WORKERS' COMPENSATION CLASSIFICATION CREDIBILITIES

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Abstract

Via excerpts from the classification section of the filing for 8/1/99 Massachusetts Workers' Compensation rates, this paper presents a practical application of the credibility ideas in "Credibility With Shifting Risk Parameters, Risk Heterogeneity and Parameter Uncertainty," PCAS 1998.

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WORKERS' COMPENSATION CLASSIFICATION CREDIBILITIES

Credibilities are an important component of calculating classification rates. The credibilities were calculated for Massachusetts Workers' Compensation classification rates using ideas from "Credibility With Shifting Risk Parameters, Risk Heterogeneity and Parameter Uncertainty," PCAS 1998.

CLASSIFICATION RELATIVITIES

In order to determine how to spread the proposed rate changes by Industry Group to individual classes, relativities within an Industry Group are determined for each class.[1]

Separate relativities are first determined for Serious, Non-Serious and Medical losses, which are then weighted together using the Industry Group pure premiums as weights. Each relativity is determined by combining three relativities: a relativity based on recent Massachusetts' experience,[2] a relativity based on recent countrywide experience, and the relativity underlying the current rates. Each relativity is determined by comparing the data for the class to its Industry Group. Examples are shown for five different classes in Exhibits 1 to 5.

Assume that the proposed average rate for an Industry Group is $3.00. Then a class in that Industry Group with a balanced formula relativity of .80, would have a proposed average rate of (.80) ($3.00) = $2.40, prior to capping. In general, the proposed average rates by Industry Group[3] are combined with the balanced formula class relativities to produce proposed average uncapped rates by class. The balancing of the relativities results in average rates by class which balance to the proposed average rate for each Industry Group.

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[1] In Massachusetts, some classes are grouped together for purposes of calculating classification rates. For example, 8810 Clerical Office Employees, and 8901 Telephone/Telegraph Company Office Employees, have the same rate.

[2] The Massachusetts Relativity is in turn a credibility weighted average of relativities based on individual years.

[3] Which result from the current average rates, the proposed overall rate change, and the Industry Group differentials

Credibility is used to weight together these different relativities. For example, the Massachusetts relativity for a class might be 1.2, the countrywide relativity 1.1 and the current relativity 1.3. Then if the Massachusetts credibility were 40%, and the countrywide were 25%, the indicated relativity would be: (1.2) (40%) + (1.1) (25%) + (1.3) (35%) = 1.21.

The calculation of these credibilities is discussed below. Generally, the more Massachusetts data for a class (or class combination), the more weight is given to the recent Massachusetts data. The smaller the class, the more weight is given to the recent countrywide data or to the current rate. Thus, for large classes the method is more responsive, while for small classes it is more stable.

CLASSIFICATION CREDIBILITIES

The manner of calculating the credibilities has been put on a more sound theoretical basis, incorporating advances in credibility theory over the last decade. Specific consideration has been given to: how quickly class relativities shift over time, the correlation between Massachusetts class relativities and those from other states, the dependence of the variance of the observed relativities on the size of the class, and the impact of the differing maturities of data.

The general features are as follows:

1. All other things being equal, larger volumes of data receive more credibility.
2. All other things being equal, a given volume of Massachusetts data receives more credibility than the same volume of data from another state.
3. All other things being equal, Massachusetts data from more recent years receives more credibility than Massachusetts data from less recent years.
4. All other things being equal, Massachusetts data from more mature years receive a little more credibility than Massachusetts data from less mature years.

5. All other things being equal, a given volume of data (as measured by expected losses) for serious losses receives less credibility than a similar volume of data for non-serious or medical losses.

An example of a very large class is 7219, Trucking. As can be seen, the most recent year of Massachusetts experience receives a lot of weight. For large classes the method is more responsive, while for smaller classes the method is more stable.

Credibilities for Class 7219

	Serious	Non-Serious	Medical
Mass., 90/91 @ 5th	7.4%	7.3%	6.6%
Mass., 91/92 @ 4th	9.1	9.7	9.2
Mass., 92/93 @ 3rd	11.1	9.9	9.5
Mass., 93/94 @ 2nd	14.6	12.9	14.0
Mass., 94/95 @ 1st	33.8	32.3	38.4
Sum for Massachusetts	76.0	72.1	77.7
Countrywide	14.1	23.4	19.7
Underlying Current Rates	9.9	4.5	2.6

An example of a small to medium size class is 3220, Can Manufacturing. In comparison to a large class, less weight is given to recent Massachusetts data and more weight is given to data from other states and to the relativity underlying the current rates.

	Credibilities for Class 3220		
	Serious	**Non-Serious**	**Medical**
Mass., 90/91 @ 5th	5.3%	3.3%	4.6%
Mass., 91/92 @ 4th	3.5	2.4	3.1
Mass., 92/93 @ 3rd	3.7	3.0	3.9
Mass., 93/94 @ 2nd	5.1	4.6	6.5
Mass., 94/95 @ 1st	4.8	5.3	8.1
Sum for Massachusetts	22.4	18.6	26.2
Countrywide	22.9	48.7	44.1
Underlying Current Rates	54.7	32.7	29.7

An example of a very small size class is 5443, Lathing. Very little weight is given to recent Massachusetts data. Almost all of the weight is given to data from other states and to the relativity underlying the current rates.

	Credibilities for Class 5443		
	Serious	**Non-Serious**	**Medical**
Mass., 90/91 @ 5th	.7%	.1%	.3%
Mass., 91/92 @ 4th	.1	.0	.0
Mass., 92/93 @ 3rd	.6	.2	.3
Mass., 93/94 @ 2nd	2.7	.9	1.6
Mass., 94/95 @ 1st	1.0	.4	.7
Sum for Massachusetts	5.1	1.6	2.9
Countrywide	32.4	50.0	49.1
Underlying Current Rates	62.5	48.4	48.0

The calculation of these credibilities is explained in detail below.

CALCULATING CREDIBILITIES

The last time the amount of credibility to be assigned to classification data was reviewed by the Bureau was in the filing for 1/1/89 rates. The results of that study have been used since

then. The Bureau has performed a new analysis of this whole subject. It differs from the analysis done a decade ago in a number of aspects:

1. It is based on updated experience, including more total years of data.[4]
2. It is based on recent developments in Bühlmann/Least Squares Credibility theory. The previous formula was based on Classical/Limited Fluctuation Credibility.
3. Specific consideration of the appropriate credibility to assign to countrywide data is included. At the time of the previous study countrywide data was not being used in Massachusetts.
4. Separate credibilities are calculated for each year of Massachusetts data.
5. The impact of the different maturities of data are taken into account.

An example of the calculation of classification credibilities is given below. Appendix A discusses the selection of the parameters that enter into this calculation. Appendix B gives a numerical example of the calculation of credibilities involving fewer years.

AN EXAMPLE OF CLASSIFICATION CREDIBILITIES

Volume of Data

As an example, take Class 3220, Can Manufacturing, using data available at the time of the filing for 7/1/96 rates.

Composite Policy Year	MA Payrolls	MA Expected Losses[5]		
		Serious	Non-Serious	Medical
88/89 @ 5th	10,048,010	$269,287	$87,418	139,667
89/90 @ 4th	6,461,890	173,179	56,218	89,820
90/91 @ 3rd	8,345,618	223,663	72,607	116,004
91/92 @ 2nd	7,243,313	194,121	63,017	100,682
92/93 @ 1st	7,283,640	195,202	63,368	101,243

[4] Fifteen years of data were analyzed, from Composite Policy Year 80/81 to Composite Policy Year 94/95.

[5] Based on reported payrolls times Pure Premiums Underlying the Current Rate of 2.68, .87 and 1.39.

Three years of "countrywide" data had 159 Serious claims and 1,801 Non-Serious claims. Translate to the equivalent Massachusetts Expected Losses per year as follows:[6]

Serious (159) (80,000)/3 = $4,240,000
Non-Serious (1,801) (6,000)/3 = $3,602,000
Medical (159 + 1,801) (7,000)/3 = $4,573,333

Assumptions

- Trying to predict Year 54 Massachusetts classification relativity.
- MA data for Years 46 to 50, at reports 5 through 1. (5 years of Massachusetts data at latest report.)
- CW data for Years 47 to 49 at reports 3 through 1. (3 years of "countrywide" data at latest report.)
- CW data is from 10 states of equal size. (An approximation to the actual situation, where there have been various numbers of states, some bigger and some smaller.)
- The current relativity is based on MA data from Years 1 to 45 at 5th report and CW data from Years 40 to 46 at 3rd report. (The Massachusetts rates have been based on Massachusetts data for at least the last 50 years. Since the rates effective 1/1/93, Massachusetts classification data has been supplemented by data from other states.)
- The annual volume of MA data for Years 1 to 45 and 54 is equal to the average annual volume of MA data for Years 46 to 50. (A simplifying assumption.)
- The annual volume of CW data for Years 40 to 46 and 54 is equal to the average annual volume of CW data for Years 47 to 49. (A simplifying assumption.)

[6] Based on $80,000 Indemnity per Serious claim, $6,000 of Indemnity per Non-Serious claim, and $7,000 Medical per Lost-time claim.

Covariance Structure

As per Sections 7.11, 8.2 and 8.6 of "Credibility with Shifting Risk Parameters, Risk Heterogeneity and Parameter Uncertainty" by Howard C. Mahler, PCAS 1998, assume the covariances between years of data are given by Equations 5.10 and 5.11.

$$\mathrm{Cov}\left[X_i, X_j\right] = r^2 \left\{\rho^{|i-j|} + \gamma^{|i-j|}\, I \Big/ \sqrt{E_i\, E_j} + \delta_{ij}\left(K \Big/ \sqrt{E_i\, E_j} + J\right)\right\} \qquad \sqrt{E_i E_j} \geq \Omega \qquad (5.10)$$

$$\mathrm{Cov}\left[X_i, X_j\right] = r^2 \left\{\rho^{|i-j|} + \gamma^{|i-j|}\, I \Big/ \Omega + \delta_{ij}\left(K \Big/ \sqrt{E_i\, E_j} + J\right)\right\} \qquad \sqrt{E_i E_j} \leq \Omega \qquad (5.11)$$

where $\delta_{ij} = \begin{cases} 0 & i \neq j \\ 1 & i = j \end{cases}$

Parameters of the Covariance Structure

Start with the parameters for Total Losses (not split into Serious, Non-Serious and Medical) from Sections 7.7 and 8.3 of Mahler (1998). Then based on the analysis in Appendix A, select parameters for Serious, Non-Serious and Medical Pure Premiums.

