Using Best Practices to Determine a Best Reserve Estimate

Paul J. Struzzieri, FCAS
Paul R. Hussian, FCAS
Milliman & Robertson, Inc.
Two Pennsylvania Plaza
Suite 1552
New York, NY 10121
(212) 279-7166

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ABSTRACT

Currently, many actuaries produce a range of reasonable reserve estimates for IBNR loss and loss adjustment expense. NAIC Issue Paper No. 55 would effectively eliminate the possibility of booking any amount except management's "best estimate" within this range. The term "best estimate" has not been defined, neither in the issue paper nor in the actuarial literature. We propose to define the best estimate by describing a set of "best practices" -- many already found in the actuarial literature -- that a reserving actuary should follow, while minimizing the number of arbitrary judgments. Central to this paper is the recently introduced Generalized Cape Cod method. Many of these best practices have been shown to lead to minimum bias results. The best estimate, therefore, will be the outcome of following the framework contained in this paper.

Keywords:

- Reserving
- IBNR
- Generalized Cape Cod
- Chain Ladder
- Bornhuetter-Ferguson

INTRODUCTION AND OVERVIEW

In Issue Paper No. 55 [10], the NAIC addresses the issue of recording a point estimate of unpaid loss and loss adjustment expense (LAE). The key element of this issue paper is the requirement that company management book its "best estimate" of the loss and LAE reserves. Presumably, management will want to consider basing their best estimate on their actuary's best estimate.

Currently, actuaries produce a range of reasonable reserve estimates. As long as management records loss and LAE reserves within this range, most actuaries would not object. Issue Paper No. 55 would effectively eliminate the possibility of booking any amount except management's "best estimate" within this range. In the instances where no point within the range is more probable than any other point, the midpoint of the range would be accrued.

The term "best estimate" has not been defined, neither in the issue paper nor in the actuarial literature. The Casualty Actuarial Society *Statement of Principles Regarding Property and Casualty Loss and Loss Expense Adjustment Reserves* refers to a "most appropriate" reserve within a range of actuarially sound estimates. This estimate is dependent on (1) the relative likelihood of estimates within the range and (2) the financial reporting context in which the reserve will be presented.

We propose to define the best estimate by describing a set of “best practices” that a reserving actuary should follow. A best estimate would result by incorporating certain best practices already found in the actuarial literature and minimizing the number of arbitrary judgments that the actuary may otherwise make. Many of these best practices have been shown to lead to minimum bias results. The use of these techniques, then, would lead to more “likely” or more “probable” estimates.

The best estimate, therefore, will be the outcome of following the framework contained in this paper, rather than selecting a “best estimate” from a range of results. Since there will always be an element of judgment within this framework, a range can then be determined by varying some of the underlying judgments and assumptions.

We hope this paper will promote dialogue among property/casualty actuaries in which best practices can be defined and refined periodically.

DESCRIPTION OF RESERVING ENVIRONMENT

The reserving practices described below are not intended to apply to all situations. For example, we do not expect that all of these best practices will apply to start-up insurance companies or to situations where reserves are being analyzed using publicly-available information (e.g. from Schedule P). These practices do apply to the scenario where (1) the data are within the control of the insurance company, (2) there is sufficient consistency within the data, and (3) there is sufficient history within the data, i.e. there is enough information about development in the “tail”. Best practices for tail factors and ULAE reserves are beyond the scope of this paper. The situation where the data is not consistent is dealt with separately in the Appendix to this paper.

In this environment, the following data is available for each line of business or homogeneous business segment:

1. paid and incurred loss development triangles, gross and net of reinsurance;
2. paid and incurred allocated loss adjustment expense (ALAE) development triangles;
3. triangles of salvage and subrogation (S&S) or other recoveries;
4. claim count triangles, preferably number reported, number open, number closed with indemnity payment (CWIP), and number closed without payment (CWOP);
5. several exposure bases, including number of policies issued, premiums earned and ratemaking exposure units;
6. historical profiles of policies, exposures and premiums by limit of liability, deductible, classification, state or territory and other categories that have the effect of changing the risk profile of the book;
7. information on changes in the rate of settlement of claims, case reserving practices, underwriting, loss control or risk management (if any);

8. summaries of ceded reinsurance programs, showing historical excess of loss and quota share retentions, company participation in excess layers, and the like.

When beginning a loss reserve study, the actuary has a wide array of tools and methods from which to choose. One school of thought says to use several methods, and average all of the methods to get to the selected result. However, some of these methods may be more biased or more variable than others. A better practice would be to exclude these methods from the average. The selected result would then be less biased and/or have less variance.

By the time that reserving actuaries have developed their best estimate, they should have already addressed several questions, including the following:

- When should loss development methods be used? When should exposure-based methods be used?
- What is the best way to weigh together different methods?
- What is the best way to determine the expected (a priori) ultimate losses?
- What items should be considered when selecting the best exposure base?
- What exposure base is generally best for estimating ultimate losses?
- What is the best method for determining ALAE reserves?
- What is the best method for determining S&S reserves?
- What are some of the best practices for picking development patterns?

- What are some of the best practices for selecting ultimate losses?
- What is a practical method for determining a reasonable range around the best estimate?

BLENDING LOSS DEVELOPMENT AND EXPECTED LOSS ESTIMATES

Before discussing our best practices, some definitions are in order. **Exposure base** is defined as a measure that is known or accurately estimated in advance and that varies directly with the quantity being estimated. Payroll, sales, and car-years are well known examples of exposure bases used for ratemaking purposes. They are equally useful for reserving purposes. A **leading indicator** is a measure that is not known in advance, but is directly correlated with the quantity being measured. Leading indicators alert the actuary to the estimated quantity's possible realized value earlier than if the actuary relied on observed experience alone. For reserving purposes, leading indicators can be exposure bases for losses also. The only difference is that the ultimate value of the leading indicator also needs to be estimated. For example, reported claim counts is a leading indicator of ultimate losses and, therefore, could be considered an exposure base for ultimate losses. This is discussed further in the BEST EXPOSURE BASE section.

The terms ***a priori*** and **expected** are used interchangeably to refer to any estimate of ultimate loss (or other quantity being projected) that is based on an exposure measure. For example, multiplying the calendar year premium by the actuary's expected loss ratio results in an exposure-based estimate.

This leads to the first best practice: Whenever an appropriate exposure base has been identified, the actuary should rely on a loss reserving method that mixes the loss development, or chain ladder, method with exposure-based expected loss methods. The most common of these blended methods in use are the Bornhuetter-Ferguson (BF) [2] and Cape Cod (CC) methods.

Actuarial literature is full of examples why the loss development result alone should not be considered the best estimate. Stanard [12] used simulation techniques to measure the expected value and the variance of the prediction error of several loss reserving methods, including loss development method and expected loss-based methods. [**Prediction error**, or **bias**, is the difference between our estimate of an unknown value, e.g. ultimate loss, and the actual realized value of that quantity. The prediction error is therefore an unknown random variable. We are also interested in the expected *squared* prediction error and the variance of the prediction error for any method; these are the same if the method is unbiased. The "best" methods have low-mean, low-variance prediction errors.]

Stanard found significantly higher prediction errors (both mean and variance) when using the loss development method. Based upon the results of the simulation model, Stanard concluded that the loss development method is clearly inferior to the methods which give weight to expected losses.

Murphy [9] came to the same conclusions, although he found that loss development techniques could be improved upon by varying the averaging methods used to select link ratios.

Besides these studies which demonstrate that blended methods reduce prediction errors, there are several other reasons why we believe such methods are superior. First, these techniques are easy to apply. Second, blending in expected losses is intuitively appealing. The less mature the loss experience, the more the weight assigned to the expected losses. As many actuaries using the loss development method have discovered, early development is unstable, not a useful predictor of ultimate losses, and will understate ultimate losses when the current evaluation is less than average and overstate when the current evaluation is greater than average. Murphy and Patrik [11] make similar observations.

A third advantage cited by Patrik is that future loss emergence predicted by these methods is correlated with an exposure measure (instead of with past loss emergence). The advantage is that external information can be incorporated; the exposure measure can be adjusted to reflect expected changes in rate level adequacy (if premiums are used) or changes in the distribution of business by class, territory, limit, etc. (if ratemaking exposures are used).

