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The Casualty Actuarial Society *Forum*
Winter 1997 Edition
Including the Ratemaking Call Papers

To CAS Members:

This is the Winter 1997 Edition of the Casualty Actuarial Society *Forum*.

It contains several new papers along with materials from various committees. In addition, four Ratemaking Call Papers are included (see the following note from the CAS Committee on Ratemaking).

The Casualty Actuarial Society *Forum* is a non-refereed journal printed by the Casualty Actuarial Society. The viewpoints published herein do not necessarily reflect those of the Casualty Actuarial Society.

The CAS *Forum* is edited by the CAS Committee for the Casualty Actuarial Society *Forum*. Members of the committee invite all interested persons to submit papers on topics of interest to the actuarial community. Articles need not be written by a member of the CAS, but the paper's content must be relevant to the interests of the CAS membership. Members of the Committee for the Casualty Actuarial Society *Forum* request that the following procedures be followed when submitting an article for publication in the *Forum*:

1. Authors should submit a camera-ready original paper, and two copies.
2. Authors should not number their pages.
3. All exhibits, tables, charts, and graphs should be in original format and camera ready.
4. Authors should avoid using gray-shaded graphs, tables, or exhibits. Text and exhibits should be in solid black and white.

The CAS *Forum* is printed periodically based on the number of articles submitted. The committee's goal is to publish two editions during the calendar year.

All comments or questions may be directed to the Committee for the Casualty Actuarial *Forum*.

I would like to thank Paul E. Lacko and Gerald T. Yeung for their time and effort in putting this edition together.

Sincerely,

Robert G. Blanco, CAS *Forum* Chairperson

The 1997 CAS Ratemaking Call Papers

Presented at the
1997 CAS Seminar on Ratemaking
March 13-14
Marriott Copley Place
Boston, Massachusetts

The Winter 1997 Edition of the CAS *Forum* is a cooperative effort of the CAS Continuing Education Committee on the CAS *Forum* and the Research and Development Committee on Ratemaking.

The CAS Committee on Ratemaking is pleased to present for discussion four papers prepared in response to its 1997 Ratemaking Call Paper Program. Topics addressed are implementing PH-transforms in ratemaking, personal auto cost drivers, pricing the earthquake exposure using modeling, and reflecting reinsurance costs in homeowners ratemaking. These papers will be discussed by the authors at the 1997 CAS Seminar on Ratemaking, March 13-14, in Boston, Massachusetts.

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Modelling Mortgage Insurance Claims Experience: A Case Study
(1996 CAS Hachemeister Prize Paper)

by Greg Taylor

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MODELLING MORTGAGE INSURANCE CLAIMS EXPERIENCE: A CASE STUDY

BY GREG TAYLOR

December 1991
Revised June 1993
A Coopers & Lybrand research report

ABSTRACT

Mortgage insurance indemnifies a mortage lender against loss on default by the borrower. The sequence of events leading to a claim under this type of insurance is relatively complex, depending not only on the credit worthiness of the borrower but also on a number of external economic factors.

Prominent among these external factors are the loan to valuation ratio of the insured loan, the disposable income of the borrower, and movements in property values. A broad theoretical model of the functional dependencies of claim frequency and average claim size on these variables is established in Sections 6 and 7. Section 8 fits these models, extended by other "internal" variables such as the geographic location of the mortgaged property, to a real data set.

Section 9 compares the fitted model with the data, and finds an acceptable fit despite extreme fluctuations in the claims experience recorded in the data set.

KEYWORDS

Mortgage insurance; housing price index; loan to valuation ratio; regression.

1. INTRODUCTION

Mortgage insurance indemnifies a mortgage lender against loss on default by the borrower. The typical sequence of events leading to the invocation of the indemnity is as follows.

The amount of the mortgage is repayable by a sequence of instalments, perhaps monthly, over a period of some years, up to perhaps 25 or in a few cases more. If a borrower fails to meet one or more of these instalments, arrears collection procedures will be instigated. If it appears that the borrower is experiencing financial difficulties which threaten his capacity to pay the scheduled instalments, the lender's initial response will usually be to attempt rehabilitation of the borrower, possibly by some form of rescheduling of the debt repayment.

In many cases this will render the borrower's difficulties temporary. In other less fortunate cases it will become clear that the borrower is quite unable to repay the debt. The lender will then force sale of the mortgaged property, and retain that part of the sale proceeds required to discharge the remaining debt. In the majority of sales, the proceeds will be sufficient for this purpose, but if they are not the mortgage insurance indemnity is invoked to reimburse the lender for the shortfall.

It is an elementary observation that inflation of property values reduces the call on mortgage insurance; the proceeds of property sales cover a greater proportion of the corresponding debts. It is also clear from the above description that a loan needs to go through several stages (healthy → in arrear → property under management → sale of property) before a mortgage insurance claim arises, and each of these stages involves some delay. As will be discussed in Section 3, each of them also depends on its own specific economic factors.

For these reasons, the underlying process generating mortgage insurance claims is complex and dependent on several variables which are exogenous to the insurance portfolio. Consequently, mortgage insurance run-off arrays, whether in terms of numbers or amounts of claims, exhibit very different characteristics from those of other lines of business. A striking example of this is given in Section 2.

These different characteristics necessitate rather different modelling techniques. The purpose of the present paper is to illustrate these techniques by means of a case study. Since this study is specific to a particular portfolio, it cannot be claimed that the modelling techniques illustrated are generally applicable. It is hoped, however, that they are fairly generally indicative of the **type** of modelling which needs to be attempted.

2. NUMERICAL EXAMPLE: PRELIMINARY DISCUSSION

The following data are given as an indication of the difficulties likely to arise if a mortgage insurance portfolio is subjected to conventional run-off analysis. More detail of the data on which this paper is based appears in Appendices E and G.

Year of loan advance	Number of claims, per 10,000 loan advances, emerging in development year (a)										
	0	1	2	3	4	5	6	7	8	9	10
1980					30	18	6	0	0	0	6
1981				116	42	31	5	0	0	0	
1982			54	27	45	36	13	13	4		
1983		25	20	20	23	9	0	3			
1984	0	13	24	55	35	5	0				
1985	1	21	134	68	15	6					
1986	0	17	30	4	2						
1987	3	1	0	2							
1988	0	0	5								
1989	0	0									
1990	0										

(a) **Development year** is defined as year of emergence of claim minus year of loan advance.

Let the term relative claims frequency denote the number of claims per 10,000 loan advances. If C_{ij} denotes the relative claim frequency in development year j of year of advance i, and A_{ij} denotes the age-to-age factor:

$$A_{ij} = \sum_{k=0}^{j+1} C_{ik} \Big/ \sum_{k=0}^{j} C_{ik}, \tag{2.1}$$

then the following table of age-to-age factors is obtained.

Year of loan advance i	Age-to-Age factor in development year j =				
	1	2	3	4	5
1984	2.86	2.50	1.38	1.04	1.00
1985	7.12	1.44	1.07	1.03	
1986	2.71	1.08	1.05		
1987	1.00	1.50			

The great instability in these ago-to-age factors is evident in the sense of variability within a development year. The basic reason for the instability is clear from the first table. It is the apparent correlation between relative claim frequency and year of emergence of claim, i.e. with the number of the diagonal in the table. Such a data structure suggests application of the separation method (TAYLOR, 1977, 1986), with the model structure:

$$E[C_{ij}] = r_j \lambda_{i+j}. \tag{2.2}$$

The separation method yields the following parameter estimates.

j	$\hat{r}_j$	k	$\hat{\lambda}_k$
0	0.00		
1	0.06		
2	0.20		
3	0.22		
4	0.14	1984	366
5	0.11	1985	167
6	0.03	1986	195
7	0.03	1987	350
8	0.02	1988	196
9	0.00	1989	48
10	0.20	1990	29

This produces the following comparison between observed and fitted relative claim frequencies.

Year of loan advance	Observed and fitted (shown in bold type) relative claim frequency in development year																							
	0		1		2		3		4		5		6		7		8		9		10		Total	
1980									30	**52**	18	**18**	6	**6**	0	**9**	0	**3**	0	**0**	6	**6**	60	**94**
1981							116	**79**	42	**24**	31	**21**	5	**11**	0	**5**	0	**1**	0	**0**			195	**140**
1982					54	**72**	27	**36**	45	**28**	36	**38**	13	**6**	13	**1**	4	**0**					193	**181**
1983			25	**21**	20	**33**	20	**42**	23	**50**	9	**21**	0	**1**	3	**1**							101	**169**
1984	0	**1**	13	**9**	24	**38**	55	**76**	35	**28**	5	**5**	0	**1**									131	**159**
1985	1	**1**	21	**11**	134	**69**	68	**42**	15	**7**	6	**3**											245	**133**
1986	0	**1**	17	**20**	30	**38**	4	**10**	2	**4**													53	**73**
1987	3	**1**	1	**11**	0	**9**	2	**6**															6	**28**
1988	0	**1**	0	**3**	5	**6**																	5	**9**
1989	0	**0**	0	**2**																			0	**2**
1990	0	**0**																					0	**0**

The table indicates that the separation method achieves a reasonable fit. No formal goodness-of-fit statistics are examined, because this model is later discarded. The difficulty is that, despite the reasonableness of the fit, the sequence of **escalation index numbers** λ_k is peculiar by normal standards. Until some explanation of this peculiarity is found, it is impossible to produce any reliable projection of the sequence into future years.

One of the major objectives of subsequent sections of this paper will therefore be to obtain such an explanation. The discussion of this aspect of the modelling problem is taken up in Section 3.

3. THE PROCESS OF CLAIM OCCURRENCE

3.1. Major financial factors

As pointed out in Section 1, a loan must traverse several stages of financial deterioration before producing a mortgage insurance claim. These stages are subject to different financial influences. Of these separate influences, two are of particular prominence:

(a) the onset of financial difficulties for the borrower; and
(b) in the event of forced sale, the extent to which the sale proceeds repay the outstanding loan.

These two factors are discussed in the following two sub-sections.

3.2. Onset of borrower's financial difficulties

Despite its importance in a borrower's budget, the mortgage payment instalment will nevertheless be to some extent a residual item in that budget. It will rank after tax and consumer expenditure on necessities (food, clothing, etc.). In addition, most past loans have been of a type whereby the amount of instalment varies with variations in current day interest rates.

It appears, therefore, that a reasonable measure of the degree of financial pressure on mortgage borrowers would be provided by an estimate of the average residual income after allowance for tax, consumer expenditure and mortgage instalment. This residual income, called here the **home affordability index** (HAI), was constructed in the following form:

Home affordability index = average weekly gross household income

minus

tax

minus

consumer expenditure

minus

mortgage instalment,

expressed as a percentage of gross income.

A baseline distribution of gross household income over these categories of expenditure was derived from a 1988/89 household expenditure survey (HES) conducted by the Australian Bureau of Statistics. The items of expenditure for this base year were adjusted to other years in various ways, indicated by the following table.

Item of income or expenditure	Adjustment from year to year according to
Gross household income	Average weekly earnings
Tax	Average weekly earnings (a)
Consumer expenditure	Consumer price index
Mortgage instalments	Average weekly earnings (b) Mortgage interest rates (b)

(a) Preliminary investigation indicated little variation in the effective average tax rate over the period concerned.

(b) The average amount of a new loan was assumed to change in proportion with average weekly earnings. These loans were assumed repayable over periods of 20 years, and the average mortgage instalment calculated on the basis of the most common interest rate charged in the year concerned in respect of the loan portfolio under analysis.

The component time series used in the construction of the HAI (at year end) are set out as Appendix F.

The resulting HAI (at mid-year) is as set out in the following table.

The rather irregular progression of this index is seen in Appendix F to derive from quite reasonable component indexes. Each of these components may be projected over future years, producing a rationally based projection of HAI. This situation may be contrasted with that which arises on application of "black box" estimates of past claims escalation, as in Section 2, and in which no guidance as to future escalation is available.

Year	Home affordability index
1979	100.0
1980	104.8
1981	111.9
1982	101.7
1983	104.1
1984	128.9
1985	128.3
1986	101.7
1987	87.4
1988	90.6
1989	81.5
1990	81.2

3.3. Recovery of outstanding loan on forced sale

The HAI of Section 3.2 provides an indication of the likelihood that an individual borrower will experience financial difficulty in a particular year. However, such difficulty, while a necessary condition, **is not** sufficient for the emergence of a mortgage insurance claim. It is quite possible the borrower's difficulties are such as to force sale of the property, but that property values will be sufficient for the entirety of the outstanding loan amount to be recovered by the lender.

Whether or not this is the case will depend mainly on movements in property values between the date of advance of the loan and the date of the forced sale. In Sydney these movements may be estimated by reference to the **Housing Price Index** (HPI) computed and published by Residex Pty Limited. The following table was derived from that index with slight modification.

Year ended 30 June	Housing price index (Sydney) at mid-year (30/6/79 = 100)
1980	115.3
1981	145.1
1982	158.6
1983	158.4
1984	168.2
1985	177.2
1986	182.4
1987	191.5
1988	245.8
1989	363.5
1990	430.7

Evidently, the greater the increase in value of properties generally, the less the chance that forced sale of a particular property will lead to a loss to the mortgage lender.

3.4. Lags in claims process

While movements in the HAI (Section 3.2) and HPI (Section 3.3) have been identified as major variables in the frequency of mortgage insurance claims, it is to be expected that there will be a lag between cause and effect in each case.

Information from the company operating the mortgage insurance portfolio discussed in this paper was that, broadly:

(a) the average period between mortgage instalments falling in arrears and the property being taken under management (if indeed this latter occurred) was about 6 months; and
(b) the average period between taking a property under management and effecting its sale was also about 6 months.

On the basis of this information, it might be reasonable to expect lags of:

(a) 12 months between movements in the HAI and the consequent movement in claim frequency; and
(b) 6 months between a movement in the HPI and its consequent movement in claim frequency.

Thus, it has been assumed in subsequent modelling that a claim frequency experienced during year t is dependent upon:

(a) the value of the home affordability index at the **middle** of year $t-1$; and
(b) the value of the HPI at the **end** of year $t-1$.

Examination of alternatives suggested that this choice of lags provided about the best fit of model to data. Further detail on the incorporation of the HAI and HPI in the model is given in Section 6.2.

4. DATA

4.1. Variables affecting claims experience

Section 3 identified the HAI and HPI as likely to be major explanatory variables of **claim frequency**. Other variables in this category include:

(a) the proportion of the original property value advanced by way of mortgage, i.e. the **loan to valuation ratio** (LVR);
(b) the geographic area of the mortgaged property (described in more detail in Section 4.2);
(c) the agreed term of the mortgage loan;

(d) the type of property mortgaged (e.g. new house, old unit, land only, etc.);
(e) the financial type of the loan (e.g. reducible loan with variable interest, interest only instalments with fixed interest rate, etc.).

In addition, it is likely that claims experience will vary with **development year**, even in the absence of movements in the HAI and HPI. This would reflect a process of natural selection operating on each year's mortgage advances, whereby the poorest risks succumb to financial pressures relatively early, and the remainder survive the mortgage term.

It is clear that the major variable affecting **claim size** will be the **size of the original loan**. In addition, the explanatory variables (a) to (e) of claim frequency potentially affect claim size also.

4.2. Form of data

As the tables of Section 2 indicate, claims experience relates to the period 1984 to 1990. In fact, the 1984 experience covers only 7 months of that year.

Data supplied in respect of these claims consisted of a claim by claim tabulation, recording in each case the relevant variables identified in Section 4.1:

(a) year of advance;
(b) amount of loan;
(c) value of property;
(d) geographic area of property;
(e) term of loan;
(f) type of property;
(g) financial type of loan;
(h) year of emergence of claim.

The tabulated geographic area was the postal code of the property. These codes were grouped into 14 broad urban and rural regions within the states of New South Wales and Australian Capital Territory, as follows:

> Metropolitan regions 1 to 5; Canberra (6); Newcastle (7); Wollongong (8); Central Coast (9); North Coast (10); South Coast (11); Blue Mountains (12); Southern Highlands (13); Other (14).

The exposure base for the study consisted of all loans advanced over the years 1980 to 1990 inclusive. These were recorded, loan by loan, according to the variables (a) to (g) listed above as potentially affecting claim frequency.

As the data described above constitute a unit record file, it is not practical to present the full detail here. It is not even practical to tabulate cells of data since there are 1499 exposure cells. However, Appendix G gives a tabulation of exposures and claims according to year of advance and development year. It is to be stressed that, while such a tabulation is interesting, it omits a great deal of the raw data.

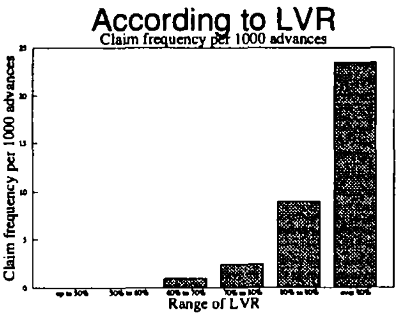
5. EXPLORATORY DATA ANALYSIS

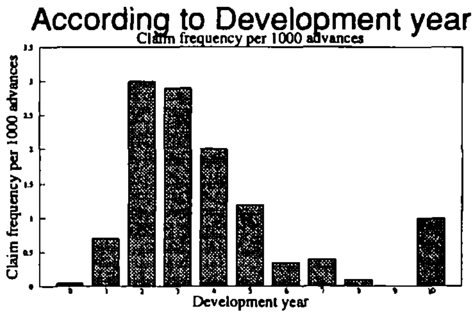
5.1. Claim frequency

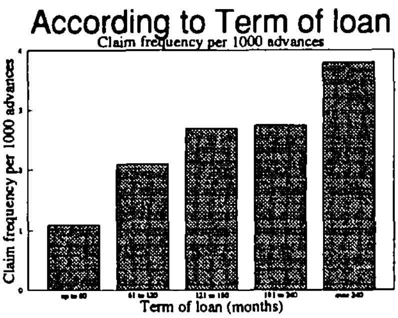
Section 4.1 identified a number of characteristics of individual loans (such as LVR, term of loan, etc.) which might have a bearing on the likelihood of those loans leading to claims. These characteristics will be referred to here as **risk variables**.

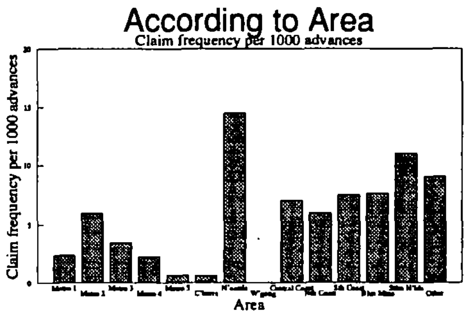
Initially, data concerning claim numbers were analysed according to the risk variables listed in Section 4.1. This provided initial guidance concerning the types of loans which were subject to high or low risk of default.

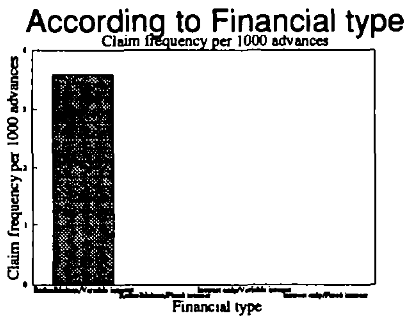
The results of this analysis are summarized in the following sequence of bar charts.

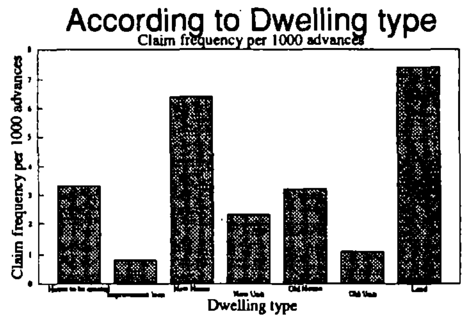












These charts raise the following possibilities:

(a) claim frequency peaks in the second, third and fourth years after the year of advance;
(b) claim frequency increases dramatically with increasing loan to valuation ratio (LVR);
(c) claim frequency increases significantly with increasing term of loan;
(d) certain geographic areas experience conspicuously higher or lower claim frequencies than average;
(e) defaults appear to be confined totally to reducible loans carrying a variable interest rate;
(f) claim frequency appears highest in relation to land, higher in relation to new properties than old, and lowest in relation to improvement loans.

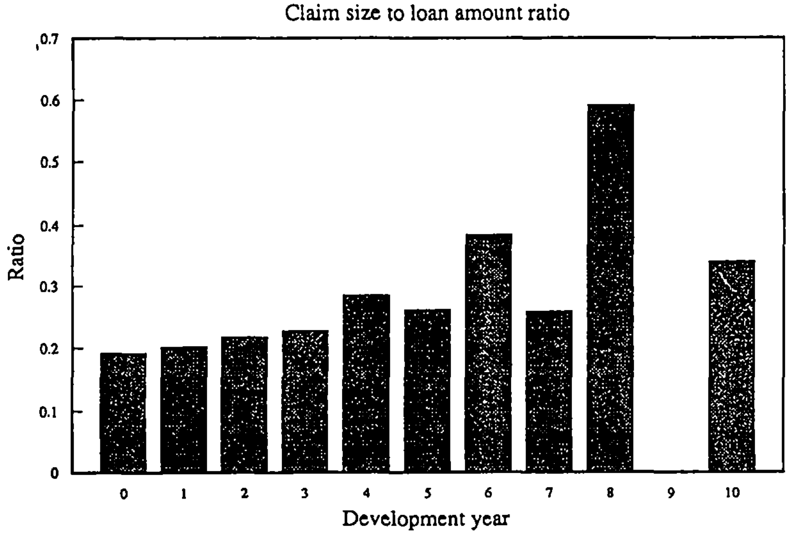
As stated, these are raised as **possibilities** only, rather than conclusions. Without further analysis, it would be impossible to determine whether all of these variables affect the default risk directly, or some of them are merely correlated with the genuinely operative risk variables.

For example, it might be the case that term of loan has no bearing on default risk, but appears to be relevant because LVR does have such a bearing and long terms are associated with high LVRs.

The question of possible correlation between risk variables is remarked upon further in Section 8.1.

5.2. Claim size

Initially, data concerning claim sizes were analysed according to the risk varibles listed in Section 4.1. This provided initial guidance concerning the



types of loans which led to larger or smaller losses when default occurred. The detailed results of this analysis are set out in Appendix D. The results indicate little variation in claim size with any of the risk variables except development year. The variation of claim size with development year is graphed in the preceding chart.

The chart suggests that the greater the time elapsed between advance of loan and default, the greater the **claim size to loan amount ratio**, i.e. the greater the loss on default expressed as a proportion of the original advance. This result is confirmed by formal regression analysis, as described in Section 8.2.

Since growth in property value generally increases with development year, this chart is consistent with the predicted form (7.2) of model.

6. FORM OF CLAIM FREQUENCY MODEL

6.1. General

In the following the basic units of tabulation of claims data will be referred to as **cells**. A cell will consist of an item of data associated with a particular combination of year of advance, development year, and any sub-set of the risk variables identified in Section 4.1.

It is reasonable that the total effect of risk variables on claim frequency should be **multiplicative**, i.e.

(6.1) expected relative claim frequency = function (development year, HAI, HPI)
×
function (risk variables, e.g. LVR, geographic area, etc.).

The form of the first of the two functions on the right will be discussed in Section 6.2. As far as the second function is concerned, a reasonable first approximation would consist of the product of a factor in respect of each of the risk variables present. Equation (6.1) then becomes:

(6.2) expected relative claim frequency = function (development year, HAI, HPI)
×
factor dependent on LVR
×
factor dependent on geographic area
×
etc.

Interactions between the factors making up this product could be added if necessary.

Expected relative claim frequency (per loan advanced) is adjusted by a factor of 7/12 in all cells whose experience relates to 1984. This allows for the fact that the data include only 7 months' claims (Section 4.2).

Some of the risk variables identified in Section 4.1, e.g. financial type of loan, are categorical by nature. Others, e.g. LVR, are continuous by nature. It was convenient for exploratory analysis of the data to convert all variables (i.e. risk variables, not HAI and HPI) to categorical form. Details appear in Section 5.1. The categorical form of data was retained in the final modelling, described in Section 8.1.

6.2. Dependence on development year and economic variables

Preliminary analysis (Section 5.1) indicated that relative claim frequency, expressed as a function of development year, was generally consistent with the shape of a **Hoerl curve**. Appendix B provides a theoretical underpinning of this observation. Consequently, the model adopted for relative claim frequency in the absence of any other effects took the form:

$$\text{const.} \times (j + \tfrac{1}{2})^{\alpha} \exp(-cj), \tag{6.3}$$

where j represents development year.

The modification of (6.3) by HAI and HPI raises some questions. Consider HAI first.

As noted in Section 3.2, the HAI may be regarded as a measure of the average borrower's residual income after payment of mortgage instalment. An individual borrower will experience difficulties in payment of mortgage instalment if this residual income turns negative. The frequency with which this occurs in the event of movements of HAI will depend on the distribution of individual residual incomes, rather than just the average of this distribution represented by HAI. There is virtually no information available in respect of this distribution.

There is, however, some evidence that individual gross incomes are subject to a Paretian distribution (MANDELBROT, 1960).

If a similar assumption is made about residual incomes after payment of mortgage instalment (i.e. HAI), then Appendix A demonstrates that, to first approximation, logged claim frequency will contain a term linear in $R(i+j)/R(i)$, where i denotes year of advance, j development year, and $R(t)$ the HAI experienced in year t. Allowance for the one year lag in the effect of HAI, as discussed in Section 3.4, modifies this term to $R(i+j-1)/R(i)$ (1 in the case $j=0$).

Because of the approximations leading to this result in Appendix A, an alternative linear term involving

$$\log[R(i+j-1)/R(i)] \qquad \text{for} \qquad j \geq 1;$$

or

$$0, \qquad \text{for} \qquad j = 0, \tag{6.4}$$

was tried. This latter form produced a slightly better fitting regression than the unlogged ratio, and has been adopted henceforth. In fact, both alternatives produced quite similar results.

Appendix B, particularly (B.10), demonstrates that, under seemingly reasonable assumptions about the accumulation of the amount of mortgage debt on default, and about property values on resale, claim frequency should also contain the following factor involving LVR and HPI:

$$L^{\nu}[H(i+j)/H(i)]^{-\nu}, \qquad \nu \text{ const.} > 0,$$

where L denotes LVR and $H(t)$ the HPI experienced in year t. In order to accommodate the lag in the effect of HPI discussed in Section 3.4, this last expression should be modified to the following:

$$L^{\nu}[H(i+j-\tfrac{1}{2})/H(i)]^{-\nu}, \qquad j \geq 1;$$

or

$$L^{\nu}, \qquad j=0, \tag{6.5}$$

where $H(t-\tfrac{1}{2})$ is interpreted a the HPI experienced at the **end** of year $t-1$.

Note that (6.5) indicates that claim frequency should include the **same power** of both LVR and HPI. However, this result was derived in Appendix B on the assumption that LVR affected the proportion of principal outstanding at default, but not the risk of default itself. In practice, it is likely that LVR is correlated with the ability of the borrower to meet financial commitments, in which case it intrinsically affects the risk of default. For this reason, (6.5) should be generalized to the following:

$$L^{\lambda}[H(i+j-\tfrac{1}{2})/H(i)]^{-\nu}, \qquad j \geq 1;$$

or

$$L^{\lambda}, \qquad j=0. \tag{6.6}$$

Combination of (6.2) to (6.4) and (6.6) yields the following model:

(6.7) expected relative claim frequency in development year j of year advance i

$$\begin{aligned} &= \text{const.} \times (j+\tfrac{1}{2})^{\alpha} \exp(-cj) \\ &\quad \times L^{\lambda}[R(i+j-1)/R(i)]^{-\rho}\, [H(i+j-\tfrac{1}{2})/H(i)]^{-\nu} \\ &\quad \times \text{factor dependent on geographic area} \\ &\quad \times \text{etc. for } j \geq 1, \end{aligned}$$

and with the two square bracketed terms removed in the case $j=0$.

Let $\mu(i,j)$ denote the expected relative claim frequency (6.7), and $E(i)$ the number of loans advanced in year i. Let $N(i,j)$ denote the number of claims emerging in development year j of year of advance i. Then the claim frequency model adopted was:

$$N(i,j) \sim \text{Poisson}\,[E(i)\,\mu(i,j)]. \tag{6.8}$$

It should be noted that the precise form of dependency of relative claim frequency on LVR and HPI in (6.7) relies upon distributional assumptions made in Appendix B. If these assumptions were varied, the form of (6.7) would change. Consequently, an alternative to (6.7) is considered in Section 8.1, in which the terms involving LVR and HAI are replaced by:

$$\exp(\lambda L) \; \exp\left[-\nu\, H(i+j-\tfrac{1}{2})/H(i)\right].$$

This alternative model turns out to be inferior to (6.7).

7. FORM OF AVERAGE CLAIM SIZE MODEL

Appendix C shows that, on the same seemingly reasonable assumptions as in Appendix B (referred to in relation to the development of (6.5)), the average claim size in respect of loans advanced in year i should progress over development years according to the following parametric form:

$$E[Q(i,j)] = \text{const.} \times H(i+j)/H(i), \tag{7.1}$$

where

$Q(i,j)$ = the claim ratio (i.e. ratio of claim size to original loan size) experienced in development year j of year of advance i;

$H(t)$ = HPI experienced during year t.

One may note the interesting effect whereby average claim size **increases** with development year even though outstanding principal is decreasing. Clearly this result derives from the assumptions made in Appendices B and C. Different assumptions would lead to a different parametric form in (7.1). However, an examination of the development of Appendix C indicates that the property of increasing $E[Q(i,j)]$ with $H(i+j)$ derives only from an assumption that the variable γ has a **decreasing failure rate**, where $\gamma = \alpha/\beta$ and

α = a random variable representing the factor by which outstanding principal has been enlarged after default by arrears of principal and interest and any other costs,

β = a random variable representing the factor by which the property value has been reduced by the forced nature of the sale and the associated expenses.

While there is no particular evidence concerning the failure rate of γ, it is interesting to note that the seemingly reasonable assumption of a Pareto distribution leads to the result (7.1) which is found in Section 8.2 to accord with experience, at least to the extent that the claim ratio trends upward with increasing property factor. However, because the Pareto assumption may be a little too specific, it is reasonable to widen the model (7.1) to the following:

$$Q(i,j) = a + b\, H(i+j)/H(i) + \text{error term}, \tag{7.2}$$

where approximately

$$\text{error term} \sim N(0, \sigma^2)\,. \tag{7.3}$$

The appropriateness of this error term is discussed further in Section 8.2.

8. FITTING THE MODEL

8.1. Claim frequency

By (6.7) and (6.8),

$$\begin{aligned} \log E[N(i,j)] = {} & \log E(i) + \text{const.} + \alpha \log (j + \tfrac{1}{2}) - cj \\ & + \lambda \log L - p \log [R(i+j-1)/R(i)] \\ & - \nu \log [H(i+j-\tfrac{1}{2})/H(i)] \\ & + \text{term dependent on geographic area} \\ & + \text{etc.},\ j \geq 1\,, \end{aligned} \tag{8.1}$$

with the two square bracketed terms on the right omitted for the case $j = 0$. This linear form, subject to the error structure (6.8), was fitted to the data using GLIM (Generalised Linear Interactive Modelling) (Royal Statistical Society, 1987). Various combinations of the potential explanatory variables listed in Section 4.1 were tried, and the main results are reported in the next table but one.

Geographic area		
Original coding (a)	First aggregation	Second aggregation
1, 4	Area 1	AREA 1
3	Area 3	
5	Area 4	
6	Area 5	
2	Area 2	AREA 2
7, 10-12	Area 6	AREA 3
9, 14	Area 7	
13	Area 9	
8	Area 8	AREA 4

(a) As set out in Section 4.2.

The results of the trial regressions are displayed in the following table.

Variable	Coefficient in variable at left (a) in Regression No.						
	1	2	3	4	5	6	7
Regression constant	−9.505	−12.18	−10.50	−9.848	−12.90	−5.776	−5.943
Development year	−1.093	− 1.143	− 1.218	−1.097	− 1.096	−1.119	−0.8536
Log (development year + ½)	4.908	5.066	4.558	4.906	4.903	5.076	4.505
LVR (d)	1.100	1.144	0.994	1.100	1.099		
Log (LVR)						8.93	8.413
Log (home affordability factor) (b)							−2.158
Property growth factor (c)	−3.039	− 3.070	− 2.036	−3.017	− 3.015		
Log (property growth factor)						−4.636	−5.658
Indicator variables (f):							
AREA 2				0.52	0.52	0.53	0.5131
AREA 3				0.87	0.87	0.87	0.8772
AREA 4				−5.24	− 5.24	−5.25	−7.254*
Area 2	0.60						
Area 3	0.16*						
Area 4	−0.35*						
Area 5	−0.26*						
Area 6	1.05						
Area 7	1.15						
Area 8	−5.33*						
Area 9	0.81						
60 ≤ Term < 120 months		3.74*					
120 ≤ Term < 180 months		2.95*					
180 ≤ Term < 240 months		2.00*					
240 ≤ Term		2.74*			3.06*		
Dwelling:							
Improvements & increases			1.33*				
All other than improvements, increases & land only			0.64*				
Dwelling type missing			7.05*				
Deviance (e)	854	549	632	611	610	593	527

(a) Dependent variable in regression log (claim frequency), as in (8.1).
An asterisk attached to a coefficient in the table indicates that this coefficient differs from zero by less than 2 standard errors.

(b) The home affordability factor is the ratio of values of HAI appearing in (8.1).

(c) The property growth factor is the ratio of values of HPI appearing in (8.1).

(d) The variable referred to here is in fact

$$10 \times \text{LVR} - 3.5.$$

The variable log (LVR) uses the genuine LVR, though grouped in ranges of 10 percentage points width. Each such range is represented by its mid-value.

(e) Deviance is a measure of goodness of fit, related to the log likelihood ratio of the model. A lower deviance implies a better fit.

(f) The variables Area k and AREA m have already been described as 0-1 indicator variables. The variables listed subsequently in the table are also of the 0-1 indicator type, taking the value 1 if the loan is subject to the risk variable displayed, 0 otherwise.

By (6.8) and (8.1), the model is multivariate Poisson with multiplicative structure of the mean. GLIM fits this by maximum likelihood. Note that the logarithmic form of (8.1) is no more than a convenience of expression. It could equally have been written in its unlogged (multiplicative) form. In particular, (8.1) does **not** imply that the observations $N(i, j)$ (many of which are zero) are to logged.

For the interpretation of this table, special reference should be made to geographic area of the mortgaged property. On the strength of the chart of Section 5.1, a number of areas, seemingly similar in claim frequency and/or physically contiguous, were aggregated. The areas at this initial level of aggregation were denoted by "Area k". These were 0-1 variables, taking the value 1 if the property lay in the relevant area, 0 otherwise.

Regression 1 in the table indicated that further aggregation was possible. The new variables resulting from this aggregation were denoted by "AREA m", and were 0-1 variables, each of which consisted of the sum of the relevant variables Area k. The key to the two aggregations is as shown in the previous table but one.

It may be noted that the trial regressions included alternative versions of (8.1) in which the terms dependent on LVR and HPI were replaced by their respective unlogged forms, as discussed at the end of Section 6.2. These alternatives were, however, inferior to (8.1) in terms of fit.

Regression 7 provided the best fit of model to data, and was adopted as the final model. This final model, expressed in non-symbolic form, was as follows:

$$\text{(8.2)} \quad \begin{array}{ll} \text{CLAIM FREQUENCY} = & 2.624\,(t+\tfrac{1}{2})^{4.505}\exp(-0.8536\,t) \\ \text{(per 1000 advances)} & \times \\ \text{IN DEVELOPMENT YEAR } t & (\text{LVR})^{8.413} \\ & \div \\ & [(\text{HOME AFFORDABILITY FACTOR})^{2.158} \\ & \times \\ & (\text{PROPERTY GROWTH FACTOR})^{5.658}] \\ & \times \\ & \begin{cases} 1 \text{ if AREA 1} \\ 1.670 \text{ if AREA 2} \\ 2.404 \text{ if AREA 3} \\ 0.0007 \text{ if AREA 4} \end{cases} \end{array}$$

where

HOME AFFORDABILITY FACTOR and PROPERTY GROWTH FACTOR are the ratios involving H and R respectively in (8.1).

The formula in the box indicates that claim frequency:

(a) moves sharply upward with increasing LVR;

(b) moves sharply downward as property values or disposable incomes after mortgage instalments increase;
(c) varies significantly by geographic area, exhibiting a particularly low value in the Wollongong district.

Because of correlations of the type discussed at the end of Section 5.1, not all of the risk variables exhibited a significant effect on claim frequency.

8.2. Average claim size

The form of the model was suggested in Section 7 as the following:

$$Q(i,j) = a + b\, H(i+j)/H(i) + \text{error term}, \tag{7.2}$$

where approximately

$$\text{error term} \sim N(0, \sigma^2). \tag{7.3}$$

This model appears unnatural to the extent that the normal error term would permit claim sizes to be negative. This would be avoided by the inclusion of an error term which was by nature positive. An example would be a lognormal error term, as would be incorporated in an alternative model of the form:

$$\log Q(i,j) = \log a + b \log [H(i+j)/H(i)] + \text{error term}, \tag{8.3}$$

where

$$\text{error term} \sim N(0, \sigma^2). \tag{8.4}$$

Equivalently,

$$Q(i,j) = a[H(i+j)/H(i)]^b \times \text{error term}, \tag{8.5}$$

where

$$\text{error term} = \text{lognormal}\,(0, \sigma^2). \tag{8.6}$$

Note that both forms (7.2) and (8.5) accommodate the theoretical form (7.1).

Ordinary regression produced the following two alternative models.

Parameter	Unlogged model (a)	Logged model (b)
a	0.1622	0.1555
b	0.0494	0.3083
σ^2	0.0257	0.8676

(a) This is the model described by (7.2) and (7.3). Of the 425 observed claim ratios, 2 large values have been excluded as outliers.
(b) This is the model described by (8.3) and (8.4).

In fact, neither of the two models considered in the preceding table produced an ideal fit to the data. Their respective residuals are tabulated in the following table.

Values of standardized residuals	Relative frequency of standardized residual in	
	Unlogged model	Logged model
	%	%
less than −3	0	1
−3 to −2	0	3
−2 to −1	12	8
−1 to 0	47	32
0 to 1	24	44
1 to 2	10	12
2 to 3	5	0
more than 3	1	0
Total	100	100

These two tabulations of standardized residuals are very much reflections of each other about the origin. While the unlogged model is somewhat skewed to the right, the logged model is about equally skewed to the left. This suggests that the correct model lies somewhere between normal and log normal. Such a model might be of the form (7.2), but with the error term strictly positive and skewed to the right but less so than log normal.

Note that the fitted values of claim ratios, according to the two alternative models, are:

(8.7)
$$EQ(i,j) = a + bH(i+j)/H(i) \text{ for unlogged model;}$$

(8.8)
$$= a[H(i+j)/H(i)]^b \exp(\tfrac{1}{2}\sigma^2) \text{ for logged model.}$$

In the event, (8.8) produced a rather heavy upward bias, about 18% in total, in fitted values of claim amount relative to observed amounts. The form of this comparison was exactly as reported in Section 9.2, but with the unlogged model used there replaced by the logged.

This result appears to indicate that the exponential scaling factor in (8.8) is not robust against the non-normality in the error term of (8.4), as was demonstrated in the above table of standarized residuals.

On the other hand, Section 9.2 indicates that the unlogged model provides an adequate fit, and accordingly it was adopted.

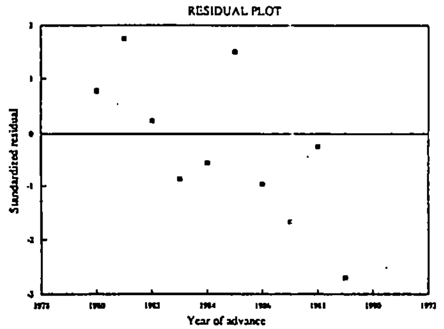
9. MODEL VERIFICATION

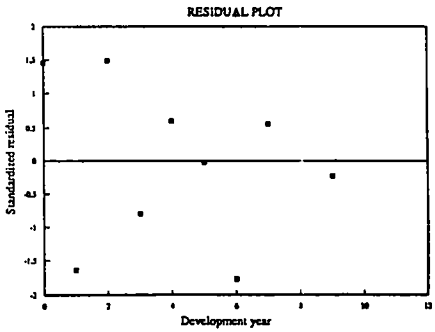
9.1. Claim frequency

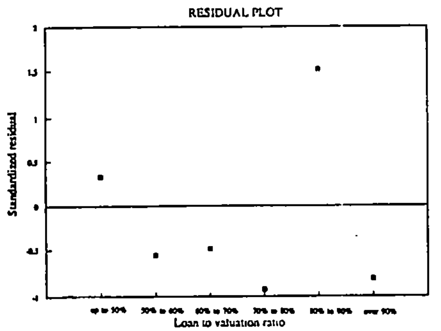
The model adopted in Section 8.1 has been used to compute standardized residuals according to several variables. The resulting residual plots appear below. Note that each residual relates to the aggregation of all experience at the value of the independent variable displayed. For example, the first residual in the first plot may be obtained from the second table of the present sub-section as:

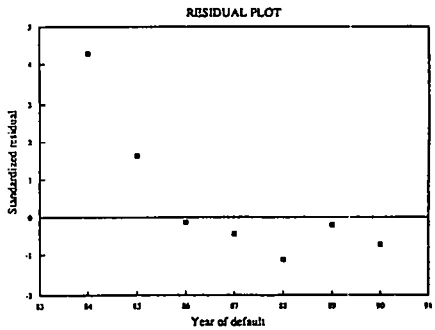
$$(8-6)/\sqrt{6} = 0.8\,.$$

A plot of the residuals of all cells (taken over all explanatory variables) would not be helpful since the great majority of cells contain very small expectations.









These plots appear generally satisfactory in terms of magnitude, with the exception of year of default 1984. This one anomaly, in the relatively distant past, involves relatively few claims (see first table below) and is insufficient to invalidate the model.

The plot against year of advance contains a downward trend. If included in the model, year of advance appears as a highly significant explanatory variable; other things equal, claim frequency declines by 29 % as between each year of advance and the next. Naturally, the effects of the other explanatory variables, particularly those which are time dependent, change.

While this model provides a superior fit to the data, the abstract nature of the year of advance effect is problematic. It might be interpreted as a factor representing improvement in underwriting. However, in this case, the total improvement over the decade of underwriting would be almost 97%, which might strain credulity.

It seems more likely that year of advance is acting as a proxy for some other unidentified explanatory variable(s). When this variable is omitted from the model, its effect is largely captured by the other explanatory variables.

Moreover, an examination of the fitted numbers of claims (using the model which omits year of advance effect) against the data suggests that the apparent trend in the residuals may not be particularly meaningful (see second table below).

The following table displays the actual and model numbers of claims underlying the above plot of standardized residuals by experience year.

Period	Number of claims emerging	
	Actual	Model
1984 (7 months)	28	13
1985	32	24
1986	.53	54
1987	168	174
1988	103	115
1989	21	22
1990	20	24
Total	425	425

The table illustrates the close agreement between actual and model numbers of claims for all experience years except 1984, despite the extreme fluctuations in numbers of claims.

More detailed information is given by the following table which tabulates experience and model simultaneously by year of advance and development year, and from which the above table may be derived.

Year of loan advance	Observed and fitted (shown in bold type) number of claims in development year											
	0	1	2	3	4	5	6	7	8	9	10	Total
1980					3 **1.8**	3 **1.5**	1 **1.2**	0 **1.2**	0 **0.3**	0 **0.0**	1 **0.0**	8 **6**
1981				13 **4.5**	8 **4.8**	6 **4.4**	1 **4.9**	0 **1.4**	0 **0.1**	0 **0.0**		28 **20**
1982			7 **4.9**	6 **7.6**	10 **8.7**	8 **11.4**	3 **3.5**	3 **0.3**	1 **0.1**			38 **37**
1983		5 **1.6**	7 **5.3**	7 **8.8**	8 **14.7**	3 **5.2**	0 **0.5**	1 **0.2**				31 **36**
1984	0 **0.1**	7 **4.3**	13 **15.5**	30 **37.7**	19 **16.8**	3 **1.8**	0 **0.8**					72 **77**
1985	1 **0.3**	16 **16.2**	104 **86.6**	53 **56.7**	12 **7.6**	5 **3.8**						191 **171**
1986	0 **0.2**	14 **17.1**	24 **24.6**	3 **4.8**	2 **3.1**							43 **50**
1987	3 **0.3**	1 **6.2**	0 **2.7**	2 **2.5**								6 **12**
1988	0 **0.4**	0 **2.7**	8 **5.6**									8 **9**
1989	0 **0.2**	0 **7.1**										0 **7**
1990	0 **0.3**											0 **0**

The following table presents these results in the same format as in Section 2, enabling comparison of the present set of results with those from the separation method.

Year of loan advance	Observed and fitted (shown in bold type) relative claim frequency in development year											
	0	1	2	3	4	5	6	7	8	9	10	Total
1980					30 **18**	18 **9**	6 **7**	0 **7**	0 **2**	0 **0**	6 **0**	60 **43**
1981				116 **41**	42 **25**	31 **23**	5 **26**	0 **7**	0 **1**	0 **0**		195 **122**
1982			54 **38**	27 **34**	45 **39**	36 **51**	13 **16**	13 **1**	4 **0**			193 **179**
1983		25 **8**	20 **16**	20 **26**	23 **43**	9 **15**	0 **1**	3 **1**				101 **109**
1984	0 **0**	13 **8**	24 **28**	55 **69**	35 **31**	5 **3**	0 **1**					131 **140**
1985	1 **0**	21 **21**	134 **111**	68 **73**	15 **10**	6 **5**						245 **220**
1986	0 **0**	17 **21**	30 **30**	4 **6**	2 **4**							53 **62**
1987	3 **0**	1 **6**	0 **3**	2 **3**								6 **12**
1988	0 **0**	0 **2**	5 **3**									5 **5**
1989	0 **0**	0 **6**										0 **6**
1990	0 **0**											0 **0**

9.2. Average claim ratio

For each claim in the experience, a fitted value of its claim ratio was calculated according to (8.7) using the values of a and b tabulated in Section 8.2. Each of these claim ratios was multiplied by the associated amount of its loan, to produce a fitted claim size.

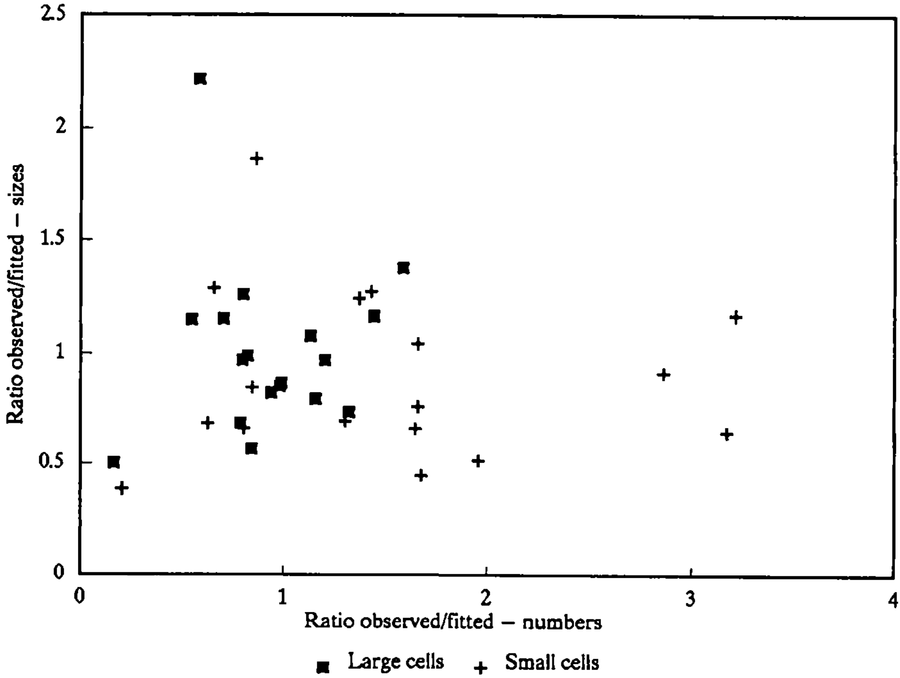
Observed and fitted claim sizes were then summarized in 2-way tabulations by year of advance and development year. These tabulations are displayed in Appendix E, and reduced to their corresponding 1-way tabulations below.

Year of advance	Amount of claims			Development year	Amount of claims		
	Observed	Fitted	Ratio: Observed / fitted		Observed	Fitted	Ratio: Observed / fitted
	$ 000	$ 000	%		$ 000	$ 000	%
1980	51	70	73	0	32	46	70
1981	294	312	94	1	425	471	90
1982	398	374	106	2	1750	1844	95
1983	354	323	110	3	1051	1133	93
1984	632	642	98	4	674	642	105
1985	1931	2063	94	5	321	301	107
1986	425	472	90	6	47	38	124
1987	46	69	67	7	31	35	88
1988	259	222	117	8	56	28	199
1989	0	0		9	0	0	
1990	0	0		10	1	7	14
Total	4388	4545	97		4388	4545	97

It should be particularly noted that the fitted amounts of claims, according to the above description are **conditional** upon the observed numbers of claims. This is a proper approach to examination of the fit of the average claim size model. Agreement between model and data appears satisfactory.

It is useful to carry out some check that the common dependence of the claim frequency and claim size models on the HPI does not lead to unwanted correlation between the two. That this does not in fact occur is indicated by the following scatter plot of the observed fitted ratios of average claim size against a similar ratio for number of claims.

Each point represents a particular combination of year of advance and development year. To give a simple indication of the significance of the plotted points, they are divided into "large cells" and "small cells". The former are those cells containing a fitted number of claims in excess of 5; otherwise the cell is "small".



9.3. Loan sizes associated with claims

While Section 9.2 models the claim size which will arise from a particular loan size **if a claim occurs**, it provides no indication of which loan sizes are likely to lead to claims.

There is no particular reason to believe that the sizes of loans associated with claims will be representative of the entire portfolio of loans advanced. Indeed, the table below indicates that, on average, it is the larger loans that lead to claims.

Care is needed here, however, as the model of claim frequency in Section 9.1 conditions on LVR and other risk factors, for which average loan sizes may differ from the portfolio average, and so without further analysis it is not clear to what extent the inclusion of these factors in the model will effectively select average loan sizes above the portfolio average. This question is also examined in the following table.

Year of advance	As a percentage of portfolio average loan size		
	average loan size associated with past claims (a)		average loan size weighted by model numbers of future claims (b)
	%		%
1980	135	(8)	96
1981	144	(28)	102
1982	119	(38)	101
1983	116	(31)	102
1984	85	(72)	102
1985	95	(191)	102
1986	144	(43)	103
1987	97	(6)	100
1988	241	(8)	98
Average	109 (c)	(425)	102 (d)

(a) The numbers of claims on which the ratios are based are shown in parenthesis. For each year of advance, the average size of loans associated with **recorded** claims has been calculated and related to the portfolio average (for that year of advance).

(b) For each combination of year of advance and risk variables, the average loan advanced and model claim frequency (according to the model of Section 8.1) are calculated. The average loan advanced, weighted by model claim frequency, is then calculated for each year of advance.

(c) Average of the entries in the column, weighted by numbers of claims shown in parenthesis.

(d) Unweighted average of the entries in the column.

The table suggests that the average loan size associated with claims of a particular cell for a particular year of advance is about 7% higher than the overall average loan size for the cell.

Thus, a forecast of future claim amount for a particular cell of development year j of year of advance i would be computed as:

$$\begin{aligned} &1.07 \times \text{average loan size in year of advance } i \\ &\times \hat{N}(i,j)\,\hat{Q}(i,j), \end{aligned}$$

where $\hat{N}(i,j)$, $\hat{Q}(i,j)$ are estimates of $N(i,j)$ and $Q(i,j)$ from Sections 9.1 and 9.2.

An alternative approach to the above would be to include loan size as an explanatory variable in the claim frequency model of Section 8.1. This might be awkward in practice, however, because it would increase very considerably the number of data cells entering into the regressions of Section 8.1.

10. CONCLUSION

Section 8 fits models to the claim frequency and claim ratio in the mortgage insurance portfolio examined. Section 9 verifies that these models provide a reasonable fit to the data.

The models therefore can be, and indeed have been, used to estimate the liability for claims still to emerge in respect of past years of loan advance. In order to carry out this estimation, one needs to project future values of the HAI and HPI. This in turn requires projection of incomes, tax rates, mortgage interest rates and growth in property values. Projections such as these are, problems of substance in their own right, but are beyond the scope of the present paper.

11. ACKNOWLEDGEMENT

I should like to acknowledge the computing assistance provided by Mr A.J. Greenfield in the preparation of this paper.

APPENDIX A

DEPENDENCE OF CLAIM FREQUENCY ON HOME AFFORDABILITY INDEX

Let X denote the random variable representing the proportion of an individual's income required for tax, consumption and mortgage instalment. Assume this variable to be Pareto distributed, i.e. with p.d.f.:

(A.1) $$f(x) = kx^{-\alpha-1}, \; k \text{ const.}$$

The borrower will experience financial difficulties if $X \geq 1$, which occurs with probability:

(A.2) $$P[X \geq 1] = kx^{-\alpha}/\alpha|_{x=1}.$$

Now, suppose that X shifts by a factor of c to $X' = cX$. Then the probability (A.2) shifts to

(A.3) $$P[X' \geq 1] = P[X \geq 1/c] = kx^{-\alpha}/\alpha|_{x=1/c}.$$

Comparison of (A.2) and (A.3) shows that the probability (A.2) has shifted by a factor of c^{α}. Now note that the scale shift of X to cX must shift the mean of X by a factor of c:

(A.4) $$E[X'] = cE[X].$$

Let

$$Y = 1 - X,$$

and note that

$$E[Y] \ \alpha \ \text{HAI}. \tag{A.5}$$

Then the factor by which HAI changes when X changes to X' is:

$$\begin{aligned} R &= \{1-E[X']\}/\{1-E[X]\} \\ &= (1-c\mu)/(1-\mu), \end{aligned} \tag{A.6}$$

where

$$\mu = E[X].$$

Inversion of (A.6) yields:

$$c = [1-R(1-\mu)]/\mu. \tag{A.7}$$

Thus, the shift in HAI by a factor of R causes the frequency with which borrowers experience difficulties to shift by a factor of:

$$c^{\alpha} = \{[1-R(1-\mu)]/\mu\}^{\alpha}. \tag{A.8}$$

Now, it is convenient to analyse log (claim frequency), which will depend on log (frequency of borrower's difficulties), and (A.8) shows that this latter will depend on an additive term of:

$$\begin{aligned} \log c^{\alpha} &= \alpha \log \{[1-R(1-\mu)]/\mu\} \\ &\sim -\alpha R(1-\mu)+\text{const.}, \end{aligned}$$

for small values of $(1-\mu)R$.

Thus, to first approximation, the model of expected log (claim frequency) should include a linear term in R, the ratio by which HAI has changed since advance of the loan(s) in question.

APPENDIX B

DEPENDENCE OF CLAIM FREQUENCY ON HOUSING PRICE INDEX, LVR AND DEVELOPMENT YEAR

Consider a loan taken at time $t=0$. Let $V(t)$ be the value of the associated property at time t, and $P(t)$ the amount of principal then outstanding. Then

$$V(t) = V(0)[H(t)/H(0)], \tag{B.1}$$

$$P(t) = P(0)f(t), \tag{B.2}$$

where

$H(t)$ = HPI at time t;

$f(t)$ = proportion of principal still to be repaid at time t.

By (B.1) and (B.2),

$$P(t)/V(t) = Lf(t)\,H(0)/H(t), \tag{B.3}$$

where

(B.4) $$L = P(0)/V(0) = \text{loan to valuation ratio}.$$

Suppose that the borrower has encountered financial difficulties at some time $s < t$. At time t sale of the property is forced. At that point, the debt in respect of the loan will be $P(t)\,\alpha(t)$, where

$\alpha(t)$ = a random variable representing the factor by which outstanding principal has been enlarged by arrears of principal and interest and any other costs.

Similarly, the net proceeds of the sale of the property will be $V(t)\,\beta(t)$, where

$\beta(t)$ = a random variable representing the factor by which the property value has been reduced by the forced nature of the sale and the associated expenses.

Then the ratio of outstanding debt to sale proceeds is:

(B.5) $$X(t) = \gamma(t)\, P(t)/V(t),$$

where

(B.6) $$\gamma(t) = \alpha(t)/\beta(t).$$

By (B.3) and (B.5),

(B.7) $$X(t) = L[H(t)/H(0)]^{-1} f(t)\,\gamma(t).$$

A claim will occur if $X(t) > 1$, i.e. if

(B.8) $$\gamma(t) > [H(t)/H(0)]\,[Lf(t)]^{-1}.$$

Now suppose that $\gamma(t)$ is Pareto distributed with d.f.

(B.9) $$F(\gamma) = 1 - (\gamma/a)^{-\nu}, \qquad \gamma > a,$$

assumed independent of t. Then, by (B.8), the probability of occurrence of a claim is:

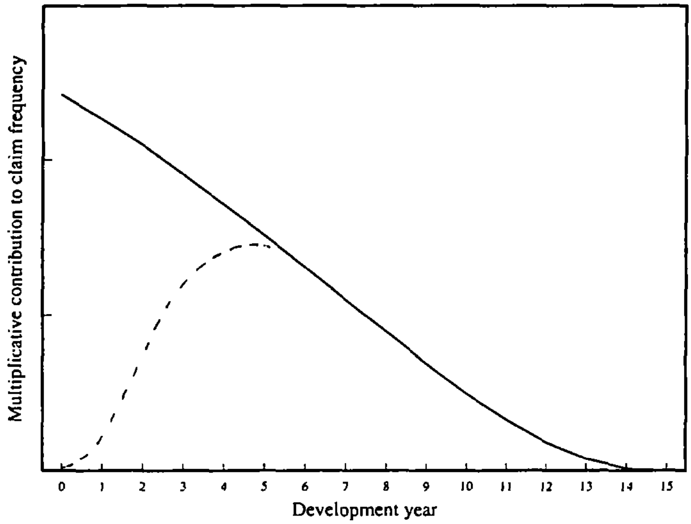
(B.10) $$P[X(t) > 1] = \{af(t)\, L[H(t)/H(0)]^{-1}\}^{\nu}.$$

Thus, expected claim frequency varies as a power of $L[H(t)/H(0)]^{-1}$. Note also that claim frequency for policies of a particular term n varies over development years t by a factor of

(B.11) $$[f(t)]^{\nu} \quad \alpha \quad [a_{\overline{n-t|}}]^{\nu},$$

which has the shape illustrated by the solid line in the following diagram.

However, note the above assumption that the distribution of the factor $\gamma(t)$ is independent of t. While perhaps largely true, it will break down as $t \to 0$ as the screening procedures of the lender force claim frequency toward zero. Hence, the curve (B.11) of frequency over development year will be modified for small t in the manner indicated by the broken line in the diagram.



When allowance is made for the variety of original terms n, the dependence of claim frequency on development year is seen to be represented by a weighted average of curves of the type illustrated in the diagram.

APPENDIX C

DEPENDENCE OF AVERAGE CLAIM SIZE ON HOUSING PRICE INDEX

As noted just prior to (B.8), the financial difficulties of a borrower will lead to a claim if $X(t)$, as defined there, exceeds 1. In fact, by the same argument as led to that result, the amount of the claim will be

$$\text{(C.1)} \qquad \begin{aligned} A(t) &= \alpha(t)\, P(t) - \beta(t)\, V(t) \\ &= \beta(t)\, V(t)\, [X(t) - 1]. \end{aligned}$$

Note that $\beta(t)$ and $\gamma(t)$ (and hence $X(t)$) will **not** be independent, even if $\alpha(t)$ and $\beta(t)$ are.. For general random variables Y and Z, let μ_Y and μ_Z denote their means, v_Y and v_Z their coefficients of variation, and ρ_{YZ} their correlation. It is straightforward to demonstrate that:

$$\text{(C.2)} \qquad E[YZ] = \mu_Y \mu_Z (1 + \rho_{YZ} v_Y v_Z).$$

By (C.1) and (C.2),

$$\text{(C.3)} \qquad E[A(t)] = V(t)\, E[X(t) - 1]_+\, \mu_\beta (1 + \rho_{\beta X} v_\beta v_X),$$

where $E[Y]_+$ denotes $E[Y | Y > 0]$.

Now, by (B.5)

$$\text{(C.4)} \qquad E[X(t) - 1]_+ = E[\gamma(t) - V(t)/P(t)]_+\, P(t)/V(t).$$

By the Pareto assumption (B.9),

$$E[y(t) - V(t)/P(t)]_+ = [V(t)/P(t)]\, v/(v-1), \tag{C.5}$$

whence (C.3) and (C.4) yield:

$$\begin{aligned} E[A(t)] &= V(t)\, \mu_\beta (1 + \rho_{\beta X} v_\beta v_X)\, v/(v-1) \\ &\alpha\ V(0)\, H(t)/H(0) \qquad \text{[by (B.1)]} \end{aligned} \tag{C.6}$$

if μ_β, v_β, v_X and $\rho_{\beta X}$ are the assumed independent of t.

Thus, the expected average claim size is directly proportional to property values, all other things equal. This has the interesting effect of causing average claim size in respect of a group of identical policies usually to **increase** with development year even though outstanding principal is decreasing.

APPENDIX D

EXPLORATORY ANALYSIS OF CLAIM SIZE

D1. Variation of claim ratio with loan to valuation ratio

Loan to valuation ratio	Number of claims	Claim to loan ratio		95% confidence limits (a)	
		Sample mean	Sample standard deviation	Lower	Upper
up to 50%	1	55.8%			
50 to 60%	1	56.9%			
60 to 70%	8	23.3%	13.7%	11.8%	34.8%
70 to 80%	36	23.9%	19.2%	17.4%	30.4%
80 to 90%	189	22.9%	18.4%	20.3%	25.6%
over 90%	191	23.5%	15.6%	21.3%	25.7%

(a) These are the symetric t-distribution confidence limits. Where the sample size is less than 2, the confidence limits do not exist.

D2. Variation of claim ratio with term

Term	Number of claims	Claim to loan ratio		95% confidence limits (a)	
		Sample mean	Sample standard deviation	Lower	Upper
months					
60 to 119	3	36.4%	14.1%	1.3%	71.4%
120 to 179	16	34.8%	29.8%	18.9%	50.7%
180 to 239	55	28.4%	20.2%	22.9%	33.9%
240 & more	352	22.0%	15.6%	20.4%	23.7%

(a) See Appendix D1.

D3. Variation of claim ratio with area

Area	Number of claims	Claim to loan ratio		95% confidence limits (a)	
		Sample mean	Sample standard deviation	Lower	Upper
M1, M4	29	16.5%	11.7%	12.0%	20.9%
M2	63	21.2%	15.0%	17.5%	25.0%
M3	77	16.5%	12.6%	13.7%	19.4%
M5	5	25.8%	14.8%	7.5%	44.1%
Canberra	4	23.1%	13.0%	2.4%	43.8%
Coastal	100	24.6%	18.2%	21.0%	28.2%
Newcastle	32	31.7%	17.2%	25.6%	37.9%
Wollongong	0				
Other	116	27.5%	19.4%	23.9%	31.1%

(a) See Appendix D1.

D4. Commentary

All pairs of confidence limits in Appendices D1 to D3 straddle the overall mean of 23.4% except in four cases. All four of these cases relate to area of residence, and are found in Appendix D3.

APPENDIX E

COMPARISON OF OBSERVED AND FITTED CLAIM AMOUNTS

The following are the amounts of claim **observed** in respect of each combination of year of advance and development year.