	Total[7]		**Serious**		**Non-Serious**		**Medical**	
	Intrastate	Interstate	Intrastate	Interstate	Intrastate	Interstate	Intrastate	Interstate
ρ	.98	.98	.99	.99	.99	.99	.99	.99
γ	.85	.85	.85	.85	.85	.85	.85	.85
r^2	1	.7	1	.7	1	.7	1	.7
I	100,000	100,000	50,000	50,000	20,000	20,000	30,000	30,000
J	.10	.05	.04	.02	.04	.02	.04	.02
K	500,000	0	500,000	0	200,000	0	200,000	0
Ω	50,000	50,000	25,000	25,000	10,000	10,000	15,000	15,000

Where r^2 is prior to adjustment for the effects of differences in maturity.

Linear Equations of Credibilities

As per Equations 8.1 in Mahler (1998):

$$\sum_{j=1}^{50} Z_j S_{ij} + \sum_{j=40}^{49} W_j U_{ij} = \frac{\lambda}{2} + S_{i,\,54} \qquad i = 1, 2, \ldots, 50$$

[7] From Mahler (1998).

$$\sum_{i=1}^{50} Z_i U_{ij} + \sum_{i=40}^{49} W_i T_{ij} = \frac{\lambda}{2} + U_{54,\,j} \qquad j = 40, 41, \ldots, 49$$

$$\sum_{i=1}^{50} Z_i + \sum_{j=40}^{49} W_j = 1$$

Where: Z_i is credibility for MA for year i

W_j is credibility for CW for year j

S_{ij} = the covariances within MA

T_{ij} = the covariances for CW

U_{ij} = the covariances between MA and CW

λ = Lagrange Multiplier (whose value has no particular importance)

Once one solves for the Z's and W's then

Z_{46} is the credibility assigned to the oldest of five years of MA data

Z_{47} is the credibility assigned to the 2nd oldest of five years of MA data

Z_{48} is the credibility assigned to the middle of five years of MA data

Z_{49} is the credibility assigned to the 2nd most recent of five years of MA data

Z_{50} is the credibility assigned to the most recent of five years of MA data

$W_{47} + W_{48} + W_{49}$ is the credibility assigned to the three years of CW data combined.

The remaining weight, $\sum_{i=1}^{45} Z_i + \sum_{j=40}^{46} W_j$ is assigned to the current relativity.

Appendix B contains a numerical example involving fewer years.

Incorporating the Effect of Differing Maturities

As per Sections 7.10-7.12 of Mahler (1998), the differing maturities of data are taken into account by reducing the covariances between data at different reports. This is equivalent to multiplying the r^2 parameter in the covariance structure by a correlation.

As per Equation 7.6 of Mahler (1998), assume this correlation depends on the size of the class and the amount of loss development between the reports.

$$\text{correlation} = \text{LDF}^{-1/(1.5 + 2.25\ \text{size/million})}$$

Where the coefficient multiplying the size has been changed from .75 to 2.25 (multiplied by three), in order to take into account the fact that the total expected losses are divided into three pieces: Serious, Non-Serious, and Medical.[8]

The following loss development factors have been selected based on Massachusetts Unit Statistical Plan Data:

	1-2	2-3	3-4	4-5
Serious	1.33	1.10	1.06	1.03
Non-Serious	1.07	1.01	1.00	1.00
Medical	1.04	1.00	1.00	1.00

Resulting Credibilities

One gets the following credibilities using the volume of data for Class 3220.

	Serious	Non-Serious	Medical
88/89 @ 5th	5.7%	4.3%	5.0%
89/90 @ 4th	3.8	3.0	3.4
90/91 @ 3rd	5.2	4.8	5.6
91/92 @ 2nd	4.8	5.1	6.3
92/93 @ 1st	4.7	6.1	8.3
Total MA (5 years)	24.2	23.3	28.6
CW (3 years)[9]	30.9	50.0	50.0
Current Relativity (Remainder)	44.9	26.7	21.4

Appendix B contains a numerical example involving fewer years.

Practical Constraints

In rare circumstances where there is a very small amount of either Massachusetts or countrywide data, the credibilities that result from the solution to the linear equations can be

[8] A class with $3 million in losses is assumed to have roughly $1 million in each of Serious, Non-Serious and Medical. Multiplying the coefficient by 3 would result in the same exponent for this class measured in terms $1 million of Serious losses or $3 million of total losses.

[9] Countrywide credibility limited to 50%.

unusual. Therefore, the following constraints will be imposed on the credibilities used for classification ratemaking:

1. Where there is less than $1,000 in average annual Massachusetts Expected Losses, the countrywide credibilities shall be the maximum of those calculated with the Expected Losses and those calculated with at least $1,000 in Massachusetts Expected Losses in each year.
2. Credibilities shall not be negative.
3. Countrywide credibilities shall be limited so that the sum of the Massachusetts and Countrywide credibilities is no more than 100%.
4. **Countrywide credibilities shall be limited to 50%.**

MASSACHUSETTS WORKERS' COMPENSATION
Calculation of Class Relativities

Class:
3220 CAN MFG

Schedule: 176
Proposed Effective: 08/01/1999
Manufacturing

Policy Period	Payroll	Adjusted On-Level Loss and Loss Adjustment Expense						Pure Premiums Per $100 of Payroll			
		Serious		Non-Serious		Medical	Total				
		Cases	Amount	Cases	Amount	Amount	Amount	Serious	Non-Serious	Medical	Total
1990/1991	11,153,663	4	192,851	20	59,934	100,306	353,091	1.73	0.54	0.90	3.17
1991/1992	7,243,313	1	70,909	7	42,932	71,565	185,406	0.98	0.59	0.99	2.56
1992/1993	7,283,640	3	117,063	4	3,855	69,301	190,219	1.61	0.05	0.95	2.61
1993/1994	9,040,649	2	199,751	7	9,761	77,489	287,001	2.21	0.11	0.86	3.17
1994/1995	8,568,800	2	167,473	4	53,506	99,790	320,769	1.95	0.62	1.16	3.74
MA Total	43,290,065	12	748,047	42	169,988	418,451	1,336,486	1.73	0.39	0.97	3.09
						Massachusetts Indicated Relativity,	1990/1991	0.777	0.650	0.733	
						Massachusetts Indicated Relativity,	1991/1992	0.601	0.720	0.870	
						Massachusetts Indicated Relativity,	1992/1993	1.265	0.067	0.951	
						Massachusetts Indicated Relativity,	1993/1994	1.850	0.142	0.956	
						Massachusetts Indicated Relativity,	1994/1995	2.114	0.937	1.447	
						Massachusetts Credibility,	1990/1991	0.053	0.033	0.046	
						Massachusetts Credibility,	1991/1992	0.035	0.024	0.031	
						Massachusetts Credibility,	1992/1993	0.037	0.030	0.039	
						Massachusetts Credibility,	1993/1994	0.051	0.046	0.065	
						Massachusetts Credibility,	1994/1995	0.048	0.053	0.081	
						Massachusetts Weighted Average Relativity:		1.361	0.521	1.058	1.072
						Sum of Massachusetts Credibilities:		0.224	0.186	0.262	
						Countrywide Pure Premium:		3.38	1.05	1.15	5.58
						Countrywide Indicated Relativity:		2.285	1.457	1.191	1.754
						Countrywide Credibility:		0.229	0.487	0.441	
						Countrywide Lost-time Claims:		48	850		
						Pure Premium Underlying Present Rate:		1.67	0.44	0.93	3.04
						Relativity to Underlying Present Rate:		1.286	0.741	1.079	1.095
						Credibility to Underlying Relativity:		0.547	0.327	0.297	
						Industry Group Adjusted On-Level Pure Premium:		1.538	0.779	1.039	3.356
						Formula Relativity:		1.532	1.049	1.123	
						Balanced Formula Relativity:		**1.539**	**1.054**	**1.133**	**1.301**

MASSACHUSETTS WORKERS' COMPENSATION
Calculation of Class Relativities

Class:
5443 LATHING-DR

Schedule: 276
Proposed Effective: 08/01/1999
Construction

		Adjusted On-Level Loss and Loss Adjustment Expense						Pure Premiums Per $100 of Payroll			
Policy Period	Payroll	Serious		Non-Serious		Medical	Total				
		Cases	Amount	Cases	Amount	Amount	Amount	Serious	Non-Serious	Medical	Total
1990/1991	79,155	0	0	1	1,330	4,155	5,485	0.00	1.68	5.25	6.93
1991/1992	10,156	0	0	0	0	0	0	0.00	0.00	0.00	0.00
1992/1993	57,790	0	0	0	0	0	0	0.00	0.00	0.00	0.00
1993/1994	237,536	0	0	0	0	0	0	0.00	0.00	0.00	0.00
1994/1995	76,564	0	0	0	0	0	0	0.00	0.00	0.00	0.00
MA Total	461,201	0	0	1	1,330	4,155	5,485	0.00	0.29	0.90	1.19
						Massachusetts Indicated Relativity,	1990/1991	0.000	0.841	1.935	
						Massachusetts Indicated Relativity,	1991/1992	0.000	0.000	0.000	
						Massachusetts Indicated Relativity,	1992/1993	0.000	0.000	0.000	
						Massachusetts Indicated Relativity,	1993/1994	0.000	0.000	0.000	
						Massachusetts Indicated Relativity,	1994/1995	0.000	0.000	0.000	
						Massachusetts Credibility,	1990/1991	0.007	0.001	0.003	
						Massachusetts Credibility,	1991/1992	0.001	0.000	0.000	
						Massachusetts Credibility,	1992/1993	0.006	0.002	0.003	
						Massachusetts Credibility,	1993/1994	0.027	0.009	0.016	
						Massachusetts Credibility,	1994/1995	0.010	0.004	0.007	
						Massachusetts Weighted Average Relativity:		0.000	0.053	0.200	0.060
						Sum of Massachusetts Credibilities:		0.051	0.016	0.029	
						Countrywide Pure Premium:		6.46	2.63	2.47	11.56
						Countrywide Indicated Relativity:		1.344	1.478	1.147	1.323
						Countrywide Credibility:		0.324	0.500	0.491	
						Countrywide Lost-time Claims:		6	69		
						Pure Premium Underlying Present Rate:		6.42	1.13	1.88	9.43
						Relativity to Underlying Present Rate:		1.343	0.860	0.967	1.152
						Credibility to Underlying Relativity:		0.625	0.484	0.480	
						Industry Group Adjusted On-Level Pure Premium:		5.019	1.864	2.224	9.107
						Formula Relativity:		1.275	1.156	1.033	
						Balanced Formula Relativity:		**1.270**	**1.153**	**1.030**	**1.187**

