A Note on the Paid Bornhuetter-Ferguson Loss Reserving Method: Recognizing Dependency on Case Reserves

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Recognizing Dependency on Case Reserves

By

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Abstract

Many actuaries use a Bornhuetter-Ferguson ("BF") loss reserving method[1] based on paid loss data. What may be overlooked is that IBNR estimated with the paid BF method depends on both paid losses and case reserves, a situation the actuary may wish to avoid when case reserves are volatile or unreliable. This paper explores the dependence of IBNR estimates on case reserves when IBNR is derived from a paid loss Bornhuetter-Ferguson method. An alternative to reduce this dependence is provided.

Introduction

While revising a prior loss reserve study that utilized both paid and reported BF methods, the author noticed that the paid BF method is dependent on case reserves. The prior loss reserve study was based upon industry expected loss ratios and industry reporting and payment patterns. The revision to the study only affected the values for actual paid and reported (defined as paid loss plus case reserves) losses. The industry-based factors were not changed. Further, in both the original and revised versions, the author had selected an ultimate loss based on the average of the paid and reported BF methods. Upon review of the results, the author discovered an interesting result. What follows is a discussion of the author's findings, which should be of interest to those who use a paid BF method for estimating IBNR reserves.

[1] 1972, Bornhuetter, R.L. and Ferguson, R.E., "The Actuary and IBNR," PCAS LIX

Analysis

The easiest way to demonstrate the paid BF method's dependence on case reserves is through a simple example. For this example, reporting and payout patterns are based on industry wide data from Schedule P Other Liability. All other data was made up.

Exhibit I shows typical calculations utilizing the BF loss reserving method. The upper third of the exhibit is a paid BF method. The middle third of the exhibit is a reported BF method. In the paid method we estimate expected unpaid losses while in the reported method we estimate expected IBNR. Actual paid and reported losses are then added to expected amounts to derive estimates of ultimate loss for each method, respectively. In the lower third of the exhibit, the estimates of ultimate loss for each method have been averaged to arrive at a selected ultimate loss. Then reported losses are subtracted to calculate indicated IBNR.

Exhibit II shows the same calculation as Exhibit I using revised actual paid and reported losses. All other factors remain unchanged. The following table summarizes the results from Exhibit I and II:

	Exhibit I: Original	Exhibit II: Revised	Change
Paid Loss	141	144	3.0
Reported Loss	248	253	5.0
Selected Ultimate Loss	372	376	4.0
IBNR	124	123	-1.0
Case Reserve	107	109	2.0

What is interesting here is that the IBNR changed by minus one half of the change in case reserves.

Algebra helps explain what is happening in this example. In what follows, we assume that reporting patterns, payout patterns, and expected losses are not changed by revisions in the reported and paid data (we discuss certain implications of this assumption later). When the paid and reported BF methods are analyzed together, the author believes it is easier to compare the different contributions to the estimated IBNR resulting from each

method. Therefore, we include weighting the paid BF method with the reported BF method in our analysis.

First, let us define the following symbols:

P_o = actual paid losses in the original data set
P_r = actual paid losses in the revised data set
R_o = actual reported losses in the original data set
R_r = actual reported losses in the revised data set
U = expected unpaid losses from paid BF method
I = expected IBNR losses from reported BF method
L_p = ultimate loss based on paid BF method
L_r = ultimate loss based on reported BF method
W = weight given to the paid method (1-W = weight given to reported method)
L = selected ultimate
IBNR = indicated IBNR based on subtracting reported losses from L

Using these symbols we can derive the following relationships:

For the **original** data set we have

$$L_p = U + P_o$$

$$L_r = I + R_o$$

$$\begin{aligned} L &= WL_p + (1-W)L_r \\ &= WU + WP_o + (1-W)I + (1-W)R_o \\ &= W(U + P_o - I - R_o) + I + R_o \end{aligned}$$

$$\begin{aligned} IBNR &= L - R_o \\ &= W(U + P_o - I - R_o) + I \end{aligned}$$

Similarly, for the **revised** data set we have

$$L = W(U + P_r - I - R_r) + I + R_r$$
$$IBNR = W(U + P_r - I - R_r) + I$$

If we then calculate the change in IBNR (ΔIBNR) equal to the revised IBNR minus the original IBNR, we have the following relationship:

$$\begin{aligned} \Delta IBNR &= W(U + P_r - I - R_r) + I - W(U + P_o - I - R_o) - I \\ &= W(P_r - R_r - P_o + R_o) \\ &= W[(R_o - P_o) - (R_r - P_r)] \qquad (1) \end{aligned}$$

The quantities in (1) inside the parentheses are the case reserves before and after the data was revised. The quantity in (1) inside the brackets represents the change in case reserves. Hence, the change in IBNR is equal to minus the change in case reserves times the weight W given to the paid method. That is, if ΔC is the change in case reserves, then

$$\Delta IBNR = -W\Delta C$$

In our example, W was ½ and ΔC was 2. The change in IBNR was -1.

