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FOREWORD

Actuarial science originated in England in 1792 in the early days of life insurance. Because of the technical nature of the business, the first actuaries were mathematicians. Eventually, their numerical growth resulted in the formation of the Institute of Actuaries in England in 1848. Eight years later, in Scotland, the Faculty of Actuaries was formed. In the United States, the Actuarial Society of America was formed in 1889 and the American Institute of Actuaries in 1909. These two American organizations merged in 1949 to become the Society of Actuaries.

In the early years of the 20th century in the United States, problems requiring actuarial treatment were emerging in sickness, disability, and casualty insurance—particularly in workers compensation, which was introduced in 1911. The differences between the new problems and those of traditional life insurance led to the organization of the Casualty Actuarial and Statistical Society of America in 1914. Dr. I. M. Rubinow, who was responsible for the Society's formation, became its first president. At the time of its formation, the Casualty Actuarial and Statistical Society of America had 97 charter members of the grade of Fellow. The Society adopted its present name, the Casualty Actuarial Society, on May 14, 1921.

The purposes of the Society are to advance the body of knowledge of actuarial science applied to property, casualty, and similar risks exposures, to establish and maintain standards of qualification for membership, to promote and maintain high standards of conduct and competence for the members, and to increase the awareness of actuarial science. The Society's activities in support of this purpose include communication with those affected by insurance, presentation and discussion of papers, attendance at seminars and workshops, collection of a library, research, and other means.

Since the problems of workers compensation were the most urgent at the time of the Society's formation, many of the Society's original members played a leading part in developing the scientific basis for that line of insurance. From the beginning, however, the Society has grown constantly, not only in membership, but also in range of interest and in scientific and related contributions to all lines of insurance other than life, including automobile, liability other than automobile, fire, homeowners, commercial multiple peril, and others. These contributions are found principally in original papers prepared by members of the Society and published annually in the *Proceedings of the Casualty Actuarial Society*. The presidential addresses, also published in the *Proceedings*, have called attention to the most pressing actuarial problems, some of them still unsolved, that have faced the industry over the years.

The membership of the Society includes actuaries employed by insurance companies, industry advisory organizations, national brokers, accounting firms, educational institutions, state insurance departments, and the federal government. It also includes independent consultants. The Society has three classes of members—Fellows, Associates, and Affiliates. Both Fellowship and Associateship require successful completion of examinations, held in the spring and fall of each year in various cities of the United States, Canada, Bermuda, and selected overseas sites. In addition, Associateship requires completion of the CAS Course on Professionalism. Affiliates are qualified actuaries who practice in the general insurance field and wish to be active in the CAS but do not meet the qualifications to become a Fellow or Associate.

The publications of the Society and their respective prices are listed in the Society's *Yearbook*. The *Syllabus of Examinations* outlines the course of study recommended for the examinations. Both the *Yearbook*, at a charge of $40 (U.S. funds), and the *Syllabus of Examinations*, without charge, may be obtained from the Casualty Actuarial Society, 1100 North Glebe Road, Suite 600, Arlington, Virginia 22201.

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NOTICE

Papers submitted to the *Proceedings* of the Casualty Actuarial Society are subject to review by the members of the Committee on Review of Papers and, where appropriate, additional individuals with expertise in the relevant topics. In order to qualify for publication, a paper must be relevant to casualty actuarial science, include original research ideas and/or techniques, or have special educational value, and must not have been previously copyrighted or published or be concurrently considered for publication elsewhere. Specific instructions for preparation and submission of papers are included in the *Yearbook* of the Casualty Actuarial Society.

The Society is not responsible for statements of opinion expressed in the articles, criticisms, and discussions published in these *Proceedings*.

Editorial Committee, *Proceedings* Editors

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THE DIRECT DETERMINATION OF RISK-ADJUSTED DISCOUNT RATES AND LIABILITY BETA

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Abstract

The development of a complete financial structure including balance sheet, income and cash flow statements, coupled with conventional accounting and economic valuation rules, provides the foundation from which risk-adjusted discount rates and liability betas can be determined. Since liability betas cannot be measured directly, a shift in focus is proposed to one based on measures more readily available and better understood, such as cost of capital, equity beta, leverage, etc. The risk-adjusted discount rate is shown as a function of these variables based on the developed financial structure and valuation framework.

The liability beta is then shown to follow as a consequence, also to be calculated as a function of these same variables. The risk-adjusted discount rates that result are less than the risk-free rate and the liability betas are negative to a greater degree than often suggested.

Several relationships are demonstrated including: risk/return versus leverage, equity beta versus liability beta, and underwriting profit margin related in turn to loss payout, investment yield, market risk premium, and leverage.

1. SUMMARY

The original Myers–Cohn "model" [11] presented basic principles of discounted cash flow, with losses risk-adjusted, for use in the determination of a "fair" premium in ratemaking. Determination of the risk adjustment to be used in discounting, a critical model parameter, was based on the liability beta. Unfortunately, determination of liability beta has proven to be both elusive and controversial, since data does not exist to support its direct measurement. As a consequence, arguments in rate hearings regarding the value of liability beta have become influenced more by subjective matters, such as one's philosophical view of the role of insurance in society, than by concrete facts. The ratemaking focus must be brought back to one based on analytics and supported by financially based, quantifiable assumptions and data. In the end, some means must be established for more rigorously incorporating underwriting risk and variability in the ratemaking process.

While elegant in many respects, what Myers–Cohn first presented was more conceptual than substantive, and it lacked many elements needed to permit its use in a ratemaking environment. Successful implementation of these concepts in a ratemaking context requires the development of a more complete and sophisticated financial model structure. At a minimum, the means to determine the rate of return implied by a particular insurance rate must be provided. In addition, the present overly subjective practice by which liability beta is selected in Massachusetts must give way to a more rigorous and quantifiable one.

The purpose of this paper is to first recap the essential changes that need to be made to the Myers–Cohn model, presented in detail in [3], to round it into a complete financial model containing the key components of total return. Second, the importance of using after-tax discount rates and the equivalency of net present value rates of return and internal rates of return that follow as a consequence is reviewed (also discussed in detail in [1], [2] and [3]). This foundation provides the critical model structure and valuation framework from which risk-adjusted discount rates and liability beta can be determined.

An important principle is introduced—that being that the risk-adjusted total rate of return must equal the risk-free rate. This fundamental principle provides a stepping stone from which a direct estimate of the liability beta becomes possible within the total return framework. Liability betas are shown in relationship to the total return to shareholders, and the linkage with equity betas demonstrated. The sensitivity of the underwriting profit margin to variations in loss payout, investment yield, market risk premium and leverage is demonstrated and discussed.

Liability betas cannot be directly measured, and Cummins and Harrington [6] and Fairley [9] presented approaches to estimate them. Kozik [10] discussed the many problematic aspects of the Capital Asset Pricing Model (CAPM) and liability beta theory, demonstrating why any estimate of liability beta is likely to be subject to much debate. It is important to keep in mind, however, that the development of a liability beta is a secondary objective to that of determining the appropriate risk-adjusted discount rate. This paper proposes a shift in focus from liability beta to one based on measures more readily available and better understood, such as cost of capital, equity beta, leverage, etc. The risk-adjusted discount rate will be shown as a function of these variables. While not essential to this ratemaking process, the liability beta which *must* follow as a consequence can be calculated as a function of these same variables, if one desires to do so.

The shift to a total return focus supported by equity betas and indicated cost of capital requirements, gives rise to the discussion of another important principle—the need to maintain consistency in financial leverage and equity beta due to the influence of leverage on the magnitude and volatility in shareholder returns.

2. TOTAL RETURN MODEL

Practitioners recognize that a more rigorous financial model framework is necessary to implement the basic Myers–Cohn principles (see [3] and [7]). A brief overview of Myers–Cohn and the "fair" premium determination is given in the Appendix. In addition to adding the missing elements needed to provide the complete total return model framework necessary to support ratemaking, some of the more critical "shortcomings" of the original Myers–Cohn presentation which must be addressed include:

1. A single period focus, utilizing the rather simplistic premium-to-surplus relationship, which avoids dealing with more involved issues that follow from the need to link surplus flows to policyholder liability flows over a multi-period timeframe.
2. The simplified view in which only losses are risky (i.e., require use of a risk-adjusted discount rate). Other uncertain underwriting cash flows, and variables such as underwriting income tax and surplus, which are dependent on losses, also require risk adjustment.
3. The reliance on a liability beta, needed within the CAPM framework to develop an estimate of the required risk adjustment, for which no direct measurement or actual data exists.

As discussed in detail in [3], several changes listed below are required in order to convert the Myers–Cohn model into a total rate of return model:

1. Introduce surplus flows into the model, including related investment income.

2. Separate and clearly delineate income from (a) underwriting, (b) investment of policyholder funds, and (c) investment of shareholder surplus.
3. Construct balance sheets and income statements, valued on both a nominal and a present value basis, given the respective cash flows. The present values of liabilities and surplus are of particular importance.
4. Discount all flows using after-tax rates, whether risk-free or risk-adjusted rates.
5. Develop rate-of-return measures from the net present value income components (underwriting income, operating income, and total income) by forming a ratio to the relevant balance sheet liability item. Display net present value calculations both *with* and *without* risk adjustment.
6. Discount surplus and underwriting taxes, also on a risk-adjusted basis, to the degree they are influenced by losses. Surplus is determined by use of a leverage ratio relative to liabilities inclusive of loss. Therefore, both surplus and underwriting taxes, which are both affected by loss, must also be risk-adjusted for the portion so affected. As in the case of losses, display net present value calculations both with and without risk adjustment.
7. Control surplus flows through a linkage with liabilities, with respect to both amount and timing.
8. Distribute operating earnings in proportion to the liability exposure over the period for which exposures exist. Essentially this rule distributes operating earnings in proportion to the loss reserve over time.

The above changes are merely those that permit Myers–Cohn to enter into the discounted cash flow/net present value family of models. The first six represent change with respect to model structure and analytics; the last two represent rules that specify the pattern of surplus flows and earnings realization based on relationships between risk and return. The Appendix provides a recap of these steps, converting Myers–Cohn into a net present value total return model. D'Arcy and Dyer [8] review many important principles with respect to discounted cash flow and other models in a broad economic context.

The introduction of surplus, via the leverage ratio, is necessary if a total rate of return is to be calculated. This provides an indication as to whether the cost of capital is being met, along with insurance costs, as specified in the actuarial ratemaking principles.

As a result of these steps, equivalency is achieved in rates of return, whether determined on a net present value, internal rate of return, or shareholder return basis. This is reviewed in the Appendix and discussed in detail in [2] and [3]. An important element in this reconciliation is the proper reflection of taxes with respect to discounting and the time value of money. This area is worthy of review.

3. AFTER-TAX DISCOUNTING

The economic value that can be realized over time by holding onto an asset is determined through the process of discounting. The reasonably risk-free, pre-tax rate at which an asset can be invested, net of the tax payable on such implied investment income, is the rate that fully reflects the economics involved.

While it is common to see models that use pre-tax discounting (and some of these introduce taxes as a last step), this is incorrect in principle. Insurance companies are tax-paying entities, obligated to pay taxes on income (including investment income) as earned. Thus insurers realize only an after-tax economic return on their investments. Just as bottom-line net income from underwriting is top-line premium less underwriting expense and tax, bottom-line net income from investment is top-line pre-tax investment income less investment expense and tax. Simply put, taxes are a significant expense that cannot be ignored.

To illustrate this point, consider a \$1,000 asset to be held for one year, with risk-free government yields available of 6%. At the end of a year \$60 of pre-tax investment income will be realized, and be subject to tax. At a 35% tax rate, only \$39 will remain, the net economic value generated from this asset. The effective earnings rate is thus 3.9%, or 6% taxed.

Now suppose a claim for \$1,000 is to be paid in one year. If one assumes that the present value can be based on a pre-tax interest rate of 6%, then only \$943 need be set aside to cover it (\$1,000 discounted with a factor of 1.06). The \$943 will grow at 6% to \$1,000; however, tax will have to be paid on the \$57 dollars of income, leaving the company short of the \$1,000. The necessary amount that must be set aside to cover the claim is actually \$962 (\$1,000 discounted with a factor of 1.039). The \$962 will earn interest of \$58 dollars, less a tax of \$20, leaving the company with the necessary \$1,000 to pay the claim. Thus the economic value associated with the \$1,000 loss payable in one year is \$38, and the discounted loss reserve is \$962 at the beginning of the year.

While models that apply taxes to calculate the final answer in a last step may be reasonably accurate and simpler to construct, this is akin to assuming a life-insurance-like inside buildup, and the degree of error will increase as the holding period extends beyond a single year.

4. DERIVATION OF RISK-ADJUSTED DISCOUNT RATE AND LIABILITY BETA

The model framework supporting the calculation of a rate of return, both with and without risk adjustment, with taxes fully reflected in the discount rate, provides the key to being able to directly estimate liability beta. The following principles will be utilized in conjunction with the rate of return model:

(i) If no adjustment is made for risk in the discount rate, then the total calculated rate of return must equal the required cost of equity, whereas,

(ii) if all risk is taken into account in the discount process, then the total calculated rate of return must equal the risk-free rate.

These principles simply state that rates of return should normally equal the cost of equity when no adjustment is made for risk in the discount rate, but that they should equal the risk-free rate in the absence of risk, as occurs when risk-adjusted discount rates are used. The first principle is simply a statement that total return should equal the cost of capital.

The second principle is at the core of the risk adjustment process with respect to rate of return. The purpose of risk adjustment is to adjust mathematically for risk such that the result becomes comparable to other such risk-adjusted rates of return. Usually this process targets the adjusted result to a common reference point represented by the risk-free rate of return. In the case of discounted cash flow calculations, this involves an economically-based formula that reflects the time value of money. The important point is that the risk adjustment to the discount rate has the effect of mathematically accounting for (i.e., eliminating) risk so that the resulting risk-adjusted total return is the risk-free rate. If this were not the result, then by definition further risk would remain and the risk adjustment process would not have been complete.

The rate of return model formulation, both with and without risk adjustment, will be used to demonstrate by way of simple examples how the required risk adjustment and liability beta can be determined directly. For simplification in the examples presented here, expenses will be assumed to be zero, premium to be fully collected at policy inception (i.e., at time 0), taxes paid without delay, and losses fully paid on a single date. Only losses will be assumed to require risk adjustment. The formulas used below for calculating net present value rates of return are presented in detail in [3], [4] and [5], and are reviewed in the Appendix.

First, given loss (L), tax (T), before-tax interest rate (R_b), loss payment date (N), liability/surplus leverage factor (F), equity beta (β_e) and the market risk premium (R_p), a premium (P) is determined that generates a total return, without risk adjustment, equal to the CAPM-based cost of equity of $R_b + \beta_e R_p$:

$$\frac{(P-L)(1-T)+L\{1-1/(1+R)^N\}}{L\{1-1/(1+R)^N\}/R}F + R = R_b + \beta_e R_p. \quad (1)$$

Second, this premium (P) is used to determine the after-tax risk adjustment (R_L) that produces a risk-adjusted total return equal to the risk-free rate:

$$\frac{(P-L)(1-T)+L\{1-1/(1+R+R_L)^N\}}{L\{1-1/(1+R+R_L)^N\}/(R+R_L)}F + R = R_b. \quad (2)$$

Finally, the implied liability beta is determined using the relationship:

$$\beta_L = R_{Lb}/R_p = \{R_L/(1-T)\}/R_p,$$

with required assumptions for:

R_b: Interest rate, before-tax

R: Interest rate, after-tax

L: Loss

F: Liability/surplus leverage factor

R_p: Market risk premium

β_e: Equity beta,

and the following derived by formula:

P: Premium

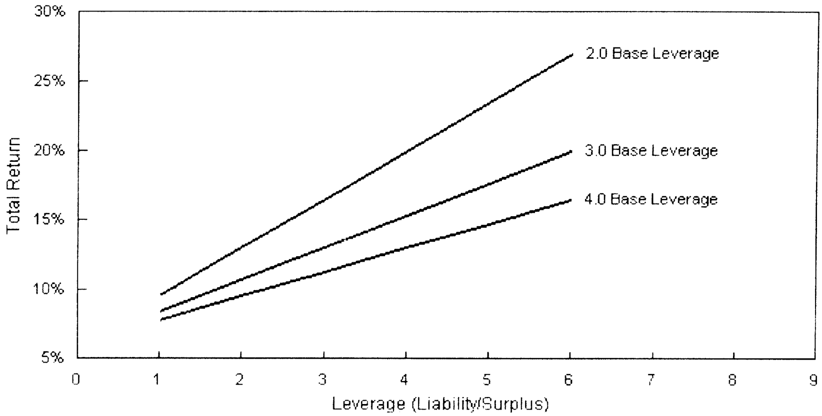
R_L: Risk discount adjustment, after-tax

β_L: Liability beta

R_{Lb}: Risk discount adjustment, before-tax.

FIGURE 1

BASELINE RISK/RETURN LINE VS. LEVERAGE



Average loss payout = 3 years
Equity beta = 1.0
Investment yield before-tax = 6%
Market risk premium = 7%

Note: Each line will produce a 13% total return when the actual leverage is the same as the base leverage was when the equity beta was determined to be 1.0.

Formula (1) expresses the sum of after-tax underwriting income and the present value of investment income on loss reserves, in ratio to the present value of balance sheet loss reserve liabilities. This is the operating return, and it is multiplied by leverage and the investment return on surplus is added to produce the total return. Formula (2) differs only by the introduction of the risk discount adjustment. These formulas are simplified due to the assumptions that premium is collected at policy inception, expenses are zero, there is no delay in tax payments, and that losses are paid in a single payment. Formulas (1) and (2) are reviewed in more detail in the Appendix.

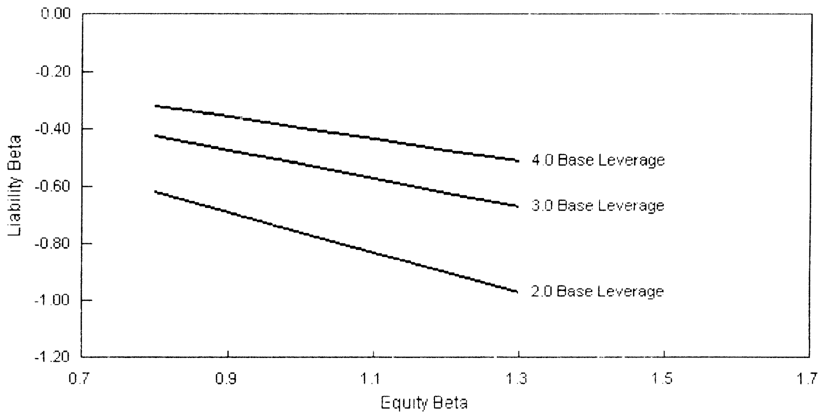
These basic relationships were used to produce Figures 1–7, which demonstrate various relationships among the variables. Figure 1 establishes a base point of reference by demonstrating the relationship between leverage and return at three given leverage levels. In practice, the measured equity beta and CAPM target cost of capital are at some “typical” leverage. If leverage were to be higher, then the required return would be higher, and if leverage were to be lower, then the required return would be lower. Presumably this would affect measured equity betas. This interplay between leverage, return and risk should be considered when solving for the target premium.

The actual leverage level is an extremely important, yet often overlooked aspect of risk and return. Leverage simultaneously and similarly affects both return and risk, as measured by the variability in return. All else being equal, higher leverage should produce greater returns (i.e., higher cost of capital) and greater variability in returns. Although one might expect higher betas to be produced when leverage is higher, this aspect is seldom considered when they are calculated and published. Given the significant impact of leverage, and since industry leverage has been declining steadily over the past several years, three specific values were selected to represent this range and to reflect this dynamic in the following discussion. The fact that insurance industry equity betas today are around 1.0 is consistent with the effect that large amounts of surplus and low leverage have in suppressing variability in return, and in making insurer returns align more closely to overall market returns. Both the cost of capital and equity beta are expected to flex with leverage changes over time.

Three base leverage levels (2.0, 3.0 and 4.0) have been assumed. These represent three possible levels of leverage in existence at the point in time when the equity beta was determined to be 1.0. Actual leverage may subsequently vary from these respective base points as shown by the three lines on the chart. Each of the lines, however, must produce a total return of 13% when the actual leverage matches the base level corresponding to the original calculation of the equity beta. This is the CAPM framework in which the cost of capital (13%) is equal to the

FIGURE 2

EQUITY VS. LIABILITY BETA



Average loss payout = 3 years
Investment yield before-tax = 6%
Market risk premium = 7%

risk-free rate (6%) plus the market risk premium (7%) times the equity beta (1.0). These lines are used in the analysis to adjust the total return target up or down if actual leverage increases or decreases from the respective base.

5. LIABILITY BETA

Following the steps discussed above, the liability betas determined by formula are shown in Figure 2 in relationship to equity betas. The example shown is for a three-year loss payout, 7% market risk premium, and 6% risk-free yield. *The liability beta is negative in all cases*. The magnitude shown here is substantially more negative than most of the literature has indicated. This is likely due to the fact that more sources of risk (i.e., variability) exist than may have been recognized by previous measures that have assumed that losses alone are risky. This narrow assumption excludes sources of risk from the variability in the timing of loss payout and the variability in the amount and timing of all other cash flows, including premium, expense, tax and investment.

One would hope that liability betas would be estimable within a more narrow range than that shown in Figure 2. Since total returns are affected by leverage, it would seem logical to expect that equity betas would flex to some degree as leverage changed, whereas liability betas should be relatively unaffected and more stable. Increasingly more negative liability betas occur when moving from the upper, higher leverage line to the lower, less leveraged line in Figure 2. More negative betas are what should be expected given the historical trends in declining industry leverage and the likely delay in market response in forcing equity betas down proportionally in line with this.

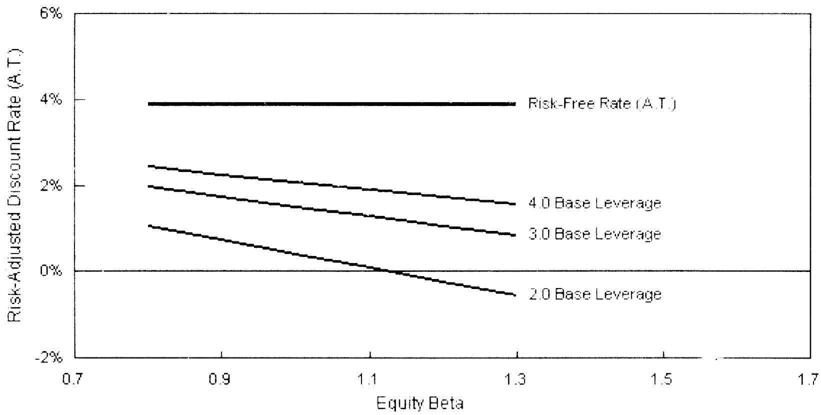
The fact that the liability beta must be negative is intuitively obvious. Suppose that a $1,000 loss payable in one year is to be reinsured (100%), with risk-free rates at 6%. If the amount and timing are both absolutely certain, it should be possible to find a reinsurer who would agree to assume the loss obligation for a premium of $962 ($1,000/1.039). If, on the other hand, losses are uncertain, the additional risk transfer that occurs from the insurer to the reinsurer requires that the reinsurer receive additional compensation. The reinsurer will require a premium *greater* than $962. In other words, *the risk-adjusted discount rate must be less than the risk-free rate*, and liability beta must be negative.

The degree to which the risk-adjusted discount rate must be less than the risk-free rate is shown in Figure 3, for the same example, and also in relation to the equity beta.

Although low leverage would not generally be associated with a large equity beta, this extreme (lower right, bottom line in Figure 3) would result in a negative risk-adjusted discount rate. In other words, the discounted liability would be greater than the nominal liability. A sufficiently large surplus base, without corresponding reductions in the equity beta and the cost of capital,

FIGURE 3

EQUITY BETA VS. RISK-ADJUSTED DISCOUNT RATE (AFTER-TAX)



Average loss payout = 3 years
Investment yield before-tax = 6%
Market risk premium = 7%

impose an unrealistic burden on insurance pricing. This is the true essence of "surplus-surplus" as discussed at times in the ratemaking context. This will be explored further below with respect to the underwriting profit margin.

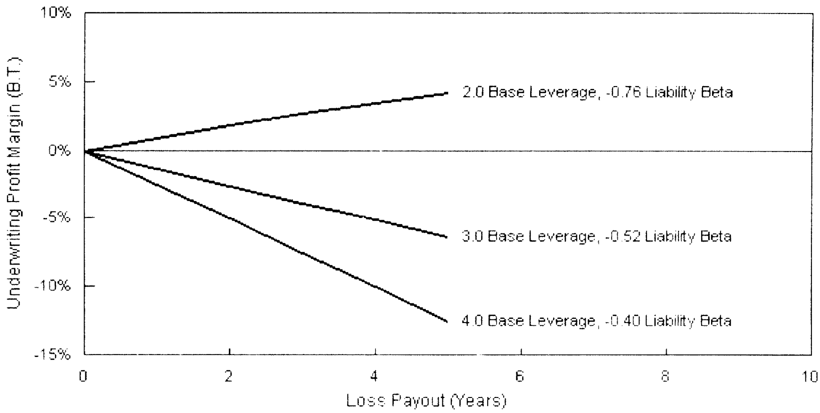
6. UNDERWRITING PROFIT MARGIN

The ultimate goal in ratemaking is to determine the premium, given assumptions on all costs and financial conditions, that in some way is "fair" to policyholders and owners of the company alike. Figures 4–7 present the premium rate as an underwriting profit margin as a function of loss payout, investment yield, market risk premium, and leverage, respectively.

In each of the examples, the base leverage cases (2.0, 3.0 and 4.0) require liability betas of approximately −0.8, −0.5, and −0.4, respectively. Underwriting profit margins typically become

FIGURE 4

UNDERWRITING PROFIT MARGIN VS. LOSS PAYOUT



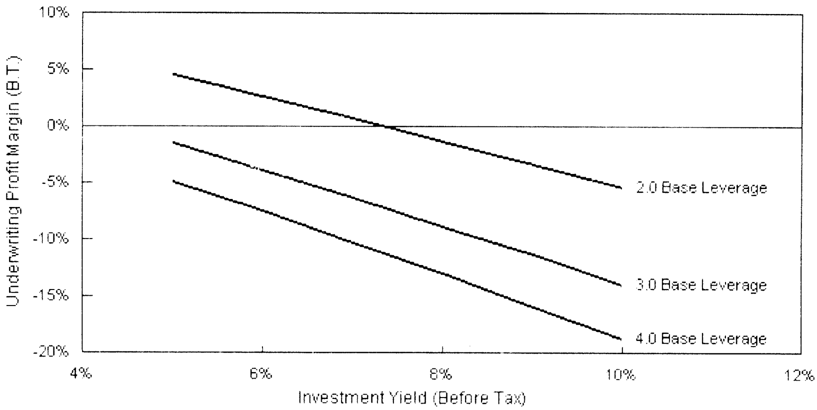
Investment yield before-tax = 6%
Market risk premium = 7%
Equity beta = 1.0

more negative (i.e., higher combined ratios) when loss payouts lengthen as shown in Figure 4, due to the greater investment income that will be generated prior to loss payment. This is shown in the lower two lines on the chart. However, as noted above, when leverage levels become so low as to create burdensome amounts of surplus, the opposite can happen if cost of capital and equity betas are not adjusted. This is the case in the upper line in Figure 4, in which the cost of equity and the equity beta have not been altered to reflect the lesser risk implied by the lower leverage. If the equity beta were to decline to at least 0.8 (and the capital target return decline to 11.6% from 13.0%) in this example, this effect would be avoided, with the resulting expected downward sloping line.

As investment yields increase, underwriting profit margins deteriorate as shown in Figure 5. While this sounds a bit like cash flow underwriting, if premium rates are to fully reflect the ben-

FIGURE 5

UNDERWRITING PROFIT MARGIN VS. INVESTMENT YIELD



Average loss payout = 3 years
Equity beta = 1.0
Market risk premium = 7.0%

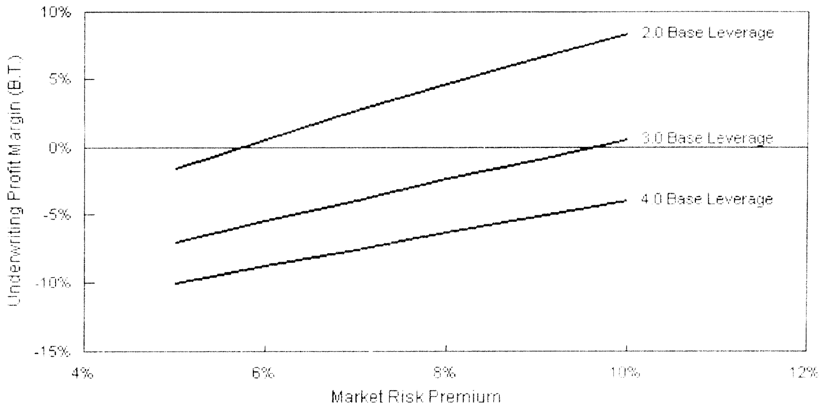
efit of higher yields, this is the result. It is important, however, that individual policy or accident periods be self-sustaining, and that historical portfolio yields not be used to subsidize new writings, truly the negative sort of cash flow underwriting that has occurred in the past in the industry.

As in the previous example, the burden on underwriting when leverage is low is shown on the top line, upper left. Here again, the equity beta and cost of capital have not responded to the lower leverage-induced risk.

The effects of the market risk premium on rates and the profit margin are shown in Figure 6. Clearly, higher required risk premiums lead to higher required total return and higher premium to achieve it. Once again, the importance of achieving consistency in the leverage and the measured equity beta and the resultant cost of capital target is evident by the wide spread between the three lines on the chart.

FIGURE 6

UNDERWRITING PROFIT MARGIN VS. MARKET RISK PREMIUM



Average loss payout = 3 years
Equity beta = 1.0
Investment yield before-tax = 6%

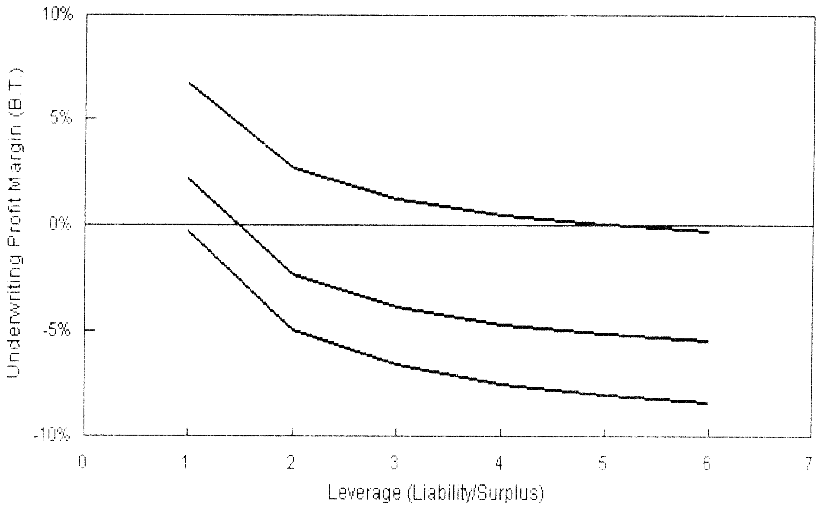
The relationship between leverage and the profit margin is shown in Figure 7. Note the severe impact caused when leverage is very low. If target returns are to be achieved when leverage declines to very low levels, significant increases in premium are required. Once again one has to question at what point surplus levels become "excessive" in relation to current writings, and whether it is reasonable to require target rates of return on the full amount of surplus beyond this point. Perhaps the current low levels of industry leverage are now creating just such a dilemma in which it is becoming increasingly difficult to generate adequate returns on the entire amount of surplus available.

7. CONCLUSION

This paper has presented a methodology for the direct determination of risk-adjusted discount rates and liability betas. It involves the utilization of a "complete" total rate of return

FIGURE 7

UNDERWRITING PROFIT MARGIN VS. LEVERAGE



Average loss payout = 3 years
Equity beta = 1.0
Investment yield before-tax = 6%
Market risk premium = 7%

model (albeit in simplified form) in which rates of return can be determined both with and without risk adjustment. The total return *without* risk adjustment must equal the target cost-of-capital-based return. The total return *with* risk adjustment must equal the risk-free rate. Within this formulation it is important that taxes be reflected by utilizing discount rates on an after-tax basis.

This formulation provides the capability to directly determine the required risk-adjusted discount rate and liability beta, given standard underwriting financials, leverage factor, market risk premium and equity beta. The risk-adjusted discount rates that result are less than the risk-free rate and the liability betas are negative, to a much greater degree than are often suggested. In no instance are they positive.

The important influence of leverage, and the need for consistency with the cost of capital and equity beta measurements, are noted. Subsequent changes in leverage require adjustment to these critical CAPM parameters. While the estimation and application of equity beta and the cost of capital are not without debate, at least there is a wide body of comparative data available to help judge the reasonableness of the results. This is not the case with respect to liability beta.

Hopefully, in the future the conceptual dialogue over risk adjustment and liability betas can be made more meaningful by combining clearly specified parameter assumptions into a concrete total return model framework, such as has been presented in this paper.

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APPENDIX

The following example provides high-level balance sheet, income and cash flow statements. These are used to demonstrate various rate of return calculations and to show the resulting equivalency between conventionally reported rates of return and net present value rates of return, assuming certain rules are followed to control the flow of surplus and to distribute profits. The net present value rate of return is shown with and without risk adjustment. Following this, the Myers–Cohn fair premium approach is briefly recapped, as modified to use after-tax discounting, shown in relation to this same example.

The following financial assumptions form the basis for the example presented:

- 103.85% combined ratio
- $9,629 premium, collected without delay
- $10,000 loss, single payment at end of year 3
- $0 expense
- 35% income tax rate, no delay in payment
- 6.0% investment interest rate before-tax, 3.9% after-tax
- No loss discount tax
- 3.0 liability/surplus ratio.

This example corresponds to the following:

- 1.0 equity beta
- 7.0% market risk premium
- 3.65% liability risk discount adjustment, before-tax
- −0.521 liability beta.

Simplified balance sheet, income and cash flow statements are shown for this example in Table A.1. The rules governing the flow of surplus are as follows: (1) the level of surplus is maintained at a 1/3 ratio with loss reserves, (2) investment income on surplus is paid to the shareholder as earned, and (3) operating earnings are distributed in proportion to the level of insurance exposures in each year, measured by loss reserve levels, relative to the total exposure. Since loss reserves are equal at $10,000 in each of the three years, operating earnings are distributed to the shareholder equally in each year.

Three "levels" of return exist within an insurance company. The first is the underwriting rate of return, which reflects what the company earns on pure underwriting cash flows, before reflecting investment income on the float. This is a "cost of funds" to the company. The second, operating return, reflects what the company earns on underwriting when investment income on the float is included. This is the "risk charge" to the policyholder for the transfer of risk to the company. The third, the total return, is the net result of underwriting and investment income from operations together with investment income on surplus.

These rates of return can be determined by either a cash-flow-based internal rate of return (IRR) calculation or by relating income earned to the amount invested. With regards to the shareholder total return perspective, the internal rate of return (IRR) based on cash flows from and to the shareholder indicates a 13.0% return over the three-year period. The income versus investment approach (i.e., ROE) relates the income over the full three-year aggregate financial life of the business to the shareholder's investment over this same period. This is shown in both nominal (i.e., undiscounted) and in present value (discounted, but without risk adjustment) dollars to produce a 13.0% rate of return on investment. Furthermore, the return realized by the shareholder via dividends is also an identical 13.0% in each year. (This attribute follows from the rules used to control the flow of

TABLE A.1

THREE-PERIOD DEMONSTRATION EXAMPLE RATE OF RETURN

	PERIOD					Present Value	
	1	2	3	4	Total	Not Risk Adjusted	Risk Adjusted
BALANCE SHEET (Beginning of Period)							
Invested Assets	13,092	13,170	13,250	0	39,512	37,072	38,808
Loss Reserves	10,000	10,000	10,000	0	30,000	27,804	29,106
Retained Earnings	−241	−164	−83	0	−488		
Surplus Contributed	3,333	3,333	3,333	0	10,000	9,268	9,702
Liabilities/Surplus	3.0	3.0	3.0	0	3.0	3.0	3.0
INCOME AFTER-TAX (During Period)							
Underwriting	−241	0	0	0	−241	−241	−241
Investment Income on Retained Earnings	−9	−6	−3	0	−19		
Investment Income on Loss Reserves	390	390	390	0	1,170	1,084	445
Total Operating	140	384	387	0	910	843	204
Investment Income on Surplus	130	130	130	0	390	361	378
Total Net Income	269	514	517	0	1,300	1,205	582
Return on Beginning Contributed Surplus	8.1%	15.4%	15.5%	0	13.0%	13.0%	6.0%
CASH FLOW (During Period)							
Operating Cash Flows							
Premium Receipt	9,629	0	0	0	9,629	9,629	9,629
Loss Payment	0	0	0	−10,000	−10,000	−8,916	−9,555
Tax Paid	130	0	0	0	130	130	130
Retained Earnings "Funding"	241	−87	−87	−87	−19	0	0
Total Underwriting Income	10,000	−87	−87	−10,087	−260		
Underwriting Return			Underwriting IRR = −0.9%				

Investment Receipts (After-Tax)		390	390	390	1,170		
Total Operating Income	10,000	303	303	–9,697	910	843	204
Operating Return			Operating IRR = 3.0%				
Surplus Cash Flows (Beginning of Period)							
Contributed Surplus	3,333	0	0	3,333	0		
Dividend							
Surplus Investment Income		–130	–130	–130	–390		
Operating Earnings		–303	–303	–303	–910		
Net Shareholder Income	3,333	–433	–433	–3,767	–1,300		
Shareholder Rate of Return			Shareholder IRR = 13.0%				
Shareholder “Dividend” Yield		13.0%	13.0%	13.0%	13.0%		

RATE OF RETURN

		(Income/Investment)		
			Net Present Value Basis	
	IRR	Nominal	Not Risk-Adjusted	Risk-Adjusted
Underwriting Return	–0.9%	–0.9% = –260/30,000	–0.9% = –241/27,804	–0.8% = –241/29,106
Operating Return	3.0%	3.0% = 910/30,000	3.0% = 843/27,804	0.7% = 204/29,106
Shareholder Total Return	13.0%	13.0% = 1,300/10,000	13.0% = 1,205/9,268	6.0% = 582/9,702

surplus.) When risk-adjusted, the total net present valued rate of return is 6.0%, which is identical to the risk-free rate.

The operating return, inclusive of underwriting and investment income, is shown to generate a cash-flow-based internal rate of return of 3.0%. Equivalently, the operating income of $910 is a 3.0% return on the "investment equivalent" of $30,000, the total balance sheet policyholder supplied float upon which these earnings were generated. Also, $843 of present valued income is 3.0% on the present valued liability of $27,804.

The formulas that can be used to directly determine the net present value based rates of return, both without and with risk adjustment, are shown in Tables A.2 and A.3, respectively. The following variables are used in Tables A.2 through A.4:

- P: Premium
- R_b: Interest rate, before-tax
- L: Loss
- R: Interest rate, after-tax
- N: Loss payment date
- R_L: Risk discount adjustment, after-tax
- T: Tax rate
- UWPT: Underwriting profit tax
- N_T: Underwriting tax payment delay
- IBT: Investable balance investment income tax
- F: Liability/surplus leverage factor
- S: Initial surplus contribution (L/F).

Myers–Cohn Fair Premium With After-Tax Discounting

The $9,629 premium shown in the example can be derived using the traditional Myers–Cohn (MC) format, as long as all

TABLE A.2

NET PRESENT VALUE INCOME, BALANCE SHEET AND RATE OF RETURN DEFINITIONS, FORMULAS AND CALCULATIONS WITHOUT RISK ADJUSTMENT

INCOME ITEMS	FORMULAS
Underwriting Income	$(P-L)(1-T)$ $(9{,}629 - 10{,}000)(1 - 0.35) = -241$
Operating Income	$\text{PV}(P)-\text{PV}(L)-\text{PV}(\text{UWPT}) = P-L/(1+R)^N - T(P-L)$ $9{,}629 - 10{,}000/(1+0.039)^3 - (0.35)(9{,}629 - 10{,}000)$ $= (P-L) - T(P-L)/(1+R)^{N_T} + L[(1-1/(1+R)^N]$ $(9{,}629 - 10{,}000) - (0.35)(9{,}629 - 10{,}000)/(1+.039)^0$ $+ 10{,}000[1 - 1/(1+0.039)^2]$ Underwriting Income + Investment Income Credit on Policyholder Liabilities − 241 + 1,084 = 843
Surplus Investment Income	R (Surplus) (0.039)(9,268) = 361
Total Income	= Operating Income + Investment Income on Surplus 843 + 361 = 1,205
BALANCE SHEET ITEMS	
Policyholder Liabilities	$L(1-1/(1+R)^N)/R$ $10{,}000(1-1/(1+0.039)^3)/0.039 = 27{,}804$
Surplus	$S(1-1/(1+R)^N)/R$ $3{,}333(1-1/(1+0.039)^3)/0.039 = 9{,}268$
RATES OF RETURN	
Underwriting Return on Liabilities (Cost of Policyholder-Supplied Funds)	Underwriting Income / Policyholder Liabilities −241/27,804 = −0.9%
Operating Return on Liabilities (Risk Charge to Policyholder)	Operating Income / Policyholder Liabilities 843/27,804 = 3.0%
Total Return on Surplus (ROS) (Shareholder Return)	Total Income / Surplus 1,205/9,268 = 13.0%
	= (Operating Return on Liabilities)(Liability/Surplus) + R 3.0%(3) + 3.9% = 13.0%

TABLE A.3

NET PRESENT VALUE INCOME, BALANCE SHEET AND RATE OF RETURN DEFINITIONS, FORMULAS AND CALCULATIONS WITH RISK ADJUSTMENT

INCOME ITEMS	FORMULAS
Underwriting Income	$(P - L)(1 - T)$ $(9{,}629 - 10{,}000)(1 - 0.35) = -241$
Operating Income	$\text{PV}(P) - \text{PV}(L) - \text{PV}(\text{UWPT}) = P - L/(1 + R - R_L)^N$ $-T(P - L)$ $9{,}629 - 10{,}000/(1 + 0.039 - 0.024)^3 - (0.35)$ $(9{,}629 - 10{,}000)$ $= (P - L) - T(P - L)/(1 + R)^{N_T} + L[(1 - 1/(1 + R - R_L)^N]$ $(9{,}629 - 10{,}000) - (0.35)(9{,}629 - 10{,}000)/(1 + 0.039)^0$ $+10{,}000[1 - 1/(1 + 0.039 - 0.024)^3]$ = Underwriting Income + Investment Income Credit on Policyholder Liabilities $-241 + 445 = 204$
Surplus Investment Income	R (Surplus) $(0.039)(9{,}702) = 378$
Total Income	Operating Income + Investment Income on Surplus $204 + 378 = 582$
BALANCE SHEET ITEMS	
Policyholder Liabilities	$L(1 - 1/(1 + R - R_L)^N)/(R - R_L)$ $10{,}000(1 - 1/(1 + 0.039 - 0.024)^3)/(0.039 - 0.024) =$ $29{,}106$
Surplus	$S(1 - 1/(1 + R - R_L)^N)/(R - R_L)$ $3{,}333(1 - 1/(1 + 0.039 - 0.024)^3)/(0.039 - 0.024) =$ $9{,}702$
RATES OF RETURN	
Underwriting Return on Liabilities (Cost of Policyholder Supplied Funds)	Underwriting Income / Policyholder Liabilities $-241/29{,}106 = -0.8\%$
Operating Return on Liabilities (Risk Charge to Policyholder)	Operating Income / Policyholder Liabilities $204/29{,}106 = 0.7\%$
Total Return on Surplus (ROS) (Shareholder Return)	Total Income / Surplus $582/9{,}702 = 6.0\%$
	= (Operating Return on Liabilities)(Liability/Surplus) + R $0.7\%(3) + 3.9\% = 6.0\%$

discounting is on an after-tax basis, and given a liability beta that is “consistent” with the equity beta. The traditional MC model format as shown in [1] is as follows:

$$P = \text{PV}(L) + \text{PV}(\text{UWPT}) + \text{PV}(\text{IBT}).$$

This states that the fair premium (P) is equal to the sum of the present value of the losses (L), the tax on underwriting profit (UWPT), and the tax on investment income derived from the investable balance (IBT). The investable balance includes all policyholder liabilities (net of premium, loss and expense) and surplus. Note that underwriting expense is combined with loss as total liabilities in the example in the cited reference. It is suggested that the discount rates be adjusted for risk (i.e., uncertainty), particularly the rate applicable to losses. No mention is made as to whether discount rates are on a before-tax or after-tax basis.

The present example differs from the model in [1] to some degree, first by extending from one to three periods, and then by assuming that taxes on underwriting and investment are paid without delay. In [1] underwriting taxes were assumed to have a one year delay in their payment. The tax loss discount (TRA 86) was excluded for simplification in both instances. In [1] S was set equal to P for the single period example presented. In the present example, surplus was set at each point in time to an amount equal to L/F, where F is the liability/surplus leverage factor. Since surplus is set as a function of liabilities, surplus is implicitly risk-adjusted as well.

Table A.4 presents the derivation of the “fair” premium that results when the Myers–Cohn approach is reformulated to use after-tax discounting and to control surplus via a linkage to liabilities over the multi-period timeframe. In this example the interest rate is 6%, the tax rate is 35%, and a risk adjustment of 3.65% before-tax (i.e., 2.4% after-tax) is made when discounting. This is the risk adjustment that results from a liability beta of –0.521. A liability/surplus ratio of 3-to-1 is used to determine the level

TABLE A.4

DERIVATION OF "FAIR" PREMIUM WITH AFTER-TAX DISCOUNTING

P = PV(L)	9,555	$L/(1+R-R_L)^N$ $10{,}000/(1+0.039-0.024)^3$
+ PV(UWPT)	−130	$T[P/(1+R)^{N_T} - L/(1+R-R_L)^{N_T}]$ $(0.35)[(9{,}629)/(1+0.039)^0 - 10{,}000/(1+0.039-0.024)^0]$
+ PV(IBT)	204	$TR_bS[(1-1/(1+R-R_L)^N)/(R-R_L)]$ $(0.35)(0.06)(3{,}333)[(1-1/(1+0.039-0.024)^3)/$ $(0.039-0.024)]$
"Fair" Premium	9,629	

Notes: Due to after-tax discounting PV(IBT) reduces to simply tax on investment income derived from the investable surplus balance.

Liability/surplus relationship implies surplus level affected by risk adjustment.

of surplus. The fair premium that results is $9,626. As stated previously, premiums and taxes are assumed to have no delay in their receipt or payment.

RISK AND RETURN: UNDERWRITING, INVESTMENT AND LEVERAGE

PROBABILITY OF SURPLUS DRAWDOWN AND PRICING FOR UNDERWRITING AND INVESTMENT RISK

RUSSELL E. BINGHAM

Abstract

The basic components of the risk/return model applicable to insurance consist of underwriting return, investment return and leverage. A pricing approach is presented to deal with underwriting and investment risk, guided by basic risk/return principles, which addresses the policyholder and shareholder perspectives in a consistent manner. A methodology to determine leverage is also presented, but as a distinct and separate element, enabling the pricing approach to be applied either with or without allocation of surplus to lines of business. Since the leverage is also developed within a total risk/return framework, the approach provides a means to integrate what are often disjointed rate and solvency regulatory activities.

Risk is controlled by a focus on the likelihood that total return falls short of the target "fair" return by an amount which results in a specified drawdown of surplus. Thus rate adequacy and solvency are dealt with simultaneously. A shift away from probability of ruin and expected policyholder deficit approaches to solvency and ratings is proposed and explained.

An "Operating Rate of Return" is defined and suggested as the appropriate rate of return measure that should be used for measuring the charge for risk transfer from the policyholder to the company, rather than other measures such as profit margin, return on premium, etc.

1. SUMMARY

Rate of return and *risk* in return represent the dimensions of expectation and uncertainty, respectively. The tradeoffs between them are real and faced by individuals and businesses frequently. The decision to invest involves a choice among alternatives having anticipated variation in both return and risk. Being generally averse to risk, individuals and businesses choose the least risky investment for a given level of anticipated return, or require a greater return when investments are riskier. The investor perspective with respect to risk tends to be one of concern with the degree to which returns might depart (or vary) from the expected level.

The policyholder perspective, as represented by regulators and rating agencies, is typically more concerned with the dimension of risk having to do with the occurrence of extreme and adverse events, and whether the level of capital available is adequate given the probability and magnitude of such events occurring. However, the risk transfer that occurs from the policyholder to the company is governed by much the same risk/return principles as those that govern the relationship between the company and the shareholder. When viewed within the risk/return context, the linkage between the policyholder and shareholder perspectives becomes clear, and the means for determining both fair premiums to the policyholder and fair returns to the shareholder is provided.

In employing its equity and setting prices, insurance company management is making an investment choice among alternative lines of business and investment asset classes based on knowledge of expected returns coincident with the risks associated with those choices. These risks reflect both the shareholder and policyholder perspectives. The assessment of the tradeoff between these risks and returns and the level of surplus either required or available, is guided by the company's desire to achieve a reasonably balanced portfolio of businesses with a controlled risk/return profile for the company in aggregate.

This paper will explain the basic components of the risk/return model applicable to insurance, as comprised of underwriting return, investment return and insurance leverage. It will discuss a pricing approach to deal with underwriting and investment risk (i.e., variability) that addresses the concerns of both the policyholders and the shareholders. A risk charge is shown as a function of underwriting and investment risk, and the sensitivity of price changes to them is demonstrated. Operating return (i.e., return on underwriting and investment of policyholder funds) coupled with the specification of "probability of surplus drawdown" (PSD) is a focal point in this approach.

The PSD is a fundamental aspect of the risk/return relationship that is applicable to both the policyholder and the shareholder. Although consistent with the probability of ruin and expected policyholder deficit concepts, it differs in that its focus is more on the degree to which returns depart from expected levels, rather than simply on the extreme adverse outcomes.

The "operating return–probability of drawdown" method presented in this paper is suggested as a replacement for the return on premium concept by an operating return measure which extends shareholder risk/return principles to the policyholder level. As a consequence, the method demonstrates how risk can be reflected in the pricing mechanism *without* varying the allocation of surplus to individual lines of business, through the focus on operating return. The result is a unified and consistent framework for establishing fair returns that reflects the transfer of risk from the policyholder to the company and from the company to the shareholder.

Importantly, issues of leverage and surplus allocation are removed from the pricing process. The need for surplus is viewed primarily as an overall company issue with respect to financial strength and ratings. The result is a mechanism for establishing prices which recognizes the policyholder and shareholder perspectives centered around their respective risk/return tradeoffs, without requiring that surplus be allocated to lines of business.

Varying leverage ratios by line of business is shown to be an optional risk adjustment step that translates rates of return to a common level, such as a specified cost of capital.

With respect to style and focus, this paper will avoid an overly-detailed and mathematically-oriented presentation in favor of simpler demonstrations focused on the most basic of principles. These principles are essentially:

1. Functionally and mathematically, insurance is composed of underwriting, investment and leverage.
2. Interactions among the policyholder, company and shareholder are governed by the fundamental risk/return relationship, in which higher risk requires higher return and vice versa.
3. The transfer of risk either from the insured to the company or from the company to the shareholder are both essentially investment-like decisions, which involve a charge for this transfer to occur. In the policyholder case, this results in a premium payment to the company; in the case of the company, this results in an expected "payment" to the shareholder via dividends or stock price appreciation (i.e., the cost of capital).
4. The amount and timing of policyholder-related liabilities and cash flows that will eventually be paid are uncertain. The price for the transfer of this underwriting risk from the policyholder to the company must be incorporated into the premium charged when insurance is sold.

These fundamental principles apply broadly to all ratemaking models. Unfortunately, unnecessary confusion exists with respect to the many ratemaking models presented in the literature, for two basic reasons. First, because the relevance of these basic risk/return principles may not be recognized in each of the models, the assumptions and parameters used in them are determined in various ways, causing their output to diverge substantially.

Second, because many of the models differ in construction and output, comparisons to one another are made difficult. It is important to note that the many ratemaking models (such as underwriting profit margin, target total rate of return, insurance capital asset pricing model, discounted cash flow, Myers–Cohn, and internal rate of return, etc.) *are all essentially equivalent*. A single well-constructed total return model, supported by the full complement of balance sheet, income and cash flow statements, and further valued both nominally and on a discounted basis, encompasses them all and will produce identical results when the same input assumptions are used (as discussed in the material in the References).

2. BACKGROUND

2.1. Rate of Return

Rate of return (often referred to more simply as return) reflects the amount of income produced on an investment in relation to the investment itself. This ratio is usually expressed as an annual rate, although the investment period may be more or less than one year. Insurance decisions to invest in underwriting operations, in particular, usually involve a multi-year commitment (e.g., losses may take many years to settle) and the rate of return that results must reflect this timeframe as well. This is much like an investment with a holding period of several years, wherein both the level of investment and return might vary over time, requiring that some form of composite annual percentage rate of return (APR) be calculated.

Insurance companies deploy (i.e., invest) their surplus in either of two essential operating activities—underwriting or investing. Each of these activities carries with it an anticipated rate of return. The amount of insurance written on the one hand and the amount of surplus/capital provided from financing activities on the other, result in an operating leverage that magnifies the underwriting and investment returns in relation to surplus. The following expression provides a simple yet accurate representation of the way that underwriting and investment return, in conjunction with their respective leverage, contribute to total return:

$$R = (Ru)(L/S) + (Ri)(L/S + 1). \qquad (1)$$

Total return on surplus (R) is the sum of the respective products of return and leverage from underwriting and investment. The return on underwriting (Ru) measures the profitability from underwriting operations (absent investment income). The return on underwriting can be measured in various ways, depending on whether the view is historical or prospective, or whether it is relative to calendar or ultimate accident year. The return on investment (Ri) is essentially a yield on total invested assets, which include assets generated from both underwriting liability "float" and surplus.

Each of these returns is magnified by the leverage employed by the company. The underwriting leverage (L/S) is the net liability to surplus ratio. Liabilities consist primarily of loss reserves, but other liabilities must be considered, such as premiums receivable (a negative liability), reinsurance balances payable, taxes, etc. Since invested assets (I) are equal to net liabilities (L) plus surplus (S), $L/S + 1$ in the above expression is equivalent to the ratio of invested assets to surplus, or investment leverage. Viewed in this way, *the total return is seen to be dependent simply on underwriting return, investment return, and insurance leverage*. (It is noted that statutory surplus and GAAP equity differ in their definitions. For purposes of risk transfer pricing and in the context of this paper, surplus is better thought of as a required risk-based "benchmark" amount. This is discussed in [3].)

Underwriting income (after-tax) is expressed as a rate of return (Ru) and can be determined in either of two ways. The first is to use a common finance tool, the internal rate of return (IRR), which is based on the *underwriting cash flows* that evolve over time. The second is to relate *underwriting income* to the *balance sheet investment* that is derived from the same insurance liabilities that produce the underwriting income. This is approximately the ratio of after-tax underwriting income to underwriting float (i.e., primarily loss reserves). Both of these alternatives are demonstrated by way of example in the Appendix, and are discussed in detail in the reference material.

Underwriting return, *Ru*, is not the same as return on premium. While return on premium may be a useful statistic, a ratio to sales does not capture the dynamics as fully as a return on funds invested statistic does, when the magnitude and time periods of the investment differ widely. Returns on premium are not comparable between short- and long-tailed lines of business, since the magnitude and time commitment of supporting policyholder funds are dramatically different. The underwriting rate of return (*Ru*) fully reflects this dimension and presents a statistic that is comparable across lines of business.

Investment return is dependent on returns (yields on fixed income investments, stock market dividends and capital gains, etc.) available in financial markets, together with the selection of various asset classes in which investments are made. In the case of fixed income investments, investment return is also affected by the maturity selected (which entails added interest rate risk as well).

Options exist within both underwriting and investment to select lines of business and/or investments that entail varying returns and associated risks. The above formula (1) refers to a single underwriting return and a single investment return when, in reality, there are numerous options within each of them.

2.2. *Risk in Return*

Risk is a measure of the uncertainty of achieving expected returns (which encompasses the possibility of a complete loss of the investment itself). The most common measure of risk is the standard deviation statistic, which provides a means of quantifying the degree of likely variation of actual return relative to the return expected. The larger the standard deviation, the greater the chance that the actual return will deviate from the expected return (either above or below it).

Underwriting and investment returns both involve a degree of uncertainty (i.e., volatility). The expression below reflects how the standard deviation in total return (σ_R) is affected by the standard deviation in underwriting return (σ_{Ru}) and the standard deviation in investment return (σ_{Ri}). This formula makes use of the square of the standard deviation (known as the variance) for simplicity:

$$\sigma_R^2 = \sigma_{Ru}^2(L/S)^2 + \sigma_{Ri}^2(L/S+1)^2 + 2r(L/S)(L/S+1)\sigma_{Ru}\sigma_{Ri}. \tag{2}$$

Leverage has a similar compounding effect on variability as it does on return. In addition, the interaction (i.e., correlation) between underwriting and investment is a critical component of the total risk, as captured by the last term in (2).

The correlation coefficient (r) measures the degree that underwriting and investment performance move in tandem with each other. Underwriting and investment returns that move together in lock step in the *same* direction, both up or both down, will have a perfect positive correlation ($r = +1$). Underwriting and investment returns that move in exact *opposite* directions, one up and the other down, will have a perfect negative correlation ($r = -1$). When underwriting and investment returns are independent of one another, there is no correlation ($r = 0$). Thus, in terms of total variability, when underwriting and investment move together (positive correlation), risk is greater. Conversely, when underwriting and investment move opposite to one another (negative correlation), risk is less. The same principles apply at a finer level among the lines of business within underwriting and among alternative investments.

In insurance circles, when the topic of a company's surplus requirements is discussed, the term covariance is often used. This is simply another term for describing the interaction among underwriting lines of business and investments, and the effect this may have on the overall need for surplus and the risk to the company as described above (i.e., the benefit of diversification).

It is important to note that of the three basic factors affecting risk and return, leverage stands alone in that it can be controlled directly by management; underwriting and investment, on the other hand, involve given levels of risk which are largely uncontrollable. (This risk can be managed to some degree through diversification.) The selected leverage at which a company chooses to operate has a significant influence on both the level and variability of reported total returns, and is subject to practical regulatory and rating agency constraints.

This process is more complex than can be reviewed here, especially if the correlations among many lines of business and alternative investments were to be considered simultaneously.

2.3. Leverage

The leverage employed by a company is subject to many constraints, including ratings, cost of capital, and most importantly in insurance, the probability of ruin. Insurance, unlike most other businesses, involves selling a product whose costs can only be estimated at the time the product is sold, and whose ultimate value has a significant potential to cause financial loss to an insurer well in excess of premiums charged. Recognizing this financial exposure and the additional limits imposed on leverage by rating agencies and financial markets, insurers have traditionally considered the probability of ruin in determination of surplus requirements. This concept results in the establishment of surplus levels in such a way as to keep to an acceptable minimum probability the chance that surplus will be exhausted by unfavorable loss or other developments. More recently, the concept of expected policyholder deficit (EPD) has been used to further quantify the amount of ruin.

Leverage plays a direct role in the risk/return tradeoff as noted previously, since it simultaneously magnifies both return and risk as shown in formulas (1) and (2). To demonstrate this relationship, it is helpful to express formula (1) differently as follows:

$$R = Ri + (Ru + Ri)(L/S). \tag{3}$$

This is the expression for a straight line, with an intercept of Ri (the return on investment) and a slope of $(Ru + Ri)$. If no insurance were written (i.e., $L/S = 0$), the only return would be on investments, with a return equal to the average yield (Ri). Assuming a consistent level of profitability, as writings and leverage increase, total return increases at a rate of $(Ru + Ri)$. This term has special meaning in that it represents the *operating return* from insurance. Operating return reflects the income from underwriting operations plus the investment income related to the assets generated from underwriting operations (i.e., insurance liability float). It excludes income from investment of surplus, captured in the above formula by the intercept Ri. The meaning and measurement of the underwriting, investment and operating returns is discussed in the reference material and recapped briefly with an example in the Appendix.

Repeating the important point—*leverage simultaneously affects both return (shown by formula 3) and variability in return (shown by formula 2)*. Apart from product or geographic diversification, returns cannot be increased by raising leverage without also increasing variability. Similarly variability cannot be reduced without also reducing returns. Since *insurance uncertainty cannot be eliminated*, some combination of policyholder and/or shareholder pricing mechanisms is needed to deal with this risk transfer.

Predominant drivers of overall variability are: (1) variability in the amount of liabilities, (2) variability in the timing of liability payments, and (3) variability in interest rates. The greater the variability in these three basic drivers, the greater the variability in return. While reinsurance and investment hedges can be used to reduce some of this variability, there will always be a degree of variability remaining which cannot be eliminated, and this should be an important input into the pricing and leverage setting processes.

The following chart (Figure 1) presents key relationships among balance sheet, income and cash flows and the risk transfer activities within the insurance firm. Within this structure the total company is delineated into policyholder versus shareholder related components. Note that the left side of the balance sheet consists of invested assets only. Non-invested assets are portrayed as a negative liability, and included within net liabilities on the right side of the balance sheet.

Several alternatives exist for setting leverage. As noted previously, controlling the probability of ruin has been a traditional approach. More recently the expected policyholder deficit (EPD) has been developed. Controlling the variability in total return, of more interest to the shareholder, is another criterion that is often addressed either by modifying the leverage ratio or by changing the target rate of return.

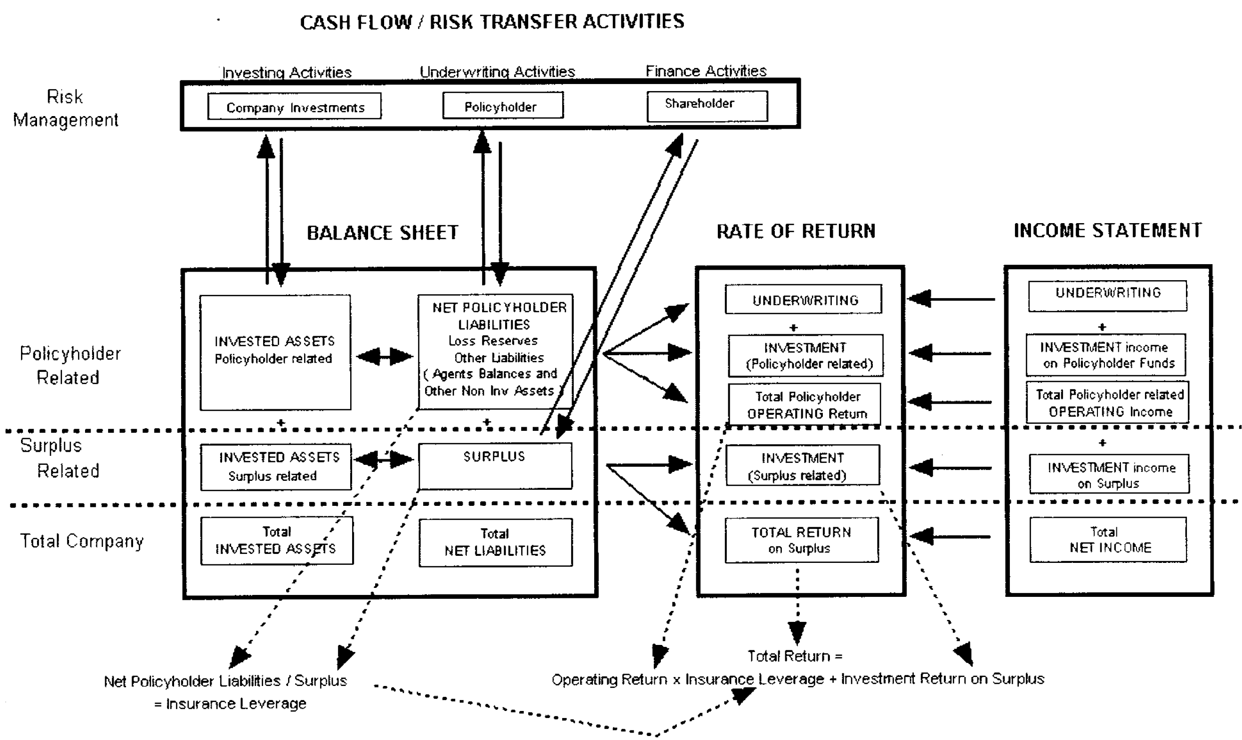
2.4. The Probability of Ruin

The probability of ruin represents the likelihood that the combined effect of variability in liabilities and variability in the timing of liability payments will cause surplus to be exhausted. To keep this probability to an acceptable minimum, surplus can be established at a level which is sufficient to cover the adverse conditions that can occur (e.g., losses larger than expected or payable sooner than anticipated) all but, say, 1% of the time in an individual line of business.

Variability in the amount of loss and variability in the timing of loss payments are most critical in terms of influencing the leverage level and the variability in total return. Variability in loss has an even greater impact, due to a tendency to be

FIGURE 1

KEY BALANCE SHEET, INCOME, CASH FLOW AND RETURN RELATIONSHIPS



skewed, with the possibility of a very large loss (e.g., a natural catastrophe). However, the probability of ruin approach has shortcomings in that it does not typically reflect the impact of taxes and other components of total net income, and may not consider sources of variability other than from losses. A large loss payable shortly after policy issuance is much more serious than is the same loss payable many years later, since, in the latter case, substantial investment income is generated in the meantime. Also, the tax credit generated by losses reduces the out-of-pocket cost to the company. Variability in premium, expense and investment are also potentially significant contributors to overall risk, which should be considered.

Furthermore, it should also be noted that control of probability of ruin does *not* result in a uniform variability in total return. Neither does it reflect the magnitude of policyholder deficit if ruin does occur. Note, for example, that a highly skewed loss distribution may result in a greater policyholder shortfall than would a normal distribution, yet have the same probability of ruin.

2.5. *Expected Policyholder Deficit (EPD)*

EPD is a broader concept than is the probability of ruin, in that it includes both the frequency and severity of extreme adverse consequence. Whereas the probability of ruin specifies the chance that company surplus may be exhausted, the EPD further estimates how much this amount is likely to be on average. Clearly policyholders and regulators are concerned with both the probability and potential magnitude of loss, should surplus be exhausted. While shareholders are concerned with the probability of ruin, EPD is of little relevance since shareholder loss is limited to the amount of their investment.

The EPD concept has gained prominence in recent years and is being incorporated in some rating agency methodologies. However, this approach will have the same shortcomings as the probability of ruin, if it does not reflect the impact of taxes and other components of total net income.

Of more serious concern, however, is a basic principle of statistics and probability distributions that cautions against use of the "tail," or low probability outcomes in frequency distributions. Most statistical methods rely on the "middle" of the distribution, where the vast majority of the values occur. The probability of ruin and EPD approaches rely on the areas of the data distribution having the least credibility and reliability. While of interest to policyholders, shareholders are instead concerned with how returns might vary from that expected (that is, with the middle of the distribution).

This shareholder perspective is one of risk versus return and is more appropriate within a context of risk transfer pricing. The probability of ruin and EPD, while important from a solvency standpoint, are not as well-suited to this end.

2.6. Variability in Return

Shareholder investments reflect a tradeoff between the level of return required and the uncertainty of such return. Shareholders expect returns commensurate with uncertainty—if returns are more variable, then investors will expect a higher absolute return, all else being equal. This in essence reflects the middle of the distribution of returns about the expected value. In this regard the shareholder perspective inherently embodies more statistical credibility.

Fortunately, the probability of ruin, EPD and variability in return viewpoints are connected. The concept of "probability of surplus drawdown" will be discussed in this regard.

2.7. Value at Risk and Probability of Surplus Drawdown (PSD)

The distribution of total return encompasses all financial components of an insurance company and the variability inherent in them. This is the distribution that is of concern to the shareholders, or investors, who provide capital to support the operations of the company. In fact the traditional probability of ruin and EPD, when expanded to include all sources of underwriting and investment income and taxes, are captured in the tail of this distribution. Ruin occurs when the total rate of return is -100% or worse, with EPD being the average magnitude of such events. Thus the first step in bridging the gap between the policyholder and shareholder measures is the conversion of ruin and EPD to a net income basis, and their expression as a rate of return.

The second step is to view the distribution of returns as a continuum from the expected value downward to the ruin threshold of -100% return. *Economic* surplus drawdown occurs along this continuum when total returns fall below the rate of return that could be achieved on alternative (typically risk-free) investments. Alternatively, this is equivalent to the point at which *operating returns* fall below 0% as shown in (3). This rate of return is most properly defined on an economic basis to reflect the point at which investors lose money in economic terms. Thus the PSD represents the likelihood that an investor will experience an economic loss, when time value is considered. This is a specific case within the more general *value at risk* approach, which deals with a reduction in surplus of any specified amount (i.e., below zero) together with the probability of its occurrence, rather than just simply the single threshold of 0% return at which surplus drawdown occurs.

It is important to note that the points of surplus drawdown and ruin, and their respective probabilities, both lie on the same distribution. Actions which alter the return distribution will simultaneously and similarly improve or worsen both the policyholder and shareholder positions. This is shown more clearly by examining the basic risk/return relationship.

2.8. The Basic Risk/Return Tradeoff

The basic risk/return relationship is shown schematically in Figure 2. As variability in return increases along the x-axis, the return required to compensate for this risk also increases. Beginning at the origin (the point of "no risk, no return"), a risk/return line exists such that the probability of a negative return, or surplus drawdown, is the same at all points along the line. This probability is controlled by increasing or decreasing the slope of this line. A higher return (steeper slope) will reduce the probability of surplus drawdown by moving the distribution at each point on the line farther up and away from negative return territory.

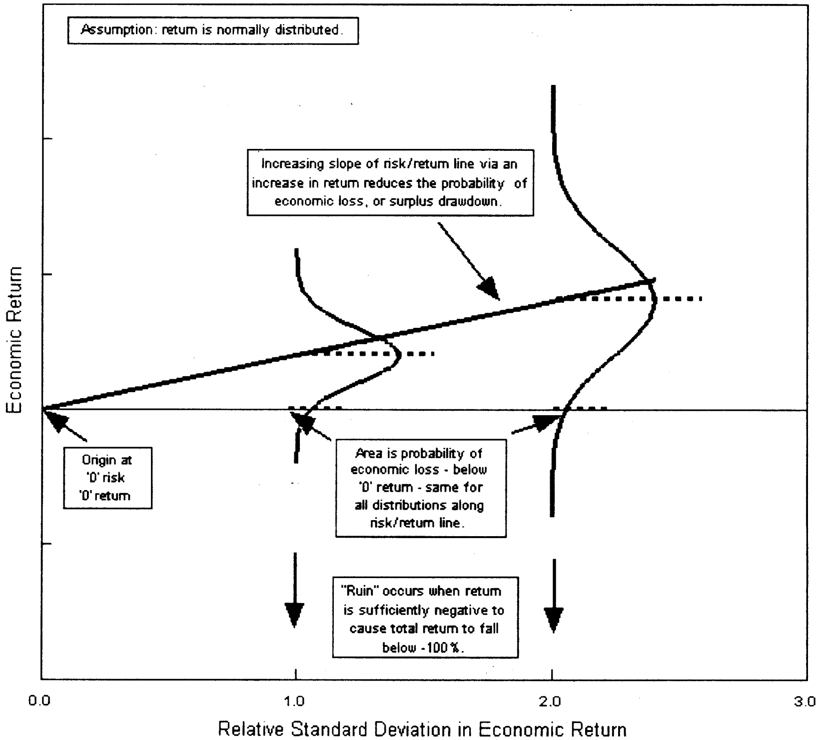
This essential relationship, that increased risk requires an increased return, is at work governing the risk transfer process that takes place between the policyholder and the company and between the company and the shareholder. Referring back to the basic relationship shown in (3), the operating return components, particularly its expected value and variability (i.e., mean and standard deviation) define the essentials of the risk/return relationship between the policyholder and the company. When leverage is applied and the investment of surplus (Ri) is included, the risk/return relationship between the company and the shareholder is established on a total return basis. Consistency in these two risk transfer pricing activities is important in order to simultaneously establish fair policyholder premiums and fair shareholder returns. A focus on operating return, in particular how risk and variability are priced, will be presented first, with total return following.

3. OPERATING RETURN–PRICING FOR RISK AND VARIABILITY

As shown previously, operating return on insurance operations, driven by both its underwriting and investment components and coupled with the magnifying effect of leverage, defines the total risk and return profile of the insurance enterprise. The particular characteristics of a line of business, such as the amount

FIGURE 2

RISK VERSUS RETURN



and variability of its loss payouts, specify its operating return profile with respect to risk and return (i.e., the two dimensions of expected value and variability). This profile has policyholder implications, with respect to risk transfer and pricing, which can be assessed separately from leverage.

3.1. *The Policyholder Risk/Return Tradeoff*

The traditional shareholder (investor) risk/return perspective is one that reflects the need to provide returns consistent with risk. Greater risk requires greater returns, which must be comparable to other investment opportunities. The essence of the policyholder risk versus return relationship can be viewed similarly as reflected in Figure 3, which portrays variability in operating return and average operating return. Regardless of the underlying underwriting or investment uncertainty, this basic relationship must hold. In fact, for a given PSD, all combinations of loss variability and business tail length are shown here to lie on the same risk/return line. That is to say that all businesses conform to a uniform risk/return relationship, regardless of the variety of characteristics possessed by them.

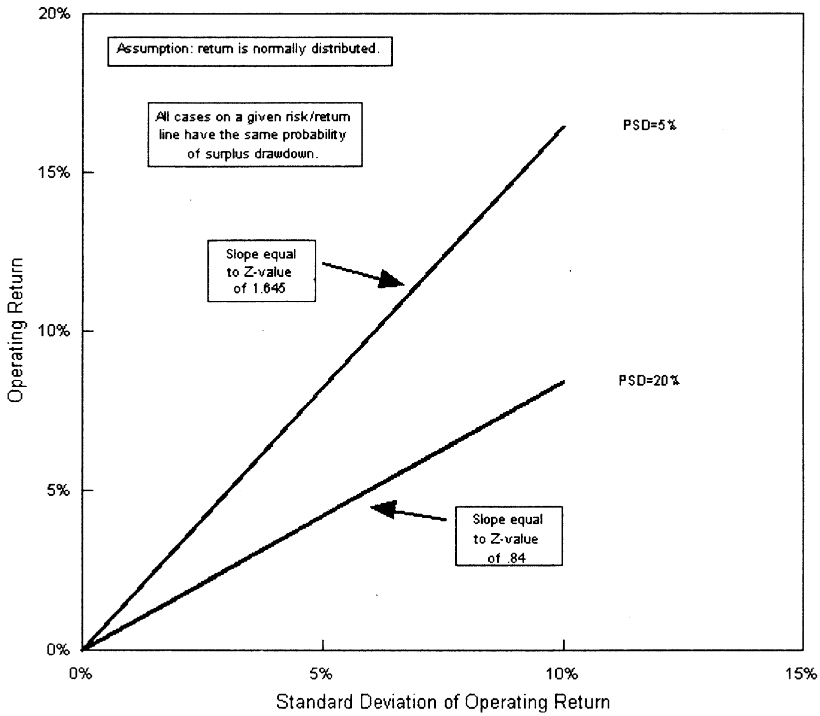
Since losses are assumed here to be normally distributed, each line has a slope equal to the normal distribution "Z-value." This is the number of standard deviations from the mean corresponding to specified probabilities from a normal distribution. For example, a Z-value of 1.645 corresponds to a 5% probability of occurrence (in each tail) from the mean. Thus using the Z-value provides an easy shortcut to determine the necessary operating return required to compensate for risk, with a specified PSD.

In practice loss distributions are typically skewed, and the standard deviation alone does not adequately describe risk. In such cases it is important that the area under the tail within each respective total return distribution be used to measure risk (i.e., the PSD), and in turn be used in the pricing process. The Z-value shortcut based on the standard deviation is not appropriate. While Figure 3 would not appear as a straight line in such cases, the approach remains valid with the downside risk to surplus controlled consistently.

If the operating return above is converted to total return by multiplying by a leverage factor and adding Ri to account for investment yield on surplus, Figure 4 emerges. In this scenario that assumes no interest rate variability, the probability of surplus drawdown is now the probability of a total return below Ri. This is the shareholder view that can be used to provide a comparison to alternative investments, and guidance as to whether rates are

FIGURE 3

POLICYHOLDER OPERATING RISK/RETURN TRADEOFF (WITH VARYING PSD)



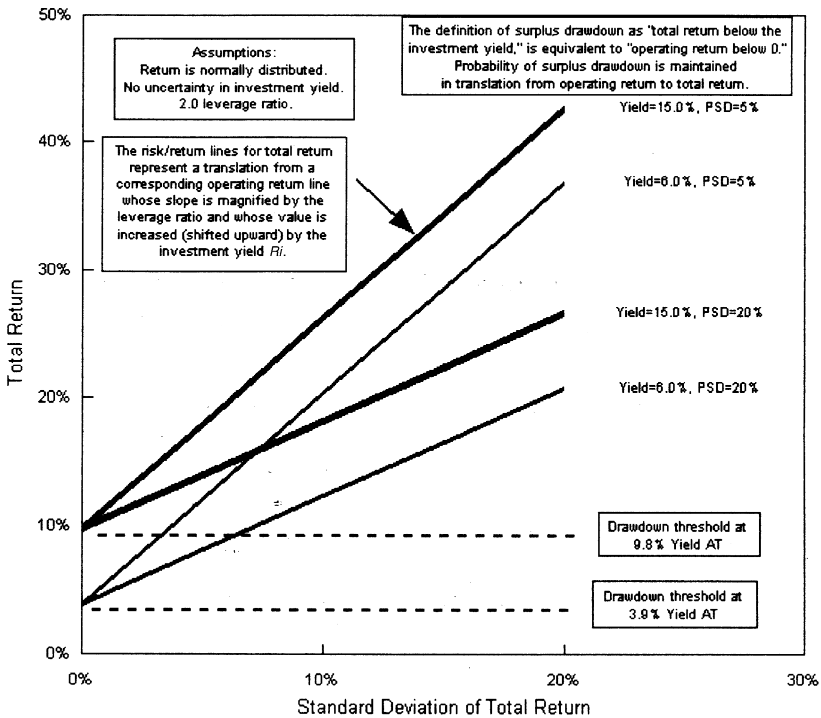
adequate from a shareholder perspective. This will be discussed in more detail later.

In practical terms these steps equate to the use of a constant Sharpe ratio to control risk. The Sharpe ratio, which is calculated by dividing the difference between the total return and the risk-free return by the standard deviation in return, is in effect a Z-value.

It is important to note that the introduction of leverage does not change the probability of drawdown. (This is not true if risky

FIGURE 4

SHAREHOLDER RISK/RETURN TRADEOFF
(WITH VARYING PSD & INVESTMENT YIELD)



investments are assumed.) *Since leverage similarly magnifies both return and risk, increasing leverage simply causes total return to move from lower left to upper right while remaining on the same line. Leverage thus becomes a factor that provides a translation from internal measures of risk-based operating return to total shareholder return, while maintaining a specified probability of surplus drawdown.*

The significance of this characteristic bears amplification, and explains why this risk pricing approach is largely independent of the level of actual company surplus and does not require surplus allocation to lines of business *as long as returns are sufficiently positive*. The premium necessary to generate a total return large enough to keep the downside risk to surplus sufficiently low is the same regardless of the leverage factor utilized, due to the balanced and simultaneous effect leverage has on both risk and return. The *stated* total return (as well as the variability in total return) will of course be higher as leverage increases, but the PSD will remain the same. *Reducing leverage does not improve the joint risk/return profile, and increasing leverage does not worsen it.*

Practically speaking, however, it is much easier to present a rate filing based on a lower rate of return than a higher one, *even if the premium is identical in both cases*. In a total return ratemaking environment, the leverage utilized must be such so as to produce a rate of return within an acceptable range while satisfying other rating criteria. This is one of the considerations in the determination of total company surplus requirements to reflect the concerns of rating agencies and regulators. Furthermore, since premiums often are not sufficient to ensure fair profits, the risk of surplus loss is increased and a greater level of supporting surplus (i.e., lower leverage) is necessary to provide an adequate ruin safety margin.

The primary goals of state regulators, fair premiums and solvency, are simultaneously addressed by this risk transfer pricing process. Fair returns are determined which simultaneously guard against the probability of loss of surplus and ruin. As noted previously, almost any reasonable risk-based level of operating return provides an adequate safety margin, and a very small probability of ruin. *Fair risk-adjusted returns provide the direct connection to solvency and the means by which solvency is ensured.*

3.2. Policyholder Pricing for Underwriting Risk

Operating return is a financial measure which reflects the basic nature of insurance—the fact that it incorporates the activities of underwriting and investment and that it is subject to substantial variability in result. Operating return quantifies the return realized by the insurance company for the transfer of risk from the insured. While some may view insurance simplistically as the spreading of risk from a single policyholder to several policyholders in order to reduce the cost to a more stable per policyholder basis, it is more than this. No matter how large the cohort of policyholders might be, a degree of uncertainty as to the total cost will remain, due to the highly variable and uncertain nature of insurance. A proper price must be determined for this transfer of risk from the insured to the insurer.

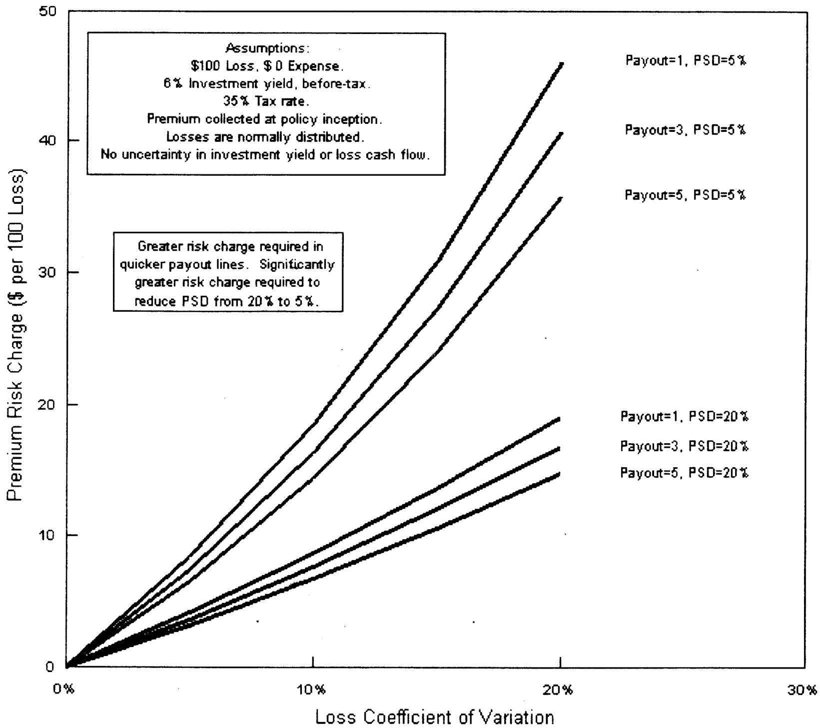
The primary financial drivers which determine the *expected* operating return are the amount and timing of cash flows related to premium, loss, expense and taxes, as well as interest rates. The *variability* in operating return is primarily driven by the variability in loss amount, timing of loss payments and interest rates. These factors must be reflected in the pricing process. The nature of the distribution of operating returns provides such a means, and one by which a degree of objectivity and consistency among lines of business can be maintained by utilizing the basic risk/return relationship.

The probability of surplus drawdown, or negative operating return, can be set at a desired level. Simultaneously the probability of ruin is altered in the same direction. Figure 5 presents the price increase required as the loss variability increases, for lines of business having average loss payouts of one, three, and five years, and for PSD levels of 5% and 20%. Note that the lines for a given drawdown scenario intersect at the origin, since no incremental risk implies no incremental return (in principle). The mathematics for this risk charge are provided in the Appendix.

In this pricing approach, the risk charge is a direct function of loss variability, subject to the specified probability of negative return (i.e., that the charge will prove to be inadequate to cover the risk). How this probability is set should consider both the policyholder and shareholder perspectives.

FIGURE 5

PRICING FOR UNDERWRITING RISK-LOSS VARIABILITY
(WITH VARYING LOSS PAYOUT & PSD)



As noted earlier, a lower operating return (and premium) will bring with it an increased probability of negative return and probability of ruin. In most instances, any reasonable price level and risk charge will have a very small probability of ruin and EPD. Clearly, long run financial strength and solvency cannot be maintained without adequate rates. In other words, adequate rates are *the true* means by which solvency is made secure, at least with respect to current business writings (i.e., excluding other balance sheet risks).

3.3. Policyholder Pricing for Investment Risk

Risks exist in both underwriting and investment. Figure 5 presents the impact of variability in underwriting (incurred loss) only. Investment risks range from a low involving government "risk-free" investments (which experience only relatively modest fluctuations in yield) to higher-risk investments which have a far greater potential to vary, as well as an exposure to loss. A further component of a risk-averse investment strategy would be to match investment maturities with the timing of expected underwriting cash flows. While higher fixed-income investment yields can be achieved by investing at longer maturities, this creates risk should cash flows not emerge as expected.

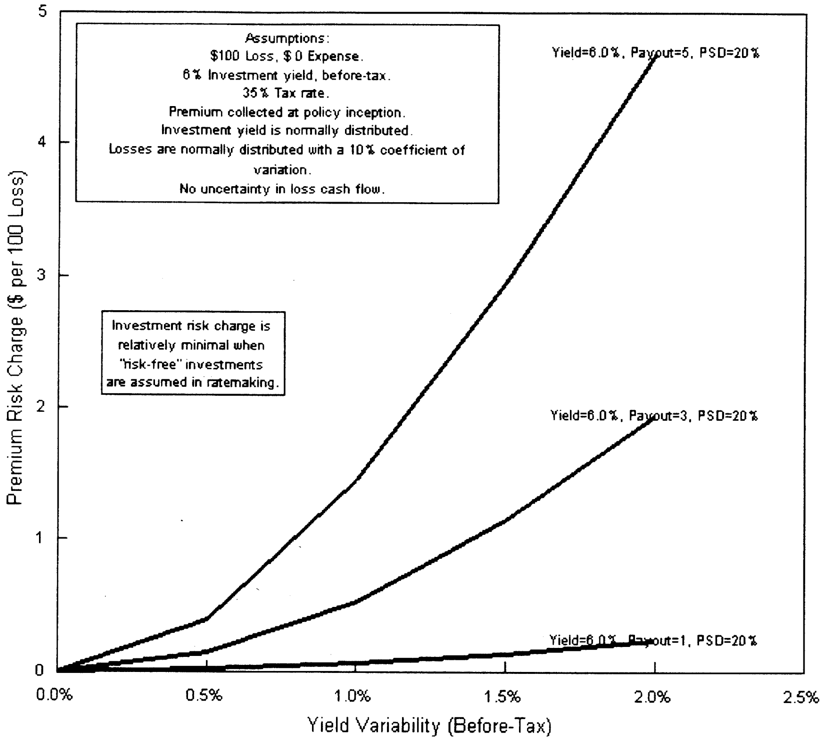
A controversial issue is whether or not insurance prices should be based on a risk-free investment strategy. Should policyholders be credited with risk-free rates or something more in line with the higher-risk investments that insurers are making. If it is the latter, then the increased yield carries with it an increase in risk. The mechanism presented here provides a framework in which the return and risk characteristics of investment can be priced along with those from underwriting.

Figure 6 presents the price increase required as the investment yield variability increases, for lines of business having average loss payouts of one, three, and five years for a PSD of 20%. The variability in yield is very small, as might be expected with risk-free investments. A maturity matching policy is assumed, and the loss variability is assumed to be 10%. Once again note that the lines for a given drawdown scenario intersect at the origin, since no incremental risk implies no incremental return (in principle). The mathematics for this risk charge are also provided in the Appendix.

When risk-free investments are assumed, the risk charge for investment risk is very minor in comparison to that required to cover underwriting risk, since such investments are subject to

FIGURE 6

PRICING FOR INVESTMENT RISK-YIELD VARIABILITY ONLY (WITH VARYING LOSS PAYOUTS–"MINIMAL" INVESTMENT RISK)

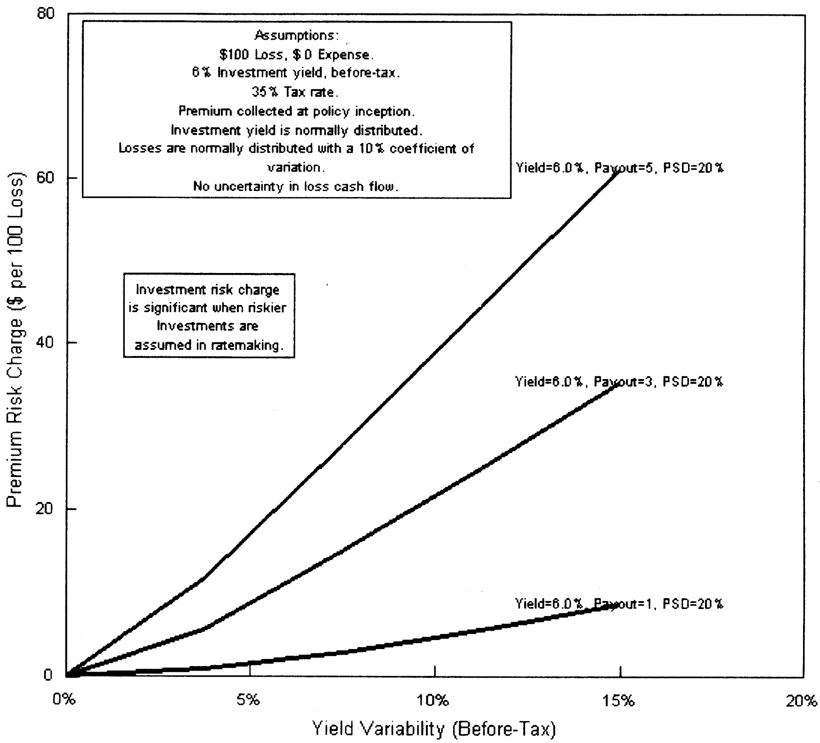


interest rate risk only. However, this picture changes dramatically if higher credit risk investments are assumed.

The charge for higher investment risk becomes substantial, as shown in Figure 7. This presents the additional premium required to reflect increases in investment risk for lines of business having average loss payouts of one, three, and five years with a PSD of 20%, when investment variability is substantial.

FIGURE 7

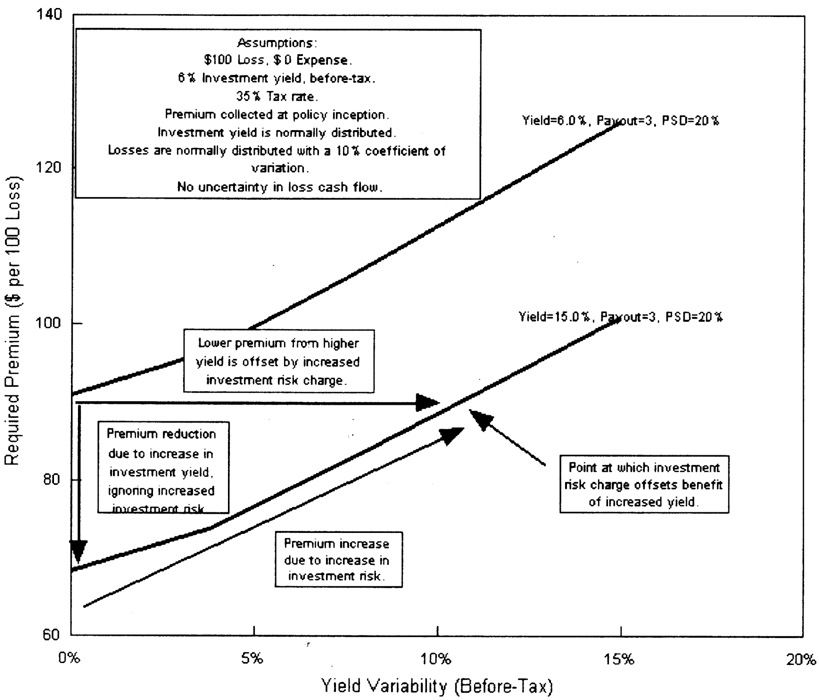
PRICING FOR INVESTMENT RISK-YIELD VARIABILITY ONLY (WITH VARYING LOSS PAYOUTS–RISKIER INVESTMENTS)



However, the key issue is to judge to what degree the increased benefit from higher yields (via a reduction in price) is offset by the increase in price due to the higher risk. Figure 8 presents an example of such an assessment. (Mismatching, which would increase risk and required premiums, has not been factored into this analysis.) A line of business with a three-year average loss payout in which yields increase from a risk-free 6% to 15% (before-tax) will lose the entire benefit from this increase

FIGURE 8

PRICING FOR INVESTMENT RISK-YIELD VARIABILITY ONLY (WITH VARYING LOSS PAYOUTS–RISK CHARGE OFFSETTING HIGHER YIELD)



if the attendant variability increases to a standard deviation of approximately 10%.