MASSACHUSETTS WORKERS' COMPENSATION
Calculation of Class Relativities

Class:
7219 TRUCKING-NOC-DR

Schedule: 320
Proposed Effective: 08/01/1999
Miscellaneous

		Adjusted On-Level Loss and Loss Adjustment Expense						Pure Premiums Per $100 of Payroll			
		Serious		Non-Serious		Medical	Total				
Policy Period	Payroll	Cases	Amount	Cases	Amount	Amount	Amount	Serious	Non-Serious	Medical	Total
1990/1991	310,879,391	253	23,886,364	1,344	5,683,578	10,726,533	40,296,475	7.68	1.83	3.45	12.96
1991/1992	297,662,403	116	10,944,340	1,147	4,193,907	6,112,181	21,250,428	3.68	1.41	2.05	7.14
1992/1993	175,644,627	86	9,719,206	515	3,254,293	3,874,080	16,847,579	5.53	1.85	2.21	9.59
1993/1994	150,072,203	65	7,509,871	335	2,656,423	2,993,179	13,159,473	5.00	1.77	1.99	8.77
1994/1995	322,229,670	75	9,150,763	832	3,721,144	5,089,702	17,961,609	2.84	1.15	1.58	5.57
MA Total	1,256,488,294	595	61,210,544	4,173	19,509,345	28,795,675	109,515,564	4.87	1.55	2.29	8.71
						Massachusetts Indicated Relativity,	1990/1991	2.157	1.794	1.852	
						Massachusetts Indicated Relativity,	1991/1992	1.419	1.095	1.126	
						Massachusetts Indicated Relativity,	1992/1993	1.974	1.240	1.249	
						Massachusetts Indicated Relativity,	1993/1994	2.057	1.464	1.314	
						Massachusetts Indicated Relativity,	1994/1995	1.580	1.182	1.232	
						Massachusetts Credibility,	1990/1991	0.074	0.073	0.066	
						Massachusetts Credibility,	1991/1992	0.091	0.097	0.092	
						Massachusetts Credibility,	1992/1993	0.111	0.099	0.095	
						Massachusetts Credibility,	1993/1994	0.146	0.129	0.140	
						Massachusetts Credibility,	1994/1995	0.338	0.323	0.384	
						Massachusetts Weighted Average Relativity:		1.766	1.291	1.289	1.522
						Sum of Massachusetts Credibilities:		0.760	0.721	0.777	
						Countrywide Pure Premium:		5.35	1.81	2.42	9.58
						Countrywide Indicated Relativity:		1.996	1.584	1.493	1.758
						Countrywide Credibility:		0.141	0.234	0.197	
						Countrywide Lost-time Claims:		2,052	24,632		
						Pure Premium Underlying Present Rate:		4.75	1.21	2.09	8.05
						Relativity to Underlying Present Rate:		1.891	1.430	1.470	1.667
						Credibility to Underlying Relativity:		0.099	0.045	0.026	
						Industry Group Adjusted On-Level Pure Premium:		2.716	1.178	1.674	5.568
						Formula Relativity:		1.811	1.366	1.334	
						Balanced Formula Relativity:		**1.850**	**1.386**	**1.368**	**1.607**

MASSACHUSETTS WORKERS' COMPENSATION
Calculation of Class Relativities

Class:
8803 AUDITOR,ACCOUNTANT,ETC-TRAVELING

Schedule: 350
Proposed Effective: 08/01/1999
Office & Clerical

		Adjusted On-Level Loss and Loss Adjustment Expense						Pure Premiums Per $100 of Payroll			
Policy		Serious		Non-Serious		Medical	Total		Non-		
Period	Payroll	Cases	Amount	Cases	Amount	Amount	Amount	Serious	Serious	Medical	Total
1990/1991	504,194,686	4	207,365	28	163,181	198,071	568,617	0.04	0.03	0.04	0.11
1991/1992	518,396,037	4	702,091	12	161,037	204,147	1,067,275	0.14	0.03	0.04	0.21
1992/1993	555,743,864	2	143,967	12	270,935	204,666	619,568	0.03	0.05	0.04	0.11
1993/1994	622,025,390	1	28,855	13	85,331	96,443	210,629	0.00	0.01	0.02	0.03
1994/1995	812,346,415	2	186,949	16	70,298	173,148	430,395	0.02	0.01	0.02	0.05
MA Total	3,012,706,392	13	1,269,227	81	750,782	876,475	2,896,484	0.04	0.02	0.03	0.09
						Massachusetts Indicated Relativity,	1990/1991	0.192	0.329	0.287	
						Massachusetts Indicated Relativity,	1991/1992	0.882	0.295	0.314	
						Massachusetts Indicated Relativity,	1992/1993	0.181	0.553	0.319	
						Massachusetts Indicated Relativity,	1993/1994	0.037	0.178	0.162	
						Massachusetts Indicated Relativity,	1994/1995	0.226	0.132	0.260	
						Massachusetts Credibility,	1990/1991	0.057	0.046	0.044	
						Massachusetts Credibility,	1991/1992	0.062	0.057	0.055	
						Massachusetts Credibility,	1992/1993	0.070	0.077	0.075	
						Massachusetts Credibility,	1993/1994	0.084	0.106	0.113	
						Massachusetts Credibility,	1994/1995	0.107	0.156	0.185	
						Massachusetts Weighted Average Relativity:		0.278	0.258	0.255	0.266
						Sum of Massachusetts Credibilities:		0.380	0.442	0.472	
						Countrywide Pure Premium:		0.07	0.05	0.06	0.18
						Countrywide Indicated Relativity:		0.483	0.578	0.536	0.524
						Countrywide Credibility:		0.135	0.240	0.250	
						Countrywide Lost-time Claims:		72	1,044		
						Pure Premium Underlying Present Rate:		0.09	0.05	0.06	0.20
						Relativity to Underlying Present Rate:		0.680	0.771	0.631	0.687
						Credibility to Underlying Relativity:		0.485	0.318	0.278	
						Industry Group Adjusted On-Level Pure Premium:		0.150	0.087	0.112	0.349
						Formula Relativity:		0.501	0.498	0.430	
						Balanced Formula Relativity:		**0.503**	**0.502**	**0.430**	**0.479**

MASSACHUSETTS WORKERS' COMPENSATION
Calculation of Class Relativities

Class:
9089 BILLIARD HALL-NO BOWLING LANES

Schedule: 362
Proposed Effective: 08/01/1999
Goods & Services

		Adjusted On-Level Loss and Loss Adjustment Expense						Pure Premiums Per $100 of Payroll			
Policy		Serious		Non-Serious		Medical	Total		Non-		
Period	Payroll	Cases	Amount	Cases	Amount	Amount	Amount	Serious	Serious	Medical	Total
1990/1991	929,258	0	0	1	268	1,963	2,231	0.00	0.03	0.21	0.24
1991/1992	1,258,286	0	0	1	76	2,552	2,628	0.00	0.01	0.20	0.21
1992/1993	2,307,572	1	86,797	1	251	37,937	124,985	3.76	0.01	1.64	5.42
1993/1994	2,440,514	1	78,031	5	24,896	32,196	135,123	3.20	1.02	1.32	5.54
1994/1995	3,059,466	1	47,463	5	5,955	12,903	66,321	1.55	0.19	0.42	2.17
MA Total	9,995,096	3	212,291	13	31,446	87,551	331,288	2.12	0.31	0.88	3.31
						Massachusetts Indicated Relativity,	1990/1991	0.000	0.032	0.164	
						Massachusetts Indicated Relativity,	1991/1992	0.000	0.007	0.184	
						Massachusetts Indicated Relativity,	1992/1993	2.842	0.014	1.544	
						Massachusetts Indicated Relativity,	1993/1994	2.818	1.472	1.514	
						Massachusetts Indicated Relativity,	1994/1995	2.342	0.287	0.590	
						Massachusetts Credibility,	1990/1991	0.011	0.014	0.014	
						Massachusetts Credibility,	1991/1992	0.016	0.024	0.024	
						Massachusetts Credibility,	1992/1993	0.032	0.047	0.053	
						Massachusetts Credibility,	1993/1994	0.038	0.059	0.070	
						Massachusetts Credibility,	1994/1995	0.050	0.083	0.103	
						Massachusetts Weighted Average Relativity:		2.144	0.493	0.967	1.340
						Sum of Massachusetts Credibilities:		0.147	0.227	0.264	
						Countrywide Pure Premium:		0.43	0.15	0.47	1.05
						Countrywide Indicated Relativity:		0.325	0.189	0.463	0.336
						Countrywide Credibility:		0.113	0.078	0.088	
						Countrywide Lost-time Claims:		1	5		
						Pure Premium Underlying Present Rate:		0.72	0.66	0.83	2.21
						Relativity to Underlying Present Rate:		0.595	1.102	0.948	0.839
						Credibility to Underlying Relativity:		0.740	0.695	0.648	
						Industry Group Adjusted On-Level Pure Premium:		1.303	0.790	1.016	3.109
						Formula Relativity:		0.792	0.893	0.910	
						Balanced Formula Relativity:		**0.793**	**0.893**	**0.912**	**0.857**

