

Integrated Financial Risk Management: Capital Allocation Issues

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Abstract

A strategic optimization model provides the ideal setting for allocating the scarce capital of a financial intermediary such as an insurance company. The goal of management is to *maximize* shareholder value. Capital allocation serves three primary purposes: to compare managerial performance across business units, to provide a risk indicator for regulators and other stakeholders, and to develop a common basis for major decisions, including investment and underwriting strategies, and setting the corporate structure. Capital allocation puts diverse activities on an equal footing by adjusting profits and revenues for risks. We show that optimal allocation rules can be readily retrieved from the solution to a special dynamic financial analysis model of the firm.

Motivation

For any financial intermediary, a major concern involves growing the company's wealth over time, consistent with taking on acceptable risks. In the same light, we are often told that senior management should render decisions so as to maximize shareholder value; it is the mantra of modern capitalism. Yet there are few practical mathematical modeling tools for assisting in the integrated optimization of a financial organization¹. We propose a dynamic financial analysis (DFA) for monitoring risks -- by allocating capital, by setting underwriting limits, and by rewarding decision makers in concert with financial benchmarks. This system identifies the goals of a financial company as benchmark targets.

A critical aspect of strategic planning is determining the underlying capital structure of a firm. Too little capital leads to excess risks for the shareholders and the public. Too much capital leads to inferior returns as compared with competing investments. There are several alternative approaches to capital allocation:

Existing banking and insurance capital requirements, such as risk based capital required by regulators (e.g. 1992 NAIC capital adequacy requirements, and the 1988 Basle accord). Mostly, these rules are based on business activity such as premium to surplus ratios, and they emphasize credit risks.

Measurements of risks based on probabilistic estimates and stochastic models:

- Value at Risk – VaR,
- Expected policyholder deficit – EPD and its generalization ETD,
- Certainty equivalent returns – CER and its approximation, target planning.

Each approach possesses limitations due to the conflicting purposes for capital allocation. Of course, a primary goal is to prevent an unacceptable loss of capital by the financial intermediary. In a large financial organization, there are multiple stakeholders who possess conflicting goals: shareholders, many of whom can diversify their financial stake, officers and employees, obligators, state and federal regulators, the ceding company, and others. Each group has an interest in seeing that the company survives economic downturns. However, the risk/reward tradeoff may be different for each group. Regulators, for example, may have less concern for higher short-term returns than do the stockholders. Many current capital allocation procedures are indirect indicators of risks since they fail to estimate the probability of losses to the surplus over a multi-period time horizon. Likewise, those methods based exclusively on historical returns may be inadequate when evaluating a new set of circumstances and future market dynamics.

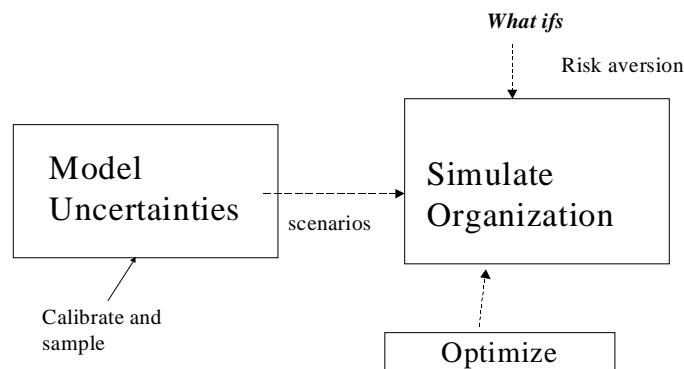
The remainder of this report is organized as follows. In the next section, we review key elements of the DFA model for the firm. Several alternative objectives are mentioned, depending upon the needs of the company and its stakeholders. In most cases, the model should display the values of the multiple objectives since there are so many ways to evaluate the health of a financial intermediary. In the third section, we propose a decentralized approach to the DFA system. In section 4, we discuss the capital allocation process. Section 5 presents a small example based on an insurance problem. Last, we make some concluding remarks.

¹ Other domains have embraced optimization methods, for instance, see Integrated Logistics (Asad, et al. 1992).

1. Integrated Financial Risk Management

This section presents the basic framework for integrating the major decisions of a financial intermediary. We propose a multi-period stochastic optimization model. The primary goal is to maximize shareholder value at the end of a long-term planning horizon, in which we propose several alternative ways to define “value”. Risk is taken into account by including a non-linear penalty for dropping below a benchmark capital level and by adding a set of risk-based constraints.

The three primary elements of a strategic DFA system are: 1) a stochastic scenario generator; 2) a set of decision rules involving investments, insurance underwriting strategies, reinsurance, and the corporate structure; and 3) a dynamic optimization module. See Figure 1. The first two elements form the corporate simulation system; these are deployed before the optimization module seeks out the best compromise decisions for the company given the relevant business, policy, and regulatory constraints. In effect, the optimization runs the simulation by searching for decisions that best fits the proposed objective function over the multi-period planning period.



Basic Technology

Figure 1

The Primary Elements of an Integrated Financial Risk Management System

In this section, we describe a tightly integrated model for the insurance firm. Major decisions are evaluated by running the system to determine the impact of the decisions on the company’s growth of capital and risks. For example, business mix decisions should be linked to the amount and type of reinsurance². All senior decision-makers should be comfortable with accessing and running the corporate planning system. Of course, the DFA model should be simple enough so that a substantial number of sensitivity analyses can be conducted (Mulvey and Madsen 1999).

