned—computer peripherals, software programs—and they quickly gained significant market shares.[17]

Nevertheless, these two models are important, for they set the bounds of the price range. If capital can be transferred to other uses, firms will not price below marginal cost.[18] And if sufficient supply is available, firms will not price above the monopoly price.

---

[16]Scherer [97, pg. 325] comments: "Even when firms produce physically identical commodities, complete homogeneity is not likely to be attained because of differences in location.... When producers are located at different points on the map, their products are said to be *spatially differentiated*."

[17]On the history of IBM's market dominance in the mainframe computer industry and the entry of competitors in peripheral equipment and software products, see Brock [20]. Government sponsored monopolies, such as municipal utilities, cable TV franchises, and telephone service until the 1980s, are different. These industries have strictly regulated rates; they do not price by supply and demand considerations.

The diversity of insurance rate regulation affords a range of insurance markets. In some states, such as Massachusetts and Texas, insurance rates are set by the regulator or by official rating bureaus. In other states, such as Illinois and pre-1989 California, the free market determines insurance prices. Insurance rate regulation is a factor (albeit a minor one) in underwriting cycle severity.

[18]Transferring capital can be difficult, and firms may price below marginal cost in a declining industry. The Personal Insurance lines present an excellent illustration of this. Over the past 40 years, direct writers have steadily garnered most of the Personal Lines market, and they have consistently attracted the better risks among the insured population. Independent agency companies have a declining market with worsening risk quality. Many of these companies are slowly moving to other lines of business (such as Commercial, Specialty, Reinsurance, and Substandard Auto), experimenting with less expensive distribution systems (such as direct mail), or trying to start joint ventures with other financial institutions (such as life insurers, health insurers, and securities brokers). Meanwhile, average Personal Lines returns for independent agency companies are below marginal cost.

This price range is wide, since the monopoly price may be well above marginal cost. So if the market is neither purely competitive nor monopolistic, what prices will actually be charged?

Economic Models

Economics is rigorous. Theorists provide the needed assumptions, then "prove" the desired conclusions. But these assumptions are invariably idealistic. The equations are mathematically perfect but of limited practical value.

We cannot proceed without a theoretical framework. We will deal with price-cost margins, Nash equilibria, entry conditions, and price elasticity of demand. However, we are interested not in formulating theorems but in understanding a business phenomenon: the underwriting cycle. So we must step gingerly over the coming terrain.

We can view this distinction from another perspective. Economic models abstract reality. They isolate some elements, and the results are determined from the assumptions. The business world is represented by succinct mathematical expressions.

Underwriting cycles, however, are complex phenomena: no two companies react identically to their course. We will not try to determine the exact duration or severity of the cycles. Rather, we seek to understand the driving forces behind insurance pricing.

We begin with an abstract model of pricing in a competitive market with a limited number of firms.[19] Our emphasis will

---

[19]In truth, there are thousands of American insurers, and dozens of new ones enter the industry each year. This is a central characteristic of the insurance model that we develop further on. For clarity of exposition, however, we begin with a model of a limited number of firms.

Supplier interdependence is enhanced by high market concentration. Some economists use four firm concentration ratios of 50% or greater, or a Herfindahl-Hirshman index of about 1,000 or greater, as indicators of possible interdependence. (See, for example, the June 1984 Justice Department merger guidelines for antitrust action.) The Personal Auto insurance industry shows a four firm concentration ratio of 40% and a Herfindahl-Hirshman index of 610 on a national basis, and corresponding average figures of 53% and 1,000 on a statewide basis. These figures depend on the definition of the market:

be on Nash equilibria and conjectural variation. We then revise the model, discarding the idealism and adding reality, to explain profit cycles in the Property/Casualty insurance industry.

Conjectural Variation

Suppose two rival firms, producing identical products, each have 50% of the market. Consumers are conscientious price shoppers with excellent information, so if either firm underprices the other it quickly captures the entire market. If the firms compete by setting prices, then a *static* microeconomic analysis implies that both firms will set prices at marginal cost.[20] If one firm prices above marginal cost, the other firm can charge slightly less, gain the other 50% of the market, and increase its total profits.

---

state versus national and individual line versus all insurance products. Inter-industry comparisons of market concentration must use similar criteria of market definition; if so, insurance shows low relative concentration. On automobile insurance, see Klein [61, pg. 12, Table 1, pp. 18–19, Table 4]; on Workers Compensation insurance, see Countryman [29, pg. 17, Table 1], Klein [62], and Appel and Gerofsky [6; 7].

[20]Firms may compete either by setting prices or by choosing the quantities they supply. Price and quantity are interrelated, since the industry demand curve sets a one-to-one relationship between them. If firms compete by choosing the quantities they supply, "Cournot competition" implies that the resulting price will exceed marginal cost. The price-cost margin varies inversely with the number of firms: one firm (pure monopoly) produces the greatest profits, and an infinite number of firms (pure competition) eliminates economic profits. See Tirole [107, pp. 218–221], or Scherer [97, pp. 152–155].

Manufacturing firms with long production cycles may compete by choosing the quantities that they supply. A Cournot analysis is appropriate for them. Insurers have almost no supply restrictions; rather, they compete on premium rates. A "Bertrand" analysis, which results in price equaling marginal cost, is the appropriate model (see below in this note). See Tirole [107, pp. 209–212] or Varian [109, pp. 461–464].

The appropriate model for insurers depends on their supply constraints. Unlimited capacity implies that firms compete by setting prices. Severe capacity constraints imply that firms compete by choosing quantities. For an analysis of the limits on insurance capacity, see Stone [102]. Stone's analysis applies to large Commercial risks, where random losses may adversely affect an insurer's income or even solvency. In practice, there are no capacity constraints in the Personal Lines or for small Commercial risks. Moreover, for some large risks, the availability of reinsurance mitigates the capacity constraints.

For a general discussion of insurance supply, see Stewart [101]. Stewart correctly notes that insurance supply is determined by psychological and financial considerations, not by plant, equipment, labor, or other physical restrictions. The ability of insurers to quickly revise quantities and prices is an essential aspect of the underwriting cycle; see the text below.

This analysis is static: it considers only a single time period. Dynamic models presume that firms respond to their rivals' competitive actions. Moreover, each firm anticipates how its rivals will respond before implementing its own strategy. Economists term this conjectural variation: "Each firm believes that its choice of price will affect the price selected by its rivals."[21]

Suppose again that two firms producing identical products and competing on price each have 50% of the market. In the static analysis, if the market price exceeds marginal cost, then either firm may slightly reduce its price and garner the entire market. In reality, the businessman wonders: "If I cut my price to increase market share, how will my rival respond?"

Clearly, the rival will match the price cut—at least if a small reduction in price enables it to retain its market share. If both firms presume that the other will match a price cut, neither will initiate the price reduction.[22]

We formulate this mathematically as follows. Let P^m be the current market price and P^c be the competitive, or marginal cost, price. Let v be the annual discount rate for future earnings (the discount rate is treated more fully below). Suppose that each firm knows that if it reduces its price below P^m, its rival will immediately charge P^c. Finally, assume that a price cut below the current market price promptly attracts the entire consumer population.[23]

The current market price, P^m, provides total industry earnings of E^m, a positive amount. The marginal cost price, P^c, provides

[21] Tirole [107, pg. 244]. For a mathematical development, see Varian [110, pp. 102–103], or Waterson [115, pp. 18–19]. Porter [93] presents a non-mathematical discussion of the strategic consideration of expected rival responses.

[22] That is, conjectural variation influences optimal business strategy. If an insurer believed that its peer companies use cost-based pricing and that they do not consider competitive pressures, it would have no disincentive to reduce rates in order to gain market share. In practice, insurers' prices are strongly affected by those of their rivals. This is most evident in the Personal Automobile market, where the major direct writers carefully examine their rivals' rates, by territory and classification, to set their own prices.

[23] These are the ideal assumptions so endearing to economists. We will return to reality in a few paragraphs.

zero economic profits; that is, $E^c = 0$. If both firms maintain the current market price, P^m, their earnings will be $1/2(E^m + vE^m + v^2E^m + \cdots)$ for each. If either firm slightly shades prices, its earnings will be E^m in the current period.[24] Since its rival quickly cuts prices to marginal cost, its earnings are 0 in all future periods.

If the firms are to be dissuaded from cutting prices, then E^m must be less than $1/2(E^m + vE^m + v^2E^m + \cdots)$. That is,

$$1 < (1 + v + v^2 + \cdots) \div 2, \qquad \text{or} \qquad v > \tfrac{1}{2}.$$

This makes sense. If v is high enough (more than one half in this instance), firms are unwilling to sacrifice future earnings for immediate profits. Conversely, if v is low, firms disregard future earnings and emphasize short-term results.[25]

Discount Rates

The discount rate measures the relative value of a dollar of future earnings compared with a dollar of present earnings. The interest rate is a part—but only a part—of this. Also important is the uncertainty about future market conditions. Perhaps consumer demand will slacken, other suppliers will enter the industry, restrictive regulations will impede price adjustments—and future profits will dissipate. Perhaps demand will grow and entry barriers will harden, increasing future profits. Perhaps rival firms will differentiate their products and segment the market.[26]

Future earnings in an inflationary economy are worth less in real dollars. In a competitive market, they are also uncertain:

---

[24]This is a theoretical model. It assumes that an infinitesimal price reduction attracts the entire market. In insurance, (1) a substantial rate reduction is required to gain market share, and (2) shifts in the insured population occur at renewal time, not continuously. The model of underwriting cycles developed below incorporates these elements.

[25]For more complete discussions, see Tirole [107, pp. 245–251], or Shapiro [98].

[26]Describing the discount rate, δ, Shapiro [98, pg. 362, note 58] writes: "Formally, δ may be thought of as the product of two terms: $\delta = \mu e^{-iT}$, where μ is the hazard rate for the competition continuing (i.e., the probability that the game continues after a given period, given that it has not previously ended), and e^{-iT} is the pure interest component of the discount factor, with period length T and interest rate i."

anticipated profits may never materialize. Business strategy, which determines the quantities supplied and the prices charged, affects the realization of future profits.

The size of the discount rate (v) needed to discourage price cutting varies with the number of competing firms. If there are two firms of equal size, v must be greater than $\frac{1}{2}$, as the equation above implies. If there are ten rival firms of equal size, v must be greater than 9/10 to discourage price cutting.[27] The insurance market has hundreds of rival firms in the major lines of business, so this simple model implies that the discount rate must be near unity to discourage price cutting. But if insurers generally price at marginal cost, why are there severe profit cycles? To answer this problem, we present a more sophisticated model. First, however, let us take another detour: How does a firm choose an "optimal" price?

Limit Pricing and Entry Barriers

The optimal price depends upon the strength of entry barriers. If entry barriers are low and profits are high, new firms enter the market. Entrants cannot gain market share if they charge the current price, so they have little to lose by price cutting.[28] Incumbent firms rarely let the market price remain high enough to attract new entrants.

The cut-off price between attracting and discouraging new entrants is termed the "limit price." But why should the limit price be any different from the competitive marginal cost price? If all firms have the same production costs, then any price exceeding marginal cost attracts new entrants.[29]

---

[27]That is, E^m must be less than $(E^m + vE^m + v^2E^m + \cdots)/10$. Thus, $1 < (1 + v + v^2 + \cdots)/10$, or $v > 9/10$.

[28]In underwriting parlance, we speak of new entrants "buying" market share. A new firm may suffer operating losses for several years before it develops a profitable book of business. This is particularly true in insurance, since new entrants attract the marginal and unprofitable risks.

[29]"Limit pricing" is a standard economic term, unrelated to the actuarial procedure of "increased limits pricing."

But firms do not all have the same production costs. In particular, new firms face a fixed (sunk) cost of entry, so the limit price exceeds the marginal cost price.[30]

In theory, there are few barriers to entry in insurance. The insurer need build no factories to manufacture its product; it may contract for the needed actuarial, underwriting, and loss adjustment skills; and statutory capitalization requirements are not excessively onerous (although they are higher than they were before the advent of risk-based capital requirements). The firm may simply "hang out a shingle" and begin writing policies.

In practice, this is not correct. In the Personal Lines market, the direct writers are profitable whereas the independent agency companies are losing money. Yet few independent agency companies have successfully switched to direct writing or exclusive agency distribution systems. The constraints on the distribution system are powerful, raising large entry barriers to the *profitable* insurance markets.[31]

The traditional barriers to entry, such as minimum efficient production scales, or the advertising budget needed to place products on retail shelves, are not important in insurance. The insurance "distribution" barrier to entry does not involve getting consumers to purchase policies. Rather, it involves getting the *better* risks to purchase policies.

We return to this topic later on, in our model of underwriting cycles. Note, however, how deceptive these barriers to entry are.

---

[30] On limit pricing, see Milgrom and Roberts [76] and Porter [93, pg. 14] (who uses the term "entry deterring price"). Insurers face few fixed costs, particularly in lines of business dominated by the independent agency distribution system. Entry into the Commercial Lines insurance marketplace is deceptively easy—new firms believe they can enter quickly. Thus, there is a short span between the marginal cost price and the limit price.

[31] Several life insurers have recently entered the Property/Casualty Personal Lines market. Although they came with strong underwriting, actuarial, and distribution systems, enormous capital, and extensive experience in Life and Health insurance, most of these firms have had trouble transforming the newly acquired Personal Lines risks into profitable books of business. The hidden barriers to entry are strong deterrents to prospective insurers.

It is easy to enter the insurance market, since there are no major capital or regulatory barriers. It is far more difficult to enter successfully.

These are the bounds postulated by industrial economics. In the long run, prices will not remain below marginal cost or above the limit price.[32] The actual prices charged depend on the number of firms, the extent of "conjectural variation," the discount rate assumed by each firm, and other factors affecting the price-cost margin.

The theoretical economist would ascribe the insurance industry's low profitability to the competitive characteristics of its market.[33] But we need a more specific analysis to understand underwriting cycles, so we ask: "How do the nature of the insurance product and the operations of the insurance carrier affect anticipated profits?"

4. INSURANCE INDUSTRY CHARACTERISTICS

An industry's structure and the characteristics of its products influence both expected profits and strategic possibilities. Three considerations particularly germane to insurance are

1. Product differentiation and substitute products,
2. Cost structures and barriers to entry, and
3. Consumer loyalty and price shopping.

We begin with these insurance attributes, in preparation for the analysis of underwriting cycles.

---

[32]In the short run, this is not true. In declining industries, prices often sink below marginal cost. In expanding industries, incumbent firms may price above the limit price, allowing new entrants even as they reap large profits. Numerous other short term exceptions are discussed in the economics literature.

[33]Plotkin [89, 90, 91, 92] has documented the relative profitability of insurers vs. other firms. See also Braithwaite [19], Banfield [9], and Bailey [10].

Product Differentiation

When firms supply products with varying attributes, such as automobiles, computer hardware, and fashion clothing, each of them may enjoy some market power and associated economic profits. When the products of competing firms do not vary much, as is true in agriculture, all firms may be constrained by the prices of the most efficient producer. In short, product differentiation increases expected profits.

Most insurance policies are indistinguishable to the average consumer. In certain lines of business, such as Workers Compensation and no-fault Auto Insurance, benefits are mandated by statute. Even where no laws impede differentiation, product diversity is hard to maintain. Improved policy forms can be copied by rivals, so advantageous innovations are transient.

The existence of close substitutes for an industry's products has a similar effect: substitutability constrains profitability. For instance, aluminum often can be substituted for steel. Aluminum prices constrain steel profitability, regardless of competition in the steel industry.

In many lines of business, there are few substitutes for insurance. The Personal Lines consumer has no choice but to purchase an auto insurance or Homeowners policy. Similarly, most small business owners must buy Workers Compensation insurance, since self insurance techniques are feasible mostly for large and sophisticated companies. The rising claims consciousness of the public, and the increasing predilection of Americans to turn to the courts, strengthens the demand for Commercial Liability products. Small businesses have no alternative other than to buy insurance protection.

In sum, the lack of product differentiation means that individual insurers have difficulty increasing prices and profits. But the lack of close substitutes for an essential product means that the industry as a whole can raise or lower premium rates without losing consumer demand. Formally, aggregate consumer demand for insurance products is inelastic with respect to price, but interfirm elasticity is high.

Cost Structures and Barriers to Entry

We distinguished above between traditional and "hidden" barriers to entry. Traditional barriers depend on cost structures: minimum efficient plant size, up-front capital requirements, the time needed to enter, and production process learning curves. Potential entrants observe these costs, which influence their willingness to join the industry.

Insurance has few traditional barriers to entry. Almost all costs, including losses, loss adjustment expenses, commissions, salaries, and premium taxes, are variable, not fixed.[34] No plants need to be built, no expensive equipment is required, and statutory capitalization requirements are manageable.[35] Most costs are paid either on the policy effective date (e.g., commissions) or after the policy is in force (e.g., losses).[36] The cash inflows from "producing" an insurance policy precede the cash outflows,

---

[34]The distinction between variable and fixed costs differs from the actuarial distinction between costs that vary directly with premium and those that do not. Salaries of nonmanagerial personnel are variable costs, though they do not vary directly with premium. The other expenditures listed in the text are both variable costs and vary directly with premium.

[35]Meyerson [74, pg. 151], writing before the advent of risk-based capital requirements, notes that "the initial capital and surplus requirements of most states are much too low under present conditions." Danzon [36] examines the relationship of state licensing statutes to entry barriers, in terms of delay of operations and cost of entry. She finds average delays of six to ten months, and an average personnel cost per state for entry expenses of $100,000. She notes that these costs are too small to serve as entry barriers. See also Klein [62, pp. 91–92], who shows high entry and exit to the Workers Compensation market.

The implementation of risk-based capital requirements in 1994 for Property/Casualty insurance companies should somewhat raise these entry barriers. For some small insurers, though, the risk-based capital requirements are not that much higher than the previous minimum capital requirements. The effect of the new capital standards is more evident for medium and large insurers. In fact, an early attempt to add a "small company charge" to the risk-based capital formula died on the conference table in 1993.

[36]Other acquisition expenses and certain administrative and underwriting costs are expended before premiums are received. The National Council on Compensation Insurance, using a 1977 study of Massachusetts Workers Compensation expenses, estimates that only 14% of "other expenses" (i.e., general expenses, other acquisition costs, and miscellaneous taxes, licenses, and fees; thus, about 2% of insurance costs) are paid before the

thereby facilitating the entry of new firms. Underwriting intricacies are not readily discernable, and many entrants believe that there is no significant learning curve. (In fact, casualty underwriting is a fine art, but new entrants sometimes seem loath to admit this.) Finally, a firm can contract for underwriting, actuarial, accounting, and loss adjustment skills, so little time is needed before writing policies.

As we noted earlier, the "hidden" barriers to entry in insurance are powerful. It is easy to enter the insurance marketplace; it is far more difficult to enter successfully. New entrants attract marginal risks, and actual insurance losses are high in early policy periods. It takes many years to obtain a profitable book of business (Conning & Co. [27]).

So new firms continuously enter the insurance market. Were earnings steady, the high rate of entry would depress expected profits. But fluctuating earnings, and the "hidden" entry barriers discussed above, impair the chances of successful operations. Many new entrants, with low quality books of business, do not last through the trough of the first underwriting cycle.

Consumer Loyalty

Price changes affect purchasing decisions. If the price for a particular brand of toothpaste rises 10%, some buyers of that toothpaste may switch to other brands.

Some goods have large "switching costs." Consumers of large electrical equipment may not change suppliers unless prices rise substantially, since such a switch would involve costs of installation, inspection, testing, retraining, and adapting other machinery. In other words, consumer loyalty to a particular brand or

---

policy's inception; see WCRIBM [117]. Mahler [68, Appendix 11, pp. 269–270] estimates that only 20% of "company expenses" (that is, general expenses, other acquisition expenses, and one half of unallocated claim expenses; thus, about 3% of insurance costs) are paid prior to the policy's inception.

supplier depends upon the costs of changing products.[37] High switching costs impede competition and raise expected profits.

Toothpaste, unlike large electrical equipment, has no "switching costs." Consumers have no constraints, either *ante hoc* or *post hoc*, on the brands they choose. When switching costs are absent, competition more easily dissipates economic profits.

Insurance seems similar. At renewal time, a consumer can purchase coverage from a competing carrier with no additional costs or gaps in coverage. This implies low expected profits in insurance.

In truth, insurance is not at all like toothpaste, particularly in the Personal Lines. Insureds rarely compare competitors' prices when their policies come up for renewal, whether or not they made such comparisons when they first obtained the coverage.[38] Only if an insurer dramatically raises its rates will policyholders begin searching for other agents or carriers.

Over the long term, insurance is no different from other goods. Higher than average prices cause a slow but steady loss of market share, which is extremely difficult to win back. But in the short term, a reputable insurer can maintain a higher than average price-cost margin without a significant loss of business.

Were insurance earnings steady, long-term expected profits would be low. The lack of product differentiation and the apparent ease of entry would force insurers to price close to marginal cost. But the lack of close substitutes, consumer loyalty, and the difficulty of successful entry facilitate short-term price fluctua-

[37] Porter [93, pg. 10] defines switching costs as "one-time costs facing the buyer of switching from one supplier's product to another's"; he adds: "Switching costs may include employee retraining costs, cost of new ancillary equipment, cost and time in testing or qualifying a new source, need for technical help as a result of reliance on seller engineering aid, product redesign, or even psychic costs of severing a relationship."

[38] Fox [47] reports that most of the auto policyholders who made cost comparisons did so at least two years prior to the survey date; see particularly his Tables 2 and 3 on page 23. Joskow [58] describes the relationship of policyholder information to insurance industry market structure.

tions. These characteristics of the insurance industry underlie the model of underwriting cycles in the following sections.

5. DYNAMICS OF THE UNDERWRITING CYCLE

Profit Cycles

Profit fluctuations may take two forms. In the first form, the market is in equilibrium during certain periods. External influences change costs, supply, or demand, and they thereby shift expected profits. Disequilibrium results until the price mechanism forces profits back to the original level. If external influences again affect the market, the fluctuations start anew.

Such profit fluctuations are rarely cyclical. For instance, weather conditions affect farm produce and profits: an unexpected frost may damage citrus fruit production, or a severe drought may lower crop supply. The affected farmers suffer from lost production, while other farmers benefit from higher prices. Prices and profits fluctuate, but the pattern is not cyclical.

Underwriting cycles take a different form: no phase is in equilibrium. Insurer strategies during profitable years drive rates down; changed strategies during poor years push rates up.

At two points in the cycle, in the upswing and the downturn, prices pass through the same point. But the underlying forces are different. One reflects a downward driving price strategy founded on high rates; the other reflects an upward driving price strategy founded on inadequate rates. This difference may be missed by an outsider looking at a snapshot of industry income. But the disparity is keenly felt by the businessman struggling for profits.

The Profitable Years

If there is no equilibrium point, there is no good place to begin analyzing the cycle. Yet we must start somewhere. So we begin, perhaps arbitrarily, at the top, as in 1977–78 or 1986–87 or 1992–94 (for Workers Compensation): income is high and insurers are satisfied.

Entry and Exit

Satisfaction breeds desire. Outside firms are enchanted by the ease of insurance operations: simply write the policy, collect the premium, and pay less in claims while you invest the assets. There are few explicit barriers to entry, so new firms join the industry.

Figure 1 shows insurance company entries and exits in the 1980s. Note the prevalence of entry into an industry earning below average profits and with low growth potential. Many of these entrants quickly failed. Insurance company exits climbed during the unprofitable 1984–85 and 1989 periods, and dipped in the profitable 1980–82 and 1987 periods.[39]

New insurers cannot sell their policies at the going market rate. Entrants must discount prices in any industry. This is all the more true in insurance, where it is hard to attract new customers. But new insurers believe that they have little to lose by charging lower rates. They have no existing business, so they do not lose money on older policyholders by cutting rates. Any price above marginal cost is profit.[40]

Price Shaving and Market Shares

New entrants charging low rates are an unwelcome thorn in the industry's side. Equally unwelcome is the change in strategy among existing insurers.

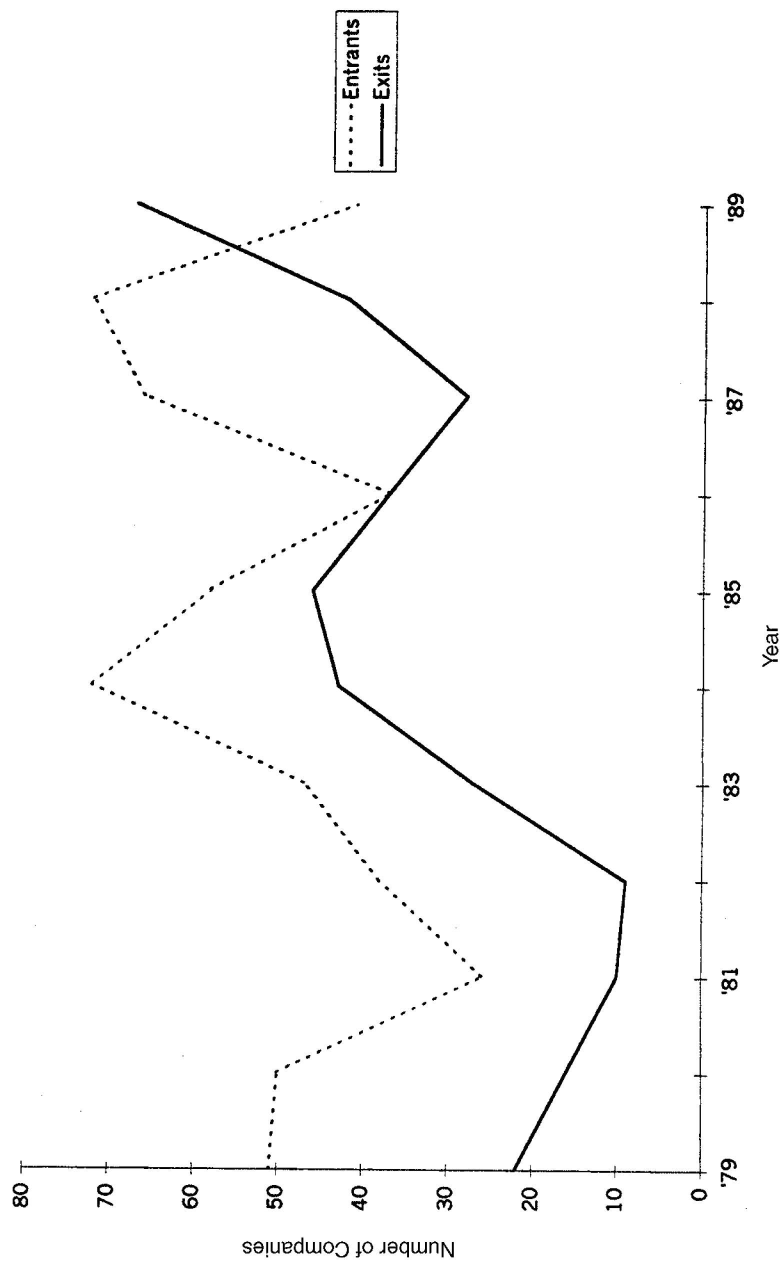
The model presented in Section 3, "Competition and Profits," assumes an equal division of the market among insurers.

[39] See Stern [100]. Nelson [85], analyzing data for 1957 through 1967, notes that the number of exits is correlated with the combined ratio with a lag of one year.

[40] Meidan [73, pg. 395], who calls this a "market challenger strategy," notes that it "is characterized by the aggressiveness of the marketing tactics. Typically insurers that follow this strategy are ambitiously trying to grow as fast as they can."

FIGURE 1

INSURANCE COMPANY ENTRIES AND EXITS



Suppose, instead, that there are ten firms: one has 50% of the market, eight have 6% of the market, and one has 2% of the market. Also assume that the appropriate discount rate is 10% per annum. Let us restore the ideal assumptions for a moment: if any firm cuts prices, it immediately attracts all consumers. Moreover, if any firm cuts prices, its competitors reduce their prices to marginal cost.

The large firm presently earns 50% of the industry's economic profits. If current pricing continues, it will earn this amount in perpetuity. Using the notation of Section 3, where E^m is annual economic profits at the market price and v is the discount rate, the present value of this profit stream is $(50\%)(E^m)(1 + v + v^2 + \cdots)$. This equals $5.5 \times E^m$ at a discount rate of 10%. If the insurer cuts prices slightly, it earns a bit below E^m in the current year, but no economic profits in all future years. The large firm has an incentive to continue its present pricing strategy.

Now consider the firm with only 2% of the market. It now earns 2% of the industry's economic profits. If conditions do not change, it will earn this amount in perpetuity. The present value of its profit stream is $(2\%)(E^m)(1 + v + v^2 + \cdots)$, or $0.22 \times E^m$ at a 10% discount rate. If it cuts prices slightly, it earns much more than this in the current year. The small but aggressive firm has a strong incentive to cut prices.[41]

Realistically, of course, the small insurer will not instantly capture the entire market with a small price reduction. Most policyholders are loyal to their current insurers, and they often ig-

[41] Harrington and Danzon [54] suggest that the aggressive marketing strategy of small firms may result from an inability to avoid the "winner's curse." In competitive bidding among suppliers, a firm which provides unbiased bids will generally win only when its offered price is too low. When its offered price is too high, another supplier will generally win. Harrington and Danzon differentiate between established and inexperienced firms: "... established firms in stable markets have learned to make formal or informal adjustments to their loss forecasts in order to avoid the curse. The availability of information from agents and brokers also may facilitate this process .... Inexperienced firms may use nonoptimal forecasts, placing too much emphasis on their own information or drawing incorrect inferences from the actions of other firms."

nore competitors' rates at renewal time. The small firm's rate decrease would slowly increase its market share: say, 10% a year. Although substantial, the gain is not overwhelming.

The large insurer expects different outcomes. A carrier with 50% of the market may have already saturated its target customer populations. Even if it desires to grow rapidly, there are few new insureds for it to attract. The large firm's rate reduction may increase its market share only 1% a year.

Rival Responses

Competitive responses to rate cuts by a small firm or a large firm also differ, particularly in insurance. Premium rates vary by classification, territory, type of coverage, and similar dimensions. Rate comparisons can be an exhausting task, especially when the classification schemes of the insurers differ. Thus, carriers do not monitor premium rates of small companies. In Personal Auto insurance, insurers analyze the rates charged by State Farm, Allstate, and a handful of other large carriers. The premiums charged by smaller insurers are revealed only in industry-wide accounting statistics. Actual rates, although publicly available in rate filings, are rarely examined.

Moreover, rivals do not react swiftly to rate cuts by small insurers. If a firm with 1% of the market has a 10% growth in business, and the new business is drawn evenly from its rivals, then the other firms suffer only a 0.1% decrease in volume. If an insurer with 50% of the market has the same growth, its rivals lose 10% of their business.

Thus, when rates are high, small insurers are tempted to cut prices aggressively.[42] Their actions may not be noticed, re-

[42]Anderson and Formisano [5], in a study of six insurance failures between 1975 and 1985, found rapid premium growth, expansion to other states, and inadequate pricing to be three of the most significant causes of the insolvencies. For instance, in the years preceding the insolvencies, Reliable Insurance Company and All-Star Insurance Company had premium growth of over 50% per annum. Wisconsin Surety Company expanded

sponses of rivals will be delayed, they may increase market share rapidly, and their revenues will climb. Large insurers, however, have less incentive to reduce rates. Their market shares increase more slowly, their actions are quickly noticed, competitors respond swiftly, and the premium lost on existing business may exceed the premium gained on new insureds.

The incentive for an incumbent insurer to reduce rates depends on the expected profits in its renewal book of business. Renewal business is generally more profitable than new business, and insurers strive to maintain policyholder loyalty. An incumbent insurer may reduce its own rates to avoid the loss of profitable renewal business to a competitor.

The profitable phase of the underwriting cycle is in disequilibrium. Some firms enjoy current earnings, others aggressively seek to grow, and entrants clamor to join the industry.

Competitive Strategies

Profits influence business strategies. As the profitable phase of the underwriting cycle continues, more firms ignore short term income and seek growth. For simplicity, let us differentiate strategies between (a) aggressive growth and (b) price maintenance. Assume that at time t, $w\%$ of firms emphasize aggressive growth and $(100 - w)\%$ of firms emphasize price maintenance.

The change in w depends upon the sign and magnitude of economic profits, labeled p here. The greater the economic profits and the longer the economic profits are expected to persist, the more firms will seek aggressive growth.[43]

---

from 2 states to 13 states in 6 years, and Eastern Indemnity Corp. expanded from 1 state to 34 states in 5 years. The aggressive marketing strategies of these insurers eventually led to their failures. As Anderson and Formisano comment (page 460): "rapid growth . . . can realistically only be accomplished by pricing below cost and taking an unreasonable proportion of poor risks." Similarly, Best's [12, pg. 39] notes that "approximately 81% of all insolvencies occurred in companies experiencing unusual growth trends, which we defined as growth outside industry norms of 5% to 25%."

[43] Actuaries are tempted to express such relationships as partial derivative equations. We might say that the partial derivatives of w with respect to both t and p are positive.

This price maintenance strategy is not sustainable. If your rivals are cutting prices and gaining market share, you must either respond or disappear. But the optimal response depends on the number of firms reducing rates. If the percentage of firms aggressively seeking market share is small, then it is reasonable to hold prices above marginal cost. The high level of policyholder loyalty to the insurer means that insurance market share growth is a slow process. For instance, suppose that 10% of firms are aggressively cutting rates, or $w = 10\%$. (For simplicity, assume that firms are of equal size, so 10% of firms means 10% of the market.) If such discounts provide a 10% annual growth in market share, then these firms will have 11% of the market after a year's time, and their rivals will remain with 89% of the market. The maintenance of high prices has led to a 1% reduction in market share—a small loss compared to current profits.

If 50% of firms are aggressively reducing prices, the outcome changes. The same 10% market share growth for these firms reduces their rivals' portion from 50% to 45%. Short term profits do not offset a 10% loss of business.

The Nadir of the Cycle

How might one respond? Following rates downward is no remedy. The insurance industry has thousands of firms, a competitive structure, and invitingly easy entry conditions. Expected profits would be extremely low if prices were left purely to market pressures.

Indeed, premium rates do not drop slowly when the cycle heads downward. Rather, prices cascade downward, to well below marginal cost. Industry Annual Statement operating

---

In truth, we lack information about expected profitability (and about expected duration of profitability), and we lack good information about business strategies. Mathematical expressions give an aura of empirical precision that is not warranted.

Perhaps one day we will have empirical data on the causes of underwriting cycles. We do not have such data, and we do not pretend to have such data. This data provides an intuitive understanding of underwriting cycles, based on types of market structures and competitive strategies found in other industries.

income was negative in 1975 and again in 1984–85. Moreover, the reported operating ratios conceal the true severity of underwriting cycles, for several reasons:

- First, accounting data does not include a "reasonable profit" margin, although the economist's marginal cost does. For instance, a 2% accounting return on equity is a severe economic loss.

- Second, most insurers desire steady earnings, particularly if their financial statements are scrutinized by government regulators or by stockholders. Insurers tend to under-reserve during poor years, thereby increasing net income. Conversely, when profits improve, insurers strengthen reserves of prior years, dampening their reported earnings.