Year of advance	Amount of claims observed in development year										
	0	1	2	3	4	5	6	7	8	9	10
	$	$	$	$	$	$	$	$	$	$	$
1980					28522	13349	7873	0	0	0	1009
1981				115151	69711	105156	3724	0	0	0	
1982			71488	29799	102851	81026	35484	20827	56169		
1983		60085	71469	61801	85959	64416	0	10110			
1984	0	45337	68811	325411	180820	11766	0				
1985	9591	161743	1060021	474840	179612	44976					
1986	0	150351	219581	28174	26638						
1987	22882	7054	0	15810							
1988	0	0	258976								
1989	0	0									
1990	0										

The following are the amounts of claims **fitted** to each combination of year of advance and development year by the procedure described in Section 9.2.

Year of advance	Amount of claims fitted in development year										
	0	1	2	3	4	5	6	7	8	9	10
	$	$	$	$	$	$	$	$	$	$	$
1980					27287	25853	9332	0	0	0	7380
1981				125940	91833	84727	9687	0	0	0	
1982			56280	43406	129344	70032	19012	27658	28253		
1983		51324	96763	63585	74571	29094	0	7572			
1984	0	68421	121228	258339	167683	26301	0				
1985	14819	185929	1089849	576994	130423	64647					
1986	0	151670	258058	41149	20740						
1987	30697	13995	0	23866							
1988	0	0	221693								
1989	0	0									
1990	0										

Each cell in this table is of the form:

actual number of claims

×

fitted average claim size.

Hence comparison of the table with the previous one examines only variation of experience from model amounts of claim.

An alternative version of the preceding table consists of cells of the form:

fitted number of claims

×

fitted average claim size.

This table is as follows.

Year of advance	Amount of claims fitted in development year										
	0	1	2	3	4	5	6	7	8	9	10
	$	$	$	$	$	$	$	$	$	$	$
1980					16472	13202	11077	**0**	**0**	**0**	52
1981				44040	55444	61935	47805	**0**	**0**	**0**	
1982			39396	55278	111986	99883	22086	2637	2910		
1983		15962	73512	80326	136558	50459	**0**	1408			
1984	**0**	41551	144634	324560	148532	15693	**0**				
1985	4668	188718	907194	617384	82395	49662					
1986	**0**	185146	264079	66099	31805						
1987	3131	86881	**0**	29785							
1988	**0**	**0**	153966								
1989	**0**	**0**									
1990	**0**										

For cells in which where are no claims observed, the procedure of Section 9.2 does not produce a fitted average claim size. These cells, **indicated in bold**, have been assigned a fitted amount of claims equal to zero.

APPENDIX F

HOME AFFORDABILITY INDEX

Year (as at 31 December)	Economic indicators			Household expenditure					
	Average weekly earnings	Consumer price index	Mortgage interest rates (a)	Gross household income (b)	Tax (b)	Consumer expenditure (b)	Mortgage instalment (b)	Residual income: Amount	Residual income: As percentage of gross income
	$		p.a.	$ per week	$ per week	$ per week	$ per week	$ per week	
1978	224.35	82.4	11.50%	562.74	118.28	326.21	64.40	53.85	9.569%
1979	246.00	91.1	11.50%	617.05	129.70	360.65	70.61	56.08	9.089%
1980	278.25	100.0	12.00%	697.94	146.70	395.89	82.26	73.10	10.473%
1981	315.90	110.2	14.50%	792.38	166.55	436.27	107.18	82.39	10.397%
1982	346.70	123.4	15.50%	869.64	182.79	488.52	123.78	74.54	8.572%
1983	375.90	130.9	14.00%	942.88	198.19	518.22	124.22	102.26	10.846%
1984	405.40	136.0	13.50%	1016.88	213.74	538.41	130.41	134.33	13.210%
1985	428.20	147.5	15.00%	1074.07	225.76	583.93	149.07	115.30	10.735%
1986	450.85	161.4	15.50%	1130.88	237.70	638.96	160.96	93.25	8.246%
1987	477.70	173.7	14.50%	1198.23	251.86	687.66	162.07	96.64	8.066%
1988	521.65	187.7	14.25%	1308.47	275.03	743.08	174.68	115.68	8.841%
1989	560.75	203.0	17.25%	1406.55	295.64	803.65	217.77	89.48	6.362%
1990	600.68	213.0	15.50%	1506.69	316.69	843.24	214.46	132.30	8.781%

(a) The most common interest rates applying to loans in the mortgage insurance portfolio under analysis.

(b) These four columns were derived in a consistent manner from the HES, as described in Section 3.2.

APPENDIX G

DATA

The data described in Section 4.2 are summarized in the following table. This should be considered in conjunction with the qualification set out in the final paragraph of Section 4.2.

Year of advance	Number of loans advanced	Number of claims (a) recorded in development year										
		0	1	2	3	4	5	6	7	8	9	10
1980	1700					3	3	1	0	0	0	1
1981	1917				13	8	6	1	0	0	0	
1982	2231			7	6	10	8	3	3	1		
1983	3426		5	7	7	8	3	0	1			
1984	5496	0	7	13	30	19	3	0				
1985	7787	1	16	104	53	12	5					
1986	8077	0	14	24	3	2						
1987	9910	3	1	0	2							
1988	17646	0	0	8								
1989	11878	0	0									
1990	13614	0										

(a) Development year is defined as year of emergence of claim minus year of loan advance. Claims emerging in 1984 represent the experience of only 7 months.

The Parameter Variance Adjustment in Lognormal Linear Models for Loss Reserves: Bayesian versus Frequentist Analysis

by Frederick L. Klinker, FCAS

The Parameter Variance Adjustment
in Lognormal Linear Models for Loss Reserves:
Bayesian vs. Frequentist Analysis

by Fred Klinker

Abstract: In lognormal linear models for loss reserve estimation, losses are assumed to be lognormally distributed, where the expectations of the logarithms of losses are assumed linear in explanatory variables. A parameter variance term appears in the exponent of the estimator for expected losses. There is disagreement regarding the sign of this term. It will be argued in this note that the sign depends on whether one adopts a Bayesian or Frequentist viewpoint. Each sign is correct within the appropriate paradigm.

A number of actuarial papers have considered lognormal linear models for loss reserve estimation, among them Verrall [11], Verrall [12], Wright [14], and Zehnwirth [15]. This list is illustrative only and is far from exhaustive. In such models, losses (generally incremental, not cumulative) are assumed to be lognormally distributed, where the expectations of the logarithms of losses are assumed linear in explanatory variables. A parameter variance term appears in the exponent of the estimator for expected losses. There is disagreement regarding the sign of this term. The disagreement is implicit rather than explicit; none of the above referenced authors appears to acknowledge the different sign in other authors' works. However, Gary Venter, in his introduction to the papers on variability in reserves included in the Spring 1994 CAS Forum, specifically in his comments on Verrall [12], notes that "...adjusting the maximum likelihood estimator of the lognormal mean for bias involves some controversy, with different authors advising upward or downward or no adjustment." (Venter [10], page 97.) It will be argued in the rest of this note that the sign of the adjustment depends on whether one adopts a Bayesian or Frequentist point of view. Each sign is correct within the appropriate paradigm.

Aside from its discussion of an admittedly technical fine point which may not interest many actuaries, this note may also serve to remind readers of the fundamental distinctions between Bayesian and Frequentist paradigms and the fact that the two do not always yield the same result. This last reminder is useful, since the statistical model most actuaries are most familiar with, the normal linear model, yields the same result whether from a Frequentist viewpoint or from a Bayesian (with uninformative prior), although the interpretation of the result differs somewhat according to viewpoint. (Regression and ANOVA are common examples of linear models. The normal linear model in a Frequentist setting assumes normally distributed errors. In a Bayesian setting, normal priors and normal errors are assumed, resulting in normal posteriors.) This happy coincidence of Bayesian and Frequentist results is not preserved in many other models, including lognormal linear models.

In subsequent sections of this note, first the general Bayesian and Frequentist paradigms are discussed, then the estimators that follow from these paradigms. Lastly, the special case of lognormal linear models is introduced.

Paradigms: Bayesian vs. Frequentist

The general problem is as follows. The state of nature is described by the parameter μ. There is a quantity of interest, y, whose expectation, conditional on μ, is a function of μ, $E[y|\mu]=f(\mu)$. If the state of nature were known to be μ, both Bayesians and Frequentists would probably agree that a good estimator for y is $f(\mu)$. However, the state of nature is not known. Data, x, either a single observation or a collection of observations, is collected in an attempt to determine μ and y. But Bayesians and Frequentists proceed differently.

First, the Frequentist approach: The state of nature, μ, is considered to be fixed, although unknown. The Frequentist refuses to quantify uncertainty surrounding μ via a probability distribution on μ. On the other hand, the Frequentist considers not only the actual outcome of the experiment, x, but also other outcomes that might have been, but weren't. The possible outcomes are described by a probability distribution on x, conditional on the fixed but unknown μ. Expectations and variances of functions of x are calculated over x, conditional on μ. The focus is on finding unbiased estimators $\hat{\mu}$ and $\hat{f}$ such that $E[\hat{\mu}(x)|\mu]=\mu$ and $E[\hat{f}(\hat{\mu}(x))|\mu]=f(\mu)=E[y|\mu]$.

Consider next the Bayesian point of view. Uncertainty surrounding the state of nature, μ, is quantified via a prior probability distribution on μ. This prior can be Objective Bayes (an uninformative prior), Subjective Bayes (based on personal estimates of probabilities), or Empirical Bayes (based on previous data from similar problems). Data, x, is observed, and, based on this data and Bayes' Rule, a posterior distribution for μ follows. All inferences are conditioned on the observed data. There is no consideration given to other outcomes that might have come to pass but didn't. The focus is no longer on unbiased estimators. Unbiasedness is a Frequentist notion which requires taking expectations over actual and possible observed data, whereas the Bayesian does not consider the randomness of the data after the data has been observed and instead conditions on that observed data. Instead, the Bayesian desires an estimator which minimizes Bayes Risk across all states of nature still considered possible after observing x. Expectations and variances are calculated over μ via the posterior distribution for μ, conditioning on x. Adopting the standard loss function (quadratic), the minimum Bayes Risk estimator for y is its posterior expectation, $E[y|x]=E[E[y|\mu]|x]=E[f(\mu)|x]$.

To summarize the key distinctions between Bayesian and Frequentist, the Frequentist considers the data, x, to be a random variable, but not μ, which is considered fixed, although unknown. The Frequentist continues to worry, even after the data is observed, about observational outcomes that could have come to pass but didn't, and considers expectations and variances over x, conditional on μ. The Bayesian conditions all inferences on the observed data, x, and considers μ to be the random variable over which posterior expectations and variances are calculated. The Bayesian steadfastly refuses to be concerned about outcomes that could have come to pass but didn't. To clear up a common misconception, it is this conditioning on x which is the heart of the Bayesian paradigm, not the invocation of Bayes' Rule. Even some Frequentist methods invoke Bayes' Rule.

Before leaving this foundational section of this note, a few clarifying comments are in order.

1) In the above Frequentist discussion, I have focused on unbiased estimators. It should at least be noted that Frequentists do occasionally invoke considerations other than unbiasedness. However, it is certainly true that unbiasedness is one of the first characteristics that a new statistics student learns and one that is invoked often.

2) In the above Bayesian discussion, by mentioning Bayes Risk and loss functions, I have implicitly adopted a decision theoretic approach to Bayesian statistics. It should be noted that Bayesian theory and statistical decision theory are not synonymous. There are practicing Bayesians who are not decision theorists, at least not knowingly. And there are decision theorists who are not Bayesian, but rather quite decidedly Frequentist. On the other hand, of those discussions of Bayesian foundations with which I am familiar, all the best seem to adopt a decision theoretic viewpoint. Once one rejects the questionable Frequentist "objectivity", one seems driven naturally towards a decision theoretic viewpoint. Statistics appears to be less a method of discovering "truth" and more an aid to rational decision making. Any Bayesian can calculate the posterior expectation, $E[y|x]$. Only the Bayesian with a decision theoretic bent knows why this might be the appropriate quantity to calculate, because it minimizes posterior Bayes Risk under the most popular loss function, expected squared error.

The above has only scratched the surface. For those interested in more, Silvey [9] is a good introduction to Frequentist inference. There is a rapidly expanding literature on Bayesian foundations. Two good discussions are Berger [2] and Efron [6]. I particularly recommend the first of the two as an excellent discussion of Bayesian philosophy. (This is the source which first pounded into me the central role played by conditioning arguments.) The second of the two compares Bayesian and Frequentist paradigms. Although more applied, Gelman and others [8] and West and Harrison [13] also have interesting insights on Bayesian foundations.

Estimators: Bayesian vs. Frequentist

Suppose, first, that the function $f(\mu)$ of the previous section of this note is linear in μ. Then the following two operations commute: 1) taking expectations and 2) evaluating the function. In the Frequentist paradigm, $E[f(\hat{\mu}(x))|\mu]=f(E[\hat{\mu}(x)|\mu])=f(\mu)$ for $\hat{\mu}(x)$ an unbiased estimator of μ. In other words, $\hat{f}=f$ and $\hat{f}(\hat{\mu}(x))=f(\hat{\mu}(x))$ is an unbiased estimator for $E[y|\mu]=f(\mu)$. In the Bayesian paradigm, $E[y|x]=E[f(\mu)|x]=f(E[\mu|x])=f(\mu_x)$, where $\mu_x=E[\mu|x]$ is the posterior expectation of μ conditional on the observed x. Comparing the Bayesian and Frequentist estimators for y, they are of the same functional form as long as we identify the Bayesian μ_x with the Frequentist $\hat{\mu}(x)$. Why is the class of linear f so important? Because the normal linear model, already mentioned in the introduction, falls into this class.

Now assume that f is non-linear and take the Taylor series expansion to second order, about μ in the Frequentist case, and about μ_x in the Bayesian. It is not suggested that this calculation produces good estimators in all situations, but second order is the lowest order in which interesting phenomena arise, which are at least suggestive of the form of adjustments required for non-linear f. Considering first the Frequentist case,

$$E[f(\hat{\mu}(x)) \mid \mu] \approx E[f(\mu) + f'(\mu)(\hat{\mu}(x)-\mu) + \frac{1}{2}f''(\mu)(\hat{\mu}(x)-\mu)^2 \mid \mu]$$

$$= f(\mu) + \frac{1}{2}f''(\mu)\, Var[\hat{\mu}(x) \mid \mu]$$

(1)

where $\hat{\mu}(x)$ is an unbiased estimator for μ, and where the variance in the last line is the variance of the estimator $\hat{\mu}(x)$ conditional on μ. This equation suggests that $f(\hat{\mu}(x))$ would not in general be an unbiased estimator for $f(\mu)$ and, further, that the following might be <u>approximately</u> unbiased.

(2) $$\hat{f}(\hat{\mu}(x)) = f(\hat{\mu}(x)) - \frac{1}{2}f''(\hat{\mu}(x))\, Var[\hat{\mu}(x) \mid \mu]$$

The unknown μ in the second derivative of f has been replaced by its unbiased estimator. The variance would also have to be estimated somehow. The unbiasedness would presumably be only approximate for a couple of reasons. First, higher order terms in the Taylor series expansion have been ignored. Second, both the μ (in the second derivative) and the variance in the variance adjustment term of equation (2) must be estimated, hence this variance adjustment term is itself a random variable, not a constant. There is no guarantee that the expectation of this random variable will be exactly numerically equal to the variance adjustment term of equation (1), barring a very judicious choice of variance estimator.

The Bayesian calculation is similar to the Frequentist.

$$E[f(\mu)|x] \approx E[f(\mu_x) + f'(\mu_x)(\mu-\mu_x) + \frac{1}{2}f''(\mu_x)(\mu-\mu_x)^2|x]$$
$$(3) \qquad = f(\mu_x) + \frac{1}{2}f''(\mu_x)\, Var[\mu|x]$$

Equation (2) is the approximately unbiased estimator for y in the Frequentist case, equation (3) the approximate estimator for y in the Bayesian. Both have an adjustment for parameter variance. As before, upon identifying the Frequentist $\hat{\mu}(x)$ with the Bayesian μ_x, the functional forms would be identical, <u>except that the signs of the parameter variance terms are opposite</u>.

<u>The Lognormal Linear Model</u>

Consider first the lognormal distribution. A random variable z is said to be lognormally distributed with parameters μ and σ if and only if the natural log of z is normally distributed with expectation μ and standard deviation σ. μ and σ^2 are therefore the expectation and process variance in the log scale. Back in the original scale, the expectation of z, conditional on μ and σ, is $E[z|\mu,\sigma]=\exp(\mu+.5\sigma^2)$. For the actuarial reader unfamiliar with the lognormal distribution, past actuarial papers, such as Bickerstaff [3] and Finger [7], have made use of this distribution and include either a brief description or technical appendix on the lognormal. Those who desire considerably more detail on the lognormal distribution may consult Aitchison and Brown [1] or Crow and Shimizu [5].

Consider now the lognormal linear model. The data, x, and the quantity of interest, y, are assumed to be lognormally distributed, with expected logs that are linear in explanatory variables. The state of nature is characterized by the expectation of log(y), μ, and the process standard deviation of log(y), σ. μ will be linear in explanatory variables and their associated regression coefficients. The parameter variance of μ will depend on variances and covariances of the estimated regression coefficients via standard regression formulas involving the process variance and the structure matrix. In what follows, the process variance and the parameter variance will be assumed known. The fact that process and parameter variances must generally be estimated from the data is a technical complication which must be considered when designing exact estimators but which contributes nothing to the discussion at the present elementary level. So we will treat σ as a known rather than unknown descriptor of the state of nature and write $E[y|\mu]=\exp(\mu)\exp(.5\sigma^2)=f(\mu)$. (The additional problems introduced by unknown process and parameter variances, which must also be estimated, are treated in Verrall [11] and Verrall [12]. These two papers further reference Bradu and Mundlak [4], a highly educational paper in itself.)

The Frequentist now considers the problem to be one of estimating μ an '[y|μ]=f(μ) from observed data x using unbiased estimators. Given the assumption that logs are normally distributed and linear in explanatory variables, standard regression analysis on the logs yields an unbiased linear estimator for μ, call it $\hat{\mu}(x)$, and an expression for the parameter variance of this estimator, $Var[\hat{\mu}(x)|\mu]$, in terms of the process variance, assumed known, and the structure matrix of the regression. Applying equation (2), an approximately unbiased estimator for $f(\mu)$ is:

$$
\begin{aligned}
\hat{f}(\hat{\mu}(x)) &\approx f(\hat{\mu}(x))\left(1 - \frac{1}{2} Var[\hat{\mu}(x)\,|\mu]\right) \\
(4) \qquad &\approx f(\hat{\mu}(x))\exp\left(-\frac{1}{2} Var[\hat{\mu}(x)\,|\mu]\right) \\
&= \exp\left(\hat{\mu}(x) - \frac{1}{2} Var[\hat{\mu}(x)\,|\mu] + \frac{1}{2}\sigma^2\right)
\end{aligned}
$$

The first approximation follows from equation (2), because, given the present definition of $f(\mu)=\exp(\mu)\exp(.5\sigma^2)$, the second derivative of f with respect to μ is equal to $f(\mu)$ itself. The second approximation follows if the parameter variance is small (because $1-x \approx e^{-x}$ if x small), which is probably the regime in which equation (2) is valid anyway. (It should be noted that equation (4), derived under the above approximations, is an exact unbiased estimator for $f(\mu)$ if the variance terms are known, rather than estimated and the distribution of the data, x, is such that the estimator, $\hat{\mu}(x)$, is not only unbiased but normally distributed.) The second term in the exponent is the adjustment for parameter variance and appears with a negative sign.

Consider now the Bayesian estimator, $E[y|x]$. After observing the data x, μ has a posterior distribution with expectation μ_x and variance $Var[\mu|x]$. Applying equation (3),

$$
\begin{aligned}
E[y|x] &= E[f(\mu)|x] \\
&\approx f(\mu_x)\left(1 + \frac{1}{2}Var[\mu|x]\right) \\
&\approx f(\mu_x)\exp\left(+\frac{1}{2}Var[\mu|x]\right) \\
&= \exp\left(\mu_x + \frac{1}{2}Var[\mu|x] + \frac{1}{2}\sigma^2\right)
\end{aligned}
\tag{5}
$$

This holds to the same level of approximation as equation (4). (Actually, if the posterior distribution for μ is normal with expectation μ_x and variance Var[μ|x], then equation (5) follows exactly, without approximation, because then exp(μ) is itself lognormally distributed.) Again, the second term in the exponent is the adjustment for parameter variance, but in the Bayesian setting it appears with a positive sign.

Note that equations (4) and (5) have the same functional form, except that the signs on the parameter variance term are reversed. Why? The Frequentist recognizes that his unbiased estimator for μ, $\hat{\mu}(x)$, has finite, non-zero variance. Because of the convex shape of the exponential function, excursions of $\hat{\mu}(x)$ above μ result in excursions of $\exp(\hat{\mu}(x))$ above exp(μ) of greater magnitude than excursions of $\exp(\hat{\mu}(x))$ below exp(μ) due to excursions of $\hat{\mu}(x)$ below μ. As an estimator of exp(μ), $\exp(\hat{\mu}(x))$ is therefore biased upward, and the bias is greater the greater the variance of the estimator $\hat{\mu}(x)$, the larger the excursions of $\hat{\mu}(x)$ from μ. The $\exp(-.5Var[\hat{\mu}(x)|\mu])$ factor removes this bias (approximately).

The Bayesian, on the other hand, estimates $E[y|x]=E[f(\mu)|x]=E[\exp(\mu+.5\sigma^2)|x]$. Again, because of the convex shape of the exponential function, excursions of μ

above μ_x=E[μ|x] have a larger impact on exp(μ) than excursions of μ below μ_x. Upward excursions of μ are more dangerous than downward excursions because of their greater impact on exp(μ), and the Bayes estimator, being a minimum risk estimator, augments the naive estimator $\exp(\mu_x+.5\sigma^2)$ with the factor exp(+.5Var[μ|x]) to protect against the more dangerous upward excursions.

In closing this section of this note, what relation do the above results bear to those of other authors? I don't see an explicit parameter variance adjustment in Zehnwirth [15]. However, I know from the manual for his ICRFS loss reserving system and from private conversations with him that Zehnwirth is solidly in the Bayesian camp and advocates, or at least at one time advocated, the positive sign on the parameter variance adjustment. Verrall [12] actually appears to advocate both signs, depending on whether he is describing an unbiased Frequentist estimator or a Bayesian estimator, but he doesn't draw attention to the change in sign.

First, Verrall's equation (4.16) provides an unbiased Frequentist estimator. (Although he doesn't refer to this estimator as Frequentist, he notes its unbiasedness, which is a Frequentist notion. Furthermore, he invokes Bradu and Mundlak [4], which is a Frequentist paper.) To establish the connection between his notation and ours, note that $\underline{Z}$ is the vector of values of explanatory variables associated with our quantity of interest, y. $\underline{\beta}$ is the vector of regression coefficients associated with these explanatory variables, or rather the true but unknown values of these coefficients. $\underline{\hat{\beta}}$ is the vector of estimates of these regression coefficients derived from the regression. $\underline{Z}\underline{\hat{\beta}}$

and $Z\hat{\beta}$ are therefore inner products representing, respectively, our μ and our $\hat{\mu}$. From Verrall's equation (4.16), the unbiased estimator for $E[y|\mu]=\exp(\mu+.5\sigma^2)$ is

$$\exp(\hat{\mu})\, g_m\left[-\frac{1}{2}Z(X'X)^{-1}Z's^2 + \frac{1}{2}s^2\right] \tag{6}$$

where X is the regression structure matrix and s^2 is an unbiased estimator for σ^2. $g_m(t)$ is defined via power series expansion in Verrall's equation (4.5). It is clear from this definition that, as m becomes large, $g_m(t)$ tends to exp(t). m becomes large when the data base on which the regression is performed becomes large, without a corresponding increase in the number of explanatory variables. In this limit, the unbiased estimator for $E[y|\mu]$ of expression (6) above becomes

$$\exp\left(\hat{\mu} - \frac{1}{2}Z(X'X)^{-1}Z's^2 + \frac{1}{2}s^2\right) \tag{7}$$

From standard regression theory, the second term in the exponent is precisely -1/2 times the variance of the estimator $\hat{\mu}$. This estimator (7) therefore reproduces equation (4) above.

Lastly, Verrall provides, the middle of page 409, Bayesian estimators for posterior expected losses for lognormally distributed losses with parameters θ (our μ) and σ, where the posterior distribution of θ is normal with expectation m (our μ_x) and variance τ^2 (our $\mathrm{Var}[\mu|x]$). Verrall's estimator is

$$\exp\left(m + \frac{1}{2}\tau^2 + \frac{1}{2}\sigma^2\right) \tag{8}$$

which reproduces equation (5) above.

Concluding Remarks

So, which is the correct estimator in a lognormal linear model setting, equation (4) or equation (5)? Do we add or subtract a parameter variance adjustment? Each is correct, within its own paradigm, Bayesian or Frequentist. Unfortunately, for the lognormal linear model, unlike for the normal linear model, the result depends on the paradigm. It is up to the actuary to select the paradigm and, hence, the sign of the parameter variance adjustment. Unfortunately, there is no clear guidance as to which is appropriate for the loss reserving problem. Neither paradigm is without problems regarding its theoretical foundations, as Efron [6] is quick to point out.

A number of observations may be appropriate in closing, first some statistical ones, then some actuarial ones.

1) While calculating the Bayesian posterior mean, $E[y|x]$, it may be worthwhile to reflect on the fact that many Bayesians consider the greatest strength of the Bayesian paradigm to be its ability to produce readily interpretable posterior distributions and confidence intervals. (See, in particular, Gelman and others [8].) These Bayesians would consider someone who went to the trouble of constructing a Bayesian analysis only to extract posterior means and nothing else to have discarded most of the information revealed by their analysis. Yet, because of the very narrow focus of this note, I have ignored posterior variances, $Var[y|x]$, posterior predictive distributions for y, and posterior intervals resulting from those distributions.

2) A hard core Bayesian who wished to remain a Bayesian and yet was troubled by the above Bayesian/Frequentist discrepancy might be able to construct a valid Bayesian decision analysis that would reproduce the Frequentist unbiased result by considering loss functions other than quadratic, resulting in minimum Bayes Risk estimators other than the Bayesian posterior expectation, $E[y|x]$. I have not investigated what loss function might bring Bayesian and Frequentist analyses into agreement, but I might guess that such a loss function would appear quite ad hoc.

3) Both the unbiasedness of the Frequentist estimator and the minimum risk of the Bayesian estimator are predicated on the selected lognormal linear model being a reasonable approximation to reality. While we debate unbiasedness vs. minimum risk (tastes great vs. less filling), let us not forget that, if our model does not adequately approximate reality (incremental losses are not lognormally distributed, or expected logs are not linear in explanatory variables, or we have failed to include in the model important explanatory variables, etc), then, relative to a more adequate model, our Frequentist estimator is quite likely to be biased, and our Bayesian estimator is unlikely to be minimum risk.

Now, a few actuarial comments.

1) The Bayesian increases the indicated loss reserve for risk; the Frequentist reduces the indicated reserve to correct for presumed bias. The Bayesian indicated reserve is more conservative than the Frequentist. The Bayesian increase is, in effect, a kind of risk load. For those model parametrizations I have seen, the

greatest effect of parameter variance, percentagewise, tends to be out in the tail, at high development ages, because age tends to be selected as one of the explanatory variables and tends to be highly leveraged at high ages. Intuitively, out in the tail, at high development ages, is where an actuary would want the greatest risk load and conservativeness, because this is where the greatest uncertainty, percentagewise, lies.

2) The Frequentist loss reserver might believe the Bayesian indicated reserve to be redundant on average, because it fails to adjust for bias. Have you, or anyone you know, ever seen a truly redundant loss reserve (or Nessie, or Bigfoot)?

3) In the presence of controversy, with no clear indication as to how to resolve that controversy, perhaps we should employ the time-honored practice of practical actuaries everywhere: compromise. Ignore the parameter variance adjustment altogether. This produces indications intermediate between the bias adjusted Frequentist indication at the low end and the risk adjusted Bayesian indication at the high end.

My first preference would be for the Bayesian estimator because of its conservativeness, and because it is most conservative in the tail, where conservativeness is most appropriate. Upon failing to get my first preference, my second preference would be to ignore the parameter variance adjustment altogether. Why make any adjustment when we can't even agree on the sign of the adjustment? I would be very loathe to quote the Frequentist indication, to reflect the downward adjustment for bias, which is probably being mis-estimated anyway because our selected lognormal linear model, on which the indicated bias is based, is likely to be an oversimplification of reality.

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An Introduction to Credibility

by Curtis Gary Dean, FCAS

This paper is derived from the presentation on basic credibility
concepts that the author has given at the
1995 and 1996 CAS Seminars on Ratemaking.

AN INTRODUCTION TO CREDIBILITY

Credibility theory provides important tools to help the actuary deal with the randomness inherent in the data that he or she analyzes. Actuaries use past data to predict what can be expected in the future, but the data usually arises from a random process. In insurance, the loss process that generates claims is random. Both the number of claims and the size of individual claims can be expected to vary from one time period to another. If $1,500,000 in losses were paid by an insurer during the past year, one might estimate that $1,500,000 would likely be paid in the current year for the same group of policies. However, the expected accuracy of the estimate is a function of the variability in losses. Using credibility theory, the actuary estimates the randomness inherent in the data and then calculates a numeric weight to assign to the data.

Here is a dictionary definition of credible:

<u>credible</u>: Offering reasonable grounds for being believed

The actuary wants to know how much to believe the data that's being analyzed. To use the data to predict the future, this "belief in the data" must be quantified so that calculations can be made. This leads us to actuarial credibility:

<u>actuarial credibility</u>: the weight to be given to data relative to the weight to be given to other data

If we cannot fully believe our data, we may call on other information or data to supplement the data at hand. The data at hand and the supplemental data are each given an appropriate numeric weight in calculating an estimate.

The variability in insurance loss data can be seen in Table 1 which shows the loss experience for a group of policies covering contractor's pickup trucks. The last column shows that the average loss per truck varies widely from one year to the next. Any one year is a poor predictor of subsequent years.

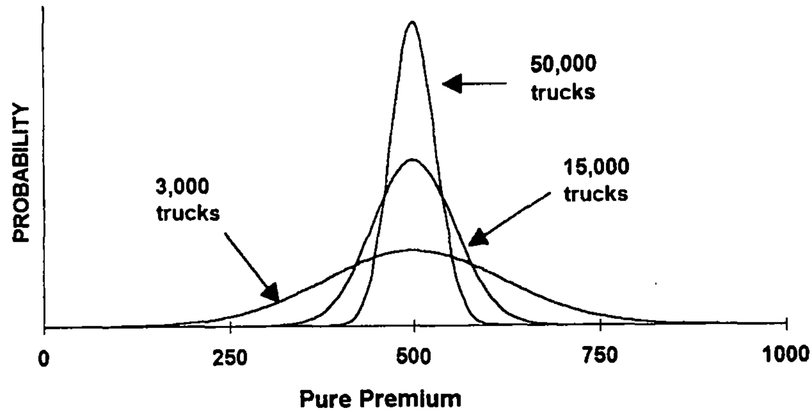
TABLE 1

Contractor's Pickup Trucks

	(1) # of Insured Trucks	(2) Incurred Losses	Pure Premium (2)/(1)
1990	2,900	$2,030,000	$700
1991	3,000	1,470,000	490
1992	3,050	1,830,000	600
1993	3,050	1,250,500	410
1994	3,200	864,000	270

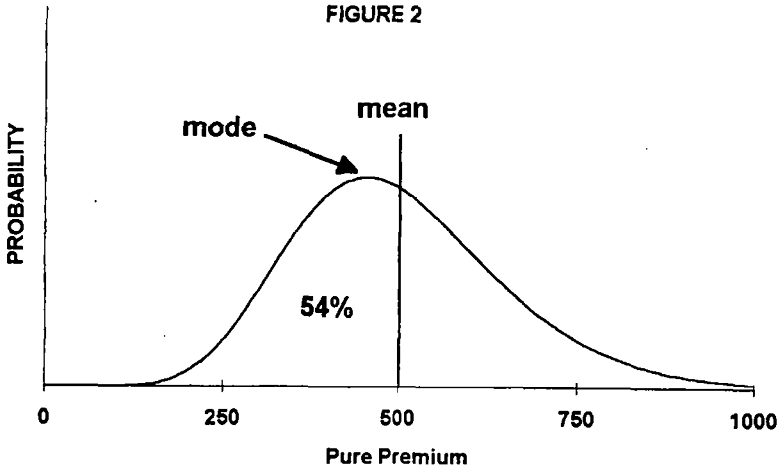
The variability in the average loss per pickup truck is depicted graphically in Figure 1. The expected average loss (pure premium) is $500 which we would observe if our body of data were infinite in size. But, for limited sample sizes, the observed average losses are randomly distributed. Note that as our sample size increases, the variability of the observed average loss decreases - the probability density curve becomes more concentrated around the $500 value. For a smaller sample size, the probability density curve flattens out. If our sample body of data consists of 50,000 trucks we can rely upon the observed average loss to estimate the true expected average loss to a much greater extent than if the data came from a smaller sample of only 3000 trucks.

FIGURE 1



The actual distribution of pure premiums is not symmetric as shown in the prior graph, but is instead skewed to the right as shown in Figure 2. More of the observations would actually fall below the mean of $500 and the mode of the distribution is less than $500. The smaller the body of data, the greater the asymmetry in the graph. In an extreme case we could consider only one truck. In most years the truck would have no losses for an observed average loss of $0 in those loss-free years. But, every few years there would be a loss or, perhaps, several losses and the observed average loss would be substantial.

FIGURE 2



This leads us to a common problem that may occur when a group of non-actuaries is reviewing average losses or loss ratios for a series of years. The data may show, for example, four years with excellent loss ratios but a fifth year with a very high loss ratio. The five-year average may be close to some target loss ratio. Unfortunately, what frequently happens is that one of the reviewers will say that the one bad year is an anomaly that was caused by several severe claims and that the bad year should be thrown out of the data. This is a big mistake! For a small body of data, this pattern in the loss ratios is exactly what we expect to see. The majority of the loss ratios will look better than average, with a few being quite large. This doesn't mean that we should ignore the few high values; it usually means that our body of data is small.

The basic formula for calculating credibility weighted estimates is:

$$\text{Estimate} = Z \times [\text{Observation}] + (1-Z) \times [\text{Other Information}],$$

$$\text{and } 0 \leq Z \leq 1.$$

If our body of data is so large that we can give full weight to it in making our estimate, then we would set Z=1. If the data is not fully credible, then Z would be a number somewhere between 0 and 1. What is the "Other Information" that we might use in our formula? That depends on what we are trying to estimate. In Table 2, the left hand column shows our observed data and the right hand column may be the "Other Information" that we might use in the above formula.

TABLE 2

Observation		**Other Information**
Pure premium for a class	←→	Pure Premium for all classes
Loss ratio for an individual risk	←→	Loss ratio for entire class
Indicated rate change for a territory	←→	Indicated rate change for entire state
Indicated rate change for entire state	←→	Trend in loss ratio

Suppose you are trying to estimate the indicated rate change for a territory within a state, but your company has a limited volume of business in the territory. An option may be to weight the indicated change from territorial data alone with the indicated change for the entire state. This way you have reflected territorial experience in your rate change to the extent that it is credible.

The loss ratios shown below in Table 3 were produced in a computer simulation that modeled the insurance random loss process. The expected loss ratio is 60 for both the small and big states, but the observed (simulated) loss ratios will randomly vary around this value. As we would expect, the variation is much larger for the small state. In the larger state the loss ratio hovers around 60 in each year. Five-year average loss ratios were calculated and then state indicated rate changes were calculated using the expected loss ratio of 60 as the permissible loss ratio. For example, in the small state -28.3% = (43/60 - 1.000). Using one of the formulas that we will discuss in a moment, credibility values Z were calculated for each state.

TABLE 3				
	Small State		**Large State**	
	Earned ($000)	**Loss Ratio**	**Earned ($000)**	**Loss Ratio**
1990	69	17	7,100	58
1991	71	109	7,120	58
1992	72	62	7,180	60
1993	74	7	7,200	58
1994	74	19	7,400	61
Total	360	43	36,000	59
Permissible Loss Ratio		60		60
State Indication		-28.3%		-1.7
Credibility		10%		100%

Perhaps this data comes from a line of insurance that has an aggressive insurance to value program such that the inflationary trend in losses is exactly offset by the annual increases in the amount of insurance. In this case the trend in our loss ratio would be 0%. (For our data, we know that the trend in the loss ratio is 0% because each year has an expected loss ratio of 60.) We will apply our complement of credibility factor (1-Z) to this information. So, we would get the following two indications:

small state: .10 X [-28.3%] + (1 - .10) X [0.0%] = -2.8%

large state: 1.00 X [-1.7%] + (1 - 1.00) X [0.0%] = -1.7%

In both cases we know the right answer! We should take a 0.0% rate change in each state because our expected loss ratios are what we used for the permissible loss ratios. But, *because of the randomness inherent in our data, our indications are slightly off the* mark.

The important thing in the prior example is that we greatly improved the accuracy of our rate indication in the small state by incorporating credibility. We gave only a 10% weight to the raw indication arising from the small state's loss ratio. This had the result of dampening the effect of the randomness. To the extent possible we would like to use our observed data to calculate our estimate rather than rely on supplementary data, but given the randomness present in our observations, we need to temper the data. Using credibility theory we weight an estimate based on limited data with data from other sources. We want to find a weight Z that allows us to rely on our limited data to the extent reasonable, but which also recognizes that our limited data is variable.

There are two widely used formulas for the credibility Z as shown side by side in Table 4. For the classical credibility formula, if n > N then Z is set equal to 1.00. In the case of Bühlmann credibility, Z asymptotically approaches 1.00 as n goes to infinity.

TABLE 4	
Classical Credibility	Bühlmann credibility
$Z = \sqrt{\frac{n}{N}}$	$Z = \frac{n}{n+K}$
Also called:	Also Called:
(1) Limited Fluctuation Credibility	(1) Least Squares Credibility (2) Empirical Bayesian Credibility (3) Bayesian Credibility

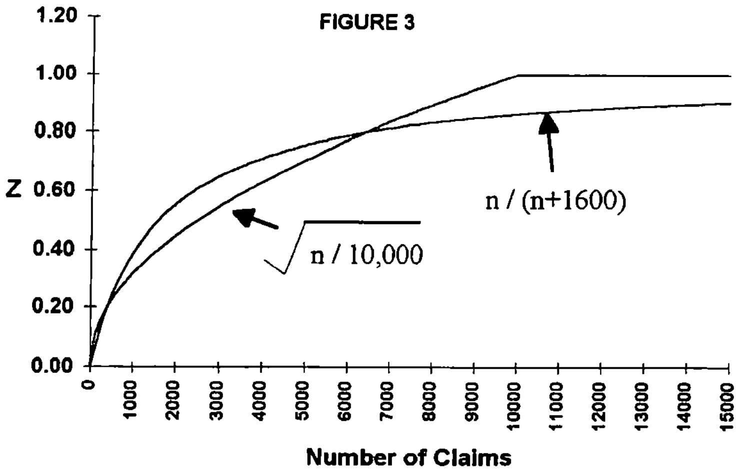
In both formulas n is a measure of the size of the body of data and is an indicator of the variability of the loss ratio or pure premium calculated from the data. n can be any of the following:

- number of claims
- amount of incurred losses
- number of policies
- earned premium
- number of insured unit-years.

These are not the only possibilities for n, but n needs to be some measure that grows directly with the size of the body of data that we have collected.

In practice both of the formulas can give about the same answer if N and K are chosen appropriately as displayed in Figure 3. Note that in the classical credibility case, when n is greater than or equal to 10,000, Z is identically 1.00.

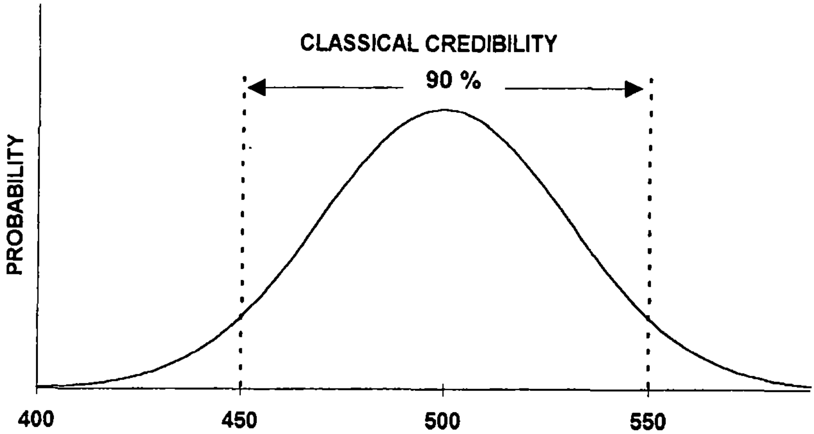
FIGURE 3



Classical Credibility

First we will discuss the classical credibility formula. Classical credibility attempts to restrict the fluctuation in the estimate to a certain range. N is calculated such that for fully credible data with n=N and Z=1.00, the observed pure premium or loss ratio will fall within a band about the expected value a specified percentage of the time. This is illustrated in Figure 4.

FIGURE 4



If N=5,200 claims, then the observed Pure Premium is within 10% of the "true" value 90% of the time.

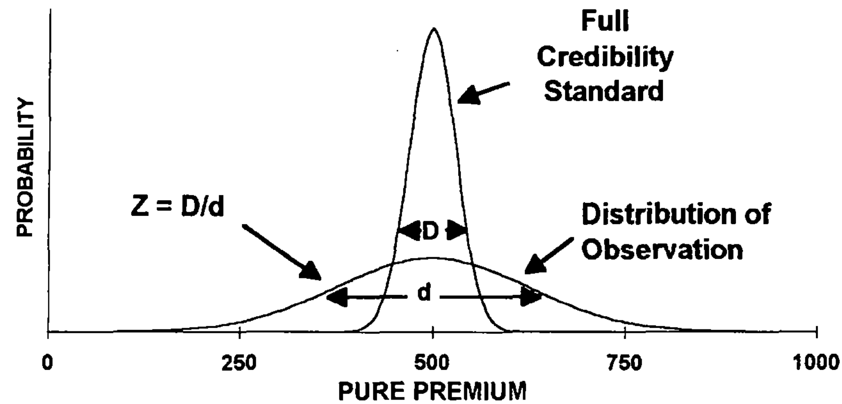
In this example the measure of the size of the body of data is the expected number of claims. When our body of data is large enough so that we expect 5,200 claims in our observation period, the observed pure premium will fall within k=10% of the true value P=90% of the time; that is, 90% of the time our pure premium calculated from our body of data will fall into the interval [450,550]. Both the 90% probability and the 10% width of the range must be selected by the ratemaker. If you wanted much less variance in your estimate you might select a P=99% probability and a k=2.5% error in your estimate. Of course, it would require a much larger body of data in the observation period to achieve this level of certainty.

The full credibility standard N is a function of the selected P and k values. A larger P value results in a larger N and a smaller k also produces a larger N. In order to calculate the N that corresponds to the selected P and k, one needs to make certain assumptions and also know something about the loss process. In classical credibility one assumes that the frequency of claims can be modeled by a Poisson distribution. Also, one needs an estimate of the average claim size and the variance in claim sizes. Using these an estimate of the variance in total losses can be computed. The next assumption is that the distribution of the total losses is normal, i.e. bell-shaped. Then, the N value can be calculated. This is all covered in much detail in the syllabus material for the actuarial exam that tests credibility theory.

One does not have to use the number of claims in the classical credibility formula, but instead can use earned premium, number of policies, or some other basis. We could convert our formula developed above to an earned premium basis. Suppose that in reviewing our data we calculate that on average there is approximately $2,500 in earned premium for each claim; that is, the ratio of earned premium to the number of claims is $2,500. A full credibility standard of (2,500 dollars/claim) x (5,200 claims) = $13,000,000 could be used in place of the 5,200 claims. Then, the credibility assigned to any data could be calculated from the earned premium of the data.

To calculate the full credibility standard, the denominator in the formula, the amount of variability acceptable in fully credible data must be defined by the selection of P and k values. For less than fully credible data the square-root formula determines the credibility Z. Figure 5 displays graphically the calculation of partial credibility.

FIGURE 5



In the graph the width of the curve representing the variability of data which just meets the standard for full credibility is represented by D. D can be considered the standard deviation of the curve. (If you prefer, D can be two standard deviations.) Likewise, d is the width corresponding to a smaller body of data that is less credible. It turns out that the credibility that should be assigned to the smaller body of data in this model is Z = D/d, the ratio of the standard deviation of the pure premium of the fully credible data to the standard deviation of the pure premium of the partially credible data. We will allow a standard deviation of size D, but if our body of data has a standard deviation of d, then we apply a weight of D/d to the data. If the pure premium (p.p.) calculated from the data is expected to have a standard deviation of d, then the quantity Z x (p.p) has a standard deviation of D, which is our target.

Bühlmann Credibility

The least-squares credibility model uses the credibility formula:

$$Z = n/(n + K)$$

K is defined by the following intimidating expression:

$$K = \frac{\textit{Expected Value of the Process Variance}}{\textit{Variance of the Hypothetical Means}}$$

A good way to think about least-squares credibility is in the context of experience rating where the rate charged to an insured is a manual rate modified to reflect the experience of the individual insured. The losses incurred by an insured are random, so an insured's loss ratio will fluctuate. The term "process variance" is the variance in the loss ratio of the risk. The "expected value of the process variance" is the average value of the variance across the risks within the population. Since each risk is unique, the expected loss ratios of the individual risks at the manual rates will vary across the population because the manual rates are based on averages calculated for groups of risks who are classified alike in the rating plan. Each risk has it's own "hypothetical mean" loss ratio. The "variance of the hypothetical means" is the variance across the population of risks of their individual hypothetical mean loss ratios.

In Figure 6 there are two risks, risk #1 and risk #2, each with its own loss ratio distribution curve. The process variance is a function of the width of the curve indicated by the [1] in the figure. As mentioned above the width of the curve can be thought of as some multiple of the standard deviation. The process variance is the square of the standard deviation. So the wider the curve, the larger the process variance. [2] marks the difference in the hypothetical means between the risks. The variance in the hypothetical means across the population is a function of the differences in the hypothetical means between the risks.

When the process variance of the risks is large in relation to the difference in the means of the risks, K is large. A large K means that the credibility $Z = n/(n + K)$ is small. Looking at the second graph in Figure 6, we see that there is a broad band where the two risks' loss ratios overlap. Since the loss ratio of each risk is so variable, it makes sense to give more weight to the manual rate calculated from the average experience of a large group of similar risks and less weight to the experience of the individual risk.

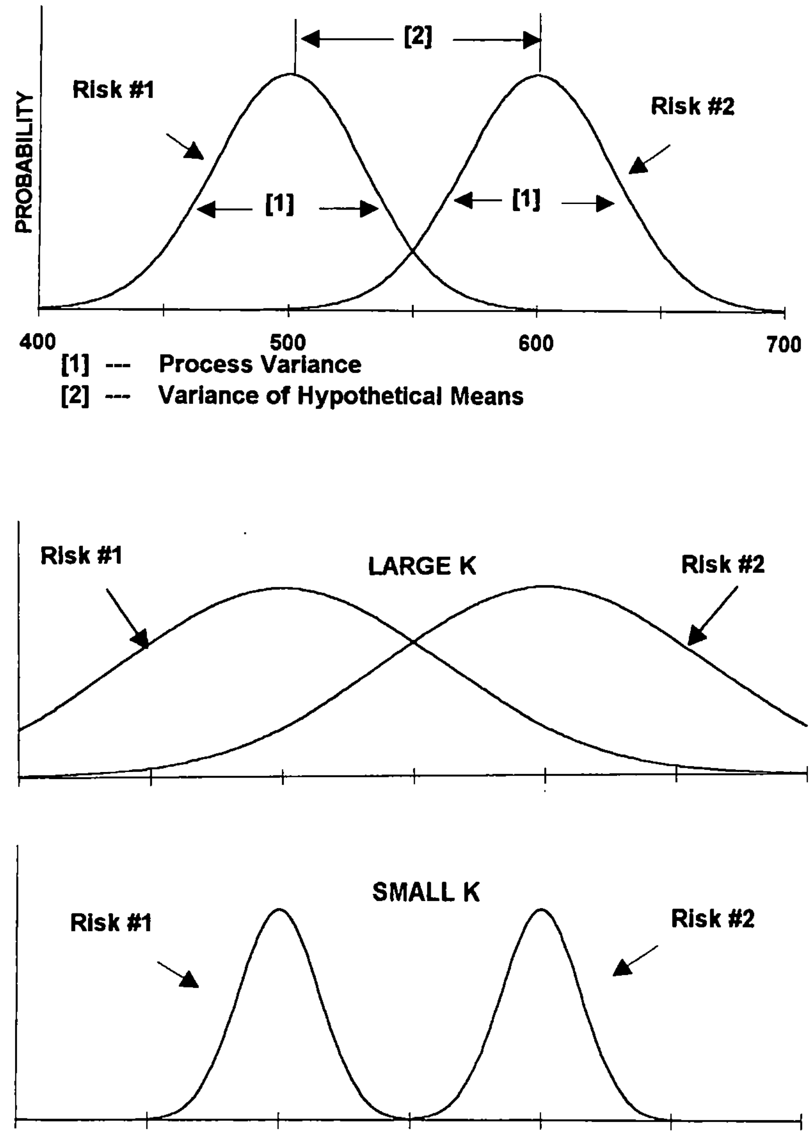
Small process variances in relation to the differences in the means of the risks results in a small K value and a larger credibility Z. This scenario is represented by the bottom graph in Figure 6. The distributions of the two risks do not overlap. The larger credibility Z means more weight is assigned to the experience of the individual risk and less, (1-Z), to the experience of the population.

Several Examples

Examples of credibility formulas developed by the Insurance Services Office are displayed in Table 5. The first set of formulas are used in Homeowners ratemaking and are based on the classical credibility model. The measure of the size of the body of data and its consequent variability is in the units of house-years; that is, one house insured for one year contributes one unit. In making a statewide change 240,000 house-years are required for full credibility, and with that large of a body of data, the observed experience should be within 5% of the actual value 90% of the time. In computing territorial changes within the state, 60,000 house-years are assigned full

FIGURE 6

CALCULATION OF K



credibility and the observed territorial experience is expected to be within 10% of the expected value of 90% of the time. As stated previously, the actuary needs to decide on the units for n, the size of the P value, and the size of the k value.

TABLE 5	
Credibility Formulas **Insurance Services Office**	
Homeowners: Owners Forms	
Statewide Changes	*Territorial Changes*
$Z = \sqrt{\frac{n}{240,000}}$	$Z = \sqrt{\frac{n}{60,000}}$
n = house years 90% confident within 5% of actual value	n = house years 90% confident within 10% of actual values
Manufacturers & Contractors	
Statewide Changes	*Relativities*
$Z = \sqrt{\frac{n}{8,000}}$	$Z = \sqrt{\frac{n}{25,000}}$
n = number of occurrences in three years 90% confident within 7% of actual value	n = number of occurrences in five years 95% confident within 5% of actual values
General Liability Experience Rating	
$Z = \frac{L}{L + \$177,000}$	
L = expected loss costs (including ALAE) at $100,000 basic limits	

The next set of formulas in Table 5 are used by ISO in Manufacturers & Contractors ratemaking. Statewide changes require 8,000 claims (occurrences) in a three-year period, and with this many expected claims, the experience of the body of data should be within 7% of the expected value 90% of the time. The full credibility standard for relativities within M&C, such as class relativities, is much tougher with 25,000 claims required for a P=95% and k=5%.

The selection of P and k is probably more art than science. If the body of data that the actuary is working with is of limited size and there is no good surrogate for the data to which to assign the complement of credibility, then the actuary may select a smaller P and larger k to produce a smaller requirement for full credibility. If the actuary wants to make the rates more responsive to current experience he or she may also select a smaller P and a larger k. If rate stability is the most important goal then larger P and smaller k may be selected.

The last formula in Table 5 is the credibility to be assigned to an individual insured's data in General Liability experience rating and it is based on the Bühlmann model. In a loss cost environment, L represents the expected loss costs (expected incurred losses and allocated loss adjustment expenses) for the individual risk. Before the advent of loss costs, premium designated by E was used instead of L. The expected loss costs included in L are $100,000 basic limits losses. ISO has recently converted from $25,000 basic limits to $100,000. At $100,000 basic limits it was necessary to increase the K value in the denominator to $177,000 from its previously smaller value that applied when $25,000 basic limits losses were used in computing the experience rating adjustment. If unlimited losses were used in the experience rating formula, then an even larger K value would be necessary because the expected value of the process variance would become even larger.

Reducing Variability of the Data

The data used by ratemakers in the insurance business arises from a random process; in fact, it is this randomness that makes insurance necessary. The ratemaker is confronted with the task of finding the proper premiums to charge insureds without knowing for sure what the cost will be to the company to provide the insurance. The ratemaker estimates the cost of future payments in insurance claims by his or her company by analyzing past costs. The ratemaker wants to use the most relevant data to estimate future costs, but he or she must also deal with the variability inherent in the data.

One way to decrease the variability in ratemaking data is to use a larger body of data. Here are several ways to do this:

- include more years in the experience period
- use Bureau data
- combine data into fewer, but larger groups

Each of these involves a tradeoff. If more years are included in the experience period then it becomes necessary to apply larger trend factors to the older data and trend can be tough to estimate. Also, the book of business to which new rates will apply may be different from the business that produced the experience years ago. The same goes for Bureau data. The insureds included in Bureau data may be very different from the average insured in the ratemaker's data. Combining the data into fewer, but larger groups, may limit a company's ability to effectively compete against competitors who can better identify the proper price to charge an insured.

Another approach to decreasing the variability in losses used in ratemaking is to:

- cap large losses
- remove catastrophes

Of course, if we do either of the above we must put something back to make up for the losses we removed. One method to cap large losses is to do basic limits ratemaking by state, territory, class, etc., and calculate basic limits rates. Then, rates for higher limits are computed using increased limits factors calculated based on the aggregate data for many states and classes. Another approach is to limit all losses at some set amount, for example $150,000, and then to prorate the excess losses amount back by state, territory, class, etc. Catastrophe losses can be removed from the data and a catastrophe load substituted in its place. This load can be computed from a very long observation period, thirty years or more for weather losses, or a computer model that attempts to model the catastrophe loss process.

An Introduction to Basic Credibility

by Howard C. Mahler, FCAS

AN INTRODUCTION TO BASIC CREDIBILITY

TALK BY HOWARD C. MAHLER

MARCH 1996

CAS RATEMAKING SEMINAR

In my talk, I will try to reinforce and expand on the ideas Gary Dean presented in his talk.

I will start off my talk by using the following set of graphs taken from my *"Student's Guide to Buhlmann Credibility and Bayesian Analysis"* to illustrate some simple credibility ideas in terms of experience rating or individual risk rating. The goal of experience rating is to use an individual insured's experience to help predict the future. Assuming the individual risk's experience were observed to be worse than average, we would predict his future experience would also be likely to be somewhat worse than average. Therefore, we would be likely to charge this insured somewhat more than average.

As mentioned by Gary Dean, credibility quantifies how much worse or better an insured's future experience is expected to be based on a particular deviation from average observed in the past. These graphs should illustrate some of the ideas Gary Dean mentioned, such as why more weight is given to an individual's experience in certain situations. Also, those of you familiar with linear regression should see much that is familiar. (With the widespread use of personal computers, anyone can do a linear regression.)

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The first graph, Exhibit 1, shows simulated claim counts for 100 insureds divided into two equal groups. In this graph, the "Good Risks" are labeled with crosses and the "Bad Risks" with circles. In both the real world and many of the subsequent graphs, the risks come without such labels attached. (If they did come with such labels, we would not need to use credibility.) Assume we have 100 insureds all in the same risk classification, territory, etc.

The 50 Bad Risks each have an expected Claim Frequency of 15 while the 50 Good Risks each have an expected Claim Frequency of 10. For each of the 100 insureds I have plotted a single prior year against a single subsequent year of claim counts. (For example, one of the Good Risks had 4 claims in the prior year and 5 claims in the subsequent year. This is indicated by a cross at the point (4, 5)). There is considerable overlap between the groups. Nevertheless, the Good Risks are more likely to be in the lower left while the Bad Risks are more likely to be in the upper right.

The next graph, Exhibit 2, shows the same 100 insureds without labels. Here we have fit a least squares regression line to the points. One could use this fitted line to predict a future year's experience based on an observation. Since the line slopes upwards, we see that a bad former year would lead one to predict a worse than average subsequent year.

So if one observed 20 claims for an insured, one might predict about 15 claims for that insured next year, compared to the overall average of 12.5. This least square's line is approximately:

$$Y = .40X + 7.6$$

This can be put in the form of the "Basic Formula" discussed by Gary Dean:

Estimate = Z (observation) + (1 - Z) (overall average)

with the credibility Z = 40%

With only 100 insureds, this result is subject to considerable random fluctuation. The simulation with many more insureds would give a credibility of 1/3. (This can be derived using methods taught on the CAS, Part 4B Exam, which were touched on by Gary Dean.)

The credibility is just the slope of the straight line.. It is the weight given to the observation.

Note the way that the fitted line passes through the point (12.5, 12.5), denoted by a plus. Average experience in the prior year yields an estimate of average experience in the subsequent year.

Note that the line Y = X, with a slope of unity, would correspond to 100% credibility, while the line Y = 12.5 with a slope of zero, would correspond to zero credibility. In general, the slope and the credibility will be between zero and one.

These general features displayed in Exhibit 2, will carry over to subsequent exhibits. The least squares line will slope upwards and pass through the point denoting average experience in the prior and subsequent period. The slope will be (approximately) equal to the credibility.

The next graph, Exhibit 3, is similar to Exhibit 2 but shows <u>three</u> years of prior experience rather than one. Note that the X-axis is now the <u>annual</u> claim frequency observed over three years. We expect three years of data to contain more useful information and thus be given more weight than would one year. In fact, when we fit a straight line we see a larger slope of about 60% (actually 58%) corresponding to a credibility of 60%. As Gary Dean noted, one way to increase the credibility of data is to increase the volume of data.

In the case of Exhibits 2 and 3, the credibility is equal to $N/(N+K)$ where N = # of years of data and $K = 2$. As mentioned by Gary Dean, this formula is used quite often, with the "Buhlmann Credibility Constant" K dependent on the statistical properties of the particular situation. Note that for Exhibit 2, $Z = \frac{1}{1+2} = \frac{1}{3}$, while in Exhibit 3, $Z = \frac{3}{3+2} = 60\%$. (In the next set of graphs, K will equal .22.)

The next graph, Exhibit 4, shows 100 risks divided this time into Excellent Risks and Ugly Risks. The Excellent Risks are shown by asterisks and the Ugly Risks by wedges. The mean frequencies are 5 and 20 rather than 10 and 15 as in the previous Exhibits. Therefore, the two groups are much more spread apart. Since there is more *dispersion between risks, each risk's data will be given more credibility than in the first* graph.

This can be seen in the next graph, Exhibit 5, where a straight line has been fit to these points. The line has a much larger slope than the first line, corresponding to higher credibility of about 82%. (Again the results of an experiment with only 100 drivers differs from the theoretical result due to random fluctuation.) So due to the larger variation in hypothetical means (holding everything else equal) in Exhibit 5 versus Exhibit 2, the credibility increased from 33% to 82%. The value of the individual risks information increased relative to the information contained in the grand mean. Conversely, the relative value of the information contained in the grand mean decreased.

The next graph, Exhibit 6, combines the four different types of insureds. This starts to approach the real world situations where risks' expected claim frequencies are along a continuous spectrum, rather than being of unique types. (One could approach a continuous situation similar to the Gamma-Poisson frequency process.) We can see plenty of overlap between the four types, although since we labeled the insureds, we can discern the grouping of different types.

The next graph, Exhibit 7, shows a line fit to all four types. There the slope of 72% is between the slopes of either 40% and 78% we got when dealing with just two groups. This makes sense since the variation of the hypothetical means is in between those two situations.

The following graphs will all involve 125 Excellent and 125 Ugly Risks, but rather than dealing with just claim frequency will deal with claim severity as well. By looking at dollars of loss rather than numbers of claims, as can be seen on the next graph, Exhibit 8, we introduce more random fluctuation. Therefore, the relative value of the observation is less compared to average; the credibility goes down. As mentioned by Gary Dean, one way to decrease the credibility of data is to increase the variability of the data.

As can be seen on the next graph, Exhibit 9, the slope of the fitted line is 51.5%. The theoretical credibility is 53% compared to 82% for the corresponding claim frequency situation. The greater random fluctuation, which is quantified by the larger "process variance" has decreased the credibility assigned to the observations.

In practical applications, one often limits the size of claims entering into Experience rating. As Gary Dean mentioned, one way to decrease the variability of the data is to cap losses. The final graph, Exhibit 10, shows the results of capping each claim at $25,000.

(This capping can be either just for the purposes of experience rating or could involve an actual policy limit.) The fitted line between prior limited losses and subsequent limited losses is 71.4%. The theoretical credibility of 70% when using limited losses compares to 53% for total losses. Capping the losses has reduced the random fluctuations, i.e., has reduced the process variance, thereby increasing the credibility assigned to the experience. (Basic limit losses are less volatile than total limits losses.) (For more on how to analyze Experience Rating Plans, see for example, *"An Analysis of Experience Rating"* by Glenn Meyers in PCAS 1985 and my discussion in PCAS 1987.)

So far my talk has illustrated the concept of using credibility for individual risk rating. As Gary Dean mentioned, credibility is also used in classification rating, reserving, trending, and other areas. Whenever an actuary wishes to make an estimate, credibility can be useful to overcome the problem of limited data.

Let X be the quantity we wish to estimate. For example, X might be the expected losses for a Workers' Compensation class relative to the statewide, i.e., X is the class relativity. In my previous example, X was a risk's future expected experience relative to average.

As shown in Exhibit 11, in the "Basic Formula" we weight together two estimates of the quantity X. In that case we usually write:

$$X = Z\,Y_1 + (1 - Z)\,Y_2$$

where Z is called the credibility and 1 - Z is called the complement of credibility. In the experience rating example, Y_1 was the risk's observed experience while Y_2 was the overall average experience.

As listed on Exhibit 12, the estimators Y_i can have many sources. (This subject is discussed in more detail in Joseph Boor's paper *"The Complement of Credibility"* in the Fall 1995 CAS Forum.)

For example:

1. The recent observation(s) of X.
2. The recent observation(s) of the same quantity as X, but for a superset.
3. The recent observation(s) of a similar quantity to X; there may be an adjustment necessary.
4. Past estimates(s) of X. There may be an adjustment for the intervening period of time.
5. The result of a model.
6. The result of judgment.

Exhibit 13 shows those rules I think will aid you in using credibility for practical applications.

Rule 1A:

Spend a lot of time and effort deciding on or choosing the Y_i. Each Y_i should be a reasonable estimate of X.

So for example, if trying to estimate a medical claim cost trend it may not make much sense to assign the complement of credibility to an estimate based on the general overall rate of inflation. It might make sense to look at some other measure of medical inflation rather than a measure of general inflation.

Rule 1B:

Spend a lot of time and effort computing, collecting data on, or estimating each Y_i.

If you are going to include a value in your weighted average, it makes sense to try to carefully quantify that value.

Rule 2A:

The procedure is generally forgiving of small "errors" in the weights. Therefore, do not worry overly much about getting the weights exactly right.

In our experience rating example, you can confirm that for most risks, small changes in the credibility do not result in major changes in the estimate of their future experience.

