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Summer 1997, Volume 2
Including the DFA Call Papers

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NOTICE

The Casualty Actuarial Society *Forum*
Summer 1997, Volume 2 Edition
Including the DFA Call Papers

To CAS Members:

This Summer 1997 Edition, Volume 2 of the Casualty Actuarial Society *Forum* contains five Dynamic Financial Analysis Call Papers and two new papers, one of which is copyrighted by the National Council on Compensation Insurance, Inc. In addition, the *Forum* contains the results of the CAS Continuing Education Survey.

The CAS *Forum* is a non-refereed journal printed by the CAS. The viewpoints published herein do not necessarily reflect those of the CAS.

The CAS *Forum* is edited by the Committee for the Casualty Actuarial Society *Forum*. Members of the committee invite all individuals to submit papers on topics of interest to the actuarial community. Articles need not be written by CAS members, but the paper's content must be relevant to the interests of the CAS membership. Members of the committee request that the following procedures be adhered to when submitting an article for publication in the *Forum*:

1. Authors should submit a camera-ready original paper, and two copies.
2. Authors should not number their pages.
3. All exhibits, tables, charts, and graphs should be in original format and camera ready.
4. Authors should avoid using gray-shaded graphs, tables, or exhibits. Text and exhibits should be in solid black and white.

The CAS *Forum* is printed periodically based on the number of call paper programs and articles submitted. The committee publishes two to four editions during each calendar year.

All comments or questions may be directed to the Committee for the Casualty Actuarial Society *Forum*.

Sincerely,

Robert G. Blanco, CAS *Forum* Chairperson

The 1997 CAS DFA Call Papers Presented at the 1997 DFA Seminar July 21-22, 1997 Sheraton Hotel and Towers Seattle, Washington

The Summer 1997 Edition, Volume 2 of the CAS *Forum* is a cooperative effort of the Committee on the CAS *Forum* and the Dynamic Financial Analysis Task Force on Variables. The Task Force was pleased to present five papers for discussion, prepared in response to its 1997 DFA Call Paper Program. These papers were discussed by the authors at the Casualty Actuarial Society's 1997 DFA Seminar, July 21-22, in Seattle, Washington.

As an adjunct to the Call for Papers, the Casualty Actuarial Society established a prize pool of up to $10,000 for papers describing a dynamic financial model that had actually been applied to a property/casualty insurer. The prizes were decided primarily upon the relevance of the concepts and applications described in the papers to the evaluation of the insurance, financial, and economic variables that should be used in the design, construction, and application of DFA models.

First prize has been awarded to Stephen P. D'Arcy, Richard W. Gorvett, Joseph A. Herbers, Thomas E. Hettinger, Steven G. Lehmann, and Michael J. Miller for "Building a Public Access PC-Based DFA Model." Second prize has been awarded to Gerald S. Kirschner and William C. Scheel for "Specifying the Functional Parameters of a Corporate Financial Model for Dynamic Financial Analysis." In this edition of the *Forum*, the two prize-winning papers appear first and the remainder of the DFA call papers appear in alphabetical order based on the last name of the first author listed.

Our thanks go to all of the authors, without whom we would have been unable to present this fine book. Our particular appreciation goes to the authors of the excellent prize-winning papers.

Gary S. Patrik,

Chairperson, CAS Dynamic Financial Analysis Task Force on Variables

CAS Dynamic Financial Analysis Task Force on Variables

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Michael J. Miller, FCAS

CAS Dynamic Financial Analysis Task Force on Variables
Call Paper Program

Building a Public Access PC-Based DFA Model

Submitted by

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Authors

Stephen P. D'Arcy is a Professor of Finance at the University of Illinois at Urbana-Champaign. He is a Fellow of the Casualty Actuarial Society, a member of the American Academy of Actuaries, and Vice President of the American Risk and Insurance Association. He received his B.A. in Applied Mathematics from Harvard College and his Ph.D. in Finance from the University of Illinois. The courses he teaches include an introduction to insurance, property-liability insurance, casualty actuarial mathematics, advanced corporate finance, and the financial risk management of insurance enterprises. He also developed and teaches a seminar on finance for the Casualty Actuarial Society. Prior to his academic career, he worked as an actuarial student at AEtna Insurance Company and as Actuary at CUMIS Insurance Society. His research interests include dynamic financial analysis, financial pricing models for property-liability insurers, catastrophe insurance futures, pension funding, and regulation.

Richard W. Gorvett is in the Department of Finance at the University of Illinois at Urbana-Champaign. He is a Fellow of the Casualty Actuarial Society, a member of the American Academy of Actuaries, and an Associate in Risk Management. He received a B.S. in Mathematics from the University of Illinois at Chicago, an MBA in Finance and Econometrics from the University of Chicago, and is completing his Ph.D. in Finance at the University of Illinois at Urbana-Champaign, where his dissertation is entitled "The Dynamic Financial Analysis of Property-Liability Insurance Companies." He has co-authored a paper entitled "A Comparison of Property-Liability Insurance Financial Pricing Models," and is continuing research into the application of financial theory to insurance. Mr. Gorvett has taught property-liability insurance and corporate risk management courses, and will be teaching investment courses as a visiting lecturer in the Fall of 1997. Prior to entering academia, he worked as an actuary and consultant with several firms in the Chicago area.

Joseph A. Herbers is an Associate of the Casualty Actuarial Society and a member of the American Academy of Actuaries. Mr. Herbers has 12 years of experience as a consultant. His practice is focused in the financial risk management and alternative market practice areas. He provides statutory letters of actuarial opinion to insurance companies, risk retention groups, syndicates, self-insured entities, and captive insurance companies. In addition, risk financing and funding studies for municipal pools and other self-insured groups constitute a significant portion of his clientele. Other focus areas are in the nonstandard automobile insurance marketplace and in the costing of automobile insurance reform measures and legislative initiatives.

He is an active member of the CAS, serving on the Committee for Program Planning and authoring several articles in professional publications.

Thomas E. Hettinger is an Associate of the Casualty Actuarial Society and a member of the American Academy of Actuaries. Mr. Hettinger has extensive experience working with insurance companies, captive insurance companies, self-insured groups, and other alternative market risks. His actuarial background includes pricing, reserving, and the development of new lines of business for insurance companies, loss cost projections, rate reviews, reserve analyses for captive insurance companies, and funding studies for self-insured groups and municipal pools. Mr. Hettinger served as a team leader for the "Cashflow" team for the Equitas project at Lloyd's of London. In addition, Mr. Hettinger's expertise includes the use and implementation of the loss reserve software ResQ and the development of residual market credit models.

Steven G. Lehmann is a Fellow of three actuarial organizations: the Casualty Actuarial Society, the Society of Actuaries, and the Canadian Institute of Actuaries. He is also a member of the American Academy of Actuaries. Mr. Lehmann has 27 years of actuarial experience, serving with State Farm Insurance for 24 years before entering the consulting field. At State Farm, he was in charge of pricing and rate filings for various jurisdictions in the United States and Canada, including providing expert witness testimony in the areas of ratemaking, risk classification, and rate of return analysis. His consulting experience has focused on development of pricing strategies including coordination of underwriting and rating programs and competitive analysis. He has served as a Vice President and a member of the Board of Directors for the CAS and has chaired a number of committees and task forces. He currently serves as Chair of the Property/Casualty Ratemaking Subcommittee of the Actuarial Standards Board and is a member of its Task Force on Rate of Return.

Michael J. Miller is a Fellow of the Casualty Actuarial Society and a member of the American Academy of Actuaries. Mr. Miller has 30 years of actuarial experience, serving first with State Farm Insurance before entering the consulting field. As a consultant, he was a Principal and Vice President in the Bloomington, Illinois office of Tillinghast/Towers Perrin. He has been a leader in the development of professional standards that guide actuaries in all areas of property/casualty actuarial practices, having served a three-year term as the Chair of the Property/Casualty Committee for the Actuarial Standards Board. He has served the CAS as the chair of both its Risk Classification and Principles of Ratemaking Committees. Mr. Miller is a past member of the Board of Directors of the CAS and former Vice President in charge of research and development for the CAS.

Abstract

This paper describes the initial version of a DFA model for U.S. property-liability insurers that will be made available to all interested parties. Those wanting to test the model will be able to obtain a copy of the program on disk or access the model over the internet. The long-term goal of this research is to develop a usable, understandable model that meets the basic needs of the industry. The model is very much a work-in-progress, and comments and suggestions are encouraged.

Introduction

In developing a Dynamic Financial Analysis (DFA) model, there are two primary problems facing the property-liability actuary. First, insurance operations are affected by an almost overwhelming number of factors, many of which deserve considerable attention. Second, the proprietary nature of most existing models limits the amount of information that has been shared and made publicly available regarding the modeling process. The project described in this paper has the long-term objective of addressing both of these problems.

In recognition of the fact that no model can successfully consider every potential source of risk, our model focuses on the *key variables* that affect the financial results and condition of a typical property-liability insurer. While the key factors affecting the underwriting and investment operations of a company are included, the number of variables actually incorporated in the model is held to a minimum. Addressing only the more important quantifiable financial risks to which property-liability insurers are exposed facilitates model comprehension and communication. Many of these same factors have been considered in the development of other models. However, certain factors, important in analyzing property-liability insurers, have been overlooked in some models that have been derived from life insurance DFA models. Differences in the essential natures of life and property-liability businesses need to be taken into account in any property-liability DFA model.

Our response to the second problem mentioned above is *public access*: our model is being made available for use by anyone interested in the process. All assumptions, techniques and calculations are explained in enough detail that other researchers and practitioners, with an appropriate understanding of the basic concepts and issues, will be able to use the model. The publication of the process should foster peer review of the model and can lead to improvements in the overall methodology. This approach should provide a valuable learning tool for individuals wanting to understand DFA for property-liability insurers. It should also help the profession deal with the issue of developing standards of practice in this emerging and important area.

Dynamic Financial Analysis is a natural outgrowth of the increasing recognition of the interdependence among all underwriting and investment facets of the insurance business. Technological advances now allow the widespread use of much more sophisticated financial models than were previously possible. While these developments represent advances for the industry, they also present challenges. In addition, the importation of DFA models previously utilized for life insurers and in other countries has resulted, sometimes, in very complex models that lack appropriate focus on risks specific to U.S. property-liability insurers. All of these factors have combined to create a significant hurdle for individuals seeking to work in this area. Thus, one goal of this research is to simplify the process by generating a usable, understandable model that can serve as a reasonable approach to Dynamic Financial Analysis for property-liability insurers.

What is DFA?

Dynamic Financial Analysis is a new term for a standard task of casualty actuaries: planning for the future. In earlier days such an analysis might have been labeled simply an "actuarial" analysis. The new term reflects, in part, the latest terminology fad, with "dynamic" representing the recognition of the stochastic, or variable, nature of insurance assets, liabilities, and operations, and 'financial' reflecting the long overdue approach of integrating assets and liabilities of insurers. In addition, the advent and dissemination of personal computers (programmed first-hand by actuaries, rather than second-hand by data processing personnel), spreadsheet programs, and other (often sophisticated) computer software allows for the performance of more timely and involved calculations. Finally, new research that provides sophisticated mathematical models of factors such as interest rates, catastrophic losses, and investment and underwriting performance allows for better modeling of complex systems of interrelationships, well beyond the precision and detail attained in earlier actuarial planning models.

The casualty actuarial profession in the United States has been a relative latecomer to the area of dynamic financial analysis. European insurers, both life and casualty, have long recognized the importance of investment risk, and incorporated financial models in their approach to actuarial work. Life insurers in the United States and Canada, after experiencing the traumatic effects of interest rate shocks during the 1980s, began to focus on interest rate risk and utilized dynamic financial models to help manage asset-liability risk. Canadian casualty actuaries, following the developments of their life industry more closely, in part as a result of regulatory requirements, also developed models in this area.

This latecomer status of U.S. casualty actuaries to DFA modeling has both benefits and disadvantages. The major benefit is that the general structure of dynamic financial analysis is already in place: there are many functioning models, used primarily in life insurance and/or in other countries, that can be used as guides and points of reference. Much of the trial-and-error and initial testing stages of model development have been accomplished, and actuaries can now benefit from that documented experience.

The potential disadvantages, however, are that the unique characteristics of U.S. property-liability insurers can be overlooked, or trivialized, when creating DFA models for this industry, simply because prior models failed to consider and incorporate them. Adopting and using inappropriate models can, in certain situations, be even worse than having no model: confidence based on improper information can sometimes be more problematic than cautiousness based on uncertainty. We explore some of the unique characteristics of our industry in the following section.

How are U.S. Property-Liability Insurers Unique?

The typical U.S. property-liability insurer writes personal and/or commercial lines, with a mixture of property and liability exposures, in a regulated environment. Property lines are subject to a range of catastrophic risks -- e.g., windstorm, earthquake, freezing, and fire -- depending on the geographic distribution of its business. (While property insurance in any country is exposed to catastrophic risks, each country – and each geographic subdivision – has different specific risk characteristics.) Liability lines are exposed to the unique vagaries of the U.S. civil justice system, which is vastly different from the European and Canadian systems. Both personal and commercial lines are subject to state insurance regulation, which can (and does) impact rates, residual market size and subsidies, policy writing requirements, entry and exit conditions, and even retroactive premium rebates. For any company, the impact of insurance jurisdictional issues, and any associated volatility in operating results, depends on both the geographical and line of business distribution of the insurer's writings.

The investment portfolio of a typical U.S. property-liability insurer is primarily bonds – of varying types and maturities -- with a smaller investment in stocks. (See Exhibits 1 through 3.) The major component of liabilities for property-liability insurers is loss reserves; typically, the second most significant liability item is the unearned premium reserve, a portion of which will, in turn, develop into reserves for losses. (See Exhibit 4.) Both assets and liabilities are subject to statutory valuation requirements that, it can be argued in some cases, defy logic and consistency. Many regulatory requirements – theoretically, for the purpose of enhancing potential solvency -- impose conservative valuation measures, such as not allowing loss reserves to reflect the time value of money. Others, such as stating bonds at an amortized value based on the initial interest rate, can be either conservative or excessive, depending on the direction of subsequent interest rate changes. These statutory valuations, despite their recognized deficiencies, do impact insurer performance, both in terms of internal decision-making and external perception. Regulatory intervention is frequently imposed as a result of indications based on statutory valuation.

U.S. property-liability insurers write short-term policies that are generally offered for renewal at the policyholder's option. While those insurers selling through independent agents are dependent on the decision of the agent as to whether the policyholder has this choice, other insurers can contact the policyholder directly with an invitation to renew the contract under the current policy terms and rates. Although the reasons for this behavior are not fully understood, policyholders do generally accept this offer, as the retention rates of most U.S. property-liability insurers are in the 90 percent level. Additionally, also for reasons not fully understood, the loss experience of renewal business is generally lower than that of new business, with experience continuing to improve as the policy is renewed a greater number of times.

Because of the improving loss ratio on renewal business, a property-liability insurer's book of business represents a significant asset to the company. However, despite the fact that renewal rates approximate, or even exceed, those for some renewable life insurance policies which are valued as assets, this asset is not reflected on the balance sheets of property-liability insurers. Nevertheless, this characteristic does need to be considered when projecting operating results for property-liability insurers.

DFA Approaches

There are two primary techniques for modeling risk factors: scenario testing (a deterministic approach), and stochastic simulation. Each has its advantages and disadvantages. Scenario testing is used to project results under specific situations. One highly visible example of this approach is the Social Security System, where several sets of economic and demographic assumptions are used to develop three different forecasts of possible funding conditions. Another example is a set of scenarios used in New York to test the ability of life insurers to withstand specific changes in interest rates and other economic conditions. However, a more common framework for DFA models is stochastic simulation, in which a series of randomly generated events, or "trials," produces a large number of different outcomes. The distribution of these outcomes then forms the basis for various indications -- for example, the proportion of simulated outcomes that are considered "unacceptable" is used as a measure of insurer risk. "Unacceptable" outcomes can be based on failing a regulatory test, incurring a ratings decline from one of the financial ratings agencies, becoming insolvent, or any other established benchmark.

Classifying Risk

The risks facing insurers can be classified into two major categories: one for items listed on the balance sheet, and the other based on continuing operations (which would appear in the operating statement). Furthermore, each of these categories can be subdivided into two further categories. Balance sheet risk consists of asset risk and liability risk. Operating risk consists of underwriting risk and investment risk.

Asset risk involves the change in value of an existing asset. For a bond, this could result from a change in interest rates, a change in the debt rating, or default on interest or principal. For an equity, asset risk involves a change in the market price, which could be caused by some of the same factors affecting bond values, or by other changes affecting company profitability or operations. Other assets, such as agents balances, are exposed to default risk.

Liability risk is primarily related to the adequacy of the loss reserves. As statutory valuation requires loss reserves to be carried as the nominal value of all future payments, this risk involves the possibility that total payments will ultimately differ from the indicated estimate. Based on market valuation of loss reserves, however, the risk also includes timing and discount rate components as well as the total payment amount. In addition, liability risk includes the adequacy of the unearned premium reserve to cover losses that will emerge on existing policies.

Underwriting risk is the risk associated with business that the insurer will write in the future, either as new business or renewals of existing policies. This risk includes pricing risk -- the ability to obtain adequate premium levels on this business -- as well as the risk associated with stochastic losses and expenses.

Investment risk relates to investment income and capital gains to be earned on existing assets and new assets resulting from continuing operations. This is dependent on interest rates and other economic conditions.

The four risk components are complexly interrelated. An increase in interest rates, for example, would lead to a decline in the value of existing assets (especially bonds), but higher investment income on new investments. Adverse development on loss reserves would generate the need for premium increases, and impact future underwriting experience. The advantage of a DFA model is that it can allow for this type of interaction. However, a drawback is that these relationships are difficult to quantify. This leads to the need to develop answers to some basic modeling questions before proceeding.

Practical Questions

Getting started building a model to forecast the future financial results of a property-liability insurance company is difficult, because so many fundamental issues must be addressed before any applications can be constructed. Three of these basic issues are:

1 **What risk exposures will be modeled?** In addition to traditional insurance risk exposures (such as pricing, reserving, reinsurance, jurisdictional, and catastrophes), property-liability insurers are also exposed to financial risk (interest rates, default, market fluctuations) and general business risk (such as fraud, mismanagement, lawsuits and off-balance sheet items). Some can be quantified (interest rate risk, pricing risk) while others defy quantification (management fraud, novel interpretation of insurance contracts).

2. **How can the exposure be quantified?** Historical data may or may not be available, either internal to the company, or from external sources. What data is available may not be of sufficient volume to be reliable for modeling purposes.

3. **How should the risk factors be modeled?** Scenario testing and stochastic simulation are alternative approaches, but the latter is generally preferred by the authors since it accounts for the stochastic nature of insurance operations. It is likely that certain risk factors may be incorporated as stochastic variables, while others are treated in a deterministic fashion. In addition to the general approach of the DFA model (i.e., deterministic vs. stochastic), the interactions between various risk exposures must be considered.

After addressing these three fundamental issues, there are a host of other fundamental questions that must be addressed as well.

First, how complex (or simple) should the model be? Given the complexity of the risk exposures facing property/casualty insurance companies, the natural tendency is to construct a rather intricate model, attempting to quantify as many risk factors as possible. This tendency should be counterbalanced by the need for a workable model that can be adequately understood and communicated. Presumably, the model will be a work in process for many years to come as additional research is conducted addressing the risk exposures and their potential treatment in DFA models.

Second, should the model incorporate any level of management intervention in which certain decisions are pre-programmed into the model, such as curtailing growth in new business when the premium-to-surplus ratio attains a particular level? Other possible management interventions include the realization of capital gains or losses depending on the profitability in a particular year, specifying certain tax elections to minimize income tax liability (such as carry-forwards and carry-backs, and the portfolio mix of taxable vs. tax-exempt securities), or withdrawing from states that are persistently unprofitable. Although many of these decisions appear reasonable, they need to be viewed in the context of the basic premise of a DFA model. The purpose of a DFA model is to provide a tool which management can use to assess the future financial condition of their business, and make better informed decisions accordingly. In this context, all management decisions should be "off the table" in the development of a DFA model. We recognize that a prudent manager would make certain decisions given the anticipated financial results in a given year. However, circumventing the outcomes of the DFA model by programming automatic decisions into the process hinders the effectiveness of the model. That is, if management will be using the model to make better informed decisions, it does not seem appropriate to incorporate any part of the decision making process into the model itself.

Third, should the modeling be done on a direct or net basis? In order to address a variety of reinsurance issues, it seems plausible to consider direct results, netted down for the impact of reinsurance. If the reinsurance program is fairly straightforward (i.e., excess of loss, quota share, aggregate excess and catastrophe type coverage), the model should be able to accommodate a direct/ceded/net approach. If, however, the reinsurance program includes financial reinsurance, clash covers, multiple line aggregates, and so on, the modeling could be extremely cumbersome. Our model uses a direct/ceded/net approach, and assumes the current reinsurance program of a company will remain in place for the projection period.

Fourth, how should data be incorporated that is external to the individual company being modeled? In particular, should the concept of credibility be factored into the modeling? If so, how should that be accomplished? We submit that in a DFA exercise, the most relevant source of information is the historical financial results of the company being modeled. However, in forecasting future results for a relatively new company, or one that has new management or some other fundamental change in its recent operations, some external data will have to be used (be it financial results of peer companies or industry aggregates). This is a question we will defer to later versions of the model. In the meantime, the model assumes the data for the company being examined are fully credible.

Key Risks

In this section, we discuss some of the key risks that need to be recognized by a property-liability DFA model.

Pricing

Property-liability insurers have the opportunity to change the premium level prior to writing new or renewal business. Thus, as expenses or expected losses change, insurers can reflect these changes in the new rate levels. However, two problems can affect the ability of insurers to charge the correct price. First, since most insurance premiums are set prior to the policy being written, the insurer may incorrectly estimate future experience, causing the price to be either inadequate or excessive. Second, the freedom of insurers to set premium levels varies by state, with some states allowing relatively unrestricted pricing and other states having extensive restrictions. Thus, there are two components to pricing risk. The first component is handled in this model by having the loss ratio (exclusive of catastrophes – see next subsection) be a random variable with the mean value and standard deviation based on company experience. Loss ratios are simulated by line, with appropriate consideration given in the simulations to correlations of contemporaneous loss experience between lines. The second component of pricing risk is handled by a factor imposing a restriction on the ability of a company to make rate changes which are indicated by changes in loss frequency or severity. In our model, a factor of 1 would represent complete freedom to adjust rates in accordance with indications, while lower values are used when companies write in states with restrictive jurisdictional forces.

Catastrophes

In addition to normal pricing risk and the inherently stochastic nature of the loss process, property-liability insurers face the risk of a catastrophic loss. Hurricanes, earthquakes, winter storms, and fires all have the potential to significantly affect the financial condition of an insurer. This risk is separated out from the normal pricing risk described above. In this model, catastrophes are handled as follows, for each simulated year:

1) The number of catastrophes (by our definition, events of any type causing industry-wide losses in excess of $25 million) during the year is determined based on a Poisson distribution, with the parameter based on historical experience.
2) Each catastrophe is assigned to a specific geographical area, or "focal point," again based on historical tendencies.
3) Once assigned to a focal point, the aggregate-industry size of each catastrophe is determined, based on a lognormal distribution. The size of the event is affected by the location, as both the type of loss and the amount of insured property exposed to a loss is a function of where the catastrophe occurred. The parameters of the lognormal distribution are based on historical industry experience, appropriately adjusted to future cost levels.

4) The geographical distribution of the event by state is determined, based on a state-by-state frequency correlation matrix determined from historical patterns.
5) The loss is allocated to the company based on market share in the lines exposed to catastrophic risk.

Loss Reserving and Development

This is the major component of liability risk, and one that distinguishes, and complicates, dynamic financial analysis for property-liability insurers. The starting value used for the loss reserve in this model should be the value indicated by an analysis of the company's historical experience, not just the loss reserve stated in the latest financial report. However, even though the loss reserve is based on an actuarial analysis, it cannot be assumed to be exact – there is likely to be some random deficiency or redundancy. In addition to the stochastic nature of the loss reserve and payout processes, a complication is the correlation between loss reserve development and interest rates, since both are correlated with inflation. However, whereas the relationship between inflation and interest rates is well recognized and has been extensively documented, the relationship between inflation and loss development is much harder to quantify. Loss reserving techniques traditionally assume that past inflation rates will continue. If inflation increases over historical (or other forecasted) levels, then future loss payments are likely to exceed the amount reserved. The relationship between inflation and loss development is one area that needs additional research.

As mentioned, loss development is subject to further variability unrelated to inflation. This variability is factored into the model by a normal random variable that allows for either favorable or adverse development. The volatility parameter is selected based on the company's size and past development patterns, as well as industry considerations (however, any tendency on the part of management – or the industry -- to consistently over- or under-reserve is considered separately, i.e., in the analysis of the appropriate beginning loss reserve level). In years in which the uncertainty regarding court decisions affecting loss payments is higher than usual or when other economic conditions generate greater volatility, this additional uncertainty would be reflected by an increase in the loss development parameters. Loss reserve development may also affect rate adequacy. Significant under-reserving, in addition to impacting surplus directly, generates the need for additional rate increases that may, depending on the jurisdictional environment (as discussed below), be difficult to obtain. Also, rate increases can affect the renewal rates on business, causing an additional effect on a company's operations.

Jurisdiction

In addition to having the potential to affect the responsiveness of rates to changes in economic conditions, the jurisdictions in which a company operates impose additional risks on insurers. Residual market subsidies, retroactive premium rebates, and benefit changes on workers compensation policies already written, are all examples of jurisdictional burdens on insurers that increase the financial risk of the company. Thus, an additional, jurisdictional, risk component, dependent upon the geographical distribution of writings, is added to the model. This risk is assumed to only have the potential for a negative impact on an insurer (an insurer is not likely to be the beneficiary of a retroactive premium surcharge on former policyholders). The number of jurisdictional "events" is simulated by a Poisson distribution, with the parameter based on the characteristics of the jurisdictional environment in which the insurer operates. The size of each simulated event is determined based on a lognormal distribution.

Interest Rates

Interest rate volatility has led to a major focus on modeling interest rates by many financial institutions, including life insurers. Extremely complex models, using multifactor stochastic variables and time series relationships, have been developed. Despite the complexity of these models, and their relative accuracy in particular situations, no single model is accepted as being correct. Each model has its shortcomings and recognized deficiencies.

Interest rates are an important factor for property-liability DFA models, as they affect asset values and investment returns, and, less directly, other economic parameters. However, the ability of property-liability insurers to re-price contracts, their lower leverage, and the generally shorter maturities of fixed income securities, make it less critical that interest rates be modeled to as high a degree of accuracy as is necessary for life insurers, banks and other financial institutions.

Duration is a measure of the sensitivity of a financial instrument to interest rate changes. For instruments whose cash flows are not affected by the interest rate change, the duration can be measured as the weighted average time to receipt of the cash flow. The sensitivity of an insurer's surplus to interest rates is determined as follows:

$$D_s S = D_A A - D_L L$$

or

$$D_s = (D_A - D_L)(A/S) + D_L$$

where

D = Duration (of the subscripted quantity),
S = Surplus,
A = Assets, and
L = Liabilities.

Property-liability insurers are much less highly leveraged than life insurers. A typical asset/surplus ratio for property-liability insurers is 3/1, whereas the typical ratio for life insurers is approximately 20/1. Assuming that property-liability insurers have a duration of liabilities of 4 and life insurers have a duration of liabilities of 10, then based on the above relationship, a two year asset-liability mismatch would lead to the following: For property-liability insurers:

$$D_s = (2)(3/1) + 4 = 10$$

For life insurers:

$$D_s = (2)(20/1) + 10 = 50$$

This means that a 1% increase in interest rates would reduce the surplus of a property-liability insurer by 10%, but the surplus of a life insurer would decline by 50%. Based on this relationship, interest rate risk, while important for property-liability insurers, is not as critical as it is for life insurers. Thus, in the trade-off between simplicity and realism, the interest rate model is selected to be one more easy to work with and explain.

The interest rate process used in this model is based on the work by Cox, Ingersoll and Ross (CIR)[1] and takes the following framework:

$$dr = a(b - r)dt + s(r^{1/2})dZ$$

where:

dr = the (instantaneous) change in the interest rate level,
a = a constant that represents the speed of adjustment in interest rates,

[1] J. C. Cox, J. E. Ingersoll and S. A. Ross, "A Theory of the Term Structure of Interest Rates," *Econometrica*, 53 (1985) 385-407.

b = the long term mean interest rate level,
r = the current short-term interest rate level,
dt = an (instantaneous) unit of time,
s = the standard deviation of the random volatility measure, and
dZ = the standard normal distribution.

This model proposes that the short-term interest rate has two components, one a deterministic factor and the other a random factor. The deterministic factor, represented by the first term, is the movement from the current interest rate level toward the long term mean, with the amount of this movement set by the speed-of-adjustment factor (if this value were 1.0, then the deterministic component would cause the interest rate level to move all the way back to the long term mean). Thus, the CIR model is a "mean-reverting" model of interest rates. The other component, represented by the second term, is the random factor, which is the product of the volatility factor, the square root of the current interest rate level (to scale the moves to the current level of interest rates and prevent negative interest rates from occurring), and the standard normal variate.

The initial values for the model, based on historical data[2], are:

a = .2339
b = .0808
r_0 = .05
s = .0854

These values reflect a discretized (specifically, annual periods) version of the continuous-time CIR model. The values resulting from this approach represent our model's simulated short-term (or T-bill, or "risk-free") interest rate for each trial year. This rate, in addition to impacting bond values and investment returns, also impacts several other simulated model values, for example inflation and equity returns. In addition, interest rates appropriate for valuing longer-term government and corporate fixed income securities can be generated by allowing for a stochastic term or default premium to be added to the basic risk-free rate. (For example, historical term yield spreads on U.S. Government instruments are displayed graphically in Exhibit 5.)

Inflation

The inflation rate for each year is a random variable that is determined after the interest rate has been simulated. In our initial version of the model, the "expected" inflation rate for a given trial year is calculated by reducing the simulated annual interest rate by a constant 2 percentage points; this

[2] K. C. Chan, G. A. Karolyi, F. A. Longstaff, and A. B. Sanders, "An Empirical Comparison of Alternative Models of the Short-Term Interest Rate," *Journal of Finance*, 47 (1992) 1209-1227.

expected value, along with a volatility parameter, then act as inputs into a normal distribution from which the "actual" inflation rate for the trial year is simulated. This approach recognizes the correlation between interest rates and inflation (see, Exhibit 6), but still allows for variability around the standard inflation-interest rate differential. Once chosen, the inflation rate affects loss experience on the current book of business, on policies to be written or renewed in the future, and the loss development patterns for current reserves. It also affects the indicated rate level changes for future years.

Future versions of the model will include an enhanced module relating the inflation rate to the contemporaneous interest rate, as well as possibly past rates. In addition, there is some empirical evidence that the level of future inflation is related to current government bond term spreads (e.g., see Exhibits 7 and 8). This and other projection techniques are currently being investigated.

Market

Equities represent risky assets whose values change over time in a largely random fashion. In our model, determining the change in equity values for each insurer is a two step process. In the first step, the change in the value of the overall equity market is simulated for each trial year. This change is a function of both historical equity risk premium patterns and contemporaneous changes in interest rates. (The latter relationship exists to the extent that equities can be priced as the present value of future dividends or free cash flow. The relationship between changes in interest rates and equity values thus tends to be negative - see Exhibit 9.) Then, once the market change is selected, the insurer's equity holdings are assumed to change in line with the Capital Asset Pricing Model, based on the beta, or systematic risk, of the insurer's equity portfolio.

Default

In addition to interest rate risk, fixed income securities pose the risk of default on interest or principal. Default rates are a function of both the underlying security (in line with the ratings assigned to the debt) and economic conditions (more volatile interest rates engender a higher level of defaults). The risk of default is included in the model.

Deterministic Values

At this point, a number of other values are assumed to be deterministic, although it is recognized that they do indeed vary in practice. However, the risk imposed by these aspects of insurance operations are considered secondary to the other elements. Specifically, the following factors are assumed not to vary from expected values: expenses, reinsurance collectible, agents balances, premium growth rates, new (and first renewal) business loss ratio penalty, and the asset allocation between stocks and bonds.

The Model

The model is set up to run in an Excel spreadsheet in conjunction with @Risk, two widely used computer software packages. The program can be run with a minimal number of mandatory inputs. Standard values are used for most variables, but these values can be replaced by alternatives if the user desires. The information that is required to run the model includes:

1. Premium written (direct and ceded) by line of business, by state
    a) Total new business by line
    b) First renewal by line (policies first written in the prior year)
    c) Second and subsequent renewals by line (policies first written two or more years prior)
2. Initial loss ratio by line
3. New business loss ratio penalty by line
4. First renewal loss ratio penalty by line
5. Growth rate by line
6. Renewal rate by line (percent of policies renewed from one year to the next)
7. Expenses by line
    a) Commissions
    b) General expenses
    c) Other acquisition expenses
    d) Premium taxes
    e) Policyholder dividends
    f) Fixed expenses
8. Assets
    a) Bonds
        i) By type of bond as listed in Schedule D - Part 1A
        ii) Par, book, cost, and market value for each maturity class (under 1 year, 1-5 years,...)
    b) Preferred stock - market value - affiliated and unaffiliated
    c) Common stock - market value - affiliated and unaffiliated

d) Other assets

9. Liabilities
   a) Loss and LAE reserves by line, by accident year
   b) Unearned premium reserves by line
   c) Other liabilities

10. Surplus

Information that may be input to override the standard values:

- i) Loss payment pattern by line
- ii) Loss ratio volatility measure
- iii) Beta of equity holdings
- iv) Interest rate parameters
- v) Catastrophe parameters
- vi) Inflation parameters
- vii) Jurisdictional parameters
- viii) Tax parameters
- ix) Interest rate sensitivity of assets

Output

The model is set up to generate 1000 runs of the next five years of experience, although the number of runs can be adjusted. The five year period was selected as a compromise, on the one hand to allow the effect of changes in operations to be apparent, but not so long that the underlying forecasts become completely unreliable. The results of each run are stored to facilitate the analysis of individual runs, but the sheer number of values available requires focusing on key factors. One summary statistic is a histogram showing the final surplus value (after five years) of all the runs. An example is shown as Exhibit 10 (which is a simplistic, single-period simulation of a fictitious company whose period-ending expected surplus was $ 350 (million)). This display facilitates an overview of the risk an insurer faces, especially if viewed in the context of the proportion of times the surplus is below a selected value. Alternatively, other financial measures can be determined. For example, the premium-to-surplus ratio in the last year of the period could be determined and displayed. Another approach would be to indicate the number of times the financial ratings from one of the insurance rating agencies is reduced by one or more levels.

An additional output of the model is a distribution of the number of Insurance Regulatory Information System (IRIS) tests that the company fails in each of the five years simulated. These twelve tests are calculated from the projected balance sheets and operating statements. Companies that fail four or more tests receive a priority classification and are subject to additional regulatory scrutiny. Thus, avoiding test failures can be a reasonable management objective.

The first three tests, Gross Premium Written to Policyholders' Surplus, Net Premium Written to Policyholders' Surplus, and Change in Net Written Premium, can all be calculated directly from values obtained from the model. The fourth test, Surplus Aid to Surplus, measures the degree to which surplus is enhanced by reinsurance transactions. Calculation of this value requires the amount of reinsurance commissions and would be based on the assumption that the reinsurance program does not change over time, which may not be the case. Thus, this estimate would be more of an approximation.

The fifth test determines if the Two Year Operating Ratio is below 100 percent, which can be calculated directly from the model values. The sixth test is based on whether the Investment Yield falls within typical guidelines, currently 4.5 to 10.0 percent. Since these guidelines change with market conditions, the model incorporates a variable guideline which is set at 2.5 percent above or below the current mid-maturity U.S. Treasury bond yield. Thus, if interest rates rose significantly, an insurer that had locked in low yielding debt would be classified as failing this test.

The next test measures Change in Policyholders' Surplus, which is determined directly from the model results. The eighth test calculates Liabilities to Liquid Assets and has as a failing value of 105 percent. Both of these quantities are determined by the model, and so the ratio can be calculated directly within the model. The ninth test is Agents' Balances to Policyholders' Surplus, which is calculated from the model output.

The last three tests are based on loss reserve adequacy: measures of One Year Loss Development, Two Year Loss Development, and the Current Estimated Deficiency, all as a percentage of Policyholders' Surplus. These calculations come out of the model results, but also require information about the carried loss and LAE reserves of the insurer, since the loss reserves input into the model are those indicated based on an actuarial analysis. Some insurers consistently over- or under-reserve, and these policies would impact the results of these tests.

The output of the IRIS tests would be a histogram indicating the proportion of the runs versus the number of failed tests, and this is provided for the current year and each of the five forecasted years.

Problem Areas

There are several areas involved with this model that require additional work and consideration. First, current tax provisions relating to the property-liability insurance industry are extremely complex. Investment income is generally taxed at a lower rate than underwriting income, reflecting capital gains, dividends, and investments in municipal bonds that are taxed at a lower rate, and that rate depends on when the securities were purchased. In addition, insurers are subject to the alternative minimum tax provisions and can have tax loss carry-forward and carry-back positions.

Our model uses the standard graduated tax rate schedule applied to total operating income to determine the tax liability. The user can adjust this calculation to apply different tax rates for underwriting and investment income. This would allow the taxation of investments in tax-favored instruments, such as municipal bonds or dividend-paying stocks, to be reflected more accurately. In order to keep the required input to the program to a manageable level, however, this model does not attempt to perform an exact tax calculation. There is proprietary software available that can be used to calculate taxes accurately, but this software must be purchased and is not available to be included in the public access model.

Second, each reinsurance program is unique, and reflecting the full effect of reinsurance would require tailoring the model specifically to each insurer. For certain types of reinsurance – e.g., catastrophe covers and quota shares – our projections of direct losses can be brought to a net basis in a straightforward manner. However, for other types of reinsurance – e.g., working excess covers – the adjustment from direct to net aggregate annual losses is much more difficult. In our model, the "net loss ratio risk" of the insurer with regard to such "problematic" reinsurance covers is selected to be one of three levels, depending on the combined effect of the size and stability of the direct business and the general characteristics of the reinsurance program. To run the model, the user need only specify whether the low, standard or high values should be applied. For a user that has a better concept of the underlying risk parameters for net losses, these values can be changed.

Standard fixed income securities, such as noncallable bonds, involve a cash flow stream that is not dependent on the level of interest rates. This type of investment represents a significant portion of the assets of most insurance companies. The change in value of these securities in relation to interest rate changes can be calculated straightforwardly. However, there are other types of assets, such as callable bonds and mortgage-related securities (including collateralized mortgage obligations), that are much more complicated. Our initial model is set up to deal with standard bonds by calculating an approximate duration from the input data. Applications involving more complex assets require the user to separately input values for interest rate sensitivity.

Future versions of our model will attempt to more precisely value the callability options of corporate bonds, as well as mortgage prepayment considerations underlying mortgage-related securities. The latter type of asset is becoming important to the property-liability insurance industry (see Exhibits 11 and 12). The prepayment rate is theoretically a function of one or more of the interest rate-mortgage coupon rate differential, the age distribution of the underlying mortgages, the characteristics of the underlying mortgage holders, the season, and the geographical distribution of the mortgaged real estate. A common industry model of the mortgage prepayment rate is the Public Securities Association (PSA) model; we are considering incorporating this approach to valuing mortgage-related securities into our model.

Using the Model

One of the most important questions relating to dynamic financial analysis is, "How do you use it?" Although relegated to a final section in work describing DFA, this question really needs to be the first thing decided before beginning a DFA project. The intended use of the model dictates the structure and content of the model.

A variety of different uses of DFA exist. One is to measure the likelihood of a company's insolvency given current operations. Fortunately, for most insurers, this value is very low, and it is hard to judge the significance of the difference between a situation that indicates the company will be insolvent 1 or 2 times in 1000 runs. However, the model allows management to examine the parameter values in the cases where the company did have financial problems in order to see if steps should be taken to reduce this risk even further.

A more widespread use of a DFA model is to examine the financial effect of different management strategies. Looking at both the range and probable outcomes from specific management decisions, such as expansion into a new line of business, withdrawing from a state or hedging investments, can provide valuable information about the potential impact of these strategies. Thus, DFA can be a useful planning tool. To facilitate this type of analysis, the model allows the user to set the volatility parameters to zero, so the model produces expected values of different strategies. The advantage of this ability is that it removes the stochastic features of the model so the outcome is not influenced by random fluctuations, and it allows the user to focus directly on the impact of the specific strategy in question.

In terms of practical issues regarding its use, our model will be publicly available via computer disk, or by accessing web pages affiliated with the authors. As mentioned before, the model utilizes Excel and @Risk spreadsheet software. Data, as specified in the section "The Model" above, is input by the user on a general input sheet. The user will also have the capability of changing the default values for a variety of variables, allowing for flexibility in modeling many different corporate or economic environments.

Conclusion

Dynamic financial analysis is becoming one of the skills casualty actuaries will need to possess in the near future. By developing a basic DFA model that can be used to understand this technique, and making the model widely available to facilitate discussion and improvement, we hope to help practitioners enhance this skill and researchers develop better models. Dynamic financial analysis has the potential to provide insurers with an opportunity to assess their risk, examine alternative strategies, and develop effective risk management approaches. Hopefully, this work will foster improvements in this field.

Appendix

Summary of DFA Model Stochastic Variables

This Appendix summarizes some of the key variables appropriate for a DFA model. The summaries of each variable below give the following information:

a) Description of the variable
b) Reason for inclusion in a DFA model
c) Some possible sources for data regarding the variable
d) Analytical approach to the variable

More extensive descriptions of several of these variables are included in the "Key Risks" section of this paper.

Asset and Investment Risks

1. Short-Term Interest Rate
   a) Rate on U.S. treasury bills (e.g., one month maturity)
   b) Short-term rates are correlated with many other financial variables (e.g., inflation, returns on other assets, insurance pricing); in our model, many other stochastic variables are simulated partly as a function of this process. In addition, T-bill market values are a function of the short-term rate
   c) Sources: Ibbotson Associates[3], Federal Reserve Bank of St. Louis FRED Database (e.g., via CAS DFA web page)
   d) Cox, Ingersoll, Ross mean-reverting short-term interest rate model

2. Term Premium
   a) Premium added onto short-term rate to longer-term U.S. government instrument rates
   b) Affects longer-maturity government bond market values
   c) Sources: Ibbotson Associates, Federal Reserve Bank of St. Louis FRED Database (e.g., via CAS DFA web page)
   d) Modeled via a statistical distribution with mean and volatility parameters derived from the time series of historical term premium values, and their relationship to the short-term rate

3. Default Premium

[3] Ibbotson Associates, *Stocks, Bonds, Bills, and Inflation: 1996 Yearbook*, 1996, Ibbotson Associates, Inc.

a) Premium added onto Treasury rates to get corporate bond rates
b) Affects corporate bond market values
c) Source: Ibbotson Associates
d) Modeled via a statistical distribution with mean and volatility parameters derived from the time series of historical default premium values, and their relationship to the short-term rate

4. Default Risk
   a) Likelihood of default by issuer of fixed-income securities
   b) Value of corporate bonds is reduced by default on interest or principal
   c) Sources: Academic studies[4]
   d) A function of the underlying securities (e.g., as reflected by debt ratings) and general economic conditions (e.g., interest rates)

5. Equity (Market) Premium
   a) Premium added onto short-term rate to get return on equity market
   b) Affects market value of equity portfolio
   c) Sources: Ibbotson Associates, Center for Research in Security Prices (CRSP), stock market links on CAS DFA web page
   d) Change in the equity market is a function of historical equity risk premium patterns and contemporaneous changes in interest rates

6. Prepayment Risk
   a) Risk of prepayment by mortgage holders
   b) Affects the market value of mortgaged-backed securities
   c) Sources: Industry and academic studies
   d) Modeled by Public Securities Association (PSA) model

Liability Risks

1. Loss Payout Pattern
   a) Percentage of ultimate losses paid in each calendar quarter
   b) The loss payout process is inherently stochastic, and impacts operating results each year through its effect on the payout of claims
   c) Sources: Individual company data, supplemented by industry statistics

[4] For example: Edward I. Altman, "Measuring Corporate Bond Mortality and Performance," *Journal of Finance*, 44 (1989) 909-922

d) A combination of two stochastic variables. The first variable is the expected historical claim payment pattern, and is modeled using a Beta distribution with parameters developed from company and industry historical experience. This variable is then combined with claims inflation (as described below) to develop calendar year payments and, in turn, estimates of reserves.

2. Loss Reserve Development (Redundancy / Deficiency)
   a) Redundancy or deficiency in the beginning loss reserve
   b) The loss process is inherently stochastic, and thus impacts operating results each year through the level of claim liabilities incurred
   c) Sources: Individual company and industry statistics
   d) Modeled via a statistical distribution with mean and volatility parameters based on company size and historical company and/or industry experience

3. Inflation
   a) Both general economic (CPI) and insurance claims inflation
   b) Affects future values of liabilities (e.g., claims payouts)
   c) Sources: Ibbotson Associates, Masterson indices (published in *Best's Review*), Federal Reserve Bank of St. Louis FRED Database (e.g., via CAS DFA web page)
   d) Future inflation a function of contemporaneous interest rates and current yield spreads; also some autoregressive properties

Underwriting Risks

1. Pricing - Rate per Exposure
   a) Premium charged per unit of exposure
   b) Affects level of premium income received
   c) Sources: Historical company and industry experience
   d) Modeled by taking into consideration the company's current average rate and the effect of interest rates, inflation rates, and the underwriting cycle. The process is similar to a prospective rate level indication.