² Retrocession coverage for reinsurance company.

In a large organization, it is impractical to expect that the major decision-makers will be able to run a single planning system. There are information and organizational barriers to a centralized approach. Most managers favor operating in a rather independent fashion from other parts of the organization. It is generally more efficient for individuals to concentrate on a well-defined business. To accomplish the goals of an integrated risk management system and to operate in a semi-independent fashion can be difficult. Figure 2 shows an organizational structure wherein headquarters focus on decisions relating to joint resources and overall capacity constraints. For example, headquarters is responsible for maintaining and deploying the proper level of risk capital.

Linking Strategic and Tactical

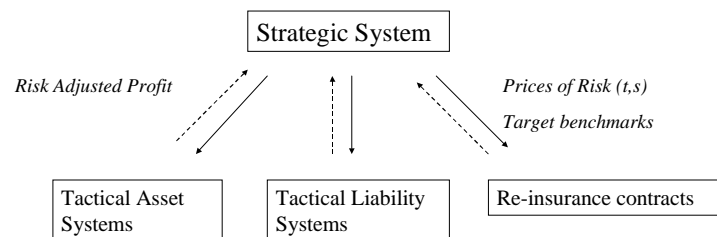


Figure 2
Structure for De-centralized Organization

2.1 Model Framework

In this section, we define the basic modeling framework in a relatively idealized setting. Many details are omitted in order to concentrate on issues relating to capital allocation. See Mulvey (1999) for a richer description of an optimization-based DFA system.

The financial planning model consists of T time stages $t = \{1, 2, 3, \dots, T\}$. The spacing of the periods will depend upon the user's needs, with small steps typically occurring early in the process. The end of the planning period T defines the horizon; it depicts a point in which the investor has some critical decision to make, such as the repayment date of a substantial liability, or a target point for planning purposes. At the beginning of each period, the model renders decisions regarding the asset mix, the liabilities, and the corporate mix of equity and debt. We assume that cashflows such as dividends and interest occur at the last instant of a time period.

We employ a system of stochastic differential equations for modeling the stochastic parameters over time. These relate a set of key economic factors to remaining components, such as asset and liability returns. As far as the model is concerned, the future at any point is the scenario sub-tree emanating out of the relevant node (Figure 4). The set of possibilities is called the representative

set of scenarios, $s \in S$. Cariño et al. (1994) and Mulvey and Thorlacius (1998) discuss scenario generators.



The Planning Period ($t = 1,2,3, \dots, T$)
Figure 3

The primary variables involve asset categories, liability-related decisions, and the capital structure:

- $X_{j,t}^s$ asset category $j \in J$, time $t \in T$, scenario $s \in S$,
- $Y_{i,t}^s$ activity category $i \in I$, time $t \in T$, scenario $s \in S$,
- $Z_{k,t}^s$ decisions regarding the firm's capital structure (e.g. stock issues), $k \in K$, time $t \in T$, scenario $s \in S$.

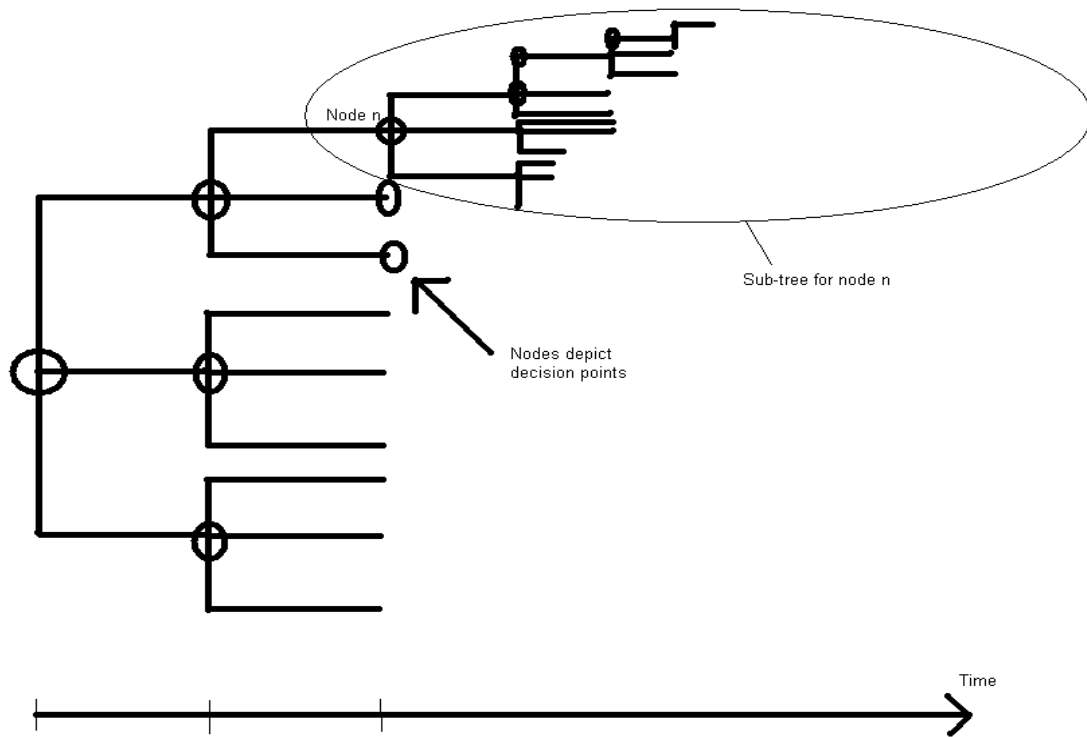
The asset and liability variables can be thought of as simple bundles of future cashflows. We are agnostic about the actual definitions themselves³, except those categories should represent a set of well-defined continuing positions, as opposed to individual securities with a fixed termination date⁴. At each time period t , the model maximizes its objective function by moving money between asset categories, adjusting liabilities⁵, and accounting for the capital structure. There are several candidates for the objective function as discussed in the next section.