This is discussed in my paper *"An Actuarial Note on Credibility Parameters"* in PCAS 1986. Exhibit 14 illustrates the effect of changing K, the Buhlmann credibility parameter, on the credibility. As can be seen, changes in K of less than a factor of 2 would result in relatively small changes in credibility. In turn, these small changes in credibility usually result in small changes in estimates of the quantities of interest.

Rule 2B:

The concept of credibility is a relative concept. A relative weight can only be assigned to any single estimator, if you know what all the other estimators are.

For example, assume you have two estimators each of which has been assigned "only" 50% credibility. This merely indicates that the two estimators are equally good or equally bad, not whether they are good or bad in some absolute sense.

Rule 2C:

The less random variation in an estimate, the more weight it should be given. In other words, the more useful information and the less noise, the more the weight. We saw that limited losses were given more weight than unlimited losses, since the limited losses had less random variation.

Rule 2D:

The more closely related to the desired quantity, the more weight an estimator should receive.

For example, observations more distant in time usually deserve less weight. A given quantity of data from the same state would probably receive more weight than data from outside the state.

Rule 3:

Cap the changes in relativities that result from the use of credibility.

A properly chosen cap may not only add stability, but may even make the methodology more accurate by eliminating extremes.

An example of a practical use of credibility involves revising the definitions of automobile insurance territories in Massachusetts. Each town's relative loss potential is determined based on four years of data and a relatively complicated credibility methodology. For frequency, the complement of credibility is given to a road density model. For severity, the complement of credibility is given to a combination of the county average severity and the state average severity. Then towns with similar estimated loss potential are grouped together. Here we will ignore the details of the procedure which are explained in Robert Conger's paper, *"The Construction of Automobile Rating Territories in Massachusetts"* in PCAS 1987, and discuss one aspect of the results of the reviews conducted over the last decade.

It has been demonstrated that use of this credibility technique produces "better" predictions on average. However, credibility is a linear process, and thus the extreme cases may not be dealt with as well as they might.

For example, Exhibit 15 shows the results of applying the same methodology consistently over time to two small towns, each with somewhere around 5,000 exposures per year.

	Estimated Loss Potential Relative to Statewide Average						
	1984 Review	1986 Review	1988 Review	1989 Review	1991 Review	1993 Review	1995 Review
Acushnet	.84	.87	.88	.87	.93	1.00	.97
Brewster	.74	.84	.70	.61	.69	.69	.58

	Indicated Territory (Prior to Capping)						
	1984 Review	1986 Review	1988 Review	1989 Review	1991 Review	1993 Review	1995 Review
Acushnet	5	6	6	6	7	8	8
Brewster	3	6	2	1	2	2	2

The results for the first town, Acushnet, are typical. The relative loss potential varies somewhat from review to review, with a change in indicated territory of plus or minus one from time to time. In this particular case there is an upward drift over time relative to the statewide average.

The results for the second town, Brewster, are not typical. In fact, Brewster was chosen as the most extreme case of fluctuating experience over the period of time from the 1984 review to the 1989 review. As you can see in Exhibit 16, the estimated relative loss potential swung up and then down. This in turn resulted in large changes in the indicated territories. This occurred in spite of relying on four years of data, so that the data periods used in the reviews overlap. This occurred in spite of the use of credibility, which ameliorated the effect of the large fluctuations in the experience of this town.

Such large swings are unlikely. However, when dealing with 350 towns, something that only has a .3% chance of happening per town, on average occurs for one town.

This problem is dealt with by capping territory movements. The actual cap chosen was to restrict movements in any one revision to at most one territory either up or down. This is an example of the third rule I discussed earlier.

Another example of a practical use of credibility, is the Workers' Compensation Experience Rating Plan. This is an individual risk rating plan conceptually similar to my first set of graphs involving Excellent, Good, Bad, and Ugly risks. Around 1989 or 1990, the National Council on Compensation Insurance made a major revision to their Workers' Compensation Experience Rating Plan. Among the changes was a major revision to the credibilities assigned to an individual insured's loss experience relative to average. This was based on an extensive and detailed study by the NCCI actuaries. (See for example, William R. Gillam's paper "Parametrizing the Workers' Compensation Experience Rating Plan" in PCAS 1992 and my discussion in PCAS 1993.) Without getting into any details, Exhibit 17 shows you the overview.

Primary Losses are the first layer of losses while Excess Losses are those above them. Z_p is the credibility assigned to primary losses. For the prior plan, it is shown by dots; for the revised plan by circles. Similarly, Z_X is the credibility assigned to excess losses. For the prior plan, it is shown by solid squares; for the revised plan, it is shown by open squares. In each case, the credibility assigned to the primary losses is greater than that assigned to the excess losses, since excess layers are more volatile than basic limits losses.

Note that the credibility varies by size of risk. The more expected losses, the more credibility is assigned to the insured's own experience and the less that is assigned to the manual rate. (Note that the maximum credibility for the revised plan is less than 100%. The credibilities for the revised plan are based on a refinement of the Buhlmann Credibility formula discussed by Gary Dean.)

Exhibit 18 shows the changes in credibilities. For smaller risks, the revised plan assigns higher credibilities than the prior plan. For larger risks, the revised plan assigns lower credibilities than the prior plan. Thus, large insureds with good experience get smaller credits under the revised plan, while large insureds with bad experience get smaller debits under the revised plan. The theoretical credibility work by the Actuaries at the National Council that led to this revision, had a major impact on thousands of businesses across the country. So "theoretical credibility" can have immense practical impact.

A final example of a practical use of credibility, is the estimation of relative average claim costs for workers compensation classes. Exhibit 19 shows the calculation of the observed average claim costs for the classes in the Office and Clerical Industry Group for one year. We divide losses by the number of claims. Then for each class we calculate the relative average claim cost by dividing the classes' average claim cost by that for the industry group. Note that I have not limited the size of claims, but that I have excluded the large lifetime claims which would produce the most random fluctuation.

So far we have not used credibility. However, since some classes have very few claims in a single year, I would not want to rely on the results of one year of observations. Exhibit 20 puts together the results of seven years of observations. We observe considerable random fluctuation in the relative claim costs. I take an average over the seven years for each class and then use credibility.

For each class its observed relative claim cost is given credibility equal to the square root of its number of claims divided by 2,500. A class with 2,500 or more claims over 7 years is assigned full credibility. The Complement of credibility is assigned to unity, an average claim cost equal to the overall average for the Industry Group. Applying the Basic Formula on Exhibit 11 to this case the estimated relative average claim cost is:

$$Z\,(\textit{observed average claim cost}) + (1 - Z)(1)$$

as shown in Column 12 of Exhibit 20.

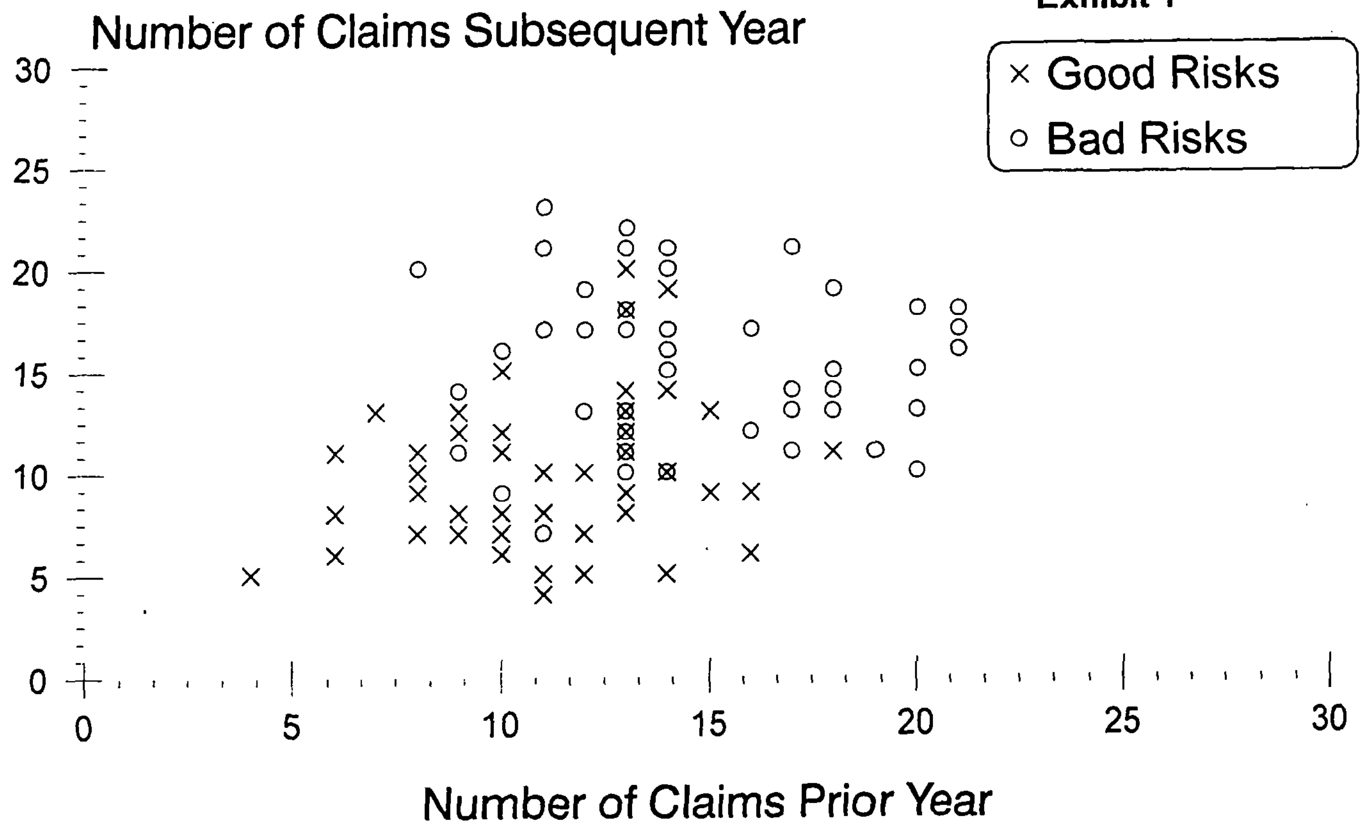
Exhibit 21 graphs the Credibility in this case. Exhibit 22 compares the credibility from the use of the square root formula to that using $Z = N / (N + K)$ with K = *350* claims. The credibilities are similar.

I have tried to illustrate a few of the many applications of credibility. I've given a number of general rules which you should find useful in your own work with credibility.

The theory behind the use of credibility can be complex. However, the use of credibility itself is set up precisely so that it can be understood by a layman. While ratemakers may differ in their knowledge of credibility theory, all ratemakers should be completely familiar with credibility practice.

Simulated Claims Experience

Exhibit 1



50 Good Risks (Poisson 10) and 50 Bad Risks (Poisson 15)

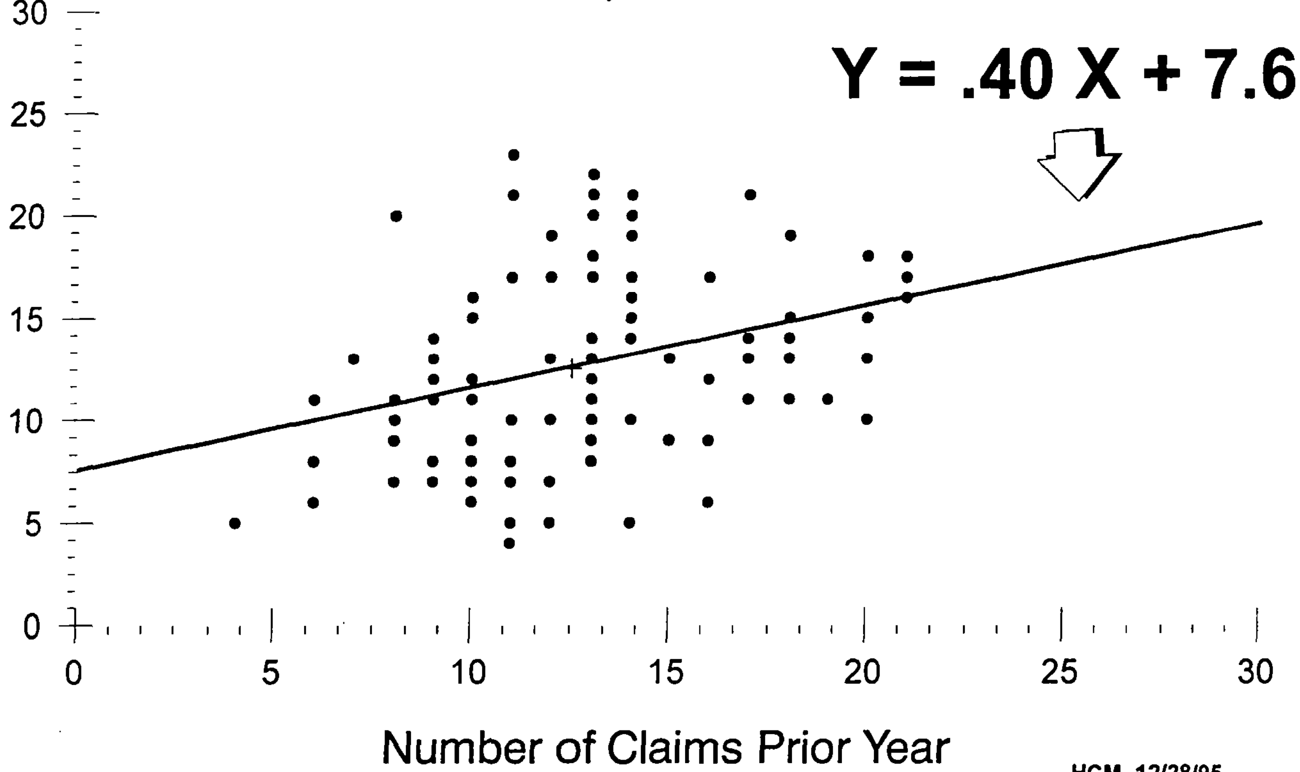
HCM 12/28/95

Simulated Claims Experience

Exhibit 2

Good & Bad Risks

Number of Claims Subsequent Year



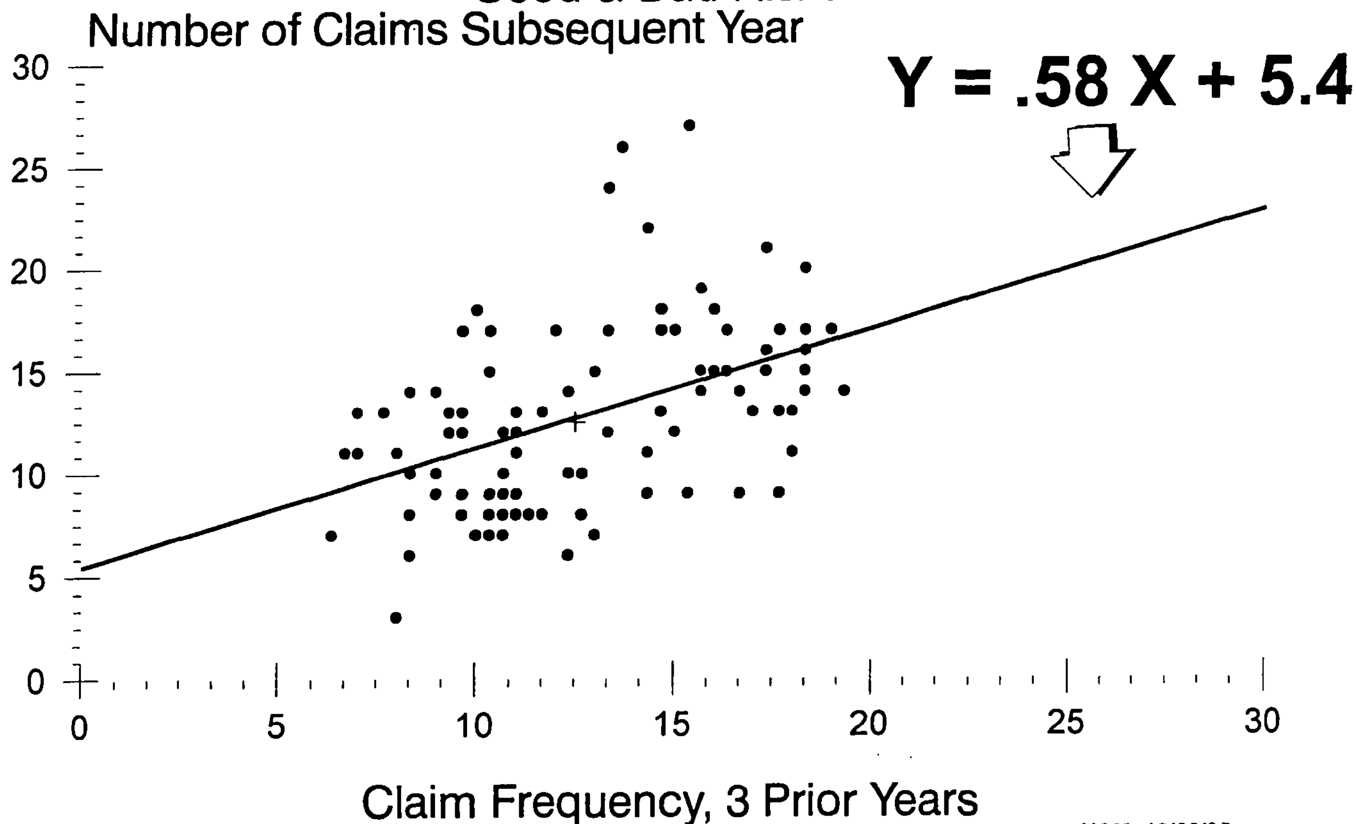
HCM 12/28/95

50 Good Risks (Poisson 10) and 50 Bad Risks (Poisson 15)

Simulated Claims Experience, 3 Prior Years

Good & Bad Risks

Exhibit 3



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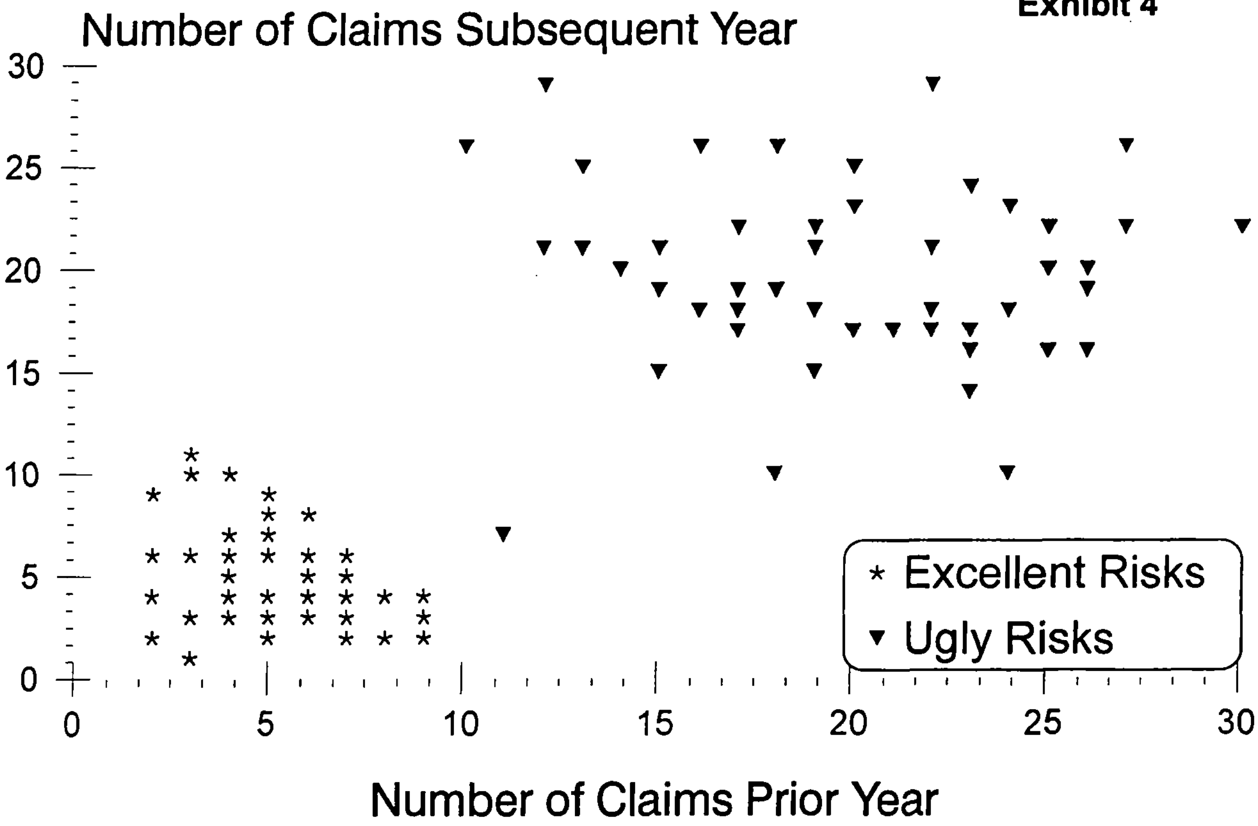
50 Good Risks (Poisson 10) and 50 Bad Risks (Poisson 15)

Simulated Claims Experience

Exhibit 4

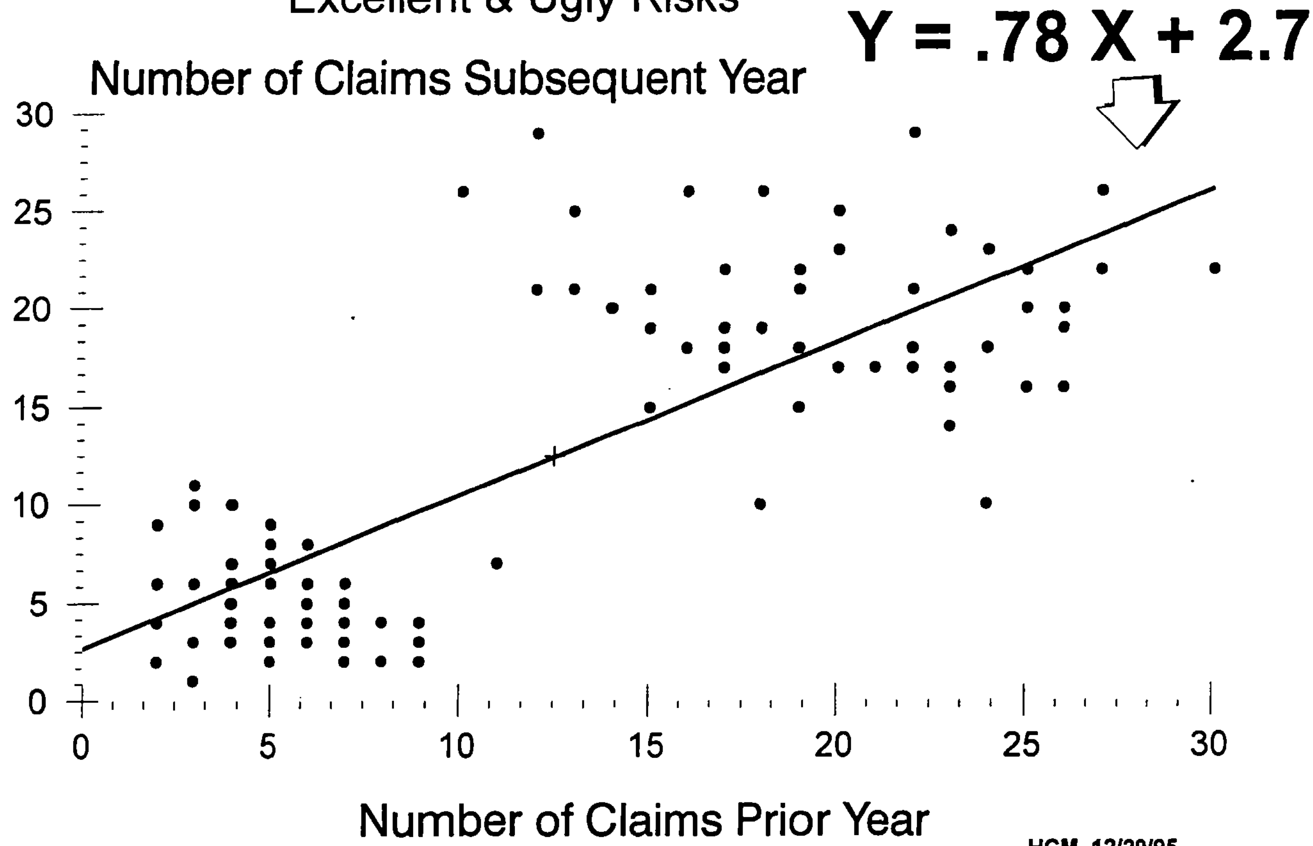


50 Excellent Risks (Poisson 5) and 50 Ugly Risks (Poisson 20)

HCM 12/28/95

Exhibit 5

Simulated Claims Experience
Excellent & Ugly Risks

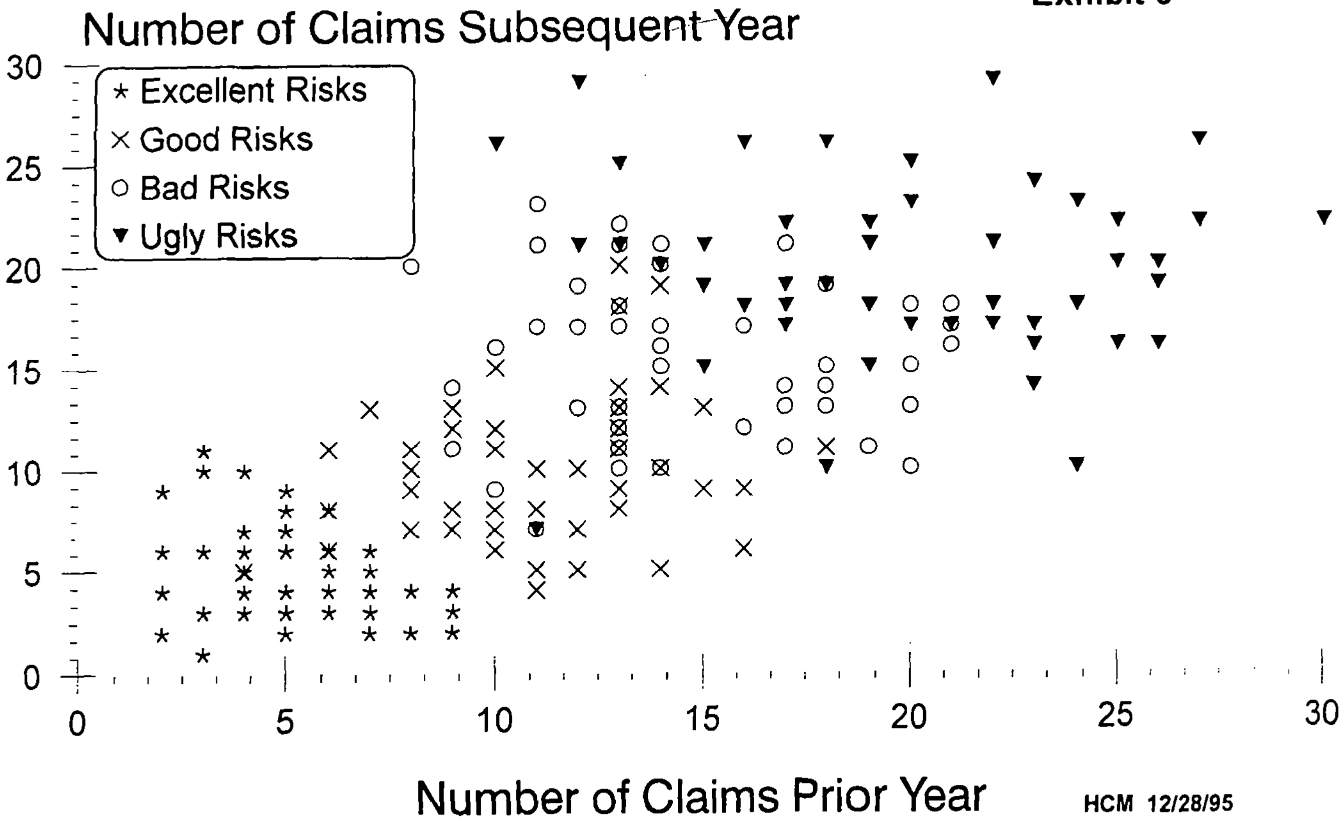


50 Excellent Risks (Poisson 5) and 50 Ugly Risks (Poisson 20)

HCM 12/29/95

Simulated Claims Experience

Exhibit 6



50 Excellent Risks (Poisson 5), 50 Good Risks (Poisson 10), 50 Bad Risks (Poisson 15), and 50 Ugly Risks (Poisson 20)

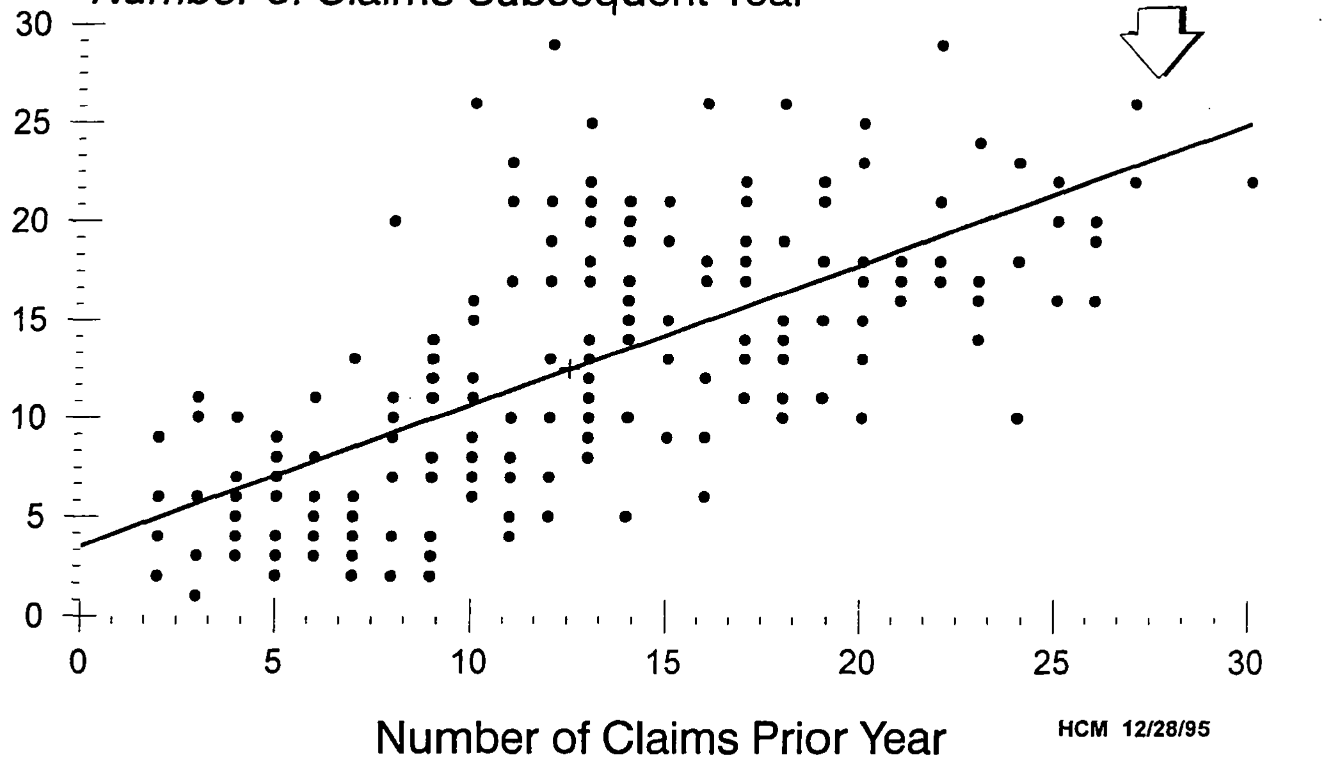
Simulated Claims Experience

Exhibit 7

Excellent, Good, Bad, & Ugly Risks

Number of Claims Subsequent Year

$$Y = .72\,X + 3.5$$



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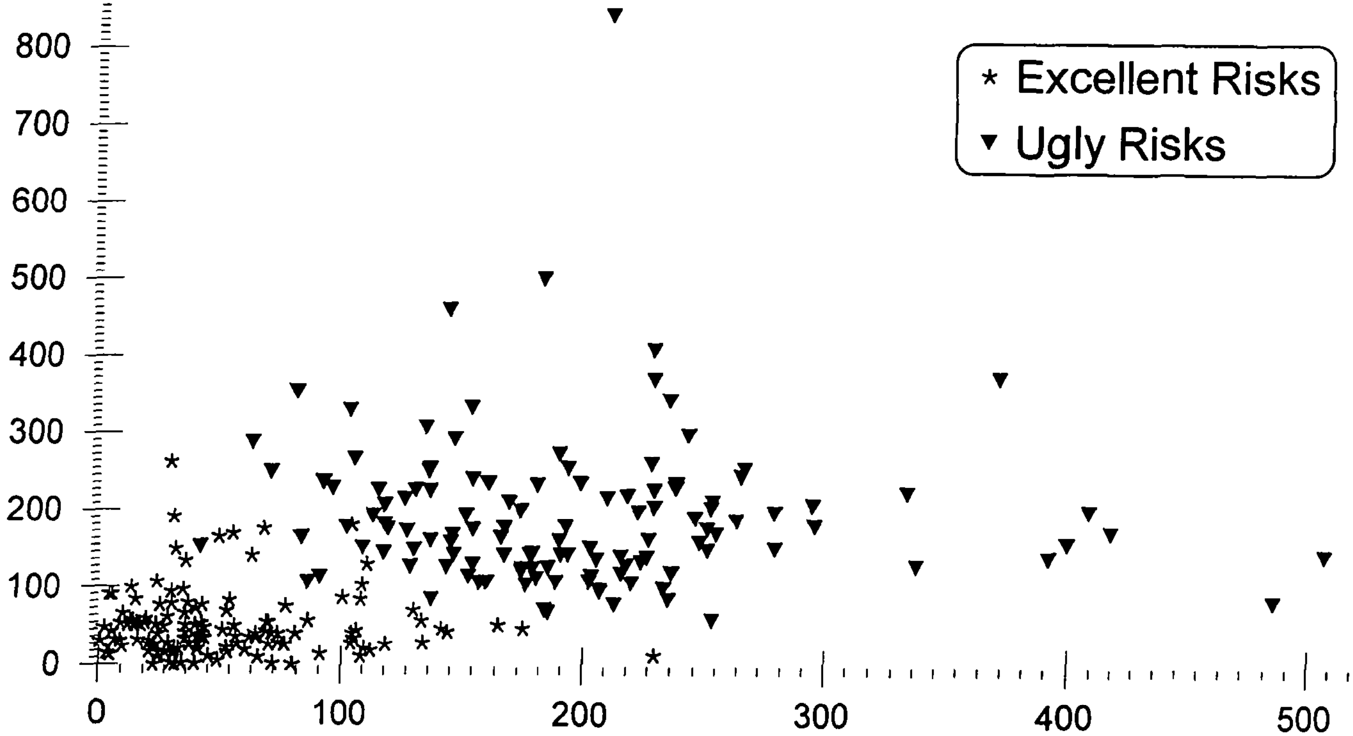
50 Excellent Risks (Poisson 5), 50 Good Risks (Poisson 10), 50 Bad Risks (Poisson 15), and 50 Ugly Risks (Poisson 20)

Simulated Loss Experience

Thousands of Dollars of Loss Subsequent Year

Exhibit 8



Thousands of Dollars of Loss Prior Year

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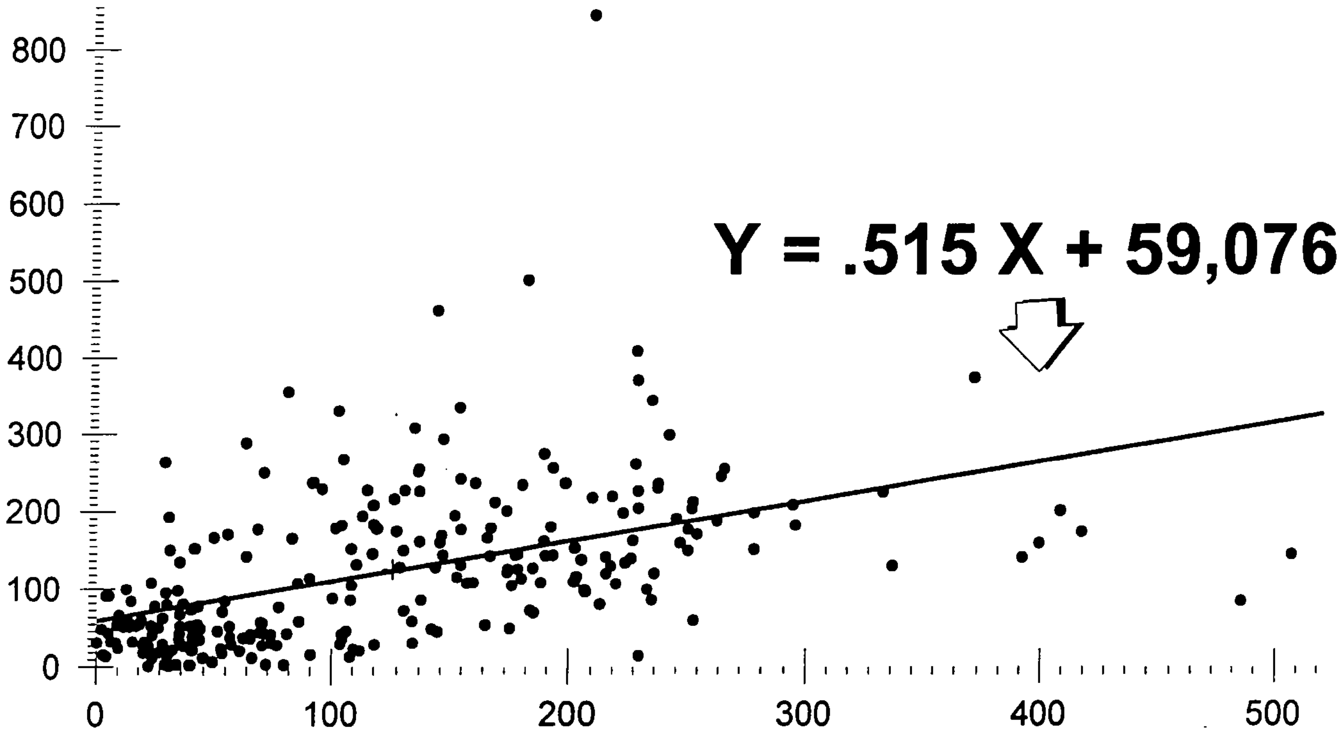
125 Excellent Risks (Poisson 5) and 125 Ugly Risks (Poisson 20), Pareto Severity (3, $20000)

Simulated Loss Experience

Exhibit 9

Thousands of Dollars of Loss Subsequent Year



Thousands of Dollars of Loss Prior Year

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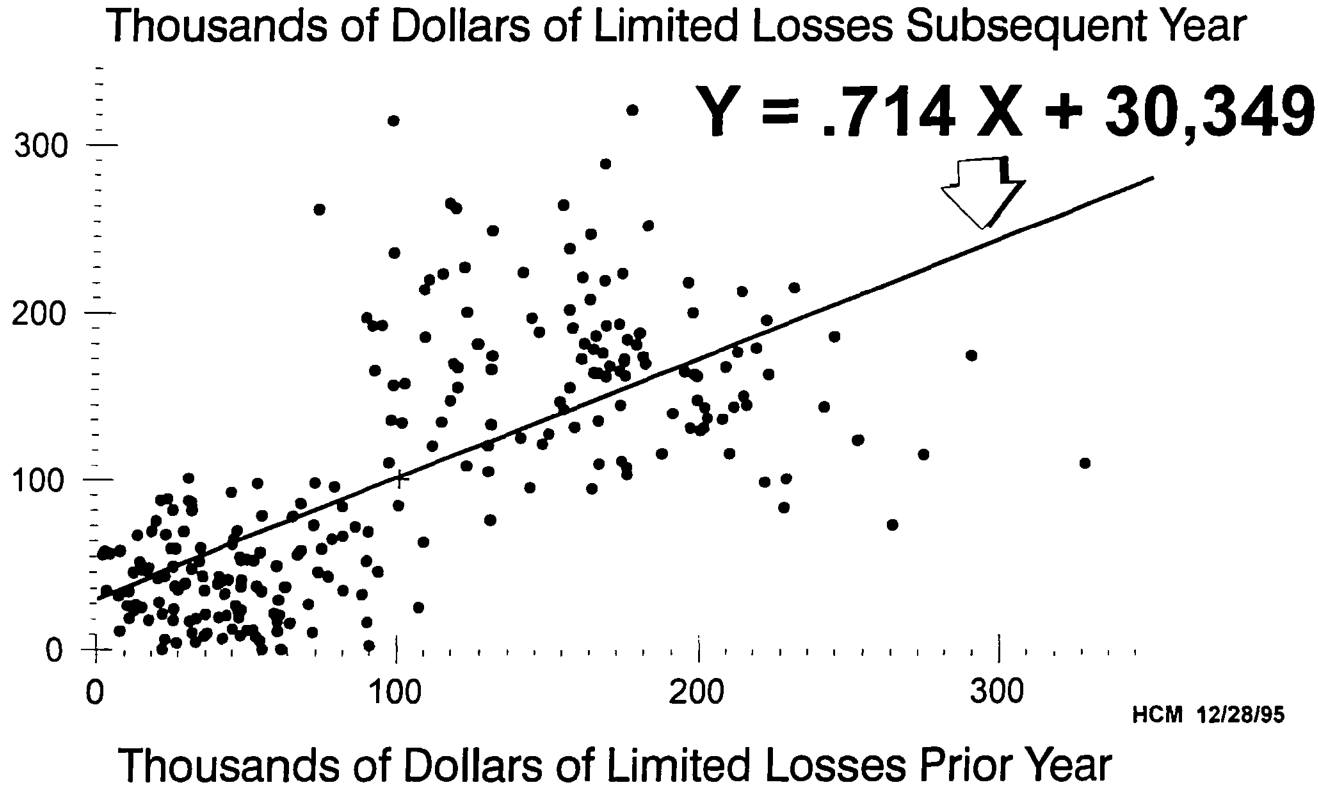
125 Excellent Risks (Poisson 5) and 125 Ugly Risks (Poisson 20), Pareto Severity (3, $20000)

Simulated Loss Experience

Each Claim Limited to $25,000 for entering into Experience Rating

Exhibit 10



125 Excellent Risks (Poisson 5) and 125 Ugly Risks (Poisson 20), Pareto Severity (3, $20000)

Exhibit 11

"BASIC FORMULA"

$$X = Z\,Y_1 + (1 - Z)\,Y_2$$

where X is the quantity to be estimated

Y_1 and Y_2 are estimators of X

Z is credibility

2555-3

The estimators Y_i can have many sources. For example:

1. The recent observation(s) of X.

2. The recent observation(s) of the same quantity as X, but for a superset.

3. The recent observation(s) of a similar quantity to X; there may be an adjustment necessary.

4. Past estimate(s) of X. There may be an adjustment for the intervening period of time.

5. The result of a model.

6. The result of judgement.

2555-4

Rule 1A:

Spend a lot of time and effort deciding on or choosing the Y_i. Each Y_i should be a reasonable estimate of X.

Rule 1B:

Spend a lot of time and effort computing, collecting data on, or estimating each Y_i.

Rule 2A:

The procedure is generally forgiving of small "errors" in the weights. Therefore, do not worry overly much about getting the weights exactly right.

Rule 2B:

The concept of credibility is a relative concept. A relative weight can only be assigned to any single estimator, if you know what all the other estimators are.

Rule 2C:

The less random variation in an estimate, the more weight it should be given. In other words, the more useful information and the less noise, the more the weight.

Rule 2D:

The more closely related to the desired quantity, the more weight an estimator should receive.

Rule 3:

Cap the changes in relativities that result from the use of credibility.

2555-5

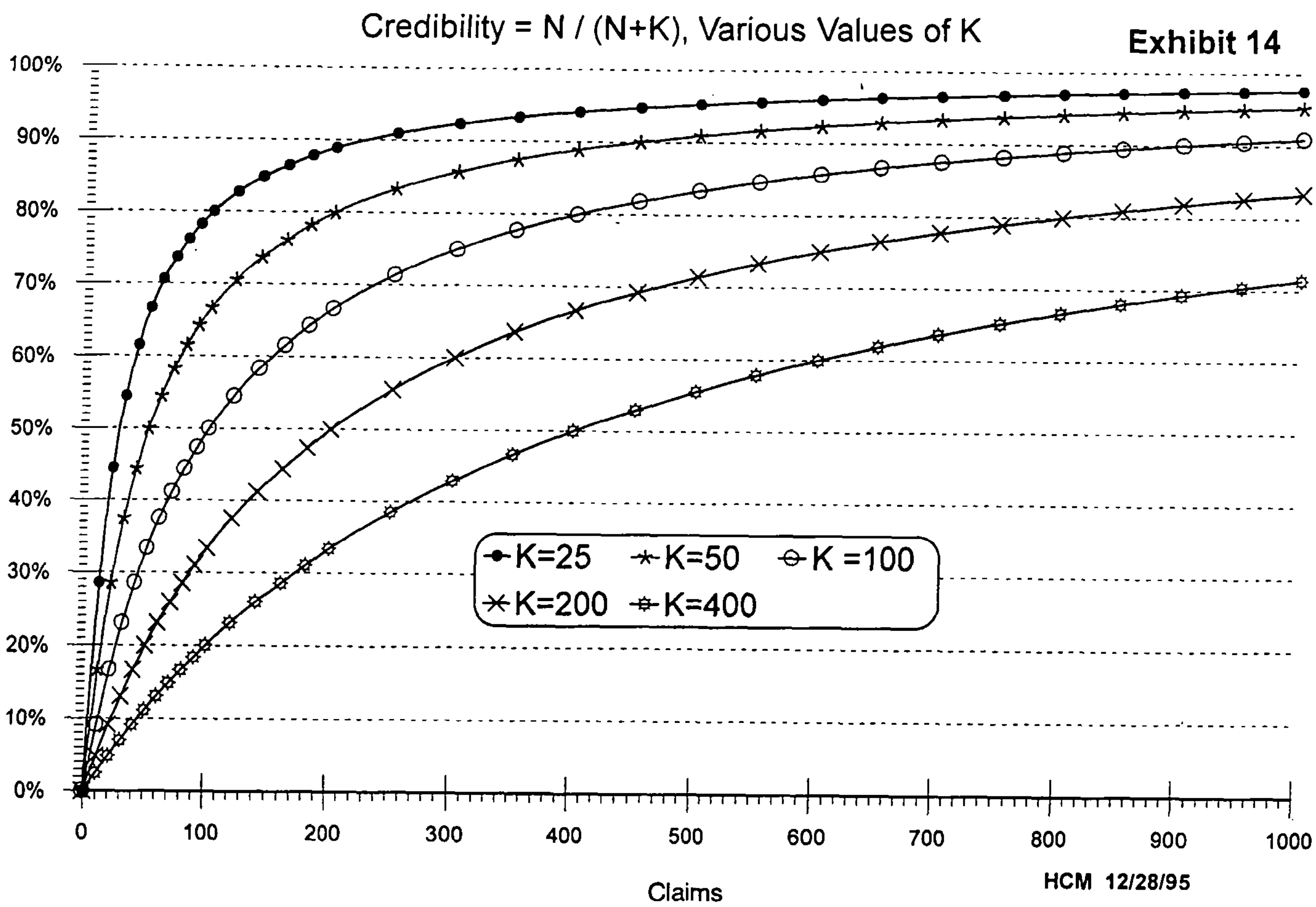

Credibility = N / (N+K), Various Values of K
Exhibit 14
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
0
100
200
300
400
500
600
700
800
900
1000
Claims
K=25
K=50
K =100
K=200
K=400
HCM 12/28/95

Exhibit 15

Massachusetts Private Passenger Automobile

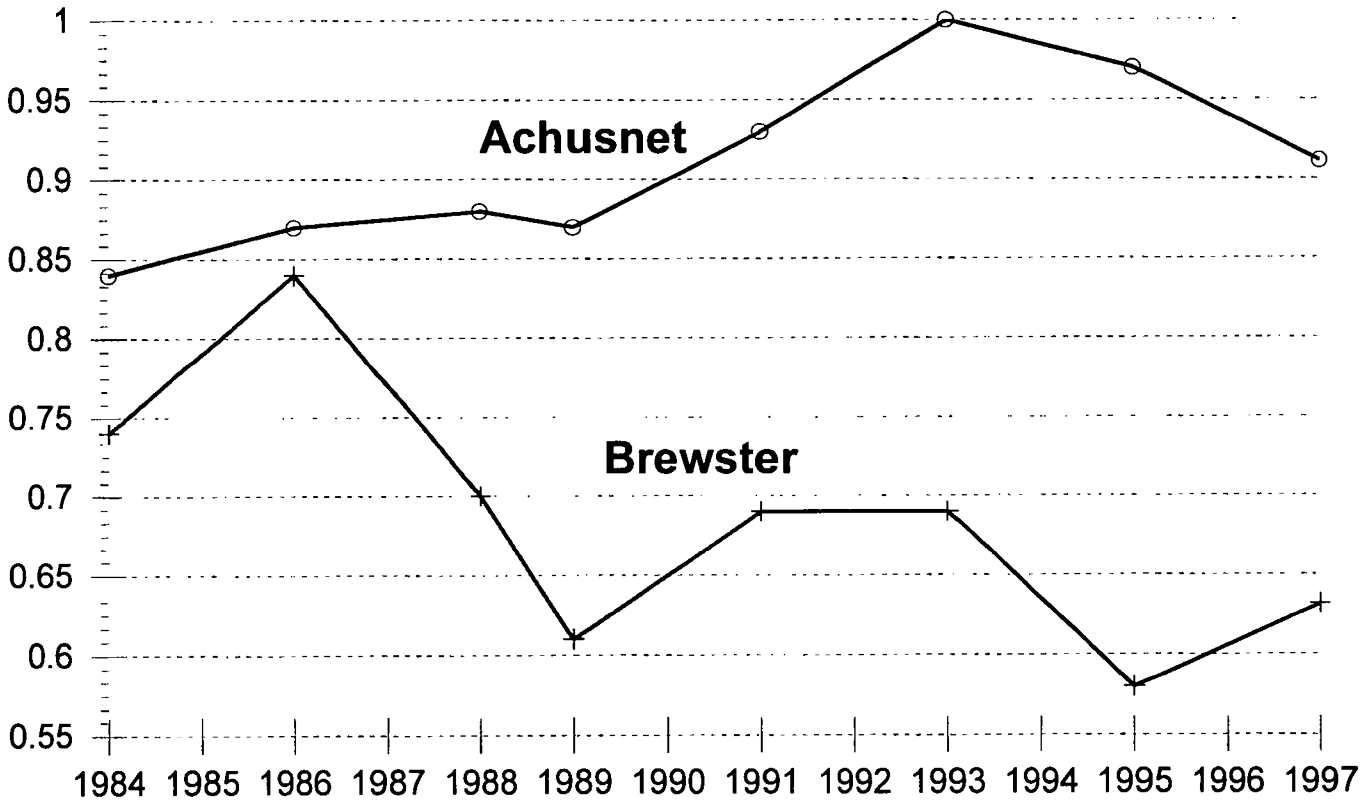
	Estimated Loss Potential Relative to Statewide Average						
	1984 Review	1986 Review	1988 Review	1989 Review	1991 Review	1993 Review	1995 Review
Acushnet	.84	.87	.88	.87	.93	1.00	.97
Brewster	.74	.84	.70	.61	.69	.69	.58

	Indicated Territory (Prior to Capping)						
	1984 Review	1986 Review	1988 Review	1989 Review	1991 Review	1993 Review	1995 Review
Acushnet	5	6	6	6	7	8	8
Brewster	3	6	2	1	2	2	2

2555-6

Estimated Loss Potential Relative to Statewide Average
Masschusetts Private Passenger Automobile

Exhibit 16

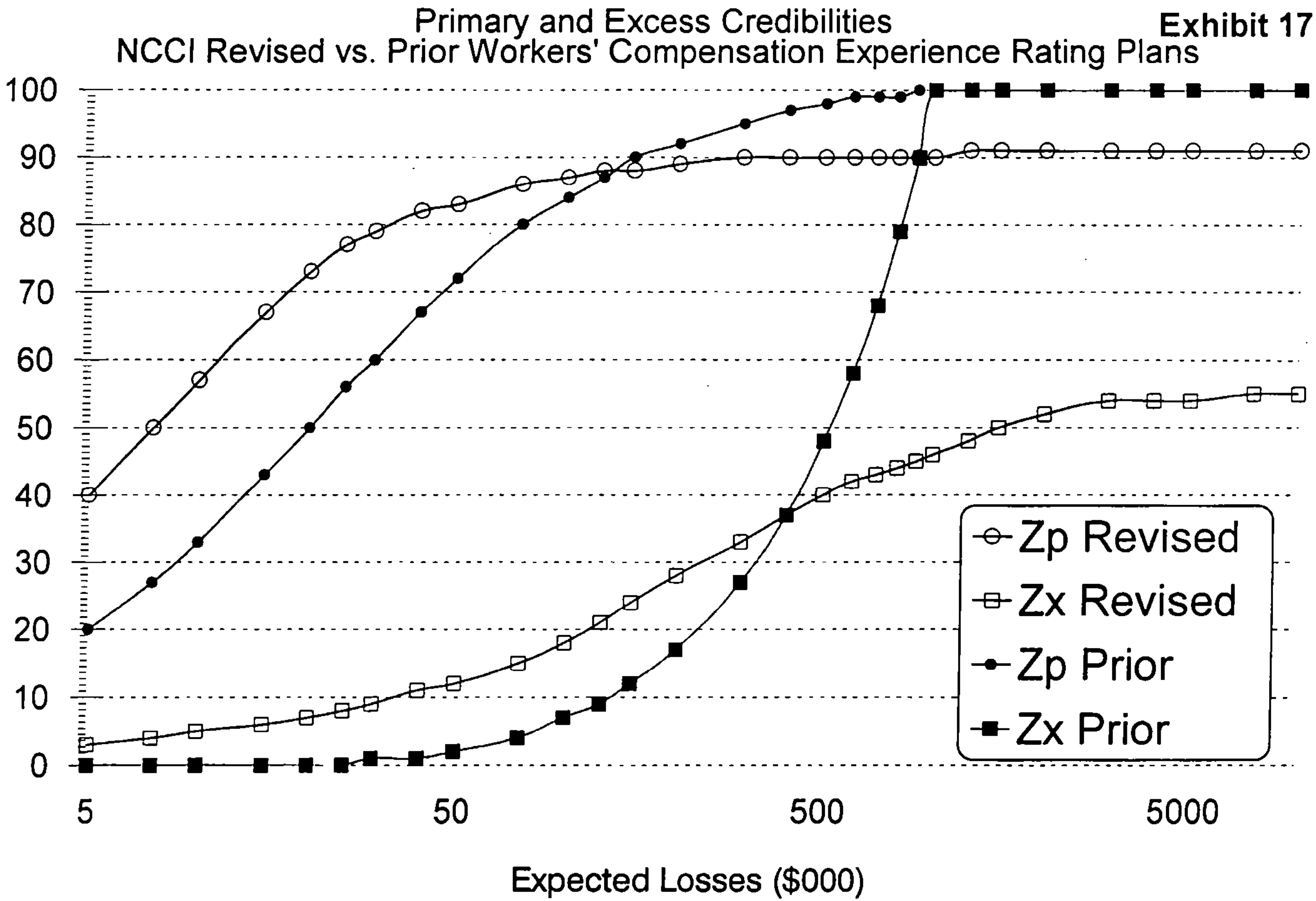


Indicated 1997 Territory: Brewster 1, Acushnet 7

Territory Review

HCM 10/2/96

Exhibit 17

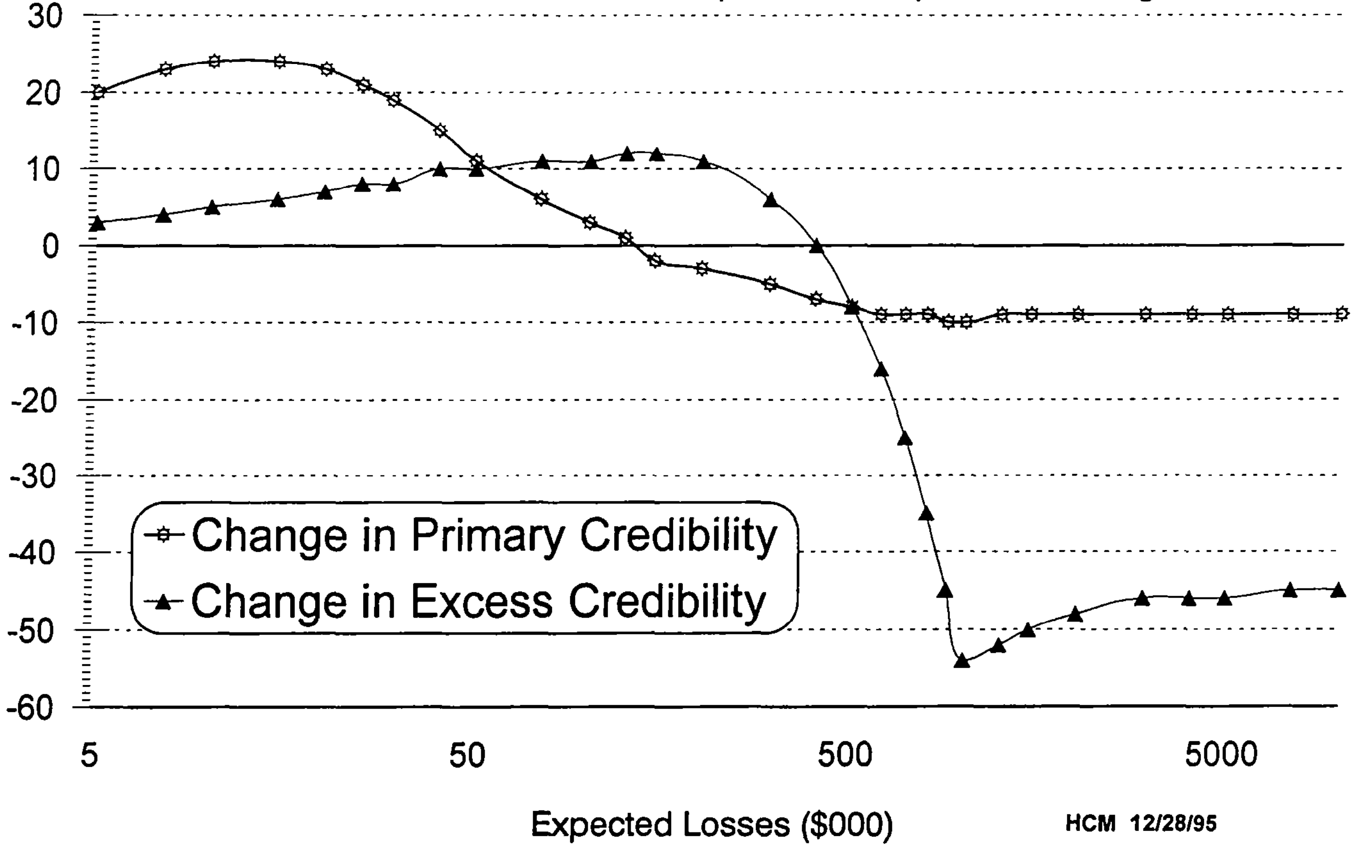


For g =2 (State Reference Point of $500,000) and Self-Rating Point of $1 million.

HCM 12/28/95

Exhibit 18

Primary and Excess Credibilities
NCCI Revised vs. Prior Workers' Compensation Experience Rating Plan



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For g =2 (State Reference Point of $500,000) and Self-Rating Point of $1 million.

Exhibit 19

MASSACHUSETTS WORKERS' COMPENSATION
Relative Average Claim Costs
Industry Group: Office & Clerical
Composite Policy Year 85/86 @2nd Report

Phraseology	(1) Class	(2) Losses (Indemnity+Medical)	(3) Number of Claims	(4) = (2)/(3) Average Claim Cost	(5) = (4)/TT(4) Relative Average Claim Cost
Photographer-All Emp-Clerical,Sales-& Dr	4361	231,122	33	7,004	0.680
Radio or TV Broadcast-All Emp,Cler-& Dr	7610	702,919	42	16,736	1.625
Engineer or Architect-Consulting	8601	1,356,461	134	10,123	0.983
Salesperson,Collector,Messenger-Outside	8742	8,771,008	703	12,477	1.211
Auto Sales or Service Agcy-Salesperson	8748	1,552,606	73	21,269	2.065
Mailing or Addressing Co-& Clerical	8800	245,229	38	6,453	0.626
Auditor,Accountant,Etc-Traveling	8803	184,289	43	4,286	0.416
Clerical Office Employees NOC	8810	24,323,122	2,404	10,118	0.982
Attorney-All Emp-Clerical,Messenger & Dr	8820	741,565	40	18,539	1.800
Physician-& Clerical	8832	1,444,953	136	10,625	1.031
Hospital-Professional Employees	8833	11,760,162	1,199	9,808	0.952
School-Professional Emp & Clerical	8868	5,263,573	634	8,302	0.806
Telephone/Telegraph Co-Office Emp & Cl	8901	146,908	14	10,493	1.019
Theatre-Players,Entertainers,Musicians	9156	131,147	26	5,044	0.490
	Total	56,855,064	5,519	10,302	

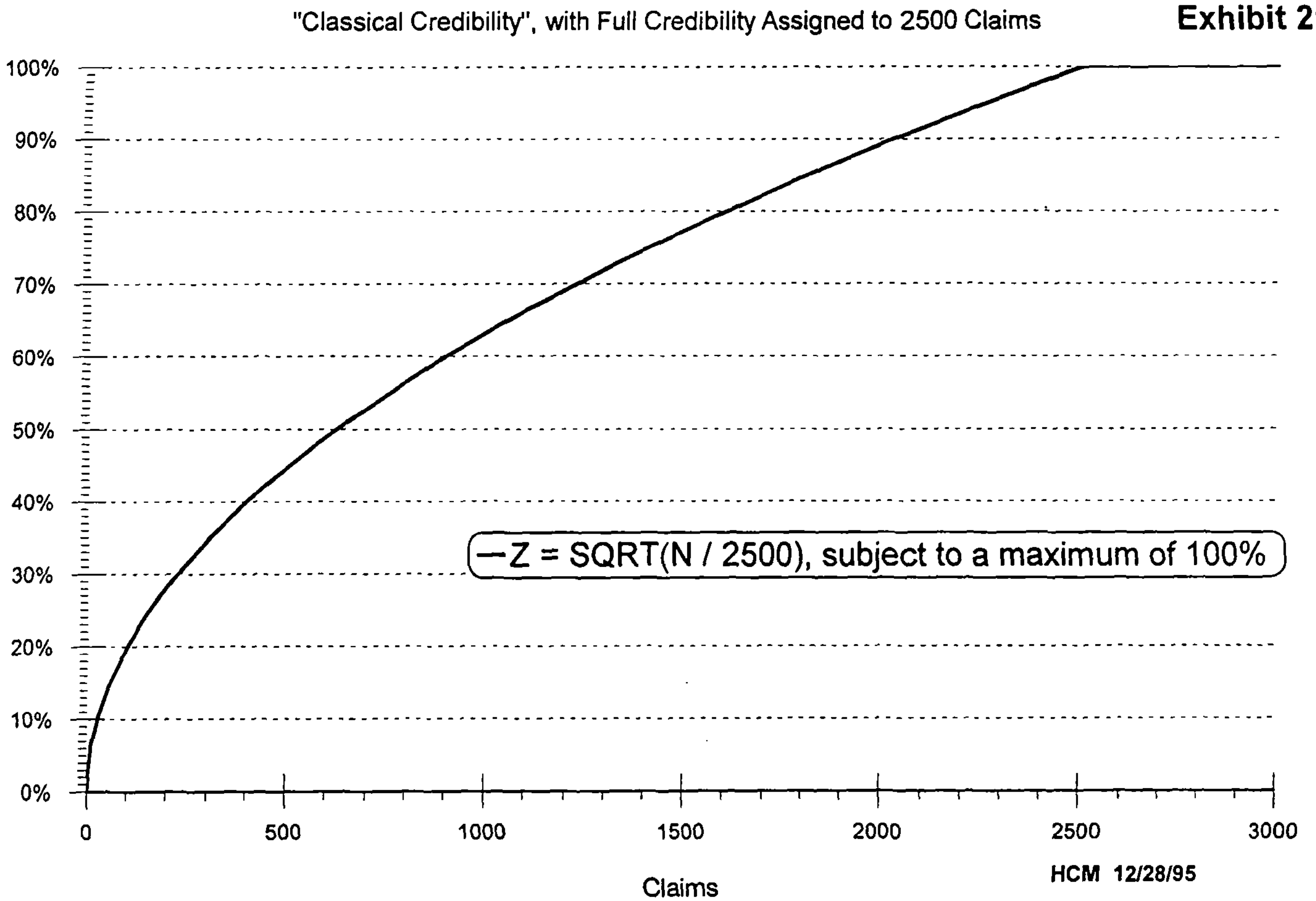
(2),(3): Losses and claims are as reported under the Unit Statistical Plan, but excluding any Fatal, Permanent Total, or Medical Only Claims. (Losses are paid plus case reserves and are neither limited nor adjusted.)