Fourth, an expected loss method can make use of loss information from all of the years in order to project any given year. Robertson [12], in his review of Stanard, discusses this advantage of the CC method. Mack [8] makes a similar observation; the loss development method uses only one data point from one accident year to project a given year, implicitly assuming that other years do not provide statistically useful information.

We believe that the use of a blended method is a best practice in all but the most extreme situations. As an example of such a situation, the loss development method may be sufficient for fast-reporting lines of business where there is very little variation in loss development factors by accident year. Even then, it is good practice to give weight to exposure-based, expected loss methods.

THE BEST WEIGHTS: BORNHUETTER-FERGUSON & ALTERNATIVE BORNHUETTER-FERGUSON

Given that the best practice is to weight together loss development and exposure-based methods, what are the best weights to use? The popular Bornhuetter-Ferguson (BF) method uses weights related to the size of the loss development factor:

$$L = D\frac{1}{LDF} + A\left[1 - \frac{1}{LDF}\right] \qquad \textbf{(Equation 1)}$$

where

L	=	estimated ultimate loss
D	=	loss development estimate
A	=	expected loss estimate
LDF	=	loss development factor.

Gluck [5] demonstrated that the BF weights are optimal (they result in the minimum variance of the prediction error) subject to certain constraints. Gluck also demonstrated that the BF weights are a specific case of the more general "best" weights: those that are inversely proportional to the variance of the prediction errors of D and A. Gluck refers to this general case as "alternative BF" weights. In the more specific BF case, the assumption is made that the LDF is proportional to the prediction error variance of the loss development method, i.e. the error using D is likely to be larger the more immature the data.

Gluck states that alternative BF weights are particularly useful when the LDF is less than unity, for example in a line such as automobile physical damage. The BF weights should not be used in this case because they will result in L being outside of the range of D and A. (D would be given greater than 100% weight.) There is still uncertainty surrounding the loss development method when the LDF is less than unity, so it should be given a weight less than 100%.

Gluck also states that, even if the LDF is greater than unity, alternative BF weights may be more appropriate if the actuary believes that the LDF approaches unity faster than the uncertainty surrounding D is eliminated. For instance, the LDF may be close to 1.0, but the incurred losses to date may include a large proportion of case reserves. The BF could assign too much weight to D in this case.

How should alternative BF weights be calculated? Gluck states that any other reasonable proxy for the variance of the loss development method may be used. In an example in his paper, Gluck uses the paid LDF in place of the incurred LDF to derive the weights. (Note that incurred LDFs are still used to derive *D*.)

Based on Gluck's work, we believe the use of the BF weights, or alternative BF weights, is a best actuarial practice for weighting together *D* and *A*. Within this discussion of best practices, when we refer to the BF method, we are referring to the weighting scheme used to blend the loss development result with the expected loss. As it is commonly applied, the BF method also refers to the method of selecting the expected losses, *A*, where *A* is often determined arbitrarily. We will approach the subject of the "best" practices for estimating *A* in the next section. There are many practices found in the literature, other than BF, that will enable us to identify a "best" expected ultimate loss estimate.

THE BEST *A PRIORI* ESTIMATE: THE GENERALIZED CAPE COD METHOD

The Traditional Cape Cod Method

The Cape Cod (CC) method was described by both Stanard and Buhlmann [3]. It is, therefore, also referred to as the Stanard-Buhlmann method. In Stanard's original presentation of the method, the exposure is assumed to be constant from year to year. Exhibit 1 shows an example of the CC method with changing exposure.

In this presentation of the CC method, the exposure is separated into two components:

1. the exposure expected to correspond to the reported losses, or “reported exposure”;
2. the component expected to correspond to the unreported losses.

The reported exposure is calculated by multiplying the exposure by the percentage of ultimate losses reported, or the inverse of the loss development factor to ultimate, for each accident year. As Stanard describes the method, the CC “averages, then adjusts” the reported losses. The reported losses for all years combined are divided by the reported exposures for all years combined to derive an expected ultimate loss-to-exposure ratio. We will refer to this ratio throughout this paper as the expected ultimate *loss ratio*, even though the exposures may be quantities other than premiums (e.g. use of ratemaking exposure units would result in an expected ultimate pure premium). The expected ultimate loss ratio is then applied to the unreported exposure for each year to derive an estimate of IBNR reserves.

A zero loss trend implicitly underlies Exhibit 1. Exhibit 2 shows how an actuary would use the CC method if he or she believed that losses increased over time at an average annual trend rate of 7.0%. The reported losses in column (4) have been adjusted to 1997 loss levels. The all-years-combined expected ultimate loss ratio in column (9), therefore, is also stated at a 1997 level. Therefore, the loss ratios in column (10) have been detrended by 7.0% per year before being multiplied by the unreported exposure for each accident year. Under the CC method, the expected loss ratio estimate, before detrending, is the same for each accident year.

The CC formula for the expected ultimate loss ratio for all accident years can be written another way:

$$Exp(LR) = \frac{\sum_i (R_i x LDF_i / E_i) x (E_i / LDF_i)}{\sum_i (E_i / LDF_i)} \qquad \textbf{(Equation 2)}$$

where

$Exp(LR)$	=	expected loss ratio estimate
R_i	=	reported trended losses for accident year i
LDF_i	=	loss development factor to ultimate for accident year i
E_i	=	exposures for accident year i.

Therefore, the expected loss ratio estimate used in the CC method can best be viewed as the weighted average of the trended, developed ultimate loss ratio for each year shown in Column (8), where the weights are based on a two dimensional weighting scheme: they are proportional to exposures and inversely proportional to development factors or, equivalently, are equal to the reported exposures in column (6) of Exhibit 2. Accident years with greater exposure levels get more weight than accident years with lower exposure levels. In addition, loss experience from more mature accident years gets more weight (in proportion to the percentage of ultimate reported at each maturity) since older years are closer to ultimate. Note that Equation 2 simplifies to the ratio of reported losses to reported exposures for all years combined.

A Modification to the Traditional Cape Cod Method

Gluck developed a modification to the CC method which introduced a third dimension to the a priori weighting scheme: *decay*. This modification is a significant departure from the traditional CC method in that the a priori loss ratio, trended to a common accident year basis, varies for each accident year. Gluck refers to this as the Generalized Cape Cod method (GCC).

In estimating the expected ultimate loss ratio for a given accident year, weight is assigned to all accident years in inverse proportion to the "distance" from the accident year in question. This is accomplished through the use of a decay factor, F, which varies between 0 and 1. Using notation similar to Gluck's, the GCC formula for the expected ultimate loss ratio in accident year i is:

$$Exp(LR_i) = \frac{\sum_j (R_j x LDF_j / E_j) x (E_j / LDF_j) x F^{|i-j|}}{\sum_j (E_j / LDF_j) x F^{|i-j|}} \qquad \textbf{(Equation 3)}$$

where

$Exp(LR_i)$ = expected loss ratio estimate for accident year i

F	=	decay factor $(0 \leq F \leq 1)$
R_j	=	reported trended losses for accident year j
LDF_j	=	loss development factor to ultimate for accident year j
E_j	=	exposures for accident year j.

Exhibit 3 shows an example of the GCC method and Exhibit 4 shows the weighting scheme for calculating the a priori loss ratios (for accident years 1994 and 1997), using a decay rate of 0.75 and a trend rate of 7.0%. A decay factor of 0.75 gives less weight to the accident years that are not immediately surrounding the accident year being calculated. In fact, as F approaches 0, the GCC method result approaches the loss development method result. As F approaches 1, the GCC method result approaches the CC method result. Therefore, the traditional loss development and CC methods can be viewed as special cases of the GCC method.

As the actuary's confidence in the loss development method increases, smaller decay factors should be used. On the other hand, for a line of business such as casualty umbrella, a decay factor close to 1.0 may be appropriate. In an appendix to his paper, Gluck describes a method for using the variances within the loss triangle (in both the development and trend directions) to determine the best decay factor, F.

Gluck demonstrated that the BF weights are optimal (they produce the minimum variance estimate) subject to certain constraints. Among these constraints is the assumption that the expected ultimate losses are known. This constraint is obviously not met in practice. Gluck has demonstrated that, if the expected ultimate loss ratio is determined as outlined in the GCC method, then the BF weights are still optimal.