  It is difficult to quantify these effects, since the "reasonable insurance profit margin" is much disputed and reserve strengthening and weakening is difficult to quantify. Nevertheless, rates were surely below marginal cost during 1974 and 1983 (in addition to 1975 and 1984–85).

- Third, the severity of the cycle differs by line. General Liability rates, for example, were below marginal cost in 1982 and perhaps in 1981 also. In other words, an accurate analysis of income adjusted for reserve changes by line of business with a reasonable profit provision shows severe price inadequacies for several years in a row.

To recapitulate: during profitable years, there are incentives for small firms to aggressively seek market share and for new firms to enter the insurance industry. The lack of product differentiation, the positive cash flow from insurance operations, and the ease of entry would normally reduce or eliminate profits from the industry.

Yet total consumer demand for insurance is inelastic with respect to price. The difficulty of price comparisons and consumer loyalty to insurers provide a large potential profit margin.

The deciding factor is business strategy. If firms aggressively seek market share by cutting rates, profits decline for all insurers. Formal agreements to maintain high prices are not sustainable in an industry as competitive as insurance. Rather, small firms and new entrants may be dissuaded from pursuing overly aggressive strategies by the competitive reactions of incumbent insurers.

Thus, the downward rate spiral is not a reflection of simple competitive pricing. Rather, it is a competitive response to aggressive strategies. By temporarily cutting rates below marginal cost, incumbent insurers hope to persuade more aggressive but short-sighted firms to modify their objectives from market share to profitability.

Changing Strategies

Indeed, as operating profitability decreases, overly aggressive insurers begin to rethink their strategy. First, low prices no longer attract additional consumers, since even the major firms have cut rates. Second, if profits remain negative, all firms suffer.

The changes in insurer strategies are revealed in the insurance trade press and trade conferences. As the cycle deepens, laments on the evils of price cutting become frequent, and exhortations to refrain from the unprofitable pursuit of premium abound. These public proclamations are disavowals of aggressive intentions. Insurers say: "We renounce the use of rate reductions to gain market share, for we see the folly of our ways."

We can model the change in strategy as follows. As the trough of the underwriting cycle continues, more firms renounce market share gains and seek profitable business. The larger the expected losses, and the longer the duration of the expected losses, the more the firms emphasize increased profitability.

Industry Discipline

When the cycle turns up, insurers who previously engaged in competitive "warfare" seem to raise rates in unison. Politicians, consumer activists, and the legal community suspect antitrust violations. But there is no collusion, no intercompany agreements, and only a general knowledge of competitors' intentions.[44]

Rather, the change in behavior reflects the change in strategy. The public exhortations during the trough of the cycle are not accompanied by rate increases. Each insurer knows that if it raises prices unilaterally, it will lose business, not return to profitability. In fact, most insurers always knew that severe rate cutting is destructive to the industry. The public statements are intended to persuade other firms to cease overly aggressive behavior. They are not explanations of any firm's current actions.[45]

Consider again the formal model. If economic profits are sufficiently negative long enough, most firms will have shifted their emphasis from market share growth to maintaining profitable rates. Yet a high price maintenance strategy is profitable only if all or most firms in the industry follow this path. Indeed, after two or three years of pricing below marginal cost, most firms are committed to writing profitable business. But how does one move from a low price situation to a high price situation?

Market Leaders

In a highly competitive and fragmented industry like insurance, firms cannot easily monitor the actions, much less the strategies, of their rivals. They need a barometer of industry feelings.

---

[44]See, for instance, the class action antitrust complaint in Van de Kamp [108] and an industry response by the Insurance Information Institute [56].

[45]Compare Porter [93, pg. 81]: "It is not uncommon for competitors to comment on industry conditions.... Such commentary is laden with signals.... As such, this discussion can be a conscious or unconscious attempt to get other firms to operate under the same assumptions and thereby minimize the chance of mistaken motives and warfare. Such commentary can also contain implicit pleas for price discipline: 'Price competition is still very harsh. The industry is doing a lousy job of passing along increased costs to the consumer.' 'The problem in this industry is that some firms do not recognize that these current prices will be detrimental to our ability to grow and produce a quality product in the long run.' Or discussions of the industry may contain ... implicit promises to cooperate if others act 'properly.'" [The quotations are from the president of the Sherwin-Williams Coating Group and from an executive of a leading commodities producer.]

Rate filings make dull newsprint. "The XYZ Insurance Company has requested a 5.1% rate increase in Arizona for Bodily Injury coverage, 4.3% for Property Damage,...." Who would ever read such details?

The *National Underwriter* periodically records State Farm's rate filings (often only State Farm's filings) in various jurisdictions. State Farm is the market leader and low cost carrier in Personal Lines coverages. It serves as the barometer of industry movement through the underwriting cycle.[46] By examining and following State Farm's actions, other firms maintain a close grasp on industry price movements, even if they lack the resources to monitor competitive rates on their own.

When other carriers see State Farm raising rates, they know that firm strategies have shifted sufficiently to allow maintenance of high prices. Insurers follow (or sometimes even anticipate) the market leader in the various jurisdictions, leading to the good years of the cycle.

In the Commercial Lines, there is no clear market leader. The major Commercial Lines insurers, such as Travelers, Hartford, CNA, AIG, and Liberty Mutual, have relatively small countrywide market shares. Other carriers do not follow AIG's General Liability rates the way they examine State Farm's Personal Auto rates. Consequently, the industry trade press rarely mentions Commercial Lines rate actions.[47]

---

[46]Moreover, State Farm has a sophisticated monitoring system to analyze the rate actions of its peer companies. Not only do State Farm's rates affect a large percentage of the insured population, but they also reflect of the strategies of other carriers.

[47]Personal Lines risks are manually rated, so State Farm's rate manual is an accurate reflection of marketplace prices. Large Commercial Lines risks may be loss rated, composite rated, schedule rated, or retrospectively rated. The rate manual is but a crude guide to actual prices. In fact, many General Liability classifications are "A-rated," so there are no manual rates to examine.

In the Personal Lines, price changes are effected by rate filings. In the Commercial Lines, prices may also be changed by varying schedule rating credits and debits, by modifying the premium payment pattern, by changing policyholder dividend plans, and by similar "non-manual" methods. Thus, rate comparisons are more difficult in the Commercial Lines of business.

Rating Bureaus

Rather, Commercial Lines insurance strategies are revealed by deviations from bureau rates or bureau loss costs. The National Council on Compensation Insurance (NCCI), and state bureaus in certain jurisdictions (e.g., California, Massachusetts, Minnesota, New York, Pennsylvania), provide loss costs for all Workers Compensation classifications. Similarly, the Insurance Services Office (ISO) provides loss cost data for the other Commercial Lines. Most insurers use NCCI or ISO rates as a benchmark, and file rate deviations or independent rates with state insurance departments.

After several years of unprofitable operations, insurers know that the industry is ready to increase rates. ISO (or another rating bureau) provides the catalyst. When private insurers follow ISO loss costs, without seeking major deviations, firms know that the industry is committed to profitable rates. The individual carriers may then curtail schedule rating credits and other price modifications, confident that their rivals are doing the same.

Profits encourage aggressive rate cutting. After one or two good years, insurer strategies begin emphasizing market share growth, and new firms are attracted to the industry. The cycle begins anew, in perpetual disequilibrium.

6. PUBLIC POLICY

As each cycle rolls through the industry, insurers ponder: "What determines the severity and frequency of underwriting cycles? What lines of business are most subject to them? When will the cycle turn? How do state regulation and statutes influence cycles?" It is time to answer these questions.

Policyholder Loyalty and Price Elasticity

The beckoning of profits leads the cycle. Why drive rates down if you cannot recoup the losses later? Firms would prefer to price at marginal cost rather than lose money over the long term.

Periods of high prices are sustainable only if consumers do not reduce their purchases of the good and do not switch to rival suppliers. In other words, the price elasticity of demand must be low enough that consumer demand will not drop substantially when suppliers raise prices.

Removing statutory requirements for Personal Automobile and Workers Compensation insurance, and curtailing judicial awards in commercial liability cases, might increase the price elasticity of demand for insurance. But the statutory insurance requirements help the victims of motor vehicle and workplace accidents. The benefits they provide outweigh the disadvantages of premium rate fluctuations.

The unpredictability of jury awards in commercial liability cases provides little social benefit, and the harm to society extends beyond insurance availability and rate fluctuation concerns. Unfortunately, the limited success of tort reform efforts in the 1980s and early 1990s highlights the intractability of this problem. To restate this: the trial bar is a powerful interest group that opposes tort reform. The results of the pervasive attorney involvement in insurance claims are bloated insurance costs and the redistribution of wealth from citizens to a particular profession (AIRAC [2; 3]). More volatile underwriting cycles are simply an additional side-effect.

Policyholder loyalty results from the difficulty of price comparisons. Personal Lines policyholders may be unaware of price slashing by competing insurers, since they rarely price shop at renewal. An insurer can maintain high prices for a short period without a major loss of market share when its competitors begin cutting rates.

Price increases, however, encourage insureds to seek better rates elsewhere. Unilateral price increases cause a loss of market share, as consumers switch to rival carriers. Industry-wide price increases are easier to sustain, since consumers cannot do better elsewhere in the marketplace. Thus, the descent to the trough of the cycle is precipitated by a small group of insurers, but the return to profitability is a uniform movement.

Greater consumer price information would reduce loyalty to the current insurer and mitigate the severity of underwriting cycles.[48] Firms would not be able to sustain high prices in the face of competitive price cutting without rapidly losing market share. Prices closer to cost would prevail over the duration of the underwriting cycle.

Life insurance regulation demonstrates the difficulty of providing price comparisons. The NAIC Life Insurance Solicitation Model Regulation requires that insurers illustrate surrender cost and net payment cost indices for 10 and 20 year durations, but few consumers examine these numbers (Black and Skipper [15]). Such comparisons are difficult, and few individuals expend the effort to understand them.

The same is true for Property/Casualty insurance. Consumers do not forgo price comparisons because the information is not available. Rather, the information is not available because the price comparisons are so distasteful.

Underwriting Cycles by Line

The history of underwriting cycles in America illustrates these relationships (see Figures 2 and 3). During the 1960s and 1970s, underwriting cycles were most pronounced for Personal Automobile and Workers Compensation insurance.[49] In the 1980s,

---

[48]Numerous studies have recommended that states make insurance price information accessible to consumers; see Virginia Bureau of Insurance [114], recommendation #5, or NAIC [81, pp. 440–441].

[49]See Stewart [101, Exhibits 5-3, 5-4, and 5-8 on pp. 290, 291, and 295]. Note how the cycles in automobile insurance mirrored those for the industry as a whole, whereas General Liability showed no clear pattern until the late 1970s. Similarly, Best's [12, pg. 33] notes that "while the majority of insolvencies during the 1970s occurred in personal lines companies, commercial lines companies accounted for the majority in the 1980s."

FIGURE 2

UNDERWRITING CYCLES
(BEST'S AGGREGATES AND AVERAGES)

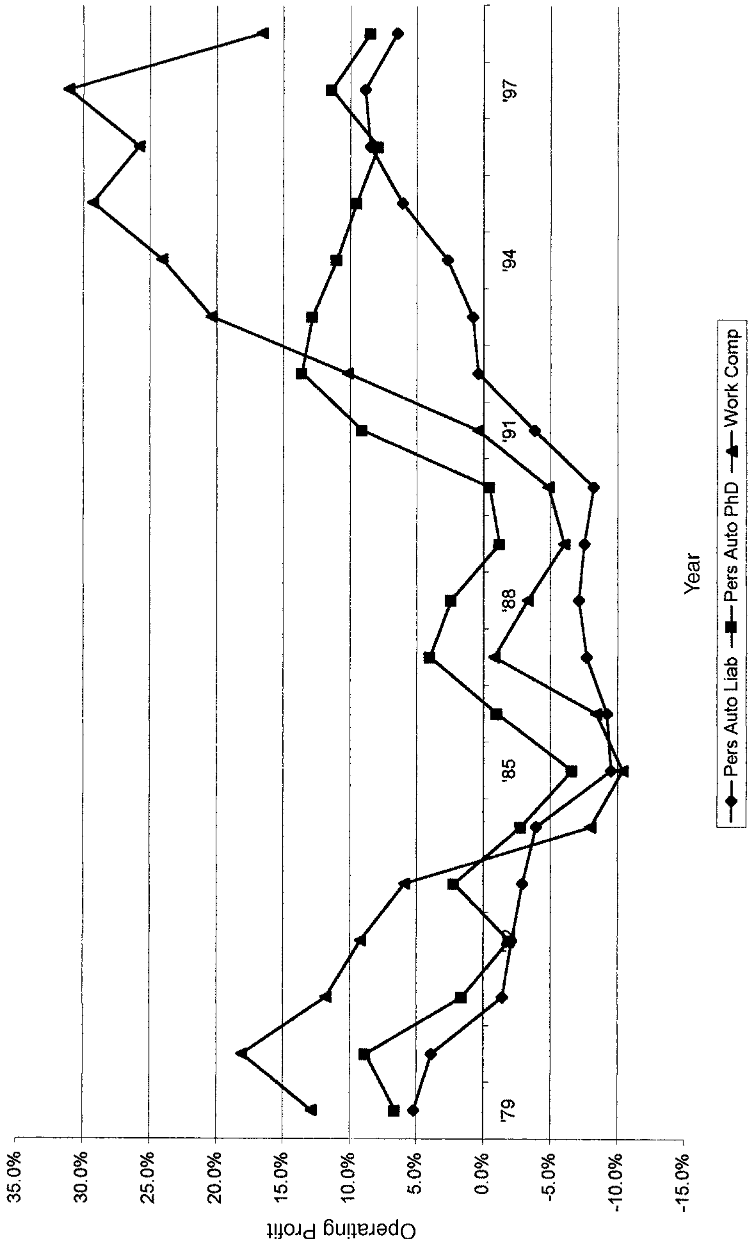
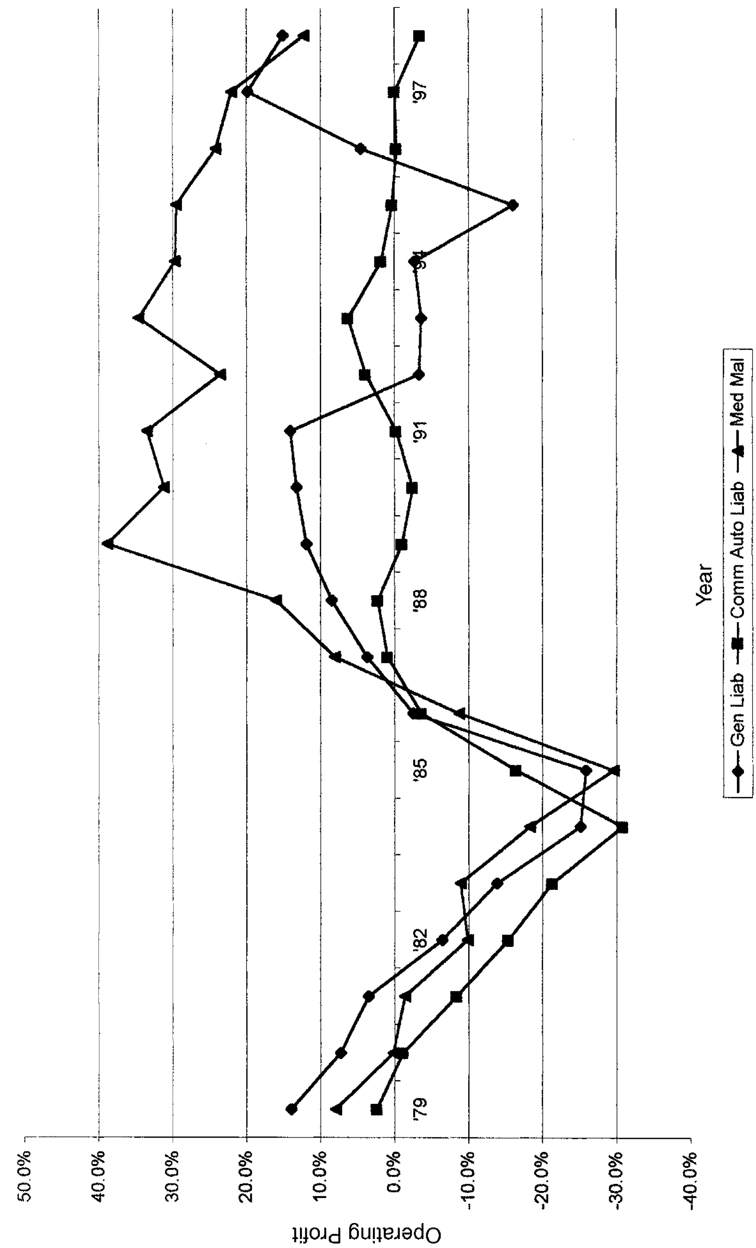


FIGURE 3

UNDERWRITING CYCLES
(BEST'S AGGREGATES AND AVERAGES)



General Liability and other Commercial Liability lines showed the greatest fluctuations in profitability: negative in 1981–1984 and highly positive in 1986–1988.

This difference is influenced by demand elasticities and barriers to entry. Personal Automobile and Workers Compensation insurance are statutorily mandated by Financial Responsibility or compulsory insurance laws. Price elasticity of demand is low.[50]

The opposite was true for General Liability until the 1970s. Believing that they had little exposure to liability hazards, many small businesses declined to purchase the coverage. Large corporations often used other risk management techniques, such as self-funding and captives.

In the 1950s and 1960s, many Personal Lines insurers used bureau rates, either as actual rates or as a baseline for pricing. By the 1980s, the low cost direct writers, such as State Farm and Allstate, had garnered most of the Personal Lines market. The efficient distribution systems of these insurers formed strong barriers to entry or expansion by other firms.

The opposite course has characterized the Commercial Liability lines of business. The major direct writers do not dominate these markets. Moreover, the lengthening tails in these lines and the rising interest rates in the 1970s increased the disparity between bureau rates and marginal cost.

---

[50]On the low price elasticity of demand, see Sherdan [99, pg. 58]; Bloom [16]; and Strain [103, pg. 448]. Strain summarizes the influences on elasticity as "The greater the tendency for the public to buy an insurance coverage without the need for sales stimulation (as to comply with financial responsibility laws, or workmen's compensation acts, or mortgage protection requirements), the more inelastic the demand for insurance." Financial Responsibility laws require a driver involved in a motor vehicle accident either to show evidence of insurance or to post a court bond (Morill [77]; Mehr and Cammack [72, pp. 308–329]; Bickelhaupt [13, pp. 646–678]). Employers must provide Workers Compensation insurance, with minor exceptions that are relating to farm employment, household work, or businesses with few workers. Employers that are financially strong enough to self-insure may provide the statutory benefits on their own. For history and detail, see Myers [79, pp. 884–900], Kulp and Hall [64, pp. 191–250], and Chamber of Commerce [25]. Many states allow group self-insurance (NAIC [82]). This increases the price elasticity of demand, since consumers have another risk management technique.

The high costs of Workers Compensation insurance in the late 1980s, exacerbated by large residual market loads in many jurisdictions, led many employers to alternative risk management techniques, such as group self-insurance and large dollar deductible policies. Price elasticity of demand increased, and a uniform increase in price would drive the better risks from the insurance market. So Workers Compensation remained unprofitable through the 1980s, until the state legislative reforms and the managed care revolution of the 1990s lowered loss costs without necessitating large rate increases.

Regulation and Social Developments

Changes in state regulation may influence underwriting cycles. During the 1960s and early 1970s, many states moved from prior approval regulation to open competition laws.[51] Competitive rating laws allow more freedom for private insurers to vary premium rates in attempts to gain market share or increase profits.

The 1980s and 1990s show ambiguous trends. California adopted prior approval regulation in November 1988, with the passage of Proposition 103, and consumer groups in other states are pushing similar legislation. Meanwhile, the low cost direct writers are driving agency companies out of the Personal Lines market. Tighter governmental regulation and increasing market concentration may dampen the severity of Personal Automobile underwriting cycles.[52]

Social developments in the 1980s and 1990s have had the opposite effect on the Commercial Liability lines. The expansion of tort law doctrines, and the increasing unpredictability of jury awards, have made coverage essential even for small firms. State

[51] See NAIC [83, pg. 310]: "It is the sense of the Subcommittee . . . that . . . reliance be placed upon fair and open competition to produce and maintain reasonable and competitive prices for insurance coverages . . . ." See also DOJ [41]).

[52] Compare Eley [43, pg. 187]: "If the likelihood of extraordinary profits during hard markets is removed, the willingness of insurers to give away insurance during soft markets will evaporate."

regulation is less restrictive, since commercial insureds can fend for themselves and do not need the governmental protection that ordinary citizens require. The major rating bureaus, such as ISO and NCCI, have changed from advisory rates to loss costs in most jurisdictions, and may soon be further transformed into quasi-consulting organizations. Commercial lines rate and form deregulation is possible in the early years of the 21st century.

Consequently, General Liability promises potential profits for the discerning insurer.[53] In the late 1970s, insurers complained vociferously about rising and unjustified liability awards. The criticism was correct: the American legal system encourages lawsuits and the redistribution of wealth from the public to the trial bar. But a secondary effect of these complaints was to impress upon businesses the need for liability coverage.

Numerous suppliers—major carriers, small firms, and new entrants—joined the fray, and insurers began positioning themselves (that is, cutting prices to build market share) for the anticipated profits. The aggressive competition threatened to eliminate the foreseen returns.

So General Liability entered the trough of a severe underwriting cycle, with firms slashing rates well below cost. The consequences were striking: when rates rose in 1985, there was an almost complete absence of aggressive price cutting.[54]

---

[53]This promise may prove illusory. Insurers who provided CGL coverage in the 1960s and 1970s are now facing enormous asbestos, pollution, and products liability litigation (Hamilton and Routman [52]; Manta and Welge [69]). Nevertheless, the potential is alluring.

[54]The power of underwriting cycles is often misunderstood. Much of the American legal community and the business public concluded that the dramatic and uniform rise in Commercial Liability insurance rates must be the result of collusion. Yet no evidence of such behavior could be found. In fact, collusion is nearly impossible in the fragmented insurance market.

Even the Attorneys General's antitrust complaint was confined to allegations of boycott in policy form development, statistical support, and coverage exclusions. Pricing in concert is never mentioned (Van de Kamp [108]). The California Attorney General's office explains that pricing in concert is protected by the McCarran-Ferguson Act and so was not contested. An alternative explanation is that the Commercial Liability insurance rate increases were characterized not by pricing in concert but by the competition driving the underwriting cycle.

And the cycle continues. The aggressive competition that precipitated the rise in rates in the mid-1980s led to price cutting a few years later. The waning influence of rating bureaus and administered pricing systems in the fragmented insurance market will lead to even more severe swings in premiums.

7. CONCLUSION

Underwriting cycles are a means of maintaining long-term profits, not a random occurrence that removes them. Insurance underwriting cycles are the display of competitive pricing in a free marketplace. To optimize the results of their companies, pricing actuaries must learn to adapt their rate setting techniques to the phases of the underwriting cycle.

REFERENCES AND BIBLIOGRAPHY

[1] AICPA, Insurance Companies Committee, Auditing Standards Division, "Discounting Claims of Insurance Enterprises," Draft Issues Paper, March 1983.

[2] All-Industry Research Advisory Council, *Attorney Involvement in Auto Injury Claims*, Oak Brook, IL: AIRAC, 1988.

[3] All-Industry Research Advisory Council, *Compensation for Automobile Injuries in the United States*, Oak Brook, IL: AIRAC, March 1989.

[4] Almagro, Manuel, and Thomas L. Ghezzi, "Federal Income Taxes—Provisions Affecting Property/Casualty Insurers," *PCAS* LXXV, 1988, pp. 95–161.

[5] Anderson, Dan R., and Roger A. Formisano, "Causal Factors in P-L Insolvency," *Journal of Insurance Regulation*, Volume 6, No. 4 (June 1988), pp. 449–461.

[6] Appel, David, and James Gerofsky, "Regulating Competition: The Case of Workers Compensation Insurance," *Journal of Insurance Regulation*, Volume 3, No. 4 (June 1985), pp. 408–425.

[7] Appel, David, and James Gerofsky, "Regulating Competition: The Case of Workers Compensation Insurance," *NCCI Digest*, Volume 2, Issue 3 (September 1987), pp. 1–18.

[8] Balcarek, Rafal J., "The Capital Investment Market and the Insurance Industry," *PCAS* LV, 1968, pp. 186–195.

[9] Banfield, Carole, "P/C Insurer Profitability: ISO Paid Claims Data," *Journal of Insurance Regulation*, Volume 5, No. 2 (December 1986), pp. 268–276.

[10] Bailey, Robert A., "A Review of the Little Report on Rates of Return in the Property and Liability Insurance Industry," *PCAS* LVI, 1969, pp. 133–162.

[11] Berger, Lawrence, "A Model of the Underwriting Cycle in the Property and Casualty Insurance Industry," *Journal of Risk and Insurance*, Volume 55 (1988).

[12] *Best's Insolvency Study: Property/Casualty Insurers 1969–1990*, Oldwick, NJ: A. M. Best Company, June 1991.

[13] Bickelhaupt, David L., *General Insurance*, Tenth Edition, Homewood, IL: Richard D. Irwin, Inc., 1979.

[14] Bierwag, G. O., George G. Kaufman, and Alden Toevs, "Duration: Its Development and Use in Bond Portfolio Management," *Financial Analysts Journal*, July–August 1983.

[15] Black, Kenneth, Jr., and Harold D. Skipper, Jr., *Life Insurance*, Twelfth Edition, Englewood Cliffs, NJ: Prentice-Hall, 1994.

[16] Bloom, Thomas S., "Cycles and Solutions," *Best's Review: Property/Casualty Edition*, Volume 86, No. 6 (October 1987), pp. 20 ff.

[17] Boor, Joseph, "A Macroeconomic View of the Insurance Marketplace," CAS Exam 5 Study Note, 1999A.

[18] Boor, Joseph, "The Impact of the Insurance Economic Cycle on Insurance Pricing," CAS Exam 5 Study Note, 1999B.

[19] Braithwaite, Paul, "P/C Profitability: The Long-Term View," *Journal of Insurance Regulation*, Volume 6, No. 1 (September 1987), pp. 96–104.

[20] Brock, Gerald W., "The Computer Industry," in Walter Adams (ed.), *The Structure of American Industry*, New York: MacMillan Publishing Company, 1986, pp. 239–260.

[21] Butsic, Robert P., "Determining the Proper Interest Rate for Loss Reserve Discounting: An Economic Approach," *Evaluating Insurance Company Liabilities*, Casualty Actuarial Society Discussion Paper Program, 1988, pp. 147–188.

[22] Butsic, Robert P., "The Effect of Inflation on Losses and Premiums for Property-Liability Insurers," *Inflation Implications for Property-Casualty Insurance*, Casualty Actuarial Society Discussion Paper Program, 1981, pp. 51–102.

[23] Butsic, Robert P., "The Underwriting Cycle: A Necessary Evil?" *The Actuarial Digest*, Volume 8, No. 2 (April/May 1989).

[24] Carlson, Thomas O., "Rate Regulation and the Casualty Actuary," *PCAS* XXXVIII, 1951, pp. 9–72; discussions by H. J. Ginsburgh, pp. 222–223; Allie V. Resony, pp. 223–225; Dudley M. Pruitt, pp. 225–230; R. W. Griffith, pp. 230–236; author's review of discussions, pp. 236–239.

[25] Chamber of Commerce of the United States, *Analysis of Workers' Compensation Laws*, Washington, D.C.: U.S. Chamber of Commerce, 1990.

[26] Cholnoky, Thomas V., and Jeffrey Cohen, *Property/Casualty 10K Loss Reserve Disclosures*, Goldman Sachs, May 31, 1989.

[27] Conning & Co., *New Business versus Renewals: The Cost of New Business in a Soft Market*, Hartford, CT: Conning & Co., June 1988.

[28] Conning and Company, *The Real Property & Casualty Cycle: Behind the Reported Results*, Conning & Co., December 1987.

[29] Countryman, Gary L., "Open Competition Rating for Workers' Compensation," *Journal of Insurance Regulation*, Volume 1, No. 1 (September 1982), pp. 9–22.

[30] Cummins, J. David, and Scott E. Harrington, "Property-Liability Insurance Rate Regulation: Estimation of Underwriting Betas Using Quarterly Profit Data," *The Journal of Risk and Insurance*, Volume 52, No. 1 (March 1985), pp. 16–43.

[31] Cummins, J. David, Scott E. Harrington, and Robert W. Klein, "Cycles and Crises," *Best's Review: Property/Casualty Edition*, Volume 92, No. 9 (January 1992), pp. 15 ff.

[32] Cummins, J. David, Scott E. Harrington, and Robert W. Klein, "Cycles and Crises in Property/Casualty Insurance: Causes and Implications for Public Policy," *Journal of Insurance Regulation*, Volume 10, No. 1 (Fall 1991), pp. 50–93.

[33] Cummins, J. David, Scott E. Harrington, and Robert W. Klein (eds.), *Cycles and Crises in Property/Casualty Insurance: Causes and Implications for Public Policy*, National Association of Insurance Commissioners, 1991.

[34] Cummins, J. David, and J. François Outreville, "An International Analysis of Underwriting Cycles in Property-Liability Insurance," *Journal of Risk and Insurance*, Volume 54, Number 2 (June 1987), pp. 246–252.

[35] Cummins, J. David, and David J. Nye, "Inflation and Property-Liability Insurance," in John D. Long and Everett D. Randall (eds.) *Issues in Insurance*, Volume I, Third Edition, Malvern, PA: American Institute for Property and Liability Underwriters, 1984.

[36] Danzon, Patricia Munch, "Rating Bureaus in U.S. Property Liability Insurance Markets: Anti or Pro-competitive?" *The Geneva Papers on Risk and Insurance*, Vol 8, No. 29 (October 1983), pp. 371–402.

[37] D'Arcy, Stephen P., "Application of Economic Theories of Regulation to the Property-Liability Insurance Industry," *Journal of Risk and Insurance*, Volume 7, No. 1 (September 1988), pp. 19–51.

[38] D'Arcy, Stephen P., and Neil A. Doherty, *The Financial Theory of Pricing Property-Liability Insurance Contracts*, Homewood, IL: Richard D. Irwin, Inc., 1988.

[39] D'Arcy, Stephen P., and James R. Garven, "Property-Liability Insurance Pricing Models: An Empirical Evaluation," *Journal of Risk and Insurance*, Volume 57, No. 3 (September 1990), pp. 391–419.

[40] Daykin, Chris D., Teivo Pentikäinen, and M. Pesonen, *Practical Risk Theory for Actuaries*, First Edition, London: Chapman and Hall, 1994.

[41] Department of Justice, "Report to the Task Force on Antitrust Immunities," *The Pricing and Marketing of Insurance*, January 1977.

[42] Douglas, Evan J., *Intermediate Microeconomic Analysis: Theory and Applications*, Englewood Cliffs, NJ: Prentice-Hall, Inc., 1982.

[43] Eley, David F., "Investment Income in Ratemaking: An Alternative to the Cycle," *Journal of Insurance Regulation*, Volume 5, No. 2 (December 1986), pp. 180–188.

[44] Ezekiel, Mordechai, "The Cobweb Theorem," *Quarterly Journal of Economics* (February 1938), pp. 255–280; reprinted in Haberler, Gottfried, (ed.), *Readings in Business Cycle Theory*, Philadelphia: The Blakiston Company, 1944, pp. 422–442.

[45] Fairley, William, "Investment Income and Profit Margins in Property-Liability Insurance: Theory and Empirical Results," *The Bell Journal of Economics* 10 (Spring 1979), pp. 192–210; reprinted in J. David Cummins and Scott E. Harrington (eds.), *Fair Rate of Return in Property-Liability Insurance*, Boston: Kluwer-Nijhoff Publishing, 1987, pp. 1–26.

[46] Feldblum, Sholom, "Asset Liability Matching for Property/Casualty Insurers," *Valuation Issues*, Casualty Actuarial Society Discussion Paper Program, 1989, pp. 117–154.

[47] Fox, Eugene H., "Automobile Insurance Analysis: The Twenty-Five and Under Age Driver vs. the Twenty-Six and Over Age Driver," *CPCU Journal*, Volume 33, No. 1 (March 1980), pp. 22–25.

[48] Friedman, James, *Oligopoly Theory*, Cambridge: Cambridge University Press, 1983.

[49] Fudenberg, Drew, and Jean Tirole, "Noncooperative Game Theory for Industrial Organization: An Introduction and Overview," in Richard Schmalensee and Robert D. Willig (eds.), *Handbook of Industrial Organization*, North-Holland: Elsevier Science Publishing Co., 1989, Volume I, pp. 159–328.

[50] Gleeson, Owen M., and Gerald I. Lenrow, "An Analysis of the Impact of the Tax Reform Act on the Property/Casualty Industry," in *Financial Analysis of Insurance Companies*, Casualty Actuarial Society Discussion Paper Program, 1987, pp. 119–190.

[51] Greene, Winfield W., and William F. Roeber, "The 'Permanent' Rate Making Method Adopted by the National Council on Compensation Insurance," *PCAS* XII, 1925–26, pp. 253–267.

[52] Hamilton, Thomas M., and Eric L. Routman, "Cleaning Up America: Superfund and Its Impact on the Insurance Industry," *CPCU Journal*, Volume 41, No. 3 (September 1988), pp. 172–184.

[53] Harrington, Scott, "The Impact of Insurance Regulation on Prices and Underwriting Results in the Property-Liability Insurance Industry: A Survey," *Journal of Risk and Insurance*, Volume 51, No. 4 (December 1984), pp. 577–623.

[54] Harrington, Scott E., and Patricia M. Danzon, "Price Cutting and Premium Volatility in Liability Insurance Markets," in J. David Cummins, Scott E. Harrington, and Robert W. Klein (eds.), *Cycles and Crises in Property/Casualty Insurance: Causes and Implications for Public Policy*, National Association of Insurance Commissioners, 1991.

[55] Hill, Raymond, "Profit Regulation in Property-Liability Insurance," *The Bell Journal of Economics*, Vol. 10, No. 1 (Spring 1979), pp. 172–191.

[56] Insurance Information Institute, *Insurance Antitrust Litigation: The Conspiracy Theory and Related Policy Issues*, April 1988.

[57] Insurance Services Office, *Changes in the Economic Environment: Insurance Implications*, October 1991.

[58] Joskow, Paul L., "Cartels, Competition, and Regulation in the Property-Liability Insurance Industry," *The Bell Journal of Economics and Management Science*, Volume 4, No. 2 (Autumn 1973), pp. 375–427.

[59] Joskow, Paul, "Competition and Regulation in the Property/Casualty Insurance Industry," *Report of the Economic Advisory Panel to the National Commission for the Review of Antitrust Laws and Procedures*, pp. 207–227.

[60] Kahane, Yehuda, "Generation of Investable Funds and the Portfolio Behavior of the Non-Life Insurers," *Journal of Risk and Insurance*, Volume 45, No. 1 (March 1978), pp. 65–77.