We represent the future as paths in a scenario tree (Figure 4). At each node of this tree, the model must decide upon the best course of action based on information available to it at the time. To do this, we must calculate the company's position in many dimensions, for instance, the market value of its assets. A single scenario is a complete path in this scenario tree. Each combination of a scenario and time point ($s \in S$ and $t \in T$) references a single node, $n \in N$ in the tree. The index set S_n identifies all scenarios that pass through node n . For practical purposes, we limit the number of nodes in the scenario tree. Statistical sampling theory, such as variance reduction provides methods for doing this.

³ We would define assets (liabilities) as cashflows normally possessing largely negative (positive) cashflows at the beginning and projected positive (negative) cashflows in the latter stages.

⁴ Securities are excluded in order to keep the model's size manageable, but they can be added with enhanced computer resources.

⁵ The level of a business should depend upon the economic environment embedded in the scenario. Thus, ongoing business assumptions will need to be made.



At each node n, we can evaluate a given product.

The Scenario Tree Figure 4

There are three basic equations to any financial planning system. These equations preserve the flow of funds so that money can be accounted for at each junction.

Equation [1] for j^{th} asset category:

$$x_{j,t+1}^s = (x_{j,t}^s + r_{j,t}^s) - q_{j,t}^s + p_{j,t}^s (1-t_j) \quad \text{for asset } j, \text{ time } t, \text{ scenario } s.$$

where $r_{j,t}^s =$ return for asset j , time t , scenario s ,
 $q_{j,t}^s =$ sales of asset j , time t , scenario s ,
 $p_{j,t}^s =$ purchase of asset j , time t , scenario s ,
 $t_j =$ transaction costs for asset j .

Equation [2] for the i^{th} liability-related category:

$$y_{i,t+1}^s = (y_{i,t}^s + r_{i,t}^s) - q_{i,t}^s + p_{i,t}^s (1-t_j) \quad \text{for asset } j, \text{ time } t, \text{ scenario } s.$$

where $r_{i,t}^s =$ return for liability i , time t , scenario s ,

$q_{i,t}^s =$ reduction in liability category i , time t , scenario s ,
 $p_{i,t}^s =$ addition to liability i , time t , scenario s ,
 $t_i =$ transaction costs for liability i .

Equation [3] for the cash flow collector nodes:

$$x_{1,t+1}^s = (x_{1,t}^s + r_{1,t}^s) - \sum q_{j,t}^s + \sum p_{j,t}^s (1 - t_j) - \sum q_{i,t}^s + \sum p_{i,t}^s (1 - t_j) + z_t^s$$

where $z_t^s =$ cash inflows or outflows at time t , scenario s , based on capital structure decisions. Note that cash is asset category 1.

The return parameters are calculated in several ways depending upon the context. To find economic value, we tally the future cashflows for the asset or liability in question and discount at the appropriate risk adjusted rate. In other situations, we employ book or regulatory value for determining the company's statutory or accounting surplus. For the core model, however, we fix the best estimates for economic value in equations 1 to 3 since these equations involve actual cashflows.

The multi-stage model avoids looking into the future in an inappropriate fashion. The model must optimize over scenarios that represent a range of plausible outcome for the future. To prevent the model from taking advantage of future information, we add special equations, called non-anticipatory conditions. The general form of the constraints is:

$$x_{j,t}^{s1} = x_{j,t}^{s2} \quad \text{and} \quad y_{i,t}^{s1} = y_{i,t}^{s2}$$

for all scenarios $s1$ and $s2$ which inherent a common scenario path up to time t , i.e. they share a sequence of arcs up to node $n(s,t)$.

The financial planning system addresses these non-anticipatory conditions, either explicitly or implicitly, and special purpose algorithms are available for solving the resulting optimization model.

Many issues complicate the decision making process, for instance, certain liabilities and assets cannot be simply bought or sold at any time or for any amount. To address these issues, we impose a set of linear constraints on the process. Some examples of the constraints are the following: limiting borrowing to certain ratios, addressing transactions costs whenever assets are bought or sold, constraining duration of the assets, or limiting the model's ability to take advantage of investment opportunities. Insurance companies often address the duration of their assets and liabilities as a critical factor in protecting their surplus. This concept can be readily incorporated, but will have less importance as companies invest in assets and take on liabilities with diminishing relationships to interest rate fluctuations.