2. Pricing - Exposures
   a) Underlying quantity of insurance sold
   b) Affects level of premium income received
   c) Sources: Various insurance industry statistics; general economic (e.g., interest rate, inflation, output, and employment)

conditions also affect exposure levels (data available, for example, via CAS DFA web page)

d) A function of rate level and general economic conditions

3. Catastrophes
   a) Risk of large losses (i.e., greater than $ 25 million on an aggregate industry basis)
   b) Impacts claims costs, and thus operating results
   c) Sources: Property Claims Services division of American Insurance Services Group, Inc. (e.g., via Insurance Information Institute[5])
   d) Five-step simulation process – see "Key Risks" section

4. Jurisdiction
   a) Insurance risk unique to geographical location – e.g., residual market subsidies, legislative / judicial / regulatory environment, level of competition
   b) Impacts operating results
   c) Sources: Aggregate insurance industry data by line by state
   d) Simulate potential variability in loss ratios that are a function of a company's distribution of business by geographic location and line of business

5. *a priori* (or Underlying) Loss Ratio
   a) Ratio of losses and allocated loss adjustment expenses to premium
   b) The loss process is inherently stochastic, and thus impacts operating results each year through the level of claim liabilities incurred
   c) Sources: Company experience adjusted for changes in rate levels, new business premium writings, and catastrophes
   d) Modeled as a function of prior years' loss ratios and inherent variability due to internal and external influences

[5] Insurance Information Institute, *1997 Fact Book: Property/Casualty Insurance Facts*, 1996, Insurance Information Institute

DISTRIBUTION OF BONDS BY TYPE

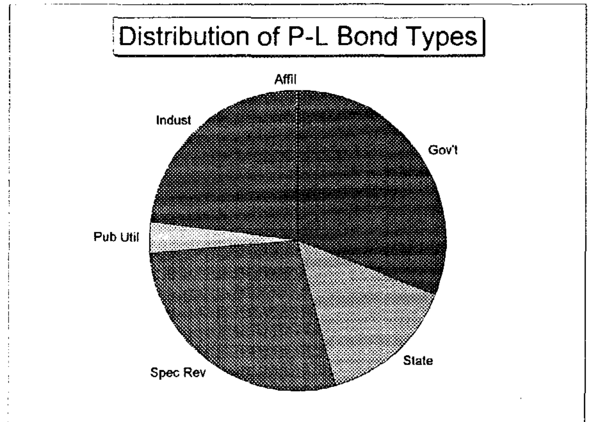
Exhibit 2

Consolidated Property-Liability Insurance Industry Totals
As Of December 31, 1995

Data Per Best's Aggregates & Averages (1996)

In Billions of Dollars

Bond Type	*Statement* Value	% of Total
Governments	154.6	30.9%
States, Territories, and Possessions	74.9	15.0%
Special Revenue	139.0	27.8%
Public Utilities	16.9	3.4%
Industrial and Miscellaneous	114.2	22.8%
Parents, Subsidiaries, and Affiliates	1.0	0.2%
	=======	=======
Total Bond Holdings	500.6	100.0%



DISTRIBUTION OF COMPANY ASSETS

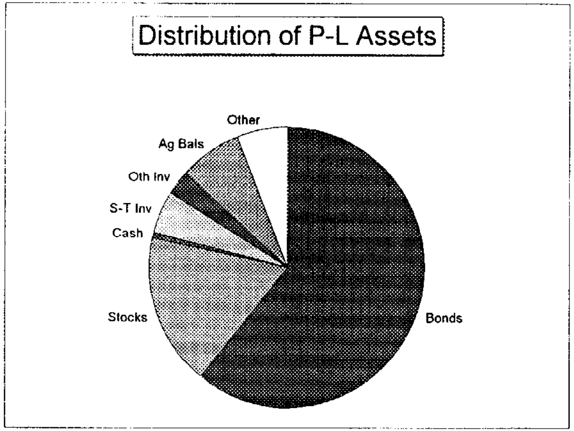
Exhibit 1

Consolidated Property-Liability Insurance Industry Totals
As Of December 31, 1995

Data Per Best's Aggregates & Averages (1996)

In Billions of Dollars

Asset Item	Statement Value	% of Total
Bonds	465.5	60.8%
Stocks	134.1	17.5%
Cash	4.9	0.6%
Short-Term Investments	37.5	4.9%
Other Invested Assets	22.0	2.9%
	----------	----------
Total Invested Assets	664.0	86.8%
Agents' Balances or Uncollected Premiums	55.4	7.2%
Other Assets	45.8	6.0%
	=======	=======
Total Assets	765.2	100.0%



DISTRIBUTION OF BONDS BY MATURITY

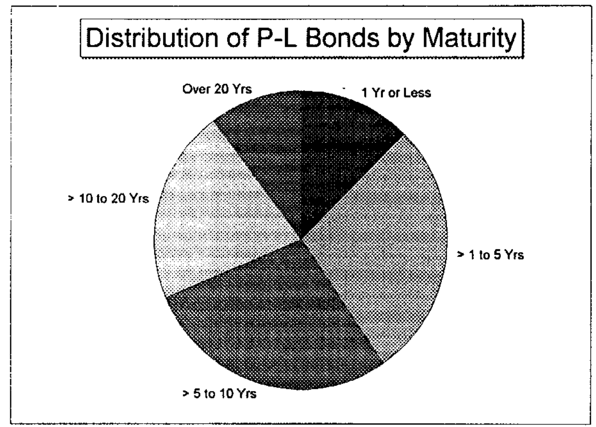
Exhibit 3

Consolidated Property-Liability Insurance Industry Totals
As Of December 31, 1995

Data Per Best's Aggregates & Averages (1996)

In Billions of Dollars

Bond Maturity	Statement Value	% of Total
1 Year or Less	62.2	12.4%
Over 1 Year, Through 5 Years	140.5	28.1%
Over 5 Years, Through 10 Years	140.9	28.1%
Over 10 Years, Through 20 Years	105.8	21.1%
Over 20 Years	51.2	10.2%
	=======	=======
Total Bond Holdings	500.6	100.0%



DISTRIBUTION OF COMPANY LIABILITIES AND SURPLUS

Exhibit 4

Consolidated Property-Liability Insurance Industry Totals
As Of December 31, 1995

Data Per Best's Aggregates & Averages (1996)

Statutory Values In Billions of Dollars

Liability or Surplus Item	Statement Value	% of Total L&S
Losses	298.9	39.1%
Loss Adjustment Expenses	62.0	8.1%
Unearned Premiums	103.9	13.6%
Other Liabilities	70.4	9.2%
	----------	----------
Total Liabilities	535.2	69.9%
Policyholders' Surplus	230.0	30.1%
	=======	=======
Total Liabilities and Surplus	765.2	100.0%

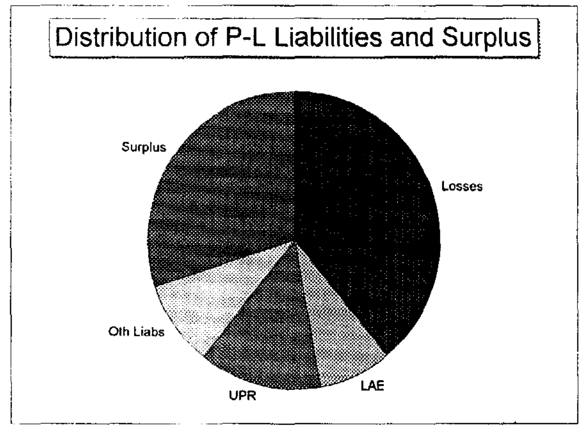
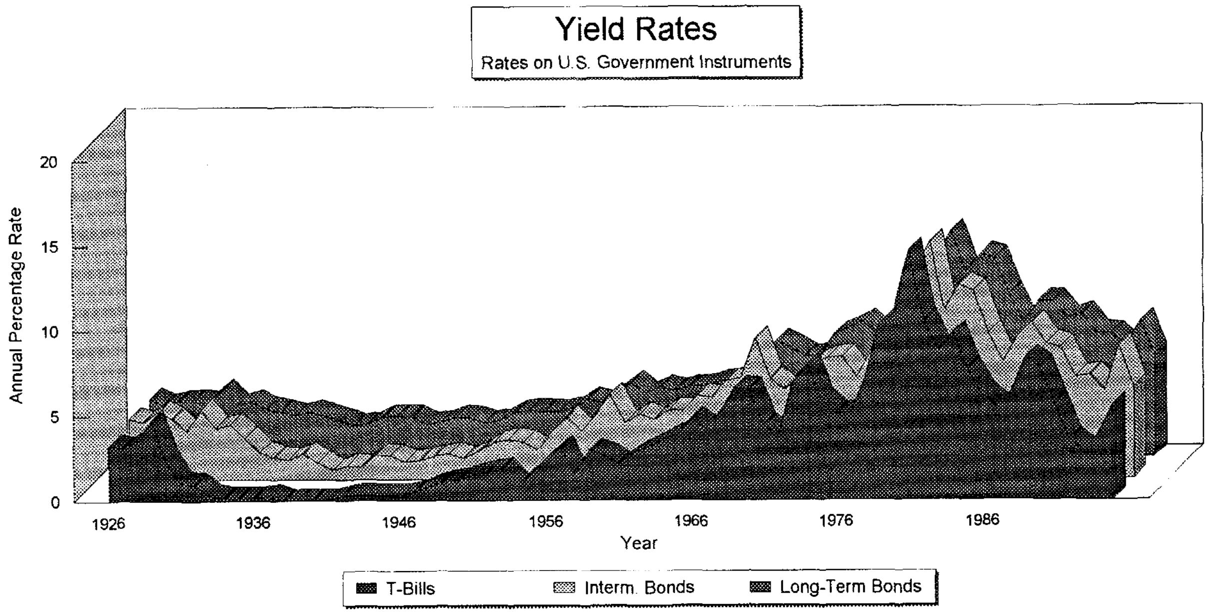
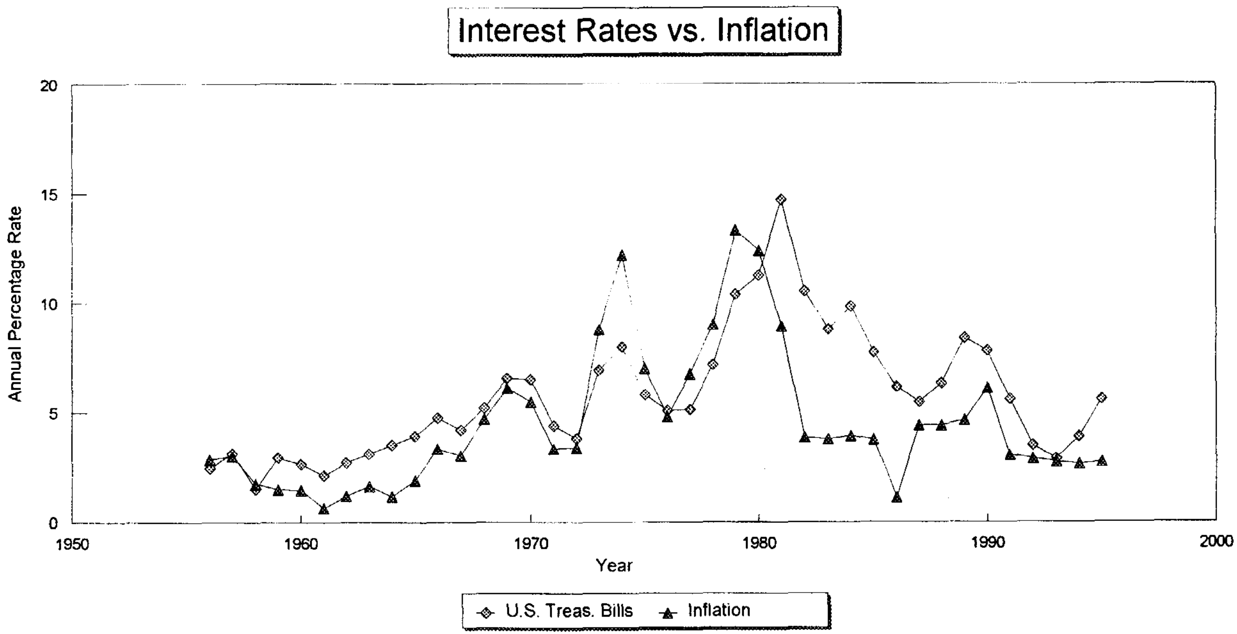


Exhibit 5



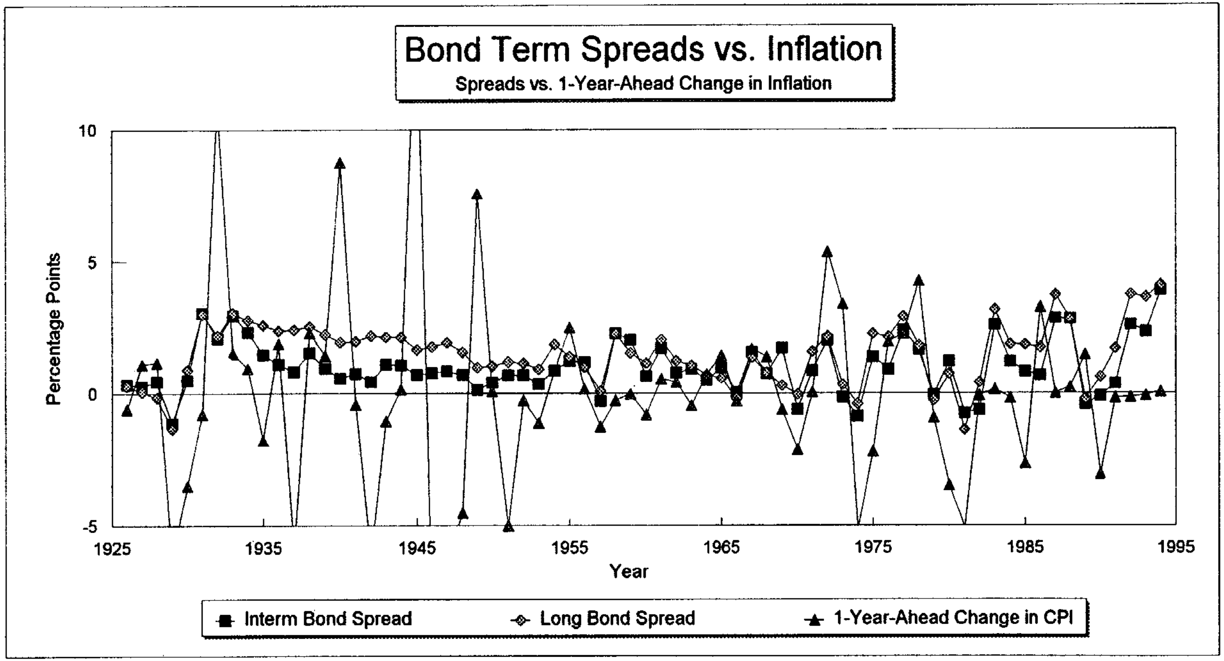
Data per Ibbotson Associates, *Stocks, Bonds, Bills, and Inflation 1996 Yearbook*

Exhibit 6



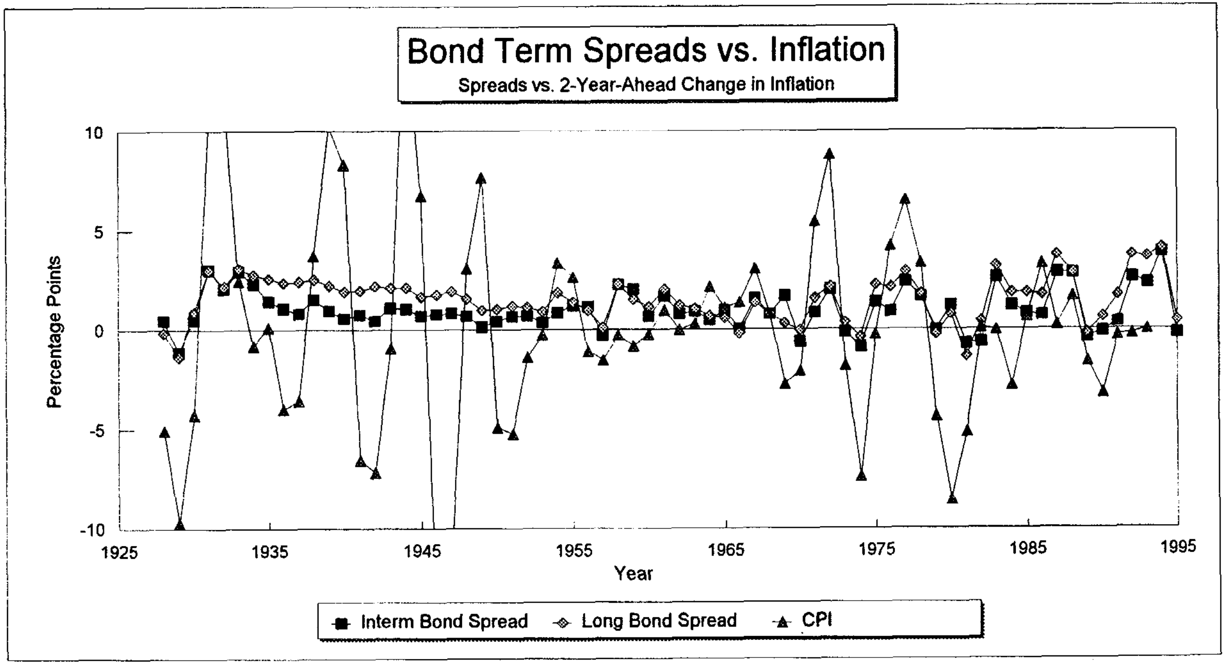
Data per Ibbotson Associates, *Stocks, Bonds, Bills, and Inflation 1996 Yearbook*

Exhibit 7



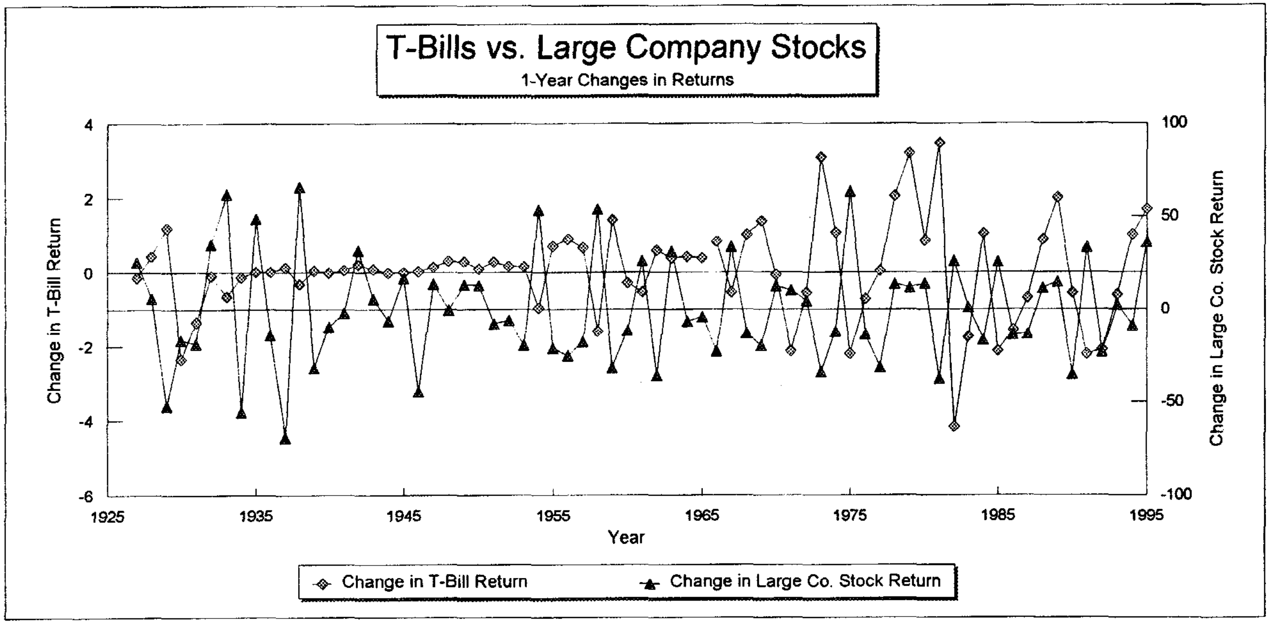
Data per Ibbotson Associates, *Stocks, Bonds, Bills, and Inflation 1996 Yearbook*

Exhibit 8



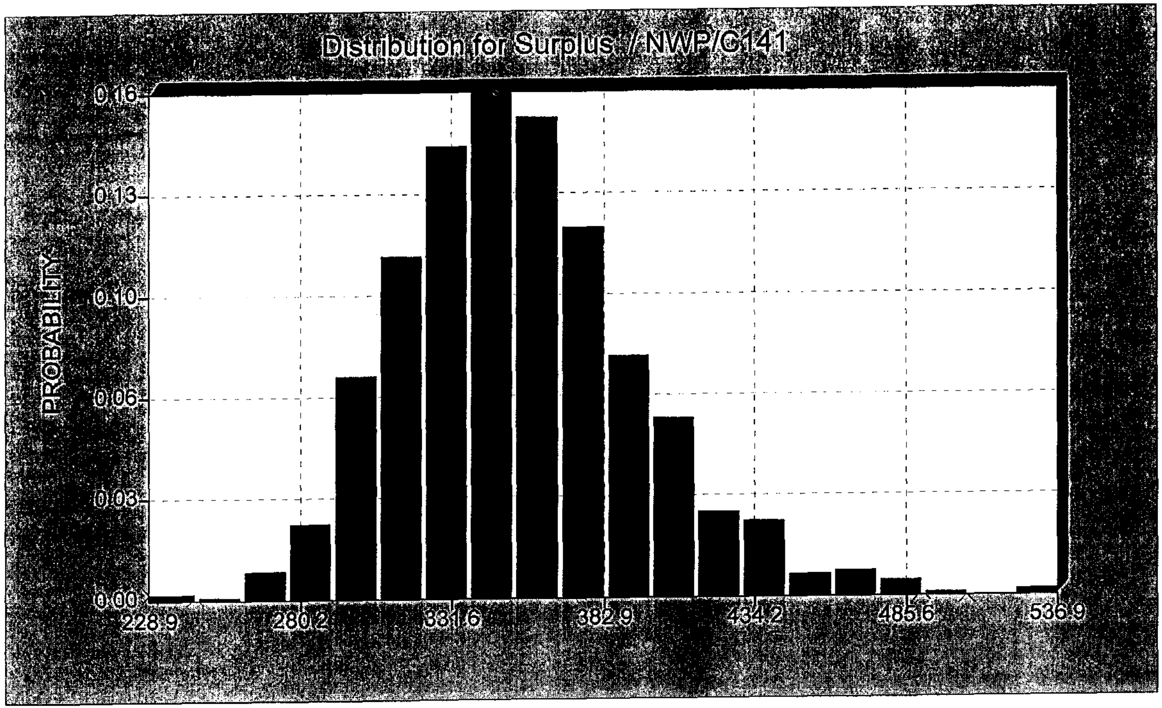
Data per Ibbotson Associates, *Stocks, Bonds, Bills, and Inflation 1996 Yearbook*

Exhibit 9



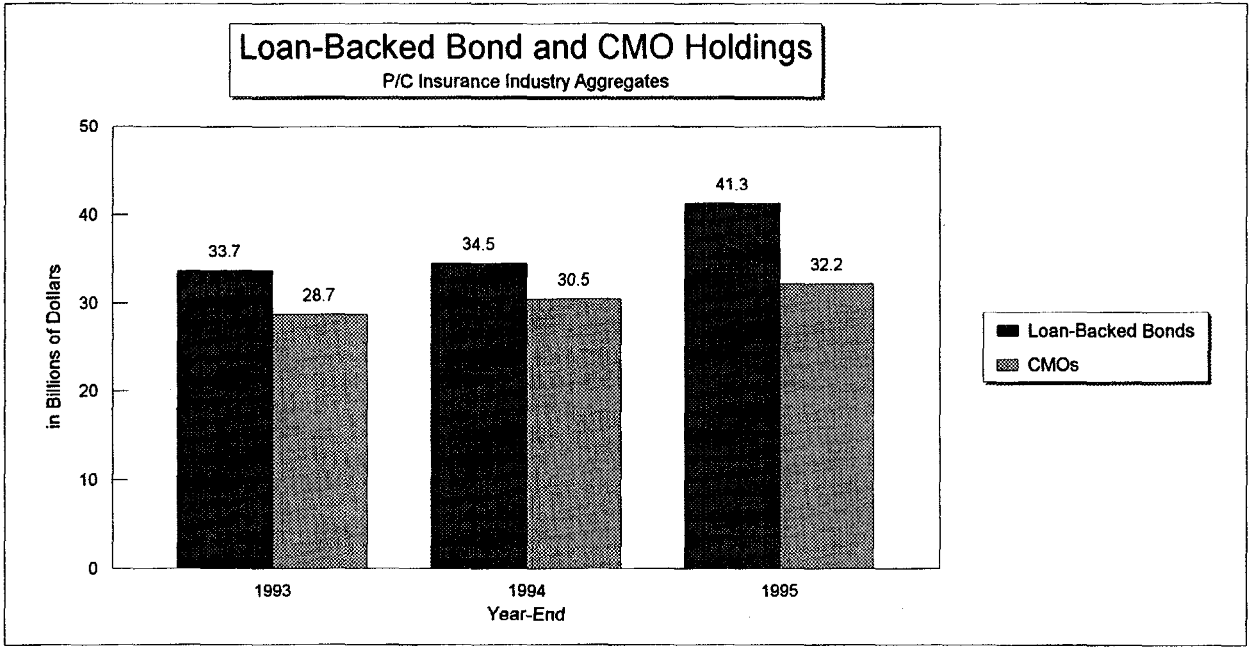
Data per Ibbotson Associates, *Stocks, Bonds, Bills, and Inflation 1996 Yearbook*

Exhibit 10



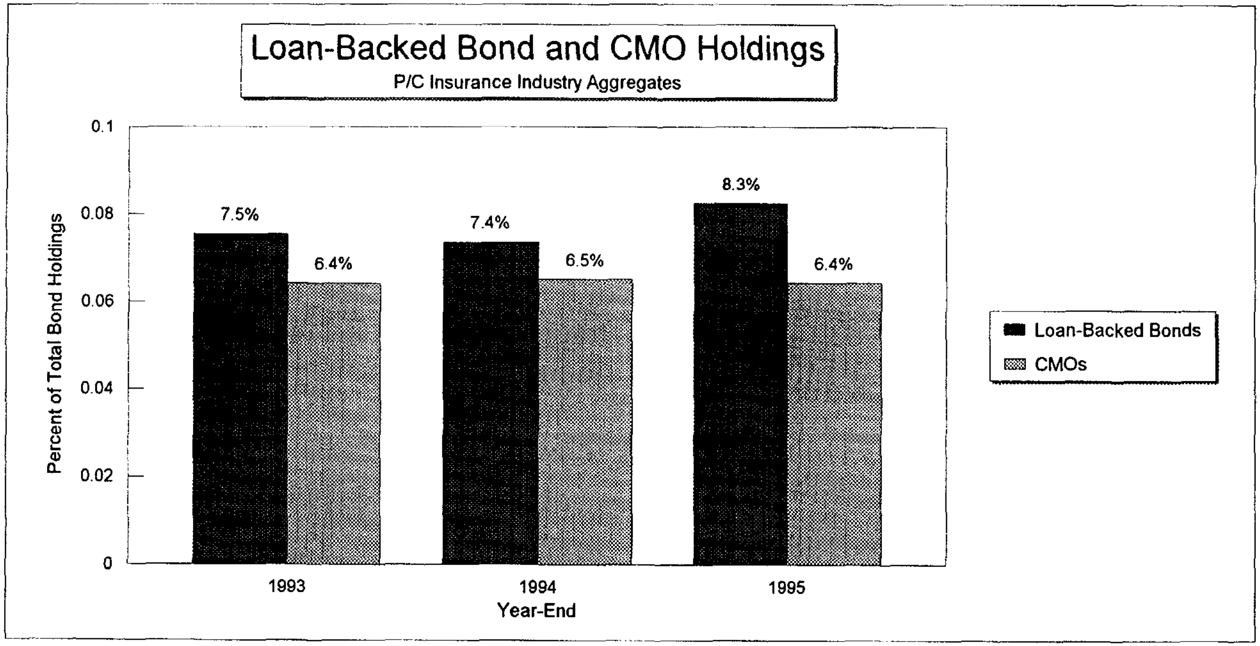
Distribution for Surplus / NWP/C141
PROBABILITY
0.16
0.13
0.10
0.06
0.03
0.00
228.9
280.2
331.6
382.9
434.2
485.6
536.9

Exhibit 11



Data per Best's Aggregates & Averages (1996)

Exhibit 12



Data per Best's Aggregates & Averages (1996)

Specifying the Functional Parameters of a Corporate Financial Model for Dynamic Financial Analysis

by Gerald S. Kirschner, FCAS, MAAA,
and
William C. Scheel, Ph.D., AIAF, ARM, CEBS, CPCU

Specifying the Functional Parameters of a Corporate Financial Model for Dynamic Financial Analysis

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Ernst & Young LLP

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SS&C Technologies, Inc.

Specifying the Functional Parameters of a Corporate Financial Model for Dynamic Financial Analysis

Gerald S. Kirschner
William C. Scheel

Abstract

When people speak of parameterizing a model, whether it be for dynamic financial analysis or otherwise, they typically discuss the ranges of values that key model elements can assume. In our paper we have broadened the concept of parameterization to include the functionality a model needs to contain in order to perform the required task. Our concept of parameterization, therefore, encompasses both the narrower definition of defining ranges of possible values for key model elements and the broader definition of describing what needs to be included in the model's design in order for it to function properly. To that end, in Section I the paper describes a model currently being used to develop property/casualty insurance company pro-forma financial statements in a dynamic modeling framework. In Section II the paper lists the key elements of variability within the modeling framework, i.e. those parameters that need to be described through probability statements rather than fixed values. Section III returns to the narrower definition of parameterization and provides some commentary regarding our experiences in developing the specific ranges of values for each of the items listed in Section II.

Biographies

Gerald S. Kirschner, FCAS, MAAA has been a consulting actuary for Ernst & Young LLP's National Actuarial Services Group since 1994. He is based in Atlanta. His primary responsibility at Ernst & Young has been to jointly develop with SS&C Technologies, Inc. a dynamic financial analysis (DFA) model for property-casualty insurance companies. Prior to joining the firm, he was employed by Aetna Life & Casualty. He is a member of the CAS Dynamic Financial Analysis Subcommittee on Models.

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Introduction

There are many facets to the concept of model parameterization. It is useful to begin with an analogy to the common actuarial problem of distinguishing between specification, parameter, and process risks. Specification risk relates to the questions "Are the model structure and the selected probability distributions correct?" Parameter risk narrows the question to "Assuming the specification is correct, are the distributional parameters correct?" Lastly, process risk is concerned with randomness, i.e. answering the question "Assuming everything else is correct, what can happen in my universe of possible outcomes?"

One might quibble between modeling loss severity with a Weibull distribution instead of a Lognormal distribution. Ferreting among the universe of possible probability distributions in model design is coping with *specification* risk. Even when this exercise is completed successfully, the student pursing this investigation still must deal with describing the parameters of the chosen process model. This second stage investigation is an exercise in *parameter* risk. The risk to the model designer is ending up choosing the wrong probability distribution or the wrong parameters. In an ideal world, the final risk, *process* risk, disappears under the weight of many, many recalculations of the model. In the real world, there could be overlooked correlations or unseen model overspecification, or combinations of the two that do not allow process risk to drop from the overall equation. Moreover, there is a corollary to this uncertainty. The specification risk may degenerate into subjective probability assessment—the knowledge set about the dynamic process may be so sparse that even a doctorate in statistics is no consolation.[1]

In financial modeling, there are many of these "risks", and the model designer should not be oblivious to them. Collectively, they constitute what we mean by parameterization problems associated with model design. The purpose of a dynamic financial model is to obtain and compare probability distributions for functions of random variables. Depending on point of view and the purpose attached to the modeling exercise, there are many risks associated with rendering these important goal or metric variables.

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[1] The mathematics describing the fitting of distributions with only sparse knowledge of the underlying risk characteristics is described in "Converting Experts' Knowledge into Dynamic Variable Distributions for Monte Carlo Simulation" by Euguene L. Filshtein in Contingencies, January/February 1996.

The model designer must leap many hurdles while formulating a corporate financial model, particularly one for dynamic financial analysis (DFA) Examples of hurdles to be overcome or pitfalls to be avoided include:

1. The model can use the wrong algebra when attempting to define causality or linkages among model constants and variables, i.e., the wrong model
2. Important components of the operational or economic environment might be omitted so that the model behavior is mischievous
3. Elements that more appropriately should be rendered in a dynamic manner are kept static
4. Model designers can be consumed by uncertainty regarding the dynamic behavior of those components deemed to be dynamic
5. The model's accounting framework may be inaccurate
6. The model could contain programming problems or other embedded and unknown deviant behavior
7. It might not be possible to achieve a consensus among decision makers about the metrics (i.e. output results) of comparison
8. Model results may not exhibit stochastic dominance[2] between different strategies under investigation
9. Model results cannot be implemented (i.e. the decision path that leads to the "best" long-term outcome is not feasible, either because it violates internal management operating constraints or regulatory boundaries).

In summary, the parameterization risks include functional mis-specification of the model, commission and omission errors in risk and process identification and failure of the accounting framework to adequately divulge the metrics needed for decision making.

In each of these looms a different dragon. Let us begin with a disclaimer to all readers who hope to find an easy recipe for defining the parameters. There is no magic bullet for alleviating either model, functional or dynamic variable mis-specification. Very often, there is not even a good place to start looking for a definition. With that in mind, we believe that (a) a definition that describes the event in question is better than no definition at all and (b) it is not worth quibbling over the finer points of parameter specification - in the overall perspective of what we are trying to model, the error introduced by using a Weibull instead of a Lognormal distribution to fit empirical claims severity data is not going to make or break our results.

[2] Stochastic dominance attempts to answer questions of choice among risky alternatives in a utility-theoretic framework – but one in which only certain limited information is known about the utility function of the decision maker. The idea of stochastic dominance is discussed in Exhibit 1. This Exhibit displays both an example of stochastic dominance and a user-defined "metric".

These thoughts are pursued in greater detail in Appendix A: Fundamentals of Dynamic Financial Analysis. We now turn to the two key concepts that form the basis for this paper:

- The model to be discussed is a *corporate financial model*, one that already has been deployed in the marketplace
- The model is *dynamic*.

By focusing on these key concepts, this paper will present:

1. An example of a model that has been built to perform dynamic financial analysis at a corporate level
2. What we have found to be some of the key parameters and model specifications that need to be described probabilistically
3. Approaches we have taken to develop specific ranges of possible values for the key parameters and model specifications.

Key Concepts

Corporate Financial Model

Day-to-day operations of a property-casualty insurance company include buying and selling assets, underwriting business, collecting premiums, administering claims and incurring the fixed costs related to running the insurance enterprise. A financial model of a property-casualty insurance enterprise needs to be able to model each of these operations separately and in conjunction with each other in order to produce realistic financial projections of the complete entity.

In order to perform a comprehensive dynamic financial analysis, a corporate financial model should have linkages and interrelationships between activity on the asset and liability sides of the business. For example, the model should:

- apply the same macroeconomic environmental conditions (i.e. interest rates, inflation rates, catastrophic events) across *all* aspects of the company
- allow investment decisions to be made after consideration of both operating needs and investment opportunities in the financial markets
- look at the risk/return tradeoffs generated by both investment and operating decisions in the context of the entire company's risk/return spectrum rather than in isolation
- provide a universal set of metrics or decision criteria by which multi-faceted company operations can be measured and managed.

These critical model components are couched in terms of one or more accounting frameworks (i.e. statutory, GAAP or economic). The accounting mechanisms serve to organize the model's projected results into a readily understood and consistent financial structure.

Dynamic vs. Static Corporate Financial Modeling

The purpose of a corporate financial model is to help company management understand how decisions made today affect the company's financial well-being tomorrow. Traditionally, corporate financial modeling has relied on static evaluations of current and future events and predetermined cause and effect relationships. Unfortunately, with static analysis, there is at best a limited ability to appreciate the sensitivity of bottom line results to changes in input variables, especially if the number of input variables is large and the interrelationships among them is complex. Yet it is critical that strategic decisions be made with the understanding of how each decision impacts the preceding ones, or how changes in the internal or external environment can alter the anticipated outcomes arising from each decision.

The essence of dynamic financial modeling is the ability to describe critical assumptions in terms of ranges of possible outcomes, rather than in terms of fixed values. Once each critical assumption is defined by a range of possible outcomes and the interrelationships among critical assumptions are mapped out, a series of model recalculations can be performed to develop ranges of results we can reasonably expect to see. The parameters used to model dynamic variables and the accounting interrelationships ultimately define the key criteria or metric variables that are of interest to management, regulators and stockholders. Differences in financial results arising from alternative strategic decisions can be evaluated by replacing one set of strategic decisions with another, re-running the modeling exercise and comparing the ranges of possible outcomes under each decision rule set.

SECTION I: MODEL STRUCTURE

The corporate financial model has been developed to include a minimum of one year of actual results and to produce pro-forma financial projections for an additional five years. For the purposes of simplification throughout the remainder of the article, it is assumed the actual results are valued as of December 31, 1996 and the projection period encompasses the years 1997-2001.

The corporate financial model has five distinct sections: invested assets, underwriting, accounting structure, tax calculations, and financial ratios.

Underwriting section

The underwriting section performs seven basic tasks:

1. It converts held loss and allocated loss adjustment expense (ALAE) reserves into calendar year payouts.
2. It converts indicated redundancies or deficiencies in held loss reserves into calendar year payouts and captures the accounting impacts of reserve redundancy or deficiency emergence. Reserve redundancies or deficiencies can arise either from variability in the held reserves (i.e. the held reserves represent the best estimate of ultimate losses, but actual loss emergence might vary in some range around the best estimate), or from deliberately holding reserves at a level other than the best estimate.
3. It calculates the inflationary impact on loss payments arising from differences between a simulated future level of inflation and a level of inflation that was implicitly (or explicitly) assumed when the held reserve level was established.
4. It allows the emergence of reserve redundancies or deficiencies into the model's accounting results to be scheduled at the same rate or faster than the redundancies or deficiencies emerge into the model's cash flows.
5. It calculates any additional premium inflows that might be derived from policies already written (i.e. audit premium, premium from retrospectively rated policies) and earns premiums on in-force and new business according to a user-defined premium earning pattern.
6. It calculates tax discounted loss reserve levels for federal income tax calculations.
7. It provides the vehicle for entering a five year underwriting plan, including future premium inflows and associated loss and variable expense outflows at a line of business level of detail. (Only variable expenses are included in the line of business section. Fixed expenses are addressed in a different section of the model.)

An example of the inter-relationship between the payout of held reserves, indicated reserve redundancy/deficiency emergence and inflationary impacts are shown in Exhibit 2.

Each line of business requires inputs—many of them, such as production volume, can be tied to economic activity that also is simulated within the model. Some of the important input items are:

- Premium volume projections for the 1997-2001 period on a direct/assumed/ceded basis
- Loss ratio projections associated with the premiums to be written between 1997 and 2001
- Variable expense projections for the future business writings
- Reserve payout patterns for loss and ALAE

- Premium earning patterns
- IRS tax discount factors applicable to held reserves, both historical and future
- Risk-based capital loss and premium factors.

Asset structure

Investment generation tracking

Assets are organized into investment year cohorts that correspond to the year in which the investments were purchased. All of the assets owned by the company at December 31, 1996 are combined into one investment year cohort. The investments purchased in 1997 will be a second cohort, the investments purchased in 1998 a third cohort, and so on. The changes in asset valuations of each investment year cohort reflect the interest rate environment projected to occur. The magnitude of a new investment year cohort is determined by many factors including asset allocation strategy, cash flow, and the operational and econometric environment at the time the investment year cohort is purchased.

The investment year cohort structure is needed to differentiate between assets purchased under different interest rate environments. The interest rate environment at the end of 1997 will most likely differ from that at the end of 1998, therefore, the characteristics of the assets purchased at the end of 1997 will most likely differ from those purchased at the end of 1998. For example, if interest rates are higher at December 31, 1997 than December 31, 1998, bonds purchased in 1997 will have higher coupon rates than those of the same time to maturity purchased in 1998. If the 1997 purchases were not maintained in a separate cohort from the 1998 purchases, the differences in their coupon rates would be lost.

Asset categories

Assets are subdivided into a number of homogenous groups for modeling. For simplicity, the structure displayed in this paper follows a statutory annual statement format. Bonds are divided into taxable and a tax-exempt groups, and further subdivided by maturity according to the divisions in Schedule D of the annual statement. Collateralized mortgage obligations can either be left in the standard bond groupings or separated into their own group. The other asset categories include preferred and common stocks, mortgage loans on real estate, real estate, cash, short term investments and other invested assets.

For each asset class and cohort, information about par, book and market values are retained. This allows the model to recalculate market values for each asset class/cohort combination based on changes in the interest rate environment from when it was purchased. The determination of unrealized gains and losses as well as various cash and accrual income effects also is enabled.

Asset rebalancing

The investment of operations and investment cash flow is done at year end in the model. Average cash balances from insurance operations are deemed to be invested at the short-term yield[3] until the end of the year when all sources of cash are combined with the market values of assets and tested for rebalancing. Depending on the rebalancing strategy, some existing assets may be sold and the pool of new money reinvested to produce approximately the proportions dictated by the strategy. The final allocations are subject to modification attributable to year-end closing transactions, primarily tax effects. The rebalancing can create capital gains or losses which are combined with operating results to determine the federal income tax liability for the year.

Asset - liability interrelationship

The model interrelates assets and liabilities in two ways. First, the amount of money available for reinvestment at any point in time is directly related to the underwriting cash flows. A severe underwriting shock such as a catastrophe will force much greater loss outflows than anticipated, with a corresponding need to liquidate assets. Second, the interest rate environment affecting asset market values is linked into the liability cash flow profiles.

This linkage is important because through it the model can stress-test the overall company financials in a variety of ways. For example, a scenario might evolve in which high interest rates with corresponding high inflation rates and an underwriting shock simultaneously occur. The high interest rates depress the market value of the bond portfolio at the same time the high inflation rate and underwriting shock are raising the calendar year loss outflows above the expected loss outflow level.

Accounting structure

The model includes both statutory and GAAP accounting structures. The accounting calculations begins with the statutory structure and applies a series of statutory to GAAP adjustments[4] to derive GAAP financials. The statutory to GAAP adjustments currently include:

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[3] By investing average cash balances at the short-term yield, the model calculates investment income earned on the average cash balance during the year.

[4] As statutory and GAAP accounting rules change, the model will need to be updated to reflect the changes.

- Restating bond valuations at market instead of amortized cost, based on the percentage of the bond portfolio that is either available for sale or available for trading
- Admitting assets that are not allowed by statutory accounting, but are for GAAP (premium receivables greater than 90 days past due, unbooked audit premium, furniture and equipment, non-admitted accounts receivable, pre-paid expenses and travel advances)
- Goodwill
- Deferred acquisition costs
- Deferred federal income taxes
- Other miscellaneous statutory to GAAP adjustments (principally investments in affiliates)
- Reclassifying ceded unearned premium reserves and loss and loss adjustment expense reserves from a contra-liability to an asset.

Although the model produces a wide variety of financial reports spanning balance sheets, income statements, reconciliations, cash flows, tax and regulator and rating agency financial measures, for this paper, only the statutory accounting exhibits will be used. Exhibits 8 and 9 provide examples of the statutory accounting exhibits produced by the model. These exhibits display just one possible financial outcome that might occur. Exhibit 10 displays graphically the ranges of results that were generated for a few of the accounting metrics when the model was run.

Tax algorithms

The model calculates both current and deferred federal income taxes.

Current income taxes

Current income taxes are calculated in accordance with insurance company tax procedures, as described in Chapter 13 of Property-Casualty Insurance Accounting.[5] Current taxes are calculated by adjusting current year statutory net income as follows:

1. Increase or (decrease) current year net income by 20% of the change in the unearned premium reserve
2. Increase or (decrease) current year net income by the difference in the amount of tax discount in held reserves[6]

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[5] Property-Casualty Insurance Accounting, Sixth Edition, July 1994, by Insurance Accounting and Systems Association, Chapter 13.

[6] The model is seeded with historical tax discount factors, either industry, company-specific or a combination of the two, depending on what tax discount factor elections were made in 1987 and

3. Decrease current year net income by 85% of the amount of tax-exempt investment income earned during the year
4. Reduce current year net income by 59.5% of the amount of dividends received from common and preferred stock (the dividends received deduction is 70%, but 15% of the deduction must be added back into net income for tax purposes)
5. Apply a 35% tax rate to the resulting taxable net income amount.

Alternative minimum taxes also are calculated for the current year by increasing taxable net income by 75% of the amount of tax-exempt investment income and dividends received deduction excluded from regular taxable net income and multiplying the resulting alternative minimum taxable net income by the 20% AMT tax rate.

These calculations develop the preliminary current year tax position. If a projection year develops an operating loss, that loss is compared against the three prior calendar years to see if it can be used to offset prior years' operating gains. If not, it is retained for possible use as an operating loss carryforward, to be applied against operating gains in a later projection year.

Deferred income taxes for GAAP accounting

The major components of the deferred income tax calculation are the tax discount in held loss reserves, deferred taxes on deferred acquisition expenses, and deferred taxes on unrealized gains or losses on equities and bonds available for sale or trade. The GAAP income statement includes the calendar year change in the portion of the deferred tax asset arising from the tax discount in held loss reserves, the deferred taxes on deferred acquisition expenses and the deferred tax asset or liability arising from unrealized capital gains or losses on that portion of the bond portfolio available for trade.

Financial ratios

Based on the accounting results for each projection years , a series of financial ratios are developed. These include:

- projections of the National Association of Insurance Commissioners ("NAIC") Insurance Regulatory Information System ("IRIS") ratios
- a selection of A.M. Best's financial ratios (operating ratios, leverage ratios, and liquidity ratios)

1992. Projected future discount rates are developed using either pre-seeded industry payout patterns or company-specific payout patterns that evolve from the line of business underwriting structure and a rolling sixty month average interest rate that is linked to the model's projected risk-free interest rate projections.

- an approximate NAIC Risk Based Capital ("RBC") indication.

Model Mechanics

Figure 1 presents a schematic of the way the corporate model operates.

Figure 1 Corporate Model Flowchart

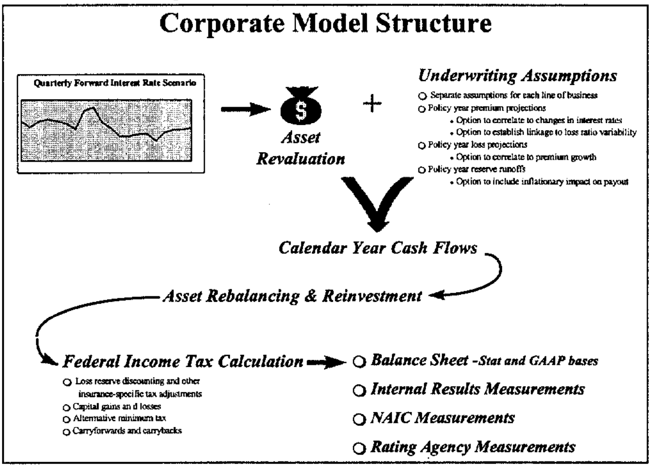

Figure 1 starts with initial conditions—the beginning balance sheet, including accident year modeling of liabilities, knowledge of accruals, tax carry backs and carry forwards, costs and valuations of assets, and so forth. The following sequence of steps is replicated many times for the entire planning horizon:

1. Stochastically generate an economic scenario (interest rates, inflation, competitive conditions, etc.) for the next period.
2. Apply the economic scenario to generate operations for the period.
3. Apply the economic scenario to value existing assets.
4. Apply endogenous effects on liabilities (e.g., correlated, random effects on loss volume or severity that are independent of economic effects).
5. Apply the economic scenario to value existing and new liabilities (e.g., inflationary impacts, shocks and other external effects).
6. Apply a reinsurance strategy based on currently liability and asset conditions or on functions of previously observed or future expected ones.
7. Apply an asset rebalancing strategy based on current liability and asset conditions or on functions of previously observed or future expected ones.
8. Rebalance the portfolio of assets (and/or liabilities), i.e., buy and sell.
9. Develop taxation effects and other fiscal period closing entries.

10. Tally assets and liabilities under the appropriate accounting scheme(s).
11. Create end-of-period financials, operating statistics and metrics.

The application of reinsurance and asset rebalancing strategies in steps six and seven presumes the model user has previously established the course or courses of action the company will take if different asset and/or liability results emerge from the model's projection horizon.

For example, reinsurance strategy options could include the purchase of less reinsurance in subsequent years if the loss ratio for previous years is better than expected, or the purchase of more quota share reinsurance if the company begins developing cash flow problems. The asset rebalancing strategy could be set up to take a higher position in tax-exempt bonds if the company is in a regular taxable income position rather than an alternative minimum taxable income position.

SECTION II: KEY DYNAMIC PARAMETERS

The table below defines what we have concluded are the most critical model elements to be modeled dynamically, both from the perspective of importance to the user and to model volatility.

Table 1: Parameters and Considerations

Parameters	Considerations
Interest rates	Is this a stochastic process and will forward yield curves be available as a by-product?
Inflation	This impact has implications for expenses, business production and retention, competitive conditions and liability payment levels.
Conversion of starting loss, loss adjustment expense and unearned premium reserves in cash outflows	Are the reserve levels on the December 31, 1996 financial a reasonable representation of the amounts that will actually be needed to meet these obligations? How volatile are the amounts actually needed? When will the obligations be paid?
Allocation of new money	What rule structure or structures should be established to tell the model how to use cash inflows for the purchase of new assets in each asset class?

Parameters	Considerations
Production of new business	Is production tied to interest rates, inflation, competitive environment, other influences? What are the correlations or functional relationships between business production and exogenous events such as interest rates or inflation? By what pattern are revenues collected and amortized? What loss ratio will the new business exhibit
Expenses	How do expenses depend on production volumes and exogenous factors? By what pattern are they paid?
Correlations	How do dynamic variables interact? Does the selection of a directional value in one dynamic variable predispose a second variable to take on different values than it otherwise might? Or are the variables independent of each other?
Risk factors in excess of risk-free interest rates that are applicable to different asset classes	What is the risk loading given by the free market to an average asset within each asset class? Is the risk loading stable or volatile? Does it change over time or with changes in the interest rate environment? How important is default risk and how should a model distinguish between it and other sources of general or specific financial risk?
Risk factors in excess of risk-free interest rates that are applicable to different liability classes	When calculating discounted reserves, what risk loading is appropriate for each liability class? Should it be a function of the volatility in the liability class payouts or a function of the length of the payout pattern?
Accounting accruals	When developing accounting entries for the model's financial statements, a substantial number of accrual items must be developed. How are these accrual items derived - are the accrual amounts based on fixed or variable relationships to other model elements?

Interest rates

The introduction of volatility in the model's projected interest rate environment has a many-tiered impact on the financial model. For example, changing interest rates are the primary driver of changes in the market value of previously purchased assets. Additionally, the model also relies on changes in interest rates to signal changes in the overall level of inflation in the insurance environment. Specific areas within the model that are directly affected by changes in interest rates include bond pricing, equity pricing, and loss reserve discounting. Other areas that can contain linkages to changes in interest rates include future premium, loss and expense levels.

Interest rates and bond pricing

A risk factor that is reflective of actual bond return over risk-free yields is specified for each bond category being modeled. The different risk factors are added to the arbitrage free interest rates to develop interest rate curves that are specifically applicable to each bond category. Since the future cash flows from each bond category are known[7] (coupon amounts and timing and principal repayment amount and timing), we can use traditional bond valuation methods to calculate changes in market values for the holdings in each bond category arising from changes in interest rates.

Interest rates and equity pricing

The model employs two alternatives for pricing equities. The choice between them is dependent on one's view of interrelationships between interest rate movements and equity prices.

One alternative bases the rate of return on equities on a normally distributed random variable with a mean market return and standard deviation based on investor expectations. This alternative uncouples equity pricing from changes in interest rates and is a conventional random walk model.

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[7] The future cash flows of bonds held at December 31, 1996 are known because the bonds themselves are known quantities. We know their coupon rate and timing, their maturity date, and their par, book, and market values. This is sufficient information to project future cash flows arising from the December 31, 1996 bond portfolio.

The future cash flows of bonds purchased during 1997-2001 are known because (a) we know the risk-free interest rate environment at the time the bonds are (will be) purchased, (b) the risk factor that is added to the risk-free interest rate for each bond category, (c) the time to maturity of the bonds that are purchased, and (d) the total dollar amount of new investments in each bond category. With this information, we can calculate an appropriate coupon rate for each dollar of investment in each bond category. We make a simplifying assumption that new bonds are purchased at par, so the new bonds' market values at the time of purchase equal their book, par and statement values. We now have sufficient information to project future cash flows arising from new bond purchases.

The second alternative relates equity returns to the projected interest rate environment through the Capital Asset Pricing Model (CAPM). Readers might recall that the CAPM formula is $R = R_f + \beta(R_m - R_f)$, where

R = the expected return on a given stock,
R_f = the risk free rate, such as the rate on Treasury bills
R_m = the overall market return
β quantifies the undiversifiable or systematic risk associated with the stock in question

In the Capital Assets Pricing Model, three hypotheses are made:

1. The expected return from a common stock is related only to the stock's systematic risk
2. The difference between the expected return from a common stock and the return on a risk-free rate is proportional to the firm's systematic risk
3. The systematic risk and the factor of proportionality are relatively constant over time.[8]

In our use of CAPM to link equity returns with interest rate movements, we have made some slight modifications to the basic CAPM formula. Our revised formula is $R = R_f + \beta_P(\alpha) + e_P$, where the terms are as follows:

R_f = short term risk free rate of return
β_P = the beta of the stock portfolio being held
α = average excess return of the market portfolio over the risk free rate[9]
e_P = stock price volatility (the random variable)

With $\beta_P = 1$, this simplifies to $R = R_f + \alpha + e_P$, which is the conventional random walk model noted above.[10]

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[8] "Pricing Insurance Policies: The Internal Rate of Return Model" by Sholom Feldblum, May 1992, page 31. Mr. Feldblum also references Portfolio Theory and Capital Markets by William F. Sharpe, New York, McGraw-Hill, 1970 and "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets", Review of Economics and Statistics, February 1965, pages 13 ff.

[9] Underlying the assertion of a constant excess return of the market portfolio over the short term risk free rate are assumptions that average risk aversion and average stock price volatility are constants. α can also be thought of as the difference between R_m and R_f, where R_m is the expected return of an index portfolio, such as the S&P 500.

[10] The use of CAPM to model equity returns is described in more detail in "Using CAPM to Generate Scenarios for an Equity Portfolio" by Vladimir Fishman and William C. Scheel in The Chalke Perspective, Second Quarter 1996, Volume 7, Issue 2.

Interest rates and loss reserve discounting

The model allows loss reserves to be discounted for accounting purposes. The discount rate can be fixed, or it can be made a function of the simulated interest rate environment. If the latter approach is used, risk loadings can be specified separately for reserve discounting by line of business.[11]

Inflation

Inflationary impacts can affect one or more model components, including loss payout amounts, future premium volumes, and future expense levels. As such, the implications of inflation as a model parameter will be discussed in the context of each affected component.

Inflation and loss payouts

The model assumes three basic factors can affect loss payouts:

1. The timing with which loss reserves will be paid out
2. Reserve redundancies or deficiencies, excluding those arising from changes in the level of inflation affecting loss payment levels[12]
3. The impact of changes in inflation on future loss payments

Changes in the inflationary environment affecting each line of business are translated into changes in calendar year loss payouts. If, for example, the stochastically generated future inflation rate is substantially greater than what existed in the past and was expected to exist in the future, future loss payouts will be greater than anticipated. Conversely, if the stochastically generated future in-

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[11] For a thorough discussion of the issues surrounding the selection of an interest rate for discounting reserves, we recommend the reader refer to "Determining the Proper Interest Rate for Loss Reserve Discounting: An Economic Approach" by Robert Butsic in Evaluating Insurance Company Liabilities, Casualty Actuarial Society Discussion Paper Program, 1988, pp. 147-188.

[12] There is typically some element of claims inflation implicit in held loss reserves. Consider reserves that are developed from a traditional actuarial analysis of a paid loss triangle. Unless the paid loss triangle is specifically detrended prior to the analysis, the loss payment amounts in that paid loss triangle include some level of inflation in the payment amounts. The assumption in component 2 is that the levels of inflation in the future will be consistent with those in the historical payment triangles, so that when I assume I will pay $100 five years from now, the inflationary pressures on loss costs will not cause me to actually pay anything other than $100. This concept, along with the concept of inflationary impacts on future loss payments, are described in much greater detail in "The Effect of Inflation of Losses and Premiums for Property-Liability Insurers" by Robert Butsic, Inflation Implications for Property-Casualty Insurance, Casualty Actuarial Society Discussion Paper Program, 1981, pp. 58-102.

flation rate is lower than what was expected to exist in the future, future loss payouts will be lower than anticipated.

By including the inflationary impacts on loss payouts, we incorporate a linkage between the macroeconomic environment affecting assets and the macroeconomic environment affecting losses. We are also in a position to examine the financial statement implications of unanticipated inflationary pressures on loss payouts.

Inflation and future premium volumes

If premium for a line of business is dependent upon an exposure base that is inflation sensitive (such as workers' compensation), a formula for projecting future premium volumes can be utilized that incorporates a linkage to changes in inflation. Volatility in future inflation rates will have a direct effect on future premium volumes.

Inflation and future loss ratios

Future loss ratio projections can also be made dependent on changes in inflation. The model has been developed with the concept in mind that there exists a "force of loss" that is independent of inflationary impacts. This force of loss describes the loss ratio that would arise if there were no other changes occurring that have an impact on the final loss ratio. Other changes might include premium rate changes, inflationary increases in the premium exposure base, or inflationary impacts on loss costs. The final projected loss ratio is developed by first randomly sampling from the probability distribution that describes this force of loss, then modifying the random sample to reflect the other changes.

Exhibit 3 provides an example of the interrelationships between premium development and loss ratio development, including inflationary and rate impact influences.

Inflation and future expense levels

Future expense levels can either be assumed to be stable with current expense levels, or the model can apply an expense growth factor. The growth factor can be predetermined by the user; it can be made partly or completely random; it can be tied to changes in interest and inflation rates; or it can be a combination of the three. As with the inclusion of an inflationary linkage in the loss reserves, by adjusting the year-to-year expense levels for inflation, we link together the assets macroeconomic environment with the macroeconomic environment affecting company operations.

Conversion of loss reserves into cash outflows

As described in the section "Inflation and loss payouts" on page 15, there are three components that can affect loss payouts. The inflationary component has already been addressed, but the other two have not.

Variability in loss payout patterns

The model is structured so that variability in the timing of loss payments does not affect the overall amount that ultimately will be paid out, but it does affect when the payments occur. We liken the imposition of variability in this area of the model to the induction of an "accordion effect" in cash flow patterns, i.e. either stretching or compressing the basic patterns. Imposing variability on the loss payout pattern stress tests the company's asset liquidity. Exhibit 4, "Accordion effect in payout patterns", provides an example of the imposition of variability in loss payout patterns.

Variability in indicated reserve levels

As sometimes occurs in actuarial analysis, it turns out that, in hindsight, held reserves were either redundant or inadequate to meet the claims obligations. Allowing for variability in the indicated (as opposed to the held) reserve levels allows the model to quantify the income statement, cash flow, and tax implications of loss reserve redundancy or deficiency. Exhibit 2, which was first described in the section "Underwriting section" starting on page 5, demonstrates the model structure used to model variability in held reserves.

Allocation of new money

How assets are rebalanced at the end of each projection period is a critical model input. This determines what assets are to be bought and sold at any point in time, and it defines a risk profile that the company is willing to assume in addition to the risk profile being determined by the company's underwriting activity. The allocation algorithm can be as simple as "maintain the same relative mix of assets next year as we had this year", or it can be a complex algorithm that adjusts next year's asset mix to better match the projected liability duration. The dynamic nature of this element is not so much in creating randomness within the rebalancing algorithm, but in crafting an algorithm that is sensitive to the changing financial projections that are emerging from the model.