There are some situations when it is prudent not to use the GCC method to calculate expected ultimate losses. For example, there may be situations when it is desirable to use external data or the actuary's own a priori expectations to derive the a priori loss ratio, such as for high excess layers where there are zero reported losses.

In most other situations, we believe it is a best practice to use the GCC method to determine the a priori ultimate losses, because:

- It uses information from all accident years in estimating any one year, thereby eliminating the need to arbitrarily determine how to select the a priori loss ratio.
- It gives more weight to surrounding years in determining a particular year's expected loss. This is appropriate because (1) insurance is subject to underwriting cycles, (2) pricing and underwriting changes are implemented gradually due to regulatory and other business constraints, and (3) the imprecision in using one year to estimate another likely increases with the distance between them.

- It gives less weight to immature years and low volume years, reducing the prediction error.
- Gluck points out that the GCC a priori loss ratio estimate is optimal -- produces the minimum bias linear estimate -- under certain conditions.
- It allows the actuary to systematically reflect internal or external changes, such as trend, coverage changes, underwriting changes, etc., through exposure base adjustments. (See BEST EXPOSURE BASE section.)
- The GCC method is a generalized case of the loss development and traditional CC methods. These other methods can be handled within the GCC framework.

BEST EXPOSURE BASE

When projecting losses, actuaries often have a choice of exposure bases. These typically would include earned premium, ratemaking exposure units and ultimate claim counts. The selection of the best exposure base should be based on the following considerations:

1. *Does there exist a leading indicator of the quantity being projected?* If so, then this exposure base will provide more information than other bases and should be considered a best exposure base.

2. *Does the exposure base require any adjustments before it can be used in the GCC method?* All other things being equal, it is best to choose an exposure base that requires the fewest adjustments.

Leading Indicators

Premiums, and ratemaking exposures upon which premiums are based, are truly a priori measures of exposure to loss. In general, they are determined for each risk in advance, before the first claim is incurred. To the extent possible, the exposures should be adjusted for changes in the underlying components of the loss process as described in the next section. While proper adjustment of the exposures underlying the loss process is necessary, it is not sufficient. A pure exposure-based approach can never model the randomness of insurance claims. In addition, there is significant parameter risk involved in using premiums or ratemaking exposures since ratemaking is not an exact science.

Therefore, we believe that the best exposure base should incorporate the latest available information. Reported claim counts, in general, emerge more quickly than losses and are leading indicators of the ultimate loss experience in most lines of business. However, ultimate claim counts themselves are not known with certainty and must be estimated. In keeping with the best practices, the GCC method should be used to estimate ultimate claim counts. Exhibit 5 provides an example.

Adjustment of Exposure Base

Littmann [7] describes an exposure-based approach to modeling the loss process. That paper developed a model of the expected change in losses by measuring the changes in the following items:

- Number of risks;
- Average size of risks;
- Policy limiting factors (limits, deductibles and net retentions);
- Class of business (including territory or state);
- Underwriting standards (including loss control and risk management initiatives);
- Claims adjustment procedures (changes in philosophy, treatment of incidents);
- Inflation;
- External factors (e.g. benefit level changes).

Breaking down the loss process into these components is a useful application for the GCC method. The GCC exposure base may need to be adjusted for changes in any of the listed factors. For example, if ratemaking exposure units are used as the exposure base, they should first be converted to a base class-equivalent basis to account for any changes in the class mix.

Alternatively, in this example, losses can be adjusted to reflect the change in distribution of exposures by classification. This adjustment to losses should be treated in the same manner as trend adjustments. That is, before expected ultimate losses are calculated for each accident year, this adjustment would have to be reversed so that each year's losses are at its actual, as opposed to common, level. This reversal is done in Column (10) of Exhibit 3.

When modeling these changes, the question to ask is: "Does this change affect the expected loss-to-exposure ratio"? If the answer is "no", then neither the exposure base nor the losses need to be adjusted to a common level. For example, if the exposure base is premium and the policy limits profile has changed dramatically, this change would not be expected to affect the expected loss ratio. As long as the increased limits factors are assumed to be correctly priced, any increase (or decrease) in loss severity would be offset by a corresponding change in premiums.

If the answer to the above question is "yes", or if the increased limits factors were not correctly priced, then either the exposure base or the losses (or both) need to be adjusted to a common level to reflect the anticipated changes. In the previous example, if the exposure base instead was ratemaking units or claim counts, then there would be a change in the expected loss per exposure.

Table 1 summarizes the adjustments that need to be made to three possible exposure bases (premiums, ratemaking exposures and claim counts) in response to changes in each of the items listed above, plus some other situations.

Table 1

Adjustments to GCC Exposure Bases

Changes in	**Exposure**	**Adjustment**
Number of Risks	Premium R/M Expos Claims	None None None
Size of Risks	Premium R/M Expos Claims	None Adjust for pure premium differences by size of risk Adjust for severity differences by size of risk
Limits, Deductibles	Premium R/M Expos Claims	None Adjust for severity differences by limit/deduct. Adjust for severity differences by limit/deduct.
Class, State Mix	Premium R/M Expos Claims	None Adjust for pure premium differences by class/state Adjust for severity differences by class/state
Underwriting Standards	Premium R/M Expos Claims	Adjust for pure premium differences Adjust for pure premium differences Adjust for severity differences
Claims Adjustment	Premium R/M Expos Claims	Adjust for pure premium differences Adjust for pure premium differences Adjust for severity differences
Inflation-Sensitive Exposure Base	Premium R/M Expos Claims	Adjust for exposure trend Adjust for exposure trend None
Rate Level Changes	Premium R/M Expos Claims	Adjust to common rate level None None

Note: R/M Expos = ratemaking exposures. Claims = claim counts.

In all cases when premiums are used as the exposure base, they should be adjusted to a common rate level, and premiums and inflation-sensitive exposure bases should also be adjusted for exposure trend. Furthermore, losses should always be adjusted to a common severity and/or frequency level -- only severity if claim counts are used as the exposure base, and also frequency if premiums or ratemaking exposures are used -- to reflect (1) inflation during the experience period and (2) external factors such as benefit level changes. (In the exhibits, it is assumed that exposures were already properly adjusted, and losses are adjusted to the basis of the most recent year via trend factors.)

All else equal, the exposure base requiring the fewest adjustments is the best choice because additional adjustments add imprecision to the process. In most situations, this would be the ultimate claim counts. Premiums have certain advantages when the mix of business by class, territory or limit has changed *as long as these items are priced correctly*. There is also the danger, in using premiums, of being unable to measure changes other than manual rate changes, e.g. changes in the average schedule debit or credit.

It should be noted that the GCC calculation will produce the same results if the exposures are adjusted to the level of the most recent year, the oldest year, or any year in between. The results are identical because only the *relative* exposure levels from year to year matter.

An Illustration

In stable environments, we have found ultimate claim counts to be the best exposure base for losses. In addition to being leading indicators of losses, they generally require the fewest adjustments.

A comparison of Exhibits 6 and 7 illustrates this point. In Exhibit 6, ultimate claim counts are used as the exposure base for estimating ultimate losses. Losses are adjusted for severity trend only. Note that the ultimate claims were derived in Exhibit 5 using a GCC approach.

In Exhibit 7, ratemaking exposure units are used as the exposure base for ultimate losses. In this situation, adjustments must be made for both severity and frequency trends. In this example, however, it is assumed that frequency trend is zero based on external indices.

Ultimate losses in Exhibit 6 are less than the ultimate losses in Exhibit 7 because the ratemaking exposure unit-based approach fails to recognize changes in claim frequency. Although it may be reasonable to assume that frequency trend is zero, the experience in Exhibit 5 indicates a decreasing frequency trend. Ultimate claim counts that are determined by a GCC method are more accurate than any externally-derived or fitted frequency trend. Remember that the purpose of the trend adjustment is to put historical years on a common level, and not to predict prospective trends.

Alternatively, the actuary could build in a negative frequency trend based on the actual claims experience. This is equivalent, however, to using the ultimate claim counts as the exposure base in the GCC method (Exhibit 6), but requires some extra steps.

Layered GCC Methods

This concept can be extended by "layering" GCC methods, using the results of one application as the exposure base for another application.

The steps should be as follows:

1. Use ratemaking exposure units (or policy counts or on-level premium) as the exposure base for projecting ultimate reported claim counts.
2. Use ultimate reported claim counts from the GCC as the exposure base for projecting ultimate losses.
3. Use ultimate losses (from the GCC) as the exposure base for projecting ultimate ALAE.
4. Use ultimate losses (from the GCC) as the exposure base for projecting ultimate salvage and subrogation (S&S).