[61] Klein, Robert, "Competition in Private Passenger Automobile Insurance," A Report to the NAIC Personal Lines (C) Committee, September 1989.

[62] Klein, Robert W., "Competitive Rating in Workers' Compensation Insurance: Michigan's Experience," *Journal of Insurance Regulation*, Volume 5, No. 1 (September 1986), pp. 79–108.

[63] Kozik, Thomas J., "Underwriting Betas—The Shadows of Ghosts," *PCAS* LXXXI, 1994, pp. 303–329.

[64] Kulp, C. A., and John W. Hall, *Casualty Insurance*, Fourth Edition, New York: John Wiley & Sons, 1968.

[65] Longley-Cook, Laurence H., "Early Actuarial Studies in the Field of Property and Liability Insurance," *PCAS* LI, 1964, pp. 140–146.

[66] Longley-Cook, Laurence H., "Underwriting Profit in Fire Bureau Rates," *PCAS* LII, 1966, pp. 305–311.

[67] Lowe, Stephen P., "A New Performance Measure for P/C Insurers," *Emphasis*, Summer 1988, pp. 8–11.

[68] Mahler, Howard C., "An Introduction to Underwriting Profit Models," *PCAS* LXXI, 1987, pp. 239–277.

[69] Manta, Joseph G., and Mark A. Welge, *Toxic Tort—Environmental Hazards Litigation Workshop*, Philadelphia, 1990.

[70] Matison, Raymond A., "Capital Cornucopia," *Best's Review: Property/Casualty Edition*, Volume 88, No. 7 (November 1987), pp. 26 ff.

[71] McGee, Robert T., "The Cycle in Property/Casualty Insurance," *Federal Reserve Bank of New York Quarterly Review*, Autumn 1986, pp. 22–30.

[72] Mehr, Robert I., and Emerson Cammack, *Principles of Insurance*, Seventh Edition, Homewood, IL: Richard D. Irwin, Inc., 1980.

[73] Meidan, Arthur, "Marketing Strategies, Organization and Performance Control in Insurance," *Journal of Risk and Insurance*, Volume 49, No. 3 (September 1982), pp. 388–404.

[74] Meyerson, Allen L., "Ensuring the Solvency of Property and Liability Insurance Companies," in Spencer L. Kimball and Herbert S. Denenberg (eds.), *Insurance, Government, and Social Policy*, Homewood, IL: Richard D. Irwin, Inc., 1969, pp. 146–190.

[75] Miccolis, Robert S., "On the Theory of Increased Limits and Excess of Loss Pricing," *PCAS* LXIV, 1977, pg. 27.

[76] Milgrom, Paul, and John Roberts, "Limit Pricing and Entry Under Incomplete Information: An Equilibrium Analysis," *Econometrica*, Volume 50 (1982), pp. 443–460.

[77] Morrill, Thomas C., "Motorists' Financial Responsibility Laws," in John D. Long and Davis W. Gregg (eds.), *Property and Liability Insurance Handbook*, Homewood, IL: Richard D. Irwin, 1965, pp. 576–599.

[78] Mowbray, Albert H., and S. B. Black, "On the Relation of Accident Frequency to Business Activity," *PCAS* II, 1915–1916, pp. 418–426.

[79] Myers, Robert J., *Social Security*, Third Edition, Homewood, IL: Richard D. Irwin, Inc., 1985.

[80] Myers, Stewart C., and Richard A. Cohn, "A Discounted Cash Flow Approach to Property-Liability Insurance Rate Regulation," in Cummins and Harrington (eds.), *Fair Rate of Return in Property-Liability Insurance*, Boston: Kluwer-Nijhoff Publishing, 1987, pp. 55–78.

[81] National Association of Insurance Commissioners, "Report of the Advisory Committee on Competitive Rating to the National Association of Insurance Commissioners," *Proceedings of the National Association of Insurance Commissioners*, 1980, Volume II, pp. 440–441.

[82] National Association of Insurance Commissioners Study Committee Report, "Regulating Workers' Compensation Groups," in John D. Long and Everett D. Randall (eds.), *Issues in Insurance*, Volume II, Third Edition, Malvern, PA: The American Institute for Property and Liability Insurance, 1984, pp. 1–110.

[83] National Association of Insurance Commissioners, *Proceedings*, 1969, pg. 310.

[84] Nash, John F., Jr., "Noncooperative Games," *Annals of Mathematics*, Volume 45 (1951), pp. 286–295.

[85] Nelson, Robert E., "Property-Liability Company Exits," *Journal of Risk and Insurance*, Volume 38, No. 3 (September 1971), pp. 357–366.

[86] Outreville, J. François, "Underwriting Cycles and Rate Regulation in Automobile Insurance Markets," *Journal of Insurance Regulation*, Volume 9, No. 2 (December 1990), pp. 274–286.

[87] Panning, William H., "Asset-Liability Matching for a Going Concern," in Edward I. Altman and Irwin T. Vanderhoof (eds.), *The Financial Dynamics of the Insurance Industry*, New York: Irwin, 1995, pp. 257–292.

[88] Pentikäinen, Bonsdorff, Pesonen, Rantala, and Ruohonen, *Insurance Solvency and Financial Strength*, Helsinki, Finland: Finnish Insurance Training and Publishing Co., 1989.

[89] Plotkin, Irving H., *Prices and Profits in the Property and Liability Insurance Industry*, report to the American Insurance Association, Cambridge, MA: Arthur D. Little, Inc., November 1967.

[90] Plotkin, Irving H., "Rates of Return in the Property and Liability Insurance Industry: A Comparative Analysis," *Journal of Risk and Insurance*, Volume 36, No 3 (June 1969), pp. 173–200.

[91] Plotkin, Irving H., *Studies on the Profitability, Industrial Structure, Finance, and Solvency of the Property and Liability Insurance Industry*, Cambridge, MA: Arthur D. Little, 1970.

[92] Plotkin, Irving H., "Total Rate of Return and the Regulation of Insurance Profits," *Total Return Due a Property/Casualty Insurance Company*, Casualty Actuarial Society Discussion Paper Program, 1979, pp. 206–245.

[93] Porter, Michael, *Competitive Strategy*, New York: The Free Press, 1980.

[94] Redington, F. M., "Review of the Principles of Life-Office Valuations," *Journal of the Institute of Actuaries*, Volume 18 (1952), pp. 286–340.

[95] Resony, Allie V., "Discussion of Thomas O. Carlson's 'Rate Regulation and the Casualty Actuary,'" *PCAS* XXXVIII, 1951, pp. 223–225.

[96] Ryan, Kevin M., and Richard I. Fein, "A Forecast for Workers Compensation," *NCCI Digest*, Volume III, Issue IV (December 1988).

[97] Scherer, Frederick M., *Industrial Market Structure and Economic Performance*, Second Edition, Boston: Houghton Mifflin Company, 1980.

[98] Shapiro, Carl, "Theories of Oligopoly Behavior," in Richard Schmalensee and Robert D. Willig (eds.), *Handbook of Industrial Organization*, North-Holland: Elsevier Science Publishing Co., 1989, Volume I, pp. 329–414.

[99] Sherdan, William A., "An Analysis of the Determinants of Demand for Automobile Insurance," *The Journal of Risk and Insurance*, Volume 51, No. 1 (March 1984), pp. 49–62.

[100] Stern, Lawrence J., "Corporate Changes 1989," *Best's Review: Property/Casualty Edition*, Volume 90, No. 11 (March 1990), pp. 20–22.

[101] Stewart, Barbara D., "Profit Cycles in Property-Liability Insurance," in John D. Long and Everett D. Randall (eds.), *Issues in Insurance*, Volume I, Third Edition, Malvern, PA: American Institute for Property and Liability Underwriters, 1984, pp. 273–334.

[102] Stone, James M., "A Theory of Capacity and the Insurance of Catastrophe Risks," *The Journal of Risk and Insurance*, Part I in Volume 40, Number 2 (June 1973), pp. 231–243, and Part II in Volume 40, Number 3 (September 1973), pp. 339–355.

[103] Strain, Robert W., "Insurance Pricing and Its Role in Relation to Economic Theory and Marketing Management," *Journal of Risk and Insurance*, Volume 33, No. 3 (September 1966), pp. 447–457.

[104] Tarbell, Thomas F., "The Effect of Changes in Values on Casualty Insurance," *PCAS* XIX, Part 1, 1932–33, pp. 1–12.

[105] Taylor, Gregory C., "Underwriting Cycles," Coopers and Lybrand, 1991.

[106] Tennyson, Sharon, "The Effect of Rate Regulation on Underwriting Cycles," *CPCU Journal*, Volume 44, No. 1 (March 1991), pp. 33–45.

[107] Tirole, Jean, *The Theory of Industrial Organization*, Cambridge, MA: The MIT Press, 1988.

[108] Van de Kamp, John K., et al., *The State of California versus Hartford Fire Insurance Company, et al.*, The United States District Court for the Northern District of California, March 1988.

[109] Varian, Hal R., *Intermediate Microeconomics: A Modern Approach*, New York: W. W. Norton and Company, 1987.

[110] Varian, Hal R., *Microeconomic Analysis*, Second Edition, New York: W. W. Norton and Company, 1984.

[111] Venezian, Emilio C., "Ratemaking Methods and Profit Cycles in Property and Liability Insurance," *Journal of Risk and Insurance*, Volume 52, Number 3 (September 1985), pp. 477–500.

[112] Victor, Richard A., "Major Challenges Facing Workers' Compensation Systems in the 1990s," in Richard A. Victor (ed.), *Challenges for the 1990s*, Cambridge, MA: Workers Compensation Research Institute, July 1990, pp. 9–23.

[113] Victor, Richard B., and Charles A. Fleischman, *How Choice of Provider and Recessions Affect Medical Costs in Workers' Compensation*, Cambridge, MA: Workers Compensation Research Institute, 1990.

[114] Virginia Bureau of Insurance, State Corporation Commission, *Competition in the Property and Casualty Insurance Industry: An Evaluation of Alternative Methods of Rate Regulation*, Richmond, Virginia: January 1978.

[115] Waterson, Michael, *Economic Theory of the Industry*, Cambridge: Cambridge University Press, 1984.

[116] Woll, Richard G., "Insurance Profits: Keeping Score," *Financial Analysis of Insurance Companies*, Casualty Actuarial Society Discussion Paper Program, 1987, pp. 446–533.

[117] The Workers' Compensation Rating and Inspection Bureau of Massachusetts, *1977 Rate Filing*, "Expense Cash Flow" section.

IS THE EFFICIENT FRONTIER EFFICIENT?

WILLIAM C. SCHEEL, WILLIAM J. BLATCHER,
GERALD S. KIRSCHNER, AND JOHN J. DENMAN*

Abstract

The paper defines plausible ways to measure sampling error within efficient frontiers, particularly when they are derived using dynamic financial analysis (DFA). The properties of an efficient surface are measured both using historical segments of data and using bootstrap samples. The surface was found to be diverse, and the composition of asset portfolios for points on the efficient surface was highly variable.

The paper traces performance of on-frontier and off-frontier investment portfolios for different historical periods. There was no clear cut superiority to the on-frontier set of portfolios, although lower risk-return on-frontier portfolios were generally found to perform better relative to comparable, off-frontier portfolios than those at higher risk levels. It is questionable whether practical deployment of optimization methods can occur in the presence of both high sampling error and the relatively inconsistent historical performance of on-frontier portfolios.

The implications of this paper for DFA usage of efficient frontiers is that sampling error may degrade the ability to effectively distinguish optimal and non-optimal points in risk-return space. The analyst should be cautious regarding the likelihood that points on an efficient frontier are operationally superior choices within that

*William C. Scheel, Ph.D., DFA Technologies, LLC; William J. Blatcher, AEGIS Insurance Services, Inc.; Gerald S. Kirschner, FCAS, MAAA, Classic Solutions Risk Management, Inc.; and John J. Denman, AEGIS Insurance Services, Inc.

space. There are many possible frontiers that optimally fit different empirical samples. Sampling error among them could cause the frontiers to traverse different regions within risk-return space, perhaps at points that are disparate in a decision sense. What is an efficient point on one frontier may be inefficient when calculated from a different sample. The paper finds the use of an efficient surface to be helpful in diagnosing the effects of such sampling error.

ACKNOWLEDGEMENT

The authors gratefully acknowledge the very constructive comments of reviewers of the paper.

1. INTRODUCTION

Companies choose among investments often with the purpose of optimizing some goal and always limited by constraints. Assets are divided among competing investment alternatives with the hope that risk will be minimized for a desired level of return, either investment return or overall return. When the allocation fulfills the goals within the boundaries of constraints, it is thought to be efficient. The allocation is deemed to be a member of the efficient set at a point on an efficient frontier. It is efficient because it dominates off-frontier, interior points in the risk-return space.

This paper investigates this popular investment allocation strategy in two ways. First, it seeks to determine what the sensitivity of the frontier is to possible sampling error in risk-return space. Second, both on-frontier and off-frontier portfolio allocations for actual series of returns are tracked for their respective performance. We begin with an apologue; it gives the reader both a rationale and definition of what we mean by an efficient surface.

1.1. A Sampling Error Apologue

I walk into a casino with shaky knees and a rather small stake. Betting doesn't come easily for me, and I expect to lose the stake. Ralph told me I would lose it. But I have a bevy of information gleaned from experiments Ralph did with a computerized simulation of a craps table. One of the items I call "knowledge" is the efficient surface he made for me. Ralph said it would help me understand the risk-return properties of the craps table and guide me in allocating my stake among the various bets that I can make.

"There are many bets you can make at the table," Ralph explained, "'Come,' 'Big-8' and lots of others. I think of the gaming as a multivariate process. Of course, it has probabilities that are objective and can be measured. Do you want me to figure out the combinatorics of the craps game and derive analytic solutions for optimal bet placement? My consulting fee might be a bit high because the math will take awhile, but I could do it."

I mentally recalculated my meager stake and replied, "Is there a less expensive way?"

Ralph shrugged and said, "Sure. I can use a computer simulation I have and take a sample of game outcomes. I'll use the sample to empirically develop a covariance matrix for some of the bets. Then, I'll figure out which combinations of bets have minimum variance for a particular payoff. You can choose which risk-return profile of bets is best for you. You'll be able to allocate your stake more efficiently. By the way, this is called an efficient frontier—it gives a profile of bets that are expected to produce a given return with minimum variance. I'll do a sample of 25 games each with a combination of various bets. This will keep the cost down."

"Well, okay," I replied, "but will this single efficient frontier really work?"

"What do you mean, 'single frontier'?" he asked.

"What if the sample your computer simulation comes up with is unusual?" Ralph scratched his head, and I continued, "You measure this thing you call a sample covariance matrix. But what if you took a different sample? You'd get a different sample covariance matrix, right?"

"Yes."

"And it might be different?"

"Yes. Even materially different."

"So, your efficient frontier (EF) is subject to sampling error—it was empirically derived from the sample of only 25 games." I then asked, "What if you had a second sample of 25 games and did another mathematical optimization. So we now have 2 different EFs; both do the same thing, but the answers are different. Which one do I use when I walk into the casino?"

Ralph exclaimed, "I'll take a sample, and then another, and another. Each will have a different EF. Then, I'll plot each point of the samples' EFs in risk-return space. I'll count the number of times the various EFs traverse a particular cell in that space. Maybe 10 EFs traverse the cell at the coordinates (10,15). Maybe only 3 EFs traverse the cell at (1,3). Don't you see? Just by counting the number of times the sample EFs traverse a region in risk-return space and normalizing the count to probabilities, I can measure an efficient surface."

I asked, "Why is the surface important?"

Ralph was now animated. He leaped to his feet. "Because, if the various sample EFs all traversed the same cells, the EFs would all be the same—there would no sampling error. What if the surface is spread out? Suppose some sectors of it are relatively flat? Then the efficiency of the EFs varies. Would you prefer to pick a point on the surface (with a particular combination of bets) that appears most often among different EFs? Probably you would. You want the surface to be tightly peaked.

In three dimensions, that's a ridge or very pointy hill; in two dimensions, it is a probability distribution with little variance."

He then went home to begin the chore of sampling and constructing an efficient surface for me. I began to think, "A single efficient frontier is measured from data. We often think of the data being a sample from a replicable experiment. If a sample of dice games is observed, the n-tuple bet outcomes for the correlated bets are the empirical data source for an optimization. It is easy to see how different samples can be drawn when talking about dice games. But the world of security returns is different from a craps table. What is a sample there? What is the meaning of sampling error, and how might it affect the way I measure efficient frontiers? Would an EF for securities really be efficient?"

These are important questions—ones addressed in this paper. It is difficult to think of how we'd repeat an experiment involving security returns. Is a series of experiments one that uses different historical periods of returns? Is it a bootstrap of a broad segment of history? These are the two approaches that are equivalent to sampling and measuring sampling error. The result of our measurements is an efficient surface.

1.2. Roadmap for the Paper

Section 2 of the paper lays the groundwork for measuring sampling error that affects efficient frontier measurement. We examine two approaches that seem particularly useful for dynamic financial analysis. We also review the literature relating to EF efficiency. Section 3 introduces the notion of an *efficient surface*—this is a construct for understanding and measuring sampling error in EFs. In this section we describe the methodology and data set used in our study.

The main body of results is presented in Sections 4, 5 and 6. We measure forecast performance of efficient frontiers in Section 4. We are particularly concerned about the performance of off-frontier portfolios. Are they really inefficient? Do on-frontier portfolios dominate performance, as we might anticipate given that they are billed as "efficient"? The evidence we present in Section 4 shows instability in EFs derived both with historical segments and bootstrap samples. This leads us to conclude later that caution should be exercised when using efficient frontiers in DFA analysis.

On the road to this conclusion, we closely examine the efficient surface in Section 5. It portrays sampling error from two different perspectives—historical and bootstrap sampling. The efficient surface is a useful construct for visualizing sampling error in EFs. We observe that such error is particularly large in the high risk-return regions of the surface. This observation is reinforced in Section 6 by observing the diversity of portfolio composition as we compare different historical segments.

The final section is devoted to conclusions and cautions on the use of EFs in DFA work. We conclude that EFs may not warrant the term *efficient.* Their best use may be as advisory measurements concerning the properties of risk-return space.

2. SCENARIO GENERATION IN DFA

Dynamic financial analysis involves scenario generation. There are many types of scenarios that are simulated so that the model builder can measure a hypothetical state-of-the-world with accounting metrics. Asset generators typically create returns for invested assets. They model exogenous economic conditions. Each modeler sees the forces of the financial markets unfolding according to a set of rules. The rule set is almost as diverse as the number of modelers.

Some DFA model builders prefer stochastic differential equations with various degrees of functional interrelatedness. The transition of returns over time, as well as the correlations among different asset components, always are represented in multiple simultaneous equations. Other DFA modelers use multivariate Normal models, which conjecture a covariance matrix of investment returns. These models do not have time-dependent transition modeling information. Such an efficient frontier, by definition, has no time transition properties. A sample taken from *any* sub-period within the time series would contain sampling error, but otherwise, the investment allocation would be unaffected.

Both approaches begin with a single instance of reality. They both purport to model it. One approach, stochastic equations, uses largely subjective methods to parameterize the process.[1] Another approach to modeling clings to assumptions that seem to be or are taken to be realistic.[2] Both produce scenarios that are deemed sufficiently similar to reality to represent it for the purpose at hand.

The efficient frontier calculation can be a constrained optimization either based on a sample from a historic series of returns or a derived series with smoothing or other ad hoc adjustment. Alternatively, an EF may be created from simulated DFA results. Both the efficient frontier and DFA asset-based modeling are using the same set of beliefs regarding the manner by which statistically acceptable parameters are used.[3] They both start with a single historic time series of returns for various component assets.

2.1. Two Viewpoints on the Use of Efficient Frontiers

The practitioner has a straightforward objective: define investment allocation strategy going forward. Today's portfolio allocation leads to tomorrow's result. The portfolio is then rebalanced relative to expectations. The new one leads to new results. The

[1] The calibration may depend on examination of stylistic facts, but there seldom is formalized, statistical hypothesis testing to judge whether the facts can be accepted as such or whether the representation of these facts in the model is really a scientific determination.

[2] Some models use multivariate Normal simulation for rendering investment returns for consecutive periods. There usually is an assumption that the covariance matrix used for multivariate Normal simulation is stationary from period to period in these models.

[3] DFA and optimization do have a critical junction. Some DFA modelers believe they understand time dependencies within period-to-period rates of return. EF attempts to optimize *expected* return. If there is a time dependence conjectured, it should be factored into the expected returns used to build the EF for any period.

cycle repeats. Where does the chicken end and the egg begin? In practice, the practitioner has only one instance of yesterday's reality and tomorrow's expectations from which to construct a portfolio and a model.

There are at least two approaches to using a DFA model to define an investment allocation. In one, a DFA analyst might set up an initial allocation of assets using an efficient frontier obtained from quadratic optimization on a prior historical period. A DFA model would be run repeatedly—a different state-of-the-world would ensue each time, and a different reading obtained for the metric. These simulations produce endpoints in the modeled risk-return space. In this approach, one beginning asset allocation leads to many different observations about endpoints. The reason they are different is that, although each starts with the same state, the model simulates various outcomes. Each hypothetical one probably leads to a different endpoint for the planning horizon.

But, another viewpoint exists.[4] We refer to it as the hybrid approach. Suppose that history serves a valid purpose in calibrating a model but should not be used to define a beginning allocation. In this viewpoint, the investment mix *is suggested by the optimizer*. DFA serves only to measure what could happen with some hypothetical starting allocation.

The optimizer deals the cards in this deck, and DFA traces where the cards lead.[5,6] The optimizer, not the modeler, submits

---

[4]Correnti, et al. review an approach similar to the hybrid model described here.

[5]The optimizer posits a trial solution; it consists of a certain portfolio allocation. This trial allocation does not depend on any prior allocation of assets. Rebalancing that ensues during the optimization period (and under the control of the DFA model) also is unknown to the optimizer. The objective value that is returned by the model is driven by the initial trial solution and model outputs that build on the trial solution.

[6]Investment rates are forecasted by the DFA model, which might use multivariate Normal simulation. There may be an overlap between what the optimizer uses and what the DFA model uses. For example, the covariance matrix used for the multivariate Normal simulation is estimated from historical data and generally is assumed to be stationary during the forecast period. It is used both by the optimizer and by the DFA model.

an initial allocation for review. In this hybrid approach, there is no initial portfolio based on optimization using prior history. In the hybrid model, the optimizer finds a portfolio, which leads to an ex post optimal result. The metric used in this optimization is part of the DFA model—it is calculated by the accounting methodology of the model as it generates future states of the world. It may be difficult to reconcile the use of efficient frontiers for investments within hybrid-DFA modeling that, on the one hand, believes there is a historically dependent component that can be used for calibration, but rejects the use of data to define a starting portfolio. Yet, on the other hand, simulations of that model are derived to construct an efficient frontier. It may appear as though history has been rejected as information for the purposes of decision making, yet *indirectly* it is used to represent the future. The starting portfolio in the hybrid approach is based at least indirectly on modeling and should represent an analyst's expectations. These expectations are in theory built into the model for return scenario generation, and that model was calibrated to history in some fashion.

In DFA work, a performance metric is chosen. This metric is measured within a risk-return space. The metric must be measurable according to the chosen accounting framework. Risk might be variance, semi-variance or some chance-constrained function of the metric. In the real world, the corporate manager is rewarded for favorable performance of the metric and often penalized by unwanted risk in the metric. The volume of investment in various stochastic components affects a metric's performance. The operational question is how should an allocation be made to investments so that performance of the metric is optimized.

In the forecast period, the modeler generates a scenario of unfolding rates of return using, say, a multivariate, time-dependent asset model. An example would be any of the multi-factor mean reversion models in use today. The simulated progression of returns for a scenario generated by one of these models is affected by an underlying mechanism that forces unusual deviations in the path back towards an expected trajectory of returns. The DFA model typically ties in some way the business operations to the simulated economic environment.[7] This economic scenario typically generates other economic rates, such as the rate of inflation. A scenario that is generated by the economic model is taken to be exogenous; it is mingled with expectations about corporate performance. The company's *operations* are tied to the exogenous influences of the economic scenario.

In the end, this modeling process is repeated many times for the optimizer in the hybrid model. The optimizer requires an answer to the question: Given an initial investment allocation, what is the end-horizon performance of the metric? The optimizer forces the model to measure the result of a simulation experiment given only an initial investment allocation. The model takes the allocation and produces an experimental point in risk-return space. All that is required of the model is its ability to measure the trajectory of the metric within the company's business plan and a beginning allocation of assets. In this regard, the hybrid model is using a sort of dynamic programming approach to optimization. The possible outcomes are considered, and the most desirable traced back to the inputs (initial allocation). The hope is that the optimized feasible set is robust relative to possible stochastic outcomes in the model trajectory. The efficient frontier traces the allocations necessary to achieve various points in this risk-return space. All of this raises the thorny question of subsequent performance dominance of the on-frontier portfolios in the hybrid model. Do EF points truly dominate the performance of off-frontier frontier points—portfolios that are thought to be inefficient and have higher risk for the same return level?

The reason that this is a hybrid approach is that DFA modeling is not deployed on an optimal asset allocation derived directly from the prior time series. Rather, DFA is combined with

[7] A typical behavioral pattern for business growth is modeling it as a function of inflation, which was generated by the economic scenario. Another is to tie severity in claims to underlying inflation as unfolded in the economic model simulation.

optimization to answer the single question: How should the portfolio be immediately rebalanced to achieve an optimal point in risk-return space over the future DFA planning horizon?

Two portfolios can be devised through optimization procedures—one is based on historical results prior to the start of the simulated future time periods. Another one involves allocations that are selected and tried by the optimizer—the DFA model is integral to this second approach. The latter hybrid optimization uses DFA-measured metrics in the optimizer goal function. If applied over the course of the simulated future time periods, and according to the plan of the DFA model, the hybrid approach would seem to yield optimal results at the end of the simulated time horizon. There is no reason to suppose that these two approaches produce the same initial portfolios. Which one is the real optimum?

During the planning horizon, the hybrid model may ignore imperfections that, in real life, might have (and probably would have) been dealt with by ongoing decision making. The EF could have been recalculated with realized data and the portfolio rebalanced. The published state-of-the art in DFA modeling is unclear in this regard; but it may be that no intra-period portfolio optimization is done by DFA models between the time the analysis starts with an allocation posited by the optimizer and when it ends, say, five years later with a DFA-derived metric. It is inconceivable that an organization would mechanically cling to an initial, EF-optimal result for an operational period of this length without retesting the waters.[8]

---

[8]There is no reason other than a few computational programming complexities why intra-period optimizations cannot be done within DFA models. The question is whether they are, or they are not, being done. For example, the DFA model can simulate a wide variety of rebalancing strategies including the real-life one that involves a rebalancing trigger for simulated portfolios whose allocation has deviated from a recent EF by some amount. Mulvey, et al. [1998, p. 160] describe an *n*-period simulation wherein such rebalancing is triggered. In addition, Mulvey, et al. describe the use of optimization constraints in a clever way to achieve an integration of strategic, long-term optimization with short-term tactical objectives. However, a DFA model that allows intra-period optimization must also capture the transaction and tax costs associated with the intra-period rebalancing and re-

2.2. *Limitations of this Study for Use of the Efficient Frontier in DFA*

We do not do a complete DFA analysis—there is neither a liability component nor a conventional DFA metric such as economic value of a business enterprise. Rather, the data are limited entirely to marketable, financial assets. Nevertheless, we believe our findings are of value to DFA work. If the efficient frontier produced solely within a traditional investment framework has unstable properties, these instabilities will apply to its use in DFA work were it to be calculated and used in a similar way.

2.3. *Other Investigations of the Efficacy of EF Analysis*

Michaud has extensively investigated the use of EFs with particular regard to general efficacy for forecasting. For example, he has shown [1998, pp. 115–126] that inclusion of pension liabilities can substantially alter the statistical characteristics of mean-variance (MV) optimization for investment portfolios.

Michaud's book [1998] examines efficient frontiers both with respect to their inherent uncertainty and what might be done to improve their worthiness. He suggests that the effects of sampling error may be improved using a methodology described as a *resampled efficient frontier*. The motivation for some kind of improvement over classical EFs is that "…optimized portfolios are 'error maximized' and often have little, if any, reliable investment value. Indeed, an equally weighted portfolio may often be substantially closer to true MV optimality than an optimized portfolio." [Michaud, 1998, p. 3].

The determination of a resampled efficient frontier is complex; Michaud has patented it. Although his book exposes the core of the method that he believes improves on forecast error, there is no empirical evidence provided in the book that a resampled efficient frontier has this desirable effect. Interested

---

optimization. See Rowland and Conde [1996] regarding the influence of tax policy on optimal portfolios and the desirability of longer term planning horizons.

readers are directed to his book. The concept of an efficient surface espoused in our paper is built on different constructs. We will readdress the important work of Michaud at a later point in the paper. We now turn to the definition and measurement of an efficient surface.

3. THE EFFICIENT SURFACE

An efficient frontier consists of points within risk-return space that have minimum risk for a return. If there were a time-stationary, multivariate probability distribution for prior history, then history is a sample from it. History, therefore, would have sampling error.[9]

The concept of a conditional marginal probability distribution either for return or risk emerges, and it, too, would have sampling error. We discuss the properties of this marginal distribution, an equi-return slice of the efficient surface, in Section 5.1.

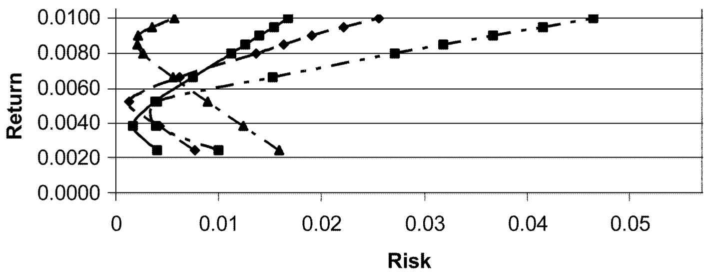
Were the instance of reality to be a sample, what is the sampling error?

Figure 1 shows efficient frontiers for random 5-year blocks of history. The EFs were derived from monthly returns beginning in January 1988. Each curve in Figure 1 requires optimizations for a 5-year history of returns. The block of monthly returns was picked at random from the entire time series. The points along each EF are obtained from separate passes through the data with the optimizer. On each pass, one of the constraints differs. That constraint is the requirement that the average portfolio return be a specified value in the return domain. The optimizer's objective function is the minimization of variance associated with that portfolio expected return.

[9]If there were conjecture, the multivariate distribution would be subjective, and the efficient frontier would be the subjective frontier. A subjectively derived EF has no sampling error, but it may lose *operational* appeal when represented in this manner, because subjectivity requires difficult reconciliation within a corporate, decision-making framework.

FIGURE 1

COMPARISON OF EFFICIENT FRONTIERS FOR DIFFERENT TIME PERIODS



Each EF in Figure 1 consists of nine points; each point involves a separate quadratic optimization. For example, one of the optimization constraints is the portfolio expected return, which is set to an equality condition. There were nine different expected returns used in the study; one was a monthly return of 0.004. An examination of the figure at this value shows a point for each of the four EFs. An empirically derived covariance matrix was determined for each of the four time series illustrated in Figure 1 as well as for hundreds of others that are not shown. The juxtaposition of the EFs displays a tangle of overlapping, crisscrossing curves.[10] This illustration can be viewed as sampling with replacement from a historical sample; it is appropriate, then, to view the figure as illustrative of a probability surface. It is a surface showing the extent of sampling error provided there has been a stationary, multivariate distribution of components' returns.[11] Figure 1 indicates that it may be hazardous to

[10]Some segments of EFs such as those shown in Figure 1 can be indeterminate. This is because the quadratic optimizer could not identify a feasible set of investment alternatives for all of the average returns chosen in the analysis. There is a small probability of overlap of data because the 5-year blocks of returns used for each EF could have overlapping sub-periods of time.

[11]The population distribution is unknown, but it is estimated from the historical record by calculation of an empirical covariance matrix for each historical block.

accept any particular segment of history as the "best estimator." This figure shows only several of the EF curves that build up an efficient surface. Examples of efficient surfaces appear later in Figures 8 and 10. The distribution of risk in a cross-sectional slice of this efficient surface also is reviewed in Section 5.

The positions and slopes of the EFs in Figure 1 are wildly different, and were other historical EFs to be included, the complexity would be greater. This lack of historical stability casts doubt on the operational validity of a particular efficient portfolio actually producing optimal performance. The figure also hints that off-frontier portfolios may perform as well as or better than on-frontier portfolios. We examine this question of forecast reliability in detail in Section 4.

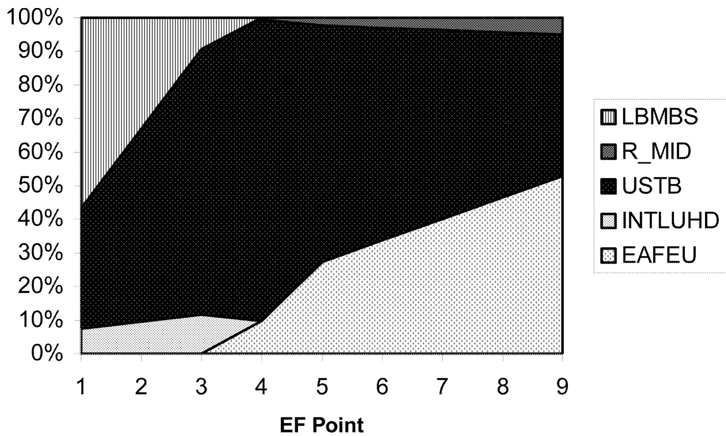
In addition to the positional changes in EFs over time, there is dramatic change in portfolio composition along the curve of each EF in Figure 1. Examples of the change in portfolio composition for EFs appear in Figures 2a and 2b. Each chart is categorical—a tic mark on the x-axis is associated with one of nine optimization points. Each chart shows a stacked area rendering of the proportion of an asset component within the efficient set. If the reader views either Figure 2a or 2b from left to right, the unfolding change, and possible collapse, of a particular component is illustrated. This type of chart is a useful way to show a component's contribution to the efficient set moving along the EF from low risk-return to high risk-return portfolios.

There is faint hope that the two different EF portfolio compositions shown in Figures 2a and 2b will operationally produce the same result when put in practice—were this to be a reasonable representation of the effects of sampling error, the operational use of efficient frontiers would be questionable; sampling error swamps operational usefulness and forecast responsiveness.

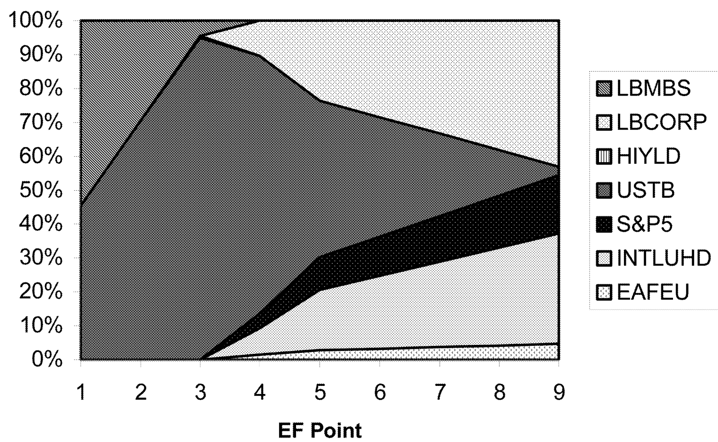
However, another illustration, Figure 3, indicates that if history is a sample from a multivariate distribution, there should be optimism that the efficient frontier evolves slowly, at least measured in monthly metrics. This figure shows EFs calculated

FIGURE 2

Portfolio Composition for Different Efficient Frontiers. (a) Composition A, (b) Composition B.