Production of new business

New business – premium volumes

New business production is a function of many different inputs. This can include the relative amount of business that is expected to be retained each year, a company's internal growth objectives, the overall insurance market conditions, and company reactions to prior year underwriting results. The interrelationship of inflation and new business volume has already been addressed. In general, it would seem that the more linkages that are established between new business production and other events being played out in the model, the better the model will be. The model then should be more reactive; it should do what the company itself might do when faced with similar circumstances. However, in some cases, the inclusion of additional dynamic elements in these linkages could lead to greater confusion in what the model is doing than is warranted by the additional realism that is gained. Each additional component has model building costs associated with it so, one must carefully reviewing both the model's objectives and the goals of company management for the model before expanding it. As Rodney Kreps and Michael Steel noted in their 1996 DFA paper, "For any of these models, a salient requirement is parsimony...there is no point in trying to model a detail whose behavior is masked by the random noise created by other terms."[13]

New business – loss ratio projections

Accompanying the mechanics for developing future premium volume projections are the processes for creating the associated loss ratios. Loss ratio projections need to consider, among other things:

- the underlying risk exposure taken on by the company
- the macroeconomic forces acting on the underlying risk exposure (i.e. inflation)
- the actions being taken by the company that may have an impact on the underlying risk exposure (such as a loss ratio's deterioration arising from reduced underwriting standards)
- and the actions being taken by the company's competitors.

An example of how these varied forces can be incorporated into the projection of future premium volumes and loss ratios is given on Exhibit 3.

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[13] "A Stochastic Planning Model for the Insurance Corporation of British Columbia" by Rodney E. Kreps and Michael M. Steel, in The Casualty Actuarial Society Forum, Spring 1996, pp. 156-157.

Expenses

Like new business production, expense projections are functions of many different inputs. Some expense projections can be tied directly to new business production, such as commission amounts. Other ones are tied directly to loss calculations. It is the year-to-year projection of fixed expenses that gives the greatest opportunities for directly including a dynamic component. For example, it can be assumed that salaries will grow five percent plus or minus one percent next year. Alternatively, including a dynamic component in expense projections must be weighed in relation to the relative benefits to be gained from the added complexity.

Correlations

This may be the most difficult set of model parameters to develop. A significant portion of the time there is no readily available information source that correlates different model components. In these situations, it falls back on the model developer's judgment to establish correlations that intuitively seem reasonable but for which there might be little or no empirical support. The area of correlation is one that current actuarial literature does not seem to address well, at least not in terms of providing statistical analysis of correlations. We would expect the amount and quality of correlation data to improve as dynamic financial modeling becomes more widespread, but that does not help current model builders. In our opinion, the best that can be done is to work judgmentally with the company to develop correlations that seem reasonable to both model developers and the party or parties for whom the model is being developed.

One data source that we have found particularly useful, at least in providing a mathematical foundation for the correlation of non-normal random variables is "A Distribution-Free Approach To Inducing Rank Correlation among Input Variables," by Ronald L. Iman and W.J. Conover.[14]. Using the mathematics described in this article, we have been able to implement an algorithm for the using pairwise rank correlations among dynamic variables. The model requires the correlation matrix to be positive definite[15]. In order to achieve this objective, the

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[14] "A Distribution-Free Approach To Inducing Rank Correlation among Input Variables," by Ronald L. Iman and W.J. Conover.[14], *Commun. Statist.-Simula. Computa.*, 11(3), 1982, pp. 311-334.

[15] Given an equation $f = ax^2 + 2bxy + cy^2$, the equation is said to be **positive definite** if for all points other than $x = y = 0$, the equation is positive. In terms of matrix mathematics and linear algebra, given a symmetric matrix $A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$

model automatically rescales the user-created (subjective) correlation matrix until a positive definite one is found. The resulting correlations may not be as strong as those initially established by the user, but they will retain the directional relationships that were established.

Accounting entries

The final category of parameters relates to the development of accounting entries from cash flow projections. It is our belief that a model must begin by quantifying cash flows— if cash can not be developed in a reasonably accurate manner, it does not matter how accurately the accounting accruals are developed. In keeping with this belief, we have concentrated on getting correct the details of the asset and liability cash flows, and have built up the balance sheet and income statement structure around the cash flows. To go from cash basis accounting to accrual accounting, we have employed a number of ratios that relate accounting accruals to annual company operations. An example of one such ratio is the relative level of premium written during the year that is due and not yet collected, which is used to quantify the agents' balance asset. These ratios can remain stable over time or can be allowed to vary, as the model user desires.

SECTION III: DEFINING PARAMETERS OF DYNAMIC RANDOM VARIABLES

Identification of model structure is arguably the most important task in dynamic financial analysis. Once model specification is laid to bed, the DFA investigator needs to establish process and parameters for the dynamic components that are modeled. The question most often dealt with in this context is, "What are the data sources?" This section of the paper addresses this question by revisiting parts of Section II and discussing how we have selected relevant parameters in some of our own models. We have omitted from this section certain items that were in Section II because either:

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satisfying any of the following tests insures the matrix A is positive definite:

- $x^T A x > 0$ for all nonzero vectors x.
- All the eigenvalues of A satisfy $\lambda_i > 0$.
- All the submatrices A_k have positive determinants.
- All the pivots (without row exchanges) satisfy $d_i > 0$.

from Linear Algebra and its Applications, 2nd edition, by Gilbert Strang, Academic Press, Inc., 1976, pp. 245-250.

a) the parameters are more appropriately defined by examination of company-specific data and discussion with company management (e.g. expense growth), or
b) the variability in the parameters is best developed in conjunction with company management, either because there is not much existing information on which to base the variability, or because the model's accuracy will not be significantly enhanced by an exhaustive analysis of the parameters' potential variability (e.g. accounting accrual percentages).

For those items that we are including in this section, a number of them can be appropriately parameterized with information from the either the DFA web site (http://dfa.risknet.com) or other Internet locations, but some can not.

There seems to be a clear dividing line between those items that can be analyzed with data available from the web site and those that can not. If a key dynamic element relates to economic issues that are not under the company's direct control, such as interest rates, inflation rates, and asset values, then the DFA web site can provide useful supporting data. On the other hand, if the control of the key dynamic element rests with the company being modeled, such as volume projections or expense growth, it is better to rely on the expectations of company management than the data from the DFA web site. With this distinction in mind, let us turn to some of the key dynamic elements from Section II.

Parameterizing an interest rate model

There has been considerable literature about the projection of interest rates. In fact, this may be the most well-documented of all DFA model parameters. The one-factor model we have implemented is closely based on the first of two interest rate generation algorithms described in a paper by James Tilley in the late 1980s[16]. It is a one-factor lognormal model that reverts interest rates to short-term expectations. In other words, projected interest rates have a tendency to move from an initial seeding (the actual December 31, 1996 interest rate level) to an equilibrium that represents historic interest rates expectations in the short-term spectrum of the yield curve. The input values needed to parameterize the Tilley model were derived from investigations using historical data and so-called stylized comparisons between model results and conventional expectations for such a model.

[16] "An Actuarial Layman's Guide to Building Stochastic Interest Rate Generators" by James A. Tilley, Transactions of the Society of Actuaries, Volume XLIV.

Asset risk parameters

Parameters describing the additional returns over the risk-free rate must be specified for each asset class. We have used information from Bloomberg databases that are available on a subscription basis to develop our selected asset class returns in excess of risk-free rates.[17] In addition, we developed the equity model volatility parameters from studies using data from the Ibbotson web site (http://ibbotson.com). Bond and equity price behavior is holding period-specific, so depending on the particular historical period examined, one can obtain wildly different indications of rate of return and volatility from the information in these studies. Judgment plays an important role in developing these parameters, however, one should choose historical values over extended periods rather than being influenced by short period of time. We have, however, modified historical information to reflect what is believed to be atypical periods of monetary authority involvement—behavior that is not likely to be repeat by the Federal Reserve Board.

Loss reserve discounting

We do not feel there is any one correct answer to the selection of an interest rate for loss reserve discounting. In the absence of strong preferences, we chose to use the short term risk-free interest rate being produced by the model's interest rate generator. This at least links liabilities to the interest rate environment that is impacting the asset side of the balance sheet.

Inflationary impacts

Impact on loss reserves

The level of inflation implicitly embedded in held loss reserves is often a difficult value for companies to quantify. It can be estimated by examining trends in claims payment patterns using a variety of loss reserving methods.

Another alternative is to use industry data to quantify historical changes in prices for different commodities. For example, the model underlying this paper includes information on the private passenger auto line of business. Exhibit 5 shows information collected from the Bureau of Labor Statistics web site (http://stats.bls.gov) on changes in the cost of automobiles and medical care. The changes in cost in the two indices were averaged to produce a 4.4% annual inflation rate, which was included in the simulation parameters.

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[17] Further information on Bloomberg services is available at their web site, http://www.bloomberg.com.

Impact on future premium volumes

For those lines whose premium base is inflation sensitive, such as workers' compensation (sensitive to payroll inflation) or homeowners (sensitive to increases in construction labor and materials costs), a component of future premium estimates can be inflation.

Historical relationships can be determined between interest rates and price indices that are applicable to the specific line of business in question. The DFA web site has links to information sources for historical interest rates. The Bureau of Labor Statistics can be used to gather information on payroll inflation or other price index changes. Once data has been collected for each, a regression equation can be established that links each line's inflationary component to an underlying interest rate level. The regression equation can then be used to relate projected future interest rates with projections of future inflationary pressures on premium volume. We leave it up to the model user to decide if he/she wants to specify the regression's error term as a random variable or to ignore it.

Impact on future expense levels

It can be argued that a company's *a priori* expectation of changes in expense levels will be more of a expense driver than will external inflationary pressures. Additionally, in this context, the issue of incremental value added must be examined. Will the model results be that much better for the inclusion of an inflation-linked expense component? Or will the added volatility just add to the noise that a dynamic model inevitably captures?

The simplest way to parameterize this component, in our opinion, is to ignore inflationary impacts all together. Instead, concentrate on the company's historical expense growth as it relates to changes in company operations. Has the company grown considerably in recent years? How have expenses changed in the same time frame? What are the company's operational expectations for the next five years? Equally rapid growth, or slower growth? We believe in this area simpler is better—link the expense changes to operational projections and (at least at first) do not confuse the issue with additional linkages to inflationary factors.

Conversion of loss reserves into cash outflows

Variability in loss payout patterns

We approach the creation of a dynamic payout pattern with the basic idea that incremental variability in a payout pattern should decrease the closer the cumulative pattern is to 100%.

When parameterizing a loss payout pattern to include variability, we have generally elected to ignore the possibility of correlations between successive incremental payout percentages. From a very unscientific sampling of data, we have found no conclusive evidence to link a higher-than-expected incremental payout in time T with either a higher-than-expected or lower-than-expected incremental payout in time T+1.

We typically try to use company-specific data as the starting point for developing the mean and ranges of variability around each incremental payout percentage. We may add in some judgment as to whether the data has sufficient variability or too much variability. For the industry-wide example in this paper, we developed the baseline payout patterns from an analysis of the industry paid loss development patterns and we based the variability on the actual observed variability in the incremental payout percentages. The development of one such variability parameter for private passenger auto is displayed in Exhibit 6.

Variability in indicated reserve levels

We address this dynamic element in a similar fashion to the way we address variability in the payout pattern. We begin by assuming that reserve variability decreases as accident years age. We do, however, expect there to be some correlation between accident year reserve redundancies or deficiencies, both within lines of business and across lines of business. The strength of the correlations is often based on expert judgment.

Exhibit 7 provides an example of a data format for determining reserve variability parameters. From this information, with some actuarial judgment thrown in, we develop final reserve variability parameters.

Allocation of new money

While the allocation of new money is a critical variable in the overall model dynamics, it is not one that we try to parameterize as a random variable. We see the allocation of new money as a management guideline, one that should be compared among competing allocation strategies. For example, we pose the question "What is the impact on our year-to-year and multi-year financial performance if we allocate more funds to taxable bonds as opposed to equities?" In this context, variable definition involves discussing investment philosophy with company management and developing the three or five or ten different broad asset allocation strategies to be evaluated.

Production of new business

Premium volumes

The amount of parameterization that is required for this variable is dependent upon the complexity with which each year's new business projection is established. If the new business projection is quantified by an analysis of retention ratios on current business in combination with expectations of writing certain volumes of completely new business, the number of variables to parameterize might be large. If, on the other hand, the new business projection is quantified by a simple growth factor applied to the prior year's level of writings, the parameterization of the growth factor could entail much less work. Either way, the data that should be used to develop the parameters needs to be specific to the company being modeled.

Loss ratios

Parameterization of future loss ratios can be a very simplistic or sophisticated analysis. The level of research and number of considerations should be commensurate with (a) the purpose for which the model is being used, (b) the amount of data available, both company-specific and industry-wide, and (c) the level of uncertainty in other key model parameters. In our experience, we have based loss ratio volatility parameters on a combination of discussions with companies, examinations of historical volatility, and historical and projected future rate adequacy.

Correlation (Revisited)

As noted in the preceding section on correlation (beginning on page 19), dynamic financial modeling must allow for correlated variates. This is particularly true for the relationship among lines of business. Correlation and causality are both important to modeling. The business environment will affect new business production—the relationship is largely causal and must be functionally built into the model. But, there also are company-specific effects such as agent activities, persistency of existing business and other phenomena that may be understood as correlated with one another. It is important, however, to distinguish between the modeling of causality and these correlated, random phenomena. Model specification must define relationships between, say, an economic or business environment scenario and production of new business. Omission of this causality consideration is an example of specification risk in model design. However, given a business scenario there still needs to be considerations of correlation among, say, loss ratios for the lines of business.

CONCLUSION

Ultimately, whether a model succeeds or fails depends on the level of trust that is placed in its parameterization. It always is preferable to build model parameters from actual data, but actuarial judgment can provide acceptable surrogates so long as the assumptions underlying the judgment are reasonable to those parties placing reliance on the model. In many cases, the data is just not available, or at least not readily available for use in model specification. In other cases, the marginal improvements that can be gained by more accurate parameter specifications are not significant enough to warrant either the work that would be needed to improve the parameterization or the model's additional complexity.

Whenever data are sparse, the need for models that reflect *subjective* reasoning or understanding will dominate choices in parameterization. With only limited, subjective information, a good choice of a distribution for process risk is one with parameters that make sense within the context of the sparse information. The Weibull distribution has merit because its parameters can be chosen with information about central tendency and chance-constrained probability estimates of the extreme tails—no other information is required to fully specify a distribution within this rich family of distributions.

It is our view that all decisions regarding model specification and model complexity should only be made after a review of:

- the additional value the parameter brings to the model results
- the ease or difficulty with which the parameter's dynamic specifications can be developed
- the amount of work that will be needed to maintain and otherwise update the parameter's specifications and
- the additional complexity engendered by the parameter's inclusion.

If, after reviewing these elements, the decision is to include the parameter in the model, there are a number of alternatives a model developer can take. Some information is readily available on the Internet, much of it already included in the DFA web site. Other data sites, including the Bureau of Labor Statistics, are also easily accessible. Still other data sites are available for a charge, such as the Insurance Services Office, AM Best and National Council on Compensation Insurance sites. Undoubtedly there are many more sites that as yet remain undiscovered by the actuarial community at large. As property-casualty insurance company dynamic financial modeling moves out of its infancy, more and more sources of information will become known.

Dynamic financial analysis is a slowly evolving area of actuarial knowledge. If the actuarial profession is patient, it will either find or develop the data sources needed to more fully address DFA issues. Until then, the best we can do is work with the limited information at our disposal, and where none exists, use our judgment to fill the gaps. If we expend our energy bemoaning what we do not have, we will not be in a position to take advantage of what we do have. Imperfect knowledge in and of itself is not a good excuse for abandoning the cause of DFA. Rather, the imperfections should be understood for what they are, and the model building should continue and purse alternatives in design that are consistent with subjective understanding. Later in time better data will become available and the imperfections can be reduced or eliminated entirely.

Appendix A: Fundamentals of Dynamic Financial Analysis

The Tenets of DFA

DFA methods are helping return and risk analysis, but the methods are still in their infancy. The computation techniques evolve with every new generation of software and computers. But, within this state of flux we find some well-anchored principles. First, DFA is very *ad hoc* and relies entirely on repetitive simulation of business events and the accounting of those events using a virtual general ledger. Second, the primary purpose of DFA is to understand with some measure of confidence the range in which general ledger-based metrics will fall. Third, DFA is interested in answering both narrow and broad questions. Narrowly, we ask: "How does ***a scenario*** measure up." Broadly, we ask: "How do we measure up in the presence of many scenarios—what is the business impact of the ***virtual scenario***?"

Accounting Frameworks

Most aspects of dynamic financial analysis ultimately are dependent on an accounting system. The primary purpose of DFA is to provide decision makers with ranges instead of point estimates.

The point estimate approach has been the bulwark of forecasting. But, static analysis leading to the point estimate is not very useful if one needs to allocate capital, choose among competing strategies with different risk profiles, or identify alternatives that optimize some goal function.

Business performance ultimately is measured by one or more accounting measurements, or metrics. These appear throughout financial statements. They range widely in their complexity and component parts:

- Balance sheet or income statement accounts
- Financial ratios,
- Complex functions of cash flows,
- Regulatory criteria, and
- Operational measurements such as business volume.

A common element to any of these metrics is the underlying accounting system from which they are derived.

It is difficult to envision DFA without an accounting system. There are many dynamic analyses that could skirt around accounting; but, if the analysis is truly financial, it will require an accounting system. For example, one might want the aggregate loss distributions for all parties to a risk transfer agreement so that each layer can be appropriately priced. The purpose of the analysis is to set prices, and it may be separated from the more interesting question of what impact a particular portfolio of reinsurance has on the solidity of an enterprise. The latter is dynamic ***financial*** analysis and certainly would require understanding how surplus and cash flows are affected by the reinsurance. Surplus is a creature of an accounting system—most objects of DFA are accounting-based metrics. However, DFA can be used in cash-based accounting exercises. Actuaries might use DFA, for example, to analyze cash flows for pricing purposes, and then use the same DFA methods in a broader context and different model. The latter usage for asset management would lead to understanding the broader implications of such pricing on surplus generation or other important metrics.

To the extent that an accounting framework is used to measure the magnitude and force of variables, its focus will greatly affect the end-product of DFA. In addition to cash-based accounting, there are statutory and GAAP bases and tax and management accounting ledgers. Each framework will be important to a particular constituency. While managerial accounting could work well for management, statutory accounting might be preferred by a regulator and GAAP accounting preferred by stockholders.

In summary, physical and financial processes are responsible for asset changes from period to period. But, the stock measurement or asset volume observed at the end of a period will be greatly influenced by the method of bookkeeping. It is important to recognize that both the magnitude and its probabilistic dispersion can be affected by the system of measurement.

Principle 1: DFA Requires an Accounting Framework

The First Principle of Financial Return and Risk measurement is to understand that DFA is conditional upon one or more accounting frameworks. Most business decisions will be made based on how they are perceived to affect results measured by an accounting system. DFA measurement results will almost always be complex accounting functions of dynamic input variables.

Static and Dynamic Financial Analysis

Traditional financial forecasting has relied on essentially static evaluations of current and future events. A traditional financial forecasting model might include a single set of assumptions (or maybe three sets of assumptions: best case, base case, worst case) about future operating results from various operating divisions or business units, an expectation of investment returns from the investment division and a projection of fixed expenses from a corporate planning division. From these inputs a financial plan is developed and critical business decisions are made.

What is missing from this picture? To begin, there is no sense of how likely it is that the base case will be achieved, or the worst case avoided. In a static forecasting environment, there is no way to quantify the variability of possible outcomes. Yet this is a critical factor is strategic decision making. It is very difficult to know which of a series of strategic options to pursue without being able to appreciate differences in both the range of possible outcomes and the most likely result to arise from each option. The essence of dynamic financial modeling is the ability to describe critical assumptions in terms of ranges of possible outcomes, rather than in terms of fixed values. Once each critical assumption is defined by a range of possible outcomes, a sophisticated modeling environment takes over, recalculating the integrated financial model again and again, returning different values each time. At the conclusion of the modeling exercise, we are left with a range of results we can reasonably expect to see, given the parameters and interrelationships that have been defined for the key variables. Differences in financial results arising from alternative strategic decisions can be evaluated by replacing one set of strategic decisions with another, re-running the modeling exercise and comparing the ranges of possible outcomes under each decision path.

Principle 2: DFA Is Communicated in the Form of Confidence Statements

The second DFA principle is how the work-product of the analysis is communicated to management. The boundaries of a metric are declared with an attached probability. This approach is not classical statistical inference regarding the probability of chance explanations of phenomena; it is the substitution of range estimates for point estimates. DFA implicitly places greater value on confidence bands than on the expected value of a distribution.

The Demeanor of Model, Process and Parameter Risk Assumptions

The demeanor of DFA for asset and liability valuation is very ugly. There are at least three faces to the problem of setting up a DFA experiment:

1. The functional relationships yielding changes in asset and liability value may be obscure and consist of both physical phenomena and accounting relationships. This inability to adequately understand and specify important functional relationships is ***DFA specification risk***.
2. The joint probability distribution of the model variables is almost always unknown. We may have some fuzzy understanding of the marginal distributions, but it may be limited to certain beliefs about central tendency and extreme behavior. We posit some degree of correlation

among variables. But, this inability to really define the joint probability distribution is a manifestation of *DFA process risk*.

3. Model designers often are lured into the belief that specification and process risk are non-existent. If only they knew the parameters of the (analytic) distribution(s), all would be well. These poor souls suffer only *DFA parameter risk*.

Collectively, we will refer to these various risks of mis-specification of the DFA experiment as ***model risk***. But, it is clear that mis-specification of a model for asset valuation almost always will occur to one degree or another.

Principle 3: DFA Has Its Own Risk Profile

> ***Although one explains DFA as a confidence measure, it fails to explain the underlying uncertainty both in the DFA model, process and parameter risk. While one might render a DFA confidence statement, there must be caveats. A well-defined accounting framework will not eliminate the subjectivity involved in the assessment of causality.***

Simulation Will Always Be Preferred for DFA Work

Simulation usually is the most tractable approach; analytic solutions usually evade us. Simulation is particularly powerful under these circumstances:

- Rule-based reasoning,
- Transformation of variables in a complex way (such as with an accounting system),
- Node events with probability distributions that are substantially empirical or subjective.

A node event is one where a probability distribution defines two or more possible outcomes, and depending on the outcome a different set of events follows.

Principle 4 DFA Must Deal with Subjectivity

> ***The elicitation of probability distributions for key variables is not like the contrivance of bets in a casino. The physical processes are rarely known. Sometimes, only a sense of central tendency and confidence in some tail point will be acknowledged as "known." This fuzzy understanding means that the choice among probability distributions should be dictated as much by the intuitiveness of their parameters as by their ability to define physical processes. Is an actuary who fits data to parameterize a lognormal distribution and declares it to be a severity distribution wiser than another who uses a Weibull distribution fit with the same data? The latter can successfully argue that he has chosen to augment his subjective understanding of central tendency and a cutoff point, and that he has chosen the Weibull because it* a priori *is a rational way to handle (and acknowledge) subjectivity.***

Spreadsheet-Centricity for DFA Work

A spreadsheet is an excellent programming venue for rules, accounting systems and scenario representation with dynamic variables. DFA models can be segmented into components—the nodes serve as logical breakpoints among strains of causality. This chained quality of the models, even with feedback and dependency relationships, is easily expressed in spreadsheet components. The components may be as small as a single cell. Or, they may be aggregations of cells or sheets within a workbook. In any case, the components can be easily modified when they are expressed as elements of a spreadsheet.

Principle 5 DFA Is A Natural Application for Spreadsheets

> ***This principle of DFA acknowledges the evolutionary nature of DFA modeling. A DFA model is never really "finished"; there are always more detailed questions that can be asked of the model and more information that can be extracted from it. A spreadsheet-based environment acknowledged the inevitable expansion of DFA models and provides a framework for implementing changes that is programmer-free.***

Stochastic Dominance: Stochastic dominance looks at the range of possible outcomes in terms of the risk/reward tradeoffs that are indicated by each. Strictly defined, stochastic dominance is an approach for choosing among risky alternatives based on certain knowledge about their cumulative probability distributions and about utility that is derived by the decision maker.[18] An evaluation of two (or more) alternative strategic directions for stochastic dominance provides the information needed to answer the question, "Which alternative has higher expected utility?" The answer to this question can be the basis for a management decisions. Graphically, it sometimes is possible to identify stochastic dominance and begin to answer the question, "Does greater reward justify greater risk," directly within a utility framework.

The picture on the following page displays the relationship between three different asset allocation strategies. The measurement, an example of a user defined metric, is the internal rate of return on the change in book value of all invested assets over the five year projection horizon plus investment income, realized and unrealized capital gains, less the difference between the market value of assets maturing and sold and those purchased during the five years.

While there is no clear cut stochastic dominance evident in this picture, it does illustrate that higher return is only achieved at the price of higher risk. The ultimate choice is a business decision; there is no alternative in this decision set that stochastically dominates the other. This finding may seem to be a bane of dynamic financial analysis—there is no mechanically driven choice within a loosely defined utility framework. However, it points out the reality underlying strategic business decisions - it is not very often that one strategic direction is clearly superior to all others.

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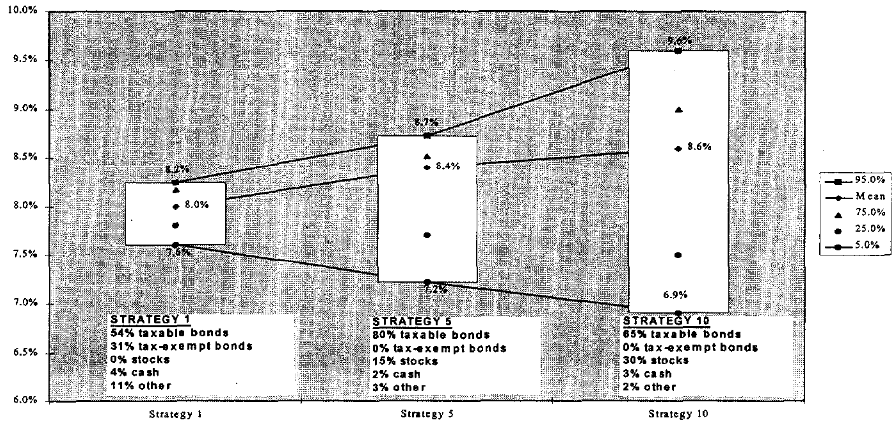
[18] Stochastic dominance also has been called a general efficiency criterion. It provides a framework for decision making under uncertainty based on sparse understanding of the decision-maker's utility function. For example, it can be shown that under the loose assumption that one prefers more to less, if $F(x) \leq G(x)$ for all x and at least one point in the domain of x is such that the strong inequality holds, one should prefer the alternative with cumulative distribution F(x). This is a typical "risk averse" profile. The reader will find a discussion of the general efficiency criterion (first degree stochastic dominance) in Haim Levy and Marshall Sarnat, *Investment and Portfolio Analysis*, John Wiley & Sons, Inc., 1972, pp. 264 ff.

Exhibit 1 continued

Comparison of Portfolio Yield with Capital Gains



Example of portfolio yield with capital gains calculation:

		1996	1997	1998	1999	2000	2001
(1)	**Investment Income**		40,000	38,000	44,000	43,000	45,000
(2)	**Realized capital gains**		2,000	-1,000	1,000	8,000	-3,000
(3)	**Unrealized capital gains**		10,000	-4,000	-10,000	26,000	-1,000
(4)	**Assets maturing or sold**		100,000	110,000	120,000	130,000	140,000
(5)	**Assets purchased**		120,000	135,000	150,000	165,000	180,000
(6) = (4) - (5)	**Net Sales**		-20,000	-25,000	-30,000	-35,000	-40,000
(7)	**Book Value**	-500,000					650,000
(8)= (1) + (2) + (3) + (6) + (7)	**Cash Flows**	-500,000	32,000	8,000	5,000	42,000	651,000
	Internal rate of return	8.66%					

Exhibit 2

The interaction of loss reserve payouts, reserve redundancy/deficiency emergence into cash flows and accounting and unanticipated inflation:

Suppose a company had one year of loss reserves on its books with the following additional information:

Held reserves: \$100,000, based on a reserve range of \$80,000 to \$105,000
Inflation level implicit in held reserves: 5%
Reserve payout pattern: 25% over each of the next four years

The model parameters incorporate the \$100,000 held reserve into the first loss triangle (payout of held reserves), and the potential for reserve variability in the second and third triangles (payout and accounting impacts of reserve redundancy/deficiency emergence). Any impacts arising from changes in the inflationary environment are captured in the calendar year financial statements only.

Let us assume the model randomly selects parameters such that the reserve redundancy is \$10,000, excluding a change in inflation, and that inflation increases to 8% in years three and four. We decide to reflect the \$10,000 reserve redundancy by weakening reserves \$5,000 in years one and two.

The model results look as follows:

	Year 0	Year 1	Year 2	Year 3	Year 4
Payout percentage		25%	25%	25%	25%
1. Expected inflation rate		5%	5%	5%	5%
2. Actual inflation rate		5%	5%	8%	8%
3. Held reserve cash flow	n/a	25,000	25,000	25,000	25,000
4. Redundancy cashflow	n/a	-2,500	-2,500	-2,500	-2,500
5. Inflationary impact	n/a	0	0	643	1,304
6. Reserve weakening	n/a	-5,000	-5,000	0	0
7. Held reserves	100,000	72,500	45,000	22,500	0
8. Net cash flow		22,500	22,500	23,592	23,804
9. Income statement impact (- gain/ + loss)		-5,000	-5,000	+643	+1,304

Row 3 formula: $\left[\left(\prod_{i=1}^{4}(1+Row2_{Year\,i})\Big/\prod_{i=1}^{4}(1+Row1_{Year\,i})\right)-1\right]*(Row3_{Year\,i}+Row4_{Year\,i})$

Example: Year 3 inflationary impact = $\left[\frac{(1.05)(1.05)(1.08)}{(1.05)(1.05)(1.05)}-1\right]*(25{,}000-2{,}500)$

Row 7 formula: $Row\ 7_{Year\,i-1} - Row\ 3_{Year\,i} + Row\ 6_{Year\,i} - Row\ 4_{Year\,i}$

Row 8 formula: $Row\ 3_{Year\,i} + Row\ 4_{Year\,i} + Row\ 5_{Year\,i}$

Row 9 formula: $Row\ 5_{Year\,i} + Row\ 6_{Year\,i}$

[Page revised: 5 March 2001]

Exhibit 3

	Actuals 1996	Projected 1997	Projected 1998	Projected 1999	Projected 2000	Projected 2001
Insurance Industry Composite						
Workers' Compensation						
INCOME STATEMENT RELATED ASSUMPTIONS						
Premiums						
1. Net written premiums prior to rate, competitive impacts:		*18,656*	*18,656*	*17,441*	*17,842*	*19,648*
2. Rate impact on net written premiums:		*n/a*	*-10.0%*	*-2.5%*	*6.7%*	*-10.0%*
3. Competitive impact on net written premiums:		*n/a*	*n/a*	*n/a*	*n/a*	*n/a*
4. Other impact on net written premiums:		*n/a*	*3.9%*	*4.9%*	*3.2%*	*2.7%*
5. Final expected net written premiums = (1) * [1+ (2)] * [1 + (4)]	*18,656*	*18,656*	*17,441*	*17,842*	*19,648*	*18,161*
Net Loss & ALAE Ratio, net of subrogation/salvage						
6. Net loss & ALAE ratio, prior to rate, inflation impacts		*71.5%*	*71.6%*	*82.4%*	*75.5%*	*79.0%*
7. Impact due to premium rate changes		*0.0%*	*9.5%*	*3.6%*	*-5.2%*	*8.4%*
8. Inflationary impact		*n/a*	*2.3%*	*2.3%*	*1.2%*	*1.2%*
9. Other impact		*n/a*	*n/a*	*n/a*	*n/a*	*n/a*
10. Final expected loss & ALAE ratio = (6) * [1+(7)] * [1 + (8)]		*71.5%*	*80.2%*	*87.1%*	*72.6%*	*86.7%*
Inflation and interest rate information						
11. Inflation rate that is implicitly embedded in premium growth levels		*3.2%*	*3.2%*	*3.2%*	*3.2%*	*3.2%*
12. Inflation rate that is implicitly embedded in loss payout pattern		*4.7%*	*4.7%*	*4.7%*	*4.7%*	*4.7%*
13. Risk free interest rate underlying asset valuations in current scenario		*7.3%*	*9.2%*	*6.0%*	*5.1%*	*3.9%*
Premium earning information						
14. Percent of written premium that is earned during first twelve months		*86.5%*	*86.5%*	*86.5%*	*86.5%*	*86.5%*
15. Percent of written premium that is earned during second twelve months		*13.5%*	*13.5%*	*13.5%*	*13.5%*	*13.5%*

Exhibit 3 Formula explanations:

2.	**Rate impact on net written premiums:** Model is assuming an expected loss ratio of 81.9%. If the prior year's loss ratio is less than 81.9%, a rate decrease is implemented. The rate decrease is the lesser of a 10% decrease and the difference between the prior year's loss ratio and 81.9%. For example, the projected 1997 loss ratio equals 71.5%, so the rate change in 1998 is the smaller of -10% and (71.5% / 81.9%), or -12.7%. A similar formula exists for rate increases if the prior year's loss ratio exceeds 81.9%. The 81.9% expected loss ratio was derived from the ten year average industry loss and ALAE ratio. The 10% cap was implemented based on judgment.
3.	**Competitive impact on net written premiums:** this premium adjustment element is not being used in this example. It could be used to incorporate an underwriting cycle element in pricing.
4.	**Other impact on net written premiums:** this is being used to quantify the impact of wage inflation on premium levels. Based on Bureau of Labor Statistics data, wage inflation has averaged 3.2% over the past ten years. For example purposes, it was assumed that the risk-free interest rate over the past ten years has averaged 6.0%. The projected wage inflation impact is equal to [(3.2% / 6.0%) * prior year projected risk free interest rates in (13)]. Example: 1999 premium inflation impact = (3.2% / 6.0%) * 9.2% = 4.9%.
6.	**Net loss and ALAE ratio, prior to rate, inflation impacts:** this is a stochastically generated loss ratio. The distributional parameters were developed from historical industry loss ratios. This is the underlying "force of loss" that is associated with the policies being earned during the year. This is the loss ratio that would develop, absent any other influences on the loss ratio, such as premium rate changes, premium inflation, and loss inflation.
7.	**Impact on loss ratio from premium rate changes:** $\frac{1}{\left\{\left[\left(1+\text{current year rate change \% from (2)}\right)*(14)\right]+\left[\left(1+\text{prior year rate change \% from(2)}\right)*(15)\right]\right\}}-1$ Example: 1998 impact = 1 / { [(1 - 2.5%)*86.5%] + [(1 - 10.0%) * 13.5%] } - 1 = 3.6%.
8.	**Inflationary impact on loss ratio:** This is the result of inflationary pressure on loss costs, partly offset by inflationary increases in premium volume. Based on Bureau of Labor Statistics data, loss inflation has averaged 4.6% over the past ten years. Again, for example purposes, it was assumed that the historical risk free interest rate over the past ten years has averaged 6.0%. Formula: $\frac{\left\{\left[\left(4.7\%/6.0\%\right)*\text{prior year risk free interest rate in (13)}\right]+1\right\}}{\left\{\left[\left(1+\text{current year prem inflation \% from (4)}\right)*(14)\right]+\left[\left(1+\text{prior year prem inflation \% from (4)}\right)*(15)\right]\right\}}-1$ Example: 1999 impact = $\frac{\left\{\left[\left(4.7\%/6.0\%\right)*9.2\%\right]+1\right\}}{\left\{\left[\left(1+4.9\%\right)*(86.5\%)\right]+\left[\left(1+3.9\%\right)*(13.5\%)\right]\right\}}-1=2.3\%$
9.	**Other impact on net written premiums:** this element is not being used in this example. It could be used as the counterpart to item (3) in the premium development calculation.
11.	**Inflation rate that is implicitly embedded in the premium growth levels:** this is the ten year average wage inflation statistic from the Bureau of Labor Statistics Producer Price Index.
12.	**Inflation rate that is implicitly embedded in the loss payout pattern:** this is an average of the ten year average wage inflation statistic from the Bureau of Labor Statistics Producer Price Index and the medical care inflation index from the Bureau of Labor Statistics Consumer Price Index.
13.	**Risk free interest rate underlying asset valuations in current scenario:** this is a stochastically generated future interest rate path, based on a one factor mean-reverting interest rate model.
14.	**Percent of written premium that is earned in first twelve months:** this was calculated from industry statistics in Best's Aggregates and Averages, 1996 edition.
15.	**Percent of written premium that is earned in second twelve months:** the complement of (14).

Accordion effect in payout patterns

The basic payout pattern is the expected pattern for a line of business. For example the basic pattern for homeowners as set forth by the Internal Revenue Service in their 1996 publication of industry tax discount factors was:

Incr:	66.9%	23.6%	2.9%	2.3%	1.7%	1.2%	0.5%	0.4%	0.2%	0.1%	0.1%	0.1%
Cum:	66.9%	90.5%	93..4%	95.7%	97.4%	98.6%	99.1%	99.5%	99.7%	99.8%	99.9%	100.0%

The accordion effect arises when volatility is allowed to occur within the incremental payout percentages. For example, one iteration of the model used in this paper gave rise to the following incremental volatility amounts for the first five homeowners payout increments:

3.8%	-3.6%	0.8%	-0.1%	-0.1%

Combining the basic pattern with the incremental volatility, we derive a new payout pattern, one that may or may not add up to 100%.

Incr:	70.7%	20.0%	3.7%	2.2%	1.6%	1.2%	0.5%	0.4%	0.2%	0.1%	0.1%	0.1%
Cum:	70.7%	90.7%	94.4%	96.6%	98.2%	99.4%	99.9%	100.3%	100.5%	100.6%	100.7%	100.8%

The following table displays the difference in the projected calendar year 1996 payouts for homeowners. The reserve levels were taken from Best's Aggregates and Averages, 1996 edition.

Accident Year	Reserve	1996 payout, based on baseline payout pattern	1996 payout, based on revised payout pattern
1991	287	132	132
1992	470	186	179
1993	628	219	216
1994	1,367	417	501
1995	4,862	3,467	3,230

The formulas for the accident year 1995 payouts are as follows:

Baseline: 4,862 * 23.6% / (100% - 66.9%) = 3,467
Revised: 4,862 * 20.0% / (100.8% - 70.7%) = 3,230

The accordion effect does not change the overall amount that will ultimately be paid out. It does, however, shift when the payouts will occur.

Exhibit 5

Producer Price Index and Consumer Price Index economic inflationary data

	Cars	Change	Med Care	Change	Average Change
Dec-87	112.2		130.1		
Dec-88	116.5	3.8%	138.6	6.5%	5.2%
Dec-89	119.0	2.1%	149.3	7.7%	4.9%
Dec-90	124.1	4.3%	162.8	9.0%	6.7%
Dec-91	127.9	3.1%	177.0	8.7%	5.9%
Dec-92	128.7	0.6%	190.1	7.4%	4.0%
Dec-93	132.9	3.3%	201.4	5.9%	4.6%
Dec-94	135.7	2.1%	211.0	4.8%	3.4%
Dec-95	138.0	1.7%	220.5	4.5%	3.1%
Dec-96	137.0	-0.7%	228.2	3.5%	1.4%
				Average:	**4.4%**

Example of data retrieval from Bureau of Labor Statistics Web site

Consumer Price Index-All Urban Consumers

Series Catalog:

Series ID : CUUR0000SA5

Not Seasonally Adjusted
Area : U.S. City Average
Item : Medical care
Base Period : 1982-84=100

Data:

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Ann
1987	126.6	127.4	128.1	128.7	129.2	129.9	130.7	131.2	131.7	132.3	132.8	133.1	130.1
1988	134.4	135.5	136.3	136.9	137.5	138.2	139.3	139.9	140.4	141.2	141.8	142.3	138.6
1989	143.8	145.2	146.1	146.8	147.5	148.5	149.7	150.7	151.7	152.7	153.9	154.4	149.3
1990	155.9	157.5	158.7	159.8	160.8	161.9	163.5	165.0	165.8	167.1	168.4	169.2	162.8
1991	171.0	172.5	173.7	174.4	175.2	176.2	177.5	178.9	179.7	[illegible]	181.8	182.6	177.0
1992	184.3	186.2	187.3	188.1	188.7	189.4	190.7	191.5	192.3	193.3	194.3	194.7	190.1
1993	196.4	198.0	198.6	199.4	200.5	201.1	202.2	202.9	203.3	[illegible]	204.9	205.2	201.4
1994	206.4	207.7	208.3	209.2	209.7	210.4	211.5	212.2	212.8	214.0	214.7	215.3	211.0
1995	216.6	217.9	218.4	218.9	[illegible]	219.8	220.8	221.6	222.1	222.9	222.5	223.8	220.5
1996	225.2	226.2	226.6	227.0	227.4	227.8	228.7	229.2	229.4	230.1	230.5	230.6	228.2
1997	231.8	232.7	233.4										

Industry Private Passenger Automobile Paid Loss & ALAE data from 1996 Best's Aggregates & Averages

PAID Percent of Ultimate Loss:

Age in Months

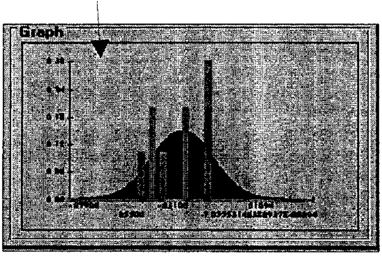
AY	12	24	36	48	60	72	84	96	108	120
1986	32.8%	65.1%	81.2%	90.2%	95.0%	97.4%	98.6%	99.2%	99.5%	99.7%
1987	32.6%	65.3%	81.3%	90.2%	95.0%	97.3%	98.6%	99.2%	99.5%	
1988	33.3%	66.3%	82.2%	90.9%	95.4%	97.7%	98.7%	99.2%		
1989	33.4%	66.7%	82.6%	91.1%	95.6%	97.7%	98.6%			
1990	34.6%	67.6%	83.3%	91.6%	95.7%	97.5%				
1991	35.0%	67.9%	83.7%	91.6%	95.5%					
1992	35.8%	68.9%	83.8%	91.3%						
1993	36.3%	69.0%	83.4%							
1994	37.0%	68.4%								
1995	36.2%									
Selected	36.2%	68.4%	83.4%	91.3%	95.5%	97.5%	98.6%	99.2%	99.5%	99.7%
Incremental										
Selected	36.2%	32.2%	15.0%	7.9%	4.2%	2.0%	1.1%	0.6%	0.3%	0.2%

Incremental Volatility

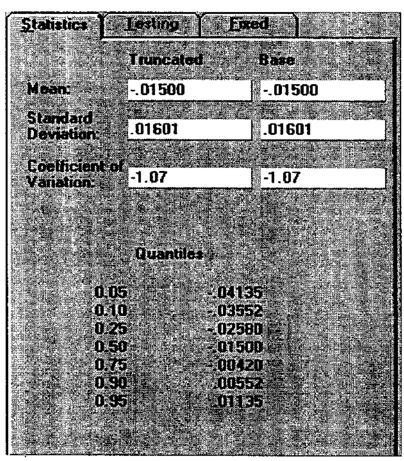
(actual incremental percentage paid amount minus selected incremental payout percentage)

AY	12	24	36	48	60	72	84	96	108	120
1986	-0.034	0.001	0.011	0.011	0.006	0.004	0.001	0	0	0
1987	-0.036	0.005	0.01	0.01	0.006	0.003	0.002	0	0	
1988	-0.029	0.008	0.009	0.008	0.003	0.003	-0.001	-0.001		
1989	-0.028	0.011	0.009	0.006	0.003	0.001	-0.002			
1990	-0.016	0.008	0.007	0.004	-0.001	-0.002				
1991	-0.012	0.007	0.008	0	-0.003					
1992	-0.004	0.009	-0.001	-0.004						
1993	0.001	0.005	-0.006							
1994	0.008	-0.008								
1995	0									

Fitting a Normal distribution to the empirical data set.



Fitting Parameters



Example of the parameter development for Private Passenger Automobile Loss Reserve Volatility

① E&Y Projection of Industry Ultimates based on historical paid loss and ALAE

AY	@12/92	@12/93	@12/94	@12/95	Industry Held Ult at 12/95
1986	26,406,610	26,427,683	26,429,159	26,413,038	26,500,698
1987	29,548,320	29,616,034	29,619,906	29,615,379	29,704,772
1988	32,656,743	32,729,085	32,711,916	32,686,971	32,774,500
1989	35,843,963	35,983,377	36,002,069	35,951,523	36,055,351
1990	38,330,949	38,498,661	38,479,624	38,388,591	38,612,934
1991	37,670,396	38,082,996	38,086,500	37,966,123	38,419,836
1992	39,615,561	40,370,102	40,291,377	40,120,162	41,011,278
1993		43,200,096	43,431,530	43,097,655	44,613,169
1994			47,556,950	46,570,975	48,197,320
1995				49,454,854	50,571,974

② E&Y Projection of Industry Ultimates based on historical incurred loss and ALAE

AY	@12/92	@12/93	@12/94	@12/95	Industry Held Ult at 12/95
1986	26,435,231	26,446,182	26,446,393	26,435,063	26,500,698
1987	29,585,085	29,629,415	29,642,891	29,631,640	29,704,772
1988	32,684,503	32,713,484	32,710,846	32,686,419	32,774,500
1989	35,848,550	35,939,792	35,949,356	35,907,663	36,055,351
1990	38,250,686	38,379,424	38,369,401	38,330,857	38,612,934
1991	37,747,473	37,976,174	37,937,228	37,856,009	38,419,836
1992	39,716,251	40,143,201	40,067,955	39,978,163	41,011,278
1993		43,058,315	43,013,254	42,859,799	44,613,169
1994			46,089,101	45,707,212	48,197,320
1995				47,001,834	50,571,974

③ Retrospective Reserve Levels (12/31/95 Held Ultimate - Paid at 12/31/XX)

AY	@12/92	@12/93	@12/94	@12/95
1986	453,854	290,590	208,933	175,515
1987	883,751	492,229	328,722	243,350
1988	1,595,112	851,005	508,254	356,636
1989	3,321,067	1,699,822	939,433	593,635
1990	6,643,782	3,454,263	1,874,086	1,169,235
1991	12,636,618	6,657,484	3,637,568	2,171,259
1992	26,667,636	13,380,267	7,407,071	4,371,781
1993		28,971,671	14,886,787	8,668,443
1994			30,978,332	16,322,168
1995				32,665,810

④ (E&Y Projection of Industry Ultimates - 12/31/95 Industry Held Ultimate) / Retrospective Reserve Level

	12	24	36	48	60	72	84	96	108	120
1995 paid	-3.4%									
1995 inc	-10.9%									
1994 paid	-2.1%	-10.0%								
1994 inc	-6.8%	-15.3%								
1993 paid	-4.9%	-7.9%	-17.5%							
1993 inc	-5.4%	-10.7%	-20.2%							
1992 paid	-5.2%	-4.8%	-9.7%	-20.4%						
1992 inc	-4.9%	-6.5%	-12.7%	-23.6%						
1991 paid		-5.9%	-5.1%	-9.2%	-20.9%					
1991 inc		-5.3%	-6.7%	-13.3%	-26.0%					
1990 paid			-4.2%	-3.3%	-7.1%	-19.2%				
1990 inc			-5.5%	-6.8%	-13.0%	-24.1%				
1989 paid				-6.4%	-4.2%	-5.7%	-17.5%			
1989 inc				-6.2%	-6.8%	-11.3%	-24.9%			
1988 paid					-7.4%	-5.3%	-12.3%	-24.5%		
1988 inc					-5.6%	-7.2%	-12.5%	-24.7%		
1987 paid						-17.7%	-18.0%	-25.8%	-36.7%	
1987 inc						-13.5%	-15.3%	-18.8%	-30.1%	
1986 paid							-20.7%	-25.1%	-34.2%	-49.9%
1986 inc							-14.4%	-18.8%	-26.0%	-37.4%

⑤ Loss Reserve Volatility

	Age 12	Age 24	Age 36	Age 48	Age 60	Age 72	Age 84	Age 96	Age 108	Age 120
Paid Obs 1	-3.4%	-10.0%	-17.5%	-20.4%	-20.9%	-19.2%	-17.5%	-24.5%	-36.7%	-49.9%
Inc Obs 1	-10.9%	-15.3%	-20.2%	-23.6%	-26.0%	-24.1%	-24.9%	-24.7%	-30.1%	-37.4%
Paid Obs 2	-2.1%	-7.9%	-9.7%	-9.2%	-7.1%	-5.7%	-12.3%	-25.8%	-34.2%	
Inc Obs 2	-6.8%	-10.7%	-12.7%	-13.3%	-13.0%	-11.3%	-12.5%	-18.8%	-26.0%	
Paid Obs 3	-4.9%	-4.8%	-5.1%	-3.3%	-4.2%	-5.3%	-18.0%	-25.1%		
Inc Obs 3	-5.4%	-6.5%	-6.7%	-6.8%	-6.8%	-7.2%	-15.3%	-18.8%		

Fitting a Normal distribution to the Age 24 reserve volatility

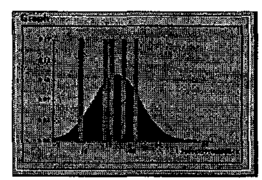


Exhibit 8

Sample Model Output: Beginning Balance Sheet

Insurance Industry Composite
Calendar year 1996 - Actual Results

Balance Sheet Assets (000):	
Bonds	*430,860*
Taxable bonds maturing < 1 year	*15,091*
Taxable bonds maturing 1-5 years	*96,036*
Taxable bonds maturing 5-10 years	*76,423*
Taxable bonds maturing 10-20 years	*26,808*
Taxable bonds maturing 20+ years	*22,964*
Tax-exempt bonds maturing < 1 year	*7,279*
Tax-exempt bonds maturing 1-5 years	*35,285*
Tax-exempt bonds maturing 5-10 years	*55,779*
Tax-exempt bonds maturing 10-20 years	*73,482*
Tax-exempt bonds maturing 20+ years	*21,714*
Stocks	
Preferred stocks	*11,694*
Common stocks	*122,377*
Mortgage loans on real estate	*2,818*
Real estate	
Properties occupied by the company	*7,201*
Other properties	*1,688*
Collateralized Mortgage Obligations	*32,243*
Cash on hand and on deposit	*4,851*
Short-term investments	*37,534*
Other invested assets	*9,885*
Aggregate write-ins for other invested assets	*498*
Subtotal, cash and invested assets	**661,648**
Agents balances < 90 days past due	*16,570*
Premiums booked but not yet due	*32,494*
Accrued retrospective premiums	*6,264*
Funds held by reinsured companies	*3,856*
Bills receivable, taken for premiums	*935*
Reinsurance recoverable on paid losses	*10,711*
Federal income tax recoverable	*0*
Computer equipment	*2,130*
Interest receivable	*8,546*
Receivable from parent, subsidiary, affiliate	*8,065*
Equities and deposits in pools and assoc.	*2,571*
Receivables relating to uninsured A&H plans	*45*
Aggregate write-ins for other assets	*7,635*
Total Net Admitted Assets	**761,473**

Balance Sheet Liabilities (000):	
Loss reserves net of subrogation and salvage	
Reserves gross of discount	*310,240*
Less tabular and nontabular discount	*12,559*
Allocated loss adjustment expense reserves	
Reserves gross of discount	*48,468*
Less tabular and nontabular discount	*278*
Reins. payable on paid Loss & LAE	*2,352*
Unallocated loss adjustment expenses	*13,740*
Contingent commissions	*2,279*
Other expenses (excl. taxes, licenses, fees)	*6,319*
Taxes, licenses, and fees	*2,560*
Federal and foreign income taxes	*716*
Borrowed money	*1,812*
Interest	*49*
Unearned premium	*103,852*
Dividends declared and unpaid	
To stockholders	*296*
To policyholders	*1,546*
Funds held under reinsurance treaties	*8,914*
Amounts retained for account of others	*4,529*
Provision for reinsurance	*3,453*
XS of statutory over statement reserves	*1,434*
Net adjust. due to foreign exchange rates	*623*
Drafts outstanding	*4,914*
Payable to parent, subsidiary, affiliates	*6,278*
Payable for securities	*1,935*
Liability for $ held under uninsured A&H	*0*
Other liabilities	*17,999*
Total liabilities	**531,472**
Write-ins for special surplus funds	*18,277*
Common capital stock	*7,367*
Preferred capital stock	*1,682*
Write-ins for other than special surplus funds	*508*
Surplus notes	*3,087*
Gross paid in and contributed surplus	*91,883*
Unassigned funds (surplus)	*107,506*
Less treasury stock, at cost	
Shares common	*282*
Shares preferred	*28*
Policyholders Surplus	**230,001**
Total Liability + Surplus	**761,473**

Exhibit 9

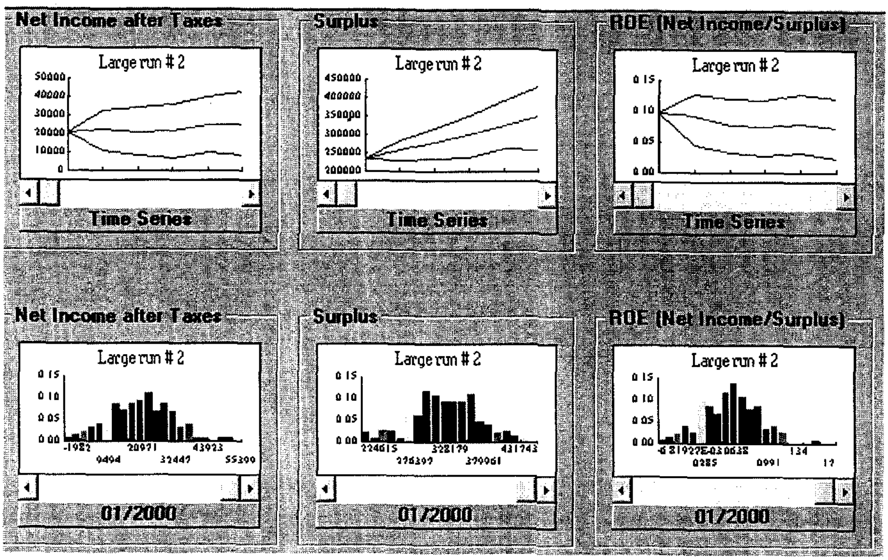
Sample Model Output: Base Case Balance Sheet at end of Five Years

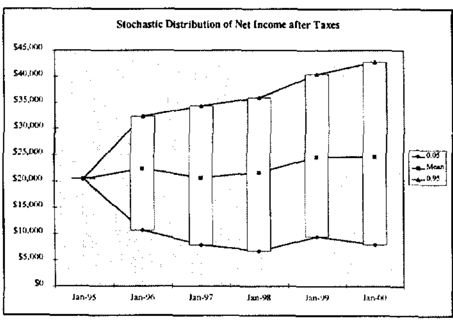
Insurance Industry Composite
Calendar year 2001 Baseline Projection

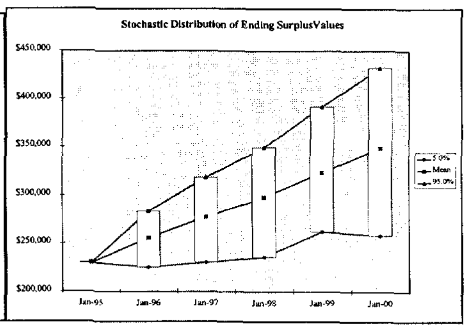
Balance Sheet ***Assets:***	
Bonds	*1,215,673*
Taxable bonds maturing < 1 year	*175,744*
Taxable bonds maturing 1-5 years	*167,062*
Taxable bonds maturing 5-10 years	*194,129*
Taxable bonds maturing 10-20 years	*71,244*
Taxable bonds maturing 20+ years	*60,640*
Tax-exempt bonds maturing < 1 year	*57,238*
Tax-exempt bonds maturing 1-5 years	*81,347*
Tax-exempt bonds maturing 5-10 years	*141,321*
Tax-exempt bonds maturing 10-20 years	*205,661*
Tax-exempt bonds maturing 20+ years	*61,286*
Stocks	
Preferred stocks	*33,720*
Common stocks	*352,869*
Mortgage loans on real estate	*8,124*
Real estate	
Properties occupied by the company	*7,201*
Other properties	*4,867*
Collateralized mortgage obligations	*19,651*
Cash on hand and on deposit	*(43,331)*
Short-term investments	*108,227*
Other invested assets	*28,501*
Aggregate write-ins for invested assets	*498*
Subtotal, cash and invested assets	**1,736,002**
Agents' balances or uncollected premium	
Agents balances < 90 days past due	*9,830*
Premiums booked but not yet due	*19,275*
Accrued retrospective premiums	*3,716*
Funds held by reinsured companies	*3,856*
Bills receivable, taken for premiums	*915*
Reinsurance recoverable on paid losses	*10,791*
Federal income tax recoverable	*0*
Computer equipment	*2,130*
Interest receivable	*18,942*
Receivable from parent, subsidiary, affiliate	*8,065*
Equities and deposits in pools and assoc.	*2,571*
Receivables relating to uninsured A&H plans	*45*
Aggregate write-ins for other assets	*7,635*
Total Net Admitted Assets	**1,823,775**

Balance Sheet ***Liabilities:***	
Loss reserves net of subrogation and salvage	
Reserves gross of discount	*469,824*
Less tabular and nontabular discount	*19,018*
Allocated loss adjustment expense reserves	
Reserves gross of discount	*60,995*
Less tabular and nontabular discount	*365*
Reins. payable on paid Loss & LAE	*2,352*
Unallocated loss adjustment expenses	*19,123*
Contingent commissions	*865*
Other expenses (excl. taxes, licenses, fees)	*4,803*
Taxes, licenses, and fees	*1,289*
Federal and foreign income taxes	*9,865*
Borrowed money	*1,812*
Interest	*49*
Unearned premium	*125,308*
Dividends declared and unpaid	
To stockholders	*(296)*
To policyholders	*2,559*
Funds held under reinsurance treaties	*8,914*
Amounts retained for account of others	*4,529*
Provision for reinsurance	*3,491*
XS of statutory over statement reserves	*736*
Net adjust. due to foreign exchange rates	*623*
Drafts outstanding	*4,914*
Payable to parent, subsidiary, affiliates	*6,278*
Payable for securities	*1,935*
Liability for $ held under uninsured A&H	*0*
Other liabilities	*17,999*
Total liabilities	**728,582**
Write-ins for special surplus funds	*18,277*
Common capital stock	*7,367*
Preferred capital stock	*1,682*
Write-ins for other than special surplus funds	*508*
Surplus notes	*3,087*
Gross paid in and contributed surplus	*91,883*
Unassigned funds (surplus)	*972,698*
Less treasury stock, at cost	
Shares common	*282*
Shares preferred	*28*
Policyholders Surplus	**1,095,193**
Total Liability + Surplus	**1,823,775**

Graphical displays of some output metrics produced by the corporate financial model, based on 250 iterations using dynamic variable parameters as described in the paper.