MASSACHUSETTS WORKERS' COMPENSATION

SERIOUS PURE PREMIUMS BY CLASS

Expected Annual Losses ($000)	Number of Classes	Expected Annual Loss Per Class ($000)	"Slope" of Regression of Correlations	"Intercept" of Regression of Correlations	Estimated *K* Parameter ($000)
Manufacturing					
10 to 30	32	17.3	.753	.128	353
30 to 100	40	60.7	1.011	.065	1,590
100 to 300	51	185	.882	.398	348
300 to 1,000	57	576	.965	.491	626
1,000 to 3,000	30	1,531	.936	.548	1,243
Goods and Services					
30 to 300	29	143	.944	.231	637
300 to 3,000	43	1,081	.957	.667	521
All Five Industry Groups					
3 to 10	24	6.6	.946	.103	172
10 to 30	44	17.3	.935	.076	630
30 to 100	66	61.0	.868	.103	964
100 to 300	91	187	.985	.394	357
300 to 1,000	117	567	.960	.524	538
1,000 to 3,000	74	1,686	.974	.676	765
3,000 to 10,000	34	5,737	.980	.920	274

Notes: Fatal and Permanent Total claims are assigned $150,000 in Indemnity Losses. Correlations between fifteen years of MA W.C. classification relativities within Industry Groups. Correlations are weighted using the percentage of a class's payroll from a given year; Per Capita Classes are excluded. The *K* Parameter is estimated using $J = .04$, $I = 50{,}000$ and $\Omega = 25{,}000$. For example, for Manufacturing with 10 to 30 thousand expected annual losses,
$K = E\{(1/Z - 1)(1 + I/\Omega) - J\} = 17.3\{(1/.128 - 1)(1 + 50{,}000/25{,}000) - .04\} = 353$, where Z is the "Intercept" of Regression of Correlations.

MASSACHUSETTS WORKERS' COMPENSATION

NON-SERIOUS PURE PREMIUMS BY CLASS

Expected Annual Losses ($000)	Number of Classes	Expected Annual Loss Per Class ($000)	"Slope" of Regression of Correlations	"Intercept" of Regression of Correlations	Estimated *K* Parameter ($000)
Manufacturing					
3 to 10	30	6.6	1.123	.026	741
10 to 30	40	18.3	.905	.167	190
30 to 100	50	64.7	.963	.257	242
100 to 300	45	181	.943	.485	206
300 to 1,000	47	516	.954	.591	350
Goods and Services					
30 to 300	42	141	.912	.522	142
300 to 3,000	32	1,130	.988	.700	448
All Five Industry Groups					
3 to 10	41	6.3	1.089	.031	591
10 to 30	58	18.7	.861	.144	229
30 to 100	82	63.9	.965	.277	216
100 to 300	99	184	.988	.438	254
300 to 1,000	94	492	.941	.650	256
1,000 to 3,000	40	1,638	.990	.754	475
3,000 to 10,000	22	4,777	1.007	.878	475

Notes: Correlations between fifteen years of MA W.C. classification relativities within Industry Groups. Correlations are weighted using the percentage of a class's payroll from a given year; Per Capita Classes are excluded. The *K* Parameter is estimated using $J = .04$, $I = 20{,}000$ and $\Omega = 10{,}000$. For example, for Goods and Services with 30 to 300 thousand expected annual losses,
$K = E\{(1/Z - 1)(1 + I/E) - J\} = 141\{(1/.522 - 1)(1 + 20/141) - .04\} = 142$, where Z is the "Intercept" of Regression of Correlations.

MASSACHUSETTS WORKERS' COMPENSATION

MEDICAL PURE PREMIUMS BY CLASS

Expected Annual Losses ($000)	Number of Classes	Expected Annual Loss Per Class ($000)	"Slope" of Regression of Correlations	"Intercept" of Regression of Correlations	Estimated *K* Parameter ($000)
Manufacturing					
3 to 10	32	6.1	1.123	.038	463
10 to 30	40	17.1	.959	.130	315
30 to 100	50	64.7	.935	.309	209
100 to 300	47	189	.960	.466	243
300 to 1,000	47	520	.984	.651	275
Goods and Services					
30 to 300	41	148	.937	.516	134
300 to 3,000	43	1,087	.997	.792	250
All Five Industry Groups					
3 to 10	43	5.9	.942	.055	304
10 to 30	57	17.4	.918	.196	194
30 to 100	84	64.6	.993	.327	192
100 to 300	104	193	.990	.635	120
300 to 1,000	93	508	.990	.640	282
1,000 to 3,000	42	1,681	.994	.837	266
3,000 to 10,000	18	4,699	.997	.985	116

Notes: Fatal and Permanent total claims are assigned $50,000 in Medical Losses. Correlations between fifteen years of MA W.C. classification relativities within Industry Groups. Correlations are weighted using the percentage of a class's payroll from a given year; Per Capita Classes are excluded. The *K* Parameter is estimated using $J = .04$, $I = 30{,}000$, and $\Omega = 15{,}000$. For example, for Manufacturing with 3 to 10 thousand expected annual losses,
$K = E\{(1/Z - 1)(1 + I/\Omega) - J\} = 6.1\{(1/.038 - 1)(1 + 30{,}000/15{,}000) - .04\} = 463$, where Z is the "Intercept" of Regression of Correlations.

APPENDIX A

SELECTION OF PARAMETERS OF THE COVARIANCE STRUCTURE

The parameters of the covariance structure will be selected using the same general framework as in Sections 7 and 8 of "Credibility With Shifting Risk Parameters, Risk Heterogeneity, and Parameter Uncertainty" by Howard C. Mahler, PCAS 1998. The parameters are: ρ, γ, r^2, I, J, K and Ω, by Serious, Non-Serious and Medical and by Intrastate and Interstate. Mahler (1998) analyzed the total pure premium, rather than the three partial pure premiums used in classification ratemaking.

Interstate Covariances

As in Mahler (1998), r^2 for the interstate covariances will be taken as .7 times r^2 for the intrastate covariances. (Data from different states is less correlated than data from the same state, all other things being equal.) Also, the covariance between data from different states has (virtually) no process variance. Therefore, the K parameter is zero and the J parameter is one-half the intrastate J parameter.

Selecting I and Ω

The parameters I and Ω are related to the impact of risk heterogeneity by size of class. It will be assumed that the impacts for a given size class are similar on the individual partial pure premiums as on the total pure premiums. Thus, I and Ω will be converted to put them in terms of Serious, Non-serious or Medical Expected Losses instead of Total Expected Losses.

Of the Expected Losses, about 49% are Serious, 21% are Non-Serious and 30% are Medical. Thus, multiplying the total I parameter of \$100,000 by these percentages would produce Serious, Non-Serious, and Medical I parameters of \$49,000, \$21,000 and \$30,000 respectively. We select \$50, \$20 and \$30 thousand for the I parameters. Similarly, the total Ω

parameter of $50,000 will be divided into Serious, Non-Serious and Medical Ω parameters of $25, $10 and $15 thousand.

Correlations

As in Mahler (1998), correlations were calculated between years of classification relativities. However, here the losses were split into Serious, Non-Serious and Medical, as in classification ratemaking. However, the claims were not capped,[1] since the detail was not readily available to do that for older years. Instead as a partial substitute, $150,000 of Serious Indemnity and $50,000 Medical Losses per fatal and permanent total claim were substituted for the reported fatal and permanent total claims.[2]

For each class, for each of 15 composite policy years[3], for Serious, Non-Serious and Medical, relativities were calculated with respect to its Industry Group. Then the correlations between different years were calculated.

For this purpose, we will restrict our attention to one size category of class at a time.[4] There are a number of ways to categorize the volume of data. We have used an estimate of the average annual expected losses for a class based on its reported payroll.[5] Other reasonable measures of volume should produce roughly similar results.

For each such size category, we estimate the covariance between any two years of observed relative pure premiums R_{ic} and R_{jc} for $c = 1, ..., k$ where there are k classes in the size category:[6]

---

[1] Currently claims are capped at $200,000 for classification ratemaking in Massachusetts.

[2] As with capping claims, this reduces the random fluctuations in the corresponding pure premiums and relativities.

[3] From 80/81 @ 5th report through 94/95 @ 1st report.

[4] Nevertheless, the pure premiums are relative to the entire Industry Group, regardless of size of class.

[5] By class, by year, Expected Losses were calculated as (payrolls ÷ 100) (Industry Group Partial P.P. for CPY) (SAWW 94/95 ÷ SAWW for CPY) (Current Class Relativity). Then average annual Expected losses were calculated by averaging over those years for which there were reported payrolls.

[6] Recall that $\mathrm{Cov}[X, Y] = E[XY] - E[X]\,E[Y]$.

$$\mathrm{Cov}[R_{ic}, R_{jc}] \approx \frac{\sum_{c=1}^{k} \sqrt{p_{ic} p_{jc}}\, R_{ic} R_{jc}}{\sum_{c=1}^{k} \sqrt{p_{ic} p_{jc}}} - \frac{\sum_{c=1}^{k} p_{ic} R_{ic}}{\sum_{c=1}^{k} p_{ic}} \quad \frac{\sum_{c=1}^{k} p_{jc} R_{jc}}{\sum_{c=1}^{k} p_{jc}}$$

The percent of a class's payroll p_{ic} reported in year i has been used as weights, in order to take into account the fact that for some classes the volume of data may be radically different by year. The variances are estimated in the same manner. Then as usual the estimated correlations are:

$$\mathrm{Corr}[R_{ic}, R_{jc}] = \mathrm{Cov}[R_{ic}, R_{jc}] \Big/ \sqrt{\mathrm{Var}[R_{ic}]\, \mathrm{Var}[R_{jc}]}$$

With 15 separate years of data, one can estimate $\frac{(15)(14)}{2} = 105$ correlations. These correlations correspond to a separation of between one year and fourteen years. We note considerable random fluctuation. Nevertheless, as the separation grows the correlations tend to decline.

Then as in Mahler (1998) one can fit a linear regression to the logs of these correlations versus the separation between years. For example, one such regression might be correlation $= (.46)\ (.94^{\text{separation}})$. Then .94 will be referred to as the "slope"; it quantifies how quickly parameters are shifting. In this example, .46 will be referred to as the "intercept"; it is an estimate of the credibility for one year of data in the absence of shifting risk parameters over time.