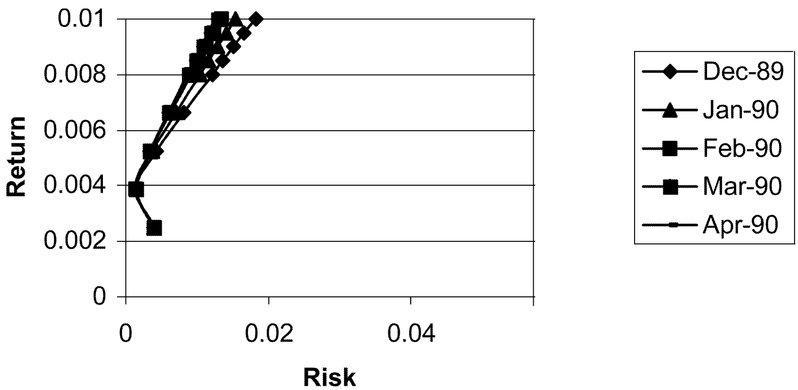
(a)



(b)

FIGURE 3

EFFICIENT FRONTIERS FOR CONSECUTIVE TIME PERIODS



from consecutive, overlapping historical blocks of time. In this case, the time interval between between consecutive EFs is one month. The stability deteriorates fastest at higher risk-return levels. The result was found to hold for a wide variety of consecutive historical blocks starting at various points since 1977. This stability may provide an operational basis for investing in an on-frontier portfolio and seeing its performance prevail over off-frontier portfolios, at least for relatively short planning horizons.

There are other ways to use the historical record. The paper shortly will turn to the use of the bootstrap as a method of measuring sampling error. First, the data and manipulation methods are described in more detail.

3.1. Data Manipulation

This study uses the time series described in Appendix A: Review of Data Sources. Except where gaps were present in the historical record, the portfolio returns are actual.[12,13]

---

[12]The data represent returns for a selected group of investment components. There was no attempt to filter or smooth the time series in any way. However, a few gaps in the historical record were interpolated.

The data were used in two ways: (1) bootstrap samples were made from the original time series in an attempt to approximate sampling error phenomena, and (2) various historical series of the data were used for performance analysis. The study examines period segmentation and the performance of efficient and inefficient portfolios for different forecast durations.

3.1.1. Historical Performance Analysis

In this section of the paper, data for an efficient frontier are extracted for a historical period and used to evaluate the efficient frontier. The on-frontier portfolios are minimum variance portfolios found using quadratic programming.[14] Off-frontier portfolios also were calculated.[15] The study is concerned with whether the performance of off-frontier portfolios really were inefficient compared to the performance of on-frontier portfolios.

3.2. Bootstrap Sampling

A bootstrap sample of a data set is one with the same number of elements, but random replacement of every element by drawing with replacement from the original set of data. When this process of empirical resampling is repeated many times, the bootstrap samples can be used to estimate parameters for functions of the data. The plug-in principle [Efron and Tibshirani, 1993, p. 35] allows evaluation of complex functional mappings from

[13]One technique for deploying efficient frontiers within DFA analysis involves removal of actual values from the data series used in optimization. These points in the actual time series may be deemed abnormalities. The efficient frontier calculation does not use all available data or uses them selectively. See Kirschner [2000] for a discussion of the hazards of historical period segmentation.

[14]All optimization was done using Frontline Systems, Inc. *Premium Solver Plus V3.5* and Microsoft Excel.

[15]It is possible to restate a portfolio optimization problem to produce off-frontier portfolios. These are asset allocations for points in risk-return space that are within the concave region defined by the set of efficient points. They are portfolios with variance greater than the minimum variance points for the same expected returns. They were found by goal equality calculation using the same constraints as were used for minimum variance optimization. However, the equality risk condition was set to a higher level than found on the efficient frontier. Non-linear optimization was used for this purpose, whereas quadratic optimization was used for minimum variance optimization.

examination of the same functional mapping on the bootstrap samples. The function $\theta = t(F)$ of the probability distribution F is estimated by the same function of the empirical distribution $\hat{F}$, $\hat{\theta} = t(\hat{F})$, where the empirical distribution is built up from bootstrap samples. This technique often is deployed for the derivation of errors of the estimate.

The plug-in features of a bootstrap enable inference from sample properties of the distribution of bootstrap samples. The plug-in properties extend to all complex functions of the bootstrap, including standard deviations, means, medians, confidence intervals and any other measurable function. The EF is one of these functions.

The bootstrap is used in this paper to illustrate the impact of sampling error on the EF.[16] EF is a complex function of the historical returns from which it was calculated. If the sample is from a larger, unknown domain, the bootstrap principles apply. In the case of correlated investment returns, a segment of history might be thought of as a sample, but it may not be operationally meaningful because of sampling error. Yet, the use of the historical data in DFA applications treats it as though it were meaningful, representative, and *not* a sample.

The behavior of the EFs for our bootstrap samples is a nonparametric technique used to evaluate the effect of sampling error, were history to be properly thought of as a sample. Because actuarial science is built largely on the precept that past history, even of seemingly unique phenomena, really is a sample, we too proceed along this slippery slope.

3.2.1. Bootstrapping n-*Tuples*

The n-tuple observation of correlated observations at time t can be sampled with replacement. This technique was used by

[16]The bootstrap has been used in connection with mean-variance optimization by Michaud and others in an attempt to improve performance of EF portfolios. See Michaud [1998].

Laster [1998]. The experiment is similar to drawing packages of colored gum drops from a production lot. Each package contains a mixture of different colors that are laid out by machinery in some correlated manner. Suppose the lot that has been sampled off the production line contains n packages. A bootstrap sample of the lot also contains n observations. It is obtained by draws, with replacement, from the original *sample* lot. The n-tuple of investment returns at time t is analogous to a package within the lot of gum drop samples. The historical sequence of correlated returns is analogous to the mix of different colors of gum drops in a package. The analogy halts because we know the lot of gum drop packages is a sample. We never will know whether the sequence of historical, n-tuple investment returns is a sample in a meaningful sense.

The data consist of a matrix of monthly returns; each row is an n-tuple of the returns *during a common interval of time* for the component assets (columns of the matrix); the value of n was ten and measures the use of the ten investment categories described in Appendix A: Review of Data Sources. The bootstrap method involves sampling rows of the original data matrix. An n-tuple describing the actual returns for asset components at an interval of time is drawn and recorded as an "observation" in the bootstrap sample. Because this n-tuple can appear in another draw, the process involves sampling with replacement. This randomized choice of an n-tuple is repeated for each observation in the original sample. When the original sample has been replaced by a replacement sampling of the sample, the result is referred to as a bootstrap sample. This process of drawing a bootstrap sample can be repeated many times, usually in excess of 2,000.

Each bootstrap sample has both a measurable covariance matrix and an efficient frontier that can be derived using that covariance matrix. It is unlikely that any two bootstrap samples will necessarily have the same covariance matrix. Each sample can be subjected to mathematical optimization to produce an efficient frontier. The study asks whether this frontier is stable across the samples. Instability is measured in two ways. First, the bootstrapped efficient frontier may fluctuate from sample to sample. This means that the distribution of risk for a return point on the EF is not a degenerate distribution that collapses to a single point. Rather, there is a range of different portfolio risks among the bootstrap samples at a given return. There is a probability distribution associated with risk, given a return among the bootstrap samples. In other words, the study attempts to measure the distribution, and the study views that distribution as a measure of sampling error in risk-return space as it affects the calculation of an efficient frontier.

Second, the portfolio allocations may diverge qualitatively among bootstraps. Were portfolio allocations to be about the same in an arbitrarily small region of risk-return space among different bootstrap samples, the practical effects of sampling error would be small.

3.2.2. Extension of the Bootstrap Sample as a DFA Scenario

The bootstrap samples can be used in the way a DFA model might have used the original historical data, including their direct use within the calculation of the DFA results as a random instance of investment results. They are the source of DFA scenarios. This paper suggests how that direct use of the bootstrap might unfold in a DFA liability-side simulation, but it does not deploy it in that manner.[17,18] The authors have a less ambitious objective of examining just the performance of the efficient frontier built from bootstrapping investment information.

---

[17] Although the n-tuple used in this paper is a cross-sectional observation of returns, it can be expanded to a cross-section of the entire business environment at time t. This includes all economic aggregates, not just rates of return. Any flow or stock business aggregate that can be measured for interval t is a candidate for the n-tuple. This would include inflation, gross domestic product, or any worldly observation of the business climate prevailing at that time. A bootstrap sample can be used as a component of a larger simulation requiring simulation of these worldly events.

[18] DFA model builders spend time modeling empirical estimates of process and parameter risk [Kirschner and Scheel, 1998]. Bootstrapping from the data removes much of this estimation work and leaves the data to speak for themselves.

3.3. *Sampling Error within Risk-Return Space*

There is no clear-cut method for estimating sampling error that may exist in risk-return space. We do not know the underlying distribution generating a historical sample. We do not know whether a population distribution, were it to exist, is stationary over any time segment. We might, however, view history as an experimental sample, particularly if we want to use it to forecast corporate strategic decisions using DFA.

Sampling error can be envisioned and approximated in different ways for this hypothetical unfolding of reality. One way is to break the actual time series into arbitrary time segments and ask whether a random selection among the subsets of time leads to different, operationally disparate results—these would be EFs based on the sub-segment of time that have portfolio allocations disparate enough to be viewed as operationally dissimilar. If they are dissimilar enough to warrant different treatment, a sampling distribution of interest is the one measured by the effects of these time-period slices.

Another approach is to envision prior history as an instantiation, period-to-period, from an unknown multivariate distribution. The sampling error in this process is driven by a multivariate distribution. Depending on our model, we may or may not place dependencies from prior realizations on this period's realization. That is, for DFA investment return generation and intra-period portfolio rebalancing, the multivariate model may be stationary or non-stationary with respect to time.

3.3.1. *Michaud's Efficient Frontier*

Michaud [1998] approaches the measurement of sampling error effects on EF in a different way. Although his approach differs, his overall conclusions are important and consistent with many of our findings. He notes [1998, p. 33], "The operative question is not whether MV optimizations are unstable or unintuitive, but rather, how serious is the problem. Unfortunately for many investment applications, it is very serious indeed." Our paper will draw a similar conclusion.

He does not refer to an efficient surface but calculates a "resampled" portfolio that seems to capture some similar properties. Michaud uses multivariate Normal simulations from the *same* covariance matrix used to calculate EFs. This covariance matrix is from a sample of data—the data observed during some historical period. Just what definition of sampling error has been accommodated in the Michaud resampled portfolio is unclear.

One of the Michaud simulations is not equivalent to a bootstrap sample used in this study. Michaud's approach does not attempt to adjust for a primary source of sampling error—sampling error in the covariance matrix. In our study, each bootstrap sample has an independently measured covariance matrix. Using the DFA jargon of Kirschner and Scheel [1998, pp. 404–408], Michaud's approach may not account for parameter risk in the underlying returns generation mechanism. The ranking mechanism used by Michaud to combine EFs derived from various multivariate Normal simulations may distort risk-return space because each EF is segmented in some non-linear fashion to identify equally ranked points in risk-return space [Michaud, 1998, p. 46, footnote 11]. The portfolio profiles for identically ranked EF points are averaged, yet it is not clear that equi-ranked points fall within the same definition of risk-return space.

3.4. Importance to DFA Scenario Generation

This paper cannot and does not attempt to rationalize the process underlying investment yields over time.[19] Rather, the model builder should be careful to design the DFA model to be in accordance with perceptions about how a sampling methodology may apply. The use of the model will invariably mimic that viewpoint.

[19]What if there were no common observable stationary probability measure for security prices? Kane [1999, p. 174] argues we must use utility measurements.

If, for example, one views history in the fashion imagined by a bootstrap of n-tuples, and if that view does observe operational differences, then one can create scenarios from bootstrap samples. No more theory is required. Hypothetical investment returns are just a bootstrap sample of actual history.

Similarly, if EFs for historical periods produce superior performance in forecasting (compared to portfolios constructed from off-frontier portfolios derived from the same data), then the use of an empirically determined covariance model and multivariate Normal simulation makes a great deal of sense.

3.5. Importance to DFA Optimization

Optimization often is used within DFA and cash flow testing models to guide portfolio rebalancing. The DFA model usually grinds through the process of business scenario and liability scenario simulations before the optimizer is deployed. But, accounting within the model often is done while the optimizer seeks a feasible solution.

The sequence of model events runs like this:

1. Independently model many instances of exogenous states of the business world (e.g., asset returns, inflation, measures of economic activity, monetary conversion rates). Number these instances, $B_1, B_2, B_3, \ldots, B_n$. Note that each of these instances is a vector containing period-specific values for each operating fiscal period in the analysis.
2. Model many instances of the company's performance. Number these instances $C_1, C_2, \ldots, C_n$. C_1 often is dependent on B_1 because it may use an economic aggregate such as inflation or economic productivity to influence C_1's business growth or loss and expense inflation. Each C is a vector spanning the same fiscal periods as B.
3. Observe that in some DFA models neither B nor C is necessarily scaled to the actual volume of business. They are unit rates of change for underlying volumes that are yet to be applied.

4. Let the optimizer search mechanism posit a vector of weights that distribute the volume of assets at t_0, the inception point for a forecast period.

5. Apply the accounting mechanisms used by the DFA model to beginning assets and account for the unit activities expressed in B and C.[20] Do this accounting for each vector pair $\{B_1,C_1\},\{B_2,C_2\},\ldots,\{B_nC_n\}$ over the range of its time span.[21]

6. Calculate the metric used for the goal and any constraints as of the end of the fiscal period, if it is a metric such as economic value or surplus. If it is a flow-based metric such as portfolio duration or discounted GAAP income, derive the metric for the holding period results. This calculation is done for each business/company scenario pair. There are n results; collectively they constitute a simulated sample.[22]

7. Return the required metrics for the sample to the optimizer. If the optimizer is deployed for EF calculation, the goal will be a sample statistic for risk, such as variance, semi-variance, or chance-constrained percentile or range. The sample average for the distribution developed in step (6) for the metric will be used within the constraint set.

8. The optimizer will repeat steps (4)–(7) until it has obtained a feasible set.

The optimizer uses a sample. The optimizer results have sampling error. Steps (1) and (2) are experiments. Let there be 10

[20] At this stage, the derivation of taxes would occur. As noted by Rowland and Conde [1996], the determination of federal income taxes is convoluted by the combined effect of discount rates, changes in loss reserves, varying underwriting results, and tax carryforwards and carrybacks.

[21] Some models may achieve computational efficiencies when economic scenarios are paired with E(C) instead of with direct pairing to $C_1,C_2,\ldots,C_n$. When this is done, however, the variance of the metric being optimized will be reduced, and the minimum variance portfolio is likely to be different.

[22] If enough pairs are used, the chance that the model will converge improves.

repetitions of this experiment. Application of steps (1)–(8) will result in 10 efficient frontiers, each derived from a different experimental sample. It is likely that they will have different characteristics.

In a DFA experiment there are many draws from the urn; each simulation is another draw. The modeler gets distributional information about the contents of the urn by the experimental grouping of all the simulations. When enough simulations within each experiment are run, convergence of the distribution of results can be achieved. Since it is unlikely for the output distribution to be known, or necessarily capable of being parameterized, no a priori estimate is available. Instead, an empirical measure of convergence must be used.

The allocation of company assets among competing investment alternatives using a single efficient frontier calculation (based on a single experimental result) may seem to be similar to betting on the allocation among balls of different colors within the urn based on a single sample from the urn containing them. One may, or may not, be lucky. But you improve your luck by increasing the number of simulations.

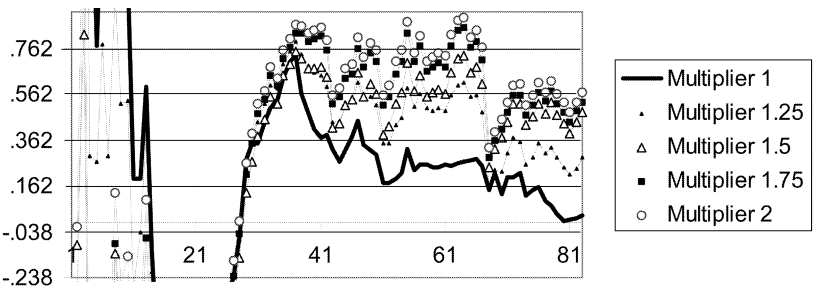
One still may become victimized by a faulty decision while ignoring sampling error. This may arise in calibrating a model to history. The historical record is a single draw from a true underlying probability distribution. We may be lucky that the number of periods in the historical realization contains sufficient information about the underlying process for unfettered decision making. But we could be victims of sampling error, which we are unable to control or even limit.

4. HISTORICAL PERFORMANCE COMPARISON

Figure 4 illustrates the performance of several portfolios over increasingly longer forecast periods. It shows results for portfolios, which, a priori, have different levels of risk for the same

FIGURE 4

COMPARISON OF PERFORMANCE FOR ON-FRONTIER AND OFF-FRONTIER PORTFOLIOS



Performance Information Ratio
Historical Period: January, 1988 - December, 1992
Forecast: January, 1993 - December, 1999
Expected annualized return=.0825

return.[23] The multipliers shown in the legend of Figure 4 are multiples of the minimum variance risk. The line for Multiplier 1 traces the performance of the on-frontier EF portfolio. Other lines in the figure with multipliers >1 show performance of portfolios with the same expected return but higher variance.

Figure 4 traces performance using a variation of the Sharpe performance measure.[24] It is known as the information ratio. The Sharpe performance ratio, which measures excess return to risk, is adjusted in the denominator of the information ratio. The denominator of the Sharpe performance indicator is changed to *excess* risk. The information ratio is given by:

$$\frac{\mathrm{E}(r_p - r_f)}{\mathrm{SD}(r_p - r_f)},$$

[23]Risk in this study is measured as the standard deviation of return.

[24]Laster [1998] created various portfolios by combining two asset components, domestic (represented by S&P 500) and foreign (represented by Morgan Stanley EAFE). His bootstrap samples of these two components were used to calculate portfolio variance,

where

r_p = monthly return on the portfolio,

r_f = monthly return on the risk free component of the portfolio,[25]

E = expectation operator, and

SD = standard deviation operator.

Although the information ratio was computed with monthly data, it is expressed as an annual measure in the paper.

4.1. EF Performance Is Better for Low Risk-Return Portfolios

The off-frontier portfolios, so-called inefficient portfolios, achieve performance that rivals or betters that of the EF portfolio.[26] There is no concept of "significance" that can be attached to the observed differences. However, it is clear that the performance differences are great and that inefficient portfolios outperform the efficient one in the Figure 4. When performance is measured by geometric return, the underperformance of the EF portfolio can be more than 100 basis points, as shown in Figure 5. The underperformance shown in Figure 5 is measured over a seven-year holding period, and there was no portfolio rebalancing during this time. Data for other time periods and the use of intervening portfolio rebalancing might materially affect this evidence of underperformance.

The performance varies considerably with the level of return and historical period. For example, Figure 6 illustrates performance for an earlier period and a lower expected return level. Here, the EF portfolio does, indeed, outperform the off-frontier

---

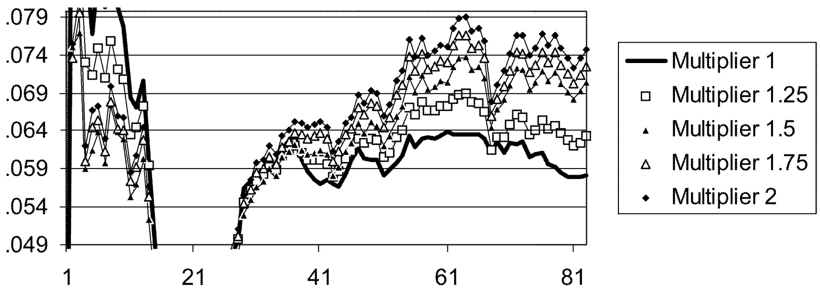
assuming various mixes. He did not separate historical and forecast periods. Instead, he measured quantiles from the bootstrap samples after constructing portfolios. He concluded that diversification into foreign equities substantially changed and improved the risk-return profiles.

[25] The 90-day Treasury bill index is used as the proxy for the risk free return.

[26] Short holding periods have performance measures calculated with few observations. The ordinal rankings among the different multipliers are volatile and should be ignored. The first six monthly periods are generally ignored in this paper.

FIGURE 5

COMPARISON OF GEOMETRIC RETURN FOR ON-FRONTIER AND OFF-FRONTIER PORTFOLIOS



Geometric Return
Historical Period: January, 1988 - December, 1992
Forecast: January, 1993 - December, 1999
Expected annualized return=.0825

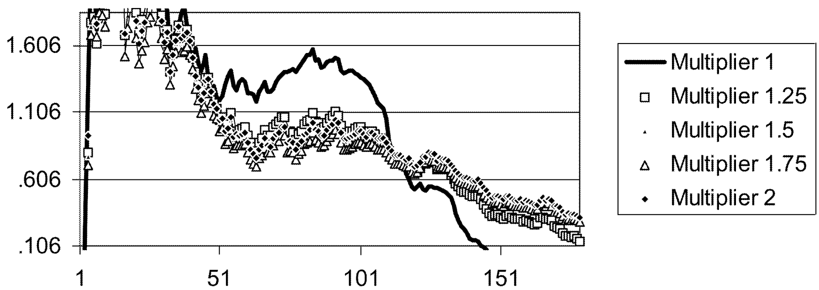
portfolios for about ten years. Thereafter, it reverses, and performance falls below off-frontier portfolios. The Figure illustrates that the contemplated holding period for use of an EF should probably not be as long. The performance variance illustrated in Figure 6 is volatile; the differences in performance in on- and off-frontier portfolios vary considerably with the choice of historical starting point and length of the holding period.

4.2. Overall Behavior of On-Frontier Portfolios for Information Ratio

The historical record was examined from several perspectives to see whether an EF portfolio continues to outperform off-frontier portfolios. Equi-return portfolios were examined. These are portfolios whose returns are the same, but they have higher risk. The forecast period immediately following the end of the historical segment was examined to determine how long the on-frontier portfolio maintained superior performance. This forecast horizon extended to the end of the data, December 1999. Historical segments consist of a 5-year block of 60 observations.

FIGURE 6

EF PORTFOLIO PERFORMANCE AT LOW RISK-RETURN LEVELS



Performance Information Ratio
Historical Period: January, 1980 - December, 1984
Forecast: January, 1985 - December, 1999
Expected annualized return=.0649

Several adjustments were made for this analysis. The first six-month period was ignored because the ratio is highly volatile and computed from few observations. The extreme low return levels also were removed from the analysis because higher ones shown in the table dominated them.[27]

Table 1 shows the relative behavior of the information ratio at the return level indicated at the top of the column. Each row block includes the time for subsequent row blocks. For example, the forecast beginning January 1980 covers the period ending December 1999. The interval of measurement is a month. All of the other blocks begin at a later point, but all forecast periods end in December 1999.[28]

---

[27]The extreme low risk-return observations occur below where the EF curve has a positive first derivative. A portfolio with a higher return for the same risk can be found above this change in the curve.

[28]*Each block of rows uses a different set of on- and off-frontier portfolios—the respective EFs are derived from optimizations on different periods.* For example, the January 1980 forecast is based on the performance of EFs derived from a historical segment covering the 5-year period, January 1975–December 1979). However, the January 1995 forecast uses EFs derived from a different period, one covering the 5-year period, January 1989–

TABLE 1

INFORMATION RATIO BEHAVIOR

Forecast Period	Return Levels					
Information Ratio (forecast begins 1/1980)	0.0066	0.0080	0.0085	0.0090	0.0095	0.0100
Periods until on-frontier point underperforms (max = 238)	6	6	6	6	6	6
Number of periods on-frontier point outperforms all others	10	142	148	153	154	151
Average on-frontier rank (5 is highest)	3.05	4.37	4.34	4.33	4.30	4.27
Information Ratio (forecast begins 1/1985)	0.0066	0.0080	0.0085	0.0090	0.0095	0.0100
Periods until on-frontier point underperforms (max = 178)	111	110	109	7	119	69
Number of periods on-frontier point outperforms all others	105	104	103	8	113	124
Average on-frontier rank (5 is highest)	3.46	3.40	3.38	1.92	3.72	3.90
Information Ratio (forecast begins 1/1990)	0.0066	0.0080	0.0085	0.0090	0.0095	0.0100
Periods until on-frontier point underperforms (max = 118)	40	6	9	9	9	9
Number periods on-frontier point outperforms all others	34	8	66	83	103	111
Average on-frontier rank (5 is highest)	4.18	4.05	4.57	4.72	4.89	4.96
Information Ratio (forecast begins 1/1993)	0.0066	0.0080	0.0085	0.0090	0.0095	0.0100
Periods until on-frontier point underperforms (max = 82)	19	6	6	6	6	10
Number of periods on-frontier point outperforms all others	15	3	5	6	2	4
Average on-frontier rank (5 is highest)	2.10	1.83	1.82	1.81	1.60	1.56
Information Ratio (forecast begins 1/1995)	0.0066	0.0080	0.0085	0.0090		
Periods until on-frontier point underperforms (max = 58)	53	56	57	Never		
Number of periods on-frontier point outperforms all others	47	50	51	53		
Average on-frontier rank (5 is highest)	4.83	4.94	4.96	5.00		

December 1994. The information in the blocks is not cumulative; the number of periods the on-frontier excels or outperforms off-frontier portfolios is a separate measurement for each row block. The row blocks show performance for portfolios constructed at different points in time.

Missing cells in Table 1 indicate that a feasible set was not found at that return level for one or more of the on- or off-frontier portfolios. There were five portfolios with risk up to two times the risk of the on-frontier point.

"Periods until on-frontier point underperforms" means the first period that an off-frontier portfolio beats the on-frontier efficient portfolio. "Number of periods on-frontier point outperforms all others" means the last period where the efficient portfolio wins. Performance tends to hold up better for lower return levels. This effect is reinforced by the larger values shown for the number of periods the on-frontier portfolio does outrank the off-frontier portfolios. In general, the on-frontier portfolio ranks well compared to the others. The average rank is generally high, above 3 out of 5. But the performance is not consistent. The on-frontier portfolio did well during the long forecast period starting January 1980 and during the shorter forecast period starting January 1995. However, the low average of the on-frontier for the January 1993 period shows that the performance is greatly influenced by the historical period and perhaps influenced by sampling error.

There also is great inconsistency in the number of periods before an off-frontier portfolio has a higher information ratio. The scan begins in period 6 of the forecast horizon, so the reversal shown in the table will either be never or a number between 6 and n. In most cases, the reversal is early, but not permanent. There are many situations where the on-frontier portfolio wavers between highest rank and something less. This latter fact is found in the rows, "Number of periods on-frontier point outperforms." In most cases this number is larger than the number of periods before reversion, indicating that the on-frontier waffles in and out of superior performance. This could be another indication of sampling error. The choice of an on-frontier point may not, and probably does not, imply superior performance.

4.3. Behavior for Other Performance Measures

The information ratio is believed to be a valid measure of performance because it adjusts for variation in the return series during the period of measurement. Were it applied to two consultants' portfolio allocation recommendations, the consultant with *lower* excess returns could be ranked higher than the other consultant, because of proportionately *lower* risk in excess return. This may be small consolation to the holder of the lower wealth portfolio recommended by the higher ranked consultant. This is why it is important to assess other characteristics beyond the appetite for risk before making an allocation decision. The manager with the higher information ratio has the better cost of risk per unit of return; yet, it is not of much use if a minimum return level or ending wealth is required.

There is considerable historic instability in the standard deviation of returns. This can be seen in Figure 7, which shows the historic progression of changes in the standard deviation of monthly returns of the portfolio components used in this study. The lines show the change in standard deviation for rolling 5-year blocks of data.[29] Any performance measure that is a function of this risk proxy, such as the information index, will be inherently sensitive to such volatility and, perhaps, exhibit similar historic instability. This volatility in risk helps to explain why historical EFs may lack forecast power.

One measure of performance that is not risk-adjusted is geometric return during a holding period. Results are arrayed in Table 2. The layout of this table is similar to Table 1.

The forecast propensity of the on-frontier allocation is markedly changed. Wealth growth appears to be unrelated to the on- or off-frontier portfolio choice, and often is worse for the on-frontier allocation. The number of holding peri-

[29]There was significant volatility in the securities markets in 10/87 ("Black Monday") and 8/98 (Long Term Capital crisis). These periods are highlighted in the figure.

FIGURE 7

VOLATILITY IN RETURN STANDARD DEVIATION

ROLLING 5-YEAR MONTHLY STANDARD DEVIATIONS

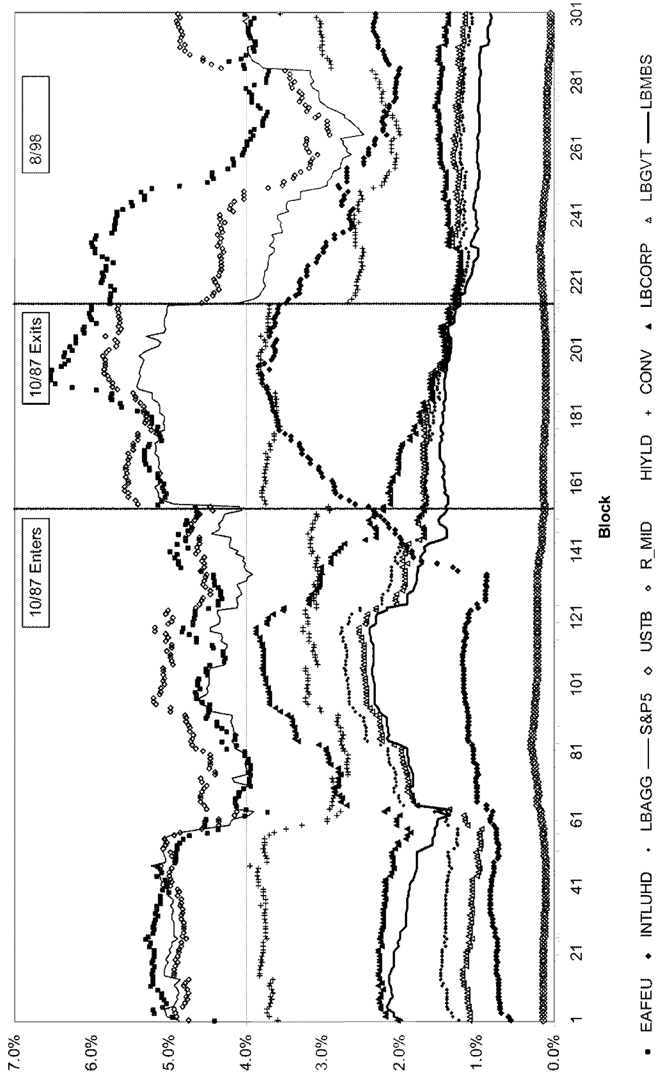


TABLE 2

Geometric Return Behavior

Forecast Period	Return Levels					
Geometric Return (forecast begins 1/1980)	0.0066	0.0080	0.0085	0.0090	0.0095	0.0100
Periods until on-frontier point underperforms (max = 239)	6	6	6	6	6	6
Number of periods on-frontier point outperforms	0	107	106	102	102	105
Average on-frontier rank (5 is highest)	2.41	3.35	3.39	3.38	3.38	3.40
Geometric Return (forecast begins 1/1985)	0.0066	0.0080	0.0085	0.0090	0.0095	0.0100
Periods until on-frontier point underperforms (max = 179)	6	6	6	6	6	6
Number of periods on-frontier point outperforms	0	0	0	0	0	1
Average on-frontier rank (5 is highest)	1.00	1.00	1.00	1.00	1.03	1.04
Geometric Return (forecast begins 1/1990)	0.0066	0.0080	0.0085	0.0090	0.0095	0.0100
Periods until on-frontier point underperforms (max = 119)	21	6	74	89	111	119
Number of periods on-frontier point outperforms	15	7	68	85	105	113
Average on-frontier rank (5 is highest)	4.11	4.06	4.60	4.75	4.92	4.99
Geometric Return (forecast begins 1/1993)	0.0066	0.0080	0.0085	0.0090	0.0095	0.0100
Periods until on-frontier point underperforms (max = 83)	15	15	16	16	16	16
Number of periods on-frontier point outperforms	11	16	18	18	14	10
Average on-frontier rank (5 is highest)	1.99	2.19	2.22	2.22	1.92	1.73
Geometric Return (forecast begins 1/1995)	0.0066	0.0080	0.0085	0.0090		
Periods until on-frontier point underperforms (max = 59)	6	6	6	6		
Number of periods on-frontier point outperforms	0	2	9	30		
Average on-frontier rank (5 is highest)	2.44	2.80	2.96	3.35		

ods the efficient frontier portfolio dominates off-frontier portfolios is generally a lower proportion of the possible number of holding periods in Table 2 than in Table 1. Michaud [1998, pp. 27–29] claims there is a portfolio within the

EF, the "critical point," below which single period mean-variance efficient portfolios are also n-period geometric mean efficient and above which single period MV efficient portfolios are not n-period geometric mean efficient.

4.4. Performance Failure within CAPM

Work with beta has led to various criticisms [Malkiel, 1996, p. 271].[30] For example, some low risk stocks earn higher returns than theory would predict. Other attacks on beta tend to mirror what we see with EF:

1. The capital asset pricing model (CAPM) predicts risk-free rates that do not measure up in practice.[31]
2. Beta is unstable, and its value changes over time.[32]
3. Estimated betas are unreliable.[33]
4. Betas differ according to the market proxy they are measured against.[34]
5. Average monthly return for low and high betas differs from predictions over a wide historical span.[35]

[30] Beta is a measure of systematic risk either for an individual security or for a portfolio. High beta portfolios, measured ex ante, in theory should have higher returns ex post than low beta portfolios.

[31] When ten groups of securities, ranging from high to low betas, were examined for the time period 1931–65, the theoretical risk free rate predicted by CAPM and actual risk free rates significantly diverged. Low risk stocks earned more and high risk stocks earned less than theory predicted [Malkiel, 1996, pp. 256–7].

[32] During short periods of time, risk and return may be negatively related. During 1957–65, securities with higher risk produced lower returns than low beta securities [Malkiel, 1996, pp. 258–60].

[33] The relationship between beta and return is essentially flat. Beta is not a good measure of the relationship between risk and return [Malkiel, 1996, pp. 267–8].

[34] Predictions based on CAPM about expected returns both for individual stocks and for portfolios differ depending on the chosen market proxy. In effect, the CAPM approach is not operational because the true market proxy is unknown [Malkiel, 1996, pp. 266–7].