What does this mean? The reported BF method explicitly produces an estimate of IBNR. To estimate IBNR using the paid BF method, we must subtract from expected unpaid losses an estimated amount for case reserves. It just happens that if we subtract actual reported loss from the paid BF ultimate loss, we use a "default" estimate of case reserves equal to the actual case reserves. *In essence, our estimate of IBNR made using the paid BF method is dependent on current case reserves.* This means that case reserves (including case reserve adequacy and volatility) become a factor in the IBNR derived by the paid BF method.

Exhibits III and IV demonstrate an alternative method to Exhibits I and II using the same original and revised data that was used above. The results from Exhibits III and IV are shown in the following table:

	Exhibit III: Original	Exhibit IV: Revised	Change
Paid Loss	141	144	3.0
Reported Loss	248	253	5.0
Selected Ultimate Loss	373	380	6.5
IBNR	125	127	1.5
Case Reserve	107	109	2.0

In Exhibits III and IV, an adjustment has been made to the paid BF method that substitutes expected reported losses for actual reported losses in the estimate of IBNR. We calculated the ultimate loss by adding the "alternative" IBNR, to the actual reported losses. Hence, when actual reported losses are subtracted from ultimate losses derived by this method, the alternative IBNR is the result.

The author's alternative method can be explained using algebra. By setting W to 1 for simplicity and examining only the paid BF method, we can derive the following IBNR formula for the alternative method:

$$\text{Original data set IBNR} = U + P_o - (1-I) \qquad (2)$$
$$\text{Revised data set IBNR} = U + P_r - (1-I) \qquad (3)$$
$$\Delta\text{IBNR} = P_r - P_o$$

The term "1-I" equals expected reported losses. The IBNR in (2) and (3) equals expected unpaid losses plus actual paid losses minus expected reported losses. This method develops an estimate of IBNR using paid losses and is independent of the current reported case reserves, as IBNR is now a function of the actual paid losses instead of case reserves. This may be a more desirable result in certain cases. For example, it gives the practitioner a method that eliminates direct dependence of IBNR on current case reserves when a paid BF method is used and current case reserves are unreliable.

The following points help put our findings in perspective:

1. Introducing reported loss information into the paid BF method in our alternative method may increase the correlation (if any) between the paid and reported methods. For example, by introducing the expected reported losses into the paid BF method, IBNR dependency on reported losses (and hence, case reserves) may still be present. In our example, the alternative paid BF method produces an answer closer to the reported BF method than the standard paid BF method.

2. In many situations, the reporting and payout patterns (and possibly the expected loss ratios as well) are derived from company data and can change as a result of revisions to reported and paid data. Hence, the relationships derived above would not be accurate, as U and I may change. In situations where data changes have modest impacts on the selection of loss development factors and expected loss ratios, the relationships derived above provide reasonable approximations. For example, where industry data is given significant weight in the selection of loss development factors, changes in U and I may be relatively small. Many actuaries use judgement in selecting payout and reporting patterns, and

minor changes to loss data will not affect those selections. Expected loss ratios may also be selected based on information independent of the company loss data currently under review.

3. Actuaries tend to utilize several methods to estimate IBNR in addition to the BF methods. Often, the resulting estimates of ultimate loss are averaged together or weighted in the process of selecting an ultimate loss. Much of the case reserve "dependency" effects noted in the above analysis may, for all practical purposes, be effectively decreased to a level that is reasonable. In most situations, case reserves may be reasonable and the standard paid BF method is fine. However, practitioners should be aware of the potential influence of case reserves on the paid BF method when it is used to derive IBNR, particularly in situations where it is the primary method used and case reserves are problematic. Careful selection of W and/or the use of the alternative paid BF method may provide alternatives in such a situation.