An important topic involves the cost of changes to the corporate structure, for example, increased borrowing costs that occur when a company's capital drops below a threshold level, say a point in which the rating agencies would decrease its bond ratings. These costs are added to the financial model for borrowing costs:

$$\text{Cost of } Z_k^s = g \text{ (capital available at current conditions under scenario } s). \quad [4]$$

The nonlinear constraints are included in the planning model along with the other general constraints. See Froot and Stein(1998) for an explanation of penalties for capital dropping below a threshold.

To directly control risks throughout a large organization, we impose an additional set of constraints on the risk capital of the firm. The next subsection takes up this issue.

2.2 Risk Based Constraints

The capital allocation issue involves in a critical manner the amount of risk that an insurance company should undertake. Greater capital requirements will reduce the chances of a negative event, but at the same time reduces the profitability of the enterprise. Several approaches are possible for addressing risks in the DFA model. For example, we could compute the amount of Value-at-Risk by setting a constraint on the quantiles of the company's profit/loss distribution. Here is an example in which we limit economic capital. The same type of equations can be employed for other definitions of capital surplus⁶.

Define the capital of the firm at node n (time t, scenario s) based on economic value:

$$C_t^s \equiv \text{market value of assets} - \text{market value of liabilities.}$$

Under this framework, capital is uncertain and depends upon the scenario at hand, scenario s. We can limit the probability that capital will drop below some threshold value at time t, say R_t with a constraint (called chance constraint) of the form:

$$\text{Probability} (C_t^s \geq R_t) \geq L \tag{5}$$

Here the parameter L depicts the critical quantile on the cumulative distribution function. We are forcing the model to allocate adequate capital so that there is a high probability (e.g. $L = 99\%$) that the firm's capital will stay above the threshold R_t under the analyzed scenarios. This approach is called VaR capital allocation.

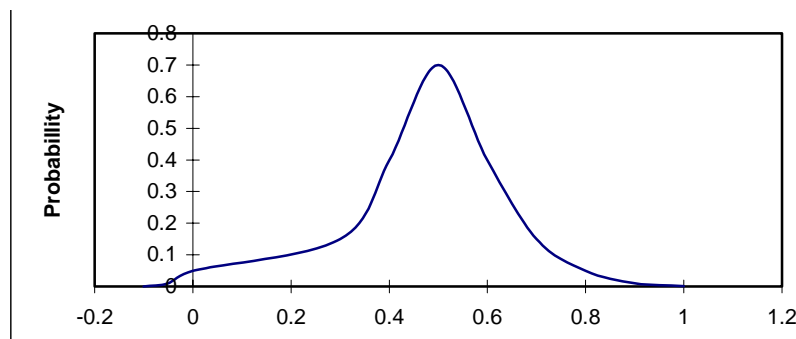


Figure 5
Probability distribution for Capital and VaR point at 1/20 level

⁶ We have omitted these details in order to keep the presentation concise; see Mulvey (1999) for further details.

An alternative approach to measuring and ultimately controlling risks is to compute the expected policyholder deficit (EPD), which determines the expected losses for the company, given that losses are great enough to outweigh investment and other income, and possibly the capital itself. Again, we can express this risk measurement as a constraint in the planning model:

$$EPD_t \leq P \quad [6]$$

where $EPD_t = E(C_t | C_t < R_t)$, where $E(\cdot)$ is expectation, and P is the maximum tolerable level of EPD losses. Typically, EPD includes allocated capital in the calculation. Generalizing EPD involves computing expected loss relative to a target benchmark. We call this Expected Target Deficit, or ETD.

EPD and ETD possess several advantages over VaR as a risk control tool. They fulfill the notion of a coherent measure of risk more closely than VaR (Artzner et al. 1998). The values can be computed more readily for a large company than the VaR number since EPD losses are sub-additive. On balance, EPD addresses a larger portion of the set of possibilities than VaR. Nevertheless, EPD possess some difficulties that are discussed in Section 5.

A third approach for addressing risk in the planning model is to consider the entire capital probability distribution, as compared with the previous methods that look only at the downside elements. The standard approach for this involves computing the company's certainty equivalent returns (CER). Here we start with a risk averse utility function, $U(C_t)$, and compute the so called certainty equivalence for the firm as:

$$\text{Certainty equivalence}_t \equiv CE_t \text{ such that } U(CE_t) = E[U(C_t)].$$

This value translates a risky set of outcomes into an equivalent riskless outcome. Expected utility corresponds to the ordinal ranking for any two alternative: x_A if preferred to x_B if and only if expected utility of x_A is greater than the expected utility of x_B . See Holloway 1979.

The CER approach accounts for the entire probability distribution. Given the certainty equivalent value, we can compute risk-adjusted profit over a period of time as:

$$CER_t = (CE_{t+1} - CE_t) / CE_t$$

As before, we set constraints for risks, CER in this instance, over the planning horizon in a manner similar to the VaR and the EPD approach. See Holmer (1998) for an application of CER at Fannie Mae. The corresponding constraint takes the form:

$$CER_t \geq S_t \quad [7]$$

where S_t is a parameter designating the minimum risk adjusted profit at time period t .