[35] The ratio of price to book value and market capitalization did a better job of predicting the structure of nonfinancial corporate share returns than beta during a 40-year period [Fama and French, 1992].

Malkiel [1996, p. 270] concludes from his survey that, “One’s conclusions about the capital-asset pricing model and the usefulness of beta as a measure of risk depend very much on how you measure beta.” This appears to be true of EFs too. The *definition* of efficiency is what is important here—perhaps more important because correct measurement requires precise definition.

The choice of an optimization mechanism couched in terms of risk-return trade-off may not lead to wealth maximization. Under these pretenses one might wish to deploy a different optimization mechanism, such as the one mentioned by Mulvey, et al. [1999, p. 153] in which the optimization seeks to maximize utility. The choice of a particular utility function may be framed in terms of absolute risk aversion—negative exponential utility works in this regard.[36] And if the behavior of security prices does not have an observable stationary probability measure [Kane, 1999], utility approaches seem to be mandatory.

The subject of what is optimal is controversial and not apt to go away. The use of optimization within hybrid models and generation of metrics by DFA models has many subtle manifestations. One is the choice of planning horizon. Michaud [1998, p. 29] argues that investors with long-term investment objectives can avoid possible negative long-term consequences of mean-variance efficiency by limiting consideration to EF portfolios at or below some critical point. There is a parallel in our paper, in what we refer to as sampling error and its effect on the shape of the efficient surface. This surface appears to have properties at the lower risk-return areas of both lower dispersion, greater similarity in portfolio composition, and better on-frontier performance among different samples (either bootstrap or historic segment).

[36]The recommendation of a utility-decision approach has great breadth in the insurance literature—beyond the use of utility as goal function in optimization, other venues find it appropriate where stochastic dominance is sought. For example, exponential utility use was suggested in rate making by Freifelder [1976]. The choice of parameters for utility functions is perhaps as much an art as the parameterization of claims generations in DFA models.

5. CHARACTERISTICS OF THE EF SURFACE

The bootstrap-generated EF surface rises within the risk-return space. Views of this surface from two different angles are shown in Figure 8.

The surface is constructed from monthly returns. Looking down on the surface of the views, one obtains a projection on risk-return space. The surface is seen to curve as the efficient frontier curves. In the low risk-return sector, the surface is more peaked. The surface flattens and broadens in the risk-return space. Imagine yourself walking along the ridge starting in the southwest and proceeding northward and then northeast. You would first be descending a steep incline, and then a vista of a vast plane would unfold along your right. This can be interpreted within the context of changes in the marginal distributions representing slices through the surface either along the risk or along the return dimensions. We refer to the latter as an equi-return slice, and its properties are examined in more detail at a latter point in the paper. In either case, the visualization is one of moving from less dispersed marginal distributions to ones with greater variance as either dimension is increased.

There is an artifact of the intervalization that results in a sudden rise in the surface at the highest risk level. This occurs because higher risk observations were lumped into this final interval. Were higher levels of risk intervalized over a broader range, this ridge would flatten.

The surface shown in either of the views in Figure 8 is built from many efficient frontiers, each produced from optimizations done on a bootstrap sample. We already have seen in Figure 1 a subset of EFs that tangle together—they can be organized to produce a surface. The surface develops the same way an empirical probability distribution is built from a sample. Repeated sampling produces points that are intervalized and counted.

FIGURE 8

VIEWS OF EF SURFACE CREATED FROM BOOTSTRAP SAMPLES

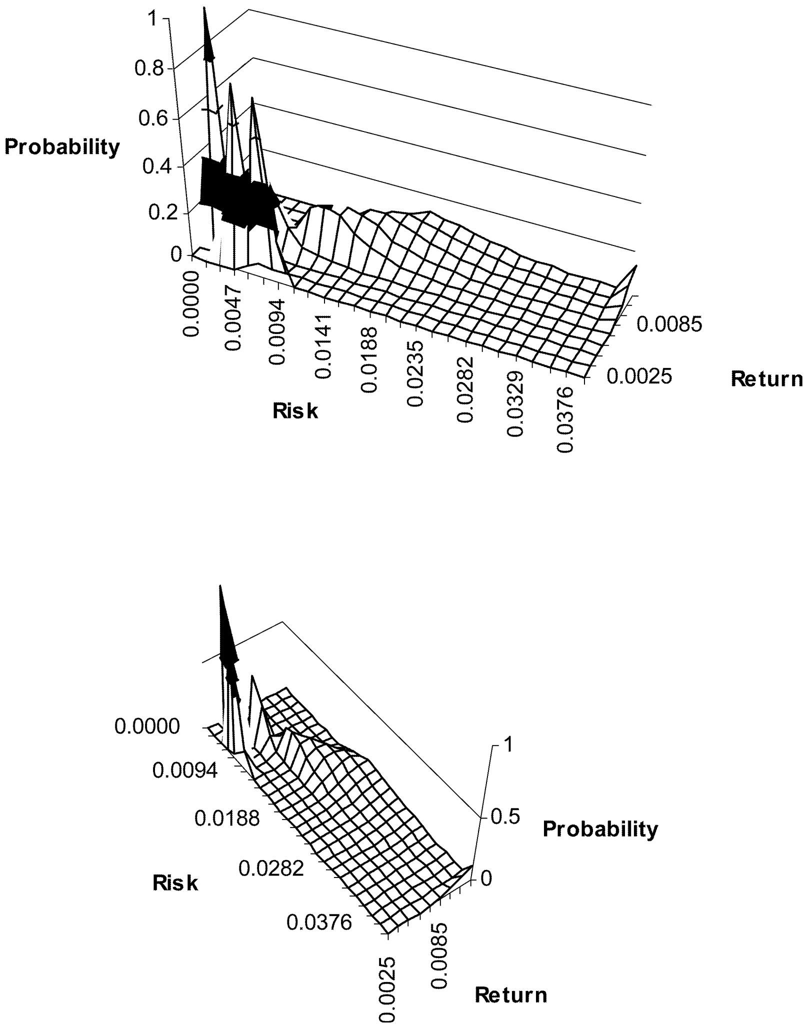
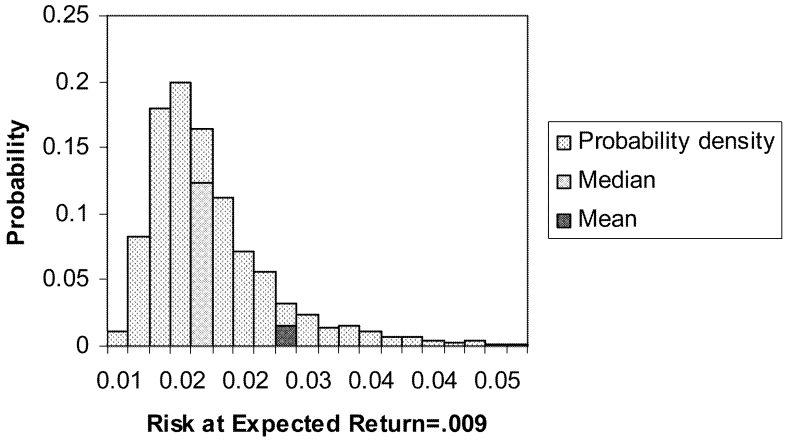


FIGURE 9

DISTRIBUTION OF RISK GIVEN A RETURN LEVEL



A frequency count can be made of observations for EFs falling within an arbitrarily small, two-dimensional region of risk-return space. An example of this mapping for 5,000 bootstrap-simulated EFs appears in Figure 8. Collectively, this mapping involves the two-dimensional intervalization of approximately 45,000 quadratic optimizations constituting the EFs for the underlying bootstrapped samples.[37]

5.1. Equi-Return Slice of the Efficient Surface

A slice through the efficient surface along the return plane produces a histogram of the minimum risk points for a given return in the EFs used for the EF Surface. As return increases, this marginal probability distribution becomes more disperse. An example appears in Figure 9.

[37]Equi-return minimum variance points for the 5,000 bootstrapped EFs were intervalized based on an overall evaluation of the range of risk among all points on all EFs. If an efficient set could not be identified for a return level, the observation was ignored. The marginal probabilities (risk-return) were normalized to the number of viable observations for that risk level. The number of viable optimizations exceeded 4,500 at each return level.

TABLE 3

STATISTICS FOR EQUI-RETURN SLICES OF THE EFFICIENT SURFACE

Statistic	Efficient Surface from Bootstrapped Efficient Frontiers						
Return Level	.0053	.0066	.0080	.0085	.0090	.0095	.0100
Mean (times 1.0E4)	.0125	.0533	3.76	8.86	17.9	33.6	50.7
Standard Deviation (times 1.0E4)	.627	3.77	38.2	58.6	81.8	109.7	131.6
Skewness (times 1.0E8)	.000123	.378	57.8	136.	275.	516.	779.

TABLE 4

STATISTICS FOR EQUI-RETURN SLICES OF THE SURFACE SHOWN IN FIGURE 10

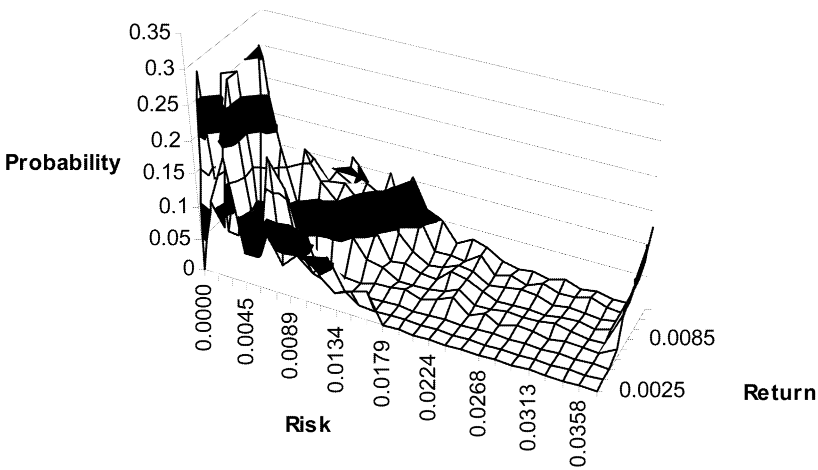
Statistic	Efficient Surface from Historical Samples						
Return Level	.0053	.0066	.0080	.0085	.0090	.0095	.0100
Mean (times 1.0E4)	.663	5.53	21.0	25.5	28.8	33.2	46.5
Standard Deviation (times 1.0E2)	.080	.451	.861	.940	.995	1.06	1.23
Skewness (times 1.0E6)	.00676	.769	2.92	3.54	4.00	4.62	6.46

The dispersion increases with return for both surfaces constructed from bootstrap samples and from randomly selected blocks of history. The distributions are positively skewed, increasingly so as return increases. The inset bars in Figure 9 identify the intervals containing the mean and median points of the distribution. Additional statistics both for bootstrapped and historical segment evaluations of sampling error appear in Tables 3 and 4.

The statistics are visually apparent in the EF surface shown in Figure 8. The surface is partially bowl-like—sloping downward in a concave fashion. Its rim encompasses a plane within the risk-return domain that is broad in the risk dimension. As one moves from low to high return, the marginal distribution of EF

FIGURE 10

EFFICIENT SURFACE FROM HISTORICAL SAMPLES

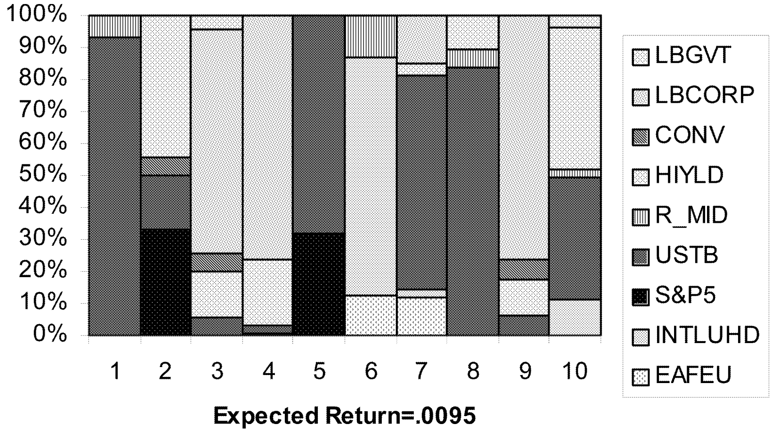


points measuring optimized risk (an equi-return slice through the surface as illustrated in Figure 9) becomes more dispersed. In a visual context as one moves from low to high risk along the EF surface and takes equi-return slices through it, one would find higher variance in the distribution of optimized EF risk points—variance shown in histogram plots such as Figure 9 is greater.

An efficient surface also can be created from EFs calculated for historical time periods. An example appears in Figure 10. The data are for 5-year overlapping blocks calculated on a monthly basis starting in 1970. The same general features are found in this representation of sample error. However, the surface is less flat than the one developed from bootstrap samples. The reduced dispersion in the surface of Figure 10 arises in part from the use of overlapping 5-year blocks used to construct the underlying EFs from which the surface is built. A statistical table similar to Table 3 was constructed for this surface. It appears in Table 4.

FIGURE 11

AVALANCHE CHART FOR HISTORICAL SEGMENTS



6. STABILITY OF PORTFOLIO COMPOSITION ALONG AN EFFICIENT FRONTIER

Portfolio allocation among component securities changes, usually dramatically, along the efficient frontier. A component may enter the feasible set at some point, increase in weight, decrease, and then drop out at other points along the EF. This effect was shown in Figure 2.

The change in composition for an equi-return level was examined among different EFs, constructed both from historical segment EFs and bootstrap EFs. We refer to this type of comparison as an avalanche chart because, when shown in an animation, the change in composition is similar to an avalanche. An example appears in Figure 11.

The vertical bars are stacked columns. Each segment within a column represents a different component of the portfolio. A br, therefore, compares the percentage value each component in the feasible set contributes across all components in the set. All bars are shown for a constant, equi-return level of an EF; but each bar is for a different historical segment. In Figure 11, each bar represents the portfolio composition for the equi-return level point on the EF, which was calculated for a 5-year block of monthly observations. The bars are for ten randomly chosen historical segments.[38] Were the blocks within the bars to consist of the same components and were they to be about the same size, the portfolio allocations would be the same regardless of the time frame. Examination of Figure 11 shows that the composition of the bars and individual component allocations varies considerably.

The portfolio composition is much more stable at lower risk-return levels. This result is in accordance with other similar findings based on the EF surface. It, too, shows less disperse results for lower return levels. This approach to measuring sampling error implies that performance of efficient frontiers may not be optimal relative to off-frontier portfolios. If the mix and composition of portfolios fluctuates considerably both with respect to historical and bootstrap sampling methods, the performance expectations of an ex ante allocation are not apt to hold ex post.

7. CONCLUSION

The behaviors shown in both Tables 1 and 2 illustrate a marked tendency towards randomness. The efficient surface built from bootstrap samples is highly variable within the risk-return domain. There appears to be some temporal dominance of on-frontier portfolios for lower risk-return levels, but the historical record is mixed. The bootstrapping of the single sample of asset returns provided by the historical data illustrates that sampling

[38]There is a small chance that two or more bars in an avalanche chart could be identical. However, there is a much larger probability that two or more bars have overlapping time periods in the calculation of their respective EFs.

error could materially affect the position and shape of the efficient frontier.

7.1. Should Efficient Frontiers Be Used in DFA Models?

There is no strong support in this paper for the practical deployment of efficient frontiers in DFA. The risk in DFA models stems from model, process, and parameter risk. It affects all aspects of DFA models of the insurance enterprise. The existence of model and process risk [Kirschner and Scheel, 1998] thwarts the usual convergence to the true underlying distributions gained by running large numbers of simulations. When all of these new risk elements are heaped on top of the sampling error derived from asset model calibration or empirically measured covariance matrices, one wonders whether EFs are really useful in DFA analysis.

The work of Michaud [1998] bears on the issue of improving the performance of EF portfolios. He defines a measure of statistical equivalence for mean-variance efficiency. Any portfolio within the efficient surface *sufficiently close* to the optimal portfolio is considered equivalent to it. The extension of his idea to the efficient frontier surface is to identify a region on it whose ex ante chance-constrained probability both can be measured and has desirable statistical properties in a forecasting sense. This is analogous to acknowledging the existence of sampling error and specifying an unknown population parameter only to within an interval of statistical confidence. Unfortunately, the definition of *sufficiently close* is constructive but difficult to implement in a rigorous manner, particularly within the context of the hybrid DFA model.

Future study will have to answer the question of whether on-frontier asset allocations that are measured from hybrid DFA models suffer a similar unreliability. But the problems with on-frontier *asset* portfolios raised in this paper are apt to be exacerbated by inclusion of known sampling error in the liability side of DFA models.

7.2. How Can EFs Be Efficiently Deployed?

Users of this construct should be aware that the term "efficient" in efficient frontiers has a good chance of being operationally false. The efficiency of portfolio composition is unlikely to be manifest in better performance of the on-frontier portfolio compared to other, off-frontier portfolios. The risk-return surface is not adequately measured by a single EF, and sampling error may lead to unwarranted conclusions about the efficacy of portfolios measured in such singular optimizations.

The user of EFs should probably view them as containing provisional, useful information about risk-return relationships. But, any single EF has limited value in understanding the risk-return surface. The conceptual basis of an efficient surface is an organized resampling of the data so that the decision process benefits from better understanding of uncertainty that might arise just because the EF is operationally derived from a sample. The misunderstanding of this uncertainty may lead to erroneous decisions, and the practitioner must be alert to potential inefficiencies of a single EF measurement. The authors recommend the elicitation of an efficient surface because the surface is apt to show a lack of statistical confidence in any single frontier on that surface. Under these circumstances, the practitioner must think in terms of confidence ranges. The sampling error shown in the efficient surface emphasizes how careful one must be when drawing inferences derived from optimization. An optimized frontier is based on an empirical covariance matrix, one that has sampling error. That error may be very important. It is easy to believe that strategic or tactical decisions motivated by so-called optimized DFA measurement will effectively move the organization to a better position in risk-return space. Unfortunately, there appears to be a broad region of "inefficiency" that may serve as well. An EF may be better than a crystal ball; but there is a good chance that it should not be taken too seriously.

REFERENCES

[1] Chopra, Vijay, and William Ziemba, "The Effect of Errors in Means, Variances, and Covariances on Optimal Portfolio Choice," *Worldwide Asset and Liability Management*, Cambridge University Press, 1998, pp. 53–61.

[2] Correnti, Salvatore, Stephen M. Sonlin, and Daniel B. Isaac, "Applying a DFA Model to Improve Strategic Business Decisions," Casualty Actuarial Society *Forum*, Summer 1998, pp. 15–52.

[3] Davison, A. C., and D. V. Hinkley, *Bootstrap Methods and Their Application*, Cambridge University Press, 1997, p. 582.

[4] Efron, Gerald F., and Robert J. Tibshirani, *An Introduction to the Bootstrap*, Chapman & Hill, 1993, p. 436.

[5] Elton, Edwin J., and Martin J. Gruber, *Modern Portfolio Theory and Investment Analysis*, John Wiley & Sons, Inc, 1995, p. 715.

[6] Fama, Eugene F., and Kenneth R. French, "The Cross-Section of Expected Stock Returns," *Journal of Finance*, June 1992.

[7] Freifelder, Leonard R., *A Decision Theoretic Approach to Insurance Ratemaking*, Irwin, 1976, p. 141.

[8] Kane, S.A., "What Happened to the Utility Functions? Imprecise Expectations of Security Prices," *International Review of Financial Analysis*, 8:2, 1999, pp. 165–175.

[9] Kirschner, Gerald S., "A Cost/Benefit Analysis of Alternative Investment Strategies Using Dynamic Financial Analysis Tools," Casualty Actuarial Society *Forum*, Summer 2000, pp. 21–54.

[10] Kirschner, Gerald S., and William C. Scheel, "The Mechanics of a Stochastic Corporate Financial Model," *PCAS* LXXXV, 1998, pp. 404–454.

[11] Laster, David S., "Measuring Gains from International Equity Diversification: The Bootstrap Approach," *The Journal of Investing*, Fall 1998, pp. 52–60.

[12] Levy, Haim, and Deborah Gunthorpe, "Optimal Investment Proportions in Senior Securities and Equities Under Alternative Holding Periods," *The Journal of Portfolio Management*, Summer 1993, p. 31.

[13] Malkiel, Burton G., *A Random Walk Down Wall Street*, W. W. Norton & Company, Inc., 1996, pp. 227–276.

[14] Michaud, Richard O., *Efficient Asset Management*, Harvard Business School Press, 1998.

[15] Michaud, Richard O., "The Markowitz Optimization Enigma: is 'Optimized' Optimal," *Financial Analysts Journal*, 45:1, pp. 32–42.

[16] Mulvey, John M., Chris K. Madsen, and François Morin, "Linking Strategic and Tactical Planning Systems for Dynamic Financial Analysis," Casualty Actuarial Society *Forum*, Summer 1998, pp. 149–167.

[17] Rowland, Vincent T., and Frank S. Conde, "Dynamic Financial Analysis Issues in Investment Portfolio Management," Casualty Actuarial Society *Forum*, Spring 1996, pp. 205–240.

[18] Sharpe, William F., "The Sharpe Ratio," *The Journal of Portfolio Management*, Fall 1994, pp. 49–58.

APPENDIX A

Review of Data Sources

This paper uses monthly time series of asset class total returns. A selection of broad asset classes typical of P&C insurance company asset portfolios was chosen for examination. The time series all begin January 1, 1970. However, certain asset classes (e.g., mortgage backed securities) do not have a history that extends back this far. For these classes, the time series were backfilled to the January 1, 1970 start date by an investment consultant. The backfill process was based on a consideration of the market conditions of the time (e.g., interest rates, fixed income spreads, inflation expectations) and how the particular sector would have performed given those market conditions. The Start Date in Table 5 refers to the date historical data begin.

TABLE 5

Asset Components

Class	Code	Source	Start Date
International Equities	EAFEU	MSCI EAFE Index	1/1970
International Fixed Income	INTLHDG	JP Morgan Non-US Traded Index	1/1970
Large Cap Domestic Equities	S&P5	S&P 500 Index	1/1970
Cash	USTB	90 Day US Treasury Bill	1/1970
Mid Cap Domestic Equities	RMID	S&P Mid Cap 400 Index	1/1982
High Yield	HIYLD	CSFB High Yield Bond Index	1/1986
Convertible Securities	CONV	CSFB Convertible Index	1/1982
Corporate Bonds	LBCORP	Lehman Brothers Corporate Bond Index	1/1973
Government Bonds	LBGOVT	Lehman Brothers Government Bond Index	1/1973
Mortgage Backed Securities	LBMBS	Lehman Brothers Mortgage Backed Securities Index	1/1986

APPENDIX B

ANNUALIZED RETURNS

The time series used in this study are monthly returns. With the exception of work relating to performance, all returns are expressed as monthly returns.

For performance measurement purposes, returns have been annualized using the following formulas.

Annualized Expected Return

$$R_p = (1 + r_p)^{12} - 1,$$

where

R_p = annualized return, and

r_p = monthly return.

Annualized Variance of Return

$$V_p = [v_p + (1 + \mu_p)^2]^{12} - (1 + M_p)^2,$$

where

V_p = annualized variance of return,

v_p = monthly variance of return,

μ_p = expected monthly return, and

M_p = expected annualized return.

Annualized Geometric Return

The growth rate, g, for a holding period of n years is given by:

$$1 + g = \left(\frac{V_n}{V_0}\right)^{1/n},$$

where

V_n = portfolio value at the end of the holding period n, and

V_0 = portfolio value at the beginning of holding period.

ADDRESS TO NEW MEMBERS—NOVEMBER 12, 2001

DAVID P. FLYNN

On behalf of the Casualty Actuarial Society, let me extend a warm welcome and congratulations to you, our new Associates and new Fellows.

The members are aware of the dedication that was required by each of you to have passed the exams and achieved the goal you've reached today for which you deserve our congratulations and best wishes. We are also aware of the support, comfort, and encouragement extended by your friends and family who have accompanied you on your journey. These friends and family have given up many hours to support you in gaining this achievement. We extend our congratulations to them as well on a job well done.

While your accomplishment is an ending of sorts, it is also the formal beginning of many wonderful opportunities for you in the years ahead. I have to admit that I'm more than a bit jealous, for I believe that the challenges and opportunities ahead of you today are far better and more interesting than ever before. Today we face a world in change brought on not only by the mass murders that occurred in September, but also by the recognition that past economic and regulatory models are in need of some adjustment. Has Alan Greenspan's magic wand lost all of its power? Ten rate adjustments in a single year so far has got to be some sort of a record. Because our industry is intrinsically bound to the wider economy, these will be interesting and challenging times, which is a great time to be an actuary!

Several months ago when Pat Grannan asked me to deliver this address, he added that he was looking forward to my comments and wondered what I was going to say.

In truth, I wondered what I was going to say as well!

What possibly could I say? What possibly could I say that would give you the appropriate insight into the guidance, support, experience, friendship, and sometimes even wisdom of the members of the Casualty Actuarial Society available to you that will be a part of your working lives for many years to come? What words would fairly communicate the new challenges that are before you in the global economy, words that would also warn you of the possible pitfalls that may lie ahead of you?

I soon decided that there must be some aids available in the great works of literature, religion, and philosophy that would guide me in communicating these thoughts. I was disappointed in my search until I recalled the existence of a more recent work by one of America's great philosophers, Dr. Theodore Geisel, better known throughout world as Dr. Seuss! He released a clever little ditty in the early '90s directed to recent graduates of all ages and stations.

To those of you who have not yet discovered this American author and philosopher, you are in for quite a treat. Francis Bacon surely could not have identified new career options now available to you any better than:

> "You have brains in your head.
> You have feet in your shoes.
> You can steer yourself
> Any direction you choose."

And today you do have many new directions to choose from. The CAS educational process has provided you with an unparalleled education in the art and science of the property and casualty risk transfer and risk management processes. No other institution even comes close in the range, depth, and practicality of its syllabus. Your talent and training is now increasingly being sought and used not only in the insurance business but in many other businesses as well. The recent revision in the banking laws added another industry in the CAS quiver. How these opportunities will emerge in the future in large part depends upon you, for you are the builders of the Casualty Actuarial Society of the future.

While this education will serve you well in the coming years, keep in mind that it is just basic training. *Your* growth and success as actuaries will depend in large measure on *you*! On your willingness to continue this learning process and to keep your mind open to the new and creative concepts available in the various seminars and programs sponsored by the CAS.

Also, keep in mind that the CAS is depending on each of you to make your contribution to these future education efforts. You owe it to those that will come after you.

In this same vein, those of you who are members of the CAS's CASNET e-mail system are aware of the recent discussions concerning some fundamental issues regarding the CAS's future direction, structure, and educational processes. With a few notable exceptions, I've generally been impressed with the thoughtfulness and quality of the discussions. My point is that CASNET has created a soapbox for you to express your opinions about CAS matters. Use it! It's a great tool.

Our learned Dr. Seuss further relates that even though your futures will likely be full of success, at times you'll fail or, in his words:

> "When you're in a Slump,
> You're not in for much fun.
> Un-slumping yourself
> Is not easily done."

There are multiple reasons for being caught in a "slump." I'm reminded of the succinct wisdom of the members of the corporate planning community, "No amount of planning beats dumb luck!" For in spite of your best plans, there will be times when things are just not going your way. Think how Alan Greenspan must feel! However, not one of those "slumps" should ever be a failure to comply with the Code of Professional Conduct. The Precepts of the Code are there not only as a warning but more importantly as a safe harbor for you, especially for those of you in the consulting community. Know the Precepts and Standards of Practice that affect you—they're your best protection!

Now that you have your head full of brains and your shoes full of feet, you're on your way!

Have fun! You're going to have a great career!

PRESIDENTIAL ADDRESS—NOVEMBER 12, 2001

PATRICK J. GRANNAN

I'd like to ask you to sit back for the next few minutes to think about the fundamental purpose of the CAS and about what we need to do to make sure it continues to meet our profession's needs. I'll do the talking for now, but your words and actions are what will determine the future course of the CAS and our profession.

In thinking about the CAS, I find it helpful to go back to the fact that it is an organization of professionals. Many professions outside the actuarial field have organizations that are more like separate businesses that actually shape the professions they serve to some extent. That's not the case for our profession. The CAS has a very talented and supportive staff, but the CAS is totally controlled by members and is headed by a Board of Directors elected by the membership. When we talk about what the CAS should do, we are talking about what we as a profession should do, not about what a separate business entity with its own objectives wants to do.

So, what do we as a profession want this organization to do? The mission we have asked it to carry out consists mainly of setting basic education standards, providing continuing education, facilitating research, and communicating about the profession, although some of the communication part is delegated to other organizations like the Academy. My view is that the CAS has been an exceptionally useful, valuable organization for our profession. I may be preaching to the choir here, but I'd like to point out a few signs of its value.

- First, compared to other professions, a very high percentage of CAS members are active in its committees, write papers, and give presentations at its meetings. Many take part in similar activities in CAS Regional Affiliates, the Academy, the Canadian Institute, and other professional groups. I believe the reason our members do this is that it's of value to them, both directly in their jobs and indirectly by helping their profession.

- A second sign of value is that the number of CAS members has been growing steadily. It doubled in the last ten years and nearly doubled in the prior ten years. This growth shows there is strong demand for the knowledge and skills of CAS members, and also that there's a good supply of people interested in joining our profession.

- Third, employers put significant value on the Associateship and Fellowship designations. If there were no CAS, there would be much less education and knowledge sharing among casualty actuaries, and people in our profession would be less valuable to their employers. I believe we as individuals have had the opportunity to do more interesting work and earn higher incomes as a result of the CAS.

I could go on, but you get the picture. I feel very good about what this organization has done for its members. I also believe that you, as CAS members, deserve to feel good about our profession's contribution to society. Although the insurance markets sometimes suffer from price swings, availability issues, and insolvencies, I believe each of those problems would be much greater if it weren't for the knowledge and skills brought to the insurance markets by casualty actuaries.

Has the CAS done everything right? Of course not. But, its occasional missteps haven't come from any lack of good intentions or efforts. I believe it will continue to meet our needs as long as its members continue to volunteer their time and energy the way they have in the past.

Two years ago when I became President-Elect, I got a fair amount of advice. All of it was helpful. But looking back, one of the better pieces of advice came from Dave Hartman. It was always to keep in mind that the volunteers are what made the

CAS what it is today, and that our profession's future depends on continued active involvement by our members.

The CAS has a lot of good work in progress in each of its functional areas, and we can count on it continuing. It's not automatic, but for the most part it's a progression that I'm confident will occur fairly naturally. What I'd like to talk about here are four changes in our environment that I think the CAS, and therefore we as members, need to focus on to make sure our profession adapts well to them. The changes I want to talk about are not revelations, and I believe the CAS is generally heading in the right direction on each of them. However, I want to suggest to each of you that you think about at least one of them and consider volunteering your help. Each of these changes requires significant attention and creative thinking in order for our profession to meet its members' needs well in the future.

The first change I want to mention is the increased competition for people who have the aptitudes and interests that lead to good actuarial work. There are more choices out there for these people today than when most of us joined the profession. We need to make sure the opportunities in our profession are well known and that our qualification process is no more of a barrier than necessary in assuring the requisite knowledge and skills. This means, among other things, that we must reduce the time it takes good actuaries to get through our examination process. The elimination of exam partitioning last year should move us in the right direction, but I believe we need to go further in order to be as competitive as we want to be with other careers. I'm optimistic that the exam system changes we are embarking on now with the help of some professional educators will result in reductions in travel time without reducing our standards. While I feel we've started in the right direction, a lot more attention and creative thinking will be needed to get to where we need to be.

The second change I want to talk about is in the skills needed from actuaries by their employers and clients. As with many other things, these changes are occurring faster today than in the past. They are creating new opportunities in many areas, and at the same time may also be causing decreased demand for some traditional actuarial work, narrowly defined. A few examples I would cite of the skills needed and forces at work are enterprise risk management, asset/liability management, securitization of risk, deregulation, and the reduction of barriers between financial services companies. The exam syllabus changes last year were designed in part to reflect these changing skill needs. The syllabus will need to evolve continually and in a way that minimizes any disruption to the lives of our candidates. At the same time, we need to facilitate the research and provide the continuing education opportunities needed by our members in order to meet their employers' current and future needs.

Third, globalization of the business world is in progress and is almost certain to continue. We need to improve the international portability of both our skills and our credentials. The importance of working with our colleagues in other countries is also growing. Actuaries in different countries need to speak the same language in an actuarial sense so we work well together for our mutual employers. To accomplish this, we as a profession need to work hard at bringing actuaries in different countries closer together through as much sharing as possible in the basic education system, continuing education programs, and research. As Allan Kaufman once said in a different context, not only do we need to do everything, but we need to do it internationally. Again, this is an area where we are headed in the right direction, but continual attention and creative thinking will be needed to get to where we want to be.

The fourth change is in the information expectations of our members and our exam candidates. As technological changes have made it easier to share information, people have come to expect more information from everywhere, including from the

CAS. I have never thought of the CAS as operating in secrecy, but some of our members and many of our exam candidates do think that. We need to go out of our way to make information about CAS activities readily available, and make every effort to obtain the input of our colleagues on important issues. This is not just more practical today. It is essential in order for many of our members to continue to feel they are part of the CAS and to want to contribute to it. In my view, we should also provide the same information to exam candidates and think of them as part of our profession. We've made some changes that move us in the right direction. For example, the Board agendas and minutes are now posted on the Web site in the "Member Services" section, which used to be called the "Members Only" section. Those and most other items in the "Member Services" section will soon be available to exam candidates and others.

I'd like to say a few words about our election this year. A lot of the discussion around the election, especially by exam candidates, had to do with what they perceived as secrecy by the CAS. Good arguments can be made both for and against changing the election system to produce a competitive election each year. Regardless of the conclusion on that question, the fact that there was a competitive election this year was very helpful in my opinion. Sholom Feldblum demonstrated that a strong candidate not only can get on the ballot, but has a significant chance of winning. It was a close election. I want to thank both Sholom Feldblum and Gail Ross publicly for all the effort they expended in the process.

In summary, there are four areas where I believe our environment is changing and our profession needs careful attention by volunteers to keep us on the right track, and creative thinking by those volunteers to get us where we need to be. The four areas are attracting talented people to our profession, meeting changing skill needs in the marketplace, improving the international portability of our skills, and sharing information more fluidly in both directions with our members and exam candidates.

THANK YOUS

The past year has been a very busy time for the CAS. I'd like to thank the hundreds of CAS members who volunteered their time in the past year for the good of our profession. I'll start by asking the Board of Directors from the past year to stand. This is a group of incredibly talented and dedicated professionals. They did a lot of extra work this year, including participation in what was probably a record number of Board conference calls.

Next I'd like to ask the Executive Council to stand. Bob Conger, Abbe Bensimon, LeRoy Boison, Dave Chernick, Gary Josephson, Mary Frances Miller, and Shelly Rosenberg—you have each done a great job for our profession and it's been a real pleasure working with each of you. Thank you.