Tables A1, A2 and A3 show the results of fitting regressions to the correlations for Serious, Non-Serious and Medical Pure Premiums. It should be noted that due to the limited data (only 15 separate years from one state) there is considerable random fluctuation. This is particularly true for the smallest categories of expected losses and for any size category with a small number of classes.

Selecting J

In the absence of shifting risk parameters, the maximum credibility assigned to a year of data from a class is $1/(1 + J)$. The intercepts of the regressions are estimates of these credibilities. For the very largest size categories these intercepts appear to get relatively close to 100%. Therefore, we will select the (intrastate) J parameter of .04, corresponding to a maximum credibility of 96%.

Selecting K

Based on Equations 4.24 and 4.25 in Mahler (1998), in the absence of shifting risk parameters over time, the credibility for one year of data is:

$$Z = \frac{E(1+I/E)}{(1+J)E+I+K} \qquad E \geq \Omega$$

$$Z = \frac{E(1+I/\Omega)}{(1+J)E+I(E/\Omega)+K} \qquad E \leq \Omega$$

Thus, given the estimated credibility, and all the parameters other than K, one can estimate K using the equations.

$$K = E\left\{\left(\frac{1}{Z}-1\right)(1+I/E)-J\right\} \qquad E \geq \Omega$$

$$K = E\left\{\left(\frac{1}{Z}-1\right)(1+I/\Omega)-J\right\} \qquad E \leq \Omega$$

One gets a corresponding estimate of K from each of the regressions fit. As shown in Tables A1, A2 and A3 there is considerable random fluctuation in the estimates of K.[7]

Table A1 would indicate for Serious losses a K parameter of around \$500,000. Table A2 would indicate for Non-Serious losses a K parameter of \$200,000 or \$300,000. However, K =

[7] One also gets considerable random fluctuation when attempting to estimate the appropriate Full Credibility Criteria as in the previous study a decade ago.

$300,000 would lead to extremely low credibilities for Non-Serious losses.[8] Table 3 would indicate for Medical losses a K parameter of around $200,000.

We will select K parameters of $500, $200 and $200 thousand for Serious, Non-Serious and Medical, respectively.

Figures A1, A2 and A3 compare the fitted credibilities (in the absence of shifting risk parameters) and the intercepts of the various regressions. The credibilities by size of class are reasonably similar to the pattern estimated by the regressions fit to correlations.[9]

Selecting ρ and γ

The parameters ρ and γ quantify the rate of shifting risk parameters. If ρ and γ are close to one, then parameters are shifting slowly. The parameter ρ mainly affects larger classes, while the parameter γ mainly affects smaller classes.

The slopes of the regressions of correlations are estimates of ρ and γ. As seen in Table A1, Table A2 and Table A3, the slopes are relatively near one; there is considerable random fluctuation. Since the small size categories tend to have smaller slopes, we will select $\gamma < \rho$. We will select $\rho = .99$ and $\gamma = .85$.

As shown in Mahler (1998)[10] the "slope" of the regression of the log correlations is expected to be about:

$$\frac{E\rho + I\gamma}{E + I} \qquad E \geq \Omega$$

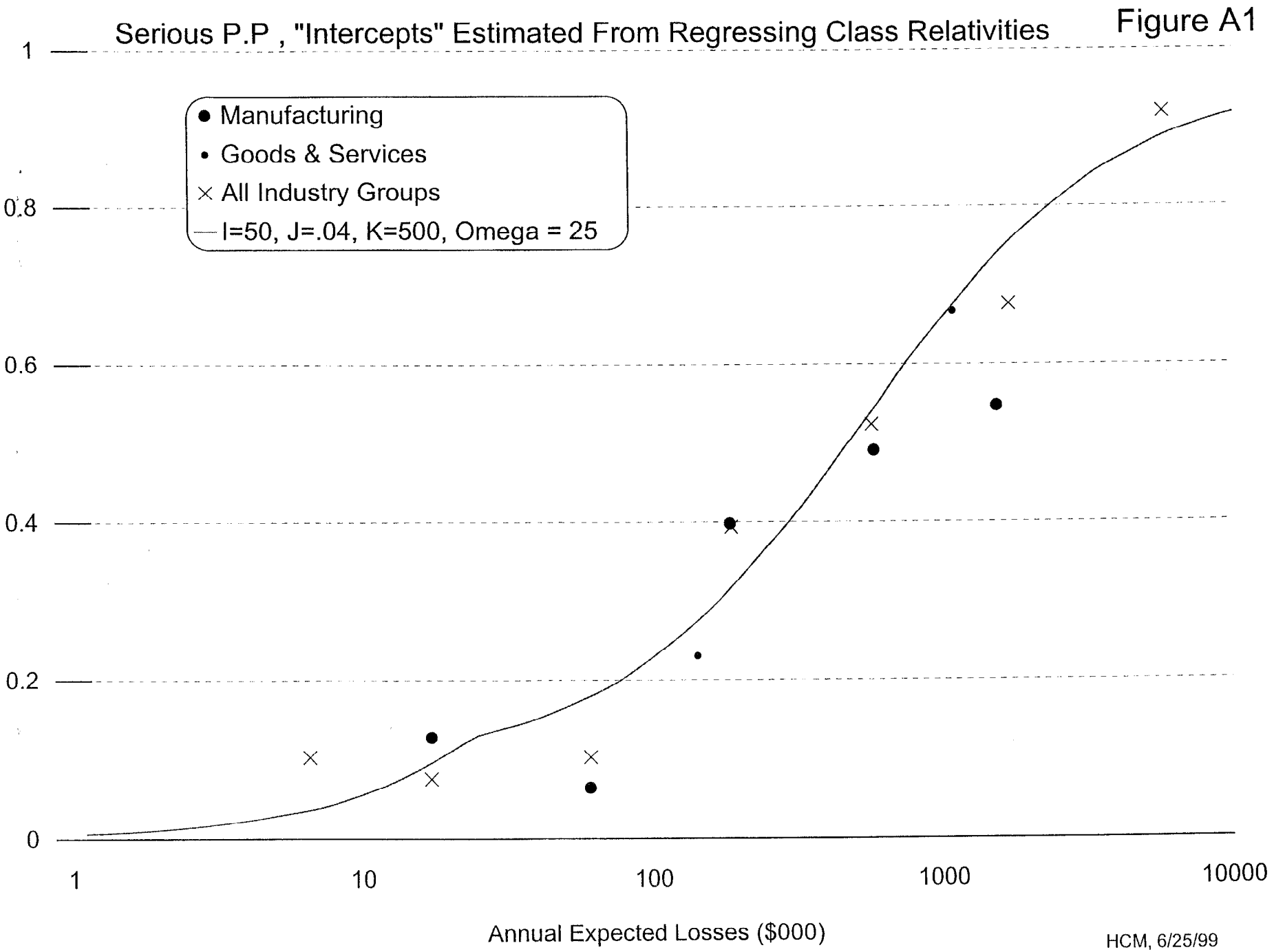
$$\frac{\Omega\rho + I\gamma}{\Omega + I} \qquad E \leq \Omega$$

[8] Generally, we have given the most credibility to Non-Serious losses, the least credibility to Serious losses, with Medical losses in between the other two.

[9] It is important to remember that a given difference in credibilities generally has a much smaller difference on the classification relativities we are trying to estimate.

[10] See Section 5.2 which derives the result for $E \geq \Omega$.

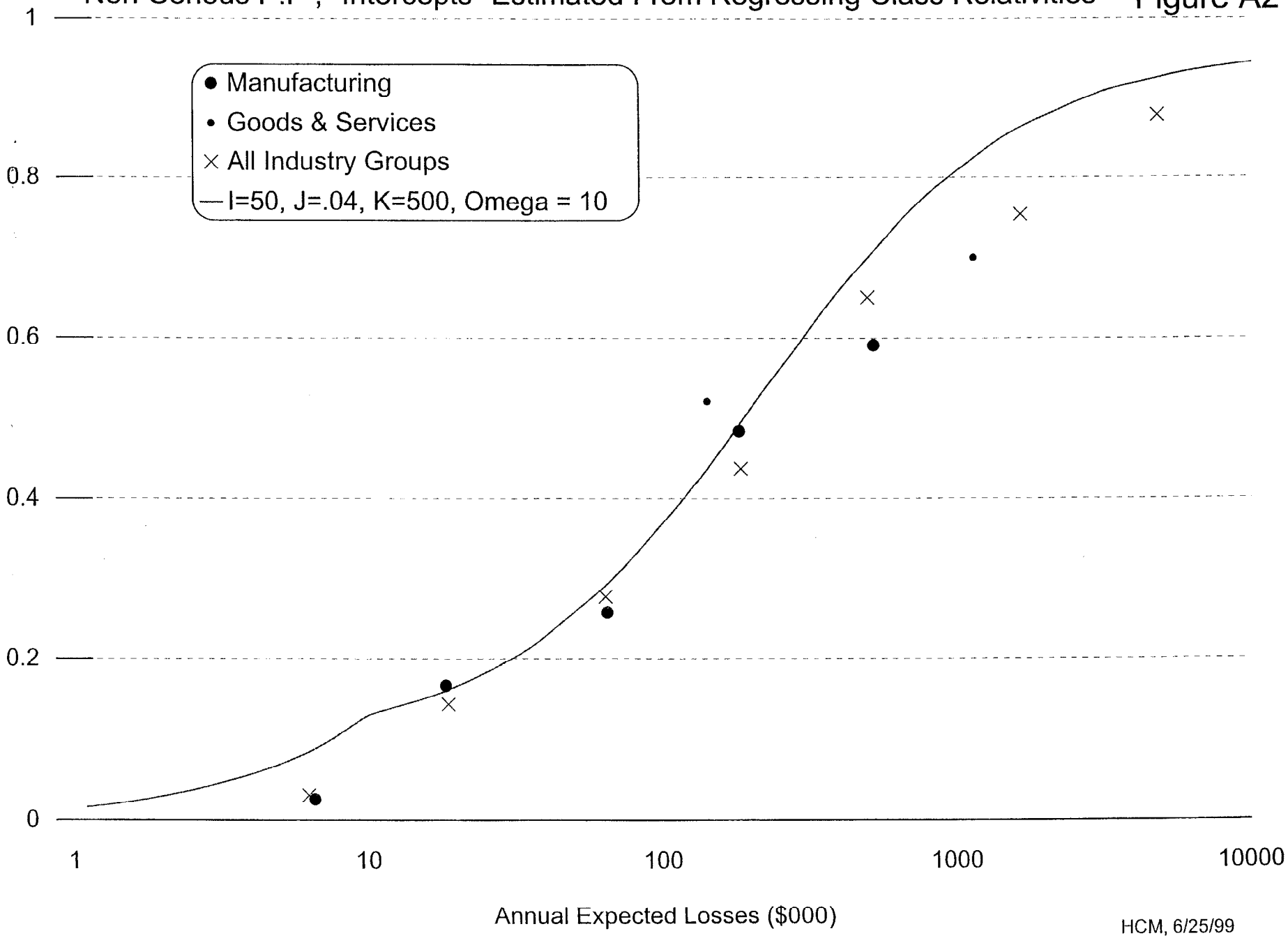
Figure A1



Estimate of the Credibility of a single year of data in the absence of shifting risk parameters