MASSACHUSETTS WORKERS' COMPENSATION
Estimated Relative Average Claim Cost
Industry Group: Office & Clerical

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12) = 1+(11)x[(9)-1]
Class Code	85/86 Relative ACC	86/87 Relative ACC	87/88 Relative ACC	88/89 Relative ACC	89/90 Relative ACC	90/91 Relative ACC	91/92 Relative ACC	Combined Relative ACC	Number of Claims	Credibility	Estimated Relative ACC
4361	0.680	0.920	0.640	0.708	1.087	0.428	1.002	0.785	323	0.359	0.923
7610	1.625	1.351	0.839	0.934	1.127	0.969	0.858	1.059	364	0.382	1.023
8601	0.983	1.440	1.169	1.069	1.026	0.919	0.915	1.100	939	0.613	1.061
8742	1.211	1.161	1.031	1.221	1.028	1.017	1.444	1.143	5,829	1.000	1.143
8748	2.065	1.747	2.151	1.967	2.130	1.626	1.215	1.895	452	0.425	1.380
8800	0.626	0.725	1.025	0.830	0.883	1.365	0.721	0.889	325	0.361	0.960
8803	0.416	1.124	0.472	1.893	0.830	1.109	2.268	1.029	188	0.274	1.008
8810	0.982	1.021	1.044	1.040	1.066	1.113	1.005	1.040	17,195	1.000	1.040
8820	1.800	1.307	1.630	1.639	1.238	1.216	1.540	1.450	426	0.413	1.186
8832	1.031	1.233	1.536	1.176	1.051	1.037	1.096	1.150	1,478	0.769	1.115
8833	0.952	0.773	0.814	0.792	0.863	0.884	0.774	0.837	6,819	1.000	0.837
8868	0.806	0.905	0.828	0.675	0.796	0.724	0.711	0.774	5,211	1.000	0.774
8901	1.019	0.556	1.128	1.068	0.788	0.567	0.386	0.817	173	0.263	0.952
9156	0.490	0.668	1.005	1.066	0.701	0.604	1.281	0.803	170	0.261	0.949

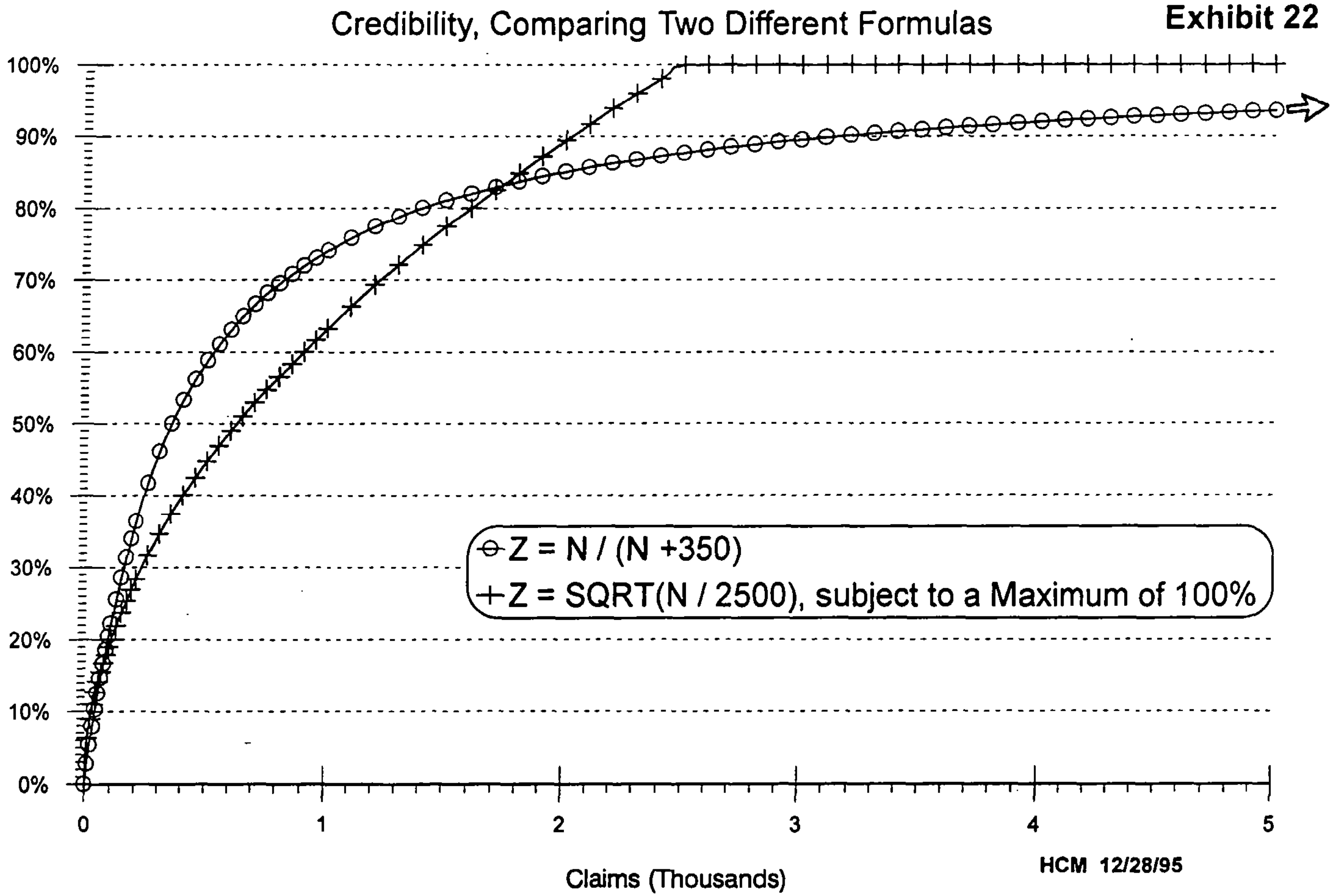
(2)-(8): *Calculated as per Exhibit 19.*
(9): Seven Years of relative average claim costs are combined by taking a weighted average using claim counts as weights.
(10): Total of Seven Years of claim counts.
(11): Credibility = square root of (7-yrs-claim-count by class / 2,500) limited to unity.
(12): Relative Average Claim Costs are credibility weighted with unity.

Exhibit 21



"Classical Credibility", with Full Credibility Assigned to 2500 Claims
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
0
500
1000
1500
2000
2500
3000
Claims
—Z = SQRT(N / 2500), subject to a maximum of 100%
HCM 12/28/95

Exhibit 22



Credibility, Comparing Two Different Formulas
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
0
1
2
3
4
5
Z = N / (N +350)
Z = SQRT(N / 2500), subject to a Maximum of 100%
Claims (Thousands)
HCM 12/28/95

Errata and Additional Material Related to "Accounting for Risk Margins" by Stephen W. Philbrick, FCAS

Originally published in the CAS *Forum*, Spring 1994 Edition,
Including Selected Papers from the
1994 Variability in Reserves Prize Program

TO CAS MEMBERS,

In the Spring 1994 edition of the CAS Forum, is the paper "Accounting for Risk Margins". That paper has been read by a number of people who have identified a few areas where formulas or numbers are either in error or potentially misleading. While several people brought this to my attention, and I am grateful to each for identifying these items, I would particularly like to thank Andrew Rippert who brought most of these items to my attention.

The following four pages summarize the appropriate corrections. On the first three pages, I have explained in narrative form most of the suggested changes. In particular this narrative contains the intellectual justification for an alternative formula for NRM_t that may be more intuitive to some.

One the last page, I have provided a quick summary which can be used as a reference, or used to make corrections to the original text.

My apologies to any who were misled by any of the errors, and my thanks again to those that took the time to bring these issues to my attention.

Stephen W. Philbrick

Errata and Additional Material related to Accounting for Risk Margins,
<u>Stephen W. Philbrick, Casualty Actuarial Society Forum, Spring 1994</u>

On page 26 and in footnote 12 on page 27, there is a reference to the coefficient of variation of the assumed aggregate distribution. The CV value used to create the example is shown as .128, which represents the actual value rounded to three decimal places. Anyone attempting to reproduce the calculations may prefer to use the value carried out to more decimal places. The assumed value of the CV is .12848, to five decimal places. (I created the portfolio from a number of individual risks, each of which had "round" values for the mean and CV. However, the individual risk detail was not relevant to the rest of the paper, so I omitted the details of the calculation of the portfolio parameters.)

Similarly, on page 36 (and subsequent calculations) the factor used to calculate the total asset need is shown to three decimal places as 1.233. This factor, carried out to six decimal places, is 1.233475. This factor is not shown explicitly on page 27, but is used to calculate the value of $359.42.

On page 27, the footnote contains two formulas. The first is stated as:

$$\int_A^\infty (z-A)dF = .003$$

This formula is correct under the assumption that the distribution has been normalized such that the expected losses are equal to 1.0. A more general formula is obtained by multiplying the right side of the equation by the mean losses. Alternatively, the mean loss amount could be placed in the denominator of the left side. Thus, we solve for A such

that: $$\int_A^\infty (z-A)dF = .003 \int_0^\infty z\,dF$$

or: $$\frac{\int_A^\infty (z-A)df}{\int_0^\infty z\,dF} = .003$$

The second formula contains an alternative representation. Unfortunately, the limits of integration were shown incorrectly. The limits of integration should be zero to A. In addition, to make the formula general rather than normalized, multiply the right side by the mean:

$$\int_0^A z\,dF + A[1-F(A)] = .997 \int_0^\infty z\,dF$$

On page 37, the formula shown as: $$NRM_t = (ROR - i)\sum \frac{BRM_t}{(1+ROR)^t}$$

Should be shown as:

$$NRM_t = (ROR - i)\sum_{j=t}^{\infty}\frac{BRM_j}{(1 + ROR)^{j+1}}$$

This formula is the easiest one to use in practice. However, an alternative formulation is easier to understand conceptually. This formula is written as:

$$NRM_t = (ROR - i)\sum_{j=t}^{\infty}\frac{SRM_j}{(1 + i)^{j+1}}$$

This formula can best be understood by thinking about the process of establishing an insurance company to take on this specific risk. The insurance company will need an amount over and above the mean(discounted) losses to account for the possibility that actual losses exceed the expected. The amount in addition to the mean losses will be contributed by both the investor and the insured, such that the investor can earn a fair rate of return on the investment.

Assuming that the insured will pay the mean losses, we now examine how the amount over and above the mean losses should be apportioned between the two parties. The investor will contribute an initial amount of surplus, SRM_0, into the company at inception. However, our losses are not fully extinguished by the end of the first year, so our investor is committed to leaving surplus in the company in subsequent years.

The surplus commitment is represented by the string of surplus values, SRM_j. (Ignore the denominator for the present.) While the investors surplus commitment is in the future for all years other than SRM_0, the loss amounts used in the calculation of the SRM_j have been calculated by discounting future losses back to time zero, so the implied surplus value is the present value of the future surplus commitment. The investor wishes to earn a return of ROR on the investment, so we must pay the investor a total of ROR times the present value of the surplus commitment. However, the surplus placed in the company will earn investment income at rate i, so we can reduce the amount required to be paid by the policyholder by this amount, hence the (ROR-i) factor. Finally, the amount paid by the policyholder (NRM_0) will be paid into the company at time zero, and this amount will earn investment income over its life. Some of the narrow risk margin will be returned to the investor at each year end, so the calculation of the investment income earned over the life of the policy is not straightforward. However, it works out that the adjustment for investment income earned by NRM_0 can be handled by dividing the SRM_j values by $(1+i)^{j+1}$

It is tempting to presume that this factor in the denominator is the factor to discount the surplus requirements back to time zero, in which case the exponent seems wrong. However, each SRM_j value is ultimately derived from L_j, which represents unpaid losses discounted back to time zero. Instead, this factor accounts for the investment income earned on the narrow risk margin over its lifetime.

On page 38, the following four quantities are shown:

$BRM_0 = .233$

$L_0 = \$131.00$

$NRM_0 = \$16.14$

$SRM_0 = \$114.87$

The last two are correct, but the first two quantities should have been shown as:

$L_0 = \$561.09$

$BRM_0 = .233 \times L_0 = \$131.00$

On page 44, in the section labeled **"Balance Sheet (Year X+1)"**, there are references to BRM_1, NRM_1, and SRM_1. These should be references to BRM_2, NRM_2, and SRM_2, respectively.

On page 79, the statement is made that $P = E(z) + R$. In the middle of the page it says "On average (or over the long run), the company will pay E(z), leaving profits of R on capital of C. Thus:

$$\frac{R}{C} = \text{return on capital"}$$

This statement is correct if we ignore investment income. The inclusion of investment income does not affect E(z), because we have defined our loss variable to be on a discounted basis. However, part of the return to the investor arises from investment income earned on surplus, as well as investment income earned on the Narrow Risk Margin. Denote these as ii_S and ii_R, respectively. Consequently, it would be more accurate to state that "On average (or over the long run), the company will pay E(z), leaving a profit consisting of R plus ii_S plus ii_R. Thus:

$$\frac{R + ii_S + ii_R}{C} = \text{return on capital"}$$

Summary of Changes and Additions

Location	Shown As	Should be
On page 26 and in footnote 12 on page 27, reference to CV	.128	.12848
Expected Deficit Assumption, page 27	2%	.3%
Factor used to calculate the total asset need, page 36 (and other pages)	1.233	1.233475
First Expected Deficit formula in Footnote 12	$\int_A^\infty (z-A)dF = .003$	$\int_A^\infty (z-A)dF = .003 \int_0^\infty zdF$
Second Expected Deficit formula in Footnote 12	$\int_A^\infty zdF + A[1-F(A)] = .997$	$\int_0^A zdF + A[1-F(A)] = .997 \int_0^\infty zdF$
Formula on page 37	$NRM_t = (ROR - i)\sum \frac{BRM_t}{(1+ROR)^t}$	$NRM_t = (ROR - i)\sum_{j=t}^{\infty} \frac{BRM_j}{(1+ROR)^{j+1}}$
Equivalent formula to the one shown on page 37	Not shown	$NRM_t = (ROR - i)\sum_{j=t}^{\infty} \frac{SRM_j}{(1+i)^{j+1}}$
Quantities on page 38	$BRM_0 = .233$ $L_0 = \$131.00$	$L_0 = \$561.09$ $BRM_0 = .233 \times L_0 = \$131.00$
Variables on page 44	BRM_1 NRM_1 SRM_1	BRM_2 NRM_2 SRM_2
Formula on page 79	$\frac{R}{C}$ = return on capital	$\frac{R + ii_S + ii_R}{C}$ = return on capital

Loss Estimates Using S-Curves: Environmental and Mass Tort Liabilities

by Bruce E. Ollodart, FCAS

Loss Estimates Using S-Curves

Environmental and Mass Tort Liabilities

Abstract

This paper discusses the application of S-Curve modeling for estimating certain environmental and mass tort liabilities. Emphasis is placed on pollution and asbestos liabilities, which are a significant component of the total environmental and mass tort liabilities for many insurance companies and manufacturers. The general concept of S-Curve modeling is discussed, followed by a technical discussion explaining its application to asbestos and pollution liabilities. Included are comments on the advantages and disadvantages of the technique.

Biography

Bruce Ollodart is a consulting actuary with the firm of Arthur Andersen LLP, and works out of their Hartford, Connecticut office. A Fellow of the Casualty Actuarial Society and Member of the American Academy of Actuaries, he has responsibility for Andersen's environmental and mass tort knowledge base in the actuarial and insurance services area.

Acknowledgments

Special thanks go to Bernard Gilden and David Rosenzweig for programming the computer models and to Robert Bear and Gus Krause for their helpful comments and peer review of the earlier drafts.

Loss Estimates Using S-Curves

Environmental and Mass Tort Liabilities

Introduction

Manufacturers, their insurers and reinsurers, as well as many other commercial enterprises have environmental and mass tort liabilities that must be estimated and managed. Such liabilities arise from many sources including environmental pollution, asbestos, medical implants, carcinogenic toxins, lead, radiation and other toxic exposures. Typically, these liabilities can be characterized by a historical period of exposure to a substance or process that produces latent health problems or property conditions that result in legal liabilities for bodily injury and/or property damage. The latency period can be many years, adding to the difficulty of estimating the exposure. For example, a chemical manufacturer legally dumped toxic wastes from 1940 to 1975 and then became legally liable for the property damage caused by these wastes as a result of 1980 superfund legislation. Similarly, a medical device manufacturer made artificial mandibular joints that were implanted in thousands of patients and later stopped sale of the devices once it was discovered they produced serious side affects for which the manufacturer was held liable.

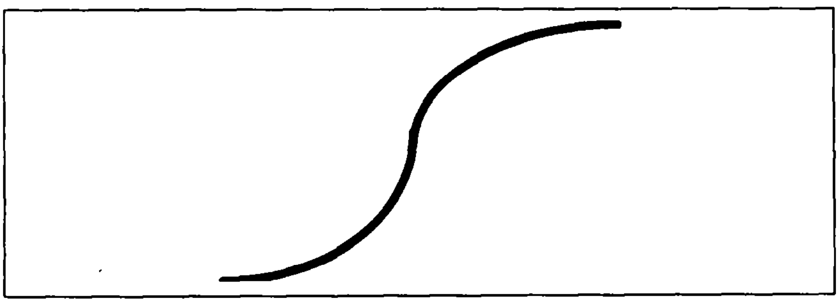
Environmental and mass tort liabilities typically arise suddenly as a result of long term exposure to a given agent or process (for example, asbestos or dumping industrial waste). Problems with data, including lack of historical precedents, poorly defined exposure periods, and improper data capture are common difficulties of estimating the value of these liabilities. Often, only calendar year data is available. Pollution claims, for example, have been attributed to multiple accident or policy periods by court decisions. Estimating the ultimate liability for these claims is often not feasible using traditional actuarial techniques, and highly sophisticated procedures involving a large number of claim by claim reviews are expensive and so time consuming that once performed, cannot be easily updated, but can quickly become outdated due to legislative and judicial changes.

The S-Curve approach, because it assumes a general pattern for loss emergence, can overcome many of these problems, is easy to apply, and can be updated readily as new information becomes available. As demonstrated in this paper, the S-Curve is a projection technique that has many of the characteristics of traditional loss development techniques.

S-Curves have been proposed by other actuaries as a method for evaluating pollution liabilities. However, technical difficulties with the sensitivity of the underlying assumptions halted most serious pursuits in this area. This paper provides techniques for overcoming these problems and increasing the objectivity, flexibility, and usefulness of the S-Curve approach for actuarial analysis.

Background

S-Curves can be used to analyze cumulative distributions for paid losses, reported losses, and claim counts. For purposes of this discussion, S-Curves will represent cumulative calendar year amounts for paid losses. The techniques and assumptions used work equally well for other cumulative forms of data. S-Curves have the following general shape:



The x-axis represents time, while the y-axis represents the cumulative amount paid. As a cumulative distribution, the first half of the curve indicates an accelerating rate of payment up to the inflection point of the curve, then the incremental payments begin to taper off and eventually stop. For a given S-Curve equation, the inflection point will be the point at which the first derivative reaches its maximum value and the second derivative changes sign. Depending on the type of exposures modeled, the representative S-Curve can be very steep in the center or almost flat. The particular S-Curve that best fits a company's historical data will depend on several factors including the length of exposure, the beginning period of exposure, the claim settlement practices of the company, the time since claims were first reported, and the legal process that affects policy coverage.

S-Curves can effectively represent the pattern of emergence for environmental and mass tort claims. A typical scenario involves detection of a health problem and/or a property condition, discovery of the agent or process that caused the situation, a period of statutory and legal developments that establish legal liability regarding the agent or process, an exodus from the production of the agent or process, a period in which policyholders and their insurers find themselves reacting to mounting claims activity related to the agent or process, a change in insurer coverage (usually eliminating future exposure to these claims), a period of increasing reserves and loss payments, then a long period of run-off of these claims. In terms of cumulative calendar year paid loss activity, it is easy to picture the resulting S shaped curve such scenarios produce.

S-Curve Functions

Previously, it has been suggested that the arc-tangent curve, because of its S shape and finite tail, be used for modeling purposes. Our research has determined that the arc-tangent is not flexible enough for environmental and mass tort liability modeling purposes. An alternative family of S-Curves based on power and gamma functions works much better and provides much more flexibility in curve selection. In this paper we deal primarily with the power functions, as they are easier to model. An example of a gamma function application is included for reference.

The general form of the power function is:

$$y=s(x-b)^p+c$$

The dependent variable y represents the cumulative paid losses, s is a scalar coefficient greater than zero; x is the year of projection (or year corresponding to the historical data), b represents the time at which the curve's inflection point occurs, p is an odd power between zero and one, and c is a constant representing the projected cumulative paid loss at time b.

The power p is typically chosen from among the family of fractional powers 1/3, 1/5, 3/5, 1/7, 3/7, 5/7, 1/9, etc. Testing of the various powers indicates that a few of them can adequately represent most of the S-Curves required for analyzing environmental and mass tort data. It is not necessary to fit all possible values of p. In our models, we fit approximately ten different values of p and select the best fits from among them.

When x is less than b, the odd power returns a negative value. When x equals b, the value of y is equal to c, which occurs at the inflection point. When x is greater than b, which occurs after the inflection point, the difference between x and b is positive. These relationships give the curve its S shape.

The s parameter determines the change in height of the curve for each time increment, and p determines the shape of the curve.

A positive c parameter is a constant that brings the curve above the x-axis and is selected such that y is equal to zero at the beginning period of claims emergence. For example, if c equals zero, then b, the inflection point, would occur where y equals zero (that is, the x-axis would cut the curve at b).

The power curve does not converge for large values of x. Therefore, a maximum number of years of run-off must be selected. Otherwise, the model will produce an infinite ultimate loss. We select our maximum number of run-off years at a point when incremental changes in the S-Curve become small, typically after about 30 years for pollution and 20 years for asbestos, a runoff period that we feel is reasonable based on other factors.

Power curves are symmetrical around the inflection point, a property that is useful when the inflection point is not observable in the data. A gamma function can be derived that is asymmetrical around the inflection point providing added flexibility to the curve fitting process.

Several actuaries have suggested fitting curves to the incremental paid data. The first derivative of the power curve, dy/dx, is given by the following equation and represents the shape of the curve corresponding to the calendar year incremental paid losses:

$$y' = ps(x-b)^{p-1}$$

This is a bell shaped curve that has an undefined value at its inflection point (where x equals b) when p is less than one. This implies that curve fitting using the incremental data cannot be achieved for the power curve for values of p less than one, as no value of b will minimize the squared error for the fit in these cases. Curve fits using other types of functions (gamma, lognormal) may work on incremental data.

Fitting S-Curves

To fit an S-Curve, numerical methods are used in our model. By minimizing the sum of the squared errors between the fitted curve and the historical data, a numerical algorithm is used to determine the best fitting parameters s, b, and c. As noted above, approximately ten values of p are selected and separate fits are made for each p value. The fit is performed on the cumulative data. Depending on the relationship between the data and the fitted S-Curve, this approach may give more weight to the squared error in the most recent data points as these points will contain the cumulative errors from all prior years. We believe this has a positive influence on the fit as it helps minimize error in the most crucial part of the curve (the most recent points). That is, precedence is given to minimizing the cumulative error over minimizing error for all points on the curve.

The S-Curve, depending on the value of p, can be very sensitive to the selection of the b parameter. To make the selection of b less subjective, we constrain the numerical algorithm as follows:

1. The year in which y first becomes positive is fixed based on the earliest date that the losses are first paid. This gives the curve a realistic starting point. This point can be varied plus or minus a few years to improve the goodness of fit, but should be within a reasonable range of the known starting date.

2. The value of b is constrained to be at least four years after the year in which payments are first made. This constraint keeps the algorithm from selecting b unreasonably close to the starting date, an outcome that may minimize squared errors but is not reasonable for projection purposes. The four year period should be used as a guide, as varying the parameter value may provide improved fit without sacrificing reasonability.

3. The parameters s and c must be positive.

For a given value of p, the other parameters are selected, subject to the above constraints, such that the sum of the squared errors is minimized.

Once a series of S-Curves have been fitted to the historical data, the best fits must be selected. Standard measures of goodness of fit do not work well with S-Curves because of their non-linearity. We developed several relative goodness of fit tests. These tests, along with graphical representations of the fit, help to determine which S-Curves provide the best fit to the data. Two of these tests are as follows:

$$R_1=\Sigma(y_f-y_d)^2/\Sigma y_f^2$$

$$R_2=1-\Sigma(\ln y_f-\ln y_d)^2/\Sigma(\ln y_d-\Sigma\ln y_d/n)^2$$

The variable y_f indicates fitted values, y_d indicates data values, and n is the number of data values in the fit. R_1 compares the squared error of the fitted values to the squared fitted values, with lower values indicating better relative fit. R_2 compares the squared error of the natural logarithms of the fitted values from the data to the squared error of the natural logarithms of the data from the average, with higher values indicating better relative fit. A third alternative, based on the R_1 statistic, is to use an absolute difference in the numerator of R_1 instead of a squared difference and drop the square in the denominator, with lower values indicating better relative fit.

In practice, we have experienced problems where two fits of the same data using the same value of p both minimize the squared error. This may occur when the data does not fit a particular S-Curve well, is extremely volatile, or is too immature. In such cases, there is enough "slack" in the shape of the curve to obtain more than one best fit. This is caused by some interdependence between the b and c parameters where, for certain data sets, several combinations of b and c can result in minimized squared error. Our numerical algorithm stops when it finds the first of these solutions. To address this limitation, we run our numerical algorithm twice. The first run determines an initial set of parameters. The second run uses the output of the first run for seed values. In almost all cases, the second fit is either identical to the first fit or is improved and subsequent fittings do not yield improved results. This approach essentially eliminates the "slack" problem.

In the final selection process, actuarial judgment must be used to determine which fits best represent the data and are reasonable for the purpose(s) intended. We typically select the best two or three fits from our analysis to determine a range of ultimate values. Consideration is also given to the quality of the underlying data and its applicability for extrapolation into the future.

Examples Using Insurance Industry Data

Power Function

To show how the S-Curve model utilizing a power function performs using actual data, we have prepared examples based on insurance industry pollution and asbestos claim information. This data is based on information from a select group of companies and does not represent an industry-wide composite. Exhibits 1 and 2 show these results for asbestos and pollution claims, respectively. The input data, the results of the numerical algorithm, best fit statistics, graphical representations of the fit, and resulting estimates of ultimate loss are shown on the exhibits.

Observations regarding these examples include:

1. The curve fits are still showing fairly large payouts at the end of our projection period. The length of the projection period could be lengthened, the curve forced to zero over a period near the end of the selected projection period, or the curve can be truncated as in our example. In certain cases, the present value of loss payments beyond our projection period will not be significant.

2. The asbestos and pollution paid losses through 1995 in the projection are approximately 60 and 40 percent of the projected ultimates, respectively.

3. The fit statistics are based on 1981 to 1995 and 1984 to 1995 for asbestos and pollution, respectively. This period was selected for practical reasons to reflect differences in the emergence of asbestos and pollution and to emphasize goodness of fit over a certain period of years. It may be more appropriate to test goodness of fit over the entire data set or a different portion of the data set depending on the application.

Gamma Function

There are cases where use of a gamma function may improve the fit or at least offer a good alternative to the power function. In practice, we found the power function to be reasonable in most cases. Cases that may be improved using a gamma function usually involve asymmetrical S-Curve shapes where the data is already fairly mature and an inflection point is clearly visible in the data. One form of the gamma function used was

$$y(\sigma,\iota)=\Gamma(\tau,\alpha,\lambda)+c=\int_0^{\tau} \lambda^{\alpha}\, x^{\alpha-1}\, e^{-\lambda x}\, dx + c$$

where λ is a scalar, α is the shape parameter, c is a constant, ι is the initial year of payment, σ is the projection year, and τ represents the number of years from the first year of payment to the projection year plus one (e.g., if the initial year of payment is 1980 and you are estimating the 1995 value, then ι is 1980, σ is 1995 and $\tau = \sigma - \iota + 1$ is 16). Both λ and α must be greater than zero. Parameters α and λ have roles in the gamma function that are comparable to the corresponding parameters p and s in the power function. The c parameter is included to improve the fit in certain cases and is optional. The inflection point for this gamma function is given by $(1-\alpha)/\lambda$, as determined by setting the second derivative equal to zero and solving for τ.

On Exhibit 3, we show a gamma function S-Curve fit to the asbestos data used in Exhibit 1. The parameter c produces a disjointed looking change in the fit near the beginning years but improves the overall fit for the latter years. The curve turns faster in the projection years than the power curve used in Exhibit 1 and runs off fairly well during the truncated projection period. The fit statistics are also comparable in quality to the power curve.

Advantages and Disadvantages of the S-Curve Approach

The following lists are based on practical application of the model as well as feedback we have received from other actuaries. The advantage or disadvantage of using this approach is dependent on the type of application involved.

The advantages of the S-Curve approach include:

1. Uses readily available data
2. Is a pure actuarial approach in the sense that it does not have to depend on claim department estimates

3. Comparable to a loss development approach as it performs aggregate loss projections rather than individual claim or policy projections
4. Can be used with paid and reported data for both dollars and counts
5. Is easy to update with more current information as the data matures
6. Provides a basis for testing the sensitivity of key assumptions including judgment concerning future changes in judicial or legislative practices
7. Can be performed fairly quickly
8. Appears to produce reasonable results for many environmental and mass tort liabilities
9. Does not require analysis and testing of a large number of assumptions and variables

The disadvantages of the S-Curve approach include:

1. May be impossible to select best fitting curves with a reasonable range of outcomes
2. Some data sets will be too immature for valid application of the model
3. Comparable to loss development methods applied to new lines of business - the ultimate pattern of runoff for the tail remains uncertain until the data becomes fairly mature

Exhibit I
Sheet 1

Asbestos Indemnity and Expense
Cumulative Paid Loss
Based on Selected Insurance Industry Data

First Year of Loss Payments = 1979
Power Curve 7
(000's)

CY	(1) Fitted Calendar Yr Cumulative Paid Loss	(2) Actual Calendar Yr Cumulative Paid Loss	(3) Fitted Calendar Yr Incremental Paid Loss	(4) Actual Calendar Yr Incremental Paid Loss
1973	0		0	
1974	0		0	
1975	0		0	
1976	0		0	
1977	0		0	
1978	0	362	0	
1979	57,426	17,918	57,426	17,556
1980	117,252	33,987	59,826	16,069
1981	179,775	84,014	62,523	50,026
1982	245,358	193,596	65,584	109,583
1983	314,454	258,994	69,095	65,397
1984	387,632	284,030	73,178	25,037
1985	465,635	324,534	78,003	40,504
1986	549,452	374,068	83,818	49,534
1987	640,459	612,636	91,007	238,568
1988	740,659	752,146	100,199	139,509
1989	853,169	898,011	112,511	145,866
1990	983,338	1,026,623	130,169	128,612
1991	1,141,855	1,259,167	158,517	232,543
1992	1,357,474	1,585,463	215,618	326,296
1993	2,095,513	2,078,939	738,040	493,476
1994	2,591,167	2,470,635	495,654	391,696
1995	2,802,383	2,835,848	211,216	365,213
1996	2,959,027		156,644	
1997	3,088,106		129,080	
1998	3,199,887		111,781	
1999	3,299,556		99,669	
2000	3,390,156		90,599	
2001	3,473,648		83,492	
2002	3,551,384		77,736	
2003	3,624,338		72,954	
2004	3,693,242		68,904	
2005	3,758,660		65,418	
2006	3,821,037		62,378	
2007	3,880,734		59,697	
2008	3,938,045		57,311	
2009	3,993,215		55,170	
2010	4,046,451		53,236	
2011	4,097,929		51,478	
2012	4,147,800		49,870	
2013	4,196,194		48,394	
2014	4,243,225		47,032	
2015	4,288,996		45,770	
2016	4,333,593		44,598	
2017	4,377,097		43,504	
2018	4,419,578		42,481	
2019	4,461,100		41,522	
2020	4,501,720		40,620	
2021	4,541,489		39,770	
2022	4,580,456		38,967	
2023	4,618,663		38,207	
2024	4,656,149		37,486	
2025	4,692,951		36,802	
2026	4,729,102		36,151	
2027	4,764,633		35,531	

Asbestos Indemnity and Expense
Cumulative Paid Loss
Based on Selected Insurance Industry Data

Exhibit I
Sheet 2

First Year of Loss Payments = 1979
Power Curve 7

Fulcrum Year* =	1993

Fit Statistic 1

CY	(1) Observed Data	(2) Fitted Data	(3) Squared Difference
1981	84,014	179,775	9,170,196,298
1982	193,596	245,358	2,679,308,189
1983	258,994	314,454	3,075,820,634
1984	284,030	387,632	10,733,322,993
1985	324,534	465,635	19,909,448,322
1986	374,068	549,452	30,759,505,110
1987	612,636	640,459	774,112,539
1988	752,146	740,659	131,949,576
1989	898,011	853,169	2,010,788,515
1990	1,026,623	983,338	1,873,600,295
1991	1,259,167	1,141,855	13,761,879,071
1992	1,585,463	1,357,474	51,978,969,483
1993	2,078,939	2,095,513	274,716,290
1994	2,470,635	2,591,167	14,528,079,463
1995	2,835,848	2,802,383	1,119,904,087
Total			162,781,600,866
	(4) Sum of value squared	25,819,226,658,823	
		(5) R_1:	0.006

Fit Statistic 2

CY	(6) Log of Observed Data	(7) Log of Fitted Data	(8) Squared Difference From Fitted	(9) Squared Difference From Mean
1981	11.339	12.099	0.579	4.2240
1982	12.174	12.410	0.056	1.4895
1983	12.465	12.659	0.038	0.8638
1984	12.557	12.868	0.097	0.7008
1985	12.690	13.051	0.130	0.4954
1986	12.832	13.217	0.148	0.3156
1987	13.326	13.370	0.002	0.0047
1988	13.531	13.515	0.000	0.0187
1989	13.708	13.657	0.003	0.0986
1990	13.842	13.799	0.002	0.2005
1991	14.046	13.948	0.010	0.4251
1992	14.276	14.121	0.024	0.7787
1993	14.547	14.555	0.000	1.3303
1994	14.720	14.768	0.002	1.7583
1995	14.858	14.846	0.000	2.1430
Total			1.090	14.8469
Ave	13.394	13.526		
			(10) R_2:	92.7%

(1) = Sheet 1, Column 2
(2) = Sheet 1, Column 3
(3) = [(1) - (2)]^2
(4) = sum of square (2)
(5) = Sum of (3) / (4)

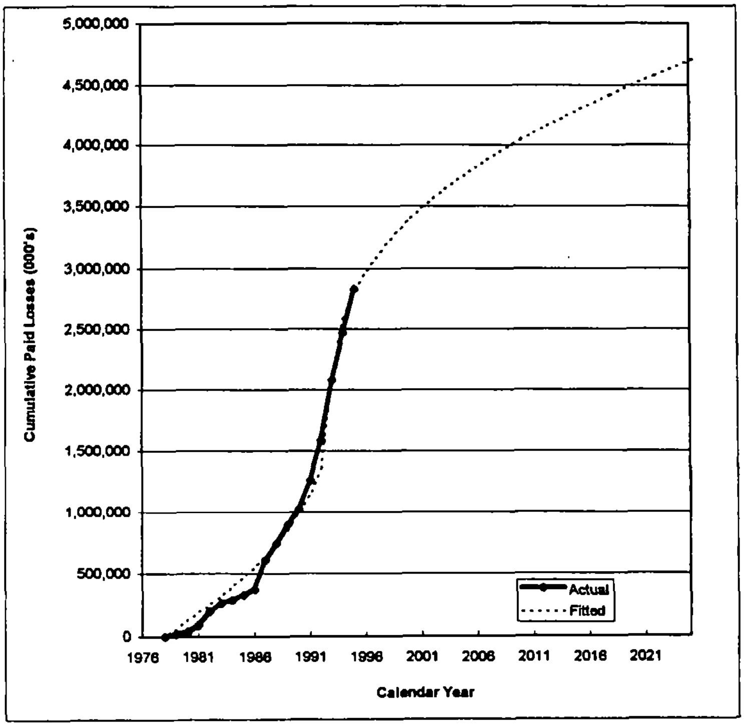
(6) = Natural Log of (1)
(7) = Natural Log of (2)
(8) = [(7) - (6)]^2
(9) = [(6) - Ave of (6)]^2
(9) = 1 - [Sum of (8) / Sum of (9)]

* The fulcrum year is the point in the power curve when the slope changes from positive to negative -- the inflection point.

Exhibit I
Sheet 3

Asbestos Indemnity and Expense
Cumulative Paid Loss
Based on Selected Insurance Industry Data

Power Curve 7



Pollution Indemnity and Expense
Cumulative Paid Loss
Based on Selected Insurance Industry Data

Exhibit II
Sheet 1

First Year of Loss Payments = 1984
Power Curve 9
(000's)

CY	(1) Fitted Calendar Yr Cumulative Paid Loss	(2) Actual Calendar Yr Cumulative Paid Loss	(3) Fitted Calendar Yr Incremental Paid Loss	(4) Actual Calendar Yr Incremental Paid Loss
1973	0		0	
1974	0		0	
1975	0		0	
1976	0		0	
1977	0		0	
1978	0		0	
1979	0		0	
1980	0		0	
1981	0		0	
1982	0		0	
1983	0	135,953	0	
1984	160,048	172,946	160,048	36,994
1985	326,616	222,134	166,567	49,188
1986	500,739	407,273	174,123	185,139
1987	683,772	579,370	183,034	172,097
1988	877,553	914,273	193,780	334,903
1989	1,084,678	1,150,537	207,126	236,263
1990	1,309,057	1,410,354	224,379	259,817
1991	1,557,103	1,613,107	248,045	202,753
1992	1,840,889	1,951,047	283,786	337,940
1993	2,189,868	2,334,475	348,979	383,428
1994	2,791,813	2,779,049	601,944	444,574
1995	3,557,863	3,373,188	766,050	594,139
1996	3,914,670		356,807	
1997	4,202,122		287,453	
1998	4,452,431		250,309	
1999	4,678,395		225,964	
2000	4,886,715		208,319	
2001	5,081,439		194,724	
2002	5,265,244		183,805	
2003	5,440,014		174,770	
2004	5,607,135		167,121	
2005	5,767,665		160,530	
2006	5,922,432		154,768	
2007	6,072,103		149,670	
2008	6,217,219		145,116	
2009	6,358,232		141,013	
2010	6,495,522		137,290	
2011	6,629,411		133,889	
2012	6,760,177		130,766	
2013	6,888,061		127,884	
2014	7,013,275		125,213	
2015	7,136,002		122,728	
2016	7,256,409		120,407	
2017	7,374,642		118,233	
2018	7,490,832		116,190	
2019	7,605,098		114,266	
2020	7,717,546		112,449	
2021	7,828,275		110,729	
2022	7,937,373		109,098	
2023	8,044,921		107,548	
2024	8,150,993		106,072	
2025	8,255,658		104,665	
2026	8,358,979		103,321	
2027	8,461,015		102,036	

Pollution Indemnity and Expense
Cumulative Paid Loss
Based on Selected Insurance Industry Data

Exhibit II
Sheet 2

First Year of Loss Payments = 1984
Power Curve 9

Fulcrum Year* =	1995

Fit Statistic 1

CY	(1) Observed Data	(2) Fitted Data	(3) Squared Difference
1981			
1982			
1983			
1984	172,946	160,048	166,353,410
1985	222,134	326,616	10,916,392,646
1986	407,273	500,739	8,735,786,395
1987	579,370	683,772	10,899,804,671
1988	914,273	877,553	1,348,405,992
1989	1,150,537	1,084,678	4,337,339,580
1990	1,410,354	1,309,057	10,261,027,087
1991	1,613,107	1,557,103	3,136,433,492
1992	1,951,047	1,840,889	12,134,793,722
1993	2,334,475	2,189,868	20,911,063,433
1994	2,779,049	2,791,813	162,916,517
1995	3,373,188	3,557,863	34,104,779,838
Total			117,115,096,782
	(4) Sum of value squared	35,572,402,910,470	
		(5) R_1:	0.003

Fit Statistic 2

CY	(6) Log of Observed Data	(7) Log of Fitted Data	(8) Squared Difference From Fitted	(9) Squared Difference From Mean
1981				
1982				
1983				
1984	12.061	11.983	0.006	3.0569
1985	12.311	12.697	0.149	2.2443
1986	12.917	13.124	0.043	0.7955
1987	13.270	13.435	0.027	0.2910
1988	13.726	13.685	0.002	0.0069
1989	13.956	13.897	0.003	0.0215
1990	14.159	14.085	0.006	0.1227
1991	14.294	14.258	0.001	0.2348
1992	14.484	14.426	0.003	0.4553
1993	14.663	14.599	0.004	0.7298
1994	14.838	14.842	0.000	1.0578
1995	15.031	15.085	0.003	1.4939
Total			0.247	10.5100
Ave	13.809	13.843		
			(10) R_2:	97.6%

(1) = Sheet 1, Column 2
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(4) = sum of square (2)
(5) = Sum of (3) / (4)

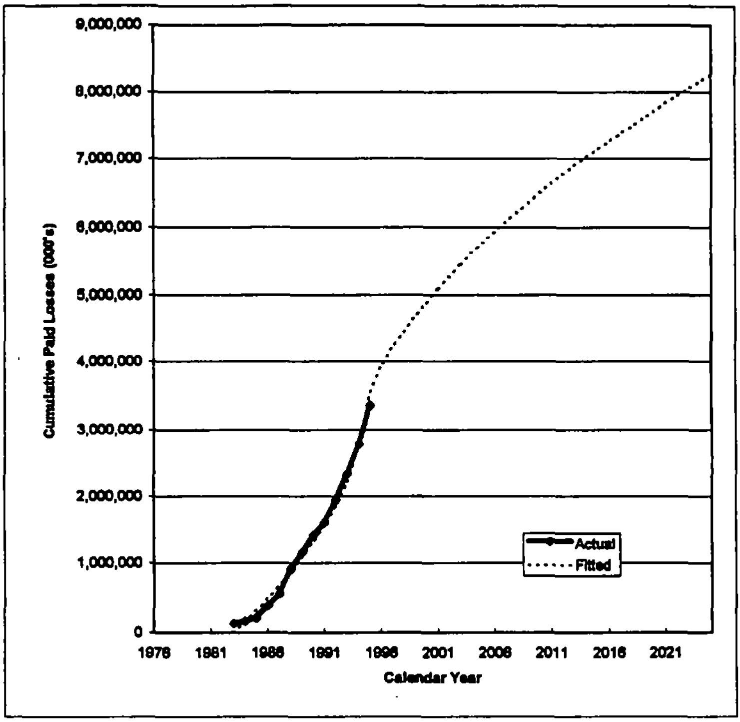
(6) = Natural Log of (1)
(7) = Natural Log of (2)
(8) = [(7) - (6)]^2
(9) = [(6) - Ave of (6)]^2
(9) = 1 - [Sum of (8) / Sum of (9)]

* The fulcrum year is the point in the power curve when the slope changes from positive to negative -- the inflection point.

Pollution Indemnity and Expense
Cumulative Paid Loss
Based on Selected Insurance Industry Data

Exhibit II
Sheet 3

Power Curve 9



Asbestos Indemnity and Expense
Cumulative Paid Loss
Based on Selected Insurance Industry Data

Exhibit III
Sheet 1

First Year of Loss Payments = 1979
Gamma
(000's)

CY	(1) Fitted Calendar Yr Cumulative Paid Loss	(2) Actual Calendar Yr Cumulative Paid Loss	(3) Fitted Calendar Yr Incremental Paid Loss	(4) Actual Calendar Yr Incremental Paid Loss
1973	0		0	
1974	0		0	
1975	0		0	
1976	0		0	
1977	0		0	
1978	0	382	0	
1979	208,151	17,918	208,151	17,556
1980	208,152	33,987	1	16,069
1981	208,193	84,014	42	50,026
1982	208,703	193,596	510	109,583
1983	211,663	258,994	2,960	65,397
1984	222,431	284,030	10,768	25,037
1985	250,870	324,534	28,439	40,504
1986	310,410	374,068	59,539	49,534
1987	414,958	612,636	104,548	238,568
1988	574,924	752,146	159,966	139,509
1989	794,069	898,011	219,145	145,866
1990	1,068,239	1,026,623	274,170	128,612
1991	1,386,219	1,259,167	317,980	232,543
1992	1,732,068	1,585,463	345,849	326,296
1993	2,088,059	2,078,939	355,992	493,476
1994	2,437,385	2,470,635	349,325	391,696
1995	2,766,116	2,835,848	328,732	365,213
1996	3,064,244		298,128	
1997	3,325,877		261,632	
1998	3,548,826		222,949	

CY	(1) Fitted Calendar Yr Cumulative Paid Loss	(2) Actual Calendar Yr Cumulative Paid Loss	(3) Fitted Calendar Yr Incremental Paid Loss	(4) Actual Calendar Yr Incremental Paid Loss
1999	3,733,847		185,021	
2000	3,883,756		149,909	
2001	4,002,598		118,843	
2002	4,094,956		92,358	
2003	4,165,435		70,479	
2004	4,218,322		52,887	
2005	4,257,398		39,076	
2006	4,285,859		28,461	
2007	4,306,314		20,456	
2008	4,320,834		14,520	
2009	4,331,022		10,188	
2010	4,338,093		7,072	
2011	4,342,952		4,859	
2012	4,346,259		3,307	
2013	4,348,489		2,230	
2014	4,349,981		1,492	
2015	4,350,971		990	
2016	4,351,623		652	
2017	4,352,049		426	
2018	4,352,325		277	
2019	4,352,504		179	
2020	4,352,618		114	
2021	4,352,691		73	
2022	4,352,738		46	
2023	4,352,767		29	
2024	4,352,785		18	
2025	4,352,796		11	
2026	4,352,804		7	
2027	4,352,808		4	

Asbestos Indemnity and Expense
Cumulative Paid Loss
Based on Selected Insurance Industry Data

Exhibit III
Sheet 2

First Year of Loss Payments = 1979
Gamma

Fulcrum Year* =	1993

Fit Statistic 1

CY	(1) Observed Data	(2) Fitted Data	(3) Squared Difference
1981	84,014	208,193	15,420,630,711
1982	193,596	208,703	228,217,129
1983	258,984	211,663	2,240,169,393
1984	284,030	222,431	3,794,433,145
1985	324,534	250,870	5,426,301,286
1986	374,068	310,410	4,052,397,325
1987	612,636	414,956	39,077,464,111
1988	752,146	574,924	31,407,582,985
1989	898,011	794,069	10,804,079,680
1990	1,026,623	1,068,239	1,731,853,967
1991	1,259,167	1,386,219	16,142,223,992
1992	1,585,463	1,732,068	21,493,136,653
1993	2,078,939	2,088,059	83,191,157
1994	2,470,635	2,437,385	1,105,540,857
1995	2,835,848	2,766,116	4,862,416,337
Total			157,869,618,729
	(4) Sum of value squared	25,488,773,081,811	
		(5) R_1:	0.006

Fit Statistic 2

CY	(6) Log of Observed Data	(7) Log of Fitted Data	(8) Squared Difference From Fitted	(9) Squared Difference From Mean
1981	11.339	12.246	0.824	4.2240
1982	12.174	12.249	0.006	1.4895
1983	12.465	12.263	0.041	0.8638
1984	12.557	12.312	0.060	0.7008
1985	12.690	12.433	0.066	0.4954
1986	12.832	12.646	0.035	0.3158
1987	13.326	12.936	0.152	0.0047
1988	13.531	13.262	0.072	0.0187
1989	13.708	13.585	0.015	0.0986
1990	13.842	13.882	0.002	0.2005
1991	14.046	14.142	0.009	0.4251
1992	14.276	14.365	0.008	0.7787
1993	14.547	14.552	0.000	1.3303
1994	14.720	14.706	0.000	1.7583
1995	14.858	14.833	0.001	2.1430
Total			1.289	14.8469
Ave	13.394	13.361		
			(10) R_2:	91.3%

(1) = Sheet 1, Column 2
(2) = Sheet 1, Column 3
(3) = [(1) - (2)]^2
(4) = sum of square (2)
(5) = Sum of (3) / (4)

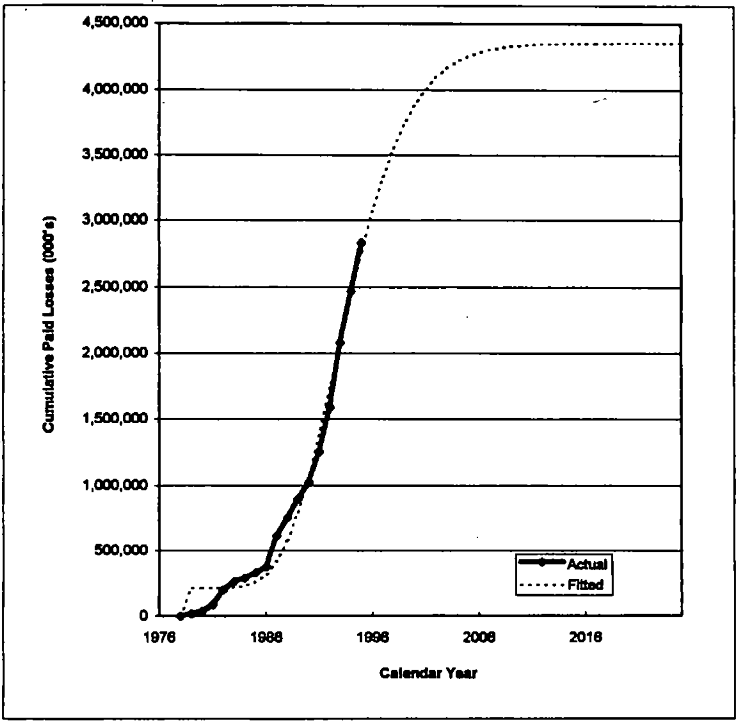
(6) = Natural Log of (1)
(7) = Natural Log of (2)
(8) = [(7) - (6)]^2
(9) = [(6) - Ave of (6)]^2
(9) = 1 - [Sum of (8) / Sum of (9)]

* The fulcrum year is the point in the curve when the slope changes from positive to negative -- the inflection point.

Asbestos Indemnity and Expense
Cumulative Paid Loss
Based on Selected Insurance Industry Data

Exhibit III
Sheet 3

Gamma



Guidance Regarding Management Data and Information

by the CAS Committee on Management Data and Information

INTRODUCTION

The CAS Committee on Management Data and Information has developed a paper entitled, **"Guidance Regarding Management Data and Information"**. The purpose of this paper is to provide guidelines to be used in designing and managing data systems in the following areas: collection of data, ensuring the quality of data, ratemaking reserving, underwriting, marketing, claims, financial analysis and investments.

The Committee is looking for comments from the membership to improve the paper as to its value as well as any suggestions to improve it.

Respectively,

CAS Committee on Management Data and Information

Arthur R. Cadorine   Chairperson
Mark S. Allaben
Randall E. Brubaker
Richard N. Gibson
Holmes M. Gwynn
Larry A. Haefner
Israel Krakowski
Richard W. Nichols

GUIDANCE REGARDING MANAGEMENT DATA AND INFORMATION

The purpose of this paper is to provide guidelines to be used in designing and managing data systems in the following areas: collection of data, ensuring the quality of data, ratemaking, reserving, underwriting, marketing, claims, financial analysis and investments.

Data needs to be managed as a critical resource. Information needed to make business decisions is best when it is timely, accurate, easily obtainable and consistent with the same information produced in other reports. To control the costs of providing this information, data, as well as systems, should not be redundant and it should be consistently defined and shareable.

Actuaries should be making significant contributions in the design and management of systems for collecting data and reporting useful and accurate management information to serve as the basis for sound decision making.

The statement consists of three parts:

I. Data Collection
II. Data Design
III. Management Information Considerations

I: DATA COLLECTION

Data collection can be separated into two areas: Data Capture and Data Quality Control. Data Capture is concerned with the what, when and how of data to capture. Data Quality Control should ensure that the data being captured, processed and reported is accurate, complete, and collected in a cost effective manner.

Before deciding what data elements should be captured, the internal (underwriters, actuaries, accountants, etc.) and external (NAIC, regulators, legislators, statistical agents, etc.) information needs must be determined, and data collection capabilities considered. Current data availability, its quality and the data collection costs need also be considered. While each of the organizations have different requirements for how the data are displayed, the system used to collect the data should be designed with each of the users' needs in mind.

A. DATA CAPTURE

Most data is captured in code. There should be an established code structure incorporating the considerations listed below. The actual use of the data and the cost to collect and store the data also need to be considered. Low redundancy of data, fast processing, flexible access to data and low storage costs may be conflicting design considerations.

1. Data requirements should be compatible and consistent to the extent possible, i.e.: monoline and multiline data.

2. Data elements should be defined to have only one meaning.

3. Common data elements should be defined similarly, regardless of line, business or function supplied.

4. Flexibility should accommodate expansion of data elements to anticipate future needs.

5. Codes should be constructed to meaningfully represent information.

6. Consideration should be given to how often the data will be updated. Will the file be on-line or changed daily, weekly, monthly, quarterly, etc.?

7. Where possible, codes which are established and understood in a wider context should be used.

B. DATA QUALITY CONTROL

Data Quality Control should ensure that the data being captured, processed and reported is accurate, complete and collected in a cost effective manner. Data Quality principles apply to the workflows for getting data into the system, the internal system checks, and the workflows for getting data out of the system.

1. A data quality control function should be established and standards of data quality should be developed and monitored within and across operational areas.

2. Critical processing points should be identified. Control procedures at these points should be developed and documented to assure that data which is transferred, translated or reproduced is complete and accurate, with appropriate backup and audit trails.

3. Edits should be installed to check accuracy, validity and reasonableness. These edits should be performed as closely as possible to the data entry source, and any errors detected should be corrected as closely as possible to the point of discovery of the error.

4. Balancing or reconciliation procedures and standards should be established in the initial project description. Special reports and techniques should be developed to test data accuracy on a selected basis.

5. The monitoring of data quality is an ongoing process. Reconciliation reports and edit error reports should be produced and examined regularly.

6. Changes made to a data field or to processing, must be thoroughly tested in order to assure data integrity is maintained. It is important adequate time is allowed to achieve this objective.

II. DATA DESIGN PRINCIPLES

Data should be managed as a critical resource. To truly control cost, data, as well as systems, should not be redundant; it should be consistently derived, consistently defined and shareable. Numerous data elements can be captured, but they are of limited value unless the data is efficiently organized in a way to maximize the use and value of the information. Every information system should be designed with flexibility to respond to different requests. The following concepts should be considered in the design of the data base.

A. CENTRAL DATA BASE

The ideal repository of data collected is a single central location. Here, all the detail collected could be stored and accessible to all report systems. Thus updates, corrections, and controls could be maintained at one location. Multiple locations of the same data elements require more stringent controls to guarantee that all data bases are updated uniformly.

B. DETAILED DATA BASE

The data base should contain all reported data elements to satisfy the needs of internal and external users.

C. DATA DICTIONARY

The existence and wide availability of a data dictionary will help assure consistency by the various users of a system. Definitions of data elements, as well as lists of codes, should be available to and commonly understood by both the providers and end users of data.

D. DATA BASE DESIGN

The design or organization of the data should address the following considerations:

1. Low redundancy of data, fast processing, flexible access to data, and low storage costs may be conflicting design considerations.

2. Run time, storage costs, volume restrictions or other processing constraints may necessitate the creation of multiple summarized or segmented data bases to fulfill different user needs. The smaller data base(s) enables report generation systems to run faster, since there are many less records to be accessed. A summarized subset of the central data base should incur lower storage cost. Summarized and/or segmented data should be updated automatically from the central source to avoid the potential control problems for updating multiple locations of the same data elements.

E. NON STANDARD REQUESTS

While many reports may be specified to extract information on a regular schedule, data bases should be flexible and organized to facilitate ad hoc report requests as well as direct user access to the central data base and/or segmented data bases.

F. STORAGE

The retention period of data in the data base depends on the number of years of data needed for meaningful analyses, legal and regulatory requirements. The form of storage depends on access requirements, such as immediate access or overnight access.

III. MANAGEMENT INFORMATION CONSIDERATIONS

How the data will be used has an effect on how the data files are managed. The basic use of the data must be well understood when designing file structure and access. Detail appropriate to the intended use of the data base should be stored.

The following examples serve to illustrate the need to capture different data in different levels of detail for different purposes. It is not intended to be a complete list of all possible Management Information System considerations. In evaluating these needs, be sure to consider any requirements for evaluating reinsurance programs.

A. RATEMAKING

There are several acceptable methods of capturing data for ratemaking purposes including calendar year, calendar/accident year, report year or policy year. The nature of the coverage being provided and data availability will determine which is most appropriate. There are three general types of data needed in any ratemaking process:

1. Premium and exposure information which could be on a written or earned basis. Adjustments to premium, such as retro adjustments, experience, schedule or other modifications, should be collected as needed. Information should be organized to monitor changes in the mix of business, such as available by class, territory, policy limit and state within each line or subline of business.

2. Loss and claim information should be collected the the same categories as premiums. In addition, historical loss development patterns of paid and incurred loss amounts, claim counts and loss adjustment expenses are needed to be available to properly estimate their ultimate values and current frequency and severity trends. Changes in the underlying loss distribution are analyzed by reviewing data segregated by size of claim and against different policy limits or deductible levels.

3. Expense information should be available to determine the appropriate provisions for various categories of expenses including unallocated loss adjustment expenses, commissions, other acquisition expenses, taxes, licenses and fees, general administrative expenses and dividends.

Insurance ratemaking takes place in the broad economic environment that affects every business. The ratemaker may supplement internal information with external economic data or industry-wide ratemaking data.

B. RESERVING

Reserves can be categorized as premium or loss reserves. Premium reserves include a variety of subcategories such as unearned premium, earned but not reported, audit, dividend, retro premium reserves, and contingent commission reserves. The techniques and data required to calculate premium reserves vary depending on the subcategory. For example, the unearned premium reserve calculation usually requires only the written premium amount, the appropriate policy effective and expiration dates, and the booking date. For other subcategories, calculations may involve the need for other premium exposure or loss information.

Information needed for the loss reserving function should be sufficient to analyze the essential characteristics of the claim reporting and settlement process. Information is usually organized in a two dimensional matrix that reflects the historical claim process in some way. The correct matching of the matrix to the reserving task is critical to the effectiveness of the reserving function. Each loss reserving matrix is usually defined by: 1) the characteristics of its dimensions, which are time related, 2) its data groupings, and 3) the statistics displayed.

1. Dimensions

   One dimension is usually accident periods, report periods, or policy periods. In other words, losses are grouped according to the date of loss, the date of reporting or the policy effective date.

   The second dimension usually reflects development of maturity levels thereby showing a particular accident or report period's history.

2. Data Groupings

   Groupings can reflect line of business, class, limit, type of loss or geographical location. Data can be configured on a gross, direct, assumed, ceded, or net basis. The degree of refinement should reflect a balancing of the possibly conflicting goals of homogeneity and credibility.

3. Statistics

   Typically, counts and dollar amounts are collected for reserve calculations. They may be displayed either cumulatively or incrementally. Some examples are:

   i. Counts - open/outstanding claims, closed claims with or without

      payment, reported claims, reopened claims.

   ii. Amounts-paid, outstanding or incurred loss and/or allocated loss

      adjustment expense.

   In addition, when evaluating reinsurance reserves, other data items may be useful such as policy retention, layer limit, and codes indicating occurrence or aggregate coverage.

C. UNDERWRITING/MARKETING

Whether the underwriting and marketing functions are handled in one or many departments, their management information needs are similar. Information is needed 1) to monitor and reevaluate marketing objectives and underwriting policy, and 2) to monitor and appraise the performance of individual producers and underwriters.

Areas that might be monitored include the following:

1. Distribution of the current book of business, and how it has changed over time. Trends in premium and loss experience.

2. Underwriting results (including expenses) by type of distribution system (agency vs. brokerage vs. direct mail), if applicable.

3. Amounts of new business, non-renewed business, cancellations, endorsements, renewal changes and hit ratios.

4. Use of experience modifications, dividends, schedule modifications, preferred rating programs, and other individual risk rating modifications to test for conformance to pricing guidelines.

5. Changes in average premium and growth of gross premium.

In each case, the reporting categories should include information on production source (agent, underwriter, branch), line of business, territory, coverage, and class.

D. CLAIMS

Management information required by the claims function generally falls into three areas: 1) claim count transactional data, 2) information on open claims, and 3) information on closed claims. The level of detail required ranges from data by individual claim adjuster to data by unit, branch, region, company, or national. Time periods covered can be weekly, monthly, quarterly, year-to-date, or the latest twelve months. Data generally should be available by type of claim, i.e., line of business, coverage, cause of loss, etc., with identification of catastrophe losses and applicable reinsurance.

1. Claim count information includes the number of claims opened, the number of claims closed with payment, the number of claims closed without payment, the number of claims reopened, and the number reclosed. Appropriate ratios between the various claim counts should be calculated. The average lag between initial reporting, establishment of a reserve, and final payment should be monitored.

2 Information on open claims can include the number of open claims, the number of pending law suits, the amount of reserves and average reserve on open claims by age since opened, the amount of reserves and average reserve on open claims by size of reserve, paid and reserved amounts for allocated loss adjustment expenses, and partial payments on pending claims.

3. Information on closed claims can include average paid claim cost (with comparisons by unit within a branch or region or state), claims closed by size of loss, claims closed by length of time to close, analysis of salvage and subrogation recoveries, and analysis of paid allocated loss adjustment expenses (by type, by adjuster, by law firm, etc.).

E. FINANCIAL ANALYSIS/INVESTMENTS

Management information needed to support the financial analysis and investment function generally breaks down into two areas: cash flow and operating results.