For everyone else in the room, I'd like to ask you to stand and remain standing if, in the past year, you served on a committee or task force of the CAS, or the Academy or the Canadian Institute, the ASB, or one of our Regional Affiliates, or wrote a paper, or gave a presentation, or contributed in any other way to our profession. You can see that our profession has a high level of participation by its members.

Will the CAS staff please stand. Most of them are probably already standing or else out of the room, keeping things running smoothly as usual. They carry out the day-to-day details for the CAS in jobs that usually aren't noticed unless something goes wrong. Since they rarely let that happen, now is our chance to notice them and thank them for the excellent job they do for our members.

I'd also like to thank my wife, Linda, and daughters Megan and Kelsey, for their support and for putting up with me—there was even more to put up with in the past year. Unfortunately, they were not able to be here today—school is important and they get to hear my views often enough.

I also want to thank my colleagues at Milliman for their support, both financially and professionally. It would have been very difficult to do this job without their support.

Finally, I'd like to thank all of you for allowing me to serve the CAS as president for the past year. It has been a real pleasure working with each of you I came in contact with. I am very confident of the CAS's future based on what I have seen in you.

MINUTES OF THE 2001 CAS ANNUAL MEETING

November 11–14, 2001

MARRIOTT MARQUIS

ATLANTA, GEORGIA

Sunday, November 11, 2001

The Board of Directors held their regular quarterly meeting from 9:00 a.m. to 5:00 p.m.

Registration was held from 4:00 p.m. to 6:00 p.m.

From 5:30 p.m. to 6:30 p.m., there was a special presentation to new Associates and their guests. All 2001 CAS Executive Council members briefly discussed their roles in the Society with the new members. In addition, Steven G. Lehmann, who is a past president of the CAS, gave a short talk on the American Academy of Actuaries' (AAA) Casualty Practice Council.

A welcome reception for all members and guests was held from 6:30 p.m. to 7:30 p.m.

Monday, November 12, 2001

Registration continued from 7:00 a.m. to 8:00 a.m.

CAS President Patrick J. Grannan opened the business session at 8:00 a.m. and introduced members of the Executive Council and the CAS Board of Directors. Mr. Grannan also recognized past presidents of the CAS who were in attendance at the meeting, including: Robert A. Anker (1996), Phillip N. Ben-Zvi (1985), Ronald L. Bornhuetter (1975), Charles A. Bryan (1990), David P. Flynn (1992), Michael Fusco (1989), Alice H. Gannon (2000), David G. Hartman (1987), Charles C. Hewitt Jr. (1972), M. Stanley Hughey (1974), Frederick W. Kilbourne (1982), Steven G. Lehmann (1998), W. James MacGinnitie (1979), Jerome

A. Scheibl (1980), Michael L. Toothman (1991), and Mavis A. Walters (1997).

Mr. Grannan also recognized special guests in the audience: Robert A. Anker, president-elect of the American Academy of Actuaries; W. James MacGinnitie, president of the Society of Actuaries; Jean-Louis Massé, president of the Canadian Institute of Actuaries; and John P. Ryan, council member of the Institute of Actuaries.

Mr. Grannan then announced the results of the CAS elections. The next president will be Robert F. Conger, and the president-elect will be Gail M. Ross. Members of the CAS Executive Council for 2001 – 2002 will be: Sheldon Rosenberg, vice president – administration; Mary Frances Miller, vice president – admissions; Roger A. Schultz, vice president – continuing education; LeRoy A. Boison, vice president – international; Christopher S. Carlson, vice president – programs and communication; and Gary R. Josephson, vice president – research and development. New members of the CAS Board of Directors are Phillip N. Ben-Zvi, Curtis Gary Dean, David G. Hartman, and Janet R. Nelson.

Gary R. Josephson announced the 92 new Associates, and Robert F. Conger announced the 116 new Fellows. The names of these individuals follow.

NEW FELLOWS

Jason R. Abrams
Stephen A. Alexander
Katherine H. Antonello
Anju Arora
Peter Attanasio
Craig Victor Avitabile
Jeremy Todd Benson
Eric D. Besman
Kristen Maria Bessette
Neil M. Bodoff
Jeremy James Brigham
Russell J. Buckley
Kevin D. Burns
Hayden Heschel Burrus
Sharon C. Carroll
Jill C. Cecchini
Richard M. Chiarini
Michael Joseph Christian
Louise Chung-Chum-Lam
Jeffrey J. Clinch
Maryellen J. Coggins
John T. Devereux
Kevin Francis Downs
Louis-Christian Dupuis
Wayne W. Edwards
Richard James Engelhuber

Jonathan Palmer Evans
Weishu Fan
Sara Frankowiak
Dustin Wayne Gary
Amy L. Gebauer
Bradley G. Gipson
Theresa Giunta
Karl Goring
Lisa N. Guglietti
Elizabeth Susan Guven
Nasser Hadidi
Brian D. Haney
Kevin B. Held
Mark D. Heyne
Glenn R. Hiltpold
Richard Michael Holtz
Susan Elizabeth Innes
Craig D. Isaacs
Patrice Jean
Weidong Wayne Jiang
Susan K. Johnston
Bryon Robert Jones
Sean M. Kennedy
David R. Kennerud
Susan E. Kent
Susanlisa Kessler
Richard F. Kohan
Richard Scott Krivo
Scott C. Kurban
Steven M. Lacke
Julie-Linda Laforce
Isabelle La Palme
Aaron Michael Larson
Dennis H. Lawton
Thomas V. Le
James P. Leise
Christian Lemay
John N. Levy
Matthew Allen Lillegard
Kathleen T. Logue
Cara M. Low
Robb W. Luck
Joshua Nathan Mandell
Jason Aaron Martin
Heather L. McIntosh
Christian Menard
Richard Ernest Meuret
Michael J. Miller
Sean Robert Nimm
Sylvain Nolet
Corine Nutting
Steven Brian Oakley
Randall William Oja
Christy Beth Olson
Rodrick Raymond Osborn
Apryle L. Oswald
Cosimo Pantaleo
John R. Pedrick
Kristin Sarah Piltzecker
Dylan P. Place
Mario Richard
John R. Rohe
Christine R. Ross
Asif M. Sardar
Parr T. Schoolman
Steven George Searle
Joseph Allen Smalley
Klayton N. Southwood
Theodore S. Spitalnick
Curt A. Stewart
Beth S. Thompson
Laura Little Thorne
Christopher S. Throckmorton
Jennifer L. Throm
Michael C. Torre
Gary S. Traicoff
Brian K. Turner
Eric Vaith
Jennifer S. Vincent
Cameron Jason Vogt
Robert J. Walling III
Kelly M. Weber
V. Clare Whitlam
Dean M. Winters
Robert F. Wolf
Kah-Leng Wong
Windrie Wong
Mary K. Woodson
Jeanne Lee Ying
Edward J. Zonenberg

NEW ASSOCIATES

Vagif Amstislavskiy
Pamela G. Anderson
Joel E. Atkins
Esther Becker
Marie-Eve J. Belanger
Kofi Boaitey
Erich A. Brandt
Maureen B. Brennan
Don J. Burbacher
James E. Calton
William Brent Carr
Ronald S. Cederburg
Hao Chai
Jennifer A. Charlonne
Alan M. Chow
Paul L. Cohen
Christopher L. Cooksey
Leanne M. Cornell
Thomas Cosenza
Michael J. Covert
Hall D. Crowder
A. David Cummings
Erik L. Donahue
Brian M. Donlan
Kiera Elizabeth Doster
Scott H. Drab
Gregory L. Dunn
Ruchira Dutta
Kyle A. Falconbury
Robin V. Fitzgerald
Patrick P. Gallagher
Genevieve Garon
Keith R. Gentile
Christopher J. Grasso
Donald B. Grimm
Jason L. Grove
Stuart J. Hayes
Scott E. Henck
Long-Fong Hsu
Katherine Jacques
Gregory O. Jaynes
Brian B. Johnson
Erik A. Johnson
Dana F. Joseph
Hye-Sook Kang
Barbara L. Kanigowski
Lawrence S. Katz
Stacey M. Kidd
Laurie A. Knoke
Anand S. Kulkarni
Stephane Lalancette
Amanda M. Levinson
Daniel A. Lowen
Sally Ann MacFadden
Teresa Madariaga Zubimendi
Jeffrey B. McDonald
Stephane McGee
Charles W. Mitchell
Matthew E. Morin
Joseph J. Muccio
Scott L. Negus
Norman Niami
Stoyko N. Nikolov
Alejandra S. Nolibos
Dianne M. Phelps
Daniel P. Post
Bill D. Premdas
John T. Raeihle
Ryan P. Royce
Giuseppe Russo
Larry J. Seymour
Brett M. Shereck
Junning Shi
Jeremy D. Shoemaker
James S. Shoenfelt
Steven A. Smith II
Anthony A. Solak
Karine St-Onge
Wei Hua Su
Christie L. Sullivan
Edward Sypher
Mary A. Theilen
Peggy J. Urness
Justin M. Van Opdorp
Gaetan R. Veilleux
Geraldine Marie L. Verano
Amy R. Waldhauer
Robert S. Weishaar
Jean P. West
William B. Wilder
Jennifer X. Wu
Run Yan

Mr. Grannan then introduced David P. Flynn, a past president of the Society, who presented the Address to New Members.

Following the address, David R. Chernick briefly highlighted the meeting's programs and thanked the CAS Program Planning Committee. Mr. Chernick then introduced Abbe S. Bensimon who announced that one *Proceedings* paper would be presented at this meeting.

Ms. Bensimon began the awards program by announcing that the 2001 Dorweiler Prize was given to two papers: "The *n*-Moment Insurance CAPM" by Thomas J. Kozik and Aaron M. Larson and "Measuring the Interest Rate Sensitivity of Loss Reserves" by Stephen P. D'Arcy and Richard W. Gorvett. Ms. Bensimon then introduced LeRoy A. Boison, vice president – international, who presented the 2001 Charles A. Hachemeister Award to Morton N. Lane for his paper, "Pricing Risk Transfer Transactions." Mr. Kozik and Mr. Larson's paper is published in this edition of the *Proceedings*. Mr. D'Arcy and Mr. Gorvett's paper is published in the 2000 *Proceedings*, and Mr. Lane's paper is published in the ASTIN *Bulletin*.

Mr. Grannan presented the 2001 CAS Matthew S. Rodermund Service Award to James R. Berquist, who was chosen for his outstanding contributions to the actuarial profession.

Mr. Grannan then requested a moment of silence in honor of those CAS members who passed away since November 2000. They are: James J. Callahan, Nathaniel Gaines, E. LeRoy Heer, James P. Jensen, Norton "Doc" Masterson, Tracey Lynn Matthew, Philip D. Miller, Harry R. Richards, Lewis H. Roberts, and Henry C. Schneiker.

In a final item of business, Mr. Grannan acknowledged a donation of $10,000 from D.W. Simpson & Company to the CAS Trust (CAST). The donation was made October 13, 2001.

Mr. Grannan then concluded the business session of the Annual Meeting and introduced the featured speaker, Dr. James Mapes.

Dr. Mapes is a management consultant and author. He is an ardent student of human behavior and communication. Dr. Mapes is dedicated to educating and encouraging his audiences to be open-minded about new options.

After a refreshment break, the first General Session was held from 10:45 a.m. to 12:15 p.m.

"A View From the Top"

Moderator: Robert V. Deutsch
Executive Vice President and CFO
CNA Insurance Companies

Panelists: Mary R. Hennessy
President and CEO
Overseas Partners Ltd.

Robert Lippincott III
CEO
AXA Corporate Solutions

Alistair Shore
Senior Vice President
Fireman's Fund Insurance Companies

Following the general session, CAS President Patrick J. Grannan gave his Presidential Address at the luncheon. At the luncheon's end, Mr. Grannan officially passed on the CAS presidential gavel to the new CAS president, Robert F. Conger.

After the luncheon, the afternoon was devoted to presentations of concurrent sessions. The panel presentations from 1:30 p.m. to 3:00 p.m. covered the following topics:

1. Introduction to the CAS Examination Committee

Moderator: Thomas G. Myers
Vice President and Actuary
Prudential Property & Casualty
Insurance Company

Panelists: J. Thomas Downey
Manager, Admissions
Casualty Actuarial Society

Floyd M. Yager
Senior Actuary
Allstate Insurance Company

Richard P. Yocius
Senior Actuary
Allstate Insurance Company

2. D & O Insurance

Moderator: John J. Lewandowski
Senior Vice President and Actuary
CNA Insurance Companies

Panelists: David F. Allen
Second Vice President
GeneralCologne Re

Mark W. Larsen
Consultant
Tillinghast-Towers Perrin

Carol A. Zacharias
Managing Director/Counsel
CNA Insurance Companies

3. The Rating Agency's View

Moderator: Eric Simpson
Senior Vice President and Chief
Financial Officer
Domestic Insurance Company

Panelists: Karen L. Davies
Vice President–Senior Analyst
Moody's Investors Service

Matthew C. Mosher
Group Vice President–P/C
A.M. Best Company

4. Loss Portfolio Transfers

Moderator: Spencer M. Gluck
Senior Managing Director, Chief Actuary
Gerling Global Financial Products

Panelists: Thomas Passante
Director
Swiss Re New Markets

Bryan C. Ware
Chief Actuary–Brokered Group
American Re-Insurance Company

5. Workers Compensation

Moderator: David M. Bellusci
Senior Vice President and Chief Actuary
Workers Compensation Insurance Rating
Bureau of California

Panelists: Jeffrey S. Estabrook
Vice President
Guy Carpenter & Company, Inc.

Robert E. Meyer
Senior Vice President
Zenith National Insurance Company

Stacy L. Mina
Associate Actuary
Liberty Mutual Group

6. Personal Lines Pricing—A New Environment

Moderator: Frank J. Karlinski
Vice President
American International Underwriters

Panelists: Claudine H. Modlin
Senior Consultant
Watson Wyatt Company

Anthony L. Alfieri
Consultant
PricewaterhouseCoopers LLP

7. Hachemeister Prize Paper: "Pricing Risk Transfer Transactions"

   Author: Morton N. Lane
   Lane Financial LLC

After a refreshment break from 3:00 p.m. to 3:30 p.m., presentations of concurrent sessions continued. A concurrent session presented earlier was repeated. Additional concurrent sessions presented from 3:30 p.m. to 5:00 p.m. were:

1. Actuarial Standard of Practice 36—An Update

   Moderator: Patricia A. Teufel
   Principal
   KPMG LLP

   Panelists: Mary D. Miller
   Actuary
   Ohio Department of Insurance

   David S. Powell
   Consulting Actuary
   Tillinghast-Tower Perrin

   James C. Votta
   Principal
   Deloitte & Touche LLP

2. Mold: The Next Looming Exposure Crisis for the Insurance Industry?

   Moderator: Jeffrey C. Kucera
   Consulting Actuary
   Miller, Herbers, Lehmann,
   & Associates, Inc.

   Panelists: David M. Golden
   Director, Commercial Lines
   National Association of Independent Insurers

   Mark Malia
   Claims Manager
   State Farm Fire & Casualty Company

	Philip O. Presley Chief Actuary Texas Department of Insurance

3. Umbrella Liability

Moderator:	Patrick J. Burns Senior Vice President–Actuarial and Financial Operations American Home Assurance Company
Panelists:	Craig J. Beardsley Second Vice President GeneralCologne Re
	Paul J. Sanchez Senior Vice President CNA Insurance Companies

4. Recruiting—Supply and Demand

Moderator/ Panelist:	Amy S. Bouska Consulting Actuary Tillinghast-Towers Perrin
Panelists:	LeNan R. Bradley Human Resources Director Fireman's Fund Insurance Company
	Patty Jacobsen Managing Partner D.W. Simpson & Company

5. ARIA Prize Paper: "Great (and not so Great) Expectations: An Endogenous Economic Explication of Insurance Cycles and Liability Crisis"

Authors:	Patrick L. Brockett University of Texas at Austin
	Hung-Gay Fung
	Gene C. Lai
	Richard MacMinn University of Nottingham

Robert C. Witt
University of Texas at Austin

An Officers' Reception for new Fellows and accompanying persons was held from 5:30 p.m. to 6:30 p.m.

A general reception for all attendees followed from 6:30 p.m. to 7:30 p.m.

Tuesday, November 13, 2001

Registration continued from 7:00 a.m. to 8:00 a.m.

The following General Sessions were held from 8:00 a.m. to 9:30 a.m.:

"From The Back Room to The Boardroom"

Moderator:	Nolan E. Asch Principal, Reinsurance Insurance Services Office, Inc.
Panelists:	Frederick O. Kist Senior Vice President and Chief Actuary Kemper Insurance Companies
	Michael A. LaMonica Product Vice President Allstate Insurance Company
	David Spiegler Senior Vice President and Chief Actuary American Re-Insurance Company

"Safer Vehicles: What Should We Know as an Insurance Industry?"

Moderator:	Gary Grant Vice President and Actuary State Farm Mutual Automobile Insurance Company
Panelists:	Frederick F. Cripe Vice President Allstate Insurance Company

Brian O'Neill
President
Insurance Institute for Highway Safety

William Shapiro
Manager, Regulatory & Product Compliance
Volvo Cars of North America, LLC

Following a break from 9:30 a.m. to 10:00 a.m., certain concurrent sessions that had been presented earlier during the meeting were repeated from 10:00 a.m. to 11:30 a.m. Additional concurrent sessions presented were:

1. Terrorism: Where Do We Go From Here?

Moderator: John J. Kollar
Vice President
Insurance Services Office, Inc.

Panelists: Matthew C. Mosher
Group Vice President
A.M. Best Company

Eric C. Nordman
Director of Research
National Association of Insurance Commissioners

David S. Powell
Consulting Actuary
Tillinghast-Towers Perrin

2. Understanding Insurance Fraud: Theory and Practice

Moderator: Richard A. Derrig
Senior Vice President
Automobile Insurers Bureau of Massachusetts

Panelists: Martin E. Ellingsworth
Director, Operations Research
Fireman's Fund Insurance Companies

Sharon Tennyson
Professor
Cornell University

3. Asbestos Claims Liabilities

Moderator: Jennifer L. Biggs
Consulting Actuary
Tillinghast-Towers Perrin

Panelists: Stephen J. Carroll
Senior Economist
RAND Institute for Civil Justice

Andrew H. Pinkowski, Esq.
Hartford Financial Services Group, Inc.
and
The Coalition for Asbestos Justice

4. American Academy of Actuaries Council on Professionalism: "Professionalism and the Reserving Actuary"

Speakers: Cara M. Blank
Consulting Actuary
Miller, Herbers, Lehmann, & Associates, Inc.

David G. Hartman
Senior Vice President and Chief Actuary
Chubb Group of Insurance Companies

Henry K. Knowlton
Vice President, Professionalism
American Academy of Actuaries

Charles L. McClenahan
Principal
MMC Enterprise Risk Consulting, Inc.

Robert W. Sturgis
Chairperson
Actuarial Board for Counseling and Discipline

5. Data Mining

Moderator/ Panelist:	Louise A. Francis Consulting Principal Francis Analytics & Actuarial Data Mining, Inc.
Panelist:	Steven J. Finkelstein Senior Manager Ernst & Young LLP

The American Academy of Actuaries held its annual meeting in conjunction with the Casualty Actuarial Society's Annual Meeting. The AAA luncheon was held from 12:00 p.m. to 2:00 p.m. Ralph Levy, Esq., a litigator with more than 25 years of experience representing actuarial, accounting, and law firms, was the keynote speaker. Following the luncheon, the AAA "Washington Insider Debate: The Tort Reform Battle — Eleanor Clift vs. Fred Barnes" was held from 2:15 p.m. to 3:30 p.m.

Entertainment and a buffet dinner were held from 7:00 p.m. to 9:30 p.m.

Wednesday, November 14, 2001

A concurrent session was repeated from 8:00 a.m. to 9:30 a.m. Additional concurrent sessions presented at this time were:

1. Capital Management

Moderator:	François Morin Consulting Actuary Tillinghast-Towers Perrin
Panelist:	Thomas A. Weidman Senior Vice President and Chief Actuary XL America, Inc.

2. NAIC Current Issues

Moderator:	Frederick O. Kist Senior Vice President and Chief Actuary Kemper Insurance Companies

Panelists: Vincent Laurenzano
Insurance Consultant
Stroock, Stroock, and Lavan

Joseph B. Sieverling
Vice President and Director of Financial Services
Reinsurance Association of America

Robert Wake
Managing Examiner
Maine Bureau of Insurance

3. Opportunities for Volunteering

Moderator: Roger A. Schultz
Assistant Vice President
Allstate Insurance Company

Panelists: Regina M. Berens
Vice President and Chief Actuary
Scruggs Consulting

Dale Porfilio
Pricing Director
Kemper Insurance Companies

Daniel G. Roth
Vice President and Chief Actuary
CNA Insurance Companies

The *Proceedings* paper presented during this time was:

1. "Is the Efficient Frontier Efficient?"

Authors: William C. Scheel
DFA Technologies LLC

William J. Blatcher
AEGIS Insurance Services

Gerald S. Kirschner
Classic Solutions Risk Management, Inc.

John J. Denman
AEGIS Insurance Services

After a break from 9:30 a.m. to 10:00 a.m., the final General Session was held from 10:00 a.m. to 11:30 a.m.

"CAS—Approaching 100"

Moderator:	Phillip N. Ben-Zvi Principal PricewaterhouseCoopers LLP
Panelists:	Michael J. Miller Principal and Consulting Actuary Miller, Herbers, Lehmann, & Associates, Inc.
	David G. Hartman Senior Vice President and Chief Actuary Chubb Group of Insurance Companies
	Jeffrey D. White Regional Actuary St. Paul Fire and Marine Insurance Company

Patrick J. Grannan officially adjourned the 2001 CAS Annual Meeting at 11:45 a.m. after closing remarks and an announcement of future CAS meetings.

Attendees of the 2001 CAS Annual Meeting

The 2001 CAS Annual Meeting was attended by 303 Fellows, 137 Associates, and 57 Guests. The names of the Fellows and Associates in attendance follow:

FELLOWS

Jason R. Abrams	Nolan E. Asch	William P. Ayres
Stephen A. Alexander	Richard V. Atkinson	Andrea C. Bautista
Terry J. Alfuth	Roger A. Atkinson	David M. Bellusci
Robert A. Anker	Peter Attanasio	Abbe Sohne Bensimon
Katherine H. Antonello	Craig Victor Avitabile	Jeremy Todd Benson
Anju Arora	Karen F. Ayres	Phillip N. Ben-Zvi

Regina M. Berens
François Bertrand
Eric D. Besman
Lisa M. Besman
Kristen Maria Bessette
Neil A. Bethel
David R. Bickerstaff
Jennifer L. Biggs
Terry J. Biscoglia
Jonathan Everett Blake
Ralph S. Blanchard
Cara M. Blank
Daniel D. Blau
Neil M. Bodoff
LeRoy A. Boison
Ronald L. Bornhuetter
Charles H. Boucek
Amy S. Bouska
Wallis A. Boyd
Paul Braithwaite
Mark L. Brannon
Yaakov B. Brauner
Jeremy James Brigham
Dale L. Brooks
Charles A. Bryan
James E. Buck
Russell J. Buckley
Kevin D. Burns
Hayden Heschel Burrus
Christopher S. Carlson
Bethany L. Cass
Jill C. Cecchini
David R. Chernick
Richard M. Chiarini
Michael Joseph Christian
Louise Chung-Chum-Lam
Gary T. Ciardiello
Jeffrey J. Clinch
Maryellen J. Coggins
Robert F. Conger
Mark Crawshaw
Frederick F. Cripe
Stephen P. D'Arcy
Karen L. Davies
Timothy Andrew Davis
Curtis Gary Dean
Kris D. DeFrain
Robert V. Deutsch
John T. Devereux
Kevin Francis Downs
Louis-Christian Dupuis
Richard D. Easton
Grover M. Edie
Gary J. Egnasko
Valere M. Egnasko
Richard James Engelhuber
Paul E. Ericksen
Janet L. Fagan
Weishu Fan
Dennis D. Fasking
Denise A. Feder
Vicki Agerton Fendley
Ginda Kaplan Fisher
William G. Fitzpatrick
Louise A. Francis
Barry A. Franklin
Sara Frankowiak
Noelle Christine Fries
John E. Gaines
Alice H. Gannon
Dustin Wayne Gary
Amy L. Gebauer
John F. Gibson
Bonnie S. Gill
Judy A. Gillam
William R. Gillam
Bradley G. Gipson
John T. Gleba
Ronald E. Glenn
Spencer M. Gluck
James F. Golz
Patrick J. Grannan
Gary Grant
Alex R. Greene
Carleton R. Grose
Victoria Grossack
Lisa N. Guglietti
Elizabeth Susan Guven
Nasser Hadidi
James A. Hall
Brian D. Haney
David G. Hartman
Jeffery Tim Hay
David H. Hays
Christopher Ross Heim
Kevin B. Held
Mary R. Hennessy
Teresa J. Herderick
Charles C. Hewitt
Mark D. Heyne
Glenn R. Hiltpold
Christopher Todd Hochhausler
Todd Harrison Hoivik
Richard Michael Holtz

Marie-Josée Huard
David Dennis Hudson
M. Stanley Hughey
Susan Elizabeth Innes
Daniel B. Isaac
Craig D. Isaacs
Patrice Jean
Marvin A. Johnson
Susan K. Johnston
Bryon Robert Jones
Gary R. Josephson
Stephen H. Kantor
Frank J. Karlinski
Clive L. Keatinge
James M. Kelly
Sean M. Kennedy
David R. Kennerud
Susan E. Kent
Susanlisa Kessler
Frederick W. Kilbourne
Chang Seob Joe Kim
Frederick O. Kist
Michael F. Klein
Leon W. Koch
Richard F. Kohan
John J. Kollar
Thomas J. Kozik
Gustave A. Krause
Richard Scott Krivo
Claudia Anita Krucher
Jane Jasper Krumrie
Jeffrey L. Kucera
Andrew E. Kudera
Kimberly J. Kurban
Scott C. Kurban
Bertrand J. LaChance
Steven M. Lacke
Julie-Linda Laforce
Dean K. Lamb
Michael A. LaMonica
Dennis L. Lange
Isabelle La Palme
Aaron Michael Larson
Pierre Guy Laurin
Dennis H. Lawton
Thomas V. Le
Marc-Andre Lefebvre
Steven G. Lehmann
James P. Leise
Christian Lemay
Jennifer McCullough Levine
John J. Lewandowski
Matthew Allen Lillegard
Kathleen T. Logue
Cara M. Low
Robb W. Luck
William R. Maag
W. James MacGinnitie
Blaine C. Marles
Heather L. McIntosh
Kelly S. McKeethan
William T. Mech
Christian Menard
Timothy Messier
Claus S. Metzner
Robert E. Meyer
Glenn G. Meyers
Robert S. Miccolis
David L. Miller
Mary D. Miller
Mary Frances Miller
Michael J. Miller
Michael J. Miller
Stacy L. Mina
Neil B. Miner
Claudine H. Modlin
Brian A. Montigney
Anne Hoban Moore
Kenneth B. Morgan
François Morin
Robert Joseph Moser
Matthew C. Mosher
Roosevelt C. Mosley
Janet R. Nelson
James R. Nikstad
Sean Robert Nimm
Ray E. Niswander
Corine Nutting
James L. Nutting
Steven Brian Oakley
Randall William Oja
Christy Beth Olson
Rodrick Raymond Osborn
David Anthony Ostrowski
Apryle L. Oswald
David J. Otto
Teresa K. Paffenback
Rudy A. Palenik
Donald D. Palmer
Jennifer J. Palo
Cosimo Pantaleo
Thomas Passante
Kathleen M. Pechan
John R. Pedrick

Kristin Sarah Piltzecker
Dale S. Porfilio
Deborah W. Price
Karen L. Queen
Kara Lee Raiguel
Mario Richard
John R. Rohe
William P. Roland
Steven Carl Rominske
A. Scott Romito
Deborah M. Rosenberg
Christine R. Ross
Gail M. Ross
Daniel G. Roth
Jerome A. Scheibl
Parr T. Schoolman
Peter J. Schultheiss
Roger A. Schultz
Allan I. Schwartz
Steven George Searle
Ollie L. Sherman
Alastair C. Shore
Lisa A. Slotznick
Joseph Allen Smalley
Klayton N. Southwood
Keith R. Spalding
Joanne S. Spalla
Angela Kaye Sparks
David Spiegler
Daniel L. Splitt
John A. Stenmark
Curt A. Stewart
Douglas N. Strommen
Robert W. Sturgis
Scott J. Swanay
Susan T. Szkoda
Karen F. Terry
Patricia A. Teufel
Beth S. Thompson
Christopher S. Throckmorton
Jennifer L. Throm
Barbara H. Thurston
Michael L. Toothman
Michael C. Torre
Gary S. Traicoff
Warren B. Tucker
Brian K. Turner
Gail E. Tverberg
Eric Vaith
William R. Van Ark
Jennifer S. Vincent
Michael A. Visintainer
James C. Votta
Kyle Jay Vrieze
Christopher P. Walker
Robert J. Walling
Michael C. Walsh
Mavis A. Walters
Bryan C. Ware
Kelly A. Wargo
Thomas A. Weidman
Scott P. Weinstein
Peter A. Weisenberger
Clifford Wess
Patrick L. Whatley
Jeffrey D. White
Dean M. Winters
Susan E. Witcraft
Robert F. Wolf
Kah-Leng Wong
Windrie Wong
Patrick B. Woods
Mary K. Woodson
Floyd M. Yager
Gerald Thomas Yeung
Richard P. Yocius
Edward J. Zonenberg

ASSOCIATES

Vagif Amstislavskiy
Pamela G. Anderson
Joel E. Atkins
David B. Bassi
Brian P. Beckman
Kofi Boaitey
Thomas S. Boardman
Erich A. Brandt
Maureen B. Brennan
Steven A. Briggs
Don J. Burbacher
James E. Calton
Janet P. Cappers
Ronald S. Cederburg
Hao Chai
Jennifer A. Charlonne
Harry Sigen Chen
Joyce Chen

Alan M. Chow
J. Paul Cochran
Paul L. Cohen
Christopher L. Cooksey
Michael J. Covert
Hall D. Crowder
A. David Cummings
Catherine L. DePolo
Gordon F. Diss
Erik L. Donahue
Brian M. Donlan
Kiera Elizabeth Doster
Scott H. Drab
Gregory L. Dunn
Alice H. Edmondson
Anthony D. Edwards
Brian A. Evans
Kyle A. Falconbury
Steven J. Finkelstein
William M. Finn
Robin V. Fitzgerald
Keith R. Gentile
Mary Jo E. Godbold
Donald B. Grimm
Christopher Gerald Gross
Jason L. Grove
Joyce G. Hallaway
Stuart J. Hayes
Philip E. Heckman
Kristina S. Heer
Hans Heldner
Scott E. Henck
Joseph A. Herbers
Gary P. Hobart
Suzanne Barry Holohan
Long-Fong Hsu
Jeffrey R. Ill
Jean-Claude Joseph Jacob
Erik A. Johnson
William Russell Johnson
Burt D. Jones
Hye-Sook (Erin) Kang
Barbara L. Kanigowski
Pamela A. Kaplan
David L. Kaufman
Daniel R. Keddie
Stacey M. Kidd
Paul E. Kinson
Laurie A. Knoke
Chung-Kuo Kuo
Frank O. Kwon
Stephane Lalancette
Todd William Lehmann
Bradley H. Lemons
Amanda M. Levinson
Elizabeth Long
Daniel A. Lowen
Sally Ann MacFadden
Teresa Madariaga Zubimendi
Scott A. Martin
John R. McCollough
Jeffrey B. McDonald
Stephane McGee
Neil L. Millman
Charles W. Mitchell
Karen M. Moritz
Michael W. Morro
Joseph J. Muccio
Scott L. Negus
Stoyko N. Nikolov
Alejandra S. Nolibos
Christopher Maurice Norman
Leigh S. Oates
Willard W. Peacock
Rosemary Catherine Peck
Claude Penland
Anthony J. Pipia
Daniel P. Post
David S. Powell
Bill D. Premdas
James E. Rech
W. Vernon Rice
Ryan P. Royce
John P. Ryan
Michael Sansevero
Susan C. Schoenberger
Peter Abraham Scourtis
Michael L. Scruggs
Larry J. Seymour
David Garrett Shafer
Jeremy D. Shoemaker
James S. Shoenfelt
L. Kevin Smith
Lee Oliver Smith
Steven A. Smith
Anthony A. Solak
Calvin C. Spence

William G. Stanfield
Wei Hua Su
Lisa M. Sukow
Mary A. Theilen
Wendy Artecona Thompson
Peggy J. Urness
Justin M. Van Opdorp
Gaetan R. Veilleux
Geraldine Marie L. Verano
Jerome F. Vogel
Roger C. Wade
Amy R. Waldhauer
Michelle M. Wass
Karen E. Watson
Kevin E. Weathers
Robert S. Weishaar
David L. Whitley
Rosemary Gabriel Wickham
Joel F. Witt
Perry Keith Wooley
Run Yan
Yin Zhang

REPORT OF THE VICE PRESIDENT–ADMINISTRATION

This report provides a one-year summary of CAS activities since the 2000 CAS Annual Meeting. I will first comment on these activities as they relate to the following purposes of the Casualty Actuarial Society as stated in our Constitution:

1. Advance the body of knowledge of actuarial science applied to property, casualty, and similar risk exposures;
2. Establish and maintain standards of qualifications for membership;
3. Promote and maintain high standards of conduct and competence for the members; and
4. Increase the awareness of actuarial science.

I will then provide a summary of other activities that may not relate to a specific purpose, but yet are critical to the ongoing vitality of the CAS. Finally, I will summarize the current status of our finances and key membership statistics.

The CAS discussion paper programs, *Proceedings*, and the *Forum* contribute to the attainment of purpose #1. The winter, spring, summer, and fall volumes of the *Forum* focused on topics in ratemaking; data management, quality, and technology; dynamic financial analysis; reinsurance; and reserving. The discussion paper program volume addressed financial and accounting systems and issues associated with globalization of insurance. The *Proceedings* papers addressed topics in stochastic claims reserving, capital asset pricing models, claim adjustment expense reserves, risk distribution, underwriting cycles and business strategies, and sampling error measurement within efficient frontiers.

The CAS Valuation, Finance, and Investments Committee (VFIC), under the leadership of Harvey A. Sherman, prepared a comprehensive note to assist the actuary considering materiality in the context of ASOP No. 36. The American Academy of Actuaries Committee on Property and Liability Financial Reporting requested that VFIC prepare the document. The note, entitled "Materiality and ASOP No. 36: Considerations for the Practicing Actuary," was published in the Winter 2001 *Forum*.

The Textbook Rewriting Committee, chaired by Robert Lowe, completed a nearly three-year project, publishing the Fourth Edition of *Foundations of Casualty Actuarial Science*, the "CAS Textbook." The Fourth Edition is a significant revision of the original version, with some new authors and refashioning all chapters to an introductory approach to basic actuarial concepts. A new chapter on risk theory was added as well.