This approach can be difficult to implement without providing a set of targets for setting penalties for shortfalls and bonuses for exceeding the target values. Also, we could include constraints based on other risks measures such as VaR.

The complete set of decisions is equal to the set: $w = (x, y, z)$ for all nodes in the scenario tree. We assume that the company selects one or more of the approaches for risk management, whether, VaR, EPD, or CER. We say that a model is feasible, $w \in W$ if it satisfies all of the constraints, including the selected risk based restriction [5], [6], or [7].

2.3 Objective Functions

We envision several objective functions for the integrated planning system. For public companies with long horizons, we might maximize expected wealth at the end of the planning horizon, time T.

$$\text{Expected wealth} = \sum_{s \in S} p_s * C_T^s \quad [8]$$

where p_s is the probability of scenario s ,
 C_T^s is the company's capital under scenario s at time T,
 S is a set of representative scenarios.

Here, the company maximizes its capital at some time in the future, subject to constraints on risk and other activities as dictated in the previous section $w \in W$. Since the horizon occurs in the long-term future, we can argue that the company's stock value will be maximized along with its capital growth. Firms that grow their business capital will be rewarded in the financial marketplace. Of course, there are other issues to consider when managing a business, but for our purposes, we can include other objectives and constraints. For example, we might include the firm's profit at the end of the next year along with the capital growth goal. The basic framework remains, with or without these other issues.

An alternative is to define a target value of capital at the end of the planning period - Target_T . The objective is to penalize deviations below the target, and to reward scenarios that exceed the target. The corresponding objective function takes the form:

$$\text{Maximize Benchmark} = \lambda_1 * \sum_s \text{Bonus}^s - \lambda_2 * \sum_s \text{Penalty}^s \quad [9]$$

Where $\text{Bonus}^s - \text{Penalty}^s \equiv C_T^s - \text{Target}_T$
 λ_1 and λ_2 are weights for deviations,
 $\text{Bonus}^s \geq 0, \text{Penalty}^s \geq 0$.

We add parameters to this framework so those penalties are more significant than bonuses. As well we can provide stepwise linear penalties so that large deviation are more significant than smaller deviations. This approach approximates the CER approach discussed above, by defining a piecewise linear function for the nonlinear utility curve.

2.4 Illustration of a DFA System

Today, the technology exists for building an integrated financial planning system such as the one proposed in the previous section (e.g. Lowe and Stanard 1996, and Correnti et al. 1998). We illustrate the results of applying an optimization-based DFA system being developed by American Re-Insurance.

A financial institution is constantly faced with the issue of how to deploy capital. In practice, new products are introduced when the expected profit exceeds a hurdle rate that reflects the opportunity cost of the capital. The opportunity cost equals the dual variable⁷ (also called shadow price or Lagrange multiplier, see Hillier and Lieberman, 1995) from the DFA model for the capital constraints, equations 5, 6, or 7. In the figure below, we show two potential product lines. The efficient frontier reflects the optimal management of the company given existing product lines and their exposures. We measure exposure as Adjusted EPD. The adjustment reflects our suggestion that risk becomes most important below a certain threshold level. This is similar to the concept of downside risk or semi-variance.

The current business is managed to a medium risk level reflected by the large white circle on the frontier. Adding product A is advantageous to the business, while product B is disadvantageous. We would thus decide that venture A is profitable relative to venture B. Clearly, venture A is superior to venture B. Any capital allocation rule should be consistent with this observation.