1. In cash flow analysis, the concern is to be able to meet current period obligations. Reports should be available to analyze current cash items such as net premiums collected, net investment income received, cash on hand and on deposit and the maturing assets. Payout of liabilities should be estimated, including expected loss and loss adjustment expenses, commissions, salaries, other expenses, stockholders and policyholders dividends, and interest payable. Besides displaying the above dollar amounts, management reports should provide analysis of trends in the various items to help maximize cash flow in the future.

2. In order to develop and analyze operating results, management information is needed which summarizes all the financial activities of the company. Data is needed which will help the company maximize total return and grow surplus while maintaining an adequate cash flow to meet expected liabilities. The types of information needed should include the following:

    i. Mix of current investments and the related interest and dividend income, including bonds (amortized and cash value), preferred stocks, common stocks, real estate, capital gains, cash, etc.

    ii. Premium income by line of business.

    iii. Loss and loss adjustment expense payments, by line of business projected by calendar year.

    iv. Stockholder and policyholder dividend requirements.

    v. Tax liabilities - Federal and State.

    vi. Expense requirements - commissions, salaries, overhead, etc.

F. FINANCIAL REPORTING

Information is required to meet financial reporting obligations. The information normally includes direct and net calendar period premium, losses, expenses and investment income. The major obligations are:

1. Statutory reporting

2. Trade associations and bureaus

3. Shareholder reporting

4. Income tax reporting

White Paper on Data Quality
by the CAS Committee on Management Data and Information

WHITE PAPER ON DATA QUALITY

The CAS Committee on Management Data and Information is pleased to present this White Paper on Data Quality. This paper presents a discussion of data quality standards applicable to actuaries and insurance data managers; expands on data quality issues faced by actuaries and insurance data managers; and, elaborates on various data quality tools and practices used in preparing actuarial analyses and work products.

This paper is the result of a joint team of insurance professionals representing the Casualty Actuarial Society and the Insurance Data Management Association. The members of the project team are:

Arthur R. Cadorine, Chairperson
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The Committee is indebted to these individuals for the production of this paper, but especially to Richard T. Schulz, who authored most of the material. The Committee thanks all the individuals from both the Casualty Actuarial Society and the Insurance Data Management Association that reviewed various drafts of the paper and provided helpful suggestions and assistance.

The Committee's charge includes furthering the development and dissemination of data management theory and principles; identifying topics for research and discussion; monitoring professional developments and regulatory activities; establishing liaisons with other organizations working in this area; and sponsoring panels, seminars, and other public forums on data management issues.

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WHITE PAPER ON DATA QUALITY

WHITE PAPER ON DATA QUALITY

I. Introduction

A. Data as an Asset

Today, more than ever before, insurers have the ability to tap into the detailed information which they collect as a result of the insurance contract. Access to this information has changed in the last decade due to the rapidly improving capabilities of computer technology, the declining cost of computer hardware & software products, and the expanding knowledge of data systems designers and programmers. The proliferation of the personal computer, compact disc (CD-ROM) storage drives, hookups to local area network (LAN) computer environments and the ever increasing pace of computer chip and data storage technology has allowed access to not only an organization's in-house detailed data but to broad based insurance aggregate data (i.e., industrywide data) and external non-insurance data useful to insurers (e.g., motor vehicle reports, geographic information, construction information).

In addition, the declining role of rate bureaus in the pricing of insurance risks has increased the need of the individual company to rely more on their own internal information in greater detail.

The concept that data is an asset means more detailed management information leading to:

- improved business opportunities (e.g., for marketing purposes);
- greater fraud detection;
- enhanced underwriting review (e.g., via motor vehicle reports);

- greater evaluation of loss control factors or risk management procedures; and,
- greater ability to use the data in actuarial analyses (e.g., for pricing, loss reserve analyses).

The need to protect and enhance the quality of data available for use is self-evident.

B. Data Quality - Actuaries & Data Managers

In July of 1993, the Actuarial Standards Board (ASB) adopted Actuarial Standard of Practice No. 23 - Data Quality. The standard adopted was the result of over three years of discussion by an Ad Hoc Data Quality Task Force of the Specialty Committee of the Actuarial Standards Board. Exposure drafts were circulated and comments solicited from members of the American Academy of Actuaries. The resulting document established a standard which provides greater consistency in actuarial practice with respect to the responsibility of the actuary regarding the quality of the data. The standard also recognizes the diversity of actuarial work, the diversity of data available in that work and the need for judgment dependent upon the intended use of the analysis.

In the summer of 1994, the Insurance Data Management Association (IDMA) formed a working group to develop a data quality opinion framework. The resulting framework and guidelines, entitled "IDMA Data Quality Certification Model for Insurance Data Management" was released on March 9, 1995. This framework is designed to give guidelines to an insurance data manager in order to monitor, measure, and, potentially, certify the quality of data in his/her organization.

Using these two documents, this White Paper attempts to broaden and merge the collective thinking on this subject for insurance actuaries and data managers. Specifically, the paper will discuss issues relating to:

1. the importance of data collection and processing;
2. reviewing the data for appropriateness, reasonableness and comprehensiveness relevant to the analysis undertaken;
3. certifying the accuracy and validity of the data;
4. materiality considerations of imperfect data;
5. the standards and procedures used to determine the extent of imperfect data; and,
6. the responsibility of certification and disclosure.

II. Data Quality Standards

A. Actuarial Standard of Practice No. 23

The stated purpose of Actuarial Standard of Practice No. 23 is to give guidance to the actuary in:

a. selecting the data which underlie the actuarial work product;
b. reviewing these data for appropriateness, reasonableness, and comprehensiveness; and
c. making appropriate disclosures.

The Standard discusses the current practices and historical issues. It then reviews and analyzes alternative practices to determine the recommended practice for an actuary in undertaking actuarial analyses. The Standard recognizes that completely accurate, appropriate, and comprehensive data is not always available. The actuary must understand the intended use of the analysis being performed in order to thoroughly evaluate the appropriateness of the data. In addition, the Standard discusses the selection of the data relevant to the reasonableness and consistency of the necessary data elements, any limitations of the data available, and the cost & feasibility of alternatives (including timeliness considerations).

By comprehensiveness of the data, the Standard refers to the availability of each data element and record needed for the analysis; that doesn't mean that every record is necessary (because a sample of records may suffice for the analysis undertaken) or that every data element in the record needs to be accessible, but it does mean that the necessary records and data elements to do a proper analysis are available.

By appropriateness, the Standard means that the data is:

1) the information needed for the analysis;

2) homogeneous so as to allow evaluation; and,

3) consistent with the purpose of the study.

By reasonableness, the Standard means that it's consistent with prior data or other information.

Taken together, the actuary must ask the following questions.

1) Is all the data necessary for the analysis, in fact, available for use in the analysis?

2) Is the quality of the data appropriate to accomplish the intended purpose of the analysis?

3) Is the data reasonable and consistent with prior data, other homogeneous data sources, and other knowledge?

The Standard leaves open the door that imperfect data may still be usable - but only after careful scrutiny. The key question is: Will incomplete, inaccurate or inappropriate data (i.e., imperfect data) result in material biases in the study's conclusions? If "yes", the data is not usable unless the bias can be quantified; if "no", the data is usable. If "maybe", then further work needs to be done. Effort must be made to identify the nature of the imperfection. Once identified, the imperfect data can be corrected, excluded, or adjusted using an appropriate mathematical or actuarial method (e.g. minimum bias techniques, confidence ranges, distributional adjustments), depending on the extent and nature of the imperfection.

Data with a known imperfection in a field not pertinent to the study undertaken, is not considered imperfect data. If, however, it affects the perception of the credibility of the data in use, the user of the data should be prepared to address the situation.

The Standard discusses the actuary's reliance on data supplied by others and concludes that the data must be accurate and complete for the analysis under study. The data must be reviewed for reasonableness and consistency. This actuarial review of the data will be based on the specific circumstances - the intended use of the data, the data available, extent of known data limitations, timeframes and other factors.

An actuary's review of the data should:

1. determine the extent of checking, verification and auditing done by the data manager/supplier;
2. identify questionable or inconsistent relationships; and,
3. determine the materiality of imperfections on the study's results.

Furthermore, the actuary should comment on the confidence, reliability and the value of the data quality procedures done by the data manager/supplier. Toward that purpose, the extent of audits and control procedures should be reviewed and noted. For instance, if the source data has been subjected to rigorous internal audits or monitoring by a Statistical Data Monitoring System (SDMS), as described later in this paper, then greater confidence in the source data may be assumed. On the other hand, if in the judgment of the actuary greater checking should be performed, then it should be done if practicable.

Standard No. 23 provides a strict disclosure standard in the actuary's report. The report should include disclosures regarding:

1. the sources of data;
2. the materiality of any biases due to imperfect data;
3. adjustments or modifications made because of imperfect data;
4. the extent of reliance on data supplied by others;
5. any resulting limitation on the use of the analysis;
6. any unresolved concerns regarding the quality of the data.

B. IDMA Data Quality Certification Model

The "Data Quality Certification Model for Insurance Data Management" released by the Insurance Data Management Association (IDMA) is intended to provide:

1. a framework for use in attesting to the data quality of an organization; and,
2. guidelines for the insurance data manager to use in controlling, monitoring and measuring the validity, accuracy, reasonableness and completeness of data.

The IDMA Certification Model makes the insurance data manager responsible for developing a commentary on the quality of the data. The commentary should include:

a. disclosure of the results of checks/tests for validity, accuracy, reasonableness and completeness of data;
b. list of the reports and monitoring tools used in ascertaining validity, accuracy, reasonableness and completeness of data;
c. review and analysis of significant data problems using the data monitoring tools;
d. action plan for correcting data problems; and,
e. certifying statement regarding the analysis and commentary.

The commentary should also include an assessment of the materiality of the data elements, including the resulting impacts and error ratios. The IDMA Certification Model holds the insurance data manager accountable for:

a. recognizing that the users are responsible for developing standards (e.g., consistent and reasonable error tolerances);
b. knowing that standards exist; and
c. prompting the establishment of standards when they do not exist.

An actuary's reliance upon an insurance data manager who has followed these practices certainly will provide him/her a degree of confidence in the source of the data.

III. Data Quality Terms

A. Ascertaining Data Quality

Most often, an assessment of data quality consists of an assessment of the following four components listed by the IDMA Certification Model:

- Validity;
- Accuracy;
- Reasonableness; and,
- Completeness.

Validity means that value of a given data element is one of all allowable ones. Data values that are valid are determined by edit checks. The most basic check is known as a field edit. For example, a State code is valid if it is one of the codes allowable under the data element "State". If two digit postal code defines the allowable values for "State" then NY would be a valid value for this data element. Validity checks also include relationship edits involving two or more fields. For example, territory code "10" may be valid in one state but not in another. Valid values are checked through the use of automated edit checks via internal and external edit packages that access tables of allowable values. Error performance reports are typically generated for review. While necessary to the data quality environment, validity checks, by themselves, can only guarantee that the field has an allowable code, not necessarily the correct one.

Accuracy means that each data transaction record or code is a true and accurate representation of what it's intended to represent. In other words, does it accurately reflect the correct information for the policy or claim it represents? A good example to illustrate accuracy is class code; the class code for a florist is accurate if the risk is a florist; it would be inaccurate if the risk were a pharmacy, however it may a valid class code (namely, the code for a florist). How do you know that you have accurate data? To ensure accurate data, a system of effective controls, including periodic audits and sampling checks at all stages of the data collection process must be established. This system of checks can only be accomplished through a thorough understanding of all data handling and collection activity in the organization. Independent comparisons with source documents, validity and other edit checks, as well as periodic audits are essential elements for ascertaining the accuracy of reported data. These essential elements are inherent in rigorous and high quality self monitoring audit programs and in the Statistical Data Monitoring System (SDMS), which are discussed later in this paper; as such, self monitoring audit programs would be a valuable aide in confirming the accuracy of the data.

Another essential component of the assessment of the quality of the data is the concept of reasonability. This component of data quality requires some summarization or aggregation of records in order to determine the data's reasonableness. For example, a single large fire loss may not look unusual by itself, but in the context of hundreds or thousands of large losses it may be an indication of a coding problem. The key questions are: Is the data reasonable compared to our prior and current knowledge? Is it reflective of prior established patterns? For example, does this quarter's territory premium distribution look similar to prior quarters? Does it jibe with our general knowledge about the data? For example, if this year's territory distribution doesn't match the profile, might it be because of a change in the company's marketing or underwriting policies? Distributional analyses and profiles, trend analyses, average rate checks, and loss ratio comparisons are examples of tests to determine the reasonableness of the data.

Completeness of data has three essential elements:

1. each transaction record contains all the necessary data for the business needs for that record (i.e., no information that's necessary or required is left blank);
2. each transaction record is consistently processed once and only once; and,
3. each transaction record is processed properly through every necessary portion of the system and only through those necessary portions.

In other words, complete data can only be realized when every area involved in the data collection and processing process handles it correctly. This requires proper coding at the source and effective controls at each step along the way. Reconciliation of statistical data to financial data helps ensure the completeness of the data since it provides a valid basis for comparison of the information. When material discrepancies arise in reconciliation results, every effort must be made to reconcile the discrepancy and take corrective action if necessary.

<u>B. Accuracy of Data</u>

Usable data can be classified in three levels or degrees of accuracy:

1. Absolute Accuracy;
2. Effective Accuracy; and,
3. Relative Accuracy (i.e inaccurate but consistent over time).

The definition of Absolute Accuracy is simply that the data is 100% correct. There are no known defects in the data. Each and every data element on each and every transaction record is properly and accurately coded. It can be used down to transaction level detail.

Most data are of the Effective Accuracy type where there are some imperfections in the data but are generally usable in most analyses. There are two categories or types of Effective Accuracy. First, where the coding of a specific data element may be incorrect, analyses not involving the incorrectly coded data element (either, in any intermediate calculations, or in the aggregate result) may be unaffected. For example, territory coding may be inaccurate, but for analyses of statewide (all territories combined) data, the data may be suitably accurate for use; however, if territory is used in calculating Premium at Present Rates (PPR) where the rate differs by territory and the analysis involves this calculated premium, then it would affect the statewide analysis. Analyses requiring a high level of detail (either, in the intermediate calculations, or in the aggregate result) need to be accurate enough to that level of detail. A second type of Effective Accuracy is dependent upon whether the imperfect data will materially impact the result. For example, returning to the territory PPR calculation above, if a small amount of territory data (relative to the overall volume included in the analysis) appears to be incorrectly coded, there may be no material effect as to the results of the analysis; on the other hand, this may indicate that there may be substantial unknown data problems. Whether it's an immaterial anomaly in the quality of the data, or an indication of additional unknown data quality problems is what the actuary needs to decide.

Defining Relative Accuracy is a bit trickier. Data coded inaccurately as to its definition but reported consistently over time are data that are relatively accurate. For example, the definition of what's included as allocated loss adjustment expenses (ALAE) may vary by company, and by statistical agent; a company may not strictly adhere to the statistical agent's definition of ALAE in reporting its statistical data, yet the data may be reported consistently over time and with proper recognition can be used in various analyses. An analogous example can be made regarding loss reserving procedures (i.e., case vs. case with a loading). With proper recognition of differences in data definition, relatively accurate data is generally usable. The problem with relatively accurate data is that when a procedural change is instituted the data will no longer be consistent over time.

IV. Data Reliability Tools

A. Reliability of Data & Data Audits

One of the key tools to ascertain the accuracy of the data is periodic auditing. The reliability of the data used in an actuarial work product will be higher if there are periodic and comprehensive internal or external audits of the data quality process.

Besides checking the accuracy and completeness of the data, audits help to:

- ensure consistent handling;
- determine the quality of systems control procedures;
- measure and improve timeliness of data; and,
- increase the reliability of results.

Successful audits, both internal and external, include the following elements:

1. are properly planned;
2. measure results according to established standards;
3. are statistically sound, regarding the sampling technique;
4. perform data checks from source to end product <u>and</u> end product back to source;
5. verify data according to their intended use and definition, including assuring that all data elements resulting from calculations, mappings and other programming algorthyms are correct as intended;
6. audit the data preparation & data entry processes, and reviews all program and output controls (assuring that the input and output data balances, as well as reconciles with prior data processed);
7. determine whether the company's entire process detects errors adequately and corrects them properly; and,finally
8. provide adequate documentation of the results with recommendations for improvement (if any) and follow-up implementation review.

B. Statistical Data Monitoring System (SDMS)

In 1982, the New York Insurance Department, acting on a commissioned analysis by an independent accounting firm, set up a system of procedures designed to control the quality of data submitted to and processed by statistical agents. The objective of this system, known as the Statistical Data Monitoring System (SDMS), is to assure the reliability of the data collection process for statistical data used in statistical and ratemaking filings. SDMS is a self-monitoring system which was adopted not only by the New York Insurance Department but subsequently by the insurance departments of Rhode Island and Connecticut. Currently, the SDMS functions for the Personal Automobile line of insurance, but the procedures inherent in the system can be applied to all lines of insurance.

The System mandates a set of procedures that must be followed by insurance companies and statistical agents. Each company is responsible for various data quality tests and documentation, with each company certifying their own data. Likewise, each statistical agent must collect and summarize specified reports from its reporting companies, carry out specified monitoring system tests and compile documentation. The statistical agents perform data quality checks on their own internal systems, as well as certify their reporting companies' monitoring activities. State regulators have overall responsibility for an effective program.

The Statistical Data Monitoring System (SDMS) has 6 basic components which jointly serve to increase the reliability of the data for statistical, ratemaking and actuarial analyses:

1. process description and review of control procedures;
2. detailed data verification via sampling tests;
3. summary data verification via reasonability reviews;

4. financial reconciliation;
5. annual review and certification;
6. review and evaluation by state examiners.

The first component, the process description and review of control procedures is accomplished by requiring system flowcharts and narratives, using standardized procedural control checklists and reviewing specific checklist functions in detail.

To accomplish the detailed data verification, a random sample representative of the data is taken for both premium and loss claim transactions. For each transaction, every data element is verified. When an error is found, the source and cause of the error are identified and corrective action taken. Sample sizes are determined such that data errors which affect more than 1% of the transactions will be discovered with a 99% probability.

Summary data verification is accomplished through a review for reasonableness of the essential data elements to be used in the actuarial ratemaking review - premiums, losses, claims - by the main components of the review - territory and coverage. The most questionable (or inconsistent) experience is then researched to determine any errors and their cause; if errors are uncovered, corrective action is taken.

As respects data reconciliation, each company must reconcile its statistical data (as reported to its statistical agent) to the company's financial data (reported in the Annual Statement).

Finally, the annual review and certification requires documentation of the monitoring activities conducted and the error incidence statistics of the data. The certification document must be signed by the company's Data Quality Officer.

Taken together, the system provides an effective self monitoring tool which allows state examiners to independently review the data quality of each company's data and the processing of it by the statistical agents. By providing a clear set of procedures, the SDMS system provides a structure on which actuaries and insurance data managers can rely on the quality of data, thereby increasing the accuracy and credibility of actuarial, ratemaking and other statistical data analyses.

V. Professional Responsibilities

A. The Responsibility of the Actuary on Reasonability

Almost all statistical data used in actuarial analyses undergo various validity checks as a matter of routine company or statistical agent procedures. Whether the data is sufficiently accurate, reasonable and complete is generally the key determinant of the quality of the data. While this paper has discussed various ways to monitor and improve the accuracy or completeness of the data, the actuary should be aware of and prepared to perform various additional summary checks, edits and tests designed to determine the reasonableness of the data. In short, a good reasonability review provides the answer to the question: Does the data make sense?

A good reasonability review starts with good judgment based on experience and supplements it with objective measures. First and foremost, does the data look right? For instance, if the actuary is performing a Statewide Rate Level analysis, the resulting current indication should make sense relative to last year's indication after accounting for various differences and factors in the ratemaking formula as well as any known experience changes (such as the effect of a major hurricane on property losses); if it doesn't make sense, then this raises further questions that should be resolved satisfactorily.

Some key tests or checks that should be considered in a review of the reasonableness of the data are:

- distributional edit review;
- consistency checks;
- statistical tests, such as, chi-square goodness of fit tests or non-parametric rank tests;
- graphical tests; and,
- industry comparisons, including reasonable range of results comparisons.

A bird's eye view of the data can be had by reviewing summary data by key field relative to a profile of that data based on prior experience. Known as a Distributional Edit Review (DER), data is compared for consistency to a prior quarter's or year's data. A DER helps detect data anomalies and inconsistencies. An extreme example would be: if coverage is sold statewide (i.e., in all territories) then a data problem resulting from data coded all under territory "001" is easily found. Of course, most data problems are more subtle than this example, so automated statistical tests should be used. For example, chi-square tests between current data and the profile can be used. These automated statistical tests help to provide the best review of the distribution of the data by providing an objective measure of the data elements that seem to match or not match the distributional profile; those with the highest chi-square values fit the distributional profile the least. Although helpful, oftentimes the actuary doesn't have the historical data to perform this type of review on the data; on the other hand, if the provider of the data does perform this type of data review before providing the source data, then the actuary may have improved confidence in the data.

An easier yet more limited check are comparison tests, done by comparing the premium/exposure/loss/claim volumes by the highest order data variables (e.g., state, coverage, year, etc.)

either to each other or to prior reported volumes. If the volumes appear inconsistent across years, or if there are divergent exposure/premium or loss/claim relationships, further review of the data may be necessary.

Range comparison tests, non-parametric rank tests or graphical views of the data can be used to supplement the reviewer's judgment. An example of a range comparison test is a test of premium-to-exposure ratios; these ratios can be compared to average rates in effect and values falling outside a reasonable range (depending on the level of summarization) can be flagged. Used far less often, non-parametric rank tests (like Kendall's Tau or Spearman's Rho) similarly can detect inconsistent or divergent patterns in the data and can provide an objective measure of the quality of the data. Graphs provide a quick, visual aid to ascertaining unusual relationships; computer software that allows pivot table calculations and graphical views of various ratios can be invaluable in spotting data problems, thereby enhancing the reasonability review of the data.

Finally, company data can be compared to industrywide data. However, this is only useful if distributional differences between the company's book of business and the industry average are reasonably expected to be similar.

In the end, the actuary must be confident that he/she can rely upon the data for the specific analysis and circumstances. He/she should document all reasonability checks and tests performed, highlighting any known or suspected deficiencies in the data.

B. Responsibilities of the Data Manager on the Quality of Data

The ability to form decisions and conclusions based on an actuarial analysis is dependent upon the quality of the data and the specifics of the analysis. Oftentimes, the underlying data of the study is imperfect in some respect. Once imperfections in the data are uncovered, the insurance data manager providing the source data should take the following steps.

1. Determine the reasons and cause(s) of the error.
2. Inform the actuary undertaking the current study and incorporate needed adjustments, modifications or corrections to the source data for use in the current analysis.
3. Stop the error by fixing the system or revising the data handling and collection process.
4. Quantify, if possible, the impact and magnitude of the error on the data underlying the current study.
5. Decide if the error may materially impact prior analyses and whether these prior analyses may need to be retroactively corrected.
6. Finally, if it is materially significant, make disclosures regarding past analyses appropriately. On an external basis, this may mean notification of insurance regulators, or insurance statistical agents. On an internal basis, company management may need to rethink financial, policy or pricing decisions.

Regarding this last step, note that in almost every situation, if the extent of imperfect data might change the conclusions or the results of the analysis using this data then there is an obligation to disclose the data imperfections to all potentially affected parties. Further, there is a duty to raise "red flags" in all situations where there are significant imperfections in the data.

VI. Concluding Remarks

A. What's Next?

There's been much discussion in various Casualty Actuarial Society (CAS), IDMA and other data quality forums regarding the use of a self monitoring audit system as a way of responding to various regulatory concerns raised by state officials and the National Association of Insurance Commissioners (NAIC). A frequent suggestion is that an industry self monitoring system, with a rigorous audit program that checks the statistical records submitted to statistical agents back to company source documents, would satisfy the various regulatory concerns. A starting point (but perhaps not the ideal model) for such a system might be the SDMS, described above. The appropriate forms and procedures necessary would be available on demand by State Financial Examiners. This approach may be advanced further in the upcoming months, but much work needs to be done regarding the details of such a self monitoring audit model, as there are divergent opinions as to its scope and necessity.

Undoubtedly, future data quality efforts will be the result of the impact of continually improving technology. The synthesis of technology and knowledge allows improved concepts in data base design and automation.

Current topics include:

1. Data Warehouse Concept - which allows broad use of data in great detail by many areas of the company;
2. Greater use of complementary databases - ZIP Code, motor vehicle reports, geographic mapping - in improving data validation and accuracy; and,
3. Pattern Recognition/Expert Systems/Fuzzy Logic Systems - that enhance automation efforts and allow graphical views of the data.

What's next? The challenge for both actuaries and insurance data managers is to keep up with the improved technology and to use it as an aid to improving data quality.

B. Conclusion

Data quality has long been a concern of the insurance industry and the regulator. However, data quality must be administered in a cost efficient manner. The more rigorous statistical plans are subject to some degree of interpretation versus financial data accounting. As technology has improved, better data quality (and better reconciliation of statistical & financial data) can be realized more economically and efficiently by both data managers and actuaries. Managements have recognized that high quality data provide them accurate controls of their businesses.

Two professional groups - data managers and actuaries - have developed formal standards to better recognize the importance of data quality. Both standards have been reviewed in this paper. The data manager's responsibility is specifically stated to go beyond the production of the data. Error detection, evaluation, and disclosure are now part of that responsibility. The actuary cannot simply accept data and rely on the work of others regarding it's quality. Data must be reviewed for reasonableness and consistency, and data imperfections must be addressed.

Formal professional education is available to both professions, and it can be expected that data quality will continue to be an issue addressed by each professional organization.

1996 CAS Geo-Coding Survey
by the CAS Committee on Management Data and Information

EXECUTIVE SUMMARY

The purpose of the 1996 Geo-Coding Survey was to assess the current usage of geo-coded data in the casualty actuarial profession, and to foster development of new actuarial techniques using such data. A total of 152 CAS members returned a completed survey. The following are the key findings of the Geo-Coding Survey:

- Nearly four in ten (36.8%) respondents reported they were currently using geo-coded data for the monitoring of catastrophe exposures, while nearly one-third (30.9%) reported current use in the definition of rating territories.
- Over one in five (21.1%) respondents reported they were currently using geo-coded data for the determination of unexpected insurance costs for specific locations, and the same number reported use for marketing/underwriting.
- Close to half (48%) of all respondents reported they were not currently using geo-coded data for any purpose.
- Zip code data was named most frequently by respondents when asked the type of geographic data they were using for listed purposes. Zip code data was the most popular response for six of the seven listed purposes, such as the monitoring of catastrophe exposures or the definition of rating territories.
- Over nine in ten (90.8%) respondents report that they believe geo-coded data will become useful in the monitoring of catastrophe exposures, while over three-fourths (77%) believe geo-coded data will become useful in the definition of rating territories.
- A clear majority of respondents believe that geo-coded will become useful in the determination of unexpected insurance costs for specific locations (63.8%) or in marketing/underwriting (59.9%), while nearly one-half believe geo-coded data will become useful in competitive analysis (48%) or policy rating (47.4%).
- Of those using geo-coded data, nearly two-thirds (62%) indicate the source of latitude/longitude to be software that determines latitude/longitude from street address.
- When asked to describe successful applications of geo-coded data they believed would be of interest to the CAS membership, respondents mentioned catastrophe related applications most often. These applications included catastrophe modeling, catastrophe analysis, and catastrophe management.
- When asked to describe any significant problems in development of geo-coded applications they believe CAS members should be made aware of, respondents mentioned data quality

issues most often. These issues included inconsistency of data gathering, accuracy of geo-coded data software, and accuracy of street addresses and zip codes.

- When asked to provide references that they knew of that may be helpful in the development of geo-coded applications, Mapinfo software and Business Geographic magazine were mentioned most often by respondents.

- Nearly half (48%) of all respondents reported interest in participating as an attendee of a panel discussion focused on development of applications for geo-coding, while close to one-fourth (23.7%) reported interest in participating as a Limited Attendance Seminar attendee.

- A clear majority (57.9%) of respondents reported a designation of FCAS, while over one-fourth (26.9%) reported having over 21 years of actuarial experience.

- Close to six in ten (59.2%) respondents reported a property/casualty primary insurance company as their type of employer.

RESULTS

Applications

Item 1:
Indicate the areas of actuarial practice for which you are currently using geo-coded data:

Area of Actuarial Practice	Percentage of Respondents Using Geo-Coded Data
• Monitoring of catastrophe exposures	36.8
• Determination of unexpected insurance costs for specific locations	21.1
• Reserving	4.6
• Definition of rating territories	30.9
• Policy rating	15.8
• Competitive analysis	16.4
• Marketing/Underwriting	21.1
• Other	3.9
• Not currently using for any purpose	48.0

Please write in the type of geographic data that you are using for the listed purposes.

Area of Actuarial Practice	Type of data in use (number of responses)
• Monitoring of catastrophe exposures	zip code (37) street address (12) county (10) latitude/longitude (6) postal code (3) exposure information (2) various (2) location (1) geo-coded (1)
• Determination of unexpected insurance costs for specific locations	zip code (22) street address (7) county (5) latitude/longitude (4) rating territory (4) postal code (2) state and county (2) exposure information (1) location (1) census tract, block, and group (1)

Applications
(continued)

Please write in the type of geographic data that you are using for the listed purposes (continued).

Area of Actuarial Practice	**Type of data in use** (number of responses)
• Reserving	state (4) street address (1) county (1)
• Definition of rating territories	zip code (37) county (14) postal code (4) state and county (1) city (1) state (1) street (1) rating territory (1) river (1) bureau definitions (1) latitude/longitude (1) census tract, block, and group (1) creation of catastrophe zones to monitor exposure (1)
• Policy rating	zip code (16) county (4) postal code (2) rating territory (2) bureau territory (1) street address (1) town code (1) latitude/longitude (1) distance to work (1) distance to coastline (1) Rand McNally database (1) various (1)

Applications
(continued)

Please write in the type of geographic data that you are using for the listed purposes (continued).

Area of Actuarial Practice	**Type of data in use** (number of responses)
• Competitive analysis	zip code (15)
	postal code (4)
	rating territory (3)
	bureau territory (1)
	state (1)
	county (1)
	exposure information (1)
	latitude/longitude (1)
• Marketing/Underwriting	zip code (14)
	street address (4)
	county (4)
	latitude/longitude (3)
	postal code (3)
	census block group (2)
	census tract (1)
	overlay of census data (1)
	town code (1)
	bureau territory (1)
	exposure information (1)
	target marketing (1)
	risk selection (1)
	wind (1)
	drive distance (1)
	local tax (1)
	fire protection (1)
• Other	
Ratemaking - homeowner (2)	exposure information, zip code
Reinsurance	zip code
Agency management	street address, block group
Identifying policy holder in catastrophe areas	latitude/longitude

Applications
(continued)

Please provide any explanatory comments on the above applications that you believe will be helpful to the CAS in assessing the current state-of-the-art for applications of geo-coding.

- I plan to use data for expected losses by location, definition of rating territories, allocation of capital, but these studies aren't underway yet.
- Using IRAS (from RMS) to assist with catastrophe capacity management; considering additional uses, much interest in this area at this time.
- Street address used to match waste sites to publicly available listings.
- Systems development and statistical plans have a long way to go - at this point were getting started on a system to access zip code data. More detailed geo-coding would require substantial changes to statistical plans.
- As a regulator, I had to review the use of geo-coded data, or the proposed use of such data.
- Recently participated in the NAIC study of insurance availability in urban areas. The survey data included policy counts and premiums by homeowners policy form for zip code.
- As a reinsurer, we get limited data, particularly operating in the broker market. We may get for catastrophe exposures, total insured value by county.
- We expect to use expected profit by location to predict profits for given group property and casualty accounts.
- I don't believe our current practice should necessarily be considered "state-of-the-art." We are a small regional insurer, just beginning a major overhaul of our data reporting systems, partly to improve our access to more detailed location data (among many other issues).
- The ratemaking process performed by EQECAT for the California Earthquake Authority is a fine example of the issues I've checked above. (monitoring of catastrophe exposures, determination of expected insurance costs, competitive analysis, marketing/underwriting, other - ratemaking HO).
- A couple of areas that might deserve mention: Use of geo-coding in Business Planning and Strategy; Use of geo-coding in Dynamic Financial Analysis.
- We are moving toward policy-specific geo-coding due to the increasing need to asses risk at the policy level.
- Do not use lat/long for anything explicitly although our mapping software uses it internally.
- We are a reinsurance company, so detailed data is sometimes difficult to obtain and sometimes too time consuming to evaluate. We don't use detailed data for reserving and don't have territory rating.
- (Not currently using, but) Please note that with my previous employer, I was using 3 digits postal code for monitoring of catastrophe exposures and definition of rating territories.
- In all the above areas, zip code level data is used, but no geo-coded data.
- We are finding that zip code is too broad for many of the above applications.
- Data is easily available for US. Very expensive or not available for rest of world.
- I think the CAS should check with major personal lines companies to determine the more sophisticated programs that may be used to define rating territories. Also, check on use of geo-coding in Neural Network Analysis (Peter Wu?).

- There is a lack of geo-coded historical data that is available to set rates, etc.

Item 2:
Indicate the areas of actuarial practice for which you believe geo-coded data will become useful:

Area of Actuarial Practice	Percentage Who Believe Geo-Coded Data Will Become Useful
• Monitoring of catastrophe exposures	90.8
• Determination of unexpected insurance costs for specific locations	63.8
• Reserving	14.5
• Definition of rating territories	77.0
• Policy rating	47.4
• Competitive analysis	48.0
• Marketing/Underwriting	59.9
• Other	7.2

(reinsurance pricing (3), research, claims, business planning, international catastrophes, agency management, more innovative design of rating plans, risk management, allocation of capitol)

Please provide any explanatory comments on the above applications that you believe will be helpful to the CAS in fostering the future state-of-the-art for applications of geo-coding.

- Extent of future possibilities for geo-coding will depend on how precise the location/information provided is! For example, will geo-coding allow me to differentiate between two apartment buildings on the same block; one at 55 Main St, the other 60 Main St?
- Uniform (throughout the industry) use of geo-coding territories would simplify many things, starting with statistical reporting. Many things would flow from this, including simplified competitive analysis. Imagine...no more zip, city, county, town, convoluted territories.
- Anything is possible - it is a wonderful rating variable if the data is available.
- Most consumers/customers don't know their address in geo-coding form so wide spread use without an inexpensive translation mechanism will slow the use of location information in this format for insurance purposes.
- The categories above are redundant (rating territories = policy rating).
- Claims - Would be helpful to have an idea of how many and where to deploy claims adjusters post-event.
- I am especially intrigued with territorial ratemaking that uses continuous surfaces and using data at a street address level.
- Eventually, territory rating may be replaced by the detailed information associated with a given location: home values, crime statistics, etc. For hurricanes and tornadoes, we should consider looking at elevation.
- Ultimately, it will be the way to do business and will be useful for all of the above.

Applications
(continued)

Please provide any explanatory comments on the above applications that you believe will be helpful to the CAS in fostering the future state-of-the-art for applications of geo-coding (continued).

- Given the complexity of working with such data, I envision only those specializing in monitoring catastrophe exposures to be using such data on a regular basis.
- Presentation at the CAS Ratemaking Seminar in Las Vegas provided the state-of-the-art ideas which were excellent.
- Knowing the location and mapping the neighboring exposures to the geo-coded risk you are writing is an invaluable resource.
- Location of insured and of losses could change rating concepts. For instance, automobile rating could depend on where you drive (to work, to grandma's house) besides just where your garage is.
- Location is an important determinant of the concentration and exposure to many perils. It will be more common to use location analysis as data and GIS software becomes widely available.
- Policy holders will not/do not know their geo-code; rate structures need to be simple enough to file and explain. Zip code is simple enough, while latitude/longitude is not.
- Geo-coded data could be particularly helpful in monitoring earthquake exposure.
- For personal auto, could be used for rating purposes to refine traditional zip code rated territories.
- Catastrophe exposures vary over short distances that are not captured any other way.

Sources

Item 3:
If you are using geo-coded data, indicate the sources of latitude/longitude that you are using:

Source	Percentage of Respondents Using Source
• Software that determines latitude/longitude from street address	62.0
• Designation of map location on computer screen	15.2
• On-site radio signals to satellites	6.3
• Other (post office publication of zip codes, all business assigned to geographic center for each zip code, zip codes from location of insured property)	6.3

Please provide comments on the above alternatives that you believe will be helpful to the CAS membership in assessing practical alternatives for development of geo-coded data.

- We use RMS's IRAS model.
- Satellites should be used in the future; e.g. a satellite photo in a hurricane aftermath could be overlaid with a map of company policyholders. Claim services could be routed and estimates could be made prior to claims being filed.
- Relationships of site to known landmarks can be of great value to underwriters, actuaries, and claims. Information such as known pollution sites, or distance to coastline or fault line.
- For determining territorial definition, we use a Rand/McNally database which is keyed by FIPS - place code, state, zip.
- IRAS earthquake model by RMS uses lat/long.

Success and Problems

Item 4:
Please describe successful applications of geo-coded data that you believe will be of interest to the CAS membership:

- Catastrophe modeling.
- Catastrophe exposure.
- Catastrophe analysis.
- Catastrophe management.
- Catastrophe modeling software by EQECAT and RMS.
- Determining catastrophe rates.
- Pricing alternate contract terms.
- Auto theft analysis from trackers.
- NY, NJ hurricane deductible zones.
- FL (FWUA) voluntary credits.
- Territory redefinition using census data and geo-coding.
- Earthquake modeling and predictive modeling.
- I think all the areas in #2 above, except reserving, will provide successful applications.
- Not quite finished, but we're putting together a system to group various postal codes together to analyze appropriate territories and changes in territories.
- Risk location/concentration and exposure determination.
- We examine loss data by postal code in order to define territories and rate policies for homeowners insurance; we review every year and move postal codes from one territory to another as dictated by either loss experience or our competitive position in the marketplace. We once examined loss data by postal code to refine our territory definitions for auto insurance; we will periodically review to make sure out territory definitions still make sense.
- Result is only as good as the initial address entry. Misspellings, etc., can have a large impact.
- Using 3-digits postal code to refine large urban territories (both property and automobile).
- IRAS earthquake loss model.
- Marketing, underwriting, rating.
- Entire book of biz geo-coded in US. GPS units in use in Asia and Latin America.
- Assessing hazard exposure is very successful using geo-coded data.
- Obviously, the area of greatest payback would be in monitoring catastrophe exposures. This has helped our company tremendously in negotiations with our catastrophe reinsurers. Also, extremely useful internally for business planning.
- Currently assigning rating territories from keyed in address; agents/customer service reps no longer need to learn territory definitions.
- I feel that zip code rating is superior.
- Coastal exposures can be problematic when trying to view exposures. Some geo-coding software are not consistent so data that may be geo-coded by one software but later used in another may appear to be out in the "water".
- Software to display physical locations of zip codes to check for contiguousness of territories. Also to view patterns in loss costs.
- Density of exposures related to hurricane or earthquake risk.

Success and Problems
(continued)

Item 4 (continued):
Please describe successful applications of geo-coded data that you believe will be of interest to the CAS membership:

- As a reinsurer, we have used it only to monitor our catastrophe exposures. We rely on the reinsurance intermediaries and/or clients to collect data and run models.
- Hurricane modeling, reinsurance exposure analysis, rating territories.
- Map of geological fault lines and location of exposure by geo-coded location with reference to fault lines.
- I use geo-coded data to monitor earthquake exposure in a region.
- Geo-coding makes it much easier for a direct writer phone operation to properly determine the rating territory since the insurer representative may not know the geographic area where the risk is located.

Item 5:
Please describe any significant problems in development of geo-coding applications you believe CAS members should be made aware of:

- Blanket rated covers enough information; may not currently be captured.
- Constant updating of new addresses.
- Addressing the large variability around the expected results.
- Risks with hundreds/thousands of locations - difficult to charge enough to cover costs of capturing all the specific location data.
- Consistency in measuring devices (accuracy).
- Several, separate policies for the same coverage written on risks at the same geo-code, e.g. earthquake coverage for renters in a high-rise.
- Major problem would be credibility associated with finely divided data.
- Data quality problems have surfaced frequently.
- Multi-location policies are especially difficult to handle.
- Distribution of geo-coded applications to agents.
- Data quality, model parameter estimating, regulatory concerns.
- Capturing accurate raw data.
- The major problems I see are dealing with what are sure to be large quantity of the data, (i.e. cut so fine that many observations are required before anything useful can be gained) and relating the data to other, known, data for verification.
- Computer software still (at least what we use) can not interpret similar spellings.
- Accuracy of TIGER, census, street information in the software.
- Quality of internal data - for example capturing billing address vs. site address.
- Inconsistency of data gathering, e.g. bad zip code in valid city.
- Difference between loss location and premium location.
- Zip codes change constantly, keeping up with them is costly.
- Lack of data quality of street address leading to low hit rates for geo software.

Success and Problems
(continued)

Item 5 (continued):
Please describe any significant problems in development of geo-coding applications you believe CAS members should be made aware of:

- We have challenges with "split zips" - a zip code which has more than one rate. Our rating system can't use the person's address to properly place them in the section of the zip; we rely on a person to look up information on a map or a descriptive table - sometimes the person just assigns "the first one" of the two or more choices.
- Unavailability of latitude/longitude coordinates for most locations. Do you know the coordinate for your home??
- Data quality - We have problems where zip codes sometimes don't map on the correct state. We find mailing address coded on all risks on one policy, rather than the actual location.
- Because of the ever changing numbers of zip codes, it is important to utilize software that is regularly updated. Data quality of the addresses being utilized is also a watch out.
- Accuracy concerns on street segments.
- Accuracy of geo-coding/underlying data.
- To match the longitude/latitude, you need exact address. Many addresses are not exactly correct, i.e. street instead of drive, north or south not included with street, etc.
- Coding of varying limits, classification information, etc. by location on a single policy.
- It is difficult to determine whether county data is "good enough" or if we need zip code or street address data.
- Credibility issue as the volume of data decreases rapidly.
- Redlining issue.
- Software needs to be continually updated for new zip code definitions.
- Regulatory acceptance; it shouldn't be a problem, but I fear that it will; change is difficult.
- Need to be careful of over-refinement of territories.
- Need to be careful when using for auto insurance because cars are mobile (unlike houses).
- Level of data needed to make geo-coded location data valuable, i.e. coverage, limits, type of construction, contents, etc.
- Credibility procedures! / Lack of credibility. / Credibility issues and techniques.
- The actual coding by street address is complicated by numerous factors. The software's street index is incomplete. Finding the location as opposed to billing address can be difficult and multiple location policies are present.
- Blanket risk coding on commercial accounts.
- Annual or more frequent shifts in zip code boundaries.
- A Canada conversion of rural addresses or postal codes to geo-code may not be accurate enough for certain applications (i.e. catastrophe analysis).
- Perils that are not discrete enough for high resolution analysis: hail, brush fires, mud slide.
- Willingness to believe model output without appreciation for inherent uncertainty.
- Errors in geo-coding and address changes.
- What is the source of this information for a property? If the insured, it can be falsified. Systems development costs, cost of capturing data, lack of interest by senior management.

References

Item 6:
Please provide any references that you know of that may be helpful in the development of geo-coding applications. Such references may be printed materials, vendor organizations, professional organizations, or any other source of information:

Multiple responses are indicated by the number in parentheses

- RMS (2)
- Strategic Mapping Inc. (2)
- ETAK (2)
- Advance Technology Corporation, Atlanta, GA, Mark Fouraker, 770-399-4343
- Toprate, Insurquote (rating services)
- Maplinx (software)
- Mike Miller (actuarial consulting)
- American Demographic magazine (2)
- Business Geographic magazine (4)
- ESRI (3)
- Mapinfo software (4)
- EQECAT
- Workers Compensation Insurance Rating Bureau (California) - data by zip code
- GIS World Magazine
- Geographic Data Technology (GDT) (2)
- ISO
- Vista Information Services
- The software we use is called IRAS from the vendor Risk Management Solutions.
- Tactician Corporation (software)
- USGS
- Compu search organization operating in Toronto

Future CAS Programs

Item 7:
Please indicate below the type of future activity focused on development of applications for geo-coding in which you would be interested in participating:

Type of Activity	Percentage of Respondents with Interest
Panel Discussion - attendee	48.0
Panel Discussion - panel member	3.9
Discussion Paper Program - author	0.0
Discussion Paper Program - reviewer	5.3
Limited Attendance Seminar - attendee	23.7
Limited Attendance Seminar - discussion leader	1.3
Other	.7

Other Comments

Item 8:
Please provide any additional comments that you believe would be helpful to the CAS in facilitating development of geo-coding applications among CAS members:

- As a regulator, I see future concerns about territorial definitions. How will they look? Will they exist? How will the laws of various states need modification to adapt to the changing technology?
- Must recognize difference in personal lines vs. commercial lines risks with hundreds of locations.
- Workshops, data source description, model building papers, etc.
- Coin the term "geode" to mean the smallest geographical unit under consideration. Geodes would be defined by regulators with industry assistance. Geodes should be along easily identifiable physical or political boundaries. Territories would be aggregations of geodes. Most Importantly: In order for computerized hurricane models to become accepted, they must utilize real exposure and loss data to calibrate the model. Exposure and loss data should be reported by geode and each geode should be small enough that we would expect that wind damage would be uniform throughout the geode.
- I'm not confident that expending resources in this direction would be fruitful for the CAS.
- ISO geog u/w system.
- Geo-coding in ratemaking will require a great deal of information, so only the largest companies will have sufficient data. To make geo-coding of interest (of practical interest to many actuaries' employers), a large database would need to be available. Perhaps the CAS would work with statistical agencies to gather this information so that it is reliable and available. This might expand the interest level among actuaries and help the small companies from being the victims of large companies that can use this information to exploit the current rating territory definitions.
- There must be other more important issues to be spending time on.
- How about an article in the Actuarial Review?
- What about the impact on companies' systems departments?
- Geo-coding has not been used by me because my job function does not.
- Not much to offer!
- Everyone benefits from a universal adoption of risk location identification. Reinsurers, regulators, statistic reporters, primary companies, etc. all seeing risk the same way would be ideal. Geo-coding (at least on the surface) would seem to offer this.
- Not to get too carried away, but will the CAS be seeking the NAIC's input on the potential for geo-coding to become mandatory?
- Wouldn't it be nice to have a regulation promulgated that actually helped an entire industry?
- Any discussion should consider practical applications.

Member Profile

Item 9:
Please include some information about yourself:

Actuarial Designation	Frequency	Percent
FCAS	88	57.9
ACAS	57	37.5
No response	7	4.6
Total	152	100.0

Years of Actuarial Experience	Frequency	Percent
0-5	10	6.6
6-10	36	23.7
11-15	31	20.4
16-20	29	19.1
21+	41	26.9
No response	5	3.3
Total	152	100.0

College Degree	Frequency	Percent
None	1	.7
BA or equivalent	109	71.7
MA or equivalent	34	22.3
Ph.D.	3	2.0
No response	5	3.3
Total	152	100.0

Type of Employer	Frequency	Percent
Property/Casualty Primary Insurance Company	90	59.2
Reinsurance Company	11	7.2
Consulting Firm	26	17.1
Insurance Broker	8	5.3
State Insurance Department	5	3.3
Other Government Entity	0	0.0
Organization serving the insurance business	4	2.6
University or college	1	.7
Other	2	1.3
No response	5	3.3
Total	152	100.0

Compilation of Variables Necessary for Performing Dynamic Financial Analysis of Insurance Companies

by James R. Garven, Ph.D.,
under the direction of the
CAS Task Force on DFA Variables

COMPILATION OF VARIABLES NECESSARY FOR PERFORMING DYNAMIC FINANCIAL ANALYSIS OF INSURANCE COMPANIES

Final Stage One Report Submitted to
the DFA Liaison Team

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Abstract. In recent years, a virtual consensus has emerged within the casualty actuarial science community that actuaries must broaden their role in insurance organizations by developing a set of tools that will enable them to render expert opinions regarding not only loss reserves but the overall value and solvency of the firm as a whole. In order to support this effort to broaden the roles of casualty actuaries, the Casualty Actuarial Society has embarked upon a many-year, multi-stage project entitled Dynamic Financial Analysis. This aim of the project is to set up a general actuarial framework for the modeling and financial evaluation of insurance companies as risk-assuming, ongoing entities. The outcome of the project will likely be general specifications for insurance company financial models, a database of important variables to support these kinds of models for the purposes of research and model design, and suggested procedures and considerations for those who would design, use and interpret these models. This is the final report for stage 1 of the Dynamic Financial Analysis project.

1. INTRODUCTION AND OVERVIEW

The Casualty Actuarial Society has embarked upon a many-year, multi-stage project known as Dynamic Financial Analysis. This aim of the project is to set up a general actuarial framework for the modeling and financial evaluation of insurance companies as risk-assuming, ongoing entities. The outcome of the project will likely be general specifications for insurance company financial models, a database of important variables to support these kinds of models for the purposes of research and model design, and suggested procedures and considerations for those who would design, use and interpret these models. Some of the specifications expected for a Dynamic Financial Analysis Model are as follows:

1. It should be able to account for and evaluate the things that are most likely to affect the value of the company.

2. It should produce probability distributions of financial outcomes.

3. It should provide enough detail to allow evaluations of outcomes on a variety of accounting bases, such as on-going, run-off, etc.

4. It should produce risk/return consequences of changes in major management decision variables.

5. It should recognize the interplay among various segments of the company and also with various external variables.

6. It should be devised as a strategic management tool, with regulatory compliances features regarded as byproducts, albeit mandatory ones.

As originally conceived, the Dynamic Financial Analysis project is expected to consist of four stages occurring over the next few years:

1. Stage 1: Identification of variables and data sources
2. Stage 2: Creation of a research database

3. Stage 3: Analysis
4. Stage 4: Specifications and feasibility plan for a permanent widely accessible database

This is the final report for Stage 1 of the CAS Dynamic Financial Analysis project. The outline of presentation will be as follows. The next section sets forth a set of general considerations that ought to be considered in dynamic financial analysis.[1] The third section of the report provides a discussion of variables and data sources. The fourth section of the report provides some recommendations for setting up the database for further research, and the fifth section discusses some possible future directions for research to complete the further stages of the project.

2. GENERAL CONSIDERATIONS

In recent years, a virtual consensus has emerged within the casualty actuarial science community regarding the future role of the profession. It is now widely believed that actuaries must "re-engineer" themselves by becoming "actuaries of the third kind" (see Bühlmann (1987) and D'Arcy (1990)). This will require developing a set of tools that will enable actuaries to render expert opinions regarding not only the value of loss reserves, but the overall value and solvency of the firm as a whole.

The need for dynamic financial analysis has been anticipated in the actuarial literature, well before terms such as the "appointed actuary" and "dynamic financial analysis" became popular. For example, D'Arcy (1990) presents the very compelling argument that factors such as the

[1]The report is generally agnostic insofar as model selection is concerned, focusing primarily upon the identification of variables and data sources. It is the primary responsibility of the CAS Committee on Valuation and Financial Analysis (VFAC) and its subcommittees to decide and advise the CAS concerning model selection and parameterization (see CAS Subcommittee on Dynamic Financial Models (1995) and Szkoda, *et al.* (1995)). The next section of the report does, however, set forth the argument that dynamic financial analysis can and should incorporate a rigorous integration of financial economics with actuarial science.

growing importance of investment performance in insurance operations, increasing volatility in financial markets and the emergence of investment-linked insurance contracts are creating the need for actuaries to develop new skills and a greater awareness of investment performance. Bühlmann (1987) refers to actuaries who understand both the asset and liability structures of insurance companies as actuaries of the "third kind".[2]

The importance of integrating actuarial science and finance has been recognized by actuaries and financial economists alike, and has resulted in the development of a literature on the convergence of the two fields. Borch (1985), Boyle and Butterworth (1982) and D'Arcy (1990) present lucid analyses from the actuarial perspective, whereas Garven (1987) and Smith (1986) approach this topic from a financial economics perspective. The reference section of this report provides a research bibliography that addresses financial theory and its applications to insurance and actuarial problems.

Besides the parallels between finance and actuarial science that have been noted to exist in published literature, common approaches in practice are also observed. For example, deterministic and stochastic techniques described in a number of CAS reports (e.g., see CAS Subcommittee on Dynamic Financial Models (1995) and Szkoda, *et al.* (1995)) bear a close resemblance to capital budgeting techniques that are presented in some of the more popular corporate finance textbooks (e.g., see Brealey and Myers (1991)).[3]

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[2]According to Bühlmann, actuaries of the first kind are life actuaries whose methods primarily involve deterministic calculations. Casualty actuaries are actuaries of the second kind, in the sense that they develop probabilistic methods for dealing with risky situations (for example, using methods such as scenario testing and Monte Carlo simulations). Actuaries of the third kind address investment and underwriting aspects of insurance companies and apply principles from financial theory to create more fully integrated models of the insurer.

[3]In the finance literature, the seminal work on the use of simulation in the evaluation of corporate capital projects was done by David Hertz (1964, 1968). In the tenth chapter of

Historically, financial research has tended to oversimplify insurance markets and institutions, whereas robustly specified actuarial models of insurance markets and institutions often lack the analytic rigor and economic foundations that have become the hallmark of financial research.[4] However, in recent years, there have been extensive applications of financial theory and empirical methods to the analysis of property-liability insurance markets and institutions. For example, there is now an extensive finance literature applying the capital asset, arbitrage pricing and option pricing models to the problem of the "fair" rate of return in property-liability insurance markets.[5,6] Option pricing models have particularly important implications for dynamic financial analysis, as they allow for a stochastic modeling framework in which asset and liability management impacts the value of the firm *and* its solvency level.

Furthermore, an extensive finance literature has developed that analyzes, both theoretically and empirically, the economics of organizational structure. In view of the significant degree of

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Brealey and Myers, the cases for and against both simulation and scenario testing are summarized. Although the finance literature champions risk analysis, it is generally very critical of simulation analysis in particular (see Lewellen and Long (1972)).

[4]Indeed, although finance is essentially a field of applied economics, it has experienced remarkable success as a scientific discipline. This has culminated in the awarding of the Nobel Prize in Economic Science five years ago to three financial economists: Merton Miller, William Sharpe, and Harry Markowitz for their seminal research on corporate capital structure, asset pricing and portfolio theory.

[5]For applications of the capital asset pricing model (CAPM) to insurance pricing, see Biger and Kahane (1978), Fairley (1979), Hill (1979), Hill and Modigliani (1987) and Myers and Cohn (1987). Kraus and Ross (1982) provide a more general framework based upon arbitrage pricing theory, and Doherty and Garven (1986), Cummins (1988b), Derrig (1989), and Phillips (1995) provide solutions to the fair return problem in a contingent claims, or option pricing framework.

[6]A particularly important paper in this literature is by Phillips (1995), who derives an option pricing model that allows for the determination of premium levels by line of business for a multi-line insurance company. He also finds empirically that insurance prices are inversely related to the riskiness of the firm, as predicted by the option model. This inverse relationship is stronger for long-tail lines of business than for short-tail lines, suggesting that the default premium increases the longer the payout tail.

cross sectional variation in ownership structures and distribution systems that exists in the property-casualty insurance industry, this is a particularly relevant literature. Of particular interest is the question concerning whether incentives exist for firms adopting different organizational features to optimally employ different risk management strategies. To date, the empirical evidence is generally consistent with testable hypotheses contributed by financial models of insurance companies. Specifically, it appears that mutual insurance companies tend to adopt more conservative investment and underwriting strategies than do stock insurers. Mutuals have been found to concentrate a larger proportion of their investments in financial assets and smaller proportions in non-financial assets than stock insurers (see Fama and Jensen (1983)). After controlling for size, stock companies write relatively more business in riskier lines of insurance (see Lamm-Tennant and Starks (1993)) and reinsure less (see Mayers and Smith (1990)) than mutuals.[7] Stock insurers also tend to be more highly leveraged and bear more interest rate risk than mutuals (see Doherty and Garven (1995)). Finally, Babbel and Staking (1989, 1990) show that the market rewards (in the form of a higher stock price) firms that match asset and liability durations. Clearly, some consideration ought to be given to the incorporation of the perspectives offered by the theoretical and empirical studies cited above in the further development of the underlying theory that supports dynamic financial analysis.

3. Important Variables

[7]Mayers and Smith (1990) find that widely held stock insurance companies cede proportionately less reinsurance than any other ownership class, including mutuals. Although they also find weak evidence that single-owner stock insurers reinsure more than mutuals, this is to be expected since risk aversion is more likely to be an important motivating factor for closely held than for widely held firms.

Appendix 1 provides an initial partial list of some of the factors relevant to insurer solvency and management planning, and was included with the original Stage 1 DFA request for proposals as an attachment. Furthermore, in the CAS Subcommittee on Dynamic Financial Models report entitled "Dynamic Financial Models of Property/Casualty Insurers" (see CAS Subcommittee on Dynamic Financial Models (1995)), attention is focused on the following classification of property-liability insurance risks:

- C-1 risk - Uncertainty surrounding cash flows from invested assets other than from uncertainty regarding interest rate risk.
- C-2 risk - Uncertainty surrounding cash flows from the obligation or underwriting aspects of an insurance company.
- C-3 risk - Uncertainty surrounding cash flows from interest rate fluctuations in the presence of a mismatch of assets and liabilities and the risk of disintermediation caused by embedded options that are sensitive to changes in interest rates.

There obviously exists a high degree of correspondence between this particular classification scheme and the list of factors provided in Appendix 1. Furthermore, the factors listed above are generally incorporated in the financial literature cited earlier.

The approach taken in Stage 1 has been to orient the research primarily around variables and data sources for which information can be obtained via the Internet. In many cases, data vendors are either moving toward Internet-based distribution or allowing licensees to create Internet-based delivery mechanisms for these data.[8] Given the "distributed" nature of the Casualty

[8]The Social Sciences Data Collection at the University of California, San Diego (UCSD) (see `http://ssdc.ucsd.edu/ssdc/econ.html`) and the Yale University Social Science Statistical Laboratory (see `http://statlab.stat.yale.edu`) provide interesting "proofs of concept" for the CAS. Although access to most of the data available from these collections is restricted to on-campus users, it would certainly be technically possible to engineer similarly secured Internet-based database systems for the Society.

Actuarial Society, the Internet constitutes the best long-run solution for creating and maintaining a permanent widely accessible database for CAS members. The costs of such a system can be allocated in such a way that the CAS and its members can acquire a very efficient and cost-effective delivery system for data that can also be virtually managed and updated as needed. In fact, because the World Wide Web makes it is possible to link sites together via the so-called hypertext transport protocol (`http`), effective site management could in principle be accomplished on either a completely centralized or decentralized basis. In the last two sections of the report, recommendations are made concerning setting up the database and possible future directions for research that presume the Internet to be the computing platform of choice.

Consistent with this "net-centric" philosophy, this report and its appendices can be accessed directly from the DFA World Wide Web home page, the address for which is `http://www.risknet.com/dfa/dfa.html`. Appendix 2 lists the information that is provided there.[9] Hypertext links to Appendices 3-9 of this report are provided in the Data Access section of the home page. These appendices provide detailed listings of variable definitions, length of time series available, cost and feasibility of data acquisition, licensing issues, and information concerning levels of aggregation.

Issues such as variable interactions are more specific to model selection than to the identification of variables and data sources, which is the focus here. Nevertheless, some observations regarding variable interactions can and should be made. The general approach to modeling variable interaction is to estimate variance-covariance matrices for variables. Since the insurer may be viewed as a portfolio of assets and liabilities, a portfolio based analysis of insurer

[9] In this and later appendices, all underlined text represents hypertext links to other documents. Unfortunately this is a feature not easily replicated in the context of a hard copy, or analog document such as this.

risk and return can provide important insights into performance measurement and solvency. Halliwell (1995) presents the mathematical theory behind portfolio analysis, while Almagro and Sonlin (1995) and Lamm-Tennant (1995) apply this method to evaluating asset allocation strategies. The Lamm-Tennant paper is an especially important paper in this regard, as it provides a very rigorous yet elegant approach to estimating variable interactions on an after-tax basis. Furthermore, it is highly recommended that readers of this report look further into the JP Morgan RISKMetrics database (listed in Appendix 4 and available for free on the World Wide Web). This database provides information on volatilities and correlations among over 300 different types of financial assets.

As one would expect, most of the data that are useful for dynamic financial analysis are available on a commercial basis, and a number of vendors are already beginning to experiment with different forms of World Wide Web-based distribution. Appendix 3 lists commercial vendors who provide comprehensive financial and economics database products. For financial analysis, the CRSP and COMPUSTAT databases are particularly useful; indeed, most of the best academic research on firm valuation uses these databases. The CITIBASE database is unparalleled in its breadth and depth of coverage of interest rate and macroeconomic data.

Unlike insurance data, there is an abundance of economic and financial market time series data already available on the Internet. Furthermore, access to many of these databases is free, although this is not universally the case. Appendix 4 lists a number of free data sources. These data sources that are the most "professionally" presented and supported are demarcated with special "NICE!" icons. These include the Federal Reserve Bank of St. Louis' *FRED* Database, Financial Markets Data from the Federal Reserve Bank of Chicago, volatility, correlation, and price index data from JP Morgan, and the EDGAR Database. All of these resources, with the exception of JP Morgan, are funded by government or quasi-government agencies, and they are intended to provide reliable access to some very high quality data sources. Indeed, much of the data listed under the *General Economic Time Series* and *Performance Measures for Investment Instruments* headings in Appendix 1 can be accessed from these resources.

Appendices 5-9 provide information about insurance data that are available from a number of commercial vendors. In addition to providing insurance data through traditional means such as diskette, tape, and CD-ROM, a number of these vendors feature proprietary online services.

4. RECOMMENDATIONS FOR SETTING UP THE DATABASE FOR FURTHER RESEARCH

Currently, a number of commercial database vendors distribute data using proprietary CD-ROM products or proprietary network interfaces. Good examples of CD-ROM products for insurance data include the A.M. Best and OneSource products, whereas proprietary (i.e., non-Internet) online insurance database products include A. M. Best's BestLink, the NAIC's InsureNet, and the NCCI's InsNet. However, World Wide Web-based access is fast becoming a preferred method of distribution for a number of reasons. Although there are important reasons to be concerned about security on the Internet, it is now widely believed that the Internet, rather than proprietary wide area networks, will increasingly become the network solution of choice for commercial organizations. Indeed, information technology has become one of the most dynamic sectors of the U.S. economy, and tremendous amounts of capital are being invested to find ways to secure the Internet.

Secondly, along with innovations in security, the speed of Internet access is increasing dramatically at a time when access costs are plummeting. This is causing the economics of a net-centric as opposed to a CD-ROM based or proprietary network-based distribution system to become very compelling. By locating the data on a central World Wide Web server, mistakes are easily and cheaply corrected, and updates to the database can be made at minimal cost. Furthermore, it matters not whether the consumer uses a computer running the DOS, Windows, Macintosh, or Unix operating system, because the World Wide Web provides a "platform-independent" system of distribution. This lowers costs even further, since all program coding can be done according to open rather than closed and proprietary standards. The World Wide Web will therefore make it possible for data vendors (and/or possibly their licensees) to create much less expensive and easier to use methods for data access and analysis. The NAIC and NCCI are already giving serious consideration to the possibility of developing net-centric approaches to distributing insurance data, and I expect that other vendors such as A. M. Best and OneSource will eventually be compelled by market forces to seriously consider the development of similar distribution systems.

Net-centric data distribution will also enhance the ability of the Society to more effectively develop and implement standards for dynamic financial analysis. CAS members can expect to interact increasingly via email and the Web for the purpose of not only accessing data, but also critically discussing and debating modeling issues. Indeed, many of the functions now performed by meetings and publications of the CAS are likely to migrate toward this environment.