Dynamic Financial Analysis of a Workers' Compensation Insurer

by David Appel, Ph.D.,
Mark W. Mulvaney, FCAS, MAAA, and
Susan E. Witcraft, FCAS, MAAA

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DYNAMIC FINANCIAL ANALYSIS OF A WORKERS' COMPENSATION INSURER

The purpose of this paper will be to identify the important financial aspects of a workers' compensation insurer and describe their incorporation into a dynamic financial model. The first section of the paper will identify and describe these financial aspects, e.g., claim frequency, claim severity, emergence patterns and investment returns. The second section of the paper will describe one or more approaches (e.g., regression on other variables, autoregression and/or distributions of random values) for incorporating these financial aspects into a dynamic financial model. The third and final section of the paper will identify the data elements needed to parameterize the models described in the second section. This final section will be presented in the format outlined in DFATFOV's Call of Papers: identification of variable, rationale for inclusion, possible source(s) of data and brief descriptions of the analytical methods presented in the second section.

DYNAMIC FINANCIAL ANALYSIS OF A WORKERS' COMPENSATION INSURER

The purpose of this paper is to identify the important financial aspects of a workers' compensation insurer and describe their incorporation into a dynamic financial model. The first section of the paper identifies and describes these financial aspects. That is, it will present brief descriptions of the drivers of workers' compensation financial results. The second section of the paper describes possible approaches for incorporating these drivers into a dynamic financial model. This section will discuss the approaches we have used in practice as well as possible alternatives and refinements to these approaches. The third and final section of the paper identifies the data elements needed to parameterize the models described in the second section. This section is meant to address the issues raised in the Call for Papers.

Key Financial Aspects

To evaluate the financial position of an insurer, it is necessary to keep track of the actual cash values as well as the booked values. The actual cash values will be dependent only on external values, whereas the booked values are dependent upon the various accounting rules that may be in effect (GAAP, statutory and tax), as well as the insurer's perception of the external environment. We have organized the key financial drivers of a workers' compensation insurer into the following categories:

- Premium - including rate level, exposure, payroll inflation, earning pattern and collectibility.
- Losses - including claim frequency, medical and indemnity severity, loss adjustment expenses, payment patterns and reserve adjustments.
- Operating expenses - including fixed and variable components.
- Reinsurance - including pricing and availability.
- Policyholder dividends.
- Investment returns.
- Residual market burdens and other assessments.

We observe that the list of drivers does not vary significantly across lines of insurance. Obviously there are some drivers, such as residual market burdens and separate analysis of medical and indemnity losses, that are significant to workers' compensation but not to all other lines. Nonetheless, an understanding of workers' compensation drivers, modeling approaches and data needs provides significant insight into the corresponding factors for models of other property-casualty lines of insurance.

Dynamic Financial Analysis

One of the refinements that dynamic financial analysis requires as compared to more traditional financial modeling is close attention to the timing of cash flows. Because many analyses focus on investment questions, more accurate projections of cash flows are required. Therefore, in each section below we reference the timing of the various revenue and cost drivers as well as their nominal and accounting values.

Premium

Obviously, premium volume is a key driver of insurer financial results. For all lines of insurance, the two primary components of premium volume are the rate level and the exposure base. For workers' compensation, the exposure base can be decomposed into changes in insured employee count and changes in wage inflation. The timing of the earning and collection of the premium, relative to when it is booked as written on the financial statements affect the income and cash flow statements, respectively.

For workers' compensation, the issues of retrospective premiums and audit premiums contribute to the complexity of modeling premium earning and collection patterns. A sophisticated model will allow premium from policies issued in one calendar year to be written, earned and collected over several years to reflect the timing of audit adjustments and retrospective premiums. We note that many models quite reasonably approximate these patterns by tracking premium booked as written in a year without tying it back to the year in which the policy was actually issued.

Losses and Loss Adjustment Expenses

The key components of workers' compensation losses and loss adjustment expenses are the frequency of claims (per whatever unit of exposure is used to project premium volume), the average cost of medical and indemnity per claim, the amount of allocated loss adjustment expenses, either per claim or per dollar of loss or indemnity, and the amount of unallocated loss adjustment expenses. As with premium, the projections from dynamic financial analyses are highly dependent upon the payment pattern assumption.

An important driver of calendar year results is of course the emergence of losses and loss adjustment expenses by calendar year for each accident or policy year. That is, analyses that are intended to provide insights regarding calendar year results must reflect not only projections of the ultimate cost of claims by accident year and their payment patterns, but also the initial amount reported by the insurer on its financial statements and the adjustments made thereto until the ultimate losses underlying the financial statements equal the actual ultimate results.

Operating Expenses

Operating expenses also affect both the income statement and cash flow results of a workers' compensation insurer. The key components of operating expenses that are often used in dynamic financial modeling include commissions, premium taxes, other acquisition expenses and general expenses. The timing of commission payments generally follows that of premium collection. Premium taxes and other acquisition expenses, on average, are incurred and paid when premium is written, whereas general expenses are usually incurred and paid over the term of the policy. Many expenses are treated differently under GAAP as compared to statutory accounting.

Reinsurance

The true profitability of a book of business can only be evaluated on a net of reinsurance basis. Whether business is modeled gross of reinsurance and netted down or modeled net of reinsurance often depends on the application and the importance of reinsurance to the book of business under review. In modeling reinsurance (or determining whether to model reinsurance separately), it is important to consider the timing and amount of reinsurance premium payments, the timing and amount of ceded loss payments and related recoveries, the impact of ceding commissions (particularly those with profit sharing features), and the collectibility of reinsurance.

Policyholder Dividends

Many workers' compensation insurers offer participating policies to insureds meeting certain size criteria. The resulting dividends are generally dependent upon the loss ratio incurred by each qualifying policyholder. Models need to incorporate the amount of such dividends as well as the timing with which they are incurred (for GAAP), declared and paid.

Investment Returns

Investment income and capital gains, both realized and unrealized, are significant contributors to workers' compensation insurers' financial results. As indicated previously, these values are dependent upon the amount and timing of cash flows from the company's underwriting operations, the economic environment in which the company operates as well as the company's investment strategy.

Residual Market Burdens and Other Assessments

Of lesser importance in the past couple of years, assessments (usually related to residual market mechanisms) can have a critical impact on the profitability of a book of workers' compensation business. As recently as the early 1990s, the residual market burden, on average, was 20% to 25% of voluntary market premium. Insurers also face other types of assessments, such as those from second injury funds and guaranty funds.

Modeling Approaches

The process used to perform a dynamic financial analysis is to first construct a base or expected value case. This base case will generally include the expectations regarding the dependencies among inputs, such as returns on various asset classes and the yield curve. The results are then tested under a relatively large number of different scenarios. For most applications, results are projected under varying economic environments. For workers' compensation, other scenarios to be tested (often simultaneously with changes in the economic environment) might include regulatory or legislative control of rates, changes in residual market burdens and significant changes to benefits.

Economic Scenarios

As indicated, most dynamic financial analyses include testing of a range of economic environments. At a minimum, the economic variables to be projected for each projection year for each scenario are:

- Change in gross domestic product.
- Consumer price inflation.
- Short-term treasury yield rate.
- Long-term treasury yield rate.
- Stock returns and dividend yields.

Assuming these economic variables are randomly generated, other asset returns are determined as functions of these variables. However, for applications focusing on analyses of asset strategies, refinements could be made to (a) include more asset classes, (b) project yield curves in more detail, allowing both convex and concave curves, (c) randomly generate the differentials between each of municipal and corporate bonds and government bonds, and (d) randomly generate the differentials between bonds of various qualities. For workers' compensation, another refinement of interest might be separate modeling of wage and medical inflation, rather than use of constants applied to consumer price inflation.

In the remainder of this section, we will discuss how each of the key drivers identified previously can be modeled in a dynamic financial analysis. For many of these drivers, the models will include formulas that incorporate components of the economic scenarios. We have assumed that randomly generated, internally consistent economic scenarios are available for use in modeling. The derivation of projections of economic variables is beyond the scope of this paper, as many such models have already been developed or can be developed based on methods and information available in economic and finance literature.

Losses and Loss Adjustment Expenses

In our work, we find it useful to first model losses and then project premium as losses divided by a modeled loss ratio. (The approach for modeling loss ratios will be discussed under the premium section.) We have applied models that have a varying range of detail for projecting future workers' compensation losses. In some instances, loss and loss adjustment expenses have been modeled in the aggregate using relatively simple equations, while in other applications we have modeled each major component of losses separately. In this paper, we describe a somewhat complex approach, thereby allowing readers to simplify the model as appropriate for their particular application.

Typically, our projections of workers' compensation losses rely on prior years' losses and changes in a number of variables:

> <u>Changes in the number of workers insured for the total market</u> - These changes can be approximated through use of the projected changes in real gross domestic product, adjusted for any significant changes in the percentage of the market that is self-insured. (We note that

changes in real gross domestic product include both changes in number of workers and changes in productivity and thereby only approximate the changes in the number of insured workers from year to year. The productivity change component of the change in real gross domestic product can be estimated using econometric methods. We are inclined to utilize a long-term average for this component.) The percentage of the market that is self-insured can be modeled based on rate adequacy; that is, when rates are high (i.e., loss ratios used in pricing are low), a greater proportion of the market is likely to be self-insured and vice versa. For a specialty insurer focusing on only a few industries, it may be appropriate to model insured exposure as a function of growth in those particular industries, rather than based on growth in total real gross domestic product.

Changes in the insurer's market share - Generally we assume that changes in market share are random with a serial correlation component. Changes in market share could be ignored entirely for relatively short projections periods. Theoretically, the insurer's market share could be modeled as a function of its expected loss ratio used in pricing relative to that underlying the pricing of the market on average. Although we have not generally done so, a sophisticated dynamic financial model could produce separate estimates of the insurer's expected loss ratio and the market expected loss ratio. Differences in these loss ratios in conjunction with evaluations of the elasticity of demand could drive models of the insurer's market share.

Alternately, insured exposure (i.e., the combination of the growth in the market place with the changes in the insurer's market share) could be based on the estimates implicit in the company's financial plan. The actuary must then identify and quantify the range of possible variations from the company's implicit projections.

Changes in the frequency of each of medical-only, small and severe indemnity claims per injured worker - The frequency of workers' compensation claims per worker has generally decreased in recent years, at least in part as a result of changes in the mix of employees by class. Depending on the insurer's class mix and changes therein relative to the market as a whole, these trends could be extrapolated into the future or adjusted as appropriate. If losses are modeled separately for small and severe claims, the frequency of large claims will be expected to increase relative to that of all claims as the result of inflationary effects. We note that an alternative to adjusting the frequency trend for large claims is to index the threshold above which claims are considered large. (Consistency in trend rates among the retention, small and large claims is critical.)

Consideration should also be given to the impact of exposure growth on both frequency and severity trends. When companies are growing rapidly, both the frequency and severity of claims are more likely to increase faster than under more stable conditions. Further, it is generally believed that claim frequency increases as real gross domestic product increases. As the economy is coming out of a recession, workers generally lengthen their hours thereby increasing the time exposed to injury per worker. Employers then expand the work force with generally less experienced employees who tend to have more injuries.

Changes in average wages - Wage inflation can be modeled either directly by the economic scenario generator or as a function of projected consumer price inflation.

Changes in indemnity benefits for each of small and severe claims - Changes in indemnity benefits arise through two sources. One is the normal adjustments to maximum and minimum benefits that are frequently made in response to changes in wages. These changes can occur automatically or by the specific act of a state legislature. We prefer to model these changes through the indemnity trend rate. Note that this treatment may result in indemnity trends different from those produced by NCCI, as those values typically represent indemnity trends adjusted to constant law level.

The second source of indemnity benefit change is benefit reform. Benefit reforms occur randomly, generally when rates are perceived to be too high relative to the benefits. These types of changes can be expected to occur sporadically and would typically have a larger impact on losses than the normal adjustments discussed above. An approach to modeling this component is to assume that indemnity benefit reform always produces a savings and occurs with low frequency (probability increases after several years of both high rates and high loss ratios). The theory here is that workers' compensation involves three parties that significantly contribute to potential legislation: insurers, employers, and labor. We assume that two of these parties must support legislation for it to pass. This criterion will usually only be met when employers complain of high cost (rates) and insurers complain of low profitability (high loss ratios).

We acknowledge that labor can achieve benefit increases, but these increases tend to be smaller and more frequent minor changes to benefit structures or administrative or court decisions that increase benefits. For modeling, we include these changes in the underlying trend rate. The combined impact of the underlying trend rate and benefit reform adjustments then produce long-term averages consistent with observed experience.

Non-benefit changes in the cost of indemnity per claim - Historically, average costs of indemnity claims have increased slightly faster than wage inflation, even after adjustment for benefit changes. Thus, indemnity claim cost trend rates are generally modeled as a function of wage inflation.

Changes in medical benefits - In a similar manner to the indemnity benefit changes discussed above, we prefer to model medical benefit changes only for benefit reforms. Even in states with medical fee schedules, we find that adjustments are typically responsive to inflationary pressures and can therefore be modeled as part of the underlying trend rate. Medical benefit reforms may include changes in administration, broad changes to managed care provisions, choice of physician, etc. These changes would be expected to occur sporadically in response to the conditions identified above for indemnity benefit changes.

Non-benefit changes in the cost of medical per medical-only, small and severe claim - The changes in the average cost of medical on medical-only, small and severe claims is generally modeled as a function of medical inflation.

We have found it valuable to model each of medical and indemnity on each of medical-only claims and small indemnity claims in the aggregate and on severe indemnity claims individually. As such, in addition to incorporating the above relationships into our models, we also incorporate random error terms for each of medical-only and small indemnity losses in the aggregate and the number of large claims. Each of these error terms, as well as the size of large claims, will be randomly selected from a user-defined distribution for each projection year for each iteration. Modeling losses in this fashion can incorporate elements of both process risk and parameter risk. Note that the goal of the modeling process at this point is to determine the initial value to be booked as losses. This value may be allowed to change over time as future economic conditions affect the values that may have been initially booked. Approaches for modeling these adjustments are discussed later in this paper.

The model of losses for a single projection year for a single scenario might be as follows:

1. Generate the number of workers insured in the market using a formula such as:

$$NW_i = NW_{i-1} \, x \, [a + b\,\Delta GDP_i] + e_i \qquad [1]$$

where
- a and b are constants;
- i refers to the policy year;
- NW refers to number of workers;
- ΔGDP is the percentage change in the gross domestic product; and
- e is a randomly generated error term.

2. Generate the insurer's market share using a formula such as:

$$MKSH_i = a + b\,MKSH_{i-1} + e_i \qquad [2]$$

where
- a and b are constants;
- $MKSH$ is market share; and
- e is a randomly generated error term.

3. Generate the frequencies per insured worker of medical-only, small and large indemnity claims as:

$$FRMO_i = a + b\,FRMO_{i-1} + c\left(\frac{\Delta GDP_i}{\Delta GDP_{i-1}} - 1\right) + d\,\Delta MKSH_i + e_i$$ **[3]**

$$FRSM_i = f + g\,FRSM_{i-1} + h\,(FRMO_i - FRMO_{i-1}) + j_i$$

$$FRLG_i = k + l\,FRLG_{i-1} + m\,(FRSM_i - FRSM_{i-1}) + n_i$$

where *a, b, c, d, f, g, h, k, l* and *m* are constants;
FRMO is frequency of medical only claims;
FRSM is frequency of small claims;
FRLG is frequency of large claims;
ΔGDP is the percentage change in the gross domestic product;
ΔMKSH is the percentage change in market share; and
e, j and *n* are randomly generated error terms.

4. Generate the average cost of medical on medical-only, small and large indemnity claims as:

$$MSVMO_i = a + b\,MSVMO_{i-1} \times (1 + c\,(emcpi_i)) \times (1 + d\,mb_i) + e_i$$ **[4]**

$$MSVSM_i = f + g\,MSVSM_{i-1} \times (1 + h\,(emcpi_i)) \times [1 + dursm(emcpi_i - emcpi_{i-1})] \times (1 + j\,mb_i) + k(MSVMO_i - MSVMO_{i-1}) + l_i$$

$$MSVLG_i = m + n\,MSVLG_{i-1} \times (1 + o\,(emcpi_i)) \times [1 + durlg(emcpi_i - emcpi_{i-1})] + q\,(MSVMO_i - MSVSM_{i-1}) + r_i$$

where *a, b, c, d, f, g, h, j, k, m, n, o, p and q* are constants;
MSVMO is the average cost of medical only claims;
MSVSM is the average cost of medical on small indemnity claims;
MSVLG is the average cost of medical on large indemnity claims;
dursm is the duration of the medical payment pattern on small indemnity claims at the average medical inflation rate;
durlg is the duration of the medical payment pattern on large indemnity claims at the average medical inflation rate;
emcpi is the expected medical trend used in pricing and is determined as:

$$emcpi_i = a + b\,mcpi_{i-1} + c\,(emcpi_{i-1})$$

mcpi is the medical component of the consumer price index;
mb is the impact on medical of medical benefit reforms; and
e, l and *r* are randomly generated error terms;

5. Generate the average cost of indemnity on small and large indemnity claims as:

$$ISVSM_i = a + b\, ISVSM_{i-1}\, x\, (1 + c\,(ew_i))\, x\, (1 + d\,ibs_i) + f\,(MSVSM_i - MSVSM_{i-1}) + g_i \qquad [5]$$

$$ISVLG_i = h + j\, ISVLG_{i-1}\, x\, (1 + k\,(ew_i))\, x\, (1 + l\,ibl_i) + m\,(ISVSM_i - ISVSM_{i-1}) + n\,(MSVLG_i - MSVLG_{i-1}) + o_i$$

where $a, b, c, d, e, f, h, j, k, l, m$ and n are constants;
$ISVSM$ is the average cost of indemnity on small indemnity claims;
$ISVLG$ is the average cost of indemnity on large indemnity claims;
ew is expected indemnity trend used in pricing and is estimated as:

$$ew_i = a + b\,w_{i-1} + c\,(ew_{i-1})$$

w is wage inflation;
ibs_i is the indemnity benefit change on small claims in year i;
ibl_i is the indemnity benefit change on large claims in year i;
$MSVSM$ is the average cost of medical on small indemnity claims;
$MSVLG$ is the average cost of medical on large indemnity claims; and
g and o are randomly generated error terms.

6. Generate the actual medical and indemnity cost of each large claim from a joint size of loss distribution with

$$mean = (MSVLG_i,\ ISVLG_i) \qquad [6]$$

Generate $FRLG_i\ x\ NW_i\ x\ MKSH_i + e_i$ of these large claims.

7. Calculate medical and indemnity losses as:

$$INDLOSS = MKSH_i\, x\, NW_i\, x\, FRSM_i\, x\, ISVSM_i + \sum_{j=1}^{FRLG_i\, x\, NW_i\, x\, MKSH_i + e_i} LARGE_{j,\,ind} \qquad [7]$$

$$MEDLOSS = MKSH_i\, x\, NW_i\, x\, (FRMO_i\, x\, MSVMO_i + FRSM_i\, x\, MSVSM_i) + \sum_{j=1}^{FRLG_i\, x\, NW_i\, x\, MKSH_i + e_i} LARGE_{j,\,med}$$

$$LOSS = INDLOSS + MEDLOSS$$

where $LARGE_{j,\,med}$ and $LARGE_{j,\,ind}$ refer to the medical and indemnity losses on the large claims generated in [6].

We usually model allocated loss adjustment expenses as a function of indemnity losses, although the recent introduction of managed care fees as part of allocated loss adjustment expenses suggests that total losses may be a more appropriate projection base in some instances. Unallocated loss adjustment expenses can be projected as a function of total losses or claim counts by type of claim (medical-only, small or large indemnity). More refined models projecting changes in average loss adjustment expenses per claim as functions of economic variables and mix of claims could also be developed.

The above process results in estimates of ultimate policy year losses and loss adjustment expenses as they are initially reported in the company's financial statements. To develop income statements, the underlying exposure must first be assigned to the year in which the related premium is earned. Premium earning patterns are discussed in a later section.

For each accident year, estimates of ultimate medical and indemnity should be adjusted over time to reflect two factors:

(1) The difference between the initially reported value and changes that are expected to occur due to economic conditions that subsequently become known.

(2) Any historically observed reporting biases that are expected to persist for future accident years.

To calculate the first adjustment, it is necessary to determine the medical and indemnity payment patterns. We find it convenient to represent the payment pattern by a zero inflation rate, then modify it by subsequently observed inflation for the components of losses that may be sensitive to inflation (medical and, if weekly benefits escalate, indemnity). Variability in the zero inflation rate payment pattern can be reflected by either randomly selecting from a set of pre-determined payment patterns with associated probabilities (say slow, medium, fast) or utilizing a distribution with randomly selected parameters. Under the latter approach, a theoretical distribution is used to model claim payments, such as a Poisson distribution with mean equal to the average lag between accident year and calendar year of payment. For a Poisson with mean 3, the payment pattern would be:

Paid in AY +	0	1	2	3	4	5	6	7	8	9
Percent	5%	15%	22%	22%	17%	10%	5%	2%	1%	1%

Obviously, a longer tailed distribution, such as Lognormal, Weibull, or Pareto, might be more appropriate for modeling unlimited workers' compensation loss payment patterns. Refinements to random selection of the parameters of the distribution include making the mean parameter dependent upon one or more of the actual loss ratio, the percentage of losses emanating from large claims or the mix of losses between medical and indemnity. We generally expect that the payment pattern will lengthen when losses are higher than expected or there are more large claims than average. Alternatively, separate payment patterns could be modeled for each of medical-only, small and large indemnity claims.

Once the zero inflation rate payment patterns have been derived, the formula for the booked reserves (for medical) at the kth evaluation date for accident year i would be:

$$BOOKED_{i,k} = MEDLOSS_i \, x \qquad [8]$$

$$\frac{\sum_{j=1}^{k} NOINFPP_{i,j} \prod_{l=1}^{j}(1+mtrend_{i+l-1}) + \left(\prod_{l=1}^{k}(1+mtrend_{i+l-1})\right)\left(\sum_{j=k+1}^{\infty} NOINFPP_{i,j}(1+emcpi_{i+k})^j\right)}{\sum_{j=1}^{\infty} NOINFPP_{i,j}\,(1+emcpi_i)^j}$$

where $MEDLOSS_i$	is the initial estimate of ultimate medical losses for Accident Year i;
$NOINFPP_i$	is the randomly selected zero inflation rate payment pattern for medical for Accident Year i;
mtrend	is the observed medical claim cost trend rate which we model as a function of the medical component of the consumer price index; and
emcpi	is the estimated medical inflation rate used in pricing.

Any historical bias could then be incorporated, as desired. For example, if a company has had the tendency to book initial reserves that lead to a redundancy of 5% of ultimate losses and reduce that redundancy over five years, booked ultimate medical losses for Accident Year i at evaluation date k could be modified as:

$$ADJBOOKED_{i,k} = \begin{cases} BOOKED_{i,k} + (0.05 - 0.01k)\, ULTMEDLOSS_i & k < 5 \\ BOOKED_{i,k} & k \geq 5 \end{cases} \qquad [9]$$

where $BOOKED_{i,k}$	is as derived in [8] above; and
$ULTMEDLOSS_i$	is the ultimate medical losses as defined below in [10].

The actual dollar amount of ultimate medical losses for any given accident year would be calculated as:

$$ULTMEDLOSS_i = MEDLOSS_i \frac{\sum_{k=1}^{\infty} NOINFFP_{i,k} \prod_{j=1}^{k} (1 + mtrend_{i+j-1})}{\sum_{k=1}^{\infty} NOINFFP_{i,k} (1 + emcpi_i)^k} \qquad [10]$$

where $MEDLOSS_i$	is the initial estimate of ultimate medical losses for Accident Year i;
$NOINFPP_i$	is the randomly selected zero inflation rate payment pattern for medical for Accident Year i;
mtrend	is the observed medical claim cost rate; and
emcpi	is the estimated medical inflation rate used in pricing.

Ultimate indemnity losses would be calculated similarly, using indemnity claim cost trend, wage inflation and indemnity trend used in pricing instead of the corresponding values for medical. The analyst must exercise care, however, to distinguish between the accident year component and the calendar year component on indemnity losses. Only the calendar year component of these values (if any) should be used to adjust the ultimate losses, booked reserves, and payment pattern.

Last, models of loss and loss adjustment expense payments need to be developed. As discussed above, we consider a two-step approach to modeling payment patterns. In the first step, parameter risk is addressed either through random selection of a pre-determined payment pattern or through random selection of the parameters of a selected distribution. The result of this first step is the zero inflation rate payment pattern. The second step incorporates adjustments for the actual inflation observed during the payment period. For medical, the kth increment of the payment pattern for the ith accident year to be applied to the actual ultimate medical losses (ULTMEDLOSS) is:

$$PAYPATT_{i,k} = \frac{NOINFPP_{i,k} \prod_{j=1}^{k} (1 + mtrend_{i+j-1})}{\sum_{l=1}^{\infty} NOINFPP_{i,l} \prod_{j=1}^{l} (1 + mtrend_{i+j-1})} \qquad [11]$$

where $NOINFPP_i$	is the randomly selected zero inflation rate payment pattern for medical for Accident Year i; and
mtrend	is the observed medical claim cost rate.

A similar calculation would be made for indemnity payments.

Premium

We model premium based on a modeled expected loss ratio implicit in the rates actually charged. Expected losses (based on the insured exposure, prior year losses, and recent inflation and interest rates) are divided by the expected loss ratio to derive premium. We note that the expected losses used in the premium determination differ from the actual modeled policy year results because these expected losses are those expected at the time that the policy is priced.

Expected losses based on information available at the time of pricing first need to be estimated. Expected losses could be calculated using the algorithm laid out in Formulas [1] through [7] with the exceptions that the current year error terms would be excluded and:

$$\sum_{j=1}^{FRLG_i x NW_i x MKSH_i + e_i} LARGE_{j,med} + LARGE_{j,ind} \qquad [12]$$

would be replaced by

$$MKSH_i \ x \ NW_i \ x \ FRLG_i \ x \ (MSVLG_i + ISVLG_i)$$

in Formula [7].

These expected losses would then be divided by an expected loss ratio underlying the insurer's market rates. This loss ratio is influenced by interest rates (as they determine the discount factor used to calculate the underwriting profit margin), prior year(s') expected loss ratios, changes in desired market share, and a random error term. If more than one autoregressive term (i.e. more than one prior year's expected loss ratio) is included in the formula, underwriting cycles can be modeled. The model for the expected loss ratio used in pricing would then take the following form:

$$ELR_i = a + \sum_{j=1}^{n} b_j ELR_{i-j} + c\,(int_{i-1} - int_{i-2}) + d\,\Delta MKSH_i + e_i \qquad [13]$$

where a, b, c and d	are constants;
ELR	is the expected loss ratio;
n	is the number of autoregressive terms in the model;
int	is the short-term government yield;
$\Delta MKSH$	is the percentage change in market share; and
e	is a randomly generated error term.

In theory, we expect that the c coefficient will approximate the duration of the loss payment pattern.

The next step in modeling premium is to allocate the policy year premium across calendar years through the use of an earning pattern. In theory, a multi-year earning pattern should be used to reflect the earning of audit and retrospective premiums. Further, the actual booking of premium as written could be lagged to reflect many insurers' practice of recording written premium on a monthly basis. In practice, we generally look at the company's statutory financial statements and compare the unearned premium reserve with calendar year written premium to derive an estimate of the percentage of a calendar year's written premium that is earned during that calendar year and the portion remaining to be earned in the subsequent calendar year. As a result of the booking of premiums as written as monthly installments are made, we generally find that 80% to 90% of workers' compensation premium written in a calendar year is also earned in that calendar year. By comparison, for other lines of business with annual policies, we generally find that approximately 50% of premium is earned in the year it is written.

Models must consider the timing and amount of premium collected. Similar to the approach used for premium earning patterns, we generally review agents' balances as a percentage of calendar year written premium to estimate the percentage of premium written in a calendar year that is also collected in the year. We generally assume that the remainder of the premium is collected in the subsequent calendar year.

For many insurers, the percentage of premium that is never collected is immaterial. For other insurers, uncollectible premium is of significant concern. For insurers in the latter category, models of the expected value of the percentage of premium that is uncollectible and variability therein can be constructed. We expect that the percentage of uncollectible premium is negatively correlated with changes in gross domestic product and positively correlated with interest rates. When the gross domestic product increases at a lower rate than average, more insureds would be expected to experience financial difficulties and therefore default on premium payments. When interest rates are high, insureds are more likely to purchase paid loss retro policies, thereby increasing insurers' credit risk. As such, the formula for the percentage of premium that is uncollectible might take the form of:

$$PU_i = a + b\,PU_{i-1} + c\,(\Delta gdp_i - \Delta gdp_{i-1}) + d\,(int_i - int_{i-1}) + e_i \qquad [14]$$

where a, b, c and d are constants;
PU_i is the percent of premium from policy year i that is uncollectible;
Δgdp is the percentage change in real gross domestic product; and
int is the short-term interest rate.

Operating Expenses

We find it practical to model the fixed and variable components of expenses separately. Occasionally, we model commissions and premium taxes separate from all other variable expenses. We have found that it is reasonable to model commission and premium tax rates as constants over time. One possible refinement is to have commission rates vary with the expected loss ratio or whatever other measure of the competitive marketplace is used. That is, in very competitive markets, insurers may pay higher than usual commission rates to maintain growth targets.

We model all other variable expenses as a constant percentage possibly with a random error term. The distribution and standard deviation of the error term can be derived from historical company experience or the experience of other insurers with similar characteristics.

Fixed expenses (actually, all expenses that are not charged as a percentage of premium) are usually dependent upon inflation rates and, to a limited extent, changes in exposure. Our fixed expense models take the form of:

$$FE_i = FE_{i-1} \; x \; (1 + cpi_i) \; x \left(a + b \frac{NW_i \; x \; MKSH_i}{NW_{i-1} \; x \; MKSH_{i-1}} \right) + e_i \qquad [15]$$

where a and b are constants;
FE is fixed expenses;
cpi is consumer price inflation;
NW is the number of workers;
$MKSH$ is market share; and
e is a randomly generated error term.

Reinsurance

Our models of ceded reinsurance have been relatively simple. We first assume that the reinsurance terms (i.e., premium rate, attachment point, participation, and commission schedule) are constant over time and across scenarios. When determining ceded losses, we apply the reinsurance terms to each of the large claims individually and, for quota share treaties, to small claims in the aggregate. We also model the payment pattern of cessions separate from the payment pattern of direct losses. Once ceded losses have been modeled, we can then calculate any sliding scale premium or commission adjustments.

Refinements to our simple models might include the incorporation of a pricing cycle for reinsurance (i.e., increases and declines in the price of reinsurance relative to ceded losses and commissions), and changes in the retention and in reinsurer quality that are sensitive to that pricing cycle. That is, as reinsurance rates increase, an insurer might increase its retention to reduce ceded premium or it might purchase reinsurance from less expensive and, presumably, lower quality reinsurers. In the latter case, the issue of collectibility of reinsurance must be addressed.

Policyholder Dividends

We model policyholder dividends as functions of the premium volume and loss ratio on policies written in each year. That is, policyholder dividends can be calculated using a formula such as:

$$PD_i = GEP_i \times (a + bLR_i) + e_i \qquad [16]$$

where a and b are constants;
PD is policyholder dividends incurred;
GEP is gross earned premium;
LR_i is the estimate of the accident year i loss ratio at the end of calendar year $i+1$; and
e is a randomly generated error term.

We incur policyholder dividends in the year premium is earned, declare them in the following year and pay them in the year after that. The timing of declaration and payment will vary across insurers. For participating or retrospectively rated policies, the above formula could be reevaluated at successive evaluation dates to more closely follow the actual flow of dividend or retrospective premium payments between insurers and insureds.

Investment Yields

We model investment yields based on the output of our economic scenario generator. The output of the economic scenario generator includes short and long term interest rates and S&P 500 returns and dividend yields. We interpolate between the short and long term rates to model yields on government bonds and apply factors for municipal and corporate bonds. We usually assume that insurers' stock portfolios are sufficiently diversified to use the S&P 500 total return and dividend yields and we control for bond defaults by using a default function that is conditional on gross domestic product. Other considerations in modeling assets depend on the mix of assets held. For example, the sensitivity of prepayment rates to interest rates is important for an insurer with a significant mortgage-backed security holding. As there is significant literature available regarding assets and the economy, we will not expand upon these relationships further in this paper.

Residual Markets

The driving factor for residual market assessments is the perception by the market place of overall rate adequacy. If it is believed that rates are inadequate, particularly when the cause is regulatory rate suppression, the size of the residual market will increase. If rates for the residual market are low (we expect that rate adequacy in the competitive and residual market are positively correlated), the loss ratio in the residual market will also grow. However, the increase in loss ratios may be offset to some extent because, as the residual market expands, the quality of insureds may improve and the loss ratio may be lower than if the residual market were smaller. Nonetheless, we generally expect that the deficit as a percentage of residual market premium will grow. There will be a compounding effect when the deficit is compared to the premium in the competitive market, because, as indicated, it is expected that the residual market will have a greater market share when these conditions are present.

We model residual market burdens as being positively correlated with both the expected loss ratio in pricing (which affects residual market size) and the difference between the actual loss ratio experienced by the insurer and the expected loss ratio from pricing (which, if assumed to also indicate that the residual market experience was better or worse than average, would be expected to be indicative of the residual market burden as a percentage of residual market premium). A further refinement is possible when insurer market share is explicitly modeled in projecting losses. In this situation, the insurer's participation in the residual market can be modeled as being positively correlated with the size of its residual market burden. The formula for the residual market burden will take the form of:

$$RM_i = MKSH_i \ x \ RMMS_i \ x \ NW_i \ x \ (a\,PPI_i + b) \ x \ (c + d\,LR_i) + e_i \qquad [17]$$

where a, b, c and d are constants;

RM_i	is the residual market burden emanating from premium earned by the residual market in year i;
$MKSH$	is the insurer's share of the insurer;
$RMMS$	is the market share of the residual market;
NW	is the number of insured workers in the market;
PPI_i	is the insurer's premium per insured worker which is presumed to bear some relationship to the residual market's average premium per insured worker;
LR_i	is the insurer's ultimate loss ratio on premium earned in year i; and
e	is a randomly generated error term

To model the timing of the cash flow impact of residual market burdens, we occasionally model residual market participation (such as business assumed from the National Pool) as a separate line of business. This approach allows for more refined projections of the timing of payment of residual market burdens.

Data Requirements

The types of data needed to develop the models underlying the dynamic financial analysis depend heavily on the level of detail used in the dynamic financial analysis. In this section of the paper, we present an inventory of the data that would be valuable in developing the parameters for a highly detailed model that incorporates all of the features described in the previous section. To the extent that a simpler model is being applied, less data are needed.

The data listed in this section would be used primarily to develop the parameters (constants) in the formulas presented and/or described in the previous section and to develop the distributions of losses by size and the distributions of the error terms. The "Formulas" column in the tables that follow provide references to the previously presented formulas whose parameters might be dependent on each data element. The "Possible Source" column indicates whether we found the desired information on the CAS DFA web site. If not, alternative sources are suggested. Of course, to the extent that the data for a specific insurer were available and credible, we would rely upon them before looking to these industry sources.

Economic Indices

	Frequency	Formulas	Possible Source
Interest rates for each annual maturity on government bonds	Annual	ESG*, [13], [14]	CAS DFA Web Site
Interest rates at each annual maturity on corporate bonds of each quality of interest for modeling	Annual	ESG	CAS DFA Web Site
Interest rates at various maturities on municipal bonds	Annual	ESG	Moody's
Annual default rates for corporate bonds of each quality	Annual	ESG	Prof. Altman-NYU Moody's/ S&P
Annual default rates for municipal bonds	Annual	ESG	Moody's/ S&P
S&P 500 total return	Annual	ESG	S&P/ Ibbotson Associates
S&P 500 dividend yield	Annual	ESG	S&P/ Ibbotson Associates
Consumer price index	Annual	ESG, [15]	CAS DFA Web Site
Wage inflation countrywide and by state	Annual	ESG, [5], [8], [10], [11]	NCCI Annual Statistical Bulletin

	Frequency	Formulas	Possible Source
Medical component of the consumer price index countrywide and by region	Annual	ESG, [4], [8], [10], [11]	M&R Health Cost Index Database
Gross domestic product	Annual	ESG, [1], [3], [14]	CAS DFA Web Site
Number of workers covered by commercially insured workers' compensation programs by state	Annual	ESG, [1]	

* ESG = Economic Scenario Generator

Losses and Loss Adjustment Expenses

	Frequency	Formulas	Possible Source
Market share of each insurer in each state and nationwide	Annual	[2], [3], [13], [15], [17]	Statutory Annual Statement Page 14 **
Number of medical-only claims per worker by state	Policy or accident year	[3]	NCCI Annual Statistical Bulletin
Number of small indemnity claims per worker by state	Policy or accident year	[3]	NCCI Annual Statistical Bulletin
Number of large indemnity claims per worker by state	Policy or accident year	[3]	NCCI Annual Statistical Bulletin

** Possibly distorted for servicing carriers by residual market premium.

	Frequency	Formulas	Possible Source
Average cost of medical-only claims by state	Policy or accident year	[4]	NCCI Annual Statistical Bulletin
Average cost of medical on small indemnity claims by state	Policy or accident year	[4], [5]	NCCI Annual Statistical Bulletin
Average cost of indemnity on small indemnity claims by state	Policy or accident year	[5]	NCCI Annual Statistical Bulletin
Average cost of medical on large indemnity claims by state	Policy or accident year	[4], [5]	NCCI Annual Statistical Bulletin
Average cost of indemnity on large indemnity claims by state	Policy or accident year	[5]	NCCI Annual Statistical Bulletin
Distribution of combined medical and indemnity losses on large indemnity claims by state	Policy or accident year	[6]	NCCI Excess Loss Premium Factor Calculation
Changes in medical benefits by state	Annual	[4]	NCCI Annual Statistical Bulletin
Changes in indemnity benefits on small indemnity claims by state	Annual	[5]	NCCI Annual Statistical Bulletin

	Frequency	Formulas	Possible Source
Changes in indemnity benefits on large indemnity claims by state	Annual	[5]	NCCI Annual Statistical Bulletin
Allocated loss adjustment expenses by state	Policy or accident year	ALAE model	Rate filings
Unallocated loss adjustment expenses by state	Calendar year	ULAE model	Rate filings
Reserve adjustments by calendar year	Accident Year	[8], [9]	Statutory Annual Statement Schedule P Part 2D
Paid medical, indemnity and allocated loss adjustment expense triangles by state	Accident Year	[8], [10]	Rate filings or NCCI Annual Statistical Bulletin

Premium

	Frequency	Formulas	Possible Source
Expected loss ratio underlying rates by state	Policy year	[13]	Rate filings
Medical and indemnity trend rates underlying pricing by state	Policy year	[8], [10], [11]	Rate filings
Direct premium uncollectible by state	Policy year	[14]	

Expense

	Frequency	Formulas	Possible Source
General expenses by insurer	Calendar year	[15]	Rate filings

	Frequency	Formulas	Possible Source
Commissions by insurer by state	Calendar year	Variable expense model	Rate filings
Premium taxes, licenses and fees by state	Calendar year	Variable expense model	Rate filings, NCCI Annual Statistical Bulletin, State agencies
Other acquisition expenses by insurer by state	Calendar year	Variable expense model	Rate filings

Reinsurance

	Frequency	Formulas	Possible Source
Projected ultimate workers' compensation ceded loss ratios for each of quota share and excess insurance (preferably for ranges of attachment points)	Accident or policy year	Ceded premium and loss models	
Development of workers' compensation ceded paid losses for excess insurance (preferably for ranges of attachment points)	Accident or policy year	Ceded loss model	
Summary of reinsurers not meeting obligations and credit rating in each of five years prior to defaulting on obligations		Ceded loss model	

Policyholder Dividends

	Frequency	Formulas	Possible Source
Policyholder dividends with corresponding policy year loss ratios for workers' compensation by insurer	Policy year	[16]	

Residual market

	Frequency	Formulas	Possible Source
Emergence of residual market burdens by state	Calendar year by policy year	[17]	National and State Pool Mgmt. Reports
Residual market premium by state	Policy year	[17]	National and State Pool Mgmt. Reports
Residual market direct loss ratio by state	Policy year	[17]	National and State Pool Mgmt. Reports

Managing the Tax Liability of a Property-Liability Insurance Company

by Richard A. Derrig, Ph.D., and Krzysztof M. Ostaszewski

MANAGING THE TAX LIABILITY OF A PROPERTY-LIABILITY INSURANCE COMPANY

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Abstract

The income tax burden placed upon a property-liability insurance company creates a variable liability with profound effects on the functioning of the enterprise. It directly affects product pricing and asset investment policies and, therefore, the potential profitability of the insurer. Recent research works have identified fuzzy sets theory as a potentially useful modeling paradigm for insurance uncertainty -- in claim cost forecasting, underwriting, rate classification, and premium determination. We view the insurance liabilities, properly priced, as a management tool of the short position in the government tax option. To implement that tool, we propose a new method of measuring uncertainty of taxes. Critical parameters of underwriting and investment are modeled as fuzzy numbers, leading to a model of uncertainty in the tax rate, rate of return and the asset-liability mix.

Keywords

Insurance, Taxes, Rate of Return, Fuzzy Sets, Investments, Swaps, Derivatives

INTRODUCTION

In this work[1], we analyze the tax management policy of a property-liability insurance company. Myers' Theorem (1984) implies that the present value of the expected tax liability , the government's tax option, is determined solely by the effective tax rate and the risk free rate. Therefore, controlling the effective tax rate of the firm is crucial in its financial management. A firm that can craft a lower effective tax rate than its competitors does enjoy a competitive advantage, but in competitive equilibrium this lowering of tax rates is achieved by all firms, and results in lower premium rates. We suggest some alternative methods of lowering the effective tax rate through the use of swaps with a life insurance firm. We also examine the uncertainty of the tax rate by proposing fuzzy sets methodology for modeling that uncertainty. Our analysis implies that uncertainty is indeed quite great, and may be underestimated under other methodologies.

MYERS' THEOREM AND ITS IMPLICATIONS

We assume that an insurance corporation holds an asset portfolio yielding a one-period investment return, and is subject to a tax liability on realized income. We also assume a simple Capital Asset Pricing Model market. Let T be the effective tax rate on the investment income, for now taken to be known with certainty.

Myers' Theorem (1984) says that the risk-adjusted present value of the tax liability on investment income from a risky investment portfolio held by a corporation is

$$PV(T\tilde{r}_A) = \frac{Tr_F}{1 + r_F} \qquad (1)$$

where $\tilde{r}_A$ is the rate of return on the risky portfolio, while r_F is the risk-free rate of return. In other words, the present value of the tax liability on the risky return is calculated as if that return were the risk free rate. The present value of the tax liability is independent of the investment strategy, and determined solely by the effective tax rate and the risk free rate.

Derrig (1994) notes that the tax liability itself is not risk free. In fact, the beta of the tax can be determined to be:

$$\beta_{TAX} = \beta_A \frac{1 + r_F}{r_F}, \qquad (2)$$

where β_A is the beta of the risky asset utilized by the company's investment strategy. Note that unless that asset is risk free, or the risk free rate equals zero, $\beta_{TAX} > \beta_A$.

[1] The second author received research funding from the College of Arts and Sciences of the University of Louisville. The authors also gratefully acknowledge the research assistance of Daniel Scala, the production assistance of Julie Jannuzzi and the comments of an anonymous referee.

The present value of the after-tax final investment holdings of the corporation equals

$$PV(1+(1-T)\tilde{r}_A) = \frac{1+(1-T)r_F}{1+r_F} \qquad (3)$$

and the after-tax beta of the risky portfolio is:

$$\beta_{AFTER-TAX} = \beta_A \frac{(1-T)(1+r_F)}{1+(1-T)r_F}. \qquad (4)$$

The implication of these results is that the effective tax rate and the risk free rate fully determine the present value of the expected investment tax liability, and when combined with the market riskiness of the investment portfolio, the after-tax, effective, riskiness of that portfolio.

Following Myers, we consider a one-period insurance company market value balance sheet at the time a policy is issued:

ASSETS	*LIABILITIES*
Asset Value	Present Value of Expected Losses and Expenses
(Premium + Equity Invested)	Present Value of Underwriting Tax
	Present Value of Investment Tax
	Present Value of Future Profits and Equity Returned

Any firm by virtue of its existence assumes a short position in a security producing cash flows of taxes payable by the firm. The government collecting the tax is long that security. One might naturally expect a firm to develop strategies to manage this short position.

In the case of tax on investment income, we see certain important implications for its management given by the Myers' Theorem. The present value of tax can be matched perfectly by investing a portion of assets given by the rate T at the risk free rate (e.g., if the effective tax rate is 35%, invest 35% of your portfolio in Treasury Bills maturing when taxes are due and use the interest earned to pay taxes). However, from the investor's perspective, the present value of the tax burden imposed on the investor's equity in the insurance firm is transferred to the policyholder through the premium charged (Myers and Cohn, 1987). An increase in the tax liability on the balance sheet, e.g., through a higher investment tax rate, results in an increase in the assets acquired from premiums.

The implication is that the effective tax rate on combined investment and underwriting income is an essential parameter in the implementation of theoretical underwriting profit models (Cummins (1990), Taylor (1994)). In this work, we will investigate two issues related to the management of the effective tax rate on investment income:

- Can a strategy of minimization of the tax liability through the use of derivative securities be rationally pursued, given the uncertainties of the firm's position, and
- Can fuzzy sets theory be used as a tool for management of uncertainty arising from forecasts of the effective tax rate and after-tax rate of return.

CRAFTING AN EFFECTIVE TAX RATE

Rational investors seek after-tax risk. In a world with taxes there is a question of whether true tax advantages exist, when all differences in risk are properly accounted for (Derrig, 1994). Stone[2] introduced the concept of a regulatory standard investment portfolio in the context of an insurance company -- that is a portfolio of zero-coupon Treasury securities whose maturities[3] are matched to the expected loss payment patterns. If this regulatory standard investment portfolio is used, computation of the effective investment tax rate is simple -- all income from Treasury securities is fully taxable at 35% corporate tax rate.[4] Further, the short position in the tax liability is fully covered by investing the portion of the policyholder premium equal to the expected tax liability in Treasury securities.

Myers (1984) posed the question whether some other investment portfolio with lower tax rates is actually superior in all relevant aspects to the regulatory standard portfolio, so that it brings about an additional value to the company holding such a portfolio. If such a portfolio exists, it must contain risky securities. In that case, the short position in the tax liability can be fully covered provided either (1) the effective tax rate of the portfolio is known with certainty, so the tax portion of the policyholder premium will exactly cover the option price of the tax liability, or (2) the uncertainty in the effective tax rate of the portfolio can be eliminated.

Cummins and Grace (1994) determined that insurers perceive a yield advantage for longer maturity tax exempt bonds, implying the existence of a portfolio with an effective tax rate lower than 35 percent. This can be justified only by a tax clientele effect -- a marginal buyer with a marginal tax rate of less that the insurers' 35% less, at a minimum, their 5.1% proration, alternative minimum tax, and capital gains income tax. Of course, the question of comparison of risk characteristics of longer maturity tax exempt bonds with the regulatory standard portfolio, or any other portfolio, remains a complicated issue to resolve.

An insurer, nevertheless, acts as a financial intermediary between, on one hand, the claimholders (policyholders, investors, government), and, on the other hand, the suppliers of securities. What Myers' Theorem implies is that:

- Claims of government (tax liabilities) are transferred to policyholders at the prevailing effective tax rates, so that an economic profit can be earned by crafting a lower effective tax rate (assuming of course this strategy is not available to, or employed by, the competitors of the firm, in which case a lower competitive premium develops);
- Investment tax liability acts to dampen the riskiness of the after-tax investment income of the insurer, so that higher expected profit can be earned by seeking higher level of risk if sufficient return compensation is available.