In regards to purpose #2, there were a number of developments in the CAS education and examination system during the last year. The Board of Directors' principal activity during 2001 concerned review of the CAS education and examination system and, in particular, revision of the new Preliminary Actuarial Exams 3 and 4, which are jointly sponsored by the CAS and SOA. The Board, at its November 12, 2000 meeting, having noted the poor performance of casualty candidates on these exams and following a review of the report of the CAS Task Force on Exams 3 and 4, had directed that short-, intermediate-, and long-term actions be taken to address problems identified in the task force report. Copies of the task force report were distributed to CAS members and candidates under cover of President Pat Grannan's letter of December 21, 2000.

As a short-term result, as requested by the CAS Board of Directors, the joint committees for Exams 3 and 4 reduced the amount of material for these examinations (especially for Exam 3) starting with the Spring 2001 exams. In addition, the Board decided that the CAS should commission study notes for Exams 3 and 4 to make the syllabus more amenable to self-study. The joint Preliminary Actuarial Examinations Syllabus and Education Committee determined that some of the loss models material should be given the highest priority in this regard, and Stuart

Klugman has drafted a study note (to supplant textbook material on loss models) for the syllabus for the Spring 2002 exam sitting.

The Board in March and May 2001 adopted "Principles of the CAS for Basic Education," a pass mark policy, and policy for joint sponsorship of exams. The Board directed in May that the CAS work with the SOA to develop creative approaches to reducing the amount of life contingencies material on Exams 3 and 4 as soon as possible.

In September, the Board reviewed the audit of the development and administration of all CAS exams by The Chauncey Group International. The CAS retained this professional education consulting firm to assist in better articulating the learning objectives for CAS exams and help train examination committee members in developing good, thinking questions that test whether the learning objectives have been met.

The Board in November took several more actions regarding examinations. For Exams 3 and 4, the Board moved that these exams are not adequately meeting the needs of the CAS. The Admissions Committees and Executive Council were charged with designing and preparing Exams 3 and 4 that are appropriate for casualty actuaries, either administered jointly with the SOA or otherwise by the CAS. The Executive Council was directed to develop an implementation plan.

In November, the Board recommended as guidance to the CAS Admissions Committees the expectation that the median time required to move from first full-time actuarial employment to attainment of Fellowship should be in the range of five to seven years.

For the longer term, the Board in September approved a new Task Force on Future Education, chaired by Mary Frances Miller. The task force is charged with evaluating the CAS admissions process and recommending appropriate changes to ensure that the process provides basic education in all areas necessary to the education of all casualty actuaries, ensures that candidates for admission have demonstrated mastery of the critical components of casualty actuarial practice, and minimizes the expected amount of time to achieve CAS membership.

A quality program of continuing education and a Code of Professional Conduct support purpose #3: "promote and maintain high standards of conduct and competence for the members."

The CAS provides educational opportunities through the publication of actuarial materials and the sponsorship of meetings and seminars. This year's sessions included the following, shown with the number of CAS members in attendance:

Meetings:

Meeting	Location	CAS Members
Spring	Miami Beach	410
Annual	Atlanta	433

Seminars:

Topic	Location	CAS Members
Ratemaking	Las Vegas	406
Understanding the Enterprise Risk Management Process	San Francisco	94
Reinsurance	Washington, DC	218
Dynamic Financial Analysis	Boston	114
Casualty Loss Reserve	New Orleans	382
Appointed Actuary—Joint CAS/CIA	Montreal, Canada	271*
Course on Professionalism—Dec '00	2 locations	80 Students
Course on Professionalism—June '01	3 locations	85 Students

*Total attendance. Separate count for CAS members is not available.

Limited attendance seminars included "Practical Applications of Loss Distributions" (two seminars) and "Reinsurance."

During 2001, the CAS participated in, promoted, and cosponsored several other actuarial organizations' events. In July, the CAS hosted the ASTIN Colloquium in Washington, DC. The CAS Reinsurance Seminar was held in conjunction with this

Colloquium, including a one-day joint session with ASTIN. The CAS also continues to support ASTIN by placing all ASTIN Bulletin papers on the CAS Web Site in the Download Library.

The American Academy of Actuaries held its Annual Meeting in conjunction with the Casualty Loss Reserve Seminar (CLRS) in New Orleans during September 2001. When the horrific attacks on the World Trade Center and Pentagon took place on September 11, the CLRS was in its second day. The remaining sessions at the Seminar were cancelled, having completed five of the scheduled eight sessions. The limited attendance seminar on Asset/Liability Management & Principles of Finance the following day in New Orleans was cancelled.

The CAS October 2001 Special Interest Seminar sessions were incorporated into a joint General Insurance Research Organization (GIRO)/CAS Convention in Glasgow, Scotland during October 3–6. The theme for this convention was "Globalization and Technology: Issues and Solutions." Thirty-eight CAS members attended (others had to cancel as a result of the September 11 events). The joint convention was considered a success and facilitated closer ties between GIRO and the CAS for joint research efforts and future participation in GIRO meetings.

The CAS continued its program to offer training in general business skills to CAS members, creating the Task Force for Delivery of General Business Skills Training, headed by Kathy Olcese. The task force's mission was researching available sources to teach sessions on general business skills and recommending a plan to offer these educational opportunities to CAS members. Two sessions on executive presentation skills, one for beginners and the other for advanced presenters, were held at the 2001 CAS Spring Meeting.

Other CAS efforts in support of purpose #3 include the appointment of two new liaison representatives. Mary Frances Miller became liaison representative of the American Academy of Actuaries Committee on Qualifications, and Donna S. Munt was appointed liaison representative to the Institute of Actuaries/Faculty of Actuaries Future Education Strategy Group.

In support of purpose #4, which is to increase the awareness of actuarial science, the CAS and SOA jointly produced a new actuarial career encouragement video. It has been distributed to all CAS University Liaisons and is available to the Regional Affiliates.

The CAS Web Site supports all four purposes. Some highlights from the past year that have not been mentioned elsewhere in this report include: (1) a redesigned home page to incorporate color branding, simplified navigation tools, and a unique online logo; (2) online registration for meetings and seminars; and (3) a new section created for Actuaries in Nontraditional Roles (five case studies are posted there).

The CAS Web Site was used to advise the membership of the status of 36 members who worked in the North and South Towers of the World Trade Center. Sadly, Philip D. Miller, FCAS 1975, was among the thousands who lost their lives in this tragedy.

OTHER CAS ACTIVITIES

During 2001, several other CAS activities contributed to the ongoing vitality of the organization. Following is a summary of these activities.

The CAS formed a search committee to find candidates for the post of CAS Executive Director. Cynthia R. Ziegler was chosen to succeed James H. “Tim” Tinsley, who retired December 31, 2001, after 11 years of service.

As recommended by the Board, the Fellows voted in June 2001 to amend Article IX, Public Expression of Professional Opinion, to authorize the Board to direct the CAS Delegate to the International Actuarial Association to cast a vote on behalf of the CAS on a proposed public expression of professional opinion to be issued by the IAA and to allow the IAA to list the CAS as a supporting organization when the vote is positive.

The Board stepped up initiatives to increase communication between the Board and the members, adopting and publishing a policy for members to attend Board meetings. Draft meeting agendas and approved minutes are now posted on the CAS Web Site.

The Executive Council in June approved a petition to establish a new CAS Regional Affiliate, Central States Actuarial Forum. Membership in this Regional Affiliate will be drawn from Colorado, Iowa, Kansas, Missouri, Nebraska, North Dakota, and South Dakota.

The lease for the CAS Office suite in Arlington, Virginia was amended to expand the space for future growth and provide appropriate work preparation facilities and extend the lease by five years to 2006 with an option to renew again for another five years.

The CAS introduced a new member service to offer forty *Proceedings* volumes, 1960 to present, on a two CD set for $24. The CDs contain advanced search and navigation features.

In an outreach effort the CAS formed the Task Force on Promoting Nonmember Attendance at CAS Seminars. Under the guidance of leader Robert N. Darby, the task force's goal was to determine the reasons for declining nonmember attendance at CAS seminars, formulate a strategy and implementation plan to boost attendance for Executive Council approval, and work with seminar committees to implement the approved strategy.

MEMBERSHIP STATISTICS

Membership growth continued with 125 new Associates, 145 new Fellows, and 3 new Affiliates. The total number of members as of November 2001 was 3,564, up 3.15% for the year.

For the first time in many years, there were two candidates for the position of president-elect for 2001–02. Gail M. Ross, the nominee of the Nominating Committee, received 55% of the votes. Sholom Feldblum, who was nominated by a petition from the Fellows, received 45% of the votes. A record 1,279 Fellows voted (58% of the total Fellows). New members elected to the Board of Directors for next year are Phillip N. Ben-Zvi, Curtis Gary Dean, David G. Hartman, and Janet R. Nelson. Robert F. Conger assumed the presidency.

The Executive Council, with primary responsibility for day-to-day operations, met either by teleconference or in person at least once a month during the year. The Board of Directors elected the following vice presidents for the coming year: Vice President–Administration, Sheldon Rosenberg; Vice President–Admissions, Mary Frances Miller; Vice President–Continuing Education, Roger A. Schultz; Vice President–International, LeRoy A. Boison; Vice President–Programs and Communications, Christopher S. Carlson; and Vice President–Research and Development, Gary R. Josephson.

FINANCIAL STATUS

The CPA firm of Langan Associates has been engaged to examine the CAS books for fiscal year 2001, and its findings will be reported by the Audit Committee to the Board of Directors in March 2002. The fiscal year ended with an audited Net Loss from Operations of $207,947 compared to a budgeted Net Loss of $351,770. Fiscal year 2001 had been budgeted for a net loss because of the strong equity position that resulted from higher than expected income in prior years.

Members' equity now stands at $2,938,698. This represents a decrease in equity of $30,181 from the amount reported last year. In addition to the net loss from operations, there was interest income of $199,305 and unrealized gain of $48,912 recorded to adjust marketable securities to market value as of September

30, 2001. There was also a total net decrease of $70,451 in various research, prize, and scholarship accounts arising from the difference between incoming funds and interest earned less expenditures. These amounts are not reflected in net income from operations.

For 2001–2002, the Board of Directors has approved a budget of approximately $4.4 million, a decrease of about $250,000 compared to the prior fiscal year. Members' dues for next year will be $310, an increase of $10, while fees for the Subscriber Program will increase by $10 to $380. A $30 discount is available to members and subscribers who elect to receive the *Forums* and *Discussion Paper Program* in electronic format from the CAS Web Site.

Respectfully submitted,
Sheldon Rosenberg
Vice President–Administration

FINANCIAL REPORT
FISCAL YEAR ENDED 9/30/2001

OPERATING RESULTS BY FUNCTION

FUNCTION	*INCOME*	*EXPENSE*	*DIFFERENCE*
Membership Services	$ 1,061,832	$ 1,483,473	$ (421,641)
Seminars	1,355,019	1,126,175	228,844
Meetings	574,678	654,658	(79,980)
Exams	2,593,427 (a)	2,530,386 (a)	63,041
Publications	34,664	32,874	1,790
TOTALS FROM OPERATIONS	$ 5,619,620	$ 5,827,566	$ (207,946)
Interest Income			199,305
Unrealized Gain/(Loss) on Marketable Securities			48,912
TOTAL NET INCOME (LOSS)			$ 40,271

NOTE: (a) Includes $1,628,025 of Volunteer Services for income and expense (SFAS 116).

BALANCE SHEET

ASSETS	*9/30/2000*	*9/30/2001*	*DIFFERENCE*
Checking Accounts	$ 30,029	$ 368,491	$ 338,462
T-Bills/Notes	3,511,251	3,102,104	(409,147)
Accrued Interest	43,006	37,791	(5,215)
Prepaid Expenses	90,789	59,492	(31,297)
Prepaid Insurance	16,719	19,737	3,018
Accounts Receivable	2,980	48,715	45,735
Textbook Inventory	3,499	174	(3,325)
Computers, Furniture	406,702	390,925	(15,777)
Less: Accumulated Depreciation	(307,174)	(297,268)	9,906
TOTAL ASSETS	$ 3,797,801	$ 3,730,160	$ (67,642)

LIABILITIES	*9/30/2000*	*9/30/2001*	*DIFFERENCE*
Exam Fees Deferred	$ 325,339	$ 466,121	$ 140,782
Annual Meeting Fees Deferred	44,605	32,345	(12,260)
Seminar Fees Deferred	42,750	1,050	(41,700)
Accounts Payable and Accrued Expenses	349,159	246,072	(103,087)
Deferred Rent	2,652	0	(2,652)
Unredeemed Vouchers	14,400	0	(14,400)
Accrued Pension	50,016	45,875	(4,141)
TOTAL LIABILITIES	$ 828,921	$ 791,462	$ (37,459)

MEMBERS' EQUITY

Unrestricted	*9/30/2000*	*9/30/2001*	*DIFFERENCE*
CAS Surplus	$ 2,561,879	$ 2,602,150	$ 40,271
Michelbacher Fund	110,185	116,245	6,060
CAS Trust	63,628	85,827	22,199
Research Fund	160,972	117,718	(43,254)
ASTIN Fund	54,910	0	(54,910)
Subtotal Unrestricted	$ 2,951,574	$ 2,921,941	$ (29,633)
Temporarily Restricted			
Scholarship Fund	$ 6,610	$ 6,475	$ (135)
Rodermund Fund	10,695	10,283	(412)
Subtotal Restricted	17,305	16,758	(547)
TOTAL EQUITY	$ 2,968,879	$ 2,938,698	$ (30,181)

Sheldon Rosenberg, Vice President–Administration

This is to certify that the assets and accounts shown in the above financial statement have been audited and found to be correct.

CAS Audit Committee: Frederick O. Kist, Chairperson;
Ralph S. Blanchard, John F. Gibson, and Anthony J. Grippa

2001 EXAMINATIONS—SUCCESSFUL CANDIDATES

Examinations for Exams 5, 7–Canada, 7–United States, and 8 of the Casualty Actuarial Society were held on April 30, and May 1 and 2, 2001. Examinations for Exams 6 and 9 of the Casualty Actuarial Society were held on October 30 and 31, 2001.

Examinations for Exams 1, 2, 3, and 4 are jointly sponsored by the Casualty Actuarial Society and the Society of Actuaries and were held in May and November 2001. Candidates who were successful on these examinations were listed in joint releases of the two Societies.

The following candidates were admitted as Fellows and Associates at the 2001 CAS Spring Meeting in May. By passing Fall 2000 CAS examinations, these candidates successfully fulfilled the Society requirements for Fellowship or Associateship designation.

NEW FELLOWS

David Matthew Biewer
David R. Border
Conni Jean Brown
Stephanie T. Carlson
Jeffrey Alan Courchene
Laura Ann Esboldt
Joseph Gerard Evleth
Emily C. Gilde
Bryan Hartigan
Kurt D. Hines
Randall Allen Jacobson
Michael G. Kerner
Kimberly J. Kurban
James P. Lynch
Daniel Patrick Maguire
Atul Malhotra
Julie Martineau
Eric Millaire-Morin
Scott Allan Miller
Michael A. Pauletti
John M. Pergrossi
Jordan J. Pitz
Sean Evans Porreca
Joseph John Sacala
Gary Frederick Scherer
Annmarie Schuster
Alastair Charles Shore
Mark Alan Verheyen
Shaun S. Wang
Mark Lee Woods

NEW ASSOCIATES

Afrouz Assadian
Sara T. Broadrick
Stephanie Anne Bruno
Hugo Corbeil
David Francis Dahl
Feifei Ford
Edward Kofi Gyampo
James Anthony Heer
Suzanne Barry Holohan
Christopher Wayne Hurst
Jamison Joel Ihrke

Shantelle Adrienne Johnson
Tricia Lynne Johnson
William Russell Johnson
Joseph E. Kirsits
Matthew Allen Lillegard
Timothy James McCarthy
Sharon D. Mott
Michael A. Onofrietti
Matthew R. Ostiguy
Chad Michael Ott
Michael Robert Petrarca
Jayne L. Plunkett
Gregory T. Preble
Jennifer L. Richard
Ellen Marie Tierney
Jennifer Anne Vezza
Cameron Jason Vogt
Scott Michael Woomer
Jimmy L. Wright
Stephanie C. Young
Michael R. Zarember
Xiangfei Zeng

The following candidates successfully completed the following Spring 2001 CAS examinations.

Exam 5

Koosh Arfa-Zanganeh
Daryl S. Atkinson
Farid Aziz Ibrahim
John D. Back
Stevan S. Baloski
Dan S. Barnett
Patrick Beaulieu
Richard J. Bell III
Stacey Jo Bitler
Kirk D. Bitu
Nathan L. Bluhm
Timothy D. Boles
Jonathan E. Bransom
Suejeudi Buehler
Amber L. Butek
Christine Cadieux
Alison S. Carter
Jennifer L. Caulder
Ronald S. Cederburg
Hao Chai
Kevin K. W. Chan
Whye-Loon Chan
Hung Francis Cheung
Wai Yip Chow
Gregory R. Chrin
Kevin J. Christy
Robert J. Collingwood
Matthew P. Collins
Cameron A. Cook
Craig A. Cooper
Thomas Cosenza
Carissa Ann Dahlen
Mari A. Davidson
Chantal Delisle
Jeremy J. Derucki
Christopher P. DiMartino
Dennis Herman Dunham
Brian Elliott
Jessica L. Elsinger
James C. Epstein
Choya A. Everett
Gina C. Ferst
Sean W. Fisher
John S. Flattum
Jeffrey R. Fleischer
Robin A. Fleming
William J. Fogarty
Peter L. Forester
Sebastien Fortin
Laurie L. Frayne
Rebecca E. Freitag
David S. Futterleib
Gina L. Gagliardi
Kareen Gaudreault
Isabelle Girard
Joel D. Glockler
Christopher J. Graham
Glenda J. K. Granowski
Ann E. Green
Karen L. Greene

Veronique Grenon
Travis J. Grulkowski
Jonathan M. Guy
Brian O Haaseth
Todd R. Hakala
Patricia W. Hardin
Robert D. Harrington
Jason B. Heissler
Gregory L. Helser
Scott E. Henck
Milton G. Hickman
Carole K. L. Ho
Kathleen Hobbs
David J. Horn Jr.
Frank E. Horn
Victoria K. Imperato
Kenneth L. Israelsen
Vibha N. Jayasinghe
Rachel Elizabeth Jenny
Paul A. Johnson
John B. Kelly
Amy Jieseon Kim
Steve C. Klingemann
Perry A. Klingman
Jonathan David Koch
John E. Kollar
Matthew R. Kuczwaj
Charles B. Kullmann
Terry T. Kuruvilla
Kristine Kuzora
Mai B. Lam
James A. Landgrebe
Thomas P. Langer
Jason A. Lauterbach
Eric T. Le
Michaela Ledlova
Kenneth L. Leonard
Julia Leung
Jenn Y. Lian
Nicole P. Libby
Herman Lim
Jia Liu
Jin Liu
Nataliya A. Loboda
Gwenette K. Lorino
PeiQing Luo
Lynn C. Malloney
Archibald G. Mattis
Michael B. McCarty
Wayne H. McClary
Robert B. McCleish IV
James P. McCoy
John D. McMichael
Sylwia S. McMichael
Charles A. Metzger
Michael E. Mielzynski
Lori A. Moore
Jason L. Morgan
Catherine A. Morse
Kyle S. Mrotek
Yuchun Mu
Joseph J. Muccio
James C. Murphy
Christopher A. Najim
Jacqueline L. Neal
Richard U. Newell
Kee Heng Ng
Tang-Tri Nguyen
William S. Ober
Liam F. O'Connor
Felix Patry
Robert Anthony Peterson
Etienne Plante-Dube
Timothy K. Pollis
David N. Prario
Lind R. Pratt
Julie-Ann Puzzo
Lynellen M. Ramirez
Monica L. Ransom
Suzanne M. Reddy
Joe Reschini
Michelle L. Rockafellow
Robert C. Roddy
Michele S. Rosenberg
John D. Rosilier
Nancy Ross
Steven M. Schienvar
Mark W. Schluesche
Thomas Schneider
Bradley J. Schroer
Mandy M. Y. Seto
Steven R. Shallcross
Peter M. Shelley
Frank W. Shermoen
Richard Sieger
Barry Dov Siegman
Paul Silberbush
Janel M. Sinacori
James M. Smieszkal
Robert K. Smith
Patrick Shiu-Fai So
Anthony A. Solak
Christa Sorola
David Chan Stanek
Michelle J. Steinborn
Natalie St-Jean
Shelley A. Stone
Alexandra R. St-Onge

Christopher J. Styrsky
Ju-Young Suh
Christie L. Sullivan
Lisa Liqin Sun
Erica W. Szeto
Mary A. Theilen
Jonas F. Thisner
Dovid C. Tkatch
Dominic A. Tocci
Michael C. Torre
Jean-François Tremblay
Lien K. Tu
Maxim Viel
Hanny C. Wai
Gary C. Wang
Timothy P. Wiebe
Nicholas J. Williamson
Jill C. Willie
Joshua C. Worsham
Christopher H. Yaure
Sung G. Yim
Bradley J. Zarn

Exam 7–Canada

Patrick Barbeau
Marie-Eve J. Belanger
Brad D. Birtz
Nathalie Charbonneau
Yvonne W. Y. Cheng
Richard Jason Cook
Gregory L. Dunn
Wayne W. Edwards
Genevieve Garon
Lisa N. Guglietti
Katherine Jacques
Stephane Lalancette
Jean-François Larochelle
Stephane McGee
Sylvain Perrier
Bill D. Premdas
Lester Pun
Asif M. Sardar
Karine St-Onge

Exam 7–United States

Jason R. Abrams
Jeffrey R. Adcock
Stephen A. Alexander
Vagif Amstislavskiy
Brian M. Ancharski
Pamela G. Anderson
Deborah Herman Ardern
Anju Arora
Kevin J. Atinsky
Joel E. Atkins
Peter Attanasio
Silvia J. Bach
John L. Baldan
Esther Becker
Andrew W. Bernstein
Kofi Boaitey
Lesley R. Bosniack
Erich A. Brandt
Elaine K. Brunner
Claude B. Bunick
Don J. Burbacher
Hayden Heschel Burrus
James E. Calton
William Brent Carr
Sharon C. Carroll
Patrick J. Causgrove
James Chang
Jennifer A. Charlonne
Alan M. Chow
Michael Joseph Christian
Paul L. Cohen
Christian J. Coleianne
Christopher L. Cooksey
Kevin A. Cormier
Leanne M. Cornell
Michael J. Covert
Hall D. Crowder
A. David Cummings
Aaron T. Cushing
John Edward Daniel
Robert E. Davis

Paul B. Deemer
Erik L. Donahue
Brian M. Donlan
Kevin P. Donnelly
Brian S. Donovan
Kiera Elizabeth Doster
Kevin Francis Downs
Scott H. Drab
George T. Dunlap IV
Ruchira Dutta
Tomer Eilam
Jonathan Palmer Evans
Kyle A. Falconbury
Kevin M. Finn
Robin V. Fitzgerald
Ellen D. Fitzsimmons
Sharon L. Fochi
Gregory A. Frankowiak
Dana R. Frantz
Patrick P. Gallagher
Anne M. Garside
Keith R. Gentile
James W. Gillette Jr.
Theresa Giunta
Andrew Samuel Golfin Jr.
Melanie T. Goodman
Karl Goring
Matthew R. Gorrell
Christopher J. Grasso
Joseph P. Greenwood
Donald B. Grimm
Jason L. Grove
Serhat Guven
David Bruce Hackworth
Nasser Hadidi
Brian D. Haney
Stuart J. Hayes
James D. Heidt
Rhonda R. Hellman
Ronald L. Helmeci
Glenn R. Hiltpold
Brook A. Hoffman
Allen J. Hope
Long-Fong Hsu
Jesse T. Jacobs
Gregory O. Jaynes
Brian B. Johnson
Erik A. Johnson
William Brian Johnson
Susan K. Johnston
Steven M. Jokerst
Daniel R. Kamen
Hye-Sook Kang
Kyewook (Gary) Kang
Barbara L. Kanigowski
Lawrence S. Katz
Stacey M. Kidd
Ziv Kimmel
Jennifer E. Kish
Jeff A. Kluck
Laurie A. Knoke
Henry Joseph Konstanty
Brandon E. Kubitz
Anand S. Kulkarni
Aaron Michael Larson
Bradley R. LeBlond
Borwen Lee
Ruth M. LeStourgeon
Amanda M. Levinson
Jonathan D. Levy
Kenneth Lin
Daniel A. Lowen
Sally Ann MacFadden
Teresa Madariaga Zubimendi
Kevin M. Madigan
Joshua Nathan Mandell
Kevin Paul McClanahan
Jeffrey B. McDonald
Lawrence J. McTaggart III
William A. Mendralla
Mitchel Merberg
Charles W. Mitchell
Matthew Kevin Moran
Matthew E. Morin
Janice C. Moskowitz
Scott L. Negus
Shannon P. Newman
Norman Niami
Michael Douglas Nielsen
Stoyko N. Nikolov
Matthew P. Nimchek
John E. Noble
Alejandra S. Nolibos
James L. Norris
Christy Beth Olson
Lowell D. Olson
Bruce G. Pendergast
Robert B. Penwick
Dianne M. Phelps

Daniel P. Post
John T. Raeihle
Kathleen M. Rahilly-VanBuren
Stephen Daniel Riihimaki
Delia E. Roberts
Benjamin G. Rosenblum
Scott I. Rosenthal
Ryan P. Royce
Bryant Edward Russell
Giuseppe Russo
Frederick Douglas Ryan
Laura Beth Sachs
Frances G. Sarrel
Teresa Marie Scharn
Jeremy N. Scharnick
Cindy R. Schauer
Doris Y. Schirmacher
Jeffery Wayne Scholl
Steven George Searle
Larry J. Seymour
Brett M. Shereck
Junning Shi
Jeremy D. Shoemaker
James S. Shoenfelt
Joseph Allen Smalley
Lora L. Smith
Steven A. Smith II
Michael William Starke
John P. Stefanek
Wei Hua Su
Beth M. Sweeney
Edward Sypher
Stephen James Talley
Laura Little Thorne
Michael C. Torre
Matthew D. Trone
Matthew L. Uhoda
Justin M. Van Opdorp
Geraldine Marie L. Verano
Brian A. Viscusi
John E. Wade
Amy R. Waldhauer
Kristie L. Walker
Matthew J. Wasta
Bethany R. Webb
Robert S. Weishaar
Thomas E. Weist
Jean P. West
Carolyn D. Wettstein
V. Clare Whitlam
Rosemary Gabriel Wickham
William B. Wilder
Duane A. Willis
Dean M. Winters
Karin H. Wohlgemuth
Robert F. Wolf
Mary K. Woodson
Jennifer X. Wu
Mihoko Yamazoe
Run Yan
Jeanne Lee Ying
Yingjie Zhang
Rita M. Zona

Exam 8

Paul D. Anderson
Katherine H. Antonello
Craig Victor Avitabile
Robert D. Bachler
Jeremy Todd Benson
Ellen A. Berning
Eric D. Besman
Kristen Maria Bessette
Neil M. Bodoff
Jeremy James Brigham
Stephanie Anne Bruno
Russell J. Buckley
Kevin D. Burns
Janet P. Cappers
Jill C. Cecchini
Todd D. Cheema
Richard M. Chiarini
Louise Chung-Chum-Lam
Jeffrey J. Clinch
Maryellen J. Coggins
Hugo Corbeil
John T. Devereux
Barry P. Drobes
Dennis Herman Dunham
Louis-Christian Dupuis
Richard James Engelhuber
Ellen E. Evans

Weishu Fan
Kathleen Marie Farrell
Benedick Fidlow
David Michael Flitman
Feifei Ford
Sara Frankowiak
Dustin Gary
Amy L. Gebauer
Charles E. Gegax
Bradley G. Gipson
Elizabeth Susan Guven
Edward Kofi Gyampo
Marc S. Hall
Kevin B. Held
Daniel D. Heyer
Mark D. Heyne
Patricia A. Hladun
Richard Michael Holtz
Susan Elizabeth Innes
Craig D. Isaacs
Patrice Jean
Weidong Wayne Jiang
Bryon Robert Jones
William Rosco Jones
Sean M. Kennedy
David R. Kennerud
Susan E. Kent
Susanlisa Kessler
Jill E. Kirby
Richard F. Kohan
Richard Scott Krivo
Scott C. Kurban
Steven M. Lacke
Julie-Linda Laforce
Jean-Sebastien Lagace
Isabelle La Palme
Peter H. Latshaw
Michael L. Laufer
Dennis H. Lawton
Thomas V. Le
James P. Leise
Christian Lemay
John N. Levy
Xiaoying Liang
Matthew Allen Lillegard
Kathleen T. Logue
Richard Paul Lonardo
Cara M. Low
Robb W. Luck
Jason Aaron Martin
David Michael Maurer
Heather L. McIntosh
Christian Menard
Richard Ernest Meuret
Vadim Y. Mezhebovsky
Michael J. Miller
Sean Robert Nimm
Sylvain Nolet
Christopher Maurice Norman
Corine Nutting
Steven Brian Oakley
Randall William Oja
Rodrick Raymond Osborn
Apryle L. Oswald
Cosimo Pantaleo
John R. Pedrick
Tracie L. Pencak
Kraig Paul Peterson
Michael Robert Petrarca
Kristin Sarah Piltzecker
Dylan P. Place
Mario Richard
Ezra Jonathan Robison
John R. Rohe
Christine R. Ross
James C. Sandor
Parr T. Schoolman
Vladimir Shander
Klayton N. Southwood
Wendy Rebecca Speert
Theodore S. Spitalnick
Curt A. Stewart
Beth S. Thompson
Christopher S. Throckmorton
Jennifer L. Throm
Gary S. Traicoff
Brian K. Turner
Eric Vaith
Jennifer S. Vincent
Cameron Jason Vogt
Robert J. Walling III
Wade Thomas Warriner
Kelly M. Weber
Kah-Leng Wong
Windrie Wong
Michael R. Zarember
Edward J. Zonenberg

The following candidates were admitted as Fellows and Associates at the 2001 CAS Annual Meeting in November. By passing Spring 2001 CAS examinations, these candidates successfully fulfilled the Society requirements for Fellowship or Associateship designation.

NEW FELLOWS

Jason R. Abrams
Stephen A. Alexander
Katherine H. Antonello
Anju Arora
Peter Attanasio
Craig Victor Avitabile
Jeremy Todd Benson
Eric D. Besman
Kristen Maria Bessette
Neil M. Bodoff
Jeremy James Brigham
Russell J. Buckley
Kevin D. Burns
Hayden Heschel Burrus
Sharon C. Carroll
Jill C. Cecchini
Richard M. Chiarini
Michael Joseph Christian
Louise Chung-Chum-Lam
Jeffrey J. Clinch
Maryellen J. Coggins
John T. Devereux
Kevin Francis Downs
Louis-Christian Dupuis
Wayne W. Edwards
Richard James Engelhuber
Jonathan Palmer Evans
Weishu Fan
Sara Frankowiak
Dustin Wayne Gary
Amy L. Gebauer
Bradley G. Gipson
Theresa Giunta
Karl Goring
Lisa N. Guglietti
Elizabeth Susan Guven
Nasser Hadidi
Brian D. Haney
Kevin B. Held
Mark D. Heyne
Glenn R. Hiltpold
Richard Michael Holtz
Susan Elizabeth Innes
Craig D. Isaacs
Patrice Jean
Weidong Wayne Jiang
Susan K. Johnston
Bryon Robert Jones
Sean M. Kennedy
David R. Kennerud
Susan E. Kent
Susanlisa Kessler
Richard F. Kohan
Richard Scott Krivo
Scott C. Kurban
Steven M. Lacke
Julie-Linda Laforce
Isabelle La Palme
Aaron Michael Larson
Dennis H. Lawton
Thomas V. Le
James P. Leise
Christian Lemay
John N. Levy
Matthew Allen Lillegard
Kathleen T. Logue
Cara M. Low
Robb W. Luck
Joshua Nathan Mandell
Jason Aaron Martin
Heather L. McIntosh
Christian Menard
Richard Ernest Meuret
Michael J. Miller
Sean Robert Nimm
Sylvain Nolet
Corine Nutting
Steven Brian Oakley
Randall William Oja

Christy Beth Olson
Rodrick Raymond Osborn
Apryle L. Oswald
Cosimo Pantaleo
John R. Pedrick
Kristin Sarah Piltzecker
Dylan P. Place
Mario Richard
John R. Rohe
Christine R. Ross
Asif M. Sardar
Parr T. Schoolman
Steven George Searle
Joseph Allen Smalley
Klayton N. Southwood
Theodore S. Spitalnick
Curt A. Stewart
Beth S. Thompson
Laura Little Thorne
Christopher S. Throckmorton
Jennifer L. Throm
Michael C. Torre
Gary S. Traicoff
Brian K. Turner
Eric Vaith
Jennifer S. Vincent
Cameron Jason Vogt
Robert J. Walling III
Kelly M. Weber
V. Clare Whitlam
Dean M. Winters
Robert F. Wolf
Kah-Leng Wong
Windrie Wong
Mary K. Woodson
Jeanne Lee Ying
Edward J. Zonenberg

NEW ASSOCIATES

Vagif Amstislavskiy
Pamela G. Anderson
Joel E. Atkins
Esther Becker
Marie-Eve J. Belanger
Kofi Boaitey
Erich A. Brandt
Maureen B. Brennan
Don J. Burbacher
James E. Calton
William Brent Carr
Ronald S. Cederburg
Hao Chai
Jennifer A. Charlonne
Alan M. Chow
Paul L. Cohen
Christopher L. Cooksey
Leanne M. Cornell
Thomas Cosenza
Michael J. Covert
Hall D. Crowder
A. David Cummings
Erik L. Donahue
Brian M. Donlan
Kiera Elizabeth Doster
Scott H. Drab
Gregory L. Dunn
Ruchira Dutta
Kyle A. Falconbury
Robin V. Fitzgerald
Patrick P. Gallagher
Genevieve Garon
Keith R. Gentile
Christie L. Gilbert
Christopher J. Grasso
Donald B. Grimm
Jason L. Grove
Stuart J. Hayes
Scott E. Henck
Long-Fong Hsu
Katherine Jacques
Gregory O. Jaynes
Brian B. Johnson
Erik A. Johnson
Dana F. Joseph
Hye-Sook Kang
Barbara L. Kanigowski
Lawrence S. Katz
Stacey M. Kidd
Laurie A. Knoke
Anand S. Kulkarni
Stephane Lalancette
Amanda M. Levinson
Daniel A. Lowen
Sally Ann MacFadden

Teresa Madariaga Zubimendi
Jeffrey B. McDonald
Stephane McGee
Charles W. Mitchell
Matthew E. Morin
Joseph J. Muccio
Scott L. Negus
Norman Niami
Stoyko N. Nikolov
Alejandra S. Nolibos
Dianne M. Phelps
Daniel P. Post
Bill D. Premdas
John T. Raeihle
Ryan P. Royce
Giuseppe Russo
Larry J. Seymour
Brett M. Shereck
Junning Shi
Jeremy D. Shoemaker
James S. Shoenfelt
Steven A. Smith II
Anthony A. Solak
Karine St-Onge
Wei Hua Su
Edward Sypher
Mary A. Theilen
Peggy J. Urness
Justin M. Van Opdorp
Gaetan R. Veilleux
Geraldine Marie L. Verano
Amy R. Waldhauer
Robert S. Weishaar
Jean P. West
William B. Wilder
Jennifer X. Wu
Run Yan

The following candidates successfully completed the following Fall 2001 CAS examinations.