5. Possible Future Directions for Research

Stages 2 through 4 envision the actual creation of a research database, analysis, and the development of specifications and a feasibility plan for a permanent widely accessible database system. Assuming that the Society is willing to embrace the Internet as its computing platform of choice, I think that the future course of the project can be modified somewhat. The next logical step would be to launch a pilot test of a distributed database system. The Society needs to identify a cadre of important and influential members who are willing to serve as "beta testers" for the pilot test. The pilot test needs to be coordinated by a Stage 2 researcher who has strong financial modeling and information technology skills. This individual will need to work closely with the beta testers for the purpose of creating proper specifications for the research database.

Besides funding a Stage 2 researcher, the budget will also require funding for the development of a World Wide Web site from which the database can be distributed. Essentially, the Stage 2 researcher will need to be an "Internet integrator" who can take a leadership role in persuading data vendors to "buy into" the pilot test by either creating their own secure and metered data feeds into the system or providing the CAS with the licensing necessary in order to administer such a system for its members.

In order for the second stage to be a success, it will require active participation from some very committed members of the Society. It will be important to include a mix of consulting, company and academic actuaries if at all possible, as this will facilitate the development of very broad and objective feedback to the Stage 2 researcher.

Finally, the CAS needs to foster and support a cultural environment that enthusiastically embraces emerging and important information technologies such as electronic mail and the World Wide Web. Not only will this facilitate the eventual development and implementation of a permanent and widely accessible database, but it will also enhance the ability of the Society and its members to compete even more effectively.

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An Initial Partial List of Some of the Factors Relevant to Insurer Solvency and Management Planning

General Economic Time Series
Inflation measures, such as CPI components, GNP/GDP deflators
Output measures
Employment measures
Interest rates, by term
Exchange rates

Performance Measures for Investment Instruments
Stock market, by segment, large vs. small, by *b*?, various exchanges
Bonds - corporate, muni, federal, by term, in various economic environments
Precious metals
Other commodities
CMO's and derivative products
Derivative products
Real estate, including rental value:

Insurance Industry Data
Premiums, losses, expenses, investment income, taxes, etc. by line. Accident year too.
Annual statement aggregates at least in detail of NAIC profitability report by line
Payout patterns by line and changes over time
Development patterns by line and changes over tune
Frequency and severity distributions by line and changes over time
Insolvencies and retirements with 5 years of data prior to
Matching group of solvent companies

Natural and Unnatural Disasters
Frequency and severity by location by type and changes over time
Impacts on insurance industry
Effectiveness of various mitigation programs for business interruption, including that of insurers

Appendix 2: DFA Home Page

(http://www.risknet.com/dfa/dfa.html)

CASUALTY ACTUARIAL SOCIETY

Dynamic Financial Analysis

Introduction

Welcome to the Casualty Actuarial Society (CAS) Dynamic Financial Analysis (DFA) Home Page. The CAS has embarked upon a many-year project entitled Dynamic Financial Analysis. It is anticipated that this prototype World Wide Web site will eventually evolve into a full scale distribution mechanism for a permanent and widely accessible research database.

The original Request for Proposals for the DFA project can be accessed by clicking here. The winning proposal for Stage 1 of the DFA project can be accessed by clicking here.

Data Access

- Financial and Economics Databases -- Click here to access information concerning commercial financial and economics databases. Click here to access information concerning free financial and economics databases.
- Insurance Industry Data -- From A. M. Best, ISO, OneSource, NAIC, and NCCI.

DFA Stage 1 Liaison Team Corner*

- DFA Discussion Archive
- DFA Stage 1 Preliminary Report
- Addendum to the DFA Stage 1 Preliminary Report
- DFA Stage 1 Final Report

*Participation limited to the principal investigator and the DFA Liaison Committee

Other CAS Web Sites

- CAS Committee on Theory of Risk*

*Participation limited to members of COTOR

Other Sites of Interest

- Important Variables Survey Form -- Friends and members of the Casualty Actuarial Society are welcome to make further suggestions about variables and data sources by filling out this survey form. Click here to view an archive of all such submissions.
- *Financial Theory and its Applications to Insurance/Actuarial Problems* -- a Research Bibliography.
- Agenda for the Limited Attendance Workshop on Financial Risk Theory, held October 1, 1995 at the Boston Marriott, Copley Place.
- Economics Data catalogued in Bill Goffe's Summer 1994 *Journal of Economic Perspectives* article entitled *Resources for Economists on the Internet* (Current version: Vol. 1, No. 12, January, 1996).
- Actuarial Resources on the Internet
- Jacobson Associates' listing of job openings for actuaries

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(http://www.risknet.com/dfa/finance/commerce.htm)

CASUALTY ACTUARIAL SOCIETY

Financial and Economics Databases

This page provides information concerning commercially available financial and economics database products.

Berkeley Options Database

The Berkeley Options Data Base is a historical record of trades and quotes, time-stamped to the nearest second, for all standardized contracts traded on the Chicago Board Options Exchange. The data base, which is derived from the CBOE's Market Data Retrieval tapes, begins in August, 1976 and is updated annually. Data are currently available through December, 1994.

Boston International Advisors

Boston International Advisors maintains a family of international stock market indices with historic returns and values beginning in 1975. The indices cover the performance of sectors of country stock markets based on growth and market capitalization. Approximately 5,000 stocks are included from over forty countries.

Citibase

The CITIBASE database contains approximately 7,000 monthly, quarterly, and annual economic and financial time series that date back to 1946 when available and end with the latest available observations. These data are collected from various government and private sources and distributed by FAME Information Services - a subsidiary of CITICORP. Monthly and Quarterly variable definitions and periods of time series are available on-line, as is a spec sheet that summarizes FAME's Financial, Index, Fundamentals, and Estimates Data Groups.

CRSP (Center for Research in Security Prices)

The Center for Research in Security Prices (CRSP) at the University of Chicago produces a number of data files on U.S stocks and government securities. The CRSP databases are very comprehensive and reliable, constituting one of the most important sources of security market data for researchers in the field of financial economics.

- The CRSP Stock Files contain stock price and return data for companies listed on the New York (NYSE), American (AMEX), and NASDAQ Stock Exchanges. Daily data are available from as early as 1962 for NYSE/AMEX securities, and 1972 for NASDAQ securities.

- The CRSP Bond Files contain term structure, bond price and return data. End-of-day price data on virtually all negotiable direct obligations of the United States Treasury are available during the period December 31, 1925, to the present.

More detailed information about these databases can be obtained by downloading and printing the 205 page manual for the CRSP Stock Files and the 75 page manual for the CRSP Bond Files. (***Important Note:*** *You will need to download and install a free program called Adobe Acrobat in order to view and print either of these documents.*)

Hoover's MasterList Plus Database

The Hoover's MasterList database was created and is maintained by The Reference Press, Inc. of Austin, Texas. This searchable database contains information on 6,700 publicly traded companies in the United States. Each company profile provides basic information needed for locating, communicating with, and evaluating the companies listed in the database.

Intex Solutions - Collateralized Mortgage Obligation data

Intex CMO Database lists over 30,000 bonds, modeled and updated every month.

Standard and Poors

Includes Comstock, J. J. Kenny Drake, Ratings Services, Platt's, MMS International, DRI/McGraw Hill, and the CUSIP Service Bureau.

Standard & Poors Compustat

COMPUSTAT provides superior accounting statement information on companies from around the world.

U.S. Commerce Department STAT-USA /Internet

The Department of Commerce gathers business and economic information from over 50 Federal Agencies and redistributes this information for a nominal subscription fee from its world wide web site. STAT-USA/Internet provides access to the
National Trade Data Bank (NTDB), the
National Economic, Social, and Environmental Data Bank (NESE-DB), the
Economic Bulletin Board, the Global Business Opportunities Service, and the
Bureau of Economic Analysis databases.

Last Updated 3/7/96.

CASUALTY ACTUARIAL SOCIETY

Free Data Sources

This page provides information concerning freely available financial and economics data sources

General Economic Data

Federal Reserve Bank of St. Louis' *FRED* Database NICE!

FRED stands for "Federal Reserve Economic Data". This free data source provides historical U.S. economic and financial data, including daily U.S. interest rates, monetary and business indicators, exchange rates, and regional economic data.

Business Cycle Data

Gordon's Business Cycle book a lengthy appendix which contains finance and macroeconomic data. It is provided in a text file (300K) in a SAS program format (not a SAS dataset) here.

Consumer Price Index. Monthly, 1913-1995

Financial Market Data

Financial Markets Data from the Federal Reserve Bank of Chicago NICE!

The Federal Reserve Bank of Chicago provides free and comprehensive financial markets datasets, including Foreign Exchange Rates, Selected Interest Rates, and Money Markets. Many datasets include daily data going all the back to 1971.

JP Morgan NICE!

JP Morgan is using the Internet to offer information needed to implement their RiskMetrics methodology and to provide data which can help managers control risk of their positions by using information on volatilities and correlations among over 300 financial assets. J.P. Morgan offers the following data for free: Commodity Index, Currency Indices, Emerging Markets Bond Index Plus, and a Government Bond Index.

Monthly Treasury Bill Rates, 1934-1995

This series provides averages of the daily closing T-Bill rate.

Treasury Bond Futures Data, 1994-95

This is an ASCII data file that contains high and low prices over 20 minute intervals on Treasury Bond futures from Jan 7 1994 to Feb 3 1995, for a total of 5347 observations. Variables reported include date, time, high price and low price. An hourly series is also available.

Term Structure Data Excel spreadsheet - 1.1 megabytes

McCulloch/Kwon US Term Structure Database

This data set offers U.S. Treasury term structure data for the period 1947-1991.

Aggregate Stock Market Information

Most of the following data are current as of year end 1995:

- All 4,417 Tickers, Company Names and SIC Codes for NYSE
- All 10,616 Tickers, Company Names and SIC Codes for OTC
- All 1,445 Tickers, Company Names and SIC Codes for AMEX
- The 500 Companies in S & P 500 Ranked by 1995 Stock Price Appreciation
- Dow Jones Industrials Performance Since 1929
- Monthly Stock Price Performance of S&P 500 since 1984 (Last 2/29/96)
- The 500 Companies in S & P 500 -- Stock Price Performance P/E Yields, etc.

New York Stock Exchange Daily Returns & Volume 1962-1992

Weekly Dow Jones Industrial Average 1900-1989

This dataset lists an important aggregate stock price index beginning in 1900. The data is in date, high, low, close, volume format. A daily version of this dataset is also available, but it nearly 2 megabytes in size and starts in 1915.

Corporate Data

EDGAR Database NICE!

EDGAR is the Electronic Data Gathering, Analysis, and Retrieval system. It is a free service provided by the Securities and Exchange Commission (SEC). EDGAR is an important source of corporate financial report data, providing online access to the complete 10Ks, 14Ds, S3s, 8Ks etc. of most public companies in the US over the last few years. Nearly three-quarters of the publicly traded domestic (U.S.) companies use EDGAR to make the majority of their filings, and all registrants will be required to do so starting May 1996.

Corporate Debt Issues, 1983-93

This Excel file (2.7MB) lists over 10,000 bonds, convertibles, Euronotes, MTNs, Warrant bonds and other issues by company and CUSIP number (where available). Click here to download the same file in comma separated value format (1MB). For more information about the data click here.

This page has been visited 0 0 0 3 1 times since March 5, 1996.
Last Updated 3/7/96.

A. M. Best

Insurance Data

This page points to insurance data available from A. M. Best Company.

More detailed information about A. M. Best Property and Casualty Insurance database products can be obtained from their world wide web site. The address for A. M. Best's home page on the World Wide Web is http://www.ambest.com.

Details on the A.M. Best Database

Like OneSource, A.M. Best is a valued added reseller of NAIC annual statement data. A.M. Best runs each company's statement data through a rigorous process of intra and inter-page cross checks to ensure accuracy. Furthermore, A.M. Best also provides other useful information that extends well beyond the data on a company's annual statement.

The two key file types of interest to researchers include the A.M. Best Statement and Product Files. Statement Files retain the basic organization structure of the NAIC annual statement. All key data items found on a given page or schedule are presented in an individual file. Product Files present selections of data provided in several of A.M. Best's print publications, such as Best's Insurance Reports, Best's Insurance News, Best's Key Rating Guide, Best's Market Guide, and Best's Experience By State (By Line). Furthermore, A.M. Best also provides a Custom Files service that supports the creation of custom data selections. Furthermore, Best's has developed its own proprietary network for online access to data called BestLink, allowing access via local access telephone numbers as well as with IBM's `advantis` network. The pricing of these services (valid as of March 1996) are as follows):

STATEMENT FILES retain the basic organization structure of the NAIC annual P statement. All key data items found on a given page or schedule are presented in an individual file.

		Statement Pages Included on File	Standard Products Available	Any Single Year of Data	Each Additional Year of Data	Five Years of Data
PC-BF-01	Balance Sheet	2,3	Tape/Disk	$525	$175	$1,050
PC-BF-02	Income Statement	4	Tape/Disk	$375	$125	$750
PC-BF-03	Cash Flow	5	Tape/Disk	$375	$125	$750
PC-BF-04	Investment Income & Capital Gains/Losses	6	Tape/Disk	$525	$175	$1,050
PC-BF-06	Premiums Written (By Line)	8	Tape/Disk	$525	$175	$1,050
PC-BF-09	General Expenses	11	Tape/Disk	$525	$175	$1,050
PC-BF-11	Stocks&Bonds-Summary	29	Tape/Disk	$555	$185	$1,110
PC-BF-12	Bonds-Quality&Maturity Distribution	30-33	Tape	$975	$325	$1,950
PC-BF-14	Loss Reserves	72-126	Tape/CD	$8,500	N/A	N/A

PC-BF-14-Z	Loss Reserves-Summary	72,73	Tape	$850	N/A	N/A
PC-BF-15	Direct Business (By State)	131	Tape/Disk	$525	$175	$1,050
PC-SF-16	Underwriting Analysis with Ratios (By Line) - IEE	IEE	Tape/CD	$1,200	$400	$2,400
PC-SF-52*	P/C Best's Statement File (with Best's Ratings)	N/A	CD	N/A	N/A	$10,000

PRODUCT FILES present selections of data provided in several of A.M. Best's Statement printed publications, such as Best's Insurance Reports, Best's Key Rating Guide, Best's Market Guide, and Best's Experience By State (By Line).

		Statement Pages Included on File	Standard Products Available	Any Single Year of Data	Each Additional Year of Data	Five Years of Data
PC-PF-01	Name & Address	N/A	Tape/Disk	$450	N/A	N/A
PC-PF-02*	P/C Exp. by State (By Line) All Sts.-Standard Lines	14:All Sts.	Tape/CD	$4,500	$1,500	$9,000
PC-PF-02A*	P/C Exp. by State (By Line) Standard Lines	14:Ea. St.	Tape	$375	$125	$750
PC-PF-03*	P/C Exp. by State (By Line) All Sts.-Combined Lines	14:All Sts.	Tape	$4,500	$1,500	$9,000
PC-PF-03A*	P/C Exp. by State (By Line) Per St.-Combined Lines	14:Ea. St.	Tape	$375	$125	$750
PC-PF-05*	P/C Key Rating Guide (with Best's Ratings)-Regular Service	N/A	Disk	N/A	N/A	$175
PC-PF-05*	P/C Key Rating Guide (with Best's Ratings)-Full Service	N/A	Disk	N/A	N/A	$535
PC-PF-05S	P/C Key Rating Guide-Supplement (2nd & 3rd Qtrs.)	N/A	Disk	N/A	N/A	S75
PC-PF-50	P/C Best's Ins. Reports (with Best's Ratings)-Regular Service . .	N/A	CD	$2,500	N/A	N/A
PC-PF-50	P/C Best's Ins. Reports (with Best's Ratings)-Full Service	N/A	CD	$2,860	N/A	N/A
PC-PF-01	Name & Address	N/A	Tape/Disk	$450	N/A	N/A

BESTLINK SERVICES is A. M. Best's proprietary online database that provides continually updated financial data on more than 3,800 insurers, as well as daily insurance-related news.

Unlimited Access Options

Users requiring frequent and extensive access to one or more BestLink databases can purchase the right to unlimited access to the file(s) by prepaying the equivalent of the basic file cost (magnetic tape or CD-ROM file) plus 30%. If you have already purchased a current data year file on tape or CD-ROM, you can be credited the tape or CD-ROM price toward the unlimited access price and charged the additional 30%. (Example: If you purchased the Statement File at $10,000, you can receive unlimited BestLink access for an additional $3,000.)* Note: The $15 per hour Connect Time charge will still apply.

	Tape/CD Price	Unlimited Online Access Charge (30%)	Total Price
Loss Reserves/Schedule P (P/C)	$8,500	$2,550	$11,050
Schedule D (P/C)	$3,060	$918	$3,978
Underwriting Analysis with Ratios (IEE)-(P/C)	$2,400	$720	$3,120
Experience By State (By Line)-Std. Lines (P/C)	$9,000	$2,700	$11,700
Experience By State (By Line)-Cmb. Lines (P/C)	$9,000	$2,700	$11,700
Insurance News	$500	$150	$650
Statement File* (P/C)	$10,000	$3,000	$13,000
Best's Company Reports	$2,500	$750	$3,250

*Best's Statement File on CD-ROM includes unlimited access to the corresponding databases on BestLink: Profile Annual, Profile Quarterly, Financial, Schedule D, and Reinsurance-Summary (P/C only).

CUSTOM FILES

To order custom data products, call A. M. Best Custom Products & Services at (908) 439-2200, extension 5383.

Insurance Services Office

Insurance Data

This page points to insurance data available from Insurance Services Office (ISO).

The Insurance Services Office section of DFAWeb is under "heavy construction". Please come back later for a more complete site.

OneSource US Insurance: Property and Casualty

Insurance Data

This page provides details concerning insurance database services available from OneSource.

More detailed information about the OneSource US Insurance: Property and Casualty products, including software demos, can be obtained from their world wide web site. The address for OneSource's home page on the World Wide Web is http://www.onesource.com.

Details on the OneSource Database

1. Definition

OneSource Information Services (OIS) offers more than 50 electronic business information titles that respond to market demands for products that combine high quality information with state-of-the-art access and manipulation software.

In the case of the OneSource US Insurance: Property and Casualty products, OIS delivers 5 annual statement information titles that include the company financials, (including the IEEs), the page 14 State and LOB information, and the full details of Schedules P, F and D. The source of the information is the National Association of Insurance Commissioners (NAIC) with whom OIS has a long term redistribution agreement.

2. How long a time series is available?

Most of the financial information is presented in an integrated 5 year historical series. The Schedule P, F and D products are current year only.

3. Cost and feasibility of obtaining

OneSource does not publish a price list to the public, but their products are delivered on a flat price annual subscription basis that varies with number of databases accessed and number of user groups. While the data are delivered on CD/ROM, OneSource "products" include customer-specific training and intensive support services. The products are Windows-based and can be run on i486 or higher PCs with configurations for stand alone computers or local area networks (LANs).

The first delivery of the annual financial information, including Schedule P, occurs at the end of March for the previous year's data. OIS refreshes the database around April 15th and around the 15th of each month there after. New information continues to flow in during the spring and early summer in consonance with the deadlines for the various filings, i.e. Combined Filings for groups, quarterly filings, etc. Schedules D and F are initially released around the 20th of April and updated two additional times in the summer and fall.

4. Legal considerations: who owns and confidentiality

Subscribers must sign and adhere to the provisions of OneSource's product license agreement. Like other software license agreements, this document requires the subscriber to acknowledge that the product is a copyrighted work, and that the data is the property of the data vendor. The agreement spells out how the product can be installed & used, what the subscriber's redistribution rights and restrictions are, and addresses the issue of indemnity.

5. Available by company or by larger groupings?

The financials and Schedule P are available for both individual companies and for the "combined" NAIC filing groups. Users can manipulate the datadase using 10,000+ different criteria to form additional groupings for peer group analysis or benchmarking.

6. Other relevant information

The products can be directly accessed from either Lotus 1-2-3 or Microsoft Excel by using the OneSource Add-in. This feature empowers the spreadsheet user who wants to develop proprietary analytical and graphical models. The needed information is tagged using controls in the add-in software and it then flows automatically into the spreadsheet from the CD/ROM.

OneSource's Schedule D holdings database includes the complete securities portfolio of every holding of every company. The holdings data can be manipulated to develop groupings based types, classes and quality, as well as many other criteria.

(http://www.risknet.com/dfa/insurance/naic/naic.htm)

National Association of Insurance Commissioners

Insurance Data

This page provides details concerning insurance database services available from National Association of Insurance Commissioners (NAIC).

The NAIC Database Products Catalog provides further descriptive information. The address for the NAIC's home page on the World Wide Web is http://www.naic.org

Details on the NAIC Database

Definition: The NAIC maintains the largest insurance industry database in the world, with over 4700 Life/Health and Property Casualty companies. This accounts for 98% of all U.S. domiciled insurance companies. The information on the NAIC database captures nearly all of the information from the statutory filings that the insurance companies are required to submit. The database also contains information filed by Title, Fraternal, and HMDI companies.

The number of insurance companies reporting to the NAIC and the availability of their data to the commercial market is as follows:

Company Type	Number of Companies	Filing Date	Data Availability
Life	1,692	3/1	4/1
Property	2,685	3/1	4/1
Fraternal	139	3/1	4/1
HMDI	119	3/1	4/1
Title	91	3/1	4/1
Combined Filings	P/C 325 L/H 234	5/1	5/15

The information on the database dates back to 1984. Any or all years of data can be extracted from the database.

Timeliness: Preliminary data for the current filing year is available as early as the first week in April, as indicated in the table above. The database is finalized and complete in the second week of June.

Formats Available: Requests for nearly any media or format are easily accommodated by the NAIC. Available media types include CD-ROM, 3.5 inch diskette, cartridge tape, or reel tape. Data can be produced in mainframe formats, comma delimited formats for use with PC's, as well as many other formats.

Legal Considerations : The NAIC requires customers to sign a Database License Agreement. This agreement is a standard contract that describes payment, shipping, and order processing terms. The contract also describes how the data may be used. The standard License Agreement specifies that the data is for internal use only and redistribution is not allowed. When necessary, a customized License Agreement can be written to accommodate certain uses not permitted in the standard contract.

Availability by individual company or company groupings: The customer has total flexibility in the selection of the type and number of companies selected. Data can be extracted for all companies in the database, or for customized lists of companies. The NAIC Database Products technical team can also extract data for companies chosen by custom selection criteria as specified by the customer.

Pricing: There are standard prices for many parts of the database. Pricing for some of the most commonly requested information is as follows:

	Life/Health	Property Casualty
Standard "A" List (see attached list of schedules)	$24,000	$34,500
Schedule D Detail (All Parts)	$ 5,000	$ 8,000
Schedule F (All Parts)		$ 8,000
Schedule S (All Parts)	$ 4,500	
State Page Data	$ 2,300	$ 3,300
State Page CD-ROM/Reports	$ 3,000	$ 4,000
Balance Sheet/Income Statement	$ 1,100	$ 1,300

Pricing for custom orders is determined by individual estimate.

National Council on Compensation Insurance

Insurance Data

This page provides details concerning insurance database services available from National Council on Compensation Insurance (NCCI).

More detailed information concerning the NCCI's InsNet Online Service and Research and Reference Products can be found on NCCI's home page on the World Wide Web, located at http://bocaraton.com/ncci/.

Details on the NCCI Databases

The National Council on Compensation Insurance, Inc. (NCCI) headquartered in Boca Raton, Florida, is the nation's largest information company serving the voluntary and involuntary Workers Compensation marketplace. The corporation provides database products, software, publications and consultation services to state funds, self insureds, independent bureaus, agents, regulatory authorities, legislatures and more than 700 insurance companies.

A description of four of NCCI's major databases follows:

Policy Issue Capture System (PICS)

The Policy Issue Capture System serves as the database of workers compensation and employers liability policies. PICS data is the information from the actual policy information page issued by the insurer to the insured. Policy data is used for controlling the submission of WCSP data, the distribution of experience ratings to insurers and for NCCI's Proof of Coverage service provided to Industrial Accident Boards and Commissions. Information on policies for the latest three years is readily available.

Financial Data Calls

NCCI collects aggregate financial data calls which are used to determine the aggregate rate or loss cost level in a state. The primary ratemaking calls are the Policy Year and Accident Year Calls for Compensation Experience by State. These calls gather collected premiums, premiums at a common level, and losses for over 15 years of aggregate financial data by state.

A major product produced from the Financial Data Calls is Loss Development Exhibits (Product Code 2911) This package of exhibits provides a history of loss development factors by state for both policy year and accident year experience. Factors are provided for four development methodologies for indemnity, medical and total losses. The development methodologies are paid, paid plus case, incurred excluding IBNR, and incurred including IBNR. Additional exhibits in the package include paid to incurred ratios and premium development factors as well as summarized financial data. This product is available in hard copy or on diskette. The price is $1,000 per state for hard copy ($350 per state for affiliates) and $2,000 per state for diskette ($700 per state for affiliates).

Workers Compensation Statistical Plan (WCSP)

Workers Compensation Plan data is the audited exposure, premium, and loss experience summarized by policy and state on a unit report for each Workers Compensation policy. The WCSP requires losses on the unit report to be valued as of the 18 months after policy effective date. Subsequent unit reports through a fifth report are required at 12 month intervals thereafter for any policies which contain open claims as of the previous submission.

A major product produced from this database is Class Experience (Schedule Z Summary Data) (Product Code 2838) Schedule Z summarizes by class the combined experience for all affiliates in a state as reported on the Workers Compensation Statistical Plan. The report provides the class experience including exposures, premiums, indemnity losses and medical losses and claim counts by injury type. Experience is furnished for the latest five policy periods available. This product is available on hard copy, diskette, or magnetic tape. Beginning approximately second quarter 1996 Class Experience will be available through InsNet, NCCI's on-line network.

Detailed Claim Information

Detailed Claim Information (DCI) collects 85 detailed data elements describing the insured, the claimant, the claim characteristics, the benefits and payment made, and the claim administration details of individual claims. The purpose of DCI is to provide insight into the underlying elements inherent in the aggregate costs of workers compensation insurance. Claims are selected based on a sampling methodology which concentrates on collecting information for major injuries. Claims are valued at six months after accident date with subsequent reports required at annual intervals up to ten reports for any claims that remain open.

New summary publications from this database will be produced in 1996. Custom data extracts are available on diskette or magnetic tape.

Other Products of Special Interest to Actuaries

The Annual Statistical Bulletin (Product Code 2845) $225 ($145 affiliates) contains a summary of the latest and most significant statistics on Workers Compensation available. Reference tables provide histories of premium and benefit level changes by state, expense data, tax provisions, benefit provision summaries, loss development factors, and claim frequency and severity exhibits. The Bulletin in published annually in hard copy format.

Economic Conditions Report (Product Code 3043) This compendium of data and forecasts from government agencies, private companies, and universities contains comprehensive statistics that cannot be found anywhere else in one source. It provides indications of the changes that are occurring in a state's economy and how those changes impact the Workers Compensation system. Available in hard copy for $250 per state per year. ($125 per state per year for affiliates).

Legal Considerations

NCCI owns the data contained in its databases and licenses it to interested persons.

Available By Company

Specific carrier and specific risk data is not available. Custom requests may be produced for specific groups of carriers as long as the individual carrier data is protected.

Other Relevant Information

NCCI affiliates receive significant discounts on most NCCI products and services. NCCI affiliation programs are available for private carriers, state funds, self insurance groups and reinsurers.

For a complete catalogue of NCCI products and services or for more information on any NCCI product call Customer Service at 800-NCCI-123 (800-622-4123) from 8 AM to 8 PM EST.

Reflecting Reinsurance Costs in Rate Indications for Homeowners Insurance

by Mark J. Homan, FCAS

Reflecting Reinsurance Costs in Rate Indications for Homeowners Insurance

by

Mark J. Homan

Biography

Mr. Homan is the Director of Personal Lines Catastrophe Management with ITT Hartford. Prior to this position, he was the Director of State Management for Agency Personal Lines and spent seven years as the Actuary and Director of Personal Property Pricing with ITT Hartford. He received a B.A. degree, *summa cum laude*, with majors in Mathematics and Quantitative Methods from the University of St. Thomas, St. Paul, MN. He is a Fellow of the Casualty Actuarial Society (1987), a Fellow of the Canadian Institute of Actuaries (1990), a Member of the American Academy of Actuaries and a Chartered Property Casualty Underwriter (1995). He has authored two other papers on property ratemaking, *Homeowners Insurance Pricing* and *Homeowners Excess Wind Loads: Augmenting the ISO Wind Procedure*.

Abstract

This paper presents the rationale for reflecting reinsurance costs explicitly in Homeowners indications. Catastrophe reinsurance has become relatively expensive and it should be reflected in rates to ensure rate adequacy. The basic concepts to adjust historical losses for the benefits of reinsurance and to reflect the reinsurance premium will be addressed. One approach for dealing with the concepts will be illustrated with some discussion of possible variations.

Reflecting Reinsurance Costs in Rate Indications for Homeowners Insurance

Overview

Reinsurance costs are widely recognized as a legitimate cost of doing business. In the past, these costs were not **explicitly** reflected in Homeowners rate level indications but were either ignored or only **implicitly** reflected. They were implicitly reflected to the extent that the loss portion of reinsurance costs was assumed to be in the direct losses. The additional transaction costs were not always getting into the indications, and then, only indirectly.

Most often reinsurance costs were simply ignored, since most of the ratemaking procedures used are based on the ISO procedures. Since ISO is a bureau, not an insurance company, they do not purchase reinsurance so they do not recognize it in their techniques. Also, now that ISO produces only loss costs rather than rates, and since reinsurance is an expense item, reinsurance costs should not be part of the ISO loss cost procedure.

In the past, companies relied on the excess wind procedure to give them an adequate loading for catastrophe events. If it were sufficient, then the companies were only overlooking the transaction costs of reinsurance. When the reinsurance costs were relatively low, the transaction costs were low, so the omission of reinsurance costs had only a small impact on the rate indications.

Now catastrophe reinsurance costs are much higher and we know that the excess wind procedure does not generate an adequate catastrophe loading. It is no longer prudent to omit reinsurance costs from explicit treatment and still expect to produce an adequate rate. Thus, the indication procedure should be changed to allow for direct reflection of reinsurance costs. In many states there is not sufficient room to fully reflect these costs implicitly, if they ever were reflected.

This paper will outline a basic approach that could be taken to reflect reinsurance costs in ratemaking. The paper discusses reflecting the cost of a property catastrophe treaty (referred throughout as catastrophe reinsurance) but the techniques could be applied to any reinsurance treaty.

Underlying Justification

In reviewing the CAS *Statement of Principles Regarding Property and Casualty Insurance Ratemaking*, one can find several items that touch on the validity of reflecting reinsurance costs in rates. Two items are of particular interest.

Principle 2 states that "a rate provides for all costs associated with the transfer of risk." Under the Considerations section, the Principles state that "Consideration should be given to the effect of reinsurance arrangements."

There are two primary impacts from a reinsurance arrangement. First is the cost for the risk transfer, the reinsurance premium, and second is the reduction in incurred losses, the loss recoveries. Part of the process of risk transfer that an insurance company uses is the transfer of a portion of their risk to other parties via reinsurance transactions. Such risk transfer is necessary to preserve the financial solvency of the insurer and protect their assets so that claims may be paid. This makes the reinsurance cost a component in the overall cost associated with the transfer of risk. Thus, the Statement of Principles does not merely allow for the reflection of reinsurance costs but compels us to consider such costs.

Some may also question whether catastrophe reinsurance is a legitimate cost of doing business. It seems that its primary function is to protect the insurance company's assets after a significant event. The arguments against catastrophe reinsurance as a legitimate cost are getting much quieter in recent years. It is clear that catastrophe reinsurance is important for a company to maintain it's ability to pay claims. Several companies become insolvent after Hurricane Andrew and the Northridge earthquake. Additional catastrophe reinsurance may have protected many of these companies. In addition, A.M. Best now reviews the catastrophe exposure and catastrophe reinsurance programs of a company as part of their rating procedure. Inadequate management of catastrophes, such as not managing exposure levels with appropriate reinsurance, will lead to a lower rating which may impact a company's marketing. Clearly catastrophe reinsurance has become a necessity for any company with significant property writings. Several states now have specific regulations allowing the reflection of reinsurance costs in ratemaking, recognizing their validity.

As stated earlier, some companies may have been implicitly reflecting reinsurance costs in their rates through the selection of a rate change based on the indications. More likely, I believe that these costs were basically ignored in the past. To reflect the costs implicitly, there must be sufficient room between the indications filed and the actual change that the company feels is necessary. This gap stems primarily from the allowable profit and contingency load and that the company truly feels it needs. However, as more states are becoming tighter on how profit loads are determined, the gap is getting smaller. At the same time, catastrophe reinsurance costs have increased to historically high levels. This leaves insufficient room in the more cat prone states to reflect these costs implicitly, leading to the need to reflect these costs explicitly, at least for catastrophe reinsurance. The smaller costs from other reinsurance programs are still ignored by most companies, or treated implicitly. In many cases, their costs may be too small to justify the effort to reflect them explicitly.

Basic Outline

At my company, we are only reflecting our catastrophe reinsurance treaty in indications at this time. This paper will only address this one treaty and not the other types of reinsurance that a company may purchase. While other forms of reinsurance could also be reflected using a similar approach to that taken for the catastrophe treaty, I will not develop all the comparable allocations of premium and loss benefit that would be needed.

These other reinsurance treaties do not represent nearly as significant a cost to Homeowners as does a catastrophe treaty. So, at this time, I have chosen to limit my discussion to reflecting catastrophe treaty costs.

A reinsurance premium contains two primary components. The first is the **loss benefit** which represents the recoveries from the reinsurer that should be expected over the long term for the coverage purchased. The second component is the reinsurer's expenses and profit, the **transaction costs**. In theory, the expected loss recoveries **should** already be reflected in the direct loss estimates in traditional indication procedures, so it is only the transaction costs for reinsurance that need to be added.

There are some catastrophe treaties that include a payback provision. In essence, this reduces the loss recovery benefit of the treaty, since the reinsurer is basically loaning the funds that will be paid back. Thus, the loss benefit should be reduced by the funds that will be paid back.

There are two possible approaches to loading in the reinsurance transaction costs. Theoretically, they both will yield the same answer, with perfect information. But the practicalities of applying the methods will drive the choice of which method to use. The first approach would be to break down the reinsurance premium into the loss and transaction cost components and then reflect only the transaction cost portion as an additional expense. However, it is extremely difficult, if not impossible, to determine this breakdown. Reinsurers do not file rates nor do they typically release such breakdowns.

In fact, catastrophe reinsurance costs are as much a function of supply and demand as they are the underlying economics. So this first approach is theoretical only and is not practically feasible.

The second approach eliminates the need to determine the breakdown. This approach reduces the projected losses used in the rate level indications to reflect the expected benefits of reinsurance and then loads the entire reinsurance premium as an expense. It is this second approach that I advocate and will present here.

Net Loss plus Reinsurance Approach

The approach that we have recently developed is referred to as the *Net Loss plus Reinsurance* Approach. The basic procedure is to determine the reinsurance premium by state, adjust the losses to a net basis (after reinsurance) and load the reinsurance cost as an expense item. The following sections will outline each step in more detail. As used herein, the term "net" refers only to net of the reinsurance treaties which costs are being explicitly loaded, not final net of all reinsurance, pools, etc. Also, the premiums are on a direct basis, not net.

Allocating Reinsurance Premium to State

The first step in reflecting the reinsurance costs in the rate indications is to determine what these costs are for each state. Most catastrophe treaties are countrywide, corporate level treaties. Therefore, we must break down the total reinsurance premium to state and line. While this allocation will vary depending on individual company circumstances, a general approach will be discussed here. An illustration of this allocation is shown in Exhibit 1. The example shown is just for one line. If multiple lines were involved, they could be treated as if they were additional states.

Exhibit 1

Reinsurance Premium Allocation to State

State	Subject Premium	Estimated Annual Loss to Treaty	Allocated Premium Based on Losses *	Residual Allocation Based on Premium	Total Reinsurance Allocation
A	18,975,000	2,345,000	3,165,750	10,071	3,175,821
B	7,650,000	0	0	4,060	4,060
C	17,325,000	1,350,000	1,822,500	9,195	1,831,695
D	11,038,000	0	0	5,858	5,858
E	650,000	0	0	345	345
F	4,650,000	980,000	1,323,000	2,468	1,325,468
G	22,950,000	1,765,000	2,382,750	12,180	2,394,930
H	4,850,000	0	0	2,574	2,574
I	4,425,000	375,000	506,250	2,349	508,599
J	1,225,000	0	0	650	650
Total	93,738,000	6,815,000	9,200,250	49,750	9,250,000

Total Reinsurance Premium	9,250,000
Residual Premium	49,750

* - Estimated Premium is Expected loss loaded by 35% for Expenses, Profit and Risk Load

The allocation is done in two stages. First, the expected losses for major events are determined for each state that has a significant exposure to large catastrophes such as hurricanes or earthquakes. We estimate these losses through the use of models. We use both an in-house single event model for hurricane and earthquake and a simulation model from an outside vendor to develop estimates. These outside vendor models are becoming widely used within the industry and all can provide loss estimates for extreme events on a state basis. Earthquake must be split separate from hurricane since not all of these losses are covered by Homeowners. In fact, the majority are covered under a separate line. Some earthquake losses are covered in certain Homeowners endorsements, such as an "all risks" contents endorsement like the ISO HO-15. This portion of the earthquake losses should be allocated to Homeowners along with the hurricane estimates. These major events represent a significant portion of the catastrophe treaty costs, since these are the events that the treaty is expected to cover.

The expected losses are then loaded by a factor to represent the reinsurer's expenses, risk load and profit. For illustration, the exhibit shows a 35% load. This converts the expected losses to an estimated premium. To the extent possible, the loading should represent that actually used by the reinsurer in the treaty. Often, this is not directly available from the reinsurer, so it must be estimated. The loading, actual or estimated, will vary based on the reinsurance market and the amount of capacity in the market relative to demand. The procedure described is somewhat sensitive to the loading selected. A higher loading will allocate more of the treaty costs based on the expected losses from major events. Some analysis has estimated this load to run as high as 50% to 100% of the expected losses for some catastrophe treaties.

There is typically some additional cost beyond the major events. This explains why there is some residual reinsurance premium to allocate beyond the major events. The residual premium is then allocated based on subject premium (the premium for lines subject to the treaty). Every state receives some allocation, even if a small one, since there can be multi-state events that will entail a reinsurance recovery. The amount of premium allocated based on subject premium should be fairly small and will depend on the expected losses and loading chosen. Using the subject premium is not completely accurate since states with similar premium volumes may have significantly different exposures to catastrophic events. Further research into the use of loss estimates from certain perils or events rather than subject premium will improve this allocation.

Coastal states will have a greater allocation than the inland states, such as the Midwest, since they have more significant catastrophe potential. In addition, the Homeowners line has more catastrophe potential than Inland Marine or Automobile Physical Damage which are also subject to the catastrophe treaty. So coastal Homeowners states will receive a catastrophe treaty allocation that is greater than the corporate average.

Adjusting Losses to a Net Basis

Since the selected procedure reflects the full reinsurance premium as an expense, we can not reflect the full loss loading. Otherwise, we would be double counting some losses; in both the reinsurance costs and in the direct losses. Therefore, we adjust the direct losses to a net basis (i.e. after catastrophe reinsurance), to eliminate any double counting. Since most large events are capped to their net basis, it is less important that they are initially estimated accurately. The amount of loss that is removed is not important. These losses are loaded through the reinsurance premium. Thus, the reinsurance premium can serve to provide the necessary loading for larger events.

The actuary should also determine whether certain events are capable of exceeding the upper limit of the treaty. If an event blows through the treaty, the company will be responsible to pay these losses with no recovery. Clearly, these additional losses beyond the treaty limit should continue to be reflected in the rates at a 100% basis.

The method discussed here is based on using an excess wind procedure to develop the underlying loss estimates. Further discussion on variations to the approach based on the method used to determine the underlying loss loading is included in Appendix A. We adjust the losses to a net basis in two ways. First, the excess wind procedure is modified so that any wind event reflected in the long term load is adjusted to a net basis. Second, any event in the 5 year indication experience period that is other than wind or hail, and thus not in the excess wind procedure, and which exceeds the treaty threshold is also capped. The catastrophe treaty threshold is determined by state.

Assuming that the treaty is corporate in nature, Homeowners losses do not need to reach the corporate attachment point to generate recoveries. Recoveries on the Homeowners line will begin once the total corporate losses exceed the attachment point. To determine the level of losses at which the catastrophe treaty will start to cover Homeowners losses, the ratio of the Homeowners reinsurance premium to the total reinsurance premium for the state is multiplied by the corporate treaty attachment point. For example, if Homeowners represents 39% of a state's reinsurance premium and the corporate treaty attachment point is $50 million; then, the threshold for Homeowners is $19.5 million. This means that that if Homeowners losses exceed $19.5 million, we expect that the corporate losses will exceed $50 million and we will then recover losses above that point from our reinsurer. However, each actual event will have a different mix of damage for each line covered. So while this may be the expected values for line by line retention, it will vary by event. Alternative approaches, such as modeling of each event, may not need to rely on this assumption.

In addition, most catastrophe treaties do not pay 100% of the losses subject to the treaty. There is some copayment by the insurer to make sure that the company is still vigilant in their loss settlement practices. For example, if a catastrophe treaty will pay 95% of the losses subject to the treaty, we should retain 5% of the losses above the threshold. The example shown in Exhibit 2 reflects a 5% copayment.

As mentioned earlier, the basic approach here is based on a variation of the ISO Excess Wind Procedure. The variation on the previous ISO methodology augments the excess wind procedure by reflecting a longer historical period through the application of modeling. A 50 year plus event is reflected to extend the historical period from the current 35 years or so. In many states, the limited history is inadequate to produce a proper loading (for catastrophes, 35 years is still inadequate). By augmenting the actual history with a projection for more extreme events, a more accurate loading can be developed. Thus, we are no longer at the mercy of what may have happened in the historical period. This event is determined from the models by taking the top two percentile of potential events and deriving an annual expected loss from such events.

We remove any actual year from the historical period any loss that exceeds the modeled 50 year plus level to avoid any overstatement or double counting of extreme events. By weighting the modeled 50 year plus loss event at 2% (once in 50 years) and the remaining history at 98%, we derive an excess wind factor that reflects extreme events. As shown in Exhibit 2, for this example, we weight the 1.030 factor from the historical period with a 1.474 factor from the modeled event to yield a final excess wind factor of 1.039. (.98*1.030 + .02*1.474 = 1.039) However, we are still not reflecting the full spectrum of events since there may be a gap between the historical events and the 50 year plus event. Yet, we are making a more accurate projection of the loading needed to cover excess wind events than is possible using the historical period only.

A sample calculation of adjusting the ISO excess wind procedure is shown in Exhibit 2. The modified excess wind procedure starts with the historical wind and total losses as before. The wind and non-wind losses are then restated to current cost levels in order to apply the current reinsurance treaty coverage. To adjust the losses to current levels, we multiply the historical wind/non-wind ratio by an average of the non-wind losses for the past three years trended to the projected cost level. This brings the wind losses from their historical level to the projected level using the non-wind losses as a cost index. The resulting wind losses are then capped for the effect of the catastrophe treaty. The wind/non-wind ratio is then recalculated and the calculation proceeds as before from this point. For a discussion of the remaining steps in the calculation, please refer to Appendix B included or to my earlier paper.[1]

The historical losses used can be either industry or company losses. The non-wind projected losses used must be a company basis to allow the reinsurance capping to be applied. The historical years are used to determine a wind/non-wind ratio to multiply the projected non-wind loss average by, on an individual year basis. Because of this, you can even mix industry experience with company experience. This may be advisable since the industry experience typically lags the experience available on a company basis. The example shown is based on company experience for all our Homeowners operations combined. The other exhibits are only for one operation so they will not balance precisely. This is similar to what one would see if we had used industry experience for the excess wind load calculation and company experience for the calculations shown on the other exhibits.

[1] Homan, Mark, *Homeowners Insurance Pricing*, CAS Discussion Paper Program, Pricing-May 1990, pg. 719

State: Example

Homeowners Insurance - Forms 1,2,3,& 5
Derivation of Excess Wind Factor

Exhibit 2

(1) Year	(2) HO Wind Losses	(3) Non-Wind Losses	(4) Wind-to-Non-Wind	(5) Wind Losses	(6) Wind Loss Adj Reins	(7) Adj Wind/ Non-Wind	(8) Excess Years*	(9) Excess Ratio	(10) Excess Losses	(11) Non-Excess Losses	(12) Non-Wind/ Non-Excess
1961	333,914	1,317,614	0.253	8,111,698	8,111,698	0.253	0.253	0.140	4494736	35625485	0.898
1962	165,136	1,381,151	0.120	3,827,073	3,827,073	0.120	.	0.000	0	35835595	0.893
1963	314,163	1,945,021	0.162	5,170,060	5,170,060	0.162	.	0.000	0	37178582	0.861
1964	300,898	2,651,451	0.113	3,632,458	3,632,458	0.113	.	0.000	0	35640979	0.898
1965	279,773	3,661,664	0.076	2,445,638	2,445,638	0.076	.	0.000	0	34454160	0.929
1966	406,828	3,907,434	0.104	3,332,608	3,332,608	0.104	.	0.000	0	35341130	0.906
1967	647,379	5,424,683	0.119	3,819,880	3,819,880	0.119	.	0.000	0	35828402	0.893
1968	718,958	6,118,069	0.118	3,761,443	3,761,443	0.118	.	0.000	0	35769965	0.895
1969	769,906	6,948,936	0.111	3,546,379	3,546,379	0.111	.	0.000	0	35554900	0.900
1970	709,614	8,283,773	0.086	2,741,950	2,741,950	0.086	.	0.000	0	34750471	0.921
1971	951,449	8,053,825	0.118	3,781,367	3,781,367	0.118	.	0.000	0	35789889	0.894
1972	1,232,940	9,173,544	0.134	4,302,000	4,302,000	0.134	.	0.000	0	36310522	0.882
1973	963,160	12,347,440	0.078	2,496,819	2,496,819	0.078	.	0.000	0	34505341	0.928
1974	2,124,450	14,700,504	0.145	4,625,726	4,625,726	0.145	.	0.000	0	36634248	0.874
1975	2,368,211	16,620,161	0.142	4,560,903	4,560,903	0.142	.	0.000	0	36569424	0.875
1976	2,117,819	12,914,435	0.164	5,249,029	5,249,029	0.164	.	0.000	0	37257551	0.859
1977	1,249,659	14,735,064	0.085	2,714,595	2,714,595	0.085	.	0.000	0	34723116	0.922
1978	1,457,036	12,079,981	0.121	3,860,733	3,860,733	0.121	.	0.000	0	35869254	0.892
1979	1,550,489	13,988,540	0.111	3,547,822	3,547,822	0.111	.	0.000	0	35556344	0.900
1980	1,357,404	18,499,926	0.073	2,348,576	2,348,576	0.073	.	0.000	0	34357098	0.932
1981	8,501,300	12,407,363	0.685	21,931,659	19,669,083	0.614	0.614	0.501	16052120	35625485	0.898
1982	1,233,589	18,804,394	0.066	2,099,794	2,099,794	0.066	.	0.000	0	34108316	0.938
1983	1,380,579	14,345,616	0.096	3,080,403	3,080,403	0.096	.	0.000	0	35088925	0.912
1984	1,846,638	17,438,451	0.106	3,389,529	3,389,529	0.106	.	0.000	0	35398051	0.904
1985	7,489,385	18,063,499	0.415	13,271,191	13,271,191	0.415	0.415	0.302	9654228	35625485	0.898
1986	1,194,155	17,772,839	0.067	2,150,649	2,150,649	0.067	.	0.000	0	34159170	0.937
1987	1,299,821	19,453,024	0.067	2,138,760	2,138,760	0.067	.	0.000	0	34147282	0.937
1988	1,592,569	25,124,761	0.063	2,028,906	2,028,906	0.063	.	0.000	0	34037427	0.940
1989	3,146,335	24,688,970	0.127	4,079,131	4,079,131	0.127	.	0.000	0	36087652	0.887
1990	1,663,199	24,388,796	0.068	2,182,827	2,182,827	0.068	.	0.000	0	34191349	0.936
1991	4,686,796	25,965,580	0.181	5,777,549	5,777,549	0.181	.	0.000	0	37786071	0.947
1992	6,000,910	20,607,290	0.291	9,320,986	9,320,986	0.291	0.291	0.178	5704023	35625485	0.898
1993	3,912,613	23,522,674	0.166	5,324,095	5,324,095	0.166	.	0.000	0	37332616	0.857
1994	2,265,676	40,016,913	0.057	1,812,257	1,812,257	0.057	.	0.000	0	33820779	0.946
Total	66,232,746	477,353,384	0.139					1.122	35905106	1206586546	30.692
Average								0.033			0.903
Projected Non-Wind Loss				32,008,522							
Median Wind/Non-Wind Ratio				0.113							
Excess Wind Factor				1.030	[1 + (0.033 * 0.903)]						
50-Year	38,563,750	32,008,522	1.205	38,563,750	20,500,688	0.640	0.640	0.527	16883725	35625485	0.898
				1.474	[1 + (0.527 * 0.898)]						
Excess Wind Factor				1.039							

*The ratio for a year must be > 1.5M and at least .250 for that year to qualify as an excess year.
Treaty Threshold: 19,550,000

The adjustment of historical losses to current costs is very important to determine the impact of the catastrophe treaty. Early events would appear to be too small for the treaty but there has been significant inflation over the past 30 years. In addition, the non-wind losses reflect the growth in exposures that the company has experienced over time. So, a similar event to one in the historical period may now cost much more since we have more values exposed. Using the non-wind losses as our cost index takes both elements into account and adjusts the wind losses to the level that we would expect if the same event occurred today in terms of both current costs and current exposures. In Exhibit 2, the year 1981 would not be capped by reinsurance if it were not adjusted to current cost and exposure levels.

I would like to make some points on the reinsurance capping. Our company uses a high layer catastrophe reinsurance program. In most years, we do not expect to trigger our reinsurance coverage. Some companies purchase coverage at a working layer that is triggered more frequently. This is a company choice that is driven by their size, desire for stability, etc. With a higher layer program, there will be fewer years that must be capped in this approach. Second, the method as outlined treats the losses in a year as a single event for capping. This is not completely accurate. In the years that must be capped, with a high layer program, we expect that there will have been a large event that would trigger coverage. However, some portion of the losses are likely from other events. If the historical data is available, one should split the losses into the large event, or events, and all other. If they are not available, which is most likely for the older years, this method may overstate the capping and thus understate the load. With a high layer program, this understatement is small and is then spread over the number of years used in the excess load calculation. We accept this understatement as slightly conservative and not truly significant. Depending on a company's catastrophe reinsurance program, the extent of this understatement should be reviewed and adjustments made if it is considered significant.

There will still be excess wind losses that fall in the range between the normal wind threshold, which is based on the median of the wind/non-wind ratio, and the catastrophe treaty threshold. Most companies purchase catastrophe reinsurance only for protection from extreme events. They should have sufficient financial resources to handle the smaller catastrophes that occur with respectively greater frequency. However, some of these smaller catastrophe events are still treated as excess wind by the excess wind procedure. So there will still be an excess wind factor. The excess wind factor after adjusting for reinsurance is always less than or equal to the excess wind factor before the reinsurance adjustment. It is equal when there are no years in the procedure that would exceed the reinsurance treaty threshold.

For catastrophe events other than wind or hail, the capping is much simpler. Any catastrophe is trended to current costs using the loss trend factors in the indication. If the event would exceed the catastrophe threshold, the loss is capped for the effect of reinsurance.

Splitting Reinsurance Premium by Form Group

Now that we have the losses adjusted to the appropriate level, we move on to the reflection of the reinsurance premium. The reinsurance premium allocated to the state must be split into the two form groups used to develop rate indications. These are the building forms; 1,2,3 and 5; and the content forms; 4 and 6. The contents forms do not represent the same exposure to the treaty as the building forms due to the type of property being covered. The reinsurance premium is split into two parts based on the values exposed. See Exhibit 3 for a sample calculation which also shows State C for comparison purposes.

Exhibit 3

Split of Reinsurance Premium to Forms

Input Items

	State A	State C
Reinsurance Allocation	3,175,821	1,831,695
Total State Premium	18,975,000	17,325,000
Average Amount of Insurance		
Forms 2,3 Average AOI	115,375	106,750
Contents Exposure Factor	63.0%	65.0%
Forms 2,3 Adj Avg AOI	188,061	176,138
Forms 4,6 Average AOI	30,466	30,185
Total Values Exposed		
Forms 2,3 Total Values	98.5%	97.3%
Forms 4,6 Total Values	1.5%	2.7%
Written Premium Split		
Forms 2,3 Premium	95.9%	94.8%
Forms 4,6 Premium	4.1%	5.2%

Calculated Items

	Reinsurance Premium	Written Premium	Reinsurance Load
State A			
Forms 2,3	3,128,183	18,197,025	17.2%
Forms 4,6	47,637	777,975	6.1%
Total	3,175,821	18,975,000	16.7%
State C			
Forms 2,3	1,782,239	16,424,100	10.9%
Forms 4,6	49,456	900,900	5.5%
Total	1,831,695	17,325,000	10.6%

For building forms, the exposed value is the building amount in Coverage A and the contents in Coverage C. A basic Form 3 provides Coverage C at 50% of Coverage A. Many companies have replacement cost on contents endorsements that increase this percentage. In the example, we are using 70% of Coverage A for the increase from the endorsement with 65% of the policies having the endorsement. This yields a contents exposure increase of 63% (.65 * 70% + .35 * 50%). For the tenants forms, there is only Coverage C exposure. For condominium policies (Form 6), there is some structural coverage, Coverage A. Historically, the amount of Coverage A on these policies has been small. However, we are starting to see this increase and we will have to reflect the total amount of exposed values from Coverage A on these policies in future calculations. After allocating the premium by exposure, the reinsurance premium for the form group is then divided by the direct premium for the form group to determine the reinsurance cost as a percentage of premium. This leads to a smaller charge for the contents forms than for the building forms.

Loading the Reinsurance Cost into the Indications

The premium charged for the catastrophe treaty is determined as a percentage of the subject premium. Since most treaties are corporate in nature, the percentage applied to the subject premium represents an average rate for all states and all property lines. Any increase in premium subject to the treaty, beyond our current levels, will increase the reinsurance cost by this corporate rate.

In the example for State A in Exhibit 4, let's assume a catastrophe program that costs 9.9% of the subject premium. Therefore, in the rate indication, the first 9.9% in any state is treated as variable and any portion above 9.9% is considered fixed cost. Any increases in subject written premium will lead to additional reinsurance charges only at the 9.9% rate. So for a sample state, the reinsurance cost for Forms 1,2,3&5 is 16.1%, of which 9.9% is variable and 6.2% fixed. For Forms 4,6, the reinsurance cost is 5.8% which is all considered variable. The variable reinsurance cost is subtracted from the PLR while the fixed portion is added to the adjusted loss ratio. A similar calculation is shown for State C as well.

There may be some shortfall in completely covering the projected reinsurance costs in using this approach, assuming that the reinsurance treaty is priced based on a percentage of the subject premium. A shortfall could occur if there was significant growth in states with lower than average reinsurance charges. The increased premium would increase the reinsurance charge at the higher corporate rate, yet the rates in the state developed by the approach presented here would be based on a lower reinsurance cost. One should be sensitive to this. However, capping the variable portion in states with higher than average reinsurance charges will not necessarily lead to shortfalls. In fact, if they were not capped, the company could collect more premium than is needed to cover the reinsurance costs. This could cause a poor competitive position in the market or possibly negative reactions from the regulators in a state.

Instead, if the reinsurance premium is based on exposure, then the only variable portion is that which adjusts for the increases in value. The remaining cost should be considered fixed. Again, before applying these techniques, an actuary should review the exact framework of the company's reinsurance treaty.

Exhibit 4

Expense Breakdown for Indications

State A	**Forms 2,3**	**Forms 4,6**
Current Expenses	28.3%	34.7%
Current PLR	71.7%	65.3%
Reinsurance Expense	17.2%	6.1%
Variable	9.9%	6.1%
Fixed *	7.3%	0.0%
Proposed PLR	61.8%	59.2%

State C	**Forms 2,3**	**Forms 4,6**
Current Expenses	28.3%	34.7%
Current PLR	71.7%	65.3%
Reinsurance Expense	10.9%	5.5%
Variable	9.9%	5.5%
Fixed *	1.0%	0.0%
Proposed PLR	61.8%	59.8%

* - Fixed portion is amount over the corporate rate on line.

Summary

Although reinsurance costs have long been recognized as a legitimate cost of doing business, they have not been explicitly reflected in rates until recently. These costs are too significant to be ignored and they must be addressed. Reinsurance costs need to be considered to ensure an adequate rate. It's in the Principles.

Appendix A

Variations in Underlying Loss Loading

The method described herein is dependent on the approach used to reflect the excess losses. There are several methods being used to reflect excess wind losses. Regardless of the method used, the basic concepts remain the same. The initial loss loading must be modified for the expected reinsurance recoveries and then the reinsurance premium can be reflected. The approach to modify the losses for anticipated recoveries will depend on how the losses are reflected.

In the paper, I have been using an excess wind procedure based on the ISO procedure. Historically, such excess wind procedures based their loss estimation only on historical data. During periods when there is a lack of hurricanes or excess wind losses, an excess wind procedure is a limited tool for developing rates since it will understate the expected losses. On the other hand, when there are more events or the presence of extreme events, the excess wind procedure can overstate the expected losses. The variation shown was designed to augment the history used in the ISO procedure with additional losses as needed to avoid understatement and to eliminate the more extreme events from the historical period to avoid overstatement. A more detailed discussion of this augmentation can be found in an earlier paper[2].

[2] Bradshaw, John and Homan, Mark, *Homeowners Excess Wind Loads: Augmenting the ISO Wind Procedure*, CAS Forum, Summer 1993, pg. 339

However, ISO is no longer using an excess wind procedure, so they no longer are updating the industry experience in that format. It may become difficult to obtain the history to use this method. Thus an alternative method has been developed which will tie into the loss distribution from a wind model.

The use of models for estimating hurricane losses has become increasingly widespread. Not all companies have access to such models and many still are uncertain whether the estimates from the models are correct. The approach discussed in this paper can alleviate much of the reliance on the accuracy of such model. Wind models provide estimated losses for the events reflected in the model. A wind model that estimates losses for each individual event is the easiest to use. Such a model allows for the estimated loss to be adjusted for reinsurance on an event by event basis. Thus, one can get the loss projection and the reinsurance adjustment at the same time. Also, since many catastrophe treaties are corporate in nature as are the models, the reinsurance adjustment can be more accurate, assuming the model is run on a corporate level. This makes the line adjustment to the treaty threshold unnecessary eliminating a potential source of error. Some wind models provide loss estimates in terms of average annual costs rather than event by event. To make the necessary adjustments for reinsurance to such models, you must work with the model designers to make the necessary changes within their formulas.

Other companies use all catastrophes in their loadings rather than just wind. Some use hurricanes only. In either case, the historical events should be adjusted to current cost levels and then adjusted for reinsurance using the current program. After adjusting the history to be net to anticipated recoveries, the reinsurance premium can be reflected.

Appendix B

Excess Wind Procedure

This appendix will provide a more detailed explanation of the modified Excess Wind procedure shown in Exhibit 2.

Columns 2 and 3 are the raw data inputs of the wind and non-wind losses. Each year is treated as a sample observation and is treated independent of the other years. The procedure relies on averages of the observed ratios rather than aggregates. This allows for a mixture of industry and company data, which will be at different loss levels. Since industry data is often not as up to date as company data, the company data can be used until industry data is available for the latest year or two.

Column 4 is the ratio of the Wind to Non-wind losses, or column 2 divided by column 3. Column 5 is the ratio from column 4 multiplied by the projected non-wind loss. The projected non-wind loss is the average of the latest three years, trended by the average cost factor used in the indication. In this case, the trend factor is 4.5% for a three year period to go from an average of 1993 to 1996. The wind losses determined by this calculation represent wind losses at current cost and exposure levels as explained in the paper. These losses are needed to determine the impact of the current catastrophe reinsurance treaty to historical losses.

Column 6 are the wind losses adjusted for the impact of reinsurance. If the recalculated wind loss for the year is greater than the treaty threshold (noted on the exhibit), than the wind loss is capped at the treaty amount plus 5% of the loss above the treaty threshold. The 5% is the copayment under the treaty.

Column 7 is the adjusted wind/non-wind ratio calculated by dividing column 6 by the non-wind projected loss. It is important to note that for most years, column 7 is the same as column 4. It is only for years that would trigger the catastrophe reinsurance coverage that the ratio will change. Also, column 7 is always equal to or less than column 4. Column 8 is the wind/non-wind ratio from column 7 for the years that are considered excess years. For a year to be considered excess, the wind/non-wind ratio must excess 1.5 times the median wind ratio and be greater than .250. The second threshold of .250 is important for states with fairly low wind activity. It keeps the excess wind adjustment small for such states so that the adjustment is truly for excess wind. In this example, the .250 is the key value not 1.5 times the median. Only four years in the historical period are considered excess.

Column 9 is the excess ratio. This is the portion of the excess wind/non-wind ratio from column 8 that is greater than the median. While it may at first seem odd that the trigger for an excess year is 1.5 times the median and that the excess portion is the amount over the median, this was intended. The same approach is taken to adjust the five years in the experience period of the indication, so it produces the proper answer.

Column 10 is the amount of the excess losses. This is column 9 times the projected non-wind losses.

Column 11 is the non-excess losses which is the sum of the projected non-wind losses and the wind losses in column 5 minus the excess losses in column 10.
Column 12 is the non-wind losses divided by the non-excess, or the projected non-wind losses divided by column 11.

This provides all the numbers needed to calculate the excess wind factor. The excess wind factor is unity plus the product of the average excess ratio from column 9 and the average non-wind/non-excess ratio from column 12. Since the excess ratio is the ratio of the excess losses to the non-wind losses, the product is the ratio of excess losses to non-excess losses. It is applied to the non-excess losses in the indication procedure, so the result is the excess losses. The unity is to retain the non-excess losses in the final figure. There is one final set of calculations that must be done for the 50-Year event situation. The wind losses used here are for any events in excess of 50 year return periods. It is derived from modeling and represents the expected wind losses from the top two percentile. The non-wind projected losses remain the same as used above. The calculation of all ratios and figures is the same for any individual year as outlined above. The one year is then used to calculate an excess wind factor for these larger events. The two excess factors are then weighted together using 98% weight on the historical period and 2% for the 50 year plus event. To eliminate any duplication, we drop any year that is in excess of the 50 year event from the historical period.

Pricing the Earthquake Exposure Using Modeling

by Debra L. Werland, FCAS
Joseph W. Pitts, FCAS

PRICING THE EARTHQUAKE EXPOSURE USING MODELING

Debra L. Werland and Joseph W. Pitts

ABSTRACT

Catastrophe hazard modeling has become an important tool for ratemaking in lines of business subject to low frequency, high severity type losses. Natural hazard events such as hurricanes, tornadoes, and earthquakes rarely occur, but their devastation can be overwhelming when they do. Few insurance companies have enough historical loss data to sufficiently price for these events. In our paper, we plan to demonstrate a methodology which details the use of a model's output in determining a statewide rate level indication for the earthquake line of business, as well as a methodology for determining more equitable territorial relativities within a state.

Our paper will outline a practical and understandable methodology for dealing with some complex issues involved in pricing the earthquake insurance exposure. The emphasis of the methodology within our paper will be on practicality and potential regulatory acceptance. Another feature of our paper will be the inclusion of a section dealing with the reflection of the net cost of reinsurance in the proposed direct rates. A final consideration is the treatment of a model's output when it is believed the modeled results are less than fully credible.