Non-Serious P.P , "Intercepts" Estimated From Regressing Class Relativities

Figure A2

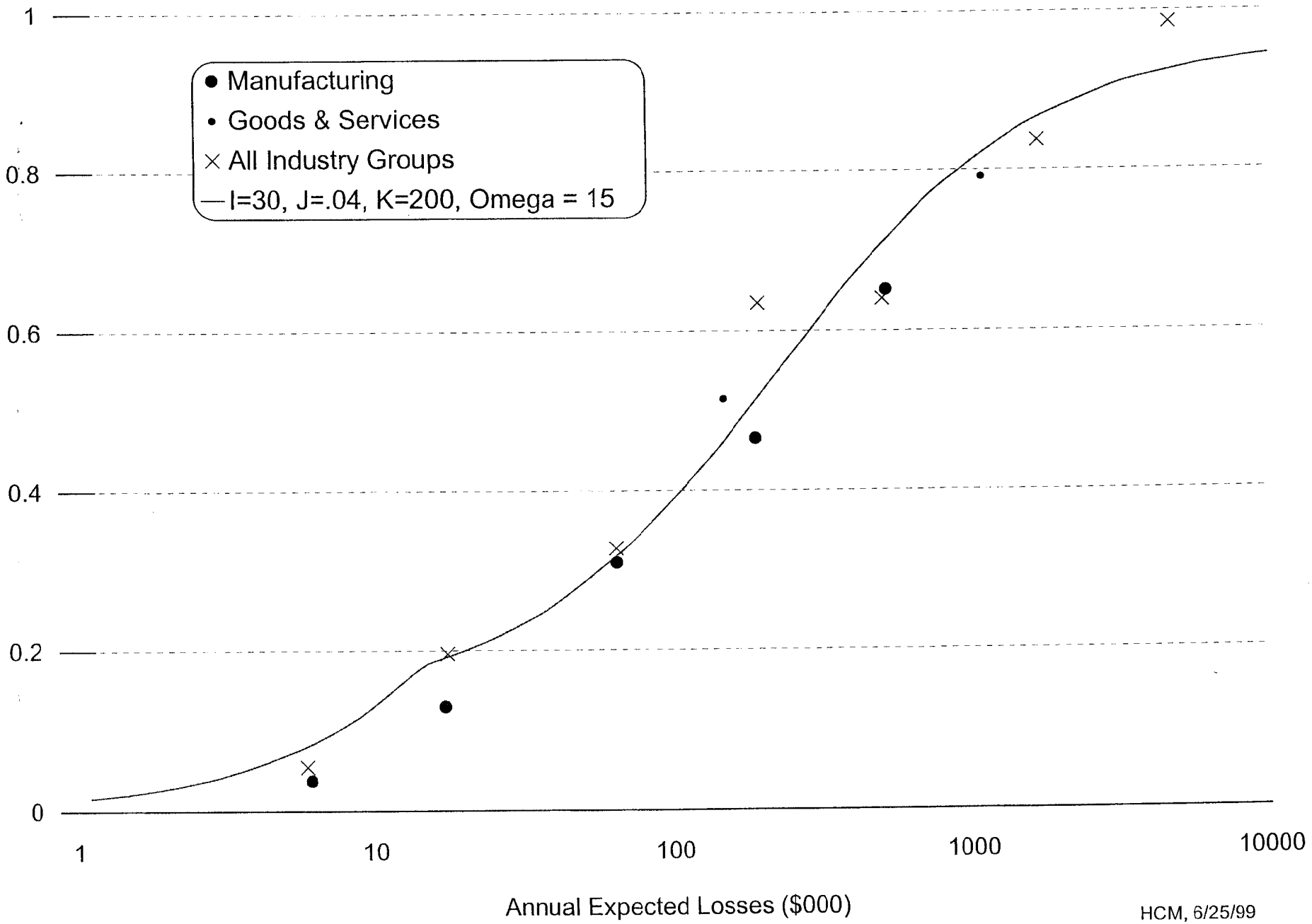


HCM, 6/25/99

Estimate of the Credibility of a single year of data in the absence of shifting risk parameters

Medical P.P , "Intercepts" Estimated From Regressing Class Relativities

Figure A3

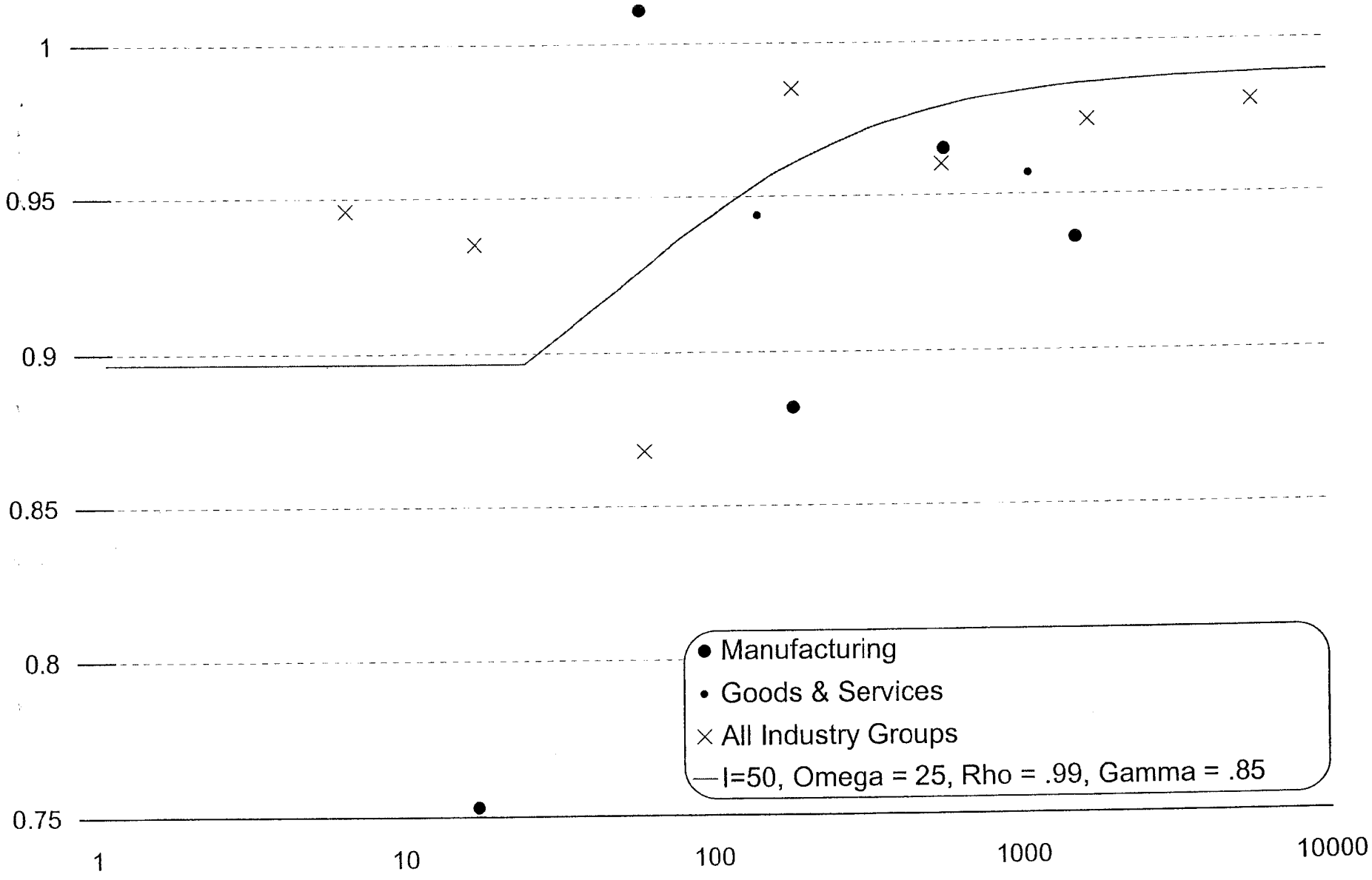


HCM, 6/25/99

Estimate of the Credibility of a single year of data in the absence of shifting risk parameters