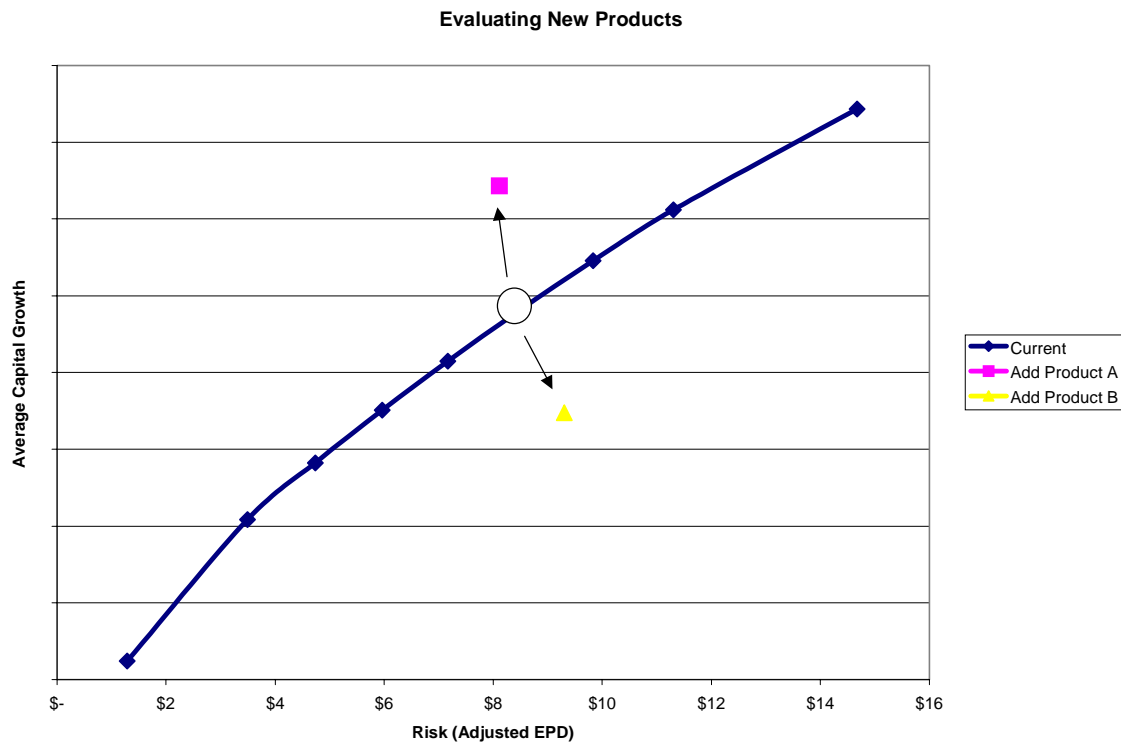


Figure 6
Efficient Frontier for Capital at end of Multi-period DFA System

⁷ The name dual variable comes from the solution to an equivalent optimization problem, called the dual model. We employ the term dual variable as interchangeable from shadow price or Lagrange multiplier.

It is important to observe the effect multiple time periods can have on the decision to add a given product. In certain instances, risk may look acceptable in a short-term basis, while the longer-term discloses unacceptable exposures. We use American Re-Insurance's (Am Re) Global Economic Model (GEM), which is a multi-period model that generates inflation and interest rate scenarios along with other economic conditions. The economic conditions are reflected in the business model through Am Re's Risk Management System (ARMS). The presented efficient frontier appears at the end of the multi-period model.

In a large company, the planning problem is complicated by organizational and information limits. So much data and time pressure exists that a single planning system cannot be easily implemented. It is difficult to control a large organization from a central location. Information must be passed to headquarters and back to the business units on a timely and concise fashion. Many people must be involved in joint decisions, complicating the decision process. As a consequence, we next turn to decentralized approaches in the next section.

3. Decentralized Approaches

In order to accommodate a decentralized system, we must agree on some goals under which the system will operate. First, the decentralized system should approximate the solution to the previous model, or at least it should converge to the solution of the centralized model. Decisions that are accepted by the centralized system should be approved by the decentralized system. Likewise, unprofitable projects should be rejected by both approaches. Second, the decentralized system should depend on information that is readily available and transmitted. Headquarters must take the responsibility for timely communications. Third, the decision process within each of the business units should be relatively similar so that consistency is maintained to the extent possible. Of course, actual decisions are unique to the business units.

Let us lay out some notation: set D defines the business units, with decisions $x(d)$ and $y(d)$ for assets and liabilities, respectively⁸. We add decision variables for the asset and liability categories, within the business units – $x_{j,t}^s(d)$, and $y_{i,t}^s(d)$, respectively. The return for a unit is then the change in net position across time periods:

$$\text{Market}_t^s(d) = \sum_s \text{Market value (assets}_t^s(d) - \text{liabilities}_t^s(d)).$$

Next, we add a constraint set to the model for allocating capital across the business units:

$$\sum C_t^s(d) \leq C_t^s \quad \text{where } C(d) \text{ indicates capital for business } d \in D. \quad [10]$$

Based on the capital allocated, we derive a risk-adjusted profit for each division as:

$$\text{Profit}_t^s(D) = \{\text{Market}_{t+1}^s(d) - \text{Market}_t^s(d)\} / C_t^s(d) \quad [11].$$

⁸ For simplicity, we are allowing business units to render both asset and liability decisions. This situation can be readily constrained to one side of the balance sheet, depending upon the particular circumstances at hand.

The profit values are adjusted for the capital needed to support the business⁹. Alternatively, decision-makers should be rewarded for beating their benchmarks and this aspect can be included in the planning system. If the profits exceed the company's hurdle rate, they are further evaluated for funding. Otherwise, the projects are rejected.

The decisions for assets and liabilities (x and y) could be carried out in a completely decentralized fashion, except that business units' capital structure would come from headquarters as satisfying constraint [10]. In effect, businesses will act as autonomous companies sharing a common capital basis. Unfortunately under such a regime, there is little information for helping headquarters set the above allocation rules, unless the businesses are truly independent of each other in all respects. This assumption seems unlikely to be satisfied in most cases since correlations are present, for instance, with respect to discount rates. Hence the decisions will be sub-optimal.

To solve this problem, we turn to the area of optimization called decomposition methods. These methods converge to the solution of the centralized problem, but require only partial information. Decomposition methods are divided into two types: price directive and resources directive. We will concentrate on price directive methods since they are easier to understand and require fewer directives from headquarters. Prices are computed for those activities that have a distinguishing impact upon the company's return and risk.