Exam 6

Sajjad Ahmad
Fernando Alberto Alvarado
John A. Annino
Richard T. Arnold
Ashaley N. Attoh-Okine
Gregory S. Babushkin
Kevin J. Bakken
John L. Baldan
Stevan S. Baloski
Dan S. Barnett
Danielle L. Bartosiewicz
Thomas C. Bates
Patrick Beaulieu
Elizabeth G. Bedard
Nathalie Belanger
Jonathan P. Berenbom
Andrew W. Bernstein
Kirk D. Bitu
Nathan L. Bluhm
Nebojsa Bojer
Donna Bono-Dowd
John R. Bower
Elaine K. Brunner
Thomas L. Cawley
Kevin K.W. Chan
Michael Tsz-Kin Chan
Whye-Loon Chan
James Chang
Hung Francis Cheung
Tracy L. Child
Julia Chou
Martin P. Chouinard
Philip A. Clancey Jr.
Marlene Marie Collins
Spencer L. Coyle
Michael B. Cunningham
Kelly K. Cusick
Carissa Ann Dahlen
Mari A. Davidson
Amy L. DeHart
David E. Dela Cruz
Chantal Delisle
David A. DeNicola
Melodee S. Dixon
Christopher A. Donahue
Brian S. Donovan
John A. Duffy
Ramakrishna Duvvuri
Jessica L. Elsinger

Jieqiu Fan
Matthew B. Feldman
Dale A. Fethke
Kevin M. Finn
Kristine M. Fitzgerald
Ellen D. Fitzsimmons
Jeffrey R. Fleischer
Robin A. Fleming
Sharon L. Fochi
Peter L. Forester
Susan J. Forray
Louise Frankland
Gregory A. Frankowiak
Andre Gagnon
Martine Gagnon
Matthew P. Gatsch
Stuart G. Gelbwasser
Gregory Evan Gilbert
Isabelle Girard
Michael F. Glatz
Joel D. Glockler
Lori A. Gordon
Matthew R. Gorrell
Matthew L. Gossell
Veronique Grenon
Stacie R. W. Grindstaff
Stephanie A. Groharing
Simon Guenette
Serhat Guven
Kimberly Baker Hand
Jason C. Harland
Robert D. Harrington
Joseph Hebert
James D. Heidt
Kandace A. Heiser
Rhonda R. Hellman
Brandon L. Heutmaker
Daniel D. Heyer
Milton G. Hickman
Carole K.L. Ho
Ryan Yin-kei Ho
Jeremy A. Hoch
Joseph H. Hohman
Melissa S. Holt
Chun Hua Hoo
David J. Horn Jr.
Gerald K. Howard
Wang Yang Hu
Jesse T. Jacobs
Julie A. Jordan
Inga Kasatkina
Susan M. Keaveny
Amy Jieseon Kim
Ziv Kimmel
Patricia Kinghorn
Scott M. Klabacha
Jeff A. Kluck
Brandon E. Kubitz
Charles B. Kullmann
Elizabeth A. Kurina
Terry T. Kuruvilla
David M. Lang
Thomas P. Langer
Annie Latouche
Nathalie M. Lavigne
Khanh M. Le
Hayden Anthony Lewis
Wei Li
Jenn Y. Lian
Kenneth Lin
Hazel J. Luckey
Eric A. Madia
John T. Maher
Steven Manilov
Luis S. Marques
Lora K. Massino
Archibald G. Mattis
William R. McClintock
Sylwia S. McMichael
Lawrence J. McTaggart III
Mea Theodore Mea
Charles A. Metzger
Thomas E. Meyer
Ryan A. Michel
Michael E. Mielzynski
James J. Moloney
Christian Morency
Alan E. Morris
Kyle S. Mrotek
John A. Nauss
Richard U. Newell
Matthew P. Nimchek
James L. Norris
Charles A. Norton
William S. Ober
Melissa A. Ogden
Lowell D. Olson
Kelly A. Paluzzi
Bruce G. Pendergast
Robert B. Penwick
Matthew J. Perkins
Andrea L. Phillips
Faith M. Pipitone
Jorge E. Pizarro

Conni A. Rader
David P. Rafferty
Danielle L. Richards
Laura D. Rinker
Michelle L. Rockafellow
Robert C. Roddy
Charles A. Romberger
Benjamin G. Rosenblum
John D. Rosilier
John C. Ruth
Teresa Marie Scharn
Steven M. Schienvar
Thomas Schneider
Monica S. Schroeter
Ronald J. Schuler
Mandy M. Y. Seto
Jin Shao
Peter M. Shelley
Summer L. Sipes
James M. Smieszkal
Douglas E. Smith
Robert K. Smith
Christopher Y. So
Michael D. Sowka
Laura T. Sprouse
Anya K. Sri-Skanda-Rajah
David Chan Stanek
Christine Seung Steer
Esperanza Stephens
Christopher J. Styrsky
Lisa Liqin Sun
Adam D. Swope
Erica W. Szeto
Dovid C. Tkatch
David A. Traugott
Jean-François Tremblay
Matthew D. Trone
William D. Van Dyke
Kevin K. Vesel
Maxim Viel
Brian A. Viscusi
John E. Wade
Hanny C. Wai
Matthew J. Walter
Gary C. Wang
Qingxian Wang
Bethany R. Webb
Duane A. Willis
Donald S. Wroe
Andrew Yershov
Janice M. Young
Jonathan K. Yu
Bradley J. Zarn
Ruth Zea
Yingjie Zhang

Exam 9

Denise M. Ambrogio
Vagif Amstislavskiy
Deborah Herman Ardern
Satya M. Arya
Patrick Barbeau
Jack Barnett
Marie-Eve J. Belanger
Jody J. Bembenek
Jason E. Berkey
Ellen A. Berning
Jean-Philippe Boucher
Maureen B. Brennan
Peter J. Brown
David C. Brueckman
Don J. Burbacher
Angela D. Burgess
Cheryl R. Burrows
Jennifer L. Caulder
Ronald S. Cederburg
Hao Chai
Wai Yip Chow
Susan M. Cleaver
Paul L. Cohen
Richard Jason Cook
Hugo Corbeil
Thomas Cosenza
Michael J. Covert
A. David Cummings
Paul B. Deemer
Peter R. DeMallie
Christopher P. DiMartino
Erik L. Donahue
Dean P. Dorman
J. Chris Dougherty
Dennis Herman Dunham
Ruchira Dutta

Jeffrey A. Dvinoff
Feifei Ford
Dana R. Frantz
Mauricio Freyre
Serge Gagne
James M. Gallagher
Genevieve Garon
John S. Giles
Patrick J. Gilhool
Andrew Samuel Golfin Jr.
Peter Scott Gordon
Isabelle Groleau
Jason L. Grove
Chantal Guillemette
Marc S. Hall
Dawn Marie S. Happ
Stuart J. Hayes
Patricia A. Hladun
Suzanne Barry Holohan
Christopher Wayne Hurst
Jamison Joel Ihrke
Ali Ishaq
Katherine Jacques
Erik A. Johnson
Tricia Lynne Johnson
Steven M. Jokerst
Derek A. Jones
Dana F. Joseph
Lawrence S. Katz
Joseph E. Kirsits
Omar A. Kitchlew
Andrew M. Koren
Anand S. Kulkarni
Michael A. Lardis
Peter H. Latshaw
Jason A. Lauterbach
Borwen Lee
Erik Frank Livingston
Richard Paul Lonardo
Teresa Madariaga Zubimendi
Sharon L. Markowski
James J. Matusiak Jr.
David Michael Maurer
Stephane McGee
John D. McMichael
Sarah K. McNair-Grove
Vadim Y. Mezhebovsky
Camilo Mohipp
Celso M. Moreira
Karen E. Myers
Scott L. Negus
Kee Heng Ng
Khanh K. Nguyen
Alejandra S. Nolibos
Joshua M. Nyros
Matthew R. Ostiguy
Ajay Pahwa
Gerard J. Palisi
Michael Thomas Patterson
Isabelle Perron
Kevin Thomas Peterson
Kraig Paul Peterson
Dianne M. Phelps
Daniel P. Post
Bill D. Premdas
Scott I. Rosenthal
Nancy Ross
Ryan P. Royce
Giuseppe Russo
Laura Beth Sachs
James C. Sandor
Daniel David Schlemmer
Larry J. Seymour
Brett M. Shereck
Junning Shi
Scott G. Sobel
Wendy Rebecca Speert
Karine St-Onge
Neeza Thandi
Lien K. Tu
Son T. Tu
Turgay F. Turnacioglu
Jennifer L. Vadney
Geraldine Marie L. Verano
Jennifer Anne Vezza
Josephine M. Waldman
Wade Thomas Warriner
Joseph C. Wenc
Mark Steven Wenger
Christopher John Westermeyer
Arthur S. Whitson
Scott Michael Woomer
Jennifer X. Wu
Huey Wen Yang
Sung G. Yim
Michael R. Zarember
Larry Xu Zhang
Lianmin Zhou
Steven Bradley Zielke

NEW FELLOWS ADMITTED IN MAY 2001

Row 1 (left to right): Jeffrey Alan Courchene, John M. Pergrossi, Michael A. Pauletti, Kimberly J. Kurban, **CAS President Patrick Grannan**, Stephanie T. Carlson, Julie Martineau, Conni Jean Brown, Shaun S. Wang, Annmarie Schuster, Daniel Patrick Maguire. **Row 2 (left to right):** Sean Evans Porreca, Mark Lee Woods, Atul Malhotra, Bryan Hartigan, Jordan J. Pitz, Joseph Gerard Evleth, Randall Allen Jacobson. **Row 3 (left to right):** David R. Border, Gary Frederick Scherer, Joseph John Sacala, Michael G. Kerner, Laura Ann Esboldt. **Row 4 (left to right):** Scott Allan Miller, Eric Millaire-Morin, Mark Alan Verheyen, Kurt D. Hines, James P. Lynch. **New Fellows not pictured:** David Matthew Biewer, Emily C. Gilde, Alastair Charles Shore.

New Associates Admitted in May 2001

Row 1 (left to right): Michael R. Zarember, Matthew Allen Lillegard, Afrouz Assadian, Sharon D. Mott, **CAS President Patrick Grannan**, Sara T. Broadrick, Jennifer L. Richard, Jennifer Anne Vezza, James Anthony Heer. **Row 2 (left to right):** Jamison Joel Ihrke, Scott Michael Woomer, Tricia Lynne Johnson, Jayne L. Plunkett. **Row 3 (left to right):** William Russell Johnson, Michael Robert Petrarca, Stephanie Anne Bruno, Stephanie C. Young, Shantelle Adrienne Johnson, Chad Michael Ott. **Row 4 (left to right):** Edward Kofi Gyampo, Christopher Wayne Hurst, Xiangfei Zeng, Matthew R. Ostiguy, Joseph E. Kirsits. **Row 5 (left to right):** Hugo Corbeil, Timothy James McCarthy, David Francis Dahl, Jimmy L. Wright, Ellen Marie Tierney, Gregory T. Preble. **New Associates not pictured:** Feifei Ford, Suzanne Barry Holohan, Michael A. Onofrietti, Cameron Jason Vogt.

New Fellows Admitted in November 2001

New Fellows, first row, from left: Steven George Searle, Richard Michael Holtz, Stephen A. Alexander, Randall William Oja, **CAS President Patrick Grannan,** Bryon Robert Jones, Dustin Wayne Gary, Jeremy Todd Benson, Jennifer S. Vincent. **Second row, from left:** Kevin D. Burns, Brian D. Haney, Michael C. Torre, Craig Victor Avitabile, Kristen Maria Bessette, Kristin Sarah Piltzecker, Richard F. Kohan, Sean M. Kennedy, Nasser Hadidi. **Third row, from left:** Mary K. Woodson, Susan Elizabeth Innes, Christopher S. Throckmorton, Klayton N. Southwood, Rodrick Raymond Osborn, Mark D. Heyne, Corine Nutting. **Fourth row, from left:** Cosimo Pantaleo, Katherine H. Antonello, Michael Joseph Christian, John R. Pedrick, Louise Chung-Chum-Lam, Cara M. Low, Susan K. Johnston. **Fifth row, from left:** Aaron Michael Larson, Robb W. Luck.

NEW FELLOWS ADMITTED IN NOVEMBER 2001

New Fellows, first row, from left: Dean M. Winters, Elizabeth Susan Guven, Heather L. McIntosh, Russell J. Buckley, **CAS President Patrick Grannan,** Maryellen J. Coggins, Richard James Engelhuber, Bradley G. Gipson, Richard Scott Krivo. **Second row, from left:** Kevin B. Held, Gary S. Traicoff, Jason R. Abrams, Sean Robert Nimm, Neil M. Bodoff, Richard M. Chiarini, Sara Frankowiak, Hayden Heschel Burrus, Thomas V. Le. **Third row, from left:** David R. Kennerud, Eric Vaith, John R. Rohe, John T. Devereux, Michael J. Miller, Joseph Allen Smalley, James P. Leise. **Fourth row, from left:** Christy Beth Olson, Lisa N. Guglietti, Apryle L. Oswald, Kathleen T. Logue, Matthew Allen Lillegard, Susan E. Kent, Christine R. Ross, Dennis H. Lawton.

NEW FELLOWS ADMITTED IN NOVEMBER 2001

New Fellows, first row, from left: Edward J. Zonenberg, Parr T. Schoolman, Eric D. Besman, Julie-Linda Laforce, **CAS President Patrick Grannan,** Steven Brian Oakley, Windrie Wong, Weishu Fan, Robert J. Walling III. **Second row, from left:** Peter Attanasio, Christian Lemay, Mario Richard, Louis-Christian Dupuis, Scott C. Kurban, Craig D. Isaacs, Kevin Francis Downs, Anju Arora. **Third row, from left:** Jeffrey J. Clinch, Jeremy James Brigham, Brian K. Turner, Glenn R. Hiltpold, Robert F. Wolf, Patrice Jean, Christian Menard. **Fourth row, from left:** Susanlisa Kessler, Isabelle La Palme, Amy L. Gebauer, Beth S. Thompson, Jill C. Cecchini, Curt A. Stewart, Kah-Leng Wong, Jennifer L. Throm. **New Fellows not pictured:** Sharon C. Carroll, Wayne W. Edwards, Jonathan Palmer Evans, Theresa Giunta, Karl Goring, Weidong Wayne Jiang, Steven M. Lacke, John N. Levy, Joshua Nathan Mandell, Jason Aaron Martin, Richard Ernest Meuret, Sylvain Nolet, Dylan P. Place, Asif M. Sardar, Theodore S. Spitalnick, Laura Little Thorne, Cameron Jason Vogt, Kelly M. Weber, V. Clare Whitlam, Jeanne Lee Ying.

NEW ASSOCIATES ADMITTED IN NOVEMBER 2001

New Associates, front row, from left: Bill D. Premdas, Larry J. Seymour, Vagif Amstislavskiy, Daniel P. Post, **CAS President Patrick Grannan**, Michael J. Covert, Kofi Boaitey, Keith R. Gentile, Barbara L. Kanigowski, **Second row, from left:** Hye-Sook Kang, Stoyko N. Nikolov, Scott L. Negus, Brian M. Donlan, Justin M. Van Opdorp, Robert S. Weishaar, Anthony A. Solak, Kiera Elizabeth Doster, Charles W. Mitchell. **Third row, from left:** Joel E. Atkins, Jason L. Grove, Hall D. Crowder, Scott E. Henck, Stuart J. Hayes, Kyle A. Falconbury, Mary A. Theilen. **Fourth row, from left:** Erich A. Brandt, Christopher L. Cooksey, Maureen B. Brennan, Teresa Madariaga, Pamela G. Anderson, Alejandra S. Nolibos, Stacey M. Kidd, Jennifer A. Charlonne. **Fifth row:** Scott H. Drab.

New Associates Admitted in November 2001

New Associates, first row, from left: Jeff B. McDonald, Geraldine Marie Verano, Erik A. Johnson, A. David Cummings, **CAS President Patrick Grannan**, Ronald S. Cederburg, Joseph J. Muccio, Jeremy D. Shoemaker, Gaetan R. Veilleux. **Second row, from left:** Laurie A. Knoke, James S. Shoenfelt, Ryan P. Royce, Paul L. Cohen, Stephane McGee, Robin V. Fitzgerald, Sally Ann MacFadden. **Third row, from left:** Amy R. Waldhauer, Daniel A. Lowen, Long-Fong Hsu, D. Joe Burbacher, Run Yan, Steven A. Smith II, Erik L. Donahue. **Fourth row, from left:** Hao Chai, Wei Hua Su, Gregory L. Dunn, Stephane Lalancette, Peggy J. Urness, James E. Calton, Alan M. Chow, Amanda M. Levinson. **New Associates not pictured:** Esther Becker, Marie-Eve J. Belanger, Brent Carr, Leanne M. Cornell, Thomas Cosenza, Ruchira Dutta, Patrick P. Gallagher, Genevieve Garon, Christie L. Gilbert, Christopher J. Grasso, Donald B. Grimm, Katherine Jacques, Gregory O. Jaynes, Brian B. Johnson, Dana F. Joseph, Lawrence S. Katz, Anand S. Kulkarni, Matthew E. Morin, Norman Niami, Dianne M. Phelps, John T. Raeihle, Giuseppe Russo, Brett M. Shereck, Junning Shi, Karine St-Onge, Edward Sypher, Jean P. West, William B. Wilder, Jennifer X. Wu.

OBITUARIES

James J. Callahan
E. LeRoy Heer
James P. Jensen
Arthur S. Leight
Tracey Lynn Matthew
Philip D. Miller
Harry R. Richards
Lewis H. Roberts
Henry C. Schneiker

JAMES J. CALLAHAN
1948–2001

James J. Callahan died April 16, 2001, of colon cancer. He was 52.

Born November 11, 1948, Callahan lived much of his life in Franklin Square, on Long Island in New York. He graduated from Manhattan College in 1970, attaining a B.S. degree with special honors in mathematics. He was a member of Pi Mu Epsilon and Mu Alpha Theta honor societies, setting a new school record for scores received in mathematical competitions sponsored by these societies. He also participated in several programs under a National Science Foundation grant. He placed 53rd in the nation out of thousands who participated in the William Lowell Putnam Mathematical Competition sponsored by the Mathematics Association of America. He later attended Brown University, under a full fellowship, to study for a combined masters and doctorate degree.

Callahan served in the U.S. Army in Vietnam during 1970 to 1972. He and several others in his unit were awarded the Bronze Star for meritorious service during several battles while in Vietnam. Callahan also received the National Defense Service Medal, the Vietnam Service Medal, the Vietnam Campaign Medal, and a Combat Infantry Badge. Callahan never mentioned receiving these awards, and friends and family were surprised to learn of them after his death.

A 1986 Associate of the Casualty Actuarial Society, Callahan spent more than 25 years of his career at the Insurance Services Office, Inc. (ISO). In 1975 he worked as an actuarial assistant in the private passenger actuarial division. In 1978 he moved to the personal lines actuarial services division, working on econometrics and reviews of class factors, territory definitions, and loss reserves. His background led him to take on more ongoing computer-related assignments involving statistical, financial, and claims data.

Callahan was known for his sharp sense of humor and an unusually quick ability to work through actuarial problems. Former colleague Christopher Diamantoukos (FCAS 1988) identified Callahan as the point man during ISO's first analysis of the adequacy of the industry property-casualty loss reserves. "His programming and data analysis prowess gave him the ability to unlock the extensive Schedule P of the entire industry and produce the first analysis of its kind," said Diamantoukos. "His problem-solving abilities were second to none." In a letter of recommendation for CAS membership, Diamantoukos wrote of Callahan: "If I had a staff of actuaries to assemble, Jim would undoubtedly be on the list."

Callahan's leisure time hobbies reflected his keen interest in solving problems. He enjoyed cryptography, puzzles, and computer games, and he was a member of a national cryptographers' society. Callahan also shared his expertise, mentoring several actuarial assistants who went on to become credentialed actuaries or experts in the information technology field.

To fulfill one of his last wishes, Callahan's family hosted a special dinner for his friends and colleagues at ISO. Callahan is survived by his mother, Catherine, of Franklin Square, New York, one sister, Kathleen, a brother-in-law, Alfred, and nephews Kevin and Christopher, all of Sandy Hook, Connecticut. A memorial scholarship fund has been established in his name at Archbishop Molloy High School in Briarwood, New York, to encourage young men and women to further develop their skills and love of mathematics and science.

E. LEROY HEER
1938–2001

E. LeRoy Heer died on March 18, 2001, at his home in Stamford, Connecticut. He was 62.

Born on August 19, 1938, in American Falls, Idaho, he was the son of Edwin Heer and the late Kathryn Heer.

Heer lived much of his early life in Alaska and received his bachelor's degree from the University of Alaska. He completed his master's degree in business administration from St. Mary's University in San Antonio, Texas, in 1976.

Heer worked as an actuary for the Insurance Company of North America in Philadelphia for five years. He moved to the United Service Automobile Association in San Antonio in 1973, where he served as assistant vice president.

He was most recently senior vice president and chief corporate actuary of the W. R. Berkley Corp. of Greenwich, Connecticut, where he worked for 12 years. He also served as a director on the boards of many of the companies owned by W. R. Berkley.

Heer became an Associate of the Casualty Actuarial Society in 1967 and a Fellow in 1969. He was a member of the Committee on Continuing Education from 1976–1982 and the Long Range Planing Committee from 1982–1984. Heer reviewed "Loss Reserving and Ratemaking in an Inflationary Environment," published in the 1981 CAS *Discussion Paper Program*. He was also a member of the American Academy of Actuaries and was a Chartered Property Casualty Underwriter.

Heer was a lifelong outdoorsman and particularly enjoyed hunting and fishing.

In addition to his father, he is survived by his wife, Judith Overton-Jones; a son, Kevin; a stepson, Edward; and three grandchildren.

JAMES P. JENSEN
1933–2001

James P. Jensen died April 7, 2001, just 12 days after being diagnosed with pancreatic cancer. He was 68.

Jensen attended Northeastern University in Boston, graduating in 1959 with degrees in mathematics and education. Jensen described one month in 1959 as one of the most memorable times of his life: one week he graduated from Northeastern, the next week he married his wife Joyce, and two weeks later he started at Liberty Mutual.

Jensen worked for Liberty Mutual Insurance Company in Boston from 1959 to 1992 in a number of positions, starting as an actuarial assistant and ultimately as director of industry and governmental relations and assistant vice president. In his later career, Jensen focused on medical malpractice issues. He represented Liberty Mutual on the board of directors of the Medical Malpractice Insurance Association (the New York medical malpractice joint underwriting association) from mid-1982 through mid-1993 and served as chairman of that board from June 1987 through May 1993.

Jensen became an Associate of the Casualty Actuarial Society in 1962 and volunteered on the CAS Public Relations Committee from 1971 to 1974.

A longtime friend and colleague, John B. Conners (FCAS 1974), said Jensen was a friend, coach, and mentor to many of the actuaries who went through Liberty Mutual. "Jim will be remembered as a good actuary and a good business person, but most of all as a good friend by those who knew him," said Conners.

In 1992 Jensen retired, splitting his time between homes in Osterville, Massachusetts on Cape Cod and Bonita Springs, Florida. He is survived by his wife Joyce and son Jonathan.

ARTHUR S. LEIGHT
1931–2001

Arthur S. Leight, 70, died on September 26, 2001. He received his bachelor's degree from New York University and his master's degree from Columbia University. He was elected to Phi Beta Kappa. Leight became a Fellow of the Society of Actuaries and an Associate of the Casualty Actuarial Society in 1959.

Leight spent his professional life in and around New York City. He began his actuarial career in 1962 with the Metropolitan Life Insurance Company, where he worked as an actuarial associate. Leight was an assistant actuary for five years with Guardian Life Insurance Company. He became a consultant in 1973.

Leight worked at such companies as Laikern, Siegel and Company; Lambert M. Huppeler Company, Inc.; Hallman & Lorber Associates Inc.; and the Federation Pension Bureau, where he worked as chief actuary.

In 1993, Leight retired in the Bronx, New York, but still kept busy with a part-time career as an income tax preparer.

Leight was an avid world traveler and visited many remote parts of the world. He was a lover of the theater, opera, and classical music.

He is survived by his brother, Lester, a niece, two nephews, and their children.

TRACEY LYNN MATTHEW
1967–2001

Tracey Lynn Matthew died March 28, 2001, in South Jersey Hospital in Elmer, New Jersey. She was 34.

Matthew was a 1985 graduate of Clearview Regional High School. She graduated in 1989 from Ursinus College in Collegeville, Pennsylvania, with a bachelor's degree in math. While at Ursinus College she was a member of the Phi Alpha Psi Sorority. She received her Associateship to the Casualty Actuarial Society in 1995.

Leslie Marlo (FCAS 1996), Donna Dowd, Lynne Peterson (FCAS 1998), and Nancy Arico (ACAS 1992), all colleagues of Matthew's at KPMG, composed the following tribute:

"During the five years Tracey was employed at KPMG, she became not only a valued coworker but, more importantly, an endeared friend to many of us. When we think of Tracey in both her professional and personal lives, what comes to mind most strongly is her commitment and dedication to whatever task was at hand. She consistently gave more than 100 percent of herself to make sure that a job would get done expertly.

"Her extremely analytic mind would look at an issue, whether an actuarial problem or a social or political problem, from all angles to make sure she understood all the facets and to make sure she made a decision that she could be comfortable with. Her coworkers and her clients certainly valued this quality.

"She always tried to improve whatever model or actuarial process she was working with. Often after working with a spreadsheet, she would propose ideas on how to streamline the work and make it more understandable for others, and these suggestions were usually right on target. It would be clear that she didn't just want to do the job to get it done; she wanted it to be the best it could be. This quality was also often exhibited when she heard about situations in people's lives that were stressful or difficult.

"Tracey left KPMG to stay at home and devote her time to her young family. She didn't feel she was able to give 100 percent to both, so she decided to give everything to her family. We were always glad to have Tracey come back to visit and have lunch with us. Although a tremendous loss for us, we were happy for Tracey when she made the decision to leave KPMG and are glad that Tracey was able to spend her last several months at home with her family, which is right where she wanted to be.

"It's not only her work ethic but her integrity and her whole outlook on life that are missed here every day. Although we had the honor of working with Tracey for only a fairly short time, she touched our lives in a very special and unique way that will keep her with us forever."

Matthew was a member of the Holy Name Church in Mullica Hill, New Jersey, and a former member of Richwood Methodist Church.

She is survived by her husband, Ronald B. Jr.; three children, Clint, Caden, and Alexis; parents, Walter and Kathleen Hitchner; brother, W. Thomas Hitchner; sister, Kara Hitchner, all of Richwood, New Jersey; sister, Therese Klodnick of Mullica Hill, New Jersey; and her grandmother, Hannah Samphone of Richwood, New Jersey.

PHILIP D. MILLER
1948–2001

Philip D. Miller died on September 11 during the attack on the World Trade Center. He was 53.

Miller graduated from City College of New York in 1968 at age 20, having already passed two CAS exams. He began a promising career that same year with the Insurance Rating Board, a predecessor of Insurance Services Office, Inc. (ISO), in its actuarial department in New York City.

According to Miller's wife, Arlene, he kept a strict study regimen, taking only one week off between sittings, and using a closet in their small New York apartment as his study space. By May 1975, Phil was a CAS Fellow with a perfect record of passing all exams on the first sitting.

The vigor with which Miller approached actuarial exams also defined his career at ISO. He moved up the corporate ladder the same way he passed exams: from actuarial student, to manager of the commercial automobile actuarial division, to ISO's first data quality officer, to vice president of data management and control, to senior vice president and chief actuary, a position he held at the time he left ISO to pursue a career as a consulting actuary.

"Phil had the ability to recognize your strengths even before you may have recognized them yourself—and he helped to nurture these strengths," recalled Rose Reindl, an ISO colleague. "While he remained focused and a classic workaholic, Phil always found the time to laugh and enjoy life. He was easy to talk to and made time to listen. To many of us, Phil was not just our boss or our colleague—he was our friend."

In 1995, Miller left ISO to pursue a career as a consultant with Tillinghast-Towers Perrin. He joined Aon in March 2001 as assistant director and actuary in their offices in the World Trade Center.

“I have known Phil over the last 25 years,” said Terry Alfuth (FCAS 1979). “I first met him at ISO when he was involved with the various committees. He was a soft-spoken leader and a person you could easily develop a friendship with. I recall many of the bus rides at the CAS meeting evening gatherings where we often talked about the future of the CAS and our individual careers. I will miss Phil as one of our Society’s true professionals and a dear friend.”

Miller volunteered his time as a member on the Education and Examination Committee (1976–1978) and the Committee on Management Data and Information (1985–1993). He also served as chairperson of the Committee on Management Data and Information (1992–1995). He most recently served on the Committee on Ratemaking.

Miller’s article, “Geographical Techniques to Review and Track Environmental Liabilities” was published in the 1994 Summer *Forum*.

Terry Pfeifer, his colleague at Aon, remembers, “Phil loved to talk about his family and the special retreat he and his wife had created in the Poconos. My picture of him will forever be the ever-smiling, tall, yet nonintimidating figure that commanded your attention with his gentle demeanor.”

Miller is survived by his wife Arlene, daughter Sheryl, and son Danny.

HARRY R. RICHARDS
1930–2001

Harry R. Richards died February 14, 2001, in his home in South Windsor, Connecticut. He was 70.

Richards was born in Sawyerwood, Ohio, on June 14, 1930, the son of the late George and Jessie Richards. He received his bachelor's degree from the University of Virginia in 1953 and became a second lieutenant in the U.S. Marine Corps, followed by a period of service in the Naval Reserve. Richards married Joan Sullivan in 1953.

He started his career at The Travelers Insurance Company of Hartford, Connecticut, serving as chief supervisor, assistant actuary, and associate actuary. During 1972 and 1973, Richards worked for the National Council on Compensation Insurance in Lyndhurst, New Jersey. He then became vice president of Berkley, Portermain & Associates in Greenwich, Connecticut and vice president and actuary of Portermain, Richards, and Davis, Inc. of South Windsor, Connecticut. He then became president of Independent Actuarial Services, Inc. After 20 years as president, he retired in South Windsor in 1999.

Richards received his Associateship to the Casualty Actuarial Society in 1960 and his Fellowship in 1963. His volunteer efforts spanned three decades. He served as a member on several committees including the Education and Examination Committee–Education, Public Relations Committee, and Committee on Loss Reserves. He also served as chairperson to the Committee on Consultants' Interests for four years.

Harry was a member of the Rotary Club of South Windsor, where he recently received the prestigious Paul Harris Fellow Award. Harry was an avid golfer and when his children were young, he was active with them in many youth programs in town. Besides his wife, Joan, he leaves three sons, a daughter, and two daughters-in-law: Michael and Sandy of Stafford, Connecticut;

David of Marblehead, Ohio; James and Barbara of Broad Brook, Connecticut; and Patricia, of Massachusetts. Richards is also survived by four sisters: Pearl Austin of Huntington, West Virginia; Janet Keller of Sun City, Arizona; Thelma Shepard of Fort Myers, Florida; Josephine Downing of Crystal River, Florida; and four grandchildren.

LEWIS H. ROBERTS
1918–2001

Lewis H. Roberts died April 21, 2001. He was 83.

Roberts earned a bachelor of science degree and received his master's degrees in business administration and mathematics from New York University. He worked as a mathematician and actuary at the National Bureau of Casualty Underwriters and the National Fire Insurance Company of Hartford. In 1963, Roberts began a 25-year stint with Woodward & Fondiller, becoming president of the actuarial consulting firm in 1979. Before retiring in 1997, he served as assistant commissioner and chief actuary for the New Jersey Department of Insurance.

During World War II, Roberts served in the Merchant Marine aboard the transport ships John Clem and Margaret Slanger in the Pacific. He was a member of the Northeast chapter of the U.S. Navy Armed Guard World War II and the Veterans of Foreign Wars.

A dedicated member of the Casualty Actuarial Society for over 40 years, Roberts received his Associateship in 1956 and his Fellowship in 1958. Among his many CAS activities, Roberts was a member of the Committee on Distribution of Losses (1964–1968), the Committee on Mathematical Theory of Risk (1964–1968), the Committee on Review of Papers (1964), the Committee on Professional Conduct (1972–1973), the Committee on Theory of Risk (1976–1987), the Committee on Consultants' Interests (1978–1980), and the Ad Hoc Committee on Memorials and Bequests (1981–1982). Roberts also served as a liaison representative on the Council of Presidents Joint Committee on the Code of Professional Conduct. In addition, Roberts was an accomplished author with six papers published in the *Proceedings of the Casualty Actuarial Society*.

Marty Adler (FCAS 1969), who worked for Roberts at Woodward & Fondiller from 1966–69, described him as a "unique individual, probably a genius" with strong personal opinions that he was not reluctant to share. "One of my favorite anecdotes relates to a time he had to work on Thanksgiving Day," said Adler. "He always underestimated the time he would need for an assignment, probably because he was a perfectionist about his work. On that day a woman sat next to him on the train from Norwalk, Connecticut, to New York City. She asked Lew where he was going. She next said it was a shame he had to work on a holiday. Then she asked if he had such a long commute (about an hour train ride) every day. Lew said yes. She said that was a bigger shame. At this point, Lew told me, he couldn't resist. He said that the thought had occurred to him when he first moved to Norwalk but after a short time he realized that it was the only time of day when he wouldn't be disturbed. The woman said nothing after that."

Roberts was the son of the late Earl Roberts and Leora Dennis Roberts and brother of the late Derwood Roberts. His wife, Alice J. Montana Roberts, died in 2000. Roberts is survived by a daughter, Christine F. Lonzello of Huntington Beach, California; a son, William J. of Gainesville, Florida; and a grandson.

HENRY C. SCHNEIKER
1928–2000

Henry C. Schneiker was born March 15, 1928. He became an Associate of the Casualty Actuarial Society in 1957.

Schneiker worked for the Mutual Insurance Rating Bureau of New York City as an associate statistician for two years before moving to the Home Insurance Company in 1960 where he worked for 25 years. He served the Home Insurance Company as assistant actuary, associate actuary, manager of the actuarial department, assistant secretary, secretary, and finally assistant vice president. He spent his retirement in Katonah and Brooklyn, New York.

Schneiker's contributions to the CAS include his discussion of the Robert L. Hurley (FCAS 1955) paper, "Commercial Fire Insurance Ratemaking Procedures." Schneiker's seminal discussion, which was published in the 1974 *Proceedings of the Casualty Actuarial Society*, highlighted the evolution of actuarial methods in fire insurance and identified a number of conspicuous problems associated with various aspects of procedures at that time.

INDEX TO VOLUME LXXXVIII

INDEX—CONTINUED

INDEX—CONTINUED

INDEX—CONTINUED