Serious P.P , "Slopes" Estimated From Regressing Class Relativities

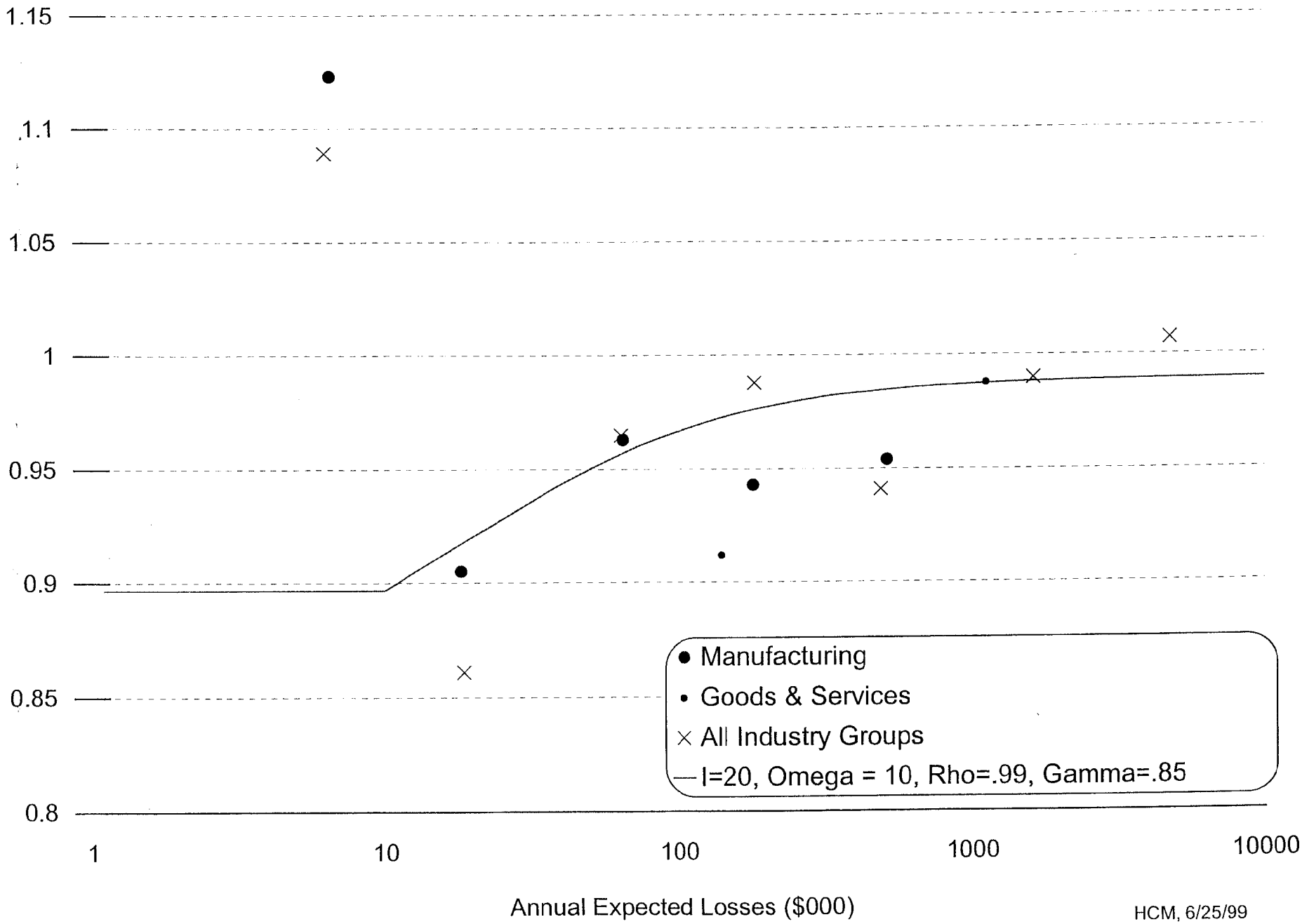
Figure A4



HCM, 6/25/99

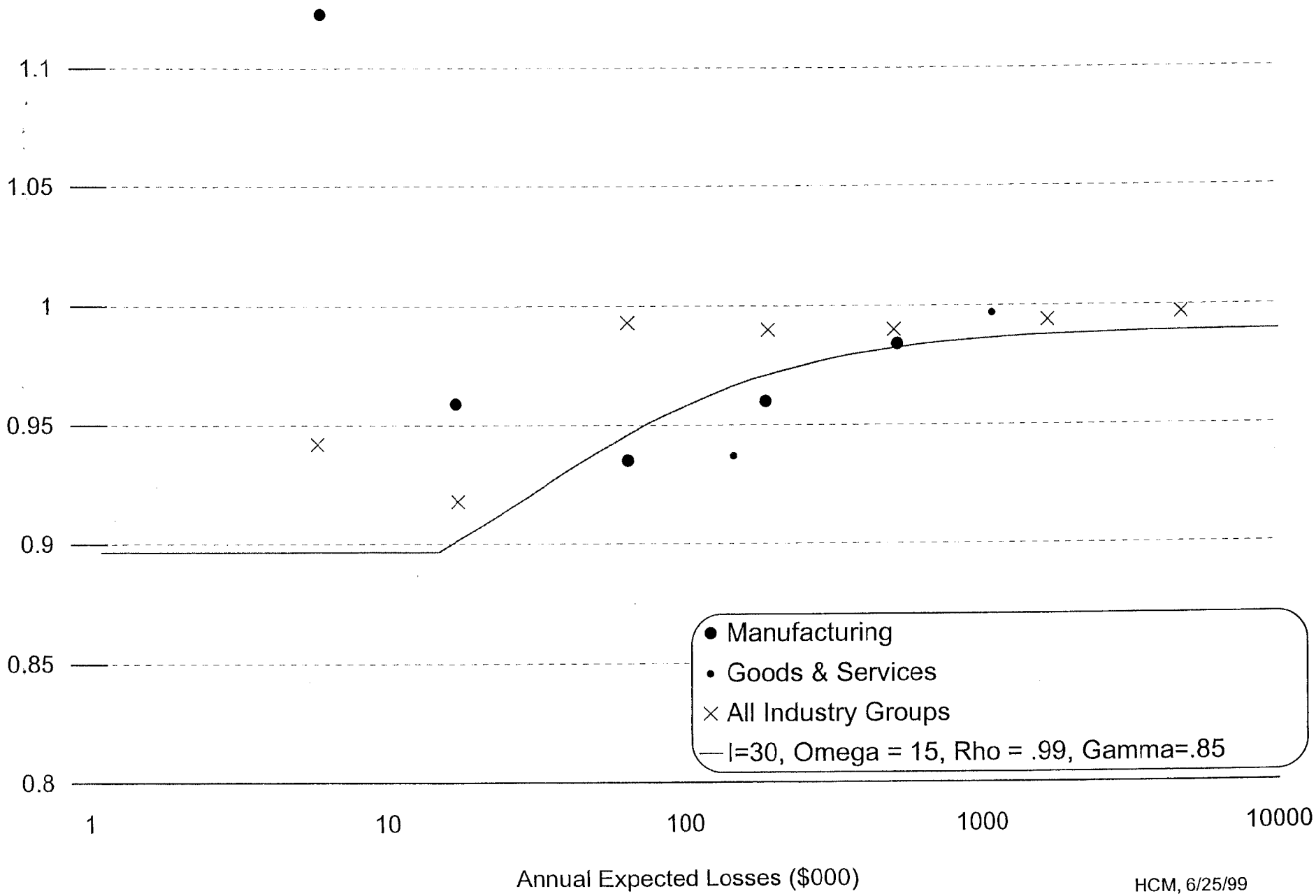
Estimate of the rate of shifting risk parameters

Figure A5



Estimate of the rate of shifting risk parameters

Figure A6



Estimate of the rate of shifting risk parameters

APPENDIX B

NUMERICAL EXAMPLES OF THE CALCULATION OF CREDIBILITIES

The calculation of credibilities will be illustrated for examples involving fewer years than in the main text.

Assumptions

1. Three years of Massachusetts Serious Losses:

   Year 48 at 3rd report, $250,000 in Expected Losses

   Year 49 at 2nd report, $150,000 in Expected Losses

   Year 50 at 1st report, $200,000 in Expected Losses

2. Three years of Countrywide Serious Losses:

   Year 47 at 3rd report, Year 48 at 2nd report, Year 49 at 1st report, for each year $60,000 in Expected Losses in each of ten non-Massachusetts states.

3. Predicting Massachusetts Year 54 at 5th report. (Expected Losses of $200,000, the average of the three years of Massachusetts data.)

4. Covariance Structure and Parameters as in the main text.

5. No weight to the overall mean, as in the main text.

Equations for Credibilities

The resulting linear equations for the credibilities are in this case the following seven equations in seven unknowns.

$$Z_{48}\,S_{48,48} + Z_{49}\,S_{48,49} + Z_{50}\,S_{48,50} + W_{47}\,U_{48,47} + W_{48}\,U_{48,48} + W_{49}\,U_{48,49} = \lambda/2 + S_{48,54}$$

$$Z_{48}\,S_{49,48} + Z_{49}\,S_{49,49} + Z_{50}\,S_{49,50} + W_{47}\,U_{49,47} + W_{48}\,U_{49,48} + W_{49}\,U_{49,49} = \lambda/2 + S_{49,54}$$

$$Z_{48}\,S_{50,48} + Z_{49}\,S_{50,49} + Z_{50}\,S_{50,50} + W_{47}\,U_{50,47} + W_{48}\,U_{50,48} + W_{49}\,U_{50,49} = \lambda/2 + S_{50,54}$$

$$Z_{48}\,U_{48,47} + Z_{49}\,U_{49,47} + Z_{50}\,U_{50,47} + W_{47}\,T_{47,47} + W_{48}\,T_{48,47} + W_{49}\,T_{49,47} = \lambda/2 + U_{54,47}$$

$$Z_{48}\,U_{48,48} + Z_{49}\,U_{49,48} + Z_{50}\,U_{50,48} + W_{47}\,T_{47,48} + W_{48}\,T_{48,48} + W_{49}\,T_{49,48} = \lambda/2 + U_{54,48}$$

$$Z_{48}\,U_{48,49} + Z_{49}\,U_{49,49} + Z_{50}\,U_{50,49} + W_{47}\,T_{47,49} + W_{48}\,T_{48,49} + W_{49}\,T_{49,49} = \lambda/2 + U_{54,49}$$

$$Z_{48} + Z_{49} + Z_{50} + W_{47} + W_{48} + W_{49} = 1$$

Ignoring Maturity

Credibilities are calculated in Exhibit B1, ignoring any impact of maturities.