Unfortunately a further complication arises in that, in the translation from operating to total return, the variability of *Ri* adds greater variability to total return as seen by the shareholder above that reflected and priced into the operating return (based on (3)). In other words, the variability in investment income on surplus itself adds variability beyond that coming from operating return, and additional total return is required to compensate the shareholder for this additional risk.

An alternative approach is to view operating returns in insurance on a risk-free investment basis, with higher-risk investment strategies being introduced incrementally after this for total return purposes. Such a step moves the risks and rewards of higher risk investments to the shareholder, and issues of risk, return and leverage are addressed separately for this component. This also provides a useful delineation between the underwriting and investment functions, permitting the investment function to be managed incrementally on a value-added risk/return basis.

The basic risk-charge mechanism functions well without introducing higher-risk investments into the equation. Furthermore, as practical policy, it is difficult to see why two identical insurance policies should be priced differently simply because the insurance companies offering them have a different investment mix, assuming that policyholders should be insulated from investment risk. A mechanism for dealing separately with investment risk will be explored further from the total return shareholder perspective.

4. LEVERAGE AND TOTAL RETURN

4.1. The Shareholder Risk/Return Perspective

Leverage magnifies returns and variability from insurance operations which, with the inclusion of investment income on the surplus itself, results in the total return as shown in (3). Once the operating return profile has been established, leverage merely provides a translation to the shareholder perspective, as shown previously in Figure 4. The probability that the total return will not achieve an economic return—a total return below the yield on surplus Ri which could be achieved without taking insurance risk—is maintained as specified during the determination of the risk charge. In other words, insurance risk is charged to the insured.

Surplus, and thus leverage, is set by balancing the policyholder-related concerns of the regulators and insurance rating agencies (i.e., lower leverage is better) with shareholder-related concerns of the investment rating agencies and analysts (i.e., reasonably higher leverage is generally better). While shareholders should receive a higher return if risk is higher, changing leverage does *not* alter the probability of a negative economic downside risk. Although a leverage increase will raise returns to the shareholder, it also increases risk at the same time, with the result that the PSD remains unchanged.

If returns are low relative to risk and not consistent with other investment alternatives available to the shareholder, then insurance companies will have difficulty raising capital. Essentially, the insurance company is not generating a sufficient return on operations to pay for the transfer of risk from the company to the shareholder under such circumstances. This scenario exists when the risk/return relationship governing the company/shareholder relationship is not supported by a similar risk/return relationship between the company and its policyholders. The only recourse is to increase the underlying policyholder risk charge to bring that risk/return relationship in line with that needed to support a total company risk/return profile comparable to other external investment choices. More specifically, the risk charge and return must be increased and the PSD reduced, so that the risk and returns are made consistent with other investments available to the shareholder.

One important benefit to the aggregate company, and thus to the shareholder, is the reduction in risk and variability that comes from underwriting (line of business) and investment diversification (i.e., covariance). Companies benefit in many ways from offsetting factors which reduce aggregate variability, and thus risk. These offsets occur: 1) in underwriting between lines of business, 2) in underwriting between variables such as expense and loss within a line of business, 3) in investment between asset classes, 4) between underwriting and investment, and 5) in reported calendar year financial results in longer-tailed lines of business (due to an averaging effect on the more volatile policy/accident period results as they flow in). While very difficult to assess, these covariance benefits are of greater benefit to the larger, more diversified insurers. Just how this effective reduction in risk is reflected in the risk transfer pricing mechanisms is a topic that must be addressed at some point.

One of the interesting aspects of this is that surplus allocation to lines of business is not necessary for purposes of pricing, as long as a uniform PSD is maintained among the various insurance products. The probability of ruin and EPD will be similarly controlled, and if prices are adequate, that probability will be sufficiently small and negligible. While this may be a bit of a simplification (since many loss distributions are skewed), the basic principles are valid.

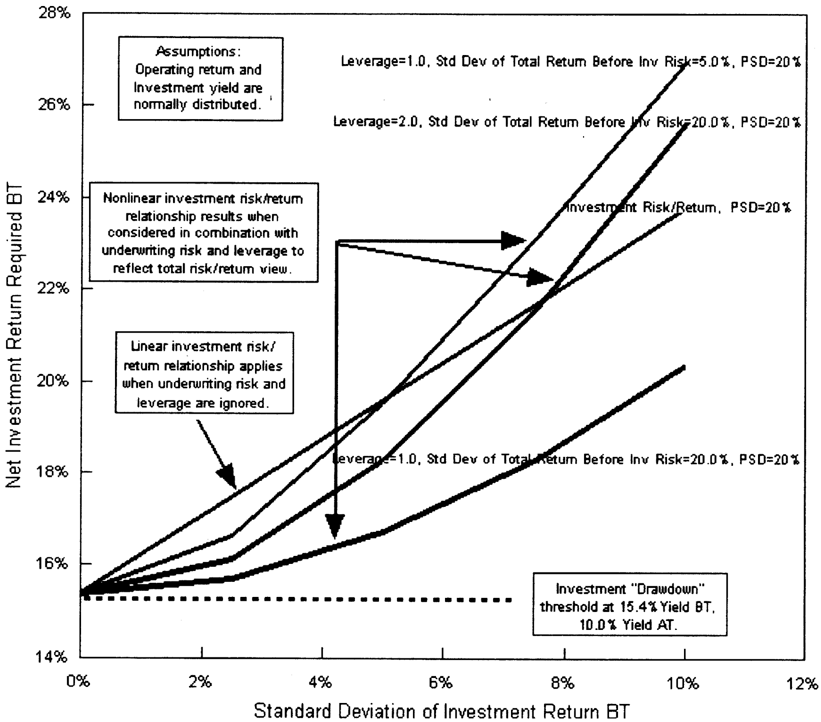
It should be noted that if risky investment strategies are included in the pricing mechanism, it is likely that the degree of risk will vary among the lines of business. For instance, longer-tailed lines might extend maturities to a greater degree than shorter-tailed lines, thus adding more risk.

4.2. Investment Pricing for Investment Risk

The use of operating return, its expected value and distribution, together with the concept of the probability of surplus drawdown provides a basis for setting fair premiums to the policyholder, while at the same time permitting a fair return to the shareholder consistent with the amount of variability in total return. The issue of investment risk remains as an additional component of overall total return variability. A mechanism for including higher investment and a related policyholder premium risk charge for the added investment risk entailed was presented earlier. An alternative approach is to base policyholder premium on an assumed risk-free investment strategy and separately reflect investment in the shareholder total return, with the risks and rewards of investment kept within the shareholder domain.

FIGURE 9

INVESTMENT RISK/RETURN
(WITH VARYING LEVERAGE & TOTAL RETURN VARIABILITY–TO MAINTAIN PSD)



This perspective recognizes that insurance company investment activities are themselves subject to the same risk/return principles that apply to policyholders and shareholders, facing the same decisions that require greater compensation in return when risk is higher. Investment activities are viewed as an incremental, value-added complement to underwriting activities, which together form insurance operations. Figure 9 presents the basic tradeoff in investment risk and return. (The mathematics for the required increase in investment return are shown in the Appendix.) Here it is assumed that the policyholder premium has been based on a risk-free investment strategy.

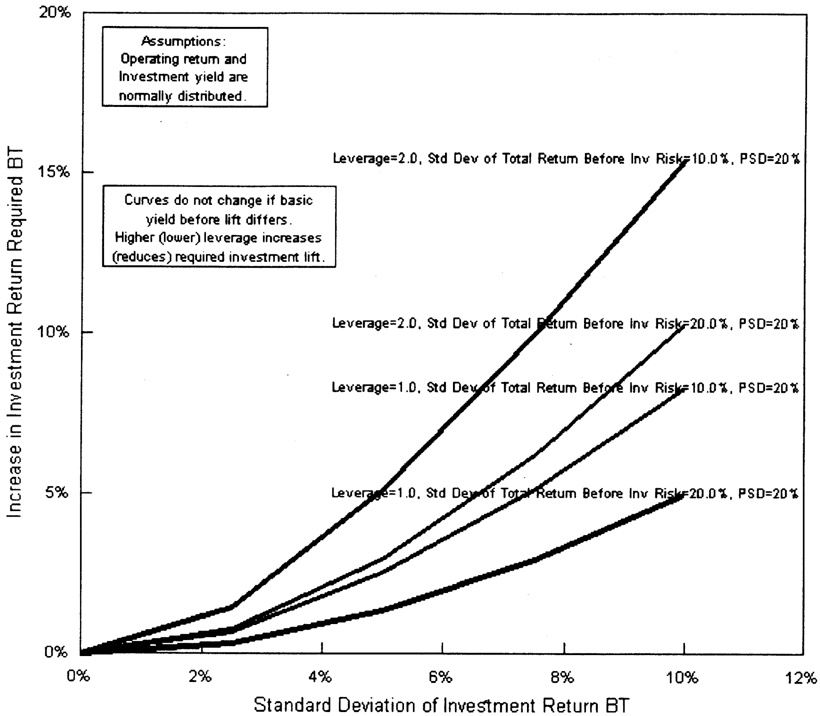
The straight line on this chart reflects the increase in return required to compensate for increase in investment risk in order to maintain a PSD of 20%, when investment is viewed apart from underwriting risk and leverage. Unfortunately, the interplay between underwriting and investment risk and the effect of leverage on total return variability must be considered. This results in the other nonlinear examples shown on the chart. Note that there is a benefit to the firm when investment risk is on the lower side, compared to the independent (i.e., linear) investment risk/return perspective. However, when investment risk continues to increase while the underlying underwriting risk is small or leverage is low, a greater investment return is required. This points out the important connection between underwriting and investment risk and financial leverage.

Figure 10 presents the increase in investment return or "lift" required to maintain a given PSD, as investment return variability increases with the connection between underwriting and investment risk and leverage considered in all cases. This figure provides a frame of reference indicating the degree by which investment returns must improve as investment risk increases. Importantly, the curves shown do not depend on the underlying level of investment yield.

If the lift in investment returns is below those indicated, then the probability of surplus drawdown is increased. If investment returns cannot be improved, then perhaps the risks are too great. Furthermore, higher leverage requires a higher lift due to the magnifying effects of leverage on variability. Thus an alternative to increasing investment return when investment risk increases is to reduce leverage. In other words, increases in investment risk may embody elements of both higher investment return and more conservative (i.e., lower) leverage.

FIGURE 10

REQUIRED INVESTMENT LIFT
(WITH VARYING LEVERAGE & TOTAL RETURN
VARIABILITY–TO MAINTAIN PSD)

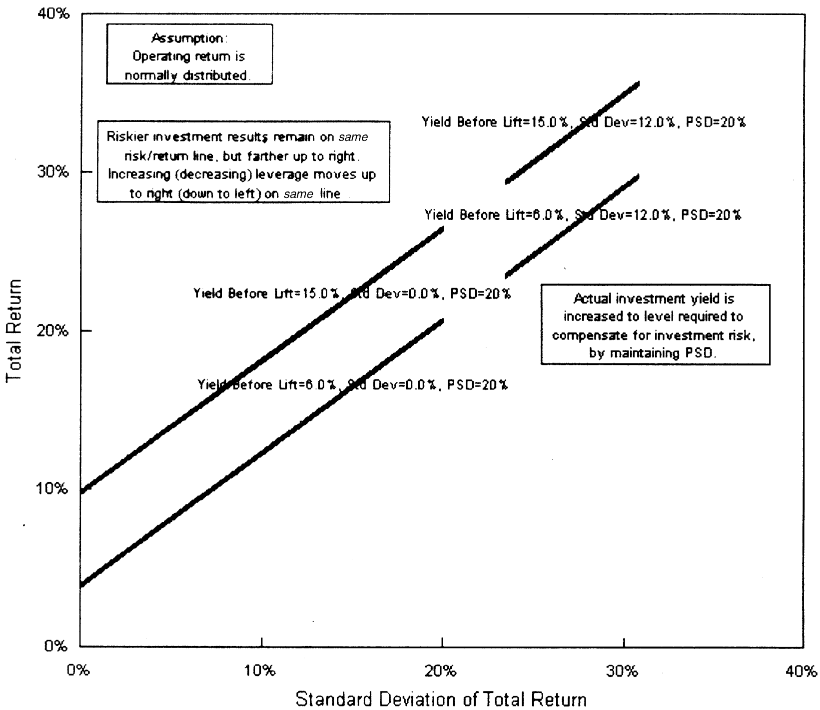


This perspective presents the investment function as subject to the same risk/return principles and PSD that have been applied elsewhere for risk transfer pricing purposes, and provides a means for managing the investment function as an incremental, value-added complement to underwriting.

Figure 11 reflects the risk versus return perspective (similar to Figure 4 shown previously) when the required investment lift is exactly achieved. Note that when investment risk increases, the

FIGURE 11

TOTAL RISK/RETURN
(WITH VARYING YIELD & VARIABILITY AFTER PSD BASED INVESTMENT LIFT)



variability in total return increases as well, but the appropriately increased investment return holds on the same risk/return line (albeit farther up and to the right). Thus the PSD is held. This figure demonstrates how increases in investment risk, followed by an increase in variability, should lead to increases in total return.

Whether actual investment returns are built in at the policy-holder or at the shareholder level, the important point is that the attendant increase in risk must be reflected. In the case of the policyholder, this means an increase in premium, possibly by enough to outweigh the benefit of the higher investment return. In the case of the shareholder, this means an increase in the overall total return which recognizes the increase in total return variability.

4.3. Surplus Requirements and Allocation to Lines of Business

A long-running debate continues with regard to the need to allocate surplus to lines of business for the purposes of ratemaking. Those not in favor of surplus allocation and the total return approach to ratemaking usually suggest use of return on premium (i.e., sales) as a preference. This statistic, however, is not a measure of return on *investment*, and it lacks a frame of reference as to what is fair and on what basis it should be set. Also problematic is the fact that it can and should differ markedly among lines of business, due to the length of the tail and the float-generating investment income that results. By way of alternative, operating return as presented in this paper has important attributes including:

1. Both of the operating return components of underwriting and investment rates of return, *Ru* and *Ri*, respectively, represent a return on supporting policyholder funds "invested." Thus operating return is truly a return on investment concept.

2. Operating return is an integral component of total return, since mathematically total return is calculated simply as the product of operating return times leverage, plus the investment return on surplus.

3. Operating rate of return fully reflects the differing magnitude and cash flow timing characteristics of individual lines of business. Operating return represents an annualized rate of return, regardless of investment horizon, comparable across all lines of business.

It is suggested here that, at a minimum, operating return be used in place of return on premium.

Arguments which favor the use of total return include the fact that it is a widely recognized benchmark (e.g., 15% ROE) which is readily comparable to other industries in terms of the risk versus return relationship. (It is also clear that every additional policy written entails an increase in risk to the insurance company, and requires some marginal increase in surplus.) The approach presented here extends the same risk (variability in return) versus return principles that govern shareholder actions to a lower operating return level within the insurance company.

In essence the "operating return–probability of drawdown" method presented in this paper is a replacement of return on premium by operating return, and an extension of shareholder risk/return principles to the policyholder level. As a consequence, the method demonstrates how risk can be reflected in the pricing mechanism *without* varying the allocation of surplus to individual lines of business, through the focus on operating return. Yet this remains as a mathematical component of the total return, made complete simply by the application of leverage and the addition of investment return on surplus. The PSD, driven by the connected variability in operating and total return, provides a unifying and consistent framework for establishing fair returns to reflect the transfer of risk from the policyholder to the company and from the company to the shareholder. Furthermore, if the PSD is made the same when pricing the individual lines of business, then leverage can be set uniformly in each line equal to the overall company average for purposes of calculating total return.

Were pricing models able to estimate all prospective financial parameters sufficiently well, then adequate pricing would lay the foundation for financial strength and lessen the need for surplus. However, many factors such as inflation, changing tort law, competitive pricing and catastrophic exposures, introduce uncertainty with respect to balance sheet value and require a substantial surplus cushion. Furthermore, these risks and resulting surplus needs are likely to differ among the lines of business.

The total surplus of an insurance company needs to be sufficient to provide an adequate financial cushion for the many balance sheet risks. The approach presented in this paper supports solvency with respect to current business writings, since the probability of ruin that results is extremely small (given any reasonable PSD and operating return).

4.4. Application Steps to Put Concepts into Practice

The following overview presents the essential steps and capabilities that are necessary to put these concepts into practice.

1. Develop a model framework that provides key balance sheet, income, and cash flow components. If ratemaking is the primary focus, then modeling a single policy period may be sufficient, in which case a single payment approach as presented in [2] and [3] may suffice. If calendar year financials are needed, then a multi-period cash flow model is needed, such as in DFA applications. Ideally this develops calendar period financials as the sum of current and prior policy/accident period contributions.

2. Develop a simulation capability built on top of this model, which can be applied to individual lines of business and then aggregated to a company total. The capability to incorporate key correlations among lines of business and variables may be important.

3. Specify the expected values of all variables and distributions of key variables as necessary. Generally interest rates and the amount and timing of losses, coupled with distributions that reflect the variability in them, are important. Although difficult to determine, key correlations

among lines of business and variables should be specified. Omitting this (i.e., assuming independence) will tend to overstate the benefit of covariance (i.e., diversification) and understate company surplus needs, since correlations are typically positive.

4. Set the (fixed) risk parameter to be used in each line of business. This is the desired probability that the total rate of return in an individual line of business will fall below the risk-free yield. A value in the range of 10% will probably be reasonable for starters. The number of lines of business, and the resultant diversification benefit, will affect this choice. The ruin probability for the total company that results should be verified as sufficiently small.

5. Beginning with underwriting risk/return, initially set a fixed leverage ratio (2 or 3 to 1 for liability to surplus) in all lines, and solve for the premium necessary to satisfy the specified risk parameter. The distributional outcomes from the simulation are used in this step. (A "risk-free" investment yield is suggested at this point.) This will indicate a required underwriting profit margin (i.e., combined ratio). At this point a risk/return line can be viewed for the modelled lines of business.

6. Adjust leverage by line of business to achieve a target total return. Premium is unchanged by this step, since the process is one of simply sliding up or down the risk/return line depending on whether the initial return is below or above the target return. If initially below the target return, leverage is increased (and decreased if above). The risk probability remains the same. The leverage ratios that result provide a risk adjustment mechanism, indicating relative line of business surplus requirements that permit all lines of business to be viewed relative to the same risk-adjusted total return target.

7. If higher risk investments are to be introduced, steps 5 and 6 are repeated for investment risk/return. After estimated investment risk and variability are increased, solve for the investment return necessary to satisfy the specified risk parameter. This will indicate a required investment margin. This should fall on the risk/return line but farther up and to the right (i.e., greater risk, greater return).

8. Adjust leverage by line of business to achieve the target total return. Required investment yields (as well as original premiums) are unchanged by this step, again since the process is one of simply sliding up or down the risk/return line, depending on whether the return is below or above the target return. The risk probability remains the same. The leverage ratios that result provide a risk adjustment mechanism, indicating the relative line of business surplus requirements for underwriting and investment risk combined. The difference from this surplus amount and that in step 6 is the amount required to compensate for investment risk. The leverage ratios that result provide a risk adjustment mechanism that permits all lines of business to be viewed on a comparable total return basis, in which both underwriting and investment risk have been reflected.

The risk-based required premium and investment yield determined in steps 5 and 7 may or may not be achievable. This then becomes part of the company's portfolio investment decision as to whether certain lines of business and/or investments should be written or undertaken.

In summary, this process provides a risk transfer pricing mechanism applicable to underwriting and investment activities, by indicating the premiums and investment returns required given their respective risks. Necessary leverage and relative surplus amounts are also indicated in order to risk-adjust to a common risk/return target.

5. CONCLUSION

It should be clear that the returns from underwriting and investment (in terms of expectations and uncertainty) together with the operating leverage employed by an insurance company, establish the essential elements of the risk/return tradeoff. This paper has presented an approach based on the application of very basic risk/return tradeoff principles to the risk transfer process that occurs between the insured and the company and between the company and the shareholder. Risk-based pricing algorithms have been presented to deal consistently with underwriting risk among lines of business and with investment risk. This process is apart from leverage, and does not require a varying surplus allocation to lines of business.

Operating return as presented in this paper is suggested as the fundamental measure that should be used to judge the risk transfer activities and pricing with respect to the policyholder. It is noted that risk is a fundamental element of insurance and it cannot be eliminated. Variability in results is expected, and simply throwing more surplus into the mix does not alter the basic risk/return relationship. Therefore, whether it is underwriting or investment based, some charge for risk transfer is needed whenever it occurs.

The PSD has been introduced as a guide by which the risk/return tradeoff can be managed similarly for both the policyholder and for the shareholder. This is suggested as the appropriate basis by which risk and return should be managed and prices set. Furthermore it is suggested that, while consistent with the probability of drawdown, the probability of ruin and EPD perspectives are different and more appropriate as a means to satisfy company solvency criteria than as a basis for risk transfer pricing. Instead, the risk transfer pricing approach presented here provides a single unified method which simultaneously satisfies regulatory concerns with respect to both setting fair risk-adjusted premiums and maintaining solvency.

Ultimately, insurance companies are faced with investment decisions with respect to the creation of optimum portfolio combinations of underwriting lines of business and investments to increase total return for a given level of risk. This involves application of the basic principles associated with the tradeoff between risk and return, and in which aggregate company diversification and covariance benefits play a role. While this paper has attempted to present concepts in as simple a manner as possible, solutions must extend into those cases which reflect the many insurance variables, how they relate to one another, and how they evolve over time.

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APPENDIX

PROBABILITY OF SURPLUS DRAWDOWN RISK-BASED PREMIUM DETERMINATION

The underwriting risk-based premium based on the PSD in which loss is the only parameter with uncertainty is:

$$P = \{(1-T)-(1-D)\}L/\{(1-T)(1-ZC)\}, \qquad \text{(I)}$$

where:

P = Premium required

Z = Standard normal deviate corresponding to desired probability of drawdown

L = Estimated loss

T = Tax rate

R = Investment interest rate, after-tax

N = Average loss payment date

σ_L = Standard deviation of loss

C = Loss coefficient of variation (σ_L/L)

D = Discount factor= $1/(1+R)^N$,

assuming:

- Expenses are 0
- Premium is collected at policy inception
- Losses are normally distributed
- Approximation using average loss payment date
- Variability in loss amount only (i.e., certain cash flows and interest rates).

The underwriting risk-based premium based on the PSD in which interest rates and losses are both uncertain and independent is found by solving the following quadratic equation:

$$P^2A + PB + C = 0, \qquad P = (-B - \sqrt{B^2 - 4AC})/(2A), \quad \text{(II)}$$

where:

$$\begin{aligned} A &= \{1 - (ZC)^2\}/L^2 \\ B &= -2(1 - R/M)/L \\ C &= \{\sigma_R^2 - (M^2 - 2RM + R^2\}/M^2 \\ M &= R(1 - T)/(1 - D). \end{aligned}$$

The investment return lift in yield required to maintain the specified PSD from the shareholder perspective is $Ra - Rf$:

$$Ra = Z\sqrt{\sigma_{Ru}^2(L/S)^2 + \sigma_{Ra}^2(L/S + 1)^2} - Z\sigma_{Ru}(L/S), \qquad \text{(III)}$$

where:

Ra = Actual yield

Rf = Risk-free yield

σ_{Ru} = Standard deviation of underwriting return

σ_{Ra} = Standard deviation of actual yield

L/S = Liability to surplus leverage ratio,

assuming policyholder premium does not include a risk charge for investment.

Formula (I) is derived by noting that $Ru = (P/L - 1)M$, and solving for P such that $(Ru + Ri) = Z\sigma_{Ru}$.

Formula (II) is derived by solving for P such that $(Ru + Ri) = Z\sqrt{\sigma_{Ru}^2 + \sigma_{Ri}^2}$.

Formula (III) is derived by solving for Ra by determining the value that results in a shift from the risk-free total return line, given by $Rf + (Ru + Rf)(L/S)$ having a standard deviation of $\sigma_{Ru}(L/S)$, to the riskier total return line, given by $Ra + (Ru + Ra)(L/S)$ having a standard deviation of:

$$\sqrt{\sigma_{Ru}^2(L/S)^2 + \sigma_{Ra}^2(L/S+1)^2},$$

in order to satisfy the same PSD (i.e., Z-value).

The formulae presented here to demonstrate the concepts do not reflect all variables. While loss is generally the key driver in terms of expected return and variability, a more complete extension of this approach should reflect the impact of all parameters and multi-period cash flows (and the relationships among them) on return.

Demonstration Example

The following financial assumptions form the basis for the example presented in Exhibit A-1:

- 101.0% Combined ratio
- $9,900 Premium, collected without delay
- $10,000 Loss, single payment after 3 years
- $0 Expense
- 35% Income tax rate, no delay in payment
- 6.0% Investment interest rate before-tax, 3.9% after-tax
- No loss discount tax
- 3.0 Liability/surplus ratio.

Simplified balance sheet, income and cash flow statements are shown for this example. The rules governing the flow of surplus are as follows: (1) the level of surplus is maintained at a 1/3 ratio with loss reserves, (2) investment income on surplus is paid to the shareholder as earned, and (3) operating earnings are distributed in proportion to the level of insurance exposures in each year (measured by loss reserve levels) relative to the total exposure. Since loss reserves are equal at $10,000 in each of the three years, operating earnings are distributed to the shareholder equally in each year.

Three "levels" of return exist within an insurance company. The first is the underwriting rate of return, which reflects what the company earns on pure underwriting cash flows before reflecting investment income on the float. This is a "cost of funds" to the company. The second, operating return, reflects what the company earns on underwriting when investment income on the float is included. This is the "risk charge" to the policyholder for the transfer of risk to the company. Finally, the total return is the net result of underwriting and investment income from operations together with investment income on surplus.

These rates of return can be determined either by a cash flow-based internal rate of return (IRR) calculation, or by relating income earned to the amount invested. With regard to the shareholder total return perspective, the IRR based on cash flows from and to the shareholder indicates a 14.9% return over the three-year period. The income versus investment approach (i.e., ROE) relates the income over the full three-year aggregate financial life of the business to the shareholder's investment over this same period. This is shown in both nominal (i.e., undiscounted) and in present value (discounted) dollars to produce a 14.9% rate of return on investment. Furthermore, the return realized by the shareholder via dividends is also an identical 14.9% in each year. (This attribute follows from the rules used to control the flow of surplus.)

The operating return, inclusive of underwriting and investment income, is most easily shown to generate a cash flow-based internal rate of return of 3.7%. Equivalently, the operating income of $1,100 is a 3.7% return on the "investment equivalent" of $30,000, the total balance sheet float upon which these earnings were generated.

EXHIBIT A-1

THREE-YEAR DEMONSTRATION EXAMPLE

	PERIOD 1	2	3	4	Total	Present Value
BALANCE SHEET (Beginning of Period)						
Invested Assets	13,268	13,289	13,311	0	39,868	37,072
Loss Reserves	10,000	10,000	10,000	0	30,000	27,804
Retained Earnings	−65	−44	−23	0	−132	
Surplus Contributed	3,333	3,333	3,333	0	10,000	9,268
Liabilities/Surplus	3.0	3.0	3.0	0	3.0	3.0
INCOME AFTER-TAX (During Period)						
Underwriting	−65	0	0	0	−65	−65
Inv Inc Retained Earnings	−3	−2	−1	0	−5	
Inv Inc Loss Reserves	390	390	390	0	1,170	1,084
Total Operating	322	388	389	0	1,100	1,019
Inv Inc on Surplus	130	130	130	0	390	361
Total Net Income	452	518	519	0	1,490	1,381
Return on Beginning Contributed Surplus	13.6%	15.5%	15.6%	0	14.9%	14.9%
CASH FLOW (During Period)						
Operating Cash Flows						
Premium Receipt	9,900	0	0	0	9,900	9,900
Loss Payment	0	0	0	−10,000	−10,000	−8,916
Underwriting Tax Paid	35	0	0	0	35	35
Retained Earnings "Funding"	65	−23	−23	−23	−5	0
Total Underwriting	10,000	−23	−23	−10,023	−70	

EXHIBIT A-1

(Continued)

	PERIOD 1	2	3	4	Total	Present Value
Underwriting Return	Underwriting IRR = −0.2%					
Investment Receipts (After-Tax)	0	390	390	390	1,170	
Total Operating	10,000	367	367	−9,633	1,100	1,019
Operating Return	Operating IRR = 3.7%					
Surplus Cash Flows (Beginning of Period)						
Contributed Surplus	3,333	0	0	−3,333	0	
Dividend						
Surplus Inv Inc	0	−130	−130	−130	−390	
Operating Earnings	0	−367	−367	−367	−1,100	
Net Shareholder	3,333	−467	−467	−3,830	−1,490	
Shareholder Rate of Return	Shareholder IRR = 14.9%					
Shareholder "Dividend" Yield		14.9%	14.9%	14.9%	14.9%	

RATE OF RETURN

	IRR	(Income/Investment) Nominal	Net Present Value Basis
Underwriting Return	−0.2%	−0.2% = −70/30,000	−0.2% = −65/27,804
Operating Return	3.7%	3.7% = 1,100/30,000	3.7% = 1,019/27,804
Shareholder Total Return	14.9%	14.9% = 1,490/10,000	14.9% = 1,381/9,268

ESTIMATING U.S. ENVIRONMENTAL POLLUTION LIABILITIES BY SIMULATION

CHRISTOPHER DIAMANTOUKOS

Things should be made as simple as possible, but not any simpler. —Albert Einstein

Abstract

The application of computer simulation to the estimation of environmental pollution costs for inactive hazardous waste sites is presented. The various modules of the pollution costs simulation model (PCSM) are described, with the flow of costs traced from remedial action at EPA and state-administered sites through to insureds in the form of potentially responsible parties (PRPs), and finally to the application of coverage defenses. Methods are presented for using precision (credibility) estimates for state averages, and for projecting costs for an insurance portfolio based on sampling proportions. Countrywide results are presented, including the characterization of variability and comparisons to published insurance industry estimates of ultimate loss and expenses.

1. INTRODUCTION

The pollution costs simulation model (PCSM) described in this paper represents in many respects a work in progress. The remediation of pollution at inactive hazardous waste sites in the United States possesses a life cycle whose components are undergoing continual change as regards their attributes and durations. A model designed to estimate the associated costs or liabilities of remediation must first learn to walk, or crawl. It is natural to expect that, over time, changes and enhancements will be made to such a model.

This paper describes the status of the PCSM at a point in time: the PCSM will have undoubtedly changed by the time this paper is read. The purpose of this paper is therefore to present the approach to solving these estimation problems through the application of simulation techniques and actuarial principles.

The model is used to perform two tasks. The first is a countrywide estimate for all environmental pollution costs and ultimate liabilities to the U.S. insurance market for abandoned sites. The second is its application to a specific portfolio of insurance contracts in order to determine the liabilities to an individual insurer or reinsurer.

It is assumed that the reader has some familiarity with the history of the creation of environmental pollution liabilities through the Comprehensive Environmental Response, Compensation and Liability Act (CERCLA), Superfund Amendments and Reauthorization Act of 1986 (SARA), and other federal, state, or local laws. A thorough discussion of the governing laws and of the history and evolution of pollution liabilities in the U.S. will not be offered in this paper. The interested reader is directed to several references that will serve to provide any needed background.

2. DESIGN OF MODEL

The PCSM is designed to trace and simulate the sequential flow of pollution liabilities from the creation of costs at an individual hazardous waste site to the estimation of insurance liabilities at a contract (policy) level. The PCSM is best characterized as an exposure model of insurance liabilities and costs, since it is based on the construction of a model that attempts to measure the insurable loss of a set of risks and then apply policy conditions in order to estimate insured losses. The model does this through the estimation of population parameters and their interaction through modeling of the constituent databases. This process creates an "exposure measure"[1] that relates the expected values and under-

[1]The term "exposure measure" reflects the usage afforded by [1, p. 5].

lying distributions of the associated random variables, starting with site remediation costs and ending with the potential attachment of insurance coverage. There are no actual insured losses used in the construction of the PCSM as presented in this paper.

This form of exposure model is in sharp contrast to an extrapolation, regression, or any other model which makes projections into the future based on historical patterns and observations of actual insured losses. Those types of models are normally applied directly to empirical observations related to the liabilities being estimated.

One population invoked by the PCSM is the totality of inactive hazardous waste sites, where the great majority of the constituent sites do not have cleanup costs specifically associated with them. Some of these sites may be or may become National Priority List (NPL) sites in the future, and can be expected to command tens of millions of dollars in cleanup costs. Tens of thousands of these sites are state sites or will be sites arising under individual state supervision that do not have costs separately identified for them.

Another population is composed of those entities identified as potentially responsible parties (PRPs), which are presumed to form the bulk of what might better be termed ultimately responsible parties because they originally created the pollution. There are undoubtedly many more entities beyond those identified to date that will bear the cost for cleanup.

The PCSM constructs random variables that model costs at the site level and that model the sharing of these costs among known PRPs. Such estimates of costs in turn create exposures to loss for policies that afford insurance to PRPs. The expectations and uncertainties of the costs modeled by the PCSM are aggregated across sites, PRPs, and the policy or incurred years over which insurance has been provided, and then extended to the entire population of known sites in order to derive estimates of ultimate cleanup costs and insured liabilities.

The statistical foundation of the PCSM is directly reflected by the measures and distributions introduced throughout the model that reflect the uncertainty associated with each cost estimate element.

The estimation of pollution liabilities from U.S. locations utilizes the inactive hazardous waste site information identified by the Comprehensive Environmental Response, Compensation and Liability Information System (CERCLIS), as well as those identified or estimated by state and territory site lists or data bases. The latter sources contain sites that did not originate with the EPA through CERCLIS. Over time, these lists will expand to include sites that originated in CERCLIS, but have been referred to individual state or local authorities for supervision and further remedial action.

The PCSM estimates cleanup costs (Capital, sometimes referred to as Remediation, and Operation and Maintenance costs) as well as transaction costs (including allocated and unallocated loss adjustment expenses for insurance coverage purposes) for non-federal sites.[2] The term "cost" will often be used to refer to "cleanup costs" while "loss adjustment expense" has been reserved for transaction costs.

The PCSM performs five separate tasks in distinct, self-contained modules during each iteration of a simulation. The first module simulates individual site cleanup costs for each of the approximately 40,000 sites on the CERCLIS database (Section 3 of this paper). The second module assigns a share of each site's liability to each PRP for those sites identified with PRPs on the Site Enforcement Tracking System (SETS)[3] database (Section 4). The third module applies state specific estimated probabilities

[2]These sites are also referred to as "private sites" (see for example [15, p. 9]) although there are private entities identified as potentially responsible parties on federal sites.

[3]SETS is no longer the term used to identify this information as the EPA has restructured its data bases, thereby integrating information that may have been provided by separate sources. The term is used here for historical reference and to convey the existence of such information.

of coverage defenses being upheld and state specific probabilities of exposure triggers in order to establish insurable costs or liabilities (Section 5). The fourth module associates loss adjustment expense with PRP cleanup costs and distributes both the individually determined PRP insurable costs and loss adjustment expense across years of potential insurance coverage (Sections 6 and 7). The fifth module performs different tasks depending on whether the PCSM is being used for a countrywide simulation or a portfolio simulation. For the countrywide simulation it estimates the costs for those sites found only on state lists (all non-CERCLIS sites in a state) or relegated to state enforcement from the CERCLIS database (Section 8). For an insurance portfolio it evaluates the potential indemnification afforded by the policies and coverages contained within (Section 9).

Any additional cost elements or modifications to any component of cost estimates associated with environmental pollution liabilities that are not specifically mentioned are not estimated by the PCSM. These include consideration of third party liabilities, collateral suit defendants, Natural Resource Damage claims, and orphan shares. As the goal of this paper is to convey concepts and approaches, such limitations of the PCSM as presented herein should not detract from the achievement of that goal.

3. SITE COST ESTIMATES

Individual site cost estimates are performed for those sites contained on the CERCLIS data base. The primary source of cost information for an individual site in CERCLIS is contained in the Records of Decisions (RODs) issued by the United States Environmental Protection Agency (EPA). Specific cost information is available for only a small portion of these sites.

The estimation of site costs from RODs information is worthy of a paper in and of itself. One of the author's associates who contributed to the construction of the PCSM (Steven Finkelstein) has written such a paper [2]. One key issue addressed therein is the extraction of cost information from the RODs in a manner that allows the transformation of present value costs, as shown in the RODs, into undiscounted (nominal) values. However, an in-depth analysis of RODs reveals other issues including the identification of interim RODs (that presumably will be supplemented by final RODs), amendment RODs, the emergence of future RODs applicable to additional operable units for a site, and the translation of cost estimates from the date of preparation (issuance) for a ROD to the time actual construction (remediation) begins. The results of Finkelstein's analysis of RODs costs are presented here in the form of cleanup costs and associated present value factors without further explanation.

Exhibit 1 contains the distribution of CERCLIS sites at the time the PCSM was constructed according to Active/Archived, Site Status, RODs information, and PRP information. Some sites actually have more than one ROD issued while those identified as Archived, or No Further Remedial Action Planned (NFRAP), are unlikely to rise to the NPL. One of the ingredients to the model is, therefore, an estimate of the respective probabilities that a Non-NPL CERCLIS site may become an NPL site in the future. It was assumed that only the balance of Non-NPL active sites are eligible. All other sites, including all archived sites, would be subject to state or local authority. Indeed, some of these sites may not require any cleanup as evidenced by the statistics shown in Exhibit 2.

The simulation of site costs is preceded by an analysis of costs by site that results in the construction of two fundamental data bases:

- Sites with Variable Cost Estimates—contains information on those CERCLIS sites where more than one cost estimate was provided in the RODs for one or more of a site's operable units.
- Sites Without Variable Cost Estimates—contains those CERCLIS sites with a single value of cost estimates from the

RODs for each of its operable units or those CERCLIS sites with no specific cost information.

Appendix A describes how these databases were used to construct site cost variability parameters. These parameters include both the analysis of the distribution of cleanup costs on a site basis, as well as the variability or uncertainty of the cleanup costs for a single site.

The variability of site costs is in addition to the EPA default accuracy guideline of -30% to $+50\%$ of the published RODs costs [16, pp. 2–10]. Costs were randomized uniformly down to -30% or uniformly up to $+50\%$ and then normalized back to unity by dividing by 1.05, the expected value of the random variable so created[4] during each iteration of the PCSM.

Appendix B includes the derivation of the frequency of future NPL sites and the associated average site costs by state and category (NPL or state authority). It is of great value to identify site characteristics that can be used in a predictive manner for estimating cleanup costs for those sites that currently do not carry such information. There were very few site characteristics sufficiently populated for this purpose at the time the PCSM was constructed. State name was always available and useful to the extent it reflected differences among states resulting from differing industrial or economic development and attitude to cleanup standards and enforcement. Precision weighting of average site cleanup costs that employed estimates of variance was used in a manner consistent with actuarial estimates of credibility.

With these preliminary analyses completed, the sequence of steps employed by the PCSM to simulate a site cost during each iteration is as follows:

[4]The expected value below unity is .85, while above unity it is 1.25, which results in an expected value of 1.05 when they are equally weighted.

1. For Sites With Variable Cost Estimates:
    a) Randomly determine an EPA accuracy factor between −30% and +50%, or between .7 and 1.5, normalized to unity for each separate cost. This is typically applied to each individual total ROD present value cost.
    b) Multiply each individual ROD total present value cost by its random EPA accuracy factor.
    c) Divide by the corresponding present value factor to obtain nominal costs.
    d) Add the individual nominal costs for each ROD to determine a total undiscounted site cleanup cost.
2. For Sites Without Variable Cost Estimates:
    a) Randomly determine an EPA accuracy factor between −30% and +50%, or between .7 and 1.5, normalized to unity.
    b) Randomly determine a site cost uncertainty relativity as described in Appendix A.
    c) Randomly assign NPL status to an individual eligible active CERCLIS site by state, assigning state authority status otherwise.
    d) Assign state authority status to all non-eligible (archived) CERCLIS sites.
    e) Apply the individual ROD total present value cost if applicable; otherwise, apply the corresponding present value average state NPL site cost or the present value average state authority site cost according to site assignment.
    f) Multiply by the random EPA accuracy factor and site cost uncertainty relativity and, if a ROD cost was not used, also apply a normalized random factor from the

TABLE 1

SIMULATION PROCEDURE FOR SITES WITHOUT VARIABLE COST ESTIMATES

Is this an NPL site?	Y	Y	Y	N	N	N
Does site have ROD present worth costs?	Y	Y	N	Y	Y	N
Does site (ROD) have specific PV Factor?	Y	N	N	Y	N	N
EPA accuracy factor	X	X	X	X	X	X
Site cost uncertainty	X	X	X	X	X	X
Average NPL site cost for state			X			
Average state authority site cost for state						X
Site variability normalized factor			X			X
ROD present worth cost	X	X		X	X	
Default PV Factor		X	X		X	X
Specific site (ROD) PV Factor	X			X		

distribution of site cleanup costs to recognize differences among site cleanup costs as described in Appendix A.

g) Divide by the estimated present value factor when available to determine the nominal (undiscounted) site cost; otherwise, divide by the separately determined average present value factor (see Appendix B).

The decision logic table above summarizes the process for simulating costs for sites without variable cost estimates. The questions on the left are answered for each site from top to bottom and left to right, until the bottom of a column is reached in the upper half of Table 1. The actions applied to a site are then identified with an "X" in that column in the bottom half of the table. Note that the first question in the table is answered after randomly assigning NPL or state authority status to the individual eligible active CERCLIS site as referred to in item 2.c) above.

One important reason for performing the simulation of costs at the site level is to provide the starting point of ground-up losses, which is used to estimate insurable losses for excess of loss coverages. It is proper actuarial procedure to compare the expected value of losses in excess of the attachment point for excess of loss coverage; it is not proper to compare the expected value of ground-up losses to the attachment point for excess of loss coverage.

The simulation of ground-up losses therefore contributes to the characterization of the uncertainty of pollution costs in the aggregate, as well as providing the vehicle to properly estimate insurable losses for excess of loss coverage.

4. LIABILITY SHARES FOR POTENTIALLY RESPONSIBLE PARTIES

The estimate of individual liability shares for PRPs is perhaps the most difficult function performed by the PCSM. Actual shares are found in settlements that are contained in litigation files, claim files, or other records, all of which are proprietary in nature. The only public information that is available for some CERCLIS sites is the actual name of a PRP from which the total number of PRPs identified to date can be determined.[5] This is subject to change, as Finkelstein's paper clearly shows how PRPs emerge both prior and subsequent to the attainment of NPL status for an individual site [2]. The analysis is based on the date a site attained NPL status and the dates of the notification letters for the associated PRPs. Indeed, Exhibits 1-F and 1-H show that only 558 sites have PRP information out of a total of 803 sites with RODs cost information, and Exhibit 1-G shows that a total of 1,191 sites have PRP information.

The Beta distribution was employed as a modeling tool to vary PRP shares by utilizing the only available crude measure of total number of PRPs. This modeling of shares was performed in lieu of assigning equal shares to all PRPs for a site. Equal shares would appear to be a reasonable assumption, due to the

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[5]The information presented in this paper related to PRPs is derived from the February 1997 version of SETS.

joint and several nature of the retroactive liability associated with pollution cleanup and the lack of any further information on the financial ability of the member PRPs to fund the cleanup efforts. Variable shares were modeled through the Beta distribution in order to simulate the phenomenon of *de minimis* PRP shares. Appendix C presents the theory behind the structure of the Beta distribution used for this purpose.

This module of the PCSM assigns a share to each PRP from the appropriate Beta distribution for the site, and then normalizes the shares so that they add to unity. Although this estimate may exhibit a large degree of uncertainty for any individual PRP at an individual site, it should offer reasonable results when considering a portfolio of sites and PRPs as would be the case from an insurance perspective. The process is performed as described in Appendix C. No attempt is made to reduce PRP shares for orphan shares, which should be considered a conservative assumption. However, this may not be so conservative as experience accrues for the funding of pollution costs. It may very well be the case that the application of joint and several liability theories serves to erode the limited savings offered by reduction for the recognition of orphan shares,[6] resulting in a re-normalizing of shares among PRPs.

Those sites without PRP information are provided an estimate of the total number of PRPs based on an analysis of Site Category. This information can be used when a portfolio provides specific information that relates a PRP to a site for which no such information appears on SETS.

One final consideration that has not been specifically modeled is the emergence of future PRPs at a given site. This should serve to reduce the shares of those PRPs currently identified at a site. Future PRPs also encompass the naming of collateral suit defendants. This phenomenon would reflect both reductions to

[6]One estimate of these shares is 18% [23, p. 33].

existing shares as well as increases resulting from the naming of a PRP as a collateral suit defendant at another site.

These latter considerations from the preceding paragraph impact the equity of allocating pollution liability exposure among PRPs rather than the total estimate of pollution costs. Such equity considerations clearly have the greatest bearing on the analysis of an individual insurance portfolio.

5. INSURANCE COVERAGE DEFENSE

The extent to which casualty insurance coverages provide indemnification for pollution claims is a phenomenon that varies with the period of time a given policy was in force. Case law has been established to varying degrees by state that has upheld or denied policy exclusions or conditions. The strongest defenses are those associated with the Absolute Pollution Exclusion introduced formally by ISO, Inc. beginning in 1986, with filings introduced among the various states over time.

The PCSM employs a Coverage Defense Module (CDM) that translates the information related to coverage exclusions or conditions into subjective estimates of probabilities that any specific coverage defense by the insurer will be upheld. The initial version of this module was based on a review of two publications ([13] and [14]).

The creation of these probabilities was founded on the basic principle that the higher the level of state court in which a ruling has been rendered, the higher the probability that the ruling will be upheld and applied in similar situations. The initial probabilities based on the type of court were as follows: 95% for supreme courts, 75% for appellate courts and 60% for any district or circuit courts. The tempering at 95% of the highest court rulings for a state supreme court was used to eliminate absolute certainty for the outcome of any particular coverage defense, thereby permitting the possibility of conditions that might cause an exception for a given situation from that state's ruling. Probabilities were also selected considering the number of cases related to the coverage defense that were similar, and the age of the cases reviewed for a particular coverage defense.

These two final inputs to the probability selections enable the CDM to be dynamic, accounting for changes in court decisions over time. Such an approach permits different probabilities depending on when site cleanup costs are estimated or when pollution insurance claims are settled. The approach can apply different levels of likely success for a coverage defense for sites with cost estimates in the past versus those sites with yet to be determined costs and shares. Another approach is to reflect the phenomenon of when insurance claims are actually presented to carriers and when they, in turn, are denied or, alternately, judged to be valid and thereby represent indemnifiable losses.

The choice of state (venue) for any particular combination of site and PRP must also be considered. The possible choices include at least the state of domicile of the insurer, the state of incorporation or domicile of the PRP, and the location of the site. The PCSM was run using the state of the site's location, on the assumption that the state has a controlling, vested interest in the remediation of property within its jurisdiction and because the location of a risk (site) is often the controlling element for the settlement of insurance claims. In the case of pollution, the author has been advised that the state of incorporation or domicile of the PRP is often used, but the issue of proper venue selection is far from settled. In specific portfolio applications, the venue itself has been simulated by the author from several plausible candidates.

The PCSM incorporates the CDM by randomly determining for each site and PRP whether or not there will have been a successful defense (a favorable outcome to the insurer) for each category analyzed for the particular state. A favorable outcome to the insurer translates to no indemnification for cleanup costs proper. Each such random determination is made by performing a Bernoulli trial (BT), with success measured by the probabilities described earlier. The interaction of several defenses must also be considered, that is, the possibility that an insurer will move to deny coverage based on several exclusions contained in the policy language. It was not practical, and perhaps not possible, during the design of the CDM to perform such an analysis of all possible interactions. Instead, a hierarchy was employed that placed greatest emphasis on the pollution exclusions and at most one other defense, *viz.*, Cleanup Costs as Damages. Three separate time periods were considered, responding to changing coverage wording, to execute the planned hierarchy:

1. 1972 and prior used the BT for Cleanup Costs as Damages defense (i.e., cleanup costs are not considered damages and should therefore not be indemnified). The default probability for Cleanup Costs as Damages was used if the state did not have a ruling.

2. 1973 through 1986 considered several defenses by employing the following hierarchy:

    a) If there was a decision on the pollution exclusion requiring a "sudden and accidental event" that predated the Absolute Pollution Exclusion, and the BT resulted in no coverage, then no other defense was considered.

    b) If the result described in a) was to provide coverage (i.e., the defense was denied), then the BT of the Cleanup Costs as Damages defense was used if available, and if not available the BT using the default probability for Cleanup Costs as Damages was used.

    c) If there was no ruling on the pollution exclusion, then the BT of the Cleanup Costs as Damages defense was used if available, and if not available the BT using the default probability for Cleanup Costs was used.

3. 1987 and subsequent focused on precedents for the Absolute Pollution Exclusion where available:

a) If the state supreme court made a ruling upholding the Absolute Pollution Exclusion, then the BT for successful denial of coverage used a probability of 95%.

b) If the result of a) was to provide coverage, then the BT of the Cleanup Costs as Damages defense was used if available, and if not available the BT using the default probability for Cleanup Costs as Damages was used.

c) If there was a lower level ruling and BT indicated no coverage, then that result was used.

d) If there was a lower level ruling and the result of c) was to provide coverage then (similar to b) above) the BT of the Cleanup Costs as Damages defense was used if available, and if not available the BT using the default probability for Cleanup Costs as Damages was used.

e) If there were no rulings on the Absolute Pollution Exclusion, the BT of the Cleanup Costs as Damages defense was used if available, and if not available the BT using the default probability for Cleanup Costs as Damages was used.

The following decision logic table (Table 2) summarizes the process used to determine a successful coverage defense, and operates in similar fashion to Table 1. The abbreviations used below are PE for pollution exclusion (sudden and accidental), APE for Absolute Pollution Exclusion, and, as used in the preceding discussion, BT for performing a Bernoulli trial.

The default cleanup cost defense probability was based on the average for those states that had respective rulings for these defenses. The reference “State (or default)” in Table 2 refers to the use of the specific state ruling if available, and using the default probability if not available.

TABLE 2

SIMULATION PROCEDURE FOR COVERAGE DEFENSE MODULE

Coverage 1972 or prior?	Y	N	N	N	N	N	N	N	N
Coverage 1973–1986?		Y	Y	Y	N	N	N	N	N
Coverage after 1986?					Y	Y	Y	Y	Y
PE ruling for state?		Y	Y	N					
PE BT results in “no coverage”?		Y	N						
APE state supreme court ruling?					Y	Y	N	N	N
APE state lower court ruling?							Y	Y	N
APE BT results in “no coverage”?					Y	N	Y	N	N
No cleanup costs indemnification		X			X		X		
State (or default) cleanup costs as damages BT	X		X	X		X		X	X

6. TRIGGER THEORIES AND ALLOCATIONS

Four policy trigger theories are included in the CDM: exposure (operation), manifestation, continuous, and injury-in-fact. As in the case of policy coverage defenses, case law has also established the degree to which policies are triggered over time. In addition to the analysis of rulings within a state, the findings from U.S. District Courts were also considered in determining trigger probabilities.

The identification of policies triggered is the fundamental prerequisite which establishes the basis upon which insurable exposure can be measured. To say that a set of policies are triggered under a particular theory means that they respond jointly and severally to the pollution loss or claim. It is another matter to measure each policy’s exposure to the pollution loss on a relative basis (i.e., how much each policy contributes to the final total indemnification). To employ the actual strategies used in pollution claim settlements to allocate coverage among insurers and policies, obtaining coverage charts for the universe of PRPs would be necessary. Coverage charts describe the commercial insurance and self-insurance programs for an insured over time.

Instead, the PCSM assigns indemnified losses and loss adjustment expenses across years based on a simulation of the trigger applied to that site and PRP combination. In the case of a manifestation trigger, only one year is involved. The year associated with manifestation is based on the date of a special notice letter, or general notice letter if the former is not applicable.

The PCSM uses a simple method of allocating pollution losses over time according to the coverage trigger simulated. It is based on the analysis of a large sample of claims at the insured and site level that provided exposure (operation) dates and date of notice to the insurer. This permitted the estimation of the distribution of exposure to pollution loss over time for exposure and continuous triggers. Valid actual dates were employed wherever possible, while simulated dates from the distribution of known valid dates were employed for CERCLIS sites without dates and on an aggregate basis for each set of state sites.

The continuous trigger distribution was translated to a conditional basis based on the manifestation year described earlier.[7] The manifestation year serves as the endpoint for continuous trigger assignments.

The resulting exposure and continuous trigger distributions were heavily weighted towards more recent years, paralleling the increased coverage afforded insureds in general as time goes by. This approach tends to create an element of conservatism to the industry estimates as greater exposure is be generated from the more recent years that afford greater insurance coverage.

The injury-in-fact trigger used the continuous trigger distribution, normalized between the first discovery date of pollution at a site and the notice date to a PRP pertaining to either a special or general letter from the EPA. This approach represents a conservative assumption as regards industry estimates, because it places the earliest date of injury at the first discovery date

[7] A more precise method would have been to decompose the distributions conditional on discovery dates, albeit with limited empirical information to do so.

of pollution rather than the earliest default operation date of 1950.

An alternative for measuring operation periods is to simply employ the operation dates of a site for those sites with RODs containing cost estimates. This does not provide for variation among PRPs for their involvement at a site. These dates and time periods must also be extrapolated over the entire population of hazardous waste sites.

The PCSM does not employ an All Sums, or "Fountain"[8] trigger. This trigger tends to create large concentrations of losses for a single insured in a single year. The losses may then be allocated or shared among other insurers or reinsurers considered to be exposed to such losses through settlement of the pollution claims. There is less case law precedent on this trigger at the state level in comparison to other triggers, thereby preventing a reliable simulation of this relatively infrequently invoked trigger. However, the trigger is indeed employed by some specific PRPs and against some insurers.

7. LOSS ADJUSTMENT EXPENSE

The concept of duty to defend deals with whether or not an insurer will incur legal expenses and other loss adjustment expenses (LAE) for a pollution claim, even though there is no indemnification due to a successful coverage defense. Case law on this subject is quite varied. The PCSM employed a conservative assumption by including all LAE as costs to the insurance industry.

A summary of the LAE analysis is presented in Exhibit 3. It is based on analyses performed by the Rand Institute [9]. The PCSM simulated the specific LAE to cleanup costs ratio uniformly within the ranges of 23.5% to 29.9%, and 29.9% to 37.0%, with equal probability associated with the lower and

[8]For example, see [8, p. 122].

upper range. The simulated value was balanced to the estimated average of 29.9% by dividing by the uniform average of 30.25% (i.e., the average of the expectation below and above the mean) in a manner analogous to the procedure employed for the EPA accuracy factor discussed earlier in Section 3.

Another Rand Institute study [10, p. 51ff] also includes information that correlates total site transaction costs with the number of PRPs. An enhancement to the modeling of LAE would be to include such variation with the additional constraint that the simulated average balance to, or be consistent with, the original estimate of the mean LAE to cleanup costs ratio.

8. COUNTRYWIDE RESULTS

In order to obtain countrywide results, the contribution from state sites must be estimated. The PCSM based these estimates on the frequency and cost figures contained in Exhibit 2. The state severities were also used for those CERCLIS sites that were deemed to be excluded from NPL status, either through simulation or through identification as NFRAP. Note that these individual sites were simulated according to the form of the distribution of site costs described earlier, but specific state average severities were employed.

Exposure triggers were weighted according to the expectation for each trigger by state, rather than selecting a single trigger for each iteration or for each state site. Costs were allocated over time based on the simulation of default dates. Each iteration of the PCSM performed one random selection by state for average site cost, each trigger default date, and the ratio of LAE to cleanup costs.

Exhibit 4 shows the estimates of pollution losses and the allocation over time. The distribution of insurance coverages was censored at 1950 (i.e., the contributions to exposure for years prior to 1950 were added into 1950). It could be argued that some relevant coverage would have been provided by commercial general liability policies as early as 1940. However, although the author is aware of exposure for one type of site (manufactured gas plants) where exposure is claimed to exist as far back as the 1840s, it is extremely unlikely that any property and casualty policies provided indemnification that far in the past.

A few remarks concerning Exhibit 4 are in order. First and foremost is the magnitude of the estimate. It is approximately $70 billion (the total of the columns from all three pages of Exhibit 4), which is higher than the $56 billion estimate published in "BestWeek."[9] The difference is likely understated as the "BestWeek" estimate presumably includes third party bodily injury claims and Natural Resource Damage claims.

Second, a comparison between the simulated mean of total cleanup costs to indemnified costs indicates that very close to 50% of costs were not insured as a result of the application of the parameters contained in the CDM. This compares with the 40% reduction in insurable costs disclosed by "BestWeek"[10] associated with successful coverage defense.

Third, no underlying limits were introduced, which would have served to reduce the estimates. This is a conservative PCSM assumption. The author has witnessed insurance settlements that belie the conventional wisdom associated with the introduction of underlying deductibles, self-insured retentions, or the use of underlying limits for excess coverages. Underlying limits have specific application to the analysis of reinsurance portfolios.

Fourth, the estimate of pollution losses over time permits one to perform a more refined exposure analysis lacking specific insured information for a portfolio.[11] For example, a market share

[9]See [21, p. P/C 6] and [22, p. 4].

[10]See [20, p. P/C 7] where Insurers' Liability %, footnote D, refers to "settlements and cases won by insurers."

[11]This is in lieu of another possible estimate based on the use of actual pollution claims, which the author has developed in a manner to remove the bias introduced by the emergence of the potentially most serious claims in the insured portfolio (nominally those related to NPL sites).

approach uses premiums as the exposure medium which can be matched against the relevant pollution estimates for the exposed years.

Finally, the simulation of costs at the state level for non-CERCLIS sites tends to contribute greater variation than that afforded by individual site simulation that is performed using CERCLIS.

9. PORTFOLIO APPLICATIONS

The countrywide analysis and simulation form the kernel of the estimation for a specific portfolio. The simulation for a portfolio is streamlined somewhat by the initial restriction of sites to those matched to the PRPs embedded in the portfolio. Specific information on such items as trigger theories, dates of operation, or PRP shares are used wherever possible at the site and PRP level.

The simulated costs by PRP, site, and year are normally matched to the coverage parameters of the policies issued for that insured. The parameters can usually be limited to an attachment point, limit or layer of coverage, and participation when direct excess insuranceor reinsurance is involved. Other coverage parameters can also be included such as aggregate retentions for excess policies. Deductibles can also be employed, although it is the author's experience that a limited amount of deductible coverages for Other Liability were offered through the mid 1980s.

The identification of the PRPs in the portfolio and the matching to those contained in SETS is the most time-consuming effort in a portfolio application. It is further complicated by the need to consider the aggregation of costs for a single insured due to liability derived from subsidiaries, and the alternate identification of an insured through aliases. The simulation proper is then seen as employing one of two possible representations of the portfolio. The first is where an exhaustive identification and matching of insureds and PRPs has been made that characterizes the portfolio. The other is where a sample of insureds within the portfolio has been identified.

In the case of a complete characterization of the portfolio, the extrapolation to an ultimate basis needs to consider the expected costs on sites that have not been specifically associated with the known PRPs identified in the portfolio. This would include estimates related to sites found only on state lists and the potential for future NPL sites to be associated with the PRPs in the portfolio. It is also prudent in such circumstances to include additional costs related to RODs to be issued in the future on known NPL sites related to portfolio PRPs. Discussion of such estimates in further detail is beyond the scope of this paper.

For the case where a sample of the portfolio has been simulated, Appendix D contains the derivation of a scalar that permits the extrapolation of the simulation results to the entire portfolio. Under these conditions the estimate will be biased downwards due to the inability to identify all PRPs embedded in a portfolio as discussed in Appendix D. This method may be enhanced by including some element of a market share exposure analysis as alluded to earlier.

10. WHERE DO WE GO FROM HERE?

There is a great deal of uncertainty surrounding the ultimate cost of pollution cleanup from inactive hazardous waste sites in this country. Unfortunately, the model presented herein, as well as any other, will for some considerable length of time suffer from model specification error of immeasurable magnitude.

The degree to which there are repeatable occurrences or events from which to base projections varies among the components and processes underlying the phenomenon of pollution cleanup. On the one hand, it took a study of only 18 Superfund sites to provide enough material for the Rand Institute to publish a leading work in this area. On the other hand, there are on the order of 1,000 CERCLIS sites with varying degrees of complete cost information; with such varying circumstances, the estimated averages must by necessity have a great deal of uncertainty.

There is considerable uncertainty in extrapolating any averages, as many of the site characteristics that could serve as predictive elements in a model are lacking from publicly available information. For example, it is of limited predictive value to employ average site cleanup costs by standard industry classification (SIC) if the thousands of sites that remain without cleanup costs are not identified by SIC.

At the end of the day, the task at hand is to project activities and costs that will take place in the future, without the familiar historical information that actuaries might use in development (extrapolation) estimates, regression models, or other methods of estimation that rely to varying degrees on the observation of actual insurance losses. Indeed, there is anecdotal evidence to show that cost figures from RODs turn out to be quite different from actual expenditures,[12] thereby casting additional doubt on the reliability of some of the harder numbers that underlie these analyses. The costs of remedial investigations and feasibility studies also increase costs and, if the decision by the California Supreme Court in the Aerojet-General case on December 29, 1997 sets a precedent, such costs will be subject to indemnification by insurance companies. Indeed, that decision also affects how pollution losses would trigger insurance policies over time when self-insurance is involved.

The point of this brief discussion is that there are many signs indicating higher ultimate paid costs, and very few that point in the other direction. The continuation of perceived downward trends in remediation costs may be altogether uncertain if funding for the requisite continued research is discontinued, or if there

[12] According to an article on page B-1 from *The Philadelphia Inquirer* of August 14, 1997, the GEMS Landfill in New Jersey will require $62 million in capital costs to cleanup, well in excess of any readily available estimates from RODs and elsewhere that are on the order of $27 million in capital costs.

are disincentives for insureds to engage in voluntary cleanup efforts at reduced costs. In addition, the distribution of losses over time for indemnification by insurance policies is a phenomenon that continues to undergo change.

From an insurance perspective, the focus of the balance sheet is on liabilities. Given an estimate of ultimate insured costs, the question that remains to be answered is how much is there left to be paid? From an economic perspective, the timing of those payments should provide insight as to their present value.

The matching of payments and ultimate costs cannot be done by simply subtracting payments to date from the ultimate estimates. It is more proper to subtract expected payments from ultimates. Failure to do so can lead to erroneous results such as in the GEMS Landfill case alluded to earlier in a footnote. Further, there is some uncertainty as to the reliability of properly matching payments against these liabilities. At a minimum this is caused by the lack of separate identification of such early costs in the records of insurers. Models could be created to fabricate an answer, but it is a topic that requires further research for resolution.

Finally, some concerted efforts may be needed to obtain updated and reliable information on state administered sites [24]. The evidence so far indicates substantial costs from this source, yet the level of available information does not approach that available for many NPL sites. Indeed, the volume of such data, if available, would present a challenge to its use in the same manner as that employed for NPL sites.

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[23] The Brookings Institution and Resources for the Future, *Footing the Bill for Superfund Cleanups*, 1995.

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EXHIBIT 1-A

Site Characteristics and Cost Analyses by State
CERCLIS Source: Archive
Number of Sites
NPL Status

State ID	NPL Site	Deleted NPL	Final NPL	Removed NPL	NFRAP Federal	NFRAP Not Federal	Total
AK	.	.	.	.	131	130	261
AL	.	.	.	.	12	563	575
AR	.	.	.	1	5	343	349
AS	.	.	.	.	2	4	6
AZ	.	.	.	2	18	699	719
CA	.	.	.	8	92	2,434	2,534
CM	.	.	.	.	.	6	6
CO	.	.	.	.	35	408	443
CT	.	.	.	.	2	327	329
DC	.	.	.	.	10	10	20
DE	.	.	.	.	1	221	222
FL	.	.	.	2	34	552	588
GA	.	.	.	.	11	723	734
GU	.	.	.	.	10	3	13
HI	.	.	.	6	20	104	130
IA	.	.	.	.	1	402	403
ID	.	.	.	1	67	134	202
IL	.	1	.	3	3	1,284	1,291
IN	.	5	.	2	2	1,433	1,442
KS	.	.	.	.	2	339	341
KY	.	.	.	.	4	487	491
LA	.	.	.	.	14	525	539
MA	.	.	.	.	8	516	524
MD	.	.	.	.	11	309	320
ME	.	.	.	.	8	103	111
MI	.	6	1	5	11	1,495	1,518
MN	.	12	.	.	2	407	421
MO	.	.	.	.	7	906	913
MS	.	.	.	2	6	374	382
MT	.	.	.	.	16	177	193
NC	.	.	.	.	8	687	695
ND	.	.	.	.	7	58	65
NE	.	.	.	.	4	235	239
NH	.	.	.	.	3	62	65
NJ	2	.	.	.	23	926	951
NM	.	.	.	.	30	266	296

EXHIBIT 1-A

(Continued)

State ID	NPL Site	Deleted NPL	Final NPL	Removed NPL	NFRAP Federal	NFRAP Not Federal	Total
NN	.	.	.	.	.	83	83
NV	.	.	.	.	18	157	175
NY	.	.	.	.	51	1,059	1,110
OH	.	.	.	1	9	1,078	1,088
OK	.	.	.	1	10	664	675
OR	.	.	.	.	30	337	367
PA	.	.	.	.	12	2,486	2,498
PR	.	.	.	.	8	179	187
RI	.	.	.	.	2	119	121
SC	.	.	.	.	5	378	383
SD	.	.	.	.	7	77	84
TN	.	.	.	.	9	580	589
TT	.	.	.	.	2	32	34
TX	.	.	.	.	35	2,367	2,402
UT	1	.	.	1	18	163	183
VA	.	.	.	.	7	440	447
VI	.	.	.	.	1	25	26
VT	1	.	.	.	2	69	72
WA	.	.	.	2	37	550	589
WI	.	2	.	1	3	336	342
WQ	.	.	.	.	.	1	1
WV	.	.	.	.	7	497	504
WY	.	.	.	.	16	121	137
Total	4	26	1	38	909	29,450	30,428

EXHIBIT 1-B

SITE CHARACTERISTICS AND COST ANALYSES BY STATE
CERCLIS SOURCE: ACTIVE
NUMBER OF SITES
NPL STATUS

State ID	NPL Site	Deleted NPL	Final NPL	Federal Not NPL	Proposed NPL	Not Federal Not NPL	Removed NPL	Total
AK	1	1	7	46	.	49	.	104
AL	.	1	12	15	1	110	.	139
AR	.	1	12	2	.	42	2	59
AS	.	1	.	.	.	1	.	2
AZ	3	1	10	11	.	162	.	187
CA	14	3	90	100	4	424	6	641
CM	.	1	.	.	.	2	.	3
CO	.	1	16	11	2	132	1	163
CT	18	1	15	3	.	409	.	446
DC	.	.	.	8	.	9	1	18
DE	.	2	18	2	.	30	3	55
FL	1	7	55	10	2	405	3	483
GA	.	1	15	6	1	243	1	267
GU	.	.	2	12	.	.	.	14
HI	6	.	4	22	.	37	.	69
IA	5	4	16	15	1	189	6	236
ID	4	1	8	17	2	29	.	61
IL	7	.	38	9	3	339	.	396
IN	.	.	30	9	1	157	1	198
KS	1	5	10	7	1	202	1	227
KY	.	.	20	13	.	90	.	123
LA	.	.	14	3	3	70	.	90
MA	7	1	30	9	.	459	.	506
MD	.	2	13	45	3	72	1	136
ME	.	.	12	.	.	88	.	100
MH	.	.	.	.	.	1	.	1
MI	.	4	71	9	2	148	1	235
MN	.	2	29	2	1	53	.	87
MO	16	1	22	29	.	347	2	417
MQ	.	.	.	1	.	.	.	1
MS	.	2	1	5	2	58	1	69
MT	.	.	8	8	1	31	1	49
NC	.	1	23	25	.	176	1	226
ND	.	.	2	1	.	7	.	10
NE	19	.	10	18	.	122	2	171
NH	8	.	18	2	.	107	.	135

EXHIBIT 1-B

(Continued)

State ID	NPL Site	Deleted NPL	Final NPL	Federal Not NPL	Proposed NPL	Not Federal Not NPL	Removed NPL	Total
NJ	6	9	106	10	.	669	2	802
NM	.	1	10	17	1	61	.	90
NN	.	.	.	.	.	68	.	68
NV	.	.	1	10	.	17	.	28
NY	.	10	78	11	1	521	.	621
OH	.	1	34	7	4	212	.	258
OK	.	.	10	18	1	106	1	136
OR	6	2	10	11	1	45	.	75
PA	.	10	100	16	2	394	2	524
PR	.	.	9	4	1	72	.	86
RI	24	.	12	7	.	150	.	193
SC	.	1	26	8	.	140	.	175
SD	.	.	3	1	1	23	.	28
TN	.	1	17	18	1	260	1	298
TT	.	1	.	.	.	4	.	5
TX	.	5	26	21	1	175	2	230
UT	.	.	12	4	4	122	2	144
VA	.	2	25	37	.	166	1	231
VI	.	.	2	1	.	8	.	11
VT	4	.	8	.	.	64	.	76
WA	52	10	48	24	2	43	.	179
WI	.	.	40	20	.	92	.	152
WQ	.	.	.	1	.	.	.	1
WV	.	1	6	1	1	57	1	67
WY	.	.	3	1	.	23	.	27
Total	202	98	1,217	723	51	8,292	46	10,629

EXHIBIT 1-C

Site Characteristics and Cost Analyses by State
Total
Number of Sites
NPL Status

State ID	NPL Site	Deleted NPL	Final NPL	Federal Not NPL	Proposed NPL	Not Federal Not NPL	Removed NPL	NFRAP Federal	NFRAP Not Federal	Total
AK	1	1	7	46	.	49	.	131	130	365
AL	.	1	12	15	1	110	.	12	563	714
AR	.	1	12	2	.	42	3	5	343	408
AS	.	1	.	.	.	1	.	2	4	8
AZ	3	1	10	11	.	162	2	18	699	906
CA	14	3	90	100	4	424	14	92	2,434	3,175
CM	.	1	.	.	.	2	.	.	6	9
CO	.	1	16	11	2	132	1	35	408	606
CT	18	1	15	3	.	409	.	2	327	775
DC	.	.	.	8	.	9	1	10	10	38
DE	.	2	18	2	.	30	3	1	221	277
FL	1	7	55	10	2	405	5	34	552	1,071
GA	.	1	15	6	1	243	1	11	723	1,001
GU	.	.	2	12	.	.	.	10	3	27
HI	6	.	4	22	.	37	6	20	104	199
IA	5	4	16	15	1	189	6	1	402	639
ID	4	1	8	17	2	29	1	67	134	263
IL	7	1	38	9	3	339	3	3	1,284	1,687
IN	.	5	30	9	1	157	3	2	1,433	1,640
KS	1	5	10	7	1	202	1	2	339	568
KY	.	.	20	13	.	90	.	4	487	614
LA	.	.	14	3	3	70	.	14	525	629
MA	7	1	30	9	.	459	.	8	516	1,030
MD	.	2	13	45	3	72	1	11	309	456
ME	.	.	12	.	.	88	.	8	103	211
MH	.	.	.	.	.	1	.	.	.	1
MI	.	10	72	9	2	148	6	11	1,495	1,753
MN	.	14	29	2	1	53	.	2	407	508
MO	16	1	22	29	.	347	2	7	906	1,330
MQ	.	.	.	1	.	.	.	.	.	1

EXHIBIT 1-C

(Continued)

State ID	NPL Site	Deleted NPL	Final NPL	Federal Not NPL	Proposed NPL	Not Federal Not NPL	Removed NPL	NFRAP Federal	NFRAP Not Federal	Total
MS	.	2	1	5	2	58	3	6	374	451
MT	.	.	8	8	1	31	1	16	177	242
NC	.	1	23	25	.	176	1	8	687	921
ND	.	.	2	1	.	7	.	7	58	75
NE	19	.	10	18	.	122	2	4	235	410
NH	8	.	18	2	.	107	.	3	62	200
NJ	8	9	106	10	.	669	2	23	926	1,753
NM	.	1	10	17	1	61	.	30	266	386
NN	.	.	.	.	.	68	.	.	83	151
NV	.	.	1	10	.	17	.	18	157	203
NY	.	10	78	11	1	521	.	51	1,059	1,731
OH	.	1	34	7	4	212	1	9	1,078	1,346
OK	.	.	10	18	1	106	2	10	664	811
OR	6	2	10	11	1	45	.	30	337	442
PA	.	10	100	16	2	394	2	12	2,486	3,022
PR	.	.	9	4	1	72	.	8	179	273
RI	24	.	12	7	.	150	.	2	119	314
SC	.	1	26	8	.	140	.	5	378	558
SD	.	.	3	1	1	23	.	7	77	112
TN	.	1	17	18	1	260	1	9	580	887
TT	.	1	.	.	.	4	.	2	32	39
TX	.	5	26	21	1	175	2	35	2,367	2,632
UT	1	.	12	4	4	122	3	18	163	327
VA	.	2	25	37	.	166	1	7	440	678
VI	.	.	2	1	.	8	.	1	25	37
VT	5	.	8	.	.	64	.	2	69	148
WA	52	10	48	24	2	43	2	37	550	768
WI	.	2	40	20	.	92	1	3	336	494
WQ	.	.	.	1	.	.	.	.	1	2
WV	.	1	6	1	1	57	1	7	497	571
WY	.	.	3	1	.	23	.	16	121	164
Total	206	124	1,218	723	51	8,292	84	909	29,450	41,057

EXHIBIT 1-D

Site Characteristics and Cost Analyses by State
CERCLIS Source: Archive
Sites with Rod Costs
NPL Status

State ID	NPL Site	Deleted NPL	Final NPL	Removed NPL	NFRAP Federal	NFRAP Not Federal	Total
AK	.	.	.	.	.	.	.
AL	.	.	.	.	.	.	.
AR	.	.	.	.	.	.	.
AS	.	.	.	.	.	.	.
AZ	.	.	.	.	.	.	.
CA	.	.	.	1	.	.	1
CM	.	.	.	.	.	.	.
CO	.	.	.	.	.	.	.
CT	.	.	.	.	.	.	.
DC	.	.	.	.	.	.	.
DE	.	.	.	.	.	.	.
FL	.	.	.	.	.	.	.
GA	.	.	.	.	.	.	.
GU	.	.	.	.	.	.	.
HI	.	.	.	.	.	.	.
IA	.	.	.	.	.	.	.
ID	.	.	.	.	.	.	.
IL	.	.	.	.	.	.	.
IN	.	2	.	.	.	.	2
KS	.	.	.	.	.	.	.
KY	.	.	.	.	.	.	.
LA	.	.	.	.	.	.	.
MA	.	.	.	.	.	.	.
MD	.	.	.	.	.	.	.
ME	.	.	.	.	.	.	.
MI	.	1	1	.	.	.	2
MN	.	6	.	.	.	.	6
MO	.	.	.	.	.	.	.
MS	.	.	.	.	.	.	.
MT	.	.	.	.	.	.	.
NC	.	.	.	.	.	.	.
ND	.	.	.	.	.	.	.
NE	.	.	.	.	.	.	.
NH	.	.	.	.	.	.	.
NJ	.	.	.	.	.	.	.
NM	.	.	.	.	.	.	.

EXHIBIT 1-D

(Continued)

State ID	NPL Site	Deleted NPL	Final NPL	Removed NPL	NFRAP Federal	NFRAP Not Federal	Total
NN	.	.	.	.	.	.	.
NV	.	.	.	.	.	.	.
NY	.	.	.	.	.	.	.
OH	.	.	.	.	.	.	.
OK	.	.	.	.	.	.	.
OR	.	.	.	.	.	.	.
PA	.	.	.	.	.	.	.
PR	.	.	.	.	.	.	.
RI	.	.	.	.	.	.	.
SC	.	.	.	.	.	.	.
SD	.	.	.	.	.	.	.
TN	.	.	.	.	.	.	.
TT	.	.	.	.	.	.	.
TX	.	.	.	.	.	.	.
UT	.	.	.	.	.	.	.
VA	.	.	.	.	.	.	.
VI	.	.	.	.	.	.	.
VT	.	.	.	.	.	.	.
WA	.	.	.	.	.	.	.
WI	.	.	.	.	.	.	.
WQ	.	.	.	.	.	.	.
WV	.	.	.	.	.	.	.
WY	.	.	.	.	.	.	.
Total	.	9	1	1	.	.	11

EXHIBIT 1-E

Site Characteristics and Cost Analyses by State
CERCLIS Source: Active
Sites with Rod Costs
NPL Status

State ID	NPL Site	Deleted NPL	Final NPL	Federal Not NPL	Proposed NPL	Not Federal Not NPL	Removed NPL	Total
AK	.	.	1	.	.	.	.	1
AL	.	1	8	.	.	.	.	9
AR	.	1	8	.	.	.	.	9
AS	.	.	.	.	.	.	.	.
AZ	.	.	5	.	.	.	.	5
CA	.	3	48	.	.	.	3	54
CM	.	.	.	.	.	.	.	.
CO	.	1	11	.	.	.	1	13
CT	.	1	5	.	.	.	.	6
DC	.	.	.	.	.	.	.	.
DE	.	1	9	.	.	.	.	10
FL	.	4	33	.	.	.	.	37
GA	.	.	9	.	.	.	.	9
GU	.	.	.	.	.	.	.	.
HI	.	.	.	.	.	.	.	.
IA	.	3	12	.	.	2	3	20
ID	.	.	4	.	.	.	.	4
IL	.	.	18	.	.	.	.	18
IN	.	.	18	.	.	.	.	18
KS	.	1	3	.	.	.	.	4
KY	.	.	13	.	.	.	.	13
LA	.	.	7	.	.	.	.	7
MA	.	1	17	.	.	.	.	18
MD	.	.	7	.	.	.	.	7
ME	.	.	6	.	.	.	.	6
MH	.	.	.	.	.	.	.	.
MI	.	4	43	.	.	.	.	47
MN	.	1	16	.	.	.	.	17
MO	.	.	13	.	.	.	1	14
MQ	.	.	.	.	.	.	.	.
MS	.	1	1	.	.	.	.	2
MT	.	.	7	.	.	.	1	8
NC	.	.	19	.	.	.	.	19
ND	.	.	2	.	.	.	.	2
NE	.	.	4	.	.	.	.	4
NH	.	.	13	.	.	.	.	13

EXHIBIT 1-E

(Continued)

State ID	NPL Site	Deleted NPL	Final NPL	Federal Not NPL	Proposed NPL	Not Federal Not NPL	Removed NPL	Total
NJ	.	5	74	.	.	.	.	79
NM	.	.	7	.	.	.	.	7
NN	.	.	.	.	.	.	.	.
NV	.	.	.	.	.	.	.	.
NY	.	5	52	.	.	1	.	58
OH	.	.	24	.	.	.	.	24
OK	.	.	9	.	.	.	.	9
OR	.	1	5	.	.	.	.	6
PA	.	6	62	.	.	.	.	68
PR	.	.	6	.	.	1	.	7
RI	.	.	8	.	.	.	.	8
SC	.	1	15	.	.	.	.	16
SD	.	.	1	.	.	.	.	1
TN	.	1	11	.	.	.	.	12
TT	.	.	.	.	.	.	.	.
TX	.	3	20	.	.	.	.	23
UT	.	.	8	.	.	.	.	8
VA	.	1	15	.	.	.	.	16
VI	.	.	.	.	.	.	.	.
VT	.	.	2	.	.	.	.	2
WA	.	4	20	.	.	1	.	25
WI	.	.	25	.	.	.	.	25
WQ	.	.	.	.	.	.	.	.
WV	.	1	3	.	.	.	.	4
WY	.	.	.	.	.	.	.	.
Total	.	51	727	.	.	5	9	792

EXHIBIT 1-F

SITE CHARACTERISTICS AND COST ANALYSES BY STATE
TOTAL
SITES WITH ROD COSTS
NPL STATUS

State ID	NPL Site	Deleted NPL	Final NPL	Federal Not NPL	Proposed NPL	Not Federal Not NPL	Removed NPL	NFRAP Federal	NFRAP Not Federal	Total
AK	.	.	1	.	.	.	.	.	.	1
AL	.	1	8	.	.	.	.	.	.	9
AR	.	1	8	.	.	.	.	.	.	9
AS	.	.	.	.	.	.	.	.	.	.
AZ	.	.	5	.	.	.	.	.	.	5
CA	.	3	48	.	.	.	4	.	.	55
CM	.	.	.	.	.	.	.	.	.	.
CO	.	1	11	.	.	.	1	.	.	13
CT	.	1	5	.	.	.	.	.	.	6
DC	.	.	.	.	.	.	.	.	.	.
DE	.	1	9	.	.	.	.	.	.	10
FL	.	4	33	.	.	.	.	.	.	37
GA	.	.	9	.	.	.	.	.	.	9
GU	.	.	.	.	.	.	.	.	.	.
HI	.	.	.	.	.	.	.	.	.	.
IA	.	3	12	.	.	2	3	.	.	20
ID	.	.	4	.	.	.	.	.	.	4
IL	.	.	18	.	.	.	.	.	.	18
IN	.	2	18	.	.	.	.	.	.	20
KS	.	1	3	.	.	.	.	.	.	4
KY	.	.	13	.	.	.	.	.	.	13
LA	.	.	7	.	.	.	.	.	.	7
MA	.	1	17	.	.	.	.	.	.	18
MD	.	.	7	.	.	.	.	.	.	7
ME	.	.	6	.	.	.	.	.	.	6
MH	.	.	.	.	.	.	.	.	.	.
MI	.	5	44	.	.	.	.	.	.	49
MN	.	7	16	.	.	.	.	.	.	23
MO	.	.	13	.	.	.	1	.	.	14
MQ	.	.	.	.	.	.	.	.	.	.

EXHIBIT 1-F

(Continued)

State ID	NPL Site	Deleted NPL	Final NPL	Federal Not NPL	Proposed NPL	Not Federal Not NPL	Removed NPL	NFRAP Federal	NFRAP Not Federal	Total
MS	.	1	1	.	.	.	.	.	.	2
MT	.	.	7	.	.	.	1	.	.	8
NC	.	.	19	.	.	.	.	.	.	19
ND	.	.	2	.	.	.	.	.	.	2
NE	.	.	4	.	.	.	.	.	.	4
NH	.	.	13	.	.	.	.	.	.	13
NJ	.	5	74	.	.	.	.	.	.	79
NM	.	.	7	.	.	.	.	.	.	7
NN	.	.	.	.	.	.	.	.	.	.
NV	.	.	.	.	.	.	.	.	.	.
NY	.	5	52	.	.	1	.	.	.	58
OH	.	.	24	.	.	.	.	.	.	24
OK	.	.	9	.	.	.	.	.	.	9
OR	.	1	5	.	.	.	.	.	.	6
PA	.	6	62	.	.	.	.	.	.	68
PR	.	.	6	.	.	1	.	.	.	7
RI	.	.	8	.	.	.	.	.	.	8
SC	.	1	15	.	.	.	.	.	.	16
SD	.	.	1	.	.	.	.	.	.	1
TN	.	1	11	.	.	.	.	.	.	12
TT	.	.	.	.	.	.	.	.	.	.
TX	.	3	20	.	.	.	.	.	.	23
UT	.	.	8	.	.	.	.	.	.	8
VA	.	1	15	.	.	.	.	.	.	16
VI	.	.	.	.	.	.	.	.	.	.
VT	.	.	2	.	.	.	.	.	.	2
WA	.	4	20	.	.	1	.	.	.	25
WI	.	.	25	.	.	.	.	.	.	25
WQ	.	.	.	.	.	.	.	.	.	.
WV	.	1	3	.	.	.	.	.	.	4
WY	.	.	.	.	.	.	.	.	.	.
Total	.	60	728	.	.	5	10	.	.	803

EXHIBIT 1-G

Site Characteristics and Cost Analyses by State
Sites with PRPs Identified
NPL Status

State ID	NPL Site	Deleted NPL	Final NPL	Federal Not NPL	Proposed NPL	Not Federal Not NPL	Removed NPL	NFRAP Federal	NFRAP Not Federal	Total
AK	.	1	2	.	.	1	.	.	.	4
AL	.	1	8	.	.	2	.	.	3	14
AR	.	1	9	.	.	1	1	.	1	13
AS	.	1	.	.	.	.	.	.	.	1
AZ	.	1	7	.	.	2	.	2	2	14
CA	1	2	45	.	2	8	2	1	30	91
CM	.	1	.	.	.	.	.	.	.	1
CO	.	1	12	.	1	14	1	.	10	39
CT	.	1	11	.	.	1	.	.	2	15
DC	.	.	.	.	.	.	.	.	.	.
DE	.	.	17	.	.	.	1	.	.	18
FL	1	6	43	.	.	2	.	.	1	53
GA	.	1	11	.	.	8	.	.	7	27
GU	.	.	.	.	.	.	.	.	.	.
HI	.	.	.	.	.	1	.	.	.	1
IA	.	3	9	.	.	12	3	.	.	27
ID	.	1	6	.	2	.	.	.	.	9
IL	.	1	18	.	1	23	.	.	2	45
IN	.	2	19	.	.	12	.	.	4	37
KS	.	4	7	.	.	5	.	.	1	17
KY	.	.	12	.	.	1	.	.	.	13
LA	.	.	9	.	.	1	.	.	.	10
MA	.	1	20	.	.	3	.	.	1	25
MD	.	1	9	.	.	.	.	.	.	10
ME	.	.	8	.	.	4	.	.	1	13
MH	.	.	.	.	.	.	.	.	.	.
MI	.	5	39	.	.	29	1	.	16	90
MN	.	4	12	.	.	3	.	.	.	19
MO	.	.	12	.	.	17	1	.	2	32
MQ	.	.	.	.	.	.	.	.	.	.

EXHIBIT 1-G

(Continued)

State ID	NPL Site	Deleted NPL	Final NPL	Federal Not NPL	Proposed NPL	Not Federal Not NPL	Removed NPL	NFRAP Federal	NFRAP Not Federal	Total
MS	.	2	.	.	1	3	2	.	3	11
MT	.	.	8	.	.	.	1	.	3	12
NC	.	1	16	.	.	8	.	.	8	33
ND	.	.	1	.	.	.	.	.	.	1
NE	.	.	6	.	.	1	.	.	.	7
NH	.	.	16	.	.	5	.	.	.	21
NJ	.	2	37	.	.	1	.	.	.	40
NM	.	.	8	.	1	.	.	.	.	9
NN	.	.	.	.	.	1	.	.	.	1
NV	.	.	1	.	.	1	.	.	.	2
NY	.	4	28	.	.	3	.	.	.	35
OH	.	1	22	.	.	35	.	.	6	64
OK	.	.	6	.	1	.	.	.	2	9
OR	.	2	6	.	.	.	.	.	1	9
PA	.	3	84	.	1	1	1	.	2	92
PR	.	.	1	.	.	.	.	.	1	2
RI	.	.	8	.	.	1	.	.	.	9
SC	.	1	17	1	.	2	.	.	2	23
SD	.	.	2	.	1	1	.	.	2	6
TN	.	1	6	.	.	2	.	.	1	10
TT	.	1	.	.	.	1	.	.	.	2
TX	.	4	20	.	1	1	1	.	2	29
UT	.	.	8	.	.	9	.	.	4	21
VA	.	2	17	.	.	.	1	.	1	21
VI	.	.	.	.	.	.	.	.	.	.
VT	.	.	8	.	.	.	.	.	1	9
WA	.	1	19	.	.	3	.	.	1	24
WI	.	.	20	.	.	8	1	.	8	37
WQ	.	.	.	.	.	.	.	.	.	.
WV	.	1	3	.	.	1	1	.	3	9
WY	.	.	2	.	.	3	.	.	.	5
Total	2	65	715	1	12	241	18	3	134	1,191

EXHIBIT 1-H

SITE CHARACTERISTICS AND COST ANALYSES BY STATE
SITES WITH PRPS AND ROD COSTS
NPL STATUS

State ID	NPL Site	Deleted NPL	Final NPL	Federal Not NPL	Proposed NPL	Not Federal Not NPL	Removed NPL	NFRAP Federal	NFRAP Not Federal	Total
AK	.	.	.	.	.	.	.	.	.	.
AL	.	1	7	.	.	.	.	.	.	8
AR	.	1	6	.	.	.	.	.	.	7
AS	.	.	.	.	.	.	.	.	.	.
AZ	.	.	5	.	.	.	.	.	.	5
CA	.	2	26	.	.	.	1	.	.	29
CM	.	.	.	.	.	.	.	.	.	.
CO	.	1	9	.	.	.	1	.	.	11
CT	.	1	5	.	.	.	.	.	.	6
DC	.	.	.	.	.	.	.	.	.	.
DE	.	.	9	.	.	.	.	.	.	9
FL	.	3	28	.	.	.	.	.	.	31
GA	.	.	8	.	.	.	.	.	.	8
GU	.	.	.	.	.	.	.	.	.	.
HI	.	.	.	.	.	.	.	.	.	.
IA	.	3	9	.	.	.	2	.	.	14
ID	.	.	3	.	.	.	.	.	.	3
IL	.	.	12	.	.	.	.	.	.	12
IN	.	1	14	.	.	.	.	.	.	15
KS	.	1	3	.	.	.	.	.	.	4
KY	.	.	10	.	.	.	.	.	.	10
LA	.	.	6	.	.	.	.	.	.	6
MA	.	1	16	.	.	.	.	.	.	17
MD	.	.	6	.	.	.	.	.	.	6
ME	.	.	5	.	.	.	.	.	.	5
MH	.	.	.	.	.	.	.	.	.	.
MI	.	5	30	.	.	.	.	.	.	35
MN	.	3	9	.	.	.	.	.	.	12
MO	.	.	6	.	.	.	1	.	.	7
MQ	.	.	.	.	.	.	.	.	.	.

EXHIBIT 1-H

(Continued)

State ID	NPL Site	Deleted NPL	Final NPL	Federal Not NPL	Proposed NPL	Not Federal Not NPL	Removed NPL	NFRAP Federal	NFRAP Not Federal	Total
MS	.	1	.	.	.	.	.	.	.	1
MT	.	.	7	.	.	.	1	.	.	8
NC	.	.	14	.	.	.	.	.	.	14
ND	.	.	1	.	.	.	.	.	.	1
NE	.	.	4	.	.	.	.	.	.	4
NH	.	.	12	.	.	.	.	.	.	12
NJ	.	2	31	.	.	.	.	.	.	33
NM	.	.	7	.	.	.	.	.	.	7
NN	.	.	.	.	.	.	.	.	.	.
NV	.	.	.	.	.	.	.	.	.	.
NY	.	1	23	.	.	.	.	.	.	24
OH	.	.	19	.	.	.	.	.	.	19
OK	.	.	5	.	.	.	.	.	.	5
OR	.	1	4	.	.	.	.	.	.	5
PA	.	2	56	.	.	.	.	.	.	58
PR	.	.	.	.	.	.	.	.	.	.
RI	.	.	6	.	.	.	.	.	.	6
SC	.	1	11	.	.	.	.	.	.	12
SD	.	.	1	.	.	.	.	.	.	1
TN	.	1	6	.	.	.	.	.	.	7
TT	.	.	.	.	.	.	.	.	.	.
TX	.	3	19	.	.	.	.	.	.	22
UT	.	.	5	.	.	.	.	.	.	5
VA	.	1	13	.	.	.	.	.	.	14
VI	.	.	.	.	.	.	.	.	.	.
VT	.	.	2	.	.	.	.	.	.	2
WA	.	1	14	.	.	1	.	.	.	16
WI	.	.	18	.	.	.	.	.	.	18
WQ	.	.	.	.	.	.	.	.	.	.
WV	.	1	3	.	.	.	.	.	.	4
WY	.	.	.	.	.	.	.	.	.	.
Total	.	38	513	.	.	1	6	.	.	558

EXHIBIT 1-I

SITE CHARACTERISTICS AND COST ANALYSES BY STATE
PRESENT WORTH
AVERAGE
NPL STATUS

State ID	NPL Site	Deleted NPL	Final NPL	Federal Not NPL	Proposed NPL	Not Federal Not NPL	Removed NPL	NFRAP Federal	NFRAP Not Federal	Total
AK	.	.	467,300	.	.	.	.	.	.	467,300
AL	.	1,283,333	28,996,964	.	.	.	.	.	.	25,917,672
AR	.	61,000	12,006,993	.	.	.	.	.	.	10,679,660
AS	.	.	.	.	.	.	.	.	.	.
AZ	.	.	17,660,300	.	.	.	.	.	.	17,660,300
CA	.	817,667	22,835,876	.	.	.	8,088,920	.	.	20,562,377
CM	.	.	.	.	.	.	.	.	.	.
CO	.	7,681,600	14,549,994	.	.	.	58,240,000	.	.	17,382,426
CT	.	263,000	11,345,400	.	.	.	.	.	.	9,498,334
DC	.	.	.	.	.	.	.	.	.	.
DE	.	466,147	15,111,456	.	.	.	.	.	.	13,646,925
FL	.	2,122,756	7,353,448	.	.	.	.	.	.	6,787,968
GA	.	.	6,305,637	.	.	.	.	.	.	6,305,637
GU	.	.	.	.	.	.	.	.	.	
HI	.	.	.	.	.	.	.	.	.	.
IA	.	2,578,633	2,459,398	.	.	13,179,500	3,424,600	.	.	3,694,073
ID	.	.	37,078,259	.	.	.	.	.	.	37,078,259
IL	.	.	13,391,110	.	.	.	.	.	.	13,391,110
IN	.	102,000	17,125,433	.	.	.	.	.	.	15,423,090
KS	.	1,638,456	7,313,338	.	.	.	.	.	.	5,894,618
KY	.	.	10,859,015	.	.	.	.	.	.	10,859,015
LA	.	.	26,362,248	.	.	.	.	.	.	26,362,248
MA	.	391,500	19,996,597	.	.	.	.	.	.	18,907,425
MD	.	.	8,481,334	.	.	.	.	.	.	8,481,334
ME	.	.	8,415,750	.	.	.	.	.	.	8,415,750
MH	.	.	.	.	.	.	.	.	.	.
MI	.	5,441,862	15,853,174	.	.	.	.	.	.	14,790,796
MN	.	6,550,213	5,341,116	.	.	.	.	.	.	5,709,102
MO	.	.	15,474,164	.	.	.	8,306,000	.	.	14,962,152
MQ	.	.	.	.	.	.	.	.	.	.