At a general level, the procedure is straightforward: business units will add penalties via costs for activities that account for contributory risks. Conversely, the same units are given bonuses when their profit comes at an opportune time – i.e. scenarios that adversely affect the overall corporate capital. Bonuses and penalties are tallied according to the company's risk measure. The capital allocation values are then adjusted to drive the company towards the desired firmwide solution. The idea is to include an additional set of factors in the calculation of capital allocation at the business level:

$$\text{Modified } C_t^s(d) = \text{standard } C_t^s(d) + \pi_t^s(d) \quad [12]$$

Where the dual variable $\pi_t^s(d)$ comes from the solution of a special DFA model at headquarters.

To compute risk prices for the business units requires an understanding of the unit's impact on the overall company. In the decentralized approach, headquarters take aggregate information from each of the businesses, as it pertains to each scenario. In other words, the units provide estimates of their profits and losses for each scenario and the corresponding capital.

The capital for a business $d \in D$ is then calculated as a probability weighted average of the scenarios:

$$E [C_t(d)] = \sum p_s C_t^s(d) \quad \text{where the probability scenario } s \equiv p_s \quad [13]$$

For this model, we need not maintain the usual risk restrictions across the total businesses as dictated in the previous section, whether VaR, EPD, or CER, since the approach is based on a price mechanism. However, for practical purposes, the imposition of risk considerations within each business unit provides a conservative feature. Imposing these constraints requires a resource

⁹ We add a charge for capital in this discussion. However, there are a variety of methods for addressing the risk reward tradeoff, such as applying a Sharpe like ratio.

directive decomposition approach (Mulvey, 1999). Also, since the overall risk constraints must be computed for the entire company, headquarters must carry out the calculations.

4. Capital Allocation Discussion

Allocating capital fulfills several functions within a financial intermediary. First, it sends signals to the various divisions/groups regarding directional improvements in their activities. Adding capital to an existing business often indicates that the managers are expected to go out and gain some top-line growth. Conversely, eliminating capital can be construed as a sign that the activity should be shrunk. In an ideal world, businesses would be totally independent upon each other, so that the capital allocation decision could be compared with standalone businesses of a similar size and complexion. In the real world, unfortunately, rarely are businesses totally independent. Correlations appear due to dependence upon the same factors (such as the level of business activity, or the competitiveness of an industry segment), due to discounting cashflows with the same discount rate, due to overlapping territories or overlapping marketing campaigns. We need to approximate the degree of overlap, and account for it in the calculation of the desired level of capital. The equations presented in Section 3 take into account overlapping correlations.

As a second advantage, notional amounts (from an allocation process) form the basis of the risk adjusted profit/loss figures. Company planners can distribute incentives to select business components, depending upon circumstances.

A third benefit of allocating capital entails the scalability of the process. Certainly, the total company must decide upon the proper level of capital. This decision hinges on the predictability of earnings, the market's perception of rewarding risk taking versus risk avoidance, and the cost of borrowing funds as compared with equity infusions. Also, regulators must be convinced of the adequacy of the capital reserves.

Once a company has found a workable approach to this global problem, the organization can emulate it for other capital budgeting decisions. It fits the standard model for building a business: first, you take a certain amount of capital, build an infra-structure to conduct the business, and then manufacture a new product (Holmer and Zenios, 1995). The process starts with the capital allocation decision.

5. An Insurance Example

This section presents a simple example with four lines of insurance and three equally likely scenarios. See the chart below. Remembering that the goal should be to develop an approach that will give approximately the same solution for the two systems, we show that the capital allocation rules dictated by EPD can be adjusted to achieve the desired goal. Note that we have computed capital allocation based on EPD and then modified the values based on the correlation with the company's overall profit. This approach approximates the procedure discussed in section 3.

In the first part, we calculate the EPD values in the standard fashion, both unadjusted and adjusted for the assigned capital. Note that the four business lines are relatively profitable on an expected value basis, but two of the lines have large losses under certain scenarios, especially the Cat line under scenario 1.

In the second part, we compute the correlation coefficients for each line with the overall profit of the company. We see that the Cat business has the highest correlation, .98. We can then adjust the allocated capital by decreasing capital for negatively correlated lines. This modification decreases capital for lines 1 and 2. We can then take the capital allocation and return to the adjusted EPD values. Note that the return on equity (ROE) values suggest that lines 1, 3 and 4 lie above 20%.

In the third part, we conduct an optimization of the four lines. We employ an exponential utility curve with two parameters, $a = 2$ and $b = .8$ for the function:

$$U(C) = a \times C - \exp(-b \times C). \quad [14]$$

See Mulvey et al. (1997) for a discussion of the advantages of this utility function. At the optimal solution, the model recommends that the first three lines be accepted, but that line four should be reduced to 44% of its original size. Taking the dual variables from this optimization model, we can adjust the capital allocation values from the standard EPD values. Here we must increase the capital for line 4 to \$9.88 M in order to account for the large losses under scenario 1. This reduces the ROE value for this line down to 10.1 % (from the previous 20%). In effect, the large loss must incur a penalty even though it occurs under one scenario due to its impact on the entire company's capital. The dual prices from the DFA system provide a direct measure for making this adjustment. In contrast, the adjustment of EPD based on correlation is unable to render enough penalty for line 4. The utility optimization takes into account the entire probability distribution.