If the ratio of claims closed with indemnity payment (CWIP) to total closed claims is changing materially, then ultimate reported claim counts are inappropriate for estimating ultimate losses. Instead, the actuary may want to use ultimate CWIPs. (This and other data inconsistencies, along with a method for deriving ultimate CWIPs, are explored in the Appendix.)

Exhibit 8 shows the GCC method as applied to reported ALAE. In this example, the GCC method derives a priori ultimate ALAE-to-loss ratios. The trend factor is used to reflect any anticipated changes in this ratio. In the example, the ratio is expected to decline in 1995. Potential causes include coverage or operational changes such as including ALAE within policy limits or decreased reliance on outside adjusters. An ALAE ratio trend factor of 0.80 is used for accident years 1993 and 1994 to adjust these years to a 1997 level in Columns (8) and (9). In Column (10), the a priori ALAE ratios are adjusted back to 1993 and 1994 for purposes of calculating unreported ALAE reserves.

BEST LOSS DEVELOPMENT PATTERNS

It is generally preferable to analyze losses separately from allocated loss adjustment expenses (ALAE), salvage and subrogation (S&S), and other quantities that vary with losses for the same reasons an actuary would separately analyze losses by line of business. Each of these quantities has its own, unique development pattern. To the extent that S&S recovery efforts have been increasing, for example, the S&S development pattern would be changing over time. If losses were reviewed net of S&S, the changes in recovery efforts might not be observable. It would be safe to combine losses, ALAE and recoveries if the actuary (1) observes these items developing consistently and (2) expects that the relative mix of losses, ALAE and recoveries to remain stable over time.

To the extent allowed by the data, then, a best development pattern should be selected for each of the following:

- reported claim counts
- incurred loss
- paid loss
- paid ALAE
- incurred ALAE
- S&S recoveries

Specific considerations for each of these patterns are addressed separately. In addition, best practices for selecting link ratio averages, which applies to all of the above patterns, is addressed in the last section.

Reported Claim Counts

Reported claim counts are defined as the sum of claims closed with payment (CWIP), closed without payment (CWOP), and open claims. These are all claims reported to the insurer by its insureds.

Reported claim counts are generally the easiest patterns to select, for many of the reasons that we are proposing using claim counts as the best exposure base for ultimate losses. The patterns are often stable and consistent, approach ultimate more quickly than losses, and are generally unaffected by insurer operational changes other than “incident” claim reporting (addressed in the Appendix). They are a function of the insureds’ and plaintiffs’ actions and societal trends, not the insurer’s internal procedures. In statistical terms, the variance of the bias is expected to be less for claim counts than for dollars of loss, resulting in greater credibility.

The actuary should also perform tests to determine whether the ratio of CWIP claims to closed claims (CWIP ratio) is changing for accident years at similar maturities. If CWIP ratios are relatively constant, then using ultimate reported claim counts as the exposure base in the GCC method is appropriate. If CWIP ratios are changing materially, then the actuary should consider using ultimate CWIP’s as the exposure base, because reported claim counts would no longer represent the true exposure to loss. In this case, a method for deriving ultimate CWIP’s is needed. Such a method is described in the Appendix.

Incurred Loss

One incurred loss development pattern should be identified as being the “best”. Unlike reported claim counts, incurred losses are materially affected by changes in insurer operational procedures. The best pattern should be determined after testing and adjusting for these operational changes. For purposes of determining a “best estimate”, the actuary would eliminate from consideration all other incurred development patterns. “High” and “low” estimates can be derived by selecting slower or faster development patterns.

The following issues should be considered when selecting the best incurred pattern:

- case reserving changes
- changing net retentions (net data)
- changing policy limits profile (gross data)
- underwriting changes
- changes in claim count definition (“incident” claims).

These issues are explored in the Appendix.

Paid Loss

In addition to the incurred development pattern, one paid loss development pattern should be identified as being the “best”. Many of the proposed best practices described for incurred development also apply to paid development, such as:

- changing net retentions (net data)
- changing policy limits profile (gross data)
- underwriting changes.

For paid loss development, the actuary should also perform diagnostic tests to determine whether there have been changes in claims settlement rates. These tests are also discussed in the Appendix.

ALAE

Before selecting the best ALAE patterns, the actuary should question the claims manager about changes that would impact ALAE development, or the ratio of ALAE to loss. Examples of such events include:

1. Shift of claim handling responsibilities between outside adjusters and inside adjusters;
2. Changes in claim settlement philosophy;
3. Changes in the rate in which lawyers and adjusters are compensated; for example, are fees billed on a monthly basis, quarterly basis, upfront, at the end of the suit, etc;
4. Changes in the claims settlement rate (may also affect paid ALAE development); and
5. Changes in case reserve adequacy (may also affect incurred ALAE development).

Salvage & Subrogation Recoveries

S&S development patterns generally lag the paid loss pattern, since the loss must be paid before a recovery can be sought. If S&S development data beyond a given maturity is thin, then the paid loss development may be lagged to estimate the S&S pattern. To be conservative, the actuary might consider using the paid loss development without any lag.

Best Link Ratio Averages

The reader may skip this section without loss of continuity. The ideas expressed in this section have been previously worked out in the actuarial literature, notably by Stanard, Peck [12], Mack and Murphy. We believe it is useful to consolidate these ideas into a framework for selecting best link ratio averages. These ideas apply to all development patterns.

Based on the cited authors' work, the actuary may choose from among the following link ratio methods:

- straight average development (SAD)
- weighted average development (WAD)
- least squares (LS)
- geometric average development (GAD).

The SAD, WAD and LS methods are all weighted averages of the observed link ratios between two maturities. The difference is in the weights: SAD applies equal weights to the observed link ratios, WAD applies weights in proportion to observed losses, and LS applies weights in proportion to the square of the observed losses. GAD differs from the other methods in that it is not a linear average of the observed link ratios; it is the *nth* root of the product of *n* observed link ratios.

Earlier work by Hachemeister and Stanard [6] indicated that WAD is superior to SAD, as the latter is likely to produce substantial additional bias. Peck, in his review of Stanard, confirmed that both GAD and LS are superior to SAD. Murphy replicated Stanard's simulation, incorporating GAD into the results, and reached the same conclusion. His results indicate that WAD, GAD and LS are superior to SAD.

Based on these authors' work, it appears that a best practice should be to *not* use SAD.

According to Mack and Murphy, the best average to use depends on the underlying process that we assume is generating losses (or ALAE, counts, etc.) For example, if the loss at a subsequent maturity depends on the loss at the previous maturity, then a statistical model of the following form may be assumed:

$$y = b\,x + e$$

where

y = the observed loss at the subsequent maturity

x = the observed loss at the current maturity (x is therefore a constant)

b = the unknown constant link ratio

e = a random error term with an expected value of zero.

Mack and Murphy prove that if the above process is the true process generating losses, then the best linear unbiased estimator of b is LS. (In general, the best weights to use are those which are inversely proportional to the variances of the link ratios.)

The loss generating models which correspond to SAD, WAD and GAD being the "best" are as follows. LS is also repeated for the reader's convenience.

If loss generating model is:	**Best average is:**
$y = b\,x + e\,x$	SAD
$y = b\,x + e\,(x)^{1/2}$	WAD
$y = b\,x + e$	LS
$y = b\,x\,e$	GAD*

*For GAD, the expected value of the error term is 1.

The actuary can determine which model applies using ideas presented by Mack. Mack describes residual plots which can be constructed to help determine whether SAD, WAD or LS is the true loss process. A GAD plot can similarly be constructed.

The actuary can also consider the following features of the models to judgmentally determine which averages to use:

- SAD assumes that the variance of the realized loss at the next observed maturity is proportional to the square of the loss at the current maturity. Equivalently, SAD assumes that the variance of the realized link ratio is constant for all observed losses at the current maturity.

- WAD assumes that the variance of the realized loss at the next observed maturity is proportional to the loss at the current maturity. Equivalently, WAD assumes that the variance of the realized link ratio is lower the greater the observed loss at the current maturity.