EXHIBIT 1-I

(Continued)

State ID	NPL Site	Deleted NPL	Final NPL	Federal Not NPL	Proposed NPL	Not Federal Not NPL	Removed NPL	NFRAP Federal	NFRAP Not Federal	Total
MS	.	2,000,000	14,180,000	.	.	.	.	.	.	8,090,000
MT	.	.	23,918,597	.	.	.	11,515,500	.	.	22,368,210
NC	.	.	10,754,436	.	.	.	.	.	.	10,754,436
ND	.	.	2,123,625	.	.	.	.	.	.	2,123,625
NE	.	.	17,861,517	.	.	.	.	.	.	17,861,517
NH	.	.	11,915,482	.	.	.	.	.	.	11,915,482
NJ	.	584,272	28,159,892	.	.	.	.	.	.	26,414,600
NM	.	.	6,950,802	.	.	.	.	.	.	6,950,802
NN	.	.	.	.	.	.	.	.	.	.
NV	.	.	.	.	.	.	.	.	.	.
NY	.	15,183,580	16,959,324	.	.	35,100,000	.	.	.	17,119,013
OH	.	.	20,425,537	.	.	.	.	.	.	20,425,537
OK	.	.	30,183,849	.	.	.	.	.	.	30,183,849
OR	.	6,707,400	3,833,041	.	.	.	.	.	.	4,312,101
PA	.	3,605,500	15,361,822	.	.	.	.	.	.	14,324,500
PR	.	.	3,661,910	.	.	8,987,800	.	.	.	4,422,752
RI	.	.	9,872,522	.	.	.	.	.	.	9,872,522
SC	.	1,032,000	7,710,604	.	.	.	.	.	.	7,293,192
SD	.	.	882,813	.	.	.	.	.	.	882,813
TN	.	990,627	13,907,438	.	.	.	.	.	.	12,831,037
TT	.	.	.	.	.	.	.	.	.	.
TX	.	1,836,333	25,480,049	.	.	.	.	.	.	22,396,086
UT	.	.	17,143,887	.	.	.	.	.	.	17,143,887
VA	.	292,000	14,670,931	.	.	.	.	.	.	13,772,248
VI	.	.	.	.	.	.	.	.	.	.
VT	.	.	7,125,650	.	.	.	.	.	.	7,125,650
WA	.	3,232,125	19,368,497	.	.	6,900,000	.	.	.	16,287,938
WI	.	.	9,020,730	.	.	.	.	.	.	9,020,730
WQ	.	.	.	.	.	.	.	.	.	.
WV	.	1,014,000	13,037,000	.	.	.	.	.	.	10,031,250
WY	.	.	.	.	.	.	.	.	.	.
Total	.	3,911,259	16,403,531	.	.	15,469,360	12,069,098	.	.	15,410,316

EXHIBIT 1-J

Site Characteristics and Cost Analyses by State
Nominal Cost
Average
NPL Status

State ID	NPL Site	Deleted NPL	Final NPL	Federal Not NPL	Proposed NPL	Not Federal Not NPL	Removed NPL	NFRAP Federal	NFRAP Not Federal	Total
AK	.	.	767,763	.	.	.	.	.	.	767,763
AL	.	2,374,500	46,415,568	.	.	.	.	.	.	41,522,116
AR	.	112,866	34,594,560	.	.	.	.	.	.	30,763,261
AS	.	.	.	.	.	.	.	.	.	.
AZ	.	.	30,563,571	.	.	.	.	.	.	30,563,571
CA	.	1,408,865	39,132,318	.	.	.	12,349,402	.	.	35,126,827
CM	.	.	.	.	.	.	.	.	.	.
CO	.	13,496,520	24,371,771	.	.	.	99,552,758	.	.	29,318,366
CT	.	445,131	20,108,342	.	.	.	.	.	.	16,831,140
DC	.	.	.	.	.	.	.	.	.	.
DE	.	812,746	26,144,967	.	.	.	.	.	.	23,611,745
FL	.	3,996,810	14,062,275	.	.	.	.	.	.	12,974,116
GA	.	.	14,930,349	.	.	.	.	.	.	14,930,349
GU	.	.	.	.	.	.	.	.	.	.
HI	.	.	.	.	.	.	.	.	.	.
IA	.	4,676,223	5,311,589	.	.	18,720,801	5,974,392	.	.	6,656,626
ID	.	.	67,505,812	.	.	.	.	.	.	67,505,812
IL	.	.	22,973,090	.	.	.	.	.	.	22,973,090
IN	.	180,613	31,098,616	.	.	.	.	.	.	28,006,816
KS	.	2,773,107	12,351,631	.	.	.	.	.	.	9,957,000
KY	.	.	19,031,256	.	.	.	.	.	.	19,031,256
LA	.	.	47,050,720	.	.	.	.	.	.	47,050,720
MA	.	738,864	32,947,010	.	.	.	.	.	.	31,157,669
MD	.	.	14,648,928	.	.	.	.	.	.	14,648,928
ME	.	.	14,626,718	.	.	.	.	.	.	14,626,718
MH	.	.	.	.	.	.	.	.	.	.
MI	.	7,992,494	28,797,108	.	.	.	.	.	.	26,674,188
MN	.	17,042,931	9,508,811	.	.	.	.	.	.	11,801,804
MO	.	.	26,291,361	.	.	.	14,771,486	.	.	25,468,513
MQ	.	.	.	.	.	.	.	.	.	.

EXHIBIT 1-J

(Continued)

State ID	NPL Site	Deleted NPL	Final NPL	Federal Not NPL	Proposed NPL	Not Federal Not NPL	Removed NPL	NFRAP Federal	NFRAP Not Federal	Total
MS	.	4,272,786	24,723,408	.	.	.	.	.	.	14,498,097
MT	.	.	34,822,980	.	.	.	33,480,232	.	.	34,655,136
NC	.	.	18,955,149	.	.	.	.	.	.	18,955,149
ND	.	.	3,685,611	.	.	.	.	.	.	3,685,611
NE	.	.	31,484,483	.	.	.	.	.	.	31,484,483
NH	.	.	20,470,613	.	.	.	.	.	.	20,470,613
NJ	.	962,901	48,938,205	.	.	.	.	.	.	45,901,793
NM	.	.	13,140,628	.	.	.	.	.	.	13,140,628
NN	.	.	.	.	.	.	.	.	.	.
NV	.	.	.	.	.	.	.	.	.	.
NY	.	28,256,708	29,422,226	.	.	59,407,183	.	.	.	29,838,732
OH	.	.	42,433,077	.	.	.	.	.	.	42,433,077
OK	.	.	51,813,119	.	.	.	.	.	.	51,813,119
OR	.	14,571,916	6,841,917	.	.	.	.	.	.	8,130,250
PA	.	6,882,585	27,419,561	.	.	.	.	.	.	25,607,475
PR	.	.	6,306,889	.	.	14,477,208	.	.	.	7,474,077
RI	.	.	17,528,495	.	.	.	.	.	.	17,528,495
SC	.	1,872,027	14,962,700	.	.	.	.	.	.	14,144,533
SD	.	.	1,682,826	.	.	.	.	.	.	1,682,826
TN	.	1,693,331	23,315,219	.	.	.	.	.	.	21,513,395
TT	.	.	.	.	.	.	.	.	.	.
TX	.	3,378,845	48,302,380	.	.	.	.	.	.	42,442,788
UT	.	.	28,397,784	.	.	.	.	.	.	28,397,784
VA	.	573,345	24,437,711	.	.	.	.	.	.	22,946,188
VI	.	.	.	.	.	.	.	.	.	.
VT	.	.	12,364,261	.	.	.	.	.	.	12,364,261
WA	.	5,573,490	30,875,449	.	.	11,794,540	.	.	.	26,063,899
WI	.	.	15,726,999	.	.	.	.	.	.	15,726,999
WQ	.	.	.	.	.	.	.	.	.	.
WV	.	1,876,163	20,343,405	.	.	.	.	.	.	15,726,594
WY	.	.	.	.	.	.	.	.	.	.
Total	.	7,655,068	29,013,618	.	.	24,624,107	21,512,526	.	.	27,296,966

EXHIBIT 1-K

SITE CHARACTERISTICS AND COST ANALYSES BY STATE
NON-FEDERAL NPL SITES ONLY
ESTIMATED NOMINAL COST
AVERAGE
NPL STATUS

State ID	Deleted NPL	Final NPL	Total
AL	2,374,500	57,706,560	49,801,980
AR	112,866	34,594,560	30,763,261
AZ	.	33,974,328	33,974,328
CA	1,408,865	38,489,931	36,017,860
CO	13,496,520	25,706,217	24,485,247
CT	445,131	20,108,342	16,831,140
DE	812,746	29,347,048	26,176,570
FL	3,996,810	14,062,275	12,974,116
GA	.	14,475,330	14,475,330
IA	4,676,223	5,311,589	5,184,516
ID	.	68,814,810	68,814,810
IL	.	22,546,175	22,546,175
IN	180,613	31,098,616	28,006,816
KS	2,773,107	12,351,631	9,957,000
KY	.	18,475,011	18,475,011
LA	.	47,050,720	47,050,720
MA	738,864	31,503,896	29,694,188
MD	.	14,335,113	14,335,113
ME	.	13,215,305	13,215,305
MI	7,992,494	28,797,108	26,674,188
MN	19,681,607	8,985,890	12,194,605
MO	.	16,710,321	16,710,321
MS	4,272,786	24,723,408	14,498,097
MT	.	34,822,980	34,822,980
NC	.	18,703,327	18,703,327
ND	.	3,685,611	3,685,611
NE	.	31,484,483	31,484,483
NH	.	19,524,436	19,524,436
NJ	962,901	50,405,342	47,152,550
NM	.	14,949,158	14,949,158
NY	28,256,708	29,828,770	29,688,407
OH	.	42,039,226	42,039,226
OK	.	55,356,542	55,356,542
OR	14,571,916	7,180,084	8,658,451

EXHIBIT 1-K

(Continued)

State ID	Deleted NPL	Final NPL	Total
PA	6,882,585	27,918,993	26,006,593
PR	.	6,306,889	6,306,889
RI	.	20,153,130	20,153,130
SC	1,872,027	14,451,264	13,612,648
SD	.	1,682,826	1,682,826
TN	1,693,331	9,215,398	8,463,191
TX	3,378,845	48,302,380	42,442,788
UT	.	18,928,753	18,928,753
VA	573,345	26,094,620	24,393,201
VT	.	12,364,261	12,364,261
WA	575,650	33,481,827	31,288,082
WI	.	15,726,999	15,726,999
WV	1,876,163	25,883,999	17,881,387
Total	7,792,408	29,091,665	27,457,750

EXHIBIT 1-L

Site Characteristics and Cost Analyses by State
Non-Federal NPL Sites Only
Estimated Nominal Cost
Standard Deviation
NPL Status

State ID	Deleted NPL	Final NPL	Total
AL	.	100,877,180	94,432,769
AR	.	51,201,687	49,254,653
AZ	.	21,799,843	21,799,843
CA	1,234,692	51,234,240	50,334,315
CO	.	18,837,644	18,175,147
CT	.	17,153,003	17,315,332
DE	.	39,533,419	38,183,729
FL	4,554,291	18,633,258	17,899,421
GA	.	13,813,427	13,813,427
IA	3,148,575	6,315,871	5,729,550
ID	.	93,384,505	93,384,505
IL	.	25,947,713	25,947,713
IN	144,468	23,172,572	23,895,760
KS	.	9,454,497	9,084,531
KY	.	27,828,592	27,828,592
LA	.	35,783,012	35,783,012
MA	.	29,116,665	29,162,812
MD	.	16,428,029	16,428,029
ME	.	4,197,908	4,197,908
MI	8,891,659	35,821,430	34,591,694
MN	16,656,700	13,731,904	15,077,081
MO	.	17,940,591	17,940,591
MS	.	.	14,460,773
MT	.	35,140,676	35,140,676
NC	.	21,042,609	21,042,609
ND	.	577,386	577,386
NE	.	46,406,366	46,406,366
NH	.	15,541,107	15,541,107
NJ	1,702,904	109,423,261	106,431,308
NM	.	14,756,017	14,756,017
NY	52,413,553	38,296,275	39,157,075
OH	.	40,106,569	40,106,569
OK	.	79,989,582	79,989,582
OR	.	7,974,755	7,656,721

EXHIBIT 1-L

(Continued)

State ID	Deleted NPL	Final NPL	Total
PA	5,105,863	37,916,318	36,661,725
PR	.	3,617,667	3,617,667
RI	.	17,108,656	17,108,656
SC	.	24,593,799	23,920,706
SD	.	.	.
TN	.	7,584,365	7,535,871
TX	1,650,193	57,319,885	55,471,504
UT	.	13,778,021	13,778,021
VA	.	32,381,314	31,891,620
VT	.	17,044,091	17,044,091
WA	.	38,743,582	38,288,823
WI	.	13,016,710	13,016,710
WV	.	6,327,381	14,565,139
Total	17,610,317	49,742,140	48,371,328

Abbreviation: Delt = Deleted

EXHIBIT 1-M

Site Characteristics and Cost Analyses by State
Estimate of Final Disposition of CERCLIS Sites

State ID	Estimated Future NPL Sites	NPL Eligible Sites	Eligible Sites Not Becoming NPL	Current NPL Sites	Proposed NPL Sites	CERCLIS NFRAP State Sites
AK	4.22	49	44.78	8	0	130
AL	6.86	110	103.14	13	1	563
AR	6.86	42	35.14	13	0	343
AS	0.53	1	0.47	1	0	4
AZ	5.80	162	156.20	11	0	699
CA	49.07	424	374.93	93	4	2,434
CM	0.53	2	1.47	1	0	6
CO	8.97	132	123.03	17	2	408
CT	8.44	409	400.56	16	0	327
DC	0.00	9	9.00	.	0	10
DE	10.55	30	19.45	20	0	221
FL	32.71	405	372.29	62	2	552
GA	8.44	243	234.56	16	1	723
GU	0.00	0	0.00	2	0	3
HI	2.11	37	34.89	4	0	104
IA	10.55	189	178.45	20	1	402
ID	4.75	29	24.25	9	2	134
IL	20.58	339	318.42	39	3	1,284
IN	18.47	157	138.53	35	1	1,433
KS	7.91	202	194.09	15	1	339
KY	10.55	90	79.45	20	0	487
LA	7.39	70	62.61	14	3	525
MA	16.36	459	442.64	31	0	516
MD	7.91	72	64.09	15	3	309
ME	6.33	88	81.67	12	0	103
MH	0.00	1	1.00	.	0	.
MI	43.26	148	104.74	82	2	1,495
MN	22.69	53	30.31	43	1	407
MO	12.14	347	334.86	23	0	906
MQ	0.00	0	0.00	.	0	.
MS	1.58	58	56.42	3	2	374
MT	4.22	31	26.78	8	1	177
NC	12.66	176	163.34	24	0	687
ND	1.06	7	5.94	2	0	58
NE	5.28	122	116.72	10	0	235
NH	9.50	107	97.50	18	0	62

EXHIBIT 1-M

(Continued)

State ID	Estimated Future NPL Sites	NPL Eligible Sites	Eligible Sites Not Becoming NPL	Current NPL Sites	Proposed NPL Sites	CERCLIS NFRAP State Sites
NJ	60.68	669	608.32	115	0	926
NM	5.80	61	55.20	11	1	266
NN	0.00	68	68.00	.	0	83
NV	0.53	17	16.47	1	0	157
NY	46.43	521	474.57	88	1	1,059
OH	18.47	212	193.53	35	4	1,078
OK	5.28	106	100.72	10	1	664
OR	6.33	45	38.67	12	1	337
PA	58.04	394	335.96	110	2	2,486
PR	4.75	72	67.25	9	1	179
RI	6.33	150	143.67	12	0	119
SC	14.25	140	125.75	27	0	378
SD	1.58	23	21.42	3	1	77
TN	9.50	260	250.50	18	1	580
TT	0.53	4	3.47	1	0	32
TX	16.36	175	158.64	31	1	2,367
UT	6.33	122	115.67	12	4	163
VA	14.25	166	151.75	27	0	440
VI	1.06	8	6.94	2	0	25
VT	4.22	64	59.78	8	0	69
WA	30.60	43	12.40	58	2	550
WI	22.16	92	69.84	42	0	336
WQ	0.00	0	0.00	.	0	1
WV	3.69	57	53.31	7	1	497
WY	1.58	23	21.42	3	0	121
	707.0	8,292		1,342	51	29,450

EXHIBIT 1-N

Site Characteristics and Cost Analyses by State
Decomposition of Cleanup Costs on Nominal (Undiscounted) Basis

State ID	Total	Known NPL Costs Info	Proposed, Future, NPL Without RODs Costs	Future State Sites From CERCLIS	Known and Suspected State Sites
AK	319,489,408	0	302,208,213	16,260,421	1,020,774
AL	800,243,696	448,217,823	346,932,805	5,093,068	0
AR	793,396,579	276,869,346	317,127,538	134,274,180	65,125,515
AS	42,730,706	0	41,142,605	1,588,101	0
AZ	1,567,135,332	169,871,642	380,810,759	471,423,694	545,029,237
CA	5,852,798,216	1,836,910,841	2,819,368,495	1,196,518,880	0
CM	43,795,977	0	41,142,605	2,653,372	0
CO	2,532,673,723	293,822,963	460,969,706	1,777,881,054	0
CT	1,221,951,570	100,986,839	463,353,987	258,348,791	399,261,954
DC	11,655,672	0	0	6,746,714	4,908,958
DE	873,297,123	261,765,698	593,984,617	17,546,808	0
FL	2,350,861,512	480,042,297	1,399,494,011	471,325,204	0
GA	991,724,244	130,277,969	373,503,457	340,019,537	147,923,281
GU	54,930,528	0	53,865,257	1,065,271	0
HI	246,615,130	0	164,570,421	49,318,319	32,726,390
IA	584,505,779	77,767,735	153,357,978	206,111,312	147,268,753
ID	691,232,055	275,259,240	350,124,976	56,193,554	9,654,285
IL	2,725,877,804	405,831,149	1,256,700,805	413,278,839	650,067,010
IN	1,842,145,754	560,136,321	1,030,338,116	251,433,534	237,783
KS	2,833,456,839	39,828,001	331,837,839	1,152,182,330	1,309,608,668
KY	1,082,270,483	240,175,141	477,323,344	201,140,050	163,631,948
LA	918,648,361	329,355,043	564,049,050	25,244,267	0
MA	1,490,890,190	534,495,385	878,774,882	77,619,923	0
MD	652,692,269	100,345,791	446,525,668	56,789,192	49,031,617
ME	318,235,836	79,291,833	207,749,057	20,801,183	10,393,764
MH	355,090	0	0	355,090	0
MI	4,118,381,348	1,307,035,217	2,274,047,991	537,298,141	0
MN	2,906,326,828	280,475,913	956,689,197	237,828,746	1,431,332,972
MO	4,735,016,944	217,234,172	558,038,707	1,820,468,365	2,139,275,700
MQ	0	0	0	0	0
MS	413,171,342	28,996,194	105,341,631	152,836,918	125,996,600

EXHIBIT 1-N

(Continued)

State ID	Total	Known NPL Costs Info	Proposed, Future, NPL Without RODs Costs	Future State Sites From CERCLIS	Known and Suspected State Sites
MT	505,140,507	243,760,857	189,432,845	53,833,697	18,113,108
NC	1,295,409,231	355,363,204	469,722,304	301,946,448	168,377,275
ND	46,565,796	7,371,221	4,706,911	22,706,163	11,781,500
NE	647,312,745	125,937,933	331,028,329	124,893,704	65,452,779
NH	730,842,884	253,817,669	379,479,281	56,637,947	40,907,987
NJ	12,893,111,192	3,725,051,465	2,808,335,071	494,821,378	5,864,903,278
NM	1,417,017,757	104,644,104	252,408,433	575,205,279	484,759,941
NN	53,618,620	0	0	53,618,620	0
NV	124,994,895	0	41,142,605	61,598,345	22,253,945
NY	4,693,766,786	1,692,239,204	2,304,957,769	544,555,733	152,014,080
OH	2,694,289,058	1,008,941,427	1,039,116,496	451,509,116	194,722,018
OK	716,930,669	498,208,881	216,360,424	2,361,364	0
OR	632,186,956	51,950,703	191,737,787	133,396,259	255,102,207
PA	5,606,039,050	1,768,448,294	2,948,889,901	887,925,859	774,996
PR	244,242,446	37,841,332	77,069,843	87,441,492	41,889,779
RI	579,759,859	161,225,043	276,174,165	93,271,066	49,089,584
SC	962,217,773	217,802,372	638,460,199	105,955,201	0
SD	341,710,585	1,682,826	130,812,764	34,946,971	174,268,025
TN	498,660,443	101,558,298	232,611,772	81,093,105	83,397,269
TT	53,738,503	0	41,142,605	12,595,897	0
TX	5,068,348,941	976,184,131	759,667,084	2,884,224,472	448,273,254
UT	1,740,513,108	151,430,026	365,378,710	686,632,206	537,072,165
VA	1,887,896,556	390,291,224	723,705,515	773,899,818	0
VI	93,628,487	0	82,285,211	11,343,277	0
VT	581,932,996	24,728,522	233,301,964	45,728,198	278,174,312
WA	2,775,542,143	750,913,962	1,975,469,916	49,158,266	0
WI	1,304,403,133	393,174,966	909,049,392	2,178,775	0
WQ	355,090	0	0	355,090	0
WV	541,352,396	71,525,548	192,602,352	195,408,523	81,815,974
WY	196,907,228	0	123,427,816	50,570,940	22,908,473
	91,944,942,173	21,589,085,765	35,287,851,183	18,839,458,067	16,228,547,158

EXHIBIT 2

State-Administered Cleanup Statistics

State ID	Total Sites	State Sites With Costs	Average State Site Cost	Total Cost State Sites	PRP Sites With Costs	Average PRP Site Cost	Total Cost PRP Sites	Known and Suspected Sites	Identified as Needing Attention
AL	69	26	7,308	190,000	0	0	0	650	125
AK	41	38	95,161	3,616,104	3	9,899	29,697	1,347	1,347
AS	3	0	0	0	0	0	0	.	.
AZ	76	54	231,929	12,524,140	7	2,802,228	19,615,593	1,620	400
AR	.	.	.	0	0	0	0	398	45
CA	850	223	407,141	90,792,505	0	0	0	4,809	1,079
CO	11	1	3,200,000	3,200,000	0	0	0	225	225
CT	642	0	0	0	0	0	0	2,440	649
DE	185	18	69,750	1,255,500	0	0	0	280	120
DC	.	.	.	0	0	0	0	30	0
FL	36	25	487,394	12,184,849	0	0	0	1,023	656
GA	.	.	.	.	.	.	.	904	82
GU	.	.	.	.	.	.	.	.	.
HI	.	.	.	.	.	.	.	200	25
ID	.	.	.	.	.	.	.	59	0
IL	906	375	167,227	62,710,275	138	461,948	63,748,843	5,000	950
IN	79	10	168,113	1,681,128	1	1,000	1,000	2,500	200
IA	.	.	.	.	.	.	.	900	200
KS	183	2	83,500	167,000	14	2,349,000	32,886,000	609	324
KY	.	.	.	.	.	.	.	1,000	600
LA	101	24	41,062	985,483	0	0	0	690	136
ME	70	29	89,742	2,602,516	2	367,500	735,000	419	92
MD	378	8	77,469	619,750	6	236,178	1,417,066	463	198
MA	3,089	33	77,390	2,553,857	0	0	0	7,500	4,500
MI	2,662	496	321,021	159,226,580	0	0	0	.	2,764
MQ	.	.	.	.	.	.	.	.	.
MN	414	89	127,562	11,353,000	89	912,045	81,172,000	3,000	215
MS	110	0	0	0	0	0	0	770	156
MO	44	3	42,667	128,000	5	2,218,000	11,090,000	1,475	200
MT	214	3	253,333	760,000	1	250,000	250,000	277	240
NN	.	.	.	.	.	.	.	.	.
NE	15	0	0	0	0	0	0	400	200

EXHIBIT 2

(Continued)

State ID	Total Sites	State Sites With Costs	Average State Site Cost	Total Cost State Sites	PRP Sites With Costs	Average PRP Site Cost	Total Cost PRP Sites	Known and Suspected Sites	Identified as Needing Attention
NV	34	0	0	0	0	0	0	136	136
NH	.	.	.	.	.	.	.	250	250
NJ	5,996	32	974,329	31,178,532	1,083	288,565	312,516,209	20,000	6,500
NM	58	2	67,500	135,000	1	5,000,000	5,000,000	278	182
NY	556	.	.	.	.	.	.	929	793
NC	873	0	0	0	0	0	0	1,029	801
ND	.	.	.	.	.	.	.	72	0
CM	.	.	.	.	.	.	.	.	.
OH	86	0	0	0	0	0	0	1,190	406
OK	40	29	2,951	85,590	0	0	0	767	162
OR	307	0	0	0	0	0	0	1,559	218
PI	.	.	.	.	.	.	.	.	.
PA	41	26	304,615	7,920,000	1	200,000	200,000	100	50
PR	.	.	.	.	.	.	.	256	256
RI	97	0	0	0	0	0	0	300	40
SC	42	35	201,034	7,036,201	0	0	0	550	120
SD	674	0	0	0	0	0	0	1,065	241
TN	244	43	51,175	2,200,526	29	155,829	4,519,053	1,270	198
TX	200	13	701,077	9,114,000	3	2,783,333	8,350,000	821	66
TT	.	.	.	.	.	.	.	.	.
UT	31	4	32,597	130,389	2	7,000,000	14,000,000	220	0
VT	.	.	.	.	.	.	.	1,700	931
VI	.	.	.	.	.	.	.	.	.
VA	21	2	1,250,000	2,500,000	0	0	0	2,015	363
WQ	.	.	.	.	.	.	.	.	.
WA	1,220	131	83,545	10,944,364	0	0	0	1,364	932
WV	.	.	.	.	.	.	.	500	0
WI	1,849	183	5,131	939,021	0	0	0	4,000	565
WY	.	.	.	.	.	.	.	140	0
	22,547	1,957		438,734,309	1,385		555,530,461	79,499	28,938

EXHIBIT 3

ENVIRONMENTAL ANALYSIS
ESTIMATED SHARE OF CLEAN-UP COST AS A SHARE OF INSURABLE COSTS

Terminology

a. Total Costs include cleanup cost and total transaction costs.[13]
b. Total Transaction Costs include declarative judgment (DJ) costs and all other (non-DJ) costs.
c. DJ Costs represent money spent relating to coverage disputes which would not be recoverable under insurance.
d. Non-DJ Transaction Costs are the total transaction costs excluding DJ costs. They include the cost of negotiating with other PRPs and the government.
e. Insurable Costs are the sum of the cleanup cost and non-DJ transaction costs.

Calculation

			Low Estimate	Average Estimate	High Estimate
(1)	Total Transaction Costs as a Percentage of Total Costs[13]:		19.0%	23.0%	27.0%
(2)	Total Transaction Costs Between 1981 & 1991	\$ 27.7 M			
(3)	Total Declarative Judgment (DJ) Costs Between 1981 & 1991	\$ 1.7 M			
(4)	Total Non-DJ Transaction Costs Between 1981 & 1991: [= (2) − (3)]	\$ 26.0 M			
(5)	Total Non-DJ Costs as a Percentage of Total Transaction Costs Between 1981 & 1991: [= (4)/(2)]	93.9%			
(6)	Total Non-DJ Transaction Costs as a Percentage of Total Costs: [= (1) × (5)]		17.8%	21.6%	25.3%
(7)	Total Cleanup Cost Between 1981 & 1991	\$106.4 M			
(8)	Total Costs Between 1981 & 1991: [= (2) + (7)]	\$134.1 M			
(9)	Total Insurable Costs Between 1981 & 1991: [= (4) + (7)]	\$132.4 M			
(10)	Ratio of Total Costs to Insurable Costs: [= (8)/(9)]	1.013			
(11)	Non-DJ Transaction Costs as a Percentage of Insurable Costs: [= (6) × (10)]		18.1%	21.9%	25.7%
(12)	Selected Percentage: Non-DJ Transaction Costs as a Percentage of Insurable Costs:			20.0%	
(13)	Selected Percentage: Cleanup Cost as a Percentage of Insurable Costs: [= 1 − (12)]			80.0%	
(14)	Selected Percentage: ALAE as a Percentage of Cleanup Costs: [= (12)/(13)]			25.0%	

[13]Data in (1) thru (4) and (7) are taken from Rand's "Private Sector Cleanup Expenditures and Transaction Costs at 18 Superfund Sites" 1993 Study [9].

EXHIBIT 4-A

Countrywide Estimates of Cleanup Costs and Insurance Liabilities Contribution from CERCLIS Sites without PRP Information

Incurred Year	Expected Insured Indemnity	Expected Insured Total LAE
1950	78,360,396	25,838,315
1951	85,610,965	28,257,789
1952	92,024,753	30,388,337
1953	99,721,482	32,956,994
1954	105,716,880	34,887,819
1955	129,700,309	42,911,990
1956	146,961,720	48,589,293
1957	166,175,314	54,927,640
1958	171,891,171	56,725,636
1959	210,012,887	69,420,829
1960	264,283,372	87,621,264
1961	263,778,913	87,324,648
1962	276,911,959	91,483,972
1963	358,320,407	118,354,873
1964	369,920,913	122,378,740
1965	431,886,515	142,733,936
1966	456,064,014	150,810,116
1967	528,178,899	174,313,437
1968	581,024,191	191,998,746
1969	611,209,148	201,852,426
1970	705,690,931	232,725,476
1971	782,649,316	258,557,443
1972	984,303,036	325,372,395
1973	705,151,793	355,230,931
1974	778,481,015	391,476,355
1975	830,139,894	417,893,986
1976	849,086,161	427,854,594
1977	906,657,766	456,487,688
1978	1,047,241,843	529,244,881
1979	1,365,183,212	842,360,896
1980	1,648,470,477	1,140,641,841
1981	1,497,911,940	1,056,043,450
1982	1,111,473,699	839,552,938
1983	986,442,947	793,599,638
1984	896,240,561	740,260,761

EXHIBIT 4-A

(Continued)

Incurred Year	Expected Insured Indemnity	Expected Insured Total LAE
1985	1,036,923,734	903,666,858
1986	747,843,073	673,930,936
1987	422,947,296	600,408,790
1988	605,131,121	486,803,933
1989	558,482,291	477,068,174
1990	474,707,704	385,374,777
1991	392,718,395	311,488,374
1992	352,867,772	277,026,963
1993	190,365,131	151,231,576
1994	139,281,095	112,522,165
	25,444,146,412	14,980,602,621

EXHIBIT 4-B

Countrywide Estimates of Cleanup Costs and Insurance Liabilities Contribution from State Sites without PRP Information

Incurred Year	Expected Insured Indemnity	Expected Insured Total LAE
1950	23,336,106	8,338,011
1951	25,358,564	9,039,625
1952	27,319,522	9,792,835
1953	29,493,937	10,612,216
1954	31,424,145	11,171,526
1955	37,357,631	13,569,170
1956	42,677,906	15,412,940
1957	48,439,574	17,333,095
1958	50,945,064	18,187,952
1959	61,135,128	21,986,674
1960	73,584,559	27,080,882
1961	75,871,208	27,244,333
1962	81,126,395	29,032,359
1963	104,268,905	37,911,136
1964	108,219,776	39,031,999
1965	125,524,641	45,205,169
1966	133,200,244	47,697,450
1967	155,302,630	55,737,992
1968	171,400,936	61,819,904
1969	179,870,661	64,908,772
1970	208,434,792	74,959,018
1971	230,057,736	82,897,716
1972	283,594,912	102,583,930
1973	261,068,409	113,006,054
1974	290,285,859	126,562,599
1975	310,339,209	133,725,246
1976	320,278,029	137,549,183
1977	348,810,469	147,153,365
1978	394,125,435	167,697,746
1979	402,299,968	191,363,415
1980	425,953,289	243,573,494
1981	393,416,023	264,859,819
1982	364,515,316	322,284,348
1983	307,133,799	305,130,024
1984	245,790,016	247,413,545

EXHIBIT 4-B

(Continued)

Incurred Year	Expected Insured Indemnity	Expected Insured Total LAE
1985	221,688,995	308,149,067
1986	167,065,353	242,846,713
1987	147,908,737	207,374,914
1988	100,667,233	178,889,358
1989	114,491,815	196,030,899
1990	103,059,409	179,970,529
1991	56,352,816	104,414,807
1992	80,054,797	121,425,425
1993	47,295,950	56,371,520
1994	28,727,347	24,120,406
	7,439,273,244	4,851,467,183

EXHIBIT 4-C

Countrywide Estimates of Cleanup Costs and Insurance Liabilities Contribution from CERCLIS Sites with PRP Information

Incurred Year	Expected Insured Indemnity	Expected Insured Total LAE
1950	36,124,737	11,703,776
1951	39,506,765	12,817,427
1952	42,448,887	13,762,347
1953	46,031,092	14,925,567
1954	48,753,261	15,791,902
1955	60,158,527	19,509,852
1956	68,064,319	22,068,248
1957	76,910,460	24,920,014
1958	79,313,796	25,698,288
1959	97,223,613	31,532,778
1960	123,313,666	40,014,747
1961	122,378,154	39,685,575
1962	128,044,863	41,521,841
1963	165,888,464	53,808,145
1964	171,093,073	55,490,106
1965	199,990,444	64,901,402
1966	210,998,983	68,455,395
1967	244,062,943	79,115,842
1968	268,320,314	87,024,981
1969	282,348,869	91,501,124
1970	325,723,414	105,569,161
1971	362,557,962	118,592,271
1972	457,508,944	149,594,461
1973	269,598,311	162,800,201
1974	297,117,698	179,762,071
1975	316,655,982	191,861,349
1976	323,322,688	196,260,119
1977	344,114,604	209,185,673
1978	399,958,117	248,706,334
1979	417,433,372	284,561,602
1980	455,593,706	348,004,661
1981	442,254,551	348,295,107
1982	521,449,186	465,396,433
1983	395,828,330	340,207,552
1984	384,771,459	336,795,444

EXHIBIT 4-C

(Continued)

Incurred Year	Expected Insured Indemnity	Expected Insured Total LAE
1985	329,896,771	397,563,712
1986	379,020,264	350,773,713
1987	157,298,911	238,182,926
1988	241,992,415	156,616,132
1989	245,765,651	187,123,419
1990	233,338,009	190,450,997
1991	185,971,174	128,735,547
1992	194,457,021	141,657,392
1993	131,508,517	79,163,605
1994	137,295,225	93,503,327
	10,461,407,514	6,463,612,567

APPENDIX A

MEASURING SITE COST VARIABILITY

There are two sources of site variability that were analyzed. The first is that caused by differences among sites (i.e., the distribution of site cleanup costs). The second is the variability in the cleanup cost associated with the uncertainty in cleanup costs at the site level. Note that the latter is in addition to the EPA accuracy guidelines of −30% to +50%.

To measure the first source of variability, a distribution of cleanup costs was needed. The chosen source was the average present worth by site from the RODs prepared by the EPA. These costs are not at nominal values. Nominal, or undiscounted values, need to be estimated from present worth values, thereby introducing additional uncertainty which detracts from the precision of the measurement of variability that is sought.

The RODs costs were extracted directly from the text of these documents. Where a range of costs was indicated then the average was chosen, with a single cost effectively representing the average for the particular site.

The RODs used at the time covered the time period from 1987 to approximately the end of 1993, with a few RODs actually issued prior to 1987. In order to combine these data on a consistent cost basis, parameters were extracted from the University of Tennessee study [15, p. 15]. The specific parameters were a 2% trend in RODs cost annually plus a 46% increase in costs to represent cost growth as measured by the same study.[14] The average present worth values were trended by 2% from the year of the date the ROD was issued to 1997. The results were then averaged by site, as there are many sites with multiple RODs.

[14]This uniform factor could have been omitted as it would not have affected the shape of the distribution nor the estimated value of σ. A colleague of the author advised as to the potential misleading nature of this cost growth factor due to a mismatching of actual paid costs and expected costs.

Figure A-1 presents the resulting histogram for the 803 sites included in the analysis. The statistic graphed is the natural logarithm of the average present worth by site. Figure A-2 presents a normal probability distribution plot for the same distribution using the observed mean of 15.8135 and standard deviation of 1.506764. Based on this plot and visual inspection of the histogram, it was assumed that these costs could be reasonably described by a lognormal distribution with a coefficient of variation (CV) determined by the standard deviation:

$$CV = (e^{\sigma^2} - 1)^{1/2} = (e^{1.506764^2} - 1)^{1/2} = 2.9466 \qquad (A.1)$$

Most importantly, the resulting CV was chosen to apply to the estimated average site cost by state. This was accomplished by multiplying the average cost by a randomly sampled value from the lognormal distribution (with the chosen CV of 2.9466 and a mean of unity) every time a CERCLIS site cost was generated as a state enforced site. Since the mean of the lognormal is equal to $e^{\mu+(\sigma^2/2)}$, the value of μ is solved directly as -1.135169 in order to have a mean of 1 for the underlying σ of 1.506764.

To measure the second source of variability, an analysis was performed of those RODs that contained either a range of estimates or several alternate cost estimates. These ranges are separate and distinct from alternative remedies, and represent contingencies and additional uncertainties with regard to cleanup cost estimates.

Figure A-3 presents a histogram of the relativities. The relativities are measured by comparing a cost estimate to the average for the ROD. A frequency is assigned to each estimate that represents the reciprocal of the number of estimates in that ROD. In this fashion, the relativities from each ROD receive a total frequency (weight) of unity. The resulting histogram is not symmetric because there are RODS with three or four estimates.

Exhibit A-1 presents the table underlying the histogram. The empirical relativities were used as shown when modeling the variability of all site costs that did not have more than one cost estimate (as described in Section 3).

FIGURE A-1

DISTRIBUTION OF SITE COSTS

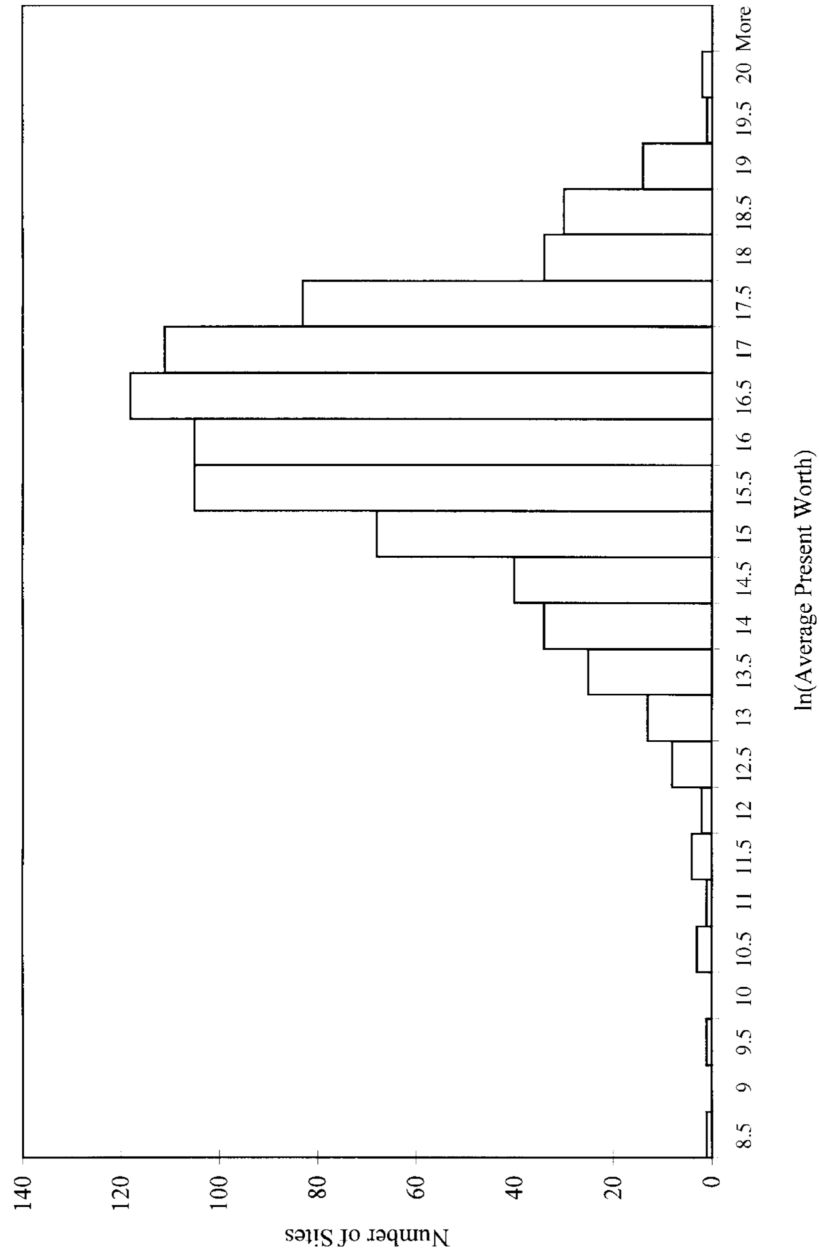

FIGURE A-2

NORMAL PROBABILITY PLOT

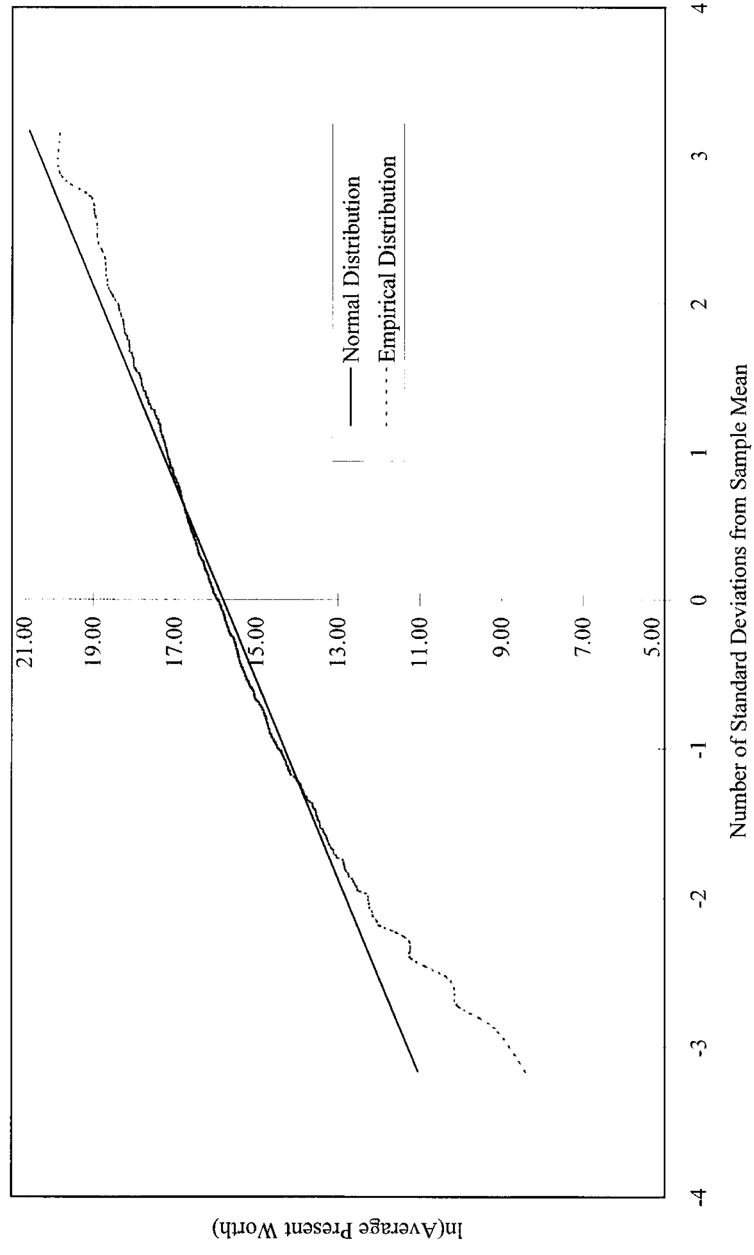

FIGURE A-3

DISTRIBUTION OF RELATIVITIES FOR MULTIPLE COST RODS

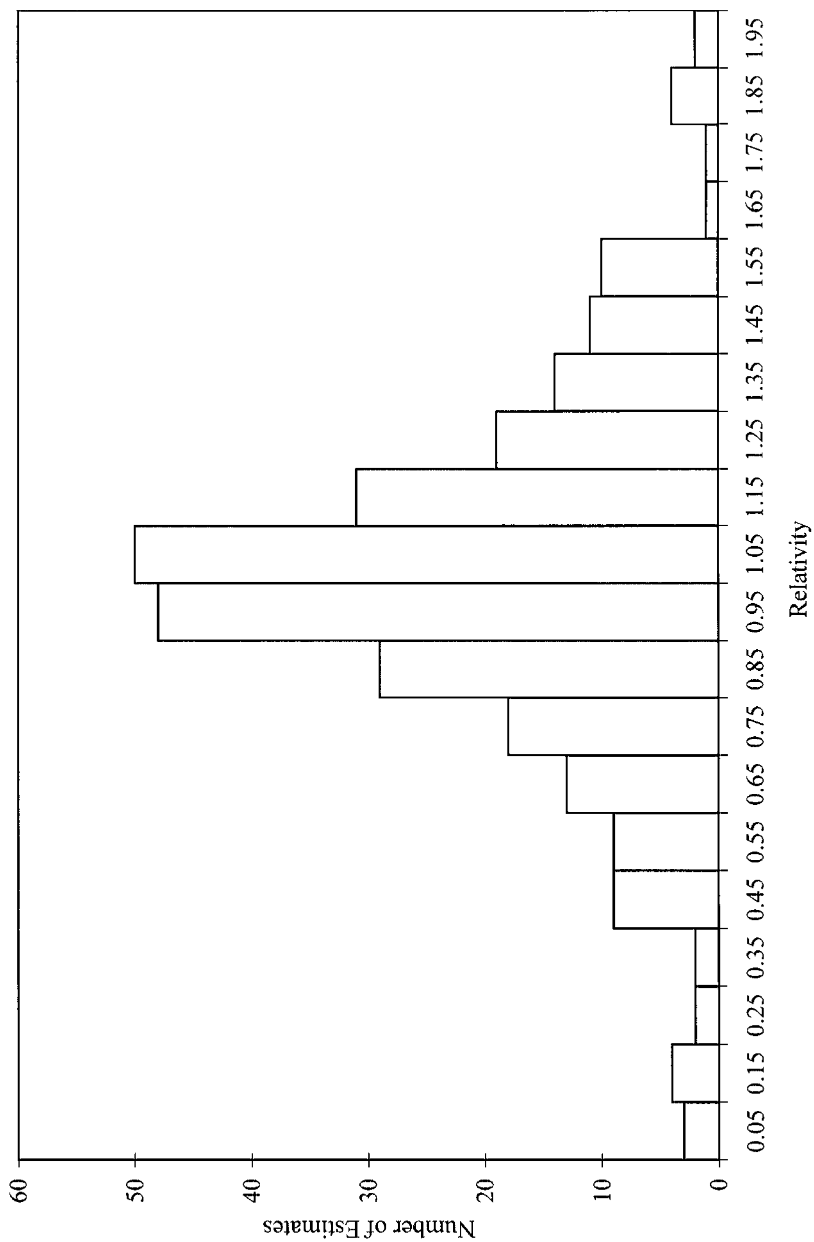

EXHIBIT A-1

Analysis of Relativities for Multiple Cost RODs

Relativity to Unity	Number of Estimates	Cumulative Number of Estimates	Cumulative Distribution of Relativity
0.05	3	3	0.0107
0.15	4	7	0.0250
0.25	2	9	0.0321
0.35	2	11	0.0393
0.45	9	20	0.0714
0.55	9	29	0.1036
0.65	13	42	0.1500
0.75	18	60	0.2143
0.85	29	89	0.3179
0.95	48	137	0.4893
1.05	50	187	0.6679
1.15	31	218	0.7786
1.25	19	237	0.8464
1.35	14	251	0.8964
1.45	11	262	0.9357
1.55	10	272	0.9714
1.65	1	273	0.9750
1.75	1	274	0.9786
1.85	4	278	0.9929
1.95	2	280	1.0000
		Weighted Average Relativity	1.000357

APPENDIX B

ESTIMATING AVERAGE SITE COSTS AND FREQUENCIES BY STATE

The initial steps in this analysis involved obtaining an understanding of the distribution of CERCLIS sites according to several characteristics. Exhibit 1 contains the tables that provide the requisite descriptive summaries.

The distribution by CERCLIS Source indicates that the great majority of sites are on the Archive file (30,428 out of a total of 41,057). All but 69 of these have been identified as NFRAP by the EPA. The other 69 sites were identified as NPL sites and have apparently been removed due to cleanup or other reasons.[15] All of these NFRAP sites are assumed to be subject to remediation under the respective state authorities.

Of the 10,629 Active sites, 1,217 are active NPL sites, 51 are proposed NPL sites, 202 are sites identified or associated with NPL sites, 98 are deleted NPL sites, 46 have been removed from the NPL, and 723 are Federal non-NPL sites. This leaves a balance of 8,292 sites that are non-Federal sites that are eligible for assignment to the NPL in the future.

The estimate of the ultimate number of NPL sites was chosen as 2,100. This selection was based on work performed by the University of Tennessee [15, p. 3] and the American Academy of Actuaries [18, p. i]. It was assumed that the future emergence of NPL sites would conform to the existing NPL distribution of sites by state. This assumption was based on the reasoning that state characteristics differ in their reflection of long-time industrial and manufacturing use or, alternatively, rapid industrialization and development since the Second World War are causal factors in the emergence of NPL sites by state to date.

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[15] One site is actually shown as an active NPL site.

Likewise, when addressing the estimate of future average NPL cleanup costs by site, it is these same states' densities of sites that are likely to affect the inclusion of sites by state underlying that average. Individual state average cleanup costs for NPL sites were precision (credibility) weighted with overall countrywide estimates in order to estimate future average NPL cleanup costs by site as described below.

Exhibit 1 also contains information on the distribution of RODs with cost information, as well as the average present worth and average nominal cost of cleanup. All of the 803 sites discussed in Appendix B have been associated with NPL sites at one time or another. The estimated nominal costs are based on the analysis of present worth values by RODs and the direct estimate of undiscounted values. The default value of .672 for the present value factor[16] was used when insufficient information was available for a ROD to perform this estimate specifically. Values were trended to 1997 at an annual inflation rate of 2%, as performed in the analysis described in Appendix B.

The values used for the estimation of future NPL cleanup costs weres based on the nominal costs for non-Federal Deleted and Final NPL sites. The averages and the standard deviation of these costs are shown on Exhibit 1. The following notation applies to the estimation of the future average NPL cleanup cost and estimate of the number of future NPL sites by state:

μ_i = average (mean) of the nominal non-Federal Deleted or Final NPL cleanup costs for state *i*,

μ_{CW} = the average of the state means of the nominal non-Federal Deleted or Final NPL cleanup costs,

σ_i = standard deviation of nominal non-Federal Deleted or Final NPL cleanup cost for state *i*,

[16]This value is the average present value factor obtained after removing outlier factors of .4 or less and of .95 or greater.

σ_{CW} = standard deviation of the state averages (mean) of the nominal non-Federal Deleted or Final NPL cleanup costs,

z_i = precision weight (credibility) estimated for state i average,

$\hat{C}_i$ = precision-weighted estimate of future average NPL cleanup cost for state i,

$\hat{M}_i$ = final balanced estimate of future average NPL cleanup cost for state i,

NPL_i = current number of NPL sites for state i. This is composed of CERCLIS sites identified as Deleted or Final NPL sites (NPL Status D or F) for non-Federal sites,

P_i = number of Proposed NPL sites for state i (NPL Status P),

Q_i = number of sites eligible for future NPL status for state i, defined as sites with current NPL Status of Q (non-Federal and not NPL), and

FNPL_i = estimated number of future NPL sites for state i.

The precision-weighted estimates of the mean future NPL site cleanup cost are estimated as follows:

$$z_i = \frac{\frac{1}{\sigma_i^2}}{\frac{1}{\sigma_i^2} + \frac{1}{\sigma_{CW}^2}}, \qquad \text{and} \tag{B.1}$$

$$\hat{C}_i = z_i \cdot \mu_i + (1 - z_i) \cdot \mu_{CW}. \tag{B.2}$$

The estimated means were balanced to the overall countrywide average by uniformly applying the ratio of the average countrywide NPL cleanup cost to the weighted average of the state-estimated average future NPL cleanup cost, using the number of NPL sites by state as weights:

$$\hat{M}_i = \hat{C}_i \cdot \frac{\sum_{\forall i} \mu_i \cdot \text{NPL}_i}{\sum_{\forall i} \hat{C}_i \cdot \text{NPL}_i}. \tag{B.3}$$

At the time of the analysis, Guam had two NPL sites and no site eligible for future NPL status. With this restriction in mind, the number of future NPL sites for Guam was set at zero while for all other states it was estimated by:

$$\text{FNPL}_i = \frac{\text{NPL}_i}{\sum_{\forall i} \text{NPL}_i - 2} \cdot \left(2100 - \sum_{\forall i} \text{NPL}_i - \sum_{\forall i} P_i\right). \tag{B.4}$$

The value of 2100 represents the estimated ultimate number of NPL sites referred to earlier. The frequency with which a status Q site becomes a future NPL site is the ratio of FNPL_i to Q_i by state.

The last input to the estimation process is provided by the limited aggregate information provided on state- and territory-administered cleanup efforts. Exhibit 2 shows the aggregate information obtained from [11] and [12]. Integrating this information with the cost information obtained from the RODs text made use of three important observations from review of this EPA state and territory study:

- Federal Total Costs averaged $1.669 million per site at that time but the expected future average is expected to be $25 million [11, p. ES-10].
- The average state cost is $300,000 and the average PRP cost is $401,000 [11, p. ES-8], the latter representing approximately one-quarter of the aforementioned Federal cost.
- The total cost for the states is $1.205 billion on 3,395 sites [11, p. ES-8].

It is important to note that these are actual paid costs that comprise a portion of ultimate total cleanup costs, the latter including operation and maintenance costs. This phenomenon is likely to influence the translation of average costs to an ultimate basis for federal NPL sites to a greater extent than sites subject to state-administered cleanup efforts. It is certainly an aspect that may point to a downward bias in total cost estimates.

This cost information includes enforcement as well as voluntary cleanup efforts, creating an average that will likely tend to be understated. Furthermore, there may be sites where both the state and PRP costs are involved, yet only one of the parties reported its costs. This would also tend to understate any average cost estimates.

The data by state as they appear in Exhibit 2 appear incomplete, and countrywide statistics were used in the estimation process. Specifically, the following identity was employed:

$$\text{Total Cost} = \text{State Average} \times \text{State Sites} + \text{PRP Average} \times \text{PRP Sites}. \quad \text{(B.5)}$$

Based on the national data summary from the EPA state and territory study [11, p. ES-8], there were a total of 3,395 sites countrywide, 2,167 state sites, and 1,385 PRP sites, the latter two quantities adding to 3,552. This relationship between the number of "total" countrywide sites (3,395 being less than 3,552, or what appears to be "the sum of its parts") reflects the fact that there is an overlap between the state sites and the PRP sites (i.e., there are sites that carry both state and PRP costs). The scalar reflecting this ratio (i.e., 3,395/3,552) was applied to the sum of the state and PRP sites by state in order to better estimate the total number of true separate and distinct sites.

Given the estimate of the number of sites by state, the average cost per site is estimated as the total cost divided by the total number of sites by state. For those states without state and PRP site count information, the countrywide average was used

($1,205,531,234/3,395 sites). The PRP insurable share is estimated as the ratio of the PRP cost to the total cost by state. For states without this cost information, the countrywide average was used ($555,530,464/$1,205,531,234). Note that this component of the model derives the insurable cost from state sites similar to the CERCLIS estimate that includes non-Federal sites only.

The average state site cost is multiplied by the applicable CERCLIS sites that do not become future NPL sites (NFRAP sites plus the remaining eligible sites that do not become future NPL sites) in order to estimate the total cleanup costs associated with state-administered programs. This average is not greater than the estimated average state PRP site cost, therefore permitting a measure of conservatism in some states for those sites arising from CERCLIS. The average state PRP site cost is multiplied by the sum of the number of known or suspected sites by state and the expected number of future state sites. The latter quantity is implied by the difference between the total sites and the estimated true number of sites, derived from the application of the countrywide site scalar mentioned above.

The dollar amounts included in this study are in nominal dollars. Average costs derived from these data are therefore not present value estimates, but rather nominal (undiscounted) costs at the level of the years in which they were spent. No inflation adjustments were made to the average cost estimates for future cleanup costs.

The final results for insurable costs are shown in Exhibit 4. The total cleanup costs were simulated with a mean of $87.9 billion. The histogram of the results is displayed in Figure B-1. The effects of the simulation from the CDM are to reduce the cleanup costs that are indemnified by approximately 50%. Given an average LAE factor to cleanup cost of 29.9% from Exhibit 3, the expected value estimate of total insured costs is approximately $73.4 billion. This compares with the simulated mean of $69.6 billion contained within the histogram of the results displayed in Figure B-2.

FIGURE B-1

SIMULATED DISTRIBUTION OF UNDISCOUNTED CLEANUP COSTS

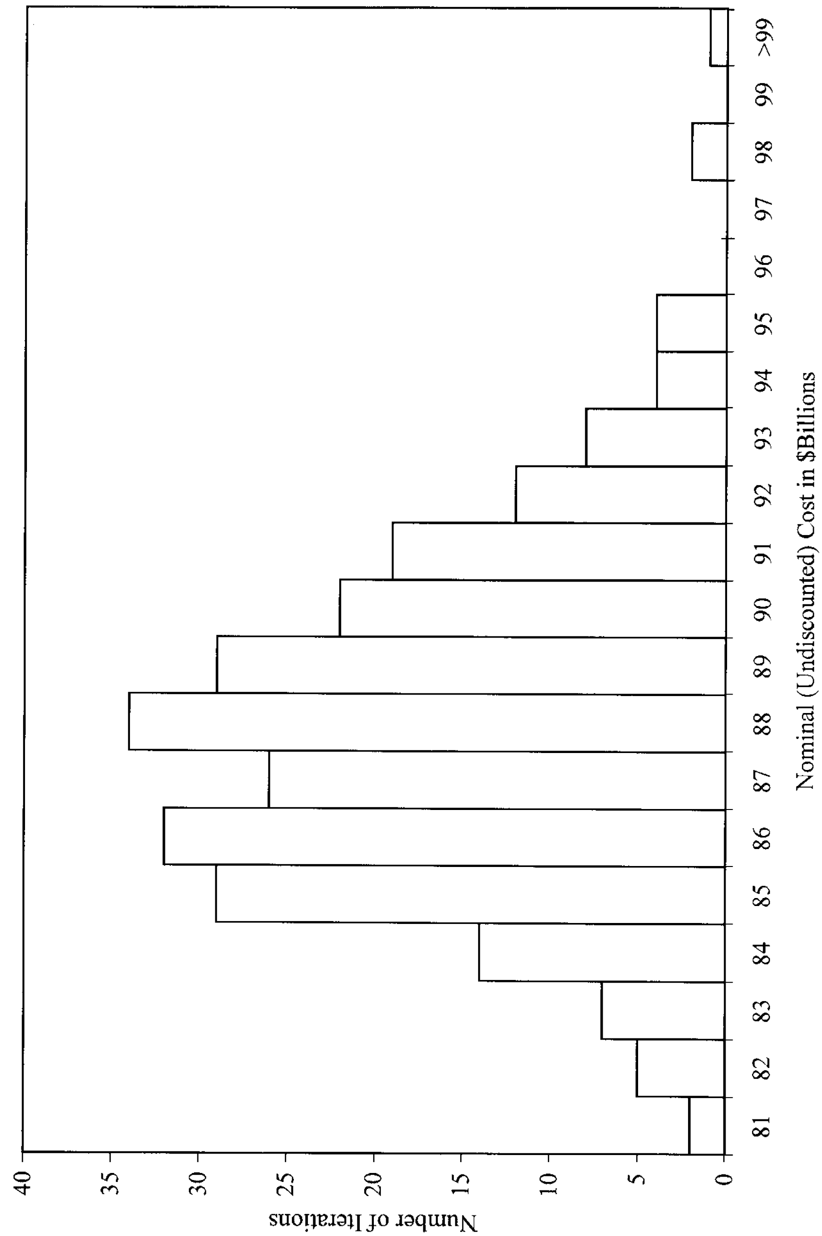
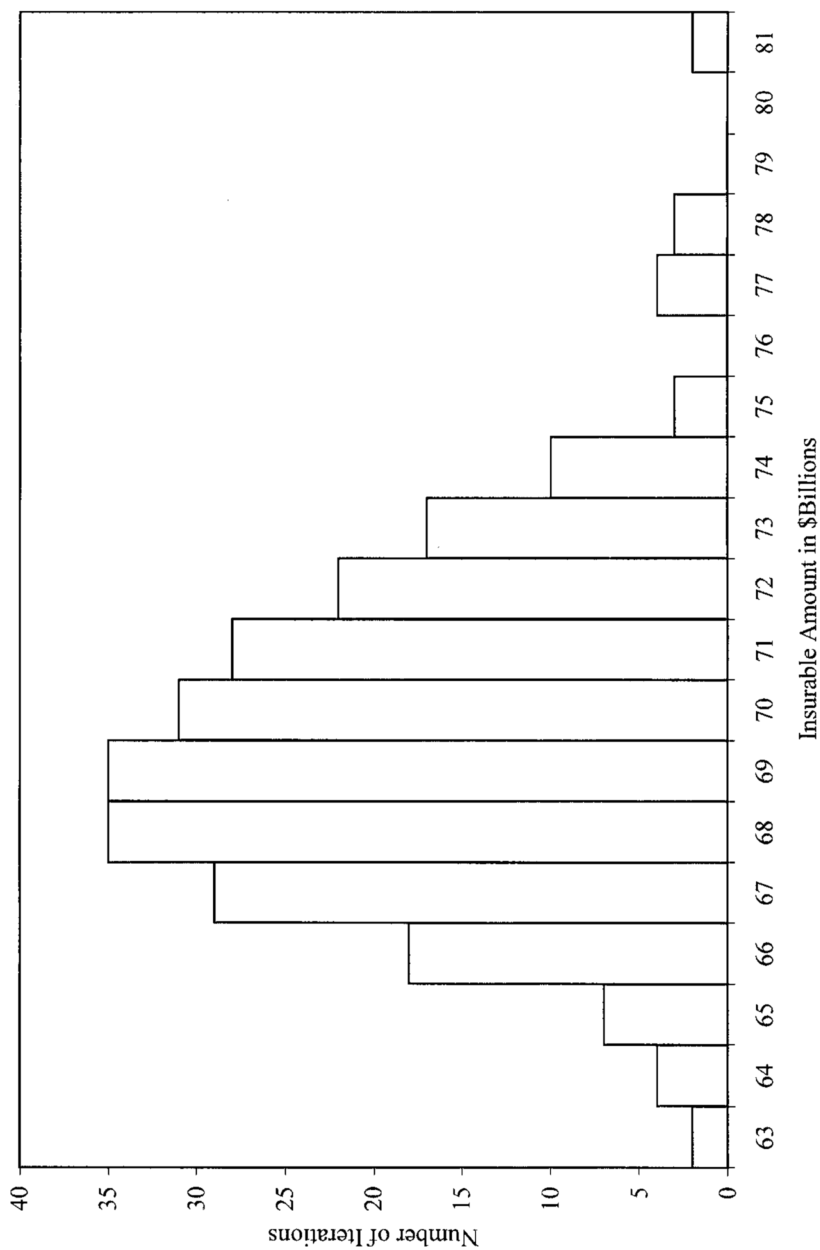


FIGURE B-2

SIMULATED DISTRIBUTION OF INSURABLE INDEMNITY LOSSES & LAE



APPENDIX C

PRP SHARES

Shares have been assigned to PRPs in one of two different ways when estimating pollution liabilities using the PCSM. In some instances, a specific share was obtained from information contained in the claim files of an insurer or reinsurer when performing a portfolio analysis. In all other cases a simulated share was assigned to a PRP by using a Beta distribution. The total of all shares for a site is balanced to unity, including both simulated and specific shares. The remainder of this appendix will discuss the simulation of PRP shares that is used for the industry estimate.

The Beta distribution used in the simulation has the following form:

$$B(x,\alpha,\beta) = \frac{\int_0^x t^{\alpha-1}(1-t)^{\beta-1}dt}{\beta(\alpha,\beta)},$$

$$\text{where} \quad \beta(\alpha,\beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha,\beta)}. \qquad \text{(C.1)}$$

The mean of this distribution is set equal to the reciprocal of the number of PRPs. In order to afford higher probabilities at or near zero, α is equal to the reciprocal of the number of PRPs whenever the number of PRPs is greater than one. The reason for creating higher probabilities at or near zero is to afford a simulation of the *de minimis* shares phenomenon. This phenomenon results from the incidence of many PRPs on a site that have been determined to be minor (*de minimis*) contributors to the pollution at the site. Setting α equal to the reciprocal of the number of PRPs results in a value less than unity, thereby providing the sought after shape of the distribution—a large mode at zero (0% share) and a much smaller secondary mode at unity (100% share).

TABLE C-1

BALANCED SIMULATED AND SPECIFIC SHARES

Individual PRPs	Beta Distribution Simulated Share	Balanced Share
1	0.2000	0.2000
2	0.8526	0.5644
3	0.0003	0.0002
4	0.0001	0.0001
5	0.2535	0.1678
6	0.0000	0.0000
7	0.0000	0.0000
8	0.0130	0.0086
9	0.0632	0.0418
10	0.0258	0.0171
Total Excluding PRP-1	1.2085	1.0000

If we let n be the number of PRPs and M be the mean of the Beta distribution, then we have the following relationships:

$$M = \frac{\alpha}{\alpha + \beta} = \frac{1}{n}, \qquad \alpha = \frac{1}{n}, \qquad \text{and} \qquad \beta = \frac{n-1}{n} = 1 - \alpha.$$

This distribution is used for each site for which shares were simulated during each iteration of the PCSM simulation for a portfolio. To illustrate the process for ten PRPs on a site, the Table C-1 reproduces a sample iteration for ten PRPs with the introduction of a fixed known share of 20% for PRP-1. PRP-1 represents a specific share assignment encountered during a portfolio analysis as mentioned earlier.

Each of the nine other PRPs have shares simulated with a total excluding PRP-1 determined. The parameters of the Beta distribution in this case are $\alpha = .1$ and $\beta = .9$. The shares of these other nine PRPs are then balanced to produce unity when including the known fixed share of PRP-1. Note that this particular example includes several PRPs with *de minimis* shares.

APPENDIX D

Derivation of Scalars For Samples

When estimating ultimate pollution losses for a portfolio, it is necessary to determine a scalar that can be applied to the costs simulated by the PCSM. This is because only a sample of sites and PRPs exists for the simulation employed for any portfolio. In the cases where there does not exist an exhaustive list of PRPs and policies exposed to pollution liabilities, which will often be the case, a pro-rata scalar can be used of the following form:

$$Ins_T(P) = \frac{CU_T}{CU_N(S)} \cdot Ins_N(S). \tag{D.1}$$

The scalar represents the ratio shown on the right-hand side of the formula above. This ratio will vary from one portfolio to another. The following notation will be employed to show how this scalar is derived:

$CU_X(Y)$ = total cleanup and operation and maintenance costs relating to portfolio Y of potentially responsible parties (PRPs) from subset X of all sites (cleanup costs),

$Ins_X(Y)$ = Insured costs related to portfolio Y of PRPs from subset X of all sites,

N = NPL and CERCLIS (national) sites with potentially responsible parties (PRPs) information,

T = all sites countrywide, including CERCLIS and state- and territory-administered sites, and

S = national sites included in the sample employed by the PCSM.

If the PRP portfolio is not designated in a term, then the entire population of PRPs is implied. In this presentation, the value of P refers to the PRPs associated with the portfolio under analysis.

The value of S refers to the sample (subset of the portfolio) of PRPs that is used in the simulation model.

It is important to note that often not all underlying insureds embedded in the portfolio can be identified. The insureds from primary policies can frequently be identified over a long period of time, although the volume of names can become overwhelming from an analytical perspective and perhaps less reliable for older policy periods. Usually, all the direct excess policies and facultative reinsurance contracts within a portfolio can be matched to the EPA SETS (albeit with no accounting of aliases and subsidiaries); however, the underlying insureds identified for excess of loss and proportional reinsurance treaties represent samples based on the identification of PRPs through an audit or review of claim files, or as captured from their identification on actual reported claims. The simulation therefore employs a sample (subset) of all national sites and their PRPs to which the portfolio is exposed, incorporating only those PRPs that could be identified from the total set of PRPs embedded in the portfolio.

The use of a PRP sample implies that an estimate of the total insured costs for the portfolio can be extrapolated from the simulated costs for the sample by employing two ratio estimates. The first ratio results when the cleanup costs of all national sites is compared to the cleanup costs from the sample. This first ratio is applied to the insured costs from the sample to derive the estimated national cleanup cost for sites associated with the portfolio. The national cleanup cost is then used to determine portfolio insured costs arising from all sites countrywide (population) by applying the second ratio. This second ratio compares all sites' countrywide insured costs to national sites' insured costs. The equation representing this estimate is:

$$Ins_T(P) = \frac{CU_N}{CU_N(S)} \cdot Ins_N(S) \cdot \frac{Ins_T}{Ins_N}. \tag{D.2}$$

This formula can be rewritten by rearranging terms as:

$$Ins_T(P) = \frac{Ins_N(S)}{CU_N(S)} \cdot Ins_T \cdot \left(\frac{CU_N}{Ins_N}\right). \tag{D.3}$$

It is assumed that the ratio of cleanup costs to insured costs is the same for national sites as it is for all sites, that is:

$$\left(\frac{CU_N}{Ins_N}\right) \approx \left(\frac{CU_T}{Ins_T}\right). \tag{D.4}$$

Note that this assumption presumes that the insurance coverage of insureds is commensurate with their hazard. As an example, a corporate entity that is not a Fortune 500 company may purchase insurance that attaches at a lower amount due to its smaller self-insurance capacity. However, that smaller entity's involvement at a site with lower cleanup costs could still result in an insured loss commensurate with, say, a Fortune 500 company's share of costs at an NPL site.

The formula now reduces to:

$$Ins_T(P) = \frac{Ins_N(S)}{CU_N(S)} \cdot Ins_T \cdot \left(\frac{CU_N}{Ins_N}\right). \tag{D.5}$$

Cancellation of total sites' insured costs results in the scalar equation cited earlier:

$$Ins_T(P) = \frac{CU_T}{CU_N(S)} \cdot Ins_N(S). \tag{D.6}$$

The scalar used for any given portfolio is the ratio from the simulation of all sites' countrywide cleanup costs to the sampled national sites' cleanup costs.

APPLICATION OF THE OPTION MARKET PARADIGM TO THE SOLUTION OF INSURANCE PROBLEMS

MICHAEL G. WACEK

DISCUSSION BY STEPHEN J. MILDENHALL

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1. INTRODUCTION

Michael Wacek's paper is based on the well-known fact that the Black–Scholes call option price is the discounted expected excess value of a certain lognormal random variable.[1] Specifically, the Black–Scholes price can be written as

$$BS = e^{-r(T-t)}\mathrm{E}[(\tilde{S}(T)-k)^+],$$

where r is the risk-free rate of interest, T is the time when the option expires, t is the current time, $\tilde{S}(T)$ is a lognormal random variable related to the stock price $S(T)$ at time T, k is the exercise price, and $x^+ := \max(x,0)$. In insurance terms, $(L-k)^+$ represents the indemnity payment on a policy with a loss of L and a deductible k. The Black–Scholes price can also be regarded as the discounted insurance charge (see Gillam and Snader [18] or Lee [25]). It is easy to compute the insurance charge under

[1]The formula is explicit in virtually all financial economics derivations, for example, Merton [27, p. 283], Cox and Ross [4, p. 154, equation 19, which is essentially the author's Equation 1.3], Harrison and Kreps [19, Corollary to Theorem 3], Karatzas and Shreve [23, p. 378], Hull [20, p. 223 (for forward contracts on a stock)], as well as more overtly actuarial works, such as Gerber and Shiu [17, p. 104] and Kellison [24, Appendix X].

the lognormal assumption to arrive at—but not to derive—the explicit Black–Scholes formula.[2]

Even without reference to the Black–Scholes formula, there are obvious analogies between insurance and options because both are *derivatives*. An insurance payment is a function of—is *derived* from—the insured's actual loss; similarly, the terminal value of an option is a function of the value of some underlying security. To the extent that options and insurance use the same functions to derive value, there will be a dictionary between the two. As Wacek points out, this is the case. For example, the excess function $(L-k)^+$ is used to derive the terminal value of a call and an insurance payment with a deductible k; $\min(L,l)$ determines the value of an insurance contract with a limit l; and $(k-L)^+ = k - \min(L,k)$ gives the terminal value of a put option as well as the insurance savings function. There are several other examples given in the paper, including a cylinder. The author explains how an insurance cylinder can be used to provide cheaper reinsurance and greater earnings stability for the cedent. The idea of regarding an insurance payment as a function of the underlying loss has been discussed previously in the *Proceedings* by Lee [25] and [26], and Miccolis [28]. The connection between insurance and options, based on the fact that both are derivatives, was also noted in D'Arcy and Doherty [9, p. 57].

Here are two other interesting correspondences between option structures and insurance. The first is the translation from put-call parity in options pricing to the relationship "one plus savings equals entry ratio plus insurance charge" from retrospective rating. The put option is equivalent to the insurance savings function and the call option to the insurance charge function (see Lee [25] and [26], which has the options profit diagrams, or Gillam and Snader [18] for more details).

The second correspondence applies Asian options to a model of the rate of claims payment or reporting in order to price catas-

[2]Kellison [24, Appendix X] gives all the details.

trophe index futures and options. This example is too involved to describe in detail here. The interested reader should look in the original papers by Cummins and Geman [7] and [8].

Insurance can also be regarded as a swap transaction. Hull [20] defines swaps generically as "private agreements between two companies to exchange cash flows in the future according to a prearranged formula." In insurance language one cash flow is the known premium payment, generally consisting of one or more installments during the policy period, and the other varies according to losses and continues for a longer period of time. Many recent securitization transactions have been structured as swaps. Indeed, in that context a swap is essentially insurance from a non-insurance company counter-party. Arguably, swaps are a better model for insurance than options because they involve a series of cash flows into the future rather than a single payment. Options, which involve a single payment when the option is exercised, are not a good model for a per occurrence insurance product that could cover many individual claims.

Despite the title of the paper, Wacek is more concerned with options *notation*—puts, calls, profit diagrams and so forth—than with the options market *paradigm*. The dictionary definition of a paradigm is a "philosophical and theoretical framework of a scientific school or discipline within which theories, laws, and generalizations…are formulated." Wacek's paper does not discuss the assumptions underlying Black–Scholes nor the derivation of the formula in any detail. Each is an important part of the options pricing paradigm. Moreover, the comments he offers on options prices tend to confuse a pure premium (loss cost) with a price (loss cost including risk charge, in this context). He rightly draws a distinction between the two but does not clearly state whether the Black–Scholes formula gives the former or the latter.

This review will focus on the theoretical framework, or paradigm, of options pricing. Section 2 will compare the Option Pricing Paradigm with the corresponding actuarial notion, and discuss how the former relies on hedging to remove risk while the latter relies on the law of large numbers to assume and manage risk. The distinction between using hedging and diversification to manage risk highlights an essential difference between the capital and insurance markets. Section 3 will determine the actuarial price for a stock option under the lognormal distribution assumption, and will compare the result to the Black–Scholes formula. Section 4 then discusses why the Black–Scholes result is different from the actuarial answer. It will also explain why the Black–Scholes formula gives a price rather than a pure premium. Section 5 will propose an application of the Option Pricing Paradigm to catastrophe insurance and discuss options on non-traded instruments. Finally, Section 6 will compare market prices with the Black–Scholes prices.

This review will only discuss applications of options pricing to individual contracts in a very limited way. The reader should be aware that there are many other important applications, including the pioneering work of Cummins [6], revolving around valuing the insurance company's option to default. The groundbreaking paper by Phillips, Cummins and Allen [30] gives an application of these ideas to pricing insurance in a multi-line company. The reader should refer to the recent literature for more information on these ideas.

2. OPTION PRICING PARADIGM AND ACTUARIALLY FAIR PRICES

The actuarial, or fair, value of an uncertain cash flow is defined to be its expected value. Insurance premiums are generally determined by loading the discounted actuarial value of the insured losses for risk and expenses. In this discussion it will be assumed that a risk charge is loaded into the pure premium by discounting at a risk-adjusted interest rate. Clearly this is neither the only choice nor is it necessarily the best choice. It will also be assumed that there are no expenses, and the word "price" will be used to refer to a risk-loaded pure premium.

The Option Pricing Paradigm defines the price of an option to be the smallest cost of bearing the risk of writing the option, which is completely different from the actuarial viewpoint. In this context, being able to *bear* the risk of writing an option (equivalent to writing insurance) means being able to respond to the holder of the option whatever contingency might occur. In actuarial-insurance language this implies a zero probability of ruin, for if there is a non-zero probability of ruin then there is a contingency under which the option writer cannot respond to the holder, and hence the writer is not able to bear the risk (according to the definition).

The insurance company approach to bearing risk is to charge a pure premium plus risk load, to have a substantial surplus, and to pool a large number of independent risks. If stock prices follow an unbounded distribution, such as the lognormal, then it is not possible to write an option and achieve a zero probability of ruin using this insurance approach to bearing risk. Thus, unlike insurance, pricing and risk bearing in the Option Pricing Paradigm do not rely on the law of large numbers—a crucial difference.

One way of bearing the risk of writing a stock option is to set up a hedging portfolio with the following four properties:

1. The portfolio consists of the stock underlying the option and risk-free borrowing or lending.
2. The terminal value of the hedging portfolio equals the terminal value of the option for all contingencies.
3. The hedging portfolio is *self-financing*: once it has been set up it generates no cash flows, positive or negative, until the option expires.
4. The hedging portfolio uses a deterministic trading strategy which only relies on information available when each trade is made. Trading only takes place between when the option is written and when it expires.

It is not clear that hedging portfolios exist. However, if they do then the Option Pricing Paradigm price of an option can be no greater than the smallest amount for which it is possible to set up (i.e., purchase) a hedging portfolio. Indeed, by setting up a hedging portfolio the writer of the option is able to bear the attendant risks, because the portfolio generates enough cash to respond to the holder no matter what contingency occurs. By definition, the price of the option is the smallest amount of money for which this is possible. Therefore, the actual option price can be no greater than the cost of the cheapest hedging portfolio.

On the other hand, if there are no arbitrage opportunities, the Option Pricing Paradigm price must be at least as large as the cost of setting up the cheapest hedging portfolio. Since the writer can bear the risk of writing the option, it must have a portfolio, purchased with the proceeds of writing the option, with an ending cash position at least as large as the terminal value of the option (and hence a hedging portfolio) in every contingency. Such a portfolio is said to *dominate* the hedging portfolio. If portfolio A dominates portfolio B then, in the absence of arbitrage, A must cost more than B. Here, the option price is used to purchase a portfolio which in turn dominates a hedging portfolio, and therefore the option price must be at least as great as the cost of a hedging portfolio.[3] Combining this with the previous paragraph shows that in the absence of arbitrage, the Option Pricing Paradigm price equals the smallest amount for which it is possible to set up a hedging portfolio.

The above argument relies on the absence of arbitrage opportunities in the market. An arbitrage is the opportunity to earn a

[3]The price of the option could simply be defined as the smallest cost of setting up a hedging portfolio. For example, in Karatzas and Shreve [23] the fair price for a contingent claim is defined as the smallest amount x which allows the construction of a hedging portfolio with initial wealth x. However, it is generally not possible for an insurer to set up a hedging portfolio because it cannot trade in the security underlying the insurance contract option. Thus, a definition in terms of hedging portfolios would not have transferred to insurance. On the other hand, "the cost of bearing the risk," albeit with a possibly weaker notion of bearing risk, makes perfect sense in an insurance setting and is equivalent to the hedging portfolio definition for options.

riskless profit. In general, the existence of arbitrage opportunities is not compatible with an equilibrium model of security prices, since informed agents would engage in arbitrage and hence modify market prices (see Dybvig and Ross [16, pp. 57–71] for an explanation of the close connections between no-arbitrage and options pricing). In options pricing, no-arbitrage is used to justify defining the price of an option as the smallest cost of a hedging portfolio; if the option sold for more or less than the cost of a hedging portfolio then risk-free arbitrage profits would be possible. Put another way, the option and the hedging portfolio are *comparables*, and no-arbitrage implies that comparables must have the same value. Since "the value of an asset is equal to the combined values of its constituent items of cash flow" [3], if two assets have the same cash flows then they are equivalent to an investor and must command the same price. The fact that one is an option and the other a synthetic option created from a portfolio of bonds and stocks is irrelevant. Obviously this only applies in a world where the Black–Scholes assumptions hold—so in particular there are no transaction costs, no discontinuous jumps in stock prices, and continuous trading. Finally, no-arbitrage is a *consequence* of the model framework, not an *assumption*; potential arbitrages are ruled out through restrictions on admissible trading strategies. This is a more advanced point; the interested reader should see Harrison and Kreps [19], Dothan [14], and Delbaen and Schachermayer [10], [11] and [12] for more detailed information.

To conclude, this section has introduced the notions of no-arbitrage and hedging portfolios, and explained how the Option Pricing Paradigm defines price to be the smallest cost of setting up a hedging portfolio. These beginnings are enough to point out some significant differences compared to actuarial methods of pricing, one of which is that option pricing does not rely on the law of large numbers. The question of whether hedging portfolios actually exist will be discussed in Section 4. First, we will look at how an actuary would price an option.

3. AN ACTUARIAL APPROACH TO OPTION PRICING

It is instructive to compare the Black–Scholes price with the actuarial price—including risk-load—for a call option. Before defining terms we must fix the notation. Assume all interest rates and returns are continuously compounded. Let r be the risk-free rate of interest, μ the expected return on the stock, $S(t)$ or S_t the stock price at time t, and r' a risk-adjusted interest rate for discounting the option payouts. To keep the notation as simple as possible, assume that the current time is $t = 0$ and that the option expires at time $t = T$. Assume also that the stock price process is a geometric Brownian motion,[4] so that $\ln(S(t)/S(0))$ is normally distributed with mean $t(\mu - \sigma^2/2)$ and variance $t\sigma^2$ for some $\sigma > 0$. Finally, let k be the exercise price.

The actuarial price for the option is the present value of the expected payouts discounted at a risk-adjusted interest rate:

$$e^{-r'T}\mathrm{E}((S(T) - k)^+). \tag{3.1}$$

With $r' = r$, Equation 3.1 is Equation 1.3 from the paper.[5] An actuary could compute Equation 3.1 after estimating appropri-

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[4]This means that over a very short time interval dt, the return on the stock dS_t/S_t satisfies the stochastic differential equation $dS_t/S_t = \mu dt + \sigma dW_t$, where W_t is a Brownian motion. By definition W_t is normally distributed with mean zero and variance $t\sigma^2$. It follows from Ito's Lemma that the solution to the stock price stochastic differential equation can be written as $S_t = S_0\exp((\mu - \sigma^2/2)t + \sigma W_t)$. Hence S_t has a lognormal distribution and $\ln(S_t/S_0)$ is normal with mean $(\mu - \sigma^2/2)t$ and variance $t\sigma^2$. Since $E(S_t) = S_0 e^{\mu t}$ it is reasonable to call μ the expected rate of return on the stock. See Hull [20], or Karatzas and Shreve [23], for more details. In particular, Hull [20, Chapter 10.3] discusses the difference between expected returns over a short period of time and the expected continuously compounded rate of return. If there is variability in the rate of return, so $\sigma > 0$, then the former, μ, is greater than the latter, $\mu - \sigma^2/2$.

[5]Wacek justifies assuming $r' = r$ using the notion of a hedging portfolio. However, in this section an actuarial viewpoint is taken instead. To the actuary—and the financial economics community as a whole prior to Black–Scholes—we should have $r' > \mu > r$, since the option is more leveraged than the stock and hence more risky. Brealey and Myers [2] point out that the option has a higher beta and a higher standard deviation of return than the underlying stock. Clearly $\mu > r$, since the stock is more risky than a risk-free bond. Note that $r' > r$ assumes the actuary is buying an option and discounting the payout as income; if the actuary were writing the option and pricing the payout as a loss, then $r' < r$ would be appropriate.

ate values for μ, σ and r'. For μ the actuary might try using a historical average return, the Capital Asset Pricing Model price, or some other suitable tool. It is easy to estimate σ from a time series of stock prices. For the discounting rate, the actuary would mystically select some number $r' > \mu$ if buying an option (as is assumed here) or $r' < r$ if writing one. Using Equation 3.1 the actuary would then arrive at the following expression for the call price:

$$e^{(\mu-r')T} S_0 \Phi\left(\frac{\ln(S_0/k) + (\mu + \sigma^2/2)T}{\sigma\sqrt{T}}\right) - e^{-r'T} k \Phi\left(\frac{\ln(S_0/k) + (\mu - \sigma^2/2)T}{\sigma\sqrt{T}}\right). \qquad (3.2)$$

This formula is identical to a pre–Black–Scholes result derived by Samuelson [31] following the same logic used here. It is also mentioned in Ingersoll [21, pp. 199–212], which includes a survey of earlier attempts to determine a formula to price options. Comparing the actuarial Equation 3.2 to the Black–Scholes equation

$$S_0 \Phi\left(\frac{\ln(S_0/k) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}\right) - e^{-rT} k \Phi\left(\frac{\ln(S_0/k) + (r - \sigma^2/2)T}{\sigma\sqrt{T}}\right), \qquad (3.3)$$

and relating the variables back to Equation 3.1 highlights two differences:

1. The Black–Scholes model appears to assume the stock earns the risk-free rate of return, that is $\mu = r$;
2. The Black–Scholes model discounts at the risk-free rate of interest, so $r' = r$.

Clearly, substituting $\mu = r$ and $r' = r$ into Equation 3.2 gives Equation 3.3. Section 6 compares option prices computed with these two equations.

The expected rate of return μ on the stock is a function of investor risk preferences. Individual investors could differ as to their opinions of μ and yet the assumption underlying the first difference above says they will agree on the option price. In order to underline how remarkable and counter-intuitive this assumption is, it is instructive to translate it into insurance language. Suppose $S(T)$ is the value of a portfolio of losses at time T. Further, suppose $S(T)$ is lognormal with parameters μ and σ. The insurance analog of the call option pricing problem is to price an aggregate stop loss on $S(T)$ with attachment k. Assume all actuaries agree on σ, that is, they agree on the coefficient of variation of aggregate losses.[6] Now, the assumption discussed above says different actuaries could disagree on μ—and hence the mean of $S(T)$—and yet agree on the price of the aggregate stop loss! Clearly there is something significant going on behind the Black–Scholes formula.

No actuary would assume $\mu = r$ and $r' = r$ in pricing since they are not reasonable in the real world. In his paper, Wacek points out that option pricing discounts at the risk-free rate, $r' = r$, but he does not mention the first point. The next section examines why the two assumptions above can be made in the Option Pricing Paradigm.

4. THE HEDGING PORTFOLIO IN DISCRETE TIME AND THE BLACK–SCHOLES FORMULA

The Black–Scholes formula is best understood by considering a discrete time example. While the example may appear simplistic, it contains all of the key ideas in the Black–Scholes derivation. Cox, Ross and Rubenstein [5] give an explanation of how to derive the full Black–Scholes result from a limit of the binomial models considered here. Their explanation is considered in more detail by Nawalkha and Chambers [29], who show that

[6]For a lognormal distribution, the coefficient of variation is $\sqrt{\exp(\sigma^2) - 1}$, a function of σ alone.

preference independence in [5] depends on a particular choice of binomial parameters.

Consider writing a call option on a stock currently priced at \$100. When the option expires, assume the stock price will be either \$120 or \$90. The one period risk-free rate of interest is 5%; also, the option expires in one period and has an exercise price of \$105. Finally, assume the stock does not pay any dividends.

At this point it is important to understand what has, and what has not, been assumed. Underlying our assumptions are two unknowns: the probability p that the stock price will end at \$120, and an expected return μ on the stock. The expected return μ is

$$\mu = \ln\left(\frac{120p + 90(1-p)}{100}\right)$$

(using continuous compounding) or, equivalently,

$$100 = e^{-\mu}(120p + 90(1-p)). \tag{4.1}$$

Equation 4.1 expresses the current price as an expected present value, discounted at a risk-adjusted interest rate. It gives one relationship between the two unknowns p and μ. It is impossible for us to know whether the current stock price is \$100 because there is a very good chance of an upward price movement (high p) but investors are all very risk-averse (giving a high μ), or because there is only a moderate chance of an upward movement in a largely risk-neutral market. The fact that the Black–Scholes formula is independent of the choice of μ and p, subject to the constraint Equation 4.1, is one of its most remarkable features and it leads to the notion of risk-neutral valuation.

Return now to pricing the \$105 call option under the Option Pricing Paradigm. From Section 2, the price of the call is the smallest cost for which it is possible to set up a hedging portfolio. An explicit hedging portfolio for the option will now be constructed, demonstrating that they exist, at least in this simple case. Suppose the hedging portfolio consists of a stocks and b

dollars in bonds. The replicating property of a hedging portfolio requires that, at expiration,

$$120a + b = 15 \qquad \text{and}$$
$$90a + b = 0.$$

The top line corresponds to an upward movement in the stock price, when the option is worth $\max(120 - 105, 0) = 15$ at expiration. The bottom line corresponds to a downward movement in the stock price, when the option is worth $\max(90 - 105, 0) = 0$. Solving gives $a = 1/2$ and $b = -45$, meaning borrowing of \$45. The cost today of setting up a portfolio which consists of half of one stock plus \$45 debt one period from now equals $100/2 - e^{-0.05}45 = 7.19$. The first term is the cost of buying half a stock and the second term is the present value of a debt of \$45 one period from now.

It is easy to confirm that this portfolio hedges the option. If the stock price moves up, then selling the half-stock yields \$60, exactly enough to pay off the \$45 debt and pay the owner of the option the \$15 terminal value. If the stock price moves down, then selling the half-stock realizes \$45, which is exactly the amount required to pay off the debt. There is nothing else to pay since the option expires worthless.

Using the hedging portfolio it is also easy to see why no-arbitrage implies \$7.19 is the appropriate price for the option. If the option sold for more than \$7.19, say \$7.25, then arbitrageurs would write (sell) the over-priced options. With \$7.19 of the proceeds they could set up a hedging portfolio, effectively closing out their option position. They would make a risk-free profit of six cents per option written.

On the other hand, suppose the option sold for only \$7.15. Then arbitrageurs would want to buy the under-price options. They could short one stock to get \$100 and use the proceeds to buy two options for \$14.30, put \$85.61 into bonds earning the risk-free rate, and skim off the remaining nine cents as arbitrage profit. If the stock price rises to \$120, they exercise the two options to yield a \$30 profit, which combined with $85.61e^{0.05} =$ \$90 in bonds gives \$120—exactly enough to close out the short position in the stock. If the stock price falls to \$90 then the options expire worthless, but the arbitrageurs still have \$90 in the bonds to close out the stock position.

To summarize, these arguments lead to essentially the same two conclusions already noted in Section 3 from comparing Black–Scholes and the actuarial option price:

1. The option price is \$7.19 regardless of the risk preferences of individuals, expressed through the unknown quantities μ and p, provided Equation 4.1 is satisfied;
2. The hedging portfolio consists of stock and risk-free borrowing only. Once it is set up there is no risk to the option writer because movements in the stock price and the option price are perfectly correlated. A portfolio consisting of the hedge and the underlying option must therefore earn, and hence be discounted at, the risk-free rate of return.