Once the risk adjusted profit figures are computed, we must decide upon the best mix of policies to write. Certainly, the returns for lines 1 and 3 (auto and liability) are superior and likely to be meet the company's hurdle rate. On the other hand, lines 2 and 4 (workers comp and catastrophe property) are much less desirable from the risk adjusted standpoint, with returns closer to 10%. The utility optimization recommends that line 4 be written only at the 44% of its total, due to the risks to the overall company capital. Alternatively, retrocessional reinsurance could be taken on. In any event, the company executives should conduct sensitivity analysis of the parameters of the utility function in order to gain confidence in the DFA model's recommendations.

Capital Allocation for Each of Four Business Lines – Basic Environment								
Insurance Lines	Premium							
Auto	\$4,500,000							
Workers Comp	\$9,200,000							
Liability	\$10,500,000							
Catastrophe Property	\$5,000,000							
Total	\$29,200,000							
Losses for each division in all scenarios.								
	Probability of Scenario Occurring							
	33.33%	33.33%	33.33%					
Division	Scenario 1	Scenario 2	Scenario 3	Expected Loss				
Auto	\$4,000,000	\$4,200,000	\$5,000,000	\$4,400,000				
Workers Comp	\$6,000,000	\$9,000,000	\$12,000,000	\$9,000,000				
Liability	\$12,000,000	\$10,000,000	\$8,000,000	\$10,000,000				
Catastrophe Property	\$10,000,000	\$1,000,000	\$1,000,000	\$4,000,000				
	\$32,000,000	\$24,200,000	\$26,000,000	\$27,400,000				
1. Capital Allocation using Expected Policyholder Deficit (EPD) at 1% level								
Unadjusted EPD Calculation								
	Scenario 1	Scenario 2	Scenario 3	EPD	EPD Ratio	Acceptable	Allocated Capital	
Auto	\$0	\$0	\$500,000	\$166,667	3.8%	1%	\$368k	
Workers Comp	\$0	\$0	\$2,800,000	\$933,333	10.4%	1%	\$2,530k	
Liability	\$1,500,000	\$0	\$0	\$500,000	5.0%	1%	\$1,200k	
Catastrophe Property	\$5,000,000	\$0	\$0	\$1,666,667	41.7%	1%	\$4,880k	
						Total	\$8,978k	
Expected Deficits Adjusted for Allocated Capital (at 1% level)								
	Scenario 1	Scenario 2	Scenario 3	EPD	EPD Ratio			
Auto	\$0	\$0	\$132,000	\$44,000	1.0%			
Workers Comp	\$0	\$0	\$270,000	\$90,000	1.0%			
Liability	\$300,000	\$0	\$0	\$100,000	1.0%			
Catastrophe Property	\$120,000	\$0	\$0	\$40,000	1.0%			

2. Adjusting EPD Allocated Capital with Correlations

EPD Calculation as Above, Modified for Correlation with Entire Portfolio

	Correlation	Corr Mult	Modified EPD	Mod EPD Ratio	Acceptable	Allocated Capital	ROE
Auto	-0.47	0.63	\$105,334	2.4%	1%	\$290k	34%
Workers Comp	-0.73	0.57	\$528,604	5.9%	1%	\$2,300k	9%
Liability	0.73	0.93	\$466,819	4.7%	1%	\$1,170k	43%
Catastrophe Property	0.98	0.99	\$1,656,424	41.4%	1%	\$4,880k	20%
					Total	\$8,640k	21%

Modified Expected Deficits Adjusted for Allocated Capital

	Scenario 1	Scenario 2	Scenario 3	EPD	Adj EPD	EPD Ratio
Auto	\$0	\$0	\$210,000	\$70,000	\$44,240	1.0%
Workers Comp	\$0	\$0	\$500,000	\$166,667	\$94,394	1.0%
Liability	\$330,000	\$0	\$0	\$110,000	\$102,700	1.0%
Catastrophe Property	\$120,000	\$0	\$0	\$40,000	\$39,754	1.0%

3. Adjusting EPD Allocated Capital based on dual prices from DFA solution

Scenario-based Penalties

Optimization Model

Cost function parameter	2
Cost function positive part	0.8
Company Surplus	\$20,000,000

Decision Variables

	Current policy	Optimization Solution	Marginal gain-adding business
Auto	1	1	80,000
Workers Comp	1	1	961,353
Liability	1	1	400,000
Catastrophe Property	1	0.44	Negative

6. Conclusions

This paper presents a practical approach for optimizing a large financial intermediary over a multi-period planning horizon. The optimization can be conducted with a tightly integrated approach, or it can be decomposed across a collection of business units. The goal is the same – maximize shareholder value by employing an optimization based DFA system.

A critical difficulty for any decentralized method involves businesses that possess correlated risks. To address overlapping risks, we adjusted the capital allocation process by adding bonuses for diversification, and penalties for concentration, for each scenario based on the DFA's optimal dual prices (Lagrange multipliers). For businesses that possess cashflows across multiple years, there is an implicit correlation through the discounting of the cashflows. Hence, we must adjust the discount factor to take into account correlated risks.

The optimization based DFA model provides an ideal framework for conducting an integrated risk analysis. The prices for risks come out of the model's solution in a natural and direct fashion. Thus, we can readily apply a decentralized system in an efficient and convergent fashion. This will improve the chances that DFA systems will be implemented in large insurance and reinsurance companies.

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