Traditionally, the pursuit of a lower effective tax rate has been performed by insurers through investments in tax exempt bonds, as indicated by Cummins and Grace (1994). Other tax-preferred

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[2] See Derrig, 1990, pp. 7-9.

[3] The Regulatory Standard Company could also utilize immunization techniques, e.g., match durations rather than maturities, but this would not produce an exact match of expected cash flows i.e., a match of the net premium, loss, expense and equity flows.

[4] The marginal corporate tax rate in the US at the time of this writing is 35 percent

strategies have been employed as well, such as the corporate dividend exemption[5], or a capital gains preferred tax rate.

The perspective suggested above implies that insurers, through their financial intermediary status, act as issuer of derivative securities (i.e., insurance contracts). The pursuit of lower effective tax rate can be enhanced by augmenting the existing derivative position with other derivatives which exploit the nature of insurer's activities.[6] The notion that insurers issue derivative securities is not new. Smith (1982) discussed it in the context of a life insurance policy. Doherty and Garven (1986) modeled the insurance transaction as a bundle of long and short call options, thereby leading to the pricing of the transaction through options pricing theory. Ostaszewski (1995) presented a generalized perspective of that nature -- that all financial intermediaries are indeed derivative securities issuers.

It should be noted that tax implications of derivative securities do depend on whether the ownership of underlying assets is considered to have been transferred. The uncertainty created by Internal Revenue Service (IRS) interpretations of whether ownership has transferred for tax purposes contributes to the uncertainty of the effective tax rate when such swapping arrangements are employed. For the purpose of this work we only assume that certain parameters of underlying securities are traded in the derivative position, while ownership remains.

At this point we want to outline investment strategies for an insurer that pursues its goal of minimizing its effective investment tax rate while maximizing investment return. An insurer should exploit any clientele effect by using its comparative advantage. We give two examples here, which we will use to craft proposed derivative strategies for insurers, and leave other strategies to the creativeness of the reader.

SWAP OPPORTUNITIES: TAX EXEMPT PERPETUALS, PERPETUAL CMT

Unlike other financial institutions (life insurers, mutual funds, banks) property-liability insurers do not receive a portion of their investment income free of taxes (for other financial intermediaries deemed to be an expense). The tax shield of underwriting can be utilized by them, but only to a limited degree. Unlike other investors in the tax exempt market (individuals), insurers have very long "life expectancy." Finally, insurers enjoy some corporate tax preferences.

It would seem, therefore, natural for property-liability insurers to pursue the following derivative strategies to seek tax exempt income by swapping other forms of income for it. The tax exempt income most desired by property liability insurers is of a long term nature, and ideally the security should pay only the tax exempt income without any capital gains or losses, or capital returns. We proceed to describe tax exempt perpetuals. At this point we only look at the management of the tax liability, while other considerations such as the duration or convexity of the portfolio may deem perpetuals less desirable, and these issues would have to be balanced in practice -- as indeed they are, even with yield advantage in tax-exempt bonds perceived by property-liability companies. Alas, tax exempt perpetuals are not issued. They can, however, be crafted by a series of forward contracts for delivery of long term tax exempt bonds. Similar perpetual series of tax preferred items, such as corporate dividends and capital gains, can be

[5] The current stock dividend exemption available to property-liability insurers is nominally 70 percent. But through the proration provision of the tax code, at least 15 percent of the excluded 70 percent is taxed at the marginal rate of 35 percent yielding an overall effective tax rate of at least 14.2 percent. Alternate minimum tax provision can drive that effective rate higher than 14.2 percent.

[6] While derivatives have received adverse publicity, such as the billion dollar losses in the Orange County/Robert Citron affair (NY Times, December 2, 1994, page D1), the value of derivatives as hedging securities, as opposed to speculative positions, remains valid.

created. In fact, preferred stocks exist precisely for that purpose -- they provide predictable dividend income enjoying corporate tax preference. Tax-exempt perpetuals proposed here could be viewed as special synthetic adjustable-rate preferred stocks (created with the use of forward contracts).

Let us then discuss one implication -- although the comparative advantage of insurers for the existing tax free bonds remains a debatable issue, one can hardly argue the fact that the theoretical clientele effect does exist for tax exempt perpetuals.

There are numerous ways of trading fully taxable income for tax preferred items. The simplest trade is designed in the following example: company A trades to company B the current capital gains on its tax exempt bond portfolio, which would be taxed (currently) at the full corporate rate, for a forward commitment to purchase new issue tax exempt bonds of the same quality as the current portfolio matures and of equal tax exempt income to company A. This is illustrated below:

COMPANY POSITION	
Company A *Property Insurer*	*Company B* *Life Insurance Company*
Asset = Long Term Tax Exempt Bond purchased at a discount	Asset = Cash

Assume that investment income of B qualifies for reserve deduction. Let now A enter with B into the following swap:

COMPANY SWAP POSITION	
Company A *Property Insurer*	*Company B* *Life Insurance Company*
Asset = Long Term Tax Exempt Bond purchased at a discount	Asset = Cash
A pays B annual amortization of tax exempt bond discount.	B pays A a forward commitment to purchase same amount of tax exempt income as A is now receiving beyond A's bond maturity.

This swap converts the fully taxable capital gain income to the property insurer A into (future) tax exempt coupon income. Thus, the capital gain portion of the government's tax claim short position is covered. In a more general sense, property and casualty insurance companies form a natural clientele for long term forward contracts for tax exempt income, and they should be willing to pay out of current taxable income for those forwards. It should be stressed that the actual portion of capital gains which would be traded this way would depend on the risk profiles of the companies involved, and we do not imply that all taxable gains should be traded.

The more promising trade can be devised by utilizing another source of taxable income of a property liability insurance firm -- premium revenue. A typical insurer has an underwriting loss which is balanced by an investment gain. Thus the operating income is already swapped for investment income for a typical company. However, capital requirements pose an additional hidden tax which penalizes an insurer with large liabilities and asset base. Disregarding for the moment reasonable solvency concerns we must admit that in view of this powerful combination of incentives, securitization of insurer's premium receivables creates a natural opportunity to trade a portion of premium receivables (possibly equal to the capital requirements) for forward delivery of long term tax exempt bonds. The remaining premium and capital requirement would be invested in taxables to offset the loss and expense payouts.

Note that annuity companies are a natural clientele for perpetual assets yielding five year constant maturity Treasuries (CMT). The natural trade here is to exchange premium cash flows crafted to match perpetual five year CMT income for tax exempt perpetuals. The trade could, of course, be settled at any point by matching to market. We should note that to the degree that underwriting losses provide a tax shield, this trade may need to be examined by comparing the value of the tax shield possibly lost and capital requirement tax released (so that, for example, only a partial swap may be desirable). The strategy also must be viewed in the context of all tax management, and asset-liability management strategies utilized. Tax management strategies are indeed prevalent in current practices of property-liability companies, and they generally rest on the two pilars brought forth above: tax shield of underwriting and the use of tax-preferred investments (Almagro and Ghezzi (1988)). We propose that a derivative of the structured note type can enhance existing practices.

Our final proposal addresses the degree of risk assumed by property liability insurers. Since insurance firm's beta is "dampened" by the investment tax, it would appear appropriate that insurance firms leverage up their investments to higher beta, in pursuit of higher returns. One such strategy would be for A to issue floating (e.g., LIBOR, or 5 year Constant Maturity Treasury) notes, to be purchased by B, a life insurance company with such floating liabilities, while A uses proceeds to purchase long term bonds. The resulting leverage ratio (from equation (4) with asset beta of one) should be

$$\frac{1+(1-T)r_F}{(1-T)(1+r_F)}. \tag{5}$$

In this case the property insurer holds a tax exempt portfolio with beta equal to that of the market, while lowering its investment tax rate by the use of the interest expense exemption. Clearly, this strategy not only increases expected return, but also the risk of the firm, and it actually exploits higher expected return for higher risk accepted. Let us add here, that in case of most floating assets there is a significant clientele which in fact pursues floating income of perpetual securities: money market funds (LIBOR), life and annuity insurance companies (five year CMT). Our main conclusion is that if the so common among property liability insurers belief in their comparative advantage in the tax exempt securities markets is valid, one should expect it to be fully utilized in swaps of floating taxable income for long term tax exempt income. In either case, the caveat of IRS interpretations of ownership remains. On the other hand, holdings of nonconvertible preferreds by corporate clientele indicate that such opportunities are perceived and utilized, albeit to a limited degree, and can be enhanced by various forms of synthetic structured notes.

It is up to the securities marketplace to determine if our proposals are valid. We must admit that the limited size of the asset base of property liability insurers, in relation to e.g., household mortgages, puts them second in line in financial engineering creativity. Time will tell.

FUZZY PARAMETERS

As we have stated above, Myers' Theorem implies that calculation of the effective investment tax rate becomes an essential part of both the ratemaking and portfolio management process. However, that calculation is not only affected by the composition of the insurer's investment portfolio, with varying rates of investment tax on tax exempt bonds, taxable bonds, preferred stock, and common stock, and insurance liabilities but also by future changes in the tax code and IRS interpretations of that code. Derrig (1994) shows how the 1986 Tax Reform Act sharply increased effective tax rates of U.S. property-liability insurers.

Clearly, the investment tax rate will vary within the range between zero percent (assuming a tax exempt bond portfolio issued completely before 1986) and 35 percent. In practice, the calculation of the effective tax rate, including the implicit tax embedded in the lower yields of tax-exempt bonds, becomes immensely complicated, especially when projecting future income and taxes, where the returns also become uncertain. We believe that we have made a case for estimation of the effective tax rate as an important tool of asset-liability management. However, we also believe that the traditional probabilistic approach may not be appropriate in this context. Uncertainty of taxes goes beyond the standard probability model, in which all outcomes of experiments are clearly defined, and future states of the world are mutually exclusive. Even legislated taxes are subject to interpretations, both in the regulatory context of the Internal Revenue Code, and in the practical terms of how the firms perceive them. Thus we propose that the management of the tax liabilities should be undertaken with the use of an alternative uncertainty model. Likewise, the choice among estimates for the expected after-tax returns on risky assets is not amenable to purely probabilistic models.[7]

We propose the use of fuzzy sets theory for estimation of the uncertainty in the tax rate and after-tax rate of return of a property-liability insurer. Lotfi Zadeh (1965) suggested a methodology for uncertainty radically different from traditional probabilistic models, including that uncertainty caused by vagueness and imprecision of human perception, or other human factors.

There may be several reasons for wanting to search for models of a form of uncertainty other than randomness. One is that vagueness is unavoidable. It is caused by the imprecision of natural language, or human perception of the phenomena observed. But also when the phenomena observed become so complex that exact measurement involving all features considered significant would be next to impossible, mathematical precision is often abandoned in favor of more workable simple, but vague, "common sense" models. Complexity of the problem may be another cause of vagueness.

These reasons were the motivation behind the development of the fuzzy sets theory (FST). This area has become a dynamic research and applications field, with success stories ranging from a fuzzy logic rice cooker to an artificial intelligence in control of the Sendai subway system in Japan.

Let us define the basic concepts of FST. Recall that a *characteristic function* of a subset E of a universe of discourse U is

$$\chi_E(x) = \begin{cases} 1 \text{ if } x \in E \\ 0 \text{ if } x \notin E \end{cases} \quad (6)$$

In other words, the characteristic function describes the membership of an element x in a set E. It equals 1 if x is a member of E, and 0 otherwise.

Zadeh (1965) suggested that there are sets whose membership should be described differently. One example would be the set of "good drivers." This is an important concept in auto insurance, yet its inescapable vagueness is obvious.

[7] Good discussions of what has become known as the equity risk premium puzzle can be found in Mehra and Prescott (1985), Ibbotson (1996), p. 151-161 and Abel (1996).

In the fuzzy sets theory, an element's membership in a set is described by the *membership function* of the set. If U is the universe of discourse, and $\widetilde{E}$ is a fuzzy subset of U, the membership function $\mu_E: U \rightarrow [0,1]$ assigns to every element x its degree of membership $\mu_E(x)$ in the set $\widetilde{E}$. We write either (E, μ_E) or $\widetilde{E}$ for that fuzzy set, to distinguish from the standard set notation E. The membership function is a generalization of the characteristic function of an ordinary set. Ordinary sets are termed *crisp sets* in fuzzy sets theory. They are considered a special case -- a fuzzy set is crisp if, and only if, its membership function does not have fractional values.

On the base of this definition, one then develops such concepts as set theoretic operations on fuzzy sets (union, intersection, etc.), as well as the notions of fuzzy numbers, fuzzy relations, fuzzy arithmetic, and approximate reasoning (known popularly as "fuzzy logic"). Pattern recognition, or the search for structure in data, provided an early impetus for developing FST because of the fundamental involvement of human perception (Dubois and Prade, 1980), and the inadequacy of standard mathematics to deal with complex and ill-defined systems (Bezdek and Pal, 1992). A complete presentation of all aspects of FST is available in Zimmerman (1991). Numerical manipulations of FST are amply described in Kaufmann and Gupta (1991).

A *fuzzy number* is a fuzzy subset of the real line such that its membership function has a value of one for at least one point, is zero outside a certain closed interval (finite support), and has a convex area under its graph. If two fuzzy numbers are given, $\tilde{A}$ with membership function μ_A, and $\tilde{B}$ with membership function μ_B, then fuzzy addition is performed by defining the membership function of $\widetilde{C} = \widetilde{A} + \widetilde{B}$ as μ_C with $\mu_c(z) = \max\{\min(\mu_A(x), \mu_B(y)) : x+y=z\}$ (Kaufmann and Gupta, 1991). Similar application of the so called maximin principle (Zadeh, 1965) allows for the creation of other fuzzy arithmetic operations. We will utilize them in the illustrations that follow.

The first recognition of FST applicability to the problem of insurance underwriting is due to DeWit (1982). Lemaire (1990) set out a more extensive agenda for FST in insurance theory, most notably in the financial aspects of the business. Under the auspices of the Society of Actuaries, Ostaszewski (1993) assembled a large number of possible applications of fuzzy sets theory in actuarial science. Cummins and Derrig (1993, 1996) complemented that work by exploring applications of fuzzy sets to property-casualty insurance forecasting and pricing problems. Derrig and Ostaszewski (1995) applied fuzzy clustering algorithms to problems of auto rating territories and fraud detection. Young (1996) modeled the rate changing decision problem in fuzzy logic terms.

In this work, we will illustrate how FST can be useful in estimation of the effective tax rate and after-tax rate of return on an insurance firm's asset and liability portfolio. Let us begin with a simple model of an insurance firm's expected investment income and tax position. Table 1 displays the expected CAPM results for a simple one period investment portfolio. We assume a bond/stock allocation of 80/20, approximately the allocation of the US property-liability industry in 1994.[8] We assume only US government bond holdings and diversified (beta=1) stock holdings. Using corporate bonds, which are taxed at the same rate as Treasuries, would only increase the expected yield (and uncertainty) and, therefore, the bond assessment weight in the tax rate calculation. Using tax-exempt bonds with implicit tax rates equal to the effective property-liability rate of less than 30 percent would be the equivalent of using Treasury securities but with a slightly higher beta than we assume here. The estimation of the effective tax rate of

[8] The actual proportion of P-L company portfolios on an annual statement (amortized bonds, market stocks) basis for 1994 QIII is 18.2 (stocks), 75.3 (bonds), 0.7 (mortgages), 4.8 (miscellaneous) and 0.9 (cash) according to the Board of Governors of the Federal Reserve System Flow of Funds Report.

tax-exempt securities with a positive tax-advantage to property-liability insurers, such as perceived by the US portfolio managers (Cummins and Grace, (1994)) is beyond the scope of this paper.

We use CAPM expected yields with a bond beta of 0.049 and stock beta of one. We use an expected market risk premium (MRP), excess of Treasury Bills, of 8.6 percent, the 1926-1993 average MRP for the US stock market (Ibbotson Associates, 1994). The expected tax rates reflect the dividend exclusion available to US property-liability companies. The capital gain marginal rate, currently equal to the marginal corporate rate, is adjusted downward to reflect the effective tax advantage of annually deferring 50 percent of the unrealized capital gains. With this set of assumptions the nominal tax rate is 32.4 percent, lower than the marginal rate of 35 percent because of the tax preferences available to stock income. Note that none of the uncertainty of the expected income or tax assumptions is reflected in Table 1.

Table 1
Non-Fuzzy Investment Tax Rate Example

Categories	(1) Assets	(2) Expected Return on Assets	(3) Expected Pre-Tax Income (1) × (2)	(4) Tax Rate	(5) Taxes (3) × (4)
US Government Bonds	800.0	5.70%	45.60	35.0%	15.96
Stocks:	200.0	13.88%			
Dividends		3.81%	7.62	14.2%	1.08
Capital Gains		10.07%	20.14	33.3%	6.71
Total	1000.0	7.34%	73.36	32.4%	23.75

Notes: Asset mix approximates US property liability company holdings (Federal Flow of Funds, 1994 QIII), Risk-Free Return of 5.28% is Cash-Flow weighted Treasury Bill and Note average yields, November 1993-October 1994. Bond and Stock Returns are CAPM with Bond Beta of .049, stock beta of 1.0, and Market Risk Premium of 8.6%; Dividend Yield is 10-Year S&P Average Yield 1984-1993; Corporate Tax Rate is 35; Dividend and Capital Gains Tax Rates reflect P-L dividend exclusions and deferral of unrealized Capital Gains of 50% per period

Fuzzy set theory gives us a way to rework Table 1 into a display that reveals the uncertainty in the various input parameters and, hence, in the tax results themselves. Table 2 portrays a version of Table 1 where the tax rates and investment income expectations are suitably uncertain. Admittedly, there are many ways to portray the parameters as fuzzy numbers by incorporating as much or as little of the random and non-random uncertainty into the membership function. Generally, we choose to illustrate the FST effect by using triangular (i.e., the shape of the graph of the membership function is triangular) fuzzy numbers, with the uncertainty pegged at plus or minus a value dependent on the uncertainty illustrated.[9] Each fuzzy member is identified by four variables (m_1, m_2, m_3, m_4) representing the left axis, left top, right top and right axis points.[10] The tax rate outcome is the fuzzy number (31.3%, 32.4%, 32.4%, 33.4%) portraying an uncertain range of about 2 percent on the tax rate, arising directly from an assumed 2 percent uncertainty range in the marginal tax rate.

[9] The "fuzziness" of stock returns in this example represents the uncertainty in the estimation of the CAPM expected, rather than actual, return. Uncertainty in the expected equity risk premium could arise for example in choosing, contrary to Ibbotson's advice, some shorter more recent time period to average equity returns excess of the risk free rate (Ibbotson, (1996) Table A16). Random variation could be illustrated by fuzzy numbers with support equal to one standard deviation about the mean.

[10] Although we do not use the illustration here, $m_2 < m_3$ describes a uniform range of uncertainty for the expected or middle values. This situation may often be the case for non-random uncertainty (Berliner and Babad, (1994)).

			Table 2 *Fuzzy Investment Tax Rate Example* *Corporate Tax Rates and Returns*				
			Investment Categories				
		Fuzzy Number	US Government Bonds	Stocks	Dividends	Capital Gains	Total
(1)	Investments		800.0	200.0			1000.0
(2)	Expected Return	m_1	4.42%	13.08%	3.59%	9.49%	6.15%
		m_2	5.70%	13.88%	3.81%	10.07%	7.34%
		m_3	5.70%	13.88%	3.81%	10.07%	7.34%
		m_4	6.98%	14.68%	4.03%	10.65%	8.52%
(3)	(1) × (2) Expected Pre-Tax Income	m_1	35.36		7.18	18.98	61.52
		m_2	45.60		7.62	20.14	73.36
		m_3	45.60		7.62	20.14	73.36
		m_4	55.84		8.06	21.30	85.20
(4)	Tax Rate	m_1	34.0%		13.8%	32.0%	31.0%
		m_2	35.0%		14.2%	33.3%	32.4%
		m_3	35.0%		14.2%	33.3%	32.4%
		m_4	36.0%		14.6%	34.7%	33.6%
(5)	(3) × (4) Taxes	m_1	12.02		0.99	6.08	19.09
		m_2	15.96		1.08	6.71	23.75
		m_3	15.96		1.08	6.71	23.75
		m_4	20.10		1.18	7.38	28.66

Notes: Investment Returns are CAPM Table 1 returns with Fuzzy Risk-Free Rates, Market Risk Premiums, and crisp Betas of .049 (Bonds) and 1 (Stocks).

	Fuzzy Parameter	
	Risk-Free	MRP
m_1	4.00%	0.061
m_2	5.28%	0.086
m_3	5.28%	0.086
m_4	6.56%	0.111

1. A Fuzzy Number is Identified by the Left Axis, Left Top, Right Top, and Right Axis points (m_1, m_2, m_3, m_4).

INCLUDING THE INSURANCE POLICY TAX HEDGE

The illustrations in Tables 1 and 2 focused on the uncertainty in insurer's investment portfolio. But tax considerations involve the interplay, and uncertainty, of the insurance or liability part of the company's entire portfolio of assets. Table 3 reworks the simple investment illustration of Table 1 to show the interaction with writing insurance liabilities and using the tax shield of those liabilities to offset some of the tax liabilities from investments. This situation, of course, assumes that property-liability insurers are writing to a nominal underwriting loss, a recent historical fact. We assume, in addition to all investment assumptions of Table 1, liabilities written at 2:1 to the surplus (net worth) of the company. We assume an expected underwriting loss of 4.07 percent, a recent value for Massachusetts private passenger automobile insurance rates. The tax rate for liability returns will be assumed to be 34.5 percent, a value lower than the marginal rate reflecting the discounting of loss reserves for tax purposes. The expected tax rate for the pre-tax income on the insurers portfolio drops to 31.1 percent from 32.4 percent because of the effect of the tax shield.

	(1)	(2)	(3)	(4)	(5)
			Expected		
			Pre-Tax		
	Portfolio	**Expected**	**Income**	**Tax**	**Taxes**
Categories	**Weights**	**Return**	**(1) × (2)**	**Rate**	**(3) × (4)**
Liabilities	-667.0	4.07%	-27.15	34.5%	- 9.36
US Government Bonds	800.0	5.70%	45.60	35.0%	15.96
Stocks:	200.0	13.88%			
Dividends		3.81%	7.62	14.2%	1.08
Capital Gains		10.07%	20.14	33.3%	6.71
Surplus/Totals	333.0	13.88%	46.21	31.1%	14.39

Table 3
Non-Fuzzy Portfolio Tax Rate Example

Notes: Investment Returns and Tax Rates as in Table 1; Expected Return on Liabilities as in expected underwriting profit margin for Massachusetts private passenger automobile liabilities, Tax Rate for Liabilities reflects discounting of Loss Reserves.

The effects of making the entire insurer portfolio fuzzy, investments and liabilities, are shown in Table 4. In addition to the fuzzy tax rate and investment returns of Table 2, we use a fuzzy underwriting return of plus or minus 10 percent of the expected.

Table 4
Fuzzy Portfolio Tax Rate Example
Corporate Tax Rates and Returns

		Fuzzy Number[1]	Investment Categories: Liabilities	US Government Bonds	Stocks	Dividends	Capital Gains	Total
(1)	Portfolio Weights		-667.0	800.0	200.0			333.0
(2)	Expected Pre-Tax Return	m_1	3.65%	4.42%	13.08%	3.59%	9.49%	9.48%
		m_2	4.07%	5.70%	13.88%	3.81%	10.07%	13.88%
		m_3	4.07%	5.70%	13.88%	3.81%	10.07%	13.88%
		m_4	4.49%	6.98%	14.68%	4.03%	10.65%	18.27%
(3)	(1) × (2) Expected Pre-Tax Income	m_1	-29.95	35.36	26.16	7.18	18.98	31.57
		m_2	-27.15	45.60	27.76	7.62	20.14	46.21
		m_3	-27.15	45.60	27.76	7.62	20.14	46.21
		m_4	-24.35	55.84	29.36	8.06	21.30	60.85
(4)	Tax Rate	m_1	33.6%	34.0%		13.8%	32.0%	28.6%
		m_2	34.5%	35.0%		14.2%	33.3%	31.1%
		m_3	34.5%	35.0%		14.2%	33.3%	31.1%
		m_4	35.4%	36.0%		14.6%	34.7%	33.0%
(5)	(3) × (4) Taxes Paid	m_1	-10.06	12.02	7.07	0.99	6.08	9.03
		m_2	- 9.36	15.96	7.79	1.08	6.71	14.39
		m_3	- 9.36	15.96	7.79	1.08	6.71	14.39
		m_4	- 8.61	20.10	8.56	1.18	7.38	20.05
(6)	(3) - (5) Expected After-Tax Income	m_1	-19.89	23.34	19.09	6.19	12.90	22.54
		m_2	-17.79	29.64	19.97	6.54	13.43	31.82
		m_3	-17.79	29.64	19.97	6.54	13.43	31.82
		m_4	-15.74	35.74	20.80	6.88	13.92	40.80
(5)	(6) ÷ (1) Expected After-Tax Return	m_1	2.36%	2.92%	9.55%	3.10%	6.45%	6.77%
		m_2	2.67%	3.71%	9.98%	3.27%	6.71%	9.56%
		m_3	2.67%	3.71%	9.98%	3.27%	6.71%	9.56%
		m_4	2.98%	4.47%	10.40%	3.44%	6.96%	12.25%

Note: Investment Returns are CAPM with Fuzzy Risk-Free Rates, Market Risk Premiums, and crisp Betas of .049 (Bonds) and 1 (Stocks).

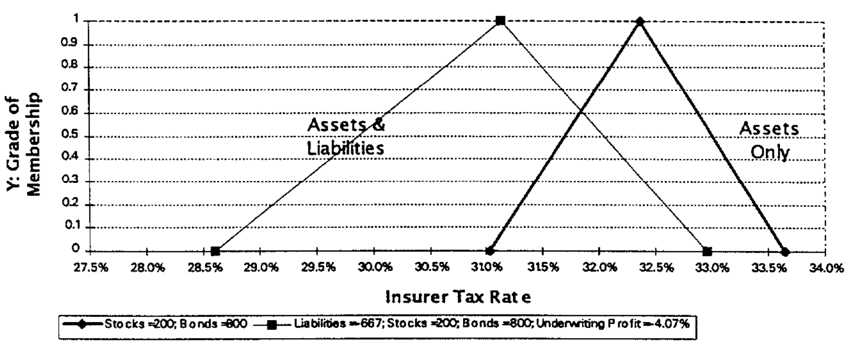
	Fuzzy Parameter: Risk-Free	MRP
m_1	4.00%	0.061
m_2	5.28%	0.086
m_3	5.28%	0.086
m_4	6.56%	0.111

1. A Fuzzy Number is Identified by the Left Axis, Left Top, Right Top, and Right Axis points (m_1,m_2,m_3,m_4).

In addition to showing the effect of these fuzzy numbers on the tax rate, we list the fuzzy expected after-tax returns. The fuzzy tax rate spans 28.6 percent to 33.0 percent, a 4.4 percent gap. While the overall expected tax rate has been reduced by the effect of the tax shield (and policyholder tax hedge), the uncertainty has increased! Likewise, the after-tax rate of return, expected to be 9.56 percent, obtains a wide fuzzy range from 6.77 percent to 12.25 percent - a gap of about 5.5 percent.

Figure 1 displays the effect of a fuzzy tax shield on the fuzzy expected tax rate.

Figure 1
Fuzzy Investment Tax Rates
Effect of Liability Tax Shield

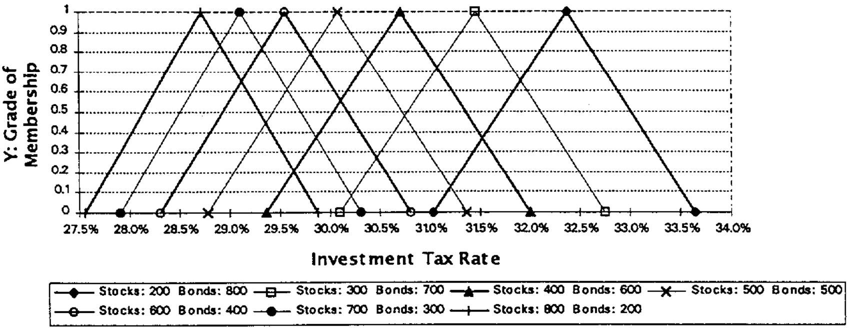


ASSET ALLOCATION

A common method of tax management in property-liability companies is to balance the trade-off of increased risk from a larger stock allocation with the decreased tax rate that emanates from the stock income preferences. Figure 2 shows the fuzzy range of tax rates as the asset allocation changes from 80/20 bond/stock to 20/80. If we measure the uncertainty of the *difference* between two fuzzy expected tax rates by the height of their intersection (the point at which they cross), one can observe the increasing uncertainty in distinguishing tax outcomes as the asset allocation moves to a larger stock position. Thus, while 80/20 and 20/80 are clearly distinct, even in the fuzzy sense, 50/50 and 40/60 retain a high degree (0.7 to 0.8) of uncertainty in differentiation of results.

Figure 2

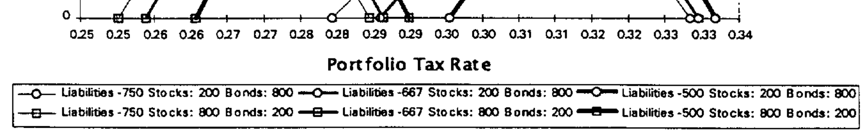
Fuzzy Investment Tax Rates with Selected Asset Mixes



The fuzzy tax effect of adding the insurance liabilities to the invested asset portfolio is demonstrated in Figure 3. Leverage ratios of 1:1 to 3:1, liabilities to surplus, provide for lower crisp expected tax rates. But those lower rates have little to distinguish them from one another on a fuzzy (uncertain) basis on either end of the assets allocation spectrum.

Figure 3

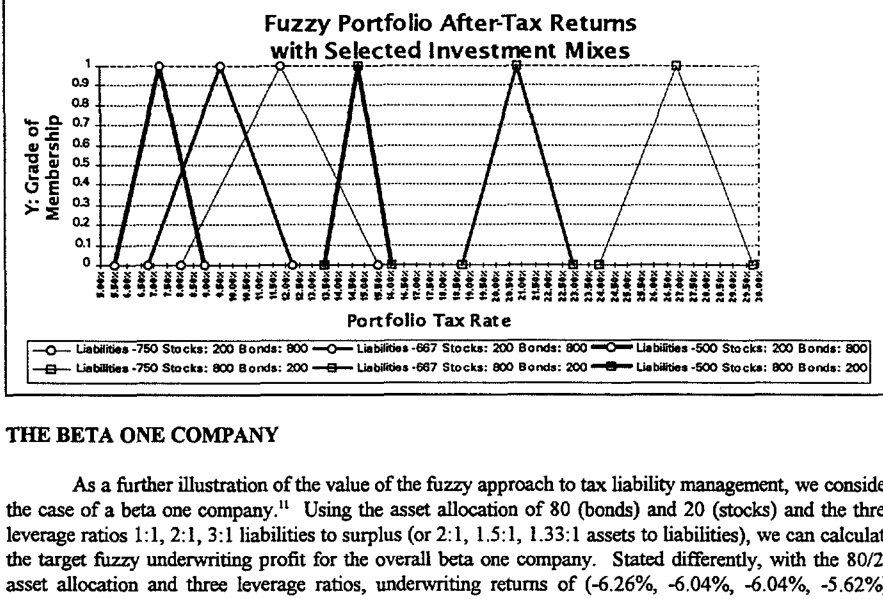
Fuzzy Portfolio Tax Rates with Selected Investment Mixes



AFTER-TAX RATES OF RETURN

The fuzzy after-tax rates of return were displayed in Table 2. They reflected, of course, the uncertainty in the tax rates, expected investment yields and in the liabilities. Figure 4 shows the portfolio effect on after-tax rates of return for different leverage ratios and the extremes of the asset allocation illustrations (80/20, 20/80). Note that the ability to distinguish the fuzzy outcomes at the low investment risk level (80/20) for different leverage ratios but not to distinguish at the high investment risk level (20/80) lends the interpretation that the fuzzy after-tax rates of return reflect *total* uncertainty.

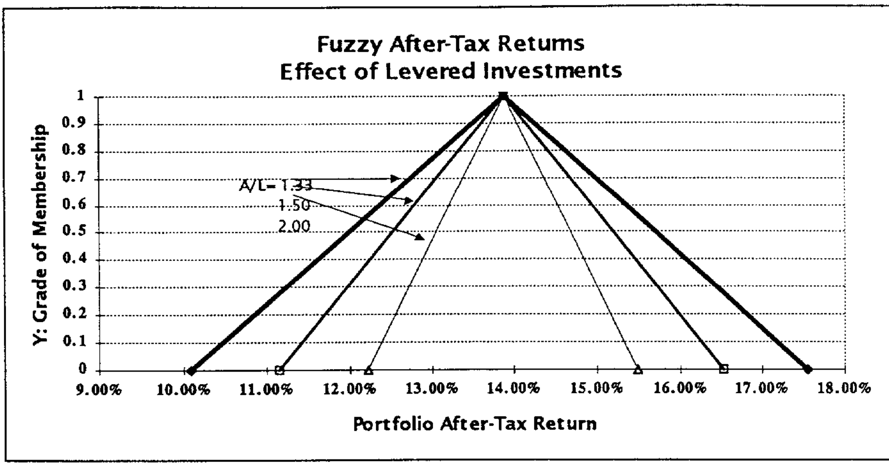
Figure 4



THE BETA ONE COMPANY

As a further illustration of the value of the fuzzy approach to tax liability management, we consider the case of a beta one company.[11] Using the asset allocation of 80 (bonds) and 20 (stocks) and the three leverage ratios 1:1, 2:1, 3:1 liabilities to surplus (or 2:1, 1.5:1, 1.33:1 assets to liabilities), we can calculate the target fuzzy underwriting profit for the overall beta one company. Stated differently, with the 80/20 asset allocation and three leverage ratios, underwriting returns of (-6.26%, -6.04%, -6.04%, -5.62%), (0.36%, 0.78%, 0.78%, 1.20%) and (2.62%, 3.04%, 3.04%, 3.46%) will result in three fuzzy after-tax returns, all "centered" on 13.88 percent - the beta one expected return. Figure 5 shows those fuzzy after-tax returns and their ranges of uncertainty. Note that the intuitive result of more uncertainty in the higher leveraged firm obtains even when the target after-tax return is the same.

[11] US property-liability companies are often thought of as being of average (beta) risk. Unfortunately, this view does not necessarily take into account the vast distribution of the capitalization of those companies. Our simplifying assumption is used regardless of leverage of the firm.



CONCLUSION

This paper has explored the management of the government's short position for tax liabilities in the context of a property-liability insurance firm. We viewed the writing of the insurance liability as covering that short position under certain circumstances. Alternative derivative (swap) positions were suggested as the beginning of possible elements in a tax hedging portfolio.

By virtue of the Myers Theorem, the tax management focus falls upon the effective tax rate of the investment portfolio. We show the ability of fuzzy set theory to illustrate not only the parametric interactions, but also the uncertainty, random and non-random, in the key parameters and outcomes. The advantages of the underwriting tax shield and the effects of parametric uncertainty on tax rate and after-tax return uncertainty were illustrated. Outcomes generally follow intuitive results; the benefit is the quantification, and graphic display, of the uncertainty of those results.

A good next step would be to expand and integrate the derivative security selection into the fuzzy set context. Better levels of uncertainty for primary and derivative assets combined may be shown through the fuzzy set paradigm. Finally, someone might undertake the formidable task of making the foregoing ideas rigorous (e.g., fuzzy partial derivatives on leverage). The richness of the fuzzy approach can only help to illuminate the problems of uncertainty.

References:

Abel, Andrew B. [1996], Risk Premia and Term Premia in General Equilibrium, Wharton School, University of Pennsylvania, working paper.

Almargo, M. and Ghezzi, T.L. [1988], Federal Income Taxes - Provisions Affecting Property/Casualty Insurers, Proceedings of the Casualty Actuarial Society, 75:95-162.

Babad, Y. and Berliner, B. [1994], The Use of Intervals of Possibilities to Measure and Evaluate Financial Risk and Uncertainty, *4th AFIR Conference Proceedings*, International Actuarial Association, April 20-22 (Orlando), 1:111-140.

Bezdek, J.C. and Pal, S.K. (editors) [1992], *Fuzzy Models for Pattern Recognition: Methods That Search for Structure in Data*, IEEE Press, New York.

Cummins, J. D. [1990], Multi-period Discounted Cash Flow Ratemaking Models in Property-Liability Insurance, Journal of Risk and Insurance, 57:79-109.

Cummins, J. D. and Grace, Elizabeth [1994], Tax Management and Investment Strategies of Property-Liability Insurers, *Journal of Banking and Finance*, 18:43-72.

Cummins, J. D. and Derrig, R.A. [1993], Fuzzy Trends in Property-Liability Insurance Claim Costs, *Journal of Risk and Insurance*, 60:429-465.

Cummins, J. D. and Derrig, R.A. [1996], Fuzzy Financial Pricing of Property-Liability Insurance, Wharton School of University of Pennsylvania, working paper.

Derrig, Richard A. and Ostaszewski, K. [1995], Fuzzy Techniques of Pattern Recognition in Risk and Claim Classification, *Journal of Risk and Insurance,* 62:447-482.

Derrig, Richard A. [1994], Theoretical Considerations of the Effect of Federal Income Taxes on Investment Income in Property-Liability Ratemaking, *Journal of Risk and Insurance*, 61:691-709.

Derrig, Richard A. [1990], The Development of Property-Liability Insurance Pricing Models in the United States 1969-1989, *1st AFIR Conference Proceedings*, International Actuarial Association, May 23-26 (Paris), 4:237-264. Reprinted in *CAS Actuarial Forum*, Fall 1991, 19-46.

DeWit, G.W. [1982], Underwriting and Uncertainty, *Insurance: Mathematics and Economics*, 1:277-285.

Doherty, N.A. and Garven, J.R. [1986], Price Regulation in Property-Liability Insurance: A Contingent Claims Approach, *Journal of Finance*, 41:1031-1050.

Dubois, D. and Prade, H. [1980], *Fuzzy Sets and Systems*, Academic Press, San Diego.

Ibbotson Associates [1994 and 1996], *Stock, Bonds, Bills and Inflation Yearbook*, Chicago, IL.

Kaufmann, A. and Gupta, M.M. [1991], *Introduction to Fuzzy Arithmetic*, Van Nostrand Reinhold, New York.

Lemaire, Jean [1990], Fuzzy Insurance, *Astin Bulletin*, 20:33-56.

Mehra, Rajnish and Prescott, Edward C. [1985], The Equity Premium: A Puzzle, *Journal of Monetary Economics*, 15:145-162.

Myers, Stewart C. and Cohn, Richard A. [1987], A Discounted Cash Flow Approach to Property-Liability Insurance Rate Regulations, Chapter 3 of J. D. Cummins and S. E. Harrington, eds., *Fair Rate of Return in Property-Liability Insurance*, Boston, Massachusetts, Kluwer-Nijhoff, pages 55-78.

Myers, Stewart C. [1984], Paper prepared for the Massachusetts Automobile rate hearings, Massachusetts Rating Bureau Filing for 1985 Automobile Rates, Boston, Mass. (December).

Ostaszewski, K. [1993], *An Investigation into the Possible Applications of Fuzzy Sets Methods in Actuarial Science,* Society of Actuaries, Schaumburg, Illinois.

Ostaszewski, K. [1995], In Defense of Derivatives, *Contingencies*, May-June 1995.

Smith, M. L. [1982], The Life Insurance Policy as an Options Package, *The Journal of Risk and Insurance*, 45:583-601.

Taylor, G. C. [1994], Fair Premium Rating Methods and the Relations Between Them, Journal of Risk and Insurance, 61:592-615.

Young, V. [1996], Rate Changing: A Fuzzy Logic Approach, Journal of Risk and Insurance, September, 63:461-484.

Zadeh, L.A. [1965], Fuzzy Sets, *Information and Control*, 8:338-353.

Zimmerman, H.J. [1991], *Fuzzy Set Theory and its Applications*, Second Edition, Kluwer Academic Publishers, Norwell, MA.

Modeling the Evolution of Interest Rates: The Key to DFA Asset Models

by Gary G. Venter, FCAS

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Modeling the Evolution of Interest Rates: The Key to DFA Asset Models

Fluctuations in short term and long term interest rates can have significant impact on insurer financial results. Hence projecting the probabilities of these possible fluctuations is an important step towards a credible dynamic financial analysis. Changes in the level of interest rates as well as shifts in the shape of the yield curve both need to be modeled. Such shape-shifting is not an unconstrained random process - there are relationships among the yields of different terms - yet a good deal of flexibility is required to be able to reproduce historical curves.

Financial theory suggests that the yield curve at any point in time is a function of the probabilities of the future values of the short-term interest rate. Thus a process that produces probabilities for the evolution of the short-term rate will also have implications for the entire term structure. Simulating probabilities for future yield curves can proceed by first simulating short-term rate probabilities over an extended horizon, and then using those to simulate yield curve probabilities for a shorter horizon.

To illustrate that procedure, this paper has four sections: first models for short-term interest rate changes will be discussed, followed by a discussion of how to produce yield curves from those models. Then estimation issues are addressed, and the final topic is adding other correlated economic variables.

1 Modeling Short-Term Interest Rates

Most models of short-term rates are expressed as stochastic differential equations involving Brownian motion. At the time of this writing that is not a topic on the CAS Syllabus, so a short deviation will be taken to explain the notation to be used.

1.1 Brownian Motion

Standard Brownian motion is a sequence of random variables X_t indexed by time t, where X_t is normally distributed with $E(X_t)=0$ and $Var(X_t)=t$. Thus the variance grows over time. A *Brownian motion with drift μ and variance σ^2* has X_t normally distributed with mean μt and variance $\sigma^2 t$.

There are other technical requirements: a Brownian motion must be a continuous process with stationary independent increments. That means that the increments $X_s - X_t$ are independent for different choices of s and t, and are stationary in the sense that the distribution of $X_s - X_t$ is the same for any s and t with a common value of s-t. One reason for the popularity of Brownian motion is that the converse is also true: a continuous stochastic process with stationary independent increments must be a Brownian motion. (See L. Brieman *Probability* Addison-Wesley 1968 ch. 12.) This can be related to the Central Limit Theorem. The sum of a lot of independent increments would tend to normality.

1.2 Stochastic Differential Equations

Methods for solving stochastic differential equations will not be addressed in this paper, but the notation will be used as a recipe for simulation. For example, let z be a standard Brownian motion, and consider the following equation for the short-term interest rate r:

$$dr = \mu dt + \sigma dz \qquad (1)$$

This can be interpreted as a way to simulate changes in r over short intervals. Say the short interval has length Δt. Then simulate the change in r (i.e., Δr) as a draw from a normal distribution with mean $\mu\Delta t$ and variance $\sigma^2\Delta t$.

1.3 Models of Short-Term Interest

It is fairly common in financial mathematics to express interest rates as continuously compounding - what actuaries call the force of interest. Thus the usual interest rate i becomes the force of interest r where $(1+i)^t = e^{rt}$. The short-term rate is the instantaneous continuously compounding rate, which can be thought of as the limit of shorter and shorter terms. Sometimes this is estimated as the one-month rate, or even the three-month rate, or as a projection backwards from a few of the rates for shorter terms.

One possible model for r is expressed in (1) above: r is a Brownian motion. Another possibility would be to let $y = \ln r$ be a Brownian motion. This is called *geometric Brownian motion,* and excludes any possibility that r could become negative. *Single factor models* are those that can be expressed using only one Brownian motion process. A number of these models are of the form:

$$dr = (a+br)dt + \sigma r^k dz \qquad (2)$$

where $0 \leq k \leq 1$. Often k is taken as ½ or 1, which would make the variance of the change in rates proportional to r or r^2. Typically a is non-negative and b is non-positive. Note that this model has four parameters to estimate, even though it is a single-factor model.

In this model it is not possible for r to become negative if a is positive, because if r gets to zero, all the terms become zero except for the positive drift, and r becomes positive in the next instant. When b is negative the process is called *mean reverting*. If $|br|$ is above a, the drift will be downward, and if below a, the drift will become upward. Thus the drift is always back towards a. A Brownian motion process with no drift that is not adjusted to be mean reverting will eventually become quite wild. The variance $\sigma^2 t$ will grow with time, so the probability of finding the process to be within a given distance of zero will diminish to the vanishing point. However non-mean-reverting processes are sometimes used in short-term forecasts of interest rates.

A mean-reverting process may display negative auto-correlation at some intervals. That is, if it is going up at some point, it is likely to be going down at some future point. Interest rates seem to display this behavior. For instance, one study found that the logs of the growth rates of short-term rates are positively correlated from one month to the next, but negatively correlated to those of six and seven months earlier (D. Becker, *Statistical Tests of the Lognormal Distribution as a Basis for Interest Rate Changes,* Transactions, Society of Actuaries, vol. XLIII).

Multi-factor models specify interest rate evolution as process involving the interaction of multiple random effects at each stage. For instance, a three factor model may specify that the interest rate evolves according to a random process in which the expected rate of change and its variance also evolve randomly. Six or seven parameters may be needed to describe these three random effects.

One apparently successful three-factor model is given by Anderson and Lund (Working Paper No. 214, Northwestern University Department of Finance):

$$dr_t = a(u_t - r_t)dt + s_t r_t^k dz_1 \qquad k>0 \tag{3}$$

$$d\ln s_t = b(p - \ln s_t)dt + v\,dz_2 \tag{4}$$

$$du_t = c(q - u_t)dt + w u_t^{1/2} dz_3 \tag{5}$$

Here there are three standard Brownian motion processes, z_1, z_2, and z_3. The rate r moves subject to different processes at different times. It always follows a mean-reverting process, with the mean at time t denoted by u_t. But that mean itself changes over time, following a mean-reverting process defined by c, q, and w. The standard deviation of r_t is $r_t^k s_t$, where s_t also varies over time via a mean reverting geometric Brownian motion process. In total there are eight parameters: a, b, c, k, p, q, v, and w.

A two-factor model by M. Tenney (The Double Mean Reverting Process™, Society of Actuaries Technical Report, 1996) takes a somewhat different approach toward keeping the interest rate positive. Let $y = \ln r$ be the ln of the interest rate rather than the interest rate itself. Tenney's model then can be expressed as:

$$dy_t = a(u_t - y_t)dt + v\,dz_1 \qquad (6)$$

$$du_t = c(q - u_t)dt + w\,dz_2 \qquad (7)$$

where z_1 and z_2 are correlated standard Brownian motion processes with correlation ρ. Thus there are six parameters: a, c, q, v, w, and ρ.

Neither of the above models is particularly easy to estimate from data, but once estimated, simulation is quite straightforward for either of them. Multi-factor models are used despite the estimation difficulties because of some of the weaknesses of single factor models. These include difficulty in capturing the movements of historical interest rates, and difficulty in matching the historical yield curves. The historical yield rates, for instance, display different rates of variation in different periods, and these do not necessarily correlate directly to the interest rate level. This could be evidence of stochastic movement of the variance, as in equation (4) above. It is also consistent with an infinite variance process, which would generate unstable measurements of observed variance in different periods. Modeling interest rates as such a process will not be addressed here, however. Historical yield curves occasionally have inversions, in which the short-term rates are higher than those for longer terms. This usually is not allowed by the single-factor models.

2 Yield Curves Implied by Short-Term Rate Models

One reason long-term rates are usually higher than short-term rates is that long-term investors take the risk that intervening events will render the accumulated earnings worth less. In the very long run, though, all things might average out, and so very long-term rates are not necessarily higher than long-term rates, and may even be lower. It is not unusual, for instance, for 20 year rates to be a little higher than 30 year rates. Some infinite term bonds have been issued in the UK at fairly low rates.

The standard method for producing yield curves from a stochastic generator of short-term rates is to change the parameters of the generator to make it generate higher short-term rates for time periods further into the future, and then to take the expected value of the future adjusted rates as the estimate of what the short-term rate will be at that future period. The short-term rates so estimated for each future period then can be put together to make the long-term rates. This general concept will be spelled out more precisely below.

Adjusting the future rates generated is equivalent to keeping the rates but adjusting the probabilities in a manner that increases the expected value of the future short-term rates. Adjusted probability methods seem intuitively reasonable, but they are also justified by arbitrage theory. Thus a short detour into arbitrage theory may be useful.

2.1 Arbitrage of Interest Rates

A common financial definition of an arbitrage opportunity is a possibility to make a net investment of zero, and end up with no probability of a loss and a positive probability of a gain. Arbitrage theory says that there are no arbitrage opportunities available. This is not universally accepted by casualty actuaries. Two types of comments are often heard:

1. Investment houses make arbitrage profits all the time. They borrow at the 3-month rate and lend out at the higher 6-month rate.
2. Investment houses make arbitrage profits all the time. They have sophisticated trading models that look for these opportunities continuously, and put up big bucks whenever they arise, which is often.

The first is not really an arbitrage profit, at least by the above definition. It may be a pretty good bet, but now and then the 3-month rate will jump while the money is still out at the now lower 6-month rate, and the investors will have a loss when they have to borrow at a higher rate than they are getting.

The second may indeed be true. But if it is happening, the big boys are taking out all the arbitrage profits before anyone else ever sees them. It would be highly unusual for the end-of-day published rates to have arbitrage possibilities in them. If they appear in the 20 minute delay quotes on-line, they are probably gone by the time they appear.

An example of an interest rate arbitrage is adapted from P. Boyle (*Options and the Management of Financial Risk*, Society of Actuaries, 1992). Suppose the yield curve is flat: all rates are 8%. In the next instant they will be flat again, with 50% probability of staying at 8%, but with 25% probability each of moving to 7% or 9%. Borrow 1000 due in 10 years, and use the $1000/1.08^{10}$ to make loans with single payments of $500/1.08^5$ due in 5 years and $500 \cdot 1.08^5$ due in 15 years. Each of those loans costs $500/1.08^{10}$ to make today, so this produces a net position of zero. But in the next instant if the interest rates go to 7% the net position is worth 0.550 (i.e., $500/[1.08^5 1.07^5] + 500 \cdot 1.08^5/1.07^{15} - 1000/1.07^{10}$). Interestingly enough, if they go to 9% it is worth 0.449. At 8% it stays at zero. This is an arbitrage opportunity by the above definition, and so it is ruled out by arbitrage theory.

Although this is a highly artificial example, it shows that certain combinations of yield curves and interest rate movements are not possible under arbitrage theory. Boyle has more seemingly realistic examples that are likewise disallowed. This raises the issue of what yield curve / interest rate movement combinations are possible without generating arbitrage opportunities.

2.2 Pricing Consistent with Arbitrage Theory

It turns out that to rule out arbitrage possibilities, securities must be priced as the expected value of their returns under some probability distribution. The probabilities do not have to be the actual probabilities of those returns. In fact, if they were, there would be no reward for risk, which is unrealistic. Thus risk-adjusted probabilities must be used. Sometimes the risk-adjusted probabilities are called *risk-neutral* probabilities. That is because when using them you act as if risk were not important - i.e., you just take expected values. But this does not mean risk is ignored: expected value pricing based on risk-neutral probabilities is a method for building risk premium into prices.

One constraint on the risk-adjusted probabilities is that they are equivalent to the actual probabilities in the sense that they give zero probability to the same set of events. This is violated in the Boyle example above, where there is positive probability of a change in interest rates, but prices are based on expected values under the assumption of no possibility of changing rates. In this situation the adjusted probabilities give no chance to the events that can lead to the actual positive profit probabilities.

Although it is complex to prove that no-arbitrage and adjusted-probability expected-value pricing are equivalent, the following heuristic argument may help make it plausible. The key to avoiding arbitrage is to ensure that prices are additive. That is, the sum of the prices of a combination of securities that always produce the same outcomes as another given security should equal the price of that security. If not, buying the cheaper and selling the dearer set will give a profit in every case. But if prices of all securities are additive, there must be some set of event probabilities that gives those prices as expected values. The key to seeing that is to define fundamental securities that relate to the specific possible events. For instance with interest rates, those securities might pay 1 if the interest rates exceed specific targets for each term, and 0 otherwise. Such securities could be defined for any combination of term interest rates. The prices of those securities would define a joint probability distribution for the interest rates, and all other securities could be priced as combinations of those, which would be like taking their expected values under the distribution so defined.

2.3 Arbitrage-Free Pricing under Interest Rate Generators

The price at time t of a zero-coupon bond maturing at time T is the discounted value of the payment. With constant interest this is no problem, but with stochastic interest this would require the expected average discount, with the discount taken with respect to the risk-adjusted probabilities. The price of a bond that pays 1 at maturity can be expressed as:

$$P(t,T) = E_t^*[\exp(-\int r_s ds)] \tag{8}$$

where the integral goes from t to T, and E^* is the mean using the risk-adjusted probabilities. From the price of the bond, the implied interest rate for that term can then be backed out. In practice, the integral is evaluated as a sum over the small intervals used in the generation of short-term rates.