Black and Scholes showed these two results are still true when the stock price is allowed to follow a more complex path in continuous time. Under the lognormal stock price assumption, they proved the option price function is given by Equation 3.3, and derived the required trading strategy to use in the hedging portfolio—the so-called "delta-hedging" strategy.

Cox and Ross [4] used the risk preference independence in the first conclusion above to argue that an option could be priced assuming investors have any convenient risk preference. The simplest selection for preferences is risk-neutrality. In a risk-neutral world, all stocks are expected to earn the risk-free return because investors do not require a premium for uncertainty. Thus $\mu = r$ is determined. Of course, stocks do not earn the risk-free return in the real, risk-averse, world. In our simple discrete model, selecting risk-neutral preferences for investors is equivalent to setting $\mu = r$, so now we can solve Equation 4.1 for p, to get $\tilde{p} = (100e^r - 90)/30 = 0.50424$. A similar result holds in continuous time: it is possible to explicitly adjust the stock price process to that which would prevail in a risk-neutral world. The adjusted process is denoted $\tilde{S}(t)$. $\tilde{S}(T)$ and $S(T)$, the distribution of the actual stock price at time T, will be different since the risk-neutral assumption does not hold in the real world.

The adjustment that takes S to $\tilde{S}$ is an adjustment of underlying probabilities. Risk preferences can be understood as a subjective assessment of probabilities for future events. Risk-averse individuals will assign greater than actual probability to bad outcomes. The adjustment we are looking for will be an assessment of these subjective probabilities. In order to reduce the return on a stock from μ to r, the probability of bad outcomes is increased and the probability of good outcomes is decreased. The adjusted probabilities are called an equivalent martingale measure, because the discounted stock price process becomes a martingale with respect to the new probabilities.

Wang's proportional hazard transform method for computing risk-loads also works by altering probabilities (see [34], [35]). Wang's method is therefore in line with modern financial economic thinking and deserves serious consideration by actuaries.

In a risk-neutral world, an option will be valued as the present value of its expected payouts, discounted at the risk-free rate. For a call option with exercise price k this would be

$$e^{-rT}\mathrm{E}((\tilde{S}(T) - k)^+). \tag{4.2}$$

Equation 4.2 gives a pricing formula very similar to the author's equation

$$e^{-rT}\mathrm{E}((S(T) - k)^+). \tag{4.3}$$

The difference between the two is the use of S, the real stock price process, versus $\tilde{S}$, the process that holds in a risk-neutral world. $\tilde{S}$ will have an expected return equal to $r < \mu$ and is certainly different from S, the "probability distribution of market prices at expiry" which Wacek uses in his version of Equation 4.3 on page 703. Section 6 gives a comparison of prices from these two formulae with actual market prices.

Finally, it is now clear that the Black–Scholes formula gives a *price* rather than a *pure premium*. Once the hedging portfolio has been set up there is no risk to the option writer; therefore there is no need for a risk load. In fact, adding a risk load to the Black–Scholes price would create an arbitrage opportunity. Wacek makes this point in his footnote 2, but then obscures the issue by characterizing the rate as a pure premium to allow for an extra risk load when the hedging argument is not available. However, he does not discuss what conditions are necessary in order to use a hedging argument. It turns out the condition is precisely that there exists an equivalent martingale measure, as discussed above (see Duffie [15] for more details on this point). Also, see Gerber and Shiu [17], Cox and Ross [4] and Delbaen, Schachermayer and Schwizier [13] for a discussion of pricing options based on stock price processes other than the lognormal used in Black–Scholes.

5. TWO ACTUARIAL APPLICATIONS OF THE OPTION PRICING PARADIGM

Catastrophe Insurance

I believe the Option Pricing Paradigm view is useful in a situation where Black–Scholes will likely never apply: catastrophe insurance and reinsurance pricing. It tells us to price by computing the cost of being able to bear the risk for the contract period and not by loading the expected loss for risk. For catastrophe reinsurance this means having access to a large, liquid pool of cash. The Option Pricing Paradigm also tells us to move away from a "bank" mentality where reinsurance is providing inter-temporal smoothing, and consider the premium *spent* during the exposure period on bearing the risk. Most other lines of insurance are based on such a point-in-time, between-insured risk sharing, rather than inter-temporal, per-insured risk sharing. As noted in Jaffee and Russell [22], many institutional problems arise for catastrophe insurance precisely because of the inter-temporal way it is currently handled.

Using the Black–Scholes model, the writer of an option uses the option premium to maintain the hedging portfolio (the hedging trading strategy for a call is buy high, sell low, so it is guaranteed to lose money). When the option expires, the initial premium has been exactly used up in stock trading losses whether the option ends up in or out of the money. Similarly, working within this framework, a catastrophe insurance premium should be spent during the policy term, perhaps on maintaining a line of credit, or paying a higher than market interest rate on a cat-bond.

Interestingly, it does not make sense to ask for a contingency reserve with this viewpoint for two reasons. First because at the end of the contract period there is no remaining premium to put into a reserve, and secondly because there is little or no taxable income produced by the product. The need for catastrophe reserves is largely a product of taxation of insurance companies. In this model, the catastrophe risk and premium would pass through the insurance company to an entity, such as a hedge fund, more economically suited to bearing the risk and providing the necessary funding after a large event.

Options on Non-Traded Instruments

The Black–Scholes approach appears to rely on the possibility of taking a position in the underlying stock. This is partially true. More important, however, is the fact that the stock represents the only source of uncertainty, or stochastic behavior, in the system. Writing an option and maintaining a hedging portfolio cancels out the pricing uncertainty, leaving a risk-free portfolio—as discussed above.

Consider an option on an untraded quantity, such as interest rates or an insurance loss index. While, by definition, it is not possible to take a position in the underlying, there is still only one source of uncertainty. Therefore, a Black–Scholes type argument can be used to construct a risk-free portfolio consisting of two options with different exercise prices or expiration dates. The portfolio will use the fact that the two options have instantaneously perfectly correlated prices to cancel all the risk (stochastic behavior). The result is a partial differential equation similar to the Black–Scholes equation involving the prices of the two options as unknowns. Unfortunately, one equation between two unknowns does not give a unique solution. When the underlying is traded, its price is known already, giving one equation in one unknown, which is soluble. However, the partial differential equation can be separated into an expression of the form

$$f(C_1) = f(C_2)$$

for some function f, where C_1 and C_2 are the unknown option prices. Since the lefthand side depends on the expiration and exercise price of C_1 but not C_2, both sides must be a function of the risk-free rate r and time t alone. This implies there are Black–Scholes-like partial differential equations for the prices of C_1 and C_2 each with one extra unknown, called the market price of risk for the underlying index. Since all the option prices depend on the same extra parameter, there are strong consistency conditions put on the prices of a set of options on one underlying instrument. This approach could be useful in an insurance context to help price derivatives off an insurance-based index. For a more detailed explanation of how the approach is applied to price interest rate derivatives, see Wilmot, Howison and Dewynne [36].

6. BLACK–SCHOLES IN ACTION

How well does the Black–Scholes formula perform in practice? It is often asserted that the model is widely used in the industry and also that traders are aware of its weaknesses; it is

TABLE 1

BLACK–SCHOLES PRICES

Exercise Price	Market Price	Traded Volume	Intrinsic Value	BS Price	Percent Error	Implied σ	Actuarial Price
750	186	714	169.77	181.06	−2.7%	32.83%	201.98
805	135	3	114.77	130.90	−3.0	27.92	150.55
890	67 1/2	10	29.77	66.89	−0.9	23.86	82.04
900	64	6	19.77	60.88	−4.9	25.27	75.32
910	59 1/4	102	9.77	55.21	−6.8	25.73	68.93
930	44 1/4	3,291	0.00	44.96	1.6	23.12	57.18
935	41 3/4	5	0.00	42.62	2.1	23.04	54.46
940	42 1/2	264	0.00	40.36	−5.0	24.64	51.83
950	36 1/4	14	0.00	36.11	−0.4	23.57	46.82
960	31 1/2	2	0.00	32.20	2.2	23.12	42.16
990	21	5	0.00	22.37	6.5	22.69	30.19
995	20	107	0.00	20.98	4.9	22.91	28.48
1,025	11	7	0.00	14.07	27.9	21.28	19.71

rarer to see comparisons with market prices. Such a comparison will be given in this section. This is not a scientific test of the model; rather it is supposed to indicate roughly how well it performs.

Table 1 gives the closing prices for all December S&P 500 European call options on September 15, 1997. The calls expired on December 19, 1997. The risk-free force of interest was about 5.12%, giving a discount factor of 0.9868. The S&P 500 closed September 15 at 919.77.

The market price shows the last trade price for each option. The intrinsic value is given by the current index price minus the exercise price, if positive. The Black–Scholes formula price (BS Price) is computed using $\sigma = 23.50\%$, an estimate derived from a contemporaneous sample of S&P daily returns. The implied volatility is calculated by setting the Black–Scholes price equal to the market price and solving for σ. The actuarial price is computed using Equation 3.2 with $r' = r$, the risk-free rate of return and $\mu = 13.98\%$ for a 15% annual return. Assuming

$r' = r$ makes Equation 3.2 exactly the same as Equation 1.3 on page 703 of the paper for a lognormal stock price distribution. Thus, the last column shows the impact on the option price of using the approximate expected rate of return of the underlying instrument rather than the risk-free rate of return; this is the difference between using $\tilde{S}$ and S as the stock price process. Clearly market prices are much closer to the Black–Scholes price. Using a higher discount rate in place of r, but leaving μ unchanged, would bring the actuarial price closer to the Black–Scholes value.

The results are really quite spectacular, especially when compared to the range of reasonable values determined by many actuarial analyses. Remember there is only one free parameter underlying all the model values, and even that is easy to estimate. As Hull [20] points out, the last option trade may have occurred well before the market closed, so the option price may correspond to a different S&P index value than the close. Hull is also a good reference for more information on the mechanics of options markets and for reasons why market prices diverge from Black–Scholes prices.

Finally, Table 1 only provides evidence that the market prices using Black–Scholes or a very similar formula. It does not necessarily follow that this is the "correct" price!

7. CONCLUSION

The thrust of Wacek's paper is that options pricing and insurance pricing are essentially the same and that it should be possible for each discipline to learn from the other. In many ways this is true, particularly on a practical level. Examples include the author's sections on rate guarantees and multi-year contracts. The philosophy of "look for the option" is an important part of modern finance, and is well illustrated by the many applications in Brealey and Myers [2]. Given its central role in finance, actuaries should understand the Option Pricing Paradigm and be able to apply it, where relevant, in their own work. Wacek's paper is valuable because it helps point actuaries in this direction.

However there are some very significant differences between the Option Pricing Paradigm and insurance which should not be glossed over. The Option Pricing Paradigm is based on arbitrage and pricing comparables, and it relies on hedging to remove risk. Insurance assumes and manages specific risk (see Turner [33]). There are typically no liquid markets or close comparables for the specific assets underlying insurance liabilities, and so option pricing techniques do not apply.[7] The specialized underwriting knowledge that insurance companies develop is a key part of their competitive advantage; what they do is bear the resulting underwriting risk, they do not hedge it away.[8] This point is discussed by Santomero and Babbel [32] in their review of financial risk management by insurers. Obviously, how an individual company chooses to manage risk does not alter the market price; the existence of a hedge-based pricing mechanism does, however, determine a market price in the absence of arbitrage.

At a detailed level, Wacek's transformation from the Black–Scholes formula to his supposedly more general Equation 1.3 (Equation 4.3 here) is inappropriate. In this discussion, I have shown how an actuarial approach to option pricing produces a result similar to the Black–Scholes formula but with two important differences: the assumed return on the stock (expected market return compared to risk-free return) and the discount rate (a rate greater than the expected market return compared to the risk-free return). The Black–Scholes argument shows that writing options is risk-free (in the conceptual model) because of the possibility of setting up a self-financing hedging strategy with the proceeds from writing an option. No-arbitrage then implies

[7] As Babbel says in [1]: "When it comes to the valuation of insurance liabilities, the driving intuition behind the two most common valuation approaches—arbitrage and comparables—fails us."

[8] Unless they can use their specialized knowledge to arbitrage the reinsurance markets!

the price of the option must be the smallest amount for which it is possible to set up a hedging portfolio. It follows that the option price is independent of an individual investor's risk preferences. Cox and Ross [4] then argued that the option can be priced assuming risk-neutrality. In a risk-neutral world stocks earn the risk-free return, thus explaining the first assumption. The hedging portfolio argument also shows the risk-free rate is appropriate for discounting, which explains the second assumption. Wacek makes the latter point but does not mention the former. As shown in Table 1, there is a significant difference between the option prices with and without the former assumption. Moreover, market prices are consistently closer to the Black–Scholes prices. The discussion of hedging and options pricing also makes it clear Black–Scholes gives a price, not a pure premium. Finally, Wacek's assertion that "the pricing mathematics is basically the same" for options and insurance is not really the case. Doubts as to this point can be dispelled by looking in any more advanced text on options pricing, such as [14].

In this review, a simple discrete time example has been given to illustrate the hedging portfolio argument. It shows how the option price is independent of risk preferences *given the current stock price*. While the example is often reproduced in finance texts, the discussion of exactly how risk preferences fit in (through Equation 4.1) is less common. A new application of the Option Pricing Paradigm to catastrophe insurance was proposed, and how the paradigm works in the case of an underlying which is not traded was discussed. Finally, a comparison of market prices, Black–Scholes prices and actuarial prices for some S&P options has been given.

The Black–Scholes option pricing formula is an important and beautiful piece of mathematics and financial economics. On the surface the formula is just the discounted expected excess value of a lognormal random variable—the tricky part is *which* lognormal variable! Understanding some of the paradigm lying behind the formula, and some of its subtleties, gets us to the core of the differences between how insurers and other financial institutions bear and manage risk. Given the current convergence between insurance and banking it is important for insurance actuaries to understand and to be able to exploit these differences—our future livelihoods could depend upon it.

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THE 1999 TABLE OF INSURANCE CHARGES

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"The problem is all inside your head,
She said to me.
The answer is easy if you
Take it logically.
I'd like to help you in your struggle
To be free;
There must be fifty ways
To leave your lover."

—Paul Simon

Abstract

This paper describes the development of the 1999 Workers Compensation Table of Insurance Charges (Table M), filed in NCCI states to be effective November 1, 1998.

It presumes the reader knows what Table M is, and how it is used in retrospective rating. Familiarity with the NCCI Retrospective Rating Plan (the Plan) is helpful.

Development of the 1999 Table M is described in steps, beginning with the sample data and how it was manipulated, followed by the algorithm used to model loss ratios, methods for developing the loss ratio distribution, graduation of the excess ratios and derivation of Expected Loss Size Ranges used in the Plan.

The impact on premium of the proposed new Table M is estimated.

1. BACKGROUND

At the heart of retrospective rating is a table of excess ratios commonly called Table M. Details on the definition and use of

Table M are provided much more fully elsewhere in the literature. See [1], [2], [3] and [4] among the references to this paper. The reader is expected to know the vocabulary and basic significance of the table.

The Workers Compensation Table of Insurance Charges, the proper name of Table M, was last changed in 1984. Derivation of the 1984 Table M was never documented in *PCAS*, as was done for the 1964 Table M by LeRoy Simon, but it served well for almost 15 years. Its passing is hereby lamented, if a bit satirically, in the song above, fittingly by another Simon. (One of fifty ways: “Don’t need to be coy, Roy.”)

Since 1984, annual updates have been made to account for inflation in the average cost per case, which was quite significant during the late 1980s. The body of Table M was not changed, but only the expected loss sizes necessary to qualify for specific columns of the table. Increasing skewness in the severity distribution, as discussed below, impacted the loss ratio distribution, and this needed to be reflected in the body of Table M; changes of this sort are not accounted for in inflationary updates. Even if the table was approximately adequate in 1998, which our research verified, the need to update the body of Table M was evident.

The changes in the loss size distribution were recognized early on and led to non-trivial updates in the calculation of excess loss factors (ELFs), also used in the retrospective rating plan. Three revisions were made (in 1987, 1992, and 1996) to the model distributions used in the calculation of ELFs. The changes are documented in [5] and [6]. In general, each step involved recognition of increasing skewness in the distribution of serious claims by size.

The changes in loss severity are a sign of the times. Underwriting results in the workers compensation line of business during the last two decades of this century are a matter of record.

Volatility in premium adequacy was driven by changes in legal rules, program administration, benefits, and salaries, as well as a generally increasing feeling of entitlement among the public starting in the 1970s. During the late 1980s, this led to the need for large rate increases, relatively more large compensation awards and a heavier tailed severity distribution.

2. OUTLINE

This is a short description of the basic steps in the creation of the 1999 Table of Insurance Charges. There is a section of the paper for each step.

A. *Sample Data*—Premium and loss information was assembled for a sample of 450,156 insureds from policy year 1988 at fifth report. The sample was grouped into 25 overlapping adjusted expected loss size ranges. Risk expected loss size is adjusted by formula: a product of standard expected loss and the appropriate state and hazard group relativity. The empirical loss ratio distribution of each group was normalized to a mean of unity.

B. *Modeling Sample Excess Ratios*—For each sample group, the empirical excess ratio distribution was fitted to excess ratios based on a Heckman–Meyers (HM) model distribution, as described in [7] and [8]. The severity distribution used in the model was exactly the one underlying the empirical fifth report, and the frequency parameters were selected to effect a fit.

C. *Development to Ultimate*—The fifth report severity distribution was replaced with one developed to ultimate. The severity uncertainty parameter was increased to account for loss ratio uncertainty, i.e., parameter risk. An ultimate excess ratio distribution was produced for 26 groups. (The 26th distribution was based on a hypothetically large expected number of claims and, as such, had no empirical sample.)

D. *Graduating the Table*—Inverse exponential polynomials were used to graduate 26 model excess ratio distributions.

E. *The 1999 Table of Insurance Charges*—This is it!

F. *Derivation of Size Ranges*—Sample risk average (formula) adjusted size and HM model frequency were used to derive nominal average severity by sample group. By selecting one average severity, we were able to assign a 1988 expected loss size, frequency times severity, to each of 26 mother curves. Interpolation was used to create boundaries for (adjusted) expected loss size ranges, indexed by charges at entry ratio unity of 0.095 to 0.975. Estimated severity trend was used to make size ranges appropriate for 1999.

G. *Estimating the Impact*—The algorithm described in [4] was used to estimate premium recovered by using Table M.

3. THE PROCESS

Sample Data

We used the latest available statistical plan data at fifth report for the review. The unit statistical information includes the following information for each policy in each state: payroll by class, manual premium (which is an extension of payroll along the respective class rates), experience modification, standard premium and loss. We were able to group this information by risk to allow the tabulation of standard expected loss (standard premium times permissible loss ratio), actual loss, loss ratio and hazard group of each risk (which can occasionally vary by state).

Exhibit 1 shows the actual policy periods by state used for this analysis. These are close to policy year 1988, but vary by state according to the filing schedules.

The exhibit also shows state and hazard group severity relativities (S&HGRs), effective 10/1/91. In order to assign a risk to a column of Table M, the risk's adjusted expected loss size is used. Starting with standard expected loss by state, the adjustment is accomplished by application of the appropriate S&HGR. This should account for known differences in scale of severity distributions and is part of the current retrospective rating plan. Ideally, we would have used values calculated for 1988, but 1991 was the first year the filing was effective and these were the earliest calculated, so we used these to adjust expected losses. We believe using the relativities is essential to be consistent with the use of Table M in the retrospective rating plan, notwithstanding the discrepancy in effective date. Experience has shown the S&HGRs, which are relativities to the average, do not change much from year to year.

Using adjusted expected losses by risk, we grouped sample risks into 25 size ranges. To maximize the number of risks in each group, we allowed the size ranges to overlap, so some risks fall into two different size ranges. Exhibit 2 shows the 1988 expected loss size ranges used. Column 1 shows the applicable indices for columns of the table. As described in [1], columns of Table M are indexed by the charge at entry ratio 1, in percent. So for the third row, applicable columns have charges at unity of 0.16 to 0.22. This corresponds to seven size ranges:

Index	1988 Expected Loss Minimum	1988 Expected Loss Maximum
16	4,386,336	5,565,157
⋮	⋮	⋮
21	1,544,131	1,872,497
22	1,281,534	1,544,130

Risks with adjusted loss within the total range formed a sample group of loss ratios from which empirical excess ratios were calculated.

Embedded in the 1988 risk data used is a fifth report severity distribution, all states combined. This distribution became an integral part of the next step.

Modeling Sample Excess Ratios

Each sample group of risks exhibited an empirical distribution of loss ratios, $F[x]$. The loss ratios x could easily be converted into entry ratios r by dividing x by the average, so that $\mathrm{E}[r] = 1.0$. The excess ratios $X(r) = \sum_{s=r}^{\infty}(s-r)f(s)$, where f is the normalized density, were calculated directly from the data. See [2] or [3] for a more detailed treatment of the topic of excess ratios.

This is a summary of the HM model as it pertains to this application: Using the collective risk model from risk theory, the HM algorithm creates a loss ratio distribution from underlying frequency and severity distributions. The algorithm uses the moment generating function of the frequency and the characteristic function for the severity to derive the characteristic function of the loss ratio distribution, then inverts it to generate the aggregate distribution. Using simplifying assumptions, the input data are an expected claim count λ and a contagion parameter c to model the frequency distribution, and a piecewise linear severity distribution with a severity uncertainty parameter b to model severity. See [7], [8], and [9] for details.

We used the fifth report severity distribution along with choices of λ, c, and b to fit the empirical sample of excess ratios.

Each sample excess ratio distribution was fit directly to one based on the HM model. Exhibit 3 shows results of the modeling process. (Each page shows results of 25 different fits at entry ratios 1.0 and 3.0.) There was no special technique used to effect the fit. It turned out that using the appropriate frequency to match the excess ratio at the entry ratio $r = 1$ assured a fairly good fit to the entire sample excess ratio distribution. We started with λ proportional to the average adjusted expected loss size of the sample. A severity uncertainty parameter $b = 0.001$ worked well in the fit. Such a small value makes sense given that the empirical sample severity distribution and normalized loss ratio are determinate. There was more room to adjust the contagion parameter. We started with $c = 0.30$ for the small size groups. To fit the distribution for larger risks, we needed to vary the contagion parameter c downward, as can be seen in Exhibit 3. We made fine-tuning adjustments to λ or c to extend the fit to all entry ratios for all 25-size groups.

Development to Ultimate

We did not change frequency parameters λ and c after fifth report, assuming that change in the claim frequency distribution is insignificant after fifth report. This is a reasonable assumption, borne out by empirical evidence of very little frequency development from fourth to fifth report. In practice, of course, some claims may close with no payment or emerge as IBNR, but this could be considered a matter of parameter risk as discussed below. We thus retained the fifth report model parameters λ and c.

The development of the claim severity distribution is another matter. We had learned a lot about loss severity development in excess loss factor (ELF) studies, first in 1992 and carried further in 1996, as described in [6]. Underlying the ELF procedure are three indemnity claim size distributions by injury group, developed to ultimate. We also have one for medical-only claims. We were able to create an ultimate severity distribution consistent with the 1988 data by weighting scaled component distributions. Each state has its own (estimated) ultimate scale and injury weight for each of these distributions.

We created an ultimate severity distribution using techniques much the same as in the ELF procedure [5]. We were able to develop average costs per case at ultimate and injury type weights by state. We used those severities to scale the underlying distributions and the weights to combine them across injury group by state and then across states.

1. Empirical average severity by state and injury type was calculated from the 1988 data.
2. Fifth to ultimate severity development factors were calculated for serious injury types by state. This was done by attributing the fifth to ultimate loss development factors from ratemaking entirely to serious injury claim size.
3. The development factors were applied respective of state and injury type.
4. Developed permanent total (PT) and major permanent partial (Major) claim types were combined, as well as fifth report minor permanent partial (Minor) and temporary total (TT), to obtain average severities for fatal, PT/Major, Minor/TT and medical-only injury groups by state.
5. Loss weights by injury group and state were calculated.
6. Average severities and loss weights by injury group were used with respective ultimate loss size distributions from the ELF procedure, and an associated distribution for medical-only, to make a weighted average severity distribution for all claims combined by state.
7. All states were weighted together to create one ultimate 1988 severity distribution. This was fit by a piecewise linear model for use in the HM algorithm.

Now, using this derived ultimate severity distribution,[1] we were able to use HM to create an ultimate loss ratio distribution with corresponding excess ratios. We did this first retaining $b = 0.001$ as at fifth report. The impact of changing to an ultimate severity distribution was considerable. This can be seen in Exhibit 4 by comparing column 5 to column 4. We saw increases of up to 5

[1]Both the Table of Insurance Charges and the underlying severity distribution are products for sale by NCCI; hence they are not included in this paper.

percentage points in excess ratios for entry ratio 1, applicable to risks in the size ranges most impacted by retrospective rating (25 to 60). We saw even larger increases at the higher entry ratios, reflecting the increased skewness of the loss ratio distribution based on ultimate severity.

To account for parameter risk, we chose to increase the severity uncertainty parameter b from 0.001 to 0.015. This is a judgment call, and based on our estimates of loss ratio uncertainty, not simply scale uncertainty. We needed to account for the fact that the expected loss ratio for each risk is only an estimate. The flexibility of the HM model to allow such an adjustment is a huge advantage of HM over competing models. Even though $b = 0.015$ represents a 12.2% uncertainty in expected loss ratio, which is large, the resulting increase in charges (about half a percentage point) did not seem that excessive. A comparison of columns 7 and 5 shows the impact of this choice is much smaller than the change from column 4 to 5.

The result of the process is 25 sample excess ratio tables based on 25 loss ratio distribution models with underlying frequency and severity distribution. A 26th sample was created using the ultimate severity distribution and enough expected claims to produce a charge at unity of less than 0.095.

Graduating the Table

Having 26 columns of an excess ratio table based on HM model loss distributions is a wonderful thing, but in practice the functional form of the associated insurance charges is more complex than practitioners may have wanted. They did want more of other qualities: ease of data entry and at least 80 columns with charges at unity in even percents. (The charges at unity of the 26 models were not necessarily integral percents.)

The 1984 table could be generated by interpolating between a sample of 19 inverse exponential polynomials, and two boundary functions. A similar algorithm to generate the new table was needed. This was accomplished in the following manner, a slightly simplified version of the prior technique.

1. Each sample HM excess ratio table was modeled. This was done by catenating three models:

   a. $X(r) = 1 - r$ for small values of r (at least for the larger size groups).

      The HM samples verified this simple expression for X, so it was better not to try and extend the fit further than necessary.

   b. $X(r) = \exp[\sum_{k=1}^{8} a_k r^k]$ for medium values for r. The coefficients are derived from a regression on the HM model excess ratios, and of course differ by size range. By limiting the fit to these critical values of r, very close approximations are possible.

   c. $X(r) = X(r_l)^{k(r)} X(r_u)^{1-k(r)}$, where

   $$k(r) = \left[\frac{r_u - r}{r_u - r_l}\right], \qquad \text{and} \qquad r_l \le r \le r_u.$$

      $X(r_u)$ is taken from the fitted curve in (b), and $X(r_l)$ is taken from the underlying HM sample tables. (r_l, r_u) is the interval of (largest) entry ratios where this simple decay works best.

   These provide sample curves for interpolating the final table. For curves 29.35 and 33.16, Exhibit 5 shows results of this modeling.

2. Two more mother curves were defined to be used as boundary values.

$$X_0(r) = \begin{cases} 1 - r & 0 \le r \le 1 \\ 0 & \text{for all} \quad r \ge 1, \\ & \text{where } r \text{ is the entry ratio} \end{cases}$$

$$X_{100}(r) = 1 \qquad \text{for all} \quad r \ge 0$$

   Now there are 28.

3. Linear interpolation between the 28 mother curves was used to generate the columns of the table with integral percent p charge at unity entry ratio. For any entry ratio r, the charge for the column indexed by p is:

$$X_p(r) = X_l(r) + \left[\frac{X_p(1) - X_l(1)}{X_u(1) - X_l(1)}\right][X_u(r) - X_l(r)]$$

Exhibit 6 is a graph of the values used to interpolate X_{32}.

The 1999 Table of Insurance Charges

After jumping through hoops, standing on our heads, and spitting wooden nickels to create this table, it is time to take a break and enjoy a picture. Exhibit 7 is a three-dimensional graph of the Table (r,X,I). From the point of view of the reader, the graph is a concave surface. The vertical X axis is the charge. The entry ratios r from 0.0 to 3.2 go from left to right coming towards the reader across the left half of the page, and the size group column shows indices from 0 to 100, going away from left to right across the right half.

The surface is flat (planar) in the upper left, where entry ratios are low and risks are large. In this region, $X(r) = 1 - r$, which implies loss ratios are always at least rE. The curved isoclines denote constant charges of $0.90, 0.80, \ldots, 0.10$ from top to bottom. Note that there is an implied isocline for $X = 0.0$.

The foreground cross-section of the surface is concave and increasingly so for larger and larger values of r, as it will tuck in closer and closer to the line $(r,0,1)$ behind the surface on the right hand side of the page. This is because for all size groups bigger than the boundary where $X(1) = 1$, the charge approaches 0 as r increases. Within the contoured surface, there is a straight line where $r = 1$, and the charge is 1/100 of the index $(1, X, 100X)$.

On Derivation of Size Ranges

Thus far we have developed a table with columns indexed by the (percent) insurance charge at unity entry ratio. As explained above, this is based on 26 loss ratio distributions, complete with frequency and severity parameters.

Table M was to be filed effective November 1, 1998. It was necessary to determine size ranges to be used for selection of the columns of the table applicable to an individual risk. These ranges would of course be adjusted going forward for trend in average severity.

The HM model severity is scale free in the sense that the loss ratio distribution, and consequent table of excess ratios, depends on the expected claim count and the shape, but not the size, of the severity distribution. If we could attach a scale to this distribution, we could use frequency times adjusted average severity to determine a dollar size corresponding to each model.

Exhibit 8 shows the first step in the estimation of the implicit average adjusted severity. We have already calculated the average adjusted expected loss of each of 25 empirical sample groups in Exhibit 2. Our modeling process assigned a frequency (expected claim count) to each group which was needed to match the sample loss ratio distribution. In Exhibit 8, we simply divide the adjusted expected losses by the expected claim count to produce an expected average adjusted severity. This of course varies with each sample group, but, except for a small upward tick in the largest size ranges, the estimated severity is remarkably flat.

We selected an average adjusted severity value consistent with the 1988 samples. Using expected claim counts from the models (including the hypothetical model), the product is the point estimate of adjusted expected loss size for the 26 samples in 1988. This is shown in Exhibit 9.

Exhibit 10 shows how we developed 1988 size ranges. In short, we used interpolation between the 26 points to estimate adjusted expected losses corresponding to the boundaries of the ranges (i.e., even percents +0.005).

We wanted to adjust these average sizes to a point in 1999. Using statistical plan data, we were able to determine that the actual severity trend between 1988 and 1994 was about +25%. Independent analysis of the most recent available data showed severity trend to be nearly flat from 1993 through 1996, so we projected no severity trend after 1994. Using the 1.25, we determined boundaries of expected loss size ranges applicable in 1998/99. This is column 7 of Exhibit 10.

Estimating the Impact

The 1999 Table of Insurance Charges was filed effective 11/1/98, replacing the 1984 Table. The aggregate impact on expected retrospective rating premium was not great.

In the body of Table M, for the low entry ratios associated with the run of the mill retrospective rating contracts sold, the changes in the table values were fairly small. The change to the expected loss size ranges, although not a simple linear inflation, was also moderate. This can be seen in Exhibit 11. For the high charge/small expected loss size columns, the inflationary impact was minimal, with less than a 7% increase in the expected loss size ranges 72 to 46. This encompasses 1999 expected losses of $17,000 to $132,500. The size of a risk needed to qualify for the lower charge columns grew significantly, so that to qualify for column 20, a risk had to be 60% larger ($5.9 vs. $3.7 million). We assume not many risks this size are written on a straight retro, even before this change.

Using methods described in [4], we were able to estimate an impact of about +1% on expected retrospectively rated premium. Exhibit 12 shows the evaluation. Assuming the new table is a correct measure of excess ratios, we began with sample plans for representatives of each range, calculated based on the old Table M and size groups. The expected retrospective premium written under those plans was evaluated with the proposed table and size ranges applicable to actual losses. The estimated shortfall is about 1% of premium which would (theoretically) be recovered if the new table is implemented.

4. CONCLUSION

The new Table of Insurance Charges is a significant improvement to the former table. This is not a matter of pricing adequacy, as the estimated overall impact is small. What matters more is the increase in individual risk equity due to the associated nonlinear update of expected loss size ranges, but even this is only part of the story.

The use of explicit underlying frequency and severity distributions is a great advance in the science, making the table more useful in new as well as standard applications. It allows for much more facile future updates, not only for inflation, but also for changes in workers compensation law, administration or environment. There must be 50 ways.

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[5] Gillam, William R., "Retrospective Rating: Excess Loss Factors, *PCAS* LXXVIII, 1991 p. 1.

[6] Gillam, William R., "Retrospective Rating: 1997 Excess Loss Factors," *PCAS* LXXXIV, 1997, p. 450.

[7] Heckman, Philip E., and Glenn G. Myers, "The Calculation of Aggregate Loss Distributions from Claim Severity and Claim Count Distributions," *PCAS* LXX, 1983, p. 22.

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[9] Klugman, Stuart A., Harry H. Panjer, and Gordon E. Willmot, *Loss Models: From Data to Decisions*, New York: John Wiley & Sons, Inc., 1998.

EXHIBIT 1

STATE AND HAZARD GROUP RELATIVITIES USED TO DETERMINE EXPECTED LOSS SIZE GROUP NCCI RETROSPECTIVE RATING MANUAL, EFFECTIVE 10/1/91

State Name	Policy Year Beginning	Relativity by Hazard Group			
		I	II	III	IV
AL	5/1/88	1.677	1.512	1.161	0.936
AK	4/1/88	1.142	0.955	0.712	0.565
AZ	3/1/88	1.264	1.139	0.867	0.706
AR	4/1/88	1.326	1.197	0.912	0.732
CO	3/1/88	1.097	0.972	0.712	0.547
CT	1/1/88	1.418	1.294	0.954	0.748
DC	4/1/88	1.322	1.160	0.859	0.682
FL	10/1/87	0.831	0.750	0.560	0.500
GA	2/1/88	1.182	1.049	0.800	0.645
HI	6/1/88	1.487	1.293	0.958	0.777
ID	3/1/88	1.350	1.203	0.923	0.754
IL	4/1/88	1.353	1.236	0.960	0.792
IN	12/1/87	1.844	1.701	1.343	1.104
IA	3/1/88	1.627	1.474	1.105	0.888
KS	1/1/88	1.432	1.280	0.967	0.775
KY	1/1/88	1.352	1.211	0.917	0.737
LA	4/1/88	0.874	0.813	0.611	0.500
ME	6/1/88	1.210	1.099	0.820	0.643
MD	4/1/88	1.398	1.276	0.974	0.801
MI	4/1/88	1.098	0.991	0.748	0.595
MS	1/1/88	1.368	1.229	0.932	0.751
MO	1/1/88	2.000	1.879	1.505	1.260
MT	11/1/87	1.066	0.968	0.727	0.579
NE	2/1/88	1.425	1.253	0.942	0.757
NH	4/1/87	1.424	1.275	0.948	0.750
NM	1/1/88	1.132	0.982	0.744	0.595
NC	4/1/88	1.575	1.438	1.107	0.900
OK	6/1/87	1.310	1.187	0.927	0.766
OR	1/1/88	1.013	0.938	0.723	0.594
RI	1/1/88	1.171	1.035	0.765	0.611
SC	1/1/88	1.353	1.235	0.939	0.760
SD	1/1/88	1.273	1.139	0.849	0.674
TN	1/1/88	1.401	1.283	1.001	0.835
UT	5/1/87	1.479	1.297	0.971	0.779
VT	4/1/88	1.521	1.337	1.003	0.806
VA	2/1/88	1.323	1.207	0.915	0.738
WI	1/1/88	1.900	1.771	1.976	1.144

EXHIBIT 2

1988 RISK ADJUSTED SAMPLE SIZE RANGES

1988 ELG Range (1)	No. of Risks in Sample (2)	Adjusted Expected Loss Range ($) (3)	Average Adjusted Expected Loss ($) (4)
5–14	98	7,152,383 – & above	110,489,802
9–19	369	2,286,901 – 49,031,955	12,000,542
16–22	646	1,281,534 – 5,565,157	2,921,544
21–26	1,248	640,985 – 1,872,497	1,385,528
24–29	1,973	398,112 – 1,069,773	742,680
28–33	3,809	219,916 – 544,970	424,437
31–36	5,961	144,215 – 341,870	252,281
35–39	6,116	107,226 – 190,702	154,055
38–42	5,584	85,218 – 125,725	111,337
41–46	8,894	63,128 – 99,263	88,321
44–49	10,965	50,529 – 79,016	67,843
48–52	11,514	40,455 – 58,604	52,322
51–56	17,564	30,005 – 46,921	41,980
54–57	13,435	27,822 – 37,557	32,220
58–63	28,663	17,465 – 27,821	22,867
63–65	18,360	14,858 – 18,908	17,458
65–69	38,026	10,599 – 16,116	13,629
69–73	50,227	7,364 – 11,554	10,584
71–77	84,492	4,932 – 9,701	8,059
75–82	106,705	2,760 – 6,688	5,478
79–85	82,613	1,824 – 4,427	3,410
84–87	37,374	1,329 – 2,419	1,980
87–90	35,669	752 – 1,562	1,227
90–95	70,888	171 – 922	586
94–99	55,485	0 – 352	177

EXHIBIT 3

Page 1

Excess Ratio Distribution Modeling Results

1988 ELG Range	λ	c	Entry Ratio 1.0 (1) Empirical Group Avg. ϕ	Entry Ratio 1.0 (2) HM-5th (b = .001) ϕ
N/A	75,000	0.040	N/A	
5–14	20,000	0.075	0.106	0.1111
9–19	2,200	0.140	0.159	0.1590
16–22	600	0.190	0.205	0.2008
21–26	310	0.190	0.229	0.2206
24–29	170	0.190	0.268	0.2479
28–33	100	0.190	0.298	0.2818
31–36	65	0.190	0.325	0.3172
35–39	42	0.190	0.362	0.3610
38–42	32	0.190	0.389	0.3922
41–46	24.25	0.190	0.427	0.4268
44–49	19.50	0.205	0.458	0.4577
48–52	14.25	0.205	0.500	0.5009
51–56	11.00	0.220	0.539	0.5390
54–57	9.55	0.220	0.559	0.5590
58–63	6.35	0.250	0.619	0.6187
63–65	5.00	0.300	0.654	0.6540
65–69	3.80	0.300	0.690	0.6895
69–73	2.75	0.300	0.729	0.7288
71–77	1.98	0.300	0.766	0.7657
75–82	1.25	0.300	0.811	0.8113
79–85	0.87	0.300	0.843	0.8425
84–87	0.56	0.300	0.874	0.8741
87–90	0.37	0.300	0.902	0.8978
90–95	0.15	0.300	0.952	0.9338
94–99	0.03	0.300	0.978	0.9743

EXHIBIT 3

PAGE 2

EXCESS RATIO DISTRIBUTION MODELING RESULTS

			Entry Ratio 3.0	
1988 ELG Range	λ	c	(1) Empirical Group Avg. ϕ	(2) HM-5th (b = .001) ϕ
N/A	75,000	0.040	N/A	
5–14	20,000	0.075	0.000	0.0001
9–19	2,200	0.140	0.002	0.0001
16–22	600	0.190	0.015	0.0020
21–26	310	0.190	0.015	0.0068
24–29	170	0.190	0.054	0.0150
28–33	100	0.190	0.048	0.0272
31–36	65	0.190	0.053	0.0434
35–39	42	0.190	0.076	0.0688
38–42	32	0.190	0.089	0.0905
41–46	24.25	0.190	0.120	0.1183
44–49	19.50	0.205	0.147	0.1455
48–52	14.25	0.205	0.198	0.1898
51–56	11.00	0.220	0.237	0.2331
54–57	9.55	0.220	0.258	0.2581
58–63	6.35	0.250	0.335	0.3378
63–65	5.00	0.300	0.384	0.3880
65–69	3.80	0.300	0.444	0.4431
69–73	2.75	0.300	0.503	0.5071
71–77	1.98	0.300	0.568	0.5698
75–82	1.25	0.300	0.648	0.6493
79–85	0.87	0.300	0.705	0.7046
84–87	0.56	0.300	0.758	0.7631
87–90	0.37	0.300	0.807	0.8088
90–95	0.15	0.300	0.903	0.8804
94–99	0.03	0.300	0.954	0.9486

EXHIBIT 4

PAGE 1

EXCESS RATIO MODEL DEVELOPMENT TO ULTIMATE

			Entry Ratio = 1.0				
(1) 1988 ELG Range	(2) λ	(3) c	(4) HM-5th (b = .001) ϕ	(5) HM-Ult. (b = .001) ϕ	(6) HM-Ult. (b = .01) ϕ*	(7) HM-Ult. (b = .015) ϕ*	(8) HM-Ult. (b = .02) ϕ*
N/A	75,000	0.040	0.0946				
5–14	20,000	0.075	0.1111	0.1144	0.1203	0.1234	0.1265
9–19	2,200	0.140	0.1590	0.1737	0.1777	0.1798	0.1819
16–22	600	0.190	0.2008	0.2278	0.2308	0.2325	0.2341
21–26	310	0.190	0.2206	0.2557	0.2584	0.2599	0.2613
24–29	170	0.190	0.2479	0.2899	0.2923	0.2936	0.2948
28–33	100	0.190	0.2818	0.3284	0.3305	0.3316	0.3327
31–36	65	0.190	0.3172	0.3660	0.3678	0.3688	0.3698
35–39	42	0.190	0.3610	0.4101	0.4117	0.4125	0.4133
38–42	32	0.190	0.3922	0.4406	0.4420	0.4427	0.4435
41–46	24.25	0.190	0.4268	0.4737	0.4749	0.4756	0.4763
44–49	19.50	0.205	0.4577	0.5027	0.5037	0.5043	0.5049
48–52	14.25	0.205	0.5009	0.5429	0.5438	0.5443	0.5448
51–56	11.00	0.220	0.5390	0.5777	0.5784	0.5789	0.5793
54–57	9.55	0.220	0.5590	0.5959	0.5966	0.5970	0.5974
58–63	6.35	0.250	0.6187	0.6489	0.6495	0.6498	0.6501
63–65	5.00	0.300	0.6540	0.6799	0.6804	0.6807	0.6810
65–69	3.80	0.300	0.6895	0.7115	0.7120	0.7123	0.7126
69–73	2.75	0.300	0.7288	0.7473	0.7477	0.7479	0.7481
71–77	1.98	0.300	0.7657	0.7816	0.7819	0.7821	0.7823
75–82	1.25	0.300	0.8113	0.8247	0.8249	0.8251	0.8252
79–85	0.87	0.300	0.8425	0.8543	0.8545	0.8546	0.8546
84–87	0.56	0.300	0.8741	0.8843	0.8844	0.8845	0.8846
87–90	0.37	0.300	0.8978	0.9063	0.9064	0.9064	0.9065
90–95	0.15	0.300	0.9338	0.9382	0.9382	0.9383	0.9383
94–99	0.03	0.300	0.9743	0.9748	0.9748	0.9748	0.9748

*b parameter is increased to account for loss ratio uncertainty.

EXHIBIT 4

PAGE 2

EXCESS RATIO MODEL DEVELOPMENT TO ULTIMATE

			Entry Ratio = 3.0				
(1) 1988 ELG Range	(2) λ	(3) c	(4) HM-5th (b = .001) ϕ	(5) HM-Ult. (b = .001) ϕ	(6) HM-Ult. (b = .01) ϕ*	(7) HM-Ult. (b = .015) ϕ*	(8) HM-Ult. (b = .02) ϕ*
N/A	75,000	0.040	0.0000				
5–14	20,000	0.075	0.0001	0.0001	0.0001	0.0001	0.0001
9–19	2,200	0.140	0.0001	0.0016	0.0018	0.0020	0.0021
16–22	600	0.190	0.0020	0.0148	0.0155	0.0159	0.0163
21–26	310	0.190	0.0068	0.0314	0.0323	0.0327	0.0332
24–29	170	0.190	0.0150	0.0537	0.0547	0.0553	0.0559
28–33	100	0.190	0.0272	0.0810	0.0822	0.0828	0.0835
31–36	65	0.190	0.0434	0.1098	0.1111	0.1118	0.1125
35–39	42	0.190	0.0688	0.1463	0.1477	0.1485	0.1492
38–42	32	0.190	0.0905	0.1733	0.1747	0.1755	0.1763
41–46	24.25	0.190	0.1183	0.2046	0.2060	0.2068	0.2076
44–49	19.50	0.205	0.1455	0.2326	0.2341	0.2349	0.2357
48–52	14.25	0.205	0.1898	0.2760	0.2774	0.2781	0.2789
51–56	11.00	0.220	0.2331	0.3162	0.3175	0.3183	0.3190
54–57	9.55	0.220	0.2581	0.3390	0.3403	0.3410	0.3417
58–63	6.35	0.250	0.3378	0.4099	0.4110	0.4116	0.4122
63–65	5.00	0.300	0.3880	0.4540	0.4550	0.4555	0.4560
65–69	3.80	0.300	0.4431	0.5023	0.5031	0.5035	0.5040
69–73	2.75	0.300	0.5071	0.5574	0.5581	0.5585	0.5588
71–77	1.98	0.300	0.5698	0.6106	0.6112	0.6115	0.6119
75–82	1.25	0.300	0.6493	0.6781	0.6787	0.6790	0.6793
79–85	0.87	0.300	0.7046	0.7271	0.7275	0.7277	0.7279
84–87	0.56	0.300	0.7631	0.7806	0.7809	0.7811	0.7813
87–90	0.37	0.300	0.8088	0.8235	0.8238	0.8239	0.8240
90–95	0.15	0.300	0.8804	0.8908	0.8909	0.8909	0.8910
94–99	0.03	0.300	0.9486	0.9508	0.9508	0.9509	0.9509

*b parameter is increased to account for loss ratio uncertainty.

EXHIBIT 5

Insurance Charges for Curve 29.35				Insurance Charges for Curve 33.16			
Entry Ratio (r)	Heckman Meyers Model	Graduated	Equation # Used	Entry Ratio (r)	Heckman Meyers Model	Graduated	Equation # Used
0.01	0.9900	0.9900	—	0.01	0.9900	0.9900	—
0.02	0.9800	0.9800		0.02	0.9800	0.9800	
0.03	0.9700	0.9700		0.03	0.9700	0.9700	
0.04	0.9600	0.9600		0.04	0.9601	0.9600	$1-r$*
0.05	0.9600	0.9500		0.05	0.9601	0.9500	
0.06	0.9501	0.9400	$1-r$*	0.06	0.9502	0.9400	
0.07	0.9401	0.9300		0.07	0.9403	0.9300	
0.08	0.9302	0.9200		0.08	0.9304	0.9200	—
0.09	0.9203	0.9100		0.09	0.9206	0.9102	
0.10	0.9104	0.9000		0.10	0.9108	0.9005	
0.11	0.9005	0.8900	—	0.25	0.7632	0.7629	
0.25	0.7669	0.7580		0.50	0.5732	0.5733	
0.50	0.5543	0.5543		0.75	0.4325	0.4326	
0.75	0.4017	0.4016		1.00	0.3316	0.3316	
1.00	0.2936	0.2935		1.25	0.2596	0.2595	
1.25	0.2187	0.2188		1.50	0.2080	0.2079	
1.50	0.1671	0.1672		1.75	0.1704	0.1704	
1.75	0.1312	0.1312		2.00	0.1427	0.1427	
2.00	0.1058	0.1058		2.25	0.1217	0.1217	
2.25	0.0874	0.0874		2.50	0.1056	0.1056	
2.50	0.0737	0.0737	IEP†	2.75	0.0930	0.0930	IEP†
2.75	0.0634	0.0634		3.00	0.0828	0.0828	
3.00	0.0553	0.0553		3.25	0.0746	0.0746	
3.25	0.0489	0.0489		3.50	0.0678	0.0678	
3.50	0.0437	0.0437		3.75	0.0620	0.0620	
3.75	0.0394	0.0394		4.00	0.0571	0.0571	
4.00	0.0357	0.0357		4.25	0.0529	0.0529	
4.25	0.0326	0.0326		4.50	0.0493	0.0493	
4.50	0.0300	0.0300		4.75	0.0460	0.0460	
4.75	0.0276	0.0276		5.00	0.0432	0.0432	
5.00	0.0256	0.0256		5.25	0.0406	0.0406	
5.25	0.0237	0.0237		5.50	0.0383	0.0383	
5.50	0.0221	0.0221		5.75	0.0362	0.0362	
5.75	0.0206	0.0206		6.00	0.0343	0.0343	—
6.00	0.0192	0.0192	—	7.00	N/A	0.0288	
7.00	N/A	0.0152		8.00	N/A	0.0242	ED‡
8.00	N/A	0.0120	ED‡	9.00	N/A	0.0204	
9.00	N/A	0.0095		10.00	N/A	0.0171	—
10.00	N/A	0.0075	—				

*Equation (2)—Straight Line (see Appendix)
†Equation (1)—Inverse Exponential Polynomial (see Appendix)
‡Equation (3)—Exponential Decay (see Appendix)

EXHIBIT 6

Page 1

1999 Table of Insurance Charges

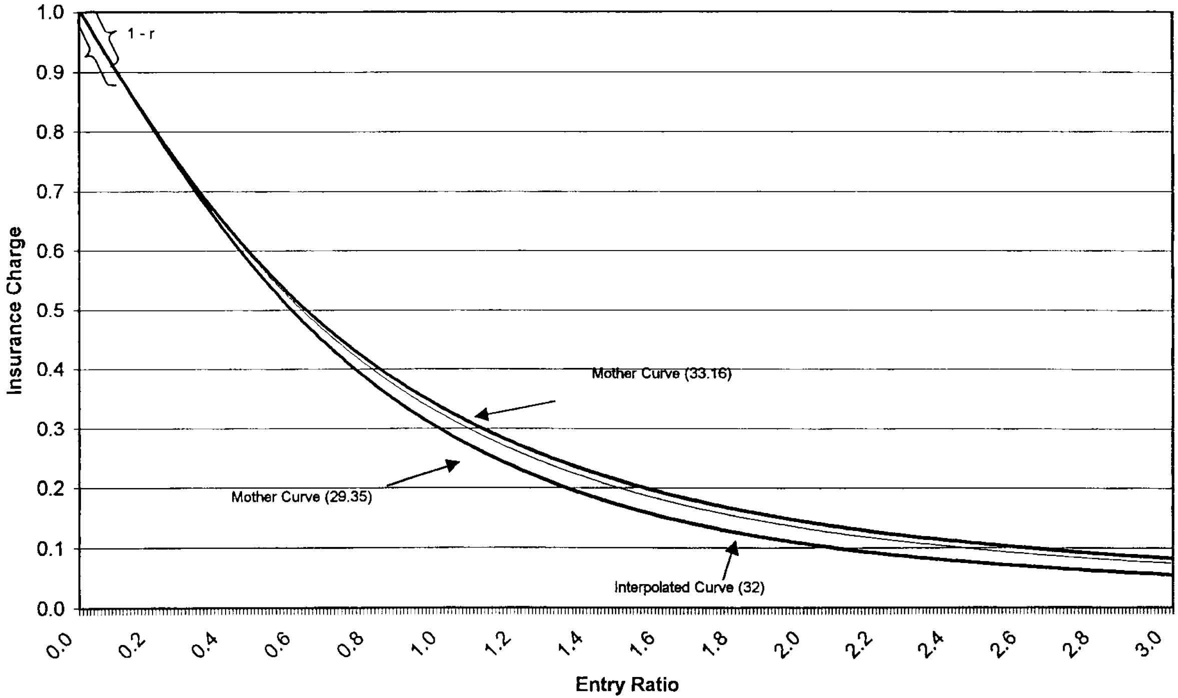

EXHIBIT 6

PAGE 2

1999 TABLE OF INSURANCE CHARGES

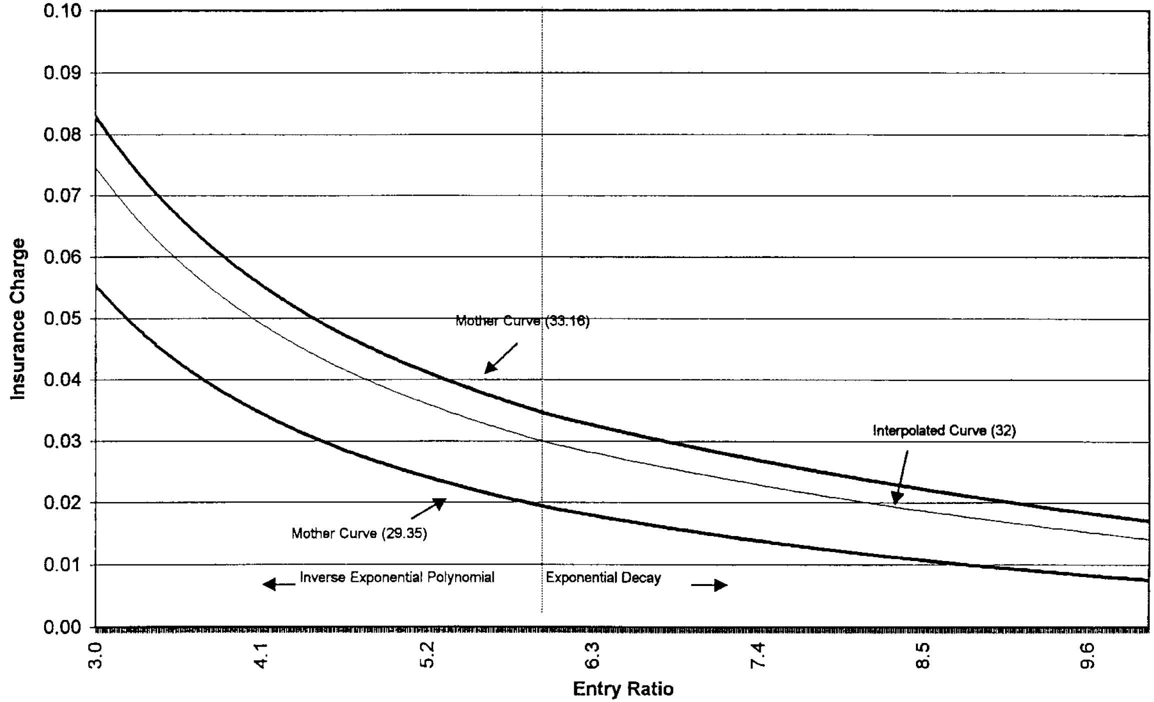

EXHIBIT 7

GRAPH OF TABLE OF INSURANCE CHARGES

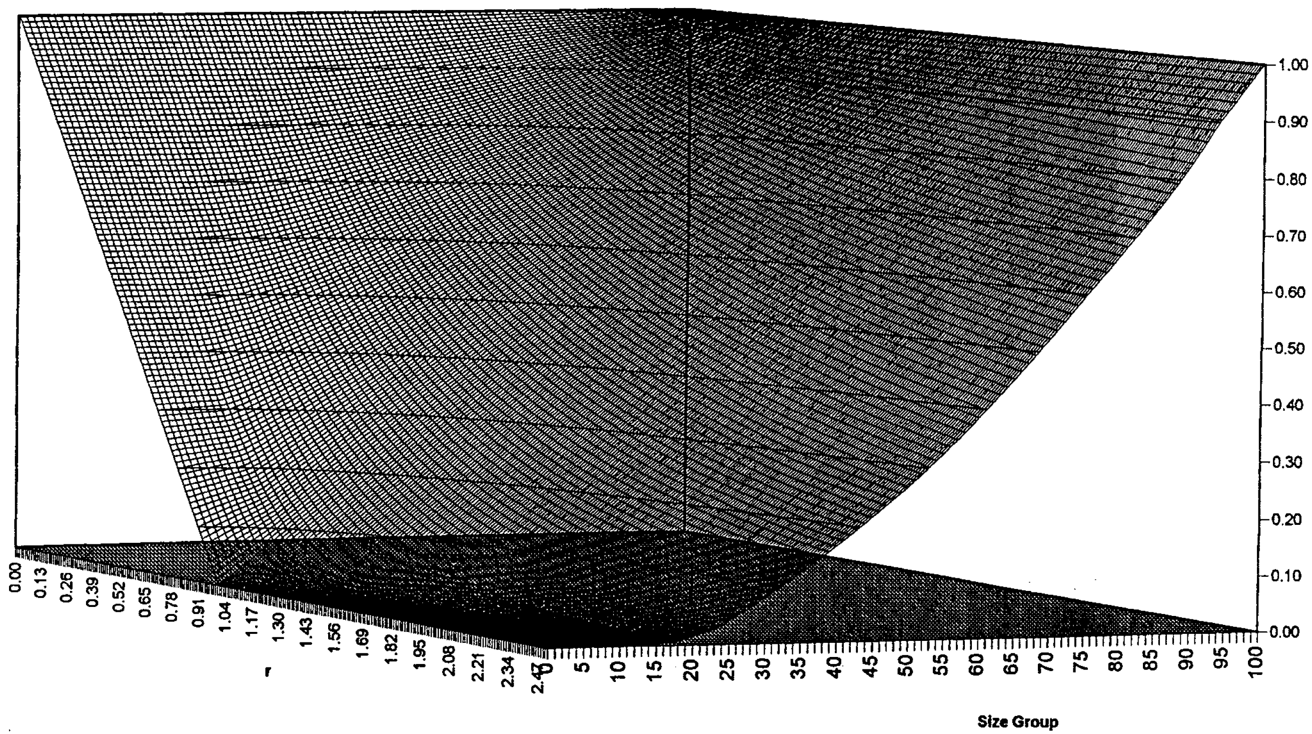

EXHIBIT 8

DETERMINATION OF 1988 AVERAGE ADJUSTED SEVERITY

1988 ELG Range (1)	Average Adjusted Expected Loss (2)	Model Expected No. of Claims (3)	Average Severity (4) = (2)/(3)
5–14	$110,489,802	20,000	$5,524
9–19	$12,000,542	2,200	$5,455
16–22	$2,921,544	600	$4,869
21–26	$1,385,528	310	$4,469
24–29	$742,680	170	$4,369
28–33	$424,437	100	$4,244
31–36	$252,281	65	$3,881
35–39	$154,055	42	$3,668
38–42	$111,337	32	$3,479
41–46	$88,321	24	$3,642
44–49	$67,843	20	$3,479
48–52	$52,322	14	$3,672
51–56	$41,980	11.00	$3,816
54–57	$32,220	9.55	$3,374
58–63	$22,867	6.35	$3,601
63–65	$17,458	5.00	$3,492
65–69	$13,629	3.80	$3,587
69–73	$10,584	2.75	$3,849
71–77	$8,059	1.98	$4,081
75–82	$5,478	1.25	$4,382
79–85	$3,410	0.87	$3,920
84–87	$1,980	0.56	$3,549
87–90	$1,227	0.37	$3,317
90–95	$586	0.15	$3,808
94–99	$177	0.03	$5,907
		Overall Avg	$4,031
		Selected	$4,000

EXHIBIT 9

Calculation of Expected Losses Corresponding to 1988 Claim Counts in Model

(1) Insurance Charge at Unity ϕ	(2) Expected Claim Count in Model	(3) = (2) × 4,000* Expected Total Losses
0.0946	75000	300,000,000
0.1234	20000	80,000,000
0.1798	2200	8,800,000
0.2325	600	2,400,000
0.2599	310	1,240,000
0.2935	170	680,000
0.3316	100	400,000
0.3688	65	260,000
0.4125	42	168,000
0.4427	32	128,000
0.4756	24.25	97,000
0.5043	19.5	78,000
0.5443	14.25	57,000
0.5790	11	44,000
0.5971	9.55	38,200
0.6501	6.35	25,400
0.6811	5	20,000
0.7127	3.8	15,200
0.7484	2.75	11,000
0.7828	1.975	7,900
0.8257	1.25	5,000
0.8552	0.87	3,480
0.8850	0.558	2,232
0.9068	0.37	1,480
0.9382	0.154	616
0.9749	0.03	120

*4,000 = Average Severity from Exhibit 8

EXHIBIT 10

INTERPOLATION OF 1998 EXPECTED LOSS RANGES

(1) Expected Loss Group Boundary	(2) Lower Point Used	(3) Upper Point Used	(4) Total Losses for Lower Point 1988	(5) Total Losses for Upper Point 1988	(6) Expected Losses 1988	(7) = (6) × 1.25* Trended Expected Losses (1998)
0.095	0.0946	0.1234	300,000,000	80,000,000	294,542,927	368,178,659
0.105	0.0946	0.1234	300,000,000	80,000,000	186,136,775	232,670,969
0.115	0.0946	0.1234	300,000,000	80,000,000	117,629,371	147,036,714
0.125	0.1234	0.1798	80,000,000	8,800,000	75,144,198	93,930,248
...	...	...	...	...	...	...
...	...	...	...	...	...	...
...	...	...	...	...	...	...
...	...	...	...	...	...	...
0.285	0.2599	0.2935	1,240,000	680,000	791,614	989,518
0.295	0.2935	0.3316	680,000	400,000	665,942	832,428
0.305	0.2935	0.3316	680,000	400,000	579,363	724,204
0.315	0.2935	0.3316	680,000	400,000	504,041	630,051
0.325	0.2935	0.3316	680,000	400,000	438,511	548,139
0.335	0.3316	0.3688	400,000	260,000	384,557	480,696
0.345	0.3316	0.3688	400,000	260,000	342,506	428,133

*severity trend from 1988 to 1998 = 1.250

EXHIBIT 11

PAGE 1

TABLE OF EXPECTED LOSS SIZE RANGE COMPARISON

Present Table			Proposed Table		
Expected Loss Group	Expected Loss Range		Expected Loss Group	Expected Loss Range	
80	6,922 –	7,773	80	7,795 –	8,670
79	7,774 –	8,690	79	8,671 –	9,646
78	8,691 –	9,681	78	9,647 –	10,645
77	9,682 –	10,747	77	10,646 –	11,720
76	10,748 –	11,891	76	11,721 –	12,904
75	11,892 –	13,130	75	12,905 –	14,180
74	13,131 –	14,452	74	14,181 –	15,525
73	14,453 –	15,878	73	15,526 –	16,996
72	15,879 –	17,406	72	16,997 –	18,609
71	17,407 –	19,042	71	18,610 –	20,314
70	19,043 –	20,802	70	20,315 –	22,158
69	20,803 –	22,679	69	22,159 –	24,168
68	22,680 –	24,696	68	24,169 –	26,204
67	24,697 –	26,849	67	26,205 –	28,304
66	26,850 –	29,162	66	28,305 –	30,573
65	29,163 –	31,635	65	30,574 –	33,021
64	31,636 –	34,280	64	33,022 –	35,665
63	34,281 –	37,114	63	35,666 –	38,519
62	37,115 –	40,149	62	38,520 –	41,603
61	40,150 –	43,404	61	41,604 –	44,933
60	43,405 –	46,884	60	44,934 –	48,540
59	46,885 –	50,612	59	48,541 –	52,483
58	50,613 –	54,610	58	52,484 –	56,666
57	54,611 –	58,894	57	56,667 –	61,055
56	58,895 –	63,490	56	61,056 –	65,784
55	63,491 –	68,426	55	65,785 –	70,879
54	68,427 –	73,720	54	70,880 –	76,640
53	73,721 –	79,406	53	76,641 –	82,891
52	79,407 –	85,521	52	82,892 –	89,654
51	85,522 –	92,100	51	89,655 –	96,966
50	92,101 –	99,181	50	96,967 –	104,636
49	99,182 –	106,809	49	104,637 –	112,895
48	106,810 –	115,032	48	112,896 –	121,865
47	115,033 –	123,912	47	121,866 –	132,583
46	123,913 –	133,498	46	132,584 –	144,243
45	133,499 –	143,873	45	144,244 –	156,928
44	143,874 –	155,101	44	156,929 –	171,488
43	155,102 –	167,271	43	171,489 –	187,645

EXHIBIT 11

PAGE 2

TABLE OF EXPECTED LOSS SIZE RANGE COMPARISON

Present Table			Proposed Table		
Expected Loss Group	Expected Loss Range		Expected Loss Group	Expected Loss Range	
42	167,272 –	180,485	42	187,646 –	205,325
41	180,486 –	194,841	41	205,326 –	226,345
40	194,842 –	210,468	40	226,346 –	250,134
39	210,469 –	227,507	39	250,135 –	276,423
38	227,508 –	246,785	38	276,424 –	305,474
37	246,786 –	283,076	37	305,475 –	339,621
36	283,077 –	325,233	36	339,622 –	381,318
35	325,234 –	374,326	35	381,319 –	428,133
34	374,327 –	431,669	34	428,134 –	480,696
33	431,670 –	498,861	33	480,697 –	548,139
32	498,862 –	577,847	32	548,140 –	630,051
31	577,848 –	671,049	31	630,052 –	724,204
30	671,050 –	781,446	30	724,205 –	832,428
29	781,447 –	912,772	29	832,429 –	989,518
28	912,773 –	1,069,714	28	989,519 –	1,183,249
27	1,069,715 –	1,258,177	27	1,183,250 –	1,414,910
26	1,258,178 –	1,485,737	26	1,414,911 –	1,744,291
25	1,485,738 –	1,762,082	25	1,744,292 –	2,219,661
24	1,762,083 –	2,099,838	24	2,219,662 –	2,824,583
23	2,099,839 –	2,515,497	23	2,824,584 –	3,609,321
22	2,515,498 –	3,030,945	22	3,609,322 –	4,618,468
21	3,030,946 –	3,675,490	21	4,618,469 –	5,909,766
20	3,675,491 –	4,488,912	20	5,909,767 –	7,562,105
19	4,488,913 –	5,525,974	19	7,562,106 –	9,676,428
18	5,525,975 –	6,863,311	18	9,676,429 –	13,273,220
17	6,863,312 –	8,609,855	17	13,273,221 –	19,630,986
16	8,609,856 –	10,923,744	16	19,630,987 –	29,034,073
15	10,923,745 –	14,039,278	15	29,034,074 –	42,941,163
14	14,039,279 –	18,312,631	14	42,941,164 –	63,509,638
13	18,312,632 –	24,300,443	13	63,509,639 –	93,930,248
12	24,300,444 –	32,901,239	12	93,930,249 –	147,036,714
11	32,901,240 –	45,622,243	11	147,036,715 –	232,670,969
10	45,622,244 –	65,106,001	10	232,670,970 –	368,178,659
9	65,106,002 –	96,243,920	9	368,178,660 –	& over
8	96,243,921 –	148,702,022			
7	148,702,023 –	243,230,605			
6	243,230,606 –	429,365,314			
5	429,365,315 –	& over			

EXHIBIT 12

Premium Impact of 1999 Table M for Retrospectively Rated Risks

Sample distribution of risks and plans

Premium Range	Risks	Average Standard Premium	Expected Loss Ratio (E)	Tax Multiplier (T)	Expense Ratio (e)	LCF (c)	Minimum Retro Prem (H)	Maximum Retro Prem (G)
$25,001– 50,000	58	$35,874	0.620	1.070	0.227	1.125	0.80	1.20
$50,001–100,000	71	$72,371	0.620	1.070	0.220	1.125	0.70	1.20
$100,001–250,000	89	$154,037	0.620	1.070	0.210	1.125	0.65	1.10
$250,001–500,000	53	$360,223	0.620	1.070	0.203	1.125	0.55	1.10
Over $500,000	27	$1,290,138	0.620	1.070	0.188	1.125	0.45	1.10
	298	$251,187						

Calculation of the Premium Impact of Changes in Retro Parameters

	Calculation of current plan rating values									*Evaluation of plan under proposed change					
Average Standard Premium	(1) ELG	(2) $r_G - r_H$	(3) $X_H - X_G$	(4) r_G	(5) r_H	(6) X_G	(7) S_H	(8) B	(9) EXP(R)	(10) E^*	(11) ELG*	(12) X_G^*	(13) S_H^*	(14) EXP(R^*)	(15) (14)/(9)-1.0
$35,874	69	0.54	0.142	0.81	0.27	0.724	0.136	0.560	0.907	0.620	69	0.7247	0.1367	0.907	0.0000
$72,371	60	0.67	0.266	0.78	0.11	0.653	0.031	0.576	0.898	0.620	61	0.6604	0.0306	0.893	−0.0056
$154,037	50	0.60	0.319	0.70	0.10	0.595	0.014	0.538	0.888	0.620	51	0.6003	0.0128	0.884	−0.0045
$360,223	39	0.74	0.443	0.87	0.13	0.435	0.009	0.423	0.881	0.620	41	0.4528	0.0066	0.866	−0.0170
$1,290,138	29	0.87	0.555	1.04	0.17	0.276	0.003	0.301	0.865	0.620	30	0.2861	0.0031	0.857	−0.0092
Average									0.877					0.868	−0.0103

(1) Expected Loss Group, based on present table; see Exhibit 11. For example (.620)(360,223) = 223,338, which is in Expected Loss Group 39.
(2) $r_G - r_H = (G - H)/cET$. See Gillam and Snader [2].
(3) $X_H - X_G = (e + E - H/T)/cE$. See Gillam and Snader [2].
(4)–(7) Solve for the set of entry ratios that match the desired differences of entry ratios in (2) and charges in (3), as described in Gillam and Snader [2].
(8) $B = e - (c - 1)E + c(X_G - S_H)E$. See Gillam and Snader [2].
(9) Represents the expected retrospective premium that would result if the current Table M were a correct model of the loss process. Exp $(R) = T(B + cE(1 - X_G + S_H))$. See Gillam [4].
(10) Actual Expected Loss Ratio (for evaluating the new table, we assume premium is adequate).
(11) Expected Loss Group, based on proposed Table M; see Exhibit 11.
(12)–(13) Charge in Maximum and Savings at Minimum, as per columns (6) and (7), but based on the proposed Table M.
(14) As per column (9), but expected retrospective premium if the proposed Table M correctly models the loss process.
(15) The premium impact of not updating the table is determined using the ratio of column (14) to column (9) minus unity.

ADDRESS TO NEW MEMBERS—MAY 8, 2000

THOUGHTS ON MY CAREER: THE SEQUEL

RUTH E. SALZMANN

When General Douglas MacArthur was relieved of his command in the Pacific Theater by President Truman, he was asked to address the United States Congress. His famous line was: "Old soldiers never die; they just fade away." In similar fashion, I can say: "Old female actuaries never die; they just lose their figures."

It is a pleasure to be here today to add my congratulations to the 14 new Fellows and the 147 new Associates. How times have changed from the "olden days;" 37 percent of you are women!

Last November, CAS Past-President LeRoy Simon addressed the new members in San Francisco. One of his quotable lines was: "It is easier to become an actuary than to be one." To that I will add: "But being one is a lot more fun!"

You now have the knowledge, at least to the satisfaction of the examination committees, and you have the credentials to meet the challenges that lie ahead in your actuarial careers. Progress will be measured by solutions and answers that become less reflexive and more reflective. It is this continuous transition from one to the other that leads to wisdom.

As I looked back over my career, I wondered what would be of interest to actuaries who are looking forward to theirs. In my case, it was the early years that set the course for a personally rewarding career. Each of you, I know, is looking forward to that same goal.

When I became a Fellow in 1947, recent legislation made it possible for one company to underwrite both casualty and fire coverages. This change allowed casualty companies to underwrite auto, fire, and theft. When the transfer was made from the fire company to the casualty company where I worked, the loss ratios in both companies went down. Our CEO was a bit troubled over that.

More importantly, the new legislation created a need for a combined annual statement blank and later a uniform classification of expenses by account, function, and line of insurance. To make this happen, many industry committees were established. Actuaries as well as insurance accountants served on these committees.

Subsequently, these same committee members served on Insurance Accounting and Statistical Association[1] (IASA) panels and workshops to communicate and discuss the new financial reporting. I served on these committees and conducted IASA presentations, and it was this involvement that shaped and helped my career. Based on this experience, I encourage all of you to serve on industry and society committees in those areas that interest you most.

In those earlier times, actuaries became regular participants on IASA panels. Let me tell you about one such panel. The subject was loss reserving methodologies for casualty lines. My presentation described the basic procedure of extrapolating a final incurred amount by accident year and then subtracting paid losses to derive the liability. The next speaker, a renowned actuary considerably senior to me, criticized that approach with a story about the village idiot who entered an archery contest. He shot his arrow into a wall and then painted a target around it. The point in his analogy was that the sequence was wrong. When the end result is determined first and the liability is then derived to support it, the objectivity of such a measurement may be questionable. He likewise would have criticized the Bornhuetter–Ferguson method, as it incorporates a predetermined expected loss ratio.

[1] Subsequently renamed Insurance Accounting and Systems Association.

The better approach, in his opinion, was to measure the IBNR independently and add it to the case reserves. This methodology was the customary practice at the time. In all fairness to this gentleman, the panel took place in the early fifties, an age that predated computers. As a result, it was difficult to produce sophisticated measurements of loss liabilities on a timely basis. Also, the average life of claims was much shorter than it is today. Because of the latter, case reserve levels were undoubtedly more dependable.

Over the years, I became a believer in the merits of estimating loss liabilities on a basis other than by extrapolation, but I also wanted to avoid the vagaries of case reserve levels. In my experience, I seldom found case reserve levels to be dependably consistent from year to year, or measurably adjustable when they are believed to be inconsistent. I did finally produce two methods[2] that independently measured the total liability, i.e., a combined liability for both reported and unreported claims. At last I found methodologies that would satisfy me as well as my copanelist, and I could put this challenge behind me. I hope this story will encourage each of you to seek involvements where different approaches are discussed. You, too, may end up with a challenge that won't let you rest until you find a solution.

Recent legislation affecting your careers is the 1999 Financial Services Modernization Act. This Act permits affiliations among all types of financial firms, including insurance. As a result, there will be newer alternative ways for customers to package all of their financial risks. What an enormous opportunity lies ahead for each of you!

It is with a bit of humor that I predict what I believe to be the biggest deterrent to mergers with insurance companies, and that is the statutory annual statement blank. When I became a Fellow, there were 34 pages in the blank. The last page in the

[2] Methods 5 and 6: Salzmann, Ruth E., *Estimated Liabilities for Losses and Loss Adjustment Expenses*, Prentice Hall, Englewood Cliffs, New Jersey, 1984.

1999 blank is numbered 136; but if you add in all the pages that are numbered in tenths, the number exceeds 200. This may be the information age, but I fear we may be putting the financial health of the trees over the financial health of the forest. Doc Masterson once said, "The annual statement needs only three pages: one for the assets, the second for the liabilities and surplus, and the third for the fingerprints of the officers."

If any of you keep a list of "reminders" that you review now and then, here are a few items to add:

1. Get involved in industry committees and meetings so as to benefit from the cross-fertilization of ideas.
2. When evaluating data, have the confidence to judge what may be random and what my not be. Paul Otteson once said, "A normal number of abnormal losses is not abnormal."
3. Keep in mind that it still takes vision and imagination to harness data and information; technology is just a tool.
4. With any actuarial methodology, ask yourself: "Why might I want to do it differently?"
5. Keep testing those assumptions.
6. Don't give up on an elusive solution; sometimes a kind of irrational overconfidence is needed.

In conclusion, your new status should give you the confidence to meet the challenges and to seek the opportunities in your futures. Current social, economic, and environmental issues will affect the measurement of financial risk and, therefore, will need the expertise and knowledge of both the actuarial scholar and the actuarial practitioner.

Good luck to each of you!

MINUTES OF THE 2000 SPRING MEETING

May 7–10, 2000

BELLAGIO

LAS VEGAS, NEVADA

Sunday, May 7, 2000

The Board of Directors held their regular quarterly meeting from 9:00 a.m. to 5:00 p.m.

Registration was held from 4:00 p.m. to 6:00 p.m.

New Associates and their guests were honored with a special presentation from 5:30 p.m. to 6:30 p.m. Members of the 2000 Executive Council discussed their roles in the Society with the new members. In addition, Robert A. Anker, who is a past president of the CAS, gave a short talk on the American Academy of Actuaries' (AAA) Casualty Practice Council.

A reception for all meeting attendees followed the new Associates reception and was held from 6:30 p.m. to 7:30 p.m.

Monday, May 8, 2000

Registration continued from 7:00 a.m. to 8:00 a.m.

The 2000 Business Session, which was held from 8:00 a.m. to 9:15 a.m., started off the first full day of activities for the 2000 Spring Meeting. Alice H. Gannon introduced the CAS Executive Council, the Board of Directors, and CAS past presidents who were in attendance, including Robert A. Anker (1996), Phillip N. Ben-Zvi (1985), Ronald L. Bornhuetter (1975), Charles A. Bryan (1990), Michael Fusco (1989), Ruth E. Salzmann (1978), LeRoy J. Simon (1971), Michael L. Toothman (1991), and Mavis A. Walters (1997).

Ms. Gannon also recognized special guests in the audience: A. Norman Crowder III, President, Society of Actuaries; Curtis

E. Huntington, Executive Director, Actuarial Education and Research Fund; Stephen R. Kern, President, American Academy of Actuaries; Michael L. Toothman, President, Conference of Consulting Actuaries; and Stuart F. Wason, President, Canadian Institute of Actuaries.

Abbe S. Bensimon, Gary R. Josephson, and Mary Frances Miller announced the 147 new Associates and Patrick J. Grannan announced the 14 new Fellows. The names of these individuals follow.

NEW FELLOWS

Amy Petea Angell
Mark E. Bohrer
Julie Burdick
Robert Neil Campbell
Mark Kelly Edmunds
Brandon Lee Emlen
Christopher Todd Hochhausler
Brandelyn C. Klenner
Elaine Lajeunesse
Diana Mary Susan Linehan
Scott A. McPhee
Kathy Popejoy
Richard A. Rosengarten
Meyer Shields

NEW ASSOCIATES

Jodie Marie Agan
Brian M. Ancharski
Kevin L. Anderson
Deborah Herman Ardern
Patrick Barbeau
Jody J. Bembenek
Ellen A. Berning
Brad D. Birtz
Mary Denise Boarman
Veronique Bouchard
Thomas Leininger Boyer II
David C. Brueckman
Angela D. Burgess
Patrick J. Charles
Harry Sigen Chen
Brian Kenneth Ciferri
Susan M. Cleaver
Richard Jason Cook
Kevin A. Cormier
Jeffrey Alan Courchene
Mary Elizabeth Frances Cunningham
Patricia A. Deo-Campo Vuong
Jean-François Desrochers
Mark Richard Desrochers
Michael Devine
Kevin George Donovan
Louis-Christian Dupuis
Donna L. Emmerling
Keith Andrew Engelbrecht
Laura Ann Esboldt
Juan de la Cruz Espadas
Farzad Farzan
Donia Burris Freese
Shina Noel Fritz
Cynthia Galvin
Michael Anthony Garcia

Hannah Gee
James Brian Gilbert
Joseph Emmanuel Goldman
Andrew Samuel Golfin Jr.
Olga Golod
Stacey C. Gotham
Mark R. Greenwood
Chantal Guillemette
James Christopher Guszcza
David Bruce Hackworth
Dawn Marie S. Happ
Jason Carl Head
Pamela Barlow Heard
Kristina Shannon Heer
Hans Heldner
Robert C. Hill
David E. Hodges
Richard Michael Holtz
Allen J. Hope
Carol Irene Humphrey
Rusty A. Husted
Michael Stanley Jarmusik
Patrice Jean
Charles Biao Jin
Steven M. Jokerst
Cheryl R. Kellogg
James F. King
Jill E. Kirby
Omar A. Kitchlew
Henry Joseph Konstanty
Darjen D. Kuo
Christine L. Lacke
Bobb J. Lackey
Jean-François Larochelle
Peter H. Latshaw
Doris Lee
Wendy Rebecca Leferson
William Scott Lennox
Joshua Yuri Ligosky
Erik Frank Livingston
Rebecca Michelle Locks
Richard Paul Lonardo
William F. Loyd III
Alexander Peter Maizys
Victor Mata
David Michael Maurer
Timothy C. McAuliffe
Richard J. McElligott
Jennifer A. McGrath
Martin Menard
Mitchel Merberg
Vadim Y. Mezhebovsky
Eric Millaire-Morin
Rebecca E. Miller
Suzanne A. Mills
Matthew Kevin Moran
Lambert Morvan
Thomas M. Mount
Ronald Taylor Nelson
Michael Dale Neubauer
Loren J. Nickel
Ajay Pahwa
Cosimo Pantaleo
Michael Thomas Patterson
Wendy Wei-Chi Peng
Jill E. Peppers
Michael C. Petersen
Kevin Thomas Peterson
Kraig Paul Peterson
Kristin Sarah Piltzecker
Sean Evans Porreca
Warren T. Printz
Stephen Daniel Riihimaki
Ezra Jonathan Robison
Bryant Edward Russell
Frederick Douglas Ryan
Laura Beth Sachs
Salimah H. Samji
Rachel Samoil
Jennifer Arlene Scher
Daniel David Schlemmer
Parr T. Schoolman
Ernest C. Segal
Michelle L. Sheppard
Paul O. Shupe
Lee Oliver Smith
Lora L. Smith-Sarfo
Scott G. Sobel
Mary Jane Sperduto
Christine Steele-Koffke

Gary A. Sudbeck
Jonathan Leigh Summers
Neeza Thandi
Tanya K. Thielman
John David Trauffer
Nathalie Tremblay
Matthew L. Uhoda
Dennis R. Unver
Richard Alan Van Dyke
Josephine M. Waldman
Colleen Ohle Walker
Kristie L. Walker
Tice R. Walker
Wade Thomas Warriner
Kelly M. Weber
Petra Lynn Wegerich
Christopher John Westermeyer
Karin H. Wohlgemuth
Terry C. Wolfe
Mihoko Yamazoe
Nora J. Young

Ms. Gannon then introduced Ruth E. Salzmann, a past president of the Society, who presented the Address to New Members.

David R. Chernick, CAS vice president-programs and communications, spoke to the meeting participants about the highlights of this meeting and what was planned in the program.

Richard I. Fein, chairperson of the Committee On Review of Papers, announced that four *Proceedings* papers and one discussion of a *Proceedings* paper would be presented at this meeting. In all, five papers were accepted for publication in the 2000 *Proceedings of the Casualty Actuarial Society*.

Robert S. Miccolis, chairperson of the Michelbacher Prize Committee, gave a brief description of this year's Call Paper Program on Insurance in the Next Century. He announced that all of the call papers would be presented at this meeting. (The papers are published in the 2000 CAS *Discussion Paper Program* and can be found on the CAS Web Site.) Mr. Miccolis presented the Michelbacher Prize to Sergei Esipov and Dajiang Guo for their paper, "Portfolio Based Pricing of Residual Basis Risk with Application to the S&P 500 Put Options." The Michelbacher Prize commemorates the work of Gustav F. Michelbacher and honors the authors of the best paper(s) submitted in response to a call for discussion papers. The papers are judged by a specifically appointed committee on the basis of originality, research, readability, and completeness.

Ms. Gannon then began the presentation of other awards. She explained that the CAS Harold W. Schloss Memorial Scholarship Fund benefits deserving and academically outstanding students in the actuarial program of the Department of Statistics and Actuarial Science at the University of Iowa. The student recipient is selected by the Trustees of the CAS Trust, based on the recommendation of the department chair at the University of Iowa. Ms. Gannon announced that Feng Sun is the recipient of the 2000 CAS Harold W. Schloss Memorial Scholarship Fund. Sun will be presented with a $500 scholarship.

Ms. Gannon then concluded the business session of the Spring Meeting.

Ms. Gannon next introduced the featured speaker, James Canton, who is one of the nation's leading futurists, a digital entrepreneur, and author.

The first General Session was held from 10:45 a.m. to 12:15 p.m.

"Enterprise Risk Management"

Moderator:	Richard I. Fein Principal PricewaterhouseCoopers LLP
Panelists:	Michael L. Albanese Group Vice President A.M. Best Company
	Jerry A. Miccolis Principal Tillinghast-Towers Perrin
	Thomas W. Wronski Risk Director Fidelity Investments

After a luncheon, the afternoon was devoted to presentations of concurrent sessions, *Proceedings* papers, and call papers. The call papers presented from 1:30 p.m. to 3:00 p.m. were:

1. "Enterprise Risk Management: A Consultative Approach"

   Authors: Edgar W. Davenport
   Advanced Risk Management Services of Willis North America

   L. Michelle Bradley
   Advanced Risk Management Services of Willis North America

2. "Enterprise Technology Projects and the Role of the Actuary"

   Author: Paul C. Martin
   Milliman & Robertson, Inc.

   Charlie Coon
   St. Paul Companies, Inc.

The concurrent sessions presented from 1:30 p.m. to 3:00 p.m. were:

1. Financial Services Regulation

   Moderator: Elise C. Liebers
   Deputy Chief Actuary
   New York State Insurance Department

   Panelists: Therese M. Vaughan
   Commissioner
   Iowa Insurance Division

   Thomas A. Oravez
   Assistant Vice President
   Federal Reserve Bank of New York

   Robert Partridge
   Director
   Standard & Poor's Rating Group

2. CAS Research Efforts

   Moderator/Panelist: Roger M. Hayne
   Consulting Actuary
   Milliman & Robertson, Inc.

Panelists: Frederick F. Cripe
Assistant Vice President
Allstate Insurance Company

Richard W. Gorvett
Assistant Professor of Actuarial Science
University of Illinois

3. CAS Membership Survey

Moderator: David Skurnick
Senior Vice President and Actuary
St. Paul Re

Panelists: Roger A. Schultz
Assistant Vice President
Allstate Insurance Company

David J. Oakden
Consulting Actuary
Tillinghast-Towers Perrin

4. Technology and Insurance

Moderator/Panelist: Robin A. Harbage
General Manager
Progressive Corporation

Panelists: Richard Bishop
Consultant
Richard Bishop Consulting

Janet K. Silverman
Supervising Casualty Actuary
New York State Insurance Department

5. Quality Assurance for the Actuarial Work Product

Moderator: James E. Buck
Principal, Actuarial Consulting
Insurance Services Office, Inc.

Panelists: Mary D. Miller
Actuary
Ohio Department of Insurance

Marc J. Adee
Chief Financial Officer and Chief Actuary
Burlington Insurance Group

Proceedings papers presented during this time were:

1. "Dirty Words: Interpreting and Using EPA Data in Actuarial Analysis of an Insurer's Superfund-Related Claim Costs"

   Author: Steven J. Finkelstein
   Ernst & Young LLP

2. "Estimating U.S. Environmental Liabilities by Simulation"

   Author: Christopher Diamantoukos
   Ernst & Young LLP

After a refreshment break, presentations of call papers, concurrent sessions, and *Proceedings* papers continued from 3:30 p.m. to 5:00 p.m. Call papers presented during this time were:

1. "The Last Few Obstacles on the Way to Digital Paradise"

   Author: Aleksey S. Popelyukhin
   Sam Sebe LLC, Commercial Risk Re

2. "Pricing Multiple Triggers—An Electrifying Example"

   Author: Lawrence Schober
   Associated Electric and Gas
   Insurance Services

3. "Pricing for the Financial Risk of Uncollateralized Deductible Policies"

   Author: Robert F. Brown
   Travelers Property Casualty

The concurrent sessions presented from 3:30 p.m. to 5:00 p.m. were:

1. Basics of Financial Risk Management

   Presenter: Richard W. Gorvett
   Assistant Professor of Actuarial Science
   University of Illinois

2. Current Accounting Issues

Moderator: Patricia A. Teufel
Principal
KPMG LLP

Panelists: Ralph S. Blanchard
Second Vice President and Actuary
Travelers Property Casualty Corporation

Andrea M. Sweeney
Manager
Arthur Andersen LLP

3. The Actuary and Earnings Management

Moderator/Panelist: Aaron Halpert
Principal
KPMG LLP

Panelist: Martha Marcon
Partner
KPMG LLP

4. Questions and Answers with the CAS Board of Directors

Moderator: Patrick J. Grannan
Principal
Milliman & Robertson, Inc.

Panelists: Frederick O. Kist
Senior Vice President and Chief Actuary
CNA

Gail M. Ross
Vice President
Am-Re Consultants, Inc.

Michael L. Toothman
Partner
Arthur Andersen LLP

5. Proposals for Managing the Industry's Catastrophe Exposure

Moderator: Wayne H. Fisher
Executive Vice President & Chief Actuary
Zurich U.S.

Panelist: Ross J. Davidson Jr.
Vice President, Industry Affairs
United Services Automobile Association

Gordon K. Hay
Actuary
SAFECO Insurance Companies

Lee R. Steeneck
Vice President & Actuary
General Reinsurance Corporation

The *Proceedings* paper presented during this time was:

1. Discussion of "Application of the Option Market Paradigm to the Solution of Insurance Problems" (by Michael G. Wacek, *PCAS* LXXXIV, 1997, p. 701)

   Discussion by Stephen J. Mildenhall
   CNA Re

A reception for new Fellows and their guests was held from 5:30 p.m. to 6:30 p.m., and the general reception for all members and their guests was held from 6:30 p.m. to 7:30 p.m.

Tuesday, May 9, 2000

Registration continued from 7:00 a.m. to 8:00 a.m.

The General Session presented from 8:00 a.m. to 9:30 a.m. was:

"The Outsider's View of the Actuary"

Moderator: Jeanne M. Hollister
Consulting Actuary
Tillinghast-Towers Perrin

Panelists: Nicholas M. Brown
President and CEO
NAC Re Corporation

Robert J. Dellinger
Executive Vice President & Chief Finance Officer
Employers Reinsurance Corporation

V. J. Dowling Jr.
Dowling & Partners Securities, LLC

Patrick W. Kenny
Executive Vice President of Finance and Chief Finance Officer
Frontier Insurance Group, Inc.

Two limited attendance workshops, "Write Up Front" and "Executive Presentation Skills," were held from 8:00 a.m. to 5:00 p.m. and from 8:00 a.m. to noon, respectively.

Certain call papers that had been presented earlier during the meeting were repeated this morning from 10:00 a.m. to 11:30 a.m. The additional call paper presented during this time was:

1. "Portfolio Based Pricing of Residual Basis Risk with Application to the S&P 500 Put Options"

   Authors: Sergei Esipov
   Centre Solutions

   Dajiang Guo
   Centre Risk Advisors and Centre Solutions

The concurrent sessions presented from 10:00 a.m. to 11:30 a.m. were:

1. Actuaries in Nontraditional Roles

   Moderator: Maribeth Ebert
   Principal
   William M. Mercer, Inc.

Panelists: James G. Evans
Investment Actuary
Prime Advisors, Inc.

Regina M. Berens
Senior Consulting Actuary
Scruggs Consulting

Robert G. Blanco
Vice President and Actuary
National Council on Compensation Insurance

David W. Simpson
Managing Principal
D. W. Simpson & Company

2. Data Standards

Moderator: Carole J. Banfield
Executive Vice President
Insurance Services Office

Panelists: Gary Knoble
Vice President
Hartford Financial Services Group

Arthur R. Cadorine
Assistant Vice President
Insurance Services Office

Beth Grossman
Director of Industry Relations
ACORD

Christine Sickierski
Vice President
Wisconsin Compensation Rating Bureau

Charles E. Wight
Managing Consultant/Senior Vice President
Marsh USA Risk & Insurance Services

3. Recent Developments in Transferring Risks

Moderator/ Panelist:	John G. Aquino Executive Vice President Aon Re Services
Panelist:	Stephen J. Mildenhall Vice President CNA Re

4. Update on e-Commerce and its Use by Insurers

Moderator:	J. Parker Boone Senior Vice President InsWeb
Panelists:	Anthony L. Alfieri Actuary eCoverage
	Charles S. Brofman President & Co-CEO Cybersettle.com
	Paul J. Ford Senior Vice President–Business Development Inslogic.com

Various CAS committees met from noon to 5:00 p.m. Certain call papers and concurrent sessions presented earlier were repeated from 1:00 p.m. to 2:30 p.m. A limited attendance workshop, “Executive Presentation Skills,” was held from 1:00 p.m. to 5:00 p.m.

All members and guests enjoyed dinner and a show at Luxor Las Vegas from 5:30 p.m. to 9:30 p.m.