The CAS ratemaking principles address data considerations used in making rates. Catastrophe hazard modeling output is an important component of "other relevant data" that is referred to in the principles [1]. A company's history of earthquake premiums and losses does not have sufficient predictive power for establishing adequate rates. Our paper will rely on the power of catastrophe hazard simulation of multiple possible events and the associated loss costs generated from these models.

Biography:

Debra L. Werland is Executive Director of Homeowners and Property Pricing Actuary for United Services Automobile Association in San Antonio, Texas. She is a Fellow of the Casualty Actuarial Society and a member of the American Academy of Actuaries. She has co-authored a previous paper entitled "Using a Geographic Information System to Identify Territory Boundaries."

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PRICING THE EARTHQUAKE EXPOSURE USING MODELING

INTRODUCTION

Pricing for an insurer's risk to hurricanes and earthquakes has never been an easy task. No insurer's loss history is adequate enough to cover the expectation of all possible type and size of events. Any ratemaking formula based on actual loss experience alone for such rare events will fail to capture the scope of possible events that could impact an insurer's financial results. Catastrophe hazard modeling represents a way of developing the scope of possible catastrophic events that can impact an insurer's book of business. The financial impact of these events is based on scientific evidence of the characteristics of the underlying peril and its interaction with the insured properties.

In this paper we will concentrate on the earthquake peril and its pricing. After an overview of earthquake modeling, we will discuss target underwriting profit provisions, reinsurance costs, and other components of developing an adequate rate per $1,000 of dwelling coverage for a typical book of Homeowners business. The credibility of the results will be addressed in the derivation of the indicated rates, along with partitioning of the state into geographic zones based on the relative difference in loss costs determined from the modeled results.

We will then discuss possible shortcomings inherent in modeling and suggest several solutions on how to handle these deficiencies in the derivation of an adequate rate. We will conclude the paper with a list of additional considerations that need further research, given the great uncertainty associated with any modeling process.

OVERVIEW OF EARTHQUAKE MODELING

Actuaries are relying more than ever on the use of modeling in accurately pricing catastrophic risks such as hurricanes and earthquakes. While we may not completely understand the intricacies of all functions and assumptions used in modeling, it is important nonetheless to present an overview of an earthquake computer simulation model. Appendix A describes the earthquake model developed by Applied Insurance Research (AIR) of Boston, Massachusetts.

The US earthquake model developed by AIR uses sophisticated mathematical techniques to estimate the probability distribution of losses resulting from earthquakes anywhere in the 48 contiguous states. The earthquake model is composed of three separate elements: an earthquake occurrence model, a shake damage model, and a fire-following model.

For ratemaking purposes, the output from the model will include loss costs applicable to a specific location, type of construction and policy form. Our interest is in a single-family dwelling as covered under a typical Homeowners policy. The loss costs generated by the AIR model are the basic building blocks in the development of an appropriate rate for this coverage. The next section will begin with those basic building blocks.

PROPOSED METHODOLOGY

The goal of this paper is to present a methodology for developing a rate per $1,000 of Earthquake coverage. We will assume that the indicated rate is based on Coverage A of a typical Homeowners single-family dwelling. That is, the modeled results include all coverages (including time element expenses), and the figures have been ratioed to Coverage A, in 1000's.

We begin with the statewide indicated rate as developed from the loss costs resulting from the model. Sections on the net cost of reinsurance and the target rate of return and proper underwriting profit provision follow. Territorial partitioning and the derivation of zone relativities conclude this section.

Statewide Indicated Rate

The statewide indicated rate is determined using the pure premium method. The first input into the methodology is the statewide modeled incurred losses stated at a base deductible level. In this example, the base deductible is 10% applicable to the dwelling limit. The annual expected losses represent the average annual amount of incurred losses an insurer could expect from writing the Earthquake line of business in State X if each insured had a 10% deductible. The modeled results are generally available on an individual state basis as well as on a zip code or county basis within the state. The annual expected losses are trended (severity only) and adjusted for LAE, then ratioed to the total trended value of insured dwellings to develop a projected pure premium which is used to determine the indicated rate as shown on Exhibit 1. (A viable alternative would be to trend the insured values first and use these trended values as input to the catastrophe model, thus yielding an estimate of trended severity within the model results). In this example, the current rate is assumed to be $2.50 per $1,000 of dwelling coverage. The indicated rate is calculated by taking the projected pure premium and grossing it up to include reinsurance costs, trended fixed expenses, and variable expenses. After completing these calculations, the indicated rate is $3.77 per $1,000 of coverage.

Exhibit 1
Sheet 1

STATEWIDE INDICATED RATE

(1)	Modeled Incurred Losses at a 10% Deductible as of 12/31/95:	$19,500,000
(2)	Total Dwelling Coverage as of 12/31/95:	10,965,281,000
(3)	Proposed Effective Date:	7/1/96
(4)	LAE Factor:	1.150
(5)	Loss Trend Factor Trended to 7/1/97:	1.250
(6)	Exposure Trend Factor Trended to 7/1/97:	1.190
(7)	State X Earthquake Share of Expected Net Cost of Reinsurance:	$7,592,703
(8)	Trended Fixed Expense Provision Per $1000 of Coverage:	0.265
(9)	Pure Premium Per $1000 of Coverage: $\left\{\{\{[(1) \times (4) \times (5)]+(7)\} \times 1000\} / [(2) \times (6)]\right\} + (8)$	$ 2.99
(10)	Variable Permissible Loss and LAE Ratio:	0.794
(11)	Indicated Rate: (9)/(10)	$3.77
(12)	Current Statewide Rate Per $1000 of Dwelling Coverage:	$2.50
(13)	Indicated Percentage Change: (11) / (12) - 1	50.8%
(14)	Proposed Change:	50.8%
(15)	Proposed Statewide Rate: (12) x [1 + (14)]	$3.77

Exhibit 1
Sheet 2

STATEWIDE INDICATED RATE
EXPLANATORY NOTES

(1) This is the main output received from the modeling firm. It is an estimate of the annual expected losses at a base deductible for an insurer, given the current book of business within the state for the Earthquake line of business.

(2) The total value of insured dwellings is provided to the modeling firm by the insurer and is used to determine the average annual expected losses per $1,000 of coverage in the pure premium method.

(3) The proposed effective date as selected by the insurer.

(4) The LAE factor is calculated based on a comparison of estimated ultimate loss adjustment expenses to estimated ultimate losses from the most recent earthquake events faced by the insurer.

(5) The modeled losses are trended using historical Homeowners severity data. Earthquake loss trend data is not used because of its instability. Losses should not be trended for frequency, unless the insurer is confident there exists an increased period of seismicity in the future.

(6) The exposure trend is based on historical changes in the average amount of insurance for the Earthquake line of business.

(7) The State X Earthquake share of the expected net cost of reinsurance is calculated as described on Exhibit 2.

(8) The trended fixed expense provision per $1,000 of coverage is calculated by trending fixed expenses to a point in time appropriate for the proposed effective date and ratioing it to trended insured value using an annualized fixed expense trend of 5%.

(9) The formula combines the modeled incurred losses with the net cost of reinsurance for the state and line of business with the trended fixed expense provision to provide an estimate of the projected pure premium to be expected during the time the proposed rates are to be in effect.

(10) The variable permissible loss and LAE ratio is calculated based on historical variable expenses and a consideration of the relative riskiness of the Earthquake line of business compared to other lines being written and the overall required return on surplus. An 18.2% underwriting profit provision was used along with 2.4% provision for variable expenses.

Net Cost of Reinsurance

An important component which we reflected in the rate indication is the net cost of reinsurance. An insurer should decide whether to include this component based on the costs and anticipated recoveries associated with its reinsurance program. This component should be included as a cost if the expected reinsurance recovery is less than the amount of premium paid to the reinsurer for reinsurance protection. This relationship will generally be the case due to the presence of transaction costs which include a margin for reinsurance risk load and profit. The expected reinsurance recovery represents the average annual amount an insurer could expect to recover from the reinsurer(s) due to insured events and can be determined using catastrophe modeling. The expected reinsurance recovery needs to be calculated considering the attachment points or quota share percentages associated with an insurer's reinsurance program. Most often, an insurer's reinsurance program is structured to provide protection against many types of hazards; however, some reinsurance contracts are designed to provide protection against only one hazard. To accurately measure the net cost of reinsurance for a particular hazard, the reinsurance premium from all programs which provide protection for the hazard should be included. If other catastrophic hazards such as hurricanes are a large proportion of an insurer's exposure to catastrophe loss, the reinsurance premium for multi-hazard contracts could be segregated for each hazard. The reinsurance premium for each hazard could then be included with each net cost of reinsurance calculation for every line of business. In the example, however, the net cost of reinsurance is allocated to the Earthquake line of business and then the appropriate state. The allocation to line of business in our example as shown on Exhibit 2 was based on model results by comparing expected Earthquake reinsurance recovery to the total expected reinsurance recovery. This ratio was applied to the net cost of reinsurance to obtain the earthquake-only net cost of reinsurance. The allocation to a state level was done using written premium. It is important to note that this allocation may introduce a distortion if the state in question has a different level of premium adequacy than the countrywide premium adequacy.

Exhibit 2
Sheet 1

ESTIMATED NET COST OF REINSURANCE

(1)	1995 Countrywide Reinsurance Premium for Contracts covering the Earthquake peril:	$37,890,000
(2)	Expected Reinsurance Recovery:	$17,481,970
(3)	Net Cost of Reinsurance: (1) - (2)	$20,408,030
(4)	Expected Earthquake Reinsurance Recovery:	$ 9,154,600
(5)	Proportion of Earthquake Recovery to Total Recovery: (4) /(2)	52.4%
(6)	Earthquake Share of Net Cost of Reinsurance: (3) x (5)	$10,693,808
(7)	1995 State X Earthquake Written Premium:	$27,271,677
(8)	1995 Countrywide Earthquake Written Premium:	$38,551,154
(9)	State X Earthquake Share of Net Cost of Reinsurance: [(7) /(8)] x (6)	$ 7,592,703

Exhibit 2
Sheet 2

NET COST OF REINSURANCE
EXPLANATORY NOTES

(1) This is the total of all reinsurance premium paid for reinsurance contracts which provide protection for earthquake losses.

(2) This is a model output number. It is determined based on the attachment point or quota share arrangement an insurer has with its reinsurer(s).

(3) The net cost of reinsurance is the difference between the reinsurance premium paid for contracts providing earthquake protection and the expected total reinsurance recovery.

(4) Model results are used to determine what portion of the expected recovery is due to earthquake.

(5) The Earthquake proportion of the total expected reinsurance recovery is expressed as a factor to be applied to the total net cost of reinsurance.

(6) The Earthquake share of the net cost of reinsurance is the proportion of the earthquake recovery to the total recovery multiplied by the total net cost of reinsurance.

(7) The latest year State X Earthquake written premium is used to allocate the Earthquake share of the net cost of reinsurance to a state level.

(8) The latest year countrywide Earthquake written premium is used to determine what proportion of the countrywide Earthquake written premium is represented by State X.

The concept of including the net cost of reinsurance in a rate indication is relatively new and will likely be challenged or subjected to additional scrutiny by regulatory agencies. However, it does represent a cost of doing business, and therefore, we have chosen to include its net costs. Reinsurance costs could also be considered in conjunction with the selected rate of return and that discussion follows.

Target Rate of Return

For purposes of developing an underwriting profit provision, we have chosen a total rate of return methodology. We are not proposing one method over another, but we have selected this particular one for the development of a reasonable profit target for the Earthquake line of business. The target rate of return on GAAP equity is developed using a Discounted Cash Flow (Dividend Yield) Method and the Capital Asset Pricing Model (CAPM). The selected rate of return, averaged from the results of these two methods, is 13.0%. From this selected rate of return we have subtracted 8.0%, which represents the post-tax investment rate of return from all investable funds. Exhibit 3 converts this difference to a pre-tax basis, using a corporate tax rate of 35%. For an insurer's total book of business this percentage is then divided by the company's premium-to-surplus ratio in order to convert the target underwriting profit provision to a percentage of premium. Although we do not endorse the divisibility of surplus or leverage ratios, we are proposing this method for calculating a reasonable Earthquake underwriting profit provision.

We have selected a company whose underwriting results resemble the years 1985-1994 for all Property and Casualty insurers writing Personal Lines Automobile, Homeowners Multi-Peril, and Earthquake coverages. (It would be appropriate for more years to be used; however, the Earthquake line of business was not segregated prior to 1985). The data can be found in Best's Aggregate and Averages, 1995 edition [2]. A company's own data can be used for this purpose as well.

Exhibit 3

TARGET UNDERWRITING PROFIT PROVISION

A. Target Rate of Return
(% of GAAP Surplus)

1. Dividend Yield Model	12.0%
2. Capital Asset Pricing Model	14.0%
3. Selected Target Rate of Return	13.0%

B. Target Underwriting Rate of Return
(% of GAAP Surplus)

1. Investment Rate of Return After Tax	8.0%
2. Target U/W Return After Tax (A3) - (B1)	5.0%
3. Target U/W Return Before Tax (B2)/(1 - 0.35)	7.7%

C. Target Underwriting Profit Provision
(% of Direct Earned Premium)

1. Net Written Premium/GAAP Surplus Ratio	1.30
2. Indicated U/W Profit Provision (B3) / (C1)	5.9%
3. Selected U/W Profit Provision	5.9%

Note: A select group of insurers were chosen that resemble the mix of business written by the filing insurer. Company betas and projected dividend yields were taken from Value Line. Both the Dividend Yield Method and the Capital Asset Pricing Model were used in determining an appropriate rate of return. The selected target rate of return is a straight average of the two methods.

Basically, a company's underwriting profit provision should vary based on the riskiness of the line of business. A measure of risk we have chosen is the coefficient of variation (measured as standard deviation/mean, σ/μ) of a series of underwriting results for each line. Since the selected period includes the effects of Hurricane Andrew and the Northridge Earthquake, we adjusted the losses so that Andrew reflects a 1-in-30 year event and Northridge a 1-in-50 year event. We did not adjust for Hurricane Hugo, although one could argue for that adjustment as well. Table 1 shows the yearly (1985-1994) underwriting gains/losses as a percent of net earned premium.

Table 1
Underwriting Results as a Percentage of Premium

Year	Private Passenger Automobile	Homeowners Multi-Peril	Earthquake
1985	-11.0%	-11.7%	60.0%
1986	- 8.3%	-3.5%	58.0%
1987	-6.0%	3.3%	44.2%
1988	-6.8%	0.0%	57.5%
1989	-8.9%	-13.9%	-42.1%
1990	-9.1%	-12.9%	43.8%
1991	-4.6%	-17.7%	55.3%
1992	-1.9%	-58.4%	61.4%
1993	-1.8%	-13.5%	68.0%
1994	-1.3%	-18.4%	-222.2%

Table 2 shows the coefficient of variation of each line, the weighted average of the CVs using the latest ten years of premium, and what we are labeling as a risk index, which is the ratio of each line's CV to the weighted CV.

Table 2

Line of Business	Premium Distribution	Coefficient of Variation*	Risk Index
Private Passenger Automobile	80.1%	0.550	0.92
Earthquake	0.5%	1.854	3.09
Homeowners Multi-peril	19.4%	0.780	1.30
Total	100.0%	0.600	1.00

* Absolute Value

Assume the company's premium-to-surplus ratio corresponds to the industry's at 1.30, so that its inverse is .77. The risk indices are used to adjust each line's surplus ratio (surplus-to-premium) in the total rate of return methodology, resulting in target underwriting profit provisions which reflect the risk of each line of business. The resulting Earthquake profit provision will be used in the derivation of the variable permissible loss and loss adjustment expense provision to follow later. Table 3 summarizes this information.

Table 3

Line of Business	Risk Index	Implied Surplus Ratio (S/P)	Target Underwriting Profit Provision
Private Passenger Automobile	0.92	0.71	5.4%
Earthquake	3.09	2.38	18.2%
Homeowners Multi-peril	1.30	1.00	7.7%
Total	1.00	0.77	5.9%

In this example, industry net underwriting results were used to determine an appropriate underwriting profit provision for the Earthquake line of business. A larger Earthquake underwriting profit provision would probably be obtained if direct results were used instead. This is due to net underwriting results having variability stripped off by the stabilization of reinsurance. Using our methodology, it is reasonable to conclude that part of the difference between underwriting profit provisions calculated using net or direct underwriting results would be due to reinsurance costs. An insurer should expect a lower net cost of reinsurance if part of the reinsurance cost is reflected in the Earthquake underwriting profit provision calculated using direct underwriting results. Efforts could be made to quantify what portion of the net cost of reinsurance is contained in an Earthquake underwriting profit provision based on direct underwriting results. One possible approach would be to compare the difference in Earthquake underwriting profit provisions calculated using net and direct underwriting results to a net cost of reinsurance as calculated in this example.

Zone Relativities

Model results can also be used to determine revised Earthquake zone definitions and Earthquake zone relativities. The data used to establish Earthquake zone definitions are model results at a five-digit zip code level. The sum of all the five-digit zip code modeled losses and dwelling insured values should balance to the statewide totals used to determine the statewide indicated rate. In the example, we are assuming the state is comprised of twenty distinct five-digit zip codes. The data on Exhibit 4 shows the data segregated by five-digit zip code. We used a SAS clustering program to determine the new Earthquake zone definitions and zone relativities. The following is a description of the SAS procedure we used as described in the SAS user's manual [2].

PROCFASTCLUS performs a joint cluster analysis on the basis of Euclidean distances computed from one or more quantitative variables. The observations are divided into clusters such that every observation belongs to one and only one cluster. The procedure is intended for use with large data sets, from approximately 100 to 100,000 observations. With small data sets, the results may be highly sensitive to the order of the observations in the data set.

PROCFASTCLUS uses a method referred to as nearest centroid sorting. A set of points called cluster seeds is selected as a first guess of the means of the clusters. Each observation is assigned to the nearest seed to form temporary clusters. The seeds are then replaced by the means of the temporary cluster, and the process is repeated until no further changes occur in the cluster.

After specifying the desired number of Earthquake zones, and using the SAS procedure, we obtained the results in Exhibit 5. The number of zones to be used in a real application will depend on the size of the insurer's Earthquake book of business, geographic spread, and the level of seismic variation that exists within the state. It is important to note that the proposed Earthquake zones will probably not be contiguous because five-digit zip codes from different parts of the state will very often fall into the same cluster in the SAS procedure. We only used twenty zip codes in our example; however, the SAS procedure has the capability to handle a much larger number of zip codes. The relativities shown in Exhibit 5 are applied to the statewide indicated rate previously calculated to determine each zone's Earthquake rate.

The resultant earthquake zone rates should probably display a wider variance, since it could be argued that risk margins should vary by geographic location for the earthquake peril. We view this as another area deserving further consideration and an important aspect of determining adequate earthquake rates.

Exhibit 4

STATE X
EARTHQUAKE MODEL RESULTS
ZIP CODE LEVEL

Five-digit Zip Code Area	Dwelling Insured Value (in $000)	Expected Annual Loss at a 10% Deductible	Loss Cost
1	$ 921,339	$ 2,303,348	$ 2.50
2	1,096,528	1,644,792	1.50
3	258,481	387,722	1.50
4	548,264	603,090	1.10
5	922,272	830,045	0.90
6	79,839	98,897	1.24
7	722,114	902,643	1.25
8	103,211	232,225	2.25
9	803,112	3,011,670	3.75
10	801,247	721,122	0.90
11	552,322	359,009	0.65
12	402,178	623.376	1.55
13	700,659	1,156,087	1.65
14	1,102,321	2,369,990	2.15
15	200,321	490,786	2.45
16	402,111	1,105,805	2.75
17	727,727	1,928,477	2.65
18	202,001	490,786	1.03
19	112,007	123,768	1.11
20	307,227	399,088	1.30
Total	$ 10,965,281	$ 19,500,000	$ 1.78

Exhibit 5

STATE X
EARTHQUAKE ZONE RELATIVITIES

Earthquake Zone	Total Dwelling Insured Value (in $000) (1)	Expected Annual Loss at 10% Deductible (2)	Loss Cost (3)	Indicated Relativity to Statewide (4)	Indicated Earthquake Zone Rate (5)
1	$ 552,322	$ 359,009	$ 0.65	0.37	$ 1.38
2	3,694,971	3,886,713	1.05	0.59	2.23
3	3,560,167	6,181,967	1.74	0.98	3.68
4	2,354,709	6,060,641	2.57	1.45	5.46
5	803,112	3,011,670	3.75	2.11	7.95
Statewide	$ 10,965,281	$ 19,500,000	$ 1.78	1.00	$ 3.77

Note: (3) = (2)/(1)
(4) = (3)/1.78
(5) = (4) x 3.77

SHORTCOMINGS INHERENT IN MODELING

Modeled results fall short of expected values for many reasons, most of which can be attributed to company issues or to adjustments not made within the models themselves. First, we will discuss company shortcomings, then follow-up with model shortcomings. Where appropriate, we will make suggestions on how to handle quantifiable and supportable adjustments to the modeled input or output. The following list is not meant to be exhaustive, but is typical of company issues. Company shortcomings include:

1. Underinsurance (homes not insured to value) or overinsurance.
2. Demand surge for labor and materials after a large catastrophic event.
3. The need for extra claims adjusters following large events.
4. No data collecting or coding for retrofitting safety features.
5. Invalid or incomplete data.

The *major company shortcoming may well rest on the problem of underinsurance.* Expected loss to a particular structure in a particular area is based on applying an average damage ratio (defined as the ratio of the repair cost of a building to its total replacement value) to the total insured value of the structure. It is assumed then that the insured value of a building represents its true replacement cost. A company would do well to estimate its underinsurance (or overinsurance) problem before providing data to a modeling firm. If, on average, it is determined that a book of business is underinsured by 10%, then all limits should be adjusted before the model is run.

The effects of demand surge can be quite significant and should be factored into all modeled results. (It is not clear to us whether this adjustment should be made by the insurer or by the modeler.) Obviously, the demand for labor and materials will vary depending on the location and magnitude of each earthquake. The additional cost probably varies between 0% and 30%, but the highest demand is associated with events that have the lowest expected probability; therefore, the effect on average annual aggregate losses should be minimal. We believe this adjustment to the modeled loss costs is important, yet is an uncertain aspect of the process. Studies should be conducted to determine the impact of demand surge factors, perhaps by studying the payout of events such as Loma Prieta and Northridge, if the data is available. Either overall average demand surge factors should be applied to the resultant loss costs, or variable demand surge factors should be determined and applied by location and event.

The need for independent claims adjusters is a very real cost of settling claims following large catastrophic events. It is not clear which loss adjustment expense (LAE) factors should be applied to the modeled expected loss costs. There has simply not been enough loss experience to determine appropriate factors. We suggest using either the ratio of LAE to losses of past events (which may understate the true ratio) or simply use the underlying policy average LAE factor, given Earthquake coverages are normally endorsed to a Homeowners or Dwelling Fire program.

Modeled results should account for retrofitting safety features of an insured structure. Average damage ratios should be adjusted for these features. It is not clear to us how their effects can be measured, but research should be conducted and insurers should encourage their installation. A strongly built and reinforced home should surely withstand the initial impact and aftershocks of an earthquake, as opposed to a home whose frame is not bolted to the foundation, for example. Most insurance companies probably do not request information on retrofitting mechanisms, nor do they store the data. We would encourage the Insurance Institute for Property Loss Reduction to study the effects of such safety features and simulate an earthquake under monitored laboratory conditions to determine the extent of damage on the structure and its contents.

Finally, there is always the possibility of invalid data, incomplete data, or no data at all. Invalid data is most prominent if zip code, county, or street address is not validated before being stored on the insurer's database. Either the data should be cleaned up before the input files are created, or the data should be eliminated from analysis. Most companies do not have enough insureds located in all areas of the state. Therefore, there will be many locations with no modeled loss costs. In these situations, modeling firms have access to an inventory of typical building structures by location: average dwelling limit, type of construction, average year of construction, building height, etc. Modeled loss costs from this "generic" inventory can supplement an insurer's results where few or no insureds reside.

There will also be locations with insufficient data. Assume for a moment that an insurer's book of business is mapped to the geographic zip code centroid of each zip code within the state. Although modeled results are assumed to be 100% credible by location, the reader could obviously question whether one, ten, or even one hundred exposures are enough to deem the results credible. An insurer's database could be complemented with the results of the generic inventory. The authors have chosen to consider data 100% credible by zip code with more than 100 exposures; otherwise, the generic inventory is given full credibility.

We now turn to shortcomings in the models themselves. These brief remarks are not intended to criticize any model or modeler, but to highlight the importance of their impact on modeled results. The following list is also not meant to be exhaustive, but does represent typical shortcomings.

1. Factor for unknown faults.
2. Inclusion of debris removal expenses.
3. Effects of aftershocks.
4. Parameter risk within the model.

The 1994 Northridge earthquake is a perfect example of an unknown fault, a blind thrust fault which does not break the earth's surface. Not even seismologists know the extent of undiscovered fault lines beneath the earth's surface. How understated could the modeled results be? No one knows for sure, and we propose no solution to handle this uncertainty. Although the models account for possible earthquakes in all historical seismic source zones, it is highly questionable if distributions in the model account for all potential seismicity. With the passage of time and advanced technology, perhaps some day these models will account for all possible faults. For now we will have to assume that a model's results may understate expected average annual losses, and hence, expected loss costs per $1,000 of coverage.

Debris removal expenses, although small, should be added to the model's expected loss costs. More prominent would be the effects of aftershocks which follow moderate to large earthquakes. Oftentimes, claims are reopened months later due to weakened structures repeatedly damaged from aftershocks. Future modifications to catastrophe models should account for this possibility.

Since catastrophe modeling is based on incomplete distributions developed from historical information, there will always exist parameter risk. This risk may lead to gross understatement (or overstatement) of potential insured losses, and as such, represents a potential shortcoming of modeling.

ADDITIONAL CONSIDERATIONS

There will always exist areas that deserve further consideration. While we have presented a practical procedure for developing adequate earthquake rates, some areas deserve additional research and attention. We will divide these topics into four categories: (1) shortcomings of models, (2) credibility of data, (3) necessary target rate of return, and (4) net reinsurance costs.

We devoted an entire section of this paper to model shortcomings and company data issues. We only repeat them here to emphasize their importance and need for further study. The cooperation of the insurance industry, modeling firms, and the IIPLR is necessary in order to quantify the impact of outstanding issues on expected loss costs. Perhaps special data calls or cooperative studies can be conducted and the results shared with all interested parties.

Computer modeling simulates thousands of possible events, and as such, its results are generally considered fully credible. The earthquake peril is so unique by location, especially in California, so there really does not exist a feasible complement of credibility to augment a local result. Perhaps a regional complement could be used, but its applicability is questionable, given local soil conditions and proximity to fault lines. We choose to believe that an industry inventory database represents the best alternative for a complement.

Insuring the Earthquake peril is much riskier than insuring Auto physical damage coverages. Due to the relationship between risk and return, a higher rate of return, and therefore, a higher underwriting profit and contingency provision, should be allowed to cover a company's earthquake exposure. As mentioned earlier, this provision should probably vary by location as well. We have presented a simplified method for deriving a reasonable profit provision, but we encourage more research in this important area.

Debate exists as to whether rates should include the costs of reinsurance on an insurer's book of business. After all, their inclusion could be viewed as a pass-through to the consumer. Also, in the long-run, neither the insurer nor the reinsurer(s) should be worse off for engaging in a reinsurance program; otherwise, neither party would enter into the contract. However, in the short-run, reinsurance costs are a legitimate expense of doing business, and we believe that all parties should share in that expense, including policyholders. Indeed, policyholders benefit from financially strong companies.

SUMMARY

Catastrophe hazard modeling has become an integral part of the ratemaking process. Actuarial ratemaking principles [1] state that "other relevant data may supplement historical experience. These other data may be external to the company or to the insurance industry ...". We have entered the realm of that other relevant data. Actuarial Standard of Practice (SOP) No. 9 [4] states that "an actuary should take reasonable steps to ensure that an actuarial work product is presented fairly ... if it describes the data, material assumptions, methods, and material changes in these with sufficient clarity that another actuary practicing in the same field could make an appraisal of the reasonableness and the validity of the report." However, with the advent of modeling the actuary must rely on the work of another person. SOP No. 9 continues by stating that "reliance on another person means using that person's work without assuming responsibility therefore." These other persons now include experts in the fields of geology, seismology, and structural engineering, just to name a few. Actuaries, however, can play a key role in contributing to the development of the models, and more importantly, the interpretation and communication of their valuable results.

Catastrophe hazard modeling has become a necessary tool for the adequate pricing of large catastrophic events such as hurricanes and earthquakes. Their frequency is so low and their severity so potentially high that not even all of the property and casualty companies in a state could have enough loss history upon which to base rates. Despite any shortcomings models may have, they hold the key to the future and the pricing of nature's perilous attacks.

REFERENCES

[1] Casualty Actuarial Society, "Statement of Principles Regarding Property and Casualty Insurance Ratemaking," as adopted May, 1988.

[2] Best's Aggregates & Averages, Property-Casualty United States, 1995 Edition, A.M. Best Company, Inc., pp. 174,176.

[3] SAS/STAT® User's Guide, Version 6, Fourth Edition, Volume 1, Cary, NC: 1989, Copyright® SAS Institute Inc. pp. 823-824.

[4] Actuarial Standards Board, "Documentation and Disclosure in Property and Casualty Insurance Ratemaking, Loss Reserving, and Valuations," as adopted January, 1991.

APPENDIX A

The model developed by Applied Insurance Research uses sophisticated mathematical techniques to estimate the probability distribution of losses resulting from earthquakes anywhere in the 48 contiguous states. The earthquake model is composed of three separate elements: an earthquake occurrence model, a shake damage model, and a fire-following model. The earthquake occurrence portion of the model uses a probabilistic simulation to generate a synthetic catalog of earthquake events that is consistent with the historical record. The shake damage estimation portion of the model uses analytical numerical techniques to calculate the distribution of losses for individual buildings given the characteristics of the event. The fire-following portion of the model uses simulation to estimate fire losses following an earthquake. Together these techniques allow the estimation of a wide range of information about potential earthquake losses in the United States. The earthquake simulation model incorporates statistical descriptions of a large number of variables which define both the originating event (the earthquake) and its effect on structures. Some of these variables are defined probabilistically, and some deterministically. This section will describe the key components of the model, the main variables affecting the outcomes, and the relationships between the primary variables.

The model is described in the following sections:

- Earthquake occurrence
- Attenuation
- Exposure characterization
- Shake damage estimation
- Fire-following loss estimation

Earthquake Occurrence

For earthquakes there are three key types of variables that describe the physical phenomenon. In broad terms, these variables describe (1) where earthquakes can occur, (2) the size of the earthquake, and (3) the likelihood of seeing an earthquake of a particular size. In other words, the variables describe where, how big, and how often earthquakes occur.

The issue of where earthquakes occur is handled by identifying *faults* or *seismic zones* where historical earthquakes have been observed. On the west coast earthquakes tend to occur along well defined geological features called faults, which are places where the surface of the earth has been ruptured by past earthquakes, and which are observable at the ground surface or by subsurface sounding techniques. Not all faults are active, which is to say that not all faults are believed capable of rupturing in the present, although they have ruptured in the distant past. Where faults are observed, and where the historical catalog of earthquakes indicate that the faults are still capable of rupturing, the surface trace of the fault defines a possible location for future earthquakes.

Not all earthquakes occur on identifiable faults, however. Many earthquakes, especially those east of the Rocky Mountains, occur on faults that are not visible at the surface. Such faults are inferred from the occurrence of earthquakes in the historical record. For these areas, a source zone is created, which is an area with fuzzy boundaries within which future earthquakes are possible.

The AIR model contains approximately 250 seismic source zones covering the 48 contiguous states. Each source zone is defined by a line on the surface of the earth with probability distributions describing the variability of potential epicenters both along and perpendicular to that line. Hence a potential earthquake is not limited to occur along a known fault line, but can with some probability occur anywhere in the vicinity of a fault, or anywhere within a seismic source zone, depending on the degree of uncertainty associated with the historical record of earthquakes in that area. The central line of the source zone does define the dominant direction of faults in the area and characterizes the orientation of the rupture surface.

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The size of an earthquake is usually measured by one of several *magnitude* scales. In the AIR model, the surface wave magnitude M_s scale is used to characterize the earthquake magnitude. For every fault and source zone, the frequency of earthquakes of different magnitudes must be described. Seismologists generally agree that, over a considerable magnitude range, the logarithm of the number of historic earthquakes that exceed a given magnitude scales linearly with magnitude. This indicates that the frequency-magnitude relationship is approximately exponential. Additionally, paleo-seismologic data have been interpreted by some researchers to indicate that the frequency-magnitude relationship for large earthquakes differs from exponential scaling, leading to the notion of characteristic earthquakes in certain geographic areas. The AIR Model incorporates a truncated exponential distribution, or truncated "Gutenberg-Richter" relationship, to represent potential seismicity in each source zone. Where appropriate we additionally incorporate a characteristic earthquake model.

The AIR earthquake model is calibrated to a catalog of historical earthquakes which is as complete as possible, and which covers the historical record from the mid-1600's to the present. Because the completeness of the catalog varies both in time and as a function of magnitude (larger earthquakes are more likely to be included in the historical record), the fitting of the frequency-magnitude distribution is adjusted to account for the variation in historical completeness.

Earthquake Attenuation

After earthquakes are simulated using the probability distributions of the different earthquake parameters, the shaking intensity of the earthquake at every location affected by the earthquake is calculated using a relationship called an attenuation function. The local intensity is then corrected to reflect local soil conditions, as some types of soil amplify the shaking intensity relative to other soil types. This section discusses the variable interrelationships required to calculate the local shaking intensity.

From the characteristics of the earthquake, the local shaking intensity is calculated using an attenuation relationship. The attenuation relationship depends on the location of the source zone, as earthquake shaking attenuates more quickly in the western U.S. than in the eastern part of the country. That is to say that the same magnitude earthquake will affect a smaller area in California than in the northeast.

The attenuation calculation starts by spreading the energy released by the earthquake over the rupture surface, and integrating over the entire rupture surface to calculate the total effect of the earthquake. In effect, energy is assumed to be released uniformly over the rupture, and each incremental piece of energy is separately attenuated to obtain the effect at some distant point. This results in contours of equal intensity that are elongated along the orientation of the rupture.

The calculation of local shaking intensity itself consists of two parts. First, a basic intensity is calculated that assumes uniform soil conditions at every location. This intensity (called a Rossi-Forel intensity) depends on the distance of the site from the earthquake rupture, the orientation of the rupture, and the earthquake magnitude and focal depth. The rupture length is calculated from the basic earthquake parameters. Second, the Rossi-Forel intensity is modified to reflect the soil conditions at the site. Soil conditions for the entire country are digitized on grids varying from 0.1 degree latitude/longitude squares to 0.5 minute latitude/longitude squares. The local soil condition can significantly affect shaking intensity. The final intensity is identified as a Modified Mercalli Intensity (MMI).

The MMI is a generally accepted unit of shaking intensity that has had wide adoption for many years. It describes, in general terms, the type of damage that might be expected to buildings of usual design, and other effects of earthquakes that would be expected at that location. As such, the MMI is a good metric for estimating damages to structures.

Exposure Characterization

In order to calculate damages from an earthquake, the AIR model incorporates an extensive description both of the structural characteristics of an exposure and of the policy conditions describing the treatment of deductibles and other factors.

The seismic performance of a building depends primarily on the structural system resisting the lateral loads, but is also affected by other factors, including, in the AIR model, the age of the building and the height of the building. The age of the building is used to determine the likely code provisions under which the building was designed and constructed. Newer buildings, which may have been built to more exacting code provisions for seismic performance, are usually expected to perform better than older buildings.

The AIR model incorporates damageability relationships for many different classes of exposures, with up to three height categories in each class. In all, there are 42 different damage relationships for each coverage type, plus several different age categories. The categories of structural types are based in part on the structural types defined in ATC-13 (Applied Technology Council, 13-member advisory project engineering panel established in 1982 to develop earthquake damage/loss estimates for facilities in California), although the actual damage relationships are modified and extended well beyond those covered in that reference.

The exposures are characterized by policy limits for four different coverages: A, building applied to the total loss or to the loss from Coverages A, B, and C. Most commonly, Coverage B is combined with Coverage A for calculation purposes, and is assumed to apply to the same structural type as coverage A. The policy limit for each coverage may be defined by both a replacement value and a policy limit. This is because the replacement value may rise in time without the policy limit being adjusted to reflect inflation. Damage is always calculated with respect to replacement value, and then is capped at the policy limit if appropriate.

The location of the risk can be defined by a latitude and longitude point or by the five digit zip code in which the risk is located. The risk can also be associated with a line of business (homeowners, renters, commercial multi-peril, etc.) in order to report losses separately in categories meaningful to the insurer.

Damage Estimation

Given the local shaking intensity in MMI units, damages to structures at that location can be calculated if sufficient information is available about the structure. Two types of damage are calculated by AIR: shake damage due to the lateral and vertical motions of the ground, and fire damage due to earthquake-induced fires.

In order to calculate shake damage, the exposure information is combined with the level of shaking intensity at the building. Information on the structural characteristics of the properties at risk are used to select an appropriate damageability relationship (also sometimes called a damage function or a fragility curve) relating the probability of different levels of damage to the local shaking intensity (MMI). The damageability relationship is a complete probability distribution of damage, ranging from no damage to complete destruction (0 to 100 percent damage), with a probability corresponding to each level of damage in between. The probability distribution is a continuous function of the local MMI level.

The earthquake damageability relationships have been derived and refined over a period of several years. They incorporate well documented engineering studies by earthquake engineers and other experts both within and outside of AIR. These damageability relationships also incorporate the results of post-earthquake field surveys performed by AIR engineers and others as well as detailed analyses of actual loss data provided to AIR by its client companies. These relationships are continually refined and validated.

Fire-Following Loss Estimation

Once the shake damages have been calculated for a particular earthquake, fire-following losses are estimated. This part of the model uses a separate simulation to estimate fire losses for each event.

First, the number of fires spawned by the earthquake is generated. The fire ignition rate is based on the local MMI intensity and the total population in the area. A number of fires is simulated for each affected zip code. The mean ignition rate increases as the MMI increases. The probability distribution of ignition rates is assumed to be uniform in some interval around the mean rate. Once the number of fires is simulated, each fire is randomly placed within a zip code and is assigned to affect either residential properties, commercial properties, and/or mobile homes.

The fire simulation then simulates the spread of the fires as well as the actions taken by local fire departments to control the fires. The fire spread rate is affected by a randomly selected wind speed appropriate for the location of the earthquake. Higher wind speeds increase the rate of spread of the fire.

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Some of the factors included in the fire simulation are the time to report the fire, the time for one or more fire engines to reach the fire, and the availability of water to fight the fire. All of these factors are affected by the local MMI, as areas experiencing high shaking intensity are more likely to have obstructed roads and broken water mains. Also, the influence of fire breaks - wide roads or other natural impediments to fire spread - is included in the simulation. Fire engines can move from fire to fire as fires are controlled.

Since the fire losses are determined by simulation, different levels of fire loss can be calculated for a given earthquake. Typically, the variability of fire losses is large, at least for the larger earthquakes, such that fire losses can vary by at least a factor of two if the same earthquake is simulated several times. This reflects the true uncertainty in fire losses for larger earthquakes.

Implementation of PH-Transforms in Ratemaking

by Shaun Wang, Ph.D.

— Discussion Paper for the 1997 Casualty Actuarial Society Ratemaking Seminar

IMPLEMENTATION OF PH-TRANSFORMS IN RATEMAKING

SHAUN WANG, Ph.D.

ABSTRACT

In this article we introduce a relatively new method for deciding contingency provisions in insurance ratemaking by the use of proportional hazard(PH) transforms. This method is easy to understand, simple to use, and supported by theoretical properties as well as economic justification. After an introduction of the PH-transform method, we show through examples how it can be used in pricing ambiguous risks, excess-of-loss coverages, increased limits, and risk portfolios with dependency risk. We also show how a minimum rate-on-line can be achieved. As well, we propose a right-tail index for insurance risks.

1 INTRODUCTION

Recently, there has been considerable interest in and extensive discussion on risk loads by Fellows of the Casualty Actuarial Society. These discussions have focused on what measures a risk and methods to arrive at a 'reasonable' risk load. Although there are diverse opinions on the appropriate measurement of risk, there is general agreement on the distinction between process risks and parameter risks, and on the importance of parameter risks in ratemaking; see Finger (1976), Miccolis (1977), McClenahan (1990), Feldblum (1990), Philbrick (1991), Meyers (1991) and Robbin (1992).

Following Venter's (1991) advocacy of adjusted distribution methods, Wang (1995) proposes using proportional hazard (PH) transforms in the calculation of risk-adjusted premiums. Although extensive discussion on the economic justifications is valuable, this paper focuses on the practical aspects of implementation of PH-transforms in ratemaking. More specifically, we will show how it can be used to quantify process risks, parameter risks and dependency risks.

Consistent with previous papers, this paper will consider only pure premiums, excluding all expenses and commissions. To utilize the PH-transform in ratemaking, a probability distribution for the insurance claims is needed. With the advent of computerized technology, a probability distribution can often be estimated from industry claim data or by computer simulations. Even though a probability distribution can be obtained from past claim data, sound and knowledgeable judgements are always required to ensure that the estimated loss distribution is valid for ratemaking.

It is safe to say that no theoretical risk-load formula can claim to be the *right* one, since subjective elements always exist in any practical exercise of ratemaking. However, a good theoretical risk-load formula can assist actuaries and help maintain logical consistency in the ratemaking process. In this respect, it is hoped that the PH-transform method offers a useful tool to practicing actuaries in insurance ratemaking.

The remainder of this paper is divided into three sections. Section 2 introduces the PH-transform method and applies it to pricing of ambiguous risks, excess-of-loss layers, increased limits and risk portfolios. Section 3 discusses two simple mixtures of PH-transforms. The first mixture can yield a minimal rate-on-line, and the second mixture suggests a new index for the right tail risk. Section 4 briefly reviews the leading economic theories of risk and uncertainty, and their relations with insurance ratemaking.

2 PROPORTIONAL HAZARD TRANSFORM

An insurance risk X refers to a non-negative loss random variable, which can be described by the decumulative distribution function (ddf): $S_X(t) = \Pr\{X > t\}$. An advantage of using the ddf is the unifying treatment of discrete, continuous and mixed-type distributions. In general, for a risk X, the expected loss can be evaluated directly from its ddf:

$$\mathrm{E}(X) = \int_0^\infty S_X(t)dt.$$

Definition 1 *Given a best-estimate loss distribution $S_X(t) = Pr\{X > t\}$, for some exogenous* **index** r *($0 \le r \le 1$), the* **proportional hazard (PH)** *transform refers to a mapping $S_Y(t) := [S_X(t)]^r$, and the* **PH-mean** *refers to the expected value under the transformed distribution:*

$$\mathrm{H}_r(X) = \int_0^\infty [S_X(t)]^r dt, \qquad (0 \le r \le 1).$$

The PH-mean was introduced by Wang (1995) to represent risk-adjusted premiums.

Example 1: The following three loss distributions

$$\begin{aligned} S_U(t) &= 1 - \tfrac{1}{2b}t, \quad 0 \le t \le 2b & \text{(uniform)} \\ S_V(t) &= e^{-\frac{t}{b}} & \text{(exponential)} \\ S_W(t) &= (\tfrac{b}{b+t})^2 & \text{(Pareto)}, \end{aligned}$$

have the same expected loss, b. One can easily verify that

$$\mathrm{H}_r(U) = \frac{2b}{1+r}, \qquad \mathrm{H}_r(V) = \frac{b}{r}, \qquad \mathrm{H}_r(W) = \begin{cases} \frac{b}{2r-1}, & r > 0.5; \\ \infty, & r \le 0.5. \end{cases}$$

Table 1: Some values of PH-mean $\mathrm{H}_r(.)$

	U	V	W
$r_1 = \frac{5}{6}$	$1.09\,b$	$1.2\,b$	$1.5\,b$
$r_2 = \frac{2}{3}$	$1.2\,b$	$1.5\,b$	$3.0\,b$

The PH-mean, interpreted as risk-adjusted premium, preserves the ordering of relative riskiness among those three distributions (see Table 1).

Example 2: When X has a Pareto distribution with parameters (α, λ):

$$S_X(t) = (\frac{\lambda}{\lambda + t})^\alpha,$$

the PH-transform $S_Y(t)$ also has a Pareto distribution with parameters $(r\alpha, \lambda)$.

When X has a Burr distribution with parameters (α, λ, τ):

$$S_X(t) = (\frac{\lambda}{\lambda + t^\tau})^\alpha,$$

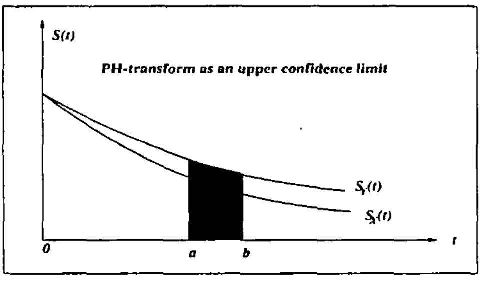
the PH transform $S_Y(t)$ also has a Burr distribution with parameters $(r\alpha, \lambda, \tau)$.

When X has a gamma (or log-normal) distribution, the PH transform $S_Y(t)$ is no longer a gamma (or log-normal). In such cases, numerical integration may be required to evaluate the PH-mean.

2.1 Pricing of Ambiguous Risks

In practice, the underlying loss distribution is seldom known with precision. There are always uncertainties regarding the best-estimate loss distribution. Insufficient data or poor-quality data often results in sampling errors. Even if a large amount of high-quality data is available, due to changes in the claim generating mechanisms, past data may not fully predict the the future claim distribution.

Figure 1: Margins for parameter uncertainty by PH-transforms



As illustrated in Figure 1, the PH-transform, $S_Y(t) = [S_X(t)]^r$, can be viewed as an upper confidence limit for the best-estimate loss distribution $S_X(t)$. A smaller index r yields a wider range between the curves S_Y and S_X. This upper confidence limit interpretation has support in a statistical estimation theory (see Appendix). The index r can be assigned accordingly with respect to the level of confidence in the estimated loss distribution. The more ambiguous the situation is, the lower the value of r should be used.

Example 3: Consider the following experiment conducted by Hogarth and Kunreuther (1992). An actuary is asked to price warranties on the performance of 10,000 units of a new line of microcomputers. Suppose that the cost of repair is \$100 per unit, and there can be at most one breakdown per period. Also, suppose that the risks of breakdown associated with any two units are independent. The best-estimate of the probability of breakdown has three scenarios:

$$\theta = 0.001, \qquad \theta = 0.01, \qquad \theta = 0.1.$$

The level of confidence regarding the best estimate has two scenarios:

Non-ambiguous: There is little ambiguity regarding the best-estimate loss distribution. Experts all agree with confidence on the chances of a breakdown.

Ambiguous: There is considerable ambiguity regarding the best-estimate loss distribution. Experts disagree and have little confidence in the estimate of the probabilities of a breakdown.

Note that the loss associated with a computer component can only assume two possible values, either zero or \$100. For any fixed $t < 100$, the probability that the loss exceeds t is the same as the probability of being exactly \$100, θ. For a fixed $t \geq 100$, it is impossible that the loss exceeds t. Thus, the best-estimate ddf of the insurance loss cost is

$$S_X(t) = \begin{cases} \theta, & 0 < t < 100; \\ 0, & 100 \leq t. \end{cases}$$

A PH-transform with index r yields a risk-adjusted premium at $100\theta^r$. If we choose $r = 0.97$ for the non-ambiguous case, and $r = 0.87$ for the ambiguous case, we get the following premium structures as in Table 2:

Table 2: The ratio of the risk-adjusted premium to the expected loss

	$\theta = 0.001$	$\theta = 0.01$	$\theta = 0.1$
Non-ambiguous ($r = 0.97$)	1.23	1.15	1.07
Ambiguous ($r = 0.87$)	2.45	1.82	1.35

In summary, the PH-transform can be used as a means of provision for estimation errors. The actuary can subsequently set up a table for the index r according to different levels of ambiguity, such as the following:

Ambiguity Level	Index r
Non-ambiguous	0.96-1.00
Slightly ambiguous	0.90-0.95
Moderately ambiguous	0.80-0.89
Highly ambiguous	0.50-0.79
Extremely ambiguous	0.00-0.49

2.2 Pricing of Excess-of-Loss Layers

Since most practical contracts contain clauses such as a deductible and a maximum limit, it is convenient to use the general language of excess-of-loss layers. A layer $(a, a+h]$ of a risk X is defined by the loss function:

$$I_{(a,a+h]} = \begin{cases} 0, & 0 \leq X < a; \\ (X-a), & a \leq X < a+h; \\ h, & a+h \leq X, \end{cases}$$

where a is the **attachment point (retention)**, and h is the **limit**.

One can verify that the loss variable $I_{(a,a+h]}$ has a ddf:

$$S_{I_{(a,a+h]}}(t) = \begin{cases} S_X(a+t), & 0 \leq t < h \\ 0, & h \leq t, \end{cases}$$

and that the average loss cost for the layer $(a, a+h]$ is

$$\mathrm{E}[I_{(a,a+h]}] = \int_0^h S_X(a+t)dt = \int_a^{a+h} S_X(t)dt.$$

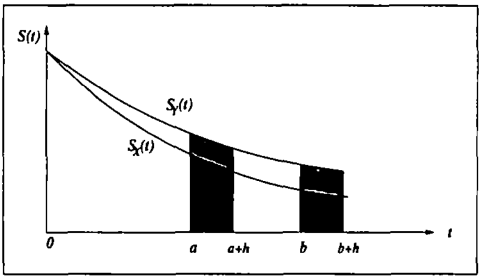
Note that $S_X(t)dt$ represents the net premium for an infinitesimal layer at $(t, t+dt]$. Thus, the ddf $S_X(t)$ plays an important role of *layer net premium density*.

Under the PH-transform $S_Y(t) = [S_X(t)]^r$, the PH-mean for the layer $(a, a+h]$ is

$$H_r(I_{(a,a+h]}) = \int_0^{\infty} [S_{I_{(a,a+h]}}(t)]^r dt = \int_0^h [S_X(a+t)]^r dt = \int_a^{a+h} [S_X(t)]^r dt.$$

In other words, the net premium and the risk-adjusted premium for the layer $(a, a+h]$ are represented by the areas over the interval $(a, a+h]$ under the curves $S_X(t)$ and $S_Y(t)$, respectively (see Figure 2).

Figure 2: Risk load by layers: an illustration



In Wang (1995), it is shown that, for $0 < r < 1$, the PH-mean has the following properties:

- Positive loading:

$$H_r(I_{(a,a+h]}) > E(I_{(a,a+h]}).$$

- Decreasing risk-adjusted premiums:

$$\text{For } a < b, \qquad H_r(I_{(a,a+h]}) > H_r(I_{(b,b+h]}).$$

- Increasing relative loading:

$$\text{For } a < b, \qquad \frac{H_r(I_{(a,a+h]})}{E(I_{(a,a+h]})} < \frac{H_r(I_{(b,b+h]})}{E(I_{(b,b+h]})}.$$

These properties are consistent with market premium structures (Patrick, 1990; Venter, 1991).

Example 4 : A risk has a 10% chance of incurring a claim, and if a claim occurs the claim size has a Pareto distribution ($\lambda = 2,000$, $\alpha = 1.2$). Putting frequency and severity together, we have

$$\begin{aligned} S_X(t) &= \Pr\{X > t\} \\ &= \text{Probability of occurrence} \times \Pr\{\text{Loss Size } > t\} \\ &= 0.1 \times (\tfrac{2000}{2000+t})^{1.2}. \end{aligned}$$

Suppose that, the actuary infers an index, say $r = 0.833$, from individual risk analysis and market conditions. The actuary may need to compare with the risk loads for other contracts with similar characteristics in the market. The PH-transform with $r = 0.833$ yields a ddf:

$$S_Y(t) = 0.1^{0.833} \times (\frac{2000}{2000+t})^{1.2\times 0.833},$$

which produces risk-adjusted layer premiums as shown in Table 3.

Table 3: Layer premiums using PH-transforms

Layer	Net Premium	Risk-adjusted Premium	Percentage Loading
(0, 1000]	77.892	119.129	53%
(5000, 6000]	20.512	39.250	91%
(10000, 11000]	11.098	23.533	112%
(50000, 51000]	1.982	5.603	183%
(100000, 101000]	0.888	2.870	223%
(500000, 501000]	0.132	0.587	345%
(1000000, 1001000]	0.058	0.294	412%

2.3 Increased Limits Ratemaking

In commercial liability insurance, a policy generally covers a loss up to a specified maximum dollar amount that will be paid to any individual loss.

It is general practice to publish rates for some standard limit called the basic limit (used to be $25,000 and nowadays $100,000). Increased limit rates are calculated using a multiple factor, called the increased limit factors (ILFs). Without risk load, the increased limit factor is the expected loss at the increased limit divided by the expected loss at the basic limit. The increased limit factor with risk load is the sum of the expected loss and the risk load at the increased limit divided by the sum of the expected loss and the risk load at the basic limit:

$$\mathrm{ILF}(\omega) = \frac{\mathrm{E}[X;\omega] + \mathrm{RL}_{(0,\omega]}}{\mathrm{E}[X;\ 100,000] + \mathrm{RL}_{(0,\,100,000]}}.$$

It is widely felt that ILFs should satisfy the following conditions (Rosenberg, 1977; Meyers, 1991; Robbin, 1992):

1. The relative loading with respect to the expected loss is higher for increased limits.

2. ILFs should produce the same price under any arbitrary division of layers.

3. The ILFs should exhibit a pattern of declining marginal increases as the limit of coverages is raised. In other words, when $x < y$,

$$\mathrm{ILF}(x+h) - \mathrm{ILF}(x) \geq \mathrm{ILF}(y+h) - \mathrm{ILF}(y),$$

In the U.S., most companies use the Insurance Service Office (ISO) published ILFs. Traditionally, only the severity distribution is used (ISO assumes a Pareto loss severity distribution) when producing ILFs. Until the mid-1980's, ISO used the variance of the loss distribution to calculate risk loads, a method proposed by Robert S. Miccolis (1977). From mid-1980's to early 1990's, ISO used the standard deviation of the loss distribution to calculate risk loads (e.g. Feldblum, 1990). Meyers (1991) presents a Competitive Market Equilibrium approach, which yields a variance-based risk load method; however, some authors have questioned the appropriateness of the variance-based risk load method for the calculation of ILFs (e.g. Robbin, 1992).

The following is an illustrative example to show how the PH-transform method can be used in increased limits ratemaking.

Example 5: Assume that the claim severity distribution has a Pareto distribution with ddf:

$$S_X(t) = (\frac{\lambda}{\lambda + t})^{\alpha},$$

with $\lambda = 5,000$ and $\alpha = 1.1$. This is the same distribution used by Meyers (1991), although he also considered parameter uncertainty.

Assume that. based on the market premium structure, the actuary feels that (for illustration only) an index $r = 0.8$ provides an appropriate provision for parameter uncertainty. When using a Pareto severity distribution, there is a simple analytical formula for the ILFs:

$$\mathrm{ILF}(\omega) = \frac{1 - (\frac{\lambda}{\lambda+\omega})^{r\alpha-1}}{1 - (\frac{\lambda}{\lambda+100,000})^{r\alpha-1}}.$$

One can then easily calculate the increased limit factors at any limit (see Table 4).

Table 4: Increased limit Factors using PH-transforms

Policy Limit ω	Expected Loss $\mathrm{E}[X;\omega]$	ILF Without RL	Risk Load	ILF With RL
100000	13124.	1.00	5251.	1.00
250000	16255.	1.24	8866.	1.37
500000	18484.	1.41	12344.	1.68
750000	19726.	1.50	14687.	1.87
1000000	20579.	1.57	16490.	2.02
2000000	22543.	1.71	21330.	2.39

2.4 Pricing of Risk Portfolios and Dependency Risk

For ratemaking based on the aggregate claims from a risk portfolio, the actuary often considers claim frequency and claim severity separately, due to the type of information available.

Let N denote the claim frequency with probability function $f_N(k) = \Pr\{N = k\}$ and ddf: $S_N(k) = f(k+1) + f(k+2) + \cdots$, $(k = 0, 1, 2, \cdots)$.

Let X denote the claim severity and let

$$Z = X_1 + X_2 + \cdots X_N = \sum_{i=1}^{N} X_i$$

represent the aggregate claims from the risk portfolio.

Depending on the available information, the actuary may have different levels of confidence in the estimates for the frequency and severity distributions. According to the level of confidence in the estimated frequency and severity distributions, the actuary can choose an index r_1 for the frequency and an index r_2 for the severity. As a result, the actuary can calculate the risk-adjusted premium for the risk portfolio as:

$$\mathrm{H}(Z) = \mathrm{H}_{r_1}(N) \times \mathrm{H}_{r_2}(X).$$

Example 6: Consider a group coverage of liability insurance. The actuary has estimated the following loss distributions: (i) the claim frequency has a Poisson distribution with $\lambda = 2.0$, and (ii) the claim severity is modeled by a log-normal distribution with a mean of \$50,000 and coefficient of variation of 3, which was used by Finger (1976) for liability claim severity distribution. Suppose that the actuary has low confidence in the estimate of claim frequency, but higher confidence in the estimate of the claim severity distribution, thus chooses $r_1 = 0.7$ for the claim frequency and $r_2 = 0.8$ for the claim severity. The premium can be calculated using numerical integrations:

$$\mathrm{H}_{0.7}(N) = 2.527, \qquad \text{and} \qquad \mathrm{H}_{0.8}(X) = 82,960.$$

Thus, the required total premium is

$$\mathrm{H}_{0.7}(N) \times \mathrm{H}_{0.8}(X) = 209,640.$$

Kunreuther et al (1993) discussed the ambiguities associated with the estimates for claim frequencies and severities. They mention that for some risks such as playground accidents, there are considerable data on the chances of occurrence but much uncertainty about the potential size of the loss due to arbitrary court awards. On the other hand, for some risks such as satellite losses or new product defects, the chance of a loss occurring is highly ambiguous due to limited past claim data, however, the magnitude of such a loss is reasonably predictable.

For some risk events such as earthquake insurance, it is more plausible to consider the dependency between claim frequency and claim severity. For instance, the Richter scale value of an earthquake may affect both the frequency and severity simultaneously; and for hurricane losses the wind velocity would affect both the frequency and severity simultaneously.

Regardless of the dependency structure, computerized simulation methods can always be used to model the total claims based on given geographic concentration.

For instance, in simulating earthquake losses, one can use the following procedures: (i) simulate some numerical values of the Richter scale; (ii) conditional on the simulated Richter scale values, run a secondary generator for the claim frequency and the claim severity (of course both the frequency and the severity depend on the Richter scale values). Once the actuary has obtained sample distributions for the claim frequencies and severities, or a sample distribution for the total claims, he or she can apply a PH-transform directly to the simulated sample distributions.

2.5 Some Properties of the PH-Mean

In general, for $0 \leq r \leq 1$, the PH-mean has the following properties:

- $\mathrm{E}(X) \leq \mathrm{H}_r(X) \leq \max(X)$. When r declines from one to zero, $\mathrm{H}_r(X)$ increases from the expected loss, $\mathrm{E}(X)$, to the maximum possible loss, $\max(X)$.
- Scale and translation invariant: $\mathrm{H}_r(aX + b) = a\mathrm{H}_r(X) + b$, for $a,\ b \geq 0$.
- Sub-additivity: $\mathrm{H}_r(X_1 + X_2) \leq \mathrm{H}_r(X_1) + \mathrm{H}_r(X_2)$.
- Layer additivity: when a risk X is split into a number of layers

  $$\{(x_0, x_1],\ (x_1, x_2], \cdots\},$$

  the layer premiums are additive (the whole is the summation of the parts):

  $$\mathrm{H}_r(X) = \mathrm{H}_r(I_{(x_0,x_1]}) + \mathrm{H}_r(I_{(x_1,x_2]}) + \cdots.$$

Pricing often assumes that a certain degree of diversification will be reached through the market efforts. In real life examples, risk-pooling is a common phenomena. It is assumed that, in a competitive market, the benefit of risk-pooling is transferred back to the policy-holders (in the form of premium reduction). In the PH-model, the layer-additivity property has already taken into account of the effect of risk-pooling.

Theoretically, in an efficient market (no transaction expenses in risk-sharing schemes) with complete information, the optimal cooperation among insurers is to form a market insurance portfolio (like the Dow Jones index), and each insurer takes a layer or quota-share of the market insurance portfolio.

In real life, however, the insurance market is *not* efficient. This is mainly because of incomplete information (ambiguity) and extra expenses associated with the risk-sharing transactions. There exist distinctly different local market climates in different geographic areas and in different lines of insurance. For instance, one can compare the automobile insurance market with the market for earthquake damage coverages in both California and Ontario. As a result, the value of the index r may vary with respect to the local market climate, which is characterized by the levels of ambiguity, risk concentration, and competition.

3 MIXTURE OF PH-TRANSFORMS

While a single index PH-transform has one parameter r to control the relative premium structure, one can obtain more flexible premium structures by using a mixture of PH-transforms:

$$p_1 \mathrm{H}_{r_1} + p_2 \mathrm{H}_{r_2} + \cdots + p_n \mathrm{H}_{r_n}, \qquad \sum_{j=1}^{n} p_j = 1, \qquad 0 \leq r_j \leq 1 \ (j = 1, \cdots, n).$$

Let $\bar{r} = \sum_{j=1}^{n} p_j r_j$ be the weighted average index. It can be verified that

- For any risk X, $p_1 \mathrm{H}_{r_1}(X) + p_2 \mathrm{H}_{r_2}(X) + \cdots + p_n \mathrm{H}_{r_n}(X) \geq \mathrm{H}_{\bar{r}}(X)$.
- For a layer $I_x = (x, x+h)$, the ratio

$$\frac{p_1 \mathrm{H}_{r_1}(I_x) + p_2 \mathrm{H}_{r_2}(I_x) + \cdots + p_n \mathrm{H}_{r_n}(I_x)}{\mathrm{H}_{\bar{r}}(I_x)}$$

  is an increasing function of x.

The PH-measure mixture can be interpreted as a collective decision-making process. Each member of the decision-making 'committee' chooses a value of r, and the index mixture represents different r's chosen by different members. It also has interpretations as (i) an index mixture chosen by a rating agency according to the indices for all insurance companies in the market; (ii) an index mixture which combines an individual company's index with the rating agency's index mixture.

For ratemaking purposes, mixtures of PH-transforms add more flexibility than a single index. In the remaining sections of this article, we shall discuss some special two-point mixtures of PH-transforms:

$$(1-\alpha)\mathrm{H}_{r_1}(X) + \alpha \mathrm{H}_{r_2}(X), \qquad 0 \leq \alpha \leq 1, \qquad r_1, r_2 \leq 1.$$

3.1 Minimum Rate-on-Line

In most practical circumstances, very limited information is available for claims at extremely high layers. In such highly ambiguous circumstances, most (re)insurers adopt a survival rule of minimum rate-on-line. The rate-on-line is the premium divided by the coverage limit, and most (re)insurers establish a minimum they will accept for this ratio (see Venter, 1991).

By using a two-point mixture of PH-transforms with $r_1 \leq 1$ and $r_2 = 0$, the premium functional

$$(1-\alpha)\mathrm{H}_{r_1}(X) + \alpha \mathrm{H}_0(X) = (1-\alpha)\mathrm{H}_{r_1}(X) + \alpha \max(X)$$

can yield a minimum rate-on-line at α.