- LS assumes that the variance of the realized loss at the next observed maturity is constant, for all observed losses at the current maturity. Equivalently, LS assumes that the variance of the realized link ratio is lower the greater the observed loss at the current maturity.

- GAD assumes that the variance of the realized loss at the next observed maturity is proportional to the square of the loss at the current maturity times the unknown link ratio. GAD assumes that the variance of the realized link ratio is constant for all observed losses at the current maturity.

The actuary can compare the above to his pre-conceived beliefs to decide on the best averages to use. In practice, use of plots of the type described by Mack may be the exception rather than the rule. We believe that WAD is the "best" approach because it has been shown to consistently produce among the "best" results and it is simple to apply.

Final Thoughts on Loss Development Factors

In addition to choosing the averaging method, the actuary must also choose the number of years to include in the link ratio averages. Many of the best practices followed earlier may help determine the years that should be discarded from averages, if any. Aside from this, it is generally considered appropriate to use averages of recent data points.

Finally, a good thought experiment to perform at the end, after the actuary has selected all the "best" patterns, is to justify to an imaginary company executive the actuarial decisions regarding development that the actuary has made, using the business changes implemented by the insurer and discovered by the actuary as support. If an actuary cannot describe a business reason behind all decisions regarding the development patterns, then the actuary should reconsider making the change until he/she can support it in such terms. For example: "I selected slower development patterns than indicated by the data because net retentions have increased significantly and case reserves were lowered two years ago."

SELECTED ULTIMATE LOSSES

The *Statement of Principles Regarding Property and Casualty Loss and Loss Expense Adjustment Reserves* states that ordinarily the actuary will examine the indications of more than one method. The various indications that are produced by following the practices outlined in this paper can be considered different methods. For example, paid and incurred variations should be considered different approaches. Additionally, the use of different exposure bases within the GCC method can be considered distinct methods.

Unless there is an obvious reason to eliminate one or the other, both paid and incurred methods should be given weight when selecting ultimate losses. Keeping with the best practices expressed in this paper, the weights assigned to the paid and incurred methods should be inversely proportional to the variance of their ultimates.

Earlier in this paper, we discussed considerations for determining the best exposure base. There may be situations where two or more exposure bases appear to be equally applicable. If several GCC methods are used, each with a different exposure base, then more weight should be given to the method with less variable developed ultimate loss ratios (in Column 7). Ultimate loss ratios that vary minimally from year to year indicate that the actuary has successfully adjusted losses and exposures for trends, distributional shifts and other explainable changes. Ideally, if losses and exposures are perfectly adjusted, then the developed ultimate loss ratios for each year should fluctuate randomly around some mean value.

POINT ESTIMATES VS. RANGES

Once the "best estimate" is determined, a range can then be determined by varying key parameters within this framework. For example, alternate loss development patterns, tail factors, trend rates, decay rates and exposure bases can be used to produce reasonable high and low reserve estimates. Great care should be exercised to ensure that too wide of a range is not created by compounding various optimistic (or pessimistic) assumptions.

The key idea, though, is to use the GCC framework to generate the range. This is a departure from the more common situation where ranges are created by using the results of various methods.

CONCLUSION

Table 2 summarizes the proposed best practices discussed in this paper.

Table 2
Proposed Best Practices

Problem or Issue	Best Practice
When should loss development methods be used? When should exposure-based methods be used?	Whenever an appropriate exposure base has been identified, the actuary should rely on a loss reserving method that mixes loss development methods with exposure-based expected loss methods.
What is the best way to weigh together different methods?	The best weights used to average the results of the loss development and expected loss methods are Bornhuetter-Ferguson weights (or alternative BF weights).
What is the best way to determine the expected ultimate loss ratio?	The expected ultimate loss ratio should be determined using the Generalized Cape Cod (GCC) method described by Gluck. The expected ultimate loss ratio is based on a combination of factors, namely: (a) maturity of data, (b) volume of data and (c) decay.
What items should be considered when selecting the best exposure base for the GCC method?	(1) The exposure base must be a leading indicator for the quantity being projected. (2) The exposure base requiring the fewest adjustments is most likely the one with the least distortion.
What exposure base is generally best for projecting ultimate losses in the GCC method?	As long as there is a stable environment, ultimate reported claim counts are generally the best exposure base when projecting ultimate losses using the GCC method.
What is the best method for determining ALAE reserves?	GCC methods should also be applied to ALAE using the ultimate losses (from the loss GCC method) as the exposure base.
What is the best method for determining S&S reserves?	GCC methods should also be applied to S&S using the ultimate losses (from the loss GCC method) as the exposure base.
What are some of the best practices for picking development factors?	Select one “best” claim count pattern, one incurred pattern, one paid, one ALAE and one S&S (after adjusting for any operational changes). Use weighted averages to determine link ratios.
What are some best practices for selecting ultimate losses?	Give weight to both paid and incurred GCC methods. If one exposure base has not been identified as the “best”, then select averages of GCC methods based on various exposure bases.
What is a practical method for determining a range around the best estimate?	A practical way to determine the reasonable range of reserves is to select reasonable alternative parameters (e.g. tail factors, development factors, trend rates, decay rates or exposure bases) and substitute these parameters into the selected methodology, e.g. Generalized Cape Cod.

The GCC method provides a framework to handle all of the best practices identified in Table 2. It is flexible and effective for nearly all types of insurers, including primary, excess, reinsurance and international. The GCC method should be an actuary's central reserving technique.

This paper is not meant to be exhaustive. It is hoped that the ideas expressed will lead to further dialogue among property/casualty actuaries and refinement of what are considered best reserving practices. Suggested areas of further study could include:

- best practices for tail factors;
- best practices for ULAE reserving.

APPENDIX

BEST PRACTICES WHEN THE DATA IS INCONSISTENT

When data has not been consistent, for example due to operational changes made by the insurer, the actuary may need to adjust the data to remove distortions caused by factors other than random “noise.”

Reported Claim Counts

There is a distortion which should be accounted for when selecting the best claim count pattern: that arising from “incident” claims. These are events which have occurred between the insured and a third party, but have not yet given rise to a claim. The insured notifies the insurer of the incident, and depending on the insurer’s data procedures, the incident is coded as a reported claim immediately or not until the claim is actually reported by the insured.

The important consideration here is consistency. If incident claims have always been counted as “claims” by the insurer, and these are a relatively constant mix of all claims, then there is no distortion. However, if the insured recently revised their treatment of such incidents, e.g. only in recent years were they coded as claims, then they should not be included in the claim count triangle until they turn into real claims. The best pattern should be selected on this basis.

Claims Closed With Indemnity Payment (CWIP)

The actuary should also perform tests to determine whether the ratio of CWIP claims to closed claims (CWIP ratio) is changing over time for accident years at similar maturities. If CWIP ratios are relatively constant, then using ultimate reported claim counts as the exposure base in the GCC method is appropriate. If CWIP ratios are changing materially, then the actuary should consider using ultimate CWIPs as the exposure base. In this case, a method for deriving ultimate CWIPs is needed.

We do not recommend applying a development method to CWIP claims, as the main advantage of using reported claims counts, namely that they develop to ultimate more quickly than losses, does not apply to CWIP development. In addition, ultimate reported claims serve as the upper bound on ultimate CWIP claims. With development methods, ultimate CWIP claims can be projected to be greater than the ultimate number of reported claims. Instead, the ultimate reported counts should be used as the basis in any method for deriving ultimate CWIP counts.

An example of such a method is shown in Exhibits 9 to 11. In Exhibit 9, the number of closed claims are subtracted from the ultimate reported claim counts (estimated earlier in Exhibit 5) to derive the total number of claims unpaid -- including open and unreported -- for each accident year as of each evaluation date. Based on the historical information, we estimate for each interval (1) the rates at which the total unpaid claims will be disposed, and (2) the percentage of those claims that will close with indemnity payment.

The first item, or “disposal ratios”, are derived in Exhibit 10 by dividing the triangle of claims closing in each interval by a triangle of unpaid claims (open + IBNR) at the beginning of each interval. The second item, or “in-period CWIP ratios”, are derived in Exhibit 11 by dividing the triangle of CWIPs in each interval by the triangle of all claims closing during the interval. By applying both of these factors to the unpaid claims for each accident year, one can estimate the number of CWIP claims in each future interval, and, therefore, the ultimate number of CWIP claims; this is shown in Exhibit 11. The ultimate CWIPs so derived are certain to be less than the ultimate reported claims.