Wednesday, May 10, 2000

Certain call papers and concurrent sessions that had been presented earlier during the meeting were repeated this morning from 8:00 a.m. to 9:30 a.m. Additional concurrent sessions presented were:

1. Integrated Products and Holistic Financial Risk Management

Moderator/ Panelist:	Shawna Ackerman Consulting Actuary Miller, Herbers, Lehmann, & Associates, Inc.
Panelists:	Ugur Koylouglu Oliver Wyman & Company
	Edmund S. Scanlon Assistant Vice President Zurich-American Specialties

2. Role of Regional Affiliates

Moderator:	Ramona C. Lee Chairperson Regional Affiliates Task Force
Panelists:	James E. Buck Vice President Casualty Actuaries of Greater New York
	Julia L. Perrine President Casualty Actuaries of Desert States
	Therese A. Klodnicki Member Regional Affiliates Task Force

Proceedings papers presented from 8:00 a.m. to 9:30 a.m. were:

1. "Risk and Return: Underwriting, Investment and Leverage Probability of Surplus Drawdown and Pricing for Underwriting and Investment Risk"

Author:	Russell E. Bingham The Hartford Financial Services Group

2. "The Direct Determination of Risk-Adjusted Discount Rates and Liability Beta"

   Author: Russell E. Bingham
   The Hartford Financial Services Group

After a refreshment break, the final General Session was held from 10:00 a.m. to 11:30 a.m.:

"The Actuary and the Insurance Industry of the 21st Century—What's Ahead?"

Moderator: Phillip Ben-Zvi
Principal-In-Charge
PricewaterhouseCoopers LLP

Panelists: Thomas N. Anderson
Principal
McKinsey & Company

Robert A. Anker
Quay Quest

Michael DeGusta
Chief Technology Officer
eCoverage

Alice H. Gannon officially adjourned the 2000 CAS Spring Meeting at 11:45 a.m. after closing remarks and an announcement of future CAS meetings.

Attendees of the 2000 CAS Spring Meeting

The 2000 CAS Spring Meeting was attended by 361 Fellows, 278 Associates, and 52 Guests. The names of the Fellows and Associates in attendance follow:

FELLOWS

Rimma Abian	Marc J. Adee	John P. Alltop
Shawna S. Ackerman	Kristen M. Albright	Dean R. Anderson
Mark A. Addiego	Stephanie J. Albrinck	Richard R. Anderson

Amy Petea Angell
Robert A. Anker
John G. Aquino
Steven D. Armstrong
Timothy J. Banick
Bruce C. Bassman
Philip A. Baum
William H. Belvin
Xavier Benarosch
Phillip N. Ben-Zvi
Regina M. Berens
Lisa M. Besman
David R. Bickerstaff
Richard A. Bill
James E. Biller
Richard S. Biondi
Everett G. Bishop
Suzanne E. Black
Wayne E. Blackburn
Ralph S. Blanchard
Robert G. Blanco
Daniel David Blau
Barry E. Blodgett
Mark E. Bohrer
LeRoy A. Boison
J. Parker Boone
Ronald L. Bornhuetter
François Boulanger
Amy S. Bouska
Nancy A. Braithwaite
Paul Braithwaite
Michael D. Brannon
Robert S. Briere
Nicholas M. Brown
Lisa J. Brubaker
James E. Buck
Julie Burdick
George Burger
Christopher J. Burkhalter
John Frederick Butcher
Robert Neil Campbell
Claudette Cantin
John E. Captain
Christopher S. Carlson
Kenneth E. Carlton
Sanders B. Cathcart
Kevin J. Cawley
Dennis K. Chan
Scott K. Charbonneau
David R. Chernick
Francis X. Corr
Michael D. Covney
Frederick F. Cripe
Alan M. Crowe
Michael Kevin Curry
Robert J. Curry
Daniel J. Czabaj
Kenneth S. Dailey
Guy Rollin Danielson
Robert N. Darby
Stephen P. D'Arcy
Edgar W. Davenport
John Dawson
Jerome A. Degerness
Jeffrey F. Deigl
Michael L. DeMattei
Christopher Diamantoukos
Anthony M. DiDonato
Michael C. Dolan
James L. Dornfeld
Robert B. Downer
Denis Dubois
Janet E. Duncan
M. L. "Butch" Dye
Maribeth Ebert
Grover M. Edie
Dale R. Edlefson
Mark Kelly Edmunds
Valere M. Egnasko
Donald J. Eldridge
Edward B. Eliason
Thomas J. Ellefson
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Paul E. Ericksen
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Phillip A. Evensen
Sylvain Fauchon
Richard I. Fein
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Wayne H. Fisher
Kirk G. Fleming
Daniel J. Flick
Barry A. Franklin
Michael Fusco
Scott F. Galiardo
Cecily A. Gallagher
Thomas L. Gallagher
Gary J. Ganci
Alice H. Gannon
Kathy H. Garrigan
James J. Gebhard
John F. Gibson
William R. Gillam
Michael Ambrose Ginnelly

Gregory S. Girard
Moshe D. Goldberg
Richard W. Gorvett
Jay C. Gotelaere
Patrick J. Grannan
Russell H. Greig
Anthony J. Grippa
David Thomas Groff
Marshall J. Grossack
Terry D. Gusler
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Lynne Karyl Wehmueller
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Russell B. Wenitsky
Christopher John Westermeyer
David L. Whitley
Miroslaw (Mirek) Wieczorek
Dean M. Winters
Karin H. Wohlgemuth
Robert F. Wolf
Windrie Wong
Rick A. Workman
Mihoko Yamazoe
Nora J. Young
Steven Bradley Zielke

Volume LXXXVII, Part 2 **No. 167**

PROCEEDINGS
November 12, 13, 14, 15, 2000

BEST ESTIMATES FOR RESERVES

GLEN BARNETT AND BEN ZEHNWIRTH

Abstract

Link ratio techniques can be regarded as weighted regressions. We extend these regression models to handle different exposure bases and modeling of trends in the incremental data, and we develop a variety of diagnostic tools for testing the assumptions of these models.

This "extended link ratio family" (ELRF) of regression models is used to test the assumptions made by standard link ratio techniques, and compare their predictive power with modeling trends in the incremental data. Most loss arrays don't satisfy the assumptions of standard link ratio techniques. The ELRF modeling structure creates a bridge to a statistical modeling framework where the assumptions are more consistent with actual data. There is a paradigm shift from standard link ratio techniques to the statistical modeling framework—the ELRF models form a bridge from the "old" paradigm to the "new."

There are three critical stages involved in arriving at a reserve figure: extracting information from the data *in terms of trends and stability, and distributions about these trends; formulating assumptions about the future leading to forecasting of distributions of paid losses; and consideration of the correlations between lines and their effect on the desired security level.*

Other benefits of the new statistical paradigm are discussed, including segmentation, credibility, and reserves or distributions for different layers.

1. INTRODUCTION AND SUMMARY

A model that is used to forecast reserves cannot include every variable that contributes to the variation of the final reserve amount. The exact future payment (being a random variable) is unknown and unknowable. Consequently, a probabilistic model for future reserves is required. If the resulting predictive distribution of reserves is to be of much use, or to have any meaning, the assumptions contained in that probabilistic model must be satisfied by the data. An appropriate probabilistic model will enable the calculation of the distribution of the reserve that reflects both the process variability producing the future payments and the parameter estimation error (parameter uncertainty).

The regression models based on link ratios developed by Brosius [2], Murphy [8], and Mack [6], [7] are described in Section 2, and are extended to include trends in both the incremental data and different exposure bases. We refer to that family of models as the extended link ratio family (ELRF). The ELRF provides both diagnostic and formal tests of the standard link ratio techniques. It also facilitates the comparison of the relative predictive power of link ratios *vis-a-vis* modeling the trends in the (log) incremental data.

Very often, for real data, even the best model within the ELRF is not appropriate, because the data doesn't satisfy the assumptions of that model. The common causes of this failure to satisfy assumptions motivate the development of the statistical modeling framework discussed in Section 3. The rich family of statistical models in that framework contains assumptions more in keeping with reality.

This statistical modeling framework is based on the analysis of the logarithms of the incremental data. Each model in the framework has four components of interest. The first three components are trends in each of the directions: development period, accident period, and payment/calendar period, while the fourth component is the distribution of the data about the trends. Each model fits a distribution to each cell in the loss development array and relates cell distributions by trend parameters. This rich family of models we call the Probabilistic Trend Family (PTF). We describe how to identify the optimal model in the statistical modeling framework via a step by step model identification procedure, and illustrate that in the presence of an unstable payment/calendar year trend, formulating assumptions about the future may not be straightforward. Because it is statistical, the modeling framework allows separation of parameter uncertainty and process variability.

It also allows us to:

- check that all the assumptions contained in the model are satisfied by the data;
- calculate distributions of reserve forecasts, including the total reserve;
- calculate distributions of, and correlations between, future payment streams;
- price future underwriting years, including aggregate deductibles and excess layers;
- easily update models and track forecasts as new data arrive.

The final part of the paper discusses how the combination of information extracted from the data and business knowledge allows the actuary to formulate appropriate assumptions for the future in terms of predicting distributions of loss reserves. Correlations between different lines and a prescribed security level are important inputs into a final reserve figure. Finally, other benefits of the statistical paradigm are alluded to, including segmentation, credibility, and the pricing of different layers.

2. EXTENDED LINK RATIO FAMILY

2.1. Introduction

Brosius [2] points out that the use of regression in loss reserving is not new, dating back to at least the 1950s, and says that using link ratio techniques corresponds to fitting a regression line without an intercept term. Mack [6] derives standard errors of development factors and forecasts (including the total) for the chain ladder regression ratios. He mentions the connection to weighted least squares regression through the origin, and he presents diagnostics that indicate that an intercept term may be warranted on the data he analyses.

Working directly in a regression framework, Murphy [8] derives results for models without an intercept (such as the chain ladder ratios), as well as models with an intercept.

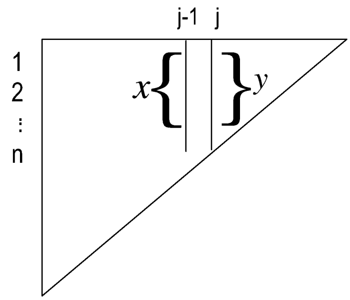
Under the assumption of heteroscedastic (i.e., with nonconstant variance) normality, we derive results for a more general family of models (ELRF) that also include accident-year trends for each development year. We discuss calculations and diagnostics for fitting and choosing between models and checking assumptions. Standard errors of forecasts for both cumulatives and incrementals are also derived.

In the current section, we analyze a number of real loss development arrays. Diagnostics, including graphs of the data and formal statistical testing, indicate that models based on link ratios suffer several common deficiencies; and frequently even the optimal model in the ELRF is inappropriate. Moreover, models based on the log incremental data have more predictive power than the optimal model in the ELRF.

The standard link ratio models carry assumptions not usually satisfied by the data. This can lead to false indications and low

FIGURE 1

CUMULATIVE LOSS ARRAY



predictive power, so that the standard errors of forecasts become meaningless. Hence, we relegate the calculation of standard errors to the appendices to this paper.

2.2. Calculating Ratios Using Regressions

Suppose $x(i)$, $i = 1,2,\ldots,n$, represent the *cumulative* values at development period $j-1$ for accident periods $i = 1,2,\ldots,n$, and $y(i)$ are the corresponding cumulative values at development period j. See Figure 1.

A graph of y versus x may appear as in Figure 2.

A link ratio $y(i)/x(i)$ is the slope of a line passing through the origin and the point $[x(i),y(i)]$, so each ratio is a trend.

Accordingly, a link ratio (trend) average method is based on the regression

$$y(i) = bx(i) + \varepsilon(i), \qquad (2.1.a)$$

where

$$\mathrm{Var}[\varepsilon(i)] = \sigma^2 x(i)^\delta. \qquad (2.1.b)$$

The parameter b represents the slope of the "best" line through the origin and the data points $[x(i),y(i)]$, $i = 1,2,\ldots,n$.

The variance of $y(i)$ about the line depends on $x(i)$, via the function $x(i)^\delta$, where δ is a "weighting" parameter. The term

FIGURE 2

CUMULATIVE LOSSES VERSUS PREVIOUS DEVELOPMENT YEAR

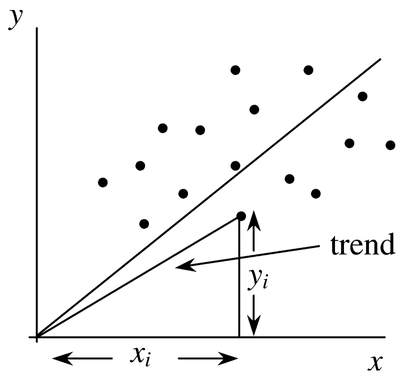
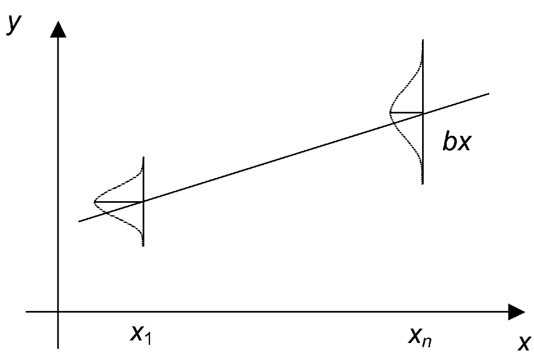


FIGURE 3

CHAIN LADDER RATIO REGRESSION



σ^2 represents an underlying level of variance (or base variance) common to the whole development period.

In Figure 3, $\text{Var}[\varepsilon(i)] = \sigma^2 x(i)^\delta$, where $\delta = 1$. It turns out that the assumption that, conditional on $x(i)$, the "average" value of $y(i)$ is $bx(i)$, is rarely true for real loss development arrays.

Consider the following cases:

CASE 1 $\delta = 1$. The weighted least squares estimator of b is

$$\hat{b} = \frac{\sum x(i) \cdot y(i)/x(i)}{\sum x(i)}. \tag{2.2}$$

This is the weighted (by volume) average ratio (i.e., the chain ladder average method, or chain ladder ratio).

CASE 2 $\delta = 2$. The weighted least squares estimator of b is

$$\hat{b} = \frac{1}{n}\sum y(i)/x(i). \tag{2.3}$$

This is the simple arithmetic average of the ratios.

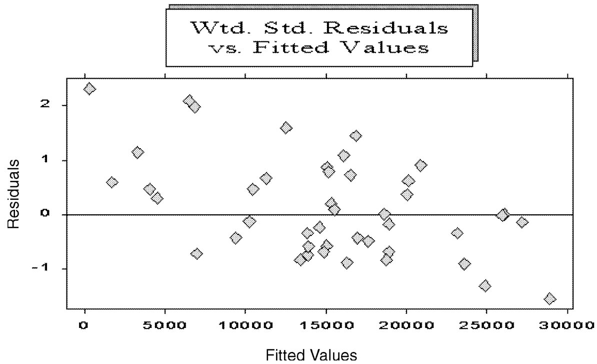
CASE 3 $\delta = 0$. This yields a weighted average (weighted by volume squared) corresponding to ordinary least squares regression through the origin.

So, by varying the parameter δ, we obtain different link ratio methods (averages).

One of the advantages of estimating link ratios using regressions is that both the standard errors of the parameters in the average method selection and the standard errors of the forecasts can be obtained. A more important advantage is that the assumptions made by the method can be tested.

One important assumption is that the standardized errors, $\varepsilon(i)/\sigma x(i)^{\delta/2}$, $i = 1,2,\ldots,n$, are normally distributed with mean 0 and standard deviation 1. Otherwise, the weighted least squares estimator of b is not necessarily efficient; and the reserve forecasts consequently may be poor estimates of the mean—they will have a large variance. The normality assumption can be checked by examining a number of diagnostic displays, including the normal probability plot, box-plot, and histogram of the weighted standardized residuals. The Shapiro–Francia test [10], based on the normality plot, is a formal test for normality of the residuals.

FIGURE 4



The link ratio method also carries with it other assumptions that should always be tested.

Another basic assumption is that

$$E(y(i)\,|\,x(i)) = bx(i). \tag{2.4}$$

That is, in order to obtain the mean cumulative at development period j, take the cumulative at the previous development period, $j-1$, and multiply it by the ratio. A quick diagnostic check of this assumption is given by the graph of $y(i)$ versus $x(i)$. Very often this shows that a (non-zero) intercept is also required. (See Figure 6.)

Equation 2.4 can be re-cast

$$E((y(i) - x(i))\,|\,x(i)) = (b-1)x(i). \tag{2.5}$$

That is, the mean incremental at development period j equals the cumulative at development period $j-1$ multiplied by the link ratio, b, minus 1. What are the diagnostic tests for this assumption?

If the assumption underlying Equation 2.4 is valid, then the weighted standardized residuals versus fitted values should appear random. Instead, what you will usually see is a downward trend like that depicted in Figure 4, representing the residuals from the chain ladder ratios model for the Mack [7] data. (See Example 1 below.)

This indicates that large values are overpredicted and small values are underpredicted, so that $E(y \mid x) = bx$ is *not* true.

Comparison of graphs of weighted standardized residuals with graphs of the data will indicate that accident periods that have "high" cumulatives are overfitted and those with "low" cumulatives are underfitted. Figure 5 shows the two displays for the Mack [7] data. Note that as a result of the equivalence of Equations 2.4 and 2.5, the residuals of the cumulative data are also the residuals of the incremental data.

If you think of the way the incrementals are generated and the fact that there are usually payment-period effects, the cumulative at development period $j-1$ rarely is a good predictor of the next incremental (after adjusting for accident period trends).

Murphy [8] suggested an extension of the regression model represented by Equation 2.1 to include the possibility of an intercept:

$$y(i) = a + bx(i) + \varepsilon(i), \tag{2.6a}$$

such that

$$\mathrm{Var}[\varepsilon(i)] = \sigma^2 x(i)^{\delta}. \tag{2.6b}$$

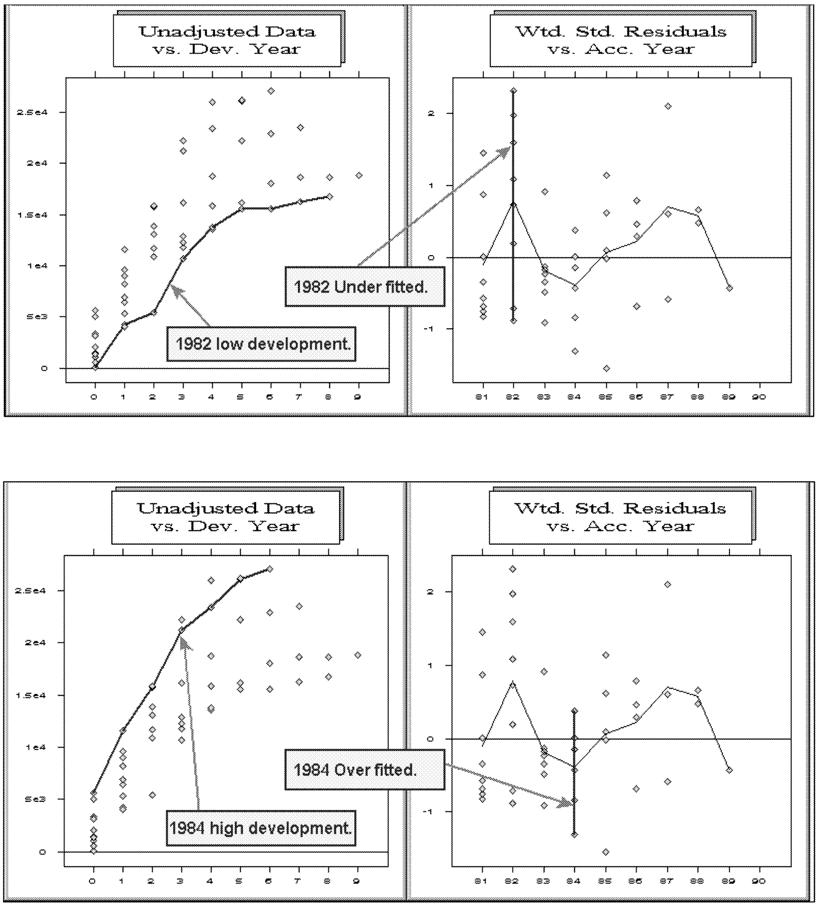
If the intercept a is non-zero and we do not include it in the regression model, then the estimate of the link ratio b (slope) is biased. Note that in the graph in Figure 6 of cumulative values at development period 1 versus cumulative values at development period 0, the intercept appears to be different from zero (the origin sits well below the graph). Indeed, it is significant between every pair of contiguous development periods. See the data of Example 1 below. We can rewrite Equation 2.6 thus:

$$y(i) - x(i) = a + (b-1)x(i) + \varepsilon(i). \tag{2.7}$$

So here, $y(i) - x(i)$ is the incremental at development period j.

FIGURE 5

RAW DATA AND RESIDUALS FROM CHAIN LADDER MODEL



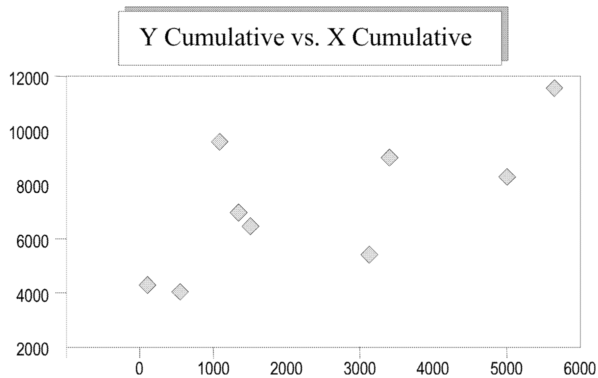
Consider the following two situations:

- $b > 1$ and $a = 0$.

Here, to forecast the mean incremental at development period j, we take the cumulative x at development period $j - 1$ and multiply it by $(b - 1)$.

FIGURE 6

CUMULATIVE IN DEVELOPMENT PERIOD 1 VERSUS CUMULATIVE IN DEVELOPMENT PERIOD 0



- $b = 1$ and $a \neq 0$.

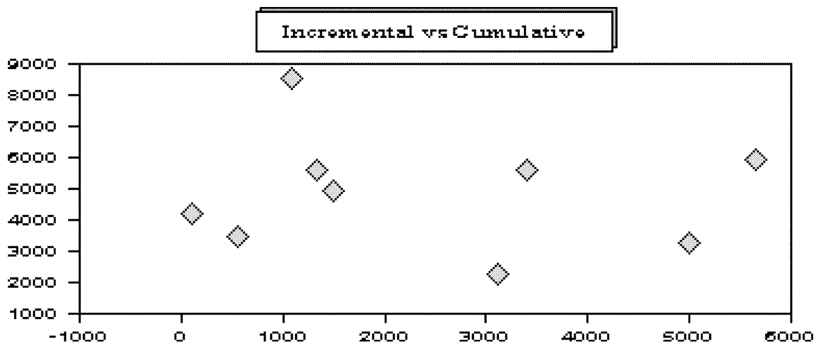
  In this case, $x(i)$ has no predictive power in forecasting $y(i) - x(i)$. The estimate of a is a weighted average of the incrementals in development period j. We would therefore forecast the next accident period's incremental by averaging the incrementals down a development period. Accordingly, the standard link ratio approach is abandoned in favor of averaging incrementals for each development period down the accident periods.

  If $b = 1$ then the graph of $y(i) - x(i)$ against $x(i)$ should be flat, as depicted in Figure 7, which represents the incrementals versus previous cumulatives (development period 0) for the Mack [7] data. It is clear that the correlation is essentially zero. This is also true for every pair of contiguous development periods.

In conclusion, if the incrementals $y(i) - x(i)$ in development period j, say, appear random, it is very likely that the graph of $y(i) - x(i)$ versus $x(i)$ is also random. That is, there is zero

FIGURE 7

INCREMENTALS IN DEVELOPMENT PERIOD 1 VERSUS CUMULATIVE IN DEVELOPMENT PERIOD 0



correlation between the incrementals and the previous cumulatives.

Now, if the incrementals possess a trend down the accident periods, it is likely that the cumulatives in the previous development period also trend down the accident periods. In this case, the estimate of the parameter b in Equation 2.7 will be significant; and so the link ratio b, together with the intercept a, will have some predictive power. In this circumstance, we should incorporate an accident period trend parameter for the incremental data; that is,

$$y(i) - x(i) = a_0 + a_1 i + (b-1)x(i) + \varepsilon(i), \qquad (2.8a)$$

where

$$\text{Var}[\varepsilon(i)] = \sigma^2 x(i)^{\delta}. \qquad (2.8b)$$

For most real cumulative loss development arrays that possess a constant trend down the development period, the trend parameter a_1 will be more significant than the ratio minus 1 (i.e., $b-1$). Indeed, more often than not, $b-1$ will be insignificant, if the trend parameter a_1 is included in the equation. That is, more often than not, the trend will have more predictive power than the ratio, and the residual predictive power of the ratio after including the trend will be insignificant.

We use the following naming convention for the three parameters:

$$a_0 = \text{intercept};$$
$$a_1 = \text{trend};$$
$$b = \text{ratio (slope)}.$$

Here are some models included in the ELRF described by Equation 2.8.

- Chain Ladder Link Ratios

  In this model, $a_0 = a_1 = 0$ and $\delta = 1$.

- Cape Cod—intercept only

  Here it is assumed that $b = 1$ and $a_1 = 0$. The Cape Cod estimates a weighted average (with weights depending on δ) of the incrementals in each development period. The forecasts are also based on a weighted average down the accident periods for each development period.

  The model can be written as:

$$y(i) - x(i) = a_0 + \varepsilon(i), \qquad (2.9a)$$

  where

$$\text{Var}[\varepsilon(i)] = \sigma^2 x(i)^\delta. \qquad (2.9b)$$

- Trend with $b = 1$

  The model estimates a weighted (depending on δ) trend (parameters a_0 and a_1) down the accident periods for each development period. The forecasts are also based on a weighted trend down the accident periods for each development period.

2.3. *Example 1: The Mack Data*

The data for the first example is from Mack [7] (see Table 2.1). The data are incurred losses for automatic facultative business

TABLE 2.1

INCURRED LOSS ARRAY FOR THE MACK DATA†

Accident Year	Development Year 0	1	2	3	4	5	6	7	8	9
1981	5012	8269	10907	11805	13539	16181	18009	18608	18662	18834
1982	106	4285	5396	10666	13782	15599	15496	16169	16704	
1983	3410	8992	13873	16141	18735	22214	22863	23466		
1984	5655	11555	15766	21266	23425	26083	27067			
1985	1092	9565	15836	22169	25955	26180				
1986	1513	6445	11702	12935	15852					
1987	557	4020	10946	12314						
1988	1351	6947	13112							
1989	3133	5395								
1990	2063									

†Note that 1982 accident year values are low.

in general liability, taken from the Reinsurance Association of America's Historical Loss Development Study [9].

We first fit the chain ladder ratios regression model; that is, we fit Equation 2.1 with $\delta = 1$ for every pair of contiguous development periods. The standardized residuals are displayed in Figure 8. Note that the equivalence of Equations 2.5 and 2.6 means that the residuals of the model for the cumulative data are identical to the residuals for the model of the incremental data.

We have already observed the downward trend in the fitted values (Figure 4), and that the high cumulatives are overpredicted whereas the low cumulatives are underpredicted. This is mainly due to the fact that intercepts are required.

So we now fit a model of the type given in Equation 2.6 to each year (i.e., with intercepts, except for the last two pairs of contiguous development periods, as there is insufficient data there). (See Table 2.2 for the regression output.) Note that none of the slope (ratio) parameters are significantly different from 1 and, if both parameters are insignificant, the slope (ratio) is less

TABLE 2.2

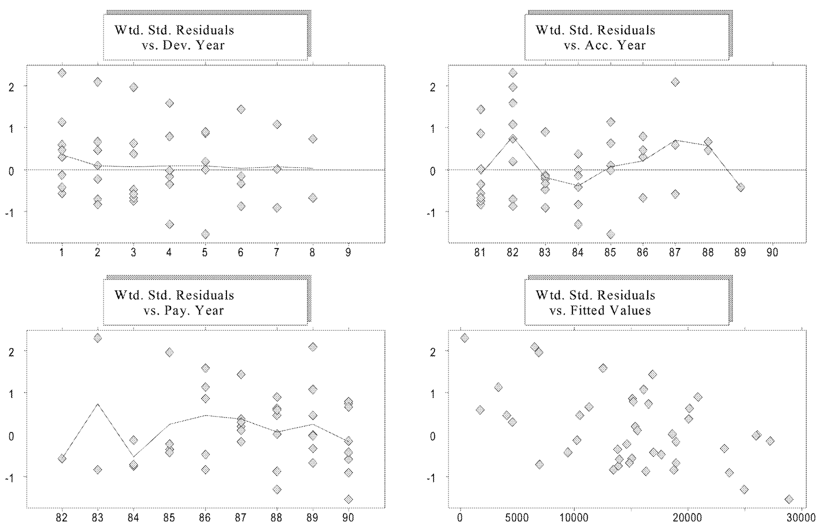
FIT OF THE MODEL WITH INTERCEPT† AND RATIO, WITH $\delta = 1$

Develop. Period	Intercept Estimate	Intercept Std. Error	Intercept *p* value	Slope Estimate	Slope−1 Estimate	Slope Std. Error	*p* value
00–01	4,329	516	0.000	1.21445	0.21445	0.42131	0.626
01–02	4,160	2,531	0.151	1.06962	0.06962	0.35842	0.852
02–03	4,236	2,815	0.193	0.91968	−0.08032	0.24743	0.759
03–04	2,189	1,133	0.126	1.03341	0.03341	0.07443	0.677
04–05	3,562	2,031	0.178	0.92675	−0.07325	0.11023	0.554
05–06	589	2,510	0.836	1.01250	0.01250	0.12833	0.931
06–07	792	149	0.118	0.99110	−0.00890	0.00803	0.467
07–08	—	—	—	1.01694	0.01694	0.01506	0.463
08–09	—	—	—	1.00922	0.00922	—	—

† Due to lack of observations in the tail, there is no intercept fitted for the last two years.

FIGURE 8

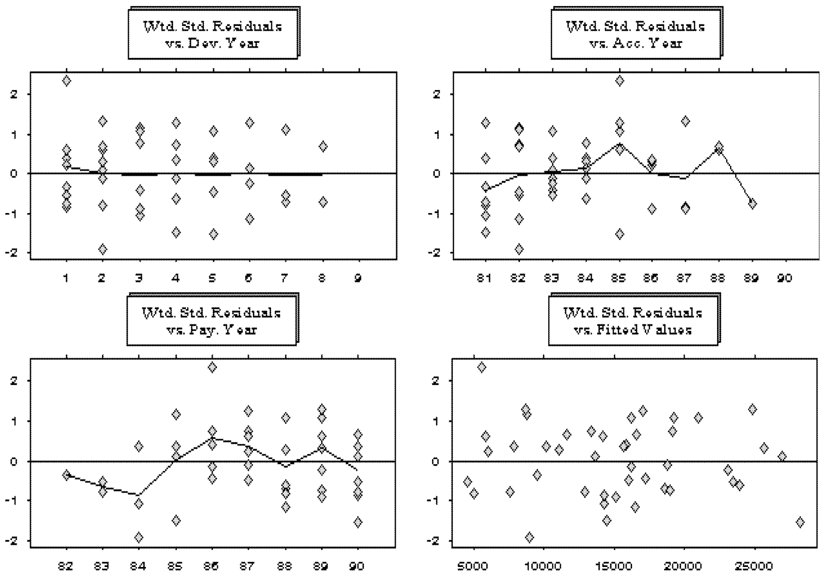
RESIDUAL PLOT FOR THE CHAIN LADDER RATIOS MODEL††



†† Note that the lines join the means at each period.

FIGURE 9

RESIDUAL PLOT FOR MODEL WITH INTERCEPTS FITTED, ALL SLOPES SET TO 1 AND $\delta = 1$



significant. This means that the previous cumulative is not really of much help in predicting the next incremental incurred loss.

The model is overparameterized (i.e., has many unnecessary parameters), so we eliminate the least significant parameter in each regression. We find that in each case the intercept is the parameter retained; that is, for every pair of contiguous development periods, the model reduces to Cape Cod. This results in the model:

$$y(i) - x(i) = a_0 + \varepsilon(i) \tag{2.10}$$

The residual plots for the reduced model (Cape Cod) are given in Figure 9.

Note that residuals versus fitted values are "straight" now and that we do not have the high-low effect in the plot of residu-

TABLE 2.3

COMPARISON OF CAPE COD COEFFICIENTS OF VARIATION WITH THOSE FOR THE CHAIN LADDER

	Cape Cod			Chain Ladder		
Accident Year	Mean Forecast	Standard Error	Coeff. of Variation	Mean Forecast	Standard Error	Coeff. of Variation
1981	0	0	—	0	0	—
1982	172	41	0.244186	155	148	0.954839
1983	483	465	0.899142	616	586	0.951299
1984	1,113	498	0.385531	1,633	702	0.429884
1985	1,941	1,218	0.512170	2,779	1,404	0.505218
1986	4,200	1,555	0.408791	3,671	1,976	0.538273
1987	6,878	1,677	0.271393	5,455	2,190	0.401467
1988	10,252	3,247	0.308234	10,934	5,351	0.489391
1989	14,874	3,657	0.253810	10,668	6,335	0.593832
1990	19,336	4,532	0.215021	16,360	24,606	1.504034
Total	59,248	8,494	0.110347	52,272	26,883	0.514291

als versus accident period. The plot of residuals versus accident year does not exhibit a trend; if we were to include a trend, by estimating

$$y(i) - x(i) = a_0 + a_1 i + \varepsilon, \qquad (2.11)$$

we would find that the estimate of a_1 would not be significantly different from zero.

We now present forecasts and coefficients of variation (mean divided by standard deviation) of forecasts based on the Cape Cod (intercept-only) model with $\delta = 1$, and compare this with the forecasts and coefficients of variation for the chain ladder ratios (see Table 2.3).

Note that, for the Cape Cod model, the standard errors are generally decreasing as a percentage of the accident-year forecast totals as we proceed down to the later years. This is because the model relates the numbers in the triangle to a certain degree—it assumes that the incremental values in the same development period are randomly drawn from the same distribution. This does not happen with the chain ladder ratios, because the model does *not* relate the incrementals in the triangle in any meaningful way. For example, how are the values in the development period 0 related? Consequently, the coefficients of variation are substantially higher for the chain ladder ratios model. Moreover, the coefficient of variation for 1990 is 150%, but for the previous year it is 59%. Note that 1990 has one more incremental value to forecast than 1989; if anything, a good model will on average have smaller coefficients of variation for totals of years with more observations. Since the 1990 accident-year total pools one more uncertain value than 1989 and the remaining values (conditionally on the first) could be expected to have similar coefficients of variation to the corresponding values from 1989, this appears to violate the fundamental statistical principle of insurance—risk reduction by pooling.

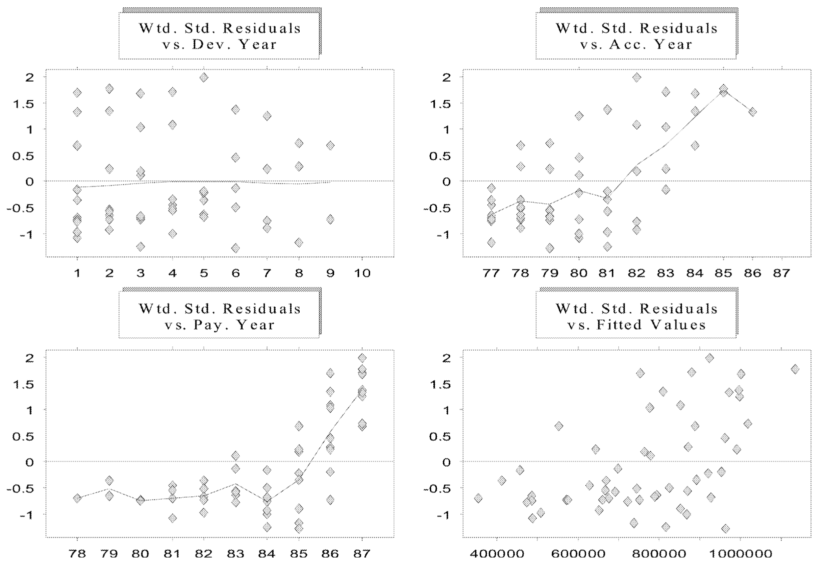
For the Mack data, the model with intercepts is reasonable, as there is no accident-year trend in the incrementals. For data where a constant trend (on a dollar scale) does exist, then the trend will be significant, but very often the ratio will not be significantly different from one.

2.4. Summary

We have so far considered two cases that can occur in real data: incrementals for a particular development period have no trend, and incrementals have trend in the accident period direction (after possibly adjusting the data by accident period exposures). In the first case, link ratios are often insignificant and so lack predictive power. In the second case, when incrementals versus accident periods for a particular development period have a constant trend, it is likely that the cumulatives in the preceding development period also exhibit a trend so that the ratio has some predictive power (equivalently, the ratio is significantly different from one). However, more often than not, the accident period trend has more predictive power than the ratio; and, once

FIGURE 10

RESIDUAL PLOTS FOR THE CHAIN LADDER RATIOS MODEL

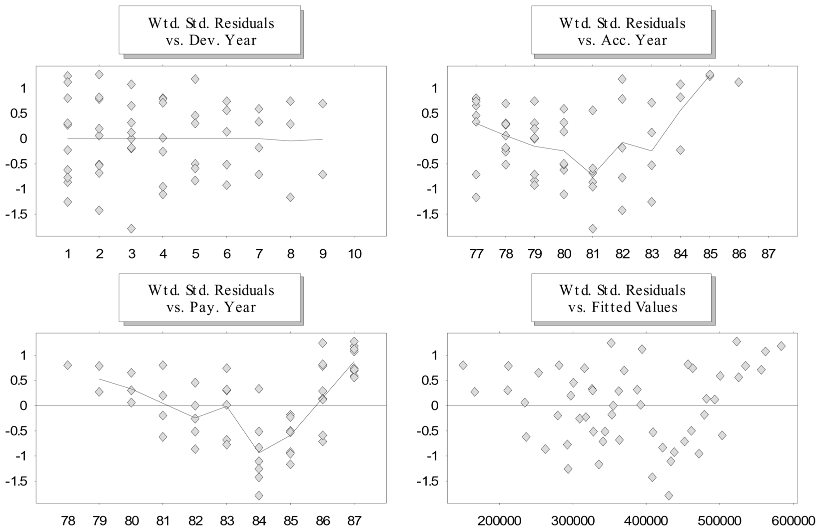


it is included in the model, the term $(b-1)$ is often insignificant (i.e., the ratio does not have any residual predictive power). The situation encountered most often in practice, however, involves a trend change along the payment/calendar periods (diagonals). This means that as you look down each development period, the change in trend will occur in different accident periods. Consequently, none of the above models in the ELRF can capture these trends.

The weighted standardized residual plots depicted in Figures 10 and 11 are those of the chain ladder ratios and Equation 2.8, respectively, applied to project ABC (Workers Compensation Portfolio) discussed in Section 3. Note that the chain ladder ratios indicate a payment-year trend change, and the model in Equation 2.8, which fits a constant trend down the accident years for each development year, indicates that the trend before pay-

FIGURE 11

RESIDUAL PLOTS FOR TREND PLUS RATIO MODEL



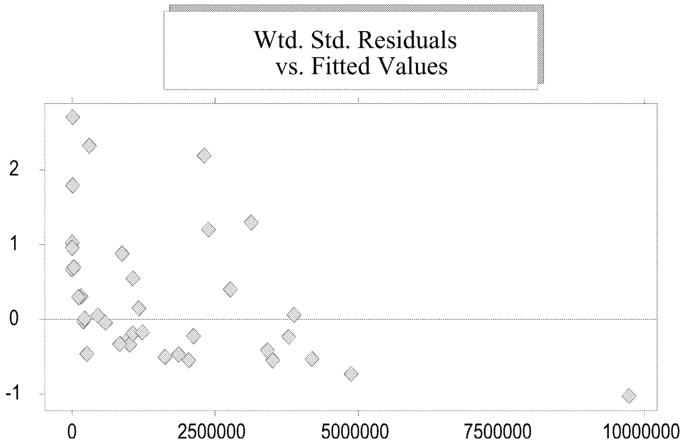
ment year 1984 is lower than the trend after 1984. This project (ABC) is analyzed in more detail in Section 3.

The models subsumed by Equation 2.8 can be used to diagnostically identify payment period trend changes but do not identify these trend changes or forecast with them. The models in the ELRF form a bridge to models that also include payment period trend parameters; that is, statistical models in the Probabilistic Trend Family (PTF) of the next section.

It is important to note that ELRF models also make the implicit assumption that the weighted standardized errors come from a normal distribution. If the assumption is true, the estimates of the regression parameters are optimal. If the assumption is not true, the estimates may be very poor. This normality assumption is rarely true for loss reserving data. In fact, the weighted standardized residuals are generally skewed to the right, suggesting that the analysis should be conducted on the

FIGURE 12

RESIDUALS VERSUS FITTED VALUES FOR THE CHAIN LADDER RATIOS



logarithmic scale. The graph in Figure 12 illustrates the skewness of a set of weighted standardized residuals based on chain ladder ratios for Project PAN6 (analyzed in detail in Example 4 of Section 3). The positive-weighted standardized residuals are further from zero than the negative ones. If the normality assumption were correct, the plot would look roughly symmetric about the zero line.

In summary, using the ELRF regression methodology you will discover that, for any type of real loss development array, the standard development factor (link ratio) techniques are frequently inappropriate. Analyzing the incrementals on the logarithmic scale with the inclusion of payment period trend parameters has more predictive power.

Finally, but importantly, the estimate of a mean forecast of outstanding (reserve) and corresponding standard deviation based on a model may be quite meaningless, unless the assumptions made by the model are supported by the data.

3. STATISTICAL MODELING FRAMEWORK

3.1. Introduction

Clearly, we require a model that is able to deal with changing trends. Trends in the data on the original (dollar) scale are hard to deal with, since trends on that scale are not generally linear but instead move in percentage terms—for example, 5% superimposed (social) inflation in early years, and 3% in later years. It is the logarithms of the incremental data that show linear trends. Consequently, we introduce a modeling framework for the logarithms of the incremental data that allows for changes in trends. The models of this type provide a high degree of insight into the loss development processes. Moreover, they facilitate the extraction of a great deal of easily communicated information from the loss development array.

The details of the modeling framework and its inherent benefits are described in Zehnwirth [12]. However, given that there is a paradigm shift from the standard link ratio methodology to the statistical modeling framework, we review the salient features of the statistical modeling framework.

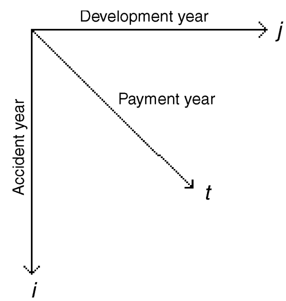
3.2. Trend Properties of Loss Development Arrays

Since a model is supposed to capture the trends in the data, it behooves us to discuss the geometry of trends in the three directions; viz., development-year, accident-year and payment/calendar-year.

Development years are denoted by j, $j = 0,1,2,\ldots,s-1$; accident years by i, $i = 1,2,\ldots,s$; and payment years by t, $t = 1,2,\ldots,s$. See Figure 13.

The payment-year variable t can be expressed as $t = i + j$. This relationship between the three directions implies that there are only two independent directions.

FIGURE 13



The two directions, development-year and accident-year, are orthogonal. That is, trends in either direction are not projected onto the other. The payment-year direction t is not orthogonal to either the development- or accident-year directions. That is, a trend in the payment-year direction is also projected onto the development-year and accident-year directions. Similarly, accident-year trends are projected onto payment-year trends.

The main idea is to have the possibility of parameters in each of the three directions—development-years, accident-years and payment-years. The parameters in the accident-year direction determine the level from year to year; often the level (after adjusting for exposures) shows little change over many years, requiring only a few parameters. The parameters in the development-year direction represent the trend from one development year to the next. This trend is often linear (on the log scale) across many of the later development years, often requiring only one parameter to describe the tail of the data. The parameters in the payment-year direction describe the trend from payment year to payment year. If the original data are inflation-adjusted (by a price or wage index) before being transformed to the log scale, the payment-year parameters represent superimposed (social) inflation, which may be stable for many years or may not be stable at all. This is determined in the analysis. We see that very often only a few parameters are required to describe the trends in the data. Consequently, the (optimal) identified model for a particular loss development array is likely to be parsimonious. This allows us to have a clearer picture of what is happening in the incremental loss process.

In this section, let $y(i,j)$ be the natural log of the incremental payment data in accident year i and development year j. This is different from our use of $y(i,j)$ in Section 2, but we do it for consistency with the literature appropriate to the models in each section. The mathematical formulation of the models in the statistical modeling framework is given by Equation 3.5. We now illustrate the geometry of trends with a simulation example.

3.3. Example 2—Simulated Data

To illustrate the trend properties of a loss development array, let us examine a situation where we know the trends (because we have selected them). Consider a set of data where the underlying paid loss (at this point without any payment-year trends or even randomness—just the underlying development) is of the form

$$y(i,j) = \ln(p_{ij}) = 11.51293 - 0.2j. \tag{3.1}$$

On a log scale, this is a line with a slope of −0.2. The accident years are completely homogeneous. Let's add some payment/calendar year trends: a trend of 0.1 from 1978 to 1982, 0.3 from 1982 to 1983 and 0.15 from 1983 to 1991. Note that a linear trend of 0.1 per year on the log scale is about a 10% *per annum* increase on the original scale.

The trends are depicted in Figure 14. Patterns of change like this are quite common in real data. Note that trends in the payment/calendar year direction project onto the other two directions, as they must. The resultant trends for the first six accident years are shown in Figure 15.

Note that each line in the graph is the resultant development-year trend for a single accident year. As you go down the acci-

FIGURE 14

DIAGRAM OF THE TRENDS ON THE LOG SCALE IN THE DATA ARRAY

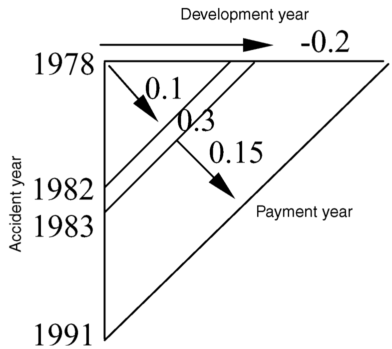


FIGURE 15

PLOT OF THE LOG(PAID) DATA AGAINST DELAY FOR THE FIRST SIX ACCIDENT YEARS

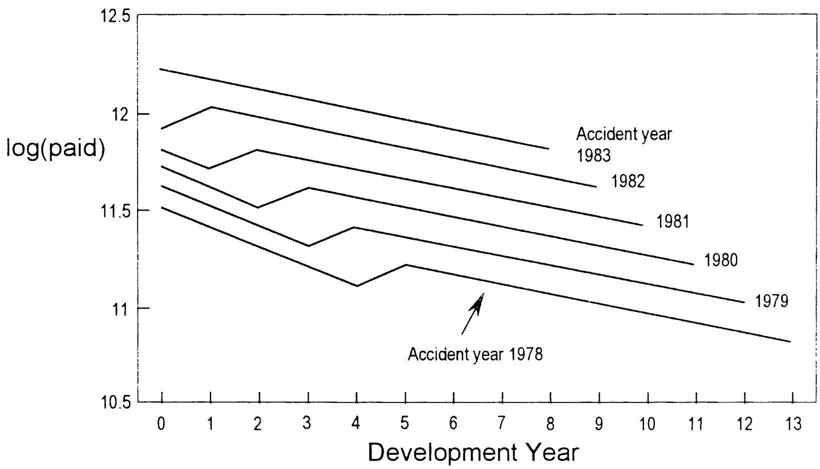
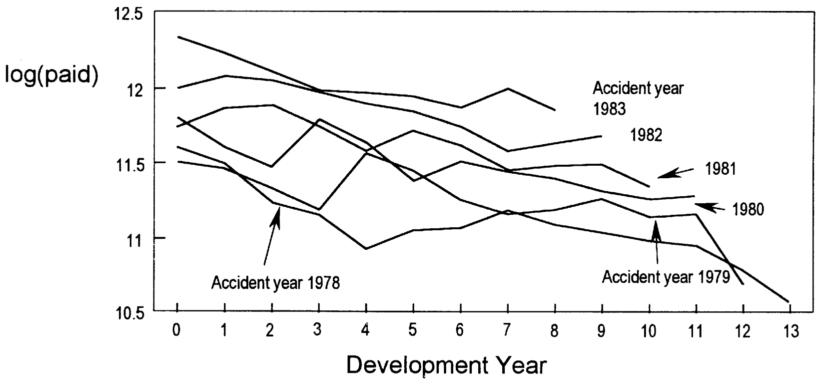


FIGURE 16

TREND PLUS RANDOMNESS FOR THE FIRST SIX ACCIDENT YEARS



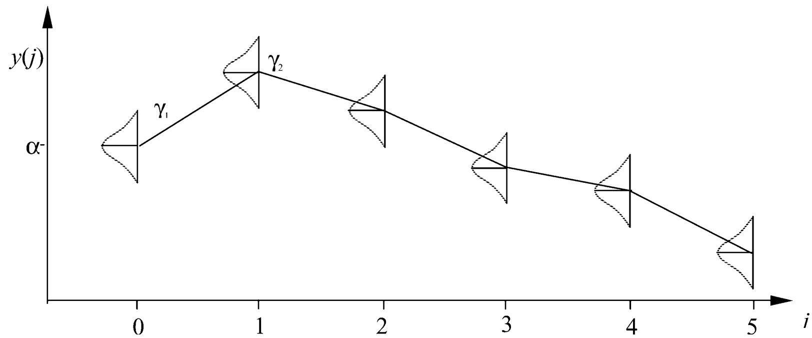
dent years (1978 to 1983), the 30% trend always kicks in one development-period earlier. The payment-year trends also project onto the accident years, which is why the early years are at the bottom and the later years are at the top. Note how the "kink" moves back as we go up to the more recent accident years. The resultant development-year trends are different for each accident year now. We can't model even this simple situation with link ratios or any other ELRF model.

Of course, real data are never so smooth. On the same log scale, we add some noise—random numbers with mean 0 and standard deviation 0.1, as shown in Figure 16.

Now the underlying changes in trends are not at all clear for two reasons—the payment-year trends project onto development years, and the data always exhibits randomness that tends to obscure the underlying trend changes. It has many of the properties we observe in real data; and yet it is plain that, even with the extensions presented there, the regression models in ELRF from Section 2 are inadequate for this data. We instead look to a model

FIGURE 17

PROBABILISTIC MODEL FOR DEVELOPMENT-YEAR TRENDS (LOG SCALE)



that incorporates the trends in the three directions and the variability about those trends, measured on a log scale.

Consider a single accident year (dropping the i subscript for the moment). We represent the expected level in the first development year by the parameter α. We can model the trends across the development years by allowing for a (possible) parameter to represent the expected change (trend) between each pair of development years. We model the variation of the data about this process with a zero-mean, normally-distributed random error, represented as:

$$y(j) = \alpha + \sum_{k=1}^{j} \gamma_k + \varepsilon_j. \qquad (3.2)$$

This probabilistic model is depicted in Figure 17 (for the first six development years).

For this probabilistic model, α is not the value of y observed at delay 0. It is the mean of $y(0)$; indeed, $y(0)$ has a normal distribution with mean α and variance σ^2. Similarly, γ_j is not the observed trend between development year $j-1$ and j, but rather, it is the mean trend between those development years—$\mathrm{E}[y(j)-y(j-1)] = \gamma_j$.

The parameters of the probabilistic model represent means of random variables. Indeed, the model (on a log scale) comprises a normal distribution for each development year, where the means of the normal distributions are related by the parameter α and the trend parameters γ_1, γ_2, and so on.

Based on the model in Equation 3.2, the random variable $p(j)$ has a lognormal distribution with

$$\text{median} = \exp\left[\alpha + \sum_{k=1}^{j}\gamma_k\right], \tag{3.3}$$

$$\text{mean} = \text{median} \times \exp[\tfrac{1}{2}\sigma^2], \tag{3.4}$$

and

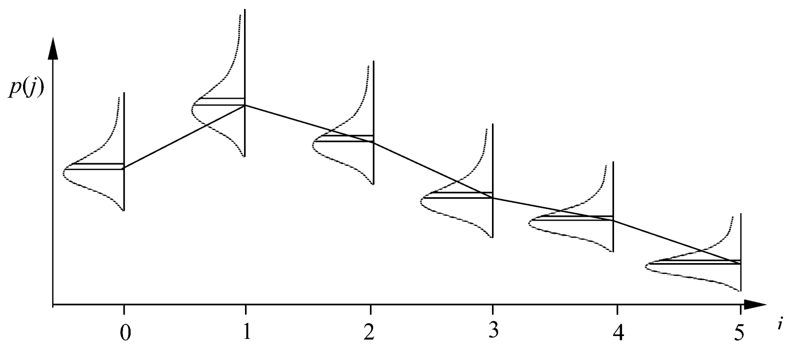
$$\text{standard deviation} = \text{mean} \times \sqrt{\exp[\sigma^2]-1}. \tag{3.5}$$

The probabilistic model for $p(j)$ comprises a lognormal distribution for each development year, where the medians of the lognormal distributions are related by Equation 3.3 and the means are related by Equation 3.4. So, in estimating the model, we are essentially fitting a lognormal distribution to each development year. The trend (on a log scale) comprising the straight line segments is only one component of the model. A principal component comprises the distributions about the trends.

Note from Equation 3.3 that exponentiating the mean on the log scale gives the median on the dollar scale. This is why the line in Figure 17, after exponentiation in Figure 18, joins the medians (the lower of the two lines on each density) not the means. We will normally use the mean as our forecast, rather than the median, but the uncertainty (measured by the standard deviation) of the lognormal distribution is just as important a component of the forecast.

FIGURE 18

MODEL FOR TRENDS ALONG A DEVELOPMENT YEAR (DOLLAR SCALE)



If we compute expected values of the logs of the development factors on the *incremental* data with this model, we obtain $E[\ln(p(j)/p(j-1))] = E[(\gamma_j + \varepsilon_j - \varepsilon_{j-1})] = \gamma_j$. That is, trend parameters also underpin this new model, but in a way that will allow it to appropriately model the trends in the incremental data (in the three directions).

The model described so far only covers a single accident year; we have not yet accounted for the payment-year and accident-year trends. Let the mean of the (random) inflation between payment year t and $t+1$ be represented by ι_t (*iota-t*).

Hence the family of models can be written:

$$y(i,j) = \alpha_i + \sum_{k=1}^{j} \gamma_k + \sum_{t=1}^{i+j} \iota_t + \varepsilon_{i,j}. \tag{3.6}$$

We call this family of models the probabilistic trend family (PTF). Note that the mean trend between cells $(i, j-1)$ and (i, j) is $\gamma_j + \iota_{i+j}$, and the mean trend between cells $(i-1, j)$ and (i, j) is $\alpha_{i+1} - \alpha_i + \iota_{i+j}$.

TABLE 3.1

PARAMETER ESTIMATES FOR THE MODEL WITH CONSTANT TRENDS

Parameter	Estimate	Standard Error	t-ratio
α	11.4256	0.0302	378.57
γ	−0.2062	0.0037	−55.08
ι	0.1563	0.0037	41.74
$s = 0.1129$		$R^2 = 97.0\%$	

A member of this family of models relates the lognormal distributions of the cells in the triangle. On a log scale, the distribution for each cell is normal, where the means of the normal distributions are related by the "trends" described by the particular model.

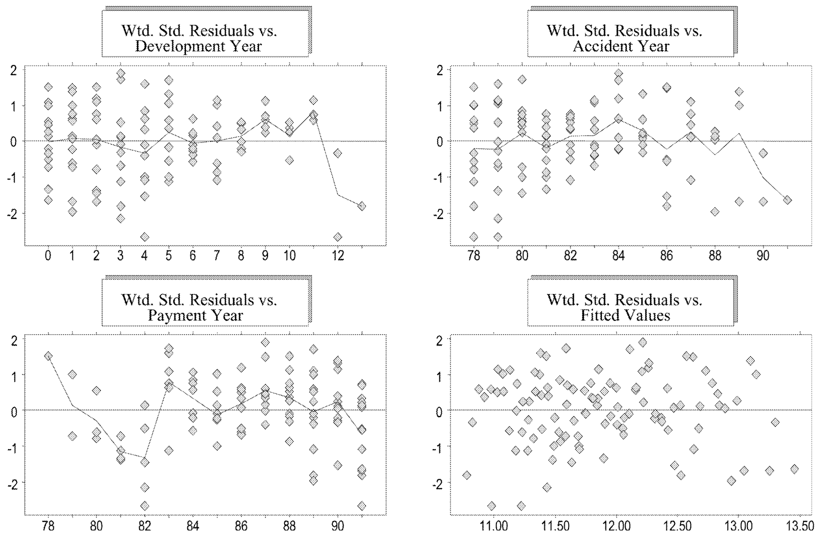
If the error terms $\varepsilon_{i,j}$ (each coming from a normal distribution with mean 0) do not have a constant variance, then the changes in variance must also be modeled. Note that there are numerous models in the PTF, even if we do not include the varying (stochastic) parameter models discussed in Section 3.7. The actuary has to identify the most appropriate model for the loss development array being analyzed. The assumptions made by the "optimal" model must be satisfied by the data. In doing so, one extracts information in terms of trends, stability of trends, and the distributions of the data about the trends.

3.3. Example 2 continued—Estimation

Let's now try to identify the model that created the data. We begin by fitting a model with all the development-year trends equal to each other (one γ), all payment-year trends equal to each other (one ι), and no accident-year trends (one α); that is, with $\gamma_k = \gamma$, $\iota_t = \iota$, and $\alpha_i = \alpha$ for all parameters. The parameter estimates are given in Table 3.1.

FIGURE 19

PLOTS OF STANDARDIZED RESIDUALS VERSUS THE THREE DIRECTIONS AND FITTED VALUES FOR THE SINGLE PAYMENT-YEAR TREND MODEL



The estimate of 0.1563 for ι (iota) is a weighted average of the three trends 0.1, 0.3 and 0.15.

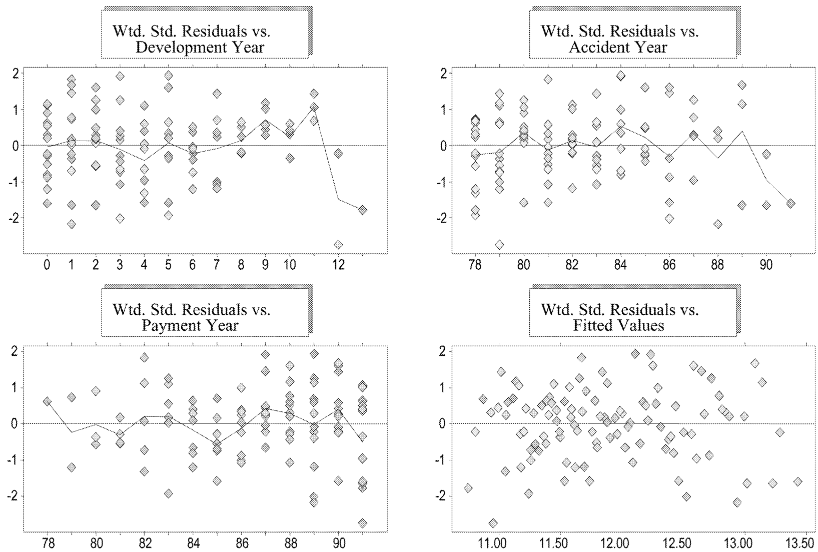
Removing constant trends makes any changes in trend more obvious; the residuals are shown in Figure 19.

The residuals need to be interpreted as the data adjusted for what has been fitted; accordingly, the residuals versus payment years represent the data minus the fitted value of 0.1563 per year.

Immediately, the changes in trends in the payment-year direction become obvious. We can see that the trend in the early years is substantially less than the estimated average of 0.1563; that the trend from 1982 to 1983 is much larger than it; and, after

FIGURE 20

PLOTS OF STANDARDIZED RESIDUALS VERSUS THE THREE DIRECTIONS AND FITTED VALUES FOR THE THREE-PAYMENT-YEAR-TRENDS MODEL



that, the trend is pretty close to the fitted trend, as 0.15 – 0.1563 is approximately zero. This suggests that we should introduce another ι (iota) parameter between 1982–1983, and a further ι parameter between 1983–1984 (that will continue to 1991).

The residuals of the model with three payment-year trends are given in Figure 20—this model seems to have captured the trends. The parameter estimates are given in Table 3.2.

Note that the estimates of the trend parameters 0.1, 0.3, 0.15 are not equal to the true values; indeed, 0.3927 (standard error 0.0442) is a bit off the mark (which is about two standard errors). The estimate is far from 0.3 because in the payment years 1982 and 1983, there aren't many data points. Given that the

TABLE 3.2

PARAMETER ESTIMATES FOR THE THREE-PAYMENT-YEAR-TRENDS MODEL

Parameter	Estimate	Standard Error	t-ratio
α	11.5321	0.0612	188.34
γ	−0.2062	0.0033	−61.91
ι: 78–82	0.0873	0.0209	4.18
ι: 82–83	0.3927	0.0442	8.90
ι: 83–91	0.1446	0.0046	31.72
$s = 0.1005$		$R^2 = 97.7\%$	

TABLE 3.3

FORECASTS, STANDARD ERRORS, TREND ESTIMATES AND THEIR STANDARD ERRORS AS THE LATER PAYMENT YEARS ARE REMOVED

Years in Estimation	N	γ (83–91)	Standard Error (γ)	ι (83–91)	Standard Error (ι)	Mean Forecast	Standard Error of Forecast
78–91	105	−0.2062	0.0033	0.1446	0.0046	23,426,542	927,810
78–90	91	−0.2075	0.0036	0.1527	0.0051	25,333,522	1,191,129
78–89	78	−0.2086	0.0042	0.1512	0.0064	24,850,972	1,526,246
78–88	66	−0.2119	0.0045	0.1575	0.0075	26,296,366	1,997,089
78–87	55	−0.2131	0.0055	0.1563	0.0103	25,894,931	2,868,948

trend of 0.15 is in the data since 1983, we would expect stability of forecasts, and trend parameter estimates as we remove years.

The forecasts are stable—if we remove the most recent data, the forecasts of this model don't change much relative to the standard error in the forecast, as we can see in Table 3.3.

Note that the estimate of γ (recall that $\gamma = -0.2$) is pretty stable, as we remove the latest years.

FIGURE 21

PREDICTION ERRORS FOR 1988–1991, FOR MODEL ESTIMATED IN 1987

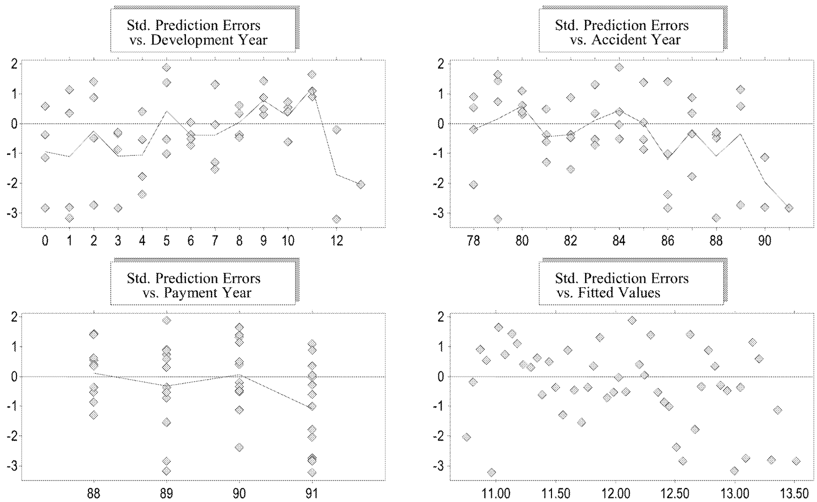


Figure 21 gives the prediction errors (on a log scale) for the four payment years 1988–1991, based on the model estimated at year end 1987.

The estimated model at the end of payment year 1987 slightly over-predicts the payment periods 1988–1991. That is because the trend estimate since 1983 (see Table 3.4) is now $15.63\% \pm 1.03\%$ (where we are writing mean $\pm$ standard deviation as shorthand), in place of $14.46\% \pm 0.46\%$ when we use all the years in the estimation. Hence the forecast of \$26M ($\pm$\$2.9M) is "higher" than \$23M ($\pm$\$0.9M). When you test for a trend change between 1987 and 1988, it is not significant (as we would expect). Note that removal of recent payment years to check the model's ability to predict them (validation analysis) is part of the model identification procedure and extraction of information process.

TABLE 3.4

VALIDATION RESULTS—PARAMETER ESTIMATES AND FORECASTS AS PAYMENT YEARS ARE REMOVED FROM THE SELECTED MODEL

Payment Years in Estimation	Estimate of gamma	Estimate of iota (83–91)	Forecast ± Standard Error ($M)
1978–91	−20.62 ± 0.33	14.46 ± 0.46	23 ± 0.9
1978–90	−20.75 ± 0.36	15.27 ± 0.51	25 ± 1.2
1978–89	−20.86 ± 0.42	15.15 ± 0.64	25 ± 1.5
1978–88	−21.19 ± 0.45	15.75 ± 0.75	26 ± 2.0
1978–87	−21.31 ± 0.55	15.63 ± 1.03	26 ± 2.9

3.4. Example 3—Real Data With Major Payment-Year Trend Instability

We now analyze a real data array as presented in Table 3.5.

This loss development array has a major trend change between payment years 1984 and 1985, even though the data and link ratios are relatively smooth. Indeed, it needs to be understood that, in general, trend instability has nothing to do with volatility or smoothness of the data and link ratios. When there is a trend change, formulation of the assumptions about the future trend will depend on the explanation for that trend change.

The individual link ratios for the cumulated data are very stable, as can be seen in Figure 22. It is very dangerous to try to make judgements about the suitability of development factor techniques from the individual link ratios on the cumulated data.

We first conduct some diagnostic PTF analysis, then show how the ELRF modeling structure also indicates payment-year trend change, indeed that any method based on link ratios is quite meaningless for this data. Consequently, there is little to be gained by forecasting any of the ELRF models. Figure 23 shows

TABLE 3.5

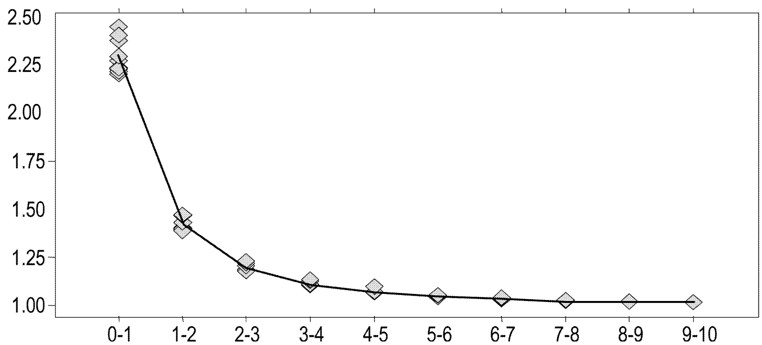
INCREMENTAL PAID LOSSES AND EXPOSURES FOR ABC

	0	1	2	3	4	5	6	7	8	9	10
1977	153,638	188,412	134,534	87,456	60,348	42,404	31,238	21,252	16,622	14,440	12,200
1978	178,536	226,412	158,894	104,686	71,448	47,990	35,576	24,818	22,662	18,000	
1979	210,172	259,168	188,388	123,074	83,380	56,086	38,496	33,768	27,400		
1980	211,448	253,482	183,370	131,040	78,994	60,232	45,568	38,000			
1981	219,810	266,304	194,650	120,098	87,582	62,750	51,000				
1982	205,654	252,746	177,506	129,522	96,786	82,400					
1983	197,716	255,408	194,648	142,328	105,600						
1984	239,784	329,242	264,802	190,400							
1985	326,304	471,744	375,400								
1986	420,778	590,400									
1987	496,200										

Accident Year	1977	1978	1979	1980	1981	1982	1983	1984	1985	1986	1987
Exposure	2.2	2.4	2.2	2.0	1.9	1.6	1.6	1.8	2.2	2.5	2.6

FIGURE 22

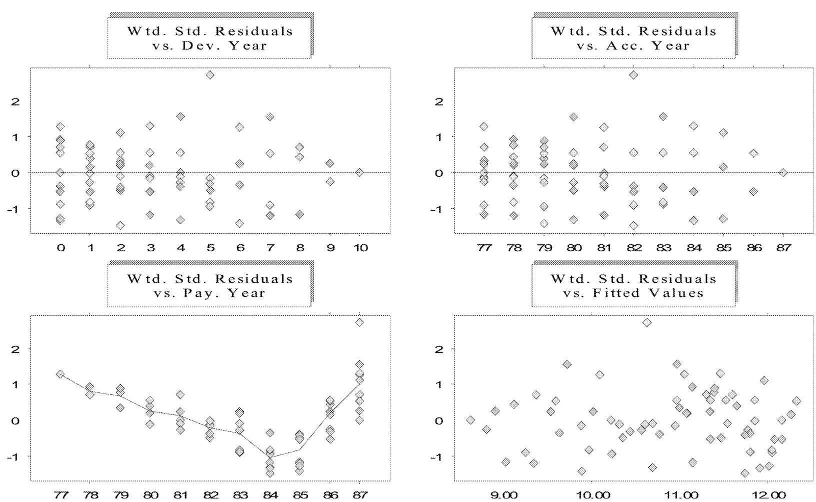
PLOT OF INDIVIDUAL LINK RATIOS BY DELAY†



† The line joins Chain Ladder ratios.

FIGURE 23

STANDARDIZED RESIDUALS OF THE STATISTICAL CHAIN LADDER MODEL



the standardized residuals of the statistical chain ladder in PTF (i.e., the statistical chain ladder fits all the gamma parameters and all the alpha parameters with no iotas). The residuals are just the data (less the average level) adjusted for the (average) trend between every pair of contiguous development periods and every pair of contiguous accident periods. This is why the plots of standardized residuals versus development years and accident years are centered on zero! We use this model only as a diagnostic tool to determine quickly whether there are payment-year trend changes that can be attributed solely to the payment years.

Contrast the smoothness of the ratios above with the plot of the residuals from this model. We can now see dramatic changes in the payment-year direction. It might be very dangerous to use forecasts from any model assuming no changes in payment-year trend, such as a model from the ELRF. There is a difference between Figures 23 and 10—the statistical chain ladder shows the payment-year trends after adjusting for the trends in the other two directions, while the chain ladder ratios (Figure 10) do not do that. But the change in trend is clear in either graph. In the current statistical modeling framework, we are able to model this change; we have a lot more control over how we incorporate the trend changes into our model and hence into the forecasts. Even the best ELRF model for this data hardly uses ratios and is deficient because it gives us no control in the payment-year direction. It turns out that the trend before 1984 is approximately 10% whereas the trend after 1984 is approximately 20%. So which trend should we assume for the future? This depends on the explanation for the change. If the trend instability is due to new legislation that applies retrospectively (to all accident periods), then one would revert to the 10%—as a change to the level of payments will be a single jump in level (possibly taking several years to be completely manifested). If there is no explanation for the trend change, except that the payments have

TABLE 3.6

PAID LOSS ARRAY FOR THE PAN6 DATA FOR EXAMPLE 4

Accident Year	Development Year 0	1	2	3	4	5
1986	194324	571621	327880	249194	524483	1724274
1987	1469	57393	485791	169614	121410	599021
1988	1860	161538	408008	314614	6744000	****
1989	23512	185604	260725	1134272	851099	2174200
1990	1044	70096	93600	1283752	1595466	913215
1991	****	3730	869959	187019	2764795	****
1992	****	443205	180064	683407	878117	
1993	****	12808	433511	118017		
1994	1431	77765	151161			
1995	51539	****				
1996	****					

increased, then calling the future in terms of trends is more difficult.

3.5. *Example 4—Volatile Data With Stable Trends*

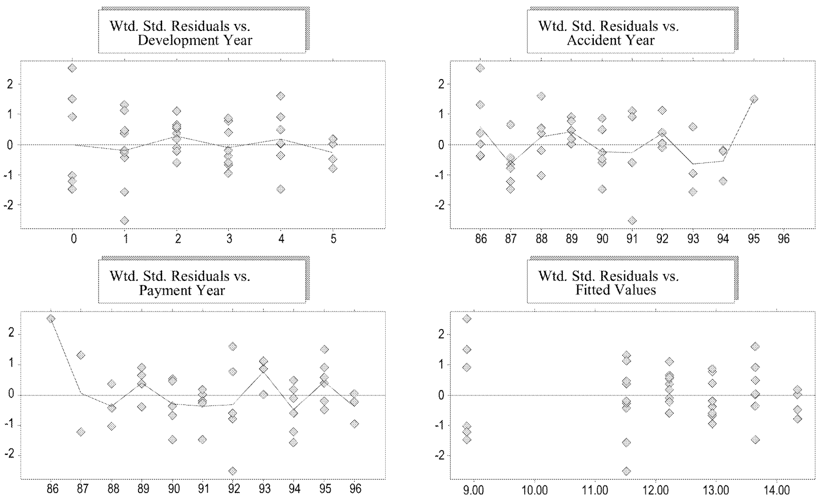
We now consider an array where the paid losses are very volatile, but the trends are stable (see Table 3.6). Recall that trend stability/instability is dependent on neither the volatility of the data nor the volatility of the link ratios. Since the random component is an integral part of the model, this model captures the behavior of this volatile data very well. We call this array PAN6.

A good model can be identified quickly for the logarithms of these data; it has no payment-year trends, and only two different development-year trends—between development years 0–1 and for all later years (the residual plot is given in Figure 24).

Note that the spread of the first two development years is wider than for the later years, and the spread for "small" fitted values is larger than the spread for "large" fitted values. If we estimate the standard deviations in the two sections, we find

FIGURE 24

PLOT OF STANDARDIZED RESIDUALS FOR THE MODEL WITH TWO GAMMA PARAMETERS AND ONE ALPHA PARAMETER

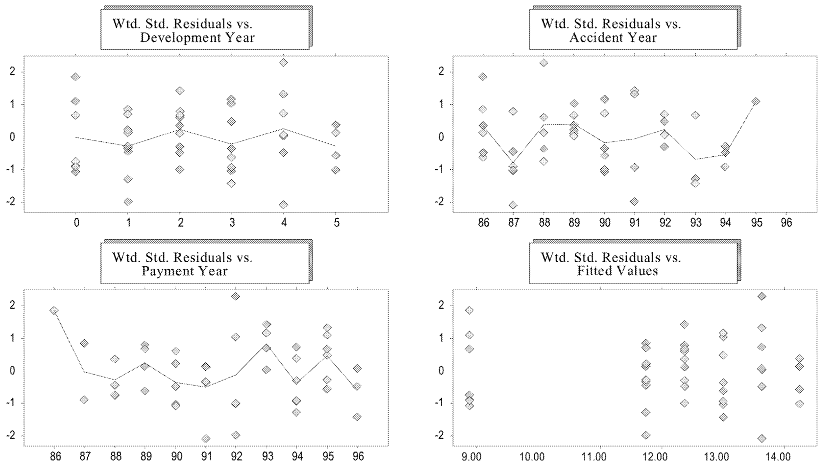


that they are 3.0177 and 0.8015, respectively. This requires a weighted regression; development years 0 and 1 are given weight $(0.8015/3.0177)^2$, and the other years (2+) have a weight of 1. The weighted, standardized residual plots now look fine (see Figure 25). A check of the plot of residuals against normal scores (not presented here) indicates that the assumption of normality of the logarithms of the data is very reasonable—the squared correlation is greater than 0.99.

The normal distributions for this model have relatively large variances—the estimate of σ^2 for development periods 0–1 is 2.923, and for development periods 2+ is 0.80346. Note that if a normal distribution has a variance σ^2, then the corresponding lognormal distribution has a coefficient variation of $\sqrt{\exp(\sigma^2) - 1} > \sigma$.

FIGURE 25

PLOT OF WEIGHTED STANDARDIZED RESIDUALS AFTER THE WEIGHTED REGRESSION



This model also has forecasts that are stable as we remove the most recent data, as we see in Table 3.7. This is a very important attribute of the identified model that captures the information in the data—if the trends in the data are stable, then so are the forecasts based on the estimated model. In this case, we were able to remove almost half the (most recent) data. The standard errors of the forecasts are large because the lognormal distributions are skewed—insurance is about measuring variance, not just means.

While the variability of the data and hence the standard errors of the forecasts are large, the message from the data has been consistent over many years. We are predicting the distribution of the data in each cell, not merely their mean and standard deviation, so a large standard deviation does not imply a bad model. Indeed, the model is very good. It captures the variances, indeed the distributions, in each cell.

TABLE 3.7

FORECASTS, FORECAST STANDARD ERRORS, FINAL TREND ESTIMATES AND THEIR STANDARD ERRORS FOR THE FINAL MODEL AS THE LATER PAYMENT YEARS ARE REMOVED

Years in Estimation	N	Trend (dev. period 1+)	Standard Error	Mean Forecast	Standard Error
86–96	44	0.6250	0.1432	20,352,011	9,136,870
86–95	41	0.6102	0.1479	21,410,781	9,839,127
86–94	35	0.6149	0.1681	21,037,520	10,654,173
86–93	29	0.5024	0.1977	19,755,944	11,647,274
86–92	25	0.5631	0.2143	18,567,664	11,529,359

The high standard errors of forecasts are due to large process variability. As we remove recent years (diagonals) from the estimation, we note forecasts are stable. This is further evidence of a stable trend in the data.

Note that at the end of year 1992, the estimated model would have predicted the normal distributions for the log(payments) in years 1993–1996 (see Figure 26), and would have produced statistically the same forecast of outstanding claims. Figure 27 indicates that the assumption of normality is reasonable.

We now turn to ELRF analysis. Since the data are extremely skewed (lognormal with large coefficient of variation), the residuals of the chain ladder (regression) ratios in ELRF are extremely skewed to the right (see Figure 28). The plot of residuals against fitted values shows a downward trend, indicating that we overpredict the large values and underpredict the small ones. The residuals also show strong indications of non-normality. Moreover, all the ratios have no predictive power (provided there is an intercept). In any event, residuals are skewed (not normal), so even the best model in ELRF—the Cape Cod ($y - x = a_0 + \varepsilon$)—is not a good one.

FIGURE 26

PREDICTION ERRORS FOR YEARS 1993–1996

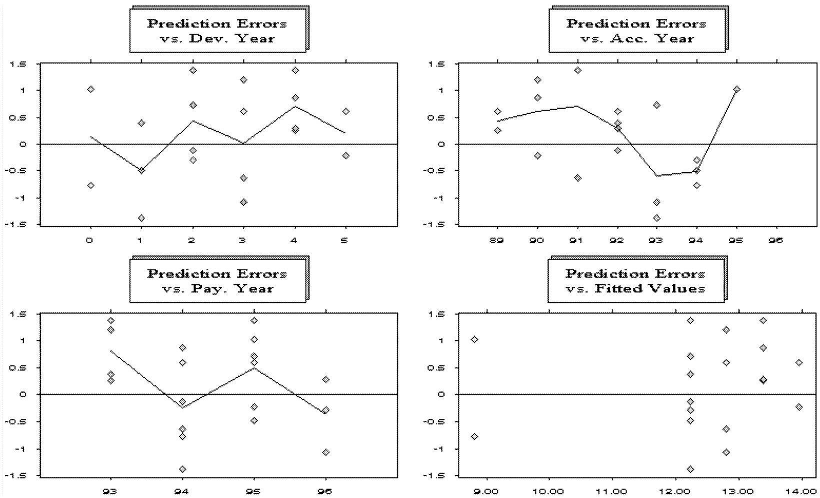


FIGURE 27

NORMALITY PLOT OF PREDICTION ERRORS FOR 1993–96
BASED ON MODEL ESTIMATED AT YEAR END 1992

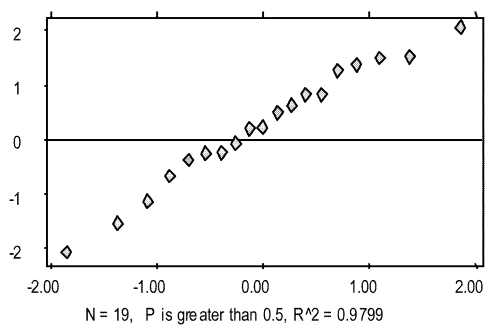
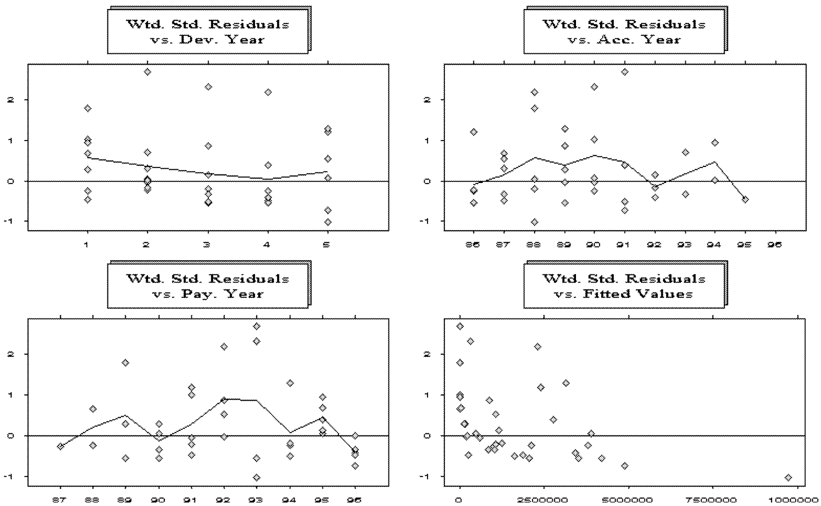


FIGURE 28

RESIDUALS OF CHAIN LADDER RATIOS REGRESSION MODEL



Recall that if model assumptions are not satisfied by the data, then forecast calculations may be quite meaningless.

3.6. Example 5—Simulated Array Based on a Model With Only Two Parameters

The following array, which we call SDF, is a simulated data set where the incremental paid losses have completely homogeneous accident years. The actual model driving the data has one alpha (α) = 10, one gamma (γ) = -0.3, and has σ^2 = 0.4. That is,

$$y(i,j) = 10 - 0.3j + \varepsilon(i,j), \qquad (3.7)$$

where the $\varepsilon(i,j)$ are independent and identically distributed from a normal distribution with mean 0 and variance 0.4.

The simulated data is presented in Table 3.8.

TABLE 3.8

INCREMENTAL PAID LOSS DATA FOR SIMULATED EXAMPLE SDF

Accident	Development Year																
Year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1978	24,307	44,260	3,900	13,393	17,731	5,802	1,975	6,294	4,758	1,529	2,383	161	760	445	640	501	255
1979	19,122	7,003	8,147	2,872	5,639	2,967	3,455	2,364	4,005	2,029	1,370	718	432	1,282	456	232	
1980	18,082	27,708	27,901	5,699	9,297	2,899	7,461	6,885	1,854	2,681	792	1,338	820	702	179		
1981	80,451	11,411	68,627	6,703	6,430	2,693	5,560	847	4,706	1,245	1,289	717	754	436			
1982	49,099	7,144	11,979	3,481	2,279	5,975	4,472	2,066	509	2,345	755	1,702	596				
1983	33,475	54,717	8,774	4,859	5,808	20,750	1,903	2,449	937	796	1,446	1,271					
1984	23,070	49,554	5,659	9,909	9,123	8,805	7,008	8,634	3,053	493	2,108						
1985	14,324	8,352	7,955	8,092	6,044	8,542	7,700	2,849	1,130	1,754							
1986	58,785	16,833	6,068	5,227	3,276	16,592	2,407	908	2,508								
1987	9,017	7,999	10,796	12,737	3,880	6,536	6,779	2,336									
1988	12,205	24,980	13,835	9,881	4,978	3,380	7,105										
1989	17,883	5,194	11,429	2,769	9,540	5,107											
1990	25,584	14,468	12,543	5,774	10,414												
1991	49,089	22,514	24,075	9,591													
1992	24,064	49,272	3,231														
1993	17,858	19,689															
1994	24,869																

The first thing to note with this data is that, once noise is added, it looks like incremental paid data for a real array, even though it was generated from a very simple model.

The relatively large σ^2 (0.4) explains the high variability in the observed paid losses. The incremental data displayed in Table 3.8 appear volatile, but the values in the same development period are independent realizations from the same lognormal distribution.

For example, in development period zero, the simulated values 80,451 and 9,017 come from a lognormal distribution with mean 26,903 and standard deviation 18,867. Since a lognormal distribution is skewed to the right, realizations larger than the mean are typically further away from the mean compared to realizations less than the mean, which are bounded below by zero (and so are closer to the mean).

The apparent volatility in the data is not due to instability in trends; indeed, the reality is quite the opposite—though volatile, the incremental paid losses have stable trends. Since we know the exact probability distributions driving the data, we can compute the exact mean and exact standard deviation for each cell in the rectangle and also the exact means and standard deviations of sums.

The exact mean of the total outstanding is \$284,125, with an exact standard deviation of \$30,970, and so the process variance is $30{,}970^2$. When we analyze the data in PTF, we identify only two significant parameters—$\hat{\alpha} = 9.9667$ (with standard error 0.0847) and $\hat{\gamma} = -0.2867$ (standard error 0.0126), and the estimate of σ^2 is 0.4085. Residuals from this estimated model are displayed in Figure 29.

Table 3.9 gives forecasts of total outstanding, including validation forecasts (note that the forecasts are stable, as expected).

We now study the cumulative array, displayed in Figure 30. Even though the incremental data was generated with homogeneous accident years, the cumulated data have each accident year

FIGURE 29

RESIDUALS BASED ON THE ESTIMATED PARAMETERS OF THE MODEL FROM WHICH THE DATA WERE GENERATED

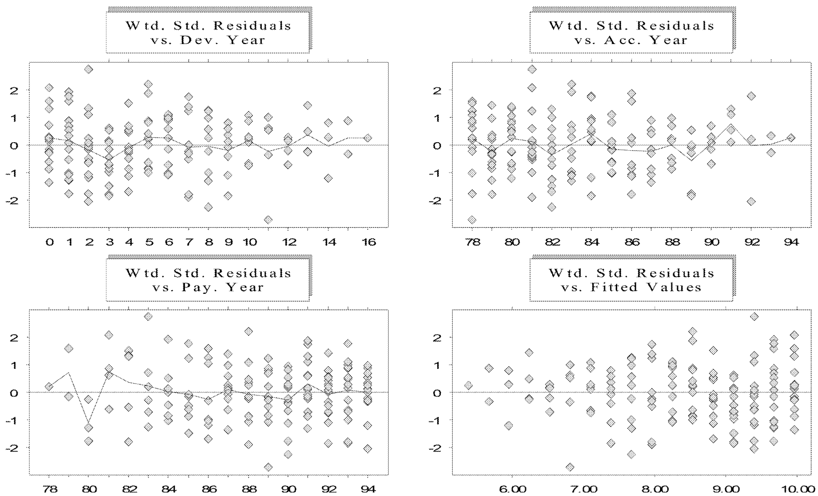


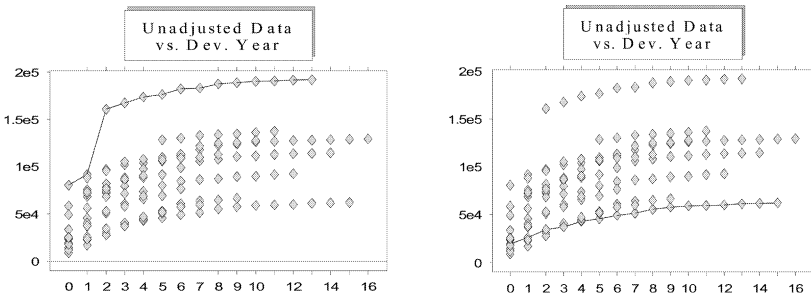
TABLE 3.9

ESTIMATES OF DEVELOPMENT-YEAR TRENDS AND TOTAL OUTSTANDING WITH STANDARD ERRORS AS YEARS ARE REMOVED FROM THE ANALYSIS

Payment Years in Estimation	Estimate of Gamma	Standard Error of Gamma	Mean Forecast	Standard Error of Forecast
1978–94	−0.2867	0.0126	299,660	35,487
1978–93	−0.2858	0.0146	303,980	37,886
1978–92	−0.2865	0.0166	302,601	38,843
1978–91	−0.2926	0.0195	304,711	42,148
1978–90	−0.2940	0.0228	296,650	43,625
1978–89	−0.2861	0.0271	313,604	50,001

FIGURE 30

CUMULATIVE DATA (SDF)†



†Accident year 1981 has high development; 1979 has low development.

at a completely different level. The plot against accident years jumps all over the place—the values along an accident year tend to be high or low. This is a common feature with cumulative arrays.

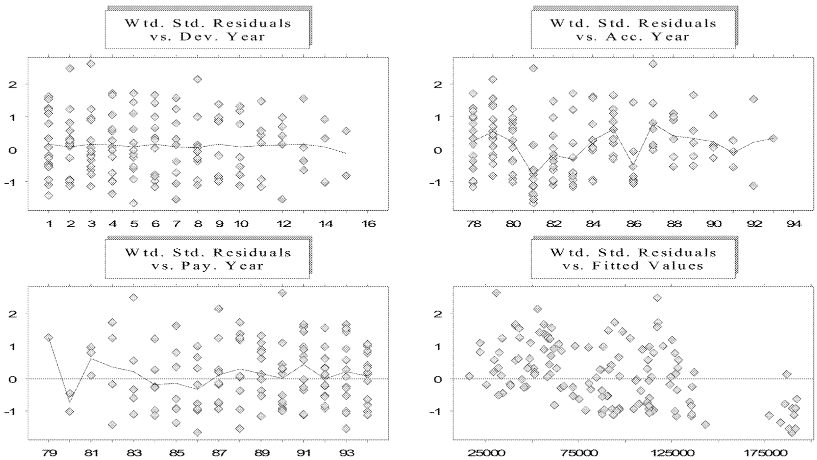
The cumulative values for 1979 lie entirely below those for 1982 (see Figure 30), yet most of the incremental payments are "close" together. One "large" incremental value from the tail of the lognormal has a major impact on the cumulative data. The link ratio techniques assume that the next incremental payment will be high if the current cumulative is high, and this looks like what is going on with the cumulative data. So the cumulatives deliver a false indication, even for data where there are no payment-year trend changes.

Note that for 1979, cumulative paid at development year 5 is \$45,750, whereas for 1981 it is \$176,315. For this array, we know that current emergence is not a predictor of future emergence.

The chain ladder ratios model gives a mean outstanding forecast of \$254,130 and a standard error of the outstanding forecast

FIGURE 31

PLOT OF WEIGHTED STANDARDIZED RESIDUALS FOR CHAIN LADDER RATIOS FOR TRIANGLE SDF



of \$59,419. The plot of residuals against fitted values makes it clear where the problem lies, as we see in Figure 31.

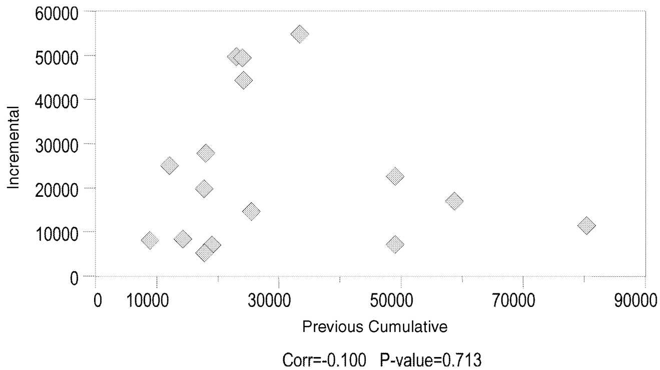
Again, we have a year with high cumulatives (1981) overpredicted, and a year with low cumulatives (1979) underpredicted.

Note how there is a distinct downward trend in the plot of fitted values. It indicates that the model overpredicts the high cumulative values and underpredicts the low values—which it will do if the cumulatives don't really contain information on the subsequent incrementals. Plots of residuals against normal scores show non-normality (not presented here). If we look at the plot of the incremental paid losses against the previous cumulative (see Figure 32), we can see that models involving ratios will be inappropriate since there is no relationship.

The best model in the ELRF sets all of the ratios to 1 and only uses intercepts (i.e., it takes averages of incrementals in

FIGURE 32

PLOT OF INCREMENTAL PAYMENTS AGAINST PREVIOUS CUMULATIVE



each development year). But, due to the non-normality, this is insufficient. At least the ELRF analysis informs us that the incrementals in a development period may be regarded as random values from the same distribution, and that these incrementals are not correlated with the previous cumulatives (just as the data were generated). It also tells us that the data are skewed, so we need to take a transformation. By way of summary, the ELRF analysis informs us that the data were created incrementally, accident years are homogeneous, and we should be modeling the logs of the incremental data—it is telling us the truth.

If you generate (simulate) data using link ratios, the ELRF will tell you that ratios have predictive power and that the data were generated cumulatively. Importantly however, for most real loss development arrays, ELRF analysis indicates that the data were generated incrementally and that ratios have much less predictive power than trends in the log incrementals. The ELRF analysis also shows when there may be payment/calendar year trend changes.

3.7. Varying (Stochastic) Parameters

In view of the trend relationships between the development-year, accident-year, and payment-year directions, a model with several parameters in the payment-year and accident-year directions may suffer from multi-collinearity problems. Zehnwirth [12, Section 7.2] discusses the importance of varying (stochastic) parameter models, especially the introduction of a varying alpha parameter (in place of adding parameters), to overcome multi-collinearity. This is akin to exponential smoothing in the accident-year direction. This approach is necessary, very powerful, and increases the stability of the model, especially if in the more recent accident years there are some slight changes in levels. The amount of stochastic variation in α is determined by the SSPE statistic, which is explained in Zehnwirth [12].