4. While our alternative to the standard paid BF method eliminates dependence of IBNR on current case reserves, dependence on current paid losses results. The practitioner should decide if this is a more appropriate method for the loss reserve data under review.

5. Using the standard paid BF method, an increase in case reserves results in a decrease in IBNR, as total unpaid losses are fixed at U. This method essentially allocates the total unpaid losses determined by U between IBNR and case reserves. Hence, increases in actual paid losses or actual case reserves have no effect on IBNR or reduce IBNR, respectively.

6. Using the alternative paid BF method, an increase in paid losses results in an increase in IBNR, as the ultimate loss increases, but the expected reported losses are fixed at "1-I". The alternative method responds directly to changes in paid losses, similar to the way the paid loss development method responds to changes in paid losses. Hence, increases in actual paid losses or actual case reserves increase IBNR or have no effect on IBNR, respectively.

7. For comparison, the reported BF method produces an estimate of IBNR that is independent of changes in current paid loss and/or case reserves.

Conclusion

The standard paid BF method uses expected unpaid losses and *actual* case reserves to estimate IBNR. This compares to the reported BF method that estimates IBNR based on expected losses and expected reporting patterns. Hence, IBNR derived by the standard paid BF method is dependent on case reserves. Case reserve dependency in the paid BF method can be eliminated by subtracting expected reported losses, instead of actual reported losses, from the standard paid BF ultimate loss to estimate IBNR. This adjustment results in IBNR that is dependent on paid losses instead of case reserves. In certain cases, the actuary may prefer IBNR estimates that are dependent on paid losses rather than case reserves, particularly if case reserves are volatile or unreliable.

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Estimates Using Standard BF Approaches
Other Liability - Original Data

Exhibit I

Paid BF Approach								
Months Maturity	Earned Premium	Expected Loss Ratio	Expected Loss	Cumulative Payout Pattern	Unpaid Percentage	Estimated Unpaid Loss	Actual Paid Loss	Estimated Ultimate Loss
12	106	0.70	74	0.099	0.901	67	8	75
24	105	0.75	79	0.238	0.762	60	15	75
36	100	0.66	66	0.403	0.597	39	28	67
48	110	0.68	75	0.556	0.444	33	37	70
60	115	0.70	81	0.675	0.325	26	53	79
Total	536		374			226	141	367

Reported BF Approach								
Months Maturity	Earned Premium	Expected Loss Ratio	Expected Loss	Cumulative Report Pattern	*IBNR* Percentage	Estimated *IBNR* Loss	Actual Reported Loss	Estimated Ultimate Loss
12	106	0.70	74	0.327	0.673	50	26	76
24	105	0.75	79	0.548	0.452	36	45	81
36	100	0.66	66	0.705	0.295	19	48	67
48	110	0.68	75	0.811	0.189	14	56	70
60	115	0.70	81	0.875	0.125	10	73	83
Total	536		374			129	248	377

BF Approach Selected Ultimate Loss and Estimated IBNR					
Months Maturity	Paid BF Method	Reported BF Method	Selected Ultimate*	Reported Losses	Indicated IBNR
12	75	76	75	26	49
24	75	81	78	45	33
36	67	67	67	48	19
48	70	70	70	56	14
60	79	83	81	73	8
Total	367	377	372	248	124

*Average of paid and reported methods

Estimates Using Standard BF Approaches
Other Liability - Revised Data

Exhibit II

Paid BF Approach

Months Maturity	Earned Premium	Expected Loss Ratio	Expected Loss	Cumulative Payout Pattern	Unpaid Percentage	Estimated Unpaid Loss	Actual Paid Loss	Estimated Ultimate Loss
12	106	0.70	74	0.099	0.901	67	10	77
24	105	0.75	79	0.238	0.762	60	13	73
36	100	0.66	66	0.403	0.597	39	29	68
48	110	0.68	75	0.556	0.444	33	38	71
60	115	0.70	81	0.675	0.325	26	54	80
Total	536		374			226	144	370

Reported BF Approach

Months Maturity	Earned Premium	Expected Loss Ratio	Expected Loss	Cumulative Report Pattern	IBNR Percentage	Estimated IBNR Loss	Actual Reported Loss	Estimated Ultimate Loss
12	106	0.70	74	0.327	0.673	50	28	78
24	105	0.75	79	0.548	0.452	36	44	80
36	100	0.66	66	0.705	0.295	19	49	68
48	110	0.68	75	0.811	0.189	14	55	69
60	115	0.70	81	0.875	0.125	10	77	87
Total	536		374			129	253	382