Example 7: Reconsider Example 4, the best-estimate loss distribution (ddf) is

$$S_X(t) = 0.1 \times (\frac{2000}{2000+t})^{1.2}.$$

By choosing a two-point mixture with $r_1 = 0.85$, $r_2 = 0$, and $\alpha = 0.02$, we get an adjusted distribution:

$$S_Y(t) = 0.98 \times 0.1 \times (\frac{2000}{2000+t})^{1.2\times 0.85} + 0.02.$$

As shown in the table below, this two-point mixture guarantees a minimum-rate-on-line at 0.02 (1 full payment out of 50 years). By comparing Table 5 with Table 3 one can see that, at higher layers, this method yield distinctly different premiums from those in Example 4.

3.2 The Right-Tail Deviation

Consider a two-point mixture of PH-transforms with $r_1 = 1$ and $r_2 = \frac{1}{2}$:

$$(1-\alpha)\mathrm{H}_1(X) + \alpha \mathrm{H}_{\frac{1}{2}}(X), \qquad 0 < \alpha < 1,$$

which can be rewritten as (noting that $\mathrm{H}_1(X) = \mathrm{E}(X)$):

$$\mathrm{E}(X) + \alpha\, [\mathrm{H}_{\frac{1}{2}}(X) - \mathrm{E}(X)],$$

which is analogous to the standard deviation method: $\mathrm{E}(X) + \alpha\, \sigma(X)$.

Now we introduce a new risk-measure analogous to the standard deviation.

Table 5: Layer premiums under an index mixture

Layer	Net Premium	Risk-adjusted Premium
(0, 1000]	77.892	131.802
(5000, 6000]	20.512	56.006
(10000, 11000]	11.098	41.363
(50000, 51000]	1.982	24.940
(100000, 101000]	0.888	22.497
(500000, 501000]	0.132	20.493
(1000000, 1001000]	0.058	20.244

Definition 2 *The* **right-tail deviation** *is defined as*

$$D(X) = H_{\frac{1}{2}}(X) - E(X) = \int_0^\infty \sqrt{S_X(t)}dt - \int_0^\infty S_X(t)dt,$$

and the **right-tail index** *is defined as*

$$d(X) = \frac{H(X)}{E(X)}.$$

Analogous to the standard deviation, the right-tail deviation $D(X)$ satisfies:

- If $\Pr\{X = b\} = 1$, then $D(X) = 0$.
- Scale-invariant: $D(cX) = cD(X)$ for $c > 0$.
- Sub-additivity: $D(X + Y) \leq D(X) + D(Y)$.
- If X and Y are perfectly correlated, then $D(X + Y) = D(X) + D(Y)$.

At very high layers, the standard deviation and the right-tail deviation converge to each other, as demonstrated in the following example.

Example 8 : Re-consider the claim distribution in Example 4 with a ddf:

$$S_X(t) = 0.1 \times (\frac{2000}{2000 + t})^{1.2}.$$

For different layers with fixed limit at 1000, we compare the standard deviation and the right-tail deviation in the following table.

Layer	Expected loss	Std-deviation of the loss	Right-tail deviation	Percentage difference
I	$\mathrm{E}(I)$	$\sigma(I)$	$\mathrm{D}(I)$	$\frac{\sigma(I)}{\mathrm{D}(I)} - 1$
(0, 1000]	77.89	256.0	200.5	27.7%
(1000, 2000]	51.56	214.3	175.2	22.3%
(10000, 11000]	11.10	103.9	94.24	10.3%
(100000, 101000]	.8879	29.76	28.91	2.93%
(1000000, 1001000]	.05754	7.584	7.528	.75%
(10000000, 10001000]	.003640	1.908	1.904	.19%
(100000000, 100001000]	.0002297	.4793	.4791	.05%
(1000000000, 1000001000]	.00001450	.1204	.1204	.01%

It can be shown that, for any small layer $[a, a+h)$, $\mathrm{D}(I_{(a,a+h]}) \leq \sigma(I_{(a,a+h]})$, $\mathrm{D}(I_{(a,a+h]})$ converges to $\sigma(I_{(a,a+h]})$ at upper layers (i.e. the relative error goes to zero when a becomes large). As a result, for any non-negative random variable X, the right-tail deviation $\mathrm{D}(X)$ is finite, if and only if, the standard deviation $\sigma(X)$ is finite.

Having stated a number of similarities, here we point out some crucial differences between the right-tail deviation $\mathrm{D}(X)$ and the standard deviation $\sigma(X)$:

- $\mathrm{D}(X)$ is layer-additive, but $\sigma(X)$ is *not* additive.
- $\mathrm{D}(X)$ preserves some natural ordering of risks such as first stochastic dominance[1], but $\sigma(X)$ does not.

3.3 Links to the Gini Index in Welfare Studies

Historically, some long-tailed distributions have an origin in income distributions (e.g. Pareto, log-normal distributions, see Arnold, 1983). In social welfare studies, a celebrated measure for income inequality[2] is the Gini index. Assume that individual's wealth level in a country (community) can be summarized by a distribution: $S_X(u) = \mathsf{Proportion}\{X > u\}$. As a measure of income inequality of a society, the Gini index is

[1]Risk X is small than risk Y in first stochastic dominance if $S_X(t) \leq S_Y(t)$ for all $t \geq 0$; or equivalently, Y has the same distribution as $X + Z$ where Z is another non-negative random variable.

[2]Here 'income inequality' refers to the polarization of the wealth distribution.

defined as

$$G(X) = \frac{2E(|X - Y|)}{E(X)},$$

where X and Y are independent and identically distributed.

An equivalent definition of the Gini index is

$$G(X) = 1 - \frac{\int_0^\infty [S_X(u)]^2 du}{\int_0^\infty S_X(u) du}.$$

The higher the Gini index is, the more polarized a society is. As a measure of welfare inequality, the Gini index has the following properties:

- Each dollar transferred from the rich to the poor will lower the Gini index.
- Adding an equal amount to all persons' wealth will decrease the Gini index.

It is noted that $d(X)$ and $G(X)$ are similar in their definition formulae. This similarity may suggest that the role of the right-tail index $d(X)$ in measuring the right-tail risk is parallel to the role of the Gini index $G(X)$ in measuring income inequalities.

Consider the following loss distributions each with the same mean(=1) and variance(=3). Without referring to higher moments, we can order them by the right-tail index $d(X)$.

Risk X_i	Distribution	$E(X_i)$	$\sigma(X_i)$	$d(X_i)$	Gini index
Pareto	$S(t) = (\frac{2}{2+x})^3$	1	$\sqrt{3}$	**3.00**	0.600
Log-normal	$\mu = -\log(2),\ \sigma = \log(4)$	1	$\sqrt{3}$	**2.46**	0.595
Inverse-Gaussian	$f(x) = \frac{\exp\{-\frac{(x-1)^2}{6x}\}}{\sqrt{6\pi x^3}}$	1	$\sqrt{3}$	**2.17**	0.632
Gamma	$\alpha = \beta = \frac{1}{3}$	1	$\sqrt{3}$	**1.96**	0.713
Bernoulli	$f(0) = \frac{3}{4},\ f(4) = \frac{1}{4}$	1	$\sqrt{3}$	**1.00**	0.750

As its name may suggest, the right-tail deviation measures the right-tail risk, as opposed to the standard deviation which measures the deviation about the mean, and as opposed to the Gini index which measures the polarization of the wealth distribution.

4 ECONOMIC THEORIES

4.1 Expected Utility Theory

Traditionally, expected utility (EU) theory has played a dominant role in modeling decisions under risk and uncertainty. To a large extent, the popularity of EU was attributed to the axiomatization of von Neumann and Morgenstern (1947). They proposed five axioms (somewhat self-evident) and showed that any decision-making which is consistent with these axioms can be modeled by using a utility function of wealth. However, due to difficulties associated with implementation, EU remains as an academic pursuit and has had little direct impact in practice.

When EU is applied to produce an insurance premium for a risk X, the minimum premium P that an insurance company will accept for full insurance is defined by $u(w) = \mathrm{E}[u(w + P - X)]$, in which u and w refer to the insurer's utility and wealth (see Bowers et al, 1986). As pointed out by Meyers (1995), EU gives lower and upper bounds of an insurance premium, without due consideration of the market setting.

The EU does have an indirect application in actuarial work via the mean-variance analysis, which is viewed by some authors as a rough approximation of utility theory (Meyers, 1995). A commonly used actuarial method for deciding risk loads is based on the first two moments. Since loss distributions are often highly skewed, the first two moments cannot accurately reflect the level of insurance risk. In fact, actuaries often find that long tailed claim distributions, such as Pareto distributions, are more appropriate to describe the potential losses for some insurance contracts (e.g. liability insurance). Even for a large risk portfolio, the total claim distribution can be highly non-normal due to correlations or ambiguities in the initial estimates of individual risks.

The inconsistency of moment-based methods in calculating layer premiums are discussed by a number of authors (e.g. Venter, 1991; Robbin, 1992).

4.2 The Dual Theory of Yaari

A new theory of decision under uncertainty has been developed in the last decade by a group of economists (e.g. Quiggin, 1982; Yaari 1987). Analogous to the development of non-Euclidean geometry, Yaari (1987) formalized an alternative set of axioms and developed a *dual* theory of decision under uncertainty. In Yaari's dual theory, risk-aversion is described by a distortion function (increasing and convex) $g : [0,1] \mapsto [0,1]$

which is applied to probability distributions. The certainty equivalent to a bounded random economic prospect V $(0 \leq V \leq m)$ is

$$\int_0^m g[S_V(t)]dt, \qquad \text{where } S_V(t) = \Pr\{V > t\}.$$

In other words, the certainty equivalent to a random economic prospect, V, is just the expected value under the distorted probability distribution, $g[S_V(t)]$.

4.3 Schemeidler's Ambiguity-Aversion

As early as 1921, John Keynes identified a distinction between the *implication* of evidence (the implied likelihood) and *weight* of evidence (confidence in the implied likelihood). Frank Knight (1921) also drew a distinction between *risk* (with known probabilities) and *uncertainty* (ambiguity about the probabilities). A famous example on ambiguity-aversion is Ellsberg's (1961) paradox which can be briefly described as follows: There are two urns each containing 100 balls. One is a non-ambiguous urn which has 50 red and 50 black balls; the other is an ambiguous urn which also contains red and black balls but with unknown proportions. When subjects are offered \$100 for betting on a red draw, most subjects choose the non-ambiguous urn (and the same for the black draw). Such a pattern of preference *cannot* be explained by EU (Quiggin, 1993, p.42).

Ellsberg's work has spurred much interest in dealing with ambiguity factors in risk analysis. Schmeidler (1989) brought to economists *non-additive probabilities* in his axiomization of preferences under uncertainty. For instance, in Ellsberg's experiment, the non-ambiguous urn, with 50 red and 50 black balls, is preferred to the ambiguous urn with unknown proportions of red or black balls. This phenomenon can be explained if we assume that one assigns a subjective probability $\frac{3}{7}$ to the chance of getting a red draw (or black draw). Since $\frac{3}{7} + \frac{3}{7} = \frac{6}{7}$ which is less than one, the difference $1 - \frac{6}{7} = \frac{1}{7}$ may represent the magnitude of ambiguity aversion.

Built on its own axiomatic system, Schmeidler's theory leads to the same mathematical formulation as that of Yaari; that is, a certainty equivalent to a random economic prospect V $(0 \leq V \leq m)$ can be evaluated as

$$\mathrm{H}(V) = \int_0^m g[S_V(t)]dt,$$

where $g : [0,1] \mapsto [0,1]$ is a distortion function and $g[S_X(t)]$ represents the subjective probabilities.

The method of using adjusted distributions is widely known by actuaries. However, actuaries often use a transformed random variable, $Y = g(X)$, which yields $S_Y(t) = S_X(g^{-1}(t))$, a different formulation from Yaari's and Schmeidler's. A key point in the theories of Yaari and Schmeidler is that one needs to transform *directly* the distribution function $S_X(t)$.

Using a market argument, Venter (1991) discussed the no-arbitrage implications of insurance pricing. He observed that in order to ensure additivity when layering a risk, it is necessary to adjust the loss distribution so that layer premiums are expected losses under the adjusted loss distribution. Inspired by Venter's insightful observation, Wang (1995, 1996a) proposed the PH-transform method, which is in agreement with the formulation in Yaari and Schmeidler, thus is supported by their economic theories.

5 SUMMARY

In this paper we have introduced the basic methodologies of the PH-transform method and have shown by example how it can be used in insurance ratemaking. We did not discuss how to decide the overall level of contingency margin, which depends greatly on market conditions. An important avenue for future research is to link the overall level of risk load with the required surplus for supporting the written contract. Some pioneer work in this direction can be found in Kreps (1990) and Philbrick (1994).

The use of adjusted/conservative life tables has long been practiced by life actuaries (see Venter, 1991). To casualty actuaries, the PH-transform method contributes a theoretically sound and practically plausible way to adjust the loss distributions. For economic interpretations and empirical tests of the PH-transform method, see Wang (1996b). For updating risk-adjusted premiums in the light of new information, see Wang and Young (1996).

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APPENDIX: Ambiguity and Parameter Risk

Most insurance risks are characterized by the uncertainty about the estimate of the tail probabilities. This is often due to data sparsity for rare events (small tail probabilities), which in turn causes the estimates for tail probabilities to be unreliable.

To illustrate, assume that we have a finite sample of n observations from a class of identical insurance policies. The empirical estimate for the loss distribution is

$$\hat{S}(t) = \frac{\#\text{ of observations } > t}{n}, \qquad t \geq 0.$$

Let $S(t)$ represent the *true* underlying loss distribution, which is generally unknown and different from the empirical estimation $\hat{S}(t)$. From statistical estimation theory (e.g., Lawless, 1982, pp. 402; Hogg and Klugman, 1984), for some specified value of t, we can treat the quantity

$$\frac{\hat{S}(t) - S(t)}{\sigma(\hat{S}(t))},$$

as having a standard normal distribution for large values of n, where

$$\sigma(\hat{S}(t)) \approx \frac{\sqrt{\hat{S}(t)[1 - \hat{S}(t)]}}{\sqrt{n}}.$$

The η% **upper confidence limit (UCL)** for the true underlying distribution $S(t)$ can be approximated by

$$\text{UCL}(t) = \hat{S}(t) + \frac{q_\eta}{\sqrt{n}}\sqrt{\hat{S}(t)[1 - \hat{S}(t)]},$$

where q_η is a quantile of the standard normal distribution: $\Pr\{N(0,1) \leq q_\eta\} = \eta$. Keeping n fixed and letting $t \to \infty$, the ratio of the UCL to the best-estimate $\hat{S}(t)$ is

$$\frac{\text{UCL}(t)}{\hat{S}(t)} = 1 + \frac{q_\eta}{\sqrt{n}}\sqrt{\frac{1 - \hat{S}(t)}{\hat{S}(t)}} \to \infty,$$

which grows without bounds as t increases.

As a means of dealing with ambiguity regarding the best-estimate, the PH-transform:

$$\hat{S}_Y(t) = [\hat{S}_X(t)]^r, \quad r \leq 1,$$

can be viewed as an upper confidence limit (UCL) for the best-estimate $\hat{S}_X(t)$. It automatically gives higher relative safety margins for the tail probabilities, and the ratio

$$\frac{[\hat{S}_X(t)]^r}{\hat{S}_X(t)} = [\hat{S}_X(t)]^{r-1} \to \infty, \qquad \text{as } t \to \infty,$$

increases without bound to infinity.

Personal Automobile: Cost Drivers, Pricing, and Public Policy

by John B. Conners, FCAS

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Personal Automobile:
Cost Drivers, Pricing, and Public Policy

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Personal Automobile:
Cost Drivers, Pricing, and Public Policy

Abstract

Traditional actuarial pricing procedures have focused on pre-accident driver attributes, vehicle characteristics, and garaging location in an effort to explain personal automobile loss cost "drivers." Although these traditional factors are important for statewide ratemaking in a static environment, they account for only part of the influences on auto insurance loss costs.

This paper draws on the industry research of the past 15 years to present a more comprehensive four dimensional framework for understanding auto insurance loss costs, comprising factors grouped into the following categories:

- Pre-accident drivers attributes and vehicle characteristics
- Post-accident factors: claimant characteristics, medical providers, and attorney representation
- External environment, such as road conditions and traffic density
- Compensation system, such a tort liability versus no-fault

As an illustration, the paper shows how territory, which is often considered a reflection of external conditions (such as road safety and traffic density), is more properly analyzed as a proxy for post-accident factors – specifically, the "treatment triangle" among claimants, medical providers, and attorneys in certain locations. The paper concludes with two proposed public policy reforms, demonstrating how the expanded four-dimensional framework for personal auto loss cost drivers facilitates the development of more efficacious methods for holding down auto insurance loss costs.

Personal Automobile:
Cost Drivers, Pricing, and Public Policy

Introduction

Actuarial ratemaking sets policy premiums to cover anticipated loss and expenses. To estimate the needed premiums, the pricing actuary examines the "cost drivers": that is, the factors that influence the expected future losses and expenses.

In the past, actuaries have concentrated on variables related to driver, vehicle, and geographic characteristics. Indeed, these are the factors most susceptible to policy rating, the traditional role of the casualty actuary.

Although this traditional perspective produces accurate rates, it does not provide a full understanding of the underlying factors that influence automobile insurance loss costs. The recent studies of the Insurance Research Council ("IRC"; formerly, AIRAC), the RAND Institute, and the Automobile Insurance Bureau of Massachusetts (AIB) illuminate a host of other factors that play significant roles in determining these costs.

This paper integrates the results of these studies into a comprehensive framework for analyzing personal automobile insurance loss costs. The framework looks at four "dimensions" that affect loss costs: (a) driver and vehicle attributes, (b) claim and claimant characteristics, (c) compensation systems, and (d) environmental characteristics. The following section shows how these four dimensions combine to influence territorial rates.

The implications for policy pricing are highlighted by comparison with the traditional "claim severity / claim frequency" paradigm, using national statistics compiled by the IRC and Massachusetts experience analyzed by the AIB. The importance of the expanded framework is further revealed by three other uses, besides policy pricing:

- Several traditional classification dimensions are reinterpreted, underscoring their true effects on insurance loss costs. The IRC studies, for instance, show how territory is shifted from a factor related to the "physical environment" to a factor related to "claimant characteristics."

- Changes in compensation systems can be more accurately priced. The AIB studies show how a simplistic prognosis of the 1989 Massachusetts no-fault reform vastly mis-estimated the true effects on loss frequency and loss severity. This is comparable to the shift in the pricing of workers' compensation statutory amendments from "direct effects" to "direct plus incentive effects."

- Public policy recommendations for lowering the cost and improving the efficiency of personal auto insurance are made more realistic and more effective.

These uses of the expanded framework for personal automobile insurance cost drivers reflect the widening role of the casualty actuary in today's insurance environment.

Framework

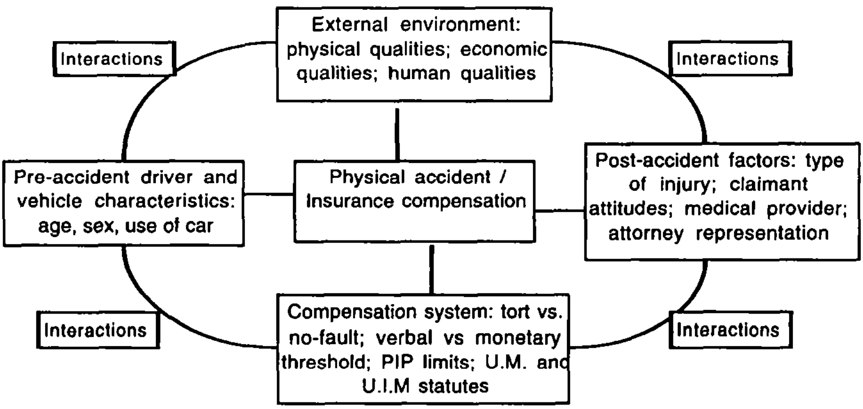
Let us begin with the fundamental question faced by the pricing actuary:

An insurer issues a personal automobile insurance policy. What factors influence the expected claim costs from this policy?

The traditional actuarial focus on ratemaking and classification systems, as well as a predilection for quantifiable data, has led to an emphasis on pre-accident factors – particularly driver, vehicle, and geographic characteristics – to the virtual exclusion of other factors that affect the insurer's payments. The likelihood and severity of an accident are considered to depend on driver attributes, vehicle characteristics, and garaging location. The amount of the claim and its monetary resolution stem directly from the physical aspects of the auto accident.

This perspective suffices for the novice actuary working in a static world with an automobile classification plan that is already optimal. It is inadequate for an actuary working with changing external conditions and compensation systems, or for an actuary refining classification plans, revising pricing procedures, or formulating public policy recommendations.

The expanded perspective in this paper groups loss cost drivers into four dimensions:



● *Pre-accident driver and vehicle characteristics*

Pre-accident characteristics refer to the elements shown on the policy application:

- *Driver attributes*, such as age, sex, marital status, driving record, driving experience, and driver education.

- *Vehicle and vehicle use characteristics*, such as make and model of the car, horsepower, mileage driven, multi-car discounts, and vehicle use (e.g., drive to work vs. pleasure).

- *Policy age*, such as new versus renewal policy.

These factors are used for setting rate relativities in existing classification schemes, since they are known to the insurer at policy inception, and they can therefore be used to rate the policy. *These factors are most important for predicting the occurrence of a physical event* (e.g., an accident). Once that event occurs, the insurance payments (if any) depend on a number of post-accident factors and on the compensation system.

❷ *Post-Accident Factors*

Studies of "classification efficiency" often fault traditional risk classification plans for failing to adequately explain the variance in insurance loss costs (see Spetzler, Casey, and Pezier [1976], Giffin, Travis, and Owen [1978], and Woll [1979]). Indeed, the factors discussed above relate primarily to the occurrence of the physical event – i.e., of cars colliding with one another. Other factors, such as the type of injury, the honesty of the claimant, attorney representation, and the type of medical treatment sought, are strong predictors of insurance claim costs.[1]

Post-accident factors relate to (i) whether an injury claim will be brought for the physical accident and (ii) the amount of the claim. These factors may be grouped into the following categories:

- *Type of injury*, such as soft-tissue injuries (back and neck sprains and strains) vs. fractures vs. more serious injuries. The topology of injury types should distinguish between injuries that are more or less susceptible to "build-up" and potential fraud. For instance, a fracture is readily discernable, and the length of needed treatment is objectively determinable. Soft-tissue injuries are harder to validate, and there is less consensus on their appropriate treatment. If claim frequency depends (in part) on "claim-filing" behavior, and if claim severity depends (in part) on "build-up," then a topology of injury

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[1] See, for instance, Weisberg and Derrig [1993], particularly Tables 2 and 3 on page 133, Table 4 on page 135, and Table 6 on page 138. Weisberg and Derrig note (page 132) that

> *For claims that involved strains or sprains, variables that reflected the seriousness of the injury explained little of the variation in medical expenses. For pure strains/sprains our model R^2 was only .04 and for mixed claims with strains/sprains and "hard" injuries, the R^2 was .21. . . . However, when variables related to treatment utilization and claimant behavior were added in, the value of R^2 for strain/sprain claims jumped to .78 and that for mixed claims to .79.*

In general, claimants are more likely to engage attorneys in more serious cases. However, even when the degree of injury is comparable, attorney represented cases are more likely to settle for higher amounts, though the benefit to the accident victim is often questionable (AIRAC [1989], IRC [1994]).

types that differentiates claims by the criteria mentioned above is most useful for forecasting loss costs.

- *Type of medical practitioner*, such as physician vs chiropractor vs physical therapist, as well as type of treatment, such as hospital admission vs. outpatient treatment in a practitioner's office.[2] The "type of injury" and "type of medical practitioner" variables have two or more values for most claims. In other words, many auto liability claims allege both a sprain/strain and another type of injury. Similarly, many claimants see two or more types of medical practitioner, such as a physician in an emergency room setting and then a chiropractor for extended visits.[3]

- Whether the insurance claim is being *represented by an attorney*. In tort liability claims, plaintiffs' attorneys are generally compensated on a contingent fee basis. That is, the attorney receives a percentage of the court award or of the insurance compensation, such as 33%.

  For BI claims, the insurance company's settlement offer is often a multiple of the economic damages (generally medical bills and wage loss) suffered by the accident victim. The

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2 The distributions of auto insurance claims by type of injury and type of medical practitioner differs from the distributions for standard health insurance. The distributions noted by Marter, Weisberg, and Derrig for claims reported in Lawrence, Massachusetts (an area suspected of widespread insurance fraud) are particularly revealing. Among the 1985-86 Lawrence claims studied by Marter and Weisberg [1991], 44 out of 48 were for sprains or strains (page 404). For these claims, moreover, 89% of the medical charges went to chiropractors, and only 10% went to physicians (page 407); see also Weisberg and Derrig [1991].

The predisposition of some actuaries is to view the lower back sprain treated by a chiropractor as a minor influence on auto insurance loss costs. On the contrary. In certain areas, such claims are the preponderant loss cost drivers. Even in the rest of the country, strains and sprains are the predominant type of auto injury in bodily injury claims, and treatment by chiropractors and physical therapists is becoming increasingly common.

3 The Insurance Research Council has documented both the multiplicity of injury and of medical practitioners as well as the trends in these statistics in recent years. In 1992, the average BI claimant reported about 2 different types of injury and was treated by about 2 different types of medical practitioners, as reported in the IRC's September 1994 volume, *Auto Injuries: Claiming Behavior and Its Impact on Insurance Costs*: "The growing share of claimants reporting multiple types of injuries also is reflected in the growth of the average number of different types of injuries reported by BI claimants. BI claimants reported an average of 1.92 types of injuries per person in 1992, up from 1.79 types of injuries per person in 1987." and "On average, BI claimants were treated by 1.95 different types of medical practitioners per person in 1992, up from 1.59 in 1987."

plaintiff's attorney has a financial incentive to encourage the "build-up" of the claim.[4] The IRC studies have consistently shown higher average costs for attorney represented claims, even when the type of injury is held constant.[5]

Perspectives regarding post-accident factors vary widely; we illustrate by two extremes. The difference in viewpoint is essential for estimating the costs of the auto insurance system and for developing reforms to reduce this cost.

Suppose an accident victim in a no-fault state with a monetary tort threshold suffers a lower back sprain, sees a chiropractor 30 times, recovers the out-of-pocket expenses from PIP coverage, and files a BI claim, which is handled by an attorney.

- The innocent (sometimes termed "naive") perspective sees the physical injury as the "loss

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[4] An illustration should clarify this. Suppose that an insurance company settles most BI cases for three times the economic damages: that is, the compensation for "pain and suffering" is about twice the medical bills. Suppose also that attorneys require 33% of the award for most BI claims.

If an accident victim without an attorney incurs $1,000 in medical bills, the total BI compensation would be $3,000, for a "net gain" of $2,000. If the claimant is represented by an attorney, who takes 33% of the award, or $1,000, the "net gain" to the claimant is only $1,000. However, if the attorney "encourages" the claimant to incur greater medical bills (perhaps by recommending a medical practitioner who sets a longer course of treatment), so that the economic damages rise to $2,000 and the insurance compensation rises to $6,000, the attorney's fee becomes $2,000 and the claimant's "net gain" remains $2,000. Many insurance company personnel and industry researchers believe that this accurately depicts the role played by many (though not all) attorneys. In other words, attorneys often drive up the cost of the system, with little benefit to claimants (assuming there are no other collateral sources of compensation, such as private medical insurance).

In no-fault states, there is a second incentive to build up claims. Many states have monetary tort thresholds, which allow accident victims to press bodily injury claims only if medical bills exceed a stated amount. [Most of these states also have verbal thresholds, which allow BI claims for "serious" injuries even if medical bills are low.] Attorneys can provide little aid in PIP recoveries. However, if by encouraging their clients to "build up" the medical bills to exceed the tort threshold they can file BI claims for "pain and suffering," both they and their clients can "profit."

[5] See AIRAC [1988] and IRC [1994]. The IRC study notes that "Attorney involvement in auto insurance injury claims has more than doubled in the last 15 years, moving from 19% in 1977 to 42% in 1992. . . . The use of attorneys results in a big cost to the auto insurance reimbursement system. Attorney-represented claimants incurred medical expenses and other economic losses averaging $14,718, compared with an average of $4,123 for claimants without attorneys." Figure 4-7 and the accompanying discussion on pages 29-33 of the IRC study show that this same pattern holds true even when claims are stratified by type of injury.

cost driver." The lower back sprain incurred in the auto accident motivates the victim to seek out a medical practitioner competent to handle such injuries. The length of the needed treatment, and the lack of reimbursement for non-economic damages under PIP coverage (such as "pain and suffering"), motivates the victim to file a BI claim. The complexity of the insurance claim process, and the uncertainties of BI compensation, motivate the victim to seek an attorney's aid. No one "profits" from the claim.

- The cynical perspective sees the "entitlement philosophy," or "claims-consciousness," or the "insurance lottery" as the "loss cost driver."[6] Whether the accident victim files an insurance claim, seeks treatment from a particular medical practitioner, or even "suffers" a lower back sprain is not dependent solely upon the physical events in the auto accident. Rather, the accident victim, seeking to profit from the event, sees an attorney, who encourages him or her to be examined by a certain medical practitioner. The medical practitioner diagnoses the lower back sprain and recommends the course of treatment. Either the chiropractor or the attorney notes that the medical expenses will be covered by PIP (as well as by other health insurance), and that the BI claim will pay for additional "pain and suffering" costs. The accident victim, the attorney, and the medical practitioner all "profit" from the claim.

The difference in perspectives leads to differing public policy recommendations. The "innocent perspective" sees injury prevention as the key to reducing insurance costs. Injury prevention efforts include mandatory seat belt laws, air bags, lower speed limits, and better policing of "driving while intoxicated" statutes. The "cynical perspective" sees the removal of the "claim lottery" incentives as the key to reducing insurance costs. Policy actions include anti-fraud units, peer review of medical practitioners, and verbal tort thresholds in no-fault states.

Compensation systems

Compensation systems may be grouped into tort liability, no-fault, and add-on systems. Tort liability systems may be subdivided by the financial responsibility limits and by the type of comparative negligence rule. No-fault compensation systems may be subdivided by the type of tort threshold: pure, verbal, and monetary. Verbal thresholds may be further classified by their definitions. Monetary thresholds may be further classified by their magnitude. No-fault systems may also be classified by the PIP limits, by the type of benefits provided, and by the compensation rate (e.g., "75% of wage loss").

The compensation system has a direct effect on claim frequency and claim severity, since a claim may be compensable under one system but not under another system. The compensation system has an "incentive" effect on claim filing (the "insurance lottery" perspective) and on

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[6] Casualty actuaries speak of "claims consciousness," which the IRC studies refer to as "claim filing behavior." "Claim consciousness" is frequently measured by BI/PD ratios; see the discussion of territory in the text. The "entitlement philosophy" is broader. Many accident victims, having paid thousands of dollars over the years for their own auto insurance, now feel that they are entitled to recover their money from the "insurance industry." The fact that their past auto premiums are unrelated to the insurance claim at issue rarely deters people from linking the two.

claim severity (e.g., the "build-up" of claims either to pass a monetary tort-threshold in a no-fault compensation system or to legitimize claims for pain and suffering awards in a tort liability system).[7]

Compensation system are most important in explaining state-by-state differences in insurance costs. Not only the insurance compensation but also the occurrence of claims and the amount of economic damages depend on the state compensation system.

❶ The *external environment*

The external environment relates to non-insurance characteristics that affect claim frequency or claim severity. We group these factors into three categories:

- *Physical qualities*, such as traffic density, road hazards and maintenance, and safety regulations (such as speed limits and seat-belt statutes). The *garaging location*, or the *rating territory*, is often thought of as reflecting physical road qualities. In truth, territory affects auto claim costs primarily by its relationship to several post-accident factors, such as attorney representation, the nature of the medical providers, and claimant characteristics. As the discussion below indicates, territory is not simply a reflection of road characteristics and traffic density.[8]

- *Economic qualities*, such as the "underwriting beta" argument that in prosperous years people drive more, purchase new vehicles, and take more vacations, leading to higher bodily injury accident frequencies.

- *Human qualities*: e.g., a higher proportion of poor residents in certain geographic areas *may* lead to more uninsured motorists and higher UM costs.

The Frequency-Severity Paradigm

The explanatory power of the expanded framework can be seen most clearly in contrast with the old "loss frequency – loss severity" paradigm. Previously, personal automobile loss cost drivers were viewed simply as inflation-induced changes in loss severity and as slow, long-

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[7] The "insurance lottery" perspective says the incentive effect on claim filing depends on the ease of pressing an insurance claim. States with strong anti-fraud statutes may greatly reduce claim frequency. The "build-up" of claims is useful only if it provides a greater "net gain" to the claimant and his or her associates. The incurral of additional medical expenses in a no-fault state with a strong verbal tort threshold is sometimes pointless, if the type of injury does not allow a tort claim to be pursued.

[8] Physical factors may be important in particular instances, such as to explain a high accident frequency at a four way intersection with stop signs but no traffic light. They are less important in the aggregate. Two cities may have similar physical characteristics but different claim frequencies.

term trends in loss frequency. The frequency trends were often modeled by econometric equations based on changes in gasoline prices, car density, and similar factors.

This paradigm is still useful for certain isolated analyses in static environments. But it provides no clue regarding why claim frequency or claim severity may be changing, or what the insurer should expect in the future. The expanded framework provides a different framework for viewing personal auto loss frequency and loss severity.

- *Frequency*: The Insurance Research Council studies of the mid-1990s show that the countrywide *property damage* claim frequency has decreased by 12% from 1987 to 1992. This is a measure of *accident* frequency, and it is consistent with fewer youthful drivers, greater public awareness of drunk drivers, and better quality cars.

  Over the same time period, the frequency of bodily injury claims increased by 16%. Given the 12% decline in *accident* frequency, this is a 32% increase in bodily injury claims per physical accident.[9]

For bodily injury, the changes in "claim filing" behavior among the public overwhelms the changes in physical accident frequency. The "loss frequency drivers" are not economic and environmental attributes like gasoline prices and car density. Rather, the drivers lie in the "claim and claimant characteristics" dimension of the expanded framework:

- *Type of injury*: the greatest increase over this period was in "soft-tissue" injuries (sprains and strains). Moreover, sprains and strains are particularly dominant in urban areas, which also have the highest ratio of BI to PD claims. In fact, the May 1994 IRC study, *Paying for Auto Injuries*, concludes that "Almost all of these additional injury claims are for difficult-to-verify injuries such as sprains and strains."

- *Type of medical practitioner*: the greatest increase over this period was in chiropractic treatment, especially for sprains and strains. Conversely, injuries requiring hospital stays have declined.

- *Attorney involvement*: between 1977 and 1992, the percentage of claims represented by lawyers rose from 31% to 46% for all injury coverages combined and from 47% to 57%

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9 Formally, 32% = [(1 + 16%) ÷ (1 − 12%)] − 1. The full IRC studies, see Insurance Research Council, *Auto Injuries: Claiming Behavior and Its Impact on Insurance Costs* (Oak Brook, Illinois, September 1994), and Insurance Research Council, *Trends in Auto Injury Claims*, Second Edition, Part One: Analysis of Claim Frequency (Wheaton, Illinois, February 1995). See also Insurance Research Council, *Paying for Auto Injuries: A Consumer Panel Survey of Auto Accident Victims* (Oak Brook, Illinois, May 1994): "More people involved in auto accidents are making claims for injuries, even though accident rates have been declining. . . . Many states enacted seat belt laws during these years, resulting in substantial increases in seat belt use. Seat belts reduce the number and severity of injuries in auto crashes. Around the same time, states passed tougher drunk driving laws in response to growing public awareness of this problem. In addition, the federal government now requires additional safety standards for vehicles that make cars safer for passengers."

for bodily injury claims.[10]

- *Law changes* In 1989, the threshold in Massachusetts for pursuing a BI liability claim was increased from $500 to $2,000. The traditional actuarial analysis would predict that the frequency of BI claims would decrease substantially, because injury claims with medical expenses between $500 and $2,000 would no longer be eligible for BI liability payments. In fact, the frequency reductions were minimal, because of incentive effects. The higher tort threshold encouraged accident victims (and their attorneys) to "build up" the medical expenses so that a bodily injury claim could be filed.

In sum, changes in claim and claimant characteristics are the key drivers for bodily injury claim frequency trends. Moreover, the claim frequency trends for BI coverage may be entirely different from the corresponding claim frequency trends for property damage liability and for collision coverage, even though all of these trends ostensibly relate to the occurrence of auto accidents.

❷ *Loss severity*: Actuaries have traditionally used two methods to project trends in loss severity.

A. Trend projections based on *internal* data fit observed average costs per claim to an exponential curve and assume that the same trend will continue in the future.

B. Trend projections based on *external* data correlate the historical average costs per claim with an economic index, such as the medical cost component of the CPI, and then estimate future claim severity based on the expected future values of the economic index.

Both methods work well in static environments. The first method works well when inflation is stable, so that past changes in loss severity are deemed to be unbiased predictors of future

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10 These statistics are from the IRC closed claim studies. Compare also the IRC consumer panel surveys, which show a similar ending point for 1992, but a lower starting point in 1977: "Attorney involvement in auto insurance injury claims has more than doubled in the last 15 years, moving from 19% in 1977 to 42% in 1992" (IRC, *Paying for Auto Injuries* [May 1994]).

Of particular importance to pricing actuaries are the relative differences by state, which are relevant for loss severity and loss frequency trends. Credibility weighting statewide loss severity and loss frequency trends with the corresponding countrywide figures is inappropriate if the statewide trends are affected by changes in (a) claim and claimant characteristics and (b) the compensation system in ways that the countrywide figures are not affected.

The same phenomenon may be seen in workers' compensation. In the past, statewide medical benefit trends were credibility weighted with countrywide trends. However, trends were lower in states with medical fee schedules, the counterpart to the "medical practitioner" dimension of the personal automobile framework here. Now, the figures assigned the "complement of credibility" in workers' compensation medical benefit trends depends on whether the state has a medical fee schedule.

Personal Auto: Cost Drivers, Pricing, and Public Policy changes. The second method works well when loss cost trends are considered to be closely linked to recognized inflation indices.

In personal automobile bodily injury insurance, loss severity trends are composed of three influences.

1. Trends in cost of treatment. This includes both (a) medical cost inflation and (b) trends in utilization rates that are independent of the personal auto compensation system.[11]

2. Trends in loss frequency. Severe automobile accidents lead to insurance claims regardless of the claim filing proclivity of the accident victim. The growing influence of attorneys and the changing "claim filing" behavior of the public lead to greater claim frequency for "minor" injuries, such as sprains and strains with no visible signs of impairment. These are often low cost claims. In other words, the factors that increase loss frequency often lead to decreases in average loss severity.[12] A change in expected claim frequency stemming from changes in claim or claimant characteristics should be *partially* offset by changes in expected claim severity.

3. Changes in compensation systems and in claim handling procedures. Compare the discussion above on the tort threshold change in Massachusetts in 1989. The new low severity projections changed dramatically because a whole cohort of cases which formerly had medical costs between $500 and $2,000 moved up to over $2,00 with higher pain and suffering awards.

Proxies

Many of the traditional classification variables used today are proxies for the true ("causative") factors affecting insurance loss costs. To clarify the difference between a causative factor and a proxy, let us contrast life insurance with automobile insurance.

- Sex and age are physiological attributes that affect expected mortality rates, so they are used as rating variables for life insurance underwriting and life annuity underwriting.

- Sex and age have equally strong correlations with auto accident frequencies, so they are used to set auto insurance rate relativities. Indeed, a 17 year old unmarried male may have about the same mortality rate as a 30 year old married female, but he may have several times the auto bodily injury claim frequency rate that she has. Yet sex and age (except at advanced

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11 For instance, the development of new medical procedures may engender greater utilization of services even when the personal auto compensation system remains unchanged.

12 The IRC studies demonstrate this phenomenon. Among the BI, PD, and PIP coverages over the 1980 to 1993 period, BI had the greatest increase in claim frequency and the smallest increase in claim severity; see especially Insurance Research Council, *Trends in Auto Injury Claims*, Second Edition, Part One: Analysis of Claim Frequency (Wheaton, Illinois, February 1995), chapters 1 and 2.

ages when bodily functions deteriorate) have little intrinsic relationship with accident propensity. Rather, they are proxies for other driver characteristics, such as maturity.

The use of territory as a proxy for external conditions, driver attributes, and claimant characteristics are discussed below.

Interactions

The factors in one dimension may interact with the factors in another dimension to determine expected loss costs. We illustrate with two examples.

- *Underwriting attributes and compensation systems:* Age, sex, and marital status may be more important as rating variables in tort liability systems, which focus on the tortfeaser's "fault," than in no-fault compensation systems, in which all accident victims are compensated. Conversely, the applicant's income and employment status may be important in no-fault compensation systems with high PIP wage-loss limits.[13]

- *Claim characteristics and compensation system:* The "padding" of claims, or "build-up," can be stimulated by a no-fault compensation system with a low or moderate monetary tort threshold. The AIB studies by Marter, Weisberg, and Derrig referenced above show how the 1989 increase in the Massachusetts tort threshold lengthened the average number of outpatient visits to chiropractors, who handled the majority of neck and back sprains and strains incurred in auto accidents.

The interactions of the four components of the expanded framework is essential for proper pricing and public policy recommendations, as discussed in the final section of this paper.

Territory as a Rating Variable

Territory is one of the chief variables used by U.S. insurers for automobile rate setting. Territory provides an excellent example for seeing how pre-accident driver characteristics, the pre-accident physical environment, post-accident characteristics, and the compensation system all affect automobile insurance loss costs.

- *Pre-Accident Driver Characteristics:* Pre-accident driver characteristics, such as age, sex, and marital status, do not generally have a direct effect on territorial relativities. Since the distributions by age and sex are relatively constant by territory, territorial relativities are not normally affected by the demographic characteristics of the drivers in

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[13] The comments in the text relate to relative importance only. Thus, age, sex, and marital status are important for no-fault compensation systems as well, since young, unmarried, male drivers are not only more likely to cause accidents, they are also more likely to be injured in accidents. Similarly, income and employment status are important for tort liability systems as well, since unemployed persons with little assets are often "judgment proof" and therefore carry low liability limits of coverage.

that territory.[14]

- *External Environment:* The physical environment in an area can raise or lower the expected number of accidents. For instance, population density and vehicle density are often cited as explanatory variables for accident frequency, on the assumption that with more cars per square mile, there will be more accidents per car. While this is true, a combination of road design, traffic controls, and law enforcement can reduce the variation caused by traffic density.

In a 1988 study, the Insurance Services Office and the National Associate of Independent Insurers compared the variation in traffic density with the variation in property damage (PD) claim frequencies.[15] Although the major cities in each state had traffic densities over ten times the statewide average, these cities had PD claim frequencies that were often only 10% higher than the statewide average.[16]

In sum, there is a tendency to overestimate the effects of traffic density on automobile claim frequencies. In theory, accident frequencies might be expected to increase proportionately with traffic densities. In practice, traffic safety devices in urban areas, such as traffic lights, stop signs, and well-designed roads, by causing traffic to move at a somewhat lower speed, keep the increase in the accident frequency to a relatively small percentage over the statewide average frequency.

Table 1 shows 1993 property damage claim frequencies by state.[17] With only 2 exceptions, the states lie in a narrow range from 20% above the countrywide average of 4 claims per 100

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[14] An exception would be communities, such as retirement communities, where a disproportionate number of senior citizens reside. This lowers the average pure premium of the territory, but the class rating system should produce the correct overall territorial rate.

[15] Traffic density, or "vehicle density," is defined in the study as car registrations per square mile.

[16] For example, the 1988 study shows a traffic density for Chicago of 5,423 cars per square mile, versus the statewide average of 152 car registrations per square mile. Nevertheless, the PD claim frequency in Chicago was only 11.7% higher than the statewide average claim frequency. More recent data (Insurance Research Council, *Trends in Auto Injury Claims*, 1995) shows a similar relativity, with the Chicago PD claim frequency being about 13% higher than the statewide average claim frequency.

[17] The data are taken from Figure 2-6 in the IRC study, *Trends in Auto Injury Claims*.

insured vehicles to 25% below the countrywide average.[18]

Table 1: *Number of PD Claims per 100 Insured Vehicles (1993)*

Massachusetts	7.13	Indiana	3.98	California	3.65	S Carolina	3.38
Dist of Colum	5.38	Nebraska	3.98	Oklahoma	3.64	Hawaii	3.38
Texas	4.76	Georgia	3.89	Kentucky	3.63	Vermont	3.36
Missouri	4.72	Alaska	3.89	Wisconsin	3.62	South Dakota	3.32
New York	4.67	Iowa	3.89	Arkansas	3.60	N Carolina	3.31
Illinois	4.35	Michigan	3.81	W Virginia	3.59	New Mexico	3.29
Rhode Island	4.23	Ohio	3.77	Virginia	3.54	Mississippi	3.26
Maryland	4.18	Nevada	3.76	Tennessee	3.54	Alabama	3.26
Connecticut	4.11	Minnesota	3.73	Colorado	3.52	North Dakota	3.26
Utah	4.09	Pennsyl	3.70	New Jersey	3.50	Maine	3.23
Louisiana	4.05	Florida	3.69	Washington	3.45	Montana	3.19
Kansas	4.03	Arizona	3.68	Oregon	3.45	Wyoming	3.02
N. Hampshire	4.02	Delaware	3.67	Idaho	3.39	*Countrywide*	*4.00*

Several other attributes of the physical environment also affect automobile insurance rates. Automobile theft rates vary by geographic location. Higher theft rates in urban areas cause higher comprehensive losses and therefore higher premiums for comprehensive coverage. Similarly, the 1988 ISO/NAII study shows substantially higher uninsured motorist costs in many urban areas, presumably resulting, at least in part, from higher levels of uninsured motorists. Finally, the cost of services provided by insurers, such as auto body shop repair costs and medical costs, vary by region, and they therefore affect territorial relativities.

➌ *Post-Accident Characteristics:* The occurrence of an automobile accident is a physical event. The decision to press a bodily injury claim once an accident has occurred, however, varies dramatically by state and even within a state.

The two dimensions of the expanded framework discussed directly above – pre-accident driver characteristics and pre-accident physical characteristics – relate to the occurrence of the accident itself. *Post-accident characteristics relate to the probability of a claim being filed given that an accident has occurred.*

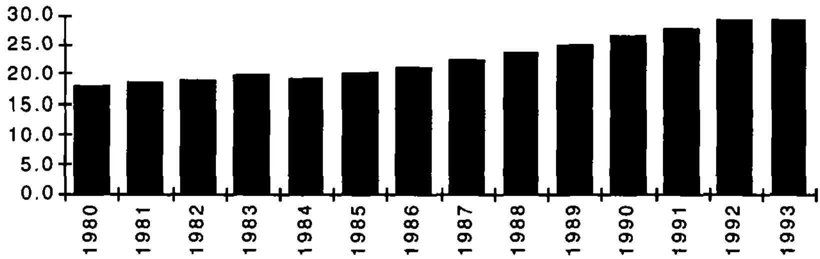
We want to measure this probability for bodily injury (BI) claims. Note carefully: we are not concerned with BI claim frequency or with automobile accident frequency. Rather, we are concerned with *the probability of a BI claim being file given that an accident has occurred where another driver could potentially be liable for damages.*

We presume that the filing of a property damage (PD) liability claim is influenced primarily by the nature of the physical accident, so relative PD claim frequency is a proxy for relative

[18] The two exceptions are the District of Columbia, which is an entirely urban area, and the Commonwealth of Massachusetts, which seems to have a statewide penchant for aggressive driving.

accident frequency where another driver could potentially be liable for damages. The ratio of bodily injury (BI) claims per 100 PD claims serves as a measure of the propensity to press personal injury claims. Table 2 shows the countrywide trend in this ratio over the past 15 years, from 18 BI claims per 100 PD claims in 1980 to over 29 BI claims in 1993.[19]

Table 2: BI Claims per 100 PD Claim



Our concern here is the relationship of this ratio to geographic location: that is, the variation in this ratio by state and by territory within state. Indeed, the BI/PD ratios vary greatly by state, as Table 3 shows. California, for instance, produces 61 BI claims for every 100 PD claims, whereas Wyoming, which is also a tort state, produces only 18 BI claims. [The effects of the compensation system are also evident from Table 3: the eight states with the lowest BI/PD ratios are all no-fault states.]

Table 3: Number of BI Claims per 100 PD Claims (1993)

California	60.7	Massachusetts	34.8	W Virginia	26.9	Nebraska	19.5
Louisiana	49.4	Oregon	34.3	Indiana	26.0	Florida	19.1
S Carolina	46.8	N Carolina	34.1	Maine	26.0	S Dakota	18.5
Nevada	45.4	Arkansas	33.9	Idaho	25.6	Wyoming	17.6
Arizona	45.3	Georgia	33.6	Alabama	25.1	New York	16.3
Rhode Island	39.7	Virginia	31.3	Connecticut	24.9	Kentucky	15.9
Oklahoma	38.9	Illinois	30.4	Montana	24.3	Hawaii	13.9
Dist of Colum	38.8	N Hampshire	29.8	Utah	22.2	Colorado	12.8
New Mexico	37.6	Delaware	29.1	Alaska	21.3	Minnesota	11.7
Washington	37.4	Ohio	28.1	New Jersey	21.2	Kansas	9.2
Texas	36.7	Tennessee	28.1	Vermont	20.9	Michigan	8.2
Maryland	35.5	Missouri	27.8	Pennsylvania	20.4	N Dakota	5.6
Mississippi	35.3	Wisconsin	27.4	Iowa	19.9	*Countrywide*	*29.3*

19 The data for the exhibits in this section derive from Insurance Research Council studies. They are from both full tort states and no-fault states. These are BI liability claims; they do not include no-fault claims.

The trends in BI/PD ratios over time and the variations by territory highlight the strong effects of post-accident characteristics on auto insurance loss costs. In California, for instance, the 61% BI/PD ratio for 1993 marks a steady climb from a 31% BI/PD ratio in 1980.

A common perception is that the accident frequencies themselves vary greatly by territory, being far higher in urban areas than in rural areas. Although such differences in accident frequencies do exist, the preceding statement confuses two issues, and it misinterprets the reasons for the territorial differences. Often, the frequency of physical accidents and of PD liability claims is only marginally greater in metropolitan areas than in the surrounding region. Once the accident occurs, however, the BI claiming pattern is substantially different in the metropolitan area than in other parts of the state.

IRC data from 1989 through 1991 illustrates this phenomenon. For instance, the PD claim frequency during these years was about 10% higher in Los Angeles than in the rest of the state, but the BI/PD ratio was 98.8% in Los Angeles, versus 45.2% in the rest of the state. In other words, it was not accident frequency differences that were driving up BI liability costs in Los Angeles, but BI claim filing patterns that were causing the difference.

Although BI/PD ratios are generally higher in large metropolitan areas, a simple urban/rural dichotomy is not always a good proxy for the actual claim filing patterns. For instance, during the 1989 through 1991 period, the state of Pennsylvania as a whole had a BI/PD ratio of 23%, the city of Pittsburgh had a ratio of 18%, and the city of Philadelphia had a ratio of 78%.

The attributes of territorial differences implicit in the discussion above have major implications for understanding auto bodily injury liability loss cost drivers:

- Loss cost differences by region are great, with some areas, whether urban centers or entire states, having high insurance costs and "affordability" concerns.

- Traffic congestion is <u>not</u> the primary determinant of these differences. In fact, the variations in PD claim frequencies are generally minor between urban areas and the statewide average.

- Differences in the BI/PD ratios account for most of the variation in BI loss costs by region, with higher cost areas having higher BI/PD ratios.

Thus, once an accident occurs, the decision of whether to over-treat the injury, or even to seek medical treatment when no injury exists, drives the major costs differences between states for bodily injury coverage.

The Treatment Triangle

The over-treatment of automobile injuries in certain locations, as well as the treatment of non-existent injuries, results from the interaction between claimants, medical providers, and attorneys, and it depends upon the type of injury and the structure of the compensation system. Our emphasis in this paper is on the lost cost drivers affecting territorial relativities. In particular, the major factors affecting territorial relativities are <u>not</u> pre-accident driver characteristics or pre-accident physical characteristics. Rather, they are the post-accident characteristics and compensation system attributes which determine how automobile accidents affect insurance payments.

Television reports on the human toll of highway accidents leave us with grisly pictures of torn metal and mangled bodies, as if most automobile accidents resulted in severe injuries. In fact, the opposite is true. About 60% of BI claimants report their only injury to be a strain or a sprain, and another 23% claim to have suffered a strain or a sprain plus another injury (IRC 1994: 19). Most strain and sprain injuries are difficult to verify, their severity is hard to measure, and radically different treatment patterns may be recommended by medical providers.

For over-treatment of injuries to occur, it is necessary that all parties deciding on the course of treatment gain from the over-treatment. For injuries and illnesses not covered by automobile liability insurance or workers' compensation insurance, the patient generally derives no financial gain from the medical treatment. Even if the patient has health insurance coverage (whether individual health insurance or employer provided group health insurance), the coverage simply reimburses the hospital costs or physicians' charges, and it often requires a co-payment from the patient.

Automobile bodily injury claims are different. BI liability awards consist of two parts: economic damages, such as medical costs or wage loss, and general damages, or "pain and suffering." Medical expenses comprise about three-fourths of economic damages. "Pain and suffering" damages are not objectively determinable on their own. Rather, the general damages are generally pegged as a multiple of the economic damages.

In sum, the medical expenses incurred by the claimant drive not only the insurance reimbursement for economic damages but also the insurance award for general damages. Each dollar of medical expenses incurred may translate into two dollars of insurance compensation.[20] In fact, many potential BI claims in the United States are not even pursued unless there is a sufficient amount of medical expense to support a "pain and suffering" claim.

In automobile accident cases, excessive treatment of "soft-tissue" injuries inure to the financial benefit of the claimant, the medical provider, and the attorney, and to the detriment of the driving public who pay the premiums that fund these loss payments. This phenomenon raises the BI/PD ratios and is a major driver of auto insurance loss costs.

Three parties are needed for excessive treatment to exist on a large scale, and the interactions of these parties is a major influence on territorial relativities:

1. Medical providers who aggressively treat even routine strain and sprain injuries in order to increase the medical expenses paid. The vast majority of medical providers, of course, do not engage in such over-treatment of minor injuries. Rather, a small coterie of medical providers who specialize in injuries covered by automobile liability and workers' compensation insurance serve this function well.

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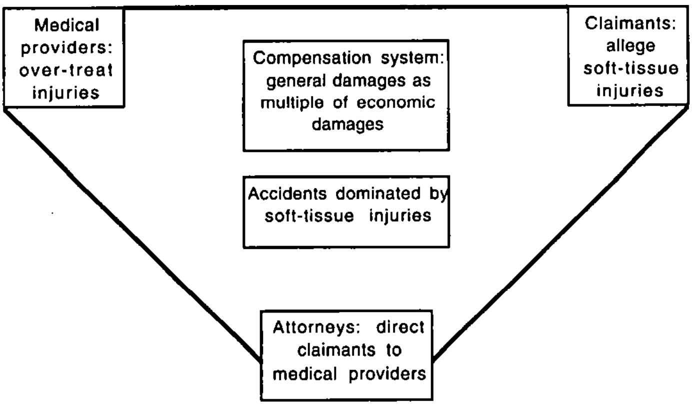
[20] The actual ratio, of course, varies by state and by year, since it is greatly influenced by the type of compensation system.

2. Accident victims willing to complain of soft-tissue injuries, even when objective medical impairment is non-existent or slight.

3. A third party who can direct a willing accident victim to the proper medical provider. Most auto accident victims are not sufficiently aware of the auto liability compensation system to take full financial advantage of the system. In the United States, a relatively small number of attorneys who specialize in strain and sprain injuries in automobile liability and workers' compensation insurance claims fulfill this function by directing potential BI claimants to medical providers willing to over-treat soft tissue claims.

This "treatment triangle" is shown schematically below.



This phenomenon is exceedingly difficult to police, even when insurers are aware of its existence in a given location. As long as the accident victim claims to be injured, the medical provider can continue the aggressive treatment pattern. To justify the recommendation of a particular medical provider, the attorney need only state that the medical provider is licensed by the state and has produced "good results." Sting operations are difficult to run, since a claimant who claims not to be injured will simply not be treated.

Evidence for over-treatment of automobile injuries is necessarily indirect, though in some locations it is compelling. We illustrate with data from Massachusetts, where a detailed claim database has been in existence for two years.

Were there no incentive to over-treat injuries, one would expect a wide dispersion of treatment costs for each provider, with some patients requiring substantial treatment while others require minimal treatment, depending on the severity of the injury. Moreover, one would expect that the number of BI claimants treated by a medical provider would be about half the number of PIP ("personal injury protection") claimants, since all injuries need treatment

(PIP) whereas a BI claim may be filed only if another driver was at fault.[21]

The automobile compensation system in Massachusetts has a $2,000 tort threshold. That is, a BI claim may be filed only if the PIP medical expenses exceed $2,000.[22] A small number of medical providers in Massachusetts have a large percentage of their patients suffering from automobile accident injuries who routinely require above $2,000 in treatment. The implication is that the course of treatment is being determined not by the type of injury but by the desire to reach the tort threshold in order to file a BI claim.

Similarly, among automobile accident victims being treated by these same medical providers, the number of BI plus uninsured motorist claimants is almost equal to the total number of PIP claimants. The implication is that patients are being referred to these medical providers for the primary purpose of building up the PIP expenses so that a liability suit can be pursued.

- *Compensation Systems and Benefit Levels:* The type of compensation system and the level of benefits are reflected in the statewide rates and the territorial relativities. Changes in state laws require an analysis of the "effectiveness" of the current law and of the proposed law. For example, in an urban area, the current tort system or monetary tort threshold in a given state may lead to substantial medical overtreatment, with resultant high rates, in comparison to a suburban or rural area, with little overtreatment. A law change that curtails this overtreatment would have a larger percentage decrease in the urban territory than in the suburban or rural territories.

Summary: Territory and the Four-Dimension Framework

Geographic location, or rating territory, has often been a difficult classification variable for the actuary to explain. Why should auto insurance policies cost more in California than in other states? Why does auto coverage cost so much more in certain urban areas?

Driver characteristics do not differ significantly from place to place. Physical conditions, such as road hazards and traffic density, have a minor effect on accident frequencies. They contribute only marginally to the observed loss cost differences by territory.

Rather, geographic location and rating territory serve as proxies for powerful but often overlooked factors that drive auto insurance loss costs. Between states, the incentive effects of compensation systems account for much of the wide variation in claim frequencies and loss costs. Within states, the "treatment triangle" phenomenon accounts for much of the variation in territorial relativities.

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21 In fact, we would expect the number of BI claimants treated by a medical provider to be less than half the number of PIP claimants, since only those cases exceeding the tort threshold can lead to a BI claim (see below in the text).

22 For certain types of severe injuries, a BI claim may be filed even if medical expenses do not exceed $2,000. However, these types of severe injuries are relatively rare in auto accidents. When they do occur, the $2,000 tort threshold is quickly reached.

Pricing and Public Policy

The framework for analyzing personal automobile loss cost drivers presented in this paper has numerous ratemaking and public policy implications, ranging from territorial relativity analysis to pricing statutory amendments. In workers' compensation, for instance, the pricing of statutory amendments is a finely honed actuarial tradition, well described in Fratello's 1955 *PCAS* paper.[23] It is also half wrong, as shown by the consistent actuarial misestimates throughout the 1980s, since it covers only the direct effects of law changes, not the incentive effects.[24]

Compensation system reforms in personal auto insurance are often accompanied by mandatory rate rollbacks. If no changes are assumed in claim filing behavior, then the cost effects of the reform may be grossly over- or under-estimated, as shown by the 1989 Massachusetts changes. It is vital for casualty actuaries to understand the complete system of personal auto loss cost drivers to order to accurately price system changes.

The availability and affordability of auto insurance are of public concern in many jurisdictions, and casualty actuaries are often called to testify on these issues. The actuary who knows only what the existing rating plan indicates, but who does not understand why rates are higher in some territories than in others, or how the compensation system affects loss costs, makes a poor prognosticator. Rather, the actuary must explain how claimant behavior and the compensation system interact with the traditional driver attributes, vehicle characteristics, and the external environment to determine the expected loss costs.

We provide two possibilities for public policy reforms to reduce automobile insurance loss costs that stem from the expanded framework in this paper. These are not the only possible reforms, but they are efficacious and practical proposals.[25]

- *Peer review of medical treatment*: The discussion above of claim characteristics and of medical treatment indicate that one of the major factors contributing to the increases in

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23 See B. Fratello, "The Workmen's Compensation Injury Table and Standard Wage Distribution Table - Their Development and Use in Workmen's Compensation Ratemaking," *Proceedings of the Casualty Actuarial Society*, Volume 42 (1955), pages 171-202.

24 See John Gardner, *Return to Work Incentives: Lessons for Policymakers from Economic Studies* (Cambridge, Massachusetts: Workers' Compensation Research Institute, 1989), as well as the numerous state specific studies form the Workers' Compensation Research Institute.

25 Other reforms would be equally effective. For instance, most auto actuaries agree that movement from a tort liability compensation system to a no-fault system with a strong verbal tort threshold, as in Michigan, would reduce overall costs. However, there are strong interest groups opposing such a move, and who support instead such changes as epitomized by California's Proposition 103: rate rollbacks, classification restrictions, and prior approval, but no attack on the real problem of overtreatment.

bodily injury loss costs over the past decade has been the "build-up" of hard-to-verify soft tissue injuries, generally with extended courses of treatment by a small number of chiropractors, physical therapists, and physicians, often orchestrated by attorneys experienced in such claims. Insurance claims adjusters are aware of the "padding" in these claims. Yet it is nearly impossible for claims adjusters to find "objective" evidence of unnecessary or inappropriateness treatment, especially on any specific case.

Peer review of medical treatment in auto insurance claims, by state panels of physicians and other medical practitioners, could succeed in eliminating the worst abuse and stemming or reversing the upward trend in bodily injury loss costs. The state insurance department would appoint a panel of medical experts to review treatment patterns by individual medical providers. A substantial database of auto injury losses would be needed to properly identify such patterns. It is generally impossible to determine over-treatment by reviewing any one specific case since the severity of any soft-tissue strain or sprain is a subjective estimate. However, by reviewing all treatment by particular medical providers, patterns of overtreatment can be recognized. Medical practitioners would be more hesitant to provide excessive treatment on a consistent basis if they knew that their actions would be subject to professional review.

- *Consumer representation*: A second factor contributing to the increase in bodily injury loss costs over the past decade has been the rapid increase in attorney representation of insurance claims. If the attorney helps build up the economic damages, there is generally no "net loss" to the claimant despite the hefty contingency fee, and sometimes even a "net gain." In addition, the attorney handles all the claim filing paperwork and negotiates with insurance loss adjusters. Both of these activities can be frightening to the average citizen, particularly in third party cases.

  State insurance departments could provide "claims representatives" to handle claim filing and negotiation on behalf of auto accident victims who need aid in insurance matters. The claims representatives would be compensated by salary, so they would have no interest in building up claims. The insurance industry would defray the costs of these claims representatives.

  All parties could gain. Claimants would have representation by state insurance officials, who could guide them through the claims process – *at minimal cost to the claimant*. Insurance companies would gain because the cost of such claims representatives is far less than the costs of claim "build-up." The general public would gain by lower insurance premiums and increased satisfaction with the insurance claim process. State insurance departments would gain because they would be offering additional and highly valued services.

Conclusion

The days of simple claim severity and frequency trends in automobile rate making are gone. The ultimate cost of automobile insurance is a complex and changing mosaic of many diverse factors. Actuaries who understand these factors will be of great value to their companies, and they may eventually help design systems to control the cost of automobile insurance.

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