3.8. Model Identification

It is important to identify a parsimonious model in PTF that separates the (systematic) trends from the random fluctuations, and moreover determines whether the trend in the payment/calendar year direction is stable.

The model identification procedure is discussed in Section 10 of Zehnwirth [12]. We start off with a model that only has one parameter in each direction, model (sequentially) the trends in the development-year direction, and follow that by looking at the trends in the payment-year or accident-year directions (depending on which direction exhibits more dramatic trend changes). Adjustments for different variances may also be necessary. Validation analysis is an integral component of model identification, extraction of information, and testing for stability of trends.

3.9. Assumptions About the Future

Stability and assumptions about the future are discussed in Section 9.6.2 and 10.2 of Zehnwirth [12]. If payment/calendar year trend has been stable in the more recent years, then the assumption about the future is relatively straightforward. For example, if the estimate in the last seven years of ι is $\hat{\iota}$ with standard error $s.e.(\hat{\iota})$, then we assume for the future a mean trend of $\hat{\iota}$, with a standard deviation of trend of $s.e.(\hat{\iota})$; we do *not* assume the trend in the future is constant. Our model includes the variability (uncertainty) in trend in the future, in addition to the process variability (about the trend).

If on the other hand, payment/calendar year trend has been unstable, as was illustrated with Project ABC, assumptions about the future will depend on the explanation for the instability—for Project ABC we revert to the 10% trend if the dramatic change is explained by new legislation. Zehnwirth [12] also cites some other practical examples where special knowledge about the business is a contributing factor in formulating assumptions about the future, especially in the presence of trend instability. Importantly, however, that special knowledge is combined with the information that is extracted from past experience.

It is not possible to enumerate all feasible cases, though several cases are discussed in Zehnwirth [12]. The more experience the actuary has with the new statistical paradigm, the better he/she is equipped to formulate assumptions about the future in the presence of unstable trends. Bear in mind, of course, that quite often trends are stable; but we only know this after performing an analysis like that described for the PTF.

3.10. How Do We Know That Real Data Triangles Can Be Generated By the Members of the PTF?

Let's conduct the following experiment. Begin with 100 data arrays (triangles) and, for each triangle, the "best" model in the PTF is given. Recall that a fitted (best) model relates the distributions of each cell in terms of trends on the log scale. These models are tested to ensure they are good models. ELRF analysis is then conducted on each triangle. Now assume that some of the triangles are real data from some companies, but some are not.

That is, for some triangles the data represent a sample path from the so-called "fitted" distributions. Which are the real data and which are simulated data from the model?

Both the real and simulated data display similar features to each other, whether we analyze them using PTF models or ELRF models. For example, both usually indicate that once you fit an intercept (and accident-year trend, if present), ratios are not needed. Both tend to display trends in the payment years. Since you cannot distinguish between real triangles and simulated triangles generated from models in the PTF, these kinds of models must be valid. That is, the rich family of models in PTF possess probabilistic mechanisms for generating real data.

Of course, the models do not represent the underlying complex generation process that is driven by many variables. However, the variables that drive the data are implicitly included in the trends and the noise (σ^2). We do the same thing when we fit a loss distribution (e.g., Pareto) to a set of severities. The estimated Pareto did *not* create the severities, but it has probabilistic mechanisms for creating the data as a sample.

Suppose we now simulate (cumulative) loss development arrays from ratio models. The ELRF methodology applied to the simulated arrays will inform us that ratios have predictive power (indeed more predictive power than using the trends in the incrementals). If we now analyze the corresponding incremental arrays using PTF models, we notice a phenomenon that is very rarely observed for real data. After removing trends in the development-year direction, there are distinct patterns in the accident-year direction—the levels jump up and down dramatically, much more so than is typical for real data. This is because if an incremental value in development period zero is low, then the subsequent incrementals for the same accident period remain relatively low, as all accident years are multiplied by the same ratios. Similarly, if the initial incremental value is high, the subsequent incrementals from the model are high. This pattern occurs even when you include considerable randomness.

Finally, suppose an actuary is presented with many development arrays, some real, and the others simulated using models based on ratio-techniques. By applying the ELRF and PTF methodologies, the actuary is very frequently able to distinguish the real arrays from the ones simulated from the ratio models.

4. THE RESERVE FIGURE

Loss reserves often constitute the largest single item in an insurer's balance sheet. An upward or downward 10% movement of loss reserves could change the whole financial picture of the company.

4.1. Prediction Intervals

We have argued for the use of probabilistic models, especially in assessing the variability or uncertainty inherent in loss reserves. The probability that the loss reserve carried in the balance sheet will be realized in the future is effectively zero, even if the loss reserve is the true mean!

Future (incremental) paid losses may be regarded as a sample path from the forecast (estimated) lognormal distributions, which include both process risk and parameter risk. Forecasting of distributions is discussed in Zehnwirth [12].

The forecast distributions are accurate provided the assumptions made about the future are, and remain, true. For example, if it is assumed that future payment/calendar year trend (inflation) has a mean of 10% and a standard deviation of 2%, and in two years time it turns out that inflation is 20%, then the forecast distributions are far from accurate.

Accordingly, any prediction interval computed from the forecast distributions is conditional on the assumptions about the future remaining true. The assumptions are in terms of mean trends, standard deviations of trends, and distributions about the trends.

It is important to note that there is a difference between a fitted distribution and the corresponding predictive distribution. A predictive distribution necessarily incorporates parameter estimation error (parameter risk); a fitted distribution does not. Ignoring parameter risk can result in substantial underestimation of reserves and premiums (see the paper by Dickson, Tedesco and Zehnwirth [4] for more details).

Under the model, the distribution of a sum of payments (e.g., accident-year outstanding payments) is the distribution of a sum of correlated lognormal variables. The lognormal variables are correlated because of the correlations between the estimated parameters describing their mean level—indeed many forecasts will even share some parameters. The distribution of the sum can be obtained by generating (simulating) samples from the estimated multivariate lognormal distributions. The same could be done for payment-year totals (important for obtaining the distributions of the future payment stream) or for the overall total. This information is relevant to Dynamic Financial Analysis. Distributions for future underwriting years can also be computed—this information is useful for pricing, including aggregate deductibles and excess layers.

An insurer's risk can be defined in many different ways. One common definition is related to the standard deviation of the risk, in particular a multiple of the standard deviation. If the reserve is based on a given percentile of the distribution of the total outstanding, the size of the loading as a multiple of the standard deviation will be dependent on the skewness of the distribution.

If an insurer writes more than one long-tail line and aims for a $100\,(1-\alpha)\%$ security level on all the lines combined, then the risk margin per line decreases the more lines the company writes, no matter which allocation principle is used. This is always true, even if there is some dependence (and so correlation) between the various lines. In the following example, the standard deviation principle is used.

Consider a company that writes n independent long-tail lines. Suppose that the standard error of loss reserve $L(j)$ of line j is $s.e.(j)$ (i.e., $s.e.(j)$ is the standard error of the loss reserve variable $L(j)$). The standard error for the combined lines $L(1) + \cdots + L(n)$ is

$$s.e.(\text{Total}) = [s.e.^2(1) + \cdots + s.e.^2(n)]^{0.5}. \tag{4.1}$$

If the risk margin for all lines combined is $k \times s.e.(\text{Total})$, where k is determined by the level of security required, then an allocation of the risk margin for line j is

$$k \times s.e.(\text{Total}) \times s.e.(j)/[s.e.(1) + \cdots + s.e.(n)] < k \times s.e.(j). \tag{4.2}$$

The last inequality is true even when $s.e.(\text{Total})$ is not given by the expression above.

If as a result of analyzing each line using the statistical modeling framework, we find that, for some lines, trends change in the same years and the changes are of similar size, then the lines are not independent. (There may also be correlations between the residuals, but that is generally only important when forecasting sums of lines.)

In that situation, if line i and j are correlated, say, then one could use $s.e.(i) + s.e.(j)$ as the upper bound of the standard error of $L(i) + L(j)$. (Based on our experience, it is not often the case that different lines are much correlated in terms of trends.)

Suppose we assume for the future payment/calendar years a mean trend of $\hat{\iota}$ with a standard deviation (standard error) $s.e.(\hat{\iota})$. Specifically, we are saying that the trend ι, a random variable, has a normal distribution with mean $\hat{\iota}$ and standard deviation $s.e.(\hat{\iota})$. Recognition of the relationship between the lognormal and normal distributions tells us that the mean payment increases as $s.e.(\hat{\iota})$ increases (and $\hat{\iota}$ remains constant). The greater the uncertainty in a parameter (the mean remaining constant), the more money is paid out. The same argument applies to the other estimated parameters in the model. This is known as Jensen's inequality (this concept is in many college finance texts—see for example Brearley & Myers [1]). It is dangerous to ignore this concept.

4.2. *Risk-Based Capital*

There are a number of misconceptions regarding risk-based capital. It is important to note the following:

- The uncertainty in loss reserves (for the future) should be based on a probabilistic model (for the future) that may bear no relationship to reserves carried by the company in the past.
- The uncertainty for each line for each company should be based on a probabilistic model, derived from the company's experience, that describes the particular line for that company. A model appropriate for one loss development array will usually not be appropriate for another.
- The company's experience may bear very little relationship to the industry as a whole.

The approach discussed here allows the actuary to determine the relationships within and between companies' experiences and their relationships to the industry in terms of simple, well-understood features of the data.

In establishing the loss reserve, recognition is often given to the time value of money by discounting. The absence of discounting implies that the (median) estimate contains an implicit risk margin. But this implicit margin may bear no relationship to the security margin sought. The risk should be computed before discounting (at a zero rate of return).

4.3. *Booking of the Reserve*

There are no hard and fast rules here, but three very important steps are critical.

Step 1

Extract information, in terms of trends, stability of trends, and distributions about trends, for the loss development array; in particular, the incremental paid losses. Information is extracted by identifying the best model in the PTF. Model identification and extraction of information necessarily involves validation analysis (re-analyzing and forecasting the array after removal of past recent payment/calendar years).

Step 2

Assumptions about the future are formulated. If payment/calendar year trend is stable, this is straightforward. If trends are unstable in more recent years, then an attempt is made to determine the cause by analyzing other data types and using any relevant business knowledge. A number of examples are given in Zehnwirth [12], but it is impossible to give an exhaustive list as each case may be different.

Step 3

Using the distributions of reserves, the security margin sought on combined lines, and the risk capital available to the company, a percentile can be selected. Incidentally, the more uncertain the trends are for the future, the higher the security margin that may be called for.

4.4. Other Benefits of the Statistical Paradigm

Finally, the statistical modeling framework has other benefits, including:

- Credibility models

  If a particular trend parameter estimate for an individual company is not fully credible, it can be formally shrunk towards an industry estimate.

- Segmentation and layers

Very often the statistical model identified for a combined array of all payment types applies to some of the segments (by the same model, we mean the same parameter structure, not that the estimates are identical). Indeed, the variance of the normal distribution for a segment is larger than for the whole—on the original scale, the coefficient of variation is larger for the components. These ideas can also be applied to territories, etc., and to layers.

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APPENDIX A

WEIGHTED LEAST SQUARES ESTIMATES WITHOUT INTERCEPT

In this appendix, we give a brief outline of the simplest case of the derivation of weighted least squares estimates. This corresponds to the no-intercept models at the start of Section 2—including the standard chain ladder.

Consider the following model:

$$y(i) = bx(i) + \varepsilon(i), \qquad \text{with} \tag{A.1a}$$

$$\varepsilon(i) \sim \text{Normal}(0, \sigma^2/w(i)). \tag{A.1b}$$

The value $w(i)$ is called the weight of observation i. Note that weights are inversely proportional to variances. Estimation of the parameters via maximum likelihood corresponds to minimizing the weighted sums of squared residuals:

$$\sum_{i=1}^{n} w(i)(y(i) - \hat{b}x(i))^2. \tag{A.2}$$

We find the minimum using calculus in a straightforward manner: taking derivatives with respect to $\hat{b}$, and setting the result equal to zero, we get

$$\sum_{i=1}^{n} -2x(i)w(i)(y(i) - \hat{b}x(i)) = 0; \tag{A.3}$$

then solving for $\hat{b}$ we obtain:

$$\hat{b} = \sum_{i=1}^{n} w(i)x(i)y(i) \Bigg/ \sum_{i=1}^{n} w(i)x(i)^2. \tag{A.4}$$

When $w(i) = x(i)^{-\delta}$, we obtain the estimates given near the start of Section 2; specifically, with $\delta = 1$ we obtain Equation 2.2; and, with $\delta = 2$, we obtain Equation 2.3.

APPENDIX B

CALCULATIONS FOR LINK RATIO MODELS WITH INTERCEPTS AND ACCIDENT-YEAR TRENDS

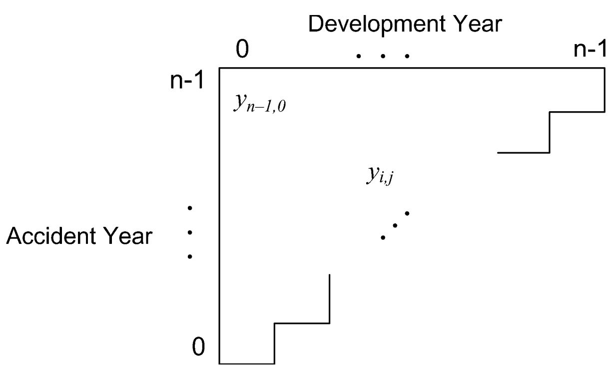
Let there be n accident years, numbering the most recent accident year as 0, and the first as $n-1$, as in Murphy [8]. Let y_{ij} be the cumulative amount paid in accident year i, development year j, $i = 0,\ldots,n-1$, $j = 0,\ldots,n-1$, as in Figure 33. This simplifies many of the formulas. Let $x_{ij} = y_{i,j-1}$, so that y_{ij}/x_{ij} is the observed development factor from $j-1$ to j in accident year i.

The only difference a more complex array shape (such as missing early payment years, or with late development years cutoff) will make is to change the limits on summations.

Now let $p_{ij} = \alpha_j + \lambda_j z_{ij} + (\beta_j - 1)x_{ij} + u_{ij}$, where p_{ij} is the incremental paid loss in accident year i at development year j, x_{ij} is the cumulative paid in accident year i, up to development year $j-1$, and z_{ij} is the count of accident years from the top, starting from 0. Since we number from the bottom, in the current

FIGURE 33

TRIANGULAR LOSS DEVELOPMENT ARRAY OF SIZE n, WITH ACCIDENT YEARS LABELED IN REVERSE ORDER



notation, $z_{ij} = n-1-i$. Denote the cumulative by y_{ij}. Note that $p_{ij} = y_{ij} - x_{ij}$. Here, α_j is an intercept (level) term, λ_j represents accident-year trend, and β_j represents the dependence on the previous cumulative. We can also write the model as $y_{ij} = \alpha_j + \lambda_j z_{ij} + \beta_j x_{ij} + u_{ij}$, and we will proceed with this formulation of the model. As before, $\mathrm{Var}(u_{ij}) = \sigma_j^2 x_{ij}^\delta$.

The regressions are independent, so most of the calculations are straightforward.

Parameter Estimates and Standard Errors

With three parameters in each regression, it will be easiest to use a standard regression routine. We now describe how to do a weighted regression with an unweighted routine.

Writing the j^{th} regression in matrix form (and dropping the j subscript), we have: $\mathbf{y} = X\boldsymbol{\beta} + \mathbf{u}$, where $\mathbf{y} = (y_{n-1}, y_{n-2}, \ldots, y_j)'$, $\boldsymbol{\beta} = (\alpha, \lambda, \beta)'$, $\mathbf{u} = (u_{n-1}, u_{n-2}, \ldots, u_j)'$,

$$X = \begin{bmatrix} 1 & z_{n-1} & x_{n-1} \\ 1 & z_{n-2} & x_{n-2} \\ \vdots & \vdots & \vdots \\ 1 & z_j & x_j \end{bmatrix} \qquad \text{and}$$

$$\mathrm{Var}(\mathbf{u}) = \sigma^2 U = \sigma^2 \begin{bmatrix} x_{n-1}^\delta & 0 & \cdots & 0 \\ 0 & x_{n-2}^\delta & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & x_j^\delta \end{bmatrix}.$$

Let $\mathbf{y}^* = U^{-1/2}\mathbf{y}$, $X^* = U^{-1/2}X$, $\mathbf{e} = U^{-1/2}\mathbf{u}$. Then we have $\mathbf{y}^* = X^*\boldsymbol{\beta} + \mathbf{e}$, with the e_i's independently normal with variance σ^2. That is, $\mathbf{y}^* = (y_{n-1}x_{n-1}^{-\delta/2}, y_{n-2}x_{n-2}^{-\delta/2}, \ldots, y_j x_j^{-\delta/2})'$ and similarly with each column of X, including the column of 1s. The parameter estimates (and parameter variance-covariance matrix) for this new (unweighted) regression are the same as that of the old regression; standard regression results give $\hat{\boldsymbol{\beta}} = (X^{*\prime}X^{*})^{-1}X^{*\prime}\mathbf{y}^{*}$ (see, for example, Cook and Weisberg [3]).

Consequently, we will simply take $\hat{\alpha}, \hat{\lambda}, \hat{\beta}$ and their estimated variances and covariances as being available. Note that in the final development years it isn't possible to fit all three parameters; usually we would choose to fit α and/or β as appropriate.

Residuals

Note that the residuals, $\hat{u}_{ij}$, are the same whether we consider incremental or cumulative values: $\hat{u}_{ij} = y_{ij} - \hat{y}_{ij} = p_{ij} - \hat{p}_{ij}$. We now derive the variance of the residuals. Let $H^{*} = X^{*}(X^{*\prime}X^{*})^{-1}X^{*\prime}$. Note that $\hat{\mathbf{y}}^{*} = X^{*}\hat{\boldsymbol{\beta}} = X^{*}(X^{*\prime}X^{*})^{-1}X^{*\prime}\mathbf{y}^{*} = H^{*}\mathbf{y}^{*}$. Then $\mathrm{Var}(\hat{\mathbf{e}}) = \mathrm{Var}(\mathbf{y}^{*} - \hat{\mathbf{y}}^{*}) = \mathrm{Var}((I - H^{*})\mathbf{y}^{*}) = (I - H^{*}) \cdot \mathrm{Var}(\mathbf{y}^{*})(I - H^{*})' = \sigma^{2}(I - H^{*})^{2}$. Note that $H^{*2} = H^{*}$. Hence $\mathrm{Var}(\hat{\mathbf{e}}) = \sigma^{2}(I - H^{*})$. This is a standard regression result (see, for example, Cook and Weisberg [3]). Consequently $\mathrm{Var}(\hat{\mathbf{u}}) = U^{1/2}\mathrm{Var}(\hat{\mathbf{e}})(U^{1/2})'$, from which we can find the variance of an individual observation. Consider the regression for development year j, and suppress that subscript. Then $\mathrm{Var}(\hat{e}_{i}) = \sigma^{2}[1 - (\mathbf{x}_{i}^{*})'(X^{*\prime}X^{*})^{-1}\mathbf{x}_{i}^{*}]$, where $\mathbf{x}_{i}^{*}$ is the i^{th} row of X^{*}. Therefore $\mathrm{Var}(\hat{u}_{i}) = \sigma^{2}[1 - (\mathbf{x}_{i}^{*})'(X^{*\prime}X^{*})^{-1}\mathbf{x}_{i}^{*}]x_{i}^{\delta}$.

Forecasts and Standard Errors

Note that all forecasts and standard error calculations given here are conditional on the observed data.

- Forecasts: Clearly $\hat{y}_{i,i+k} = \hat{\alpha}_{i+k} + \hat{\lambda}_{i+k}z_{i,i+k} + \hat{\beta}_{i+k}\hat{y}_{i,i+k-1}$, where $\hat{y}_{ii} = y_{ii}$.
- Standard Errors:

$$\begin{aligned}
\mathrm{Var}(\hat{y}_{i,i+k} - y_{i,i+k}) &= \mathrm{Var}(\hat{y}_{i,i+k} - \mu_{i,i+k} + \mu_{i,i+k} - y_{i,i+k}) \\
&= \mathrm{Var}(\hat{y}_{i,i+k} - \mu_{i,i+k}) + \mathrm{Var}(y_{i,i+k} - \mu_{i,i+k}) \\
&= v_{i,i+k}^{p} + v_{i,i+k}^{e}. \qquad \text{(B.1)}
\end{aligned}$$

The first term is what Murphy [8] calls the parameter variance, and the second the process variance. These may be thought of as the variability of the *predictions* about the true model and the variability of the *data* about the true model, respectively. Both must be estimated. Now:

$$\begin{aligned} v^p_{i,i+k} &= \mathrm{Var}(\hat{y}_{i,i+k}) \\ &= \mathrm{Var}(\hat{\alpha}_{i+k} + \hat{\lambda}_{i+k} z_{i,i+k} + \hat{\beta}_{i+k}\hat{y}_{i,i+k-1}) \\ &= \mathrm{Var}(\hat{\alpha}_{i+k}) + 2z_{i,i+k}\mathrm{Cov}(\hat{\alpha}_{i+k}, \hat{\lambda}_{i+k}) + z^2_{i,i+k}\mathrm{Var}(\hat{\lambda}_{i+k}) \\ &\quad + 2\hat{y}_{i,i+k-1}\mathrm{Cov}(\hat{\alpha}_{i+k}, \hat{\beta}_{i+k}) \\ &\quad + 2z_{i,i+k}\hat{y}_{i,i+k-1}\mathrm{Cov}(\hat{\lambda}_{i+k}, \hat{\beta}_{i+k}) \\ &\quad + \mathrm{Var}(\hat{\beta}_{i+k}\hat{y}_{i,i+k-1}). \end{aligned} \tag{B.2}$$

Note that if X and Y are independent random variables, with means μ_X and μ_Y, respectively, then $\mathrm{E}[(X-\mu_X)^2(Y-\mu_Y)^2] = \mathrm{E}[(X-\mu_X)^2]\mathrm{E}[(Y-\mu_Y)^2]$. Expanding the left hand side, using the elementary properties of expectation and rearranging, we readily obtain $\mathrm{Var}(XY) = \mathrm{Var}(X)\mathrm{Var}(Y) + \mathrm{Var}(X)\mu_Y^2 + \mathrm{Var}(Y)\mu_X^2$.

Consequently,

$$\begin{aligned} \mathrm{Var}(\hat{\beta}_{i+k}\hat{y}_{i,i+k-1}) &= \beta^2_{i+k}\mathrm{Var}(\hat{y}_{i,i+k-1}) + y^2_{i,i+k-1}\mathrm{Var}(\hat{\beta}_{i+k}) \\ &\quad + \mathrm{Var}(\hat{\beta}_{i+k})\mathrm{Var}(\hat{y}_{i,i+k-1}), \end{aligned} \tag{B.3}$$

which we estimate by

$$\hat{\beta}^2_{i+k}\,\hat{\mathrm{Var}}(\hat{y}_{i,i+k-1}) + \hat{y}^2_{i,i+k-1}\,\hat{\mathrm{Var}}(\hat{\beta}_{i+k}) + \hat{\mathrm{Var}}(\hat{\beta}_{i+k})\,\hat{\mathrm{Var}}(\hat{y}_{i,i+k-1}). \tag{B.4}$$

Hence we estimate

$$\begin{aligned}\hat{v}^{p}_{i,i+k} &= \hat{\text{Var}}(\hat{\alpha}_{i+k}) + 2z_{i,i+k}\,\hat{\text{Cov}}(\hat{\alpha}_{i+k},\hat{\lambda}_{i+k}) + z^{2}_{i,i+k}\,\hat{\text{Var}}(\hat{\lambda}_{i+k})\\ &\quad + 2\hat{y}_{i,i+k-1}\,\hat{\text{Cov}}(\hat{\alpha}_{i+k},\hat{\beta}_{i+k}) + 2z_{i,i+k}\hat{y}_{i,i+k-1}\,\hat{\text{Cov}}(\hat{\lambda}_{i+k},\hat{\beta}_{i+k})\\ &\quad + \hat{y}^{2}_{i,i+k-1}\,\hat{\text{Var}}(\hat{\beta}_{i+k}) + [\hat{\beta}^{2}_{i+k} + \hat{\text{Var}}(\hat{\beta}_{i+k})]\hat{v}^{p}_{i,i+k-1}. \qquad \text{(B.5)}\end{aligned}$$

Note that v^{p}_{ii} is zero, since we are conditioning on the data. Further,

$$\begin{aligned}v^{e}_{i,i+k} &= \text{Var}(y_{i,i+k} - \mu_{i,i+k})\\ &= \text{Var}(\alpha_{i+k} + \lambda_{i+k}z_{i,i+k-1} + \beta_{i+k}y_{i,i+k-1} + u_{i,i+k})\\ &= \beta^{2}_{i+k}\text{Var}(y_{i,i+k-1}) + \text{Var}(u_{i,i+k})\\ &= \beta^{2}_{i+k}\text{Var}(y_{i,i+k-1} - \mu_{i,i+k-1}) + \sigma^{2}_{i+k}x^{\delta}_{i,i+k}\\ &= \beta^{2}_{i+k}v^{e}_{i,i+k-1} + \sigma^{2}_{i+k}x^{\delta}_{i,i+k}, \qquad \text{(B.6)}\end{aligned}$$

which we estimate as:

$$\begin{aligned}\hat{v}^{e}_{i,i+k} &= \hat{\text{Var}}(y_{i,i+k})\\ &= \hat{\beta}^{2}_{i+k}\,\hat{\text{Var}}(y_{i,i+k-1}) + \hat{\sigma}^{2}_{i+k}\hat{\text{E}}(y^{\delta}_{i,i+k-1})\\ &= \hat{\beta}^{2}_{i+k}\hat{v}^{e}_{i,i+k-1} + \hat{\sigma}^{2}_{i+k}\hat{f}^{\delta}_{i,i+k-1}, \qquad \text{(B.7)}\end{aligned}$$

where $f^{\delta}_{i,j} = \text{E}(y^{\delta}_{i,j})$. We have

$$\hat{f}^{\delta}_{ij} = \begin{cases} 1, & \delta = 0\\ \hat{y}_{ij}, & \delta = 1\\ \hat{y}^{2}_{ij} + \hat{\text{Var}}(y_{ij}), & \delta = 2 \end{cases}, \qquad \text{(B.8)}$$

just as with Murphy [8]. With the normality assumption we can obtain estimates at other values of δ, but we omit details here.

Forecasts and standard errors on the incrementals can be obtained in similar fashion.

Forecasts and Standard Errors of Development-Year Totals

Let D_j be the unknown future development-year total forecast, so

$$D_j = \sum_{i=0}^{j-1} y_{ij}, \tag{B.9}$$

and

$$\hat{D}_j = \sum_{i=0}^{j-1} \hat{y}_{ij}. \tag{B.10}$$

Note that

$$\begin{aligned}
\mathrm{Var}(\hat{D}_j - D_j) &= \mathrm{Var}\left(\sum_{i=0}^{j-1} \hat{y}_{ij} - y_{ij}\right) \\
&= \mathrm{Var}\left(\sum_{i=0}^{j-1} \hat{y}_{ij} - \mu_{ij}\right) + \mathrm{Var}\left(\sum_{i=0}^{j-1} y_{ij} - \mu_{ij}\right) \\
&= V_j^p + V_j^e.
\end{aligned} \tag{B.11}$$

Taking $Z_j = \sum_{i=0}^{j-1} z_{ij}$, then

$$\begin{aligned}
V_j^p &= \mathrm{Var}\left(\sum_{i=0}^{j-1} \hat{y}_{ij}\right) \\
&= \mathrm{Var}\left(\sum_{i=0}^{j-1} \hat{\alpha}_j + \hat{\lambda}_j z_{ij} + \hat{\beta}_j \hat{y}_{i,j-1}\right) \\
&= \mathrm{Var}(n_j \hat{\alpha}_j + Z_j \hat{\lambda}_j + \hat{\beta}_j [\hat{D}_{j-1} + y_{j-1,j-1}])
\end{aligned}$$

$$
\begin{aligned}
&= n_j^2\mathrm{Var}(\hat{\alpha}_j) + 2n_jZ_j\mathrm{Cov}(\hat{\alpha}_j,\hat{\lambda}_j) + Z_j^2\mathrm{Var}(\hat{\lambda}_j)\\
&\quad + 2n_j[\hat{D}_{j-1} + y_{j-1,j-1}]\mathrm{Cov}(\hat{\alpha}_j,\hat{\beta}_j)\\
&\quad + 2Z_j[\hat{D}_{j-1} + y_{j-1,j-1}]\mathrm{Cov}(\hat{\lambda}_j,\hat{\beta}_j)\\
&\quad + \mathrm{Var}(\hat{\beta}_j[\hat{D}_{j-1} + y_{j-1,j-1}])\\
&= n_j^2\mathrm{Var}(\hat{\alpha}_j) + 2n_jZ_j\mathrm{Cov}(\hat{\alpha}_j,\hat{\lambda}_j) + Z_j^2\mathrm{Var}(\hat{\lambda}_j)\\
&\quad + 2[\hat{D}_{j-1} + y_{j-1,j-1}][n_j\mathrm{Cov}(\hat{\alpha}_j,\hat{\beta}_j) + Z_j\mathrm{Cov}(\hat{\lambda}_j,\hat{\beta}_j)]\\
&\quad + [D_{j-1} + y_{j-1,j-1}]^2\mathrm{Var}(\hat{\beta}_j) + [\beta_j^2 + \mathrm{Var}(\hat{\beta}_j)]\mathrm{Var}(\hat{D}_{j-1}).
\end{aligned}
\tag{B.12}
$$

Note that the last term, $\mathrm{Var}(\hat{D}_{j-1})$ is just V_{j-1}^p. We estimate V_j^p by replacing D_{j-1}, β_j, and the variance and covariance terms by their estimates, which have either been defined or are immediately available from standard regression calculations. Also,

$$
\begin{aligned}
V_j^e &= \mathrm{Var}\left(\sum_{i=0}^{j-1} y_{ij}\right)\\
&= \mathrm{Var}\left(n_j\alpha_j + Z_j\lambda_j + \beta_j\sum_{i=0}^{j-1} y_{i,j-1} + \sum_{i=0}^{j-1} u_{i,j}\right)\\
&= \mathrm{Var}(\beta_j[D_{j-1} + y_{j-1,j-1}]) + \mathrm{Var}\left(\sum_{i=0}^{j-1} u_{i,j}\right)\\
&= \beta_j^2\mathrm{Var}(D_{j-1}) + \sigma_j^2\sum_{i=0}^{j-1} x_{ij}^{\delta}\\
&= \beta_j^2 V_{j-1}^e + \sigma_j^2\left(y_{j-1,j-1}^{\delta} + \sum_{i=0}^{j-2} y_{i,j-1}^{\delta}\right).
\end{aligned}
\tag{B.13}
$$

Due to independence across accident years, $\mathrm{E}(\sum_{i=0}^{j-2} y_{i,j-1}^{\delta}) = \sum_{i=0}^{j-2} \mathrm{E}(y_{i,j-1}^{\delta}) = \sum_{i=0}^{j-2} f_{i,j-1}^{\delta}$, so the process variance term is estimated by $\hat{V}_j^e = \hat{\beta}_j^2 \hat{V}_{j-1}^e + \hat{\sigma}_j^2 (y_{j-1,j-1}^{\delta} + \sum_{i=0}^{j-2} \hat{f}_{i,j-1}^{\delta})$. The estimated standard error of $\hat{D}_j$ is then $\sqrt{\hat{V}_j^p + \hat{V}_j^e}$.

APPENDIX C

EXPOSURES AND FORECASTING

Let us extend the notation of the previous section. Let y^o_{ij} be the observed cumulative at accident year i, development year j; let y^n_{ij} be the corresponding normalized-for-exposures cumulative. Similarly, let p^o_{ij} and p^n_{ij} be the corresponding incremental values.

Let c_i be the exposure for accident year i. Then $y^n_{ij} = y^o_{ij}/c_i$, and $p^n_{ij} = p^o_{ij}/c_i$. We fit the ELRF model to y^n, but we use it to forecast y^o.

Individual Forecasts and Standard Errors (all models)

Clearly, $\hat{y}^o_{ij} = c_i\hat{y}^n_{ij}$, and similarly for p, $\hat{p}^o_{ij} = c_i\hat{p}^n_{ij}$; also $\mathrm{Var}(\hat{y}^o_{ij} - y^o_{ij}) = c_i^2\mathrm{Var}(\hat{y}^n_{ij} - y^n_{ij})$ and $\mathrm{Var}(\hat{p}^o_{ij} - p^o_{ij}) = c_i^2\mathrm{Var}(\hat{p}^n_{ij} - p^n_{ij})$. So individual observed forecasts and standard errors are just the corresponding normalized values, multiplied by the exposure.

Cumulative Development-Year Totals

$$\mathrm{Var}(\hat{D}^o_j - D^o_j) = V_j^{op} + V_j^{oe}, \tag{C.1}$$

where

$$\begin{aligned}
V_j^{op} &= \mathrm{Var}\left(\sum_{i=0}^{j-1}\hat{y}^o_{ij}\right)\\
&= \mathrm{Var}\left(\sum_{i=0}^{j-1}c_i\hat{y}^n_{ij}\right)\\
&= \mathrm{Var}\left[\sum_{i=0}^{j-1}c_i(\hat{\alpha}_j + \hat{\lambda}_j z_{ij} + \hat{\beta}_j\hat{y}^n_{i,j-1})\right]\\
&= \mathrm{Var}\left[\hat{\alpha}_j\left(\sum_{i=0}^{j-1}c_i\right) + \hat{\lambda}\left(\sum_{i=0}^{j-1}c_i z_{ij}\right) + \hat{\beta}_j\sum_{i=0}^{j-1}\hat{y}^o_{i,j-1}\right]
\end{aligned}$$

$$= \text{Var}\left[\hat{\alpha}_j C_j + \hat{\lambda}_j Z_j^* + \hat{\beta}_j \sum_{i=0}^{j-1} \hat{y}_{i,j-1}^o\right]$$

$$= C_j^2 \text{Var}(\hat{\alpha}_j) + 2C_j Z_j^* \text{Cov}(\hat{\alpha}_j, \hat{\lambda}_j) + (Z_j^*)^2 \text{Var}(\hat{\lambda}_j)$$

$$+ 2C_j(y_{j-1,j-1}^o + \hat{D}_{j-1}^o)\text{Cov}(\hat{\alpha}_j, \hat{\beta}_j)$$

$$+ 2Z_j^*(y_{j-1,j-1}^o + \hat{D}_{j-1}^o)\text{Cov}(\hat{\lambda}_j, \hat{\beta}_j)$$

$$+ (y_{j-1,j-1}^o + D_{j-1}^o)^2 \text{Var}(\hat{\beta}_j) + [\beta_j^2 + \text{Var}(\hat{\beta}_j)]\text{Var}(\hat{D}_{j-1}^o)$$

$$= C_j^2 \text{Var}(\hat{\alpha}_j) + 2C_j Z_j^* \text{Cov}(\hat{\alpha}_j, \hat{\lambda}_j) + (Z_j^*)^2 \text{Var}(\hat{\lambda}_j)$$

$$+ 2(y_{j-1,j-1}^o + \hat{D}_{j-1}^o)[C_j \text{Cov}(\hat{\alpha}_j, \hat{\beta}_j) + Z_j^* \text{Cov}(\hat{\lambda}_j, \hat{\beta}_j)]$$

$$+ (y_{j-1,j-1}^o + D_{j-1}^o)^2 \text{Var}(\hat{\beta}_j) + [\beta_j^2 + \text{Var}(\hat{\beta}_j)]V_{j-1}^{op}, \tag{C.2}$$

and

$$V_j^{oe} = \text{Var}\left(\sum_{i=0}^{j-1} y_{ij}^o\right)$$

$$= \text{Var}\left(\sum_{i=0}^{j-1} c_i y_{ij}^n\right)$$

$$= \text{Var}\left[\sum_{i=0}^{j-1} c_i(\alpha_j + \lambda_j z_{ij} + \beta_j y_{i,j-1}^n + u_{ij})\right]$$

$$= \text{Var}\left(\sum_{i=0}^{j-1} c_i u_{ij}\right) + \beta_j^2 \text{Var}\left[\sum_{i=0}^{j-1} y_{i,j-1}^o\right]$$

$$= \sum_{i=0}^{j-1} \text{Var}(c_i u_{ij}) + \beta_j^2 \text{Var}\left[\sum_{i=0}^{j-1} y_{i,j-1}^o\right]$$

$$= \sigma_j^2 c_{j-1}^2 (y_{j-1,j-1}^n)^\delta + \sum_{i=0}^{j-2} c_i^2 v_{i,j-1}^{ne} + \beta_j^2 V_{j-1}^{oe}$$

$$= \sigma_j^2 c_{j-1}^{2-\delta} (y_{j-1,j-1}^o)^\delta + \sum_{i=0}^{j-2} v_{i,j-1}^{oe} + \beta_j^2 V_{j-1}^{oe}, \qquad \text{(C.3)}$$

where again we estimate this variance by replacing the D's, β's, variances and covariances by their estimates.

APPENDIX D

LIKELIHOOD AND CONDITIONAL REGRESSIONS

Let $y(j)$ be the vector of data in development year j, and $\theta(j)$ be all the parameters for that development year. Let $\mathbf{y} = (y(0)',\ldots,y(n-1)')'$, so the development years are stacked one on top of the other, and $\theta = (\theta(1)',\ldots,\theta(n-1)')'$. Then, straightforward application of conditional probability and some simplification gives us:

$$\begin{aligned}
L(\theta \mid \mathbf{y}) &\propto p[\mathbf{y} \mid \theta] \\
&\propto p[y(n-1) \mid \theta(n-1), y(n-2), y(n-3),\ldots,y(0)] \\
&\quad \cdot p[y(n-2) \mid \theta(n-2), y(n-3), y(n-4),\ldots,y(0)] \\
&\quad \vdots \\
&\quad \cdot p[y(1) \mid \theta(1), y(0)] \\
&\quad \cdot p[y(0)] \\
&\propto p[y(n-1) \mid \theta(n-1), y(n-2)] \\
&\quad \cdot p[y(n-2) \mid \theta(n-2), y(n-3)] \\
&\quad \cdots \cdot p[y(1) \mid \theta(1), y(0)] \cdot p[y(0)]. \qquad \text{(D.1)}
\end{aligned}$$

Since, for each regression, we are conditioning on the data from previous development years, the fact that the previous development data is stochastic and not fixed is not an issue—the conditional likelihoods still correspond to ordinary regressions.

The likelihood for $y(0)$ doesn't contain any of the parameters. At any value for θ, then, the likelihood of $y(0)$ is just a constant; consequently, it cannot affect the location of the maximum of the likelihood, nor its curvature there. So the way that the forecasts depend on the parameters isn't affected by $y(0)$, apart from the way it enters the regression for $y(1)$.

The model for the data says that the values in future development years depend on the earlier development years. We've observed the whole of $y(0)$, so we know exactly how it will impact the future runoff, because the model describes that. Of course, the model may be wrong (and we argue that it is); but, given the model, the regressions may all be performed as ordinary regressions.

The forecasts are made conditionally on the data. We've argued above that even the stochastic nature of $y(0)$ can be ignored in the forecasting because the model fully describes its impact on the future observations. However, this is not an important point—if an argument were made that the stochastic nature of $y(0)$ should somehow affect the forecasts, it would not affect any of our arguments about the unsuitability of these models.

APPENDIX E

DESIGN MATRICES FOR THE MODELS DESCRIBED IN SECTION 3

Readers may wish to fit the regression models described in Section 3 of this paper. The models described there can be fitted to data in any of the common statistical packages, or using spreadsheet software such as Excel. Here we briefly describe what the various predictors look like. We begin by describing the full model (which is not used in practice, as it's overparameterized, but is the general case of which all the useful models are special cases) and then some of the more common simpler models.

Table E.1 displays the expected values in each cell in the log(incremental) array under the general model using the notation of Section 3.

The vector of observations may be produced by stacking up the development years one on top of another: $\mathbf{y} = (y(0)', y(1)', \ldots, y(n-1)')'$, as in the previous appendix. Similarly, there is a column in the X-matrix for each parameter, and the parameters become a column with rows in the same order as the corresponding columns of the X-matrix (design matrix). Note that α is already an intercept parameter, so we don't add an intercept (i.e., the regression is written $\mathbf{y} = X\boldsymbol{\beta} + \boldsymbol{\varepsilon}$). A good approach is to do all the α's, then all the γ's, and finally all the ι's.

For $n = 4$, this corresponds to the X-matrix in Table E.2 (the zeroes have been suppressed to make the patterns more clear).

In general, the (i, j) row for an array of size n would have a 1 for the column for α_j, it would have 1's for the columns for γ_k (where $k \leq j$), and it would have 1's for the columns for ι_r (where $r \leq i + j$), with zeroes everywhere else.

Setting some of the parameters to be equal is simply a matter of adding together columns from the full design matrix. For ex-

TABLE E.1

Expected Values of Log(Incremental) Under the General Model

		Development Year					
		0	1	…	J	…	$n-1$
Accident	0	α_0	$\alpha_0+\gamma_1+\iota_1$	⋯	$\alpha_0+\sum_{k=1}^{j}\gamma_k+\sum_{r=1}^{j}\iota_r$	⋯	$\alpha_0+\sum_{k=1}^{j}\gamma_k+\sum_{r=1}^{j}\iota_r$
Year	1	$\alpha_1+\iota_1$	$\alpha_1+\gamma_1+\sum_{r=1}^{2}\iota_t$	⋯	$\alpha_1+\sum_{k=1}^{j}\gamma_k+\sum_{r=1}^{j+1}\iota_r$	⋯	
	⋮	⋮	⋮		⋮	⋰	
	i	$\alpha_{iI}+\sum_{r=1}^{i}\iota_r$	$\alpha_1+\gamma_1+\sum_{r=1}^{i+1}\iota_r$		$\alpha_i+\sum_{k=1}^{j}\gamma_k+\sum_{r=1}^{i+j}\iota_r$		
	⋮	⋮	⋮	⋰			
	$n-1$	$\alpha_{n-1}+\sum_{r=1}^{t}\iota_r$					

TABLE E.2

Design Matrix (X-Matrix) for the Full Model for a Triangle With 4 Years' Data

	α_0	α_1	α_2	α_3	γ_1	γ_2	γ_3	ι_1	ι_2	ι_3
y(0,0)	1									
y(1,0)	1				1			1		
y(2,0)	1				1	1		1	1	
y(3,0)	1				1	1	1	1	1	1
y(0,1)		1						1		
y(1,1)		1			1			1	1	
y(2,1)		1			1	1		1	1	1
y(0,2)			1					1	1	
y(1,2)			1		1			1	1	1
y(0,3)				1				1	1	1

TABLE E.3

Design Matrix (X-Matrix) for a Simple Model for a Triangle With 4 Years' Data

y(0,0)	1			
y(1,0)	1	1		1
y(2,0)	1	1	1	2
y(3,0)	1	1	2	3
y(0,1)				1
y(1,1)		1		2
y(2,1)		1	1	3
y(0,2)				2
y(1,2)		1		3
y(0,3)				2

ample, Table E.3 shows the design matrix for the array of size 4, with one level of log payments for all years, two development-year trends (0–1, and all later years), and a single payment-year trend—that is, all α's equal, $\gamma_2 = \gamma_3$, and all ι's equal.

APPLICATIONS OF RESAMPLING METHODS IN ACTUARIAL PRACTICE

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Abstract

Actuarial analysis can be viewed as the process of studying profitability and solvency of an insurance firm under a realistic and integrated model of key input random variables such as loss frequency and severity, expenses, reinsurance, interest and inflation rates, and asset defaults. Traditional models of input variables have generally fitted parameters for a predetermined family of probability distributions. In this paper we discuss applications of some modern methods of non-parametric statistics to modeling loss distributions, and possibilities of using them for modeling other input variables for the purpose of arriving at an integrated company model. Several examples of inference about the severity of loss, loss distributions percentiles and other related quantities based on data smoothing, bootstrap estimates of standard error and bootstrap confidence intervals are presented. The examples are based on real-life auto injury claim data and the accuracy of our methods is compared with that of standard techniques. Model adjustment for inflation and bootstrap techniques based on the Kaplan–Meier estimator, useful in the presence of policies limits (censored losses), are also considered.

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1. INTRODUCTION

In modern analysis of the financial models of property-casualty companies the input variables can be typically classified into financial variables and underwriting variables (e.g., see D'Arcy, Gorvett, Herbers and Hettinger [6]). The financial variables generally refer to asset-side generated cash flows of the business, and the underwriting variables relate to the cash flows of the liabilities side. The process of developing any actuarial model begins with the creation of probability distributions of these input variables, including the establishment of the proper range of values of input parameters. The use of parameters is generally determined by the use of parametric families of distributions. Fitting of those parameters is generally followed either by Monte Carlo simulation together with integration of all inputs for profit testing and optimization, or by the study of the effect of varying the parameters on output variables in sensitivity analysis and basic cash flow testing. Thus traditional actuarial methodologies are rooted in parametric approaches, which fit prescribed distributions of losses and other random phenomena studied (e.g., interest rate or other asset return variables) to the data. The experience of the last two decades has shown greater interdependence of basic loss variables (severity, frequency, exposures) with asset variables (interest rates, asset defaults, etc.), and sensitivity of the firm to all input variables. Increased complexity has been accompanied by increased competitive pressures, and more frequent insolvencies. In our opinion, in order to properly address these issues one must carefully address the weaknesses of traditional methodologies. These weaknesses can be summarized as originating from either ignoring the uncertainties of inputs, or mismanaging those uncertainties. While early problems of actuarial modeling could be attributed mostly to ignoring uncertainty, we believe at this point the uncertain nature of model inputs is generally acknowledged. Note that Derrig and Ostaszewski [9] used fuzzy set techniques to handle the mixture of probabilistic and non-probabilistic uncertainties in asset/

liability considerations for property-casualty claims. In our opinion it is now time to proceed to deeper issues concerning the actual forms of uncertainty. The Central Limit Theorem and its stochastic process counterpart provide clear guidance for practical uses of the normal distribution and all distributions derived from it. But one cannot justify similarly fitting convenient distributions to, for instance, loss data and expect to easily survive the next significant change in the marketplace. What does work in practice, but not in theory, may be merely an illusion of applicability provided by powerful tools of modern technology. If one cannot provide a justification for the use of a parametric distribution, then a nonparametric alternative should be studied, at least for the purpose of understanding the firm's exposures. In this work, we will show such a study of nonparametric methodologies applied to loss data, and will advocate the development of an integrated company model with the use of nonparametric approaches.

1.1. Loss Distributions

We begin by addressing the most basic questions concerning loss distributions. The first two parameters generally fitted to the data are average claim size and the number of claim occurrences per unit of exposure. Can we improve upon these estimates by using nonparametric methods?

Consider the problem of estimating the severity of a claim, which is, in its most general setting, equivalent to modeling the probability distribution of a single claim size. Traditionally, this has been done by means of fitting some parametric models from a particular continuous family of distributions (e.g., see Daykin, Pentikainen, and Pesonen [7, Chapter 3]). While this standard approach has several obvious advantages, we should also realize that occasionally it may suffer some serious drawbacks:

- Some loss data has a tendency to cluster about round numbers like \$1,000, \$10,000, etc., due to rounding off the claim amount and thus in practice follows a mixture of continuous and discrete distributions. Usually, parametric models simply ignore the discrete component in such cases.

- The data is often truncated from below or censored from above due to deductibles and/or limits on different policies. In particular, the presence of censoring, if not accounted for, may seriously compromise the goodness-of-fit of a fitted parametric distribution. On the other hand, trying to incorporate the censoring mechanism (which is often random in its nature, especially when we consider losses falling under several insurance policies with different limits) often leads to a creation of a very complex model which is difficult to work with.

- The loss data may come from a mixture of distributions depending upon some known or unknown classification of claim types.

- Finally, it may happen that the data simply does not fit any of the available distributions in a satisfactory way.

It seems, therefore, that there are many situations of practical importance where the traditional approach cannot be utilized, and one must look beyond parametric models. In this work we point out an alternative, nonparametric approach to modeling losses and other random parameters of financial analysis, originating from the modern methodology of nonparametric statistics based on *the bootstrap* or *resampling* method.

To keep things in focus we will be concerned here only with applications to modeling the severity of loss, but the methods discussed may be easily applied to other problems such as loss frequencies, asset returns, asset defaults, and the combination of variables into models of Risk Based Capital, Value at Risk, and general Dynamic Financial Analysis (DFA), including Cash Flow Testing and Asset Adequacy Analysis.

1.2. The Concept of Bootstrap

The concept of bootstrap was first introduced in the seminal piece of Efron [10], and relies on the consideration of the discrete empirical distribution generated by a random sample of size n from an unknown distribution F. This empirical distribution assigns equal probability to each sample item. In the discussion which follows, we will write $\widehat{F}_n$ for that distribution. By generating an independent, identically distributed (IID) random sequence (resample) from the distribution $\widehat{F}_n$ or its appropriately smoothed version, we can arrive at new estimates of various parameters and nonparametric characteristics of the original distribution F. This idea is at the very root of the bootstrap methodology. In particular, Efron [10] points out that the bootstrap gives a reasonable estimate of standard error for any estimator, and it can be extended to statistical error assessments and to inferences beyond biases and standard errors.

1.3. Overview of the Article

In this paper, we apply bootstrap methods to two data sets as illustrations of the advantages of resampling techniques, especially when dealing with empirical loss data. The basics of bootstrap theory are covered in Section 2, where we show its applications in estimating standard errors and calculating confidence intervals. In Section 3, we compare bootstrap and traditional estimators for quantiles and excess losses using some truncated wind loss data. The important concept of smoothing the bootstrap estimator is also covered in that section. Applications of bootstrap to auto bodily injury liability claims in Section 4 show loss elimination ratio estimates together with their standard errors in a case of lumpy and clustered data (the data set is enclosed in Appendix B). More complicated designs that incorporate data censoring and adjustment for inflation appear in Section 5. Sections 6 and 7 provide some final remarks and conclusions. The Mathematica 3.0 programs used to perform bootstrap calculations are provided in Appendix A.

2. BOOTSTRAP STANDARD ERRORS AND CONFIDENCE INTERVALS

As we have already mentioned in the previous section, the central idea of bootstrap lies in sampling the empirical cumulative distribution function (CDF) $\widehat{F}_n$. This idea is closely related to the following, well-known statistical principle, henceforth referred to as the "plug-in" principle. Given a parameter of interest $\theta(F)$ depending upon an unknown population CDF F, we estimate this parameter by $\hat{\theta} = \theta(\widehat{F}_n)$. That is, we simply replace F in the formula for θ by its empirical counterpart $\widehat{F}_n$ obtained from the observed data. The plug-in principle will not provide good results if $\widehat{F}_n$ poorly approximates F, or if there is information about F other than that provided by the sample. For instance, in some cases we might know (or be willing to assume) that F belongs to some parametric family of distributions. However, the plug-in principle and the bootstrap may be adapted to this latter situation as well. To illustrate the idea, let us consider a parametric family of CDF's $\{F_\mu\}$ indexed by a parameter μ (possibly a vector), and for some given μ_0, let $\widehat{\mu}_0$ denote its estimate calculated from the sample. The plug-in principle in this case states that we should estimate $\theta(F_{\mu_0})$ by $\theta(F_{\widehat{\mu_0}})$. In this case, bootstrap is often called parametric, since a resample is now collected from $F_{\widehat{\mu_0}}$. Here and elsewhere in this work, we refer to any replica of $\hat{\theta}$ calculated from a resample as "a bootstrap estimate of $\theta(F)$" and denote it by $\hat{\theta}^*$.

2.1. The Bootstrap Methodology

Bickel and Freedman [2] formulated conditions for consistency of bootstrap, which resulted in further extensions of Efron's [10] methodology to a broad range of standard applications, including quantile processes, multiple regression and stratified sampling. They also argued that the use of bootstrap did not require theoretical derivations such as function derivatives, influence functions, asymptotic variances, the Edgeworth expansion, etc.

Singh [19] made a further point that the bootstrap estimator of the sampling distribution of a given statistic may be more accurate than the traditional normal approximation. In fact, it turns out that for many commonly used statistics the bootstrap is asymptotically equivalent to the one-term Edgeworth expansion estimator, usually having the same convergence rate, which is faster than the normal approximation. In many more recent statistical texts the bootstrap is recommended for estimating sampling distributions and finding standard errors and confidence sets. The extension of the bootstrap method to the case of dependent data was considered for instance by Künsch [15], who suggested a *moving block bootstrap* procedure which takes into account the dependence structure of the data by resampling blocks of adjacent observations rather than individual data points. More recently, Politis and Romano [16] suggested a method based on circular blocks (i.e., on wrapping the observed time series values around the circle and then generating the blocks of the bootstrap data from the circle's "arcs"). In the case of the sample mean this method, which is known as *circular bootstrap*, again was shown to accomplish the Edgeworth correction for dependent, stationary data.

The bootstrap methods can be applied to both parametric and non-parametric models, although most of the published research in the area is concerned with the non-parametric case since that is where the most immediate practical gains might be expected. Let us note though that a simple, non-parametric bootstrap may often be improved by other bootstrap methods taking into account the special nature of the model. In the IID non-parametric models, for instance, the smoothed bootstrap (bootstrap based on some smoothed version of $\widehat{F}_n$) often improves the simple bootstrap (bootstrap based solely on $\widehat{F}_n$). Since in recent years several excellent books on the subject of resampling and related techniques have become available, we will not be particularly concerned here with providing all the details of the presented techniques, contenting ourselves with making appropriate references to more technically detailed works. Readers interested in gaining some basic background in resampling are referred to Efron and Tibisharani [11]. For a more mathematically advanced treatment of the subject, we recommend Shao and Tu [17].

2.2. Bootstrap Standard Error Estimate

Arguably, one of the most important applications of bootstrap is to provide an estimate of the standard error of $\hat{\theta}$ ($se_F(\hat{\theta})$). It is rarely practical to calculate it exactly; instead, one usually approximates $se_F(\hat{\theta})$ with the help of multiple resamples. The approximation to the bootstrap estimate of standard error of $\hat{\theta}$ (or BESE) suggested by Efron [10] is given by

$$\widehat{se}_B = \left\{ \sum_{b=1}^{B} [\hat{\theta}^*(b) - \hat{\theta}^*(\cdot)]^2/(B-1) \right\}^{1/2}, \qquad (2.1)$$

where $\theta^*(\cdot) = \sum_{b=1}^{B} \hat{\theta}^*(b)/B$, B is the total number of resamples (each of size n) collected with replacement from the plug-in estimate of F (in the parametric or non-parametric setting), and $\hat{\theta}^*(b)$ is the original statistic $\hat{\theta}$ calculated from the bth resample ($b = 1,\ldots,B$). By the law of large numbers

$$\lim_{B\to\infty} \widehat{se}_B = \mathrm{BESE}(\hat{\theta}),$$

and, for sufficiently large n, we expect

$$\mathrm{BESE}(\hat{\theta}) \approx se_F(\hat{\theta}).$$

Let us note that B, the total number of resamples, may be as large as we wish since we are in complete control of the resampling process. It has been shown that for estimating the standard error, one should take B to be about 250, whereas for different resampled statistics this number may have to be significantly increased in order to reach the desired accuracy (see [11]).

2.3. *The Method of Percentiles*

Let us now turn to the problem of using the bootstrap methodology to construct confidence intervals. This area has been a major focus of theoretical work on the bootstrap, and several different methods of approaching the problem have been suggested. The "naive" procedure described below is not the most efficient one and can be significantly improved in both rate of convergence and accuracy. It is, however, intuitively obvious and easy to justify, and seems to be working well enough for the cases considered here. For a complete review of available approaches to bootstrap confidence intervals, see [11].

Let us consider $\hat{\theta}^*$, a bootstrap estimate of θ based on a resample of size n from the original sample $X_1,\ldots,X_n$, and let G_* be its distribution function given the observed sample values

$$G_*(x) = P\{\hat{\theta}^* \leq x \mid X_1 = x_1,\ldots,X_n = x_n\}.$$

Recall that for any distribution function F and $p \in (0,1)$ we define the pth quantile of F (sometimes also called pth percentile) as $F^{-1}(p) = \inf\{x : F(x) \geq p\}$. The *bootstrap percentiles method* gives $G_*^{-1}(\alpha)$ and $G_*^{-1}(1-\alpha)$ as, respectively, lower and upper bounds for the $(1-2\alpha)$ confidence interval for $\hat{\theta}$. Let us note that for most statistics $\hat{\theta}$, the distribution function of the bootstrap estimator $\hat{\theta}^*$ is not available. In practice, $G_*^{-1}(\alpha)$ and $G_*^{-1}(1-\alpha)$ are approximated by taking multiple resamples and then calculating the empirical percentiles. In this case the number of resamples B is usually much larger than for estimating BESE; in most cases $B \geq 1000$ is recommended.

3. BOOTSTRAP AND SMOOTHED BOOTSTRAP ESTIMATORS VS TRADITIONAL METHODS

In making the case for the usefulness of bootstrap methodology in modeling loss distributions, we would first like to compare its performance with that of the standard methods of inference as presented in actuarial textbooks.

3.1. Application to Wind Losses: Quantiles

Let us consider the following set of 40 losses due to wind-related catastrophes that occurred in 1977. These data are taken from Hogg and Klugman [12], where they are discussed in detail in Chapter 3. The losses were recorded only to the nearest \$1,000,000 and data included only those losses of \$2,000,000 or more. For convenience they have been ordered and recorded below.

2,	2,	2,	2,	2,	2,	2,	2,	2,	2
2,	2,	3,	3,	3,	3,	4,	4,	4,	5
5,	5,	5,	6,	6,	6,	6,	8,	8,	9
15,	17,	22,	23,	24,	24,	25,	27,	32,	43

Using this data set we shall give two examples illustrating the advantages of applying the bootstrap approach to modeling losses. The problem at hand is a typical one: assuming that all the losses recorded above have come from a single unknown distribution F, we would like to use the data to obtain some good approximation for F and its various parameters.

First, let us look at an important problem of finding the approximate confidence intervals for the quantiles of F. The standard approach to this problem relies on the normal approximation to the sample quantiles (order statistics). Applying this method, Hogg and Klugman [12] have found the approximate 95% confidence interval for the 0.85th quantile of F to be between X_{30} and X_{39}, which for the wind data translates into the observed interval $(9,32)$. They also have noted that "this is a wide interval but without additional assumptions this is the best we can do." Is that really true? To answer this question let us first note that in this particular case the highly skewed binomial distribution of the 0.85th sample quantile is approximated by a symmetric normal curve. Thus, it seems reasonable to expect that normal approximation could be improved here upon introducing some form of correction for skewness. In the standard normal approximation theory this is usually accomplished by considering, in addition to the normal term, the first non-normal term in the asymptotic Edgeworth expansion of the binomial distribution. The resulting formula is messy and requires the calculation of a sample skewness coefficient, as well as some refined form of the continuity correction (e.g., see Singh [19]). On the other hand, the bootstrap has been known to make such a correction automatically (Singh [19]) and hence we could expect that a bootstrap approximation would perform better here.[1] Indeed, in this case (in the notation of Section 2) we have $\theta(F) = F^{-1}(0.85)$ and $\hat{\theta} = \widehat{F}_n^{-1}(0.85) \approx X_{(34)}$, the 34th order statistic, which for the wind data equals 23. For sample quantiles the bootstrap distribution G_* can be calculated exactly (Shao and Tu [17, p.10]) or approximated by an empirical distribution obtained from B resamples as described in Section 2. Using either method, the $(1-2\alpha)$ confidence interval calculated using the percentile method is found to be between $X_{(28)}$ and $X_{(38)}$ (which is also in this case the exact confidence interval obtained by using binomial tables). For the wind data this translates into the interval (8,27), which is considerably shorter than the one obtained by Hogg and Klugman [12].

3.2. The Smoothed Bootstrap and its Application to Excess Wind Losses

As our second example, let us consider the estimation of the probability that a wind loss will exceed a \$29,500,000 threshold. In our notation that means that we wish to estimate the unknown parameter $(1-F(29.5))$. A direct application of the plug-in principle gives the value 0.05, the nonparametric estimate based on relative frequencies. However, note that the same number is also an estimate for $(1-F(29))$ and $(1-F(31.5))$, since the relative

[1]This turns out to be true only for a moderate sample size (here, 40); for a binomial distribution with large n (i.e., large sample size) the effect of the bootstrap correction is negligible. In general, the bootstrap approximation performs better than the normal approximation for large sample sizes only for continuous distributions.

frequency changes only at the threshold values present in reported data. In particular, since the wind data were rounded off to the nearest unit, the nonparametric method does not give a good estimate for any non-integer threshold. This problem with the same threshold value of \$29,000,000 was also considered in [12, Example 4 on p. 94 and Example 1 on p. 116]. As indicated therein, one reasonable way to deal with the non-integer threshold difficulty is to first fit some continuous curve to the data. The idea seems justified since the clustering effect in the wind data has most likely occurred due to rounding off the records. In their book Hogg and Klugman [12] have used standard techniques based on method of moments and maximum likelihood estimation to fit two different parametric models to the wind data: the truncated exponential with CDF

$$F_\mu(x) = 1 - e^{-(x-1.5)/\mu}, \qquad 1.5 < x < \infty \tag{3.1}$$

for $\mu > 0$, and the truncated Pareto with CDF

$$F_{\alpha,\lambda}(x) = 1 - \left(\frac{\lambda}{\lambda + x - 1.5}\right)^{\alpha}, \qquad 1.5 < x < \infty \tag{3.2}$$

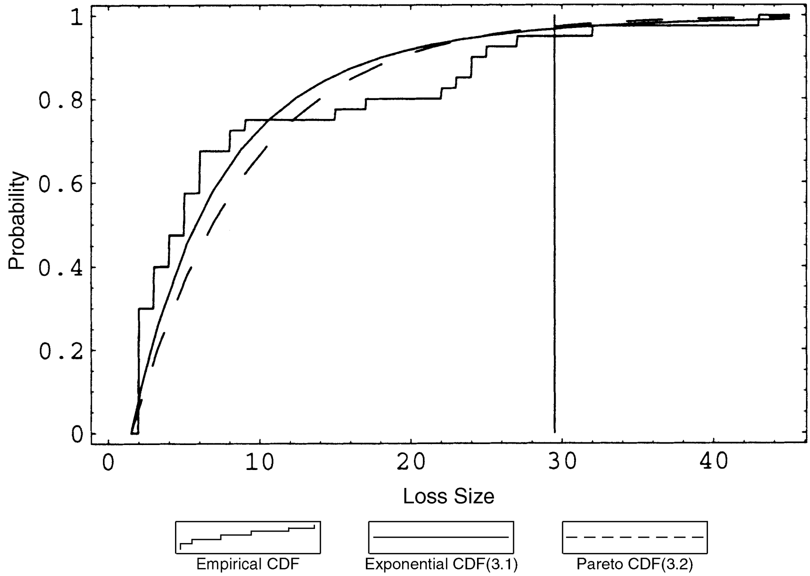
for $\alpha > 0,\ \lambda > 0$.

For the exponential distribution the method of moments estimator as well as maximum likelihood estimator (MLE) of μ was found to be $\hat{\mu} = 7.725$. The MLE's for the Pareto distribution parameters were $\hat{\lambda} = 28.998$ and $\hat{\alpha} = 5.084$; similar values were obtained using the method of moments.

The empirical distribution function for the wind data along with two fitted maximum likelihood models are presented in Figure 1. The solid smooth line represents the curve fitted from the exponential family (3.1), the dashed line represents the curve fitted from the Pareto family (3.2), and a vertical line is drawn for reference at $x = 29.5$. It is clear that the fit is not good at all, especially around the interval $(16, 24)$. The reason for the bad fit is the fact that both fitted curves are consistently concave down

FIGURE 1

EMPIRICAL AND FITTED CDF'S FOR WIND LOSS DATA



for all the x's and F seems to be concave up in this area.[2] The fit in the tail seems to be much better.

Once we determine the values of the unknown model parameters, MLE estimators for $(1 - F(29.5))$ may be obtained from (3.1) and (3.2). The numerical values of these estimates, their respective variances and their 95% confidence intervals are summarized in the second and third row of Table 1. All the confidence intervals and variances for the first three estimates shown in the table are calculated using the normal theory approximation. The variance and confidence intervals for the fourth estimate based on the moving-average smoother are calculated by

[2]In practice, this drawback could be possibly remedied by fitting a mixture of the distributions shown in (3.1) and (3.2). However, this approach could considerably complicate the parametric model and seems unlikely to provide much improvement in the tail fit, which is of primary interest here.

TABLE 1

COMPARISON OF THE PERFORMANCE OF ESTIMATORS FOR $(1 - F(29.5))$ FOR THE WIND DATA

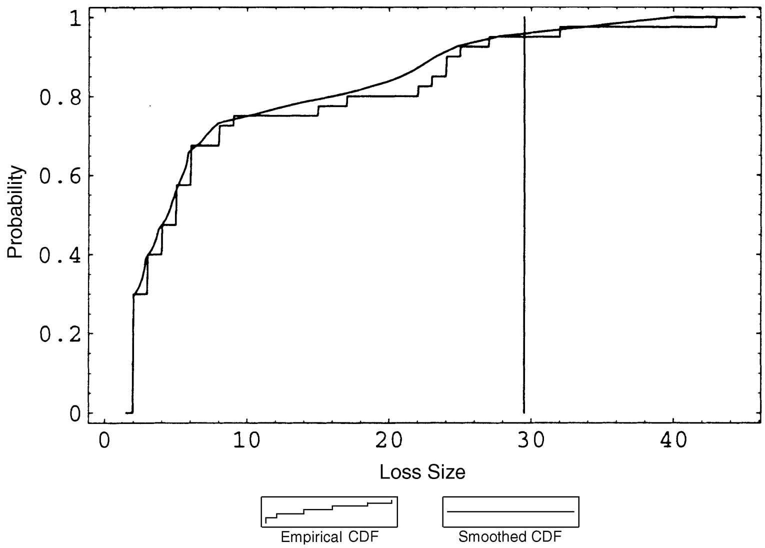
Fitted Model	Estimate of $(1 - F(29.5))$	Approx. s.e.	Approx. 95% c.i. (two sided)
Non-parametric (Plug-in)	0.05	0.034	$(-0.019, 0.119)$
Exponential	0.027	0.015	$(-0.003, 0.057)$
Pareto	0.036	0.024	$(-0.012, 0.084)$
3-Step Moving Average Smoother	0.045	0.016	$(0.013, 0.079)$

means of the approximate BESE and bootstrap percentile methods described in Section 2. In the first row the same characteristics are calculated for the standard non-parametric estimate based on relative frequencies. As we may well see, the respective values of the point estimators differ considerably from model to model and, in particular, both MLE's are quite far away from the relative frequency estimator. Another thing worth noticing is that the confidence intervals for all three models have negative lower bounds—they are obviously too long, at least on one side. This also indicates that their true coverage probability may in fact be greater than 95%.

In order to provide a better estimate of $(1 - F(29.5))$ for the wind data, we will first need to construct a smoothed version of the empirical CDF. In order to do so we employ the following data transformation widely used in image and signal processing theory, where a series of raw data $\{x_1, x_2, \ldots, x_n\}$ is often transformed to a new series of data before it is analyzed. The purpose of this transformation is to smooth out local fluctuations in the raw data, so the transformation is called *data smoothing* or a *smoother*. One common type of smoother employs a linear transformation and is called a linear filter. A linear filter with weights $\{c_0, c_1, \ldots, c_{r-1}\}$ transforms the given data to weighted averages $\sum_{j=0}^{r-1} c_j x_{t-j}$ for $t = r, r+1, \ldots, n$. Notice that the new data set has

FIGURE 2

EMPIRICAL AND SMOOTHED CDF'S FOR WIND LOSS DATA



length $(n - r - 1)$. If all the weights c_k are equal and they sum to unity, the linear filter is called an r-term moving average. For an overview of this interesting technique and its various applications, see Simonoff [18]. To create a smoothed version of the empirical CDF for the wind data, we have first used a three-term moving average smoother and then linearized in between any two consecutive data points.

The plot of this linearized smoother along with the original empirical CDF is presented in Figure 2. A vertical line is once again drawn for reference at $x = 29.5$. Let us note that the smoother follows the "concave-up-down-up" pattern of the data, which was not the case with the parametric distributions fitted from the families (3.1) and (3.2).

Once we have constructed the smoothed empirical CDF for the wind data, we may simply read the approximate value of

$(1 - F(29.5))$ off the graph (or better yet, ask the computer to do it for us). The resulting numerical value is 0.045. What is the standard error for that estimate? We again may use the bootstrap to answer that question without messy calculations. An approximate value for BESE (with $B = 1000$, but the result is virtually the same for $B = 100$) is found to be 0.016, which is only slightly worse than that of the exponential model MLE and much better than the standard error for the Pareto and empirical models. Equivalently, the same result may be obtained by numerical integration. Finally, the 95% confidence interval for $(1 - F(29.5))$ is found by means of the bootstrap percentile method with the number of replications set at $B = 1000$. Here the superiority of the bootstrap is obvious, as it gives an interval which is the second shortest (again exponential MLE model gives a shorter interval) but, most importantly, is bounded away from zero. The results are summarized in Table 1. Let us note that the result based on a smoothed empirical CDF and bootstrap dramatically improves that based on the relative frequency (plug-in) estimator and standard normal theory. It is perhaps of interest to note also that the MLE estimator of $(1 - F(29.5))$ in the exponential model is simply a parametric bootstrap estimator. For more details on the connection between MLE estimators and bootstrap, see [11].

4. CLUSTERED DATA

In the previous section we have assumed that the wind data were distributed according to some continuous CDF F. Clearly this is not always the case with loss data, and in general we may expect our theoretical loss distribution to follow some mixture of discrete and continuous CDF's.

4.1. Massachusetts Auto Bodily Injury Liability Data

In Appendix B we present the set of 432 closed losses due to bodily injuries in car accidents, under bodily injury liability (BI) policies reported in the Boston Territory (19) for calendar year

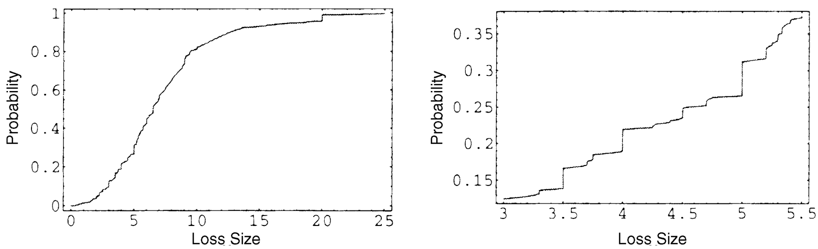
1995 (as of mid-1997). The losses are recorded in thousands and are subject to various policy limits but have no deductible. Policy limits capped 16 out of 432 losses which are therefore considered right-censored. The problem of bootstrapping censored data will be discussed in the next section; here we would like to concentrate on another interesting feature of the data. Massachusetts BI claim data are of interest because the underlying behavioral processes have been analyzed extensively. Weisberg and Derrig [20] and Derrig, Weisberg and Chen [8] describe the Massachusetts claiming environment after a tort reform as a "lottery" with general damages for non-economic loss (pain and suffering) as the prize. Cummins and Tennyson [5] showed signs of similar patterns countrywide, while Carroll, Abrahamse and Vaiana [3] and the Insurance Research Council [13] documented the pervasiveness of the lottery claims in both tort and no-fault state injury claim payment systems. The overwhelming presence of suspected fraud and buildup claims[3] allow for distorted relationships between the underlying economic loss and the liability settlement. Claim negotiators can greatly reduce the "usual" non-economic damages when exaggerated injury and/or excessive treatment are claimed as legitimate losses. Claim payments in such a negotiated process with discretionary injuries tend to be clustered at some usual mutually-acceptable amounts, especially for the run-of-the-mill strain and sprain claims. Conners and Feldblum [4] suggest that the claim environment, rather than the usual rating variables, are the key elements needed to understand and estimate relationships in injury claim data. All the data characteristics above tend to favor empirical methods over analytical ones.

Looking at the frequencies of occurrences of the particular values of losses in Massachusetts BI claim data, we may see that several numerical values have especially high frequency. The loss

[3]In auto insurance, fraudulent claims are those in which there was no injury or the injury was unrelated to the accident, whereas buildup claims are those in which the injury is exaggerated and/or the treatment is excessive.

FIGURE 3

APPROXIMATION TO THE EMPIRICAL CDF FOR THE BI DATA ADJUSTED FOR THE CLUSTERING EFFECT



of \$5,000 was reported 21 times (nearly 5% of all the occurrences), the loss of \$20,000 was reported 15 times, \$6,500 and \$4,000 losses were reported 14 times, a \$3,500 loss was only slightly less common (13 times), and losses of size \$6,000 and \$9,000 occurred 10 times each. There were also several other numerical values that have occurred at least five times. The clustering effect is obvious here and it seems that we should incorporate it into our model. This may be accomplished for instance by constructing an approximation to the empirical CDF, which is linearized in between the observed data values except for the ones with high frequency, where it behaves like the original, discrete CDF. In Figure 3 we present such an approximate CDF for the BI data. We have allowed our adjusted CDF to have discontinuities at the observed values which occurred with frequencies of five or greater. The left panel of Figure 3 shows the graph plotted for the entire range of observed loss values (0, 25). The right panel zooms in on the values from 3.5 to 5.5. Discontinuities can be seen here as the graph's "jumps" at the observed loss values of high frequency: 3.5, 4, 4.5, 5.

4.2. Bootstrap Estimates for Loss Elimination Ratios

To give an example of statistical inference under this model, let us consider a problem of eliminating part of the BI losses by purchasing a reinsurance policy that would cap the losses at some level d. Since the BI data is censored at \$20,000, we would consider here only values of d not exceeding \$20,000. One of the most important problems for the insurance company considering purchasing reinsurance is an accurate prediction of whether such a purchase would indeed reduce the experienced severity of loss and if so, by what amount. Typically this type of analysis is done by considering the loss elimination ratio (LER) defined as

$$\text{LER}(d) = \frac{E_F(X,d)}{E_F X}, \tag{4.1}$$

where $E_F X$ and $E_F(X,d)$ are, respectively, expected value and limited expected value functions for a random variable X following a true distribution of loss F. Since LER is only a theoretical quantity unobservable in practice, its estimate calculated from the data is needed. Usually, one considers the empirical loss elimination ratio (ELER) given by the obvious plug-in estimate

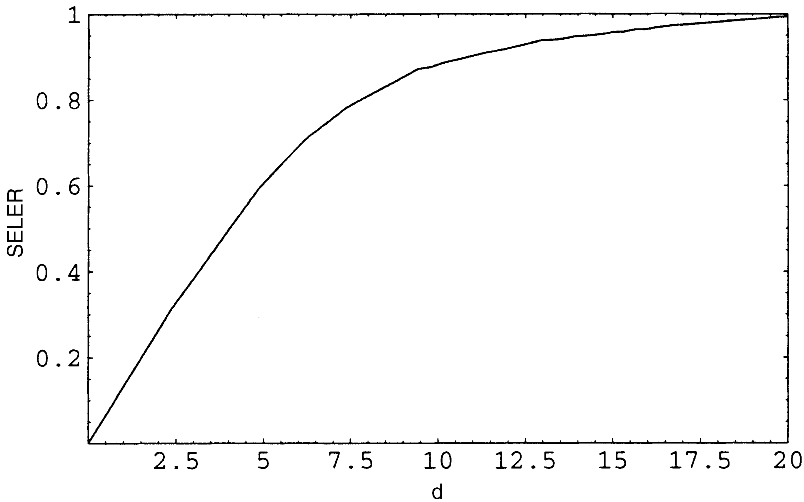
$$\text{ELER}(d) = \frac{E_{\widehat{F}_n}(X,d)}{E_{\widehat{F}_n} X} = \frac{\sum_{i=1}^{n} \min(X_i,d)}{\sum_{i=1}^{n} X_i}, \tag{4.2}$$

where $X_1,\ldots,X_n$ is a sample.

The drawback of ELER is in the fact that (unlike LER) it changes only at the values of d equal to the observed values of $X_1,\ldots,X_n$. It seems, therefore, that in order to calculate an approximate LER at different values of d, some smoothed version of ELER (SELER) should be considered. SELER may be obtained from Equation 4.2 by replacing the empirical CDF $\widehat{F}_n$ with its smoothed version, obtained for instance by applying a linear smoother (as for the wind data considered in Section 3) or a cluster-adjusted linearization. Obviously, the SELER formula may become quite complicated and its explicit derivation may be tedious (as would be the derivation of its standard error). Again, the bootstrap methodology can be applied here to facilitate the

FIGURE 4

APPROXIMATE GRAPH OF SELER(d)



computation of an approximate value of SELER(d), its standard error and confidence interval for any given value of d. In Figure 4 we present the graph of the SELER estimate for the BI data calculated for the values of d ranging from 0 to 20 (lowest censoring point) by means of a bootstrap approximation. This approximation was obtained by resampling the cluster-adjusted, linearized version of the empirical CDF (presented in the left panel of Figure 3) a large number of times (B = 300) and replicating $\hat{\theta}$ = SELER each time. The resulting sequence of bootstrap estimates $\hat{\theta}^*(b)$ for $b = 1,\ldots,B$ was then averaged to give the desired approximation of SELER. The calculation of standard errors and confidence intervals for SELER was done by means of BESE and the method of percentiles, as described in Section 2. The standard errors and 95% confidence intervals of SELER for several different values of d are presented in Table 2. The approximate BESE and bootstrap percentile methods

TABLE 2

VALUES OF SELER(d)

d	SELER(d)	Standard Error	95% Confidence Interval (two-sided)
4	0.505	0.0185	(0.488, 0.544)
5	0.607	0.0210	(0.597, 0.626)
10.5	0.892	0.0188	(0.888, 0.911)
11.5	0.913	0.0173	(0.912, 0.917)
14	0.947	0.0127	(0.933, 0.953)
18.5	0.985	0.00556	(0.98, 0.988)

described in Section 2 were used to calculate the standard errors and confidence intervals for the BI data in Table 2.

5. EXTENSIONS TO MORE COMPLICATED DESIGNS

So far in our account we have not considered any problems related to the fact that often in practice we may have to deal with truncated (e.g., due to deductible) or censored (e.g., due to policy limit) data. Another frequently encountered difficulty is the need for inflation adjustment, especially with data observed over a long period of time. We will address these important issues now.

5.1. Bootstrapping Censored Data for Policy Limits and Deductibles

Let us consider again the BI data presented in Section 4. There were 432 losses reported, of which 16 were at the policy limits.[4] These 16 losses may therefore be considered censored from above (or right-censored), and the appropriate adjustment for this fact should be made in our approach to estimating the loss distribution F. Whereas 16 is less then 4% of the total number of observed losses for the BI data, these censored observations are crucial in order to obtain a good estimate of F for the large loss values.

[4]Fifteen losses were truncated at $20,000 and one loss was truncated at $25,000.

Since the problem of censored data arises naturally in many medical, engineering, and other settings, it has received considerable attention in statistical literature. For the sake of brevity we will limit ourselves to the discussion of only one of the several commonly used techniques, the so-called Kaplan–Meier (or product-limit) estimator.

The typical statistical model for right-censored observations replaces the usual observed sample $X_1,\ldots,X_n$ with the set of ordered pairs $(X_1,\delta_1),\ldots,(X_n,\delta_n)$, where

$$\delta_i = \begin{cases} 0 & \text{if} \quad X_i \text{ is censored,} \\ 1 & \text{if} \quad X_i \text{ is not censored} \end{cases}$$

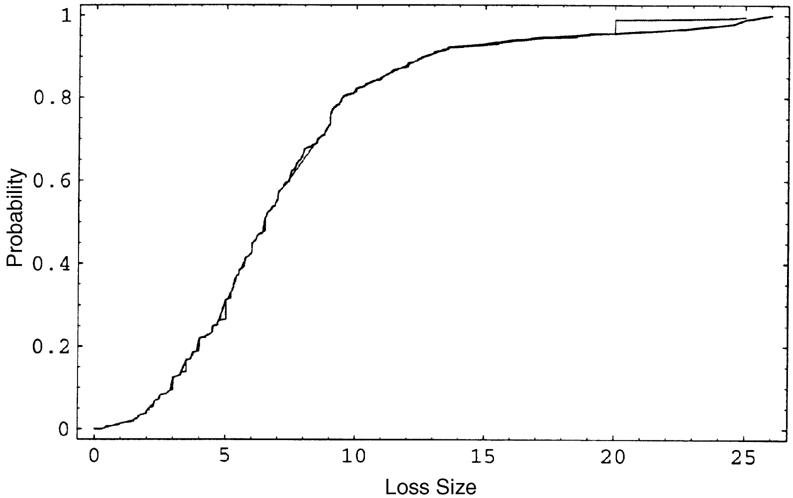
and the recorded losses are ordered $X_1 = x_1 \leq X_2 = x_2 \leq \cdots \leq X_n = x_n$ (with the usual convention that in the case of ties the uncensored values x_i ($\delta_i = 1$) precede the censored ones ($\delta_i = 0$)). The Kaplan–Meier estimator of $1 - F(x)$ is given by

$$\widehat{S}(x) = \prod_{i:x_i \leq x} \left(\frac{n-i}{n-i+1}\right)^{\delta_i}. \tag{5.1}$$

The product in the above formula is that of i terms, where i is the smallest positive integer less than or equal to n (the number of reported losses) and such that $x_i \leq x$. The Kaplan–Meier estimator, like the empirical CDF, is a step function with jumps at those values x_i that are uncensored. In fact, if $\delta_i = 1$ for all i, $i = 1,\ldots,n$ (i.e., no censoring occurs), it is easy to see that Equation 5.1 reduces to the complement of the usual empirical CDF. If the highest observed loss x_n is censored, Equation 5.1 is not defined for the values of x greater then x_n. The usual practice is to then add one uncensored data point (loss value) x_{n+1} such that $x_n < x_{n+1}$, and to define $\widehat{S}(x) = 0$ for $x \geq x_{n+1}$. For instance, for the BI data the largest reported loss was censored at 25 and we had to add one artificial "loss" at 26 to define the Kaplan–Meier curve for the losses exceeding 25. The number 26 was picked quite arbitrarily; in actuarial practice a more precise guess of

FIGURE 5

THE KAPLAN–MEIER ESTIMATOR



the maximum possible value of loss (e.g., based on past experience) should be easily available. The Kaplan–Meier estimator enjoys several optimal statistical properties and can be viewed as a generalization of the usual empirical CDF adjusted for the case of censored losses. Moreover, truncated losses or truncated and censored losses may be easily handled by some simple modifications of Equation 5.1. For more details and some examples, see Klugman, Panjer and Willmot [14, Chapter 2].

In the case of loss data coming from a mixture of discrete and continuous CDF's as, for instance, the BI data, the linearization of the Kaplan–Meier estimator with adjustment for clustering seems to be appropriate. In Figure 5 we present the plots of a linearized Kaplan–Meier estimator for the BI data and the approximate empirical CDF function (which was discussed in Section 4), not corrected for the censoring effect. It is interesting to note that the two curves agree very well up to the first censoring point (20), where the Kaplan–Meier estimator starts to correct for the effect of censoring. It is thus reasonable to believe that, for instance, the values of SELER calculated in Table 2 should be close to the values obtained by bootstrapping the Kaplan–Meier estimator. This, however, does not have to be the case in general. The agreement between the Kaplan–Meier curve and the smoothed CDF of the BI data is mostly due to the relatively small number of censored values. The estimation of other parameters of interest under the Kaplan–Meier model (e.g. quantiles, probability of exceedance, etc.) as well as their standard errors may be performed using the bootstrap methodology outlined in the previous sections. For more details on the problem of bootstrapping censored data, see Akritas [1].

5.2. *Inflation Adjustment*

An adjustment for the effect of inflation can be handled quite easily in our setting. If X is the random variable modeling the loss which follows CDF F, when adjusting for inflation we are interested in obtaining an estimate of the distribution of $Z = (1+r)X$, where r is the uniform inflation rate over the period of concern. If Z follows a CDF G, then obviously

$$G(z) = F\left(\frac{z}{1+r}\right) \tag{5.1}$$

and the same relation holds when we replace G and F with the usual empirical CDF's or their smoothed versions.[5] In this setting, bootstrap techniques described earlier should be applied to the empirical approximation of G.

6. SOME FINAL REMARKS

Although we have limited the discussion of resampling methods to the narrow scope of modeling losses, we have presented

[5]Subclasses of losses may inflate at different rates (soft tissue versus hard injuries for the BI data is an example). The theoretical CDF G may be then derived using multiple inflation rates as well.

only some examples of modern statistical methods relevant to the topic. Other important areas of application which have been purposely left out here include kernel estimation and the use of resampling in non-parametric regression and auto-regression models. The latter includes, for instance, such important problems as bootstrapping time-series data, modeling time-correlated losses and other time-dependent variables. Over the past several years some of these techniques, like non-parametric density estimation, have already found their way into actuarial practice (e.g., Klugman, Panjer and Willmot [14]). Others, like bootstrap, are still waiting. The purpose of this article was not to give a complete account of the most recent developments in non-parametric statistical methods, but rather to show by example how easily they may be adapted to real-life situations and how often they may, in fact, outperform the traditional approach.

7. CONCLUSIONS

Several examples of the practical advantages of the bootstrap methodology were presented. We have shown by example that in many cases the bootstrap technique provides a better approximation to the true parameters of the underlying distribution of interest than the traditional, textbook approach relying on the MLE and normal approximation theory. It seems that bootstrap may be especially useful in the statistical analysis of data which do not follow any obvious continuous parametric model (or mixture of models) or/and contain a discrete component (like the BI data presented in Section 4). The presence of censoring and truncation in the data does not present a problem for the bootstrap which, as seen in Section 5, may be easily incorporated into a standard non-parametric analysis of censored or truncated data. Of course, most of the bootstrap analysis is typically done approximately using a Monte Carlo simulation (generating resamples), which makes the computer an indispensable tool in the bootstrap world. Even more, according to some leading bootstrap theorists, automation is the goal

[11, p. 393]:

> One can describe the ideal computer-based statistical inference machine of the future. The statistician enters the data…the machine answers the questions in a way that is optimal according to statistical theory. For standard errors and confidence intervals, the ideal is in sight if not in hand.

The resampling methods described in this paper can be used (possibly after correcting for time-dependence) to handle the empirical data concerning all DFA model input variables, including interest rates and capital market returns. The methodologies also apply to any financial intermediary, such as a bank or a life insurance company. It would be interesting, indeed it is imperative, to make bootstrap-based inferences in such settings and compare their effectiveness and applicability with classical parametric, trend-based, Bayesian, and other methods of analysis. The bootstrap computer program (using Mathematica 3.0 programming language; see Appendix A) that we have developed here to provide smooth estimates of an empirical CDF, BESE, and bootstrap confidence intervals could be easily adapted to produce appropriate estimates in DFA, including regulatory calculations for Value at Risk and Asset Adequacy Analysis. It would also be interesting to investigate further all areas of financial management where our methodologies may hold a promise of future applications. For instance, by modeling both the assets side (interest rates and capital market returns) and the liabilities side (losses, mortality, etc.), as well as their interactions (crediting strategies, investment strategies of the firm), one might create nonparametric models of the firm and use such a whole-company model to analyze value optimization and solvency protection in an integrated framework. Such whole company models are more and more commonly used by financial intermediaries, but we propose an additional level of complexity by adding the bootstrap estimation of their underlying random structures. This methodology is immensely computationally intensive, but it holds great promise not just for internal company models but also for regulatory supervision, hopefully allowing for better oversight and avoiding problems such as the insolvencies of savings and loans institutions in the late 1980s, the insolvencies of life insurance firms such as Executive Life and Mutual Benefit, or the catastrophe-related problems of property-casualty insurers.

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APPENDIX A

MATHEMATICA BOOTSTRAP FUNCTIONS

The following computer program written in Mathematica 3.0 programming language was used to calculate bootstrap replications, bootstrap standard errors estimates (BESE) and bootstrap 95% confidence intervals using the method of percentiles.

(* Here we include the standard statistical libraries to be used in our bootstraping program *)

```
<<Statistics'DataManipulation'
<<Statistics'ContinuousDistributions'
```

(* Here we define resampling procedure "boot" as well as empirical cdf functions: usual empirical cdf "empcdf" and its smoothed version "cntcdf". Procedure "inv" is used by "boot" *)

(* Arguments for the procedures are as follows:

"boot" has two arguments: "lst" (any data list of numerical values) and , "nosam" (number of resamples, usually nosum=Length[lst]

"empcdf" and "cntcdf" both have two arguments "lst" (any data list of numerical values) and "x" -the numerical argument of function *)

```
inv[x_, lstx_] :=
Module[{nlx=Length[lstx]},
  If [x == 0 , lstx[[1]],
    If[x == 1, lstx[[nlx]], k=Floor[(nlx - 1) x];
    ((nlx - 1) x - k ) (lstx[[k+2]] -
lstx[[k+1]])+lstx[[k+1]]
    ]
  ]
];
```

```
boot[lx_, nosam_] := Module[{tt, i, a, n, lstx},
lstx=Sort[lx]; n=Length[lx];
lstx=Flatten[{{2 lstx[[1]] - lstx[[2]]},
lstx, {2 lstx[[n]] - lstx[[n - 1]]}}];
  tt=RandomArray[UniformDistribution [0, 1], nosam];
    For[i=1, i <= nosam , i++, a[i] = inv[tt[[i]],
lstx]];
      Table[a[i], {i, 1, nosam}]
      ];

cntcdf[lst_, x_] := Module[{ll=Sort[lst],
n=Length[lst], i=1},
    ll=Flatten[{{2 ll[[1]] - ll[[2]]}, ll, {2 ll[[n]] -
ll[[n - 1]]}}];
While[i <= n+2 && x > ll[[i]], i++];
  If[i == 1, 0,
If[i == n+3,
1, ((x - ll[[i - 1]])/(ll[[i]] - ll[[i - 1]])+(i -
2))/(n+1)]]
  ];

empcdf[lst_, x_] :=Module[{ll=Sort[lst], n=Length[lst],
i=1},
While[i <= n && x > ll[[i]], i++];
  If[i == 1, 0, (i - 1)/ n]
  ];
```

(* Here we define the bootstrap replications of statistic theta. Procedure “theta” calculates a statistic from the list of data “lst”. Procedure “replicate” replicates the statistic “theta” “norep” number of times using procedure “boot “ with parameters “lst” and “nosam”. As a result of this procedure we obtain a list of replicated values of “theta” *)

```
theta[lst_] := 1; (* define your Theta statistic here*)
```

```
replicate[lst_, norep_, nosam_] := Module[{i, ll = },
For [i=1, i <= norep, i++,
    ll =  Flatten[{ll, theta[boot[lst, nosam]]}]
    ]; ll
  ];
```

(*Here we calculate BESE and 95% confidence interval based on the method of percentiles for 1000 replications *)

(* run “replicate” procedure, store the results in variable “listofrep” *)

```
listofrep=replicate[lst, norep, nosam];
```

(* BESE*)

```
Variance[listofrep]
```

(* 95% confidence interval for number of replications (norep)= 1000 *)

```
95ci = {listofrep[[25]], listofrep[[975]]}
```

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APPENDIX B

MASSACHUSETTS BI DATA

The table below presents a set of 432 closed auto BI losses in Boston Territory (19) for calendar year 1995 (as of mid-1997). For each loss we have provided the injury type classification code along with the actual payment amount, as well as the corresponding policy limit. A description of the injury codes is provided on the last page of the appendix.