For example, the covariance between MA data in years 48 and 50:

$$S_{50,48} = (1)\left\{(.99^2) + (.85^2)(50{,}000)/\sqrt{(250{,}000)(200{,}000)}\right\} = 1.1417$$

The covariance between MA data in year 48 and CW data in year 47:

$$U_{48,47} = (.7)\left\{(.99) + (.85)(50)/\sqrt{(250)(60)}\right\} = .9359$$

The covariances between Countywide data are the sum of two terms, 1/10 times the covariance within each non-Massachusetts state, plus 9/10 times the covariance between each non-Massachusetts state. For example, for year 49 with year 49, the covariance within each non-Massachusetts state is:

$$(1)\left\{.90^0 + .85^0\,(50)/\sqrt{(60)(60)} + (500)/\sqrt{(60)(60)} + .04\right\} = 10.2067.$$

The covariance between each non-Massachusetts state is:

$$(.7)\left\{.99^0 + .85^0\,(50)/\sqrt{(60)(60)} + .02\right\} = 1.2973.$$

Combining the two terms

$$T_{49,49} = (1/10)\,(10.2067) + (9/10)\,(1.2973) \approx 2.1883$$

The resulting credibilities (ignoring the impact of differing maturity) are:

MA Year 48:	Z_{48}	=	20.3%
MA Year 49:	Z_{49}	=	11.9%
MA Year 50:	Z_{50}	=	19.0%

CW Year 47: W_{47} = 16.2%

CW Year 48: W_{48} = 14.3%

CW Year 49: W_{49} = 18.2%

The Impact of Maturity

Credibilities are calculated in Exhibit B2, including the impact of maturity.

For example, the covariance between MA data for year 48 @ 3rd and year 50 @ 1st is multiplied by a factor of:

$$LDF^{-1/(1.5+2.25\,size/million)} = \{(1.33)(1.10)\}^{-1/\left(1.5+2.25\left(\sqrt{(.25)(.20)}\right)\right)} = 1.463^{-.4992} = .8270.$$

Thus, taking into account maturity, $S_{50,\,48}$ = (.8270) (1.1417) = .9442.

The following seven linear equations are used to solve for the seven unknowns:

$$\begin{bmatrix} 3.2400 & 1.1514 & 0.9442 & 0.9359 & 0.9475 & 0.7554 & -0.5 \\ 1.1514 & 4.7067 & 1.0623 & 0.9011 & 1.0066 & 0.9169 & -0.5 \\ 0.9442 & 1.0623 & 3.7900 & 0.7040 & 0.7788 & 0.9646 & -0.5 \\ 0.9359 & 0.9011 & 0.7040 & 2.1883 & 1.1696 & 0.9152 & -0.5 \\ 0.9475 & 1.0066 & 0.7788 & 1.1696 & 2.1883 & 1.0413 & -0.5 \\ 0.7554 & 0.9169 & 0.9646 & 0.9152 & 1.0413 & 2.1883 & -0.5 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} Z_{48} \\ Z_{49} \\ Z_{50} \\ W_{47} \\ W_{48} \\ W_{49} \\ \lambda \end{bmatrix} = \begin{bmatrix} 0.9818 \\ 0.9794 \\ 0.8581 \\ 0.7178 \\ 0.7019 \\ 0.6175 \\ 1 \end{bmatrix}$$

The resulting credibilities, including the impact of differing maturity are:

Z_{48} = 22.3%

Z_{49} = 11.8%

Z_{50} = 15.6%

W_{47} = 20.9%

W_{48} = 14.9%

W_{49} = 14.4%

Massachusetts Workers' Compensation Serious Losses
Example of Calculation of Credibilities
Ignoring the Impact of Maturity

Expected Losses

Year	MA	Countrywide *
47		60,000
48	250,000	60,000
49	150,000	60,000
50	200,000	
54	200,000 **	

MA Intrastate Covariances

	S_{ij}	j (Year) 48	49	50	54
i (Year)	48	3.2400	1.2095	1.1417	1.0258
	49	1.2095	4.7067	1.2354	1.0791
	50	1.1417	1.2354	3.7900	1.0911

MA vs. Countrywide Covariances

	U_{ij}	j (Countrywide Year) 47	48	49
i (MA Year)	48	0.9359	0.9998	0.9359
	49	0.9526	1.0066	1.0829
	50	0.8754	0.9169	0.9646
	54	0.7549	0.7795	0.8075

Countrywide Covariances ***

	T_{ij}	j (Year) 47	48	49
i (Year)	47	2.1883	1.2398	1.1550
	48	1.2398	2.1883	1.2398
	49	1.1550	1.2398	2.1883

Countrywide Intrastate Covariances

	C'_{ij}	j (Year) 47	48	49
i (Year)	47	10.2067	1.6983	1.5822
	48	1.6983	10.2067	1.6983
	49	1.5822	1.6983	10.2067

Countrywide Interstate Covariances

	D'_{ij}	j (Year) 47	48	49
i (Year)	47	1.2973	1.1888	1.1075
	48	1.1888	1.2973	1.1888
	49	1.1075	1.1888	1.2973

SOLUTION:

$\lambda/2$	0.4583

Credibilities

Z_{48}	**20.3%**	W_{47}	**16.2%**
Z_{49}	**11.9%**	W_{48}	**14.3%**
Z_{50}	**19.0%**	W_{49}	**18.2%**
Total	51.2%	Total	48.8%

Linear Equations for Credibilities

i	(1) $\sum_{j=48}^{50} Z_j S_{ij}$	(2) $\sum_{j=47}^{49} W_j U_{ij}$	= (1) + (2) $S_{i,54} + \frac{\lambda}{2}$	j	(3) $\sum_{i=48}^{50} Z_i U_{ij}$	(4) $\sum_{i=47}^{49} W_i T_{ij}$	= (3) + (4) $U_{54,j} + \frac{\lambda}{2}$
48	1.0186	0.4655	1.4841	47	0.4700	0.7432	1.2132
49	1.0414	0.4960	1.5374	48	0.4973	0.7405	1.2378
50	1.1004	0.4490	1.5494	49	0.5025	0.7632	1.2658

Notes: See Exhibit B3 for Covariance Equations and Parameters.

* Expected Losses for each of ten states.

** Average of MA Expected Losses for Years 48, 49, 50.

*** Where $T_{ij} = (0.1 \times C'_{ij}) + (0.9 \times D'_{ij})$

Massachusetts Workers' Compensation Serious Losses
Example of Calculation of Credibilities
Including the Impact of Maturity

Expected Losses

Year @ Report	MA	Year @ Report	Countrywide *
		47 @ 3rd	60,000
48 @ 3rd	250,000	48 @ 2nd	60,000
49 @ 2nd	150,000	49 @ 1st	60,000
50 @ 1st	200,000		
54 @ 5th	200,000 **		

MA Intrastate Covariances

		j (Year)			
	S_{ij}	48	49	50	54
i (Year)	48	3.2400	1.1514	0.9442	0.9818
	49	1.1514	4.7067	1.0623	0.9794
	50	0.9442	1.0623	3.7900	0.8581

MA vs. Countrywide Covariances

		j (Countrywide Year)		
	U_{ij}	47	48	49
i (MA Year)	48	0.9359	0.9475	0.7554
	49	0.9011	1.0066	0.9169
	50	0.7040	0.7788	0.9646
	54	0.7178	0.7019	0.6175

Countrywide Covariances ***

		j (Year)		
	T_{ij}	47	48	49
i (Year)	47	2.1883	1.1696	0.9152
	48	1.1696	2.1883	1.0413
	49	0.9152	1.0413	2.1883

Countrywide Intrastate Covariances

		j (Year)		
	C'_{ij}	47	48	49
i (Year)	47	10.2067	1.6022	1.2537
	48	1.6022	10.2067	1.4265
	49	1.2537	1.4265	10.2067

Countrywide Interstate Covariances

		j (Year)		
	D'_{ij}	47	48	49
i (Year)	47	1.2973	1.1215	0.8776
	48	1.1215	1.2973	0.9986
	49	0.8776	0.9986	1.2973

SOLUTION:

$\lambda/2$	0.4716

Credibilities

Z_{48}	**22.3%**	W_{47}	**20.9%**
Z_{49}	**11.8%**	W_{48}	**14.9%**
Z_{50}	**15.6%**	W_{49}	**14.4%**
Total	49.8%	Total	50.2%

Linear Equations for Credibilities

i	(1) $\sum_{j=48}^{50} Z_j S_{ij}$	(2) $\sum_{j=47}^{49} W_j U_{ij}$	= (1) + (2) $S_{i,54} + \frac{\lambda}{2}$	j	(3) $\sum_{i=48}^{50} Z_i U_{ij}$	(4) $\sum_{i=47}^{49} W_i T_{ij}$	= (3) + (4) $U_{54,j} + \frac{\lambda}{2}$
48	1.0076	0.4458	1.4534	47	0.4256	0.7638	1.1894
49	0.9804	0.4706	1.4510	48	0.4524	0.7212	1.1735
50	0.9276	0.4022	1.3297	49	0.4278	0.6614	1.0891

Notes: See Exhibit B3 for Covariance Equations, Parameters, and Loss Development Factors.

* Expected Losses for each of ten states.

** Average of MA Expected Losses for Years 48, 49, 50.

*** Where $T_{ij} = (0.1 \times C'_{ij}) + (0.9 \times D'_{ij})$

Massachusetts Workers' Compensation Serious Losses
Example of Calculation of Credibilities

Covariance Equations

$$\mathrm{Cov}\left[X_i, X_j\right] = r^2\left\{\rho^{|i-j|} + \gamma^{|i-j|} I / \sqrt{E_i E_j} + \delta_{ij}\left(K / \sqrt{E_i E_j} + J\right)\right\} \qquad \sqrt{E_i E_j} \geq \Omega$$

$$\mathrm{Cov}\left[X_i, X_j\right] = r^2\left\{\rho^{|i-j|} + \gamma^{|i-j|} I / \Omega + \delta_{ij}\left(K / \sqrt{E_i E_j} + J\right)\right\} \qquad \sqrt{E_i E_j} \leq \Omega$$

$$\textit{where} \quad \delta_{ij} = \begin{cases} 0 & i \neq j \\ 1 & i = j \end{cases}$$

Selected Parameters for Serious Losses

	ρ	γ	r^2	I	J	K	Ω
Intrastate	0.99	0.85	1	50,000	0.04	500,000	25,000
Interstate	0.99	0.85	0.7	50,000	0.02	0	25,000

Selected Loss Development Factors for Serious Losses

Report	1-2	2-3	3-4	4-5
LDF	1.33	1.10	1.06	1.03

Adjustment Equation for Loss Development Factors

correlation = $\mathrm{LDF}^{-1/(1.5 + 2.25\ \mathrm{size/million})}$