The ultimate CWIP ratios in Exhibit 11 are declining, alerting the actuary that ultimate CWIPs should serve as the exposure base for the ultimate loss GCC method. (Note that the actuary could have determined this simply by examining the declining CWIP ratios in Exhibit 11, Section (A).) If the actuary were to use ultimate reported claims as the exposure base, the GCC method could overstate ultimate losses, since an important inconsistency in the data -- the fact that the percentage of claims that are “CWIPing” is declining -- would be hidden from the actuary.

Incurred Losses

The following issues should be considered when selecting the best incurred pattern(s):

- case reserving changes
- changing net retentions (net data)
- changing policy limits profile (gross data)
- underwriting changes
- changes in claim count definition (“incident” claims)

Case Reserving Changes

The first step in the process is to perform certain diagnostic tests of the data. Based on these tests, the actuary would draw conclusions as to whether there have been changes in the level of average relative case reserve adequacy (CRA) and, if so, adjust the data to put all years onto the current year’s basis. These diagnostics and the corresponding adjustments are described by Berquist and Sherman [1] and are not repeated here.

Changes in CRA are most often a calendar year phenomenon, rather than an accident year or policy year phenomenon. For example, when a claims department implements a new case reserving procedure, it generally applies to all open cases from all prior accident years, as well as future accident years. This means it affects all accident/policy years at different maturities and to differing extents. The CRA adjustments, if any, should reflect this.

If the actuary determines that changes in CRA are observed in the data, then a best practice should be to contact the claims personnel of the insurer to confirm that operational changes have occurred. The questions asked by the actuary should address the year in which changes in CRA occurred, and whether it was a calendar year or accident year implementation; this should then be confirmed for consistency with the data.

The information being provided by the data and by the claims department may be inconsistent, however. For example, if the data indicates weakening CRA in recent calendar years, the actuary may determine after interviewing claims personnel that there have been no changes in case reserving procedures. Instead, underwriting changes, declining net retentions, or declining policy limits may be the cause behind declining average claim sizes on open claims, rather than specific claims department actions. In this case, the CRA diagnostics will give a false reading of weakening case reserves. The actuary should consider tempering the CRA diagnostics and adjustments when claims personnel do not corroborate phenomena observed in the data.

This will alert the actuary of the need to interview appropriate personnel to discuss changes in net retentions, policy limits or underwriting procedures, to verify the observed changes in relative case reserves.

Net Retentions & Policy Limits Profile

This is a policy year or reinsurance underwriting year phenomenon rather than an accident year or calendar year phenomenon. Changing net retentions affect the net development patterns, and changing policy limits impact direct development. Declines in these values should serve to shorten development patterns, and similarly increases will lengthen the development. The practitioner should also be aware of significant changes to deductibles.

Even if net retentions or the mix of policy limits is not changing in nominal terms, the real retentions and policy limits are declining due to claim cost trend. Although an actuary could construct statistical methods to adjust for this, they are beyond the scope of this paper. The actuary should make a determination whether he/she believes this effect to be material, based on severity trends in the line of business, the average severity in the book of business, and the limits written by the insurer. The closer the limits are to the average severity, the greater the dampening effect will be, i.e. recent accident years will develop more quickly than older accident years.

We believe it is a best practice to request the net retention and policy limits information at the same time as the loss and exposure data, which may eliminate the need to discuss these changes with the insurer. The actuary should request detailed information about the insurer’s ceded reinsurance program -- including excess of loss and quota share retentions by line of business -- for as many years as are available. The requested policy limits information should show written premium, or policy counts, by policy year and limit for as many years as are available.

The actuary may choose to select different development patterns for different groups of accident years, depending on the extent of the changes in retentions and policy limits. This may be a judgmental determination rather than a statistical construction.

Underwriting Changes

After discussing the claims issues with the insurer and examining the retention and limits profiles, the actuary may conclude that neither of those provide verification of the observed CRA changes in the data. In this case, changes in underwriting should be explored. For practical purposes, the term "underwriting" would also include risk management, loss control or engineering changes.

This is often a policy year, not a calendar year, phenomenon. If there have been underwriting changes, and the book of business has significantly turned over in recent years, then it may not be possible to measure the relative adequacy of the reserves in recent accident years to those from older accident years. If claims personnel indicate no changes in case reserving, the actuary should consider not making any adjustments to the data and assume that all years have consistent relative CRA.

Different patterns by year may still be appropriate, however, if the changing mix of types of insureds due to underwriting results in shorter or longer development patterns in recent years. This effect may be observed in the actual link ratios, or may have to be judgmentally accounted for. Discussions with underwriters may provide enlightenment here.

Incident Claims

If the definition of a "claim count" has changed due to the changing treatment of "incident" claims, then Berquist-Sherman type adjustments due to changes in case reserve adequacy may not be possible because there will appear to be more (or fewer) open claims for recent accident years. This will give the illusion of changing case reserve adequacy as incident claims are usually reserved for at some low value. Such incident claims should be removed from the data as described earlier, to allow meaningful CRA diagnostics and adjustments. If not, then CRA methods cannot be used.

Paid Losses

For paid loss development, the actuary should first perform diagnostic tests to determine whether there have been changes in claims settlement rates (CSR). These diagnostics are also described by Berquist and Sherman. The actuary would select paid development factors that reflect his or her beliefs about changes in CSR by adjusting the data to the current year's basis. For purposes of determining a "best estimate", the actuary would eliminate from consideration all other paid development patterns.

The CSR diagnostics determine if there have been changes in the rate at which claims are settled by expressing closed claims at a given maturity as a percentage of ultimate reported claims, separately by accident year. Alternatively, CWIP's may be related to ultimate CWIP's. The choice of method is dependent on whether CWIP ratios have been changing over the course of the historical data period. This was examined by the actuary as part of the incurred loss development best practices described earlier.

If CWIP ratios have not materially changed, then the actuary may use closed claims and ultimate reported claims to measure changes in CSR. Otherwise, CWIP's and ultimate CWIP's should be used. The actuary would have derived ultimate CWIP's as the exposure base for the GCC method after determining that CWIP ratios were changing.

Before performing CSR diagnostics, the actuary should remove incident claims from the data as described earlier.

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Sample Insurance Company
Cape Cod Method

Trend Rate = 0.0%

Accident Year	(1) Exposures	(2) Reported Losses	(3) Trend Factor	(4) Trended Reported Losses	(5) Percent Reported	(6) Reported Exposure	(7) Unreported Exposure
1993	7,000	3,600	1.000	3,600	85%	5,950	1,050
1994	8,000	4,000	1.000	4,000	75%	6,000	2,000
1995	9,000	4,800	1.000	4,800	60%	5,400	3,600
1996	10,000	3,600	1.000	3,600	45%	4,500	5,500
1997	11,000	2,800	1.000	2,800	25%	2,750	8,250
Total	45,000	18,800		18,800		24,600	20,400

Accident Year	(8) Trended Developed Loss Ratio	(9) Expected Ultimate Loss Ratio	(10) Detrended Expected Loss Ratio	(11) IBNR Reserve	(12) Ultimate Losses
1993	60.5%	76.4%	76.4%	802	4,402
1994	66.7%	76.4%	76.4%	1,528	5,528
1995	88.9%	76.4%	76.4%	2,751	7,551
1996	80.0%	76.4%	76.4%	4,203	7,803
1997	101.8%	76.4%	76.4%	6,305	9,105
Total	76.4%			15,590	34,390

Notes:

(1) Can be premiums, claim counts, ratemaking exposures, etc.
(2) Can also be claim counts, paid losses, ALAE, salvage & subrogation, etc.
(3) Can also reflect other adjustments to losses.
(4) = (2) x (3)
(5) = 1.0/(development factor to ultimate)
(6) = (1) x (5)
(7) = (1) x [1.0 - (5)]
(8) = (4)/(6)
(9) = total of (8). Trended to 1997.
(10) = (9)/(3)
(11) = (7) x (10)
(12) = (2) + (11)