BF Approach Selected Ultimate Loss and Estimated IBNR

Months Maturity	Paid BF Method	Reported BF Method	Selected Ultimate*	Reported Losses	Indicated IBNR
12	77	78	77	28	49
24	73	80	76	44	32
36	68	68	68	49	19
48	71	69	70	55	15
60	80	87	84	77	7
Total	370	382	376	253	123

*Average of paid and reported methods

Estimates Using Adjusted BF Approaches
Other Liability - Original Data

Exhibit III

Paid BF Approach											
Months Maturity	Earned Premium	Expected Loss Ratio	Expected Loss	Cumulative Payout Pattern	Unpaid Percentage	Estimated Unpaid Loss	Actual Paid Loss	Cumulative Reporting Pattern	Expected Reported Loss	Actual Reported Loss	Estimated Ultimate Loss*
12	106	0.70	74	0.099	0.901	67	8	0.327	24	26	77
24	105	0.75	79	0.238	0.762	60	15	0.548	43	45	77
36	100	0.66	66	0.403	0.597	39	28	0.705	47	48	69
48	110	0.68	75	0.556	0.444	33	37	0.811	61	56	66
60	115	0.70	81	0.675	0.325	26	53	0.875	70	73	82
Total	536		374			226	141		245	248	370

Reported BF Approach								
Months Maturity	Earned Premium	Expected Loss Ratio	Expected Loss	Cumulative Report Pattern	IBNR Percentage	Estimated IBNR Loss	Actual Reported Loss	Estimated Ultimate Loss
12	106	0.70	74	0.327	0.673	50	26	76
24	105	0.75	79	0.548	0.452	36	45	81
36	100	0.66	66	0.705	0.295	19	48	67
48	110	0.68	75	0.811	0.189	14	56	70
60	115	0.70	81	0.875	0.125	10	73	83
Total	536		374			129	248	377

BF Approach Selected Ultimate Loss and Estimated IBNR					
Months Maturity	Paid BF Method	Reported BF Method	Selected Ultimate**	Reported Losses	Indicated IBNR
12	77	76	76	26	50
24	77	81	79	45	34
36	69	67	68	48	20
48	66	70	68	56	12
60	82	83	82	73	9
Total	370	377	373	248	125

* Expected unpaid loss + actual paid loss - expected reported loss + actual reported loss
**Average of paid and reported methods

Estimates Using Adjusted BF Approaches
Other Liability - Revised Data

Exhibit IV

Paid BF Approach

Months Maturity	Earned Premium	Expected Loss Ratio	Expected Loss	Cumulative Payout Pattern	Unpaid Percentage	Estimated Unpaid Loss	Actual Paid Loss	Cumulative Reporting Pattern	Expected Reported Loss	Actual Reported Loss	Estimated Ultimate Loss*
12	106	0.70	74	0.099	0.901	67	10	0.327	24	28	81
24	105	0.75	79	0.238	0.762	60	13	0.548	43	44	74
36	100	0.66	66	0.403	0.597	39	29	0.705	47	49	71
48	110	0.68	75	0.556	0.444	33	38	0.811	61	55	66
60	115	0.70	81	0.675	0.325	26	54	0.875	70	77	87
Total	536		374			226	144		245	253	378

Reported BF Approach

Months Maturity	Earned Premium	Expected Loss Ratio	Expected Loss	Cumulative Report Pattern	IBNR Percentage	Estimated IBNR Loss	Actual Reported Loss	Estimated Ultimate Loss
12	106	0.70	74	0.327	0.673	50	28	78
24	105	0.75	79	0.548	0.452	36	44	80
36	100	0.66	66	0.705	0.295	19	49	68
48	110	0.68	75	0.811	0.189	14	55	69
60	115	0.70	81	0.875	0.125	10	77	87
Total	536		374			129	253	382

BF Approach Selected Ultimate Loss and Estimated IBNR

Months Maturity	Paid BF Method	Reported BF Method	Selected Ultimate**	Reported Losses	Indicated IBNR
12	81	78	79	28	51
24	74	80	77	44	33
36	71	68	70	49	21
48	66	69	67	55	12
60	87	87	87	77	10
Total	378	382	380	253	127

* Expected unpaid loss + actual paid loss - expected reported loss + actual reported loss
**Average of paid and reported methods