Thus the term structure is tied to the future paths of the short-term rate. Once an interest rate generator is available, what needs to be specified is how the risk-adjusted probabilities are to be defined. What is usually done is to change the generator so that it produces higher interest rates over time. Strictly speaking this gives a higher rate at each probability, but this then produces higher probabilities for the higher rates, which is what arbitrage theory is looking for.

A typical adjustment is to add something to the drift terms. For instance, Anderson and Lund to (3) - (5) above add $\lambda s_t r_t dt$ to the r diffusion and $\kappa u_t dt$ to the u diffusion. This gives a new process for generating risk-adjusted short-term rates, as below:

$$dr_t = a(\lambda s_t r_t + u_t - r_t)dt + s_t r_t^k dz_1 \qquad \lambda,k>0 \tag{9}$$

$$d\ln s_t = b(p - \ln s_t)dt + v dz_2 \tag{10}$$

$$du_t = c(\kappa u_t + q - u_t)dt + w u_t^{1/2} dz_3 \qquad \kappa>0 \tag{11}$$

Thus both r and u increase at higher rates, on the average, in the risk-adjusted process. The rate scenarios generated by (9) - (11) are used to evaluate the expected value of the integral in (8) to give bond prices, which are essentially the discount rates for the various terms.

Tenney similarly increases the drifts, but also changes the rates of mean reversion. The adjusted process from (6) and (7) is:

$$dy_t = \phi a(\lambda + u_t - y_t)dt + v dz_1 \quad \lambda>0,\ \phi>1 \tag{12}$$

$$du_t = \varphi c(\kappa + q - u_t)dt + w dz_2 \quad \kappa>0,\ 1>\varphi>0 \tag{13}$$

Note that in the u diffusion the mean reversion is slower than in the unadjusted process. This increases the variability of u.

3 Estimation of Parameters

Since the same parameters predict both the movements of short-term rates and the term structure, fitting can be done to either or both. If the fit is going to emphasize the term structure, equation (8) can be fit via simulation. However, this could require that a simulation be carried out a each step of a parameter search, which can be quite calculation intensive. Thus closed form or otherwise more tractable forms are usually sought for the zero-coupon bond prices at each maturity. Even if the fit is going to emphasize movements of the short-term rate, further fitting to the bond prices is needed to get the risk adjustments.

The usual approach to bond price formulation is to develop a stochastic differential equation for the price of the bond. This can then be solved explicitly or numerically. Developing such equations typically uses Ito's Lemma, which is the chain rule for stochastic calculus.

3.1 Ito's Lemma

Brownian motion is continuous, but is very jumpy at small scales, so is not differentiable. However a method of integration of these processes has been developed. This allows the use of differential notation, but the usual rules of derivative calculus, such as the chain rule, do not apply. However an analogue of the chain rule has been developed, and is known as Ito's Lemma. Suppose a process x can be expressed by $dx = y\,dt + s\,dz$, where z is a standard Brownian motion. If f is a twice differentiable real-valued function, then:

$$df(x) = \tfrac{1}{2}f''(x)s^2dt + f'(x)dx \qquad (14)$$

The second term is the usual chain rule, while the first is sometimes called the *convexity* term.

For example, suppose the change in r is proportional to the current level of r:

$$dr = \mu_t r\,dt + \sigma r\,dz \qquad (15)$$

Let $y = f(r) = \ln r$. Then $f'(r)=1/r$ and $f''(r)=-1/r^2$, so:

$$dy = -\tfrac{1}{2}\sigma^2dt + \mu_t dt + \sigma dz \qquad (16)$$

Thus the two methods illustrated above for keeping r positive - namely Brownian motion proportional to a power of r and geometric Brownian motion - are closely related. When converting a lognormal mean to the normal mean you add $\tfrac{1}{2}\sigma^2$ before exponentiating, which corresponds to the extra term in (16).

3.2 Solving for Bond Price

The price of a bond is some function f of the interest rate r. Thus Ito's Lemma can be used to derive a differential equation for the bond price. The usual approach in the single factor setting is to specify a risk-adjusted diffusion for r like:

$$dr_t = u_t(r_t)dt + \sigma_t(r_t)dz \qquad (17)$$

Then the price at time t of a bond maturing at time T, expressed as $P(r_t,t,T)$, can be shown to follow:

$$rP = P_t + uP_r + \frac{1}{2}\sigma^2 P_{rr} \qquad (18)$$

which is a differential equation with boundary condition $P(r,T,T) = 1$.
For example, see Vetzal *A Survey of Stochastic Continuous Time Models of the Term Structure of Interest Rates*, Insurance Mathematics and Economics (14), 1994.

In the multi-factor setting a similar differential equation can be derived, with partial derivatives of the price entering from all factors. In general if Y is the vector of factors, u is the vector of risk-adjusted drifts for the factors, and σ is the vector of variances, then the bond price P satisfies:

$$rP = P_t + u^T P_Y + \frac{1}{2}tr[\sigma\sigma^T P_{YY}] \qquad (19)$$

It turns out that the discount formula (8) above is a solution to (19), sometimes called the *Feynman-Kac* solution. Thus using simulation to solve (8) does solve (19). The advantage of going to (19) directly is that it can sometimes be solved in closed form, as in the example below, or by numerical methods that are less intensive than simulation. This is the approach taken by Tenney, for example, to estimate the parameters in the geometric Brownian motion two-factor model (6)-(7) above. However, Anderson and Lund solve for the diffusion parameters directly, which requires using (8) and a lot of computation or (19) and some derivation and numerical methods to get the risk terms.

3.3 A Simplified Model

Since the model parameters affect both the evolution over time of the short-term rate and the term structure at each point, both effects can be used to evaluate the goodness of fit. Thus both will influence the choice of parameters - i.e., parameters are needed that fulfill both roles. In the models above it is difficult to illustrate this interaction, as the generation of the term structure is complex.

Thus to illustrate these general concepts, parameter estimation will be discussed for a somewhat simplified three-factor model of the short-term interest rate - one with a closed form solution for the yield curve. This is the model of Kraus and Smith (*A Simple Multifactor Term Structure Model*, The Journal of Fixed Income, March 1993).

K&S postulate that the term structure at any point in time can be described as a function of three factors r, μ, and α. These factors evolve over time through Brownian motion, according to the equations below.

$$dr = \mu dt + \sigma dz_1 \tag{20}$$

$$d\mu = m dt + s dz_2 \tag{21}$$

$$d\alpha = dm - d(\sigma^2) = b dt + v dz_3 \tag{22}$$

So r is just a Brownian motion process, but the drift and variance both change over time. Here the time subscripts on the variables that change over time are omitted, but are implied. The drift μ is itself subject to a Brownian motion. The variance of the r diffusion, σ^2, is linked with the drift of the drift, m, as a single process $\alpha = m - \sigma^2$. This turns out to simplify the term structure formulation. However the variance itself could follow some unspecified process, like mean-reverting geometric Brownian motion.

On empirical and practical grounds it seems reasonable to set b to zero. Doing so leaves only two parameters - s and v. It is problematic that the model is not mean reverting, and interest rates are allowed to become negative. However, this may be a reasonable model to use for short-term projections. The advantage of its formulation is in the simplicity and flexibility of the resulting term structure. With some additional assumptions, the term structure turns out to be a simple polynomial form in the term T, with coefficients that are linear functions of the parameters and factors of the model. Thus long-range simulations are not needed to generate a term structure distribution for the near future.

K&S assume that the yield rates are linear functions of the factors (but not of the term). They also introduce three risk-adjustment coefficients, one for each factor, that are similar to the λ and κ risk terms in the models above. They then derive, using a no-arbitrage argument, a polynomial form for the term structure. This proceeds by setting up a differential equation for bond prices, from (19):

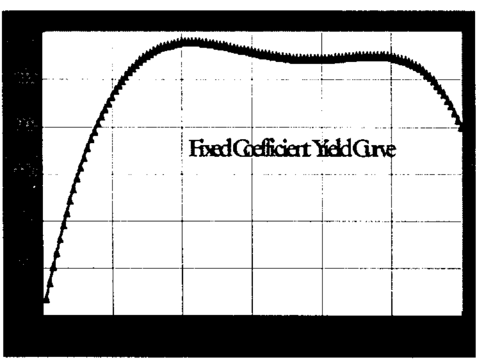
$$rP = P_t + (\mu_j+\lambda_r)P_r + (m+\lambda_\mu)P_\mu + \lambda_\alpha P_\alpha + \tfrac{1}{2}[\sigma^2 P_{rr} + s^2 P_{\mu\mu} + v^2 P_{\alpha\alpha}] \qquad (23)$$

where λ_Φ is the risk factors for factor Φ. Here an additive constant risk factor has been added to each drift coefficient. This equation has a closed form solution. Let $y_j(T)$ denote the yield for term T at time j. The assumptions then give:

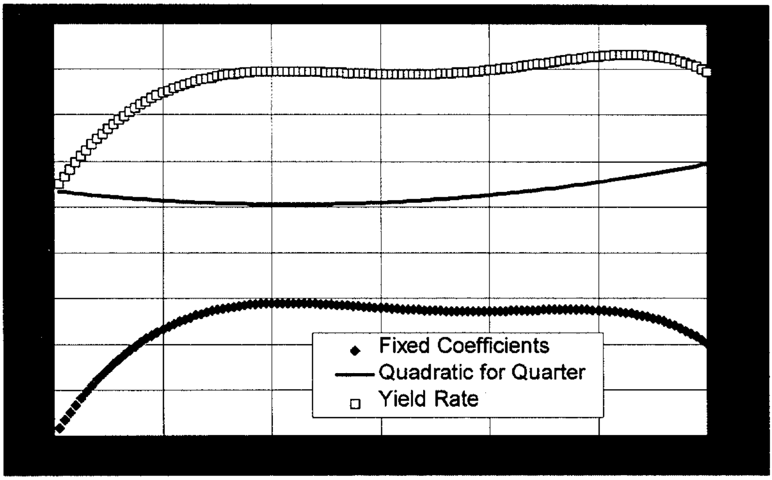
$$y_j(T) = r_j + T(\mu_j+\lambda_r)/2 + T^2(\alpha_j+\lambda_\mu)/6 + T^3\lambda_\sigma/24 - T^4 s^2/40 - T^6 v^2/504 \qquad (24)$$

Note that the equation is a sixth degree polynomial in the term, and the higher order coefficients are negative. This implies that for long enough terms the yield rate will decrease. For the parameters estimated below, the thirty year rate is often less than the twenty-five year rates, which is often the case in the data as well. Of course this also implies that very long term rates will be negative, which is not realistic. The model clearly should not be used for very long terms.

The first three (quadratic) terms of the polynomial vary with time. An interpretation is then that there is a fixed sixth degree polynomial for the standard yield curve, and this gets shifted up and down by a quadratic over time. The graph to the left shows the polynomial defined by the estimates of the fixed terms of (17) - i.e., all the elements without subscripts, namely $T\lambda_r/2 + T^2\lambda_\mu/6 + T^3\lambda_\sigma/24 - T^4 s^2/40 - T^6 v^2/504$.



The quadratic shift gives quite a bit of flexibility to the shape of the yield curve. Reversals and other unusual shapes are readily produced. This is in contrast to single factor models, which typically allow only parallel shifts. The quadratic terms for a selected quarter and the resulting yield curve are shown below.



3.4 Results and Discussion

The fitting approach described below gave the following estimates based on quarterly data from 82:4 to 95:4.

λ_r	λ_μ	λ_σ	s^2	v^2
.0146	-3.75E-3	4.82E-4	6E-6	3.8E-8

The fit was reasonably good, which to some extent justifies the assumptions. However it seems unusual that λ_μ is negative, in that the risk adjustments are supposed to push rates up.

3.5 Fitting Parameters

Parameters were fit using term structure data for 1982:Q4 - 1995:Q4. For quarterly observations the evolution equations become:

$$\Delta r = \mu/4 + \varepsilon_1\sigma/2 \quad (25)$$

$$\Delta\mu = m/4 + \varepsilon_2 s/2 \quad (26)$$

$$\Delta\alpha = \varepsilon_3 v/2 \quad (27)$$

Here the ε's are random draws from the standard normal distribution. The term structure is what is observable, so the fitting is based on fitting equation (24) to the data for each quarter. The evolution equations (25) - (27), however, put constraints on the parameters. There are at least these constraints:

1. The T^4 and T^6 coefficients must be negative.
2. v^2 in the T^6 coefficient is the variance of the changes in the T^2 coefficients α.
3. s^2 in the T^4 coefficient is the variance of the changes in the T coefficients μ.
4. The average change in the T coefficient is m which is imbedded in the T^2 coefficient.
5. The constant term r_i changes by an average of μ_I from the T term and by a variance of σ_I, which is imbedded in the T^2 term.

The basic approach is to get coefficients of T^i by regression, subject to the constraints. Suppose we have a preliminary estimate of the coefficients of the T^i. Then some of the constraints can be used to separate regression coefficients into components, and then these can be used to check other constraints. For instance, the constant term r_i should change by an average of μ_i and by a variance of σ_i. These relationships can be used to estimate λ_r and σ_i. To see this in greater detail, to estimate λ_r, by adding up the changes in r we get:

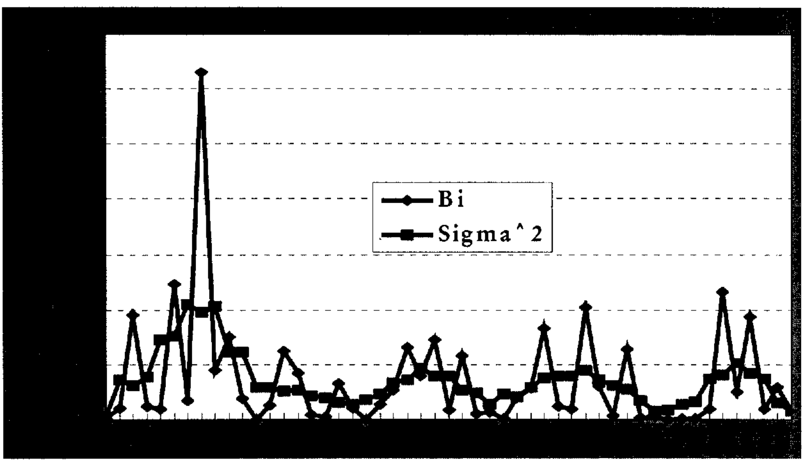
$$r_n - r_1 \cong \Sigma\mu_i/4 \quad (28)$$

Then, since $n\lambda_r$ can be expressed as $\Sigma(\mu_i+\lambda_r) - \Sigma\mu_i$, (28) can be used to estimate λ_r, as the first sum can be calculated from the T coefficients.

Having estimated λ_r, the T coefficients then give the μ_i's. To estimate σ_i, use that:

$$E(r_{i+1}-r_i-\mu_i/4)^2 = \sigma_i^2/4 \quad (29)$$

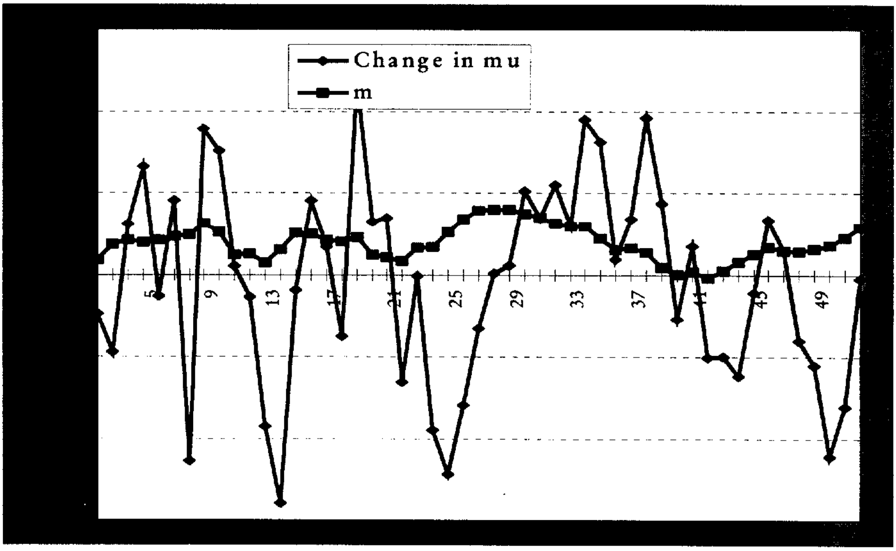
The expression inside the expectation on the left-hand side of this equation can now be calculated for each i. Call it B_i. The following ad hoc method is one way to estimate the expectation at each i. Take a seven term centered moving average of the B_i at each point, with the middle three points getting double weight. To reduce the effect that extreme observations have on this average, trim each point to a maximum of twice its own centered seven point average. Then re-average the points to estimate the expected B_i at each point, and use that as the estimate of $\sigma_i^2/4$ from (22). There are clearly other ways to estimate the σ_i^2. Since these do not directly impact the term structure, their estimation is not critical to the overall fit. The smoothing of the B_i to estimate $\sigma_i^2/4$ is shown in the graph below.



A similar procedure can be used to split out m_i and λ_μ from the T^2 coefficient. Since m_i is just $\alpha_i - \sigma_i$, subtracting σ_i from the coefficient just leaves $m_i + \lambda_\mu$. Using the fact that μ_i changes by an average of m_i and with variance s^2 (known from T^4) gives a way to estimate λ_μ. From the constraint (26) the following should hold:

$$s^2/4 = E(\mu_{i+1}-\mu_i-m_i/4)^2 \qquad (30)$$

Each value of λ_μ implies values for the m_i's, so λ_μ can be estimated as the value that would give the resulting m_i's that satisfy (30). The m's that result from matching this variance are graphed along with the change in μ that they are meant to average in the graph below. The variance is quite large, so the fit may be reasonable.



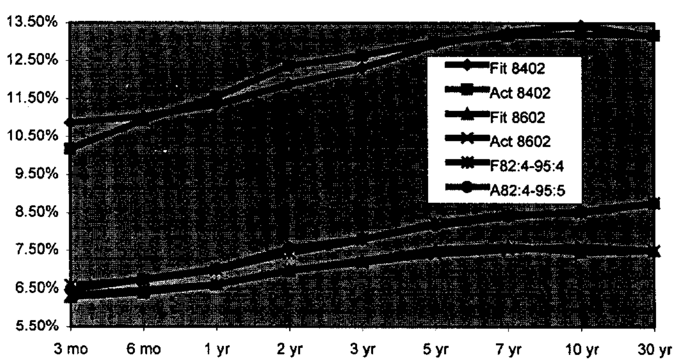
Another requirement is $\mu_n - \mu_1 \cong \Sigma m_i/4$, seen by summing (26) over all periods. This has not been used to estimate anything, so it was used as a test of the fit. The relationship $E(\alpha_{i+1}-\alpha_i)^2 \cong v^2$, from (27) was used as a constraint. The right side of this is the T^6 coefficient, and the left side is called the *implied* v^2.

A search procedure (simplex based) was used to fit the parameters. The search is looking for the three higher order coefficients defined by λ_σ, s, and v. What is minimized is the sum of squared errors between the actual and fitted yield rates at each period plus a weighting constant times the difference between v^2 and the implied v^2. In the minimization, for each trial λ_σ,s,v triplet, the sum of the higher terms, i.e., $T^3\lambda_\sigma/24-T^4s^2/40-T^6v^2/504$, is subtracted from the yield rates. This leaves a quadratic expression for the yields so adjusted for each quarter. The three quadratic coefficients are fit by regression for each time period to the observed interest rates for the terms used. These were 3 month, 6 month, 1 year, 2 year, 3 year, 5 year, 7 year, 10 year, and 30 years. This gives estimates for the coefficients r_j ,$(\mu_j+\lambda_r)/2$, and $(\alpha_j+\lambda_\mu)/6$ for each period j. Then the above approach is used to split these out into μ_j, λ_r, α_j, λ_μ, m_j, and σ_j. The sum of squared errors and the difference between the trial and implied v^2's are computed. New triplets are tried until the sum is minimized. That gave the parameters above. The resulting λ_μ of -0.00375 is roughly in the ballpark of the value of -0.00303 need to equalize μ_n-μ_1 and $\sum m_i/4$.

3.6 Goodness of Fit

The graph below shows the actual and fitted interest rates by term for a bad fitting quarter, a good fitting quarter, and the average of all the periods. The bad fit was actually somewhat exceptional, as the good fits were more typical.

The Best of Fits and the Worst of Fits (and the Average)



Although the fitting was simplified due to the closed form of the yield curve, the relationships between the yields and the movement of interest rates would hold in the more complex cases as well. Thus this example illustrates the interrelationships to be preserved in interest rate fits.

4 OTHER ECONOMIC VARIABLES

The term structure of interest rates incorporates investors' anticipations of future rates and thus implicitly of future levels of prices and economic activity. In recent years a number of articles have been published which attempt to forecast economic variables based on the term structure. For instance, see Eugene F. Fama *Term Structure Forecasts of Interest Rates, Inflation, and Real Returns,* Journal of Monetary Economics 25 (1990) pp. 59-76. It appears from this research that aspects of the yield curve do correlate with future economic activity, and so these correlations need to be taken into account when generating economic scenarios.

Forecasts will of course not be perfect, so when forecasting economic series from interest rates the prediction distributions will need to be taken into account. In a simulation context, for a given time frame the interest rate generator will produce yield curve scenarios, and then from each of those a prediction can be made of the other economic variables. Then a random draw from the prediction distribution can be made to produce a specific simulated scenario that includes interest rates and other series. This procedure should produce scenarios that are realistic over the time period chosen and with reasonable relative probabilities.

4.1 Examples of Prediction of Economic Variables from Yield Curves

To illustrate this process, two economic series are estimated from the term structure: the Consumer Price Index (CPI) and the Wilshire 5000 Index (W5). These both could have significant impact on insurer financial results.

Measures of the term structure typically are the interest rates for different terms as well differences between interest rates for different terms, e.g., the 10 year rate minus the 3 year rate. These are used at various lags. In this exercise all rates and lags are in multiples of a calendar quarter, so for notational purposes the time periods will be expressed as quarters. Notation such as 3L40:12 will denote the third lag of the difference between the 40 quarter and 12 quarter interest rates, i.e., the 10 year rate less the 3 year rate seen 9 months ago. Without the colon 0L40 is just the 10 year rate for the current quarter. Thus the notation comes with an actuarial spirit.

4.2 Consumer Price Index

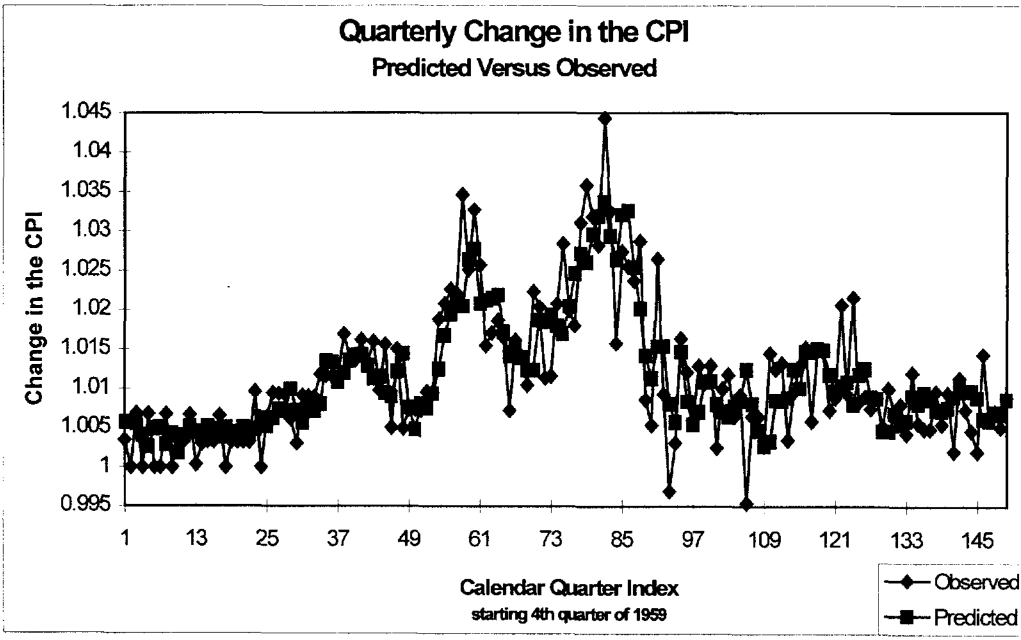
The variable estimated here, denoted qccpi, is the ratio of the CPI for a quarter to that for the previous quarter. The variables used in the fit along with indications of their significance are shown in the table below. The data used is from the fourth quarter of 1959 to first quarter 1997, as this was available from pointers within the CAS website.

Change in CPI

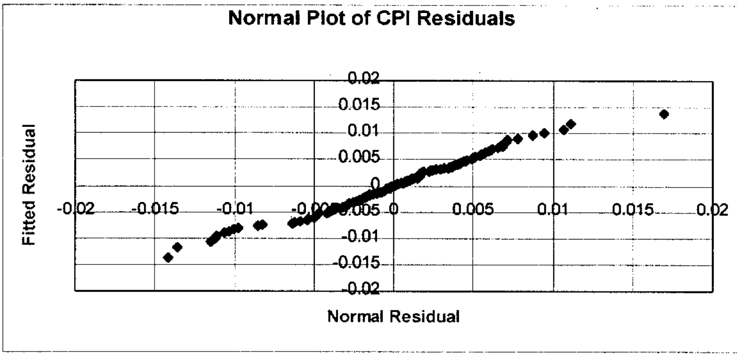
Variable	Estimate	T-statistic	Significance Level
1:4Lqccpi	0.9994	1649.4	<.01%
0L40:4	-0.2668	-5.3349	<.01%
2L40:20	0.8486	4.6411	<.01%
3L2:1	0.7182	3.4663	.07%

The most important indicator of inflation is recent inflation. The variable used to represent this, denoted 1:4Lqccpi, is the average of qccpi for the past four quarters. The coincident variable, 0L40:4 has a negative coefficient. This may be due to inflation influencing current interest rates, but with a greater impact on short term than long term rates, thus flattening the yield curve. At lag 2 quarters, the coefficient for 2L40:20 is positive and at lag 3 quarters that for 3L2:1 is positive. These indicate a general tendency for a steeper yield curve to anticipate future inflation. Other yield spreads also appear to have significant impact on inflation, but only a few can appropriately be included in any one regression. Interest rate series are highly correlated, and many that do not enter the formulation will still end up having significant correlations with the inflation rates produced. The coefficients suggest that over 80% of any increase in these yield spreads will be reflected in subsequent inflation.

The r-squared, adjusted for degrees of freedom, is 65%. The standard error of the estimate is 0.0051. Thus the typical predicted quarterly change is accurate to about half a percentage point. The standard error is the standard deviation of a residual normally distribution around the predicted point, which can be used to draw the scenario actually simulated. The actual vs. fit is graphed below. The series can be seen to be fairly noisy, but the model does pick up the general move-



ments over time. The residuals are graphed on a normal scale below. Normality looks to be reasonably consistent with the observed residuals.



4.3 Wilshire 5000

The variable modeled, qcw5, is the ratio of the W5 at the end of a quarter to that at the previous quarter end. In this case the CPI percentage change variable qccpi was included in the regression as an explanatory variable. This allows creation of scenarios that have simulated values of W5 that are probabilistically consistent with the CPI value for the scenario.

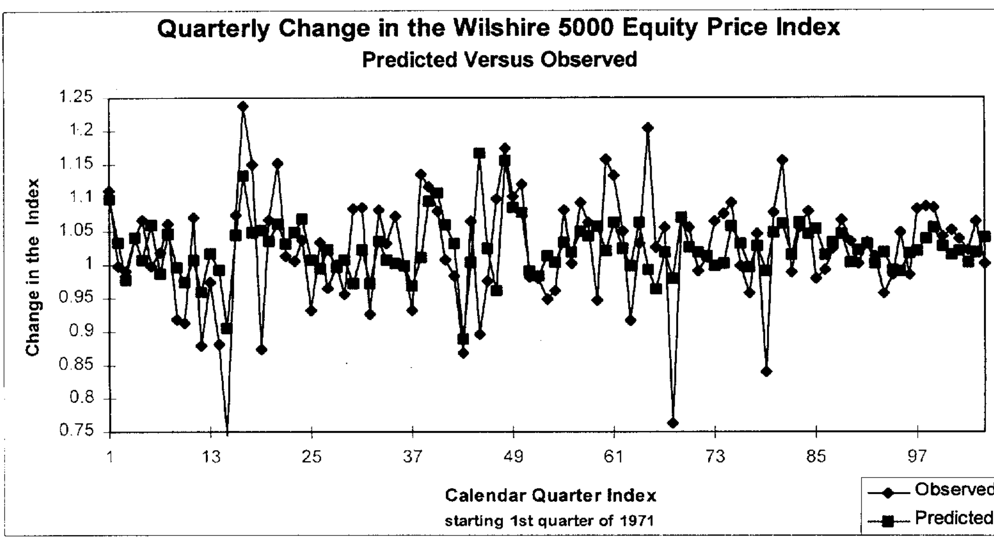
The fitted equation for quarter ending data 1971 through first quarter 1997 is shown in the table below. In this regression only two variables were used, but they are composite series. The first, denoted 0-4Lqccpi, is the increase in qccpi over the last year, i.e., the current rate less the rate a year earlier. This variable has a negative coefficient, indicating that an increase in inflation is bad for equity returns. The other variable is denoted qcrelsprd. It represents the previous quarter's increase in the long-term spread less this quarter's increase in the short-term spread. Here the long-term spread is the difference between 10-year and 5-year rates, and the short-term spread is the difference between 6-month and 3-month rates. The increases noted are the quarter-to-quarter arithmetic increases in these spreads.

The coefficient on qcrelsprd is positive. This variable is positive if the increase in the short-term spread is less than the previous increase in the long-term spread, or if its decrease is greater. Either could suggest moderating inflation and interest rates, and thus be positive for equity returns.

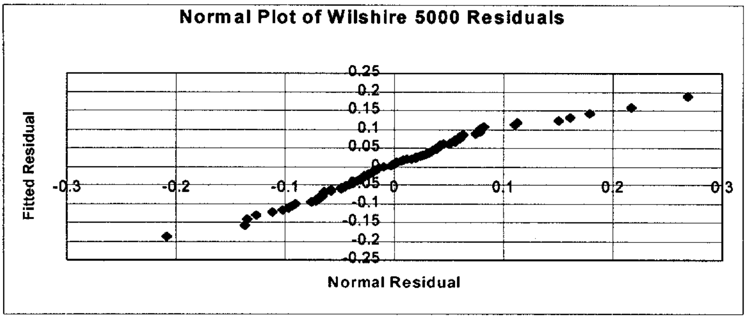
Quarterly Change in Wilshire 5000

Variable	Estimate	T-statistic	Significance Level
0-4Lqccpi	-2.7113	-3.1936	0.2%
qcrelsprd	11.869	4.5273	<.01%
constant	1.02316	145.311	<.01%

The adjusted-r-squared is only 24% for this regression, indicating that the fit is not particularly good. The residual standard deviation is .0721, which allows a

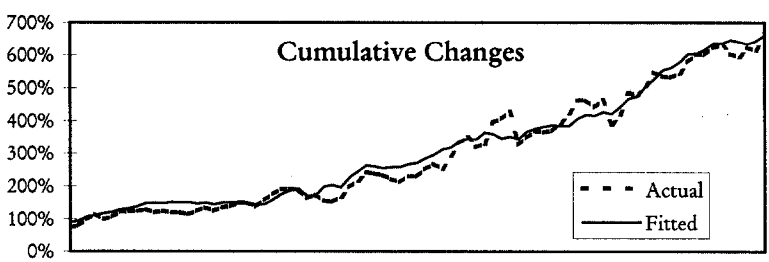


fairly wide deviation from the model. The actual vs. fit is graphed below.
The residuals also appear to be more heavy-tailed than for a normal distribution. They are graphed on a normal scale below.

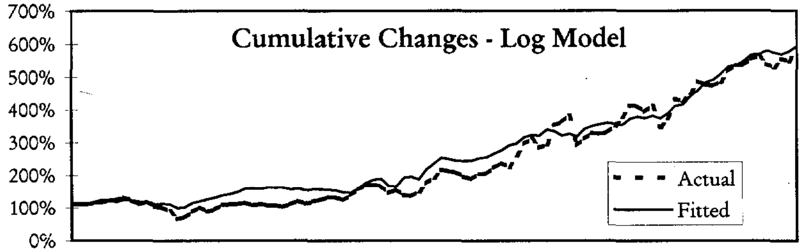


An alternative would be to simulate equity returns independently of interest rates and inflation. However, even the weak relationship found here would reflect the correlations that are likely among these variables, and would thus be preferable to assuming independence. More research into appropriate models for equity returns would be worthwhile.

Fitting percentage changes typically gives low r-squareds. The fit is usually better when translated to cumulative. The graph below shows cumulative products of the actual and fitted changes since 1974. The fit appears better on this basis.



The fit is not unbiased for products of factors over the entire horizon, so the above graph begins at a point selected to give a horizon where it is unbiased. A fit in the logs of the change ratios is unbiased for products when exponentiated. The graph below shows this fit for the same variables used in the original regression. The fit is actually not quite as good as the original. Both are fairly close cumulatively for the past ten years, however.



5 Applications

Duration matching, which seems to work well for life insurers, is problematic for P&C carriers, who have shorter duration liabilities, and so would have to give up expected return to match. Simulation studies have suggested that going longer on assets provides a margin to P&C insurers, which can compensate for duration mismatch. Realistic stochastic asset generators may help quantify this trade-off. Even duration matching and its refinement to convexity matching do not provide complete hedges against interest rate movements. As an alternative, the robustness of investment strategies can be tested against the whole range of possible outcomes, by probability level, by measuring against simulated assets.

Asset simulations can be tied to liability simulations as well, e.g., by linking inflation movements to loss trends. The total risk of assets and liabilities can thus be quantified simultaneously by such dynamic financial analysis.

Acknowledgment: The valuable assistance of John Gradwell in gathering data and fitting models is gratefully acknowledged.

Appendix - Summary Evaluation of Variables

Variable Wilshire 5000 Equity Price Index

Rationale Broad-based indicator of value of equity investments

Source Downloaded from the website "Wilshire Index History", address wilshire.com/home/products. Could not find a pointer on the CAS website.

Method of Analysis Multiple regression based on series already modeled, which in this case were Treasury yields and the CPI.

Variable Consumer Price Index

Rationale Inflation measure that covarys with interest rates, trend factors, and other economic series.

Source Downloaded from the CAS DFA website, following the pointers "Data Access", "financial and economics databases", "Consumer Price Index", at the address http://205.230.252.34. However this site ends with data from first quarter 1995.

Method of Analysis Multiple regression based on series already modeled, which in this case were Treasury yields.

Variable US Treasury yields for 3 months, 6 months, 1 year, 2 years, 3 years, 5 years, 7 years, 10 years, and 30 years

Rationale Many insurers invest in Treasury securities, and when these are carried at market value their price will depend specifically on the interest rates. Other economic variables are correlated to interest rate movements.

Source Downloaded from the website maintained by the Saint Louis Federal Reserve Bank's "FRED Database" , address www.stls.frb.org/fred/, which has a pointer from the CAS DFA website.

Method of Analysis Simulation based on multi-factor arbitrage-free diffusion processes fit to historical interest rate movements and yield curves.

The Effect of Residual Market Depopulation on Loss Ratio

by Christopher J. Poteet

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The Effect of Residual Market Depopulation on Loss Ratio

Introduction

The workers compensation residual market has been shrinking in size. Now that rate adequacy has improved, insurance carriers are willing to voluntarily write some of the risks which in previous years would have had to seek coverage in the residual market. The loss ratios for the risks leaving the residual market are, on average, higher than for the risks which are already written in the voluntary market, but lower than for the risks which remain in the residual market. This depopulation has the effect of increasing the loss ratio for the remaining group of residual market risks and increasing the loss ratio for the new group of voluntarily written risks. This study quantifies the effect of depopulation on the loss ratio of the residual market.

Data

A study of the impact of depopulation on loss ratios requires that market status and loss experience be tracked over time on a risk by risk basis. This is possible using statistical plan data. Eight states demonstrated consistent significant depopulation for the latest policy years of data available from this database (1992, 1993, 1994). Risk Identification Number is used to identify risks because this number does not change over time. Unfortunately, this excludes the small risks because only experience rated risks (those with premiums greater than $5000) have a risk ID. Fortunately, experience rated risks account for most of the data. Risks that were in the database for all three years were included in the study. Losses are first report undeveloped paid plus case reserve unlimited losses. Premium is manual premium times experience mod. This does not include premium credits or ARAP surcharge. It is before premium discounts (or removal of discounts for assigned risks) and before expense constant. The data is attached.

Methodology

This study takes an empirical look at the average loss ratio for the risks in the residual market. It does not make any assumption about the distribution of loss ratios.

For a given particular state, risks were grouped according to market status in each of the three years. For example, AAV refers to the group of risks which were assigned to the residual market in 1992 and 1993 and found coverage in the voluntary market in 1994.

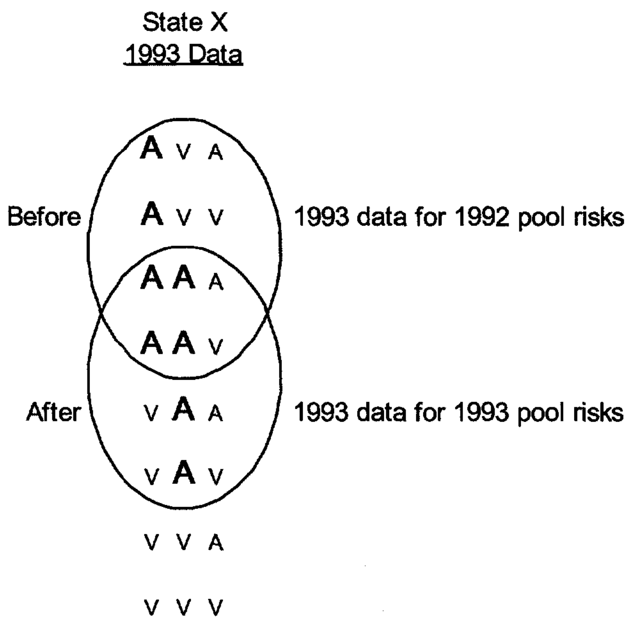
Risk Groups

Risks grouped according to market status in each year

Year 1992	Year 1993	Year 1994
A	A	A
A	A	V
A	V	A
A	V	V
V	A	A
V	A	V
V	V	A
V	V	V

A = Assigned Risk
V = Voluntary

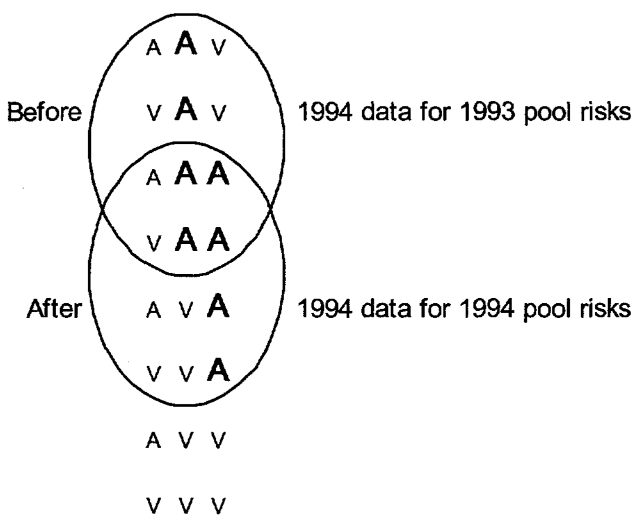
Risks were further grouped according to market status in 1992 versus market status in 1993. The "Before" group is the group of risks which were assigned risks in 1992. The "After" group is the group of risks which were assigned risks in 1993. Loss ratios were determined for policy year 1993 experience for each of these groups. Using data from just one year eliminates any change in experience, trend, changes in rate adequacy, and effects of changes in cost containment. This regrouping of the data helps to isolate the impact of depopulation.



The State X, 1993 surcharge was added in for the AVA and AVV groups to determine what the data would be if they were still in the residual market (pool). The surcharge was taken out for the VAA and VAV groups to determine what the data would be if they were still in the voluntary market.

Risks were also grouped according to market status in 1993 versus market status in 1994. The "Before" group is the group of risks which were assigned risks in 1993. The "After" group is the group of risks which were assigned risks in 1994. Loss ratios were determined for policy year 1994 experience for each of these groups.

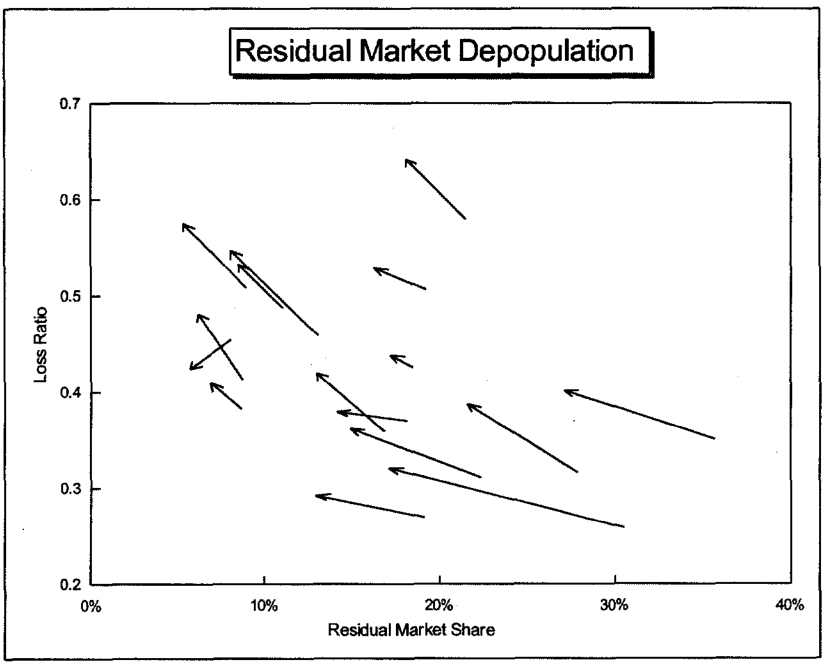
State X
1994 Data



The State X, 1994 surcharge was added in for the AAV and VAV groups to determine what the data would be if they were still in the residual market (pool). The surcharge was taken out for the AVA and VVA groups to determine what the data would be if they were still in the voluntary market.

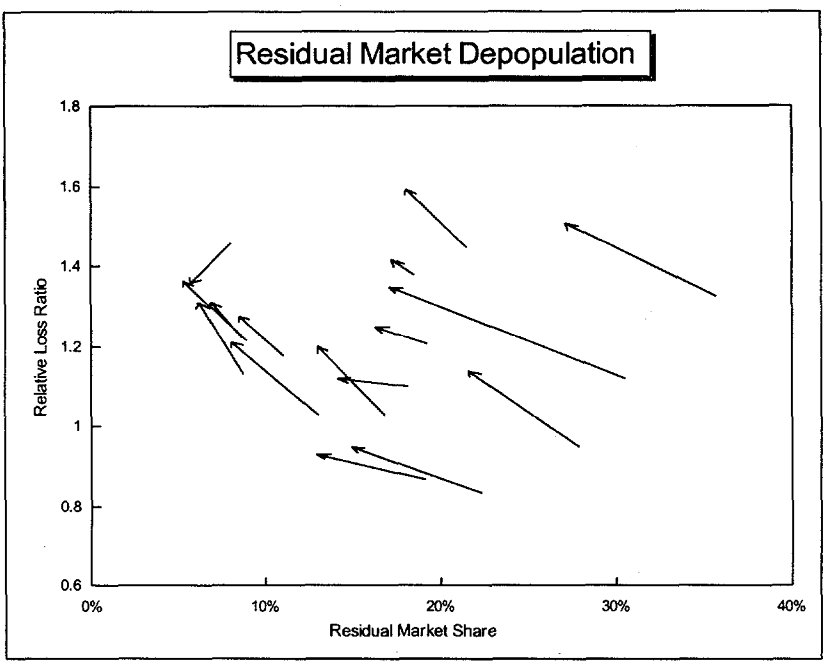
Initial Analysis

The residual market loss ratio versus residual market share, before and after depopulation, was graphed for each state based on policy year 1993 data. An arrow was drawn connecting the "before" data point to the "after" data point (arrowhead). This was also done with 1994 data. The slope of each resultant line segment is the change in residual market loss ratio for the given change in residual market share. If the slopes were similar, then this could be used to draw a general conclusion about the effect of depopulation on the loss ratio for the countrywide residual market pool. Not all slopes are similar, however. There is a curved pattern evident in the graph.



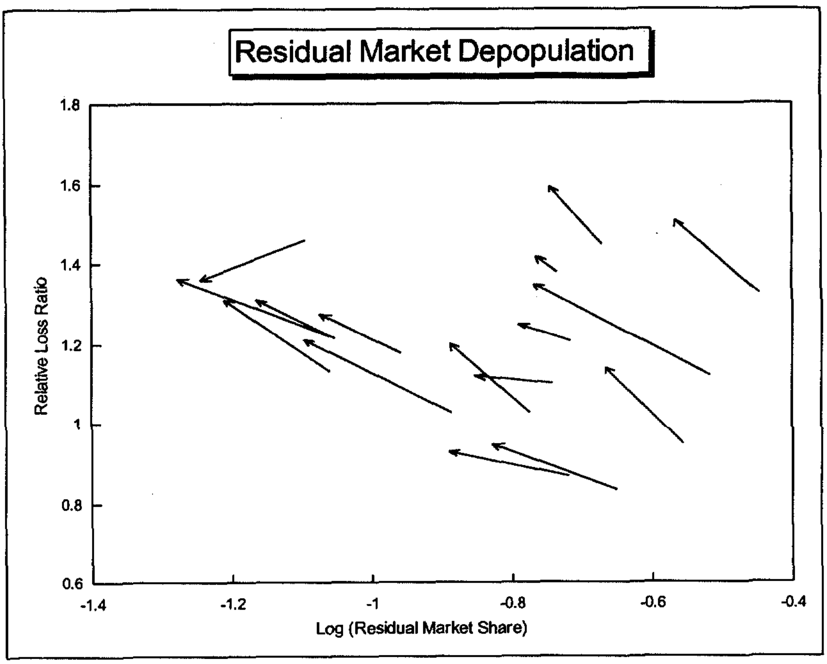
Using Relative Loss Ratios

Residual market loss ratios relative to statewide (residual market and voluntary market combined) were determined. This eliminates the need for on-level factors or loss development. Different states can be directly compared. Possible effects of unknown extraneous variables (such as relative levels of rate adequacy) are reduced. After this adjustment to the data is made, the points regraphed and the arrows redrawn, there appears to be two distinct groups (low market share and high market share) which have similar slopes within the groups.



Using Logarithm of Market Share

Since the line segments are flatter and longer at higher market shares, taking the logarithm of the market share might make the slopes more similar. The logarithm (base 10) of the market share was computed. Relative loss ratio was compared to the logarithm of market share. The graph exhibits a distinct similarity in slopes.



Comparison of approaches

The coefficient of variation of slopes was used to determine that the aforementioned data transformations improved the quantification of the slope (i.e. narrowed the confidence interval). The slopes were approximately lognormally distributed for each scenario. The two highest and two lowest observations were excluded before the average slope and coefficient of variation of slope was computed. The median is close to the mean when these outliers are excluded. The outliers did not tend to be any particular state or any particular year which indicates that there is not a bias in the results with regard to state or year.

Method	Coefficient of Variation of Slope	Average Slope	
Loss Ratio & Market Share	-0.468	-1.103	
Relative Loss Ratio & Market Share	-0.413	-2.842	
Relative Loss Ratio & Log (Market Share)	-0.366	-0.930	☜ use this

Slope Equation

Variables:

m = slope

R_1 = Relative Loss Ratio (compared to statewide) ***before*** depopulation
R_2 = Relative Loss Ratio (compared to statewide) ***after*** depopulation

S_1 = Residual Market Share ***before*** depopulation
S_2 = Residual Market Share ***after*** depopulation

Equation:

$$m = \frac{R_2 - R_1}{\log(S_2) - \log(S_1)}$$

$$m = \frac{R_2 - R_1}{\log\left(\frac{S_2}{S_1}\right)}$$

$$R_2 = R_1 + m \log\left(\frac{S_2}{S_1}\right)$$

A change in market share from 50% to 25% will have the same additive adjustment to the loss ratio as a change in market share from 10% to 5%. Both changes are quite dramatic. The change from 50% to 25% will have a big impact on the loss ratio because this is a high volume change. The change from 10% to 5% will also have a big impact on the loss ratio because when the residual market is so small the average loss ratio for the risks which remain in the residual market is much greater than the average loss ratio for the risks which depopulate.

Property:

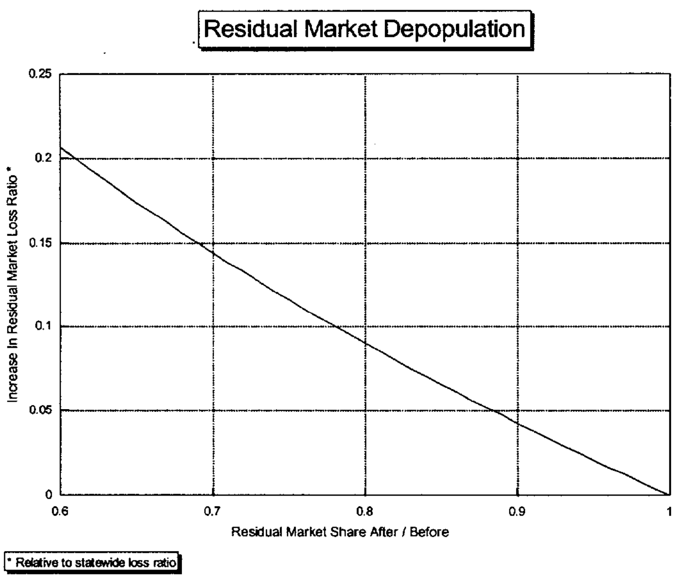
$$R_3 = R_2 + m \log\left(\frac{S_3}{S_2}\right)$$

$$R_3 = R_1 + m \log\left(\frac{S_2}{S_1}\right) + m \log\left(\frac{S_3}{S_2}\right)$$

$$R_3 = R_1 + m \log\left(\frac{S_3}{S_1}\right)$$

The model has the desirable property that two subsequent changes in market share will have the sum total adjustment to the loss ratio equal to the adjustment that would be made if the changes in market share were combined into one.

The relationship $R_2-R_1 = -.930 \log (S_2/S_1)$ is graphed for typical amounts of depopulation. The curve would take a sharp upward turn on the left if the graph were extended to include extreme amounts of depopulation. This study did not include any states which experienced extreme depopulation or states with very low residual market shares and therefore extrapolation of the results of this study for such states would be questionable.



Example

Policy Year 1994 Residual Market Loss Ratio = .653

Policy Year 1994 Statewide Loss Ratio = .600

1994 Residual Market Share = 23.4%

Residual Market Share for estimated year = 16.5%

Policy Year 1994 Residual Market Loss ratio adjusted to reflect estimated depopulation =

$.653 + .600[-.930 \log(.165/.234)] = .738$

This additive adjustment should be made before other loss ratio adjustments such as trend, change in benefits and premium level changes. This adjustment can be thought of as a regrouping of the data to reflect depopulation and is not a movement forward in time to a different policy year.

Applying the results to an individual state

The results of this study have already been applied in two states to project a residual market loss ratio and the assigned risk surcharge needed with an assumed amount of depopulation. Following are some guidelines to be used in applying the results of this study.

• Look at a range of scenarios for a state

The confidence interval around the average slope of -.930 is sufficiently narrow so that the model can predict the impact of depopulation on the loss ratio for the pool with reasonable accuracy. The impact of depopulation for an individual state cannot be predicted as precisely, because of the uncertainty in calculating a state specific slope. Several scenarios should be considered. The average slope of -.930 should be considered. The two state specific slopes can also be considered for those states which were included in the study. Each of these slopes will yield a predicted impact on the residual market loss ratio. The range of predictions can aid in selecting an impact. A chart is provided showing the individual state slopes for the Relative Loss Ratio versus Log of Market Share method.

STATE&YR	SLOPE
B94	0.666
A94	-0.167
F94	-0.357
H93	-0.604
E94	-0.640
D94	-0.661
B93	-0.849
D93	-0.864
C93	-0.869
G94	-0.918
C94	-1.187
A93	-1.222
H94	-1.478
G93	-1.509
E93	-1.745
F93	-1.916

• Use a statewide loss ratio consistent with the residual market loss ratio

The statewide loss ratio is one of the inputs used to determine the adjustment to the residual market loss ratio. Statewide losses and premium should be developed and on-level consistent with the residual market loss ratio.

• Be consistent with market shares

Since the ratio of market shares S_2/S_1 is used, market shares do not have to be based on manual premium times mod but they do have to be consistent. Use assigned risk premium on the same basis as voluntary premium (e.g. they both include expenses).