Sample Insurance Company
Cape Cod Method

Trend Rate = 7.0%

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Accident Year	Exposures	Reported Losses	Trend Factor	Trended Reported Losses	Percent Reported	Reported Exposure	Unreported Exposure
1993	7,000	3,600	1.311	4,719	85%	5,950	1,050
1994	8,000	4,000	1.225	4,900	75%	6,000	2,000
1995	9,000	4,800	1.145	5,496	60%	5,400	3,600
1996	10,000	3,600	1.070	3,852	45%	4,500	5,500
1997	11,000	2,800	1.000	2,800	25%	2,750	8,250
Total	45,000	18,800		21,767		24,600	20,400

	(8)	(9)	(10)	(11)	(12)
Accident Year	Trended Developed Loss Ratio	Expected Ultimate Loss Ratio	Detrended Expected Loss Ratio	IBNR Reserve	Ultimate Losses
1993	79.3%	88.5%	67.5%	709	4,309
1994	81.7%	88.5%	72.2%	1,445	5,445
1995	101.8%	88.5%	77.3%	2,782	7,582
1996	85.6%	88.5%	82.7%	4,548	8,148
1997	101.8%	88.5%	88.5%	7,300	10,100
Total	88.5%			16,783	35,583

Notes:

(1) Can be premiums, claim counts, ratemaking exposures, etc.
(2) Can also be claim counts, paid losses, ALAE, salvage & subrogation, etc.
(3) Can also reflect other adjustments to losses.
(4) = (2) x (3)
(5) = 1.0/(development factor to ultimate)
(6) = (1) x (5)
(7) = (1) x [1.0 - (5)]
(8) = (4)/(6)
(9) = total of (8). Trended to 1997.
(10) = (9)/(3)
(11) = (7) x (10)
(12) = (2) + (11)

Sample Insurance Company
Generalized Cape Cod Method

Decay Rate = 0.75
Trend Rate = 7.0%

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Accident Year	Exposures	Reported Losses	Trend Factor	Trended Reported Losses	Percent Reported	Reported Exposure	Unreported Exposure
1993	7,000	3,600	1.311	4,719	85%	5,950	1,050
1994	8,000	4,000	1.225	4,900	75%	6,000	2,000
1995	9,000	4,800	1.145	5,496	60%	5,400	3,600
1996	10,000	3,600	1.070	3,852	45%	4,500	5,500
1997	11,000	2,800	1.000	2,800	25%	2,750	8,250
Total	45,000	18,800		21,767		24,600	20,400

	(8)	(9)	(10)	(11)	(12)
Accident Year	Trended Developed Loss Ratio	Expected Ultimate Loss Ratio	Detrended Expected Loss Ratio	IBNR Reserve	Ultimate Losses
1993	79.3%	86.1%	65.7%	690	4,290
1994	81.7%	87.4%	71.3%	1,427	5,427
1995	101.8%	89.7%	78.3%	2,819	7,619
1996	85.6%	89.8%	84.0%	4,618	8,218
1997	101.8%	90.9%	90.9%	7,499	10,299
Total	88.5%			17,052	35,852

Notes:

(1) Can be premiums, claim counts, ratemaking exposures, etc.
(2) Can also be claim counts, paid losses, ALAE, salvage & subrogation, etc.
(3) Can also reflect other adjustments to losses.
(4) = (2) x (3)
(5) = 1.0/(development factor to ultimate)
(6) = (1) x (5)
(7) = (1) x [1.0 - (5)]
(8) = (4)/(6)
(9) Trended to 1997. See Exhibit 4.
(10) = (9)/(3)
(11) = (7) x (10)
(12) = (2) + (11)

Sample Insurance Company
Generalized Cape Cod Method
Calculation of a-Priori Ultimate Loss Ratios

Decay Rate = 0.75 (a)
Trend Rate = 7.0%

Calculation of a-priori loss ratio for: **1994** (b)

(1) Accident Year	(2) Reported Exposure	(3) Lag	(4) Decay Weight	(5) Weighted Exposure	(6) Trended Ultimate Loss Ratio
1993	5,950	1	0.750	4,463	79.3%
1994	6,000	0	1.000	6,000	81.7%
1995	5,400	1	0.750	4,050	101.8%
1996	4,500	2	0.563	2,531	85.6%
1997	2,750	3	0.422	1,160	101.8%
Total	24,600			18,204	87.4%

Calculation of a-priori loss ratio for: **1997** (b)

(1) Accident Year	(2) Reported Exposure	(3) Lag	(4) Decay Weight	(5) Weighted Exposure	(6) Trended Ultimate Loss Ratio
1993	5,950	4	0.316	1,883	79.3%
1994	6,000	3	0.422	2,531	81.7%
1995	5,400	2	0.563	3,038	101.8%
1996	4,500	1	0.750	3,375	85.6%
1997	2,750	0	1.000	2,750	101.8%
Total	24,600			13,576	90.9%

Notes:

(2) From Exhibit 3, Column (6).
(3) = absolute value of difference between (b) and (1)
(4) = (a) raised to power of (3)
(5) = (4) x (2)
(6) From Exhibit 3, Column 8 for individual years. Total is weighted average of values in (6) using (5) as weights.
(a) Selected judgmentally, based on considerations described in text.

Sample Insurance Company
Generalized Cape Cod Method
Ultimate Claim Counts

Decay Rate = 0.75
Freq. Trend Rate = 0.0%

Accident Year	(1) Ratemaking Exposure Units	(2) Reported Claim Counts	(3) Frequency Trend Factor	(4) Trended Claim Counts	(5) Percent Reported	(6) Reported Exposure	(7) Unreported Exposure
1993	3,500	400	1.000	400	95%	3,325	175
1994	4,000	420	1.000	420	90%	3,600	400
1995	4,500	450	1.000	450	75%	3,375	1,125
1996	5,000	340	1.000	340	60%	3,000	2,000
1997	5,500	200	1.000	200	40%	2,200	3,300
Total	22,500	1,810		1,810		15,500	7,000

Accident Year	(8) Trended Developed Frequency	(9) Expected Ultimate Frequency	(10) Detrended Expected Frequency	(11) IBNR Claim Counts	(12) Ultimate Claim Counts	(13) Ultimate Claim Frequency
1993	12.03%	11.88%	11.88%	21	421	12.02%
1994	11.67%	11.86%	11.86%	47	467	11.69%
1995	13.33%	11.87%	11.87%	134	584	12.97%
1996	11.33%	11.62%	11.62%	232	572	11.45%
1997	9.09%	11.34%	11.34%	374	574	10.44%
Total	11.68%			809	2,619	

Notes:

(1) Can also be policy counts or on-level premium.
(2) Given.
(3) Based on selected frequency trend rate.
(4) = (2) x (3)
(5) = 1.0/(development factor to ultimate)
(6) = (1) x (5)
(7) = (1) x [1.0 - (5)]
(8) = (4)/(6)
(9) Trended to 1997. See Eq. 3 in text.
(10) = (9)/(3)
(11) = (7) x (10)
(12) = (2) + (11)
(13) = (12)/(1)

Sample Insurance Company
Generalized Cape Cod Method
Ultimate Loss - Using Claim Counts as Exposure Base

Decay Rate =	0.75
Severity Trend Rate =	7.0%

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Accident Year	Ultimate Claim Counts	Reported Losses	Severity Trend Factor	Trended Reported Losses	Percent Reported	Reported Exposure	Unreported Exposure
1993	421	3,600	1.311	4,719	85%	358	63
1994	467	4,000	1.225	4,900	75%	351	117
1995	584	4,800	1.145	5,496	60%	350	233
1996	572	3,600	1.070	3,852	45%	258	315
1997	574	2,800	1.000	2,800	25%	144	431
Total	2,619	18,800		21,767		1,460	1,159

	(8)	(9)	(10)	(11)	(12)
Accident Year	Trended Developed Severity	Expected Ultimate Severity	Detrended Expected Severity	IBNR Reserve	Ultimate Losses
1993	13.2	14.4	11.0	694	4,294
1994	14.0	14.6	11.9	1,396	5,396
1995	15.7	15.0	13.1	3,052	7,852
1996	15.0	15.2	14.2	4,468	8,068
1997	19.5	15.5	15.5	6,689	9,489
Total	14.9			16,298	35,098

Notes:

(1) From Exhibit 5, Column (12).
(2) Given.
(3) Based on severity trend rate..
(4) = (2) x (3)
(5) = 1.0/(development factor to ultimate)
(6) = (1) x (5)
(7) = (1) x [1.0 - (5)]

(8) = (4)/(6)
(9) Trended to 1997. See Eq. 3 in text.
(10) = (9)/(3)
(11) = (7) x (10)
(12) = (2) + (11)