No.	Injury Type	Total Amount Paid	Policy Limit
1	5	$393	$20,000
2	1	$500	$20,000
3	6	$500	$20,000
4	8	$900	$20,000
5	6	$1,000	$20,000
6	5	$1,000	$20,000
7	5	$1,250	$20,000
8	5	$1,500	$20,000
9	5	$1,500	$20,000
10	5	$1,525	$20,000
11	5	$1,631	$100,000
12	4	$1,650	$20,000
13	5	$1,700	$20,000
14	5	$1,700	$20,000
15	5	$1,800	$20,000
16	5	$1,950	$20,000
17	5	$2,000	$20,000
18	5	$2,000	$25,000
19	5	$2,007	$20,000
20	5	$2,100	$20,000
21	5	$2,100	$20,000
22	5	$2,100	$20,000
23	5	$2,250	$20,000
24	5	$2,250	$20,000
25	5	$2,250	$20,000
26	5	$2,250	$20,000
27	5	$2,270	$20,000
28	5	$2,300	$20,000
29	6	$2,300	$20,000
30	5	$2,375	$20,000
31	5	$2,450	$20,000

No.	Injury Type	Total Amount Paid	Policy Limit
32	5	$2,500	$20,000
33	5	$2,500	$100,000
34	5	$2,500	$20,000
35	6	$2,500	$20,000
36	1	$2,600	$20,000
37	5	$2,750	$20,000
38	5	$2,800	$20,000
39	5	$2,813	$20,000
40	5	$2,900	$20,000
41	5	$3,000	$20,000
42	5	$3,000	$20,000
43	5	$3,000	$20,000
44	5	$3,000	$20,000
45	5	$3,000	$20,000
46	5	$3,000	$20,000
47	5	$3,000	$20,000
48	6	$3,000	$20,000
49	6	$3,000	$50,000
50	99	$3,000	$20,000
51	6	$3,000	$20,000
52	5	$3,000	$20,000
53	5	$3,000	$20,000
54	4	$3,000	$20,000
55	5	$3,150	$20,000
56	5	$3,250	$20,000
57	5	$3,300	$20,000
58	5	$3,300	$20,000
59	5	$3,300	$20,000
60	4	$3,500	$20,000
61	4	$3,500	$1,000,000
62	5	$3,500	$20,000
63	1	$3,500	$20,000
64	5	$3,500	$20,000
65	5	$3,500	$20,000
66	5	$3,500	$20,000
67	5	$3,500	$20,000
68	5	$3,500	$20,000
69	4	$3,500	$20,000
70	5	$3,500	$20,000
71	5	$3,500	$50,000
72	99	$3,500	$20,000
73	5	$3,650	$20,000
74	5	$3,700	$20,000
75	5	$3,700	$20,000
76	5	$3,700	$20,000

No.	Injury Type	Total Amount Paid	Policy Limit
77	5	$3,750	$20,000
78	5	$3,750	$20,000
79	5	$3,750	$20,000
80	5	$3,750	$20,000
81	6	$3,900	$20,000
82	5	$4,000	$20,000
83	5	$4,000	$1,000,000
84	5	$4,000	$20,000
85	5	$4,000	$20,000
86	5	$4,000	$20,000
87	4	$4,000	$20,000
88	6	$4,000	$20,000
89	5	$4,000	$20,000
90	5	$4,000	$20,000
91	5	$4,000	$20,000
92	5	$4,000	$20,000
93	5	$4,000	$20,000
94	1	$4,000	$20,000
95	5	$4,000	$25,000
96	5	$4,250	$20,000
97	6	$4,250	$20,000
98	6	$4,278	$50,000
99	5	$4,396	$25,000
100	5	$4,400	$20,000
101	5	$4,476	$20,000
102	5	$4,500	$20,000
103	5	$4,500	$20,000
104	5	$4,500	$25,000
105	5	$4,500	$20,000
106	10	$4,500	$20,000
107	5	$4,500	$20,000
108	5	$4,521	$20,000
109	5	$4,697	$20,000
110	5	$4,700	$20,000
111	5	$4,700	$20,000
112	5	$4,700	$20,000
113	4	$4,725	$20,000
114	5	$4,750	$20,000
115	5	$5,000	$20,000
116	5	$5,000	$100,000
117	5	$5,000	$20,000
118	5	$5,000	$20,000
119	5	$5,000	$20,000
120	5	$5,000	$20,000
121	5	$5,000	$20,000

No.	Injury Type	Total Amount Paid	Policy Limit
122	4	$5,000	$20,000
123	5	$5,000	$20,000
124	5	$5,000	$20,000
125	5	$5,000	$20,000
126	5	$5,000	$20,000
127	5	$5,000	$20,000
128	6	$5,000	$20,000
129	4	$5,000	$20,000
130	1	$5,000	$20,000
131	5	$5,000	$20,000
132	5	$5,000	$20,000
133	5	$5,000	$20,000
134	5	$5,000	$100,000
135	5	$5,000	$20,000
136	6	$5,100	$20,000
137	5	$5,200	$20,000
138	5	$5,200	$20,000
139	5	$5,200	$20,000
140	5	$5,200	$20,000
141	5	$5,200	$20,000
142	5	$5,200	$20,000
143	5	$5,200	$20,000
144	5	$5,225	$20,000
145	5	$5,250	$20,000
146	5	$5,250	$20,000
147	5	$5,292	$20,000
148	5	$5,296	$20,000
149	5	$5,300	$20,000
150	5	$5,300	$20,000
151	4	$5,300	$20,000
152	5	$5,333	$20,000
153	5	$5,333	$20,000
154	5	$5,333	$20,000
155	5	$5,333	$20,000
156	4	$5,344	$20,000
157	5	$5,366	$20,000
158	4	$5,400	$30,000
159	5	$5,400	$20,000
160	5	$5,415	$20,000
161	5	$5,497	$100,000
162	4	$5,500	$20,000
163	5	$5,500	$20,000
164	5	$5,500	$20,000
165	5	$5,500	$20,000
166	6	$5,500	$20,000

No.	Injury Type	Total Amount Paid	Policy Limit
167	5	$5,566	$20,000
168	5	$5,600	$25,000
169	5	$5,714	$20,000
170	5	$5,714	$20,000
171	5	$5,714	$20,000
172	5	$5,714	$20,000
173	5	$5,714	$20,000
174	5	$5,714	$20,000
175	5	$5,714	$20,000
176	5	$5,725	$20,000
177	6	$5,750	$20,000
178	5	$5,750	$100,000
179	5	$5,750	$20,000
180	5	$5,852	$20,000
181	6	$5,898	$20,000
182	5	$5,900	$20,000
183	5	$5,964	$20,000
184	6	$5,990	$20,000
185	5	$6,000	$25,000
186	5	$6,000	$20,000
187	5	$6,000	$20,000
188	5	$6,000	$20,000
189	1	$6,000	$20,000
190	5	$6,000	$20,000
191	5	$6,000	$20,000
192	5	$6,000	$20,000
193	5	$6,000	$20,000
194	5	$6,000	$20,000
195	4	$6,077	$20,000
196	5	$6,078	$20,000
197	5	$6,131	$20,000
198	5	$6,166	$20,000
199	5	$6,166	$20,000
200	5	$6,169	$20,000
201	5	$6,171	$20,000
202	5	$6,208	$20,000
203	5	$6,243	$20,000
204	5	$6,318	$20,000
205	5	$6,399	$20,000
206	5	$6,413	$20,000
207	5	$6,500	$20,000
208	5	$6,500	$20,000
209	5	$6,500	$20,000
210	5	$6,500	$20,000
211	5	$6,500	$20,000

No.	Injury Type	Total Amount Paid	Policy Limit
212	5	$6,500	$20,000
213	5	$6,500	$20,000
214	5	$6,500	$20,000
215	99	$6,500	$20,000
216	5	$6,500	$20,000
217	5	$6,500	$50,000
218	5	$6,500	$25,000
219	5	$6,500	$20,000
220	5	$6,500	$50,000
221	5	$6,519	$20,000
222	4	$6,536	$20,000
223	5	$6,549	$20,000
224	1	$6,558	$25,000
225	6	$6,600	$20,000
226	5	$6,600	$20,000
227	6	$6,620	$20,000
228	5	$6,700	$20,000
229	6	$6,703	$20,000
230	1	$6,743	$25,000
231	5	$6,750	$20,000
232	5	$6,800	$20,000
233	4	$6,870	$20,000
234	5	$6,893	$50,000
235	5	$6,898	$50,000
236	5	$6,907	$20,000
237	5	$6,933	$20,000
238	5	$6,935	$100,000
239	5	$6,977	$100,000
240	5	$7,000	$100,000
241	5	$7,000	$20,000
242	5	$7,000	$20,000
243	5	$7,000	$20,000
244	5	$7,000	$20,000
245	5	$7,000	$20,000
246	5	$7,000	$20,000
247	5	$7,014	$20,000
248	4	$7,043	$20,000
249	5	$7,079	$20,000
250	5	$7,118	$20,000
251	5	$7,163	$20,000
252	5	$7,191	$20,000
253	5	$7,200	$20,000
254	5	$7,200	$20,000
255	5	$7,250	$20,000
256	4	$7,252	$20,000

No.	Injury Type	Total Amount Paid	Policy Limit
257	5	$7,304	$20,000
258	1	$7,412	$25,000
259	1	$7,425	$100,000
260	5	$7,432	$20,000
261	5	$7,444	$50,000
262	5	$7,447	$20,000
263	5	$7,500	$20,000
264	5	$7,500	$20,000
265	5	$7,500	$25,000
266	5	$7,500	$20,000
267	5	$7,500	$20,000
268	5	$7,500	$20,000
269	99	$7,500	$20,000
270	1	$7,564	$20,000
271	5	$7,620	$20,000
272	18	$7,629	$20,000
273	5	$7,657	$20,000
274	1	$7,670	$20,000
275	5	$7,671	$20,000
276	4	$7,696	$100,000
277	4	$7,700	$100,000
278	5	$7,750	$20,000
279	5	$7,754	$20,000
280	5	$7,820	$20,000
281	4	$7,859	$20,000
282	5	$7,868	$20,000
283	1	$7,873	$25,000
284	5	$7,920	$100,000
285	5	$7,922	$20,000
286	5	$7,945	$20,000
287	5	$7,954	$20,000
288	5	$7,961	$20,000
289	5	$8,000	$100,000
290	5	$8,000	$100,000
291	5	$8,000	$20,000
292	10	$8,013	$50,000
293	5	$8,073	$20,000
294	5	$8,200	$20,000
295	1	$8,298	$25,000
296	6	$8,300	$20,000
297	1	$8,420	$20,000
298	5	$8,485	$20,000
299	5	$8,500	$50,000
300	5	$8,500	$20,000
301	99	$8,500	$20,000

No.	Injury Type	Total Amount Paid	Policy Limit
302	5	$8,500	$20,000
303	5	$8,515	$20,000
304	5	$8,612	$20,000
305	5	$8,634	$100,000
306	5	$8,686	$20,000
307	5	$8,785	$20,000
308	5	$8,786	$20,000
309	5	$8,794	$20,000
310	5	$8,805	$20,000
311	5	$8,815	$20,000
312	5	$8,856	$20,000
313	5	$8,861	$20,000
314	6	$8,882	$20,000
315	5	$8,911	$20,000
316	5	$8,914	$20,000
317	5	$8,988	$20,000
318	5	$9,000	$100,000
319	5	$9,000	$20,000
320	5	$9,000	$20,000
321	5	$9,000	$20,000
322	5	$9,000	$20,000
323	5	$9,000	$0
324	5	$9,000	$20,000
325	5	$9,000	$20,000
326	5	$9,000	$20,000
327	5	$9,000	$20,000
328	5	$9,009	$20,000
329	5	$9,020	$20,000
330	5	$9,030	$25,000
331	5	$9,051	$20,000
332	5	$9,053	$20,000
333	5	$9,073	$100,000
334	5	$9,100	$20,000
335	1	$9,129	$20,000
336	5	$9,200	$20,000
337	5	$9,208	$20,000
338	5	$9,300	$20,000
339	5	$9,355	$20,000
340	5	$9,356	$20,000
341	5	$9,392	$20,000
342	5	$9,395	$100,000
343	5	$9,423	$20,000
344	5	$9,428	$20,000
345	5	$9,451	$100,000
346	5	$9,500	$20,000

No.	Injury Type	Total Amount Paid	Policy Limit
347	5	$9,500	$20,000
348	5	$9,602	$20,000
349	5	$9,710	$20,000
350	4	$9,881	$25,000
351	5	$9,909	$20,000
352	8	$10,000	$20,000
353	6	$10,000	$20,000
354	5	$10,000	$100,000
355	6	$10,000	$20,000
356	4	$10,106	$20,000
357	5	$10,229	$20,000
358	5	$10,330	$20,000
359	5	$10,331	$20,000
360	5	$10,400	$20,000
361	5	$10,505	$100,000
362	4	$10,555	$20,000
363	1	$10,645	$20,000
364	8	$10,861	$20,000
365	5	$10,968	$20,000
366	5	$11,000	$50,000
367	4	$11,000	$100,000
368	5	$11,032	$20,000
369	5	$11,144	$20,000
370	5	$11,166	$20,000
371	1	$11,262	$25,000
372	5	$11,344	$50,000
373	99	$11,353	$20,000
374	5	$11,385	$20,000
375	1	$11,500	$20,000
376	5	$11,626	$20,000
377	5	$11,835	$20,000
378	99	$11,986	$20,000
379	5	$11,991	$20,000
380	4	$12,000	$20,000
381	5	$12,000	$20,000
382	5	$12,000	$20,000
383	5	$12,214	$100,000
384	5	$12,274	$20,000
385	5	$12,374	$20,000
386	99	$12,380	$20,000
387	3	$12,500	$20,000
388	5	$12,509	$20,000
389	5	$12,621	$100,000
390	5	$12,756	$20,000
391	5	$12,859	$20,000

No.	Injury Type	Total Amount Paid	Policy Limit
392	5	$12,988	$20,000
393	7	$13,000	$20,000
394	5	$13,009	$20,000
395	5	$13,299	$50,000
396	4	$13,347	$20,000
397	5	$13,500	$20,000
398	5	$13,570	$20,000
399	99	$13,572	$100,000
400	4	$14,181	$20,000
401	5	$14,700	$20,000
402	5	$14,953	$20,000
403	5	$15,500	$20,000
404	5	$15,500	$100,000
405	5	$15,765	$20,000
406	18	$16,000	$20,000
407	5	$16,668	$20,000
408	5	$16,794	$20,000
409	4	$17,267	$100,000
410	99	$18,500	$20,000
411	99	$18,500	$20,000
412	18	$19,000	$20,000
413	5	$19,012	$20,000
414	99	$20,000	$20,000
415	5	$20,000	$20,000
416	7	$20,000	$20,000
417	8	$20,000	$20,000
418	8	$20,000	$20,000
419	7	$20,000	$20,000
420	7	$20,000	$20,000
421	3	$20,000	$20,000
422	6	$20,000	$20,000
423	16	$20,000	$20,000
424	5	$20,000	$20,000
425	6	$20,000	$20,000
426	5	$20,000	$20,000
427	9	$20,000	$20,000
428	5	$20,000	$20,000
429	1	$22,692	$100,000
430	5	$24,500	$50,000
431	99	$25,000	$25,000
432	2	$25,000	$100,000

INJURY CODE DESCRIPTION

Injury Type	Description	Injury Type	Description
1	MINOR LACERATIONS/ CONTUSIONS	13	PARALYSIS/PARESIS
2	SERIOUS LACERATION	14	JAW JOINT DYSFUNCTION
3	SCARRING OR PERMANENT DISFIGUREMENT	15	LOSS OF A SENSE
4	NECK ONLY SPRAIN STRAIN	16	FATALITY
5	BACK OR NECK & BACK SPRAIN/STRAIN	17	DENTAL
6	OTHER SPRAIN/STRAIN	18	CARTILAGE/MUSCLE/TENDON/ LIGAMENT INJURY
7	FRACTURE OR WEIGHT BEARING BONE	19	DISC HERNIATION
8	OTHER FRACTURE	20	PREGNANCY RELATED
9	INTERNAL ORGAN INJURY	21	PRE-EXISTING CONDITION
10	CONCUSSION	22	PSYCHOLOGICAL CONDITION
11	PERMANENT BRAIN INJURY	30	NO VISIBLE INJURY
12	LOSS OF BODY PART	99	OTHER

MEASURING THE INTEREST RATE SENSITIVITY OF LOSS RESERVES

STEPHEN P. D'ARCY AND RICHARD W. GORVETT

Abstract

In order to apply asset-liability management techniques to property-liability insurers, the sensitivity of liabilities to interest rate changes, or duration, must be calculated. The current approach is to use the Macaulay or modified duration calculations, both of which presume that the cash flows are invariant with respect to interest rate changes. Based on the structure of liabilities for property-liability insurers, changes in interest rates—given that interest rates are correlated with inflation—should affect future cash flows on existing liabilities. This paper analyzes the effect that interest rate changes can have on these cash flows, shows how to calculate the resulting effective duration of these liabilities, and demonstrates the impact of failing to use the correct duration measure on asset-liability management for property-liability insurers.

1. INTRODUCTION

Property-liability insurance companies are exposed to a wide variety of risks. However, the focus of most insurers and reinsurers has been primarily on traditional insurance risks, such as legal, regulatory or catastrophic exposures. It is widely recognized that the potential impact of natural catastrophes on property-liability insurers is so severe that this area has been given extensive attention by the industry: sophisticated models have been developed to quantify catastrophe exposure and securitized insurance products are being designed to facilitate the trading of such risks through the capital markets. Extensive attention has also been paid to quantifying and predicting the underwriting cycle, although with considerably less success.

However, insurers are also exposed to a variety of financial risks that have not received the same level of attention, despite the success that other financial services firms have achieved in this venue. For example, with the rising level of globalization in the insurance industry, the risk of fluctuations in foreign exchange rates is becoming an increasing concern for insurers. Nevertheless, insurers lag well behind other financial institutions in foreign currency hedging activity. Another critical area of risk faced by insurers involves fluctuations in value due to interest rate movements. Banks, life insurers and other financial institutions have developed sophisticated approaches to attempt to deal with interest rate risk. Most property-liability insurers have neither adopted the approaches of other financial institutions nor adapted those models to reflect the unique characteristics of this industry. This article seeks to address this area of concern. While interest rate risk is not as significant for the property-liability insurance industry as, for example, catastrophe risk, it does represent an important source of risk and is one that can be effectively dealt with through the use of accepted risk management techniques.

Similar to any other financial institution, the values of an insurer's assets and liabilities can be affected by changes in future interest rates. The reason for this is that the economic value of a financial asset or liability is the discounted value of its future cash flows. Thus, if interest rates increase, the economic value of future cash flows will decrease; if interest rates decrease, economic value will increase. The direction of the movement in values of both the assets and the liabilities, according to this principle, will be the same. The problem, however, is that asset and liability values will generally not move by the same amount in response to a particular change in interest rates (unless specifically and accurately set up to do so). If they do not move similarly, the net worth of an insurer will change over time due to the volatility of interest rates.

Asset-liability management (ALM), as used in the insurance industry, is a process by which insurers attempt to evaluate and adjust the exposure of the net value of the company (assets minus liabilities) to interest rate changes. Although, in theory, the volatility of other factors (e.g., catastrophes, changes in unemployment rates[1]) can also affect both asset and liability values, the current focus of ALM for insurers, as for most other financial institutions, is on interest rate risk. Life insurers were the first in the industry to apply ALM techniques, since they have significant exposure to interest rate risk due to the long payout patterns of losses and their high leverage. However, this approach is now being applied to the property-liability insurance industry as well.

The general approaches used by life insurers to measure the sensitivity of assets to interest rate risk are applicable to property-liability insurers to the extent that they have similar asset portfolios. In general, property-liability companies invest more heavily in equities and less in mortgages, but the overall structure of the investment portfolio is roughly similar. However, the *liabilities* of property-liability insurers are different enough that the approaches used by life insurers are simply not applicable to them, and new techniques must be developed.

Duration is a measure of the interest rate sensitivity of a financial instrument. The term duration, which seems to signify more a measure of time than of interest rate sensitivity, is derived from early work on fixed income assets in which the interest rate sensitivity was found to correspond closely to a weighted average time value. The basic approach of ALM involves measuring the durations of assets and liabilities, and then adjusting one or both until the insurer is not significantly affected by interest rate changes (essentially, this involves setting the duration of surplus equal to zero). If the duration of liabilities is measured incorrectly, then an insurer trying to immunize itself from inter-

[1]For example, an increase in the unemployment rate is likely to increase the severity of workers compensation losses and also alter the prepayment patterns on mortgage-backed securities.

est rate risk based on the incorrect measure will actually still be exposed to interest rate risk. Much research has been done on determining the duration of complex financial instruments held by insurers, such as collateralized mortgage obligations (Fabozzi [12], Chapter 27) and corporate bonds with callability provisions. Attention has also been given to determining the appropriate duration measure of life insurance liabilities (Babbel [3]). However, much less attention has been paid to the duration of liabilities of property-liability insurers. (The issue has been briefly discussed or alluded to in, for example, Butsic [6]; D'Arcy [8]; Ferguson [14]; and Noris [23].) The general approach to measuring the duration of liabilities for property-liability insurers has been to calculate a weighted average of the time to payment for loss reserves (Campbell [7], Hodes and Feldblum [16], and Staking and Babbel [26]). This approach is patterned after the work by Macaulay [20], which determined that the sensitivity of the price of non-callable fixed income securities to changes in interest rates was approximated by this duration measure:

$$\text{Macaulay Duration} = \sum_{t=1}^{n} \frac{t(\text{PVCF}_t)}{\text{PVTCF}}, \tag{1.1}$$

where

PVCF_t = the present value now of the cash flow at time t,

PVTCF = the present value of the total cash flow, and

t = time to payment of the cash flow.

Additional analysis (Panning [24]) has been based on the modified duration measure (Fabozzi [11]), which is the Macaulay duration value divided by $1 + r$ (where r is the current interest rate):

$$\text{Modified Duration} = \frac{\text{Macaulay Duration}}{1 + r}, \tag{1.2}$$

or alternatively a measure of the slope of the price versus yield curve.

To illustrate Macaulay and modified duration, consider a bond with a \$1000 face value and an 8% annual coupon that matures in 10 years. If interest rates are currently 8%, then the price of the bond would be \$1000. The Macaulay duration of this bond is 7.25 and the modified duration is 6.71. To use duration to measure interest rate sensitivity, the expected change in the value of a bond is equal to the negative of the change in interest rates times the modified duration (or Macaulay duration divided by $(1 + r)$). If interest rates were to increase slightly to 8.01%, then the price of the bond would drop to \$999.33, which is a decline of 0.0671%. The predicted change in price based on duration would be the negative of the change in interest rates, −.0001, times 6.71, or −0.0671%. For such a small change in interest rates for a bond with a fixed cash flow, duration measures the interest rate sensitivity fairly accurately.

Both the Macaulay and modified duration calculations are only accurate measures of interest rate sensitivity under the following conditions:

- the yield curve is flat
- any change in interest rates is a parallel yield curve shift
- the cash flows do not change as interest rates change.

In practice, none of these conditions is likely to be met. A number of researchers have examined the effect of the first two conditions in general (see Klaffky, Ma, and Nozari [18]; Ho [15]; and Babbel, Merrill, and Panning [4]). In addition, the issue of variable cash flows has been widely recognized for specific classes of assets. Bonds with embedded options (such as call provisions) and mortgage-backed securities (where prepayments depend on the interest rate level) are examples of assets on which the expected cash flows change as interest rates change. A measure termed effective duration has been developed to express the sensitivity of the present value of the expected cash flows with respect to interest rate changes; this measure specifically reflects the fact that the cash flows can change as interest rates change (Fabozzi [11]). For assets with variable cash flows, it is appropriate to calculate the effective duration rather than the modified duration.

The liabilities of property-liability insurers also vary with interest rates, due to the correlation of interest rates with inflation. As explained by Hodes and Feldblum [16, p. 558],

> "Personal auto loss reserves are at least partially inflation sensitive. Medical payments in tort liability states, for instance, depend in part upon jury awards at the date of settlement. The jury awards, in turn, are influenced by the rate of inflation, which is correlated (at least in the long run) with interest rates."

Thus, the appropriate measure of interest rate sensitivity of the liabilities of property-liability insurers is one that reflects this interest rate-inflation relationship, or effective duration. Hodes and Feldblum [16, p. 559] indicate that "A mathematical determination of the loss reserve (effective) duration is complex." This is the task that is addressed in the remainder of this paper. The focus of this research is to develop a method to quantify the sensitivity of economic surplus to parallel shifts in the yield curve.

In order to accommodate non-parallel yield curve shifts, stochastic interest rate models must be used. This approach has been advocated for insurance applications by Tilley [28], Reitano [25], and Briys and de Varenne [5]. However, as pointed out by Litterman and Scheinkman [19], parallel shifts explain over 80% of historical yield curve movements. Although hypothetical portfolios can be constructed that show significant differences in duration values under parallel versus non-parallel yield curve shifts, these differences are likely to be far less important than the impact of variable cash flows for the asset and liability portfolios of typical property-liability insurers. Thus, this paper focuses on analyzing liability cash flows that vary with interest rate changes by recognizing that interest rate changes are impacted by changes in the inflation rate. Further research will explore the impact of stochastic interest models for both assets and liabilities for representative property-liability insurers.

Section 2 of this paper discusses the nature and relative significance of property-liability insurance company liabilities. Section 3 examines the three major liability items, and discusses the timings of cash flows for each of these items. The natures of the cash flows have important implications for the type and level of impact on liability durations of changes in interest rates. Section 4 provides a mathematical derivation of a closed-form effective duration formula in a highly simplified framework. Section 5 describes a more detailed numerical model used to estimate effective durations. Section 6 summarizes the results of empirical estimates and sensitivity tests of effective duration measures. Section 7 demonstrates the impact on asset-liability management of using modified versus effective duration measures of liabilities.

2. THE LIABILITIES OF PROPERTY-LIABILITY INSURERS

The three major balance sheet liability items of property-liability insurers are the loss reserve, the loss adjustment expense reserve, and the unearned premium reserve. As of 12/31/97, for the industry in aggregate, these components totaled 84.8% of liabilities (A.M. Best [1]). All of these three reserves are subject to change, via inflationary pressures, as interest rates change. The remaining liabilities of property-liability insurers consist primarily of expenses payable, including taxes, reinsurance, contingent commissions, and declared dividends. These cash flows are not likely to be affected by interest rate changes so the interest rate sensitivity of these liabilities can be measured by Macaulay or modified duration.[2]

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[2]Panning [24] proposes that the present value of future business be considered in the asset-liability management of an insurer. This approach, though, is contrary to accepted accounting standards, both statutory and GAAP, and introduces significant, unverifiable

Since loss and loss adjustment expense reserve estimates are based on historical development patterns, and the historical development patterns are affected by historical economic variables such as interest rates and inflation, the accuracy of the loss and loss adjustment expense reserves are, in essence, path dependent with respect to those economic variables. In other words, the level of loss and loss adjustment expense reserves calculated at any point in time will depend upon how economic variables have performed in prior years. However, it is not the accuracy of the current estimate that is of concern in measuring the effective duration, but how future cash flow patterns are influenced by future interest rate changes, which are in turn driven by changes in inflation. Reserving techniques that attempt to isolate the inflationary component from the other effects have been proposed by Butsic [6] and Taylor [27], but these approaches are not widely used currently.

Similarly, although the unearned premium reserve is calculated based on the portion of written premiums that apply to unexpired policy terms, the cash flows that will emanate from the unearned premium reserve are essentially losses and loss adjustment expenses on claims that occur after the evaluation date but during the remaining policy term. Since these events have not yet occurred, they are completely sensitive to changes in inflation affecting the value of these future losses.

An added complication to the measurement of the sensitivity of insurer assets and liabilities to interest rate changes is the statutory accounting conventions of the insurance industry. Specifically, bonds are valued on a book, or amortized, basis. Also, loss liabilities are not discounted to reflect the time value of money until payment. Thus, *statutory* valuations are often not directly

judgement factors about future premiums, losses, retention rates and pricing policies. Thus, this approach is not included here. The next step in asset-liability management for property-liability insurers should be to measure existing assets and liabilities accurately by recognizing the interest rate sensitivity of the cash flows from loss reserves, which is the focus of the rest of this paper.

affected by changes in interest rates. However, the *economic* values of these assets and liabilities *are* affected by interest rate changes. (Under GAAP accounting, bonds not held to maturity are reported at market value and therefore would also be affected by interest rates.) It is the economic values that are considered here, since these reflect the true worth of the company to its owners.

Each of the three major liability items is discussed in greater detail below. More specifically, Section 3 sets the groundwork for evaluating the impact of future interest rate changes and inflation on the liabilities of property-liability insurers.

3. THE TIMING OF PROPERTY-LIABILITY INSURER LIABILITIES

Loss Reserves

A company's aggregate loss reserve represents the total amount to be paid in the future on all claims that have already been incurred. However, a variety of different situations can exist with respect to these claims:

1. A loss reserve can reflect a claim on which the insurer is in the process of issuing a check—the claim has already been fully investigated, and the insurer has agreed to a settlement amount with the claimant. The nominal value of the claim amount will not be affected by changes in interest rates, although the present value would change slightly.
2. Alternatively, a loss reserve can represent a claim that has caused a known amount of damage to property or to a person (the medical bills are complete). Thus, the amount of the loss to the claimant is determined and will not change. However, the insurer and the claimant are still in dispute over whether the incident is covered, or over the extent of the insurer's liability for payment. Again, the nominal amount of the payment should not

change if interest rates change.[3] However, the economic value of the loss would change, since the future cash flow would be discounted by a different interest rate.

3. A third type of loss reserve is for damages that have yet to be discovered. The insurer will be liable for the loss when the claimant experiences it, but the value of the loss will only be known in the future. On an occurrence-based policy, this could apply for medical malpractice to a person who has not yet suffered the adverse consequences of an injury caused by a negligent physician (e.g., improper diagnosis, long term adverse consequences from prescribed medication, surgical errors that will lead to future complications). Or, in the case of workers compensation, if a former employee exposed to a work-related environmental hazard first manifests the ailment at some future date, the claim will be assigned to policies in effect during the period of employment. For these claims, the nominal value of the loss payment will be affected by interest rate changes to the extent that the interest rate change is correlated with inflation on the goods or services related to the cost of the claim (property damage, medical expenses). The economic value of these losses will also change with interest rates.

4. The most common type of loss reserve is for losses on which some of the damages have already been fixed in value, but the remainder has yet to be determined. In addition, the question of the extent of the insurer's liability may not have been settled. This could apply to an automobile accident involving property damage and bodily injury in which the policyholder of the insurer may be liable. The damage to the claimant's vehicle is prede-

[3]One way this could happen is if the insurer's claim settlement philosophy were to change with interest rates (e.g., if the financial condition of the insurer were to become impaired in conjunction with an interest rate change and the company had to alter its claim settlement approach).

termined. The injured person has received some medical care, but that care will continue at least up until the settlement of the claim and perhaps beyond. The nominal value of a portion of these losses, termed “fixed,” will not be affected by interest rate changes, but the remaining portion of the losses will be affected by future inflation.

Calculating the effect of inflation on *tangible* losses, such as medical expenses, wage losses, and property damage, although complicated, is relatively straightforward once the appropriate inflation indices are determined. However, quantifying the effect of inflation on the value of *intangibles* in a liability claim, termed “general damages” in a legal context, presents additional challenges. These components include items such as pain and suffering, loss of consortium, and hedonic losses. It is difficult to determine exactly how these values are established. Are they based on the value at the time of the loss or the time of the verdict in a jury trial? Is the pain and suffering of a broken arm that occurred in 1986 evaluated the same as, or less than, a similar broken arm that occurred in 1996, if both are being settled at the same time?

Due to the difficulty in putting a numerical value on an intangible such as pain and suffering, general rules of thumb arise that try to relate the pain and suffering award to the medical expenses incurred by the patient. Thus, a broken arm that generated $15,000 in medical bills is worth roughly three times as much as another broken arm that generated only $5,000 in medical bills. (This does not mean that the pain and suffering from a soft-tissue injury, such as a sore neck, which generated $15,000 in medical expenses would be worth as much as a broken arm with the same amount of medical expenses.) On this basis, the general damages on liability claims will be impacted by interest rate changes to the same extent that medical expenses are affected. However, a typical question asked by a plaintiff’s attorney in a bodily injury case is how much a member of the jury would require to be willing to undergo the same pain that the client has experienced. Since this is asked, rhetorically, near the end of the claim settlement process, conceivably the jury will implicitly adjust the value of the claim to the then-current cost of living. In this case, the entire loss reserve for general damages would be sensitive to future inflation changes.

Determining the effective duration of reserves will, therefore, depend on a model for dividing the future payments into a *fixed* component, which is not sensitive to future inflation, and an *inflation sensitive* component, which will vary with subsequent inflation. This model is developed and described in Section 4 below.

Loss Adjustment Expense Reserves

Loss adjustment expense reserves are established for future payments in a manner similar to loss reserves. These expenses will be paid over the time during which the remaining losses are settled. Loss adjustment expenses are assigned to the accident year in which the loss that generated these expenses occurred; they are assigned either directly (for allocated loss adjustment expenses) or indirectly (for unallocated loss adjustment expenses). The same approach used for determining the proportion of loss reserves that are fixed in value can be used for loss adjustment expense reserves. However, since the rate with which these expenses become fixed in value can differ from the loss itself, they may be modeled separately using different parameter values.

Loss adjustment expenses are different from loss reserves in the following respect. As an insurer generates loss adjustment expenses, such as by hiring outside adjusters, it would generally pay these expenses shortly after the work is completed. The loss adjustment expense reserve, then, represents costs that are fixed in value to a much lower degree than loss reserves. Also, the legal costs associated with defending a claim that goes to court will not be established until the very end of the loss settlement process. In addition, the allocation process for unallocated loss adjustment expenses assigns a portion of the general claim department's expenses to the accident year of the claim when the loss is paid. Thus, for loss adjustment expense reserves, few of these costs will be fixed in value when the claim occurs and a relatively high portion of the total costs will be based on the cost of living when the claim is finally settled.

Unearned Premium Reserves

Since the unearned premium reserve essentially represents exposure to losses that have not even occurred yet, this liability is fully sensitive to future inflation. The expected cash flow emanating from the unearned premium reserve will shift to the extent that any change in interest rates is correlated with inflation. If it is assumed that the insurer writes policies with terms not more than one year, then all of the claims emanating from the unearned premium reserve will occur in the next accident year. The payments on these losses will follow the claim payout pattern of the insurer, except that losses will occur approximately in the middle of the *first* half of the year (assuming annual policies written evenly throughout the year), as opposed to in the middle of the full year as would be assumed for accident year data. Thus, the duration of the unearned premium reserve at the end of a full year would be the weighted average of the time until payment of the most recent accident year, plus 3/4 of a year. For example, the unearned premium reserve as of 12/31/99 covered losses that occurred, on average, on 4/1/00. For the loss reserve for accident year 1999, the average loss would have occurred at the middle of the year, or 7/1/99. Thus, the duration of the unearned premium reserve as of 12/31/99 is 3/4 of a year more than the duration of the accident year 1999 loss reserves.

4. MATHEMATICAL MODEL OF THE EFFECTIVE DURATION OF RESERVES

In Section 5, we will present a detailed numerical model for determining effective duration. In this section, we develop a simplified mathematical model of an effective duration formula based on the assumption of proportional decay of reserve liabilities. This assumption allows for a closed-form solution for duration when inflation is recognized. This formula will provide a method to determine the general value of the effective duration of insurance liabilities, as well as a point of reference for the more detailed calculations discussed later. It should be noted that other decay patterns are possible, but most would not lead to a closed-form solution, so caution should be used when this approach is applied in practice.

In this section, it is assumed that all payments are fully sensitive to inflation. In this case, the price level at which an insurer makes a claim payment depends only upon the date of that payment. Put in the context of "fixed" costs described in the last section, here it is assumed that there are no fixed costs. This provides a framework in which a closed-form solution can be easily derived, assuming an appropriate payment pattern. The measurement of duration assuming partial fixed costs will be derived in Section 5.

Assume that the payout over time of property-liability reserves is represented by a "proportional decay" model—each year, proportion c of the beginning reserve is paid out.[4] Thus,

$$R_t = (1-c)R_{t-1}, \tag{4.1}$$

where

R_t = the (correct) nominal reserve at time t,

c = the (constant) annual payout ratio, and

r = the relevant interest rate.

[4]Theoretically, this assumes that payouts are made forever, although after some years they become negligible in size. Finite-length payout patterns are considered in Section 5.

Under this assumption, the present value of the initial reserve is expressed as

$$\mathrm{PV}(R_0) = \sum_{t=1}^{\infty} \frac{(1-c)^{t-1} c R_0}{(1+r)^t} = \frac{cR_0}{1-c} \sum_{t=1}^{\infty} \left(\frac{1-c}{1+r}\right)^t = \frac{cR_0}{r+c}, \tag{4.2}$$

where the final form of the equation is derived from the formula for an infinite geometric progression.[5] Now, we can derive an expression for the Macaulay duration by multiplying the numerator of each term in the present value calculation by t, and dividing the new summation by the original present value:

$$\text{Macaulay Duration} = D_0 = \frac{\displaystyle\sum_{t=1}^{\infty} \frac{(1-c)^{t-1} c R_0 t}{(1+r)^t}}{\mathrm{PV}(R_0)}. \tag{4.3}$$

By again using the properties of infinite geometric progressions, the numerator of the Macaulay duration formula reduces to:

$$\frac{cR_0(1+r)}{(r+c)^2}. \tag{4.4}$$

Dividing by the previous expression for $\mathrm{PV}(R_0)$, the Macaulay duration is

$$D_0 = \frac{1+r}{r+c}. \tag{4.5}$$

Since the modified duration is the Macaulay duration divided by $(1+r)$, we have

$$\text{Modified Duration} = \mathrm{MD}_0 = \frac{1}{r+c}. \tag{4.6}$$

In order to determine the *effective* duration of property-liability insurer liabilities, we must calculate the present value of those liabilities in three different ways: with the original interest rate, with an increased interest rate, and with a decreased

[5]For $0 < x < 1$, the value of $x + x^2 + x^3 + \cdots = x/(1-x)$.

interest rate. Under this approach, after calculating the present value assuming the original interest rate, we assume that the interest rate increases (e.g., by 100 basis points), and then that the interest rate decreases (e.g., by 100 basis points). The effective duration is then calculated as:

$$\text{Effective Duration} = \text{ED}_0 = \frac{\text{PV}_{-} - \text{PV}_{+}}{2\text{PV}_0(\Delta r)}, \qquad (4.7)$$

where

PV_{-} = the present value of the expected cash flows if interest rates decline by Δr,

PV_{+} = the present value of the expected cash flows if interest rates increase by Δr, and

PV_0 = the initial present value of the expected cash flows.

The key in calculating the effective duration is to account for the impact of hypothetical changes in the interest rate on the future cash flows emanating from the liability items. For property-liability reserves, the primary impact on cash flows of a change in interest rates is due to the change in the inflation rate: since interest rates are correlated with inflation, and inflation increases future nominal claim payments, changes in interest rates will affect the level of future cash outflows, and thus the present value of those outflows. Therefore, in order to calculate the effective duration, we need to adjust the formulas above to reflect this inflationary impact.

Define the following additional variables:

$r_{+\text{ or }-} = r +/- \Delta r$ = the increased or decreased interest rate, and

$i_{+\text{ or }-}$ = the inflationary adjustment after the change in interest rate.

The inflationary adjustment contemplates the correlation between changes in interest rates and inflation (actually, not just overall inflation, but claim inflation for the specific type of insurance at issue).

We can now adjust the initial present value equation introduced in this section in preparation for calculating the effective duration:

$$\mathrm{PV}_+(R_0) = \sum_{t=1}^{\infty} \frac{(1-c)^{t-1} c R_0 (1+i_+)^t}{(1+r_+)^t}$$

$$= \frac{cR_0}{1-c} \sum_{t=1}^{\infty} \left(\frac{(1-c)(1+i_+)}{1+r_+} \right)^t = \frac{cR_0(1+i_+)}{r_+ + c + ci_+ - i_+}. \qquad (4.8)$$

A similar equation applies for the present value of reserves under the assumption of an interest rate decrease. Thus, we derive the following formula for the effective duration:

$$\mathrm{ED}_0 = \frac{r+c}{2\Delta r} \left(\frac{1+i_-}{r_- + c + ci_- - i_-} - \frac{1+i_+}{r_+ + c + ci_+ - i_+} \right). \qquad (4.9)$$

These formulas can be used to indicate the relative magnitudes of the various duration measures. For example, assume the following illustrative parameter values: $r = 0.05$, $\Delta r = 0.01$, $c = 0.40$, and the correlation between interest rate and inflation changes is 0.50 (thus, $i_+ = 0.005$, and $i_- = -0.005$). Given these values, the formulas above provide the following duration measures: $D_0 = 2.333$, $\mathrm{MD}_0 = 2.222$, and $\mathrm{ED}_0 = 1.056$. This example illustrates the potentially significant differences between effective duration and the more common, traditional measures of duration.

5. MODELING THE INTEREST RATE SENSITIVITY OF LOSS AND LAE RESERVES

One of the difficulties in measuring the interest rate sensitivity of liabilities is the need for extensive data. What information is available, either publicly or within the company, to determine the impact of interest rate changes on the cash flows of losses? For the loss and loss adjustment expense reserve, the expected nominal cost of these amounts at the end of each year are reported in aggregate, by accident year, by line of business, in the Annual Statement. Internally, actuaries have access to this same information on a more frequent and more detailed basis. Although the expected payment dates for future payments are not generally recorded, the actual payments made in each historical year—categorized by accident year (or month) and by line of business (or finer breakdown)—are available. This allows a comparison of the actual payments with the expected payments and permits the generation of a profile of when the aggregate loss reserves are likely to be paid in the future. However, there is no public information, and frequently not even any information within a company, on when the value of an unpaid loss is set in value. To obtain such information, claim files would need to record the date when each expenditure relating to a claim is made by the claimant, not just when the insurer pays the claim. Since few, if any, insurers currently maintain such detail, the only way to obtain this information is to perform a special study, as detailed in the next section of the paper. Given the lack of data to measure this effect precisely, this relationship needs to be modeled.

For this model, the following assumptions are made. At the time the loss occurs, proportion k of the eventual cost of the claim is "determined" (i.e., a proportion of the future cost is "fixed" and no longer open to change from interest rate and inflationary changes). In addition, proportion m of the loss will not be determined until the time the claim is settled. Examples of loss costs that will go into k are medical treatment sought immediately after the loss occurs, the wage loss component of a bodily injury claim, and property damage. Examples of loss costs that will go into m are medical evaluations that are done immediately prior to determining the settlement offer, general damages to the extent they are based on the cost of living at the time of settlement, and loss adjustment expenses connected with settling the claim.

The remaining $(1-k-m)$ portion of the expenses are modeled in three ways, to allow for differing rates at which the claim values could become fixed: these expenses could be fixed in value linearly over the time period from loss to settlement, or in a manner that would represent either a concave function or a convex function. Figure 1 illustrates the three different functions proposed for the proportion of loss reserves that are fixed in value, and therefore not subject to inflation, over time.

A representative function that displays these attributes is:

$$f(t) = k + \{(1-k-m)(t/T)^n\}, \qquad (5.1)$$

where

$f(t)$ = the proportion of ultimate paid claims "fixed" at time t,

k = the proportion of the claim that is fixed in value immediately,

m = the proportion of the claim that is not fixed in value until the claim is settled,

$n = 1$ for the linear case,

$n < 1$ for the concave case,

$n > 1$ for the convex case, and

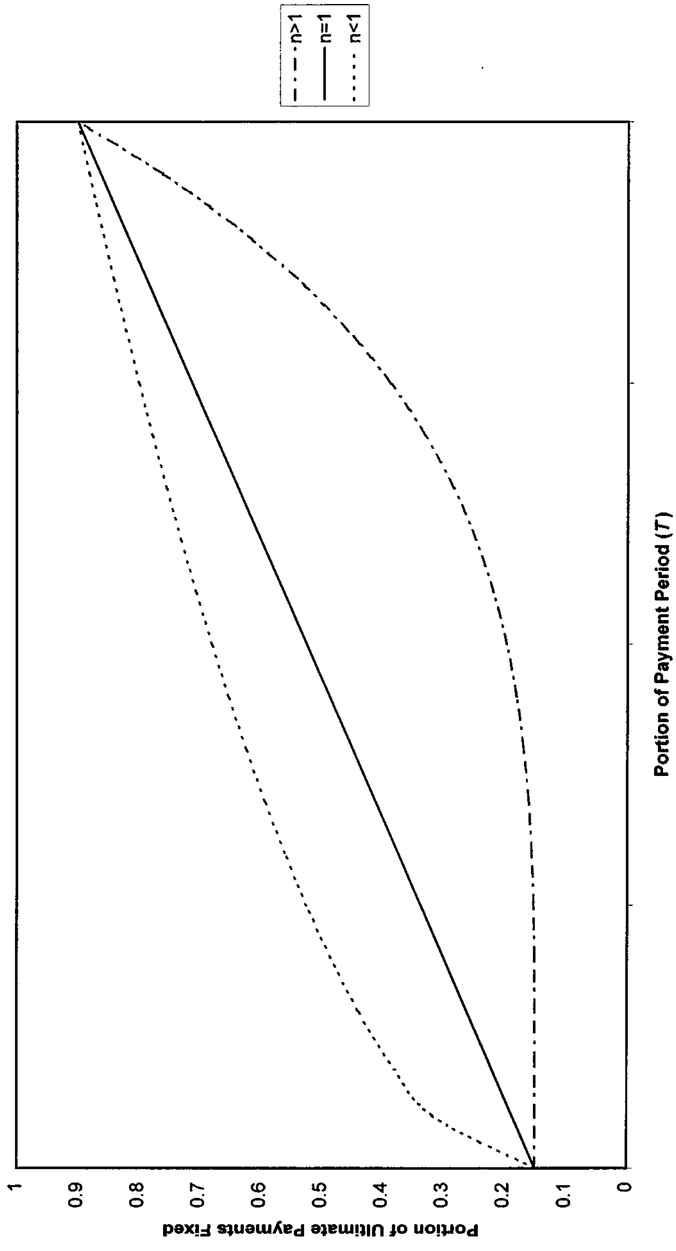
T = the time at which the claim is fully and completely settled.

For example, assume an insured causes an automobile accident in the middle of 1997, and the victim requires immediate

FIGURE 1

FORMULA FOR "FIXED" COSTS

$f(t) = k + [(1 - k - m)(t/T)^n]$



medical attention. This is the k portion of the claim that is predetermined immediately; assume that it represents 15% of the total cost of the claim. Further, assume that m is zero. After the accident, the victim receives medical care on an ongoing basis until the claim is eventually settled in the middle of the year 2000. These continuing care expenses will be influenced by inflation. At the end of 1997, half of a year of continuing expenses has been obtained. The total length of time before the claim will be settled is three years (2000–1997). Thus, for the linear case ($n = 1$),

$$f(0.5) = 0.15 + \{(1 - 0.15)(0.5/3)^1\}.$$

In this case, $f(0.5) = 0.292$, meaning that at the end of 1997, 29.2% of the loss reserve for this 1997 accident year claim is fixed in value, with the remainder subject to future inflation.

This approach can be applied whether a particular claim has been reported or whether it is a component of IBNR. As long as the claim has been incurred, then some of portion of the loss is fixed in value, some portion will not be fixed in value until the claim is settled, and the remaining portion is becoming fixed in value over the intervening time. For example, even though the insurer does not know of a particular injury on which it will be liable, the victim is likely to have received medical treatment at the time of the loss.

6. DURATION MEASURES FOR INSURER LIABILITIES

Empirical Estimates

In order to implement our model of effective duration, values of several parameters must be determined:

- Loss payout pattern
- Economic parameters
    - Interest rate
    - Correlation between interest and inflation

  - Growth rate of insurance writings (g)
- Cost determination parameters
  - k (the proportion of claim value that is fixed immediately)
  - m (the proportion of claim value that is not fixed until the claim is settled)
  - n (the shape parameter of the fixed-claim-proportion function).

Each of these parameter values is discussed in greater detail below.

A key component to determining effective duration is identifying the future cash flows. For property-liability insurance, this involves determining the timing of future loss payments as loss reserves run off. For a particular corporate application of this effective duration procedure, the company's historical loss payment information by line of business can be used as a basis for estimating future claim payouts. For purposes of this paper, we used aggregate industry information available from A.M. Best [1]. Due to their size and importance, two lines of business were used in our analysis: private passenger auto liability (PPAL) and workers compensation (WC). An additional advantage of using these two lines of business is that their cash flows have different timing characteristics: WC pays out more slowly, in general, than PPAL. This distinction allows us to test the potential impact of calculating effective duration under different payout environments.

Aggregate industry payout data for PPAL and WC were each used in two different ways. First, the raw empirical data were used. Empirical loss payment patterns were generated from an actuarial analysis of historical calendar and accident year payment data. The second approach was to fit statistical distributions

TABLE 1

CUMULATIVE PROPORTION OF ULTIMATE ACCIDENT YEAR LOSSES PAID
(Based on Age After Beginning of Accident Year)

	PPA Liability		Workers Compensation	
Age (Years)	Empirical	Smoothed	Empirical	Smoothed
1	.386	.398	.225	.362
2	.701	.672	.486	.496
3	.843	.827	.635	.588
4	.919	.909	.727	.658
5	.958	.953	.785	.713
6	.977	.976	.822	.757
7	.986	.988	.847	.793
8	.991	.994	.867	.823
9	.994	.997	.880	.848
10	.995	.998	.891	.869

to the raw empirical payment patterns.[6] For both PPAL and WC, a gamma distribution was used for illustrative purposes as the "smoothed alternative" to the raw empirical payment pattern.

The loss payment patterns used in our tests were as shown in Table 1. This table reflects the payout patterns through ten years, which is the timeframe in which aggregate industry data is available in any particular edition of A.M. Best's *Aggregates and Averages*. For our purposes, the WC patterns are extrapolated out to 30 years, and the PPAL patterns to 15 (empirical) and 19 (smoothed) years.

The selected economic parameters are based largely on current and historical economic relationships. A "base case" 5% interest rate was selected in accordance with the level of short-term government rates in effect during the late 1990s. A 40% relationship between interest rates and claim inflation was selected

[6] In this case, the curve fitting was done using software called "BestFit" (a product of Palisade Corporation), which provides best-fit parameter values to sample data for a variety of theoretical distributions.

based on the historical relationship between these two economic variables.[7] Finally, a 10% growth rate (g) is assumed, based on judgment. This parameter reflects the fact that a typical insurance company carries reserves for a number of different accident years. The distribution of reserves by accident year is a function of the growth rate in ultimate accident year incurred losses, and the runoff patterns. The 10% growth assumption assumes that ultimate accident year losses are growing at 10% per year, which reflects the growth in both the number of policies written and claim cost inflation.

The selection of cost determination parameters is very difficult. Publicly available loss development information (e.g., Best's *Aggregates and Averages* or the NAIC data tapes) includes loss payments made each year, by accident year, on a by-line basis. This is not sufficient to determine the fixed and variable portions of loss reserves. Even within a company, the data needed to determine these relationships is not generally maintained in an easily accessible format. To address this issue, several large insurers were approached and asked to participate in a study to help estimate the parameters used in this model. These companies were asked to report information on a small sample of claims that were settled several years after the date of loss. None of the companies could provide an answer to the question of when the general damages portion of a claim is fixed in value. It appears that there is simply too much uncertainty about the process used to establish this figure to know if it is based on costs at the time of the loss, the time of the settlement, or some interim time.

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[7] The selected relationship is based upon the long-term (1926 through 1995) correlation coefficient between U.S. Treasury Bill returns and the Consumer Price Index (CPI). Correlations and regressions were also estimated over other time periods, and between Treasury bill returns and a variety of inflation indices: CPI, private passenger auto bodily injury liability claim inflation, auto physical damage claim inflation, and other line of business inflation series. The correlation and regression coefficients varied greatly—by both magnitude and statistical significance—according to the type of inflation and the period being tested. The 0.40 relationship in the text is used for illustrative purposes only; the value used in any specific effective duration analysis would require further investigation and would depend upon the particular application. (Insurance claim inflation data were taken from Masterson [21]; T-bill and CPI data were taken from Ibbotson [17].)

One company did provide especially detailed reports on a sample of auto liability insurance claims. These reports showed all the medical, wage loss, and property damage costs associated with the claims, the date any of these expenses were incurred by the claimant, and the total claim payment made by the company. For most of these cases, the final claim paid exceeded the total costs the claimant had incurred. This is expected, since the itemized expenses represented special damages, and the final payment would also include the intangible general damages. However, there was one case in which the policyholder was not fully liable for the claim and the total payment was less than the plaintiff's expenses.

The general pattern of the expenses was as follows. At the time of the loss, the plaintiff incurred significant medical expenses, property damage, and wage loss. After the initial medical treatment, the plaintiff incurred some continuing medical expenses, either for additional treatment or for rehabilitation. These expenses most frequently ended before the claim was finally settled. This would suggest that the function for the value of the fixed claim is concave ($n < 1$), at least for the special damages portion of the claim.

The results of this sample indicate that a more extensive and detailed examination of this process would be very helpful in determining the appropriate parameters for measuring effective duration. For purposes of getting initial empirical estimates of effective duration, we have chosen to begin with $k = 0.15$, $m = 0.10$, and $n = 1.0$. These values will be varied in the next subsection, in order to determine the potential sensitivity of effective duration results to the magnitude of these parameters.

Based on these selected parameters, a Δr of 100 basis points, and using a spreadsheet model to implement the calculations, the effective duration indications in Table 2 were derived. The essential finding is that effective duration measures—which properly account for the inflationary impact of interest rate changes on

TABLE 2

SUMMARY OF DURATION MEASURES FOR LOSS RESERVES
(Based on "Base Case" Parameter Assumptions)

	PPA Liability		Workers Compensation	
	Empirical	Smoothed	Empirical	Smoothed
Macaulay Duration	1.516	1.511	4.485	4.660
Modified Duration	1.444	1.439	4.271	4.438
Effective Duration	1.089	1.085	3.158	3.285
Convexity	5.753	5.214	50.771	45.060
Effective Convexity	1.978	1.807	16.038	14.383

future loss reserve payments—are approximately 25% below their modified duration counterparts. This relationship appears to be consistent, based on the illustrative PPAL and WC tests above, regardless of line of business, or whether empirical or smoothed payout patterns are utilized.

In addition to duration, another quantity that is important to asset-liability management—convexity—is also displayed in Table 2. Just as the impact of inflation on future cash flows must be measured via *effective* duration, the second derivative of the price/interest rate relationship is appropriately measured by *effective* convexity in an inflationary environment. The results in Table 2 show that there is a significant difference between the traditional and effective measures of convexity. The effective convexity formula used to derive the values in Table 2 was:

$$\text{Effective convexity} = \frac{PV_{-} + PV_{+} - 2PV_0}{PV_0(\Delta r)^2}. \qquad (6.1)$$

Sensitivity of Effective Duration to Parameter Values

As indicated above, effective duration measures can provide significantly different evaluations of property-liability insurer interest rate sensitivity than the traditional modified duration measures. Use of the appropriate effective duration measure is therefore critical when utilizing asset-liability management techniques. Similarly, it is important to have an understanding of which parameter values have the greatest impact on the magnitude of the effective duration calculation. In Table 3, various parameters have been changed—one at a time—to demonstrate the level of sensitivity of effective duration values with respect to those parameters. (Since the empirical and smoothed pattern results were so similar above, to promote clarity only the empirical patterns were used for each line of business.)

The main result from Table 3 is the significant sensitivity of effective duration to the interest rate-inflation relationship. In particular, this parameter expresses how much inflationary pressure is associated with a 100 basis point change in interest rates. If there is no correlation between interest rates and inflation, the modified duration and effective duration are the same. If the correlation is as high as 80%, the effective duration is approximately one-half the modified duration. The relationship between changes in interest rates and changes in inflation—both CPI and line of business claim inflation—has historically been very volatile. Our results suggest that additional efforts to determine reasonable values for this relationship parameter would be worthwhile.

Another observation from the table is that the results are not overly sensitive to some of the cost determination parameters. Given the difficulties mentioned above of determining values for the parameters, this is a somewhat comforting finding. For companies undertaking asset-liability management, simply using effective duration measures of their liabilities is more important than having the exact parameter values. However, these companies should be encouraged to collect data that will allow them to monitor the sensitivity of their results to different cost determination function specifications.

TABLE 3

ANALYSIS OF THE SENSITIVITY OF EFFECTIVE DURATION MEASURES OF LOSS RESERVES
(Based on Single-Parameter Changes From "Base Case" Values*)

		PPAL Empirical	WC Empirical
Macaulay Duration**		1.516	4.485
Modified Duration**		1.444	4.271
Effective Duration			
Base Case		1.089	3.158
Inflation-Interest Relationship:			
	80%	0.733	2.036
	60%	0.911	2.596
	40%	1.089	3.158
	20%	1.267	3.721
	0%	1.445	4.286
$k =$	0.25	1.128	3.284
	0.20	1.108	3.221
	0.15	1.089	3.158
	0.10	1.069	3.095
	0.05	1.049	3.032
$m =$	0.20	1.067	3.104
	0.15	1.078	3.131
	0.10	1.089	3.158
	0.05	1.099	3.185
	0.00	1.110	3.212
$n =$	1.40	1.045	3.040
	1.20	1.065	3.092
	1.00	1.089	3.158
	0.80	1.120	3.245
	0.60	1.160	3.362
$g =$	0.20	1.070	2.849
	0.15	1.079	2.985
	0.10	1.089	3.158
	0.05	1.101	3.367
	0.00	1.116	3.589

*Base case values are: $k = 0.15$, $m = 0.10$, $n = 1.00$, $g = 0.10$ (where g represents the insurer's growth rate), a 5% interest rate, and a 40% relationship between interest rate and inflation movements.
**These duration figures reflect base case parameter values. When parameter g is changed according to the range above, Macaulay and modified durations also change slightly:

$$PPAL: \quad D_O = 1.501 \quad \text{to} \quad 1.540, \qquad \text{and} \qquad \text{MD}_O = 1.429 \quad \text{to} \quad 1.466$$

$$WC: \quad D_O = 4.128 \quad \text{to} \quad 4.910, \qquad \text{and} \qquad \text{MD}_O = 3.932 \quad \text{to} \quad 4.676$$

7. USE OF EFFECTIVE DURATION IN ASSET-LIABILITY MANAGEMENT

In previous sections, the deficiencies of traditional measures of duration in an inflationary world were identified, and an alternative measure—effective duration—was described. In this section, the impact of using effective, as opposed to modified, duration on a company's asset-liability management process is illustrated. The example used is a hypothetical workers compensation insurer; it is assumed that this company has asset and liability values which are related in a manner consistent with aggregate industry balance sheet figures.

The effective duration analysis in the prior section concentrates on loss and allocated loss adjustment expense reserves and runoffs. A complete asset-liability management analysis would also consider unallocated loss adjustment expenses and unearned premium reserves (the timings of which are described in Section 3 of this paper). For simplicity, and because they represent a relatively small part of an insurer's liabilities, unallocated loss adjustment expenses are considered together with losses and ALAE in the illustrative example in this section. However, the reasonableness of this assumption would need to be evaluated in any specific corporate application of asset-liability management.

The duration of the unearned premium reserve was described in Section 3. The one adjustment that must be made with respect to asset-liability management is to only consider the portion of the unearned premium reserve (UPR) which is associated with future losses and loss adjustment expenses—it is only this portion which represents a liability for future cash flows which may be impacted by inflation. The duration for this portion of the UPR is calculated by determining the duration of the loss and LAE reserve for the most recent accident year, and adding 0.75. The other portion of the UPR—the "equity" in the UPR—represents prepaid expenses associated with prior writings of insurance policies, and is essentially an accounting construct which is unrelated to *future* cash flows. Thus, this portion of the UPR is not considered in the following illustration.

For illustrative purposes, all other liability items on the insurer's balance sheet are considered to have a Macaulay duration of 1.0 (and thus, at an interest rate of 5%, a modified duration of 0.952).

The duration of an insurer's surplus, D_S, is as follows (Staking and Babbel [26]):

$$D_S S = D_A A - D_L L, \tag{7.1}$$

where

S = surplus,

D = duration,

A = assets, and

L = liabilities.

In order to immunize its surplus (setting $D_s = 0$) from interest rate risk,[8] an insurer needs to set the duration of its assets as follows:

$$D_A = D_L \frac{L}{A}. \tag{7.2}$$

Thus, the appropriate determination of the duration of liabilities is critical for asset-liability management.

Based on the aggregate industry balance sheet figures for WC insurers reported in A.M. Best [1], Table 4 shows the liability distribution for an insurer with assets of $1 billion.[9] The liability

[8]In some cases, management would prefer to accept interest rate risk if an adequate return were provided for taking this risk. This alternative approach is to balance the generally higher returns from a longer term portfolio of assets against the risk of this position. Regardless of whether an insurer is attempting to immunize its portfolio or balance the risk-return trade-off, an accurate measure of duration for assets and liabilities is needed.

[9]Workers compensation insurers tend to have a slightly higher proportion of their liabilities in loss and loss adjustment expenses, and a much lower proportion in the unearned premium reserve, than other insurers. In applications of this technique, the actual values for these liabilities and the actual relationship between assets and liabilities for the company should be used.

TABLE 4

EXAMPLE OF ASSET-LIABILITY MANAGEMENT FOR A HYPOTHETICAL WORKERS COMPENSATION INSURER
($ figures are in millions)

	Dollar Value	Modified Duration	Effective Duration
Loss and LAE Reserves	590	4.271	3.158
UPR (portion for losses and LAE only)	30	3.621	1.325
Other Liabilities	90	0.952	0.952
Total Liabilities	710	3.823	2.801
Total Assets	1,000		
Indicated Asset Duration to Immunize Surplus:		2.714	1.989

durations were calculated as described above and in Section 6 based on the empirical WC payout pattern. The resulting overall (value-weighted) liability modified duration is 3.823, while the effective duration of total liabilities is 2.801.

If the insurer wanted to immunize surplus from interest rate swings based on modified duration, the duration of assets would need to be 2.714. However, based on effective duration, the duration of assets should be 1.989. An insurer that attempted to immunize its exposure to interest rate risk by matching the duration of assets with the modified duration of liabilities, instead of effective duration, would find that it still would be exposed to interest rate risk. Based on these values, the insurer would have a duration of surplus of 2.501: each 1 percentage point increase in the interest rate would decrease surplus by 2.501 percent (where surplus here is defined as the economic value of statutory surplus plus the equity in the unearned premium reserve).

8. CONCLUSION AND FUTURE RESEARCH

This paper has demonstrated a method for determining the effective duration and convexity of property-liability insurer liabilities, and has provided some general estimates of these values. Based on the results derived, it appears that there can be significant differences between the traditional measures of duration (i.e., Macaulay and modified duration) and effective duration. Of these measures, only effective duration is capable of properly accounting for the impact of inflationary pressures on liability cash flows that are associated with potential changes in interest rates. This means that effective duration is the appropriate tool for measuring the sensitivity of the liabilities of property-liability insurers to interest rates when performing asset-liability management. Use of the wrong duration measure can lead to an unintended mismatch of assets and liabilities, and an unwanted exposure to interest rate risk.

In addition to inflation, interest rate changes may also be correlated with other financial and economic variables. For example, a decrease in interest rates is often—on average—associated with an increase in stock prices (since the discount rate on future dividends and capital gains is lower). Similarly, changes in interest rates in the U.S. may certainly impact international financial relationships. To the extent to which these other variables are factors in a jury's damage award considerations, they must also be contemplated in an effective duration framework. For example, if the stock market has increased in value significantly between the time of an accident and the final jury verdict, a well-structured comment from the plaintiff's attorney to the jury may lead to a higher award on the grounds that the plaintiff could have invested the monies lucratively if they had been available at the time of the accident.[10] These types of issues are beyond the analytical scope of this paper, and are left for future research.

In this paper, we have approached the measurement of effective duration from the standpoint of a shift in a constant interest rate. Future research should examine the impact of a stochastic

[10]The appropriate analytical framework in this case may involve option pricing theory—it is possible that the jury award may depend on the maximization of alternatives involving such considerations as inflationary environment, stock market performance, etc.

interest rate model on effective duration and asset-liability management. Interesting and important work in the non-insurance literature on effective duration, yield curves, and stochastic interest rates (e.g., Babbel, Merrill, and Panning [4]) has significant future applicability to the issues addressed in this paper. In addition, stochastic interest rate models are beginning to appear in the property-liability insurance industry, especially within the context of dynamic financial analysis (D'Arcy and Gorvett, et al [9 and 10]). DFA models can connect underwriting experience, as well as loss development, to stochastically generated interest rate paths. In analyses in which assets are valued according to a stochastic rate assumption, it is appropriate to value liabilities on the same basis. These will be an important areas for future research.

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ADDRESS TO NEW MEMBERS—NOVEMBER 13, 2000

CHARLES C. HEWITT JR.

In preparing for this talk, I asked myself the question, "Is it realistic for a person my age and so long away from actuarial work to be giving advice to the successful young people who are here this morning?"

I think not!

An axiom by which many of us "older statesmen" live is: "If you have a question about your personal computer or the Internet, call the grandchildren." However, there is a subject about which I have come to know a great deal: "What is it like to grow old?"

Growing Old

Recently my wife has been making a *novena*. For the benefit of non-Catholics, a novena is a series of nine church services where each attendee has an *intention.* The intention is a wish or prayer that something will or will not happen.

Coming out of the service one day she recognized an extremely elderly man and nodded to him. They struck up a conversation and at one point she said to him, "What is your intention for this novena?"

He replied as follows: "Recently I've heard that medical science may reach the time in this century when people will live to be two hundred years old! I am asking the Lord, 'Please do not let me live beyond one hundred thirty!'"

Growing old is a boring subject to many. However, if you're going to live to 200 it's still important. I'll try to touch on it lightly.

First the bad news. Any knowledgeable physician will tell you that the aging process, in many ways, reverses the growth process. Aptitudes that you acquired as a child growing up will weaken or disappear—both the physical and the mental and even the emotional. I'll give you an example from my own experience.

In earlier years I was blessed with "instant recall." So much so that I actually qualified for and appeared on one of the ill-fated TV quiz shows in the late fifties. And thereby hangs a tale.

When I was much younger and I did have instant recall, my mother-in-law persuaded me to go on the quiz show *21*. This is the one in which a contestant, Charles Van Doren, was fed the answers and, when caught, lied about it to a grand jury and was convicted of perjury as a result.

My part in all of this was insignificant. I lost in my very first turn to a woman named Elfrida Von Nardroff.

Shortly afterward I was sent as a consultant by Bowles, Andrews and Towne (a predecessor to Tillinghast) to Saudi Arabia. The Arabian-American Oil Company had requested a study of workmen's compensation rates there.

While I was in Saudi Arabia, the quiz-show scandal broke. (As some of you may remember, a movie was made about this whole matter.)

The *New York Daily News* decided to do a follow-up with all of the contestants to see who else, if anyone, had been prepped with answers before the show. My wife took the phone call from the *Daily News*, and explained that I was not home because my firm had sent me to Saudi Arabia on a consulting job.

The following day the *Daily News* reported in essense that "we tried to reach Mr. Hewitt about this scandal but he had already left the country!"

Today I find that the "iters"—passages in the brain—do not always respond as quickly as they used to. I'm not talking about memory loss; my memory will ultimately respond. The bit of memory is still there—the forgetfulness is only temporary. In Florida we call this having a "senior moment" but it is very common among older persons. Furthermore, I don't listen "as fast" as I used to, and I'm not referring to hearing loss. That happens too.

Now the good news, growing old is not all bad. First there is something called grandchildren and even great-grandchildren, who somehow manage to bring one pure joy.

Next there is the ability to look at one's grown-up children—adult sons and daughters—and realize that the thing most responsible for their well-being and self-assurance is the love that they received from their mother. Do you realize what a powerful force love is in this world of ours?

And, next to love, the most important thing we give to our children is discipline. And I'm not talking about physical punishment. Discipline, properly exercised, promotes self-control. And once one achieves self-control, self-assurance follows. Don't take my word for it. This comes straight from the modern Dr. Spock, Dr. Berry Brazelton, who, incidentally, happens to be a college classmate.

Enough about "old people"!

Pursuit of Excellence

Instead of advice, I'm going to issue to you two challenges. The first challenge has to do with the "pursuit of excellence" and is chosen because of the fact that our principal speaker this morning is George Will.

George knows about excellence and the pursuit thereof. In fact, he has written a book about it entitled *Men at Work*. The book is about the game of baseball and those who play it or coach.

Now, the pursuit of excellence has a handmaiden, and it is PREPARATION, PREPARATION, PREPARATION! Ask any successful courtroom lawyer what the secret of trial work is and he will tell you it is "preparation". Ask any successful football coach how he expects to win next week's game, and he will give you the same answer. There is no substitute for preparation unless it is raw talent and, if you are blessed with raw talent, so be it!

But why am I telling you, today's honorees, this? It was preparation for exams that got you here today, unless you are one who is blessed with raw talent. If so, will you please stand and be recognized.

Excellence is hard to define, but like the Supreme Court justice said about pornography, you'll know it when you see it. Every so often I see a tape of the great athlete, Wilma Rudolph, winning the Olympic 100-meter dash by more than ten meters. The hair on the back of my neck stands on end every time!

If you're looking for nonsports-related evidence, try Sir Alec Guiness in *The Bridge on the River Kwai* or the less familiar *Tunes of Glory*. Or, how about Sir John Gielgud in *anything*!

In recent years we have been blessed with people like Michael Jordan and Tiger Woods. Are you thinking to yourself, "raw talent"? Maybe so, but then why does Tiger spend two hours on the practice range after shooting a 66? Or, why does Pete Sampras wonder how Michael Jordan got himself up emotionally for every game?

My first challenge today is that you continue to pursue excellence in your actuarial careers. That means, among other things, not accepting an important assignment without the determination and the will to prepare yourselves fully for the task.

Planting of Trees (An Allegory)

My second challenge to you graduates is in the form of an allegory having to do with the planting of trees.

There is an old Greek proverb that says: "A society grows great when old men plant trees in whose shade they will never sit."

I will repeat: "A society grows great when old men plant trees in whose shade they will never sit."

Now think about that one in a world in which the business executive asks, "What have you done for me lately?" Or where the media deals in "thirty second sound bites." Or where politicians govern by yesterday's poll results. Or where the general public asks to be entertained rather than to be informed. In the early part of the twentieth century, the British historian, Arnold Toynbee, suggested that we are in the declining period of our civilization. He pointed to the similarities between our own times and ancient Rome with its circuses with gladiatorial combats, Christians being thrown to the lions, and other forms of what was then entertainment by public spectacle. What would Toynbee have thought about World Championship Wrestling or Jerry Springer?

I repeat: "A society grows great when old men plant trees in whose shade they will never sit."

Recently I found this proverb in the valedictory address given in May of this year by a graduating senior at a well-known liberal arts university. I want to share a small portion of the valedictorian's effort with you.

Now, in a sense, what we have here today is a graduation so, with a stretch, I will be your valedictorian.

As I speak from my platform you are seated on a large lawn in front of the administration building. I call your attention to our surroundings—handsome academic buildings and beautiful old trees. I point to the dean's house.

Behind that house over there are two of the oldest trees on campus. They were planted in 1766 to commemorate the repeal by the British of the Stamp Act.

I was so taken with the beauty of our campus that I became interested in the care of the trees on campus and investigated the "tree situation." I found that maintenance required a staff of 20 groundsworkers and two full-time arborists. I worried about what special steps were taken in times of drought and was told that, because the older trees have deeper roots and can draw water from well below the surface, it was the young trees that were nurtured by whatever water was available. There we now have an allegory within an allegory!

And now comes the exhortation to your graduates.

With all that you have been given, now is the time to rise and to give of yourselves. It is only fitting at the moment of your glory to thank all of those who have "nurtured" you throughout your whole lives—your parents, for their endless love and inspiration; your teachers (at every level of your education), for their willingness to cultivate your minds; your spouses or other special persons who have encouraged you in your dreams and in your preparation for this day by forgiving the many hours spent away from them in that preparation; and finally, the "old men" who have gone before and who are not here any longer to join you and me in the shade in which we are living.

Now, I break the spell and remove my imaginary mortarboard and tell you that the young person who gave that address was an engineering major. Amazing!

So my second challenge is this: That some time during your careers or your lifetimes, you will stop and, figuratively speaking, plant a tree under whose shade you will never sit.

So I congratulate all of you and your families upon the accomplishment being recognized today.

I can assure you that for the balance of your lives you will look back on this accomplishment as one for which you will always be proud!

Thank you for allowing an old actuary to be part of this celebration.

PRESIDENTIAL ADDRESS—NOVEMBER 13, 2000

THE MAIN THING

ALICE H. GANNON

"In any endeavor, the main thing is to keep the main thing the main thing. But you can't keep the main thing the main thing if you don't know what the main thing is." Steve Goldberg, my friend and colleague of 25 years and my boss for the last 11 years, shared these words of wisdom with me shortly after he heard them at the opening ceremony of Black History Month at USAA many years ago. They were the theme of the speech by the featured speaker, an educator from Colorado, whose name is unfortunately long forgotten.

I believe these words to be true, both with regard to the focus that should be given to one's primary goal or objective and to the importance of knowing what that goal or objective is. While it is possible for undirected activity to accidentally lead to a desired result, we are much more likely to achieve what we want to achieve when we know what that is and work deliberately toward it.

I began my preparation for this, my presidential address, by reading a number of CAS documents. I read the original and current CAS constitution and bylaws. I read for the second dozenth time this year our current strategic plan. I read the three historical accounts of the CAS: "The First 25 Years" written by Francis Perryman; "The First 50 Years" written by Dudley Pruitt; and "The First 75 Years" written by Stanley Hughey. Following the tradition of many CAS presidents, I read about three dozen or so presidential addresses from prior years. I then reflected on what I had read in light of my experiences serving as president of the CAS—a job, which, at least for someone as inefficient as I am, entails spending part of every day, seven days a week, working on or worrying about some CAS activity or issue.

I came to three major conclusions about the CAS. (Actually I came to a lot of conclusions about the CAS but only three that I want to share with you today.) The first two concern our past and the third our future. My first conclusion is that the CAS has enjoyed a remarkable record of success throughout our 86-year history. I will not take the time to provide even a partial list of the accomplishments that lead me to that conclusion. Any of you who doubt it should read the documents I named, and I am confident that your conclusion will be the same as mine. In fact, I would encourage all members of the CAS to try to find time to read the historical accounts of the CAS. You will be impressed either once again or for the first time by how rich a legacy we have inherited. All of us can take great pride in being members of an organization that has made so many contributions to actuarial science, to the actuarial profession, to employers and clients of actuaries, and to the public. We indeed have a heritage worth celebrating!

My second conclusion is that an essential contributor to the CAS's success has been that throughout those 86 years, the CAS has maintained an understanding of its purpose and has kept that purpose largely at the forefront of its planning and its activities. There has always been a statement of purpose in our constitution. What is really important, however, is that a large number of active CAS members have frequently engaged in discussion about the meaning and implications of the CAS's purpose, such that, despite linguistical limitations, the CAS's purpose as an organization has been well understood by most of the active members. In turn the leaders and volunteers of the CAS have, for the most part, maintained focus on that well-understood and accepted purpose as strategic directions have been debated and activities have been planned and carried out. In other words, for 86 years the CAS has known what its main thing is and has kept its main thing the main thing.

Now my third conclusion, the one most relevant to us today, is that in recent years it has become more difficult for the CAS to know and understand its purpose and keep that purpose as the focus for our activities. And it will be even more difficult in the years ahead. Knowing and keeping the main thing the main thing will be as important to our continued success as it has been instrumental to the achievement of our past success. It is going to be more difficult to do than ever before, however. I believe we must become deliberate about keeping our purpose known and understood among our membership and very deliberate about keeping it in focus as we carry out our work as a society.

Now let me be clear that I do not advocate that the CAS forever cling to its current purpose. The CAS has very wisely through the years revised and expanded both the written statement of its purpose and its understanding of what that written statement means. While strongly rooted in our origins, the CAS is nevertheless not only a different organization today than what it was in 1914, but it is much different and broader in scope than what our visionary founders ever imagined it would become. It will be important for us and those who follow us to continue to explore new applications and opportunities for our science and our profession. To paraphrase a statement from Bob Anker's address: Change is our friend and we should embrace it. We should (forgive the cliches) push ourselves to think outside the box, to break down old paradigms, and to boldly go where no actuary has gone before! To do so effectively we must do so within the context of knowing who we are and what our current purpose and capabilities are, so that we can make well-considered and conscious changes to our direction when they are appropriate.

I am reminded of a discussion at a Board meeting earlier this year. I forget which agenda item we were on but, as is sometimes the case at Board meetings, we had wandered off into a discussion of whether we should launch a marketing campaign to convince employers and potential employers of all the wonderful skills and abilities actuaries possess, which the employers should use to address a far greater range of business problems than what is currently the case. We perhaps got a little carried away in our description of actuarial abilities. At one point Steve Lowe spoke up and provided us with a reality check. He pointed out that a dog who tries to convince someone else that he is a cat will probably only convince them that he is a foolish dog. At that point we got back to our agenda item.

So why do I think that it is getting increasingly difficult to know our purpose and keep it at the center of our activities? In part it is a function of the growth in the size and diversity of our membership. There were 97 charter members of the CAS. Today we have almost 3,500 members and are adding about 200 new members a year.

In 1914 the members of the CAS were all men, 60 percent had a business address in New York City and another 20 percent in Connecticut or Massachusetts. Only six had an address west of the Mississippi. None had an address outside the U.S. As best I can tell they all seemed to work directly for or with the insurance industry in some way.

Today a sizeable portion of our members are women and we are much more geographically diverse. We have a significant number of members who live outside the U.S., particularly Canada, but a growing number of members who live outside of North America as well. While the insurance industry still dominates our members' business activities, there is a growing number of members who are spending at least part of their time working on other than insurance matters.

The larger we become and the more diverse our membership, the more difficult it is to conduct the kind of communication necessary to achieve an ongoing consensus understanding of our purpose, especially as that purpose evolves. How to keep a full range of input from our diverse members as part of the evolution of our purpose, and how to keep the majority of our members aware of and accepting of our evolving purpose will be a challenge that grows as our membership grows in numbers and diversity.

There is also another reason why I believe it will be increasingly difficult for us to know and understand our purpose and keep it the focal point of our activities. During the last 86 years, our purpose has grown in scope and in complexity. I believe that is a trend that will continue. The broader and more complex our purpose becomes, the harder it will be to keep it defined and well understood by even a small homogeneous group, let alone the membership as a whole. Thus, it will be harder for our volunteers to keep our purpose the focal point of activities.

Let me elaborate for a moment on two areas in which the CAS's role is currently evolving and which potentially adds both breadth and complexity to our purpose—nontraditional practice areas and globalization.

In 1999 the Task Force on Nontraditional Practice Areas addressed the issue of how the CAS can better support its members who are currently working or who wish to work in nontraditional practice areas. Their excellent report was completed in September 1999 and is available on the CAS web site. As a result of their work we have launched four new advisory committees that will help identify and direct significant new activity for the CAS in nontraditional practice areas. I believe that the action the CAS has taken so far is very consistent with the CAS's purpose statement as it currently exists and is understood by the majority of our members. I had to give it some careful thought at the time, but I believe that these new activities are within the scope of "actuarial science as applied to property, casualty and similar risk exposures." However, with this step I believe we are headed in a direction that will ultimately challenge our current understanding of the definition of "similar risk exposures."

In the background section of the task force's report there is a discussion of the issue of defining "nontraditional practice areas." The task force reported that after some attempt to do so, they finally agreed that they would not define the term as it was not critical to their charge. In fact they found that it is very difficult to define "nontraditional practice area." Some of the difficulty is over what is traditional versus nontraditional. However, difficulty also arises over what falls within the scope of casualty actuarial practice and what does not. What potential practice areas are consistent with the purpose of the CAS and what practice areas are not? That question becomes more complex as we move further away from the insurance context that was our historical focus.

Likewise, as the CAS tries to develop a clearer vision of its role in the worldwide actuarial profession, we are likely to confront further complexity in defining our purpose. It took decades for the CAS to change from an organization whose purpose was solely focused on the U.S. casualty actuary to one that also focuses on the Canadian casualty actuary. I could not determine when the CAS first admitted a Canadian to membership, but I know it was long before 1987, the first year that the CAS's exam syllabus differentiated U.S. and Canadian specialties. Going from a U.S.-only organization to a U.S./Canadian organization certainly increased the complexity of our mission. Even today we struggle with how best to support the needs of casualty actuaries in both countries. Despite the complications involved, I think it has been highly beneficial for all parties for the CAS to become the learned body for casualty actuaries in both the U.S. and Canada. It was clearly the right thing to do despite the complication added to our purpose. Likewise, I strongly believe that the CAS should develop a significant and active role within the worldwide actuarial profession. We have so much to offer and there is such a tremendous need for our knowledge and skills in so many other countries. We are also observing that more and more of the current CAS members' employers and clients have business interests in several countries, a trend that seems inevitable to continue. Therefore, to best support even our U.S. and Canadian members properly, the CAS will need to develop and maintain a prominent worldwide position in the profession. Yes, I think the CAS needs to be involved globally. But the added complexity of that should not be underestimated.

And so as my term as president comes to an end, my advice to all future leaders of the CAS is to be mindful of the importance of keeping the CAS's purpose defined and well understood by the membership, and keeping that purpose at the center of the CAS's activities.

As the CAS moves into the new millennium (I had to work in a new millennium reference somewhere), it does so with confidence and with purpose. We have an impressive record of achievement to build upon. We have a current solid understanding of our mission. And we have a wealth of committed and capable volunteers dedicated to achieving that mission. I envision a CAS in the year 2086 that is even stronger and more successful than the CAS of today. And I envision that the CAS president of 2086 will be able to say what I have said today. The CAS is an organization that has always known what its main thing is and has kept the main thing the main thing.

Now before I step down from the podium I would like spend a few more moments expressing my gratitude to a number of people.

First I want to thank the Executive Council (EC). Pat, Gary Dean, Abbe, Dave, Gary Josephson, LeRoy, and Mary Frances, you have all worked incredibly hard this past year on behalf of all CAS members. You are outstanding examples of what servant leadership is all about. Thank you for your service.

Thanks also to the CAS Board of Directors. Many CAS members may not know how hard our Board works. They don't just get together four times a year to rubberstamp whatever the EC puts before them. They spend significant time and energy developing and debating all the policies and strategic direction of the CAS. As a Board they have been pretty tough on the EC this year, holding our feet to the fire. But as individuals they have all pitched in to help time and time again. They are a great group of CAS volunteers and leaders.

I, of course, need to thank the hundreds of CAS members who have worked on committees, spoken at CAS seminars or meetings, conducted research and wrote papers, or who served the CAS in some other way. I cannot name you all today but you know who you are and you are indeed the heart and soul of the CAS.

Earlier today I recognized and thanked the CAS staff members but there is one staff member that I must mention again—Tim Tinsley, our outstanding executive director. There are not words fine enough to describe Tim Tinsley. He is intelligent, highly organized, hard working, diplomatic, thorough, patient, cool under pressure, and I could go on and on. The CAS owes him our thanks and appreciation. I also owe Tim my personal thanks. In his quiet but firm way, he kept me on track this past year and on schedule. He often made me look more prepared and in control than I really was, and he never failed to provide me wise council when I was smart enough to ask for it. He has become not only a highly respected colleague but a dear, dear friend as well. Thank you, Tim.

If you will bear with me a few more moments I have a few other personal thanks to offer. To my outstanding executive assistant, Debbie Seales, who is here with me today. She has endured much from me this last year and has been invaluable in helping me juggle my CAS responsibilities with my paid employment responsibilities. Thank you, Debbie. I am also grateful to my boss, Steve Goldberg, for his patience and support and to four actuaries in my department at USAA who covered for me many times this past year when I was out on CAS business: Catherine Taylor, Rhonda Aikens, Geoff Werner and Tim Ungashick. Thanks for all your help.

I also need to thank some individuals whose advice on CAS matters was invaluable and whose encouragement often helped me over the rough spots. Thanks to Mavis Walters who never let me forget it's supposed to be fun to be the president of the CAS—and there were times I needed reminding. Thanks to

Chuck Bryan, Bob Anker, Jerry Degerness, Gail Ross, John Kollar, Pat Grannan, Mike Toothman, Dave Hartman, and so many others who provided me with great advice and assistance. What would I have done without you?

And thanks to my wonderful family. To my beloved husband, David, and to the two best children a mother could have, Andrew and Zoe. This past year they were often without a wife and a mother but they never complained and were always supportive. Now if I can just figure out how to get them to keep doing all the laundry and other household chores I used to share in.

And finally thanks to God and thanks to all CAS members who gave me the opportunity to serve as president of the CAS. It has been such an honor and a privilege and an experience I will always cherish. Thank you very much.

MINUTES OF THE 2000 CAS ANNUAL MEETING

November 12–15, 2000

JW MARRIOTT

WASHINGTON, D.C.

Sunday, November 12, 2000

The Board of Directors held their regular quarterly meeting from 9:00 a.m. to 5:00 p.m.

Registration was held from 4:00 p.m. to 6:00 p.m.

From 5:30 p.m. to 6:30 p.m., there was a special presentation to new Associates and their guests. All 2000 CAS Executive Council members briefly discussed their roles in the Society with the new members. In addition, Steven G. Lehmann, who is a past president of the CAS, gave a short talk on the American Academy of Actuaries' (AAA) Casualty Practice Council.

A welcome reception for all members and guests was held from 6:30 p.m. to 7:30 p.m.

Monday, November 13, 2000

Registration continued from 7:00 a.m. to 8:00 a.m.

CAS President Alice H. Gannon opened the business session at 8:00 a.m. and introduced members of the Executive Council and the CAS Board of Directors. Ms. Gannon also recognized past presidents of the CAS who were in attendance at the meeting, including: Robert A. Anker (1996), Phillip N. Ben-Zvi (1985), Ronald L. Bornhuetter (1975), Charles A. Bryan (1990), Michael Fusco (1989), David G. Hartman (1987), Charles C. Hewitt Jr. (1972), Frederick W. Kilbourne (1982), Steven G. Lehmann (1998), Michael L. Toothman (1991), Mavis A. Walters (1997), and Michael A. Walters (1986).

Ms. Gannon also recognized special guests in the audience: Robert L. Brown, president of the Society of Actuaries; Kenneth A. Kent, president-elect of the Conference of Consulting Actuaries; W. James MacGinnitie, president-elect of the Society of Actuaries; and David J. Oakden, president of the Canadian Institute of Actuaries.

Ms. Gannon then announced the results of the CAS elections. The next president will be Patrick J. Grannan, and the president-elect will be Robert F. Conger. Members of the CAS Executive Council for 2000–2001 will be: Sheldon Rosenberg, Vice President–Administration; Mary Frances Miller, Vice President–Admissions; Abbe Sohne Bensimon, Vice President–Continuing Education; LeRoy A. Boison, Vice President–International; David R. Chernick, Vice President–Programs and Communication; and Gary R. Josephson, Vice President–Research and Development. New members of the CAS Board of Directors are Ralph S. Blanchard III, Janet L. Fagan, Michael J. Miller, and Deborah M. Rosenberg.

Curtis Gary Dean announced the 33 new Associates and Patrick J. Grannan announced the 134 new Fellows. The names of these individuals follow.

NEW FELLOWS

John Scott Alexander
Michele S. Arndt
Carl Xavier Ashenbrenner
David Steen Atkinson
Emmanuil Theodore Bardis
Michael William Barlow
Keith M. Barnes
Patrick Beaudoin
Nicolas Beaupre
Andrew S. Becker
Michael J. Bluzer
Sherri Lynn Border
Veronique Bouchard
Erik R. Bouvin
Tobe E. Bradley
Robert F. Brown
Hugh E. Burgess
Allison F. Carp
Joseph G. Cerreta
Patrick J. Charles
Kin Lun (Victor) Choi
Andrew K. Chu
Kuei-Hsia Ruth Chu
Wei Chuang
Steven A. Cohen
Larry Kevin Conlee
Kathleen T. Cunningham
Mary Elizabeth Cunningham
Jonathan Scott Curlee
Loren Rainard Danielson
Kris D. DeFrain
Michael Brad Delvaux
Sean R. Devlin

Tammi B. Dulberger
Louis Durocher
Sophie Duval
Kevin M. Dyke
Jane Eichmann
Gregory James Engl
Vicki A. Fendley
Kenneth D. Fikes
Chauncey E. Fleetwood
Hugo Fortin
Ronnie Samuel Fowler
Noelle C. Fries
Susan I. Gildea
Todd Bennett Glassman
Sanjay Godhwani
Rebecca N. Hai
Kenneth Jay Hammell
Alex A. Hammett
Michelle Lynne Harnick
Jeffery Tim Hay
Qing He
Amy Louise Hicks
Jay T. Hieb
Amy L. Hoffman
Todd Harrison Hoivik
Nancy Michelle Hoppe
Walter L. Jedziniak
Charles B. Jin
Philippe Jodin
Lori E. Julga
Elina L. Koganski
Claudia A. Krucher
Chingyee Teresa Lam
Travis J. Lappe
Robin M. LaPrete
Lewis Y. Lee
Neal M. Leibowitz
Charles Letourneau
Dengxing Lin
Shu C. Lin
Michelle Luneau
Andrea Wynne Malyon
Ian John McCracken
Ain Milner
Paul D. Miotke
Benoit Morissette
Kari S. Mrazek
Kevin T. Murphy
Seth Wayne Myers
Kari A. Nicholson
Mihaela Luminita O'Leary
Richard A. Olsen
Richard D. Olsen
Rebecca Ruth Orsi
Nathalie Ouellet
Michael G. Owen
Mark Paykin
Julie Perron
Christopher Kent Perry
Daniel B. Perry
Anthony George Phillips
Richard Matthew Pilotte
Glen-Roberts Pitruzzello
Igor Pogrebinsky
Gregory J. Poirier
Ricardo A. Ramotar
Christopher David Randall
Leonid Rasin
Peter S. Rauner
Ellen J. Respler
Rebecca L. Roever
David A. Rosenzweig
Romel G. Salam
James C. Santo
Jason Thomas Sash
Stuart A. Schweidel
William Harold Scully III
Ernest C. Segal
Christopher M. Steinbach
Carol A. Stevenson
Roman Svirsky
Karrie Lynn Swanson
Chester J. Szczepanski
Varsha A. Tantri
Jonathan Garrett Taylor
Michael J. Tempesta
Robert M. Thomas
Michael J. Toth
Michael C. Tranfaglia
Laura M. Turner
Kieh Tsung Ty
Martin Vezina
Nathan K. Voorhis
Claude A. Wagner
William B. Westrate
Jerelyn S. Williams
Kendall P. Williams
Laura Markham Williams
Brandon L. Wolf
Jonathan Stanger Woodruff
Yuhong Yang

NEW ASSOCIATES

Katherine H. Antonello
Maura Curran Baker
Jeremy T. Benson
Neil M. Bodoff
Peter R. DeMallie
William M. Finn
Dustin W. Gary
Amy L. Gebauer
Patrick J. Gilhool
Mark D. Heyne
Kurt D. Hines
Patricia A. Hladun
Derek R. Hoyme
Craig D. Isaacs
Gregory K. Jones
Sean M. Kennedy
David R. Kennerud
Susanlisa Kessler
Jing Liu
Kathleen T. Logue
Julie Martineau
James J. Matusiak Jr.
John R. McCollough
Rodney S. Morris
Brian C. Neitzel
Sean R. Nimm
Sylvain Nolet
Rodrick R. Osborn
Christopher S. Throckmorton
Mark A. Verheyen
Shaun S. Wang
Ya-Feng Wang
Eric Zlochevsky

Ms. Gannon then introduced Charles C. Hewitt Jr., a past president of the Society, who presented the Address to New Members.

Following the address, David R. Chernick, Vice President–Programs and Communications, briefly highlighted the meeting's programs and thanked the CAS Program Planning Committee. Mr. Chernick then introduced John M. Kulik, Vice Chairperson of the CAS Committee on Review of Papers, who announced that two *Proceedings* papers would be presented at this meeting.

Mr. Kulik began the awards program by announcing that the 2000 Woodward-Fondiller Prize was given to Stephen J. Mildenhall for his discussion of "Application of the Option Market Paradigm to the Solution of Insurance Problems" by Michael G. Wacek. Mr. Kulik then introduced LeRoy A. Boison, Vice President–International, who presented the 2000 Charles A. Hachemeister Award to Uwe Schmock for his paper, "Estimating the Value of the WINCAT Coupons of the Winterthur Insurance Convertible Bond: A Study of the Model Risk." Mr. Mildenhall's paper is published in this edition of the *Proceedings*. Mr. Schmock's paper is published in the ASTIN *Bulletin*.

Ms. Gannon presented the 2000 CAS Matthew S. Rodermund Service Award to Charles F. Cook, who was chosen for his outstanding contributions to the actuarial profession.

Ms. Gannon then requested a moment of silence in honor of those CAS members who passed away since November 1999. They are: J. Edward Faust Jr., Olaf E. Hagen, Phillip B. Kates, Paul S. Liscord Jr., Thomas E. Murrin, John H. Rowell, Irwin T. Vanderhoof, and James M. Woolery.

In a final item of business, Ms. Gannon acknowledged a donation of $10,000 from D.W. Simpson & Company to the CAS Trust (CAST). The donation was made October 13, 2000.

Ms. Gannon then concluded the business session of the Annual Meeting and introduced the featured speaker, George Will. Mr. Will is one of America's leading political observers and is seen weekly on ABC's *This Week* and read nationally in *Newsweek* and in his syndicated newspaper columns.

After a refreshment break, the first General Session was held from 10:45 a.m. to 12:15 p.m.:

"Product Distribution in a Changing Business Environment"

Moderator:	Charles A. Bryan Senior Vice President–Chief Actuary Nationwide Insurance Company
Panelists:	Mark Benson Executive Vice President Cybercomp
	Mark Cis Vice President, Strategy Development CUNA Mutual Group
	Steven L. Groot President Allstate International

Waymon L. Lynch
Agency Owner
Nationwide Insurance Company

Following the general session, CAS President Alice H. Gannon gave her Presidential Address at the luncheon. At the luncheon's end, Ms. Gannon officially passed on the CAS presidential gavel to the new CAS president, Patrick J. Grannan.