State A

	# policies	1993 losses	1993 premium	1994 losses	1994 premium
AAA	1,775	18,444,133	37,147,231	15,044,939	35,345,905
AAV	361	4,608,273	13,311,095	4,456,882	11,473,837
AVV	297	4,983,475	13,334,758	5,616,873	13,937,226
AVA	16	152,178	197,039	253,356	335,475
VAV	88	1,268,452	4,290,207	1,592,135	3,520,868
VAA	258	2,666,603	6,750,059	1,083,214	5,661,589
VVA	200	2,277,879	3,878,297	1,101,216	4,655,397
VVV	4,910	76,483,975	278,470,508	81,777,325	251,569,097

	1993 surcharge	1994 surcharge
	15.5%	25.6%

1993

	losses	premium	loss ratio
A**	28,188,059	66,087,552	0.427
V**	82,696,909	291,907,477	0.283
			0.310
A	26,987,461	61,498,592	0.439
V	83,897,507	295,880,602	0.284
			0.310

18.5% → 17.2% residual market share

1994

	losses	premium	loss ratio
A	22,177,170	59,840,843	0.371
V	88,748,770	269,479,947	0.329
			0.337
**A	17,482,725	45,998,366	0.380
**V	93,443,215	280,501,028	0.333
			0.340

18.2% → 14.1% residual market share

A = Assigned Risk
V = Voluntary
* = either A or V

State B

	# policies	1993 losses	1993 premium	1994 losses	1994 premium
AAA	1,138	9,654,614	22,332,855	9,256,262	20,661,729
AAV	298	3,680,784	10,205,587	8,190,822	14,443,915
AVV	318	5,078,009	15,784,512	5,704,421	13,362,129
AVA	20	511,218	1,142,118	370,972	581,586
VAV	90	1,208,224	3,098,353	641,570	2,729,149
VAA	191	1,473,758	3,455,754	937,274	3,229,607
VVA	236	3,373,054	4,544,202	1,997,733	5,158,458
VVV	11,365	153,292,196	509,555,689	134,744,125	458,216,175

1993 surcharge	1994 surcharge
0.0%	4.6%

1993

	losses	premium	loss ratio	residual market share
A**	18,924,625	49,465,072	0.383	8.7%
V**	159,347,232	520,653,998	0.306	
			0.313	
A	16,017,380	39,092,549	0.410	6.9%
V	162,254,477	531,026,521	0.306	
			0.313	

1994

	losses	premium	loss ratio	residual market share
A	19,025,928	41,854,361	0.455	8.1%
V	142,817,251	477,065,918	0.299	
			0.312	
**A	12,562,241	29,631,380	0.424	5.7%
**V	149,280,938	488,751,368	0.305	
			0.312	

A = Assigned Risk
V = Voluntary
* = either A or V

State C

	# policies	1993 losses	1993 premium	1994 losses	1994 premium
AAA	3,481	51,560,296	84,396,298	44,557,947	90,884,971
AAV	926	17,999,541	38,324,443	17,302,822	39,755,862
AVV	1,079	28,784,826	64,691,956	27,662,685	73,778,907
AVA	48	1,089,113	2,727,110	2,037,140	4,033,340
VAV	222	7,539,751	21,256,577	5,203,612	21,672,392
VAA	403	5,742,670	11,751,219	5,853,745	11,557,013
VVA	442	15,215,417	11,716,406	7,173,892	17,279,689
VVV	33,510	640,051,700	1,607,436,564	627,377,170	1,747,475,833

1993 surcharge	1994 surcharge
20.0%	20.0%

	1993 losses	1993 premium	1993 loss ratio	residual market share
A**	99,433,776	203,623,620	0.488	11.0%
V**	668,549,538	1,646,659,467	0.406	
			0.415	
A	82,842,258	155,728,537	0.532	8.5%
V	685,141,056	1,686,572,036	0.406	
			0.417	

	1994 losses	1994 premium	1994 loss ratio	residual market share
A	72,918,126	176,155,889	0.414	8.7%
V	664,250,887	1,839,015,598	0.361	
			0.366	
**A	59,622,724	123,755,013	0.482	6.2%
**V	677,546,289	1,882,682,994	0.360	
			0.367	

A = Assigned Risk
V = Voluntary
* = either A or V

State D

	# policies	1993 losses	1993 premium	1994 losses	1994 premium
AAA	1,638	11,888,105	20,636,259	13,516,344	20,469,725
AAV	551	8,239,524	16,571,780	10,525,854	19,449,755
AVV	624	12,790,971	28,124,000	12,408,768	29,394,934
AVA	47	661,951	677,100	198,154	717,244
VAV	118	1,813,303	3,182,520	1,241,088	3,363,612
VAA	180	2,345,250	4,106,017	1,670,046	4,019,310
VVA	176	3,910,853	6,428,236	2,667,688	6,207,385
VVV	17,800	208,314,605	475,375,142	207,786,929	509,503,229

1993 surcharge	1994 surcharge
25.0%	25.0%

	1993			
	losses	premium	loss ratio	residual market share
A**	33,580,551	73,209,414	0.459	13.1%
V**	216,384,011	487,634,208	0.444	
			0.446	
A	24,286,182	44,496,576	0.546	8.0%
V	225,678,380	510,604,478	0.442	
			0.450	

	1994			
	losses	premium	loss ratio	residual market share
A	26,953,332	53,005,744	0.508	8.9%
V	223,061,539	544,437,866	0.410	
			0.418	
**A	18,052,232	31,413,664	0.575	5.3%
**V	231,962,639	561,711,530	0.413	
			0.422	

A = Assigned Risk
V = Voluntary
* = either A or V

State E

	# policies	1993 losses	1993 premium	1994 losses	1994 premium
AAA	1,128	9,977,821	26,780,660	9,208,462	26,057,745
AAV	276	4,052,651	13,034,682	4,341,669	11,461,955
AVV	366	5,025,093	14,285,735	5,846,386	14,313,966
AVA	11	31,422	532,996	127,599	312,094
VAV	68	692,675	1,945,646	425,969	2,393,270
VAA	129	3,091,838	4,124,614	1,234,270	3,583,898
VVA	58	1,761,417	2,329,360	924,855	1,806,423
VVV	3,066	47,773,986	149,406,468	59,316,177	153,531,771

	1993 surcharge	1994 surcharge
	38.0%	38.0%

1993

	losses	premium	loss ratio
A**	19,086,987	60,265,191	0.317
V**	53,319,916	156,134,567	0.341
			0.335
A	17,814,985	45,885,602	0.388
V	54,591,918	166,554,559	0.328
			0.341

Residual market share: 27.8% → 21.6%

1994

	losses	premium	loss ratio
A	15,210,370	48,761,854	0.312
V	66,215,017	169,380,894	0.391
			0.373
**A	11,495,186	31,760,160	0.362
**V	69,930,201	181,700,962	0.385
			0.381

Residual market share: 22.4% → 14.9%

A = Assigned Risk
V = Voluntary
* = either A or V

State F

	# policies	1993 losses	1993 premium	1994 losses	1994 premium
AAA	969	19,459,557	21,022,186	7,192,841	23,211,615
AAV	294	3,270,952	12,649,709	3,166,217	11,073,141
AVV	275	3,132,432	8,909,387	2,908,688	8,999,598
AVA	9	169,822	319,349	66,713	433,722
VAV	64	502,401	1,387,493	455,804	1,436,884
VAA	125	826,403	2,441,134	732,855	3,277,952
VVA	85	1,507,714	1,382,712	343,203	1,490,004
VVV	6,589	55,106,180	160,142,906	54,712,118	170,643,350

	1993 surcharge	1994 surcharge
	22.1%	30.1%

1993

	losses	premium	loss ratio	residual market share
A**	26,032,763	44,940,182	0.579	21.4%
V**	57,942,698	164,661,267	0.352	
			0.401	
A	24,059,313	37,500,522	0.642	18.0%
V	59,916,148	170,754,354	0.351	
			0.403	

1994

	losses	premium	loss ratio	residual market share
A	11,547,717	42,765,110	0.270	19.1%
V	58,030,722	181,121,600	0.320	
			0.311	
**A	8,335,612	28,413,293	0.293	12.9%
**V	61,242,827	192,152,973	0.319	
			0.315	

A = Assigned Risk
V = Voluntary
* = either A or V

State G

	# policies	1993 losses	1993 premium	1994 losses	1994 premium
AAA	976	14,810,880	28,246,446	9,038,192	28,178,911
AAV	656	4,746,490	20,105,785	4,806,808	20,235,102
AVV	473	3,864,645	17,141,331	3,158,768	16,944,457
AVA	19	329,721	430,084	183,926	382,287
VAV	65	502,611	1,712,401	303,140	1,713,267
VAA	66	403,064	780,667	473,838	840,690
VVA	44	521,440	1,615,885	189,572	1,362,151
VVV	3,731	25,037,156	117,882,960	24,741,745	110,409,025

	1993 surcharge	1994 surcharge
	10.0%	25.1%

1993	losses	premium	loss ratio	residual market share
A**	23,751,736	67,680,788	0.351	35.7%
V**	26,464,271	121,765,270	0.217	
			0.265	
A	20,463,045	50,845,299	0.402	27.1%
V	29,752,962	137,070,260	0.217	
			0.267	

1994	losses	premium	loss ratio	residual market share
A	14,621,978	56,477,011	0.259	30.5%
V	28,274,011	128,747,917	0.220	
			0.232	
**A	9,885,528	30,764,039	0.321	17.1%
**V	33,010,461	149,301,851	0.221	
			0.238	

A = Assigned Risk
V = Voluntary
* = either A or V

State H

	# policies	1993 losses	1993 premium	1994 losses	1994 premium
AAA	567	5,625,193	8,705,494	4,253,886	9,565,398
AAV	119	922,623	2,866,509	724,018	3,534,064
AVV	106	1,337,910	3,468,114	1,415,204	3,968,740
AVA	3	36,682	44,028	13,854	55,886
VAV	26	199,775	682,217	177,322	611,329
VAA	61	187,416	872,972	371,624	1,071,238
VVA	54	1,173,722	909,654	304,246	1,077,020
VVV	2,920	24,694,365	63,314,666	24,616,615	71,049,610

1993 surcharge 15.0%

1994 surcharge 15.0%

1993	losses	premium	loss ratio	residual market share
A**	7,922,408	15,610,966	0.507	19.2%
V**	26,255,278	65,576,658	0.400	
			0.421	
A	6,935,007	13,127,192	0.528	16.2%
V	27,242,679	67,736,462	0.402	
			0.423	

1994	losses	premium	loss ratio	residual market share
A	5,526,850	15,403,838	0.359	16.9%
V	26,349,919	76,003,486	0.347	
			0.349	
**A	4,943,610	11,769,542	0.420	12.9%
**V	26,933,159	79,163,743	0.340	
			0.351	

A = Assigned Risk
V = Voluntary
* = either A or V

Independent Claim Report Lags and Bias in Forecasts Using Age-To-Age Factor Methodology

by Stewart H. Gleason, Ph.D., ACAS

Independent Claim Report Lags and Bias in Forecasts Using Age-To-Age Factor Methodology

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I. Introduction

In his 1985 Proceedings paper "A Simulation Test of Prediction Errors of Loss Reserve Estimation Techniques", J. Stanard [1] pointed out an apparent bias in forecasts of ultimate claims when commonly used reserving methods were applied to simulated data. The approach was to specify a stochastic model of claims emergence and use it to generate data to be used as input to various reserving methods. One of the methods selected was the familiar age-to-age factor method and it was found to produce overstated forecasts of ultimate claims in certain cases.

Stanard's simulation model assumes that the report lag of each claim is independent. This hypothesis has been put forth in other work, particularly that of E. Weissner [2], [3]. The work presented here will show analytically that when report lags are assumed to be independent, the age-to-age factor method is biased.

This will be shown in two special cases of claim count development. First, it will be assumed that the ultimate number of claims for an accident period has a Poisson distribution. In this case, the assumption of independent report lags implies the independence of the total number of claims reported in any two periods. This is a special case of what will here be called the assumption of independent increments. A general argument may then be given to show that the age-to-age factor methodology gives biased results when the underlying process is known to have independent development increments.

The situation where the ultimate number of claims has a negative binomial distribution is also addressed and is in fact the model specified by Stanard. In this case, assuming that report lags are independent does not imply that increments are independent and a somewhat different argument is required.

The arguments presented here will make use of Jensen's Inequality. Stanard notes in Appendix A of his paper that the observed bias is likely due to the fact that the expected value of the quotient of two random variables is not necessarily equal to the quotient of their expected values, i.e.

$$\frac{E[X]}{E[Y]} \neq E\left[\frac{X}{Y}\right].$$

Jensen's Inequality may be used to show that, when the right conditions are specified,

$$E\left[\frac{X}{Y}\right] > \frac{E[X]}{E[Y]}$$

These quotients will arise in what follows as the usual claims development or age-to-age factors. In addition, it will be demonstrated that weighted average forecasts exhibit a smaller bias than straight average estimates.

II. Preliminaries

Notation and Assumptions

For simplicity, claims activity segmented into n consecutive, non-overlapping time periods of equal length will be considered. $X_{i,j}$ will denote the number of incidents occurring in period i which are reported as claims in period $i+j-1$ (or with lag $j-1$). The incremental development triangle at the end of the nth period is displayed as:

Number of Accident Period *i* Claims Reported With Lag *j*-1

				Lag + 1			
Accident Period	1	2	...	n-i+1	...	n-1	n
1	X_{11}	X_{12}	...	$X_{1,n-i+1}$	...	$X_{1,n-1}$	$X_{1,n}$
2	X_{21}	X_{22}	...	$X_{2,n-i+1}$	...	$X_{2,n-1}$	
⋮	⋮				⋰		
i	$X_{i,1}$			$X_{i,n-i+1}$			
⋮	⋮		⋰				
n-1	$X_{n-1,1}$	$X_{n-1,2}$					
n	$X_{n,1}$						

This data is more commonly summarized as a cumulative development triangle

Number of Accident Period *i* Claims Reported With Lag ≤ *j*-1

				Lag + 1			
Accident Period	1	2	...	n-i+1	...	n-1	n
1	S_{11}	S_{12}	...	$S_{1,n-i+1}$	...	$S_{1,n-1}$	$S_{1,n}$
2	S_{21}	S_{22}	...	$S_{2,n-i+1}$	...	$S_{2,n-1}$	
⋮	⋮				⋰		
i	$S_{i,1}$			$S_{i,n-i+1}$			
⋮	⋮		⋰				
n-1	$S_{n-1,1}$	$S_{n-1,2}$					
n	$S_{n,1}$						

where

$$S_{i,j} = \sum_{k=1}^{j} X_{i,k} \, .$$

The assumptions will be stated in terms of the $X_{i,j}$.

The basic problem for data given in this format is to deduce the number of incidents occurring in each accident period from the number reported through period n and from the pattern, consistent from period to period, in which they are reported. It is sufficient for what is intended here to consider only the problem of forecasting the next reporting increment.

There are two assumptions which will be imposed on the claims process. First, one assumes that the increments at the same age of development for different accident periods are independent, identically distributed and nonnegative random variables:

(A) For each j, the $X_{i,j}$ are independent, identically distributed and nonnegative.

One also assumes to begin with that, for a given accident period i, the development increments are independent of what has taken place up to that point in time:

(B) For each i, $X_{i,j}$ is independent of $X_{i,k}$ for $k<j$.

These independence assumptions are sufficient to demonstrate bias in the age-to-age factor estimates. Later, it will be shown condition (B) is satisfied if the ultimate claim count distribution is Poisson.

Jensen's Inequality

Proved here is a special case of a key analytical tool to be used in the demonstration. Readers familiar with the *Actuarial Mathematics* text [4] will recall a version of this for functions of one real variable. References for the multivariate statement used here may be found in [5] and [6].

Jensen's Inequality Let f be a function defined on a set $A \subseteq \mathbf{R}^n$ which takes only positive real values. Let μ be a probability measure on A with

$$E[f(\mathbf{X})] \equiv \int_A f(x_1,\ldots,x_n)\, d\mu(x_1,\ldots,x_n)$$

finite and non-zero. Provided f is not constant on every set of non-zero probability,

$$E\left[{}^{1}\!/_{f(\mathbf{X})}\right] > {}^{1}\!/_{E[f(\mathbf{X})]}.$$

Proof. Let $\gamma = E[f(\mathbf{X})]$. Consider the tangent line to the curve $s = 1/t$ at $t = \gamma$ which has the equation

$$s = -\frac{1}{\gamma^2}t + \frac{2}{\gamma}.$$

This line is always below the graph of $s = 1/t$ and so

$$\frac{1}{t} \geq -\frac{1}{\gamma^2} t + \frac{2}{\gamma}.$$

The range of f is such that, for each $\mathbf{x}$ in A,

$$\frac{1}{f(\mathbf{x})} \geq -\frac{1}{\gamma^2} f(\mathbf{x}) + \frac{2}{\gamma}.$$

Integrating each side with respect to μ gives

$$\int_A {}^{1}\!\!\Big/\!{}_{f(x_1,\ldots,x_n)}\, d\mu(x_1,\ldots,x_n) \geq -\frac{1}{\gamma^2} \int_A f(x_1,\ldots,x_n) d\mu(x_1,\ldots,x_n) + \frac{2}{\gamma} = \frac{1}{\gamma}.$$

If equality held in this expression, then it would be the case that

$$\frac{1}{f(\mathbf{x})} = -\frac{1}{\gamma^2} f(\mathbf{x}) + \frac{2}{\gamma} \text{ or } f(\mathbf{x}) = \gamma$$

for all $\mathbf{x}$ except in a set having probability zero. This situation was ruled out and the result is now clear.

III. The Basic Argument

Using the familiar weighted average forecast, the age-to-age methodology predicts the next cumulative value as

$$\hat{S}_{i,n-i+2} = \frac{\sum_{k=1}^{i-1} S_{k,n-i+2}}{\sum_{k=1}^{i-1} S_{k,n-i+1}} \cdot S_{i,n-i+1}$$

provided that $\sum_{k=1}^{i-1} S_{k,n-i+1}$ and $S_{i,n-i+1}$ are non-zero. It is easier to work with the implied forecast of the change:

$$\begin{aligned}
\hat{X}_{i,n-i+2} &\equiv \hat{S}_{i,n-i+2} - S_{i,n-i+1} \\
&= \left(\frac{\sum_{k=1}^{i-1} S_{k,n-i+2}}{\sum_{k=1}^{i-1} S_{k,n-i+1}} - 1 \right) \cdot S_{i,n-i+1} \\
&= \frac{\sum_{k=1}^{i-1} X_{k,n-i+2}}{\sum_{k=1}^{i-1} S_{k,n-i+1}} \cdot S_{i,n-i+1}
\end{aligned}$$

Theorem 1. Given the independence conditions (A) & (B) stated above, the expected value of the weighted average forecast $\hat{X}_{i,n-i+2}$ is always greater than the expected value of the actual change. That is,

$$E\left[\frac{\sum_{k=1}^{i-1} X_{k,n-i+2}}{\sum_{k=1}^{i-1} S_{k,n-i+1}} \cdot S_{i,n-i+1} \middle| \sum_{k=1}^{i-1} S_{k,n-i+1} > 0, S_{i,n-i+1} > 0\right] > E\left[X_{i,n-i+2} \middle| \sum_{k=1}^{i-1} S_{k,n-i+1} > 0, S_{i,n-i+1} > 0\right]$$
$$\equiv E\left[X_{\cdot,n-i+2}\right]$$

Proof. To see this, one observes that

$$E\left[\frac{\sum_{k=1}^{i-1} X_{k,n-i+2}}{\sum_{k=1}^{i-1} S_{k,n-i+1}} \cdot S_{i,n-i+1} \middle| \sum_{k=1}^{i-1} S_{k,n-i+1} > 0, S_{i,n-i+1} > 0\right]$$
$$= E\left[S_{i,n-i+1} \middle| \sum_{k=1}^{i-1} S_{k,n-i+1} > 0, S_{i,n-i+1} > 0\right]$$
$$\cdot E\left[\frac{\sum_{k=1}^{i-1} X_{k,n-i+2}}{\sum_{k=1}^{i-1} S_{k,n-i+1}} \middle| \sum_{k=1}^{i-1} S_{k,n-i+1} > 0, S_{i,n-i+1} > 0\right]$$
$$= E\left[S_{i,n-i+1} \middle| S_{i,n-i+1} > 0\right] \cdot E\left[\frac{\sum_{k=1}^{i-1} X_{k,n-i+2}}{\sum_{k=1}^{i-1} S_{k,n-i+1}} \middle| \sum_{k=1}^{i-1} S_{k,n-i+1} > 0\right]$$

due to the independence of accident periods. Because of the independence of increments, it is also true that

$$E\left[S_{i,n-i+1} \middle| S_{i,n-i+1} > 0\right] \cdot E\left[\frac{\sum_{k=1}^{i-1} X_{k,n-i+2}}{\sum_{k=1}^{i-1} S_{k,n-i+1}} \middle| \sum_{k=1}^{i-1} S_{k,n-i+1} > 0\right]$$
$$= E\left[S_{i,n-i+1} \middle| S_{i,n-i+1} > 0\right] \cdot (i-1) \cdot E\left[X_{k,n-i+2}\right]$$
$$\cdot E\left[\frac{1}{\sum_{k=1}^{i-1} S_{k,n-i+1}} \middle| \sum_{k=1}^{i-1} S_{k,n-i+1} > 0\right]$$

Using Jensen's Inequality, with $f(\mathbf{x}) = x_1 + \cdots + x_{i-1}$, one deduces that

$$E\left[\frac{1}{\sum_{k=1}^{i-1} S_{k,n-i+1}} \middle| \sum_{k=1}^{i-1} S_{k,n-i+1} > 0\right] > \frac{1}{E\left[\sum_{k=1}^{i-1} S_{k,n-i+1} \middle| \sum_{k=1}^{i-1} S_{k,n-i+1} > 0\right]}$$

This may be strengthened slightly by noting that

$$E\left[\sum_{k=1}^{i-1} S_{k,n-i+1} \middle| \sum_{k=1}^{i-1} S_{k,n-i+1} > 0\right] \leq E\left[\sum_{k=1}^{i-1} S_{k,n-i+1} \middle| S_{k,n-i+1} > 0, k = 1,\dots,i-1\right]$$
$$= (i-1) \cdot E\left[S_{i,n-i+1} \middle| S_{i,n-i+1} > 0\right]$$

Combing these steps produces

$$E\left[\frac{\sum_{k=1}^{i-1} X_{k,n-i+2}}{\sum_{k=1}^{i-1} S_{k,n-i+1}} \cdot S_{i,n-i+1} \middle| \sum_{k=1}^{i-1} S_{k,n-i+1} > 0, S_{i,n-i+1} > 0\right]$$
$$> E\left[S_{i,n-i+1} \middle| S_{i,n-i+1} > 0\right] \cdot E\left[X_{k,n-i+2}\right] / E\left[S_{i,n-i+1} \middle| S_{i,n-i+1} > 0\right]$$
$$\equiv E\left[X_{i,n-i+2}\right]$$

This completes the proof.

Readers will observe at this point that the result is really only a statement of fact regarding ratios of independent variables and does not make much use of the underlying process. This should not be surprising since *the age-to-age factor methodology doesn't either.* Intuition provides a guide in the construction of forecasts in a natural way by relying on the identical distributions by "lag". The conclusion to be drawn here is not that the age-to-age factor method is biased absolutely but rather that it is not compatible with a claims process assumed to have independent increments.

IV. Independent Increments From Independent Claims Lags: The Poisson Case

It will now be shown that condition (B) holds when the report lags are independent and when the distribution of ultimate accident period claims is Poisson with mean λ. This in turn relies on the observation that, in this case, the number of claims reported with lag j-1 is also Poisson with mean $p_j\,\lambda$, where p_j is the probability that a claim from accident period i is reported in period i+j-1.

Proposition 1. When the distribution of ultimate claims is Poisson with mean λ and the report lags are independent, the number of claims reported with lag j-1 is also Poisson with mean $p_j\,\lambda$.

Proof. Let N be the ultimate number of claims and N_j be the number reported with lag j-1, then

$$\begin{aligned}\Pr\{N_j = J\} &= \sum_{n=0}^{\infty} \Pr\{N_j = J | N = n\} \cdot \Pr\{N = n\} \\ &= \sum_{n=J}^{\infty} \frac{n!}{J!(n-J)!} p_j^J (1-p_j)^{n-J} \cdot e^{-\lambda} \frac{\lambda^n}{n!} \\ &= e^{-\lambda} \frac{(\lambda p_j)^J}{J!} \sum_{n=0}^{\infty} \frac{[\lambda(1-p_j)]^n}{n!} \\ &= e^{-\lambda p_j} \frac{(\lambda p_j)^J}{J!} .\end{aligned}$$

Showing that N_j and N_k are independent is accomplished by a similar calculation.

Proposition 2. When the distribution of ultimate claims is Poisson with mean λ and the report lags are independent, the number of claims reported with lag j-1 and with lag k-1 are independent.

Proof. It need only be shown that

$$\Pr\{N_j = J, N_k = K\} = \Pr\{N_j = J\} \cdot \Pr\{N_k = K\}.$$

To this end, one proceeds as before and sees that

$$\begin{aligned}\Pr\{N_j = J, N_k = K\} &= \sum_{n=0}^{\infty} \Pr\{N_j = J, N_k = K | N = n\} \cdot \Pr\{N = n\} \\ &= \sum_{n=J+K}^{\infty} \frac{n!}{J!K!(n-J-K)!} p_j^J p_k^K (1-p_j-p_k)^{n-J-K} \cdot e^{-\lambda} \frac{\lambda^n}{n!} \\ &= e^{-\lambda} \frac{(\lambda p_j)^J}{J!} \frac{(\lambda p_k)^K}{K!} \sum_{n=0}^{\infty} \frac{[\lambda(1-p_j-p_k)]^n}{n!} \\ &= e^{-\lambda p_j} \frac{(\lambda p_j)^J}{J!} \cdot e^{-\lambda p_k} \frac{(\lambda p_k)^K}{K!} \\ &= \Pr\{N_j = J\} \cdot \Pr\{N_k = K\} .\end{aligned}$$

One now knows that the Poisson case satisfies condition (B) and the hypothesis of Theorem 2. One obtains as an implication

Theorem 2. When the distribution of ultimate claims is Poisson and the report lags of individual claims are independent, the weighted average forecast $\hat{X}_{i,n-i+2}$ is biased.

V. The Negative Binomial Case

It will be shown presently that bias is present in Stanard's "Claim Counts Only" scenario as well. In his paper, claim count triangles are generated by drawing a number of claims from (a normal approximation to) a negative binomial distribution and then drawing for each claim a report period. The latter is determined by drawing a value from the convolution of a uniform time-to-accident distribution and an exponential report lag distribution. The exact form of the report lag distribution is not important here.

Proposition 1 has a counterpart when ultimate claims have a negative binomial distribution. The form of the negative binomial distribution that will be used for a random variable M is given by

$$\Pr\{M=m\} = \frac{\beta^\alpha}{\Gamma(\alpha)}\int_0^\infty \frac{e^{-\lambda}\lambda^m}{m!}\lambda^{\alpha-1}e^{-\beta\lambda}d\lambda$$
$$\equiv \frac{\Gamma(\alpha+m)}{\Gamma(\alpha)\cdot m!}\left(\frac{\beta}{\beta+1}\right)^\alpha\left(\frac{1}{\beta+1}\right)^m.$$

Proposition 3. When the distribution of ultimate claims is negative binomial with parameters α and β and the report lags are independent, the number of claims reported with lag j-1 is also negative binomial with parameters α and $\beta_j = \dfrac{\beta}{p_j}$.

Proof. Let N be the ultimate number of claims and N_j be the number reported with lag j-1, then

$$\Pr\{N_j = J\} = \sum_{n=0}^\infty \Pr\{N_j = J | N = n\}\cdot\Pr\{N=n\}$$
$$= \sum_{n=J}^\infty \frac{n!}{J!(n-J)!}p_j^J(1-p_j)^{n-J}\cdot\frac{\beta^\alpha}{\Gamma(\alpha)}\int_0^\infty \frac{e^{-\lambda}\lambda^n}{n!}\lambda^{\alpha-1}e^{-\beta\lambda}d\lambda$$
$$= \frac{\beta^\alpha p_j^J}{\Gamma(\alpha)J!}\int_0^\infty\left\{\sum_{n=0}^\infty\frac{\left[(1-p_j)\lambda\right]^n}{n!}\right\}\lambda^{\alpha+J-1}e^{-(\beta+1)\lambda}d\lambda$$
$$= \frac{\beta^\alpha p_j^J}{\Gamma(\alpha)J!}\int_0^\infty \lambda^{\alpha+J-1}e^{-(\beta+p_j)\lambda}d\lambda$$
$$= \frac{\Gamma(\alpha+J)}{\Gamma(\alpha)\cdot J!}\left(\frac{\beta/p_j}{\beta/p_j+1}\right)^\alpha\left(\frac{1}{\beta/p_j+1}\right)^J.$$

Unfortunately, Proposition 2 has no analogue as the increments are not independent as demonstrated in the following

Proposition 4a. When the distribution of ultimate claims is negative binomial variate M with parameters α and β, then

$$\Pr\{N_j = J | N_k = K\} = \frac{\Gamma(\alpha + J + K)}{\Gamma(\alpha + K) \cdot J!} \left(\frac{\beta + p_k}{\beta + p_j + p_k} \right)^{K+\alpha} \left(\frac{p_j}{\beta + p_j + p_k} \right)^J$$

Proof. The following calculation suffices.

$$\Pr\{N_j = J | N_k = K\} = \Pr\{N_j = J, N_k = K\} / \Pr\{N_k = K\}$$

$$= \frac{\sum_{n=J+K}^{\infty} \frac{n!}{J!K!(n-J-K)!} p_j^J p_k^K (1 - p_j - p_k)^{n-J-K} \cdot \frac{\Gamma(\alpha+n)}{n!\Gamma(\alpha)} \left(\frac{\beta}{\beta+1}\right)^{\alpha} \left(\frac{1}{\beta+1}\right)^n}{\sum_{n=K}^{\infty} \frac{n!}{K!(n-K)!} p_k^K (1-p_k)^{n-K} \cdot \frac{\Gamma(\alpha+n)}{n!\Gamma(\alpha)} \left(\frac{\beta}{\beta+1}\right)^{\alpha} \left(\frac{1}{\beta+1}\right)^n}$$

$$= \frac{p_j^J}{J!} \left(\frac{1}{\beta+1}\right)^J \frac{\sum_{n=0}^{\infty} \frac{(1-p_j-p_k)^n}{n!} \Gamma(\alpha+n+J+K) \left(\frac{1}{\beta+1}\right)^n}{\sum_{n=0}^{\infty} \frac{(1-p_k)^n}{n!} \cdot \Gamma(\alpha+n+K) \left(\frac{1}{\beta+1}\right)^n}$$

$$= \frac{\Gamma(\alpha + J + K)}{\Gamma(\alpha + K) \cdot J!} \left(\frac{\beta + p_k}{\beta + p_j + p_k} \right)^{K+\alpha} \left(\frac{p_j}{\beta + p_j + p_k} \right)^J$$

Of this the following is direct consequence.

Proposition 4b. When the distribution of ultimate claims is negative binomial variate M with parameters α and β, then

$$E[N_j | N_k] = \frac{(N_k + \alpha) p_j}{(\beta + p_k)}$$

Two very similar expressions for the conditional probability and expectation are also required for the case where N_k is not given but is known to be non-zero.

Proposition 5a. When the distribution of ultimate claims is negative binomial variate M with parameters α and β, then

$$\Pr\{N_j = J | N_k > 0\} = \frac{\Gamma(\alpha + J)}{\Gamma(\alpha) \cdot J!} p_j^J \left\{ \frac{1}{(\beta + p_j)^{\alpha+J}} - \frac{1}{(\beta + p_j + p_k)^{\alpha+J}} \right\} \Big/ \left\{ \frac{1}{\beta^{\alpha}} - \frac{1}{(\beta + p_k)^{\alpha}} \right\}$$

Proof. Proceeding in a now familiar fashion, one sees that

$$\Pr\{N_j = J | N_k > 0\} = \sum_{K=1}^{\infty} \Pr\{N_j = J, N_k = K\} / \Pr\{N_k > 0\}$$

$$= \frac{\sum_{K=1}^{\infty} \sum_{n=J+K}^{\infty} \frac{n!}{J!K!(n-J-K)!} p_j^J p_k^K (1-p_j-p_k)^{n-J-K} \cdot \frac{\beta^\alpha}{\Gamma(\alpha)} \int_0^\infty \frac{e^{-\lambda}\lambda^n}{n!} \lambda^{\alpha-1} e^{-\beta\lambda} d\lambda}{\left\{1 - \left(\frac{\beta}{\beta+p_k}\right)^\alpha\right\}}$$

$$= \frac{\frac{\beta^\alpha p_j^J}{\Gamma(\alpha)J!} \int_0^\infty \left\{\sum_{K=1}^{\infty} \frac{(\lambda p_k)^K}{K!} \sum_{n=0}^{\infty} \frac{[(1-p_j-p_k)\lambda]^n}{n!}\right\} \lambda^{\alpha+J-1} e^{-(\beta+1)\lambda} d\lambda}{\left\{1 - \left(\frac{\beta}{\beta+p_k}\right)^\alpha\right\}}$$

$$= \frac{\frac{\beta^\alpha p_j^J}{\Gamma(\alpha)J!} \left\{\int_0^\infty \lambda^{\alpha+J-1} e^{-(\beta+p_j)\lambda} d\lambda - \int_0^\infty \lambda^{\alpha+J-1} e^{-(\beta+p_j+p_k)\lambda} d\lambda\right\}}{\left\{1 - \left(\frac{\beta}{\beta+p_k}\right)^\alpha\right\}}$$

$$= \frac{\Gamma(\alpha+J)}{\Gamma(\alpha)\cdot J!} p_j^J \left\{\frac{1}{(\beta+p_j)^{\alpha+J}} - \frac{1}{(\beta+p_j+p_k)^{\alpha+J}}\right\} \Big/ \left\{\frac{1}{\beta^\alpha} - \frac{1}{(\beta+p_k)^\alpha}\right\}$$

This leads immediately to

Proposition 5b. When the distribution of ultimate claims is negative binomial variate M with parameters α and β, then

$$E[N_j | N_k > 0] = \frac{\alpha p_j}{\beta(\beta+p_k)} \left\{\frac{(\beta+p_k)^{\alpha+1} - \beta^{\alpha+1}}{(\beta+p_k)^\alpha - \beta^\alpha}\right\}$$

Proof. It is straightforward to sum the expression from Proposition 5a over J:

$$E\left[N_j \middle| N_k > 0\right] = \sum_{J=1}^{\infty} \frac{\Gamma(\alpha+J)}{\Gamma(\alpha)\cdot(J-1)!} p_j^{\,J} \left\{ \frac{1}{(\beta+p_j)^{\alpha+J}} - \frac{1}{(\beta+p_j+p_k)^{\alpha+J}} \right\} \Big/ \left\{ \frac{1}{\beta^{\alpha}} - \frac{1}{(\beta+p_k)^{\alpha}} \right\}$$

$$= \alpha p_j \frac{\left\{ \begin{array}{l} \dfrac{1}{\beta^{\alpha+1}} \sum_{J=1}^{\infty} \dfrac{\Gamma(\alpha+1+J-1)}{\Gamma(\alpha+1)\cdot(J-1)!} \left(\dfrac{\beta}{\beta+p_j}\right)^{\alpha+1} \left(\dfrac{p_j}{\beta+p_j}\right)^{J-1} \\ -\dfrac{1}{(\beta+p_k)^{\alpha+1}} \sum_{J=0}^{\infty} \dfrac{\Gamma(\alpha+1+J-1)}{\Gamma(\alpha+1)\cdot(J-1)!} \left(\dfrac{\beta+p_k}{\beta+p_j+p_k}\right)^{\alpha+1} \left(\dfrac{p_j}{\beta+p_j+p_k}\right)^{J-1} \end{array} \right\}}{\left\{ \dfrac{1}{\beta^{\alpha}} - \dfrac{1}{(\beta+p_k)^{\alpha}} \right\}}$$

$$= \frac{\alpha p_j}{\beta(\beta+p_k)} \left\{ \frac{(\beta+p_k)^{\alpha+1} - \beta^{\alpha+1}}{(\beta+p_k)^{\alpha} - \beta^{\alpha}} \right\}$$

The final task may now be addressed.

Theorem 3. When the distribution of ultimate claims is negative binomial, the weighted average forecast $\hat{X}_{i,n-i+2}$ is biased. That is,

$$\Omega \equiv E\left[\frac{\sum_{k=1}^{i-1} X_{k,n-i+2}}{\sum_{k=1}^{i-1} S_{k,n-i+1}} \cdot S_{i,n-i+1} \middle| \sum_{k=1}^{i-1} S_{k,n-i+1} > 0, S_{i,n-i+1} > 0 \right] > E\left[X_{i,n-i+2} \middle| S_{i,n-i+1} > 0\right]$$

Proof. First, due to independence between accident periods, one may write

$$\Omega \equiv E\left[\frac{\sum_{k=1}^{i-1} X_{k,n-i+2}}{\sum_{k=1}^{i-1} S_{k,n-i+1}} \cdot S_{i,n-i+1} \middle| \sum_{k=1}^{i-1} S_{k,n-i+1} > 0, S_{i,n-i+1} > 0 \right]$$

$$= E\left[\frac{\sum_{k=1}^{i-1} X_{k,n-i+2}}{\sum_{k=1}^{i-1} S_{k,n-i+1}} \middle| \sum_{k=1}^{i-1} S_{k,n-i+1} > 0 \right] \cdot E\left[S_{i,n-i+1} \middle| S_{i,n-i+1} > 0\right]$$

In the proof of Theorem 1, it was possible to separate the expectation operator containing the quotient. As has been shown, however, independence of increments does not hold here and some other mechanism must be employed. To this end, one fixes the $S_{k,n-i+1}$ and computes the expectation in successive steps.

$$\Omega = E\left[E\left[\sum_{k=1}^{i-1} X_{k,n-i+2} \middle| S_{k,n-i+1}, k=1,\ldots,i-1\right] \middle/ \sum_{k=1}^{i-1} S_{k,n-i+1} \middle| \sum_{k=1}^{i-1} S_{k,n-i+1} > 0\right] \cdot E\left[S_{i,n-i+1} \middle| S_{i,n-i+1} > 0\right]$$
$$= E\left[\sum_{k=1}^{i-1} E\left[X_{k,n-i+2} \middle| S_{k,n-i+1}\right] \middle/ \sum_{k=1}^{i-1} S_{k,n-i+1} \middle| \sum_{k=1}^{i-1} S_{k,n-i+1} > 0\right] \cdot E\left[S_{i,n-i+1} \middle| S_{i,n-i+1} > 0\right]$$

Proposition 4b may now be applied to show that

$$\Omega = \frac{p_{n-i+2}}{\beta + p_*} \cdot E\left[\sum_{k=1}^{i-1} \left(S_{k,n-i+1} + \alpha\right) \middle/ \sum_{k=1}^{i-1} S_{k,n-i+1} \middle| \sum_{k=1}^{i-1} S_{k,n-i+1} > 0\right] \cdot E\left[S_{i,n-i+1} \middle| S_{i,n-i+1} > 0\right]$$
$$= \frac{p_{n-i+2}}{\beta + p_*} \cdot \left(1 + (i-1)\alpha E\left[1 \middle/ \sum_{k=1}^{i-1} S_{k,n-i+1} \middle| \sum_{k=1}^{i-1} S_{k,n-i+1} > 0\right]\right) \cdot E\left[S_{i,n-i+1} \middle| S_{i,n-i+1} > 0\right]$$

where $p_* = p_1 + \cdots + p_{n-i+1}$. It is at this stage when Jensen's Inequality may again be utilized and one observes that as in the case of independent increments that

$$\Omega > \frac{p_{n-i+2}}{\beta + p_*} \cdot \left(1 + (i-1)\alpha \middle/ E\left[\sum_{k=1}^{i-1} S_{k,n-i+1} \middle| \sum_{k=1}^{i-1} S_{k,n-i+1} > 0\right]\right) \cdot E\left[S_{i,n-i+1} \middle| S_{i,n-i+1} > 0\right]$$
$$\geq \frac{p_{n-i+2}}{\beta + p_*} \cdot \left(1 + \alpha \middle/ E\left[S_{\bullet,n-i+1} \middle| S_{\bullet,n-i+1} > 0\right]\right) \cdot E\left[S_{i,n-i+1} \middle| S_{i,n-i+1} > 0\right]$$

One makes use of Proposition 3 to see that

$$E\left[S_{\bullet,n-i+1} \middle| S_{\bullet,n-i+1} > 0\right] = \frac{\alpha p_*}{\beta\left(1 - \left(\frac{\beta}{\beta + p_*}\right)^{\alpha}\right)}$$

which may be substituted into the previous expression. Doing so produces

$$\Omega > \frac{\alpha}{\beta} \frac{p_{n-i+2}}{\beta + p_*} \cdot \left(\frac{p_* + \beta\left(1 - \left(\frac{\beta}{\beta + p_*}\right)^{\alpha}\right)}{\left(1 - \left(\frac{\beta}{\beta + p_*}\right)^{\alpha}\right)}\right)$$
$$= \frac{\alpha}{\beta} \frac{p_{n-i+2}}{\beta + p_*} \cdot \left(\frac{(\beta + p_*)^{\alpha+1} - \beta^{\alpha+1}}{(\beta + p_*)^{\alpha} - \beta^{\alpha}}\right)$$

Proposition 5b identifies the final expression as being precisely $E\left[X_{i,n-i+2} \middle| S_{i,n-i+1} > 0\right]$. Therefore,

$$\Omega > E\left[X_{i,n-i+2} \middle| S_{i,n-i+1} > 0\right].$$

VI. Straight Average Factors

In this section, it will be demonstrated that the "straight" average estimator

$$\overline{X}_{i,n-i+2} \equiv \frac{1}{(i-1)} \sum_{k=1}^{i-1} \frac{X_{k,n-i+2}}{S_{k,n-i+1}} \cdot S_{i,n-i+1}$$

cannot reduce or eliminate the bias seen in the weighted average estimator. For brevity, attention is restricted to the case of independent increments.

Theorem 4. When both are defined, the expected value of the unweighted average prediction is greater than the expected value of the weighted average prediction. That is,

$$E\left[\frac{1}{i-1} \sum_{k=1}^{i-1} \frac{X_{k,n-i+2}}{S_{k,n-i+1}} \cdot S_{i,n-i+1} \middle| S_{k,n-i+1} > 0, k = 1,\ldots,i\right] \geq E\left[\left(\frac{\sum_{k=1}^{i-1} X_{k,n-i+2}}{\sum_{k=1}^{i-1} S_{k,n-i+1}}\right) \cdot S_{i,n-i+1} \middle| S_{k,n-i+1} > 0, k = 1,\ldots,i\right]$$

Proof. Again making use of the independence and symmetry between the periods, showing this is equivalent to proving that

$$\frac{1}{i-1} \sum_{k=1}^{i-1} E\left[\frac{X_{k,n-i+2}}{S_{k,n-i+1}} \middle| S_{k,n-i+1} > 0\right] \geq \sum_{k=1}^{i-1} E\left[\frac{X_{k,n-i+2}}{\sum_{k=1}^{i-1} S_{k,n-i+1}} \middle| S_{k,n-i+1} > 0, k = 1,\ldots,i-1\right]$$

or

$$E\left[\sum_{k=1}^{i-1} \frac{1}{S_{k,n-i+1}} \middle| S_{k,n-i+1} > 0, k = 1,\ldots,i-1\right] \geq (i-1)^2 \cdot E\left[\frac{1}{\sum_{k=1}^{i-1} S_{k,n-i+1}} \middle| S_{k,n-i+1} > 0, k = 1,\ldots,i-1\right]$$

After rewriting the left hand side and arranging terms to one side, this becomes

$$E\left[\frac{\sum_{k=1}^{i-1} \prod_{l=1, l\neq k}^{i-1} S_{l,n-i+1}}{\prod_{k=1}^{i-1} S_{k,n-i+1}} - \frac{(i-1)^2}{\sum_{k=1}^{i-1} S_{k,n-i+1}} \middle| S_{k,n-i+1} > 0, k = 1,\ldots,i-1\right] \geq 0 .$$

Thus it is the goal to determine that the quantity inside the brackets is non-negative. After cross multiplication, the resulting numerator is

$$\sum_{k=1}^{i-1}\left(\sum_{m=1}^{i-1} S_{m,n-i+1}\right)\prod_{l=1,l\neq k}^{i-1} S_{l,n-i+1} - (i-1)^2 \prod_{k=1}^{i-1} S_{k,n-i+1}$$
$$= \sum_{k=1}^{i-1}\sum_{m=1,m\neq k}^{i-1}\left(S_{m,n-i+1}\prod_{l=1,l\neq k}^{i-1} S_{l,n-i+1} - \prod_{k=1}^{i-1} S_{k,n-i+1}\right)$$
$$= \sum_{k=1}^{i-1}\sum_{m=1,m\neq k}^{i-1}\left(\prod_{l=1,l\neq k,m}^{i-1} S_{l,n-i+1}\right)\left(S_{m,n-i+1}{}^2 - S_{k,n-i+1}S_{m,n-i+1}\right)$$

The inner sum may be broken into two steps first summing from m=1 to k-1 and then from k+1 to i-1. For the latter, one interchanges the order of summation and interchange the roles of k and m to find that the numerator may be written as

$$\sum_{k=2}^{i-1}\sum_{m=1}^{k-1}\left(\prod_{l=1,l\neq k,m}^{i-1} S_{l,n-i+1}\right)\left(S_{m,n-i+1} - S_{k,n-i+1}\right)^2$$

which is clearly non-negative.

VII. Conclusion

It is not the purpose of this paper to advocate one set of assumptions regarding the independence of report lags over another. Indeed, if one believes that expected development increments are directly proportional to the accumulated total claims at a given point in time, then one might conclude that methods based on independent increment assumptions produce understated results.

It is, however, apparent that Stanard's simulation test of the development method produced the correct observation. If one believes that individual report lags are independent, then the loss development methods will produce overstated results. One thing that the analytical work presented here does not show is the magnitude of the bias. Stanard's work produced measures of that in specific cases. The key point is that there is a fundamental incompatibility between loss development techniques and methods relying on independent report lags.

Bibliography

[1] Stanard, J. "A Simulation Test of Prediction Errors of Loss Reserve Estimation Techniques", PCAS LXXII, 1985.

[2] Weissner, E., "Estimation of the Distribution of Report Lags by the Method of Maximum Likelihood", PCAS LXV, 1978.

[3] Weissner, E., "Evaluation of IBNR on a Low Frequency Book Where the Report Development Pattern is Still Incomplete", Casualty Loss Reserve Seminar Transcripts, 1981.

[4] Bowers, N.L. , et. al., *Actuarial Mathematics*, Society of Actuaries, Chicago.

[5] Wheeden, R. and Zygmund, A., *Measure and Integral: An Introduction to Real Analysis*, Marcel Dekker, Inc., New York, 1977.

[6] Royden, H., *Real Analysis*, Macmillan Publishing Co., New York, 1968.

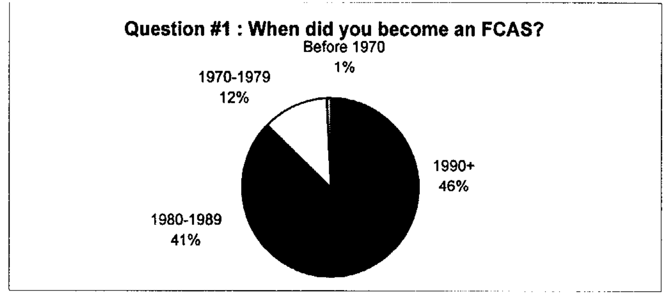
Results of the 1997 CAS Continuing Education Survey
by the CAS Committee on Continuing Education

CAS Continuing Education Survey : Graphical Analysis of Results

General Questions

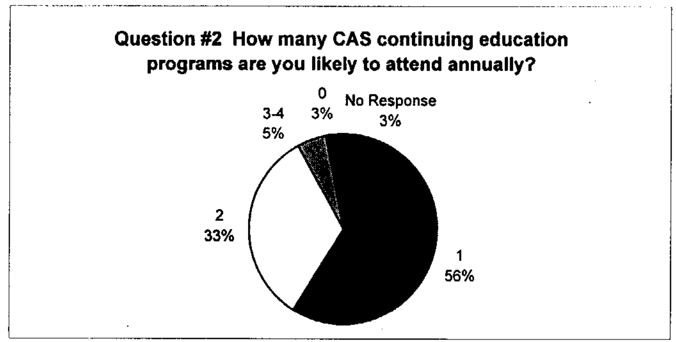
Question #1 : When did you become an FCAS?

Date	%
1990+	47
1980-1989	41
1970-1979	12
Before 1970	1



Question #2 How many CAS continuing education programs are you likely to attend annually?

Number	%
0	3
1	56
2	33
3-4	5
No Response	3

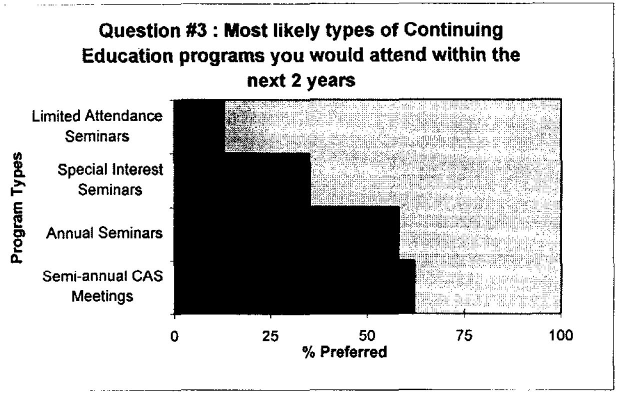


Analysis : Only Fellows of the CAS were surveyed. Of those, a significant percentage of answers came from members who received their fellowships prior to 1990, and have the most need to use continuing education opportunities. Most members who responded are likely to attend one (1) to two (2) continuing education programs each year, but not likely to attend more than two (2) each year.

General Questions (continued)

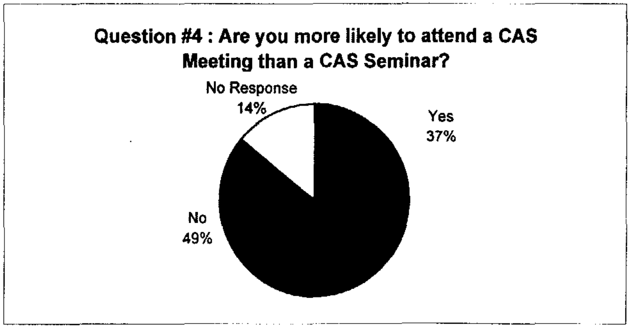
Question #3 : Most likely types of Continuing Education programs you would attend within the next 2 years

Type of Program	%
Semi-annual CAS Meetings	62
Annual Seminars	58
Special Interest Seminars	35
Limited Attendance Seminars	13



Question #4 : Are you more likely to attend a CAS Meeting than a CAS Seminar?

Response	%
Yes	37
No	49
No Response	14



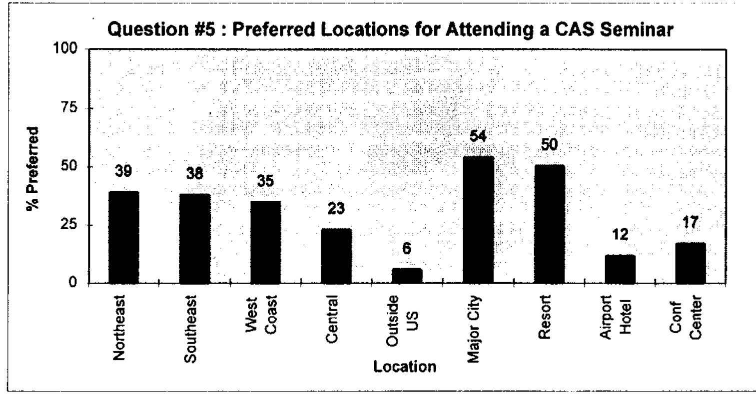
Analysis : While semi-annual CAS meetings and annual seminars (such as the CLRS and Ratemaking seminars) are popular with the membership, a significant percentage find Special Interest Seminars and Limited Attendance Seminars appealing.

CAS Continuing Education Survey : Graphical Analysis of Results

General Questions (continued)

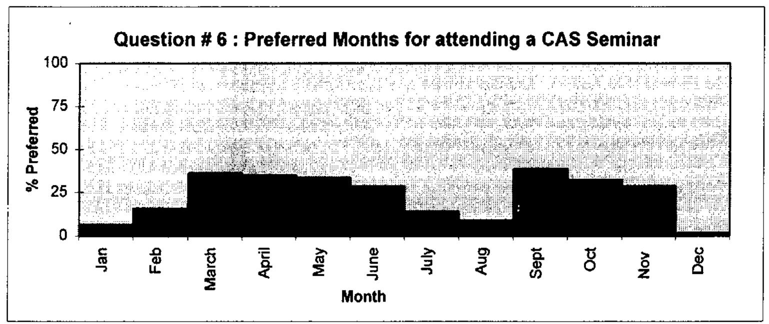
Question #5 Please indicate your top three preferred locations for attending a CAS Seminar

Locations	%
Northeast	39
Southeast	38
West Coast	35
Central	23
Outside US	6
Major City	54
Resort	50
Airport Hotel	12
Conf Center	17



Question #6 Please indicate your top 3 preferred months of the year for attending a CAS Seminar

Month	%
Jan	7
Feb	16
March	36
April	35
May	34
June	29
July	14
Aug	9
Sept	39
Oct	32
Nov	29
Dec	2

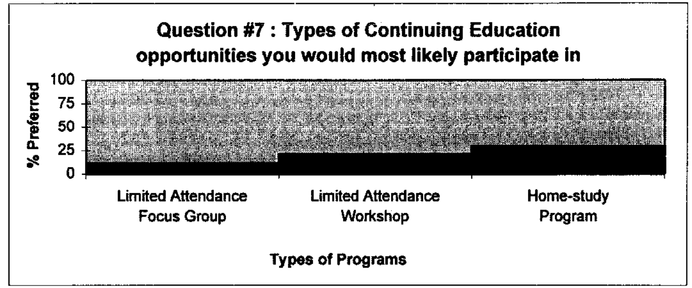


Analysis : Preferences for geographical locations for seminars are uniform across the country, and there appears to be no preference for major city sites versus resorts. Not surprisingly, members prefer to avoid December, January, July and August for attending seminars.