Sample Insurance Company
Generalized Cape Cod Method
Ultimate Loss - Using Ratemaking Exposures as Exposure Base

Decay Rate =	0.75
Pure Prem. Trend Rate =	7.0%

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Accident Year	Ratemaking Exposure Units	Reported Losses	Pure Prem. Trend Factor	Trended Reported Losses	Percent Reported	Reported Exposure	Unreported Exposure
1993	3,500	3,600	1.311	4,719	85%	2,975	525
1994	4,000	4,000	1.225	4,900	75%	3,000	1,000
1995	4,500	4,800	1.145	5,496	60%	2,700	1,800
1996	5,000	3,600	1.070	3,852	45%	2,250	2,750
1997	5,500	2,800	1.000	2,800	25%	1,375	4,125
Total	22,500	18,800		21,767		12,300	10,200

	(8)	(9)	(10)	(11)	(12)
Accident Year	Trended Developed Pure Prem.	Expected Ultimate Pure Prem.	Detrended Expected Pure Prem.	IBNR Reserve	Ultimate Losses
1993	1.59	1.72	1.31	690	4,290
1994	1.63	1.75	1.43	1,427	5,427
1995	2.04	1.79	1.57	2,819	7,619
1996	1.71	1.80	1.68	4,618	8,218
1997	2.04	1.82	1.82	7,499	10,299
Total	177.0%			17,052	35,852

Notes:

(1) From Exhibit 5, Column (1).
(2) From Exhibit 6, Column (2).
(3) Based on pure premium trend rate.
(4) = (2) x (3)
(5) From Exhibit 6, Column (5).
(6) = (1) x (5)
(7) = (1) x [1.0 - (5)]
(8) = (4)/(6)
(9) Trended to 1997. See Eq. 3 in text.
(10) = (9)/(3)
(11) = (7) x (10)
(12) = (2) + (11)

Sample Insurance Company
Generalized Cape Cod Method
Ultimate ALAE

Decay Rate = 0.75
Trend Rate = 0.0%

Accident Year	(1) Ultimate Losses	(2) Reported ALAE	(3) ALAE Ratio Trend Factor	(4) Trended Reported ALAE	(5) Percent Reported	(6) Reported Exposure	(7) Unreported Exposure
1993	4,294	1,320	0.800	1,056	80%	3,435	859
1994	5,396	1,500	0.800	1,200	70%	3,777	1,619
1995	7,852	1,350	1.000	1,350	55%	4,319	3,533
1996	8,068	950	1.000	950	40%	3,227	4,841
1997	9,489	600	1.000	600	20%	1,898	7,591
Total	35,098	5,720		5,156		16,656	18,443

Accident Year	(8) Trended Developed ALAE Ratio	(9) Expected Ultimate ALAE Ratio	(10) Detrended Expected ALAE Ratio	(11) Unreported ALAE	(12) Ultimate ALAE
1993	30.7%	31.0%	38.8%	333	1,653
1994	31.8%	31.1%	38.8%	629	2,129
1995	31.3%	31.0%	31.0%	1,094	2,444
1996	29.4%	30.8%	30.8%	1,492	2,442
1997	31.6%	30.9%	30.9%	2,345	2,945
Total	31.0%			5,893	11,613

Notes:

(1) From Exhibit 6, Column (12).
(2) Given.
(3) Reflects changes in expected ALAE-to-loss ratio.
(4) = (2) x (3)
(5) = 1.0/(development factor to ultimate)
(6) = (1) x (5)
(7) = (1) x [1.0 - (5)]
(8) = (4)/(6)
(9) Trended to 1997. See Eq. 3 in text.
(10) = (9)/(3)
(11) = (7) x (10)
(12) = (2) + (11)

Exhibit 9

Sample Insurance Company
Claim Count Triangles
As of 12/31/97

(A) Number of Claims Closed With Loss Payment ("CWIPs")

Accid. Year	12	24	36	48	60
1993	31	73	130	159	171
1994	45	76	131	161	
1995	38	78	158		
1996	45	84			
1997	49				

(B) Number of Closed Claims

Accid. Year	12	24	36	48	60	Estimated Ultimate No. of Claims (D) ULT
1993	93	202	329	390	412	421
1994	144	236	364	431		467
1995	129	254	448			584
1996	162	291				572
1997	196					574

(C) Number of Open & IBNR Claims

Accid. Year	12	24	36	48	60
1993	328	219	92	31	9
1994	323	231	103	36	
1995	455	330	136		
1996	410	281			
1997	378				

Notes:

	(A), (B)	From Sample Insurance Company.
	(C)	= (D) - (B)
	(D)	From Exhibit 5, Column 12.

Exhibit 10

Sample Insurance Company
Projected Number of Closed Claims
As of 12/31/97

(A) In-Period Disposal Ratios

Accid. Year	12:24	24:36	36:48	48:60	60:ULT
1993	33.2%	58.0%	66.3%	71.0%	
1994	28.5%	55.4%	65.0%		
1995	27.5%	58.8%			
1996	31.5%				
Weighted Average	30.0%	57.6%	65.6%	71.0%	
Selected	30.0%	57.6%	65.6%	71.0%	100.0%

(B) Projected Number of Open & IBNR Claims by Age

Accid. Year	12	24	36	48	60
1993	328	219	92	31	9
1994	323	231	103	36	10
1995	455	330	136	47	14
1996	410	281	119	41	12
1997	378	265	112	39	11

(C) Projected Number of In-Period Claims Closed by Age

Accid. Year	0:12	12:24	24:36	36:48	48:60	60:ULT
1993	93	109	127	61	22	9
1994	144	92	128	67	26	10
1995	129	125	194	89	33	14
1996	162	129	162	78	29	12
1997	196	113	152	74	27	11

Notes: Boxed numbers are projections.

(A) From Exhibit 9, Section (B) and Section (C)
= in-period closed claims / number of open claims at beginning of period.

(B) Historical values from Exhibit 9, Section (C).
Projected values
= [(B) from previous period] x [1 - selected disposal ratio from Section (A)].

(C) Historical values = in-period closed claims from Exhibit 9, Section (B).
Projected values
= [(B) at start of period] x [selected disposal ratio from Section (A)].

Sample Insurance Company
Projected Ultimate Number of Claims Closed with Loss Payment
As of 12/31/97

(A) In-Period Closed With Payment Ratios

Accid. Year	0:12	12:24	24:36	36:48	48:60	60:ULT
1993	33.3%	38.5%	44.9%	47.5%	54.5%	
1994	31.3%	33.7%	43.0%	44.8%		
1995	29.5%	32.0%	41.2%			
1996	27.8%	30.2%				
1997	25.0%					
Weighted Average	28.7%	33.4%	42.8%	46.1%	54.5%	
Selected	28.7%	33.4%	42.8%	46.1%	54.5%	60.0%

(B) Projected Number of In-Period Claims Closed With Loss Payment by Age

Accid. Year	0:12	12:24	24:36	36:48	48:60	60:ULT
1993	31	42	57	29	12	5
1994	45	31	55	30	14	6
1995	38	40	80	41	18	8
1996	45	39	69	36	16	7
1997	49	38	65	34	15	7

(C) Summary

Accident Year	(1) Ultimate Number of Reported Claims	(2) Closed Claims To Date	(3) CWIP Claims To Date	(4) Estimated Remaining CWIP Claims	(5) Estimated Ultimate Number of CWIP Claims	(6) Estimated Ultimate CWIP Ratio
1993	421	412	171	5	176	41.9%
1994	467	431	161	20	181	38.8%
1995	584	448	158	67	225	38.6%
1996	572	291	84	128	212	37.1%
1997	574	196	49	159	208	36.2%

Notes: Boxed numbers are projections.

(A) From Exhibit 9, Section (A) and Section (B)
= in-period CWIP claims / in-period closed claims.

(B) Historical values = in-period CWIP claims from Exhibit 9, Section (A).
Projected values
= [Exhibit 10, Section (C)] x [selected CWIP ratio from Section (A)].

(C) (1) From Exhibit 5, Column 12.
(2) From Exhibit 9, Section (B).
(3) From Exhibit 9, Section (A).
(4) Sum of projected CWIP claims from Section (B).
(5) = (3) + (4)
(6) = (5) / (1)