After the luncheon, the afternoon was devoted to presentations of concurrent sessions. The panel presentations from 1:30 p.m. to 3:00 p.m. covered the following topics:

1. New Classes of Claims/Megatort Update

Moderator: Jennifer L. Biggs
Consulting Actuary
Tillinghast-Towers Perrin

Panelists: Philip D. Miller
Consulting Actuary
Tillinghast-Towers Perrin

Barbara K. Murray
Assistant Vice President
Argonaut Insurance Company

2. Emerging Nontraditional Products—Loss Mitigation Underwriting, Reps, and Warranties

Moderator/Panelist: Gary Blumsohn
Vice President
St. Paul Re

Panelists: Chris E. Nelson
Vice President
CNA Re

Marvin Pestcoe
Head of Actuarial Services
Swiss Re New Markets

3. Non-U.S. DFA: Are They Doing That Too?

Moderator: Susan E. Witcraft
Consulting Actuary
Milliman & Robertson, Inc.

Panelist: Stavros Christofides
Consultant
Bacon & Woodrow

4. The Actuary's Role in Due Diligence

Moderator: Stuart G. Sadwin
Vice President & Actuary
CGU Insurance Companies

Panelists: Gayle E. Haskell
Chief Financial Officer and Chief Actuary
Providence Washington Insurance Company

Gail M. Ross
Vice President
Am-Re Consultants

5. Actuaries in an e-World

Moderator: John F. Gibson
Principal
PricewaterhouseCoopers LLP

Panelists: John S. Peters
Director of Business Development
OneShield.com

B. C. Verniero
Vice President of Marketing and Internal Development
Homesite Insurance

Nancy P. Watkins
Consulting Actuary
Milliman & Robertson, Inc.

6. The View from Overseas

Moderator:	John C. Narvell Chief Actuary Winterthur International
Panelist:	John C. Burville Chief Actuary ACE Bermuda

7. Hachemeister Prize Paper: "Estimating the Value of the WINCAT Coupons of the Winterthur Insurance Convertible Bond: A Study of the Model Risk"

Author:	Uwe Schmock ETH Zurich

After a refreshment break from 3:00 p.m. to 3:30 p.m., presentations of concurrent sessions continued and *Proceedings* papers were presented. Certain concurrent sessions presented earlier were repeated. Additional concurrent sessions presented from 3:30 p.m. to 5:00 p.m. were:

1. Prescription Drugs and the Workers Compensation System

Moderator/ Panelist:	Alex Swedlow Consultant California Workers Compensation Institute
Panelist:	Laura Gardner, MD Axiomedics Research, Inc.

2. Current Trends in D&O and E&O Insurance

Moderator/ Panelist:	Mark W. Larsen Consultant Tillinghast-Towers Perrin
Panelists:	Bernard R. Horovitz Assistant Vice President and Actuary Chubb Executive Risk

Cynthia Traczyk
Vice President
CNA

3. AAA Securitization Task Force

Moderator: Frederick O. Kist
Chief Actuary
Kemper Insurance Companies

Panelists: Glenn G. Meyers
Chief of Actuarial Research and Assistant Vice President
Insurance Services Office, Inc.

William F. Dove
Vice President
Centre Solutions

Proceedings papers presented during this time were:

1. "Applications of Resampling Methods in Actuarial Practice"

Authors: Richard A. Derrig
Automobile Insurers Bureau of Massachusetts

Krzysztof M. Ostaszewski
University of Illinois

Grzegorz A. Rempala
University of Louisville

2. "Best Estimates for Reserves"

Authors: Glen Barnett
Insureware Pty. Ltd.

Ben Zehnwirth
Insureware Pty. Ltd.

An Officers' Reception for New Fellows and Accompanying Persons was held from 5:30 p.m. to 6:30 p.m.

A general reception for all attendees followed from 6:30 p.m. to 7:30 p.m.

Tuesday, November 14, 2000

Registration continued from 7:00 a.m. to 8:00 a.m.

The following General Sessions were held from 8:00 a.m. to 9:30 a.m.:

"Auto Safety, Engineering, and Insurance—As We Begin the 21st Century"

Moderator:	Steven F. Goldberg Senior Vice President United Services Automobile Association
Panelists:	Brian O'Neill President Insurance Institute of Highway Safety
	Robert Shelton Executive Director National Highway Traffic Safety Administration
	Rob Strassburger Vice President, Vehicle Safety Harmonization Alliance of Automobile Manufacturers

"Financial Services Modernization: Information vs. Privacy Issues"

Moderator:	Michael A. Walters Consulting Actuary Tillinghast-Towers Perrin
Panelists:	Debra Ballen Executive Vice President, Public Policy Management American Insurance Association

Jeffrey S. Duncan
Legislative Director, Representative
Edward J. Markey

Kevin Rampe
Special Deputy Superintendent and
General Counsel
New York Insurance Department

Two limited attendance workshops, "Negotiation Skills" and "Executive Presentation Skills," were held from 8:00 a.m. to 5:00 p.m. and from 8:00 a.m. to 12:00 p.m., respectively.

Following a break from 9:30 a.m. to 10:00 a.m., certain concurrent sessions that had been presented earlier during the meeting were repeated from 10:00 a.m. to 11:30 a.m. Additional concurrent sessions presented were:

1. The e-volving CAS

Moderator/ Panelist:	Therese A. Klodnicki Committee On Online Services
Panelist:	Janet L. Dauber Webmaster Casualty Actuarial Society

2. Report of CAS Advisory Committee on Enterprise Risk Management

Moderator:	Andrew T. Rippert Senior Vice President Deloitte & Touche LLP
Panelists:	Lawrence F. Marcus Director & Associate Actuary ACE USA
	Chris E. Nelson Vice President CNA Re

3. The Actuary and Earnings Management

   Moderator: Marc F. Oberholtzer
   Principal Consultant
   PricewaterhouseCoopers LLP

   Panelists: Matthew J. Adams
   Partner
   PricewaterhouseCoopers LLP

   William M. Wilt
   Vice President–Senior Analyst
   Moody's Investors Service

4. NAIC Redefinitions of Loss Adjustment Expense

   Moderator: Charles F. Cook
   Consulting Actuary
   MBA Inc.

   Panelists: Richard Carris
   Senior Claims Consultant
   Ernst & Young LLP

   W. H. Odell
   President
   Odell & Associates, Inc.

   John A. Stenmark
   Vice President
   Southern Farm Bureau Casualty Company

Various committee meetings were held from 12:00 p.m. to 5:00 p.m. Certain concurrent sessions that had been presented earlier during the meeting were also repeated from 1:00 p.m. to 2:30 p.m. The additional concurrent session presented at this time was:

1. Actuaries in Nontraditional Practice Areas—Case Studies

   Moderator: Robert F. Wolf
   Chairperson
   External Communications Committee

Panelists: David Fishbaum
Principal
William M. Mercer, Inc.

David Molyneux
Assistant Vice President
Zurich Re

Rade T. Musulin
Vice President–Actuary
Florida Farm Bureau Insurance Companies

Following the concurrent sessions, a special Capitol Hill Briefing on Catastrophe Issues was held from 2:30 p.m. to 4:00 p.m.

Entertainment and a buffet dinner were held from 7:00 p.m. to 10:00 p.m.

Wednesday, November 15, 2000

Certain concurrent sessions were repeated from 8:00 a.m. to 9:30 a.m. Additional concurrent sessions presented at this time were:

1. Actuarial Statements of Opinion and ASOP 36

Moderator: Patricia A. Teufel
Principal
KPMG LLP

Panelists: Joseph A. Herbers
Principal and Consulting Actuary
Miller, Herbers, Lehmann, & Associates, Inc.

Robert S. Miccolis
Senior Vice President
Reliance Reinsurance Corporation

Robert H. Wainscott
Principal
Ernst & Young LLP

2. New Fellows' Perspectives

Moderator:	Robert F. Conger Consulting Actuary Tillinghast-Towers Perrin
Panelists:	Sean R. Devlin Assistant Vice President American Re-Insurance Company
	Chester J. Szczepanski Chief Actuary Pennsylvania Insurance Department

After a break from 9:30 a.m. to 10:00 a.m., the final General Session was held from 10:00 a.m. to 11:30 a.m.:

"Worshipping at the Altar of Shareholder Value"

Moderator:	John J. Kollar Vice President Insurance Services Office, Inc.
Panelists:	Weston M. Hicks Managing Director J. P. Morgan Securities, Inc.
	David B. Kelso Chief Financial Officer Chubb Group of Insurance Companies
	Stephen P. Lowe Chief Actuary Tillinghast-Towers Perrin
	Kim D. Thorpe Chief Financial Officer FPIC Insurance Group, Inc.

Alice H. Gannon officially adjourned the 2000 CAS Annual Meeting at 11:45 a.m. after closing remarks and an announcement of future CAS meetings.

Attendees of the 2000 CAS Annual Meeting

The 2000 CAS Annual Meeting was attended by 353 Fellows, 118 Associates, and 60 Guests. The names of the Fellows and Associates in attendance follow:

FELLOWS

Ralph L. Abell
Jonathan D. Adkisson
Martin Adler
Rebecca C. Amoroso
Richard B. Amundson
Scott C. Anderson
Robert A. Anker
Michele Segreti Arndt
Nolan E. Asch
Carl Xavier Ashenbrenner
Richard V. Atkinson
Emmanuil Theodore Bardis
Michael William Barlow
Betty H. Barrow
Andrea C. Bautista
Patrick Beaudoin
Nicolas Beaupre
Andrew Steven Becker
Phillip N. Ben-Zvi
Abbe Sohne Bensimon
Cynthia A. Bentley
Regina M. Berens
David R. Bickerstaff
William P. Biegaj
Jennifer L. Biggs
Terry J. Biscoglia
Ralph S. Blanchard
Michael P. Blivess
Carol Blomstrom
Gary Blumsohn
Michael J. Bluzer
LeRoy A. Boison
Joseph A. Boor
Sherri Lynn Border
Ronald L. Bornhuetter
Veronique Bouchard
Theresa W. Bourdon
Amy S. Bouska
Tobias E. Bradley
James F. Brannigan
Yaakov B. Brauner
Paul J. Brehm
Ward M. Brooks
Robert F. Brown
Ron Brusky
Charles A. Bryan
Gary S. Bujaucius
Hugh Eric Burgess
Jeanne H. Camp
Douglas A. Carlone
Allison Faith Carp
Joseph Gerald Cerreta
Lisa G. Chanzit
Patrick J. Charles
David R. Chernick
Kin Lun (Victor) Choi
James K. Christie
Kuei-Hsia Ruth Chu
Andrew K. Chu
Wei Chuang
Rita E. Ciccariello
Mark M. Cis
Steven A. Cohen
Robert F. Conger
Larry Kevin Conlee
Eugene C. Connell
Charles F. Cook
Mark Crawshaw
Patrick J. Crowe
Kathleen T. Cunningham
Mary Elizabeth Frances Cunningham
Jonathan Scott Curlee
Ronald A. Dahlquist
Charles Anthony Dal Corobbo
Loren Rainard Danielson
Lawrence S. Davis
Timothy Andrew Davis
Thomas J. DeFalco
Kris D. DeFrain
Curtis Gary Dean

Jerome A. Degerness
Camley A. Delach
Michael Brad Delvaux
Sean R. Devlin
Patrick K. Devlin
Kurt S. Dickmann
Behram M. Dinshaw
William F. Dove
Michael C. Dubin
Diane Symnoski Duda
Judith E. Dukatz
Tammi B. Dulberger
Louis Durocher
Sophie Duval
Kevin M. Dyke
Richard D. Easton
Bob D. Effinger
Gary J. Egnasko
Jane Eichmann
Martin A. Epstein
John S. Ewert
Michael A. Falcone
Vicki Agerton Fendley
Kenneth D. Fikes
Russell S. Fisher
Beth E. Fitzgerald
Chauncey Edwin Fleetwood
Claudia S. Forde
John R. Forney
Hugo Fortin
Ronnie Samuel Fowler
Noelle Christine Fries
John E. Gaines
Alice H. Gannon
Robert W. Gardner
David B. Gelinne
Eric J. Gesick
John F. Gibson
Susan I. Gildea
Todd B. Glassman
Sanjay Godhwani
Steven F. Goldberg
Annette J. Goodreau
Gregory S. Grace
Patrick J. Grannan
Eric L. Greenhill
Linda M. Groh
Steven L. Groot
Rebecca N. Hai
Leigh Joseph Halliwell
Alexander Archibold Hammett
Michelle Lynne Harnick
David G. Hartman
Gayle E. Haskell
Jeffery Tim Hay
Qing He
Christopher Ross Heim
Kirsten Costello Hernan
Charles C. Hewitt
Amy Louise Hicks
Jay T. Hieb
Amy L. Hoffman
Todd Harrison Hoivik
Nancy Michelle Hoppe
Robert J. Hopper
Bertram A. Horowitz
Mary T. Hosford
David Dennis Hudson
Daniel B. Isaac
Richard M. Jaeger
Stephen Jameson
Walter Leon Jedziniak
Charles B. Jin
Philippe Jodin
Jennifer Polson Johnson
Eric J. Johnson
Warren H. Johnson
Thomas S. Johnston
Gary R. Josephson
Lori Edith Julga
Frank J. Karlinski
Clive L. Keatinge
Anne E. Kelly
Steven A. Kelner
Allan A. Kerin
Frederick W. Kilbourne
Mary Jean King
Frederick O. Kist
Joel M. Kleinman
Elina Koganski
John J. Kollar
Israel Krakowski
Sarah Krutov
Andrew E. Kudera
Ronald T. Kuehn
Kay E. Kufera
John M. Kulik
Jason Anthony Kundrot
Robin M. La Prete
Michael A. LaMonica
David A. Lalonde

Chingyee Teresa Lam
Dean K. Lamb
John A. Lamb
Travis J. Lappe
Lewis Y. Lee
Merlin R. Lehman
Steven G. Lehmann
Neal Marev Leibowitz
Elizabeth Ann Lemaster
Charles Letourneau
Kenneth A. Levine
Siu K. Li
Peter M. Licht
John J. Limpert
Shu C. Lin
Dengxing Lin
Orin M. Linden
Stephen P. Lowe
Michelle Luneau
William R. Maag
W. James MacGinnitie
Andrea Wynne Malyon
Lawrence F. Marcus
Leslie R. Marlo
Kelly J. Mathson
John W. McClure
Michael F. McManus
William T. Mech
David L. Menning
Matthew P. Merlino
Claus S. Metzner
Stephen J. Meyer
Glenn G. Meyers
Robert S. Miccolis
Philip D. Miller
Mary Frances Miller
David L. Miller
David L. Miller
Ain Milner
Neil B. Miner
Camille Diane Minogue
Paul David Miotke
David Molyneux
Bruce D. Moore
Benoit Morissette
Kari Sue Mrazek
Robert V. Mucci
Raymond D. Muller
Todd B. Munson
Donna S. Munt
John A. Murad
Kevin T. Murphy
Thomas G. Myers
Seth Wayne Myers
John C. Narvell
Chris E. Nelson
Aaron West Newhoff
Kari A. Nicholson
G. Chris Nyce
Margaret O'Brien
Mihaela Luminita S. O'Leary
David J. Oakden
Marc F. Oberholtzer
Richard Alan Olsen
Richard D. Olsen
Rebecca Ruth Orsi
Nathalie Ouellet
Timothy A. Paddock
Rudy A. Palenik
Robert G. Palm
Donald W. Palmer
Joseph M. Palmer
Mark Paykin
Harry Todd Pearce
Kathleen M. Pechan
Daniel Berenson Perry
Christopher Kent Perry
Marvin Pestcoe
John S. Peters
Mark W. Phillips
Richard Matthew Pilotte
Glen-Roberts Pitruzzello
Arthur C. Placek
Igor Pogrebinsky
Gregory John Poirier
Dale S. Porfilio
Virginia R. Prevosto
Michael David Price
Deborah W. Price
Regina Marie Puglisi
Eduard J. Pulkstenis
Mark S. Quigley
Ricardo Anthony Ramotar
Christopher David Randall
Leonid Rasin
Ellen J. Respler
Rebecca L. Roever
William P. Roland
A. Scott Romito
Deborah M. Rosenberg
David A. Rosenzweig

Gail M. Ross
James V. Russell
Stuart G. Sadwin
Romel G. Salam
James C. Santo
Jason Thomas Sash
Letitia M. Saylor
Joseph R. Schumi
Stuart A. Schweidel
Gregory R. Scruton
William Harold Scully
Ernest C. Segal
Meyer Shields
Jeffrey Parviz Shirazi
Roy G. Shrum
Lisa A. Slotznick
Daniel L. Splitt
Douglas W. Stang
Christopher M. Steinbach
Phillip A. Steinen
John A. Stenmark
Carol A. Stevenson
Deborah L. Stone
Edward C. Stone
Stuart B. Suchoff
Roman Svirsky
Jeanne E. Swanson
Karrie Lynn Swanson
Adam M. Swartz
Chester John Szczepanski
Susan T. Szkoda
Christopher Tait
Varsha A. Tantri
Catherine Harwood Taylor
Jonathan Garrett Taylor
Michael Joseph Tempesta
Patricia A. Teufel
Robert M. Thomas
Kevin B. Thompson
John P. Tierney
Glenn Allen Tobleman
Michael L. Toothman
Michael J. Toth
Cynthia Traczyk
Philippe Trahan
Michael C. Tranfaglia
Everett J. Truttmann
Laura M. Turner
Martin Vezina
Gerald R. Visintine
Steven M. Visner
Robert H. Wainscott
Michael A. Walters
Mavis A. Walters
Nancy P. Watkins
William Boyd Westrate
Patricia Cheryl White
Mark Whitman
William Robert Wilkins
Laura Markham Williams
Kendall P. Williams
Jerelyn S. Williams
William M. Wilt
John J. Winkleman
Michael L. Wiseman
Susan E. Witcraft
Brandon L. Wolf
Jonathan Woodruff
Yuhong Yang
Joel D. Yatskowitz
Charles J. Yesker
Ralph T. Zimmer

ASSOCIATES

Anju Arora
Martha E. Ashman
Maura Curran Baker
Jeremy Todd Benson
Thomas S. Boardman
Neil M. Bodoff
David R. Border
Karen Ann Brostrom
Robert L. Brown
Kenrick A. Campbell
John A. Canetta
Donald L. Closter
J. Paul Cochran
Brian Roscoe Coleman
Thomas P. Conway
Kenneth M. Creighton
Daniel A. Crifo
Robert E. Davis

Peter R. DeMallie
Sara P. Drexler
Sharon C. Dubin
Alice H. Edmondson
Ellen E. Evans
Farzad Farzan
William M. Finn
Brian C. Fischer
Sean Paul Forbes
Mauricio Freyre
Timothy J. Friers
Kai Y. Fung
Dustin Wayne Gary
Amy L. Gebauer
Patrick J. Gilhool
Bradley G. Gipson
Stewart H. Gleason
Terry L. Goldberg
Christopher David Goodwin
Bruce H. Green
Monica A. Grillo
Nasser Hadidi
Brian D. Haney
Susan Wadman Hayes
Thomas F. Head
Joseph A. Herbers
Mark D. Heyne
Kurt D. Hines
Patricia A. Hladun
Jason N. Hoffman
Bernard R. Horovitz
Derek Reid Hoyme
David D. Hu
Jeffrey R. Ill
Craig D. Isaacs
Brian J. Janitschke
Brian E. Johnson
Gregory K. Jones
James W. Jonske
Edwin G. Jordan
Scott A. Kelly
Sean M. Kennedy
David R. Kennerud
Susanlisa Kessler
David Neal Kightlinger
Jeffrey D. Kimble
Linda S. Klenk
Therese A. Klodnicki
Karen Lee Krainz
Steven M. Lacke
Stephen E. Lehecka
Todd William Lehmann
Jing Liu
Kathleen T. Logue
William F. Loyd
Julie Martineau
James J. Matusiak
John R. McCollough
Douglas H. Min
Rodney S. Morris
Rade T. Musulin
Brian C. Neitzel
Sean Robert Nimm
Sylvain Nolet
Leigh S. Oates
W. H. Odell
Leo Martin Orth
Prabha Pattabiraman
Willard W. Peacock
Claude Penland
Sasikala Raman
Thomas O. Rau
James E. Rech
Michael Sansevero
Joshua Stewart Sawyer
Michael Robert Schummer
Steven George Searle
Barbara A. Seiffertt
Ahmad Shadman
Halina H. Smosna
David C. Snow
Calvin C. Spence
Michael William Starke
Avivya Simon Stohl
Frederick M. Strauss
Eugene G. Thompson
Joseph O. Thorne
Christopher S. Throckmorton
Frederick A. Urschel
Eric Vaith
Mark Alan Verheyen
Mary Elizabeth Waak
Roger C. Wade
David G. Walker
Felicia Wang
Gregory S. Wanner
Thomas J. White
Robert J. White
Robert F. Wolf
Nora J. Young
Yin Zhang

REPORT OF THE VICE PRESIDENT–ADMINISTRATION

This report provides a one-year summary of CAS activities since the 1999 CAS Annual Meeting. I will first comment on these activities as they relate to the following purposes of the Casualty Actuarial Society as stated in our Constitution:

1. Advance the body of knowledge of actuarial science applied to property, casualty, and similar risk exposures;
2. Establish and maintain standards of qualifications for membership;
3. Promote and maintain high standards of conduct and competence for the members; and
4. Increase the awareness of actuarial science.

I will then provide a summary of other activities that may not relate to a specific purpose, but yet are critical to the ongoing vitality of the CAS. Finally, I will summarize the current status of our finances and key membership statistics.

The CAS discussion paper programs, *Proceedings*, and the *Forum* contribute to the attainment of the first purpose. The winter, summer, and fall volumes of the *Forum* focused on topics in ratemaking, reserving, dynamic financial analysis, and health and managed care. The discussion paper program volume addressed insurance in the next century. The *Proceedings* papers addressed topics in workers compensation, residual markets, and Superfund-related claims costs, as well as mathematical/modeling techniques including the relationship between minimum bias and generalized linear models, a semiparametric model for loss distributions, and bias in loss development factors.

During the past year, members were able to receive the *Forum* and discussion paper publications electronically via the CAS Web Site for the first time in lieu of hard copy. The cost savings were shared with the members who elected this option by a reduction in their annual dues.

The Task Force on Fair Value of Liabilities, under the leadership of Ralph Blanchard, prepared a comprehensive report discussing the issues surrounding fair valuing of property/casualty insurance liabilities, particularly in the United States. This is a timely subject because the issues are currently being reviewed by the Financial Accounting Standards Board and the International Accounting Standards Committee (IASC). Casualty actuaries are qualified through their education and experience to offer valuable input on this topic and this report will support the CAS representative to the International Actuarial Association, the international actuarial body working with the IASC.

In regards to purpose 2, there were a number of developments in the CAS education and examination system during the last year. Beginning with the Spring 2000 examinations, the CAS instituted a new system requiring nine exams for Fellowship and seven exams for Associateship. The first four exams are jointly administered with the Society of Actuaries (SOA); partial exams were discontinued. There was a rearrangement of material among exams on the syllabus and topics were updated and sometimes changed to reflect the current needs for CAS education. The new examination Part 8, "Investments and Financial Analysis," will provide CAS actuaries with much broader exposure to the area of investments and financial analysis.

The final report of the Task Force on Education and Examination Process and Procedures, chaired by John Kollar, was presented to the Executive Council and Board of Directors early in the year. The task force reviewed the current education and examination system to assess its strengths and weaknesses and made recommendations to improve the process. Admissions committees were assigned responsibility for implementing the recommendations. A professional educational consultant has been hired on a contract basis to assist the CAS with defining learning objectives and exam blueprints.

The CAS Task Force on Exams 3 and 4, chaired by Howard Mahler, reevaluated examinations 3 and 4 addressing (1) whether the learning objectives are appropriate for casualty actuaries and for all actuaries and (2) whether the readings are appropriate to teach the learning objectives. A draft report was provided to the Board in November 2000. The Board directed the Executive Council (EC) to work with the SOA to make improvements in time for the Spring 2001 exams. The Board also directed the EC to commission the development of study notes for exams 3 and 4 that are relevant to CAS learning objectives and appropriate for self-study.

The Board approved waivers for CAS exams 1–4 for candidates who have received credit for specified Institute of Actuaries' exams. A reciprocal waiver policy for the Institute exams has been extended to the CAS.

A quality program of continuing education and a Code of Professional Conduct support purpose 3: "promote and maintain high standards of conduct and competence for the members."

The Board adopted a revised Code of Professional Conduct to be effective January 1, 2001. This revised code has also been adopted by the other four U.S. actuarial organizations and is the result of the work of the Joint Committee on the Code of Professional Conduct, whose goal was to produce a uniform code for the U.S. actuarial profession.

The CAS provides educational opportunities through the publication of actuarial materials and the sponsorship of meetings and seminars. This year's sessions included the following, shown with the number of CAS members in attendance:

Meetings:

	Location	CAS Members
Spring	Las Vegas	639
Annual	Washington, D.C.	471

Seminars:

Topic	Location	CAS Members
Ratemaking	San Diego	282
Valuation of Insurance Operations	St. Louis	77
Reinsurance	Boston	179
Dynamic Financial Analysis	New York	123
Casualty Loss Reserves	Minneapolis	316
Funding Catastrophe Risks	Providence	74
Appointed Actuary—Joint CAS/CIA	Toronto	357*
Course on Professionalism—Dec '99	3 locations	82 students
Course on Professionalism—June '00	2 locations	43 students

*Total attendance. Separate count for CAS members is not available.

Limited attendance seminars covered the following six topics: (1) Practical Applications of Loss Distributions, (2) Managing Asset and Investment Risk, (3) Principles of Finance, (4) Reinsurance, (5) Advanced Dynamic Financial Analysis, and (6) (Basic) Dynamic Financial Analysis. The CAS co-sponsored a special interest seminar in London with the Institute of Actuaries in June 2000.

Four new advisory committees in nontraditional practice areas (i.e., nontraditional for CAS members) were formed: (1) Asset/Liability Management and Investment Policy, (2) Valuation of P&C Insurance Companies, (3) Enterprise Risk Management, and (4) Securitization/Risk Financing. These committees will identify research and education initiatives.

A program to offer training in general business skills to CAS members was initiated at the 2000 Spring Meeting and repeated at the Fall meeting. Sessions on executive presentation skills and business writing skills were held.

To increase the awareness of opportunities in actuarial science, the CAS created a new web site, jointly sponsored with the SOA, devoted entirely to recruiting and career development: www.BeAnActuary.org.

The CAS web site, now in its fifth year of existence, supports all four purposes. Here are some highlights from the past year that have not been mentioned elsewhere in this report: (1) registrations for meetings and payments of dues can be processed online; (2) a collection of over 70 volumes and 600 articles from the ASTIN Bulletin was placed on the web site; and (3) a quick search function for the home page and all second-level pages was added.

The CAS exercised an option to expand the size of the current CAS Office to provide needed facilities and for future staff growth. The CAS also exercised an option to extend the lease for five years beyond the current expiration date in February 2001.

The Task Force on CAS Election Process chaired by John Purple submitted its report in February 2000. The Board of Directors acted on the recommendations in February and May, in time to implement approved changes for the CAS 2000 elections. The changes included providing additional biographical information on candidates for election in the ballot package, along with candidates' responses to two questions proposed by the Nominating Committee. The questions asked for the 2000 elections were: (1) why do you want to be a member of the CAS Board?; and (2) what particular qualities and experience would you bring to the Board? The Board also charged the EC with developing a communication plan to educate the membership about the election process.

A Board Question & Answer Forum on the CAS web site was approved by the Board in September to enhance communication between the membership and the Board.

A new CAS Committee on Investments was established following Board agreement that the CAS investment strategy should be expanded to allow investment of long-term funds in equities using low expense, broadly diversified index funds. This represents a shift from the conservative prior policy of investing in intermediate-term U.S. Treasury fixed-income securities with staggered maturity dates.

Membership growth continued with 183 new Associates, 149 new Fellows, and 8 new Affiliates. (Affiliate is a nonvoting membership category for qualified actuaries who have attained their actuarial credentials in another actuarial organization.) The total number of members as of November 2000 was 3,455, up 5.3% for the year.

New members elected to the Board of Directors for next year are Ralph S. Blanchard III, Janet L. Fagan, Michael J. Miller, and Deborah M. Rosenberg. The membership elected Robert F. Conger to the position of President-Elect, while Patrick J. Grannan assumed the presidency.

The Executive Council, with primary responsibility for day-to-day operations, met either by teleconference or in person at least once a month during the year. The Board of Directors elected the following Vice Presidents for the coming year: Vice President–Administration, Sheldon Rosenberg; Vice President–Admissions, Mary Frances Miller; Vice President–Continuing Education, Abbe S. Bensimon; Vice President–International, LeRoy A. Boison; Vice President–Programs and Communications, David R. Chernick; and Vice President–Research and Development, Gary R. Josephson.

The CPA firm of Langan Associates has been engaged to examine the CAS books for fiscal year 2000 and its findings will be reported by the Audit Committee to the Board of Directors in February 2001. The fiscal year ended with an unaudited net loss from operations of $183,280 compared to a budgeted net loss of $186,242. Fiscal year 2000 had been budgeted for a net loss because of the strong equity position that resulted from higher than expected income in prior years.

Members' equity now stands at $2,968,879. This represents a decrease in equity of $105,980 over the amount reported last year. In addition to the net loss from operations, there was an unrealized gain of $17,766 recorded to adjust marketable securities to market value as of September 30, 2000. There was also a total net increase of $59,534 in various research, prize, and scholarship accounts, arising from the difference between incoming funds and interest earned less expenditures. These amounts are not reflected in net income from operations.

For 2000–2001, the Board of Directors has approved a budget of approximately $4.7 million, an increase of about $400,000 over the prior fiscal year. Members' dues for next year will be $300, an increase of $10, while fees for the Subscriber Program will increase by $10 to $370. A $30 discount is available to members and subscribers who elect to receive the *Forum* and *Discussion Paper Program* publications in electronic format from the web site.

Respectfully submitted,
Curtis Gary Dean
Vice President–Administration

FINANCIAL REPORT
FISCAL YEAR ENDED 9/30/2000

OPERATING RESULTS BY FUNCTION

FUNCTION	*INCOME*	*EXPENSE*	*DIFFERENCE*
Membership Services	$ 1,218,006 (a)	$ 1,366,188	$ (148,182)
Seminars	921,739	1,030,025	(108,286)
Meetings	767,087	807,849	(40,762)
Exams	2,984,072 (b)	2,861,526 (b)	122,546
Publications	39,594	30,424	9,170
TOTAL	$ 5,930,498	$ 6,096,012	$ (165,514)

NOTES: (a) Includes gain of $17,766 to adjust marketable securities to market value (SFAS 124).
(b) Includes $1,865,955 of Volunteer Services for income and expense (SFAS 116).

BALANCE SHEET

ASSETS	*9/30/1999*	*9/30/2000*	*DIFFERENCE*
Checking Accounts	$ 134,490	$ 30,029	$ (104,461)
T-Bills/Notes	3,537,154	3,511,251	(25,903)
Accrued Interest	51,708	43,006	(8,702)
Prepaid Expenses	72,451	90,789	18,338
Prepaid Insurance	16,871	16,719	(152)
Accounts Receivable	11,255	2,980	(8,275)
Textbook Inventory	8,174	3,499	(4,675)
Computers, Furniture	386,873	406,702	19,829
Less: Accumulated Depreciation	(256,384)	(307,174)	(50,790)
TOTAL ASSETS	$ 3,962,594	$ 3,797,801	$ (164,793)

LIABILITIES	*9/30/1999*	*9/30/2000*	*DIFFERENCE*
Exam Fees Deferred	$ 500,444	$ 325,339	$ (175,105)
Annual Meeting Fees Deferred	29,355	44,605	15,250
Seminar Fees Deferred	27,441	42,750	15,309
Accounts Payable and Accrued Expenses	263,779	349,159	85,380
Deferred Rent	9,018	2,652	(6,366)
Unredeemed Vouchers	19,800	14,400	(5,400)
Accrued Pension	37,896	50,016	12,120
TOTAL LIABILITIES	$ 887,735	$ 828,921	$ (58,814)

MEMBERS' EQUITY

Unrestricted	*9/30/1999*	*9/30/2000*	*DIFFERENCE*
CAS Surplus	$ 2,727,393	$ 2,561,879	$ (165,514)
Michelbacher Fund	105,861	110,185	4,324
Dorweiler Fund	1,911	0	(1,911)
CAS Trust	36,616	63,628	27,012
Research Fund	133,207	160,972	27,765
ASTIN Fund	52,046	54,910	2,864
Subtotal Unrestricted	$ 3,057,034	$ 2,951,574	$ (105,460)
Temporarily Restricted			
Scholarship Fund	$ 6,738	$ 6,610	$ (128)
Rodermund Fund	11,087	10,695	(392)
Subtotal Restricted	17,825	17,305	(520)
TOTAL EQUITY	$ 3,074,859	$ 2,968,879	$ (105,980)

C. Gary Dean, Vice President–Administration

This is to certify that the assets and accounts shown in the above financial statement have been audited and found to be correct.

CAS Audit Committee: Charles A. Bryan, Chairperson;
Anthony J. Grippa; Frederick O. Kist; and Richard W. Lo

2000 EXAMINATIONS—SUCCESSFUL CANDIDATES

Examinations for Exams 5, 7–Canada, 7–United States, and 8 of the Casualty Actuarial Society were held on May 1, 2, and 3, 2000. Examinations for Exams 6 and 9 of the Casualty Actuarial Society were held on October 31 and November 1, 2000.

Examinations for Exams 1, 2, 3, and 4 are jointly sponsored by the Casualty Actuarial Society and the Society of Actuaries and were held in May and November 2000. Candidates who were successful on these examinations were listed in joint releases of the two Societies.

The following candidates were admitted as Fellows and Associates at the 2000 CAS Spring Meeting in May. By passing Fall 1999 CAS examinations, these candidates successfully fulfilled the Society requirements for Fellowship or Associateship designation.

NEW FELLOWS

Amy Petea Angell
Mark E. Bohrer
Julie Burdick
Robert Neil Campbell
Mark Kelly Edmunds
Brandon Lee Emlen
Christopher Todd Hochhausler
Brandelyn C. Klenner
Elaine Lajeunesse
Diana Mary Susan Linehan
Scott A. McPhee
Kathy Popejoy
Richard A. Rosengarten
Meyer Shields

NEW ASSOCIATES

Jodie Marie Agan
Brian M. Ancharski
Kevin L. Anderson
Deborah Herman Ardern
Patrick Barbeau
Jody J. Bembenek
Ellen A. Berning
Brad D. Birtz
Mary Denise Boarman
Veronique Bouchard
Thomas Leininger Boyer II
David C. Brueckman
Angela D. Burgess
Patrick J. Charles
Harry Sigen Chen
Brian Kenneth Ciferri
Susan M. Cleaver
Richard Jason Cook
Kevin A. Cormier
Jeffrey Alan Courchene
Mary Elizabeth Frances Cunningham
Patricia A. Deo-Campo Vuong

Jean-François Desrochers
Mark Richard Desrochers
Michael Devine
Kevin George Donovan
Louis-Christian Dupuis
Donna L. Emmerling
Keith A. Engelbrecht
Laura Ann Esboldt
Juan Espadas
Farzad Farzan
Donia Burris Freese
Shina Noel Fritz
Cynthia Galvin
Michael Anthony Garcia
Hannah Gee
James Brian Gilbert
Joseph Emmanuel Goldman
Andrew Samuel Golfin Jr.
Olga Golod
Stacey C. Gotham
Mark R. Greenwood
Chantal Guillemette
James Christopher Guszcza
David Bruce Hackworth
Dawn Marie S. Happ
Jason Carl Head
Pamela Barlow Heard
Kristina S. Heer
Hans Heldner
Robert C. Hill
David E. Hodges
Richard Michael Holtz
Allen J. Hope
Carol Irene Humphrey
Rusty A. Husted
Michael Stanley Jarmusik
Patrice Jean
Charles B. Jin
Steven M. Jokerst
Cheryl R. Kellogg
James F. King
Jill E. Kirby
Omar A. Kitchlew
Henry Joseph Konstanty
Rebecca Michelle Kristal
Darjen D. Kuo
Christine L. Lacke
Bobb J. Lackey
Jean-François Larochelle
Peter H. Latshaw
Doris Lee
William Scott Lennox
Joshua Yuri Ligosky
Erik Frank Livingston
Richard Paul Lonardo
William F. Loyd III
Alexander Peter Maizys
Victor Mata
David Michael Maurer
Timothy C. McAuliffe
Richard J. McElligott
Jennifer A. McGrath
Martin Menard
Mitchel Merberg
Vadim Y. Mezhebovsky
Eric Millaire-Morin
Suzanne A. Mills
Matthew Kevin Moran
Lambert Morvan
Thomas M. Mount
Rebecca E. Mozi
Ronald Taylor Nelson
Michael Dale Neubauer
Loren J. Nickel
Ajay Pahwa
Cosimo Pantaleo
Michael Thomas Patterson
Wendy Wei-Chi Peng
Jill E. Peppers
Michael C. Petersen
Kevin Thomas Peterson
Kraig Paul Peterson
Kristin Sarah Piltzecker
Sean Evans Porreca
Warren T. Printz
Stephen Daniel Riihimaki
Ezra Jonathan Robison
Bryant Edward Russell
Frederick Douglas Ryan
Laura Beth Sachs

Salimah H. Samji
Rachel Samoil
Jennifer Arlene Scher
Daniel David Schlemmer
Parr T. Schoolman
Ernest C. Segal
Michelle L. Sheppard
Paul O. Shupe
Lee Oliver Smith
Lora L. Smith-Sarfo
Scott G. Sobel
Wendy Rebecca Speert
Mary Jane Sperduto
Christine Steele-Koffke
Gary A. Sudbeck
Jonathan Leigh Summers
Neeza Thandi
Tanya K. Thielman
John David Trauffer
Nathalie Tremblay
Matthew L. Uhoda
Dennis R. Unver
Richard Alan Van Dyke
Josephine M. Waldman
Colleen Ohle Walker
Kristie L. Walker
Tice R. Walker
Wade Thomas Warriner
Kelly M. Weber
Petra Lynn Wegerich
Christopher John Westermeyer
Karin H. Wohlgemuth
Terry C. Wolfe
Mihoko Yamazoe
Nora J. Young

The following candidates successfully completed the following Spring 2000 CAS examinations.

Exam 5

Keith P. Allen
Tanuja S. Alwar
Vagif Amstislavskiy
Julie A. Anderson
Amy J. Antenen
Katherine H. Antonello
Melissa J. Appenzeller
Kevin J. Atinsky
Joel E. Atkins
Nicki C. Austin
Gregory K. Bangs
Danielle L. Bartosiewicz
Thomas C. Bates
Rick D. Beam
Esther Becker
Chad M. Beehler
James H. Bennett
Andrew W. Bernstein
Nebojsa Bojer
Joseph V. Bonanno Jr.
Jean-Philippe Boucher
Judy L. Boutchee
Bernardo Bracero Jr.
Melissa L. Brewer
Claude B. Bunick
Don J. Burbacher
Scott W. Carpinteri
Simon Castonguay
Thomas L. Cawley
John Celidonio
Phyllis B. Chan
James Chang
Jennifer A. Charlonne
Benjamin W. Clark
Wesley G. Clarke
Paul L. Cohen
Marlene Marie Collins
Christopher L. Cooksey
Kathleen M. Cooper
David C. Coplan
Leanne M. Cornell
Michael J. Covert
Sandra Creaney
Russell A. Creed
Arthur D. Cummings
David W. Dahlen
Rich A. Davey
David E. Dela Cruz
David A. DeNicola

Romulo N. Deo-Campo Vuong
Ryan M. Diehl
Erik L. Donahue
Brian M. Donlan
Kevin P. Donnelly
Pamela G. Doonan
Crisanto A. Dorado
Jeffrey S. Ernst
Brian A. Fannin
Wendy A. Farley
Robert E. Farnam
Kevin M. Finn
Ellen D. Fitzsimmons
Christine M. Fleming
Louise Frankland
Gregory A. Frankowiak
Jeffrey J. Fratantaro
François Fugere
Andre Gagnon
Patrick P. Gallagher
William J. Gerhardt
Gregory Evan Gilbert
John S. Giles
William G. Golush
Jennifer Graunas
Stacie R.W. Grindstaff
Stephanie A. Groharing
Jason L. Grove
Kanwal Hameed
Kimberly Baker Hand
Sunny M. Harrington
Susan M. Harris
Guo Harrison
Stuart J. Hayes
Joseph Hebert
William S. Hedges
Rhonda R. Hellman
Kathryn E. Herzog
Brandon L. Heutmaker
Daniel D. Heyer
Stephen J. Higgins Jr.
Joseph S. Highbarger
Michael F. Hobart
Jeremy A. Hoch
Melissa S. Holt
Elizabeth J. Hudson
Craig D. Isaacs
Katherine Jacques
Gregory O. Jaynes
Julie A. Jordan
Hye-Sook Kang
John J. Karwath
Lawrence S. Katz
Susan M. Keaveny
Douglas H. Kemppainen
Young Y. Kim
Patricia Kinghorn
Jennifer E. Kish
Anne Marie Klein
Jeff A. Kluck
Laurie A. Knoke
Steven M. Koester
Anand S. Kulkarni
Gregory E. Kushnir
François Lacroix
Heather D. Lake
Michael A. Lardis
Michael L. Laufer
Nathalie M. Lavigne
Damon T. Lay
Anh Tu Le
Patricia Lee
Ruth M. LeStourgeon
Amanda M. Levinson
Jonathan D. Levy
Hayden Anthony Lewis
Wei Li
Monika Lietz
Eric F. Liland
David Grant Lim
Steven R. Lindley
Laura J. Lothschutz
Daniel A. Lowen
Hazel J. Luckey
Teresa Madariaga
Steven Manilov
Luis S. Marques
Joseph W. Mawhinney
Stephane McGee
Shaun P. McGovern
Hernan L. Medina
Charles W. Mitchell
Camilo Mohipp
Celso M. Moreira
Alan E. Morris
Matthew D. Myshrall
John A. Nauss
Scott L. Negus
Jennifer Y. Nei
Shannon P. Newman
Lester M. Y. Ng
Khanh K. Nguyen
Norman Niami

Stoyko N. Nikolov
Matthew P. Nimchek
Alejandra S. Nolibos
Charles A. Norton
Tom E. Norwood
Joy-Ann C. Payne
Dianne M. Phelps
Daniel P. Post
Stephen R. Prevatt
Lester Pun
Lovely G. Puthenveetil
Edward L. Pyle
Michael J. Quigley
William C. Reddington
Neil W. Reiss
Peggy-Anne K. Repella
Danielle L. Richards
Laura D. Rinker
Joseph L. Rizzo
Charles A. Romberger
Ryan P. Royce
Giuseppe Russo
Doris Y. Schirmacher
Brett M. Shereck
Junning Shi
Jimmy Shkolyar
Jeremy D. Shoemaker
Summer L. Sipes
Douglas E. Smith
Michael D. Sowka
Laura T. Sprouse
Karine St-Onge
Wei Hua Su
Adam D. Swope
Michelle M. Syrotynski
Ming Tang
Hugh T. Thai
Christian A. Thielman
Matthew D. Trone
David S. Udall
Stephen H. Underhill
Mary Vacirca
Paul A. Vendetti
Brian A. Viscusi
Natalie Vishnevsky
Cameron Jason Vogt
Matthew J. Walter
Bethany R. Webb
Robert S. Weishaar
Jean P. West
Carolyn D. Wettstein
Shannon A. Whalen
Joel D. Whitcraft
Paul D. Wilbert
Chun Shan Wong
Walter R. Wulliger
Run Yan
Huey Wen Yang
Joshua A. Youdovin
Janice M. Young
Jonathan K. Yu
Grace Zakaria
Larry Xu Zhang
Lianmin Zhou

Exam 7—Canada

Genevieve L. Allen
Anna Marie Beaton
Patrick Beaudoin
Chantal Guillemette
Patricia A. Hladun
Isabelle La Palme
Julie-Linda Laforce
Christian Lemay
Julie Martineau
Ian John McCracken
Martin Menard
Lambert Morvan
Sylvain Nolet
Nathalie Ouellet
Cosimo Pantaleo
Julie Perron
Sylvain Renaud
Claude A. Wagner

Exam 7—United States

Afrouz Assadian
Maura Curran Baker
Phil W. Banet
Emmanuil Theodore Bardis
Nicolas Beaupre
Jody J. Bembenek
Jeremy Todd Benson
David Matthew Biewer
Linda Jean Bjork
Michael J. Bluzer
Neil M. Bodoff
Daniel R. Boerboom
Christopher David Bohn
Jerelyn S. Boysia
Tobias E. Bradley
Maureen B. Brennan
Jeremy James Brigham
Sara T. Broadrick
Karen Ann Brostrom
Stephanie Anne Bruno
Hugh Eric Burgess
Anthony Robert Bustillo
Ronald S. Cederburg
Joseph Gerald Cerreta
Hao Chai
Kin Lun (Victor) Choi
Andrew K. Chu
Kuei-Hsia Ruth Chu
Christopher J. Claus
Susan M. Cleaver
Jeffrey J. Clinch
Hugo Corbeil
Thomas Cosenza
Jeffrey Alan Courchene
David Francis Dahl
Mujtaba H. Datoo
Kris D. DeFrain
Peter R. DeMallie
John T. Devereux
Sean R. Devlin
Dean P. Dorman
Sharon C. Dubin
Louis-Christian Dupuis
Sophie Duval
Kevin M. Dyke
Jane Eichmann
James Robert Elicker
Richard James Engelhuber
Gregory James Engl
Weishu Fan
Kenneth D. Fikes
William M. Finn
Chauncey Edwin Fleetwood
Sean Paul Forbes
Feifei Ford
James M. Gallagher
Dustin Wayne Gary
Christie L. Gilbert
Patrick J. Gilhool
Bradley G. Gipson
Stacey C. Gotham
Robert A. Grocock
Edward Kofi Gyampo
Rebecca N. Hai
Kenneth Jay Hammell
Alexander Archibold Hammett
Michael S. Harrington
James Anthony Heer
Kristina S. Heer
Kevin B. Held
Scott E. Henck
Mark D. Heyne
Kurt D. Hines
Amy L. Hoffman
Suzanne Barry Holohan
Derek Reid Hoyme
Christopher Wayne Hurst
Jamison Joel Ihrke
Weidong Wayne Jiang
Philippe Jodin
Shantelle Adrienne Johnson
Tricia Lynne Johnson
William Russell Johnson
Bryon Robert Jones
Dana F. Joseph
Sean M. Kennedy
David R. Kennerud
Susanlisa Kessler
Joseph E. Kirsits
Scott C. Kurban
Elizabeth A. Kurina
Chingyee Teresa Lam
Peter H. Latshaw
Thomas V. Le

Doris Lee
Michael E. Lewis
Matthew Allen Lillegard
Dengxing Lin
Shu C. Lin
Jing Liu
Kathleen T. Logue
Elizabeth Long
Michelle Luneau
Atul Malhotra
Sharon L. Markowski
James J. Matusiak Jr.
Timothy J. McCarthy
William R. McClintock
John R. McCollough
Christian Menard
Ulysis V. Mensah-Bonsu
Vadim Y. Mezhebovsky
Ryan A. Michel
Scott Allan Miller
Rodney S. Morris
Sharon D. Mott
Kari Sue Mrazek
Joseph J. Muccio
Brian C. Neitzel
Kari A. Nicholson
Sean Robert Nimm
Kathleen C. Odomirok
Richard Alan Olsen
Michael A. Onofrietti
Rodrick Raymond Osborn
Matthew R. Ostiguy
Chad Michael Ott
Michael Guerin Owen
Robert A. Painter
Mark Paykin
Kraig Paul Peterson
Michael Robert Petrarca
Christopher A. Pett
Andrea L. Phillips
Anthony George Phillips
Kristin Sarah Piltzecker
Jorge E. Pizarro
Dylan P. Place
Jayne L. Plunkett
Igor Pogrebinsky
Gregory John Poirier
Mitchell S. Pollack
Gregory T. Preble
Warren T. Printz
Ni Qin-Feng
Ricardo Anthony Ramotar
Leonid Rasin
Jennifer L. Richard
Mario Richard
Romel G. Salam
Rachel Samoil
James C. Sandor
Gary Frederick Scherer
Daniel David Schlemmer
Parr T. Schoolman
Annmarie Schuster
Stuart A. Schweidel
Kelvin B. Sederburg
Tina Shaw
Michelle L. Sheppard
Alastair Charles Shore
Jeffery J. Smith
Anthony A. Solak
Wendy Rebecca Speert
Christopher M. Steinbach
Mark Richard Strona
Jonathan Garrett Taylor
Neeza Thandi
Mary A. Theilen
Christopher S. Throckmorton
Ellen Marie Tierney
Michael J. Toth
Michael C. Tranfaglia
Brian K. Turner
Kieh Tsung Ty
Dennis R. Unver
Peggy J. Urness
William D. Van Dyke
Gaetan R. Veilleux
Mark Alan Verheyen
Martin Vezina
Jennifer Anne Vezza
Nathan Karl Voorhis
Josephine M. Waldman
Tice R. Walker
Felicia Wang
Shaun S. Wang
Douglas M. Warner
David W. Warren
Kelly M. Weber

Christopher John Westermeyer
Apryle L. Williams
Brandon L. Wolf
Stephen K. Woodard
Mark Lee Woods
Scott Michael Woomer
Jimmy L. Wright
Linda Yang
Stephanie C. Young
Michael R. Zarember
Xiangfei Zeng
Yin Zhang
Eric Zlochevsky

Exam 8

Jason R. Abrams
John Scott Alexander
Michele Segreti Arndt
Anju Arora
Carl Xavier Ashenbrenner
David Steen Atkinson
Michael William Barlow
Keith Michael Barnes
Andrew Steven Becker
David R. Border
Sherri Lynn Border
Veronique Bouchard
Erik R. Bouvin
Conni Jean Brown
Robert F. Brown
David C. Brueckman
John C. Burkett
Stephanie T. Carlson
Allison Faith Carp
Patrick J. Charles
Kin Lun (Victor) Choi
Wei Chuang
Steven A. Cohen
Larry Kevin Conlee
Kathleen T. Cunningham
Mary Elizabeth Frances Cunningham
Jonathan Scott Curlee
Loren Rainard Danielson
Michael Brad Delvaux
Kevin Francis Downs
Tammi B. Dulberger
Louis Durocher
Laura Ann Esboldt
Jonathan Palmer Evans
Joseph Gerard Evleth
Vicki Agerton Fendley
Hugo Fortin
Ron Fowler
Noelle Christine Fries
Cynthia Galvin
Emily C. Gilde
Susan I. Gildea
Isabelle Gingras
Theresa Giunta
Todd B. Glassman
Sanjay Godhwani
Matthew R. Gorrell
Lisa N. Guglietti
Nasser Hadidi
Brian D. Haney
Dawn Marie S. Happ
Michelle Lynne Harnick
Bryan Hartigan
Jeffery Tim Hay
Qing He
Amy Louise Hicks
Jay T. Hieb
Glenn R. Hiltpold
Todd Harrison Hoivik
Nancy Michelle Hoppe
Walter Leon Jedziniak
Charles B. Jin
Susan K. Johnston
Lori Edith Julga
Elina Koganski
Claudia Anita Krucher
Kimberly J. Kurban
Robin M. LaPrete
Travis J. Lappe
Aaron Michael Larson
Borwen Lee
Lewis Y. Lee
Neal Marev Leibowitz
Charles Letourneau
James P. Lynch
Kelly A. Lysaght
Jason K. Machtinger
Daniel Patrick Maguire

Andrea Wynne Malyon
Joshua Nathan Mandell
Michael E. Mielzynski
Eric Millaire-Morin
Ain Milner
Paul David Miotke
Benoit Morissette
Kevin T. Murphy
Seth Wayne Myers
Mihaela Luminita S. O'Leary
Richard D. Olsen
Christy Beth Olson
Rebecca Ruth Orsi
Michael A. Pauletti
John M. Pergrossi
Christopher Kent Perry
Daniel Berenson Perry
Richard Matthew Pilotte
Glen-Roberts Pitruzzello
Jordan J. Pitz
Sean Evans Porreca
Christopher David Randall
Peter S. Rauner
Ellen J. Respler
Rebecca L. Roever
David A. Rosenzweig
Joseph J. Sacala
James C. Santo
Jason Thomas Sash
William Harold Scully III
Steven George Searle
Ernest C. Segal
Joseph Allen Smalley
James M. Smieszkal
Anya K. Sri-Skanda-Rajah
Carol A. Stevenson
Roman Svirsky
Karrie Lynn Swanson
Chester John Szczepanski
Varsha A. Tantri
Michael Joseph Tempesta
Robert M. Thomas II
Michael C. Torre
Laura M. Turner
William B. Westrate
V. Clare Whitlam
Kendall P. Williams
Laura Markham Williams
Dean M. Winters
Jonathan Woodruff
Yuhong Yang

The following candidates were admitted as Fellows and Associates at the 2000 CAS Annual Meeting in November. By passing Spring 2000 CAS examinations, these candidates successfully fulfilled the Society requirements for Fellowship or Associateship designation.

NEW FELLOWS

John Scott Alexander
Michele Segreti Arndt
Carl Xavier Ashenbrenner
David Steen Atkinson
Emmanuil Theodore Bardis
Michael William Barlow
Keith Michael Barnes
Patrick Beaudoin
Nicolas Beaupre
Andrew Steven Becker
Michael J. Bluzer
Sherri Lynn Border
Veronique Bouchard
Erik R. Bouvin
Jerelyn S. Boysia
Tobias E. Bradley
Robert F. Brown
Hugh Eric Burgess

Stephanie T. Carlson
Allison Faith Carp
Joseph Gerald Cerreta
Patrick J. Charles
Kin Lun (Victor) Choi
Andrew K. Chu
Kuei-Hsia Ruth Chu
Wei Chuang
Steven A. Cohen
Larry Kevin Conlee
Kathleen T. Cunningham
Mary Elizabeth Frances Cunningham
Jonathan Scott Curlee
Loren Rainard Danielson
Kris D. DeFrain
Michael Brad Delvaux
Sean R. Devlin
Tammi B. Dulberger
Louis Durocher
Sophie Duval
Kevin M. Dyke
Jane Eichmann
Gregory James Engl
Vicki Agerton Fendley
Kenneth D. Fikes
Chauncey Edwin Fleetwood
Hugo Fortin
Ron Fowler
Noelle Christine Fries
Susan I. Gildea
Todd B. Glassman
Sanjay Godhwani
Rebecca N. Hai
Kenneth Jay Hammell
Alexander Archibold Hammett
Michelle Lynne Harnick
Jeffery Tim Hay
Qing He
Amy Louise Hicks
Jay T. Hieb
Amy L. Hoffman
Todd Harrison Hoivik
Nancy Michelle Hoppe
Walter Leon Jedziniak
Charles B. Jin
Philippe Jodin
Lori Edith Julga
Elina Koganski
Claudia Anita Krucher
Chingyee Teresa Lam
Travis J. Lappe
Robin M. LaPrete
Lewis Y. Lee
Neal Marev Leibowitz
Charles Letourneau
Dengxing Lin
Shu C. Lin
Michelle Luneau
Andrea Wynne Malyon
Ian John McCracken
Ain Milner
Paul David Miotke
Benoit Morissette
Kari Sue Mrazek
Kevin T. Murphy
Seth Wayne Myers
Kari A. Nicholson
Mihaela Luminita S. O'Leary
Richard Alan Olsen
Richard D. Olsen
Rebecca Ruth Orsi
Nathalie Ouellet
Michael Guerin Owen
Mark Paykin
Julie Perron
Christopher Kent Perry
Daniel Berenson Perry
Anthony George Phillips
Richard Matthew Pilotte
Glen-Roberts Pitruzzello
Igor Pogrebinsky
Gregory John Poirier
Ricardo Anthony Ramotar
Christopher David Randall
Leonid Rasin
Peter S. Rauner
Ellen J. Respler
Rebecca L. Roever
David A. Rosenzweig
Romel G. Salam
James C. Santo
Jason Thomas Sash
Stuart A. Schweidel
William Harold Scully III
Ernest C. Segal

Christopher M. Steinbach
Carol A. Stevenson
Roman Svirsky
Karrie Lynn Swanson
Chester John Szczepanski
Varsha A. Tantri
Jonathan Garrett Taylor
Michael Joseph Tempesta
Robert M. Thomas II
Michael J. Toth
Michael C. Tranfaglia
Laura M. Turner
Kieh Tsung Ty
Martin Vezina
Nathan Karl Voorhis
Claude A. Wagner
William B. Westrate
Kendall P. Williams
Laura Markham Williams
Brandon L. Wolf
Jonathan Woodruff
Yuhong Yang

NEW ASSOCIATES

Katherine H. Antonello
Maura Curran Baker
Jeremy Todd Benson
Neil M. Bodoff
Peter R. DeMallie
William M. Finn
Dustin Wayne Gary
Amy L. Gebauer
Patrick J. Gilhool
Mark D. Heyne
Kurt D. Hines
Patricia A. Hladun
Derek Reid Hoyme
Craig D. Isaacs
Gregory K. Jones
Sean M. Kennedy
David R. Kennerud
Susanlisa Kessler
Jing Liu
Kathleen T. Logue
Julie Martineau
James J. Matusiak Jr.
John R. McCollough
Rodney S. Morris
Brian C. Neitzel
Sean Robert Nimm
Sylvain Nolet
Rodrick Raymond Osborn
Christopher S. Throckmorton
Mark Alan Verheyen
Felicia Wang
Shaun S. Wang
Eric Zlochevsky

The following candidates successfully completed the following Fall 2000 CAS examinations.

Exam 6

Vera E. Afanassieva
Vagif Amstislavskiy
Jonathan L. Ankney
Koosh Arfa-Zanganeh
Afrouz Assadian
Kevin J. Atinsky
Joel E. Atkins
Rick D. Beam
Marie-Eve J. Belanger
David Matthew Biewer
Jean-Philippe Boucher
Erick A. Brandt
Maureen B. Brennan
Sara T. Broadrick
Stephanie Anne Bruno
Don J. Burbacher
James E. Calton
Mary Ellen Cardascia
William Brent Carr
Jennifer L. Caulder
Phyllis B. Chan

Yves Charbonneau
Jennifer A. Charlonne
Alan M. Chow
Benjamin W. Clark
Kevin M. Cleary
Paul L. Cohen
Cameron A. Cook
Christopher L. Cooksey
Hugo Corbeil
Leanne M. Cornell
Michael J. Covert
Hall D. Crowder
Arthur D. Cummings
Keith R. Cummings
Aaron T. Cushing
David W. Dahlen
Willie L. Davis
Paul B. Deemer
Romulo N. Deo-Campo Vuong
Ryan M. Diehl
Christopher P. DiMartino
Laura S. Doherty
Erik L. Donahue
Brian M. Donlan
Kevin P. Donnelly
Pamela G. Doonan
Crisanto A. Dorado
Dennis Herman Dunham
Gregory L. Dunn
Ruchira Dutta
Jeffrey A. Dvinoff
Tomer Eilam
Julie A. Ekdom
Kyle A. Falconbury
Brian A. Fannin
Wendy A. Farley
William J. Fogarty
Feifei Ford
Dana R. Frantz
Patrick P. Gallagher
Genevieve Garon
Keith R. Gentile
William G. Golush
Christopher J. Grasso
Donald B. Grimm
Isabelle Groleau
Jason L. Grove
Edward Kofi Gyampo
Brian P. Hall
Barbara Hallock
Guo Harrison
Eric A. Hatch
Stuart J. Hayes
James Anthony Heer
Suzanne Barry Holohan
Cheng-Chi Huang
Christopher Wayne Hurst
Jamison Joel Ihrke
Katherine Jacques
Brian B. Johnson
Erik A. Johnson
Shantelle Adrienne Johnson
Tricia Lynne Johnson
William Russell Johnson
Hye-Sook Kang
Barbara L. Kanigowski
Kathryn E. Keehn
Stacey M. Kidd
Joseph E. Kirsits
Anne Marie Klein
Laurie A. Knoke
Robert A. Kranz
Anand S. Kulkarni
Gregory E. Kushnir
Kristine Kuzora
François Lacroix
Heather D. Lake
James A. Landgrebe
Michael A. Lardis
Francis A. Laterza
Jason A. Lauterbach
Geraldine Marie Z. Lejano
James J. Leonard
Amanda M. Levinson
Jonathan D. Levy
Michael E. Lewis
Matthew Allen Lillegard
Nataliya A. Loboda
Daniel A. Lowen
Tai-Kuan Ly
Teresa Madariaga
Richard J. Manship
Laura A. Maxwell
Timothy J. McCarthy
Jeffrey B. McDonald
John D. McMichael
Scott Allan Miller
Charles W. Mitchell

Camilo Mohipp
Matthew E. Morin
Sharon D. Mott
Yuchun Mu
Jacqueline L. Neal
Shannon P. Newman
Kee Heng Ng
Lester M. Y. Ng
Norman Niami
Alejandra S. Nolibos
Tom E. Norwood
Miodrag Novakovic
Nancy Eugenia O'Dell-Warren
Michael A. Onofrietti
Matthew R. Ostiguy
Chad Michael Ott
Robin V. Padwa
Robert A. Painter
Isabelle Perron
Michael Robert Petrarca
Jeffrey J. Pfluger
Dianne M. Phelps
Jayne L. Plunkett
Daniel P. Post
Gregory T. Preble
Lester Pun
Julie-Ann Puzzo
John T. Raeihle
Kathleen M. Rahilly-VanBuren
William C. Reddington
Jennifer L. Richard
Nancy Ross
David A. Royce
Ryan P. Royce
Giuseppe Russo
Doris Y. Schirmacher
Brett M. Shereck
Junning Shi
Jimmy Shkolyar
Jeremy D. Shoemaker
James S. Shoenfelt
Steven A. Smith II
Thomas M. Smith
Karine St-Onge
Lisa C. Stanley
Jason D. Stubbs
Wei Hua Su
Ellen Marie Tierney
Malgorzata Timberg
Michael C. Torre
Tamara L. Trawick
Stephen H. Underhill
Jennifer L. Vadney
Gaetan R. Veilleux
Paul A. Vendetti
Jennifer Anne Vezza
Amy R. Waldhauer
Keith A. Walsh
Robert S. Weishaar
Thomas E. Weist
Jean P. West
Carolyn D. Wettstein
Scott Michael Woomer
Joshua C. Worsham
Jimmy L. Wright
Jennifer X. Wu
Huey Wen Yang
Sung G. Yim
Stephanie C. Young
Michael R. Zarember
Xiangfei Zeng
Larry Xu Zhang

Exam 9

Stephen A. Alexander
Brian M. Ancharski
Paul D. Anderson
Anju Arora
Peter Attanasio
Jeremy Todd Benson
Brad D. Birtz
Tony Francis Bloemer
Christopher David Bohn
David R. Border
Jeremy James Brigham
Conni Jean Brown
Hayden Heschel Burrus
Fatima E. Cadle
Todd Douglas Cheema
Wanchin W. Chou
Jeffrey Alan Courchene
John Edward Daniel
Patricia A. Deo-Campo Vuong
Scott H. Drab

Sara P. Drexler
Louis-Christian Dupuis
James Robert Elicker
Richard James Engelhuber
Laura Ann Esboldt
Joseph Gerard Evleth
Weishu Fan
Dustin Wayne Gary
Amy L. Gebauer
Christie L. Gilbert
Emily C. Gilde
Theresa Giunta
Stacey C. Gotham
Stephanie Ann Gould
Joseph P. Greenwood
Jacqueline Lewis Gronski
John A. Hagglund
David Lee Handschke
Michael S. Harrington
Bryan Hartigan
Kevin B. Held
Hans Heldner
Scott E. Henck
Mark D. Heyne
Kurt D. Hines
Richard Michael Holtz
Eric J. Hornick
Derek Reid Hoyme
Li Hwan Hwang
Susan Elizabeth Innes
Randall Allen Jacobson
Daniel R. Kamen
John J. Karwath
Douglas H. Kemppainen
David R. Kennerud
Michael G. Kerner
Susanlisa Kessler
Henry Joseph Konstanty
Kimberly J. Kurban
Julie-Linda Laforce
Jean-Sebastien Lagace
Stephane Lalancette
Jean-François Larochelle
Aaron Michael Larson
Kan Yuk A. Lau
Michael L. Laufer
Kathleen T. Logue
James P. Lynch
Kelly A. Lysaght
Daniel Patrick Maguire
Atul Malhotra
Julie Martineau
Jennifer A. McGrath
Christian Menard
Eric Millaire-Morin
Lisa J. Moorey
Lambert Morvan
Ethan Charles Mowry
Joseph J. Muccio
Michael Douglas Nielsen
Stoyko N. Nikolov
Christopher Maurice Norman
Rodrick Raymond Osborn
Cosimo Pantaleo
Michael A. Pauletti
Jeremy Parker Pecora
John M. Pergrossi
Sylvain Perrier
Kristin Sarah Piltzecker
Jordan J. Pitz
Sean Evans Porreca
Mario Richard
Brad E. Rigotty
Stephen Daniel Riihimaki
Delia E. Roberts
Ezra Jonathan Robison
Joseph J. Sacala
Gary Frederick Scherer
Doris Y. Schirmacher
Jeffery Wayne Scholl
Annmarie Schuster
Michelle L. Sheppard
Alastair Charles Shore
Joseph Allen Smalley
Anya K. Sri-Skanda-Rajah
Avivya Simon Stohl
Stephen James Talley
Mary A. Theilen
Beth S. Thompson
Jennifer L. Throm
Michael C. Torre
Richard Alan Van Dyke
Mark Alan Verheyen
Cameron Jason Vogt
Mary Elizabeth Waak

Shaun S. Wang
David W. Warren
Karen E. Watson
Kelly M. Weber
William B. Wilder
David S. Wolfe
Windrie Wong
Mark Lee Woods
Rick A. Workman
Run Yan
Gene Q. Zhang

NEW FELLOWS ADMITTED IN MAY 2000

Row 1 (bottom to top): Amy Petea Angell, Richard A. Rosengarten, Meyer Shields, Brandon Lee Emlen, Christopher Todd Hochhausler, and Mark Kelly Edmunds. **Row 2 (bottom to top):** Elaine Lajeunesse, Kathy Popejoy, Diana Mary Susan Linehan, Julie Burdick, Brandelyn C. Klenner, Mark E. Bohrer, and Robert Neil Campbell. **Row 3: CAS President Alice H. Gannon. New Fellow not pictured:** Scott A. McPhee.

New Associates Admitted in May 2000

Row 1 (bottom to top): Cosimo Pantaleo, Shina Noel Fritz, Doris Lee, Jill E. Peppers, and Richard Michael Holtz. **Row 2 (bottom to top):** Josephine M. Waldman, Lora L. Smith-Sarfo, Kelly M. Weber, Michelle L. Sheppard, Richard Paul Lonardo, Petra Lynn Wegerich, Jody J. Bembenek, and Karin H. Wohlgemuth. **Row 3 (bottom to top):** Cynthia Galvin, Patricia A. Deo-Campo Vuong, Susan M. Cleaver, Victor Mata, Jodie Marie Agan, Steven M. Jokerst, Michael Dale Neubauer, and Mark Richard Desrochers. **Row 4 (bottom to top):** Nora J. Young, Kevin George Donovan, Lee Oliver Smith, Dennis R. Unver, Michael Devine, Michael Anthony Garcia, James Brian Gilbert, and Joshua Yuri Ligosky. **Row 5: CAS President Alice H. Gannon.**

New Associates Admitted in May 2000

Row 1: CAS President Alice H. Gannon. Row 2 (bottom to top): Warren T. Printz, Sean Evans Porreca, Henry Joseph Konstanty, Brad D. Birtz, Cheryl R. Kellogg, Ajay Pahwa, and Allen J. Hope. **Row 3 (bottom to top):** Parr T. Schoolman, Wendy Rebecca Leferson, Kristina Shannon Heer, David Bruce Hackworth, and Brian Kenneth Ciferri. **Row 4 (bottom to top):** Kevin L. Anderson, Mihoko Yamazoe, Patrick J. Charles, Ernest C. Segal, Andrew Samuel Golfin Jr., and Christine L. Lacke. **Row 5 (bottom to top):** David E. Hodges, Tanya K. Thielman, Angela D. Burgess, Kristin Sarah Piltzecker, Carol Irene Humphrey, and Frederick Douglas Ryan.

NEW ASSOCIATES ADMITTED IN MAY 2000

Row 1: CAS President Alice H. Gannon. Row 2 (bottom to top): Stephen Daniel Riihimaki, Tice R. Walker, Martin Menard, Erik Frank Livingston, Jonathan Leigh Summers, Donna L. Emmerling, Gary A. Sudbeck, and Nathalie Tremblay. **Row 3 (bottom to top):** Jennifer Arlene Scher, Rachel Samoil, Lambert Morvan, Kristie L. Walker, Alexander Peter Maiszys, Scott G. Sobel, and Pamela Barlow Heard. **Row 4 (bottom to top):** Salimah H. Samji, Colleen Ohle Walker, Timothy C. McAuliffe, Daniel David Schlemmer, Stacey C. Gotham, Dawn Marie S. Happ, Ezra Jonathan Robison, and Olga Golod. **Row 5 (bottom to top):** Joseph Emmanuel Goldman, Bobb J. Lackey, Ellen A. Berning, Brian M. Ancharski, Omar A. Kitchlew, Paul O. Shupe, James Christopher Guszcza, and John David Trauffer.

New Associates Admitted in May 2000

Row 1: CAS President Alice H. Gannon. Row 2 (bottom to top): Donia Burris Freese, Rebecca E. Miller, Jennifer A. McGrath, Juan de la Cruz Espadas, William Scott Lennox, Thomas L. Boyer II, and Peter H. Latshaw. **Row 3 (bottom to top):** Mary Jane Sperduto, Laura Anne Esboldt, Michael Stanley Jarmusik, Mary Denise Boarman, Mary Elizabeth Frances Cunningham, David C. Brueckman, and Ronald Taylor Nelson. **Row 4 (bottom to top):** Charles Biao Jin, Matthew L. Uhoda, Jason Carl Head, Hans Heldner, and Matthew Kevin Moran. **Row 5 (bottom to top):** Richard Jason Cook, Farzad Farzan, Christopher John Westermeyer, Richard Alan Van Dyke, Vadim Y. Mezhebovsky, and Wade T. Warriner.

NEW ASSOCIATES ADMITTED IN MAY 2000

Row 1 (bottom to top): Chantal Guillemette, Jean-François Larochelle, Jean-François Desrochers, Patrice Jean, Thomas M. Mount, and Keith A. Engelbrecht. **Row 2 (bottom to top):** Veronique Bouchard, Jill E. Kirby, Michael Thomas Patterson, Mitchel Merberg, Eric Millaire-Morin, and Kraig Paul Peterson. **Row 3 (bottom to top):** Louis-Christian Dupuis, Laura B. Sachs, Deborah Herman Ardern, Kevin Thomas Peterson, David Michael Maurer, Kevin A. Cormier, and Terry C. Wolfe. **Row 4: CAS President Alice H. Gannon. New Associates not pictured:** Patrick Barbeau, Harry Sigen Chen, Jeffrey Alan Courchene, Hannah Gee, Mark R. Greenwood, Rusty A. Husted, James F. King, Darjen D. Kuo, Rebecca Michelle Locks, William F. Loyd III, Richard J. McElligott, Suzanne A. Mills, Loren J. Nickel, Wendy We-Chi Peng, Michael C. Petersen, Bryant Edward Russell, Christine Steele-Koffke, and Neeza Thandi.

New Fellows Admitted in November 2000

First row, from left: Chingyee Teresa Lam, Todd Bennett Glassman, Nathalie Ouellet, Kari A. Nicholson, **CAS President Alice H. Gannon**, Amy Louise Hicks, Lewis Y. Lee, Jerelyn S. Boysia, Ernest C. Segal. **Second row, from left:** Sophie Duval, Kevin M. Dyke, Leonid Rasin, Kendall P. Williams, Michael William Barlow, Michael J. Tempesta, Charles Letourneau, Rebecca Ruth Orsi, Steven A. Cohen. **Third row, from left:** Martin Vezina, Gregory J. Poirier, Jonathan Stanger Woodruff, Brandon L. Wolf, Jay T. Hieb, Philippe Jodin, Louis Durocher, Todd Harrison Hoivik.

New Fellows Admitted in November 2000

First row, from left: Tobe E. Bradley, Ricardo A. Ramotar, Michael C. Tranfaglia, Kuei-Hsia Ruth Chu, **CAS President Alice H. Gannon**, Walter L. Jedziniak, Ain Milner, Mark Paykin, Lori E. Julga. **Second row, from left:** Christopher David Randall, Paul D. Miotke, Kin Lun (Victor) Choi, Qing He, Joseph G. Cerreta, Kevin T. Murphy, Richard A. Olsen, Jonathan Garrett Taylor, Hugh E. Burgess, Jason Thomas Sash. **Third row, from left:** Jane Eichmann, Robin M. LaPrete, Jeffery Tim Hay, Emmanuil Theodore Bardis, Seth Wayne Myers, Roman Svirsky, Richard Matthew Pilotte, Chester J. Szczepanski.

NEW FELLOWS ADMITTED IN NOVEMBER 2000

First row, from left: Nancy Michelle Hoppe, Michele S. Arndt, Michael J. Bluzer, Andrea Wynne Malyon, **CAS President Alice H. Gannon**, David A. Rosenzweig, Vicki A. Fendley, Sean R. Devlin, Amy L. Hoffman. **Second row, from left:** Carl Xavier Ashenbrenner, Laura Markham Williams, Susan I. Gildea, Neal M. Leibowitz, Hugo Fortin, Jonathan Scott Curlee, Chauncey E. Fleetwood, Elina L. Koganski, Alex A. Hammett. **Third row, from left:** Rebecca L. Roever, Larry Kevin Conlee, Carol A. Stevenson, Patrick Beaudoin, Michelle Lynne Harnick, Nicolas Beaupré, Michelle Luneau, Michael J. Toth, Shu C. Lin, Kathleen T. Cunningham.

NEW FELLOWS ADMITTED IN NOVEMBER 2000

First row, from left: Sherri Lynn Border, Veronique Bouchard, Stuart A. Schweidel, Andrew S. Becker, **CAS President Alice H. Gannon**, Daniel B. Perry, Ellen J. Respler, Andrew K. Chu, Christopher M. Steinbach. **Second row, from left:** Dengxing Lin, Romel G. Salam, Mary Elizabeth Cunningham, Sanjay Godhwani, Igor Pogrebinsky, Richard D. Olsen, William B. Westrate, Travis J. Lappe. **Third row, from left:** Charles B. Jin, Varsha A. Tantri, Loren Rainard Danielson, Robert F. Brown, Patrick J. Charles, Christopher Kent Perry, Benoit Morissette, Kenneth D. Fikes, James C. Santo, Allison F. Carp.

New Fellows Admitted in November 2000

First row, from left: William Harold Scully III, Mihaela Luminita O'Leary, Rebecca N. Hai, **CAS President Alice H. Gannon**, Kari S. Mrazek, Yuhong Yang, Noelle C. Fries. **Second row, from left:** Michael Brad Delvaux, Glen-Roberts Pitruzzello, Karrie Lynn Swanson, Robert M. Thomas II, Ronnie Samuel Fowler. **New Fellows admitted in November 2000 who are not pictured:** John Scott Alexander, David Steen Atkinson, Keith M. Barnes, Erik R. Bouvin, Stephanie T. Carlson, Wei Chuang, Kris D. DeFrain, Tammi B. Dulberger, Gregory James Engl, Kenneth Jay Hammell, Claudia A. Krucher, Ian John McCracken, Michael G. Owen, Julie Perron, Anthony George Phillips, Peter S. Rauner, Laura M. Turner, Kieh Tsung Ty, Nathan K. Voorhis, Claude A. Wagner.

NEW ASSOCIATES ADMITTED IN NOVEMBER 2000

First row, from left: Peter R. DeMallie, Susanlisa Kessler, Amy L. Gebauer, Kathleen T. Logue, **CAS President Alice H. Gannon**, Patrick J. Gilhool, Felicia Wang, John R. McCollough, Jing Liu. **Second row, from left:** Christopher S. Throckmorton, Patricia A. Hladun, Sylvain Nolet, Mark D. Heyne, Derek R. Hoyme, Kurt D. Hines, James J. Matusiak, Jr., Sean R. Nimm, Mark A. Verheyen, Maura Curran Baker. **Third row, from left:** Craig D. Isaacs, Dustin W. Gary, Neil M. Bodoff, Brian C. Neitzel, William M. Finn, David R. Kennerud, Sean M. Kennedy, Julie Martineau, Jeremy T. Benson, Rodney S. Morris.

New Associates admitted in November 2000 who are not pictured: Katherine H. Antonello, Gregory K. Jones, Rodrick R. Osborn, Shaun S. Wang, Eric Zlochevsky.

OBITUARIES

OLAF E. HAGEN
PHILLIP B. KATES
NORTON "DOC" MASTERSON
THOMAS E. MURRIN
JOHN H. ROWELL
IRWIN T. VANDERHOOF
JAMES M. WOOLERY

OLAF E. HAGEN
1909–2000

Olaf E. Hagen was born December 28, 1909.

He received his Associateship to the Casualty Actuarial Society in 1939 and his Associateship to the Society of Actuaries in 1950. In 1949, he was working for the Metropolitan Life Insurance Company in New York City. He remained with Met Life for at least 35 years, working as a senior assistant actuarial supervisor.

In 1976, he retired to Closter, New Jersey. He died in January 2000 at the age of 90.

PHILLIP B. KATES
1919–2000

Phillip B. Kates, Sr. of Tavernier, Florida passed away August 28, 2000 in Jacksonville. He was 81.

Kates was born January 14, 1919 in Texas and spent his boyhood years in the Mississippi Gulf Coast. Born during troubling economic times, Kates went to work at the age of 7 to contribute to the family's household income. This early discipline would follow him as he progressed in his career, establishing him to be a leader.

Kates began his career in insurance in 1942 with The Southern Fire and Casualty Company of Knoxville, Tennessee. While working there he met CAS Fellow Joe Linder, who guided him into a career as an actuary.

Kates' career path was interrupted by World War II. He joined the Marine Corps and volunteered to be a turret gunner and radar operator in dive-bombers. As a staff sergeant, Kates flew over 300 combat missions, including battles of Guadalcanal, and was shot down during the Battle of Iwo Jima. He received numerous decorations, which include the Air Medal and Combat Action Ribbon.

After the war, he returned to the insurance industry and enrolled in the University of Tennessee, selling vacuum cleaners on the side to help with finances. After two years at Tennessee, he transferred to the University of Michigan, graduating in 1951 with a BBA in actuarial science.

In 1946, he married Sara Louise Brogden, whom he met on a blind date. Together they had two children.

Kates' actuarial career progressed quickly, and in 1957 he became vice president and actuary of Southern Fire and Casualty Company in Knoxville and earned his Fellowship to the Casualty Actuarial Society. Kates served the CAS as a meeting panelist, member of the Committee on Annual Statement, and member and chairperson of the Committee on Sites.

In 1969, he founded Independent Fire Insurance Company and became the chairman of the board and chief executive officer in 1970. Close friend, Robert Bailey, speculated that Kates was possibly the first ever CAS member to hold such positions in an insurance company.

In 1987, Kates retired to Jacksonville, Florida, later moving to the Florida Keys where he enjoyed golf and fishing.

Survivors include his wife, Sara; daughter, Carolyn and her husband Skip Dreps of Seattle, WA; son, Phillip B. Kates Jr. and his wife Laura Kates of Jacksonville, FL; and five grandchildren.

NORTON "DOC" MASTERSON
1902–2000

Norton "Doc" Masterson died December 22, 2000, at St. Michael's Hospital in Stevens Point, Wisconsin. He was 98.

Masterson was born October 17, 1902, on a farm near St. Croix Falls in northwestern Wisconsin. He was the son of the Norton J. and Ulricka (Stenberg) Masterson. The senior Masterson was nicknamed "Doc" according to Irish folklore in which the seventh son of a seventh son is to be a healer or doctor, though he was a farmer. When Masterson's father died, the nickname was passed onto the younger Masterson, even though he became an actuary.

Masterson, who became an important voice in the school administration system in Wisconsin, began his education in a two-room schoolhouse. He graduated from high school in 1920 and in 1924 received his bachelor's degree from Lawrence University, graduating Phi Beta Kappa. In 1935, he earned a master's degree in business administration from Harvard.

For 37 years, Masterson worked in Stevens Point, Wisconsin for Hardware Mutual Casualty Company, the forerunner of Sentry Insurance. He worked as a statistician, actuary, and finally vice president and actuary. In 1947 the company expanded to Hardware Mutual Casualty Company and Hardware Dealers Mutual Fire Insurance Company. Masterson retired from Sentry Insurance-Hardware Mutuals Group in 1967, where he worked for three years as a vice president and actuary. For many years since his retirement, he operated an actuarial consulting business for clients in the United States and United Kingdom.

In addition to his actuarial work, Masterson was an educator and school board administrator. He taught statistics at the University of Wisconsin–Stevens Point from 1962 to 1963, and served as president on the Stevens Point Board of Education from 1935 to 1952. He was also on the Wisconsin Association of School

Boards, serving as president. He received the Wisconsin Outstanding School Board Award in 1949. He was involved on many statewide educational committees and appointed by several Wisconsin governors to committees for education reform. He served as a board member for his alma mater, Lawrence University, from 1955 to 1961.

A long-time member of the Casualty Actuarial Society, Masterson received his Associateship in 1926 and his Fellowship in 1927. A dedicated volunteer and past president of the society (1955–56), Masterson wrote several papers, including four presidential addresses. He served on the Committee on Government Statistics and the ASTIN Organizing Committee. He was a Delegate to ASTIN, chairperson for the Committee on Admissions, and CAS vice president (1949–1950).

He was a charter member of the American Academy of Actuaries, as well as a member and past officer of the National Association of Business Economists, International Actuarial Association, American Risk and Insurance Association, and Wisconsin Actuaries. Masterson also compiled and published an economic index to measure insurance claims costs, which is named the Masterson Index.

Masterson was very much involved in the community. He was an elder in the Frame Memorial United Presbyterian Church and a charter member of both the Stevens Point Country Club and the Curling Club. Masterson was a member of the Stevens Point Rotary Club since 1928, and had served as its president and historian. In 1978, the Rotary Club honored him as a Paul Harris Fellow for contributions made in his name to the Rotary International Foundation. He was believed to be the oldest living, active Rotarian out of the 32,000 Rotary Clubs throughout the world.

He married Cathryn A. Wolfe on October 16, 1926. She predeceased him August 2, 1957. He married Emma C. Turner on October 10, 1959.

In Masterson's eulogy, the Reverend Ed Hunt of Frame Memorial Church recalled Masterson's commitment to education and his sense of humor: "Doc…would mention that he was still coming to church at his age because he was cramming for his finals," said Hunt. "Doc has taken his final exams, and he has graduated summa cum laude."

Masterson is survived by his wife, Emma; two daughters, Cathryn (Wayne) Weinfurter of Land O'Lakes, Minnesota, and Meridith Masterson of Palatine, Illinois; four grandsons; and one great-granddaughter and one great-grandson.

THOMAS E. MURRIN
1923–2000

Thomas E. Murrin, a former Casualty Actuarial Society president, died on July 18, 2000. He was 76.

Murrin was born on September 12, 1923 and graduated magna cum laude from St. John's University in New York. After working many years in corporate America, he went on to complete the Stanford University Graduate School's Business Executive Program in 1968.

Murrin served as an officer in the United States Navy from 1944 to 1946. In 1955, he began his insurance career with the National Bureau of Casualty Underwriters in New York. In 1961, he joined the American Insurance Company in Newark, New Jersey as vice president and actuary. Murrin made the move to the West Coast to assume the same position of vice president and actuary in the San Francisco office of Fireman's Fund in 1963. While with Fireman's Fund, he was promoted to senior vice president and actuary. From 1977 to 1984, Murrin was executive vice president for Insurance Services Office, Inc. in New York. Murrin moved back to the San Francisco Bay in 1984 to work as an executive consultant in the Casualty Actuarial Risk Management Practice of Coopers & Lybrand. He retired to Novato, California in 1993.

Murrin became an Associate of the Casualty Actuarial Society in 1950 and a Fellow in 1954. A dedicated CAS volunteer and leader, Murrin served as CAS president from 1963 to 1964, as well as chairperson to the Committee on Programs in 1964, the Committee on Professional Status from 1965 to 1968, and the Committee on Professional Conduct from 1969 to 1970. Throughout the late 1960s and early 1970s, Murrin served on various CAS committees including the Constitution Committee, the Committee to Review Election Procedures, the Nominating Committee, and the Committee on the Future Course of the Society.

Murrin, who was the American Academy of Actuaries' second president, was much involved with the organization's early development and formation. In a 1995 interview with *Actuarial Update*, the Academy's newsletter, Murrin commented on the Academy's early structural issues and its current direction. "Policy-makers had no idea how the profession could help them in crafting legislation and regulations," said Murrin. "It's very gratifying to see the increased visibility the profession now enjoys as a source of objective analysis as a result of the Academy's efforts." Murrin was Academy president-elect from 1965 to 1966, the first year of the organization's existence, and served as Academy president from 1966 to 1967.

He was also a member of the International Actuarial Association.

Murrin is survived by his brother, Vincent of Medford, New York; his children Maureen Goodin, Tom Murrin, Rosemary O'Neill, Ann Finnegan, Patricia Eckhardt, Elizabeth Roney, Jim Murrin, and Marguerite Soldavini; and his 18 grandchildren.

JOHN H. ROWELL
1917–2000

John Holden Rowell was born May 25, 1917. He died June 6, 2000 at the age of 83.

Rowell attended the University of Pennsylvania's Wharton School, graduating in 1939 with a degree in economics. While in college he met and married his "best girl," Goldie. She and Rowell's father supported him during his school years. After graduation, the school dean recommended Rowell for his first job as examiner of insurance for the commonwealth of Pennsylvania. His salary enabled his wife to resign from her job and pursue her studies at the Wharton School. Goldie Rowell was the first woman admitted to the prestigious school.

A mentor encouraged him to pursue the actuarial exams. Rowell studied three hours a night, five days a week for six years. He received his Associateship to the Casualty Actuarial Society in 1946 and his Fellowship in 1947.

Rowell's early actuarial career included work at General Life Insurance Company in Hartford, Connecticut and Lumbermens Mutual Casualty Company in Chicago. In 1952 Rowell moved his family to California, where he spent four years working with the California Inspection Rating Bureau in San Francisco and then as vice president and chief actuary of Freedom Insurance Company in Berkeley.

In 1958, Rowell moved back to Illinois where he lived for the remainder of his life. He worked as an actuary at Health Service Inc., Medical Indemnity of America, Inc. for two years until he began a long association with Marsh and McLennan Inc. In his nine years with Marsh and McLennan Inc., Rowell served as actuary, assistant vice president, and vice president. Rowell then worked for Frank B. Hall and Company as account executive and vice president from 1974 to 1978 and as senior consultant from 1979 to 1980.

In 1978 he started his own business in Glencoe, Illinois, John Rowell and Associates. The company specialized in deferred compensation. Later on, Rowell's son Brenton joined him in the business.

Rowell enjoyed his work with retirement income and financial planning, working 10- to 12-hour days. When he had some spare time, he enjoyed fishing and his bridge club.

Rowell had two children, a son Brenton Rowell and a daughter, Robin Rowell Smith.

In an autobiographical sketch for his 50th class reunion, Rowell wrote: "As I re-read this story, it is clear that I never did anything alone. Someone else has always influenced me. I believe the someone elses in my life have been influenced by God."

IRWIN T. VANDERHOOF
1927–2000

Irwin T. Vanderhoof died on September 24, 2000, at St. Clare's Hospital in Denville, New Jersey after a long illness. He was 72.

Born December 4, 1927, Vanderhoof earned a B.S. in physics from Worcester Polytechnic Institute. He later earned a doctorate in finance from New York University and attended advanced courses at Clark University and the New School for Social Research. He earned his Associateship to the Casualty Actuarial Society in 1964.

Among Vanderhoof's many accomplishments is the unique adaptation of the Monte Carlo method, which was originally used to develop the hydrogen bomb. The quasi-Monte Carlo has been used for determining the worth of financial derivatives as well as in a variety of applications, ranging from finance to physics. Vanderhoof and two others were awarded a patent for the adaptation in 1999.

Vanderhoof spent most of his professional career in and around the New York area. In 1959, he helped found Standard Security Life Insurance Company of New York where he spent 14 years, the majority of which as senior vice president and chief actuary. He ended his stint at Standard Security Life as executive vice president and treasurer. Vanderhoof then served ten years as senior vice president at The Equitable Life Insurance Society. In 1993, he became president of Actuarial Investment Consulting, Inc. in New Jersey where he stayed until 1996.

Vanderhoof's teaching career spanned 28 years and included work at the College of Insurance in New York and as professor of finance at the Stern School of Business at New York University. He coedited four finance books for Stern. His last book, in press at the onset of his illness, has since been dedicated to him. He was an associate editor for the insurance journal *The Actuary*, the actuarial journal *Contingencies*, and the scientific publication *Journal of Spirochetal and Tick-Borne Diseases*.

An author of numerous papers, Vanderhoof penned perhaps his most heartfelt article, "Lyme Disease: Cost to Society," which appeared in *Contingencies*. Vanderhoof cowrote the article with his daughter, Karen, whose son Jamie fell victim to the disease.

Vanderhoof was an active Fellow of the Society of Actuaries and served as a board member as well as chairperson of the Life Research, Planning, and Research Committees. He was also a member of the International Congress of Actuaries and frequently presented papers at national and international meetings. He was a member of the American Academy of Actuaries, the American Academy for the Advancement of Science, Fellow of the Life Management Institute, Chartered Life Underwriter, Chartered Financial Analyst, Associate of the Institute of Actuaries (London), and senior analyst of the New York Society of Security Analysts.

He is survived by wife Ruth Green Vanderhoof; son Thomas (Dutch) Vanderhoof and his wife Tricia; daughter Karen Vanderhoof-Forschner and her husband Thomas Forschner of Connecticut; and granddaughter Christy Vanderhoof-Forschner.

JAMES M. WOOLERY
1901–1999

James M. Woolery, whose contributions to the insurance industry spanned over 60 years, died January 11, 1999.

Woolery was born in Harrison County, Kentucky on November 5, 1901, the son of James Bascom Woolery and Frances Belle McCracken Woolery. Educated in the public schools of Falmouth, Kentucky, Woolery was valedictorian of his high school class. He entered college at Transylvania University and the University of Kentucky in Lexington, Kentucky, where he graduated magna cum laude with degrees in mathematics and economics. He earned a masters degree in actuarial science from the University of Michigan.

While working in Birmingham, Alabama he met Grace Godfrey. The two were married more than 63 years until her death in August 1992.

Woolery became an Associate of the Casualty Actuarial Society in 1925 and a Fellow of the Society of Actuaries in 1935. He worked with the U.S. Treasury Department on Social Security in 1936 and helped establish the North Carolina/Virginia health plan for old age.

During the 1920s and 1930s, he worked in various companies throughout the South including Protective Life Insurance Company in Birmingham; Inter-Southern Life Insurance Company and Kentucky Home Life Insurance Company in Louisville, Kentucky; and Southeastern Life Insurance Company in Greenville, South Carolina. In 1936, Woolery settled in Raleigh, North Carolina, where he worked as an actuary for the North Carolina Department of Insurance. In 1945, Woolery made a move to New York City and to the Union Labor Life Insurance Company, but the next year he returned to Raleigh where he would spend the remainder of his life.

In 1947 Woolery began an 18-year stint with Occidental Life Insurance Company in Raleigh, first as vice president and actuary and in 1964 as senior vice president and actuary. In 1968 he began his independent consulting work. He served on the boards of directors for British American Insurance Company of Nassau, Bahamas; Occidental Life Insurance Company; and Peninsular Life Insurance Company in Jacksonville, Florida. He retired in 1987.

Woolery enjoyed the mountains surrounding his home in Blowing Rock, North Carolina. An avid sports enthusiast, Woolery played baseball in college and in his later years enjoyed mountain hiking and jogging. He belonged to a jogging club at North Carolina State for more than 30 years and ran the Great Raleigh Road Race when he was over 80 years old.

Woolery was very active in his community, serving as a member of the Southern Rite Masons, Pi Kappa Alpha fraternity, the Carolina Country Club, Lions Club, the Church of Good Shepherd, and as a Kentucky Colonel. Among his other activities, Woolery was an Enrolled Actuary, chairman of Domestic Life Companies for North Carolina, and member and president of the Middle Atlantic Actuarial Club. He was also a member of the Southeast Actuaries Club and served on the club's board of directors.

Woolery is survived by a daughter, Martha Woolery Sneed of Pensacola, Florida; a son, James Godfrey Woolery of Raleigh, North Carolina; and two grandchildren.

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