General Questions (continued)

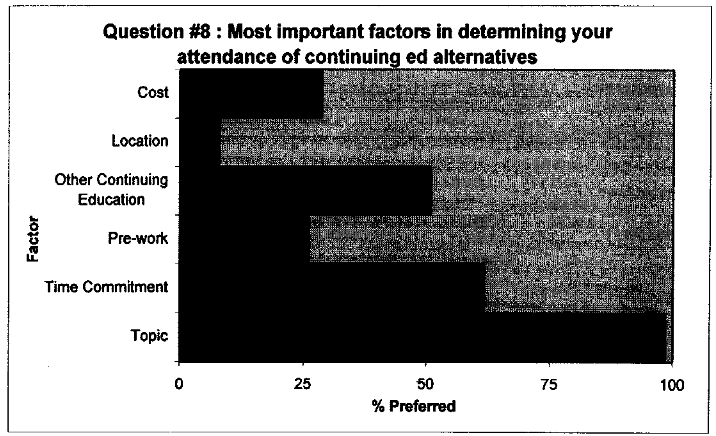
Question #7 : Types of Continuing Education opportunities you would most likely participate in

Type of Program	%
Limited Attendance Focus Group	11
Limited Attendance Workshop	22
Home-study Program	30



Question #8 : Most important factors in determining your attendance of continuing ed alternatives

Critical/Very Important Factors	%
Topic	99
Time Commitment	62
Pre-work	26
Other Continuing Education opportunities	51
Location	8
Cost	29



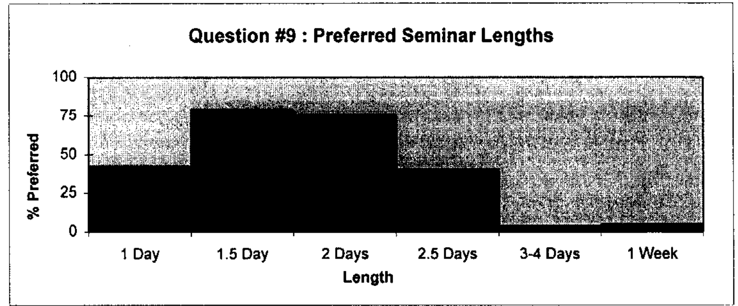
Analysis : There does not appear to be sufficient interest in focus groups compared to workshops or self-study programs. Topic and time commitment are the most important factors in deciding which alternative to select, compared to the cost or the level of pre-work required.

CAS Continuing Education Survey : Graphical Analysis of Results

Limited Attendance Programs

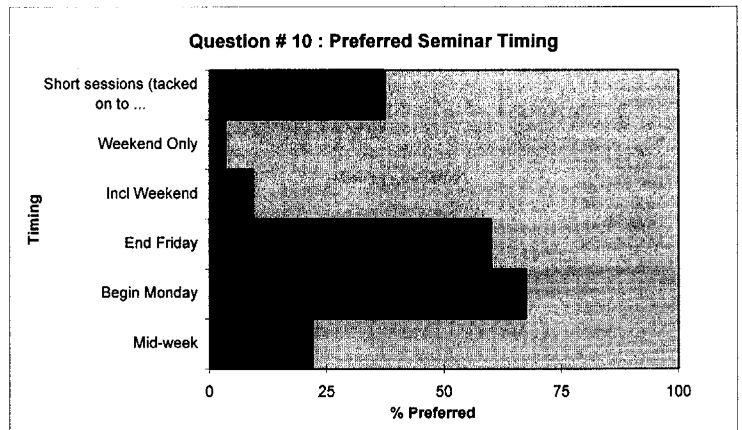
Question # 9 : Preferred Length

Preferable/Desirable Lengths	%
1 Day	42
1.5 Day	79
2 Days	75
2.5 Days	40
3-4 Days	4
1 Week	5



Question # 10 : Preferred Seminar Timing

Preferable/Desirable Timing	%
Mid-week	22
Begin Monday	67
End Friday	60
Incl Weekend	9
Weekend Only	3
Short sessions (tacked on to ... other meetings/events)	37

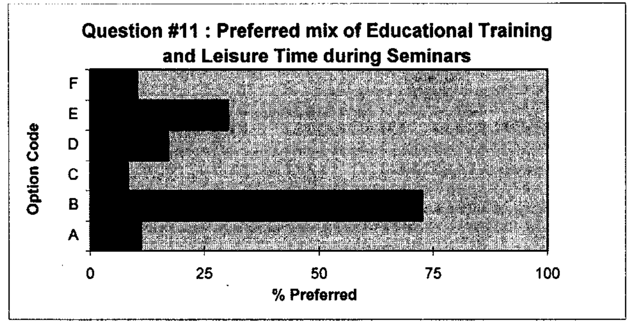


Analysis : One and a half to two days is the preferred length for limited attendance seminars. Beginning or ending the business week are the most preferred times. A significant percentage (37%) prefer to see these programs tacked onto other CAS meetings.

Limited Attendance Programs (continued)

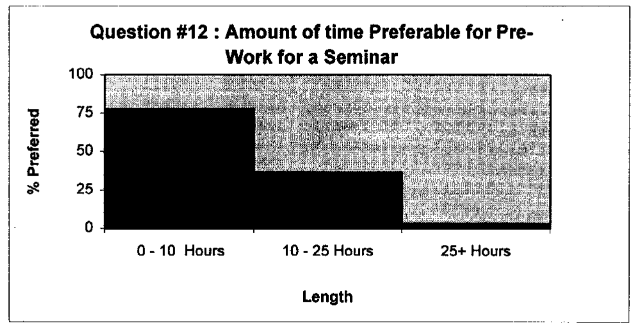
Question #11 : Preferred mix of Educational Training and Leisure Time during Seminars

Preferable/Desirable Options	Code	%
Intense training all day & evening	A	11
Intense training all day, no evening classes	B	73
Free AM with afternoon & evening classes	C	8
Free afternoon with morning & evening classes	D	17
At least 1 full day free	E	30
Half day program with rest of day free	F	10



Question #12 : Amount of Time you are Willing to Commit for pre-work for a seminar

Preferable/Desirable Lengths	%
0 - 10 Hours	78
10 - 25 Hours	37
25+ Hours	3

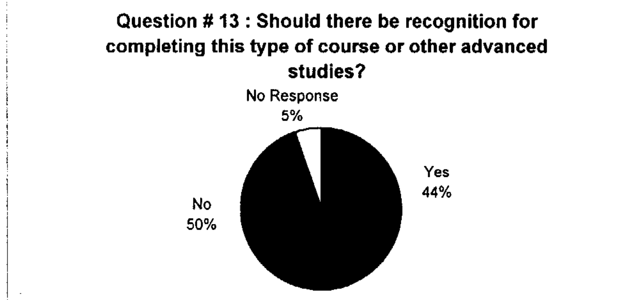


Analysis : Members prefer not to mix leisure time with limited attendance seminars. The majority of members are willing to commit up to 10 hours for pre-work for a seminar, with a smaller segment willing to put in up to 25 hours.

Limited Attendance Programs (continued)

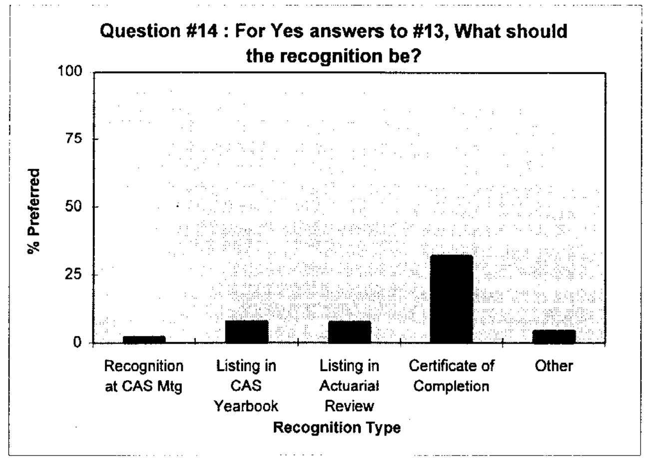
Question #13 : Should there be recognition for completing this type of course or other advanced studies?

Response	%
Yes	44
No	50
No Response	5



Question #14 : For Yes answers to #13, What should the recognition be?

Recognition Type	%
Recognition at CAS Mtg	2
Listing in CAS Yearbook	8
Listing in Actuarial Review	8
Certificate of Completion	32
Other	4



Analysis : Members are divided on whether there should be recognition for completing these types of courses. Of those that feel that recognition is warranted, the most appropriate recognition would be a certificate of completion.

Topics for Continuing Education

Question #15 : What topics would you most like to see offered in a Limited Attendance Focus Group or workshop?

Topic	% Choosing
Rate of Return	54.3
Dynamic Solvency	49.6
Managing CAT Exposure	42.9
Assets & Investments	41.6
Financial Modeling	62.8

Question #21 : What topics would you most like to see offered in a Home Study Format?

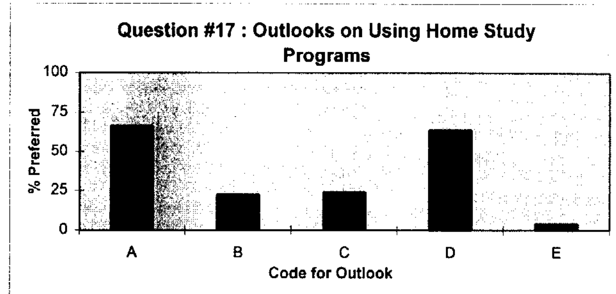
Topic	% Choosing
Rate of Return	57.4
Dynamic Solvency	49.4
Loss Distributions	42.9
Managing CAT Exposure	42.1
Assets & Investments	49.6
Financial Modeling	61.5

Analysis : There is widespread interest in continuing education opportunities on financial and catastrophe issues, particularly dynamic analyses.

Home Study Programs

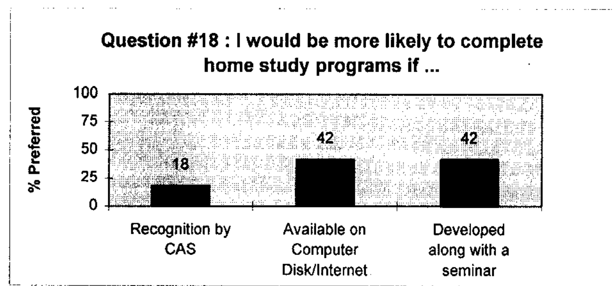
Question #17 : Outlooks on Using Home Study Programs

Chart Key Code	%	Answer
A	66	Definitely complete topics I found interesting or relevant to my job
B	23	Complete topics only to fulfill my continuing education requirements
C	24	Look at what was offered, but realistically probably wouldn't complete it
D	64	Consider using them to train staff
E	4	Interested in developing a program for the CAS



Question #18 : I would be more likely to complete home study programs if ...

Choices	%
Recognition by CAS	18
Available on Computer Disk/Internet	42
Developed along with a seminar	42

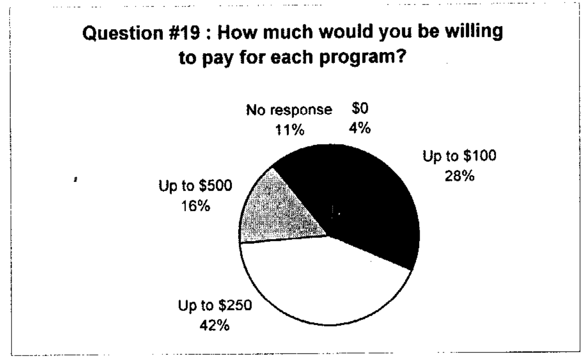


Analysis : While there is significant interest in using home study programs, especially in training staff, there is very little

Home Study Programs (continued)

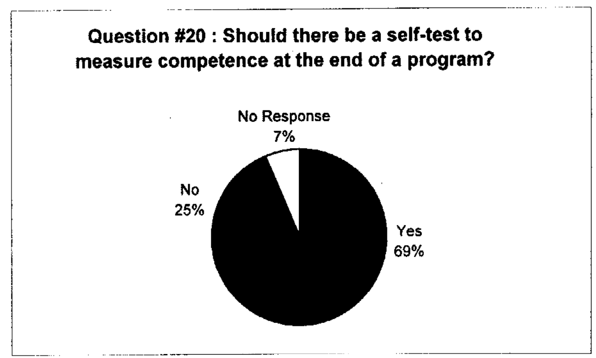
Question #19 : How much would you be willing to pay for each program?

Amount	%
$0	4
Up to $100	28
Up to $250	42
Up to $500	16
No response	11



Question #20 : Should there be a self-administered test to measure competence at the end of the program?

Response	%
Yes	69
No	25
No Response	7

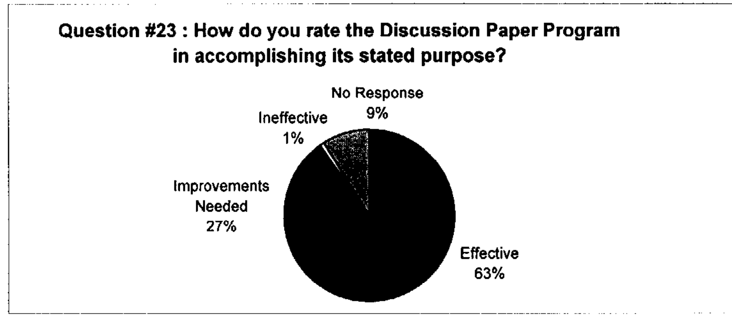


Analysis : Members prefer a cost range of $100 to $250 for home study programs, and a majority agreed that there should be a self-test to measure competence at the end of the program.

Discussion Paper Program

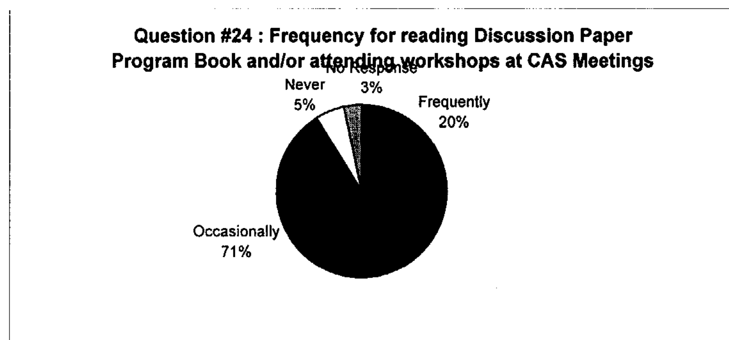
Question #23 : How do you rate the Discussion Paper Program in accomplishing its stated purpose?

Rating	%
Effective	63
Improvements Needed	27
Ineffective	1
No Response	9



Question #24 : How often have you read the papers in the Discussion Paper Program Book and/or attended the workshops at the CAS Meetings?

Frequency	%
Frequently	20
Occasionally	71
Never	5
No Response	3

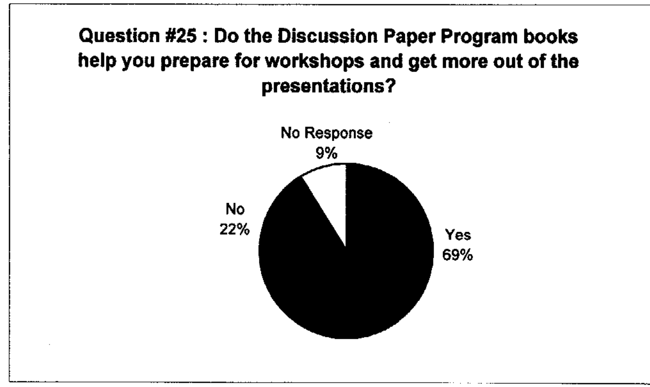


Analysis : A majority of those responding find the Discussion Paper Program effective, and frequently or occasionally attend the workshops at the CAS meetings.

Discussion Paper Program (continued)

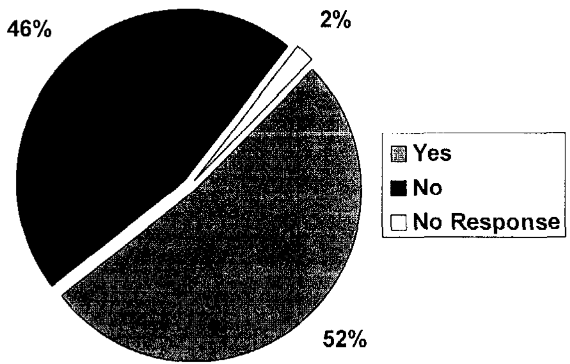
Question #25: Do the Discussion Paper Program books help you prepare for the workshops and get more out of the author's presentation?

Response	%
Yes	69
No	22
No Response	9



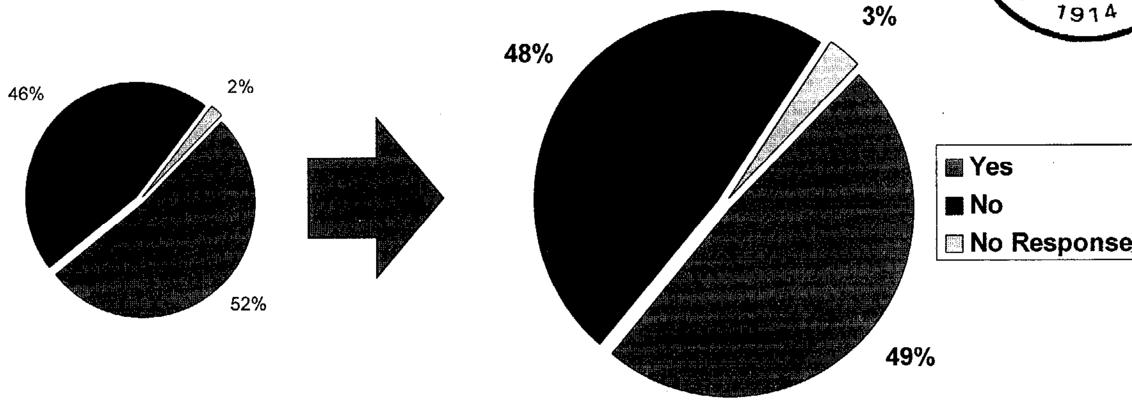
Analysis : Most members find it helpful to have papers in advance to prepare for workshops at CAS meetings.

Question 27: "Are you aware of the June 5, 1996 AAA exposure draft on revisions to the Qualification Standards for Public Statements of Actuarial Opinion?"



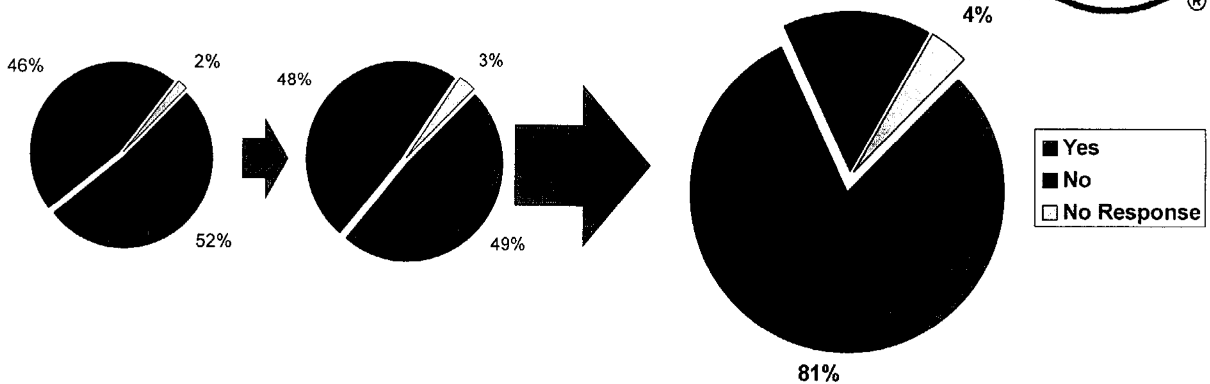
The purpose of this question was to determine the membership's awareness of the proposed revisions to the Qualification Standards. ***Only half of the respondents were aware of the Qualification Standard exposure draft.***

Question 28: "Are you aware of the provision regarding professionalism education?"



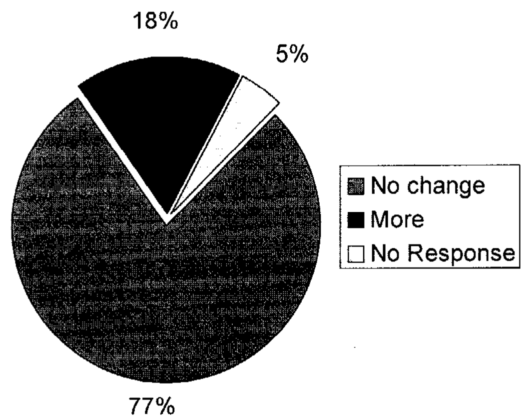
The purpose of this question was to determine the membership's awareness of the proposed revisions regarding professionalism education. ***Nearly all respondents who were aware of the Qualification Standard exposure draft recognized the revision regarding professionalism education.***

Question 29: "Do you support the provision allowing professionalism education to count towards your continuing education requirements?"



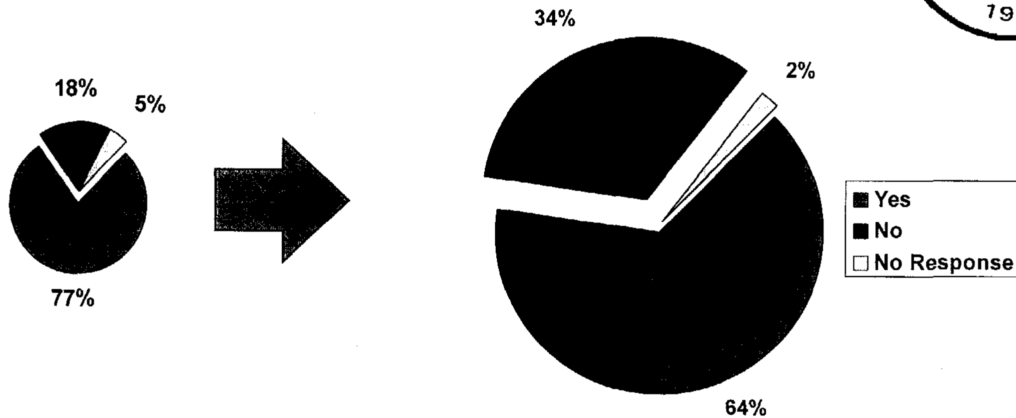
The purpose of this question was to determine how the membership felt about professionalism education. ***A vast majority of respondents felt that professionalism education should count toward the continuing education requirement.***

Question 30: "If the promulgated standard contains a provision allowing professionalism education to count towards continuing education, how will this impact your attendance at the professionalism sessions offered at conventions and/or seminars?"



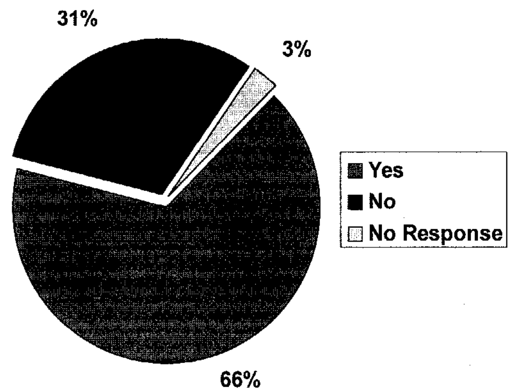
The purpose of this question was to determine the demand for future professionalism education at meetings. ***There will NOT be a significant change in the demand for professionalism education, even if these sessions were to count towards continuing education.***

Question 31: "Have you ever attended a concurrent session, general session or break-out session on professionalism or ethical issues at any meeting or seminar?"



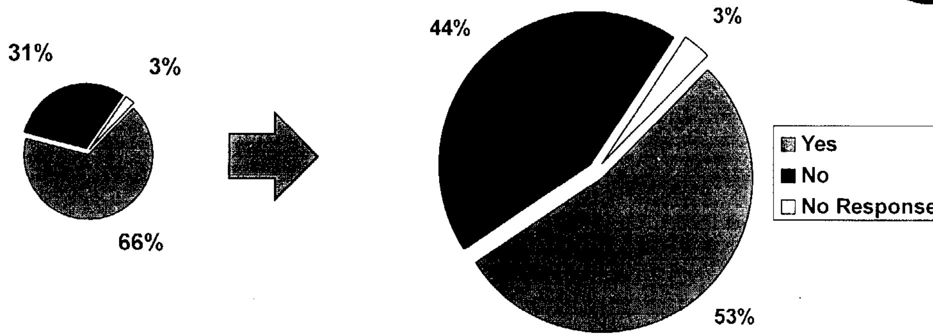
The purpose of this question was to determine how much of the membership previously had interest in professionalism education. ***Fully a third of the respondents has NEVER been to any session of professionalism at a meeting or seminar.***

Question 34: "Do you think the CAS should require all members to have education on professionalism and/or ethical responsibilities?"



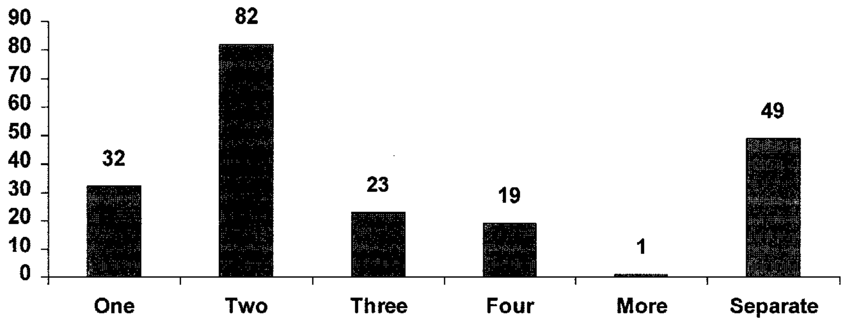
The purpose of this question was to determine how the membership feels about requiring professionalism education for members. ***While a majority of members believe we SHOULD REQUIRE this education, there is also a significant minority of members who do not.***

Question 35: "Would you like to see the AAA and/or CAS require that its members attend a minimum number of hours on professionalism education in order to make public statements of actuarial opinion?"



The purpose of this question was to determine how the membership feels about requiring professionalism education for members who make public statements. ***While most respondents feel we should require professionalism education for everyone, fewer (a scant majority) members believe this education SHOULD BE REQUIRED for members making public statements. These responses are inconsistent.***

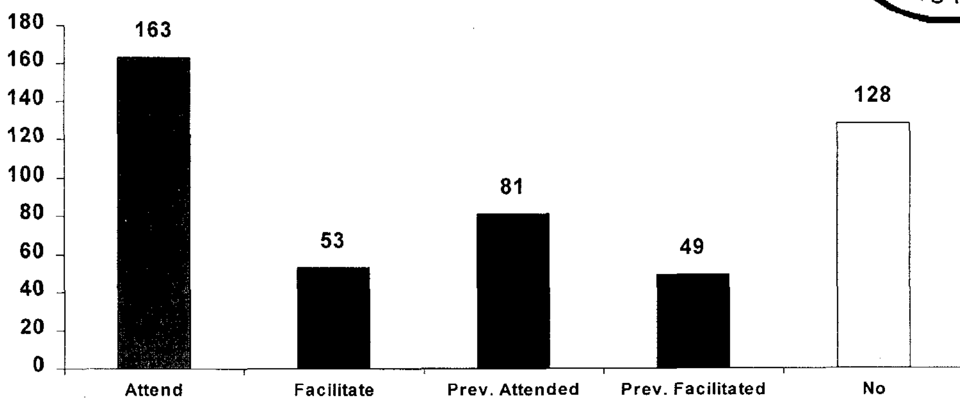
Question 36: "Currently, the AAA requirement for continuing education is 24 hours over two years. Of these, how many hours should be dedicated to professionalism education?"



The purpose of this question was to determine how much professionalism education the membership feels is sufficient. ***Most seem to feel about 10% of the continuing education requirement should be fulfilled by professionalism education. This amounts to about one session a year. There is only modest interest (about 25% of respondents) in creating a separate requirement.***

Question 32: "Would you be interested in attending or facilitating a Course on Professionalism?"

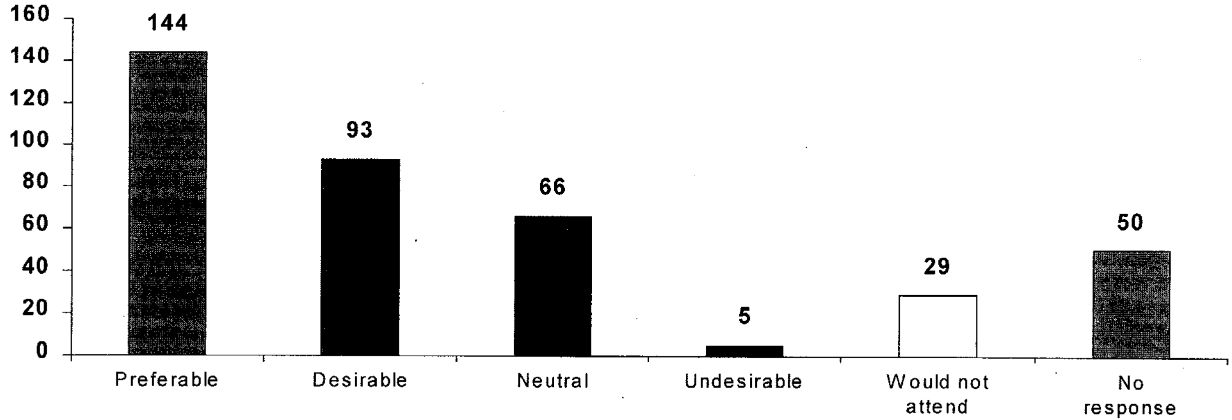
Check all that apply. (Average number of responses is 1.22)



The purpose of this question was to assess the existing membership's interest in attending or contributing to the Course on Professionalism. ***Over 40% of respondents are interesting in attending the COP, another 14% would facilitate. Over a third have already attended or facilitated. Another third want nothing to do with the course.***

Question 33a: "Please indicate the importance of the following regarding your desire to attend a Course on Professionalism:

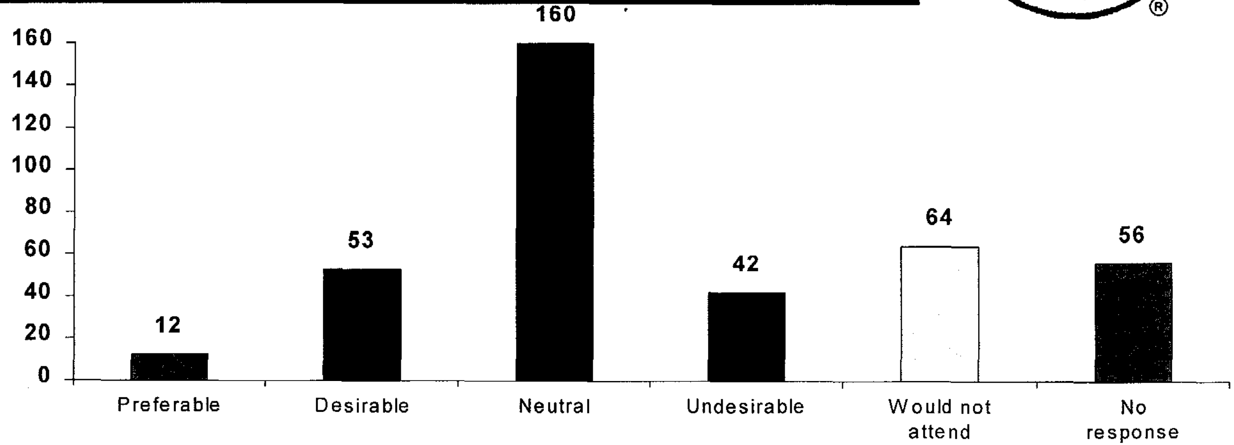
A course offered locally."



The purpose of this question was to determine the membership's requirements regarding a Course on Professionalism. ***There is strong preference for a local course.***

Question 33b: "Please indicate the importance of the following regarding your desire to attend a Course on Professionalism:

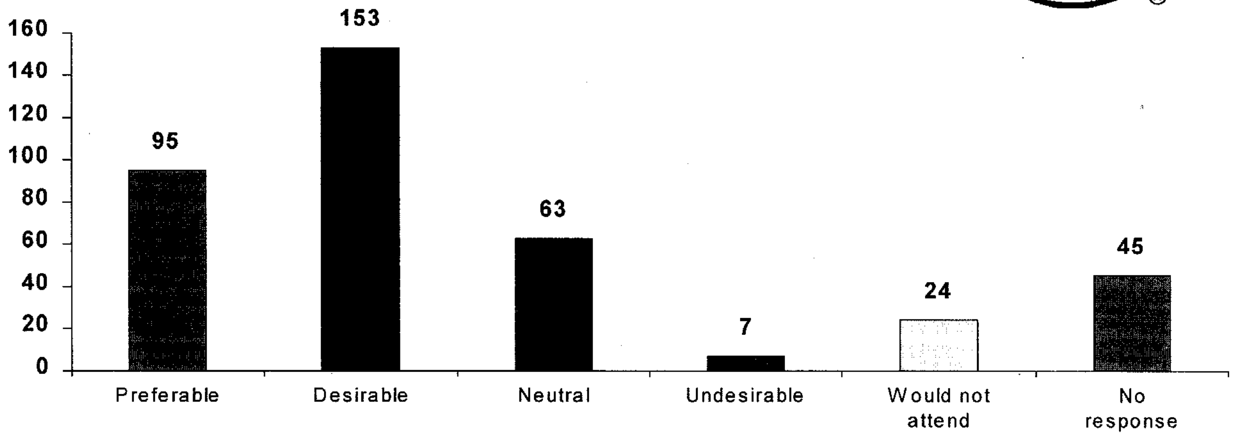
A course offered at a resort."



The purpose of this question was to determine the membership's requirements regarding a Course on Professionalism. ***The type of facility for the course is not that important to the membership.***

Question 33c: “Please indicate the importance of the following regarding your desire to attend a Course on Professionalism:

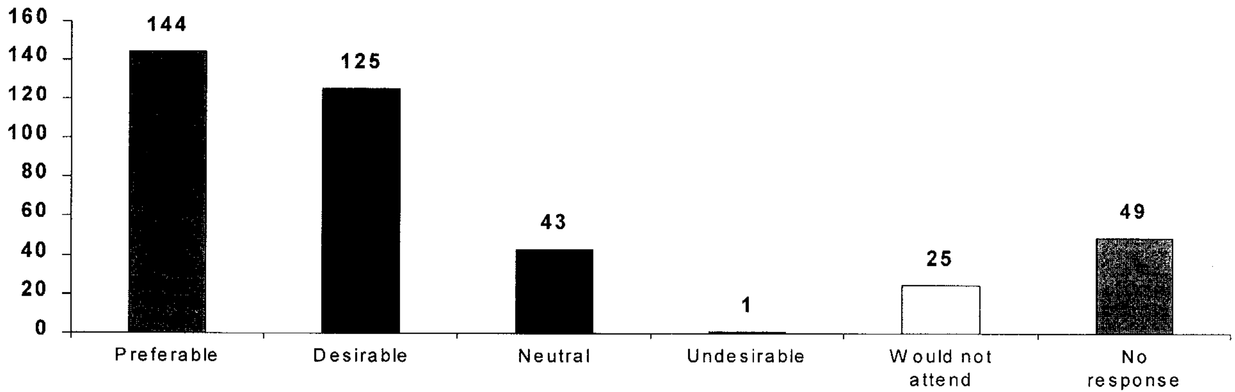
A course held in conjunction with another CAS program.”



The purpose of this question was to determine the membership’s requirements regarding a Course on Professionalism. ***There is strong desire to tie the course into another event.***

Question 33d: "Please indicate the importance of the following regarding your desire to attend a Course on Professionalism:

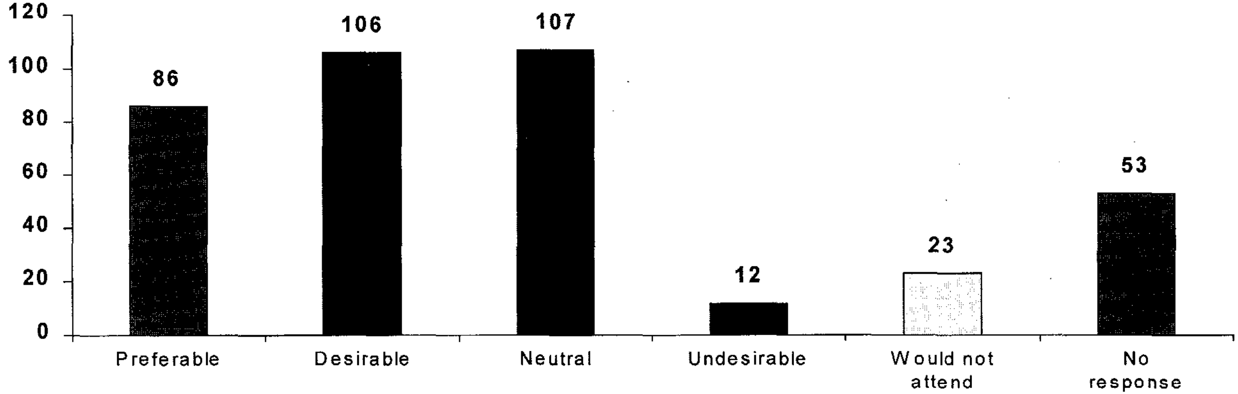
A course tailored to an audience of existing CAS members (instead of pre-Associate students)."



The purpose of this question was to determine the membership's requirements regarding a Course on Professionalism. ***The Course content MUST be tailored to the audience.***

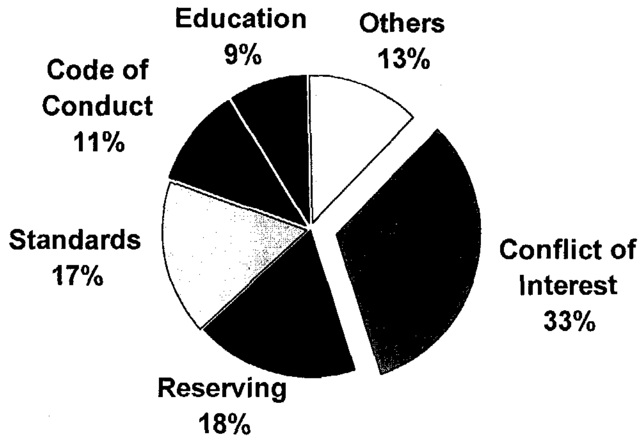
Question 33e: "Please indicate the importance of the following regarding your desire to attend a Course on Professionalism:

The course counts toward my continuing education requirements."



The purpose of this question was to determine the membership's requirements regarding a Course on Professionalism. ***There is strong interest in having the course count toward continuing education.***

Question 37: "In your opinion, what are the major professionalism and ethical issues faced by the CAS that you would like to see addressed by the Committee on Professionalism Education?"



The purpose of this question was to determine the membership's education priorities. ***A 33% of the responses related to some sort of issue related to conflict of interest between professional obligations and other interests. Another 46% (18%+17%+11%) related to application of professionalism in day-to-day work.***

Continuing Education Survey
Comments - Q. 37

- Conflict of Interest - 72 total comments
  - With company management/clients/business pressure - 37 comments
  - Maintaining independence/objectivity - 10 comments
  - With other actuaries/advocacy - 9 comments
  - With reward systems - 6 comments
  - General - 10 comments

Continuing Education Survey

Comments - Q. 37

- Statement of Opinion/Reserve Adequacy - 40 total comments
  - Bad practice - 12 comments
  - Relating to technicalities in completing them - 9 comments
  - General statement - 19 comments

Continuing Education Survey

Comments - Q. 37

- Standards of Practice - 38 total comments
  - ✗ Application of /adherence to them - 11 comments
  - ✗ What is "reasonable" and how does one handle the "range of reasonability"? - 7 comments
  - ✗ Qualification issues - 5 comments
  - ✗ Documentation/Disclosure - 4 comments
  - ✗ Need for them - 4 comments
  - ✗ Use of judgment - 4 comments
  - ✗ Expert testimony SOP - 2 comments
  - ✗ "Politicization" - 1 comment

Continuing Education Survey

Comments - Q. 37

- Code of Conduct - 24 comments
  - ✗ Discipline process - 10 comments
  - ✗ Ethical behavior - 5 comments
  - ✗ Honesty - 4 comments
  - ✗ Conflicts with legal requirements - 4 comments
  - ✗ Proprietary information - 1 comment

Continuing Education Survey

Comments - Q. 37

- General Education - 19 total comments
  - ✗ Existing membership - 10 comments
  - ✗ Don't need COPE/COP - 5 comments
  - ✗ Company management - 2 comments
  - ✗ External audiences - 2 comments

Continuing Education Survey

Comments - Q. 37

- Other Issues
  - ✗ Regulatory specific - 4 comments
  - ✗ Actuary in non-traditional role - 4 comments
  - ✗ Peer Review - 3 comments
  - ✗ Rate filing / certification - 2 comments
  - ✗ Role/impact of technology - 2 comments
  - ✗ Communication - 2 comments

Continuing Education Survey

Comments - Q. 37

- Other Issues (continued)
  - ✗ Reliance - 2 comments
  - ✗ Materiality - 2 comments
  - ✗ Public responsibility - 3 comments
  - ✗ Valuations - 1 comment
  - ✗ Product prices - 1 comment
  - ✗ "Consultants" - 1 comment
  - ✗ Guidance - 1 comment

Continuing Education Survey

Representative Comments - Q. 37

- Conflict of Interest
  - With company management/clients/business pressure
    - ☑ "When and how to present adverse comments to the clients of a consulting actuary, and when to be flexible in such circumstances."
    - ☑ "Balancing the responsibility to corporate management vs. the stockholders and the public in general."
    - ☑ "Accurate reserving under pressure to under-reserve/release reserves."
    - ☑ "Balancing professionalism and business operations in competitive environment."
    - ☑ "There are FCAS's who 'bend' the truth to fit client needs. This is really a violation of integrity standards but it is impossible to prove."
    - ☑ "Attempted manipulation/coercion by senior management of actuarial positions."
    - ☑ "The temptation faced by small consulting firms (one person) to compromise their ethics to keep a client."

Continuing Education Survey

Representative Comments - Q. 37

- Conflict of Interest
  - ✗ Maintaining independence/objectivity
    - ☑ "Objectivity in client reports, company reserving."
    - ☑ "Requiring unbiased opinions."
    - ☑ ""I sometimes am disturbed by an appearance of an analysis being slanted toward "proving" or supporting a position held by the requester of the analysis."
  - ✗ With other actuaries/advocacy
    - ☑ "Handling conflicts with other actuaries."
    - ☑ "Credibility issues with actuaries disagreeing."
    - ☑ "Too many actuaries are advocates for company or client -- they have abdicated professionalism to an alarming degree."

Continuing Education Survey
Representative Comments - Q. 37

- Conflict of Interest
  - With reward systems
    - ☑ "Conflict between bonus plans and setting reserves."
    - ☑ "Impact of downsizing and lack of job security on the quality and accuracy of actuarial opinions."
    - ☑ "The actuary is heavily persuaded by who is paying him."
  - General
    - ☑ "I believe that many actuaries are faced with issues/situations that test their ethics. I would like to known when to draw the line in the sand and how do I do that!"
    - ☑ "Educating actuaries on what their options are when faced with professional/ethical dilemmas."

Continuing Education Survey

Representative Comments - Q. 37

- Statement of Opinion/Reserve Adequacy
  - ✗ Bad practice
    - ☑ "Actuaries (often consultants) who do sloppy job on small company opinions and/or delegate them to lower staff."
    - ☑ "The key issue is signing off on reserve statements when the actuary has not fulfilled reasonable standard of practice (fiduciary responsibility)."
    - ☑ "Vastly increased amount of 'shopping for opinions' in the last couple of years."
    - ☑ "People are still signing off on loss reserves being adequate when they know there is a strong likelihood the reserves are substantially deficient. The biggest problem is the small, independent actuarial consulting firms."
    - ☑ "Proper supervision by ABCD, mandatory review of actuary responsible for insolvent company reserves."

Continuing Education Survey
Representative Comments - Q. 37

- Statement of Opinion/Reserve Adequacy
  - Relating to technicalities in completing them
    - ☑ "I find the direction provided by the COPLFR on opinions to be so general as to be not useful. I'd like more specifics to be addressed, perhaps through the Professionalism Committee."
    - ☑ "Reserving for 'unquantifiable' liabilities."
    - ☑ "How to opine, especially for reserves, or 'paint estimates' when there is a wide range of error."
    - ☑ "Reserve opinions especially when reasonable discount would imply adequate reserves."
    - ☑ "Setting reserve levels and product prices."

Continuing Education Survey

Representative Comments - Q. 37

- Standards of Practice
  - Application of /adherence to them
    - ☑ "Inability to prove differences between reasonable actuarial 'opinion' and dishonesty."
    - ☑ "More practical and precise guidelines. Right now there is a huge grey area, which probably is intentional."
    - ☑ "Making sure members continually comply with CAS and AAA standards of practice."

Continuing Education Survey

Representative Comments - Q. 37

- Standards of Practice
  - ✗ What is "reasonable" and how does one handle the "range of reasonability"?
    - ☑ "Clarifying what is meant by a 'reasonable' provision for unpaid losses."
    - ☑ "How to determine the boundaries within which 'reasonable provisions' fall."
    - ☑ "Actuarial studies have a broad range of results. If the procedures are actuarially sound, then what direction can be given for the quality of the calculations, procedures and results? Subsequent research studies should always result in improvement in analysis and procedures. They will probably also find some faulty logic in the prior analysis. We need some guidance in these areas."

Continuing Education Survey

Representative Comments - Q. 37

- Standards of Practice
  - Qualification issues
    - ☑ "Criteria required by standard of practice of an actuary before he/she is considered to have the expertise necessary to opinionate on a subject."
    - ☑ "Actuaries practicing outside of their expertise."

Continuing Education Survey

Representative Comments - Q. 37

- Code of Conduct
  - Discipline process
    - ☑ "Members should be made aware of all cases dealt with by the ABCD so that they realize what these issues involve (all names should be kept confidential)."
    - ☑ "I believe most actuaries do not perceive a very big axe over their heads compelling them to adhere to the strictest and highest level of professionalism sought by the Society."
    - ☑ "What to do when aware that others in own organization are intentionally violating rate filings and intent of various insurance programs."
    - ☑ "Shoddy work by consulting actuaries."

Continuing Education Survey

Representative Comments - Q. 37

- Code of Conduct
  - ✗ Ethical behavior
    - ☑ "Guidelines on what constitutes ethical behavior with respect to applying judgment, accepting gifts, etc.."
  - ✗ Honesty
    - ☑ "Issues include honesty and full disclosure in statements of public opinion and expert testimony."
  - ✗ Conflicts with legal requirements
    - ☑ "Arbitrary decisions made by regulatory bodies and how actuaries can work for those bodies."
    - ☑ "Regulators promulgating rules in deference to AAA Standards of practice."

Continuing Education Survey
Representative Comments - Q. 37

- General Education
  - Existing membership
    - ☑ "I am concerned that as the CAS membership increases, it will be tougher for the CAS to maintain consistent professionalism among its members. I strongly support/encourage all forms of professionalism education for all members."
    - ☑ "Older fellows are not aware of the requirements and need to be better informed."
    - ☑ "At this point, I believe a broad approach should be developed."

Continuing Education Survey

Representative Comments - Q. 37

➢ General Education

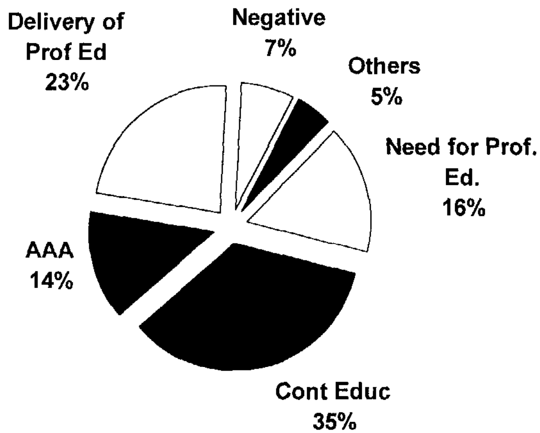
✗ Don't need COPE/COP

☑ "I believe the greatest service the CAS could do is to eliminate its Committee on Professionalism Education. A definition of a professional actuary is that he/she gets paid to do the job -- and if he/she doesn't continue to do a professional job, he/she wouldn't continue to get paid much longer. ... All COPE will do, if it continues to exist, is create bureaucracy and dogmas."

☑ "I don't think it is a big issue. People generally have a sense of what is right or wrong. A course is not going to change the basic values of an individual."

☑ "Give a self-test, if you like, of situations. Forcing people to do what they should be doing anyway is absurd. Why do we need this course? Are you telling me ACAS and FCAS don't know what is right and wrong??? BALONEY!"

Question 38: "Do you have any additional comments on professionalism education?"



The purpose of this question was to allow the respondents to let us know what other thoughts they had on this topic. ***The comments received mostly offered suggestions or considerations about the design of professionalism education.***

Continuing Education Survey

Comments to Q. 38

- Need for professionalism - 7 total comments
  - Must have it - 5 comments
  - I would like it - 2 comments
- Related to continuing education requirements - 15 total comments
  - Only need one time, not ongoing - 5 comments
  - Won't change behavior - 4 comments
  - Other - 6 comments
- American Academy of Actuaries - 6 comments
  - Discipline process - 3 comments
  - Other - 3 comments

Continuing Education Survey

Comments to Q. 38

- Educational Delivery - 10 total comments
  - Real world issues/cases - 4 comments
  - Considerations for delivery - 4 comments
  - Other comments - 2 comments
- Negative Comments - 3 total comments
- Other Issues
  - International issue - 1 comment
  - Survey too long - 1 comment

Continuing Education Survey

Representative Comments - Q. 38

- Need for professionalism
  - Must have it
    - ☑ "It's required to help maintain the professional respect and reliance placed on actuaries and the profession."
    - ☑ "With issues like Florida self-insured funds (PCA) and Golden Eagle in California, we need to be <u>very clear</u> about what we expect from ourselves and what we want the public to expect from us, <u>promulgate</u> it, and then <u>hold ourselves to it</u>, for credibility."

Continuing Education Survey

Representative Comments - Q. 38

- Related to continuing education requirements
  - Only need one time, not ongoing
    - ☑ "If a requirement were imposed on existing members, a one-time requirement should be implemented rather than a continuing requirement."
    - ☑ "I don't believe you need to cover this subject every few years. A good program or two, one for experienced fellows and another for new associates, would be sufficient. The experienced fellow course could be offered shortly after attaining fellowship. The focus could be case studies."

Continuing Education Survey

Representative Comments - Q. 38

- Related to continuing education requirements
  - ✗ Won't change behavior
    - ☑ "I can't imagine what you could say during a one or two day meeting that would change someone from unethical to ethical."
    - ☑ "A fascinating area. It can warn people but perhaps we have already had our individual morality/character formed."
    - ☑ "Most egregious acts would not be prevented by education."

Continuing Education Survey

Representative Comments - Q. 38

- American Academy of Actuaries
  - ✗ Discipline process
    - ☑ "Greater encouragement of actuaries to repute bad work products and behavior to the ABCD."
    - ☑ "I believe that the fact that only one CAS member in recent history was expelled from the CAS - and that was after he was found guilty of criminal charges - makes it look like ethics aren't really taken seriously. Another CAS member has been expelled from the CIA (for good reason) but there has been no known action by the ABCD even though he practices in the US."

Continuing Education Survey

Representative Comments - Q. 38

- Educational Need
  - Real world issues/cases
    - ☑ "Would like to hear more real world experience descriptions from experienced actuaries when their professionalism standards were tested and the outcome."
    - ☑ "The content should not be overly simplistic. Situations where people are blatantly unethical tend not to occur in real life."

Continuing Education Survey

Representative Comments - Q. 38

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- Educational Need
  - Considerations for delivery
    - ☑ "Offering mini courses "on-site" at a company with large actuarial staffs might be good."
    - ☑ "There ought to be a standard of procedure or equivalent that focuses specifically and comprehensively on the topic - something we all can refer to and cite and discuss among the members. ... A real good reference is what I would like to see."
    - ☑ "Can't force all actuaries to attend, but the CAS should facilitate education for all those with questions."